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INTEREST RATE SWAPS:

WHY DO THEY EXIST AND HOW SHOULD THEY BE PRICED?

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ABSTRACT

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This thesis applies the contingent claims analysis to investigate the reasons for the development and the pricing of interest rate swaps. I show that an interest rate swap transaction involves an exchange of credit risk between two firms and is not an arbitrage of market imperfections or inefficiencies as suggested by existing literature. The difference in credit risks gives firms comparative advantages by borrowing in different credit markets. Two firms can reduce their borrowing costs through interest rate swaps by trading their comparative advantages. This exchange of credit risks helps to complete the securities markets by expanding the opportunities for risk allocation, that is, by creating opportunities which are not yet provided by existing securities. Hence, an interest rate swap is not a redundant security and its contributions to reduce firms' borrowing costs and to complete the securities markets provide stronger reasons for its continuing development. The credit risks of interest rate swaps are not dealt with properly by existing literature that either replicate an interest rate swap by using either a series of forwards or futures or by the exchange of a fixed rate bond for a floating rate note. I show that the contingent claims analysis on the payoffs of firms' liabilities and interest rate swaps can better deal with the credit risk of an interest rate swap.

In this thesis I make the following contributions to the research of interest rate swaps:

1) I show that quality spread differentials exist under the Black-Scholes-Merton option pricing models that assume perfect and efficient market conditions. The model shows that financial leverage and volatility of earnings asset values are the two major factors in determining the quality spread differential. The quality spread differential allows two firms to lower their borrowing costs through interest rate swaps.
2) Relaxing the standard assumptions of Black-Scholes-Merton models by including the coupon paying debts and allowing the default free interest rate to be variable, I develop a simulation model to price the default risky corporate fixed and floating rate debts. The results of the model show that quality spread differentials exist.
3) With the application of the Arrow-Debreu security analysis, I show that interest rate swaps help to complete the market under the debt and swap priority rules. By contrast, interest rate swaps under the cross default rule cannot complete the market.
4) I write option-like equations for the payoffs of interest rate swaps under different settlement rules based on the state contingent payoff analysis. The equations reflect the credit risks of both participating firms and serve as a foundation for the pricing of interest rate swaps.

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Preface

This thesis is the result of the work done mainly while I was in registered candidature.

Earlier versions of Chapters 2, 3 and 5 have been presented at various conferences. Chapter 2 was presented at the 9th Annual Australian Finance & Banking Conference organised by the University of New South Wales in Sydney, Australia in December 1996. The Revised Version of Chapter 2 is under the review process for publication at the time of submitting this thesis. Chapter 3 was presented at the 1999 British Accounting Association Annual Conference organised by the University of Glasgow in Glasgow, Scotland in March 1999. Chapter 5 was presented at the 3rd Annual Asia Pacific Economic Law Forum organised by the University of Canterbury in Christchurch, New Zealand in December 1997.

This thesis was written as a set of articles rather than a monograph. Chapter 1 serves as the introduction and the overall literature review of the thesis. As a result, there is some repetition between the brief literature reviews of Chapters 2, 3, 4 and 5 and the main literature review contained in Chapter 1.

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To my family and all my friends in Southampton

Chapter 1

Introduction

1.1 Background of study and research problems

Interest rate swaps have evolved as one of the most successful financial innovations in the last two decades. In its simplest form, an interest rate swap consists of an agreement between two firms (called counterparties) to exchange in the future two streams of cash flows. One stream of cash flows is based on a floating interest rate while another stream is based on a fixed interest rate. From their inception in the early 1980s, the global market has grown to the notional amount of US$36.3 trillion in 1998. Table 1.1 and figure 1.1 show the year end notional amount of outstanding interest rate swaps, currency swaps and interest rate options from 1982 to 1998 extracted from the market survey of the International Swaps and Derivatives Association (ISDA). We can see that both the notional amount outstanding and the annual growth rate of interest rate swaps are always exceeding those of the other two, i.e. currency swaps and interest rate options. Beginning in June 1998, the Bank for International Settlements (BIS) started the semi-annual statistical survey of the global over-the-counter (OTC) derivatives market.[1] The BIS statistics for end-June 1999 show that in terms of notional amount outstanding, interest rate instruments remain by far the largest component of the OTC market (66%), followed by foreign exchange products (18%) and those based on equities and commodities (with 2% and 0.5%, respectively). Amongst the interest rate instruments, swaps remain the most important with 71% of the total notional amount outstanding of interest rate contracts, followed by options (16%)

[1] The BIS survey covers the OTC derivatives exposure of major banks and dealers in the G10 countries in the four main categories of derivatives contracts: foreign exchange, interest rate, equity and commodity.

and forwards (13%). In fact, the interest rate swap market is the largest market amongst all OTC derivatives markets.

The remarkable development of interest rate swaps has induced great interest of finance researchers to investigate the reasons why interest rate swaps evolved as such a successful financial innovation. One of the most debatable and not yet resolved arguments for the development of interest rate swaps is that interest rate swaps can reduce the borrowing costs of the swap participating firms. Market participants advocate that firms with different level of default risk have a comparative advantage when borrowing in different credit markets. Two firms can reduce their borrowing costs by borrowing in the market in which they have a comparative advantage and swap with each other. Finance researchers tend to argue that the reduction in borrowing costs achieved by using interest rate swaps derives mainly from the arbitrage of market imperfections or inefficiencies. The conjecture is: as the market becomes more perfect or efficient, interest rate swap activities should decline. This conjecture notwithstanding, the rate of growth of the interest rate swap market is still increasing. The arbitrage arguments dominate the literature on the development of interest rate swaps; however, they cannot explain satisfactorily this development given that arbitrage cannot last long when the securities market becomes more efficient. This is one of the research problems this thesis attempts to address.

As the interest rate swap activities develop, there has been a growing concern for the proper pricing of interest rate swaps. The proper pricing of interest rate swaps is important for at least three types of swap participants. Banks holding portfolios of interest rate swaps need to be able to measure the value of interest rate swaps net of default risk. Bank regulators require a consistent way of measuring the potential default risk so that they can set appropriate capital requirements. Corporations borrowing and using swaps to transform their liabilities should include an allowance for default risk in their comparison of the cost of direct and synthetic borrowing. In order to price interest rate swaps, the common practice is to replicate the swap using basic securities. One approach replicates an interest rate swap as a series of forwards or futures and another approach replicates an interest rate swap as an exchange of a fixed rate bond for a floating rate note. However, neither of the approaches is able to appropriately deal with the credit risk of interest rate swaps and the price obtained cannot fully reflect the credit risk involved. This thesis will also deal with this research problem.

Addressing the above research problems, this thesis shows that the sources of comparative advantages of two firms borrowing in different credit markets come from the differences in financial leverage or volatility of assets value between two firms. Applying the option pricing models in the pricing of firms' debt obligations, I show that two firms with different financial leverage or different volatility of assets value can lower their borrowing costs through interest rate swaps under perfect and efficient market conditions. The results of this thesis imply that an interest rate swap transaction is not necessarily an arbitrage of market imperfections or inefficiencies. Instead, interest rate swaps represent an exchange of credit risk between two firms. Moreover, an interest rate swap transaction will create two more different securities and a new payoff pattern of the liabilities of the firms in the swap. I show that interest rate swaps help to complete the market in the sense that they expand the opportunities of risk allocation by creating new payoff patterns of firms' liabilities that are not available without the swaps. In this context, an interest rate swap is not a redundant security and possesses specific characteristics that cannot be replicated by existing securities. Due to the important feature of exchange of credit risk between two firms in an interest rate swap, I assert that the pricing of the swap should always take into account the credit risk of both firms in the swap. By means of payoff analysis, I show that the price of an interest rate swap will depend on the settlement rule of firms' liabilities.

Since the literature on the reasons for the development of interest rate swaps raises so much debate and unresolved issues, a more detailed literature review is necessary in order to facilitate the understanding of the arguments presented throughout this thesis. The existing literature on the pricing of interest rate swaps tends to focus on the mathematical elegance of the pricing models, thereby overlooking the important characteristics of credit risk involved in interest rate swaps. Many pricing models, e.g. Cooper and Mello's (1991), assume one counterparty to be default free, therefore, the price obtained is unable to reflect the fact that interest rate swaps are normally arranged between two default risky firms. Most importantly, any pricing model that does not explicitly deal with the default risk of both firms participating in a swap cannot reflect the important feature of exchange of credit risk between the firms engaged in an interest rate swap transaction. I discuss and comment the existing literature on the development and pricing of interest rate swaps in the next section.

1.2 A review of the literature on the development and pricing of interest rate swaps

<u>The reasons for the development of interest rate swaps</u>

Market participants always advocate that financial innovation is induced by the desire of individuals and firms to increase profits or reduce risks. Changes in the economic environment such as changes in tax and regulation, increased volatility of interest rates, exchange rates, commodity prices and others, and improvement in computer technology and telecommunication will all stimulate the search for financial innovations that are likely to be beneficial to market participants. For an innovative security to be successful it must provide benefits either to the issuer or to a clientele of investors that would not otherwise be interested. Specifically, the security must enable an investor to realise a higher rate of return or enable the issuer to realise a lower cost of funds that would not have been possible before the introduction of the security without changing the level of risk. Changes in tax and regulation will create profitable arbitrage opportunities between markets. Increased volatility of interest rates, exchange rates and commodity prices will increase the demand for hedging. Improvement in computer technology and telecommunication helps financial markets to operate more efficiently, which induces the design of new financial products not feasible with the old technology. It has been suggested that interest rate swaps are popular because they provide an efficient means to hedge against interest rate risk, to exploit advantages inherent to market anomalies or simply to make changes in a balance sheet in response to environmental changes. However, economists argue that if the benefits provided by the innovative security are purely based on exploiting the market imperfections and inefficiencies, the innovative security will create no new value to the society and transactions in this innovative security will fade away when the market imperfections or inefficiencies disappear. For financial innovations such as interest rate swaps, the major market imperfections and inefficiencies in which economists focus on are financial distress costs, tax and regulatory costs, agency costs, and the pricing inefficiency caused by the information asymmetry problem. For example, early explanations of the development of interest rate swaps rely on market segmentation caused by tax and regulation. The market segmentation creates opportunities for firms to reduce borrowing costs through an interest rate swap transaction. However, economists argue that a financial innovation will have a zero or negative social value if it is designed only to avoid or lessen the constraints of existing regulations or taxes because it only represents a zero or negative sum games of wealth transfers within a society that can only increase the total cost of achieving the intended objectives of the regulations or taxes. In case of pricing, many empirical studies show that most capital markets such as those in UK and US are in the semi-strong form of pricing efficiency[2]. Unless one holds some insider information, one cannot exploit the pricing inefficiency based on information. However, most capital markets prohibit the use of insider information for profit-making transactions. Moreover, if any security is used to exploit any piece of information not yet known to the market, the

[2] Fama (1970) first introduced the definition of weak form, semi-strong from and strong form pricing efficiency. Ever since, there have been a large number of empirical studies on the efficiency of capital markets such as those in UK and US, which suggested anomalies to the market efficient theory. However, a recent paper by Fama (1998) shows that the anomalies are anecdotal and market efficiency holds.

use of such security will introduce a signalling effect that will eliminate the benefit previously brought by the pricing adjustment in the market. In conclusion, for a financial innovative product to provide positive contribution to the society, it must do so by improving economic efficiency in either one of the following aspects:-

a) lowering transactions costs or increasing liquidity by improving the operational efficiency of the market;
b) reducing 'agency' costs caused by either asymmetric information between trading parties or principals' incomplete monitoring of their agents' performance, in other words, to improve the informational efficiency of the market;
c) 'completing the markets' with expanded opportunities for risk-sharing, risk-pooling, hedging and intertemporal or spatial transfers of resources that are not already available.

I shall discuss the research on the economic rationale for the development of interest rate swaps according to the above three aspects as follows: in section 1.2.1, I discuss the costs reduction issue. The existing literature focus on the argument that interest rate swaps help to reduce the borrowing costs of firms instead of suggesting that interest rate swaps lower transaction costs or increase liquidity. The argument that interest rate swaps help to lower the borrowing costs of firms is the most debatable one in the research of the development of interest rate swaps. I shall discuss in detail the different arguments proposed in this debate. In section 1.2.2, I discuss the problems of information asymmetry and agency costs. These problems are related to the argument of lowering borrowing costs in that some researchers claim that the information asymmetry and agency costs problems can explain the borrowing costs reduction in interest rate swaps. Lastly, I discuss the market completeness issue in section 1.2.3.

1.2.1 The debate of lowering borrowing costs through interest rate swaps

Loeys (1985) and Bicksler and Chen (1986) were amongst the first to point out the existence of quality spread differentials between two firms in the fixed rate and the floating rate markets respectively. When there exists a quality spread differential between two firms, it implies that one firm has a comparative advantage in borrowing fixed rate while the other has a comparative advantage in borrowing floating rate. Two firms can reduce their borrowing costs by borrowing in the market in which they have the comparative advantage and then swap. The idea of quality spread differential between a firm in the fixed rate market and a firm in the floating rate market and how two firms can reduce their borrowing costs through the sharing of this quality spread differential by engaging in an interest rate swap can be illustrated with the following example. Instead of using a numerical example as commonly done in the existing literature, I use an example with general notation and discuss the conditions of cost savings in interest rate swaps.

Suppose there are two default risky firms, A and B and their costs of borrowing in the floating rate and the fixed rate markets are represented by the following notation:

	Firm A	Firm B
Floating rate borrowing	$RF(t) + \pi_A(t)$	$RF(t) + \pi_B(t)$
Fixed rate borrowing	$RF_N + \pi_{AN}$	$RF_N + \pi_{BN}$

where

$RF(t)$	the default free floating interest rate;
RF_N	the default free fixed interest rate for N-period maturity;
$\pi_A(t)$, $\pi_B(t)$	the default risk premium firm A and firm B need to pay over the default free floating interest rate for their floating rate borrowing respectively;
π_{AN}, π_{BN}	the default risk premium firm A and firm B need to pay over the default free fixed interest rate for their N-period fixed rate borrowing respectively.

The default risk premia firm A and firm B need to pay over the default free interest rates either for their floating rate or fixed rate borrowings are termed 'quality spread'. Quality spread differential (QSD) is defined as the difference in quality spreads between two firms in the floating rate and the fixed rate debt markets. Assume firm B is riskier than firm A, quality spread differential exists if ($\pi_B(t)$ - $\pi_A(t)$) is not equal to (π_{BN} - π_{AN}). For a default risk premium increasing along the maturity of the debt (π_{BN} - π_{AN}) is greater than ($\pi_B(t)$ - $\pi_A(t)$) and

$$QSD = (\pi_{BN} - \pi_{AN}) - (\pi_B(t) - \pi_A(t)) \quad (1)$$

In this case, firm A has a comparative advantage in borrowing fixed rate whereas firm B has a comparative advantage in borrowing floating rate. Two firms can reduce their borrowing costs by borrowing in the markets in which they have a comparative advantage and then swap with each other.

The mechanism of a plain vanilla fixed-for-floating interest rate swap is that the floating rate payer will pay the periodic default free rates whereas the fixed rate payer will pay the fixed swap rate. The fixed swap rate is determined at the initiation of the swap so that the initial value of the swap is zero. Since an interest rate swap is an exchange of cash flows between two default risky firms, the fixed swap rate will also include a default risk premium, termed the 'swap spread', over the default free rate. If we denote F_N as the fixed swap rate of a swap for N periods and π_{SN} as the swap spread, then

$$F_N = RF_N + \pi_{SN} \tag{2}$$

Firm A can create a synthetic floating rate debt with an interest rate swap with firm B as follows:

Borrowing fixed rate	$RF_N + \pi_{AN}$
Swap with firm B where	
Firm A pays floating rate to firm B	$RF(t)$
Firm A receives fixed rate from firm B	$(RF_N + \pi_{SN})$
Effective rate of the synthetic floating rate debt	$RF(t) + (\pi_{AN} - \pi_{SN})$

Compared to the direct floating rate debt, firm A can reduce the borrowing cost with the synthetic floating rate debt if $(\pi_{AN} - \pi_{SN}) < \pi_A(t)$.

Similarly, firm B can create a synthetic fixed rate debt with an interest rate swap as follows:

Borrowing floating rate	$RF(t) + \pi_B(t)$
Swap with firm A where	
Firm B pays fixed rate to firm A	$RF_N + \pi_{SN}$
Firm B receives floating rate from firm A	$(RF(t))$
Effective rate of the synthetic fixed rate debt	$RF_N + (\pi_B(t) + \pi_{SN})$

Compared to direct fixed rate debt, firm B can reduce the borrowing cost with the synthetic fixed rate debt if $(\pi_B(t) + \pi_{SN}) < \pi_{BN}$.

Rearranging the terms, the conditions that both firm A and firm B can reduce borrowing costs with synthetic borrowings are

$\pi_{AN} - \pi_A(t) < \pi_{SN} < \pi_{BN} - \pi_B(t)$ or

$0 < \pi_{SN} - (\pi_{AN} - \pi_A(t)) < QSD$

Therefore, the swap spread should be set so that it is greater than $(\pi_{AN} - \pi_A(t))$ but smaller than the quality spread differential between firm A and firm B.

What are the causes of quality spread differentials? Finance researchers argue that in the absence of market imperfections or inefficiencies, there should be no pricing inconsistencies of financial securities that give rise to profitable arbitrage opportunities. Researchers on interest rate swaps tend to view the quality spread differential as a pricing inconsistency between the floating rate and the fixed rate markets and they try to identify what kinds of market imperfections or inefficiencies lead to such pricing inconsistencies. Since quality spread differentials are related to the pricing of firms' debt securities, it has been suggested that the following market imperfections or inefficiencies in corporate finance may lead to quality spread differentials:

a) taxes and regulation that leads to market segmentation;

b) bankruptcy risk and bankruptcy costs;

c) information asymmetry;

d) agency costs.

Based on the classification of the economic rationale for the development of interest rate swaps discussed before this section, I shall discuss points a) and b) below while points c) and d) will be discussed in section 1.2.2.

a) Tax and regulatory arbitrage

The early development of interest rate swaps relies on the arbitrage of quality spread differentials that derive from the differences in tax and regulation across countries, especially between the Euro and the US markets. Arnold (1984) observes that the short-term interest rates in the US floating rate market are usually lower. The reason is that the suppliers of floating rate funds are mainly US banks whose deposits are generally covered by deposit insurance. US firms have a comparative advantage in borrowing floating rate in the US short-term market because the US banks know them better. Foreign firms do not have this advantage. At the meantime, the registration or disclosure requirement for issuing new bonds in the Euro market is less strict than in the US market. Loeys (1985) notes that Securities and Exchange Commission (SEC) registration requirements increase the cost of issuing bonds in the US bond market by approximately 80 basis points over the cost of issuing bonds in the Eurobond market. However, not all US firms have access to the Eurobond market, especially the smaller firms. Foreign firms or banks, which are better known in the Eurobond market, have a comparative advantage in borrowing long-term fixed rate in this market. As a result, foreign firms and US firms can borrow in the market in which they have a comparative advantage and swap to share the benefit. Smith, Smithson and Wakeman (1986) point out that the tax treatment of firms' debts is different between US and other countries. It gives rise to the difference in the borrowing costs of firms based in different countries that may also lead to quality spread differentials. Smith, Smithson and Wakeman (1986) summarise the reasons for the existence of quality spread differentials as tax and regulatory arbitrage. However, the savings that firms make by avoiding or lessening the constraints of existing regulations or taxes represent a zero or negative sum games of wealth transfers that can only increase the total cost of achieving the intended objectives of the regulations or taxes. For regulatory arbitrage, while it is correct to argue that disclosure requirements are less stringent in the Euro market than in the US market, it is not clear that this should always give rise to arbitrage opportunities. When pricing bonds, investors would presumably take due note of the lack of full disclosure of information. Even if there is such arbitrage opportunity arose, it would be traded away as an increasing number of firms operate in both the Euro and the US markets. Moreover, as the swap statistics show, interest rate swaps denominated in US dollars are mainly transacted between firms within the US. Thus, arbitrage of tax and regulatory differences across countries cannot provide a satisfactory explanation for the growth of interest rate swap activities in the US today. It is more applicable to explain the growth of currency swaps, which are still mainly made by two firms from different countries.

b) <u>Differences in bankruptcy risk and bankruptcy costs</u>

It is interesting to note that Wall and Pringle (1988, 1989) suggest that the differences in bankruptcy risk between two firms can explain the existence of quality spread differentials but at the same time claim that quality spread differentials are not exploitable. Suppose that the probability of bankruptcy of a firm with a lower credit rating at some point during the next ten years is twenty times that of its probability of bankruptcy over the next year. A higher credit rated firm, on the other hand, has a probability of bankruptcy over the next ten years is only eleven times its probability of failing next year. Both the lower credit-rated firm and the higher credit-rated firms will pay a greater risk premium on ten-year bonds than on one-year notes. However, Wall and Pringle (1988, 1989) suggest that the lower credit-rated firm will have to pay a proportionately greater risk premium because its risk of bankruptcy increases at a faster rate.

Wall and Pringle (1988, 1989) argue that quality spread differentials are not exploitable for the following reason. Suppose the lower credit-rated firm uses the combination of short-term debt and an interest rate swap as a substitute for long-term debt. Initially the firm will have lower interest expenses than would be the case if it had financed with long-term debt. However, as the firm's probability of bankruptcy increases, so will its cost of short-term debt. The swap will not compensate the firm for changes in firm-specific risk premiums. The expected value of interest payments if the lower credit-rated firm issues a short-term debt and then swaps will equal the expected value of a long-term debt issue. The higher credit-rated firm may appear to have an interest saving in this example, but in fact its reduced expense is merely compensation for the risk that the lower credit-rated firm will not perform its part of the swap payment.

Nonetheless, Wall and Pringle's (1988, 1989) assertion is rather vague. First, it is not clear what they mean that quality spread differentials are not exploitable. They can only be right if they mean that quality spread differentials cannot be arbitraged risklessly as suggested by market participants. In fact, I point out in this thesis that an interest rate swap transaction is not an arbitrage process but a trading between two firms. It is not necessary to guarantee the firms that their gains from interest rate swaps will exactly and actually equal the quality spread differential in order to induce them to swap. As long as there exists a quality spread differential between two firms, the expected savings from a swap transaction will be a sufficient incentive for two firms to swap. Second, Wall and Pringle (1988, 1989) do not explain clearly why the firm's probability of bankruptcy must increase over the time period. They do not explain which are the factors affecting a firm's probability of bankruptcy or the firm's risk premium. This thesis identifies that the financial leverage and the business risk as reflected by the volatility of a firm's asset value are the major factors determining the risk premium of firms' debts. However, it is important to note that for a given maturity, the risk premium is a function of only two variables: a) the volatility of the firm's asset value, and b) the financial leverage as measured by the ratio of the present value of the promised debt payment at the risk free rate to the current value of the firm. The risk premium does not necessarily increase unless there is a dramatic change in either of the two factors, i.e. the nature of a firm's business or the financial leverage of the firm. This is not usual for the majority of firms participating in interest rate swaps because they are normally rated investment grade or higher. Even though, theoretically, we cannot totally exclude such possibility, this kind of risk should have been already considered by the firms in pricing the alternatives of synthetic borrowing through an interest rate swap and direct borrowing. If the risk is always higher than the savings, interest rate swap activities should decline. However, the growth of interest rate swap activities suggests otherwise. This thesis also shows that a quality spread differential can exist based on the difference of either the volatility of firm's asset value or the financial leverage rather than the bankruptcy costs factor as suggested by Litzenberger (1992) and Titman (1992). This thesis clarifies and refutes Wall and Pringle's (1988, 1989) assertion that bankruptcy risk cannot explain the growth of interest rate swaps.

1.2.2 Information asymmetry and agency costs problems

Whenever the market is not informational efficient and information asymmetry exists, the party that has better information can always exploit the interests of the party that does not have the same information. Within a corporation, the limited liability characteristic of shareholders creates opportunities of investment or financing projects that are beneficial to shareholders but detrimental to the interests of debtholders. The corporation is controlled by shareholders and they always have incentive to undertake projects that are beneficial to them. Debtholders try to protect their interests by imposing various kinds of control that ultimately incur additional costs. These costs are termed 'agency costs' by Jensen and Meckling (1976) in their seminal paper on agency theory. I discuss below the literature on the development of interest rate swaps, which suggest that such swaps can reduce borrowing costs based on the information asymmetry and agency costs.

Arak, Estrella, Goodman and Silver (1988) highlight the main difference between direct long-term fixed rate debt and synthetic long-term fixed rate debt obtained through an interest rate swap. In the former, the risk free interest rate, as well as the firm-specific risk premium, is fixed for the period of the maturity of the debt. However, with synthetic long-term debt created by issuing short-term debt and swapping into fixed rate payment, only the risk free interest rate is fixed. Arak et al (1988) assert that a firm will prefer synthetic long-term fixed rate debt if it has a more optimistic expectation of its own future credit risk premium. By creating a synthetic long-term fixed rate debt and leaving the credit risk premium floating, a firm can lower its borrowing costs when its credit quality improves over the life of the debt. Arak et al (1988) do not explain why the firm and the market might have different expectations. Titman (1992) asserts that the different expectations between the firm and the market are due to the information asymmetry on the firm's credit quality. Titman (1992) further asserts that synthetic long-term fixed rate debt will particularly be preferred when there is interest rate uncertainty and financial distress costs. Flannery (1986) was the first to propose the idea that in the absence of interest rate uncertainty there is a tendency for firms with private information to prefer short-term borrowing. This tendency arises because those firms with the most favourable information prefer not to be locked into a fixed borrowing rate, since they expect to be able to borrow under more favourable terms in the future when their information is publicly revealed. This argument implies that those more favourably informed, in any given pool of potential long-term borrowers, can lower their expected borrowing costs by switching to short-term borrowings. However, Titman (1992) points out that when there is interest rate uncertainty and financial distress costs, the firms with favourable information will face a dilemma. Borrowing long-term forces them to pool with their less creditworthy counterparts, which increases their borrowing costs, while borrowing short-term subjects the firm to interest rate risk and hence to a greater likelihood of financial distress. Titman (1992) shows that interest rate swaps can solve this dilemma. By borrowing short-term and swapping a floating rate obligation for a fixed rate obligation, a firm with an optimistic expectation can achieve the benefits of short-term borrowing without the higher expected costs of financial distress. Titman (1992) employs a signalling model of one lower credit-rated firm, say a single-A firm, with two types (i.e. a good type and a bad type) to formalise the above argument. In his model, a single-A firm with good type would issue short-term debt to gain upgrading on its undergraded credit and swap into fixed, long-term liabilities to hedge short-term default free interest rate volatility. While a bad type single A firm would issue fixed rate bonds to lock in its overgraded credit without entering into a swap.

This line of argument has two weaknesses. First, it can only explain the motive of one side of swap participant, i.e. the fixed rate payer. The motive of the counterparty, i.e. the floating rate payer, is not yet explored. One possible explanation is that the favourably informed firm is willing to share the gain from swapping with the counterparty. Second, it has to rely on the information inefficiency of the market that cannot distinguish good type and bad type firms. However, a firm that wishes to borrow long-term fixed rate and expects its credit quality to improve should prefer the market to be efficient enough to reflect their true credit quality. As such, the firm can borrow long-term at the fair price directly rather than involving additional costs and risks in engaging in an interest rate swap.

Wall and Pringle (1989) and Wall (1989) suggest that agency costs can be an explanation for quality spread differentials and the development of interest rate swaps. They focus on the agency problems between creditors and shareholders of a firm and exclude the agency problem involving managers. They claim that agency costs can explain the existence of quality spread differentials without relying on market inefficiencies. Wall (1989) points out that quality spread differentials reflect the differences in agency costs associated with long-term debt between two firms. Wall's (1989) analysis suggests that a firm with a lower credit rating may find a combination of short-term debt and interest rate swaps to have lower agency costs than with long-term, non-callable debt, even if no risk is shifted to the firm's owners. The counterparty in the swap, the firm with a higher credit rating, is willing to participate in the swap because the lower credit-rated firm shares part of its gain.

The agency problems associated with long-term debts are mainly the under-investment and the investment risk shifting. The under-investment incentive exists because shareholders cannot fully capture the benefits of future investment. Part of the benefit goes to existing creditors via an increase in the value of their debt claims due to the reduction in the probability of bankruptcy. This share of the net present value of the profits of the investment between shareholders and bondholders lead to an incentive to under-invest. The higher the risk of pre-existing debt, the more the impact of profitable investment on the market value of that debt; the greater the fraction the net present value of new projects captured by debtholders, the stronger the adverse incentive to under-invest. Bondholders suffer from under-investment because the firm foregoes an opportunity to improve interest coverage and thereby to increase the value of the bondholders' claim. Bondholders attempt to protect themselves against under-investment by adding a risk premium proportional to the perceived risk of under-investment. The greater the risk of the firm, the greater the agency cost component in the risk premium. Hence agency costs due to under-investment can give rise to quality spread differentials.

With respect to risk shifting, firms have an incentive to shift toward higher-risk projects after issuing non-callable bonds because bondholders bear a portion of the increase in downside risk, while shareholders receive all of the upside potential. Hence, a zero net present value transaction that shifts toward higher risk may actually increase the value of the shareholders' claim, while reducing commensurately that of the long-term creditors' claim. The higher the risk of the firm, the more the bond price is depressed by additional risk, and the more the shareholders have to gain from the asymmetric effects of a risky project. Again, bondholders protect themselves by increasing the risk premium and by imposing costly covenants and monitoring arrangements. The higher the risk of the firm, the greater the need for protection. Hence, as with the under-investment problem, agency costs increase with risk and contribute to a quality spread differential.

Short-term debt financing reduces both types of adverse incentives by allowing creditors to adjust the risk premium or, in the extreme case, not renewing the debt in response to changes in investment policy and other developments. By financing with short-term debt, a firm avoids the agency costs associated with long-term debt. The substitution of long-term debt by a combination of short-term debt and interest rate swaps enables a firm to save the agency costs associated with long-term debt as well as to avoid facing the interest rate risk. The riskier firm (lower credit rating) issues short-term, then swaps to pay a fixed rate and receive floating rate. The short-term debt remains outstanding. There exists no adverse incentive to under-invest because gains from investment accrue to shareholders, nor is there any incentive to shift toward higher-risk investment because such a shift would be penalised by an increase in the risk premium on the still-outstanding short-term debt. The swap thus insulates the firm from changes in the level of interest rates, but the firm still pays a premium for any increase in firm-specific risk. The swap would reduce debt-financing costs without a fully offsetting disadvantage, such as an increase in the equity capitalisation rate. In this case, Wall and Pringle (1989) and Wall (1989) claim that interest rate swaps can provide real savings for both participating firms which are not the result of market inefficiencies or segmentation. Moreover, savings on agency costs could explain the continuing growth of the swap market, since the use of short-term debt and a swap by one firm does not reduce the potential gains for another firm.

Wall and Pringle's (1989) and Wall's (1989) agency cost argument draw mainly from the discussion of agency theory in Jensen and Meckling's (1976) seminal paper. Jensen and Meckling (1976) use the agency framework to analyse the resolution of conflicts of interest between stockholders, managers and bondholders of the firm. Wall and Pringle (1989) and Wall (1989) focus their discussion on the agency problems between stockholders and bondholders. While I am aware that the agency theory constitutes a research area in the analysis of the control of incentive conflicts in contractual relations and the implications for the capital structure of corporations, I adopt the state contingent claims framework to analyse interest rate swaps and assume no agency problems in the contingent claims models. Theoretically, the agency cost theory does not contradict the behaviour of bankruptcy risk. The under-investment and investment risk shifting problems should indirectly increase the bankruptcy risk. The risk premium adjusted to the bankruptcy risk should, therefore, include the agency costs as well. A quantitative model measuring the default risk of firms can provide a more straightforward analytical result.

1.2.3 Completing the market

In a more complete securities market investors would have the choice of reducing uncertainty and obtaining a desirable outcome regardless of the state of the world. The early work of Arrow (1964) and Debreu (1959) suggests that markets become more complete as contingent claim securities are added to the economy. In the limit, the number of securities in a complete market would equal the number of possible states of the world. If there is no limit on the trading of securities, investors can allocate the risk in the payoff distribution they prefer. There is an extensive literature discussing the role of organised exchange traded options, another financial innovation, in completing the market.[3] To date, unfortunately, there is no discussion about interest rate swaps and their contribution towards completing the market using the state contingent claims framework. Smith et al (1986) and Arak et al (1988) were amongst the first to suggest market completeness as an explanation for the development of interest rate swaps, but their analyses were

[3] See Ross (1976), Cox and Rubinstein (1985). I shall discuss them in more detail in chapter 4.

not based on state contingent claims framework and suffered from various weaknesses which I shall discuss below.

Smith et al (1986) assert that the interest rate swap market contributes to the integration of financial markets by allowing market participants to fill the gaps left by missing markets. At the time of mid 1980s, there were no interest rate forward contracts available. Since an interest rate swap behaves like a series of forward contracts, a swap could be used in place of the missing forward contract. Hence, the swap market can be used as a way of synthetically 'completing' the financial markets. Subsequent researchers suggest that this argument may be weakened by the fact that today interest rate forward contracts are available for both short and long maturities. Arak et al (1988) divide the borrowing costs of firms into two components, namely, the risk free rate and the credit spread. They suggest that interest rate swaps help to complete the market by providing a combination of the risk free rate and the credit spread that is not available otherwise. Prior to the introduction of swaps, the instruments available to firms were long-term fixed rate, long-term floating rate, and short-term debts. These instruments differ in their combinations of fixed and floating risk free rates and credit spread. A long-term fixed rate bond has both the risk free rate and the credit spread fixed. A long-term floating rate note has the credit spread fixed but allows the risk free rate to float. A short-term debt allows both the risk free rate and the credit spread to float. It is interesting to note that the combination of fixed risk free rate and floating credit spread is not available from the traditional debt instruments. Arak et al (1988) show that a synthetic borrowing obtained by combining short-term debt with a long position in an interest rate swap (pay fixed and receive floating) provides a combination of fixed risk free rate and floating credit spread. In this context, interest rate swaps help to complete the market by providing a combination of borrowing costs that is not available from the existing debt instruments. Unfortunately, Arak et al (1988) rely on information asymmetry to explain the conditions under which a firm would prefer a synthetic borrowing having the risk free rate fixed and the credit spread floating.

In this thesis, I apply the Arrow-Debreu (1964, 1959) pure securities analysis to show that interest rate swaps help to complete the market by expanding the state contingent payoff distributions of a firm's assets. I show that interest rate swaps can produce state contingent payoffs not yet provided by existing securities and, therefore, expand the opportunities for risk allocation.

The debate regarding the reduction of borrowing costs through interest rate swaps dominated the literature on the reasons for the development of interest rate swaps in the 1980s and 1990s. As discussed above, most of the literature tends to argue that the borrowing cost reduction achieved through interest rate swaps derives mainly from the arbitrage of market imperfections or inefficiencies. Due to the scarcity of data on the usage of interest rate swaps, empirical studies dealing with the reasons for the use of interest rate swaps are not found until the mid 1990s. In 1990, the Financial Accounting Standards Board (FASB) in US issued a Statement of Financial Accounting Standards (SFAS 105) mandating the disclosures of swap usage which made empirical research on the usage of interest rate swaps possible. Samant (1996) carries out an empirical study of interest rate swap usage by nonfinancial corporate business in the US. His results suggest that, compared to non-swap-users, fixed rate payers have more leverage, greater profitability, more growth options, less operating risk, lower ratios of fixed to total assets, and more divergent earnings estimates. On the other hand, floating rate payers do not seem to have financial and operating characteristics significantly different from non-swap-users. Samant's (1996) findings provide supportive evidence to the view that interest rate swaps are usually arranged between firms of different credit risk. Recently, Saunders (1999) performs an empirical study to test the different explanations of the borrowing cost reduction through interest rate swaps. His findings support the comparative advantage, information asymmetry and agency cost explanations as the reasons for such reduction.

To summarise, I argue that the borrowing cost reduction function together with the contribution to complete the market are the major reasons for the development of interest rate swaps. The borrowing cost reduction is derived from the existence of quality spread differentials between firms borrowing in different credit markets. This thesis shows that the factors causing the existence of quality spread differentials are not necessarily market imperfections or inefficiencies.

Pricing of interest rate swaps

The literature on the pricing of interest rate swaps is grouped according to the focus. For instance, one stream of swap pricing literature is done from the viewpoint of banks that act as swap dealers. This stream was motivated by a joint study made between the US Federal Reserve and the Bank of England in 1987 which investigated the credit exposure of banks to interest rate swaps with the purpose of determining the appropriate capital requirement for banks acting as swap dealers. Similar to the loan transactions, bank regulators focus on the analysis of the potential loss in swaps caused by the default of counterparty so that they can form the basis for the determination of capital requirement. Effort was therefore spent on estimating the swap value based on different methods of predicting the future interest rates after the swap initiation. One popular method used is the Monte Carlo simulation of the future course of interest rates to estimate the values of interest rate swaps under different interest rate scenarios. Neal and Simons (1988) and Simons (1989, 1993) carry our their studies along this approach. Neal and Simons (1988) calculate the average credit exposure of a portfolio of interest rate swaps containing 20 matched pairs of swaps, each with an original maturity of five years and notional principal of US$10mn. They use the randomly generated interest rates that exhibited the same volatility as observed in historical rates from 1987 to 1991. Several thousand interest rate scenarios were generated and the credit exposure resulting from each scenario was calculated. Neal and Simons (1988) find that the average credit exposure of the interest rate swap portfolio is about 1.85% in the period between 1987 to 1991. Their findings also show that the credit exposure of interest rate swaps depends on the volatility of interest rates and on the remaining maturity of swap contracts. Simons (1989, 1993) carries out a similar analysis for different time periods and matched pairs of swaps with different maturities. His findings show that the credit exposure of interest rate swaps is close to zero near the beginning and the end of the term to maturity, but tend to be higher around the middle of the terms. Comparing swaps with different maturities, Simons (1989, 1993) finds that swaps of longer maturities reach higher levels of exposure than those of shorter maturities. Although the studies carried out by Neal and Simons

(1988) and Simons (1989, 1993) are interesting for bank capital requirement purposes, they cannot reflect properly the nature of credit risk involved in swaps. First, the studies only deal with the magnitude of credit risk but the probability of default, which is related to individual firm's specific factors is not addressed. Second, by estimating the credit exposure of banks to swaps in case of default by a counterparty, Neal and Simons (1988) and Simons (1989, 1993) implicitly ignore the default risk of banks. As a result, the important feature of exchange of credit risk between two firms in an interest rate swap transaction is not reflected.

Another stream of literature on the pricing of interest rate swaps adopts the replicating portfolio approach. Based on the nature of interest rate swaps as an exchange of fixed cash flows for variable cash flows, the literature replicates an interest rate swap either as a series of forward / futures contracts or the exchange of fixed rate bond for a floating rate note. Smith, Smithson and Wakeman (1988), McNulty (1990) and Alworth (1993) price interest rate swaps by replicating them as a series of forward or futures contracts. For example, in a plain vanilla interest rate swap, the net cash flow to the fixed rate payer is the difference between the floating and fixed rate payments at each settlement date. This payoff is equivalent to the payoff on a basic forward or futures contract on the floating interest rate with a delivery price equal to the fixed interest rate. Smith et al (1988), McNulty (1990) and Alworth (1993) then replicate the cash flows of an interest rate swap by constructing a portfolio of consecutive short-term either interest rate forward or futures contracts that span for the life of the swap. In another approach of replicating an interest rate swap as an exchange of a fixed rate bond for a floating rate note, Bicksler and Chen (1986) and Smith et al (1988) replicates the cash flows of an interest rate swap through a long position in a fixed rate bond and a simultaneous short position in a floating rate note, with both securities noncallable and equal to the swap in par value and maturity. Again, neither of the replicating approaches can reflect properly the nature of credit risk involved in interest rate swaps in the swap price obtained. Recently, Minton (1997) examines the empirical implications of these two approaches. His results show that neither of the approaches is completely consistent with the implications of differential counterparty risks involved in swaps. In the case of the replicating portfolio using Eurodollar futures, Minton (1997) finds that the over-the-counter (OTC) swap rates do not move one-for-one with analogous swap rates derived from Eurodollar futures prices. He finds that the difference between short-term OTC swap rates and swap rates derived from Eurodollar futures prices are positively related to proxies for counterparty default risk. Minton (1997) explains this phenomenon as a result of the clearinghouse of the futures exchange acting as the counterparty to every transaction, and because all futures positions are marked to market daily, counterparty default risk is effectively non-existent in the futures market. Thus, while the basic payoff profiles of swaps and corresponding Eurodollar strips are similar, the amount of counterparty default risk in a swap is greater than that in a futures contract.

To test the pricing models in which interest rate swaps are priced as portfolios of bonds, Minton (1997) examines the relationship between the determinants of corporate bond prices and the levels of par US interest rate swap spreads. He finds that the par swap rates are related to factors that are proxies for the shape of the yield curve. However, the swap rates do not move one-for-one with the corporate bond yields which suggests that default risk in a swap, while greater than that in a Eurodollar futures contract, is less than that in a bond because of the settlement features of the swap. In short, Minton's (1997) study provides supportive evidence that the replicating approaches cannot price properly the credit risk involved in interest rate swaps. The theoretical problems with the replicating approaches and the pricing models that assume one counterparty as default free in an interest rate swap transaction will be discussed in more detail in chapter 5.

Based on the discussions above, I state the propositions and research objectives of this thesis in section 1.3 below.

1.3 Propositions and research objectives

The aim of this thesis is to develop a theoretical explanation for the development and the pricing of interest rate swaps. I posit that interest rate swaps can bring benefits to the swap participating firms for reasons other than the arbitrage of market inefficiencies as suggested by the existing literature. For an interest rate swap to be a viable tool for reducing financing costs, one party must have access to comparatively cheap fixed rate funding but desire floating rate funding while another party must have access to comparatively cheap floating rate funding but desire fixed rate funding. I conjecture that the comparative advantage between two firms in different credit markets can exist in perfect and efficient market conditions. The savings in financing costs by two firms through an interest rate swap do not necessarily represent an arbitrage of market imperfections or inefficiencies. Instead, I posit that interest rate swaps should be treated as a trading of credit risks between two firms. Analogous to international trade between two countries, cost savings derived from comparative advantages and preferences are two necessary and sufficient conditions for interest rate swaps to occur. First, it is necessary that interest rate swaps enable two firms to reduce financing costs. Second, it must also be sufficient that interest rate swaps produce a combination of risk and return that improves the position of both firms. Since interest rate swaps involve an exchange of credit risk between two firms, investigations of the development and the pricing of this kind of swaps should always take into account the credit risk of both firms.

In summary, the research objectives of this thesis are to investigate:

a) the conditions under which a comparative advantage exists between two firms borrowing in different credit markets. Such conditions are conjectured to be different from market imperfections or inefficiencies;

b) the way in which interest rate swaps can produce a risk and return combination that improves the preferences position of both firms, i.e. the market completeness issue;

c) the pricing of interest rate swaps which appropriately reflects the credit risk of both participating firms.

In investigating the above issues, I apply mainly the state contingent claims framework. First, I develop the option pricing models to examine the conditions for the existence of a comparative advantage in perfect and efficient market conditions. Second, I apply the state contingent claims analysis to show that interest rate swaps help to complete the market. Lastly, I develop the state contingent payoff equations for the pricing of interest rate swaps. The theme and major contributions of each chapter and the organisation of this thesis will be discussed in section 1.4 below.

1.4 Organisation of the thesis

This thesis was written as a set of articles rather than a monograph. Four individual papers were developed, each of which comprises a separate chapter. The last chapter contains the overall conclusions and discussions of this thesis. The major objectives and findings of each chapter and their relationships are briefly described below.

Chapter 2 Interest rate swap – an option pricing approach

Based on the observation that interest rate swaps generally involve two firms with different credit ratings and the suggestion that through the swap the two firms may reduce their borrowing costs by sharing the quality spread differential, this chapter applies the Black-Scholes-Merton option pricing models to investigate the factors giving rise to quality spread differentials. I apply the option pricing model because the model assumes perfect and efficient market conditions. If the results show that quality spread differentials exist, they can serve as evidence against the argument of the existing literature that quality spread differentials must derive from market imperfections or inefficiencies. Merton's (1974) paper on the risk structure of interest rate depicts the behaviour of risk premia of default risky debts along the time to maturity. However, Merton (1974) does not analyse explicitly the difference in risk premia between two firms with different credit ratings for different maturities. In addition, the results in Merton's (1974) paper are mathematically too complicated to determine explicitly the conditions under which quality spread differentials exist. I modify Merton's (1974) approach by using guarantee costs of firms' debt as a measurement of risk premia. If guarantee costs between the debts of two firms differ along different maturities, then a quality spread differential would exist. The results of this chapter support this conjecture. The underlying factors giving rise to quality spread differentials are found to be firms' financial leverage and volatility of earnings asset value. This chapter further improves the analysis of the risk structure of interest rates by explicitly determining the conditions under which a quality spread differential exists for different values of financial leverage and volatility of earnings asset value.

Chapter 3 Valuation of default risky fixed and floating rate debt obligations:- implications for interest rate swaps

There are two major criticisms on the application of Black-Scholes-Merton option pricing model in analysing the behaviour of risk premia of default risky debts. One is that the Merton's (1974) model dealt with discount bonds but most of the firms' debts are coupon paying. Another is the assumption of fixed default free interest rate. In practice, it is uncommon to find a flat term structure of interest rates. Subsequent research has attempted to improve the performance of Merton's (1974) model by dealing with the above two issues. However, the resulting models are either intimidating or mathematically complex. This chapter relaxes the assumptions of Merton's (1974) model by analysing the default risky coupon paying debts under a variable default free interest rate condition. No closed formed solutions for firms' coupon paying debts could be derived under the relaxed assumptions. I develop simulation models to determine the values of risk premia for both fixed rate and floating rate default risky coupon paying debts for different values of firms' leverage and volatility of earning assets. My results show that quality spread differentials also exist for some values of firms' leverage and volatility of earning assets. The results of this chapter further reinforce the arguments and results of chapter 2.

Chapter 4 The contribution of interest rate swaps to market completeness

Based on my proposition that an interest rate swap is analogous to international trade between two countries, cost savings derived from comparative advantages and preferences are two necessary and sufficient conditions for interest rate swaps to occur. Chapters 2 and 3 show that quality spread differentials exist for reasons other than market imperfections or inefficiencies. Two firms can save borrowing costs by sharing the quality spread differential through an interest rate swap. In terms of preferences, I investigate the possibility of interest rate swaps completing the securities market by expanding the opportunities for risk allocation in this chapter. If interest rate swaps help to complete the securities market there will be two implications. First, an interest rate swap is not a redundant security. Second, an interest rate swap expands the state contingent payoff distributions of firms' liabilities that may improve the preference of firms' stakeholders. In order to investigate these issues, I apply the Arrow-Debreu pure security analysis on firms' liabilities and interest rate swaps. My results show that interest rate swaps under the debt priority and swap priority rules help to complete the securities market while an interest rate swap under the cross default rule does not. To my knowledge and at the time of writing this thesis, there was no research paper that analysed interest rate swaps with the Arrow-Debreu pure security analysis. This chapter contributes to provide a new insight into the market completeness function of interest rate swaps. The contributions of interest rate swaps towards the reduction of borrowing costs reported in chapters 2 and 3 and towards market completeness reported in this chapter provide stronger arguments for the development of interest rate swaps than the arguments found in the existing literature that are based mainly on market imperfections and inefficiencies.

Chapter 5 The pricing of default risky interest rate swaps

To complete the analysis of interest rate swaps, this chapter develops the state contingent payoff analysis to value interest rate swaps. Since an interest rate swap always involves two firms of different default risk, the price of interest rate swaps should reflect properly these risks. Based on the state contingent payoff analysis of firms' liabilities and interest rate swaps under different settlement rules, I express interest rate swaps as a combination of different kinds of options on the swap participating firms' assets. The kind of options will depend on the settlement rule. The results of this chapter serve as a foundation for the development of the valuing equation for the pricing of interest rate swaps. This can be done by inputting the value generating process of firms' assets and default free interest rate, which is mainly a mathematical task. This chapter also improves the existing literature on the pricing of interest rate swaps by dealing explicitly with the default risk of both swap participating firms.

Chapter 6 Conclusions

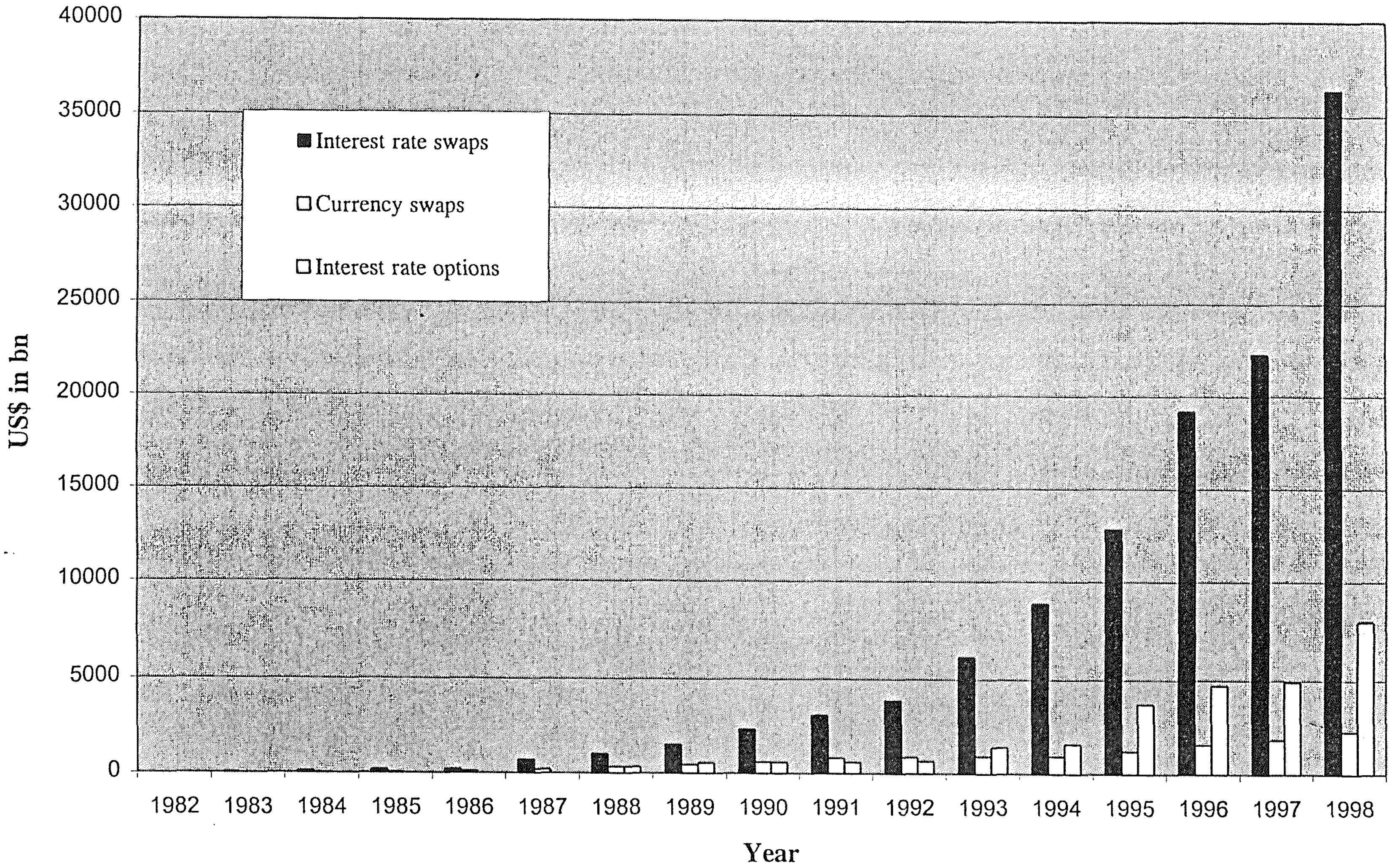
This chapter contains the overall conclusions and discussions of this thesis. I conclude that interest rate swaps help to lower firms' borrowing costs and complete the securities market in perfect and efficient market conditions. These two contributions of interest rate swaps to the economy are consistent with the criteria for successful financial innovative products and explain the continuing development of interest rate swaps without relying on the market imperfections or inefficiencies arguments. In an interest rate swap transaction, two firms are in fact exchanging their credit risks between themselves. The price of an interest rate swap should therefore reflect properly the credit risks of both firms in the swap. The direction of future research in interest rate swaps is also discussed in this chapter.

Table 1.1 Total outstandings at year end

Year	Interest rate swaps	Annual growth rate (%)	Currency swaps	Annual growth rate (%)	Interest rate options	Annual growth rate (%)	Total	Annual growth rate (%)
1982	2		3				5	
1983	30	1400	5	67			35	600
1984	90	200	19	280			109	211
1985	175	94	50	163			225	106
1986	190	9	100	100			290	29
1987	683	259	182	82			865	198
1988	1010	48	316	74	327		1653	91
1989	1503	49	435	38	537	64	2475	50
1990	2312	54	577	33	561	4	3450	39
1991	3065	33	807	40	577	3	4449	29
1992	3851	26	860	7	635	10	5346	20
1993	6177	60	900	5	1398	120	8475	59
1994	8816	43	915	2	1573	13	11304	33
1995	12811	45	1197	31	3705	136	17713	57
1996	19171	50	1560	30	4723	27	25454	44
1997	22291	16	1824	17	4920	4	29035	14
1998	36362	63	2253	24	7997	63	46512	60

Source of data: ISDA market survey summary statistics (notional principal in billions of US dollars)

Figure 1.1 Total outstandings at year end



Chapter 2

Interest Rate Swap – an Option Pricing Approach

Abstract

Interest rate swaps generally involve two firms with different credit ratings. A quality spread differential is observed to exist at different maturities for firm debts with different credit ratings. The quality spread differential allows two firms with different credit ratings to lower their borrowing costs through interest rate swaps by utilising the comparative advantage of borrowing in different markets. The credit ratings of firms are determined by credit risk factors such as leverage and volatility of earnings asset value. This chapter investigates the effect of leverage and volatility on the behaviour of risk premia between the debts of firms with different credit ratings by using the contingent claims analysis. My results show that the quality spread differential can be explained by the differences in leverage and volatility. Thus, two firms with different leverage and volatility of earnings asset value will benefit from interest rate swaps. However, it is found that the duration within which the quality spread differential exists is limited by the values of the leverage and the volatility of two firms.

2.1 Introduction

The argument for swap arrangements rests on the well-known economic principle of comparative advantage. For instance, with interest rate swaps, it is found that a higher credit-rated firm has a comparative advantage in borrowing long-term, fixed-rate while a lower credit-rated firm has a comparative advantage in borrowing short-term, floating-rate. Both firms can lower their borrowing costs by engaging in a fixed/floating interest rate swap. This chapter seeks to identify the source of comparative advantage as the default or credit risks resulting from differences in leverage and volatility of earnings asset value of the two firms engaged in the swap. The analysis will be carried out in terms of contingent claims theory.

Since the swap arrangement between the World Bank and IBM in 1981, swaps have become one of the most important tools for corporations to improve their financial performance. Financial managers use swaps to reduce borrowing costs, to increase asset returns or to hedge risk. Major market participants include commercial and investment banks, securities firms, savings and loan institutions, corporations, and government agencies. While there are no publicly available data on the users of swaps, a study by Levich (1998) finds that by the end of 1994, 39% of the end users of swaps are corporations, 51% are financial institutions that are not swap dealers, and 10% are government agencies and supranationals. Banks are active in the market as both dealers and end-users.

Swap markets have grown rapidly in the last two decades. By the end of 1982, the aggregate of swap contracts outstanding was US$5 billion. By the end of

1998, contracts outstanding exceeded US$36 trillion. The compound annual growth rate was about 68%. On the one hand, such a tremendous growth of the swap market opens new opportunities for financial institutions to expand their business. On the other hand, the reasons giving rise to swap transactions, their pricing, and their risk have not yet been fully clarified.

In this chapter, I find that two firms with different leverages or volatilities can lower their costs of borrowing by dealing in an interest rate swap. However, the time to maturity of the interest rate swap is limited by the values of firm leverage and asset value volatility. The chapter is organised as follows: in the next section, I discuss the principle of the comparative advantage in interest rate swaps. Section 3 shows how the quality spread differential can enable two firms to reduce borrowing costs through interest rate swaps. In section 4, I discuss the contingent claims analysis employed to analyse the quality spread and the quality spread differential. I develop the pricing equations in sections 5 and 6 and find out the effect of leverage, volatility, and time to maturity on the quality spread and the quality spread differential. In section 7 I examine the conditions under which the quality spread differential exists. Finally, I draw my conclusion in section 8.

2.2 Reasons for interest rate swaps

A swap is an agreement between two parties to exchange cash payments based on a particular formula for a period of time. Although different kinds of swaps have been developed, interest rate swaps remain the most popular and account for most of the swap business. By the end of 1998, interest rate swaps alone exceeded US$36 trillion out of the US$39 trillion total swap contracts outstanding in the global swap markets.

The most prevalent form of interest rate swap is the 'Plain Vanilla'. Here two counterparties agree to swap interest payments involving the same currency on equal principal amounts. No principal ever changes hands. A higher credit-rated firm borrows at a fixed rate and swaps its fixed rate loan for a floating rate loan booked by a lower credit-rated firm. After the swap, the higher credit-rated firm has effectively exchanged its fixed rate loan for a floating rate loan while the lower credit-rated firm has done the converse. Why does the higher credit-rated firm not borrow floating or the lower credit-rated firm borrow fixed directly but engage instead in a swap? Since interest rate swaps are voluntary market transactions by two parties, both parties must obtain some economic benefit in order to have an incentive to engage in a swap. Bicksler and Chen (1986) point out that the economic benefits in an interest rate swap are the result of the principle of the comparative advantage.

A comparative advantage arises because one party can borrow in one market relatively cheaply whereas another party can do the same in a different market. Such phenomenon may exist between two firms with different balance sheet structures and different credit ratings. A lower credit-rated firm usually has to borrow at a higher cost than a higher credit-rated firm. Quality spread is the difference in the costs of borrowing for a given maturity between two firms. In both the short-term floating-rate and the long-term fixed-rate markets, a lower credit-rated firm has to pay a quality spread over that paid by a higher credit-rated firm. Thus, the latter firm has the absolute advantage in raising funds both at the short-term and the long-term markets. However, Bicksler and Chen (1986) observe that the quality spreads in the long-term fixed-rate market and in the short-term floating-rate market are not identical. The quality spread is narrower in the floating-rate market than in the fixed-rate market. It implies that a lower credit-rated firm has a comparative advantage in borrowing floating-rate while a higher credit-rated firm has a comparative advantage in borrowing fixed-rate. This quality spread differential presents an opportunity in which each firm can borrow in the market where it has the comparative advantage and swaps the borrowing with each other. A lower credit-rated firm borrows at floating-rate and then swaps its floating-rate loan for a fixed-rate loan borrowed by a higher credit-rated firm. Through this transaction both firms succeed in lowering their borrowing costs.

Many researchers have attempted to explain the quality spread differential. Bicksler and Chen (1986) assert that information differential and institutional restrictions between two markets such as the US bond market and the Eurobond market are the major factors contributing to the quality spread differential. Smith, Smithson, and Wakeman (1988) show that the quality spread differential reflects differences in the terms of the debt contracts between the US and the Euro markets. For instance, there is always a call option to prepay the debt at some future date for long-term securities. The issuing firm pays for the call option via a higher interest rate. Unlike US issues, Eurobond debt contracts often adjust call prices for changes in market interest rates, thereby lowering the option value of the call option. Thus the quality spread between two firms in the US and the Euro markets will be higher for long-term debt than for short-term debt. These arguments explain the quality spread differential in terms of structural differences in different markets. Within the same market, Smith, et al (1988) argue that the call option value to prepay the debt is greater for lower credit-rated firms than higher credit-rated firms. The difference in the call option value for long-term debt represents the quality spread differential. Wall and Pringle (1989) point out that quality spread differential is due to differences in expected bankruptcy costs. The expected bankruptcy costs are included in the risk premium a firm has to pay for its debt. If the risk of bankruptcy of a lower credit-rated firm increases at a faster rate than a higher credit-rated firm, the quality spread between the two firms will differ at different maturities.

The bankruptcy cost argument can be developed further by analysing the credit risk of firms engaged in interest rate swaps. However, little research has been devoted to analysing the credit risk characteristics of the borrowing firm itself. Since the credit risk of firms is reflected in their credit ratings, factors contributing to the differences in credit ratings will help to explain the reasons for the existence of swaps. Factors such as leverage and volatility of a firm's earnings asset values can be either observed or estimated directly. By using option pricing techniques, this chapter investigates whether differences in the leverage and the volatility of a firm's earnings asset values will cause quality spread differentials to occur.

2.3 Borrowing cost savings through interest rate swaps

Figure 2.1 shows a graph of the costs of borrowing faced by two firms, A and B, assuming that firm A has a higher credit rating than firm B. R_{At} and R_{Bt} represent the costs of borrowing of firms A and B for different times to maturity, t. The quality spreads are represented by (R_{B1} - R_{A1}) and (R_{B2} - R_{A2}). (R_{B1} - R_{A1}) represents the quality spread of short-term borrowing whereas (R_{B2} - R_{A2}) represents that of long-term borrowing. It shows that firm A has the absolute advantage in borrowing because it can borrow at lower costs in both short-term and long-term. However, it is important to note that as long as (R_{B2} - R_{A2}) is greater than (R_{B1} - R_{A1}), a quality spread differential arises and a swap will be beneficial to both firms. Thus, the quality spread differential (QSD) is represented by

$$QSD = (R_{B2} - R_{A2}) - (R_{B1} - R_{A1}) \qquad (1)$$

If there is a quality spread differential, both firms can lower their borrowing costs through interest rate swaps. The total gain in interest rate swaps will not be equal to the quality spread differential due to transaction costs, which are mainly the fee charged by the financial intermediary. How the two firms divide the quality spread differential between themselves depends on the relative creditworthiness and bargaining power of the two firms. For simplicity, the procedures of how the quality spread differential is shared through interest rate swap are shown in Table 2.1 by assuming that the quality spread differential is shared equally between two firms at no transaction costs.

Firm A has effectively converted its fixed-rate borrowing to floating-rate and it can save QSD/2 through swap less than borrow in floating-rate market directly itself. Through equation (1), it can be shown that $R_{B1} + R_{A2} + QSD/2 - R_{A1} = R_{B2} - QSD/2$. Thus, firm B has effectively converted its floating-rate borrowing to fixed-rate through swap and it also saves QSD/2. Therefore, the total cost is less than borrowing in the fixed-rate market directly itself. Figure 2.2 shows the direction of cash flow in the interest rate swap.

If the quality spread between firm A and firm B is expressed in terms of the first derivative against time, $\partial(R_{Bt} - R_{At}) / \partial t$, then quality spread differential occurs when $\partial(R_{Bt} - R_{At}) / \partial t > 0$. In order to investigate if quality spread differentials can exist in efficient market conditions, I value the firms' debts with option pricing model that assumes market efficiency. A review of the option pricing theory is given in the following section.

2.4 Valuation of financial claims using option pricing technique

On the debt maturity date, if the value of the firm's assets is less than the promised debt payment, then the firm is insolvent. The creditors can force the firm to liquidate and take over the firm's assets. Assuming no bankruptcy costs, the value of debt will be equal to the value of the firm's assets which is less than the promised payment. The shareholders need not make up for the deficiency because of the limited liability provision. If the value of the firm's assets is higher than the promised debt payment, then it is in the interests of the shareholders for the firm to make the payment. Shareholder equity remains positive. Black and Scholes (1973) first point out the possibility of using the contingent claims theory, i.e. option pricing theory, to analyse the financial claims issued by firms. Subsequently, many researchers, notably Ingersoll (1987) and Merton (1990) have developed the option pricing approach on the pricing of the firm's debt.

For the simple case where a firm has only a single homogeneous debt issue which promises to pay a total of B dollars on the maturity date T, the payoffs of the debt and the shares on the debt maturity date will be as follows:

If $V_T > B$: $D_T = B$ (2)

$E_T = V_T - B$ (3)

If $V_T < B$: $D_T = V_T$ (4)

$E_T = 0$ (5)

where D_T = value of the debt on maturity date T;
E_T = value of the shares on maturity date T;
V_T = value of the assets on maturity date T.

The value of the debt and the shares on the debt maturity date can be expressed as:

$$D_T = B + \mathrm{Min}\,(V_T - B, 0) \tag{6}$$

$$E_T = \mathrm{Max}\,(V_T - B, 0) \tag{7}$$

The value of the shares possesses the same characteristic of a call option. Furthermore, it is because of the identity, Asset = Liabilities + Equities, the value of the debt can also be evaluated through the option as follows:

$$D_T = V_T - \mathrm{Max}\,(V_T - B, 0) \tag{8}$$

The following sections analyse the quality spreads and the quality spread differential by using the option pricing technique.

2.5 Analysis of the quality spread by using the option pricing technique

I follow the assumptions made by Merton (1974) in his analysis of corporate debt pricing as a basis for my quality spread analysis. The firm has only two classes of claims: a) a single homogeneous class of debt, **D**, and b) the shareholder equity, E. The indenture of the debt issue contains the following provisions and restrictions: a) the firm promises to pay a total of B dollars to the creditors on the maturity date T; b) in the event that this payment is not met, the creditors immediately take over the firm's assets and the shareholders receive nothing; c) the firm cannot issue any new senior (or of equivalent rank) claims on the firm nor can it pay cash dividends or do share repurchase prior to the maturity of the debt. Thus, the debt can be looked at as a discount bond with a face value of B and a maturity period of time T. At time t = 0, the value of the debt D_0 = Bexp(-RT), where R is the interest rate the firm has to pay for its debt which is related to the firm's risk of default. In the following paragraphs, I examine the reasons for differences in R which are the basis for comparative advantage.

Under the restrictions of the debt indenture, the funds raised through issuing debt and shares are all used in investing in the firm's assets, V. Assume the value of the firm's assets is generated by the Ito process,

$$dV = \rho V dT + \sigma V dZ \tag{9}$$

where dZ is the Wiener process,

$$dZ = \varepsilon\sqrt{dT} \tag{10}$$

ε is a standard normal deviate (i.e. $E(\varepsilon) = 0$ and $V(\varepsilon) = 1$), ρ is the net instantaneous return and σ is the asset price volatility. The solution to equation (9) is obtained through the application of Ito's Lemma,

$$\ln\left(\frac{V_T}{V_0}\right) = \left(\rho - \frac{\sigma^2}{2}\right)T + \sigma\varepsilon\sqrt{T} \tag{11}$$

Given limited liability provisions, the value of the firm's equity at the end of period T, E_T can be modelled as a call option on the firm's assets with terminal condition expressed by equation (7), $E_T = \text{Max}\ (V_T - B, 0)$. The expected value of the equity, E_T at time T will be

$$E(E_T) = \int_B^{\infty} (V - B)L'(V)dV \tag{12}$$

where $L'(\cdot)$ = the density function of the log-normal distribution.

Given my assumption about the process generating V_T (equation 11), I can apply the Cox and Ross (1975) procedure of assuming risk neutrality and set ρ equal to the risk free rate of interest r and discount equation (12) by r to yield

$$E_0 = V_0\phi(X_1) - B\exp(-rT)\ \phi(X_2) \tag{13}$$

where

$$X_1 \equiv \frac{\ln\left(\frac{V_0}{B}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \tag{14}$$

$$\equiv \frac{\frac{\tau}{2} - \ln(d)}{\sqrt{\tau}}$$

$$X_2 \equiv X_1 - \sigma\sqrt{T} \tag{15}$$

$$\equiv X_1 - \sqrt{\tau}$$

where $d \equiv B\exp(-rT) / V_0$, $\tau \equiv \sigma^2 T$ and ϕ is the cumulative normal function.

The ratio $d \equiv B\exp(-rT) / V_0$ can be regarded as the 'quasi' leverage of the firm because the promised loan payment is discounted at risk free rate rather than at the risk adjusted rate paid by the firm. Although it is a biased-upward estimate of the actual debt-to-asset value ratio, it serves as a good indicator in comparing the leverage of different firms.[1] The variable τ is a measure of the volatility of the firm's earnings asset value over the life of the loan.

From the accounting identity V = D + E and equation (13), we can write the value of the debt issue as:

$$D_0 = V_0\phi\,(-X_1) + B\exp(-rT)\phi\,(X_2) \tag{16}$$

[1] d is a biased-upward estimator of leverage for individual firm because Bexp(-rT) > Bexp(-RT). However, for comparing the relative difference in the leverage of different firms, d will always be higher for a firm with higher leverage and vice versa.

ϕ $(-X_1)$ is simply equal to 1 - ϕ (X_1) based on the property of cumulative normal distribution curve. Since D_0 is also equal to Bexp(-RT), then,

$$\text{Bexp}(-RT) = V_0\phi(-X_1) + \text{Bexp}(-rT)\,)\phi(X_2) \tag{17}$$

Dividing both sides of equation (17) by Bexp(-rT), taking logarithm of both sides, and rearranging the terms, we obtain

$$R - r = -\frac{1}{T}\ln\left[\frac{V_0}{B\exp(-rT)}\phi(-X_1) + \phi(X_2)\right] \tag{18}$$

As discussed before, R is the interest rate the risky firm has to pay for its debt that is related to the firm's default risk. Default will occur when a firm is insolvent, i.e. V < B. Default risk is the probability that a firm will become insolvent. If we let prob(s) and prob(f) be the probability of solvency and insolvency, respectively, then the default risk can be written as:

$$\begin{aligned} prob(f) &= 1 - prob(s) \\ &= 1 - \phi(X_2) \end{aligned} \tag{19}$$

where

$$prob(s) = \phi(X_2)$$

$$= \int_B^\infty L'(V)dV \tag{20}$$

R will be higher for a higher default risk. R - r represents the default risk premium the risky firm has to pay over the risk free rate. The higher the R, the higher the premium.

From equation (18) we can see that, for a given maturity, the risk premium is a function of only two variables: a) the volatility of the firm's asset value, σ^2; and, b) the ratio of the present value of the promised debt payment at the risk free rate to the current value of the firm, d. For identical loan contract, the quality spread between two firms is due to their different default risk. Therefore, the quality spread between two firms can also be attributed to these two variables, σ^2 and d. As long as σ^2 or d of the two firms are not identical, there will exist a quality spread between the two firms.

For interest rate swaps, we are interested in the quality spread differential, which is the change in the quality spread along different maturities. If we let H = R - r, then $\Delta H = H_B - H_A$ will be the quality spread between two firms, A and B. The quality spread differential will be represented by $\partial(\Delta H)/\partial T$ which equals $\partial H_B / \partial T - \partial H_A / \partial T$. From equation (18) we can write

$$\frac{\partial H}{\partial T} = \frac{\ln(P) + \frac{\sigma\sqrt{T}}{2dP}\phi'(-X_1)}{T^2} > or < 0 \tag{21}$$

where

$$P = \frac{D_0}{B\exp(-rT)} \tag{22}$$

$$= \phi(X_2) + \frac{1}{d}\phi'(-X_1)$$

where $\phi'(\cdot)$ denotes the first derivative.

$P = D_0 / B\exp(-rT)$ is the price today of a risky dollar promised at a future time T in terms of a dollar delivered at that date with certainty, and it is always less than or equal to 1. $\partial H / \partial T$ can be either positive or negative because ln(P) is always less than or equal to 0. Merton (1974) shows that for $d \geq 1$, $\partial H / \partial T$ will be negative. From equation (21) it follows that it is very difficult to analyse the effect of σ^2, d and T on $\partial(H) / \partial T$ and $\partial(\Delta H) / \partial T$. Appendix 2A shows the mathematical analysis and the difficulty in determining the conditions of $\partial H / \partial T$ and $\partial(\Delta H) / \partial T > 0$ or < 0 explicitly. In order to ameliorate such difficulty, it is necessary to express the quality spread differential in a simpler way.

2.6 Analysis of the quality spread differential by using guarantee cost as substitute for the risk premium

Merton (1977) shows that the risk premium can be related to the cost of a guarantee on the borrowing of a firm. Suppose that at the time of borrowing the firm purchases a guarantee on the debt from a third party which can guarantee that the promised payment, B, will be paid at the time of maturity. The debt of the firm then becomes default-free and will yield the risk free rate of interest only. The firm must bear the cost of this guarantee that depends on the risk premium it must originally pay. The funds that the firm will get at the time of borrowing equals to Bexp(-rT) - G_0, where G_0 is the cost of guarantee. It must be the same as Bexp(-RT). Then,

$$G_0 = B\exp(-rT) - B\exp(-RT) \tag{23}$$

The value of the guarantee can, thus, be used as a substitute for the risk premium. In the quality spread analysis, we are interested in the cross-sectional difference of guarantee costs between two firms at a point in time.

The arrangement of the guarantee is that if at the debt maturity date, $V_T > B$, then the guarantee will simply be left to expire. If $V_T < B$, the guarantor takes over the assets of the firm and pay the promised payment B to the creditors. The deficiency is the loss to the guarantor. The guarantee will possess a pay-off structure as follows:

if $V_T > B$: $G_T = 0$ (24)

if $V_T < B$: $G_T = -(V_T - B)$ (25)

Therefore,

$$G_T = -\text{Min}\,(V_T - B, 0)$$

$$= \text{Max}\,(B - V_T, 0) \tag{26}$$

The payoff structure of the loan guarantee is identical to that of a put option. By purchasing a loan guarantee, the firm has effectively purchased a put option on its assets which gives the firm the right to sell the assets for B dollars on the maturity date of the debt. The guarantee is a put option on the firm's assets with exercise price B. Following the same arguments in the valuation of equity (a call option of the firm's assets), we can derive a formula for the value of the guarantee, and it can be written as:

$$G_0 = B\exp(-rT)\phi\,(-X_2) - V\phi(-X_1) \tag{27}$$

Since we are interested in the cross-sectional difference of guarantee costs amongst firms at a point in time, it will shed more light on the characteristic of this difference to work with the ratio $g \equiv G_0 / B\exp(-rT)$ rather than with the absolute cost level G. Thus, g is the cost of the guarantee per dollar of the guarantee loan payment discounted at risk free rate and it is always less than or equal to 1.

For $g \equiv G_0 / B\exp(-rT)$, equation (27) can be rewritten as:

$$g = \phi(-X_2) - \frac{1}{d}\phi(-X_1) \tag{28}$$

Equation (28) shows that g is a function of the two variables d and τ. The effect of a change in either variable on g can be inspected by taking the first derivative

$$\frac{\partial g}{\partial d} = \frac{\phi'(X_1)}{d^2} > 0 \tag{29}$$

$$\frac{\partial g}{\partial \tau} = \frac{\phi'(X_1)}{2d\sqrt{\tau}} > 0 \tag{30}$$

where $\phi'(\cdot)$ denotes the first derivative.

The change in the guarantee cost is an increasing function of both variables d and τ. We will see how d and τ affect the credit ratings and the quality spreads in the following paragraphs.

Standard and Poor's (1992) defines a credit rating as "an assessment of an issuer's ability to pay interest and repay capital in a timely manner." Thus, a credit rating is a measure of the default risk of the debt issuer. In practice, the credit-rating agencies will assign a high credit rating to a firm with low default risk and a low credit rating to a firm with high default risk. The risk premium required by investors will be higher for high default risk than for low default risk. Ederington and Yawitz (1987) find that there is persistent existence of such a quality spread between lower and higher credit-rated firms.

Financial variables such as coverage, profitability and leverage are found to be important determinants of ratings. In the case of leverage, Ederington and Yawitz (1987) empirically document an inverse relationship between ratings and leverage. The higher the leverage, the lower the credit rating and vice versa. Although volatility is seldom used in the rating process, we can easily observe the inverse relationship between ratings and the volatility of a firm's earnings asset value. A high volatility of earnings asset value implies a high volatility of profitability and coverage. As such, the default risk will also be higher with implied higher volatilities. As a result, the credit rating will be lower. The difficulty in using the volatility of the firm's earnings asset value is that it is not observable empirically. However, Christie (1982) shows that the volatility of the firm's asset value can be inferred from the volatility of the market value of equity, which is easily observable. We can conclude that the higher the leverage or the volatility, the lower the credit rating and vice versa. From the results of equations (29) and (30), we can see that a lower credit-rated firm has to pay a higher guarantee cost for its borrowing which is equivalent to the quality spread it has to pay over that of a higher credit-rated firm. Therefore, the option pricing technique suggests that the existence of a quality spread between two firms with different ratings at a point in time can be explained by the different leverage levels and volatility of earnings asset values between the firms.

2.7 Conditions under which quality spread differentials occur

On the basis of the analysis provided in the previous section, the problem has become one of examining the change in the difference in guarantee costs between two firms against the change in the times to maturity of the loan. Let Δg be the difference of the guarantee costs between two firms, A and B, i.e. $\Delta g = g_B - g_A$; if the Δgs are not identical for different times to maturity, T, then a quality spread differential exists. Two firms will gain through the sharing of the quality spread differential by engaging in an interest rate swap.

Quality spread differential occurs when

$$\frac{\partial[\Delta g]}{\partial T} = \frac{\partial[g_B - g_A]}{\partial T} > 0$$

From equation (28), the first derivative of g with respect to T will be

$$\frac{\partial g}{\partial T} = \frac{\sigma}{2d\sqrt{T}}\phi'(X_1) \qquad (31)$$

Then

$$\frac{\partial[\Delta g]}{\partial T} = \frac{\sigma_B}{2d_B\sqrt{T}}\phi'(X_{1B}) - \frac{\sigma_A}{2d_A\sqrt{T}}\phi'(X_{1A}) \qquad (32)$$

Equation (32) shows that the change in the difference in guarantee costs for different times to maturity is also a function of the variables d, σ and T.

Assume that firm A has a higher credit rating than firm B. From the previous arguments regarding the relationship between credit ratings and leverage and volatility, it follows that either the leverage or the volatility of firm B will be higher than that of firm A, i.e. $d_B > d_A$ or $\sigma_B > \sigma_A$ or both. I would like to identify the conditions under which $\partial[\Delta g] / \partial T > 0$ so that a quality spread differential occurs for some values of d and σ^2. The results are as follows:

a) σ^2 of two firms are identical, $d_A < d_B$ and $d \le 1$

$\partial[\Delta g] / \partial T > 0$ for the values of T:

$$-\frac{[\ln(d_A)+\ln(d_B)]}{\sigma^2} > T$$

b) d of two firms are identical and $d \le 1$, $\sigma_A^2 < \sigma_B^2$

$\partial[\Delta g] / \partial T > 0$ for the values of T:

$$\left|\ln(\frac{\sigma_B}{\sigma_A})\right| > \left|\frac{(\sigma_B^2-\sigma_A^2)\left(\ln(d)+\frac{\sigma_A\sigma_B T}{2}\right)\left(\ln(d)-\frac{\sigma_A\sigma_B T}{2}\right)}{2\sigma_A^2\sigma_B^2 T}\right|$$

Mathematical derivations are shown in Appendix 2B. Appendix 2C presents one of the TSP programmes that generates figures 2.3 to 2.6. Table 2.2 shows the maximum value of T within which $\partial[\Delta g] / \partial T > 0$ for some representative values of d and σ^2. Figures 2.3 and 2.4 plot Δg against T for some different values of d and σ^2. Several points call for further comment.

First, I ignore the case where $d > 1$. A firm with $d > 1$ is technically insolvent and may go bankrupt at any time. The default risk of this firm is extremely high and it may not get any credit rating at all. Even if there is any credit rating for this firm, it will be classified into speculative grade which is below BBB- in Standard & Poor's ratings scale. Swap markets are generally credit risk averse and firms that are able to deal in these markets are mainly confined to those with high credit rating, i.e. investment grade of BBB- or above. In fact, it is now common for swap contracts to include a rating trigger which is designed to shorten or terminate the swap transaction in the event the rating of a counterparty be downgraded. This aims at reducing the potential for credit losses due to counterparty bankruptcy. I, therefore, focused my analysis on $d \leq 1$.

Second, I find that the quality spread differential does not increase indefinitely but only for some limited values of T, the time to maturity of debt. The traditional argument that the quality spread between two firms with different credit ratings will become greater as time to maturity, T, increases is valid only up to a certain value of T. Merton (1974) uses the option pricing technique to analyse and depict the graphs of corporate risk premia against the time to maturity of debt. This work was later refined by Lee (1981) and Pitts and Selby (1983). Their results show that for $d < 1$, the term structure of risk premia is humped for medium leveraged firms and upward sloping for low leveraged firms. The risk premia for a higher leveraged firm is always greater than that for a lower leveraged firm. Nevertheless, the difference will become smaller as time to maturity increases. I perform a similar analysis by using the guarantee costs as substitutes for risk premia. Figure 2.5 shows the guarantee costs of firms with different leverage levels for different times to maturity of the debt. Figure 2.6 shows the guarantee costs of firms with different volatilities. All guarantee costs are upward sloping and the gap between them first increases, then decreases, and finally approaches to zero as time to maturity increases. The changes of the gap between the guarantee costs explain why the quality spread differential is humped shape as depicted in Figures 2.3 and 2.4. It means that the quality spread differential increases up to a certain value of T and then decreases. It applies not only to firms with different leverage levels (as shown in Figure 2.3), but also to firms with different volatilities (as shown in Figure 2.4) as long as the leverage is lower than 1. The results of my analysis of the behaviour of quality spreads are consistent with the analysis of risk structure of corporate debt by Merton (1974), Lee (1981) and Pitts and Selby (1983). Moreover, I improve these studies by providing additional information on the behaviour of quality spreads between firms with different volatilities.

Third, the value of T for $\partial[\Delta g] / \partial T > 0$ depends on the values of d and σ^2. T will be longer for lower values of d or σ^2 (as shown in Table 2.2). A traditional fixed/floating interest rate swap, where a higher credit-rated firm borrows fixed and a lower credit-rated firm borrows floating, will take place only when the maturity falls within the limit of T. It implies that short-term interest rate swaps will happen more frequently or that long-term interest rate swaps will be used mainly by firms with low leverage levels and volatilities. In 1993, the US Commodity Futures Trading Commission found that there was a shift toward shorter maturities for interest rate swaps over the period of 1987 to 1991. Interest rate swaps with maturities of less than one year have grown at an annualised rate of 58%, compared to the growth rate of 15% for swaps with maturities exceeding 8 years. This evidence supports my analysis too.

2.8 Conclusion

The principle of the comparative advantage in trade theory asserts that international trade is beneficial whenever, in the absence of trade, there is a difference in the opportunity costs of production between countries. An interest rate swap transaction is analogous to international trade in that both counterparties can benefit from swap whenever there is a difference in the quality spreads between short-term and long-term borrowing. The existence of the quality spread differential between two firms forms one of the sources of economic benefits in that both firms can lower their costs of borrowing through the sharing of the quality spread differential by engaging in swaps.

In this chapter, I have identified the differences in leverage and volatility of earnings asset value as the sources of the quality spread differential. The difference in the balance sheet structure between two firms forms the basis of comparative advantage. My results show that the behaviour of the difference in risk premia of debts between firms with different leverage or volatility along different maturities gives rise to the quality spread differential. Given that the leverage and the volatility of asset value are important factors in credit ratings, the results of this chapter explain why interest rate swaps are usually transacted between two different firms with different credit ratings.

The fact that interest rate swaps generally involve two firms with different credit ratings has a further implication on the pricing and default risk of interest rate swaps. The probability of default is usually higher for a firm with a lower credit rating than for a firm with a higher credit rating. In dealing with a lower credit-rated firm in interest rate swaps, the higher credit-rated firm will take on more risk. It will then be interesting to see how the latter firm determines the price of the interest rate swap in order to compensate for the higher risk. This will be especially important for banks that act as swap dealers since they are usually the party with the higher credit rating. The default risk of interest rate swaps has also an impact on the regulatory requirements for banks that have swap contracts in their assets. Further research on the pricing and the default risk of interest rate swaps which especially focus on individual firms' balance sheet structures will contribute to shed light on the sources of the comparative advantage underlying swaps.

Table 2.1. Sharing of quality spread differential through interest rate swap

<u>Firm A</u>

Borrow in long-term (fixed-rate) market at	R_{A2}
Swap, where	
swap payment to firm B at floating	R_{A1}
swap receipt from firm B at fixed	$(R_{A2} + QSD/2)$
Effective cost of borrowing	R_{A1} - $QSD/2$

<u>Firm B</u>

Borrow in short-term (floating-rate) market at	R_{B1}
Swap, where	
swap payment to firm A at fixed	$R_{A2} + QSD/2$
swap receipt from firm A at floating	(R_{A1})
Effective cost of borrowing	$R_{B1} + R_{A2} + QSD/2 - R_{A1}$

Table 2.2 Maximum values of T within which $\partial[\Delta g] / \partial T > 0$

a) For identical σ^2, $d_A < d_B$

σ^2	d_A	d_B	T
0.05	0.2	0.4	50
	0.5	0.7	21
	0.7	0.9	9
0.10	0.2	0.4	25
	0.5	0.7	11
	0.7	0.9	5
0.25	0.2	0.4	10
	0.5	0.7	4
	0.7	0.9	2

b) For identical d, $\sigma_A^2 < \sigma_B^2$

d	σ_A^2	σ_B^2	T
0.2	0.05	0.07	98
	0.10	0.15	48
	0.25	0.30	21
0.5	0.05	0.07	75
	0.10	0.15	36
	0.25	0.30	16
0.7	0.05	0.07	69
	0.10	0.15	34
	0.25	0.30	15

Note: If d and σ^2 are year measurements, then T will be expressed in terms of number of years.

Figure 2.1 Quality spreads over time

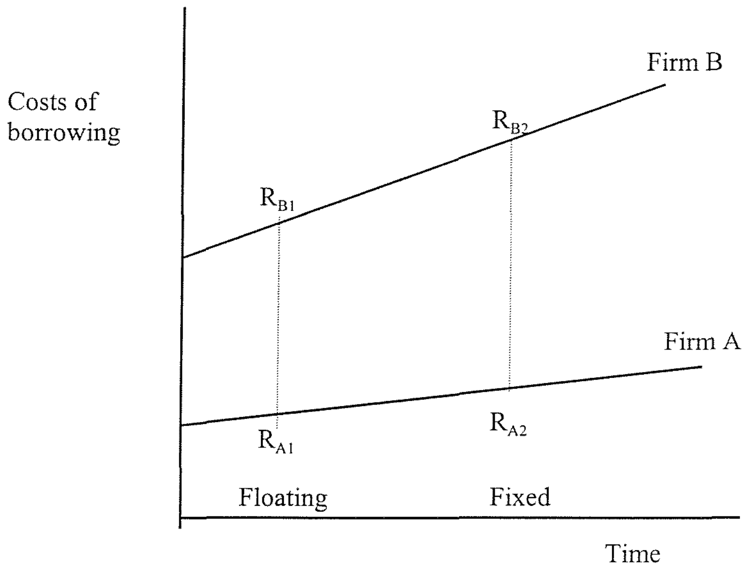


Figure 2.2 Fixed / Floating interest rate swap

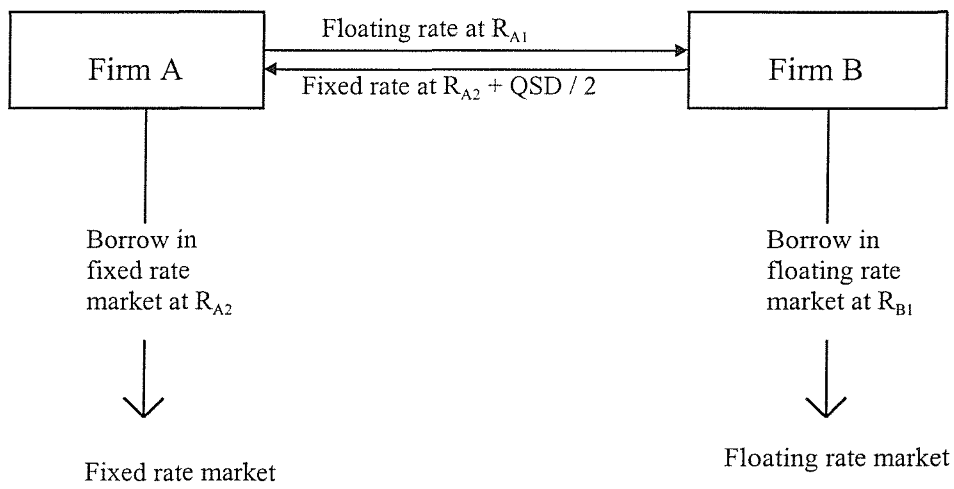
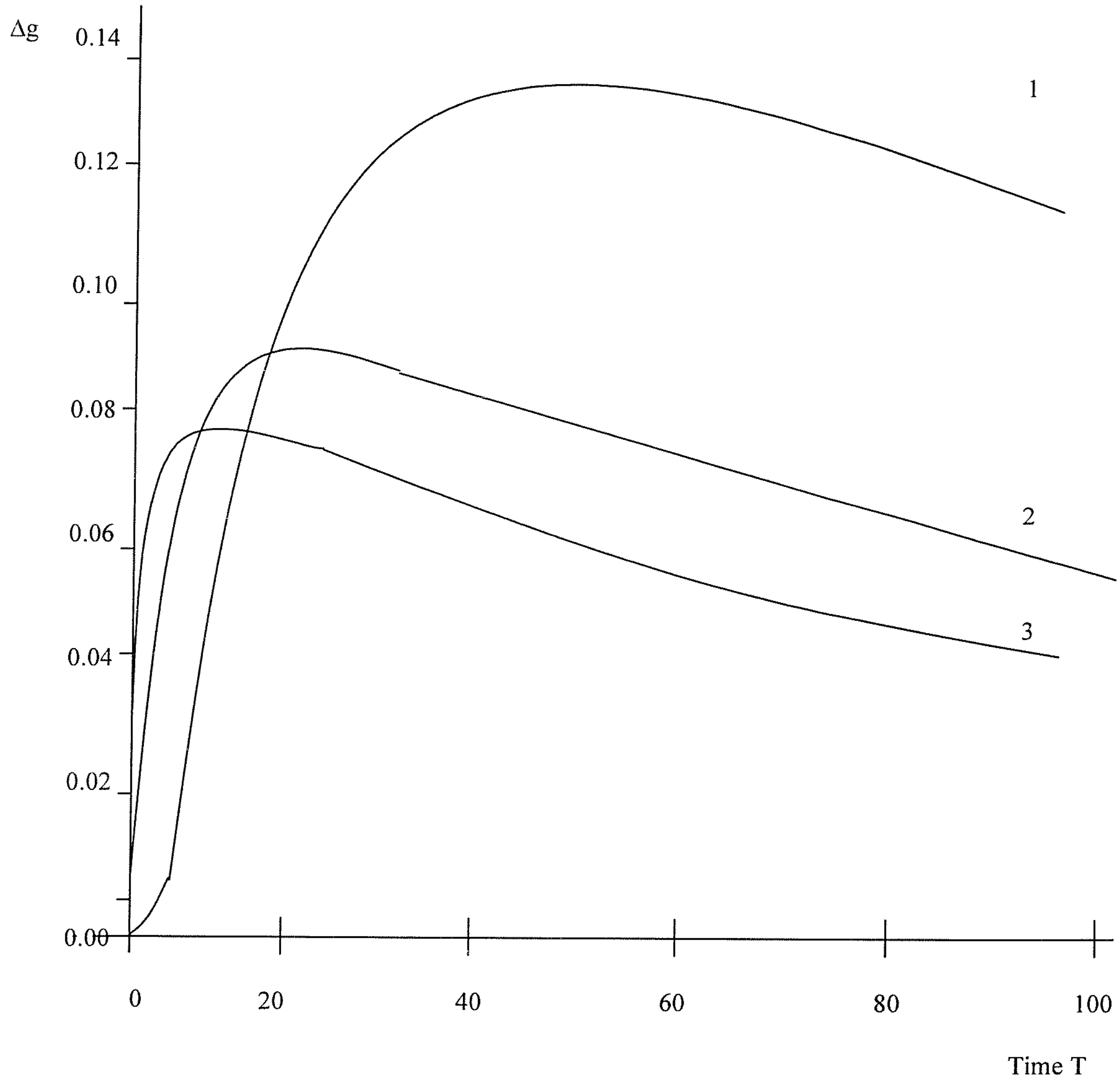
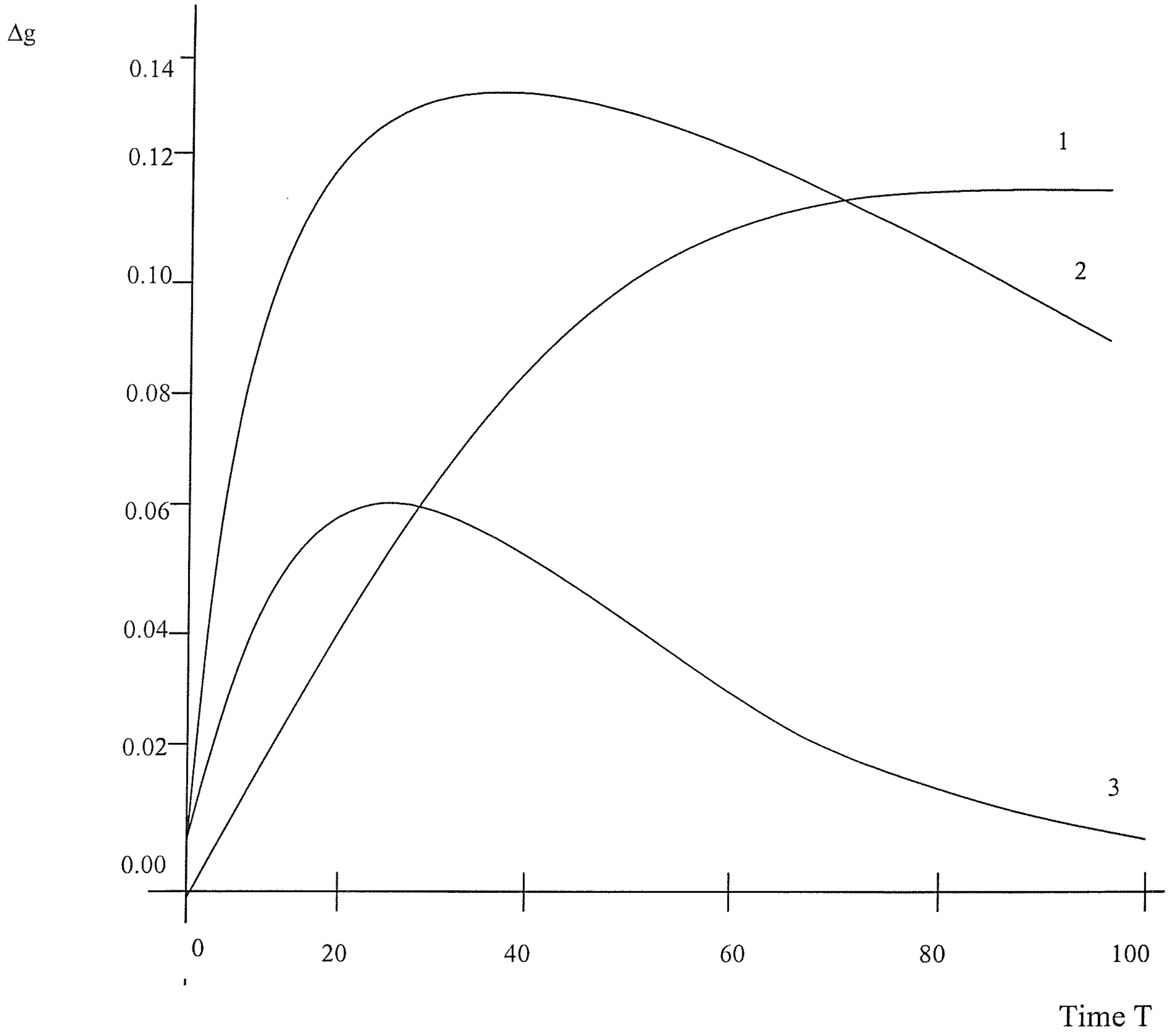


Figure 2.3 Quality spread differential Δg against time T between two firms of identical σ^2, $d_A < d_B$



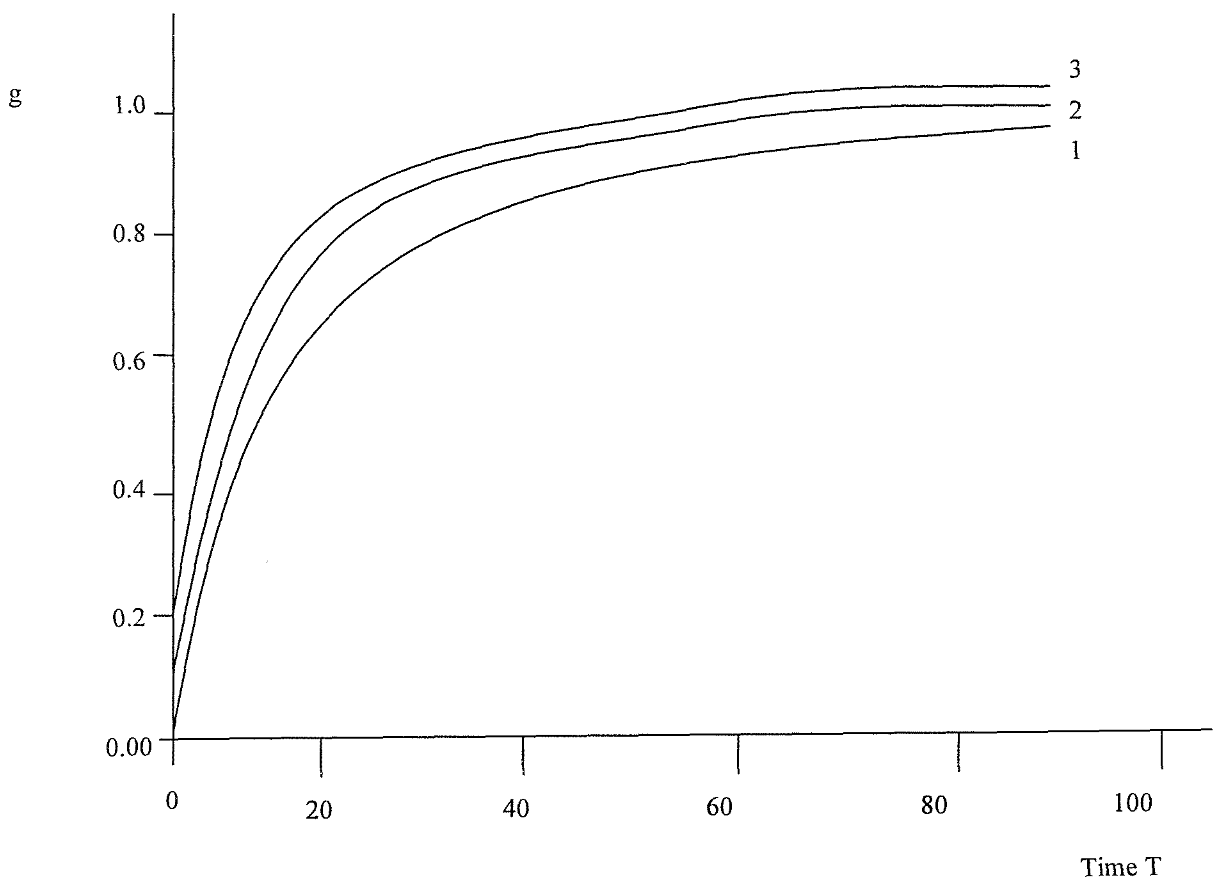
	σ^2	d_A	d_B
1	0.05	0.2	0.4
2	0.05	0.5	0.7
3	0.05	0.7	0.9

Figure 2.4 **Quality spread differential Δg against time T between two firms of identical d, $\sigma_A^2 < \sigma_B^2$**



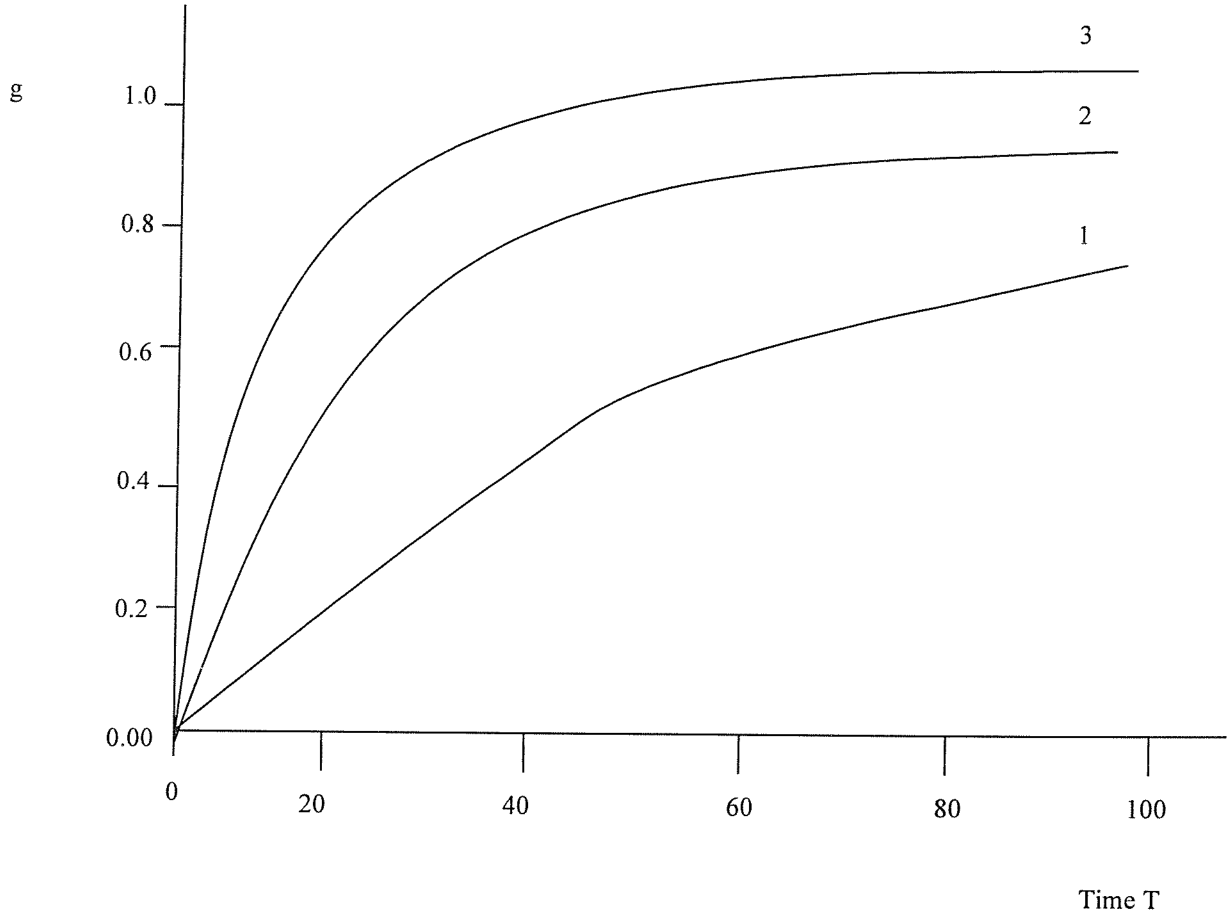
	d	σ_A^2	σ_B^2
1	0.5	0.05	0.07
2	0.5	0.10	0.15
3	0.5	0.25	0.30

Figure 2.5 Guarantee costs g against time T of firms of same σ^2 but different d



	σ^2	d
1	0.25	0.4
2	0.25	0.7
3	0.25	0.9

Figure 2.6 **Guarantee costs g against time T of firms of same d but different σ^2**



	<u>d</u>	<u>σ^2</u>
1	0.5	0.07
2	0.5	0.15
3	0.5	0.30

Appendix 2A

This appendix was the collaboration with two colleagues in the Department of Applied Mathematics of the Hong Kong Polytechnic University. A modified version of this paper that includes this appendix was under the review process for publication at the time of submitting this dissertation.

If we let $H = R - r$, then for firm A, the default risk premium function H_A is

$$H_A = R_A - r = -\frac{1}{T}\ln\left[\frac{1}{d_A}\phi(-x_{1A}) + \phi(x_{2A})\right], \tag{A.1}$$

and for firm B, H_B is

$$H_B = R_B - r = -\frac{1}{T}\ln\left[\frac{1}{d_B}\phi(-x_{1B}) + \phi(x_{2B})\right]. \tag{A.2}$$

Then the difference of the default risk premium function between two firms ΔH is given by

$$\begin{aligned} \Delta H &= H_B - H_A \\ &= \frac{1}{T}\left\{\ln\left[\frac{1}{d_A}\phi(-x_{1A}) + \phi(x_{2A})\right] - \ln\left[\frac{1}{d_B}\phi(-x_{1B}) + \phi(x_{2B})\right]\right\}, \end{aligned} \tag{A.3}$$

and

$$\frac{\partial(\Delta H)}{\partial T} = \frac{\partial H_B}{\partial T} - \frac{\partial H_A}{\partial T}. \tag{A.4}$$

Hence, $\frac{\partial(\Delta H)}{\partial T}$ can be worked out explicitly as in equation (A.5) below,

$$\begin{aligned}\frac{\partial(\Delta H)}{\partial T} &= \frac{1}{T^2}\ln\left(\frac{P_B}{P_A}\right)+\frac{1}{T}\left[\frac{1}{P_A}\left(\frac{r}{d_A}\phi(-x_{1A})-\frac{\sigma_A}{2\sqrt{2\pi}\sqrt{T}}e^{-\frac{x_{1A}^2}{2}}\right)\right]\\ &-\frac{1}{T}\left[\frac{1}{P_B}\left(\frac{r}{d_B}\phi(-x_{1B})-\frac{\sigma_B}{2\sqrt{2\pi}\sqrt{T}}e^{-\frac{x_{1B}^2}{2}}\right)\right] \qquad (A.5)\end{aligned}$$

where $\phi(x) = \phi'(-x) = \frac{1}{\sqrt{2\pi}}e^{\frac{-x^2}{2}}$ is the standard normal density function. It is difficult to show that $\frac{\partial(\Delta H)}{\partial T} > 0$ because $\frac{\partial(\Delta H)}{\partial T}$ depends on several variables. The exact relations are very complicated mathematically. However we may examine $\frac{\partial(\Delta H)}{\partial T}$ in some special cases. From equation (A.1), we can write by dropping suffice A

$$\frac{\partial H}{\partial T} = \frac{1}{T^2}\ln P - \frac{1}{TP}\left[\frac{r}{d}\phi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}\sqrt{T}}e^{-\frac{x_2^2}{2}}\right], \qquad (A.6)$$

where

$$\begin{aligned}P &= \frac{D_0}{B\exp(-rT)}\\ &= \frac{D_0\exp(rT)}{B}\\ &= \phi(x_2)+\frac{1}{d}\phi(-x_1).\end{aligned}$$

Equation (A.6) is either one of the component of equation (A.3) by dropping the suffix A or B.

Let $S = \frac{V}{B}$ and $f(S,T) = Se^{r\tau}\phi(-x_1) + \phi(x_2)$. Then $H(S,T) = -\frac{1}{\tau}\ln f(S,T)$ and

$$x_1 = \frac{S + \left(r+\frac{\sigma^2}{2}\right)T}{\sigma\sqrt{\tau}} \quad \text{and} \quad x_2 = \frac{S + \left(r-\frac{\sigma^2}{2}\right)\tau}{\sigma\sqrt{\tau}}. \qquad (A.7)$$

The first partial derivative of $H(S,T)$ with respect to T is given by

$$\frac{\partial H(S,T)}{\partial T} = \frac{1}{T^2}\ln f(S,T) - \frac{1}{T}\frac{1}{f(S,T)}\frac{\partial}{\partial T}f(S,T), \tag{A.8}$$

that is

$$T^2 f(S,T)\frac{\partial H(S,T)}{\partial T} = f(S,T)\ln f(S,T) - T\frac{\partial}{\partial T}f(S,T). \tag{A.9}$$

In equation (A.9), the sign of $\frac{\partial H(S,T)}{\partial T}$ is the same as $f(S,T)\ln f(S,T) - T\frac{\partial}{\partial T}f(S,T)$, if $f(S,T) > 0$. So in order to discuss the sign of $\frac{\partial H(S,T)}{\partial T}$, we must consider the ratio $S = \frac{V}{B}$. S is the ratio of the firm's initial asset value V_0 and the face value B of the debt.

Since

$$rT - \frac{x_1^2}{2} = -\frac{x_2^2}{2} - \ln S, \tag{A.10}$$

we have

$$\begin{aligned}
\frac{\partial f(S,T)}{\partial T} &= rse^{rT}\phi(-x_1) + \frac{1}{2\sqrt{2\pi}\sigma T^{3/2}} \\
&\quad \left[e^{-\frac{x_2^2}{2}}\left(\left(r-\frac{\sigma^2}{2}\right)T - \ln S\right) - e^{rT-\frac{x_1^2}{2}}S\left(\left(r+\frac{\sigma^2}{2}\right)T - \ln S\right)\right] \\
&= rse^{rT}\phi(-x_1) + \frac{1}{2\sqrt{2\pi}\sigma T^{3/2}}e^{-\frac{x_2^2}{2}} \\
&\quad \left[\left(r-\frac{\sigma^2}{2}\right)T - \ln S - e^{\ln S}\left(S\left(r+\frac{\sigma^2}{2}\right)T - \ln S\right)\right] \\
&= rse^{rT}\phi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}\sqrt{T}}e^{-\frac{x_2^2}{2}}.
\end{aligned} \tag{A.11}$$

In particular, if $r = \frac{\sigma^2}{2}$, we have

$$\frac{\partial f(S,T)}{\partial T} = rse^{rT}\varphi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}\sqrt{T}}\exp\left\{-\frac{(\ln S)^2}{2\sigma^2 T}\right\}. \tag{A.12}$$

From the definition of $f(S,T)$, we have the followings:

(a) for $S \geq 1$, $\lim_{T\to 0} f(S,T) = 1$,

(b) for $S < 1$, $\lim_{T\to 0} f(S,T) = S$,

(c) Since

$$\begin{aligned}
\lim_{T\to\infty} e^{rT}\phi(-x_1) &= \lim_{T\to\infty} \frac{\phi(-x_1)}{e^{-rT}} \\
&= \lim_{T\to\infty} \frac{\left(r+\frac{\sigma^2}{2}\right)T - \ln S}{2\sqrt{2\pi}r\sigma T^{3/2}} \exp\left\{rT - \frac{x_2^2}{2}\right\} \\
&= S\lim_{T\to\infty} \frac{\left(r+\frac{\sigma^2}{2}\right)T - \ln S}{2\sqrt{2\pi}r\sigma T^{3/2}} e^{-\frac{x_2^2}{2}} = 0, \qquad \text{(A.13)}
\end{aligned}$$

we deduce, for any $S > 0$,

$$\begin{aligned}
\lim_{T\to\infty} f(S,T) &= \lim_{T\to\infty} \phi(x_2) \\
&= \lim_{T\to\infty} \phi\left(\frac{\ln S + (r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right) = \begin{cases} 0, & r < \frac{\sigma^2}{2}, \\ \frac{1}{2}, & r = \frac{\sigma^2}{2}, \\ 1 & r > \frac{\sigma^2}{2}. \end{cases} \qquad \text{(A.14)}
\end{aligned}$$

We can also obtain that, for any $T > 0$,

$$\lim_{S\to\infty} f(S,T) = \lim_{S\to\infty} \phi\left(\frac{\ln S + (r-\frac{\sigma^2}{2})T}{\sigma\sqrt{T}}\right) = 1. \qquad \text{(A.15)}$$

Further we discuss the monotonic behaviour of $f(S,T)$ with respect to T for any fixed $S > 0$. For $T > \ln \frac{S}{(r+\frac{\sigma^2}{2})}$,

$$\begin{aligned}
\frac{d}{dT}(e^{rT}\phi(-x_1)) &= re^{rT}\phi(-x_1) - e^{rT}\frac{(r+\frac{\sigma^2}{2})T - \ln S}{2\sqrt{2\pi}\sigma T^{3/2}} e^{-\frac{x_1^2}{2}} \\
&= e^{rT}\left[r\phi(-x_1) - \frac{(r+\frac{\sigma^2}{2})T - \ln S}{2\sqrt{2\pi}\sigma T^{3/2}} e^{-\frac{x_1^2}{2}}\right] \\
&\leq \frac{1}{\sqrt{2\pi}} e^{rT - \frac{x_1^2}{2}} \left[\frac{r}{x_1} - \frac{(r+\frac{\sigma^2}{2})T - \ln S}{2\sigma T^{3/2}}\right] \\
&= \frac{S}{\sqrt{2\pi}} e^{-\frac{x_2^2}{2}} \frac{-(r+\frac{\sigma^2}{2})T^2 + (\ln S)^2}{2\sigma T^{3/2}[(r+\frac{\sigma^2}{2})T + \ln S]},
\end{aligned}$$

which implies that, for $T > \ln \frac{S}{(r+\frac{\sigma^2}{2})}$, then $\frac{d}{dT}(e^{rT}\phi(-x_1)) > 0$. This means that $e^{rT}\phi(-x_1)$ is a strictly decreasing function with respect to T for $T > \ln \frac{S}{(r+\frac{\sigma^2}{2})}$ and for $S > 0$.

Since $\lim_{T\to\infty} e^{rT}\phi(-x_1) = 0$, we have $\lim_{T\to\infty} T\phi(-x_1) = 0$, and by equation (A.13),

$$\begin{aligned}
\lim_{T\to\infty} Te^{rT}\phi(-x_1) &= \lim_{T\to\infty} \frac{T\phi(-x_1)}{e^{-rT}} \\
&= -\lim_{T\to\infty} \frac{\phi(-x_1)}{re^{-rT}} + \lim_{T\to\infty} \frac{(r+\frac{\sigma^2}{2})T - \ln\frac{v}{F}}{2\sqrt{2\pi}r\sigma^{3/2}} \\
&\quad \exp\left\{\frac{(r-\frac{\sigma^2}{2})^2T^2 + 2(r+\frac{\sigma^2}{2})T\ln S + (\ln S)^2}{2\sigma^2 T}\right\} \\
&= 0 \qquad \text{(A.16)}
\end{aligned}$$

and

$$\lim_{T\to\infty} \sqrt{T}e^{-\frac{x_2^2}{2}} = \lim_{T\to\infty} \sqrt{T}\phi\left(\frac{\ln S - \left(r - \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}}\right) = \begin{cases} 0, & r \neq \frac{\sigma^2}{2}, \\ \infty, & r = \frac{\sigma^2}{2}. \end{cases} \qquad \text{(A.17)}$$

Equations (A.16) and (A.17) imply that, for any fixed $S > 0$,

$$\begin{aligned}
\lim_{T\to\infty} T\frac{\partial f(S,T)}{\partial T} &= rs\lim_{T\to\infty} Te^{rT}\phi(-x_1) - \frac{\sigma}{2\sqrt{2\pi}}\lim_{T\to\infty}\sqrt{T}e^{-\frac{x_2^2}{2}} \\
&= \begin{cases} 0, & r \neq \frac{\sigma^2}{2}, \\ -\infty, & r = \frac{\sigma^2}{2}. \end{cases} \qquad \text{(A.18)}
\end{aligned}$$

By equations (A.14) and (A.18), we deduce that, for any fixed $\delta > 0$,

$$\lim_{T\to\infty}\left[f(S,T)\ln f(S,T) - T\frac{\partial f(S,T)}{\partial T}\right] = \begin{cases} 0, & r \neq \frac{\sigma^2}{2}, \\ \infty, & r = \frac{\sigma^2}{2}. \end{cases} \qquad \text{(A.19)}$$

Also equations (A.11), (A.12) and (A.14) yield that for any fixed $T > 0$,

$$\lim_{S\to\infty}\left[f(S,T)\ln f(S,T) - T\frac{\partial f(S,T)}{\partial T}\right] = 0. \qquad \text{(A.20)}$$

Let $F(S,T) = f(S,T)\ln f(S,T) - T\frac{\partial f(S,T)}{\partial T}$. It is easy to prove that

$$\lim_{\sigma\to\infty} F(S,T) = 0. \qquad \text{(A.21)}$$

To show that whether $\frac{\partial(\Delta H)}{\partial T} > 0$, it suffices show that $\frac{\partial H_A(S,T)}{\partial T}$ and $\frac{\partial H_B(S,T)}{\partial T}$ are of different signs within a certain time period. However, it is difficult to determine under what conditions that $\frac{\partial H(S,T)}{\partial T} > 0$ or < 0 explicitly. Therefore we must resort to another method of measuring default risk premium in order to obtain results for swapping of interest rates so that both firms are of mutual benefit.

Appendix 2B

$$\frac{\partial[\Delta g]}{\partial T} = \frac{\sigma_B}{2d_B\sqrt{T}}\phi\,'(X_1)_B - \frac{\sigma_A}{2d_A\sqrt{T}}\phi\,'(X_1)_A$$

a) For identical σ, $d_A < d_B$ and $d \leq 1$

By factorising,

$$\frac{\partial[\Delta g]}{\partial T} = \frac{\sigma}{2d_A\sqrt{T}}\phi\,'(X_{1A})\left[\frac{d_A}{d_B}\frac{\phi\,'(X_{1B})}{\phi\,'(X_{1A})} - 1\right] \qquad (B1)$$

ϕ' (X_1) denotes the first derivative of the cumulative normal distribution ϕ' (X_1) with respect to X_1, where by definition,

$$\phi\,'(x_1) = \frac{1}{\sqrt{2\pi}}e^{\frac{-X_1^2}{2}}$$

Therefore,

$$\frac{\phi\,'(X_{1B})}{\phi\,'(X_{1A})} = e^{\frac{X_{1A}^2 - X_{1B}^2}{2}}$$

where

$$X_1 \equiv \frac{\ln(d) - \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}}$$

Since the terms σ, d, T and $\phi'(X_{1A})$ are always positive, the sign of $\partial[\Delta g] / \partial T$ is determined by the terms within the bracket in equation A1. That is,

$$\text{if} \quad \frac{d_A}{d_B}\frac{\phi'(X_{1B})}{\phi'(X_{1A})} > 1, \quad \text{then} \quad \frac{\partial[\Delta g]}{\partial T} > 0 \quad \text{or vice versa.}$$

$$\text{If we let } \pi = \frac{d_A}{d_B}\frac{\phi'(X_{1B})}{\phi'(X_{1A})}, \quad \text{then if } \pi > 1 \quad \text{or} \quad \ln(\pi) > 0, \text{ then } \frac{\partial[\Delta g]}{\partial T} > 0.$$

By algebra,

$$\ln(\pi) = \frac{1}{2\sigma^2 T}\{[\ln(d_A) + \ln(d_B) + \sigma^2 T][\ln(d_A) - \ln(d_B)]\}$$

For $d_A < d_B$ and $d \leq 1$, then $\ln(\pi) > 0$ for the values of T:

$$-\frac{\ln(d_A) + \ln(d_B)}{\sigma^2} > T$$

b) For identical d, $d \leq 1$ and $\sigma_A < \sigma_B$

By factorising,

$$\frac{\partial[\Delta g]}{\partial T} = \frac{\sigma_A}{2d\sqrt{T}}\phi'(X_{1A})\left[\frac{\sigma_B}{\sigma_A}\frac{\phi'(X_{1B})}{\phi'(X_{1A})} - 1\right] \qquad (B2)$$

Since the terms σ_A, d, T and $\phi'(X1A)$ are always positive, the sign of $\partial[\Delta g] / \partial T$ is determined by the terms within the bracket in equation A2. That is,

$$\textit{if} \quad \frac{\sigma_B}{\sigma_A}\frac{\phi'(X_{1B})}{\phi'(X_{1A})} > 1, \quad \textit{then}\ \frac{\partial[\Delta g]}{\partial T} > 0 \quad \textit{or vice versa.}$$

$$\textit{If we let}\ \pi = \frac{\sigma_B}{\sigma_A}\frac{\phi'(X_{1B})}{\phi'(X_{1A})}, \quad \textit{then if}\ \pi > 1\ \textit{or}\ \ln(\pi) > 0,\ \textit{then}\ \frac{\partial[\Delta g]}{\partial T} > 0.$$

By algebra,

$$\ln(\pi) = \ln\left(\frac{\sigma_B}{\sigma_A}\right) + \frac{\left(\sigma_B^2 - \sigma_A^2\right)\left(\ln(d) + \frac{\sigma_A\sigma_B T}{2}\right)\left(\ln(d) - \frac{\sigma_A\sigma_B T}{2}\right)}{2\sigma_A^2\sigma_B^2 T}$$

For $d \leq 1$ and $\sigma_A < \sigma_B$, then $\ln(\pi) > 0$ for the values of T:

$$\left|\ln\left(\frac{\sigma_B}{\sigma_A}\right)\right| > \left|\frac{\left(\sigma_B^2 - \sigma_A^2\right)\left(\ln(d) + \frac{\sigma_A\sigma_B T}{2}\right)\left(\ln(d) - \frac{\sigma_A\sigma_B T}{2}\right)}{2\sigma_A^2\sigma_B^2 T}\right|$$

Appendix 2C TSP program for QSD analysis

```
OPTIONS CRT;
FREQ N;
SMPL 1 1;
?
set dax(1) = 0.2;
set dax(2) = 0.5;
set dax(3) = 0.7;
set dbx(1) = 0.4;
set dbx(2) = 0.7;
set dbx(3) = 0.9;
?
do j = 1 to 3;
set da = dax(j);
set db = dbx(j);
?
set sigma2a = 0.05;
set sigma2b = 0.05;
do i = 1 to 100;
set t = i;
?
set x1a = (log(da) - sigma2a*t/2)/sqrt(sigma2a*t);
set x2a = x1a + sqrt(sigma2a*t);
?
set x1b = (log(db) - sigma2b*t/2)/sqrt(sigma2b*t);
set x2b = x1b + sqrt(sigma2b*t);
?
set ga = cnorm(x2a) - (1/da)*cnorm(x1a);
set gb = cnorm(x2b) - (1/db)*cnorm(x1b);
?
set deltag = gb - ga;
?
set nrow = 100;
set ij = (j -1)*nrow + i;
a(ij) = deltag;
enddo;
enddo;
?
do i = 1 to 100;
Period(i) = i -1;
?
set j = 1;
set ij = (j-1)*nrow + i;
set Graph1(i) = a(ij);
```

```
?
set j = 2;
set ij = (j-1)*nrow + i;
set Graph2(i) = a(ij);
?
set j = 3;
set ij = (j-1)*nrow + i;
set Graph3(i) = a(ij);
?
enddo;
smpl 1 100;
?
?
graph(device=epson, ymin = 0, xmin = 0, line, preview,
title = 'Figure 3')Period, Graph1, Graph2, Graph3;
?
?
stop;
end;
```

Chapter 3

Valuation of Default Risky Fixed and Floating Rate Debt Obligations: Implications for Interest Rate Swaps

Abstract

One of the arguments most frequently advocated by market participants for the development of interest rate swaps is the reduction in the borrowing costs obtained by exploiting the comparative advantage each firm has in different credit markets. It is argued that some firms have a comparative advantage in borrowing fixed rate while others have a comparative advantage in borrowing floating rate. The comparative advantage is reflected by the 'quality spread differential', which is the difference in the borrowing costs between two firms in different credit markets. Both firms can lower their borrowing costs by sharing the quality spread differential through a fixed-for-floating interest rate swap.

However, this comparative advantage argument has not received unanimous support by academics. Many researchers claim that the source of comparative advantage comes primarily from market inefficiencies. As such, they claim that the cost reduction resulting from creating a synthetic liability with an interest rate swap cannot last long, as the market becomes more efficient. This claim notwithstanding, the interest rate swap market is still growing fast. Nowadays, many firms claim that they can reduce borrowing costs through interest rate swaps.

Employing the Black-Scholes-Merton option pricing models in the valuation of a firm's debt, it is found that quality spread differentials exist. However, the Black-Scholes-Merton models suffer from two major shortcomings. First, they consider discount bonds only and second, they assume fixed risk free interest rate. Neither of the assumptions is prevalent in practice. This chapter relaxes the assumptions of Black-Scholes-Merton models by analysing the coupon paying debts in a variable default-free interest rate environment. I develop a simulation model based on contingent claims theory to evaluate the coupon interest that should be paid by the fixed and floating rate default risky corporate debts. My model shows that there exist quality spread differentials in the fixed and floating rate debt markets. The source of the quality spread differentials comes mainly from the firms' balance sheet structure and the nature of the business. My results show that interest rate swaps can benefit both participating firms for reasons other than market inefficiencies.

3.1 Introduction

One of the arguments most frequently advocated by market participants for the development of the interest rate swap market is the reduction in the borrowing costs obtained by exploiting the comparative advantage that each firm has in different credit markets. It is argued that some firms have a comparative advantage in borrowing long-term, fixed rate while others have a comparative advantage in borrowing short-term, floating rate. Two firms can lower their borrowing costs by engaging in a fixed / floating interest rate swap.

Bicksler and Chen (1986) and subsequent researchers suggest that a quality spread differential exist between long-term fixed rate and short-term floating rate debt markets. The short-term floating rate debt is obtained by renewing the short-term borrowings. Recently, Ungar (1996) shows that a quality spread differential can also exist between a fixed rate coupon bond and a floating rate note.[1] Therefore, the study of how interest rate swaps help to reduce firms' borrowing costs should also be extended to the fixed rate and floating rate coupon paying debt markets. The difference in the borrowing costs between two default risky firms may be reflected in the difference in the coupon payments of the debts issued by the firms. Such difference should be mainly derived from the difference in credit risk between the firms.

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[1] See Ungar (1996) for illustration of quality spread differential between fixed rate coupon bond and floating rate note.

The purpose of this chapter is to analyse the credit spreads of default risky fixed rate coupon paying bonds and floating rate notes by using the contingent claims theory. Moreover, I allow the default free interest rate to be variable in the pricing models of the fixed rate and floating rate coupon paying debts. This is consistent with the fact that interest rate swap activities are also stimulated by an environment characterised by an increasing volatility of interest rates. The model derived from the contingent claims theory is based on efficient market. I find that there exist differences in borrowing costs in fixed and floating rate credit markets, which can be explained by the firm's balance sheet structure and the nature of the business. My results show that interest rate swaps can benefit the participating firms for reasons other than market inefficiencies. This chapter reinforces the results of chapter 2 by incorporating the coupon payment and variable default free rate factors. The structure of this chapter is as follows. Section 2 discusses the problems in the valuation of default risky coupon paying debt securities. Section 3 presents my valuation framework and derives a contingent claims equation for the value of default risky coupon paying debt. Section 4 discusses the simulation model and procedures. Section 5 presents the simulation results and concludes the chapter.

3.2 Valuation of debt securities

The traditional method of valuing debt securities is done by discounting future expected cash flows as specified under the debt indenture at the risk-adjusted rate of return. The risk-adjusted rate of return will incorporate all the risks related to the investment on the debt security. One approach to determine the appropriate discount rate for an individual debt security is to use the Capital Asset Pricing Model (CAPM) to price the risks with the market portfolio. However, the problem of using CAPM is that we inevitably have to use the parameters of expected return and risk preference of investors, which are either unobservable or difficult to evaluate.[2] Black and Scholes (1973), in their development of the option-pricing model, also recognise that the insights of option pricing theory can be applied to the pricing of corporate liabilities. This is mainly because of the limited liability provision for the shareholders allowed by most corporations nowadays. On debt maturity date or any interest payment date, it can be shown that the corporate equity can be treated as a call option on the assets of the corporation with exercise price of the debt payment needed to be satisfied. Based on the put-call parity, the risk premium of a default risky corporate debt can be treated as a put option with the same parameters as the equity call option that the debtholders have sold to the equityholders. Using the option pricing approach, Merton (1974) carries out the analysis of the default spread between corporate and US Treasury discount securities. His results show that the default spread is a function of a) the level of leverage; b) the volatility of the corporate assets; and, c) the time to maturity of the corporate debt. Merton's (1974) analysis acts as a catalyst for research on corporate liabilities with the application of option pricing or contingent claims theory. However, there are two major problems with Merton's (1974) model. First, Merton (1974) analyses the default spread on pure discount securities but in practice

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[2] It is not saying that CAPM cannot be used to value the firm's securities. However, in practice, we still have to use historical values of variables such as β and share prices.

most corporate securities promise coupon payment. In fact, zero coupon securities are relatively rare. Therefore, it is difficult to apply Merton's (1974) model for the analysis of most corporate debts. The difficulty of valuing corporate coupon paying debt is that it will involve a series of options on the assets of the corporation on each debt payment date, each option being dependent on the outcome of the previous option. Second, Merton (1974) assumes the risk free rate is fixed. However, the values of both the default free Treasury and default risky corporate debt are known to be significantly influenced by interest rate risk. Subsequent research has attempted to improve the performance of Merton's (1974) model by dealing with the above two issues. However, the resulting models provide either extremely complicated mathematical solutions or only expressions without closed form solutions.

For example, Geske (1977) first suggests that a series of coupon payments can be treated as compound options in deriving the valuation equation for the coupon-paying bond. However, his results involve a multivariate normal distribution function that is both intimidating and mathematically complex as a consequence of his assumption on the financing of coupon payments. Before discussing the problems with Geske's (1977) model, I would like to point out the important factors affecting the models resulting from using contingent claims technique which attempt to value coupon paying debt securities.

The most important factors affecting the resulting contingent claims models to value default risky coupon paying debt securities are how the event of default or insolvency and the recovery value of the debts in the event of default are defined. It is generally accepted that, in the case of debt securities, default occurs when the borrower is unable to make the contractual payments due on the security at any time during the life of the contract. The assumption of how the contractual payments are financed is important because it affects the balance sheet structure of the borrowing firm and, thus, the derived valuation model. Geske (1977) assumes that the firms finances each coupon payment through a rights issue and states that the firm will find no takers for the stock whenever the value of equity, after the coupon payment, is less than the value of the firm's debts. It is assumed that in this situation the firm is insolvent and default occurs. Based on this assumption, Geske (1977) derives a valuation equation that involves a multivariate distribution function, which is difficult to apply to the study of the risk premium of default risky coupon paying debt securities. The complexity mainly arises from the assumed solvency condition that the value of the firm's assets should exceed the coupon payment as well as the market value of the debt which itself is a contingent claim on the firm's assets.

Recently, Longstaff and Schwartz (1995) derive simple closed-form expressions for both risky fixed-rate and floating-rate debt based on a continuous-time option valuation framework. Their model incorporates both default risk and interest rate risk. However, their model explicitly allows for deviations from strict absolute priority and firm insolvency may occur before the contractual coupon payment dates whenever the firm's assets values fall below a pre-defined threshold value. As a result, their model is also mathematically complicated. This chapter differs from the papers discussed above by adhering to the more common rule that insolvency can occur only when a firm cannot satisfy the debt service payment due and assuming that the firm finances the debt service payments by selling off its assets. I discuss my valuation framework in the next section.

3.3 The valuation framework

I adopt the following notation in developing my valuation framework:

t = number of time periods from 1 to T, with T as the maturity date of the debt;

V_t = the value of the firm's assets at period t;

BF = the face value of the debt;

B_{0T} = the present value of default risk free debt at t = 0 with maturity T;

D_{0T} = the present value of default risky debt at t= 0 with maturity T;

D_t = the payoff of the debt at period t;

XF = the fixed coupons paid at the end of each period t;

XX_t= the variable coupons paid at the end of each period t;

E_t = the payoff of equity at period t;

E_{0T} = the present value of equity at t = 0 with the debt's maturity T;

r_v = the instantaneous expected return of the firm's assets value;

σ_v^2= the instantaneous variance of the return of the firm's assets value;

r = the instantaneous expected return of the default risk free bond;

σ_B^2= the instantaneous variance of the return of the default risk free bond;

RF_t= the default risk free rate at period t.

Z = the standard Wiener process.

In addition to the assumptions conventionally made in contingent claims literature,[3] I make specific assumptions on the evolution of default free interest rate and the firm's financial structure and payoff conditions of liabilities in developing my valuation model.

<u>Evolution of default free interest rate</u>

A1. I allow the default free interest rate to be uncertain and the short-term rate is stochastic. This assumption specifically distinguishes my analysis from previous models which assume a fixed risk free interest rate. For simplicity, I assume that the default free rate varies over the life of the corporate debt in such a way that the return of a default free discount bond can be expressed as an Ito's process:[4]

$$dB / B(T) = r\,dt + \sigma_B(T)\,dz_B(t, T) \tag{1}$$

$\sigma_B(T)$ is assumed to be constant.

The result of this assumption is that the investment in both default free bonds and default risky bonds is subject to the interest rate risk, i.e. the change of market interest rate. This assumption is more appropriate given the actual market environment and enables the model to depict more explicitly the effect of default risk.

---

[3] 1. There are no transaction costs.
2. Modigliani and Miller's proposition 1 that the value of firm is invariant to its capital structure applies.
3. There are no penalties for short selling.
4. Investors can borrow and lend at the same rate of interest.
5. Trading in assets takes place continuously in time.

[4] See Merton (1973)

Firm's financial structure and payoff conditions of firm's liabilities

A2. The firm has only two classes of claims: a) a fixed rate coupon-paying bond or a floating rate note; and b) the residual claim, equity.

A3. a) The indenture of the bond issue contains the following provisions and restrictions: i) in the case of fixed rate coupon paying bonds, the firm promises to pay a fixed coupon, XF, at the end of each period t and the coupon and face value BF at the bond's maturity date, T; and ii) in the case of floating rate notes, the coupon payment at each period t will be

$$XX_t = BF\ (RF_{t-1} + MK) \qquad (2)$$

where RF_{t-1} is the default free rate set one period before the coupon payment date;

MK is the credit risk premium a default risky FRN must pay over a default free FRN and is assumed to be fixed. It is a common market practice of setting periodic interest payments for floating rate notes.

b) in the event of any payment not being met, the bondholders immediately take over the firm and the shareholders receive nothing. However, the firm is limited liability and the shareholders need not compensate for the deficiency of asset value and debt;

c) the firm cannot issue any new senior (or of equivalent rank) claims on the firm nor can it pay cash dividends or do share repurchase throughout the life of the debt;

d) the firm finances the debt payments by selling its assets.

A4. The value of the firm's assets, V, follows the Ito's process:

$$dV \,/\, V(T) \;=\; r_v\,dt \;+\; \sigma_v\,dz_v \tag{3}$$

For simplicity, I assume that there is no correlation between the firm's assets value and the default free rate, i.e. dz_v and dz_B are independent.

Given these assumptions and payoff conditions, the firm's liabilities can be depicted as contingent claims on the firm's assets. For a firm with a fixed coupon bond outstanding, the payoffs of the firm's liabilities will be as follows:

For $t = 1$ to $T - 1$;

If $V_t > XF$ $\qquad D_t = XF$ (4)

$\qquad E_t = V_t - XF$ (5)

the firm is solvent and will go on to another period $t + 1$.

If $V_t < XF$ $\qquad D_t = V_t$ (6)

$\qquad E_t = 0$ (7)

the firm is insolvent, dissolved and taken over by the debtholders.

On the debt maturity date, the debt service payment due will be $XF + BF$. The payoffs of the firm's liabilities will be:

For $t = T$;

If $V_T > XF + BF$ $\qquad D_T = XF + BF$ (8)

$\qquad E_T = V_T - (XF + BF)$ (9)

If $V_T < XF + BF$ $\qquad D_T = V_T$ (10)

$\qquad E_T = 0$ (11)

I start my valuation analysis of fixed rate coupon bonds with a two-period model. The two-period model is used to describe in a simple way the essential theoretical arguments. I show later that my analysis can be extended to a T-period model by a process of induction and the values of fixed coupon rates can be obtained through a simulation model. Given the payoff conditions of debt stated above, the value of debt and equity in the two periods can be written as:

<u>Period 1, t = 1</u>

$$D_1 = \mathrm{MIN}\,(\,V_1,\,XF\,) \tag{12}$$

$$E_1 = \mathrm{MAX}\,(\,V_1 - XF\,,\;0\,) \tag{13}$$

<u>Period 2, t = 2</u>

It is important to note that the value of debt and equity in period 2 is based on the condition that the firm is solvent in period 1, i.e. $V_1 > XF$. If not, the firm will be wound up by the bondholders and cease to exist after period 1.

Conditional upon $V_1 > XF$,

$$D_2 = \mathrm{MIN}\,(\,V_2\,,\;XF + BF) \tag{14}$$

$$E_2 = \mathrm{MAX}\,(\,V_2 - (XF + B_F),\;0\,) \tag{15}$$

Equation (12) to (15) serve as the boundary conditions for the valuation of debt and equity. Equation (13) and (15) appear as two call options with the important difference that the payoff to the period 2 option is conditional upon the payoff to the period 1 option. This is derived from the payoff provisions of the coupon bond.

Based on my assumptions, the expression for the expected value of E_1 can be written in a format that can be generalised subsequently:

$$E(E_t) = V_0 \exp(r\,t) \cdot N(d_t) - D_t \cdot N(h_t) \qquad \text{for } t = 1 \qquad (16)$$

where

$$d_t = [\log(V_0 / D_t) + (r + \sigma_V^2 / 2)\,t\,] / \sigma_V \sqrt{t} \qquad \text{for } t = 1, 2 \qquad (17)$$

$$h_t = d_t - \sigma_V \sqrt{t} \qquad \text{for } t = 1, 2 \qquad (18)$$

The term $N(h_t)$ is of great importance for our presentation since it indicates the conditional probability that $V_t > D_t$. It applies to all the periods subsequent to period 1. In fact, the expected value of equities in periods subsequent to period 1 can also be written in the format of equation (16), but multiplied by the conditional probability $N(h_t)$. Hence, the expected value of equity at period 2 can be written as:

$$E(E_2) = N(h_1)\,[\,V_0 \exp(2\,r) \cdot N(d_2) - D_2 \cdot N(h_2)\,] \qquad (19)$$

Valuation equations can be obtained once we solve the equation for period 1. The equations for subsequent periods are obtained using the same procedure. For period 1, the present value of equity is the present value of the call option. Given my assumptions, the problem becomes the valuation of a call option on the firm's assets under the condition of a stochastic risk-free interest rate. Following the risk neutrality argument of Cox and Ross (1976), we can write the equation of the present value of equity for different periods as:

<u>1 period</u>

$$E_{01} = V_0 \cdot N(d_1) - XF \cdot \exp(-r) \cdot N(h_1) \qquad (20)$$

where

$$d_1 = [\log(V_0 / XF) + r + \sigma_v^2 / 2] / \sigma_v \tag{21}$$

$$h_1 = d_1 - \sigma_v \tag{22}$$

<u>For 2 periods</u>

$$E_{02} = N(h_1)[V_0 \cdot N(d_2) - (BF + XF)\exp(-2r) \cdot N(h_2)] \tag{23}$$

where

$$d_2 = [\log(V_0 / (BF + XF)) + 2r + \sigma_v^2] / \sigma_v \sqrt{2} \tag{24}$$

$$h_2 = d_2 - \sigma_v \sqrt{2} \tag{25}$$

By making use of the balance sheet identity, we can write the present value of the coupon bond, D_{02}, as:

$$\begin{aligned} D_{02} &= (V_0 - E_{01}) + N(h_1)(V_0 - E_{02}) \qquad (26) \\ &= V_0(1 - N(d_1)) - D_1\exp(-r)N(h_1) + N(h_1)[V_0(1 - N(d_2)) - D_2\exp(-2r)N(h_2)] \end{aligned}$$

In order to value a T-period coupon bond, let us define a variable h_0 such that $N(h_0)$ equals one. I create a set of variables defined as follows:

$$Z_t = \prod_{0}^{t-1} N(h_t) \tag{27}$$

Z_t indicates the conditional probability that the borrowing firm will meet its debt service payments at t, thus, survive into period t + 1. Through a process of induction beginning from equation (26) we can calculate the current value of the T-period coupon bond as follows:

$$D_{0T} = \sum_{t=1}^{T} Z_t (V_0(1 - N(d_t)) - D_t \exp(-r\,t) N(h_t)) \tag{28}$$

Equation (28) shows that the key variables affecting the present value of the coupon bond, D_{0T}, are the firm's leverage, the firm's assets volatility, the risk free rate, the face value of the bond and the time to maturity. Comparing two bonds issued by two different firms, the difference in credit spreads should be explained by the firm's leverage and assets volatility.

The valuation logic for floating rate note is the same as above with only XF replaced by XX_t on each interest payment date.

3.4 The simulation model

3.4.1 Fixed rate coupon bond

I can simulate the payoffs of debt on each interest payment date and the debt maturity date based on the value generating processes of the firm's assets and default free rate. I perform a 2-period and a 10-period simulation exercises, i.e. t = 1 to 2 and t = 1 to 10, respectively. Nonetheless, there is no limit in the number of periods used in the simulation exercise. If we choose the annualised figures for the parameters, it implies that the life of the bond is 2 and 10 years. The life of the bond is divided into T steps as $\{ 0 \equiv t < t + 1 < \ldots < T \equiv 10\}$. This means that $\Delta t = 1$. The value of the firm's assets can then be written in discrete form as:

$$\ln V_t = \ln (V_{t-1} - D_{t-1}) + (r_v - \sigma_v^2 / 2)\, \Delta t + \sigma_v \sqrt{\Delta t}\, z_v \tag{29}$$

where z_V is a standard normal variable. Under the risk-neutral probability principle, we can change the probability measure and write the relevant pricing distribution as:

$$\ln V_t = \ln(V_{t-1} - D_{t-1}) + (r - \sigma_V^2 / 2)\,\Delta t + \sigma_V \sqrt{\Delta t}\, z_V \tag{30}$$

That is, the expected return of the firm's assets, r_V, is replaced by the risk free rate, r, since in a risk neutral world, all returns should be the risk free rate. I allow the risk free rate to be stochastic and, therefore, I have to simulate the risk free rates for each time period. Based on my assumptions and the risk neutrality argument, the value of a default free bond in discrete form can be written as:

$$\ln B_t = \ln B_{t-1} + (r - \sigma_B^2 / 2)\,\Delta t + \sigma_V \sqrt{\Delta t}\, z_B \tag{31}$$

The stochastic default risk free rate, RF_t, for each period is given by

$$RF_{t-1} = \ln (B_t / B_{t-1}) \tag{32}$$

Using a random number generator for the values of z_B and z_V, and determining the payoff conditions of the firm's liabilities as above[5], I can now generate independent paths for the values of RF_t, V_t, D_t, and E_t for the time periods from 1 to T. By running the simulation 5000 times, the resulting values should approach a normal distribution. Then the sum of the discounted mean values of debt payoffs, D_t, will constitute the present value of the fixed rate coupon bond. Since the debt payoffs can be written as contingent claims on the firm's assets, we can apply the risk neutral pricing argument and use the risk free rate as the discount rate. The present value of the default risky bond should be given by

[5] See Section 3.3 of this paper.

$$D_{0T} = \sum_{t=1}^{T} \frac{E(D_t)}{\prod_{t=1}^{T}(1+RF_t)} \tag{33}$$

Instead of calculating D_{0T}, I set D_{0T} = BF and calculate XF so that D_{0T} = BF. The calculated XF of a default risky coupon bond is then compared with that of a default free coupon bond, which should represent the credit spread of a default risky coupon bond.

3.4.2 Floating rate note

The simulation exercise for the floating rate note is very similar to that of the fixed rate coupon bond except that the coupon payments of the floating rate note are not fixed but stochastic. According to my assumptions, the floating coupon payments at each period are given by

$$XX_t = BF\ (RF_{t-1} + MK) \tag{34}$$

I then run simulation processes as those described in 3.4.1 for the values of RF_t, V_t, D_t, and E_t. The present value of the floating rate note at t = 0 is obtained by discounting all the expected values of D_t at the risk free rate. Similarly, by setting D_{0T} = BF, we can obtain the value of MK, which should represent the credit risk premium of the default risky floating rate note.

The technical details of the simulation programs for the fixed rate coupon paying bond and the floating rate note are included in Appendices 3A and 3B.

3.5 Simulation results and conclusions

The purpose of the simulation exercise carried out in this chapter is to study the behaviour of risk premia of fixed coupon bonds and floating rate notes along different times to maturity. The results are expected to provide evidence on the existence of quality spread differentials so that two firms can reduce their borrowing costs through interest rate swaps. I discuss these results below.

Firstly, I check if the results are consistent with the valuation theories of firms' debts. I focus on the behaviour of the fixed coupon rate and the floating rate relative to a firm's leverage and assets volatility, given the default free rate. Table 3.1 shows a selection of fixed rates values and the mark-up over the default free floating rates. Both the fixed rates and the mark-up increase with the firm's leverage and assets volatility in both short-term and long-term maturity. When the firm's leverage becomes high, e.g. 70% of debt to assets ratio or higher, the fixed rates and the mark-up will increase sharply. Similarly, when the firm's assets volatility becomes high, the fixed rates and the mark-up will increase. However, the impact of an increased leverage on the fixed rates and the mark-up is higher than that of an increased assets volatility. Both the firm's leverage and assets volatility are important factors determining the default risk of the firm. The higher the firm's leverage or assets volatility, the higher the default risk. My results are consistent with debt valuation theories in that a higher fixed coupon rate or mark-up over the default free floating rate for default risky fixed rate bonds or floating rate notes is required when default risk becomes higher.

Secondly, I examine whether or not quality spread differentials exist between two firms with different leverage levels or assets volatility. Tables 3.2 to 3.4 show the values of quality spread differentials under three different default free rate volatility environments. I investigate if quality spread differentials exist between the following different groups of firms: a) firms with same assets volatility, but different leverage; b) firms with same leverage, but different assets volatility; and c) firms with different assets volatility and different leverage levels. The results given in tables 3.2 to 3.4 show that quality spread differentials are insignificant in an environment with low volatility of the default free rate whereas significant quality spread differentials are found in an environment with high volatility of the default free rate. In an environment of high interest rate volatility, firms are more willing to find ways to hedge against interest rate risk and the existence of quality spread differentials provides a great incentive for firms to choose interest rate swaps amongst other financial instruments. The results given in table 3.4 also show that quality spread differentials are more significant between firms in groups a) and c) than in group b). This is consistent with the results shown in table 3.1 in that the impact of leverage is higher than that of assets volatility on the default risk as well as the quality spread differentials between firms.

My valuation model on the default risky fixed and floating borrowing rate sheds light on the source of quality spread differentials. The results show that one of the reasons quality spread differentials exist is the difference in leverage or assets volatility between two firms. The results in this chapter reinforce the borrowing costs reduction argument for the development of interest rate swaps by extending the analysis to the more practical situation in which the default free interest rate is variable and the firm's debts are coupon paying. In addition, my model can be extended in a variety of ways. For instance, the model could take into account the different interest rate processes or the influence of different terms of borrowing.

Such extension would then provide a theoretical basis for empirical studies on the behaviour of borrowing costs of default risky firms and their interaction with interest rate swaps.

Table 3.1 Representative Values of Default Risky Fixed and Floating Rates

For $r = 3\%$ $\sigma_B = 3\%$

Time to Maturity $T = 2$

d*	σ_V	XF	MK
20	20	3.84	0.03
40	"	3.84	0.04
50	"	3.89	0.08
70	"	4.82	0.96
100	"	47	50
30	10	3.84	0.03
"	20	3.84	0.03
"	30	3.86	0.06
"	50	5.05	1.23

Time to Maturity $T = 10$

20	20	3.52	0.65
40	"	3.78	0.89
50	"	4.08	0.29
70	"	5.30	1.47
100	"	16.07	13.13
30	10	3.51	0.64
"	20	3.60	0.72
"	30	4.24	0.46
"	50	7.66	3.86

*d = debt / asset ratio in %

Table 3.2 Quality spread differential between default risk firms under low volatility of default free rate

For $r = 3\%$ $\sigma_B = 1\%$

Time to maturity = 2

Firm	d	σ_V	XF	MK
1	20	20	3.99	0.09
2	50	"	4.03	0.14
3	70	"	4.93	1.03
4	30	10	3.99	0.09
5	"	20	3.99	0.15
6	"	50	5.19	1.29

Time to maturity = 10

1	20	20	3.96	0.08
2	50	"	4.54	0.65
3	70	"	5.83	1.94
4	30	10	3.95	0.07
5	"	20	4.04	0.15
6	"	50	8.11	4.23

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	-1	1
Firm 1 vs Firm 3	0	1
Firm 2 vs Firm 3	-1	0

b) Firms with same leverage but different assets volatility

Firm 4 vs Firm 5	0	1
Firm 4 vs Firm 6	0	0
Firm 5 vs Firm 6	0	0

c) Firms with different assets volatility and different leverage

Firm 1 vs Firm 6	0	0
Firm 3 vs Firm 4	0	1

Table 3.3 Quality spread differential between default risk firms under medium volatility of default free rate

For $r = 3\%$ $\sigma_B = 3\%$

Time to maturity = 2

Firm	d	σ_V	XF	MK
1	20	20	3.84	0.03
2	50	"	3.89	0.12
3	70	"	4.82	0.96
4	30	10	3.84	0.03
5	"	20	3.84	0.17
6	"	50	5.05	1.23

Time to maturity = 10

Firm	d	σ_V	XF	MK
1	20	20	3.52	0.65
2	50	"	4.08	0.29
3	70	"	5.30	1.47
4	30	10	3.51	0.64
5	"	20	3.60	0.73
6	"	50	7.66	3.86

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	0	2
Firm 1 vs Firm 3	5	6
Firm 2 vs Firm 3	5	4

b) Firms with same leverage but different assets volatility

	T = 2	T = 10
Firm 4 vs Firm 5	0	0
Firm 4 vs Firm 6	1	3
Firm 5 vs Firm 6	1	3

c) Firms with different assets volatility and different leverage

	T = 2	T = 10
Firm 1 vs Firm 6	1	3
Firm 3 vs Firm 4	5	6

Table 3.4 Quality spread differential between default risk firms under high volatility of default free rate

For $r = 3\%$ $\sigma_B = 5\%$

Time to maturity = 2

Firm	d	σ_V	XF	MK
1	20	20	3.53	0.84
2	50	"	3.59	0.89
3	70	"	4.57	0.85
4	30	10	3.53	0.84
5	"	20	3.53	0.84
6	"	50	4.76	1.13

Time to maturity = 10

1	20	20	2.60	0.87
2	50	"	3.12	0.43
3	70	"	4.20	0.51
4	30	10	2.58	-0.94
5	"	20	2.68	-0.02
6	"	50	6.72	3.09

QSD between firms (in basis points, bp)

a) Firms with same assets volatility but different leverage

	T = 2	T = 10
Firm 1 vs Firm 2	1	6
Firm 1 vs Firm 3	13	16
Firm 2 vs Firm 3	12	10

b) Firms with same leverage but different assets volatility

Firm 4 vs Firm 5	0	2
Firm 4 vs Firm 6	4	11
Firm 5 vs Firm 6	4	9

c) Firms with different assets volatility and different leverage

Firm 1 vs Firm 6	4	10
Firm 3 vs Firm 4	13	17

Appendix 3A

```
'THIS IS A PROGRAM FOR VALUATION OF MULTI-PERIOD FIXED RATE
COUPON BOND
'
DECLARE FUNCTION CNORM! (E!)
DEFINT A-Z
'
CLS
FIXDIGITS = 5
MZ = 10000
N1 = 5000
NT = 10
DIM HUGE B!(1 TO MZ)
DIM V!(0 TO NT)
DIM B1!(0 TO NT)
DIM B2!(0 TO NT)
DIM B10!(0 TO NT)
DIM B20!(0 TO NT)
DIM BX!(1 TO NT)
DIM V1!(0 TO NT)
DIM V2!(0 TO NT)
DIM E!(0 TO NT)
DIM AP1!(0 TO NT)
DIM AP2!(0 TO NT)
DIM Z0!(0 TO NT)
DIM Z1!(0 TO NT)
DIM Z2!(0 TO NT)
DIM X!(0 TO NT)
DIM XF!(0 TO NT)
DIM DF!(0 TO NT)
DIM N(0 TO NT)
DIM NK1(0 TO NT)
DIM NK2(0 TO NT)
DIM RF!(0 TO NT)
DIM RZZ!(0 TO NT)
DIM RESB!(0 TO NT)
'
'The next commands load the random variables
'
B$ = CURDIR$ + "\RAN5.DAT"
OPEN B$ FOR INPUT AS #1
FOR I = 1 TO MZ
INPUT #1, B!(I)
NEXT I
CLOSE #1
```

```
'
'Assumed parameter values
'
'No. of time periods
NT = 10
NT1 = NT - 1
'Starting value for firm assets
V!(0) = 100!
V1!(0) = V!(0)
V2!(0) = V!(0)
AP1!(1) = 0
'Par value of bonds
BF! = 1!
'No. of bonds
NB = 50
'Coupon rate of risky bond required
COUP1! = 0.03083
'Initial instantaneous return of default free bond
RD! = .03
RF!(0) = RD!
'Initial price of default free bond
D!(0) = 1
'Coupon rate of default free bond required
COUP0! = 0.02507
'Volatility for underlying asset
SIGMA! = 0.2
SIG2! = SIGMA! * SIGMA!
'Volatility for default free bonds
SIGMAD! = .03
SIGMAD2! = SIGMAD! * SIGMAD!
'Total volatility
SIGX2! = SIG2! + SIGMAD2!
SIGMAX! = SQR(SIGX2!)
SSIG@ = SIGMA!
'
FOR K = 0 TO NT
B1!(K) = 0
B2!(K) = 0
B10!(K) = 0
NK1(K) = 0
NK2(K) = 0
ZZ1!(K) = 0
ZZ2!(K) = 0
RZZ!(K) = 0
NEXT K
'
```

```
'Payoff to default free and risky coupon bond
FOR K = 1 TO NT
DF!(K) = BF! * COUP0!
XF!(K) = NB * COUP1!
NEXT K
DF!(NT) = DF!(NT) + BF!
XF!(NT) = XF!(NT) + NB * BF!
'
'
RANDOMIZE 1000
'
FOR J = 1 TO N1
'
'Randomly chooses NT variates from datafile
FOR K = 1 TO NT
KJ = INT(RND(1) * MZ) + 1
KZ = MZ - KJ + 1
RES!(K) = B!(KJ)
RESB!(K) = B!(KZ)
NEXT K
'
'Default free interest rate generating process
D!(0) = 1
FOR K = 1 TO NT
K1 = K - 1
D!(K) = D!(K1) * EXP((RD! - SIGMAD2! / 2) + SIGMAD! * RESB!(K))
RF!(K1) = LOG(D!(K) / D!(K1))
NEXT K
'
NX1(0) = 1
NX2(0) = 1
FOR K = 1 TO NT
NX1(K) = 0
NX2(K) = 0
NEXT K
'
'MODE 1: Simple Coupon Bond
'
FOR K = 1 TO NT
K1 = K - 1
V1!(K) = V1!(K1) * EXP((RF!(K1) - SIG2! / 2) + SIGMA! * RES!(K))
V1M! = V1M! + V1!(NT)
VXF! = V1!(K) - XF!(K)
PX! = 0
IF VXF! < 0 THEN PX! = VXF!
IF PX! = 0 AND NX1(K1) > 0 THEN NX1(K) = 1
```

```
IF PX! = 0 AND NX1(K1) > 0 THEN V1!(K) = VXF!
IF PX! < 0 THEN V1!(K) = 0
AP1!(K) = XF!(K) + PX!
NK1(K) = NK1(K) + NX1(K)
NEXT K
'
FOR I = 1 TO NT
I1 = I - 1
FOR K = I TO NT
B1!(I1) = B1!(I1) + AP1!(K) / ((1 + RF!(K1)) ^ (K - I1))
NEXT K
NEXT I
IF V1M! >= NB*BF! THEN B1M! = NB* BF!
IF V1M! < NB * BF! THEN B1M! = V1M!
'
B1!(NT) = B1!(NT) + B1M!
'
'Value of default free bond
'
FOR I = 1 TO NT
I1 = I - 1
FOR K = I TO NT
B10!(I1) = B10!(I1) + DF!(K) / ((1 + RF!(K1))^(K - I1))
NEXT K
NEXT I
B10!(NT) = B10!(NT) + DF!(NT) - BF! * COUP0!
'
NEXT J
'
NK1(0) = N1
NK2(0) = 0
A$ = CURDIR$ + "\TABLE5A1.DAT"
OPEN A$ FOR OUTPUT AS #1
PRINT #1, A$
PRINT #1, ""
D$ = DATE$
S$ = TIME$
'
PRINT , ""
PRINT , TAB(5); "Sigma ="; TAB(15); SSIG@; "Risky Coup ="; COUP1!;
TAB(45); "Default free coup ="; COUP0!;
PRINT , ""
PRINT , TAB(15); "Coupon"; TAB(25); "Prob. of"; TAB(35); "MODE 1A";
TAB(45);
PRINT , TAB(15); "Bond"; TAB(25); "Default"; TAB(35); "Bond"; TAB(45);
"Discount";
```

```
PRINT ,
'
'
PRINT #1, ""
PRINT #1, TAB(5); "Sigma ="; TAB(15); SSIG@; "Risky Coup ="; COUP1!;
TAB(45); "Default free coup ="; COUP0!;
PRINT #1, ""
PRINT #1, TAB(15); "Coupon"; TAB(25); "Prob. of"; TAB(35); "MODE 1A";
TAB(45);
PRINT #1, TAB(15); "Bond"; TAB(25); "Default"; TAB(35); "Bond"; TAB(45);
"Discount";
PRINT #1,
'
FOR K = 0 TO NT
B1X@ = B1!(K) / (N1 * NB)
B2X@ = (N1 - NK1(K)) / N1
B5X@ = B10!(K)/ N1
B7X@ = B5X@ - B1X@
'
PRINT, TAB(5); K; TAB(15); B1X@; TAB(25); B2X@; TAB(35); B5X@;
TAB(45); B7X@;
'
PRINT #1, TAB(5); K; TAB(15); B1X@; TAB(25); B2X@; TAB(35); B5X@;
TAB(45); B7X@;
NEXT K
'
print, ""
print, tab(5); d$; tab(20); s$;
'
PRINT #1, ""
PRINT #1, TAB(5); D$; TAB(20); S$;
CLOSE #1
PRINT "Finished"
STOP

'
FUNCTION CNORM! (E!)
IF E! > -13! THEN
  GOTO LINE10
  ELSE
  CNORM! = 0
  END IF
  GOTO LINE19
LINE10:
IF E! < 13! THEN
  GOTO LINE11
```

```
    ELSE
    CNORM! = 1
    END IF
    GOTO LINE19
LINE11:
B! = 1! / (1! + .2316419 * ABS(E!))
C! = .3565638
F! = .3193815
ZX! = 1.330274
Q! = ((((ZX! * B! - 1.82126) * B! + 1.781478) * B! - C!) * B! + F!) * B!
D! = .3989423 * EXP(-E! * E! / 2!)
ZZ! = Q! * D!
IF E! > 0 THEN
   CNORM! = 1! - ZZ!
   ELSE
   CNORM! = ZZ!
   END IF
LINE19:
END FUNCTION
```

Appendix 3B

```
THIS IS A PROGRAM FOR VALUATION OF MULTI-PERIOD FLOATING
RATE NOTE
'
DECLARE FUNCTION CNORM! (E!)
DEFINT A-Z
'
CLS
FIXDIGITS = 5
MZ = 10000
N1 = 5000
NT = 10
DIM HUGE B!(1 TO MZ)
DIM V!(0 TO NT)
DIM B1!(0 TO NT)
DIM B2!(0 TO NT)
DIM B10!(0 TO NT)
DIM B20!(0 TO NT)
DIM BX!(1 TO NT)
DIM V1!(0 TO NT)
DIM V2!(0 TO NT)
DIM E!(0 TO NT)
DIM AP1!(0 TO NT)
DIM AP2!(0 TO NT)
DIM Z0!(0 TO NT)
DIM Z1!(0 TO NT)
DIM Z2!(0 TO NT)
DIM X!(0 TO NT)
DIM XF!(0 TO NT)
DIM DX!(0 TO NT)
DIM DF!(0 TO NT)
DIM COUP2!(0 TO NT)
DIM N(0 TO NT)
DIM NK1(0 TO NT)
DIM NK2(0 TO NT)
DIM RF!(0 TO NT)
DIM RRF!(0 TO NT)
DIM RZZ!(0 TO NT)
DIM RESB!(0 TO NT)
'
'The next commands load the random variables
'
B$ = CURDIR$ + "\RAN5.DAT"
OPEN B$ FOR INPUT AS #1
FOR I = 1 TO MZ
```

```
INPUT #1, B!(I)
NEXT I
CLOSE #1
'
'Assumed parameter values
'
'No. of time periods
NT = 10
NT1 = NT - 1
'Starting value for firm assets
V!(0) = 100!
V1!(0) = V!(0)
V2!(0) = V!(0)
AP2!(1) = 0
'Par value of bonds
BF! = 1!
'No. of bonds
NB = 50
'Mark-up for default-free note required
MK0! = -0.00362
'Mark-up for default risky note required
MK! = 0.00187
'Initial instantaneous return of default free bond
RD! = .03
RF!(0) = RD!
'Initial price of default free bond
D!(0) = 1
'
'Volatility for underlying asset
SIGMA! = 0.2
SIG2! = SIGMA! * SIGMA!
'Volatility for default free bond
SIGMAD! = .03
SIGMAD2! = SIGMAD! * SIGMAD!
'Total volatility
SIGX2! = SIG2! + SIGMAD2!
SIGMAX! = SQR(SIGX2!)
SSIGD@ = SIGMAD!
SSIG@ = SIGMA!
'
FOR K = 0 TO NT
B1!(K) = 0
B2!(K) = 0
B20!(K) = 0
NK1(K) = 0
NK2(K) = 0
```

```
ZZ1!(K) = 0
ZZ2!(K) = 0
RZZ!(K) = 0
NEXT K
'
'
RANDOMIZE 1000
'
FOR J = 1 TO N1
'
'Randomly chooses NT variates from datafile
FOR K = 1 TO NT
KJ = INT(RND(1) * MZ) + 1
KZ = MZ - KJ + 1
RES!(K) = B!(KJ)
RESB!(K) = B!(KZ)
NEXT K
'
'Default free interest rate generating process
D!(0) = 1
FOR K = 1 TO NT
K1 = K - 1
D!(K) = D!(K1) * EXP((RD! - SIGMAD2! / 2) + SIGMAD! * RESB!(K))
RF!(K1) = LOG(D!(K) / D!(K1))
RRF!(K1) = RRF!(K1) + RF!(K1)
NEXT K
'
'
'Payoff to default-free and risky floating rate note
'
FOR K = 1 TO NT
K1 = K - 1
DX!(K) = BF! * (RF!(K1) + MK0!)
'
X!(K) = NB * BF! * (RF!(K1) + MK!)
NEXT K
'
DX!(NT) = DX!(NT) + BF!
X!(NT) = X!(NT) + NB * BF!
'
'
NX1(0) = 1
NX2(0) = 1
FOR K = 1 TO NT
NX1(K) = 0
NX2(K) = 0
```

```
NEXT K
'
'Value of default risky floating rate note
'
FOR K = 1 TO NT
K1 = K - 1
V2!(K) = V2!(K1) * EXP((RF!(K1) - SIG2! / 2) + SIGMA! * RES!(K))
V2M! = V2M! + V2!(NT)
VXF! = V2!(K) - X!(K)
PX! = 0
IF VXF! < 0 THEN PX! = VXF!
IF PX! = 0 AND NX1(K1) > 0 THEN NX1(K) = 1
IF PX! = 0 AND NX1(K1) > 0 THEN V2!(K) = VXF!
IF PX! < 0 THEN V2!(K) = 0
AP2!(K) = X!(K) + PX!
NK1(K) = NK1(K) + NX1(K)
NEXT K
'
FOR I = 1 TO NT
I1 = I - 1
FOR K = I TO NT
B2!(I1) = B2!(I1) + AP2!(K) / ((1 + RF!(K1)) ^ (K - I1))
NEXT K
NEXT I
IF V2M! >= NB*BF! THEN B2M! = NB* BF!
IF V2M! < NB * BF! THEN B2M! = V2M!
'
B2!(NT) = B2!(NT) + B2M!
'
'Value of default free floating rate note
'
FOR I = 1 TO NT
I1 = I - 1
FOR K = I TO NT
B20!(I1) = B20!(I1) + DX!(K) / ((1 + RF!(K1))^(K - I1))
NEXT K
NEXT I
NT1 = NT - 1
B20!(NT) = B20!(NT) + DX!(NT) - BF! * (RF!(NT1) + MK0!)
'
NEXT J
'
NK1(0) = N1
NK2(0) = 0
A$ = CURDIR$ + "\TABLE5C1.DAT"
OPEN A$ FOR OUTPUT AS #1
```

```
PRINT #1, A$
PRINT #1, ""
D$ = DATE$
S$ = TIME$
'
PRINT , ""
PRINT , TAB(5); "Sigma D ="; TAB(15); SSIGD@; TAB(30); "Mark-up =";
MK0!;
PRINT , ""
PRINT , TAB(5); "Sigma V ="; TAB(15); SSIG@; TAB(30); "Mark-up ="; MK!;
PRINT ,
PRINT , TAB(15); "Risky FRN"; TAB(30); "Prob. of"; TAB(45); "D-free";
TAB(55); "Risk free";
PRINT , TAB(30); "Default"; TAB(45); "FRN"; TAB(55); "Rate";
PRINT ,
'
'
PRINT #1, ""
PRINT #1, TAB(5); "Sigma D ="; TAB(15); SSIGD@; TAB(30); "Mark-up =";
MK0!;
PRINT #1,
PRINT #1, TAB(5); "Sigma V ="; TAB(15); SSIG@; TAB(30); "Mark-up =";
MK!;
PRINT #1,
PRINT #1, TAB(15); "Risky FRN"; TAB(30); "Prob. of"; TAB(45); "D-
free";TAB(55); "Risk free";
PRINT #1, TAB(30); "Default"; TAB(45); "FRN"; TAB(55); "Rate";
PRINT #1,
'
FOR K = 0 TO NT
B1X@ = B2!(K) / (N1 * NB)
B2X@ = (N1 - NK1(K)) / N1
B5X@ = B20!(K)/ N1
B7X@ = RRF!(K) / N1
'
PRINT, TAB(5); K; TAB(15); B1X@; TAB(30); B2X@; TAB(45); B5X@;
TAB(55); B7X@;
'
PRINT #1, TAB(5); K; TAB(15); B1X@; TAB(30); B2X@; TAB(45); B5X@;
TAB(55); B7X@;
NEXT K
'
print, ""
print, tab(5); d$; tab(20); s$;
'
PRINT #1,
```

```
PRINT #1, TAB(5); D$; TAB(20); S$;
CLOSE #1
PRINT "Finished"
STOP

'
FUNCTION CNORM! (E!)
IF E! > -13! THEN
  GOTO LINE10
  ELSE
  CNORM! = 0
  END IF
  GOTO LINE19
LINE10:
IF E! < 13! THEN
  GOTO LINE11
  ELSE
  CNORM! = 1
  END IF
  GOTO LINE19
LINE11:
B! = 1! / (1! + .2316419 * ABS(E!))
C! = .3565638
F! = .3193815
ZX! = 1.330274
Q! = ((((ZX! * B! - 1.82126) * B! + 1.781478) * B! - C!) * B! + F!) * B!
D! = .3989423 * EXP(-E! * E! / 2!)
ZZ! = Q! * D!
IF E! > 0 THEN
  CNORM! = 1! - ZZ!
  ELSE
  CNORM! = ZZ!
  END IF
LINE19:
END FUNCTION
```

Chapter 4

The Contribution of Interest Rate Swaps to Market Completeness

Abstract

In addition to the savings in borrowing costs, hedging and restructuring firms' debt structure are suggested to explain the usage of interest rate swaps. However, these other reasons cannot explain satisfactorily why interest rate swaps are the most popular amongst all interest rate derivatives. This chapter shows that interest rate swaps help to complete the market in a cheaper and more efficient way than other alternatives such as mergers of firms or complex options. I argue that the contribution of interest rate swaps to the completeness of the securities market is another reason to explain the ever-growing activity in interest rate swaps. Interest rate swaps complete the market by expanding the opportunities for risk sharing, pooling and hedging, which make them not a redundant security in an incomplete market. In this chapter, I apply the Arrow-Debreu pure securities analysis to the payoffs of firms' liabilities with interest rate swaps. I show that, with a one-period state contingent claims model, interest rate swaps help to complete the market under the debt and swap priority rules. The payoffs of firms' liability securities are derived from the state contingent payoffs of the assets of firms. As such, firms' liability securities can be viewed as options on firms' assets. In completing the market, firms' liability securities or portfolios of them are limited in the cases when there are indistinguishable payoffs of firms' assets between some states. The limitation cannot be solved when firms' assets are not marketable and portfolios of them are restricted. Interest rate swaps help to solve this problem by creating two more linear independent securities under the debt and swap priority rules. Under the cross default rule, the interest rate swap is redundant because the payoffs of a swap can be replicated by existing marketable securities. The contribution of interest rate swaps in completing the market provides a stronger reason than the existing literature, which relies on market imperfections or inefficiencies to explain the ever-growing development of the interest rate swap market.

4.1 Introduction

It has been well argued that interest rate swaps, like other innovative financial products, develop on the basis of providing benefits to both participants in the swap transaction. Since a swap is a voluntary contract between two firms to exchange a series of cash flows based on a pre-determined formula, it is natural to presume that the participants in the swap enjoy some economic benefits in order to have an incentive to engage in the swap. Early research focuses on the cost benefits introduced by interest rate swaps. Based on the comparative advantage argument, if the difference in the borrowing costs between two firms which participate in different credit markets i.e. fixed and floating rate markets, is not constant, then the firms can lower their borrowing costs by creating a synthetic debt with interest rate swaps. However, this line of argument raises a debate of whether the cost savings is illusory or not and whether the source of cost savings comes from market imperfections and inefficiencies or not. For example, Wall and Pringle (1988) suggest a number of market imperfections such as differences in bankruptcy costs, differences in tax and regulatory costs, and the agency costs between creditors and shareholders to explain the borrowing cost differences between two firms borrowing in different credit markets. Litzenberger (1992) and Titman (1992) suggest that the inefficiency of financial markets in reflecting the credit risk of firms gives firms the opportunity to develop debt financing package at a lower cost than when the credit risks are fully known to the market. However, this market imperfection and inefficiency argument cannot produce a satisfactory explanation to the facts that financial markets have become more efficient since the inception of interest rate swaps and that interest rate swap activities continue to expand.

Another benefit introduced by interest rate swaps, which is suggested by existing literature is that interest rate swaps may complete the market by providing the function not yet available from existing securities. Smith, Smithson and Wakeman (1986) suggest that interest rate swaps filled in the missing gap during the mid-1980s by hedging activities when there were no forward interest rate contracts available. This market completeness argument was later criticised by the suggestion that today interest rate swaps can be replicated by a series of either forward or future contracts. However, in this paper, I show that interest rate swaps are different from a series of interest rate forward or future contracts where no exact replication can be done. Arak, Estrella, Goodman and Silver (1988) suggest that interest rate swaps can help to create a debt financing package with the default free rate component fixed but the credit spread component floating. Such a package is not available from either individual or portfolios of different debt securities in the market. They argue that interest rate swaps will benefit firms when there are different expectations on the credit spread between firms and the market. However, it is difficult to explain why there are different expectations if not because of market inefficiencies as suggested by Litzenberger (1992) and Titman (1992). I posit in this chapter that interest rate swaps help to complete the market by expanding the opportunities for risk sharing, pooling and hedging. In the context of market completeness literature, the expansion of risk sharing, pooling and hedging opportunities is represented by the spanning of state contingent payoffs that are not yet provided by existing securities. I show in this chapter that interest rate swaps can produce the state contingent payoffs not yet provided by the existing securities in a market without the imperfections and inefficiencies mentioned before. In this context, an interest rate swap is not a redundant security.

The ability of interest rate swaps in expanding the state contingent payoffs derives from the fact that there is an exchange of credit risk between two firms with limited liability in the swap transaction. Firms' liability securities are 'derived securities' in the sense that their payoffs are derived from the state contingent payoffs of the productive assets of the firms. Besides, the derivation of payoffs of individual liability securities depends on the contract terms and settlement rules. One of the most important features of the modern firms is the limited liability of shareholders. Although debts have priority over equities in the settlement process, shareholders need not make up for the deficiency if the assets value is less than the value of the debt. Therefore, debtholders face the default risk of firms where the promised payoffs of debts may not be fully satisfied. In the case of shareholders, they own the firm and enjoy the unlimited potential of positive payoffs from the profits and the limited loss in ownership of the firm resulting from the limited liability protection. Based on this feature, Black and Scholes (1973) first pointed out that firms' liabilities can be treated as options. As a result, contingent claims theory can be applied to the analysis of firms' liabilities. Unlike debts and equities issued by public firms, which are marketable securities, an interest rate swap is a private contract between two firms. It is not issued to the public. However, engaging in an interest rate swap will change the payoff characteristics of firms' existing debt and equity securities. The effect of the change will depend on the settlement rules of the swap contract. As a result, the firms can produce different sets of payoff patterns of firms' liability securities that may improve the positions of the holders of such securities. In this chapter, I apply the Arrow-Debreu (1964, 1959) pure securities analysis to the payoffs of firms' liabilities and swaps. My results show that interest rate swaps under the debt priority and swap priority rules are not redundant securities in that they help to complete the market. On the other hand, an interest rate swap under the cross default rule does not contribute to market completeness. These findings add a stronger argument on the development of interest rate swaps to the existing literature which is based on market imperfections or inefficiencies. They may also help to explain the growth of interest rate swaps despite the fact that securities markets are nowadays more efficient than when interest rate swaps were first introduced twenty years ago.

This chapter is organised as follows: section 2 discusses the market completeness arguments for the development of interest rate swaps suggested by Smith et al (1986) and Arak et al (1988). Section 3 reviews the state preference theory and its application to market completeness analysis. This section serves as a theoretical background for my model. Section 4 discusses the assumptions used in the model and how I formulate my analytical model. I apply the Arrow-Debreu pure securities analysis to show that interest rate swaps may help to complete the market under perfection and efficient market conditions. Section 5 discusses the results of the payoff analysis and the settlement rules under which interest rate swaps help to complete the market. It is interesting to note that interest rate swaps do not always complete the market in all situations. While an interest rate swap can complete the market under the debt priority and swap priority rules, it cannot complete the market under the cross default rule. This result implies that we have to pay attention to the swap contract terms in using the swaps for intended purposes. I also compare the payoff patterns of firms' liability securities before and after swap. When interest rate swap completes the market, it is possible to improve the welfare of one stakeholder of the firm without impairing the welfare of another. Finally, section 6 concludes the chapter.

4.2 Market completeness arguments for the development of interest rate swaps

Smith et al (1986) and Arak et al (1988) were amongst the first suggesting the argument of market completeness to explain the development of interest rate swaps. Smith et al (1986) assert that the interest rate swap market contributes to the integration of financial markets by allowing market participants to fill the gaps left by missing markets. In the mid-1980s, there were no forward interest rate contracts available. It is because an interest rate swap behaves like a series of forward contracts, that it could be used in place of the missing forward contract. Hence, the swap market was used as a way of synthetically 'completing' the financial markets. Subsequent research on interest rate swaps suggest that this argument may be weakened by the fact that today interest rate forward contracts are available for both short and long maturities. However, I put more emphasis on the fact that swaps and forwards are in fact two different kinds of financial securities. An interest rate swap is a single contract between two firms whereas a series of forward contracts are different, independent contracts either between two firms or between a firm and many other firms. Comparing interest rate forwards and swaps, forwards will have more uncertainties and risks involved for the same term to maturity. First, the interest rate swap fixes the fixed rate payments throughout the life of the swap contract, while the one-period forward contracts need to re-negotiate the new forward rate for the next contract when the existing contract expires. The series of one-period forward rates will only be constant under the situation of flat term structure of interest rates which is very rare in practice. A historical review of the term structure of interest rates shows that in the late 1970s and early 1980s, the yield curves of US Treasury securities were downward sloping while upward sloping yield curves had generally persisted since 1982. By the late 1980s, the yield curves were somewhat flat but such phenomenon only lasted for a short period of time. In the early and middle 1990s, the yield curves exhibited a steep upward slope.[1] Therefore, a series of one-period forwards will still face the uncertainty of subsequent forward rates. Second, even if we replicate the interest rate swap with a portfolio of forward contracts with same fixed forward rates but different maturities, the default risks involved will also be different. In case of interest rate swaps, any default on a single swap payment will automatically terminate the swap contract and the position of the firms in the swap transaction will be determined. However, a series of forward contracts comprises independent contracts and default on any single forward contract may not necessarily affect the validity of other forward contracts. This is especially the case if the defaulting firm makes a series of

[1] See Jeff Madura (1998) Financial Markets and Institutions, South Western, Cincinnati, Ohio, pp66.

forward contracts with more than one counterparty. In addition, if a firm engages in the forward contracts with more than one counterparty, the default risk faced by this single firm will be more complicated than in the case of swap because it involves different degrees of default risk from different counterparty firms. In sum, interest rate forwards and swaps are different kinds of securities each one with its own advantage. On the one hand, forwards provide more flexibility in that either the forward rates in subsequent periods can be adjustable or a firm can make a series of forward contracts with more than one counterparty. On the other hand, interest rate swaps are characterised by fixing both the fixed interest rate in all periods and the counterparty involved, features which firms find particularly useful for managing interest rate risk of long term nature. A more important implication is that the payoffs of interest rate swaps cannot be replicated by a series of interest rate forwards with the same level of risk.

Arak et al (1988) divide the borrowing costs of firms into two components, namely, the risk free rate and the credit spread. They suggest that interest rate swaps help to complete the market by providing a combination of the risk free rate and the credit spread that is not available prior to swaps. Prior to the introduction of swaps, the instruments available to firms were long-term fixed rate, long-term floating rate, and short-term debts. These instruments differ in having different combinations of fixed and floating risk free rate and credit spread. A long-term fixed rate bond has both the risk free rate and the credit spread fixed. A long-term floating rate note has the credit spread fixed but leaving the risk free rate to float. A short-term debt has both the risk free rate and the credit spread floating. It is interesting to note that the combination of fixed risk free rate and floating credit spread is not available from the traditional debt instruments. Arak et al (1988) show that a synthetic borrowing by combining short-term debt with a long position in interest rate swap (pay fixed and receive floating) provides a combination of fixed risk free rate and floating credit spread. In this sense, interest rate swaps help to complete the market by providing a choice of borrowing costs combination that is not available from existing debt instruments.

A question naturally follows then is under what conditions will a firm prefer a synthetic borrowing method in having the risk free rate fixed and credit spread floating. Again, Arak et al (1988) rely on the savings in borrowing costs as a major reason for firms preferring the combination of an interest rate swap and short-term debt. They argue that the borrowing firm and the market (the lenders) will have different expectations and risk attitudes towards the risk free rate and the credit spread. They assert that if the borrowing firm has a higher expectation or is more risk averse towards a component of the borrowing costs than the market, then the borrowing firm will fix that component and pay according to the market's expectation. If the borrowing firm's expectation or risk preference towards a component is the same as or lower than that of the market, then the borrowing firm will leave that component floating and pay according to the firm's expectation. A problem in interpreting the results of Arak et al (1988) is that they do not explain why the borrowing firm and the market have different expectations towards different components of the borrowing costs and how the borrowing firm can secure the savings in borrowing costs based on its expectations. More recently, Titman (1992) and Sharma (1994) suggest that the different expectations of borrowing costs between the borrowing firm and the market are the result of information asymmetry. Titman (1992) argues that a firm that expects its credit quality to improve based on some superior information about the credit quality of the firm not yet disclosed to the market will borrow short-term and buy an interest rate swap. The combination of short-term borrowing and a long position in interest rate swap enables the firm to fix the risk free rate component and leave the credit spread component to float. Later, when the information of better credit quality is released to the market, the firm can have a lower credit spread and save in borrowing costs. However, while these explanations provide the motivation for firms to borrow short-term and buy an interest rate swap, the motivation for the swap counterparty to short an interest rate swap is not yet fully explored. In addition, the realisation of the borrowing cost savings relies on the assumption that the interest rate swap is default free. However, this assumption does not represent actual market situations. In asserting that interest rate swaps help to complete the market, we do not require different expectations from different agents in the market. The payoff patterns of swaps and other firms' liability securities are all known to the agents in the market. Later, I show with my model that interest rate swaps expand the payoff distribution not available before the swap and complete the market even under the conditions of full information and homogenous expectation from all agents in the market.

This chapter applies the Arrow-Debreu (1964, 1959) pure securities analysis to the payoffs of firm's liabilities with interest rates swaps. I show in my payoff analysis that an interest rate swap is not a redundant security in that it helps to complete the market and that no existing marketable security can replicate the payoffs of the interest rate swap. My results differ from the existing literature on interest rate swaps in that they provide a stronger argument in explaining the development and growth of interest rate swap activities, which does not rest on the common market imperfection or inefficiency rationale discussed earlier. The idea is that because not all time state-contingent spaces are spanned by existing marketable securities, it may be possible that interest rate swaps create a pattern of returns that spans the unfilled space. The benefit of having a complete market is that different agents (firms and individuals) can buy or sell combinations of securities that pay off in all desired states. In this case, agents have a full range of risk / return choices. Investment opportunities are presented in the form of basic components, which the investor can assemble on a customised basis to conform a personal utility function. I discuss the state preference theory and the models employed in analysing the payoffs of securities in the following sections.

4.3 The state preference theory and its application to complete market analysis

In an uncertain economy, investment decisions invariably involve risk. It is not surprising that one of the major functions of the securities market is to allocate risk related to interest rates, stock prices, exchange rates, commodity prices, and others. The securities market provides a wide range of securities or institutional arrangements to either diversify risks or to allocate the undiversifiable part of the risks amongst individuals and firms, i.e. risk sharing. In order to achieve an unconstrained Pareto-efficient allocation of risk within a competitive market system, securities markets must provide sufficient opportunities to trade and price the various kinds of risk. Market prices help individuals and firms as decentralised economic units to target the amount of risk they are willing to bear which in turn helps to provide an optimal allocation of resources. However, it is always claimed that traditional securities are not sufficient to carry out the risk sharing function and that innovative securities such as interest rate swaps and options have been introduced to the market to fill the gap of sufficiency. Why are traditional securities not sufficient to allocate or share risk efficiently and how can innovative securities help the securities market move towards a more efficient risk allocation and sharing level? The state preference model developed by Arrow (1964) and Debreu (1959) provides a systematic analysis of these questions.

The state preference model provides a useful way of analysing risk allocation and risk sharing under uncertainty. The future of the world is described by a set of possible states that will occur. For example, next year's economic situation can simply be described by three states, depression, normalcy and prosperity; or more precisely as numerous possible states such as: GNP will increase by 5%, 10% ...., remain no change, or drop by 5%, 10%..... The uncertainty about the future is that the state which will occur is not known exactly, although the full set of possible states is. However, defining a set of states helps to describe the characteristics of securities since any security can be regarded as a contract which pays a certain amount depending on the particular state that will occur. Given the state dependence characteristic of securities, individuals can have their intertemporal allocation of funds across various states by investing in various securities. Thus, individuals can diversify the risk across various states. Arrow (1964) and Debreu (1959) develop the idea of 'complete market' where there is at least one security available that pays off in every state. In other words, a market is complete if the number of securities with non-redundant payoffs equals the number of states. In describing a complete market, Arrow and Debreu (1964, 1959) introduce the concept of state (pure) securities which has a unit payoff if a given state occurs and nothing otherwise. The principal characteristic of a complete market is that the entire set of pure securities can be constructed with portfolios of existing securities. The unit matrix describes the payoff matrix of the entire set of pure securities across the various states. Within this framework, the risk of a specific security is characterised by the distribution of payoffs across states, and the allocation of risk is achieved by allocating portfolios of pure securities between individuals. As a result, individuals are able to achieve any desired risk allocation pattern in terms of payoff distributions across states if there is no restriction of trading in the pure securities. This implies an unconstrained Pareto efficient allocation of risk. Ross (1976) and Cox and Rubinstein (1985) illustrate with payoffs analysis how exchange traded options help to complete the market.

In the analysis of how exchange traded options help to complete the securities market, Ross (1976) and Cox and Rubinstein (1985) examine how options written on basic securities expand the set of possible patterns of payoffs. The basic securities are marketable capital securities such as stocks and bonds. It is assumed that there are a finite number of these securities but portfolios which include them can be formed easily in a perfect market. The state payoffs of the basic securities are well defined and given. The state payoffs of options written on the basic securities are then derived from the state payoffs of the basic securities and the option terms, i.e. whether it is an ordinary call or put option and depending on the exercise price. Ross (1976) develops two important proofs of the power of different kinds of options in completing the market. Cox and Rubinstein (1985) further illustrate the idea with simple examples.[2] While Ross's (1976) results are very important to my analysis, my approach to define the states and to derive the payoffs of securities is different. An examination of an example used by Cox and Rubinstein (1985) may help to perceive how options can contribute to complete the market. Then, I will discuss the difference between their and my approach in the market completeness analysis.

In a market with three possible states and only one basic security, S, available with state payoffs S = (1, 2, 3) in each state, the market is clearly not complete. S itself cannot span the states and complete the market. However, we can see that the states can be spanned by creating call options on S with exercise prices $X_1 = 1$ and $X_2 = 2$. Then we have two more securities with payoffs $C_1 = (0, 1, 2)$ and $C_2 = (0, 0, 1)$. Taking these three securities together, we have a

payoff matrix: $$\begin{pmatrix} S & C_1 & C_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 3 & 2 & 1 \end{pmatrix}$$

[2] See Cox , J. and M. Rubinstein (1985) Options Market Prentice Hall, New Jersey, pp436-437.

Clearly, it can been seen that the payoffs of these three securities are linear independent, they cannot replicate each other, and the market is complete in the sense that all payoff states can be spanned. In fact, different portfolios of these three existing securities, as shown below, can form pure securities in each state.

$$\left(S-2C_1+C_2, \quad C_1-2C_2, \quad C_2\right)=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}$$

The analysis can be extended to more complicated situations where there is a larger number of states or where there is more than one basic security, which taken on their own cannot be distinguished between each state. However, in a situation where there are securities with indistinguishable payoffs between states, ordinary calls and puts cannot complete the market; more sophisticated kinds of options, called complex options, have to be introduced into the market. Ross(1976) provides two important results relating to the power of options in completing the market:

i. for a given set of basic securities, ordinary puts and calls span the same space as all simple options; and

ii. for a given set of basic securities, there exist an 'efficient' portfolio such that puts and calls on this portfolio span the same space as all complex options.

Simple options refer to options whose payoff is a deterministic function of the value of a single underlying security. Examples are ordinary calls and puts. By contrast, complex options refer to options whose payoff is a deterministic function of the value of a given set of underlying securities. Examples are calls and puts on a portfolio of basic securities.

Before discussing my analytical model, it is important to state the differences between my approach to derive the state payoffs of securities and those of Ross (1976) and others. Instead of treating the state payoffs of basic securities as given, I treat the state payoffs of firms' liability securities as derived from the state payoffs of firms' assets and the nature of the security, i.e. whether it is a debt or an equity. Only the state payoffs of firms' assets are given. The state payoffs of firms' liability securities are then determined based on the settlement rule. In perfect market conditions, the sum of the payoffs of the liability securities should be equal to the payoffs of the assets. In a simple balance sheet structure where a firm's liabilities consist only of debt and equity and based on the debt priority and the limited liability feature of the firm, Black and Scholes (1973) illustrate that a firm's equity can be regarded as a call option on the firm's assets whereas the firm's debt can be regarded as a portfolio of default free debt and put option on the firm's assets.

The difference of my approach has several implications. First, in a multi-firm situation and in order to satisfy the mutually exclusive and exhaustive criteria to define the states in the state preference model, the number of states has to be expanded to include the different solvency states of firms. Instead of defining the states as a single factor event, i.e. the outcome of the economy, as used by Ross (1976) and others, we need to define the states more precisely by including multi-factors such as solvency states of firms and movement of interest rates. I will discuss the definition of states in my model in more detail later. Second, the payoffs of firms' liability securities are inter-dependent in that a new security issued by a firm will affect the payoffs of existing firms' liability securities. The effect depends on the settlement rule within the firm. It is unlike that the market creates options where the introduction of a new option will not affect the payoffs of the existing ones or the payoffs of basic securities. Third, while portfolios of basic securities can be formed easily as in Ross's (1976) analysis, portfolios of firms' assets are restricted especially in the case of prohibition of merger and take-over activities between firms. Merger and take-over will introduce complicated issues on control of firms' activities and different kinds of agency problems which most of the firms try to avoid. I assume in my model that merger and take-over activities are prohibited. We can only form portfolios of firms' liability securities, which are limited by the supply of such securities. These differences distinguish my model from those of Ross (1976) and others.

4.4 One-period state preference model for the analysis of interest rate swaps

4.4.1 <u>Notation</u>:-

V_i asset value of firm i, i = A, B;

D_i debt value of firm i;

E_i equity value of firm i;

F fixed rate debt payment;

X floating rate debt payment;

r default free interest rate;

S_{AB} swap payment due from firm A to B;

S_{BA} swap payment due from firm B to A.

Since I analyse the payoffs of firms' liabilities and swap in a one-period model, I simplify the notation by omitting the time subscript. All the payoff values discussed later are the payoffs in period 1 unless stated otherwise.

4.4.2 Assumptions:-

A1. The economy is perfect and efficient. This assumption allows us to concentrate on the issue of market completeness.

A2. Firms are production agents and have given production technologies. Each firm is characterised by a production set, which contains all state-dependent input-output variables that are technologically feasible. Each firm owns and controls the technology and the inputs of production which are represented by the assets on the balance sheet of the firm. The firm finances its input of production in period 0 (the present) by issuing marketable liabilities securities such as debt and equity. The payoffs of the liabilities securities in period 1 (the future) are derived from the output of the firm's production. As such, the assets of firms are 'primitive assets' while the liabilities securities are 'derived securities' in the context of state contingent analysis.

A3. There are two firms, A and B in the economy. The asset values of the two firms are state contingent and characterised by the binomial states of either success or failure in period 1. If the firm succeeds, the asset value will be

V(1s) that is large enough to settle the entire firm's liabilities including the swap. If the firm fails, the asset value will be V(1f), which is so small that the firm cannot even fully settle the smallest liability, i.e. the net swap payment. As such, the creditors with the first priority will take over the firm and other creditors or equityholders will obtain nothing. This assumption is used to reflect the default risk involved in the swap and simplifies the complications of settlements of liabilities in case of bankruptcy.

A4. There is a default free, but risky rate, r, that is characterised by the binomial states of either up, r_u or down, r_d in period 1. The default free rate is used to determine the floating rate debt payment due.

A5. Firm A issues a fixed rate debt of debt payment F due in period 1. Firm B issues a floating rate debt of debt payment X due in period 1 depending on the state of the default free rate. When the default free rate rises, X will be Xu. When the default free rate falls, X will be Xd. The rest of the firms are financed by equities. All firms are of limited liability. The liabilities of firms are 'derived securities' in the sense that their payoffs are derived from the state dependent payoffs of the assets of firms and the default free interest rate. The priority of settlement of different liabilities depends on the swap settlement rule, which will be discussed in a posterior section.

A6. Firm A is selling the swap while firm B is buying it. I assume the two firms are swapping debt payments on a net basis in this one-period model. If the default free rate rises, firm A will pay Xu - F to firm B. If the default free rate falls, firm B will pay F - Xd to firm A. Although in practice financial institutions such as banks will usually act as the intermediaries in the swap, I assume in the model that the swap is arranged directly by the firms. This assumption explicitly reflects the exchange of risk between the two firms in the swap. Nonetheless, the analysis could be extended to the more complex case of a swap involving a bank.

A7. Default free fixed rate debt with payment F due in period 1 and floating rate debt with payment of either Xu or Xd also due in period 1 are available in the market.

A8. There is no correlation amongst the changes in the values of the default free rate and the assets of the two firms.

A9. No merger or take-over activities are allowed between firms.

4.4.3 Definition of states in the state preference model

As discussed in the previous section, the state preference model requires the states to be mutually exclusive. The major conceptual difficulty is that there is an infinite number of possible states in the real world, which makes it difficult to develop a comprehensive and general analytical model. Ross (1976) and others simplify the problem by relating the definition of states to a single factor only, i.e. the state of the economy. It does not affect their illustration of the power of options in completing the markets. The definition of states therefore depends mainly on the purpose the particular state preference model intends to achieve. My model intends to capture the most important characteristics of interest rate swaps in a simple setting that is rich enough to demonstrate how they can span the states unfilled. In a swap transaction between two firms with limited liability, one of the most important characteristics is that there is an exchange of default risk between the two firms. In an interest rate swap transaction between two firms, the payoffs of firms' liability securities and swap depend on the interest rate and the solvency states of the firms. The deterministic factors in the definitions of states are thus the state of the default free, but risky rate and the solvency states of the firms. Independently I define the states with each variable in a binomial setting where the default free rate may be either up or down and the firms may either succeed (be solvent) or fail (be insolvent). If we look at the payoffs of individual firms' liability securities or the default free debts independently, we need only to consider the states determining the payoffs of that security. For example, the payoffs of firm A's fixed-rate debt depend on the solvency states of firm A only. Either interest rate is up or down or firm B is solvent or not is irrelevant to the payoffs of firm A's fixed-rate debt. According to my assumptions, the state preference model for firm A's liabilities is just a simple two states model. We can clearly see that firm A's debt and equity are two linear independent securities with state contingent payoffs as below

	$\underline{D_A}$	$\underline{E_A}$
Firm A succeeds (solvent)	F	$V_A(1s) - F$
Firm A fails (insolvent)	$V_A(1f)$	0

In a market with firm A only, the market is complete by issuance of firm A's fixed-rate debt and equity. Investors can create any patterns of payoffs by forming different portfolios of D_A and E_A.

However, the derivation of the payoff patterns of a portfolio of different securities or a security involving more than one firm will be much more complicated than is the case of a single firm. When we introduce an interest rate swap transaction between two firms in the market, the payoffs of the swap and the firms' liability securities will depend on all factors of the states of the interest rate and the solvency states of the participating firms. For example, to determine the net swap payment that firm A (floating rate payer) needs to pay in any particular state, we need to consider whether the default free fixed rate is higher or lower than the default free floating rate and the solvency states of both firms A and B. The state of each payoff of the swap and the firms' liabilities depend on three factors. Based on my assumptions, I need to combine the six original independent states to produce 2 x 2 x 2 = 8 mutually exclusive and exhaustive states in order to analyse the payoff patterns of the swap and the firms' liabilities. The definition of states in my model is illustrated in Table 4.1.

4.5 Payoffs analysis and findings

I show with my payoff analysis that when two firms engage in an interest rate swap, two more securities will be created and the payoffs of firms' liabilities will be affected. In the case of both firms being solvent, only the payoffs of the firms' equities will be affected. Effectively, firm A has changed its fixed rate debt for floating rate debt while firm B has changed its floating rate debt for fixed rate debt. In the case of either one or both firms being insolvent, the changes of the payoffs of firms' liabilities depend on the settlement rules. I analyse three possible settlement rules of swaps namely, a) debt priority, b) cross default, and c) swap priority. The definitions of the three settlement rules are as follows:

a) debt priority	- debt payments are in priority over swap payments;
	- if the value of the swap is positive and becomes an asset to the firm, it is added to the asset value of the firm. Settlement of debt payments takes priority over equity.
b) cross default	-swap payments are made only if both counterparties are solvent prior to the swap payments;
	- the order of settlement is: 1) debt; 2) swap; and 3) equity.
c)swap priority	- the net swap payments are made before any payments are paid to the debtholders of the firm.

The different settlement rules should reflect the difference in the nature and the risk of interest rate swaps under each rule. I address two issues with my payoff analysis: 1) the settlement rules under which interest rate swaps complete the market, and 2) the effect that each settlement rule has on the welfare of existing stakeholders. My findings are discussed below.

4.5.1 The settlement rule under which interest rate swaps complete the market

I analyse the payoffs of the swap, the firms' liabilities and the default free debts using the definition of states as given earlier. In order to show the contribution of interest rate swaps to market completeness, I compare the market situation before and after the swap. Before two firms engage in an interest rate swap, there would be six linear independent securities under the assumptions of my model. The payoffs of all the securities in period 1 are shown in Table 4.2. It is worth noting that the payoffs of the securities in period 1, which represent the returns of the security holders, will always be positive or zero at the minimum. This is due to nature of the securities and the limited liability feature of firms. For debt securities, the holders are creditors of the firms and they will never incur any additional monetary liabilities on their holdings. Similarly, for equity securities, equityholders will not need to pay for the deficiency of firm's assets because of the protection of limited liability. For default free debts, the payoffs are always guaranteed. However, I shall show later that a swap is a very special security in that the payoffs of the swap can either be positive or negative to the swap holders.

In order to check if a market is complete, Ross (1976) and Cox and Rubinstein (1985) show that if the number of linear independent marketable securities equals the number of states, then the market is complete. All pure securities can then be derived from portfolios of different combinations of marketable securities. I adopt Ross's (1976) and Cox and Rubinstein's (1985) approach to test if a market is complete. Clearly, Table 4.2 shows that the market is not complete before the swap because there are only six securities in a world of eight states. In addition, there are indistinguishable payoffs within a security between some states. For example, the fixed rate debt issued by firm A has the same payoffs F in four states when the firm is solvent and the same payoffs $V_A(1f)$ in the remaining four states when it is insolvent. According to Ross (1976) and Cox and Rubinstein (1985), it will not be sufficient to introduce simple options in the market in order to achieve completeness when there are indistinguishable payoffs between states. It is because when there are indistinguishable payoffs for a given security between two states, the payoffs of simple options on this security will also have indistinguishable payoffs. A unit matrix of pure securities for all states cannot be formed with this security and simple options written on it. One possible alternative in this case is to form a portfolio with the liability securities of all firms and the default free debts and then write different options on this portfolio. These options are complex options as discussed by Ross (1976) and Cox and Rubinstein (1985). More importantly, Ross (1976) suggests that complex options will be both a sufficient and necessary security to complete the market when there are indistinguishable state contingent payoffs within a security between states. However, the construction and trading of complex options on a portfolio consisting of all basic securities will involve complicated operational and regulatory issues that will lead to high transaction costs.

In fact, the idea of complex options written on a portfolio consisting of all basic securities is similar to the 'supershare' proposed by Hakansson (1976). Hakansson (1976) suggests that a new kind of financial intermediary, called 'superfund', can be formed. Theoretically, the superfund can be devised as a mechanism to complete the securities market. The asset structure of this superfund is similar to an ordinary mutual fund, which consists of a portfolio of basic securities. However, the liabilities of this superfund are different from those of an ordinary mutual fund in that the superfund issues a special kind of security, called supershare. Hakansson (1976) defines supershare as a security, which on its expiration date entitles its owner to a given dollar value proportion of the assets of the superfund, provided the value of those assets on that date lies between a lower and an upper value. Otherwise, the supershare expires worthless. We can see that the nature of supershare is very similar to that of the pure security suggested by Arrow (1964) and Debreu (1959). The only difference is that a supershare takes as the relevant state-of-the-world the value of the portfolio of basic securities on which it is written. Since this portfolio can assume a continuous number of values between any two boundaries, the return of a supershare, given that it pays off, is to some extent uncertain. However, this uncertainty can be made as small as one would like by setting the lower and upper boundaries sufficiently close. As a result, securities market can be completed with the introduction of the superfund and the supershare. Unfortunately, Hakansson's (1976) innovative concept of superfund and supershare still remains at the academic discussion level and yet no superfund or supershare has been introduced into the securities market. A lot of operational and regulatory issues are needed to be dealt with before the superfund and the supershare can be put into practice.[3]

I show with my model that an interest rate swap can be another alternative to complete the market. Compared with complex options, an interest rate swap transaction involves much less operational and regulatory problems. The International Swap and Derivatives Association (ISDA) has already set up many standardised documents which enable an interest rate swap transaction to be

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[3] See Cox, J. and Rubinstein, M (1985) Option Market, Prentice Hall, New Jersey, p466-467.

completed within a very short period of time. It makes an interest rate swap a much simpler and more efficient means to move the market into completeness. From my payoff analysis, I find that interest rate swaps under debt priority and swap priority rules can contribute to complete the market while swaps under the cross default rule cannot. I discuss my analysis below.

Tables 4.3A, 4.4A and 4.5A show the payoffs of all securities after firm A and firm B have engaged in an interest rate swap under the debt priority, cross default and swap priority settlement rules, respectively. Special attention has to be paid on the fact that a swap can be decomposed into two linear independent securities. This is due mainly to the nature of swap contracts. Unlike debt and equity securities where the holders expect to receive the cash flows from the issuer but not the opposite, swaps are contracts between two firms to exchange cash flows. The value of a swap to the holder thus depends on whether the cash payment is greater than the cash receipt from the swap or not. In the case of a fixed-for-floating interest rate swap, the floating rate payer (firm A) is in effect selling a floating rate cash flow to its counterparty (firm B) and simultaneously buying a fixed rate cash flow. The situation is vice versa for the fixed rate payer (firm B). If we denote the payment from firm A to firm B as S_{AB} and the payment from firm B to firm A as S_{BA}, firm A is actually longing a security S_{BA} and shorting another security S_{AB} at the same time. The same case applies to firm B, but with reverse situations for S_{BA} and S_{AB}. The payoffs of these two securities depend on the difference between the fixed and floating default free rates and the solvency states of the firms. Moreover, unlike debts and equities, S_{BA} and S_{AB} are not marketable securities but are only written and held by the firms themselves. Another important feature of interest rate swaps is that they are transactions between two firms only, but with the outcome affecting the interests of the existing stakeholders of the firms. Tables 4.3B, 4.4B and 4.5B compare the changes in the payoffs of firms' liabilities before and after interest rate swap. Such changes will have implications on the welfare of existing holders of the firms' liability securities. I will discuss this issue in section 4.5.2.

Let us now concentrate on the issue of market completeness. From Tables 4.3A, 4.4A and 4.5A, it can be shown that there are eight securities in a world of eight states under all the settlement rules. To see if the market is complete with these eight securities, we check whether or not they are linear independent. This can be done using Linear Algebra. However, instead of going through the complicated and tedious mathematics, we can perform a simpler test that yields the same result. If there are n securities in the world with n states, given the payoffs of the securities in all states, a non-zero determinant of the payoff matrix of the security will imply linear independence of securities. Thus, the market is complete.

I perform the test with the following parameter values of firms' liability and default free debt securities in period 1.

Security	F	Xu	Xd	VA(1s)	VA(1f)	VB(1s)	VB(1f)
Payoffs in period 1	1	1.5	0.5	4	0.02	5	0.05

The parameter values are chosen to reflect the important characteristics in the interest rate swap transactions. The interest rate swap is engaged by two firms of different assets and liabilities characteristics that are reflected by the payoffs of the firms' assets and debts. Firm A and firm B have different payoff patterns in assets in period 1 reflecting that they are firms in different business. Firm A has issued a fixed rate debt while firm B has issued a floating rate debt. Based on the payoff patterns of F, Xu, Xd, it implies that firm A and firm B have different leverage and level of default risk. The interest rate risk is reflected by the payoffs of the default free floating rate debt in period 1. When interest rate rises, the payoff of the floating rate debt is larger than that of the fixed rate debt. When interest rate falls, the situation is reverse. These are the most common characteristics we can find in an interest rate swap transaction between two firms.

I now focus on the issue of market completeness. It is sufficient to know the payoff distributions of the securities under each state but not the probabilities of each state in order to analyse whether or not the market is complete. Nevertheless, my analysis can be extended to include the probabilities of each state, but that is left to future research on the pricing of interest rate swaps. Table 4.6 shows the results of the payoff matrix and the matrix determinants for all the settlement rules. Given the above parameter values, the matrix determinants of the payoff matrix under the debt priority and swap priority rules are negative while that under the cross default rule is zero. The results show that for the same set of parameter values, interest rate swaps under the debt priority and swap priority rules can complete the market while interest rate swaps under the cross default rule cannot. It implies the terms of the swap contract will be a deterministic factor of the contribution of interest rate swaps in completing the market.

4.5.2 The welfare effect on the existing holders of firms' liability securities

In order to conclude the payoff analysis, we have to look at the effect of interest rate swaps on the payoffs of the firms' existing liabilities. Unlike the exchange-traded options analysed by Ross (1976) and Cox and Rubinstein (1985) where the issue of one option will not affect the payoffs of existing options, the creation of an interest rate swap will change the payoffs of the existing liabilities of both participating firms. When both firms are solvent, the payoffs of the firms' debts are not affected. The payoffs of firms' equities will change as if the firms had issued a fixed rate debt instead of a floating rate debt or vice versa. When either one or both firms are insolvent, the payoffs of firms' liabilities and swap will be done according to the swap settlement rule. The payoffs of firms' debts and equities will be different whether they have engaged in an interest rate swap or not. I compare the payoffs of firms' liabilities before and after engaging in the swap contract under each settlement rule, as shown in tables 4.3B, 4.4B and 4.5B. From these payoff tables, I derive equations for the firms' liabilities before and after the firms have engaged in the swap (tables 4.3C, 4.4C and 4.5C). Since the payoffs of both firms' liabilities and interest rate swaps are derived from the payoffs of firms' assets and the difference between the default free fixed and floating rate debts, the payoff equations will all consist of option components on the assets of the firms and the default free fixed and floating rate debts. I discuss the welfare issue under the three different settlement rules below.

a) debt priority

From table 4.3B, we can see that the payoffs of firms' debts are at least better off in one state after swap. For the fixed rate debt of firm A, the payoff is improved by F - Xd in state 6, although the firm is insolvent, since it receives a net swap payment of F - Xd from firm B. Likewise, the payoffs of the floating rate debt of firm B improve by Xu - F in state 3, when the firm is insolvent, because it receives a net swap payment of Xu - F from firm A. The result is a consequence of the debt priority rule and the assumptions of my model. First, the swap is subordinated to the debt and, therefore, it cannot expropriate any rights of the debtholders. Second, the default rule for the swap requires the paying firm to pay even when the receiving firm is insolvent. By inspecting the payoff equations in table 4.3C, the debts of firm A and firm B can be written as:

<u>Before swap</u>

$$\begin{aligned} D_A &= \text{MIN}(F, V_A) \text{ or} \\ &= F - \text{MAX}(F - V_A, 0) \end{aligned} \tag{1}$$

$$\begin{aligned} D_B &= \text{MIN}(X(t), V_B) \text{ or} \\ &= X(t) - \text{MAX}(X(t) - V_B, 0) \end{aligned} \tag{2}$$

<u>After swap</u>

$$D_A = \text{MIN}\,[\,F,\ V_A + \text{MAX}(\,S_{BA} - S_{AB}\,,\ 0)\,]\ \text{or}$$

$$= F - \text{MAX}\,\{\,F - [\,V_A + \text{MAX}\,(\,S_{BA} - S_{AB}\,,\ 0\,)\,]\,,\ 0\,\} \qquad (3)$$

$$D_B = \text{MIN}\,[\,X(t),\ V_B + \text{MAX}\,(\,S_{AB} - S_{BA}\,,\ 0\,)\,]\ \text{or}$$

$$= X(t) - \text{MAX}\,\{\,X(t) - [\,V_B + \text{MAX}\,(\,S_{AB} - S_{BA}\,,\ 0\,)\,]\,,\ 0\,\} \qquad (4)$$

The swap raises the value of the total assets over which the debtholders have a claim, because V_A + MAX ($S_{BA} - S_{AB}$, 0) and V_B + MAX ($S_{AB} - S_{BA}$, 0) are always higher than V_A and V_B, respectively. Does it imply a wealth transfer to the debtholders since the debts are made better off in terms of payoffs? Before answering this question, we have to inspect the payoffs of firms' equities first. Deducting the debt and swap payment from the payoffs of firms' assets derives the payoffs of firms' equities. The payoff equations of firms' equities can be written as follows:

<u>Before swap</u>

$$E_A = \text{MAX}\,(\,V_A - F\,,\ 0\,) \qquad (5)$$

$$E_B = \text{MAX}\,(\,V_B - X(t)\,,\ 0\,) \qquad (6)$$

<u>After swap</u>

$$E_A = \text{MAX}\,[\,V_A - F - (\,S_{AB} - S_{BA}\,)\,,\ 0\,] \qquad (7)$$

$$E_B = \text{MAX}\,[\,V_B - X(t) - (\,S_{BA} - S_{AB}\,)\,,\ 0\,] \qquad (8)$$

From table 4.3B, we can see that the effectiveness of a firm in converting a fixed rate debt into a floating rate debt or vice versa with an interest rate swap depends on the default risk of the counterparty in the swap. For example, if there is no default from firm B, firm A can always convert its fixed rate debt into floating rate debt.

The payoffs of firm A's equity are the same as if the firm had issued floating rate debt instead of fixed rate debt. However, in state 7 when firm B defaults on the swap contract, the payoffs of firm A's equity will be the same as if the firm had issued a fixed rate debt.

Based on the assumptions that the market is efficient and that the swap is agreed at the same time the firms' debts are issued, there will be no wealth transfer effect. The prices of all securities will be determined to reflect their payoffs. However, another question that arises is why the firms do not issue the debts they prefer directly but create synthetic debts with an interest rate swap. One possible answer to this question is that synthetic debts with an interest rate swap can prove useful to reduce their debt financing costs. The possibility of lowering debt financing costs may induce the equityholders of firms to accept the uncertainty of the payoffs caused by the swap. The reduction in debt financing costs is a pricing issue which would require my model to be expanded by including the value generating process of firms' assets and the default free interest rates. This issue, however, is beyond the scope of this chapter. On the other hand, my result that an interest rate swap under the debt priority rule contributes to market completeness has an important implication. An important theorem of financial economics is that a complete market is always Pareto-efficient irrespective of the nature of investors, while an incomplete market must be Pareto-inefficient in some circumstances. In a Pareto-efficient financial market, no other set of securities can make some investors better off without making at least one other investor worse off given the social totals of return in each state. However, in a Pareto-inefficient financial market, it may be possible that the introduction of a new security increases the interest of some investors without impairing the interest of other investors. In fact, if the new security helps to move the market into completeness, the welfare of all investors should improve. It implies that when the market is incomplete before swap, the market must be Pareto-inefficient in some circumstances. In fact, when the interest rate swap moves the market into completeness, it can improve the overall welfare of investors, which is not necessarily a zero sum game. My results provide a stronger reason for the development of the interest rate swap market different from those relying on zero-sum game arguments.

b) cross default

An interest rate swap under the cross default rule will be executed only when both firms are solvent. If either one or both firms are insolvent, the swap will be terminated and no swap payment will be required. By inspecting the payoffs in table 4.4B and the payoff equations in table 4.4C, it can be noticed that there is no effect on the payoffs of firms' debts. The payoffs of firms' equities will be affected only when both firms are solvent. In the latter case, the payoffs of firm A's equity will be the same as if firm A had issued a floating rate debt instead of a fixed rate debt while the contrary is true for firm B's equity. Although the payoffs of debts are not affected, the probabilities of the states of firms' insolvency may be affected by the introduction of an interest rate swap. Once again, this is a pricing issue. Nonetheless, my results that the matrix determinant is zero with an interest rate swap under the cross default rule implies that an interest rate swap under the cross default rule is a redundant security. In particular, if we look at the payoffs of S_{AB}

and S_{BA} more closely in tables 4.4A and 4.6, we can see that S_{AB} and S_{BA} are in fact pure securities. It derives from the contractual nature of the cross default rule that S_{AB} and S_{BA} will only have payoffs in one state and nothing otherwise. In case of S_{AB}, it will have payoff of Xu - F only in the state when interest rate goes up and both firms are solvent. The payoffs of S_{AB} are all zero in other states where the settlement condition of the cross default rule is not satisfied. Similarly S_{BA} will have a payoff of F - Xd only in the state when interest rate goes down and both firms are solvent. In this eight-state world where we have two pure securities and six other basic securities with indistinguishable payoffs in some states, the matrix determinants are always zero for all different parameter values of default free interest rate and firms' asset. Interest rate swaps cannot complete the market under the cross default rule. In fact, the cross default rule is the least common amongst the different settlement rules in interest rate swaps.

c) swap priority

An interest rate swap under the swap priority rule gives the swap payment priority over the debt payment. When the firms are solvent, there will be no effect on the payoffs of firms' debts. When the firms are insolvent, the payoffs of firms' debts may be better or worse off depending on whether the firm is due to pay or receive the swap payment. If the firm is insolvent and is due to pay the swap payment, the debt is worse off because the swap payment has priority. If the firm is insolvent but is due to receive the swap payment, the debt is better off after the swap payment has been received. The situations of the payoffs of firms' equities are similar to those in debt priority rule except the state when the firm is solvent and is due to receive the swap payment for the insolvent party. The payoffs of firms' equities are better off with the insolvent firm's asset value in the case of swap priority. Similar to the case of debt priority, the interest rate swap under the swap priority rule also completes the market. The difference between the two settlement rules lies in the pricing of the interest rate swap. This can be reflected by the payoff equations of swap payments.

Debt priority

$$S_{AB} = \text{MAX}\{X(t) - F - \text{MAX}[X(t) - F - \text{MAX}(V_A - F, 0), 0]\} \quad (9)$$

$$S_{BA} = \text{MAX}\{F - X(t) - \text{MAX}[F - X(t) - \text{MAX}(V_B - X(t), 0), 0]\} \quad (10)$$

Swap priority

$$S_{AB} = \text{MAX}\{X(t) - F - \text{MAX}[X(t) - F - V_A, 0], 0\} \quad (11)$$

$$S_{BA} = \text{MAX}\{F - X(t) - \text{MAX}[F - X(t) - V_B, 0], 0\} \quad (12)$$

V_A and V_B always exceed MAX $(V_A - F, 0)$ and MAX $(V_B - X(t), 0)$, respectively. Therefore, swap payments under the swap priority rule will be better than those under the debt priority rule when the paying firm is insolvent. This should be reflected in the different prices of interest rate swaps under the two different rules. It will serve as a topic for future research on the pricing of interest rate swaps under different settlement rules.

4.6 Conclusion

For a financial innovative security such as an interest rate swap to be successful, it must improve the economic welfare of the financial system. One way that interest rate swaps improve the economic welfare is by providing more opportunities for risk allocation so as to move the market into a higher level of completeness. A complete market is desirable in an economy with uncertainty. If there is unconstrained trading in securities in a complete market, individual investors are able to achieve any desired risk allocation pattern by constructing different portfolios of securities in the market. This implies an unconstrained Pareto efficient allocation of risk.

I show in this chapter that an interest rate swap can be decomposed into two distinct securities that are bought and sold by the swap participating firms simultaneously. At the same time, interest rate swaps also change the payoff characteristics of swap-participating firms' liabilities. The change depends on the swap settlement rule. Further, I find that interest rate swaps can complete the market under the debt and swap priority rules. By contrast, an interest rate swap under the cross default rule cannot complete the market. These findings are interesting and important in interpreting the economic functions to which interest rate swaps contribute. Whether an interest rate swap is a redundant security or not depends greatly on the settlement rule of the swap. The settlement rule also has important implications on the pricing of an interest rate swap. It is possible that the merger of two firms or options on the portfolio of all liability securities of two firms and default free securities obtain the payoffs provided by interest rate swaps. However, interest rate swaps offer the advantage of enabling the swap participating firms to retain the control of their business without involving the complicated agency issues in the case of merger. Compared to options on the portfolio of all liability securities of two firms and default free securities, interest rate swaps provide a much more efficient and cheaper means of achieving a complete market.

Table 4.1 **Definition of states when two firms engage in an interest rate swap**

Events in period 1			**Definition of states in period 1**
Interest rate movements	**Solvency situations of firms**		
Interest rate up	B successful	A successful	S1 - Interest rate up, firm A successful, firm B successful
		A failure	S2 - Interest rate up, firm A failure, firm B successful
	B failure	A successful	S3 - Interest rate up, firm A successful, firm B failure
		A failure	S4 - Interest rate up, firm A failure, firm B failure
Interest rate down	B successful	A successful	S5 - Interest rate down, firm A successful, firm B successful
		A failure	S6 - Interest rate down, firm A failure, firm B successful
	B failure	A successful	S7 - Interest rate down, firm A successful, firm B failure
		A failure	S8 - Interest rate down, firm A failure, firm B failure

Table 4.2 **Payoffs of firms' liabilities and default free debts before swap in period 1**

	Firm A		Firm B		Default free debt	
States	DA(1)	EA(1)	DB(1)	EB(1)	F	X(1)
S1	F	VA(1s) - F	Xu	VB(1s) - Xu	F	Xu
S2	VA(1f)	0	Xu	VB(1s) - Xu	F	Xu
S3	F	VA(1s) - F	VB(1f)	0	F	Xu
S4	VA(1f)	0	VB(1f)	0	F	Xu
S5	F	VA(1s) - F	Xd	VB(1s) - Xd	F	Xd
S6	VA(1f)	0	Xd	VB(1s) - Xd	F	Xd
S7	F	VA(1s) - F	VB(1f)	0	F	Xd
S8	VA(1f)	0	VB(1f)	0	F	Xd

Table 4.3A Payoffs of firms' liabilities and default free debts after swap in period 1 (debt priority)

	Swap		Firm A		Firm B		Default free debt	
	SAB(1)	SBA(1)	DA(1)	EA(1)	DB(1)	EB(1)	F	X(1)
S1	Xu - F	0	F	VA(1s) - Xu	Xu	VB(1s) – F	F	Xu
S2	0	0	VA(1f)	0	Xu	VB(1s) - Xu	F	Xu
S3	Xu - F	0	F	VA(1s) - Xu	VB(1f) +(Xu - F)	0	F	Xu
S4	0	0	VA(1f)	0	VB(1f)	0	F	Xu
S5	0	F - Xd	F	VA(1s) - Xd	Xd	VB(1s) - F	F	Xd
S6	0	F - Xd	VA(1f) + (F - Xd)	0	Xd	VB(1s) - F	F	Xd
S7	0	0	F	VA(1s) - F	VB(1f)	0	F	Xd
S8	0	0	VA(1f)	0	VB(1f)	0	F	Xd

Debt priority rule

1. Debt payments are in priority over swap payments.
2. If the value of swap is positive and becomes an asset to the firm, it is added to the asset value and the liabilities are settled in the following order: a) debt and, b) equity.

Table 4.3B Payoffs of firms' liabilities before and after swap (debt priority)

	Firm A				Firm B			
States	**DA(1)**	*DA(1)*	**EA(1)**	*EA(1)*	**DB(1)**	*DB(1)*	**EB(1)**	*EB(1)*
S1	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xu*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - F*
S2	**VA(1f)**	*VA(1f)*	**0**	*0*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - Xu*
S3	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xu*	**VB(1f)**	*VB(1f) +(Xu - F)*	**0**	*0*
S4	**VA(1f)**	*VA(1f)*	**0**	*0*	**VB(1f)**	*VB(1f)*	**0**	*0*
S5	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xd*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - F*
S6	**VA(1f)**	*VA(1f) + (F - Xd)*	**0**	*0*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - F*
S7	**F**	*F*	**VA(1s) - F**	*VA(1s) - F*	**VB(1f)**	*VB(1f)*	**0**	*0*
S8	**VA(1f)**	*VA(1f)*	**0**	0	**VB(1f)**	*VB(1f)*	**0**	*0*

Payoffs before swap are shown in BOLD.

Payoffs after swap are shown in ITALIC.

Table 4.3C Payoff equations of firms' liabilities before and after swap (debt priority)

Before Swap	After Swap
FIRM A	
$D_A = F - MAX\ (F - V_A,\ 0)$	$D_A = F - MAX\ \{ F - [V_A + MAX\ (S_{BA} - S_{AB},\ 0)],\ 0 \}$
$E_A = MAX\ (V_A - F,\ 0)$	$E_A = MAX\ [V_A - F - (S_{AB} - S_{BA}),\ 0]$
	$S_{AB} = MAX\ \{ (X(t) - F) - MAX\ [(X(t) - F) - MAX\ (V_A - F,\ 0),\ 0] \}$
FIRM B	
$D_B = X(t) - MAX\ (X(t) - V_B,\ 0)$	$D_B = X(t) - MAX\ \{ X(t) - [V_B + MAX\ (S_{AB} - S_{BA},\ 0)],\ 0 \}$
$E_B = MAX\ (V_B - X(t),\ 0)$	$E_B = MAX\ [V_B - X(t) - (S_{BA} - S_{AB}),\ 0]$
	$S_{BA} = MAX\ \{ (F - X(t)) - MAX\ [(F - X(t)) - MAX\ (V_B - X(t),\ 0),\ 0] \}$

Table 4.4A Payoffs of firms' liabilities and default free debts after swap in period 1 (cross default)

States	Swap: SAB(1)	Swap: SBA(1)	Firm A: DA(1)	Firm A: EA(1)	Firm B: DB(1)	Firm B: EB(1)	Default free debt: F	Default free debt: X(1)
S1	Xu - F	0	F	VA(1s) – Xu	Xu	VB(1s) - F	F	Xu
S2	0	0	VA(1f)	0	Xu	VB(1s) - Xu	F	Xu
S3	0	0	F	VA(1s) – F	VB(1f)	0	F	Xu
S4	0	0	VA(1f)	0	VB(1f)	0	F	Xu
S5	0	F – Xd	F	VA(1s) – Xd	Xd	VB(1s) - F	F	Xd
S6	0	0	VA(1f)	0	Xd	VB(1s) - Xd	F	Xd
S7	0	0	F	VA(1s) – F	VB(1f)	0	F	Xd
S8	0	0	VA(1f)	0	VB(1f)	0	F	Xd

Cross default rule
1. Debt payments are in priority over swap payments.
2. Whenever either firm is insolvent, the swap contract is automatically void.
3. $S_{AB} = X_U - F$ and $S_{BA} = F - X_d$ when both firms are solvent; otherwise $S_{AB} = S_{BA} = 0$.

Table 4.4B Payoffs of firms' liabilities before and after swap (cross default)

	Firm A				Firm B			
States	**DA(1)**	*DA(1)*	**EA(1)**	*EA(1)*	**DB(1)**	*DB(1)*	**EB(1)**	*EB(1)*
S1	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xu*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - F*
S2	**VA(1f)**	*VA(1f)*	**0**	*0*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - Xu*
S3	**F**	*F*	**VA(1s) - F**	*VA(1s) - F*	**VB(1f)**	*VB(1f)*	**0**	*0*
S4	**VA(1f)**	*VA(1f)*	**0**	*0*	**VB(1f)**	*VB(1f)*	**0**	*0*
S5	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xd*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - F*
S6	**VA(1f)**	*VA(1f)*	**0**	*0*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - Xd*
S7	**F**	*F*	**VA(1s) - F**	*VA(1s) - F*	**VB(1f)**	*VB(1f)*	**0**	*0*
S8	**VA(1f)**	*VA(1f)*	**0**	*0*	**VB(1f)**	*VB(1f)*	**0**	*0*

Payoffs before swap are shown in BOLD.

Payoffs after swap are shown in ITALIC.

Table 4.4C Payoff equations of firms' liabilities before and after swap (cross default)

Before Swap	After Swap If both firms are solvent	If either or both firms are insolvent
FIRM A		
$D_A = F - MAX\ (F - V_A, 0)$	$D_A = F - MAX\ (F - V_A, 0)$	$D_A = F - MAX\ (F - V_A, 0)$
$E_A = MAX\ (V_A - F, 0)$	$E_A = MAX\ (V_A - F - (S_{AB} - S_{BA}), 0)$	$E_A = MAX\ (V_A - F, 0)$
	$S_{AB} = X_u - F$	$S_{AB} = 0$
FIRM B		
$D_B = X(t) - MAX\ (X(t) - V_B, 0)$	$D_B = X(t) - MAX\ (X(t) - V_B, 0)$	$D_B = X(t) - MAX\ (X(t) - V_B, 0)$
$E_B = MAX\ (V_B - X(t), 0)$	$E_B = MAX\ (V_B - X(t) - (S_{BA} - S_{AB}), 0)$	$E_B = MAX\ (V_B - X(t), 0)$
	$S_{BA} = F - X_d$	$S_{BA} = 0$

Table 4.5A Payoffs of firms' liabilities and default free debts after swap in period 1 (swap priority)

	Swap		Firm A		Firm B	
States	SAB(1)	SBA(1)	DA(1)	EA(1)	DB(1)	EB(1)
S1	Xu - F	0	F	VA(1s) - Xu	Xu	VB(1s) - F
S2	VA(1f)	0	0	0	Xu	VB(1s) - Xu + VA(1f)
S3	Xu - F	0	F	VA(1s) - Xu	VB(1f) + (Xu - F)	0
S4	VA(1f)	0	0	0	VB(1f) +VA(1f)	0
S5	0	F - Xd	F	VA(1s) - Xd	Xd	VB(1s) - F
S6	0	F - Xd	VA(1f) + (F - Xd)	0	Xd	VB(1s) - F
S7	0	VB(1f)	F	VA(1s) - F + VB(1f)	0	0
S8	0	VB(1f)	VA(1f) + VB(1f)	0	0	0

Swap priority rule

1. Net swap payments are paid before debt payments.
2. If the value of swap is positive and becomes an asset to the firm, it is added to the asset value and the liabilities are settled in the following order: a) debt and, b) equity.

Table 4.5B Payoffs of firms' liabilities before and after swap (swap priority)

	Firm A				Firm B			
States	**DA(1)**	*DA(1)*	**EA(1)**	*EA(1)*	**DB(1)**	*DB(1)*	**EB(1)**	*EB(1)*
S1	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xu*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - F*
S2	**VA(1f)**	*0*	**0**	*0*	**Xu**	*Xu*	**VB(1s) - Xu**	*VB(1s) - Xu + VA(1f)*
S3	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xu*	**VB(1f)**	*VB(1f) + (Xu - F)*	**0**	*0*
S4	**VA(1f)**	*0*	**0**	*0*	**VB(1f)**	*VB(1f) +VA(1f)*	**0**	*0*
S5	**F**	*F*	**VA(1s) - F**	*VA(1s) - Xd*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - F*
S6	**VA(1f)**	*VA(1f) + (F - Xd)*	**0**	*0*	**Xd**	*Xd*	**VB(1s) - Xd**	*VB(1s) - F*
S7	**F**	*F*	**VA(1s) - F**	*VA(1s) – F + VB(1f)*	**VB(1f)**	*0*	**0**	*0*
S8	**VA(1f)**	*VA(1f) + VB(1f)*	**0**	*0*	**VB(1f)**	*0*	**0**	*0*

Payoffs before swap are shown in BOLD.

Payoffs after swap are shown in ITALIC.

Table 4.5C Payoff equations of firms' liabilities before and after swap (swap priority)

Before Swap	After Swap
FIRM A	
$D_A = F - MAX\ (F - V_A, 0)$	$D_A = F - MAX\ \{F - [V_A - (S_{AB} - S_{BA})], 0\}$
$E_A = MAX\ (V_A - F, 0)$	$E_A = MAX\ [V_A - (S_{AB} - S_{BA}) - F, 0]$
	$S_{AB} = MAX\ \{(X(t) - F) - MAX\ [(X(t) - F) - V_A, 0], 0\}$
FIRM B	
$D_B = X(t) - MAX\ (X(t) - V_B, 0)$	$D_B = X(t) - MAX\ \{X(t) - [V_B - (S_{BA} - S_{AB})], 0\}$
$E_B = MAX\ (V_B - X(t), 0)$	$E_B = MAX\ [V_B - (S_{BA} - S_{AB}) - X(t), 0]$
	$S_{BA} = MAX\ \{(F - X(t)) - MAX\ [(F - X(t)) - V_B, 0], 0\}$

Table 4.6 Payoff matrix and matrix determinants given parameter values under different settlement rules

Parameter values

Security	F	Xu	Xd	VA(1s)	VA(1f)	VB(1s)	VB(1f)
Payoffs in period 1	1	1.5	0.5	4	0.02	5	0.05

a) Debt priority

	SAB(1)	SBA(1)	DA(1)	EA(1)	DB(1)	EB(1)	F	X(1)
S1	0.5	0	1	2.5	1.5	4	1	1.5
S2	0	0	0.02	0	1.5	3.5	1	1.5
S3	0.5	0	1	2.5	0.55	0	1	1.5
S4	0	0	0.02	0	0.05	0	1	1.5
S5	0	0.5	1	3.5	0.5	4	1	0.5
S6	0	0.5	0.52	0	0.5	4	1	0.5
S7	0	0	1	3	0.05	0	1	0.5
S8	0	0	0.02	0	0.05	0	1	0.5

Matrix determinant **-1.23**

b) Cross default

	SAB(1)	SBA(1)	DA(1)	EA(1)	DB(1)	EB(1)	F	X(1)
S1	0.5	0	1	2.5	1.5	4	1	1.5
S2	0	0	0.02	0	1.5	3.5	1	1.5
S3	0	0	1	3	0.05	0	1	1.5
S4	0	0	0.02	0	0.05	0	1	1.5
S5	0	0.5	1	3.5	0.5	4	1	0.5
S6	0	0	0.02	0	0.5	4.5	1	0.5
S7	0	0	1	3	0.05	0	1	0.5
S8	0	0	0.02	0	0.05	0	1	0.5

Matrix determinant **0**

c) Swap priority

	SAB(1)	SBA(1)	DA(1)	EA(1)	DB(1)	EB(1)	F	X(1)
S1	0.5	0	1	2.5	1.5	4	1	1.5
S2	0.02	0	0	0	1.5	3.52	1	1.5
S3	0.5	0	1	2.5	0.55	0	1	1.5
S4	0.02	0	0	0	0.07	0	1	1.5
S5	0	0.5	1	3.5	0.5	4	1	0.5
S6	0	0.5	0.52	0	0.5	4	1	0.5
S7	0	0.05	1	3.05	0	0	1	0.5
S8	0	0.05	0.07	0	0	0	1	0.5

Matrix determinant **-0.92**

Chapter 5

The Pricing of Default Risky Interest Rate Swaps

Abstract

The pricing of interest rate swaps is commonly done by replicating an interest rate swap with some basic securities. One approach replicates the interest rate swap as a series of forwards or futures contracts and another approach replicates the interest rate swap as an exchange of a fixed-rate bond for a floating-rate note. However, neither of the approaches has been able to deal with the credit risk of interest rate swaps properly and the price obtained cannot fully reflect the credit risk involved. Existing research on the pricing of interest rate swaps which focuses on swap credit risk tends to assume that one counterparty in the swap is default risk free. Such assumption has the shortcoming that the model cannot reflect the credit risk characteristics of both counterparties in the swap and, thus, the price obtained will be biased. In this chapter, I develop a contingent claims model to analyse the payoff of firm's liabilities and interest rate swaps under different settlement rules and assuming that both counterparties are default risky. Valuation equations for the interest rate swaps are obtained from the payoff analysis. My valuation equations for the swaps show that the credit risk of swap can be expressed as different kinds of options on the firms' assets. The kind of option depends on the settlement rule. Firms participating in interest rate swaps should consider the effect of these option values in determining the price of the swap.

5.1 Introduction

Interest rate swaps are usually arranged between two risky firms and the default risk of firms should normally be taken into account in the pricing of swaps. Although the rate of default on swap contracts has been low, the rapid expansion of the market naturally raises concerns about credit risk.[1] As swap volume increases, the probability that two firms enter into more than one swap contract with each other increases. Swaps also tend to accumulate from the practice of entering into an additional swap to reverse a position, rather than simply closing out a swap prior to maturity. However, current market practice does not price the default risk of swaps properly. Swap participants tend to mitigate the default risk of swaps by dealing only with AAA-rated counterparties or requiring collateral from the counterparties. In this context, they price the interest rate swaps as if they were default free. With the growth of interest rate swaps and the increasing number of firms with lower credit ratings coming into the market, there is a greater need to understand and price properly the default risk of swaps. Participants in the interest rate swap market are most interested in finding some methods to determine the price of interest rate swaps that can fairly reflect the degree of default risk.

The bilateral relationship between the firms engaged in an interest rate swap transaction should lead to the consideration of the default risk borne by each in the pricing process. To date, unfortunately, there is no complete pricing model dealing with the default risks of both counterparties. Early research attempt to price an

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[1] Although there is not an independent survey on the default of interest rate swaps, a survey conducted by the International Swap Dealers Association (ISDA) in 1992 estimates that the cumulative total of losses was only 0.0115% of the notional principal value of the swaps. See ISDA (1992).

interest rate swap by replicating its payoffs by a series of forwards or futures contracts or by using the exchange of a fixed-rate bond for a floating-rate note. However, the credit risk involved in an interest rate swap or in a portfolio of these securities is very different. I will discuss the weaknesses of using the replicating approach to price interest rate swaps in section 3 of this chapter. Recent research has attempted to price explicitly the credit risk of interest rate swaps making use of different pricing models. However, one common weakness of these models is that they usually assume one counterparty in the interest rate swap to be default risk free. A typical example is the work of Cooper and Mello (1991) who develop a contingent claims analysis of an interest rate swap between a risk free firm and a risky firm. Based on a contingent claims model, Cooper and Mello (1991) determine the equilibrium swap rate. However, the swap rate derived by assuming one of the counterparties in an interest rate swap to be default risk free will be incorrect because, in practice, both counterparties are default risky. In fact, interest rate swaps are commonly transacted between two firms with different credit ratings. By assuming that one of the counterparties is default risk free, the derived swap rate would be lower than the rate that should have been paid by the firm with the higher credit rating, but it would be higher than that should been paid by the firm with the lower credit rating. The lower credit-rated firm will suffer by paying a higher price for the swap.

In this chapter, I perform the contingent claims analysis by assuming both firms engaged in an interest rate swap to be default risky. I deduce the payoffs of firm's liabilities and swap contingent on the firm's assets value in various states of the world and under different settlement rules. I show that the settlement rule will determine the default risk of the swap. Compared to Cooper and Mello's (1991) results, mine can reflect the default risk of an interest rate swap more appropriately. A pricing model based on my results should be able to determine the fair swap rate that should be paid by the counterparties in an interest rate swap.

The structure of this chapter is as follows: section 2 describes the mechanism of an interest rate swap and the market practice in price quotation; section 3 reviews the literature on the pricing of interest rate swaps; section 4 discusses the nature of credit risk of interest rate swaps and the gaps in the existing literature regarding the pricing of credit risk; section 5 develops a contingent claims analysis of the firm's liabilities and swap contracts under the assumption that both firms engaged in the swap are default risky; the payoffs under different states of the world and settlement rules are derived and compared with those derived under the assumption that one counterparty is default risk free in section 6; section 7 concludes the chapter.

5.2 Mechanism of an interest rate swap and the price quotation practice

A 'plain vanilla' interest rate swap is a bilateral agreement to exchange a sequence of interest payments based on a notional principal amount that is never exchanged. One party's payment is based on a fixed rate that remains constant throughout the life of the swap. The counterparty's payment is based on a floating rate that is set in advance for each period and paid in arrears. The floating rate is linked to some reference rate such as LIBOR (London Interbank Offered Rate), which is the case for most US$ interest rate swaps. Both sides have the same payment frequency. In practice, instead of making a gross exchange of fixed and floating interest payments, a net payment is made and the direction is determined by the relative values of the fixed and floating rates that are pertinent to that payment date.

The maturity of swaps can vary from 1 year to over 10 years.

In practice, an interest rate swap is always priced by quoting the floating rate based on some reference rate without any mark-up, i.e., LIBOR flat in most cases. Therefore, the floating rate payment can be regarded as a series of floating risk-free interest payments. The pricing of interest rate swaps involves the determination of the fixed rate, usually called the swap rate or swap coupon, so that the initial value of the swap contract is zero to both counterparties. It then becomes a question of determining the value of a series of fixed payments so that a firm is willing to exchange for a series of uncertain payments. Figure 5.1 shows the mechanism and common terminology of a 'plain vanilla' interest rate swap. In practice, interest rate swaps are usually done through dealers but for simplicity, I assume in figure 5.1 that two firms arrange the swap contract directly between themselves.

5.3 Pricing of interest rate swaps

Based on the nature of the cash flows of the interest rate swaps, there are two common approaches to the valuation of swaps. The first one is to treat the interest rate swap as a series of forwards or futures contracts. The second one is to treat the interest rate swap as an exchange of a fixed-rate bond for a floating rate note. The basic argument of these two approaches is that the cash flows of interest rate swaps can be replicated by either a series of forwards or futures contracts or by an exchange of fixed-rate and floating-rate loans. However, I will show later that although the cash flows are similar, there are unique features associated with interest rate swaps. One of the major problems is that the credit risk is different amongst these financial alternatives which causes the valuation of interest rate swaps by the above two approaches to be inexact. Research carried out on these two approaches are discussed below.

5.3.1 Interest rate swap as a series of forwards / futures

Smith, Smithson and Wakeman (1988) were the first to point out that the cash flows of a par swap can be replicated by the cash flows of a portfolio of consecutive forwards, each maturing at different settlement dates. At each settlement date, the cash flow is determined by the difference between the fixed and floating rate which is same as the gain or loss realised in the currently maturing forwards. If the interest rate re-set and settlement dates coincide in a swap, then it can be replicated by a series of forwards. A number of banks use this approach to value or hedge interest rate swaps. In practice, in the case of US$ interest rate swaps, Eurodollar futures contracts are used because of the efficient and highly liquid futures market where futures prices are readily available. The cash flows to the seller of a Eurodollar strip are equivalent to the cash flows to a fixed-rate payer in a swap because each is obliged to sell a series of LIBOR cash flows at a predetermined price. Similarly, the cash flows to a Eurodollar strip buyer are equivalent to the cash flows to a swap's floating rate payer. Several researchers follow this approach in developing pricing models for interest rate swaps and find that swap rates are closely related to forward rates or futures prices.[2]

However, the price obtained from this approach cannot be exact when credit risk is considered. In the case of forwards and futures, credit risk is not significant. For forward contracts, banks will usually require the contracts to be fully back up by collateral and, hence, the credit risk is fully covered. In case of futures contracts, the

[2] For examples, see McNulty (1990), Alworth (1993) and Minton (1997).

market arrangement of clearing houses and mark-to-market requirement also reduces the credit risk to a minimum. For interest rate swaps, requirement of collateral is uncommon and the validity of such arrangement is legally uncertain because it has not been tested in a court of law yet. An interest rate swap is a fixed commitment between two counterparties throughout the life of the swap contract. Forwards or futures can be negotiated separately at each settlement date and in case the creditworthiness of a counterparty deteriorates, the other counterparty can simply stop either the continuation or the renewal of the contracts. As such, there should be a risk premium added to the swap rate over the default-free forward rate, which then depends on the creditworthiness of the counterparties involved in the swap.

5.3.2 Interest rate swap as an exchange of debt instruments

Bicksler and Chen (1986), Smith, Smithson and Wakeman (1988) show that the valuation of an interest rate swap can be achieved by viewing the swap as an exchange of a fixed-rate bond for a floating-rate note. The fixed-rate payer in an interest rate swap is effectively selling a fixed-rate bond and simultaneously buying a floating-rate note from the floating-rate payer. The opposite situation applies to the floating-rate payer. The equilibrium price of a swap can thus be thought of as being the difference in value between these two instruments. The valuation of the fixed-rate bond can be carried out using the standard bond pricing formula. The valuation of the floating-rate note is more complicated because of the stochastic nature of the movement of future interest rates. Cox, Ingersoll and Ross (1980) and Ramaswamy and Sundaresan (1986) have developed arbitrage-free models of pricing floating-rate payments. Along this line of research, Sundaresan (1989) further develops a valuation model on the floating-rate side of a swap that takes into account more complicated features of swaps such as the lag between rate re-set and payment dates and in which the floating-rate payments depend on averages of past rates rather than a single rate. Their work on the pricing of swaps concentrates on the valuation of the floating-rate side which is different from the market practice of concentrating on the valuation of the fixed rate side in deriving the fixed swap rate.

However, pricing a swap by considering it as an exchange of fixed-rate and floating-rate instruments again cannot help to deal with the credit risk of the swap appropriately. This time, the credit risk is overestimated because the principal is involved in the exchange of two debt instruments while a swap is just an exchange of interest rate payments without any principal being involved. Presumably, the credit risk of a swap should be lower than that of an exchange of two debt instruments.

In summary, while the two approaches in pricing swaps - a) an interest rate swap as a series of forwards / futures; b) an interest rate swap as an exchange of two debt instruments - can properly deal with the interest rate risk component of the swap, they cannot price correctly the credit risk component of the swap. The nature of credit risk of interest rate swaps should be correctly addressed in the pricing of swaps, which will be discussed in the next section.

5.4 Credit risk of interest rate swaps

It is the nature of credit risk of interest rate swaps that distinguishes them from other financial instruments, especially debt instruments. In a loan transaction, it is a unilateral lender-borrower relationship in which only the credibility of the borrower needs to be assessed. In the case of interest rate swaps, the relationship becomes bilateral because each counterparty owes payment to the other. Such payment depends on the future movement of floating rates against the fixed rate. Although interest rate swaps are usually transacted between a higher credit-rated firm which may be a bank acting as a swap dealer and a lower credit-rated firm which may be a commercial firm as a swap end-user, there has not yet been a case in which either counterparty is completely default risk free. Therefore, the credit risk of both counterparties needs to be assessed. If the market practice is to set the risk-free floating rate against the fixed swap rate, then the determination of the fixed swap rate should take into account the credit risk of both counterparties in the swap.

The credit risk of interest rate swaps can be divided into two components: the magnitude of loss (exposure) and the probability of default. If a counterparty were to default, the maximum loss to the other counterparty would be the value of the swap at that time. The value of the swap consists of the payment that is going to be received on that payment date and the replacement value of a swap with the same terms for the remaining life of the original swap. The replacement value is determined by the current swap market rates and is uncertain at the time of initiation of swap because of the stochastic nature of the movement of interest rates. The actual loss to the non-defaulting party may be less than the maximum from realising the asset of the defaulting party but it depends on the settlement rule of the firm's liabilities. I will show later that the situation will vary greatly for different priorities in the settlement of debts and swaps.

The probability of default depends on the financial soundness of firms in various states of the world, which can be reflected by the credit rating of firms. However, in order to assess the default risk involved in an interest rate swap, a joint exercise of estimating that the swap will become negative to one counterparty and the probability that such counterparty will actually default has to be done. The swap value depends on the future movement of interest rates. The probability of default depends, first, on the sensitivity of a firm's asset value to the movement of interest rates and, second, on the firm's reasons for using an interest rate swap. If the firm's asset value is sensitive to interest rate and the interest rate moves on an unfavourable direction, the probability of that firm becoming insolvent will increase. However, the probability of insolvency will also be affected by the purpose of the use of the swap. If the firm uses interest rate swaps for hedging purposes, it will gain from interest rate swap when it losses in its assets value due to adverse interest rate movement. Interest rate swaps will most likely become an asset to the firm in this case and the default risk of the firm will not be an issue. If the firm uses interest rate swaps for speculation purposes, it might be possible that the firm loses both in its assets value and in the interest rate swap value. Evaluation of this firm's credit risk should, thus, take into account both the sensitivity of the firm's assets value to interest rates movements and the margin. The latter is used to assess the ability of the firm to cover the loss in case it makes a wrong bet on the movement of interest rates.

Recently some studies have attempted to develop models that can deal with both the magnitude and probability of default risk. Cooper and Mello (1991) apply the contingent claims approach to analyse the payoffs of swaps under different settlement rules. Using the swap payoffs and based on the assumptions of the value generating process of firm's assets value and of the value of the interest rate, Cooper and Mello (1991) solve for the spread of an interest rate swap over the risk free rate. However, a major shortcoming of this study is the assumption that one counterparty in the swap is default risk free. As a result, the swap spread derived will be biased and cannot reflect the credit risk of both counterparties in the swap transaction. By assuming one firm as default risk free, the fixed rate that this default risk free firm is willing to pay (receive) against the receipt (payment) of the risk free floating rate will certainly be lower (higher) than the rate to be paid by a default risky firm. Rendleman (1993) develops a multidimensional binomial pricing model to value a swap that is subject to default risk. However, he also makes the assumption of one counterparty being default risk free and suffers from the same weaknesses as Cooper and Mello's (1991) study. This chapter attempts to overcome these weaknesses by taking into account the credit risk of both counterparties in an interest rate swap.

5.5 Contingent claims analysis of firm's liabilities with interest rate swap

Cooper and Mello (1991) develop a contingent claims model to analyse the exchange of financial claims for risky swaps. Cooper and Mello (1991) show that the settlement rule of financial claims on a firm's assets in case of default is a major factor that determines the magnitude of the equilibrium swap rate. However, Cooper and Mello (1991) assume that one swap counterparty is default risk free throughout their analysis which causes problems in interpreting the results when both counterparties in a swap are risky. In this chapter, I apply contingent claims analysis assuming a probability that both firms will default on the swap. My results show that the swap claim should take into account the default risk of each counterparty and that the equilibrium swap rate should be lower than that derived by Cooper and Mello (1991) using the default-risk-free counterparty assumption. For debts and equities, the payoffs will also be different from those of Cooper and Mello's (1991) analysis. One reason for firms engaging in interest rate swaps is to convert fixed-rated debt into floating-rate debt or vice versa. My results show that this objective may not always be achievable when counterparties are default risky.

An important result from the assumption of one default-risk-free counterparty is that the swap payment owed by this counterparty is absolutely certain. Such a default risk free swap payment will cause two major phenomena that cannot occur when the swap payment is default risky. First, the equity of the default risky counterparty may become a call option with an exercise price equal to the swap liability. For instance, a firm with a variable rate liability can convert it into fixed-rate by swapping fixed for floating. Before the swap, the equity of the firm can be written as a call option on the assets of the firm with the variable rate debt as exercise price. After the swap, the exercise price will become the fixed-rate payment in the swap. Such a conversion can make the analysis of the wealth effect on the equity be done by comparing the call option values before and after the swap. However, this only happens when one counterparty in the swap is default risk free. When both counterparties in the swap are risky, the payoffs of the equity will not always be equal to those of a simple option and the valuation of equity cannot be done in a similar straight forward manner. The same problem arises in the analysis of debt and swap claims.

Second, by assuming one default-risk-free counterparty, the equilibrium swap rates derived will only be the extreme values. For example, when the default-risk-free counterparty swaps floating for fixed, it could ask for the highest fixed swap rate as compared to the case when it is default risky. Likewise, the default-risk-free counterparty can always pay the lowest fixed swap rate when it swaps fixed for floating. As a result, the equilibrium swap rate derived under the assumption of one default-risk-free counterparty will be mis-priced. The equilibrium swap rate when both counterparties are default risky should fall between these two extreme values.

Following Cooper and Mello (1991), I develop a single period analysis on the payoffs of firm's liabilities and swap claims. I adopt their assumptions on capital markets and the structure of an interest rate swap, with the important difference that I assume that both firms are default risky. The assumptions are stated below:

A1: Capital markets are perfect and competitive. There are no dead-weight costs to bankruptcy.

A2: There are two firms in the swap, firm A and firm B. Both have real assets which values, V, are random variables. Firm A has issued a fixed-rate debt, which will pay F at maturity. Firm B has issued a variable rate debt, which will pay X at maturity. X is a random variable. The remainder of the firm's assets is financed with equity. Equity pays no dividend prior to the debt maturity.

A3: Firm A contracts with firm B a swap which pays the amount (X - F) from firm A to firm B at the maturity of the swap. If X - F is negative, the amount F - X will be paid to firm A by firm B.

A4: V and X are sufficiently variable to give rise to a positive probability in every state of the world.

Although the above assumptions make the interest rate swap transaction much simpler than a real one, this simple interest rate swap structure contains all the important features of an interest rate swap and can serve the purpose of showing the default risk effect on swaps and the effect of swaps on a firm's liabilities. In this case, firm A is the floating-rate payer while firm B is the fixed-rate payer. The swap payment is settled on a net basis, i.e. the difference between the fixed and variable payments. Effectively, firm A has converted its fixed rate liability into variable rate while firm B has converted its variable rate liability into fixed rate through the interest rate swap. However, I will show later that the effectiveness of this conversion is greatly affected by the default risk of the firms. At the end of the period, there will be the following possibilities:

a) $F >$ or $< X$ determines who is the net payer;

b) $F >$ or $< V_A >$ or $< X$ determines the solvency status of firm A;

c) $X >$ or $< V_B >$ or $< F$ determines the solvency status of firm B.

Since I allow for the default risk of both firms in the swap, the solvency status of each of them needs to be considered. It makes my analysis different from Cooper and Mello's (1991) who assume firm A to be default risk free. At each state, the payoffs of the debt, the equity and the swap are contingent on the firm's assets value. It is a standard application of the contingent claims theory on the valuation of a firm's liabilities with the common practice of limited liability for the equityholders. I will show that the payoffs depend on the settlement rule of the swap. To make the discussion of the payoffs analysis easier, I adopt the following notation:

V_i the asset value of firm i at the end of the period;

D_i the debt value of firm i at the end of the period;

E_i the equity value of firm i at the end of the period;

Z_i the swap claim against firm i at the end of the period; for positive payoffs of Z_i, firm i is going to pay the swap payment Z_i to the counterparty; for negative payoffs of Z_i, firm i is going to receive the swap payment Z_i from the counterparty;

$C(V_i, X)$ a call option on V_i with exercise price X;

$P(V_i, X)$ a put option on V_i with exercise price X;

$PX(V_i, X, F)$ a put option on the maximum of V_i and X with exercise price F.

For a two-firm transaction, i = A, B.

Since I am analysing a single period, I do not utilise the time subscript and all the variables denote the end of period values unless stated otherwise.

5.6 Payoffs of a firm's liabilities and swap under different settlement rules

I develop the payoffs analysis under three different settlement rules:

a) Debt priority

In this case, liabilities of a firm are settled in the order: debt, swap and equity. If the value of the swap is positive and becomes an asset to the firm, it is added to the asset value and the liabilities are settled in the order of debt and equity.

b) Cross default

Under the cross default rule, the swap payment is made only if both counterparties are solvent. The order of liabilities settlement remains debt, swap and equity. This arrangement is similar to the current practice of credit trigger in an interest rate swap contract where the swap contract will automatically be terminated when either firm becomes insolvent.

c) Swap priority

In this case, the settlement of the firm's liabilities is in the order of swap, debt and equity. If the value of swap is positive and becomes an asset to the firm, it is added to the asset value and the liabilities are settled in the order of debt and equity.

I depict the payoffs of the liabilities and the swap of firm A and firm B in tables 5.1 to 5.3. Table 5.1 shows the payoffs under the debt priority rule. Tables 5.2 and 5.3 show the payoffs under the cross default rule and the swap priority rule respectively. Additionally, in order to facilitate the comparison with the results of Cooper and Mello (1991), I depict in tables 5.4, 5.5 and 5.6 the payoffs of the liabilities and the swap where one firm is default risk free and the other is default risky. Each table corresponds to a different settlement rule. Firm S is default risk free and firm 1 is default risky. Examining the payoffs, we can write the valuation equations of the swaps under different situations. I summarise the valuation equations in table 5.7. These equations represent the payoff equations for the swaps under different settlement rules. The pricing of interest rate swaps under different settlement rules should be based on these payoff equations. We can note that $Z_A = -Z_B$ and $Z_S = -Z_1$. It is the natural result of the net settlement characteristics of the swap contract. When interest rate rises, firm A and firm S will need to pay X - F to firm B and firm 1 which are denoted by Z_A and Z_S. When interest rate drops, firm B and firm 1 will need to pay F - X to firm A and firm S which are denoted by Z_B and Z_1. Therefore, when Z_A or Z_B is positive, it represents a swap payment to be made by a firm and is a liability. When Z_A or Z_B is negative, it represents a swap payment to be received by a firm and is an asset. The same applies to Z_s and Z_1. However, except for the payment from firm S, which is assumed to be default free, the payments from firms A, B and 1 are all subject to the default risk of the firms. Cooper and Mello (1991) price interest rate swaps by assuming one counterparty to be default free. By contrast, I take into account the default risk of both counterparties and provide a more complete picture than that of Cooper and Mello (1991). I discuss my results under the three different settlement rules below.

a) Debt priority

Equations (1A) and (1B) contain put options relating the assets values of firms and swap payments which are different from the one derived by Cooper and Mello (1991), as shown by equations (4A) and (4B). The difference lies mainly on the treatment of the default risk of the floating rate payer, i.e. firm A and firm S. When $X > F$, firm A and firm S need to pay $X - F$ to firm B and firm 1. Comparing equation (1A) with (4A), the former includes a term $PX(V_A, F, X)$ which represents the default risk of firm A on the swap payment but the latter does not include this term on Z_S. This is the result from assuming that firm S is default risk free. When $F > X$, firm A and firm S receive $F - X$ from firm B and firm 1 which are represented by the negatives of equations (1A) and (4A). In this case, the default risk of the swap payer firms, firm B and firm 1, are both included (as represented by $PX(V_B, X, F)$ and $PX(V_1, X, F)$). In sum, the difference between Cooper and Mello's (1991) and my analysis is that I have included the default risk of the floating rate payer whenever he needs to make a swap payment. However, when the floating rate payer receives a swap payment from the fixed rate payer, the default risk of the latter is included in both Cooper and Mello's (1991) and my analysis. Based on these differences, we can find that the equilibrium swap rate, F, calculated from equation (1A) will always be lower than that from equation (4A). We can easily prove this by assuming firm B and firm 1 to be the same and denoting F_{1A} and F_{4A} as the equilibrium swap rates derived from equation (1A) and (4A), respectively. Equations (1A) and (4A) can then be rewritten as follows

$$Z_A = X - F_{1A} - PX(V_A, F_{1A}, X) + PX(V_B, X, F_{1A}) \qquad (1A)$$

$$Z_S = X - F_{4A} + PX(V_B, X, F_{4A}) \qquad (4A)$$

The properties of the put options are:

if $X > F$ $PX(V_B, X, F) = 0$, and

$PX(V_A, F, X) > 0$ for all positive values of V_A, F and X.

if $X < F$ $PX(V_A, F, X) = 0$, and

$PX(V_B, X, F) > 0$ for all positive values of V_B, X and F.

According to the swap pricing mechanism shown in figure 5.1, our task is to determine the swap fixed rate, F, so that the initial value of the swap, Z_i, is zero. By setting $Z_A = Z_S = 0$ and subtracting equation (1A) from (4A), we get $F_{4A} - F_{1A} = PX(V_A, F_{1A}, X)$. Therefore, the equilibrium swap rate derived from equation (4A) is always higher than that from equation (1A). The implication is that by assuming himself to be default free, the floating rate payer will always ask for a higher swap fixed rate from the fixed rate payer. Similarly, the argument can also be applied to the case of assuming the fixed rate payer to be default free. In that case, the fixed rate payer will always offer a lower swap fixed rate. As a result, the swap rate obtained from any model that assumes one counterparty default risk free unfavours the other counterparty when, in practice, both counterparties are usually default risky.

b) Cross default

Under the cross default rule, the swap contract will be effective only when both firms are solvent. If either counterparty is insolvent, the swap contract is declared void and there will be no swap claim, i.e. $Z_A = Z_B = 0$. Therefore, the swap claims represented by equations (2A) and (2B), and (5A) and (5B) will exist only if $V_A > F$, $V_B > X$, and $V_I > X$. The swap claim is subject to the default risk of the paying counterparty, which is represented by the put options. This happens when the paying counterparty is not able to pay for the swap after its debt settlement. Once more, equations (2A) and (2B) contain one more option on the assets of firm A than equations (2B) and (5B). Based on the same argument as in the case of debt priority, the swap fixed rate derived from equation (2A) will always be lower than that from equation (5A) by $F_{5A} - F_{2A} = P(V_A, X)$. $P(V_A, X)$ is always a non-negative term.

c) Swap priority

Under the swap priority rule, the default risk of the swap payment is much smaller than that under the debt priority rule. We can see this by comparing the terms $P(V_A, (X - F))$ and $P(V_B, (F - X))$ with $PX(V_A, F, X)$ and $PX(V_B, X, F)$. By inspecting the payoffs of these options, we can find that the former two option values will always be lower than the latter two. However, the swap fixed rate derived from assuming one counterparty as default free will again be biased by the difference $F_{6A} - F_{3A} = P(V_A, X - F)$.

In short, the equilibrium swap rates determined by Cooper and Mello (1991) under all different settlement rules are always in favour of firm S because they assume that firm S is default free although in practice it usually is default risky.

5.7 Conclusion

In this chapter, I consider the default risk of both counterparties participating in an interest rate swap to analyse the credit risk of swaps. Applying contingent claims theory, I obtain the valuation equations for the swap under different settlement rules. I show that the credit risk of swaps can be expressed as options on the assets of both counterparties. My results improve existing pricing models, which assume one of the counterparties in a swap to be default risk free. My payoff analysis can serve as a foundation for any pricing model that attempts to solve for the 'fair' rate of a default risky interest rate swap. I show that the default risk of swaps will be different under different settlement rules and that participating firms need to take this into consideration when they negotiate the swap contract.

Figure 5.1 Mechanism and common terminology of a 'plain vanilla' interest rate swap

$$\begin{tikzcd}[column sep=huge] \text{Firm A} \arrow[r, shift left, "\text{Floating rate payment: X}"] & \text{Firm B} \arrow[l, shift left, "\text{Fixed rate payment: F}"] \end{tikzcd}$$

Counterparties

Firm A: Floating rate payer, seller (short) of a swap.
Firm B: Fixed rate payer, purchaser (long) of a swap.

Swap rates

r_x: the floating rate is set equal to a benchmark/index rate, e.g. LIBOR without any mark up and re-set periodically.

r_f: the fixed rate (swap coupon rate) is determined at the start of the swap contract so that the initial value of swap is zero and fixed throughout the life of the swap.

Notional principal

A notional principal sum, P, is agreed between the swap counterparties in order to determine the swap payments but the principal will not be exchanged.

Life of swap

From 1 year to over 10 years but 2-5 years are the most common.

Swap payments

Fixed-rate payment: $F = P \times r_f$

Floating-rate payment: $X = P \times r_x$

The floating-rate payment is determined one period before the payment date, i.e. $X_{(t)} = P \times r_{x(t-1)}$ where t is the time period.

Table 5.1 Payoffs under debt priority rule: Both firms are default risky

Firm A

State		D_A	E_A	Z_A	$PX(V_A, F, X)$	$PX(V_B, X, F)$
$X > F$						
$V_A > X > F$		F	$V_A - X$	$X - F$	0	0
$X > V_A > F$		F	0	$V_A - F$	$X - V_A$	0
$X > F > V_A$		V_A	0	0	$X - F$	0
$F > X$						
$V_B > F > X$						
$V_A > F > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > V_A > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > X > V_A$		$V_A + (F - X)$	0	$-(F - X)$	0	0
$F > V_B > X$						
$V_A > F > X$		F	$(V_A-F)+(V_B-X)$	$-(V_B - X)$	0	$F - V_B$
$F > V_A > X$;	$V_A+(V_B-X)>F$	F	$(V_A-F)+(V_B-X)$	$-(V_B - X)$	0	$F - V_B$
$F > V_A > X$;	$V_A+(V_B-X)<F$	$V_A + (V_B-X)$	0	$-(V_B - X)$	0	$F - V_B$
$F > X > V_A$		$V_A + (V_B-X)$	0	$-(V_B - X)$	0	$F - V_B$
$F > X > V_B$						
$V_A > F > X$		F	$V_A - F$	0	0	$F - X$
$F > V_A > X$		V_A	0	0	0	$F - X$
$F > X > V_A$		V_A	0	0	0	$F - X$

Firm B

State		D_B	E_B	Z_B	$PX(V_A, F, X)$	$PX(V_B, X, F)$
$F > X$						
$V_B > F > X$		X	$V_B - F$	$F - X$	0	0
$F > V_B > X$		X	0	$V_B - X$	0	$F - V_B$
$F > X > V_B$		V_B	0	0	0	$F - X$
$X > F$						
$V_A > X > F$						
$V_B > X > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > V_B > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > F > V_B$		$V_B + (X-F)$	0	$-(X - F)$	0	0
$X > V_A > F$						
$V_B > X > F$		X	$(V_B-X)+(V_A-F)$	$-(V_A - F)$	$X - V_A$	0
$X > V_B > F$;	$V_B+(V_A-F)>X$	X	$(V_B-X)+(V_A-F)$	$-(V_A - F)$	$X - V_A$	0
$X > V_B > F$;	$V_B+(V_A-F)<X$	$V_B + (V_A-F)$	0	$-(V_A - F)$	$X - V_A$	0
$X > F > V_B$		$V_B + (V_A-F)$	0	$-(V_A - F)$	$X - V_A$	0
$X > F > V_A$						
$V_B > X > F$		X	$V_B - X$	0	$X - F$	0
$X > V_B > F$		V_B	0	0	$X - F$	0
$X > F > V_B$		V_B	0	0	$X - F$	0

Table 5.2 Payoffs under cross default rule: Both firms are default risky

Firm A

State	D_A	E_A	Z_A	$P(V_A, X)$	$P(V_B, F)$
$X > F$					
$V_A > X > F$; $V_B > X$	F	$V_A - X$	$X - F$	0	0
$V_B < X$	F	$V_A - F$	0	0	0
$X > V_A > F$; $V_B > X$	F	0	$V_A - F$	$X - V_A$	0
$V_B < X$	F	$V_A - F$	0	$X - V_A$	0
$X > F > V_A$	V_A	0	0	$X - V_A$	0
$F > X$					
$\underline{V_B > F > X}$					
$V_A > F > X$	F	$V_A - X$	$-(F - X)$	0	0
$F > V_A > X$	V_A	0	0	0	0
$F > X > V_A$	V_A	0	0	0	0
$\underline{F > V_B > X}$					
$V_A > F > X$	F	$(V_A - F) + (V_B - X)$	$-(V_B - X)$	0	$F - V_B$
$F > V_A > X$	V_A	0	0	0	$F - V_B$
$F > X > V_A$	V_A	0	0	0	$F - V_B$
$\underline{F > X > V_B}$					
$V_A > F > X$	F	$V_A - F$	0	0	0
$F > V_A > X$	V_A	0	0	0	0
$F > X > V_A$	V_A	0	0	0	0

Firm B

State	D_B	E_B	Z_B	$P(V_A, X)$	$P(V_B, F)$
$F > X$					
$V_B > F > X$; $V_A > F$	X	$V_B - F$	$F - X$	0	0
$V_A < F$	X	$V_B - X$	0	0	0
$F > V_B > X$; $V_A > F$	X	0	$V_B - X$	0	$F - V_B$
$V_A < F$	X	$V_B - X$	0	0	$F - V_B$
$F > X > V_B$	V_B	0	0	0	$F - V_B$
$X > F$					
$\underline{V_A > X > F}$					
$V_B > X > F$	X	$V_B - F$	$-(X - F)$	0	0
$X > V_B > F$	V_B	0	0	0	0
$X > F > V_B$	V_B	0	0	0	0
$\underline{X > V_A > F}$					
$V_B > X > F$	X	$(V_B - X) + (V_A - F)$	$-(V_A - F)$	$X - V_A$	0
$X > V_B > F$	V_B	0	0	$X - V_A$	0
$X > F > V_B$	V_B	0	0	$X - V_A$	0
$\underline{X > F > V_A}$					
$V_B > X > F$	X	$V_B - X$	0	0	0
$X > V_B > F$	V_B	0	0	0	0
$X > F > V_B$	V_B	0	0	0	0

Table 5.3 Payoffs under swap priority rule: Both firms are default risky

Firm A

State		D_A	E_A	Z_A	$P(V_A, X\text{-}F)$	$P(V_B, F\text{-}X)$
$X > F$						
$V_A > X > F$		F	$V_A - X$	$X - F$	0	0
$X > V_A > F$;	$V_A > X\text{-}F$	$V_A - (X\text{-}F)$	0	$X - F$	0	0
	$V_A < X\text{-}F$	0	0	V_A	$(X\text{-}F) - V_A$	0
$X > F > V_A$;	$V_A > X\text{-}F$	$V_A - (X\text{-}F)$	0	$X - F$	0	0
	$V_A < X\text{-}F$	0	0	V_A	$(X\text{-}F) - V_A$	0
$F > X$						
$V_B > F > X$						
$V_A > F > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > V_A > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > X > V_A$		$V_A + (F\text{-}X)$	0	$-(F - X)$	0	0
$F > V_B > X$; $V_B > (F\text{-}X)$						
$V_A > F > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > V_A > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > X > V_A$		$V_A + (F\text{-}X)$	0	$-(F - X)$	0	0
$F > V_B > X$; $V_B < (F\text{-}X)$						
$V_A > F > X$		F	$V_B + V_A - F$	$-V_B$	0	$(F\text{-}X) - V_B$
$F > V_A > X$;	$V_A + V_B > F$	F	$V_B + V_A - F$	$-V_B$	0	$(F\text{-}X) - V_B$
	$V_A + V_B < F$	$V_A + V_B$	0	$-V_B$	0	$(F\text{-}X) - V_B$
$F > X > V_A$		$V_A + V_B$	0	$-V_B$	0	$(F\text{-}X) - V_B$
$F > X > V_B$; $V_B > (F\text{-}X)$						
$V_A > F > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > V_A > X$		F	$V_A - X$	$-(F - X)$	0	0
$F > X > V_A$		$V_A + (F\text{-}X)$	0	$-(F - X)$	0	0
$F > X > V_B$; $V_B < (F\text{-}X)$						
$V_A > F > X$		F	$V_B + V_A - F$	$-V_B$	0	$(F\text{-}X) - V_B$
$F > V_A > X$;	$V_A + V_B > F$	F	$V_B + V_A - F$	$-V_B$	0	$(F\text{-}X) - V_B$
	$V_A + V_B < F$	$V_A + V_B$	0	$-V_B$	0	$(F\text{-}X) - V_B$
$F > X > V_A$		$V_A + V_B$	0	$-V_B$	0	$(F\text{-}X) - V_B$

Table 5.3 (continued)

Firm B

State		D_B	E_B	Z_B	$P(V_A, X\text{-}F)$	$P(V_B, F\text{-}X)$
$F > X$						
$V_B > F > X$		X	$V_B - F$	$F - X$	0	0
$F > V_B > X$;	$V_B > F\text{-}X$	$V_B - (F\text{-}X)$	0	$F - X$	0	0
	$V_B < F\text{-}X$	0	0	V_B	0	$(F\text{-}X) - V_B$
$F > X > V_B$;	$V_B > F\text{-}X$	$V_B - (F\text{-}X)$	0	$F - X$	0	0
	$V_B < F\text{-}X$	0	0	V_B	0	$(F\text{-}X) - V_B$
$X > F$						
$\underline{V_A > X > F}$						
$V_B > X > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > V_B > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > F > V_B$		$V_B + (X\text{-}F)$	0	$-(X - F)$	0	0
$\underline{X > V_A > F;\ V_A > X\text{-}F}$						
$V_B > X > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > V_B > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > F > V_B$		$V_B + (X\text{-}F)$	0	$-(X - F)$	0	0
$\underline{X > V_A > F;\ V_A < X\text{-}F}$						
$V_B > X > F$		X	$V_A + V_B - X$	$-V_A$	$(X\text{-}F) - V_A$	0
$X > V_B > F$;	$V_A + V_B > X$	X	$V_A + V_B - X$	$-V_A$	$(X\text{-}F) - V_A$	0
	$V_A + V_B < X$	$V_B + V_A$	0	$-V_A$	$(X\text{-}F) - V_A$	0
$X > F > V_B$		$V_B + V_A$	0	$-V_A$	$(X\text{-}F) - V_A$	0
$\underline{X > F > V_A;\ V_A > X\text{-}F}$						
$V_B > X > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > V_B > F$		X	$V_B - F$	$-(X - F)$	0	0
$X > F > V_B$		$V_B + (X\text{-}F)$	0	$-(X - F)$	0	0
$\underline{X > F > V_A;\ V_A < X\text{-}F}$						
$V_B > X > F$		X	$V_A + V_B - X$	$-V_A$	$(X\text{-}F) - V_A$	0
$X > V_B > F$;	$V_A + V_B > X$	X	$V_A + V_B - X$	$-V_A$	$(X\text{-}F) - V_A$	0
	$V_A + V_B < X$	$V_B + V_A$	0	$-V_A$	$(X\text{-}F) - V_A$	0
$X > F > V_B$		$V_B + V_A$	0	$-V_A$	$(X\text{-}F) - V_A$	0

Table 5.4 Payoffs under debt priority rule: Firm S: default risk free and firm 1: default risky

Firm S

State	D_S	E_S	Z_S	$PX(V_S, F, X)$	$PX(V_1, X, F)$
$X > F$					
$V_S > X > F$	F	$V_S - X$	$X - F$	0	0
$F > X$					
$V_1 > F > X$	F	$V_S - X$	$-(F - X)$	0	0
$F > V_1 > X$	F	$(V_S-F)+(V_1-X)$	$-(V_1 - X)$	0	$F - V_1$
$F > X > V_1$	F	$V_1 - F$	0	0	$F - X$

Firm 1

State	D_1	E_1	Z_1	$PX(V_S, F, X)$	$PX(V_1, X, F)$
$F > X$					
$V_1 > F > X$	X	$V_1 - F$	$F - X$	0	0
$F > V_1 > X$	X	0	$V_1 - X$	0	$F - V_1$
$F > X > V_1$	V_1	0	0	0	$F - X$
$X > F$					
$V_1 > X > F$	X	$V_1 - F$	$-(X - F)$	0	0
$X > V_1 > F$	X	$V_1 - F$	$-(X - F)$	0	0
$X > F > V_1$	$V_1 + (X-F)$	0	$-(X - F)$	0	0

Table 5.5 Payoffs under cross default rule: Firm S: default risk free and firm 1: default risky

Firm S

State	D_S	E_S	Z_S	$P(V_S, X)$	$P(V_1, F)$
$X > F$					
$V_1 > X$	F	$V_S - X$	$X - F$	0	0
$V_1 < X$	F	$V_S - F$	0	0	0
$F > X$					
$V_1 > F > X$	F	$V_S - X$	$-(F - X)$	0	0
$F > V_1 > X$	F	$(V_S-F)+(V_1-X)$	$-(V_1 - X)$	0	$F - V_1$
$F > X > V_1$	F	$V_S - F$	0	0	0

Firm 1

State	D_1	E_1	Z_1	$P(V_S, X)$	$P(V_1, F)$
$F > X$					
$V_1 > F > X$;	X	$V_1 - F$	$F - X$	0	0
$F > V_1 > X$;	X	0	$V_1 - X$	0	$F - V_1$
$F > X > V_1$	V_1	0	0	0	$F - V_1$
$X > F$					
$V_1 > X > F$	X	$V_1 - F$	$-(X - F)$	0	0
$X > V_1 > F$	V_1	0	0	0	0
$X > F > V_1$	V_1	0	0	0	0

Table 5.6 Payoffs under swap priority rule: Firm S: default risk free and firm 1: default risky

Firm S

State		D_S	E_S	Z_S	$P(V_S, X\text{-}F)$	$P(V_1, F\text{-}X)$
$X > F$						
$V_S > X > F$		F	$V_S - X$	$X - F$	0	0
$F > X$						
$V_1 > F > X$		F	$V_S - X$	$-(F - X)$	0	0
$F > V_1 > X$;	$V_1 > (F\text{-}X)$	F	$V_S - X$	$-(F - X)$	0	0
	$V_1 < (F\text{-}X)$	F	$V_1 + V_S - F$	$-V_1$	0	$(F\text{-}X) - V_1$
$F > X > V_1$;	$V_1 > (F\text{-}X)$	F	$V_S - X$	$-(F - X)$	0	0
	$V_1 < (F\text{-}X)$	F	$V_1 + V_S - F$	$-V_1$	0	$(F\text{-}X) - V_1$

Firm 1

State		D_1	E_1	Z_1	$P(V_S, X\text{-}F)$	$P(V_1, F\text{-}X)$
$F > X$						
$V_1 > F > X$		X	$V_1 - F$	$F - X$	0	0
$F > V_1 > X$;	$V_1 > F\text{-}X$	$V_1 - (F\text{-}X)$	0	$F - X$	0	0
	$V_1 < F\text{-}X$	0	0	V_1	0	$(F\text{-}X) - V_1$
$F > X > V_1$;	$V_1 > F\text{-}X$	$V_1 - (F\text{-}X)$	0	$F - X$	0	0
	$V_1 < F\text{-}X$	0	0	V_1	0	$(F\text{-}X) - V_1$
$X > F$						
$V_1 > X > F$		X	$V_1 - F$	$-(X - F)$	0	0
$X > V_1 > F$		X	$V_1 - F$	$-(X - F)$	0	0
$X > F > V_1$		$V_1 + (X\text{-}F)$	0	$-(X - F)$	0	0

Table 5.7 Valuation equations of interest rate swaps

Both firms A and B are default risky

Debt priority

$Z_A = X - F - PX(V_A, F, X) + PX(V_B, X, F)$ (1A)

$Z_B = F - X - PX(V_B, X, F) + PX(V_A, F, X)$ (1B)

Cross default

For $V_A > F$ and $V_B > X$

$Z_A = X - F - P(V_A, X) + P(V_B, F)$ (2A)

$Z_B = F - X - P(V_B, F) + P(V_A, X)$ (2B)

For $V_A < F$ or $V_B < X$; $Z_A = Z_B = 0$

Swap priority

$Z_A = X - F - P(V_A, X - F) + P(V_B, F - X)$ (3A)

$Z_B = F - X - P(V_B, F - X) + P(V_A, X - F)$ (3B)

Firm S: default risk free and firm 1: default risky

Debt priority

$Z_s = X - F + PX(V_1, X, F)$ (4A)

$Z_1 = F - X - PX(V_1, X, F)$ (4B)

Cross default

For $V_1 > X$

$Z_s = X - F + P(V_1, F)$ (5A)

$Z_1 = F - X - P(V_1, F)$ (5B)

For $V_1 < X$, $Z_s = Z_1 = 0$

Swap priority

$Z_s = X - F + P(V_1, F - X)$ (6A)

$Z_1 = F - X - P(V_1, F - X)$ (6B)

Chapter 6

Conclusions

6.1 Summary of findings

This thesis has developed some theoretical explanations for the development and pricing of interest rate swaps employing contingent claims analysis. The basic proposition is that two firms must foresee economic benefits for them to have an incentive to engage in a swap. Existing literature asserts that the gains from interest rate swaps may be illusory if they are based on the arbitrage of market imperfections or inefficiencies and if the risk involved is not taken into account properly. In this work, I show that the gains from interest rate swaps may stem from the differences in firms' basic characteristics which exist in any market condition and that the risk involved arises mainly from the credit risk of the participating firms. The findings can be summarised as follows:

a) Quality spread differentials are found to exist when applying the basic Black-Scholes-Merton option pricing model to the analysis of firms' liabilities. The same results are obtained using a more advanced simulation model that relaxed the Black-Scholes-Merton model assumptions in the valuation of coupon paying debts in a variable default free interest rate environment.

b) A firm's financial leverage and volatility of earning asset values are the two major factors determining the quality spread differential. At the same time, they are deterministic factors of a firm's credit rating. This explains why interest rate swaps are usually arranged between two firms holding different credit ratings.

c) It is found that the duration within which the quality spread differentials exist is contingent upon the values of the financial leverage and the volatility of earning assets of two firms.

d) The payoffs of interest rate swaps are determined by, first, the level of interest rates and, second, the solvency states of firms. According to the first condition, an interest rate swap may be either an asset or a liability for the participating firms. The second condition confers importance to the swap settlement rule in the determination of the resulted payoffs of the swap. Applying the Arrow-Debreu pure security analysis to a one-period model of the payoffs of firms' liabilities, this dissertation shows that interest rate swaps contribute to the completeness of securities markets under the debt and swap priority rules. Nonetheless, this is not the case under the cross default rule.

e) Based on a one-period contingent claims model of firms' liabilities and interest rate swaps, it is shown that the pricing of interest rate swaps may be carried out by using valuation equations which contain different kinds of options on the swap participating firms' assets and the default free interest rates. The kind of option depends on the settlement rule.

The implications of these findings in relation to the existing literature on interest rate swaps are discussed below.

6.2 Implications of research findings and contributions

The findings of this research have some important implications for the development and the pricing of interest rate swaps which contrast with the existing literature. First, existing literature on the reasons for the development of interest rate swaps tends to consider an interest rate swap transaction as an arbitrage activity. The most common argument is that the quality spread differential, which is the basis for two firms borrowing in different credit markets to have a comparative advantage, derives mainly from some form of market imperfection or inefficiency. Therefore, the savings on the borrowing costs obtained by sharing the quality spread differential between the two firms engaged in an interest rate swap are just the result of the arbitrage of market imperfections or inefficiencies. From the viewpoint of the economy, the arbitrage argument claims that interest rate swaps have no contribution to cost reduction and that such reduction represents just a wealth transfer within the economy. According to this line of research, interest rate swaps activities should decrease when arbitrage opportunities disappear, that is when the market becomes more perfect and efficient. This claim, however, is at odds with the fact that interest rate swap activities continue to expand and no better explanations have as yet been provided. The finding of this research which shows that quality spread differentials exist in the option pricing framework implies that such differential can exist in perfect and efficient market conditions. The main factors determining the quality spread differential are found to be a firm's financial leverage and volatility of earning asset values. These two factors are basic characteristics of firms that exist in any market condition. Financial leverage determines the financial risk of firms whereas volatility of earning asset values reflects business risk. This thesis shows that a quality spread differential exists between two firms with either different levels of financial leverage or different volatilities of earning assets for a certain range of parameter values of these two factors. The results also show that the existence of quality spread differential does not necessarily rely on market imperfections or inefficiencies.

Second, the savings in borrowing costs achieved through interest rate swaps are shared by two participating firms which represent different credit risks. The fact that an interest rate swap is transacted between two firms of different credit risk implies that the cash flows in the swap are always default risky. Thus, it is not appropriate to view an interest rate swap as a 'pure' arbitrage. Instead, the results of this dissertation show that an interest rate swap involves an exchange of credit risk between two firms. Based on the contingent claims analysis, the payoffs of firms' liabilities are derived from the state contingent payoffs of firms' assets. An interest rate swap affects the payoffs of firms' liabilities because the payoffs of the swap also depend on the state of default free interest rates. This makes the interest rate swap either an asset or a liability for the swap participating firms. As a result, two more linearly independent securities are created through interest rate swap. The one-period state contingent claims model suggests that interest rate swaps under the debt and swap priority rules contribute to securities market completeness. In a more complete market, opportunities of risk allocation are expanded so that they can satisfy the preferences of different investors at a higher level.

This work's results indicating that interest rate swaps help to reduce firms' borrowing costs in perfect and efficient market conditions and that interest rate swaps help to complete the securities market are consistent with the criteria for successful financial innovative products. Cost reduction and market completeness are two contributions of interest rate swaps to the economy. These results provide an alternate outlook on interest rate swap activities. Analogous to international trade, cost reduction and satisfaction of preferences at a higher level are two necessary and sufficient conditions for two firms to engage in an interest rate swap. The possibility of two firms lowering their borrowing costs and producing a combination of risk and return that improves their positions through interest rate swaps are some of the reasons why interest rate swap activities keep growing. They explain the continuing development of interest rate swaps without relying on the market imperfections or inefficiencies arguments.

6.3 Future research

In this thesis I have applied the contingent claims framework and option pricing models to analyse interest rate swaps. I started with the simplest case based on the specific assumptions of my models. The results comprise the basis for the analysis of interest rate swaps applying contingent claims theory. Future research in interest rate swaps with the application of option pricing models can be extended to the following areas:

a) A multi-period option pricing model may be developed to price an interest rate swap. The payoffs of a firm's liabilities and interest rate swaps were analysed in Chapter 5. Based on the payoff results, a multi-period simulation model may be developed specifying the value generating process for firms' assets and default free interest rates. It will require further work on mathematics and computer programming.

b) The correlation between the value generating process of firms' assets and the default free interest rate can be explored. It will contribute towards the understanding of the pricing of interest rate swaps.

This thesis has focused on the identification of the factors that give rise to quality spread differentials and affect the price of interest rate swaps. Consistent with existing research on the pricing of firms' debts, I have shown in Chapter 3 that both the interest rate risk and the default risk have to be taken into account to better value firms' debts. To better reflect the market practice of treatment of default of firms' debts, research can be extended to:

c) Incorporate a bankruptcy-triggering mechanism that allows for the possibility of early default. Research on firms' debts suggests that it has become common practice to state in a loan document that a firm's bankruptcy occurs when the firm's assets value falls below a pre-determined threshold value. This safety covenant is intended to provide more protection to debtholders. This alternate definition of bankruptcy will also have an impact on the gains and pricing of interest rate swaps. Models that take into account different bankruptcy-triggering mechanisms can better reflect the current market practice.

d) Allow for deviations from the strict-priority rule. Violations of the strict priority rule can be modelled to better reflect the bargaining game between the stakeholders of a firm on default.

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