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CORRECTION FACTORS FOR SPRINKLER LATERALS

Volume I

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Doctor of Philosophy
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ABSTRACT

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CORRECTION FACTORS FOR SPRINKLER LATERALS

by Arif Aziz Anwar

Irrigation laterals are unique pipelines as there is a decrease of flow along the pipeline. This makes the hydraulics of laterals an interesting problem

Laterals can be broadly classified into two categories:

a) Laterals where the flow decreases in direct proportion to the length of the lateral (linear variation). This is typical of sprinkler systems with fixed, periodic move or linear displacement laterals.
b) Laterals where the flow decreases in proportion to the square of the length of the lateral (non-linear variation). This is typical of center-pivots.

Although it is conceptually feasible to have a variety of functions relating discharge along a lateral to length, non-linear variation in lateral discharge, other than (b) above, have no practical application and are not considered further.

The friction correction factor F can only be applied to pipelines of a single diameter and without outflow i.e. flow at the downstream end of the lateral. The first section of this thesis presents a new friction correction factor G. Factor G can be applied to laterals with outlets and downstream outflow. The second section in this thesis develops the adjusted friction correction factor G_a. This is analogous to the adjusted friction factor F_a although G_a can be applied to laterals with downstream outflow. In section three, average correction factors F_{AVG} and G_{AVG} are developed. Section four extends these concepts to develop adjusted average correction factors; $F_{a\ AVG}$ and $G_{a\ AVG}$. Average correction factor $G_{a\ AVG}$ is demonstrated as a generic average correction factor.

Section five of this thesis rexamines the earlier work on friction factors for center-pivots. A new conceptual model is proposed. New friction correction factors are developed which are dependent on number of outlets and the friction equation used. The final section develops new correction factors for head loss caused by friction and pressure head distribution in center pivots with end guns using the concept of end gun ratio.

LIST OF CONTENTS

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I would like to express my sincerest gratitude to Professor T.W. Tanton, Professor M.McDonald and Professor R. Stoner. They were instrumental in my joining the Department of Civil & Environmental Engineering as a lecturer, for which I am deeply indebted. I would like to mention Professor Tanton in particular who was my mentor. His easy and friendly attitude has always been very helpful and encouraging.

I would also like to thank all my other colleagues in the Department, in particular those at the Institute of Irrigation and Development Studies. I thank them for their patience with me while I ranted and raved over my correction factors.......proclaiming they were the best thing since the proverbial sliced-bread.

I must also thank my mother for her infinite patience and blessings. In memory of my late father. He always believed the most precious gift parents could give their children is the best possible education. Throughout his short life he always endeavored to do so. I now truly appreciate the wisdom of that belief.....and to Anjie for painstakingly correcting the draft.

Arif Anwar
Southampton, UK.
March, 2000.

PREFACE

PREFACE

This thesis is a compilation of peer reviewed publications by the author and is submitted under staff candidature regulations. Each published journal paper is presented as an individual chapter and is self-contained. A footnote at the beginning of each chapter gives details of the journal, and date of publication. Each paper has been reproduced verbatim; however to comply with the Faculty of Engineering and Applied Science guidelines, word-processed versions, rather than typeset offprints, have been included. Since each chapter was developed as a paper to be submitted for publication, where necessary, the publisher's guidelines have taken priority over Faculty guidelines. As a result, the structure of this thesis may appear unconventional. For example, chapters have individual literature review sections and reference sections, a practical application section also appears in every chapter, etc. Similarly the length of each chapter has been dictated by the publisher's word limit of 10,000 equivalent words for an individual paper. The use of US english was also mandatory.

Appendix A contains comments by the reviewers on individual papers and the ranking of the papers. Appendix B includes a discussion paper and closure to the discussion by the author. The author has taken the liberty of including appendices A and B in order to allow examiners access to this information. Wherever possible, the comments received from the reviewers were incorporated into each paper prior to publication. Furthermore the direction in which this work developed was strongly influenced by the reviewers' comments.

The *Regulations for Members of Staff in Candidature for the Degrees of Master of Philosophy and Doctor of Philosophy*[1] requires

> (c) that the published material is bound together with an abstract and a statement of the candidate's aims and of the nature of the research indicating the contributions to it of the works submitted.

The abstract precedes this preface, the remainder of this preface presents the statement of the candidate's aims and the nature of the research.

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[1] University of Southampton Calendar 1999/2000

The overall aim of this work is to develop additional/new tools for the analysis and design of sprinkler laterals, in particular for tapered sprinkler laterals and for center-pivots with end guns. To date all correction factors have been developed for laterals without outflow at the downstream end. However in a tapered lateral, the first reach of the lateral can be considered as a lateral with outflow at the downstream end. Similarly a center pivot lateral with an end gun is again an example of a lateral with outflow at the downstream end. The papers submitted in this thesis present a series of new correction factors that can be used for this purpose. Furthermore, the development of these factors has highlighted anomalies in earlier work and led to a better understanding of correction factors.

The friction correction factor F is widely used in estimating the head loss due to friction in a sprinkler lateral. Using factor F eliminates the need to analyze a sprinkler lateral using a stepwise approach. Although the latter approach has become quite easy using spreadsheets, factor F is still more convenient and simple to use. An assumption in developing factor F is that there is no flow downstream of the last sprinkler i.e. the downstream end of the lateral is blocked. The first chapter of this thesis presents factor G as a sequel to factor F. However, factor G can be applied to laterals with or without outflow at the downstream end of the lateral. In fact, if the downstream outflow is reduced to zero, then factor G reduces to factor F. Hence factor G is a more generic friction correction factor for sprinkler laterals. The development of factor G also highlights a subtle but important point. Factor G (and indeed the close approximation of factor F) is a summation expression which can be expanded to an infinite series using the Euler-McLaurin summation formula. This infinite series becomes finite if and only if the velocity exponent in the friction formula is two ie. for turbulent flow. For all other values of the velocity exponent in the friction formula, terminating the series after the second term is an approximation. For such exponents, a better estimate of the friction correction factors can be obtained using the summation form rather than the expansion form of the equation. A practical application of factor G has been demonstrated in the analysis of a tapered sprinkler lateral.

A second assumption in the development of factor F is that the first (most upstream) sprinkler is at a full spacing from the inlet of the lateral. This is often not the case since the length of a field cannot be expected to be an exact multiple of sprinkler spacings. This has led to the development of the adjusted friction correction factor F_a which allows the first sprinkler to be at a fractional spacing from the inlet. In this thesis, the second paper extends the friction correction factor G to the adjusted friction correction factor G_a. Factor G_a is a generic friction correction factor for sprinkler laterals. If the outflow is reduced to zero, factor G_a reduces to F_a. If the first sprinkler is at a full spacing from the inlet, the adjusted friction correction factor G_a reduces to G. Finally if both the outflow is reduced to zero, and the first sprinkler is at a full spacing from the inlet, factor G_a reduces to F.

In a sprinkler lateral there will be head loss caused by friction and hence pressure head variation along a lateral. For a lateral on the horizontal or on an incline, the pressure head will be minimum at the downstream end. For ordinary (non pressure compensating) sprinkers on a lateral, this variation in pressure head creates a variation in the discharge from sprinklers along the lateral. Sprinkler laterals are designed with an inlet pressure head such that the average of the discharge of all sprinklers along a lateral is equal to the required average sprinkler discharge. To date in all literature this inlet pressure of a sprinkler lateral is quoted as the sum of average sprinkler pressure head and 25% of head loss caused by friction. The third paper in this thesis introduces for the first time an average correction factor F_{avg}. This paper shows how for a reasonable number of sprinklers the average correction factor F_{avg} approaches 25%. This paper expands this idea further and develops the average correction factor G_{avg}. This factor can be applied to sprinkler laterals with or without outflow at the downstream end of the lateral. To apply these average friction correction factors, a concept of length weighted average correction factors has been developed. To demonstrate the practical application, a tapered sprinkler irrigation lateral has been analyzed and the required inlet pressure has been determined using these factors.

The fourth paper in this thesis extends the average correction factor to develop the adjusted average correction factor $G_{a\ \text{avg}}$. As with the adjusted friction correction factor, the adjusted average correction factor is a generic correction factor which reduces to one of the factors described earlier under specific conditions.

Center pivot laterals differ from conventional sprinkler laterals. In the latter the discharge through the lateral decreases linearly with length. However, with center pivot laterals the discharge decreases parabolically with length. The most notable work on friction factors for center pivot laterals considered the lateral to have an infinite number of small sprinklers. This was extended by considering discrete sprinklers along the center pivot lateral. The fifth paper in this thesis shows the anomaly of this extension and presents an alternative method of modelling such a center pivot lateral. In this paper, two friction correction factors are presented for center pivot laterals with constant spacing sprinklers and center pivot laterals with constant discharge sprinklers. Pressure distribution factors for both these laterals are also developed. The pressure distribution factors can be determined for any exponent of the velocity term in the friction equation used. This was hitherto not possible.

The final paper of this thesis analyzes center pivot laterals with outflow. This represents a center pivot with an end gun. Friction correction factors and pressure distribution factors are developed for two cases: a center pivot with an infinite number of small sprinklers; and, a center pivot with a discrete number of small sprinklers. These new factors remove the need to analyze center pivots using the arbitrary term of effective radius.The pressure distribution factor developed in this paper shows a higher pressure towards the center of the lateral than previously predicted.

A brief end note concludes this thesis and outlines how the ideas presented might be developed further.

1

FACTOR *G* FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

FACTOR *G* FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

By Arif A. Anwar[1]

ABSTRACT: A factor *G* for pipelines with equally spaced multiple outlets and outflow at the downstream end is derived. The proposed factor is a function of the number of outlets along the pipeline and also a function of the friction formula used. Factor *G* allows head loss in such pipelines to be computed directly provided the first outlet is one outlet spacing distance from the pipeline inlet. Under conditions of zero outflow at the downstream end of the pipeline, factor *G* reduces to the well known Christiansen's factor *F*. Factor *G* allows the design of segments of pipelines with multiple outlets. It may find application with irrigation engineers in designing sprinkler and trickle irrigation laterals and manifolds with multiple diameter sizes. It also may be used in trickle line hydraulics in flushing mode.

INTRODUCTION

The head loss caused by friction in a pipeline with multiple outlets along its length will be less than the head loss caused by friction in a pipeline without outlets, because of the decreasing discharge along the length of the pipeline. The estimation of head loss caused by friction in pipelines with multiple outlets requires a stepwise analysis starting from the most downstream outlet, working upstream and computing the head loss caused by friction in each segment. Christiansen (1942) developed a friction factor *F* to avoid the cumbersome stepwise analysis. Computing the head

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loss in a pipe considering the entire discharge to flow through the entire length and multiplying by factor F allows the head loss through a single-diameter pipeline with multiple outlets to be estimated. Factor F was derived assuming the following: (1) no outflow at the downstream end of the pipeline; (2) all outlets are equally spaced (constant outlet spacing); (3) all outlets have equal discharge; and (4) the distance between the pipe inlet and the first outlet is equal to the outlet spacing. Factor F is a function of the friction formula used and the number of outlets.

In many situations, the first outlet cannot be located in a full spacing from the pipeline inlet. Jensen and Fratini (1957) derived an adjusted factor F, which allows for calculating head loss in single-diameter pipelines with multiple equally spaced outlets, where the first outlet is one-half an outlet spacing from the pipeline inlet. However this expression does not allow for any outflow at the downstream end of the pipeline. Chu (1978) modified the adjusted factor F of Jensen and Fratini (1957) and claimed this modified factor F could be considered as a constant for five or more outlets without introducing any significant error. This work also assumes no outflow at the downstream end of the pipeline beyond the last sprinkler.

Scaloppi (1988) derived the adjusted factor F_a, which allows for direct calculation of head loss caused by friction in a single-diameter pipeline with multiple equally spaced outlets and the first outlet at any distance from the pipeline inlet. Scaloppi (1988) also assumes zero outflow past the most downstream outlet.

For a single-diameter pipeline with multiple outlets, factor F or the adjusted factor F_a allows rapid calculation of head loss caused by friction. However, if multiple-diameter pipelines are used, factor F or the adjusted factor F_a cannot be applied directly to the entire length of the pipeline. If for analytical purposes the pipeline is divided into reaches based on pipeline diameter, then again factor F cannot be applied directly to any except the most downstream pipe reaches. Other reaches of the pipeline would have outflow at the downstream end. To resolve this problem, indirect methods of using factor F to design pipelines with two pipe diameters or graphical methods for multiple pipe sizes have been developed (Keller and Bliesner 1990).

The factor *G* would allow for head loss in a pipeline with multiple equally spaced outlets and any outflow at the downstream end past the last outlet. Hence any multiple-diameter pipeline with uniformly spaced outlets can be divided into reaches based on diameter for analytical purposes. Factor *G* can be applied to each segment to calculate head loss caused by friction. Factor *G* will reduce to factor *F* if outflow at the downstream end is set to zero; hence it can be applied equally well to the most downstream reach of the lateral. Such a factor may find application in the design of pipelines with multiple equally spaced outlets using multiple pipe diameters.

ANALYSIS

Christiansen's factor *F* can be written as

$$F = \frac{1}{m+1} + \frac{1}{2N} + \frac{(m-1)^{0.5}}{6N^2} \tag{1}$$

where F = Christiansen's factor F; m = velocity exponent in the formula used for the computation of head loss caused by friction; and N = the number of outlets along the pipeline.

This factor was developed assuming the first outlet is one outlet spacing from the inlet of the pipeline. Further, Christiansen (1942) assumed that the outlets along the pipeline have equal discharge. In a pipeline with multiple outlets, there will be energy losses caused by the coupler and structure of the outlet. However, there also is gradual reduction in velocity head as flow passes the outlet and this will cause an increase in pressure, which will balance losses caused by turbulence at outlet couplings (Scallopi 1988). Hence exact procedures to calculate pressure losses in pipelines with multiple outlets cannot be justified (Pair et al. 1975). These assumptions also underline the present work.

Consider a pipeline with multiple outlets and inflow and outflow at the downstream end as illustrated in Fig. 1. The flow into the pipeline is given by

$$Q_I = Nq + Q_O \tag{2}$$

where Q_I = discharge into the pipeline at the inlet; N = number of outlets along the pipeline; q = discharge of each outlet; and Q_O = outflow discharge at the downstream end of the pipeline beyond the last outlet.

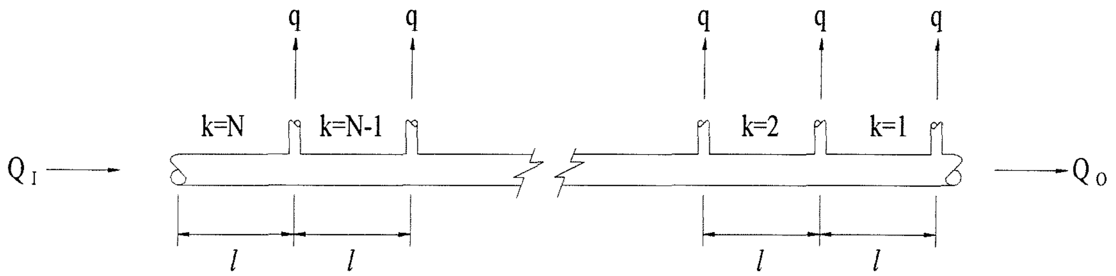

FIG.1. Pipeline with Multiple Equally Spaced Outlets and Outflow

Let the ratio of the outflow discharge to the total discharge through the outlets along the pipeline be denoted by r. Hence

$$r = \frac{Q_O}{Nq} \tag{3}$$

where $r \geq 0$. Alternatively, the outflow discharge can be expressed as

$$Q_O = rNq \tag{4}$$

Substituting in (2)

$$Q_I = Nq(1+r) \tag{5}$$

or

$$q = \frac{Q_I}{N(1+r)} \tag{6}$$

The discharge through the pipeline at any section k along the pipeline is given by

$$Q_k = kq + rNq \tag{7}$$

where Q_k = discharge in the pipeline at the given section k; k = an index representing the successive section of pipeline length between outlets with k=1 the most downstream section increasing up to k=N at the most upstream segement adjacent to the pipeline inlet ($1 \leq k \leq N$).

The head loss caused by friction in any given segment k (Christiansen 1942) can be written as

$$H_{fk} = \frac{CKQ_k^m l}{D^{2m+n}} \qquad (8)$$

where H_{fk} = head loss caused by friction in any given section k of the pipeline; C = units coefficient; K = friction factor based on the friction equation used; Q_k = discharge at the given section k of the pipe length; l = length of each pipe section, D = internal diameter of the pipeline; and m and n = exponents of the average flow velocity in the pipeline and internal pipeline diameter, respectively, which in turn depend on the friction formula used.

Substituting (7) in (8) yields

$$H_{fk} = \frac{CK}{D^{2m+n}} \left[q(k+Nr)\right]^m l \qquad (9)$$

The length of each pipe section can be written in terms of the total length as

$$l = \frac{L}{N} \qquad (10)$$

Substituting (6) and (10) in (9) yields

$$H_{fk} = \frac{CK}{D^{2m+n}} \left(\frac{Q_I}{N(1+r)} \right)^m (k+Nr)^m \frac{L}{N}$$

rearranging

$$H_{fk} = \frac{CKQ_I^m L}{D^{2m+n}} \frac{1}{N^{m+1}(1+r)^m} (k+Nr)^m \qquad (11)$$

The friction along the entire pipeline of length L can be calculated by

$$H_{fL} = \sum_{k=1}^{N} H_{fk} = \frac{CKQ_I^m L}{D^{2m+n}} \frac{1}{N^{m+1}(1+r)^m} \sum_{k=1}^{N} (k+Nr)^m \qquad (12)$$

Note that under conditions of zero outflow for which Christiansen's factor F was developed, r=0 and (12) reduces to the form

$$H_{fL} = \sum_{k=1}^{N} H_{fk} = \frac{CKQ_I^m L}{D^{2m+n}} \frac{1}{N^{m+1}} \sum_{k=1}^{N} k^m \qquad (13)$$

In which the term

$$\frac{1}{N^{m+1}}\sum_{k=1}^{N}k^{m} = \left(\frac{1}{m+1}+\frac{1}{2N}+\frac{m}{12N^{2}}\right) \tag{14}$$

according to Detar (1982), which is approximated closely by Christiansen's factor F given in (1) (Scaloppi 1988).

In (12), the term $\sum_{k=1}^{N}(k+Nr)^{m}$ can be solved using the Euler-Maclaurin Summation formula, the general form of which is given by

$$\begin{aligned}\sum_{x=1}^{N-1}f(x) = \int_{0}^{N}f(x)\,dx - \frac{1}{2}\{f(0)+f(N)\} + \frac{1}{12}\{f'(N)-f'(0)\}\\ - \frac{1}{720}\{f'''(N)-f'''(0)\} + \frac{1}{30{,}240}\{f^{v}(N)-f^{v}(0)\} +.....\\ (-1)^{p-1}\frac{B_{p}}{(2p)!}\{f^{(2p-1)}(N)-f^{(2p-1)}(0)\} +...\end{aligned} \tag{15}$$

where $f(x)$ = general form for function of a variable x; B_p = Bernoulli number; and p = the p^{th} term in the expansion of the series (Spiegel 1968).

In (8), the exponent of the average flow velocity in the pipeline m typically assumes the value of 1.85 for the Hazen-Williams friction formula or 2.00 for the Darcy-Weisbach friction formula. For m=2, all terms that include third and greater derivatives of $f(x)$ in (15) are zero. Applying (15) to the term $\sum_{k=1}^{N}(k+Nr)^{m}$ gives

$$\begin{aligned}\sum_{k=1}^{N}(k+Nr)^{m} = \frac{1}{m+1}\{\,[N(1+r)+1]^{m+1}-[Nr]^{m+1}\}\\ -\frac{1}{2}\{\,[N(1+r)+1]^{m}+[Nr]^{m}\}\\ +\frac{1}{12}\{\,m[N(1+r)+1]^{m-1}-m[Nr]^{m-1}\}\end{aligned} \tag{16}$$

Substituting (16) in (12)

$$\begin{aligned}H_{fL}=\frac{CKQ_{I}^{m}L}{D^{2m+n}}\,\frac{1}{N^{m+1}(1+r)^{m}}\,\Big(\frac{1}{m+1}\{\,[N(1+r)+1]^{m+1}-[Nr]^{m+1}\}\\ -\frac{1}{2}\{\,[N(1+r)+1]^{m}+[Nr]^{m}\}\\ +\frac{1}{12}\{\,m[N(1+r)+1]^{m-1}-m[Nr]^{m-1}\}\,\Big)\end{aligned} \tag{17}$$

or let

$$G = \frac{1}{N^{m+1}(1+r)^m} \sum_{k=1}^{N} (k+Nr)^m = \frac{1}{N^{m+1}(1+r)^m} \Big(\frac{1}{m+1} \{ [N(1+r)+1]^{m+1} - [Nr]^{m+1} \} - \frac{1}{2} \{ [N(1+r)+1]^m + [Nr]^m \} + \frac{1}{12} \{ m[N(1+r)+1]^{m-1} - m[Nr]^{m-1} \} \Big) \tag{18}$$

for m=2.00, hence, (17) becomes

$$H_{fL} = \frac{CKQ_I^{\,m}L}{D^{2m+n}} G \tag{19}$$

where G = factor G as defined by (18). Table 1 shows values of factor G for a pipeline with up to 100 outlets for various ratios of outflow r and m=1.85. Notice for r=0, which represents a pipeline with no outflow, the factor G becomes identical to the approximation of factor F calculated by (14). These values closely approximate factor F for a pipeline with the first outlet at one outlet spacing from the pipeline inlet calculated using (1), (James 1988).

The expansion of the summation form of factor G in (18) is only strictly valid m=2.00 and is an approximation for other values of m. Hence in Table 1 where m=1.85, values for factor G >1.00 can be observed. A more accurate estimate of G can be obtained for m=1.85 using the summation form of factor G rather than its Euler-Mclaurin expansion, although the latter form lends itself to calculation more readily. Table 2 shows values of factor G for a pipeline with up to 100 outlets for various ratios of outflow and m=2.00. This table can be prepared using either forms of factor G given in (18).

APPLICATION

A numerical example is used to illustrate the application of factor G for calculating head loss in a pipeline with multiple outlets and outflow at the downstream end.

Example

Calculate the head caused by friction in a sprinkler lateral which is 288 m in

TABLE 1: Values of Factor G for $m = 1.85$ Using Eq.(18)

Number of outlets (N)	r							
	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1.005	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.639	0.670	0.721	0.750	0.774	0.794	0.810	0.825
3	0.535	0.573	0.636	0.673	0.703	0.729	0.750	0.769
4	0.486	0.527	0.595	0.635	0.669	0.697	0.721	0.741
5	0.457	0.500	0.570	0.613	0.648	0.678	0.703	0.725
6	0.438	0.482	0.555	0.598	0.635	0.666	0.692	0.714
7	0.425	0.470	0.543	0.588	0.625	0.657	0.683	0.706
8	0.416	0.461	0.535	0.580	0.618	0.650	0.677	0.701
9	0.408	0.454	0.528	0.574	0.613	0.645	0.673	0.696
10	0.402	0.448	0.523	0.570	0.608	0.641	0.669	0.693
11	0.398	0.443	0.519	0.566	0.605	0.638	0.666	0.690
12	0.394	0.439	0.515	0.562	0.602	0.635	0.663	0.687
13	0.390	0.436	0.513	0.560	0.599	0.632	0.661	0.685
14	0.387	0.433	0.510	0.557	0.597	0.630	0.659	0.684
15	0.385	0.431	0.508	0.555	0.595	0.629	0.657	0.682
16	0.383	0.429	0.506	0.553	0.593	0.627	0.656	0.681
17	0.381	0.427	0.504	0.552	0.592	0.626	0.655	0.680
18	0.379	0.426	0.503	0.551	0.591	0.625	0.654	0.678
19	0.378	0.424	0.501	0.549	0.590	0.624	0.653	0.678
20	0.376	0.423	0.500	0.548	0.588	0.623	0.652	0.677
22	0.374	0.420	0.498	0.546	0.587	0.621	0.650	0.675
24	0.372	0.419	0.496	0.545	0.585	0.620	0.649	0.674
26	0.370	0.417	0.495	0.543	0.584	0.618	0.648	0.673
28	0.369	0.416	0.494	0.542	0.583	0.617	0.647	0.672
30	0.368	0.414	0.493	0.541	0.582	0.617	0.646	0.671
35	0.365	0.412	0.490	0.539	0.580	0.615	0.644	0.670
40	0.363	0.410	0.489	0.538	0.579	0.613	0.643	0.669
50	0.361	0.408	0.486	0.536	0.577	0.612	0.641	0.667
100	0.356	0.403	0.482	0.531	0.573	0.608	0.638	0.664

length. Sprinklers are installed at 12 m intervals. The first 144 m (starting at the inlet) of the lateral has an internal pipe diameter of 100 mm and the next 144 m of the lateral has an internal diameter of 75 mm. There are a total of 24 sprinklers on the lateral with each discharging 0.5 L/s. The first sprinkler is 12m from the inlet to the lateral.

TABLE 2: Values of Factor G for $m = 2.00$ Using Eq.(18)

Number of outlets (N)	r							
	0.00	0.20	0.40	0.60	0.80	1.00	1.20	1.40
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.625	0.670	0.707	0.736	0.761	0.781	0.799	0.813
3	0.519	0.573	0.618	0.656	0.687	0.713	0.735	0.754
4	0.469	0.527	0.576	0.617	0.651	0.680	0.704	0.725
5	0.440	0.500	0.551	0.594	0.630	0.660	0.686	0.708
6	0.421	0.482	0.535	0.579	0.616	0.647	0.674	0.697
7	0.408	0.470	0.523	0.568	0.606	0.638	0.665	0.689
8	0.398	0.461	0.515	0.560	0.598	0.631	0.659	0.683
9	0.391	0.454	0.508	0.554	0.593	0.626	0.654	0.678
10	0.385	0.448	0.503	0.549	0.588	0.621	0.650	0.674
11	0.380	0.443	0.498	0.545	0.584	0.618	0.647	0.671
12	0.376	0.439	0.495	0.541	0.581	0.615	0.644	0.669
13	0.373	0.436	0.492	0.539	0.578	0.612	0.642	0.667
14	0.370	0.433	0.489	0.536	0.576	0.610	0.640	0.665
15	0.367	0.431	0.487	0.534	0.574	0.609	0.638	0.663
16	0.365	0.429	0.485	0.532	0.573	0.607	0.636	0.662
17	0.363	0.427	0.483	0.531	0.571	0.606	0.635	0.661
18	0.362	0.426	0.482	0.529	0.570	0.604	0.634	0.660
19	0.360	0.424	0.480	0.528	0.569	0.603	0.633	0.659
20	0.359	0.423	0.479	0.527	0.568	0.602	0.632	0.658
22	0.356	0.420	0.477	0.525	0.566	0.600	0.630	0.656
24	0.354	0.419	0.475	0.523	0.564	0.599	0.629	0.655
26	0.353	0.417	0.474	0.522	0.563	0.598	0.628	0.654
28	0.351	0.416	0.472	0.521	0.562	0.597	0.627	0.653
30	0.350	0.414	0.471	0.520	0.561	0.596	0.626	0.652
35	0.348	0.412	0.469	0.518	0.559	0.594	0.624	0.651
40	0.346	0.410	0.467	0.516	0.557	0.593	0.623	0.649
50	0.343	0.408	0.465	0.514	0.555	0.591	0.621	0.648
100	0.338	0.403	0.460	0.510	0.551	0.587	0.618	0.645

Solution

For calculating head loss caused by friction for a pipeline without outlets, the Hazen-Williams formula will be used assuming the friction coefficient for the Hazen-Williams formula=130 for aluminium pipes with couplers approximately every 10 m (Keller and Bliesner 1990). For microirrigation submain design an appropriate friction coefficient for the Hazen-Williams formula needs to be selected. Alternatively, the Darcy-Weisbach equation may be used for a more rational characterization of the friction factor.

The lateral has the following two segments:

- Segment 1: The downstream segment with an internal diameter $D = 75$ mm, length $L = 144$ m, and no outflow
- Segment 2: The upstream segment with an internal diameter $D = 100$ mm, length $L = 144$ m, and outflow into segment 1.

For segment 1, number of sprinklers along lateral $N = 12$; therefore discharge $Q = 12 \times 0.5 = 6$ L/s

Using the Hazen-Williams formula (Keller and Bliesner 1990)

$$H_f = 1.212\times10^{12}\left(\frac{Q}{C_{HW}}\right)^{1.852} D^{-4.87}\frac{L}{100} \tag{20}$$

where $H_f =$ head loss caused by friction through a pipeline (m) ; $Q =$ discharge through the pipeline (L/s); $C_{HW} =$ friction coefficient for Hazen-Williams formula; D = internal pipe diameter (mm); and, $L =$ length of pipeline (m)

Using (20) for segment 1

$$H_f = 1.212\times10^{12}\left(\frac{6}{130}\right)^{1.852} 75^{-4.87}\frac{144}{100} = 4.33 \text{ m}$$

The velocity (or discharge) exponent in (20) is 1.852 $\approx$1.85. Therefore m=1.85 in (18). Alternatively, Table 1 can be used. Because there is no outflow past the last sprinkler in segment 1, r= 0. From Table 1, $G_{r=0,\,N=12} = 0.394$, and

$$H_{f1} = 4.33\times0.394 = 1.71m$$

where H_{f1} = head loss caused by friction in segment 1 of the lateral.

For segment 2, the outflow at the downstream end of this segment is the discharge into the first segment or

$$Q_O = 6 \text{ L/s}$$

where Q_O = outflow from segment 2. The number of sprinklers along segment 2, N = 12, using (2)

$$Q_I = Nq + Q_O$$
$$= 12\times0.5 + 6 = 12 \text{ L/s}$$

and from (3)

$$r = \frac{Nq}{Q_O} = \frac{12\times0.5}{6} = 1.0$$

Using Table 1 or from (18) $G_{r=1.0, N=12} = 0.635$

For segment 2, from (20)

$$H_f = 1.212\times10^{12}\left(\frac{12}{130}\right)^{1.852} 100^{-4.87}\frac{144}{100} = 3.85 \text{ m}$$

and

$$H_{f2} = 3.85\times0.635 = 2.44\text{m}$$

where H_{f2} = head loss caused by friction in segment 2 of the lateral. The total head loss in the sprinkler lateral is therefore given by

$$H_{f(1+2)} = H_{f1} + H_{f2} = 2.44 + 1.71 = 4.15 \text{ m}$$

where $H_{f(1+2)}$ = total head loss in the sprinkler lateral.

The same problem can be solved in a stepwise manner, starting computation at the downstream end and proceeding toward the inlet of the lateral. These computations are shown in Table 3. The head loss caused by friction using a stepwise analysis is 4.146 m and is comparable with 4.15 m using factor G. The error introduced because (16) is the Euler-Maclaurin summation formula for p=2.00, whereas in using the Hazen-Williams formula the exponent of the velocity term is m=1.85 rather than 2.00 (where m corresponds to the term p in the Euler-Maclaurin summation formula) is insignificant.

CONCLUSION

This research presents factor G as a sequel to the well-known and widely used

TABLE 3: Stepwise Solution to Example

Section no. (k) (1)	Discharge (L/s) (2)	Pipe int. diameter (mm) (3)	H_{fk} Eq. (20) (m) (4)	ΣH_{fk} (m) (5)
1	0.5	75	0.004	0.004
2	1.0	75	0.013	0.017
3	1.5	75	0.028	0.044
4	2.0	75	0.047	0.092
5	2.5	75	0.071	0.163
6	3.0	75	0.100	0.263
7	3.5	75	0.133	0.396
8	4.0	75	0.170	0.566
9	4.5	75	0.212	0.778
10	5.0	75	0.257	1.035
11	5.5	75	0.307	1.342
12	6.0	75	0.361	1.703
13	6.5	100	0.103	1.806
14	7.0	100	0.118	1.924
15	7.5	100	0.134	2.059
16	8.0	100	0.151	2.210
17	8.5	100	0.169	2.380
18	9.0	100	0.188	2.568
19	9.5	100	0.208	2.776
20	10.0	100	0.229	3.005
21	10.5	100	0.251	3.256
22	11.0	100	0.273	3.529
23	11.5	100	0.297	3.825
24	12.0	100	0.321	4.146

Christiansen's factor *F* for direct computation of head loss caused by friction in a pipeline with multiple equally spaced outlets. Factor *G* is a more generalized form of factor *F* in that it allows for outflow at the downstream end of the pipeline beyond the last outlet. If, for a particular case, the outflow at the downstream end is set to zero, then factor *G* reduces to factor *F*. Factor *G* can be used for calculating head loss caused by friction in pipelines with outlets and multiple diameters.

Application of factor G has been demonstrated with a worked example. When a friction formula such as the Darcy-Weisbach equation is used, in which the exponent of the velocity (or discharge) term is 2.00, factor G can be used with high precision. A slight error is introduced when a friction formula such as the Hazen Williams equation is used where the exponent of the velocity term is less than 2.00, because of an inherent assumption in the expansion of the summation function in the Euler-Maclaurin summation formula.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

B_p = Bernoulli number;
C = units coefficient;
C_{HW} = friction coefficient for the Hazen Williams formula;
D = internal pipeline diameter;
F = Christiansen's correction factor for pipelines with multiple equally spaced outlets with the first outlet at one outlet spacing from the pipeline inlet without downstream outflow;
$f(x)$ = general mathematical notation for function of any variable x;
G = correction factor for pipelines with multiple equally spaced outlets with the first outlet at one outlet spacing from the pipeline inlet with/without downstream outflow;

H_{fL} = head loss caused by friction in pipeline of length L;
H_{fk} = head loss caused by friction in the kth section of a pipeline with multiple equally-spaced outlets;
H_f = head loss caused by friction in a pipeline;
H_{f1} = head loss caused by friction in segment 1 of a two-segment pipeline;
H_{f2} = head loss caused by friction in segment 2 of a two-segment pipeline;
$H_{f(1+2)}$ = head loss caused by friction in segments 1 and 2 of a two-segment pipeline;
K = friction factor based on friction formula used;
k = integer representing pipe section under consideration, from $k=1$ for the downstream most section to $k=N$ for the upstream most section adjacent to the pipe inlet;
L = total length of the pipeline;
l = length of each section of the pipeline between outlets;
m = exponent of the velocity or discharge term in the friction formula used;
N = total number of outlets along the pipeline;
n = part of the exponent of the diameter term in the friction formula used;
p = integer 1,2,3,4......;
Q = discharge in the pipeline ;
Q_I = inflow into the pipeline;
Q_k = discharge in the kth section of the pipeline;
Q_O = outflow from the pipeline at the downstream end;
q = discharge of the outlet;
r = ratio of the outflow discharge to total outlet discharge; and
x = general mathematical notation for any variable.

2

ADJUSTED FACTOR G_A FOR PIPELINES WITH MULTIPLE OUTLETS AND OUTFLOW

ADJUSTED FACTOR G_a FOR PIPELINES WITH MULTIPLE OUTLETS AND OUTFLOW

By Arif A. Anwar[1]

ABSTRACT: The adjusted factor G_a is a generic friction loss correction factor for pipelines with multiple outlets. The adjusted factor G_a can be applied to pipelines with or without outflow at the downstream end. Furthermore, this factor can be applied to a pipeline where the first outlet is at a full outlet spacing or a fractional outlet spacing from the pipeline inlet. When the outflow at the downstream end is reduced to zero, the adjusted factor G_a reduces to the adjusted factor F_a. If the first outlet is positioned one outlet spacing from the pipeline inlet, the factor G_a reduces to G. Finally, if both the outflow is zero and the first outlet is one outlet spacing from the pipeline inlet, the adjusted factor G_a reduces to a close approximation of the well known factor F. The adjusted factor G_a is a function of the number of outlets along the pipeline, the location of the first outlet from the pipeline inlet, the outflow ratio, and the velocity exponent of the head loss formula.

INTRODUCTION

Pipelines with multiple outlets are used for irrigation under various types of surface, sprinkle, and trickle irrigation systems. In solid set, periodic move or linear move sprinkle systems, the outlets are uniformly spaced along the pipeline and are assumed to have uniform discharge. These characteristics also apply to most trickle

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irrigation systems and gated pipes for surface irrigation (Scaloppi and Allen 1993). In a pipeline with multiple outlets, the discharge through the pipeline decreases along the length of the pipe. As a result, the head loss caused by friction in a pipeline with multiple outlets will be less than that in a similar pipeline without outlets. To compute the head loss caused by friction requires calculating the head loss caused by friction in a stepwise manner. Christiansen (1942) developed the widely used factor *F* to avoid the cumbersome stepwise analysis required to calculate head loss in pipelines with outlets. The following assumptions are made in developing the factor *F*:

- There is no outflow past the downstream outlet.
- All outlets are equally spaced and have equal discharge.
- The distance between the pipeline inlet and the first outlet is equal to one full outlet spacing.
- Hydraulic characteristics (e.g., pipe friction factor and pipe diameter) remain constant along the length of the pipeline.

Velocity head is neglected in developing the factor *F*. This assumption has been criticized by Smith (1990), particularly for low pressure pipelines. However, for the operating pressures of typical sprinkler systems, this assumption has been shown to be acceptable (Scaloppi and Allen 1993). Furthermore, the increase in pressure head past each outlet (caused by reduction in velocity) is assumed to equal the head loss caused by turbulence associated with each outlet (Pair et al. 1975).

Factor *F* is a dimensionless factor and is a function of the friction formula used and the number of outlets along the pipeline. The head loss in a pipeline without outlets can be calculated using any of the well-known friction formulas such as Darcy-Weisbach, Hazen-Williams, and others. This is then multiplied by the factor *F* to calculate the head loss caused by friction in a pipeline with multiple outlets.

Many field layouts will not permit the first outlet on a pipeline to be located a full spacing from the pipeline inlet. Jensen and Fratini (1957) addressed this issue by developing an adjusted factor F. This factor permits calculating the head loss caused by friction in pipelines with multiple outlets, with the first outlet at one-half an outlet spacing from the pipeline inlet. Other assumptions are the same as those by Christiansen (1942). Chu (1978) modified the adjusted factor F of Jensen and Fratini (1957) and suggested this modified factor F could be considered constant for five or more outlets. Chu (1978) demonstrated a negligible error by undertaking this assumption. More recently, Scaloppi (1988) derived an expression for the adjusted factor F_a. This expression allows the adjusted factor to be calculated for a pipeline with multiple outlets and the first outlet at any fraction of a spacing from the pipeline inlet. If the first outlet is one-half an outlet spacing from the pipeline inlet, the adjusted factor by Scaloppi (1988) is identical to that by Jensen and Fratini (1957).

Factor F and adjusted factor F_a provide a very convenient tool for calculating head loss caused by friction in a pipeline with multiple outlets. However, these factors can only be used directly for pipelines with a single diameter. For pipelines with outlets and reaches of different diameters, the factor F can only be applied directly to the most downstream reach of the pipeline. To calculate the total head loss in such multiple diameter (tapered) pipelines requires using the factor F in an indirect method. Alternatively, graphical methods for multiple pipe sizes have been developed (Keller and Bliesner 1990).

Anwar (1999) developed a factor G which permits calculating the head loss caused by friction in pipelines with multiple outlets and outflow at the downstream end. Factor G can be applied to each reach within a tapered pipeline to calculate the head loss caused by friction more directly. However, factor G is limited in that it assumes the first outlet is one full spacing from the pipeline inlet. In the current work, an adjusted factor G_a is developed that will allow for the first outlet to be any fraction of an outlet spacing from the pipeline inlet. The following assumptions are made in the theoretical development of adjusted factor G_a:

- The outlets are equally spaced and of uniform discharge.
- The pipe friction factor remains constant along the pipeline length.

- The velocity head can be neglected.
- The increase in pressure past each outlet caused by decrease in the flow is equal to the head loss caused by turbulence associated with each outlet.
- Head loss at the change in pipe diameter is ignored.

ANALYSIS

Fig. 1 shows a pipeline with multiple outlets and outflow at its downstream end. All outlets are equally spaced except for the first, which is at some fraction of the outlet spacing from the pipe inlet. Scaloppi (1988) expressed the total length of the pipeline by

$$L = (N-1)l + xl \tag{1}$$

where L = total length of the pipeline; N = number of outlets along the pipeline; l = outlet spacing; and x = ratio of the distance between the inlet and first outlet to the outlet spacing ($0 < x \leq 1$).

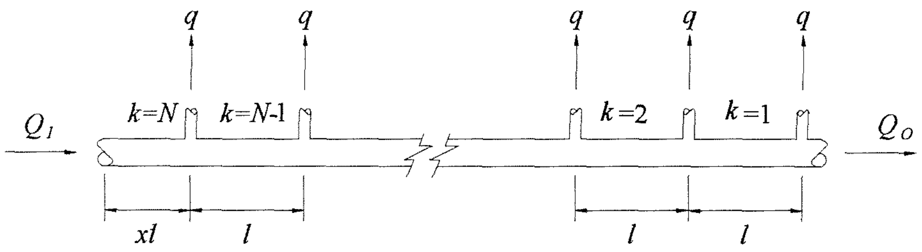

FIG. 1. Pipeline with Multiple Outlets

Rearranging (1) gives

$$l = \frac{L}{N-1+x} \tag{2}$$

The flow into the pipeline can be represented by

$$Q_I = Nq + Q_O \tag{3}$$

where Q_I = discharge at the pipeline inlet; q = discharge of each outlet; and Q_o = outflow discharge at the downstream end of the pipeline (beyond the last outlet). Let

$$r = \frac{Q_O}{Nq} \tag{4}$$

where r = ratio of the outflow discharge to the total discharge through the outlets ($r \geq 0$). Substituting (4) into (5) and rearranging

$$q = \frac{Q_I}{N(1+r)} \tag{5}$$

The discharge through the pipeline at any segment k along the pipeline is given by

$$Q_k = kq + rNq \tag{6}$$

where Q_k = discharge in the k th segment of the pipeline; and, k = an integer representing the successive segments of the pipeline. At the upstream end of the pipeline, adjacent to the inlet, $k = N$, and decreasing to $k = 1$ at the most downstream segment.

The head loss caused by friction at the given segment k can be written as follows (Christiansen 1942):

$$H_{fk} = \frac{CKQ_k^m l}{D^{2m-n}} \tag{7}$$

where H_{fk} = head loss caused by friction in any given segment k of the pipeline ; C = units coefficient; K = friction factor based upon the friction equation; Q_k = discharge in the k th segment of the pipeline; l = outlet spacing, D = internal diameter of the pipeline; and m and n = exponents of the average flow velocity in the pipeline and internal pipeline diameter, respectively, which in turn depend on the friction formula. Substituting (2), (5) and (6) into (7)

$$H_{fk} = \frac{CK}{D^{2m+n}} \left(\frac{Q_I}{N(1+r)} \right)^m \frac{L}{N-1+x} (k+Nr)^m \tag{8}$$

rearranging

$$H_{fk} = \frac{CKQ_I^m L}{D^{2m+n}} \cdot \frac{1}{N^m (1+r)^m} \cdot \frac{1}{(N-1+x)} (k+Nr)^m \tag{9}$$

The head loss caused by friction along the entire length of the pipeline is therefore

$$H_{fL} = \sum_{k=1}^{N} H_{fk} - \frac{CKQ_I^m}{D^{2m+n}} (1-x) l \tag{10}$$

where H_{fL} = head loss caused by pipe friction along the entire pipeline. In (10), the first term on the right-hand side is simply the summation of head loss in each segment of the pipeline. The second term on the right-hand side is subtracted from the first term. This corrects the over estimation of head loss caused by friction by the summation for the fractional spacing of the first outlet. Substituting for H_{fk} from (9) into (10)

$$H_{fL} = \frac{CKQ_I^m}{D^{2m+n}} \cdot \left(\frac{l}{(N-1+x)} \sum_{k=1}^{N} (k+Nr)^m - (1-x)l \right) \tag{11}$$

rearranging and substituting for L from (2)

$$H_{fL} = \frac{CKQ_I^m}{D^{2m+n}} \cdot \frac{L}{(N-1+x)} \cdot \left(\frac{N}{N^{m+1}(1+r)^m} \sum_{k=1}^{N} (k+Nr)^m - (1-x) \right) \tag{12}$$

but

$$G = \left(\frac{1}{N^{m+1}(1+r)^m} \sum_{k=1}^{N} (k+Nr)^m \right) \tag{13}$$

where G = friction correction factor for pipelines with multiple equally spaced outlets and outflow (Anwar 1999). Therefore, (12) becomes

$$H_{fL} = = \frac{CKQ_I^m L}{D^{2m+n}} \cdot \frac{1}{(N-1+x)} \cdot (NG-(1-x)) \tag{14}$$

or more appropriately

$$H_{fL} = = \frac{CKQ_I^m L}{D^{2m+n}} \cdot \frac{NG+x-1}{N+x-1} \tag{15}$$

One can recognize the similarity between (15) and that by Scaloppi (1988). Defining

$$G_a = = \frac{NG+x-1}{N+x-1} \tag{16}$$

then

$$H_{fL} = \frac{CKQ_I^m L}{D^{2m+n}} \cdot G_a \tag{17}$$

where G_a = adjusted friction correction factor for pipelines with multiple equally spaced outlets and outflow; and G is given by (13).

In (16), if the first outlet is located a full outlet spacing from the pipeline inlet (i.e., x=1), then

$$G_{a\ (for\ x=1)} = G \tag{18}$$

Factor G from (13) can be solved using the Euler-Maclaurin summation formula (Spiegel 1968), for $m = 2.00$ to give

$$G = \frac{1}{N^{m+1}(1+r)^m} \Big(\frac{1}{m+1}\{ [N(1+r)+1]^{m+1} - [Nr]^{m+1}\} - \frac{1}{2}\{ [N(1+r)+1]^{m} + [Nr]^{m}\} + \frac{1}{12}\{ m[N(1+r)+1]^{m-1} - m[Nr]^{m-1}\} \Big) \tag{19}$$

DISCUSSION

Fig. 2 shows G_a for m=2.00 and outflow ratios of: 0.20, 0.40, 0.60 and 0.80, and for first segment fractional length x ranging from 0.2 to 1.0. From (16), for x=1.0, factor G_a reduces to G. Furthermore, for an outflow ratio r=0, G reduces to a very close approximation of F (Anwar 1999). In (16), substituting the factor G with F, the right-hand side of (16) reduces to the form given by Scaloppi (1988) for F_a Therefore, for an outflow ratio r=0, G_a reduces to F_a.

G_a is a more generic friction correction factor. It can be applied to pipelines with multiple outlets and outflow (including the condition of no outflow). Adjusted factor G_a can also be applied for pipelines where the first outlet is a fraction of a full outlet spacing from the pipeline inlet (including the condition where the first outlet is a full spacing from the pipeline inlet).

APPLICATION

G_a can be used in the design of tapering laterals and manifolds. The application of G_a is best demonstrated by a numerical example to calculate the head loss caused by friction in a pipeline. In Table 1, G_a has been calculated from (16) and (19) for an outflow ratio of r=1.00 and a range of first segment length fraction x from 0.125 to 1.000. Table 1 will be used in the numerical example that follows.

Table 1: Adjusted Factor G_a for m =2.00 and Outflow Ratio r =1.00

Number of outlets N (1)	First Segment Length Fraction x							
	0.125 (2)	0.250 (3)	0.375 (4)	0.500 (5)	0.625 (6)	0.750 (7)	0.875 (8)	1.000 (9)
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
2	0.611	0.650	0.682	0.708	0.731	0.750	0.767	0.781
3	0.595	0.617	0.637	0.656	0.672	0.687	0.700	0.713
4	0.590	0.606	0.620	0.634	0.647	0.658	0.669	0.680
5	0.588	0.600	0.611	0.622	0.632	0.642	0.651	0.660
6	0.587	0.597	0.606	0.615	0.623	0.632	0.639	0.647
7	0.586	0.594	0.602	0.610	0.617	0.624	0.631	0.638
8	0.586	0.593	0.600	0.606	0.613	0.619	0.625	0.631
9	0.585	0.591	0.598	0.603	0.609	0.615	0.620	0.626
10	0.585	0.591	0.596	0.601	0.606	0.612	0.616	0.621
11	0.585	0.590	0.595	0.600	0.604	0.609	0.613	0.618
12	0.585	0.589	0.594	0.598	0.602	0.607	0.611	0.615
13	0.584	0.589	0.593	0.597	0.601	0.605	0.609	0.612
14	0.584	0.588	0.592	0.596	0.600	0.603	0.607	0.610
15	0.584	0.588	0.592	0.595	0.598	0.602	0.605	0.609
16	0.584	0.588	0.591	0.594	0.598	0.601	0.604	0.607
17	0.584	0.587	0.590	0.594	0.597	0.600	0.603	0.606
18	0.584	0.587	0.590	0.593	0.596	0.599	0.602	0.604
19	0.584	0.587	0.590	0.592	0.595	0.598	0.601	0.603
20	0.584	0.587	0.589	0.592	0.595	0.597	0.600	0.602
22	0.584	0.586	0.589	0.591	0.594	0.596	0.598	0.600
24	0.584	0.586	0.588	0.591	0.593	0.595	0.597	0.599
26	0.584	0.586	0.588	0.590	0.592	0.594	0.596	0.598
28	0.584	0.586	0.588	0.589	0.591	0.593	0.595	0.597
30	0.584	0.586	0.587	0.589	0.591	0.592	0.594	0.596
35	0.584	0.585	0.587	0.588	0.590	0.591	0.593	0.594
40	0.584	0.585	0.586	0.588	0.589	0.590	0.591	0.593
50	0.584	0.585	0.586	0.587	0.588	0.589	0.590	0.591
100	0.583	0.584	0.584	0.585	0.586	0.586	0.587	0.587

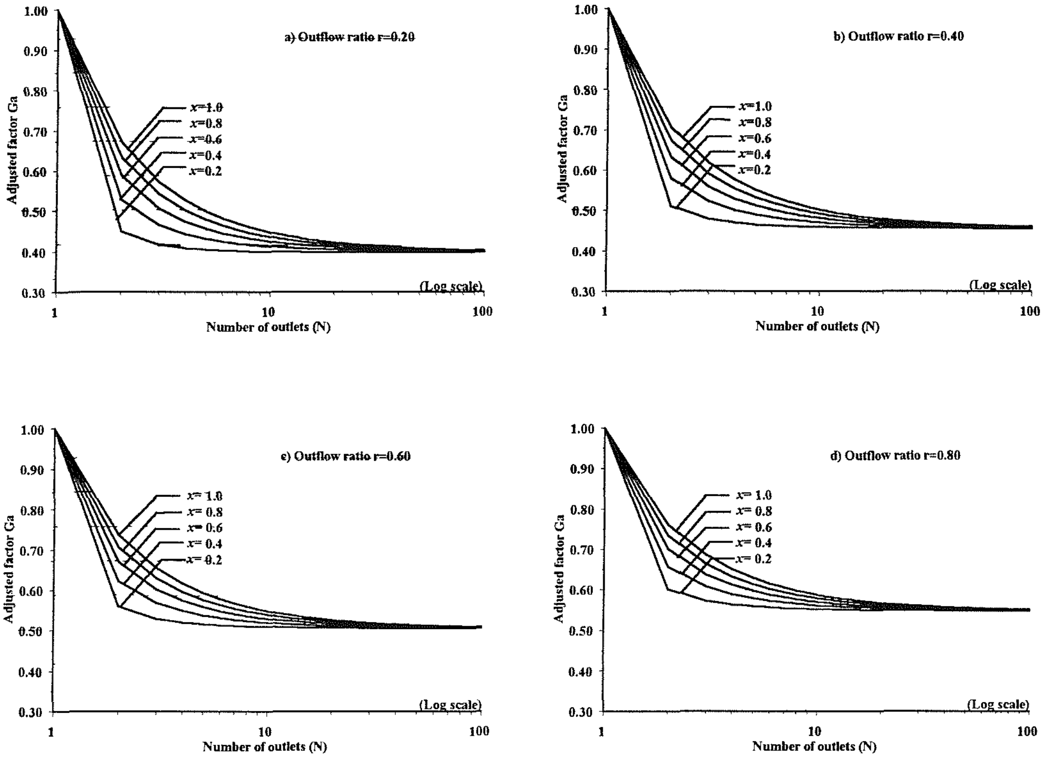

FIG. 2. Adjusted Factor G_a for $m = 2.00$ and Various Outflow Ratios

Example

An aluminium sprinkler lateral has a total length of 213 m. The upstream reach of the pipeline (starting at the pipeline inlet) is 105 m long with the first sprinkler at 9 m from the pipeline inlet and the remaining 8 sprinklers at 12 m spacing. The internal diameter of this reach of the lateral is 100 mm. The downstream reach of the pipeline is 108 m long with 9 sprinklers at 12 m spacing. This reach has an internal diameter of 75 mm. Calculate the head loss caused by friction in the pipeline, if the average sprinkler discharge is 0.5 L/s. (Assume for water at 15°C the kinematic viscosity $\nu = 1.14\times10^{-6}$ m^2/s, and a pipe roughness ϵ = 0.127mm).

Solution

The Darcy-Weisbach equation will be used to calculate head loss caused by friction given by

$$H_{fL} = CK\frac{L}{D}\frac{V^2}{2g} = 0.0826C\frac{Q^2L}{D^5} \tag{20}$$

where H_{fL} = head loss caused by friction in the pipeline; V = velocity of flow through the pipeline; g = acceleration caused by gravity; and, Q= discharge through the pipeline. Comparing (20) with (7), K=0.0826, m=2 and n=1. The Churchill equation (Churchill 1977) will be used to calculate the friction factor C given by

$$C = 8\left[\left(\frac{8}{R}\right)^{12} + \frac{1}{(\alpha+\beta)^{1.5}}\right]^{\frac{1}{12}} \tag{21}$$

where R = Reynolds number, and α and β = coefficients, given by

$$\alpha = \left\{2.457\ \ln\left[\frac{1}{\left(\frac{7}{R}\right)^{0.9} + 0.27\left(\frac{\epsilon}{D}\right)}\right]\right\}^{16} \tag{22}$$

and

$$\beta = \left(37,\frac{530}{R}\right)^{16} \tag{23}$$

Table 2 summarizes the properties of the pipeline. Based on the internal diameter of the pipe, the pipeline can be divided into two reaches: reach 1, the larger diameter pipeline starting at the pipeline inlet; and reach 2, the smaller diameter pipeline at the downstream end of the lateral.

TABLE 2: Reach Details of Sprinkler Lateral

Reach (1)	Internal diameter (mm) (2)	Length of reach (m) (3)	No. of sprinklers on reach (4)	Length of first segment (m) (5)
1	100	105	9	9
2	75	108	9	12

Pipeline reach 2

For reach 2, because there is no outflow, r=0. The first sprinkler is a full spacing from the inlet to this reach (i.e., x=1). Using (13), alternatively from appropriate tables [e.g., Anwar (1999)], $G = 0.391$. From (16), $G_a = 0.391$.

The discharge into reach 2 is 4.5 L/s. The average velocity at the first segment of reach is

$$V_2 = \frac{Q}{A} = 1.02\,\mathrm{m/s}$$

where V_2 = average velocity of flow in the first segment of reach 2; and A = cross-sectional area of the pipeline.

$$R = \frac{VD}{\nu} = 67{,}012$$

Using (21) - (23) C=0.0253. From (17)

$$H_{f2} = \frac{CKQ^{m}L}{D^{2m+n}}\cdot G_a = \frac{0.0826\times 0.0253\times(0.0045)^2\times 108}{(0.075)^5}\times 0.391 = 0.753\mathrm{m}$$

where H_{f2} = head loss caused by friction in reach 2.

Pipeline reach 1

For reach 1, the outflow from this segment is equal to the inflow to reach 2 (i.e., Q_O = 4.5 L/s, and for reach 1, Nq = 4.5 L/s).

From (4), r= 1.00, and because the first outlet is 9 m from the inlet to this segment, therefore x= 9/12 = 0.75. Using (13), or appropriate tables [e.g., Anwar (1999)], G= 0.626. From (16), G_a = 0.6153. Alternatively more directly from Table 1, G_a=0.615.

The discharge into reach 1 is 9.0 L/s. The average velocity at the first segment of reach 1 is

$$V_1 = \frac{Q}{A} = 1.15\text{m/s}$$

where V_1 = average velocity of flow in the first segment of reach 1; and A = cross-sectional area of the pipeline

$$R = \frac{VD}{\nu} = 100{,}519$$

Using (21) - (23) C=0.0232. From (17)

$$H_{f1} = \frac{CKQ^{m}L}{D^{2m+n}}{\cdot}G_a = \frac{0.0826{\times}0.0232{\times}(0.009)^2{\times}105}{(0.10)^5}{\times}0.615 = 1.004\text{m}$$

where H_{f1} = head loss caused by friction in reach 1. Total head loss in the lateral is

$$H_{f(1+2)} = H_{f1} + H_{f2} = 1.004 + 0.753 = 1.757\text{m}$$

The same problem can be solved in a stepwise manner - starting computation at the downstream end and proceeding towards the inlet of the lateral. These computations are shown in Table 3. The stepwise calculation also ignores velocity head and assumes a constant pipe friction factor for each reach. As would be expected the total head loss in the lateral is identical for both methods of calculations.

CONCLUSION

The adjusted factor G_a is presented as a sequel to adjusted factor F_a. G_a is a generic form of the friction correction factor for pipelines with equally spaced outlets and outflow and also if the first outlet at any fraction of whole outlet spacing from the pipeline inlet. G_a can be used to calculate the head caused by friction in such pipelines. When the outflow is reduced to zero, G_a reduces to F_a. If the first outlet is positioned at a full spacing from the pipeline inlet, G_a reduces to G. Finally, if both outflow is reduced to zero, and the first outlet is positioned one full spacing from the pipeline inlet, G_a reduces to the well-known Christiansen's F (1942).

G_a may find application in the design of tapered sprinkle and trickle irrigation pipelines. To demonstrate the application of factor G_a, a simple numerical example is presented. The results are compared with the same example solved in a step-wise manner.

TABLE 3: Step-wise Solution to Numerical Example

Segment (1)	Length (m) (2)	Discharge (L/s) (3)	Pipe int. diameter (mm) (4)	H_{fk} (m) (5)	ΣH_{fk} (m) (6)
1	12	0.50	75	0.0026	0.003
2	12	1.00	75	0.0106	0.013
3	12	1.50	75	0.0238	0.037
4	12	2.00	75	0.0423	0.079
5	12	2.50	75	0.0660	0.145
6	12	3.00	75	0.0951	0.240
7	12	3.50	75	0.1295	0.370
8	12	4.00	75	0.1691	0.539
9	12	4.50	75	0.2140	0.753
10	12	5.00	100	0.0576	0.811
11	12	5.50	100	0.6970	0.880
12	12	6.00	100	0.0829	0.963
13	12	6.50	100	0.0973	1.060
14	12	7.00	100	0.1128	1.173
15	12	7.50	100	0.1295	1.303
16	12	8.00	100	0.1474	1.450
17	12	8.50	100	0.1664	1.617
18	9	9.00	100	0.1399	1.757

APPENDIX I: REFERENCES

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APPENDIX II: NOTATION

The following symbols are used in this paper:

C = units coefficient;
D = internal pipeline diameter;
F = Christiansen’s correction factor for pipelines with multiple equally spaced outlets;
F_a = adjusted friction correction factor;
G = friction correction factor for pipelines with multiple equally spaced outlets and outflow;
G_a = adjusted friction correction factor for pipelines with multiple equally spaced outlets and outflow;
H_{fk} = head loss caused by friction between the downstream end of the pipe up to (and including) segment k;
H_{fL} = head loss caused by friction in a pipeline;
H_{f1} = head loss caused by friction in reach 1;
H_{f2} = head loss caused by friction in reach 2;
$H_{f(1+2)}$ = head loss caused by friction in reaches 1 and 2;
K = friction factor based on friction formula used;
k = integer representing successive segments of pipeline;
L = total length of the pipeline;
l = outlet spacing;
m = exponent of velocity term in the friction formula used;
N = number of outlets along the pipeline;
n = exponent of the diameter term in friction formula used;
Q = discharge through pipeline;
Q_I = discharge at pipeline inlet;
Q_k = discharge in kth segment of pipeline;

Q_O = outflow from pipeline at downstream end;
q = discharge of each outlet;
R = Reynolds number;
r = ratio of outflow discharge to total outlet discharge;
V = average velocity of flow in pipeline;
V_1 = average velocity of flow in first segment of reach 1;
V_2 = average velocity of flow in first segment of reach 2;
x = ratio of distance between the inlet and first outlet to outlet spacing;
α = empirical parameter used in Churchill equation;
β = empirical parameter used in Churchill equation;
ϵ = pipe roughness;
and
υ = kinematic viscosity.

3

INLET PRESSURE FOR TAPERED HORIZONTAL LATERALS

INLET PRESSURE FOR HORIZONTAL TAPERED LATERALS

By Arif A. Anwar[1]

ABSTRACT: Analytical equations are presented for two average pressure correction factors developed for linear displacement laterals with or without outflow at the downstream end. The average correction factor for laterals without downstream outflow, when applied to a relatively large number of outlets is in good agreement with earlier work. For relatively small number of outlets, the average correction factor presented is more accurate. The average correction factor for laterals with outflow reduces to that for laterals without outflow when the outflow ratio is reduced to zero. For a relatively large number of outlets, this average correction factor is primarily a function of the outflow ratio. For both large outflow ratios and large outlet numbers, the average correction factor is almost a constant. To apply the average correction factor to design a tapered lateral, an expression relating lateral inlet head to required average head and friction head loss has been developed. The expression can be applied to a lateral with any number of reaches with different diameters. A practical application has been demonstrated through an example.

BACKGROUND

A typical lateral consists of multiple outlets along its length. To analyze such a lateral requires a stepwise computational approach that can be cumbersome. Christiansen (1942) introduced the widely used friction correction factor that allows direct computation of friction head loss in a lateral. The friction correction factor is a function of the number of outlets and the exponent of the velocity term in the friction

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formula used. Christiansen (1942) assumed the most upstream outlet to be a full outlet spacing from the lateral inlet. Jensen and Fratini (1957) derived the adjusted friction correction factor, which can be applied to laterals, where the most upstream outlet is half an outlet spacing from the lateral inlet. Chu (1978) developed a modified adjusted friction correction factor and suggested that it was constant if the number of outlets along a lateral exceeded four. Scaloppi (1988) developed a more generic form of the adjusted friction correction factor that allows for the most upstream outlet to be any fraction of a full outlet spacing from the lateral inlet. Scaloppi and Allen (1993a) developed expressions for calculating head loss in laterals considering an infinite number of outlets along the lateral. These expressions correlate closely to previous work when applied to a large number of outlets. Scaloppi and Allen (1993a) also included the velocity head in their analysis and verified that for gated pipe system, the velocity head is significant and needs to be considered. However, for sprinkler and drip irrigation system, the velocity head can be ignored without introducing a significant error. Smith (1990) also recommended considering velocity head especially in low pressure systems such as gated pipelines. Anwar (1999) developed a friction correction factor for laterals with outlets and outflow at the downstream end of lateral. Anwar (1999) demonstrated the application of this friction correction factor to calculate friction head loss in tapered laterals.

The friction correction factor by Christiansen (1942) and its subsequent improvements were developed for fixed, periodic or linear displacement laterals. It assumes the discharge through the lateral decreases linearly with the length of the lateral. In center-pivot laterals, the discharge through the lateral does not decrease linearly with length, and therefore the friction correction factor by Christiansen (1942) is not applicable. This was reported by Heerman and Hein (1968). Kincaid and Heerman (1970) verified the stepwise computational approach for center-pivots. Chu and Moe (1972) developed a friction correction factor for center-pivots. Chu and Moe (1972) assumed an infinite number of sprinklers along the lateral and derived a friction correction factor as a function of the exponent of the velocity term in the friction formula used. Reddy and Apolayo (1988) developed a friction correction factor for center-pivots considering a discrete number of outlets, which, as noted by Gilley (1989), should not be compared to the friction correction factor of Christiansen (1942). Scaloppi and Allen (1993 a,b) analyzed center-pivot laterals taking into account velocity head and slopes, verifying earlier work.

The correction factor approach to analyzing laterals has been widely used as an alternative approach to stepwise computation. Although the stepwise computation has been greatly assisted with the use of spreadsheets, correction factors continue to be used [e.g., James (1988); and Keller and Bliesner (1990)]. The correction factors allow simple and direct analysis. The correction factors can be used with accuracy, provided the assumptions in developing the correction factors are considered and attention is given to avoid misinterpreting the results. This paper introduces two average correction factors and demonstrates how these factors can be used to calculate the inlet pressure for fixed, periodic or linear displacement tapered laterals.

INTRODUCTION

Consider a horizontal lateral with multiple outlets. As a result of friction and other losses, the head along the lateral will decrease from the inlet to the downstream end. Typically, the discharge through an outlet is of the form

$$q = C_d a \sqrt{H} \tag{1}$$

where q = discharge through the outlet; C_d = discharge coefficient; a = opening area of outlet; and H = head at the outlet. To ensure that every outlet has the same discharge would either require a pressure regulator at each outlet or for each outlet to have a different opening area. These solutions are often not practical. Hence a designer would attempt to design a lateral such that the average outlet discharge along the lateral is approximately equal to the discharge from an outlet operating at the average outlet head (Keller and Bliesner 1990); that is

$$q_a = \frac{1}{N} \sum_{i=1}^{N} q_i \tag{2}$$

where q_a = average outlet discharge; N = number of outlets along the lateral; i = integer (1,2,3....,N); and q_i = outlet discharge of the i th outlet (starting the count from the downstream end of the lateral). Assuming the coefficient of discharge C_d to be constant for all outlets, substituting appropriately for q_a and q_i from (1) in (2)

$$H_a = \left(\frac{1}{N} \sum_{i=1}^{N} \sqrt{H_i} \right)^2 \tag{3}$$

where H_i = head at the i th outlet; and H_a = average head required to produce the average outlet discharge.

To maintain a system coefficient of uniformity of about 97%, the total pressure variation in a lateral with outlets is limited to typically 20% (Keller and Bliesner 1990). Hence (3) can be approximated by

$$H_a \approx \frac{1}{N} \sum_{i=1}^{N} H_i \tag{4}$$

(for $H_N - H_1 \leq 0.20 H_a$, and given the lateral is horizontal) where H_N = head at the N th outlet; and H_1 = head at the first (most downstream) outlet. Fig.1 shows a horizontal lateral with multiple outlets. The head at the inlet of the lateral is given by

$$H_l = H_n + H_f \tag{5}$$

where H_l = head at the inlet of the lateral; H_n = minimum head that corresponds to the head at the most downstream outlet on the lateral; and H_f = friction head loss (ignoring other minor losses) in a lateral with multiple outlets. Defining the average correction factor for laterals with multiple outlets as the ratio of the average friction head loss to the total friction head loss, the average correction factor can be written as follows:

$$F_{AVG} = \frac{H_{f_{AVG}}}{H_f} \tag{6}$$

where F_{AVG} = average correction factor; and $H_{f_{AVG}}$ = average friction head loss at each outlet along the lateral. From Fig.1

$$H_a = H_n + H_{f_{AVG}} \tag{7}$$

Substituting (6) and (7) in (5)

$$H_l = H_a + (1 - F_{AVG}) H_f \tag{8}$$

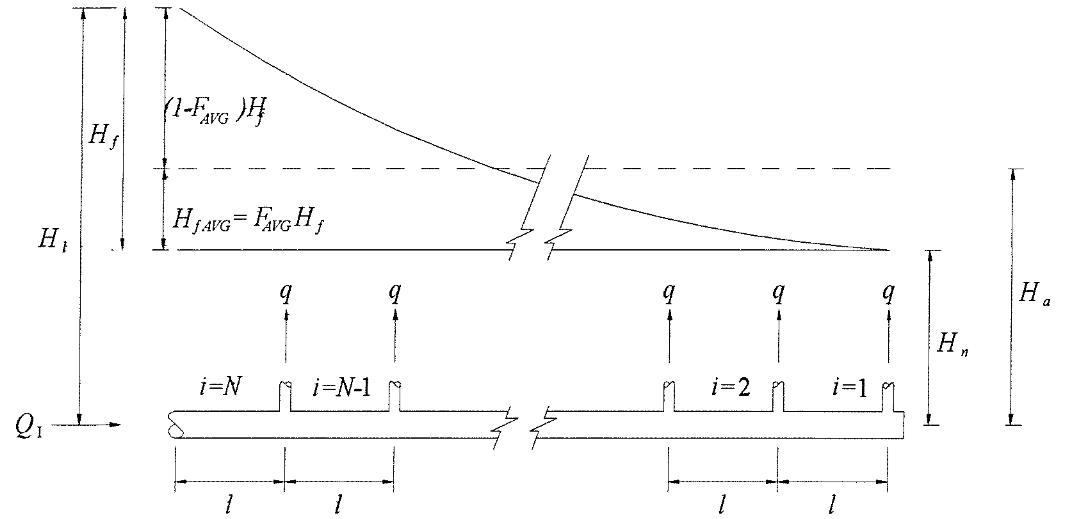

FIG.1 Lateral with Multiple Outlets and No Downstream Outflow

Scaloppi and Allen (1993a) developed a similar equation assuming uniform outlet discharge along the pipe length. For horizontal pipes and ignoring the velocity head losses, the equation developed by Scaloppi and Allen (1993a) reduces to

$$H_l = H_a + \frac{3}{4}H_f , \qquad \text{(for } m=2\text{)} \tag{9}$$

where m = exponent of the velocity term in the friction formula used. A similar equation is also given by Keller and Bliesner (1990). Comparing (8) and (9), F_{AVG} = 0.25.

The following section of this work develops average correction factors for laterals considering discrete outlets. First, the average correction factor for laterals without outflow at the downstream end, based on friction correction factor F, will be determined (Christiansen, 1942). Second, the average correction factor for laterals with outflow, based on friction correction factor G, will be examined Anwar (1999). Third, a general expression to apply these average correction factors to tapered (multiple-diameter) laterals will be developed. The assumptions made in developing these factors are as follows: the friction factor is constant along the lateral; the velocity head is negligible; an increase in pressure head caused by reduction in velocity head past each outlet is balanced by the head loss caused by turbulence at each outlet; and minor losses, for example, at reduction of pipe diameter are negligible.

ANALYSIS

Average Correction Factor F_{AVG} for Single Diameter Lateral with Multiple Outlets

Fig. 1 shows a lateral with multiple outlets. The most upstream outlet is a full outlet spacing from the lateral inlet, and there is no flow past the most downstream outlet. The outlets are uniformly spaced, and the outlet spacing (or segment length) is

$$l = \frac{L}{N} \tag{10}$$

where l = length of each segment of the lateral; N = number of segments (and/or number of oultets) along the lateral; and L = total length of the lateral. Assuming a constant discharge of each outlet by limiting the head variation along the lateral as discussed earlier

$$q = \frac{Q}{N} \tag{11}$$

where q = the discharge of each outlet; and, Q = total discharge of the lateral at the inlet. For analysis purposes, the i th outlet and i th segment along the lateral will be considered, where i is an integer (1,2,3...,N). The most downstream segment of the lateral is considered to the be first segment. Similarly the most downstream outlet is considered to be the first outlet, and subsequent outlets are numbered sequentially upstream to the Nth outlet at one full outlet spacing from the lateral inlet.

The friction head loss (ignoring minor losses) in the lateral immediately downstream of the (i+1) th outlet is the sum of head loss in all i segments upto the (i+1) th outlet. The average friction head loss can be represented by

$$H_{f_{AVG}} = \frac{1}{N}\left(H_{f_1}+H_{f_2}+....+H_{f_{N-1}}\right) \tag{12}$$

where H_{f_1} = friction head loss in the first segment of the lateral; H_{f_2} = friction head loss in the first and second segment; H_{f_3} = friction head loss in the first, second and third segment; and $H_{f_{(N-1)}}$ = friction head loss in the first, second, third, fourth..... up to and including the $(N-1)$th segment of the lateral. Alternatively, (12) can be written as follows:

$$H_{f_{AVG}} = \frac{1}{N} \sum_{i=1}^{N-1} H_{f_i} \tag{13}$$

where H_{f_i} = friction head loss in the lateral in first, second, third.... up to and including the i th segment of the lateral. The friction head loss in a lateral with i outlets is given by Christiansen (1942) as

$$H_{f_i} = \frac{CKQ_i^m}{D^n} L_i F_i \tag{14}$$

where C = units coefficient; K = friction factor based on the friction formula used; Q_i = discharge at the inlet of all i segments of the lateral; m = exponent of the velocity terms in the friction formula used; D = internal diameter of the lateral; n = exponent of the diameter term in the discharge equation used; L_i = total length of the i segments of lateral; and F_i = friction correction factor for i outlets. Substituting (14) in (13)

$$H_{f_{AVG}} = \frac{1}{N} \sum_{i=1}^{N-1} \frac{CKQ_i^m}{D^n} L_i \; F_i \tag{15}$$

but

$$L_i = il \tag{16}$$

Substituting for l from (10)

$$L_i = \frac{iL}{N} \tag{17}$$

and also

$$Q_i = iq \tag{18}$$

Substituting for q from (11)

$$Q_i = \frac{iQ}{N} \tag{19}$$

Substituting (17) and (18) in (15) and rearranging

$$H_{f_{AVG}} = \frac{CKQ^m}{D^n} L \; \frac{1}{N^{m+2}} \sum_{i=1}^{N-1} \; i^{m+1} F_i \tag{20}$$

Using the friction correction factor for N outlets in a lateral of length L in the form of (14)

$$H_f = \frac{CKQ^m}{D^n} L F_N \tag{21}$$

where F_N = friction correction factor for N outlets. Substituting for (20) and (21) in (6)

$$F_{AVG} = \frac{1}{F_N} \frac{1}{N^{m+2}} \sum_{i=1}^{N-1} i^{m+1} F_i \tag{22}$$

Christiansen (1942) defined the friction correction factor F for i outlets as:

$$F_i = \frac{1}{m+1} + \frac{1}{2i} + \frac{(m-1)^{0.5}}{6i^2} \tag{23}$$

DeTar (1982) derived a similar friction correction factor which for i outlets can be written as follows:

$$F_i = \frac{1}{i^{m+1}} \sum_{j=1}^{i} j^m \tag{24}$$

where j = integer $(1,2,3....,i)$. DeTar (1982) showed (24) can be solved to a very close approximation of (22). This is also reported by Scaloppi (1988) and can be verified by using the Euler-Mclaurin summation formula, [e.g., Spiegel (1968)]. Substituting for F_i from (24) and appropriately for F_N in (22)

$$F_{AVG} = \frac{1}{\left[\frac{1}{N^{m+1}} \sum_{i=1}^{N} i^m\right]} \left(\frac{1}{N^{m+2}} \sum_{i=1}^{N-1} i^{m+1} . \left[\frac{1}{i^{m+1}} \sum_{j=1}^{i} j^m \right] \right) \tag{25}$$

which can be simplified to

$$F_{AVG} = \frac{1}{N} \frac{\sum_{i=1}^{N} (N-i) i^m}{\sum_{i=1}^{N} i^m} \tag{26}$$

Fig. 2 shows the variation of F_{AVG} with the number of outlets N for various m values in commonly used friction formulas. As the number of outlets increases, F_{AVG} approaches a value of 0.25. Table 1 compares the expression $(1- F_{AVG})$ for N=500 using (26) with the results of Scaloppi and Allen (1993a). For the purpose of this comparison, in the expression by Scaloppi and Allen (1993a), the velocity head has been ignored and the pipe slope set to zero (i.e., horizontal laterals). It should also be noted that the work by Scaloppi and Allen (1993a) is for an infinite number of outlets along the lateral. Despite this difference, the results can be seen to agree closely. Intuitively, for the extreme case of a lateral with only one outlet, in (8) one would expect $F_{AVG}=0$ as is demonstrated by (26) in Fig.2, for any value of m.

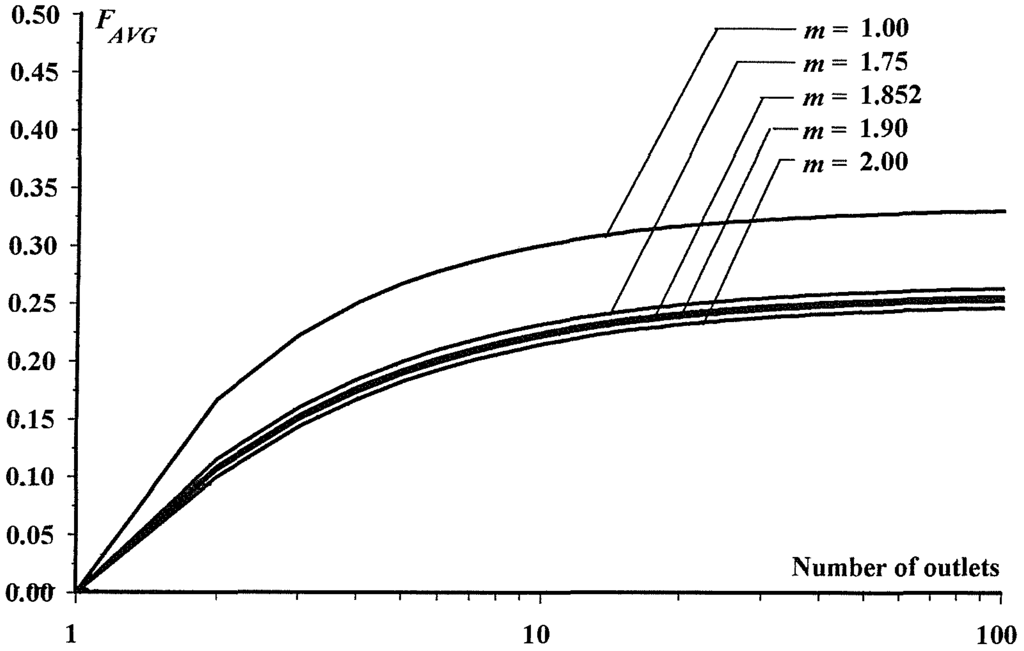

FIG.2. Average Correction Factor F_{AVG}

For laterals with less than about 10 outlets, (26) presents a more accurate estimation of the the average correction factor. For laterals with fewer than 10 outlets, Scaloppi and Allen (1993a) also recommended the use of more adequate correction factors based on discrete outlets. Scaloppi and Allen (1993a) recommended their equations should not be used for laterals with six or fewer outlets. In Fig. 2, for $N \geq 10$ and for the typical values of the exponent m in the commonly used friction formula (i.e., m = 1.75 - 2.00), a constant value for the average correction factor $F_{AVG} = 0.25$ can be used without much error. This is also reported by Keller and Bliesner (1990).

TABLE 1. Comparison of Eq.(26) with Results of Scaloppi and Allen (1993a)

Expression	Velocity Exponent in Friction Formula m				
	2.000	1.900	1.852	1.750	1.000
(1)	(2)	(3)	(4)	(5)	(6)
(1-F_{AVG}) from (26) for N=500	0.751	0.744	0.741	0.734	0.667
Scaloppi and Allen (1993a) $N = \infty$	0.750	0.744	0.740	0.733	0.667

Average Correction Factor G_{AVG} for Single Diameter Lateral with Multiple Outlets, with or without Outflow at the Downstream End of the Lateral.

Fig. 3 shows a lateral with multiple outlets and outflow at the downstream end. The upstream outlet is a full outlet spacing from the lateral inlet. The length of each segment of the lateral is given by (10). Anwar (1999) developed a friction correction factor for laterals with outlets and outflow at the downstream end of the lateral, in which the outflow ratio was defined as

$$r_N = \frac{Q_O}{Nq} \tag{27}$$

where r_N = outflow ratio for a lateral with N outlets; Q_O= outflow at the downstream end of the lateral; N = number of outlets along the lateral; and q = outlet discharge. Analagous to (6), the average correction factor for laterals with outflow can be expressed as

$$G_{AVG} = \frac{H_{f_{AVG}}}{H_f} \tag{28}$$

where G_{AVG}= average correction factor for laterals with oulflow; $H_{f_{AVG}}$ = average of the friction head loss at each outlet as given by (12); and H_f= friction head loss in a lateral with outlets and outflow as given by Anwar (1999) as

$$H_f = \frac{CKQ_I^m}{D^{2m+n}} LG_{N,r_N} \tag{29}$$

where Q_I= flow at the lateral inlet; and $G_{N,\, r_N}$= friction correction factor G for laterals with N outlets and an outflow ratio of r_N.

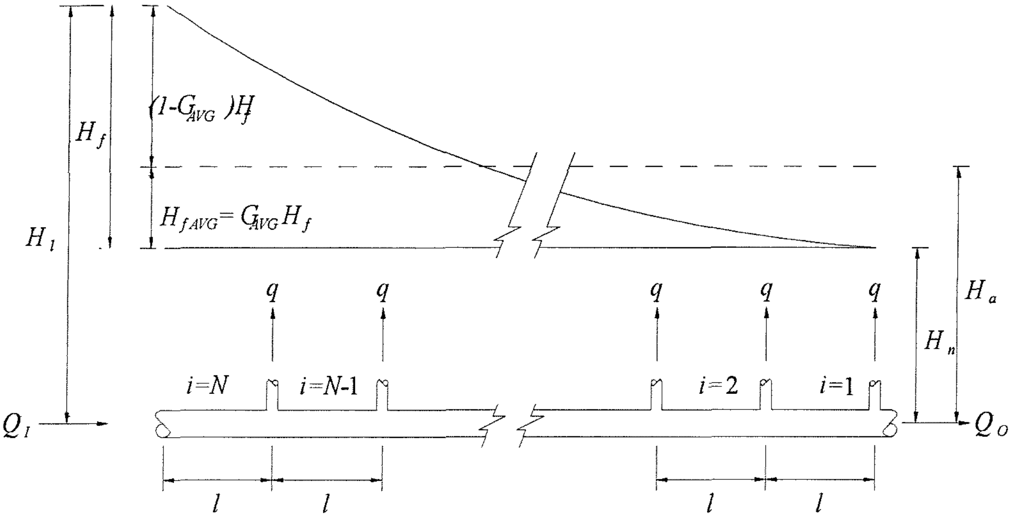

FIG.3. Lateral with Multiple Outlets and Downstream Outflow

In (28), the term $H_{f_{AVG}}$ is as given by (13) but with H_{f_i}= friction head loss in a lateral with i outlets expressed as

$$H_{f_i} = \frac{CKQ_i^m}{D^{2m+n}} L_i G_{i,r_i} \tag{30}$$

where H_{f_i}= friction head loss in a lateral with i outlets; G_{i,r_i}= friction correction factor for a lateral with i outlets and an outflow ratio r_i; and r_i = outflow ratio, given by

$$r_i = \frac{Q_O}{iq} \tag{31}$$

The friction correction factor developed by Anwar (1999), for i outlets can be expressed as

$$G_{i,r_i} = \frac{1}{i^{m+1}(1+r_i)^m} \sum_{j=1}^{i} (j+ir_i)^m \tag{32}$$

where j = integer (1,2,3...,i). From (31), the discharge in the i th segment of the lateral is

$$Q_i = iq(1+r_i) \tag{33}$$

where Q_i= discharge in the ith segment of the lateral.

Substituting for Q_i from (33), L_i from (17), and l from (10) in (30)

$$H_{f_i} = \frac{CKq^m}{D^{2m+n}} L \frac{i^{(m+1)}}{N} (1+r_i)^m G_{i,r_i} \tag{34}$$

The discharge at the lateral inlet can be expressed as follows Anwar (1999):

$$Q_I = Nq(1+r_N) \tag{35}$$

$$q = \frac{Q_I}{N(1+r_N)} \tag{36}$$

Substituting for q from (36) and H_{f_i} from (34) in (13)

$$H_{f_{AVG}} = \frac{CKQ_I^m}{D^{2m+n}} \frac{1}{N^{m+2}(1+r_N)^m} \sum_{i=1}^{N-1} i^{m+1}(1+r_i)^m G_{i,r_i} \tag{37}$$

Substituting (29) and (37) in (28)

$$G_{AVG} = \frac{1}{G_{N,\,r_N}} \frac{1}{N^{m+2}(1+r_N)^m} \sum_{i=1}^{N-1} i^{m+1}(1+r_i)^m \; G_{i,r_i} \tag{38}$$

where G_{i,r_i} is given by (32) and $G_{N,\,r_N}$ Anwar (1999) is given by

$$G_{N,r_N} = \frac{1}{N^{m+1}(1+r_N)^m} \sum_{i=1}^{N} (i+Nr_N)^m \tag{39}$$

from (27) and (31)

$$r_i = \frac{Nr_N}{i} \tag{40}$$

Substituting (32) and (39) in (38) and from (40)

$$G_{AVG} = \frac{1}{N} \frac{\sum_{i=1}^{N} (N-i)(i+Nr_N)^m}{\sum_{i=1}^{N} (i+Nr_N)^m} \tag{41}$$

For the condition of zero outflow, $r_N = 0$ (41) reduces to (26) and $G_{AVG} = F_{AVG}$. Fig. 4. shows G_{AVG} for m=2.00 and a range of outflow ratios. As would be expected, for values of $r_N > 0$, G_{AVG} differs from F_{AVG}. However, as with F_{AVG} for $N > 10$, G_{AVG} is primarily a function of outflow ratio only. Furthermore, for larger values of N and r_N, G_{AVG} almost becomes a constant.

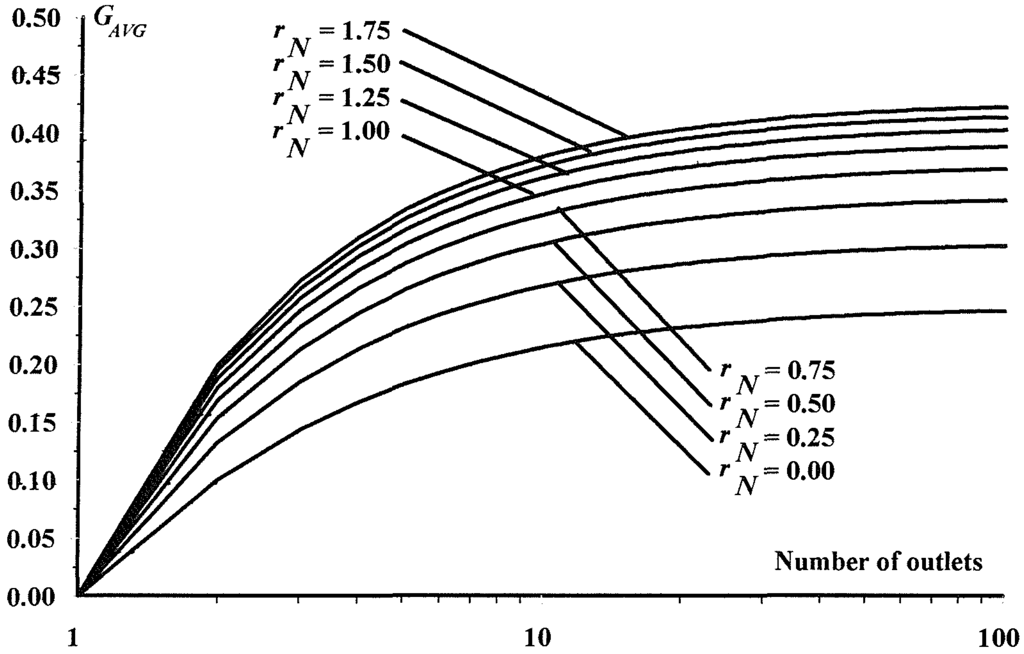

FIG.4. Average Correction Factor G_{AVG} for $m = 2.00$

Average Inlet Head for Tapered Laterals

In the preceeding section the average correction factor G_{AVG} was developed and was demonstrated to be applicable to laterals with and without outflow past the most downstream outlet. Although G_{AVG} can be applied directly to such a lateral, it can not be applied directly to a tapered (multiple-diameter) pipe. Fig. 5 shows a lateral with outlets. It consists of two reaches, the downstream reach has a smaller diameter than the upstream reach. The average head in the lateral needs to be equal to the sum of the length weighted average head in each reach, or

$$H_a(L_1+L_2) = (H_{a_1}L_1) + (H_{a_2}L_2) \tag{42}$$

where H_a = average head; L_1 = length of reach 1 (the downstream reach); L_2 = length of reach 2 (the upstream reach); H_{a_1} = average head in reach 1; and, H_{a_2} = average head in the reach 2.

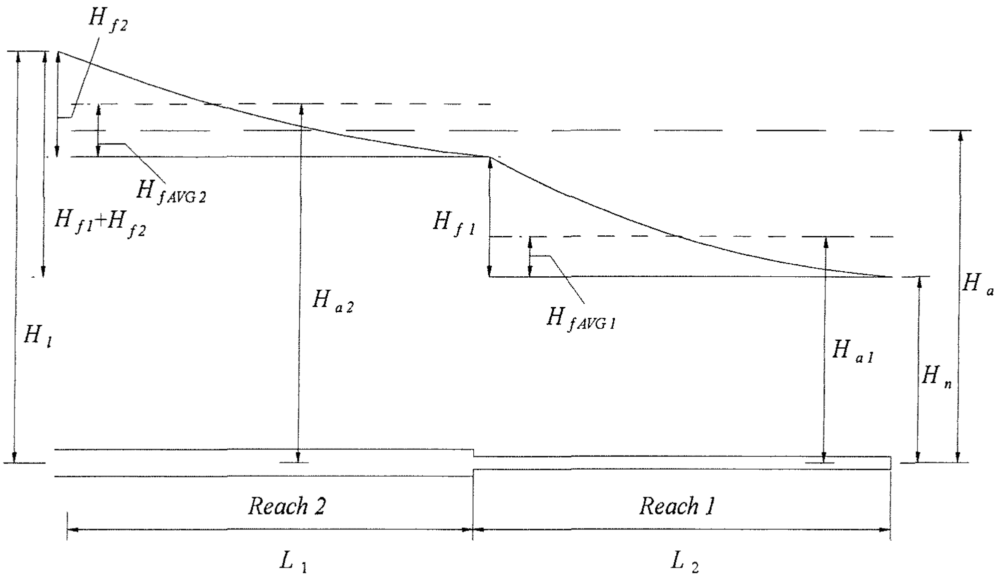

FIG.5. Tapered Lateral with Two Reaches

From Fig. 5

$$H_{a_1} = H_n + H_{f_{AVG_1}} \tag{43}$$

where $H_{f_{AVG\,1}}$ = average friction head loss in reach 1. Substituting appropriately for $H_{f_{AVG\,1}}$ from (28)

$$H_{a_1} = H_n + H_{F_1} G_{AVG_1} \tag{44}$$

where H_{F_1} = friction head loss in reach 1; and G_{AVG_1} = average correction factor for reach 1. Similarly, from Fig. 5

$$H_{a_2} = H_n + H_{F_1} + H_{f_{AVG_2}} \tag{45}$$

where $H_{f_{AVG_2}}$ = average friction head loss in reach 2. Substituting appropriately for $H_{f_{AVG_2}}$ from (28)

$$H_{a_2} = H_n + H_{F_1} + H_{f_2} G_{AVG_2} \tag{46}$$

where G_{AVG_2} = average correction factor for reach 2. Substituting (44) and (46) in (42) and rearranging

$$H_n = H_a - \frac{L_1\left(H_{F_1} G_{AVG_1}\right) + L_2\left(H_{F_1} + H_{f_2} G_{AVG_2}\right)}{\left(L_1 + L_2\right)} \tag{47}$$

From Fig. 5, the head at the inlet can be written as follows:

$$H_{l_2} = H_{F_1} + H_{F_2} + H_n \tag{48}$$

where H_{l_2} = head at the inlet of the reach 2, which for a lateral consisting of two reaches is also the head at the inlet of the lateral. Substituting for H_n from (47) in (48)

$$H_{l_2} = \left(H_{F_1} + H_{F_2}\right) + H_a - \frac{L_1\left(H_{F_1} G_{AVG_1}\right) + L_2\left(H_{F_1} + H_{F_2} G_{AVG_2}\right)}{\left(L_1 + L_2\right)} \tag{49}$$

Eq. (47) can be generalized for a lateral with x reaches to

$$H_n = H_a - \frac{\sum_{j=1}^{x} L_j \left(\sum_{k=0}^{j-1} H_{F_k} + H_{F_j} G_{AVG_j} \right)}{\sum_{j=1}^{x} L_j} \tag{50}$$

where x = number of reaches in the lateral; L_j = length of the j th reach; H_{F_j} = friction head loss in the j th reach; G_{AVG_j} = average correction factor for the j th reach, H_{F_k} = friction head loss in the k th reach; and j and k = integers. In (7), k=0 refers to an imaginary zeroth reach, for which H_{F_0}=0, and G_{AVG_0}=0. Finally (49) can be generalized to

$$H_{l_x} = \sum_{j=1}^{x} H_{F_j} + H_a - \frac{\sum_{j=1}^{x} L_j \left(\sum_{k=0}^{j-1} H_{F_k} + H_{F_j} G_{AVG_j} \right)}{\sum_{j=1}^{x} L_j} \tag{51}$$

where H_{l_x} = head at the inlet of the x th reach, which for a lateral with x reaches is also the inlet to the lateral. It can be demonstrated that for a single diameter lateral (a lateral with only one reach), x = 1 and (51) reduces to (8).

APPLICATION

An example is used to illustrate the application of average factors F_{AVG} and G_{AVG} for calculating the head required at the inlet of a sprinkler lateral.

Example

Calculate the head required at the inlet of a tapered aluminium sprinkler lateral that is 288m long. Outlets are installed at 12m intervals. The first reach (starting from the closed end) is 144m long and has an internal diameter of 75mm. The second reach of the lateral has an internal lateral diameter of 100mm. The lateral is to be designed for an average outlet discharge of 0.5 L/s, operating under an average head of 35m.

Solution

The Darcy-Weisbach equation will be used for calculating head losses caused by friction. The lateral relative roughness is assumed as 0.127mm and for water at 15°C, kinematic viscosity of water, 1.14×10^{-6} m^2/s. Table 2 summarizes the details of the lateral. The Churchill equation Churchill (1977) will be used to calculate the friction factor K.

TABLE 2. Summary of Details of Lateral

Reach (1)	Length (m) (2)	Diameter (m) (3)	Number of outlets (4)	Outlet spacing (m)	Discharge at inlet (L/s) (5)
1	144	75	12	12.00	6.00
2	180	100	15	12.00	13.50

Reach 1

$Q_{o_1} = 0.0$ L/s

From (27) $r_1 = 0$

From (35) $Q_{I_1} = 12 \times 0.5 = 6.0$ L/s

From the Churchill equation $K_1 = 0.0247$

From Anwar (1999), for $N = 12$, $r_1 = 0$ $G_1 = 0.376$

Using the Darcy-Weisbach equation and factor G to calculate friction head loss in a lateral with outlets

$$H_{F_1} = \frac{0.0826 K_1 Q_{I_1}^2 L_1}{D^5} G_1 = \frac{0.0826 \times 0.0247 \times 0.006^2 \times 144}{0.075^5} \times 0.376 \,,$$

$H_{F_1} = 1.68\text{m}$

From (41), for $N = 12$, $r_1 = 0$ $G_{AVG_1} = 0.220$

Reach 2

$Q_{o_2} = 6.0$ L/s

From (27) $r_2 = 0.80$

From (35) $Q_{I_2} = 15 \times 0.50(1 + 0.8)$ $Q_{I_2} = 13.50$ L/s

From the Churchill equation $K_2 = 0.0226$

From Anwar (1999), for $N = 15$, $r_2 = 0.8$ $G_2 = 0.574$

Using the Darcy-Weisbach equation and factor G to calculate friction head loss in a lateral with outlets

$$H_{F_2} = \frac{0.0826 K_2 Q_{I_2}^2 L_2}{D^5} G_2 = \frac{0.0826 \times 0.0226 \times 0.0135^2 \times 180}{0.100^5} \times 0.574 \,,$$

$H_{F_2} = 3.51\text{m}$

From (41), for $N = 15$, $r_2 = 0.80$ $G_{AVG_2} = 0.348$

$H_{F_1} + H_{F_2} = 1.68 + 3.51 = 5.19 \text{ m} \approx 15\% \, H_a$

This is < 20%; therefore, friction head loss is not excessive and (4) can be deemed valid. From (49)

$$H_{l_2} = (1.68 + 3.51) + 35 - \frac{144(1.68 \times 0.220) + 180(1.68 + 3.51 \times 0.348)}{(144 + 180)}$$

$$H_{l_{\#2}} = \mathbf{38.88m}$$

Using the expression by Keller and Bliesner (1990), for tapered horizontal laterals

$$H_l' = H_a + \frac{5}{8}H_f = 35 + \frac{5}{8}\times 5.19 ,$$

$$H_l' = \mathbf{38.24m}$$

where H_l' = head at inlet of a horizontal tapered lateral using the expression by Keller and Bliesner (1990). The two results are comparable.

The validity of these calculations can also be checked with using a stepwise procedure and solving iteratively (i.e., adjusting the inlet pressure until the required lateral discharge is obtained). The problem was solved using a spreadsheet and its built-in iterative equation solver. For the stepwise procedure, the friction factor of each segment was estimated using the Churchill equations rather than assuming a constant friction factor for each reach. Velocity head and minor losses at the reduction in lateral diameter were ignored. Using this iterative stepwise approach, an inlet head of 38.41m is required. The error in using the average correction factor as compared to the iteratiave step-wise approach is 1.23%. Errors of similar order of magnitude were reported by Scaloppi and Allen (1993a) in their work when simplified equations were used. Scaloppi and Allen (1993a) considered errors of such order of magnitude to be within the range of accuracy of pressure gauges commonly used in sprinkler irrigation systems.

CONCLUSIONS

This paper introduces two new correction factors; the average correction factor F_{AVG} for laterals without outflow and average correction factor G_{AVG} for laterals with or without outflow. When average correction factor F_{AVG} is calculated for a large number of outlets, it closely approximates published estimates for laterals with an infinite number of outlets. For laterals with <10 outlets, this paper presents more accurate values for this correction factor. The concepts used to develop F_{AVG} were extended to laterals with or without outflow to develop G_{AVG}. For conditions of zero outflow the two average correction factors are identical (i.e. $G_{AVG} = F_{AVG}$).

These average correction factors cannot be directly applied to the design of a tapered lateral. Therefore an expression relating the inlet head of a tapered lateral to the required average head and friction head loss is presented.

The application of the average correction factors is demonstrated with the design of a tapered lateral. The results are comparable to those obtained using the expression by Keller and Bliesner (1990). The results using the method presented in this paper have also been verified using a stepwise iterative solution. The error in using the average correction factors is of the same order of magnitude as the simplified approach by Scaloppi and Allen (1993a). The design method proposed in this method can be readily applied to a tapered pipe with any number of reaches with different diameters and should find application amongst irrigation engineers.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

a = cross-sectional area of outlet opening;
C = units coefficient;
C_D = discharge coefficient;
D = internal pipeline diameter;
F = Christiansen's correction factor for laterals with multiple equally-spaced outlets without downstream outflow;
F_{AVG} = average correction factor for lateral with mulitple outlets and no outflow;
F_i = Christiansen's factor F for i outlets;
F_N = Christiansen's factor F for N outlets;
G = average correction factor for pipelines with multiple equally spaced outlets with/without downstream outflow;
G_{AVG} = average correction factor for laterals with/without outflow;
G_{AVG_j} = average correction factor for reach j of lateral;
$G_{AVG_{\#0}}$ = average correction factor forimaginary zeroth reach of lateral;
G_{AVG_1} = average correction factor for reach 1 of lateral;
G_{AVG_2} = average correction factor for reach 2 of lateral;
$G_{i,\, r_i}$ = average correction factor for a lateral with i outlets withoutflow ratio of r_i;
$G_{N,\, r_N}$ = average correction factor for a lateral with N outlets with outflow ratio of r_N;
H = head at an outlet;
H_a = average head required to produce average discharge at outlet;
H_{a_1} = average head in reach 1 of the lateral;

H_{a_2} = average head in reach 2 of the lateral;
H_f = friction head loss;
$H_{F_{AVG}}$ = average friction head loss;
H_{F_i} = friction head loss in lateral with i outlets;
H_{F_j} = friction head loss in reach j of lateral;
H_{F_k} = friction head loss in reach k of lateral;
$H_{f_{N-1}}$ = friction head loss in first, second up to $(N - 1)$ th segment;
H_{F_0} = friction head loss in an imaginary zeroth reach of the lateral;
H_{f_1} = friction head loss in first segment;
H_{F_1} = friction head loss in reach 1 of lateral;
H_{f_2} = friction head loss in first and second segment;
H_{F_2} = friction head loss in reach 2 of lateral;
H_i = head at the i th outlet;
H_I = head at the inlet of the lateral;
$H_I^{/}$ = head at inlet of the lateal based on expression by Keller and Bliesner (1990);
H_{I_1} = head at the inlet of reach 1 of lateral;
H_{I_2} = head at the inlet of reach 2 of lateral;
H_{I_x} = head at inlet of reach x of the lateral;
H_n = minimum head in the lateral;
i = integer representing pipe segment or outlet number;
j = integer representing reach number in tapered lateral
K = friction factor based on friction formula used;
L = total length of lateral;
L_i = length of i segments of lateral;
L_j = length of reach j of lateral;
L_1 = length of reach 1 of lateral;
L_2 = length of reach 2 of lateral;
l = length of each segment of pipeline between any two outlets;
m = exponent of velocity term in friction formula used;
N = total number of outlets along the pipeline;
n = exponent of diameter term in friction formula used;
Q = discharge at inlet lateral for lateral without outflow;
Q_I = discharge at inlet of lateral for a lateral with outflow;
Q_{I_1} = inlet discharge for reach 1 of lateral;
Q_{I_2} = inlet discharge for reach 2 of lateral;
Q_i = discharge in i th segment of lateral;
Q_O = outflow discharge of lateral;
Q_{O_1} = outflow discharge for reach 1 of lateral;
Q_{O_2} = outflow discharge for reach 2 of lateral;
q = discharge of an individual outlet;
q_a = average discharge of outlets;
q_i = discharge of the ith outlet;
r = ratio of the outflow discharge to total outlet discharge;
r_i = ratio of outflow discharge to outlet discharge for i outlets;
r_N = ratio of outflow discharge to outlet discharge for N outlets;

r_1 = ratio of outflow discharge to total outlet discharge for reach 1;
r_2 = ratio of outflow discharge to total outlet discharge for reach 2; and
x = number of reaches of different diameter.

4

ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

By Arif A. Anwar[1]

ABSTRACT: This paper is the fourth in a series on friction factors for sprinkler laterals. The widely used friction correction factor F was developed by Christiansen for the hydraulic analysis of sprinkler laterals. A significant modification to this factor was the adjusted friction correction factor F_a. The adjusted friction correction factor can be used when the first sprinkler is a fraction of a full spacing from the lateral inlet. To design laterals with outlets and outflow at the downstream end, friction correction factor G was developed with the corresponding adjusted friction correction factor G_a. To calculate the average pressure head along a lateral the average correction factors F_{AVG} and G_{AVG} were developed. These average correction factors can be used where friction correction factors F and G are used to analyze a lateral. This paper introduces two final adjusted average correction factors $F_{a AVG}$ and $G_{a AVG}$, which can be used to determine the average pressure head in laterals analysed using F_a or G_a. Use of these factors is demonstrated in an example.

INTRODUCTION

In a horizontal lateral, the pressure head decreases along the length of the lateral from the inlet end to the downstream end caused by friction and other minor losses. The discharge of any outlet along the lateral is a function of the head. To maintain a constant discharge at all outlets along a lateral would either require installing pressure regulators at each outlet or varying the characteristics, e.g. area of

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every outlet. Both these solutions are impractical. Therefore, in typical installations, an allowable head variation in the lateral is permitted. Usually this is around 20% of the average head (Keller and Bliesner, 1990). The inlet head at a lateral is designed such that the average discharge of all outlets along the lateral is approximately equal to the discharge from an outlet operating at the average outlet head, (Keller and Bliesner, 1990). Keller and Bliesner (1990) proposed the following equation for calculating the head at the inlet of a single diameter horizontal lateral

$$H_l = H_a + \frac{3}{4} H_f \qquad (1)$$

where H_l = head at the inlet of the lateral; H_a = head required to produce the average discharge at an outlet; and, H_f = friction head loss. Scaloppi and Allen (1993) developed the following relationship between average head and the head at the inlet of the lateral

$$H_a = H_l - \frac{3}{4} H_f + \frac{2}{3} H_v - \frac{1}{2} H_z \qquad (2)$$

where H_v = velocity head; and, H_z = change in elevation from beginning to end of the lateral. For a horizontal lateral, $H_z = 0$. Scaloppi and Allen (1993) demonstrated that for typical operating head of sprinkler systems the velocity head can be ignored and therefore (2) is identical to (1). Anwar (2000) introduced the average correction factor given by

$$F_{AVG} = \frac{H_{f_{AVG}}}{H_f} \qquad (3)$$

where F_{AVG} = average correction factor; and $H_{f_{AVG}}$ = average of the friction head loss at each outlet. Anwar (2000) demonstrated that the head at the inlet of the lateral is given by

$$H_l = H_a + (1 - F_{AVG}) H_f \qquad (4)$$

and the average correction factor was given as

$$F_{AVG} = \frac{1}{N} \frac{\sum_{i=1}^{N} (N-i) i^{m}}{\sum_{i=1}^{N} i^{m}} \qquad (5)$$

where N = number of outlets along the lateral; i = integer 1,2,3.....N representing the segment or outlet number; and, m = exponent of the discharge term in the friction formula used.

For laterals with outlets and outflow at the downstream end, Anwar (2000) introduced the average correction factor defined as

$$G_{AVG} = \frac{H_{f_{AVG}}}{H_f} \tag{6}$$

where G_{AVG} = average correction factor for laterals with outflow at the downstream end. For laterals with outlets and downstream outflow, (4) becomes

$$H_l = H_a + (1 - G_{AVG})H_f \tag{7}$$

and the average correction factor was given as

$$G_{AVG} = \frac{1}{N} \frac{\sum_{i=1}^{N} (N-i)(i+Nr_N)^m}{\sum_{i=1}^{N} (i+Nr_N)^m} \tag{8}$$

where r_N = outflow ratio defined as

$$r_N = \frac{Q_O}{Nq} \tag{9}$$

where Q_O = outflow from the lateral; and, q = outlet discharge .

While developing the average correction factors given by (5) and (8), Anwar (2000) considered the first outlet to be a full outlet spacing from the lateral inlet. In this paper, this constraint has been removed and the first outlet can be at any fraction or full spacing from the inlet. This is broadly comparable to the adjusted friction factor F_a by Scaloppi (1988) as an extension of the friction correction factor F by Christiansen (1942). Similarly, for laterals with outlets and downstream outflow, Anwar (1999a) developed the friction correction factor G. Anwar (1999b) extended the application of factor G to a lateral with the first outlet at any position and developed the adjusted friction correction factor G_a. The assumptions made in developing the adjusted average correction factors are: (1) friction factor is constant along the lateral, (2) velocity head is negligible, (3) increase in head past each outlet is balanced by the head loss caused by turbulence at each outlet. Similar assumptions have been made by others in developing correction factors for laterals;. (Christiansen, 1942; Scaloppi, 1988; Reddy and Apolayo,1988).

ANALYSIS

Adjusted Average Correction Factor $F_{a\,AVG}$ for a Single Diameter Pipeline with Multiple Outlets

Fig.1 illustrates a lateral with multiple equally spaced outlets. The most upstream (N th) outlet is a fraction of a full spacing from the lateral inlet. The total length of the lateral was given by Scaloppi (1988) as

$$L = (N-1)l+xl \tag{10}$$

or

$$l = \frac{L}{N+x-1} \tag{11}$$

where L = total length of the lateral; l = length of each segment of the lateral; and, x = distance between most upstream outlet and lateral inlet, expressed as a fraction of a full outlet spacing ($0<x\leq 1$).

The friction head loss from the downstream end of the lateral to the ith outlet is the sum of the friction head loss in all (i-1) segments of the lateral downstream of the ith outlet. From Fig. 1 the average of the head at each outlet is given by

$$H_a^* = \frac{1}{N}\left[H_n + (H_n+H_{f_1}) + (H_n+H_{f_2})+\ldots..+(H_n+H_{f_{N-1}})\right] \tag{12}$$

where H_a^* = average of the head at each outlet; H_n = minimum head at the downstream end of the lateral; H_{f_1} = friction head loss in the 1st segment of the lateral; H_{f_2} = friction head loss in the 1st and 2nd segments; and $H_{f_{N-1}}$ = friction head

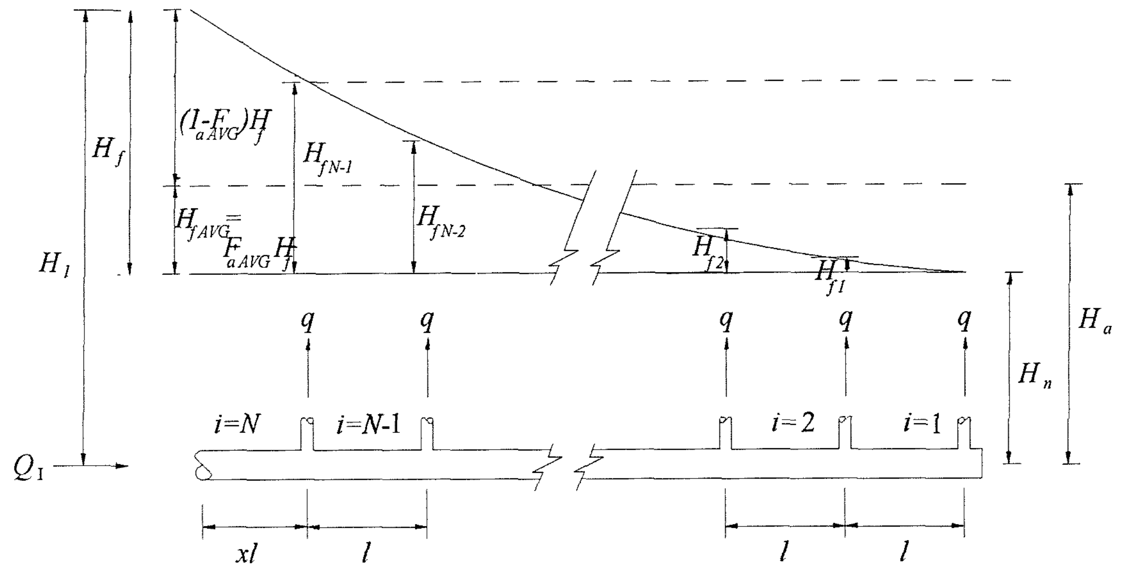

FIG. 1 Lateral with multiple outlets and no downstream outflow

loss in the 1st, 2nd, 3rd, 4th.....N-1th segment. (12) can be written as

$$H_a^* = H_n + H_{f_{AVG,\,x}} \tag{13}$$

where $H_{f_{AVG,\,x}}$ = average of the friction head loss at each outlet with first outlet fraction x given by

$$H_{f_{AVG,\,x}} = \frac{1}{N}\left(H_{f_1} + H_{f_2} + \ldots.. + H_{f_{N-1}}\right) \tag{14}$$

Alternatively (14) can be written as

$$H_{f_{AVG,\,x}} = \frac{1}{N}\sum_{i=1}^{N-1} H_{f_i} \tag{15}$$

where H_{f_i} = friction head loss in the lateral in 1st, 2nd, 3rd....i th segment. Anwar (2000) demonstrated that $H_a \approx H_a^*$ assuming the variation in discharge between outlets is not excessive. For a horizontal lateral this assumption is only reasonable if the friction head loss in the lateral is not excessive. A maximum of 20% of operating head is an often quoted figure (Keller and Bliesner, 1990). Scaloppi and Allen (1993) demonstrated that using simplified equations based on this assumption of quasi-uniform discharge from outlets, and ignoring velocity head, the error in calculating the average head in a lateral with 32 outlets was in the range of 0.20% to 1.92% as compared to a stepwise approach.

The friction head loss in a lateral with i outlets is given by Christiansen (1942) as

$$H_{f_i} = \frac{CKQ_i^m}{D^n} L_i F_i \tag{16}$$

where C = units coefficient; K = friction factor based on the friction formula used; Q_i = discharge in the ith segment of the lateral; D = internal diameter of the lateral; n = exponent of the diameter term in the friction formula used; L_i= length of the i segments of lateral; and F_i = Christiansen's friction correction factor for i outlets. Substituting (16) in (15)

$$H_{f_{AVG,\,x}} = \frac{1}{N}\sum_{i=1}^{N-1} \frac{CKQ_i^m}{D^n} L_i \, F_i \tag{17}$$

but

$$L_i = il \tag{18}$$

and

$$Q_i = iq \tag{19}$$

where the outlet discharge and is given by

$$q = \frac{Q}{N} \tag{20}$$

where Q = discharge at the inlet of the lateral. Substituting for q from (20) in (19)

$$Q_i = \frac{iQ}{N} \tag{21}$$

Substituting for l from (11) in (18)

$$L_i = i\left(\frac{L}{N+x-1}\right) \tag{22}$$

Substituting for Q_i from (21) and L_i from (22) in (17)

$$H_{f_{AVG,\,x}} = \frac{CKQ^m}{D^n} \frac{1}{N^m} \frac{L}{N+x-1} \frac{1}{N} \sum_{i=1}^{N-1} F_i \; i^{m+1} \tag{23}$$

The friction head loss in a lateral where the most upstream outlet is at a fraction of a full spacing from the inlet is given by Scaloppi (1988) as

$$H_{f_x} = \frac{CKQ^m L}{D^n} F_a \tag{24}$$

where F_a = adjusted friction correction factor. Defining the adjusted average correction factor for laterals without downstream outflow as

$$F_{a_{AVG}} = \frac{H_{f_{AVG,\,x}}}{H_{f_x}} \tag{25}$$

where $F_{a_{AVG}}$ = adjusted average correction factor. Substituting for $H_{f_{AVG,\,x}}$ from (23) and H_{f_x} from (24) in (25)

$$F_{a_{AVG}} = \frac{1}{F_a}\frac{1}{N^{m+1}(N+x-1)}\sum_{i=1}^{N-1} F_i\, i^{m+1} \tag{26}$$

Scaloppi (1988) derived the following expression for the adjusted friction correction factor:

$$F_a = \frac{NF_N+x-1}{N+x-1} \tag{27}$$

where F_N = Christiansen's friction correction factor for N outlets. DeTar (1982) showed that the friction correction factor developed by Christiansen (1942) can be closely approximated by

$$F_i = \frac{1}{i^{m+1}}\sum_{j=1}^{i} j^{m} \tag{28}$$

where j = integer 1,2,3.....i representing the segment number. For N outlets along a lateral (28) becomes

$$F_N = \frac{1}{N^{m+1}}\sum_{i=1}^{N} i^{m} \tag{29}$$

Substituting for F_N from (29) in (27)

$$F_a = \frac{\frac{1}{N^{m}}\sum_{i=1}^{N} i^{m}+x-1}{N+x-1} \tag{30}$$

Substituting for F_i from (28) F_a from (30) in (26)

$$F_{a_{AVG}} = \frac{(N+x-1)}{\frac{1}{N^{m}}\sum_{i=1}^{N} i^{m}+x-1}\times\frac{1}{N^{m+1}(N+x-1)}\sum_{i=1}^{N-1} i^{m+1}\times\frac{1}{i^{m+1}}\sum_{j=1}^{i} j^{m} \tag{31}$$

which can be simplified to

$$F_{a_{AVG}} = \frac{1}{N} \frac{\sum_{i=1}^{N} (N-i) i^{m}}{\sum_{i=1}^{N} i^{m} + (x-1) N^{m}} \tag{32}$$

For $x = 1$, (32) becomes identical to (5) i.e. $F_{a_{AVG}} = F_{AVG}$.

If the Darcy-Weisbach friction formula is used then $m = 2.00$. Fig.2 shows the variation of the adjusted average correction factor calculated using (32) for various values of x. For a relatively large number of outlets the adjusted average correction factor approaches 0.25, alternatively $\left(1 - F_{a_{AVG}}\right)$ approaches 0.75. Scaloppi (1988) reports similar findings for the adjusted friction correction factor. Scaloppi (1988) suggested that the discrepancy between the adjusted friction correction factor and the friction correction factor is negligible for laterals with more than about ten outlets. The same is valid for the adjusted average correction factor. Equation (32) can also be written as

$$F_{a_{AVG}} = \left(\frac{N F_{N}}{N F_{N} + x - 1} \right) F_{AVG} \tag{33}$$

Adjusted Average Correction Factor $G_{a\ AVG}$ for a Single Diameter Lateral with Multiple Outlets, with Outflow at the Downstream End of the Lateral.

Fig.3 shows a lateral with multiple equally spaced outlets and outflow at the downstream end. The most upstream outlet is a fraction of a full spacing from the lateral inlet. The length of this lateral is given by (10). For a lateral with i segments, Anwar (1999a) expressed the friction head loss as

$$H_{f_i} = \frac{C K Q_i}{D^{n}} L_i \; G_{i,\, r_i} \tag{34}$$

where $G_{i,\, r_i}$ = friction correction factor for a lateral with i outlets and an outflow ratio of r_i.

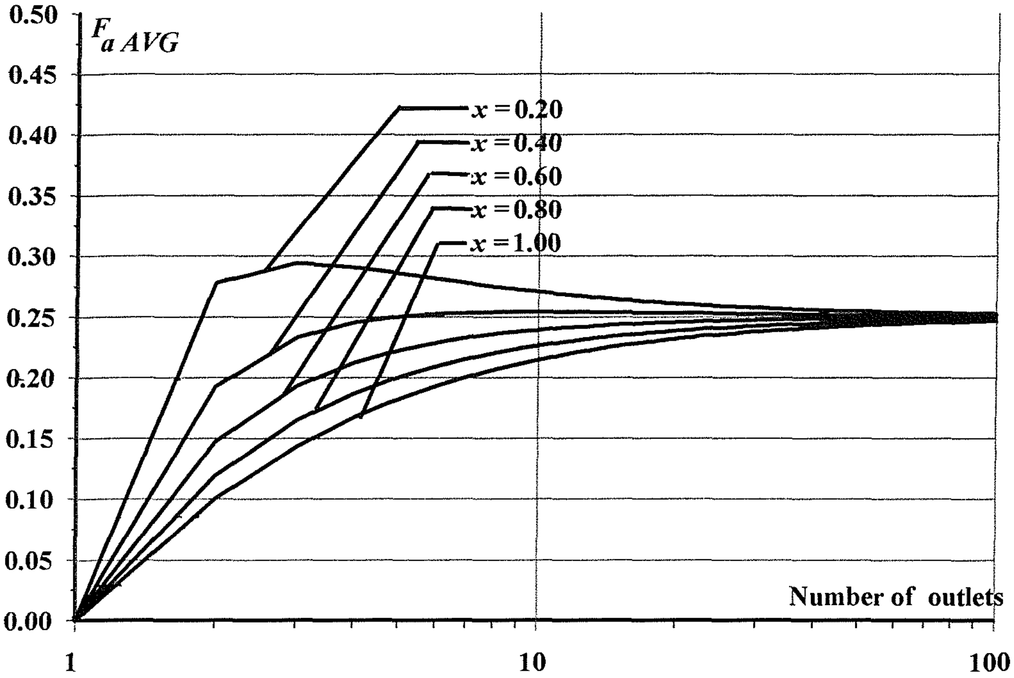

FIG. 2 Adjusted average correction factor F_{aAVG}

The outflow ratio is defined as

$$r_i = \frac{Q_O}{iq} \tag{35}$$

where r_i = outflow ratio for i outlets. Anwar (1999a) developed the following expression for the friction correction factor

$$G_{i,\, r_i} = \frac{1}{i^{m+1}(1+r_i)^m} \sum_{j=1}^{i} (j+ir_i)^m \tag{36}$$

For a lateral with outflow at the downstream end

$$Q_i = iq + Q_O \tag{37}$$

or, substituting for Q_O from (35) in (37)

$$Q_i = iq(1+r_i) \tag{38}$$

Similarly, for a lateral with N outlets

$$Q_I = Nq(1+r_N) \tag{39}$$

where Q_I = inflow in the lateral.

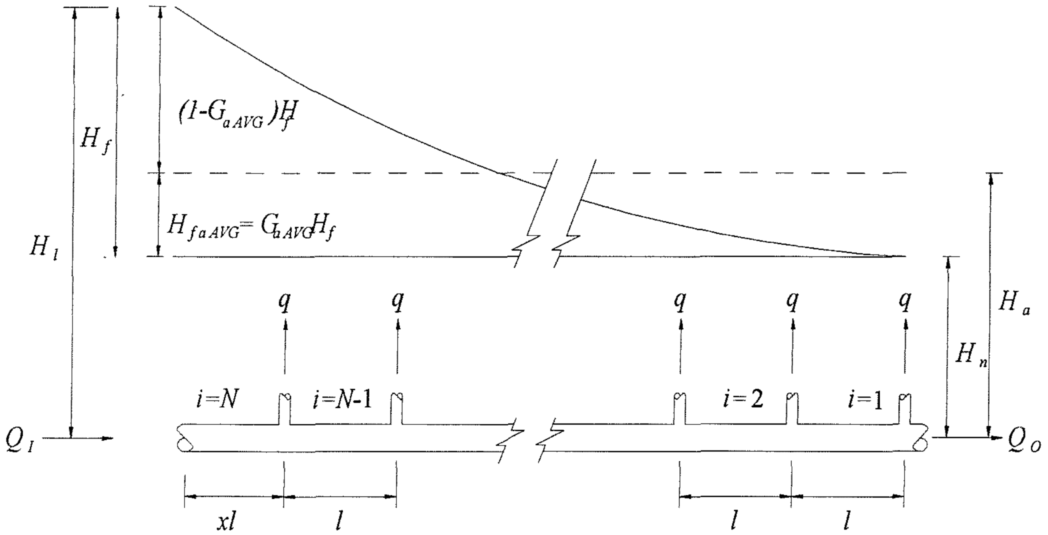

FIG.3. Lateral with Multiple Outlets and Downstream Outflow

Rearranging (39)

$$q = \frac{Q_I}{N(1+r_N)} \tag{40}$$

Substituting for q from (40) in (38)

$$Q_i = \frac{iQ_I}{N}\frac{(1+r_N)}{(1+r_i)} \tag{41}$$

Substituting for L_i from (22) and Q_i from (41) in (34)

$$H_{f_i} = \frac{CKQ_I^m}{D^n}L\frac{1}{N^m(1+r_N)^m}\frac{i^{m+1}(1+r_i)^m}{(N-1+x)}G_{i,r_i} \tag{42}$$

Substituting for H_{f_i} from (42) in (15)

$$H_{f_{AVG,\,x}} = \frac{CKQ_I^m}{D^n}L\frac{1}{N^{m+1}(1+r_N)^m\ (N-1+x)}\sum_{i=1}^{N-1} i^{m+1}(1+r_i)^m G_{i,r_i} \tag{43}$$

The friction head loss in a lateral with multiple outlets, outflow at the downstream end and with the most upstream outlet at a fraction of a full outlet spacing from the lateral inlet can be written as

$$H_f = \frac{CKQ_I^m}{D^n}G_{a,\,r_N,\,x} \tag{44}$$

where $G_{a,\,r_N,\,x}$ = adjusted friction correction factor for the lateral with an outflow ratio of r_N and first outlet fractional length x. This factor was developed by Anwar (1999b) and is given by

$$G_{a,\, r_N,\, x} = \frac{N G_{N,\, r_N} + x - 1}{(N+x-1)} \tag{45}$$

where $G_{N,\, r_N,\, x}$= friction correction factor for a lateral with N outlets, an outflow ratio of r_N and first outlet fractional length x. Substituting for $G_{N,\, r_N,\, x}$ appropriately from (36) in (45)

$$G_{a,\, r_N,\, x} = \frac{\dfrac{1}{N^m(1+r_N)^m} \displaystyle\sum_{i=1}^{N} (i+Nr_N)^m + x - 1}{(N+x-1)} \tag{46}$$

Analogous to (6), the adjusted average correction factor for laterals with multiple outlets, outflow at the downstream end and the first outlet at a fraction of a full outlet spacing from the lateral inlet can be written as

$$G_{a\,AVG} = \frac{H_{f_{AVG}}}{H_f} \tag{47}$$

where $G_{a\,AVG}$ = adjusted average correction factor for laterals with outflow. Substituting (43) and (44) in (47)

$$G_{a\,AVG} = \frac{1}{G_{a,\, r_N,\, x}} \frac{1}{N^{m+1}(1+r_N)^m\ (N+x-1)} \sum_{i=1}^{N-1} i^{m+1}(1+r_i)^m G_{i,r_i} \tag{48}$$

Substituting (36) and (46) in (48)

$$G_{a\,AVG} = \frac{(N-1+x)}{\dfrac{1}{N^m(1+r_N)^m} \displaystyle\sum_{i=1}^{N} (i+Nr_N)^m + x - 1} \frac{1}{N^{m+1}(1+r_N)^m\ (N-1+x)} \times \sum_{i=1}^{N-1} i^{m+1}(1+r_i)^m \frac{1}{i^{m+1}(1+r_i)^m} \sum_{j=1}^{i} (j+ir_i)^m \tag{49}$$

From (9) and (35)

$$r_i = \frac{Nr_N}{i} \tag{50}$$

Substituting (50) in (49) and simplifying

$$G_{a\ AVG} = \frac{1}{N}\ \frac{\sum_{i=1}^{N} (N-i)(i+Nr_N)^m}{\sum_{i=1}^{N} (i+Nr_N)^m + (x-1)N^m(1+r_N)^m} \tag{51}$$

Eq.(51) can also be written as

$$G_{a\ AVG} = \left(\frac{NG_N}{NG_N + x - 1} \right) G_{AVG} \tag{52}$$

For a lateral without outflow at the downstream end i.e. $r_N = 0$, (51) reduces to (32) i.e. for $r_N = 0$, $G_{aAVG} = F_{aAVG}$. For a lateral with the most upstream outlet at a full outlet spacing from the inlet i.e. x=1, (51) reduces to (8) i.e. for x=1, $G_{aAVG} = G_{AVG}$. Finally if both $r_N = 0$ and x=1, then (51) reduces to (5) i.e. for $r_N = 0$ and x=1, $G_{a\ AVG} = F_{AVG}$. Therefore analogous to the adjusted friction correction factor G_a, the adjusted average correction factor is a generic correction factor that reduces to more specific correction factors under particular conditions.

Fig 4 shows the adjusted average correction factor for outflow ratios of; 0.2, 0.4, 0.6 and 0.8 developed with a simple computer program using (51). The effect of the first outlet fractional length x decreases with increasing number of outlets. For a large number of outlets the adjusted average correction factor approaches the adjusted correction factor. Fig.4 can be used as a design chart as demonstrated in the subsequent section.

PRACTICAL APPLICATION

A numerical example is used to demonstrate application of the adjusted average correction factor. In the example presented, the average pressure head, and subsequently the lateral inlet pressure head for a tapered sprinkler lateral is estimated. For tapered laterals, the average head is taken as the length weighted average head of each reach as developed by Anwar (2000).

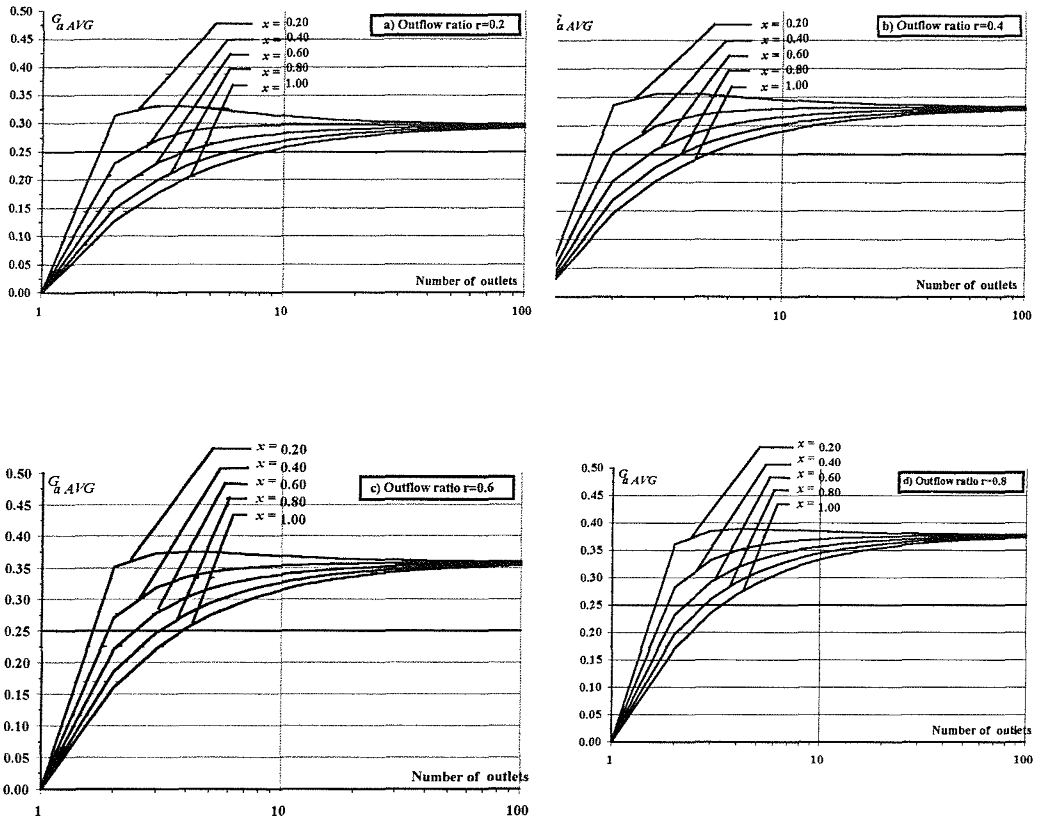

FIG.4 Adjusted average correction factor G_{aAVG}

Example

A horizontal tapered aluminium sprinkler lateral is 285 m long with outlets at 12 m intervals. The most upstream outlet is at 9 m from the lateral inlet. The upstream reach is 141 m long and has an internal diameter of 100 mm. The downstream reach of the lateral has a diameter of 75 mm. The lateral is to be designed for an average outlet discharge of 0.5 L/s operating under an average head of 35 m.

The lateral relative roughness is assumed as 0.127 mm and for water at 15°C, kinematic viscosity of water, 1.14×10^{-6} m^2/s. The Churchill equation, Churchill (1977) will be used to calculate the friction factor K

Solution

***Reach* 1**

$Q_{o_1} = 0.0$ L/s

From (35) $r_1 = 0$

From (39) $Q_{I_1} = 12 \times 0.5 = 6.0$ L/s

From the Churchill equation, $K_1 = 0.0247$

From Anwar (1999a) for $N = 12$, $r_1 = 0$ $G_{a_1} = 0.376$

Using the Darcy-Weisbach equation and factor G_{a_1} to calculate friction head loss in a lateral with outlets

$$H_{f_{L1}} = \frac{0.0826K_1Q_{I_1}^2L_1}{D^5}G_1 = \frac{0.0826\times0.0247\times0.006^2\times144}{0.075^5}\times0.376$$

$H_{f_{L1}} = 1.68$ m

From (51), for $N = 12$, $r_1 = 0$, x=1.00 $G_{a\,AVG_1} = 0.220$

***Reach* 2**

$Q_{o_2} = 6.0$ L/s

From (35) $r_2 = 1.0$

From (39) $Q_{I_2} = 12\times0.50(1+1)$ $Q_{I_2} = 12.00$ L/s

From the Churchill equation, $K_2 = 0.0227$

From, Anwar (1999b) for $N = 12$, $r_2 = 1.0$, $G_{a_2} = 0.607$

x=0.75

Using the Darcy-Weisbach equation and factor G_{a_2} to calculate friction head loss in a lateral with outlets

$$H_{f_{L2}} = \frac{0.0826K_2Q_{I_2}^2L_2}{D^5}G_{a_2} = \frac{0.0826\times0.0227\times0.012^2\times141}{0.100^5}\times0.607$$

$H_{f_{L2}} = 2.32$ m

From (51), for $N = 12$, $r_2 = 1.0, x = 0$. $G_{a\,AVG_2} = 0.367$

$H_{f_{L1}} + H_{f_{L2}} = 1.68 + 2.32 = 4.00 \text{ m} \approx 11.5\%\, H_a$

Since the lateral is horizontal, the friction head loss is also the maximum pressure head variation along the lateral. This is less than 20% therefore friction head loss is not excessive and (15) can be considered valid. From Anwar (2000)

$$H_{L2} = (1.68+2.32) + 35 - \frac{144(1.68\times0.220) + 141(1.68+2.32\times0.367)}{(144+141)}$$

H_{L2} = **37.60 m**

Solving the same problem using the back-step method and performing an iterative calculation, the inlet pressure is estimated to be 37.47 m.

SUMMARY AND CONCLUSION

This paper presents a generic adjusted average correction factor, that can be used to determine the inlet pressure for sprinkler laterals. The factor developed can be used whether or not the first sprinkler is at a full outlet spacing from the lateral inlet. Furthermore it can also be used for laterals with or without outflow at the downstream end. The expression for the adjusted average correction is explicit and is easily determined using a programmable calculator or spreadsheet. Use of this factor is demonstrated through a simple example of a tapered lateral with two reaches, however this can equally be applied to laterals with more than two reaches.

The expressions developed here need to be used appropriately for laterals on slopes. For laterals on slopes the pressure head variation is the sum of friction head losses and change of elevation (ignoring velocity head). It is important to recognize that this total pressure head variation along the lateral should not exceed approximately 20% of the operating head before the expressions developed in this paper are applied.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

C = units coefficient;

D = internal diameter of the lateral;

F_{AVG} = average correction factor;

F_N = Christiansen's friction correction factor for N outlets;

F_{aAVG} = adjusted average correction factor;

F_a = adjusted friction correction factor;

F_i = Christiansen's friction correction factor for i outlets;

G_{a_1} = adjusted friction correction factor for reach 1;

G_{a_2} = adjusted friction correction factor for reach 2;

$G_{a,\, r_N,\, x}$ = adjusted friction correction factor for the lateral, an outflow ratio of r_N and first outlet fractional length x;

$G_{i,\, r_i}$ = friction correction factor for a lateral with i outlets, and an outflow ratio of r_i;

$G_{N,\, r_N,\, x}$ = friction correction factor for a lateral with N outlets, an outflow ratio of r_N and first outlet fractional length x;

G_{AVG} = average correction factor for laterals with outflow;

G_{aAVG} = adjusted average correction factor for laterals with outflow;

$G_{a\,AVG_1}$ = adjusted average correction factor for reach 1 of the lateral;

$G_{a\,AVG_2}$ = adjusted average correction factor G for reach 2 of the lateral;

H_a = head required to produce average discharge at an outlet;

H_a^* = average of the head at each outlet
H_f = friction head loss;
$H_{f_{AVG}}$ = average of the friction head loss at each outlet;
$H_{f_{AVG,\,x}}$ = average of the friction head loss at each outlet with first outlet fraction x;
H_{f_1} = friction head loss in the 1st segment of the lateral;
H_{f_2} = friction head loss in the 1st and 2nd segments;
$H_{f_{L1}}$ = friction head loss in the reach 1;
$H_{f_{L2}}$ = friction head loss in the reach 2;
H_{f_i} = friction head loss in the 1st, 2nd, ... i th segment;
$H_{f_{N-1}}$ = friction head loss in the 1st, 2nd, ... N-1 th segment;
H_n = minimum head at the downstream end of the lateral;
H_l = head at the inlet of the lateral;
H_{L2} = head at the inlet of reach 2 of the lateral;
H_v = velocity head;
H_z = change in elevation from beginning to end of the lateral;;
i = integer 1,2,3...N representing segment or outlet number;
j = integer 1,2,3..i representing reach number;
K = friction factor based on friction formula used;
K_1 = friction factor for reach 1;
K_2 = friction factor for reach 2;
L = total length of the lateral;
L_i = length of i segments of the lateral;
L_1 = total length of reach 1;
L_2 = total length of reach 2;
l = length of each segment of the lateral;
m = exponent of the discharge term in the friction formula used;
n = exponent of the diameter term in the friction formula used;
N = number of outlets along the lateral;
Q = discharge at the inlet of the lateral;
Q_I = inflow in the lateral;
Q_i = discharge in the i th segment of the lateral;
Q_{I_1} = inlet discharge for reach 1 of the lateral;
Q_{I_2} = inlet discharge for reach 2 of the lateral;
Q_O = outflow from the lateral;
Q_{O_1} = outflow from reach 1 of the lateral;
Q_{O_2} = outflow from for reach 2 of the lateral;
q = outlet discharge;
r_i = outflow ratio for i outlets;
r_N = outflow ratio for N outlets;
r_1 = ratio of outflow discharge to total outlet discharge for reach 1;
r_2 = ratio of outflow discharge to total outlet discharge for reach 2; and,
x = distance between most upstream outlet and lateral inlet, expressed as a fraction of a full outlet spacing.

5

FRICTION CORRECTION FACTORS FOR CENTER-PIVOTS

FRICTION CORRECTION FACTORS FOR CENTER-PIVOTS

by Arif A. Anwar[1]

ABSTRACT: Analytical equations for friction correction factors for center-pivot laterals without end guns are developed. This work illustrates a discrepancy when earlier equations are applied to center-pivots with small numbers of outlets. Earlier equations were also limited to center-pivots with constant outlet spacing. Equations presented in the current work are developed for center-pivots with constant outlet spacing and also for center-pivots with constant outlet discharge. When the equations developed in the current work are applied to center-pivots with a large number of outlets, the results are in good agreement with previous work for center-pivot laterals with an infinite number of outlets. When applied to smaller number of outlets the equations presented here provide a more precise estimate of the friction correction factor. Using the current equations, the friction correction factor for center-pivots with constant outlet spacing was found to be very similar to the friction correction factor for center-pivots with constant outlet discharge. Useful simple equations are also presented for calculating the discharge of each outlet or for calculating the spacing between outlets.

INTRODUCTION

When water flows through a lateral pipeline with multiple outlets the head loss caused by friction is less than that of an equivalent pipeline without outlets caused by the decreasing discharge in the lateral with outlets. To compute the head

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loss caused by friction through a lateral requires a stepwise analysis starting from the downstream-most outlet, moving upstream and computing the head loss in each segment. Christiansen (1942) developed a friction correction factor *F*, which allows a more direct computation of head loss caused by friction in a lateral. Several subsequent improvements have been made more notably by Jensen and Fratini (1957) and Scaloppi (198).

Factor *F* (Christiansen 1942) and its improvements can only be applied to fixed, periodic move or linear move irrigation systems, where the discharge in the lateral decreases linearly with length. In center-pivot systems, the lateral moves in a circular manner about a pivot. For any rotation of the center-pivot, the outer end of the lateral must irrigate a greater area than the inner (pivot) end of the lateral, i.e. the discharge in the lateral does not decrease linearly with length of the lateral. Chu (1980) described three methods of achieving this non linear decrease in discharge in a center-pivot lateral:

1. A constant spacing system: Whereby the lateral has outlets at a constant spacing but the outlet discharge increases towards the outer (moving) end of the lateral.
2. A constant discharge system: Whereby the lateral has outlets of constant discharge but the outlet spacing decreases towards the outer (moving) end of the lateral.
3. A spray nozzle system: This system is identical to the constant discharge system except that spray nozzles are used instead of sprinklers. Spray nozzle systems shall not be considered further in this work.

Center-pivot laterals with a constant spacing of outlets or a constant discharge of outlets represent two ends of the design spectrum. Actual installations may consist of a mix of the two extremes.

Kincaid and Heermann (1970) described a stepwise computational process to determine the head loss caused by friction in a center-pivot lateral. Chu and Moe

(1972) developed a friction correction factor for center-pivots. Chu and Moe (1972) approximated both constant spacing systems and constant discharge systems as described above, by a lateral with an infinite number of tiny sprinklers. The area irrigated by any one sprinkler for one revolution of the center-pivot is given by Chu and Moe (1972) as

$$dA = 2\pi r \; dr \tag{1}$$

where dA = area irrigated by the sprinkler; r = radial distance between the sprinkler and the pivot; and dr = the infinitesimal spacing between adjacent sprinklers. Based on this assumption, Chu and Moe (1972) developed the following friction correction factor for center-pivot laterals

$$F_{cp} = \frac{1}{2} \, \beta(m+1,0.5) \tag{2}$$

where F_{cp} = friction correction factor for center-pivots (Chu and Moe 1972); β = beta function; and m = velocity exponent in the friction equation used. Eq. (2) does not contain any term referring to the number of outlets on the center-pivot lateral, because it assumes an infinite number of outlets. Scaloppi and Allen (1993b) arrived at a similar friction correction factor as (2), assuming an infinite number of outlets. Reddy and Apolayo (1988) developed the following friction correction factor for center-pivots for a finite number of outlets. Using the Hazen-Williams formula where m=1.852

$$F_{cp}(N) = \frac{1}{N} + \sum_{i=2}^{N} \frac{1}{N} \left[1 - \frac{2}{N^2} \sum_{j=1}^{i-1} j \right]^{1.852} \tag{3}$$

where $F_{cp}(N)$ = friction correction factor for center-pivots with N outlets; N = number of outlets along lateral (the 1st outlet is that closest to the pivot); i = integer (2,3,4....N); and j = integer (1,..., i-1). In developing (3), Reddy and Apolayo (1988) assumed a constant outlet spacing, and also that

$$q_i = \frac{2Q \, r_i \, l}{L^2} \tag{4}$$

where q_i = discharge of the i th outlet ; Q = discharge into the center-pivot lateral at the pivot end; r_i = radial distance of the i th outlet from the pivot; l = spacing of the outlets (sprinklers); and L = length of the center-pivot lateral, with

$$r_i = i\,l \tag{5}$$

$$l = \frac{L}{N} \tag{6}$$

Eq. (4) was developed by Chu (1980) with the assumptions of (1), and also that the depth of water applied over the circular area irrigated by the center-pivot is uniform.

Although (4) can be considered valid for an infinite number of very small outlets, it is inaccurate for a small finite number of outlets. To demonstrate this point, consider the extreme case of two outlets (N=2) on a center-pivot lateral of length L and total discharge into the lateral at the pivot Q.

From (6), the spacing l between the two outlets is given by $L/N = L/2$. From (5), for the 1st outlet (i.e., the one closest to the pivot); $r_1 = L/2$, and from (4), $q_1 = Q/2$. Similarly, for the 2nd outlet (the one at the downstream end), $r_2 = L$ and again from (4), $q_2 = Q$, then

$$\sum_{i=1}^{N} q_i = \frac{3}{2}Q \neq Q \tag{7}$$

In the present work, the anomaly of (7) is avoided by assuming that the i th outlet applies water to the irrigated area between the i-1 th outlet and the i th outlet, (the i-1 th outlet is the outlet immediately upstream of the i th outlet). Therefore, the width of the circle irrigated by the i th sprinkler is given by $r_i - r_{i-1}$, where; r_i and r_{i-1} are the radial distances of the i th and i-1 th outlet from the pivot, respectively. For a finite number of relatively large outlets, (1) becomes

$$A_i = \pi\left(r_i^2 - r_{i-1}^2\right) \tag{8}$$

where A_i= area irrigated by the i th outlet Eq. (4) then becomes

$$q_i = \frac{Q(r_i^2 - r_{i-1}^2)}{L^2} \tag{9}$$

Eq. (9) is used for the extreme case of a center-pivot with only two outlets $q_1 = Q/4$ and $q_2 = 3Q/4$. Although (9) is only applicable to center-pivots with a constant outlet spacing (and varied outlet discharge), as defined by (6), the present work also investigates center-pivots with constant outlet discharge (and varied outlet spacing). Throughout this work, as with the original work on friction correction factors by Chu and Moe (1972), the velocity head is neglected. This assumption has been criticized by Smith (1990), particularly for low-pressure pipelines. However, for the typical operating pressures of center-pivots this assumption has been shown to lead to a maximum deviation of less than 1.2% (Scaloppi and Allen 1993a). The increase in pressure head caused by the gradual reduction of the velocity head as the flow in the lateral decreases past each outlet is assumed to be equal to head loss caused by turbulence at each outlet (Pair et al. 1975).

ANALYSIS

Outlets with Constant Spacing and Varied Discharge

Fig. 1 shows a center-pivot lateral of constant diameter, with N outlets along its length and without an end-gun. The outlets are numbered from the upstream end, i.e., the first outlet is that adjacent to the pivot. The discharge at each outlet is $q_1, q_2 \ldots q_{i-1}, q_i \ldots q_N$ representing the discharge from the 1st, 2nd, ...(i -1)th, i th.....and N th outlet, respectively. The outlet spacing is given by (6).

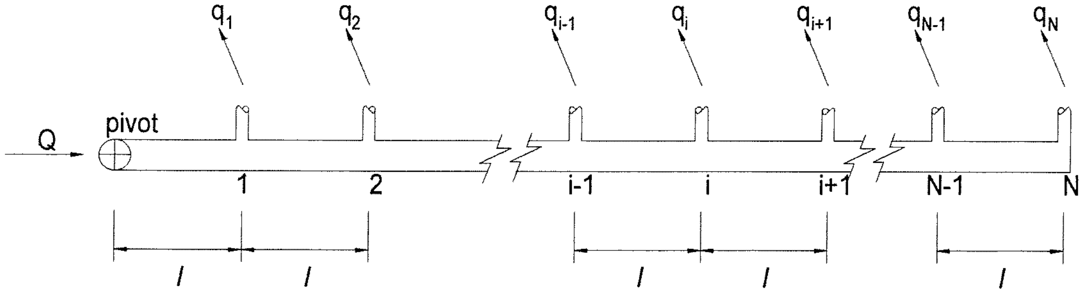

FIG. 1. Center-pivot Lateral with Constant Outlet Spacing and Varied Outlet Discharge

The head loss caused by friction in the i th segment can be written as (Christiansen 1942)

$$h_{fi} = \frac{CKQ_i^m l}{D^{2m+n}} \tag{10}$$

where h_{fi}= head loss caused by friction in the i th segment; C= friction factor based on the friction equation used; K= units coefficient; Q_i= discharge in the i th segment of the lateral; D= internal pipe diameter; and m and n = exponents of the average flow velocity in the pipeline and internal pipe diameter, respectively, which in turn depend on the friction formula used. For the Darcy-Weisbach equation, $m = 2$ and $n = 1$.The total head loss caused by friction in the lateral is given by the summation of head loss in each segment. From (10)

$$H_f = \sum_{i=1}^{N} \frac{CKQ_i^m l}{D^{2m+n}} \tag{11}$$

where H_f = total head loss caused by friction in a center-pivot lateral with constant outlet spacing. Substituting (6) in (11) and rearranging

$$H_f = \frac{CKL}{D^{2m+n}} \frac{1}{N} \sum_{i=1}^{N} Q_i^m \tag{12}$$

Based on the assumptions used in developing (9)

$$\frac{Q-Q_i}{\pi\ [(i-1)l]^2} = \frac{Q}{\pi[Nl]^2} \tag{13}$$

or

$$Q_i = Q\left[1-\left(\frac{i-1}{N}\right)^2\right] \tag{14}$$

Since

$$q_{i1} = Q_i - Q_{i+1} \tag{15}$$

where q_{i1}= discharge of the i th outlet - the first outlet closest to the pivot; Q_i= discharge in the i th segment of the lateral; and Q_{i+1} = discharge in the i+1 th segment of the lateral. Substituting for Q_i from (14) and Q_{i+1} appropriately from (14) in (15) gives

$$q_{i1} = \frac{Q}{N^2}(2i-1) \tag{16}$$

Eq. (16) gives the discharge of any outlet along the center-pivot lateral. It is essentially identical to (9) derived earlier. Substituting (14) in (12)

$$H_f = \frac{CKQ^m L}{D^{2m+n}} \frac{1}{N^{2m+1}} \sum_{i=1}^{N} \left[N^2-(i-1)^2\right]^m \tag{17}$$

alternatively, (17) may be written in the form given by Reddy and Apolayo (1988) as

$$H_f = \frac{CKQ^m L}{D^{2m+n}} F_{cp^1} \tag{18}$$

in which

$$F_{cp^1} = \frac{1}{N^{2m+1}} \sum_{i=1}^{N} \left[N^2-(i-1)^2\right]^m \tag{19}$$

where F_{cp^1} = friction correction factor for center-pivot irrigation with constant spacing (and varied outlet discharge). If the outlets are numbered from the downstream end, then

$$F_{cp^2} = \frac{1}{N^{2m+1}} \sum_{i=1}^{N} (2N-i)^m \; i^m \tag{20}$$

where F_{cp^2}= friction correction factor for center-pivot irrigation with constant spacing (and varied outlet discharge) - outlets numbered from the downstream end, and

$$q_{i^2} = \frac{Q}{N^2}(2N-2i+1) \tag{21}$$

where:q_{i^2} = discharge of the i th outlet - outlets numbered from the downstream end. It can be demonstrated that for N any integer value ≥ 1 replacing i in either (19) or (20) with (N-i+1)

$$\frac{1}{N^{2m+1}} \sum_{i=1}^{N} \left[N^2-(i-1)^2\right]^m \equiv \frac{1}{N^{2m+1}} \sum_{i=1}^{N} (2N-i)^m \; i^m \tag{22}$$

and also from (16) and (21)

$$\frac{Q}{N^2}(2i-1) \equiv \frac{Q}{N^2}(2N-2i+1) \tag{23}$$

Table 1 shows values for friction factors calculated using (19) or (20) for m=1.852 and compared with values given by (3), (Reddy and Apolayo 1988). The fifth column in Table 1 shows values given by Keller and Bliesner (1990). The authors of the latter work determined these friction correction factors using a stepwise iterative computational process. Keller and Bliesner (1990) do not specify whether they assumed that turblence losses are equal to velocity head increase along the lateral. Nor do they specify whether their correction factors have been calculated for center-pivots with a constant outlet spacing or for center-pivots with a constant outlet discharge. However the subsequent section of this paper will illustrate that friction correction factors are almost identical for either arrangement of outlets.

Eq. (19) and (20) give identical results. These also approximate the values given by Keller and Bliesner (1990) more closely than values given by equations developed by Reddy and Apolayo (1988), particularly for small values of N. For large values of N, (19) and (20) approach the function $0.5\ \beta(m+1,0.5)$ (Chu and Moe 1972); Scaloppi and Allen (1993a) developed for an infinite number of very small sprinklers along a center-pivot lateral. Estimates of the friction correction factor using (3), (Reddy and Apolayo 1988) also approach this function as the inherent assumptions in the latter work are the same as those of Chu and Moe (1972).

TABLE 1: Friction Correction Factor for Center-Pivots,Outlets with Constant Spacing

Number of outlets	Friction correction factor			
	$F_{cp}{}^1$	$F_{cp}{}^2$	$F_{cp}(N)$ Reddy and Apolayo (1988)	Keller and Bliesner (1990)
(1)	(2)	(3)	(4)	(5)
1	1.000	1.000	1.000	1.000
2	0.793	0.793	0.638	0.790
3	0.713	0.713	0.586	0.710
4	0.673	0.673	0.569	0.670
5	0.648	0.648	0.561	0.650
6	0.631	0.631	0.557	0.630
7	0.620	0.620	0.555	0.620
8	0.611	0.611	0.553	0.610
9	0.604	0.604	0.552	0.598
10	0.598	0.598	0.551	-

Outlets with Constant Discharge and Varied Spacing

Fig.2 shows a center-pivot lateral with N outlets along its length at varied spacing. The lateral does not have any end-gun attached at the downstream end. Outlets are numbered from the upstream end. The inflow into the center-pivot lateral is given by

$$Q = Nq \tag{24}$$

where Q = discharge into the lateral at the pivot end; N = number of outlets along the lateral; and q = discharge of each outlet.

Similarly, in the i th segment, the discharge is given by

$$Q_i = [N-(i-1)]q \tag{25}$$

where Q_i = discharge in the i th segment of the lateral; and i = an integer (1,2,3.....N).

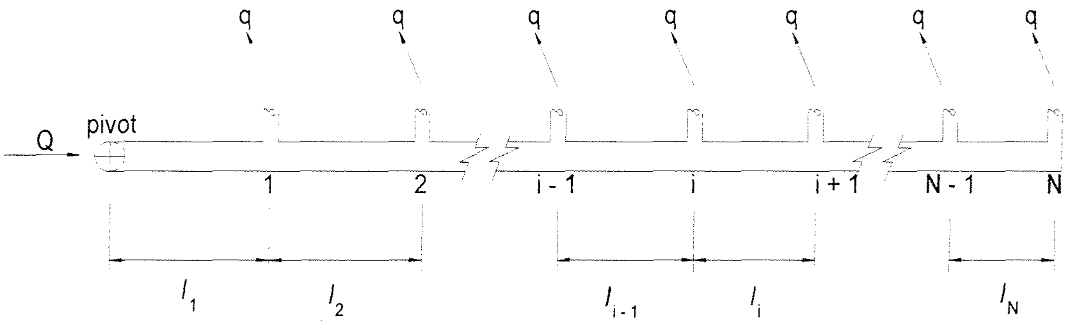

FIG. 2. Center-Pivot Lateral with Constant Outlet Discharge and Varied Outlet Spacing

The head loss caused by friction in the ith segment Christiansen (1942) can be written as

$$h_{fi} = \frac{CKQ_i^{\ m} l_i}{D^{2m+n}} \tag{26}$$

where l_i = length of the i th segment of the lateral. The total head loss in the lateral is therefore

$$H_{f'} = \sum_{i=1}^{N} \frac{CKQ_i^{\ m} l_i}{D^{2m+n}} \tag{27}$$

where H_f' = total head loss caused by friction in center-pivot lateral with constant outlet discharge. Substituting (24) and (25) in (27)

$$H_{f'} = \frac{CKQ^m}{D^{2m+n}} \frac{1}{N^m} \sum_{i=1}^{N} (N-i+1)^m \; l_i \tag{28}$$

The length of the i th segment can also be expressed as

$$l_{i1} = r_i - r_{i-1} \tag{29}$$

where r_i and r_{i-1} = radial distance between the pivot and the i th and i-1 th outlet respectively. Based on the assumptions used to develop (9)

$$\frac{iq}{\pi r_i^2} = \frac{Nq}{\pi L^2} \tag{30}$$

or

$$r_i = L \sqrt{\frac{i}{N}} \tag{31}$$

likewise

$$r_{i-1} = L \sqrt{\frac{i-1}{N}} \tag{32}$$

Substituting (34) and (35) in (32), yields

$$l_{i1} = \frac{L}{\sqrt{N}} \left(\sqrt{i} - \sqrt{i-1}\right) \tag{33}$$

For a center-pivot with outlets with constant discharge and variable outlet spacing, (33) gives the length of the i th segment of the lateral. The first segment of the lateral is that between the pivot and the first outlet. Substituting (33) in (28) and rearranging

$$H_{f'} = \frac{CKQ^m}{D^{2m+n}} \frac{L}{N^{m+0.5}} \sum_{i=1}^{N} (N-i+1)^m \left(\sqrt{i} - \sqrt{i-1}\right) \tag{34}$$

Expressing in terms of friction correction factor for center pivots, (34) becomes

$$H_{f'} = \frac{CKQ^m L}{D^{2m+n}} F_{cp^3} \tag{35}$$

where F_{cp^3} = friction correction factor for center pivots with constant discharge outlets (and varied spacing) - outlets numbered from the upstream end, given by

$$F_{cp^3} = \frac{1}{N^{m+0.5}} \sum_{i=1}^{N} (N-i+1)^m \left(\sqrt{i} - \sqrt{i-1}\right) \tag{36}$$

If the outlets are numbered from the downstream end, then

$$l_{i2} = L\left[\sqrt{1-\left(\frac{i-1}{N}\right)} - \sqrt{1-\left(\frac{i}{N}\right)}\right] \tag{37}$$

where: l_{i2}= length of the ith segement of the lateral with outlets numbered from the downstream end, and

$$F_{cp4} = \frac{1}{N^{m+0.5}}\sum_{i=1}^{N} i^{m}\left(\sqrt{N-i+1} - \sqrt{N-i}\right) \tag{38}$$

where F_{cp4} = friction correction factor for center-pivots with constant discharge outlets (and varied spacing) with outlets numbered from the downstream end.

Table 2 compares the friction correction factor for center-pivots with constant outlet spacing as determined from (19) or (20) against the friction correction factor for center-pivots with constant outlet discharge as determined from (36) or (38), using m=1.852

TABLE 2: Comparison of Friction Correction Factors for m = 1.852

Number of outlets	Friction correction factor	
	With constant spacing	With constant discharge
(1)	(2)	(3)
1	1.000	1.000
2	0.793	0.788
3	0.714	0.714
4	0.673	0.676
5	0.648	0.652
6	0.631	0.636
7	0.620	0.625
8	0.611	0.616
9	0.604	0.609
10-19	0.598 - 0.574	0.603 - 0.578
20-29	0.573 - 0.565	0.577 - 0.568
30-39	0.565 - 0.561	0.568 - 0.563
40-49	0.561 - 0.558	0.563 - 0.560

Table 2 shows that there is an insignificant difference between the friction correction factor for center-pivots with constant outlet spacing and that for center-pivots with constant outlet discharge. However, the analysis of center-pivot laterals with constant outlet discharge yields useful expressions for estimating the spacing between outlets.

RESULTS AND DISCUSSION

Fig. 3 illustrates the friction correction factor for both constant outlet spacing and constant outlet discharge as developed in this work. These are compared against values given by Keller and Bliesner (1990) and also against (3) (Reddy and Apolayo 1988). There is good correlation between the expressions developed in this work and that given by Keller and Bliesner (1990). The expressions developed in the present work do not result in an anomaly, even when applied to the limiting condition of two outlets along the lateral.

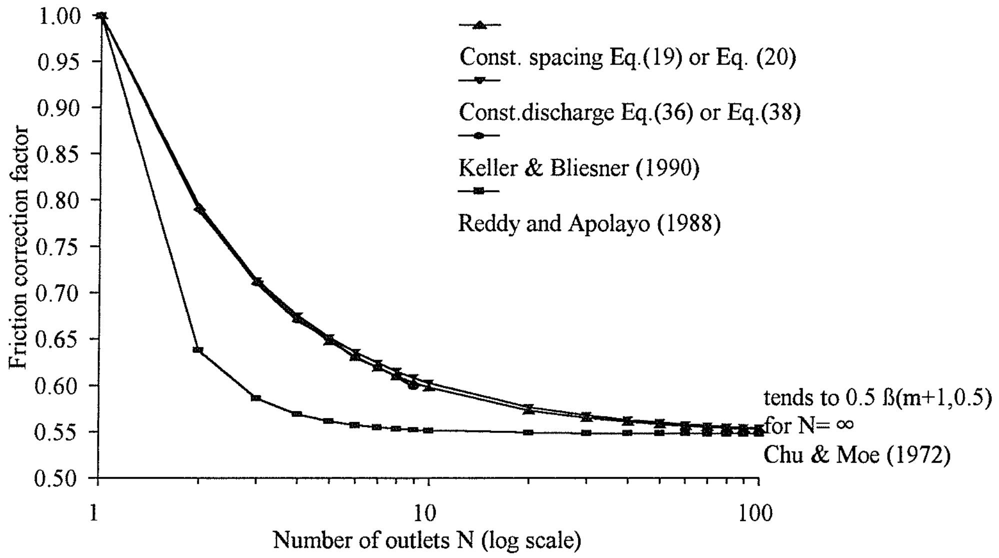

FIG.3. Comparison of Friction Correction Factors Using $m = 1.852$

Gilley (1989), in a discussion on the work by Reddy and Apolayo (1988) comments that the friction correction factor is independent of the number of sprinklers on the lateral and depends only on the conventional design criteria that the system apply a uniform depth of water over the length of the lateral. Gilley (1989) makes this comment based on (2) developed by Chu and Moe (1972). Although (2) does not contain any term referring to the number of sprinklers, in the development of the equation it has been assumed that there are an infinite number of tiny sprinklers. (2) only predicts the friction correction factor accurately for an infinite number of outlets. The accuracy of this estimate decreases as the number of sprinklers decreases (Scaloppi and Allen, 1993a,b).

Gilley (1989) points out that there would be at least eight sprinklers for even high-pressure systems, with most systems having at least 20 sprinklers and those with reduced pressure water application devices having over 200 sprinklers. Keller and Bliesner (1990) suggest center-pivots would have 73 outlets. Table 3 shows the friction correction factor assuming constant outlet spacing, i.e., using (19) or (20), and these are compared with estimates using (2) (Chu and Moe 1972). The comparison is made using different *m* values i.e,. using different friction formulas

TABLE 3: Comparison of Friction Correction Factor Estimates

Friction formula	*m*	Chu and Moe (1972)	Eq.(19) or Eq. (20)			
			$N = 8$	$N = 20$	$N = 73$	$N = 200$
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Darcy-Weisbach	2.000	0.533	0.596	0.558	0.540	0.536
Scobey	1.900	0.543	0.606	0.568	0.550	0.546
Hazen-Williams	1.852	0.548	0.611	0.573	0.555	0.550

A further comment made by Gilley (1989) in his discussion of the work by Reddy and Apolayo (1988) is that (2) by Chu & Moe (1972) is more general because it does not depend on a constant spacing of the outlets as assumed by Reddy and Apolayo (1988). Eqs. (19) and (20) also assume constant outlet spacing and may be criticized in this respect. However, in this work, equations were also developed for center-pivots with varied outlet spacing. These equations were shown to correlate closely to those developed for center-pivots with constant outlets spacing. The analysis of center-pivots with varied outlet spacing yields useful expressions to calculate the spacing between outlets.

It may be possible to expand (19) or (20) and (36) or (38) using the Euler-McLaurin summation formula (Spiegel 1968). This would remove the summation from these equations, allowing the friction correction factor to be calculated more directly. A similar approach has been used by DeTar (1982) in developing a friction correction factor for fixed/linear move laterals. Scaloppi and Allen (1993b) use a binomial expansion for a friction correction factor for center-pivots. Expansion has not been attempted in this work for two principle reasons:

- The Euler-McLaurin summation formula is an infinite series, which may become finite only under particular conditions, e.g., typically the expansion is finite only for m=2.00. For other values of *m* because higher terms of the expansion have to be neglected and the expansion becomes an approximation.
- It is unlikely that any expression developed by expansion using the Euler-McLaurin summation will be more concise than those presented in (19) or (20) and (36) or (38). Evaluation of these equations with their summation should not pose a problem, even with a simple programmable calculator. Alternatively, the friction correction factor can be expressed as an empirical hyperbolic equation of the form

$$F_{cp\ emp} = a + \frac{b}{N} \tag{39}$$

where $F_{cp\ emp}$ = empirical friction correction factor for center-pivots; *a* and *b* = empirical coefficients determined using nonlinear regression and are given in Table 4. These coefficients were determined from (19) and (36) for *N*=200. The coefficient of determination for this empirical equation is also shown in Table 4.

TABLE 4: Coefficients a and b Determined by Nonlinear Regression

Center pivots	Velocity Coefficient m for Friction Equation Used		
	2.00	1.900	1.852
(1)	(2)	(3)	(4)
With constant outlet spacing	$a = 0.5338$ $b = 0.4772$ $r^2_{coeff} = 0.9994$	$a = 0.5439$ $b = 0.4699$ $r^2_{coeff} = 0.9991$	$a = 0.5489$ $b = 0.4662$ $r^2_{coeff} = 0.9990$
With constant outlet discharge	$\mathrm{a} = 0.5356$ $b = 0.4812$ $r^2_{coeff} = 0.9982$	$\mathrm{a} = 0.5454$ $b = 0.4714$ $r^2_{coeff} = 0.9982$	$\mathrm{a} = 0.5503$ $b = 0.4666$ $r^2_{coeff} = 0.9981$

PRACTICAL APPLICATION

A numerical example is used to illustrate the application of the equations presented in this paper for calculating the head loss caused by friction in a center-pivot lateral. The equations developed will also be used to calculate the discharge of each outlet, for a constant spacing center-pivot lateral, and for comparision purposes, the spacing between outlets for a constant discharge center-pivot lateral.

Example

Calculate the head loss caused by friction in a sprinker lateral that is 402 m in length with 67 outlets along its length. The lateral is constructed of galvanized steel with an inside diameter of the lateral D=168 mm. The center-pivot is required to apply a depth of water d=8 mm in a 24-h period of time. It operates continuously and completes one revolution in 24 h. (Assume a relative roughness in the internal wall $\epsilon = 0.15$mm and the kinematic viscosity for water at 15°C $\nu = 1.14\times10^{-6}\ \mathrm{m^2s^{-1}}$.)

The total discharge for the center-pivot $Q = \pi L^2 d \dfrac{1}{24\times60^2} = 0.047\ \mathrm{m^3s^{-1}}$

The velocity at the pivot end of the lateral $V = \dfrac{Q}{\dfrac{\pi}{4}D^2} = 2.12\ \mathrm{ms^{-1}}$

Reynolds number $N_R = \frac{VD}{\nu} = 313{,}513$

Using the Churchill equation to calculate friction factor C for the Darcy-Weisbach equation (Churchill, 1977).

$$C = 8\left[\left(\frac{8}{N_R}\right)^{12} + \frac{1}{(A+B)^{1.5}}\right]^{\frac{1}{12}} \tag{40}$$

where A and B are given by

$$A = \left\{2.457\ \ln\left[\frac{1}{\left(\frac{7}{N_R}\right)^{0.9} + 0.27\left(\frac{\epsilon}{D}\right)}\right]\right\}^{16} \tag{41}$$

$$B = \left(\frac{37530}{N_R}\right)^{16} \tag{42}$$

From (40) - (42), $C = 0.0202$. Using the Darcy-Weisbach equation the head loss in a pipeline without outlets $H_{fL} = C\frac{L}{D}\frac{V^2}{2g} = 11.10\text{m}$

Outlets with Constant Spacing and Varied Discharge

From (19), $F_{cp\,1} = 0.5408$, and therefore $H_f = 6.01$m. An identical result is obtained if a stepwise calculation is used: With the individual outlet discharge calculated using (16), ignoring velocity head and recovery of pressure head past each outlet, and using a constant friction factor C [an underlying assumption of the friction formula of (10)] (Christiansen 1942).

If the Churchill equation is used to determine the friction factor for individual segments in the stepwise approach, the total head loss caused by friction is 6.07m. Alternatively, using the more approximate empirical relationship of (39), $F_{cp\ emp} = 0.5409$, and $H_f = 6.01$m.

For 67 equally spaced outlets on a lateral 402m long, the outlet spacing is

6.00m. Numbering the outlets from the pivot end, the discharge of each outlet from Eq. (16) is

Outlet no.1	$q_1 =$	0.02 Ls^{-1}
Outlet no.2	$q_2 =$	0.04 Ls^{-1}
Outlet no.3	$q_3 =$	0.06 Ls^{-1}
•	•	•
•	•	•
•	•	•
Outlet no.66	$q_{66} =$	1.38 Ls^{-1}
Outlet no.67	$q_{67} =$	1.40 Ls^{-1}
	$\Sigma =$	0.045 m^3s^{-1}

Outlets with Constant Discharge and Varied Spacing

From (36), F_{cp3}= 0.5426, and therefore $H_{f'}$= 6.03m. Alternatively using the more approximate empirical relationship of (39), $F_{cp\ emp}$ = 0.5427, and therefore, $H_{f'}$= 6.03m. For 67 outlets with equal discharge, the discharge of each outlet is 0.70 L/s. Numbering the outlets from the pivot end, the spacing between outlets or length of each segment from (33) is

Segment no.1	$l_1 =$	49.11 m
Segment no.2	$l_2 =$	20.34 m
Segment no.3	$l_3 =$	15.61 m
•	•	•
•	•	•
•	•	•
Segment no.66	$l_{66} =$	3.03 m
Segment no.67	$l_{67} =$	3.01 m
	$\Sigma =$	402 m

In the former design, the discharge of outlet No.67 is quite high, and a designer may choose to increase the number of outlets. Similarly in the latter design, the length of segment No.1 is excessive well beyond the wetted radius of most sprinklers. A larger number of outlets would need to be selected. The comparison between the two designs illustrates the use of equations developed in this paper.

CONCLUSION

This paper presents equations for a friction correction factor for center-pivots without end guns. The factor presented is a function of the number of outlets and the velocity exponent of the friction formula used. This papers compares the friction correction factor for center-pivots with constant outlet spacing against the friction correction factor for center-pivots with constant outlet discharge. The two friction correction factors are almost identical. For a very large number of outlets the friction correction factors estimated by equations presented in this work approach the estimates using the equation developed by Chu and Moe (1972) - the latter was developed for an infinite number of tiny outlets. For a small number of outlets, the equations presented here correlate closely with the values determined from a stepwise iterative computational approach.

Equations have been developed to determine the discharge of each outlet for a center-pivot with constant spacing and varied discharge. Equations have also been developed to determine the spacing of outlets for a center-pivot with constant discharge and varied spacing. The application of the equations developed in this work are demonstrated with a simple numerical example.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A = Empirical parameter used in Churchill equation for computing Darcy-Weisbach friction factor;

A_i = Area irrigated by the i th outlet;

B = Empirical parameter used in Churchill equation for computing Darcy-Weisbach friction factor;

b = Empirical coefficient;

C = Friction factor based on friction formula used;

D = Internal diameter of lateral;

F_{cp} = Friction correction factor for center pivots, Chu and Moe (1972);

$F_{cp\,emp}$ = Friction correction factor for center pivots based on an empirical hyperbolic equation;

$F_{cp}(N)$ = Friction correction factor for center pivots, Reddy and Apolayo (1988);

$F_{cp}{}^{1}$ = Friction correction factor for center pivots with constant outlet spacing, outlets numbered from the upstream end;

$F_{cp}{}^{2}$ = Friction correction factor for center pivots with constant outlet spacing, outlets numbered from the downstream end;

$F_{cp}{}^{3}$ = Friction correction factor for center pivots with constant outlet discharge, outlets numbered from the upstream end;

$F_{cp}{}^{4}$ = Friction correction factor for center pivots with constant outlet discharge, outlets numbered from the downstream end;

g = acceleration due to gravity;

H_f = Total head loss due to friction in center pivot lateral with constant outlet spacing;

$H_{f'}$ = Total head loss due to friction in the center pivot lateral with constant outlet discharge;

H_{fL} = Head loss due to friction in pipeline of length L without outlets;

h_{fi} = Head loss due to friction in the i th segment of lateral;

i = Integer representing segments of lateral (1...,N);

j = Integer representing segments of lateral (2..,.i);

K = Units coefficient for friction formula used;

L = Total length of center pivot lateral;

l = Length of segment (outlet spacing) for center pivots with constant outlet spacing;

l_{i1} = Length of i th segment for center pivots with varied outlet spacing, outlets numbered from the upstream end;

l_{i2} = Length of i th segment for center pivots with varied outlet spacing, outlets numbered from the downstream end;

m = Velocity exponent in friction formula used;

N = Number of outlets along center pivot lateral;

N_R = Reynolds number;

n = Diameter exponent in friction formula used;

Q = Total discharge of center pivot at pivot end;

Q_i = Discharge in i th segment of center pivot lateral;

q = Discharge of each outlet for center pivots with constant outlet discharge;

q_{i1} = Discharge of i th outlet for center pivots with varied outlet discharge, outlets numbered from upstream end;

q_{i2} = Discharge of i th outlet for center pivots with varied outlet discharge, outlets numbered from downstream end;

r = Radial distance of tiny sprinkler from pivot;

r^2_{coeff} = Coefficient of determination

r_i = Radial distance of i th outlet from pivot;

r_{i-1} = Radial distance of (i-1)th outlet from pivot;

r_{i+1} = Radial distance of (i+1)th outlet from the pivot;

β = Beta function;

ν = Kinematic viscosity;

π = pi; and

Σ = Summation.

6

CORRECTION FACTORS FOR CENTER-PIVOTS WITH END GUNS

CORRECTION FACTORS FOR CENTER PIVOTS WITH END GUNS

By Arif A. Anwar[1]

ABSTRACT: The end-gun discharge of center pivots is expressed as a ratio of the discharge at the pivot. Using this ratio, equations are developed for the friction correction factor and pressure distribution factor. If end-gun discharge is reduced to zero, then these equations reduce to the well-established equation for the friction correction factor and pressure distribution factor. For an end-gun ratio of unity, the friction correction factor also becomes unity, reflecting that the lateral is in fact a pipeline without outlets. The pressure distribution factor becomes linear, reflecting that head loss varies linearly with length. For a lateral of constant diameter and typical end-gun discharge there is a significant increase in head loss due to friction. However, there is insignificant difference in the estimate using either this technique or the effective radius technique. The pressure distribution factor is slightly higher, indicating that in laterals with end guns the pressure head toward the center of the lateral is higher. The equations presented can be used to design center-pivot laterals with end guns or the first segment of a tapered center pivot lateral.

INTRODUCTION

In sprinkler laterals, the discharge decreases along the length of the lateral. Laterals can be broadly classified into two categories according to the variation of discharge along the length of the lateral:

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1. Laterals where the discharge decreases proportionally with length (linear variation). This is characteristic of fixed, periodic move, side-roll or linear systems, where a rectangular field is irrigated.
2. Laterals where the discharge decreases proportionally with square of the length (nonlinear variation). This is characteristic of center-pivot systems.

Although it is conceptually feasible to have a variety of functions relating discharge along a lateral to length, nonlinear variation other than that encountered in center-pivot laterals do not have any practical application. Scaloppi and Allen (1993a) described these two categories as uniform outflow condition and nonuniform outflow condition respectively.

Kincaid and Heermann (1970) demonstrated that center pivots could be analyzed in a stepwise method by summation of friction losses calculated for each segment between outlets. Chu and Moe (1972) considered a center pivot to consist of continuous outlets rather than the discrete outlets described by Kincaid and Heermann (1970). Chu and Moe (1982) developed friction correction factors for center pivots that allow direct computation of head loss due to friction in a manner similar to that developed by Christiansen (1942) for laterals with linear variation of discharge. Chu and Moe (1972) also developed a pressure head distribution factor that allows computation of pressure head at any point along the lateral.

Reddy and Apolayo (1988) extended the model proposed by Chu and Moe (1972) to discrete outlets to develop a modified friction correction factor. This work demonstrated that the effect of the number of outlets on the friction correction factor becomes negligible for anything more than 10 outlets. Reddy and Apolayo (1988) compared their modified friction correction factor to the Christiansen (1942) factor *F*. Gilley (1989) pointed out that the two friction factors are expected to be significantly different because of the different function relating discharge along the lateral to length, as described earlier.

Several researchers, e.g., Christiansen (1942), Chu and Moe (1982), and

Scaloppi (1986) have ignored the velocity head in their analysis of laterals. At every outlet, as the discharge in the lateral decreases, the velocity head decreases and the pressure head would increase. But there is a certain amount of turbulence resulting in head loss at each outlet. Pair et al. (1975) suggested that the increase in pressure head would be balanced by the head loss due to turbulence and therefore exact procedures to calculate pressure losses in pipelines with multiple outlets cannot be justified. Smith (1990) argued that this assumption may not be valid for low-pressure pipelines with multiple outlets, although it can be justified at the higher operating pressures of typical sprinkler laterals. Scaloppi and Allen (1993a) investigated the effects considering and ignoring velocity head and compared this with results obtained by a stepwise computation. They concluded that, for most situations, the simplified equations ignoring velocity head were adequately accurate.

Center-pivot laterals are often equipped with end guns to reduce the unirrigated area. For a square field, approximately 21.5% of the area remains unirrigated if a center pivot is not equipped with an end gun (Von Bermuth 1983). Chu and Moe (1972) applied their friction correction factor by using the irrigated radius rather than the lateral length. This approach has been used by others (e.g., Scaloppi and Allen 1993b; Keller and Bliesner 1990) where the ratio of the lateral length to irrigated radius is assumed ≤ 0.94. Solomon and Kodoma (1978) presented a definition of the end sprinkler effective radius, which is a function of not just the sprinkler, but is also affected by locally appropriate agronomic criteria.

The present work investigates the friction correction factor and pressure distribution factor by considering the end-gun discharge as a ratio of the inlet discharge. This ratio was first proposed by Von Bermuth (1983). This eliminates the need to define the effective radius and also any limiting ratio between lateral length and effective radius. In this work the velocity head has been ignored, and the friction factor is assumed to be constant throughout the length of the lateral. Changes in elevation are not considered here.

ANALYSIS

Friction Correction Factor

Continuous Outlet Discharge along Lateral

Fig. 1 represents a center-pivot lateral with an end gun. The discharge at the downstream end of the lateral represents the end-gun discharge.

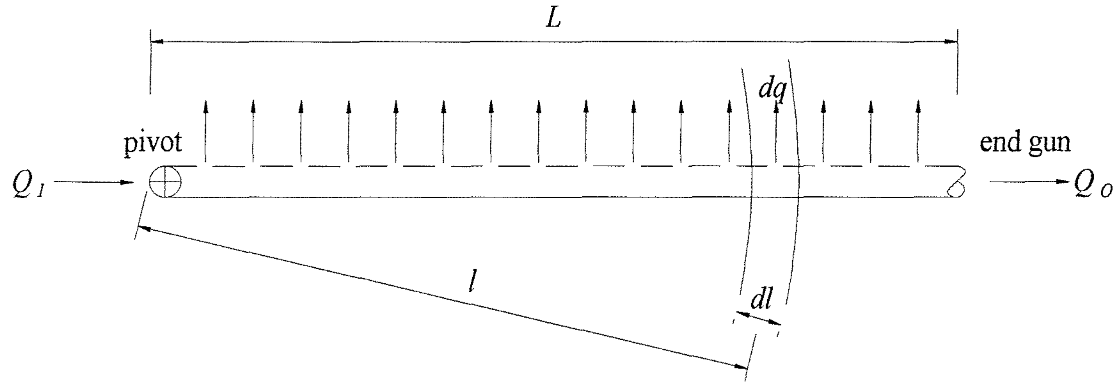

FIG. 1 Center Pivot with Continuous Outlets and End Gun

Using the model proposed by Chu and Moe (1972), the lateral is assumed to consist of an infinite number of small outlets; i.e., there is a continuous outlet discharge along the length of the lateral. Each of these outlets is assumed to irrigate an annular area given by

$$dA = 2\pi l dl \tag{1}$$

where dA = area irrigated by any one outlet; l = distance from the pivot end to the outlet; and dl = width of the annular area. If the depth of water applied over the entire circular area irrigated by the center pivot is uniform, then

$$\frac{dq}{2\pi l dl} = \frac{Q_I - Q_O}{\pi L^2} \tag{2}$$

where dq = discharge of the outlet; Q_I = discharge at the inlet; Q_O = end gun discharge; and L = length of the lateral. Von Bermuth (1983) defined the end-gun ratio as

$$R = \frac{Q_O}{Q_I} \tag{3}$$

where R = end gun ratio $(0 \le R \le 1)$. Substituting $Q_O = RQ_I$ from (3) in (2) gives

$$dq = \frac{1}{L^2} 2Q_I(1-R)l\,dl \qquad (4)$$

The discharge in the lateral at a distance *l* from the pivot end is given by

$$Q_l = Q_O + \int_l^L dq \qquad (5)$$

where Q_l = discharge in the lateral at *l* from the pivot end. From (3), (4), and (5):

$$Q_l = Q_I \left[(1-R)\left(1 - \frac{l^2}{L^2} \right) + R \right] \qquad (6)$$

The energy gradient at point *l* along the lateral is approximated by

$$\frac{dE}{dl} = \frac{d}{dl}\left(H + \frac{v^2}{2g} + Z \right) \qquad (7)$$

where dE/dl = energy gradient at a point *l*; *H* = pressure head; $v^2/2g$ = velocity head (assuming velocity head correction coefficient =1.0); and *Z* = elevation. For a horizontal lateral, and neglecting the velocity head, (7) simplifies to

$$\frac{dE}{dl} = \frac{dH}{dl} \qquad (8)$$

The general equation for head loss caused by friction in a lateral can be approximated by the expression derived by Christiansen (1942):

$$\frac{dE}{dl} = -\frac{CKQ_l^m}{D^{2m+n}} \qquad (9)$$

where *C* = friction factor based on the friction formula used; *K* = units coefficient ; *m* = velocity exponent in the friction formula used; *D* = internal diameter of the lateral; and *n* = diameter exponent in the friction formula used. From (6) and (9):

$$\frac{dE}{dl} = -\frac{CKQ_I^m}{D^{2m+n}} \left[(1-R)\left(1 - \frac{l^2}{L^2} \right) + R \right]^m \qquad (10)$$

from (8), the head loss over the entire length of the lateral is

$$H_f = \frac{CKQ_I^m L}{D^{2m+n}} \left\{ -\int_L^0 \left[(1-R)\left(1-\frac{l^2}{L^2}\right)+R \right]^m \frac{dl}{L} \right\} \tag{11}$$

where H_f = head loss caused by friction over the entire length L of the lateral. Alternatively (11) can be expressed as

$$H_f = \frac{CKQ_I^m L}{D^{2m+n}} G \tag{12}$$

where G = friction correction factor for center-pivot laterals with end guns (continuous outlets), given by

$$G = \int_0^L \left[(1-R)\left(1-\frac{l^2}{L^2}\right)+R \right]^m \frac{dl}{L} \tag{13}$$

The condition R=0 represents a lateral without an end-gun and (13) reduces to the friction correction factor F developed by Chu and Moe (1972), Scaloppi and Allen (1993a). For R= 1, which represents a pipeline without any outlets, G= 1, i.e. (12) becomes the Darcy-Weisbach friction formula for pipelines.

For rough turbulent flow $m = 2$ as in the Darcy-Weisbach formula. For laminar flow $m = 1$ and for the Hazen-Williams formula, $m = 1.852$. Assuming rough turbulent flow, (13) can be simplified to

$$G=\frac{8}{15}(1-R)^2+\frac{4}{3}(1-R)R+R^2 \tag{14}$$

Discrete Outlet Discharge along Lateral

Reddy and Apolayo (1988) extended the model described by Chu and Moe (1972) to discrete outlets along a lateral. This model of discrete outlets is now developed further to a center-pivot lateral with an end-gun, as shown in Fig. 2. The outlets are numbered consecutively from the pivot end of the lateral. All the outlets are uniformly spaced and the outlet spacing is given by

$$s = \frac{L}{N} \tag{15}$$

where s = outlet spacing, and N = number of outlets along the lateral. The friction loss in the i th segment of the lateral can be written as (Christiansen 1942)

$$h_{fi} = \frac{CKQ_i^m s}{D^{2m+n}} \tag{16}$$

where h_{fi} = head loss caused by friction in the i th segment; Q_i = discharge in the i th segment of the lateral; and i = integer (1,2,3...N). The total head loss caused by friction in all N segments of the lateral is

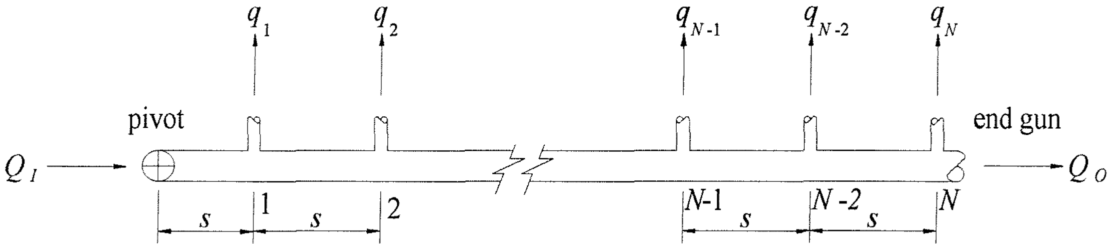

FIG. 2 Center Pivot with Discrete Outlets and End Gun

$$H_f = \sum_{i=1}^{N} \frac{CKQ_i^m s}{D^{2m+n}} \tag{17}$$

Substituting (15) in (17) gives

$$H_f = \frac{CKL}{D^{2m+n}} \frac{1}{N} \sum_{i=1}^{N} Q_i^m \tag{18}$$

The depth of application over the circular area irrigated by the center pivot is assumed to be uniform; therefore

$$\frac{q_j}{2\pi l_j s} = \frac{Q_I - Q_O}{\pi L^2} \tag{19}$$

where q_j = discharge of the jth outlet along the lateral; and l_j= radial distance of the j th outlet from the pivot, where j is an integer. But $l_j = js$, also from (15) $L=Ns$ and from (3) $Q_O=RQ_I$. Substituting in (19) brings

$$q_j = \frac{2Q_I\,(1-R)\,j}{N^2} \tag{20}$$

The discharge through the *i*th segment of a center pivot lateral with an end gun is given by

$$Q_i = Q_I - \sum_{j=0}^{i-1} q_j \tag{21}$$

From (19) and (20):

$$Q_i = Q_I\left(1 - \frac{2(1-R)}{N^2}\sum_{j=0}^{i-1} j\right) \tag{22}$$

Substituting (22) in (18):

$$H_f = \frac{CKQ_I^{\,m}L}{D^{2m+n}}G_{cp} \tag{23}$$

where G_{cp} = friction correction factor for center-pivots with end guns (discrete outlets) and is defined by

$$G_{cp} = \frac{1}{N^{2m+1}}\sum_{i=1}^{N}\left(N^2 - 2(1-R)\sum_{j=0}^{i-1} j\right)^m \tag{24}$$

For R=0, which represents a center pivot without an end gun, $G_{cp} = F_{cp}(n)$ as defined by Reddy and Apolayo (1988). For R=1, which represents essentially a pipeline, G_{cp}= 1.00 and again (23) reduces to the Darcy-Weisbach equation.

Von Bermuth (1983) suggested that typical end-gun ratios range from 5% to 20%. In Table 1, (24) is used to calculate friction correction factor G_{cp} for this range of end-gun ratios with m=2. Eq. (14) is also used for comparison, and although (14) corresponds to N=∞, Table 1 shows that for $N \geq 50$, $G = G_{cp}$. Although (24) is cumbersome, it can be applied to a finite number of outlets and also to values of m other than 2.00. Fig. 3 shows the influence of considering discrete outlets on the friction correction factor. For typical values of N found in practical center pivot installations, there is negligible difference between G and G_{cp}. However there is a significant increase in the friction factor as R increases.

TABLE 1 Comparison between G_{cp} and G with m=2.00

Outlet number (1)	End gun ratio R 0.00 (2)	 0.05 (3)	 0.10 (4)	 0.15 (5)	 0.20 (6)
1	1.000	1.000	1.000	1.000	1.000
2	0.625	0.638	0.651	0.665	0.680
3	0.572	0.586	0.600	0.615	0.631
4	0.555	0.568	0.583	0.599	0.615
5	0.547	0.561	0.575	0.591	0.608
6	0.543	0.557	0.571	0.587	0.604
7	0.540	0.554	0.569	0.585	0.601
8	0.539	0.552	0.567	0.583	0.600
9	0.537	0.551	0.566	0.582	0.599
10	0.537	0.551	0.565	0.581	0.598
15	0.535	0.549	0.563	0.579	0.596
20	0.534	0.548	0.563	0.579	0.595
25	0.534	0.548	0.563	0.578	0.595
50	0.533	0.547	0.562	0.578	0.595
100	0.533	0.547	0.562	0.578	0.595
200	0.533	0.547	0.562	0.578	0.595
G using (14)	0.533	0.547	0.562	0.578	0.595

Head Distribution Factor

Continuous Outlet Discharge along Lateral

Chu and Moe (1972) defined the distribution factor as

$$H = \frac{H_l - H_L}{H_I - H_L} \tag{25}$$

where H = distribution factor; H_l = pressure head at a length r from the upstream end; ($0 \leq r \leq L$); H_L = pressure head at the downstream end of the lateral; and, H_I = pressure head at the upstream (inlet) end of the lateral.

This can also be written as

$$H = 1 - \frac{H_I - H_l}{H_I - H_L} \tag{26}$$

but H_I - H_L is by definition equal to H_f given by (11). Similarly H_I - H_l is the head loss between the inlet and a section at *l* from the inlet. Substituting appropriately from (11) in (26) gives

$$H = 1 - \frac{-\int_{r}^{0}\left[(1-R)\left(1-\frac{l^2}{L^2}\right)+R\right]^m \frac{dl}{L}}{-\int_{L}^{0}\left[(1-R)\left(1-\frac{l^2}{L^2}\right)+R\right]^m \frac{dl}{L}} \qquad (27)$$

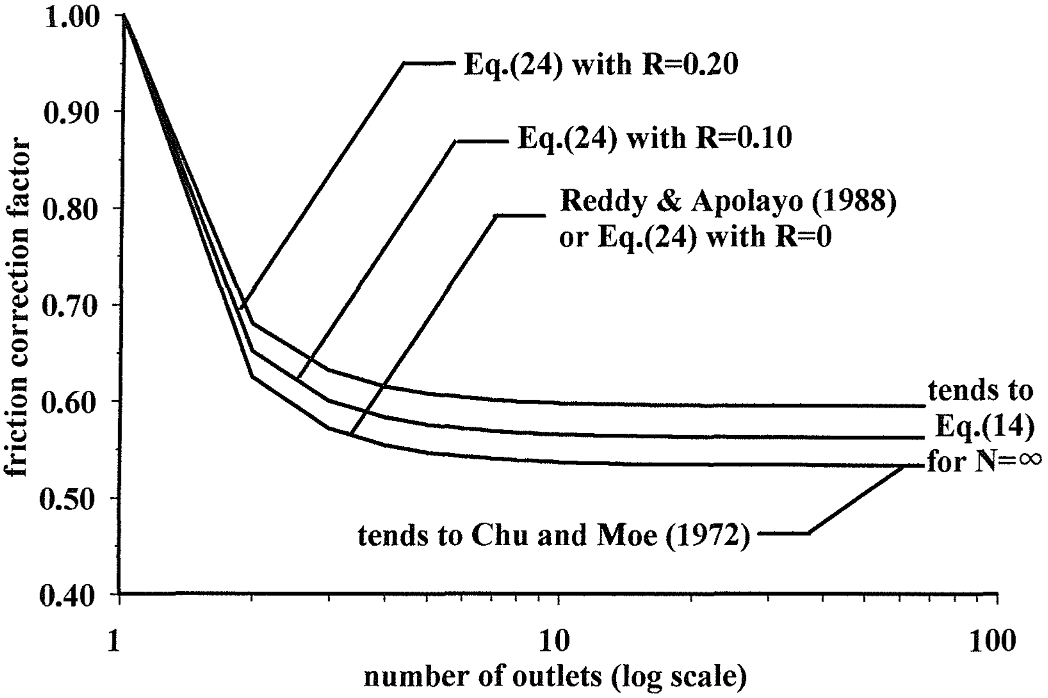

Fig. 3 Friction Correction Factor with $m = 2.00$

Chu and Moe (1972) pointed out that expressions such as (27) are awkward to solve for values of *m* other than 2.00. For m=2.00, (27) simplifies to

$$H = 1 - \frac{(1-R)^2\left(x-\frac{2x^3}{3}+\frac{x^5}{5}\right)+2R(1-R)\left(x-\frac{x^3}{3}\right)+R^2x}{\frac{8}{15}(1-R)^2+\frac{4}{3}R(1-R)+R^2} \qquad (28)$$

where x = relative length defined by l/L. For the condition of R=0, (28) reduces to the form given by Chu and Moe (1972):

$$H = 1-\frac{15}{8}\left(x-\frac{2x^3}{3}+\frac{x^5}{5}\right) \tag{29}$$

Discrete Outlet Discharge along Lateral

For a finite number of outlets the distribution factor can be written as

$$H_{cp} = \frac{H_i-H_L}{H_I-H_L} \tag{30}$$

where H_{cp} = distribution factor for a center pivot with discrete outlets; and, H_i = pressure head at the i th outlet along the lateral. Alternatively, (30) may be written as

$$H_{cp} = 1-\frac{H_I-H_i}{H_I-H_L} \tag{31}$$

In (31), the term $(H_I - H_L)$ is the total head loss in the lateral given by (23). The term $(H_I - H_i)$ in (31) is simply the head loss due to friction in a center-pivot lateral with i outlets. Rewriting (19) for the kth outlet, where k is an integer (1,2,3,...i)

$$\frac{q_k}{2\pi r_k s} = \frac{Q_I-Q_O}{\pi L^2} \tag{32}$$

where q_k = discharge of the k th outlet along the lateral, and r_k= radial distance of the k th outlet from the pivot. But $r_k = ks$, also, from (15) L=Ns and from (3) Q_O=RQ_I. Substituting in (32) gives

$$q_k = \frac{2Q_I\,(1-R)\;k}{N^2} \tag{33}$$

The discharge through the j th segment of a center pivot lateral with an end gun is given by

$$Q_j = Q_I-\sum_{k=0}^{j-1} q_k \tag{34}$$

From (33) and (34)

$$Q_j = Q_I\left(1-\frac{2(1-R)}{N^2}\sum_{k=0}^{j-1} k\right) \tag{35}$$

and

$$H_I - H_i = \frac{CKQ_I^m L}{D^{2m+n}} \frac{1}{N^{2m+1}} \sum_{j=1}^{i} \left(N^2 - 2(1-R)\sum_{k=0}^{i-1} k \right)^m \tag{36}$$

substituting for $(H_I - H_L)$ from (23) and $(H_I - H_i)$ from (36) in (31)

$$H_{cp} = 1 - \frac{1}{G_{cp}} \left(\frac{1}{N^{2m+1}} \sum_{j=1}^{i} \left[N^2 - 2(1-R)\sum_{k=0}^{j-1} k \right]^m \right) \tag{37}$$

Fig. 4 shows the variation distribution factor *H* from (28) against the relative length, for various end gun ratios. For *R*=1, the distribution factor reduces to (1-*x*), i.e., pressure head decreases linearly with length along a pipeline without outlets (Table 2). This can also be verified from (28). Fig. 4 shows that for typical end gun ratios up to 0.20, the effect of the end-gun ratio on the distribution factor is limited.

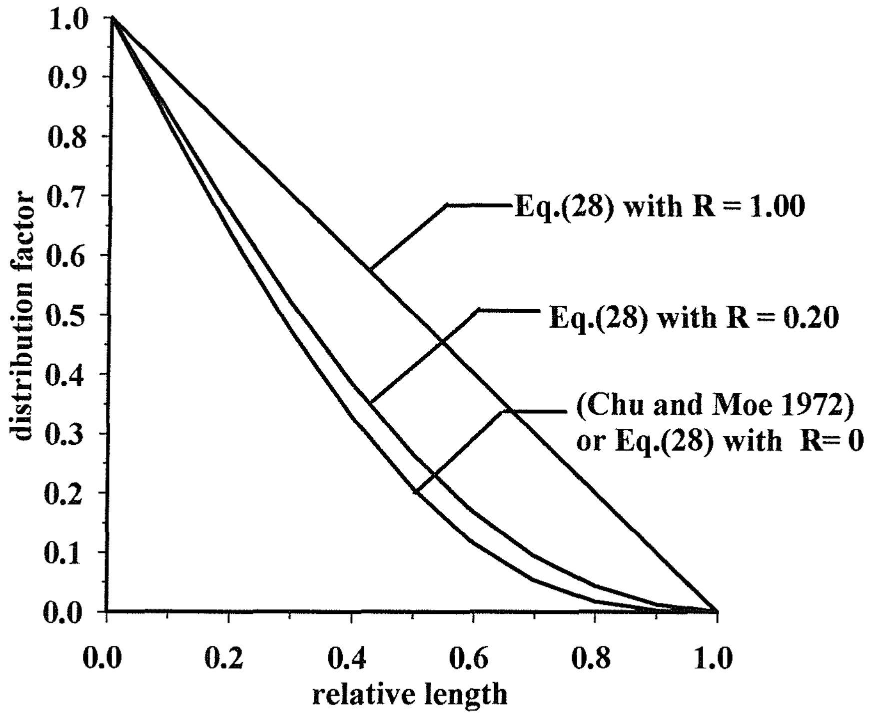

FIG. 4 Pressure Distribution Factor with $m = 2.00$

For a center-pivot with discrete outlets, the pressure distribution factor H_{cp} can be calculated using (37). If the relative length is approximated by i/N, then H_{cp} can be compared with H. At each discrete outlets, there is insignificant difference between H_{cp} and H even for very small values of N. Therefore, although the total head loss caused by friction is a function of the number of outlets, the distribution of that head loss along the lateral is largely independent of the number of outlets.

TABLE 2: Distribution Factor H

Relative length (1)	End gun ratio R 0.00 (2)	0.05 (3)	0.10 (4)	0.15 (5)	0.20 (6)
0	1.000	1.000	1.000	1.000	1.000
0.1	0.814	0.818	0.823	0.828	0.833
0.2	0.635	0.644	0.653	0.662	0.671
0.3	0.470	0.482	0.494	0.507	0.519
0.4	0.326	0.340	0.354	0.368	0.383
0.5	0.207	0.221	0.235	0.249	0.265
0.6	0.116	0.128	0.141	0.154	0.168
0.7	0.053	0.062	0.072	0.083	0.094
0.8	0.017	0.022	0.029	0.036	0.043
0.9	0.002	0.004	0.007	0.010	0.013
1	0.000	0.000	0.000	0.000	0.000

APPLICATION

A numerical example is used to illustrate the application G to calculate the head loss caused by friction and H to calculate the pressure head at any outlet.

Example

A center pivot lateral is 252 m long and has 42 outlets spaced equally at 6 m. The lateral has an internal diameter of 127 m and is constructed of galvanized steel (effective roughness assumed as 0.15 mm). The system is to apply 8 mm per revolution and the center pivot completes one revolution in 24 h. The end gun has a discharge of 3.5 L/s, and requires a pressure head of 30 m.

Solution

The Darcy-Weisbach equation will be used to solve this problem (m=2, n=1). To estimate the friction factor, the Churchill equation will be used (Churchill 1977). Scaloppi and Allen (1993b) demonstrated that the Churchill equation provides a good estimate of the Darcy Weisbach friction factor.

The discharge required to irrigate the circular area (excluding the end gun area) is given by

$$Q_I\text{-}Q_O = \frac{8\times10^{-3}\times\pi\times252^2}{24\times60^2} = 18.472\times10^{-3}\ \mathrm{m^3/s}$$

Because

$$Q_O = 3.5\times10^{-3}\ \mathrm{m^3/s}$$

therefore

$$Q_I = 21.973\times10^{-3}\ \mathrm{m^3/s},\ \text{and}\ R = 0.16$$

From (14): $G = 0.581$

alternatively, with m=2.00 and N=42.

From (24): $G_{cp} = 0.581$

Using the Churchill equation (Churchill 1977) for a relative roughness of 0.15 mm C = 0.020

From (12)

$$H_f = \frac{0.02\times(8/\pi^2 g)\times\left(21.973\times10^{-3}\right)^2\times252}{\left(127\times10^{-3}\right)^5}\times0.581 = 3.62\ \mathrm{m}$$

$$H_I = H_f + H_L = 33.62\ \mathrm{m}$$

Table 3 shows the discharge and pressure head at each outlet, allowing appropriate sprinklers to be selected from a catalogue.

This problem can also be solved using the original equations developed by Chu and Moe (1972), using the "irrigated radius." The calculated head loss is almost identical for both methods, but the current method predicts a slightly higher pressure head in the middle reach of the lateral. This difference in pressure increases if the end-gun ratio increases.

TABLE 3: Discharge and Pressure Head at Each Outlet

Outlet number (1)	Outlet discharge using (2) $\times 10^{-3}$ (m^3/s) (2)	Relative length $x = i/N$ (3)	Distribution factor H using (28) (4)	Pressure head at outlet using (31) (m) (5)
Pivot			1.000	33.62
1	0.021	0.02	0.959	33.47
2	0.042	0.05	0.918	33.32
3	0.063	0.07	0.877	33.18
4	0.084	0.10	0.837	33.03
5	0.105	0.12	0.797	32.88
•				
•				
40	0.837	0.95	0.003	30.01
41	0.858	0.98	0.001	30.00
42	0.879	1.00	0.000	30.00

CONCLUSIONS

This paper extends the friction correction factor and pressure distribution factor developed by Chu and Moe (1972) to center pivots with end guns. It also investigates the friction correction factor and pressure distribution factors for center pivots with end guns considering discrete outlets. For a small number of outlets ($N \leq 10$) the friction correction factor considering discrete outlets is greater than that considering indiscrete outlets. However, this may be of only academic interest since most practical center- pivot installations will have a far greater number of outlets. The pressure distribution factor was found to be relatively independent of the number of outlets along the lateral, and is only a function of the relative position of the outlet along the lateral and the end-gun ratio.

Although some of the equations developed in this paper appear cumbersome, they are all implicit and can easily be solved using a programmable calculator or a spreadsheet. Practical application of the correction factors developed is demonstrated through an example. This work shows that, for center-pivot laterals of uniform diameter and typical end-gun discharges in practical applications, the original expressions by Chu and Moe (1972) using the irrigated radius are sufficiently accurate.

In situations of high end gun ratios and small outlet numbers, the analysis presented in this paper may be considered. Such an application would, for example, be a tapered lateral where the lateral consists of an upstream segment with a larger diameter followed by a downstream segment with a relatively smaller diameter. The upstream segment would have an outflow and therefore can be analyzed as presented here. The factors developed here can not be applied directly to the downstream segment of a tapered lateral because the downstream segment does not start at the pivot - an underlying assumption in this work. This could be the subject of further work.

APPENDIX I. REFERENCES

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Pair, C.H., Hinz, W.W., Reid, C., and Frost, K.R., eds. (1975). *Irrigation,* 5th ed., Irrigation Association, Fairfax, VA.

Reddy, J.M., and Apolayo, H. (1988). "Friction correction factor for center pivot irrigation systems." *J. Irrig. and Drain. Engrg.*, ASCE 114(1), 183-185.

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von Bermuth, R.D. (1983). "Nozzling consideration for center pivots with end guns." *Trans. Am. Soc. Agric. Engrs.*, 26(2), 419-422.

APPENDIX II. NOTATION

The following symbols are used in this paper:

C = friction factor based on friction formula used;
D = internal diameter of lateral;
dA = area irrigated by any one outlet;
dE = change in energy;
dH = change in pressure head;
dl = width of annular area;
dq = discharge of outlet;
$F_{cp}(n)$ = friction correction factor for center pivots (Reddy and Apolayo 1988);
G = friction correction factor for center pivot laterals with end guns;
G_{cp} = friction correction factor for center pivots with end guns (discrete outlets);
H = distribution factor;
H_{cp} = distribution factor for center pivot with discrete outlets;
H_f = total head loss caused by friction in center-pivot lateral;
H_I = pressure head at pivot end of center-pivot;
H_i = pressure head at i th outlet;
H_L = pressure head at end-gun of center-pivot;
H_l = pressure head at distance l from pivot end;
h_{fi} = head loss caused by friction in i th segment of lateral;
i = integer;
j = integer;
K = units coefficient;
L = length of the center pivot lateral;
l = length of lateral measured from pivot end;
l_j = radial distance of j th outlet from pivot;
l_k = radial distance of k th outlet from pivot;
m = velocity exponent in friction formula used;
N = number of outlets along lateral;

n = diameter exponent in friction formula used;
Q_I = discharge at inlet;
Q_i = discharge in the i th segment of lateral;
Q_l = discharge in the lateral at l from pivot end;
Q_O = end gun discharge;
q_j = discharge of the j th outlet along lateral;
q_k = discharge of the k th outlet along lateral;
R = end-gun ratio;
s = outlet spacing;
v = average velocity;
x = relative length;
Z = elevation;
$\sum$ = summation; and
π = pi.

ENDNOTE

ENDNOTE

The first friction correction factor for conventional sprinkler laterals was developed by J.E. Christiansen[1] (1942) and continues to be used today, over half a century later, despite all the advances in computer technology. The most notable work on center pivot laterals is by Chu and Moe[2] (1972) and this work has also become the standard for text books and designs. It is anticipated that the new correction factors presented in this thesis will contribute to that arsenal of tools for analyzing and designing sprinkler laterals. Furthermore, the development of these factors has led to further insight and understanding of the scope and limitations of the correction factors approach in the design of sprinkler laterals.

Work on correction factors is by no means complete. Using the concept of sprinkler laterals with outflow, it may be possible to investigate sprinkler laterals on undulating slopes, with each constant slope section designed as an individual reach. A similar situation may apply for center pivots. For center pivots further investigation and development could include design of tapered center pivot laterals. More recently, Valiantzas[3] (1998) has suggested that for conventional drip laterals, the discharge should be considered a power function of length of the lateral - rather than a linear function. Valiantzas (1998) considered laterals without outflow, therefore it may be possible to develop these ideas further for laterals with outflow.

---

[1] Christiansen J.E. (1942) "Irrigation by Sprinkling." *California Agric. Experiment Station Bull. No.670*, University of California, Davis, Calif.

[2] Chu, S.T. and Moe, D.L. (1972) "Hydraulics of a Center Pivot System." *Trans. Am. Soc. Agric. Engrs.*, 15(5) 894-896.

[3] Valiantzas J.D. (1998) "Analytical Approach for Direct Drip Lateral Hydraulic Calculation" *J. Irrig. and Drain. Engrg.*, ASCE 124(6) 300-305.

A1

Referees comments on:

FACTOR *G* FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

Ⓐ ✓

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR

File No. 017715-IR

Author(s) ANWAR, ARIF A.

Title FACTOR 'G' FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

Paper XXX

Technical Note _____

New Submission XXX

Resubmission _____

Re-review _____

****

RECOMMENDATION

__X__ Accept in present form

_____ Accept, consider suggested revisions

_____ Tentatively accept, revisions required
Re-review required? _____

_____ Decline, encourage revision and resubmittal

_____ Decline as paper, consider resubmittal as technical note

_____ Decline

****

CRITIQUE (See instructions on reverse side)

ATTACHED COMMENTS.

****

EVALUATION (Circle One)

Poor	Below Average	Average	Above Average	Exceptional
1	2	[✓ 3]	4	5
			Possible Award Quality	Award Quality

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Ⓐ

Comments

The author's primary contribution is to derive an analytical formulation that may be used to obtain head loss estimates in multiple diameter pipelines. The work advances the current state-of-the-art by accommodating multiple diameter pipes (instead of assuming a uniform diameter). Formulation is simple and easy to understand. This research could find applications in the design of sprinkler systems or trickle irrigation laterals and manifolds etc. The paper is interesting and concise; and, is acceptable for publication in the Journal of Irrigation and Drainage Engineering.

Although no re-review is required, the following comments may be used for further improving the presentation to a technical but general audience:

1. Introduction: first sentence "The head loss due to friction..... without outlets." Follow it up by a sentence indicating why is it so?

2. Introduction: last sentence first para: "Factor F was derived assuming no outflow ....is one outlet spacing from the inlet to the pipeline". Would help if you define "one outlet spacing" before you introduce this statement. Introducing Figure 1 in the first para itself might be one way to go about it.

3. Page 3 regarding: Gradual reduction of velocity head as flow passes the outlet and increase in pressure by a "certain amount" balancing the losses. This is, at best, highly approximate. A few sentences clarifying the deviation from the real situation would be insightful.

4. I have not verified Eqs 11-17 however, the baseline example (r=0) showing the solution becomes equivalent to Christiansen's case is a good test.

5. How about replacing/supplementing Table 1 with a graph? Graph would provide more physical insight than the table.

6. The uniqueness of the work is to be able to handle multiple diameter pipes. Since this is so, it needs to be mentioned both in the abstract as well as the conclusion!

B

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR

File No. 017715-IR

Author(s) ANWAR, ARIF A.

Title FACTOR 'G' FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

Paper XXX

Technical Note ______

New Submission XXX

Resubmission ______

Re-review ______

****

RECOMMENDATION

X Accept in present form

____ Decline, encourage revision and resubmittal

____ Accept, consider suggested revisions

____ Decline as paper, consider resubmittal as technical note

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline

****

CRITIQUE (See instructions on reverse side)

GREAT EXAMPLE OF PRACTICAL CONTRIBUTIONS TO WORKING ENGINEERING CONCERNS. I COMPLEMENT THE AUTHOR ON MEETING THIS NEED. PAPER IS WELL STRUCTURED AND MOVES LOGICALLY FROM INTRODUCTION TO THEORY TO PRACTICAL EXAMPLE. SUGGEST HE CONSIDER A SECOND PAPER GIVING MODIFIED "G" FACTORS USEFULL IN CENTER PIVOT WITH END GUN DESIGN. ALSO USEFUL IN CALCULATING THE HYDRAULICS OF DRIP LATERALS IN A FLUSHING MODE.

****

EVALUATION (Circle One)

Poor 1	Below Average 2	Average 3	Above Average 4 Possible Award Quality	[✓ Exceptional 5 Award Quality]

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Ⓒ

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ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR
File No. 017715-IR

Author(s) ANWAR, ARIF A.
Title FACTOR 'G' FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW

Paper XXX
Technical Note ______
New Submission XXX
Resubmission ______
Re-review ______

****

RECOMMENDATION

_____ Accept in present form

_____ Decline, encourage revision and resubmittal

_____ Accept, consider suggested revisions

_____ Decline as paper, consider resubmittal as technical note

X Tentatively accept, revisions required
Re-review required? Yes

_____ Decline

****

CRITIQUE (See instructions on reverse side)

This paper is technically correct, clear, concise, and is useful to design of irrigation systems. There is one major item that should be included, however, and that is a verification of the derivation to measured data from a prototype irrigation system that the model claims to be able to model.

Minor editorial comments include (also see marked manuscript):

1. ✓ A hyphen should be included in such phrases as "equally-spaced outlets" throughout the text.
2. ✓ The reference Chu (1978) is not included in the reference list.
3. ✓ A period is missing in the reference Pier et al. (1975) on page 3.
4. ✓ After Eq. (3) it would read better if it said "$r \geq 0$" instead of "$0 \leq r$", since you are talking about r.
5. ✓ In Eq. (10) put "L/N" after the rest of the equation in keeping with the direct substitution of Eq. (10) into Eq. (9).
6. Putting a reference in commas after talking about it seems like poor grammar. Use phrases like "according to," or place the reference in parentheses.
7. ✓ The discussion on p. 5 regarding the exponent "m" should be clarified. Should it read "In Eq. 8, the exponent of the average flow velocity in the pipeline, m, typically assumes the value of 1.85 and 2.00 for the Hazen-Williams friction formula and the Darcy Weisbach friction formula, respectively"? Or do you mean "In Eq. 8, the exponent of the average flow velocity in the pipeline, m, typically assumes the value of 1.85-2.00 for both the Hazen-Williams friction formula and the Darcy Weisbach friction formulas"? Note also that commas should be around the "m" and that the word "formula" has been misspelled.
8. ✓ Explain why you cut off the infinite series of Eq. 15 after 3 terms? Why not 2 or 4 terms?
9. ✓ At the top of p. 8, the value quoted from Table 1, $G_{r=1.0,\,N=12}$ should equal 0.635, not 0.641 and all subsequent calculations corrected.
10. ✓ The reference of Jensen and Fratini has the word "Adjusted" spelled incorrectly in the title.
11. The names of books should be underlined, not placed in Italics. Not according to [illegible]
12. ✓ The notation "F" is also used in Eq. 15 as well as being Christiansen's correction factor. A note should be made in Appendix II as to what "F" means in Eq. 15.

****

EVALUATION (Circle One)

Poor 1	Below Average 2	[✓ Average 3]	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

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A2

Referees comments on:

FACTOR G_A FOR PIPELINES WITH MULTIPLE OUTLETS AND OUTFLOW

A

MAR

**** Individual Review ****

ASCE JOURNAL PAPER REVIEW FORM

Division: **IR**

File No. **019595-IR**

Title: **ADJUSTED FACTOR G FOR PIPELINES WITH OUTLETS AND OUTFLOW**

Author(s): **Anwar, Arif A.**

Paper **xx** Technical Note ____

New Submission **xx** Resubmission ____ Re-review ____

****

RECOMMENDATION

____ Accept in present form

X Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper*, consider resubmittal as technical note

____ Decline

****

CRITIQUE

Paper reads well and procedures are accurate. The value of the work, however, is questionable. Procedures for tapered manifolds as suggested by Keller & Bliesner (1990) are simple and reasonably accurate. Most designs nowadays use computer software, that are more versatile and accurate than the procedure suggested in this manuscript.

****

EVALUATION (Circle One)

Poor 1

Below Average 2

[✓ Average 3]

Above Average 4 Possible Award Quality

Exceptional 5 Award Quality

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MAR [illegible]

****** Individual Review ******

ASCE JOURNAL PAPER REVIEW FORM

Division: **IR**

File No. **019595-IR**

Title: **ADJUSTED FACTOR G FOR PIPELINES WITH OUTLETS AND OUTFLOW**

Author(s): **Anwar, Arif A.**

Paper __xx__ Technical Note _______

New Submission __xx__ Resubmission _____ Re-review _______

****

RECOMMENDATION

_____ Accept in present form

_X_ Accept, consider suggested revisions

_____ Tentatively accept, revisions required
Re-review required? _____

_____ Decline, encourage revision and resubmittal

_____ Decline as paper*, consider resubmittal as technical note

_____ Decline

****

CRITIQUE

Marked Paper

****

EVALUATION (Circle One)

Poor 1   Below Average 2   [✓ Average 3]   Above Average 4 Possible Award Quality   Exceptional 5 Award Quality

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A3

Referees comments on:
INLET PRESSURE FOR HORIZONTAL TAPERED LATERALS

SUMMARY

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MAY 06 1999

ivision: **IR** BA **File No. 020183-IR**

itle: **INLET PRESSURE FOR HORIZONTAL TAPERED LATERALS**
uthor(s): **ARIF A. ANWAR**
Paper **xx** Technical Note ______

ew Submission **xx** Resubmission ______ Re-review _____

****

RECOMMENDATION

____ Accept in present form

X Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

The Paper is accepted. Please consider the suggested revisions.

****

EVALUATION (Circle One)

Poor 1 Below Average 2 [✓ Average 3] Above Average 4 Possible Award Quality Exceptional 5 Award Quality

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A

Individual

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File No. **020183-IR**

Title: **INLET PRESSURE FOR HORIZONTAL TAPERED LATERALS**
Author(s): **ARIF A. ANWAR**
Paper xx
Technical Note ____

New Submission xx
Resubmission ____
Re-review ____

****

RECOMMENDATION

____ Accept in present form

X Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

The manuscript is relatively long for the scope of work. I suggest condensing sections 1 & 2 in the analysis through reference to Anwar (1999). Manuscript reads well, but with recent advances in computer software, the value of the work is questionable.

****

EVALUATION (Circle One)

Poor 1	[✓ Below Average 2]	Average 3	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

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Title: **INLET PRESSURE FOR HORIZONTAL TAPERED LATERALS**
Author(s): **ARIF A. ANWAR**
Paper **xx** Technical Note ____

New Submission **xx** Resubmission ____ Re-review ____

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RECOMMENDATION

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✓ Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

****

EVALUATION (Circle One)

Poor	Below Average	Average	Above Average	Exceptional
1	2	3	[✓ 4]	5
			Possible Award Quality	Award Quality

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***** SUMMARY *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

DEC 2 0 1999

ivision IR File No. 021471-IR

uthor(s) ANWAR, ARIF A.

itle ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

Paper XXX Technical Note ________

New Submission XXX Resubmission ________ Re-review ________

****

RECOMMENDATION

_____ Accept in present form

_____ Accept, consider suggested revisions

[✓] Tentatively accept, revisions required
Re-review required? NO

_____ Decline, encourage revision and resubmittal

_____ Decline as paper, consider resubmittal as technical note

_____ Decline

****

CRITIQUE

The Paper is tentatively accepted. Revisions are required. A re-review is not required.

Please note the comments of the reviewers in revising the Paper.

****

EVALUATION (Circle One)

Poor	Below Average	[✓ Average]	Above Average	Exceptional
1	2	[✓ 3]	4	5
			Possible Award Quality	Award Quality

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A4

Referees comments on:

ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

A

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR File No. 021471-IR

Author(s) ANWAR, ARIF A.
Title ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

Paper XXX Technical Note ____
New Submission XXX Resubmission ____ Re-review ____

****

RECOMMENDATION

____ Accept in present form

____ Decline, encourage revision and resubmittal

____ Accept, consider suggested revisions

____ Decline as paper, consider resubmittal as technical note

X Tentatively accept, revisions required
Re-review required? No

____ Decline

****

CRITIQUE (See instructions on reverse side)

This paper indicates an interesting hydraulic analysis of sprinkler systems which is more robust than previously published techniques. In this sense it makes a unique contribution to the literature. There are specific editori corrections marked on the manuscript. Please note that it strongly recommended that vertical and horizontal grid lines added to figures for better interpretation of results. The hor should consider ~~gl~~ globally replacing "discharge" with volumetric flow rate" which seems more appropriate. The uthor should add a paragraph ~~indicating~~ quantifying the relative value of his adjustment compared to discharge computed without the djustment. The assumption for the analysis presented is that ll outlets have equal discharge (p. 5) which of course is not

****

EVALUATION (Circle One)

Poor 1	Below Average 2	[✓ Average 3]	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

exact since all outlets do not have equal pressure (p. 8).

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B

DEC 2

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR File No. 021471-IR

Author(s) ANWAR, ARIF A.

Title ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

Paper XXX Technical Note ______

New Submission XXX Resubmission ______ Re-review ______

****

RECOMMENDATION

______ Accept in present form

______ Accept, consider suggested revisions

______ Tentatively accept, revisions required
Re-review required? ______

______ Decline, encourage revision and resubmittal

X Decline as paper, consider resubmittal as technical note

______ Decline

****

CRITIQUE (See instructions on reverse side)

Most of the proposed procedure constitutes a repetition of previous procedures published in the listed references. The only new fact in this approach is the estimate of friction head losses in multiple outlet pipes with outflow at the downstream end. In fact, the proposed procedure seems to be more complicated and exhibiting the same limitations of the traditional analytical approach to perform that estimate. The assumption of constant outflow through the outlets in all reaches does not bring any advantage compared to the traditional approach. Finally, it should be considered the precision and convenience that simple computer programs can provide in performing this kind of computation.

****

EVALUATION (Circle One)

Poor	Below Average	Average	Above Average	Exceptional
1	2	[✓ 3]	4	5
			Possible Award Quality	Award Quality

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C

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR

File No. 021471-IR

Author(s) ANWAR, ARIF A.

Title ADJUSTED AVERAGE CORRECTION FACTORS FOR SPRINKLER LATERALS

Paper XXX Technical Note ____

New Submission XXX Resubmission ____ Re-review ____

****

RECOMMENDATION

____ Accept in present form

✓ Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE (See instructions on reverse side)

The paper is well written and is worthwhile

<u>Suggestions</u>

1. References by Anwar xxxxa, and b need to changed to the expected date of publicatio

2. How Eqs. 30 and 49 were obtained from Eqs 29 and 48 respectively needs to be clear. Specifically how were the <u>numerators</u> ~~were~~ obtaine. It will help to put one more step in between.

3. In eqs. 12 and 13 it would be good to explain why $H_{f avg,x}$ does not include loss due to <u>xl</u>. If it is assumed negligible it should be stated.

****

EVALUATION (Circle One)

Poor 1

Below Average 2

[✓ Average 3]

Above Average 4 Possible Award Quality

Exceptional 5 Award Quality

*Note to Reviewer: Please sign reverse side to permit direct copying while retaining anonymity.

[redacted]

A5

Referees comments on: Anwar A.A., (1999).

FRICTION CORRECTION FACTORS FOR CENTER PIVOTS

Ⓐ

****** Individual Review ******

ASCE JOURNAL PAPER REVIEW FORM

FEB 0 [illegible]

Division: **IR**   File No. 019661-IR

Title: **FRICTION CORRECTION FACTORS FOR CENTER-PIVOTS**
Author(s): **Anwar, A.A.**

Paper xx   Technical Note _______

New Submission xx   Resubmission _____ Re-review _______

****

RECOMMENDATION

____ Accept in present form

____ Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

X Decline

****

CRITIQUE

See attached page.

****

EVALUATION (Circle One)

Poor	Below Average	Average	Above Average	Exceptional
1	[✓ 2]	3	4	5
			Possible Award Quality	Award Quality

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019661-IR

A

FEB 05 1999

12 Dec., 1998
ASCE IR manuscript review
Friction correction factors for center-pivots
by: A. A. Anwar

The apparent main purpose of the paper is to develop a friction coefficient for pivots with a small number of outlets. However, the assumption upon which Eq. 9 is based is less valid than previous methods. No one would design a pivot using Eq. 9 because it assumes the area served by a sprinkler is the area between it and the next inner (upstream) sprinkler. For design purposes, one should calculate the area served by a sprinkler from the midpoints of the space between adjacent outlets (i. e. Eq 4 for constant spacing). Keller and Bliesner (1990) (equation 14.20a, page 347) also recommend this method of calculating outlet flows.

Apparently, Keller and Bliesner used the assumption of Eq. 9 to calculate their correction factors, since they agree with the present paper. I did the calculations using the assumption of Eq. 4, and obtained values which agree with the Reddy and Apolayo paper. The method of equation 9 tends to overestimate friction loss because, in effect, water is transported to the outer edge of each wetted subarea, and thus must travel farther than necessary before being applied.

For a small number of outlets, or where end guns are used, an iterative calculation method is preferable because modifications to the inner and outermost outlet discharges (or spacing) usually must be made to obtain good application uniformity over the wetted area.

Although the paper is well written and the analysis is correct for the given assumptions, in the opinion of this reviewer, the method used in the present paper is less valid than the method of previous authors (Chu and Moe, Reddy and Apolayo, etc.). For the reasons stated above, I do not recommend publishing this paper.

3

FEB [illegible]

****** Individual Review ******

ASCE JOURNAL PAPER REVIEW FORM

Division: IR

File No. 019661-IR

Title: **FRICTION CORRECTION FACTORS FOR CENTER-PIVOTS**
Author(s): **Anwar, A.A.**

Paper xx

Technical Note ____

New Submission xx

Resubmission ____ Re-review ____

****

RECOMMENDATION

____ Accept in present form

✓ Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

The author is congratulated for a straightforward presentation of an improvement in certain center-pivot hydraulic calculations. My only suggestion for improvement would be to have the manuscript edited closely by a technical writer for grammar, sentence structure and an occassional awkward phrase.

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EVALUATION (Circle One)

Poor 1	Below Average 2	[✓ Average 3]	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

Would be higher except for the relatively narrow focus of this specific result.

*<u>Note to Reviewer</u>: Please sign reverse side to permit direct copying while retaining anonymity.

C

****** Individual Review ******

ASCE JOURNAL PAPER REVIEW FORM

Division: IR

File No. 019661-IR

Title: **FRICTION CORRECTION FACTORS FOR CENTER-PIVOTS**
Author(s): **Anwar, A.A.**

Paper xx

Technical Note ______

New Submission xx

Resubmission _____ Re-review _______

****

RECOMMENDATION

____ Accept in present form

____ Accept, consider suggested revisions

✓ Tentatively accept, revisions required
Re-review required? No

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

See Attached

****

EVALUATION (Circle One)

Poor 1	Below Average 2	Average [✓ 3]	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

*Note to Reviewer: Please sign reverse side to permit direct copying while retaining anonymity.

C

ASCE Journal Review
File No. 019661-IR

Friction Correction Factors for Center-Pivots
Anway, A.A.

The author presents a theoretically rigorous analysis of methodology to estimate the friction loss in center pivot irrigation system pipelines. This analysis overcomes some of the shortcomings of previous analyses in that it is applicable to systems that have few as well as many outlets. However, it retains the assumption of previous authors that the turbulent losses at a pipe outlet are equal to the increase in pressure due to reduced velocity head downstream of the outlet. He cites a respected author to justify this assumption. However, this reveiwer has personally measured pressures in the mainlines of center pivots on level terrain in which the minimum pressure occurs some distance from the end of the pipeline and mainline pressure head increases by more than 1 meter toward the outer end. Thus, I believe that the error introduced by the turbulence/velocity head assumption is probably greater than any improvement to theoretical calculation achieved by these new equations.

The above notwithstanding, the contribution of the present author is still worthwhile from an academic standpoint, and I do not suggest rejection on the basis of this assumption. The author develops equations for cases of both uniform outlet spacing and uniform outlet discharge, neither of which is particularly realistic. He needs to emphasize the point that these two cases represent the two ends of the design spectrum, and since the results are the same for practical purposes, it doesn't matter which case (or perhaps a mix of the two) exists. What may matter, however, and is not addressed (probably should be disclaimed) is that the common practice of attaching a large discharge sprinkler to the outboard end (q_N = 5 to 10 times q_{n-1}) cannot be addressed with the current equations.

In assessing the accuracy of the authors' equations, he presents values of r^2, which I presume represent the correlation between the authors' results and those of Keller and Bliesner (1990) which are assumed to be the true values (?). This needs clarification, and further acknowledgment that the coefficients of determination are subject to accuracy of the questionable turbulence/velocity head assumption.

Although not difficult to follow, the authors' development through 53 equations is tedious to follow, and a portion is unnecessary. I suggest showing the development of both the constant spacing and constant discharge equations, with outlet N at the outer end, then state something like "in a similar manner, it can be shown that" and show the final equation for the case of numbering from outside toward pivot.

Numerous editorial comments are made directly on the manuscript. Most notable are a slight tendency to mix British with American spelling (center vs centre). I don't care which you use, but it looks cleaner to be consistent. Also, it is hardly justified to show the results of calculation of this type to 4 decimal places. There is no way you can determine friction factor, pipe diameter, Q, or even nozzle diameter to 4 to 6 significant figures.

A6

Referees comments on:

CORRECTION FACTORS FOR CENTER PIVOTS WITH END GUNS

***** SUMMARY *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM JUL 06 1999

Division IR File No. 020655-IR

Author(s) ANWAR

Title CORRECTION FACTORS FOR CENTER PIVOTS WITHEND GUNS

Paper XXX Technical Note ____

New Submission XXX Resubmission ____ Re-review ____

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RECOMMENDATION

____ Accept in present form

X Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

****

CRITIQUE

The Paper is accepted; please consider the suggested revisions.

The reviewers expressed some concern as to how much new knowledge the Paper presents. Can you address this point?

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EVALUATION (Circle One)

Poor 1 Below Average 2 [✓ Average 3] Above Average 4 Possible Award Quality Exceptional 5 Award Quality

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A

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

JUL 06 1999

Division IR

File No. 020655-IR

Author(s) ANWAR

Title CORRECTION FACTORS FOR CENTER PIVOTS WITHEND GUNS

Paper XXX

Technical Note _____

New Submission XXX

Resubmission _____

Re-review _____

****

RECOMMENDATION

_____ Accept in present form

__X__ Decline, encourage revision and resubmittal

_____ Accept, consider suggested revisions

_____ Decline as paper, consider resubmittal as technical note

_____ Tentatively accept, revisions required Re-review required?

_____ Decline

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CRITIQUE (See instructions on reverse side)

See attached sheet. See notes on manuscript.

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EVALUATION (Circle One)

Poor 1	[✓ Below Average 2]	Average 3	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

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A

020655-

The author has performed an interesting analysis of an old problem: center pivots with end guns. However, I don't think there is sufficient novelty in what he has presented to merit publication given the limitations. Here is a summary.

The author says he has extended the work of Chu and Moe, but as Gilley pointed out in his note regarding the Scaloppi and Allen paper, this author has chosen not to use the Beta function solution and that choice limits the velocity exponent to 2. Here are the limitations as I see them to the author's solution:

1. Velocity exponent limited to 2. That excludes the empirical methods commonly used (Hazen-Williams, Scobey). Chu and Moe were not limited with Beta function solution.
2. Exact spacing of sprinklers can't be used. The step method commonly used in the industry and investigated by Scaloppi and Allen does allow that.
3. Elevation differences can't be inserted. The step method does allow that.

The appeal of the formulation in this paper is its elegance. However, it is less elegant than Chu and Moe, and only contributes end gun calculations beyond their work. Scaloppi and Allen added the end gun factor, but limited themselves by not choosing the Beta function. So...what you have is a formulation that has limitations and doesn't extend the knowledge much.

It is difficult for me to believe that anyone choosing to calculate the friction in a center pivot with any configuration of end gun would use anything other than the step method. It has none of the limitations (exponent, elevation, or spacing) that other methods have and it is very easy to program into a spreadsheet. So...the appeal of this formulation can't be based upon application.

Here is my suggestion. Rework the paper with the Beta function--which shouldn't be hard to do--and then you have a formulation with the appeal of elegance without the limitations. Compare it to the step method for different exponents.

In summary, I liked the work, but it should have novelty, elegance, or application sufficient to merit publication, and I don't think it does.

B

JUL 0 6 1999

***** Individual *****

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR

File No. 020655-IR

Author(s) ANWAR

Title CORRECTION FACTORS FOR CENTER PIVOTS WITHEND GUNS

Paper XXX

Technical Note ______

New Submission XXX

Resubmission ______

Re-review ______

****

RECOMMENDATION

______ Accept in present form

[✓ X] Accept, consider suggested revisions

______ Tentatively accept, revisions required

Re-review required? ______

______ Decline, encourage revision and resubmittal

______ Decline as paper, consider resubmittal as technical note

______ Decline

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CRITIQUE (See instructions on reverse side)

Paper is well written and understandable. Reviewer has provided some minor editorial changes on the draft.

The biggest concern with the paper is that it does not contribute significantly to new knowledge. Conclusions state that the results did not differ from earlier authors including Chu and Moe. Reviewer recommends that the author formulate the analysis addressed in the last paragraph of the paper, specifically to analyze the relationships for a tapered pipeline. Completion of this task and incorporation into the manuscript would make the paper a significant contribution to the knowledge base.

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EVALUATION (Circle One)

Poor 1 | Below Average 2 | [✓ Average 3] | Above Average 4 Possible Award Quality | Exceptional 5 Award Quality

*Note to Reviewer: Please sign reverse side to permit direct copying while retaining anonymity.

***** Individual *****

JUL 0 6 1999 C

ASCE JOURNAL MANUSCRIPT REVIEW FORM

Division IR File No. 020655-IR

Author(s) ANWAR
Title CORRECTION FACTORS FOR CENTER PIVOTS
WITHEND GUNS

Paper XXX Technical Note ____
New Submission XXX Resubmission ____ Re-review ____

****

RECOMMENDATION

____ Accept in present form

X Accept, consider suggested revisions

____ Tentatively accept, revisions required
Re-review required? ____

____ Decline, encourage revision and resubmittal

____ Decline as paper, consider resubmittal as technical note

____ Decline

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CRITIQUE (See instructions on reverse side)

Paper is generally well written, short and adequately supported with Tables + Figures. It further amplifies previous work, especially the pioneering effort of Chu and Moe (1972). Specific comments are provided on the paper. The author should specifically address the observations regarding the examples chosen. Also explain that the solutions are in reality infinite series, etc.

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EVALUATION (Circle One)

Poor 1	Below Average 2	[✓ Average 3]	Above Average 4 Possible Award Quality	Exceptional 5 Award Quality

*Note to Reviewer: Please sign reverse side to permit direct copying while retaining anonymity.

B1

Patrick J. Purcell (2000) "Discussion on 'Factor *G* for Pipelines with Equally Spaced Multiple Outlets and Outflow' by Arif A. Anwar." and closure by author. *J. Irrig. and Drain. Engrg.* ASCE 126(2), 138-140.

FACTOR *G* FOR PIPELINES WITH EQUALLY SPACED MULTIPLE OUTLETS AND OUTFLOW[a]

Discussion by Patrick J. Purcell[2]

The discusser would like to compliment the author on his interesting paper and on his modeling efforts. The key advance proposed by the author is the development of an analytical solution to the problem of calculating the head loss in a pipe manifold with outflow at the downstream end. However, some elements of the overall solution deserve further attention and will be addressed here. The following assumptions made in the paper are discussed:

- Friction factor (*K*) remains constant along the pipe manifold.
- Equal discharge (*q*) at each outlet port.
- Energy losses at each outlet port are balanced by the pressure rize downstream of the port.

The discusser will comment on these assumptions by referring to measurements made by him on a laboratory-scale pipe manifold apparatus, schematically illustrated in Fig. 1.

The data relevant to the system illustrated in Fig. 1 and to a typical experimental run are as follows: internal pipe diameter, 27.66 mm; pipe material, PVC; orifice diameter, 5.95 mm; orifice type, square-edged; orifice spacing, 150 mm; and Q_I, 0.446 L/s.

In relation to the modeling of friction, the author makes a fundamental, but unstated, assumption that the friction factor *K* remains constant along the pipe manifold. In the case of the Hazen-Williams friction formula, this assumption is correct (although other errors are introduced into the *G* factor formulation because of the exponent of the velocity term of 1.85, as pointed out by the author). In the case when pipe friction is being modeled using the Darcy-Weisbach friction formula, *K* cannot clearly have a constant value, since it is a function of the velocity, which is changing from one end of the pipe to the other. The assumption underlying (12), that

$$\sum_{k=1}^{N} K(k + Nr)^{m} = K \sum_{k=1}^{N} (k + Nr)^{m}$$

is therefore, strictly, not correct when friction is being modeled by the Darcy-Weisbach formula.

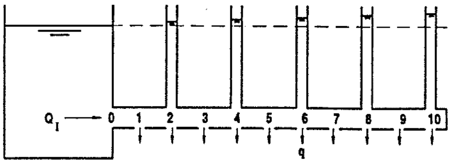

FIG. 1. Schematic Layout of Laboratory Manifold Apparatus

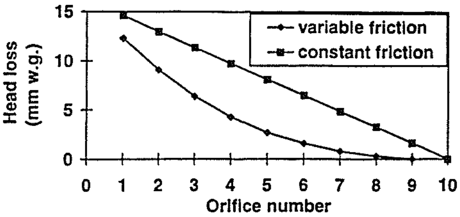

FIG. 2. Computed Head Loss due to Friction along Pipe Manifold

The discusser presents a comparison in Fig. 2, for the laboratory-scale pipe manifold system shown in Fig. 1, of the computed head loss along the pipe manifold, using the factor *G* method (*f* calculated using Q_I, referred to as "constant friction" in Fig. 2) and using the manifold flow rate at each orifice to calculate the head loss in the pipe segment between consecutive orifices (referred to as "variable friction" in Fig. 2).

Examination of Fig. 2 shows that the *G* factor method, using a constant friction factor, overestimates the head loss in the pipe manifold. The inclusion of a variable friction factor requires a stepwise analysis starting from the most downstream outlet, working upstream and computing the head loss caused by friction in each pipe segment.

Regarding the assumption of equal discharge (*q*) at each outlet port, clearly the primary objective in manifold design is the achievement of a nearly uniform discharge rate through the outlets of the manifold system. In general, this objective can be achieved by ensuring that the ratio of total head variation in the manifold system to the head loss across individual outlets is kept low (Casey 1992). Rawn et al. (1961) found that a nearly uniform orifice discharge could be achieved by ensuring that the sum of all the orifice areas is less than the cross-sectional area of the pipe. Examination of Fig. 3 shows the difficulty in practice of achieving nearly uniform discharge; although the ratio of the total orifice area to the pipe cross-sectional area is only 0.44, there is still a considerable variation in orifice discharge (±4% from the mean orifice flow rate) along the pipe manifold.

In relation to the pressure distribution along the pipe manifold, the author states that "in a pipeline with multiple outlets, there will be energy losses caused by the coupler and structure of the outlet. However, there also is a gradual reduction in velocity head as flow passes the outlet and this will cause an increase in pressure, which will balance losses caused by turbulence at outlet couplings." The implication of this statement is that there is always a net reduction in pressure head from the upstream end of the pipe manifold to the downstream end, the loss of head being due to pipe friction. This is not always the case, as illustrated in Fig. 4, which shows the pressure head increasing in the direction of flow for the laboratory pipe manifold shown in Fig. 1.

Whether the hydraulic grade line rises or falls from one end of the pipe manifold to the other depends upon the relative magnitude of the following contributory effects:

1. Loss of pressure head due to pipe friction along each reach of pipe between successive outlet ports
2. Recovery in pressure head downstream of each outlet port due to a reduction in velocity head caused by outlet discharge

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[a] January/February 1999, Vol. 125, No. 1, by Arif A. Anwar (Paper 17715).

[2] Lect., Dept. of Civ. Engrg., Univ. College of Dublin, Dublin, Ireland. E-mail: PPurcell@Iveagh.UCD.IE

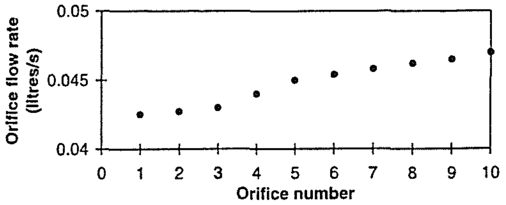

FIG. 3. Measured Orifice Discharges along Pipe Manifold

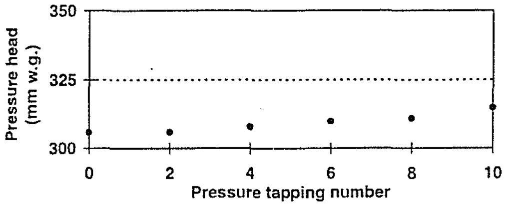

FIG. 4. *Measured Pressure along Pipe Manifold*

3. Loss of pressure head at each outlet port due to form losses at the outlet port

In the case of pipe manifolds with orifice discharges, McNown (1954) has demonstrated experimentally that the form head loss at such outlets (third effect above) is negligibly small. In the reach of manifold pipe between a pair of outlets, the pressure head must clearly fall in the direction of flow (first effect), with a step increase in pressure downstream of each outlet (second effect), as demonstrated experimentally by Acrivos et al. (1959).

In conclusion, the discusser suggests that factor G, due to the simplifying assumptions invoked, should be considered a simple but approximate method of calculating head loss in pipe manifold systems. If the upstream manifold flow rate is used as the basis for calculating friction factor K, the method proposed by the author is likely to result in conservative estimates of head loss in the pipe manifold. More accurate estimates require a stepwise analysis, which can be easily programmed using modern spreadsheet programs, as demonstrated by Pretorius (1997).

APPENDIX. REFERENCES

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Closure by Arif A. Anwar[3]

The writer would like to express his gratitude to Dr. Purcell for his interest in the paper and for raising and clarifying important issues. The discusser has presented three pertinent issues, which invite further discussion.

The discusser quite correctly points out that in the development of factor G, the friction factor is assumed constant along the lateral. This is also an assumption in the development of factor F (Christiansen 1942) and the adjusted factor F_a (Scaloppi 1988). If the Darcy-Weisbach friction formula is used, then the friction factor is, in fact, a variable. Kincaid and Heerman (1970) used a stepwise computation to analyze head loss and pressure distribution of a center pivot using the Darcy-Weisbach formula. They showed that using typical roughness values for a steel pipe gave slightly lower total head losses than were measured experimentally, and that assuming higher roughness gave better agreement with the data. Therefore, even using the Darcy-Weisbach equation, one needs to choose a pipe roughness appropriately. Alternatively, one could assume a constant friction factor provided that an appropriate value for the friction factor is selected, i.e., not necessarily the same as that for pipelines without outflow. The friction factor at the head of the pipeline may not be appropriate for the entire pipeline. What constitutes an appropriate friction factor could be the subject of further work. The discusser presents Figure 2 to demonstrate that assuming a constant friction factor overestimates the head loss in the lateral. However, Kincaid and Heerman (1970) showed that in center pivot laterals, using a variable friction factor actually underestimated head loss as compared to experimental data. The writer is of the opinion that the accuracy of factor G should be compared against experimental data for full-scale sprinkler laterals.

The discusser has also raised the issue of equal discharge at each outlet. In sprinkler lateral design, variation in discharge at outlets is minimized by keeping head loss less than 20% of the outlet operating head (Keller and Bliesner 1990). Smith (1990), in his discussion of adjusted factor F_a, has also pointed out that the head may actually increase along the length of the pipe, which leads to an increase in outlet discharge. Scaloppi (1990) conceded that, for low pressure pipes as investigated by Smith (1990), there is a significant change in velocity head along the pipe and no balance between pressure gains and losses as flow passes each outlet; however, this is not the case for high pressure systems. Scaloppi and Allen (1993) have shown that for sprinkler irrigation laterals, simplified equations ignoring velocity head (i.e., assuming balance between gains and losses at outlets) provides acceptable approximations (error 0.99–3.49%). Scaloppi and Allen (1993) have supported the argument by Smith (1990) that velocity head plays an important role in computations involving low pressure head systems. In such systems the pressure may increase along the pipeline, as shown in Fig. 4, and therefore discharge of outlets increases, as shown in Fig. 3. In contrast, Valiantzas (1998) has considered the discharge of drip emitters to decrease along the length of the lateral, and therefore assumed the variation of discharge along a drip lateral varies as a power function. This is clearly an area that would merit further research.

The discusser concludes with the point that spreadsheets can be easily used to analyze laterals. Smith (1990) and Scaloppi (1990) have made similar remarks. Friction correction factors continue to be presented in textbooks on irrigation system design, e.g., James (1988), Cuenca (1989), Keller and Bliesner (1990), as a simple method of analyzing laterals. Friction factors also avoid the use of iterative calculations otherwise needed to calculate the inlet pressure (Anwar 1999), although one may also argue that spreadsheets can be used for iterative calculations. Factor G is presented as a more generic friction factor to factor F. The assumptions made in its development are the same as those made in developing factor F, and it should be used subject to the same limitations. Its use was demonstrated as an alternative method to factor F in designing tapered laterals, which can also be done using stepwise calculations on the spreadsheet. The writer hopes it will be seen simply as another tool for engineers to use if they wish to, whether it is for design, a field check, or a quick check of the correctness of a spreadsheet or any other computer program.

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