

UNIVERSITY OF SOUTHAMPTON

**THIN FRP COMPOSITE PANELS UNDER HIGH
TRANSVERSE PRESSURE**

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ABSTRACT

FACULTY OF ENGINEERING AND APPLIED SCIENCE

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There is increasing interest in the use of fibre-reinforced plastic (FRP) composite materials for large plated structural applications in ship and civil construction sectors. Under high transverse pressure, the thin composite panels undergo large deflections and develop various kinds of damage. To understand the progressive failure behaviour of the panel experimental investigations and theoretical analyses have been carried out in this research.

On the experimental side, a new square panel has been tested in more detail, taking advantage of experience gained in the earlier tests. A number of coupons at various angles to the warp direction were also tested to obtain stress-strain diagrams, stiffnesses and strengths of the composite.

On the theoretical side, to predict the mechanical properties of composite material, a simple theoretical model was presented in this research. To understand the effect of undulation and interlacing of woven rovings on the stress distribution inside the woven composite, the complicated architecture of the composite was modelled using finite element method. The study is very helpful in identifying failure initiation and understanding failure mechanism of the woven composite.

To model progressive failure and damage accumulation of the woven composite panels, a methodology for progressive failure analysis of woven composite panels was developed. The analyses were conducted in the ABAQUS finite element package on the IBM SP2 parallel processing computer. The results from the progressive failure analysis provided a good explanations of the actual panel behaviour.

For quick prediction of the panel behaviour a simplified finite element analysis was presented. Two extreme cases were studied in the analysis. One is called *Resin Effective*, the other is called *Resin Ineffective*. The two extreme cases provide bounds to the behaviour of the panel.

Finally, for the purpose of *engineering design*, very simple approximate methods have been developed to calculate approximate maximum deflection, membrane tensile force, local bending moment and failure pressure of the panel.

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DEDICATION

TO MY WIFE

AND

MY SON

Xiaoyun Wu & Da Feng

CONTENTS

Abstract

Acknowledgement

Dedication

Contents i

List of Figures vii

List of Tables xvi

Abbreviations and Notation xx

Chapter 1 Introduction 1

Chapter 2 Literature review 5

2.1 Introduction 5

2.2 Elastic properties of unidirectional composite material 7

2.3 Elastic properties of woven composite material 10

2.4 Strength of unidirectional composite material 16

2.5 Strength of woven composite material 17

2.6 Progressive failure analysis of composite structures 19

2.6.1 Definition of progressive failure analysis 20

2.6.2 Failure criteria 20

2.6.3 Stiffness reduction scheme 24

2.6.4 Progressive failure analysis of composite structures 24

2.7 Geometrical non-linear analysis of laminated composite panels 27

2.7.1 Numerical solution procedures 27

2.7.2 Geometrical non-linear analysis of
laminated composite panels 28

Chapter 3	Experiments on composite coupons	32
3.1	Introduction	32
3.2	Test equipment and preparation	32
3.3	Burning test results	35
3.4	Tensile test results	38
3.5	Bending test results	41
3.6	Scanning electron microscope analysis	46
Chapter 4	Experiments on composite panels	50
4.1	Introduction	50
4.2	Test rig and equipment	51
4.3	Summary of panel tests of laminates A, B and C	54
4.3.1	Panel deflections	54
4.3.2	Strain development	54
4.3.3	Small in-plane edge displacement	55
4.3.4	Progressive damage and failure mechanism	55
4.3.5	The effect of aspect ratio	56
4.3.6	The effect of materials	56
4.4	Test preparation and strategy for Panel D	63
4.4.1	Material preparation	63
4.4.2	Test preparation and strategy	63
4.4.3	Small error in loading system	64
4.5	Summary of panel test of laminate D	71
4.5.1	Panel behaviour at low pressure	71
4.5.2	Strain distribution and development	73
4.5.3	In-plane edge displacement	77
4.5.4	Progressive failure of Panel D	82
4.6	Conclusions	83

Chapter 5	Predictions of elastic properties of composite materials	84
5.1	Introduction	84
5.2	Theoretical model	84
5.3	Stiffness matrix of fibre/resin layer in lamina i	87
5.4	Transformation for non-aligned fibres in fibre/resin layer in lamina i	92
5.5	Stiffness matrix of a pure resin layer	94
5.6	Elastic properties of a woven lamina and a woven laminate	94
5.7	Comparisons between the theoretical model and tests	97
5.8	Theoretical model for orthotropic fibres and comparisons with existing models	101
5.9	Conclusions	105
Chapter 6	Stress concentrations in a woven composites	106
6.1	Introduction	106
6.2	Geometry of a plain woven composite	107
6.3	Finite element modelling	110
6.3.1	Introduction to ABAQUS on the IBM SP2 parallel processing computer	111
6.3.2	Solid model for the finite element analysis	113
6.3.3	Finite element meshes	113
6.3.4	Boundary conditions	115
6.4	Stress distributions and concentrations	116
6.5	Parametric studies	118
6.5.1	The effect of waviness ratio	118
6.5.2	The effect of roving gap	119
6.5.3	The effect of different fibre contents of the roving	120
6.5.4	The effect of different materials	121
6.5.5	The effect of boundary conditions	122
6.5.6	The effect of mesh refinement	122
6.6	Conclusions	123

Chapter 7	Progressive failure analysis of thin composite panels	
	under high transverse pressure	128
7.1	Introduction	128
7.2	Prediction of the behaviour of Panel D at low pressure and adjustment of the test results	130
7.3	Methodology for progressive failure analysis of woven composite panels under high transverse pressure	133
7.4	Stiffnesses and strengths of a nominal unidirectional composite	136
7.5	Failure criterion	139
7.6	Stiffness reduction	140
7.7	Finite element modelling and implementation	140
7.8	Modelling of in-plane edge displacement	144
7.9	Analysis of the results	148
7.9.1	Damage patterns and final failure predicted by the PDM models	148
7.9.2	Comparisons of deflections between models and test	151
7.9.3	Comparisons of strains between models and test	155
7.10	Parametric Studies	167
7.10.1	Number of load increments	167
7.10.2	Element refinement	168
7.11	Conclusions	170

Chapter 8	Stiffness-modelling of thin composite panels under	
	high transverse pressure	171
8.1	Introduction	171
8.2	Stiffness-modelling of composite panels	172
8.2.1	Introduction to ANSYS	172
8.2.2	Element types available	172
8.2.3	Element type used in the analysis	173
8.2.4	Stiffness reduction scheme in the analysis	174
8.2.5	Finite element implementation and	

	modelling in-plane edge displacement	175
8.3	Analysis of the results.....	176
8.3.1	Comparison of deflections	177
8.3.2	Comparison of strains	177
8.4	Conclusions	178

Chapter 9 Approximate Calculations for thin composite panels under

	high transverse pressure	188
9.1	Introduction	188
9.2	Approximate calculation of maximum deflection and tension	188
9.2.1	Membrane theory.....	188
9.2.2	Calculation formulae	190
9.2.3	Relation of pressure q on the panel and p on the strip.....	193
9.2.4	Results from the approximate method	196
9.2.5	Use of the approximate method to determine the spring stiffness used in FE analysis.....	196
9.3	Approximate calculation of maximum bending moment	200
9.4	Approximate calculation of final failure pressure	204
9.5	Conclusions	206

Chapter 10	Conclusions	208
10.1	Retrospect	208
10.2	Main conclusions	209
10.3	Proposals for future work	211

References	213
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Appendices 223

Appendix 1 Detailed coupon test results of laminate D 223

Appendix 2 Detailed derivation of equations in Chapter 5 226

**Appendix 3 Program for calculation of tensile and bending elastic
 properties by using theoretical model 228**

**Appendix 4 Typical user-defined subroutine used in progressive
 failure analysis 235**

Appendix 5 Dimensions of the elements for a square panel mesh 238

**Appendix 6 Program for calculation of maximum deflection,
 membrane tension and local bending moment 239**

LIST OF FIGURES

	Page No.
Figure 2.1	Demonstration of woven fabric patterns
	6
Figure 2.2	Ishikawa's models: (a) Mosaic model;
	(b) Fibre undulation model; (c) Bridging model
	12
Figure 2.3	Repeating unit element (RUE) of a plain woven composite
	13
Figure 3.1	The set-up for a coupon test on laminate D
	34
Figure 3.2	Cycled load-strain curves measured from coupon tests
	(laminate D, 0 and 90 degrees to warp)
	39
Figure 3.3	Cycled load-strain curves measured from coupon tests
	(laminate D, 22.5 degree to warp)
	39
Figure 3.4	The tested samples of laminate D at various angles to warp
	40
Figure 3.5	Typical tensile stress-strain relations at various angles
	for laminate D
	41
Figure 3.6	(a) 4-point bending set-up; (b) 3-point bending set-up
	44
Figure 3.7	Coupon in three-point bending test
	45
Figure 3.8	Typical load vs. central deflection curves at various angles
	for laminate D
	45

Figure 3.9	Section of warp/weft direction where picture was taken	46
Figure 3.10	Architecture of laminate D - section of warp direction	48
Figure 3.11	Architecture of laminate D - section of weft direction	48
Figure 3.12	Architecture of laminate B - section of warp direction	49
Figure 3.13	Architecture of laminate B - section of weft direction	49
Figure 4.1	Isometric view of test rig, after reference [12]	52
Figure 4.2	Demonstration of the loading system (See also Figure 4.14)	53
Figure 4.3	The locations and numbering of strain gauges (a), and transducers (b)	58
Figure 4.4	Transducer at edge for detecting in-plane edge displacement (See also Figure 4.13)	59
Figure 4.5	Typical non-linear deflection vs. pressure curves in panel tests	59
Figure 4.6	Typical edge strain (SG4 and SG8) vs. pressure curves (Panel 30A)	60
Figure 4.7	Typical central strain (Z3 and R3) vs. pressure curves (Panel 30A)	60
Figure 4.8	Typical in-plane edge displacement (Panel 3C)	61
Figure 4.9	Final failure of Panel 31A (aspect ratio 1:1)	61

Figure 4.10	Final failure of Panel 15B (aspect ratio 1:1.5)	62
Figure 4.11	Final failure of Panel 7C (aspect ratio 1:2)	62
Figure 4.12	The locations and numbering of strain gauges (a), and transducers (b) (Panel D)	65
Figure 4.13	Transducers placed inside the panel to detect in-plane edge displacement (See also Figure 4.4)	66
Figure 4.14	Demonstration of loading system with water manometer for Panel D (See also Figure 4.2)	66
Figure 4.15	The set-up for test on Panel D	67
Figure 4.16	Test rig and data logger used in the test of Panel D	68
Figure 4.17	Loading system used in the test of Panel D	69
Figure 4.18	Top view of test rig - many transducers	70
Figure 4.19	Central deflection (T2) of Panel D at low pressure in a typical cycle	72
Figure 4.20	Central membrane and bending strains (SG7/SG8) at low pressure in a typical cycle	72
Figure 4.21	Edge membrane and bending strains (SG1/SG2) at low pressure in a typical cycle	73
Figure 4.22	Strain distribution along the x-axis at unloaded surface (See Figure 4.12a)	74

Figure 4.23	Strain distribution along the x-axis at loaded surface	75
Figure 4.24	Strain distribution along the y-axis at unloaded surface	75
Figure 4.25	Strain distribution along the y-axis at loaded surface	76
Figure 4.26	Strain distribution along the x and y axes at unloaded surface	76
Figure 4.27	Strain distribution along the x and y axes at loaded surface	77
Figure 4.28	Inside measurement method - effect of panel rotation	79
Figure 4.29	The pattern of edge displacements of the panel at 1 bar (a) before adjustment; (b) after adjustment	80
Figure 4.30	The pattern of edge displacements of the panel at failure (a) before adjustment; (b) after adjustment	80
Figure 4.31	Development of edge displacement directly measured from the transducers (TC and TG)	81
Figure 4.32	Development of edge displacement directly measured from the transducers (TA and TE)	81
Figure 4.33	Final failure of Panel D	82
Figure 5.1	(a) A woven lamina in theoretical model (b) A woven laminate in theoretical model	86
Figure 5.2	Demonstration of a fibre/resin layer	87

Figure 5.3	Shear deformation in fibre/resin layer	90
Figure 5.4	Material principal system and global system	92
Figure 6.1	Geometry of the RUE of a plain woven composite	108
Figure 6.2	Shape of the rovings taken from laminate D	109
Figure 6.3	Finite element meshes of the RUE (a) the rovings; (b) the pure resin regions	114
Figure 6.4a	Contour plotted stress distribution for woven fabrics	125
Figure 6.4b	Longitudinal stress (s_{11}) in warp rovings	125
Figure 6.4c	Longitudinal stress (s_{22}) in weft rovings	125
Figure 6.4d	Transverse stress (s_{11}) in weft rovings	125
Figure 6.4e	Transverse stress (s_{22}) in warp rovings	126
Figure 6.4f	Stress (s_{11}) in pure resin region	126
Figure 6.5a	Contour plotted stress distribution for woven fabrics	127
Figure 6.5b	Longitudinal stress (s_{11}) in warp rovings	127
Figure 6.5c	Transverse stress (s_{11}) in weft rovings	127
Figure 6.5d	Stress (s_{11}) in pure resin region	127
Figure 7.1	Central deflection (T2) vs. pressure at low pressure	132

Figure 7.2	Central membrane strain (SG7/SG8) vs. pressure at low pressure (See Figure 4.12)	132
Figure 7.3	Edge membrane strain (SG1/SG2) vs. pressure at low pressure (See Figure 4.12)	133
Figure 7.4	A five-layer woven composite panel is simplified as 10-layer cross-ply unidirectional composite	135
Figure 7.5	The springs around the edges of Panel 18B: springs, element mesh and deflections	146
Figure 7.6	Demonstration of the calculation of the stiffness of spring attached at a node	147
Figure 7.7	Demonstration of matrix crack pattern in Panel D predicted by the PDM-spring Model	150
Figure 7.8	In-plane edge displacement of Panel D (woven)	152
Figure 7.9	In-plane edge displacement of Panel 30A (woven)	152
Figure 7.10	In-plane edge displacement of Panel 18B (non-woven)	153
Figure 7.11	Central deflection (T2) vs. pressure of Panel D (woven)	153
Figure 7.12	Central deflection (T2) vs. pressure of Panel 30A (woven)	154
Figure 7.13	Central deflection (T2) vs. pressure of Panel 18B (non-woven)	154

Figure 7.14	Central strain in the x-axis at unloaded surface of Panel D (woven) (See Figure 4.12 about strain gauge location for Panel D)	156
Figure 7.15	Central strain in the x-axis at loaded surface of Panel D (woven)	157
Figure 7.16	Central strain in the y-axis at unloaded surface of Panel D (woven)	157
Figure 7.17	Central strain in the y-axis at loaded surface of Panel D (woven)	158
Figure 7.18	Central strain (Z3) in the y-axis at unloaded surface of Panel 30A (woven) (See Figure 4.12 about strain gauge location for Panel D)	158
Figure 7.19	Central strain (R3) in the y-axis at loaded surface of Panel 30A (woven)	159
Figure 7.20	Central strain (Z3) in short span at unloaded surface of Panel 18B (non-woven)	159
Figure 7.21	Central strain (R3) in short span at loaded surface of Panel 18B (non-woven)	160
Figure 7.22	Strain (SG5) in middle of half span at unloaded surface of Panel D (woven) (strain at undamaged element)	160
Figure 7.23	Strain (SG6) in middle of half span at loaded surface of Panel D (woven) (strain at undamaged element)	161
Figure 7.24	Edge strain (SG3) (50 mm away from the edge) in the x-axis	

	at unloaded surface of Panel D (woven)	162
Figure 7.25	Edge strain (SG4) (50 mm away from the edge) in the x-axis at loaded surface of Panel D (woven)	163
Figure 7.26	Edge strain (SG13) (50 mm away from the edge) in the y-axis at unloaded surface of Panel D (woven)	163
Figure 7.27	Edge strain (SG14) (50 mm away from the edge) in the y-axis at loaded surface of Panel D (woven)	164
Figure 7.28	Edge strain (SG1) (10 mm away from the edge) in the x-axis at unloaded surface of Panel D (woven)	164
Figure 7.29	Edge strain (SG2) (10 mm away from the edge) in the x-axis at loaded surface of Panel D (woven)	165
Figure 7.30	Edge strain (SG11) (10 mm away from the edge) in the y-axis at unloaded surface of Panel D (woven)	165
Figure 7.31	Edge strain (SG12) (10 mm away from the edge) in the y-axis at loaded surface of Panel D (woven)	166
Figure 8.1	Demonstration of finite element mesh for square panels	180
Figure 8.2	Central deflection vs. pressure of Panel D	181
Figure 8.3	Central deflection vs. pressure of Panel 30A	181
Figure 8.4	Central deflection vs. pressure of Panel 18B	182
Figure 8.5	Central strain (SG7) in the x-axis at unloaded surface of Panel D	182

Figure 8.6	Central strain (SG8) in the x-axis at loaded surface of Panel D	183
Figure 8.7	Central strain (SG15) in the y-axis at unloaded surface of Panel D	183
Figure 8.8	Central strain (SG16) in the y-axis at loaded surface of Panel D	184
Figure 8.9	Central strain (Z3) in the y-axis at unloaded surface of Panel 30A	184
Figure 8.10	Central strain (R3) in the y-axis at loaded surface of Panel 30A	185
Figure 8.11	Central strain (Z3) in short span at unloaded surface of Panel 18B	185
Figure 8.12	Central strain (R3) in short span at loaded surface of Panel 18B	186
Figure 8.13	Edge strain (SG1) in the x-axis at unloaded surface of Panel D	186
Figure 8.14	Edge strain (SG2) in the x-axis at loaded surface of Panel D	187
Figure 9.1	A rectangular membrane	190
Figure 9.2	Demonstration of the strip under the pressure	191
Figure 9.3	A beam with distributed transverse load and fixed ends	200
Figure 10.1	A flowchart of research structure	211

LIST OF TABLES

		Page No.
Table 3.1	Lay-up of the composite materials	34
Table 3.2	Detailed information about the composite materials	37
Table 3.3	Tensile test results of the composites	40
Table 3.4	Bending test results of the composites	43
Table 4.1	Panel numbering and key results of panel tests, after reference [12]	57
Table 4.2	Key results of the final test of Panel D	82
Table 5.1	Elastic properties of fibre and resin and related coefficients	99
Table 5.2	Overall tensile elastic properties predicted by the theoretical model	100
Table 5.3	Overall bending elastic properties predicted by the theoretical model	101
Table 5.4	Elastic properties of fibres and resins used in references [32] and [35]	104
Table 5.5	Comparisons between the theoretical model and the models in references [32] and [35]	104

Table 6.1	Material properties and geometrical parameters used in the basic model	116
Table 6.2	The effect of waviness ratio on the SVF	119
Table 6.3	The effect of roving gap on the SVF	120
Table 6.4	Material properties of yarns with different fibre contents of the roving	120
Table 6.5	The effect of different fibre contents of the roving on the SVF	121
Table 6.6	Material properties of Carbon/Epoxy material [53]	121
Table 6.7	The effect of different materials on the SVF	122
Table 6.8	The effect of different boundary conditions on the SVF	122
Table 6.9	The effect of different mesh refinement on the SVF	123
Table 7.1	Overall elastic stiffness of Panel D	130
Table 7.2	Important data for calculating the stiffnesses and strengths of the nominal unidirectional composite	137
Table 7.3	Stiffnesses and conversion strengths of the nominal unidirectional composite in different panels	138
Table 7.4	Stiffness reduction scheme at integration point	140
Table 7.5	Dependence of material properties on the field variables	142

Table 7.6	Important data of the composites and test results of the panels	143
Table 7.7	The stiffness of the spring used in analysis	147
Table 7.8	Comparisons of failure pressure between predictions and tests	149
Table 7.9	Key results of the calculations and comparisons between 500 load increments and 1000 load increments	168
Table 7.10	Key results of the calculations and comparisons between 225 elements and 900 elements	169
Table 8.1	The stiffness of composite materials used in finite element analysis	175
Table 8.2	The stiffness of springs used in RE and RI	176
Table 9.1	The effect of relationship of p and q	195
Table 9.2	Comparison between the approximate method and the finite element analysis of Panel 30A (aspect ratio 1:1) (Maximum deflection and tension)	197
Table 9.3	Comparison between the approximate method and the finite element analysis of Panel 18B (aspect ratio 1:2) (Maximum deflection and tension)	198
Table 9.4	Comparison between the approximate method and the finite element analysis of Panel D (aspect ratio 1:1) (Maximum deflection and tension)	199
Table 9.5	Comparison between the approximate method and	

	the finite element analysis of Panel 30A (Maximum local bending moment)	203
Table 9.6	Comparison between the approximate method and the finite element analysis of Panel 18B (Maximum local bending moment)	204
Table 9.7	Comparison between the approximate method and the finite element analysis of Panel D (Maximum local bending moment)	204

Abbreviations and Notation

The symbols used in this thesis are usually fully defined where they first appear. However, some of these symbols have alternative meanings. To avoid any confusion, the list given below contains the definitions of the most important symbols used.

Abbreviations

<i>CCA</i>	composite cylinder assemblage
<i>CLT</i>	classical laminate theory
<i>FE</i>	finite element
<i>FRP</i>	fibre reinforced plastic
<i>Panel nX</i>	panel with its number <i>n</i> and material <i>X</i>
<i>PDM</i>	progressive damage model
<i>RE</i>	resin effective
<i>RI</i>	resin ineffective
<i>RIM</i>	resin injection moulding
<i>RUE</i>	repeating unit element
<i>SEM</i>	scanning electron microscope
<i>SVF</i>	stress variation factor

Notation

V_f	fibre content by volume in composite
V_m	resin content by volume in composite
E_f, G_f, ν_f	material properties of the fibre
E_m, G_m, ν_m	material properties of the resin
$E_1, E_2, G_{12}, \nu_{12}$	material properties in principal material coordinate directions
$E_x, E_y, G_{xy}, \nu_{xy}$	material properties in xy - coordinate directions

X, Y, S, R	in-plane ultimate strengths of material
$\varepsilon_1, \varepsilon_2, \gamma_{12}$	strain components in principal material coordinate directions
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$	strain components in xy - coordinate directions
$\sigma_1, \sigma_2, \tau_{12}$	stress components in principal material coordinate directions
$\sigma_x, \sigma_y, \tau_{xy}$	stress components in xy - coordinate directions
σ_{ult}	ultimate tensile strength of coupon
σ_{ult}^b	ultimate bending strength of coupon
μ	pure resin fraction in composite
S_f	fibre content by volume of the roving
S_m	resin content by volume of the roving
ξ_1	lamina fibre fraction
ξ_2	warp fibre fraction
$\varepsilon_{bending}$	maximum bending strain
ε_{loaded}	strain measured on the loaded face of the panel
$\varepsilon_{membrane}$	maximum membrane strain
$\varepsilon_{unloaded}$	strain measured on the unloaded face of the panel
k	stiffness of spring
M	bending moment
p	pressure allocated on the strip
q	pressure on the panel
T	tensile force
t	thickness of the panel
w	deflection

Chapter 1 Introduction

Fibre reinforced plastics (FRP) is a heterogeneous material which consists of fibres embedded in a matrix. The fibre contributes to the high strength and stiffness of the material and the matrix provides the integrity of the material and the efficient action of the reinforcement.

The glass fibre commonly used as reinforcement is E-glass. Carbon fibres are somewhat stronger but are also more expensive [1]. The resins of the greatest application in reinforced plastics, especially in shipbuilding today, are of the unsaturated polyester type. Polyester resins are attractive because they are cheap, they cure at room temperature and they have a reasonable resistance to water absorption. Epoxy types are also commonly used. They have good heat and chemical resistance but they are more expensive than polyesters. Other resins in commercial use include vinylesters, phenolics, acrylics and furanes, all of which may be used where special properties are required. Phenolic resins particularly are considered to be promising because of their excellent performance in fire. All these resins are thermosetting compounds. Polyesters require the addition of a catalyst to promote the chemical reaction which achieves cure. An accelerator can also be added to control the working and setting time of the resin. Epoxy resins consist of a 'resin' and 'hardener' which are mixed and cured together to form the final product.

Composite materials can be tailored to meet a very wide range of requirements. They have been widely used for many years in the construction of aerospace and automotive structures. The demand for high-speed vessels, including lifeboats, has increased interest in the use of the FRP composite for large plated structural applications to replace conventional metallic structures [2]. The present work is related to such composites for large plated structures in ship construction under high transverse pressure.

The behaviour of a composite panel under high transverse pressure is less well understood than that of its metallic counterparts, such as steel and aluminium [3]. There are also relatively fewer procedures or accepted sets of databases to support the design of composite panels under high transverse pressure. For steel and aluminium structures the behaviour of plates with small and large deflections has been well studied theoretically. This work has been supported by experimental evidence which has extended into the elastic-plastic domain. Several such test results have been analysed and, when coupled with finite element analysis, these have resulted in design curves for steel plates [4-9]. With regard to composite plates, the theoretical and experimental works are incomplete. FRP is an anisotropic material and during loading composite plates suffer progressive failure because of various kinds of damage. Early theoretical work on laminated plates was based on so-called classical laminate theory (CLT) [10-11], which is the extension of Kirchhoff's classical plate theory [5]. As the governing equilibrium equations of laminated plates are very complicated few problems can be solved directly from them. Alternative methods for analysing composite structures include numerical methods, of which the finite element method is the most important. As to progressive failure analysis of the composite, it is so complicated that there are no simple equations to describe the problem. The effective way to study damage accumulation in the composite is to use the finite element method. There are several commercial finite element packages available today, such as ANSYS, ABAQUS, ADINA and so on. Recently research papers have appeared in which material stiffness degradation due to damage in the composite has been taken into account [59-69]. On the experimental side, there is no existing systematic test for investigating the behaviour of the composite panels under high transverse pressure. Therefore there is insufficient information to calibrate any analytical modelling of the behaviour of the composite panels and to form practical design graphs. Thus research is necessary to study the behaviour of composite panels under extreme transverse pressure through experimentation and numerical modelling, and to provide engineering design graphs.

Since 1993 this department has started systematic tests on the thin composite panels under high transverse pressure supported by an EPSRC project. More than 25 different

thin composite panels have been tested. A large amount of precise test data has been collected to create a database for panel behaviour. Most of the test results have been reported in reference [12]. The research reported here includes a part of the test scheme and a comprehensive theoretical study of the composite panels.

On the experimental side, a new square panel (a fourth type) has been tested in considerable detail, taking advantage of experience gained in the earlier tests. Numerous strain gauges and displacement transducers were used to collect important information during cycles of loading and unloading. In addition, a number of coupons at various angles to the warp were tested obtain stress-strain diagrams, stiffnesses and strengths of the composite.

On the theoretical side, material properties, the micromechanical analysis of woven composite, the progressive failure analysis of the panel and simplified calculations of the panel were studied. These are summarised as follows.

- To predict the mechanical properties of composite, a simple theoretical model was presented in this research, which can be used for woven and non-woven composites.
- To understand the effect of undulation and interlacing of fibre bundles on the stress distribution inside the woven composite, the complicated architecture of the composite was modelled using the finite element method.
- To model progressive failure and damage accumulation of the panel, a methodology for progressive failure analysis of woven composite panel was developed. The method took different damage modes, large deflections of the panel and possible in-plane edge displacements into account. The results provided a good explanation of the actual panel behaviour. The finite element modelling of progressive failure of the panels was carried out in ABAQUS on the IBM SP2 parallel processing computer.

- The progressive failure analysis was expensive in computer time. For quick prediction of the panel behaviour two extreme cases were studied. One is called *Resin Effective* which assumes that resin in the composite material is fully effective and linear elastic. Another is called *Resin Ineffective* in which it is assumed that the resin is no longer effective and all of the load had been transferred to the reinforcement. The Resin Effective and the Resin Ineffective solutions should provide limits or bounds to the behaviour of the panel.
- For the purpose of *engineering design*, very simple methods have been suggested to calculate approximate deflection, maximum membrane and moment, and failure pressure of the panel.

Chapter 2 Literature review

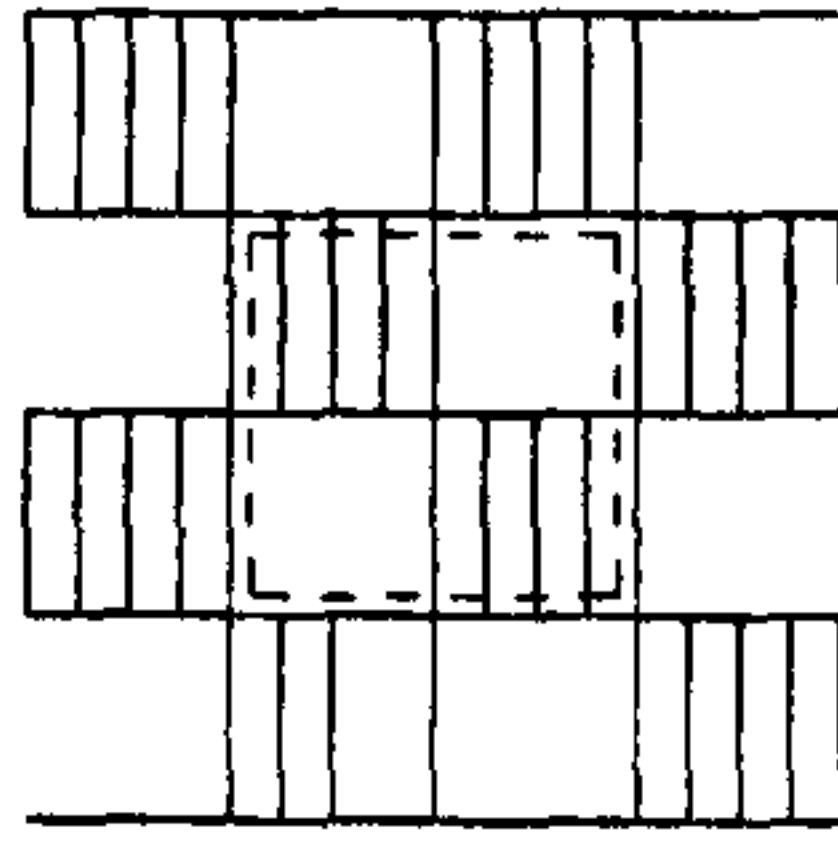
2.1 Introduction

A fibre-reinforced composite material can be considered as a stack of laminae. Each lamina is a two-dimensional arrangement of fibres, which may be unidirectional, bi-directional (usually woven), or random. Each lamina has its own principal axes, and the principal axes of successive laminae in the stack may lie in different directions, though they are often coincident. In this research three types of panel were made of woven roving composite materials, and one type of panel was a laminated composite containing stacks of unidirectional laminae.

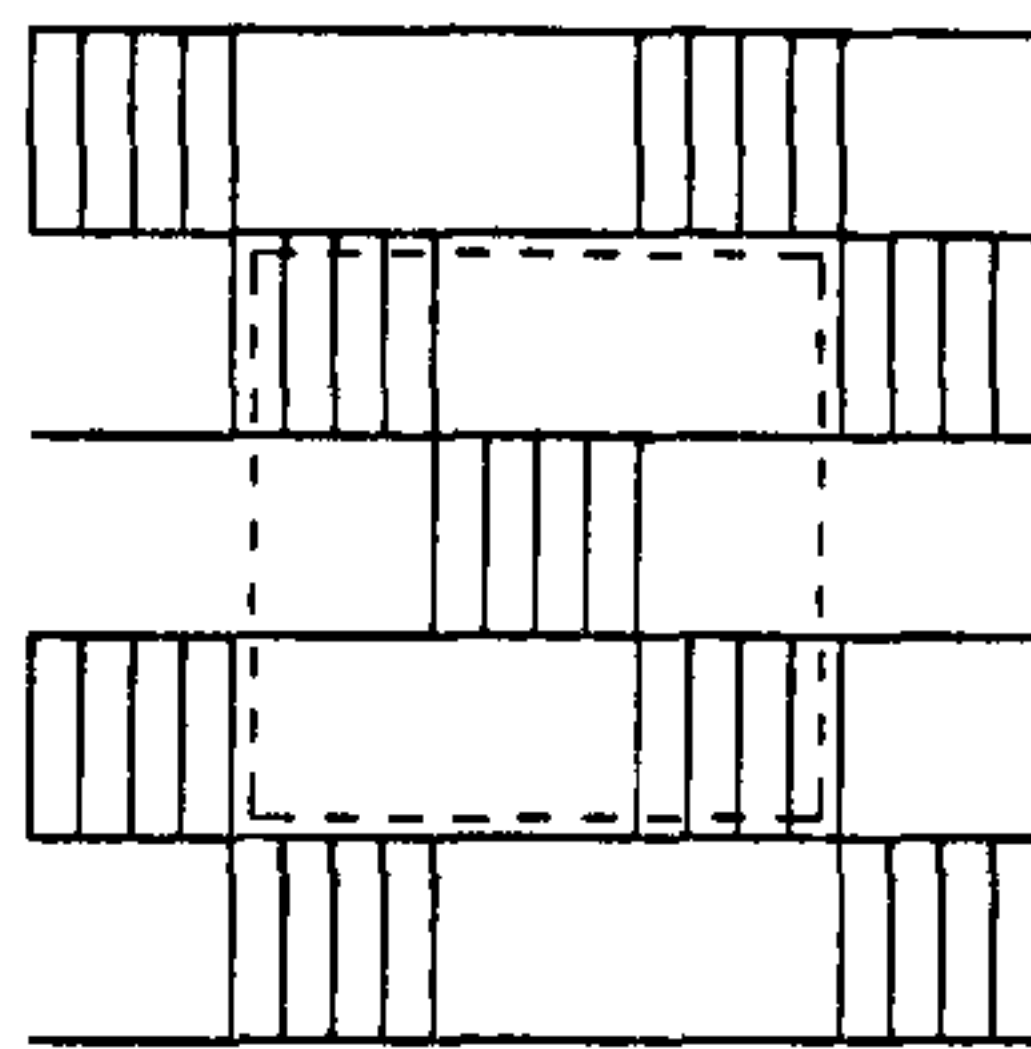
In a woven composite, the woven fabrics are formed by interlacing two sets of orthogonal *rovings* (or *yarns*) *. If one imagines the fabric on a vertical loom, the set of vertical rovings is called the warp, and the set of horizontal rovings is known as the weft (or fill). Any weave repeats on a certain number of warp and weft rovings, defined by geometrical quantities n_g . Based on the repeat the weaves are classified as plain ($n_g = 2$), satin ($n_g \geq 3$), as shown in Figure 2.1.

The following sections will review researches on prediction of mechanical properties of composite material, progressive failure analysis and non-linear analysis of the composite panel.

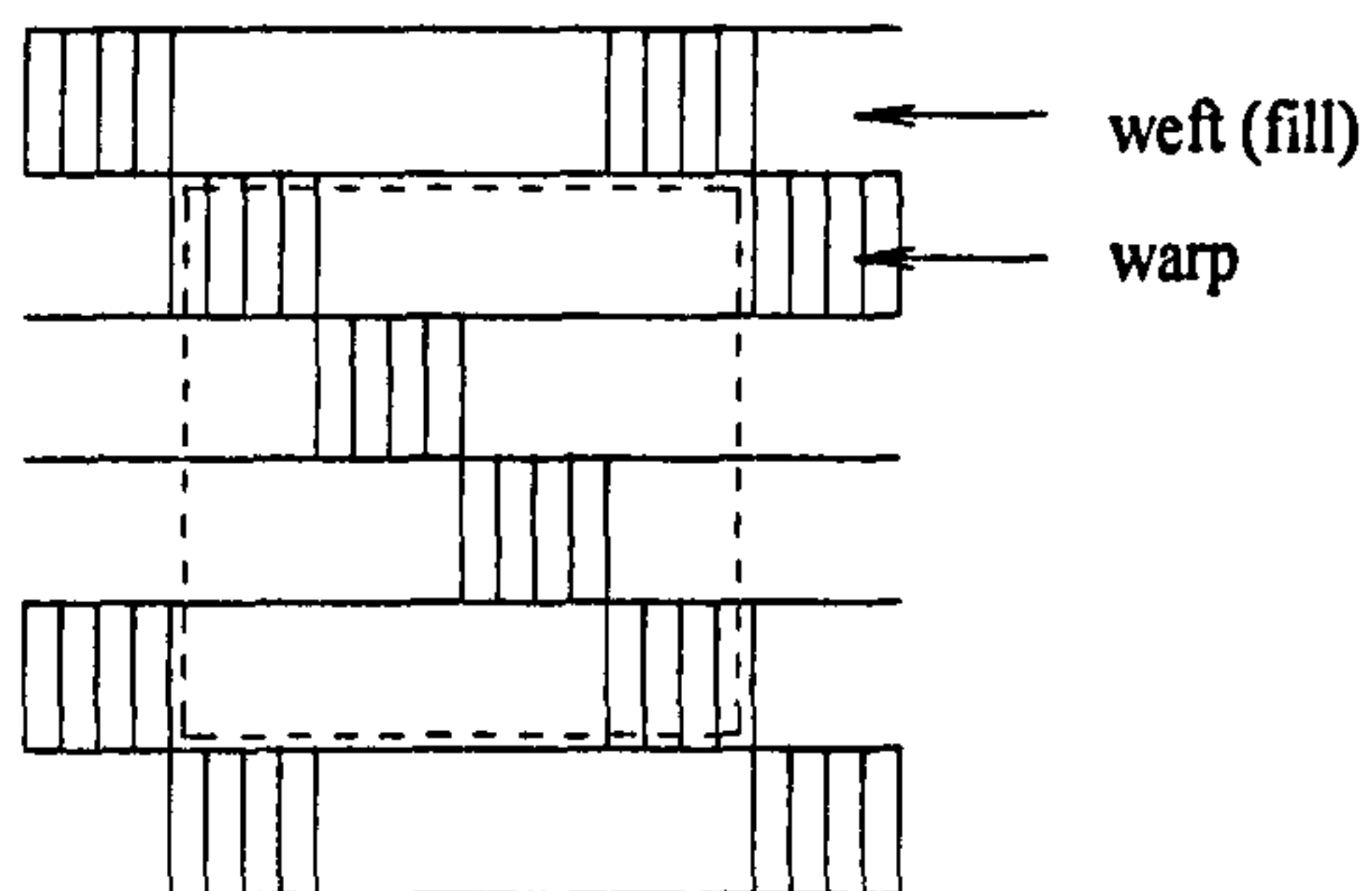
* Strictly speaking, a *yarn* is a spun fibre, which implies that it contains fibres that have been *twisted* together. Glass yarn may be woven to form *glass fabrics*. More commonly, however, the *glass fabrics* used in composites are woven from *untwisted* bundles of *strands* known as *rovings*. The strands are themselves bundles of *filaments*. Fabrics woven from rovings are known as *woven rovings*. In this research, all the woven composite materials were made from *woven rovings*. It is noted that in some literature the term *yarn* was misused to represent both twisted and untwisted fibre bundles. However, when we review the literature, we still keep the term the author used.



(a) Plain weave ($n_g = 2$)



(b) Satin weave ($n_g = 3$)



(c) Satin weave ($n_g = 4$)

Figure 2.1 Demonstration of woven fabric patterns

2.2 Elastic properties of unidirectional composite material

A laminated composite containing stacks of unidirectional laminae is one of the most commonly used composite forms. Researches on determining the material properties of the unidirectional composite have been very sophisticated [13-24]. Under the assumption that the fibres in the composite are regularly spaced and aligned in the matrix [22], various theoretical models were developed to calculate the material properties. Only significant models are discussed here.

Jones [10] described a very simple two-dimensional method for prediction of the elastic properties of unidirectional composite, usually called the rule of mixtures. The expressions of four independent elastic properties are:

Elastic Young's modulus in fibre direction

$$E_1 = E_f V_f + E_m V_m \quad (2.1a)$$

Elastic Young's modulus in the direction transverse to the fibre

$$E_2 = \frac{E_f E_m}{V_m E_f + V_f E_m} \quad (2.1b)$$

Poisson's ratio

$$\nu_{12} = V_f \nu_f + V_m \nu_m \quad (2.1c)$$

In-plane shear modulus

$$G_{12} = \frac{G_m G_f}{V_m G_f + V_f G_m} \quad (2.1d)$$

in which E_f , G_f and ν_f are elastic properties of the fibre; E_m , G_m and ν_m are elastic properties of the matrix. V_f is fibre content by volume and V_m is matrix content by volume. Compared with experimental results, Jones concluded that the elastic Young's modulus E_1 and Poisson's ratio ν_{12} predicted in the method have good accuracy, but E_2 and shear modulus G_{12} predicted are overestimated. The method is widely accepted as quick and approximate method of calculating the material properties.

More accurate prediction can be obtained by using so-called the exact method, in which elasticity theory is used to analyse the material properties. The well-known analysis model, the *composite cylinder assemblage* (CCA) model, was presented by Hashin [23]. The model treated the unidirectional composite as a long composite cylinder consisting of an inner circular fibre and an outer concentric matrix shell. Complicated formulae for predicting the material properties can be obtained by using elasticity theory. Compared with experimental results, Hashin indicated that the CCA model can provide good prediction of the elastic properties of the unidirectional composite.

The most important and generally accepted formulae for predicting the elastic properties of the unidirectional composite are probably the equations of Halpin-Tsai [24]. The exact methods for prediction of the material properties are of great value, but they are too complicated in practice. For design purpose it is often desirable to have simple and rapid method for estimating the material properties. Halpin and Tsai developed an interpolation procedure that is an approximate representation of more complicated micro-mechanical results. The procedure is accurate for design purposes if the fibre volume fraction V_f does not approach one. The equations of Halpin-Tsai are:

Elastic modulus in fibre direction

$$E_1 = E_f V_f + E_m V_m \quad (2.2a)$$

Poisson's ratio

$$\nu_{12} = \nu_f V_f + \nu_m V_m \quad (2.2b)$$

For other elastic constants

$$\frac{M}{M_m} = \frac{1 + \xi \eta V_f}{1 - \eta V_f} \quad (2.2c)$$

where

$$\eta = \frac{(M_f / M_m) - 1}{(M_f / M_m) + \xi} \quad (2.2d)$$

in which

M represents the composite modulus E_2 , or G_{12} , or G_{23}

M_f represents the corresponding fibre modulus E_f , or G_f , or ν_f

M_m represents the corresponding matrix modulus E_m , or G_m , or ν_m

ξ is a measure of fibre reinforcement of the composite that depends on the fibre geometry, packing geometry, and loading conditions. Reliable estimates for ξ can be obtained by comparison of above equations with the micromechanical solutions obtained by employing elasticity theory. Halpin and Tsai demonstrated that for circular fibres in a square array ξ can be chosen as 2 for the calculation E_2 and 1 for calculation G_{12} . It is obvious that the expressions of E_1 and ν_{12} are the same as those in the rule of mixtures.

As stated above, all the models were derived under the assumption that the fibres in the composite are regularly spaced and aligned in the matrix. They are therefore only suitable for predicting the material properties of unidirectional composite with a regular fibre arrangement. However, the accuracy of the results will be affected if the models are used to calculate the material properties of unidirectional composite with irregular fibre arrangement, such as a composite containing a pure resin region. In addition, they can not be used to predict *overall* elastic properties of laminated composite layered by unidirectional laminae directly because, firstly, the laminated composite may contain

many pure resin regions; secondly, the unidirectional laminae in a composite may lie in different directions.

2.3 Elastic properties of woven composite material

The methods used to predict the elastic properties of woven composites are still developing. The actual architecture of woven fabric is much more complicated than that of unidirectional composite. To date a number of analytical and numerical techniques have been developed to analyse the material properties of woven composites [25-44].

Halpin et al. [25] used laminate analogy for predicting the elastic stiffness of woven composites. The warp or weft yarns in a lamina were modelled as unidirectional composite layers containing crimped fibres, which were then stacked to form balanced and symmetrical laminates. The elastic constants for each unidirectional layer were calculated by using the Halpin-Tsai equations then, considering of the effect of geometric parameters of woven cloth structure such as fibre crimp and the space of two fibre yarns, the stiffness of the woven composite can be calculated. The theoretical results for the woven lamina were shown to be only qualitatively correct.

Ishikawa and Chou [26-29] made an important contribution to the prediction of the material properties of woven composites during the 1980s. They presented three models to analyse woven composite. They gave the names as the *mosaic model*, the *fibre undulation model* (crimp model) and the *bridging model* (Figure 2.2).

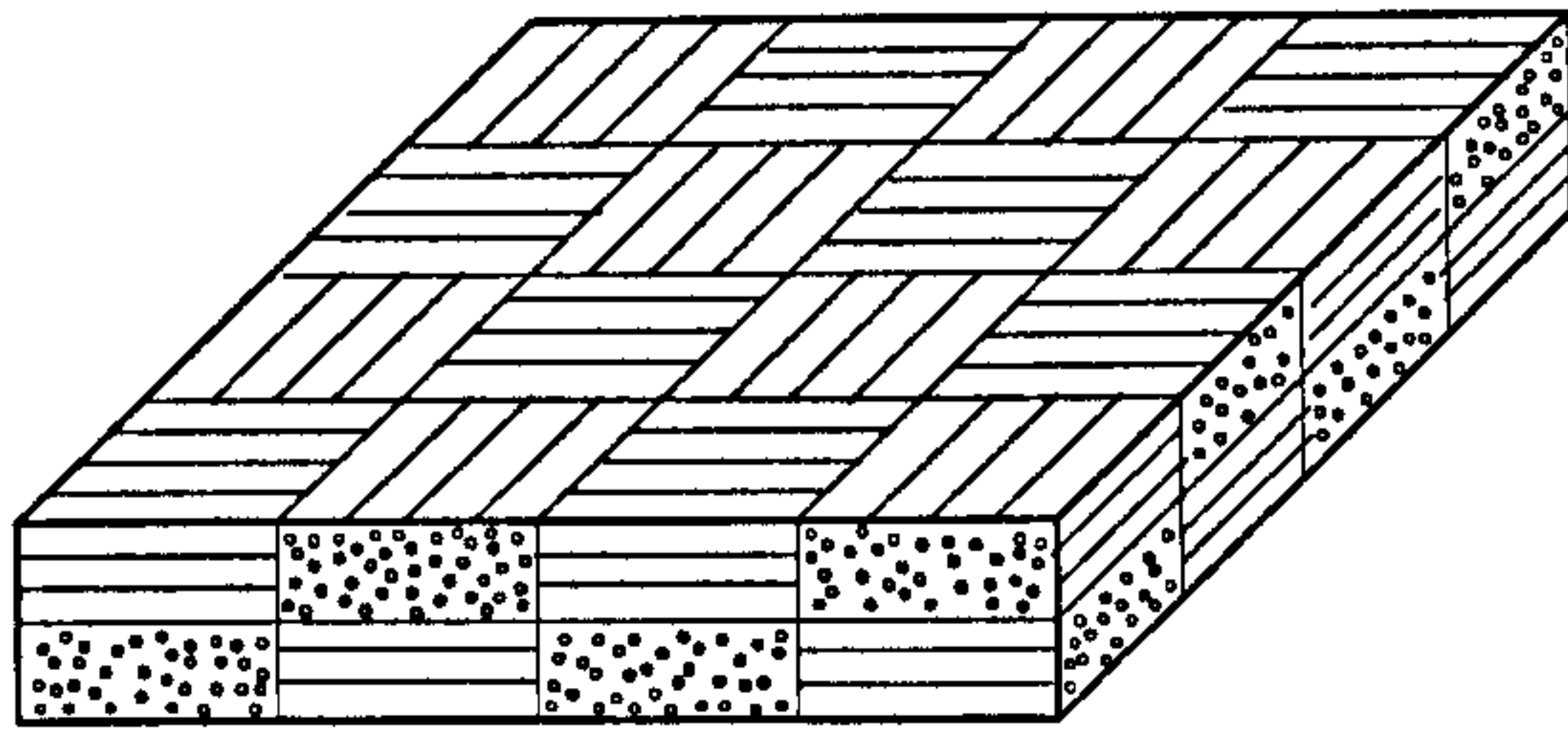
In the *mosaic model* the continuity of fibres in the yarn direction is neglected and the woven lamina is idealised as two layers with discontinuous fibres in orthogonal directions to form a mosaic pattern. The undulation of the fibres is neglected. Thus the woven lamina is regarded as an assemblage of pieces of asymmetrical cross-ply ([0/90] and [90/0]) laminates. Then the two-dimensional extent of the lamina is analysed by using two simplified models: the *parallel model* and the *series model*. It is considered

that the parallel model and the series model give upper and lower bounds of in-plane stiffness of the composite.

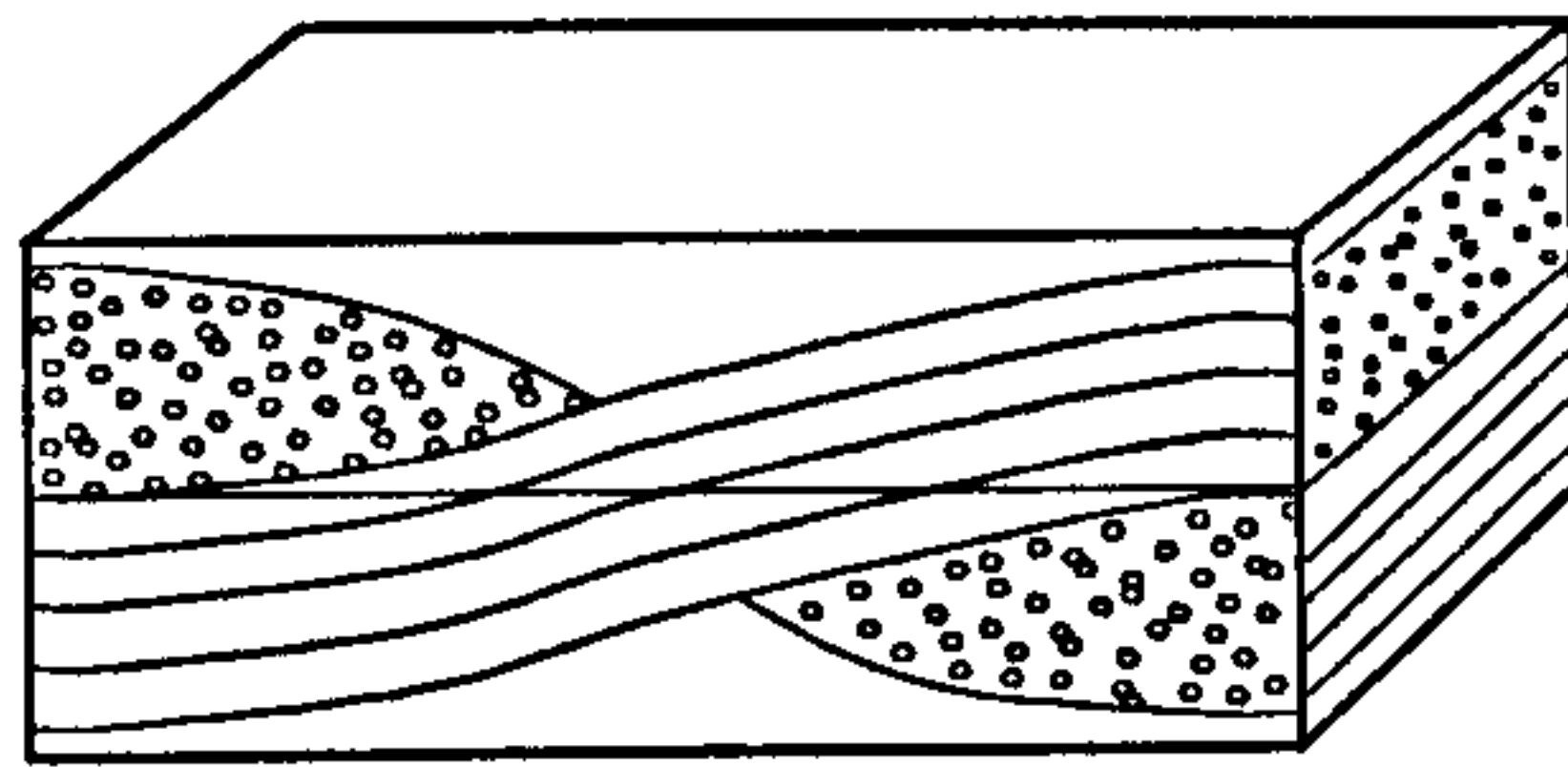
In the *fibre undulation model* the continuity and undulation of fibres in a fabric are considered. This model is particularly suitable for plain weave and weaves with low values of n_g because the interlaced regions for these weaves are comparable to the area of non-interlaced regions. The shape of the warp yarn is described analytically by assuming that the fibre undulation is a sinusoidal form. The fibre undulation model is one-dimensional model so that only fibre undulation in the weft direction is considered. The results showed that the in-plane stiffness constants obtained from the fibre undulation model were lower than those obtained by the mosaic series model.

The *bridging model* was developed to analyse woven lamina with satin weave fabric with $n_g \geq 4$. The undulation and continuity of yarns are considered only along the loading direction. The interlaced regions in a satin fabric are separated from one another by the non-interlaced regions. Since the regions with straight yarns surrounding an interlaced region have higher in-plane stiffness than the latter, they carry higher loads and play the role of load transferring bridges.

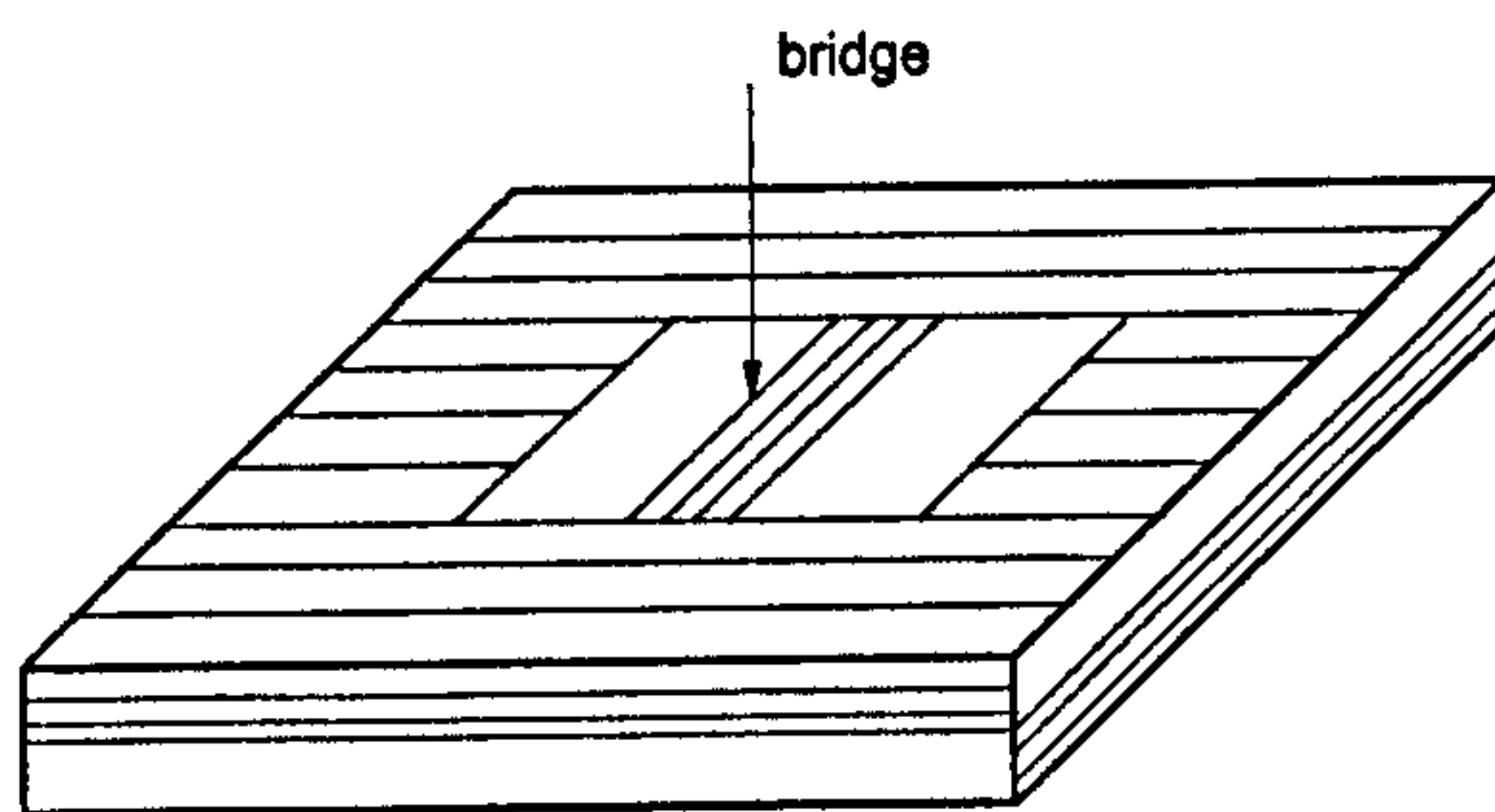
In reference [29] Ishikawa et al compared these models with experimental results. Tests were conducted on coupons cut from carbon/epoxy plain and satin woven composite plates. The elastic moduli of the composite were determined from the strain measurements. Comparisons of the experimental results with the theoretical models revealed that the experimental results of satin composite coincided well with the *bridging model* predictions. There existed some discrepancy between theoretical models (the *mosaic model* and the *fibre undulation model*) and experimental results.



(a) Mosaic model



(b) Fibre undulation model



(c) Bridging model

Figure 2.2 Ishikawa's models

More recently, several papers have been published in which a detailed model of the architecture of the woven composite has been used in the prediction of the stiffness of the composite. Naik [30-32] developed two-dimensional models for a plain woven lamina. From the symmetry of the plain woven fabric lamina, one fourth of it was analysed in the model, which is sufficient to represent the plain fabric architecture. This small part was termed a *repeating unit element* (RUE), as shown in Figure 2.3. This term is also used in the following sections.

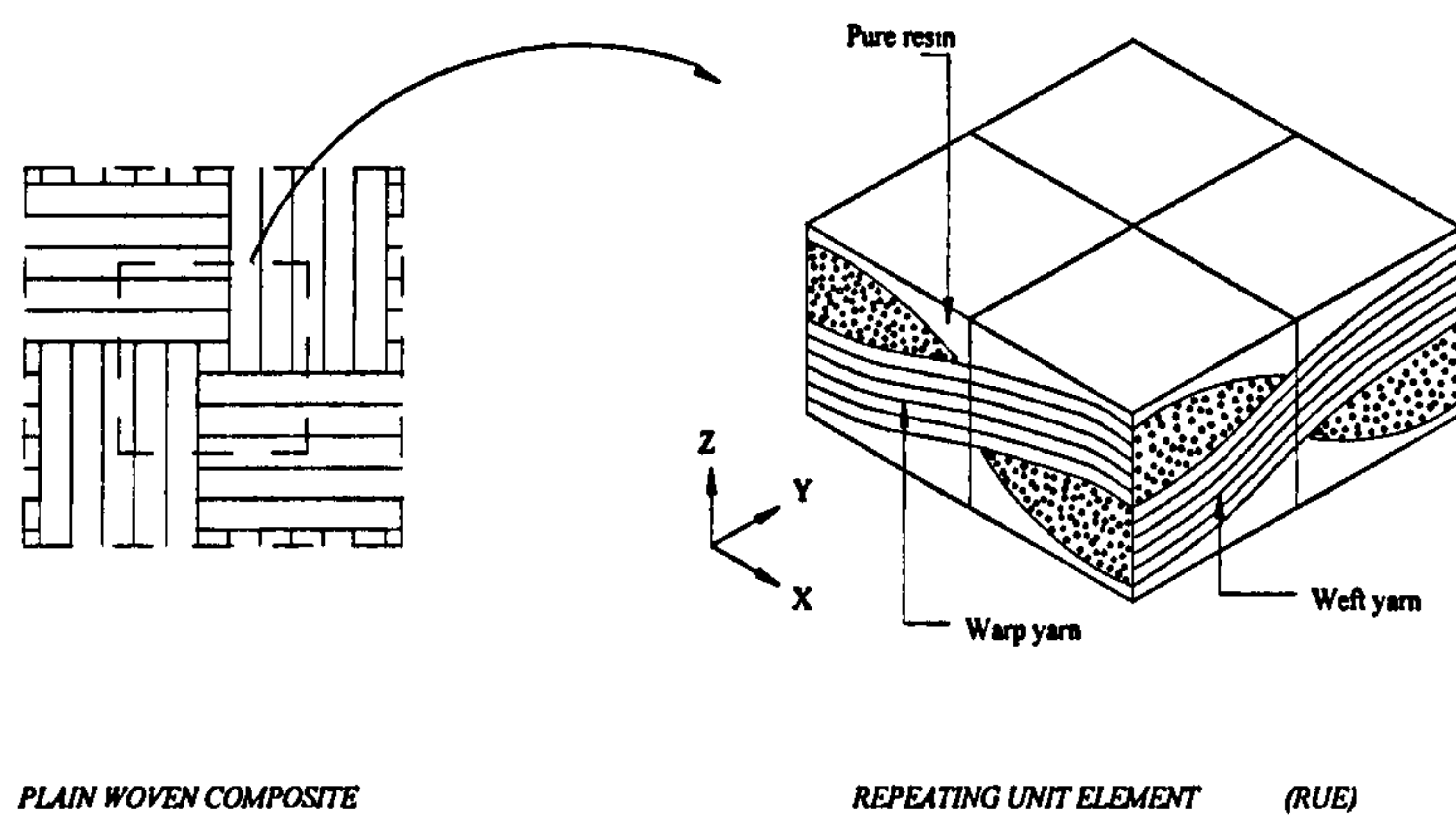


Figure 2.3 Repeating unit element (RUE) of a plain woven composite

The models took the actual fabric structure into account by considering the fibre undulation and continuity along both warp and weft directions, the possible presence of a gap between adjacent yarns and the actual cross-sectional geometry of the yarn. The undulation of the warp and weft yarns was assumed as a sinusoidal form. The elastic constants of warp and weft yarns were determined by using the CCA model. Then the classic laminated theory was used to assemble the infinitesimal pieces of a section, which may contain pure matrix region, warp and weft yarn regions. According to the

way of assemblage of the infinitesimal pieces, two schemes were used to calculate the stiffness of the lamina: a Series-Parallel (SP) model and a Parallel-Series (PS) model. Detailed analysis of woven architecture created very complicated expressions for the prediction of material properties. The authors discussed the effects of undulation length, lamina thickness on the elastic properties of the composite. The results showed that with the increase of the undulation length, the elastic moduli increase; and as the lamina thickness increases, the elastic moduli decrease. Experiments were also carried out on the coupons of carbon/epoxy and E-glass/epoxy plain weave composites to verify the analytical models. The comparisons showed that the results from two-dimensional PS model were fairly good. The models were extended to the prediction of elastic properties of the plain woven composite laminates. Reasonable results from PS model were also obtained.

In the model presented by Kabelka [33], a woven lamina was modelled as an assemblage of three layers: the warp yarn, the weft yarn and the pure resin. The elastic constants of the warp and weft yarns were calculated from the CCA model. Then the influence of the off-axis angle of the warp undulation was considered by using formulae suggested by Lekhnitskii [34]. The components of the stiffness for three layers were found by using the classic laminated theory. Elastic properties, then, were calculated finally. In this analysis, undulation of the weft yarn was not considered. No numerical or experimental results were given in this paper.

Polynkine and Kenlen [35] developed an analytical model which can consider the influence of the yarn deformation due to the woven composite forming process. In this model a woven lamina was modelled as a four-layer asymmetrical laminate. It was assumed that before the forming process was carried out, the laminate consisted of two undulated warp and weft layers in cross-ply configuration between two pure matrix layers. During the forming process, the warp and weft yarns were rotated with respect to each other by the angle corresponding to the in-plane shear deformation. The elastic properties of the warp and weft yarns were calculated by using the CCA model. Then the influence of the off-axis angle of the warp and weft undulations was considered. The components of the stiffness for four layers were obtained by using the classic

laminated theory, and elastic properties of the composite were calculated finally. The author gave several examples which showed that the model was in agreement with other models.

Naik [36] provided a general purpose micro-mechanics analysis to predict overall three-dimensional material properties of woven and braided fabric composites. In the calculation of three-dimensional effective stiffness, the woven lamina was treated as a spatially oriented fibre composite composed of yarn slices with transversely isotropic material properties. The elastic constants of warp and weft yarn were estimated by using the rule of mixtures or the micro-mechanics analysis developed by the author himself. The average stresses in the woven lamina were found by transforming yarn slice stresses to the global coordinate and computing the volume average of the stresses in all the yarn slices, then the overall stiffness matrix was calculated. The elastic properties of woven composite predicted in this model were compared with test ones, and good agreement was achieved.

Though detailed theoretical analysis in above models generated complicated expressions of material properties, it was able to study the effects of the architecture of the fabric on the material properties, for instance, how the material properties decreased as the undulation (waviness) increased. The studies showed that if waviness is small, the effect of the waviness on the material properties is not significant. This would happen if woven roving is a flat ellipse in shape, like the woven composite used in this research (See Figures 3.10-3.13 in Chapter 3)

In addition to these theoretical analyses, numerical methods such as finite element method were used to analyse the elastic properties of woven composite. Ishikawa and Chou [27] applied the finite element method to the mosaic model and the fibre undulation model. Zheng and Harding [37] and Foye [38] used the finite element analysis to calculate the elastic properties of woven lamina. Whitcome et al. [39] developed a new element type to account for the microstructure of the woven lamina within a single element. Tests of the element showed good performance for modelling the global deformation behaviour of woven composite.

Glaessgen et al [40] made a finite element model for analysis of woven composite material at microstructural level. The detailed architecture of an RUE in a plain woven composite was modelled in the finite element analysis. The distribution of strain energy density inside the composite was examined. The finite element analysis was expensive in computer time.

2.4 Strength of unidirectional composite material

Composite material made from fibres embedded in a plastic matrix is orthotropic. The characterisation of the strength therefore requires the measurement of a minimum of five independent strength parameters. They are the longitudinal tensile strength, the longitudinal compressive strength, the transverse tensile strength, the transverse compressive strength and the in-plane shear strength. The theoretical methods for the prediction of strengths are far less sophisticated than those for the prediction of stiffness, because of the complicated failure mechanism in the composite. In practice, some of the strengths can be determined by physical experiments, but some cannot be measured easily. In this research simple and widely accepted formulae for predicting the strengths of the unidirectional composite presented by Chamis [44] are introduced. The formulae were derived from the microstructural analysis of the unidirectional composite. The author demonstrated that the formulae can provide reasonably good strength predictions in comparison with the tests. The formulae are listed as follows.

Longitudinal tension:	$X_T = V_f S_{ft}$	
Longitudinal Compression:		
Fibre compression:	$X_C = V_f S_{fc}$	
Delamination/shear:	$X_C = 10 S + 2.5 S_{mt}$	
Transverse tension:	$Y_T = [1 - (\sqrt{V_f} - V_f) (1 - E_m / E_f)] S_{mt}$	(2.3)
Transverse compression:	$Y_C = [1 - (\sqrt{V_f} - V_f) (1 - E_m / E_f)] S_{mc}$	
In-plane shear:	$S = [1 - (\sqrt{V_f} - V_f) (1 - G_m / G_f)] S_{ms}$	

in which X_T , X_C , Y_T , Y_C and S are longitudinal tensile, longitudinal compressive, transverse tensile, transverse compressive and in-plane shear strengths of the composite, respectively. S_{ft} and S_{fc} are tensile and compressive strengths of the fibre. S_{mt} , S_{mc} and S_{ms} are tensile, compressive and shear strengths of the matrix. E_f and G_f are elastic moduli of the fibre; E_m and G_m are elastic moduli of the matrix. V_f is fibre content by volume of the composite.

2.5 Strength of woven composite material

A woven composite consists of pure resin regions, and regions with interlaced warp and weft yarns, as can be seen in Figure 2.3. The strength of the woven composite is more or less determined by the strength of a laminate containing two orthogonal unidirectional laminate. There are few theoretical methods in literature which can provide prediction formulae as simple as the expression (2.3). However, several attempts have been made to explore the relationships between tensile and shear strengths and strain using the finite element method and a progressive failure analysis technique [45-55].

Ishikawa and Chou [45] and Ishikawa et al [46] extended their *fibre undulation model* and *bridging model* into material non-linear analysis and the prediction of the strength of a woven composite. In the *fibre undulation model* the non-linearity of the weft and matrix regions was considered as the effect of the non-linear shear deformations of the weft yarns and extensional deformations of the pure matrix. The non-linear analysis theory developed by Hahn and Tsai [47] was used to account for the non-linear shear effect, then a modification was made to discuss the extensional non-linearity of the matrix material. The non-linear behaviour in the warp direction was considered as due to transverse cracking in the warp region. The progressive cracking which occurred as the load increases resulted in a non-linear stress-strain relationship. The treatment of the non-linearity due to transverse cracking required the use of an *effective stiffness reduction scheme*, that is, the weakening of the warp region led to a deterioration of the

initial elastic stiffness of the warp material. The prediction of the strength of the composite was based on an iterative process for each increment of applied stress. At each step the stress resultant was assumed to be constant. Initially, the stress-strain relation was evaluated from linear elastic material properties. Then the material non-linearity and transverse cracking were assumed to reduce the initial elastic properties. The iteration terminated if the ratio of the initial strain and revised strain exceeded a certain value, which indicted the failure of the composite. The authors emphasised that the increment of the applied stress should be carefully controlled. Numerical analyses were carried out for two types of composite, glass/polyimide and graphite/epoxy, to get non-linear stress-strain relationship of the composites.

Ganesh and Naik [48-50] extended the work [30-32] in the prediction of failure strength of two-dimensional plain weave fabric composite under on-axis uniaxial static tensile loading. Different stages of failure such as warp strand transverse failure, weft strand shear/ transverse failure, pure matrix region failure were considered. The authors simplified the plain weave lamina as a series of idealised laminate configurations and the studies were then carried out on these configurations. A good correlation was obtained between the predicted and the tested results.

Kriz [51] used the finite element method to study the effect of transverse cracking on the stiffness and internal stresses of a plain weave composite. In this paper the author focused on the stress distribution near to and far from the crack tip, instead of the non-linear stress-strain relationship. Stanton and Kipp [52] developed a non-linear model for plain weave carbon-carbon composites which accounted for the differences in tension and compression response, warp-weft crossover damage and multi-axial stress interactions.

Blackketter et al [53] modelled damage propagation in a plain woven composite material subjected to tension or shear loading by using the finite element method. The progressive damage analysis in the finite element method can model the non-linear stress-strain relationship of a composite material and can estimate the effect of damage propagation by stiffness reduction. The microstructure of the plain woven composite

was modelled in the analysis. The effect of progressive damage on the stress-strain relation was accounted for by using incremental-iterative process. Incremental-iterative meant that when iteration was performed at each load or displacement increment, a small increment was used to ensure stability and to maintain accurate prediction of non-linear response. The maximum stress failure criterion was used to examine the stress in pure resin, warp and weft regions. The strengths for pure resin, warp and weft yarns were determined by test. During the incremental loading, if the stress exceeded the strength in a certain failure mode, degradation was modelled by reducing the corresponding the stiffness of the composite to small value at that particular Gaussian integration point. Final failure of the entire model resulted in large displacements (several orders of magnitude larger than previous values) and program was terminated. The non-linear tensile and shear stress-strain relationships predicted by this paper for graphite/epoxy woven composite agreed with test ones well. Whitcomb and Sriengan [54] also used the same technique to predict progressive failure in a plain weave composites.

Naik [55] expended his work [35] in the prediction of non-linear stress-strain response of the woven and braided fabric composite materials. Non-linear shear response and yarn bending and straightening/wrinkling were accounted for. Maximum stress/strain failure criteria were used to detect the material failure. The effect of progressive damage in the composites was evaluated by using the same stiffness reduction scheme as Blacketter et al's in reference [53].

2.6 Progressive failure analysis of composites structures

The progressive failure analysis technique has been examined in woven composites at the micro-structural level, as mentioned above. This technique provides a most important tool for the prediction of damage and failure of large composite structures under load. In the last decade, considerable efforts have been made to understand the initiation and evolution of various damage modes for a laminated composite consisting of stacks of unidirectional laminae. These researches have provided a better

understanding of the behaviour of the composite structures. However, little work has been reported on progressive failure analysis in woven composite structures, such as laterally-loaded panels.

Failure of a composite is a result of damage accumulation, which leads to progressive failure. The presence of damage can affect the mechanical properties and the structural responses of the composite. The term *first ply failure* is used to describe the initial failure of a single layer in a laminate either in the fibre direction or in the direction normal to the fibres. The term *last ply failure* or *final failure* occurs when, due to the progressive damage accumulation, the composite has degraded to the point where it is no longer capable of carrying additional load.

2.6.1 Definition of the progressive failure analysis

Progressive failure analysis is a special technique for damage analysis of composite structures. It is performed in the finite element method at integration points in the element at each increment of load. Stresses at each integration point are checked by comparing them with strength parameters in a specific failure criterion. When an allowable value of a particular parameter is exceeded, a certain type of failure occurs. This leads to a reduction of the stiffness of the composite. The accumulated damage is accounted for by reducing the material stiffness by a certain percent of its initial value. The progressive failure analysis needs an incremental-iterative technique, which means that iteration is performed at each small load or displacement increment to detect possible damage in the integration points at each load increment. In progressive failure analysis there are two important steps. The first is to choose proper failure criteria; and the second is to choose proper stiffness reduction of the material after initial failure of the composite.

2.6.2 Failure criteria

There are five commonly used failure criteria [56]: the *maximum stress criterion*, the *maximum strain criterion*, the *Hoffman criterion*, the *Tsai-Hill criterion* and the *Tsai-*

Wu criterion. They are categorised into two: independent and interactive (or quadratic polynomial) criterion. An independent criterion (the *maximum stress criterion* or the *maximum strain criterion*) is simple to use and tells the mode of failure but it neglects the effect of stress interactions in the failure mechanism. An interactive criterion (the *Tsai-Wu criterion*, the *Hoffman criterion* or the *Tsai-Hill criterion*) includes stress interactions in the failure mechanism, but it does not tell the mode of failure.

The *maximum stress criterion* assumes that if any of the following conditions is satisfied then failure has occurred.

$$\sigma_1 \geq X_T, \quad \sigma_6 \geq S, \quad \sigma_3 \geq S, \quad \sigma_2 \geq Y_T, \quad \sigma_4 \geq R \quad (2.4)$$

where σ_1, σ_2 are normal stress components, $\sigma_4, \sigma_3, \sigma_6$ are shear stress components, X_T, Y_T are the normal strengths of the composite in the 1, 2 directions and R, S are the shear strengths in the 23 and 12 planes. When σ_1, σ_2 are of a compressive nature they should be compared with X_C, Y_C which are normal strengths in compression along the 1, 2 directions.

The *maximum strain criterion* assumes that if any of the following conditions is satisfied the failure has occurred.

$$\varepsilon_1 \geq X_{\varepsilon T}, \quad \varepsilon_6 \geq S_{\varepsilon}, \quad \varepsilon_3 \geq S_{\varepsilon}, \quad \varepsilon_2 \geq Y_{\varepsilon T}, \quad \varepsilon_4 \geq R_{\varepsilon} \quad (2.5)$$

where $\varepsilon_1, \varepsilon_2$ are the normal strain components, $\varepsilon_4, \varepsilon_3$ and ε_6 are the shear strain components, $X_{\varepsilon T}, Y_{\varepsilon T}, S_{\varepsilon}, R_{\varepsilon}$ are ultimate strains of the composite. When $\varepsilon_1, \varepsilon_2$ are negative, they should be compared with $X_{\varepsilon C}, Y_{\varepsilon C}$, which are ultimate strains in compression.

The *Tsai-Wu criterion*, which is the most general quadratic polynomial criterion, can be expressed as,

$$F_i \sigma_i + F_{ij} \sigma_i \sigma_j \geq 1 \quad i, j = 1, 2, \dots, 6 \quad (2.6)$$

where F_i and F_{ij} are second and fourth order lamina strength tensors related to the material strengths.

$$F_1 = \frac{1}{X_T} - \frac{1}{X_C}, \quad F_2 = \frac{1}{Y_T} - \frac{1}{Y_C}, \quad F_3 = \frac{1}{Z_T} - \frac{1}{Z_C}$$

$$F_{11} = \frac{1}{X_T X_C}, \quad F_{22} = \frac{1}{Y_T Y_C}, \quad F_{33} = \frac{1}{Z_T Z_C}$$

$$F_{44} = 1/R^2, \quad F_{55} = 1/S^2, \quad F_{66} = 1/S^2$$

$$F_{12} = -\frac{1}{2\sqrt{X_T X_C Y_T Y_C}}, \quad F_{13} = -\frac{1}{2\sqrt{X_T X_C Z_T Z_C}}, \quad F_{23} = -\frac{1}{2\sqrt{Y_T Y_C Z_T Z_C}}$$

Since the Tsai-Wu criterion does not describe the modes of failure, the modes of failure can be only determined by the strength tensors associated with normal stresses or shear stresses, whichever provide the most contribution to a failure.

The Hoffman criterion, which was proposed by Hoffman for brittle orthotropic materials, can be expressed as,

$$\begin{aligned} & \frac{1}{2} \left(-\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C} + \frac{1}{Z_T Z_C} \right) (\sigma_2 - \sigma_3)^2 + \frac{1}{2} \left(\frac{1}{X_T X_C} - \frac{1}{Y_T Y_C} + \frac{1}{Z_T Z_C} \right) (\sigma_3 - \sigma_1)^2 + \\ & \frac{1}{2} \left(\frac{1}{X_T X_C} + \frac{1}{Y_T Y_C} - \frac{1}{Z_T Z_C} \right) (\sigma_1 - \sigma_2)^2 + \left(\frac{1}{X_T} - \frac{1}{X_C} \right) \sigma_1 + \left(\frac{1}{Y_T} - \frac{1}{Y_C} \right) \sigma_2 + \\ & \left(\frac{1}{Z_T} - \frac{1}{Z_C} \right) \sigma_3 + \left(\frac{\sigma_4}{R} \right)^2 + \left(\frac{\sigma_5}{S} \right)^2 + \left(\frac{\sigma_6}{S} \right)^2 \geq 1 \end{aligned} \quad (2.7)$$

The *Tsai-Hill criterion* can be expressed as,

$$\begin{aligned} & \left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_3}{Z}\right)^2 - \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2}\right)\sigma_1\sigma_2 - \left(-\frac{1}{X^2} + \frac{1}{Y^2} + \frac{1}{Z^2}\right)\sigma_2\sigma_3 - \\ & \left(\frac{1}{X^2} - \frac{1}{Y^2} + \frac{1}{Z^2}\right)\sigma_1\sigma_3 + \left(\frac{\sigma_4}{R}\right)^2 + \left(\frac{\sigma_5}{S}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 \geq 1 \end{aligned} \quad (2.8)$$

The values of X , Y , Z are taken as either X_T , Y_T , Z_T or as X_C , Y_C , Z_C depending on the sign of σ_1 , σ_2 , σ_3 respectively.

The failure criteria mentioned above were originally used in metallic materials, and subsequently adapted for use in fibre reinforced plastic composites. Composite materials are different from metallic materials. There are several failure modes in the composite as mentioned before; some kinds of damage (for example, the transverse matrix failure) do not completely destroy the load bearing capability of the material. Therefore it is important for the failure criterion to indicate both damage and the orientation of the damage. It is widely accepted that the Hashin failure criterion [57] is suitable for describing the different failure modes in the composite. The *Hashin criterion* consists of three subsets of criteria for predicting three modes of in-plane failure: matrix cracking, in-plane shear failure, and fibre breakage.

$$\begin{aligned} \text{Matrix cracking:} \quad & \left(\frac{\sigma_2}{Y}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 \geq 1 \\ \text{In-plane shear failure:} \quad & \left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_6}{S}\right)^2 \geq 1 \\ \text{Fibre breakage:} \quad & \left(\frac{\sigma_1}{X}\right) \geq 1 \end{aligned} \quad (2.9)$$

The values of X , Y are taken as either X_T , Y_T or as X_C , Y_C depending on the sign of σ_1 , σ_2 , respectively. Obviously, if the maximum stress in the fibre direction exceeds the strength of the fibre, the expression (2.9) will indicate the mode of *fibre breakage*.

2.6.3 Stiffness reduction scheme

The failure criteria are used to determine the damage in a given composite structure. After the damage occurs, the structure may still carry load but the stiffness of the composite is reduced. The progressive failure analysis depends on the stiffness reduction scheme used. Different approaches have been suggested in reference [58-59].

- *Total discount approach* - the stiffness and the strength of a failed ply are reduced to zero, although the ply is still physically present. This scheme is often used in the failure criteria which are not able to describe the mode of failure, such as the Tsai-Wu criterion.
- *Limited discount approach* - the reduction of stiffness depends on the mode of failure in action. For fibre failure, the total discount method is applied. For matrix failure, a zero stiffness and strength are just assigned to the failed ply for the transverse and shear modes, while the stiffness and the strength in fibre direction are not changed. This scheme is often used in the failure criteria which are able to describe the mode of failure, such as the Hashin criterion.
- *Residual property approach* - in this model a continuum damage model is used to predict progressive damage and the stiffness drop in the composite. Unlike the other two approaches, this method is able to consider the residual stress carrying capacity at failed ply.

2.6.4 Progressive failure analysis of composite structures

Echaabi et al [60] studied the progressive damage and the failure modes of graphite/epoxy non-woven composite *beams* in three-point bending theoretically and experimentally. Three failure criteria were used: the maximum stress criterion, the Tsai-Hill criterion and the Tsai-Wu criterion. The progressive damage analysis was calculated by using the total discount approach in the stiffness reduction scheme. Greif and Chapon [61] also studied the same problem. Two failure criteria were used: the

maximum stress criterion and the Tsai-Wu criterion. The total discount approach in the stiffness reduction scheme was employed. Both of the researches achieved good prediction of progressive damage of the beams.

Kim et al [62] developed a beam element which could be used to analyse the progressive failure of a laminated *beam* in bending. The element was able to calculate normal and shear stresses at each layer. The maximum stress criterion and the Tsai-Wu criterion and the limited discount approach for stiffness reduction scheme were used in the analysis. Compared with the experimental results they concluded that the Tsai-Wu criterion provides more reliable results of final failure load than the maximum stress criterion.

Liechi and Tang [63] tried to predict the ultimate load capacity of wood composite I-beams using the Tsai-Wu failure criterion in ADINA. Failure analyses were conducted in the ADINA post-processor. The stress values from the ADINA output were checked using failure criterion. Failure was indicated by points lying outside the failure envelope. However, the ADINA program is not able to reduce the stiffness of failed elements, and the analysis was unable to include progressive damage effects.

Shahid and Chang [64] developed a model for predicting the effect of accumulated damage on the response and failure of a multi-directional laminated composite plate subjected to in-plane loads. In order to predict damage accumulation and the corresponding response of the composite plate, a damage model containing constitutive modelling and damage accumulation prediction was proposed. The constitutive modelling established the relationship between ply properties and the damage of the laminate. The accumulated damage in the laminate as a function of the applied loads was estimated. In this paper the Hashin failure criterion was selected as the damage accumulation criterion, and residual property approach was used in the stiffness reduction scheme. A finite element code was developed based on the model and the numerical analysis agreed with the experimental results well.

Tolson and Zararas [65] presented a model for determining the ultimate strength of an arbitrary laminated plate under in-plane tensions or transverse pressure. Different failure criteria, including the maximum stress, the Tsai-Wu and the Hashin failure criteria, were used to determine the initial damage. The total discount approach was used in the stiffness reduction scheme. Several numerical examples were calculated. The results for a composite plate subjected to uniaxial tension agreed with the experimental ones well. However, the experimental data from the composite plate subjected to biaxial tension were so scattered that the results from test and the numerical method were not comparable. For the composite plate under transverse pressure, no experimental data were available for comparison.

Kam and sher [66] studied the non-linear behaviour, initial damage and last ply failure of centrally-loaded laminated plates with semi-clamped edges by using the finite element method. In the analysis the von Karman large deflection theory was used to allow for membrane effects. In the stiffness reduction scheme the total discount approach was used. The five commonly used failure criteria were used to determine initial damage. The experiment on laminated plates was carried out to construct experimental load-deflection curves for the plates. To investigate the accuracy of these failure criteria the AMS3 acoustic emission system was mounted on the plate, which allowed for capturing initial damage signal from the test. Comparing theoretical results with tests, the author pointed out that none of the initial damage loads predicted from the various failure criteria was comparable with the test results. The ability of these commonly used failure criteria to predict initial damage load was doubtful. However, the final failure pressures of composite plates predicted by this paper agreed with the experimental ones well provided that central deflection-to-thickness ratio was less than 6. For the case of a deflection-to-thickness ratio greater than 6, the author concluded that the von Karman large deflection theory was insufficient to analyse such very large deflection and suggested that a more sophisticated non-linear method must be used. The similar work was also reported by Reddy and Reddy [67].

In recent years NASA has supported several research programmes on progressive failure analysis of laminated composites. Lo et al. [68] developed a method for

progressive failure analysis of laminated composites and implemented it in the NASA COMET finite element code. The damage-dependent constitutive equations were developed at the ply level in order to take the effect of microcracks in the ply on the behaviour of composite structures into account. These equations were then used in the development of the lamination equations. The method has been demonstrated well in a number of examples. Sleight et al [69] developed a progressive failure analysis methodology to predict the non-linear response and failure of laminated (non-woven) composite structures. This progressive failure analysis used C^1 plate and shell elements based on classical laminated theory to calculate the in-plane stresses. Various of failure criteria were used to determine the failure of the composite. The analysis was implemented in the NASA COMET finite element code. Results on two laminated composite panels predicted from the method agreed with test results.

2.7 Geometrical non-linear analysis of laminated composite panels

Broadly, non-linear behaviour may arise because of (1) geometrical non-linearity. (2) material non-linearity or (3) a combination of these. In general, no analytical solution of the equations is available and a numerical solution will therefore be necessary.

2.7.1 Numerical solution procedures

A number of numerical solution techniques are available for non-linear problems [70]. The commonly used numerical solution techniques are *incremental techniques* and *iterative techniques*.

In the *incremental techniques*, the load is applied in a sequence of sufficiently small increments so that the structure can be assumed to deform linearly during each increment. The initial geometry of the structure is used for the first load increment. The updated geometry is then used to obtain a further solution for the second load increment and so on. The trouble with a pure incremental approach is that it

accumulates error with each load increment, causing the final results to be out of equilibrium.

The *iterative technique* is the most commonly used numerical solution technique. In this the resulting non-linear system of equations of the discretized structure is solved iteratively - starting from an initial estimate of the solution, an improved solution which accounts for the non-linear effects is obtained using a linearised set of equations. The successful application of this class of solution methods depends upon the accuracy of the initial estimate of the solution. The Newton-Raphson equilibrium iteration is very accurate and converges very quickly for any realistic initial estimate of the solution.

2.7.2 Geometrical non-linear analysis of laminated composite panels

The geometrical non-linear theory of laminated composite plates was derived from von Karman non-linear theory of plates. Chia [71] described this theory in detail in his book. The fundamental formulae are summarised here.

Consider a thin laminated composite plate with a number of homogeneous orthotropic layers perfectly bonded together. Assume that the thickness of the plate is small in comparison with its width, and hence Kirchhoff's hypothesis is valid. The in-plane displacements u , v and transverse deflection w at an arbitrary point of the plate in the x , y and z directions are

$$u(x,y,z) = u^o - z \frac{\partial w^o}{\partial x}$$

$$v(x,y,z) = v^o - z \frac{\partial w^o}{\partial y}$$

$$w(x,y,z) = w^o(x,y)$$

in which u^o , v^o and w^o are the values of u , v and w at the reference plane. The geometrical relationship between displacements and strains are in the expression (2.10).

$$\begin{aligned}
\varepsilon_x &= \varepsilon_x^0 + z\kappa_x \\
\varepsilon_y &= \varepsilon_y^0 + z\kappa_y \\
\varepsilon_{xy} &= \varepsilon_{xy}^0 + z\kappa_{xy}
\end{aligned}
\tag{2.10}$$

in which

$$\begin{aligned}
\varepsilon_x^0 &= \frac{\partial u^0}{\partial x} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \\
\varepsilon_y^0 &= \frac{\partial v^0}{\partial y} + \frac{1}{2} \left(\frac{\partial w}{\partial y} \right)^2 \\
\varepsilon_{xy}^0 &= \frac{\partial u^0}{\partial y} + \frac{\partial v^0}{\partial x} + \frac{\partial w}{\partial x} \frac{\partial w}{\partial y}
\end{aligned}$$

and κ_x , κ_y , and κ_{xy} are the plate curvatures given by

$$\kappa_x = -\frac{\partial^2 w}{\partial x^2} \quad \kappa_y = -\frac{\partial^2 w}{\partial y^2} \quad \kappa_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

Under Kirchhoff's hypothesis the plate is in an approximate state of plane stress, by use of the plane-stress constitutive equation, for the m -th layer the stress-strain relation is

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{12} & k_{22} & k_{23} \\ k_{13} & k_{23} & k_{33} \end{bmatrix}_{(m)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix}
\tag{2.11}$$

in which k_{ij} are the stiffnesses of the m -th layer. As in the classical plate theory, the stress resultants and couples are defined by

$$[N_x, N_y, N_{xy}] = \int_{-h_1}^{h_2} [\sigma_x, \sigma_y, \sigma_{xy}]^m dz$$

$$\begin{aligned}
[Q_x, Q_y] &= \int_{-h_1}^{h_2} [\sigma_{xz}, \sigma_{yz}]^m dz \\
[M_x, M_y, M_{xy}] &= \int_{-h_1}^{h_2} [\sigma_x, \sigma_y, \sigma_{xy}]^m z dz
\end{aligned} \tag{2.12}$$

in which N_x, N_y, N_{xy} are membrane forces per unit width, Q_x, Q_y are transverse shear forces per unit width, and M_x, M_y, M_{xy} are bending moments, and h_1 and h_2 are the upper and lower boundary of the plate. Substituting equation (2.11) into (2.12), and taking equations (2.10) into account, yields the constitutive relations of the plate

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon^o \\ \kappa \end{Bmatrix} \tag{2.13}$$

where the elements in the matrix are given by

$$(A_{ij}, B_{ij}, D_{ij}) = \int_{-h_1}^{h_2} k_{ij}^m (1, z, z^2) dz$$

For a laminated composite plate subjected to the transverse pressure q the governing equations of large deflection analysis are given by

$$\begin{aligned}
\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} &= 0 \\
\frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} &= 0 \\
\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + q + N_x \frac{\partial^2 w}{\partial x^2} + N_y \frac{\partial^2 w}{\partial y^2} + 2N_{xy} \frac{\partial^2 w}{\partial x \partial y} &= 0 \\
\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x &= 0 \\
\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - Q_y &= 0
\end{aligned} \tag{2.14}$$

Using these equations, Chia solved several simple problems and gave the closed form of the solutions [71]. But, in general, it is difficult to get a direct solution to a problem. The finite element method is therefore often preferred for the analysis of geometrically non-linear composite panels [72-73].

In the derivation of the foregoing plate equations the basic assumption is Kirchhoff's hypothesis in which the effect of the transverse shear deformation is neglected. The thinner the plate, the more accurate the hypothesis. However, it is expected that the transverse shear effect on the composite plates may be greater than that on homogeneous plates, because of shear deformations in the unreinforced matrix [74-76].

Using the thick plate theory in his paper, Gorji [75] considered the effect of transverse shear deformation on laminated plates undergoing large deflection. In the solution the non-linear terms of the lateral displacement were considered as an additional set of lateral loads acting on the plate, and the complicated equations of the plate can be treated as an equivalent plate with small deflection. The numerical examples indicated that the shear deformation resulted in higher values of the lateral deflection than the classical laminated theory. Kant and Kommineni [76] used a higher order displacement field to account for the effect of the shear deformation in the analysis of angle-ply composites and sandwich panels. The numerical examples showed that the method could predict displacement and stresses more accurately than using the thick plate theory.

Chapter 3 Experiments on composite coupons

3.1 Introduction

Experiments on composite coupons can provide important information such as fibre content, overall stiffness and strength of the composite. Systematic studies of the composite panels started in 1993 in this department. Initially, three types of composite were used, which were referred to as laminates A, B and C here. The coupon test results for these three composite materials have been partly reported in reference [6]. In this research a new composite material was introduced, which was referred to as laminate D. This chapter reports the coupon tests and summarises the test results.

3.2 Test equipment and preparation

An Instron universal machine was used for all of the coupon tests. An extensometer was used to monitor the strains in the longitudinal direction in the tensile specimen. The extensometer had a gauge length of 150 mm, which was long enough to average out any local variations of strain. The design of the extensometer ensured that the output signal represented the mean strain over the cross-section. The electrical outputs from the extensometer and from the load cell of the Instron machine were used to drive an X-Y plotter. This procedure was used in preference to a data logger so that the minutest irregularities of the load-strain curve could be observed as the tests progressed. For some selected samples in laminate D foil strain gauges with 10 mm length and 150 Ω resistance were mounted in both sides in longitudinal and transverse directions. This permitted the calculation of Poisson's ratio of the material and provided a check on the longitudinal strains.

The manufacturer provided basic information about the composites, as listed in Table 3.1. Among the composites, laminates A and D are satin woven composites, laminate C

is plain woven composite and laminate B is non-woven composite. For all the composites, the reinforcement is E-glass fibre; and the matrix is polyester resin.

Laminates B and C were manufactured by using wet lay-up (or hand lay-up) method. In this method the dry reinforcement was placed on a flat mould and the appropriate quantity of resin was applied. The wet composite was then rolled by hand to distribute the resin evenly and to remove air pockets. Another layer of reinforcement was then laid on top and more resin was poured and rolled over the reinforcement. This sequence was continued until the desired thickness was reached. The layered material was then left to harden and cure under normal laboratory conditions.

Laminate A was produced by using a vacuum-assisted Resin Injection Moulding technique (RIM). The reinforcement was cut to size and placed in the flat mould. The mould was then vacuum-sealed with a flexible covering and catalysed resin was sucked into the mould using a vacuum pump. After the correct quantity had been injected the vacuum was maintained until the laminate had cured. The vacuum seal was then removed and the edges were trimmed as necessary.

Laminate D was made long afterwards and designated to have the same reinforcement and resin as laminate A. However, wet lay-up manufacturing method was used rather than a vacuum-assisted resin injection-moulding used in laminate A. This resulted in laminate D thicker than laminate A. Also the catalyser used in laminate D was different from laminate A. This made laminate D more brittle than laminate A.

Coupons with dimensions of 25*300 mm (or longer) were cut from the composites at angles of 0, 22.5, 45, 67.5 and 90 degree to the warp direction. All the specimens were accurately measured for thickness and width at three points along the length. Before the test, all the equipment were set up and calibrated. Figure 3.1 shows the typical set-up for a coupon test on laminate D.

* Strictly speaking, each layer of fabric is woven with 90.1% of the fibres in the warp direction (see Section 3.3). In effect these warp fibres are virtually straight (see Figures 3.12 and 3.13).

Table 3.1 Lay-up of the composite materials

Laminate	Reinforcement	Resin	Accelerator	Catalyst
	E-glass	Polyester		
A	5-layer	Scott Bader	AK20NL49P	Triganox 67
RIM	satin woven roving 0/90 degree	Crystic 489	0.75 %	2%
B	5-layer	Scott Bader	AK20NL49P	Butanox
Hand lay-up	'unidirectional cloth' 0/45/90/-45/0 degree	Crystic 489	0.75 - 1 %	M50 1.5%
C	4-layer	Scott Bader	AK20NL49P	Butanox
Hand lay-up	plain woven roving 0/90 degree	Crystic 489	0.75 - 1 %	M50 1.5%
D	5-layer	Scott Bader	AK20NL49P	Butanox
Hand lay-up	satin woven roving 0/90 degree	Crystic 489	0.75 - 1 %	M50 1.5%

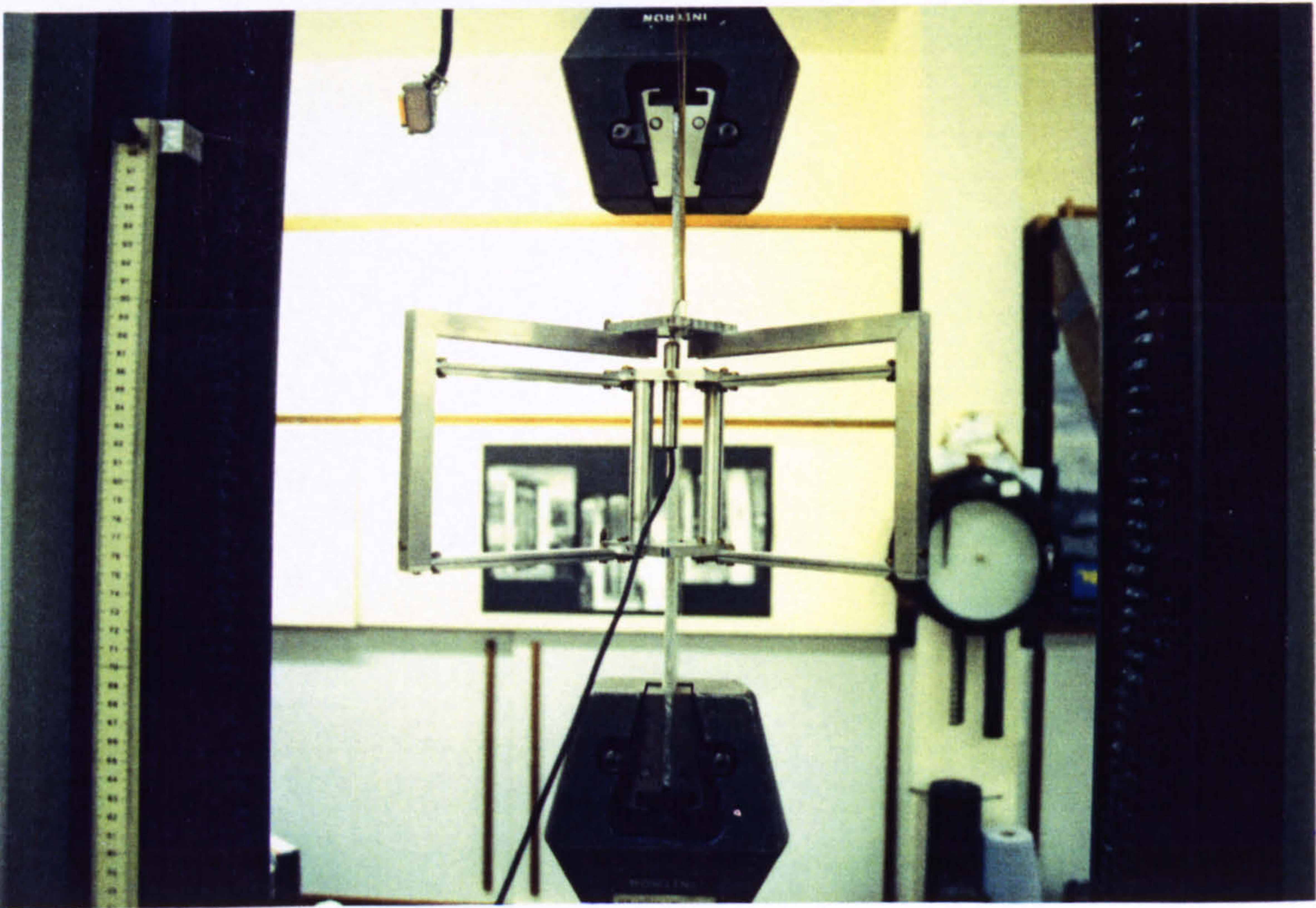


Figure 3.1 The set-up for a coupon test on laminate D.

3.3 Burning test results

In order to obtain the detailed make-up of the composite materials, pieces were cut from the coupons. For laminate D two samples with dimensions of 33 mm * 50 mm and 50 mm * 45 mm were used. These were weighed, placed in crucibles, weighed again, held for two hours in a muffle furnace at 620 C°, and then weighed again. This established the weight of the resin lost during the burning process, in comparison with the weight of the original samples. Thus the fibre content by volume V_f and the resin content by volume V_m can be calculated. In the calculation a nominal *specific gravity* of the fibre 2.55 g/cm³ and the resin 1.22 g/cm³ were used.

For laminates C and D, more detailed measurements were carried out. The fabric bundles in each woven lamina were carefully peeled off from the burnt laminates and were weighed. In comparison with the weight of all of the fibres, the *lamina fibre fraction* can be obtained. A symbol ξ_l was used to represent the lamina fibre fraction.

The fibres in the warp direction and the fibres in the weft direction in each lamina were also separated and weighed. In comparison with the weight of the lamina, the warp fibre fraction and the weft fibre fraction in each lamina can be obtained. A symbol ξ_2 was used to represent the *warp fibre fraction* in each lamina. Obviously, 1- ξ_2 represents the weft fibre fraction in each lamina. Table 3.2 lists the detailed information about the composites used in this research. From the table the main characteristics of the composites can be summarised as follows.

- *Laminate A* is a balanced 5-layer satin woven fabric composite, each of square satin-weave roving, three over, one under. The fibres are aligned at 0 and 90 degrees only. The overall fibre content by volume (V_f) is 0.547. No measurement was taken to decide the fibre content in each lamina and each direction. Hence it is assumed that each lamina has the same amount of the fibres; and in each lamina, the weight of the fibres in the warp direction is the same as that in the weft direction.

- *Laminate B* has 5 identical layers of so-called 'unidirectional' cloth at 0, 45, 90, -45 and 0 degrees. In each layer about 90.1% of the fibres are concentrated in one direction and 9.9% are in the transverse direction; these hold fabric together. V_f is 0.3077, which is the smallest among the four composites but the fibres are placed more evenly. No measurement was taken to determine the amount of fibres in each layer. Hence it is assumed that each layer has the same amount of the fibres.
- *Laminate C* is a balanced 4-layer plain-woven fabric composite with slightly different amount of the fibres in each layer. The fibres are only placed in 0 and 90 degrees. V_f is 0.4431. It is assumed that in each layer, the weight of the fibres in the warp direction is the same as that in the weft direction.
- *Laminate D* has 5 virtually identical layers of satin woven fabric composite, each of square satin-weave roving, three over, one under. In each layer the fibres played in both orthogonal directions are slightly different. The fibres are aligned in 0 and 90 degrees only. V_f is 0.3649. Laminates D and A are similar except laminate A has more fibre content by volume than laminate D.

Table 3.2 Detailed information about the composite materials

Laminate	Parameters	Layer 1	Layer 2	Layer 3	Layer 4	Layer 5
(woven)		0°/90°	0°/90°	0°/90°	0°/90°	0°/90°
A	ξ_1	0.2	0.2	0.2	0.2	0.2
V_f	0.547	ξ_2	0.5	0.5	0.5	0.5
t	2.84 mm	$1-\xi_2$	0.5	0.5	0.5	0.5
C	ξ_1	0.249	0.246	0.256	0.249	
V_f	0.4431	ξ_2	0.5	0.5	0.5	
t	2.926 mm	$1-\xi_2$	0.5	0.5	0.5	
D	ξ_1	0.199	0.198	0.200	0.202	0.201
V_f	0.3649	ξ_2	0.517	0.515	0.513	0.512
t	4.15 mm	$1-\xi_2$	0.483	0.485	0.487	0.488
Laminate	Parameters	Layer 1	Layer 2	Layer	Layer 4	Layer 5
(non-woven)		0°/90°	45°/-45°	3	-45°/45°	0°/90°
				90°/0°		
B	ξ_1	0.2	0.2	0.2	0.2	0.2
V_f	0.3077	ξ_2	0.901	0.901	0.901	0.901
t	3.431 mm	$1-\xi_2$	0.099	0.099	0.099	0.099

V_f fibre content by volume

t coupon thickness

3.4 Tensile test results

In the tensile tests all the samples were loaded firstly a small amount of load (for most of tests 5 KN was used) to determine the overall elastic material properties of the composite. The load was usually cycled a few times between zero and small value. Young's modulus of the coupon was obtained from the initial gradient of each load-strain graph. Figures 3.2 and 3.3 show the typical load-strain graphs plotted during the coupon tensile-test of laminate D. Figure 3.2 shows the test curves for coupons at 0 degree to warp and 90 degree to warp and Figure 3.3 shows the test curves for coupons at 22.5 degree to warp. Then, the samples were loaded up to failure to measure the strength of the materials.

Three coupons in each angle group were tested. The measured values, such as thickness, stiffness and strength from the three coupons were averaged. The averaged ones were used as the final results for this group. Table 3.3 lists the averaged stiffness and strength results for the composites. The detailed results of individual tests were listed in Appendix 1. The table shows that among the composites, laminate A has the biggest stiffnesses and strengths in the warp and the weft directions because of its highest fibre content. It is noted that although laminate B has the lowest fibre content among the composites, it has greater uniformity of stiffnesses and strengths at various angles; and has higher strengths at 22.5, 45 and 67.5 degrees than other composites due to its even distribution of fibres. The table also shows that the woven composites have almost balanced stiffnesses and strengths.

Figure 3.4 illustrates the tested samples of laminate D at various angles to the warp direction. Figure 3.5 illustrates the typical stress-strain relations measured at various angles for laminate D. It is obvious that at fibre direction (0 or 90 degree to warp) the composite is stiff and almost linear-elastic. At other angles, however, after a short initial linear response the stress-strain curves are almost horizontal. The non-linearity is significant. This non-linear behaviour may be caused by the plasticity of the resin and progressive damage.

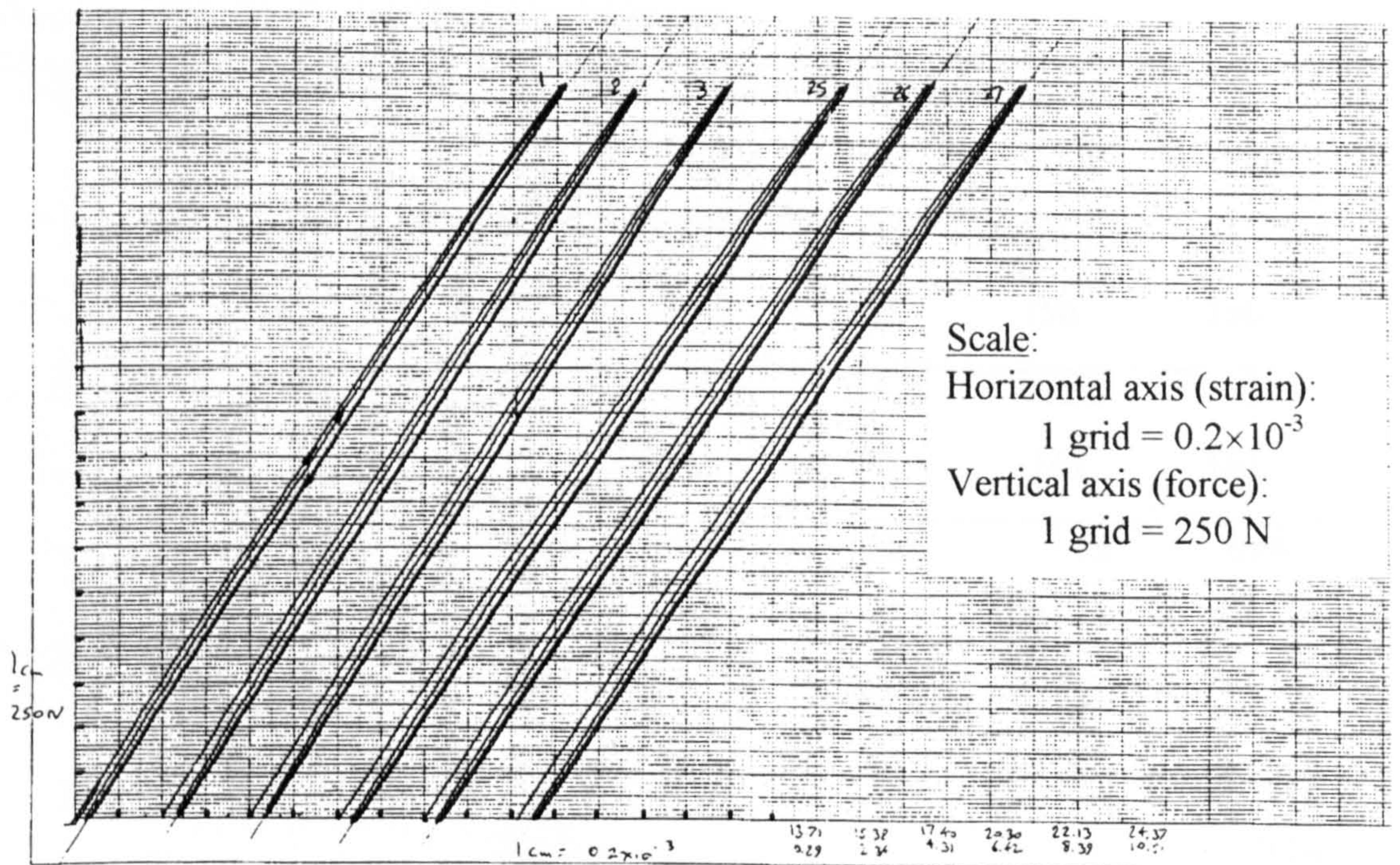


Figure 3.2 Cycled load-strain curves measured from coupon tests
(laminate D, 0 and 90 degrees to warp)

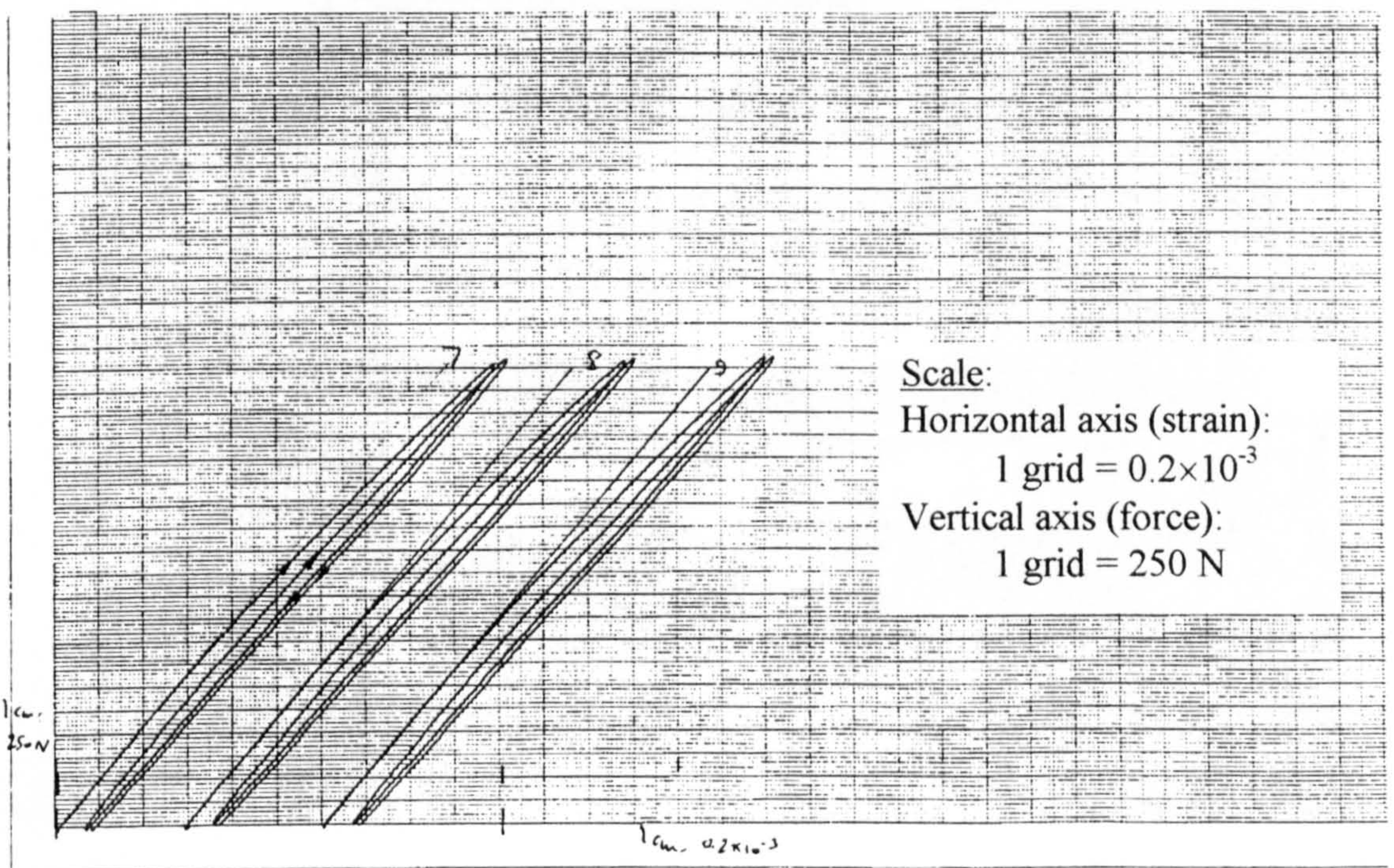


Figure 3.3 Cycled load-strain curves measured from coupon tests
(laminate D, 22.5 degree to warp)

Table 3.3 Tensile test results of the composites

Laminate	Property	Inclination to the warp direction				
		0°	22.5°	45°	67.5°	90°
A	E kN/mm ²	27.1	19.2	15.6	20.7	27.0
	σ_{ult} N/mm ²	340	124	103	156	356
B	E kN/mm ²	16.2	14.8	13.2	12.9	12.9
	σ_{ult} N/mm ²	257	162	175	161	163
C	E kN/mm ²	21.8	15.3	11.5	14.8	20.6
	σ_{ult} N/mm ²	300	119	87	117	330
D	E kN/mm ²	18.5	13	10.2	13.3	17.5
	σ_{ult} N/mm ²	257	131	109	116	229
	ν	0.14	--	--	--	0.14

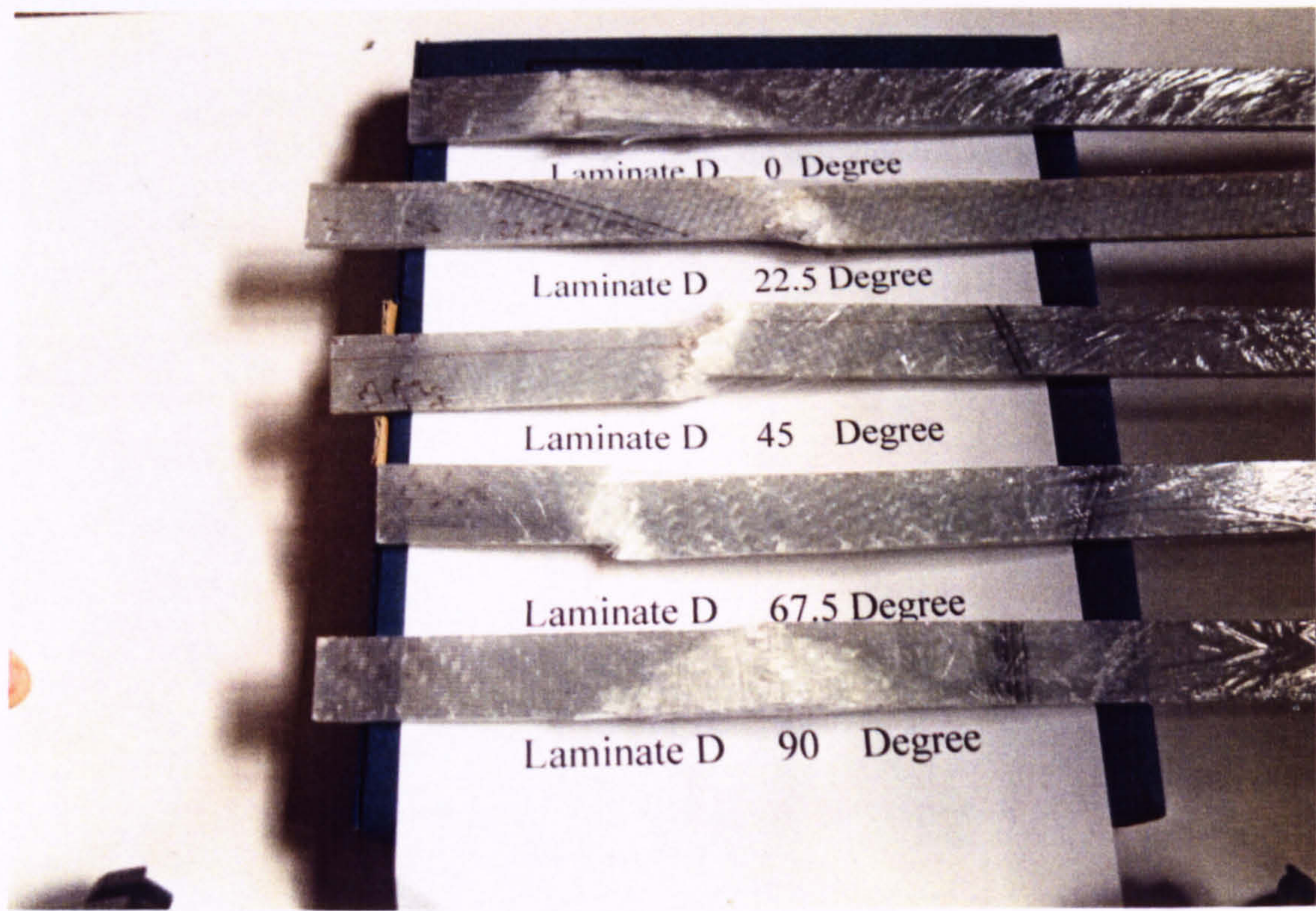


Figure 3.4 The tested samples of laminate D at various angles to warp

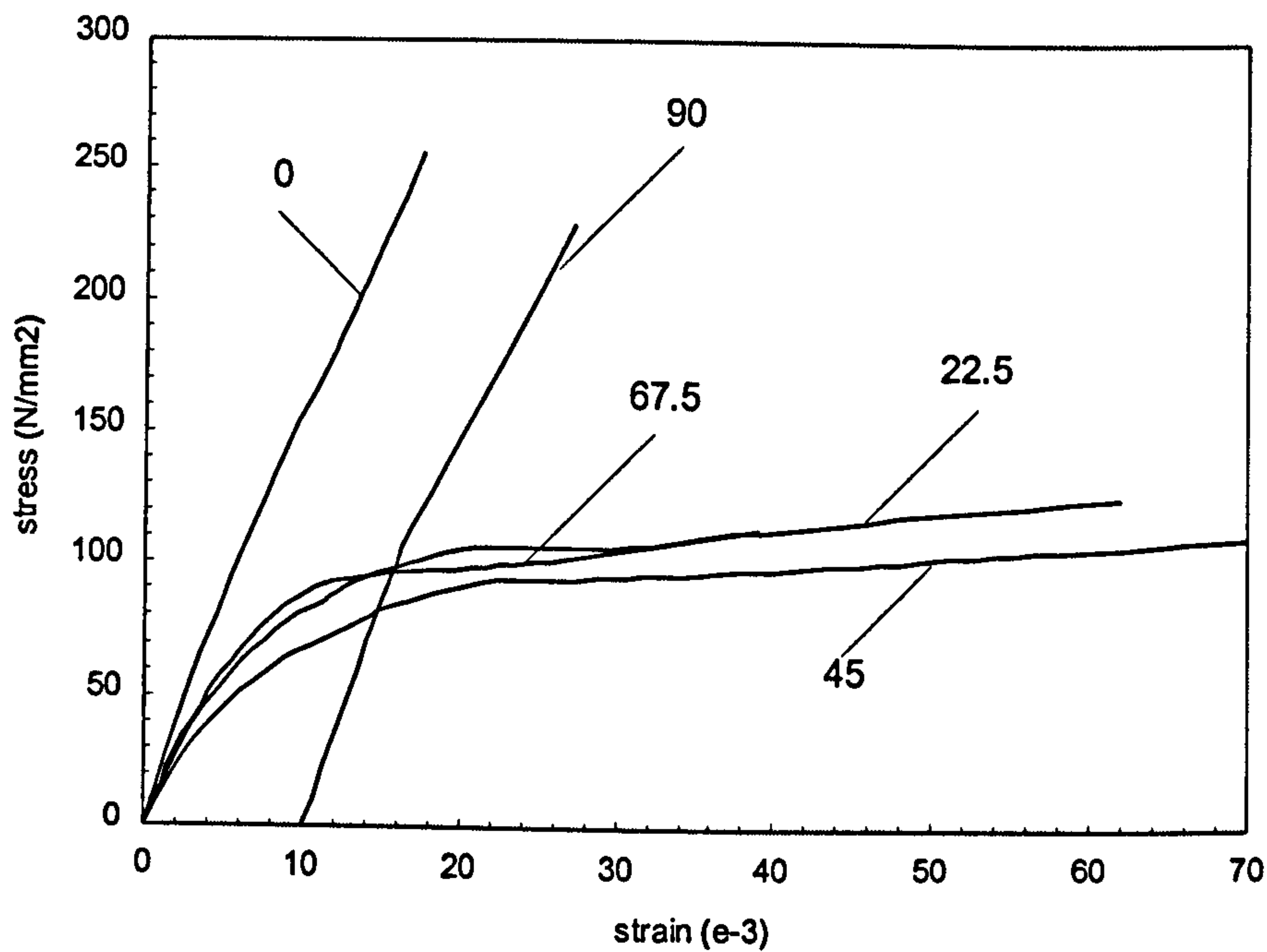


Figure 3.5 Typical tensile stress-strain relations at various angles for laminate D

3.5 Bending test results

4-point bending tests were carried out to determine the bending stiffnesses for laminates A and B and D. As the bending moduli depend on the lay-up of the composite, each sample group was tested on '*smooth side in tension*' and '*smooth side in compression*' in the bending tests.

For laminate D, two groups of coupons in the warp and the weft directions with strain gauges mounted on both smooth and rough sides of the coupons were initially tested to determine the bending moduli in the warp and the weft directions. The bending Young's modulus was then calculated by using the formula

$$E^b = \frac{M}{I} \cdot \frac{t}{\epsilon_{total}} \quad (3.1)$$

where M is bending in pure bending area; I is second moment of area; t is the thickness of the coupon, and $\varepsilon_{total} = |\varepsilon_{smooth}| + |\varepsilon_{rough}|$. ε_{smooth} and ε_{rough} are strains measured from the strain gauges at smooth side and rough side respectively. Figure 3.6a shows the set-up for an 4-point bending test.

Later, 3-point bending tests were carried out to measure ultimate bending strengths for laminate D at various angles. The ultimate bending strength was estimated by using the formula [77]:

$$\sigma_{ult}^b = \frac{6M_{ult}}{t^2b} \quad (3.2)$$

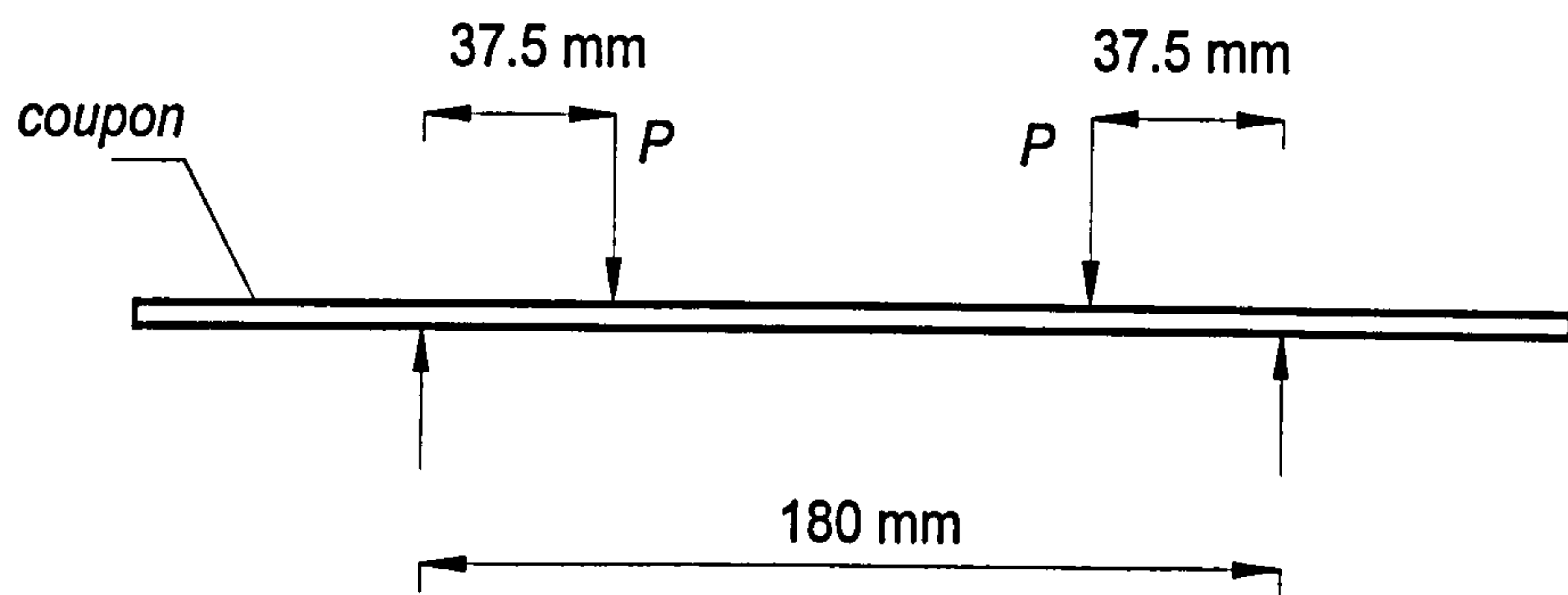
in which M_{ult} is maximum bending moment at failure of the coupon, b is the width of the coupon. Figure 3.6b shows the set-up for an 3-point bending test.

Table 3.4 lists the averaged bending test results. The detailed results of individual tests were listed in Appendix 1. The results show that the stiffnesses for the woven composites A and D are not sensitive to '*smooth side in tension*' and '*smooth side in compression*'. While the stiffness for non-woven composite B at 22.5° shows big difference between '*smooth side in tension*' and '*smooth side in compression*'. Figure 3.7 illustrates a coupon in a three-point bending test. Figure 3.8 shows the typical load vs. central deflection curves measured at various angles for laminate D.

Table 3.4 Bending test results of the composites

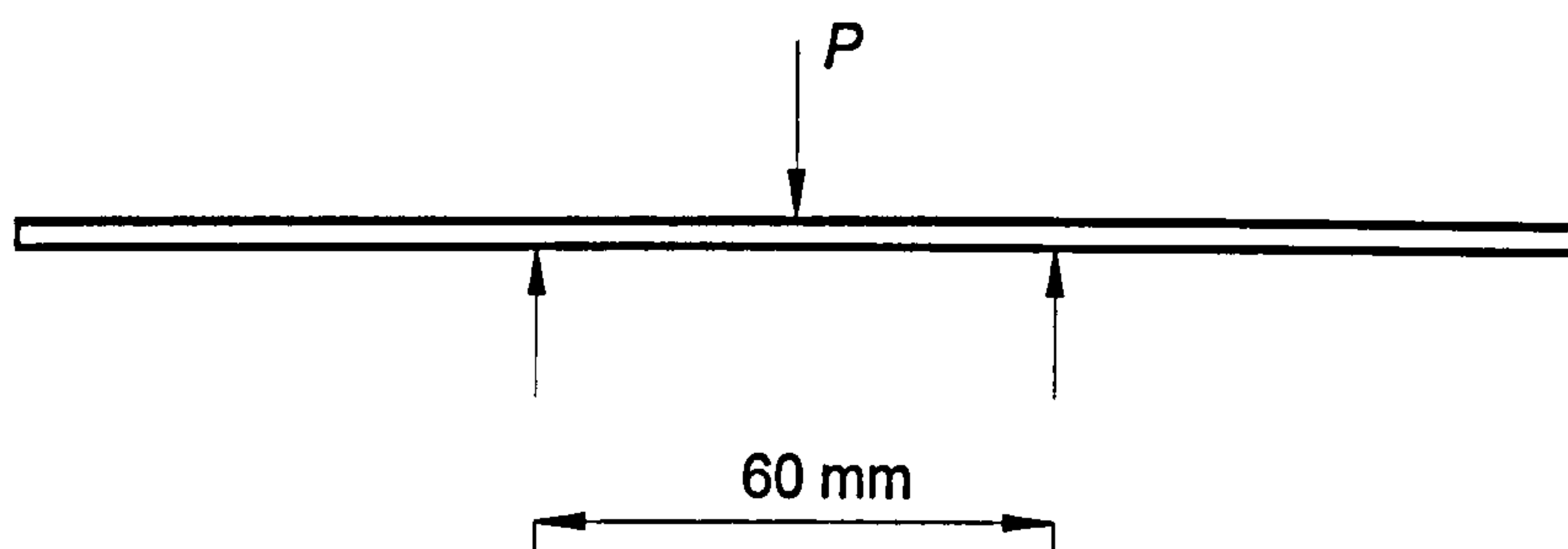
Laminate	Property	Side	Inclination to the warp direction				
			0°	22.5°	45°	67.5°	90°
A	E^b	SST	28.6	22.1	15.8	23.8	29.9
	(KN/mm ²)	SSC	29.6	22.6	16.8	23.8	30.3
		<i>Average</i>	<i>29.2</i>	<i>22.3</i>	<i>16.3</i>	<i>23.8</i>	<i>30.1</i>
B	E^b	SST	15.9	12.6	10.8	8.6	8.2
	(KN/mm ²)	SSC	14.7	17.2	9.1	8.8	8.4
		<i>Average</i>	<i>15.3</i>	<i>14.9</i>	<i>9.95</i>	<i>8.7</i>	<i>8.3</i>
C	--	SST	---	---	---	---	---
	--	SSC	---	---	---	---	---
D	E^b	SST	17.2	---	---	---	16.1
	(KN/mm ²)	SSC	17.1	---	---	---	15.9
		<i>Average</i>	<i>17.15</i>	--	--	--	<i>16.0</i>
	σ_{ult}^b	<i>Average</i>	<i>397</i>	<i>311</i>	<i>224</i>	<i>290</i>	<i>377</i>
	(N/mm ²)						

SST smooth side in tension
SSC smooth side in compression



Width of sample: 25 mm
Overhang length: 110 mm

(a)



Width of sample: 25 mm

(b)

Figure 3.6 (a) 4-point bending set-up; (b) 3-point bending set-up

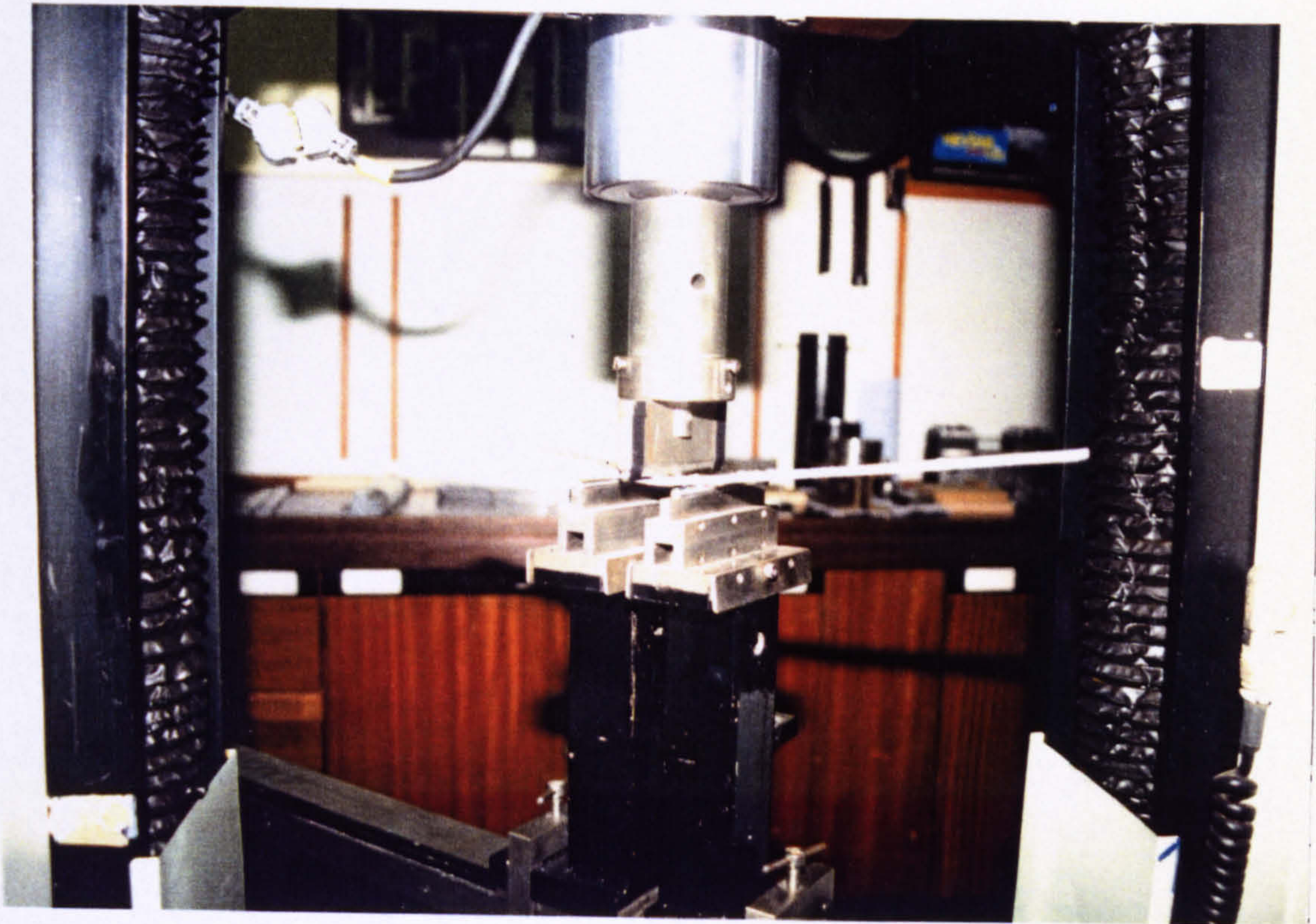


Figure 3.7 Coupon in three-point bending test

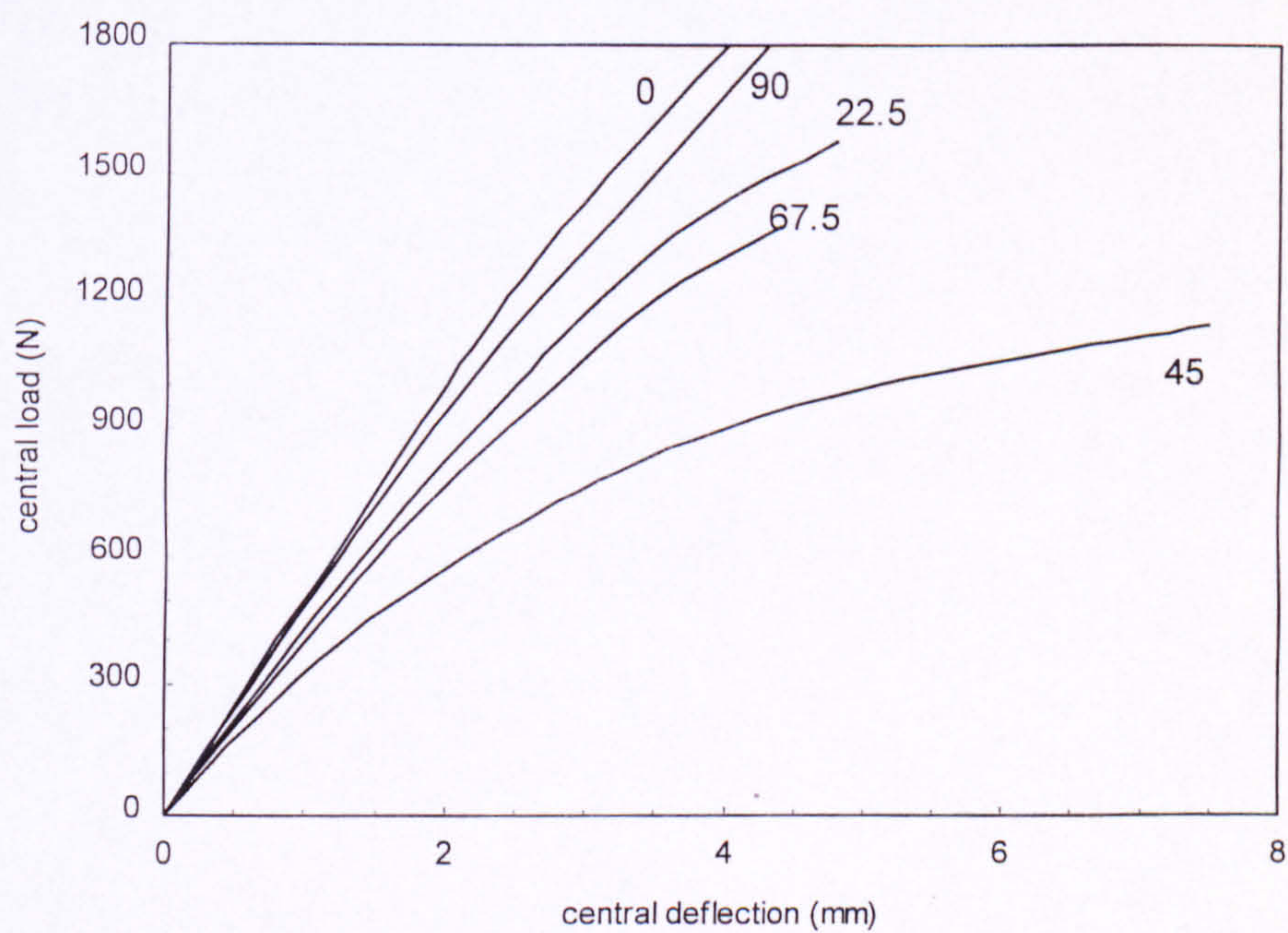


Figure 3.8 Typical load vs. central deflection curves at various angles for laminate D

3.6 Scanning electron microscope analysis

In a woven composite two orthogonal bundles of fabrics are interlaced each other. This interlacing has an effects on the behaviour of the composite. A scanning electron microscope (SEM) analysis has been carried out to investigate in detail the architecture of the composite used in this research. Two kinds of composite were selected. One is laminate D which is a woven composite; and another is laminate B which is a composite containing 'unidirectional' layers. The SEM studies provided visual proof of the waviness of the fabric bundles and of the existence of pure resin layers in the composite.

Pieces cut from the warp section and the weft section (Figure 3.9) of laminates D and B were set in resin and polished, and then put into the SEM machine to be observed in detail. Finally optical photomicrographs of the composite sections were taken as shown in Figure 3.10-3.13.

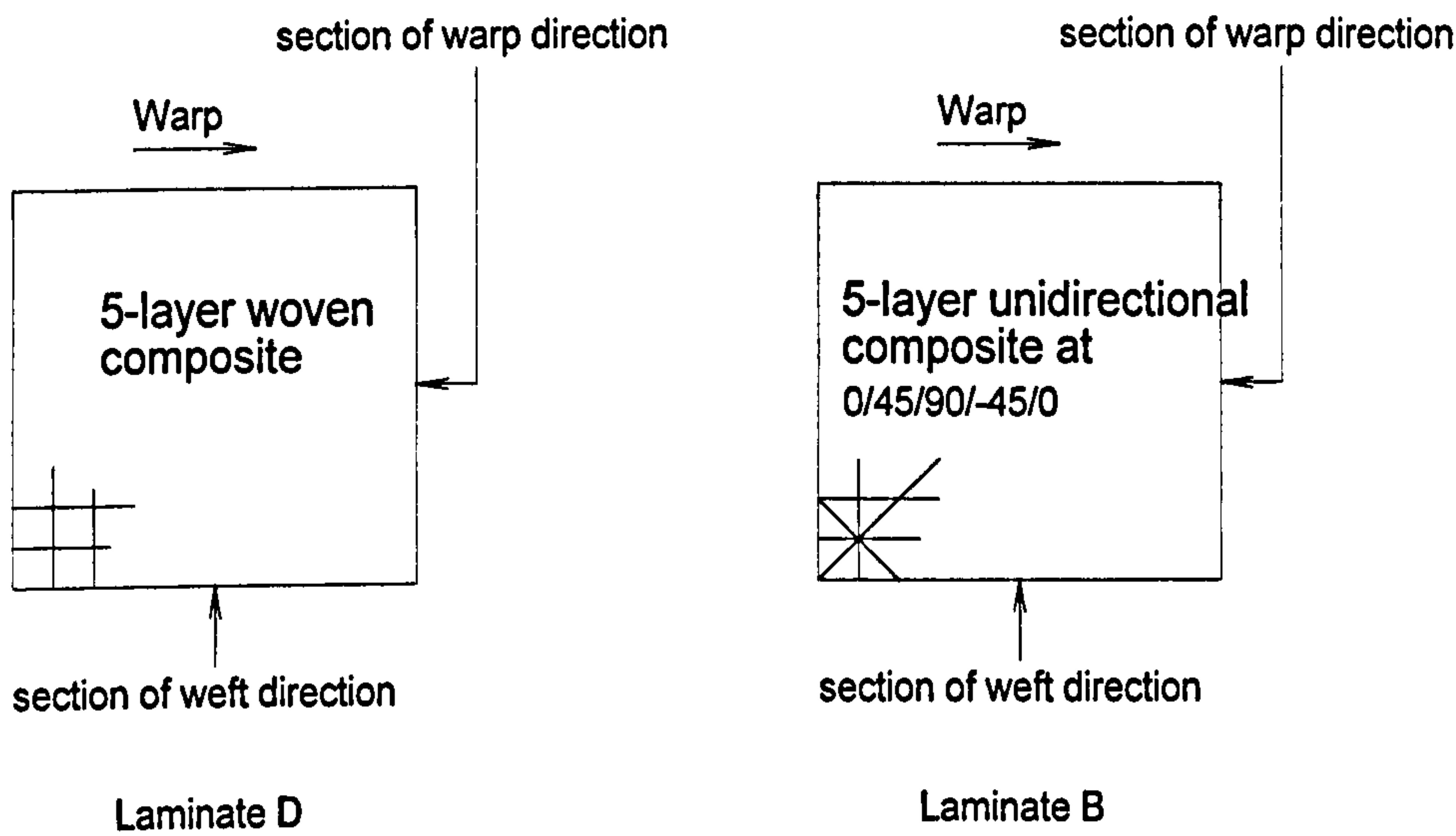


Figure 3.9 Section of warp/weft direction where picture was taken

Figures 3.10 and 3.11 illustrate the architecture of laminate D. The cross-section of each fibre bundle is a very flat ellipse. The waviness of the fabric can be seen in the interlacing regions. It is clear that part of the resin is mixed with fibres in the fabric bundles, but most of the resin is in unreinforced layers between the fabrics.

Figures 3.12 and 3.13 illustrate the architecture of laminate B (with 'unidirectional' fibres). The fabric bundles are also elliptical in cross-section. It can also be seen that the small amount of fibres which is used to hold the main fabric bundles together wave through the main bundles. Layers of pure unreinforced resin can again be seen between the layers of fabric.

In both laminates several small voids were seen in the SEM observations. However, they cannot be seen clearly in the pictures.

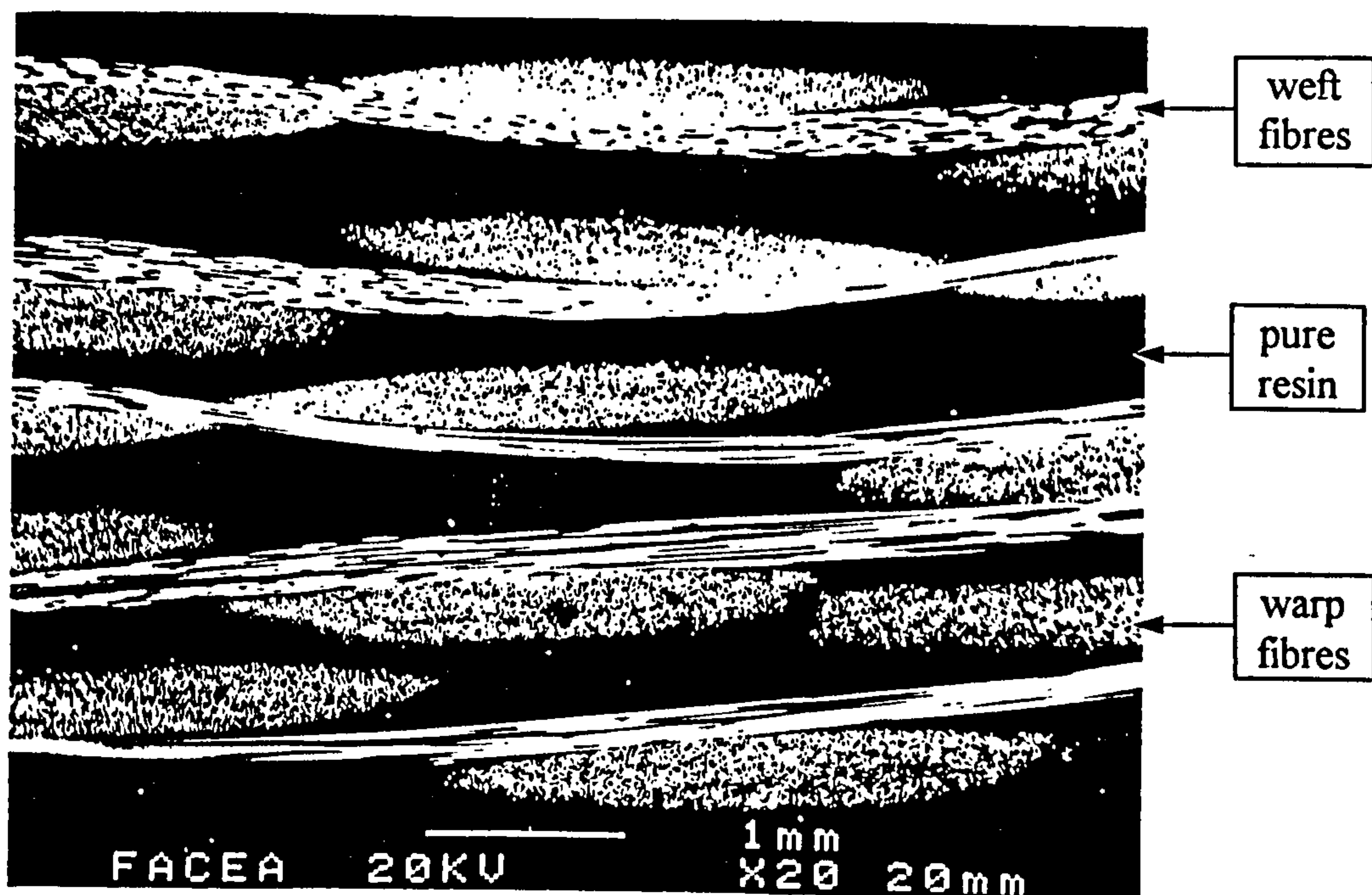


Figure 3.10 Architecture of laminate D - section of warp direction

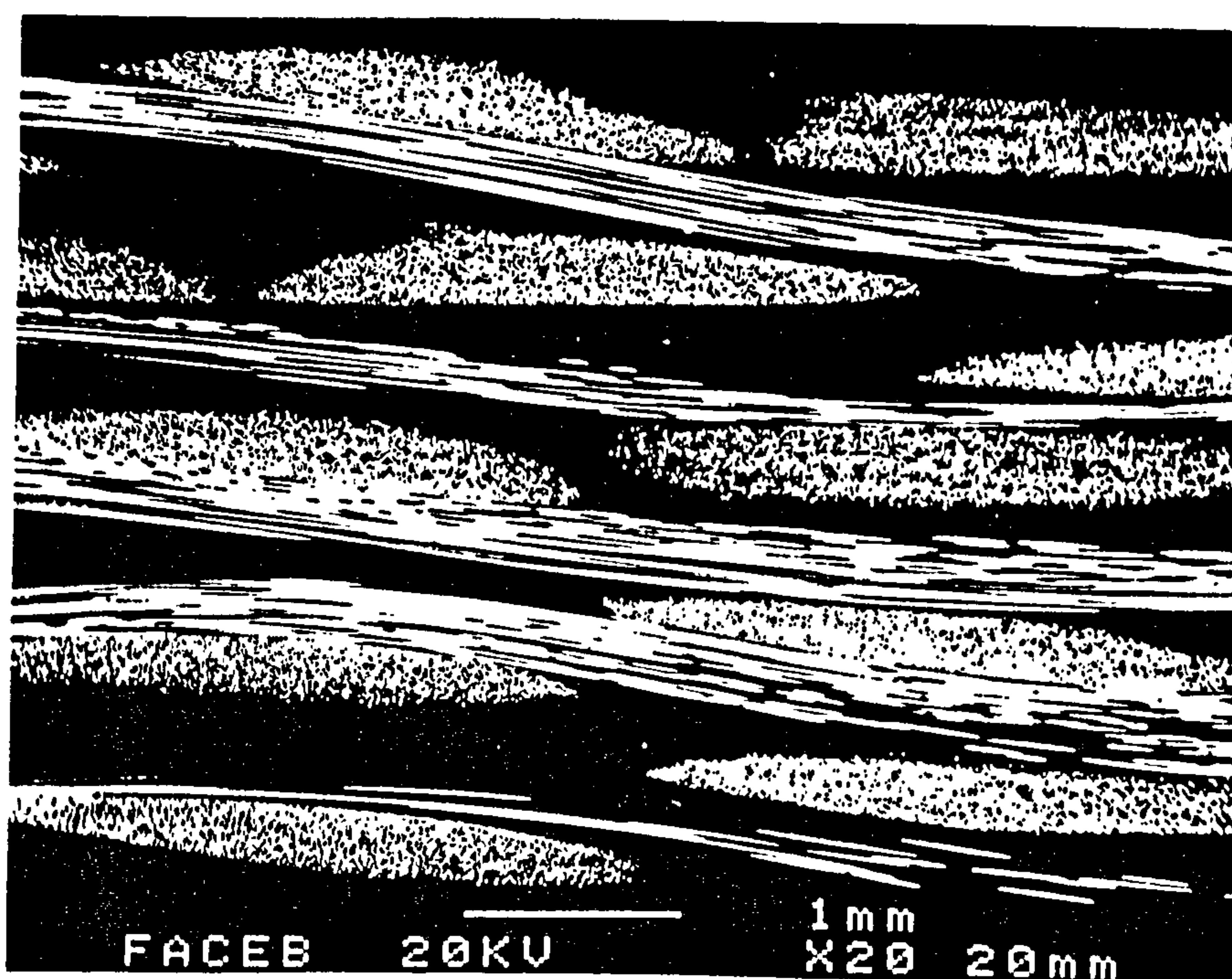


Figure 3.11 Architecture of laminate D - section of weft direction

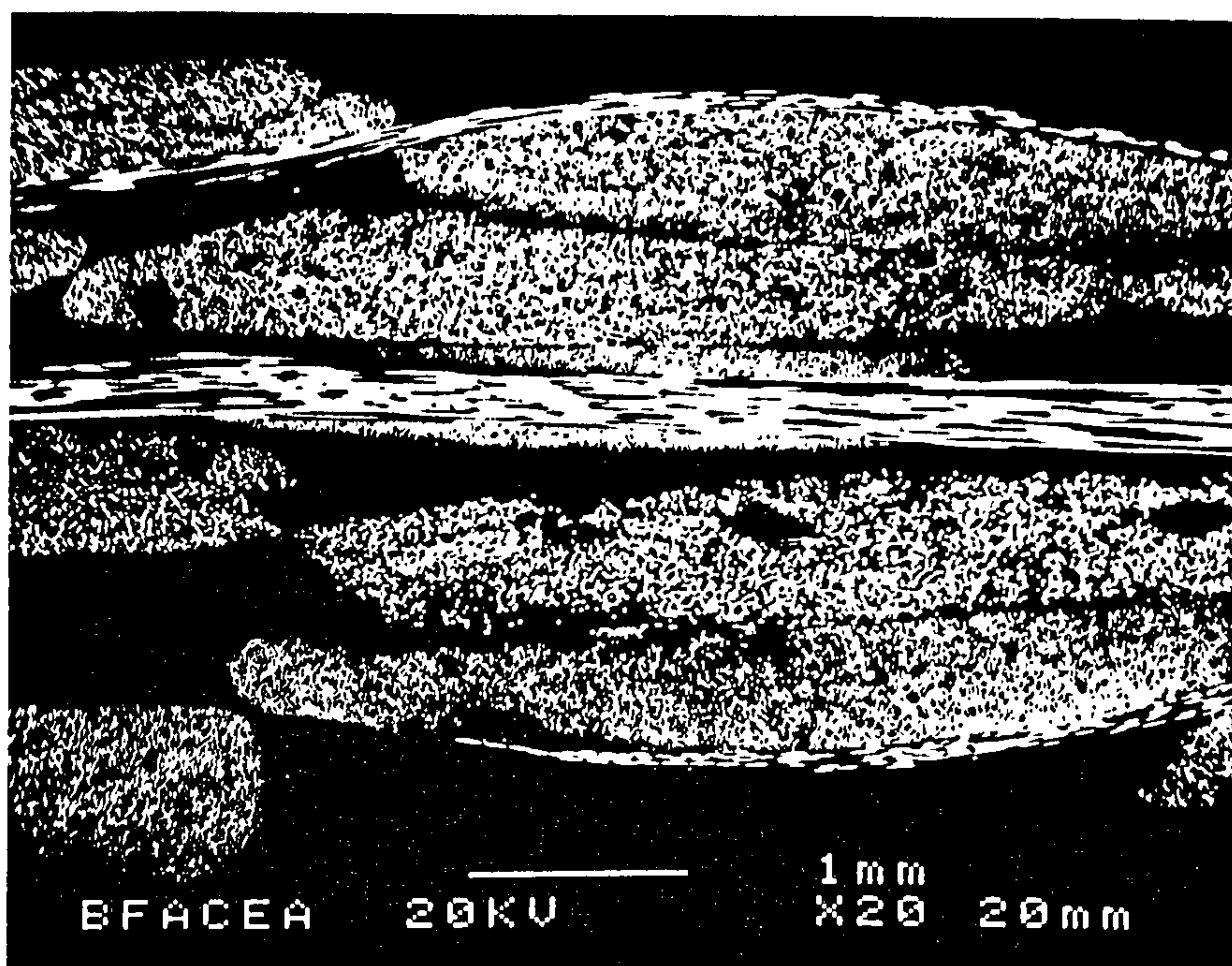


Figure 3.12 Architecture of laminate B - section of warp direction

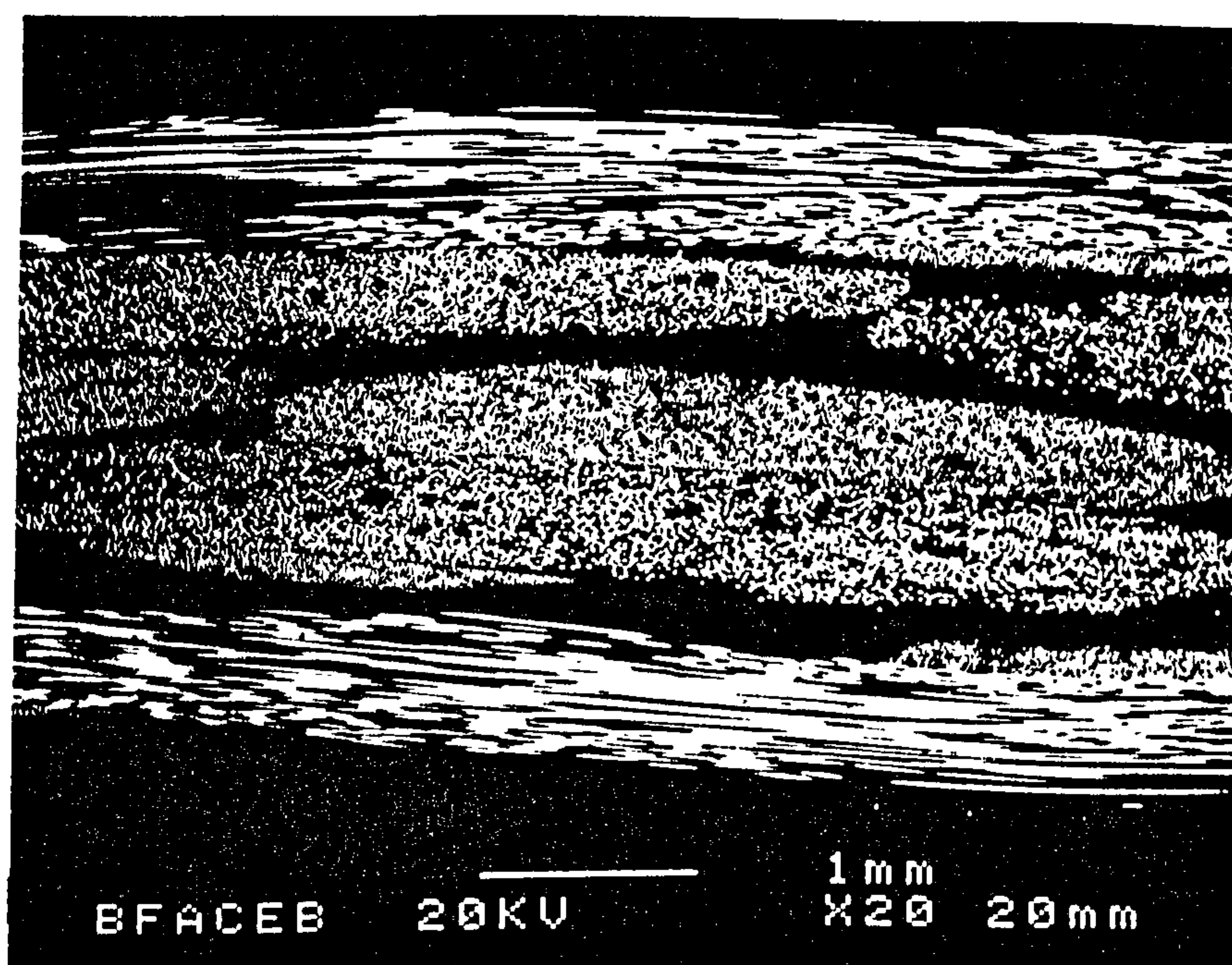


Figure 3.13 Architecture of laminate B - section of weft direction

Chapter 4 Experiments on Composite Panels

4.1 Introduction

No systematic test studies of thin composite panels under high transverse pressure have been reported. This situation directly affects the practical use of such composite panels, especially in ship construction. It is important to carry out panel tests to understand the behaviour of panels under high pressure. As mentioned before, systematic studies of composite panels under high transverse pressure were launched by an EPSRC-supported project in which the Civil Engineering and Ship Science Departments at Southampton University collaborated. During 1993-1994 more than 25 thin composite panels were tested in this department. Three different materials were used (laminates A, B and C described previously) and the panels were of aspect ratios varying from 1:1 to 1:2. A large amount of data was collected. The test results provided precise information for design purposes. All panel test results using laminates A, B and C were reported in detail in reference [12].

During initial finite element modelling of panels of laminates A, B and C, it was found that the deflections calculated from the finite element analysis did not match the test results well unless the small in-plane edge displacement (which was measured in the test) was taken into account. It was realised that the small in-plane edge displacement was very important in the theoretical analysis. However, information about the edge displacement in previous tests was not adequate (only two displacement transducers were used in the middle of one pair of opposite edges). In addition, the strain gauges used in previous tests were not enough to provide clear information about the strain distributions within panel.

For those reasons, it was necessary to carry out a new panel test to investigate the panel behaviour in more detail. A square panel of laminate D was therefore specially designed and tested, taking advantage of experience gained in the earlier tests. More

displacement transducers and strain gauges were employed in this new panel and special care was devoted to the control and measurement of edge displacements. Detailed information about the test set-up of the new panel will be described in Section 4.4

In this chapter panel tests of laminates A, B and C are summarised first, then, the panel test and result discussions of laminate D are reported.

As there are more than 25 panels, it is convenient to refer to the panels in a systematic way. Thus a notation '*Panel nX*' is introduced. The letter *n* represents the sequence number of the tested panel, letter *X* represents the material type of the panel. For example, *Panel 12B* means panel number 12, which is made from laminate *B*.

4.2 Test rig and equipment

All the panel tests (laminates A, B, C and D) were carried out in the test rig as shown in Figure 4.1. It consisted of identical upper and lower steel frames. The test panel was sandwiched between the frames which were then bolted tightly together by two lines of bolts all round. The heavy steel frames gave in-plane and rotational restraint to the edges of the panel and were originally intended to simulate fixed boundary conditions. The positions of the shorter members in each frame could be changed to give the three aspect ratios 1:1, 1:1.5 and 1:2. These were expected to cover the extremes of panel behaviour. Figure 4.9 shows a panel removed from the frames after a test.

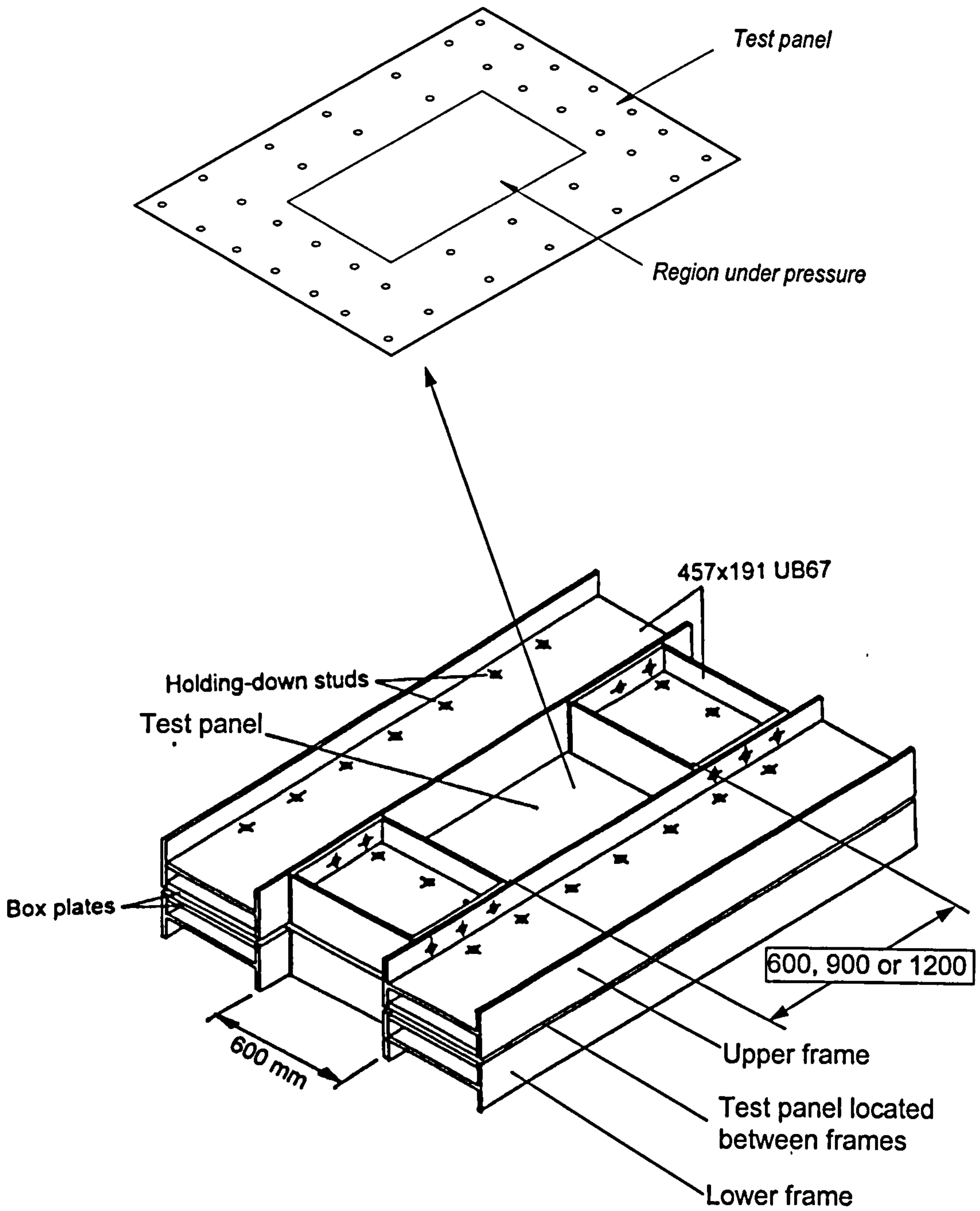


Figure 4.1 Isometric view of test rig, after reference [12]

The pressure was provided through a synthetic bag placed between the specimen and the lower steel frame. Before a test the bag was filled with water from a rigid pressure vessel. Pressure was increased by introducing compressed air into the top of the pressure vessel and was controlled manually and measured by a pressure transducer and a mechanical pressure gauge (Figure 4.2).

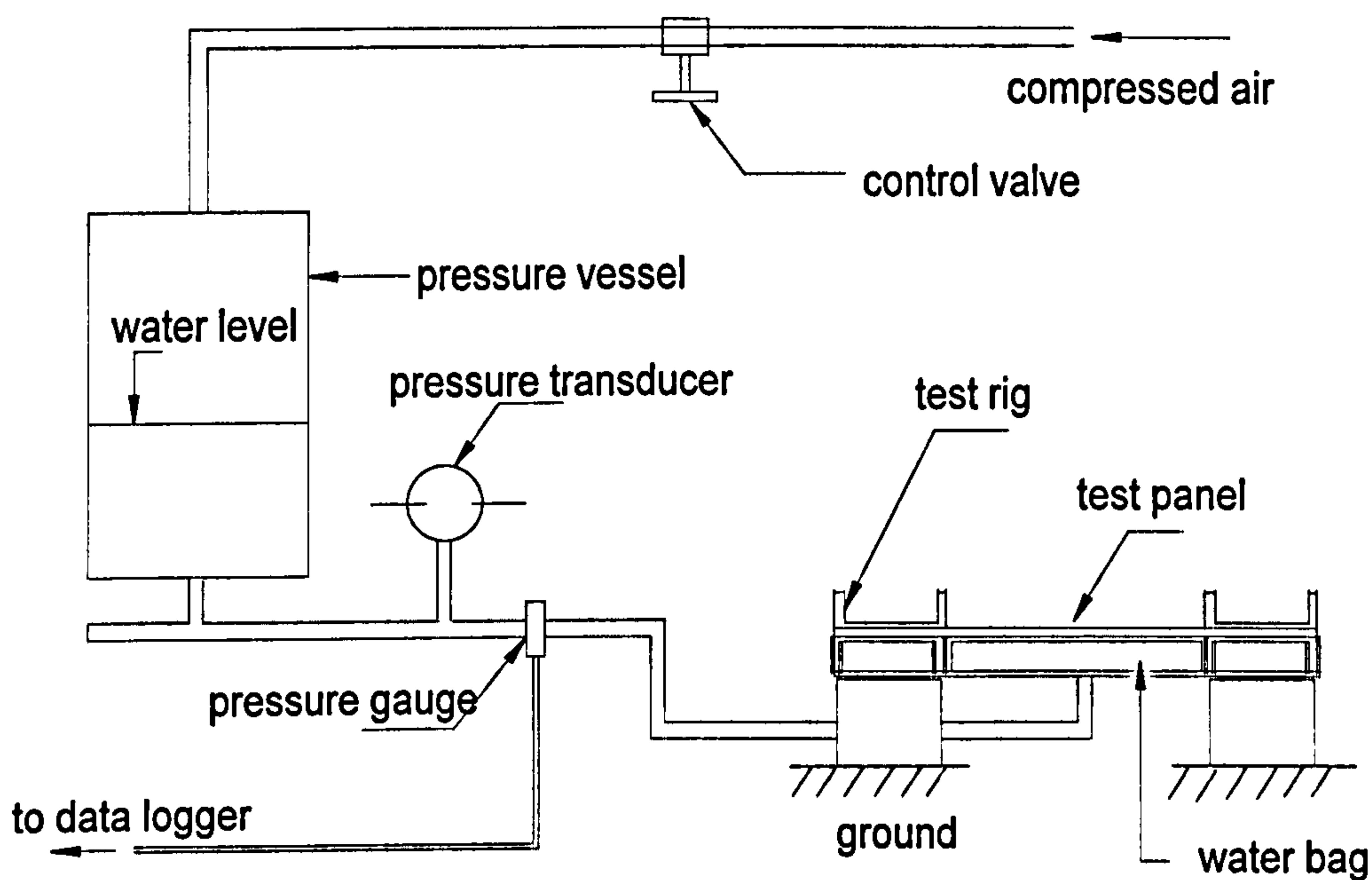


Figure 4.2 Demonstration of the loading system (See also Figure 4.14)

4.3 Summary of tests on panels of laminates A, B and C

Although the test results for panels of laminates A, B and C have been reported in reference [12], the most important results and conclusions are summarised here.

A total of 25 thin composite panels were tested in previous tests. Table 4.1 lists key results and numbering. For some panels strain gauges were mounted on both unloaded and loaded surfaces. The location of strain gauges is shown in Figure 4.3 (a). For all the panels, displacement transducers were placed on the unloaded side of the panel, and two extra transducers were placed at middle of the long edges of the panel to monitor the possible in-plane edge displacement, as shown in Figure 4.3 (b). Figure 4.4 illustrates the side-view of the extra transducer.

4.3.1 Panel deflections

The panels tested were thin, with a short-span/thickness ratio of approximately 150. As would be expected, the panels behaved non-linearly under high transverse pressure. Figure 4.5 shows typical deflection - pressure curves (Panel 32A, aspect ratio 1:1 ; Panel 33A, aspect ratio 1:1.5 ; Panel 34A, aspect ratio 1:2). At low pressure the panels acted as plates with rapid increases in deflections. With the increase of the pressure the panels became stiffer as the membrane action developed, and deflections increased at a lower rate.

4.3.2 Strain development

Denoting the measured strains on the unloaded and loaded faces $\epsilon_{unloaded}$ and ϵ_{loaded} respectively, the membrane and bending strains can be taken as

$$\epsilon_{membrane} = (\epsilon_{unloaded} + \epsilon_{loaded}) / 2$$

$$\epsilon_{bending} = (\epsilon_{unloaded} - \epsilon_{loaded}) / 2$$

Typical strain vs. pressure curves can be seen in Figures 4.6 and 4.7, taking square Panel 30A as an example. Figure 4.6 shows the strains near the middle of one edge (SG4 and SG8). Initially there was a rapid increase in bending strain. However, the membrane strain was increased rapidly. While the bending strain started to decrease. The membrane strain continued to increase, but at a lower rate. Figure 4.7 shows the strains at the centre. After initial increase the bending strain increased very slowly while the membrane strain increased continuously.

4.3.3 Small in-plane edge displacement

Although the edges of the panel were intended to be fully clamped in the test rig, it was found that there were small in-plane edge displacements. These were measured from two transducers placed at middle of the long edges during the test. Figure 4.8 demonstrates the measured in-plane edge displacements of Panel 3C. As can be seen, the edge displacements took the form of slippage. Initially, there was no edge displacement, but, as the pressure increased, the membrane tensions became big enough to overcome the friction between panel and test rig and eventually to pull the panel inwards.

4.3.4 Progressive damage and failure mechanism

Initial damage was invariably resin cracking. This was characterised by noise emitted during loading. Table 4.1 gives the pressures at which noise was first detected by ear. Background noise in the laboratory made it difficult to identify when cracking started.

Visible damage took the form of whitening of the resin. There was more damage around the edges than in other areas of the panels. For panels of laminates A and C with aspect ratios 1:1 and 1:1.5 cracks occurred parallel to the edges in both directions. For the panels of laminate B there were cracks parallel to the edges and also at 45° to the edges. In every case at aspect ratio 1:2 there were only cracks parallel to the long edges - an indication of one-way spanning.

Final failure occurred at the edges of the panels and usually on a long side. There was little warning of failure in terms of rapid increases in deflection or strain. The failure process appeared to be complicated. The initial bending at the edges of the panel caused cracks in the resin which ran along, particularly, the long edges of the panel. At higher loads the bending stresses produced large curvature of the panel around the edges and the tensile membrane stresses kept the crack open. This was accompanied by some of the reinforcement pulling out of the resin until the reinforcement was broken. Figures 4.9-4.11 show photographs of the failure in selected panels. Figure 4.9 shows the failure of Panel 31A (aspect ratio 1:1). Individual rovings pulled out through the resin. Figure 4.10 shows that rovings pulled out in different fibre directions for the non-woven composite in Panel 15B (aspect ratio 1:1.5). The large failure area is distinct in the picture. The failure of Panel 7C (aspect ratio 1:2) is similar to that of Panel 31A.

4.3.5 The effect of aspect ratio

Table 4.1 and Figure 4.5 show that for a given material the larger the aspect ratio, the larger the deflection and lower the failure pressure. Generally it is assumed that for aspect ratios 1:2 and above the panel spans in one direction. The test results indicate that this assumption is true but that the disposition of reinforcement could affect the failure mechanism.

4.3.6 The effect of materials

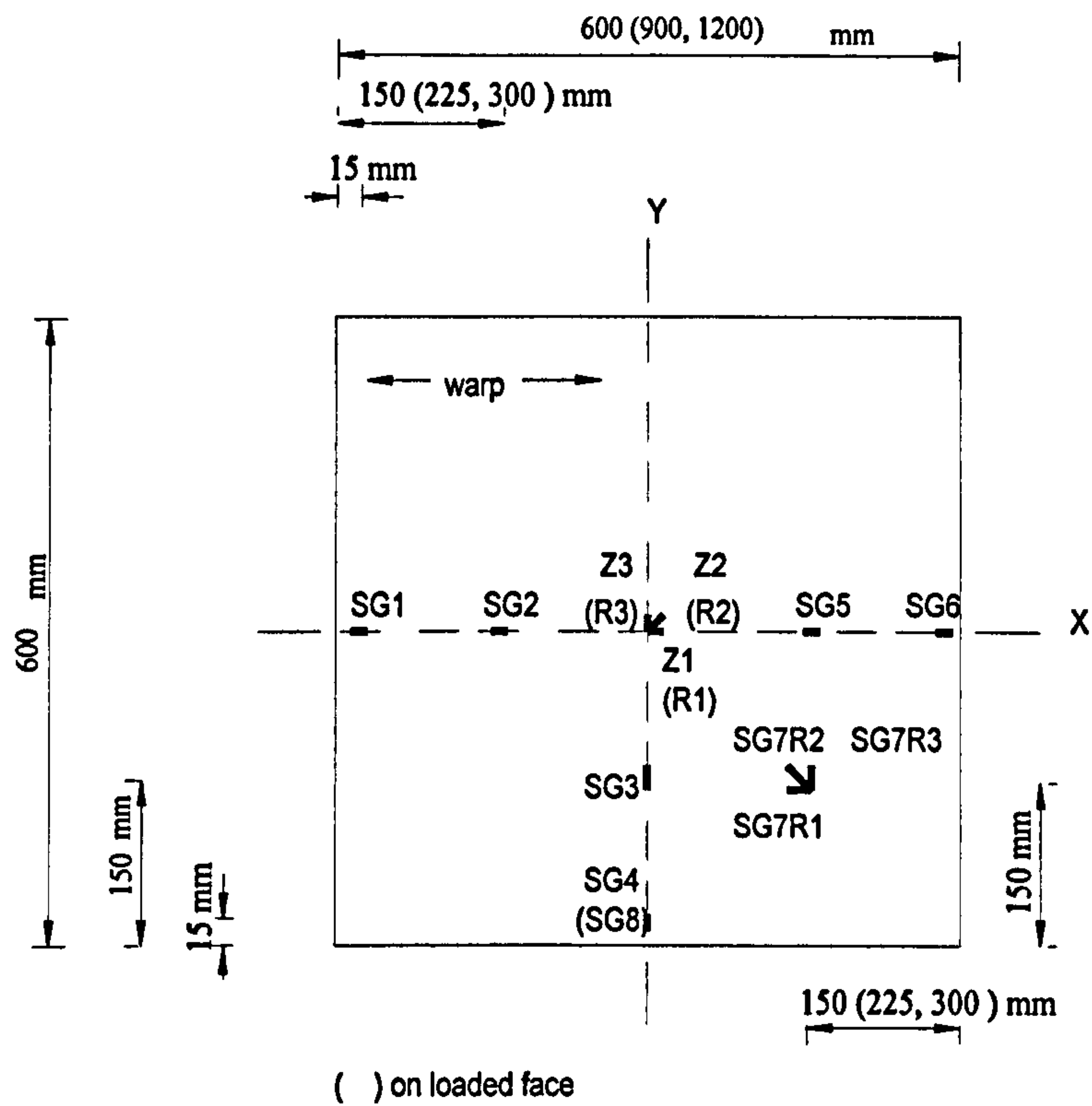
The coupon tests in Chapter 3 show that laminates A and C have higher fibre content by volume than laminate B; and laminates A and C used woven composite materials and the fibres were arranged at 0 and 90 degrees only, while laminate B was made of 'unidirectional cloth' and the fibres were arranged at 0, 45, 90, -45, 0 degrees. Table 4.1 shows that panels of laminate B, although it had lowest fibre content by volume, were the strongest and stiffest, while panels of laminates A and C produced similar panels in terms of both strength and stiffness. This indicated that laminate B with a more even disposition of fibres was more efficient for transverse loading.

Table 4.1 Panel numbering and key results of panel tests, after reference [12]

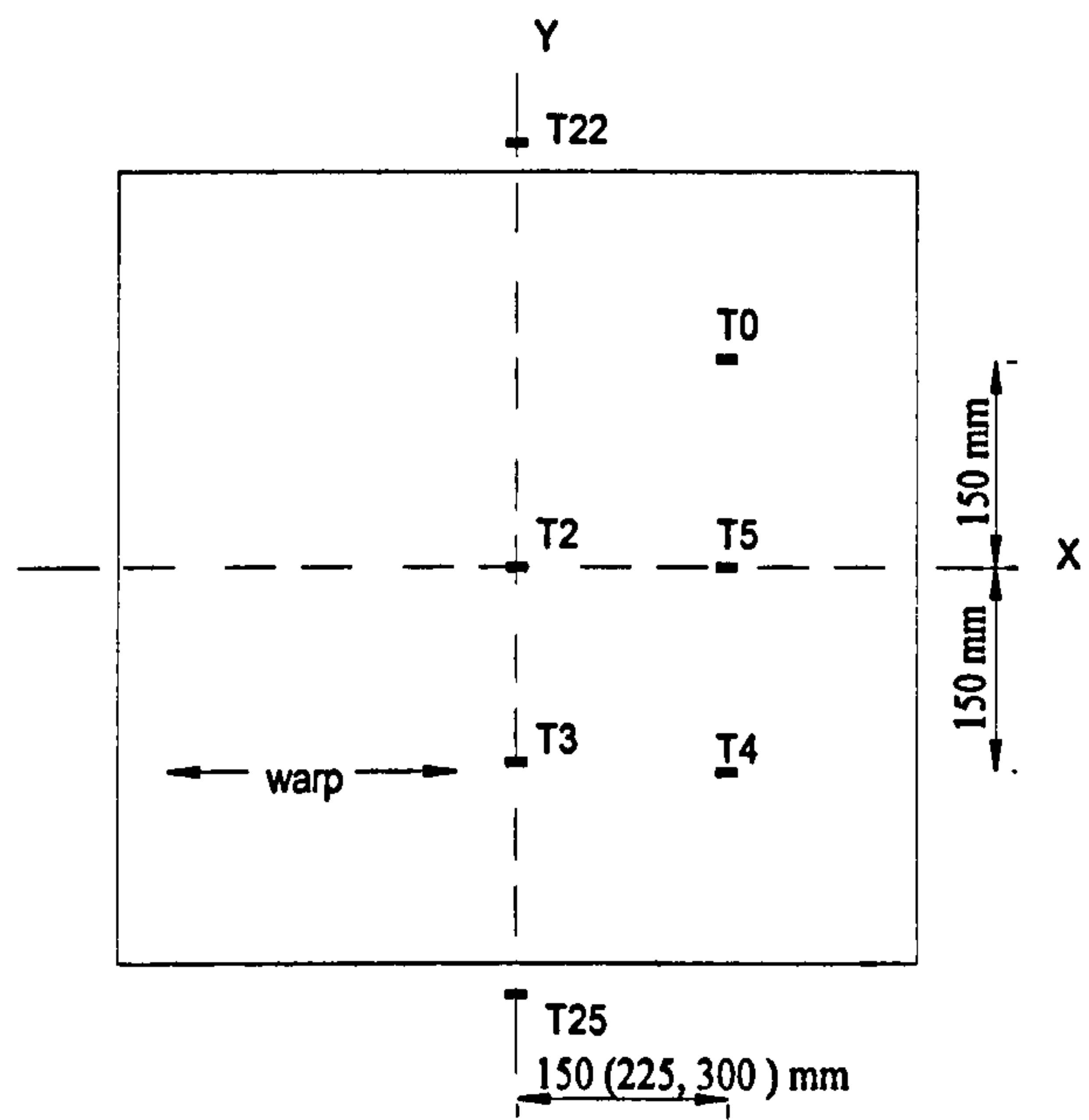
Laminate	Panel	Aspect ratio	Failure Pressure N/mm ²	Failure Deflection mm	Max. edge movement mm	Audible Cracking N/mm ²	Comment
C	1		0.446	34.1	-	0.13	
	2	1	0.449	36.8	-	0.14	
	3		0.414	37.5	1.43	0.14	
	4		0.412	44.0	-	0.03	
	5	1.5	0.381	51.9	-	0.07	
	6		0.380	49.9	2.90	-	
	7		0.350	44.0	-	0.07	
	8	2	0.348	51.9	-	-	
	9		0.330	49.9	-	0.04	
B	10		0.500	41.1	-	-	
	11	1	0.600	42.3	-	0.035	
	12		0.600	41.7	1.14	0.035	FSG
	13		0.600	53.3	-	0.04	
	14	1.5	0.627	54.3	-	0.035	
	15		0.337*	40.5	1.8	0.03	FSG
	16		0.459	55.4	-	-	
	17	2	0.495	59.6	-	0.04	
	18		0.376*	56.3	3.01	0.04	FSG
A	28		0.425	34.2	-	-	
	29		0.39	35.3	-	-	
	30	1	0.424	35.9	0.8	-	FSG
	31		0.370	29.9	0.85	0.14	FSG
	32		0.451	32.1	0.95	-	
	33	1.5	0.387	41.0	-	0.14	
	34	2	0.330	54.8	7.53	-	

* Water bag burst before panel failed, retest was carried out afterwards.

FSG - fully strain gauged, as shown in Figure 4.3a.



(a)



(b)

Figure 4.3 The locations and numbering of strain gauges (a), and transducers (b)
(For panels of laminates A, B and C)

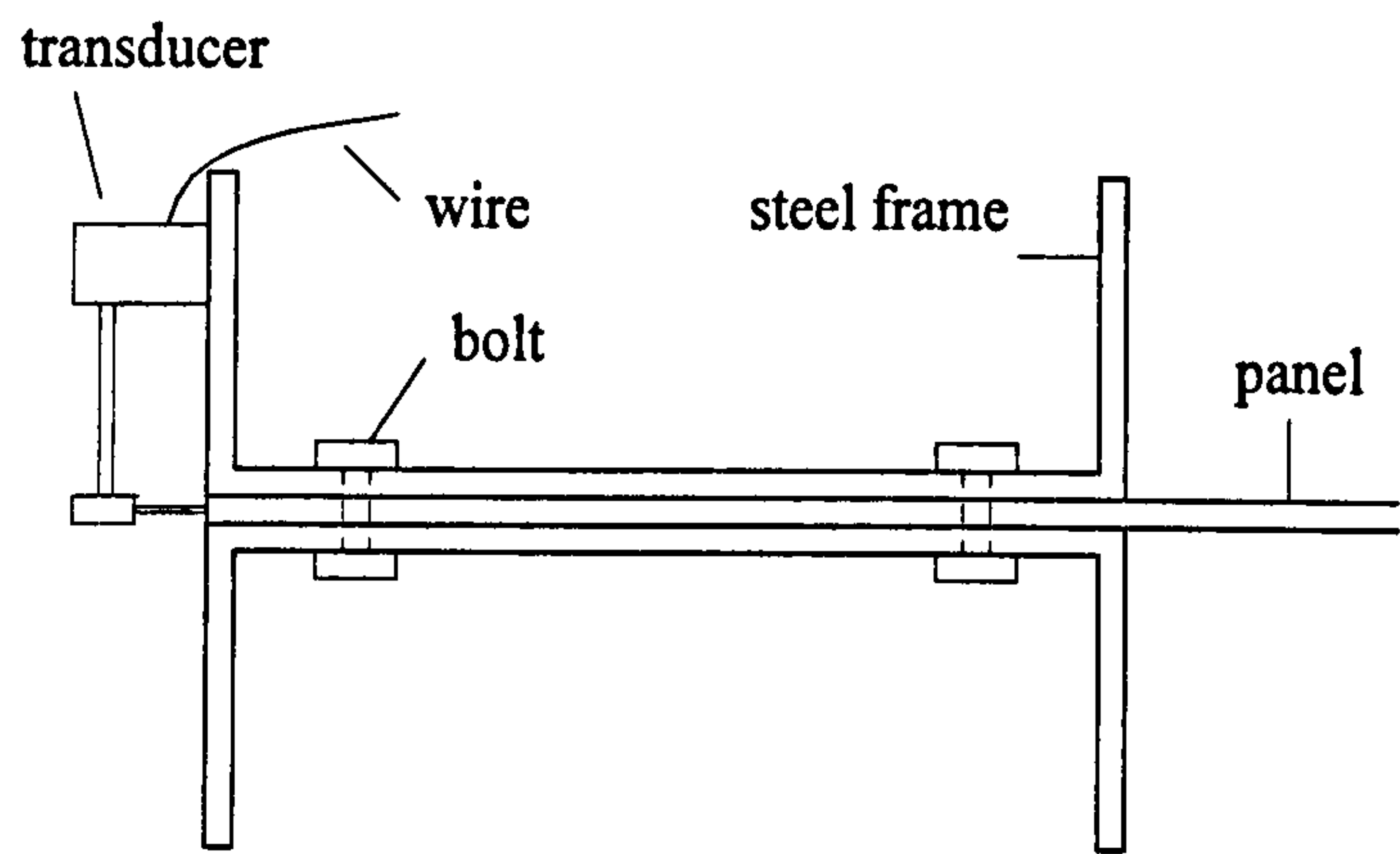


Figure 4.4 Transducer at edge for detecting in-plane edge displacement
(See also Figure 4.13)

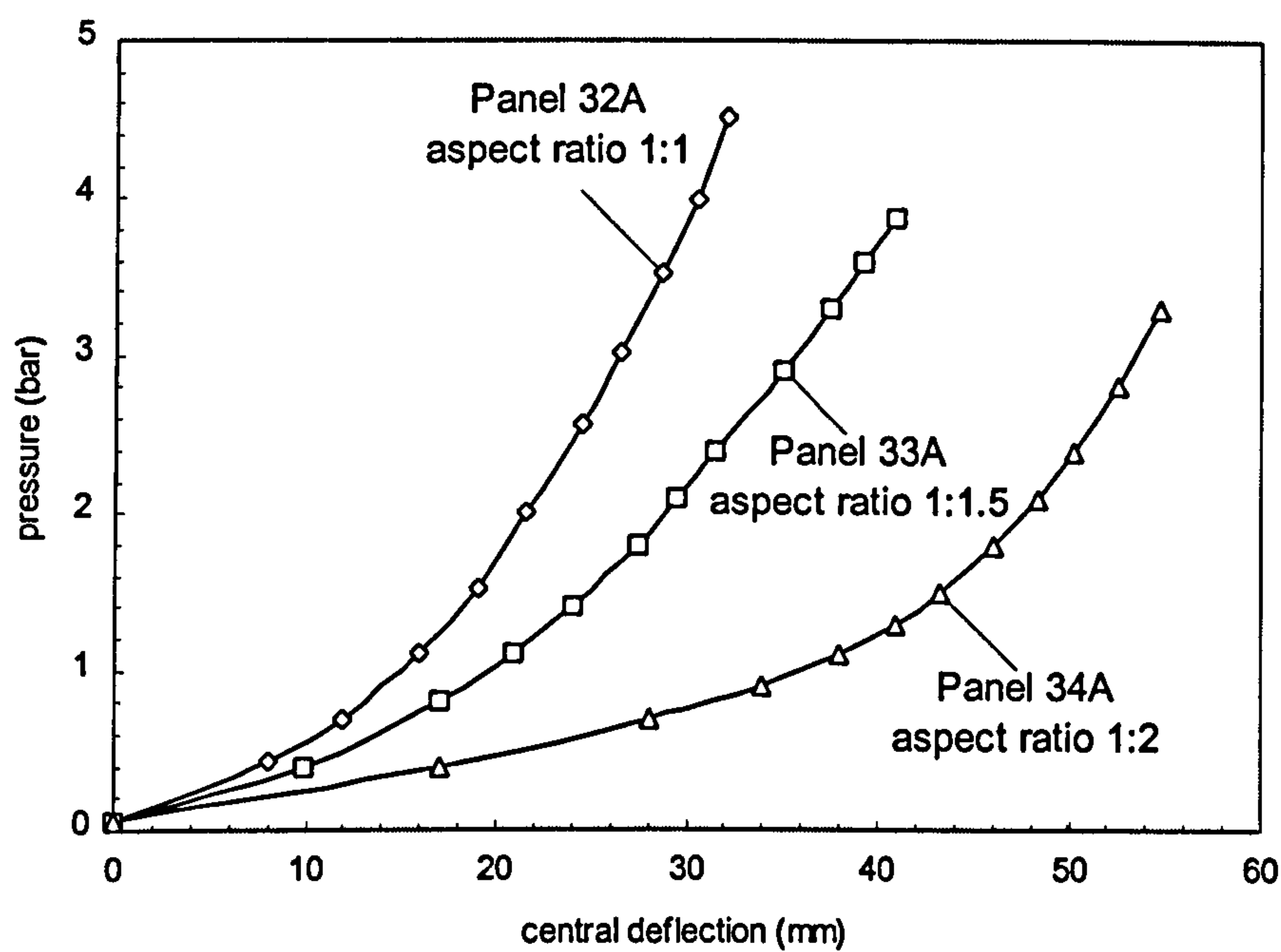


Figure 4.5 Typical non-linear deflection vs. pressure curves in panel tests

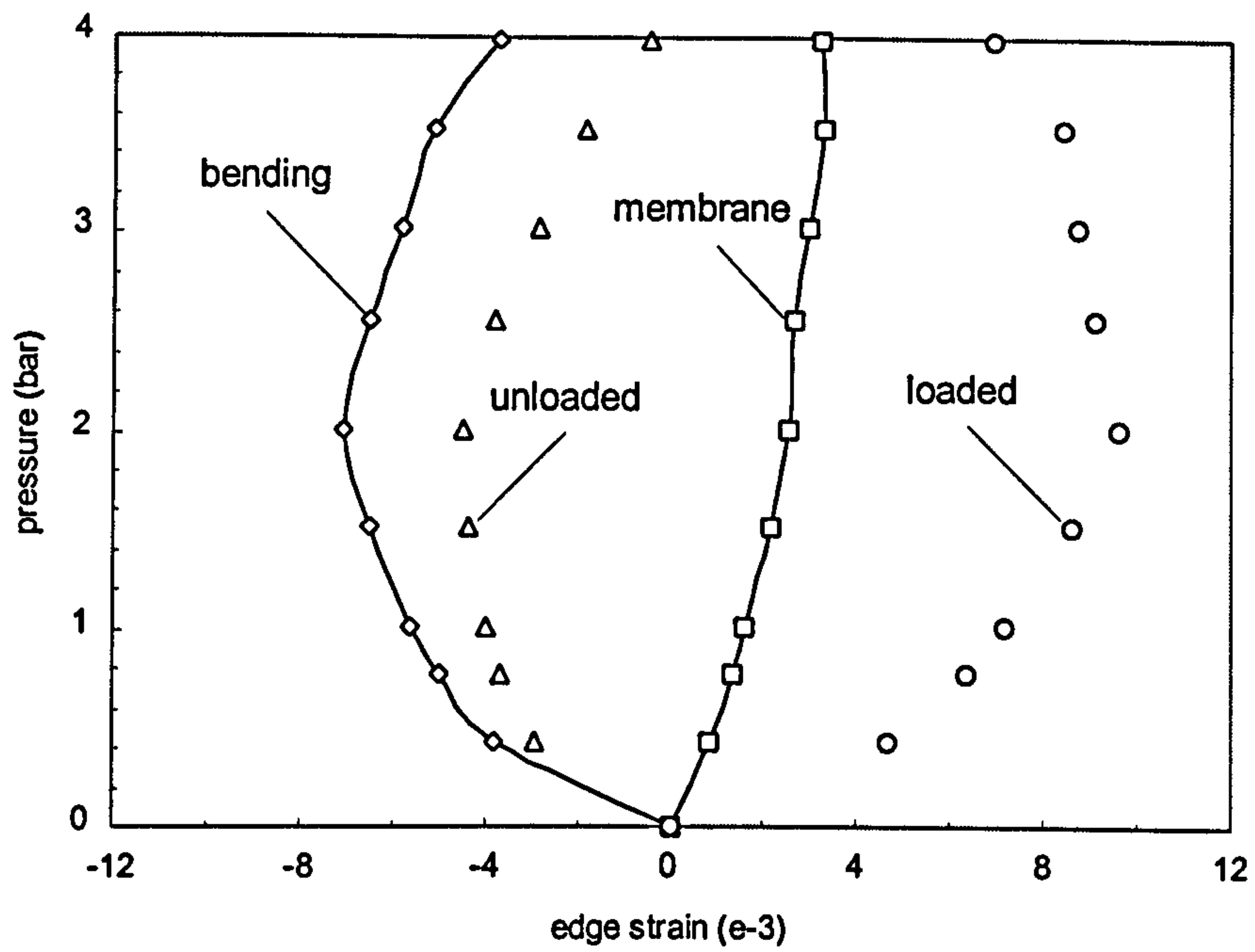


Figure 4.6 Typical edge strain (SG4 and SG8) vs. pressure curves (Panel 30A)

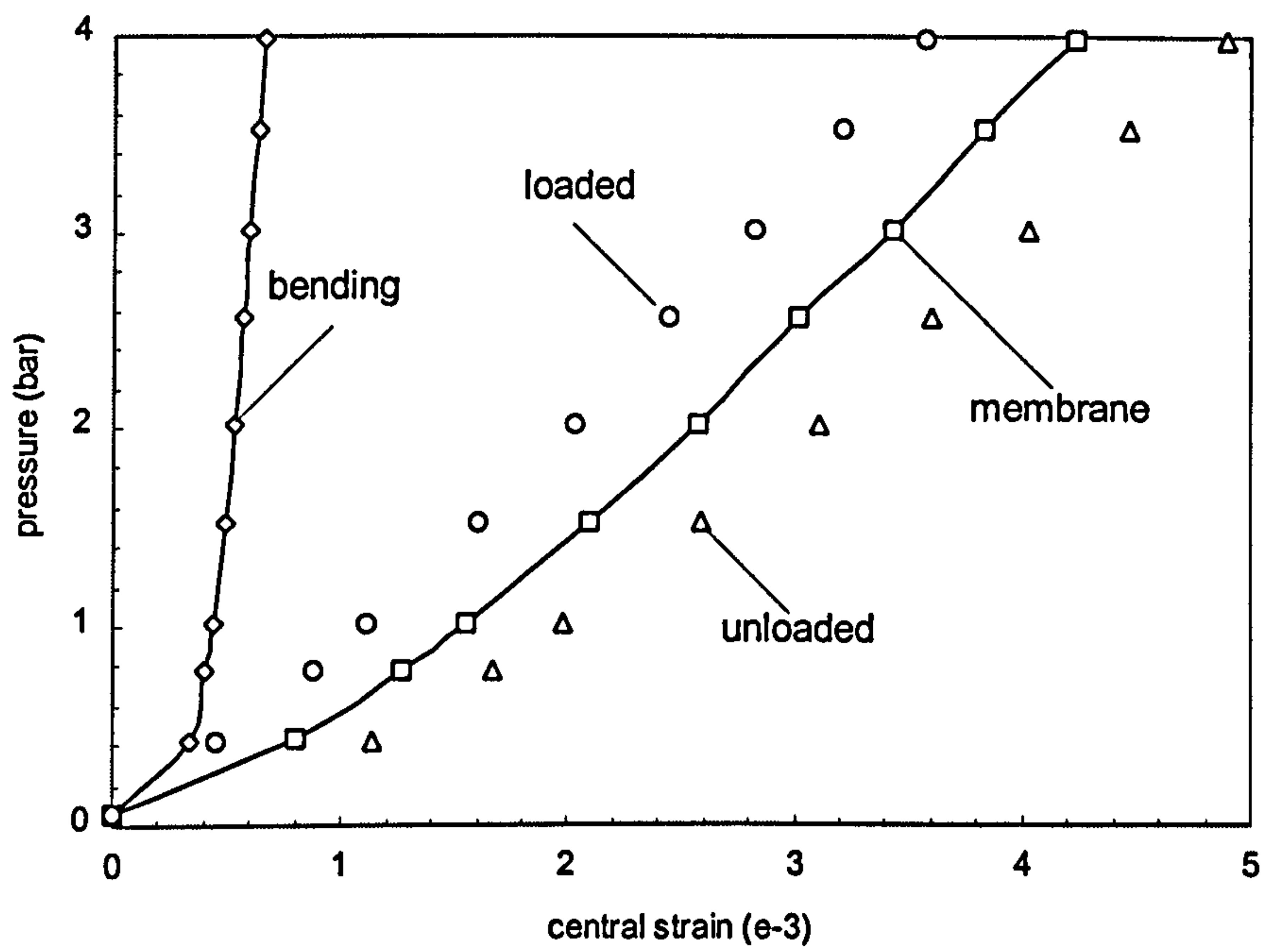


Figure 4.7 Typical central strain (Z3 and R3) vs. pressure curves (Panel 30A)

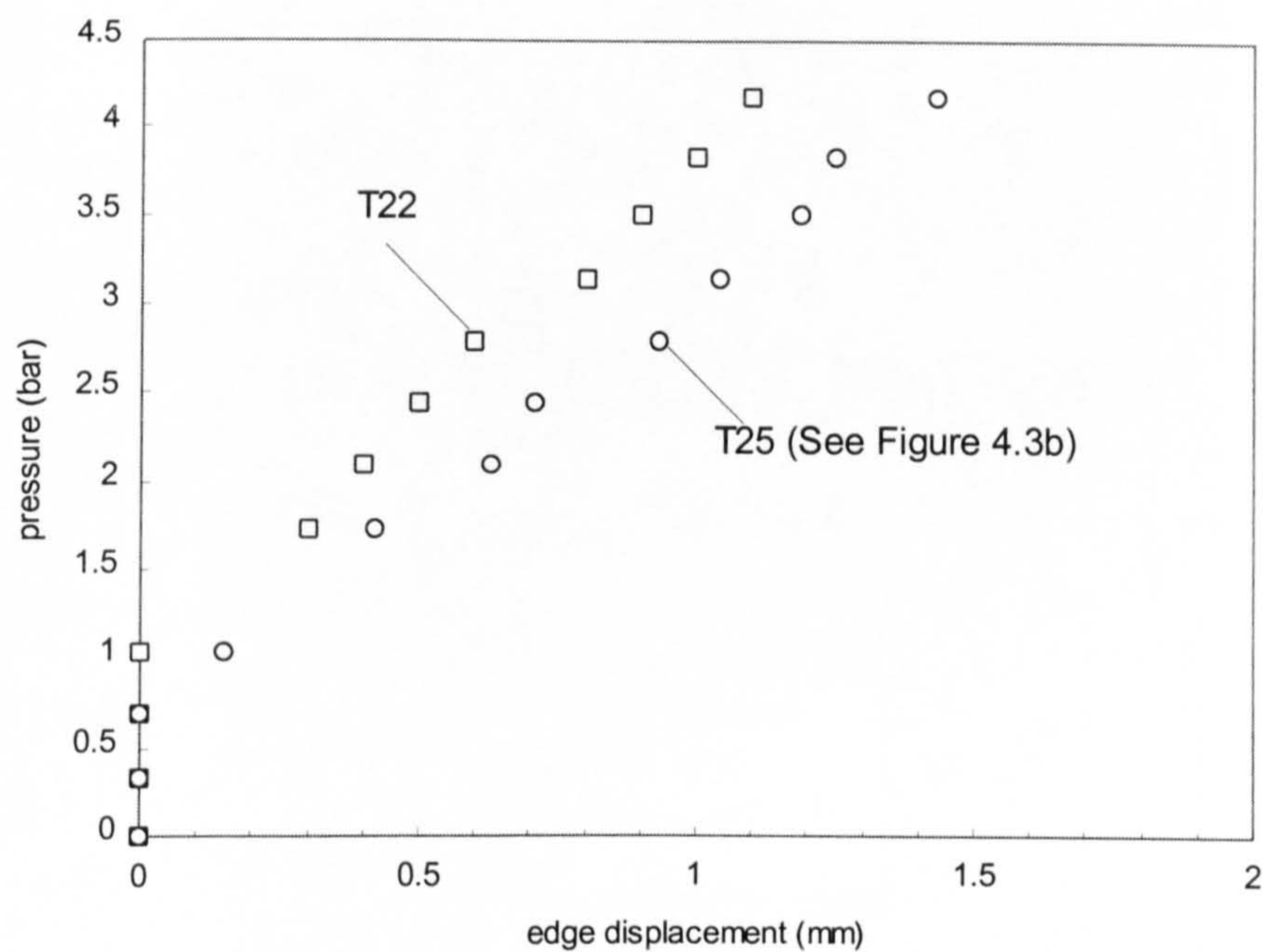


Figure 4.8 Typical in-plane edge displacement (Panel 3C)

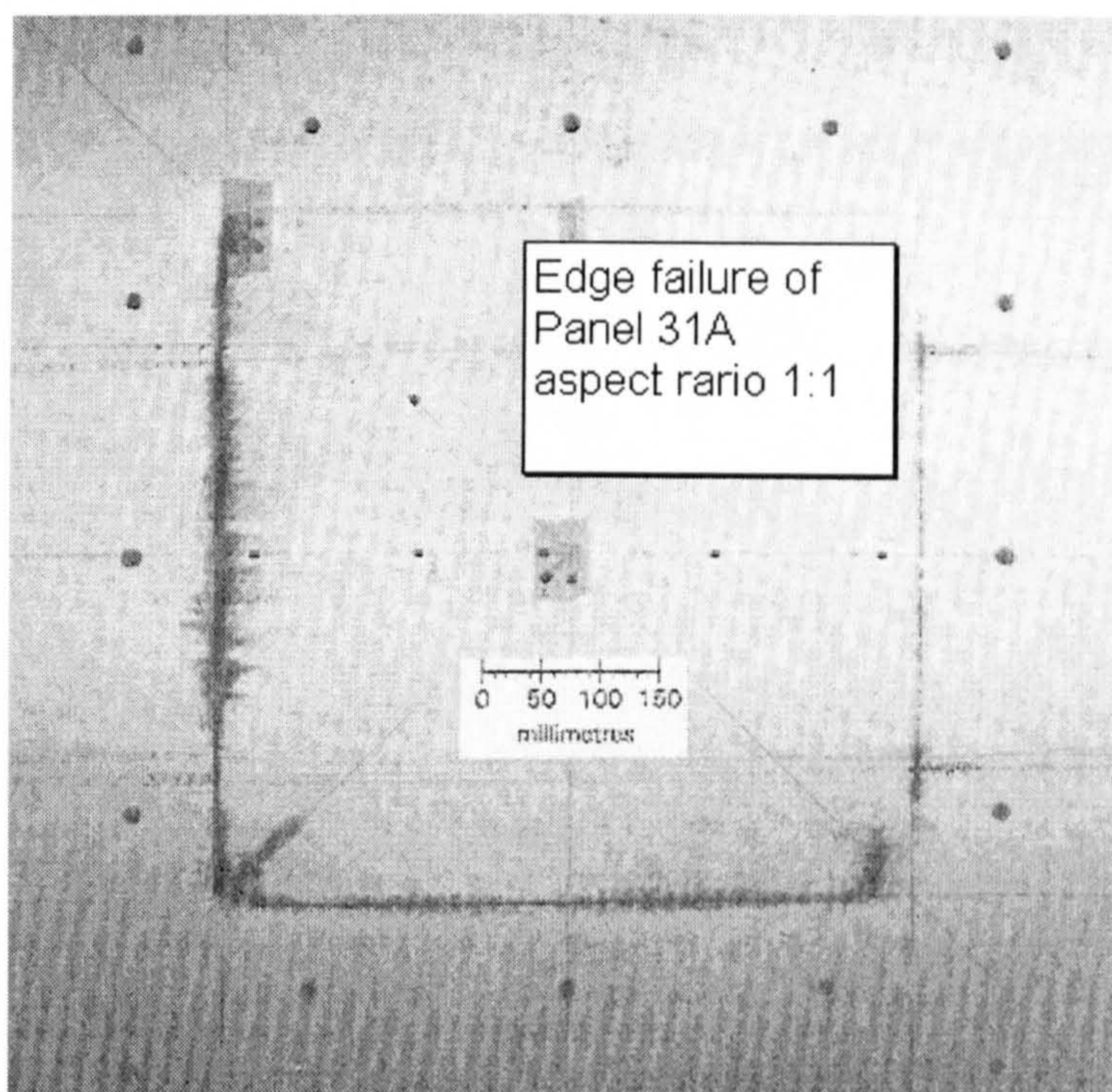


Figure 4.9 Final failure of Panel 31A (aspect ratio 1:1)

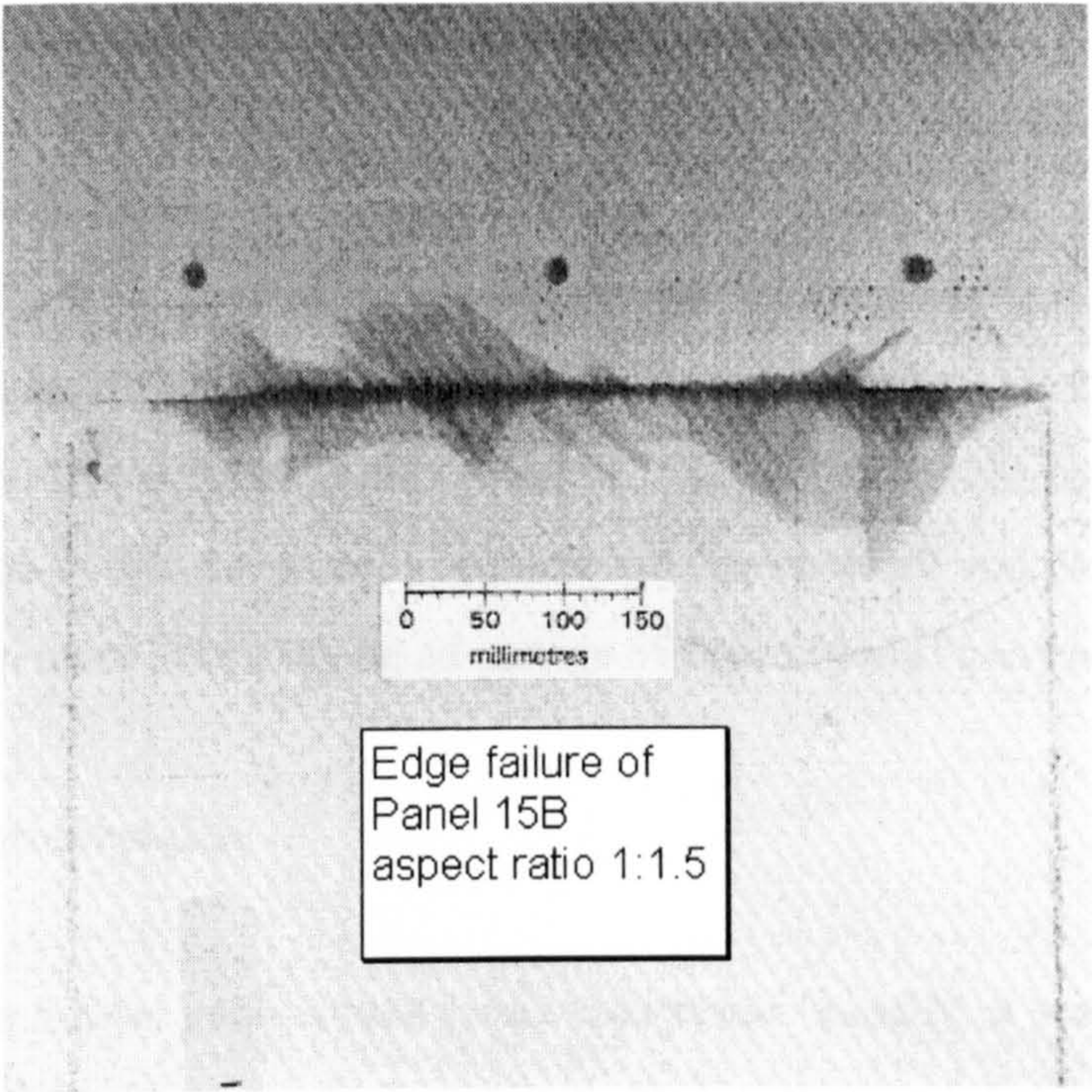


Figure 4.10 Final failure of Panel 15B (aspect ratio 1:1.5)

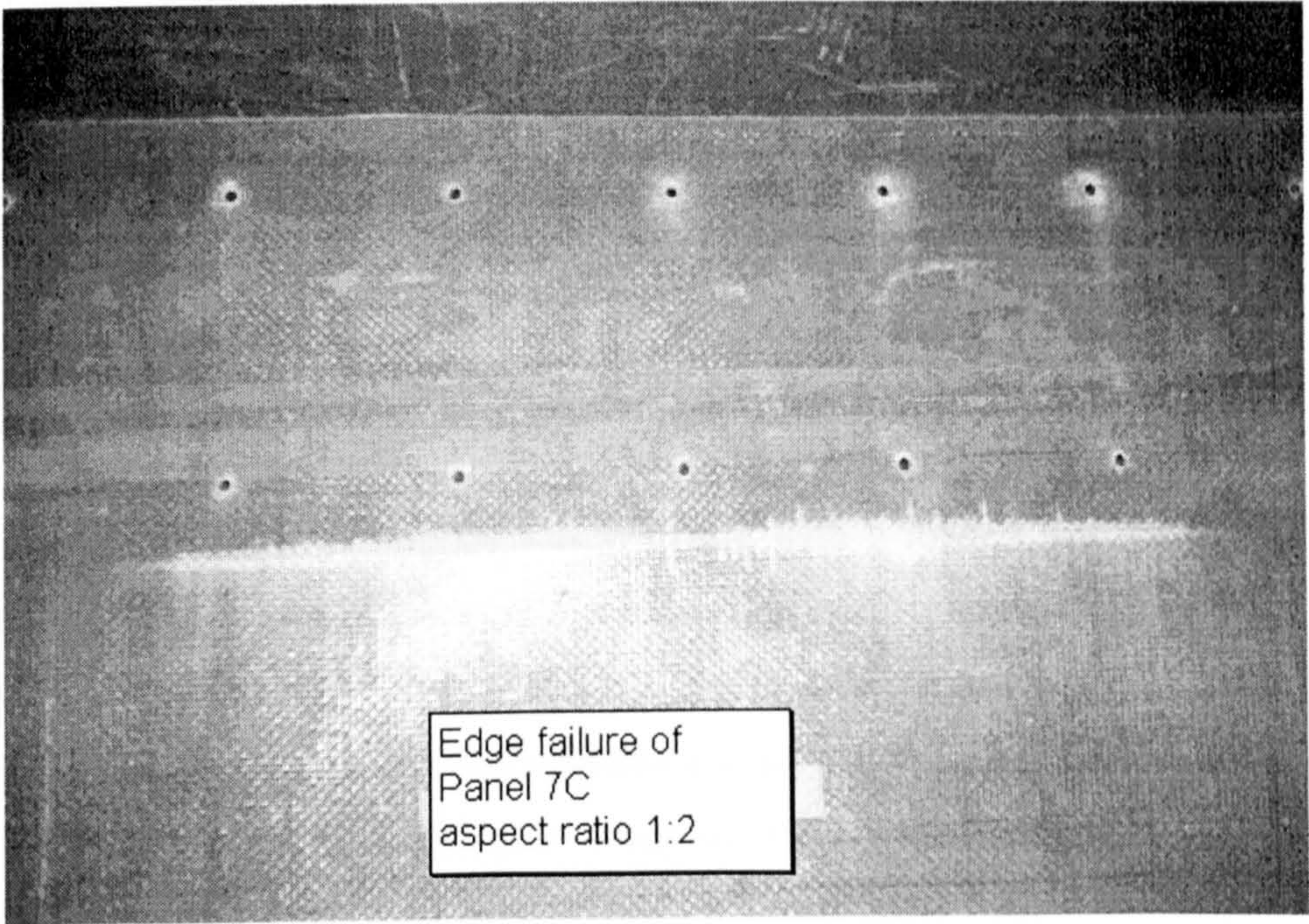


Figure 4.11 Final failure of Panel 7C (aspect ratio 1:2)

4.4 Test preparation and strategy for Panel D

Previous tests of panels of laminates A, B and C provided precise data for understanding the behaviour of the panels. It was also realised that the information about in-plane edge displacements and strains was not sufficient. Hence, a new square panel of laminate D was tested in this research. The purpose of this test is to study the strain distribution within the panel, in-plane edge displacement and panel behaviour at small pressure in more detail, taking advantage of experience of previous tests.

4.4.1 Material preparation

Laminate D is a 5-layer satin woven fabric composite. Initially, it was intended to be made the same as laminate A, which was made by Vospers (a manufacturer). However, laminate D was made by different manufacturer, Martec. Although laminates A and D had the same fibre and resin, Martec used a different fabricating process, hand lay-up rather than the resin injection moulding technique (SCRIMP), and different catalyst (See Table 3.1) during the production. Because of this, laminate D had a higher resin content and was thicker than laminate A. Also the composite was more brittle, which was possibly because of the catalyst or the high resin content. The fibre content and thickness of laminate D are 0.3649 and 4.15 mm, while these of laminate A are 0.547 and 2.84 mm.

4.4.2 Test preparation and strategy

A panel of laminate D (Panel D) was prepared with a loaded area of 600*600 mm (aspect ratio 1:1). It was tested in the same test rig and equipment as the other panels (Figures 4.1 and 4.2). Great care was taken to improve the test strategy. Firstly, more strain gauges were employed on both loaded and unloaded surfaces of the panel. Figure 4.12(a) shows the location and numbering of these strain gauges. Secondly, more displacement transducers were used to obtain better information about the in-plane edge displacement. The transducers were placed at middle of four edges and at four corners *inside* the panel close to the edges (Figures 4.12(b) and 4.13). A dial gauge was also used to obtain independent deflection readings at a point about 2 inches from the

centre of the panel. Finally, a water manometer about 10 feet high was connected to the pressure vessel in the loading system (Figure 4.14) to enable the test to be controlled more easily at low pressure.

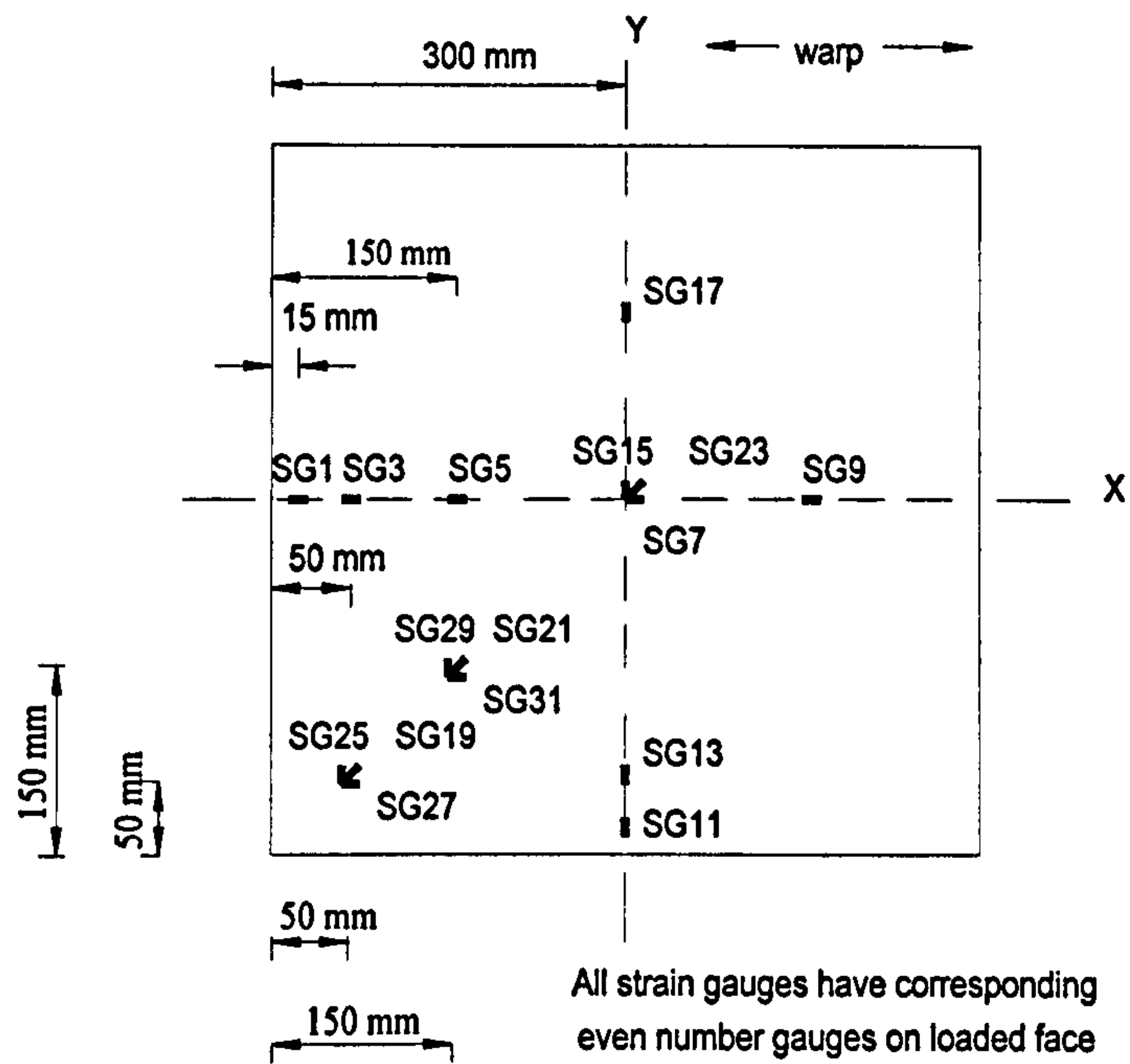
Figure 4.15 shows the overall set-up of the test of Panel D. Figures 4.16 and 4.17 show the colour pictures of close views of the test rig and equipment. Figure 4.18 show a top view of the test rig. As can be seen, a large number of transducers were used to measure lateral deflections and in-plane edge displacements.

Before the test all the strain gauges and transducers were calibrated carefully. A slip gauge with standard thickness 2 mm was put under each of the transducers. Measured data were printed from data logger, which were compared with the thickness of the slip gauge. Each strain gauge was calibrated by placing a standard resistor (50,000 Ω) in parallel with the gauge in order to generate a precise change of resistance.

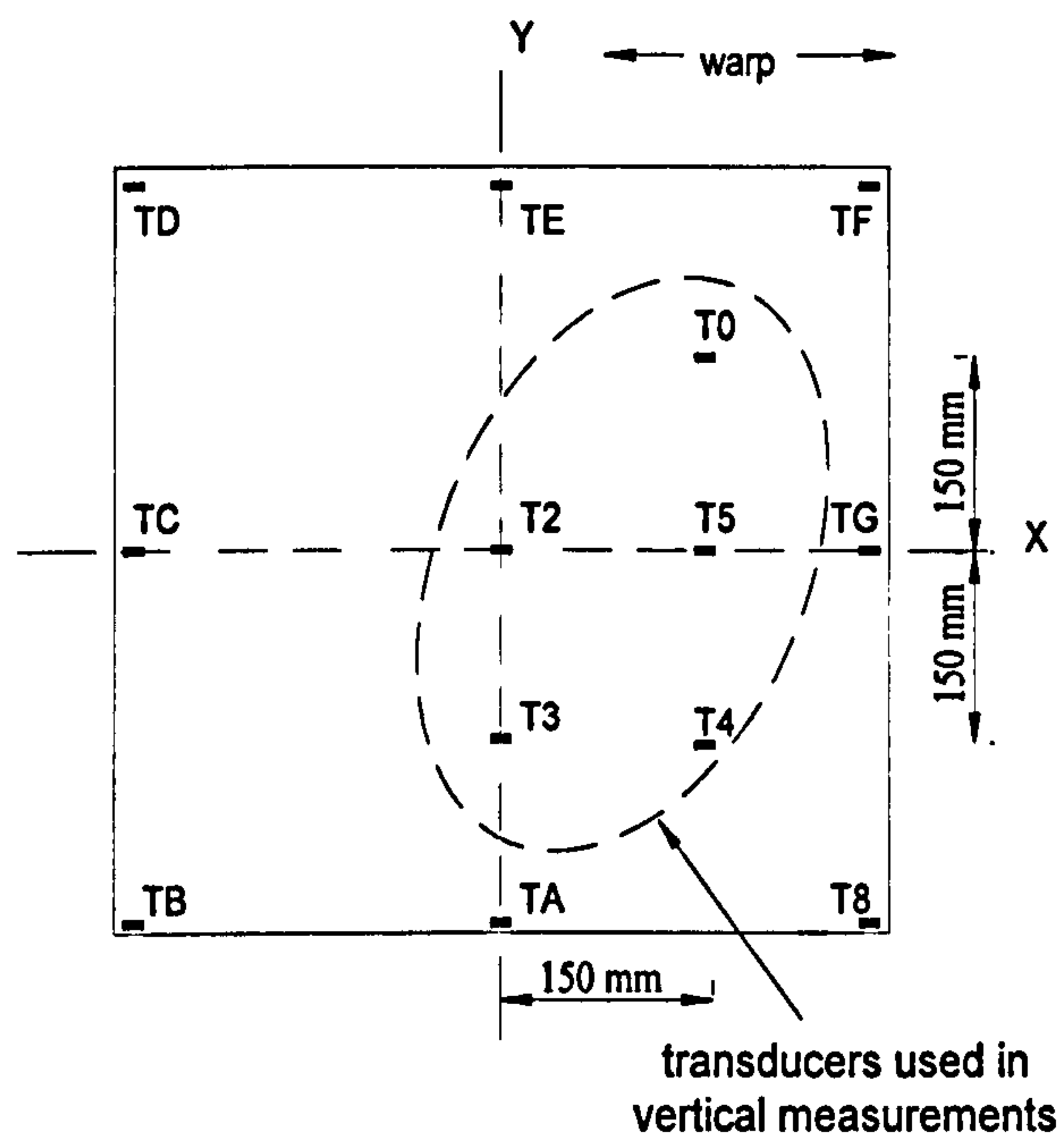
The loading procedure of the test was divided into three stages. In the *first stage* the pressure was limited to 0.3 bar (about 10 feet water head). This enabled the pressure to be monitored from the water manometer so that the behaviour of the composite panel could be studied at low pressure and the equipment could be checked without damaging the panel. Many cycles of loading were applied in stage one in an effort to perfect the techniques before proceeding to stage two. In the *second stage* the pressure was limited to 1 bar, which was controlled first by the manometer, then by pressure transducer. In the *final stage* the panel was loaded to failure.

4.4.3 Small error in loading system

With careful observation of the water head in the water manometer in loading system Figure 4.14, it was noticed that the effective zero pressure was not at the same level of the tested panel, but with a small water head of 1.5 feet (about 0.04 bar). This indicated that the small amount of pressure was pre-loaded on the panel before the test started. For the thin panel this small pre-pressure affected the panel behaviour and measurements. The influence will be discussed in Chapter 7.



(a)



(b)

Figure 4.12 The locations and numbering of strain gauges (a), and transducers (b)
(Panel D)

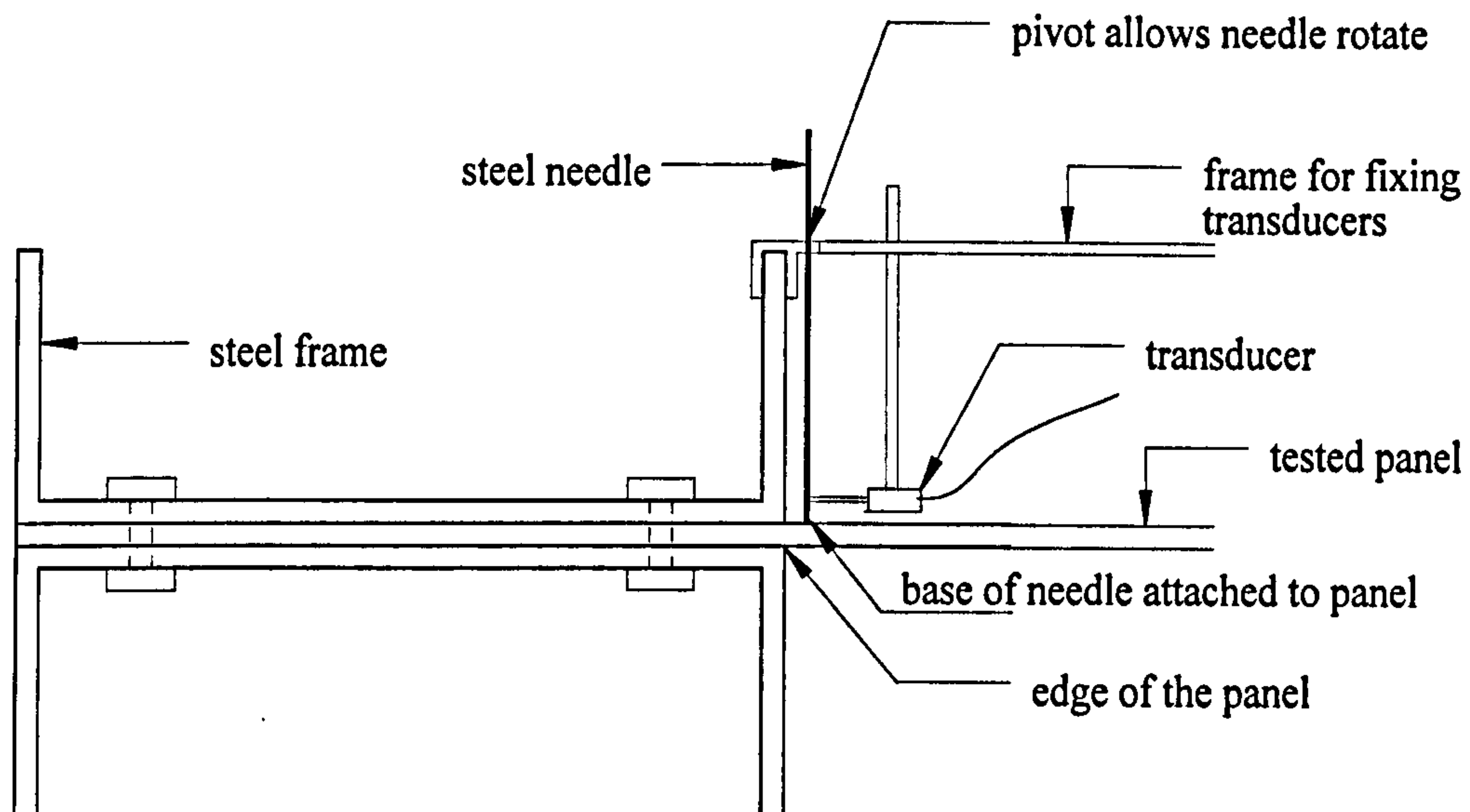


Figure 4.13 Transducer placed inside the panel to detect in-plane edge displacement
(See also Figure 4.4)

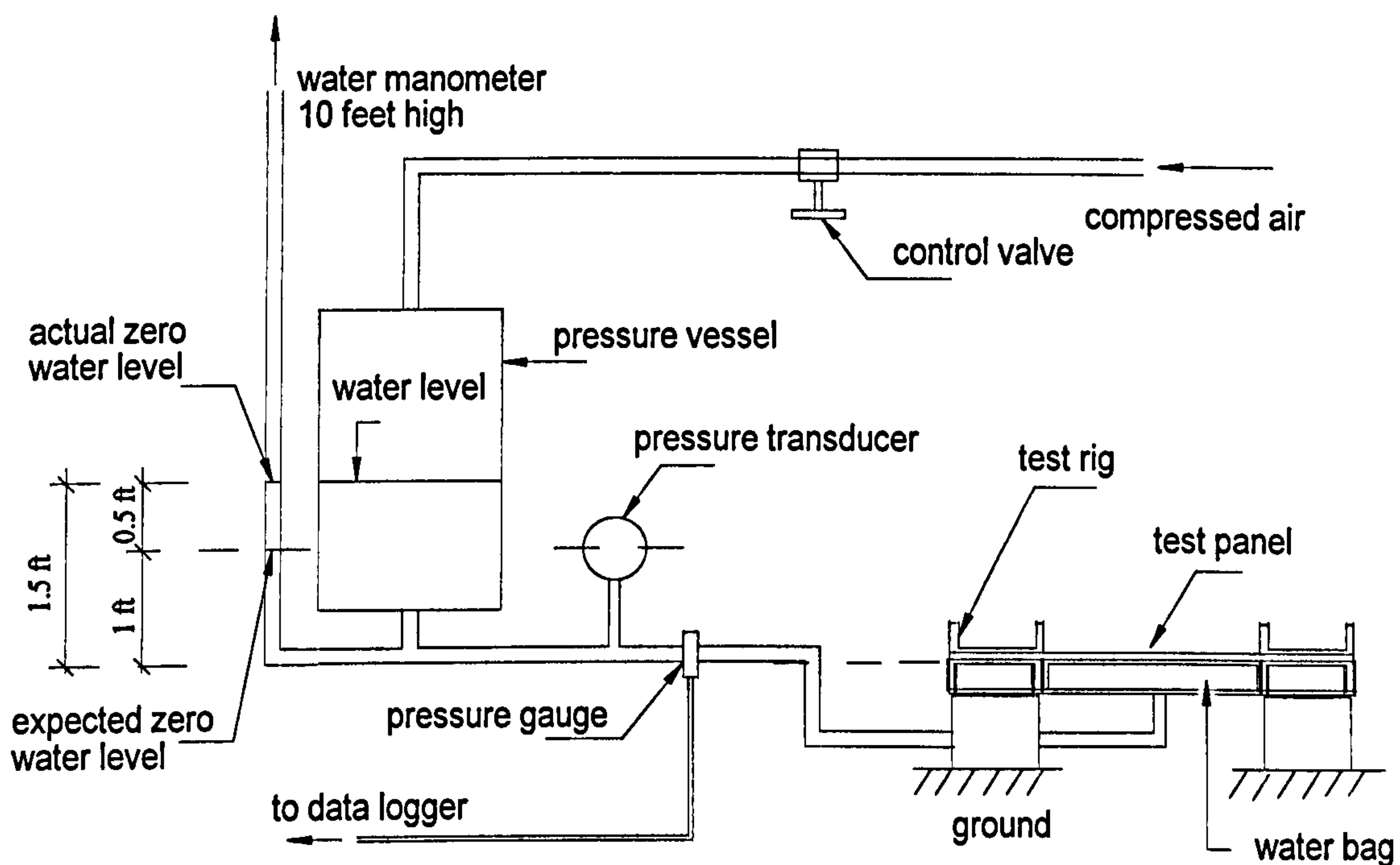


Figure 4.14 Demonstration of loading system with water manometer for Panel D
(See also Figure 4.2)

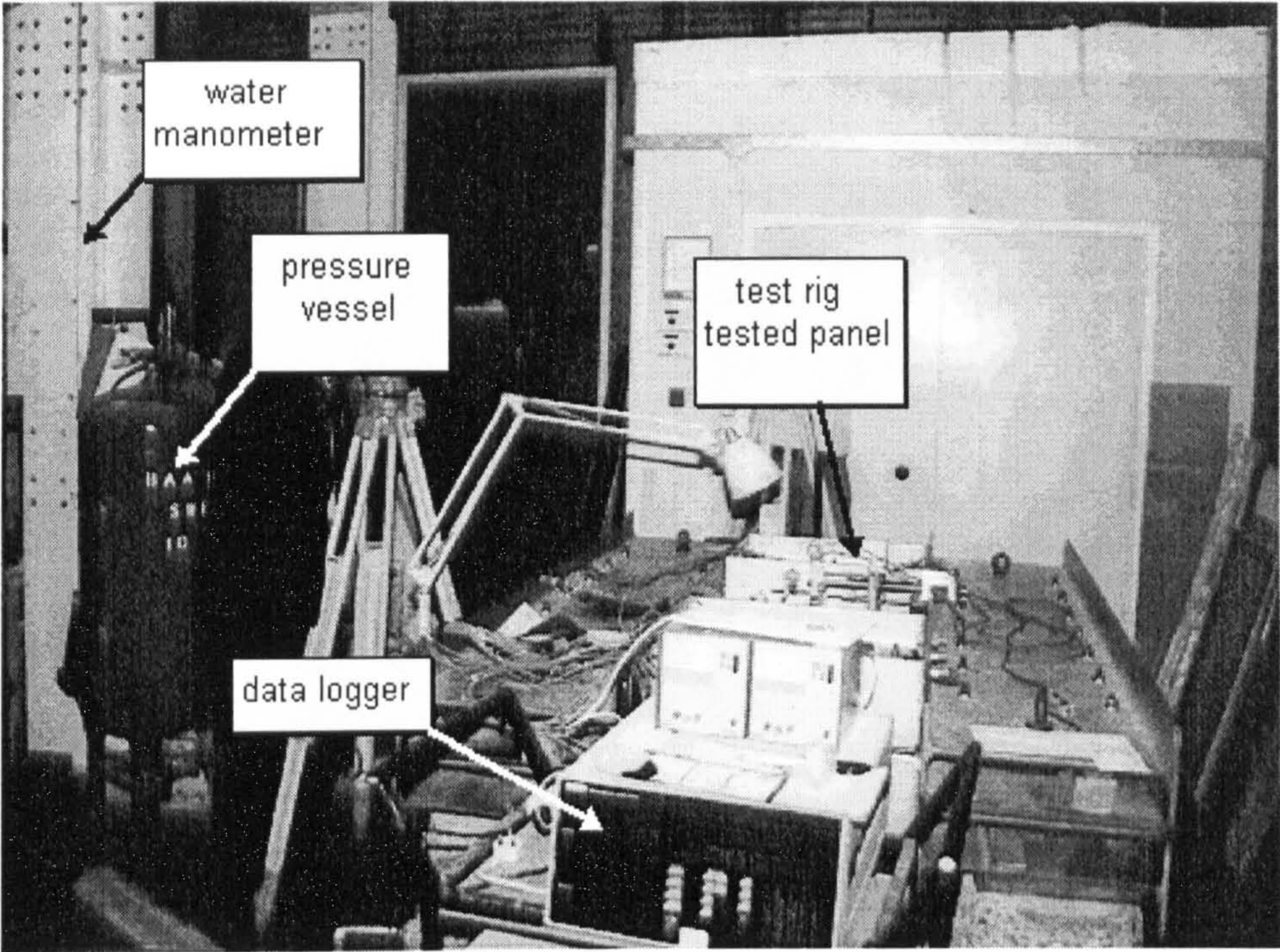


Figure 4.15 The set-up for test on Panel D



Figure 4.16 Test rig and data logger used in the test of Panel D

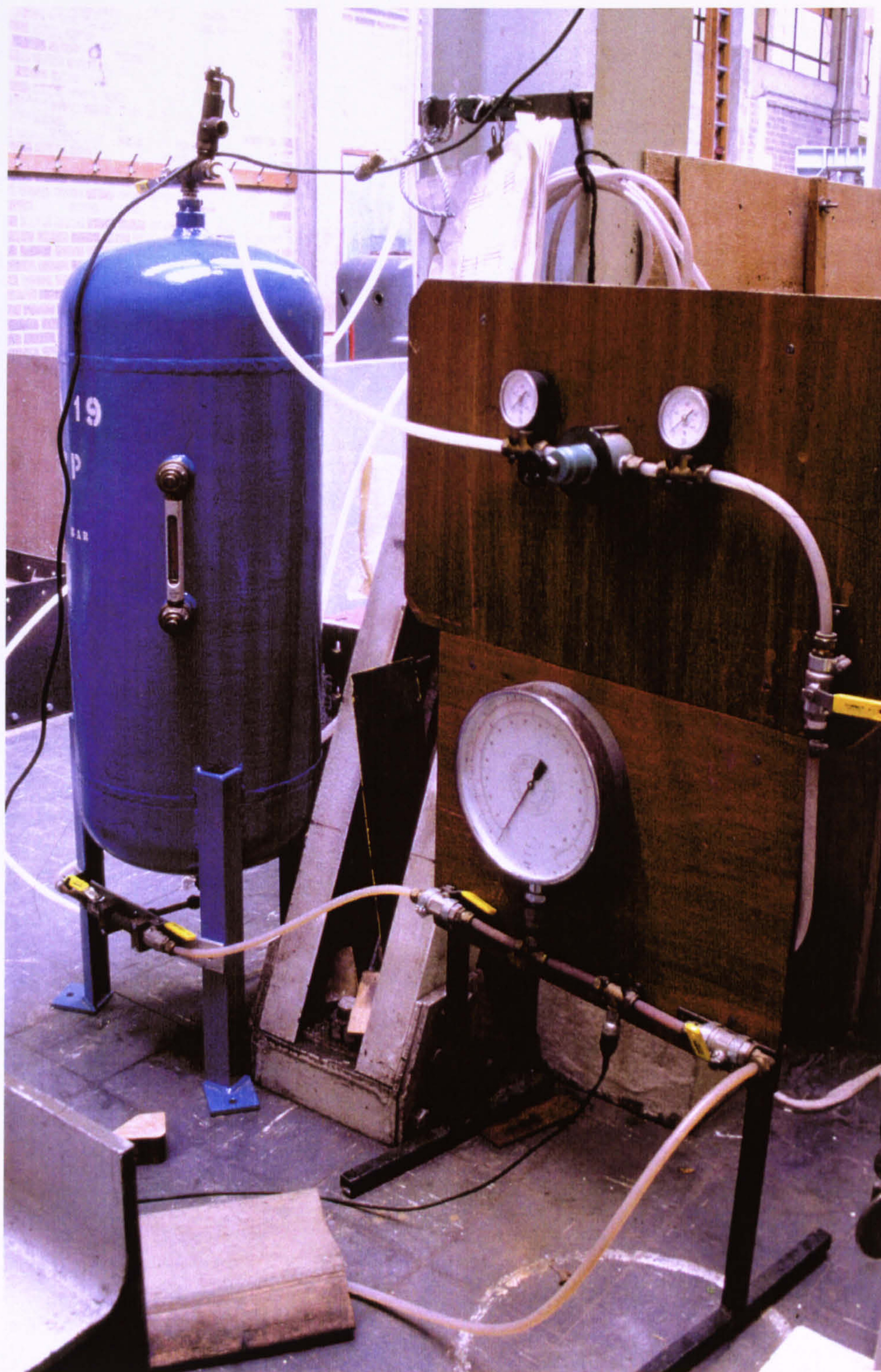


Figure 4.17 Loading system used in test of Panel D

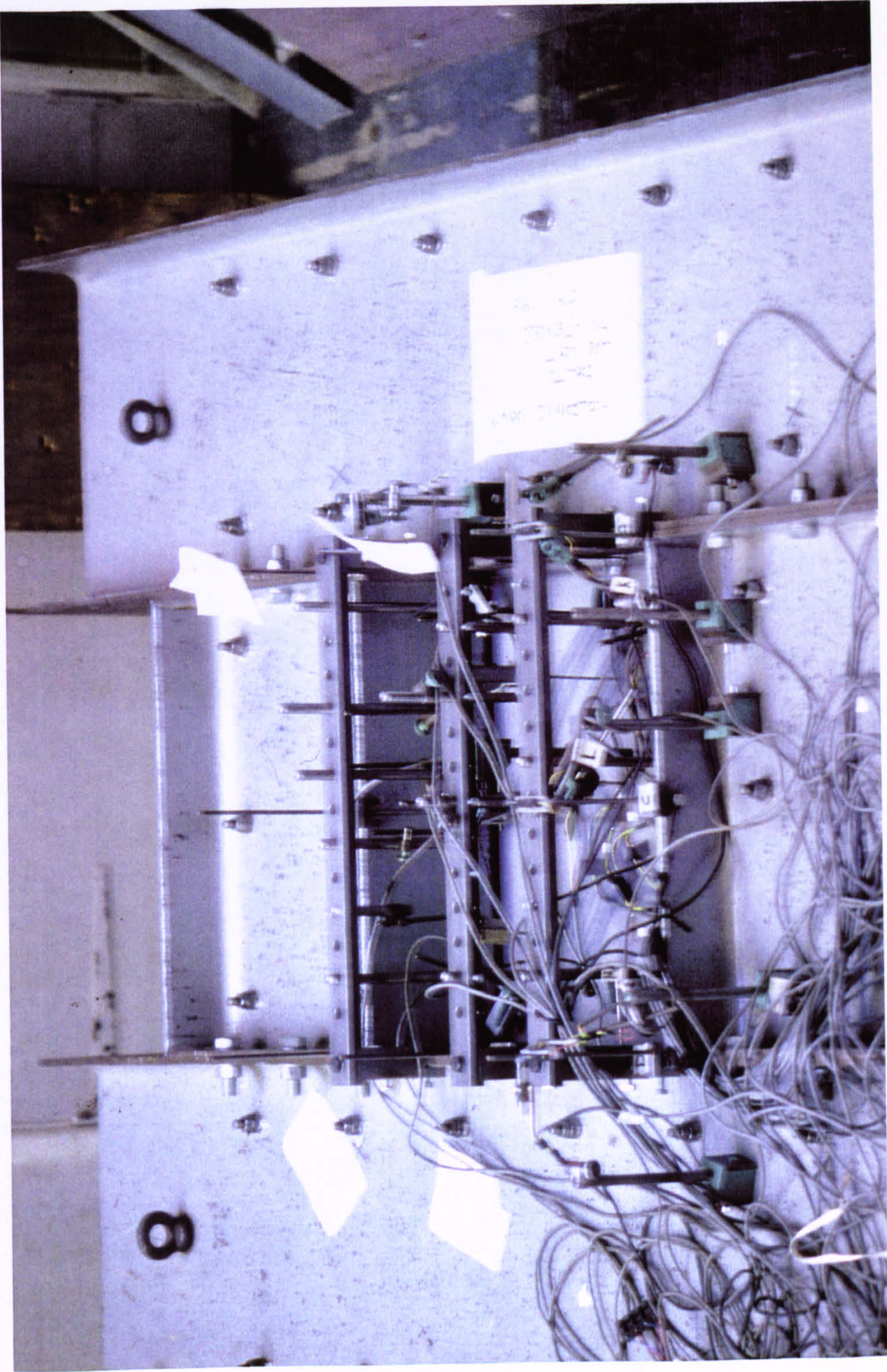


Figure 4.18 Top view of test rig - many transducers

4.5 Summary of panel test of laminate D

14 cycles of loading and unloading were carried out in the first stage of the test, in which the pressure was limited to 0.3 bar. The equipment, strain gauges and transducers were carefully checked and calibrated during this stage. 2 cycles were carried out in the second stage, in which the pressure was limited to 1 bar. Finally, the panel was loaded to failure.

4.5.1 Panel behaviour at low pressure

The pressure was controlled by watching the water manometer and adjusting the pressure valve manually. At each pressure, the deflections and the strains were recorded by a data logger first. Then the pressure was held about five minutes and the data were recorded again. A dial gauge was also used to monitor the deflections near the centre of the panel during each of the cycles. The pressure was taken to 0.3 bar in about eight steps and then reduced to zero in approximately the same steps. Figure 4.19 shows the central deflection - pressure curve measured in a typical cycle in the test. At small pressure the non-linearity was already apparent. Figures 4.20 and 4.21 show the central and the edge strain - pressure curves in a typical cycle. The central bending strain was much smaller than the central membrane strain. This indicated that the membrane effect had become important even at small pressure. At the edge the bending strain was large and increased continuously. Figures 4.19-4.21 also show that unloading curve and loading curve almost overlap each other; and this kind of the curve was repeatable in different cycles in the test. It is noted that all the curves in the figures start at about 0.04 bar in the vertical axis. This was caused by small error in the loading system, in which about 1.5ft water head was pre-loaded on the panel, as stated before. The influence will be discussed in Chapter 7.

At this stage no apparent in-plane edge displacement was recorded.

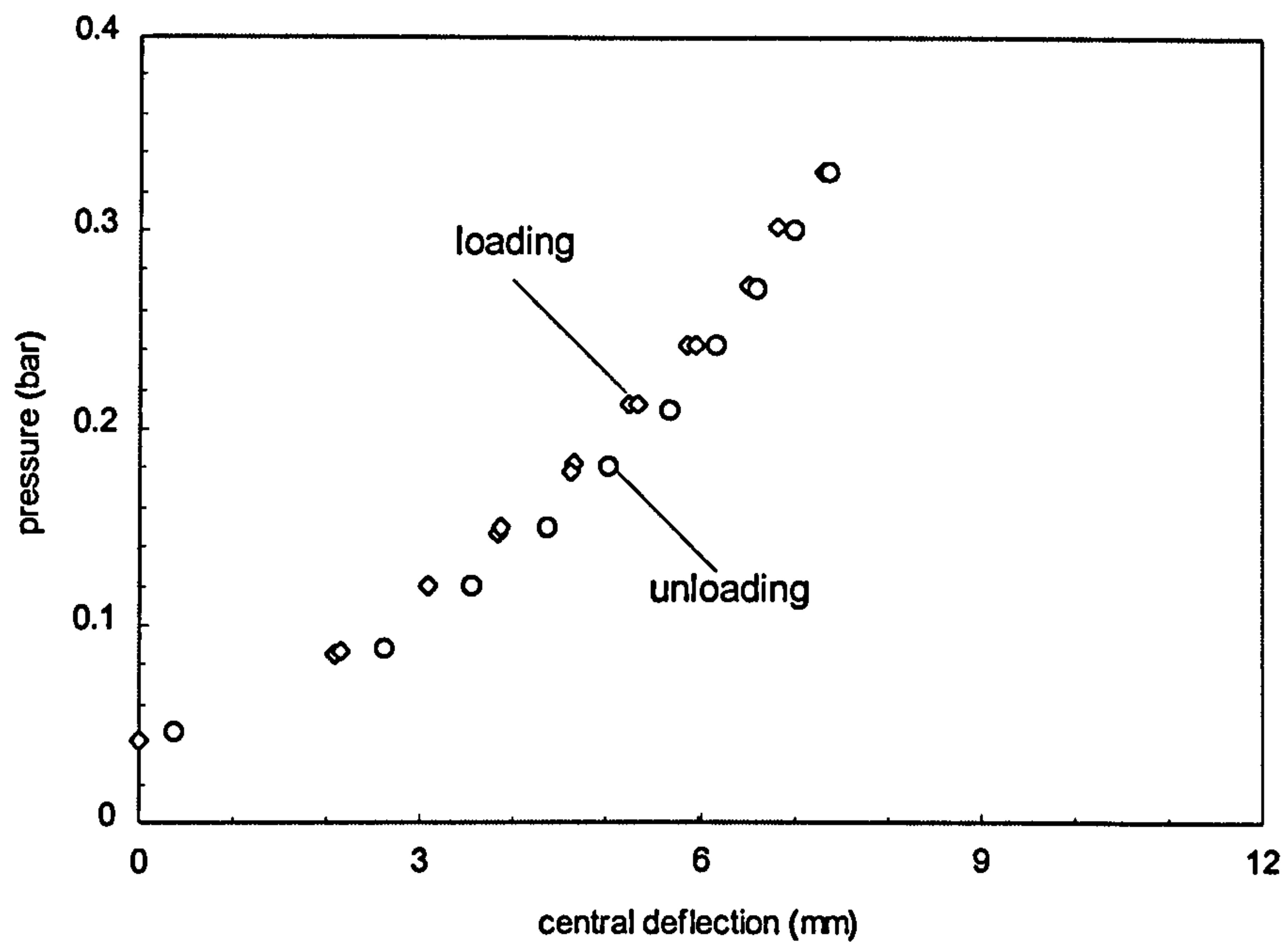


Figure 4.19 Central deflection (T2) at low pressure in a typical cycle

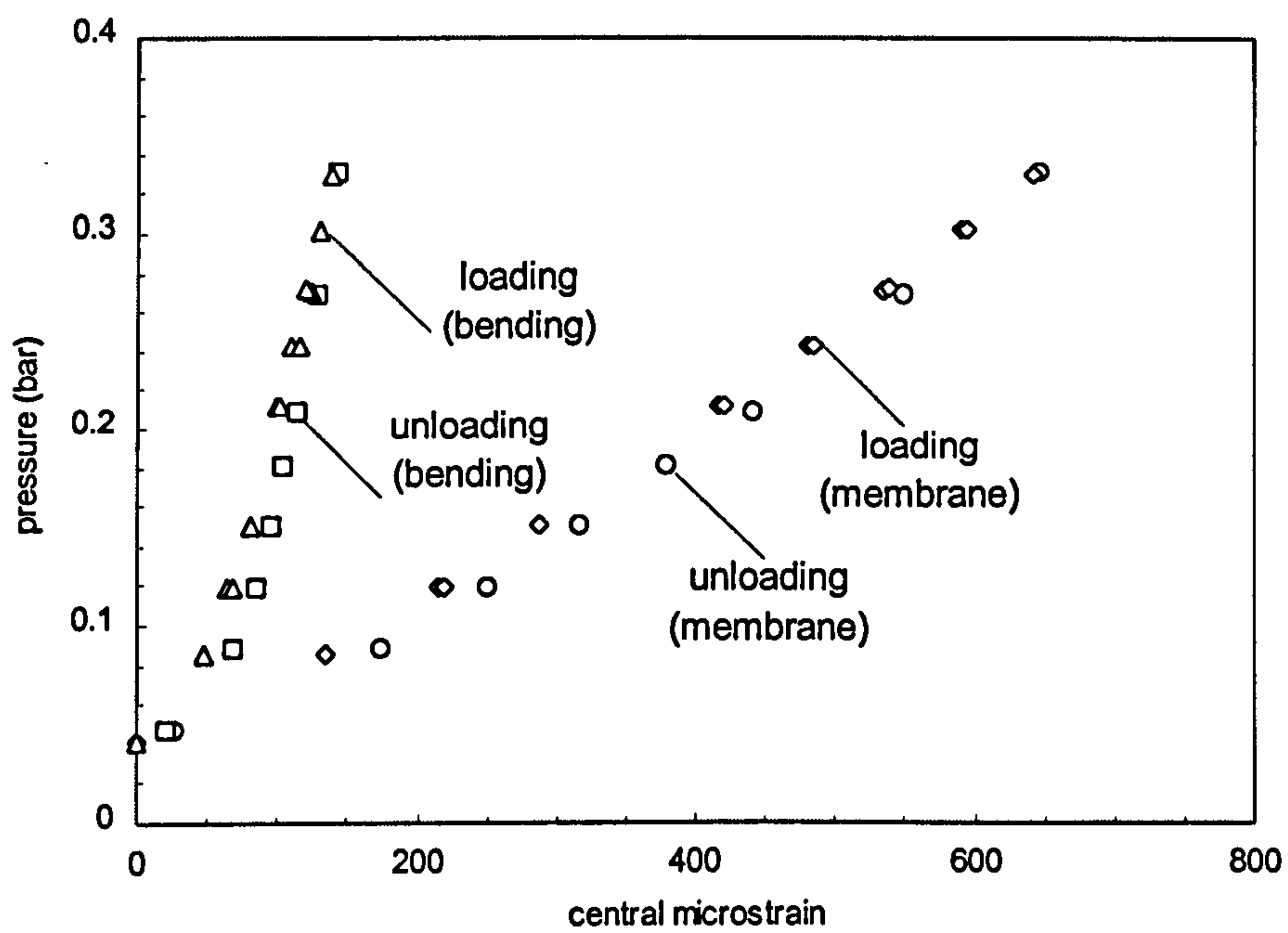


Figure 4.20 Central membrane and bending strains (SG7/SG8) at low pressure in a typical cycle

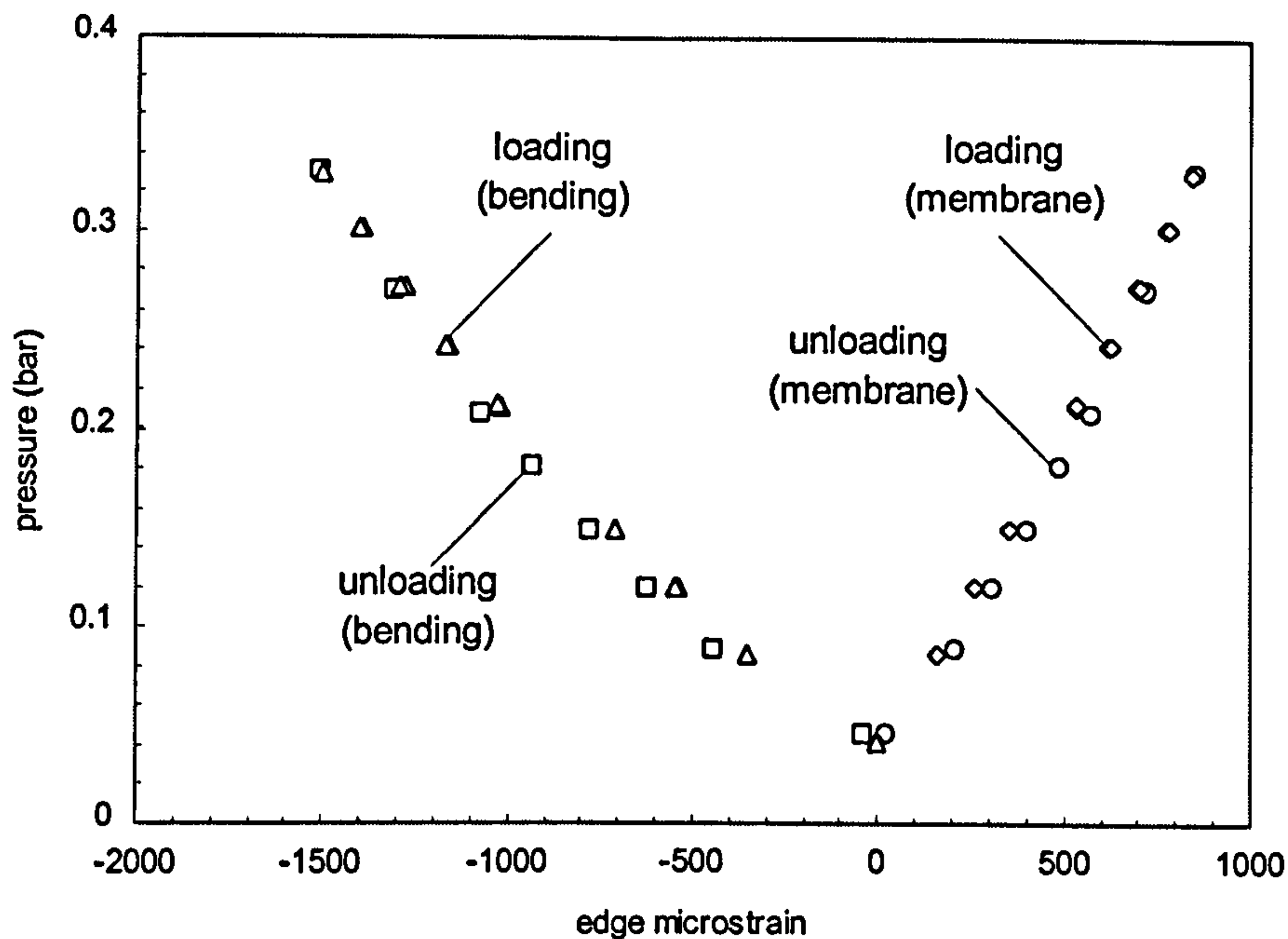


Figure 4.21 Edge membrane and bending strains (SG1/SG2) at low pressure in a typical cycle

4.5.2 Strain distribution and development

As about 24 strain gauges were placed on both loaded and unloaded surfaces of the panel, it was possible to study the strain distribution within the panel. Figures 4.22 and 4.23 illustrate the strain distribution and development along the x-axis (warp direction) at unloaded and loaded surfaces. In Figure 4.22 (unloaded surface) the strain at $x = 150$ mm point (SG9) is bigger than its mirror point at $x = -150$ mm (SG5). While in Figure 4.23 (loaded surface) strain distribution looks to be symmetrical. Figures 4.24 and 4.25 show the strain distribution and development along the y-axis (weft direction) at loaded and unloaded surfaces.

To study the symmetry of the strain between the x and y axes (warp and weft directions), Figures 4.22 and 4.24, 4.23 and 4.25 are superimposed and plotted in Figures 4.26 and 4.27. The figures show that the strains in the x-axis are bigger than those in y-axis, particularly at higher pressure.

The results indicate that although the panel is considered to be symmetrical in the x-y plane, the strain distributions are not quite symmetrical as expected. This is probably because of unsymmetrical distribution of small in-plane edge displacements (See Figures 4.29 and 4.30).

The development of the strain can also be studied. At unloaded surface (Figure 4.22), edge strain (SG1) was initially negative (in compression) and remained almost unchanged with increase of the pressure. Apart from the edge the strains in the surface became positive and increased constantly. At loaded surface (Figure 4.23), edge strain (SG2) was initially positive (in tension) and grew quickly with the increase of the pressure. The strains apart from the edge were also increased but at lower rate. These results show once again that the thin panel acts as a membrane as a whole, except at edges where localised bending strain is developed.

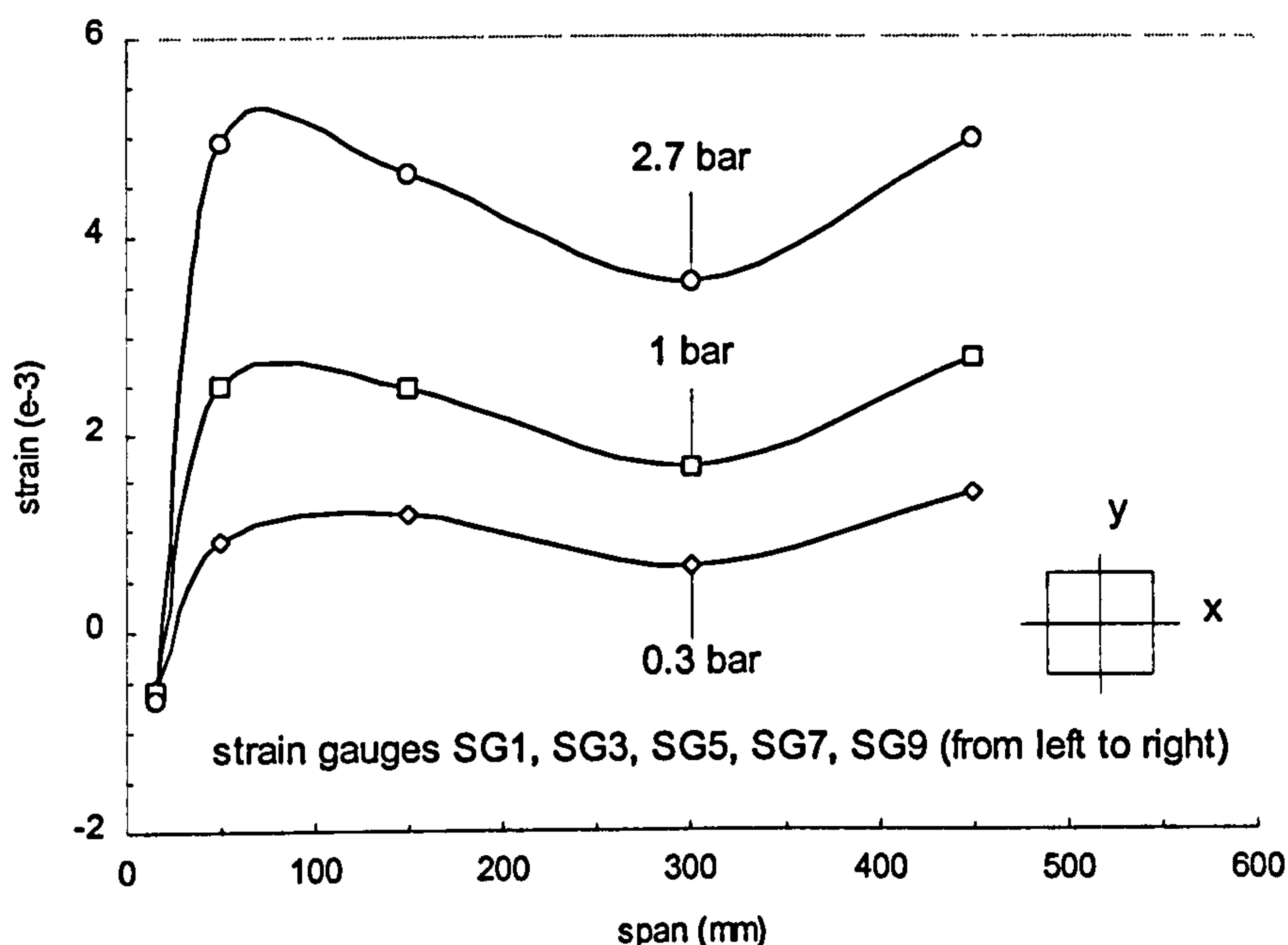


Figure 4.22 Strain distribution along the x-axis at unloaded surface
(See Figure 4.12a)

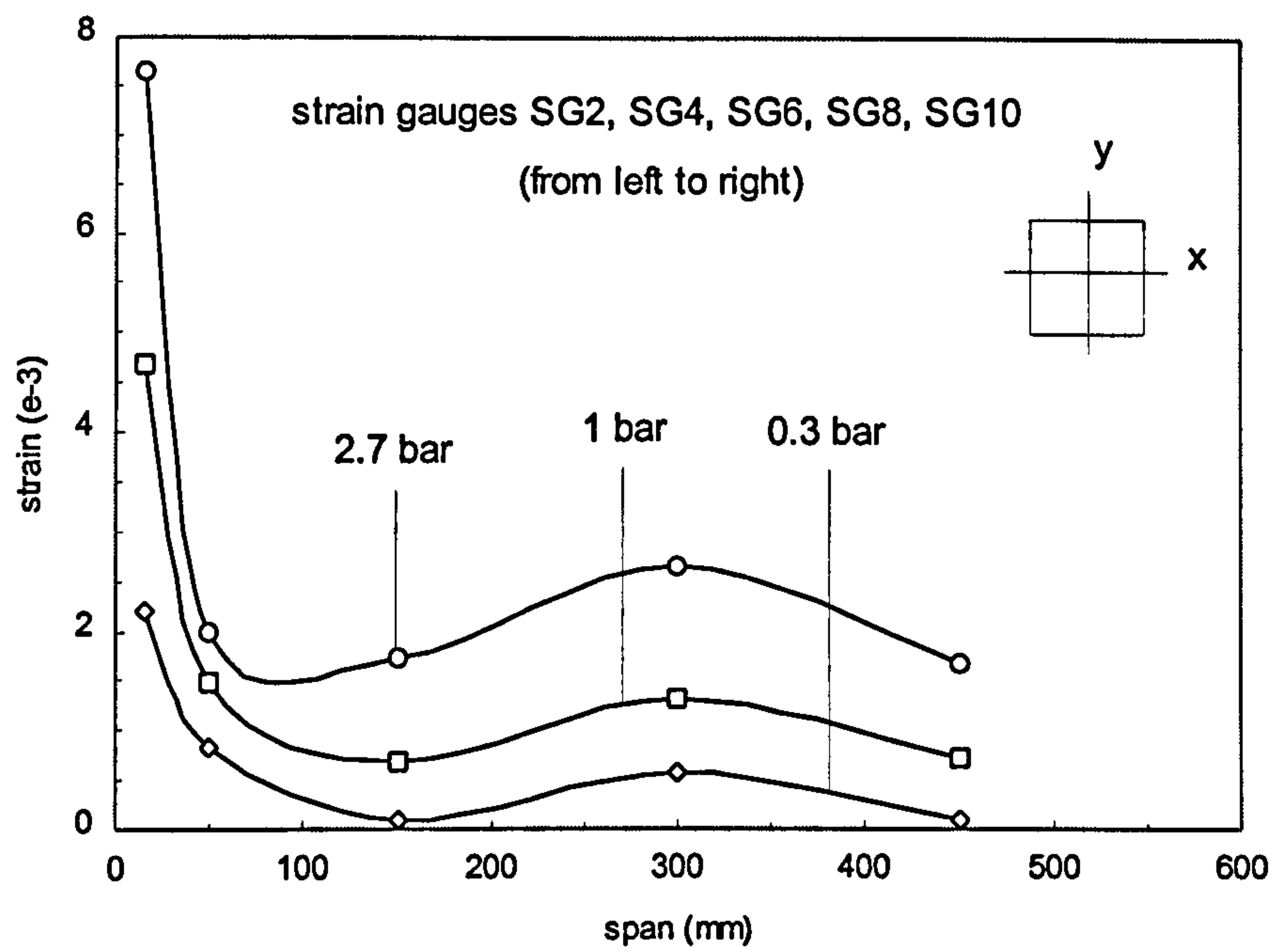


Figure 4.23 Strain distribution along the x-axis at loaded surface

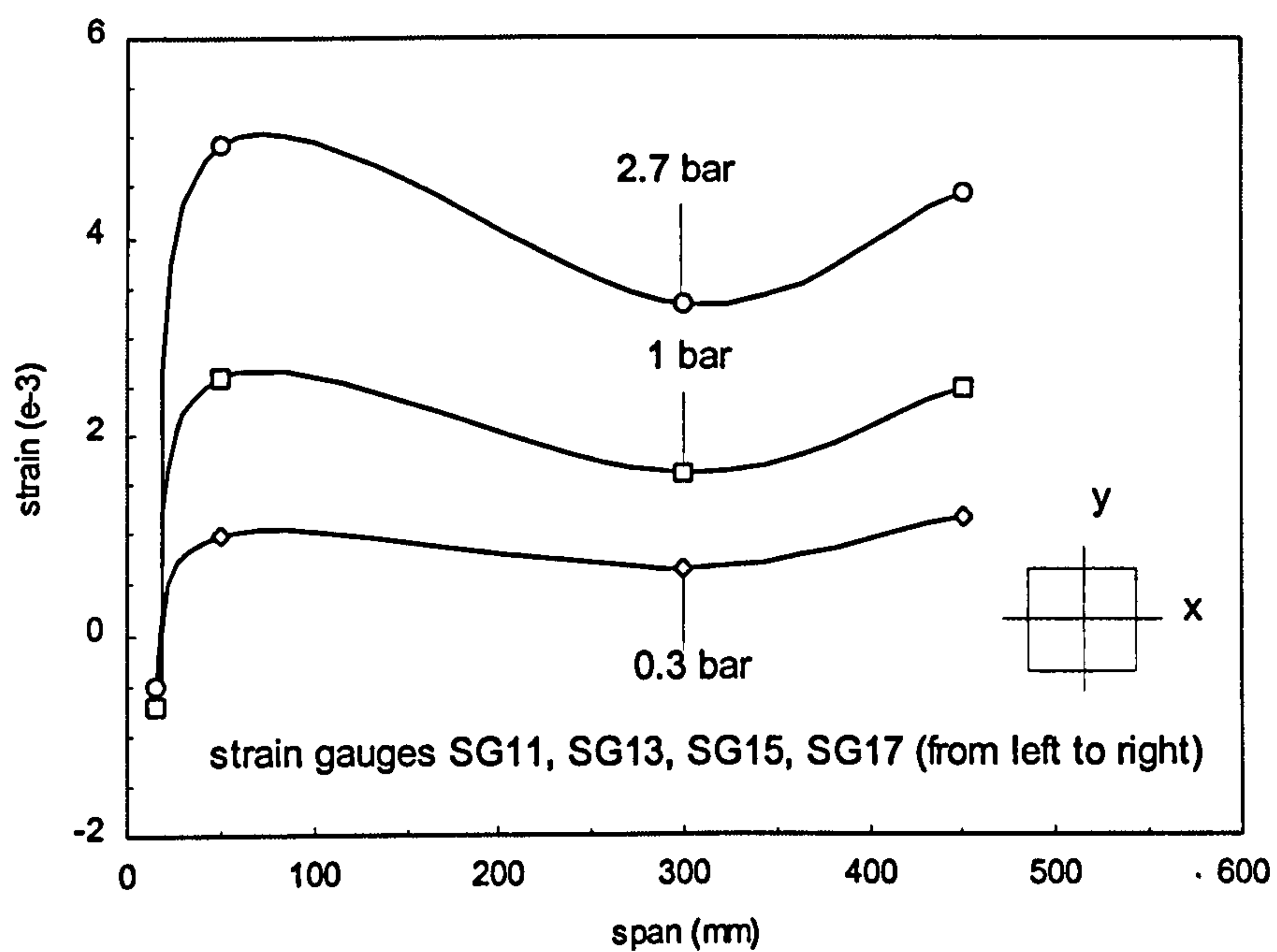


Figure 4.24 Strain distribution along the y-axis at unloaded surface

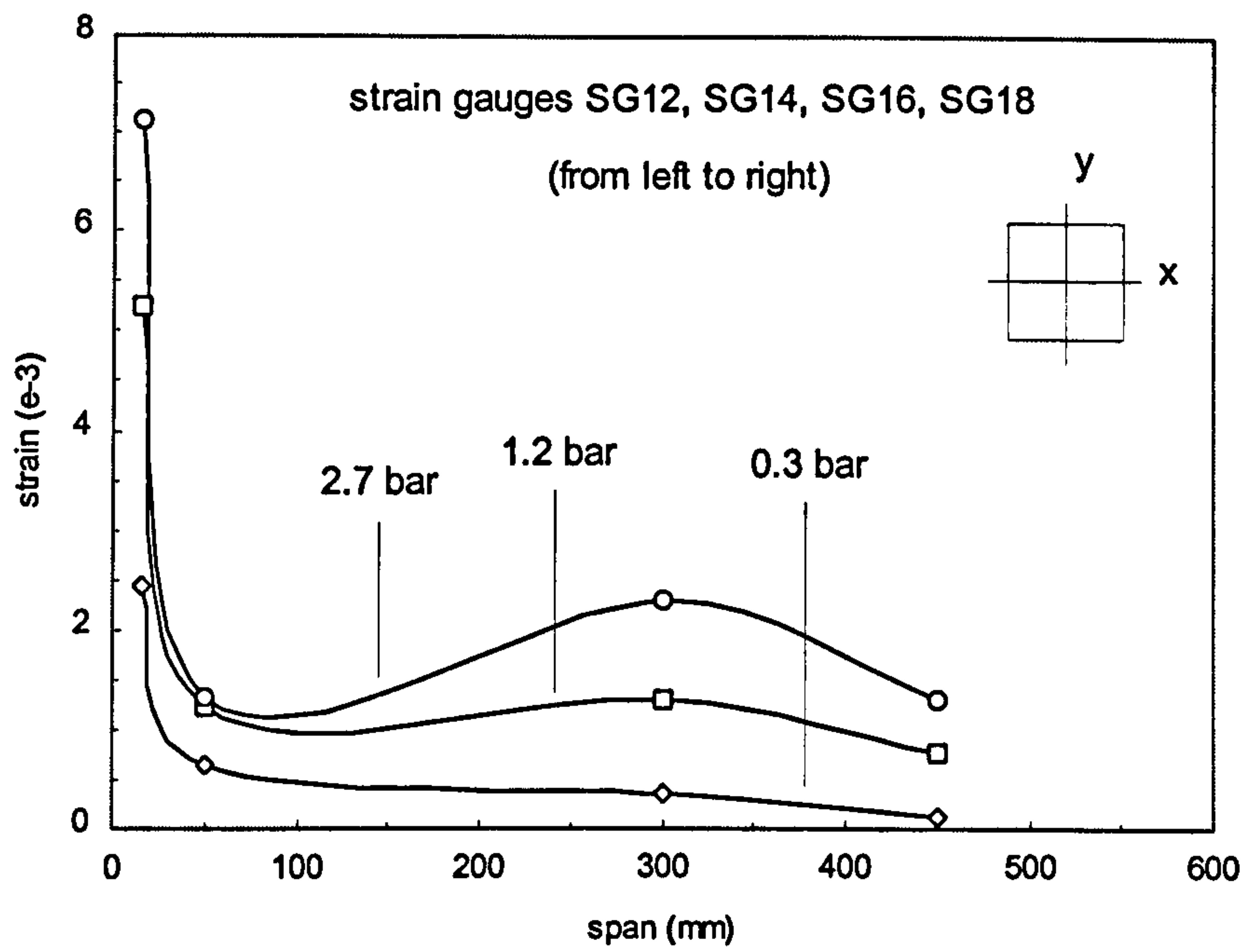


Figure 4.25 Strain distribution along the y-axis at loaded surface

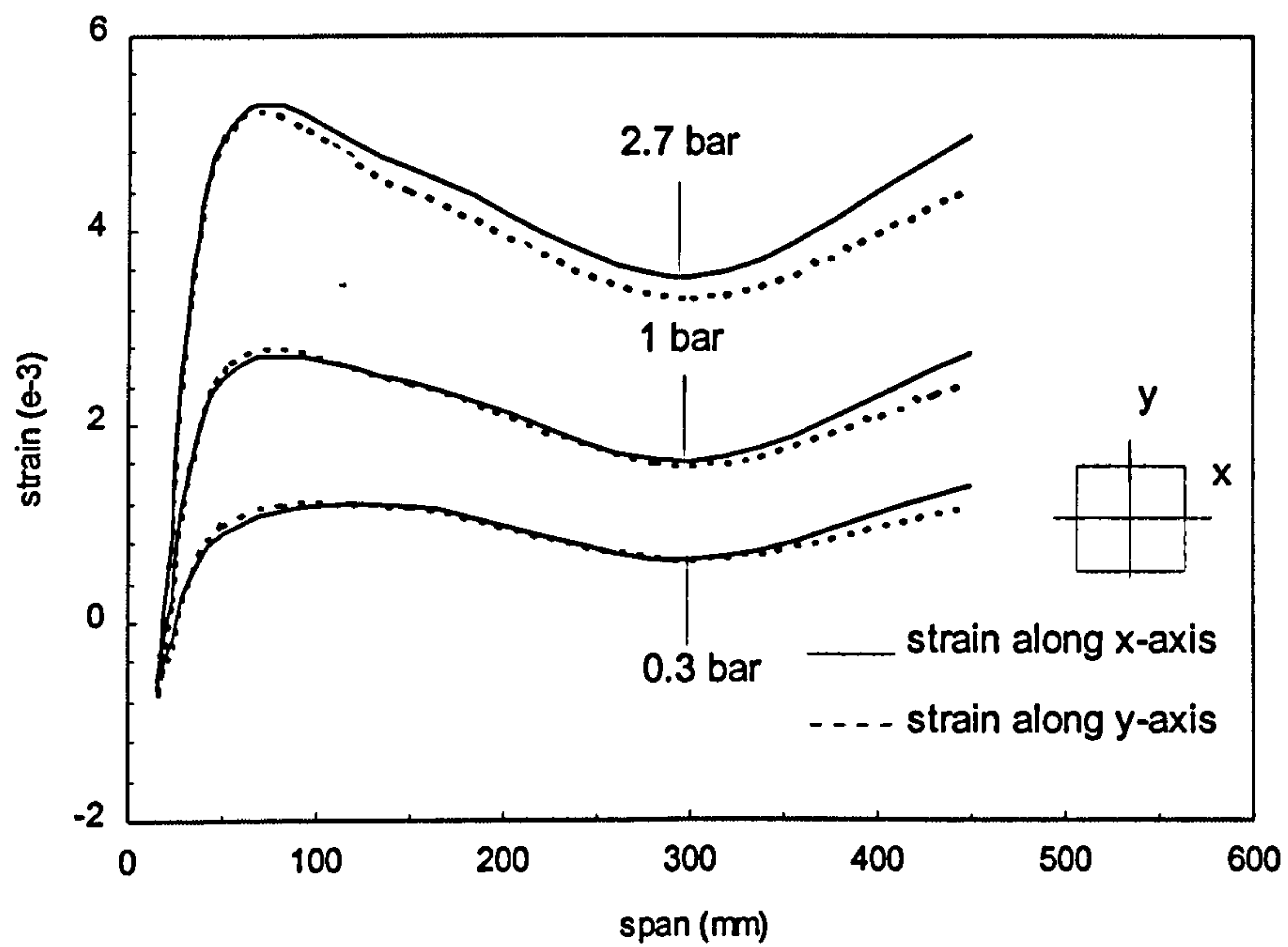


Figure 4.26 Strain distribution along the x and y axes at unloaded surface

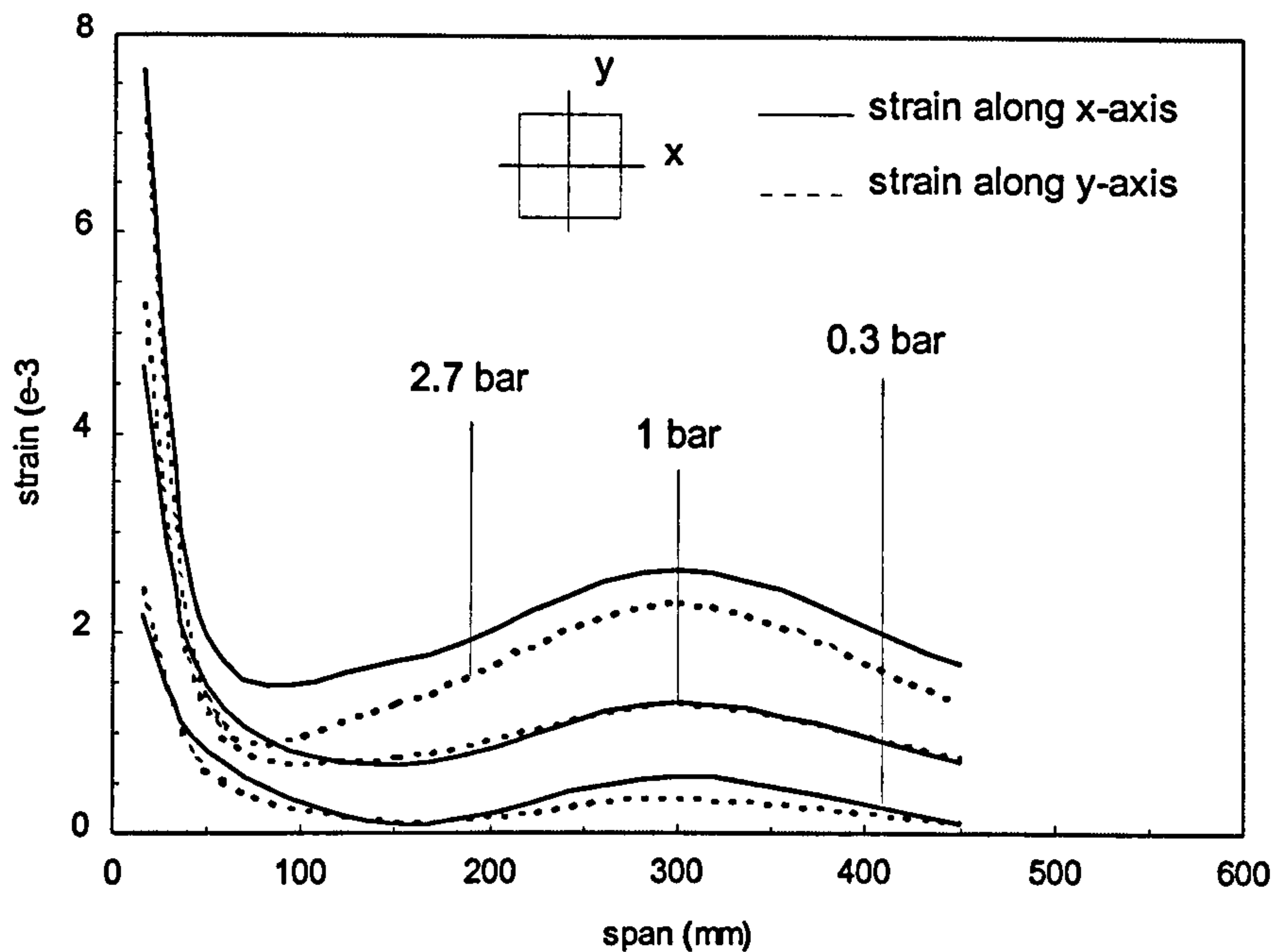


Figure 4.27 Strain distribution along the x and y axes at loaded surface

4.5.3 In-plane edge displacement

As stated earlier, initial finite element analysis on the panels showed that edge displacement can have a dominant effect on overall behaviour of the panel. However, in previous tests only two transducers were used to monitor edge displacement at the middle at each of the long edges. There was no information at the other two edges. In addition, the transducers were placed outside the panel (Figure 4.4), where the measurement could be affected by friction between frames and panel and bolts. This measuring method was not able to monitor edge displacement in the corners.

In this new panel test a great effort was made to improve the measurement method and to increase the number of measurement points. Exact measurement at the edges seemed to be almost impossible. How to measure effectively the edge displacement was discussed again and again. Finally, it was decided to take measurements using the inside-measurement system shown in Figure 4.13. This had the advantage of allowing measurements to be taken in the corners.

However, this inside measurement also had shortcomings. The main problem was that the measurement was affected by the rotation of the panel, as shown exaggeratedly in Figure 4.28. The measuring point was about 10 mm apart from the edge and it was noticed that there was sharp local bending around the edge. The measurement reading should therefore include the edge displacement and the displacement due to the rotation. Hence, the actual edge displacement (Δ) should be:

$$\Delta \approx \text{Measurement Reading} + \text{Rotation Angle } (\theta) * \text{Thickness} / 2 \quad (4.1)$$

The rotation angle was not measured in the test. The only way to estimate the edge displacement was to use the angle calculated from theoretical analysis, for instance, FE analysis. Hence, in the initial stages, the study of the in-plane displacement was aimed at qualitative analysis, that is, the distribution of the movement around the edges and the effect of the tightness of the bolts.

In the test study of Panel D, the bolts were tightened initially to a torque of 55 N-m, which was usually used in previous tests to fasten the bolts. The pressure was taken up to 1 bar (second stage). The data from the edge and corner transducers were recorded. The data showed that the distribution of the edge displacement was irregular (Figure 4.29). The edge displacement at the middle of one edge was apparently larger than at other three middle points, and small movements were also measured at some corners. Figure 4.29a demonstrates the pattern of the edge displacement measured directly from transducers at 1 bar. Because of the effect of panel rotation, measurements for some points appeared to be negative movements (assume that inwards movement is positive). By considering the effect of rotation, which was about 0.08 mm at 1 bar at middle points of the edges calculated from the expression (4.1), Figure 4.29b shows adjusted pattern of the edge displacement. It was assumed that there was no rotation at the corners, so that no adjustment was taken at the corners.

In the final stage of the panel test, the bolts were first loosened to allow the panel to settle down. Then, the bolts were tightened again to a torque of 65 N-m, which was higher than the torque (55 N-m) used in first two stages. The panel was eventually

loaded to failure. The recorded data showed that the distribution of the edge displacement was still irregular, as shown in Figure 4.30. At one pair of opposite edges (TC and TG), the edge displacements were bigger than another pair (TA and TE). In addition, the edge movement at TC was larger than that at TG, while the displacements at TA and TE were close each other. The values at the middle of the edges were also estimated by accounting for the contribution of the rotation of the panel (about 0.15 mm) according to the expression (4.1). However, there was no apparent displacement recorded at the four corners at the final stage.

Figures 4.31 and 4.32 illustrate the development of the edge displacements directly measured at four middle of the edges. All initial readings were negative because of the effect of the rotation of the panel. With the development of edge displacement, the curves change direction towards positive.

The test results showed that the distribution of the edge displacement was complicated and it was difficult to find certain regularity. However, tightening the bolts as much as possible would help to reduce the movement, especially at the corners.

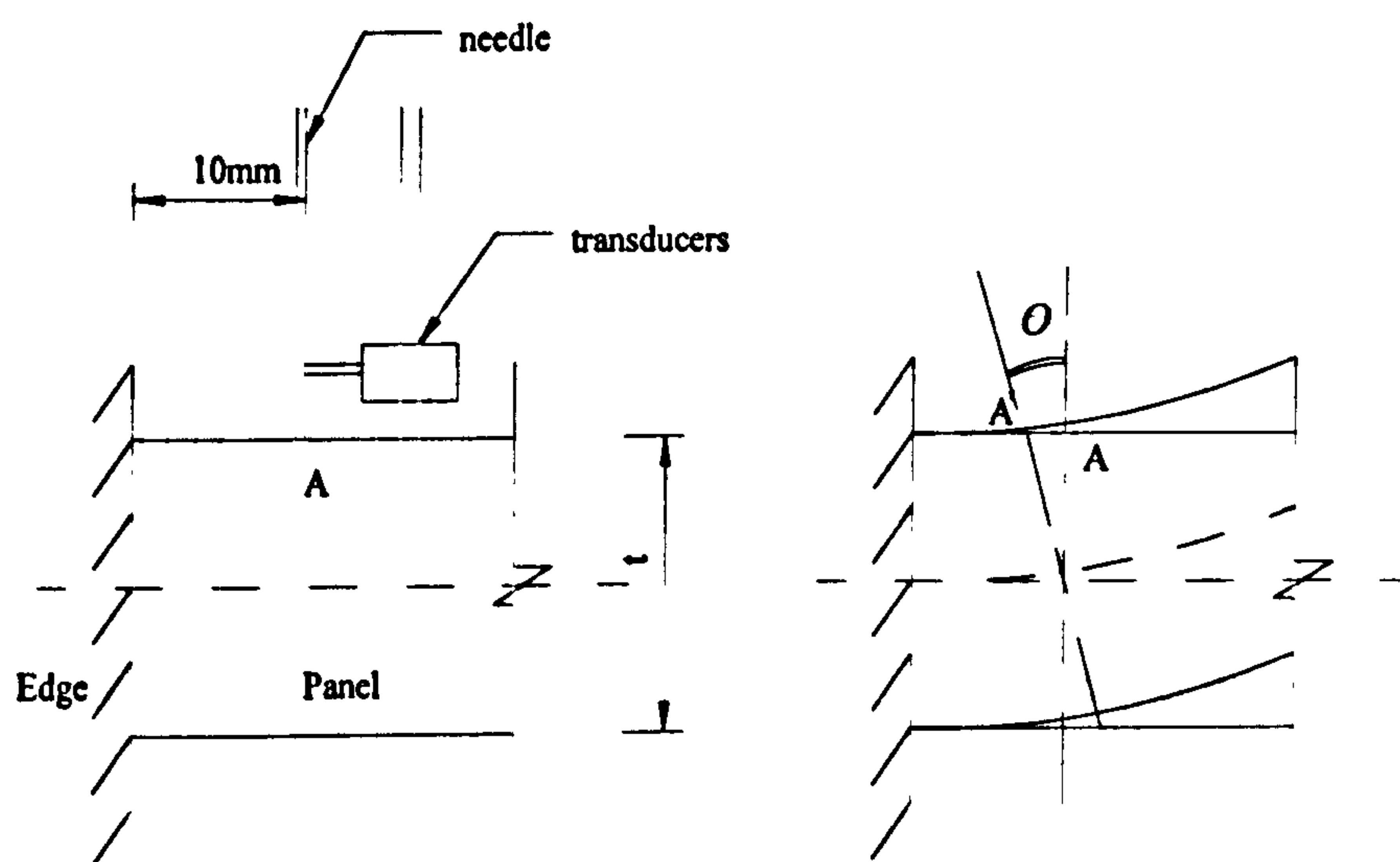


Figure 4.28 Inside measurement method - effect of panel rotation

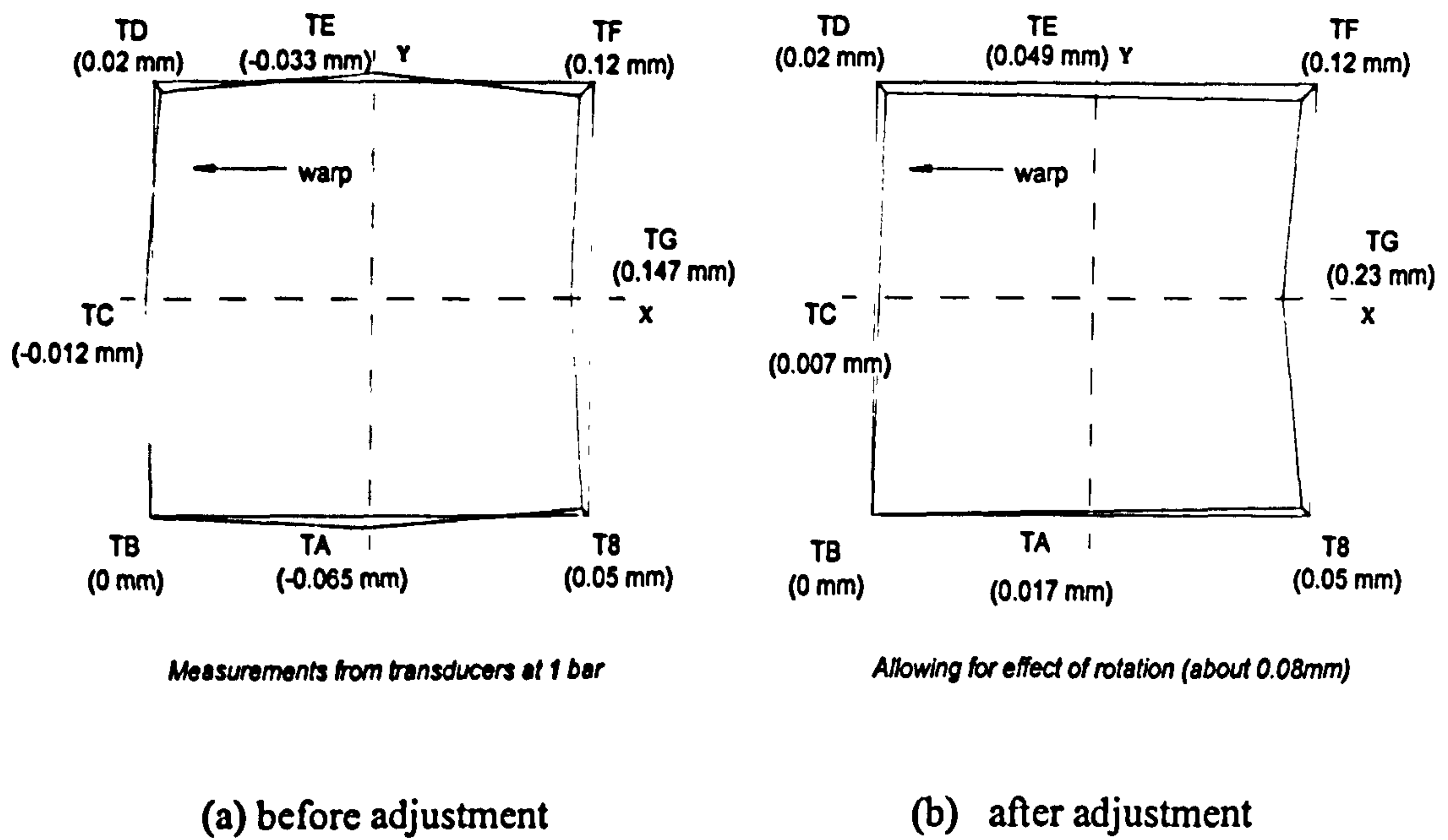


Figure 4.29 The pattern of edge displacements of the panel at 1 bar

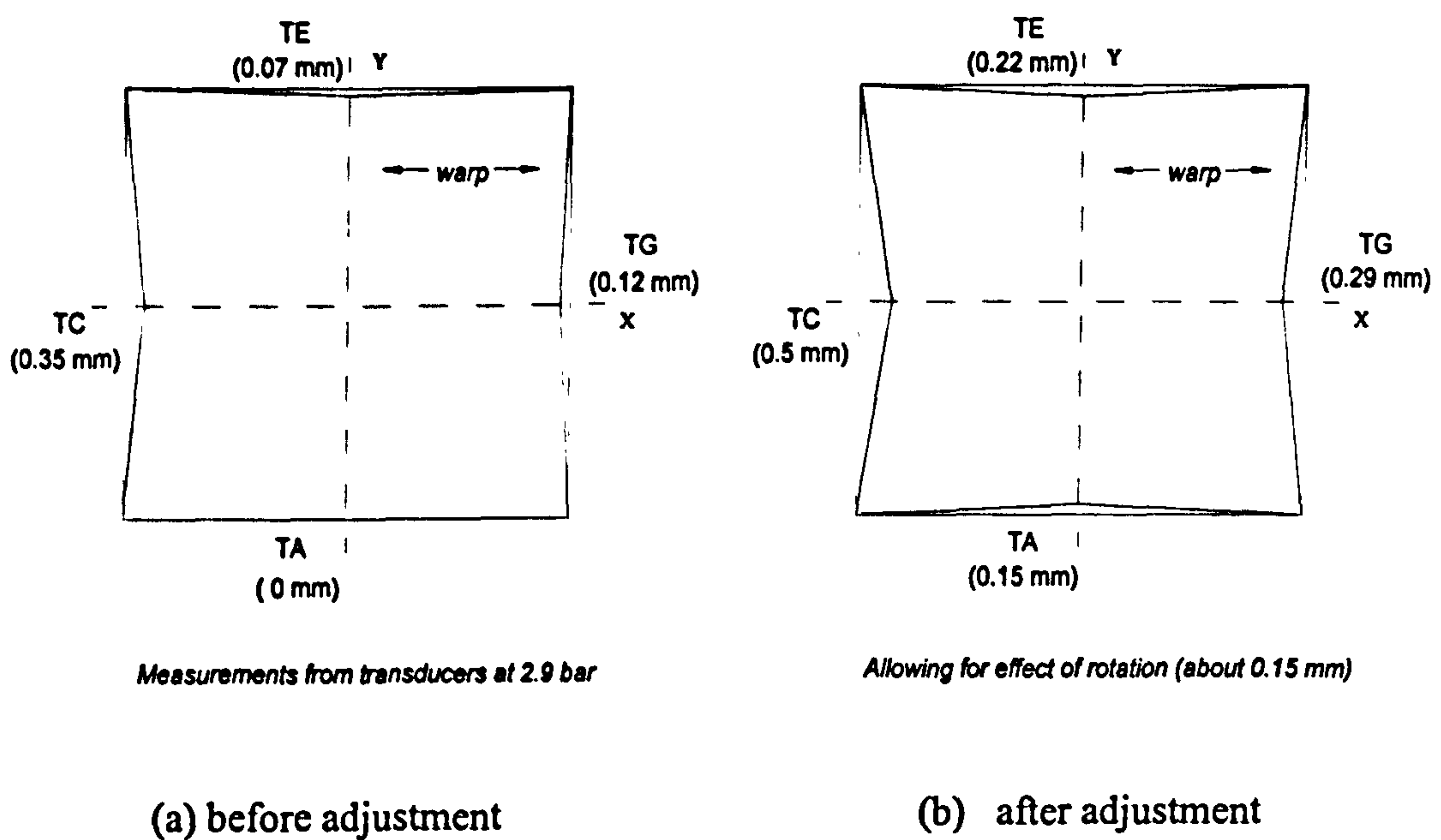


Figure 4.30 The pattern of edge displacements of the panel at failure

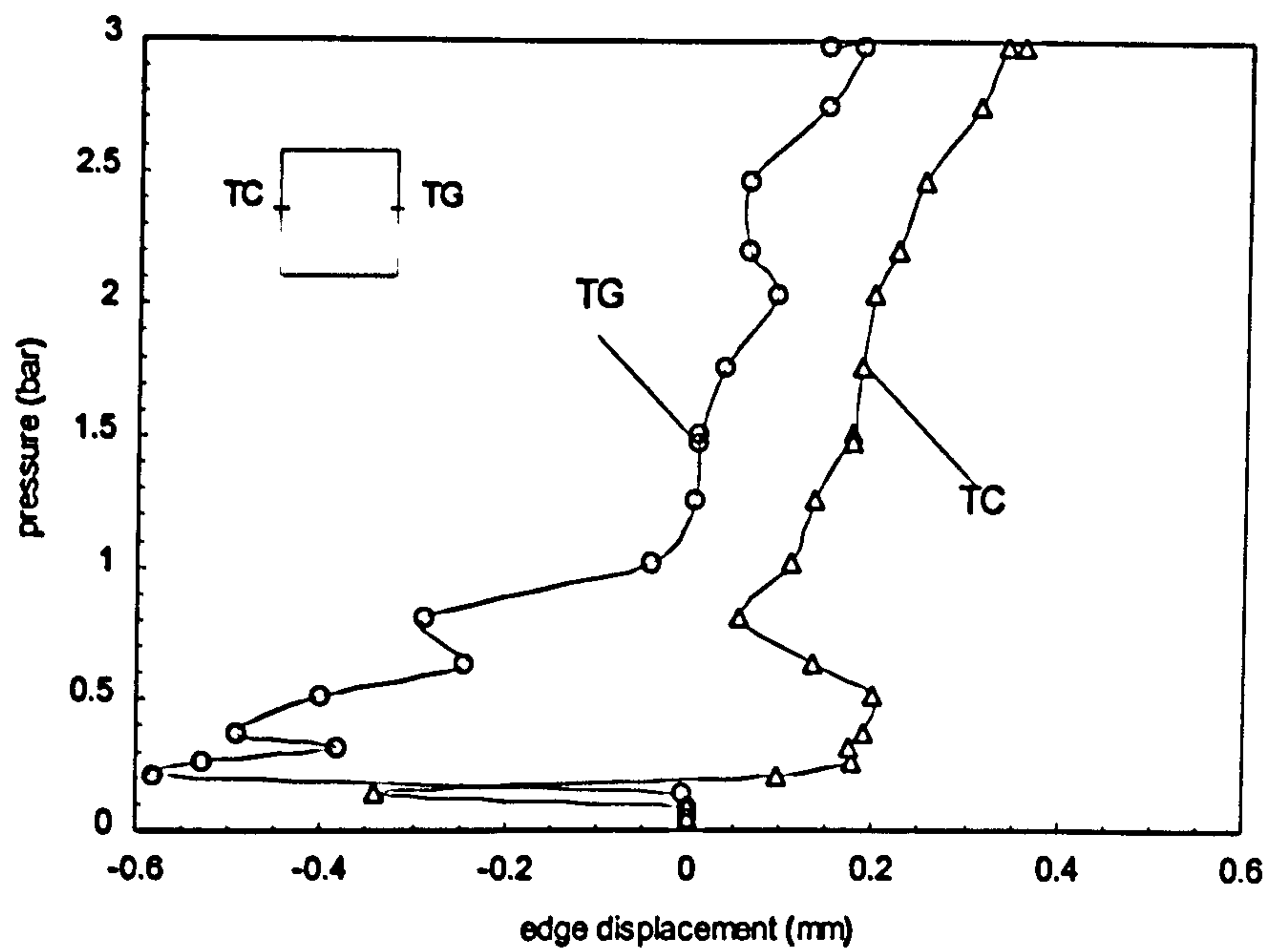


Figure 4.31 Development of edge displacement directly measured from the transducers (TC and TG)

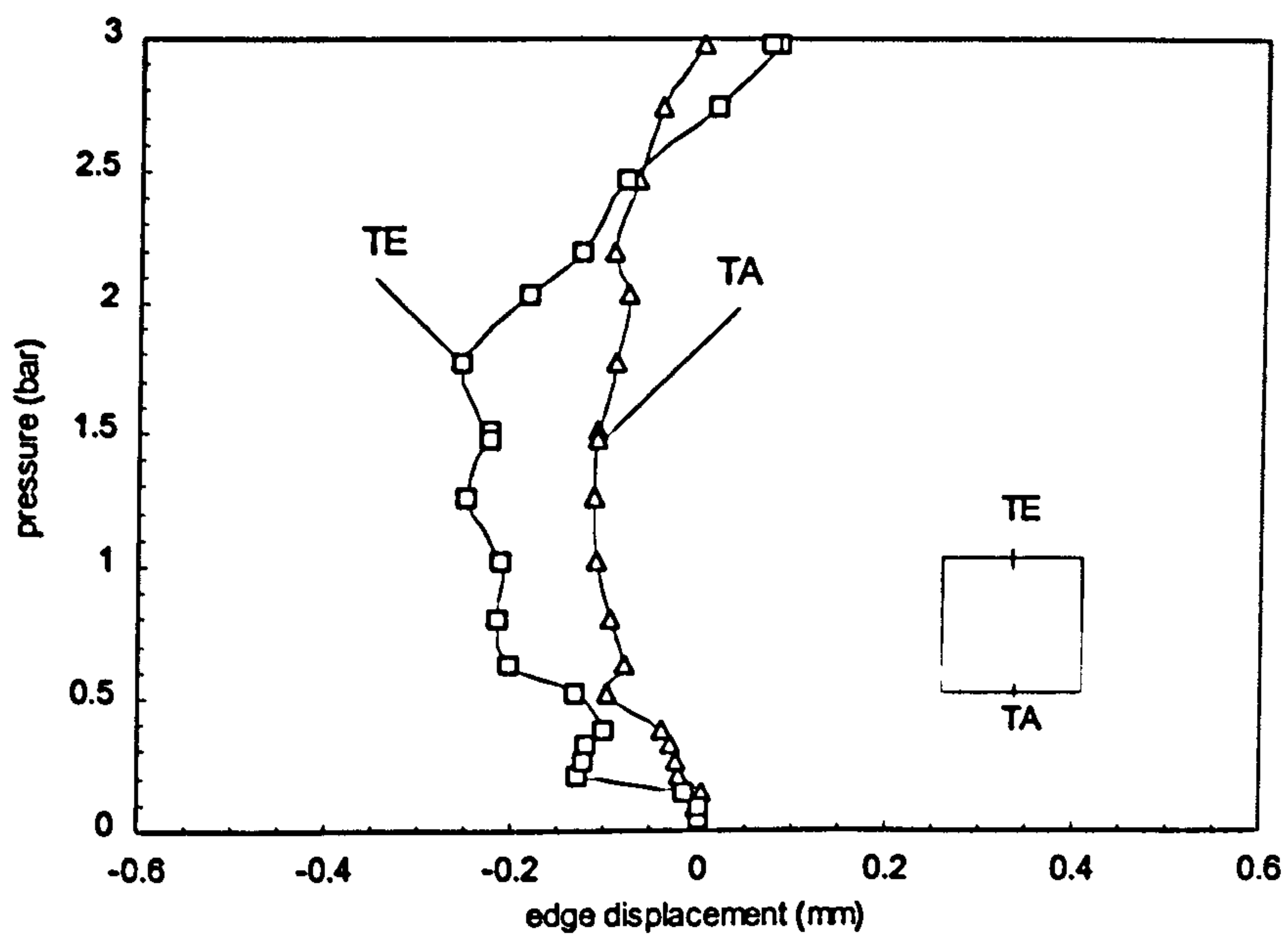


Figure 4.32 Development of edge displacement directly measured from the transducers (TA and TE)

4.5.4 Progressive failure of Panel D

The panel started to crack at about 1.3 bar as determined by noise emitted. Around the corners and at the middle of the edges there was visible whitening as the pressure increased. The panel failed by fibre breakage along the one of the edges at about 3 bar. Faint and discontinuous cracking noises suddenly became loud and continuous cracking noises, and deflections rapidly increased around the middle of one edge at which failure occurred. At failure the panel lost its symmetrical shape. Figure 4.33 shows the failed panel. Table 4.2 lists the key results of the final panel test.

Table 4.2 Key results of the final test of Panel D

Panel	Aspect ratio	Thickness mm	Failure pressure N/mm ²	Failure deflection mm	Audible cracking N/mm ²
D	1	4.15	0.297	24.75	0.13

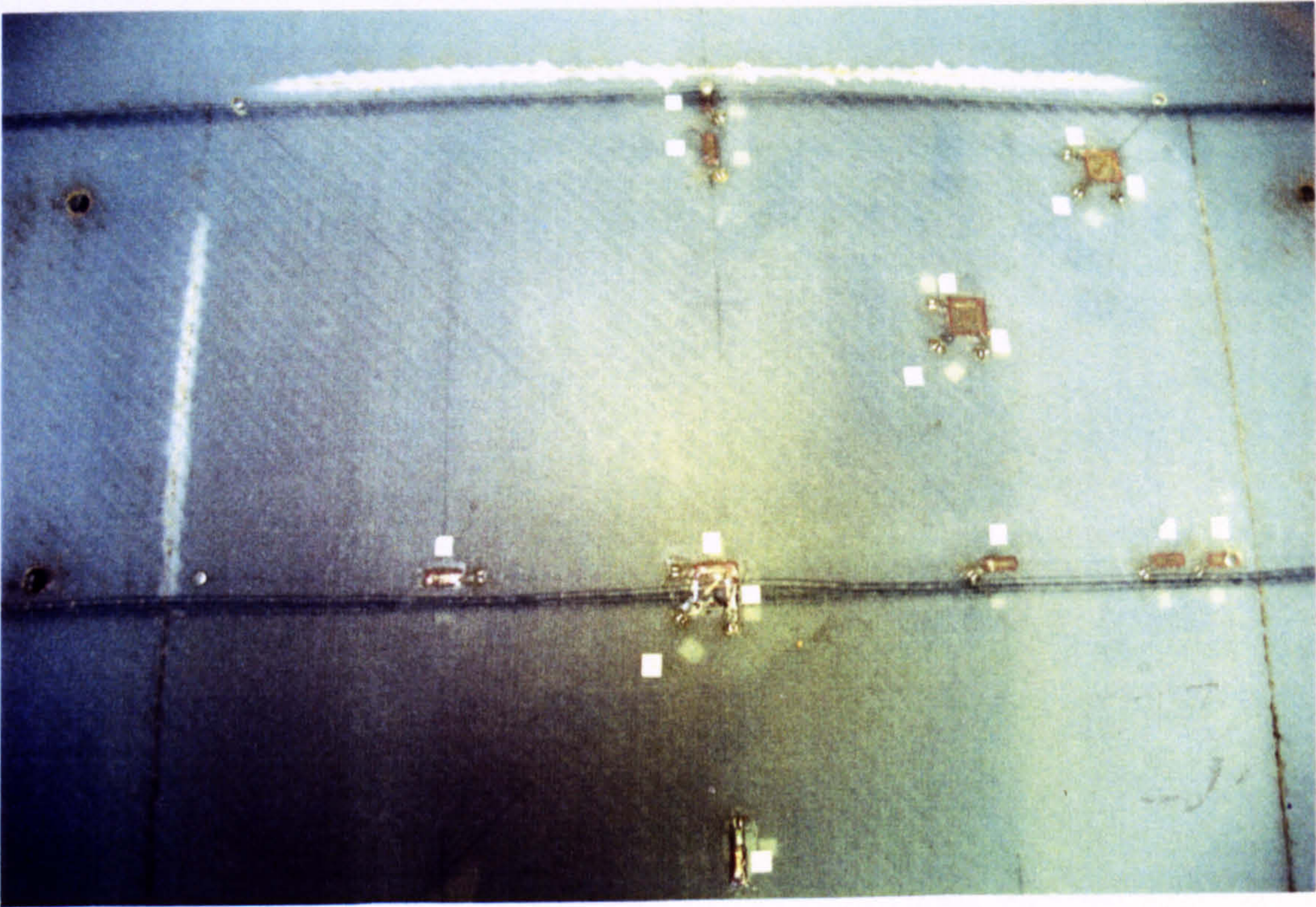


Figure 4.33 Final failure of Panel D

4.6 Conclusions

Some important conclusions can be drawn from the tests on panel.

- (1) Except at the edges, the panel acts as a membrane, even at very low pressure.
- (2) At the edges there is intense local bending, producing significant strains that are superimposed on the membrane strains.
- (3) Despite efforts to clamp the edges securely, inward in-plane displacements occur at the edges. These displacements are not exactly symmetrical and they have an irregular relationship with pressure. They are negligible at the corners.
- (4) At low pressure (up to 0.3 bar) the strain and displacements are repeatable and the loading and unloading curves coincide.
- (5) The strain distributions lack symmetry in the warp or the weft directions, although the panel was considered to be symmetrical in x-y plane.
- (6) The damage in the panel started at the edges in the form of resin cracking. The cracks developed as the pressure increased. The panel finally failed due to the fibre breakage along one of the edges.

Chapter 5 Prediction of elastic properties of composite materials

5.1 Introduction

Panel tests described in Chapter 4 provided precise data about the behaviour of composite panels under transverse pressure. In order to evaluate the performance of the panel theoretically (by means of the finite element method), it is first necessary to have a sound representation of the stiffness of the *woven composite* material in the plane of the panel (most of the materials used in this research were woven composites). Several models for the prediction of material properties of woven composite are available in the literature, as discussed in Chapter 2. However, most of them are quite complicated. In this research a simple and reasonably accurate model, which is based on methods developed by Professor Allen twenty years ago, is presented. The model has now been modified to predict the elastic properties of woven and non-woven laminae as well as the overall elastic properties of the laminate.

5.2 Theoretical model

In the theoretical model, a repeating unit element with width d in a woven composite (referred to as *woven fabric lamina* or *woven lamina*) is simplified as a four-layer laminate which consists of upper and lower layers of pure resin, and two orthogonal fibre/resin layers sandwiched between the two pure resin layers, as shown in Figure 5.1a. The pure resin layers represent the coverings on the top and bottom of the rovings and the resin in the gap between two adjacent rovings. The pure resin can be seen in the SEM microphotographs of the composite in Figures 3.10-3.13. The fibre/resin layers represent the warp roving and the weft roving, in which the fibre and the remainder of the resin are mixed together. In the same way, a *woven laminate* (containing several *woven laminae*) will be considered as several such four-layer laminates stacked

together, as shown in Figure 5.1b. The model can also be used to predict the elastic properties of composites reinforced with non-woven fibres.

The stiffness matrix of each of the layers in the theoretical model can be calculated from first principles. On a macroscopic scale all the layers share the same strain pattern, so the stiffness matrices of the various layers can be added to form the stiffness matrix of the composite. Then, the stiffness of the woven lamina and/or laminate can be calculated. It is convenient to list the notation used in the following sections.

Properties of the composite as a whole.

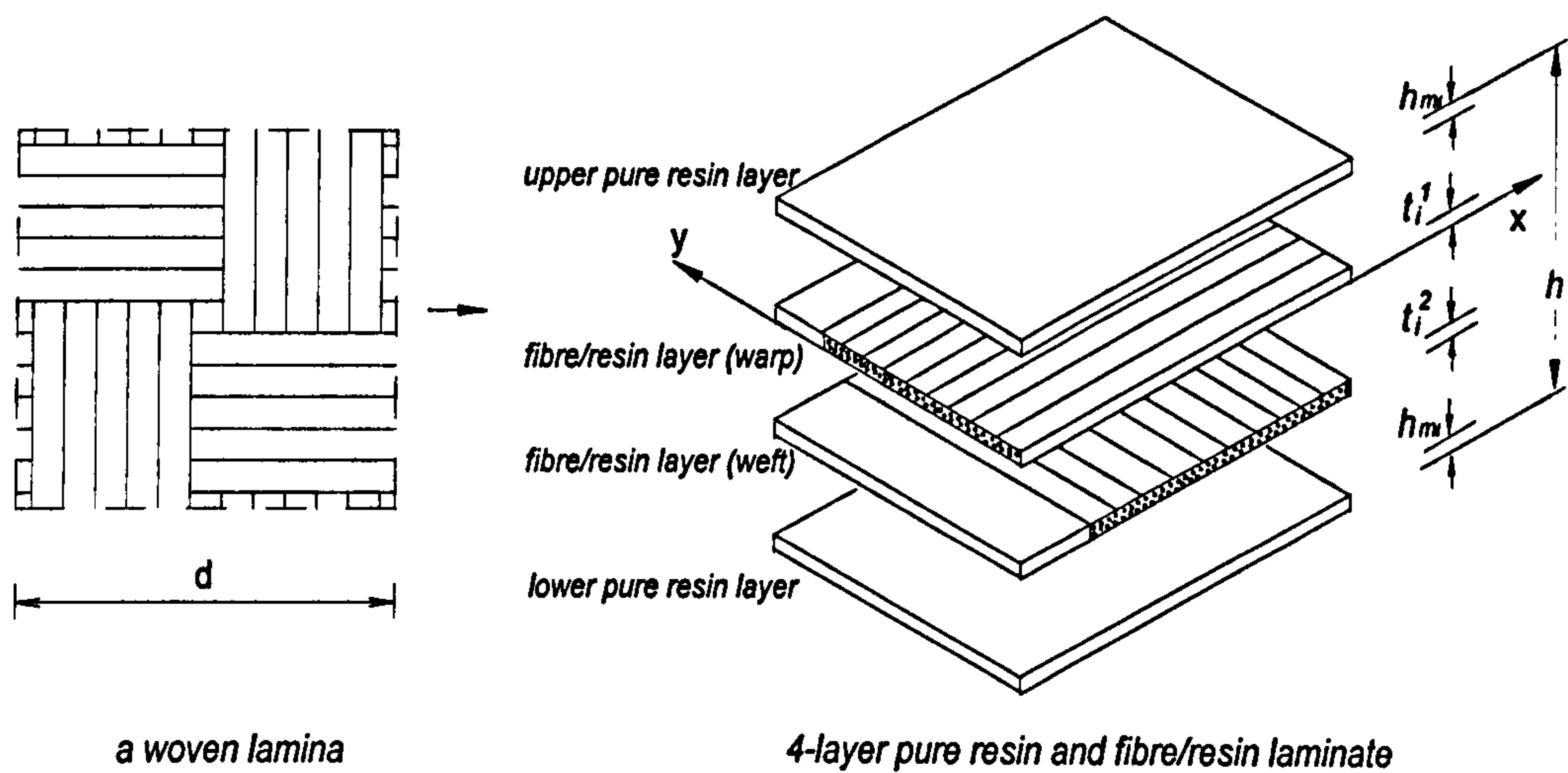
t	Overall thickness
V_f	Overall fibre content by volume
E_f, E_m	Young's moduli, fibre and resin respectively
ν_f, ν_m	Poisson's ratios, fibre and resin respectively
G_f, G_m	Shear moduli, fibre and resin respectively
$g_f = 1 - \nu_f^2$	and $g_m = 1 - \nu_m^2$

Properties of the woven lamina. (Assume pure resin is distributed evenly in each lamina. Thus, the thickness of top pure resin layer is equal to bottom pure resin layer. See Figure 5.1(a).)

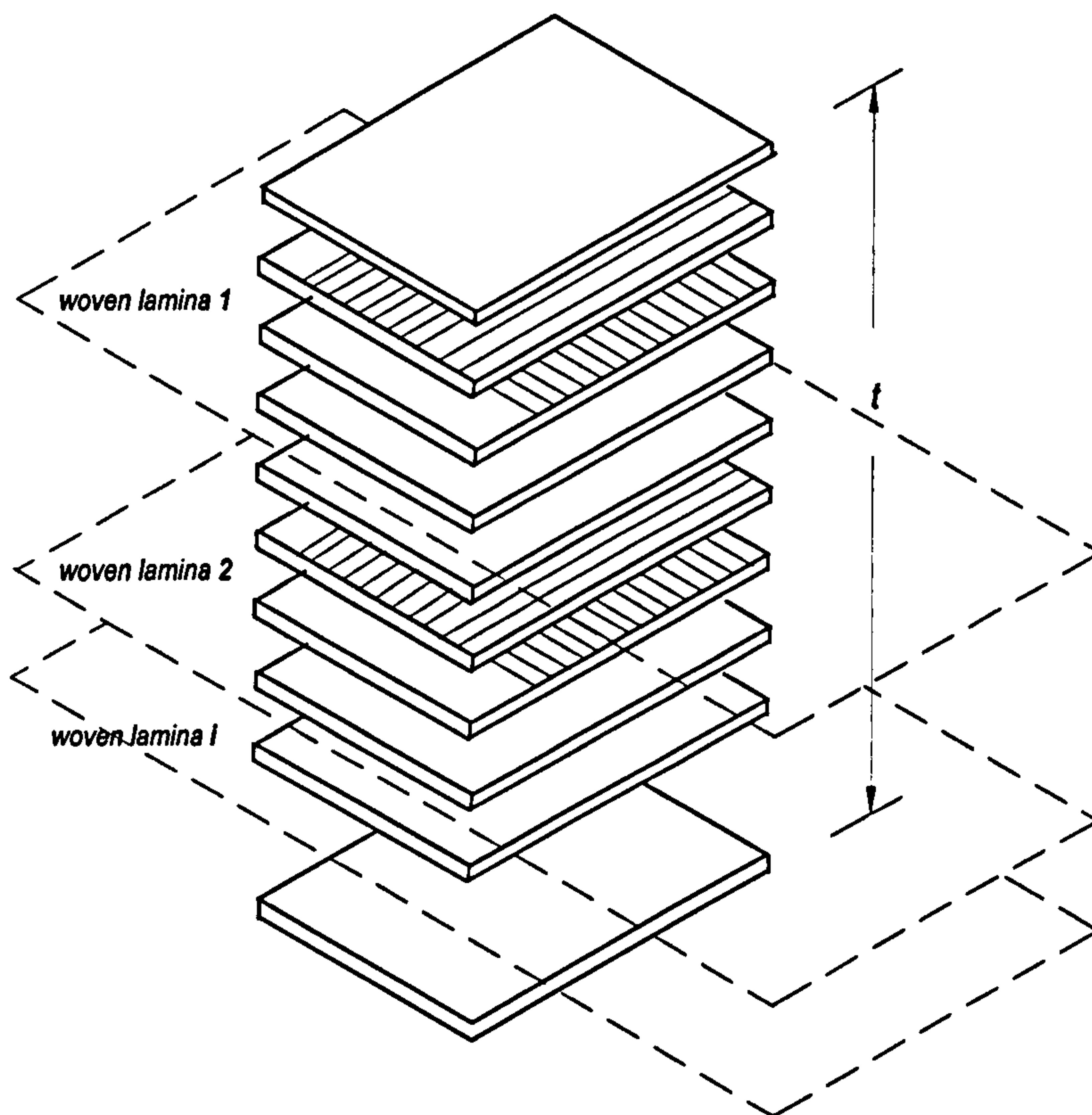
h	Thickness of lamina i
h_{mi}	Thickness of <i>top</i> or <i>bottom</i> pure resin layer in lamina i
t_i^1, t_i^2	Thickness of fibre/resin layer (<i>warp</i> and <i>weft</i>) in lamina i
S_{fi}	Fibre content in fibre/resin layer (<i>warp</i> or <i>weft</i>) in lamina i ,
S_{mi}	Resin content in fibre/resin layer (<i>warp</i> or <i>weft</i>) in lamina i , ($S_{mi} = 1 - S_{fi}$)

The way in which material is distributed between layers

μ	Fraction of resin (by volume) in pure resin layers
ξ_{1i}	Lamina fibre fraction in lamina i , (See Chapter 3 - Burning test results)
ξ_{2i}	Warp fibre fraction in lamina i , (See Chapter 3 - Burning test results)



(a)



A woven composite laminate can be simplified as many pure resin and fibre/resin layers

(b)

Figure 5.1 (a) A woven lamina in theoretical model
(b) A woven laminate in theoretical model

5.3 Stiffness matrix of fibre/resin layer in lamina i

Figure 5.2 shows a plan view of a typical fibre/resin layer, containing a band of fibres in parallel with a band of resin. Consider first a fibre/resin layer with material principal axes 1-2, in which the fibre lies along the 1-axis. There is perfect adhesion between the fibre and the resin. The dimension d is arbitrary and it does not appear in the final results. The thickness of the layer is uniform throughout.

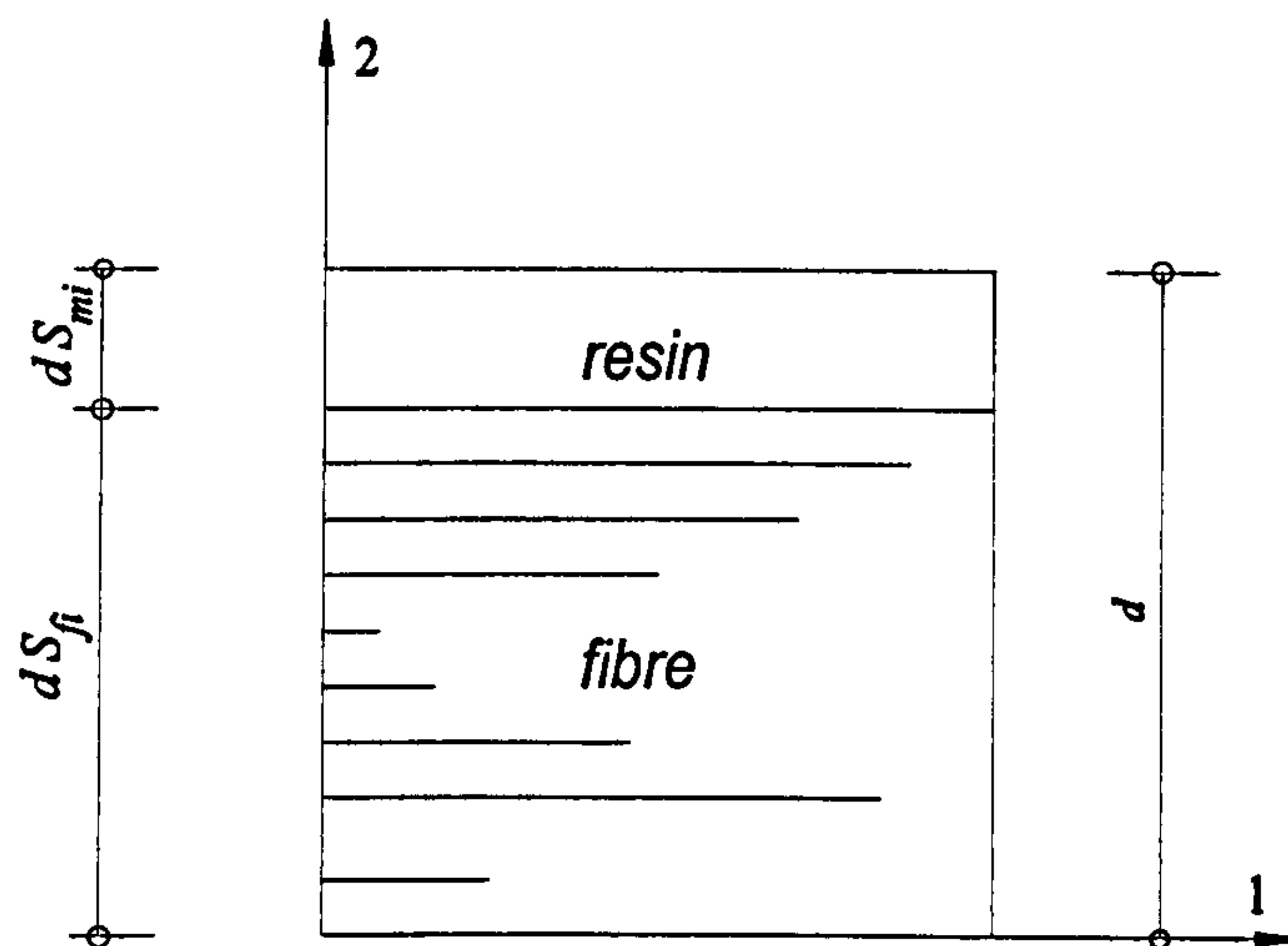


Figure 5.2 Demonstration of a fibre/resin layer

Case (1). $\varepsilon_1 \neq 0$, $\varepsilon_2 = 0$, $\gamma_{12} = 0$

In this case, all fibre and resin undergo the same strain, ε_1 , and carry the same stress, σ_1 . However, the strain in the 2-direction in the resin is different from the strain in the 2-direction in the fibre. Also, the stress in the 1-direction in the resin is different from the stress in the 1-direction in the fibre.

Stresses in the fibre are

$$\begin{bmatrix} \sigma_{1f} \\ \sigma_{2f} \end{bmatrix} = \begin{bmatrix} \frac{E_f}{g_f} & \frac{\nu_f E_f}{g_f} \\ \frac{\nu_f E_f}{g_f} & \frac{E_f}{g_f} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_{2f} \end{bmatrix} \quad (5.1)$$

Stresses in the resin are

$$\begin{bmatrix} \sigma_{1m} \\ \sigma_{2m} \end{bmatrix} = \begin{bmatrix} \frac{E_m}{g_m} & \frac{\nu_m E_m}{g_m} \\ \frac{\nu_m E_m}{g_m} & \frac{E_m}{g_m} \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_{2m} \end{bmatrix} \quad (5.2)$$

the overall strain ε_2 is zero, so the dimension ' d ' does not change.

$$\varepsilon_{2f} \cdot d \cdot S_f + \varepsilon_{2m} \cdot d \cdot S_m = 0 \quad (5.3)$$

The stress in the 2-direction is uniform:

$$\sigma_{2f} = \sigma_{2m} = \sigma_2 \quad (5.4)$$

The force in the 1-direction applied to the fibre and resin can be defined in term of the mean stress, σ_1 :

$$\sigma_1 \cdot d \cdot t_l = \sigma_{1f} \cdot t_l \cdot d \cdot S_f + \sigma_{1m} \cdot t_l \cdot d \cdot S_m \quad (5.5)$$

By suitable manipulations (detailed derivation is showed in Appendix 2) it can be shown that

$$\frac{\sigma_1}{\varepsilon_1} = k_{11}' = \frac{E_f}{g_f} \left\{ \frac{1 + S_m S_f [\lambda g_f + g_m / \lambda - 2(1 - \nu_f \nu_m)]}{\lambda S_m + S_f} \right\} \quad (5.6)$$

and

$$\frac{\sigma_2}{\varepsilon_1} = k_{21}' = \frac{E_f}{g_f} \left(\frac{\nu_m S_{mi} + \nu_f S_{fi}}{\lambda S_{mi} + S_{fi}} \right) \quad (5.7)$$

in which

$$\lambda = \frac{E_f}{E_m} \cdot \frac{g_m}{g_f} \quad (5.8)$$

Case (2). $\varepsilon_1 = 0$, $\varepsilon_2 \neq 0$, $\gamma_{12} = 0$

In this case, the strain ε_1 is everywhere zero, and the stress σ_2 is uniform throughout because the force generated by ε_2 in the fibre must be equal to the force in the resin, so:

$$\sigma_{2f} = \frac{E_f}{g_f} \varepsilon_{2f} ; \quad \sigma_{2m} = \frac{E_m}{g_m} \varepsilon_{2m} \quad (5.9)$$

$$\sigma_2 = \sigma_{2f} = \sigma_{2m} \quad (5.10)$$

The mean strain in the 2-direction is ε_2 . The width d of the fibre/resin therefore expands by a distance:

$$d \cdot \varepsilon_2 = d \cdot S_{mi} \cdot \varepsilon_{2m} + d \cdot S_{fi} \cdot \varepsilon_{2f} \quad (5.11)$$

The force in the 1-direction applied to the fibre/resin of width d is

$$\sigma_1 \cdot d \cdot t_i = \sigma_{1f} \cdot t_i \cdot d \cdot S_{fi} + \sigma_{1m} \cdot t_i \cdot d \cdot S_{mi} \quad (5.12)$$

By suitable manipulations (detailed derivation is showed in Appendix 2) it can be shown that:

$$\frac{\sigma_2}{\varepsilon_2} = k_{22}' = \frac{E_f}{g_f} \cdot \frac{1}{\lambda S_{mi} + S_{fi}} \quad (5.13)$$

$$\frac{\sigma_1}{\varepsilon_2} = k_{12}' = \frac{E_f}{g_f} \left(\frac{\nu_m S_{mi} + \nu_f S_{fi}}{\lambda S_{mi} + S_{fi}} \right) = k_{21}' \quad (5.14)$$

Case (3). $\varepsilon_1 = 0$, $\varepsilon_2 = 0$, $\gamma_{12} \neq 0$

Figure 5.3 shows the same fibre/resin layer of width d under shear strain. The shear stress all around the edge is τ . The mean shear strain is γ_{12} . But the shear strain in the resin and the fibre are γ_m and γ_f respectively.

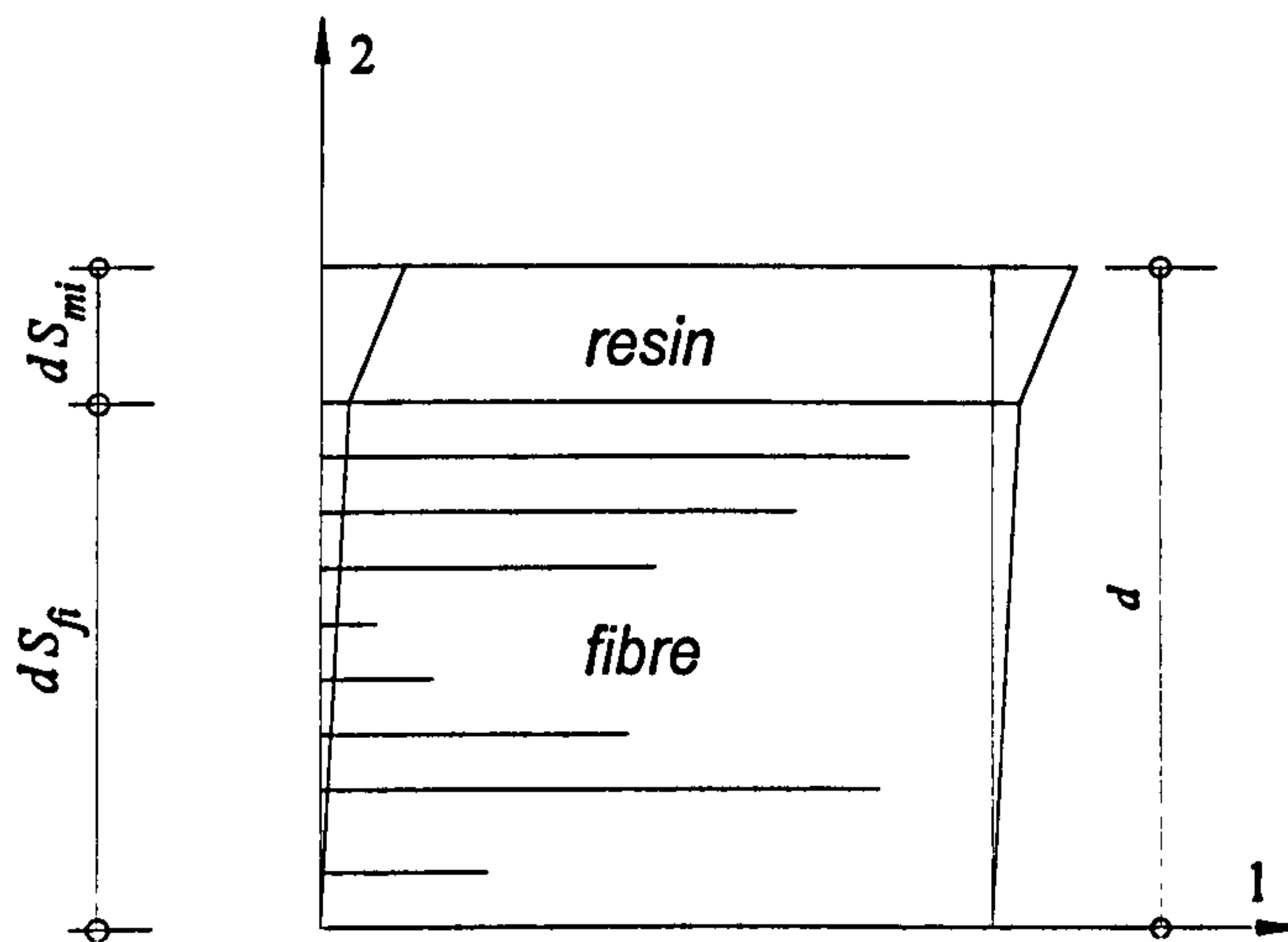


Figure 5.3 Shear deformation in fibre/resin layer

From the material properties,

$$\gamma_m = \frac{\tau}{G_m} ; \quad \gamma_f = \frac{\tau}{G_f} \quad (5.15)$$

Equating displacements,

$$d \cdot \gamma_{12} = d \cdot S_f \cdot \gamma_f + d \cdot S_{mi} \cdot \gamma_m \quad (5.16)$$

By suitable manipulations it can be shown that:

$$\frac{\tau}{\gamma_{12}} = k_{33}' = \frac{1}{\frac{S_{mi}}{G_m} + \frac{S_f}{G_f}} \quad (5.17)$$

General stress-strain relationship for fibre/resin layer in lamina i

Cases (1), (2) and (3) may be superimposed to define the general stress-strain relationship for the fibre/resin layer:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} k_{11}' & k_{12}' & 0 \\ k_{21}' & k_{22}' & 0 \\ 0 & 0 & k_{33}' \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (5.18a)$$

or

$$\{\sigma\} = [k']_i \{\varepsilon\} \quad (5.18b)$$

In this equation, $[k']_i$ is the stiffness matrix for the fibre/resin layer in lamina i in the principal material direction, and the elements k_{ij}' are defined by equations (5.6), (5.7), (5.13) and (5.17). Note that the expression (5-18) is suitable for both warp and weft fibre/resin layers in lamina i .

5.4 Transformation for non-aligned fibres in fibre/resin layer in lamina i

In general the fibres in the fibre/resin layer will lie at an angle θ to the x-axis in the global coordinate system (Figure 5.4), and the stress-strain relationship for the fibre/resin layer is required in the form:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{12} & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (5.19a)$$

or

$$\{\sigma\} = [k]_i \{\varepsilon\} \quad (5.19b)$$

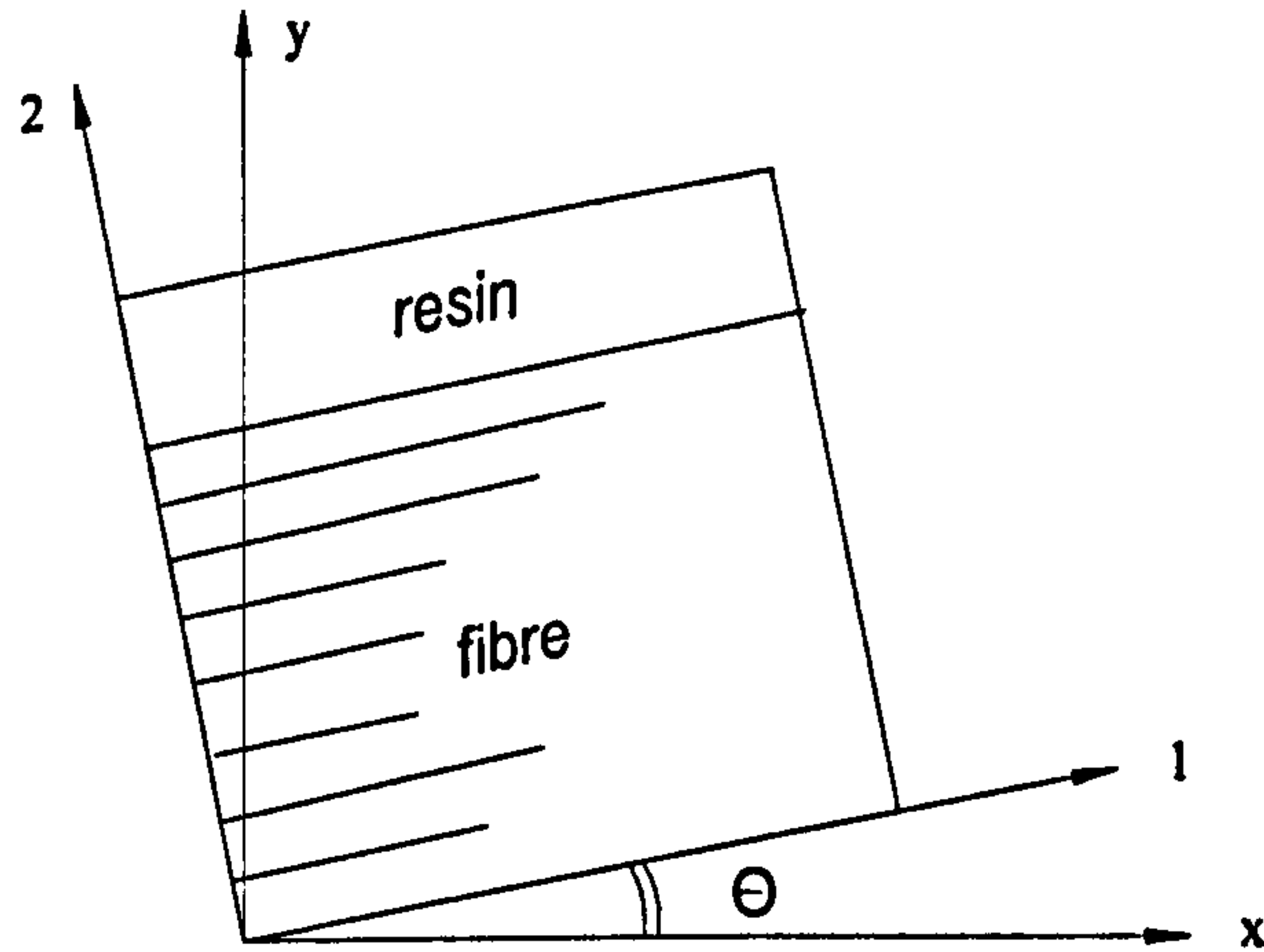


Figure 5.4 Material principal system and global system

where $[k]_i$ is the stiffness matrix for the fibre/resin layer in lamina i with fibre at an angle θ to the x-axis. Having the stress-strain relationship in the principal material directions, $[k]_i$ can be obtained by the standard transformation [10].

$$[k]_i = [T_1]_i [k']_i [T_2^{-1}]_i \quad (5.20)$$

where $[T_1]$ is the stress transformation matrix:

$$[T_1] = \begin{bmatrix} c^2 & s^2 & 2sc \\ s^2 & c^2 & -2sc \\ -sc & sc & c^2 - s^2 \end{bmatrix} \quad (5.21a)$$

and $[T_2]$ is the strain transformation matrix:

$$[T_2] = \begin{bmatrix} c^2 & s^2 & sc \\ s^2 & c^2 & -sc \\ -2sc & 2sc & c^2 - s^2 \end{bmatrix} \quad (5.21b)$$

$$\text{and } s = \sin\theta, \quad c = \cos\theta$$

The inverses of these matrices are:

$$[T_1^{-1}] = \begin{bmatrix} c^2 & s^2 & -2sc \\ s^2 & c^2 & 2sc \\ sc & -sc & c^2 - s^2 \end{bmatrix} \quad [T_2^{-1}] = \begin{bmatrix} c^2 & s^2 & -sc \\ s^2 & c^2 & sc \\ 2sc & -2sc & c^2 - s^2 \end{bmatrix} \quad (5.22)$$

Thus the inverse of $[T_1]$ is the transpose of $[T_2]$ and vice versa. If the shear stress and strain are replaced by $\tau_{xy}/2$ and $\gamma_{xy}/2$ respectively, the coefficient '2' in equations (5.21a), (5.21b) and (5.22) is replaced by '1', and $[T_1] = [T_2] = [T]$, and $[T^{-1}] = [T^T]$. This would permit some simplification of the equations but we have chosen to use the familiar engineering definitions of τ_{xy} and γ_{xy} in the stress and strain matrices respectively.

5.5 Stiffness matrix of a pure resin layer

The pure resin layer can be considered as isotropic. The stress-strain relationship of this layer in global coordinate system can be written as:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E_m}{g_m} & \frac{\nu_m E_m}{g_m} & 0 \\ \frac{\nu_m E_m}{g_m} & \frac{E_m}{g_m} & 0 \\ 0 & 0 & \frac{E_m}{2(1+\nu_m)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \quad (5.23)$$

5.6 Elastic properties of a woven lamina and a woven laminate

As stated above, a woven lamina is idealised as two pure resin layers and two orthogonal fibre/resin layers. Once the stiffness matrix for each single layer is computed, the stiffness matrix of the composite can be determined using the classical laminate theory. The general constitutive equation for the woven lamina can be written in the form [10]:

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} \varepsilon \\ \kappa \end{bmatrix} \quad (5.24)$$

where N and M represent membrane tensile and moment resultants and ε and κ are mid-plane strains and curvatures.

$$A_{ij} = \sum_{m=1}^4 k^m_{ij} (z_m - z_{m-1}) \quad \text{extensional stiffness} \quad (5.25)$$

$$B_{ij} = \frac{1}{2} \sum_{m=1}^4 k^m_{ij} (z_m^2 - z_{m-1}^2) \quad \text{coupling stiffness} \quad (5.26)$$

$$D_{ij} = \frac{1}{3} \sum_{m=1}^4 k_{ij}^m (z_m^3 - z_{m-1}^3) \quad \text{bending stiffness} \quad (5.27)$$

$$(i, j = 1, 2, 3)$$

where the sequence numbers $m=1-4$ represent the layers of upper pure resin, fibre/resin in warp, fibre/resin in weft and lower pure resin. z_m and z_{m-1} are distances from the top and bottom of the m -th layer to the middle plane of the lamina. k_{ij}^m are the stiffness components of the pure resin layers or of the fibre/resin layers.

To obtain the tensile elastic properties of a *woven lamina*, the in-plane stiffness matrix is selected from the equation (5.24). If the lay-out of the composite is symmetric the in-plane stress-strain relationship can be expressed as:

$$[\sigma] = \left[\frac{N}{h} \right] = \frac{1}{h} [A][\epsilon] \quad (5.28)$$

in which h is the thickness of the lamina. If assuming that *the warp and the weft directions in the woven lamina lie in x-axis and y-axis respectively*, then, the tensile elastic properties in the principal material direction can be obtained using well-known formulae [10]:

$$E_1 = \frac{1}{h} A_{11} \left(1 - \frac{A_{12}^2}{A_{11} A_{22}} \right)$$

$$E_2 = \frac{1}{h} A_{22} \left(1 - \frac{A_{12}^2}{A_{11} A_{22}} \right) \quad (5.29)$$

$$G_{12} = \frac{1}{h} A_{33}$$

$$\nu_{12} = \frac{A_{12}}{A_{22}}$$

To obtain the flexural elastic properties of a *woven lamina*, the bending stiffness matrix is selected from the equation (5.24). If assuming that the coordinate x-y lie in the middle plane of the lamina, the bending elastic properties in the principal material direction can be obtained using following formulae.

$$E_1 = \frac{12}{h^3} D_{11} \left(1 - \frac{D_{12}^2}{D_{11} D_{22}}\right)$$

$$E_2 = \frac{12}{h^3} D_{22} \left(1 - \frac{D_{12}^2}{D_{11} D_{22}}\right) \quad (5.30)$$

$$G_{12} = \frac{6}{h^3} D_{33}$$

$$\nu_{12} = \frac{D_{12}}{D_{22}}$$

In the same way, the overall elastic properties of a *woven laminate* can be also calculated provided that the laminate can be treated as an orthotropic material as a whole. As mentioned before, the woven laminate consists of several laminae, while each lamina is treated as four pure resin and fibre/resin layers. Hence, the woven laminate can be considered as multi-layer laminate containing such pure resin and fibre/resin layers (Figure 5.1b). The expression of (5.29) can be therefore extended to determine the overall elastic properties of the woven laminate. Provided that the x and y axes lie in the middle plane of the laminate, the expression of (5.30) can also be extended to determine the overall flexural elastic properties of the composite. In both cases, in the expressions (5.29) and (5.30) the thickness of lamina, h , should be replaced by overall thickness of the composite, t ; and the stiffness coefficients A_{ij} and D_{ij} are calculated by accounting for all the pure resin layers and fibre/resin layers in the woven laminate.

Once the elastic properties in principal material direction have been determined, the elastic properties in non-principal directions can be worked out using following formulae [10][78]:

$$\begin{aligned}\frac{1}{E_x} &= \frac{1}{E_1} \cos^4 \theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \sin^4 \theta \\ \frac{1}{E_y} &= \frac{1}{E_1} \sin^4 \theta + \left(\frac{1}{G_{12}} - \frac{2\nu_{12}}{E_1} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{E_2} \cos^4 \theta\end{aligned}\tag{5.31}$$

$$\nu_{xy} = E_x \left(\frac{\nu_{12}}{E_1} (\sin^4 \theta + \cos^4 \theta) - \left(\frac{1}{E_1} + \frac{1}{E_2} - \frac{1}{G_{12}} \right) \sin^2 \theta \cos^2 \theta \right)$$

$$\frac{1}{G_{xy}} = 2 \left(\frac{2}{E_1} + \frac{2}{E_2} + \frac{4\nu_{12}}{E_1} - \frac{1}{G_{12}} \right) \sin^2 \theta \cos^2 \theta + \frac{1}{G_{12}} (\sin^4 \theta + \cos^4 \theta)$$

5.7 Comparisons between theoretical model and test

In the practical calculation of the elastic properties, the thickness of each of the pure resin and the fibre/resin layers must be decided in advance. Assume that μ is the proportion of the resin that lies in the unreinforced layers, which can be estimated from image analysis of the SEM microphotographs of the composite; and the pure resin is distributed evenly in each of the laminae. Then, important relationships between defined properties can be obtained as follows.

(1) Thickness of top or bottom pure resin layer in lamina i

Total thickness of pure resin is $\mu(1-V_f)t$.

The amount allocated to each lamina is $\mu(1-V_f)t / (n)$.

So the thickness of top or bottom pure resin layer is

$$h_{mi} = \mu(1-V_f)t / (2n) \quad (5.32)$$

in which t is overall thickness of the composite; n is the number of the woven laminae in the composite.

(2) Equivalent thickness of fibre in fibre/resin layer (warp or weft) in lamina i

Equivalent thickness of all fibres is $V_f t$.

The amount allocated to each fibre/resin layer (warp or weft) in lamina i is

$$\xi_{ii} \xi_{2i} V_f t \quad (\text{warp fibre/resin layer})$$

$$\xi_{ii} (1-\xi_{2i}) V_f t \quad (\text{weft fibre/resin layer})$$

where ξ_{ii} is fraction of fibre contained in lamina i and ξ_{2i} is warp fibre fraction in lamina i , which can be obtained from the burning test results in Chapter 3. .

(3) Equivalent thickness of resin in fibre/resin layer (warp or weft) in lamina i

Equivalent thickness of resin in all the fibre/resin layers is $(1-\mu)(1-V_f)t$.

The amount allocated to each fibre/resin layer (warp or weft) in lamina i is

$$\xi_{ii} \xi_{2i} (1-\mu)(1-V_f)t \quad (\text{warp fibre/resin layer})$$

$$\xi_{ii} (1-\xi_{2i}) (1-\mu)(1-V_f)t \quad (\text{weft fibre/resin layer})$$

(4) Equivalent thickness of fibre/resin layer (warp or weft) in lamina i

Equivalent thickness of fibre/resin layer (warp or weft) in lamina i can be obtained by adding the equivalent thickness of the fibre and the equivalent of the resin together.

$$t_i^1 = (1 - \mu(1-V_f)) \xi_{1i} \xi_{2i} t / 2 \quad \text{(warp fibre/resin layer)} \quad (5.33a)$$

$$t_i^2 = (1 - \mu(1-V_f)) \xi_{1i} (1-\xi_{2i}) t / 2 \quad \text{(weft fibre/resin layer)} \quad (5.33b)$$

(5) Fibre content of fibre/resin layer (warp or weft) in lamina i , S_{fi} (also called fibre content by volume of the roving)

$$S_{fi} = \frac{V_f}{1 - \mu(1-V_f)} \quad (5.34)$$

To calculate the material properties of the composite, the elastic properties of the fibre and the resin must be known in advance. Typical values can be found in the composite materials handbook [79] and data sheets from the manufacturers. The elastic properties of the *fibre* and the *resin* as well as *fraction of pure resin* , μ , are chosen in the way to ensure that predicted overall elastic properties of composite materials have best fit with the results of the tests on coupons. Table 5.1 lists all these and related coefficients used in the calculations.

Table 5.1 Elastic properties of fibre and resin and related coefficients

Property	Laminate A (0/90)	Laminate B (0/45/90/-45/0)	Laminate C (0/90)	Laminate D (0/90)
$E_f KN/mm^2$	70.0	74.0	70.0	67.5
ν_f	0.3	0.3	0.3	0.3
$E_m KN/mm^2$	3.5	3.75	3.5	3.4
ν_m	0.3	0.3	0.3	0.3
μ	0.8	0.89	0.8	0.875
<i>thickness (mm)</i>	2.84	3.431	2.926	4.15
V_f	0.547	0.3077	0.4431	0.3649

A FORTRAN program (Appendix 3) was developed to calculate the elastic properties at various angles to the warp using the formulae in above sections. The program can be used for prediction of material properties of either a woven lamina and a woven laminate, or a single unidirectional layer and a laminate containing stacks of the unidirectional layers. Table 5.2 and Table 5.3 give the overall tensile and flexural elastic properties of the composite materials predicted by this theoretical model at various angles. In the calculation of flexural properties, it is noted that the bending modulus is very sensitive to the physical position of the fibre and resin through the thickness. Changing the physical position of the fibre and resin can change the results of flexural properties significantly. The test results are also listed in the tables in *italic* form. As can be seen, the results predicted by the theoretical model have very good agreement with the tests, not only in main fibre direction, but also in other directions.

Table 5.2 Overall tensile elastic properties predicted by the theoretical model

Laminate	Property	Inclination to the warp direction				
		0°	22.5°	45°	67.5°	90°
A (woven)	$E \text{ KN/mm}^2$	27.2	19.8	15.6	19.8	27.2
	<i>test E</i>	<i>27.1</i>	<i>19.2</i>	<i>15.6</i>	<i>20.7</i>	<i>27.0</i>
	$G \text{ KN/mm}^2$	5.1	7.1	11.8	7.1	5.1
	ν_{xy}	0.157	--	--	--	0.157
B (non-woven)	$E \text{ KN/mm}^2$	16.0	14.6	13.0	13.0	13.4
	<i>test E</i>	<i>16.2</i>	<i>14.8</i>	<i>13.2</i>	<i>12.9</i>	<i>12.9</i>
	$G \text{ KN/mm}^2$	4.9	5.4	5.9	5.4	4.9
	ν_{xy}	0.304	--	--	--	0.257
C (woven)	$E \text{ KN/mm}^2$	21.8	15.0	11.5	15.0	21.8
	<i>test E</i>	<i>21.8</i>	<i>15.3</i>	<i>11.5</i>	<i>14.8</i>	<i>20.6</i>
	$G \text{ KN/mm}^2$	3.7	5.3	9.5	5.3	3.7
	ν_{xy}	0.142	--	--	--	0.142
D (woven)	$E \text{ KN/mm}^2$	18.35	13.10	10.20	13.0	17.80
	<i>test E</i>	<i>18.5</i>	<i>13.0</i>	<i>10.2</i>	<i>13.3</i>	<i>17.5</i>
	$G \text{ KN/mm}^2$	3.37	4.7	7.9	4.7	3.37
	ν_{xy}	0.15	--	--	--	0.15
	<i>test ν_{xy}</i>	0.14				0.14

Table 5.3 Overall bending elastic properties predicted by the theoretical model

Laminate	Property	Inclination to the warp direction				
		0°	22.5°	45°	67.5°	90°
A	E^b KN/mm ²	30.9	22.3	15.6	22.3	30.9
	<i>test E^b</i>	29.1	22.4	15.8	23.8	30.1
	G KN/mm ²	5.7	8.0	13.4	8.0	5.7
	ν_{xy}	0.158	--	--	--	0.158
B	E^b KN/mm ²	20.2	14.9	10.5	10.0	10.5
	<i>test E^b</i>	15.3	14.9	10.0	8.7	8.3
	G KN/mm ²	4.0	4.9	6.3	4.9	4.0
	ν_{xy}	0.316	--	--	--	0.16
C	E^b KN/mm ²	21.0	14.5	11.1	14.5	21.0
	G KN/mm ²	3.6	5.16	9.2	5.16	3.6
	ν_{xy}	0.143	--	--	--	0.143
D	E^b KN/mm ²	17.9	12.9	10.0	12.7	17.4
	<i>test E^b</i>	17.1	--	--	--	16.0
	G KN/mm ²	3.3	4.6	7.7	4.6	3.3
	ν_{xy}	0.15	--	--	--	0.15

5.8 Theoretical model for orthotropic fibres and comparison with existing models

In practical use there are some kinds of fibre whose material properties are not isotropic but anisotropic, such as carbon fibres. These fibres are considered as transversely isotropic in three dimensions, but for two dimensional analysis they can be treated as orthotropic materials. Thus the composites made of these fibres, such as carbon/epoxy, are a combination of orthotropic fibre and isotropic matrix. The theoretical model above can be easily extended to analyse such composites.

Assume that the fibre is orthotropic and has the stress-strain relationship as follows:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} \frac{E_{f1}}{g_f} & \frac{\nu_{f12}E_{f2}}{g_f} & 0 \\ \frac{\nu_{f21}E_{f1}}{g_f} & \frac{E_{f2}}{g_f} & 0 \\ 0 & 0 & G_f \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (5.35)$$

where $g_f = 1 - \nu_{f12} \nu_{f21} \cdot E_{f1}$ and E_{f2} are Young's moduli along the fibre direction and perpendicular to the fibre direction, respectively. ν_{f12} , ν_{f21} are Poisson's ratios and G_f is in-plane shear modulus. For the orthotropic material there is the relation of $\nu_{f12} E_{f2} = \nu_{f21} E_{f1}$.

The stress-strain relationship of the resin is still the expression of (5.23). The derivation of the theoretical model with the orthotropic fibre is the same as that with the isotropic fibre. The only difference is to replace isotropic fibre material properties to orthotropic ones in the fibre/resin layer. This replacement has been done and the general stress-strain relationship for the orthotropic fibre/resin layer should be:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{bmatrix} = \begin{bmatrix} k_{11}' & k_{12}' & 0 \\ k_{21}' & k_{22}' & 0 \\ 0 & 0 & k_{33}' \end{bmatrix} \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{bmatrix} \quad (5.36)$$

in which

$$k_{11}' = \frac{E_{f2}}{g_f} \left\{ \frac{(\nu_{f12} S_{fi}^2 / \nu_{f21} + S_{mi}^2) + S_{mi} S_{fi} [\lambda g_f + g_m / \lambda + 2 \nu_{f12} \nu_m]}{\lambda S_{mi} + S_{fi}} \right\} \quad (5.37)$$

$$k_{12}' = k_{21}' = \frac{E_{f2}}{g_f} \left(\frac{\nu_m S_{mi} + \nu_{f12} S_{fi}}{\lambda S_{mi} + S_{fi}} \right) \quad (5.38)$$

$$k_{22}' = \frac{E_{f2}}{g_f} \frac{1}{(\lambda S_{mi} + S_{fi})} \quad (5.39)$$

$$k_{33}' = \frac{1}{\frac{S_{mi}}{G_m} + \frac{S_{fi}}{G_f}} \quad (5.40)$$

where

$$\lambda = \frac{E_{f2}}{E_m} \frac{g_m}{g_f} \quad (5.41)$$

The elastic properties of the composites with different types of the fibre predicted by this theoretical model are compared with those predicted by the models in references [32] and [35]. Table 5.4 gives the elastic properties of the fibre and the resin, and the parameters of the *woven lamina* used in their models. With knowing the fibre fraction of the roving, the fraction of pure resin μ can be obtained from the expression of (5.34). Then the elastic properties of the composites can be predicted using the theoretical model. The comparisons between the theoretical model and existing models are listed in Table 5.5. As can be seen from the table, Young's moduli predicted by the theoretical model agree with those from tests and the existing models.

It is noted that the results from the existing models were obtained by using very complicated prediction formulae, which accounted for the detailed architecture of woven composite. In the theoretical model described here, however, the waviness of the woven roving is ignored. By analysing microphotographs of the woven lamina listed in the examples (Reference [32]), it is found that the fabric bundles in the woven lamina are very flat ellipses in cross-section. Thus, the comparisons indicate that a small amount of waviness in a woven composite may have no significant influence on the elastic properties.

This is to be expected if the angle of inclination of the undulating fibres (relative to the plane of the composite as a whole) is small, since and change of apparent length depends on the square of the angle. In any case, gross changes in the alignments of the fibres cannot take place without disruption of the matrix, which is not observed at low applied stress.

Table 5.4 Elastic properties of fibres and resins used in references [32] and [35]

Properties	Material			
	<i>T-300 carbon</i>	<i>E-glass</i>	<i>Graphite</i>	<i>Epoxy (resin)</i>
E_1 KN/mm ²	230	72	388	3.5
E_2 KN/mm ²	40	72	7.2	3.5
ν_{12}	0.26	0.3	0.23	0.35
G_{12} KN/mm ²	24	27.7	6.8	1.3
V_f	0.44	0.42	0.41	--
S_f	0.78	0.7	0.8	--

Table 5.5 Comparisons between the theoretical model and the models in references [32] and [35]

Material	Models	E_x	E_y	ν_{xy}	G_{xy}
		KN/mm ²	KN/mm ²		KN/mm ²
T-300 Carbon/ epoxy	Experiment	49.3	60.3	--	--
	Naik [32]	52.4	58.9	0.07	5.1
	Polynkine [35]	51.5	57.3	0.06	4.8
	<i>Present model</i>	<i>51.8</i>	<i>61.2</i>	<i>0.05</i>	<i>3.4</i>
E-glass/ epoxy	Experiment	19.3	19.3	--	--
	Naik [32]	20.3	20.3	0.23	3.7
	Polynkine [35]	20.0	20.0	0.17	4.0
	<i>Present model</i>	<i>20.6</i>	<i>20.6</i>	<i>0.14</i>	<i>3.0</i>
Graphite/ epoxy	Naik [32]	80.1	80.1	0.02	2.3
	<i>Present model</i>	<i>83.3</i>	<i>83.3</i>	<i>0.02</i>	<i>2.5</i>

5.9 Conclusions

This chapter presents a simple theoretical model to predict the elastic properties of woven and non-woven composites. The model enables the complete two-dimensional stiffness matrix to be estimated from a knowledge of the Young's moduli of the fibres and resin, the fibre content, pure resin content and the arrangement of the various layers of reinforcement.

With proper selection of the values of the *properties of the fibre, resin and pure resin content*, the model provides very accurate predictions of the material properties in tension. Compared with coupon test results of laminates A, B, C and D. Good correlation can be found not only at 0 and 90 degrees to the warp direction, but also at 22.5, 45 and 67.5 degrees to the warp. The model is also verified by comparison with data on composites taken from the literature [32] and [35].

Compared with measured bending stiffness of the coupons, the model also provides good predictions for laminates A, C and D at various angles to the warp. For laminate B there is big error between the model and test at 0 degree to the warp.

In the theoretical model the undulation in the fibres is ignored, because the rovings are flat types and the angle of inclination to the plane of the composite is very small. This is supported by evidence from scanning electron microscope images (Figures 3.10-3.13). The good correlation between the theoretical model and the models in the literature [32] and [35] (in which detailed structure of the woven composite, including undulation in the fibres is considered) also reveals that small undulations have no significant effect on the stiffness (see Table 5.5).

Chapter 6 Stress concentrations in a woven composite

6.1 Introduction

Woven composites are used for structural applications in ship and aircraft construction because of their ease of handling, low fabrication costs and high structural performance. The interlacing of the woven fibre bundles can provide many obstacles to the growth of crack damage. Accordingly, a woven composite has better-balanced properties in the plane of the fabric and higher impact resistance than a non-woven composite. However, there are also negative effects due to the interlacing and the undulation of the fabric bundles. The undulation of the roving may induce many local stress concentrations which can result in early initiation of diffuse damage, particularly in the matrix. This in turn, reduces the stiffness and strength of the composite material. Therefore much effort needs to be devoted to gaining a better understanding of effects of the detailed architecture of the woven composite on material properties and on the failure mechanism.

The complicated geometry of the woven composite makes a detailed analysis quite challenging. Early stress analyses were based on laminate theory and on a simplified model of the woven composite, such as the mosaic model and the bridging model in references [26-29]. In recent years there have been a few attempts to model the architecture in detail, to predict internal stress states, and hence to model progressive failure. As mentioned in Chapter 2, Glaessgen et al. [40] made finite element models of a plain woven composite at microstructural level. The distribution of strain energy density inside the composite was predicted. However, little information was provided on stress concentrations and the effects of various parameters on them. Blackketter et al. [53] presented a progressive failure analysis of a plain woven composite at microstructural level. The results showed that the response of the composite was almost linear for in- plane extension.

In this chapter the stress distributions and concentrations inside the woven composite were studied by modelling the architecture in detail by the finite element method. The stress concentrations due to the interlacing and waviness of the roving can be seen very clearly in the contour plots presented below. The term *stress variation factor* was defined to study the stress concentrations. The effects of various characteristics of the composite, such as waviness ratio, fibre content of the roving, etc., on the stress variation factor were studied. Several important conclusions were drawn from the study. The results were very helpful in identifying failure initiation and in studying the failure mechanism of the woven composite.

6.2 Geometry of a plain woven composite

Woven fabrics are formed by interlacing two sets of orthogonal rovings (or yarns), the warp and the weft. The manner in which the rovings are interwoven can create different kinds of weave. The commonest weave is plain woven composite, which is taken as an example in the study of the stress concentration of woven composite in this chapter. From the symmetry of the repeating pattern of a plain woven composite, one fourth of it is modelled in detail and is defined as a *repeating unit element* (RUE). Figure 6.1 demonstrates the geometry of RUE of a plain woven composite.

The composite consists of *resin impregnated rovings* and regions of pure resin. Because of the presence of the pure resin regions, the fibre content in a resin impregnated roving (or called *fibre content by volume of the roving* (S_f)) is higher than the *overall* fibre content by volume (V_f). The fibre content by volume of the roving is defined as the volume of fibre in the roving divided by the volume of the roving. The overall fibre volume fraction can be determined experimentally. Using V_f and the roving geometry, the fibre content by volume of the roving can be determined from the expression as follows.

$$S_f = \frac{V_f}{1 - \mu(1 - V_f)} \tag{6.1}$$

where μ is fraction of pure resin in the composite, which is defined as the volume of pure resin regions divided by whole volume of the composite.

$$\mu = \frac{V_{resin}}{V} \tag{6.2}$$

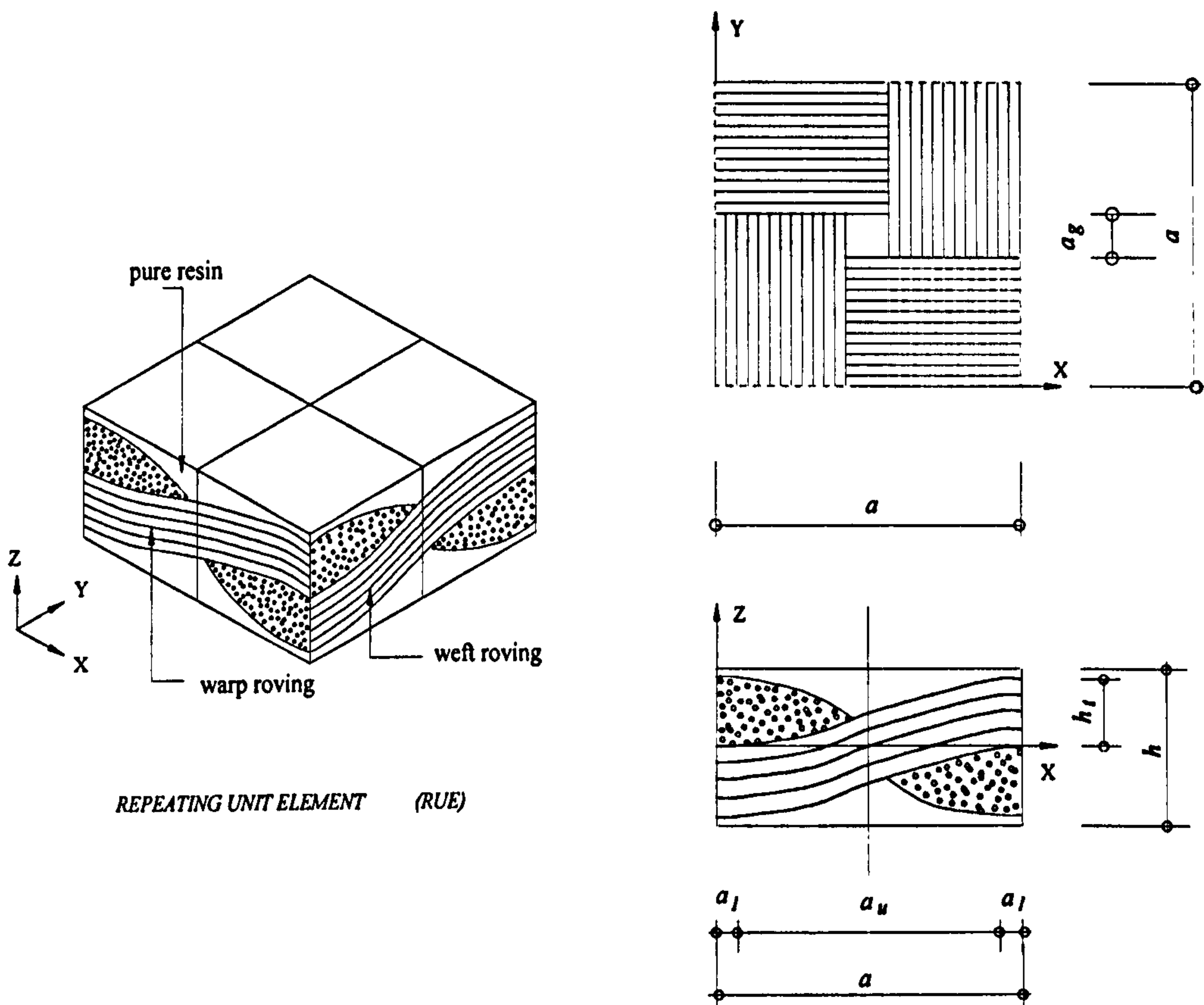


Figure 6.1 Geometry of the RUE of a plain woven composite.

The roving can consist of straight and undulating parts. The straight part is called the straight length, a_s ; and the undulating part has a length, a_u . The waviness ratio is then defined as the maximum thickness of impregnated roving (h_i) divided by a_u . Because two adjacent rovings may not attached each other, the gap between two roving is called the roving gap, a_g . The main geometrical parameters used to describe the architecture of the woven composite are summarised as follows (also see Figure 6.1).

Overall thickness of the RUE	h
Overall width of the RUE	a
Roving maximum thickness	h_i
Undulating length	a_u
Straight length	a_s
Roving gap	a_g

To implement the geometrical modelling of the woven lamina in the finite element method, the shape function of the roving needs to be defined beforehand. Figure 6.2 shows the shape of the rovings in one woven lamina taken off from the photomicrograph of laminate D in Figure 3.10.



Figure 6.2 Shape of the rovings taken from laminate D

Based on the photomicrographs of the cross-section of the fabric bundles, the shape function of the roving is assumed to be cosine function:

$$\begin{aligned}
 z &= 0 & 0 \leq x < a_1 \\
 z &= \frac{h_t}{2} \left[1 - \cos \frac{\pi(x - a_1)}{a - 2a_1} \right] & a_1 \leq x < a - a_1 \\
 z &= h_t & a - a_1 \leq x < a
 \end{aligned} \tag{6.3}$$

Depending on the location of the roving the exact expression may vary. In finite element modelling the control points of the shape of the roving were calculated from the expression above. In this research some assumptions are made to simplify the geometrical modelling as follows.

- (1) A single repeating unit element is modelled in the finite element analysis.
- (2) Warp and weft rovings perfectly conform to each other at all interlacing areas, forming a continuous contact surface. There is no pure resin wedge between warp and weft rovings.
- (3) The thickness of warp roving is the same as the thickness of weft roving.

6.3 Finite element modelling

The ABAQUS finite element package was used to model the plain woven composite and to construct the finite element meshes. The parameters used to describe the geometry of the composite were input first. In this way, it was very convenient to vary the geometrical modelling in the same model just by changing the parameters.

6.3.1 Introduction to ABAQUS on the IBM SP2 parallel processing computer

The IBM SP2 is a distributed memory parallel machine. It has a number of nodes (processors), each with its own memory and disk. The nodes are connected together by a 'High Performance Switch'. Parallel programs run on a number of different nodes that communicate with each other using a message passing library. The SP2 at this university has 23 nodes, 16 'thin1' nodes with 128 Mbyte RAM in each node; 6 'thin2' nodes with 256 Mbyte RAM in each node, 1 'wide' node with 256 Mbyte RAM. The 'wide' node acts as a job submission/development node. The remaining 22 nodes are used to run jobs submitted. ABAQUS standard (Version 5.6) is installed on the SP2.

ABAQUS [80] is a suite of powerful engineering simulation programs, based on the finite element method, that can solve linear and non-linear simulations. ABAQUS contains an extensive library of elements that can model virtually any geometry. It has an equally extensive list of material models that can simulate the behaviour of most typical engineering materials, such as metals, composites, and so on. In ABAQUS the most complicated problems can be modelled easily by combining various option blocks. For example, problems for the progressive analysis of composite structures can be modelled by assembling the geometry model block, the material defining block, the user-defined subroutine block (used to check stress level at each integration point), the geometrical non-linearity and the load increment control block, and the boundary condition block.

To create a geometrical model, the ABAQUS pre-processor 'Abapre' can be used. This is a limited version of PATRAN, which is a powerful professional pre-processor program. During the modelling, the pre-processor creates a *journey-file* to record each command input. Once the model is finished, the pre-processor generates a *input-file* which contains correct information about the modelling. Alternatively, when the problem is simple, the commands can be directly input into *input-file* to create the geometry of the problem.

The *input-file* can be edited and modified according to the problem described and ABAQUS code. For instance, one can modify material-defining block and change the



material properties of the problem without changing its geometry. The great advantages of ABAQUS are the various user-subroutines built into the program, which provide extremely powerful and flexible ways for analysis. One of the user-subroutine employed in this research is ' USDFLD ', which allows the user to define field variables at a material point as functions of the material properties. In this way the material properties can be controlled during the solution according to the damage of the material.

Once the *input-file* is ready, the job can be submitted to that main part of ABAQUS, which is the ABAQUS solver in the IBM SP2 parallel processing computer. This is done by indicating the number of the nodes required and the maximum time that the job will run for, for example:

```
spsub -abaqus -d 3@thin1 -t 600 abaqus j=jobname
```

The ' *-d 3@thin1* ' says that 3 'thin1' nodes are required; and ' *-t 600* ' specifies the maximum time that the job will run for, in this case 600 minutes. Then, the SP2 parallel computer will provide quick calculation for the problem.

Compared with ABAQUS/Pre, ABAQUS/Post is not very convenient to use. The post-processor in ABAQUS is command driven. The user has to type in every single command while the results are being interpreted. Some of the key commands are listed below:

SET, FILL=ON	required for contour rather than vector plot
VIEW, VIEWPOINT = (x,y,z)	changes of viewpoint
DRAW, DISPLACED	draw displaced shape
DETAIL, ELSET = name	select a element set
CONTOUR, VARIABLE = s_{ij}	contour-plot stress components, using with DETAIL command will plot stress in the specified element set
SET, HARDCOPY=ON	screen output is sent to the plot file

6.3.2 Solid model for the finite element analysis

To model a woven composite, the cross-section of the roving was input as a series of discrete points calculated from a shape function such as the expression (6.3). The maximum thickness and overall width of the roving were measured approximately from Figure 6.2. The length of the straight part of the roving and the roving gap were assumed to be zero in accordance with the microphotograph of laminate D. These parameters were input into the shape function of the expression (6.3), and nine control points were calculated. These control points were then fitted to Loft Spline curves in ABAQUS commands to create the outlines of the cross-section desired. The roving centre line was created in the same manner. Area Group commands were used to form the area of the cross-section. Then, the cross-section was slid along the centre line to develop a 'solid object' of a roving by using Solid/Glide commands. The procedures for creating additional rovings were the same.

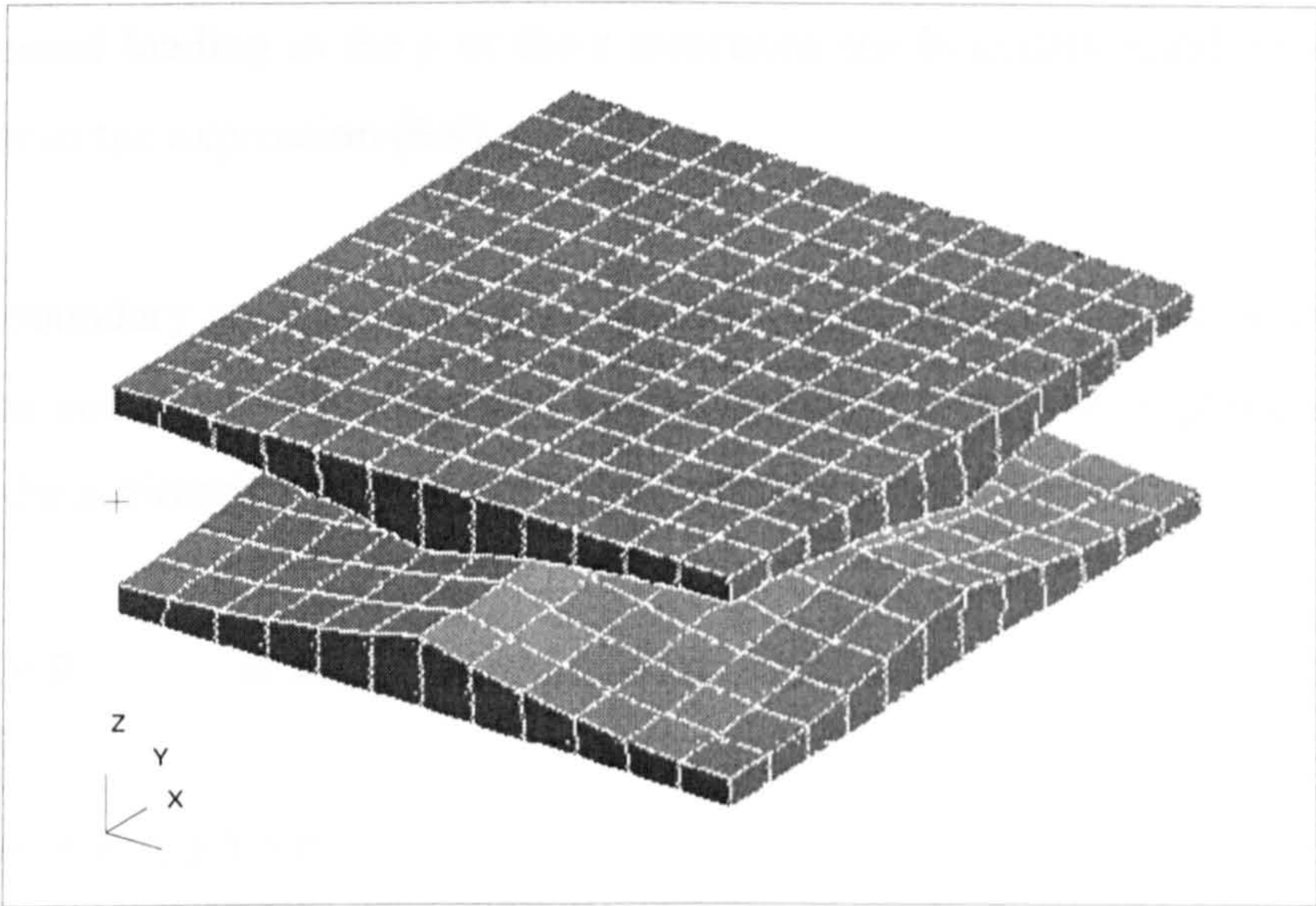
The outer boundaries of the RUE were pure resin regions. Boolean commands were used to subtract the roving solids from the outer block. The remaining objects represented the surrounding resin contained within the RUE. An alternative method to form the outer pure resin solid was to create outer boundary lines and areas, then to translate the areas to solid directly.

6.3.3 Finite element meshes

Once the solid model of the RUE was formed, the finite element meshes were set up. As this was a three-dimensional model, it was desirable to use as few elements as possible. Ideally, hexahedral elements should be used for this purpose. However, the shape at the roving tip was so sharp that the hexahedral elements were not suitable. Fortunately, the mesh generator in ABAQUS can execute mixed hexahedral and triangular prism element meshes through the roving solids automatically. All the element were checked by using the 'element check' command and were guaranteed to have good shapes and aspect ratios. The final element mesh contained about 576 elements (C3D6 and C3D8) in the RUE. Figure 6.3 shows the element meshes of the RUE.



(a)



(b)

Figure 6.3 Finite element meshes of the RUE
(a) the rovings; (b) the pure resin regions

6.3.4 Boundary conditions

The real woven composite may consist of only one layer (woven lamina) or several layers stacked together. Each of these layers is subjected to different boundary conditions. Therefore the response of a RUE to a given load depends on its position within the overall structure. Two extreme cases have been considered in this research. One of the boundary conditions was suitable for an RUE embedded in the inner layer of the composite. In this case, for in-plane extensional loading in the x -direction, the boundary conditions were described as:

$$u(0, y, z) = 0 \quad u(a, y, z) = \text{specified constant value}$$

$$v(x, 0, z) = v(x, a, z) = 0 \quad (6.4)$$

$$w(x, y, -h/2) = w(x, y, h/2) = 0$$

For extensional loading in the y or the z directions the boundary conditions would be very similar to the expression (6.4)

The other boundary conditions may be suitable for an RUE in woven lamina or outer layer of the composite with top and bottom surfaces free. For in-plane extensional loading in the x -direction, these boundary conditions were described as:

$$u(0, y, z) = 0 \quad u(a, y, z) = \text{specified constant value}$$

$$v(x, 0, z) = v(x, a, z) = 0 \quad (6.5)$$

6.4 Stress distributions and concentrations

The finite element analysis of the RUE was used to investigate the stress distributions and concentrations inside the composite. In the *basic model* the RUE was subjected to an axial displacement in the " 1 " direction (warp direction) as illustrated in the following figures, with the boundary conditions of expression (6.4). The material prototypes were woven composites of laminates A and D, which have similar reinforcement but different fibre contents by volume. The geometrical parameters were estimated from the SEM microphotographs of laminate D shown in Figure 6.2. The material properties of the roving fabric were calculated by using the theoretical model described in Chapter 5. The fibre content of the roving S_f was obtained from the expression (6.1). The material properties and geometrical parameters used in the *basic model* are given in Table 6.1.

Table 6.1. Material properties and geometrical parameters used in the basic model

Property	Roving Fabric	Pure resin
E_x (GPa)	60.6	3.5
$E_y = E_z$ (GPa)	20.2	3.5
$G_{xy} = G_{xz}$ (GPa)	7.38	2.1
G_{yz} (GPa)	7.76	2.1
$\nu_{xy} = \nu_{xz}$	0.3	0.3
ν_{yz}	0.3	0.3
Geometrical parameters		
roving gap a_g	0	
straight length a_l	0	
waviness ratio (h_l / a_u)	0.125 (1/8)	
fibre content of the roving	0.858	

Figures 6.4a-6.4f show the contour-plotted stress distributions and concentrations inside the warp and weft rovings and the pure resin. To study the concentration of stress, the term *stress variation factor* (SVF) is defined as:

$$SVF = \text{Maximum stress} / \text{Median stress}$$

in which

$$\text{Median stress} = (\text{Maximum Stress} + \text{Minimum Stress}) / 2$$

Figure 6.4a provides an overall view of an RUE and a typical stress distribution diagram. Figure 6.4b shows the distribution of the stress *along* the fibres in the *warp* rovings. Stress concentrations occur at the interlacing areas. The peak stress is about 60 percent higher than lowest stress. The SVF in the warp rovings is about 1.23. The results indicate that (a) the failure of the warp rovings in longitudinal tension would start at the interlacing areas; (b) due to the stress concentrations, fibres in a woven composite may fail earlier than those in a unidirectional composite. As the failure of the roving represents the ultimate failure of the composite, this information becomes important in design of a woven composite structure.

Figure 6.4c shows the distribution of the stress *along* the fibres in the *weft* rovings. The maximum value of the longitudinal stress in the weft rovings is about 14% of the longitudinal stress in the warp rovings. From this one may conclude that failure is unlikely to happen in the fibre direction of the weft roving if in-plane extension is applied in the warp direction, although there are the stress concentrations in the weft rovings.

Figure 6.4d demonstrates the distribution of the stress *across* the fibres in the *weft* rovings. Stress concentrations are at the tip of the weft roving in the interlacing areas. The peak transverse stress is about 70 percent higher than lowest stress. The SVF in the weft rovings is about 1.27. As the failure strength of composite in transverse direction is smaller than that in longitudinal direction in tension, the transverse cracks may occur

in the tip of the weft rovings because of the intensive transverse stress. Transverse failure (cracks) in the weft rovings does not necessarily mean that the composite cannot carry load. The transverse cracks will result in the stiffness degradation of the composite.

Figure 6.4e shows the distribution of the stress *across* the fibres in the *warp* rovings. The maximum value of the transverse stress in the warp rovings is about 13% of the transverse stress in the weft rovings. Therefore, failure may occur later in the transverse direction of the warp rovings than it does in the weft rovings.

Figure 6.4f shows the distribution of the stress in the pure resin regions (isotropic body). The stress is concentrated at the interlacing areas. The peak stress is about 60 percent higher than lowest stress. The SVF in the pure resin regions is about 1.22. This confirms that the undulation induces the local stress concentrations, and that the pure resin cracks around the interlacing areas first.

The same analyses can be carried out for the shear stress distributions of the composite. However, the critical information is the stress concentrations in the longitudinal stress in warp rovings, transverse stress in weft rovings and the principal stress in the pure resin regions. For this reason, only these three stresses are studied in the following sections.

6.5 Parametric studies

The effects of various parameters on the SVF were studied. These parameters include the waviness ratio, the roving gap, different fibre contents of the roving, different materials, boundary conditions and mesh refinement.

6.5.1 The effect of waviness ratio

Three waviness ratios were studied. One was zero. This represented a laminate which consists of four unidirectional layers: top and bottom pure resin sheets, two orthogonal

unidirectional flat rovings sandwiched between pure resin sheets. In fact, this was the theoretical model described in Chapter 5. The other waviness ratios were 0.125 (*basic model*) and 0.3. The material properties for three cases were the same as the *basic model* in Table 6.1. Figures 6.5a-6.5d show the stress distributions for the case of the waviness ratio = 0.3, in which the stress distributions in the longitudinal direction in the warp rovings, the transverse direction in the weft roving and the pure resin are very similar to the *basic model* (waviness ratio = 0.125). Table 6.2 lists the effect of the waviness ratio on the SVF.

The effect of waviness ratio on the SVF is significant. The bigger the waviness ratio, the bigger the SVF. From this results one can imagine that if the strengths of the roving fabric were the same in the three cases, under the same loading condition the strength loss for the case of waviness ratio = 0.3 would be higher than 50% compared with the case of waviness ratio = 0. This result indicates that the waviness ratio of roving fabric is so critical that must be controlled in the woven composite structures.

Table 6.2 The effect of waviness ratio on the SVF

Waviness ratio	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
0	1	1	1
0.125 (<i>basic model</i>)	1.23	1.27	1.22
0.3	1.55	1.38	1.5

6.5.2 The effect of roving gap

A woven composite with the roving gap $a_g / a = 0.1$ was analysed. The material properties and waviness ratio were the same as the *basic model* in Table 6.1. The results were compared with these from the *basic model* without the roving gap. Table 6.3 lists the effect of the roving gap on the SVF. The comparison shows that small roving gap has no significant effect on the SVF.

Table 6.3 The effect of roving gap on the SVF

Roving gap a_g/a	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
0 (<i>basic model</i>)	1.23	1.27	1.22
0.1	1.26	1.28	1.29

6.5.3 The effect of different fibre contents of the roving

A woven composite with the same fibre and resin, but with a smaller fibre content of the roving than the *basic model* was analysed. With a smaller fibre content of the roving, the stiffnesses of the rovings are smaller as well. Table 6.4 lists the material properties of the roving with fibre contents of the roving 0.7 and 0.858 (*basic model*). Table 6.5 lists the effect of different fibre contents of the roving on the SVF. The results show that the smaller the stiffness of the roving, the less the SVF in the warp and weft rovings but the greater the SVF in the pure resin. This is probably because pure resin carries more load in the composite with a less stiff roving than in the composite with a stiffer roving. However, the SVF is not greatly affected by changes of the fibre contents.

Table 6.4 Material properties of roving with different fibre contents of the roving

Property	Fibre content of the roving	
	0.7	0.858
E_x (GPa)	50	60.6
$E_y = E_z$ (GPa)	11.2	20.2
$G_{xy} = G_{xz}$ (GPa)	4.0	7.38
G_{yz} (GPa)	4.3	7.76
$\nu_{xy} = \nu_{xz}$	0.3	0.3
ν_{yz}	0.3	0.3

Table 6.5 The effect of different fibre contents of the roving on the SVF

Fibre content of the roving	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
0.7	1.22	1.20	1.26
0.858 (<i>basic model</i>)	1.23	1.27	1.22

6.5.4 The effect of different materials

A woven composite with Carbon/Epoxy material (fibre content of the roving 0.7 and waviness ratio 0.125) was analysed. The stiffness of a Carbon/Epoxy roving is much greater in the fibre direction than in the transverse direction, as shown in Table 6.6. Table 6.7 shows that there is a significant increase of the SVF in the fibre direction, compared with E-glass/Polyester material with the same geometry. This indicates that the stress concentration is more important in a Carbon/Epoxy than in an E-glass/Polyester composites.

Table 6.6 Material properties of Carbon/Epoxy material [53]

Property	Roving Fabric	Pure resin
E_x (GPa)	151	4.4
$E_y = E_z$ (GPa)	10.1	4.4
$G_{xy} = G_{xz}$ (GPa)	5.7	1.7
G_{yz} (GPa)	3.4	1.7
$\nu_{xy} = \nu_{xz}$	0.24	0.34
ν_{yz}	0.5	0.34

Table 6.7 The effect of different materials on the SVF

Material	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
E-glass/Polyester	1.22	1.20	1.26
Carbon/Epoxy	1.49	1.24	1.36

6.5.5 The effect of boundary conditions

The *basic model* was also calculated under the boundary conditions in the expression (6.5). The difference between the expressions (6.4) and (6.5) is the restriction at the top and bottom surfaces. In the expression (6.5) the top and bottom surfaces are free. The stress variation factors for the two boundary conditions are listed in Table 6.8. The SVF in the pure resin for the boundary conditions (6.5) is increased significantly. This indicates that the pure resin near the top and bottom free surfaces of the composite may crack earlier than that inside the composite.

Table 6.8 The effect of different boundary conditions on the SVF

Boundary Conditions	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
Expression (6.4)	1.23	1.27	1.22
Expression (6.5)	1.30	1.29	1.57

6.5.6 The effect of mesh refinement

The element mesh in the *basic model* was refined from 576 to 2304 elements. The sensitivity of the SVF to the mesh refinement was studied. The results in Table 6.9

show that the stress variation factors in rovings and pure resin for 2304 elements are increased slightly, compared with these for 576 elements. The SVF is not very sensitive to the mesh refinement in this study.

Table 6.9 The effect of different mesh refinements on the SVF

Element number	SVF		
	Longitudinal stress in warp	Transverse stress in weft	Principal stress in pure resin
576 (<i>basic model</i>)	1.23	1.27	1.22
2304	1.28	1.32	1.26

6.6 Conclusions

From the analyses some important conclusions can be drawn.

(1) Due to interlacing and waviness of the roving, there are significant stress concentrations inside the rovings and the pure resin regions. Figures 6.4a-6.4f provide clear pictures of these stress distributions and concentrations. The stress concentrations will result in resin cracking, transverse damage and fibre breakage in a woven composite earlier than in a unidirectional laminate made of the same materials. In other words, a woven composite will be weaker than an equivalent non-woven composite. To measure quantitatively the weakness of the material because of the stress concentration, the stress variation factor (SVF) is defined in this research. This idea will be used in progressive damage analysis of woven composite panels in next chapter.

(2) The architecture of the woven composite has various effects on the stress concentration. The results show that the waviness ratio has a significant effect on the stress concentrations. The larger the waviness ratio, the greater the SVF. Therefore the waviness should be controlled in the manufacture of woven composites. The results

also show that a small roving gap (0.1 in the example) does not have a large influence on the stress concentrations.

(3) The material construction also has effect on the stress concentration. A carbon fibre composite has higher stiffness in the fibre direction than in the transverse direction. The results show that the undulation of a woven composite made of carbon fibres causes higher stress concentrations in the fibre direction than a geometrically similar composite made of E-glass fabrics. Special attention therefore needs to be paid to the stress concentration in a carbon fabric composite. The results also show that the different fibre contents of the roving do not have a large influence on the stress concentrations (in the example, the same material was examined with fibre contents of the roving 0.7 and 0.858).

(5) A woven composite with top and bottom surfaces free has higher stress concentrations in the pure resin regions than a composite with top and bottom restricted. This suggests that the outer pure resin may crack before an inner layer of pure resin.

ABAQUS

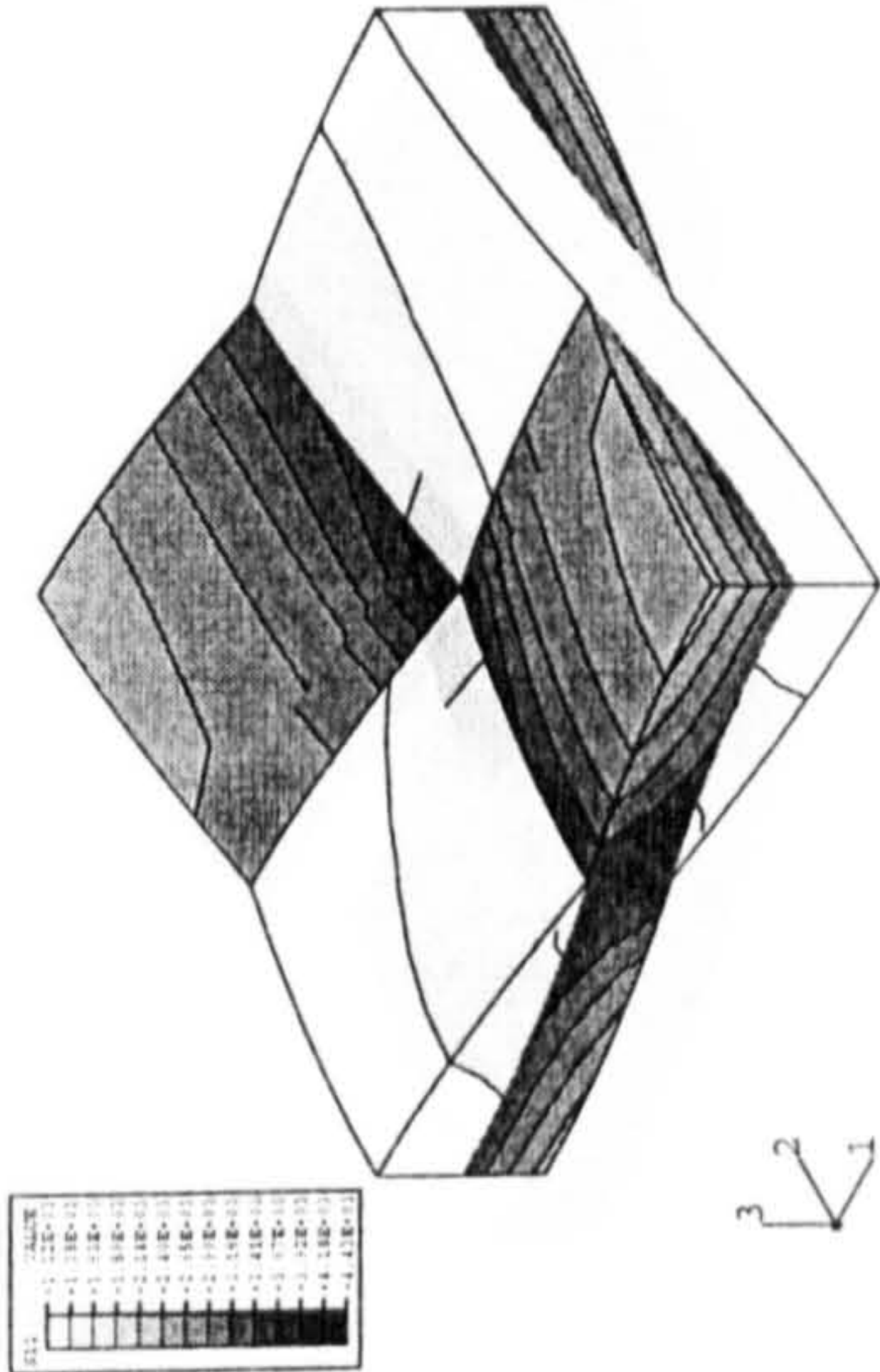


Figure 6.4a Contour plotted stress distribution for woven fabrics

ABAQUS

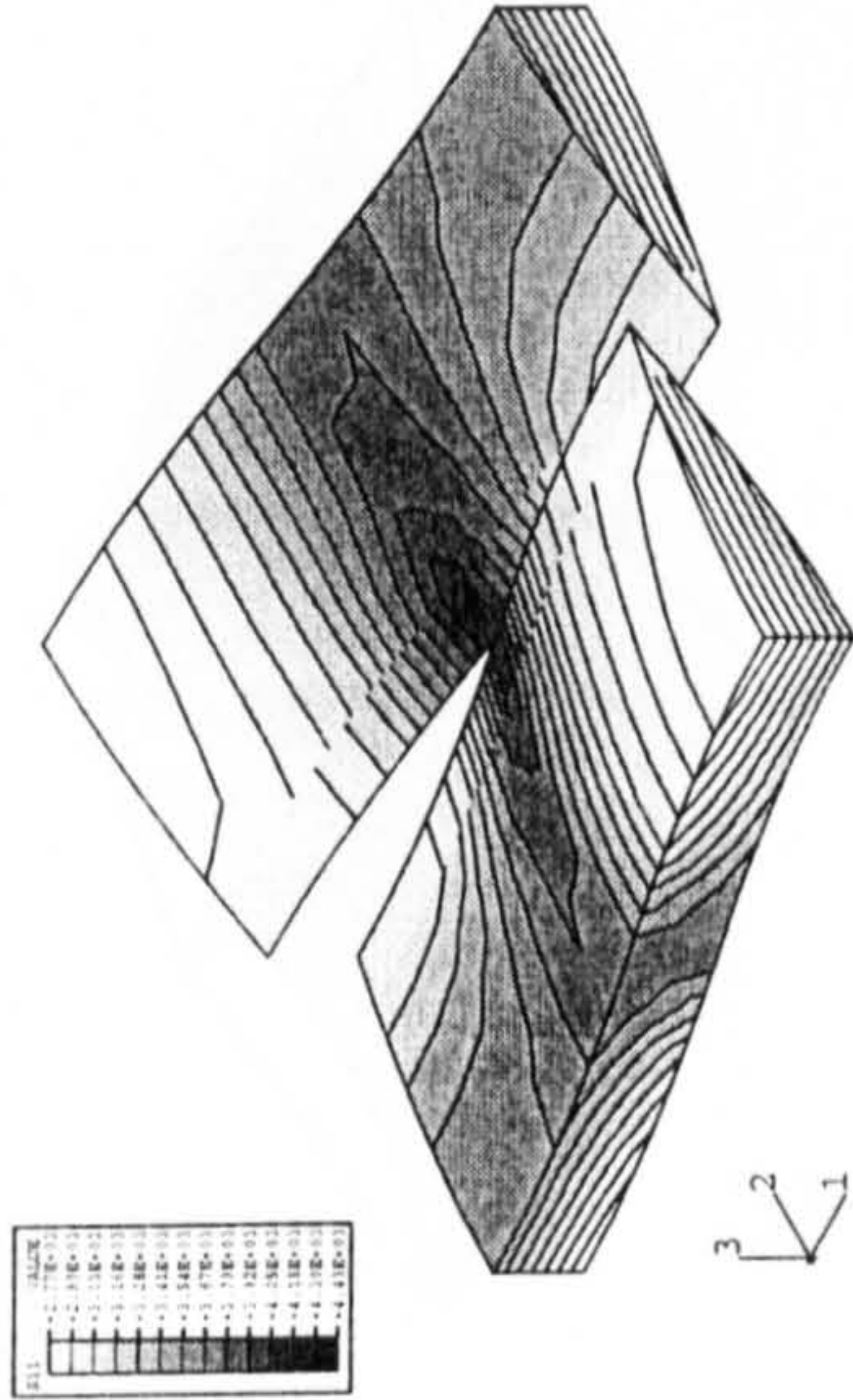


Figure 6.4b Longitudinal stress (s_{11}) in warp rovings

ABAQUS

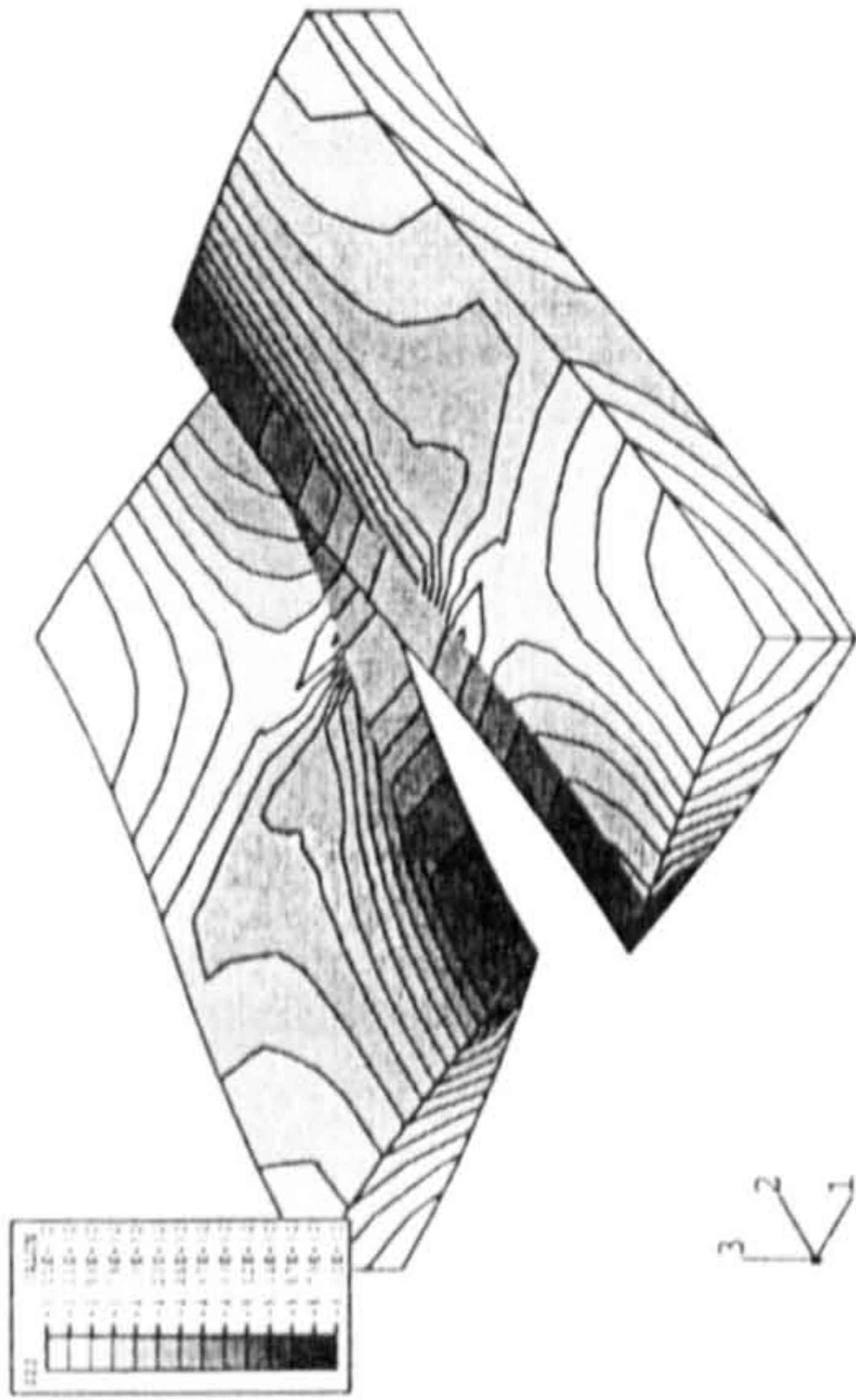


Figure 6.4c Longitudinal stress (s_{22}) in weft rovings

ABAQUS

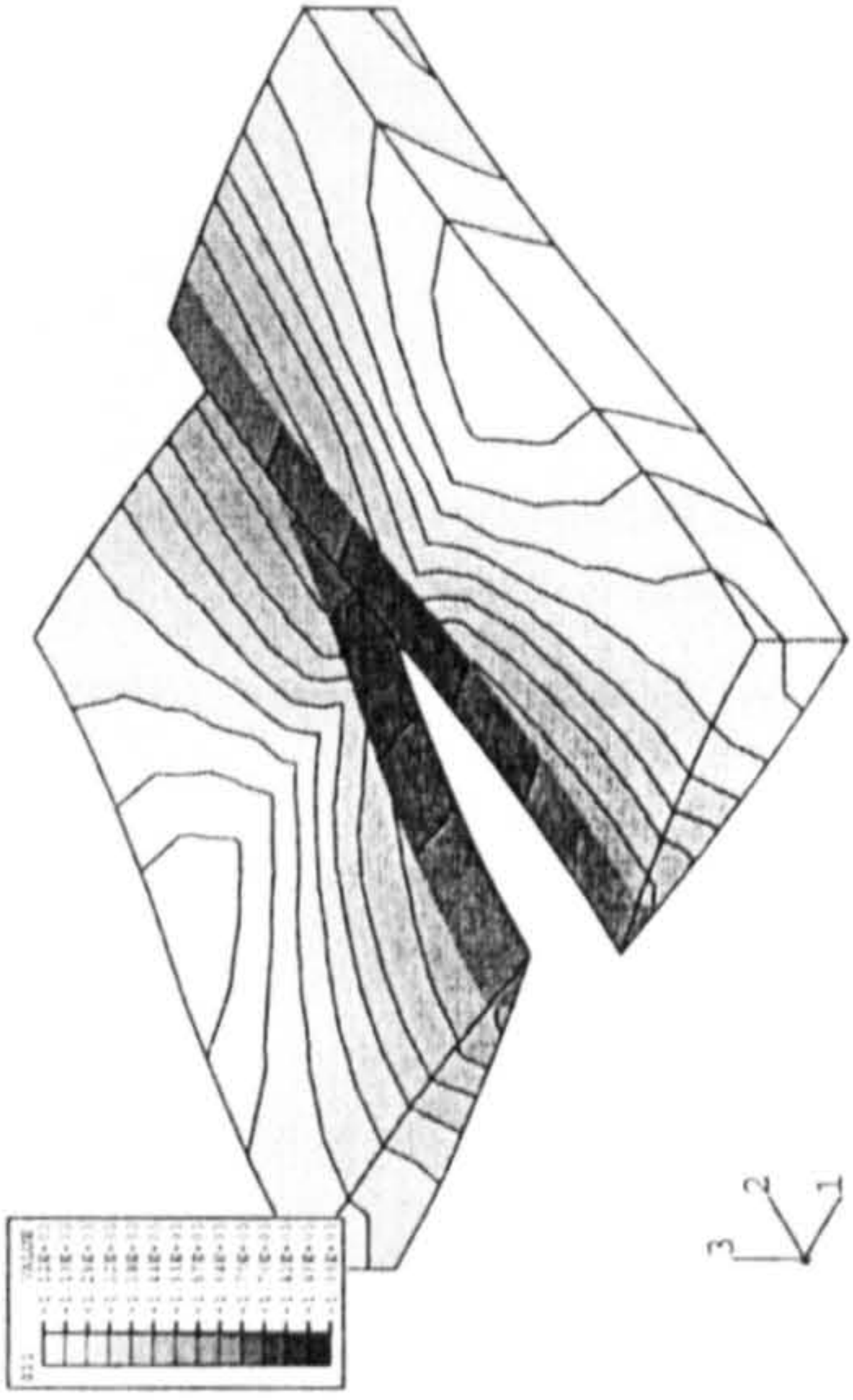


Figure 6.4d Transverse stress (s_{11}) in weft rovings

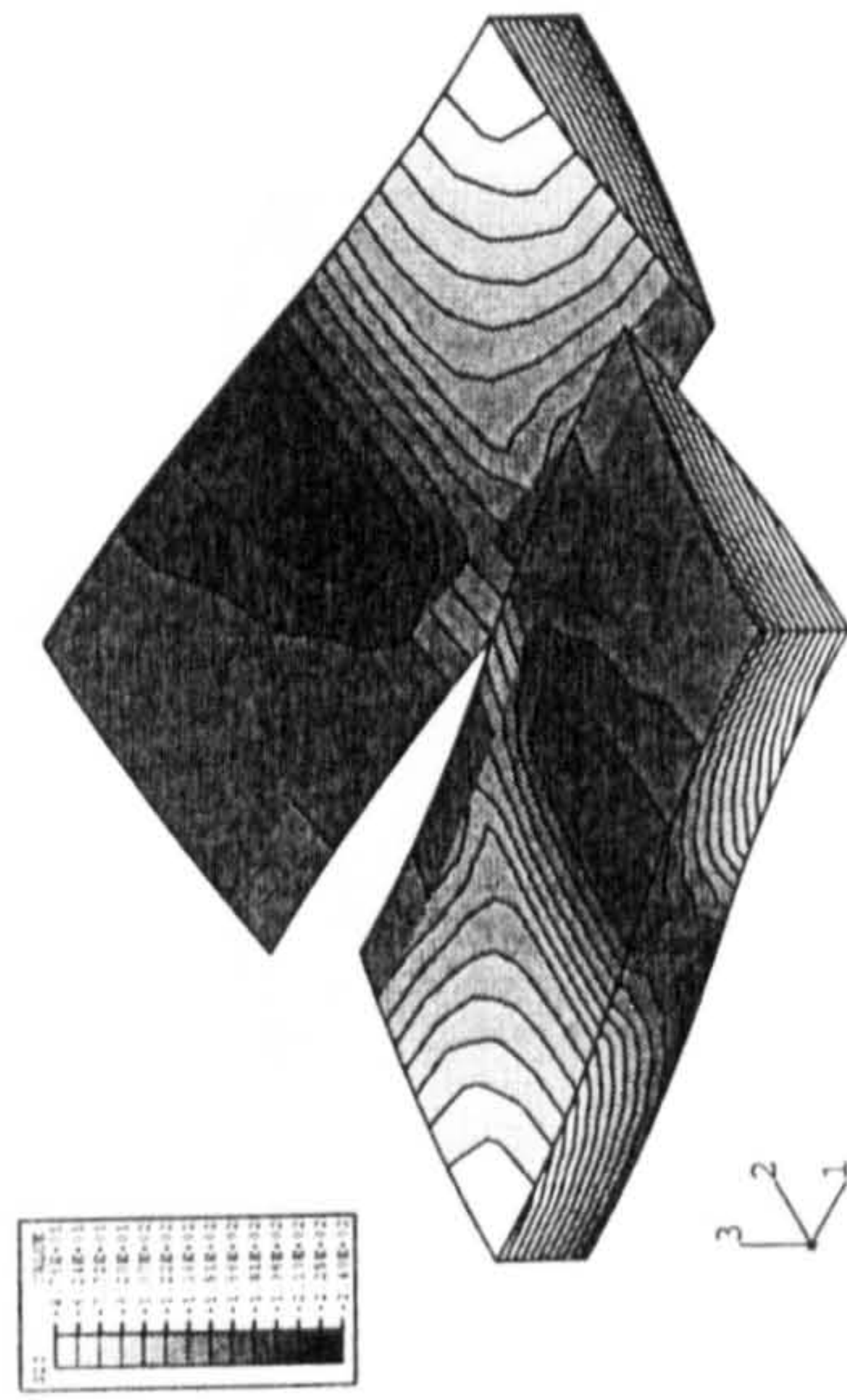


Figure 6.4e Transverse stress (s_{22}) in warp rovings

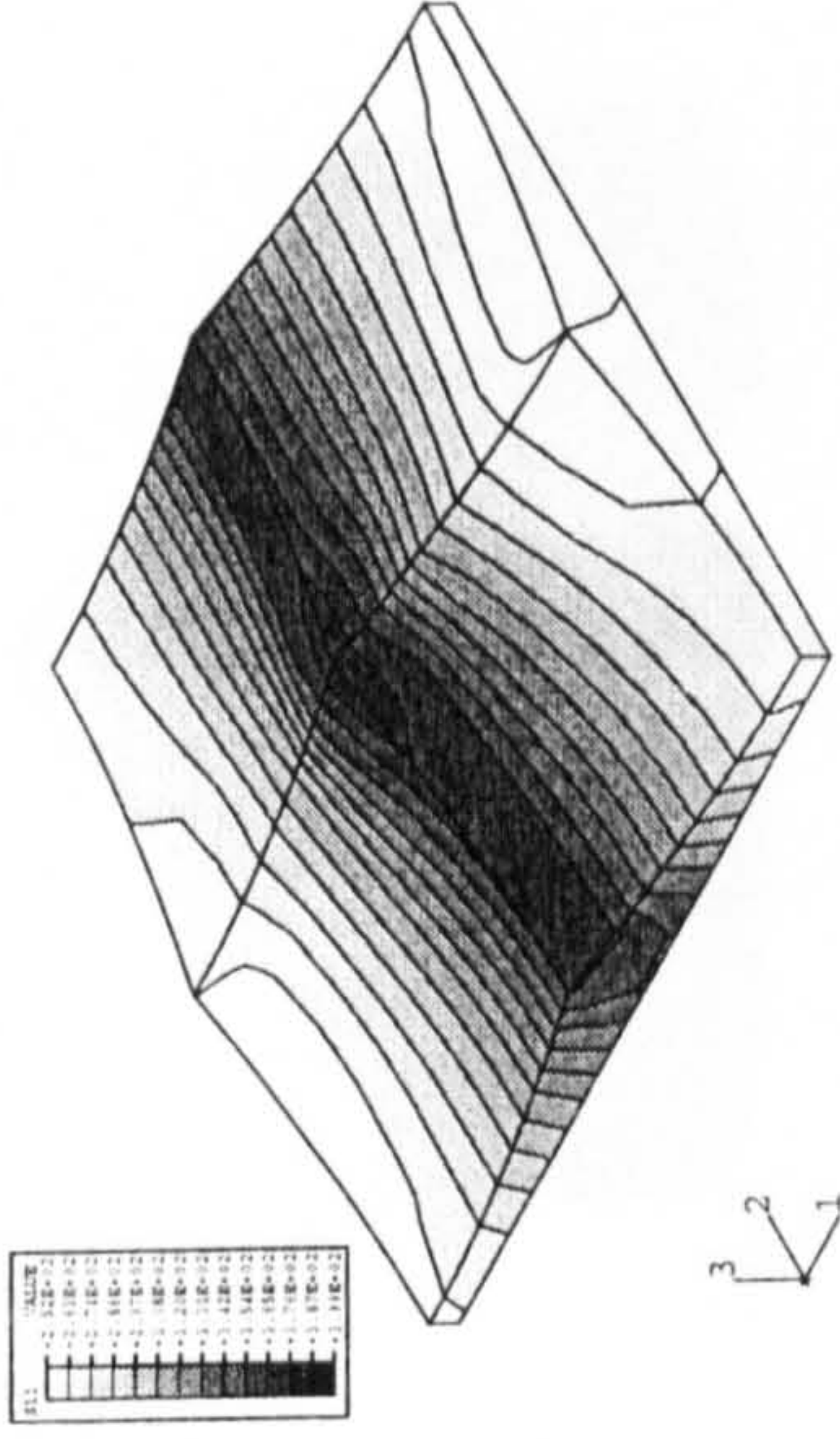


Figure 6.4f Stress (s_{11}) in pure resin region

ABAQUS

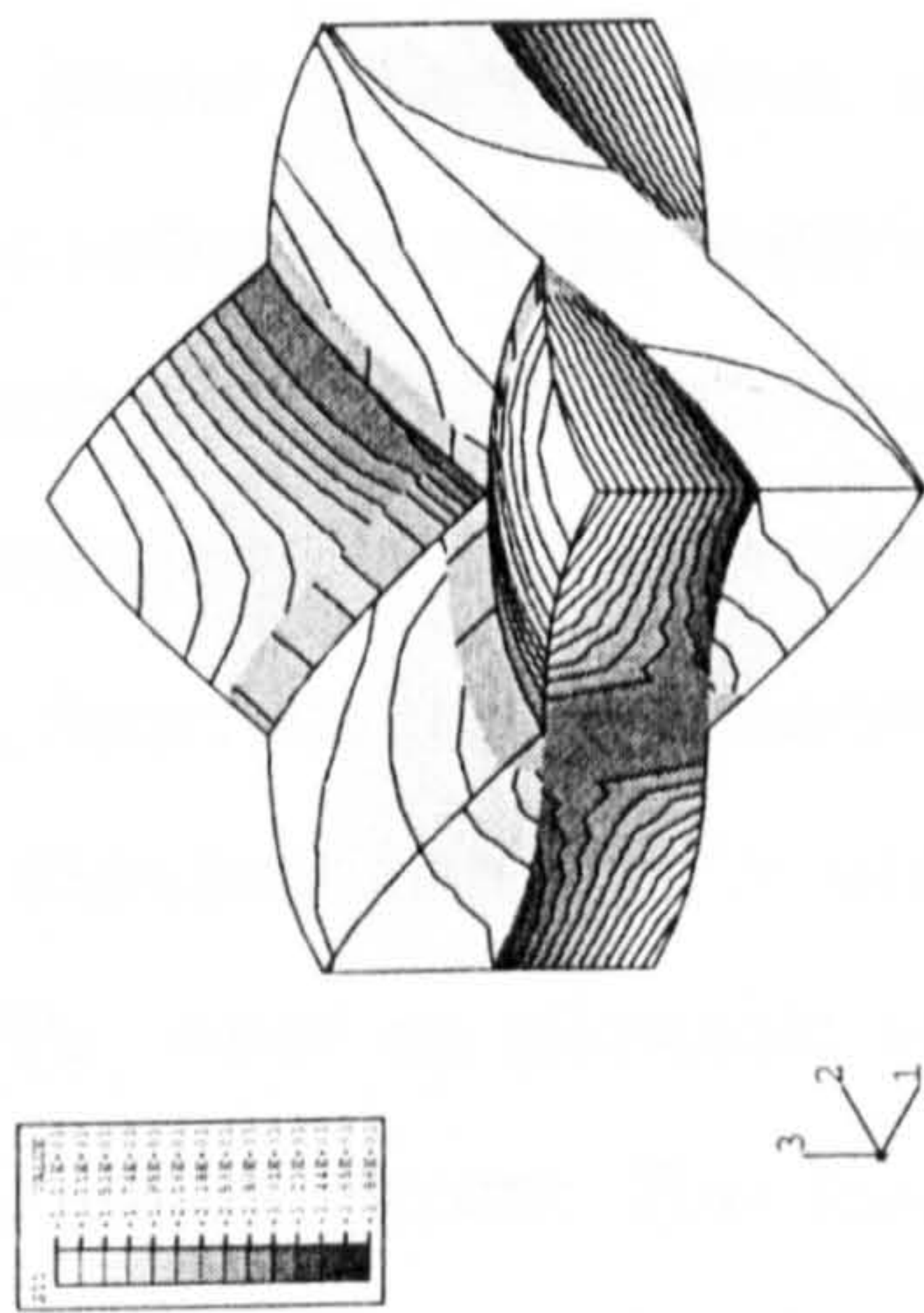


Figure 6.5a Contour plotted stress distribution for woven fabrics

ABAQUS

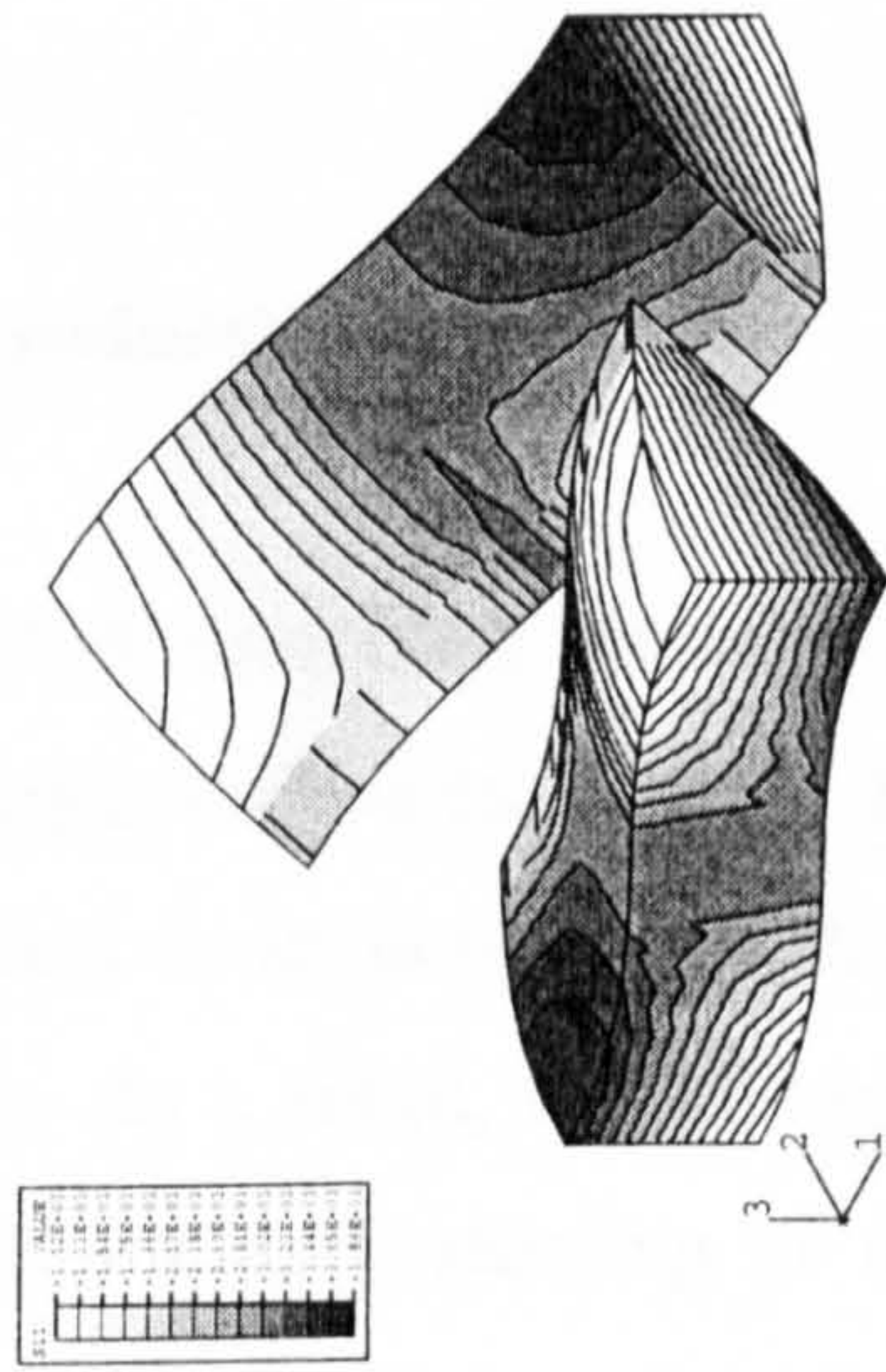


Figure 6.5b Longitudinal stress (s_{11}) in warp rovings

ABAQUS

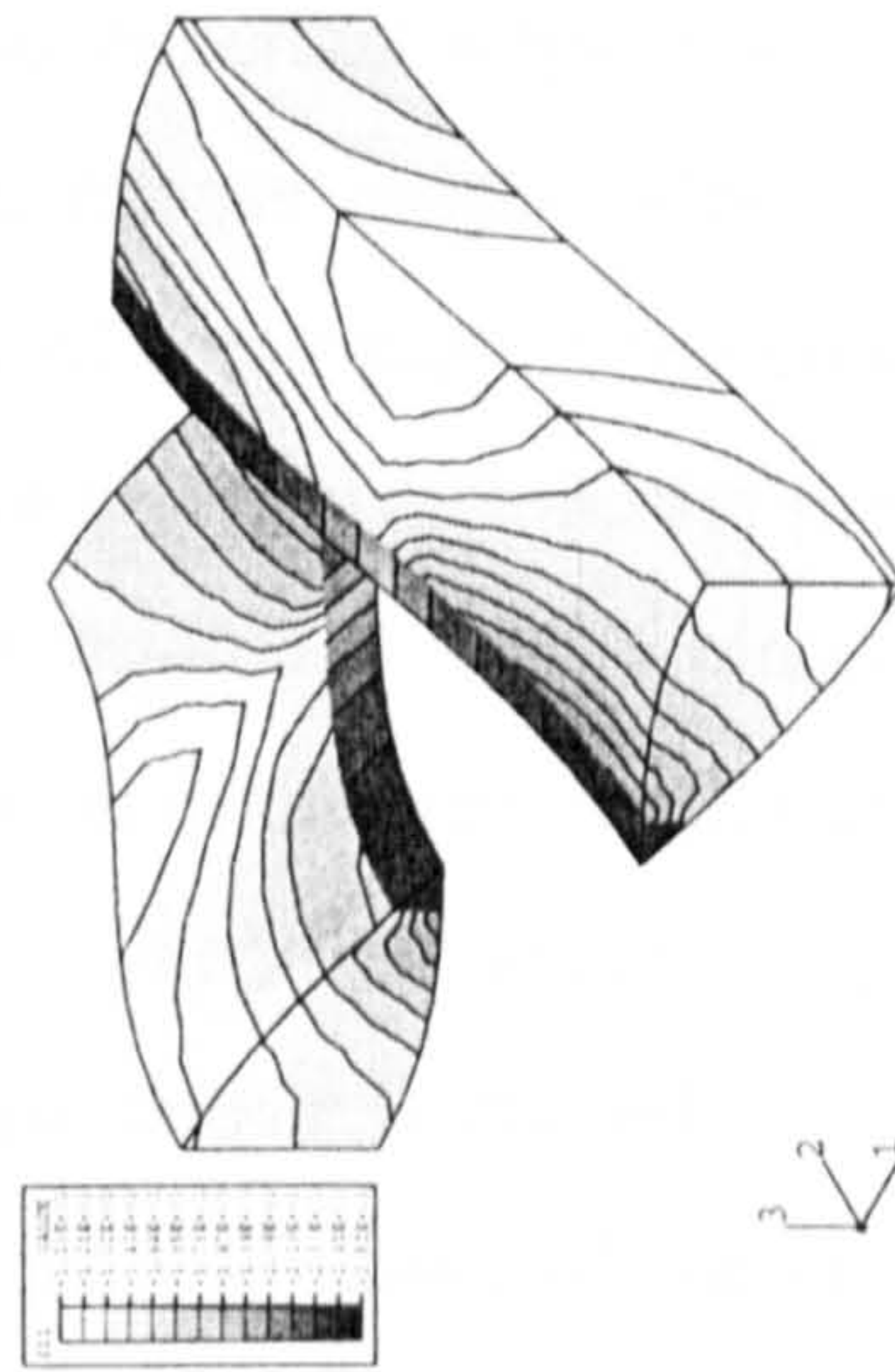


Figure 6.5c Transverse stress (s_{22}) in weft rovings

ABAQUS

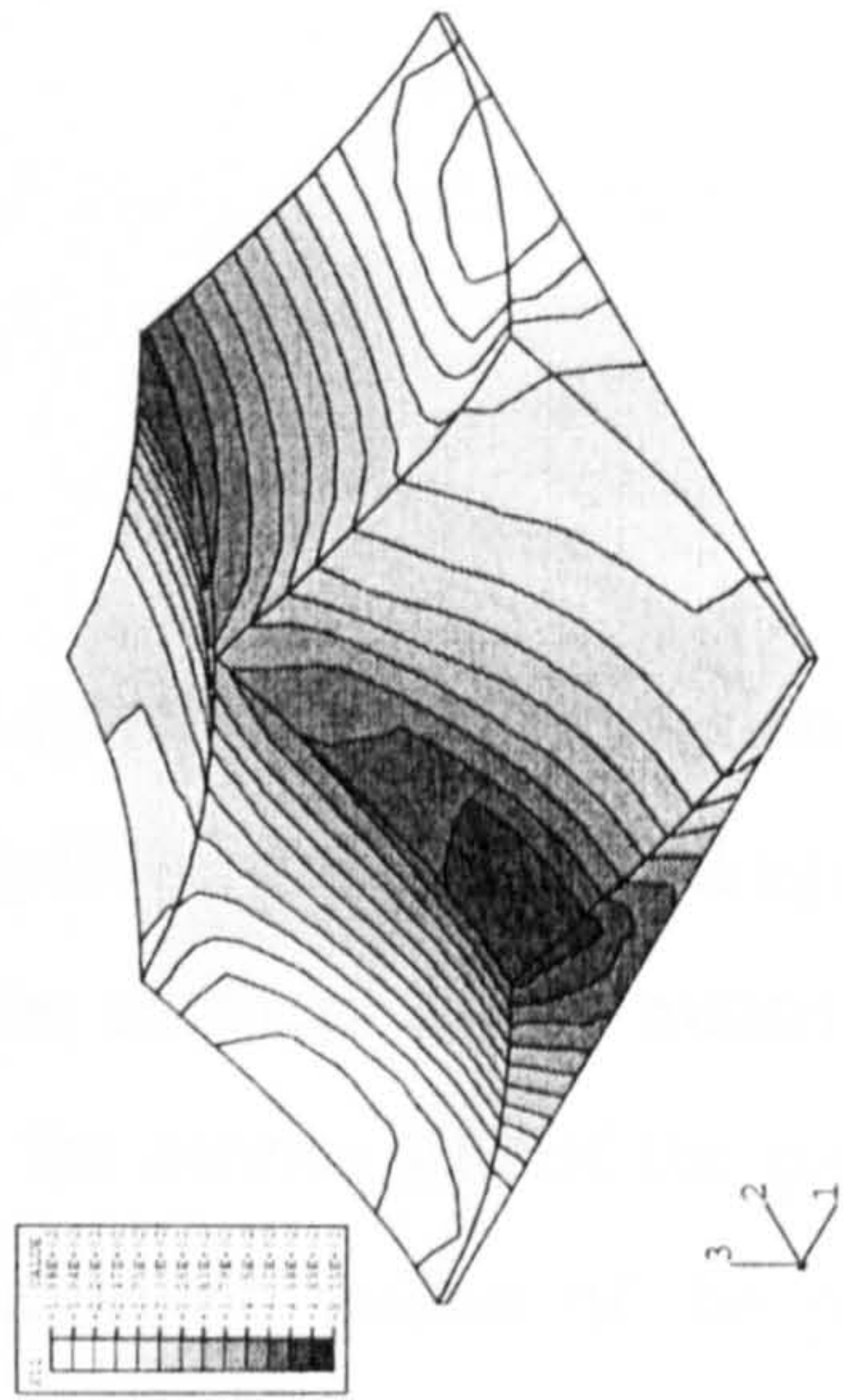


Figure 6.5d Stress (s_{11}) in pure resin region

Chapter 7 Progressive failure analysis of thin composite panels under high transverse pressure

7.1 Introduction

Under high transverse pressure, laminated composite panels may suffer various kinds of damage, such as cracking of the matrix, de-bonding (fibre pull-out), breakage of the fibres and de-lamination. As damage accumulates, the stiffness of the material may degrade. As a result, the load-carrying capacity and the service life of the panel are reduced. An understanding of the response and progressive failure of the panel is necessary for the design of a safe structure.

In the last decade, considerable progress has been made in understanding the initiation and evolution of various damage modes for a laminated composite consisting of stacks of unidirectional laminae [59-69], as reviewed in Chapter 2. These researches have achieved better understanding of the progressive failure for this kind of laminated composite. In progressive failure analysis there are two important steps. The first is to choose proper failure criteria; and the second is to choose proper allowances for stiffness reduction of the material after initial failure of some part of the composite. The commonly used failure criteria are the maximum stress criterion, the maximum strain criterion, the Hoffman criterion, the Tsai-Wu criterion and the Hashin criterion. Among these, the Hashin criterion allows for a succession of different modes of failure and is therefore particularly suitable for a fibre reinforced composite [64]. In the composite, when an allowable value of a particular parameter is exceeded, a certain type of failure occurs. This leads to a reduction of the stiffnesses of the composite according to a particular degradation method. The commonly used degradation methods are the total discount method, the limited discount method and the residual property method. The effects of damage are accounted for by reducing the stiffnesses of the materials by a certain amount. The stiffness reduction is usually carried out

within a layer and within a single finite element at selected integration points. However, the application of progressive failure analysis of this kind to composite panels with woven fabric reinforcement is rarely reported.

In Chapter 4, test results from more than 25 composite panels (woven and non-woven) under high transverse pressure were summarised. The test results can be used to calibrate an analytical approach to the progressive development of damage in the panels. This chapter introduces a progressive failure approach by means of the finite element method which can be used to predict the response and progressive damage sequence of the panel as a whole. The objective is to provide a methodology for progressive failure analysis of a woven composite panel. This will be referred to as the Progressive Damage Model (PDM). This method incorporates the results from micro-structural analysis of a woven composite unit (Chapter 6) into the macro analysis of the composite panel. The effect of stress concentrations due to interlacing and undulation of the woven fabrics on the strength of the material was considered. The model is especially adapted to allow for in-plane edge displacements, which were observed during the tests and which were found to have a major effect on the performance of the panels.

Three panels were chosen for analysis:

- (a) Panel D: square panel with square satin weave and high resin content, made from material D.
- (b) Panel 30A: square panel with the same reinforcement as Panel D but lower resin content, made from material A.
- (c) Panel 18B: 'non-woven' reinforcement at 0, 45, 90, -45, 0 degrees, made from material B, aspect ratio 1:2.

Of these, Panel D had by far the most comprehensive instrumentation as described in Chapter 4.

7.2 Prediction of the behaviour of Panel D at low pressure and adjustment of the test results

As described in Chapter 4, a water manometer was connected with the loading system (Figure 4.14); this allowed for the test to be controlled more accurately at low pressure (up to 0.3 bar). At these low pressures no damage of the panel and *no* in-plane edge displacement were observed. As Panel D has a symmetrical arrangement of the fibres, at low pressure the panel can be therefore treated as an *overall elastic orthotropic* panel with *fully fixed* boundary conditions.

Sophisticated analytical and numerical methods already exist for the analysis of such a panel. Here, the finite element method (FE) in ANSYS is used to model the behaviour of the panel (the detailed ANSYS finite element package will be described in Chapter 8). Element type Shell63 was used and the geometrical non-linear option (large deflection) was switched on. The overall elastic stiffnesses of the panel were calculated from the theoretical model developed in Chapter 5, which agreed well with the coupon test results; they are listed in Table 7.1. Using the symmetry of the panel, a quarter of the panel was modelled. No edge displacement was considered in this analysis.

Table 7.1 Overall elastic stiffnesses of Panel D

	Elastic Stiffness	Values
Panel D	$E_x \text{ KN/mm}^2$	18.35
	$E_y \text{ KN/mm}^2$	17.80
	$G \text{ KN/mm}^2$	3.37
	ν_{xy}	0.15

Figure 7.1 shows the central deflection-pressure curves from the prediction and test. The curve predicted from the FE analysis is apparently different from the curve measured from the test. It is noted that the predicted curve is similar in shape to the tested curve, but with a shift along deflection axis.

Detailed study showed that the design of the pressurization system was such that a head of 1.5 ft of water (about 0.04 bar) was applied to the panel when the water manometer read zero. This can be seen in Figure 4.14. The panel was therefore pre-loaded. Moreover, as the panel was thin, the displacement was extremely sensitive to small changes of pressure near the origin of the graph. Consequently the horizontal position of the zero point on the test curve was not clearly defined.

In order to make a meaningful comparison between the graphs from the tests and from the FE analysis, the test curve was shifted horizontally to match the FE curve at a pressure of 0.2 bar. This pressure was selected because it was well within the low pressure range (within which the test results were repeatable and the loading and unloading curves overlapped), yet it was well above the pressures at which the uncertainties about the horizontal axis occurred. Figure 7.1 shows that, with the adjustments, the test curve corresponds closely to the FE predictions.

The effect of these adjustments on the strains is also very clear. Figures 7.2 and 7.3 show the differences in the central and the edge membrane strains between the FE analysis and the test. However, good agreement can be achieved if tested curves are shifted along strain axis to match the predictions at 0.2 bar.

Hence, all the test results of deflections and strains are adjusted to match the predicted results from the finite element analysis at low pressure of 0.2 bar. Then, the shifted results are considered as ' tested results ' to be used in comparisons in the following sections.

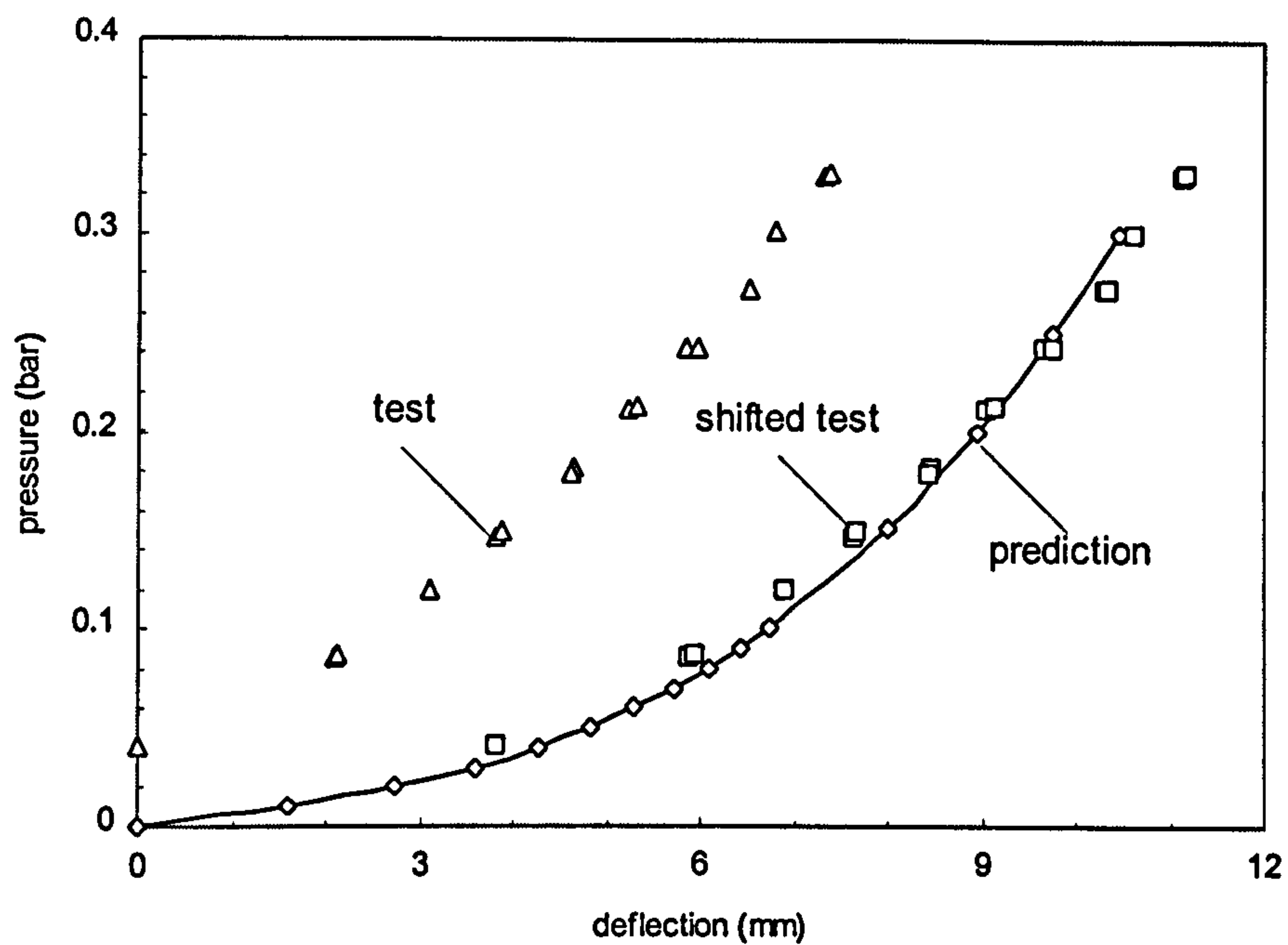


Figure 7.1 Central deflection (T2) vs. pressure at low pressure

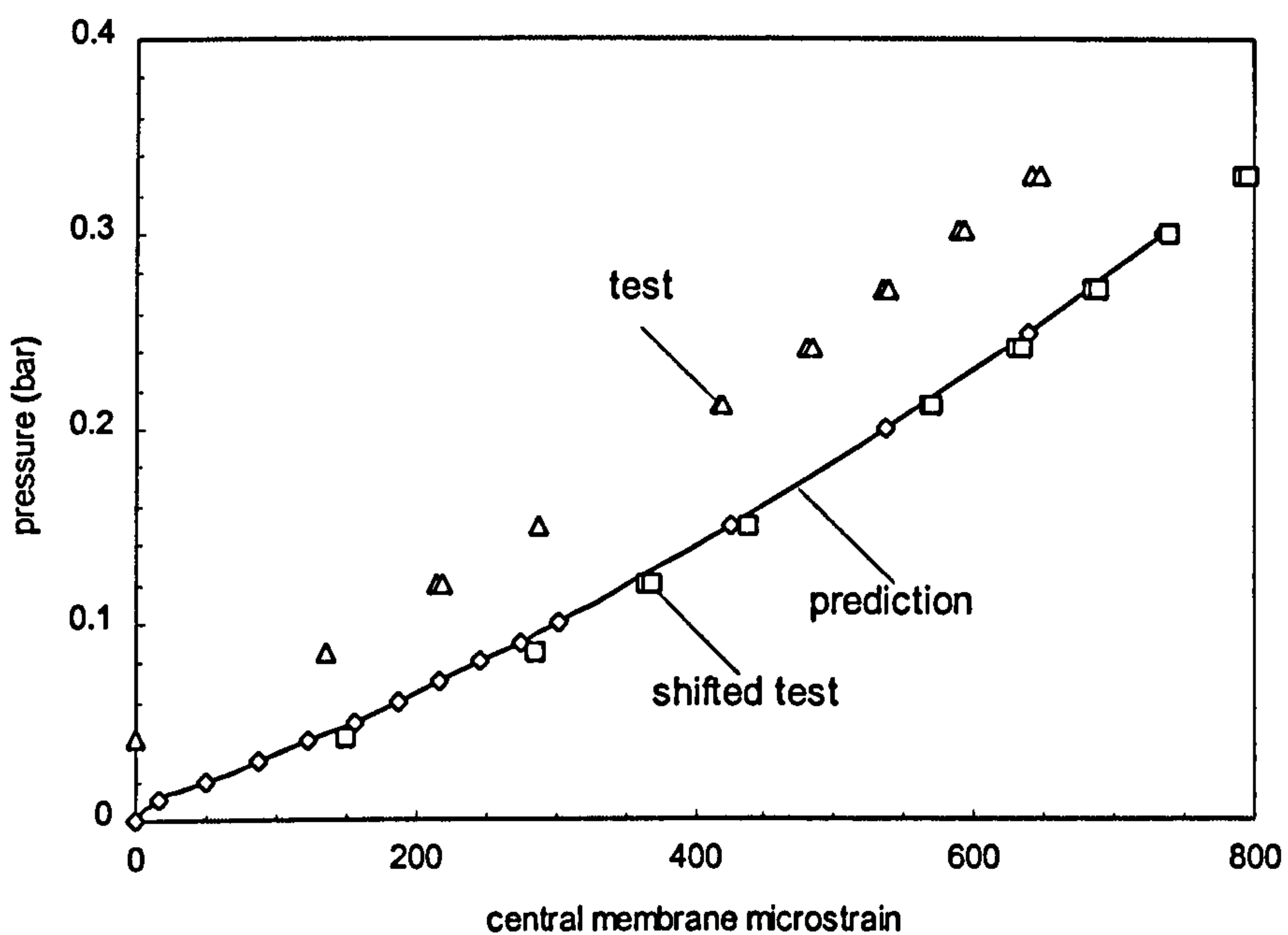


Figure 7.2 Central membrane strain (SG7/SG8) vs. pressure at low pressure
(See Figure 4.12)

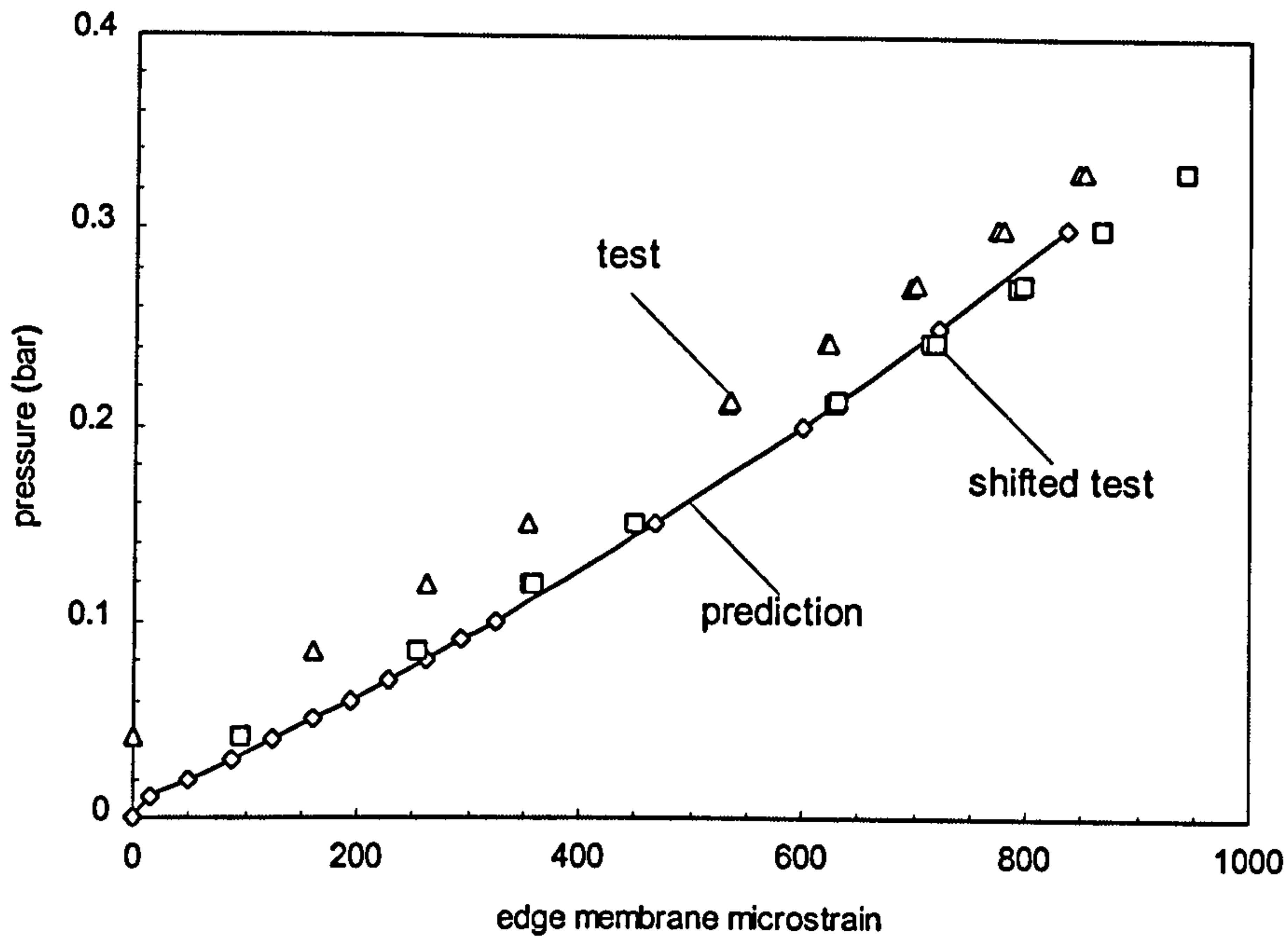


Figure 7.3 Edge membrane strain (SG1/SG2) vs. pressure at low pressure
(See Figure 4.12)

7.3 Methodology for progressive failure analysis of woven composite panels under high transverse pressure

At high pressure, test results show that the composite panel sustained progressive damage, and the small in-plane edge displacements were also observed at the middle of the edges. The damage leads to a reduction of the stiffness of the composite. As mentioned above, few works have been reported on progressive failure analysis for woven composite panels. The main difficulties probably are how to consider the effect of the interlacing and undulation of woven fabrics on the panel analysis; and how to define the failure modes in the individual woven fabric laminae.

When a micromechanical analysis of a repeating unit element (RUE) of a woven composite was performed using finite element method (Chapter 6), it was realised that the interlacing and undulation of the woven fabrics induced many stress concentrations

in the rovings and in the pure resin regions. The stress concentrations cause damage earlier in woven composites than in unidirectional composites. Research shows that the non-uniformity of the stress in the woven rovings and pure resin regions within the woven composite can be conveniently expressed in term of the *stress variation factor* (SVF). For example, SVF in the fibre in the load direction is more than 1.2, which means that the maximum longitudinal stress is 20% higher than the median stress. This suggests that failure of the *woven* composite material may occur at a lower stress than expected. However in a macro-level analysis (i.e. panel analysis), if a woven fabric lamina is taken as an analytical unit, the effects of the stress concentrations on the strength of the composite cannot be estimated easily.

It is also difficult to define failure modes for a single woven composite lamina. Stress concentrations in the direction perpendicular to the roving and in the pure resin regions results in damage to the composite at an earlier stage of loading than without stress concentration. However, it is believed that the interlacing of the fabrics provides the obstacles to the growth of the damage [81]. There is no existing proper failure criterion to represent the complicated failure modes for a woven composite. Hence, there is a real need for this criterion to be studied in the future.

In this research, in order to simulate the woven composite panels of Panel D and Panel 30A, a methodology for progressive failure analysis of the woven composite panel has been developed. The method is to simplify a single woven fabric lamina as two orthogonal unidirectional layers (the unidirectional layer is termed the *nominal unidirectional composite* in later sections) (Figure 7.4). This allows failure behaviour in the woven composite to be studied in rather simple way, treating it as a composite with unidirectional layers. The influences of the interlacing and the undulation of woven fabric are considered by reducing the strength of the *nominal unidirectional composite* according to the stress variation factor (SVF) calculated from the micromechanical analysis in Chapter 6. For this purpose, the term *conversion strength* is introduced to represent the reduced strengths in the *nominal unidirectional composite*. The *conversion strength* (X') in the fibre direction can be obtained from:

$$X' = X / SVF \quad (7.1)$$

where X is the strength of the *nominal unidirectional composite*. For the strength in transverse direction and the shear strength, assuming that negative effects (stress concentrations) and positive effects (obstacles for damage growth due to the fabric interlacing) tend to cancel each other, the conversion strengths are:

$$Y' = Y \tag{7.2}$$

$$S' = S \tag{7.3}$$

where Y and S are the transverse and shear strengths of the *nominal unidirectional composite*.

Once the woven composite panel is simplified and the *conversion strengths* for the *nominal unidirectional composite* are determined, the progressive failure analysis of the woven composite panel can be examined simply by analysing this particular composite panel with unidirectional layers.

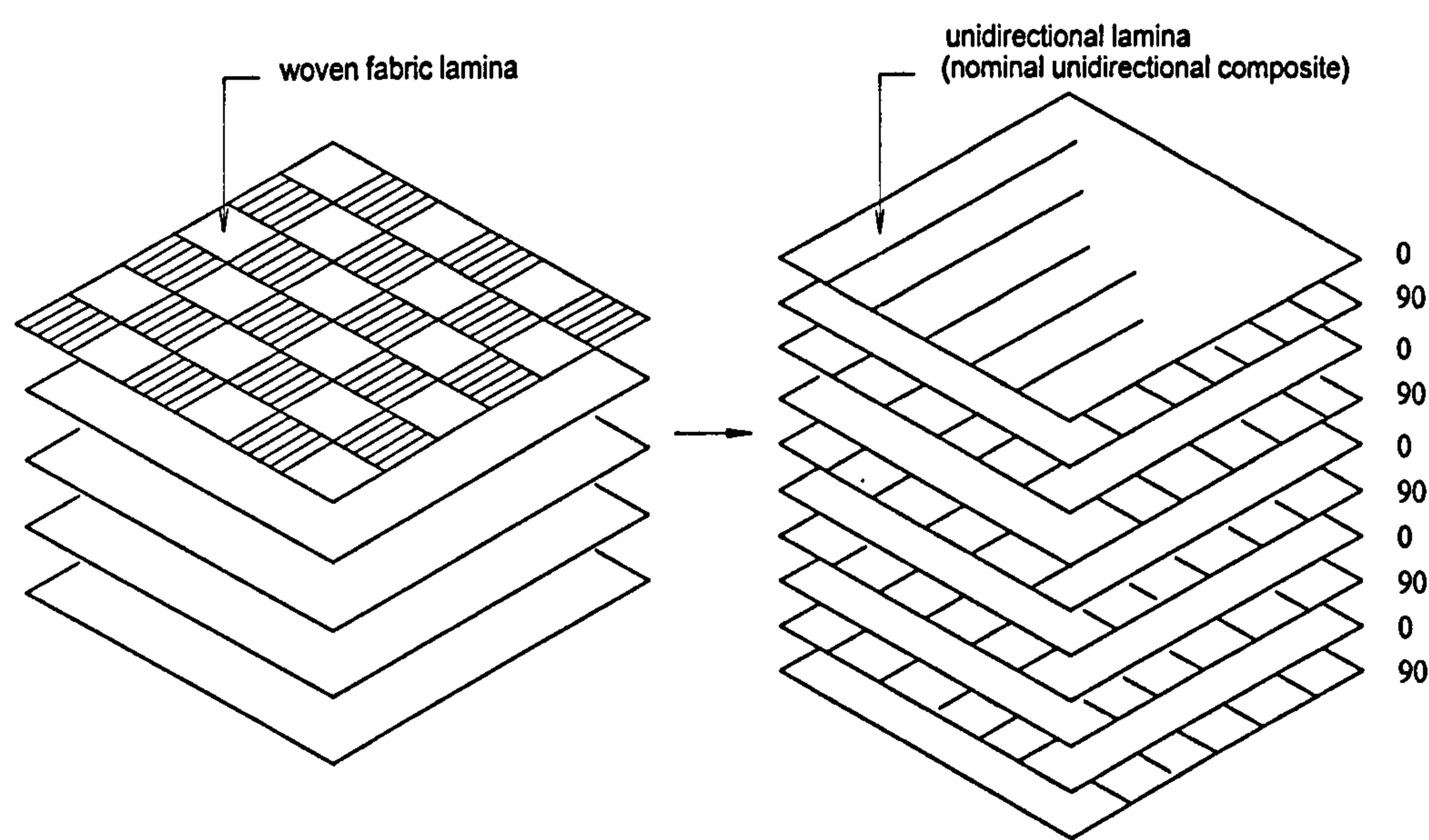


Figure 7.4 A five-layer woven composite panel is simplified as 10-layer cross-ply unidirectional composite

7.4 Stiffnesses and strengths of a nominal unidirectional composite

As a woven fabric lamina is idealised as two orthogonal unidirectional layers (each layer is referred to as the *nominal unidirectional composite*) in the progressive failure analysis, the stiffnesses and *conversion strengths* for the *nominal unidirectional composite* are needed. The stiffnesses can be determined by using the theoretical model developed in Chapter 5. The strengths of the *nominal unidirectional composite* can be calculated approximately by using formulae from reference [44], which are listed in expression (2.3) in Chapter 2. In the formulae, the stiffnesses and strengths of the fibre and the resin are required. In this case the stiffnesses of the fibre and the resin, which are listed in Table 5.1 in Chapter 5 and used to calculate overall elastic properties of the material, are also used in the calculations of the stiffnesses of the *nominal unidirectional composite*. The strengths of the fibre and the resin are chosen according to the suggestions for E-glass fibre and polyester resin in reference [82]. The fibre content by volume in the *nominal unidirectional composite* is considered to be the same as the overall fibre content by volume, based on the assumption that resin is evenly distributed in each of the layers. Table 7.2 lists the important data for the calculations of the stiffnesses and strengths of the *nominal unidirectional composite*.

Research in Chapter 6 shows the SVF for the woven composite with the same geometrical architecture of the woven roving as Panel D and Panel 30A is about 1.23. Although Panel D and Panel 30A are satin woven composite rather than plain woven composite, which is used as an example in the study of stress concentration of woven composite, the stress concentration in interlacing area in a satin weave should be very similar to that in a plain weave. From this point of view, the stress variation factor 1.23 is therefore used in the analysis of Panel D and Panel 30A.

With the strengths for the *nominal unidirectional composite* and the SVF, the *conversion strengths* for Panel D and Panel 30A used in the progressive failure analysis can be estimated by using the expressions (7.1)-(7.3). Panel 18B is a 'non-woven' composite. Hence, the *conversion strengths* for Panel 18B are equal to the strengths of the *nominal unidirectional composite*. Table 7.3 lists the stiffnesses and *conversion strengths* of the *nominal unidirectional composite* in different panels.

Table 7.2 Important data for calculating the stiffnesses and strengths of the nominal unidirectional composite

	Fibre content by volume V_f	Properties	Fibre	Resin
Panel D	0.3649	Stiffness (MPa)	$E_f = 67500$	$E_m = 3400$
			$G_f = 25962$	$G_m = 1307$
		Strength (MPa)	$S_{ft} = 2400$	$S_{mt} = 65$
			$S_{fc} = 2160$	$S_{mc} = 130$ $S_{ms} = 65$
Panel 30A	0.547	Stiffness (MPa)	$E_f = 70000$	$E_m = 3500$
			$G_f = 26923$	$G_m = 1346$
		Strength (MPa)	$S_{ft} = 2400$	$S_{mt} = 65$
			$S_{fc} = 2160$	$S_{mc} = 130$ $S_{ms} = 65$
Panel 18B	0.3077	Stiffness (MPa)	$E_f = 74000$	$E_m = 3750$
			$G_f = 28462$	$G_m = 1442$
		Strength (MPa)	$S_{ft} = 2400$	$S_{mt} = 65$
			$S_{fc} = 2160$	$S_{mc} = 130$ $S_{ms} = 65$

* S_{ft} , S_{fc} are tensile and compressive strengths of the fibre; and S_{mt} , S_{mc} are tensile and compressive strengths of the resin, S_{ms} is shear strength of the resin.

Table 7.3 Stiffnesses and conversion strengths of the nominal unidirectional composite in different panels

	Stiffness	Value	Strength	Value
Panel D	E_x MPa	26750	X_T MPa	710
	E_y MPa	9270	X_C MPa	640
	ν_{xy}	0.3	Y_T MPa	50
	G_{xy} MPa	3350	Y_C MPa	100
			S MPa	50
Panel 30A	E_x MPa	39875	X_T MPa	1050
	E_y MPa	14181	X_C MPa	950
	ν_{xy}	0.3	Y_T MPa	55
	G_{xy} MPa	5127	Y_C MPa	110
			S MPa	55
Panel 18B	E_x MPa	23800	X_T MPa	740
	E_y MPa	10500	X_C MPa	670
	ν_{xy}	0.25	Y_T MPa	50
	G_{xy} MPa	3200	Y_C MPa	100
			S MPa	50

7.5 Failure criterion

For the fibre-reinforced composite (either unidirectional or woven composite) damage can occur among the fibres running transversely to the direction of the load because of matrix failure without breaking longitudinal fibres. This transverse failure does not completely destroy the load bearing capability of the material. Therefore it is important for the failure criterion to indicate both damage (failure) and the orientation of the damage. As mentioned in Chapter 2, it is widely accepted that the Hashin failure criterion is suitable for describing the different failure modes in the fibre-reinforced composite. Hence, in this paper the Hashin criterion was used in the progressive failure analysis. The Hashin criterion consists of three subsets of criteria for predicting three modes of in-plane failure; matrix cracking, fibre-matrix shearing, and fibre breakage.

$$\text{Matrix cracking:} \quad \left(\frac{\sigma_2}{Y'} \right)^2 + \left(\frac{\sigma_6}{S'} \right)^2 \geq 1$$

$$\text{Fibre-matrix shear-out:} \quad \left(\frac{\sigma_1}{X'} \right)^2 + \left(\frac{\sigma_6}{S'} \right)^2 \geq 1 \quad (7.4)$$

$$\text{Fibre breakage:} \quad \left(\frac{\sigma_1}{X'} \right) \geq 1$$

where σ_1 , σ_2 and σ_6 are the longitudinal, transverse and shear stresses in the composite layer, respectively. X' , Y' and S' are the longitudinal, transverse and shear conversion strengths of the nominal unidirectional composite. The values of X' , Y' are taken as either X'_T , Y'_T or as X'_C , Y'_C depending on the sign of σ_1 , σ_2 , respectively.

7.6 Stiffness reduction

The stiffness of the laminated composite decreases when damage occurs in the laminae. To simulate damage, material properties at each 'failed' integration point are reduced to the values representing the particular type of damage that has occurred. This allows that the loads carried by the damaged region to be redistributed to surrounding regions. For the different damage modes the stiffness reduction can be executed according to the scheme [81] in Table 7.4.

Table 7.4 Stiffness reduction scheme at integration point

Failure modes	E_x	E_y	ν_{xy}	G_{xy}
No failure	E_x	E_y	ν_{xy}	G_{xy}
Transverse matrix failure	E_x	0	0	G_{xy}
Fibre/matrix failure	E_x	E_y	0	0
Fibre breakage	0	0	0	0

7.7 Finite element modelling and implementation

The ABAQUS finite element code was used in the progressive failure analysis of the woven composite panels. The analyses were implemented on the IBM SP2 *parallel processing computer* at Southampton University as described previously.

The thin panel exhibits large-deflection behaviour under high transverse pressure. From experimental observation, the panel acted mainly as a membrane except at the edges where there were large but localised bending moments. During the loading various kinds of damage occurred in the composite. Thus, the problem involves both geometrical and material non-linearity.

In ABAQUS, finite element modelling of a certain problem is executed in terms of various option blocks, as discussed in Chapter 6. In geometry and element blocks, the elements were meshed intensively around the edges in order to model the high gradient

of the bending stresses and to capture the failure information accurately in these areas. The thin shell element S4R was used, which is a 4-node element and is recommended to be used in thin shell problem. The 'composite' was specified for the element by means of

***SHELL SECTION, COMPOSITE**

followed by the parameters for each of the layers, which include layer thickness, number of integration points to be used through the layer, name of the material forming the layer, and orientation angle of the fibres.

To simulate the progressive failure of the composite, the material properties were made dependent on *three field variables*. Field variable (FV) is a special term in ABAQUS. The first field variable represents the matrix failure index, the second the fibre-matrix shear failure (de-bonding) index, and the third the fibre breakage failure index. In the computer program, if a particular field variable is equal to 1, the corresponding failure mode occurs; if it is zero, no damage happens. The dependence of the material properties on the field variables is shown in Table 7.5, which is, in fact, a translation of the stiffness reduction scheme into computer variables. Then, material properties are defined as a function of the field variables in the material property block. Typical commands are as follows.

***ELASTIC, TYPE=LAMINA, DEPENDENCIES=3**

E_x , E_y , ν_{xy} , G_{xy} , G_{xz} , G_{yz} , 0, 0,	
0 , 0	 No damage
E_x , 0 , 0 , G_{xy} , G_{xz} , G_{yz} , 0, 1,	
0 , 0	 Transverse matrix failure
E_x , E_y , 0 , 0 , G_{xz} , G_{yz} , 0, 0,	
1 , 0	 Fibre/matrix shear failure
E_x , 0 , 0 , 0 , G_{xz} , G_{yz} , 0, 1,	
1 , 0	 Matrix failure & fibre/matrix
.....	

Table 7.5 Dependence of material properties on the field variables

Failure mode	Material Properties				FV1	FV2	FV3
No Failure	E_x	E_y	ν_{xy}	G_{xy}	0	0	0
Matrix failure	E_x	0	0	G_{xy}	1	0	0
Fibre/matrix shear failure	E_x	E_y	0	0	0	1	0
Matrix failure & fibre/matrix	E_x	0	0	0	1	1	0
Fibre breakage	0	0	0	0	0	0	1
Matrix failure & fibre breakage	0	0	0	0	1	0	1
Fibre/matrix & fibre breakage	0	0	0	0	0	1	1
All failure modes	0	0	0	0	1	1	1

Because the panels were thin, the possibility of damage (delamination) through the thickness was considered unlikely. So G_{xz} and G_{yz} were given large values. To determine the value of the field variable, the damage or failure at each integration point was detected by comparing the current stress states with the corresponding strengths in the specific failure criterion. This procedure was executed in a user-defined subroutine USDFLD in ABAQUS, which was called in main program in the form of

***USER DEFINED FIELD**

A typical user-defined subroutine used in this research is listed in Appendix 4. If damage occurred, the corresponding field variable was given to the value of 1 and the stiffnesses of the composite were reduced according to Table 7.5, and the material was then no longer linear-elastic. Within an individual finite element the material properties at integration points may be elastic, inelastic or failed.

Modelling of non-linear material properties required an iterative solution procedure with load increments, and the load increments had to be sufficiently small to ensure convergence. In the analysis 500 load increments were chosen. In order to model

geometrical non-linearity, the large-deflection option in ABAQUS was switched on. These two options were implemented in ABAQUS as follows.

```
*STEP, INCREMENT=500, NLGEOM
```

In this way, progressive damage (material non-linearity) and geometrical non-linearity of the panel were modelled.

To verify the method of the progressive failure analysis, Panel D, Panel 30A and Panel 18B were selected and calculated as examples. Important parameters and test results of the panels are therefore re-listed in Table 7.6.

Table 7.6 Important data of the composites and test results of the panels

	Panel D	Panel 30A	Panel 18B
Overall thickness (mm)	4.15	2.84	3.431
Fibre content by volume	0.3649	0.547	0.3077
Dimensions of panel (mm)	600×600	600×600	600×1200
Composite type	five woven laminae	five woven laminae	non-woven (0/45/90/-45/0)
Failure pressure (bar)	2.93	4.24	4.5
Failure deflection (mm)	29.2	35.9	59.5
In-plane edge displacement (mm) (target value)	0.3*	0.8**	3***

* average value of four measured edge displacement at middle of the edges
** measured at pressure of 4 bar
*** measured at pressure of 3.75 bar

7.8 Modelling of in-plane edge displacement

Although the edges in the panel tests were intended to be fully clamped, small in-plane edge displacements were still observed (Table 7.6) because the large membrane tensile forces caused slippage despite the heavy bolted restraints around all four edges of the panel. This edge displacement had a significant effect on the behaviour of the panels. However, as discussed in Chapter 4, the in-plane edge displacement is complicated as shown in Figure 4.8. At a certain stage of loading, the membrane tensions were big enough to overcome the friction between panel and steel frame (test rig) and eventually pulled the panel inwards. Chapter 4 also shows that the edge displacements may not have the same values at two opposite edges (Figure 4.30). It is therefore very difficult to model them exactly in the finite element analysis.

To simplify the modelling of the in-plane edge displacement, several assumptions were made in this research.

- (1) A parabolic shape of the in-plane displacement along the edges is assumed.
- (2) The edge displacements at the mid-points of the four edges are proportional to the length of the edge. The longer the edge, the larger the in-plane displacement. There is no movement at the corners.
- (3) The edge displacements at two opposite edges are treated as the same.

Two models have been built. One was a progressive damage model with fully fixed boundary conditions, referred as the *PDM model*. The other was a progressive damage model with in-plane spring support at the edges, referred as the *PDM-spring model*. The PDM-spring model still had boundaries fixed against rotation, but springs were used to make allowance for the edge displacement. In this way the edge was allowed to move slightly in the plane of the panel, but rotation at the edge was held at zero. For this purpose, spring element type SPRINGA was activated in ABAQUS.

***ELEMENT, TYPE=SPRINGA, ELSET= name**

One end of the spring was attached to the node in the plane of the panel. The other end of the spring was fixed to the 'ground'. The PDM-spring model was used to model the panel with in-plane edge displacement. Figure 7.5 illustrates Panel 18B with surrounding springs.

The stiffness of the spring in the PDM-spring model must be determined in such a way that the in-plane edge displacement in the finite element analysis is comparable with the measured one at a certain pressure (called the 'target value'). For Panel 30A and Panel 18B this pressure was chosen around the failure pressure. For Panel D, however, since measured edge displacements at four middle of the edges were different, a mean value of them (0.3 mm) was used as the target value in the modelling of edge movement.

There is no direct way to determine the value of the stiffness of the spring. The trial-and-error method directly used in the finite element analysis is a possible but time-expensive choice. Alternatively, in this research the trial-and-error method is used in a quick approximate method to estimate the stiffness of the spring. This quick approximate method will be described in detail in Chapter 9. Table 7.7 lists the stiffnesses of the springs adopted for the panels. The unit of the stiffness of spring in the table is N/mm². The actual stiffness of the spring attached at the node in the analysis should be the *product* of the stiffness in the table and average lengths of the elements which share the same node. For instance, in Figure 7.6, edge node n is shared by two elements with element length of 30 mm and 20 mm, respectively, then the stiffness of spring attached at node n is $380 \times (30+20)/2 = 9500$ N/mm for Panel 30A.

In the PDM-spring model 225 shell elements and 42 spring elements were used for Panel D and Panel 30A. Because of the symmetrical arrangement of the fibres in the plane of the *lamina* and the symmetrical edge displacement assumed, a quarter of the panel was modelled in Panel D and Panel 30A. However, the fibre arrangement in Panel 18B was not symmetrical through the thickness in which one layer lies at 45° direction and another lies at -45° direction, and the whole panel was therefore modelled. 640 shell elements and 108 spring elements were therefore used. Figure 7.5 also demonstrates the element meshes in Panel 18B.

ABAQUS

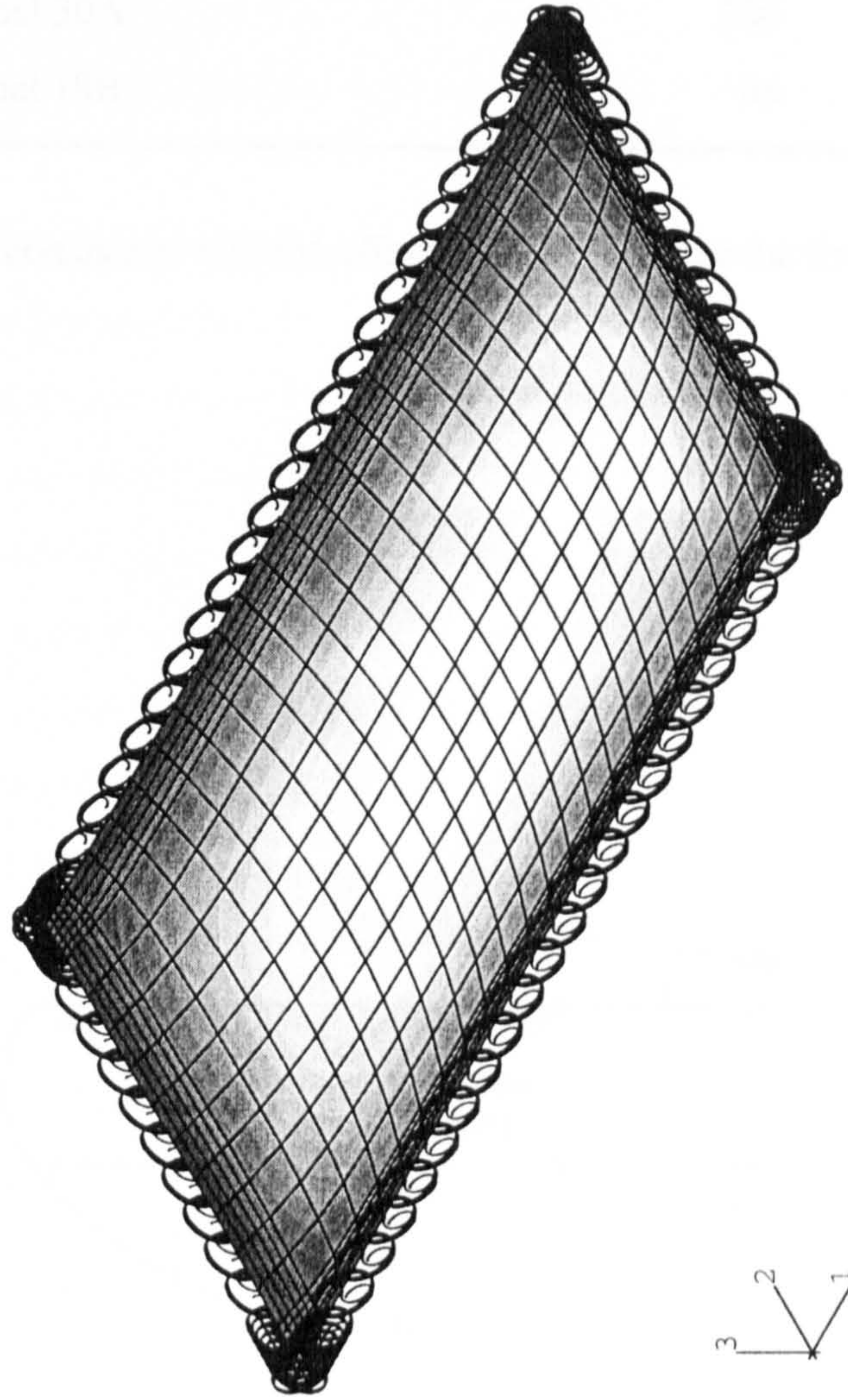


Figure 7.5 The springs around the edges of Panel 18B: springs, element mesh and deflections

Table 7.7 The stiffness of the spring used in analysis

Panel	Spring Stiffness (N/mm ²)
Panel D	1100 *
Panel 30A	380
Panel 18B	95

* This high value is consistent with the effort made to improve the fixity of the edges in the test of Panel D

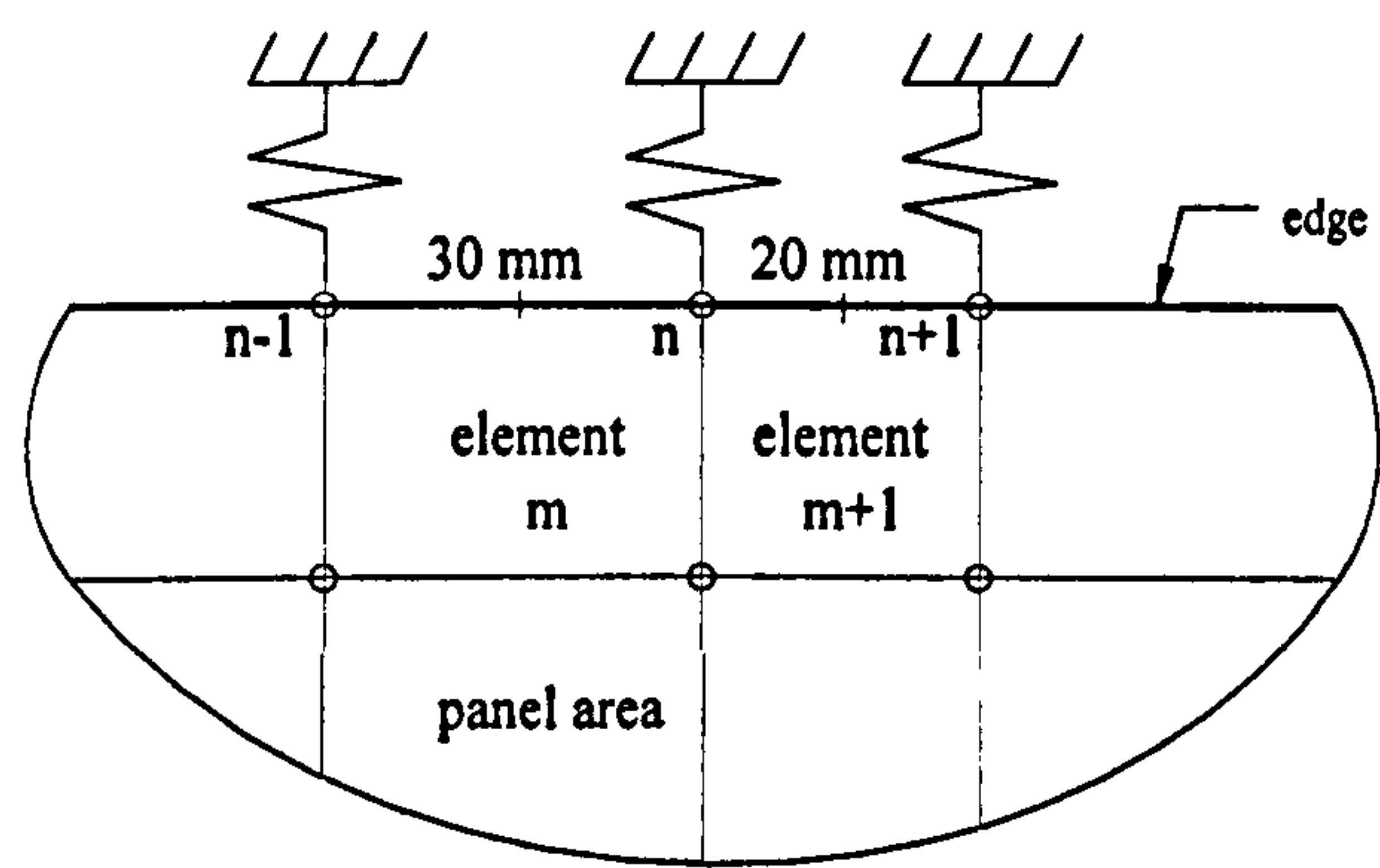


Figure 7.6 Demonstration of the calculation of the stiffness of spring attached at a node

7.9 Analysis of the results

The PDM model and the PDM-spring model were used to predict the progressive damage of the panels. Many results and diagrams were obtained. To save space only important results are shown here. By analysing the results some important information can be found.

7.9.1 Damage patterns and final failure predicted by the PDM models

The information about the damage of the panel is given by field variables FV. The load corresponding to the first non-zero FV is considered as initial damage pressure. Final failure pressure is obtained when the program is terminated due to singularity. The progressive failure sequence predicted by the *models* showed that the thin panels started to crack around the fixed edges in the matrix at a small pressure. In Panel D and Panel 30A, which were woven composites, the matrix cracking was the dominant damage pattern during loading. More extensive matrix cracking occurred around the edges as the load increased. Final failure of the panels was in the form of fibre breakage at the edges. In Panel 18B, which was a non-woven composite layered at 0/45/90/-45/0, after initial transverse matrix damage, shear damage occurred in the four corners and then extended along the edges. The shear damage and matrix cracking were the dominant damage patterns. Final failure was due to extensive de-bonding around the edges. The predicted failure patterns for all three panels coincide with the failure patterns actually observed in the tests.

Figure 7.7 demonstrates the predicted matrix crack pattern near failure at unloaded and loaded surfaces in Panel D. As can be seen in the figure, at the unloaded surface, a large amount of matrix cracking occurs in the central area of the panel where the panel is subjected to the membrane tensile force and bending. At the loaded surface, because the compressive force induced by bending cancels a part of membrane tensile force, the central matrix crack area in the loaded surface is smaller than in the unloaded surface. While many cracks around the edge occurred in the loaded surface. This matrix crack pattern coincides with that observed in the test.

Table 7.8 lists the predicted progressive failure sequence and tested one. The predicted final failure pressures for three panels agree well with the tested ones; while the predicted first-damage pressures are smaller than those observed in the tests. In the tests, the first cracking noises were taken to be evidence of the initiation of damage. This may be affected by background noise in the laboratory. The results also show that the final failure pressures from the PDM and the PDM-spring are almost the same. This indicates that small in-plane edge displacements do not have a big effect on the final failure of the panels.

The predictions indicate that the main damage mode for thin woven composite panel under high transverse pressure is matrix damage, and final failure is due to the fibre breakage. For the thin composite panel with fibre arranged at different directions (Panel 18B), the main damage modes may combine matrix damage with fibre/matrix shear damage (de-bonding), and final failure is related to the intensive de-bonding around the edges.

Table 7.8 Comparisons of failure pressure between predictions and tests

Panel	Damage Pressure	Prediction		Test
		PDM	PDM-spring	
Panel D	Initial Damage	0.5 bar	0.5 bar	1.3 bar *
	Final Failure	3.1 bar	3.15 bar	2.93 bar
Panel 30A	Initial Damage	0.2 bar	0.2 bar	-
	Final Failure	4.0 bar	4.1 bar	4.24 bar
Panel 18B	Initial Damage	0.1 bar	0.1 bar	0.4 bar *
	Final Failure	4.6 bar	4.7 bar	4.5 bar

* determined by hearing of the cracking noise

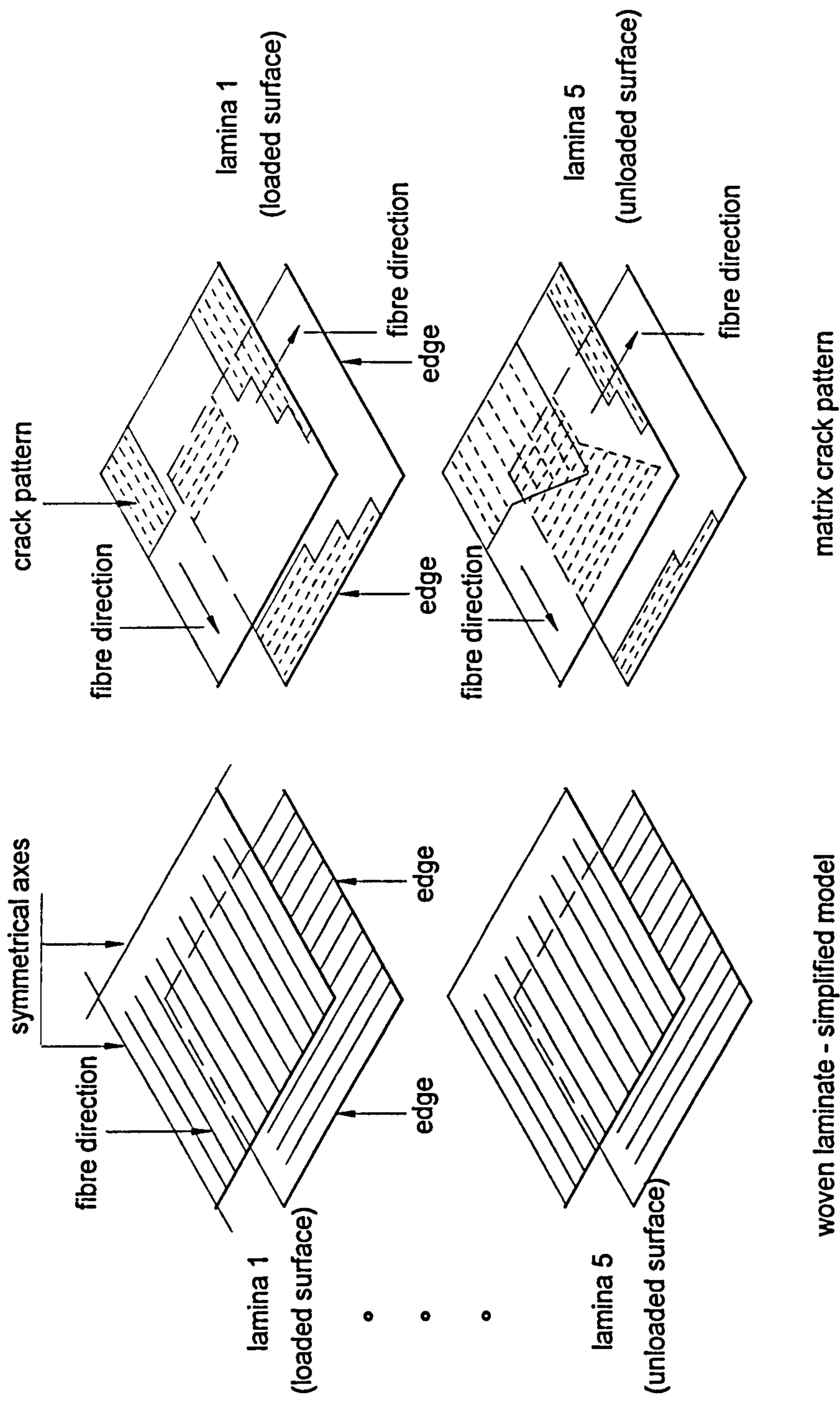


Figure 7.7 Demonstration of matrix crack pattern in Panel D predicted by the PDM-spring Model

7.9.2 Comparison of deflections between models and test

Deflection reflects the overall behaviour of the panel. The tested panels were thin and the geometric non-linear behaviour of the panel was strong. The panels therefore acted mainly as membranes. The ratio of the failure deflection to thickness was in the range 7 to 16.

Figures 7.8-7.10 demonstrate the in-plane edge displacement vs. pressure curves of the three panels calculated from the PDM-spring model and measured from the tests. As expected, the predicted edge displacement agrees with the test result at the *target value* as listed in Table 7.6. The spring support in the PDM-spring model provides reasonable modelling of the in-plane edge displacement. However, because of the complexity of the measured edge displacement, the predicted curve cannot match the whole test path well, especially at small pressure. This is because the spring in the model starts to move once membrane stresses are generated, while real edge displacement takes the form of slippage, as discussed in Section 7.8.

Figures 7.11-7.13 show that the central deflection vs. pressure curves for the three panels. The results show that deflections predicted from the *PDM-spring model* agree very well with the test results. However, the deflections from the *PDM model* are apparently far away from the tests, especially in Panel 18B, in which the in-plane edge displacement is considerable (Table 7.6). The effect of the edge displacement on the behaviour of the panel is significant. The comparisons indicate that without considering this effect one cannot realistically explore the actual behaviour of the panel.

The analysis indicates that the *progressive damage behaviour*, *geometrical non-linear behaviour* and *small in-plane edge displacement* are three important characteristics for the thin composite panel under high transverse pressure. The model predicts overall behaviour of the thin composite panel under high transverse pressure very well.

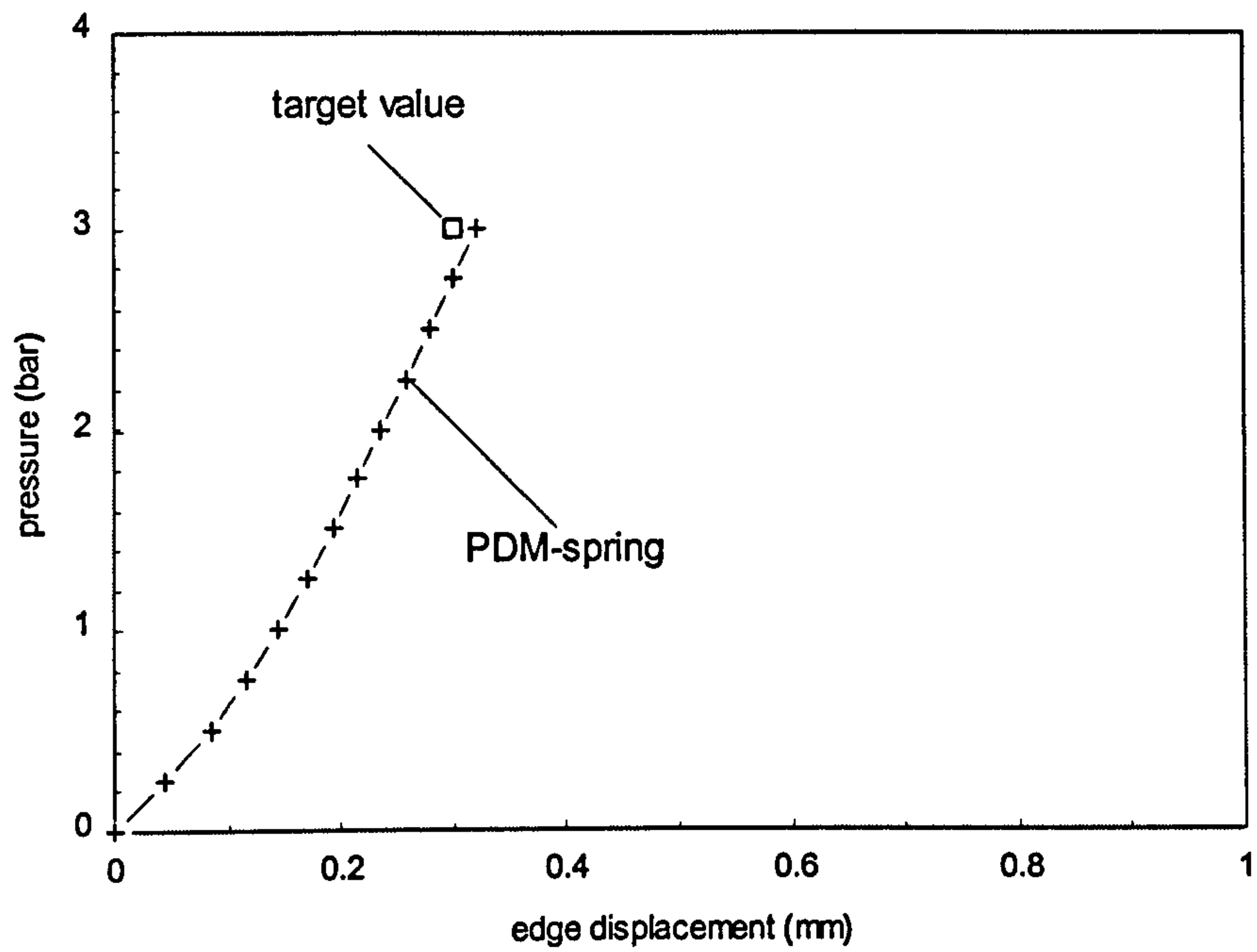


Figure 7.8 In-plane edge displacement of Panel D (woven)

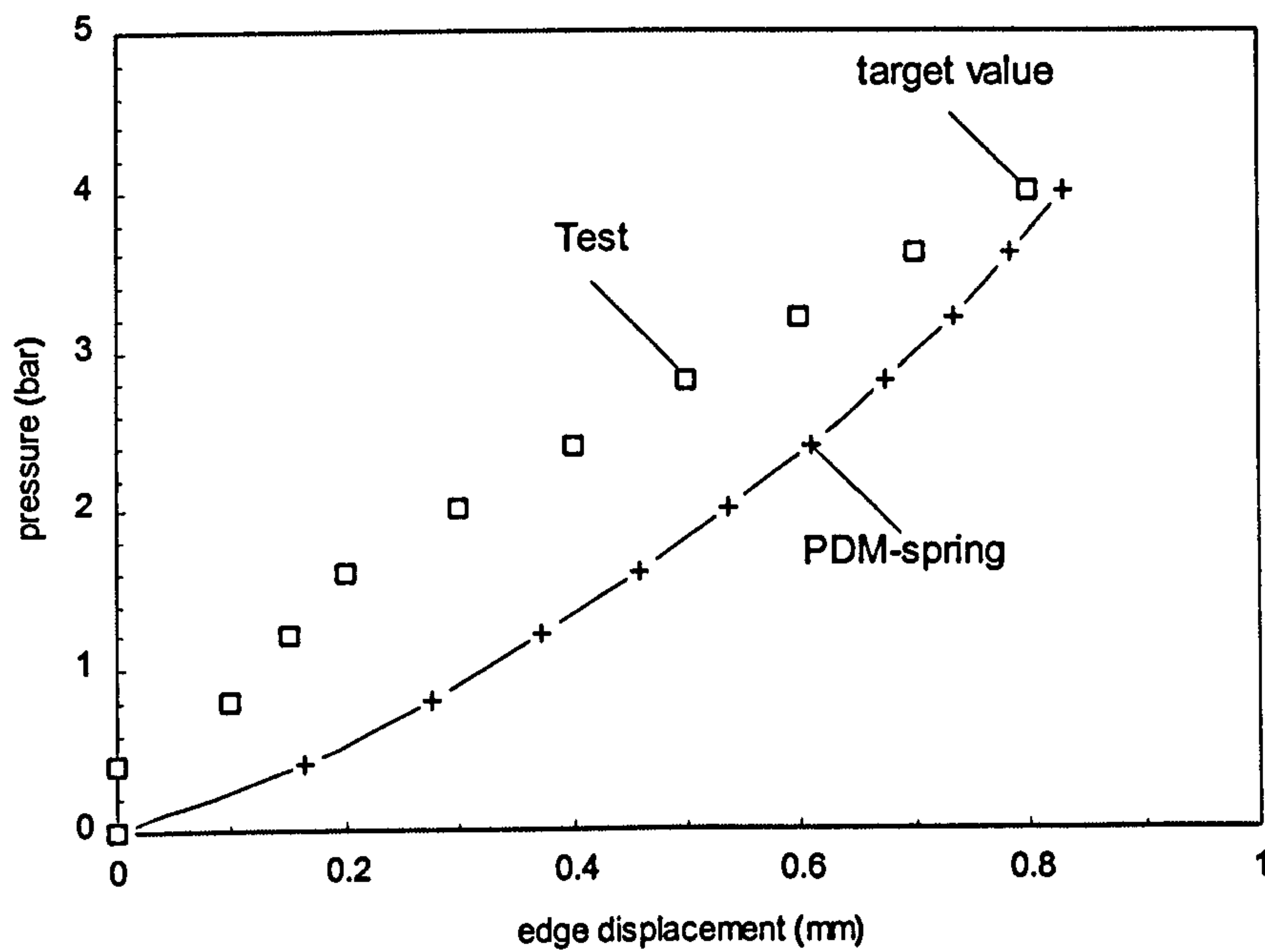


Figure 7.9 In-plane edge displacement of Panel 30A (woven)

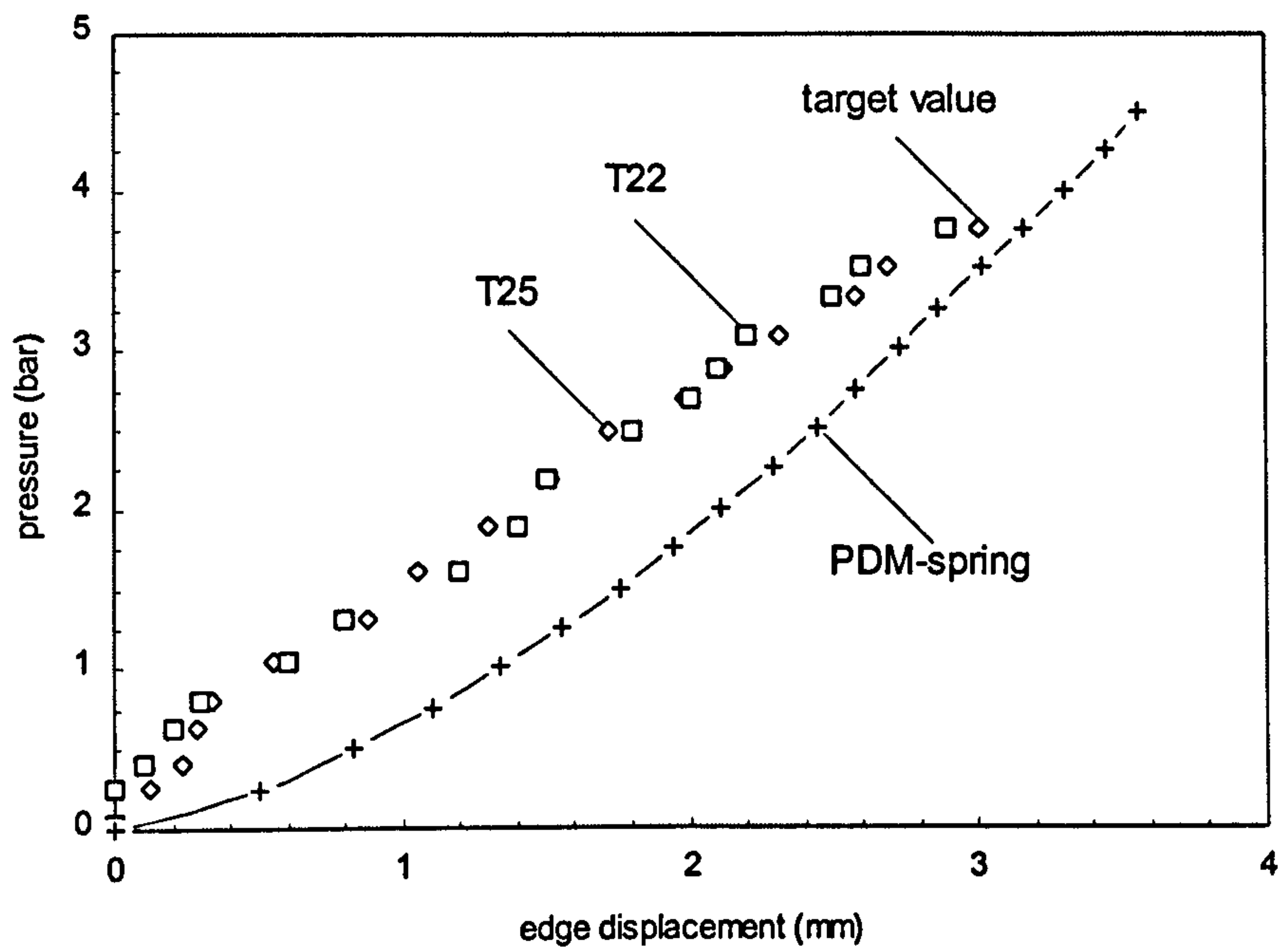


Figure 7.10 In-plane edge displacement of Panel 18B (non-woven)

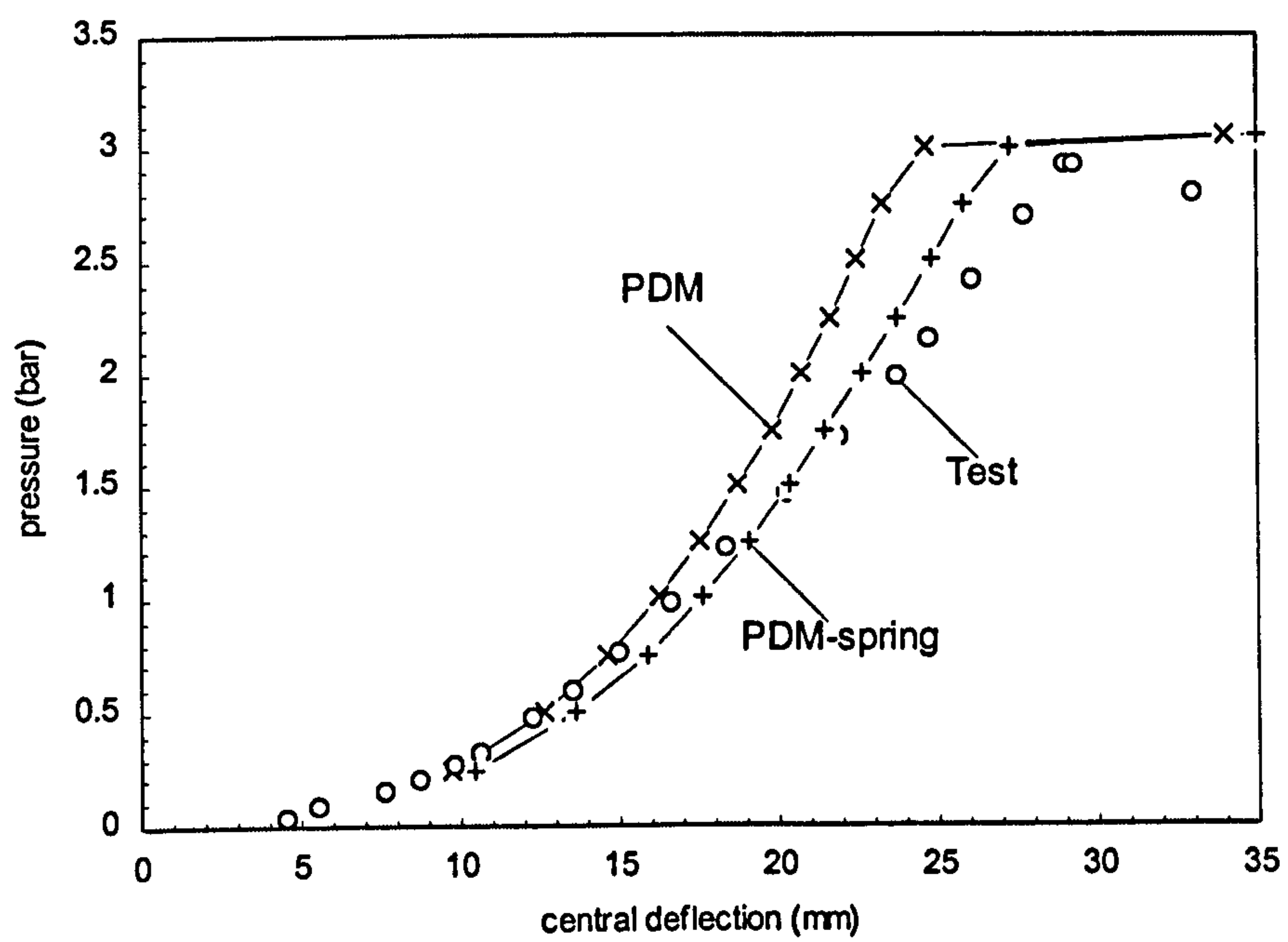


Figure 7.11 Central deflection (T2) vs. pressure of Panel D (woven)

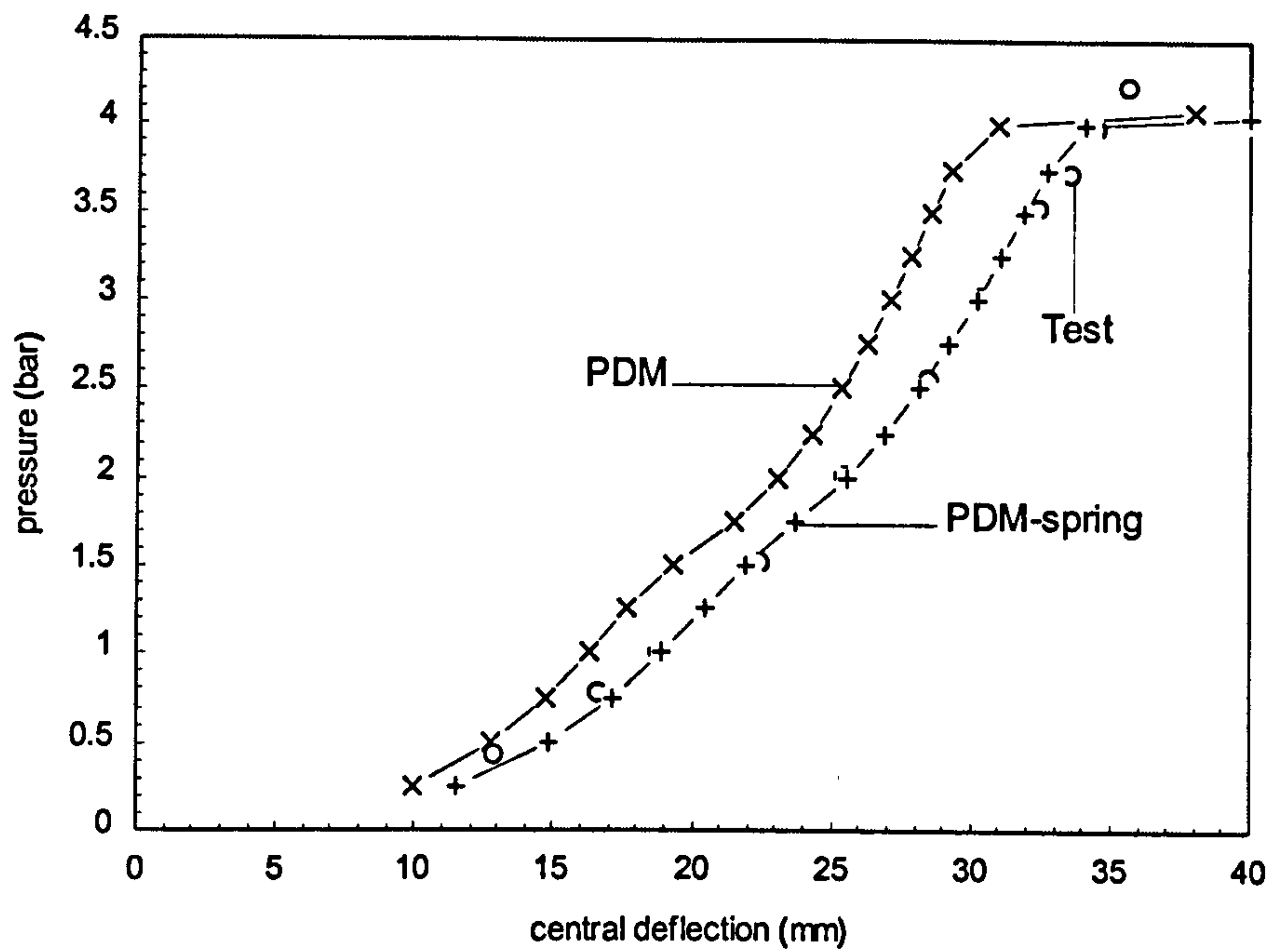


Figure 7.12 Central deflection (T2) vs. pressure of Panel 30A (woven)

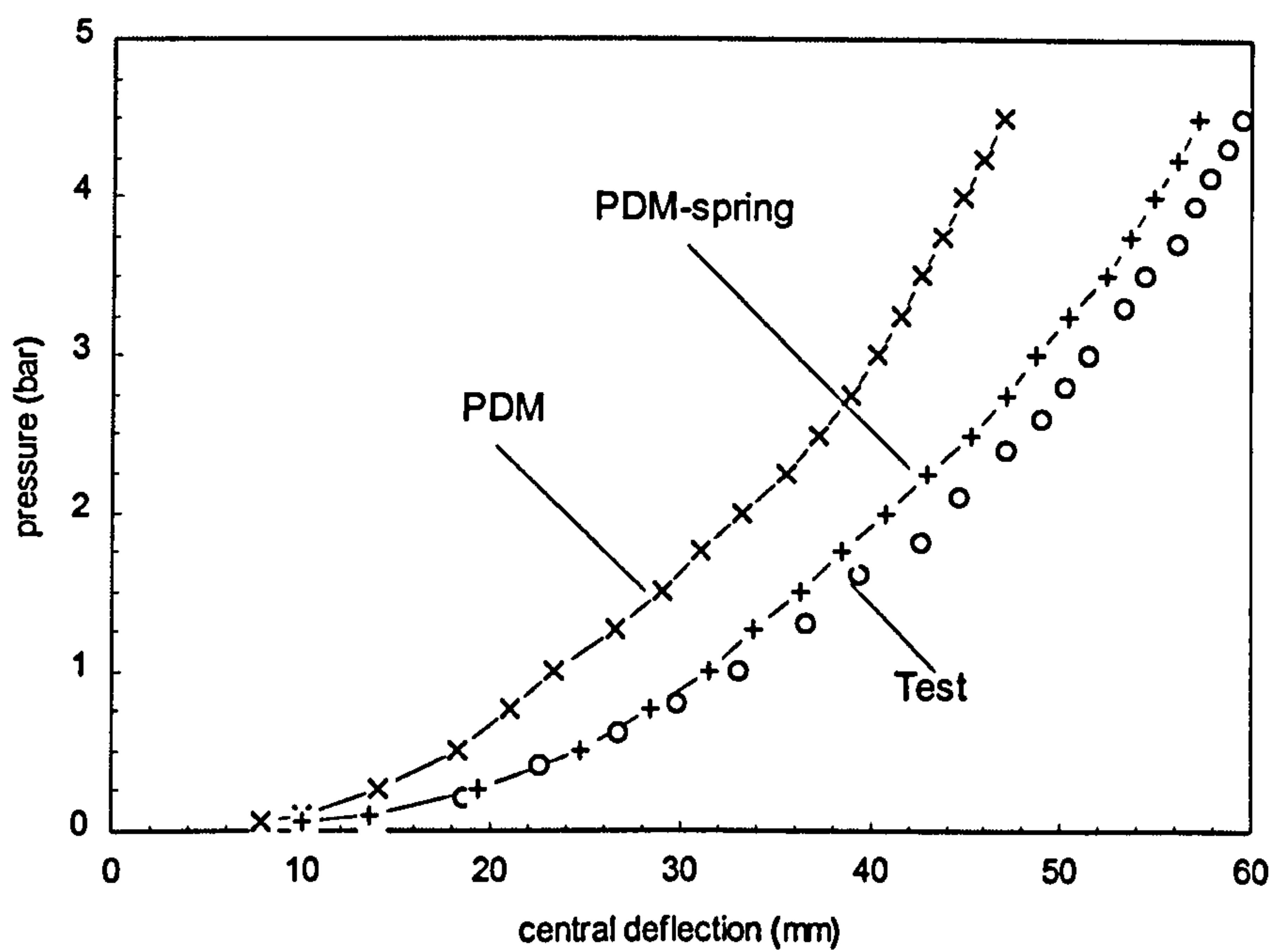


Figure 7.13 Central deflection (T2) vs. pressure of Panel 18B (non-woven)

7.9.3 Comparison of strains between models and test

Strain reflects the local behaviour of the panel. Strain gauges used in the tests was 10 mm long and were mounted on both loaded and unloaded surfaces of the panel.

Figures 7.14-7.17 show that the central strain vs. pressure curves for Panel D. As can be seen, very good predictions for central strains on both unloaded and loaded surfaces in the x-axis (the warp direction) and the y-axis (the weft direction) are achieved in the PDM-spring model. In the figures, the curves from the PDM-spring model are closer to the tests than the PDM model. This is particularly true of the curves in y-axis (the weft direction).

However, in Figures 7.18 and 7.19, which show the central strain vs. pressure curves at unloaded and loaded surfaces for Panel 30A, good agreement between the PDM-spring model and the test can only be seen below a pressure of 1.5 bar. After that a jump in the strain occurs. Detailed analysis shows that transverse matrix damage in the models happens in the *central element* at 1.5 bar in Panel 30A when the jump occurs. It is therefore probable that with the damage in the central element, the stiffness in the element is reduced suddenly in the model and this leads to a jump in the strain. In Panel D, the central element in the models sustains damage at about 2.75 bar. One can see that the jump also happens there (Figures 7.14-7.17). As it is close to the final failure, the jump can be overlooked. The jump can also be seen in the strain curves of Panel 18B when damage occurs in the element where the strain is calculated (Figures 7.20 and 7.21).

It is also noticed that the strains predicted by the progressive failure analysis in undamaged elements match the tests well. Figures 7.22 and 7.23 show the strains in middle of half-span at unloaded and loaded surfaces of Panel D, where no damage is detected in the models. Good agreement between the PDM-spring and test can be seen.

However, the jump does not happen in the load-strain curves obtained in the tests. The jump is a characteristic (shortcoming) of the progressive damage analysis model.

Failure in the model is checked at the integration point of the element. When the stress level at an integration point exceeds the strength in the particular failure mode, the corresponding stiffness for the whole element is reduced to a small value immediately. The element is two dimensional. One can imagine that after damage has occurred the panel has many two dimensional parts without stiffness. On the other hand, the damage in the actual panel is always in the form of small cracks scattered across the panel. The strain gauge is 10 mm long, and straddles damaged and undamaged parts. The strain gauge reading is an average of strain values in that length. This may be the reason that the jumps do not appear in the strain curves from the tests.

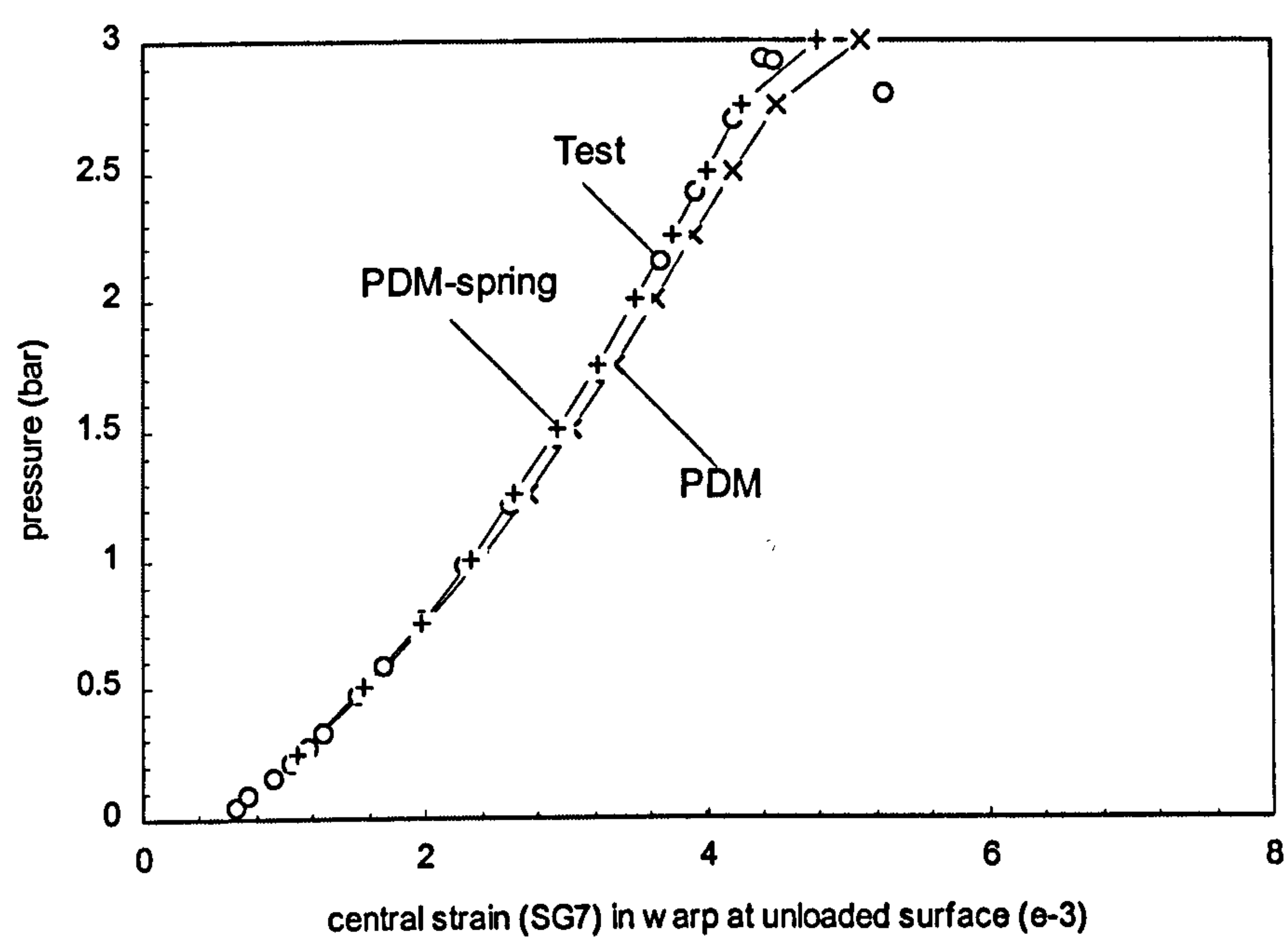


Figure 7.14 Central strain in the x-axis at unloaded surface of Panel D (woven)

(See Figure 4.12 about strain gauge location for Panel D)

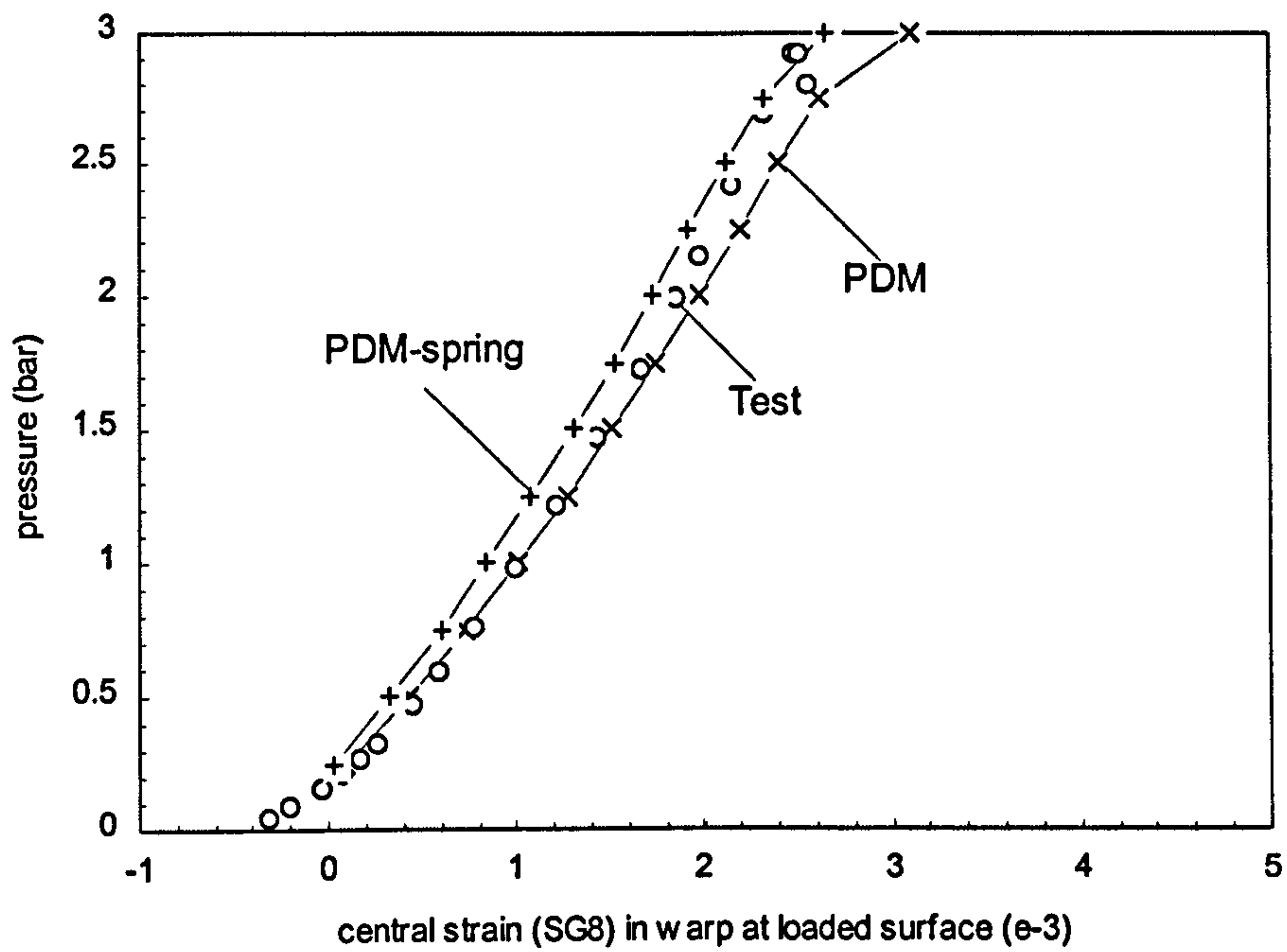


Figure 7.15 Central strain in the x-axis at loaded surface of Panel D (woven)

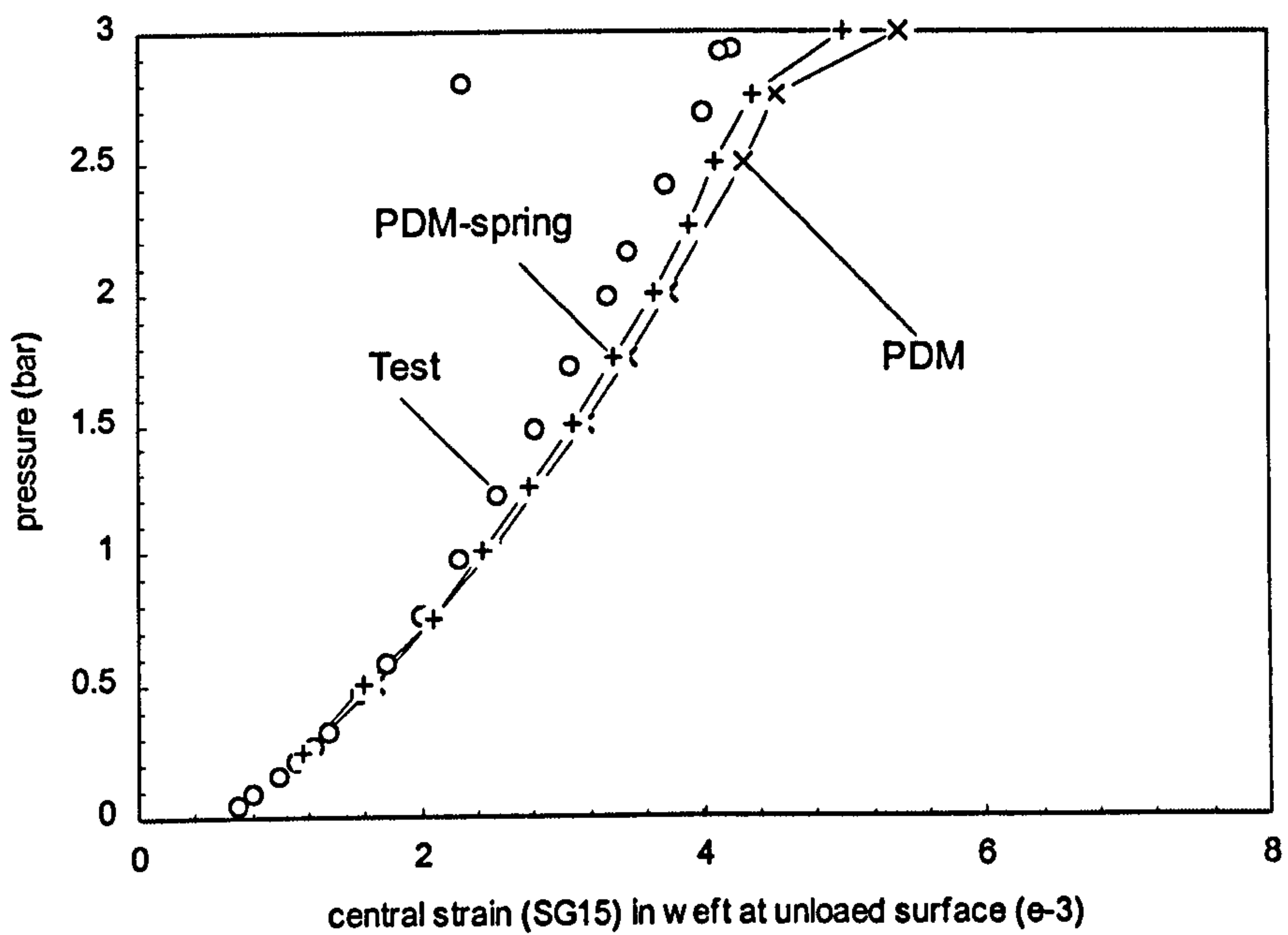


Figure 7.16 Central strain in the y-axis at unloaded surface of Panel D (woven)

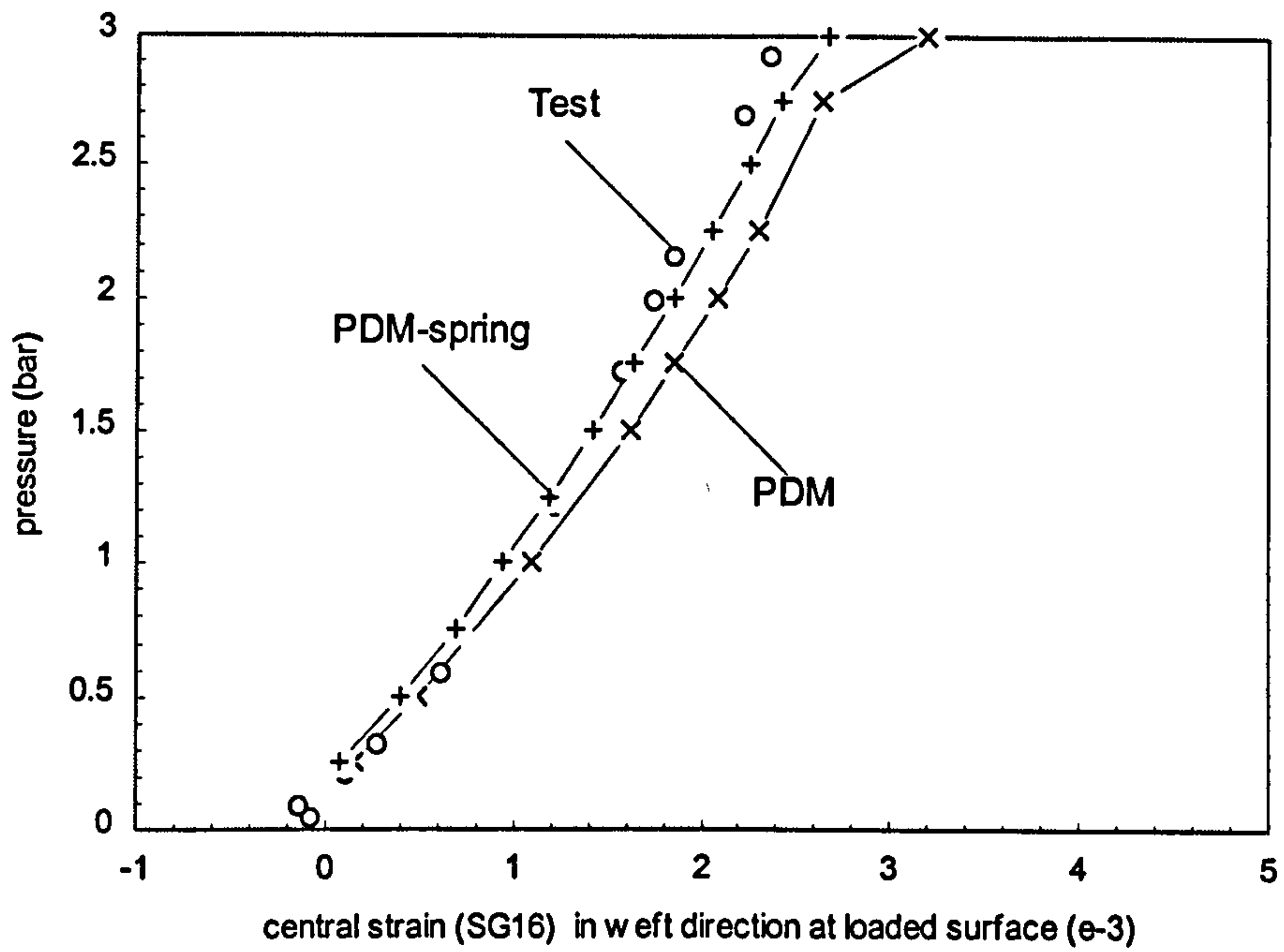


Figure 7.17 Central strain in the y-axis at loaded surface of Panel D (woven)

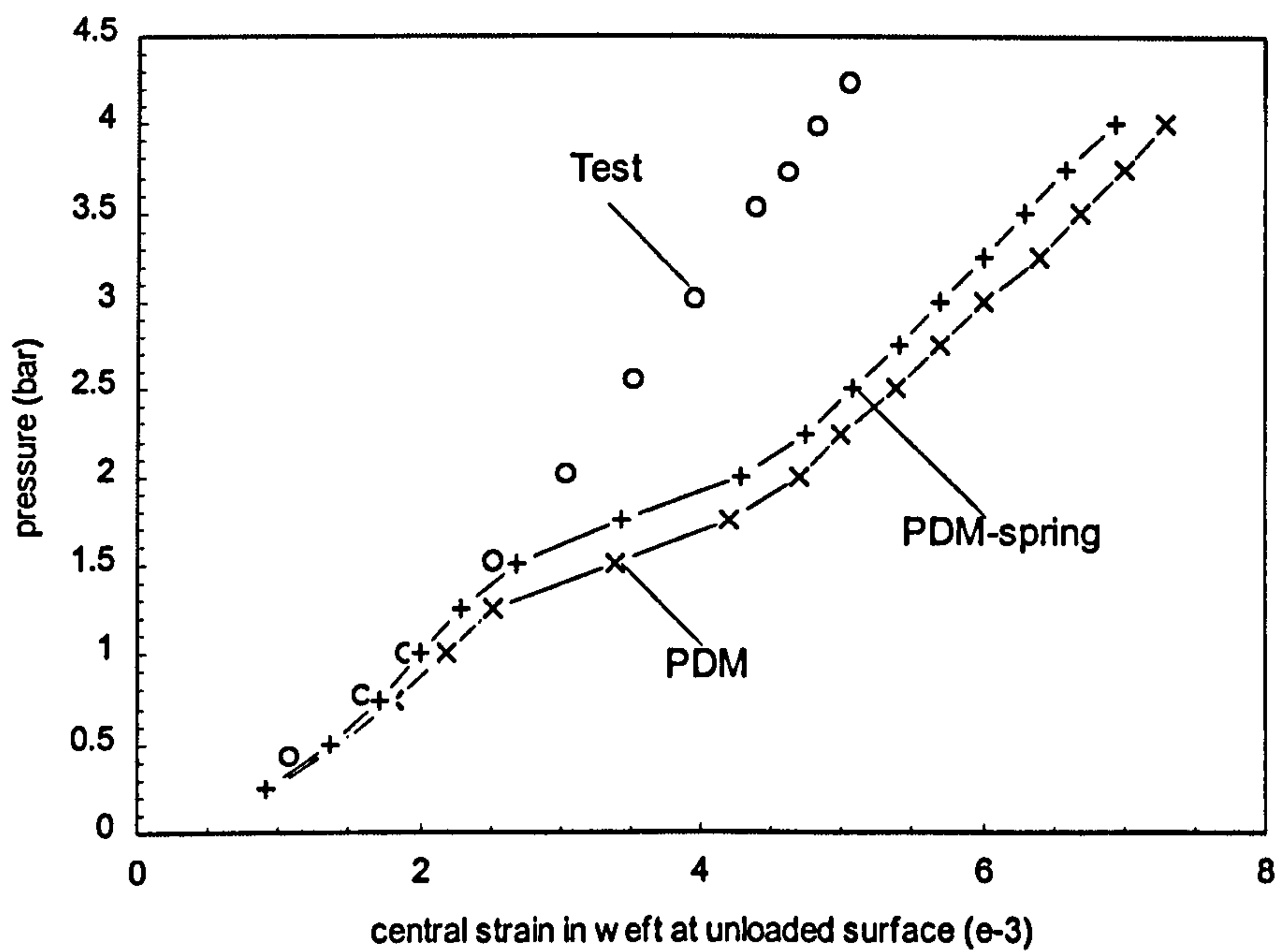


Figure 7.18 Central strain (Z3) in the y-axis at unloaded surface of Panel 30A (woven)
(See Figure 4.3 about strain gauge location for Panel 30A and Panel 18B)

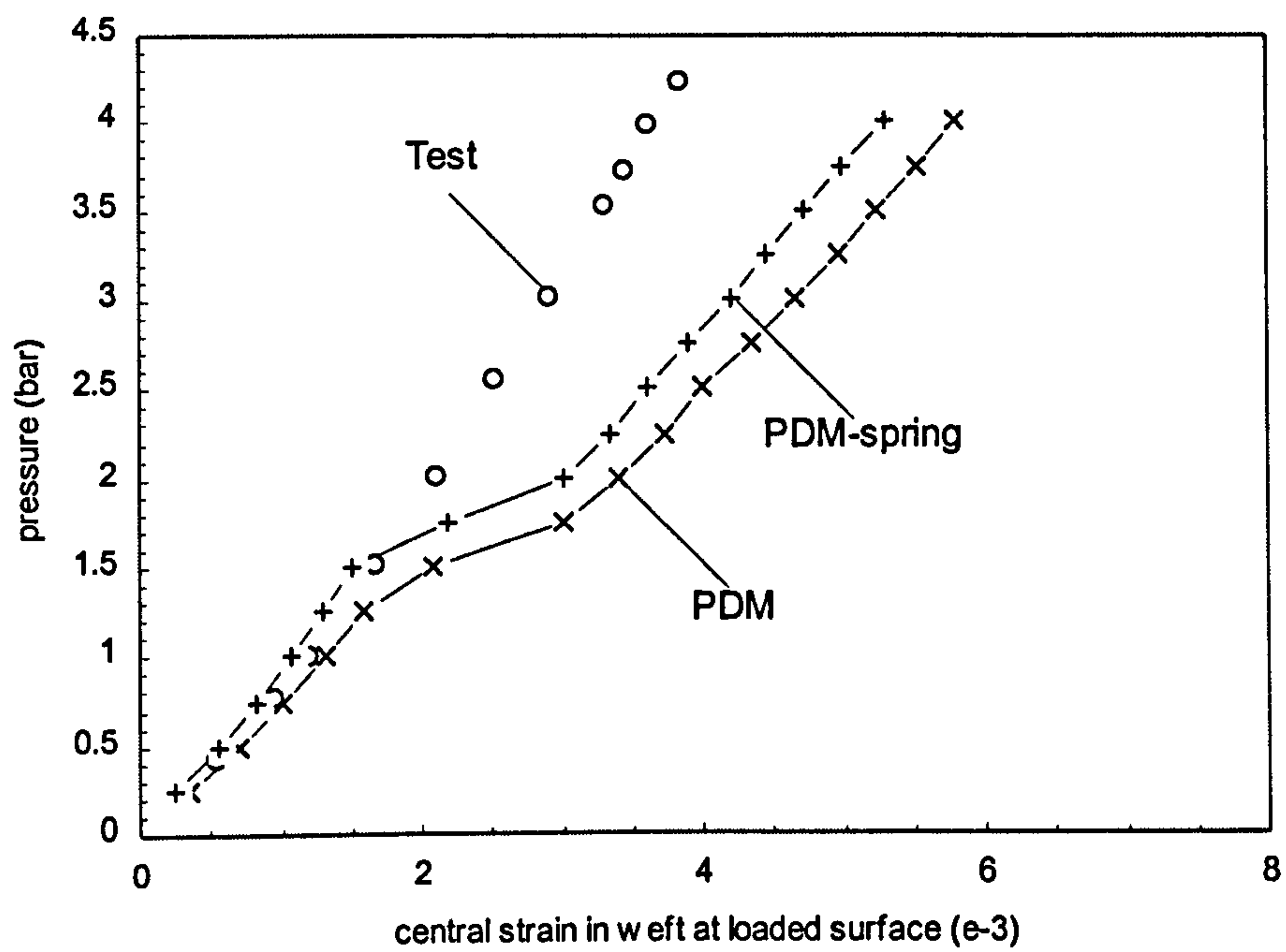


Figure 7.19 Central strain (R3) in the y-axis at loaded surface of Panel 30A (woven)

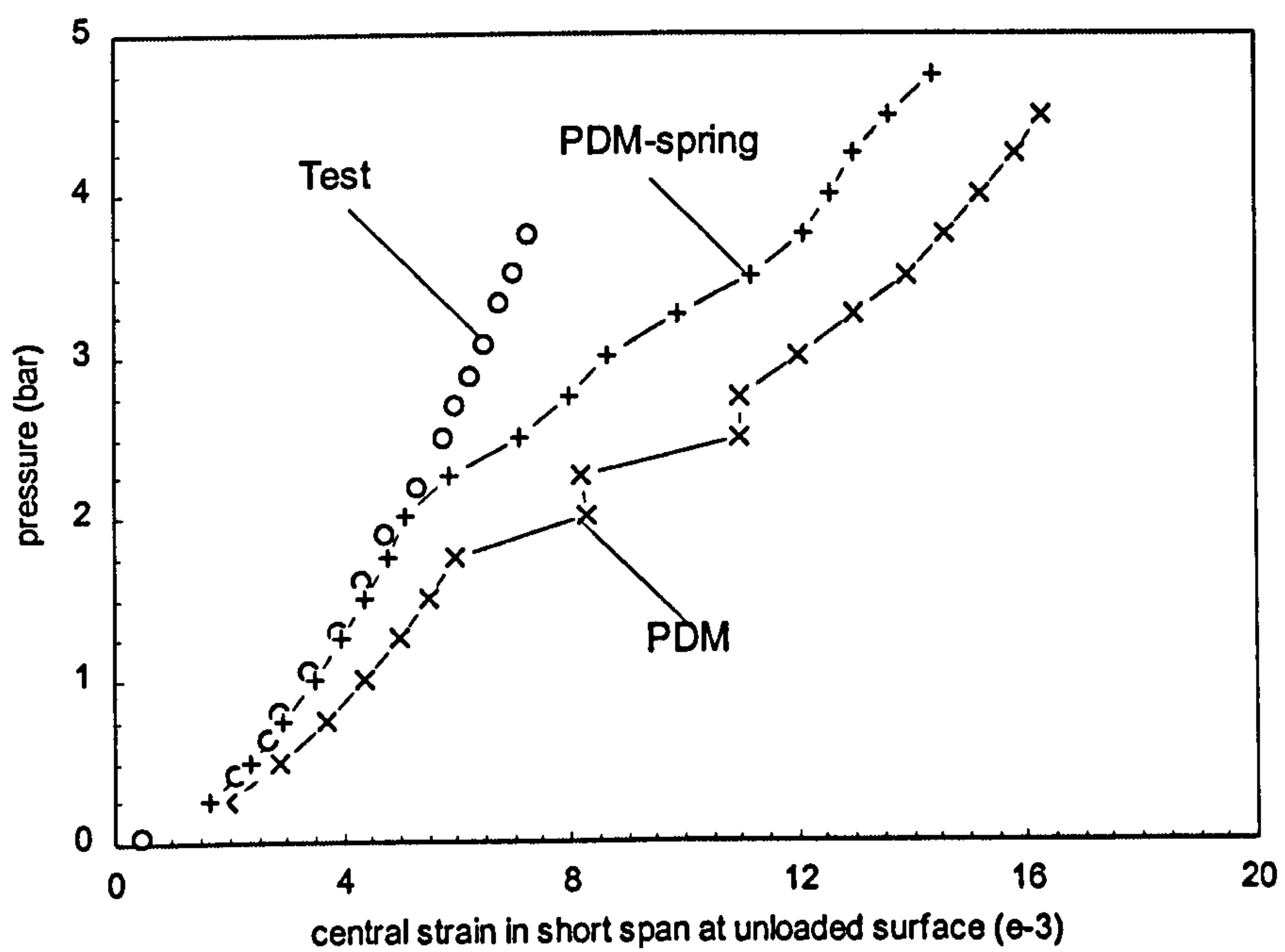


Figure 7.20 Central strain (Z3) in short span at unloaded surface of Panel 18B (non-woven)

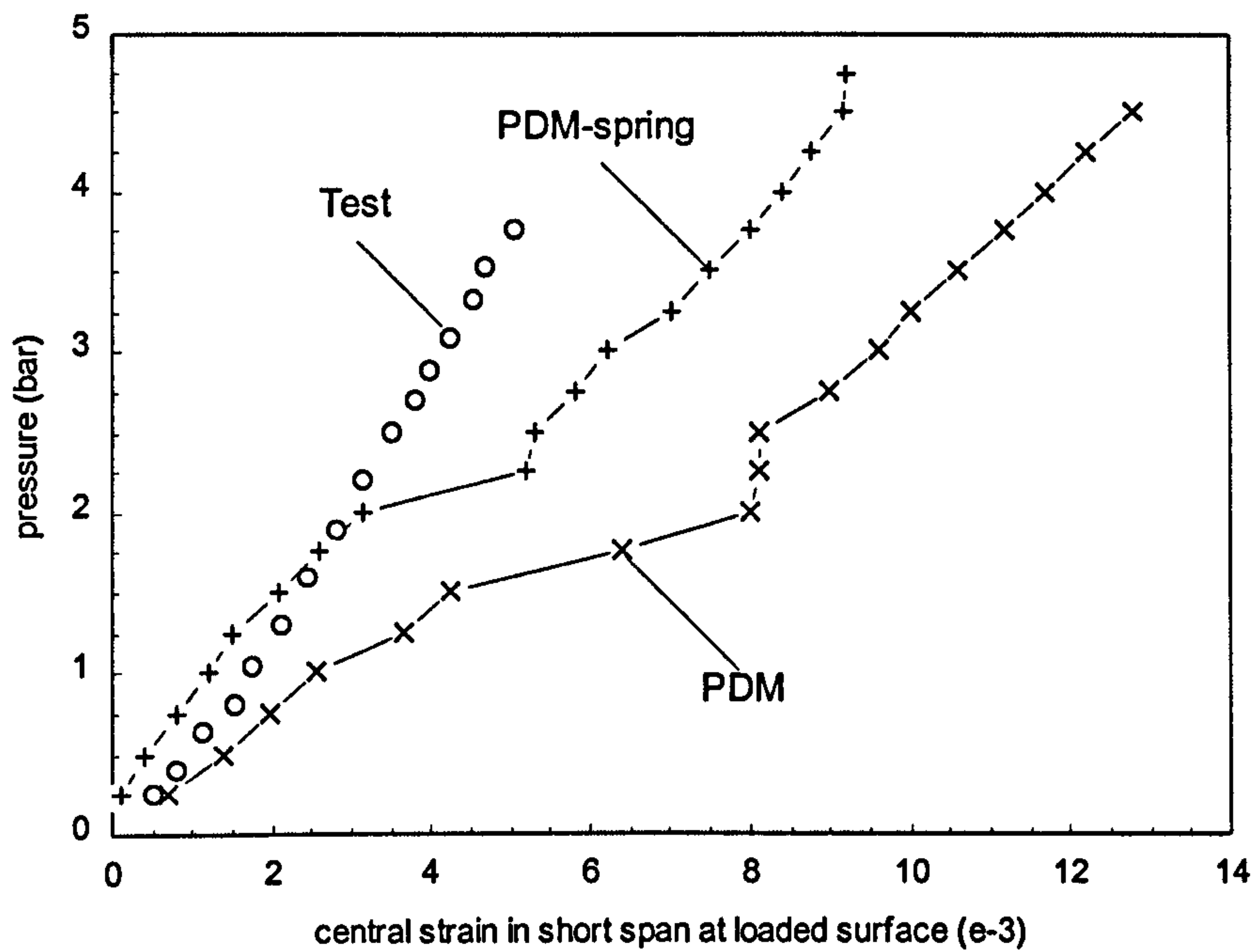


Figure 7.21 Central strain (R3) in short span at loaded surface of Panel 18B (non-woven)

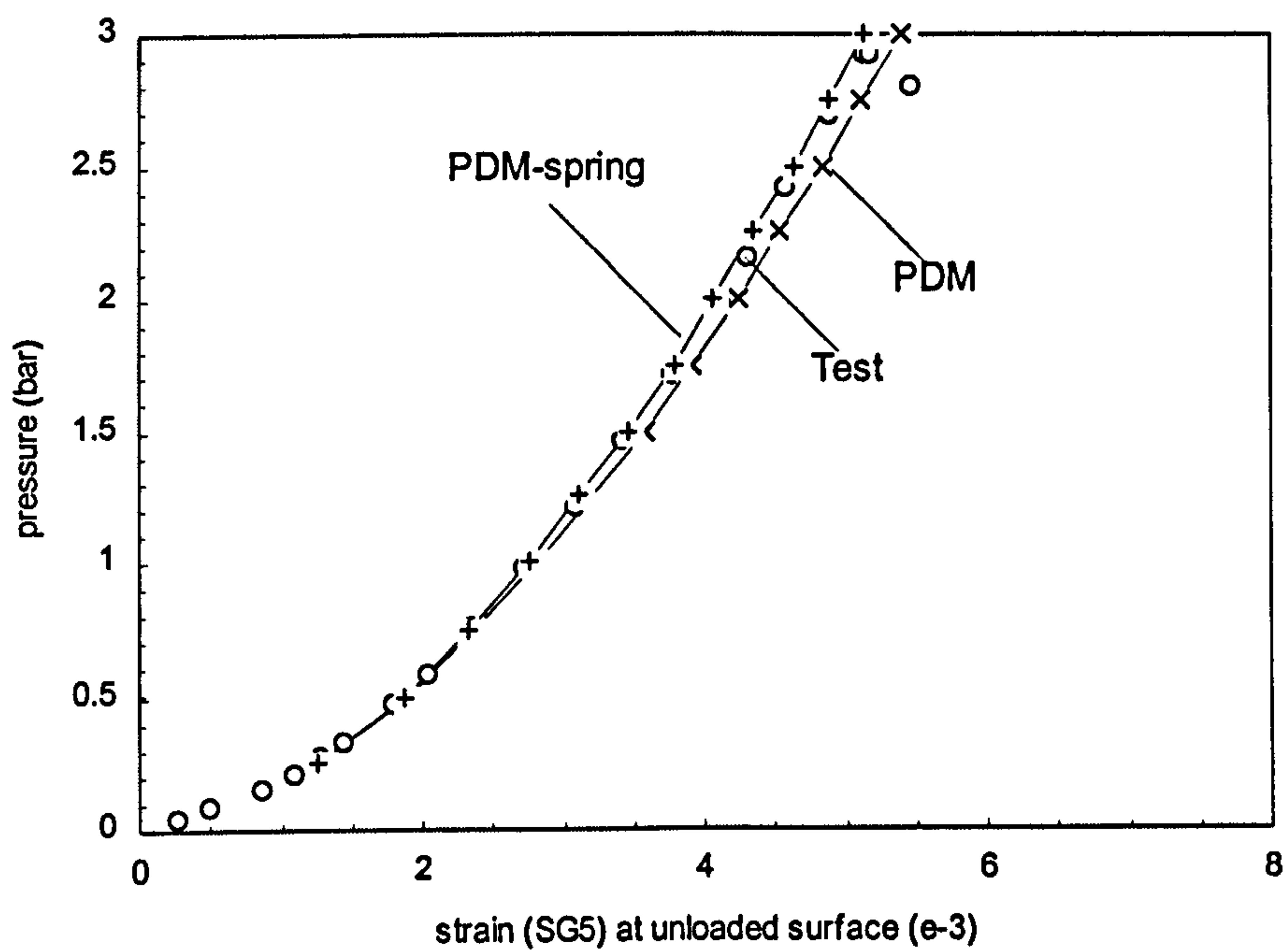


Figure 7.22 Strain (SG5) in middle of half span at unloaded surface of Panel D (woven) (strain at undamaged element)

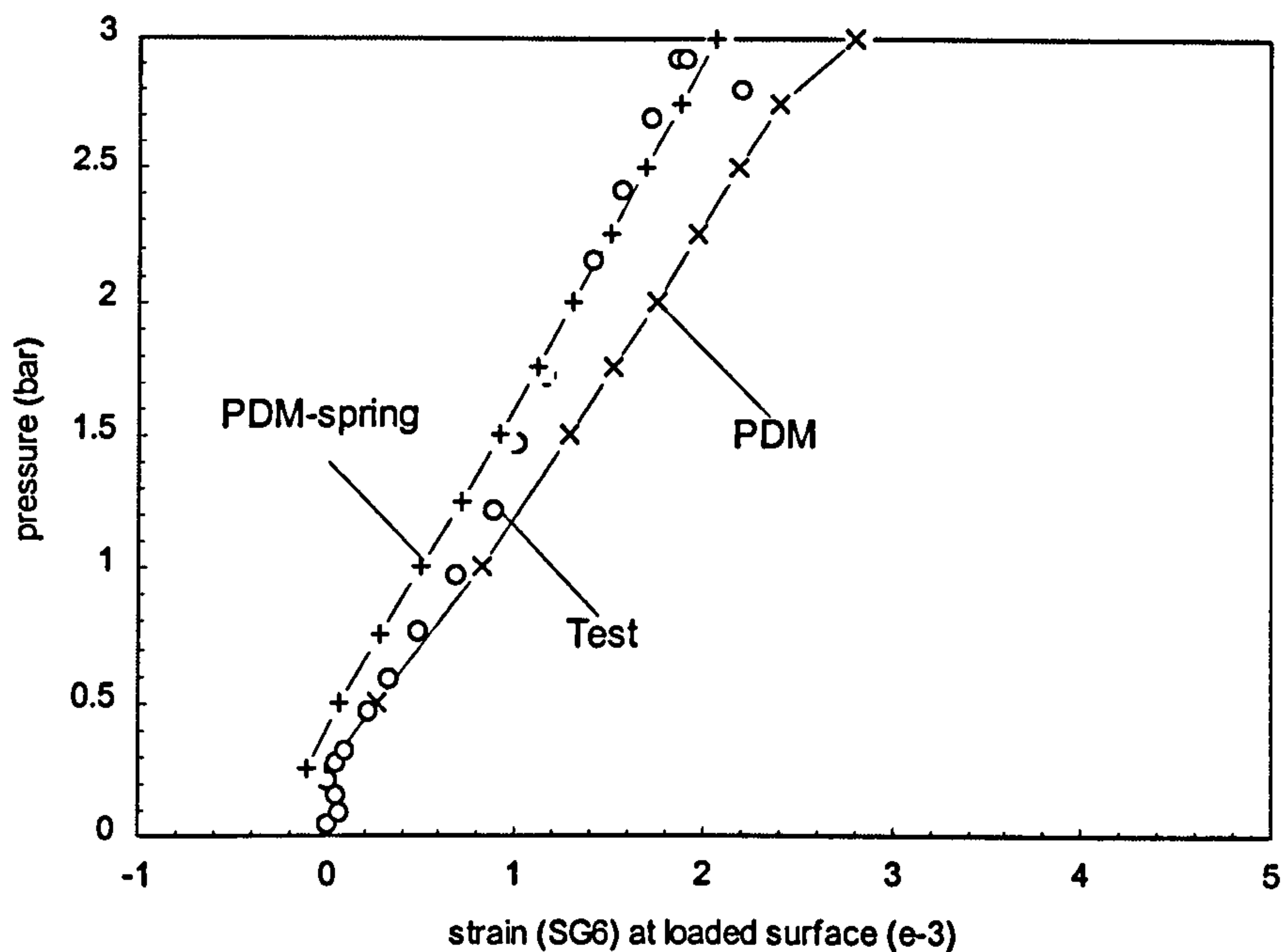


Figure 7.23 Strain (SG6) in middle of half span at loaded surface of Panel D (woven) (strain at undamaged element)

Figures 7.24 and 7.25 demonstrate the edge strains (50 mm away from the edge) in the x-axis at unloaded and loaded surfaces of Panel D. 50 mm away from the edge is considered as an area where panel behaviour is transferred from strong local bending around the edge to membrane. The strains predicted from the PDM-spring model at both unloaded and loaded surfaces agree with measured strains well. Figures 7.26 and 27 show the edge strains (50 mm away from the edge) in the y-axis at unloaded and loaded surfaces. As can be seen from Figure 7.26, the strains at unloaded surface predicted from the PDM-spring model also agree with the test results. However, the strains at loaded surface predicted from the PDM-spring model have large difference from the tested ones. This is probable because of the effect of the symmetrical edge displacement assumed in the models, while this is not true in the test. By comparing Figure 7.27 with Figure 7.25, it is clear that the strain curves at loaded surface in both the x and y axes predicted from the models have similar shapes and values. While the strains measured from the test lack symmetry between the x and y axes, as discussed in Chapter 4. This is particularly true at the point 50 mm away from the edge (see Figure 4.24).

Figures 7.28-7.31 show the edge strains at the unloaded and loaded surfaces in the x and y axes from the models and the tests for Panel D. In the predictions the transverse matrix damage occurs initially at the edge at small pressures and becomes extensive with the increase of the loading due to strong local bending and membrane tension. In the predicted edge strain curves, after the initial jump, the strains at unloaded surface become almost constant as observed in the test; and the strains at loaded surface increase slowly. The tendency of the strain growth in the predictions is similar to the test measurement.

The comparisons between predictions and tests above show that the methodology for progressive failure analysis of woven composite panel is successful. The method can provide good prediction of overall behaviour of the panel under high transverse pressure, and provide reasonable prediction of local behaviour of the panel.

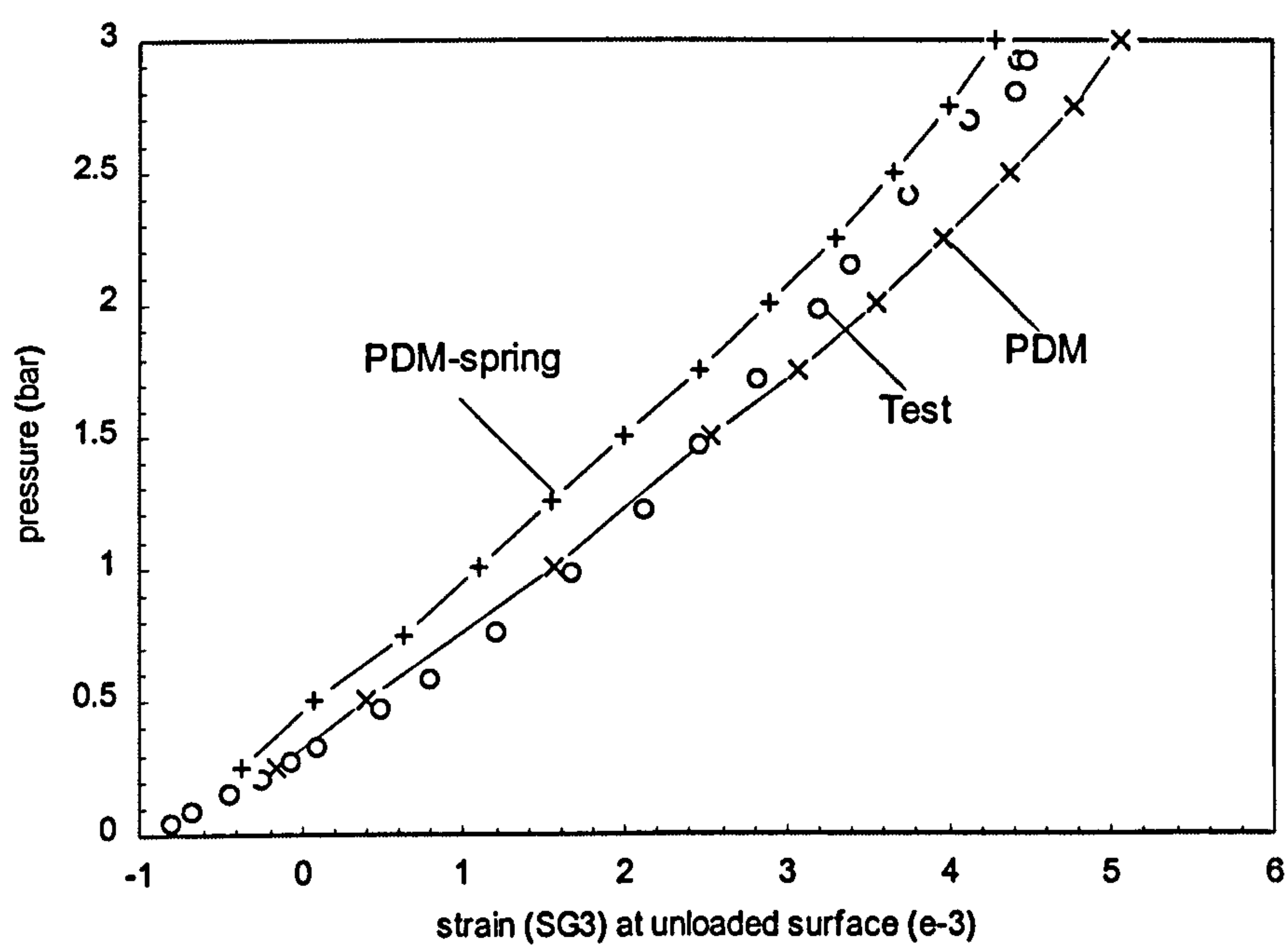


Figure 7.24 Edge strain (SG3) (50 mm away from the edge) in the x-axis at unloaded surface of Panel D (woven)

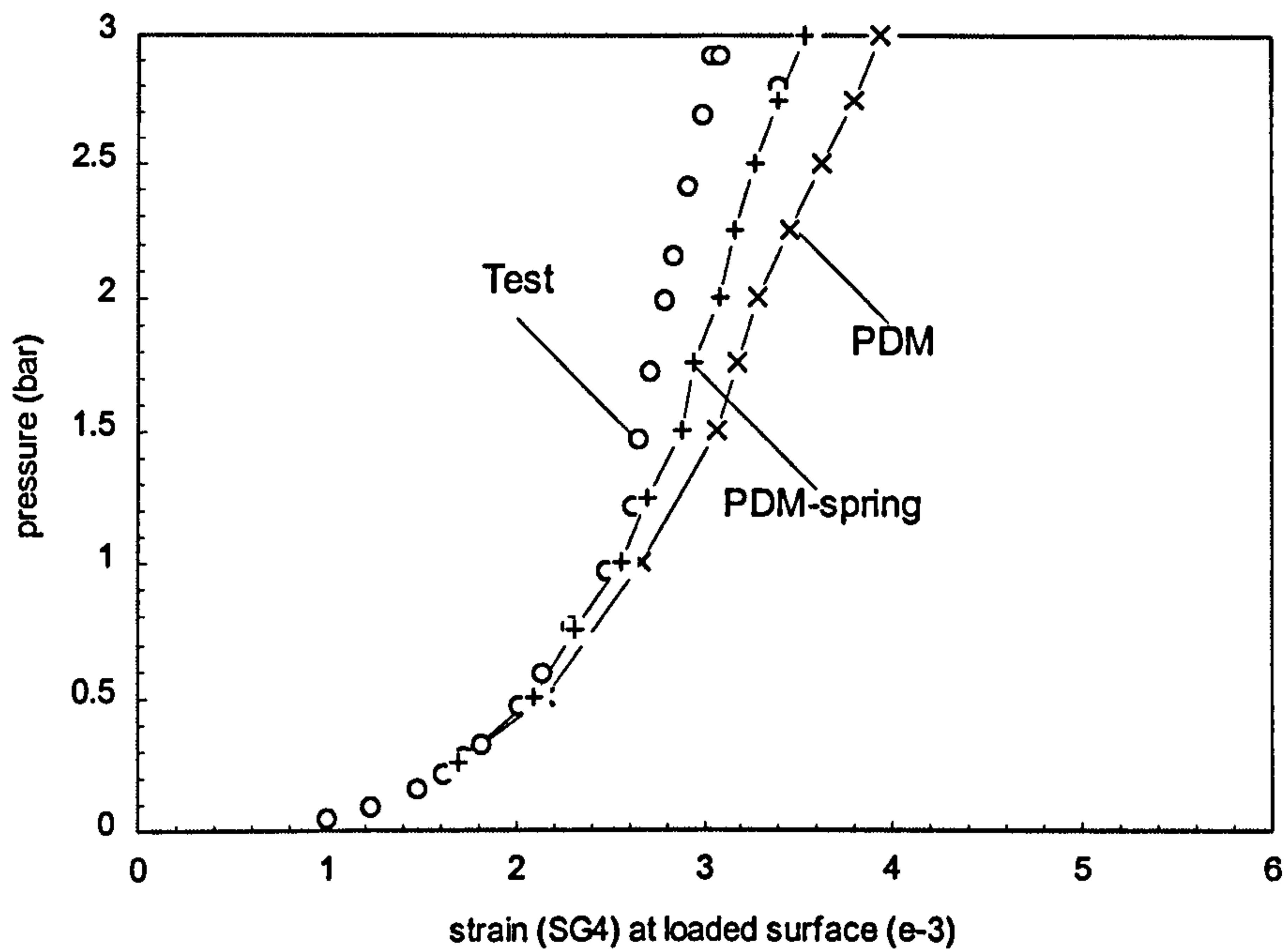


Figure 7.25 Edge strain (SG4) (50 mm away from the edge) in the x-axis
at loaded surface of Panel D (woven)

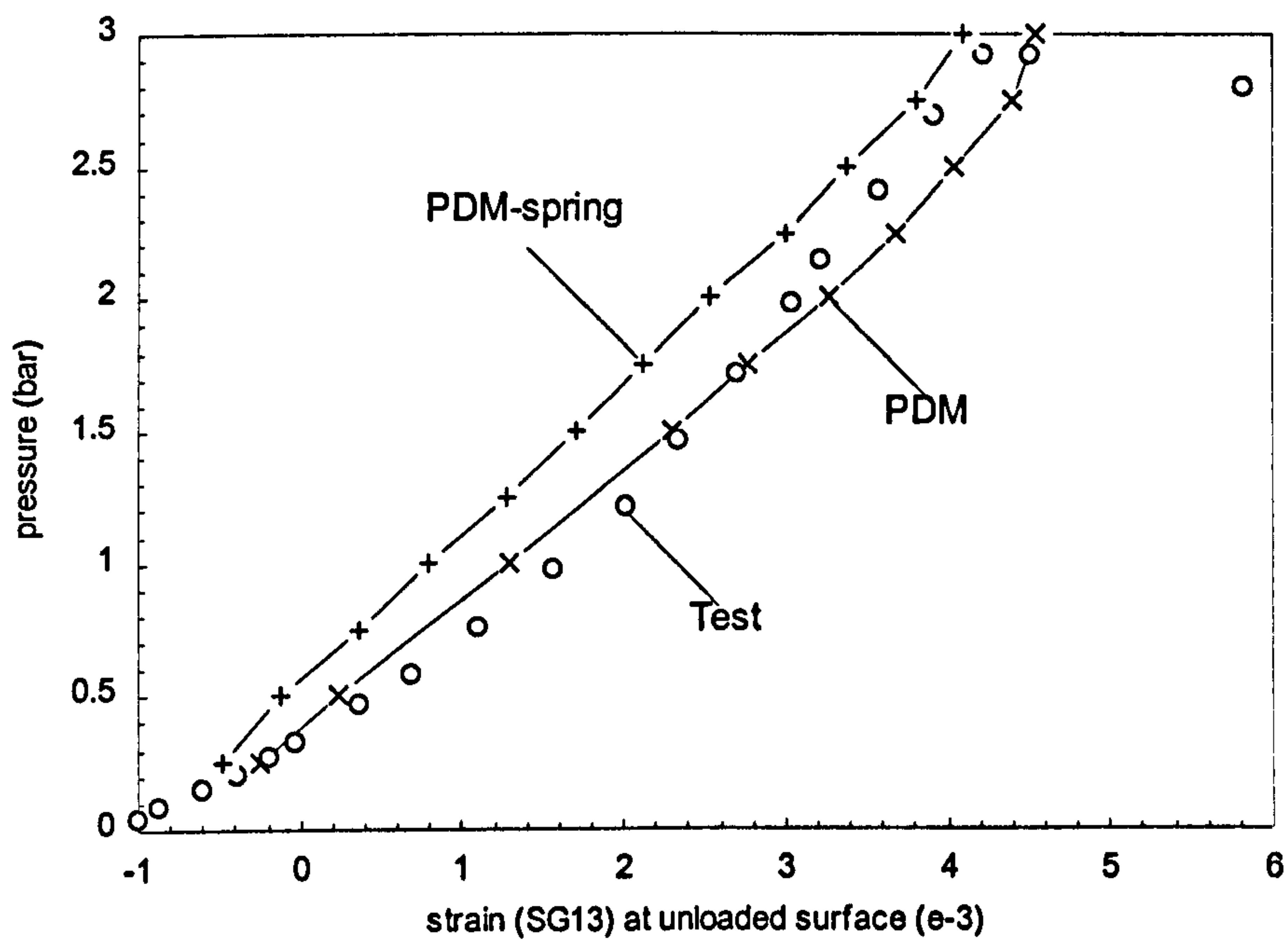


Figure 7.26 Edge strain (SG13) (50 mm away from the edge) in the y-axis
at unloaded surface of Panel D (woven)

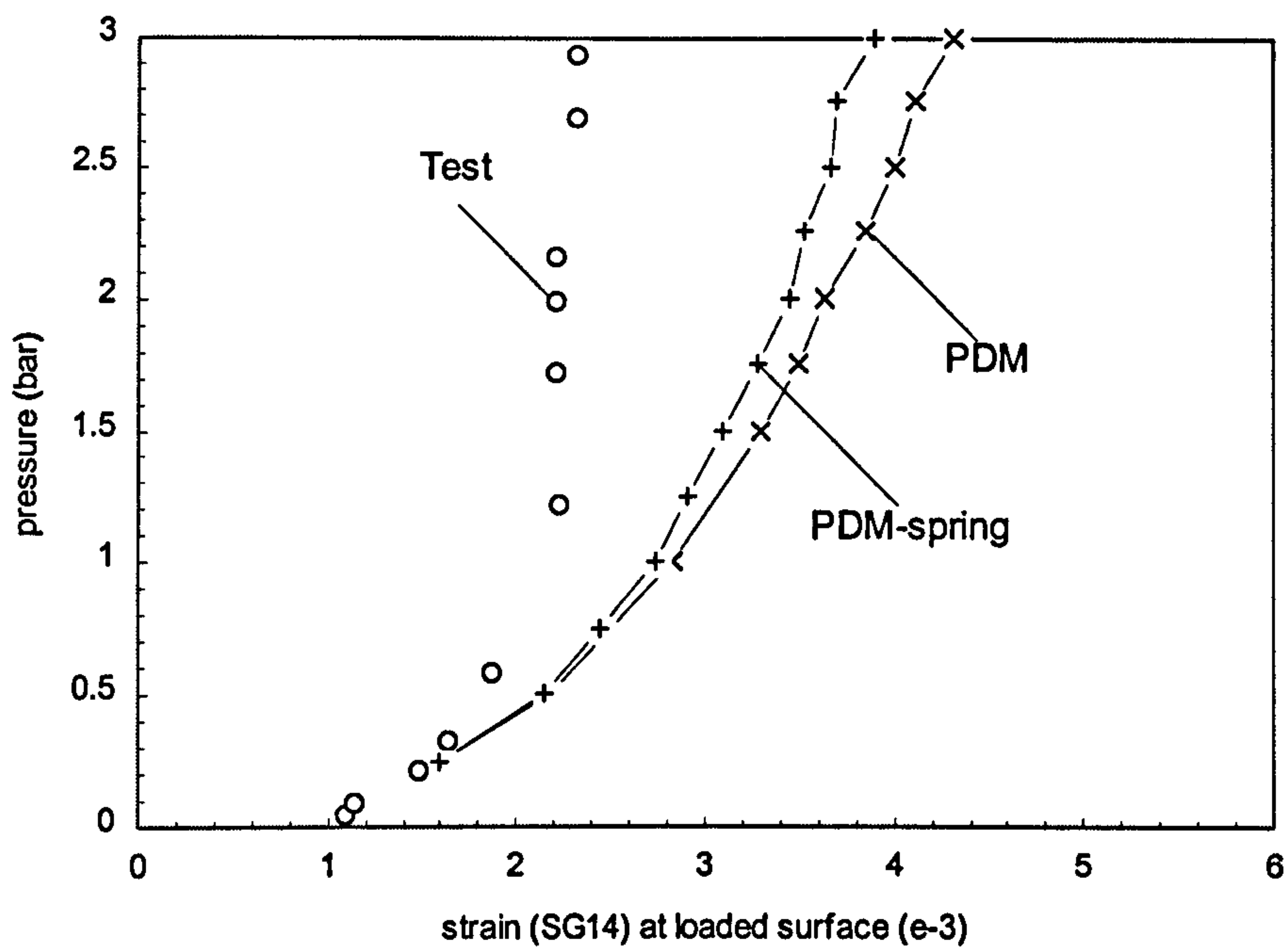


Figure 7.27 Edge strain (SG14) (50 mm away from the edge) in the y-axis at loaded surface of Panel D (woven)

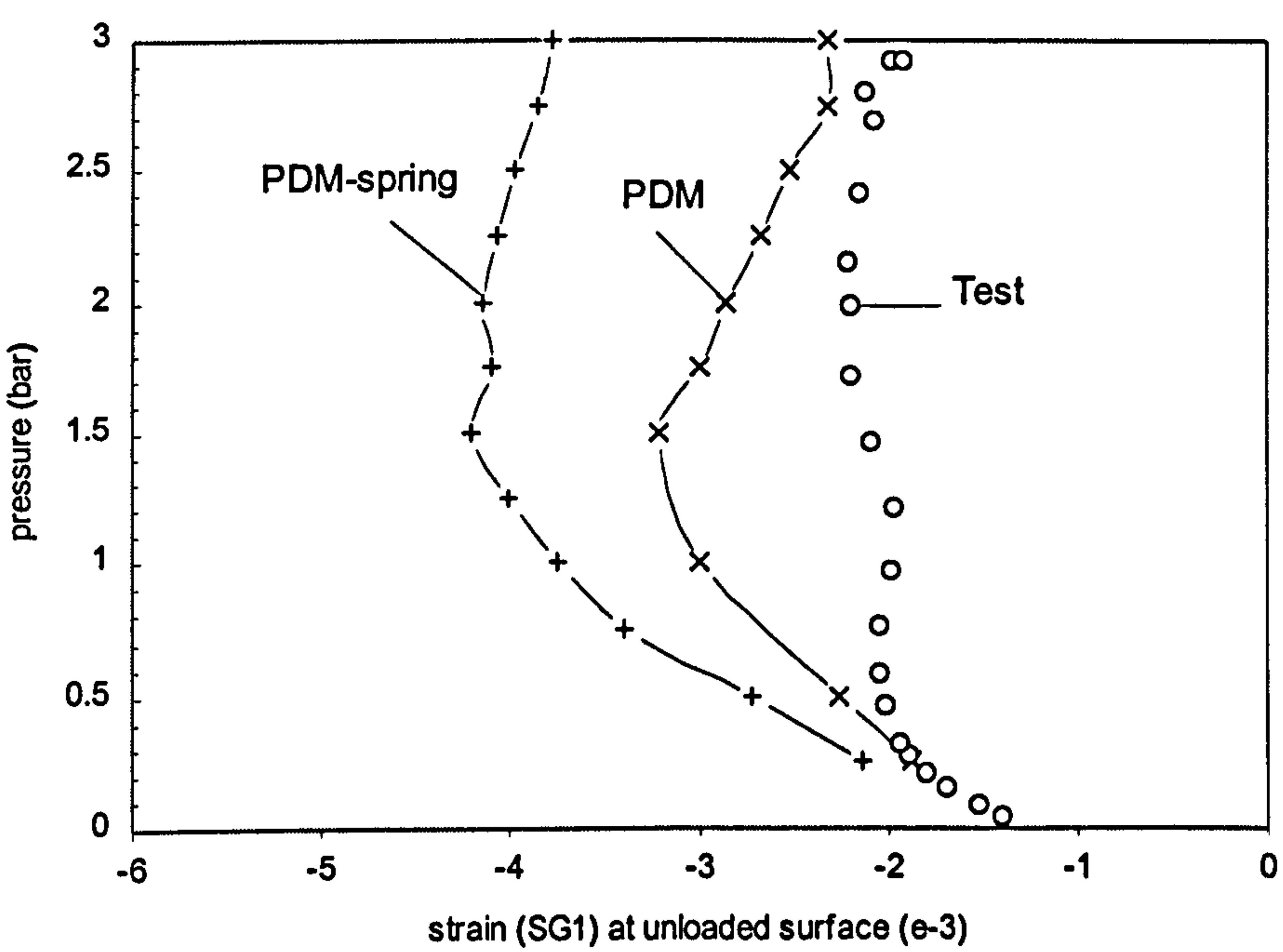


Figure 7.28 Edge strain (SG1) (10 mm away from the edge) in the x-axis at unloaded surface of Panel D (woven)

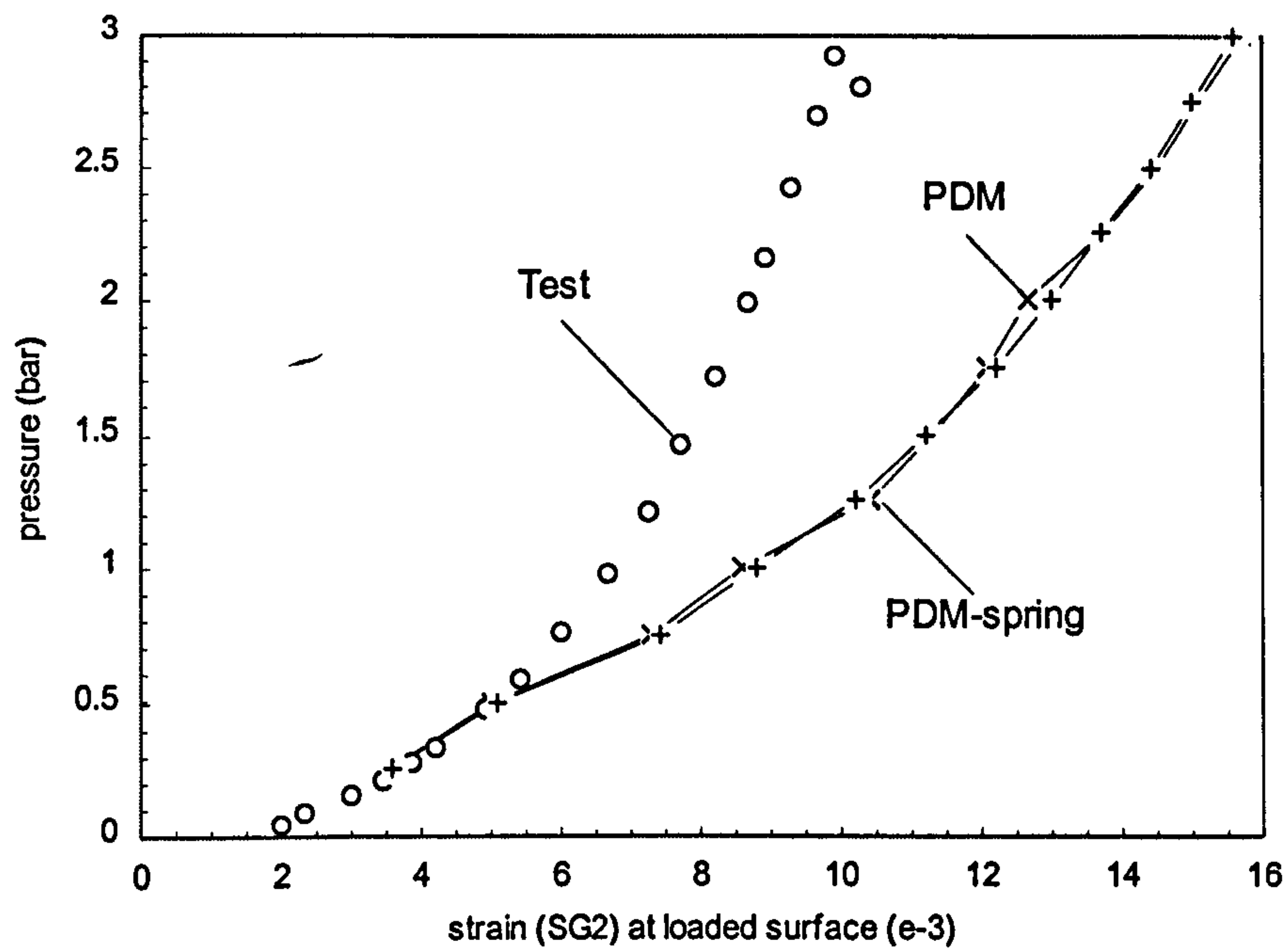


Figure 7.29 Edge strain (SG2) (10 mm away from the edge) in the x-axis at loaded surface of Panel D (woven)

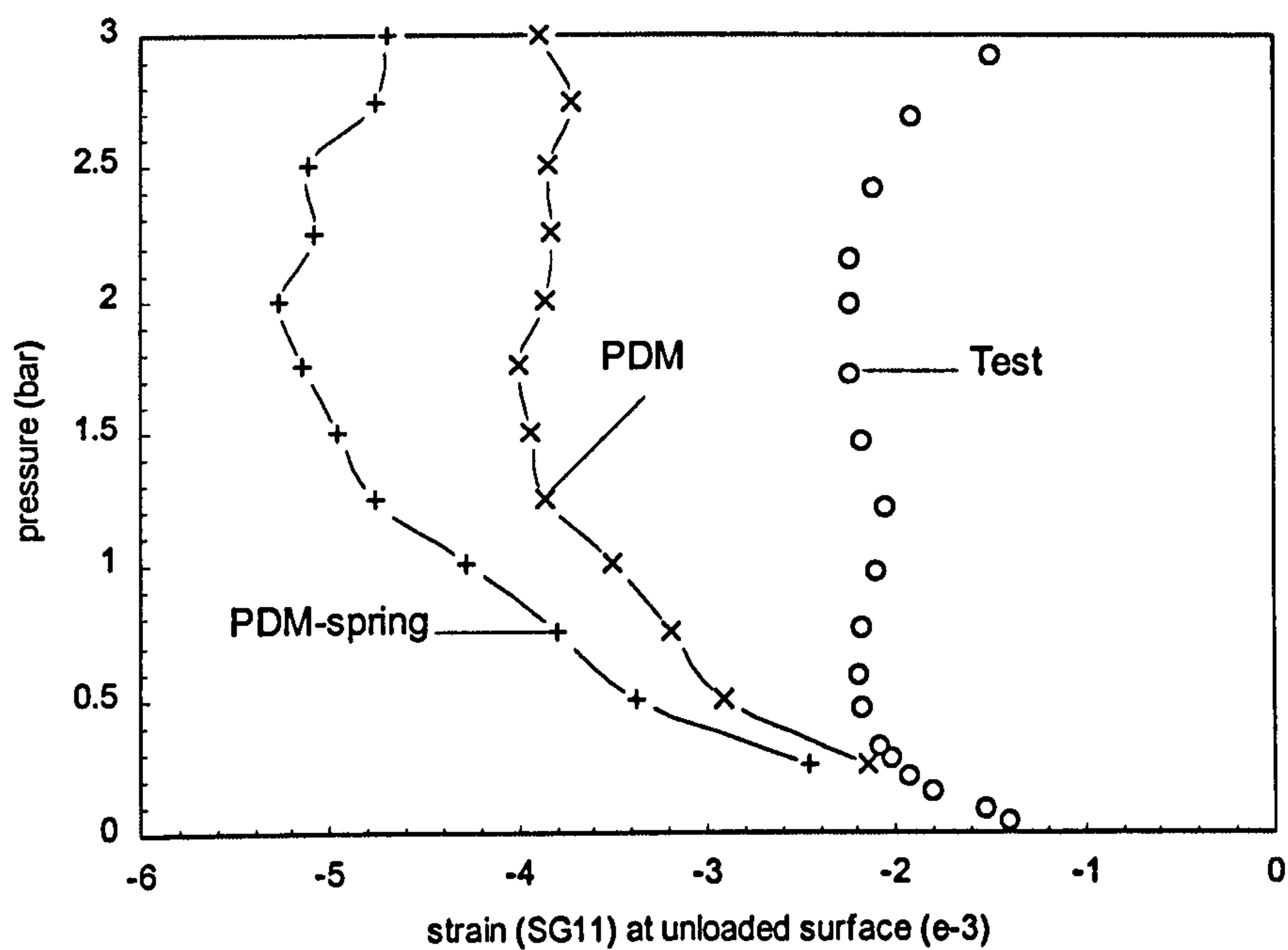


Figure 7.30 Edge strain (SG11) (10 mm away from the edge) in the y-axis at unloaded surface of Panel D (woven)

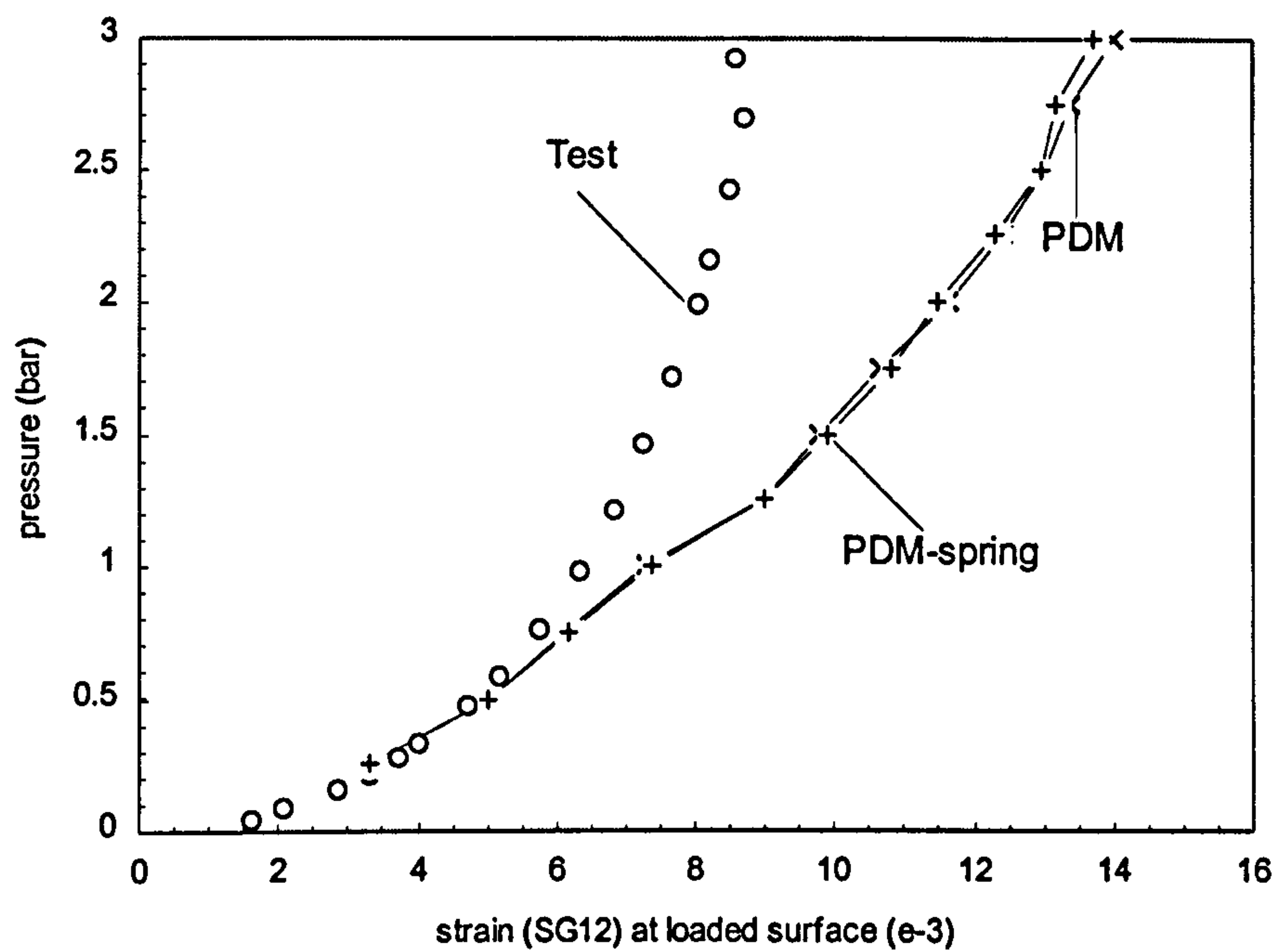


Figure 7.31 Edge strain (SG12) (10 mm away from the edge) in the y-axis at loaded surface of Panel D (woven)

7.10 Parametric Studies

The effects of two parameters, the number of elements and the number of load increments, on the damage behaviour of the panel were also studied on the *PDM model* for Panel D.

7.10.1 Number of load increments

Progressive failure analysis requires that the load increments are sufficiently small in order to ensure convergence and to capture possible damage in the panel. The more load increments, the more computer time is needed. In the initial progressive failure analysis, 500 load increments were used. The analysis of Panel D took about 8 hours CPU time in parallel processing computer, using 3 nodes with 256 Mbytes RAM in each node. In the parametric study the number of load increments was doubled. The same calculation is carried out on Panel D by using the PDM model. It is expected that the computer time in this analysis is also doubled. Table 7.9 lists some key results of the calculations and comparisons between 500 load increments and 1000 load increments.

The deflection results show that before any damage (0.5 bar), the deflections for both cases are the same. After the damage occurs, with more load increments, the deflections are slightly bigger and the pressure at failure is slightly reduced. This is probably that a larger number of increments allows the damage accumulation to be modelled more accurately. The strain jump still occurs in the analysis.

On the other hand, the comparisons show that 500 load increments may be sufficient to provide reasonably accurate results.

Table 7.9 Key results of the calculations and comparisons between 500 load increments and 1000 load increments

PDM model (<i>Panel D</i>)	Final failure (bar)	Pressure (bar)	Central Deflection (mm)	Central strain at unloaded surface in warp ($\times 10^{-3}$)	Central strain at loaded surface in warp ($\times 10^{-3}$)
500 load increments	3.1	0.5	12.6	1.6	0.43
		1	16.23	2.39	1.01
		1.5	18.75	3.05	1.52
		2	20.75	3.64	1.97
		2.5	22.45	4.19	2.4
		3	24.6	5.1*	3.1*
1000 load increments	3.06	0.5	12.6	1.6	0.43
		1	16.3	2.39	1.01
		1.5	18.8	3.05	1.52
		2	20.8	3.64	1.97
		2.5	22.8	4.43	2.49
		3	25.4	5.94*	4.58*

* a jump in strain occurs

7.10.2 Element refinement

To discuss the effect of the element refinement on the results, two analyses of Panel D were performed by using the PDM model, one with 225 shell elements, the other with 900. The re-meshed elements were obtained by splitting each element in initial mesh into four elements (that is, $4 \times 225 = 900$). It is obvious that more computer time is needed in the case with more elements. Table 7.10 lists some key results of the calculations and comparisons between 225 elements and 900 elements.

The deflection results show that the deflections for both cases are very similar to each other up to 2.5 bar; at near failure (3 bar), the deflection of the case with refinement is

slightly bigger than the deflection of the case with 225 elements. The final failure pressure for the refinement is slightly lower than that for 225 elements. The strain jump still occurs at near failure in the more refined analysis. The results indicate that the 225 elements used in the analysis can provide reasonable good results.

Table 7.10 Key results of the calculations and comparisons between 225 elements and 900 elements.

PDM model (<i>Panel D</i>)	Final failure (bar)	Pressure (bar)	Central Deflection (mm)	Central strain at unloaded surface in warp ($\times 10^{-3}$)	Central strain at loaded surface in warp ($\times 10^{-3}$)
225 elements	3.1	0.5	12.6	1.6	0.43
		1	16.23	2.39	1.01
		1.5	18.75	3.05	1.52
		2	20.75	3.64	1.97
		2.5	22.45	4.19	2.4
		3	24.6	5.1*	3.1*
900 elements	3.04	0.5	12.6	1.6	0.43
		1	16.23	2.39	1.01
		1.5	18.75	3.05	1.52
		2	20.75	3.64	1.97
		2.5	22.6	4.36	2.51
		3	25.2	5.71*	4.64*

* a jump in strain occurs

7.11 Conclusions

From the results of the analyses the following conclusions can be drawn.

- (1) Progressive failure analysis can predict damage initiation and the progressive failure sequence. This is verified by comparison with the damage patterns observed in the tests and predicted in the model. It is therefore an efficient way to understand the damage behaviour of thin composite panels under high transverse pressure.
- (2) The finite element modelling of the panels shows that the small in-plane edge displacement has a significant effect on the deflections and strains of the composite panel. The examples show that the deflections and strains predicted from the finite element modelling cannot match the test measurements unless the edge displacement is considered. The results also reveal that the edge displacement has little influence on the final failure pressure of the panel. The progressive damage model with springs around the edges (the PDM-spring model) in this research takes the edge displacement into account, enabling good correlation between the PDM-spring model and test measurements.
- (3) The analyses indicate that the *progressive damage behaviour*, *geometrical non-linear behaviour* and *small in-plane edge displacement* are three important characteristics for the thin composite panel under high transverse pressure.
- (4) The parametric studies show that the load increments and the element mesh used in the analysis are adequate to provide reasonable results of the panel.
- (5) The strain predicted in the progressive damage model is very sensitive to local damage. This is a shortcoming of the model and must be refined as the model is developed further.
- (6) Progressive failure analysis is very computer intensive. Although the analyses presented in this research were carried out on the IBM SP2 parallel processing computer, very large computer times were still needed.

Chapter 8 Stiffness-modelling of thin composite panels under high transverse pressure

8.1 Introduction

Chapter 7 presented a complete finite element model of the progressive failure behaviour of thin composite panels under high transverse pressure using the methodology for progressive failure analysis of a woven composite. The model provided sound predictions of the behaviour of the panels and a better understanding of the failure mechanism and failure sequence of the panels. The PDM model and the PDM-spring model were presented as useful tools to replace expensive panel tests.

Good predictions in progressive failure analysis require a large amount of computer time and a high-performance computer, such as the IBM SP2 parallel processing computer in this research. A typical job in this research needs about 400 minutes in the parallel computer using 3 nodes with 256 Mbyte RAM in each node.

From the engineering design point of view, an analysis should have sufficient accuracy with the least possible calculation time, and it should be possible to run it on a normal office-computer rather than a high-performance computer. The following two chapters are concerned with this approach. In this chapter, a simplified finite element model of thin composite panels under high transverse pressure is presented, and the geometrical non-linearity and the material non-linearity are separated. The geometrical non-linearity is modelled by means of large deflection option in the finite element package. The material non-linearity is modelled by two extreme cases, referred as 'Resin Effective' (RE) and 'Resin Ineffective' (RI). The model is concerned with stiffness-modelling so that there is no function in the model to predict the failure mechanism and strength-related behaviour of the panel. However, the model predicts the stiffness-related behaviour of the panel, such as deflections and strains. In this chapter, all the

analyses were carried out in the ANSYS finite element package in this university and were also compared with the tested panels of Panel D, Panel 30A and Panel 18B.

8.2 Stiffness-modelling of composite panels

8.2.1 Introduction to ANSYS

ANSYS is another large-scale general-purpose finite element package used in this university. Unlike ABAQUS, ANSYS (version 5.1) is installed on local *SiliconGraphics* Unix workstations rather than on the high-performance parallel processing computer. Therefore, running the same job in ANSYS is much cheaper but takes a longer time than in ABAQUS.

Modelling a problem in ANSYS is also divided into three stages: pre-processing, solution and post-processing. The self-contained pre-processor in ANSYS provides an easy way to model the geometry of the problem. In the solution phase, the user defines analysis options such as static or dynamic, linear or non-linear problem, applies loads and specifies load steps. Then the problem can be solved.

The post-processor in ANSYS is very convenient and user-friendly. In the general post-processor POST1, the results in a certain load increment can be reviewed. In the time history post-processor POST26, the results during the time-history can be examined. All commands are operated simply by clicking corresponding buttons on the screen.

8.2.2 Element types available

There are different element types available for the modelling of composite materials in ANSYS [83]: *Shell99*, *Shell91*, and *Solid46* , as follows.

Shell99 is an 8-node, 3-D shell element with six degrees of freedom at each node. It is designed to model thin to moderately thick layered plate and shell structures with a side-to-thickness ratio roughly 10 or greater. The *Shell99* element allows a total of 100 uniform-thickness layers, with failure criterion calculations and with geometrical non-linear calculations.

Shell91 is similar to *Shell99* except that it allows a maximum of 16 layers and no failure criteria are computed. However, *Shell91* allows one to model non-linear material behaviour, whereas *Shell99* does not.

Solid46 is a layered 8-node, 3-D solid element with three degrees of freedom per node. It is designed to model layered shells or layered solids and allows up to 100 uniform-thickness layers per element. An advantage with this element type is that one can stack several elements to model more than 100 layers to allow through-the-thickness discontinuities of gradient.

In addition, there are two shell element types. *Shell63* and *Shell93* can be used to model orthotropic shells. *Shell63* is an 4-node shell element with six degrees of freedom at each node. The element permits geometrical and material non-linear analysis. *Shell93* basically has the same function of the *Shell63* but it is a 8-node shell element, and particularly well suited to model curved shells.

8.2.3 Element type used in the analysis

In this simplified finite element analysis, the main objective is to obtain the deflection and strain responses of the panel, rather than a detailed progressive failure response. Therefore, no failure criterion is needed to examine any failure in each of the layers. If the laminated composite panel is considered as a single-layered panel, Panel D and Panel 30A can be treated as orthotropic panels because of the symmetrical disposition of the reinforcement in the plane of the panels. If Panel 18B is considered as a single-layered panel, it can be also treated as a orthotropic panel.

From this point of view, in the stiffness-modelling of the composite panels the Shell63 element can be used in the simplified finite element analysis, instead of using complicated layered element types.

8.2.4 Stiffness reduction scheme in the analysis

As mentioned above, in the material non-linearity modelling in this simplified finite element analysis, a simple stiffness reduction scheme was introduced in which two extreme conditions of the composite were considered. One is called the *Resin Effective* (RE), and the other is called the *Resin Ineffective* (RI). In the RE the resin is assumed to be fully effective and linear elastic. This matches the initial stiffness of the composite panel at a low stress level before any damage occurs. In the RI the resin is assumed to be ineffective and the elastic modulus of the resin is therefore set to very small value. This assumes that near failure the contribution of the resin to the stiffness of the composite material become small because of the resin cracking; all of the load has been transferred to the reinforcement. It is considered that the *Resin Effective* and the *Resin Ineffective* solutions should provide limits or bounds to the actual behaviour of the composite panels under extremely high pressure.

The overall stiffnesses for both the RE and the RI of the composite material in this scheme can be calculated by using the theoretical model presented in Chapter 5. For the RE case, the elastic properties of the composite predicted in the theoretical model were confirmed by the initial stiffnesses in various directions measured in the coupon tests. For the RI case, the stiffness of the resin was set to small value (10 N/mm²) in the theoretical model to avoid the difficulty in computation and this provided the reduced stiffnesses of the composite panel. The material properties of actual composite panel should lie between the stiffnesses of the two extreme conditions. Table 8.1 lists the *Resin Effective* and *Resin Ineffective* stiffnesses for different composite materials used in the finite element analysis.

Table 8.1 The stiffnesses of composite materials used in finite element analysis

	Property	Initial Stiffness (RE)	Reduced Stiffness (RI)	Stiffness Reduction <i>RI/RE</i> (%)
Panel 30A	E_x MPa	27.2	19.2	30
	E_y MPa	27.2	19.2	30
	G MPa	5.1	0.02	--
	ν_{xy}	0.157	0.0008	--
Panel 18B	E_x MPa	16.0	10.1	35
	E_y MPa	13.4	6.7	50
	G MPa	4.9	2.3	--
	ν_{xy}	0.304	0.208	--
Panel D	E_x MPa	18.35	13.1	29
	E_y MPa	17.80	12.1	30
	G MPa	3.37	0.01	--
	ν_{xy}	0.15	0.0007	--

8.2.5 Finite element implementation and modelling in-plane edge displacement

To model the geometrical non-linear behaviour of the panel, the large deflection option in ANSYS was switched on. To model material non-linear behaviour of the panel, two extreme cases of the RE and RI were considered. The finite element mesh for the panels was similar to that in Chapter 7, in which the elements were meshed intensively around the edges. Figure 8.1 shows the finite element mesh for square panels. The dimensions of the elements are shown in Appendix 5.

As in-plane edge displacement was important in the modelling, two models, the RE with spring support (RE-spring) and RI with spring (RI-spring), were calculated in ANSYS. The springs were attached the edges in the same way as described in Chapter 7 and allowed the edges to move slightly in the plane of the panel, but rotation at the edge was held at zero. The stiffness of the spring for the RE and the RI of the panels are listed in Table 8.2. It is noted that the spring stiffness for the RE is slightly different from that for the RI in the table. This is probably because of the way in which the spring stiffness is chosen to match the measured edge displacement in the trial-and-error method.

From the symmetry of the panel and the boundary conditions assumed, only a quarter of the panel is modelled.

Table 8.2 The stiffness of springs used in RE and RI

Panel	Spring Stiffness (N/mm ²)	
	Resin Effective	Resin Ineffective
Panel D	1100	970
Panel 30A	380	300
Panel 18B	95	80

8.3 Analysis of the results

In this simplified finite element analysis, only 20 load increments (or fewer) were required to ensure convergence. Computer time was therefore much less than the progressive failure analysis in Chapter 7 if the program was run in the same computer. The RE model here corresponds to the progressive failure analysis at low pressure when no damage has occurred. After the matrix cracking, the stiffness of the composite material is reduced. At the extreme case, the matrix cracks everywhere in the panel, and

this is represented by the RI model. The progressive failure results should therefore lie between the results of the RE model and the RI model. From this point of view, the RE-spring model and the RI-spring model are compared with the progressive failure analysis model with spring supports (the PDM-spring) and tests.

8.3.1 Comparison of deflections

Figures 8.2-8.4 show the comparisons of central deflections among the *Resin Effective* with spring supports (the RE-spring), the *Resin Ineffective* with spring supports (the RI-spring), the *Progressive Damage Model* with spring supports (the PDM-spring) and tests for Panel D, Panel 30A and Panel 18B. As can be seen, the RE-spring and the RI-spring provide bounds of the test as well as the PDM-spring. The test and the PDM-spring curves develop from the RE bound to the RI bound. This process illustrates how the panel changes from perfect undamaged panel to damaged one.

The comparison shows that the simple finite element analysis of the RE and RI cases can provide good predictions of the deflections of the composite panels under high transverse pressure.

8.3.2 Comparison of strains

Figures 8.5-8.8 demonstrate the central strains in the x-axis (the warp direction) and the y-axis (the weft direction) at unloaded and loaded surfaces among the models and test for Panel D. As mentioned in Chapter 7, the central element does not crack until the pressure of 2.75 bar, therefore, there is no apparent strain jump in the strain curve. It can be seen that the strain curve from the *Resin Effective* is always close to the test and the PDM-spring curves, while the strain curve from the *Resin Ineffective* is far away from the test and the PDM-spring curves. This indicates that the *Resin Ineffective* case may exaggerate the cracking in this panel.

Figures 8.9 and 8.10 show the central strains in the y-axis at unloaded and loaded surfaces for Panel 30A. From the progressive failure analysis of Panel 30A in Chapter 7, the matrix in the central element cracks at about 1.5 bar. After the damage, a strain jump in the curve occurs. The figures illustrate that the strain curves from the PDM-spring jump from the curve from the RE-spring to the curve from the RI-spring at about 1.5 bar - 2 bar.

The results reveal that the strain is local behaviour. The strain in the damaged element is close to the strain from the *Resin Ineffective* case. While the strain in the undamaged element is close to that from the *Resin Effective* case. The results also confirms that the RE-spring and the RI-spring provide the bounds of the PDM-spring.

However, the test curve does not have the strain jump; it is always close to the curve from the *Resin Effective*. This interesting result is probably because central part of the *tested panel* has less damage than the edges, and the small cracks are scattered across the panel, as analysed in Chapter 7.

Figures 8.11 and 8.12 show the central strain in short span at unloaded and loaded surfaces for Panel 18B. The same situation can be seen.

As to the edge strains, the situation is complicated because of the intensive cracks around the edges. Figures 8.13 and 8.14 demonstrate the edge strains at unloaded and loaded surfaces for Panel D. After damage, which occurs at early stage of loading, the strain curve from the PDM-spring develops in zigzag shape but with the same trend as the test. The edge strain curves from the RE-spring and the RI-spring also have the same tendency as the test. The comparisons of the edge strains between the models and the tests show that the *Resin Effective* and the *Resin Ineffective* can only predict the trend of the edge strain development, as the PDM-spring model does.

8.4 Conclusions

The finite element analysis described in this chapter shows that the complicated progressive failure analysis can be simplified by analysing two extreme cases of the panel: *Resin Effective* and *Resin Ineffective*. The results show that the *Resin Effective* and the *Resin Ineffective* cases provide bounds of those from the progressive failure analysis and tests. To calculate the same problem, the computer time used in this simple FE analysis is much less than that used in the progressive failure analysis.

The results show that the deflection curves from the test and the PDM-spring model follow a transition curve from the *Resin Effective* at low load to the *Resin Ineffective* at high load.

The results also show that the central strain vs. pressure curves from the test are close to the corresponding curves from the RE-spring model. The edge strain vs. pressure curves from the RI-spring and RE-spring models have the same trends as the edge strains from the tests and the PDM-spring model.

Unlike the progressive damage model, this simple FE analysis is unable to predict the progressive damage sequence and the strength of the panel. However, the *Resin Effective* and the *Resin Ineffective* models can be used as a quick analysis tool to predict deflections and strains in the panel, if high performance computer facilities and a large amount of computer time are not available.

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FEB 17 1998
09:59:18
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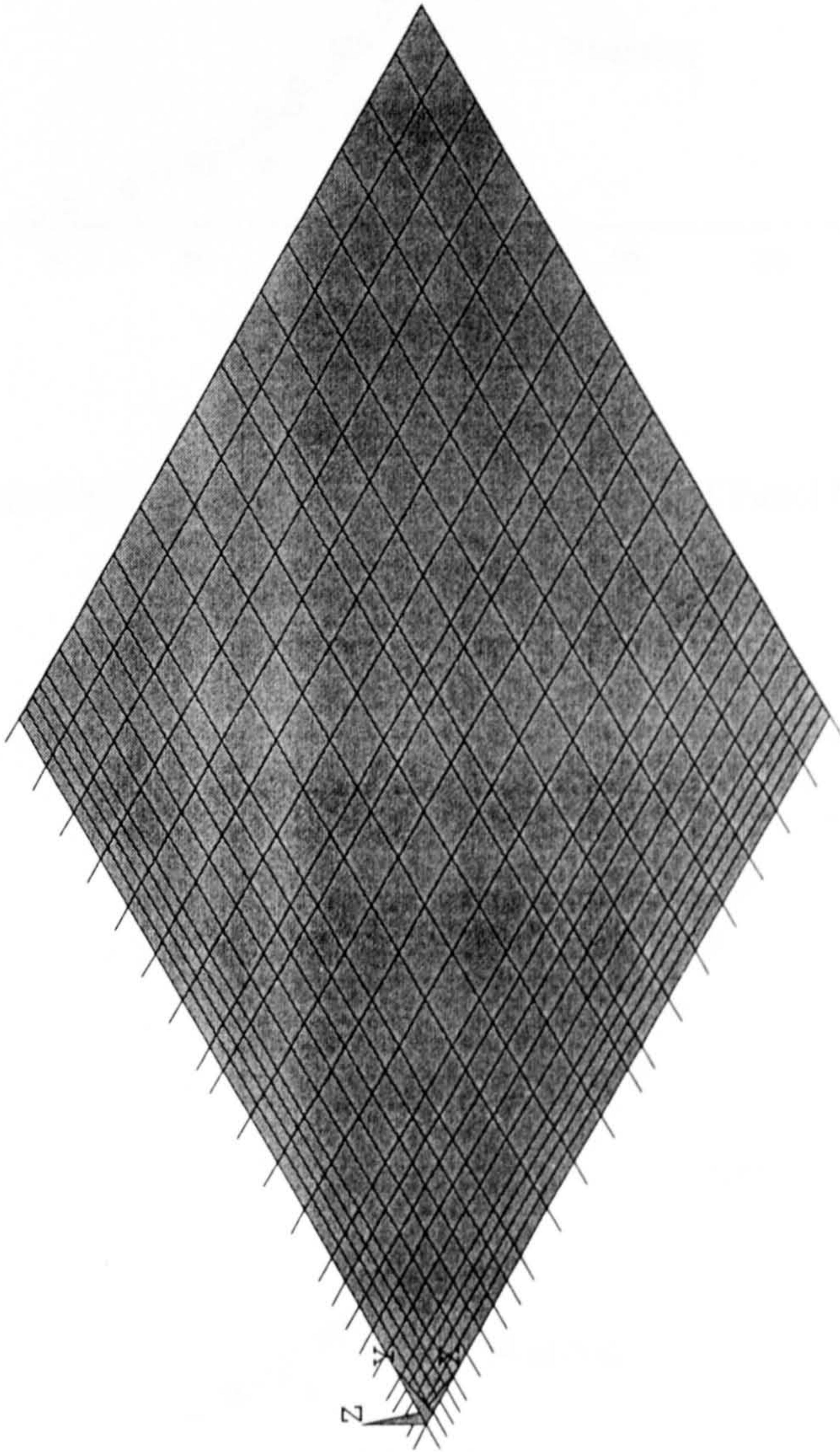


Figure 8.1 Demonstration of finite element mesh for square panels

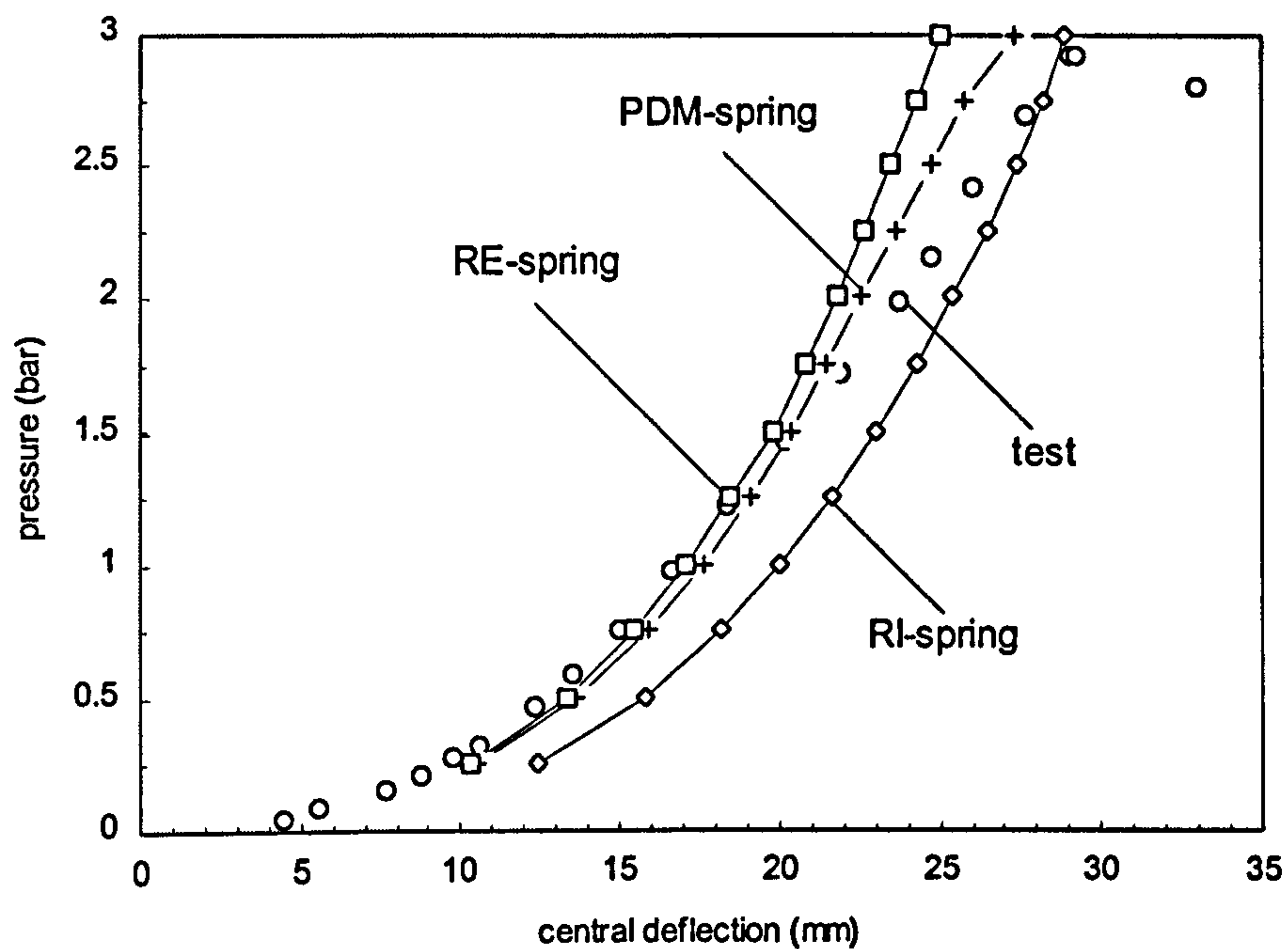


Figure 8.2 Central deflection vs. pressure of Panel D

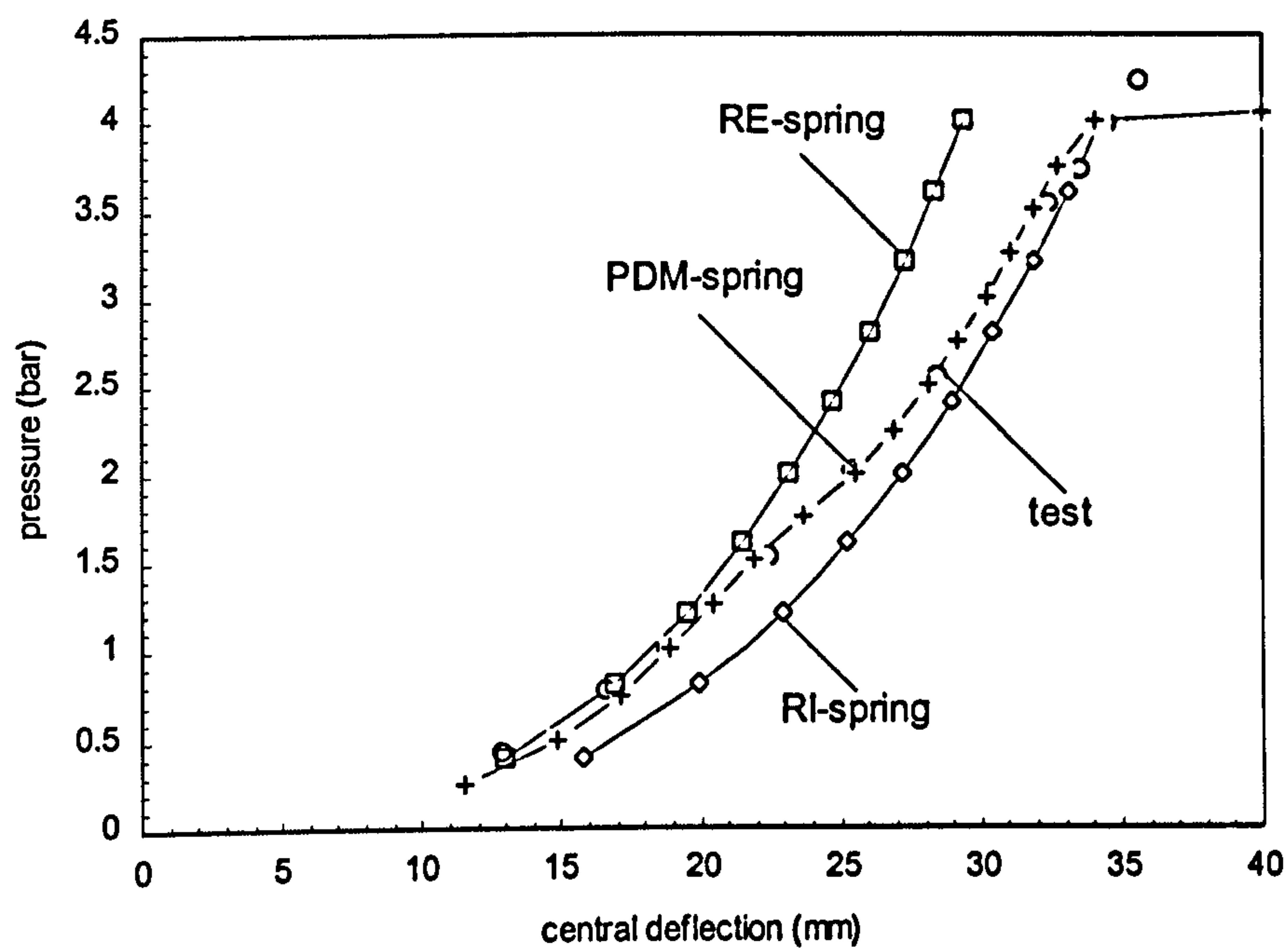


Figure 8.3 Central deflection vs. pressure of Panel 30A

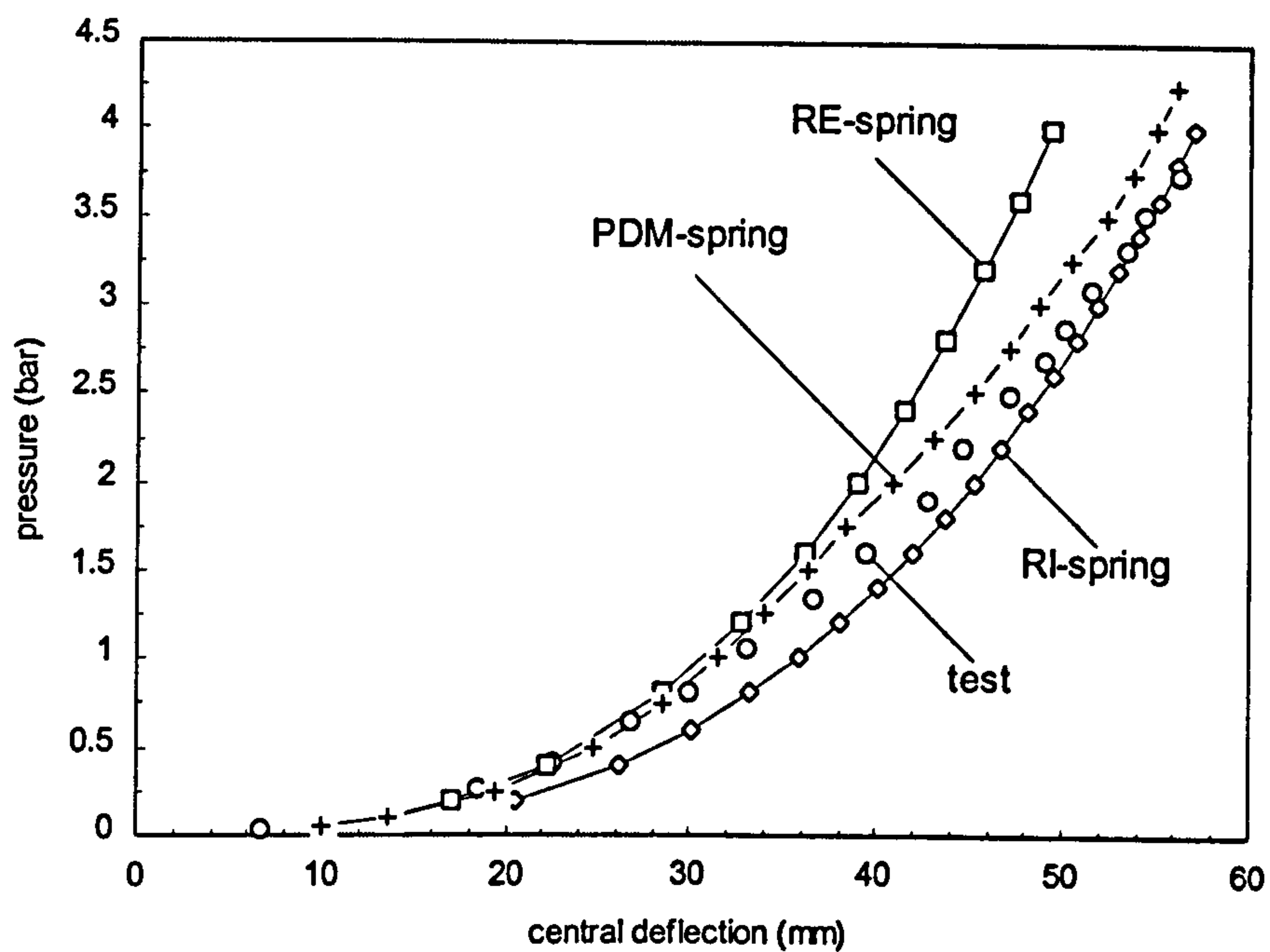


Figure 8.4 Central deflection vs. pressure of Panel 18B

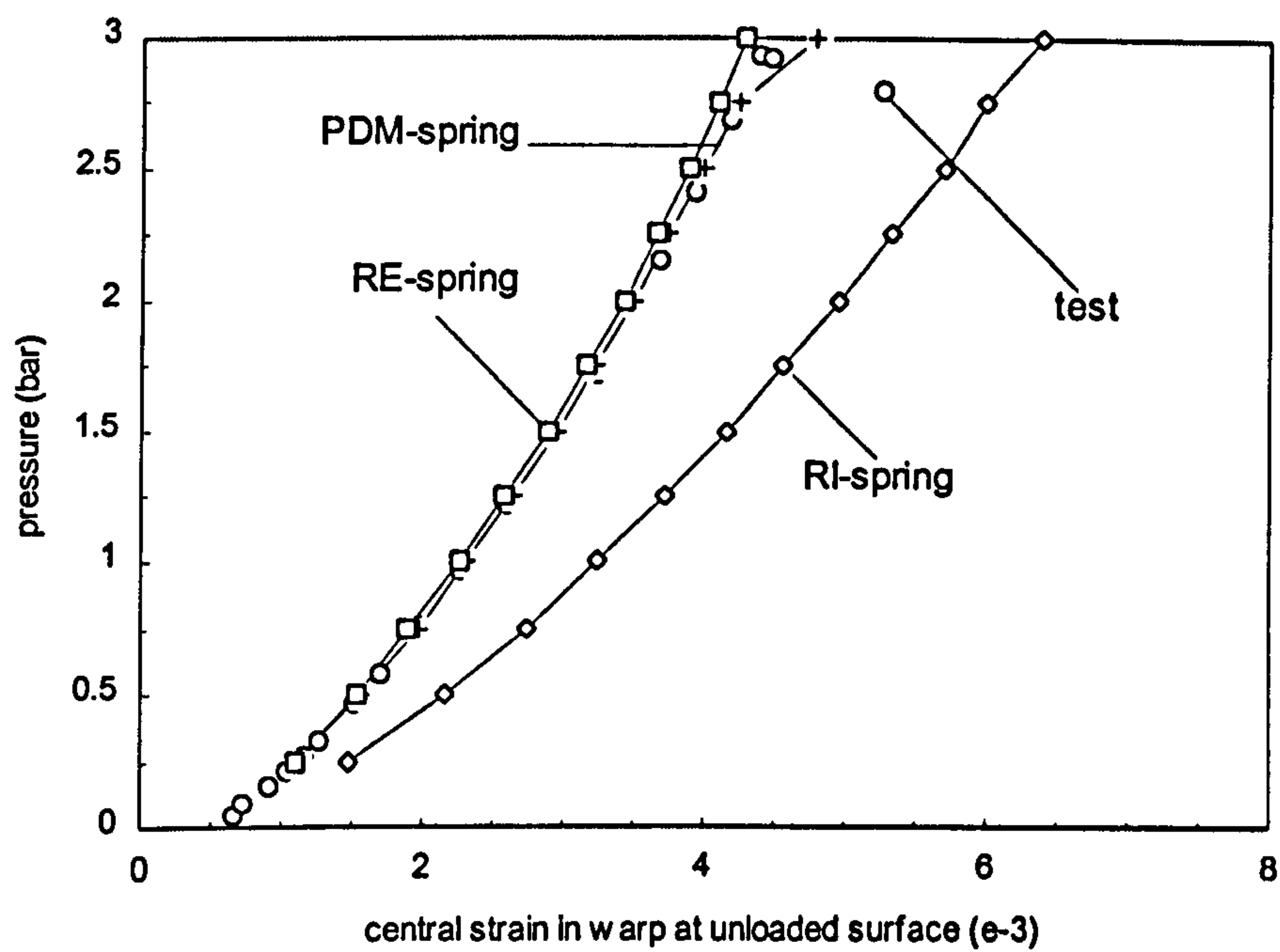


Figure 8.5 Central strain (SG7) in the x-axis at unloaded surface of Panel D

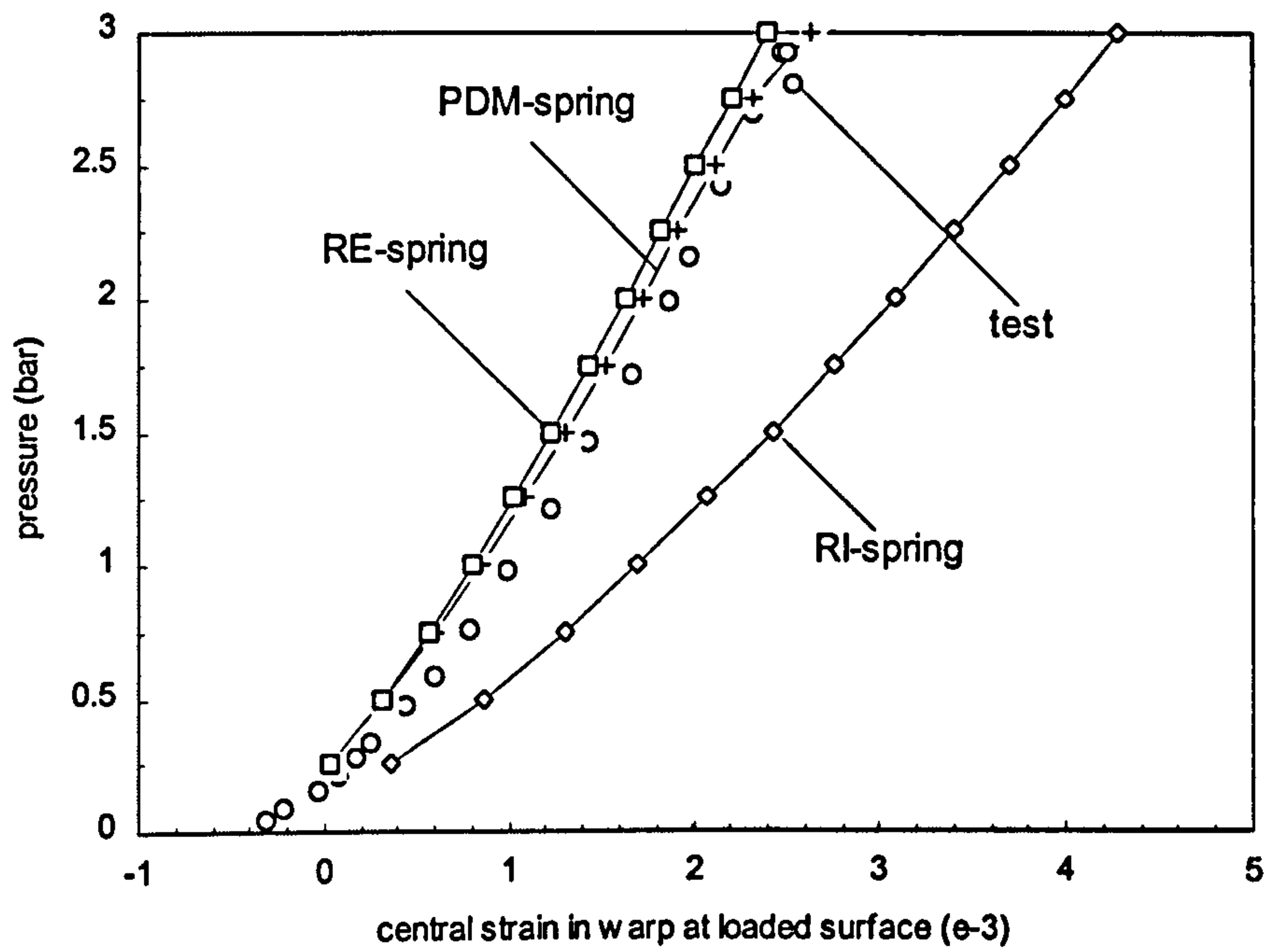


Figure 8.6 Central strain (SG8) in the x-axis at loaded surface of Panel D

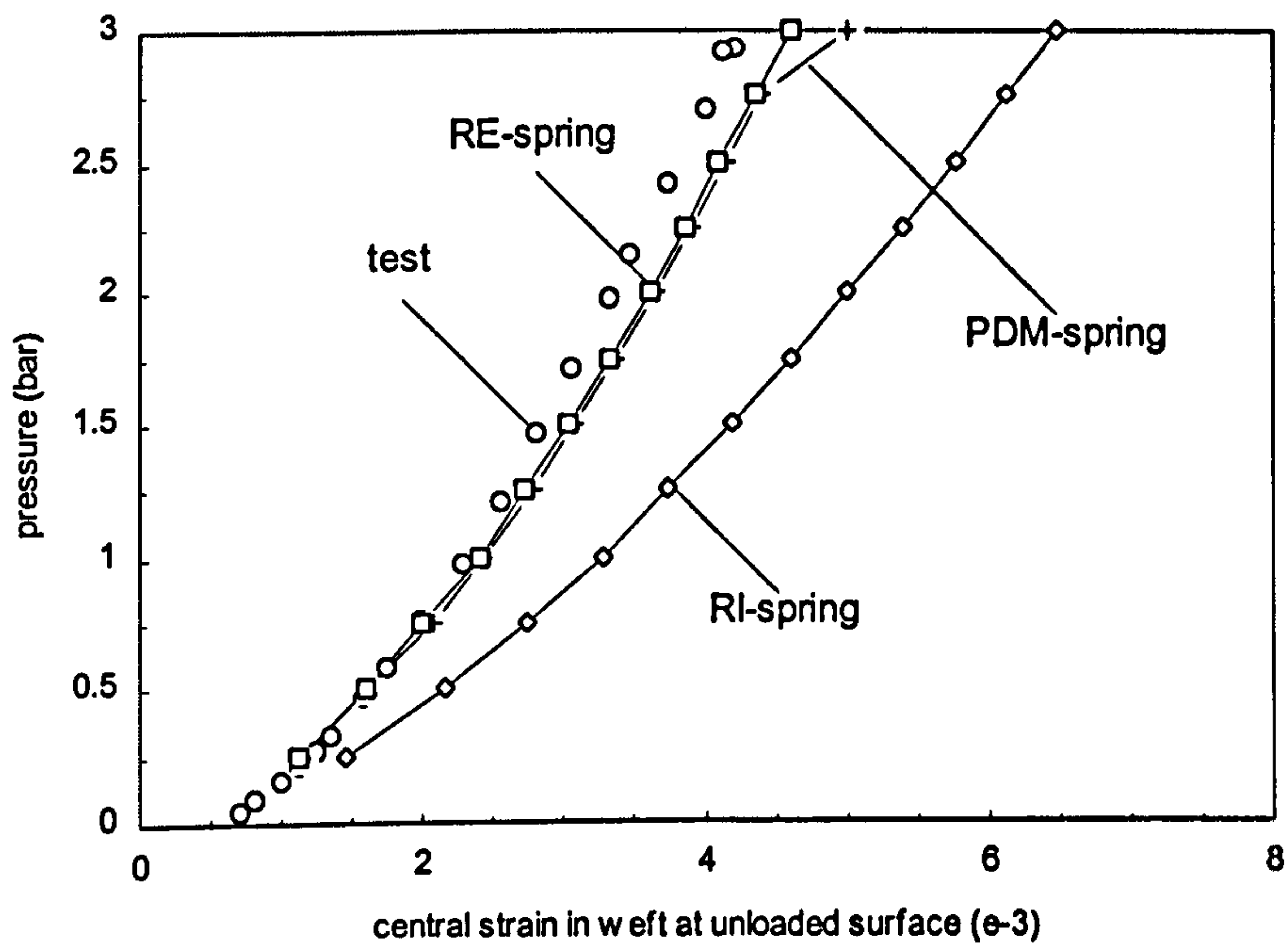


Figure 8.7 Central strain (SG15) in y-axis at unloaded surface of Panel D

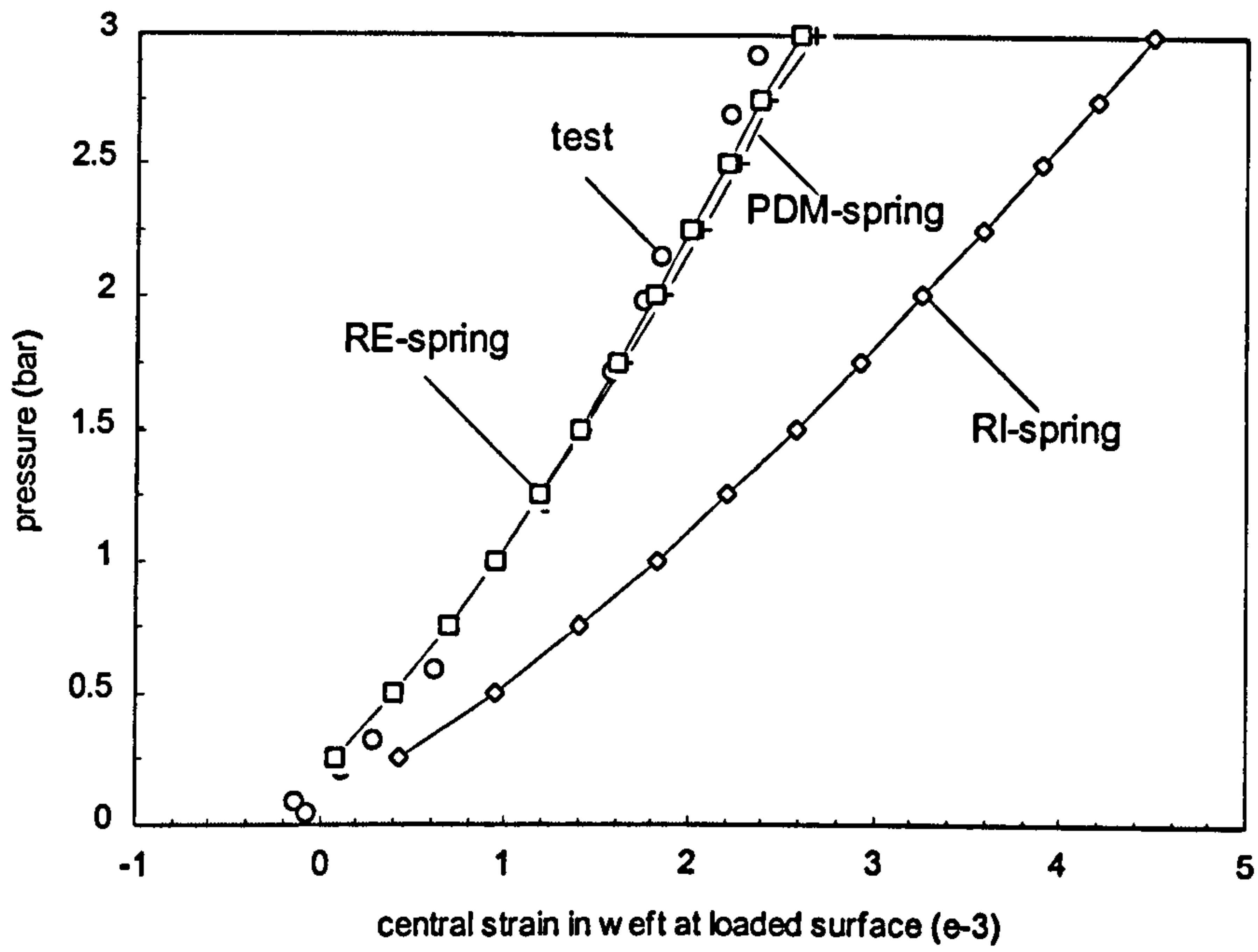


Figure 8.8 Central strain (SG16) in the y-axis at loaded surface of Panel D

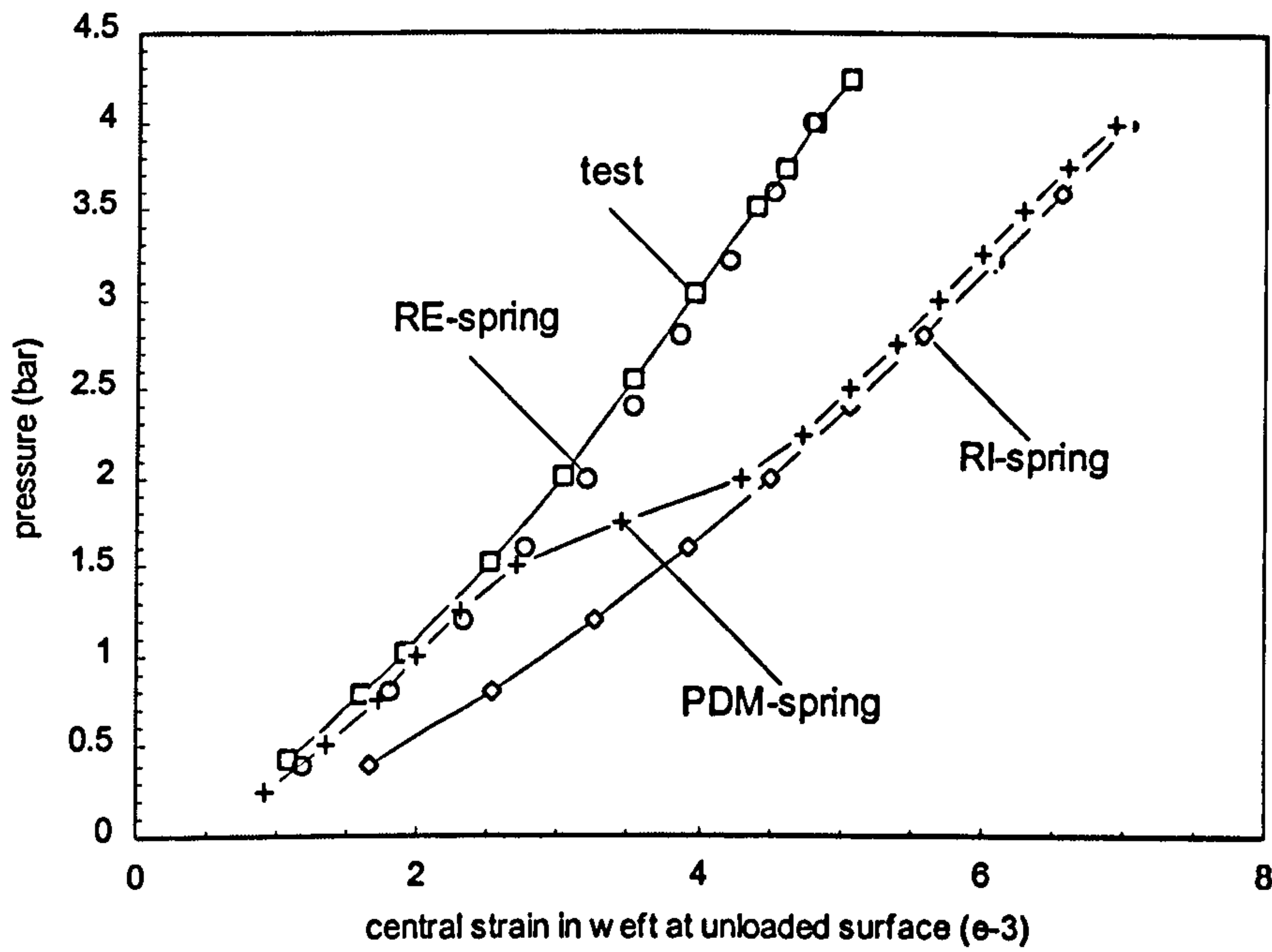


Figure 8.9 Central strain (Z3) in the y-axis at unloaded surface of Panel 30A

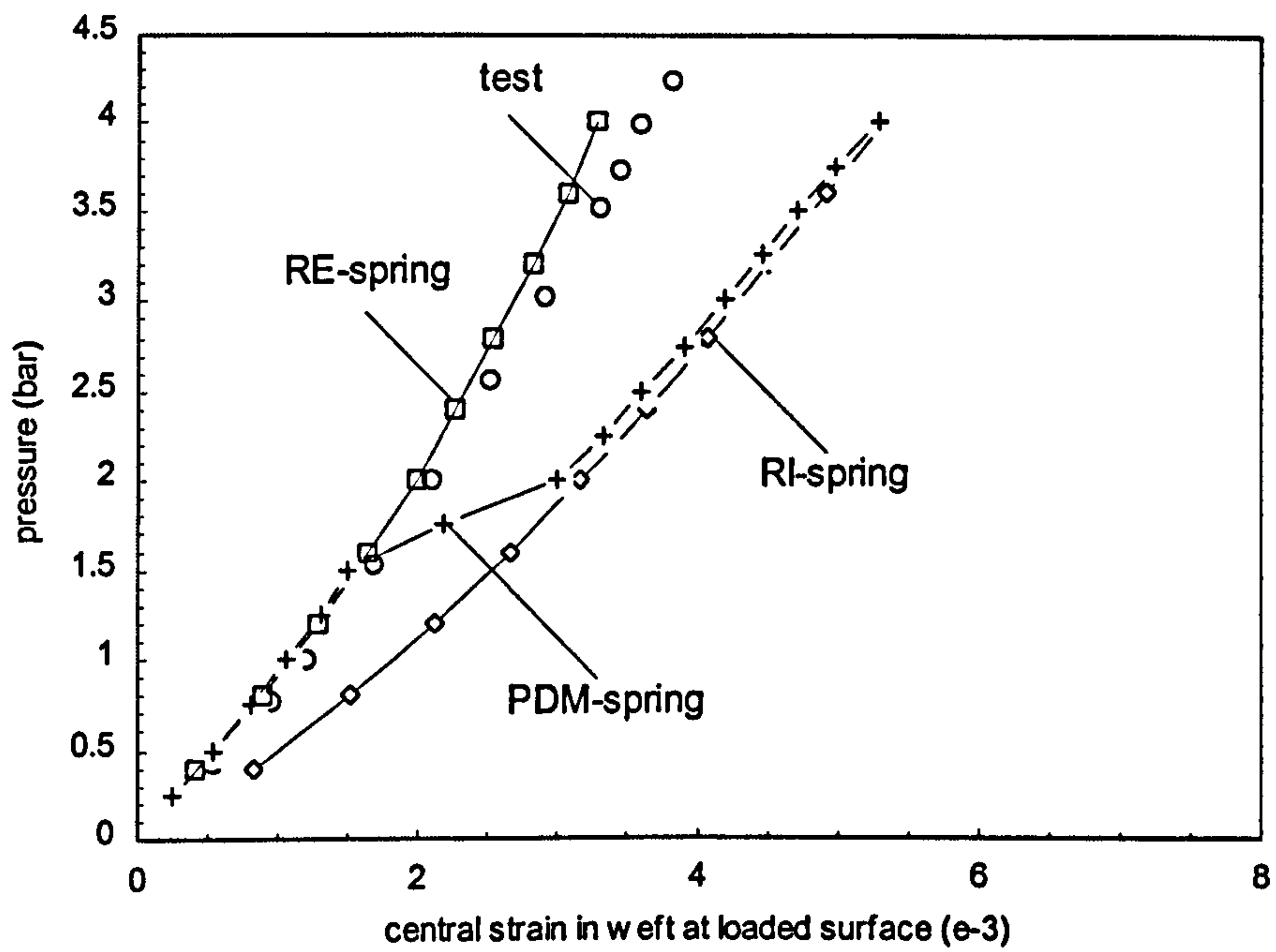


Figure 8.10 Central strain (R3) in the y-axis at loaded surface of Panel 30A

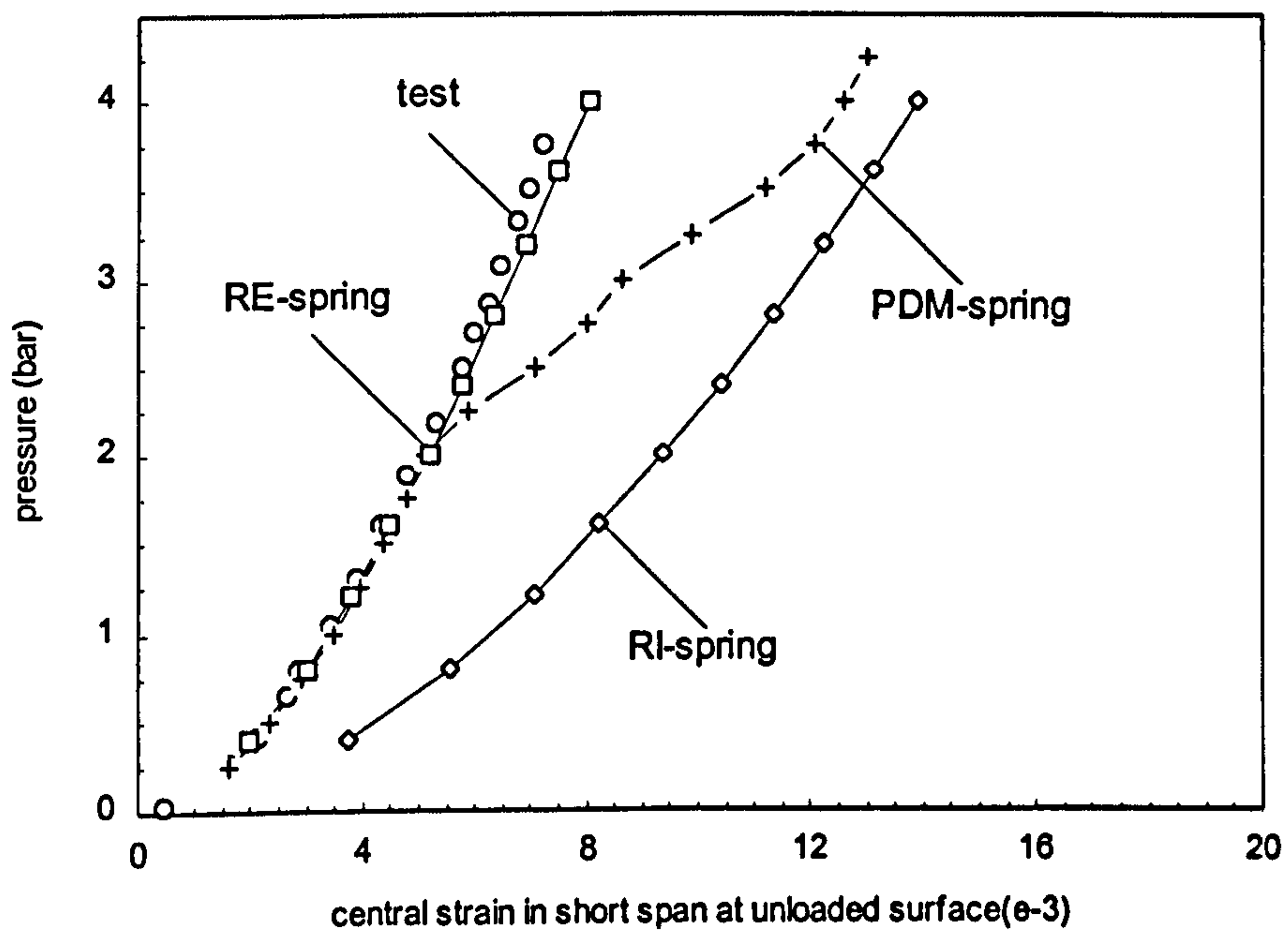


Figure 8.11 Central strain (Z3) in short span at unloaded surface of Panel 18B

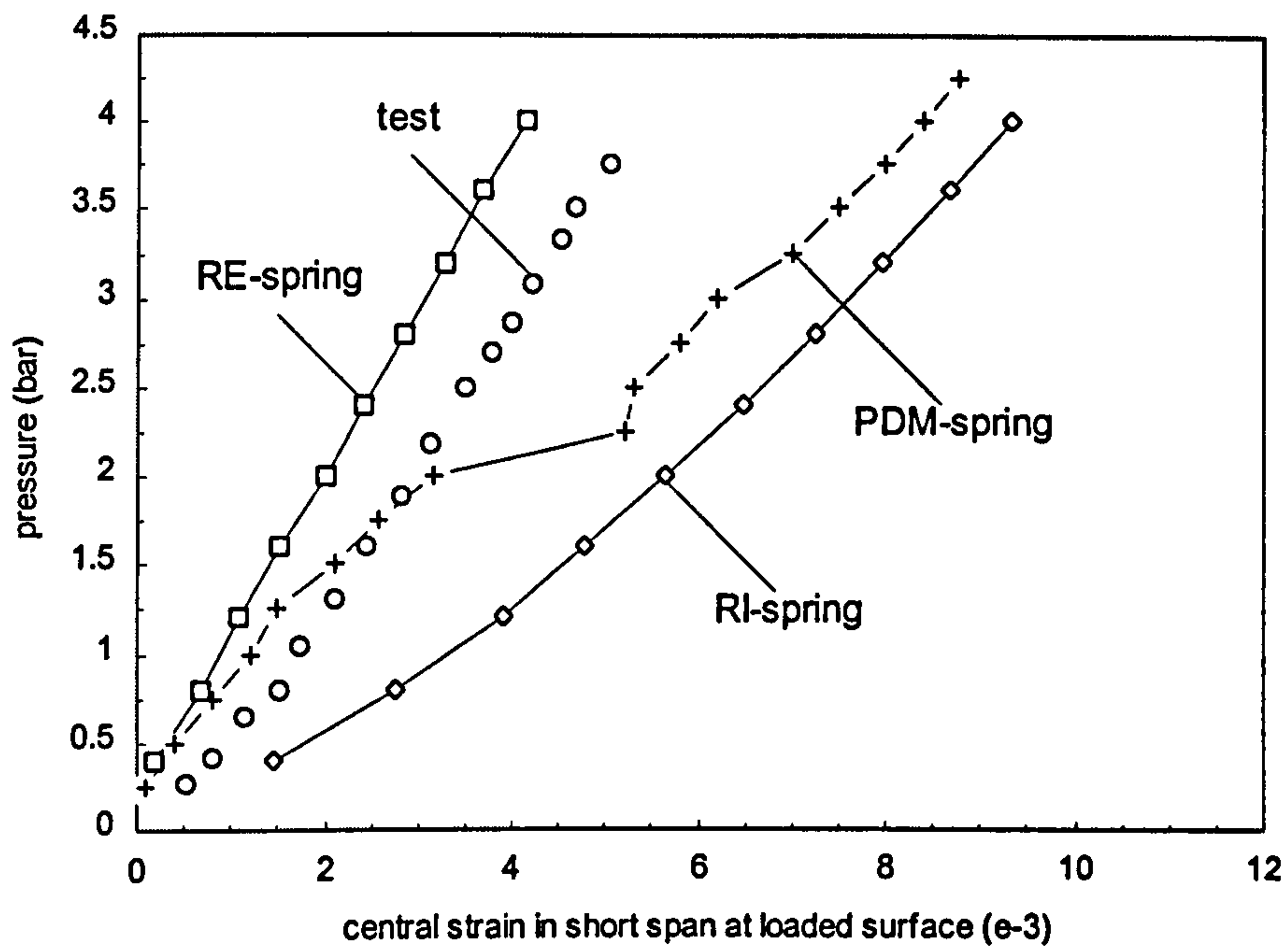


Figure 8.12 Central strain (R3) in short span at loaded surface of Panel 18B

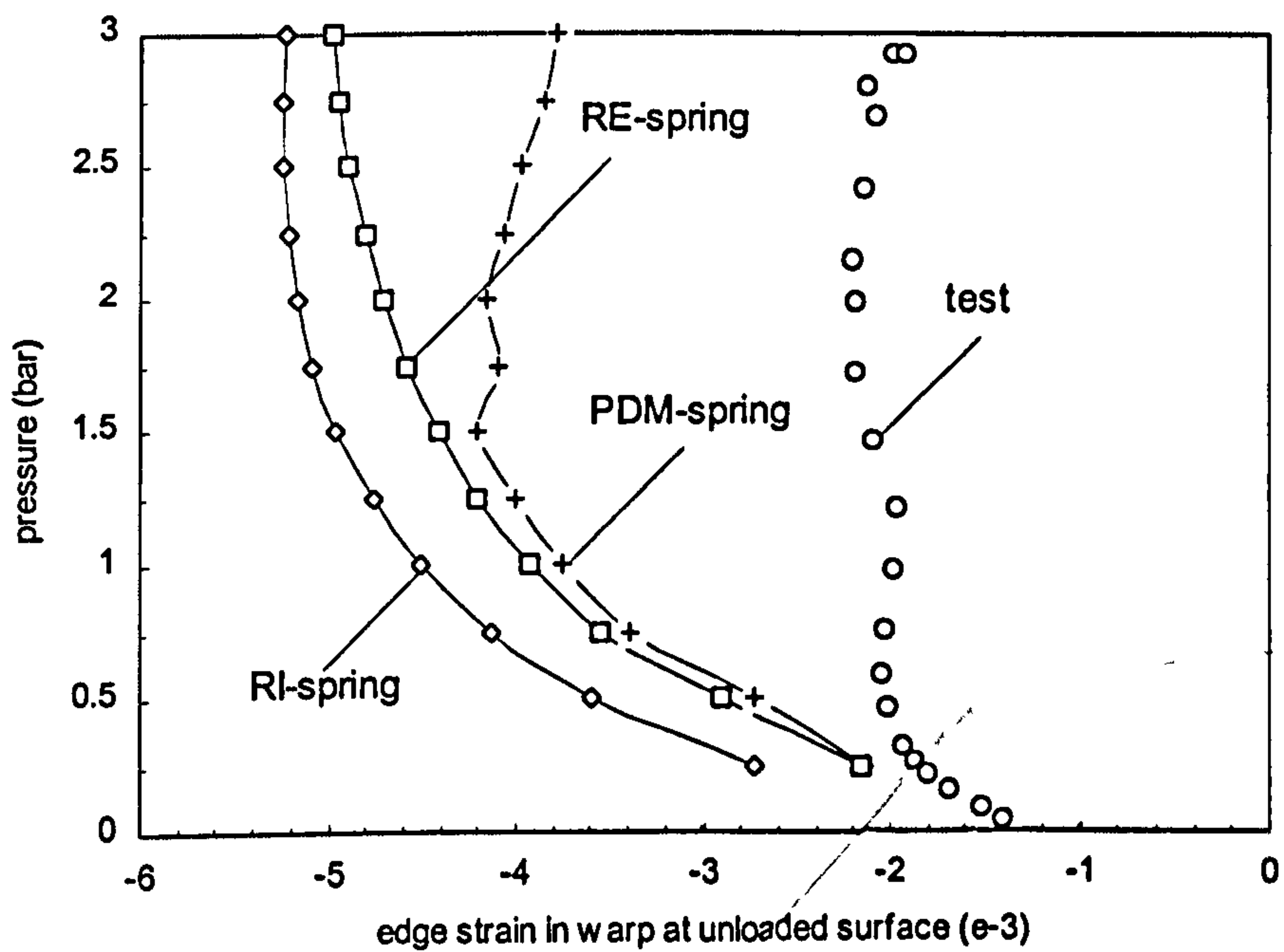


Figure 8.13 Edge strain (SG1) in the x-axis at unloaded surface of Panel D

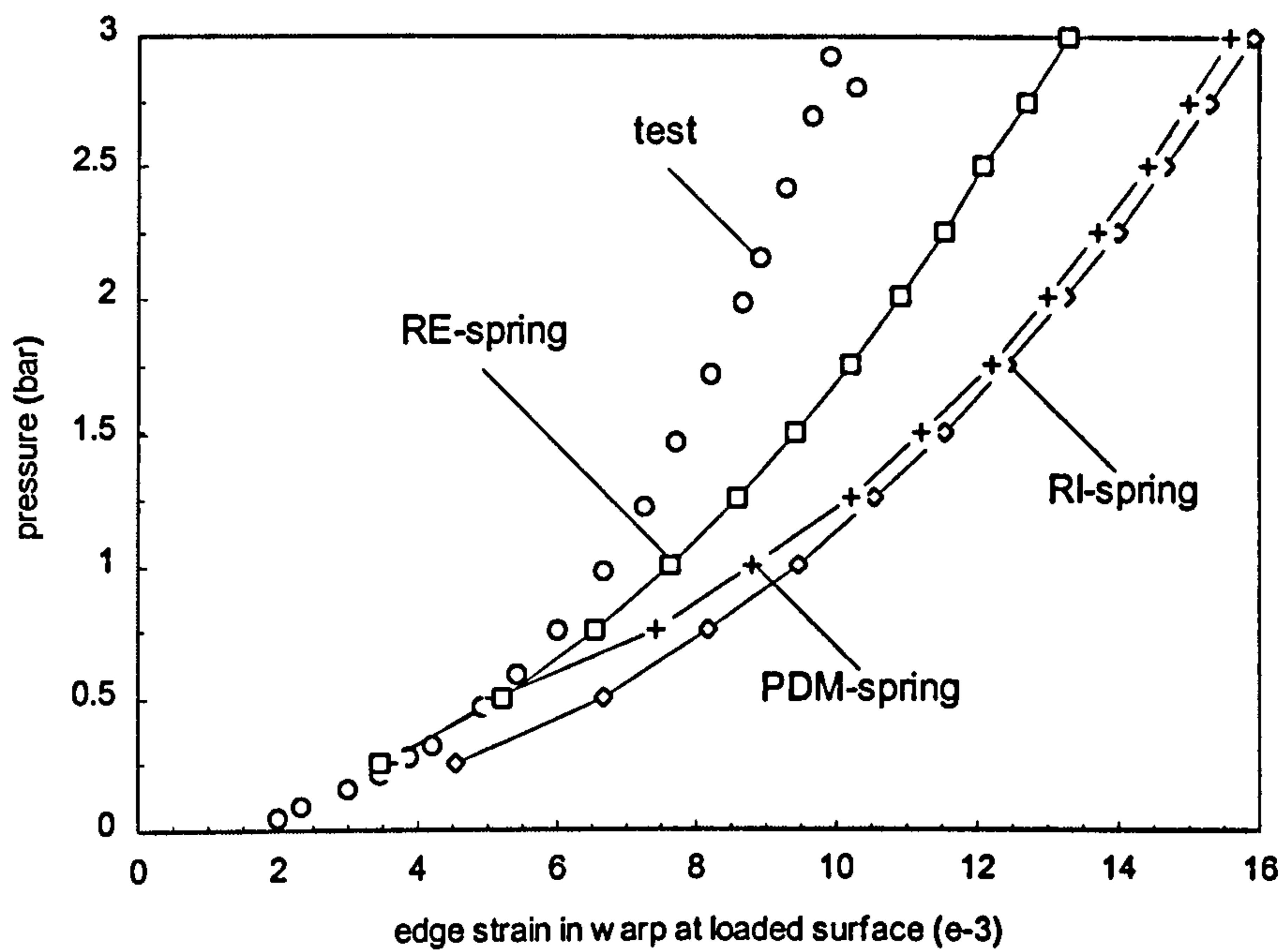


Figure 8.14 Edge strain (SG2) in the x-axis at loaded surface of Panel D

Chapter 9 Approximate calculations for thin composite panels under high transverse pressure

9.1 Introduction

In the early stages of a design, simple methods of calculation are needed so that preliminary estimates of the response of the composite panel to pressure can be made. This enables decisions to be made about the dimensions of the panel and the properties of the materials required. This chapter shows how this can be done.

9.2 Approximate calculation of maximum deflection and tension

From observations of the panel tests and the finite element analysis it is confirmed that under high pressure the panel acts mainly as a membrane except at the clamped edges where there are large but localised bending moments. In this section the panel is simulated as a membrane

9.2.1 Membrane theory

Assume that a membrane is subjected to a uniform transverse pressure q which is balanced by uniform tension T per unit width. The equation of equilibrium of the membrane is [84]:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = -\frac{q}{T} \quad (9.1)$$

where w is the deflection of the membrane.

For the rectangular membrane (Figure 9.1) with boundary conditions such that $w = 0$ at $x = \pm a$ and $y = \pm b$, the exact solution of the membrane equation is given in reference [84] as follows.

$$w = \frac{16qa^2}{T\pi^3} \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{\cosh \frac{n\pi y}{2a}}{\cosh \frac{n\pi b}{2a}} \right) \cos \frac{n\pi x}{2a} \quad (9.1)$$

The maximum deflection is at the centre of the membrane for $x=0$ and $y=0$.

$$w_{max} = \alpha \frac{16qa^2}{T\pi^3} \quad (9.2)$$

in which

$$\alpha = \sum_{n=1,3,5}^{\infty} \frac{1}{n^3} (-1)^{\frac{n-1}{2}} \left(1 - \frac{1}{\cosh \frac{n\pi b}{2a}} \right) \quad (9.3)$$

The expression α (9.3) is the function of the aspect ratio (b/a) of the rectangular membrane.

This accurate result will be the basis of the following approximate method.

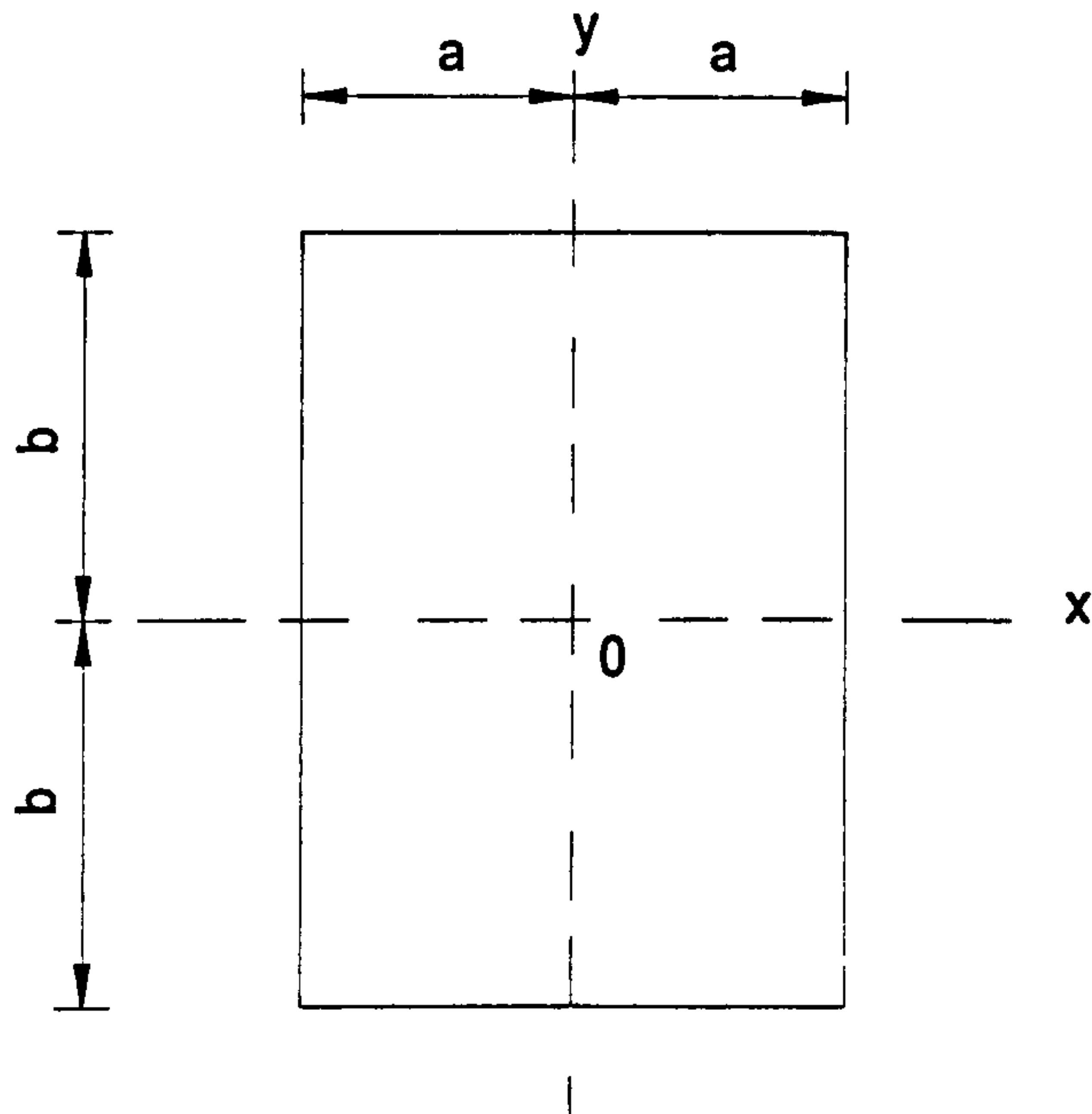


Figure 9.1 A rectangular membrane

9.2.2 Calculation formulae

In equation 9.1 the deflection of the membrane was expressed as the function of pressure q and tensile force T per unit width. In order to determine T , horizontal boundary conditions must be introduced which may or may not allow in-plane edge displacement. The exact analysis of this idea will lead to complicated expressions. In the approximate method the panel can be characterised by dividing it into *two sets of narrow strips*, one set of strips parallel with the short edge, the other set parallel with the long edge. The transverse pressure is shared between these two strips according to the aspect ratio of the panel. The strips in the short-span direction will carry the larger part of the load. The deflected shape of each strip is assumed to be a *circular arc*. Allowance must be made for the axial extensions associated with the membrane tension, and for any in-plane displacements at the boundaries. When seeking the maximum deflection and tensile force, only the *short-span strip in the middle of the long edge* needs to be analysed. Very simple formulae can be derived to determine approximately the deflection and the tension.

Consider a narrow strip of unit width attached at each end to a linear spring of stiffness k (N/mm^2 or $N/mm/mm$, spring stiffness per unit width) (Figure 9.2). If the spring stiffness is very large, the in-plane displacement of the support is prevented. Assume that the strip is subjected to pressure p which is balanced by membrane tension T per unit length. The pressure produces deflection w_{max} at the centre of the strip and a rotation $\theta/2$ at each end. The relationship between w_{max} , θ and pressure p can be obtained as follows.

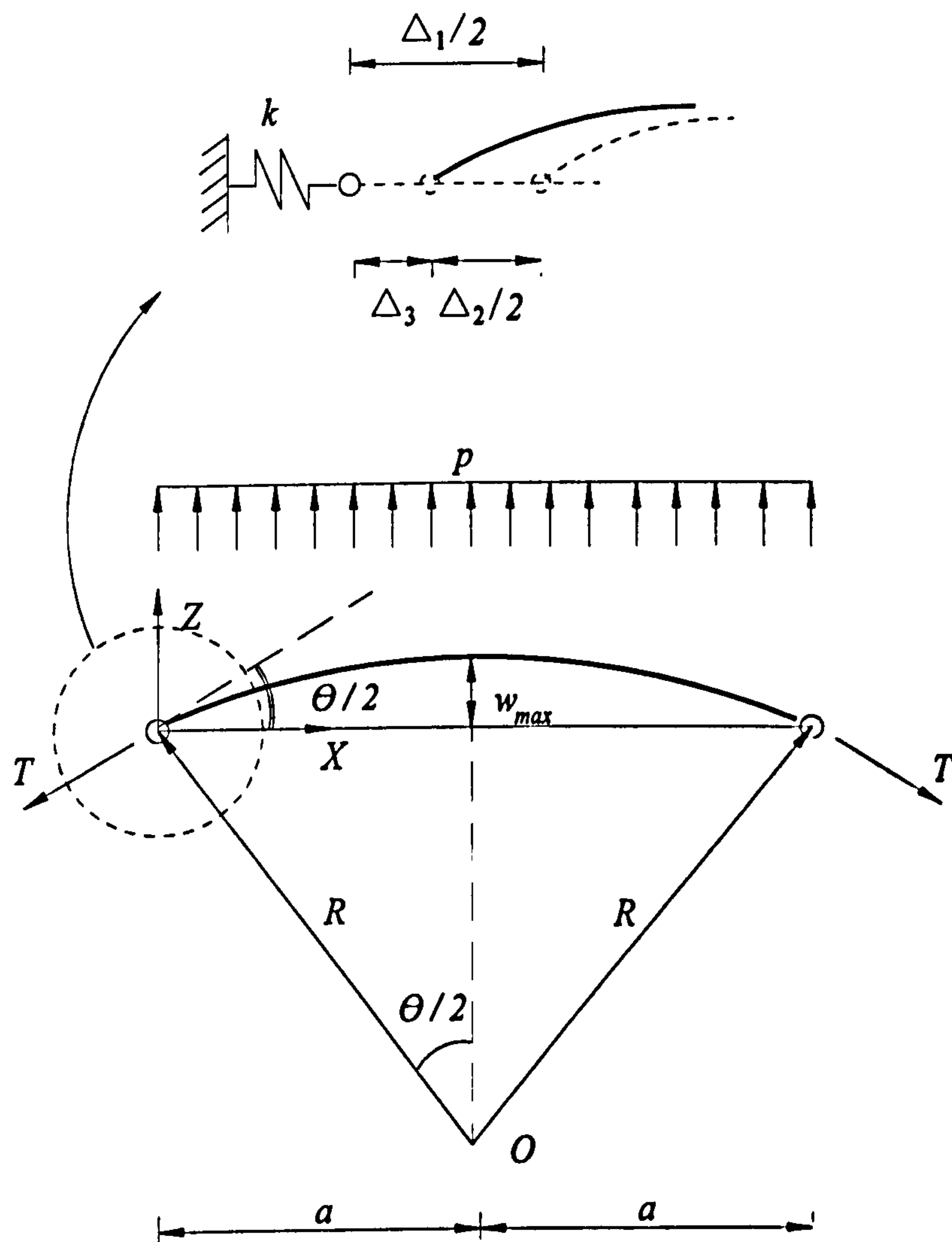


Figure 9.2 Demonstration of the strip under the pressure

(1) *Equilibrium of the strip* (vertically in Figure 9.2)

$$T = pbR \quad (9.4)$$

where b is width of the strip, and R is the radius of curvature.

(2) *Geometrical relations*

$$a = R \sin \frac{\theta}{2} \quad (9.5)$$

$$w_{max} = R - R \cos \frac{\theta}{2} \quad (9.6)$$

where a is the half length of the short span, and w_{max} is maximum deflection of the strip or the panel.

(3) *Geometrical shortening (Δ_1)*

Under the pressure the strip deflects transversely and it shortens in horizontal direction geometrically. This is referred as geometrical shortening.

$$\Delta_1 = R\theta - 2R \sin \frac{\theta}{2} \quad (9.7)$$

(4) *Elastic extension (Δ_2)*

Under the tension the strip will extend elastically.

$$\Delta_2 = \frac{T \cdot 2a}{Ebt} \quad (9.8)$$

where E is Young's modulus of the panel in short span direction and t is the thickness of the panel.

(5) Elastic deformation of restraining spring (Δ_3)

The in-plane extension of the spring under tension T at each end is:

$$\Delta_3 = \frac{T \cos \theta / 2}{k} \quad (9.9)$$

where k is the spring stiffness.

For compatibility of deformation at the boundary

$$\Delta_1 - \Delta_2 = 2\Delta_3 \quad (9.10)$$

Then the tension force in the strip can be evaluated:

$$T = \frac{\frac{R\theta - 2R \sin \theta / 2}{2a} + \frac{2 \cos \theta / 2}{k}}{\frac{Ebt}{k}} \quad (9.11)$$

For the case without edge displacement, $k = \infty$. With the increase of the rotation angle θ of the strip the maximum deflection w_{max} and tensile force T can be calculated from the expressions of (9.6) and (9.11).

9.2.3 Relation of pressure q on the panel and p on the strip

In order to find the relationship between the deflection w_{max} , the tensile force T and the pressure q on the panel, it is first necessary to determine the relationship between the pressure q on the panel and the pressure p allocated to the strip.

There are several factors which affect the relationship between q and p . For example, the aspect ratio of the panel, the stiffness of the material in both x and y directions, and the position of the strip in the panel, etc. The effects of the aspect ratio and the position of the strip are assumed to predominate. However, even in a square panel any difference between the stiffnesses of the material in the x -direction and the corresponding stiffness in the y -direction must be also be taken into account.

From the equations (9.4) - (9.11), it is impossible to link directly the pressure p allocated to the strip with the pressure q on the panel. Usually, the relationships between p and q are assumed as:

$$p = 0.5 q \quad (\text{square panel}) \quad (9.12a)$$

$$p = q \quad (\text{panel with aspect ratio greater than 2}) \quad (9.12b)$$

The expression (9.12a) means that the load is divided equally between the x and y directions; the expression (9.12b) means that the load is carried totally by the strip in the short-span direction. Initial calculations showed that the results (maximum deflection w_{max} and tensile force T) agreed with the FE analysis only for the panel with aspect ratio 1:2. For the square panel, large error was found in tensile force (Table 9.1). The reason is probably that the effect of the position of the strip in the panel is ignored in the expression (9.12).

To find a suitable relationship between p and q , the expressions of (9.4), (9.6), and (9.11) (three equations about p , w_{max} and T) are substituted into the analytical maximum deflection expression of (9.2) (involving q). This approach assumes *approximately* that the maximum deflection and tension obtained in (9.6) and (9.11) are coincident with the membrane theory results. In this way, the pressure p allocated to the strip *in the middle of the edge* can be determined approximately:

$$\text{for aspect ratio } b/a \geq 3 \quad p = q$$

$$\text{for aspect ratio } b/a = 2 \quad p = 0.95q$$

for aspect ratio $b/a = 1.5$

$p = 0.8 q$

(9.13)

for aspect ratio $b/a = 1$

$p = 0.6 q$

It is obvious that for a narrow rectangular panel the load is carried entirely by the strip in the short-span direction. Thus it is possible to neglect the effect of the short sides. This conclusion is coincident with the calculations above (the case of $b/a \geq 3$). For the square panel the load is carried in both ways equally. However, the strips near the middle of the edge should carry more pressure than those near the edge. The coefficient 0.6 for square panel is obtained to account for this effect. Table 9.1 shows results for a square panel from approximate method using $p = 0.6 q$ agree with the FE analysis well.

Table 9.1 The effect of relationship of p and q

	App. Method	App. Method	FE analysis	Error analysis	
	$(p = 0.5 q)$	$(p = 0.6 q)$			
	(1)	(2)	(3)	(3)-(1)/(3)	(3)-(2)/(3)
w_{max} (mm)	25	26.6	26	3.8%	-2.3%
T (N)	359	410	430	16.5%	4.6%

(The example is Panel 30A with *fully fixed* (no spring at the ends) boundary conditions, resin effective, at 4 bar pressure)

9.2.4 Results from the approximate method

A short FORTRAN program (Appendix 6) was developed to calculate the deflection and tension according to the formulae above. A loop of θ gradually increases the angle of rotation of the strip, and the relationship of deflection, tension vs. pressure q on the panel can be worked out.

The results from the approximate method were compared with those from the finite element method of the *RE-spring* model and *RI-spring* model discussed in Chapter 8, as shown in Tables 9.2-9.4. The results show that for the three examples (two square panels and one panel with aspect ratio 1:2) the error is not more than 6% for both maximum deflection and tensile force. This result reveals that the approximate method has a reasonably good accuracy, and it can be therefore used as quick calculation tool in the early stages of design.

9.2.5 Use of the approximate method to determine the spring stiffness used in FE analysis

In Chapter 7 and 8, the springs were attached to the panel in order to consider the in-plane displacements in the finite element analysis. The stiffness of the spring needs to be determined beforehand. The approximate method presented above was used to calculate the stiffness of the spring by trial-and-error to allow the spring extension to be equal to experimental value at certain pressure (target value, see Chapter 7). The stiffness of the spring in Table 7.7 and Table 8.2 were determined in this way. Assume the stiffness of the spring k is represented as

$$k = \beta E b t / a \quad (9.14)$$

where β is a stiffness parameter of the spring. In the calculation only β needs to be changed each time. As the formulae above are very simple, it is very easy to calculate the best stiffness of the spring to fit the edge displacement measured in the tests.

Table 9.2 Comparison between the approximate method and the finite element analysis of Panel 30A (aspect ratio 1:1) (Maximum deflection and tension)

<i>Central deflection results (mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	18.6	19.6	-5.3	21.4	21.3	0.5
2	24.0	24.9	-3.8	27.2	27.1	0.4
3	27.8	28.2	-1.4	31.2	31.1	0.3
4	30.4	31.2	-2.6	34.3	34.2	0.3
<i>Average tension results (N/mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	135	133	1.5	128	124	3.1
2	221	214	3.2	209	200	4.3
3	295	284	3.7	276	265	4.0
4	360	348	3.3	335	323	3.6

FE -- Finite element analysis in Chapter 8

App. -- Approximate method using membrane analogy

Error = (FE-App)/FE

Table 9.3 Comparison between the approximate method and the finite element analysis of Panel 18B (aspect ratio 1:2) (Maximum deflection and tension)

<i>Central deflection results (mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	30.6	30.6	0.0	35.8	34.8	2.8
2	39	38.8	0.5	45.3	44.4	2.0
3	44.8	44.7	0.2	51.9	50.9	1.9
4	49.3	49.3	0.0	57.1	56.3	1.4
<i>Average tension results (N/mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	126	135	-7.1	114	118	-3.5
2	206	219	-6.3	187	194	-3.7
3	274	291	-6.2	246	255	-3.7
4	336	356	-5.9	304	313	-3.0

Table 9.4 Comparison between the approximate method and the finite element analysis of Panel D (aspect ratio 1:1) (Maximum deflection and tension)

<i>Central deflection results (mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	17.6	18.4	-4.5	20.1	19.7	2.0
2	21.8	22.3	-2.3	25.4	24.9	2.0
3	25.1	26.3	-4.8	29.2	28.9	1.0
<i>Average tension results (N/mm)</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	156	153	1.9	133	131	1.5
2	237	230	3.0	216	211	2.3
3	316	314	0.6	289	284	1.7

9.3 Approximate calculation of maximum bending moment

In this section the flexural stiffness of the panel is considered. A strip with unit width, the same as the strip in the membrane analysis in above section, is cut along the symmetrical axis of short span of the panel. The strip is considered as a non-linear flexural-tension *beam* which carries the pressure p and tension T obtained from the approximate method from above section. Thus the flexural-tension or beam-column theory can used in the approximate analysis.

Figure 9.3 demonstrates that the flexible beam carries a uniformly distributed load p and the axial tension T , in which the rotation at the ends is prevented by the couples M_e . The bending moment at a distance x from the left-hand end is the equation (9.15)

$$M = -Tw + \frac{pL}{2}x - \frac{px^2}{2} + M_e \tag{9.15}$$

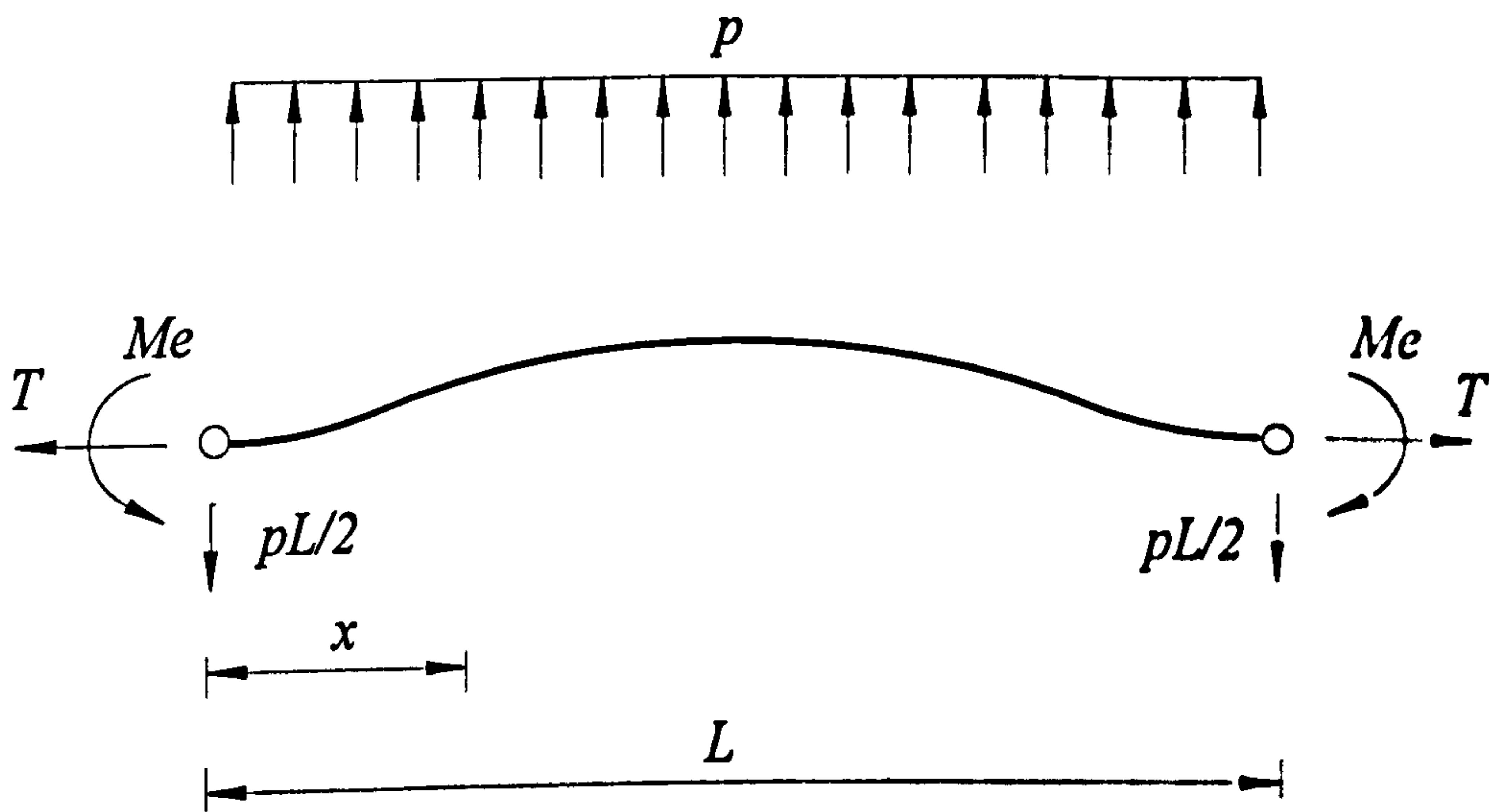


Figure 9.3 A beam with distributed transverse load and fixed ends

Since the deflections of the beam are small compared with the length of the beam the differential equation of the beam can be expressed as [85-86]:

$$\frac{d^2w}{dx^2} + \lambda^2 w = \frac{p}{EI} \left(\frac{x^2}{2} - \frac{xL}{2} \right) - \frac{M_e}{EI} \quad (9.16)$$

where $\lambda^2 = -\frac{T}{EI}$, and EI is the flexural stiffness of the beam. The solution is

$$w = A \sin \lambda x + B \cos \lambda x - \frac{p}{T} \left(\frac{x^2}{2} - \frac{xL}{2} - \frac{1}{\lambda^2} \right) + \frac{M_e}{T} \quad (9.17)$$

The boundary conditions are $w = w' = 0$ at $x = 0$; $w = w' = 0$ at $x = L$, the unknowns A , B and M_e can be determined:

$$A = \frac{ipL}{2T\lambda}, \quad B = \frac{pL}{2T\lambda} \coth \frac{\lambda L}{2} \quad (9.18)$$

$$M_e = \frac{pL}{2\lambda} \left(\frac{2}{\lambda L} - \coth \frac{\lambda L}{2} \right) \quad (9.19)$$

where $i = \sqrt{-1}$.

M_e is the edge moment desired in this approximate method. Thus the expression (9.19) can be used to estimate the local bending moment of the panel. The axial tension T in this expression is considered to be known from the calculation of the approximate method using membrane analogy in Section 9.2. The pressure p allocated on the beam can be obtained from the expression of (9.13), which links p with the pressure q on the panel.

The formula was used to calculate the maximum local bending moment of the panels, and the results were compared with the finite element analysis results, which are listed

in Tables 9.5-9.7. The results for the panel with aspect ratio 1:2 (Panel 18B) agree with the finite element analysis results well. For the panel with aspect ratio 1:1 (both Panel D and Panel 30A) the error is about 15%. However, in the early stages of design this error may be tolerable.

Table 9.5 Comparison between the approximate method and the finite element analysis of the Panel 30A (Maximum local bending moment)

<i>Local moment results (KN*mm/mm) Panel 30A 1:1</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	0.369	0.32	13.3	0.332	0.290	12.7
2	0.604	0.520	13.9	0.538	0.461	14.3
3	0.806	0.689	14.5	0.710	0.613	13.7
4	0.989	0.842	14.9	0.863	0.744	13.8

FE -- Finite element analysis in Chapter 8
App. -- Approximate method using beam-column theory
Error = (FE-App)/FE

Table 9.6 Comparison between the approximate method and the finite element analysis of the Panel 18B (Maximum local bending moment)

<i>Local moment results (KN*mm/mm) Panel 18B 1:2</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	0.482	0.461	4.3	0.351	0.357	-1.7
2	0.782	0.749	4.2	0.578	0.584	-1.0
3	1.035	0.997	3.7	0.761	0.766	-0.7
4	1.262	1.213	3.9	0.929	0.936	-0.8

Table 9.7 Comparison between the approximate method and the finite element analysis of the Panel D (Maximum local bending moment)

<i>Local moment results (KN*mm/mm) Panel D 1:1</i>						
Pressure (bar)	<i>Resin Effective</i>			<i>Resin Ineffective</i>		
	FE	App.	Error (%)	FE	App.	Error (%)
1	0.516	0.454	12.0	0.436	0.384	11.9
2	0.792	0.690	12.9	0.718	0.628	12.6
3	1.104	0.954	13.5	0.975	0.844	13.4

9.4 Approximate calculation of final failure pressure

With the formulae for the approximate calculation of the maximum deflection, membrane tension and local bending moment, the designer can decide the dimensions of the panel and the properties of the materials required. It is also possible to check the strength of the structure roughly by using an approximate method in the early stage of design.

As stated earlier, from observation of the panel tests and finite element analysis, thin composite panels under high transverse pressure behave mainly as membranes except at the edges where there are large but localised bending moments. All panels in the tests failed at the edges because of the combination of the large local bending moment and membrane tension. Thus the local bending moment and membrane tension can be considered as two key factors which determine the strength of the panel. According to this theory, a simple failure criterion is suggested to determine the final failure of the thin panel under high transverse pressure. The criterion for failure is:

$$\frac{M}{M_u} + \frac{N}{N_u} > 1 \quad (9.20)$$

where M and N are bending moment and membrane tension in the panel, and M_u and N_u are ultimate bending moment and tensile force which can be determined from the coupon tests.

This idea was examined by comparing the final failure pressure calculated from the expression (9.20) with test results for *Panel D*.

In the calculation, M and N were obtained from the approximate methods presented in this chapter above. Assume that at failure the resin is fully ineffective. Hence, M and N from the *Resin Ineffective* case were used.

M_u and N_u can be obtained by using ultimate bending stress σ_{ult}^b and ultimate tensile stress σ_{ult} listed in Table 3.4 and Table 3.3 in Chapter 3. The ultimate bending moment and ultimate tensile force in fibre direction can be calculated as follows:

$$M_u = \frac{t^2}{6} \sigma_{ult}^b = 1140 \text{ N} \cdot \text{mm/mm}$$

$$N_u = t * \sigma_{ult} = 1070 \text{ N/mm}$$

where t is the thickness of the panel. From Tables 9.4 and 9.7, at 3 bar the local bending moment and membrane tension for the *Resin Ineffective* case are $M = 844 \text{ N} \cdot \text{mm/mm}$; $N = 284 \text{ N/mm}$, thus

$$\frac{M}{M_u} + \frac{N}{N_u} = \frac{844}{1140} + \frac{284}{1070} = 1.003 > 1$$

However, at 2.6 bar, the local bending moment and membrane tension for the *Resin Ineffective* case are $M = 767 \text{ N} \cdot \text{mm/mm}$; $N = 256 \text{ N/mm}$, thus

$$\frac{M}{M_u} + \frac{N}{N_u} = \frac{767}{1140} + \frac{256}{1070} = 0.912 < 1$$

This indicates that final failure pressure is probably between 2.6 bar and 3 bar. The actual failure pressure of Panel D is 2.93 bar. Thus the approximate calculation provides a reasonably good prediction of the failure pressure. Although the local bending moment from the approximate method has certain error compared with the FE analysis, this approximate calculation is sufficiently accurate for the early stages of design.

9.5 Conclusions

This chapter provides a series of approximate methods to estimate the response of the panel to the load. In practice, the calculation can be carried out as follows.

- (a) Take a strip of unit width from the middle of the long edge. Calculate the pressure allocated to the strip according to the expression (9.13).
- (b) Gradually increase the angle of rotation (θ) of the strip and find the relationships between the pressure and maximum deflection and tension according to equations (9.4), (9.6) and (9.11).
- (c) If edge displacement needs to be considered, assume the spring stiffness in the form of expression (9.14) and repeat step (b) (change the value of β in (9.14) in each calculation) until the calculated edge displacement matches the target value as taken from measurement.
- (d) Using the expression (9.19), estimate the local bending moment.
- (e) To estimate the strength of the panel in the expression (9.20), the tested ultimate tensile and bending strengths of the panel are needed in advance. Having these strengths, the maximum tension and bending moment calculated from steps (b) and (c) from the *Resin Ineffective* case can be used to examine the strength of the panel according to (9.20).

Compared with the finite element analysis, the approximate method for predicting the maximum deflection and tension has a reasonably good accuracy. The given examples show the maximum error of the approximate method is not more than 4% .

Compared with the finite element analysis , the approximate method for predicting the maximum local bending moment has a good accuracy for the panels with aspect ratio

greater than 2. For the square panels, the error of the method is about 15%. Nevertheless, it is still acceptable for the use in the early stages of design.

The example shows that the approximate method for predicting the final failure pressure provides bounds of real final pressure measured in the test of Panel D.

The advantage of the approximate methods presented above is that they are very simple and easy to calculate. They are therefore very useful for designer to estimate the response of the composite panel to pressure in the early stages of design.

Chapter 10 Conclusions

10.1 Retrospect

Before presenting the main conclusions, it may be useful briefly to review the various aspects of the research. Figure 10.1 shows a flowchart of the research programme.

(1) *Coupon Tests*: experimental studies were conducted to investigate the material properties of the composite panels. A large number of coupons were tested in tension and in bending to obtain the stiffness and strength of the materials in different directions.

(2) *SEM*: the Scanning Electron Microscope (SEM) was used to analyse the structure of laminate. This revealed the very flat nature of the ribbons of reinforcement and the existence of the pure resin in the composite.

(3) *Panel Tests*: a fully detailed experimental study was carried out on a square panel. The strain gauges were mounted on both unloaded and loaded surfaces in many places in the panel. Many displacement transducers were also used to measure lateral deflections. A great effort was made to measure in-plane edge displacements.

(4) *Theoretical Model*: a two-dimensional theoretical model was developed to predict the elastic properties of woven and non-woven composite materials.

(5) *Micromechanical Analysis*: ABAQUS was used to study the stress distribution and concentration in a woven composite in microstructural level. The term of *stress variation factor* was defined to quantify the stress concentration.

(6) *Progressive Failure Analysis*: following from (5), ABAQUS was used to study the progressive failure behaviour of the composite panels under high transverse pressure. A

methodology for progressive failure analysis of woven composite panels was developed. The method incorporates the results from micro-structural analysis of woven composite into macro analysis of the woven composite panel. The method provided good predictions of the behaviour of the panels.

(7) *Stiffness-Modelling*: ANSYS was used in a simplified finite element analysis of the composite panels, in which two extreme cases (Resin Effective and Resin Ineffective) were analysed. The method can save computer time and provide reasonably good results.

(8) *Approximate Panel Calculations*: for the purpose of engineering design, approximate calculations were developed to estimate the response of the panel to high pressure.

10.2 Main conclusions

(1) The theoretical model presented in Chapter 5 predicts the elastic properties of woven and non-woven composite materials well. It is verified by comparing the results predicted by the model with the measured stiffnesses of samples cut from composite panels in various directions.

(2) Detailed study of a woven composite in micro-structural level reveals that due to interlacing and waviness of the roving fabric, there are significant stress concentrations inside the rovings and the pure resin regions. The stress concentrations will result in resin cracking, transverse damage and fibre breakage in the woven composite earlier than in a unidirectional laminate made of the same materials as the woven composite. In other words, a woven composite will be weaker than an equivalent non-woven composite. The stress concentration can be quantified in terms of the *stress variation factor*.

(3) It is well-known that progressive failure analysis for woven composite *panel* is very complicated, and no published report has been found. This research makes a start on this complicated area. The method for the progressive failure analysis of the woven composite panel presented in Chapter 7 takes the stress concentration into account. The results show that progressive damage sequences and deflections of the panels predicted from the progressive damage model correlate well compared with the test measurements. The strains predicted from the model agree reasonably well with the test results. The method provides a useful tool to investigate the behaviour of the composite panel under high transverse pressure.

(4) The finite element modelling of the panels shows that the small in-plane edge displacement has a significant effect on the deflections and strains of the composite panel. The examples show that the deflections and strains predicted from the finite element modelling cannot match the test measurements unless the edge displacement is considered. The results also reveal that the edge displacement has little influence on final failure pressure of the panel.

(5) The finite element analysis in Chapter 8 shows that the complicated progressive failure analysis can be simplified by analysing two extreme cases of the panel: *Resin Effective* and *Resin Ineffective*. The results show that the *Resin Effective* and the *Resin Ineffective* cases provide bounds to those from the progressive failure analysis and tests. The analysis also needs much less computer time than the progressive failure analysis.

(6) Both test and the finite element results show that the thin composite panels under high transverse pressure act mainly as membranes except at the edges where there are large localised bending moments.

(7) Conclusion (6) enables the panel analysis to be further simplified. Approximate methods in Chapter 9 provide good predictions of the maximum deflection, tension and final failure pressure of the panel. The error for the prediction of local bending moment

is also not more than 15%. As the approximate methods are very simple, they are very useful for designers to estimate the response of the panel in the early stages of design.

10.3 Proposals for future work

The present study provides a good understanding of the behaviour of the thin FRP composite panels under high transverse pressure. However, more research remains to be done as follows.

(1) The work reported here focused on the *thin* composite panels. To form a complete data base for composite panels, further information is required on *thicker* composite panels, where bending stiffness and transverse shear stress have a greater influence than in the thin panels. To do this, more testing of thicker panels is needed, and/or the progressive failure analysis presented in this research can be extended to predict the behaviour of such panels.

(2) The progressive failure analysis presented here predicts the progressive failure sequence and deflection of the panel well. However, the strains predicted from the model are sensitive to the element damage. This may be due to the lack of residual stress carrying capacity in the element after the failure criterion is exceeded. It would be necessary to extend the current model to overcome the sensitivity.

(3) The work in this thesis concentrated on methods of analysis of composite panels. It would be particularly interesting to extend the simplified analysis, especially, to complete structures made of stiffened panels, and to develop design charts for these. This may be studied by simplifying the stiffened panels as a continuous beam.

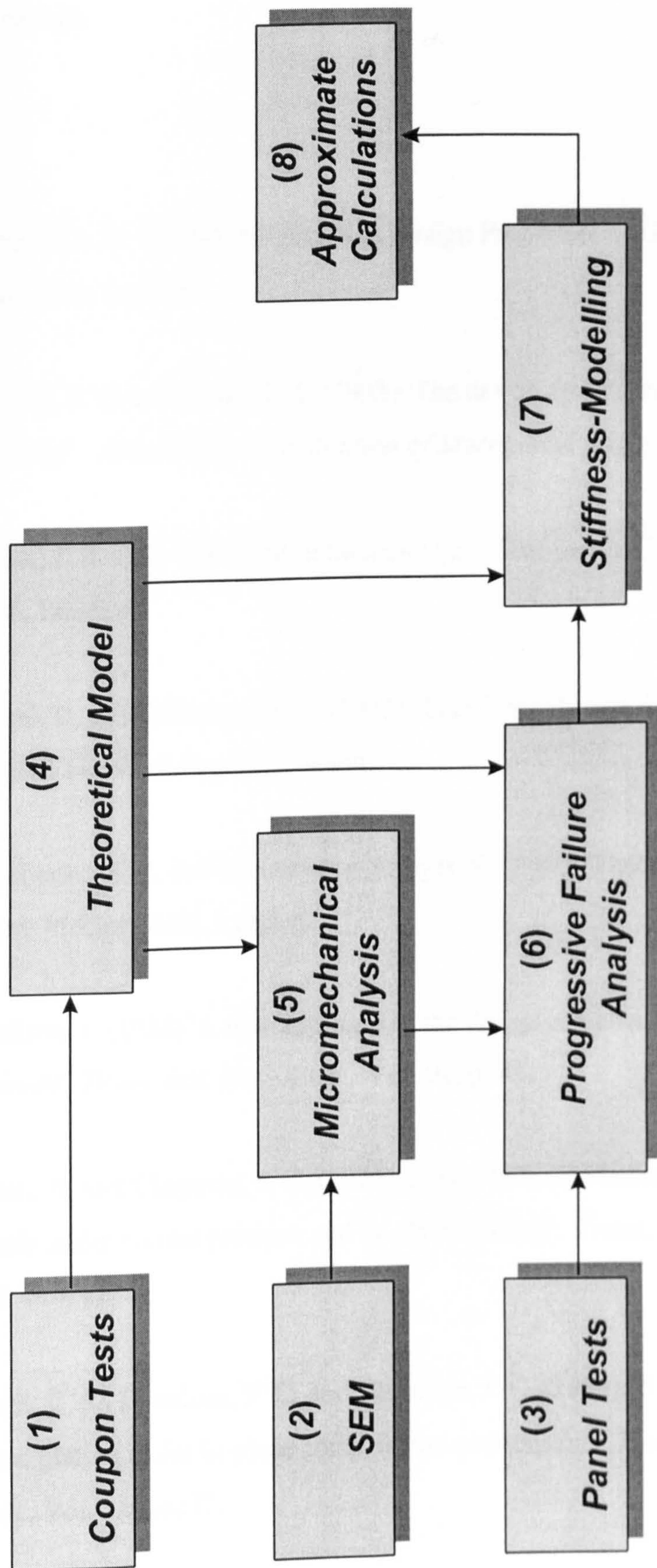


Figure 10.1 A flowchart of research structure

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Appendices

Appendix 1 Detailed coupon test results of laminate D

(1) Tensile Tests (Stiffness)

Angle to Warp	Specimen No.	Thickness (mm)	Width (mm)	Tensile Stiffness (MPa)
0°	1	4.123	24.391	18525
	2	4.170	24.913	18457
	3	4.147	24.950	18458
22.5°	7	4.247	24.570	12846
	8	4.140	24.643	13055
	9	4.140	24.483	13208
45°	13	4.350	24.983	10339
	14	4.347	25.137	10168
	15	4.333	25.167	10234
67.5°	19	4.057	25.113	13285
	20	4.073	25.073	13218
	21	4.050	24.607	13259
90°	25	4.217	24.591	17369
	26	4.163	24.810	17617
	27	4.170	24.772	17463

(2) Tensile Tests (Strength)

Angle to Warp	Specimen No.	Thickness (mm)	Width (mm)	Tensile Strength (MPa)
0°	4	4.143	24.770	256.3
	5	4.197	24.840	256.1
	6	4.180	24.771	257.9
22.5°	7	4.247	24.570	126.5
	8	4.140	24.643	131.3
	9	4.140	24.483	136.1
45°	16	4.323	25.030	110.4
	17	4.333	25.163	106.9
	18	4.300	25.040	108.2
67.5°	22	4.020	24.760	114.5
	23	3.990	24.683	115.8
	24	4.030	24.607	117.6
90°	28	4.130	24.730	228.1
	29	4.160	25.020	229.6
	30	4.147	24.820	230.3

(3) Four Points Bending Tests (Stiffness)

Angle to Warp	Spec. No.	Thickness (mm)	Width (mm)	Test Side	Bending Stiffness (MPa)
0°	1	4.123	24.391	SST	17176
				SSC	16964
	2	4.170	24.913	SST	17221
				SSC	17096
90°	25	4.217	24.591	SST	16171
				SSC	15861
	26	4.163	24.810	SST	16342
				SSC	15895

(4) Three Points Bending Tests (Strength)

Angle to Warp	Spec. No.	Thickness (mm)	Width (mm)	Test Side	Bending Strength (MPa)
0°	1	4.123	24.391	SST	412
				SSC	380
	2	4.170	24.913	SST	424
				SSC	374
22.5°	10	4.140	24.593	SST	319
				SSC	298
	11	4.183	24.547	SST	320
				SSC	309
45°	31	4.093	25.243	SST	230
				SSC	215
	32	4.087	25.013	SST	220
				SSC	230
67.5°	19	4.057	25.113	SST	281
				SSC	298
	20	4.073	25.073	SST	278
				SSC	304
90°	25	4.217	24.591	SST	373
				SSC	387
	26	4.163	24.810	SST	373
				SSC	376

Appendix 2 Detailed derivation of equations in Chapter 5

(1) Derivation of Equation (5.6) - (5.8)

From equation (5.3)

$$\varepsilon_{2m} = -\frac{S_{fi}}{S_{mi}} \varepsilon_{2f} \quad (\text{A2-1})$$

Introduction of σ_{2f} and σ_{2m} from expressions (5.1) and (5.2) and (A2-1) into (5.4) yields

$$\varepsilon_{2f} = \varepsilon_1 \left(\frac{\frac{\nu_m E_m}{g_m} - \frac{\nu_f E_f}{g_f}}{\frac{E_f}{g_f} + \frac{E_m S_{fi}}{g_m S_{mi}}} \right) \quad (\text{A2-2})$$

Again, using $\sigma_{2f} = \sigma_2$ in the equation (5.4)

$$\sigma_2 = \sigma_{2f} = \frac{\nu_f E_f}{g_f} \varepsilon_1 + \frac{E_f}{g_f} \varepsilon_{2f}$$

then substitute for ε_{2f} from (A2-2), the equations (5.7) and (5.8) can be obtained.

$$\frac{\sigma_2}{\varepsilon_1} = \frac{E_f}{g_f} \left(\frac{\nu_m S_{mi} + \nu_f S_{fi}}{\lambda S_{mi} + S_{fi}} \right) \quad (5.7)$$

in which

$$\lambda = \frac{E_f}{E_m} \cdot \frac{g_m}{g_f} \quad (5.8)$$

From equation (5.5) and (A2-1)

$$\begin{aligned}\sigma_1 &= \left(\frac{E_f}{g_f} \varepsilon_1 + \frac{\nu_f E_f}{g_f} \varepsilon_{2f} \right) S_{fi} + \left(\frac{E_m}{g_m} \varepsilon_1 + \frac{\nu_m E_m}{g_m} \varepsilon_{2m} \right) S_{mi} \\ &= \left(\frac{E_f}{g_f} S_{fi} + \frac{E_m}{g_m} S_{mi} \right) \varepsilon_1 + \left(\frac{\nu_f E_f}{g_f} S_{fi} - \frac{\nu_m E_m}{g_m} S_{fi} \right) \varepsilon_{2f}\end{aligned}\quad (\text{A2-3})$$

Substitute for ε_{2f} from (A2-2), the equation (5.6) can be obtained.

$$\frac{\sigma_1}{\varepsilon_1} = \frac{E_f}{g_f} \left\{ \frac{1 + S_{mi} S_{fi} \left[\lambda g_f + g_m / \lambda - 2(1 - \nu_f \nu_m) \right]}{\lambda S_{mi} + S_{fi}} \right\} \quad (5.6)$$

(2) Derivation of Equation (5.13)

From equations (5.9) and (5.10), the relationship between ε_{2f} and ε_{2m} can be obtained.

$$\varepsilon_{2m} = \frac{E_f}{E_m} \cdot \frac{g_m}{g_f} \varepsilon_{2f} = \lambda \varepsilon_{2f} \quad (\text{A2-4})$$

From equation (5.11)

$$\varepsilon_2 = (\lambda S_{mi} + S_{fi}) \varepsilon_{2f} \quad (\text{A2-5})$$

Again, from equation (5.10) and (A2-5), the equation (5.13) can be derived.

$$\frac{\sigma_2}{\varepsilon_2} = \frac{E_f}{g_f} \cdot \frac{1}{\lambda S_{mi} + S_{fi}} \quad (5.13)$$

Appendix 3 Program for calculation of tensile and bending elastic properties by using theoretical model

PROGRAM CMAT -- LAMINATE D

C... This program calculates the tension stiffness and bending
C... stiffness of two dimensional composite, not only unidirectional
C... but also woven fabric composite.

```
REAL MCV,MU,NUF,NUM,LAMDA,HI,H,KTOT,KK
```

```
DIMENSION THETA(30),H(30),HH(30),TF(30),TM(30),TTPT(30),  
+KTOT(3,3),TKTOT(3,3),KK(3,3),SF(30),SM(30),TI1(30),  
+T1(3,3),T3(3,3),Q(3,3),P(3,3),Z(30),  
+TKTOT1(3,3),KK1(3,3),P1(3,3),Q1(3,3)
```

C... *****

C... Data for the problem

```
OPEN(6,FILE='F.PRN')  
WRITE(*,*) 'PURE RESIN FRACTION='  
READ(*,*) A  
WRITE(*,*) 'FIBRE YOUNGS MODULUS='  
READ(*,*) B  
WRITE(*,*) 'RESIN YOUNGS MODULUS='  
READ(*,*) C
```

C...

```
PI=3.1415926  
TOA =4.15      ! Thickness overall  
RHO=0.3649     ! Fibre content by volume  
MU=A          ! Fraction of matrix  
INL=5          ! Number of layers (other than layer 0)
```

```
THETA(1)=0      ! Angle, degrees, of fibres in layer 1  
H(1) = 0.517    ! Fraction of fibre/resin in warp in layer 1  
HH(1)=0.199     ! Fraction of fibre/resin in woven layer 1  
THETA(2)=90  
H(2)=0.483      ! Fraction of fibre/resin in west in layer 1  
HH(2)=0.199  
THETA(3)=0      ! Angle, degrees, of fibres in layer 2  
H(3) = 0.515    ! Fraction of fibre/resin in warp in layer 2  
HH(3)=0.198     ! Fraction of fibre/resin in woven layer 2  
THETA(4)=90  
H(4)=0.485      ! Fraction of fibre/resin in west in layer 2  
HH(4)=0.198  
THETA(5)=0      ! Angle, degrees, of fibres in layer 3  
H(5) = 0.513    ! Fraction of fibre/resin in warp in layer 3
```

HH(5)=0.2 ! Fraction of fibre/resin in woven layer 3
 THETA(6)=90
 H(6)=0.487 ! Fraction of fibre/resin in weft in layer 3
 HH(6)=0.2
 THETA(7)=0 ! Angle, degrees, of fibres in layer 4
 H(7) = 0.512 ! Fraction of fibre/resin in warp in layer 4
 HH(7)=0.202 ! Fraction of fibre/resin in woven layer 4
 THETA(8)=90
 H(8)=0.488 ! Fraction of fibre/resin in weft in layer 4
 HH(8)=0.202
 THETA(9)=0 ! Angle, degrees, of fibres in layer 5
 H(9) = 0.508 ! Fraction of fibre/resin in warp in layer 5
 HH(9)=0.201 ! Fraction of fibre/resin in woven layer 5
 THETA(10)=90
 H(10)=0.492 ! Fraction of fibre/resin in weft in layer 5
 HH(10)=0.201

EF= B ! Young's modulus, fibre
 EM= C ! Young's modulus, matrix
 NUF=0.3 ! Poisons ratio, fibre
 NUM=0.3 ! Poisons ratio, matrix

C... *****
 C... Calculate constants for material as a whole.

FCV= RHO ! Fibre content by volume (entire laminate)
 MCV=1-RHO ! Matrix content by volume (entire laminate)

BF=1-NUF**2 ! = gf
 BM=1-NUM**2 ! = gm

GF=EF/(2*(1+NUF)) ! Shear modulus, fibre
 GM=EM/(2*(1+NUM)) ! Shear modulus, matrix

LAMDA=(EF/EM)*(BM/BF)

C... *****
 C... For pure resin layer

KK1(1,1)=EM/BM
 KK1(1,2)=NUM*KK1(1,1)
 KK1(1,3)=0

KK1(2,1)=KK1(1,2)
 KK1(2,2)=KK1(1,1)
 KK1(2,3)=0

KK1(3,1)=0
 KK1(3,1)=0
 KK1(3,3)=GM

TM0=MU*(1-RHO)*TOA ! Total thickness of pure resin layer
 TM1=TM0/(2*INL) ! Pure resin thickness allocated in each layer
 Z(1)=TOA/2
 Z(2)=TOA/2-TM1

```

C... *****
C... Add layer 0 to stiffness matrix of the laminate
      DO 100 I=1,3
      DO 110 J=1,3
      TKTOT(I,J)=TM0*KK1(I,J)
      TKTOT1(I,J)=KK1(I,J)*(Z(1)**3-Z(2)**3)/3
110  CONTINUE
100  CONTINUE

C... *****
C... Calculate each layer in turn and add to composite...

      DO 709 IL=1,2*INL
      TI1(IL)=TOA*H(IL)*HH(IL)*(1-MU*(1-RHO)) ! Thickness of warp/weft
709  CONTINUE

      Z(3)=Z(2)-TI1(1)
      Z(4)=Z(3)-TI1(2)
      Z(5)=Z(4)-2*TM1
      Z(6)=Z(5)-TI1(3)
      Z(7)=Z(6)-TI1(4)
      Z(8)=Z(7)-2*TM1
      Z(9)=Z(8)-TI1(5)
      Z(10)=Z(9)-TI1(6)
      Z(11)=Z(10)-2*TM1
      Z(12)=Z(11)-TI1(7)
      Z(13)=Z(12)-TI1(8)
      Z(14)=Z(13)-2*TM1
      Z(15)=Z(14)-TI1(9)
      Z(16)=Z(15)-TI1(10)
      Z(17)=Z(16)-TM1

      DO 111 K=2,6
      DO 112 I=1,3
      DO 113 J=1,3
      IF(K.EQ.2) TKTOT1(I,J)=TKTOT1(I,J)+KK1(I,J)*(Z(4)**3-Z(5)**3)/3
      IF(K.EQ.3) TKTOT1(I,J)=TKTOT1(I,J)+KK1(I,J)*(Z(7)**3-Z(8)**3)/3
      IF(K.EQ.4) TKTOT1(I,J)=TKTOT1(I,J)+KK1(I,J)*(Z(10)**3-Z(11)**3)/3
      IF(K.EQ.5) TKTOT1(I,J)=TKTOT1(I,J)+KK1(I,J)*(Z(13)**3-Z(14)**3)/3
      IF(K.EQ.6) TKTOT1(I,J)=TKTOT1(I,J)+KK1(I,J)*(Z(16)**3-Z(17)**3)/3
113  CONTINUE
112  CONTINUE
111  CONTINUE

      WRITE(*,*) 'Coordination of each layer'
      DO 707 J=1,3*INL+2
      WRITE(*,*) Z(J)
707  CONTINUE
      READ(*,80)

C... *****
C... Calculate stiffness of each fibre/resin layer

      DO 500 IL=1,2*INL
      TI=TI1(IL)
      TFI=RHO*TOA*H(IL)*HH(IL)
      SFI=TFI/TI ! Vol fibre/total vol of this layer

```

SMI=1-SFI ! vol matrix/total vol of this layer

```
KK(1,1)=(EF/BF)*(1+SMI*SFI*(LAMDA*BF + BM/LAMDA -2*(1-NUF*NUM)))/
+(LAMDA*SMI + SFI)
KK(1,2)=(EF/BF)*(NUM*SMI + NUF*SFI)/(LAMDA*SMI + SFI)
KK(1,3)=0
KK(2,1)= KK(1,2)
KK(2,2)= (EF/BF)/(LAMDA*SMI + SFI)
KK(2,3)=0
KK(3,1)=0
KK(3,2)=0
KK(3,3)= 1/((SMI/GM)+(SFI/GF))
```

C... *****

```
S=SIN(THETA(IL)*PI/180)
C=COS(THETA(IL)*PI/180)
```

C... TRANSFORM LAYER IL

C... Form T1, which is the stress transformation matrix.

```
T1(1,1)=C*C
T1(1,2)=S*S
T1(1,3)=2*S*C

T1(2,1)=T1(1,2)
T1(2,2)=T1(1,1)
T1(2,3)=-T1(1,3)

T1(3,1)=-S*C
T1(3,2)=-T1(3,1)
T1(3,3)=C*C - S*S
```

C... Form T3, which is the inverse of T2 (the strain transformation
C... matrix)

```
T3(1,1)=C*C
T3(1,2)=S*S
T3(1,3)=-S*C

T3(2,1)=T3(1,2)
T3(2,2)=T3(1,1)
T3(2,3)=-T3(1,3)

T3(3,1)=2*S*C
T3(3,2)=-T3(3,1)
T3(3,3)=C*C - S*S
```

C... Form P = KK* T3

```
DO 200 I=1,3 ! For each row (I) of KK
DO 210 J=1,3 ! For each column (J) of T3
P(I,J)=0
DO 220 K=1,3
P(I,J)=P(I,J)+KK(I,K)*T3(K,J)
220 CONTINUE
210 CONTINUE ! Next column J
```

```

200 CONTINUE      ! Next row I

C... Form Q=T1*P

      DO 300 I=1,3    ! For each row (I) of T1
      DO 310 J=1,3    ! For each column (J) of P
      Q(I,J)=0
      DO 320 K=1,3
      Q(I,J)=Q(I,J) + T1(I,K)*P(K,J)
320 CONTINUE
310 CONTINUE      ! Next column J
300 CONTINUE      ! Next row I

C... Q is now the transformed stiffness matrix.
C... *****
C... Add layer i to the composite matrix

      DO 400 I=1,3
      DO 410 J=1,3
      TKTOT(I,J)=TKTOT(I,J) + TI*Q(I,J)  ! CHECK THE USE OF THICKNESS

      IF(IL.EQ.1) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(2)**3-Z(3)**3)/3
      IF(IL.EQ.2) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(3)**3-Z(4)**3)/3
      IF(IL.EQ.3) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(5)**3-Z(6)**3)/3
      IF(IL.EQ.4) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(6)**3-Z(7)**3)/3
      IF(IL.EQ.5) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(8)**3-Z(9)**3)/3
      IF(IL.EQ.6) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(9)**3-Z(10)**3)/3
      IF(IL.EQ.7) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(11)**3-Z(12)**3)/3
      IF(IL.EQ.8) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(12)**3-Z(13)**3)/3
      IF(IL.EQ.9) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(14)**3-Z(15)**3)/3
      IF(IL.EQ.10) TKTOT1(I,J)=TKTOT1(I,J)+Q(I,J)*(Z(15)**3-Z(16)**3)/3
410 CONTINUE
400 CONTINUE
C... *****

500 CONTINUE      ! Next layer

C... *****
C... Display results

      WRITE(*,70) 'FINAL RESULTS OF TENSILE STIFFNESS'
      WRITE(*,70) ' '
      WRITE(*,71) 'TOA = ',TOA

      WRITE(*,70) ' '
      WRITE(*,71) 'TKTOT(1,1) = ',TKTOT(1,1)
      WRITE(*,71) 'TKTOT(1,2) = ',TKTOT(1,2)
      WRITE(*,71) 'TKTOT(1,3) = ',TKTOT(1,3)

      WRITE(*,70) ' '
      WRITE(*,71) 'TKTOT(2,1) = ',TKTOT(2,1)
      WRITE(*,71) 'TKTOT(2,2) = ',TKTOT(2,2)
      WRITE(*,71) 'TKTOT(2,3) = ',TKTOT(2,3)

      WRITE(*,70) ' '

```

```
WRITE(*,71) 'TKTOT(3,1) = ',TKTOT(3,1)
WRITE(*,71) 'TKTOT(3,2) = ',TKTOT(3,2)
WRITE(*,71) 'TKTOT(3,3) = ',TKTOT(3,3)
```

```
read(*,80)
```

```
WRITE(*,70) 'FINAL RESULTS OF BENDING STIFFNESS'
WRITE(*,70) ' '
WRITE(*,71) 'TKTOT1(1,1) = ',TKTOT1(1,1)
WRITE(*,71) 'TKTOT1(1,2) = ',TKTOT1(1,2)
WRITE(*,71) 'TKTOT1(1,3) = ',TKTOT1(1,3)
```

```
WRITE(*,70) ''
WRITE(*,71) 'TKTOT1(2,1) = ',TKTOT1(2,1)
WRITE(*,71) 'TKTOT1(2,2) = ',TKTOT1(2,2)
WRITE(*,71) 'TKTOT1(2,3) = ',TKTOT1(2,3)
```

```
WRITE(*,70) ''
WRITE(*,71) 'TKTOT1(3,1) = ',TKTOT1(3,1)
WRITE(*,71) 'TKTOT1(3,2) = ',TKTOT1(3,2)
WRITE(*,71) 'TKTOT1(3,3) = ',TKTOT1(3,3)
```

```
READ(*,80)
```

C... *****

C... Calculate material properties at 0 and 90 degree.

C... Tension results

```
TN1=TKTOT(2,1)/TKTOT(1,1)      ! Poisson's ratio Vxy
TN2=TKTOT(1,2)/TKTOT(2,2)      ! Poisson's ratio Vyx
TEX=TKTOT(1,1)*(1-TN1*TN2)/TOA ! Young's modulus in 0 degree
TEY=TKTOT(2,2)*(1-TN1*TN2)/TOA ! Young's modulus in 90 degree
TGXY=TKTOT(3,3)/TOA           ! Shear modulus
```

C... Bending results

```
BN1=TKTOT1(2,1)/TKTOT1(1,1)
BN2=TKTOT1(1,2)/TKTOT1(2,2)
BEX=TKTOT1(1,1)*(1-BN1*BN2)*12/(TOA**3)
BEY=TKTOT1(2,2)*(1-BN1*BN2)*12/(TOA**3)
BGXY=TKTOT1(3,3)*12/(TOA**3)
```

C... *****

C... Calculation material properties at arbitrary angles

```
DO 700 TNGLE=0,90,22.5 ! 0, 22.5, 45, 67.5, 90 degrees
S=SIN(TNGLE*PI/180)
C=COS(TNGLE*PI/180)
```

C... Tension results

```
EX=1/(C**4/TEX+S**4/TEY+S*S*C*C*(1/TGXY-2*TN1/TEX))
EY=1/(S**4/TEX+C**4/TEY+S*S*C*C*(1/TGXY-2*TN1/TEX))
GXY=1/(2*(2/TEX+2/TEY+4*TN1/TEX-1/TGXY)*S*S*C*C+(S**4+C**4)/TGXY)
FN1=EX*(TN1*(S**4+C**4)/TEX-(1/TEX+1/TEY-1/TGXY)*S*S*C*C)
```

C... Bending results

```

EX1=1/(C**4/BEX+S**4/BEY+S*S*C*C*(1/BGXY-2*BN1/BEX))
EY1=1/(S**4/BEX+C**4/BEY+S*S*C*C*(1/BGXY-2*BN1/BEX))
GXY1=1/(2*(2/BEX+2/BEY+4*BN1/BEX-1/BGXY)*S*S*C*C+(S**4+C**4)/BGXY)
FN11=EX1*(BN1*(S**4+C**4)/BEX-(1/BEX+1/BEY-1/BGXY)*S*S*C*C)

```

```

WRITE(*,*) 'TENSILE', TNGLE,EX,EY,FN1,GXY
WRITE(*,*) 'BENDING', TNGLE,EX1,EY1,FN11,GXY1

```

```

READ(*,80)

```

```

700 CONTINUE ! Next angle

```

```

70 FORMAT(' ',A40)
71 FORMAT(' ',A20,F20.4)
80 FORMAT(A)

```

```

STOP
END

```

.

Appendix 4 Typical user-defined subroutine used in progressive failure analysis

USER-DEFINED FIELD

*USER SUBROUTINES

SUBROUTINE USDEFLD (FIELD, STATEV, PNEWDT, DIRECT, T,
CELENT, TIME, DTIME, CMNAME, ORNAME, NFIELD, NOEL,
NPT, LAYER, KSPT, KSTEP, KINC, NDI, NSHR)

C

C

INCLUDE ' ABA_PARAM.INC '

C

C

C material strength parameters

C

PARMATER (YT=50, XC=640, XT=710, YC=100, SC=50)

CHARACTER*8 CMNAME, ORNAME, FLGRAY (15)

C

DIMENSION FIELD(NFIELD), STATEV(NSTATV), DIRECT(3,3)

DIMENSION T(3,3), TIME(2), ARRAY(15), JARRAY(15)

C

C initial failure flags from STATV

C

EM = STATEV(1)

EFS = STATEV(2)

EFBRK = STATEV(3)

C

C get the stresses at integration point

C

CALL GETVRM (' S ', ARRAY, JARRAY, FLGRAY, JRCD)

S11 = ARRAY(1)

S22 = ARRAY(2)

S12 = ARRAY(4)

C

C get the three field variables

C

CALL GETVRM (' FV ', ARRAY, JARRAY, FLGRAY, JRCD)

FV1 = ARRAY(1)

FV2 = ARRAY(2)

FV3 = ARRAY(3)

C

C matrix tensile/compressive failure (matrix cracking)

C

IF (EM .LT. 1) THEN

IF (S22 .LT. 0) THEN

EM = SQRT ((S22 / YC)**2 + (S12 / SC)**2)

ELSE

EM = SQRT ((S22 / YT)**2 + (S12 / SC)**2)

ENDIF

STATEV (1) = EM

```

ENDIF
C
C fibre-matrix shear failure
C
      IF ( EFS .LT. 1 )      THEN
            IF ( S11 .LT. 0 ) THEN
                  EFS = SQRT ( ( S11 / XC )**2 + (S12 / SC)**2 )
            ELSE
                  EFS = SQRT ( ( S11 / XT )**2 + (S12 / SC)**2 )
            ENDIF
      STATEV (2) = EFS
ENDIF

```

```

C
C
C  fibre breakage
C
C
      IF (EFBRK .LT. 1) THEN
          IF (S11 .LT. 0) THEN
              EFBRK = SQRT ((S11 / XC)**2)
          ELSE
              EFBRK = SQRT ((S11 / XT)**2)
          ENDIF
      STATEV (3) = EFBRK
      ENDIF

```

FV1: MATRIX CRACKING
FV2: FIBRE-MATRIX SHEAR FAILURE
FV3: FIBRE BREAKAGE

		FV1	FV2	FV3		E1	E2	NU12	G
(0)	NO DAMAGE	0	0	0	----	E1	E2	NU12	G
(1)	MATRIX CRACKING	1	0	0	----	E1	0	0	G
(2)	SHEAR FAILURE	0	1	0	----	E1	E2	0	0
(3)	MATRIX/SHEAR	1	1	0	----	E1	0	0	0
(4)	F/BREAKAGE	0	0	1	----	0	0	0	0
(5)	MATRIX/BREAK	1	0	1	----	0	0	0	0
(6)	SHEAR/BREAK	0	1	1	----	0	0	0	0
(7)	ALL DAMAGE	1	1	1	----	0	0	0	0

UPDATE FIELD VARIABLES

FIELD (1) = 0
FIELD (2) = 0
FIELD (3) = 0

```

IF ( EM .GT. 1 )           FIELD (1) = 1
IF ( EFS .GT. 1 )         FIELD (2) = 1
IF ( EFBRK .GT. 1 )       FIELD (3) = 1

```

C

C

IF (EM .GT. 1)
IF (EFS .GT. 1)
IF (EFBRK .GT. 1)

STATEV (71) = EM
STATEV (72) = EFS
STATEV (73) = EFBRK

C

C

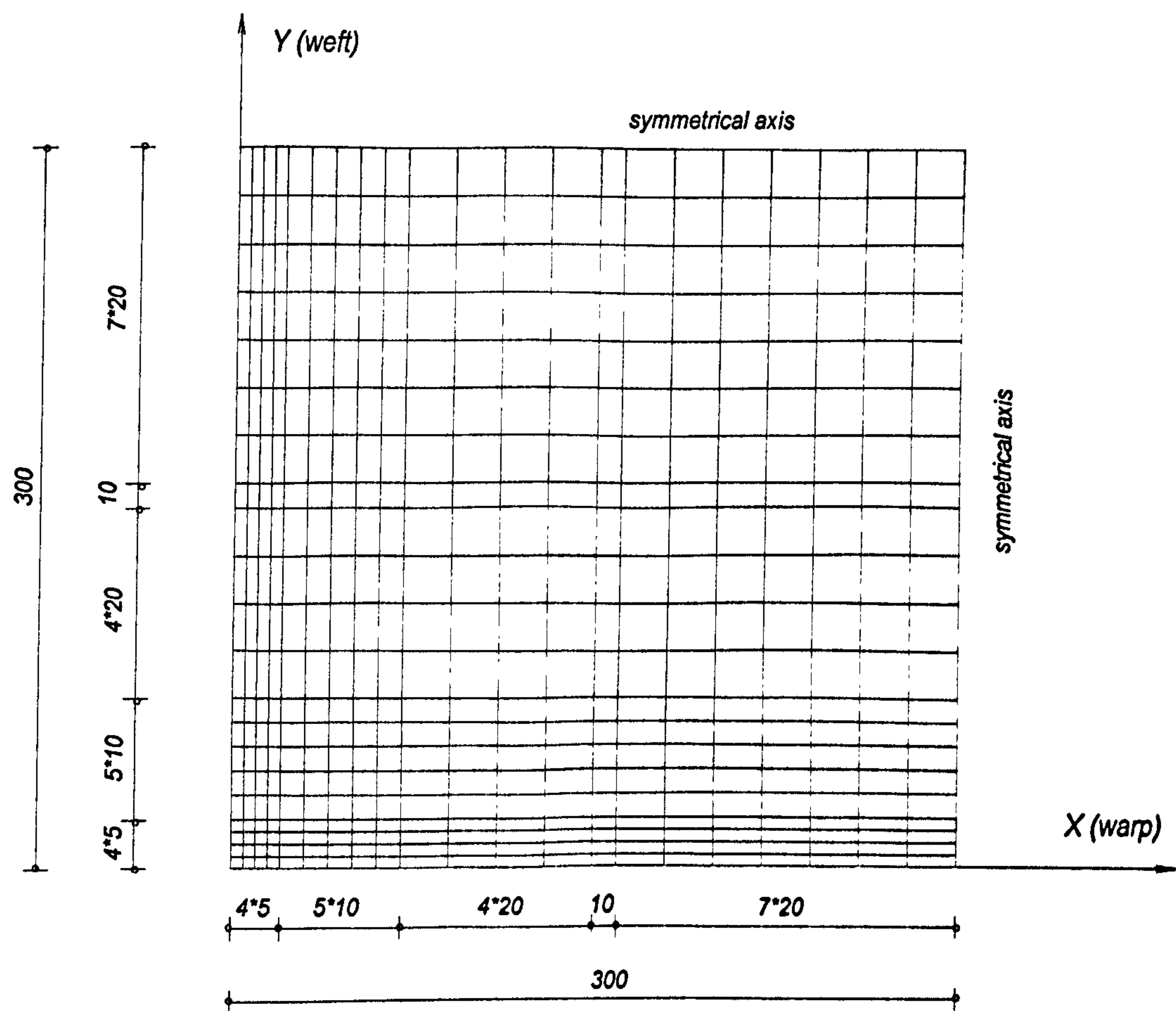
RETURN
END

C

C

Appendix 5 Dimensions of the elements for a square panel mesh

Following figure is a demonstration of the element mesh of a quarter of Panel D in the finite element analysis in Chapter 8.



Appendix 6 Program for calculation of maximum deflection, membrane tension and local bending moment

C... PROGRAM APP1

C... Approximate method for panels of laminate A, B, C and D with

C... and without in-plane edge displacement.

C... both resin effective and ineffective cases, spring support

OPEN(6,FILE='F.PRN')

C... READ PARAMETERS *****

WRITE(*,*)'EFF=, 1 for RI; 2 for RE'

WRITE(*,*)'PANEL=, 1-4 for A-D'

WRITE(*,*)'AA= 0.95, 0.8,0.6 for ASPECT RATIO 2, 1.5,1'

WRITE(*,*)'PARA=,SPRING STIFFNESS, try'

WRITE (*,*)'PANEL D: PARA=4.1 RE; PARA=5.1 RI'

WRITE (*,*)'PANEL A: PARA=1.69 RE; PARA=2.2 RI'

WRITE (*,*)'PANEL B: PARA=0.77 RE; PARA=1.32 RI'

WRITE(*,*)'N8= LOOP NUMBER, 30-40 '

WRITE(*,*)'

READ(*,*) EFF,PANEL,AA,PARA,N8

C... EFF=1, RESIN INEFFECTIVE; EFF=2, RESIN EFFECTIVE

C...

C... PANEL=1, LAMINATE A; PANEL=2, LAMINATE B

C... PANEL=3, LAMINATE C; PANEL=4, LAMINATE D

C...

C... AA--COEFFICIENT OF PREESURE ON STRIP

C... AA=1, ASPECT RATIO 2; AA=0.8 ASPECT RATIO 1.5

C... AA=0.6 ASPECT RATIO 1;

C... PARA--SPRING STIFFNESS;

C... PANEL D: PARA=4.1 RE; PARA=5.1 RI

C... IF WITHOUT EDGE DISPLACEMENT, PARA IS CHOSEN LARGE NUMBER

C... N8--LOOP NUMBER FOR THETA

C... MATERIAL PROPERTIES *****

IF (PANEL.GT.1) GOTO 2

C... PANEL OF LAMINATE A *****

TH=2.84 ! THICKNESS OF THE PANEL

IF (EFF.GT.1) GOTO 6

E=19200 ! YOUNG'S MODULUS resin ineffective

GOTO 10

6 E=27230 ! YOUNG'S MODULUS resin effective

GOTO 10

```

C... *****

2  IF (PANEL.GT.2) GOTO 3

C... PANEL OF LAMINATE B *****

      TH=3.431      ! THICKNESS OF THE PANEL

      IF (EFF.GT.1) GOTO 7
      E=6700        ! YOUNG'S MODULUS resin ineffective
      GOTO 10
7   E=13000        ! YOUNG'S MODULUS resin effective
      GOTO 10

C... *****

3  IF (PANEL.GT.3) GOTO 4

C... PANEL OF LAMINATE C *****

      TH=2.94      ! THICKNESS OF THE PANEL

      IF (EFF.GT.1) GOTO 8
      E=15500      ! YOUNG'S MODULUS resin ineffective
      GOTO 10
8   E=21800      ! YOUNG'S MODULUS resin effective
      GOTO 10

C... *****

4  CONTINUE

C... PANEL OF LAMINATE D *****

      TH=4.15      ! THICKNESS OF THE PANEL

      IF (EFF.GT.1) GOTO 9
      E=13100      ! YOUNG'S MODULUS resin ineffective
      GOTO 10
9   E=18350      ! YOUNG'S MODULUS resin effective

C... *****

10 CONTINUE

      PI= 3.1415926
      B=1          ! UNIT WIDTH, MM
      A=300        ! HALF-SPAN, MM

C..  CALCULATIONS BEGIN *****

      WRITE(6,74) 'DELTA1','DELTA2','DELTA3','P','W','T','ME'

C... CALCULATE DEFLECTION AND TENSION *****

      K=PARA*E*B*TH/A          ! SPRING STIFFNESS
      EI=E*TH**3/12           ! BENDING STIFFNESS

```

DO 100 THETAD = 1,N8 ! RANGE OF ANGLES, DEGREES

THETA = THETAD*PI/180

THETA3 = THETA**3

R = A/SIN(THETA/2) ! RADIUS OF CURVATURE

W = R - SQRT(R**2 - A**2) ! DEFLECTION

R1=R*(THETA-2*SIN(THETA/2))

R2=(2*A/(E*TH) + (2*COS(THETA/2))/K)

T=R1/R2 ! TENSION

DELTA2= T*2*A/(E*B*TH) ! ELASTIC EXTENSION

DELTA1= R*(THETA - 2*SIN(THETA/2)) ! GEMOETRICAL SHORTENING

DELTA3= 2*T/K ! SPRING DEFORMATION

P1=T/(B*R) ! PRESSURE ON STRIP

P=P1/AA ! PRESSURE ON PANEL

C... CALCULATION LOCAL MOMENT *****

U=SQRT(T/EI)

UL=U*600/2

EX1=EXP(UL)

EX2=EXP(-UL)

COTH1=(EX1+EX2)/(EX1-EX2)

TME=P1*600*(1/UL-COTH1)/(2*U) ! MOMENT AT EDGE

C... PRINT RESULTS *****

WRITE(*,73) DELTA1,DELTA2,DELTA3,P,W,T,TME

100 CONTINUE ! Next step in loop 100

C... *****

WRITE(*,74) 'DELTA1','DELTA2','DELTA3','P','W','T','ME'

73 FORMAT (3F8.3, 5F10.2)

74 FORMAT (3A8,4A10)

END