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Current Controversies in the Literature on Time Inconsistency and Monetary Policy

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ABSTRACT

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CURRENT CONTROVERSIES IN THE LITERATURE ON TIME INCONSISTENCY AND MONETARY POLICY
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This thesis comprises of four major essays concerning current controversies in the literature on time inconsistency and monetary policy. Taken together, the essays suggest that some of the most debatable issues can be resolved by means of refinements of both definitions and setting of the standard framework. The soundness of the theory in ensuring a rigorous analysis of the complex interaction between private agents and monetary institutions and the role played by credibility in this relationship is reinforced by my findings. However, I argue that the overly simplified analytical framework sets the limit of the explanatory power of the time-inconsistency paradigm for historical episodes of stagflation. I contribute to the existing literature by providing new theoretical confirmation that the positive implications of the time-inconsistency paradigm are effectively based on a too stylised model. On the other hand, I suggest also alternative formalisations reflecting stylised facts on central bank's behaviour that provide a firmer foundation of understanding on which to proceed on some major questions of theoretical ambiguity.

I examine the importance of the inflationary bias of time consistent monetary policy by using an extended version of the simple Barro-Gordon framework featuring important aspects of actual policy making. The model developed provides a counterexample to the standard theory as it yields the result that a deflationary bias may be possible as well. I identify the rationale for this surprising result in the distortion caused by instrument uncertainty in the trade-off between the costs and benefits associated with surprisingly lower interest rates faced at the margin by the policy maker.

A key debate in monetary policy is that on the importance of following systematic behaviours. I revisit the debate on rules versus discretion focusing on the design of instrument rules in a manner that push discretionary policy choices in the direction of the commitment equilibrium. I demonstrate that an instrument rule with an optimal degree of monetary inertia may render negligible the inflationary bias associated with discretion without necessarily implying a trade-off between flexibility and commitment. In particular, I make a substantial contribution on the puzzling issue of the optimality of interest-rate smoothing by showing that gradualism may enhance credibility as it contrasts the incentive to exploit employment and output gains deriving from inflation surprises.

The real effectiveness of monetary delegation in overcoming the problem of time inconsistency has been recently questioned. Jensen has shown that optimal policy can be credible under delegation only if reappointment costs are prohibitive. Conversely I make evident that, when delegation is not considered as an alternative, but rather as supplementary, to reputation and is conducive to reputation building for the central banker, the circumstances under which optimal monetary delegation can be credible need not be so extreme.

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Preface

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The thesis is dedicated to the memory of Giuliana.

Chapter 1

Introduction

During the last decade there has been a great resurgence of interest in the credibility of monetary policy. One of the symptoms of this phenomenon is the enormous amount of publications on the topic. Another is that many central banks of industrialised countries have been involved with important institutional reforms aimed at improving their performance in attaining and maintaining price stability.

It is not an exaggeration to say that central bankers have become obsessed with credibility. To the aim of improving their accountability, they have adopted measures for increasing the transparency of policy decisions. Nowadays, several central banks publish inflation reports, which contribute to a widespread understanding of ex post outcomes and provide central bank's forecasts on inflation. They started also to publish voting details with minutes from their meetings on monetary policy decisions. Many central banks have now price stability as primary goal in the Law concerning them, in sharp contrast with the previous feature of not having a clear and specific mandate.

The background to the development of the credibility literature is one where central banks succeed in maintaining price stability whilst stabilising the real economy. The literature has been influential in emphasising the potential benefits of credibility in policy regimes aimed at low inflation and defining rigorously what are the key ingredients for enhancing the credibility of monetary policy.

Private sector behaviour depends on both the expected evolution of monetary policy and the assessment of the current policy. In the relationship between private agents and monetary authorities credibility can play a crucial role either from the point of view of those agents who infer from past behaviour future inflation outcomes, or from the stand point of those agents who evaluate whether price stability is really the central bank's goal and if existing institutional arrangements represent an obstacle to achieving this goal. Low and stable inflation expectations can be interpreted as good credibility for low and stable inflation periods. The main contribution of the credibility literature is that of examining what are the options available to policy makers for dealing with potential situations of inertia of expectations to a suboptimally high level of inflation due to a lack of credibility of low-inflation policies.

The credibility literature is based on the concept of time inconsistency (or dynamic inconsistency), introduced by Kydland and Prescott and examined in detail for monetary policy by Barro and Gordon. This concept dates back to the end of the 70s. In the last decade many new results have been added and have contributed to enrich the policy recommendations of this literature. This thesis provides an assessment of the state of the art. I argue that a number of significant controversial issues still remain and need to be examined more deeply. In chapter 2, I organise synthetically the contributions relative to some of the main controversies recently emerging in the literature. I highlight three major controversial topics, that in my opinion represent also the most promising areas of research. These controversial topics are the object of the analysis presented in the subsequent three chapters of the thesis.

The first controversy that I examine is that of the importance of the inflationary bias implied by the issue of dynamic inconsistency. The notion of dynamic inconsistency is one of the answers given by economists to the existence of an inflationary bias. However, despite its popularity the issue of time inconsistency seems to be quite controversial. There is a widespread agreement on the existence of an inflationary bias as in most countries inflation has risen above any thinkable optimal level during their economic history. On the contrary the importance of the concept of time inconsistency in explaining it does not seem to share the same degree of consensus. There are several reasons for dissenting.

On one side it has been argued that it seems unconvincing that a discretionary solution would prevail in situations where the superiority of the optimal rule is generally understood. Hence the inflationary bias may not be quantitatively relevant and it does not necessarily need to be removed.

On the other side the criticism has been raised that the Barro-Gordon model is not a plausible positive model of inflation as the absence of precommitment technologies does not preclude a central banker from behaving in a committed way and private agents to rationally expect such optimal behaviour.

Another kind of criticism concerns the specification of the Barro-Gordon model. In particular it has been shown that the finding of excessively high average inflation under discretionary monetary policy might be a consequence of the simplicity of the model considered. Here, it has been shown that, if greater realism about the conduct of monetary policy is introduced in the Barro-Gordon setting, the dimension of the inflationary bias is considerably reduced and the explanatory power of the time-inconsistency paradigm is undermined. Moreover it has been shown also that by introducing explicit micro-foundations or asymmetric preferences in the analytical framework we may even have a deflationary bias. Obviously with this finding the question of the importance of the time-inconsistency concept in explaining the manifest inflationary bias of economic policy ceases to exist.

The inflationary bias of time consistent monetary policy is re-examined in Chapter 3. To this end I consider an extended version of the simple Barro and Gordon framework featuring important aspects of actual policy making such as imperfect instrument control, overlapping wage contracts, policy lags and interest rate control. The model developed provides a counterexample to the standard theory as it yields the result that a deflationary bias may be possible as well. The rationale for this surprising result is found in the distortion caused by instrument uncertainty in the trade-off between the costs and benefits associated with surprisingly lower interest rates faced at the margin by the policy maker. If the size of uncertainty is relatively large the distortion created may imply an optimal choice for the instrument which trades off the marginal benefit of lower deflation against the marginal cost of higher than optimal output. In this chapter I discuss also the implications of imprecise instrument control for welfare.

A second important controversy concerns the theoretical debate on rules versus discretion in monetary policy. Barro and Gordon interpreted the time inconsistency explanation of the inflationary bias as a case for rules over discretion. With the policy maker committed to follow a policy rule designed to achieve low inflation it is not possible to fool the private sector by inflating after nominal contracts are set. Hence the problem of dynamic inconsistency is eliminated.

However, when the Barro-Gordon framework is extended to incorporate shocks the ex-ante optimal monetary policy rule becomes contingent on the state of the world. Unfortunately, real world policy makers cannot easily commit to a state-contingent monetary policy rule. Commitment to simple rules is still possible but in this case we have a trade-off between the benefits of avoiding the inflationary bias of discretionary policy and the potential costs of being bound to follow a monetary rule that is no longer appropriate.

Some economists have tried to find compromises in order to get over this dilemma. In this perspective the formulation of simple rules with explicit escape clauses can be seen as a useful compromise. Some other economists increasingly have viewed rules not as constraints imposed on central banks externally, but as time-consistent means of operating internally - for example, as explicit starting points for consideration of current policy options. So in that sense there need not necessarily be a trade-off between flexibility and commitment.

An alternative view on the rules versus discretion dilemma is provided by Taylor. He observes that in practice a policy rule can be defined more broadly as a systematic behaviour and therefore there is no need to follow mechanically an algebraic formula. In this perspective the rules versus discretion dilemma is more a semantic issue. However McCallum shows formally using the Barro-Gordon framework that the presence or absence of a systematic behaviour is not enough to distinguish between rule-like behaviour and discretion. Policy makers may follow a systematic behaviour in the setting of the monetary instrument also under a discretionary regime but this choice does not prevent the economy from giving rise to an inflationary bias.

In chapter 4 I revisit the debate on rules versus discretion. In particular I examine McCallum's criticism to Taylor's proposal of refining the definition of policy rules beyond the idea of a fixed mechanical formula. Here I focus on the design of instrument rules in a manner that push discretionary policy choices in the direction of the commitment equilibrium, keeping at the same time some flexibility in the conduct of monetary policy for stabilising shocks. Instrument rules are compared with target rules, interpreted as an alternative to instrument rule, and are examined in terms of the rules versus discretion debate.

The analysis is based on the introduction of a particular rule for setting the interest rate which presents features consistent with some stylised facts on central banks behaviour. First, it is in agreement with the recurrent use of operative targets specified in terms of a reaction function where macroeconomic variables are related to the interest rate via some parameters, whose values are optimally chosen. Second, it incorporates some degree of monetary inertia which reflects the widespread practice of interest-rate smoothing by central banks.

One of the main finding of the analysis is that an optimally designed instrument rule may render unimportant the inflationary bias associated with discretion without implying a trade-off between flexibility and commitment. Moreover the model developed sheds some light on the puzzling issue of the optimality of interest-rate smoothing.

A third controversial topic is that of the real effectiveness of delegation in overcoming dynamic-inconsistency problems. Despite its recent positive achievements the literature on monetary delegation has been object of an important criticism by McCallum, who argues that there is always the temptation for the government to renege on the chosen monetary institution. Hence the institutional remedies proposed by this approach do not fix the problem of time inconsistency but merely relocate it. The answer to this criticism has been that of assuming the presence of high costs of changing monetary institutions for the government. However, an important contribution by Jensen has shown that the presence of reappointment costs worsens the credibility of optimal monetary policy under delegation compared to the case when monetary policy is conducted directly by the government. This result implies a negative view of the delegation approach in the sense that the benefits of optimal monetary delegation appear to be overestimated.

In chapter 5 I examine the current debate on the real effectiveness of delegation in overcoming the problem of time inconsistency that afflicts discretionary monetary policy. Jensen has shown also that when the government is unable to credibly carry out optimal policy and delegates monetary policy to a central banker with an announced incentive scheme, optimal policy can be credible only if reappointment costs are prohibitive. This finding is questioned in the present analysis. In particular I show that, when delegation is not considered as an alternative, but rather as supplementary to reputation and is conducive to reputation building for the central banker, the circumstances under which optimal delegation can be credible need not be so extreme. This different result is based on the constraint that the central banker's reputation for low inflation imposes on the government's temptation to deviate from its announcements and on the role played by incentive schemes in strengthening the central banker's reputation.

Chapter 6 concludes and provides some suggestions for further developments arising directly from the issues examined in this research. Here I discuss also the policy implications of the research.

Chapter 2

A survey of the literature on time inconsistency and monetary policy with some recent controversies

2.1 Introduction

During the last forty years, average inflation in most countries has been variable and highly persistent. It was low in the 60s, very high in 70s, while it came down in the last two decades. However, economists were particularly struck by the strong first-order serial autocorrelation that inflation has exhibited during these years. Since sustained inflation is possible only if the money supply also increases in a sustained way, persistent inflation would not have been possible if policy makers had not allowed or tolerated the persistent expansion of national money supplies. Thus an explanation of these stylised facts implies an explanation of monetary authorities behaviour. During the past decade the literature on credibility and independence of a central bank has focused on the role of dynamic inconsistency (or time inconsistency) in explaining the inflationary bias determined by the behaviour of a central bank under a discretionary policy regime.

In the present chapter, after recalling briefly the main explanations for the existence of an inflationary bias, I summarise all the different theoretical approaches proposed for solving the inflationary bias problem associated to the issue of dynamic inconsistency. Furthermore I examine in greater detail some recent controversial topics that have raised a certain amount of interest in the literature. These topics are the object of the analysis that will be developed in the next chapters. In the last part of the present chapter I review the empirical literature that tests the predictions of the theoretical literature.

2.2 The inflationary bias and its explanations

There are basically three explanations for an inflationary bias in economic policy: seigniorage, the notion of dynamic inconsistency and political business cycle.[1]

The most obvious explanation for an inflationary bias is seigniorage. Seigniorage revenues are defined as the amount of real resources bought by the government through new base money injections. Thus monetary expansion is an alternative way of financing government expenditures. The ability of government to borrow from the central bank is often restricted. But the extent to which the government is allowed to borrow from the central bank varies across countries.

The second explanation of an inflationary bias, i.e. the notion of dynamic inconsistency, is due to the pioneering work of Kidland and Prescott (1977). It forms the basis of most recent models of central bank behaviour. In these game theoretic models the crucial issue is that when policy is discretionary, the rate of inflation is excessively high

[1]Nominal interest rate targeting has also been considered as an explanation for the inflationary bias in the 1970s. But it can be shown that nominal interest rate targeting may produce an indeterminate inflation rate if inflation expectations are adaptive, but will not do so if expectations are rational. More important, combining any nominal anchor with a nominal interest rate target keeps the price level determinate (Blanchard and Fischer (1989, pp. 577-580)). Thus a central bank that fixes its nominal interest rate no matter what happens, may produce an accelerating inflation; whilst a central bank that increases the target nominal rate when inflation increases, not necessarily will suffer the same problem.

Cukierman (1992) offers four reasons for an inflationary bias: the employment or short-run Phillips curve motive; a fiscal revenue or seigniorage motive; interest rate smoothing or financial stability motive; and a balance of payments motive or the desire to devalue to improve a current account deficit. On the contrary, here we treat the revenue motive and the political business cycle motive as potentially separate causes of an inflationary bias.

because of dynamic inconsistency problems. Dynamic inconsistency occurs when the optimal policy planned currently for some future period is no longer the optimal when that period arrives. This issue will be further developed more deeply.

A third explanation of the inflationary bias comes from the literature on political business cycle models. The basic argument in this kind of approach is that monetary policy may be influenced to some extent by the political process in democratic societies. In particular these types of models, exemplified by Nordhaus (1975), Minford and Peel (1982) and Alesina (1987), predict that governments will raise inflation before elections in order to achieve employment gains.

2.3 Dynamic inconsistency

This section analyses in greater detail the issue of dynamic inconsistency as developed by Barro and Gordon (1983a) in the framework of a game theoretic model of monetary policy. In this set up the private sector behaves according to an expectations-augmented Phillips curve, while the government - a single centralised policy maker - behaves according to a loss function to be minimised. What matters in this loss function is that positive inflation is costly for the government whereas employment above the natural level is welfare improving .[2]

Conventional wisdom in macroeconomics implies that employment and output can be influenced by unanticipated inflation and therefore by unanticipated monetary growth. This can result from either the existence of Fischer (1977) - Taylor (1980) contracts or a Lucas (1973) type short-run Phillips curve. To the extent that monetary policymakers

[2] Some existing distortions or externalities in the economy, such as distortionary taxation (e.g. income taxes that cannot be levied on returns from leisure) and unemployment benefits, cause the natural level of employment to be lower than that which would be optimally preferred and would be achieved in the absence of such distortions. Another crucial assumption in this loss function is that concerning the target level of employment with respect to witch deviations of the actual level of employment are considered. Namely the target level of employment is assumed to be constant. This is an important assumption and, as observed by Persson and Tabellini (1990), here is a point where the lack of microfoundations becomes troublesome.

find the natural level of employment too low, they may be tempted to create monetary surprise in order to push employment above its natural level even at the cost of some inflation.

The private sector has the same preferences as the government, so it also agrees in maximising the described social welfare function. But since private agents are atomistic, each agent disregards any effect of his own choices on economy-wide variables and they are only interested in forecasting accurately inflation using all the available information in terms of a rational expectations formation mechanism.

In this model it is precisely because desired employment is higher than the natural rate that a conflict of interest arises between the government and the private sector. In particular this conflict derives from the design to minimise the output and employment costs of a disinflationary program by the government and the consequent inability to commit itself to a disinflationary program. Only if it is possible to commit policy in advance the announced disinflationary program will be believed by the public and the government will be able to internalise the effects of its decision rule on expectations in its problem of minimising the intertemporal social loss function. On the contrary in a discretionary monetary regime the possibility that the government can fool the public will imply a loss of control over the private sectors expectations on inflation. Thus the mere presence of an ex post incentive to deviate from the equilibrium policy rule and create surprise inflation will make non credible the announced disinflationary program, regardless of what the government does in the future.

The comparison between the discretionary equilibrium and the commitment equilibrium shows that the inability to commit results in a higher inflation rate, or inflationary bias, but leaves employment unchanged. Thus the policy game described results in a Prisoners dilemma type of outcome, in which non co-operative equilibria need not be Pareto optimal.

One of the most appealing feature of this model is that it provides, within a theory of the role of monetary policy, a rigorous definition of credibility: a policy is credible when it is dynamically consistent, and not credible if it is dynamically inconsistent.

In the following sections we review the main answers given by economists to the question of how societies can deal with potentially dynamically inconsistent situations.

2.4 Central bank independence

2.4.1 "Weight-conservative" central banker

Delegation of monetary policy to an central banker independent from elected governments is one of solution proposed in the literature for dealing with the credibility problem. Rogoff (1985) proposed appointing a conservative central banker who dislikes inflation more than everyone else in the society. There are two important issues about the equilibrium obtained in the Rogoff model. First, a conservative central banker generates a lower inflationary bias but does so by not stabilising the economy in a socially optimal fashion. Second, there is an optimal degree of inflation aversion on the part of the central banker, which means that the central banker can be excessively inflation averse .

Formally, both the central banker and society are assumed to prefer inflation and output levels that are close to a common target level, but the central banker weights deviations of inflation from target relative to output deviations more heavily than society does. The central bankers aversion to inflation reduces the average inflation rate, even if he still has the discretion to conduct stabilising countercyclical policy. But in this model flexibility is not gained without a cost: a trade-off between the reduction in the inflation bias and the increase in the variability of output and employment, higher than that attainable under a socially optimal policy, is implied.

For the conservative central bankers policies to be credible, society must believe that he cannot be removed ex-post by the current government. Thus, the central banker must have some degree of independence to pursue policies that are not desired by the current administration.

Subsequent research by Lohmann (1992) showed that complete independence is not socially optimal for certain bad states of the world. In particular Lohmann introduces the case when the conservative central banker can be overruled by the government at a cost. This yields a non-linear rule in which the central bank reacts relatively more intensively to large rather than to small perturbations, in a manner that the government never actually overrules the central banker. The outcome reached under this rule is better than that under the Rogoff solution.

Finally, it has been shown that there is no empirical support for Rogoff's trade-off between average inflation and output variability. Alesina and Summers (1993) found that both increased central bank independence and emphasis on price stability are positively correlated with lower average inflation without increased output variability.

2.4.2 Principal-agent approach

The second main strand of theory that have added precision to the analytic argument for central bank independence is the principal-agent approach developed by Persson and Tabellini (1993), Fratianni, Waller and Von Hagen (1993) and Walsh (1995a).

In this approach, a principal (the government, with the same preferences of society) with well-defined targets has to design a contract that will motivate an agent (the central bank) to act according to the principals objectives. In general the agent has an information advantage with respect to the principal. Walsh (1995a) and Persson and Tabellini (1993) have shown that a contract between the government and the central banker in which the inflation penalty on the central bankers remuneration is linear in inflation can attain the first best equilibrium (or more exactly the second best equilibrium, as the first best one requires elimination of the distortion in the labour market). With this type of contract the central banks countercyclical policy is optimally active whilst dealing with the inflationary bias of monetary policy. In this framework the basic assumption is that the central bank has the identical loss function as the government. The target inflation rate in the considered contract should be dependent on any shocks that affect the optimal dynamically consistent inflation rate. The model requires the implicit assumption that it is costly to change the contract: so it is possible to avoid the temptation for the principal to behave in a dynamically inconsistent way by changing the contract ex-post. In the principal-agent approach the Rogoff's trade-off disappears and optimal stabilisation policy with no inflationary bias can be achieved by a contract that depends only on publicly observable variables.

These two prevailing approaches to setting up institutions that push the discretionary policy in the direction of the commitment policy, point to different forms of central bank independence. Debelle and Fischer (1994) and Fischer (1995) introduce the distinction between goal independence and instrument independence, in order to make clear the difference between the two approaches considered. Following Fischer a central bank that is given the control over the levers of monetary policy and allowed to use them has instrument independence; a central bank that sets its own policy goals has goal independence. In the Rogoff approach the central banker has both goal and instrument independence, while in the principal-agent approach he has no goal independence but does have instrument independence. Furthermore, he stresses that the most important conclusion of both the theoretical and empirical literature is that a central bank should have instrument independence, but should not have goal independence. Rather, the central bank should be given a clearly defined goal or set of goals, and the power to achieve them, and should be held accountable for doing so. Accountability is needed for two reasons: first, to set incentives for the central bank to meet its goals and explain its actions; and second, to provide a democratic oversight of a powerful political institution.

Svensson (1997a) and Lockwood (1997) extend the contracting solution to the case of unemployment persistence.

The principal-agent approach has been implemented in both Canada and New Zealand, where a formula is provided to adjust the inflation target if there are supply shocks and if indirect taxes are introduced. Recently also the United Kingdom can also be thought of as pursuing the principal-agent approach, provided the penalty for excess inflation is interpreted (realistically) as the central bankers loss of reputation. There are also elements of the principal-agent approach in all countries where the central bank has reasonably clearly defined goals, such as in Germany where the Bundesbank is given the task of safeguarding the currency.

2.4.3 Inflation targeting regimes and "target-conservative" central banker

Relative to the implementation of the principal-agent approach to reality there are some controversial aspects. For instance according to Walsh (1995b) even the relationship between the central bank and the government in New Zealand is not exactly an inflation contract of the type proposed in the literature. This implies that the contracting solution, while theoretically appealing, is rarely observed in the real world. This contrasts with the pervasive adoption of inflation target regimes by developed countries. New Zealand (1989) and Canada (1991), along with United Kingdom (1992), Sweden (1993) and Finland (1993), belongs to the group of countries that has recently adopted inflation target regimes.

As explained by Leiderman and Svensson (1995) the introduction of explicit quantitative inflation targets can be seen as consistent with a three-part strategy for improving monetary policy performance which includes: price stability as the primary goal for monetary policy; central bank independence in choosing the means for achieving the goal of monetary policy; central bank accountability for achieving the goal. In other words an inflation target regime can be interpreted as an ideal form of delegation of monetary policy to a central bank. With such type of delegation, the central banker has instrument independence rather than goal independence.

In Svensson (1997a) the performance of inflation target regimes is examined with and without persistence in unemployment.[3] He shows that delegating monetary policy to a

[3] His analysis draws on earlier work by Lockwood, Miller and Zhang (1994) and Lockwood and Philippopoulus (1994) which extends the static Barro-Gordon framework to a dynamic setting with unemployment persistence.

central banker with an inflation target lower than that socially optimal may remove the inflationary bias that affects discretionary policy. The main lesson from his setting is that inflation targets may on average be exceeded and they may have imperfect credibility. However this kind of delegation may be useful in reducing the inflationary bias, with the advantage to be much easier to implement than the contract implied by the principal-agent approach. Nevertheless in the real world only the New Zealand inflation target regime is consistent with the three-part strategy for improving monetary policy performance, as required by Leiderman and Svensson. In other countries with inflation targets, the accountability of the central banks for achieving the targets is far more ambiguous, because it is based on unilateral declarations of central banks.

The subsequent research has added some important qualifications to several of the most recent findings of the delegation approach.

First, both the contracting and the inflation target approaches eliminate the inflationary bias and, hence, they are superior to Rogoff's solution. However, the importance of the conservative-central-banker approach can be restored by adding more details in the structure of the economy. This result is derived in the article of Herrendorff and Lockwood (1997) by allowing for the private sector to have an information advantage regarding the structure of the economy with respect to the government at the date of delegation.

Second, Svensson has shown formally that an optimal inflation target under a regime with a "target-conservative" central banker can be equivalent to an optimal linear inflation contract along Walsh-Persson-Tabellini lines. But as shown by Beetsma and Jensen (1998) the equivalence breaks down when uncertainty about the central banker's preferences is introduced in the analysis.

Muscatelli (1998) is another contribution that examines uncertain central banker preferences, with a comparison of the alternative solutions proposed in the delegation approach. Another interesting aspect of this type of analysis is that, in contrast to the standard theory, it implies a restriction of the circumstances under which delegation can be optimal. This finding supports the idea that delegation may have benefits as well as costs when there are asymmetries between the preferences of central bankers and society.

2.5 Reputational enforcement

Reputation for non-inflationary behaviour is another way of dealing with dynamic inconsistency and the alleged inflationary bias. This strand of theory examines the situation when repeated interactions among the players take place. Policymakers by acting consistently over long periods can build up a reputation that will cause the private sector to believe their announcements.

In particular Barro and Gordon (1983b) have shown that, if the game is repeated for an infinite number of periods and the private sector adopts a punishment strategy triggered by any observed deviation from the optimal policy, it is possible that the future cost for the government of losing its reputation for being committed to zero inflation may more then outweigh the current gain from deviating. The optimal policy is more likely to be sustainable as a reputational equilibrium when the government does not discount the future too heavily. Technically speaking, reputation is defined as the set of discount factors securing that the reputational enforcement is greater than the temptation to deviate from optimal policy and create inflationary surprises.

The framework of Barro and Gordon (1983b) suffers from many criticisms: the existence of a multiplicity of reputational equilibria; the unresolved question of how a decentralized economy, with atomistic agents who are precluded from acting collusively, happens to coordinate upon one particular equilibrium. Al-Nowaihi and Levine (1994) try to deal with these problems. They consider a particular credibility condition, called chisel-proof, and assume the existence of a single trade union. Herrendorff (1998) extends their analysis to the case when there is imperfect monitoring of the policy maker's action.

2.6 Understanding the role of information structure

2.6.1 Secrecy

If we assume that the realisation of shocks cannot be observed by the public, a natural question arises. What is the optimal information structure for the central bank if it publicly reveals his information to the private sector? And in particular how can the central bank be induced to reveal the correct information to the private sector? These questions have been addressed by Stein (1989) and Garfinkel and Oh (1995).

Actually, as shown formally by Persson and Tabellini (1990), secrecy about the realisation of the shocks can be beneficial because enables the policymaker to stabilise employment while he loses this capacity by revealing his private information on observation of shocks. But the model used is too simple to really address the welfare consequences of secrecy. According to Persson and Tabellini the only general argument for secrecy is that it enables the policymaker to time its policy surprises when they are most valuable to him.

In addressing the above questions about secrecy aspects, Stein (1989) shows that, by making imprecise announcements, the central bank can credibly reveal some ranges for the privately observed variable. The specific mechanism of communicating some information about its goals adopted by Stein is that of the cheap talk mechanism of Crawford and Sobel: making announcements that are imprecise, and only giving ranges within which these goals may lie. Whereas Garfinkel and Oh (1995) study the role of noisy or imprecise announcements in mitigating the credibility problem in monetary policy. Based on a positive analysis of verbal communications and secrecy, these papers show that there is an important role for communication in monetary policy .[4]

[4]Other papers that explicitly focus on the effects of policy announcements are Andersen (1989), Cukierman and Liviatan (1991) and Schultz (1994).

2.6.2 Reputation

Secrecy is most closely related to the literature on reputation and time-consistent monetary policy under asymmetric information. This is exemplified by the works of Backus and Driffill (1985) and Barro (1986), where it is assumed that the policymakers preferences about inflation are private information and a problem of communication or signalling arises.

In particular, the model of Backus and Driffill rationalises why a government which looks some period ahead, may rationally stick to low inflation despite the short-run temptations to inflate.[5] The main point is that a little bit of uncertainty regarding the government preferences causes the public to use past observations to predict future behaviour. This is a kind of threat, because this behaviour on belief of the public induces the government to stick to the optimal policy without the assistance of external constraints, like the assistance of an independent central bank, law enforcement or precommitment. In that sense the idea of reputation is a substitute for precommitment.

Again, one of the most appealing aspect of this approach is that it provides a rigorous definition of reputation: it is a time dependent state variable measuring the private sectors subjective probabilistic belief that the policymaker is a type who is never tempted to create surprise inflation and being updated according to Bayes rule.

The model of Backus and Driffill has been extended by Vickers (1986) to show that if both tough and weak types of government care about employment, tough governments can be even more restrictive than in the case analysed by Backus and Driffill, where the tough government cares only about unemployment.

Further research on the possible effects of announcements on reputation by Schultz (1994) has shown that announcements can be important in helping governments to build a reputation for non-inflationary behaviour, even if the preferences of the government are common knowledge. In the Backus and Driffill framework the public's uncertainty about policymakers preferences implies that a policymaker who assigns a relatively low cost to

[5] Here in contrast to Barro and Gordon (1983b) reputational framework the horizon is finite.

inflation may find it optimal to mimic the actions of a more inflation-averse policymaker in order to build a reputation. Conversely in Schultz model all agents know the preferences of the policy maker, so there is no scope for mimicking an inflation-averse policy maker. But there is uncertainty about the honesty of the policymaker in announcing that he will behave in a non-inflationary manner.

The strength of the reputational enforcement has been questioned by Canzoneri (1985). He shows that when the information structure on the monetary instrument used is incomplete, so that the public cannot verify whether the policymaker has not intentionally fooled their expectations, the strength of the reputation mechanism is weakened. The presence of private information invalidates any commitment technology to force the policymaker to carry out the optimal policy, unless there is a separate device to force the policymaker to reveal its private information. Given these problems, Canzoneri (1985) suggests to follow a legislative approach in order to solve the credibility problem when the policymaker has private information. The work of Canzoneri has been extended by Garfinkel and Oh (1993) in order to investigate a multiperiod monetary targeting procedure as a possible solution to the credibility problem when there is asymmetric information.

2.6.3 Learning as a discipline device for the central bank

This approach is based on the work of Cukierman and Meltzer (1986), and extended in Cukierman (1992). It emphasises circumstances in which the central bank has private information about its own evolving preferences concerning inflation. The basic idea here is that individuals are constantly learning about the shifting objectives of policymakers and that policymakers are aware of this process of learning. Cukierman (1992) criticises the reputation approach based on a deterrence mechanism (like trigger strategies) as being inappropriate in the monetary policy context and considers a much more plausible starting point learning as a discipline device for the central bank. He argues that the central bank has private information (usually taken to be its own preferences), and the central banks actions (partially) reveals this information to the public. In this way, today's policy choice affects the public's future expectations as the public updates its beliefs about the central banks private information. This learning process acts to constrain the behaviour of the central bank. For example, inflating today might lead the public to revise its beliefs about the preferences of the central bank (towards believing the central bank is a weak inflation type), and therefore raising the future costs to the central bank of inflating. In this approach changes in the reputation of policymakers occur as a by-product of learning by the public, instead by means of an exogenously specified punishment strategy as for example in the Barro and Gordon (1983b) framework.

However, economists are generally skeptical about theories in which shifts in preferences play an important role. Tastes are typically treated as given for the simple reason that it is too easy to explain observations if tastes are allowed to change arbitrarily.

2.7 Political-business-cycle approach

There is a more eclectic approach based on the introduction of political business cycle elements in the standard framework used for analysing dynamic inconsistency. Examples of this approach are Minford (1995), who solves the inflationary bias problem with electoral punishment strategies, and Alesina and Gatti (1995), who raise the question of what is the optimal degree of independence of a central bank.

In particular Alesina and Gatti (1995) try to provide a theoretical formalisation to the findings of Alesina and Summers (1993) that in the OECD the degree of independence of central banks is positively correlated with inflation, but uncorrelated with output or unemployment variability. As we observed before these findings contradict Rogoff's trade-off between the average inflation and output variability. Alesina and Gatti (1995) distinguish between two sources of output variability. One is the economic variability due to standard exogenous shocks that monetary policy is supposed to stabilise. The second source of variability is political or, more general, policy-induced. This last type of source derives from the uncertainty about the future development of policy. The appointment of a conservative central banker implies two positive outcomes: inflation is kept low and stable; politically induced output variability is eliminated, since monetary policy is independent of government with changing preferences. It is uncertain whether the elimination of this last type of variability offsets the increased output variability determined by the fact that the conservative central banker dislikes inflation more than society does.

Other relevant examples of this eclectic approach are: Waller and Walsh (1996), where central bank independence is examined in terms of partisanship and term length; Jonsson (1995), where the implications of unemployment persistence are examined.

2.8 Extensions of the Barro-Gordon framework to the open economy

2.8.1 Understanding the role of flexible exchange rates

The literature considered above examines the notion of time inconsistency in a standard closed-economy model. However there exists a vast body of literature that extends the Barro-Gordon framework to the open-economy case.

One of the main differences with respect to the closed-economy case is that the inflationary-bias problem may become less important in an open economy that takes world monetary policy as exogenous. Rogoff (1985b) has shown that the tendency for the exchange rate to depreciate after unanticipated monetary expansions may reduce the policy maker's incentive to fool the private sector by creating inflation surprises. If the price index that the policy maker seeks to stabilise in his objective function includes foreign goods, real currency depreciations worsen the inflation cost of monetary expansions. At the same time, if wages are partially indexed to the price index or if foreign goods are included as intermediate goods in the production function, the output gain of monetary expansions is weakened when the real exchange rate depreciates. Thus, ceteris paribus, the policy maker has less incentive to inflate and the inflationary bias associated with discretionary monetary policy is lower.

An exception to this line of reasoning is of course the case when unanticipated monetary expansions are not unilateral but are introduced at the same time by all countries.

Rogoff's finding on the relationship between inflation and openness has been confirmed by the subsequent research. Romer (1993) analyses discretionary monetary policy in a simple two-good model which extends to an open economy the standard models of the macroeconomics of imperfect competition with sticky prices. He shows that the rate of inflation is inversely related to the degree of openness of the economy. The rationale for this result is found in the fact that the benefits of a monetary surprise in terms of the gain in real output are smaller the more trade-oriented the economy is. Lane (1997) argues that the existence of nominal price rigidity and imperfect competition in the non-traded goods sector, and not the terms of trade effect suggested by Romer, account for the inverse relationship between inflation and openness.

Another important difference with respect to the closed-economy case is that for an open economy the conventional wisdom regarding the beneficial effect of appointing a weight-conservative central banker may be questioned. In particular Laskar(1989) shows, by extending Rogoff's (1985b) two-country setting, that if the degree of conservatism is chosen cooperatively by countries the presence of symmetric shocks makes the appointment of a conservative central banker less beneficial for society. On the contrary the presence of asymmetric shocks reinforces its beneficial effect. In the case of the non-cooperative choice of the degree of conservatism by countries the lack of international cooperation will lead to an inefficient degree of conservatism.

A confirmation of this welfare-decreasing effect of the choice of a conservative central banker in open economies is provided by Currie, Levine and Pearlman (1996). They consider also the case of a monetary union with delegation, where this type of arrangement is seen as a particular form of monetary policy coordination.

2.8.2 Fixed exchange rates versus flexible exchange rates

We have seen that what is required to avoid the inflationary bias, implied by the dynamic inconsistency problem, is some commitment device that makes government announcements of low inflation credible. One possibility is building a reputation for pursuing low inflation, although this takes time. Where governments tend to have short lives such a reputation may be difficult to sustain.

Another solution is to delegate monetary policy to an independent central bank that has the mandate of pursuing zero inflation. A fixed exchange rate is an indirect way of doing this, by pegging the currency to a country with an established reputation for pursuing non-inflationary monetary policies. For example, in the case of the EMS this virtuous country was Germany.

A fixed exchange rate peg is a form of monetary rule in the sense that leaves very little room for discretionary policy if the peg is taken seriously. Thus a country that pegs to a stable currency can solve its inflationary bias problem. Giavazzi and Pagano (1988) study a continuous-time extension of the Barro and Gordon (1983b) model where the central bank has an objective function specified not only over inflation and employment but also over bilateral real exchange rate within the EMS, as a measure of competitiveness with respect other EMS members. Their model supports the argument that the exchange rate agreements give more credibility to the central bank the longer the period between realignments and the smaller the realignments. In this particular model the EMS membership is welfare improving with respect to the alternative case of flexible exchange rate.

But exchange rate pegging is not danger-free. In any fixed exchange rate system, the key countries need to find a way of solving their own inflationary bias problem. As the historical experience of Bretton Woods system and the European Monetary System (EMS) show, sometimes the key country acts in a way that makes impossible to maintain the peg.

Obstfeld (1991) interprets the EMS as an institution that allowed individual countries to sustain an exchange rate rule with an escape clause. He shows that such an institution can also lead to equilibria under limited credibility far worse than with an irrevocably fixed exchange rate, with excessively high real wages, real interest rates and unemployment. The existence of an escape clause can complicate efforts to stabilise nominal exchange rates even if the authorities accept to devalue only under exceptional circumstances. The reason is simply that the circumstances in response to which the escape clause may be invoked are often unobservable, efforts to peg nominal exchange rates tend to be destabilised by uncertainty about whether or not those circumstances occur.

Further research by Drazen and Masson (1994), using an escape clause model, investigates the implications of persistence in unemployment for the credibility of policy makers maintaining the rule of a fixed parity.

2.8.3 The European monetary union and the creation of a European central bank

Fixed exchange rates within a monetary union are potentially different from fixed exchange rates among countries whose central bank can make independent monetary policy decisions. As we have mentioned above, Obstfeld shows that pegged exchange rates between national currencies are never perfectly credible. Hence they substitute imperfectly for monetary unification.

Adopting a common currency and having monetary policy being set by a common central bank, like in the case of EMU, is an almost irrevocable form of delegation of monetary policy. Such an arrangement may bring with it gains in the form of a lower inflation rate for a country that faces a domestic credibility problem. The merits of an independent Euro-Fed have been discussed within the Barro-Gordon framework by Alesina and Grilli (1991) and Van der Ploeg (1991). However, several papers find that the attractiveness of entering a monetary union or admitting a new participant decreases with the number of participants (see for ex. Alesina and Grilli (1993)).

2.9 Some recent controversial topics

2.9.1 Introduction

Now we will discuss some of the main controversies recently emerged in the literature, that in my opinion constitute also the most promising area of research. My outline does not pretend to be exhaustive. Anyway, many of the other controversial issues related to the literature have been already mentioned in the previous summary.

2.9.2 The importance of the inflationary bias implied by dynamic inconsistency

As previously observed, the notion of time inconsistency is one of the explanations given by economists for the apparent inflationary bias of economic policy. However, despite its popularity the issue of time inconsistency seems to be quite controversial. There is general agreement on the presence of an inflationary bias as in most countries inflation has risen above any conceivable optimal rate during their economic history.[6] On the contrary the importance of the issue of time inconsistency in explaining it does not seem to share the same level of agreement. The reasons for disagreement are various.

On one hand Taylor (1982, 1983) has suggested that, as societies have found solutions to the time inconsistency problems arising in other areas (e.g. patent law), then it is likely that the credibility problem of monetary policy might not be particularly severe or even present. It seems unlikely that a time consistent solution would prevail in situations where it is widely recognised the superiority of the optimal rule. In other words the inflationary bias may not be quantitatively relevant and therefore it does not need to be

[6] A zero inflation rate target is usually considered optimal based on the many costs of inflation (crf. Driffill, Mizon and Ulph 1990). However Fischer (1994), for example, discusses the cases in favor of a socially optimal inflation rate target between 1-3 per cent. Recently Akerlof, Dickens and Perry (1996) and Groshen and Schweitzer (1997) also support the idea that the optimal inflation rate should be positive as this would help the policymaker to adjust real shocks more easily in presence of a downward rigity of nominal wages. Conversely Feldstein (1996) has suggested that the optimal inflation rate might even be negative, as the tax distortions created by inflation may reduce permanently the level of output.

tackled and eliminated.

On the other hand McCallum (1995, 1997a) has suggested that even if the inflationary bias might be quantitatively relevant (i.e. if societies have not found a solution to it) central bankers can be trusted not to be tempted to create inflation surprise as they know that this will lead to a worse equilibrium. According to this kind of criticism the Barro and Gordon model is not a plausible positive model of inflation because the absence of precommitment technologies does not prevent a central banker from behaving in a committed fashion and private individuals to rationally expect such optimal behaviour.

Another kind of criticism concerns the specification of the Barro and Gordon model. In particular it has been shown that the finding of excessively high average inflation under discretionary monetary policy might be related to the simplicity of the model used. Here we have three major contributions.

First the inflationary bias associated with time consistent monetary policy has been questioned on the ground of lack of realism of the model used. Goodhart and Huang (1998) show that if, in to the Barro and Gordon framework, lags are introduced in the transmission of the effects of monetary policy, the inflationary bias disappears completely. The inflationary bias reappears only when overlapping nominal wage contracts are incorporated in the analysis and contracts with a length greater than the length of the policy lag are pervasive. But in this latter case the scale of the inflationary bias is considerably reduced and the explanatory power of the time-inconsistency answer to the apparent inflationary proclivities of industrialised countries is weakened.

Second, the Barro an Gordon model has been questioned also on the ground of lack of microfoundations. Nicolini (1998) has shown that in a general equilibrium monetary model the divergence between average inflation in equilibrium and the socially optimal level of inflation may not necessarily be of positive sign. In his framework the policy maker may find optimal to deviate by choosing inflation rates lower than expected and hence a disinflationary bias may arise. Clearly this striking result eliminates completely the issue of the importance of the time-inconsistency answer to the apparent inflationary bias of economic policy.

A third point has been made by Nobay and Peel (1998). In particular, they argue against the use of quadratic or linear preferences for examining optimal policy, as is done by Barro and Gordon. By exploiting a procedure used in Bayesian analysis, the Linex form, they analyse the implications of asymmetric preferences. It is shown that in this case the standard inflationary bias result under discretion does not hold unambiguously and there might be a deflationary bias as well. Moreover, a deflationary bias prevails unambiguously under precommitment and in this case the deviation of inflation from the socially optimal level is larger than under the case of discretion.

These criticisms have been partially left unanswered. Canzoneri (1985) has shown that Taylor's criticism does not hold when the policymaker has private information and his action cannot be monitored perfectly. The criticism concerning the size of the inflationary bias has been answered for example by Walsh (1998). In examining the issue of the importance of the inflationary bias when there are policy lags and overlapping nominal wage contracts he observes that the presence of a reduced inflationary bias does not mean that the issue of time inconsistency is unimportant. The simple model used in the time inconsistency literature may not explain all observed inflation but nevertheless it raises the important issue of the incentives to deviate from optimal rules faced by policy makers.

In chapter 3 we explore more deeply the third kind of criticism described above and related to specification issues.

2.9.3 The theoretical debate on rules versus discretion and the importance of following systematic behaviours

Real world central bankers do not seem to believe in the desirability of tying their hands by fixing policy choices according to a given formula for setting the instrument. Blinder (1998, p.41), speaking at the same time as a practitioner and as an academic, expresses clearly this view when he writes: "the real world cure to the alleged "inflation bias" problem did not come from adopting rigid precommitment ("rules") or other institutional changes, as Kydland-Prescott and Barro-Gordon suggested. It came from determined but discretionary application of tight money." However, despite of this reluctance to commit to follow fixed rules, there exists strong empirical evidence supporting the existence of a rule-like behaviour by central banks in the setting of interest rates.

Barro and Gordon viewed the time inconsistency explanation of the inflationary bias as an argument for rules over discretion, along Friedman lines.[7] With the policy maker committed to a fixed monetary policy rule it is not possible to create surprise inflation and the problem of dynamic inconsistency disappears.

But there is an important distinction from Friedman's case for a constant growth rate of money, who believed that the presence of long and variable lags between adjustments to monetary policy instruments and their real effects on the economy implies potentially destabilising impacts of an active monetary policy. Actually, the Barro and Gordon argument for a monetary policy rule can be also maintained in a framework where there is room for an activist feedback monetary policy rule. If we allow for the presence of information asymmetry between policy maker and private sector regarding the realisations of supply shocks, then the equilibrium policy rule in the commitment regime involves a potential role for an active monetary policy, offsetting shocks and therefore helping to stabilise inflation and output.

However, as observed by Alesina (1988), Persson and Tabellini (1990) and Lohmann (1992), in this Barro-Gordon framework extended to incorporate supply shocks the ex-ante optimal monetary policy rule is contingent on the state of the world. Unfortunately, in practice policy makers cannot easily commit to a state-contingent monetary policy rule. If, instead of a state contingent rule, a simple rule is pursued (e.g. a Friedman type rule), then it will dominate a discretionary rule only if output shocks are small and rare. In unstable periods there is more scope for stabilisation policies and a discretionary

[7] In page 589 of Barro and Gordon (1983a): "The value of these commitments - which amount to long-term contracts between the government and the private sector - underlies the argument for rules over discretion."

behaviour is preferable.

The idea that there is a trade-off between the benefits of avoiding the inflationary bias of discretionary policy and the potential costs of being bound to follow a monetary rule that is no longer appropriate has led some economists to find some compromises. Flood and Isard (1989) have proposed, for instance, the formulation of simple rules with explicit escape clauses. They argue that society might improve on the outcomes achieved under a discretionary regime, by motivating the central bank to follow a hybrid policy: in normal times the central bank follows a simple rule, while it responds to unusual circumstances at its discretion.

Some other economists increasingly have viewed rules not as constraints imposed on central banks externally, but as time-consistent means of operating internally - for example, as explicit starting points for consideration of current policy option. So in that sense there need not necessarily be a trade-off between flexibility and commitment. Recent efforts in this direction are those of Feldstein and Stock (1994), Hall and Mankiw (1994), and McCallum (1994). They have proposed alternative monetary rules to be considered by central banks whose policies are not strictly limited by exchange rate commitments. In particular McCallum formulates a more sophisticated monetary feedback rule, which takes account of changes in the velocity of circulation. He proposes a rule that sets the growth rate of money base at 3 per cent annum, with adjustments for the change in base velocity over the past four years and also for the deviation of nominal GNP from a defined steady noninflationary path. This rule, which treats nominal GNP as the target variable and the monetary base as the instrument, is shown to produce good inflation and output performance in several small econometric models. In the context of fixed rules, the McCallum type can provide a useful compromise with flexibility.

An alternative view on the argument is provided by Taylor (1993). According to him the rules versus discretion dilemma is rather a semantic issue, in the sense that in practice a policy rule can be defined more generally as a systematic behaviour. Hence there is no need to follow mechanically an algebraic formula. Moreover, as Taylor concludes:

"with this broader definition of policy rules, comparing the performance of different rule becomes more challenging".

In his celebrated article he formulates the so called Taylor rule: a representative interest rate rule that captures relatively well the Fed's behaviour during the 1987-1992 period. In order to make operational this rule, which obviously is not practical to follow in a rigid way, he considers two possibilities. One is to include the specific formula in the list of key elements that form the basis for monetary policy decisions. A second is that of making use of general characteristics of the given rule without referring explicitly to the algebraic formula for the central bank's decision-making process.

McCallum (1993), in answering to Taylor's proposal of broadening the definition of policy rules beyond a specific formula, shows that the presence or absence of systematic behaviour is not sufficient for separating between discretion and a behaviour based on an rule. What is also required is that "the policy authority [....] must also design the systematic response pattern to take account of the private sector's expectational behaviour". By making use of the analytical distinction between commitment and discretion on which is based the time-inconsistency literature he shows that also under a discretionary regime the policy maker may follow a systematic behaviour. In particular he expresses the policy maker problem in terms of choosing the parameter values of a fixed formula. Nevertheless the presence of this systematic behaviour under a discretionary regime does not prevent the economy from the arising of an inflationary bias.

In chapter 4 we will inspect this controversy more in detail providing a contribution to the theoretical debate. In particular, we will focus on the criticism of McCallum to Taylor's definitional issue and explore the importance of adopting systematic behaviours in the setting of monetary instruments.

2.9.4 The real effectiveness of delegation in overcoming dynamic-inconsistency problems

The proposal to change monetary institutions to deal with the issue of time inconsistency and the alleged inflationary bias is the core idea behind the approach of monetary delegation started by Rogoff. Recently this approach has been extended by the work of Persson and Tabellini (1993), Walsh (1995a), and Svensson (1997a) among others with the introduction of incentive schemes or policy targets in order to completely remove the inflationary bias.[8] Despite these positive achievements the literature on monetary delegation has been recently criticised by McCallum (1995, 1997a), who argues that as there is always the temptation for the government to renege on the chosen monetary institution.[9] Hence the institutional remedies proposed by this approach do not fix the problem of time inconsistency but merely relocate it.[10]

Posen (1995) also criticises the delegation approach and provides some empirical findings that, in contrast to the previous empirical evidence, suggest that the relationship between central bank independence and disinflationary credibility is not supported. He concludes that central bank independence alone is not sufficient. What is also needed is the presence of a coalition in society committed to protecting the central bank independence as necessary for achieving low inflation. In particular he concludes that, as the

[8] Persson and Tabellini (1997) and Walsh (1998) are splendid examples of the most recent available reviews on the literature on monetary delegation, with insightful discussions on the issue of the credibility of optimal monetary delegation.

[9] Beetsma and Jensen (1999) observe that McCallum's criticism may be particularly relevant in the case of state-contingent delegation arrangements. If it is possible to adjust for example the inflation target or the contract for the central banker before private sector's expectations are formed, then it is more likely that they can be changed also subsequently in order to fool the private sector.

[10] McCallum refers his criticism only to the Walsh contracting approach. Also Obstfeld and Rogoff (1996) make the same criticism referring to the Walsh solution. There is the view that the Rogoff's approach is immune to McCallum's criticism. On that regard Alesina (1995, p.289) wrote: 'it is institutionally harder to dismiss a "conservative" central banker than it is for the policymaker simply to renege on a policy announcement made without the independent conservative agent'. However in his argument there is the implicit assumption that the presence of reappointment costs will deter the government from over-ruling the banker or sacking him and appointing a less conservative one. If, following Jensen (1997), we interprete the principal-agent approach as a complex institutional arrangement based on a structure of incentives costly to change, then Alesina's argument should hold also for the Walsh approach.

financial community is the critical constituency that influences the central bank, the outcomes of monetary policy will predominantly reflect that of the monetary policy desired by the financial community. However, no formal analysis is presented and as shown by Alesina (1995) his empirical findings are controversial.

Persson and Tabellini, amongst others, have replied to this kind of criticism (1997, p.32) by observing that: "in the model that dominates the literature, what is needed is a high cost for changing the institution within the time horizon of existing nominal contracts" and "the cost of suddenly changing the institution could also be a loss of reputation".

This premise, which has been implicitly assumed in the standard literature on delegation, has been recently challenged by Jensen (1997). He explicitly introduces the delegation stage in the government policy choice and adds a quadratic cost for reappointments. In the static one-shot game version higher costs of reappointment reduce the inflationary bias but never remove it. An exception is represented only by the extreme and unrealistic case where the weight on reappointment costs in the government's loss function is infinite. In this situation optimal monetary delegation is not subject to a credibility problem, but all that matters in the loss function are reappointment costs. Moreover if the game is repeated over an infinite horizon, along the lines of Barro and Gordon (1983b), the presence of reappointment costs worsens the credibility of optimal monetary policy under delegation compared to the case when monetary policy is conducted directly by the government.

These results imply a negative view of the contracting solution and in general of the monetary delegation approach. Jensen concludes (quite drastically) by suggesting that too much emphasis has been given to the approach of monetary delegation and that research should focus on other directions, in particular on the relationships between time inconsistency and structural policies.

Al-Nowaihi and Levine (1996) and Herrendorf (1998) provide some opposing results based on the assumption of transparency of the delegation process. They show that if the action of the policymaker cannot be perfectly monitored, but the conditions under which monetary policy is delegated are publicly observed, relocation of the time inconsistency problem can allow the government to commit credibly to the announced institutional design. However, Jensen (1997) argues that in the case of imperfect monitoring of the policymaker's action delegation improves the credibility of monetary policy merely because delegation modifications are defined to be transparent and the issue of secrecy is ruled out of the analysis by definition.[11] Thus it is not a result of delegation per se.

In chapter 5, by using the same model as Jensen we will raise some objections to his analysis.

2.10 Empirical evidence on the predictions of the time-inconsistency literature

2.10.1 Does the time-inconsistency notion explain the behaviour of inflation?

Inflation in most of the G-7 countries ratcheted up in the 60s and 70s. By the end of the 70s, the inflation rate of United Kingdom, Italy and Canada was higher than any conceivable account of the costs and benefits of inflation could justify. No wonder that economists felt called upon to explain the apparent inflationary bias of economic policy. As we have discussed previously the Barro-Gordon paradigm provides an explanation for the presence of an excessively high level of inflation in terms of the incentives faced by policy makers and the interaction of a bias toward expansions.

Nevertheless an important empirical issue is whether the time-consistency notion does explain the actual behaviour of inflation. In fact, one of the most important characteristics of inflation during the last 40 years in the developed economies is that it varies.

[11] Obstfeld and Rogoff (1996) make a similar point in criticising the transparency of the contracting approach. They argue that it is always possible that ex-post the government offers the central banker incentives, explicitly or secretly, that can compensate him for the loss from inflating.

Average inflation changes and displays a high degree of persistence.

An attempt at testing the predictions of the Barro-Gordon model is provided by Ireland (1999), by using post-war US data. After extending the standard framework in order to account for the fact that US unemployment rate is nonstationary, the restrictions imposed by Barro and Gordon's theory on a bivariate time-series model for inflation and unemployment are derived and tested. The findings show that the data are consistent with the implications of the theory for the long-run behaviour of the two variables. However the short-run dynamics that appears in the data is not satisfactorily captured by the specified model. This last result suggests that further extensions of the standard model are required.

Using again post-war US data, Broadbent and Barro(1997) provide also estimates of the weights on price surprises and inflation in the policy maker's objective function. The results are derived by interpreting the equilibrium expressions of the key variables obtained under a discretionary regime in terms of a positive theory that can be used for describing the actual behaviour of those variables. The revealed preferences for the period examined indicate that the weight on the stabilisation of inflation is relatively higher than that corresponding to price surprises. However, the absence of previous empirical estimates available for a comparison and the lack of intuition on the plausible size of the weights placed by the policy maker to the achievement of the given objectives limit the evaluation of the findings.

2.10.2 Central bank independence: do monetary institutions matter?

The preceding discussion suggests that the inflationary bias of time consistent monetary policy can be eliminated by means of isolating or making independent the central bank from political pressures.

Alesina and Summers (1993), Cukierman, Webb and Neyapti (1992), and Grilli, Masciandaro and Tabellini (1991), Alesina and Gatti (1995), among many others, have examined the relationship between long-run industrial-country inflation rates and various indicators of central bank independence. The main approach in this empirical literature is to construct an index of central bank independence based on institutional, legal and sometimes behavioural features.

There are two key findings from this type of analysis. First, high degree of central bank independence is correlated with low-inflation performance. The second key finding, which is striking given our previous discussion on the desirability of an independent and weight-conservative central banker, is that higher degree of central bank independence implies lower inflation, but no worse performance in terms of real variable. Alesina and Summers (1993), Alesina and Gatti (1995) find that independent central banks imply low inflation with no higher variability in growth or unemployment.

A number of authors have questioned whether results such as these really imply a strong causal link from central bank independence to low inflation. First, this empirical correlation does not extend easily beyond the set of industrialised countries. It may also be possible that central bank independence and low inflation arise from a common source.

Posen (1995) argues that the political influence of a country's financial sector is uniquely positioned to provide the political support necessary for central bank independence and that central banks are most independent where financial sector support has been strongest. Monetary institutions are endogenous rather than exogenous, reflecting the preferences of powerful groups in the society. Posen's argument is supported by the empirical evidence based on an index of effective financial sector opposition to inflation, which attempts to measure both the strength of the financial sector and the extent to which the political system is such that their preferences may be translated into strong monetary institutions. Once the newly constructed index is introduced in an inflation regression for the same sample of OECD countries considered by Grilli, Masciandaro and Tabellini (1991) central bank independence loses its explanatory power.

Another problem of simple regressions of average inflation on central bank independence is that they do not account for country-specific factors that may influence inflation and that at the same time are correlated with central bank independence. One approach is to correct for potential misspecification problems by including other determinants of inflation. Campillo and Miron (1997) have shown that central bank independence has no explanatory power for cross-country differences in average inflation if other determinants of inflation are included in the regression. They have found that both the debt-to-GDP ratio and the degree of openness are an important explanatory variables of cross-country variation of inflation. This last result on the relationship between openness and inflation is consistent with the previous findings of Romer (1993).[12]

Because institutions are endogenous in the long run, the critics who view inflation and central bank independence as jointly determined have a point. Hence, at this stage, the empirical evidence on the link between central bank independence and low inflation cannot be regarded as decisive. A better understanting of the relationship between central bank independence and inflation, even if the observed correlation cannot be taken as indicating causation, will require a better understanding of the factors that have determined modifications in central bank independence across countries.

2.11 Conclusion

The debate on central banking has been for long time focused on the inflationary tendency inherent in the conflict between the short- and long-run effects of monetary expansion and on the temptations of monetary financing of government spending. More recently it has also been focused on the conflict between the benefits of a central bank independent from political pressures and of the accountability to the public of the policymakers decisions.

The new game-theoretic approach to macroeconomic policy has the important merit to have radically changed the academic analysis of policy making. For the first time, economists can talk analytically about such key issues as credibility, rules versus discretion,

[12] See also the comment by Terra (1998) and the response by Romer (1998). Terra argues that Romer's empirical results are driven largely by the behaviour of heavily-indebted countries.

reputation and central bank independence. For the first time, the complex interaction between policymakers and private agents has been widely explored.

The implications for modern central banking of this new literature can be, roughly but effectively, summarised by the Reserve Bank of New Zealand's Act of 1989, which states that the primary function of the Bank is to formulate and implement monetary policy directed to the economic objective of achieving and maintaining stability in the general level of prices. This is in sharp contrast with the previous widespread practice of not setting out any specific mandate or goals for the central banks. A similar consideration can be made also for the Maastricht Treaty, where the goals for the recently instituted European Central Bank are specified. Here price stability is considered as the primary goal and other goals can be pursued only after previous achievement of the primary one.

Moreover, the practical importance of the dynamic-inconsistency literature can be inferred also from the great weight given by central banks in the last decade to communication. In fact transparency of policy decision is strictly related to the issue of accountability. Transparency simplifies external monitoring and improves the effort of central banks in meeting their targets. The Reserve Bank of New Zealand, Bank of England and Sveriges Riksbank publish inflation reports, which contribute to a widespread understanding of ex post outcomes and provide central bank's forecasts on inflation. Bank of England and Sveriges Riksbank also publish voting details with minutes from their meetings on monetary policy decisions.

During the 90s the literature on dynamic inconsistency has witnessed a renewed interest with several important contributions that have enhanced the comprehension of both the normative and positive implications of the Barro-Gordon paradigm. Nevertheless there are still some important controversial issues that require further research. I have identified three major topics where leading economists have started to question ideas and results that were taken for solid or granted. A number of significant gaps remain in our understanding along the path of this literature. If we want to provide a firmer foundation of understanding on which to proceed it is necessary to back up and fill the gaps relative to mentioned controversial issues. In the next three chapters we attempt to move towards this goal.

Chapter 3

Time consistent monetary policy reconsidered: may we have a deflationary bias too?

3.1 Introduction

We have seen in chapter 2, section 2.9.2, that one of the main criticisms raised against the Barro-Gordon model concerns the simplicity of its specification. For instance, Goodhart and Huang (1998) have shown that if there are lags in the transmission of the effects of monetary policy, the inflationary bias vanishes. The inflationary bias re-enters in the picture only when overlapping nominal wage contracts are incorporated in the analysis and contracts with a length greater than the length of the policy lag are diffuse. But in this situation the inflationary bias is quantitatively less important and the explanatory power of the time-inconsistency answer to the apparent inflationary predisposition of industrialised countries is reduced.

In the present chapter we investigate more deeply this kind of criticism related to specification issues. In particular we focus on the issue of the lack of realism of the Barro and Gordon framework. Thus we will examine how robust is the time-inconsistency explanation of the apparent inflationary bias of economic policy to modifications in the model's specification.

Following Goodhart and Huang we extend the simple model of Barro and Gordon by incorporating policy lags and overlapping wage contracts. However, in contrast to them we focus on optimal interest rate rules, rather than optimal money growth rules, and introduce also the issue of instrument uncertainty. Therefore our framework is closer to actual policy making, where central banks adjust their short-term interest rate in response to deviations of inflation and output from given targets and the control of monetary aggregates has been progressively abandoned.[1] Moreover a further degree of realism is added with the introduction of instrument uncertainty. Real world policy makers are subject to considerable uncertainty about the effects of policy on target variables and on instruments themselves. Therefore there is also non negligible uncertainty about policy multipliers.

One of the main results of our analysis is that under certain circumstances it is possible that a disinflationary bias may emerge as well as an inflationary one. Hence our model provides a confirmation for the findings of Nicolini (1998) and Nobay and Peel (1998). This surprising result implies that the time-inconsistency explanation for the apparent positive inflationary bias becomes a qualified one. In the subsequent sections we will show in detail under which circumstances Barro and Gordon's celebrated result still holds in our framework.

In section 3.2 we describe the model. In section 3.3 the equilibrium values are determined. Section 3.4 derives and discusses the main results of the analysis and compares them with those obtained in the previous literature. Finally, in section 3.5 some concluding observations are made.

[1] In the words of Blinder (1998, pp.26-29): "Returning to Poole's dichotomy [on the choice of monetary instrument]...in the end, real-world events, not theory, decided the issue. Ferocious instabilities in estimated LM curves in the United States, United Kingdom, and many other countries, beginning in the 1970s and continuing to the present day, led economists and policymakers alike to conclude that money-supply targeting is simply not a viable option....So interest rates won by default".

3.2 The model

Following Fischer (1977) we assume two-period overlapping nominal wage contracts, which imply an aggregate supply function of the form

$$y_t = y_n + \frac{\alpha}{2}\left(\pi_t - E_{t-1}\pi_t\right) + \frac{\alpha}{2}\left(\pi_t - E_{t-2}\pi_t\right); \tag{3.1}$$

where y_n is the natural level of output, π_t is the realised rate of inflation and $E_{t-1}\pi_t$ and $E_{t-2}\pi_t$ are wage setters' inflation expectations.[2] Expectations are formed rationally using all available information at the end of period $t-1$ and $t-2$ respectively. The aggregate demand is given by a standard IS function

$$y_t = y_n - \beta\left(r_{t-1} - E_{t-1}\pi_t - \rho\right); \tag{3.2}$$

where ρ is the long-run real interest rate, r_{t-1} is the nominal interest rate. Here it is assumed that the interest rate, which is the instrument used for conducting monetary policy, affects output with a one-period lag.[3] Moreover we assume for simplicity that the monetary authority sets the instrument in terms of deviations of the nominal interest rate from the constant long-run real interest rate, $i_{t-1} = r_{t-1} - \rho$. This assumption ensures that when instrument uncertainty is introduced planned and actual level of the nominal interest rate are with high probability non negative . [4]

[2]As discussed in the introduction Goodhart and Huang (1998) and Walsh (1998) examine also the case of overlapping nominal wage contracts (along the lines of Fischer 1977) with a one-period lag in the effect of monetary policy. But in contrast to the present analysis they consider money supply as the instrument and the effects of interest rate changes on aggregate demand are ignored. Moreover the monetary authority is assumed to control perfectly the money supply and the issue of instrument uncertainty does not arise.

[3]This specification of the IS curve has been adopted for example by Ball (1997) and Svensson (1997b) in a recent analysis of optimal interest rate rules when the are lags in the effect of monetary policy. Unlike the present analysis they assume also that monetary policy affects inflation with a two-period lag . This assumption complicates the analysis as it implies that in order to have a time-inconsistency problem we should introduce three-period labor contracts. However this complication would not change the basic insights of the analysis.

[4]In the present framework the nominal interest rate can be negative for a small number of periods depending on the realisations of the stochastic control shocks. The issue of a non-negative nominal interest rate constraint is discussed in Lebow (1993), Cecchetti (1997) and Rudebush and Svensson

As we want to examine the case of instrument uncertainty we assume that the actual level of the instrument is determined in the following way:

$$i_t = \varphi_t i_t^P + \psi_t; \tag{3.3}$$

which says that the monetary authority does not control the instrument perfectly and the planned level will differ from the actual level due to the presence of multiplicative control errors, φ_t, and additive control errors ψ_t. The assumption that the monetary authority does not control the instrument perfectly may reflect the fact that the interest rate that can be controlled more accurately is typically a short-term interest rate whereas in the aggregate IS-curve the relevant interest rate is of long-term, which may not be determined only by short-term interest rate movements.

At the same time the presence of control errors in the choice of the instrument may also reflect the possibility that there is uncertainty about the effects of the policy variable. This view is related to the stochastic optimization literature with uncertainty about policy effects started by Brainard (1967). In particular Brainard has shown that uncertainty about the parameters in the relationship between the policy variable and the target variable leads to a conservative use of the policy instrument.

In order to introduce a role for stabilisation policy, we assume that before the planned level of the interest rate is chosen the monetary authority makes an optimal forecast, ε_t, of the shock ψ_t.[5] Hence we have

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(1998). In practice a negative nominal interest rate is not feasible but theoretical analysis usually does not exclude it, mainly in order to avoid the complexity of introducing a non-negativity constraint. An example is Clark, Huang and Goodhart (1999) where the level of the interest rate in the optimal decision rule for the monetary authority is not constrained to be non-negative.

Following Rudebush and Svensson (1998), we examine the potential power of central banks in conducting expansionary monetary policy and assume that there are always other instruments available (e.g. unsterilised interventions or increasing liquidity by means of open market purchases of Treasury securities at all maturities) when the nominal interest rate is near to zero.

[5]In the present framework in order to keep the analysis as simple as possible we ignore aggregate demand and supply shocks. As the model considered is static and assuming that monetary authorities at period t have no advance information about the shocks in period $t+1$, then the role of aggregate supply and demand shocks would be similar to that of the unforecastable component of additive control shocks. The monetary authorities are not able to stabilise these shocks and the optimal instrument feed-back

$$\psi_t = \varepsilon_t + \nu_t. \tag{3.4}$$

We assume for analytical convenience that the shocks follow a multivariate normal distribution

$$\begin{pmatrix} \varphi_t \\ \varepsilon_t \\ \nu_t \end{pmatrix} \sim N_3 \left(\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\varphi^2 & 0 & 0 \\ 0 & \sigma_\varepsilon^2 & 0 \\ 0 & 0 & \sigma_\nu^2 \end{pmatrix} \right);$$

where multiplicative control shocks have mean 1 and variance σ_φ^2 while additive shocks have mean zero and variances σ_ε^2 and σ_ν^2.[6]

The timing of the actions is the following: the first half of the workers fix their nominal wages for two periods using information gathered at the end of period $t-2$; at the beginning of period $t-1$ the monetary authority makes an optimal forecast of the additive control error and sets the planned level of the interest rate; subsequently the multiplicative and additive control errors are realised; finally the second half of the workers fix their nominal wages for two periods using information up to the end of period $t-1$. Thus in the present model the monetary authority can potentially fool only the proportion of workers that fix their nominal wage at the end of period $t-2$, before monetary policy is chosen. The workers that fix their nominal wages at the end of period $t-1$ can observe perfectly the actual level of the interest rate.

Now equating output in (3.1) and (3.2) we have

$$\pi_t = -\frac{\beta}{\alpha} i_{t-1} + \frac{1}{2} E_{t-1}\pi_t + \frac{\beta}{\alpha} E_{t-1}\pi_t + \frac{1}{2} E_{t-2}\pi_t. \tag{3.5}$$

From equation (3.5) we can derive the relationship between inflation and the policy

rule will not depend on them.

[6]See Schellekens (1998) for the same assumption on the stochastic structure of the shocks in the context of Brainard uncertainty and time inconsistency. Differently from here he considers also the possibility that control shocks are not independent from each other with the covariance being different from zero.

instrument. As there are no current shocks, current inflation is affected one period in advance by the lagged interest rate, inflation expectations of half of the wage setters based on available information at $t-2$ and inflation expectations formed at the end of period $t-1$. This implies that at the end of period $t-1$ wage setters and investors can predict exactly the level of the period t inflation rate. The implication for monetary policy is that now the policy maker can no longer take expectations formed at the end of period $t-1$ as given. Actually the policy maker's choice made at $t-1$ can affect expectations immediately and therefore his ability to create surprise inflation is weakened. Thus, after taking expectations at $t-1$, (3.5) becomes

$$\pi_t = -\frac{2\beta}{\alpha - 2\beta} i_{t-1} + \frac{\alpha}{\alpha - 2\beta} E_{t-2}\pi_t; \tag{3.6}$$

with $\beta \neq \alpha/2$.

Considering (3.3) and (3.4) and taking expectations at $t-2$ of (3.6) we obtain

$$E_{t-2}\pi_t = E_{t-2} i_{t-1}^P. \tag{3.7}$$

Substituting (3.7) back into (3.6) we can express inflation in terms of the control variable i_t^P and the wage setters' expectations of the level of the control variable itself. Shifting one period forward we have

$$\pi_{t+1} = -\frac{2\beta}{\alpha - 2\beta}\left(\varphi_t i_t^P + \varepsilon_t + \nu_t\right) + \frac{\alpha}{\alpha - 2\beta} i_t^{Pe}; \tag{3.8}$$

where $i_t^{Pe} = E_{t-1} i_t^P$, in order to simplify the notation.

Substituting (3.6) and (3.7) in (3.2) and shifting one period forward, output can be expressed as

$$y_{t+1} = y_n + \frac{\alpha\beta}{\alpha - 2\beta}\left[i_t^{Pe} - \left(\varphi_t i_t^P + \varepsilon_t + \nu_t\right)\right]. \tag{3.9}$$

By examining (3.8) and (3.9) it is possible to see that the effects of changes in the interest rate on output and inflation are ambiguous and depend on the assumed parameter values in the model. In particular, an increase in the actual level of the interest rate will reduce (increase) both output and inflation if $\beta < \alpha/2$ (if $\beta > \alpha/2$). On the contrary an increase in the level of the interest rate expected by the private sector will increase (reduce) both output and inflation if $\beta < \alpha/2$ (if $\beta > \alpha/2$).

Nevertheless it is possible to show that the system represented by equations (3.1) and (3.2) will be stable and converge to the long-run equilibrium only if $\beta < \alpha/2$. To confirm this take expectations through equations (3.1) and (3.2) at $t-1$, with the policy action i_{t-1}^P and $E_{t-2}\pi_t$ both taken as fixed. In this way we have that expected output $E_{t-1}y_t$ is an increasing function of expected inflation $E_{t-1}\pi_t$ in both the supply and demand equations. Since the equilibrium is a RE equilibrium, it is clear that there is a unique equilibrium and that it occurs at the intersection of the two curves, no matter whether β exceeds or is less than $\alpha/2$. This is illustrated, for example, in figures 3.1 and 3.2.

Now one can see from figure 3.1 that, if the expected demand curve (with slope equal to β) is flatter than the expected supply curve (with slope equal to $\alpha/2$), then higher than equilibrium expected inflation induces expected excess supply. Conversely it is possible to see from figure 3.2 that, if the expected demand curve is steeper than the expected supply curve, then higher than equilibrium expected inflation induces expected excess demand. Both cases imply adjustments over time of the inflation rate towards its market-clearing value.

However the dynamic process of inflation will converge to its long-run equilibrium value and at the same time lead to a dampening of the output gap only if $\beta < \alpha/2$. Assuming that the economy is composed of many small firms, with each producer setting its own price, in presence of market excess demand each firm would wish to charge a higher than market price; on the contrary with excess market supply. Price setting by individual traders only leads to a stable RE equilibrium if higher expected price reduces expected excess demand.

The convergence requirement implies that the parameter that determines the effect of the expected real interest rate on goods demanded, β, must be sufficiently small compared to the elasticity of goods supplied with respect to inflation surprises, α.[7] In the following analysis we assume that the convergence requirement holds. Hence we have also that the impacts of monetary policy on the target variables are the same of the standard IS-LM analysis: an increase in the actual level of interest rate reduces both inflation and output.

Finally the model is closed by the preferences of the monetary authority. The policy maker chooses i_t^P to minimise the following period loss function

$$L_{t+1} = \pi_{t+1}^2 + \lambda \left(y_{t+1} - \overline{y} \right)^2 ; \tag{3.10}$$

which is the standard objective function considered in the literature on the issue of time inconsistency in monetary policy. As usually, society is assumed to have the same preferences as the policy maker. Thus (3.10) represents also society's period loss function. The policy maker weights deviations of inflation and output from specified target values with $\lambda > 0$. Here a standard assumption is that the policy maker has an output target greater than the natural level of output, i.e. $\overline{y} > y_n$.

3.3 Time consistent equilibrium

The discretionary solution of the model is derived by minimising the expected value of the loss function (3.10) with respect to i_t^P, conditional on the information set of the policy maker when i_t^P is chosen,

$$\min_{i_t^P} E \left[L_{t+1} \mid \varepsilon \right] ; \tag{3.11}$$

subject to (3.8) and (3.9) and taking as given the private sector's expectations i_t^{Pe} .

[7] The empirical literature on the expectations augmented Phillips curve has evidenced some problems in isolating a significantly positive effect of price surprises on goods supplied. See for example the recent empirical analysis provided by Barro and Broadbent (1997). Moreover, to our knowledge, Fischer's model of overlapping nominal wage contracts has not been empirically tested. So the empirical literature does not provide clear insights into the relative importance of the two parameters examined.

We have the following first order condition:

$$\left(-\frac{2\beta}{\alpha-2\beta}\left(\varphi_t i_t^P+\varepsilon_t+\nu_t\right)+\frac{\alpha}{\alpha-2\beta}i_t^{Pe}\right)\left(-\frac{4\beta\varphi}{\alpha-2\beta}\right)$$

$$+\lambda\left(y_n+\frac{\alpha\beta}{\alpha-2\beta}\left(i_t^{Pe}-\varphi_t i_t^P-\varepsilon_t-\nu_t\right)-\overline{y}\right)\left(-\frac{2\beta\alpha\varphi}{\alpha-2\beta}\right)=0; \qquad (3.12)$$

which becomes, after taking expectations conditional on the optimal forecast ε_t and simplifying:

$$2\left(-\frac{2\beta}{\alpha-2\beta}\left(\left(1+\sigma_\varphi^2\right)i_t^P+\varepsilon_t\right)+\frac{\alpha}{\alpha-2\beta}i_t^{Pe}\right)$$

$$+\lambda\alpha\left(y_n+\frac{\alpha\beta}{\alpha-2\beta}\left(i_t^{Pe}-\left(1+\sigma_\varphi^2\right)i_t^P-\varepsilon_t\right)-\overline{y}\right)=0; \qquad (3.13)$$

where we have used the fact that $E\left[\varphi^2\right]=1+\sigma_\varphi^2$. Rearranging the first order condition we get the monetary authority reaction function

$$i_t^P = \frac{(2\beta-\alpha)\,\lambda\alpha\left(\overline{y}-y_n\right)}{\beta\left(4+\lambda\alpha^2\right)\left(1+\sigma_\varphi^2\right)}+\frac{\alpha\left(2+\lambda\alpha\beta\right)}{\beta\left(4+\lambda\alpha^2\right)\left(1+\sigma_\varphi^2\right)}i_t^{Pe}-\frac{1}{1+\sigma_\varphi^2}\varepsilon_t. \qquad (3.14)$$

The private sector's expectations are found by taking expectations at time $t-1$ over the policy maker's reaction function which yields

$$i_t^{Pe}=\frac{(2\beta-\alpha)\,\lambda\alpha\left(\overline{y}-y_n\right)}{2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)}. \qquad (3.15)$$

Thus the optimal instrument feed-back rule will be

$$i_t^P=\frac{(2\beta-\alpha)\,\lambda\alpha\left(\overline{y}-y_n\right)}{2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)}-\frac{1}{1+\sigma_\varphi^2}\varepsilon_t; \qquad (3.16)$$

which implies that after the control error shocks are realised the actual level of the nominal interest rate will be

$$r_t = \rho + \frac{\varphi_t \left(2\beta - \alpha\right) \lambda\alpha \left(\overline{y} - y_n\right)}{2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2 \left(4 + \lambda\alpha^2\right)} - \frac{\left[\varphi_t - \left(1 + \sigma_\varphi^2\right)\right]}{1 + \sigma_\varphi^2}\varepsilon_t + \nu_t. \quad (3.17)$$

Now we are able to find the equilibrium values for inflation and output. Substituting (3.15) and (3.16) back into the expressions for inflation and output we get

$$\pi_{t+1} = \frac{\left(2\beta\varphi_t - \alpha\right) \lambda\alpha \left(\overline{y} - y_n\right)}{2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2 \left(4 + \lambda\alpha^2\right)} - \frac{2\beta\left[\varphi_t - \left(1 + \sigma_\varphi^2\right)\right]}{\left(2\beta - \alpha\right)\left(1 + \sigma_\varphi^2\right)}\varepsilon_t + \frac{2\beta}{2\beta - \alpha}\nu_t; \quad (3.18)$$

and

$$y_{t+1} = \frac{\alpha\beta\left(\varphi_t - 1\right) \lambda\alpha \left(\overline{y} - y_n\right)}{2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2 \left(4 + \lambda\alpha^2\right)} - \frac{\alpha\beta\left[\varphi_t - \left(1 + \sigma_\varphi^2\right)\right]}{\left(2\beta - \alpha\right)\left(1 + \sigma_\varphi^2\right)}\varepsilon_t + \frac{\alpha\beta}{2\beta - \alpha}\nu_t. \quad (3.19)$$

3.4 Implications of multiplicative instrument uncertainty

Here we investigate the implications of the introduction of multiplicative uncertainty comparing the results obtained in the previous section with the case when there is only additive uncertainty. When the policy maker faces imperfect information about the shocks hitting the economy it should respond on the basis of its best forecast of these shocks. However Brainard (1967) showed that this is no longer true when there is multiplicative uncertainty in the parameters of the relationship between the level of the policy instrument and the goal variable. In this case uncertainty implies that the policy choices affect the shape of the distribution of the goal variable and that it is optimal to adjust less than completely to the disturbances.

As in Cukierman and Meltzer (1986) and Swank (1994) and Letterie (1997), in the next sections it is assumed that σ_φ^2 is an institutional feature of the implementation of monetary policy. The volatility of multiplicative control errors is chosen by the policy maker ex ante and cannot be modified after the private sector's expectations about the instrument are set. This assumption is based on the idea that, unlike policy choices, institutional operating procedures can be changed only with a time lag which is greater than the horizon of existing nominal contracts.

3.4.1 Inflation

Following the same algorithm used for deriving the discretionary solution it is easy to find that without multiplicative uncertainty the optimal instrument feed-back rule and the equilibrium inflation rate are given respectively by

$$i_t^{AD} = \frac{\lambda\alpha\left(\overline{y} - y_n\right)}{2} - \varepsilon_t; \tag{3.20}$$

$$\pi_{t+1}^{AD} = \frac{\lambda\alpha\left(\overline{y} - y_n\right)}{2} + \frac{2\beta}{2\beta - \alpha}\nu_t; \tag{3.21}$$

where i_t^{AD}, π_{t+1}^{AD} are the level of the monetary instrument and the inflation rate when there is only additive uncertainty.

By taking unconditional expectations we can get the inflationary bias in the two cases examined.[8] We have

[8] The bias in average inflation under a time consistent monetary policy is defined relative to a hypothetical regime where the policy maker is able to credibly precommit in advance to a rule for setting the monetary instrument. The precommitment solution can be found assuming that the policy maker sets the interest rate according to the following rule: $i_t^P = \overline{\phi} + \phi\varepsilon_t$. The policymaker minimises his expected loss with respect to both the systematic and the state contingent components of the rule, respectively $\overline{\phi}$ and ϕ. In contrast to a discretionary regime, in a regime with precommitment the policy maker internalises in its optimisation problem the effects of its decision rule on expectations by setting $i_t^{Pe} = \overline{\phi}$. Deriving the first order conditions and taking unconditional expectations it is possible to see that in

$$E\left[\pi^{AD}\right] = \frac{\lambda\alpha\left(\overline{y} - y_n\right)}{2}; \tag{3.22}$$

and

$$E\left[\pi\right] = \frac{\left(2\beta - \alpha\right)\lambda\alpha\left(\overline{y} - y_n\right)}{2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2\left(4 + \lambda\alpha^2\right)}. \tag{3.23}$$

The variances of inflation are given by

$$Var\left[\pi^{AD}\right] = \left(\frac{2\beta}{2\beta - \alpha}\right)^2 \sigma_\nu^2; \tag{3.24}$$

and

$$\begin{aligned} Var\left[\pi\right] &= \frac{\left[2\beta\lambda\alpha\left(\overline{y} - y_n\right)\right]^2 \sigma_\varphi^2}{\left[2\left(2\beta - \alpha\right) + \beta\sigma_\varphi^2\left(4 + \lambda\alpha^2\right)\right]^2} + \frac{\left(2\beta\right)^2 \sigma_\varphi^2}{\left(1 + \sigma_\varphi^2\right)\left(2\beta - \alpha\right)^2}\sigma_\varepsilon^2 \\ &\quad + \left(\frac{2\beta}{2\beta - \alpha}\right)^2 \sigma_\nu^2; \end{aligned} \tag{3.25}$$

where we used that $Var\left[\pi\right] = E\left[\pi^2\right] - \left(E\left[\pi\right]\right)^2$ and given the properties of the trivariate normal distribution considered here we can compute the following joint moments $E\left[\varphi^2\varepsilon\right] = E\left[\varepsilon\varphi\nu\right] = 0$; $E\left[\varphi\varepsilon^2\right] = \sigma_\varepsilon^2$ and $E\left[\varphi^2\varepsilon^2\right] = \left(1 + \sigma_\varphi^2\right)\sigma_\varepsilon^2$.

From (3.22) and (3.23) it is possible to derive one of the main results of the present analysis:

Proposition 3.1 *Multiplicative uncertainty has an ambiguous effect on average inflation: if the amount of multiplicative uncertainty is relatively large (small) it implies a deflationary (inflationary) bias.*

From (3.23) it is straightforward to verify that, for

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equilibrium average inflation will be equal to zero, independently of whether multiplicative uncertainty is present or not. However it is possible to see that, if the socially optimal level of inflation is greater than zero, in the case of commitment we may have a deflationary bias too.

$$\sigma_{\varphi}^{2} > 2(\alpha - 2\beta)/\beta\left(4 + \lambda\alpha^{2}\right), \tag{3.26}$$

average inflation under a time consistent monetary policy will be negative; otherwise it will always be positive.

The intuition for this surprising result is as follows. The result obtained is independent of the additive component of the control errors. So assume first that there is no instrument uncertainty and then introduce only multiplicative uncertainty. If there is no instrument uncertainty both the planned level of the instrument (here equal to the actual level) and average inflation are positive and are respectively given by (3.20), without the stochastic term, and (3.22). From the first order condition (3.13) it is possible to see that when $\sigma_{\varphi}^{2} = 0$ and $\varepsilon_t = 0$ the policymaker will set the planned level of the instrument at the point where the marginal cost of higher inflation with a lower (surprise) interest rate compensates exactly the marginal benefit (with negative sign) of higher output with a lower (surprise) interest rate.

Now introduce multiplicative uncertainty with σ_{φ}^{2} sufficiently large. In this case both the planned level of the instrument and average inflation have negative sign and are respectively given by (3.16), without the stochastic term, and (3.23). From the first order condition it is possible to see that the introduction of a small increase of σ_{φ}^{2} distorts at the margin both the costs of higher inflation and benefits of higher output from creating surprisingly lower interest rates. In order to compensate these distortions a further reduction of the instrument with respect to the given expectations is required. If σ_{φ}^{2} is sufficiently large the policy maker will end up trading-off the marginal benefits of lower deflation with the marginal costs of higher than optimal output deriving from surprisingly lower levels of the instrument. Thus, in the end, the reason for our new result is that multiplicative instrument uncertainty distorts the trade-off faced at the margin by the policy maker when choosing the optimal level of the instrument.

Still from (3.23) we can prove the following proposition:

Proposition 3.2 *Multiplicative uncertainty always worsens the stabilisation role of* *the instrument but the effect on the credibility of monetary policy is ambiguous: if the amount of multiplicative uncertainty is relatively large (small) it improves (deteriorates) credibility.*

Comparing the instrument feed-back rules (3.16) and (3.20) is clear that multiplicative uncertainty makes it optimal for the policymaker to stabilise less the forecast additive control error and to adopt greater caution in conducting monetary policy. The effect of multiplicative uncertainty on average inflation can be analysed from the following first derivative

$$\frac{\partial E\left[\pi\right]}{\partial \sigma_{\varphi}^{2}}=-\frac{2\beta\sigma_{\varphi}^{2}\left(2\beta-\alpha\right)\left(4+\lambda\alpha^{2}\right)\lambda\alpha\left(\overline{y}-y_{n}\right)}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_{\varphi}^{2}\left(4+\lambda\alpha^{2}\right)\right]^{2}}>0; \tag{3.27}$$

with

$$\sigma_{\varphi}^{2}\neq 2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^{2}\right). \tag{3.28}$$

Here we have the following cases:

i) for $0<\sigma_{\varphi}^{2}<2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^{2}\right)$ the inflationary bias under multiplicative uncertainty is increased as σ_{φ}^{2} increases;

ii) for $\sigma_{\varphi}^{2}>2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^{2}\right)>0$ the deflationary bias under multiplicative uncertainty is reduced as σ_{φ}^{2} increases.

This ambiguous effect of multiplicative uncertainty on the credibility of monetary policy is new in the time inconsistency literature. The standard result, as exemplified by Swank (1994), Pearce and Sobue (1997), Letterie (1997), Letterie and Lippi (1997) and Schellekens (1998), is that multiplicative uncertainty unambiguously improves the credibility of discretionary monetary policy by inducing a more cautious stance of monetary policy. Thus the standard result is that imperfect monetary control constrains the temptations of the policymaker to surprise the private sector. However in all these models the policy instrument is the supply of money and the issues of lags in the transmission of the effects of monetary policy and overlapping nominal wage contracts are excluded from the analysis. Here we show that in a more complex model, closer to actual policy making, the effect of multiplicative uncertainty on credibility is ambiguous and depends on the level of the volatility of multiplicative control errors. This ambiguity stems from the possibility in our model of having under the time consistent monetary policy both a deflationary bias and an inflationary bias.

Finally we consider the implications of the introduction of multiplicative uncertainty for the variance of inflation. We have the following proposition:

Proposition 3.3 *The introduction of a marginal increase of multiplicative uncertainty unambiguously increases the volatility of inflation.*

This proposition follows from the first derivative of (3.25) which can be showed to be

$$\left.\frac{\partial Var[\pi]}{\partial \sigma_\varphi^2}\right|_{\sigma_\varphi^2=0} = \frac{\beta^2\left[(\lambda\alpha)^2(\overline{y}-y_n)^2+4\sigma_\varepsilon^2\right]}{(2\beta-\alpha)^2} > 0. \tag{3.29}$$

3.4.2 Output

Without multiplicative uncertainty equilibrium output is

$$y_{t+1}^{AD} = y_n - \frac{\alpha\beta}{\alpha-2\beta}\nu_t. \tag{3.30}$$

Let's take unconditional expectations of (3.19) and (3.30). In both cases expected output will be equal to the natural level

$$E[y] = E\left[y^{AD}\right] = y_n; \tag{3.31}$$

while the variances of output are given by

$$Var\left[y^{AD}\right] = \left(\frac{\alpha\beta}{\alpha-2\beta}\right)^2\sigma_\nu^2; \tag{3.32}$$

and

$$Var\left[y\right] = \frac{\left[\beta\alpha\lambda\alpha\left(\overline{y}-y_n\right)\right]^2\sigma_\varphi^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^2}+\frac{\left(\alpha\beta\right)^2\sigma_\varphi^2}{\left(1+\sigma_\varphi^2\right)\left(2\beta-\alpha\right)^2}\sigma_\varepsilon^2 + \left(\frac{\alpha\beta}{2\beta-\alpha}\right)^2\sigma_\nu^2. \tag{3.33}$$

Here we have the following proposition:

Proposition 3.4 *The introduction of a marginal increase of multiplicative uncertainty unambiguously increases the volatility of output.*

This proposition follows from the first derivative of (3.33) which can be shown to be

$$\left.\frac{\partial Var\left[y\right]}{\partial\sigma_\varphi^2}\right|_{\sigma_\varphi^2=0} = \frac{\beta^2\alpha^2\left[\left(\lambda\alpha\right)^2\left(\overline{y}-y_n\right)^2+\sigma_\varepsilon^2\right]}{\left(2\beta-\alpha\right)^2} > 0. \tag{3.34}$$

3.4.3 Social welfare

Let's analyse the implications of multiplicative uncertainty for social welfare. The unconditional expectation over society's period loss function can be expressed in the following convenient way

$$E\left[L\right] = \left(E\left[\pi\right]\right)^2 + Var\left[\pi\right] + \lambda\overline{y}^2 + \lambda Var\left[y\right]. \tag{3.35}$$

After substituting the relative expressions for the variances and unconditional expectations under the two cases here considered we have

$$E\left[L\right] = \frac{\left[\left(2\beta-\alpha\right)^2+\beta^2\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]\left[\alpha\lambda\left(\overline{y}-y_n\right)\right]^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^2}+\lambda\overline{y}^2 + \frac{\left(4+\lambda\alpha^2\right)\beta^2\sigma_\varphi^2}{\left(1+\sigma_\varphi^2\right)\left(\alpha-2\beta\right)^2}\sigma_\varepsilon^2+\frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(\alpha-2\beta\right)^2}\sigma_\nu^2; \tag{3.36}$$

and

$$E\left[L^{AD}\right] = \frac{\left[\lambda\alpha\left(\overline{y}-y_n\right)\right]^2}{4} + \lambda\overline{y}^2 + \frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(\alpha-2\beta\right)^2}\sigma_\nu^2. \tag{3.37}$$

Now it is possible to show the following proposition:

Proposition 3.5 *If the amount of multiplicative uncertainty is relatively small its introduction is welfare decreasing. On the contrary if its amount is sufficiently large multiplicative uncertainty has an ambiguous effect on the expected social loss which depends on the relative importance of the credibility problem with respect to the flexibility problem: the larger is the credibility problem with respect to the flexibility problem the more likely multiplicative uncertainty improves social welfare.*

The overall effect of multiplicative uncertainty on social loss can be examined from the following first derivative

$$\begin{aligned}\frac{\partial E\left[L\right]}{\partial\sigma_\varphi^2} &= -\frac{\left[2\left(\beta-\alpha\right)\left(2\beta-\alpha\right)+\beta^2\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]\left(4+\lambda\alpha^2\right)\beta\left[\alpha\lambda\left(\overline{y}-y_n\right)\right]^2}{\left[2\left(2\beta-\alpha\right)+\beta\sigma_\varphi^2\left(4+\lambda\alpha^2\right)\right]^3} \\ &\quad + \frac{\left(4+\lambda\alpha^2\right)\beta^2}{\left(1+\sigma_\varphi^2\right)^2\left(2\beta-\alpha\right)^2}\sigma_\varepsilon^2;\end{aligned} \tag{3.38}$$

with

$$\sigma_\varphi^2 \neq 2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^2\right). \tag{3.39}$$

Here we have two cases depending on the dimension of multiplicative uncertainty:
i) for $\sigma_\varphi^2 < 2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^2\right)$, we have

$$\frac{\partial E\left[L\right]}{\partial\sigma_\varphi^2} > 0; \tag{3.40}$$

ii) for $\sigma_\varphi^2 > 2\left(\alpha-2\beta\right)/\beta\left(4+\lambda\alpha^2\right)$, we have

$$\frac{\partial E\left[L\right]}{\partial\sigma_\varphi^2} \lesseqqgtr 0 \mathit{iff}\ \frac{\overline{y}}{\sigma_\varepsilon} \gtreqqless \Delta\left(\sigma_\varphi^2\right); \tag{3.41}$$

with

$$\Delta\left(\sigma_{\varphi}^{2}\right) \equiv \frac{\sqrt{\beta\left[2\left(2\beta-\alpha\right)+\beta\sigma_{\varphi}^{2}\left(4+\lambda\alpha^{2}\right)\right]^{3}}}{\alpha\lambda\left(1+\sigma_{\varphi}^{2}\right)\left(\alpha-2\beta\right)\sqrt{2\left(\alpha-2\beta\right)\left(\alpha-\beta\right)+\beta^{2}\sigma_{\varphi}^{2}\left(4+\lambda\alpha^{2}\right)}}. \tag{3.42}$$

Thus, if the credibility problem is large enough relatively to the flexibility problem, the introduction of sufficient multiplicative uncertainty is likely to reduce the social loss compared to the case when there is only additive instrument uncertainty. A similar result has been found, for example, also by Schellekens (1998), Letterie (1997) and Letterie and Lippi (1997). They found contrary to Swank (1994), where multiplicative uncertainty always improves social welfare, that the effect of multiplicative uncertainty on social welfare is ambiguous depending on the size of the credibility problem. Devereux (1987) has provided the same result also for the case when there is only additive uncertainty. Using a model with endogenous wage indexing, along the lines of Gray (1976), he has shown that uncertainty is more likely to increase welfare when the credibility problem becomes more important. The main difference of our analysis with respect to the previous literature is that now when multiplicative uncertainty becomes more advantageous at the same time the occurrence of a deflationary bias is more likely.

3.5 Conclusion

In the present analysis we have tried to re-examine the issue of the inflationary bias associated with discretionary monetary policy by using an extended version of the Barro and Gordon framework, closer to actual policymaking. The model developed has yielded some results that question the previous findings. In particular we have shown that time inconsistency does not necessarily imply an inflationary bias, but may yield a deflationary one instead. In this respect our framework under specified circumstances provides a counterexample to Barro and Gordon's famous result.

This surprising finding implies that the current use of the time-inconsistency paradigm as a possible explanation of episodes of persistent and excessively high inflation rates should be more cautious. Actually, our model predicts that economies which feature a relatively large incentive to increase output above its long-run level are more likely to be plagued by a deflationary bias. In this case the implementation of policy should be characterised by a relatively more imprecise control of the policy instrument. An Inflationary bias is more likely to be present in economies where the credibility problem is relatively less serious. Here the implementation of policy should be characterised by a relatively better control of the policy instrument.

Some recent works by Nicolini, Nobay and Peel have shown that Barro and Gordon's inflationary bias result is not robust to interesting modifications of the original framework. In particular they have examined the implications of the introduction of microfoundations, with a general equilibrium model, and asymmetric central bank preferences. Our analysis has confirmed the above finding.

However, in contrast to Nicolini we have used a framework closer to that used by Barro and Gordon, while unlike Nobay and Peel we do not obtain a deflationary bias under a regime with commitment and the superiority of the ex ante optimal monetary policy still holds in our model. The latter result crucially depends on the assumptions about the socially optimal level of inflation. If we assume an inflation target greater than zero in the policy maker's loss function, we may have in our framework a deflationary bias also in the case of commitment.

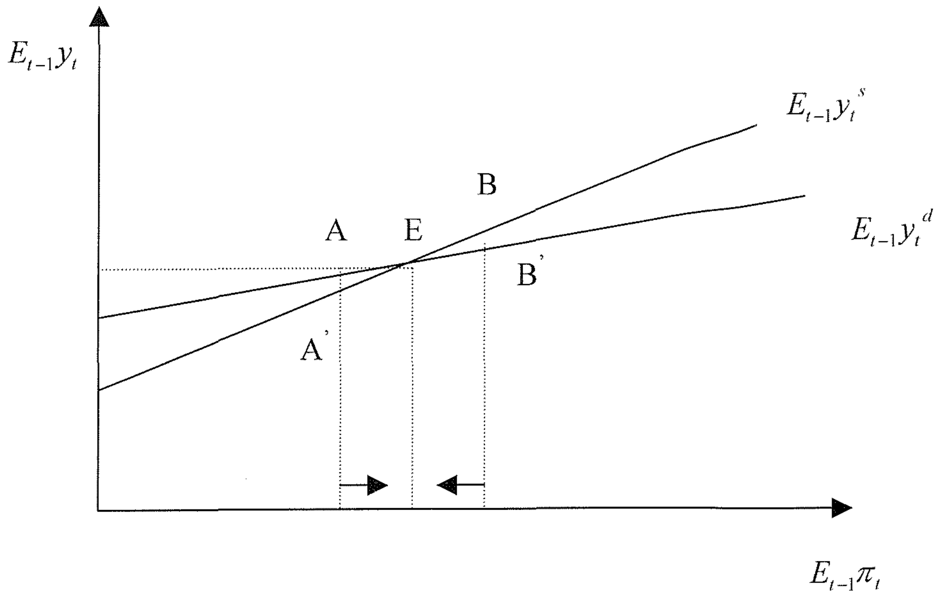

Figure 3.1 – Dynamic stability when $\alpha/2>\beta$.

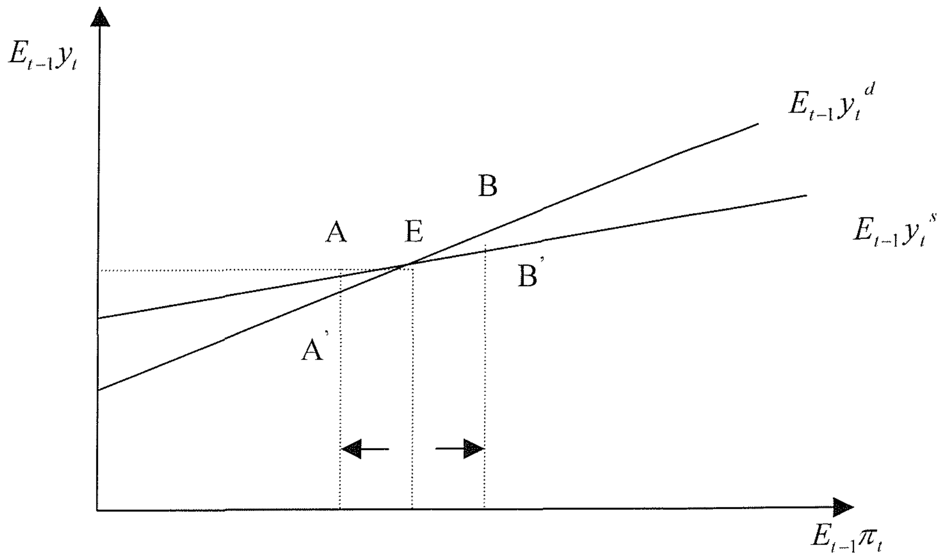

Figure 3.2 – Dynamic stability when $\alpha/2<\beta$.

Chapter 4

Designing instrument rules for monetary stability: the optimality of interest-rate smoothing

4.1 Introduction

In chapter 2, section 2.9.3, we have discussed Taylor's view on the debate on rules versus discretion in practice. He argues that the rules versus discretion dilemma is indeed more a semantic issue: if a policy rule is interpreted more generally as a systematic behaviour there need not necessarily be a trade-off between flexibility and commitment.

This view on policy rules is questioned by McCallum by using the Barro-Gordon framework. He shows that the presence or absence of a systematic behaviour is not enough to distinguish between rule-like behaviour and discretion as the presence of a systematic behaviour under a discretionary regime does not eliminate the emerging of an inflationary bias.

In this chapter the criticism of McCallum is questioned. In particular we will show that under specified circumstances optimally designed instrument rules may render negligible the inflationary bias associated with time consistent monetary policy without prejudice to stabilisation of shocks. Furthermore, our framework sheds also some light on the puzzling issue of the inertial behaviour followed by central bankers in the setting of interest rates, i.e. the practice of interest-rate smoothing.

In section 4.2 we provide an analytical distinction between instrument rules and target rules. Section 4.3 discusses the empirical evidence on interest rate rules. In section 4.4 the model is illustrated. In section 4.5 we consider the simple case of no interest-rate smoothing and replicate McCallum's finding. The case of sluggish adjustments of the interest rate is examined in section 4.6. Here the most innovative results of the analysis are shown. Section 4.7 concludes.

4.2 Instrument rules versus target rules

In our analysis we will focus our attention on instrument rules and in particular on interest rate rules. Thus a first step should be to define formally an instrument rule. With the aim of defining rigorously what are instrument rules it might be useful first to compare them with an alternative kind of rules: target rules.

Svensson (1996) proposes the following distinction: "Setting the instrument to make the inflation forecast equal to the inflation target is an example of a *target rule* which, if applied by the monetary authority, would result in an endogenous optimal reaction function expressing the instrument as a function of the available relevant information. This is different from an *instrument rule* that directly specifies the reaction function for the instrument in terms of the current information."

This dichotomy is not generally accepted. McCallum (1997b), for example, argues that the distinction between instrument rules and target rules is merely theoretical. Judging from a practical perspective, the importance of a target rule that is not expressed in terms of a feasible instrument variable is debatable. Consequently, he proposes the following definition: "a monetary policy rule is a formula that specifies instrument settings, with the choice of a target variable and path constituting only one ingredient".

In the time-inconsistency literature there is a standard formalisation of instrument rules and target rules. Instrument rules are identified with a fixed formula that specifies the decision rule of the policy maker and are interpreted in terms of the analytical distinction between commitment to a rule and discretion. The definition of target rules is found instead within the normative side of the literature, rather than in the descriptive one.

Rogoff's (1985) is considered the pioneer of strategic delegation in monetary policy. The proposal of changing monetary institutions for dealing with the issue of time inconsistency and the alleged inflationary bias is the core idea behind the approach of monetary delegation. Recently the delegation approach has been extended by the work of Walsh (1995a), Persson and Tabellini (1993) and Svensson (1997a) among others, with the introduction of incentive schemes or policy targets in order to completely remove the inflationary bias. As pointed out by Rogoff the optimality of delegation of monetary policy to a "weight-conservative" central banker suggests also an alternative interpretation of the delegation process. In particular he shows that this kind of solution to the inflationary bias problem can be interpreted also as an inflation targeting scheme based on punishments and rewards. Similar considerations have been made on Walsh contracting solution too. For example, Persson and Tabellini (1998) have stressed the close relationship between inflation targeting schemes and contracts based on penalties conditional on realised inflation.[1] Also Svensson's "target-conservative" central banker can be indirectly related to an inflation targeting scheme as it is possible to show that an optimal inflation target is equivalent to an optimal linear inflation contract.

Other types of target rules have been considered by Rogoff: monetary targeting, interest-rate targeting, etc.. But in all the cases considered the target rule has been modelled by including in the central bank's objective function some weight on achieving the specified target.

In the following sections we will consider a general definition of an instrument rule

[1] See also Walsh (1997) for a broader discussion on inflation targeting regimes.

for the interest rate which yields some interesting insights on the role that can be played by instrument rules in monetary policy.

4.3 The empirical evidence on interest rate rules

Despite the great importance of the interest rate as a policy instrument only recently the empirical and theoretical literature has focused on interest rate rules.[2] The path breaking work of Taylor (1993) started the debate. There has been a large body of literature describing the macroeconomic implications of interest-rate smoothing, which assumes the presence of an interest-rate smoothing or alternatively an interest-rate targeting objective in the loss function of the policy maker.[3] However, in this line of research interest-rate smoothing is not related to an explicit rule for setting the interest rate.

The empirical evidence on the interest rate rule followed by central banks in the last two decades can be summarised by the following form

$$r_t = \gamma r_{t-1} + (1-\gamma)\,\bar{r}_t; \tag{4.1}$$

with

$$\bar{r}_t = \chi_0 r^* + \chi_1 (\pi_t - \pi^*) + \chi_2 (y_t - y^*)\,; \tag{4.2}$$

where $0 < \gamma < 1$, r_t is the nominal interest rate and the variable $\bar{r}_t$ is an operative target given by expression (4.2). In this specification the operative target is a function of both inflation π_t and output y_t expressed as deviations from trend levels; r^* is trend nominal interest rate and χ_0, χ_1, χ_2 are positive parameters. This policy rule implies that

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[2] In the words of Blinder (1998, pp.26-29): "Returning to Poole's dichotomy [on the choice of monetary instrument]...in the end, real-world events, not theory, decided the issue. Ferocious instabilities in estimated LM curves in the United States, United Kingdom, and many other countries, beginning in the 1970s and continuing to the present day, led economists and policymakers alike to conclude that money-supply targeting is simply not a viable option....So interest rates won by default".

[3] Reviews on this literature are provided by Cukierman (1992), Goodhart (1996), Walsh (1998), Clarida, Galì and Gertler (1999), Sack and Wieland (1999).

the interest rate reacts to inflation and output gap but there is only partial adjustment to these variables due to presence of an interest-rate smoothing component. The common feature of all estimates of the degree of inertia in the central bank's response, γ, is that they are large and highly significant, normally close to one.[4]

Using the words of Clarida, Galì and Gertler (1999, p.1688): "The existing theory, by and large, does not readily account for why the central bank should adjust rates in such a sluggish fashion. Indeed, understanding why central banks choose a smooth path of interest rates than theory would predict is an important unresolved issue".

One important exception is constituted by Woodford (1998). He provides a rationale for a central banker with an interest-rate smoothing objective in terms of an optimal monetary delegation problem. Among other relevant contributions, his model provides also a New Keynesian perspective of the issue of time inconsistency.[5] However his analysis is based on two crucial assumptions. He postulates that in the social loss there is an interest-rate targeting motive and that there exists a central banker that prefers for interest rate to deviate farther from its target. In this latter case, as observed by Woodford, the interest rate target can hardly be interpreted as a target. These assumptions are needed to show that it can be advantageous for society to delegate monetary policy to a central banker who includes in his loss function an interest-rate smoothing objective.

Unfortunately these assumptions seem quite *ad hoc*. As we will see later on, our analysis provides also an alternative view on the inertia observed in the response by central banks to changes in macroeconomic variables, without using the restrictive assumptions considered by Woodford.

[4]In general the estimated value of γ is on the order of .9. See the review in Clarida, Galì and Gertler (1999). Sack (1998) estimates for γ a value of 0.63, with a standard error of 0.08. Higher values are found by Orphanides (1998) and Clarida, Galì and Gertler (1998a,b).

[5]See also Clarida, Galì and Gertler (1999) for a similar attempt.

4.4 The model

The analysis is based on a stochastic rational expectations IS-LM model. Aggregate supply is given by a standard expectations augmented Phillips curve

$$y_t = y_n + \alpha\left(\pi_t - \pi_t^e\right) + v_t; \tag{4.3}$$

where y_t is the level of output, π_t the inflation rate and v_t is a random disturbance normally distributed with mean zero and variance σ_v^2. Private sector's inflation expectations, π_t^e, are formed rationally. The parameter α is positive.

Aggregate demand is given by

$$y_t = y_n - \beta\left(r_t - \pi_t^e - \rho\right) + u_t; \tag{4.4}$$

where r_t is the nominal interest rate, ρ is the long-run equilibrium real interest rate and u_t is a stochastic disturbance normally distributed with mean zero and variance σ_u^2 . The stochastic disturbances u_t and v_t are assumed to be independent of each other. The parameter β is positive.[6]

By equating (4.3) and (4.4), after some manipulations we can obtain an expression for inflation as a function of the nominal interest rate, inflation expectations and exogenous variables

$$\pi_t = \frac{\beta}{\alpha}\rho + \frac{\alpha+\beta}{\alpha}\pi_t^e - \frac{\beta}{\alpha}r_t - \frac{1}{\alpha}v_t + \frac{1}{\alpha}u_t. \tag{4.5}$$

This equation can be expressed also in terms of interest rate expectations in the following way. After taking expectations of (4.5) we get

$$\pi_t^e = r_t^e - \rho. \tag{4.6}$$

[6]The specification of the aggregate demand with current period inflation expectations has been used also by Clark, Goodhart and Huang (1999).

Substitution of (4.6) in (4.5) implies

$$\pi_t = -\rho + \frac{\alpha+\beta}{\alpha} r_t^e - \frac{\beta}{\alpha} r_t - \frac{1}{\alpha} v_t + \frac{1}{\alpha} u_t; \tag{4.7}$$

consequently the aggregate demand equation can be rewritten as

$$y_t = y_n - \beta \left(r_t - r_t^e \right) + u_t. \tag{4.8}$$

Turning to preferences, as usual it is assumed that the government's preferences coincide with those of society. The preferences are represented by the government's loss function

$$V = E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{4.9}$$

where δ, with $0 < \delta < 1$, is the discount factor and the government's period loss function L_t is given by

$$L_t = \pi_t^2 + \lambda \left(y_t - \overline{y} \right)^2; \tag{4.10}$$

where the parameter λ is a relative weight. In order to introduce the issue of time inconsistency it is assumed that the government wishes to increase output above the natural level, with $\overline{y} > y_n$.

Monetary policy is delegated to a central banker whose preferences are given by:

$$V^b = E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{4.11}$$

where it is assumed that the central banker's period loss function is identical to expression (4.10).

Last but not least, an important novel element of our model concerns the way monetary policy is implemented. The interest rate is the instrument used by the monetary authority for achieving its goals. In the following analysis we will consider efficient instrument rules for setting the nominal interest rate. The general specification of these instrument rules will be

$$r_t = \gamma \widehat{r}_{t-1} + (1-\gamma)\,\overline{r}_t; \tag{4.12}$$

where the interest rate level chosen by the central banker is expressed as a convex combination of two components, with $0 < \gamma < 1$. One is a targeting component, in the sense that the variable $\overline{r}_t$ is an operative target which expresses the level of the interest rate in terms of a specified formula. The other is a partial adjustment term, with the interest rate r_t dependent upon past values of $\widehat{r}_t$, a component of $\overline{r}_t$ to be defined.

This general specification for an interest rate rule is consistent with the empirical evidence on interest-rate smoothing, as expressed by (4.1). But in contrast with the specifications usually considered in the literature it is based only on a component of the lagged interest rate. Later on we will compare more in detail this alternative specification with the one usually postulated in empirical analysis.

Now a crucial parameter in the following analysis will be γ, which expresses the fixed degree of inertia in the central bank's response. We suppose that monetary authorities act gradually with a certain degree of monetary inertia for some reasons. Candidate reasons that have been adduced in the literature are several: forward-looking behaviour by private agents, measurement errors associated with macroeconomic variables, uncertainty about structural parameters, concern for the financial stability of the banking system, adverse reactions of financial markets to frequent modifications in the direction of short-term interest rates.[7]

In contrast to the standard approach for modelling an interest-rate smoothing motive, we do not include in the central bank's loss function an objective of minimising the deviations of current level of interest rate from previous period levels, as done for instance by Woodford (1998). This latter approach would imply that the central bank's period

[7] For a recent review see for example Sack and Wieland (1999).

loss function should be modified in the following way

$$L_t = \pi_t^2 + \lambda (y_t - \overline{y})^2 + \varphi (r_t - r_{t-1})^2 ; \tag{4.13}$$

where φ is the weight assigned by the central banker to the interest-rate smoothing objective. The parameter φ is usually assumed to be common knowledge among players.

On the contrary, we suppose that this interest-rate smoothing motive is explicitly incorporated in the policy rule adopted by the central bank. The reason for this different approach is rather simple. The idea is to see whether there might be a superiority of instrument rules over target rules in providing an explanation for the optimality of interest-rate smoothing. This seems to be a promising route as in principle instrument rules present some advantages with respect target rules. First target rules modify the policy maker's objective function while instrument rules not. Second, the specification of instrument rules can be relatively more flexible than in the case of target rules.

How plausible is this alternative formalisation? Sack (1998) for instance says that: "many empirical studies of monetary policy incorporate an explicit interest-rate smoothing incentive in the objective function of the Fed. However, introducing this argument has little justification beyond matching the data. Furthermore, the above statistics provide evidence of gradualism only if the Fed would otherwise choose a random-walk policy in absence of an interest-rate smoothing objective. Therefore, while establishing that the funds rate is not a random walk, these statistics do not necessarily provide evidence of gradualism in monetary policy." Moreover Sack and Wieland (1999) offer several arguments, discussing the empirical evidence supporting them, that explain why central banks may have an incentive to smooth interest rates without assuming an interest-rate smoothing objective in their loss function.

We interpret interest-rate smoothing as an institutional feature of the implementation of monetary policy in practice and not as an object in the policy maker's loss function. While gradualism is a stylised fact, on the contrary there is no evidence of a central bank's having as goal or as intermediate target the minimisation of the deviations of the current level of interest rate from previous period levels.

Hence, in the present framework we assume that the central bank's behaviour is characterised institutionally by some degree of monetary inertia and that this stylised fact is incorporated in the policy rule followed, with the parameter γ expressing the specific degree of inertia. Moreover we assume also that the policy maker can choose ex ante the specific fixed value of γ (within the defined range) in order to stabilise inflation and output optimally. This means that the presence of inertia is introduced exogenously and justified by the above arguments, but the specific degree of gradualism in the adjustment of the interest rate is chosen endogenously by the central banker.

It is interesting to notice that the described process for setting the monetary instrument is very similar to that examined in chapter 3, where we considered the case when the monetary authority does not control perfectly the instrument. Following Cukierman and Meltzer and others we assumed that the variance of multiplicative control errors, like the parameter γ, is an institutional feature of the implementation of monetary policy that can be chosen ex-ante optimally. Nevertheless, the case of instrument uncertainty and that of interest-rate smoothing must be kept separated: a conservative use of the monetary instrument, due to the presence of control errors, does not necessarily mean gradualism.[8]

A crucial assumption is that the systematic behaviour followed by the policy maker and embodied in the policy rule (4.12) is generally understood by the public. In order to understand how the specific value of γ can be inferred by private agents we need to complete the specification of the interest rate rule considered and compare it with the standard specification used in the empirical literature. So let's see what can be a plausible specification for $\overline{r}_t$ and $\widehat{r}_t$. Substituting the policy rule (4.12) in (4.7) and (4.8), by using $r^e_t = E_{t-1}r_t$ and $\overline{r}^e_t = E_{t-1}\overline{r}_t$, we get

[8] For instance, this point is stressed by Stock (1999).

$$\pi_t = -\rho + \gamma \widehat{r}_{t-1} + \frac{(\alpha+\beta)(1-\gamma)}{\alpha}\overline{r}^e_t - \frac{\beta(1-\gamma)}{\alpha}\overline{r}_t - \frac{1}{\alpha}v_t + \frac{1}{\alpha}u_t, \tag{4.14}$$

and

$$y_t = y_n - \beta(1-\gamma)(\overline{r}_t - \overline{r}^e_t) + u_t. \tag{4.15}$$

Because the objective function is linear quadratic we can restrict our attention, without loss of generality, on specifications of $\overline{r}_t$ that are linear in the state variables appearing in the reduced forms (4.14) and (4.15). Thus we can write

$$\overline{r}_t = \mathsf{a} + \mathsf{b}\overline{r}^e_t + \mathsf{c}\widehat{r}_{t-1} + \mathsf{m}v_t + \mathsf{n}u_t. \tag{4.16}$$

Now suppose that the policy maker's optimisation problem consists in choosing the parameters a, b,m and n. If the degree of monetary inertia is zero, that is $\gamma = 0$, and $\widehat{r}_{t-1}$ is eliminated from (4.16) we get an expression which is analogous to that examined by McCallum (1993) for discussing the importance of the presence of a systematic behaviour in order to distinguish between rule-like and discretionary behaviour. [9]

The operating procedure described above is supposed to be an institutional feature of monetary policy and is common knowledge among all the players of the policy game examined. Moreover, this procedure is operative under both discretion and commitment. Of course the optimal parameter values may change according to the given monetary regime under which the central banker operates. However, it bears repeating that the described policy rule operates despite the fact that the policy regime is discretionary. As suggested by McCallum, the presence of this policy rule simply reflects the idea that the

[9] McCallum (1993) does not consider explicitly shocks in his policy rule but the line of reasoning is the same: the systematic procedure followed in the implementation of policy consists in choosing optimally the value of the parameters which relates the relevant state variables to the level of the control variable.

policy maker adopts a systematic behaviour in setting the monetary instrument.

The question McCallum poses is whether the presence of this systematic behaviour under a discretionary regime may imply outcomes that are distinct from the case where this systematic behaviour is absent and, in particular, if these outcomes are the same as those pertaining to a regime with commitment. We investigate the same question but with the introduction of an additional element concerning the specification of the policy rule examined by McCallum. In particular we consider the possibility that the policy maker also adopts a systematic inertial behaviour in choosing the level of the interest rate.

Relative to the partial adjustment term $\widehat{r}_t$ we consider the following specification:

$$\widehat{r}_t = \mathsf{a} + \mathsf{b}\overline{r}_t^e + \mathsf{c}\widehat{r}_{t-1}; \tag{4.17}$$

where we assume that the inertial component in the policy rule is constituted only by the systematic component of the operative target $\overline{r}_t$. The rationale for this restriction is rather intuitive: smoothing interest rate in proportion to r_{t-1} or $\overline{r}_{t-1}$ would affect the stabilisation of shocks in a suboptimal way, while the time-inconsistency problem we are seeking to tackle is related only to the systematic component of the policy rule.

Let's compare our specification of the instrument rule for the interest rate with the standard one used in the literature. Using (4.16) and (4.17), it is possible to show that (4.12) can be expressed also as

$$r_t = \gamma r_{t-1} + (1-\gamma)\,\overline{r}_t + \gamma^2\left(\widehat{r}_{t-1} - \widehat{r}_{t-2}\right) - \gamma\,(1-\gamma)\left(\mathsf{m}v_{t-1} + \mathsf{n}u_{t-1}\right). \tag{4.18}$$

This result is derived formally in Appendix A. Expression (4.18) resembles expression (4.1). Actually only the last two terms of (4.18) are not included in (4.1). If the degree of inertia in the central bank's response γ is close to one - and hence past changes of $\widehat{r}_t$ are relatively small - estimates of γ inferred from the specification (4.18) should be very close to those derived from (4.1). In the following analysis we will see that under a discretionary regime it is optimal for the central banker and society to have a degree of gradualism γ close to one, which is consistent with the empirical evidence on interest-rate smoothing.

An important aspect of the present analysis, that needs to be clarified, is the exogenous introduction of dynamics - by means of the specified instrument rule - in a static model of the economy. As the model is static, interest-rate smoothing becomes relevant and the nature of the model becomes dynamic only if there exists an optimal path along which the nominal interest rate passes from an excessively high initial level, due to the presence of an inflationary bias, to a lower level with inflation closer to the socially optimal rate. We will see in the subsequent analysis that the existence of such optimal path depends crucially on the presence of a sufficiently high level of γ. If γ is too low the specified rule for the interest rate is no longer optimal as in this case the specification of the optimal rule should be static, without any lagged variables.

Furthermore, the specified policy rule reflects the recurrent use of operating targets for the interest rate controlled by central banks. However a typical specification for the operating target would be to express $\overline{r}_t$ in terms of the current level of inflation and a proxy for real activity, as in equation (4.2). Here we consider instead an alternative specification for $\overline{r}_t$, which takes account of all the relevant state variables present in the model. Nevertheless, using the first order condition of the policy maker's optimisation problem, it is possible to show that in equilibrium the specification of $\overline{r}_t$ resembles more closely the specification of the operative target for the interest rate popularised by Taylor (1993).

In the following sections we will suppose that the implementation of monetary policy follows three stages. In the first stage, the government delegates monetary policy to a central banker and instructs him to follow an assigned instrument rule, given by (4.12). In other words the choice made by the government consists to direct the central banker to follow a systematic behaviour in the setting of interest rates. In the second stage, the central banker makes the ex-ante choice of γ, that is he chooses the fixed degree of monetary inertia introduced in the economy. In the third stage, the parameter values in $\bar{r}_t$ are derived from the optimisation problem of the central banker. Of course the stages become two in the case of no interest rate-smoothing.

4.5 No interest-rate smoothing

4.5.1 Commitment

In order to replicate McCallum's results, we start first with the case when the monetary authority does not change smoothly the interest rate, i.e. $\gamma = 0$ in the instrument rule (4.12). In this case we have that the interest rate is given by

$$r_t = \bar{r}_t; \tag{4.19}$$

with (4.16) replaced by

$$\bar{r}_t = \mathsf{a} + \mathsf{b}\bar{r}_t^e + \mathsf{m}v_t + \mathsf{n}u_t, \tag{4.20}$$

where there is no lagged variable.

In our framework, due to the presence of an instrument rule of the type of (4.12), the actual choice variable of the monetary authority is $\bar{r}_t$. Hence the relevant expectations are those that the private sector forms on $\bar{r}_t$. From (4.19) we have

$$r_t^e = \bar{r}_t^e. \tag{4.21}$$

Following Svensson (1997a) and Clark, Goodhart and Huang (1999), in the commitment solution the central banker internalises in his minimisation problem the impact of his decisions on the interest rate on private sector's expectations. So let's formalise the optimization problem of the central banker when he is able to commit in advance to a decision rule for the choice variable $\bar{r}_t$. We have:

$$\min_{\bar{r}_t, \bar{r}_t^e} E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{4.22}$$

$$\text{s.t. Eqs. } (4.7), (4.8), (4.10), (4.19), (4.21) \text{ and } \bar{r}_t^e = E_{t-1}[\bar{r}_t];$$

which is equivalent to solve the static problem of minimising the expected period loss function

$$\min_{\bar{r}_t, \bar{r}_t^e} E_{t-1} L_t; \tag{4.23}$$

$$\text{s.t. Eqs. } (4.7), (4.8), (4.10), (4.19), (4.21) \text{ and } \bar{r}_t^e = E_{t-1}[\bar{r}_t];$$

Differentiating L_t with respect to $\bar{r}_t$ and $\bar{r}_t^e$ we get the following first order conditions

$$\begin{aligned} -2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta(y_t - \overline{y}) + \Omega_{t-1} &= 0; \\ E_{t-1}\left[2\frac{\alpha+\beta}{\alpha}\pi_t + 2\lambda\beta(y_t - \overline{y}) - \Omega_{t-1}\right] &= 0; \end{aligned} \tag{4.24}$$

where Ω_{t-1} is the Lagrange multiplier of $\bar{r}_t^e = E_{t-1}[\bar{r}_t]$. Eliminating the multiplier yields

$$-2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta(y_t - \overline{y}) + 2\frac{\alpha+\beta}{\alpha}E_{t-1}\pi_t + 2\lambda\beta(E_{t-1}y_t - \overline{y}) = 0. \tag{4.25}$$

Substitution of (4.20) in (4.25) implies that for optimality we must have the following values for the coefficients of the decision rule (4.20)

$$\begin{aligned} \mathsf{a} &= \frac{\rho\alpha^2}{(1+\lambda\alpha^2)\,\beta^2}; \\ \mathsf{b} &= \frac{(1+\lambda\alpha^2)\,\beta^2-\alpha^2}{(1+\lambda\alpha^2)\,\beta^2}; \\ \mathsf{m} &= -\frac{1}{(1+\lambda\alpha^2)\,\beta}; \\ \mathsf{n} &= \frac{1}{\beta}. \end{aligned} \tag{4.26}$$

The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker, which gives

$$\bar{r}_t^{e,c} = \rho. \tag{4.27}$$

Substituting the private sector's expectation in the central banker's reaction function gives the equilibrium value of $\bar{r}_t$ under a regime with commitment

$$\bar{r}_t^c = \rho - \frac{1}{(1+\lambda\alpha^2)\,\beta}v_t + \frac{1}{\beta}u_t. \tag{4.28}$$

Inserting (4.28) in (4.19) gives the equilibrium value of the interest rate r_t^c. While substitution of $\bar{r}_t^c$ and $\bar{r}_t^{e,c}$ back in equations (4.7) and (4.8) gives the equilibrium values of inflation and output:

$$\pi_t^c = -\frac{\alpha\lambda}{1+\lambda\alpha^2}v_t; \tag{4.29}$$

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{4.30}$$

From (4.19) and using (4.28), it is possible to see that average nominal interest rate is equal to the long-run equilibrium real interest rate, ρ. While from (4.29) we can see that under a regime with commitment average inflation is equal to zero.

4.5.2 Discretion

Now consider the optimization problem of the central banker when he chooses $\bar{r}_t$ in a discretionary manner. In this case the coefficients in $\bar{r}_t$ are chosen period by period, rather than once and for all, and private sector's expectations are no longer a control variable. Here the policy maker solves the following problem

$$E_{t-1} \min_{\bar{r}_t} L_t; \tag{4.31}$$

$$\text{s.t. Eqs. } (4.7), (4.8), (4.10), (4.19), (4.21);$$

where, as observed by Persson and Tabellini (1998), it is possible to conclude that the expectations operator becomes redundant.

Differentiating L_t with respect to $\bar{r}_t$ we get the following first order condition

$$-2\frac{\beta}{\alpha}\pi_t - 2\lambda\beta\left(y_t - \overline{y}\right) = 0. \tag{4.32}$$

Again, we postulate that the central banker's reaction function is represented by (4.20). This yields the following optimal values for the coefficients of the decision rule considered

$$\begin{aligned}
\mathsf{a} &= -\frac{\alpha\left[\rho + \alpha\lambda\left(\overline{y} - y_n\right)\right]}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{b} &= \frac{\left(1+\lambda\alpha^2\right)\beta + \alpha}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{m} &= -\frac{1}{\left(1+\lambda\alpha^2\right)\beta}; \\
\mathsf{n} &= \frac{1}{\beta}.
\end{aligned} \tag{4.33}$$

Comparing the optimality conditions (4.33) with those correspondent to the case of commitment, we can observe that now a is negative instead of positive, while in the numerator of b the term $(\beta - \alpha)$ is disappeared. The private sector's expectations are found by taking expectations at $t-1$ over the reaction function of the central banker, which gives

$$\bar{r}_t^{e,d} = \rho + \alpha\lambda\left(\overline{y} - y_n\right). \tag{4.34}$$

Repeating all the substitutions made in the case of commitment we can get the following equilibrium values for the case of discretion

$$\bar{r}_t^d = \rho + \alpha\lambda\left(\overline{y} - y_n\right) - \frac{1}{\left(1+\lambda\alpha^2\right)\beta}v_t + \frac{1}{\beta}u_t; \tag{4.35}$$

Inserting (4.35) in (4.19) gives the equilibrium value of the interest rate r_t^d. While substitution of $\bar{r}_t^d$ and $\bar{r}_t^{e,d}$ back in equations (4.7) and (4.8) gives the equilibrium values of inflation and output:

$$\pi_t^d = \alpha\lambda\left(\overline{y} - y_n\right) - \frac{\alpha\lambda}{1+\lambda\alpha^2}v_t; \tag{4.36}$$

$$y_t^d = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{4.37}$$

From (4.19) and using (4.35), it is possible to see that average nominal interest rate is higher than in the case of commitment, as it is equal to $\left[\rho + \alpha\lambda\left(\overline{y} - y_n\right)\right]$. While from (4.36) we can derive the famous result that under a regime with discretion average inflation is characterised by an inflationary bias, equal to $\left[\alpha\lambda\left(\overline{y} - y_n\right)\right]$. Thus, as observed by McCallum, the presence or absence of systematic behaviour is not enough to distinguish between rule-like behaviour and discretion, as the presence of a systematic behaviour under a discretionary regime does not remove the inflationary bias.

4.6 Interest-rate smoothing

4.6.1 Commitment

Let's consider the case of interest-rate smoothing. Consider again first a regime where a commitment strategy is available to the central banker. In this case the instrument rule we will consider for the interest rate is (4.12), with (4.16) and (4.17).

The optimization problem of the central banker when he is able to commit in advance to a decision rule for the choice variable $\bar{r}_t$ can be expressed as

$$\min_{\bar{r}_t, \bar{r}_t^e} E_0 \sum_{t=1}^{\infty} \delta^{t-1} L_t; \tag{4.38}$$

$$\text{s.t. Eqs. } (4.14), (4.15), (4.10) \text{ and } \bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right].$$

This is now a dynamic programming problem with two control variables, $\bar{r}_t$ and $\bar{r}_t^e$, and one state variable, the lagged variable $\widehat{r}_{t-1}$. As shown by Lockwood and Philippopoulus (1994) and Lockwood, Miller and Zhang (1994) and (1998), the solution can be obtained by solving the following equation with the value function $V\left(\widehat{r}_{t-1}\right)$:

$$V\left(\widehat{r}_{t-1}\right) = \min_{\bar{r}_t, \bar{r}_t^e} E_{t-1}\left[L_t^b + \delta V\left(\widehat{r}_t\right)\right]; \tag{4.39}$$

$$\text{s.t. Eqs. } (4.14), (4.15), (4.10) \text{ and } \bar{r}_t^e = E_{t-1}\left[\bar{r}_t\right].$$

As we have a linear-quadratic problem, $V\left(\widehat{r}_t\right)$ must also be quadratic. Without loss of generality, we can set

$$V\left(\widehat{r}_t\right) = \theta_0 + 2\theta_1 \widehat{r}_t + \theta_2 \widehat{r}_t^2. \tag{4.40}$$

Now using the fact that $\widehat{r}_t = \bar{r}_t - (\mathsf{m}v_t + \mathsf{n}u_{t,})$, we have

$$V_{\bar{r}}(\widehat{r}_t) = 2(\theta_1 + \theta_2 \widehat{r}_t). \tag{4.41}$$

Differentiating $V(\widehat{r}_{t-1})$ with respect to $\bar{r}_t$ and $\bar{r}_t^e$ we get the following first order conditions

$$\begin{aligned} -2\frac{\beta}{\alpha}(1-\gamma)\pi_t - 2\lambda\beta(1-\gamma)(y_t - \overline{y}) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t + \Omega_{t-1} &= 0; \\ E_{t-1}\left[2(1-\gamma)\frac{\alpha+\beta}{\alpha}\pi_t + 2\lambda\beta(1-\gamma)(y_t - \overline{y}) - \Omega_{t-1}\right] &= 0; \end{aligned} \tag{4.42}$$

where Ω_{t-1} is the Lagrange multiplier of $\bar{r}_t^e = E_{t-1}[\bar{r}_t]$. Eliminating the multiplier yields

$$\begin{aligned} 0 &= -2\frac{\beta}{\alpha}(1-\gamma)\pi_t - 2\lambda\beta(1-\gamma)(y_t - \overline{y}) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t \\ &\quad +2(1-\gamma)\frac{\alpha+\beta}{\alpha}E_{t-1}\pi_t + 2\lambda\beta(1-\gamma)(E_{t-1}y_t - \overline{y}). \end{aligned} \tag{4.43}$$

Substitution of (4.16) and (4.17) in (4.43) implies that for optimality we must have the following values for the coefficients of the decision rule (4.16)

$$\begin{aligned} \mathsf{a} &= \frac{\alpha^2[\rho(1-\gamma) - \delta\theta_1]}{(1+\lambda\alpha^2)\beta^2(1-\gamma)^2 + \delta\theta_2\alpha^2}; \\ \mathsf{b} &= \frac{(1-\gamma)^2(\beta^2 - \alpha^2 + \lambda\beta^2\alpha^2)}{(1+\lambda\alpha^2)\beta^2(1-\gamma)^2 + \delta\theta_2\alpha^2}; \\ \mathsf{c} &= -\frac{\alpha^2\gamma(1-\gamma)}{(1+\lambda\alpha^2)\beta^2(1-\gamma)^2 + \delta\theta_2\alpha^2}; \\ \mathsf{m} &= -\frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}; \end{aligned}$$

$$\mathsf{n} = \frac{1}{(1-\gamma)\beta}. \tag{4.44}$$

The private sector's expectations are found by taking expectations at $t-1$ over $\bar{r}_t$:

$$\bar{r}_t^{e,c} = \frac{\rho(1-\gamma)-\delta\theta_1}{(1-\gamma)^2+\delta\theta_2} - \frac{\gamma(1-\gamma)}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1}. \tag{4.45}$$

Substituting the private sector's expectations in the central banker's reaction function gives the equilibrium value of $\bar{r}_t$ under a regime with commitment:

$$\bar{r}_t^c = \frac{\rho(1-\gamma)-\delta\theta_1}{(1-\gamma)^2+\delta\theta_2} - \frac{\gamma(1-\gamma)}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1} - \frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}v_t + \frac{1}{(1-\gamma)\beta}u_t. \tag{4.46}$$

Inserting (4.46) in (4.12) gives the equilibrium value of the interest rate r_t^c. While substitution of $\bar{r}_t^c$ and $\bar{r}_t^{e,c}$ back in equations (4.14) and (4.15) gives the equilibrium values of inflation and output:

$$\pi_t^c = -\frac{\delta[\rho\theta_2+(1-\gamma)\theta_1]}{(1-\gamma)^2+\delta\theta_2} + \frac{\gamma\delta\theta_2}{(1-\gamma)^2+\delta\theta_2}\widehat{r}_{t-1} - \frac{\lambda\alpha}{1+\lambda\alpha^2}v_t; \tag{4.47}$$

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{4.48}$$

From (4.47) emerges an important difference with respect to the static case, without interest-rate smoothing. We can observe from the equilibrium value of inflation that interest-rate smoothing allows the policy maker to smooth inflation over a number of periods.

Following Svensson (1997a), in order to find θ_1 and θ_2 we can apply the envelope theorem on (4.39), which combined with (4.40) implies

$$V_{\widehat{r}}\left(\widehat{r}_{t-1}\right) = 2\left(\theta_1 + \theta_2 \widehat{r}_{t-1}\right) = E_{t-1}\left[2\gamma\left(\pi_t\right)\right]; \tag{4.49}$$

or

$$2\left(\theta_1 + \theta_2 \widehat{r}_{t-1}\right) = -2\delta\left\{\frac{\gamma\left[\theta_1\left(1-\gamma\right) + \rho\theta_2\right]}{\left(1-\gamma\right)^2 + \delta\theta_2} - \frac{\gamma^2\theta_2}{\left(1-\gamma\right)^2 + \delta\theta_2}\widehat{r}_{t-1}\right\}. \tag{4.50}$$

Identification of θ_1 and θ_2 gives

$$\theta_1 = -\frac{\delta\theta_2\gamma\rho}{\left(1-\gamma\right)\delta\gamma + \left(1-\gamma\right)^2 + \delta\theta_2}; \tag{4.51}$$

$$\theta_2 = 0; \tag{4.52}$$

$$\theta_2 = \frac{\delta\gamma^2 - \left(1-\gamma\right)^2}{\delta}. \tag{4.53}$$

If $\theta_2 = 0$, then also $\theta_1 = 0$ and the value function (4.40) becomes equal to a constant. Hence in the case of $\theta_2 = 0$ the optimization problem is not any more a dynamic programming problem but is reduced to a static one-period issue. In this case interest-rate smoothing cannot be optimal. The same consideration can be made for the second value of θ_2 when γ has the following values

$$\gamma_1 = \frac{1-\sqrt{\delta}}{1-\delta}; \tag{4.54}$$

and

$$\gamma_2 = \frac{1+\sqrt{\delta}}{1-\delta}; \tag{4.55}$$

where the value γ_2 given by (4.55) can be excluded by definition as, for $0 < \delta < 1$, it is greater than one. Thus it cannot be a possible value of γ, being it defined as $0 < \gamma < 1$.

Let's see how we can select the relevant solution.

For convenience we focus the analysis on the parameters of $\bar{r}_t$. Without loss of generality, we can write the expression of the equilibrium value of $\bar{r}_t$ as

$$\bar{r}_t = \phi_0 + \phi_1 \widehat{r}_{t-1} + \phi_2 v_t + \phi_3 u_t. \tag{4.56}$$

Using (4.46), it is possible to see that the values of θ_2 given by (4.52) and (4.53) imply respectively the following values for the coefficient of $\widehat{r}_{t-1}$in (4.56):

$$\phi_1 = -\frac{\gamma}{1-\gamma}; \tag{4.57}$$

and

$$\phi_1 = -\frac{1-\gamma}{\delta\gamma}. \tag{4.58}$$

In order to eliminate the solution (4.57), which implies that interest-rate smoothing is not optimal, we can consider the argument used by Lockwood and Philippopoulos (1994), Svensson (1997a), Clark, Goodhart and Huang (1999). They recommend to choose the smaller solution as the relevant one. This is based on the fact that the smaller solution has the property to be stable, in the sense that when a small disequilibrium deviation is introduced the system returns to the equilibrium value of ϕ_1 under a revision rule which is consistent with the recursive nature of the optimization problem.

Now, it is possible to see that when $\gamma > \gamma_1$ the solution (4.58) is lower in absolute value than that expressed by (4.57) and the opposite occurs when $\gamma < \gamma_1$. Moreover, if $\gamma > [1/(1+\delta)] > \gamma_1$, the solution (4.58) will be less than one in absolute value; the same occurs for the solution (4.57) when $\gamma < .5$. In Appendix B it is shown what is the revision rule that can be used for analysing the stability requirement.

In the following sections we will consider the solution (4.58) as the relevant one, assuming for the moment that the central banker chooses ex ante a degree of monetary inertia included between the range $[1/(1+\delta)] < \gamma < 1$. Later on, we will show that it is indeed optimal for the central banker to choose γ in the specified range as it is possible to achieve the same steady state equilibrium that would be achieved without monetary inertia. Thus assuming that at a given moment in time we have an inflationary bias and a disinflationary programme is announced, under a commitment regime the central banker is indifferent from the point of view of the steady state between smoothing the interest rate over a number of periods or reducing suddenly the interest rate to the lower new equilibrium value in the first period.

Substitution of (4.51) and (4.53) in expressions (4.46), (4.47) and (4.48) yields

$$\widehat{r}_t^c = \frac{[(1-\gamma)+\delta\gamma]}{\gamma\delta}\rho - \frac{(1-\gamma)}{\gamma\delta}\widehat{r}_{t-1} - \frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}v_t + \frac{1}{(1-\gamma)\beta}u_t; \tag{4.59}$$

$$\pi_t^c = -\frac{\left[\delta\gamma^2-(1-\gamma)^2\right]}{\gamma\delta}\rho + \frac{\left[\delta\gamma^2-(1-\gamma)^2\right]}{\gamma\delta}\widehat{r}_{t-1} - \frac{\lambda\alpha}{1+\lambda\alpha^2}v_t; \tag{4.60}$$

and

$$y_t^c = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{4.61}$$

Here it is possible to see that average inflation and interest rate under a commitment regime with monetary inertia are the same as under a commitment regime without monetary inertia, that is

$$E\left[\pi_t^c\right] = 0, \tag{4.62}$$

$$E\left[r_t^c\right] = \rho. \tag{4.63}$$

The steady state values or unconditional expectations of inflation and the nominal interest rate can be found in the following way. Inserting the equilibrium expressions (4.56) and

$$\bar{r}_t^e = \phi_0 + \phi_1 \widehat{r}_{t-1} \tag{4.64}$$

in the expression of inflation given by (4.14), we can write the equation of inflation in terms of the coefficients ϕ in the following way

$$\begin{aligned} \pi_t &= [(1-\gamma)\phi_0 - \rho] + [\gamma + (1-\gamma)\phi_1]\widehat{r}_{t-1} - \frac{1+(1-\gamma)\beta\phi_2}{\alpha}v_t \\ &\quad -\frac{(1-\gamma)\beta\phi_3 - 1}{\alpha}u_t. \end{aligned} \tag{4.65}$$

As in equilibrium we have that

$$\widehat{r}_t = \phi_0 + \phi_1 \widehat{r}_{t-1}, \tag{4.66}$$

we can express $\widehat{r}_t$ as

$$\widehat{r}_t = \phi_1^t \widehat{r}_0 + \phi_0 \sum_{i=0}^{t-1} \phi_1^i. \tag{4.67}$$

Substituting (4.67) in (4.65) allows to compute steady state inflation as

$$E[\pi_t] = \frac{\phi_0 - \rho(1-\phi_1)}{1-\phi_1}. \tag{4.68}$$

Now, using (4.46), (4.51) and (4.53), it is possible to see that in the case of commitment we have the following values

$$\phi_0 = \frac{[(1-\gamma)+\delta\gamma]}{\gamma\delta}\rho, \tag{4.69}$$

$$\phi_1 = -\frac{(1-\gamma)}{\gamma\delta}, \tag{4.70}$$

which after substitution in (4.68) yield the steady state value of inflation given by expression (4.62).

Following the same procedure used above, we can compute the average interest rate. In this case we have the following expression

$$E\left[r_t^c\right] = \gamma\left(\frac{\phi_0}{1-\phi_1}\right) + (1-\gamma)\left[\phi_0 + \phi_1\left(\frac{\phi_0}{1-\phi_1}\right)\right] = \rho. \tag{4.71}$$

Finally, from the comparison of (4.60) and (4.61) with the analogous equilibrium expressions for the case of commitment without interest-rate smoothing, it is possible to observe that stabilisation of shocks is still optimal.

4.6.2 Discretion

The optimization problem of the central banker when a commitment technology is not available can be formalised as

$$V\left(\widehat{r}_{t-1}\right) = E_{t-1}\min_{\bar{r}_t}\left[L_t + \delta V\left(\widehat{r}_t\right)\right]; \tag{4.72}$$

$$\text{s.t. Eqs. } (4.14), (4.15), (4.10).$$

Differentiating $V\left(\widehat{r}_{t-1}\right)$ with respect to $\bar{r}_t$ we get the following first order condition

$$-2\frac{\beta}{\alpha}(1-\gamma)\pi_t - 2\lambda\beta(1-\gamma)(y_t - \overline{y}) + 2\delta\theta_1 + 2\delta\theta_2\widehat{r}_t = 0. \tag{4.73}$$

Substitution of (4.16) and (4.17) in (4.73) implies that for optimality we must have the following values for the coefficients of the decision rule (4.16)

$$\mathsf{a} = -\frac{\beta\alpha(1-\gamma)\left[\alpha\lambda\left(\overline{y}-y_n\right)+\rho\right] + \delta\theta_1\alpha^2}{\beta^2(1-\gamma)^2\left(1+\lambda\alpha^2\right) + \delta\theta_2\alpha^2};$$

$$
\begin{aligned}
\mathsf{b} &= \frac{\beta(1-\gamma)^2(\lambda\beta\alpha^2+\beta+\alpha)}{\beta^2(1-\gamma)^2(1+\lambda\alpha^2)+\delta\theta_2\alpha^2};\\
\mathsf{c} &= \frac{\beta\gamma\alpha(1-\gamma)}{\beta^2(1-\gamma)^2(1+\lambda\alpha^2)+\delta\theta_2\alpha^2};\\
\mathsf{m} &= -\frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta};\\
\mathsf{n} &= \frac{1}{(1-\gamma)\beta}.
\end{aligned} \tag{4.74}
$$

The private sector's expectations are found by taking expectations at $t-1$ over $\bar{r}_t$:

$$
\bar{r}_t^{e,d} = -\frac{\beta(1-\gamma)[\alpha\lambda(\overline{y}-y_n)+\rho]+\alpha\delta\theta_1}{\alpha\delta\theta_2-\beta(1-\gamma)^2} + \frac{\beta\gamma(1-\gamma)}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}. \tag{4.75}
$$

Following the same substitutions made previously for the case of commitment we can get the equilibrium values of $\bar{r}_t$, inflation and output:

$$
\begin{aligned}
\bar{r}_t^{d} &= -\frac{\beta(1-\gamma)[\alpha\lambda(\overline{y}-y_n)+\rho]+\alpha\delta\theta_1}{\alpha\delta\theta_2-\beta(1-\gamma)^2} + \frac{\beta\gamma(1-\gamma)}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}\\
&\quad -\frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta}v_t + \frac{1}{(1-\gamma)\beta}u_t;
\end{aligned} \tag{4.76}
$$

$$
\begin{aligned}
\pi_t^{d} &= -\frac{\alpha\delta[\rho\theta_2+(1-\gamma)\theta_1]+\lambda\alpha\beta(1-\gamma)^2(\overline{y}-y_n)}{\alpha\delta\theta_2-\beta(1-\gamma)^2} + \frac{\gamma\alpha\delta\theta_2}{\alpha\delta\theta_2-\beta(1-\gamma)^2}\widehat{r}_{t-1}\\
&\quad -\frac{\alpha\lambda}{1+\lambda\alpha^2}v_t;
\end{aligned} \tag{4.77}
$$

$$
y_t^d = y_n + \frac{1}{1+\lambda\alpha^2}v_t. \tag{4.78}
$$

From (4.77), we can conclude that also here interest-rate smoothing allows the policy maker to smooth inflation over a number of periods.

In order to find θ_1 and θ_2 we can apply the envelope theorem on (4.39) which combined with (4.40) implies

$$\begin{aligned} V_{\widehat{r}}(\widehat{r}_{t-1}) &= 2(\theta_1 + \theta_2 \widehat{r}_{t-1}) = \\ & 2\gamma \frac{\left[\delta\theta_2\alpha^2 + \beta^2(1-\gamma)^2\right]}{\alpha\left[\delta\theta_2\alpha - \beta(1-\gamma)^2\right]}\pi_t + 2\lambda\gamma \frac{\beta^2(1-\gamma)^2}{\delta\theta_2\alpha - \beta(1-\gamma)^2}(y_t - \overline{y}); \end{aligned} \tag{4.79}$$

identification of θ_1 and θ_2 gives

$$\theta_1 = -\frac{\gamma\delta\theta_2\left\{\beta(1-\gamma)^2\left[\beta\rho + \alpha\lambda(\beta+\alpha)(\overline{y} - y_n)\right] + \rho\delta\theta_2\alpha^2\right\}}{\alpha\delta\theta_2(1-\gamma)(\delta\alpha\gamma + 2\gamma\beta - 2\beta) + \beta^2(1-\gamma)^3(1-\gamma+\delta\gamma) + \alpha^2\delta^2\theta_2^2}; \tag{4.80}$$

$$\theta_2 = 0; \tag{4.81}$$

$$\theta_2 = \frac{\delta\gamma^2\alpha + 2\beta(1-\gamma)^2 + \gamma\sqrt{\delta\left[\delta\gamma^2\alpha^2 + 4\beta(1-\gamma)^2(\beta+\alpha)\right]}}{2\delta\alpha}; \tag{4.82}$$

$$\theta_2 = \frac{\delta\gamma^2\alpha + 2\beta(1-\gamma)^2 - \gamma\sqrt{\delta\left[\delta\gamma^2\alpha^2 + 4\beta(1-\gamma)^2(\beta+\alpha)\right]}}{2\delta\alpha}. \tag{4.83}$$

Here again the case of $\theta_2 = 0$ implies $\theta_1 = 0$ and, hence, the value function (4.40) becomes a constant. In this case interest-rate smoothing is not optimal and the central banker's minimisation problem can be expressed in terms of a static one-period optimisation. Using the general expression (4.56) of the equilibrium value of $\overline{r}_t$ and the coefficient values in (4.76), the values of θ_2 given by (4.81), (4.82) and (4.83) imply respectively the following values for the coefficient of $\widehat{r}_{t-1}$

$$\phi_1 = -\frac{\gamma}{1-\gamma}; \tag{4.84}$$

and

$$\phi_1 = -\frac{\sqrt{\delta}\alpha\gamma - \sqrt{\delta\gamma^2\alpha^2 + 4\beta\left(1-\gamma\right)^2\left(\alpha+\beta\right)}}{2\sqrt{\delta}\left(\alpha+\beta\right)\left(1-\gamma\right)}; \tag{4.85}$$

and

$$\phi_1 = -\frac{\sqrt{\delta}\alpha\gamma + \sqrt{\delta\gamma^2\alpha^2 + 4\beta\left(1-\gamma\right)^2\left(\alpha+\beta\right)}}{2\sqrt{\delta}\left(\alpha+\beta\right)\left(1-\gamma\right)}. \tag{4.86}$$

It is easy to see that the value of ϕ_1 given by (4.85) is in absolute value always smaller than that implied by (4.86). Hence the choice is between the values given by (4.84) and (4.85).

We can eliminate the solution (4.84), which implies that interest-rate smoothing is not optimal, by using the stability argument adopted previously. Now, by using l'Hopital's Rule, it is straightforward to show that

$$\lim_{\gamma\to 1}\left(-\frac{\sqrt{\delta}\alpha\gamma - \sqrt{\delta\gamma^2\alpha^2 + 4\beta\left(1-\gamma\right)^2\left(\alpha+\beta\right)}}{2\sqrt{\delta}\left(\alpha+\beta\right)\left(1-\gamma\right)}\right) = 0. \tag{4.87}$$

Hence it is always possible to find a value of γ that makes the solution (4.85) smaller than that expressed by (4.84), as this latter solution becomes greater than one in absolute value for $\gamma > .5$.

Substituting (4.80) and (4.82) in equations (4.76), (4.77) and (4.78) allows us to find the equilibrium values of the interest rate, inflation and output. In general average inflation will not be zero. However, an interesting property of interest-rate smoothing emerges if we take the limit of average inflation for $\gamma \to 1$. It is possible to see that we in this case have

$$\lim_{\gamma\to 1}\left[E\pi_t^d\right] = 0. \tag{4.88}$$

To understand why we have this effect on steady state inflation as $\gamma \to 1$ it is useful to consider the limit of the average interest rate. It is possible to show that we have

$$\lim_{\gamma \to 1} E\left[r_t^d\right] = \rho. \tag{4.89}$$

This implies that as $\gamma \to 1$, the average interest rate under discretion with monetary inertia tends to the average value prevailing under commitment without monetary inertia.

The steady state values or unconditional expectations of inflation and nominal interest rate under discretion can be derived in the following way. Recalling expression (4.68), previously derived, it is possible to see that in the case of discretion the expression of average inflation is more complicate as we have that

$$\phi_0 = -\frac{\beta(1-\gamma)\left[\alpha\lambda(\overline{y}-y_n)+\rho\right]+\delta\theta_1\alpha}{\delta\theta_2\alpha-\beta(1-\gamma)^2}; \tag{4.90}$$

$$\phi_1 = \frac{\beta\gamma(1-\gamma)}{\delta\theta_2\alpha-\beta(1-\gamma)^2}; \tag{4.91}$$

where θ_1 and θ_2 are given respectively by expressions (4.80) and (4.82). Taking the limit of (4.68) for $\gamma \to 1$, after inserting expressions (4.90) and (4.91), yields the steady state value of inflation given by (4.88).

In order to find the limit of this complicated expression we have used Maple©. However this result can be derived also using the following simpler limits

$$\lim_{\gamma \to 1} \theta_1 = -\rho; \tag{4.92}$$

$$\lim_{\gamma \to 1} \theta_2 = 1; \tag{4.93}$$

$$\lim_{\gamma \to 1} \phi_1 = 0; \tag{4.94}$$

$$\lim_{\gamma \to 1} \phi_0 = \rho. \tag{4.95}$$

Following the same procedure used above, it is possible to compute the steady state level of the interest rate under discretion with monetary inertia. In this case we have

$$\lim_{\gamma \to 1} E\left[r_t^d\right] = \lim_{\gamma \to 1} \left\{ \gamma \left(\frac{\phi_0}{1-\phi_1} \right) + (1-\gamma) \left[\phi_0 + \phi_1 \left(\frac{\phi_0}{1-\phi_1} \right) \right] \right\} = \rho. \tag{4.96}$$

Moreover from the comparison of (4.77) and (4.78) with the analogous equilibrium expressions for the case of discretion without interest-rate smoothing it is possible to observe that stabilisation of shocks is still optimal.

Hence, if the fixed degree of monetary inertia chosen ex ante is sufficiently high, the inflationary bias associated with time consistent monetary policy becomes negligible without implying the arising of a trade-off between credibility and flexibility. The rationale for this striking result can be found in the contrasting effect played by interest-rate smoothing on the incentive to create surprise inflation by reducing suddenly interest rates within the time horizon of existing nominal contracts.

In particular, the introduction of intertemporal considerations on the current level of the interest rate, due to the presence a partial adjustment mechanism in the specified instrument rule, restricts the possibility of changing the past level of the monetary instrument. In the present framework this new element can play a crucial role as there is an incentive to create surprise inflation only if the gain deriving from reducing suddenly the interest rate more than outweighs at the margin the cost deriving from higher inflation. Thus if the variation of the interest rate required for creating surprise inflation is not consistent with the given degree of gradualism, then the existence of monetary inertia may inhibit the central banker's temptation to deviate from the announced disinflationary programme.

Finally, it is interesting to see that from the first order condition of the policy maker's optimisation problem it is possible to derive in equilibrium an expression for $\overline{r}_t$ that resembles the Taylor rule. Rearranging expression (4.73) we can get

$$\overline{r}_t = \mu_0 + \mu_1 \pi_t + \mu_2 (y_t - y_n) + \xi_t, \tag{4.97}$$

with

$$\begin{aligned} \mu_0 &= -\left[\frac{\theta_1}{\theta_2} + \mu_2 (\overline{y} - y_n)\right] \\ \mu_1 &= \frac{\beta (1-\gamma)}{\alpha \delta \theta_2} \\ \mu_2 &= \frac{\lambda \beta (1-\gamma)}{\delta \theta_2} \\ \xi_t &= -\frac{1}{(1-\gamma)(1+\lambda\alpha^2)\beta} v_t + \frac{1}{(1-\gamma)\beta} u_t. \end{aligned} \tag{4.98}$$

As $\lim_{\gamma \to 1} \theta_1 = -\rho$ and $\lim_{\gamma \to 1} \theta_2 = 1$, $\mu_0 \to \rho$ as γ tends to one. Moreover, recalling expression (4.82), μ_1 and μ_2 are positive as $\theta_2 > 0$.

4.7 Conclusion

Our analysis has shown that it may be advantageous for society and government to delegate monetary policy to a central banker with an optimally designed instrument rule. The result obtained hinges on the particular instrument rule for setting the interest rate introduced in the analysis. This instrument rule is based on the idea that central banker's current decisions on the interest rate are a function of both past decisions and current information. Moreover, the optimal level of the interest rate is found by choosing the parameters in the specified instrument rule. Following this systematic behaviour in the implementation of monetary policy may increase the credibility of disinflationary programmes without necessarily introducing a trade-off between commitment and flexibility. We show that this favourable circumstance is associated with the presence of a sufficiently high degree of monetary inertia introduced institutionally ex ante by the policy maker.

The idea that with interest-rate smoothing optimal monetary policy can be more credible than in the case without monetary inertia is new in the literature. As observed, for example, by Walsh (1998): "Central banks have often been criticized, however, for smoothing interest rates. During the late 1960s and 1970s, the Fed's attempts to prevent interest rates from rising in the face of increasing inflation served to exacerbate subsequent inflation. Thus, an understanding of the consequences of interest-rate smoothing is important".

In the present analysis we provide a theoretical support for the optimality of interest-rate smoothing. The intuition for the surprising result that gradualism enhances credibility can be found in the view, expressed by the time-inconsistency literature, that the main problem of monetary policy is the excessive activism of central bankers seeking to exploit employment and output gains deriving from inflation surprises. In this perspective gradualism can be optimal as under specified circumstances it can contrasts the incentive to fool private sector by reducing suddenly the interest rate.

Chapter 5

The credibility of optimal monetary delegation: do we really need prohibitive reappointment costs?

5.1 Introduction

The analysis presented in this chapter focuses on the current debate on the real effectiveness of delegation in overcoming the problem of time inconsistency that afflicts discretionary monetary policy. As we have observed in chapter 2, section 2.9.4, Jensen (1997) has shown that, when the government is unable to credibly carry out optimal policy and delegates monetary policy to a central banker with an announced incentive scheme, optimal policy can be credible only if reappointment costs are prohibitive. Here we will question his finding. In particular, we will examine under which circumstances the presence of relatively high but not necessarily prohibitive costs of reappointment may ensure that optimal monetary delegation is credible. Section 5.2 exposes the model. Section 5.3 recalls briefly the main results of Jensen's analysis. Section 5.4 contains the results of our analysis and compares them with Jensen's results. Section 5.5 concludes.

5.2 The model

Apart from few minor changes in the notation and definitions the model is the same as that used by Jensen. Therefore we will summarise only briefly the key expressions and refer to the original version of the model for a more detailed discussion. Our main purpose here is not to criticise the model and assumptions used by Jensen but to develop the analysis.[1]

The supply function is given by the standard expectations-augmented Phillips curve

$$y_t = \alpha \left(\pi_t - \pi_t^e\right), \tag{5.1}$$

where for simplicity the natural level of output is normalised to zero; π_t,π_t^e are the actual and expected inflation rate respectively.

The government's loss function is expressed by

$$L_t = \left[\pi_t^2 + \lambda \left(y_t - \overline{y}\right)^2 + \varphi \left(f_t - f_t^a\right)^2\right], \tag{5.2}$$

where deviations of output and inflation from the socially optimal targets are relatively weighted with $\lambda > 0$. As usual in the time inconsistency literature the output target is assumed to be greater than the natural level, $\overline{y} > 0$. But in contrast with the previous literature on monetary delegation, there is a new additional cost on the reappointment of the central banker expressed by the difference between the announced incentive scheme with the penalty f_t^a and the realised one. In particular if $f_t \neq f_t^a$ we will say that the central banker has been reappointed, which in the present framework will happen at some cost to the government. The parameter φ reflects the distaste for reappointment

[1] Jensen himself addresses a series of weakness in his assumptions, among which the most relevant is perhaps the issue of wage contracts of longer duration than the frequency with which a new incentive scheme can be chosen. From a more game-theoretic point of view the model used by Jensen suffers from the criticism generally made to reputational models with trigger strategies; see for example Backus and Driffill (1985) who first pointed out the problems inherent in the game-theoretic framework used by Barro and Gordon (1985b), on which Jensen's analysis is based. On the contrary the models of Herrendorf (1998) and al-Nowaihi and Levine (1996), that also examine the credibility of optimal monetary delegation, use a more satisfactory game-theoretic framework for modelling reputation.

costs relative to the other costs in the loss function. When monetary policy is delegated we have $\varphi > 0$, otherwise φ is equal to zero. As observed by Jensen, if f_t and f_t^a are understood not simply as the contract proposed to the central banker but as referring to a complex system of monetary regulations, it seems natural to assume that a small change is less costly than a bigger one. This may justify the use of a quadratic cost of reappointments with an assigned weight φ.[2]

Monetary policy is delegated by the government to a central banker whose loss function is the following

$$L_t^b = \left[\pi_t^2 + \lambda\left(y_t - \overline{y}\right)^2 + 2f_t\pi_t\right]. \tag{5.3}$$

Here the central banker is fined with the penalty $2f_t$ for inflation rates greater than zero. As we will see later on, the optimal incentive scheme that allows the government to eliminate the inflation bias is $f_t = \lambda\alpha\overline{y}$.

In each period the timing of moves is the following. In stage zero, the government delegates monetary policy to a central banker and announces an incentive scheme f_t^a. In stage 1, the private sector forms expectations about inflation and sets wages. In stage 2, the government sets actual conditions f_t for monetary policy. Finally, in stage 3, the CB sets actual inflation. The timing of moves considered in Jensen's analysis is illustrated in figure 5.1.

In the discretionary regime the central banker minimises the discounted value of his loss function, $\sum_{\tau-t}^{\infty}\beta^{\tau-t}L_t^b$, subject to (5.1) by taking inflation expectations and actual conditions for monetary policy f_t as given. The parameter β is the discount factor of the central banker. It is assumed that the central banker and the government have the same discount factor. However unlike Jensen, where $0 < \beta < 1$, we assume that the discount factor of the central banker is defined in the range $.5 < \beta < 1$. This weak hypothesis has

[2] An alternative way of modelling reappointment costs would be to assume that if the government reneges on his announcement it will also incur a fixed cost and that this fixed component is relatively more important than that dependent on the size of the modifications of the given institutional arrangement. This idea is captured, for example, in the work of Lohmann (1992).

been used, for example, also by Barro (1986) and al-Nowaihi and Levine (1996).

From the first order condition we obtain the central banker's reaction function

$$\pi_t = \frac{\lambda\alpha^2\pi_t^e + \lambda\alpha\overline{y} - f_t}{\Lambda}, \tag{5.4}$$

where $\Lambda = (\lambda\alpha^2 + 1)$.When choosing actual monetary conditions for monetary policy, the government must take its prior announcements and inflation expectations as given but incorporates the behaviour of the central banker in it's decision problem. Thus it minimises the discounted value of its loss function, $\sum_{\tau-t}^{\infty} \beta^{\tau-t} L_t$, with respect to f_t subject to (5.1) and (5.4). The minimisation yields the following optimal incentive scheme

$$f_t = \frac{\varphi\Lambda}{1+\varphi\Lambda} f_t^a, \tag{5.5}$$

where f_t^a is the announcement chosen by government. As observed by Jensen the assumption of prohibitive costs of reappointment when monetary policy is delegated by the government to a central banker eliminates by definition the issue of the credibility of optimal monetary delegation. From (5.5) one can see that announcements will always be fulfilled if the government's only concern is reappointment costs, i.e. when $\varphi \to +\infty$.

The private sector's inflation expectations are obtained by substituting (5.5) into (5.4). After taking expectations we get

$$\pi_t^{e,NCD} = \lambda\alpha\overline{y} - \frac{\varphi\Lambda}{1+\varphi\Lambda} f_t^a. \tag{5.6}$$

Finally the government chooses the optimal announcement. When making this choice the government internalises the effects of its decision on the central banker's behaviour, on its own behaviour when choosing actual monetary conditions, and on the private sector's expectations, Minimising the government's loss function with respect to f_t^a subject to (5.4), (5.5) and (5.6) yields

$$f_t^{a,NCD} = \frac{\Lambda\left(1+\varphi\Lambda\right)\lambda\alpha\overline{y}}{1+\varphi\Lambda^2}. \tag{5.7}$$

Expression (5.7) implies that

$$f_t^{NCD} = \frac{\varphi\Lambda^2\lambda\alpha\overline{y}}{1+\varphi\Lambda^2}. \tag{5.8}$$

Here we can observe that if φ tends to infinite we have $f_t^{a,NCD} = \lambda\alpha\overline{y}$ and $f_t^{NCD} = f_t^{a,NCD}$. Thus, if reappointment costs are prohibitive, in the static one-shot game version of Jensen's model optimal monetary delegation is not subject to a credibility problem. However, the more realistic case is when these costs are not all that matters in the government's loss function, or in other words when in expression (5.2) the weight φ is not infinite.

The equilibrium inflation rate will be under the discretionary regime with delegation

$$\pi_t^{NCD} = \frac{\lambda\alpha\overline{y}}{1+\varphi\Lambda^2}. \tag{5.9}$$

If the government does not delegate monetary policy, i.e. $\varphi = 0$, it is straightforward to show that if the government behaves in a discretionary manner the equilibrium inflation rate would be $\pi_t^{NC} = \lambda\alpha\overline{y}$. From expression (5.9) we can see that delegation reduces the inflation bias but does not remove it. On the contrary if the government could idealistically precommit to an announced policy rule before expectations are formed then the government would not need to delegate monetary policy in order to eliminate the inflation bias and the optimal policy rule, or the precommitment policy rule, would be in this deterministic case to set $\pi_t^{PR} = 0$. Comparing the government's losses under the equilibrium with precommitment and the equilibrium with discretion it is possible to see that in the case of delegation the loss is lower than in the case when the government conducts monetary policy directly and behaves in a discretionary manner, but is greater than in the precommiment equilibrium.

In the subsequent sections we will consider the situation when the policy game is repeated for an infinite number of periods in order to study the precommitment technology where the private sector punishes deviations by a one-period reversion to expectations given by the discretionary solution.

5.3 Delegation as conducive to reputation building for the government

Let's examine the situation when repeated interactions among the players take place. In particular assume that the game is repeated for an infinite number of periods. In this case Barro and Gordon (1983b) have shown that, if the private sector adopts a punishment strategy triggered by any observed deviation from optimal policy and the government does not discount the future too heavily, it is possible that the future cost for the government of losing its reputation for being committed to zero inflation may more then outweigh the current gain from deviating.

By assuming that the private sector reverts for one period to the discretionary solution whenever a deviation from optimal policy is observed, Jensen has found that the minimal requirement for the patience of the government is given by $\beta \geq \widehat{\beta} \equiv 1/\Lambda$. If β is sufficiently high optimal monetary policy is a perfect Nash equilibrium and therefore it is also credible. Alternatively if the discount factor is not sufficiently high, optimal monetary policy is not credible. In order to achieve the precommitment solution the government might consider delegating monetary policy to a central banker with the optimal incentive scheme $f_t = \lambda\alpha\overline{y}$ and try again to maintain a reputation for low inflation. Also in this case the credibility of optimal monetary delegation, where credibility is understood as the ability to carry out optimal monetary policy, can be studied by examining simple punishment strategies based on a one-period reversion to the discretionary solution.[3] Consider the following strategy combinations:

Government plays:

[3] As observed by Jensen it is not necessary to analyse explicitly the cases when the announcement of the government is $f_t^a \neq \lambda\alpha\overline{y}$ as we can rule them out through a reversion to the discretionary solution for any value of the discount factor.

$$\begin{aligned} f_t &= f_t^a = \lambda\alpha\overline{y} \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{ and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \end{aligned} \tag{5.10}$$

Private sector plays:

$$\begin{aligned} \pi_t^e &= 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ \pi_t^e &= \pi_t^{e,NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \end{aligned} \tag{5.11}$$

The expressions of $\pi_t^{e,NCD}$ and f_t^{NCD} are found by substituting $f_t^{a,NCD}$ in expressions (5.5) and (5.6) respectively. If there is a deviation from the announced optimal delegation the government minimises the loss function with respect to f_t subject to $\pi_t^e = 0$ and $f_t^{a,DD} = \lambda\alpha\overline{y}$. This yields the following values:

$$\begin{aligned} \pi_t^{DD} &= \frac{\lambda\alpha\overline{y}}{(1+\varphi\Lambda)\Lambda}; \\ f_t^{DD} &= \frac{\varphi\Lambda\lambda\alpha\overline{y}}{1+\varphi\Lambda}. \end{aligned} \tag{5.12}$$

According to the above strategies the condition of no deviation for the government will be

$$L_t^{PR} - L_t^{DD} \leq \beta\left(L_{t+1}^{NCD} - L_{t+1}^{PR}\right), \tag{5.13}$$

which implies that

$$\beta \geq \widehat{\beta}^D(\varphi) \equiv \frac{1+\varphi\Lambda^2}{\Lambda(1+\varphi\Lambda)}. \tag{5.14}$$

Now we can compare the condition for the credibility of optimal monetary delegation with the condition for the credibility of optimal monetary policy when monetary policy is conducted directly by the government. With this aim Jensen has proved the following proposition:

Proposition 5.1 *For $\beta < \widehat{\beta}$ and all $\varphi > 0$, (i) $\partial \widehat{\beta}^D(\varphi) / \partial \varphi > 0$ and (ii) $\widehat{\beta}^D(\varphi) > \widehat{\beta}$.*

Proof. If part (i) of proposition 5.1 is true then the credibility of optimal monetary policy will be harder to support the more important reappointment costs are. Moreover as $\lim_{\varphi \to 0} \widehat{\beta}^D(\varphi) = \widehat{\beta}$ and given (i) it follows that $\widehat{\beta}^D(\varphi) > \widehat{\beta}$ for all $\varphi > 0$. □

Thus the premise made by the standard literature on delegation that it is the presence of reappointment costs that makes delegation to an independent central banker more credible than the conduct of monetary policy itself must be considered false according to Jensen's analysis. The intuition for this result is the following. The punishment subsequent to a deviation becomes weaker the higher is the weight on reappointment costs. Also the gain from deviating decreases with φ but less than the reduction in the cost deriving from the loss of reputation. The reason is that the reduction of the gain from deviating results from several opposing forces which mitigate the effect of an increase in φ.

5.4 An alternative view on the process of delegation: delegation as conducive to reputation building for the central banker

Jensen's analysis does not take into account the possibility that an independent and far-sighted central banker might try to establish a reputation for being committed to low inflation and that this possibility might influence the behaviour of the government. Thus the question that we ask here is whether the opportunity for a central banker, institutionally independent, to credibly behave in a committed fashion may also have any effect on the credibility of optimal delegation. In order to answer this question in the present analysis we assume in contrast to Jensen that in the government's delegation problem the central banker's reputation plays a key role.[4]

As we have seen before, in each period when the government chooses the optimal announcement f_t^a it incorporates in its decision process the behaviour of the central banker and the private sector and its behaviour when he chooses actual monetary conditions. Furthermore, when the government chooses f_t it takes into account only the behaviour of the central banker and takes as given private sector's expectations and its announcements. So in both cases the government will also incorporate in its decision problem the possibility that the central banker may be able to maintain a reputation for being committed to zero inflation.

In the present framework, in each period the timing of moves is the following. In stage 0 the government delegates monetary policy to an independent central banker and announces an incentive structure in order to make him accountable for the outcome of monetary policy. Again, as before, the government repeats in each period the announcement made in the first period of delegation f_0^a, or period $t = 0$, and has the choice of either sticking to the announcement or deviating from it. The crucial difference is the following. In stage 1, after the announcement of the government and before the private sector's expectations are formed, the central banker announces that he will establish a reputation for being committed to a rule for setting inflation independently of the government's behaviour. More exactly the central banker's announcement can be thought of as a costless announcement of an inflation target π_0^a.[5] The subsequent stages are the

[4]A similar approach, where the central banker's reputation plays a key role in the delegation process, has been followed also by Lockwood, Miller and Zhang (1996) in order to extend Rogoff's delegation to a weight conservative central banker to a reputational framework. But they do not analyse the credibility of optimal delegation when the option for the government of reneging on the announcements made is explicitly considered. On the contrary the approach of Jensen in defining the delegation problem faced by the government in terms only of the reputational enforcement of the government itself has been adopted, for example, also by al-Nowaihi and Levine (1996), for examining the renegotiation proveness of the Walsh contract, and Herrendorf (1998), for studying Svensson's inflation targeting regime as a substitute for an explicit precommitment.

[5]Driffill (1997, 1994) argues that the introduction of inflation targets, in the form of costless announcements, by focusing the attention on a particular reputational solution for inflation may solve the problems of coordination and multiple equilibria that affect reputational models with trigger strategies.

same as considered before just shifted one stage forward: in stage 2 expectations are set; in stage 3 actual monetary conditions are chosen by the government; in stage 4 the central banker chooses the inflation rate. The new timing of moves is illustrated in figure 5.2.

Thus if the central banker announces an inflation target the private sector will expect it to be achieved no matter what the government does - both before and after expectations are set - and whether the central banker is able to establish a reputation for achieving the given inflation target will depend on the usual condition derived from the comparison of the temptation to deviate with the reputational enforcement. In particular if the announced inflation target corresponds to the pre-commitment inflation rate then it must be enforceable simultaneously under both deviation and no deviation of the government from the announced incentive scheme. If this is not true then the central banker will commit to the lowest enforceable inflation rate simultaneously under both deviation and no deviation of the government.

The choice we have made in the analysis of focusing only on the central banker's reputation corresponds first of all to the need to establish clearly the differences relative to the analysis of Jensen. More importantly, the choice made reflects also the idea that when an independent central banker tries to establish a reputation for low inflation it seems realistic to postulate that the central banker will try to resist any possible influence from the government that would undermine the credibility of his announcements on the inflation target. This implies that the announced inflation target must be sustainable as a reputational equilibrium whether the government deviates or not. Moreover as we will show in our analysis, if this behaviour of the central banker is common knowledge among the players, the central banker has an incentive to behave in this way because then it is possible for him to constrain the government to make credible announcements without

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This idea is followed by Miller (1997) in extending Rogoff's delegation approach to the reputational framework of Barro and Gordon (1985b). Moreover Miller, considering the same model used in Lockwood, Miller and Zhang (1996), discusses the case when there are observable shocks in the economy and the announced target should have a range around it for stabilisation purposes.

relying on its reputation for not reneging on the announced incentive scheme.

Similarly to Jensen's analysis we will rule out, by means of a reversion to the discretionary solution, feasible strategy combinations which include announcements that do not imply the achievement of the precommitment solution.

The delegation problem for the government is the following. Given that the discount factor β is not sufficiently high and that establishing a reputation for low inflation is not an available option, the government decides to delegate monetary policy and must choose an initial announcement f_0^a that minimises its expected losses. Now in order to avoid a reversion to the discretionary solution, triggered by an inflation target announced by the central banker different from zero, the government will consider only announcements $f_t^a > 0$ that imply that the central banker is able to maintain a reputation for being committed to the precommitment inflation rate, independently of whether or not the government deviates from the announced incentive structure. The announcements that satisfy this requirement are optimal for the government and, as we will see later on, another difference relative to Jensen's analysis is that here the number of optimal announcements for the government can be greater than one depending on the assumed value of φ.

So our initial task is to find the set of optimal announcements for the government that ensure the achievement of optimal monetary policy by the central banker. The problem is complicated by the fact that when the central banker commits to the announced inflation target he does not know whether the government will deviate or not. It is possible to show that the key to the solution of the delegation problem for the government is to eliminate the uncertainty about its own behaviour and ensure that the central banker expects that the announcement made by the government will always be fulfilled. Actually, in the present framework the government has no interest in introducing this uncertainty as it is optimal for it to induce the central banker to announce a zero inflation target and avoid any reversion to the discretionary solution.

Now suppose the central banker has announced an inflation target equal to zero and that this target is sustainable in the case when the government sticks to its announced incentive scheme. Consider a deviation by the government. Due to the presence of reappointment costs a necessary condition for a deviation by the government is that the central banker deviates after a deviation by the government and creates unexpected inflation. Only in this case may the government have an incentive to deviate from the announced institutional arrangement as the gain from surprise inflation can more than compensate the reappointment costs. Assuming again that this is common knowledge among the players, the fact that the central banker deviates after a deviation by the government and creates unexpected inflation implies necessarily that the announced inflation target is not sustainable as a reputational equilibrium. But this contradicts the claim that in order to avoid a reversion to the discretionary solution, triggered by an inflation target announced by the central banker different from zero, the government will consider only announcements that imply that the central banker is able to maintain a reputation for being committed to the precommitment inflation rate.

So it must be the case that, if the announcement of the government is optimal, the necessary condition for a deviation by the government is never satisfied, i.e. if a deviation by the government occurs the central banker never deviates from optimal monetary policy. Notice that in Jensen's analysis the necessary condition for a deviation by the government discussed here is always satisfied as the central banker is assumed to behave only in a discretionary manner. On the contrary, in the present framework the presence of the central banker's reputation combined with the presence of reappointment costs constrains the behaviour of the government and introduces an incentive for the government to always fulfil its announcements and eliminate the uncertainty about its behaviour.

The working of this mechanism will appear more clearly after the formalisation of the necessary conditions for the existence of the reputational equilibrium in which the precommitment inflation rate is sustainable.

5.4.1 The set of optimal announcements for the government

Before starting with the analysis of the credibility of optimal monetary delegation we need to find the set of optimal announcements for the government. For expositional reasons we separate the set of announcements of the government after which if it does not deviate the central banker is able to sustain zero inflation as a reputational equilibrium, from the set of announcements after which if it deviates the central banker is able to sustain zero inflation as a reputational equilibrium. From the above discussion it is clear that the set of optimal announcements for the government will be the set that is the intersection of the two above distinct sets.

In order that a solution to the delegation problem of the government exists and a reversion to the discretionary solution is avoided, the two set considered must not be mutually exclusive. This latter condition will be examined in the next section. Here we find only the optimal announcements for the government. In the following analysis it is understood that if the discount factor of the government is sufficiently high, i.e. $\beta \geq \widehat{\beta}$, the government does not delegate monetary policy. We will examine the credibility of optimal monetary delegation only in the case when the government is not able to build a reputation for low inflation. Thus here the delegation solution is not alternative to the reputational solution, as claimed in the standard theory of delegation, but rather supplementary.

5.4.2 No deviation by the government

If we assume that the government does not deviate, the announcement $f_t^a > 0$ chosen by the government will be within the set of announcements that imply that the central banker is able to maintain a reputation for being committed to zero inflation. Thus in each period the government will announce $f_t^a = f_0^a \in \Theta$, where f_0^a is the initial announcement and Θ is the set of announcements that ensure that the precommitment solution can be sustained as a reputational equilibrium when the government does not deviate from the announcement made. Consider the following strategy combinations:

Central Banker plays:

$$\begin{aligned} \pi_t &= 0 \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ \pi_t &= \pi_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e. \end{aligned} \tag{5.15}$$

Private Sector plays:

$$\begin{aligned} \pi_t^e &= 0 \text{ if } \pi_{t-1} = 0; \\ \pi_t^e &= \pi_t^{e,NCD} \text{ if } \pi_{t-1} \neq 0. \end{aligned} \tag{5.16}$$

Government plays:

$$\begin{aligned} f_t &= f_t^a = f_0^a \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{ and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e; \end{aligned} \tag{5.17}$$

where $f_t^{a,NCD}$ and f_t^{NCD} are the same as in (5.10).

The condition of no deviation from zero inflation for the central banker when the government sticks to the announcement made will be

$$L_t^{b,PR} - L_t^{b,DD} \leq \beta \left(L_{t+1}^{b,NCD} - L_{t+1}^{b,PR} \right). \tag{5.18}$$

The above condition implies that

$$\beta \geq \widehat{\beta}^{b*} (f_0^a; \varphi) \equiv \frac{(\lambda \alpha \overline{y} - f_0^a)^2 (1 + \varphi \Lambda^2)^2}{\Lambda \left[(\lambda \alpha \overline{y})^2 (1 + 2\varphi \Lambda^2) \right]}. \tag{5.19}$$

Considering condition (5.19) with equality and f_0^a as the unknown term, it is possible to see that there are two values of f_0^a that satisfy this equation:

$$\underline{\theta}(\varphi) = \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2-\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{(1+\varphi\Lambda^2)} > 0; \tag{5.20}$$
$$\overline{\theta}(\varphi) = \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2+\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{(1+\varphi\Lambda^2)} > 0.$$

In Appendix C it is shown that for the assumed parameter values $\underline{\theta}(\varphi) > 0$. Now, inspection of condition (5.19) yields the following proposition:

Proposition 5.2 *For* $f_0^a = \theta \in \Theta(\varphi)$, *with* $\Theta(\varphi) \equiv \left[\underline{\theta}(\varphi), \overline{\theta}(\varphi)\right]$, *(i)* $\beta \geq \widehat{\beta}^{b*}(f_0^a;\varphi)$; *(ii)* $\Theta(\varphi) \supset \Theta(\varphi')$, *with* $\varphi < \varphi'$ *;(iii)* $\partial\widehat{\beta}^{b*}(f_0^a;\varphi)/\partial\varphi > 0$.

Part (i) of this proposition says that if the government's announcement is chosen between $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ the condition (5.19) is satisfied. According to part (ii) the set $\Theta(\varphi)$, i.e. the set of announcements under which the precommitment solution can be sustained as a reputational equilibrium when the government does not deviate, with given φ becomes smaller as reappointment costs become more important. Finally part (iii) says that, for a given government's announcement, increasing the weight φ on reappointment costs makes condition (5.19) harder to fulfil. Let's proof proposition 5.2.

Proof. In the present framework, for given φ, the government's announcement f_0^a is the choice variable in the government's delegation problem. Now considering condition (5.19) with equality and given φ, we can draw (see figure 5.3) the quadratic function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ as a parabola having a global minimum at $f_0^a = \lambda\alpha\overline{y}$ with $\widehat{\beta}^{b*}(\lambda\alpha\overline{y};\varphi) = 0$. The function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ will be equal to β for two values of f_0^a, which correspond to $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$. It is possible to see that the two values $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ are respectively lower and greater than $\lambda\alpha\overline{y}$ and both tend to $\lambda\alpha\overline{y}$ as $\varphi \to +\infty$.

From the above discussion it follows that $\beta \geq \widehat{\beta}^{b*}(f_0^a;\varphi)$ for all values of f_0^a included between $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$. Thus the set $\Theta(\varphi)$ is defined between these two extreme values. With a minor abuse of notation, in proposition 5.2 we have assumed that the function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ is defined also for $f_0^a = \lambda\alpha\overline{y}$. Formally it would be more correct to say that this function is not defined in that point as the central banker always chooses zero inflation independently of the value of β.

Part (ii) can be proved using the following first derivative:

$$\frac{\partial I^*(\varphi)}{\partial\varphi} = -\frac{2\varphi\Lambda^4\sqrt{\Lambda\beta}}{(1+\varphi\Lambda^2)^2\sqrt{1+2\varphi\Lambda^2}} < 0; \tag{5.21}$$

with

$$I^*(\varphi) \equiv \overline{\theta}(\varphi) - \underline{\theta}(\varphi) = \frac{2\sqrt{\beta\Lambda(1+2\varphi\Lambda^2)}}{1+\varphi\Lambda^2}; \tag{5.22}$$

which shows that for the assumed parameter values the interval $I^*(\varphi)$ is always reduced by higher values of φ. As the limit of $\underline{\theta}(\varphi)$ and $\overline{\theta}(\varphi)$ for $\varphi \to +\infty$ is in both cases $\lambda\alpha\overline{y}$, it follows that as φ increases the set $\Theta(\varphi)$ becomes smaller and will shrink to the element $\lambda\alpha\overline{y}$.

We can now turn to part (iii) of proposition 5.2. Here we can consider the following first derivative

$$\frac{\partial\widehat{\beta}^{b*}(f_0^a;\varphi)}{\partial\varphi} = \frac{2(\alpha\lambda y - f_0^a)^2(1+\varphi\Lambda^2)\Lambda^3\varphi}{(\lambda\alpha\overline{y})^2(1+2\varphi\Lambda^2)^2} > 0; \tag{5.23}$$

which for the assumed parameter values is always positive. Thus for a given announcement f_0^a as φ increases the condition (5.19) will be harder to fulfil. This result is a consequence of part (ii) of proposition 5.2. To see this consider figure 5.3. If part (ii) holds, then an increase of φ shrinks the parabola constituted by the function $\widehat{\beta}^{b*}(f_0^a;\varphi)$ to the vertical axis passing through $f_0^a = \lambda\alpha\overline{y}$. Thus, as it is possible to observe from figure 5.3, for given f_0^a the value of the function will be higher. □

5.4.3 Deviation by the government

Consider now the case of a deviation by the government from the announced f_t^a. In stage 1 the central banker does not know whether or not the government will deviate. However the announced inflation target must be enforceable also under a deviation by the government.

As reappointments with $f_t \neq f_t^a$ and $\varphi > 0$ are costly, the government may deviate from the announced incentive structure if and only if the central banker follows the government's deviation and deviates as well. So a necessary condition for a deviation by the government is $f_0^a \notin \Phi$, where Φ is the set of announcements under which the precommitment solution can be sustained as a reputational equilibrium when the government deviates. On the contrary, if the central banker is able to maintain a reputation for low inflation after a deviation by the government, then the government, due to the presence of reappointment costs, has never an incentive to deviate and the announcement $f_0^a = \phi \in \Phi$ will always be fulfilled.

Notice that the fact that the announcement is always fulfilled does not imply that it is also optimal for the government to announce it because, as we have seen previously, it is possible that the central banker might not be able to sustain the precommitment inflation rate as a reputational equilibrium when the government does not deviate.

Now, as implied by the above discussion, the announcements $f_0^a = \phi \in \Phi$ can be derived from the necessary condition for a deviation by the government assuming that the central banker has announced a zero inflation target. In this case the strategies of the central banker and the private sector are the same as in the previous case, given by (5.15) and (5.16). On the contrary, the strategy of the government is now based on the assumption that also the central banker will deviate after its deviations. We have:

Government plays:

$$\begin{aligned} f_t &\neq f_t^a = f_0^a \text{ if } \pi_{t-1} = \pi_{t-1}^e; \\ f_t^a &= f_t^{a,NCD} \text{and } f_t = f_t^{NCD} \text{ if } \pi_{t-1} \neq \pi_{t-1}^e; \end{aligned} \tag{5.24}$$

where $f_t^{a,NCD}$ and f_t^{NCD} are the same as in (5.10). When the government deviates the first time from the announcement it chooses $f_t = f_t^{DD}$, which is the same of expression

(5.12).

Here the minimal condition for the patience of the central banker implies that

$$\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi) \equiv \frac{\left[\lambda\alpha\overline{y}\left(1+\varphi\Lambda\right)-f_0^a\varphi\Lambda\right]^2\left(1+\varphi\Lambda^2\right)^2}{\left(1+\varphi\Lambda\right)^2\Lambda\left(\lambda\alpha\overline{y}\right)^2\left(1+2\varphi\Lambda^2\right)}. \tag{5.25}$$

Again it is possible to show that there are two values of f_0^a that satisfy the condition (5.25) taken with equality. Here we have

$$\begin{aligned} \underline{\phi}(\varphi) &= \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2-\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{\left(1+\varphi\Lambda\right)^{-1}\left(1+\varphi\Lambda^2\right)\varphi\Lambda} > 0; \\ \overline{\phi}(\varphi) &= \lambda\alpha\overline{y}\frac{1+\varphi\Lambda^2+\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)}}{\left(1+\varphi\Lambda\right)^{-1}\left(1+\varphi\Lambda^2\right)\varphi\Lambda} > 0. \end{aligned} \tag{5.26}$$

Again in Appendix C it is shown that for the assumed parameter values $\underline{\phi}(\varphi) > 0$. Following the discussion made before, the necessary condition for a deviation of the government is given by $\beta < \widehat{\beta}^{b**}(f_0^a;\varphi)$. On the contrary if (5.25) holds, the government does never have an incentive to deviate and the announcement made by the government will always be fulfilled. Inspection of condition (5.25) yields the following proposition:

Proposition 5.3 *For* $f_0^a = \phi \in \Phi(\varphi)$, *with* $\Phi(\varphi) \equiv \left[\underline{\phi}(\varphi), \overline{\phi}(\varphi)\right]$, *(i)* $\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi)$; *(ii)* $\Phi(\varphi) \supset \Phi(\varphi')$, *with* $\varphi < \varphi'$; *(iii)* $\partial\widehat{\beta}^{b**}(f_0^a;\varphi)/\partial\varphi \gtreqless 0$.

Part (i) of this proposition says that if f_0^a is chosen between $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$, then for given φ condition (5.25) is satisfied. According to part (ii) the set Φ, i.e. the set of announcements under which the precommitment solution can be sustained as a reputational equilibrium when the government deviates, becomes smaller as reappointment costs become more important. Finally part (iii) says that the weight φ on reappointment costs has an ambiguous effect on condition (5.25). Let's proof proposition 5.3.

Proof. Again we consider for given φ the government's announcement f_0^a as the choice variable in the government's delegation problem. Taking condition (5.25) with equality we can draw (see figure 5.4) the quadratic function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ as a parabola having a global minimum at $\sigma(\varphi) = \lambda\alpha\overline{y}\left[(1+\varphi\Lambda)/\varphi\Lambda\right]$, with $\widehat{\beta}^{b**}(\sigma;\varphi) = 0$. The function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ will be equal to β for two values of f_0^a which correspond to $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$. The two values $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$ are respectively lower and greater than $\sigma(\varphi)$ and, as well as $\sigma(\varphi)$, both tend to $\lambda\alpha\overline{y}$ as $\varphi \to +\infty$. The above discussion implies that $\beta \geq \widehat{\beta}^{b**}(f_0^a;\varphi)$ for all values of f_0^a included between $\underline{\phi}(\varphi)$ and $\overline{\phi}(\varphi)$. Thus the set Φ of announcements is defined between these two extreme values. As before, with a minor abuse of notation, in proposition 5.3 we have assumed that the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ is defined also for $f_0^a = \sigma(\varphi)$. However in this case $f_t = f_t^a\left[\varphi\Lambda/(1+\Lambda\varphi)\right] = \lambda\alpha\overline{y}$ and therefore it would be more correct to say that this function is not defined in that point as the central banker always chooses zero inflation independently from the value of β.

Part (ii) can be proved using the following first derivative:

$$\frac{\partial I^{**}(\varphi)}{\partial\varphi} = -\frac{2\sqrt{\beta}\left(1+3\varphi\Lambda^2+3\varphi^2\Lambda^4+\varphi^3\Lambda^5\right)}{\varphi^2\sqrt{\Lambda\left(1+2\varphi\Lambda^2\right)}\left(1+\varphi\Lambda^2\right)^2} < 0; \tag{5.27}$$

with

$$I^{**}(\varphi) \equiv \overline{\phi}(\varphi) - \underline{\phi}(\varphi) = \frac{2\left(1+\varphi\Lambda\right)\sqrt{\beta\left(1+2\varphi\Lambda^2\right)}}{\varphi\sqrt{\Lambda}\left(1+\varphi\Lambda^2\right)}. \tag{5.28}$$

The above derivative is always negative for the assumed parameter values. This implies that as φ increases the set Φ becomes smaller and will shrink to the element $\sigma(\varphi)$, i.e. the intermediate element between the lower and upper bound of the set Φ. Thus as φ increases the parabola constituted by the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$ shrinks to the vertical axis passing through $\sigma(\varphi)$. Moreover it is possible to see that the limit of $\underline{\phi}(\varphi)$, $\overline{\phi}(\varphi)$ and $\sigma(\varphi)$for $\varphi \to +\infty$ is in all three cases $\lambda\alpha\overline{y}$. Thus, as shown in figure 5.4, the parabola shifts also to the left in the cartesian coordinate plane together with the vertical axis passing through $\sigma(\varphi)$.

Now considering part (iii) of proposition 5.3, we can see from the following first derivative

$$\frac{\partial \widehat{\beta}^{b**}(f_0^a;\varphi)}{\partial \varphi}$$
$$= -2\frac{f_0^a + f_0^a\left[\varphi\Lambda^2\left(3+\varphi^2\Lambda^3+3\varphi\Lambda^2\right)\right] - \lambda\alpha\overline{y}\left[\varphi\Lambda^2\left(\Lambda+\varphi^2\Lambda^3+2\varphi\Lambda^2\right)\right]}{\left[\lambda\alpha\overline{y}\left(1+\varphi\Lambda\right)-f_0^a\varphi\Lambda\right]^{-1}\left(1+\varphi\Lambda^2\right)^{-1}\left(1+\varphi\Lambda\right)^3\left(\lambda\alpha\overline{y}\right)^2\left(1+2\varphi\Lambda^2\right)^2}, \tag{5.29}$$

that a marginal increase in φ will have an ambiguous effect depending on the value of f_0^a. It is possible to show that the derivative becomes negative for

$$\gamma(\varphi) < f_0^a < \sigma(\varphi); \tag{5.30}$$

with

$$\gamma(\varphi) = \lambda\alpha\overline{y}\frac{\varphi\Lambda^3\left(1+\varphi\Lambda\right)^2}{\left[1+\varphi\Lambda^2\left(3\varphi\Lambda^2+\varphi^2\Lambda^3+3\right)\right]} < \lambda\alpha\overline{y}. \tag{5.31}$$

For $f_0^a > \sigma(\varphi)$ or $f_0^a < \gamma(\varphi)$the sign of the derivative will be positive. This result is a consequence of part (ii) of proposition 5.3. The ambiguous sign of the derivative is determined by the fact that now an increase of φ both restricts and shifts to the left in the cartesian coordinate plane the parabola implied by the function $\widehat{\beta}^{b**}(f_0^a;\varphi)$. □

5.4.4 The analysis of the credibility of optimal monetary delegation

As observed above the government's announcement is optimal if it belongs both to the set of announcements after which, if the government does not deviate, the central banker is able to sustain zero inflation as a reputational equilibrium and to the set of announcements after which, if the government deviates, the central banker maintains his reputation too. Using propositions 5.2 and 5.3, it follows that an announcement ψ that belongs to the set $\Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi)$ is optimal. The set of optimal announcements is illustrated in figure 5.5, where the two curves intersect each other for a given value of φ.

In the present framework examining the credibility of optimal delegation - where credibility is understood as the ability to carry out optimal monetary policy - implies to study the circumstances under which there exists an optimal announcement for the government, i.e. the set Ψ is not empty. It is possible to show that, if the weight on reappointment costs is sufficiently high but not necessarily infinite, there is always at least one optimal announcement $\psi \in \Psi \neq \emptyset$ available for solving the government's delegation problem. To see this we consider the following proposition:

Proposition 5.4 *If $\beta \geq \widehat{\beta}$ the government conducts directly monetary policy, whereas if $\beta < \widehat{\beta}$ the government delegates monetary policy to an independent central banker and announces an optimal incentive scheme f_0^a. For $f_0^a = \psi \in \Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi)$ and $\varphi \geq \widehat{\varphi}$,(i) $\Psi(\varphi) \neq \emptyset$ and (ii) $0 < \widehat{\varphi} < +\infty$.*

This proposition says that if the weight φ is greater or equal to the threshold value $\widehat{\varphi}$ the set of optimal announcements for the government $\Psi(\varphi)$ is not empty and there exists at least one announcement $\psi \in \Psi(\varphi)$ available at the delegation stage. Moreover, in the most important part of this proposition, it claims that this threshold value is not infinite and therefore reappointment costs need not be prohibitive in order to ensure that delegation credibly delivers the same outcomes of the precommitment equilibrium.

Proof. Consider first part (i). As we said above there exists an optimal $f_t^a = \psi \in \Psi(\varphi)$ as long as $\Psi(\varphi) \equiv \Theta(\varphi) \cap \Phi(\varphi) \neq \emptyset$. From propositions 5.2 and 5.3 it is possible to see that, for $0 < \varphi < +\infty$, $\overline{\phi}(\varphi) > \underline{\theta}(\varphi)$(as $\overline{\phi}(\varphi) > \sigma(\varphi)$ and $\underline{\theta}(\varphi) < \lambda\alpha\overline{y}$). Moreover it is possible to see that

$$\begin{aligned} \overline{\phi}(\varphi) - \overline{\theta}(\varphi) &= \frac{\overline{\phi}(\varphi)}{1+\varphi\Lambda} > 0; \\ \underline{\phi}(\varphi) - \underline{\theta}(\varphi) &= \frac{\underline{\phi}(\varphi)}{1+\varphi\Lambda} > 0. \end{aligned} \tag{5.32}$$

It follows that $\Psi(\varphi) \equiv \left[\underline{\phi}(\varphi), \overline{\theta}(\varphi)\right]$. Now $\Psi(\varphi) \neq \emptyset$ if and only if

$$\overline{\theta}(\varphi) - \underline{\phi}(\varphi) \geq 0. \tag{5.33}$$

So we need to find the values of φ that solve this expression. Unfortunately this is a third order polynomial. By using, for instance, Maple© we can get three roots which are very complicated to study. This implies that, considering part (ii), in order to show that $0 < \widehat{\varphi} < +\infty$ we need to perform a very complicated numerical simulation.

Let's follow a simpler route. After some simplifications the weak inequality (5.33) can be rewritten as

$$\frac{(1+2\varphi\Lambda)\sqrt{\beta\Lambda(1+2\varphi\Lambda^2)}}{(1+\varphi\Lambda^2)} \geq 1. \tag{5.34}$$

If the government delegates monetary we must have $\beta < 1/\Lambda$. This implies that $\beta\Lambda < 1$. Moreover, the inequality $\beta < 1/\Lambda$ implies also that under delegation we have $1 < \Lambda < 2$, as $.5 < \beta < 1$. Taking the limit for $\varphi \to 0$ we obtain

$$\sqrt{\beta\Lambda} \geq 1; \tag{5.35}$$

which is never true if $\beta\Lambda < 1$. So the threshold value for φ must be greater than zero. If $\varphi \to +\infty$, the term on the left-hand side of the weak inequality (5.34) tends to infinity. Moreover the first derivative with respect to φ of this term is

$$\frac{\Lambda\sqrt{\beta\Lambda}\left[2+\varphi\Lambda^2\left(6-\Lambda+2\varphi\Lambda^2\right)\right]}{(1+\varphi\Lambda^2)^2\sqrt{1+2\varphi\Lambda^2}} > 0; \tag{5.36}$$

which is always positive for $1 < \Lambda < 2$. Hence, there always exists a value of $\widehat{\varphi}$, such that $0 < \widehat{\varphi} < +\infty$, that satisfies the weak inequality (5.33). The weak inequality is satisfied also by all $\varphi \geq \widehat{\varphi}$.

In order to have an idea of the range of $\widehat{\varphi}$ it is possible to proceed in the following way. Inspection of (5.34) yields some useful information. In particular we can see that, if $\Lambda < 2$, then

$$\frac{(1+2\varphi\Lambda)}{(1+\varphi\Lambda^2)} > 1. \tag{5.37}$$

Moreover we have that

$$\sqrt{\beta\Lambda\left(1+2\varphi\Lambda^2\right)} \geq 1; \tag{5.38}$$

if

$$\varphi \geq \widetilde{\varphi} = \frac{1-\beta\Lambda}{2\beta\Lambda^3}. \tag{5.39}$$

Expression (5.39) constitutes a sufficient condition for the existence of an optimal announcement for the government, as we clearly have $\widetilde{\varphi} > \widehat{\varphi}$. It is possible to see that the highest possible value of $\widetilde{\varphi}$ is for $\Lambda \rightarrow 1$ and for $\beta \rightarrow .5$. In this case we find that $\widetilde{\varphi} \rightarrow .5$. So we can conclude that $0 < \widehat{\varphi} < .5$. □

So proposition 5.4 shows that under delegation optimal monetary policy can be more credible than under the conduction of monetary policy directly by the government. If the weight on reappointment costs is sufficiently high, but not necessarily infinite, there always exists an announcement available for the government such that the central banker is able to sustain zero inflation as a reputational equilibrium no matter whether the government deviates or not from the announcement made. As the central banker behaviour cannot be influenced and reappointments are costly, the government never deviates and will stick to the announcement made. Thus, McCallum's criticism of the delegation approach does not hold provided that the costs of changing monetary institutions are sufficiently high. Reappointment costs play a crucial role in the delegation process but in contrast to the contracting approach to delegation, based on the static one-shot game framework, also the central banker's reputation for being committed to low inflation has a fundamental role for the credibility of optimal delegation.

5.4.5 Comparison with Jensen's analysis

Now, in order to compare our analysis with that of Jensen we examine the case where the government focuses only on the incentive scheme that would be optimal in the static one-shot game and announces $f_0^a = \lambda\alpha\overline{y}$. Thus in our framework the requirement for the patience of the central banker that must be satisfied for the credibility of optimal delegation is only (5.25), which now becomes

$$\beta \geq \widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) \equiv \frac{(1+\varphi\Lambda^2)^2}{(1+\varphi\Lambda)^2\Lambda(1+2\varphi\Lambda^2)}. \tag{5.40}$$

Even if condition (5.40) is referred to the central banker instead of the government, it is similar to the condition analysed by Jensen (which is given in our framework by (5.14)). Its fulfilment implies that, if the government delegates monetary policy to a central banker announcing the incentive scheme $f_0^a = \lambda\alpha\overline{y}$, optimal monetary policy will be credible. The main difference is that in our framework the fulfilment of the government's announcement is related to the central banker's reputation for low inflation. On the contrary in Jensen's analysis the fulfilment of the government's announcement is related to the government's reputation for low inflation.

Let's compare the condition for the credibility of optimal monetary policy when the government conducts by itself monetary policy with the condition that secures the credibility of optimal monetary policy under delegation.

From condition (5.40) it is possible to derive the following corollary:

Corollary 5.1

$$\begin{aligned}
\lim_{\varphi\to+\infty}\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) &= 0;\\
\lim_{\varphi\to 0}\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) &= \widehat{\beta}.
\end{aligned}$$

Moreover it is possible to prove the following proposition, which is analogous to proposition 5.1:

Proposition 5.5 *For $\beta < \widehat{\beta}$ and all $\varphi > 0$, (i) $\partial \widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi)/\partial\varphi < 0$ and (ii) $\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) < \widehat{\beta}$.*

PROOF. From condition (5.40) follows that

$$\frac{\partial\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi)}{\partial\varphi} = -\frac{2\left(1+\varphi\Lambda^2\right)\left[\varphi\Lambda^2\left(3-\Lambda+\varphi\Lambda^2\right)+1\right]}{\left(1+\varphi\Lambda\right)^3\left(1+2\varphi\Lambda^2\right)^2} < 0 \tag{5.41}$$

for all $\varphi > 0$ and $1 < \Lambda < 2$ (implied by $\beta < \widehat{\beta}$). Using corollary 5.1, it follows immediately also that $\widehat{\beta}^{b**}(\lambda\alpha\overline{y};\varphi) < \widehat{\beta}$. □

So, in contrast with what is stated in proposition 5.1, proposition 5.5 establishes that the condition for the credibility of optimal monetary policy under delegation becomes weaker as φ increases.

The intuition for this different result is the following. In our framework it is possible to see that the punishment (for the central banker) following a deviation at t, $\left(L_{t+1}^{b,NCD} - L_{t+1}^{b,PR}\right)$, becomes weaker the higher is φ. As in the case of Jensen (but referred to the government's loss) the reason is because $L_{t+1}^{b,PR}$ is independent of φ and $L_{t+1}^{b,NCD}$ is a decreasing function of φ. But the penalty for the central banker from inflating is twice higher then the cost for the government from deviating from the announcement made at t. We can see that under the discretionary solution we have $2f_{t+1}^{NCD}\pi_{t+1}^{NCD} = 2\varphi\left(f_{t+1}^{NCD} - f_{t+1}^{a,NCD}\right)^2$. Thus the punishment will be higher in our case for a given value of φ.

Now consider the gain from deviation in period t. Here again $L_t^{b,PR}$ is independent of φ while $L_t^{b,DD}$ increases with φ. Consequently, the temptation to deviate, expressed by $\left(L_t^{b,PR} - L_t^{b,DD}\right)$, decreases also with φ. However, the increase of $L_t^{b,DD}$ is higher than in the case of Jensen (again referred to the government's loss) as $2f_{t+1}^{DD}\pi_{t+1}^{DD} = 2\varphi\left(f_{t+1}^{DD} - f_{t+1}^{a,DD}\right)^2$. So in our case for a given value of φ the incentive to deviate will be lower than in the case considered by Jensen.

These differences relative to Jensen's analysis ensure that if reappointment costs become more important the decrease in the temptation to deviate will be higher than the decrease in the punishment from deviating.

5.5 Conclusion

The analysis developed has shown that, contrary to Jensen's analysis, institutional arrangements based on incentive structures which delegate monetary policy to an independent and far-sighted central banker and are costly to change, under certain circumstances which are not extreme, may be more credible than the conduct of monetary policy without delegation. In particular, our results suggest that if the weight assigned to reappointment costs in the loss function of the government is relatively high, but not necessarily infinite, McCallum's criticism of the delegation approach does not hold. This result is due to the presence of reappointment costs but there is an important distinction with respect to the standard theory of monetary delegation, based on the static one-shot game. It crucially depends on the influence on the behaviour of the government of the central banker's reputation for being committed to low inflation.

The recent literature on the credibility of optimal monetary delegation when delegation can be changed, as exemplified by the works of Jensen (1997), Herrendorf (1998), and al-Nowaihi and Levine (1996), has shown that the delegation solution for time inconsistency can be conducive to reputation building for the government and hence is not an alternative to the reputational solution, as is usually claimed in the standard theory, but is at best supplementary. However this new body of literature, by assuming that the central banker behaves always in a discretionary fashion, has focused exclusively on the reputational enforcement of the government for being committed to the announced institutional arrangements or to low inflation. This assumption is based on the view, formalised in the standard theory, that incentive schemes or policy targets are introduced in order to constrain the behaviour of the central banker according to the objectives of the government.

The view of the delegation process formalised in our analysis is quite different, as one of the main effects of the introduction of an incentive structure that is costly to change is to enhance the central banker's reputation. Moreover, via his commitment to the announced inflation target the central banker indirectly constrains the behaviour of the government. Hence we agree with McCallum (1997a, p.109) when he insightfully argues about the possible positive implications of contract or incentive arrangements for central bankers: ".. the main effect of such arrangements [as those of New Zealand's] is not principally to constrain the central bank to act in accordance with the government's objectives, but rather to constrain the government by increasing the difficulty of its bringing pressure to inflate upon the central bank.... Arrangements such as those of New Zealand's, therefore, give the central banks an increased opportunity to behave in a rule-like, committed fashion".

As clarified by our analysis, the role played by incentive schemes in strengthening the central banker's reputation is crucial for the importance of reappointment costs. In particular the constraint that the central banker's reputation imposes on the government's temptation to deviate from the announced incentive scheme may significantly reduce the amount of reappointment cost required for disciplining the government's behaviour.

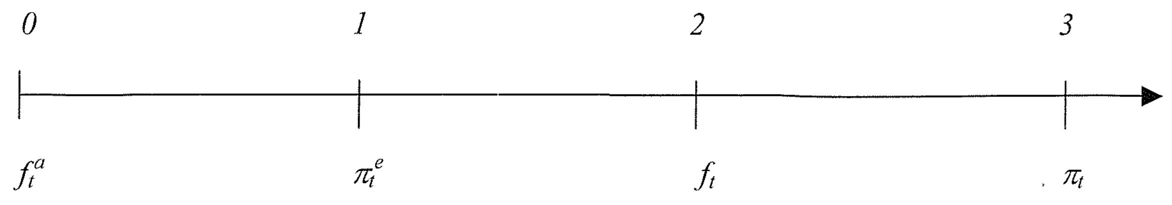

Figure 5.1 – Timing of moves for each period t in Jensen's analysis.

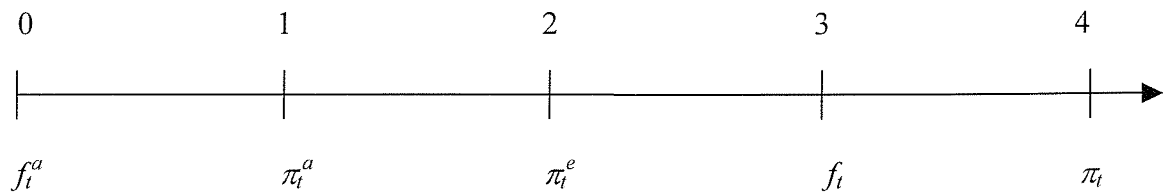

Figure 5.2 – Timing of moves for each period t in our analysis.

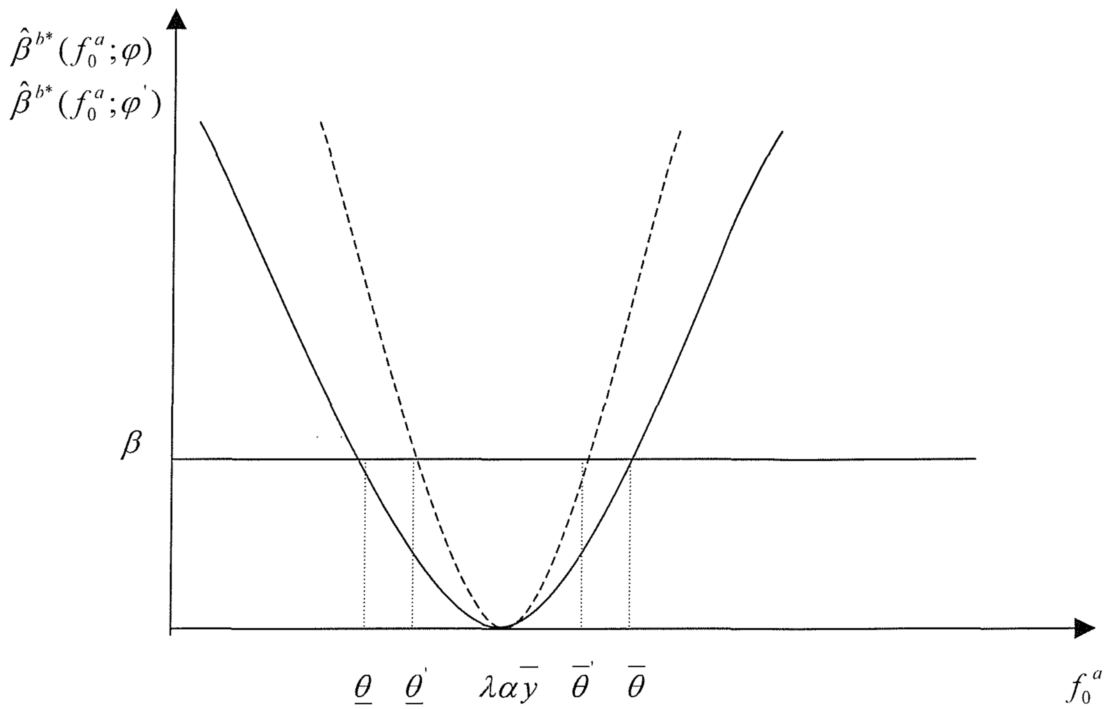

Figure 5.3 - An increase of the weight on reappointment costs.

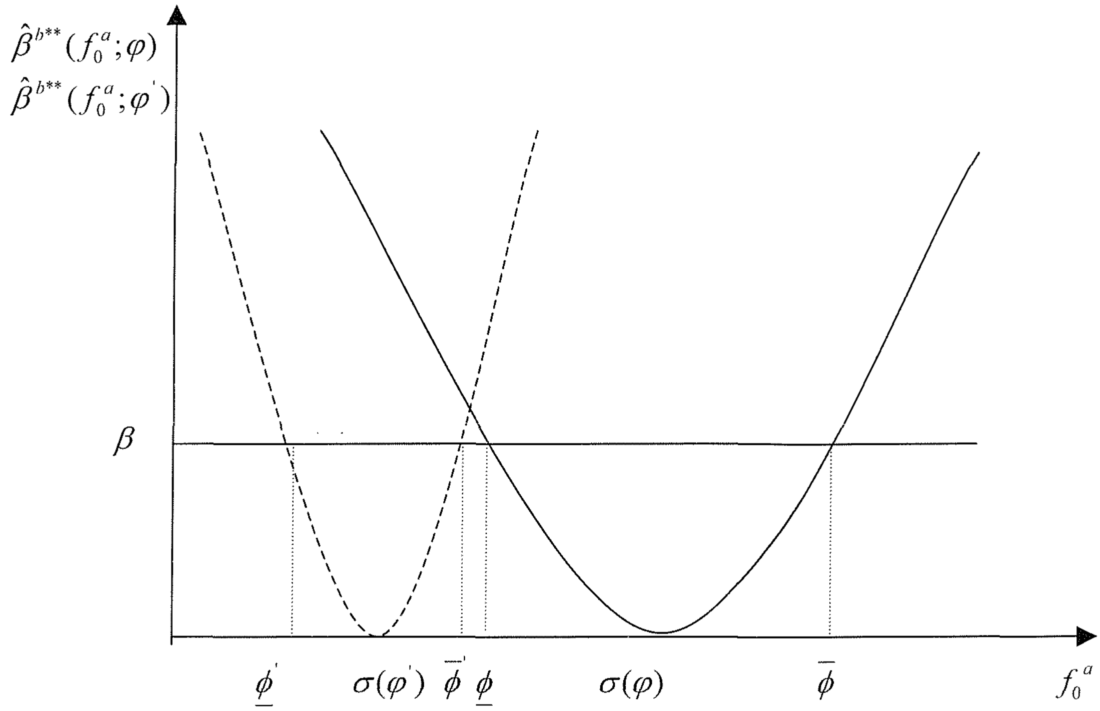

Figure 5.4 - An increase of the weight on reappointment costs.

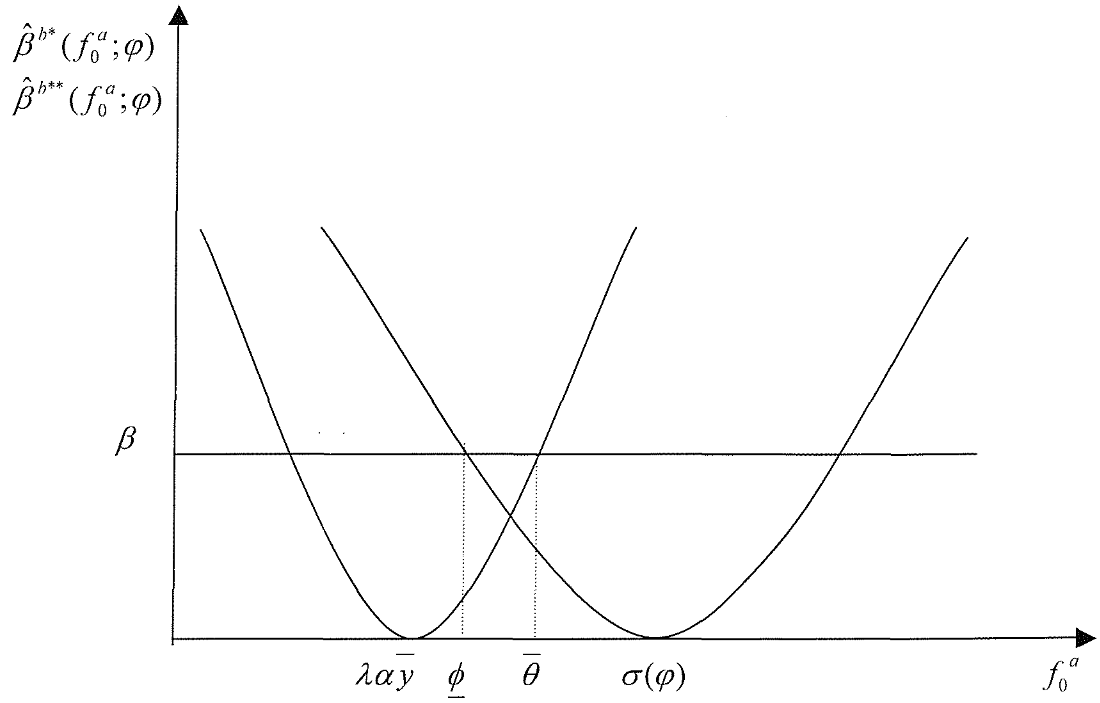

Figure 5.5 - The set of optimal announcements available for the government.

Chapter 6

Concluding remarks

The last decade has witnessed a development of the credibility literature paralleled by a renewed interest on the credibility of monetary policy in practice. The credibility literature has played an influential role by ensuring a rigorous analysis of the complex interaction between private agents and monetary institutions, which constitutes the dominant theoretical framework for discussing the design of institutional arrangements aimed at monetary stability. However, as I have argued, a number of gaps still remain in our understanding despite the progress of the literature. The survey in chapter 2 identifies some of the main questions that are still unsettled. These issues are studied in greater depth in the remaining chapters.

This thesis has reinforced the soundness of the credibility literature showing that some of the most controversial aspects can be resolved by means of refinements of both definitions and setting of the standard framework. On the other hand, the thesis has also destructive repercussions as it has been shown that the implications of the Barro-Gordon positive theory are effectively based on a too stylised framework. In both cases the present analysis has provided interesting insights and analytical tools that open new frontiers for research.

In chapter 3 I examine the controversial issue of the importance of the inflationary bias associated with discretionary monetary policy. I consider an extended version of the

Barro and Gordon framework, closer to actual policymaking. In particular, I introduce in the standard framework overlapping wage contracts, interest rate control, lags and uncertainty in the transmission of the effects of monetary policy.

The model developed provides a counterexample to Barro and Gordon's alleged inflationary bias as the time consistent monetary policy may yield a deflationary bias as well. This surprising finding implies that the current use of the time-inconsistency explanation of the apparent inflationary predisposition of industrialised countries becomes a qualified one. My model predicts that economies which feature a relatively large incentive to increase output above its long-run level are more likely to be affected by a deflationary bias. In this case the implementation of policy should feature a relatively more imprecise control of the policy instrument. An inflationary bias is more likely to emerge in economies where the credibility problem is relatively less important. Here the implementation of policy should be characterised by a relatively better control of the policy instrument.

My findings imply that the explanatory power of the dynamic-inconsistency paradigm for historical episodes of stagflation is considerably weakened. However they do not suggest that the issue of dynamic inconsistency is irrelevant. Some economists have expressed concern for the presence of a deflationary bias. For example Fischer (1994) observes that central bankers can easily develop a deflationary bias in situations where they are too shielded from public opinion. In these cases their anti-inflationary inclination can prevail upon the possibility of stabilising the real economy when it does not prejudicate the achievement of price stability. Moreover, even if the model in not able to explain all observed inflation, it draws the attention on important issues such as the incentives faced by policy makers and the interaction between credibility and private agents expectations that surely contribute together with other factors to the explanation of inflationary episodes.

The analysis can be extended in various directions. The results of the analysis could be elaborated by exploring the extent to which the deflationary bias could be ameliorated by means of a linear inflation contract, as described by Walsh, or by a linear inflation target, as discussed by Svensson. It would be of interest to see what are the policy recommendations in the case of a deflationary bias for the mentioned institutional arrangements.

In chapter 4 another important controversy is examined: the commitment versus flexibility dilemma that is associated with the adoption of policy rules. In the literature an instrument rule is commonly identified with a fixed algebraic formula to which the choices of the policy maker are mechanically tied. However there is no need to interpret instrument rules more restrictively than target rules, which are usually formalised as a constraint limiting discretionary choices.

On the contrary I interpret instrument rules as a systematic behaviour followed by the policy maker in the setting of the instrument. In particular the policy maker's optimisation problem consists in choosing some parameters of the specified instrument rule that relates macroeconomic variables to the level of the instrument. The definition of instrument rules used is in logical agreement with Taylor's alternative view on the rules versus discretion debate. He observes that in practice a policy rule can be defined more broadly as a systematic behaviour and there is no need to follow mechanically an algebraic formula. This intuition of Taylor is questioned by McCallum by using the Barro-Gordon framework. He shows that an inflationary bias will still emerge under a discretionary regime even if monetary authorities follow a systematic behaviour in the setting of the instrument.

Contrary to McCallum, I show that it may be advantageous for society to delegate monetary policy to a central banker following a systematic behaviour in the setting of the monetary instrument. I express the systematic pattern in the implementation of policy in terms of an optimally designed instrument rule for setting the interest rate. The crucial feature of the instrument rule examined is the presence of a certain degree of monetary inertia. In particular it is postulated that monetary authorities smooth interest rates by means of a partial adjustment mechanism where past decisions constitute an important determinant of the current level of the interest rate.

The analysis developed demonstrates that the presence of this systematic inertial behaviour might be optimal. If the degree of inertia is sufficiently high the inflationary bias associated with time consistent monetary policy becomes negligible without implying a trade-off between commitment and flexibility. The rationale for this surprising finding is found in the disciplining effect played by interest-rate smoothing on the incentive to create surprise inflation by reducing suddenly interest rates within the time horizon of existing nominal contracts. If the degree of gradualism is high it may enhance the credibility of optimal monetary policy as it contrasts the incentive to fool the private sector.

The analysis could be extended in a number of ways. Allowing for errors in the control of the instrument, for forward-looking behaviour by the private sector and for some persistence in inflation or unemployment would add greater realism. It would be of interest to explore whether in these cases it would be possible to derive an optimal policy rule with the same features of the popular rule examined by Taylor. However, the mentioned extensions are likely to add the complexity of the analysis without affecting the finding that regimes based on instrument rules aimed at monetary stability may represent an alternative to regimes based on target rules.

My findings have interesting policy implications. The well established international empirical evidence on instrument rules may suggest that from the point of view of implementation following instrument rules is likely to be more feasible than the adoption of the other solutions proposed in the literature for eliminating the inflationary bias. In the real world only New Zealand's monetary regime is closest to the kind of delegation formalised by the contracting approach. While Svensson's "target-conservative" central banker has been criticised as been unrealistic, as in practice the countries that have adopted an inflation targeting regime do not seem to set their inflation targets below the socially optimal rate of inflation. Moreover, Rogoff's "weight-conservative" central banker apart from not being supported by the empirical evidence on central banks independence has also been questioned for its real feasibility. In practice the government might find it difficult to appoint a central banker with exactly the right degree of conservatism.

In chapter 5 another consolidated view is questioned. Usually optimal monetary delegation is seen as an alternative to reputation building for solving the inflationary bias problem. I show that in order to survive Jensen's criticism, based on McCallum's negative view of the contracting approach, we need to consider delegation as supplementary, rather than as an alternative, to reputation.

Contrary to Jensen's analysis, I show that if delegation of monetary policy to an independent and far-sighted central banker is costly to change, optimal policy may be more credible than the conduct of monetary policy without delegation. The circumstances under which this holds are not extreme as we do not necessarily need prohibitive reappointment costs. The rationale for this different result is found in the presence of reappointment costs but there is an important distinction with respect to the standard theory of monetary delegation, based on the static one-shot game. The result hinges on the influence on the behaviour of the government of the central banker's reputation for being committed to low inflation.

My framework is based on an alternative view of the delegation process: the main effect of the introduction of an incentive structure that is costly to change is to enhance the central banker's reputation. On the contrary, the new literature on the credibility of optimal monetary delegation has shown that the institutional solution for the time-inconsistency issue can be instead conducive to reputation building for the government. This new body of literature, by assuming that the central banker behaves always in a discretionary manner, has concentrated the attention only on the reputational enforcement of the government for being committed to the announced institutional arrangement aimed at low inflation. This view is based on the idea, formalised in the standard theory, that incentive schemes or policy targets are introduced in order to discipline the behaviour of the central banker for achieving the goals of the government.

However, as shown by my analysis, the role played by incentive schemes in strengthening the central banker's reputation is crucial for the importance of reappointment costs. Moreover, via his commitment to the announced inflation target the central banker may be able to influence positively the behaviour of the government.

A natural extension of the analysis would be to consider the case when there is imperfect monitoring of the central banker's action, along the line of Canzoneri (1985). In this case it could be of interest to see what are the implications of the presence of incentive schemes costly to change for the frequency of inflationary reversions modelled by Canzoneri.

Appendices

Appendix A

Here we derive expression (4.18) reported in the text. Using expressions (4.16) and (4.17) we can rewrite the instrument rule (4.12) as

$$r_t = \gamma \widehat{r}_{t-1} + (1-\gamma)\,\widehat{r}_t + (1-\gamma)\,(mv_t + nu_t)\,. \tag{A1}$$

Expression (A1) taken at period $t-1$ becomes

$$r_{t-1} = \gamma \widehat{r}_{t-2} + (1-\gamma)\,\widehat{r}_{t-1} + (1-\gamma)\,(mv_{t-1} + nu_{t-1})\,, \tag{A2}$$

and expliciting it with respect to $\widehat{r}_{t-1}$ we can rewrite it as

$$\widehat{r}_{t-1} = \frac{1}{1-\gamma} r_{t-1} - \frac{\gamma}{1-\gamma} \widehat{r}_{t-2} - (mv_{t-1} + nu_{t-1})\,. \tag{A3}$$

Substituting (A3) in (4.12) we obtain

$$r_t = \frac{\gamma}{1-\gamma} r_{t-1} - \frac{\gamma^2}{1-\gamma} \widehat{r}_{t-2} + (1-\gamma)\,\overline{r}_t - \gamma\,(mv_{t-1} + nu_{t-1})\,, \tag{A4}$$

which can be rewritten as

$$r_t = \gamma r_{t-1} + \frac{\gamma^2}{1-\gamma} r_{t-1} - \frac{\gamma^2}{1-\gamma} \widehat{r}_{t-2} + (1-\gamma)\,\overline{r}_t - \gamma\,(mv_{t-1} + nu_{t-1})\,. \tag{A5}$$

Now multiplying both sides of (A.2) by $\gamma^2/\,(1-\gamma)$ we have

$$\frac{\gamma^2}{1-\gamma} r_{t-1} = \frac{\gamma^3}{1-\gamma} \widehat{r}_{t-2} + \gamma^2 \widehat{r}_{t-1} + \gamma^2\,(mv_{t-1} + nu_{t-1})\,. \tag{A6}$$

After substituting (A6) in (A5) we can get the expression (4.18) discussed in the text.

Appendix B

In this appendix we provide the revision rule used in chapter 4 for analysing the stability of the multiple solutions described in the text. In particular here we follow the approach of Clark, Goodhart and Huang (1999).[1]

First we consider the general form for the equilibrium decision rule of $\overline{r}_t$

$$\overline{r}_t = \phi_0 + \phi_1 \widehat{r}_{t-1} + \phi_2 v_t + \phi_3 u_t. \tag{B.1}$$

From (B.1) we have

$$\overline{r}_t^e = \phi_0 + \phi_1 \widehat{r}_{t-1}. \tag{B.2}$$

Now substituting both equations in $E_{t-1}V\left(\widehat{r}_{t-1}\right)$ and using the value function

$$V_{t-1}\left(\widehat{r}_{t-1}\right) = \theta_{0,t-1} + 2\theta_{1,t-1}\widehat{r}_{t-1} + \theta_{2,t-1}\widehat{r}_{t-1}^2, \tag{B.3}$$

we can derive the general relationship between $\theta_{0,t-1}, \theta_{1,t-1}, \theta_{2,t-1}$ and $\phi_{0,t}, \phi_{1,t}, \phi_{2,t}, \phi_{3,t}$. Identification of $\theta_{2,t-1}$ leads to

$$\theta_{2,t-1} = \frac{\left[\left(1-\gamma\right)\phi_{1,t} + \gamma\right]^2}{1 - \delta\phi_{1,t}^2}. \tag{B.4}$$

This expression is general and holds for both the values of $\phi_{1,t}$ obtained under commitment and discretion. They are given by

$$\phi_{1,t}^c = -\frac{\gamma\left(1-\gamma\right)}{\left(1-\gamma\right)^2 + \delta_b\theta_{2,t}}; \tag{B.5}$$

and

$$\phi_{1,t}^d = -\frac{\beta\gamma\left(1-\gamma\right)}{\beta\left(1-\gamma\right)^2 - \delta_b\theta_{2,t}\alpha}. \tag{B.6}$$

[1] They thank in a note Lars Svensson for suggesting them this approach.

Now, by inserting (B.4) in the expressions (B.5) and (B.6) taken at $t-1$, ϕ_1 can be revised by iteration backward as t goes to $-\infty$.

Appendix C

This appendix provides the derivation of some results used in chapter 5. Here we prove that $\underline{\phi}$ and $\underline{\theta}$ are positive. We first recall that the assumptions about the parameters are the following:

$$
\begin{aligned}
.5 &< \beta < 1; \\
\beta &< \widehat{\beta} = \frac{1}{\Lambda}; \\
\Lambda &= 1 + \lambda\alpha^2;
\end{aligned}
\tag{C.1}
$$

which imply that

$$
\begin{aligned}
1 &< \Lambda < 2; \\
\beta\Lambda &< 1.
\end{aligned}
\tag{C.2}
$$

Given these assumptions, $\underline{\theta}$ and $\underline{\phi}$ are always positive if we can prove that

$$1 + \varphi\Lambda^2 - \sqrt{\beta\Lambda\left(1 + 2\varphi\Lambda^2\right)} > 0. \tag{C.3}$$

This inequality can be rewritten as

$$1 + \varphi\Lambda^2 - \sqrt{\beta\Lambda\left[\left(1 + \varphi\Lambda^2\right)^2 - \left(\varphi\Lambda^2\right)^2\right]} > 0. \tag{C.4}$$

As from the assumed parameter values we have that $\sqrt{\beta\Lambda} < 1$, it is clear that the term outside the square root is always greater than that inside. Hence the inequality is always satisfied.

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