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WITH A SINGLE SNAPSHOT A TARGET IS LOCATED WITH HYDROPHONES IN UNCERTAIN LOCATIONS, WITH NSR = 10xlog (NUMBER OF HYDROPHONES)

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With a single snapshot a target is located with hydrophones in uncertain locations, with NSR$= 10 \times \log$(number of hydrophones)

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Abstract: At an instant in time an acoustic pressure measurement is taken from each hydrophone. A wavelength is divided up into equal parts called 'space phase bins'. The spectrum of the target can be recovered if the measured pressures are correctly assigned to the bins with which, for the wavelengths, direction and range of interest, the hydrophones are associated. The algorithm is resistant to noise at the rule-of-thumb NSR$= 10 \times \log$(number of hydrophones). A straight string of hydrophones is regarded as a singular case, where the target's range cannot be estimated if it is at end-fire, and where the method is invalid if the target is exactly at broadside and sufficiently far away for plane waves to be received at the string. Otherwise, the only conditions are on the number of hydrophones and the Nyquist condition (distance between hydrophones)$< \frac{\lambda}{2}$.

Example: A target is found to within 2% accuracy by a single string of $1,000$ hydrophones at a range of $10\times$(length of the string) with a hydrophone location uncertainty of two parts in a thousand and with NSR$= 30$.

Conclusion: Uncertainty about hydrophone location is not a problem, and noise is not a problem.

Recommendation: As many hydrophones as possible are better distributed over two dimensions than one. The algorithm works with one string of hydrophones, where better two-dimensional resolution is achieved if the string is grossly deformed. Processing speed and accuracy would increase appreciably if the algorithm were to be implemented with more than one string.□

Introduction

This algorithm is designed specifically to locate the target from one instant to the next. It is possible to 'latch on' to the target and to follow it about. And if the power of the target's acoustic radiation changes, it can noted at once. With a time history, on the other hand, it must be difficult to interpret data in these respects.

The algorithm is an effective Fourier Transform for uncertain sampling intervals. There is no theoretical basis for applying the Discrete Fourier Transform to data taken at irregular, unknown intervals. An important difference is that the Discrete Fourier Transform cannot cope with a target at a finite distance. This algorithm can.

If we make the right selection of bins, the data will reinforce each other. But if we make a wrong selection of bins, the data will tend to cancel each other out. A bin is a band of phase points.

We must have at least three hydrophones per wavelength. Three bins per wavelength is optimal in the sense that three is the minimum number of bins required to corroborate a wavelength, and the minimum number of bins will allow the maximum number of data points per bin, thus maximising the noise-rejection.

If d is the distance between adjacent hydrophones and λ is the wavelength, the condition

$$d < \frac{\lambda}{2}$$

applies. It is the Nyquist condition.

Although uncertainty about a hydrophone's location increases with distance, provided that it is known that a hydrophone is near enough a distance d from its neighbour on the string, gross deformations of the string are unimportant. Indeed, gross deformations of the string are desirable, because they increase the two-dimensional resolution. And in fact the worst case is where the target is off the broadside in the middle of a straight string, for then, with far-field plane waves, only one bin is available, and more than one bin becomes available with wave-front curvature only when, for a straight string of length l and target at a distance of n times l and number of bins I,

$$l > \frac{\lambda}{I(\sqrt{n^2 + \frac{1}{4}} - n)}$$

is observed.

With end-fire condition, uniform binning is required (see Figure 1 below). But end-fire is a singular case. In general, the string will be deformed, and so circular binning must be applied (see Figure 2). With gross deformations of a single string or, alternatively, with an array of many strings, binning can be done along the y-axis as well as the x-axis, thus improving the resolution of the algorithm (see Figure 3).

The binning power integral $P_{\lambda_k,r,\theta}$ is constructed as follows. There are added together, in the ith bin ($i = 1, 2, ..., I$), successively each jth sample, i.e. samples $s^{i,j}_{\lambda_k,r,\theta}$ thought to have arisen from a target with K wavelengths λ_k, at range r and direction θ, with the total number $J^i_{\lambda_k,r,\theta}$ of samples in the ith bin. With optimal binning $I = 3$. Then the bin average $b^i_{\lambda_k,r,\theta}$ is taken, thus

$$b^i_{\lambda_k,r,\theta} = \frac{1}{J^i_{\lambda_k,r,\theta}} \sum_{j=1}^{J^i_{\lambda_k,r,\theta}} s^{i,j}_{\lambda_k,r,\theta} \quad ,$$

square it and add it to the other $b^i_{\lambda_k,r,\theta}$ to construct the space binning integral

$$P_{\lambda_k,r,\theta} = \sum_{i=1}^{I} (b^i_{\lambda_k,r,\theta})^2 \quad .$$

The $P_{\lambda_k,r,\theta}$ convey the power associated with wavelengths λ_k at range r and angle θ.

Theoretical Foundations of the Method

The basis of the method lies in the theory of rational approximation. For instance, with a Fourier Transform the understanding is that what is in general an irrational number may be approximated by a rational one. While in general a period of a signal cannot be measured in terms of a rational number, the notional periods with which it is compared can only be conceived of in terms of rational numbers. The consequences of the mismatch are seen when a Fourier Transform is applied to data taken over many periods. Increasingly the convolvent and the convolved diverge, and a periodogram begins to lose resolution and to display aliasses.

The Fourier Transform is itself a member of the class of binning algorithms in the sense that an arbitrarily small distance between an irrational convolved and a rational convolvent itself constitutes a bin, albeit one of the narrowest kind. On the other hand, in accepting wider bins than that, the present algorithm allows a tolerance of the uncertainties about hydrophone locations. It is because of those uncertainties that the binning approach is called for rather than a point-by-point convolution.

The phase binning method was developed when pulsars were first observed in the optical. A very quick way had to be found in order to estimate frequency. Each successive sample was added simultaneously to the contents of different bins, and a watch was kept on the various totals. A rising total suggested a correct binning, while a flat one suggested that the data were cancelling each other out and that, therefore, the wrong binning had been chosen. It is proposed to employ just such a technique in order to latch on to a target and to monitor its progress from one snapshot to the next.

The difference between the present algorithm and its progenitors in astronomy is that it is designed for use in two dimensions rather than one. It is possible to extend its use to three dimensions. However, with the present version, circular binning has been applied in order to locate a target in two dimensions.

Finally, the verb 'locate' has been used rather than 'localize' because although the method might seem to belong to the activity of 'optimization', the location is being estimated with respect to only one variable, the wavelength. Usually, in optimization, more than one variable is involved.

Example

Graph 1 below shows the spectrum of a target not at infinity taken with end-fire condition in the absence of noise on the signal but with hydrophone location uncertainty of $\frac{2}{1000}\times$(distance of hydrophone from the towing vessel).

The spike at $\lambda = 40$ is an alias of $\lambda = 20$. The efficacy of the method is not restricted to the range of wavelengths shown in the example. Graph 2 shows the spectrum of the same target at 45^o in the far-field in the absence of noise with the same end-fire binning as used for Graph 1.

Graph 3 shows the spectrum of the target at 45^o in the far-field with uniform binning allowing for that angle and with NSR= 30. The choice of 45^o is arbitrary. The reader is invited to verify that although several other spikes have now appeared on the periodogram, the signature of the target of interest is still manifest amongst them.

Graph 4 shows the periodogram for 45^o and a target range of $10\times$ (length of the string) with uniform binning and NSR= 30. It can be seen that uniform binning is no longer appropriate at that range. Similarly, Graph 5 shows the periodogram for the same angle but with a target range of $6\times$(the length of the string) with uniform binning and NSR= 30. Notice how, with uniform binning, the spectrum breaks up as the target approaches the hydrophones.

But Graph 6 shows the accuracy of a circular binning for 45^o and target range of $10\times$(the length of the string). The wavelength analysed is $\frac{\lambda}{d}$ =73. The accuracy is about 2%.□

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