UNIVERSITY OF SOUTHAMPTON

Modelling a Change of Classification in Economic Time Series Data

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Master of Philosophy

FACULTY OF SOCIAL SCIENCES

DEPARTMENT OF ECONOMICS

July 2000

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

The change of classification problem for economic sectoral time series data is examined by a *conversion matrix* approach. State space representations are proposed both for data reconstruction and modelling a change of classification. The Doran (1992) methodology of constraining the Kalman filter to satisfy time varying restrictions is applied to show how to handle both limited information and aggregation constraints. We explore the implications of this approach for what will be, perhaps, the most important change of classification in sectoral data: the new European System of National Accounts. Results of an experimental application to Italian Quarterly Accounts are provided.

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Preface

As a Researcher of the Italian National Statistical Office (ISTAT) and, in particular, working in the National Accounts Department and Economic Research, the last two years have been spent on the transition towards the new European System of National Accounts (Eurostat, 1996). As a result, a general revision of the System has been carried out. In this activity, my interest has been captured by the econometric implications that the introduction of new Classification Standards has involved in compiling Accounts by Sector.

The National Accounts System consists of a wide set of economic figures compiled at different and detailed sectoral levels over a long time period. The System provides an "internationally compatible accounting framework for a systematic and detailed description of a total economy (that is a region, country or group of countries), its components and its relations with other total economies" (Eurostat, 1996, p.1).

Under normal conditions, each Accounts is compiled by aggregating data from the related survey which is coherent in terms of standards and definitions. Estimates are produced just for more recent time periods (quarters or years), updating previous estimates to the last available time period.

When a general revision occurs National Accounts have to be completely refounded. Estimates have to be recompiled over all the sample period, following the new framework to be introduced. As an implication, long and consistent time series of related surveys for the Accounts to be reconstructed are needed. But, it is a matter of fact that surveys are periodically revised to follow changes in investigated phenomena so that time series suffer from structural breaks. Furthermore, over a long sample period surveys are often available under different sectoral classifications with respect to the standards to be introduced.

Consequently, when new standards are introduced, reconstruction of sectoral Accounts proceeds first through a benchmark producing new levels from a given time period and, secondly, by giving to data new coherence in a time series sense. As a result of the first step, two different measures of the same sectoral aggregates are observed: the former, for a longer period of time, in terms of old classification standards and the latter, just from the benchmark, in terms of new standards. At the second step, retrapolation techniques are adopted to gain new Accounts even in the past.

In this thesis a framework for a conversion of sectoral time series from *old* to *new* classification standards is provided. This is based on the definition of a *conversion matrix* to express time-varying compositional effects among different sectoral definitions. State space representations are presented to handle data reconstruction and modelling change of classification. A new approach for data reconstruction is suggested. The time series to be reconstructed is considered as an unobserved variable in a state space model: the estimates are so obtained satisfying the restrictions imposed by the few available observations.

The Kalman filter provides a well-established procedure to obtain optimal parameter estimation of state space forms. This has been largely used in this work obtaining reliable results on a preliminary experimental application based on Italian Quarterly Accounts.

Acknowledgements

First of all, as my Supervisor, I am grateful to Jan Podivinsky for helpful and precise comments and for the patience demonstrated over all the period involved in the preparation of this Thesis. Among the Staff of the Economics Department of the University of Southampton very helpful comments on computational aspects, concentrating a likelihood function and GLS estimation came from Ray O'Brien.

For a deep and at the same time practical approach to Dynamic Models and Regime Switching, my attendance at the Ph.D. course in Econometrics held by James D. Hamilton and S. Hylleberg in June 1997 in Aarhus, Denmark, has been very important.

Finally, I would like to thank Howard E. Doran and Alicia L. Rambaldi for the Gauss codes on the Singular Value Decomposition approach to constrain the Kalman filter to satisfy time-varying restrictions.

This material is based upon work supported by Consiglio Nazionale delle Ricerche under the scholarship posiz.204.4317, prot.057951, announcement n.203.10.32 of July 3, 1995.

The responsibility for any errors is, of course, only mine and the views expressed in the work are those of the author and do not indicate concurrence by ISTAT.

1 Introduction

The goal of this thesis is to provide a discussion on time series reconstruction. In particular, the problem for economic sectoral time series when a change of classification in economic activities occurs is approached.
In fact, a given economic variable (e.g. industrial production, GDP, consumption, income) gives a different composition among sectors depending on which sectoral standards is adopted. If by a given time period the way Statistical Agencies collect data changes, problems of comparing pre- and post-change time series arise.
In the following chapters a new approach for data reconstruction is suggested. The time series to be reconstructed is considered as an unobserved variable in a state space model: the estimates are so obtained satisfying the restrictions imposed by the few available observations. State space representations are even provided to handle modelling a change of classification.
A two step procedure to achieve both data reconstruction and parameter estimation of a change of classification model is suggested. It is shown as the Kalman filter provides the instrument to carry out optimal parameter estimation of the suggested state space forms even when models are subject to time-varying restrictions.

1.1 Motivations

An opportunity for statistics to capture economic structural change is to upgrade the methods, classification standards and definitions underlying their construction. Historically, developed economies have been going through a fast and significant de-industrialization: the service sectors, notably distribution, banking, business services and communications, have been growing very rapidly. At the same time we have observed the relative decline of agriculture, extractive industries and manufacturing. Note that qualitative changes have been as important as quantitative changes: in fact growth goes with a large diversification of economic activities, commodities and services in such a way that new ones have been created and others have disappeared. To perform surveys adequately representative of structural changes, International Statistical Agencies propose periodically to National Offices new classification standards, schemes and methods to be followed both for commodities and monitoring of firms.

Generally, new classifications introduce more accuracy and detail in the specification of economic activities. Because progress has introduced a relative diversification of services strictly linked with traditional processes (i.e. Agriculture and Industry), new classifications split up some activities previously belonging only to the latter processes. As a result, grouping economic activities into industrial sectors or into economy-wide aggregates incurs compositional incoherences when a comparison among different standards is performed.

Furthermore, a change of classification produces a structural break in sectoral time series and an historical reconstruction has to be realized. Usually, when new standards are introduced, reconstruction of sectoral data proceeds first through a benchmark producing new levels from a given time period and, secondly, by giving to data new coherence in a time series sense.

Considering the introduction of the new European System of National Accounts (Eurostat, 1996) these aspects will be particularly important. By that date new sectoral classifications will be adopted by those Countries joining the European Unification and a problem of historical reconstruction of Sectoral National Accounts will arise. In fact, two different measures of the same sectoral aggregates will be observed: the former, for a longer period of time, in terms of old classification standards and the latter, just from the benchmark, in terms of new standards.

In the following chapters a framework for a conversion of sectoral time series from *old* to *new* classification standards is provided. This is based on the definition of a *conversion matrix* to express time-varying compositional effects among different sectoral definitions. State space representations are presented to handle data reconstruction and modelling change of classification.

The Kalman filter provides a well-established procedure to obtain optimal parameter estimation of state space forms. Moreover, the Doran (1992) and Doran and Rambaldi (1996) methodologies of constraining the Kalman filter to obey time varying restrictions is revealed as an useful instrument to obtain efficient *smoothed* estimates. These allow us to incorporate contemporaneous aggregation constraints and available observations into the model.

1.2 Plan of the Thesis

The plan of this thesis is as follows. In chapter 2 an introduction of the Kalman filter as an instrument for data reconstruction has given: the main tools on the Kalman filter are provided in order to get the material of this thesis self contained. After the description of a general state space form, the typical recursions of the Kalman filter and initial conditions, section 2 provides the basic issues on the maximum likelihood estimation; particular attention has given to the developments involved in the Kalman filter when some parameters to be estimated are concentrated out of the likelihood; furthermore, available solutions to the problem of missing observations are shortly mentioned. The smoothing algorithm is the issue of section 3, while section 4 introduces the discussion on constraining the Kalman filter to obey time varying restrictions: the Doran (1992) and Doran and Rambaldi (1996) methodology is summarized. Finally, in section 5 a new approach to the problem of data reconstruction is shown by proposing a state space form for a simplified data generating process.

Chapter 3 provides the framework for a change of classification by a formal exposition. In section 1, basic concepts on classification standards give a preliminary introduction; then deterministic preliminaries and the definition of the conversion matrix are provided. The extension to a dynamic model is the subject of section 2, where a state space representation for modelling the change of classification is proposed.

In chapter 4 results of a preliminary application on Italian quarterly accounts are shown. After a first section where data in terms of *new* classification standards are artificially generated, sections 2 and 3 provide a two stage procedure to gain both data reconstruction and modelling a change of classification.

Finally, chapter 5 provides a concluding summary discussion.

2 The Kalman Filter as an Instrument for Data Reconstruction

2.1 The Kalman Filter

2.1.1 State Space Form

Let z_t be a *p-vector* of observed variables at time t. A general *state space form* relates z_t in the sample period $t = 1, 2, .., T$ with a possibly unobserved *k-vector* μ_t by the following system[1]:

$$z_t = X_t \mu_t + w_t, \tag{2.1}$$

$$\mu_{t+1} = F\mu_t + \rho + v_{t+1}, \tag{2.2}$$

where (2.1) and (2.2) are, respectively, the *observation equation* and the *state equation.* In equation (2.1), X_t is a $(p \times k)$ matrix of exogenous or predetermined variables and w_t is a *p-vector* of serially uncorrelated disturbances with mean 0 and covariance matrix H. In equation (2.2), μ_t is generated by a first order *vector-autoregressive* process, with the *k-vector* ρ as a *drift* and v_t as a *k-vector* of white noise with mean 0 and covariance matrix Q. The disturbances w_t and v_t are assumed to be uncorrelated at any lags.

[1]The overview of the Kalman filter given in this and in sections 2 and 3 follows the more recent literature surveys on the topic by Harvey (1990) and Hamilton (1994a, 1994b, chapter 13).

Usually, the system (2.1)-(2.2) is used to describe a finite series of observations $\{z_1, z_2, ..z_T\}$, so that assumptions about the initial value of the state vector μ_t are needed. In particular, μ_1 is assumed to be uncorrelated with any realizations of v_t or w_t. Moreover, the mean of μ_1 is assumed to be equal to m_1 with covariance matrix P_1.

The system (2.1)-(2.2) is enough flexible and it can be easily generalized to a system in which the matrices H, F, ρ and Q are *time-varying*.

2.1.2 Recursions

A model like (2.1)-(2.2) is suitable for the application of the Kalman filter. This is a recursive procedure for computing the optimal estimator of the state vector μ_t based upon the information available at time t.

Let m_{t-1} denote the optimal estimator of μ_{t-1} based on all the observations up to and including time $(t-1)$. Let $Z_{t-1} = \{z_{t-1}, z_{t-2}, .., z_1,\ X_{t-1}, X_{t-2}, .., X_1\}$ denote this information, so that

$$m_{t-1} = E\left[\mu_{t-1}/Z_{t-1}\right]. \tag{2.3}$$

The covariance matrix of the estimation error is denoted as P_{t-1}, i.e.

$$P_{t-1} = E\left[\left(\mu_{t-1} - m_{t-1}\right)\left(\mu_{t-1} - m_{t-1}\right)'\right]. \tag{2.4}$$

It can be proved from the law of iterating projections[2] that, given m_{t-1} and P_{t-1}, the optimal estimator of μ_t is given by

$$m_{t/t-1} = Fm_{t-1} + \rho, \tag{2.5}$$

with covariance matrix of the estimation error

$$P_{t/t-1} = FP_{t-1}F' + Q. \tag{2.6}$$

[2]See, for example, Hamilton (1994b, p.379).

Equations (2.5) and (2.6) are known as *prediction equations*.
Once new observations z_t become available the estimator of μ_t, m_t and its covariance matrix P_t, can be updated by the following *updating equations*

$$\begin{aligned} m_t &= m_{t/t-1} + P_{t/t-1}X_t' \left(X_t P_{t/t-1} X_t' + H\right)^{-1} \left(z_t - X_t m_{t/t-1} - \rho\right), &(2.7)\\ P_t &= P_{t/t-1} - P_{t/t-1}X_t' \left(X_t P_{t/t-1} X_t' + H\right)^{-1} X_t P_{t/t-1}. &(2.8) \end{aligned}$$

From starting values specified both in terms of m_0 and P_0 or $m_{1/0}$ and $P_{1/0}$, the Kalman filter produces the optimal estimator of the state vector as each new observation becomes available. At time T the filter yields optimal estimator of the current state vector based on the full information set. Moreover, optimal predictions of future values of both m_t and z_t can be performed.
In particular, computing predictions $\hat{z}_{t/t-1}$ of z_t conditional on Z_{t-1} and X_t gives

$$\hat{z}_{t/t-1} = E\left[z_t/X_t, Z_{t-1}\right] = X_t m_{t/t-1}. \tag{2.9}$$

Note that X_t contains no information about μ_t beyond that contained in Z_{t-1}, because it is assumed predetermined or exogenous. As a result, $E\left[\mu_t/X_t, Z_{t-1}\right] = E\left[\mu_t/Z_{t-1}\right] = m_{t/t-1}$ and equation (2.9) can be derived by the law of iterated projections. The mean square error of this forecast is

$$G_t = E\left[\left(z_t - \hat{z}_{t/t-1}\right)\left(z_t - \hat{z}_{t/t-1}\right)'\right] = X_t P_{t/t-1} X_t' + H. \tag{2.10}$$

2.1.3 Initial Conditions

In the state space form (2.1)-(2.2) the starting values for the Kalman filter can be given by the mean and covariance of the unconditional distribution of the state vector denoted, respectively, as m and Σ, only if eigenvalues of the matrix F are all inside the unit circle. In other terms, only if μ_t is covariance-stationary.

This being the case, taking expectations of both sides of (2.2) and considering that $E(\mu_t) = E(\mu_{t-1}) = m$ because of stationarity of μ_t, it yields

$$m = E(\mu_t) = (I - F)^{-1}\rho, \tag{2.11}$$

where the matrix $(I - F)$ is nonsingular because no eigenvalue of F equals to 1.

The unconditional variance of μ_t, i.e. Σ, can be derived by the following steps: postmultiplying

$$(\mu_{t+1} - E[\mu_{t+1}]) = F(\mu_t - E[\mu_t]) + v_{t+1}$$

by its transpose and taking expectations it yields

$$Var(\mu_{t+1}) = F \cdot Var(\mu_t) \cdot F' + Q. \tag{2.12}$$

Because the process μ_t is covariance-stationary, $Var(\mu_{t+1}) = Var(\mu_t) = \Sigma$ and it can be shown[3] that solution is given by

$$vec(\Sigma) = [I_{k^2} - (F \otimes F)]^{-1} vec(Q), \tag{2.13}$$

where $vec(\cdot)$ is the operator that transforms a matrix in a vector by stacking the columns, $\otimes$ indicates the *Kronecker product* and I_{k^2} is the k^2 identity matrix.

If, instead, some eigenvalues of F are on or outside the unit circle, the *state equation* is not stationary and the unconditional distribution of μ_t is not defined. The distribution of μ_1 is given by genuine prior information or by a *diffuse prior*; $m_{1/0}$ is replaced with the best guess for the initial value of μ_1 and $P_{1/0}$ is a positive definite matrix summarizing the confidence in this guess.

[3] See, for example, Magnus and Neudecker (1988).

2.2 Maximum Likelihood Estimation

2.2.1 Computing the Likelihood Function

As stressed in the previous section, the forecasts $m_{t/t-1}$ and $\hat{z}_{t/t-1}$ calculated by the Kalman filter are linear functions of (X_t, Z_{t-1}). Furthermore, if the initial state vector $m_{1/0}$ and the disturbances (v_t, w_t) are multivariate Gaussian, $m_{t/t-1}$ and $\hat{z}_{t/t-1}$ are optimal among any function of (X_t, Z_{t-1}). Then, the distribution of z_t conditional on (X_t, Z_{t-1}) is also Gaussian with mean given by equation (2.9) and variance by equation (2.10).

Gaussian assumption, therefore, allows the construction of the sample log likelihood L starting from the usual expression of a multivariate normal distribution, that is

$$L = \frac{pT}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|G_t| - \frac{1}{2}\sum_{t=1}^{T}\left(z_t - \hat{z}_{t/t-1}\right)' G_t^{-1}\left(z_t - \hat{z}_{t/t-1}\right), \quad (2.14)$$

where $\hat{z}_{t/t-1}$ and G_t are values defined in equations (2.9) and (2.10), respectively. Maximization of L with respect to the unknown parameters in the matrices F, ρ, H and Q can be found by a numerical optimization routine.

The log likelihood function L can be computed iterating on the Kalman filter from proper starting values $m_{1/0}$ and $P_{1/0}$ and initial guesses of F, ρ, H and Q. Employing an optimization routine (e.g. the Newton-Raphson method), these initial guesses are gradually improved until equation (2.14) is maximized.

2.2.2 Generalized Least Squares

Running an optimization routine to maximize the likelihood function (2.14) with respect to the unknown parameters could be very risky when the parameter space is high; difficulties can arise in locating the global maximum of L. So, it becomes very important to exploit any linearities in the state space form, in order to reduce the dimension of the search.

In other terms, it is convenient to find proper reparametrizations of the state space form, so that a concentrated likelihood function could be computed.

This section provides an explanatory example of Generalized Least Squares (GLS) estimation of ρ.

For convenience, let's rewrite here the general state space model (2.1)-(2.2):

$$z_t = X_t \mu_t + w_t, \tag{2.15}$$

$$\mu_{t+1} = F\mu_t + \rho + v_{t+1}. \tag{2.16}$$

When the *state-vector* μ_t is covariance-stationary, equation (2.11) has shown that

$$\rho = (I - F)\, m.$$

Defining $\xi_t = \mu_t - m$, the system (2.15)-(2.16) can be put as follow:

$$z_t = X_t \xi_t + X_t m + w_t, \qquad t = 1, .., T, \tag{2.17}$$

$$\xi_{t+1} = F\xi_t + v_{t+1}. \tag{2.18}$$

This allows the following *regression form* representation:

$$z_t = X_t m + u_t, \tag{2.19}$$

$$u_t = X_t \xi_t + w_t. \tag{2.20}$$

If it is assumed that ξ_0 has a mean of zero and a bounded covariance matrix P_0, then the expected value of u_t is zero for all t but is, in general, serially correlated and heteroskedastic. In this form, the *GLS* estimation of m can be performed by the generalized formula

$$\hat{m} = \left[\sum_{t=1}^{T} X_t^{*'} G_t^{-1} X_t^{*}\right]^{-1} \sum_{t=1}^{T} X_t^{*'} G_t^{-1} z_t^{*}, \tag{2.21}$$

where G_t is the variance-covariance matrix of u_t.

In a Kalman filter framework, $\hat{m}$ results from a concentrated likelihood function, without any need to evaluate, as usual, the Cholesky decomposition of the variance-covariance matrix of u_t.

However, a brief explanation of equation (2.21) is needed:

z_t^* and X_t^* result from the Cholesky decomposition of the variance covariance matrix that the Kalman filter implicitly performs. In fact, decomposing X_t into k *n-vectors* such that

$$X_t = [x_{1t}\ x_{2t}\ ....\ x_{kt}]$$

and applying $k+1$ times the Kalman filter where the state equation is given from the same (2.18) and the measurement equations are given separately from

$$\begin{aligned} z_t &= X_t \xi_t^z + w_t^z, \qquad (2.22)\\ x_{1t} &= X_t \xi_t^{x_1} + w_t^{x_1}, \\ & \quad ....., \\ x_{kt} &= X_t \xi_t^{x_k} + w_t^{x_k}, \end{aligned}$$

serially uncorrelated *innovations* z_t^*, $X_t^* = [x_{1t}^*\ x_{2t}^*\ ....\ x_{kt}^*]$ with identical covariance matrix G_t can be performed. In particular, z_t^* and X_t^* are given from

$$z_t^* = z_t - X_t \hat{\xi}_{t/t-1}^z, \tag{2.23}$$

$$
\begin{aligned}
x_{1t}^{*} &= x_{1t} - X_t \hat{\xi}_{t/t-1}^{x_1}, \\
&\ldots.., \\
x_{kt}^{*} &= x_{kt} - X_t \hat{\xi}_{t/t-1}^{x_k},
\end{aligned}
$$

where $\hat{\xi}_{t/t-1}^{z}$, $\hat{\xi}_{t/t-1}^{x_1}$, ..., $\hat{\xi}_{t/t-1}^{x_k}$ are obtained from the following recursions

$$
\begin{aligned}
\hat{\xi}_{t/t-1}^{z} &= F\hat{\xi}_{t-1/t-1}^{z}, \\
\hat{\xi}_{t/t-1}^{x_1} &= F\hat{\xi}_{t-1/t-1}^{x_1}, \\
&\ldots\ldots, \\
\hat{\xi}_{t/t-1}^{x_k} &= F\hat{\xi}_{t-1/t-1}^{x_k},
\end{aligned} \tag{2.24}
$$

with

$$
\begin{aligned}
\hat{\xi}_{t/t}^{z} &= \hat{\xi}_{t/t-1}^{z} + P_{t/t-1}X_t^{'}G_t^{-1}z_t^{*}, \\
\hat{\xi}_{t/t}^{x_1} &= \hat{\xi}_{t/t-1}^{x_1} + P_{t/t-1}X_t^{'}G_t^{-1}x_{1t}^{*}, \\
&\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots, \\
\hat{\xi}_{t/t}^{x_k} &= \hat{\xi}_{t/t-1}^{x_k} + P_{t/t-1}X_t^{'}G_t^{-1}x_{kt}^{*}.
\end{aligned} \tag{2.25}
$$

The matrix $P_{t/t-1}$ is the mean squared error of ξ_t, iteratively computed from the following equations:

$$
P_{t/t-1} = E\left[\left(\xi_t - \hat{\xi}_{t/t-1}\right)\left(\xi_t - \hat{\xi}_{t/t-1}\right)^{'}\right] = FP_{t-1/t-1}F^{'} + Q, \tag{2.26}
$$

$$
P_{t/t} = E\left[\left(\xi_t - \hat{\xi}_{t/t}\right)\left(\xi_t - \hat{\xi}_{t/t}\right)^{'}\right] = P_{t/t-1} + P_{t/t-1}X_t^{'}G_t^{-1}X_tP_{t/t-1}. \tag{2.27}
$$

Note that recursions of $P_{t/t-1}$ and $P_{t/t}$ are fully independent with respect to $\hat{\xi}_{t/t-1}$ and $\hat{\xi}_{t/t}$, allowing the calculations above.

Finally, the variance-covariance matrix G_t is estimated as

$$
G_t = X_tP_{t/t-1}X_t^{'} + H. \tag{2.28}
$$

As a result of the formalization above, m can be concentrated out of the likelihood. Then, estimation of the parameters F, H and Q are obtained by a concentrated likelihood function L_c such that

$$L_c = -\frac{Tn}{2}\log 2\pi - \frac{1}{2}\sum_{t=1}^{T}\log|G_t| - \frac{1}{2}\sum_{t=1}^{T}\hat{\varepsilon}_t' G_t^{-1}\hat{\varepsilon}_t, \tag{2.29}$$

in which G_t is estimated by equation (2.28), residuals $\hat{\varepsilon}_t$ are defined as

$$\hat{\varepsilon}_t = z_t^* - X_t^*\hat{m} \tag{2.30}$$

and, conditional on given values of F, H and Q, $\hat{m}$ is performed by equation (2.21).

2.2.3 Missing Observations

All observations are assumed to be available in the discussion provided so far. In fact, some observations may be missing or subject to contemporaneous aggregation. It means that the full *p-vector*, now denoted as $z_t^\dagger$, is not necessary equal to the *p_t-vector* of observations z_t. Possible solutions to this problem are introduced in this sub-section for the following three cases:

1. $p_t \geq 1$ *for all* t $\rightarrow$ only some components of $z_t^\dagger$ are missing or contemporaneously aggregated. In this case the identity

$$z_t = W_t z_t^\dagger, \quad t = 1, ..., T, \tag{2.31}$$

is defined, where W_t is a $p_t \times p$ matrix of fixed weights. The measurement equation is now given by combining equation (2.1) with (2.31). The main difference is that the dimension of z_t is *time-varying*, without particular consequences on the Kalman filter and the *prediction error decomposition* of the likelihood function.

2. $p_t = 0$ *for some* $t \rightarrow$ no observations are available for certain t. In this case, equation (2.31) is no longer defined, so that it is assumed that observations are available only at the points t_τ, τ = *1, .., T*, where the t_τ*'s* are integers such that $0 < t_1 < t_2 \cdots < t_T$. So, equation (2.31) is replaced by

$$z_\tau = W_\tau z^\dagger_{t_\tau}, \quad \tau = 1, ..., T. \tag{2.32}$$

The system generates t_T values of $z^\dagger_t$ at unit intervals, but observations on this vector are only made in T *not-evenly* time periods. In this case as well, the particular form of the system does not affect the prediction error decomposition. Prediction errors associated with the observations z_τ, τ = *1, .., T* can be obtained by *skipping* the Kalman filter, updating equations for the state space form of $z^\dagger_t$ at the points where there are no observations (Jones, 1980). Thus, if missing observations are at *t=n*, values given by the *updating equations* (2.7) and (2.8) are simply substituted by their corresponding *prediction equations*:

$$m_n = m_{n/n-1}, \quad P_n = P_{n/n-1}. \tag{2.33}$$

3. $p_t = 0$ *for some* $t \rightarrow$ no observations are available for certain t. In this case the same problem as point 2. is handled in an alternative way: a value of zero is given to a missing observation and a dummy variable is introduced into the model. The dummy variable takes a value of unity at the point where missing occurs and zero elsewhere. The likelihood is then constructed for the full sample period but it needs to be maximised with respect to the coefficient of the dummy variable as well. When several observations are missing, problems could arise because of the high number of parameters to be estimated. For this reason this method seems suitable just for handling a small number of observations.

2.3 Smoothing

If filtering performs the expected value of the state vector μ_t conditional on information at time t, smoothing concerns an inference on μ_t based upon information available after time t. Let's denote Z_τ the information up to and including time τ, for $\tau > t$. Then, like the notation in previous sections suggests, the smoothed estimator of μ_t can be expressed as

$$m_{t/\tau} = E\left[\mu_t / Z_\tau\right] \quad for\ \tau > t, \tag{2.34}$$

with covariance matrix denoted by

$$P_{t/\tau} = E\left[\left(\mu_t - m_{t/\tau}\right)\left(\mu_t - m_{t/\tau}\right)'\right]. \tag{2.35}$$

When $\tau = T$, then $m_{t/T}$ is called *fixed-interval smoother*. If $t = \tau - j$ for $j = 1, .., M$, where M is some maximum lag, then $m_{\tau-j/\tau}$ is called *fixed-lag smoother*. Finally, *fixed-point smoothing* is the algorithm concerning the estimation of the state vector μ_t at some fixed point in time.

In economic literature the fixed interval smoothing is the most spread, so that only a short introduction of this algorithm is provided in this section[4]. It consists of a set of backward recursions for time t = *T-1, T-2, .., 1*, starting from the final estimates m_T and P_T of the standard Kalman filter. $m_{t/T}$ and $P_{t/T}$ are given by the following 3 recursive equations:

$$\begin{aligned} m_{t/T} &= m_t + J_t\left(m_{t+1/T} - F m_t\right), \\ P_{t/T} &= P_t + J_t\left(P_{t+1/T} - P_{t+1/t}\right) J_t', \\ J_t &= P_t F'\, P_{t+1/t}^{-1}, \end{aligned} \tag{2.36}$$

with $m_{T/T} = m_T$ and $P_{T/T} = P_T$.

[4]For a full detailed illustration of fixed-lag and fixed-point smoothing algorithm see Anderson and Moore (1979). A concise introduction to the latter can be also found in Harvey (1990).

2.4 Incorporating Time-Varying Restrictions

When exogenous informations provide estimates of μ_t for some t, its estimation by the Kalman filter using the methodologies introduced in the previous sections is efficient only if the original model (2.1)-(2.2) is appropriately constrained by the observed values. Such *extraneous information* can be incorporated into the model in the form of linear constraints that the parameters of the model should satisfy.

A property of the Kalman filter which is of fundamental importance at this purpose is that *time-varying* restrictions in the linear form

$$R_t \mu_t = r_t \tag{2.37}$$

can be incorporated into the model (2.1)-(2.2) so that estimates $\hat{\mu}_t$ can be obtained to satisfy equation (2.37). Considering that additional information, aggregation constraints or specific hypothesis could be available, methodologies to handle constrained estimates have to be applied.

Two different approaches can be mentioned. The more general one (Doran, 1992), consists of augmenting the observation equation (2.1) defining

$$z_t^* = \begin{bmatrix} z_t \\ r_t \end{bmatrix}, \; X_t^* = \begin{bmatrix} X_t \\ R_t \end{bmatrix}, \; w_t^* = \begin{bmatrix} w_t \\ 0 \end{bmatrix}, \tag{2.38}$$

with

$$E\left(w_t^* w_t^{*\prime}\right) = \begin{bmatrix} H & 0 \\ 0 & 0 \end{bmatrix}, \tag{2.39}$$

so that the observation equation

$$z_t^* = X_t^* \mu_t + w_t^* \tag{2.40}$$

can be associated with the *state-equation* (2.2). The usual Kalman filter methodology applied to equations (2.40) and (2.2) provides optimal *smoothed* estimates of μ_t which satisfy linear time-varying constraints. Note that this approach is extremely flexible because no mention of the row dimension of R_t and r_t is given: it can actually vary across time. This allows the incorporation of non-homogeneous information into the model whatever the linear form of equation (2.37).

Anyhow, as stressed in Doran and Rambaldi (1997), practical problems arise when computation of the Kalman filter is performed with an high number of parameters to be estimated. Difficulties in locating the global maximum of the likelihood function may occur. Thus, it becomes important to find a proper reparametrization of the state space model (2.40)-(2.2), in order to reduce the dimension of the parameter-space. Simple reparametrizations are available which allow ρ to be estimated by generalized least squares, computing a concentrated likelihood function.

The second approach (Doran and Rambaldi, 1996), more computationally efficient but less flexible, can be applied only when the row dimensions of R_t and r_t are constant over time. Instead of *augmenting* the observation equation, time-varying constraints are *substituted out*, reducing the dimension of the parameter space. The Singular Value Decomposition (SVD) Theorem (see for example Magnus and Neudecker, 1988, p.18) is used to achieve a convenient reparametrization of the model (2.1)-(2.2).

Suppose that the row dimension of R_t is J for all t. Then, by definition, the rank of R_t is J. The SVD states that two square matrices U_t and V_t of dimension J and k, respectively, corresponding to the *left* and *right* eigenvectors of R_t exist such that

$$R_t = U_t S_t V_t', \tag{2.41}$$

where S_t is a $(J \times k)$ *diagonal* matrix with non-zero singular values s_{1t}, s_{2t}, ..., s_{Jt} on the principal diagonal. Alternatively, the following standard result can be obtained:

$$P_t R_t V_t = [I_J \; 0_{J,k-J}], \tag{2.42}$$

where $P_t = S_{1t}^{-1} U_t'$, and $S_{1t} = diag(s_{1t}, s_{2t}, ..., s_{Jt})$, I_J is the J identity matrix and $0_{J,k-J}$ a $(J \times k - J)$ null matrix.

From equation (2.42) it is possible to reparametrize the constrained model given by equations (2.1) and (2.37) recognizing that equation (2.37) can alternatively be written as

$$\begin{aligned} r_t &= \left[P_t^{-1} \; 0_{J,k-J}\right] V_t^{-1} \mu_t \\ &= \left[P_t^{-1} \; 0_{J,k-J}\right] \mu_t^*, \end{aligned} \tag{2.43}$$

with $\mu_t^* = V_t^{-1} \mu_t$. If the partition of μ_t^* such that $\mu_t^* = \left[\mu_{1t}^{*'} \; \mu_{2t}^{*'}\right]'$ is considered, we obtain

$$\mu_{1t}^* = P_t r_t \tag{2.44}$$

and the following reparametrization of (2.1) is achieved:

$$\begin{aligned} z_t &= X_t V_t V_t^{-1} \mu_t + w_t \\ &= X_t^* \mu_t^* + w_t \\ &= X_{1t}^* \mu_{1t}^* + X_{2t}^* \mu_{2t}^* + w_t, \end{aligned} \tag{2.45}$$

where X_{1t}^* and X_{2t}^* are the first J and the remaining k-J columns of X_t^* respectively.

Finally, substituting equation (2.44) into (2.45),

$$z_t - X_{1t}^* P_t r_t = z_t^* = X_{2t}^* \mu_{2t}^* + w_t, \tag{2.46}$$

it should be observed that it is enough to estimate the parameter vector μ_{2t}^* to obtain estimates of the original parameter μ_t.

2.5 Data Reconstruction

Assume that in the model (2.1)-(2.2) z_t is observed only for the last l time periods (for $t = T\text{-}l\text{+}1, \ldots, T$). Restricted estimation of the parameters ρ, F, H, Q in order to obtain *smoothed* estimates of μ_t for $t = 1, 2, \ldots, T$ cannot be directly reached by the Kalman filter. In fact, the *skipping approach* (Jones, 1980) for which all the missing observations of z_t are substituted by the Kalman filter *updated estimates* cannot gain reliable results when the number of observed values of z_t (i.e. l) is too small with respect to the full sample (i.e. T). For the same reason, handling missing observations by giving them a value of zero and introducing *dummy* variables into the model is also inefficient. Moreover, serious theoretical problems occur when missings are located at the beginning of the sample period.

As a solution, alternative state space representations could be set up when many observations are missing or subject to contemporaneous aggregation. The Kalman filter together with the smoothing algorithm can be applied to gain efficient estimates of missing observations[5]. Another possible choice has given by *retrapolation* procedures, even if less flexible in accommodating restrictions which change in each time period.

In this section a new approach for data reconstruction is suggested. The time series to be reconstructed is considered as an unobserved state variable in a state space model and the few available observations as *time-varying* restrictions. The methodology of section 4 is then applied to get efficient

[5]For a complete survey of the development in the literature on *missing observations* and related topics see Harvey (1990). For more recent contributions in a Kalman filter framework and for a computer program performing estimation, forecasting and interpolation of regression models with missing observations and ARIMA errors see Gomez and Maravall (1994, 1996).

smoothed estimates of z_t satisfying restrictions given by those observations of z_t which are available.

Reconstruction for the period t = *1, 2, ..., T-l* of each element z_{it} (i = *1, 2, ...,* p) of z_t is obtained estimating, separately, parameters of p different state space models, one for each element of z_t. In order to obtain the reconstruction, it is supposed that an *n-vector* y_t of related stochastic indicators of z_t are available over all the sample period[6].

Let $y_t^{s_i}$ be a κ_i*-vector* of selected indicators from the *n-vector* y_t, that is the most convenient selection of elements of y_t to reach the reconstruction of z_{it}. Then, the following state space representation can be considered:

$$
\begin{aligned}
y_t^{s_i} &= G_i \xi_{it} + \varepsilon_{it}, & (2.47)\\
\xi_{it+1} &= \xi_{it} + \lambda_i + \eta_{it+1}. & (2.48)
\end{aligned}
$$

G_i is a $(\kappa_i \times 2)$ matrix of parameters with a κ_i*-vector* of *ones* as first column; ξ_{it} is a *bivariate state vector* such that

$$
\xi_{it} = \left[d_{it}\ z_{it}^{\dagger}\right]'. \qquad (2.49)
$$

$z_{it}^{\dagger}$ and d_{it} express, respectively, the unobserved *i-th* element of z_t to be reconstructed and an identical *time-varying* coefficient; ε_{it} and η_{it} are the usual *i.i.d.* Gaussian *white noise errors* uncorrelated with each other at any lags and with covariance matrix, respectively, H_i^{ε} and Q_i^{η}; finally, λ_i is a *bivariate drift* on the *state equation* (2.48).

Observed values for z_{it} enter into the model as time-varying restrictions on ξ_{it}; notably, for t = *T-l+1, ...,* T, constraints imply

[6]An application to population projections can be found in Doran (1996). Nevertheless, that paper concerns with the use of the Kalman filter as a technique to gain *interpolations* which should obey time-varying linear constraints.

$$[0\ 1]\,\xi_{it} = z_{it}. \tag{2.50}$$

Restricted maximum-likelihood estimation of $G_{i,}$ $\lambda_{i,}$ H_i^{ε}, Q_i^{η} and an optimal estimation of ξ_{it} by the Kalman filter is carried out through the methodologies explained in the previous sections.

In practice, the *observation equation* (2.47) is a *generalized* regression with an unobserved regressor $z_{it}^{\dagger}$ and a *time-varying* coefficient d_{it} to fit the difference between each element of $y_t^{s_i}$ and $z_{it}^{\dagger}$.

Assuming non-stationarity of y_t, reconstructed values of z_t should be non-stationary too. Then, the *state equation* (2.48) captures the dynamics of $z_{it}^{\dagger}$ and d_{it} in terms of a random walk where the drift λ_i affects the *direction* of the random movement of $z_{it}^{\dagger}$ and d_{it} over the time. Since restrictions in the form of equation (2.50) are imposed, the estimation of λ_i is strongly dependent on the observed value of z_t in the last l time periods.

Assumptions behind the state space form of equations (2.47)-(2.48) refer to a basic but significant case: in fact each element z_{it} of z_t is assumed to be equally generated by a trend component only which is additive with respect to the trend of $y_t^{s_i}$. In particular this trend component is assumed to be generated by a random walk with drift process. Cycle or seasonal components are not considered, even if the state space form (2.47)-(2.48) could be easily extended to incorporate more complicated data generating processes of z_t.

3 A Framework for a Change of Classification

3.1 Basic Concepts

Generally, statistical information is collected from National Statistical Agencies using conventional procedures. Among countries, harmonized survey schemes allow the collection of economic statistics directly from firms or individuals (*Economic Activity Unit, EAU)* and International standards are provided in order to harmonise sectoral definitions.

Classification standards univocally define the economic activities, their number and aggregation levels. According to the last international revision (United Nations, 1989), 5 aggregation levels are considered. Notably, 874 *categories* of economic activities, make up 512 *classes*, 222 *groups*, 60 *divisions* and 17 *sections*.

A larger degree of detail has been introduced with respect to the previous standards: only 675 *categories* and 4 aggregation levels (United Nations, 1969). The goal has been to guarantee more accuracy of coverage in the diversification of economic activities, sampling previously non-existent activities.

Not only the introduction of new activities, but even the implicit *split* within different aggregations that new classification implies lets the comparison in terms of *old* and *new* standards be formalized. It is possible that new standards split up economic activities in such a way that some *EAUs* joining

some *old* activities fall into different *new* aggregates.
In practice, because of the historical de-industrialization process, new standards attempt to capture economic structural change from a qualitative point of view. Qualitative change means new commodities, services and a diversification of economic activities. The diversification goes from *traditional* to *service-oriented* activities.
Assume that over a given time period *EAUs* available for a given economic variable are classified with respect to two different classification standards. Following both the definitions, aggregating on economic activities in *classes, groups* and so on determines a *compositional effect* among aggregates. As a limit case, a uniform *aggregation* in *Agriculture, Industry* and *Services* (i.e. *macro sectors*) gives the advantage to let the *compositional effect* express in terms of differences among uniform aggregates.
In the following subsections a formal exposition of the problem is attempted.

3.1.1 Deterministic Preliminaries

Let x_t^* denote a m_t-*vector* of *EAUs* values for a given economic variable at time t. It is assumed that x_t^* is evenly sampled at a given frequency and it is available for the time period $t = 1, 2, \ldots, T$. Its dimension is *time-variant* because of the variability over time in the number of sampled *EAUs*. Aggregation of x_t^* into n ($n < m_t$ for all t) *sectors* is obtained defining a binary *0/1* $(n \times m_t)$ *aggregation matrix* A_t such that

$$y_t = A_t x_t^*. \tag{3.1}$$

y_t represents a n-*vector* of aggregated data in n *sectors*[7]. A_t is a full row rank matrix where every column sums to unity and it is unique for each

[7]For instance, n could be the number of considered sectors or *branches* for the National Accounts estimates.

classification standard. A scalar sum Y_t of the elements of y_t (e.g. GDP) is also defined.

In these terms, a different classification standard means a new $(p \times m_t)$ aggregation matrix B_t (with $n \leq p < m_t$ for all t) such that

$$z_t = B_t x_t^*, \tag{3.2}$$

where z_t represents the new p-vector of aggregated data in p *sectors* and B_t, as A_t, is a full row rank matrix where every column sums to unity. Moreover a scalar Z_t as sum of the elements of z_t can be defined with the same meaning of Y_t.

3.2 Conversion Matrix

Suppose now that the new classification modifies only the composition, without any changes for the total aggregate so that $Y_t = Z_t$. Then, a $(p \times n)$ *conversion matrix* C_t is uniquely defined given A_t, B_t and x_t such that

$$z_t = C_t y_t, \tag{3.3}$$

where

$$C_t = B_t x_t^{*d} A_t' \left(A_t x_t^{*d} A_t' \right)^{-1} \tag{3.4}$$

and x_t^{*d} is a m_t-*square matrix* defined as

$$x_t^{*d} = diag\left(x_{1t}^*, x_{2t}^*, .., x_{m_t t}^* \right). \tag{3.5}$$

In detail, equations (3.3) and (3.4) can be obtained as follows: denoting as i_n' a row of ones, the equality $i_n' A_t = i_m'$ is given as the columns of A_t sum to unity. Taking the transposes, it implies $A_t' i_n = i_m$ from which, by repeated substitutions,

$$
\begin{aligned}
x_t^d i_m &= x_t, & (3.6)\\
y_t &= A_t x_t = A_t x_t^d i_m = A_t x_t^d A_t' i_n, & (3.7)\\
\left(A_t x_t^d A_t'\right)^{-1} y_t &= i_n, & (3.8)\\
A_t' \left(A_t x_t^d A_t'\right)^{-1} y_t &= A_t' i_n = i_m, & (3.9)\\
x_t^d A_t' \left(A_t x_t^d A_t'\right)^{-1} y_t &= x_t^d i_m = x_t, & (3.10)\\
B_t x_t^d A_t' \left(A_t x_t^d A_t'\right)^{-1} y_t &= B_t x_t = z_t. & (3.11)
\end{aligned}
$$

B_t is an allocation matrix as well, then $i_p' B_t = i_m'$ and the equality $Y_t = Z_t$ results from previous results, given that

$$Z_t = i_p' z_t = i_p' B_t x_t = i_m' x_t = X_t = i_n' A_t x_t = i_n' y_t = Y_t. \qquad (3.12)$$

C_t is a full column rank matrix and, as for A_t and B_t, every column sums to 1. Each element of C_t, c_t^{ij} ($i = 1, 2, .., p$ and $j = 1, 2, .., n$) is bounded between zero and one

$$0 \leq c_t^{ij} \leq 1; \qquad i = 1, 2, .., p; \; j = 1, 2, .., n.$$

In other terms, each element c_t^{ij} of the *conversion matrix* C_t gives a *transition weight* from a *sector* of the old classification to a *sector* of the new one. If the definition of the *i-th* new *sector* (in terms of either joined economic activities or *EAUs*) is precisely the same as the *j-th* sector of the old classification $c_t^{ij} = 1$; if there is a *split* from the *j-th old* sector into more than one *new* sectors, then $0 < c_t^{ij} < 1$; finally, if there are no linkages $c_t^{ij} = 0$. Then, for every i, j such that $c_t^{ij} = 0$ or $c_t^{ij} = 1$, this holds for all t ($t = 1, 2, ...T$) and *time-invariant* restrictions on C_t have to be imposed.
Note that by equation (3.3) it results a mapping from y_t to z_t by the matrix C_t which is defined only for given A_t, B_t and x_t. In fact, when the available

information is referred only to y_t and z_t the matrix C_t is not uniquely defined since equation (3.3) is a system of p equations into $p \times n$ unknowns. Anyhow equation (3.3) should be meant as a relation among aggregated data in terms of a matrix of weights, for which the information on detailed sectoral allocations is definitely lost.

A more convenient representation of equation (3.3) in order to eliminate *time-invariant* restrictions in terms of zero elements of C_t is the following:

$$z_t = (y_t' \otimes I_p)\, vec\,(C_t)\,, \tag{3.13}$$

where $\otimes$ denotes the *Kronecker-product*, I_p the *p-identity matrix*, and $vec(\cdot)$ the *vec-operator* that transforms a $(p \times n)$ matrix in a *np-vector* by stacking the columns. Then, using a $(k^* \times np)$ *selection matrix* S_β[8], a k^**-vector* β_t can be defined such that

$$\beta_t = S_\beta\ vec(C_t) \tag{3.14}$$

or, equivalently,

$$S_\beta' \beta_t = \ vec(C_t). \tag{3.15}$$

Notably, S_β is a *block diagonal* matrix given by

$$S_\beta = diag\,(S_{\beta 1}, S_{\beta 2}, .., S_{\beta n})\,, \tag{3.16}$$

where $S_{\beta j}$ $(j = 1, 2, .., n)$ is the proper $(k_j^* \times p)$ *selection matrix* for *time-varying* coefficients of each column of C_t. Obviously $k_j^* \leq p$; $k^* = \sum_j k_j^*$ and S_β is a full rank matrix.

[8]A $((k-d) \times k)$ *selection matrix* S is an operator such that

$$x^* = S\ x,$$

where x is a *k-vector* and x^* a *(k - n)-vector* equal to selected elements of x. S is an identity matrix without those rows corresponding to elements of x to be eliminated.

Then, through a $(n \times k^*)$ *block diagonal* matrix R_β defined by

$$R_\beta = diag\left(1'_{k_1^*}, 1'_{k_2^*}, .., 1'_{k_n^*}\right), \tag{3.17}$$

where $1_{k_j^*}$ is a $(k_j^* \times 1)$ vector of *1s*, it is possible to express, in terms of β_t, the property that every column of C_t sums to one as the following:

$$R_\beta \beta_t = 1_n, \tag{3.18}$$

where 1_n denotes a *n-vector* of ones

The analytical framework of a change of classification given so far is extremely useful when full information in terms of the matrices A_t, B_t and x_t is available[9]. Diversification of economic activities and split within *sectors* have been expressed in a *matrix notation* so that the conversion matrix C_t can be analytically computed.

Nevertheless, homogeneous and very detailed data sets are rarely available across several years. Whenever they are available, classifying the *EAUs* with respect to different standards is possible only for more recent observations, since new standards have typically been introduced. As a result, if A_t, B_t and x_t^* are given only for t = *T - l + 1, ..., T*, then β_t is available only for the last l time periods. Then, assuming that y_t and z_t are observed for t = *1, 2, ..., T*, a model to capture the dynamics of β_t has to be considered.

3.3 Dynamics

The dynamics of the *conversion matrix* can be represented through a *state space model* in which β_t is the *unobserved state variable* and the *measurement equation* is a generalization of the deterministic equality given by (3.13). In particular, the following system is considered:

[9] Sometimes Statistical Agencies compute conversion matrices for a significant economic variable (e.g. employment) to have a first criterion for conversion of other variables.

$$
\begin{aligned}
z_t &= X_t \mu_t + w_t, && (3.19) \\
\mu_{t+1} &= F \mu_t + \rho + v_{t+1}. && (3.20)
\end{aligned}
$$

Equation (3.19) is the *observation equation* in which z_t is the *p-vector* of sectoral composition in terms of the *new* classification as in (3.2); X_t is a $(p \times k)$ matrix, where $k = p + k^*$ and such that

$$X_t = \left(y_t^{*\prime} \otimes I_p\right) S_\mu', \tag{3.21}$$

with y_t^* the *(n+1)-vector* defined by

$$y_t^* = \left[1 \;\; y_t'\right]', \tag{3.22}$$

in which y_t is the *n-vector* of sectoral composition in terms of the *old* classification as in equation (3.1), I_p is the *p identity matrix*, S_μ the $(k \times (p + np))$ *selection matrix* such that

$$S_\mu = \begin{bmatrix} I_p & 0 \\ 0 & S_\beta \end{bmatrix}, \tag{3.23}$$

with I_p the *p-identity matrix* and S_β as defined in equation (3.16); μ_t is the following *k-vector*

$$\mu_t = \left[\bar{\mu}_{1t} \; \bar{\mu}_{2t} \ldots . \bar{\mu}_{pt} \; \beta_t'\right]', \tag{3.24}$$

in which β_t represents the k^**-vector* as in equation (3.14); finally, w_t is a *p-vector* of *i.i.d. Gaussian white noise errors* with mean 0 and

$$E\left[w_t w_\tau'\right] = \begin{cases} H & for \; t = \tau \\ 0 & otherwise \end{cases}, \tag{3.25}$$

where H is a $(p \times p)$ *positive definite and symmetric matrix.*
From the deterministic latent variable framework of Section 3.2 a more general device has been provided with the state space model of equations

(3.19)-(3.20). This is justified by considering that a flexible fitting method with quadratic objective is needed to obtain an estimate of unrestricted elements of C_t.

Note that with respect to the deterministic representation given in equation (3.13), in each equation of (3.19) a *time-varying* constant term $\bar{\mu}_{it}$ has been added in order to take into account scale effects in measuring sectoral aggregates. So, Y_t is allowed to differ from Z_t[10].

Equation (3.20) is the *state equation* in which ρ is a *drift* on μ_t, F is a $(k \times k)$ matrix of parameters and v_t is a *k-vector* of *i.i.d. Gaussian white noise errors* with mean 0 and

$$E\left[v_t v_\tau'\right] = \begin{cases} Q & for\ t = \tau \\ 0 & otherwise \end{cases}, \tag{3.26}$$

where Q is a $(k \times k)$ *positive definite and symmetric matrix.* The disturbances w_t and v_t are assumed to be uncorrelated at all lags.

In equation (3.20) the dynamics of μ_t are modelled as a first order *vector-autoregressive* process. The first p elements of μ_t are free, whereas each element of β_t is positive, satisfying the restrictions given by equation (3.18).

Such a representation is appropriate to capture the dynamics of the compositional effect within sectors that is a characteristic of economic development. The conversion matrix C_t is assimilated to a stochastic process that switches over time from a matrix with weights close to 1 concentrated

[10]This is a not irrelevant aspect when revision of sectoral aggregates is referred to National Accounts. Often new goods, new services, specific transactions or the introduction of new methodologies have to be considered so that *new macro* sectors implyes additive terms. Then, the *p-vector* μ_t summarizes the effects on the system given by situations in which classification changes are accompanied by inclusion of relevant variables. In this form the sum over the p elements of μ_t represents the difference between Y_t and Z_t.

on the principal diagonal, to one with lower and more distributed weights among all the elements of C_t. Actually, higher weights on the principal diagonal of C_t correspond to a low conversion effect within classifications, whereas distribution of weights among all the elements of C_t corresponds to a higher compositional effect.

As $t \to 0$, the old classification standards should be considered fully adequate to represent the structure among economic activities: any conversion effect should be taken into account. For the limit case when p=n, C_0 can be assumed equal to the $(p \times p)$ *identity matrix* I_p.

As $t \to \infty$ the properties of C_t depend on F and on the restrictions given in equation (3.18). Provided that the eigenvalues of F are all inside the unit circle the process for μ_t in (3.20) is covariance stationary and a *steady-state value*[11] μ of μ_t can be obtained. Taking the expectations of both sides of equation (3.20), rearranging the terms and defining $\mu = E[\mu_t]$ produces

$$(I - F)\,\mu = \rho. \tag{3.27}$$

Observing that equation (3.18) can be rewritten as

$$\begin{bmatrix} 0_{n,p} & R_\beta \end{bmatrix} \mu = 1_n, \tag{3.28}$$

where $0_{n,p}$ is a $(n \times p)$ *null matrix* and μ_t has been replaced with μ, combining (3.27) and (3.28) the following expression is reached:

$$A_\mu \mu = \begin{bmatrix} \rho \\ 1_n \end{bmatrix}, \tag{3.29}$$

where A_μ is the $((n+k) \times k)$ full column rank matrix such that

$$A_\mu = \begin{bmatrix} (I - F) \\ 0_{n,p} \quad R_\beta \end{bmatrix}. \tag{3.30}$$

[11]For a discussion of the steady-state Kalman filter see Hamilton (1994b).

The solution for μ is found pre-multiplying (3.29) by $\left(A'_\mu A_\mu\right)^{-1} A'_\mu$, so that

$$\mu = \left(A'_\mu A_\mu\right)^{-1} A'_\mu \begin{bmatrix} \rho \\ 1_n \end{bmatrix}. \tag{3.31}$$

Alternatively, if some eigenvalues of F lie on or outside the unit circle, then A_μ is singular. Unique information as $t \to \infty$ is given by equation (3.18) which provides the limit value β of β_t through the *MP-inverse* of R_β[12]:

$$\beta = R'_\beta \left(R_\beta R'_\beta\right)^{-1} 1_n. \tag{3.32}$$

[12]For a definition of the *MP-inverse* of a matrix and its properties see, for instance, Magnus and Neudecker (1988).

4 An Application

In this section results of an experimental application are provided. A *two step* procedure to achieve data reconstruction and a parameter estimation of the change of classification model is suggested.

Let the following system be the analytical extension of the observation equation (3.19):

$$
\begin{aligned}
z_{1t} &= \bar{\mu}_{1t} + c_t^{11} y_{1t} + .. + c_t^{1n} y_{nt} + w_{1t} \\
z_{2t} &= \bar{\mu}_{2t} + c_t^{21} y_{1t} + .. + c_t^{2n} y_{nt} + w_{2t} \\
& \quad \ldots\ldots\ldots\ldots\ldots\ldots \\
z_{pt} &= \bar{\mu}_{pt} + c_t^{p1} y_{1t} + .. + c_t^{pn} y_{nt} + w_{pt} \qquad for\ t = 1, ..., T.
\end{aligned} \tag{4.1}
$$

The example considers the actual Italian quarterly value added at market prices as the *n-vector* $y_t = (y_{1t} \ ... \ y_{nt})'$, observed for the period *1970.1-1996.4* (*T=108*). Data are at constant prices for *1990* and *seasonally adjusted.* The dimension of y_t is *n=3* corresponding to the *old* sectors *Agriculture, Industry* and *Services.* $z_t = (z_{1t} \ ... \ z_{pt})'$ is the *p-vector* of sectoral value added in terms of the *new* classification. The example tries to model the conversion among same sectors of two different classifications, so that $p = n = 3$ and z_{1t}, z_{2t} and z_{3t} are, respectively, the *new* definitions of the same sectors as y_{1t}, y_{2t} and y_{3t}.

$\bar{\mu}_t = (\bar{\mu}_{1t} \ ... \ \bar{\mu}_{pt})'$ is the *p-vector* summarizing the scale effect of the introduction of new accounting methods; c_t^{ij}, for $i = 1, .., p$ and $j = 1, .., n$, are the element of the *time-varying conversion matrix* C_t. Finally, $w_t = (w_{1t}$

... $w_{pt})'$ is a *p-vector* of *i.i.d. Gaussian white noise errors* with mean 0 and *variance-covariance* matrix *H*.

4.1 The Generating Process of z_t

The application requires that z_t is provided for the last l observations. For l = *20* a simulation of z_t is performed so that a short series in terms of *new* standards is available for the period *1992.1-1996.4* (t = *89, ..., 108*).

The generating process of z_t for t = *89, ..., 108* starts from an arbitrary guess of the (3×3) *conversion matrix* $C_t^{(0)}$ and the (3×1) *vector* $\bar{\mu}_t^{(0)}$ at time t = *89*[13]. Since no *crossing* among definitions of *old Agricultural* economic activities and *new Services* has been found (United Nations, 1989), the element c_t^{13} of C_t is equal to zero for all t. Eliminating out this *time-invariant* restriction of C_t, the combination in an only vector of $\bar{\mu}_t$ and each element of C_t, column by column, produces the *k-vector* μ_t, with $k = 3 + 3 \times 3 - 1 = 11$. A first *non-restricted sampling* $\mu_t^{(1)}$ of μ_t for t = *89, ...108* can be performed by the state equation (3.20)

$$\mu_{t+1}^{(1)} = F\mu_t^{(1)} + \rho^{(0)} + v_t^{(0)}. \tag{4.3}$$

In this exercise standard errors $v_t^{(0)}$ have been drawn from a normal distribution with mean *0* and *variance-covariance matrix* $Q^{(0)}$, where $Q^{(0)}$ is *diagonal* with identical values for each sector given, respectively, by the

13

$$\begin{aligned} C_{89}^{(0)} &= \begin{pmatrix} .9855 & .0067 & 0 \\ .0140 & .9449 & .0101 \\ .0005 & .0484 & .9899 \end{pmatrix}, \\ \bar{\mu}_{89}^{(0)} &= (1.03)\, y_{89} - C_{89} y_{89}. \end{aligned} \tag{4.2}$$

For $\bar{\mu}_{89}^{(0)}$ an identical revaluation of 3% among sectors has been assumed for an hypothetical introduction of *new* accounting methods.

sample standard deviations of y_{1t}, y_{2t} and y_{3t} over the full time period (t = *1, ..., 108*). Notably,

$$Q^{(0)} = diag\left(\hat{\sigma}_{y_1}, \hat{\sigma}_{y_2}, \hat{\sigma}_{y_3}, \hat{\sigma}_{y_1}, \hat{\sigma}_{y_2}, \hat{\sigma}_{y_3}, \hat{\sigma}_{y_1}, \hat{\sigma}_{y_2}, \hat{\sigma}_{y_3}, \hat{\sigma}_{y_2}, \hat{\sigma}_{y_3}\right)^2. \tag{4.4}$$

For the *drift*, $\rho^{(0)}$ has been determined as follows:

$$\rho^{(0)} = \frac{\mu_{89}^{(0)} - \mu_1^{(0)}}{89 - 1}, \tag{4.5}$$

with

$$\mu_1^{(0)} = (0\ 0\ 0\ 1\ 0\ 0\ 0\ 1\ 0\ 0\ 1)'. \tag{4.6}$$

Equation (4.5) represents the *slope* of 11 straight lines passing through the points $\mu_{1i}^{(0)}$ and $\mu_{89i}^{(0)}$, for i = *1, ..., 11*. Note that $\mu_{1i}^{(0)}$ is a vector that implies *non-conversion* among classifications at time t = *1* and *non-scale effect*: in practice $z_1 = y_1$.

Assuming that each element of $\mu_t^{(1)}$ is *non-stationary*, the parameter matrix F is equal to the *identity matrix*. Starting from $\mu_{89}^{(0)}$, by iterative substitutions equation (4.3) can be rewritten as

$$\mu_t^{(1)} = \mu_{89}^{(0)} + \rho^{(0)}(t - 89) + \sum_{i=89}^{t} v_{i-89+1}^{(0)}, \quad for\ t = 89, .., 108. \tag{4.7}$$

For t = *89, ..., 108*, z_t is obtained by substituting each element of $\mu_t^{(1)}$ into equation (4.1), where the *variance-covariance matrix* H of w_t is assumed equal to the null matrix. Finally, for the same time period, restricted sampling values of μ_t have been generated running the Kalman filter iterations by *augmenting* equation (4.1) following the Doran (1992) methodology.

Restrictions on μ_t, in the form $R_t\mu_t = r_t$, have been fixed for t = *89*

$$R_t = [0_{8,3}\ I_8]\,,\ r_t = \left(\mu_{4,89}^{(0)}...\mu_{11,89}^{(0)}\right)'\,, \tag{4.8}$$

and for t = *90, ..., 108*

$$R_t = \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix},\ r_t = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}', \tag{4.9}$$

where $0_{8,3}$ is a (8×3) null matrix and I_8 is the (8×8) *identity matrix.* Note that, for t = *89*, equation (4.8) constraints the conversion matrix C_t to be equal to the given initial guess $C_{89}^{(0)}$. For t = *90, .., 108*, R_t equation (4.9) restricts every column of C_t to sum to unity. Finally, the coefficients of the (3×1) *vector* $\bar{\mu}_t$ are free for every t.

4.2 Reconstruction of z_t

Reconstruction of z_t for the period *1970.1-1991.4* (t = *1, ..., 88*) is carried out by the Kalman filter estimating, separately, parameters of three univariate state space models as in equations (2.47)-(2.48). In particular, $y_t^{s_i}$ in the observation equation (2.47) is 100-times the logarithm of y_{it} for i = *1, 2, 3* and t = *1, ..., 108.* Furthermore $G_1 = G_2 = G_3 = [1\ 1]$ and $H_1^{\varepsilon} = H_2^{\varepsilon} = H_3^{\varepsilon} = 0$[14], so that the system (2.47)-(2.48) becomes

$$\begin{aligned} y_{it} &= d_{it} + z_{it}^{\dagger} && (4.10) \\ \begin{pmatrix} d_{i,t+1} \\ z_{i,t+1}^{\dagger} \end{pmatrix} &= \begin{pmatrix} d_{it} \\ z_{it}^{\dagger} \end{pmatrix} + \begin{pmatrix} \lambda_i^d \\ \lambda_i^z \end{pmatrix} + \begin{pmatrix} \eta_{i,t+1}^d \\ \eta_{i,t+1}^z \end{pmatrix}. && (4.11) \end{aligned}$$

[14] In fact, estimation of these parameters gives as results values not significatively different from zero.

The *variance-covariance matrix* Q_i^η of $\eta_{it} = \left(\eta_{it}^d \ \eta_{it}^z\right)'$ is diagonal for every i, such that

$$Q_i^\eta = \begin{pmatrix} \sigma_i^{\eta^d} & 0 \\ 0 & \sigma_i^{\eta^z} \end{pmatrix}^2 . \tag{4.12}$$

Restrictions in the form

$$[0 \ 1] \begin{pmatrix} d_{it} \\ z_{it}^\dagger \end{pmatrix} = z_{i,t} \tag{4.13}$$

are imposed for $t = 1$, $z_{i1}^\dagger = y_{i1}$, that implies *non-conversion* among classifications; and for $t = 89, .., 108$, $z_{it}^\dagger = z_{it}$ because observations are available. Restricted estimates of z_t for $t = 2, .., 88$ are obtained by the Kalman filter *augmenting* the observation equation (4.10).

Results of estimation are shown in Table 1. For each sector ($i = 1, 2, 3$), $\hat{L}$ is the maximum of the sample log likelihood reached by the Newton-Raphson optimization routine; $\hat{\lambda}_i^d$ and $\hat{\lambda}_i^z$ are the estimates for the *bivariate drift* of the *state equation* (4.11); $\hat{\sigma}_i^{\eta^d}$ and $\hat{\sigma}_i^{\eta^z}$ are the estimates of the diagonal terms of Q_i^η. In brackets are the standard errors of the estimates[15].

Graphic results of the reconstruction of z_t are in figure 1, where the *smoothed estimates* (dashed line) for each *new* sector is shown together with the actual value added (solid line). For the last *20* observations generated values of z_t are considered. Both y_t and z_t are *seasonally adjusted.* Values are in *billions of Italian lira* at *1990* prices.

The ordinary Kalman filter has been used for estimating the state vector. Because of *non-stationarity* of (4.11) the iterations cannot be started with

[15]Standard errors of the estimates are obtained by square root of diagonal terms of the *information matrix*, estimated by second derivatives of the sample log likelihood function. For a discussion see Hamilton (1994b, p.143).

Table 1: Results of reconstruction for the period 1970-91 of the Italian value added following a simulated new sectoral classification. Quarterly seasonally adjusted data at constant prices.

Sector	$\hat{L}$	$\hat{\boldsymbol{\lambda}}_i^d$	$\hat{\boldsymbol{\lambda}}_i^z$	$\hat{\boldsymbol{\sigma}}_i^{\eta^d}$	$\hat{\boldsymbol{\sigma}}_i^{\eta^z}$
1.*Agriculture*	−230.6	−.0340	.2268	.1565	3.9737
		(.0152)	(.4369)	(.0237)	(.2721)
2.*Industry*	−144.0	−.0532	.5753	.2640	1.5724
		(.0257)	(.1519)	(.0403)	(.1104)
3.*Services*	−33.3	−.0376	.7362	.2032	.5414
		(.0190)	(.0523)	(.0316)	(.0422)

Note: $\hat{L}$ is the restricted maximum of the sample log likelihood for the model (4.10)-(4.11); $\hat{\lambda}_{i1}$, $\hat{\lambda}_{i2}$, $\hat{\sigma}_{i1}^{\eta}$ and $\hat{\sigma}_{i2}^{\eta}$ are, respectively, the estimates for the bivariate drift and for the standard errors of the state equation. In brackets are the standard errors.

the unconditional mean and variance of $\begin{pmatrix} d_{it} & z_{it}^{\dagger} \end{pmatrix}'$. Then, the starting values $\begin{pmatrix} d_{i1} & z_{i1}^{\dagger} \end{pmatrix}'$ have been arbitrary drawn from the following normal distribution:

$$\begin{pmatrix} d_{i1} \\ z_{i1}^{\dagger} \end{pmatrix} \sim N \left(\begin{bmatrix} 0 \\ y_{i1} \end{bmatrix}, 10^2 \begin{bmatrix} \hat{\sigma}_i^{\eta^d} & 0 \\ 0 & \hat{\sigma}_i^{\eta^z} \end{bmatrix}^2 \right), \tag{4.14}$$

where the factor 10^2 registers the *prior* for the relative uncertainty about the true value of $\begin{pmatrix} d_{i1} & z_{i1}^{\dagger} \end{pmatrix}'$.

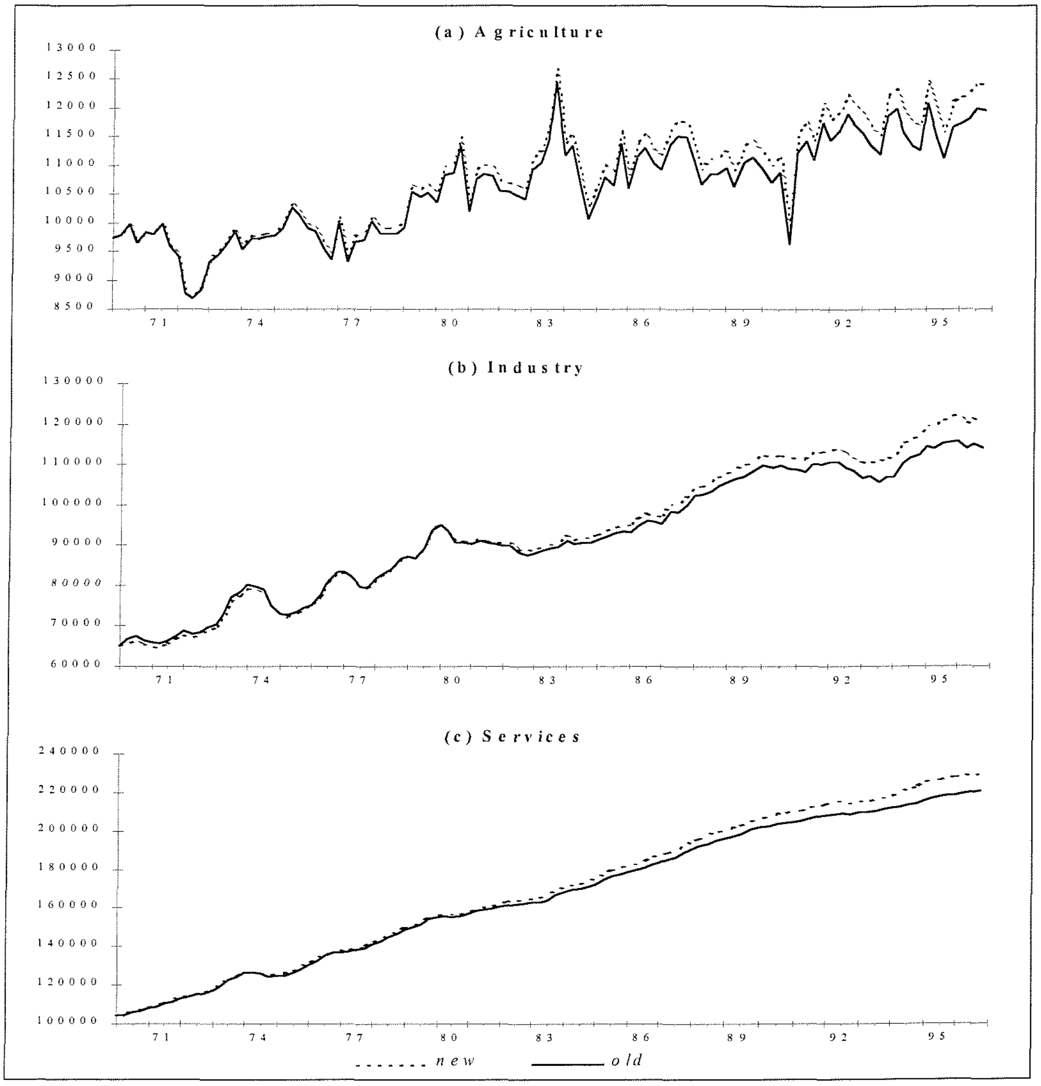
No significant difference has been found between *smoothed estimates* and simulated observations of z_t $(t = 89, .., 108)$. Over the reconstruction time period $(t = 1, .., 88)$ the pattern of z_t is very accurate, respecting the sample path of y_t. The implicit interpolation between the first observation and the last *20s* seems well fitted, distributing gradually over the time the difference among *new* and *old* classification.

Crucial has been the choice of the *prior* distribution of the initial state vector: this strongly affects the *smoothed estimates* for the first observations. In practice, constraining the Kalman filter to obey *time-varying* restrictions often generates breaks over unobserved values of z_t. Because in this exercise a model to fit the generating process of z_t is based only on the last *20* observations and a restriction is imposed on $t = 1$, it can happen that iterating back the *smoothing* algorithm, the *free* path does not converge towards constraints. The result is a *break* in the time series. A delicate *starting-prior* is the only way to handle the problem.

4.3 Change of Classification Model

For convenience, we rewrite here the state space representation (3.19)-(3.20):

Figure 1: Results of reconstruction. Seasonally adjusted Italian value added following new and old classification standards. Values in billion of lira at 1990 prices.



$$z_t = X_t\mu_t + w_t, \tag{4.15}$$

$$\mu_{t+1} = F\mu_t + \rho + v_{t+1}. \tag{4.16}$$

z_t, X_t, μ_t, w_t, F, ρ and v_t hold the same definitions set out so far. Now X_t and z_t are available for the full sample period (t = *1, ..., 108*) and a restricted maximum likelihood estimation of F, ρ and *variance-covariance* matrices H and Q, respectively, of w_t, and v_t is attempted.

Restrictions on μ_t, in the usual form $R_t\mu_t = r_t$, are referred only to the conversion matrix. Then, for t = *1, .., 88* restrictions regard the sum to unity of every column of the conversion matrix as equation (4.9). For t = *89, .., 108*, since μ_t is *hypothetically* observed, μ_t is constrained as equation (4.8). Notably,

$$R_t = [0_{8,3}\ I_8]\,,\ r_t = (\mu_{4,t}...\mu_{11,t})'. \tag{4.17}$$

Optimal estimation of F, ρ, H and Q is achieved by using a numerical optimization routine. A practical problem in using such optimizers for estimating *multivariate models* is the high number of parameter to be estimated. It gets into difficulties in seeking the global maximum of the likelihood function.

A reparametrization of the model (4.15)-(4.16) can be considered to overcome a large parameter space. Provided that the eigenvalues of F are all inside the unit circle, if we set $\mu_t^* = \mu_t - \mu$, where μ is the average or steady-state value of μ_t, from equation (3.27) $\rho = (I - F)\,\mu$ and the system (4.15)-(4.16) becomes

$$z_t = X_t\mu + X_t\mu_t^* + w_t, \tag{4.18}$$

Table 2: Results of a change of classification model on the Italian value added. Quarterly seasonally adjusted data at prices of 1990.

Parameter	value	st.error
$\hat{\rho}^{\bar{\mu}_1}$	-2.512E-4	1.251E-5
$\hat{\rho}^{\bar{\mu}_2}$	9.926E-3	7.493E-4
$\hat{\rho}^{\bar{\mu}_3}$	5.731E-3	1.044E-3
$\hat{\rho}^{c^{11}}$	-1.664E-4	1.251E-5
$\hat{\rho}^{c^{12}}$	1.621E-4	6.199E-4
$\hat{\rho}^{c^{22}}$	-6.068E-4	6.112E-4
$\hat{\rho}^{c^{23}}$	5.309E-4	6.112E-4
$\hat{\rho}^{c^{33}}$	-1.504E-4	6.015E-4
$\hat{q}_1$	1.240E-4	7.220E-6
$\hat{q}_2$	7.516E-3	3.287E-4
$\hat{q}_3$	1.065E-2	4.027E-4

Note: Rescaled values of z_t and y_t by the factor 10^{-5}. Estimation of F and H in equations (4.15)-(4.16) is restricted, respectively, to the identity and null matrices. $\hat{L}$= 3096.3.

$$\mu^*_{t+1} = F\mu^*_t + v_t. \tag{4.19}$$

The advantage is that μ can be estimated by *generalized least squares* separate from the optimization routine as stressed in section 2.2.2. Nevertheless μ is not defined if F is equal to the *identity matrix*: the state equation (4.16) changes in a *multivariate random walk with drift* ρ. ρ is the *slope* over which μ_t randomly runs. Since every column of the conversion matrix sums to one, the sum of ρ is constrained too. In particular, among the elements of ρ representing the conversion matrix (p *+ 1, ...,* k), every p elements sum to zero. Instead of estimating k parameter of ρ it is enough to consider only k - p parameters in the optimization routine.

Results of estimation with respect to values of z_t and y_t rescaled by the factor 10^{-5} are shown in Table 2. Estimation of F and H is restricted, respectively, to the *identity* and *null matrices*[16] so that, considering that $c_t^{31} = 0$ for all t, the observation equation (4.15) becomes

$$\begin{aligned} z_{1t} &= \bar{\mu}_{1t} + c_t^{11} y_{1t} + c_t^{12} y_{2t} + c_t^{13} y_{3t} \\ z_{2t} &= \bar{\mu}_{2t} + c_t^{21} y_{1t} + c_t^{22} y_{2t} + c_t^{23} y_{3t} \\ z_{3t} &= \bar{\mu}_{3t} + \qquad\quad c_t^{32} y_{2t} + c_t^{33} y_{3t} \end{aligned} \tag{4.20}$$

and the state equation (4.16)

$$\begin{pmatrix} \bar{\mu}_{1,t+1} \\ \bar{\mu}_{2,t+1} \\ \bar{\mu}_{3,t+1} \\ c_{t+1}^{11} \\ c_{t+1}^{12} \\ c_{t+1}^{13} \\ c_{t+1}^{21} \\ c_{t+1}^{22} \\ c_{t+1}^{23} \\ c_{t+1}^{32} \\ c_{t+1}^{33} \end{pmatrix} = \begin{pmatrix} \bar{\mu}_{1t} \\ \bar{\mu}_{2t} \\ \bar{\mu}_{3t} \\ c_t^{11} \\ c_t^{12} \\ c_t^{13} \\ c_t^{21} \\ c_t^{22} \\ c_t^{23} \\ c_t^{32} \\ c_t^{33} \end{pmatrix} + \begin{pmatrix} \rho^{\bar{\mu}_1} \\ \rho^{\bar{\mu}_2} \\ \rho^{\bar{\mu}_3} \\ \rho^{c^{11}} \\ \rho^{c^{12}} \\ -\rho^{c^{11}} - \rho^{c^{12}} \\ -\rho^{c^{22}} - \rho^{c^{23}} \\ \rho^{c^{22}} \\ \rho^{c^{23}} \\ -\rho^{c^{33}} \\ \rho^{c^{33}} \end{pmatrix} + \begin{pmatrix} v_{t+1}^{\bar{\mu}_1} \\ v_{t+1}^{\bar{\mu}_2} \\ v_{t+1}^{\bar{\mu}_3} \\ v_{t+1}^{c^{11}} \\ v_{t+1}^{c^{12}} \\ v_{t+1}^{c^{13}} \\ v_{t+1}^{c^{21}} \\ v_{t+1}^{c^{22}} \\ v_{t+1}^{c^{23}} \\ v_{t+1}^{c^{32}} \\ v_{t+1}^{c^{33}} \end{pmatrix}, \tag{4.21}$$

because for ρ every *tern* of elements representing the conversion matrix $(\rho^{c^{11}}, .., \rho^{c^{33}})$ sum to zero. The *variance-covariance matrix* Q of v_t has been assumed *diagonal* with identical *standard deviations* $\hat{q}_1$, $\hat{q}_2$, $\hat{q}_3$ for each sector. This parametrization allows the sample log likelihood L depending only on *8 + 3 = 11* unknown parameters. The maximum $\hat{L}$ reached by

[16] Results and tests of estimations considering F and H as *free diagonal matrices* have not been reported here. Anyhow, significant difference from the identity and the null matrices, respectively, has not been observed.

the Newton-Raphson optimization routine is *3096.3*. Standard errors of the estimates are obtained by the *second derivative method* (Hamilton, 1994b, p.143).

For each sector, -first, second and third row of equation (4.20)- *smoothed estimates* of $\bar{\mu}_{it}$, c_t^{i1}, c_t^{i2} and c_t^{i3} for $i = 1, 2, 3$ are represented, respectively, in figures 2, 3 and 4. For the last *20* observations generated values are considered. The starting values μ_1 have been arbitrarily drawn from the following normal distribution:

$$\mu_1 \sim N\left(\mu_1^{(0)}, 10 \times \hat{Q}\right), \tag{4.22}$$

where the mean $\mu_1^{(0)}$ reflects the hypothesis of no conversion among classification for t = *1*, see equation (4.6), and the factor *10* registers the *prior* for the relative uncertainty about the true value of μ_1.

Among different sectors, no significant differences have been observed between *smoothed estimates* and simulated observations of μ_t, i.e. when t = *89, .., 108*. With regard to the reconstruction period (i.e. t = *1, .., 88*), aggregation constraints among conversion parameters have always been respected but with different performances among sectors:

for the *Industry sector* (figure 3), $\bar{\mu}_{2t}$, $\hat{c}_t^{21}$, $\hat{c}_t^{22}$and $\hat{c}_t^{23}$ well interpolate the actual observations starting from hypothetical points *0* or *1* at the beginning of the sample. For these observations a small but significant break has been observed only for the conversion parameters $\hat{c}_t^{22}$and $\hat{c}_t^{23}$, revealing the difficulties stressed in section 4.2 in fitting the smoothed estimates to the initial constraints;

for the *Agriculture* and *Service* sectors (figure 2 and 4) the exercise seems to be particularly complicated: conversion parameters are highly irregular and very close to the boundary limits. Difficulties have been encountered

Figure 2: Smoothed estimates of time-varying coefficients for the Agriculture sector.

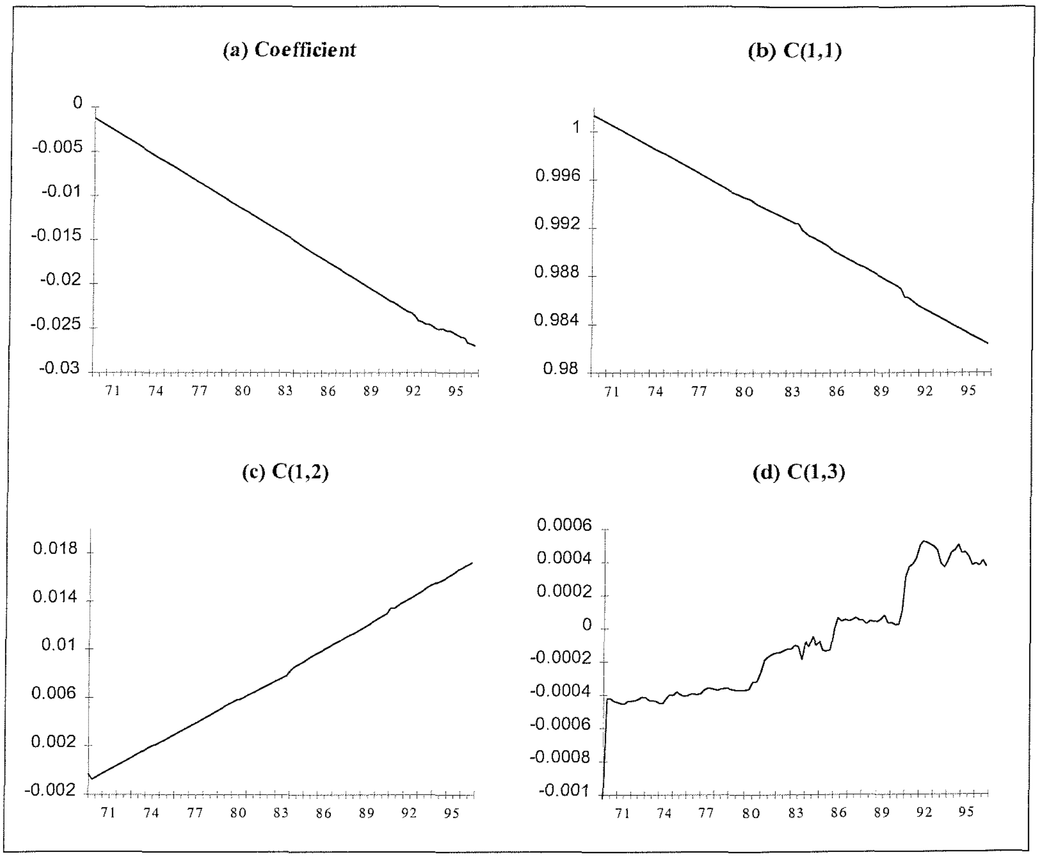


Figure 3: Smoothed estimates of time varying coefficients for the Industry sector.

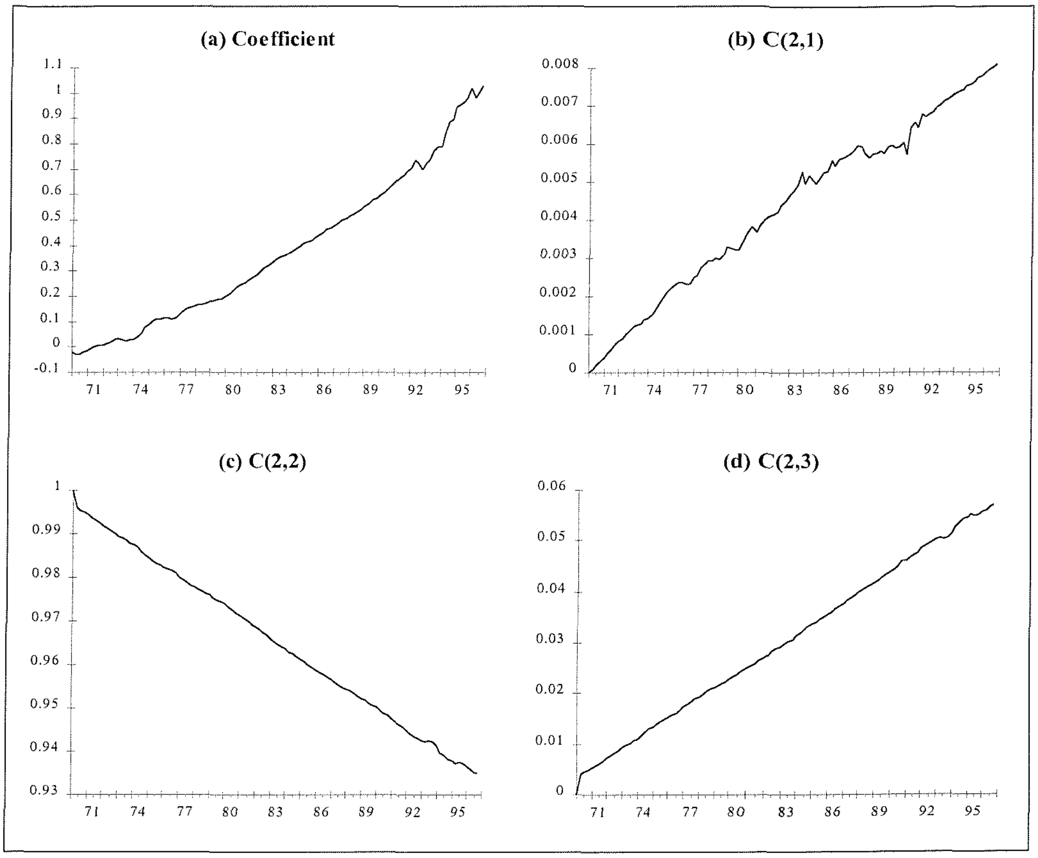
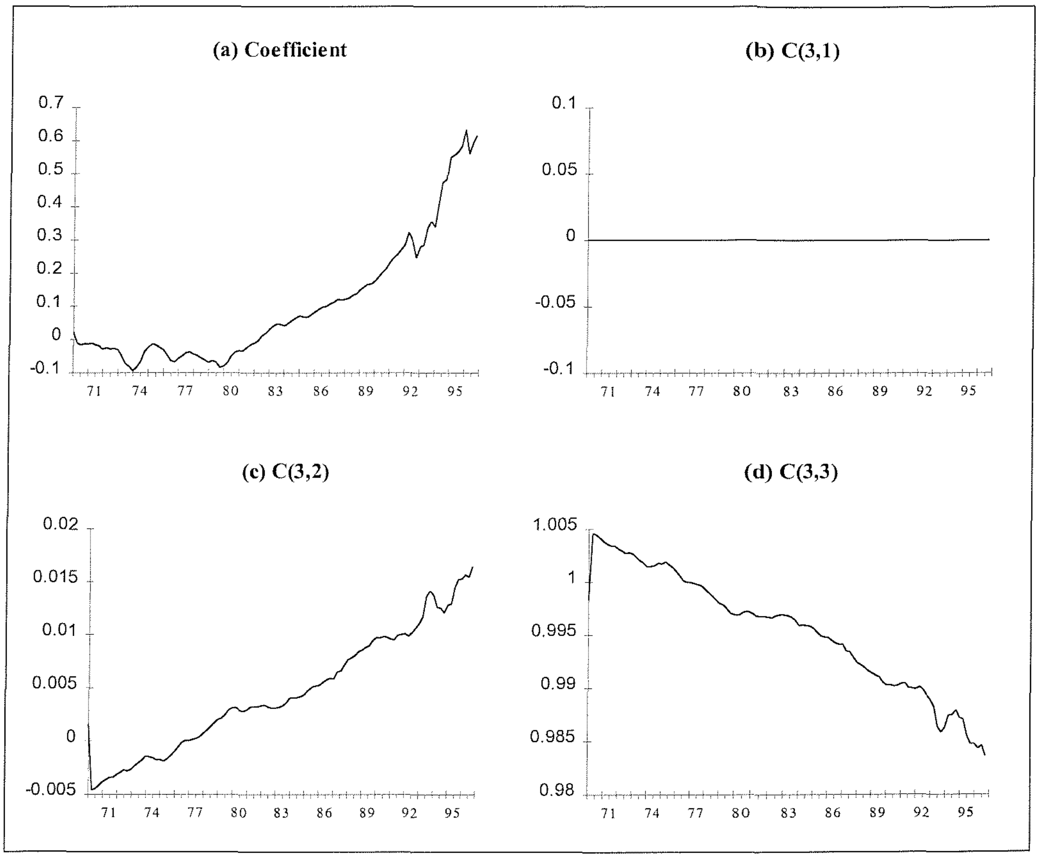


Figure 4: Smoothing estimates of time varying coefficients for the Service sector.



since estimates of the state vector give often values which tend to be less than zero or greater than one, even if never more than 0.5%. In fact, by using the ordinary Kalman filter estimates are not guaranteed to be inside a defined interval or to satisfy non-linear constraints[17]. Estimates against boundary conditions always involve first observations, revealing problems in the definition of the initial conditions of the Kalman filter.

[17] A way to handle the problem could be the *extended Kalman filter*, which allows *non-linear* state space forms in order to incorporate *sign-restrictions* on the state-vector. For an introduction to the issue see Harvey (1990, pp.160-162).

5 Conclusions

In this thesis a framework for a conversion of sectoral time series from *old* to *new* classification standards has been suggested. This is based on the definition of a *conversion matrix* to express time-varying compositional effects among different sectoral definitions.

The change of classification is an important practical problem considering European Unification. By that date all European countries will adopt National Accounts obeying new sectoral standards, causing problems of comparing pre- and post-change time series.

State space representations have been presented to handle historical reconstruction and modelling change of classification. The Doran (1992) and Doran and Rambaldi (1996) methodology of constraining the Kalman filter to satisfy time varying restrictions has provided a flexible instrument to obtain efficient *smoothed* estimates.

A two step experimental application has provided the Italian Value Added reconstruction and parameter estimation of a three-sector model. The proposals of Doran and Rambaldi (1996, 1997) to reparametrise the original model in order to reduce the parameter space have not been applied because of *non-stationarity* of original time series and time variability in the dimension of restrictions. A simpler reparametrization of the state equation has been effective in overcoming the usual convergence problems associated with numerical search procedures.

Reconstructed *smoothed* estimates have shown a good fit, well interpolating over time the difference among *new* and *old* classifications. On the other

hand, the pattern across the first observations has shown a strong dependence on the arbitrary prior distribution of the initial state vector.
The fit of the *smoothed* conversion matrix estimates is revealed to be good, always respecting aggregation constraints. Nevertheless, the exercise has stressed difficulties in restricting reconstructed values to vary within defined intervals. In particular this behaviour seems to involve the observed conversion parameters which are highly irregular and close to the boundary limits. Such a problem could be solved by considering the extended instead of the ordinary Kalman filter. Then, extended state space forms could be formulated to incorporate non linear constraints which are appropriate to the definition of the conversion matrix.

Appendix A: Data

Data used for the application of chapter 4 are shown in this appendix. These correspond to the Italian quarterly value added figures at market prices for the period *1970.1-1996.4*. The seasonally adjusted release in terms of billion of Italian lira at 1990 prices is considered.

Agriculture

	Q1	Q2	Q3	Q4
1970	9731	9771	9983	9649
1971	9833	9792	9972	9594
1972	9406	8769	8674	8833
1973	9325	9449	9613	9851
1974	9543	9727	9721	9759
1975	9779	9897	10276	10140
1976	9904	9846	9541	9361
1977	10005	9311	9673	9710
1978	10023	9808	9805	9811
1979	9917	10546	10446	10531
1980	10358	10840	10873	11351
1981	10189	10755	10846	10820
1982	10565	10538	10485	10405
1983	10916	11049	11436	12462
1984	11175	11335	10597	10064
1985	10454	10795	10657	11384
1986	10600	11159	11307	11025
1987	10926	11358	11507	11487
1988	11108	10662	10847	10843
1989	10966	10623	11043	11136
1990	10924	10704	10878	9627
1991	11235	11441	11106	11760
1992	11451	11608	11913	11727
1993	11563	11367	11188	11869
1994	12000	11579	11343	11274
1995	12080	11512	11128	11672
1996	11726	11815	12002	11961

Industry

	Q1	Q2	Q3	Q4
1970	65218	66646	67602	66513
1971	66088	65630	66309	67474
1972	68696	68069	68332	69607
1973	70342	72922	76997	78336
1974	80128	79762	78848	75114
1975	72906	72607	73384	74427
1976	75083	77513	81007	83362
1977	83427	82041	79804	79567
1978	81677	82989	84000	86698
1979	87289	86723	89293	93641
1980	94958	93329	90552	90662
1981	90274	91145	90534	90360
1982	89921	89832	88228	87490
1983	88052	88802	89347	89490
1984	91216	90237	90496	90575
1985	91337	92268	92972	93493
1986	93277	94858	96130	95905
1987	95321	98351	98152	100033
1988	102184	102534	103342	104653
1989	105538	106406	106923	108464
1990	109722	109251	109733	108923
1991	108639	108304	110065	110127
1992	110444	110451	109361	108388
1993	106769	107072	105609	106912
1994	106837	110288	111474	112381
1995	114340	113853	115202	115534
1996	115787	113823	114871	114013

Service

	Q1	Q2	Q3	Q4
1970	104342	104528	105892	106736
1971	107993	108894	110317	111287
1972	113128	113953	115356	115852
1973	117208	119681	122619	124353
1974	126620	126454	126092	124630
1975	124932	124787	126407	128155
1976	130154	132640	135390	136633
1977	136688	137408	138297	139340
1978	141214	142891	144892	146725
1979	149026	150078	151608	154226
1980	155139	155797	155619	155721
1981	156875	158452	159042	159609
1982	160698	161565	161676	162484
1983	162731	163125	164267	166501
1984	168549	169441	170014	170939
1985	172137	174648	176774	177704
1986	178605	179618	180725	182561
1987	183772	185147	186331	188733
1988	190507	192202	193516	195176
1989	195949	197038	198440	200934
1990	202009	202652	203483	204283
1991	204614	205393	206373	207172
1992	207930	208762	209114	208748
1993	209508	209685	210247	211068
1994	211593	212365	213559	214103
1995	215657	216738	217772	218047
1996	218394	219198	220089	220354

Appendix B: Codes

In this appendix codes to perform the application of Chapter 4 are provided. All the codes are compiled in Gauss, version 3.2. The material, quite complicated, is arranged in different sections, where *main programs* and *procedures* are shown separately.
Following the organization of Chapter 4, first codes on the generating process of z_t are considered: the main program *GENX1* presents the instructions to control the simulation of data in terms of a new hypothetical classification. Then, the program *GENXMAI1* performs the reconstruction of z_t as it is shown in section 4.2. Finally, *TVC8MAIN* is the main program to control the Kalman filter and the Maximum Likelihood estimation of the parameters of the state space form concerning the change of classification model of section 4.3.
The three mentioned *main programs* need specific *Gauss-procedures*, which are compiled separately. These are:
TVC8KF, which is the general *routine* to control the Kalman filter and to evaluate the Likelihood Function;
AUGMENT, to augment the state space form in terms of the Doran (1992) methodology;
INTERP, to perform a deterministic interpolation over the time of matrices provided at two given periods.

Gauss codes for the generating process of z_t

```
/* ==========================================
Filename: GENX1
Author: Filippo Moauro
Date: 24/11/1997
Type: Gauss main program
Description:
Simulate data by the Kalman filter under time-varying constraints, starting
from a synthetic conversion matrix. General state space form like:

[1]          y_t = ast(Xs_t) + Hst(Xs_t) * chsi_t + w_t,
[2]          chsi_t+1 = Fst(Xs_t) * chsi_t + mu + v_t+1,

with t = 1, ..., capt,
y = (capt x n) matrix of endogenous variables,
XX = (capt x k) matrix of exogenous variables,
chsi_t = rx-state vector, mu = drift,

[3]          Xs_t = XX_t ⊗ I_n (n x (n*k)) matrix,
[4]          ast(Xs_t) = 0,
[5]          Hst(Xs_t) = Xs_t,
[6]          Cov(w_t) = R = 0,
[7]          Fst(Xs_t) = I_n,
[8]          Cov(v_t) = Q (diagonal).

Restrictions like:

[9]          RR_t * chsi_t = cn_t.

Include:
INTERP => deterministic matrix interpolation,
TVC8KF => Kalman filter and likelihood evaluation,
AUGMENT=> Augmentation of the measurement equation.

========================================== */
@ Set global variables and Kalman filter control parameters @

n = 3; @ dimension of observation vector @
k = n+1; @ dimension of exogenous vector @
rx = n*k; @ dimension of state-space (eventually Sbb correction) @
cap0 =108;@ n.observations dataset @
capt = 20; @ sample size generated y_t @
```

```
sigc = .00001; @ coefficient on standard errors for every sector @
idgp = .03; @ arbitrary percentage of increasing of GDP @
scdata = 1; @ scale factor on dataset @
scal = 1; @ scale factor on y @
prior = 1e+3; @ diffuse prior on P_1/0 @
ind= seqa(1992.25,0.25,capt); @ time sequence @
Sbb ={1 0 0 0 0 0 0 0 0 0 0 0,
0 1 0 0 0 0 0 0 0 0 0 0,
0 0 1 0 0 0 0 0 0 0 0 0,
0 0 0 1 0 0 0 0 0 0 0 0,
0 0 0 0 1 0 0 0 0 0 0 0,
0 0 0 0 0 1 0 0 0 0 0 0,
0 0 0 0 0 0 1 0 0 0 0 0,
0 0 0 0 0 0 0 1 0 0 0 0,
0 0 0 0 0 0 0 0 1 0 0 0,
0 0 0 0 0 0 0 0 0 0 1 0,
0 0 0 0 0 0 0 0 0 0 0 1}; @ selection matrix for time invariant constraints @
rx = rx - (rows(Sbb')-rows(Sbb)); @ dimension-correction @
Smu = {1 0 0 0 0 0 0 0,
0 1 0 0 0 0 0 0,
0 0 1 0 0 0 0 0,
0 0 0 1 0 0 0 0,
0 0 0 0 1 0 0 0,
0 0 0 -1 -1 0 0 0,
0 0 0 0 0 -1 -1 0,
0 0 0 0 0 1 0 0,
0 0 0 0 0 0 1 0,
0 0 0 0 0 0 0 -1,
0 0 0 0 0 0 0 1} @ selection matrix for restrictions on mu @;
chsi = zeros(capt,rx); @ filter inferences chsi_t/t @
P = zeros(capt,rx^2); @ filter variances P_t/t @
start1 = 1; @ quarter 70.1 @
startob = 89; @ quarter 92.1 @
C1 = eye(n); @ hypothesized conversion matrix for 70.1 @
Cob={.985498549317 .006719179210 .0,
.014001400665 .944885039408 .010105882315,
.000500050018 .048395781382 .989894117685}; @ obs. conversion matrix @

/* ======================================= */
@ Read dataset @
```

```
@ value added agriculture, industry and service - old classification @
load XXX[cap0,n] = va3sec.prn;
XX = XXX[cap0-capt+1:cap0,.]/scdata;
GDPold = sumc(XX'); @ GDP old classification @
XX = ones(capt,1) ~XX; @ add constant @

/* ======================================= */
@ CC1 and CCob @

#include interp;
y1 = (1 + idgp)*XX[1,2:4]'/scdata;
constob = y1 - Cob*XX[1,2:4]'; @ observed constants in t = startob @
CC1 = zeros(n,1) ~C1; @ conversion matrix in t = 70.1 @
CCob = constob ~ Cob; @ constant extended conv.matrix @
CCinterp = interp(CC1,CCob,start1,startob,cap0); @ interpolation @
CCinterp = CCinterp[cap0-capt+1:cap0,.];
XXinterp = CCinterp*(XX.*.eye(n))'.*(eye(capt).*.ones(1,n))*
(ones(capt,1).*.eye(n));

/* ======================================= */
@ Simulated parameter values @

mu = inv(Smu'*Smu)*Smu'*Sbb*(vec(CCob) - vec(CC1))/(startob - start1);
sigv = sigc*stdc(XX[.,2:4]); @ variance-covariance matrix of v_t @
th = vec(mu)|vec(sigv);

/* ======================================= */
@ Read in and translate parameters into standard state-space matrices @

proc(2) = readin(it, y, XX, n, Sbb);
    local Xs, ydp;
    Xs = XX[it,.] .*. eye(n); @ as in [3] @
    Xs = Xs*Sbb';
    ydp = y[it,.]';
retp(ydp, Xs); endp;

proc(1) = ast(th,rx);
    local A;
    A = 0; @ as in [4] @
retp(A); endp;
```

```
proc(1) = Hst(Xs);
    local HH;
    HH = Xs'; @ as in [5] @
retp(HH); endp;

proc(1) = Rst(th, n);
    local R;
    R = zeros(n,n); @ as in [6] @
retp(R); endp;

proc(1) = Fst(th, rx);
    local Fx;
    Fx = eye(rx); @ as in [7] @
retp(Fx); endp;

proc(1) = must(th, rx);
    local aver;
    aver = Smu*th[1:rx-n,1];
retp(aver); endp;

proc(1) = Qst(th, rx);
    local Q, sigvsq;
    sigvsq = th[rx-n+1:rx,1]^2;
    sigvsq = Sbb*(vec(ones(1,k).*.sigvsq));
    Q = diagrv(zeros(rx,rx),sigvsq); @ as in [8] @
retp(Q); endp;

/* ======================================== */
@ Time-varying constrains @

int89 = 1;
RR89 = zeros(rx-n,n) ~ eye(rx-n);
cn89 = scal*vec(Cob);
cn89 = Sbb[4:rx,4:n*k]*cn89;

intob = zeros(2,1);
intob[1,1] = 2; intob[2,1] = capt;
RRob = zeros(n,n) ~ eye(n).*.ones(1,n);
RRob = RRob*Sbb';
cnob = scal*ones(n,1);

skipcon = 1; @ flag 0-1 to constraint the Kalman filter @

proc(3) = constr(it, Xs, ydp, R);
```

```
    local smacn;
    {Xs, ydp, R} = augment(cn89, RR89, Xs, ydp, R, it, int89);
    {Xs, ydp, R} = augment(cnob, RRob, Xs, ydp, R, it, intob);
retp(Xs, ydp, R); endp;

/* ======================================== */
@ Generate y before constraints @

dseed = 162443;
v_t = rndns(capt,rx,dseed)*Qst(th, rx);
slope = vec(CCob - CC1)/(startob - start1);
slope = Sbb*slope;
drift = seqa(0,1,capt).*.slope';
CCgenr = drift + ones(capt,1).*.(Sbb*vec(CCob))' + cumsumc(v_t);
CCgenr = CCgenr*Sbb;
y = ((CCgenr*(XX.*.eye(n))').*(eye(capt).*.ones(1,n)))*(ones(capt,1).*.eye(n));

/* ======================================== */
@ Results @

Fx = Fst(th, rx);
#include tvc8kf;
#include augment;
z=-ofn(th);
convob = chsi*Sbb;
yyy = convob*(XX.*.eye(n))'.*(eye(capt).*.ones(1,n))*
(ones(capt,1).*.eye(n));
save yyy;
save convob;
```

Gauss codes for the reconstruction of z_t

```
/* ========================================
Filename: GENXMAI1
Author: Filippo Moauro
Date: 25/11/1997
Type: Gauss main program
Description:
The program controls the Kalman filter for data reconstruction. For each
sector the model is:

[1]            y_t = [1 1] [a_t x_t]' + w_t,
[2]            [a_t+1 x_t+1]' = [a_t x_t]' + [a x]' + v_t+1,

with t = 1, ..., capt,
y_t = log of value added old-classification i-th sector,
x_t = log of value added new-classification i-th sector to be reconstructed.

[3]            Cov(w_t) = 0,
[4]            Cov(v_t) = Q (diagonal),

Restrictions like:

[5]            x_1= y_1
[6]            x_t = x*_t for t = startob, startob+1, .., capt

Include:
TVC8KF => Kalman filter and likelihood evaluation
OPTMUM => Gauss numerical optimizer
AUGMENT=> Augmentation of the measurement equation

======================================== */
@ Set global variables and Kalman filter control parameters @

startob = 89; @ 92.1 @
sec = 1; @ select sector (1, 2, 3) @
n = 1; @ dimension of observation vector @
k = n+1; @ dimension of exogenous vector @
rx = 2; @ dimension of state-space (eventually Sbb correction) @
capt = 108; @ sample size @
scal = 100; @ scale factor @
prior = 1e+6; @ diffuse prior on P_1/0 @
ind= seqa(1970.25,0.25,capt); @ time sequence @
```

```
chsi = zeros(capt,rx); @ filter inferences chsi_t/t @
chsif = zeros(capt,rx); @ forecasted inferences chsi_t+1/t @
chsis = zeros(capt,rx); @ smoothed inferences chsi_t/T @
P = zeros(capt,rx^2); @ filter variances P_t/t @
Pf = zeros(capt,rx^2); @ forecasted variances P_t+1/t @
Ps = zeros(capt,rx^2); @ smoothed variances P_t/T @
output file=junk reset;

/* ======================================== */
@ Read dataset @

@ value added agriculture, industry and service - old classification @
load vagg[capt,3] = va3sec.prn;
load yyy; @ generated series new class. from 92.1 (see GENX1) @
ly = scal*ln(vagg[.,sec]);
lx = zeros(startob-1,1) | scal*ln(yyy[.,sec]);

/* ======================================== */
@ Guess initial parameter values @

mu = -.2396 -.0025;
sigv = -.5161, 1;
th = vec(mu) | vec(sigv);
th0 = th; @ backup @
proc startval; @ This defines starting value for iteration to be th @
retp(th); endp;

/* ======================================== */
@ Read in and translate parameters into standard state-space matrices @

proc(2) = readin(it, y, XX, n, Sbb);
    local Xs, ydp;
    Xs = HH';
    ydp = ly[it,.];
retp(ydp, Xs); endp;

proc(1) = ast(th,rx);
    local A;
    A = 0;
retp(A); endp;
```

```
proc(1) = Hst(Xs);
    local HH;
    HH = {1, 1}; @ as in [1] @
retp(HH); endp;

proc(1) = must(th, rx); @ as in [2] @
    local mean;
    mean = th[1,1] | th[2,1];
retp(mean); endp;

proc(1) = Rst(th, n); @ as in [3] @
    local R;
    R = 0;
retp(R); endp;

proc(1) = Qst(th, rx); @ as in [4] @
    local Q, sigvsq;
    sigvsq = th[3:4,1]^2;
    Q = diagrv(zeros(rx,rx),sigvsq);
retp(Q); endp;

proc(1) = Fst(th, rx); @ as in [2] @
    local FF;
    FF = eye(rx);
retp(FF); endp;

/* ======================================= */
@ Time-variant constrains @

int1 = 1;
RR1 = {0 1};
cn1 = ly[1,1]; @ as in [5] @

intob1 = zeros(2,1); intob1[1,1] = startob; intob1[2,1] = capt;
RRob1 = RR1;
cnob1 = lx; @ as in [6] @

skipcon = 1; @ flag 0-1 to constraint the Kalman filter @

proc(3) = constr(it, Xs, ydp, R);
    local Rcn;
    {Xs, ydp, Rcn} = augment(cn1, RR1, Xs, ydp, R, it, int1);
    {Xs, ydp, Rcn} = augment(cnob1[it,.]', RRob1, Xs, ydp, R, it, intob1);
retp(Xs, ydp, Rcn); endp;
```

```
/* ======================================= */
@ Echo initial parameter values @

format /rds 10,6;
"starting values of th as follows"; th;
#include tvc8kf;
#include augment;
"Value of log likelihood"; z=-ofn(th);z;
format /m1;
"Do you wish to continue (y or n)?";;
zzs = cons;
if zzs $== "n"; end; endif;

/* ======================================= */
@ Set parameters to use Gauss numerical optimizer @

library optmum;
#include optmum.ext;
__btol = 1.e-06; @ This controls convergence criterion for coefficients @
__gtol = 1.e-06; @ This controls convergence criterion for gradient @
__algr = 1; @ This chooses BFGS optimization @
__miter = 400; @ This controls the maximum number of iterations @
__output = 1; @ This causes extra output to be displayed @
__covp = 0; @ This speeds up return from OPTMUM; note that the program
makes a reparameterization to calculate std. errors @
output off;
{x,f,g,h} =optmum(&ofn,startval); @ GAUSS numerical optimizer @
output file=junk on;
"";"";"MLE as parameterized for numerical optimization ";
"Coefficients:";x';
"";"Value of log likelihood:";;-f;
"";"Gradient vector:";g';
h = (hessp(&ofn,x));
va = eigrs(h);
call ofn(x);
if minc(eigrs(h)) <= 0;
    "Negative of Hessian is not positive definite";
    "Either you have not found local maximum, or else estimates are up "
    "against boundary condition. In latter case, impose the restricted "
    "params rather than estimate them to calculate standard errors";
```

```
else;
    h = invpd(h);
    std = diag(h)^.5;
    "standard errors";std';
    "variance-covariance matrix";
    format /m3; h;
    format /m1;
endif;

R = Rst(x, n);
FX = Fst(x, rx);
Q = Qst(x, rx);

"prior:"; format /rds 20,0; prior;
"Rst:"; format /rds 20,16; R;
"Fst:"; format /rds 20,16; FX;
"Qst:"; format /rds 20,16; Q;
"";"------------------------------------------";"";
output file=junk off;
```

Gauss codes for a change of classification model

```
/* ==========================================
Filename: TVC8MAIN
Author: Filippo Moauro
Date: 18/10/1997
Type: Gauss main program
Description:
Modelling a change of classification for sectoral time series via a conversion
matrix approach. The program controls the Kalman filter estimation of a
time-varying regression model under time-varying constraints (Doran,1992).
General model like:

[1]            y_t = ast(Xs_t) + Hst(Xs_t)' * chsi_t + w_t
[2]            chsi_t+1 = Fst(Xs_t) * chsi_t+ mu + v_t+1

with t = 1, ..., capt,

[3]            Cov(w_t) = Rst(Xs_t)
[4]            Cov(v_t) = Qst(Xs_t)

y_t = sectoral Italian value added old-classification
Xs_t = sectoral Italian value added new-classification. It is assumed that
Xs_t is observed only for a given period at the end of the sample (t = startob,
startob+1, .., capt). Interpolation via the Kalman filter to reconstruct the
previous period
Restrictions like:

[5]            RR_t * chsi_t = cn_t

Include:
TVC8KF => Kalman filter and likelihood evaluation
OPTMUM => Gauss numerical optimizer
AUGMENT=> Augmentation of the measurement equation

========================================== */
@ Set global variables and Kalman filter control parameters @

n = 3; @ dimension of observation vector @
k = n+1; @ dimension of exogenous vector @
rx = n*k; @ dimension of state-space (eventually Sbb correction) @
capt = 108; @ sample size @
scdata = 1e+4; @ scale factor on dataset @
```

```
scal = 1; @ scale factor on y @
prior = 1e+2; @ diffuse prior on P_1/0 @
ind= seqa(1970.25,0.25,capt); @ time sequence @
load Sbb; @ selection matrix for time-invariant constraints (see GENX1) @
rx = rx - (rows(Sbb')-rows(Sbb)); @ dimension-correction @
load Smu; @ selection matrix for restrictions on mu @
chsi10pr = scal*Sbb*(zeros(n,1) | vec(eye(n))); @ prior on chsi_1/0 @
chsi = zeros(capt,rx); @ filter inferences chsi_t/t @
chsif = zeros(capt,rx); @ forecasted inferences chsi_t+1/t @
chsis = zeros(capt,rx); @ smoothed inferences chsi_t/T @
P = zeros(capt,rx^2); @ filter variances P_t/t @
Pf = zeros(capt,rx^2); @ forecasted variances P_t+1/t @
Ps = zeros(capt,rx^2); @ smoothed variances P_t/T
y = capt x n matrix of observations on endogenous variables
XX = capt x k matrix of observations on exogenous variables @
start1 = 1; @ quarter 70.1 @
startob = 89; @ quarter 92.1 @
CC1 = zeros(n,1)~eye(n); @ simulated conversion matrix for 70.1 @
load convob; @ observed conversion matrices for 92.1-96.4 @
output file=junk reset;

/* ==================================== */
@ Read dataset @

@ value added agriculture, industry and service - old classification @
load XX[capt,n] = va3sec.prn;
XX = XX/scdata;
XX = ones(capt,1)~XX; @ add constant @
load agr11pr6; @ reconstructed series - agriculture (see GENXMAI1) @
load ind11pr6; @ reconstructed series - industry (see GENXMAI1) @
load ser11pr8; @ reconstructed series - service (see GENXMAI1) @
y = (agr11pr6~ind11pr6~ser11pr8)/scdata;

/* ==================================== */
@ OLS estimation of time-invariant coefficients (bb0).
Starting values for the maximum likelihood estimation @

Cob = reshape(convob[1,n+1:n*k],n,n)';
constob = y[startob,.]' - Cob*XX[startob,2:4]'; @ observed constants in
t=startob @
```

```
CCob = constob~Cob; @ constant extended conv.matrix @

y = scal*y;
_output=0;
bb = zeros(n*k,1); @ Time-invariant coefficients @
stderr = zeros(n*k,1); @ Coefficient standard errors @
sighat = zeros(n,1); @ Standard errors of regressions @
rsq = zeros(n,1); @ R^2 of regressions @
dw = zeros(n,1); @ Durbin-Watson statistics @

ii=1; ip=1; do until ii > n;
    id=ii*k;
    {vnam, m, bols, stb, vc, ste, sh, cx, r_sq, resid, dws} =
    ols("", y[.,ii], XX[.,2:k]);
    bb[ip:id,1] = bols;
    stderr[ip:id,1] = ste;
    sighat[ii,1] = sh;
    rsq[ii,1] = r_sq;
    dw[ii,1] = dws;
    ii=ii+1;
    ip=id+1;
endo;

bb = vec(reshape(bb,n,k));
stderr = vec(reshape(stderr,n,k));
bb0=bb;
__output=1;

/* ======================================== */
@ Guess initial parameter values @

mu = inv(Smu'*Smu)*Smu'*Sbb*(vec(CCob) - vec(CC1))/(startob - 1);
sigv = sighat;
th = vec(mu) | vec(sigv); @ parameters to be estimated @
proc startval; @ This defines starting value for iteration to be th @
retp(th); endp;

/* ======================================== */
@ Read in and translate parameters into standard state-space matrices @

proc(2) = readin(it, y, XX, n, Sbb);
    local Xs, ydp;
```

```
        Xs = XX[it,.] .*. eye(n);
        Xs = Xs*Sbb';
        ydp = y[it,.]';
retp(ydp, Xs); endp;

proc(1) = ast(th,rx);
        local A;
        A = 0; @ as in [1], ast(Xs_t) = 0 @
retp(A); endp;

proc(1) = Hst(Xs);
        local HH;
        HH = Xs'; @ as in [1], Hst(Xs_t) = Xs @
retp(HH); endp;

proc(1) = Rst(th, n);
        local R;
        R = zeros(n,n); @ as in [3], Rst(Xs_t) = 0 @
retp(R); endp;

proc(1) = Fst(th, rx);
        local Fx;
        Fx = eye(rx); @ as in [2], Fst(Xs_t) = I_rx @
retp(Fx); endp;

proc(1) = must(th, rx); @ mean of state vector @
        local aver;
        aver = Smu*th[1:rx-n,1];
retp(aver); endp;

proc(1) = Qst(th, rx);
        local Q, sigvsq;
        sigvsq = th[rx-n+1:rx,1]^2;
        sigvsq = Sbb*(vec(ones(1,k).*.sigvsq));
        Q = diagrv(zeros(rx,rx),sigvsq); @ as in [4], Qst(Xs_t) diagonal @
retp(Q); endp;

/* ========================================= */
@ Time-varying constrains @

int2 = zeros(2,1);
int2[1,1] = 1; int2[2,1] = startob-1;
RR2 = zeros(n,n) eye(n).*.ones(1,n);
```

```
RR2 = RR2*Sbb';
cn2 = scal*ones(n,1);

intob = zeros(2,1);
intob[1,1] = startob; intob[2,1] = capt;
RRob = RR1;
cnob = zeros(startob-1,n^2) | scal*convob[.,n+1:n*k];
cnob = cnob*Sbb[n+1:rx,n+1:n*k]';

skipcon = 1; @ flag 0-1 to constraint the Kalman filter @

proc(3) = constr(it, Xs, ydp, R);
    local smacn;
    {Xs, ydp, R} = augment(cn2, RR2, Xs, ydp, R, it, int2);
    {Xs, ydp, R} = augment(cnob[it,.]', RRob, Xs, ydp, R, it, intob);
retp(Xs, ydp, R); endp;

/* ======================================= */
@ Echo initial parameter values @

format /rds 10,6;
"OLS estimation of a time-invariant conversion matrix"; reshape(bb0,k,n)';
"sum for column"; sumc(reshape(bb0,k,n)')';
"with coefficient standard errors"; reshape(stderr,k,n)';
"standard errors of regression"; sighat';
"R-squared"; rsq';
"and Durbin-Watson statistics"; dw';
"starting values of th as follows"; th;
#include tvc8kf;
#include augment;
"Value of log likelihood"; z=-ofn(th);z;
format /m1;
"Do you wish to continue (y or n)?";;
zzs = cons;
if zzs $== "n"; end; endif;

/* ======================================= */
@ Set parameters to use Gauss numerical optimizer @

library optmum;
#include optmum.ext;
_btol = 1.e-06; @ This controls convergence criterion for coefficients @
```

```
__gtol = 1.e-06; @ This controls convergence criterion for gradient @
__algr = 1; @ This chooses BFGS optimization @
__miter = 400; @ This controls the maximum number of iterations @
__output = 1; @ This causes extra output to be displayed @
__covp = 0; @ This speeds up return from OPTMUM; note that the program
makes a reparameterization to calculate std. errors @
output off;
{x,f,g,h} = optmum(&ofn, startval); @ GAUSS numerical optimizer @
output file=junk on;
"";"";"MLE as parameterized for numerical optimization ";
format /rds 14,9;
"Coefficients:";x';
"";"Value of log likelihood:";;-f;
"";"Gradient vector:";g';
h = (hessp(&ofn,x));
va = eigrs(h);
call ofn(x);
if minc(eigrs(h)) <= 0;
    "Negative of Hessian is not positive definite";
    "Either you have not found local maximum, or else estimates are up "
    "against boundary condition. In latter case, impose the restricted "
    "params rather than estimate them to calculate standard errors";
else;
    h = invpd(h);
    std = diag(h)^.5;
    "standard errors";std';
    "variance-covariance matrix";
    format /m3; h;
endif;

R = Rst(x, n);
FX = Fst(x, rx);
mu = must(x,rx);
Q = Qst(x, rx);
bbbf = chsif*Sbb; @ beta t+1/t @
bbb = chsi*Sbb; @ beta t/t @
yyyf = bbbf*(XX.*.eye(n))'.*(eye(capt).*.ones(1,n))*
(ones(capt,1).*.eye(n)); @ yhat_t+1/t @
yyy = bbb*(XX.*.eye(n))'.*(eye(capt).*.ones(1,n))*
(ones(capt,1).*.eye(n)); @ yhat_t/t @
```

```
"Rst:"; format /rds 14,9; R;
"mu:"; format /rds 14,9; mu;
"Qst:"; format /rds 14,9; Q;
"scal:"; format /rds 14,0; scal;
"prior:"; format /rds 14,0; prior;
"scdata:"; format /rds 14,0; scdata;
"Sbb:"; format /rds 4,0; Sbb;
"Smu:"; format /rds 4,0; Smu;
"";"------------------------------------";"";
output file=junk off;
```

Gauss procedure to perform the Kalman filter and to evaluate the likelihood function

```
/* ==========================================
Filename: TVC8KF
Author: Filippo Moauro
Date: 30/09/1997
Type: Gauss procedure - ofn
Description:
The proc ofn(th) performs Kalman filter and evaluates likelihood function for
general model like:

[1]          y_t = ast(Xs_t) + Hst(Xs_t)' * chsi_t + w_t
[2]          chsi_t+1 = Fst(Xs_t) * chsi_t + mu + v_t+1

with t = 1, ..., capt,

[3]          Cov(w_t)=Rst(Xs_t)
[4]          Cov(v_t)=Rst(Xs_t)

Generalized version in order to allow Time Varying Restrictions via
Augmentation of the Measurement Equation (Doran, 1992).

Restrictions like:

[5]          RR_t * chsi_t = cn_t

Input: th = starting values for coefficients to be estimated
Output: f0 = maximum value of likelihood function
Global variables:
rx = dimension of state-space
n = dimension of observation vector
k = dimension of exogenous vector
capt = sample size
XX = (capt x k) matrix of observations on exogenous variables
y = (capt x n) matrix of observations on endogenous variables
chsi = (capt x r) matrix in which chsi_t/t is stored
chsif = (capt x r) matrix in which chsi_t+1/t is stored
chsis = (capt x r) matrix in which chsi_t/T is stored
P = (capt x r^2) matrix in which P_t/t is stored
Pf = (capt x r^2) matrix in which P_t+1/t is stored
Ps = (capt x r^2) matrix in which P_t/T is stored
scal = scale factor on y
Sbb = selection matrix for exogenous variables
```

(Sbb=eye(rx) if all the exogenous are considered for each endogenous)
RR# = time-varying constrains matrix
cn# = time-varying constrains vector

```
========================================= */
proc ofn(th);
local
FX, @ transition matrix @
Q, @ variance-covariance matrix of v_t @
R, @ variance-covariance matrix of w_t @
it, @ index of the iteration @
ydp, @ y[it,.]' augmented of cn @
Xs, @ (Xs augmented of RR @
A, @ from ast(Xs_t) @
H, @ from Hst(Xs_t) @
chsi10, @ chsi_t/t-1 @
chsi11, @ chsi_t/t @
P10, @ variance-covariance matrix of chsi10 @
P11, @ variance-covariance matrix of chsi11 @
yvar, @ yvar = (H'*P10*H + R) @
yvarinv, @ inv(yvar) @
yhat, @ estimated dependent variable @
eps, @ prediction errors @
f0; @ likelihood function @

@ read in and translate time-invariant parameters into standard state-space
matrices @

FX = Fst(th, rx);
Q = Qst(th, rx);
A = ast(th, rx);
/*—————————— likelihood function ——————————*/

f0 = 0; @ f0 will be the log likelihood function @
it = 1; @ it will index the iteration @
do until it > capt;

@ read in and translate time-varying parameters into standard state-space
matrices @

{ydp, Xs} = readin(it, y, XX, n, Sbb);
R = Rst(th, n);
```

```
if skipcon == 0;
      goto aftcon;
endif;
{Xs, ydp, R} = constr(it, Xs, ydp, R); @ time varying constraints @
aftcon:

H = Hst(Xs);

@ set initial value for filter @

if it >= 2;
      goto after;
endif;

chsi10 = must(th, rx);
P10 = prior*Q;

if det(P10) <= 0; @ This corrects initial variance to be robust
      for case of explosive eigenvalues in FX @
      P10 = prior*Q[1,1]*eye(rx);
      P10 = reshape(P10',rx,rx);
endif;

after:
chsif[it,.] = chsi10';
Pf[it,.] = vec(P10)';
yhat = A + H'*chsi10;
yvar = (H'*P10*H + R);
yvarinv = inv(yvar);
eps = ydp - yhat;
f0 = f0 - ln(det(yvar)) - eps'*yvarinv*eps;
chsi11 = chsi10 + P10*H*yvarinv*eps;
chsi[it,.] = chsi11';
chsi10 = FX*chsi11 + must(th, rx);
P11 = P10 - P10*H*yvarinv*H'*P10;
P[it,.] = vec(P11)';
P10 = FX*P11*FX' + Q;
it = it +1;
endo;
f0 = -(capt*n/2) * log(2*pi) + f0/2;
retp(-f0);
endp;
```

Gauss procedure to augment the Kalman filter

```
/* ==========================================
Filename: AUGMENT
Author: Filippo Moauro
Date: 13/10/1997
Type: Gauss procedure
Description:
Given a general state-space model and a set of restrictions (like in GENX1)
the procedure augments the measurement equation following Doran (1992).
Input:
    cn = vector of restrictions
    RR = matrix of restrictions
    Xs = matrix of regressors
    ysm = vector of dependent variables
    R = variance-covariance matrix
    it = t-th iteration of the Kalman filter
    intr = time-interval of the restriction
    (scalar or 2-vector, if zero no augmentation)
Output:
    Xstar = augmented Xs
    ystar = augmented ysm
    Rstar = augmented R

========================================== */
proc(3) = augment(cn, RR, Xs, ysm, R, it, intr);
    local Xstar, ystar, Rstar, maxint, minint;
    minint = minc(vecr(intr));
    maxint = maxc(vecr(intr));
    Xstar = Xs;
    ystar = ysm;
    Rstar = R;
    if it >= minint and it <= maxint;
        Xstar = Xs|RR;
        ystar = ysm|cn;
        Rstar = R~zeros(rows(R),rows(RR));
        Rstar = Rstar|zeros(rows(RR),cols(Rstar));
    endif;
retp(Xstar, ystar, Rstar);
endp;
```

Gauss procedure for deterministic interpolation

```
/* ==========================================
Filename: INTERP
Author: Filippo Moauro
Date: 10/10/1997
Type: Gauss-procedure
Description:
Deterministic interpolation of a matrix over the time given two observed
conditions
Input:
    M1: (n x k) matrix: first observed matrix
    M2: (n x k) matrix: second observed matrix
    t1: (n x k) matrix: observation period of M1
    t2: (n x k) matrix: observation period of M2
    capt: period of interpolation
Output:
    Mint: (capt x (n*k)) matrix (interpolated matrixes over the
    time in vec-form)

========================================== */
proc(1) = interp(M1, M2, t1, t2, capt);
    local Mint, it, a, b, n, k, beta;
    n = rows(M1);
    k = rows(M1');
    a = vec(M1)';
    b = vec(M2)';
    Mint = zeros(capt,n*k);
    beta = (b - a)/(t2 - t1); @ slope of interpolation @
    it = 1;
    do until it > capt;
        Mint[it,.] = a + (it - t1)*beta;
        it = it + 1;
    endo;
retp(Mint);
endp;
```

Gauss codes on the smoothing algorithm

```
/* ======================================== */

chsis[capt,.] = chsi[capt,.];
Ps[capt,.] = P[capt,.];
it = 1;
do until it > capt - 1;
    ii = capt - it ;
    PPs = reshape(Ps[ii+1,.],rx,rx)';
    Ptt = reshape(P[ii,.],rx,rx)';
    Pt_tt = reshape(Pf[ii+1,.],rx,rx)';
    J = Ptt*FX'*inv(Pt_tt);
    chsis[ii,.] = chsi[ii,.] + (chsis[ii+1,.] - chsif[ii+1,.])*J';
    PPs = Ptt + J*(PPs - Pt_tt)*J';
    Ps[ii,.] = vec(PPs)';
    it = it +1;
endo;

/* ======================================== */
```

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