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ESSAYS ON THE INTERACTIONS BETWEEN POPULATION AND HUMAN CAPITAL, AND CONSEQUENCES TO ECONOMIC GROWTH

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ABSTRACT

FACULTY OF SOCIAL SCIENCES
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This thesis comprises of three major chapters concerning the effects of demographic factors on aggregate human capital and through it on economic growth.

Although human capital can exist only embodied in individuals, this embodiment property has been relatively neglected in the literature. Chapter 3 aims in showing that exactly because of this property, the demographic features of an economy are very important for this economy's aggregate human capital and through it economic growth. In particular, instead of assuming an aggregate accumulation function of human capital, as in the literature, in chapter 3 I rather aggregate the education decisions of the individual economic agents. The result is that the demographic features of an economy affect its human capital accumulation in various ways, missed when one aggregates the accumulation function instead of the education decisions of the economic agents.

Although the endogenous technology literature recognises human capital as the power that drives technology and through it economic growth, it usually treats it as exogenous and fixed. As a result, it finds a linear relationship between the population size or growth rate and technological improvement. In chapter 4 I introduce human capital investment in an endogenous technology framework. It is shown that even without the effects of the previous chapter, population affects technological improvement both directly and through the stream of human capital, with technology also having a feedback effect on the latter. Multiple are therefore the effects of demographic factors on innovation and economic growth, which can explain certain facts, the most important of which is the growth patterns of the last two centuries.

Borrowing constraints on households have been found to have positive effects on physical and negative on human capital accumulation and economic growth. However, fertility is too expected to be affected by borrowing constraints, while it also interacts with the accumulation of both types of capital. The effects of borrowing constraints under assumptions of endogenous population is exactly what chapter 5 studies. The main results are that when fertility is endogenous a borrowing constraint has a negative effect on it, while by reducing fertility it may affect economic growth positively rather than negatively.

Overall, the thesis contributes in the understanding of the effects of demographic factors on the economy, although it is impossible to answer all questions on such a large subject.

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Chapter 1

Introduction

The twentieth century is the one that brought the most unprecedented demographic changes at least in the developed world. In particular, the fertility and growth rate of population dropped drastically while both the life expectancy and average age increased. Given such changes the interest of economists for the interactions between population and the economy is not surprising.

This thesis comprises of three essays on the interaction between demographic factors and human capital, and through it economic growth. My approach is an aggregate supply one, in the sense that I assume efficient use of the economy's resources and study how their stock is affected by demographic factors.

In chapter 2 I review the literature on which I build my research. The empirical studies on the relation between demographic and economic variables well prove the existence of a relation between the two. This relation has been explained in the literature through two— very roughly divided— streams of causality: The first such stream studies the effects of the economy on demographic variables, especially child bearing. The second studies the effects of demographic variables on the economy. Finally, chapter 2 also reviews the theories of human capital formation, as human capital is the special link between population and the economy that I study in this thesis.

In the first essay of the thesis— chapter 3— I concentrate on the embodiment nature of human capital. In particular, I stress the fact that as embodied to people, human capital is indispensable from the demographic features of the economy and contrary to the physical capital, no altruism or other assumption can make it inheritable. Consequently both its average level and accumulation decisions are affected by the demographic factors of the economy. I next develop a growth model where human capital is the only production factor. The assumption of perfect altruism ensures that human capital is built on the members of the society on which it has the highest returns, while the absence of physical capital rules out the capital dilution effect of population.

As was shown by the authors of the 1970s that studied the problem of human capital investment, the returns to the human capital of an individual economic agent as well as their human capital investment depend on their life expectancy. I therefore argue that the popular approach, that assumes an aggregate human capital accumulation function, is incorrect because it ignores this factor. Instead, I disaggregate human capital investment to the level of the individuals and I aggregate only the optimal solutions. In this way the demographic features of an economy are found to affect its aggregate human capital in three different ways, at least two of which have been missed by the literature: First, a fastly growing population drains the resources of an economy, with the result of less investment in human capital. Second, as long as the new generations start at a human capital level that is lower than the average of the economy, they imply a human capital dilution effect, which is higher the larger the size of the new generation. The third and most neglected effect of population consists on the life cycle theory of human capital investment. In particular, since the young generations invest in human capital more than the old the higher the portion of the young in the economy the higher the average investment.

In chapter 4 I introduce human capital investment in a framework of endogenous technology without scale effects. One common feature among endogenous technology studies is that they treat human capital as exogenous, although they recognise it as the driving force of innovation and technical progress. This way they find a linear relationship between the population size or growth rate and technological improvement. However, the absence of human capital investment conceals the fact that the relationship is not from population, but from aggregate human capital to innovation and economic growth. This aggregate human capital depends indeed on the population size, but on the per capita human capital as well. By allowing therefore the second to be increased with human capital investment, multiple effects from population to technological progress emerge, even if one assume away the effects of population on human capital investment that were described in chapter 3.

In particular, it is found that the growth rate of population ceteris paribus boosts the profits of the R&D firms and consequently the aggregate R&D effort, a result consistent with the findings of the endogenous technology studies. However, the faster productivity growth generated this way encourages investment in human capital, which reinforces the initial effect of population on R&D investment. On the negative side though, population growth congests the economy's resources which reduces human capital investment, which in turn lessens or even inverts the original growth effect of population growth.

Equally important with population growth are other, more specific demographic variables. Lower mortality for instance increases human capital investment, by increasing the returns to education. The age structure of population is also very important for human capital investment and through it technological progress, through the streams analysed in chapter 3. It is argued that all these effects combined can explain certain empirical findings, like the positive effect of a generation's size on its education attainment found by various authors, or the growth rates of the last two centuries.

Chapter 5 studies an endogenous fertility model with borrowing constraints. Borrowing constraints on households have been found to have positive effects on physical and negative on human capital accumulation and economic growth. It was also found that as far as economic growth is concerned it is the second effect that dominates, or in other words the effect of the borrowing constraint is negative. Population however has been well established in the literature to affect accumulation in both types of capital, while in an endogenous population framework one would expect fertility choice to be affected too by borrowing constraints. A model with both borrowing constraints and endogenous fertility is therefore developed, and the effects of the constraint with both exogenous and endogenous population are derived and compared. It is found that when facing a borrowing constraint the economic agents reduce both the number of their children and their education expenditure. Yet the reduction of the latter is smaller than when the fertility is exogenous. Further, the lower fertility implies lower rental cost of physical capital and, depending on how intensively the latter is used in the human capital industry, education attainment may in fact increase, despite the reduction of the total expenditure on it. With the engine of growth in the model being exactly human capital, the above result means that if fertility is endogenous the borrowing constraints may have a positive rather than negative effect on economic growth. Even when this is not the case, the endogeneity of population still mitigates the negative growth effect of the constraint. In addition, this effect is an "enlargement" of the mitigating effect of the endogeneity of population on education. Noteworthy finally is that these results are generated without any of the assumptions of the previous two chapters.

Finally chapter 6 summarises the main results of the thesis.

Chapter 2

Review of the literature on human capital and population economics

2.1 Empirical evidence of the relation between population and economic growth

To understand the relationship between demographic and economic variables several authors went as back to the past as the dawn of the human kind. Kremer (1993) for instance studied the relation between population and technological change from 1,000,000 BC. He argued that for most of the human history the Malthusian theory of population had been the case, as per capita output had essentially remained to the subsistence level. Therefore the author concluded that population growth reflected the growth of productivity, thus of technology. He then tested the hypothesis that population growth depends on the population size, and found that this was indeed the case for most of the human history. This was considered by the author as convincing evidence in support of the scaling effect of population in the production of new technology. The author didn't fail to spot the demographic transition of the last decades, but didn't make any attempt to explain it.

An explanation for this demographic transition was offered by Galor and Weil (1999), who distinguished between three periods in human history: first was the Malthusian era, where per capita income was constant and technological progress was reflected entirely to the size of the population. Next came the post-Malthusian regime, where per capita income was growing and population growth and income were positively correlated. Finally, there is the modern era, where population growth and income are negatively correlated. The authors explained the transition from the one period to the other by means of the industrial revolution, which increased both income and the cost of children. According to the authors in the post-Malthusian period dominated the first effect while in modern times dominates the second, that's why the demographic trends were those described.

Enhrich and Lui (1997) stressed a very interesting fact, that is, that the demographic transition occurred not only in Europe, but recently in the just developed far east countries as well. Demographic transition therefore has to be brought about when a certain level of development is reached.

In the remaining of this section I concentrate on the empirical relationship between population and economic growth within the "modern era", that of the demographic transition. For this relationship there is no general agreement, although Simon (1989) was rather assertive that there is no significant relationship between the growth rates of population and per capita income.

Barlow (1994) attributed this finding to autocorrelation of fertility. In particular, he argued that current fertility should have negative effects on growth, for reasons such as increase in population and reduction of the saving rate. Yet lagged fertility has positive effects since it increases labour supply. His regressions most of the times confirmed this intuition, especially with respect to the positive growth effects of lagged fertility. These results were robust even when additional explanatory variables of growth were included. Since fertility is highly autocorrelated, a single regression of population growth on economic growth gives no relation between the two. As additional evidence the author finally mentioned the far east countries, which growth rate is very high, and have high lagged and low current fertility.

Similar is the approach of Kelley and Schmidt (1995), who used both contemporaneous and lagged birth and death rates in the growth equation. They found that both fertility and mortality have negative effects on growth, yet the effect of lagged fertility is positive. Interesting also is their finding that the growth effects of the demographic variables depend on the level of economic development. This implies that not only the growth rate but also the level of income interacts with fertility. With this result also agreed Galor and Weil (2000), who argued that poorer countries have higher fertility and population growth rate.

Different however was the finding of Brander and Dowrick (1994), who found that population growth is insignificant for economic growth. The authors explained their finding with the argument that population growth affects output growth through various streams, including scale economies, and that's why its overall result is insignificant. With this argument disagreed Yip and Zhang (1997) though, who argued instead that ceteris paribus higher fertility is associated with lower per capita growth. The reason this negative relationship did not emerge in empirical studies is according to the authors the exclusion of important exogenous variables that affect both fertility and growth in the same way.

Poverty and fertility in the developing countries was the object of Eastwood and Lipton (1999). Since however poverty is also affected by growth, they considered the relation between fertility and growth as well. Their result is that fertility has a strong negative effect on growth: 4 less births in 1000 people would have increased growth by 1.1% in the median developing country. They concluded with the statement that growth and equality in the long run would both reduce fertility, leading to a virtuous circle of low fertility, high growth, and low inequality.

Interesting finally is the work of Barro and Sala-i-Martin (1995), who regressed growth on many variables, fertility and population growth among them. They found that the first has a negative effect while the effect of the latter is insignificant.

2.1.1 Growth effects of the population age structure

The study of the above literature reveals an interesting fact: Population growth does not appear to affect economic growth, but its elements— that is, current and lagged fertility and mortality— do affect growth. Since these elements affect not only population growth but also its age structure, it may well be the latter rather than the former that affects economic growth.

Relatively few however are the authors that measured directly the effects of the population age structure on growth. Perhaps Denison (1962) is the first such author. In particular, he argued that the quality of the labour force depends among other factors on its age and sex composition, and constructed an index of effective labour supply, in which the age and sex composition of the labour force was also taken into account. At his growth accounting exercise he found that the age-sex composition of the US labour force resulted to an additional 0.1% of annual growth between 1929-1957, yet predicted that the growth effect of this factor between 1960-1980 would be −0.01%. Later on (1979) he repeated this exercise and found that the age-sex composition of the labour force had a negative effect on US growth from as early as 1953, which was topped between 1969-76 where it reached −0.4%. Yet he also predicted that this factor would become favourable in the 1980s. There's no need to stress the consistency of this finding with the productivity slowdown of the 1970s and the recovery of the 1980s.

In the same spirit with Denison, Krueger (1968) used a human capital index to explain the income differences between developed and developing countries. Important is that population age structure was also included in that index. She found that the index accounted for more than half of income differences between developed and developing countries. Next, she estimated the effects of each particular factor included in the index. The population age structure was found very important, yet mostly because it affects educational attainment. Finally, it should be mentioned that this result was, as the author herself stressed, due to population age structure and not to population growth.

More directly estimated the growth effects of the population age structure Sarel

(1995). The author used as dependent variable the per capita growth of 121 countries over five years' periods between 1960 and 1990. Regarding the independent variables, he followed the "traditional" decomposition of growth to factor inputs, but argued that the labour input depends on its age structure. A second order Almon polynomial was used to capture this effect of the population age structure; the result was that the productivity of the individuals follows an inverted-U age profile, with its peak at the age of 55. An important finding also is that productivity is very low during youth while it declines very slowly in old age. Next he estimated the growth rates of the sample countries when "purified" from demographic effects. He found that in some extreme cases the demographic effect on growth is as high as 0.6 percentage points in absolute value. Further, the growth difference between US and Japan was found to be partly due to the Japanese advantage in demographic terms.

Malmberg (1994) used time series of the Swedish economy and regressed the age structure on growth and savings. He found that the 50% of the country's growth can be explained by the population age structure. This study extended later on Lindh and Malmberg (1999) to all OECD economies. In particular, they used pooled cross-section and time series data of economic growth for these economies. Dummy variables for four age groups where included in the list of the independent variables, with the missing group being the youngest one (0-14 years old). They found that the percentages of the two young adult groups have a small positive effect on growth while the middle-aged have a strong positive effect and the effect of the old on economic growth is negative and strong[1]. Important also is that the non-demographic variables had the expected effects on growth, while the effects of the age structure were robust under alternative specifications. The authors next argued that according to their results the population age structure had adverse effects to productivity growth from as early as 1965, and this steadily deteriorated till the 1980s. Interesting is the similarity of this result with that of Denison, although the latter studied the US while Lindh and Malmberg studied all

[1] Of course all these effects are net of that of the missing 0-14 age group.

OECD economies. Lindh and Malmberg finally conducted an out of sample projection and found that the age structure alone reduced productivity growth in 1990-1995 by 0.2%.

To conclude this brief review of the empirical evidence, there is a disagreement on whether the population growth rate has a negative or zero effect on per capita growth. Relatively few are the studies of the growth effects of the population age structure. Yet these studies appear to unanimously support that the latter is indeed important for the former. They also agree on the inverted-U shape of the effects of the population age on economic growth, although there is disagreement on the peak and the slope of this function.

2.2 The theory of human capital formation

Two are the dominant theories of human capital accumulation: Learning-by-doing, and accumulation through a separate production procedure (education). Although both theories are equally important the second will be reviewed more thoroughly, as this is the theory I adopt throughout the thesis. Before however proceeding with the theory a brief review of the empirical evidence on human capital and growth is deemed necessary.

2.2.1 Empirical evidence on human capital and growth

It's impossible to review the entire literature on the issue. Therefore only a sample was chosen, which however is sufficient to establish empirically the relationship between human capital and economic growth.

Azariades and Drazen (1990) measured human capital as the literacy ratio of a country. With the use of a simple 2-dimensional scatter graph they found that there is no country with high growth and high illiteracy. In addition, their graphs showed that the countries with the highest growth rates had very high literacy, comparing to their initial GDP per capita. Given this evidence, the authors argued that there is a "growth possibility frontier", that is, a country's GDP to literacy ratio defines an upper bound for the growth rate this country can achieve in the near future. Regarding finally the countries that performed much below their growth possibility frontier (mostly LDC), the authors claimed that this might have been due to emigration of skilled individuals or to high fertility rate. Next the authors regressed the growth rates of the periods 1960-80 on initial literacy and per capita GDP. They found that the first has a strong positive effect on economic growth while the effect of the second is negative.

The growth rate estimated Barro (1991) as well, only he included additional independent variables in his regression, to rule out the possibility of spurious regression between growth and human capital. Further, instead of literacy he used school enrolment rates in primary and secondary education as proxies for human capital. His results are very similar with those of Azariades and Drazen: he too found that the initial per capita GDP has a negative effect on growth, while both human capital variables were found to have a strong positive effect. The author also found a strong correlation between the initial per capita GDP and human capital, which he pointed out as the culprit for the observed lack of relation between initial GDP and growth.

In later work with Sala-i-Martin, Barro (1995) developed the model further and included more independent variables, many of which were related to education and human capital. In particular, he used both school enrolment and education attainment, which he further separated to male and female. Further, he also included higher education, along with primary and secondary. His results with respect to human capital were very similar to those of his previous study: The effect of primary education was found insignificant, but the male secondary and higher education were found to have a strong positive growth effect. In addition, the effect of higher education is much stronger, implying according to the author that human capital is a convex function of education. Life expectancy was also included as a human capital variable, because as the author argued it is related with the intertemporal supply of skills— for any given educational attainment. Negative as before was found the effect of the initial per capita GDP, only now this effect was also found to be reinforced by the human capital variables. However, puzzling was the finding that female education was found to affect growth negatively. The explanation the author offered is that low female education shows backwardness, therefore high growth potential, through the stream of the initial per capita GDP just described.

Positive found the effects of human capital Mankiw, Romer and Weil (1992) as well, who tested the convergence property of the Solow exogenous growth model, which they amended to include human capital. In particular, they assumed a production function in labour and physical and human capital, that is

$$Y_t = K_t^a H_t^\beta (A_t L_t)^{1-a-\beta}$$

This production function exhibits diminishing returns with respect to the two capital goods. They then estimated the cumulative growth rate between 1960-85 for most market economy countries. As independent variables they took investment ratios in both types of capital, as well as population and technology growth rates. They found a significant positive effect of human capital, although it was smaller than that of physical capital. Further, the results did not reject the hypothesis of Solow convergence.

Total factor productivity rather than per capita growth is the focus of Engelbrecht (1997). In particular, the author did not deny that human capital is an important production input, but also stressed the argument of Romer (1990), that the level of human capital affects productivity growth. His empirical results showed that although human capital is not significant for TFP growth it is nevertheless for domestic innovation and productivity catch-up.

Finally the work of Bils and Klenow (2000) must be mentioned, who stressed that growth is more correlated to initial school enrolment rather than attainment, and argued that this implies a causality from growth to schooling. Their empirical results show that the "expected growth causes schooling" channel yields higher relationship between schooling and growth than the "schooling causes growth" one.

To sum up the above discussion, the empirical evidence suggests a strong effect of human capital to economic growth. The factors that affect the formation of this important production factor is the objective of the rest of this section.

2.2.2 Human capital accumulation by Learning-by-Doing

Two are the dominant theories of human capital accumulation: the first is that it is the result of a separate production procedure, with time invested in education being the most important input. The second is that human capital is built on work, by acquiring experience (learning-by-doing). Although it is the first theory that is adopted throughout this thesis, a brief mention to the theory of LBD is deemed necessary.

The first that introduced the concept of LBD was Arrow (1962). In particular he took cumulative investment as an index of experience. Next he assumed that the usefulness of this experience consists in reducing the labour input required to match a unit of physical capital in the production of the final output. Yet he also argued that this is the case only for physical capital built after the experience has been acquired. Despite the rather complicated and indirect way LBD is introduced in Arrow's model, it was very useful by motivating subsequent researchers. For instance Sheshinski (1967) followed in essence Arrow's footsteps, by assuming a labour augmenting technology which depends on cumulative investment— only contrary to Arrow he assumed that the implementation of this technology does not require investment in modern physical capital.

Directly addressed the issue Echaus (1963) and Rapping (1965). The first argued that by providing LBD firms are in the same time private schools. He also argued that quite often the training is firm-specific and lost if the worker change employer— a theory that was developed later by other authors. Rapping on the other hand tested the LBD assumption empirically on the ship construction during WWII, that is, the very example Arrow used. He assumed a Cobb Douglas production function with the technology coefficient growing exogenously. When he made this coefficient dependent on cumulative output the fit of the model was considerably improved and the cumulative output was found statistically significant even when a trend variable was included. The author considered this finding as supportive for the learning-by-doing hypothesis.

The LBD assumption also adopts Rosen (1972), who assumes a production function with respect to knowledge and "other inputs". Regarding knowledge, the author assumes too that it depends on cumulative output. The contribution of Rosen however consists in stressing that although a by-product of production, knowledge is not a free good. In particular he argued that employees are paid below their marginal productivities, exactly because they also enjoy the additional benefit of learning. For this same reason jobs with higher learning opportunities are paid less. He concluded that when workers choose jobs they also take into account their learning opportunities, which they can later on cash by moving to a more skills-requiring and therefore better paid job. Finally, the author argued that LBD is not a costless procedure for the firm either, since the time of an experienced worker is often needed as an input.

Later authors studied the interactions between technology and the learning-by-doing formation of human capital. Chari and Hopenhayn (1991) for instance assumed technology specific LBD. That is, they assumed that workers acquire experience in one technology by working on it for one period (as unskilled). They can then decide either to stay in the same technology as skilled or to work in a more advanced as unskilled. Imperfect substitution between skilled and unskilled labour is also assumed. The most important result is that because of the technology-specific nature of learning there is a lag between the time a new technology is invented and its peak usage. This lag is higher if unskilled and skilled workers are complements in the production function.

Increasing variety of goods rather than more advanced technologies assumed Lucas (1993). In his model LBD is industry specific, but exerts an externality on the production of newly introduced goods. In particular, the level of knowledge of a new good is assumed to be a weighted average of experience in all existing goods. The author argued that free trade and the implied specialisation condemn a poor country to the production of low quality goods, yet in the long run they have positive growth effects.

Bounded LBD assumed on the other hand Young (1993). In particular, he assumed that learning is accumulated at a decreasing rate, till it eventually reaches its maximum. Growth in this model is achieved by the introduction of new goods with low experience on them, but also with their production requiring less labour once the maximum learning has been reached. The development of new goods is also assumed to have a positive feedback on the learning of the older ones. The main result of the paper is that the maximum productivity is not achieved for the most modern good, and this learning gap can hold an economy back.

Finally, the work of Jovanovic and Nyarko (1996) must be mentioned, who assumed that learning is technology specific and takes the form of updating in a Bayesian manner the distribution of a random term in the production function. This formulation implies both bounded and technology specific LBD. The authors found that if the knowledge is very technology-specific and the eventual returns to the new technology are low, then individuals (or countries) with high knowledge in one technology will stick to it while the others will be frequently switching to newer technologies. In this case knowledge impedes long run progress. If however the knowledge is rather general or the eventual returns to the new technology are high then all agents will be switching.

2.2.3 Life cycle choice of education

The rival to the learning-by-doing theory is that human capital accumulation is the result of a separate production procedure. According to this approach, time invested in education is the most important input for human capital. The cost of this input consists in the current income that would have been generated if this time was devoted to labour instead. This approach was first studied in a microeconomic context, in which the economic agents maximize their objective function, taking into account the costs and benefits of investing in own education. Because of its importance this background theory is briefly reviewed in this subsection.

Ben-Porath (1967) assumed that individuals maximize their lifetime earnings, which consist on labour income minus expenditure for human capital formation. The labour income was assumed proportional to human capital, which was assumed to be accumulated according to the function (own notation)

$$\dot{K}_t = \beta_0 \left(s_t K_t\right)^{\beta_1} D_t^{\beta_2} - \delta K_t \qquad\qquad (\beta_1 + \beta_2 < 1)$$

The cost of human capital accumulation is

$$I_t = a_0 s_t K_t + P_d D_t$$

where K_t is human capital, D_t stands for material inputs, s_t is the fraction of human capital allocated to human capital production[2], while δ is depreciation rate and a_0 and P_d stand for the rent for human capital and the cost of the material inputs respectively.

The result is that the demand for education is positive for the entire lifepath of the individual. This is due to the fact that the marginal productivity of education in the human capital industry is infinite at $s_t = 0$. However, the finite lifetime ensures that the demand for education is also decreasing and at the time of death (or retirement) it smoothly falls to zero. The author also found that the interest rate affects the education choice negatively, while positive is the effect of the relative price of human capital.

Subsequent authors, i.e., Ryder, Stafford and Stephan (1976), Blinder and Weiss (1976) and Heckman (1976), studied the issue of optimal lifetime education in a utility rather than earnings' maximisation context, in which they also included leisure. These authors generally assumed an intertemporal utility function with respect to consumption and leisure. They didn't alter considerably the human capital production function of Ben-Porath, yet they also allowed for accumulation in real assets. Despite their (minor) differences these authors agree in their conclusion that when leisure is included the lifetime education choice is not necessarily as smooth as Ben-Porath found. Ryder, Stafford and Stephan for instance found that for certain initial conditions it may be optimal to

[2]This definition allows for joint production of human capital and earnings. If the possibility of joint production is assumed away then s_t is the time devoted to human capital production.

end one's life with a period of high labour, although education still dies out. In addition, Blinder and Weiss found that four different phases may occur during an individual's lifetime: a period with zero work (thus with only schooling and leisure), a period with all three, a period with no education, and a period of retirement (that is, neither education nor work). Further, it was shown that the sequence of these periods is not standard; for instance impatient agents start from retirement while cycling may also occur. Driffill (1980) however showed that if individuals finish with retirement then they do not cycle, while the result of Ben-Porath, that the lifetime education choice is declining, is restored. The author also showed that the time allocation is the same for all agents regardless of their initial wealth, only the poorer do everything later than the wealthier.

Interesting finally is the work of Killingsworth (1982), who assumed a human capital accumulation function that ecompasses both education and LBD assumptions. His results are not very different from that of Blinder and Weiss. In particular, he too found the same four possible phases in one's lifetime. As a new result though, it can be mentioned that in Killingsworth's model cycling occur only between the "no work" and "all three" cases.

Finally, as a common point among all the above authors, it must be mentioned that they all assume that the level of human capital has a positive effect on its further accumulation.

2.2.4 Human capital and education in a macroeconomic framework

The recent trend in macroeconomics has been the construction of macro models from micro foundations. Following this trend, macro models assume that human capital is accumulated according to an aggregate human capital production, the core of which resembles the human capital production function of the life cycle models just reviewed.

The first author that followed this approach was Uzawa (1965). In his model three are the production factors: physical capital, human capital, and labour. Human capital accumulation depends on the time invested in education, that is

$$\dot{A}_t = \varphi(1 - u_t)A_t$$

where u_t is the time allocated to work, A_t is the level of human capital and $\varphi(...)$ is a concave function. The author derives a balanced growth path where the ratio $\frac{K_t}{A_t L_t}$ is constant. Essentially same is the work of Razin (1972), who however presented it in a more articulate manner.

This approach followed much later Lucas (1988), who assumed a physical output production function of the type

$$Y_t = AK_t^{\beta}\left(u_t h_t N_t\right)^{1-\beta} h_{at}^{\gamma}$$

while an individual's human capital is accumulated according to

$$\dot{h}_t = \delta h_t^{\zeta}(1 - u_t)$$

where u_t is the time spend in labour activities, N_t is population size, and h_t and h_{at} are the human capital of an individual and the average human capital of the economy respectively. The latter implies an externality of human capital. The optimality conditions of the model imply first that consumption and investment in physical capital have the same value, and second that time has the same value in both physical output and human capital industries. He finally derived a balanced growth path, in which all variables grow at constant rates, which however is smaller for human capital if $\gamma > 0$.

The model was further developed by Mulligan and Sala-i-Martin (1993) who assumed general forms for the production functions of both physical output and human capital. In particular, they assumed that both physical and human capital are required in both industries, albeit their factor intensities are different. In addition, the average stocks of both types of capital exert positive externalities in both industries. Thus the economic agents decide on the portions of their physical and human capital that they invest in each industry. Of course in equilibrium the rent for each capital type is the same across industries. The authors then derived a necessary condition for endogenous growth, which can be summarised as "decreasing returns in one sector must be offset by increasing returns in the other". Next they derived the balanced growth path (BGP) for the case of endogenous growth, as well as the transitional dynamics towards it. They identified three forces that drive these dynamics:

1.Solow or imbalance effect: If the physical/human capital ratio is not at its BGP value then the returns to the capital good in shortage are high, and so is the growth rate.

2.Consumption smoothing effect: The stock of the capital good in shortage can be increased to its optimal value through savings (which for human capital take the form of less labour and more education). How fast this process will be depends on the intertemporal elastility of substitution[3].

3.Wages effect: If the production of human capital is human capital intensive then the low wages implied by a relatively low stock of physical capital motivate agents to move from the physical output to the education sector.

Very similar is the model of Caballe and Santos (1993), who however assumed that physical capital is not required for the production of human. The authors studied the transitional dynamics in more detail. They found that by discouraging human capital accumulation (that's the wage effect) a high stock of physical capital can result to a transition period with physical and human capital *decumulation.* Quite the contrary, abundance of human capital stimulates accumulation in both capital types. They finally studied the case of inverse factor intensities, that is, the case where human capital is physical capital intensive. In this case the wages effect disappears, thus a relative abundance of the one capital type implies a transition period with high growth rate, as dictated by the Solow effect alone.

[3]The authors mentioned this effect only for the case of low *physical* capital. However the argument applies equally well for a human capital shortage.

Leisure in the utility function introduced Lucas (1990). In his model the economic agents allocate their time between education, work and leisure. A BGP was found, where consumption, government spending, and physical and human capital grow at the same constant rate. In this BGP the time allocation between the three activities is constant. He then studied the question of the optimal distortionary taxation. The result was that optimality requires zero taxes on the income of physical capital, since taxes on wages do no distort the time allocation decisions.

All the above studies assumed that the scale effects of human capital in the human capital industry do not change qualitatively (that is, they can be either constant, increasing, or diminishing, but their nature is the same all the time). With this assumption disagreed Azariades and Drazen (1990), who argued that human capital is exactly the sector where alternating diminishing and increasing returns are likely to occur. If this is indeed the case then multiplicity of equilibria may occur. This is according to the author an important cause of underdevelopment. Noteworthy is the similarity of this result with that of Cabelle and Santos, who studied as said the transitional dynamics in economies with two capital goods. These authors showed that low human capital discourages investment in both types of capital, with the consequence of low growth. However this effect is transitory and dies out as the physical/human capital ratio approaches its long run value. With alternating scale economies though underdevelopment is permanent, and only external forces can drive an economy out of it.

Externalities of human capital to the human capital industry obviously depend on the relations between people. With that in mind Rosenzweig and Wolpin (1994) assumed that the mother's human capital has a positive effect on that of the children. In their model an altruistic mother takes that into account when deciding on her own education. The authors concluded that there are intergenerational increasing effects in the production of human capital. Given that the human capital stock has a positive effect on its further accumulation, education has higher returns for individuals with educated parents, who because of that take more education. This according to the authors exaggerates the effect of own schooling on own human capital in empirical studies.

Both family and economy-wide externalities assumed Galor and Tsiddon (1997). They further assumed that the first externality yields alternating increasing-decreasing returns, as in Azariades and Drazen. Consequently, with the average human capital of the economy fixed, the human capital of a dynasty may have one or two stable equilibria. In the second case it is the initial conditions that determine which of the two equilibria will be reached. However the average human capital of the economy is not fixed and depends on the human capital of all families. Thus as the latter converge towards their long run equilibria the equilibria themselves change as a result of the changing average human capital. The authors concluded that this effect may eventually eliminate the lower equilibrium, leaving only the higher one. They also argued that in the case of a developing economy this is more likely to happen if the initial human capital is very unequally distributed among families.

All the above studies emphasise on the role of human capital while their attitude towards the physical is rather dismissive. The role of physical capital restored Graca, Jafarey and Philippopoulos (1995) who developed a model were both capital types have a positive externality in human capital accumulation. If human capital is very low then its production is very costly and because of that abandoned. Yet the more the economy accumulates in physical capital the more productive the human capital industry becomes, due to the externality of the former to the latter. Thus the economy may at some time reach a point where investment in human capital becomes positive. This is the "take off case" of the authors, where physical capital accumulation eventually drives accumulation in both types of capital.

2.3 The theory of endogenous population

2.3.1 The demand for children

The origins of the theory of endogenous population go back to Malthus, but in the years that followed this theory was neglected till it was revived by Becker (1960). Becker emphasised the failure of the malthusian theory to account for the demographic changes of the twentieth century, and replaced it with a new theory of fertility choice. He stressed that children are in fact consumer durables. But the main contribution of Becker was the concept of the quality of children, the money spent on children implied by this term. According to Becker parents derive utility not only from the number of children they have (quantity) but also from the quality of these children. Although he mentioned this property in 1960, he developed it formally only in 1981. In particular, he assumed a utility function in quantity and quality of children, and other goods. Innovative also was his budget constraint, which was given by

$$p_c qn + \pi_z Z = I$$

where π_z and Z stand for price and quantity of "other goods". q and n are children quality and quantity while p_c is the cost for one unit of quality for one child. Important in this specification is that the budget constraint is not linear; in fact increasing one of q and n increases the price of the other.

The author then emphasised that the children quality has higher income elasticity and this is the main reason that fertility (children quantity) did not increase in the twentieth century, despite the increase of income. This result was also intensified by the fact that the increased demand for quality had an increasing effect on the price of quantity, as described above.

Becker's theory has been generally accepted. In fact the subsequent authors didn't revise it, but rather analysed certain aspects of it. In particular, they usually tried either to make Becker's utility function more specific, or to explain what lies behind his unit cost p_c.

Schultz (1969) for instance argued that parents have a family goal, which depends on the utility and cost of children. He identified several economic factors that affect fertility. As such, the value of women's time was considered an important part of the cost. Permanent income affects according to the author the family plan, while temporary fluctuations of income affect the timing of fertility. Children mortality increases fertility, while uncertainty towards births is important as well. Important finally are institutions like pensions, child labour, and education. The latter increases the cost of children even if provided publicly. Regarding finally the preferences, although the author identified them as an individual matter he also argued that the education of parents affects their preferences towards both quantity and quality of children. This last argument developed Kremer and Chen (1999), who argued that it is towards quality rather than quantity that educated parents are more inclined to. As a result their children are also educated, have too preferences towards the quality rather than quantity of their own children, which leads according to the authors to a multiplier effect of education.

Rosenzweig (1990) studied the problem of quality versus quantity of children both theoretically and empirically. In particular he assumed a utility function in consumption, children's income, and children's human capital. He also assumed that the cost of children is material, while that of education consists in both time and schooling inputs. Among his results is that if quality and quantity of children are Hicksian substitutes then anything that increases the cost of the one lowers the demand for it and increases the demand for the other. The author also studied the problem empirically for a small number of developing countries. His empirical results prove the trade-off between quantity and quality of children. In particular, children wages—which consist a negative part of the cost of children— were found to have positive effects on fertility and negative on education attainment. Growth was also found to affect fertility negatively and schooling positively, the latter because it increases the returns to education, as argued by the author. Interesting also was his finding that the parents' income affects schooling positively, while male income has positive and female negative effect on fertility.

The fertility choice was studied empirically by various other authors. Butz and Ward (1979) for instance used time series data of the US economy and found that the US fertility was pro-cyclical before the 2nd world war, but counter-cyclical afterwards. The authors' explanation is that before WWII very few women worked, with the opposite being the case after the war. Therefore, before the war the business cycle had only an income effect on fertility— through the husband's income. When women entered the labour force though a substitution effect emerged through the mechanism of women's time: in a recession less women work and therefore can afford to spend time in child rearing.

Shields and Tracy (1986) used US time series too and regressed fertility on income, lagged infant mortality, female labour force participation, and age structure of population. The latter to account for the fact that fertility is different by age, while lagged infant mortality was included to capture the time adjustment mechanism in family planning. They found positive effects of income, infant mortality and population in the age group of 18-24, while the effect of female labour force participation was found negative.

The British fertility from as early as 1860 studied instead Tzannatos and Symons (1989). The specification of the authors is that children are a consumption good, which price is proportional to the wife's wage, since child rearing has mainly been a female business. Thus they found that the husband's earnings have a positive effect on fertility, capturing the income effect on the demand for children. The wife's earnings on the other hand have a negative effect on fertility as they affect both income and the price of children. Negative was also found the effect of female education, similarly to Willis (1973). Negatively finally were the effects of the unemployment rate and the sex discrimination act on fertility, while employment in agriculture had a positive effect, although the authors could not say whether it was due to different preferences or lower cost of children.

Different to the above studies is that of Panopoulou and Tsakloglou (1999) in the sense that they used cross-country rather than time series data. They found that income and infant mortality have positive effects on fertility while the effects of female education and of urbanisation are negative. Puzzling however was the fact that female labour force participation was found insignificant. The authors attribute this finding to simultaneity, that is, that fertility also feedbacks to labour force participation. Collinearity between female education and labour force participation is also probable.

Innovative also is the work of Wolpin (1984), who modelled fertility behaviour as a life-cycle decision and estimated it from Malaysian census data. Among his results is that children are gross substitutes to consumption, that the mother's education has a strong negative effect on fertility and that income has a small positive effect on fertility, although this effect grows stronger the higher the income is. Unlike the previous literature, he also studied the timing of income and found that an increase in income has a higher effect if it come early in life, while rising income profile delays births slightly. Finally he found that the survival probability of children has a positive effect on fertility.

The Easterlin hypothesis

Different to the above authors is the approach of Easterlin (1966 and 1978), who focused on preferences towards fertility and argued that they are in fact affected by the demographic cycle. In particular, Easterlin introduced the concept of "desired consumption level" and argued that expenditure on children, and therefore fertility, is the difference between income and the desired consumption level. Regarding this "desired consumption level" he adopted the "relative income hypothesis" and argued that people want to maintain the standard of living they used to enjoy in their parents' household, while they also want to achieve for their children the "quality" they themselves enjoyed. Because of this, the author argued that there are fertility cycles, which follow the business cycle. He further argued that the income of young adults had been declining since 1955, which was the main reason for the declining fertility.

The "Easterlin hypothesis" has been studied empirically with two different approaches: cohort size and relative income effects on fertility. By the second the individuals' income comparing to that of their parents is meant. The idea behind the first approach is that a large generation will be worse off and to make up for it will bear less children. Macunovich (1998) surveyed these empirical studies. She reached the conclusion that the results generally support the Easterlin hypothesis, regardless of which of the two approaches has been used. Of all this literature though I wish to make special mention to Abeysinghe (1991), who found that parental income is much more important for fertility decisions than own income!

The old age support hypothesis

Regardless of their differences, all the above studies have in common the assumption that children are consumer durables. To this approach opposed the authors that follow the "old age security hypothesis". According to this hypothesis children are capital goods instead. This is the argument of Cain (1983), who stressed that in developing countries the absence of capital markets or welfare institutions leave children as the only means of saving for the old age. The author also compared the two alternative theories (utility and old age security) and argued that they yield very different fertility behaviour. In particular, he argued that reduction in the cost of children will increase the demand for them under the utility hypothesis, but probably not under the old age security hypothesis.

Nugent (1985) supported the hypothesis too, and gave a list of economic conditions that are likely to make old age support an important motive for having children. These conditions can be summarised to uncertainty about the future and undeveloped assets' markets. The author also discussed the empirical studies on fertility choice and argued that although they don't support the old age security hypothesis this is mainly because they usually didn't concentrate on populations where the old age support motive would be strong.

The old age support hypothesis adopts Eswaran (1998) as well, who concentrates on children mortality, which implies risk for the parent-investor. As a means of portfolio diversification the parent prefers to have many uneducated children rather than few educated. The author also argues that as children mortality falls the diversification incentive faints while the return to education increases. As a result, fertility falls as well (may be more than mortality), and education increases. Finally Enhrich and Lui (1997) must be mentioned, for their argument that the development of social security reduced the role of children as capital goods.

Summary

To sum up the above discussion, there is a trade-off between quality and quantity of children. The demand for each has the same properties with the demand for any other good, that is, it depends on preferences, income and prices. The preferences are also affected by one's education and "relative income". The prices depend on infant and child mortality, the value of parents' time and the expected return from children. Unique in the demand for children however is that the prices for both quality and quantity depend on the demand itself, in the sense that higher demand for quality increases the price of quantity, and vice versa. Important finally is the time adjustment mechanism to the family plan.

Regarding the empirical evidence, there appears to be a general agreement that the male income has a positive effect on fertility while the female labour force participation and income affect fertility negatively. Negative also is the effect of urbanisation, as well as that of female education. Although the latter is usually attributed to its effect on female income, important also is that education has an additional negative effect through higher preferences for quality rather than quantity of children. This was also stressed by Becker (1991), who argued that because of that, empirical studies tend to exaggerate the significance of the mother's time. The evidence also appears to support that infant mortality affects fertility positively, although there is no general agreement on that. Finally, important also is the income of the previous generation because it affects the current generation's preferences towards fertility.

2.3.2 The timing of fertility

The timing and spacing of births is for the population growth rate and age structure as important as total fertility is. This question was addressed by several authors:

Heckman and Walker (1990) for instance studied the issue of the timing of births from Swedish longitudinal data. They found that higher male income or lower female wages[4] accelerate family formation, hasten all conceptions, and increase fertility. Also the effects of these economic variables are strongest for the timing of the first birth. Regarding finally the final fertility, the authors found that it is the decision to have a third child that is mostly affected by the male income and female wages[5].

The relationship between the timing of childbearing and investment in human capital studied Blackburn, Bloom and Neumark (1993). By using longitudinal data of US women the authors found that women that delayed their first birth generally enjoy higher wages. They argued that this is due to more human capital investment by these women, and tested their claim empirically. Their empirical results showed that late childbearers indeed tend to invest more heavily in human capital.

Arroyo and Zhang (1997) found that both female income and education delay all births and reduce their number. Opposite is the effect of male wages. This result is consistent with that of Heckman and Walker previously mentioned.

Finally, Iyigun (2000) studied the relation between education and the timing of childrearing theoretically. In his model, the economic agents live for three periods, of which they acquire education in the first two and work and have children in the last two. Their utility is a function of period three consumption and total number of children, which as

[4]Noteworthy is that the authors used male "income" but female "wages". This is quite reasonable; given that child rearing mainly requires women's time, the alternative cost of children consists on the *wages* of women.

[5]This is, as the authors explained, equivalent to the decision to have or not the "final" child, since having more than three children is extremely unusual in Sweden.

already said they can allocate between the last two periods of their lives in any way they like. The result is that since both children and education require time, the economic agents tend to invest in human capital in the second period and postpone their child-bearing for the last. Because of his assumption that parental human capital also has a positive effect on the productivity of time in the human capital industry, the author also found that the higher the human capital of one's parents the more she will postpone her fertility and the more education she will acquire. Multiplicity of steady state can therefore occur, that is, one with low human capital and high and early fertility, and one with late and low fertility and high human capital.

2.3.3 Fertility in a general equilibrium context

A growing approach in the literature is to include fertility decisions in models of intertemporal maximisation. According to this approach individuals derive utility from consumption and fertility, while they're also altruistic towards their offsprings, that is, in deciding on the number of their children they take into account the fact that the higher their numbers the lower their welfare. This approach has the advantage that studies fertility in a macroeconomic framework and derives a general equilibrium for both demographic and economic variables.

Perhaps the first such study was that of Becker and Barro (1988), who postulated a utility function of the type

$$U_0 = v(c_0, n_0) + a(n_0)n_0U_1$$

that is, the utility of the individuals depends on their consumption, number of children, and these children's welfare. It is however assumed that $a'(n_0) < 0$, that is, the more the number of their children, the less the individuals care about them. The authors then developed a general equilibrium and found that in it the patriarch of the dynasty is indifferent in which generation to spend a current pound, while the marginal cost of an additional child equals its marginal benefit. They also developed a steady state where per capita consumption, capital stock, and fertility are all constant. They analysed it further to find the following results:

1. Fertility is lower when technological progress is faster, a result consistent with post war evidence in developed countries.

2. With endogenous population the Ricardian equivalence does not hold any more. This result is consistent with that of Batina (1987), who found that with endogenous fertility a consumption tax is not neutral anymore. The reason why is that the prices of quality and quantity of children are interlinked, as mentioned earlier, which makes the design of a neutral consumption tax impossible. Similar is the result of Lapan and Enders (1990), who argued that public debt increases the cost of children with the result of lower fertility. Finally Enhrich and Lui (1997) stressed a different stream through which the public sector affects fertility, that is, through social security. What these authors argued is that social security creates externalities of children, which distort fertility decisions.

Wang, Yip and Scotese (1994) used continuous rather then discrete time while they also included leisure along with consumption and fertility in the utility function:

$$U = \int_0^\infty e^{-\rho t} \left[u(c_t) + v(x_t, \mu_t) \right] dt$$

where x_t and μ_t stand for leisure and fertility respectively. Having in the utility function the average consumption of the dynasty rather than the personal of the parents implies that the latter are altruistic, which implicitly takes children quality into account[6]. Then additional assumptions about the utility function were made, and the effects of productivity and utility shocks were studied. It was found that a utility shock (that is, higher preference for fertility rather than leisure) reduces steady state consumption and capital while it increases fertility. A productivity shock on the other hand increases consumption and capital and has an ambiguous effect on fertility.

[6]Athough the authors assumed away human capital, therefore children quality.

Continuous time used Palivos (1995) as well, who solved an intertemporal maximisation problem with consumption and fertility in the utility function. His also assumed that the cost of children (in units of parental time) is increasing with respect to their number. He argued that the net return to physical capital is a sum of three: first, the direct return through the marginal productivity of capital. Since however in the author's framework physical capital is also the cost of fertility, higher physical capital reduces fertility. Finally, physical capital increases wages and hence labour shifts from the child to the goods-producing sector. As a result, with endogenous fertility the total marginal return to capital is not always decreasing which may yield multiple equilibria: one with high capital and low fertility and one with high fertility and low capital.

Yip and Zhang (1997) amended the framework of Palivos with endogenous growth, through physical capital externalities. Children still cost parental time. They found that depending on the parameter values the long run equilibria can be one, two, or none, yet in any case no more than one equilibrium will be stable. In this stable equilibrium, there is a negative relationship between long run fertility and per capita growth. Yet the authors stressed that although the relationship between fertility and growth is negative if all exogenous parameters are the same, the latter are never the same when comparing different economies. Next they studied an exogenous technological change. They found that its effects are positive on fertility and ambiguous on per capita growth, therefore in case of a technological change the relation between fertility and per capita growth is indeterminate. The authors concluded that when all the exogenous variables are controlled for the relationship between fertility and growth is negative, and the reason why this is not what the empirical studies had found is exactly that the exogenous variables had not been controlled for.

Becker, Murphy and Tamura (1990) added the dimension of human capital to the framework of Becker and Barro (1988). They also assumed that human capital is human capital intensive. In this specification the quality of children does not enter the utility function directly, but it does matter for the altruistic parents, because it affects the future income of the children. The authors argued that the production of human capital may exhibit increasing returns to scale for some interval. This leads to multiplicity of equilibrium: one with low human capital and high fertility, and one with high human capital and low fertility.

The trade off between fertility and human capital was also the interest of Zhang (1997), who assumed a utility function of the type

$$V_t = \ln c_t + \rho \ln n_t + aV_{t+1}$$

where V_{t+1} is the utility of one's children. The children's human capital was assumed to depend positively on the amount of goods invested in their education, while an externality from the parental human capital was also assumed. Again, since the parents are interested for their children's utility, they're indirectly interested for their human capital as well. Within this rather common framework the author studied the effects of subsidies to children and to education. It was found that a subsidy on education increases human capital and reduces fertility, which both promote economic growth. Opposite are the effects of a subsidy on children, since it makes the quality more expensive relatively to quantity.

Finally the study of Galor and Weil (2000) must be mentioned, who studied the interaction between population and the economy from the beginning of human history. Two are the basic elements of their model: first a utility function with respect to consumption and quantity and quality of children. To this they added the constraint that consumption cannot fall below a subsistence level. Second, the size of population was assumed to have a positive effect on the production of new technology. Assuming that in the beginning both population and technology were low, the subsistence consumption constraint was binding while low was technological progress as well, feedbacking to population growth. Yet as population grows technological progress becomes faster and faster, and so does population growth. At some point though the subsistence consumption constraint ceases to be binding. As a result, the parents start investing in their children's education. However, because their resources increase as well they at first increase both the quantity and quality of their children. This is the "post-Malthusian regime" where both income and fertility increase. Eventually though, further improvements of technology do not affect the time devoted to children whereas they affect the returns to education. As a result, population growth falls while education and technical progress rise. This is the "modern regime".

To sum up the above discussion, when fertility choice is studied in a macroeconomic framework interesting links between growth, fertility, and human capital are revealed: multiplicity of long run equilibria may occur in models that otherwise exhibit uniqueness of equilibrium. When multiplicity occurs, there is a "low" equilibrium with high fertility and low human capital and growth, and a "high" equilibrium with the opposite features. Fertility and through it population size may also affect the growth rate through scale effects. The growth rate then feedbacks to population and fertility. Interesting finally is that with endogenous population, well established policy results like the Ricardian equivalence, do not hold any more.

To complete finally the picture of fertility choice in a macroeconomic framework reference need to be made to the less popular approach, that children are capital goods rather than consumer durables. A representative of this approach is Chakrabarti (1999), who studied fertility in the context of the old age security hypothesis. Under this assumption he found a general equilibrium in which the returns to children and physical capital are equal.

2.4 Population age structure and the economy

The twentieth century was the one that brought unprecedented demographic changes. With respect to the population age structure these changes refer to an increased portion of the old. Indeed as Russell (1982) stressed, the age distribution of the US at the beginning of the century corresponded more or less to a stable population. However births declined during the Great Recession and the 2nd world war, to boom after the war and fall at the late 1960s to levels lower than even the negative records of the 1930s. The author also stressed that this was roughly the case in all developed countries. To this one should add the statement of authors like Samuelson (1975a) that the increased percentage of the old in developed countries is almost entirely due to reduced fertility rather than increased life expectancy.

Given this unprecedented evolution it is not surprising that the economic consequences of the changing age structure of population have been studied by many authors and from various points of view.

2.4.1 The question of the active population

The problem of an ageing population was first studied in relation to the issue of the active population and the dependency ratio. These depend not only on the population age structure, but also on employment by age group, as argued by Johnson (1996). The evidence shows that the evolution of both factors has pushed the dependency ratio upwards.

Nam (1968) for instance claimed that the working life has been reduced, due to increased education, increased expected life and the tendency for premature retirement. He also stressed that the propensity to work has been reduced for both the young and the old. Recent literature emphasise on the role of social security and retirement benefits for the reduced working life. This is shown theoretically by authors like Lazear (1986) and Weil (1999), while Karteyn and de Vos (1998), Supan and Schnabel (1998) and Lee (1998) describe the negative effects of social security on labour force participation in various developed countries.

An insight of the dynamics of the dependency ratio give Brander and Dowrick (1994), by stressing that a fertility decline results in the short run to a decline in the dependency ratio, due to the smaller portion of dependent youngs. In the long run however there is an one to one relationship between fertility decline and dependency ratio while in the middle run the dependency ratio over-responds to demographic variables because the latter also affect the ratio of the population that cares for the dependent.

However the employment per age is unlikely to remain constant if demographic changes occur. The reason why is that these demographic changes affect the factors that determine education, labour supply and retirement decisions. This fact increases the role of the population age structure in determining the dependency ratio on an economy.

2.4.2 Public finance issues and generational accounting

The dependency ratio is strongly relevant to the question of survival and solvency of social security systems, as well as the intergenerational redistribution implied by these systems. As said, the labour force participation of both the old and the young has declined. However what matters for public economics is the old since it is mainly the family the maintains the young and the state that maintains the old (Johnson [1996]). In addition, as Fuchs (1998) argued, it has mainly been because of the increase in spending per old person rather than because of the increase in the number of the old that medicare expenditure had soared. The author predicted hard times for public economics when the increased number of the old will couple the increased spending per old person.

Similar is the prediction of Russell (1982) who argued that the Pay-As-You-Go system was very convenient during the baby boom but will be a real problem when the baby boomers retire. This is also shown by Lee and Tuljapurkar (1998) who used projections of fertility and mortality to simulate the US economy. What they found is that the US retirement funds are expected to reach zero at 2026 unless the system is reformed. They also found that even when economic growth and interest rates are taken into account it is the demographic factors- mainly fertility- that are most important in the long run. In short, the Pay-As-You-Go (PAYG) system is said to generate an implicit public debt, consisting on pensions and other provisions promised to future pensioners. This debt is due to the rapidly increasing portion of the old in developed countries.

Similarly to the formal, the implicit public debt implies transfer of income from current to future generations. Given the PAYG system, the higher the portion of the old in the population the higher the implicit debt and consequently the higher the redistribution from the young to the old. Supan and Schnabel (1998) for instance claim that in Germany the returns to pension contributions are declining and will be negative for those born in 1980. Raffelhuschen (1999) on the other hand calculated the distribution consequences of the pension systems and the welfare programmes in Europe. He then simulated under alternative hypotheses the tax increases required to restore intergenerational balance. Probably the most significant of his results is that the required tax increase is four times what would be required if the age structure of population was constant at the levels of 1995— a result that clearly shows the significance of the age structure of population.

The **transfer wealth** and its welfare implications are also studied by Lee and Lapkoff (1988). In their model they include net transfers into the individuals' budget constraint. The result is that due to the transfers average consumption and production ages are different. Important however is that these ages depend not only on life-cycle consumption and income but also on the population age structure, since the latter affects the path of net transfers— thus the budget constraint of the economic agents. It is because of this gap between average consumption and production ages that changes in fertility affect the indirect utility. This effect on the individuals' utility is of the same sign of the difference between production and consumption average ages. In other words, when transfers are from the younger to the older the average consumption age is higher and a higher fertility will increase the average welfare, by exactly reducing the gap.

2.4.3 Age structure effects on aggregate savings and investment

The negative consequences to savings and investment from a high proportion of the young stressed Palivos and Scotese (1996). According to the authors, provisions to children inevitably divert resources away from physical capital accumulation. Obviously these provisions are higher the higher the fertility rate is. Yet the life cycle theory implies that not only the proportion of the young but rather the entire population age structure matters for aggregate savings. According to this theory the young save while the old dissave. Consequently aggregate savings and through them investment and economic growth are affected by age structure and life cycle income. Two factors are therefore important: marginal propensity to save by age, and income by age.

Since the old dissave, the higher their proportion and their income the less the aggregate saving. The income is also affected by the implicit public debt, as described above. That is, similarly to the former the implicit public debt constitutes wealth for its "holders", or as it is often termed, additional to their real wealth the individuals have a **transfer wealth** which consists of expected provisions at their old age (Lee and Miller [1994]). Both types of debt are liabilities from the unborn to the already born and need not add to zero for the currently alive. Of course the individuals do not care about the form of their wealth. However this does matter for the economy: Similarly to public debt transfer wealth cannot produce output, but in an overlapping generations context it crowds out private saving and investment with detrimental consequences to economic growth[7].

The effects of the population age structure on aggregate saving and investment have also been studied empirically and the evidence appears to show that the population age structure indeed affects the saving rate of an economy. As Maddison (1992) for instance says, the universal decline of savings after 1973 may be among other reasons due to the higher population age, which- coupled by the fact that the individuals have less children to care about- reduces the incentives to save. This effect is according to the author reinforced by increased social security.

More directly studied the issue other authors. Horioka (1991) for instance studied empirically the saving rate of Japan in a model where the ratios of young and old to total population were taken as independent variables, along with economic variables which are

[7]The idea goes back to Samuelson (1975b) who argued that a PAYG pension system may improve steady state welfare if the economy is dynamically inefficient. Although he did not detail the mechanism, he implied that the PAYG has on saving and investment exactly the same effects with formal debt.

usually included in models that estimate aggregate savings. In all alternative models he used, both demographic variables were found negative and significant, with the negative effect of the portion of the old much stronger than that of the portion of the young. Moreover, the author found that the population age structure was the factor that affected mostly the path of the Japanese savings. Noteworthy also is that the results with respect both to demographic and pure economic variables (income, wealth, unemployment, etc) were very similar to those acquired from cross-country data, while the fit of the model was also satisfactory. Finally the model predicts sharp decline of Japan's saving rate in the future due to adverse changes in the age structure.

Lahiri (1989) studied empirically the saving ratio of eight Asian countries. The independent variables of the model include the dependency ratio as well as pure economic variables. The most important result is that in the long run changes in the ratio of the active population affect the Average Propensity to Save by a factor of 1.6.

Finally, the work of Malmberg (1994) must be mentioned. The author used as many as 8 rather than two age group variables, and tested empirically the effects of the groups' size on various variables of the Swedish economy. Regarding the saving rate[8], he found that the age group coefficients follow an inverted-U shape, as dictated by the life cycle hypothesis.

Although these studies are sufficient to establish the causality of the population age structure on aggregate savings, they say nothing on whether most important is the marginal propensity to save by age (life cycle hypothesis) or the income by age (transfer wealth). Undoubtfully this depends on the country's policies towards retirement and old age. Kotlikoff and Summers (1981) provide and answer to this question for the US economy. In particular, the authors found that its aging population is indeed very relevant to the country's declining savings, but mostly through the stream of intergenerational transfers rather than that of life cycle behaviour. Gokhale et. al. (1996) also examined the post war decline in the US saving rate and found that the marginal propensity to

[8]Of the other findings of the paper reference is made on the relevant sections of the present review.

consume of the elderly had increased even if medical care is assumed away; it is also claimed that their resources have been increased as well, due mainly to income redistribution. These two factors coupled with the increased portion of the old are considered as the culprits for the decline of the saving rate. The paper concludes that the saving rate of the US will continue to decline.

2.4.4 Population age structure and human capital

Similarly to physical, human capital can be increased by saving and investment. Since however the human capital has to be built on the members of the population, its formation is very dependent on demographic factors. With that in mind Tu (1969) argued that "the more children born, the greater pressure they'll exert on educational facilities and the budget". Further, individual choice on the formation of human capital depends as reviewed earlier on life cycle behaviour, which makes the aggregate human capital dependent on the population age structure.

With that in mind van Imhoff (1988) and (1989) studied an economic model where a central planner faces two investment decisions: investment in physical and in human capital. This well-known framework was enhanced by disaggregating labour supply to age group vintages. By doing this he allowed for different human capital among age groups while he also took into account life cycle considerations of investing in human capital. As the life cycle theory emphasised, the older the individuals the less their life expectancy, the less the return to their human capital and therefore the less they invest in it. By aggregating this individual behaviour, the author found that the optimal education and consequently the optimal human capital level at any given age is decreasing with respect to the birth rate. Consequently the human capital of the economy is dependent on the age structure of the population.

Park (1997) also argued that human capital accumulation depends on the age structure but used an alternative approach. In particular he assumed that the young workers learn by working with experienced ones. Yet the more the experienced workers per young the more the young learn, and consequently a larger cohort or a rapidly growing population is less educated.

The effects of the population age structure on education and human capital have also been studied empirically. Malmberg (1994) for instance found that the population age structure has a high impact on the Swedish growth rate. He also found that this impact was not solely due to age dependent saving behaviour and argued that the residual effect was due to life cycle human capital accumulation.

More directly studied the issue Jeon and Berger (1996), who used Korean data and found that a larger generation will tend to take more education. The authors explain their finding with the argument that young and old workers are imperfect substitutes and therefore a large generation enjoys lower wages. As a consequence, this generation acquires more education to offset the negative effect of its size on their earnings. Opposite is the finding of Connelly and Gottschalk (1995) who found that the size of an individual's generation has a negative effect on this individual's probability of attending higher education. The difference between the two studies may be due on the one hand to the different economies studied[9] and on the other to the fact that Connelly and Gottschalk also included important social factors in their regression, while Jeon and Berger considered only the relative cohort size. In particular, Connelly and Gottschalk included the father's education. Since this was found to have a positive effect on one's education decision while within the sample it had an increasing trend, omitting it may well distort the results.

Connelly and Gottschalk explain their finding with the argument that the larger a generation the lower the return to human capital thus the lower education they take. Their finding appear to adopt Kosai et.al. (1998), who claim that declining population won't necessarily slow economic growth in the far east, because it will encourage human capital accumulation.

[9] Connelly and Gottschalk studied the US economy.

2.4.5 Age dependent productivity

Another stream through which the age structure of population affects the economy is productivity, which depends among other factors on a worker's age. Denison (1962) was probably the first that took this factor into account. According to his own words "the average quality of the labour force as a whole is affected by its composition in terms of age and sex". He then expressed the aggregate labour input in terms of "adult male equivalents", an adjustment taking into account exactly the age and sex composition of the labour force. As weights for productivity by age or sex he used their wages. He then used his index to find that the age and sex composition of the labour force had significant effects on the US growth performance. With his views appears to agree Spengler (1968) who found that most employees in the US had their salaries topped at the age of 30. Further, judging from the life time path of wages, the author reached the conclusion that productivity is at its maximum at the ages of 35-54 to decline at an annual rate of 1% afterwards. With this agrees Fuchs (1998), who argues that the decline of earning power during old age is on the one hand attributable to obsolescence of skills, on the other to physiological changes like dexterity, stamina and cognitive functions.

Finally Kotlikoff and Gokhale (1992) addressed directly the age productivity issue by estimating the productivity of the employees of a large corporation. Using again wages as a proxy of productivity they found that productivity follows an inverted-U pattern, although its peak and slope differ between professions. Interesting also is that the peak comes sooner and the decline is more rapid for managers, that is, the professionals that do the hardest mental work.

Although the above studies are sufficient to establish the dependence of labour productivity from the age structure of the labour force, they remain purely empirical. In fact very few theoretical work has been done on the issue. Perhaps the only exception is van Imhoff and Ritzen (1988). The authors considered a labour productivity function that is inverted-U shaped with respect to age, in consistency with the previous empirical studies. With this amendment they solved a "traditional" intertemporal optimisation problem. The most interesting of their findings is the important role of the average ages of population and of labour, the latter defined as the mean age of all workers, weighted by their productivity. In particular, they found that if the average population age is higher than the average labour age then the population is very unproductive and a higher fertility may be beneficial. The authors finally studied the economic consequences of a demographic transition, which affects the economy exactly through the average ages of labour and population.

2.4.6 Imperfect substitution between age groups

Another stream of authors argue that workers of different age groups are not perfect substitutes in the production of physical output. Consequently there is an optimal age structure of the labour force and by corollary of population, and any deviation from that— such as a high ratio of either the young or the old— is detrimental.

The idea goes back to Welch (1979) and Freeman (1979) who argued that the baby boom generation had lower income and higher unemployment exactly because of their size. With that in mind Ferguson (1986) studied the question of substitutability or complementarity of age groups with both each others and physical capital. His argument is that if young workers learn the job from the elder or if the young have academic knowledge while the knowledge of the old is learning-by-doing then they are likely to be complements. He also studied the issue empirically to find both substitutability and complementarity among various age groups, while all groups are complements to physical capital. Of course regardless of whether substitutes or complements, as long as there is no perfect substitutability between age groups a generation's size affects negatively their marginal product and consequently their relative wages, as stressed by Lam (1989).

Finally Denton, Mountain and Spencer (1996) concentrate on the aggregate production function where the labour input consists of different tasks. Each task uses different age-sex groups which are non-perfect substitutes for each others. Given the non-perfect substitutability between tasks, there is no perfect substitution between age-sex groups in the "production" of effective labour. Simulations show that both a high and a low proportion of the young have negative effects to both effective labour and per capita output since they consist deviations of the age structure of the labour force from its optimal composition. Important also is that this effect of the age structure to output was established by imperfect substitution alone, since age-specific capital accumulation, saving and innovation, were ruled out by the assumed non-existence of physical capital.

One step ahead goes Connelly and Gottschalk (1995), by including human capital accumulation in this context. In particular, they assume a production function with imperfect substitutability between not only age groups but also workers with different levels of education. The most important result is that a larger generation will not only enjoy lower wages, but also acquire less education. This is so because the wages for educated workers have too been suppressed by the cohort's size.

2.4.7 The question of optimal population

Given all the above theories about the economic consequences of the population age structure the question of whether there is an optimal age structure and by corollary an optimal population growth rate emerges.

The earlier growth literature implied that unless the population size enters the utility function, minus infinity is the optimal growth rate of population; the reason why is that the faster a population grows the more resources have to be invested to maintain a given capital/labour ratio. Probably Samuelson (1975a) was the first to argue that the optimal population growth is finite. The author added to the capital dilution effect of population the need to support the retired. As this need becomes more intense when the population growth falls, the optimal rate was found to be finite. The same steps followed Blanchet (1988) who amended the traditional Ramsey growth model with an exogenous retirement age. Thus population growth has two effects on the steady state: the well known capital dilution effect on the one hand which is negative, and a positive effect through a reduced dependency ratio. The two effects are opposite, so there does exist a finite optimal fertility rate.

Van Imhoff and Ritzen (1988) argued as reviewed in section 2.4.5 that the individuals' productivity is an inverted-U function of their age. Under this assumption the per capita effective labour is an inverted-U function of the birth rate, due to the high ratio of unproductive olds or youngs implied by a low or a high birth rate respectively. This in turn implies that the optimal population growth is finite. Rodriguez (1988) maintains the assumption of productivity which follows and inverted-U age pattern, but enriches this framework from many aspects. These include general form of the survival ratios and Bentham utility function[10]. The result is that for realistic demographic features the optimal population growth is positive. Finally, Van Imhoff (1989) mentions the less education opportunities implied by a fast growing population, as an additional reason for which the optimal population growth is finite.

[10]That is, the size of the population appears as an argument in the utility function.

Chapter 3

Effects of the population age structure on aggregate human capital

3.1 Introduction

Recent empirical evidence (i.e., Sarel [1995], Lindh and Malmberg [1999], etc), has shown that the population age structure is very important for economic growth. Several streams have been proposed in the literature through which the age structure of population affects the economy. The most obvious among them is probably that of the active population (i.e., Blanchet [1988]). Authors like Lee and Lapkoff (1988) on the other hand emphasised on the intergenerational transfers generated by social security, since the size of these transfers depends on the age structure of population. One step ahead went Lee and Miller (1994), by arguing that the transfers promised to the old consist an "implicit public debt", which similarly to the conventional has a negative effect on saving and investment. The effects of the population age structure on saving is also the aim of authors like Horioka (1991), who stressed the role of life cycle saving behaviour: According to these authors, since the young save and the old dissave a younger population saves and invests more.

Finally, another stream of authors (Ferguson [1986], Denton, Mountain and Spencer [1996], etc) argued that different age groups are not perfect substitutes for each other in the aggregate production function, and consequently there is an optimal age structure of the labour force, and deviations from it have negative effects to the aggregate output.

Despite their certain validity, all these approaches will be assumed away and I will concentrate to the probably most neglected among them, that is, the dependence of human capital investment on the age structure of population. Indeed, as embodied to individuals, human capital is indispensable from the demographic features of the economy. Although one can for instance assume perfect altruism and rule out the life cycle saving behaviour or the transfer effects of social security, human capital will always die with the individual and contrary to the physical capital, no altruism or other assumption can make it inheritable.

This embodiment property of human capital was also neglected in the literature that studied economic growth with both physical and human capital (i.e., Caballe and Santos [1993], Mulligan and Sala-i-Martin [1993], etc); these authors made the assumption that human capital behaves in a way similar to the physical. Lucas (1988) in fact identified some of the demographic aspects of human capital investment, but argued that they can be dealt with by simply assuming that the new generations start with a human capital level proportional to the average of the economy. However, this assumption— sufficient to yield a positive growth rate of the average human capital— does not purify it from demographic influences; human capital investment still depends on life cycle considerations, while it is still not inheritable: although the new generations start from a human capital level that is proportional to the average, this is so because of externality rather than strictly "inheriting" the old generations. Contrary again to the inheritable nature of physical capital, there is no rivalty in the above human capital externality; the initial human capital of a new generation is independent of that generation's size. The objective of this chapter therefore is to study these demographic influences on human capital and through it, on economic growth.

A representative extended family is assumed, which consists of members of all generations. The family allocates the time of its members between labour and education in order to maximise an intertemporal utility function with respect to the members' average consumption. The fact that the family is interested only on the average consumption implies perfect altruism among its members. This is sufficient to rule out all other streams proposed in the literature through which the population age structure affects the economy. It is in other words the effects of the population age structure on the stock of the economy's resources rather than the distortions it causes to their efficient use, that are studied.

The assumption of Lucas (1988) is also adopted, that is, the newly born are assumed to start from a human capital level that is proportional to the average. Yet it is shown that this cannot purify human capital formation from demographic influences: in deciding on the education of its members the family has to take into account their expected life, and the assumption of altruism has nothing to do with that. Three effects of population on human capital investment and on economic growth are thus identified: The first has to do with the growth rate of population; a fastly growing population increases the current needs of the family and inevitably reduces its human capital investment. Second, as long as the new generations start at a human capital level that is proportional to the average but lower than that, they imply a human capital dilution effect, which is higher the larger the new generation. The third effect of population consists on the life cycle human capital investment. In particular, since young generations invest in human capital more than the old, the higher the portion of the young in the economy the higher the average investment. This final result is the one mostly neglected in the literature (van Imhoff [1988] and [1989] is one of the few exceptions), and contrary to the other two depends on the birth rate positively rather than negatively. The total growth effect of the birth rate is thus shown to be either positive or negative.

The structure of the chapter is as follows: The model is presented in the next section and in section 3 the general equilibrium and steady state are derived. The comparative statics are studied in section 4 and emphasis is given to the steady state effects of the birth rate, which are realised through the population growth rate and age structure. Finally section 5 summarises the main results.

3.2 Description of the model

A closed economy is assumed, consisting of "many" identical extended families. By "extended family" all agents with a common ancestor, whether this ancestor still lives or not, is meant. The term identical means that all families have the same age structure and population dynamics on the one hand and the same preferences on the other. The economy produces a single good, on labour alone. Yet although physical capital is assumed away, there also exists a capital market where loans are traded under conditions of perfect competition. Next the elements of the model economy are presented in more detail:

3.2.1 Households

The assumption that the economy consists of many identical extended families allows one to speak about a "representative family". This representative family is assumed to maximise an intertemporal utility function with respect to the average consumption of its members, which is given by

$$U = \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt \tag{3.1}$$

where c_t is per capita consumption. Noteworthy in this utility function is the equal weight of all members since it is only the per capita consumption that matters. This equal weight can be justified by assuming either the existence of a family planner or perfect altruism among the members of the extended family, in the sense that they regard consumption by the others as important as consumption by themselves. Under such altruism assumptions the individual members of the family would take by themselves exactly the same decisions with a family planner.

The intertemporal budget constraint of the family is given as

$$\dot{q}_t = (r_t - n_t)\, q_t + c_t - y_t \tag{3.2}$$

where r_t and n_t are the interest and population growth rates at time t, and q_t, c_t and y_t stand for the per capita real assets, consumption, and labour income of the family.

3.2.2 Population

Regarding the population of the representative family, it is assumed that at any point t of time, $\beta_t N_t$ new members are born— where N_t stands for the family's total population. Each member of the family is assumed to live for T periods. Under these assumptions the size of a generation born at time s is equal to $\beta_s N_s$ if $t - s <$T, and zero otherwise. With the growth rate of the population given by n_t, it is $N_s = N_t e^{\int_t^s n_v dv}$, which yields for the size of the generation s

$$N_{st} = N_t \beta_s e^{\int_t^s n_v dv} \qquad \text{for } t - s < \text{T}$$

In the above expression, N_{st} stands for the size of generation s at time t[1]. The relative size, n_{st}, of this generation is then given by

$$n_{st} = \beta_s e^{\int_t^s n_v dv} \tag{3.3}$$

By definition the relative sizes of all generations have to add to unity, that is,

$$1 = \int_{t-T}^{t} \beta_s e^{\int_t^s n_v dv} ds \tag{3.4}$$

[1]In what follows a variable x_{st} will denote the value of variable x at time t for the generation born at time s.

Equation (3.4) gives the population dynamics of the representative family, and therefore of the economy. In what follows it is assumed that the birth rate β_t is constant. Under this assumption constant as well is the population growth rate n_t, and is given implicitly by

$$1 = \beta \int_{-T}^{0} e^{nx} dx \tag{3.5}$$

while its implicit derivative with respect to the birth rate is

$$\frac{dn}{d\beta} = -\frac{\int_{-T}^{0} e^{nx} dx}{\beta \int_{-T}^{0} x e^{nx} dx} > 0 \tag{3.6}$$

3.2.3 Human capital and labour supply

The labour income of the family, mentioned previously, depends on the wage rate and the amount of labour the family supplies to the market— measured in effective units. This "effective labour" depends on the human capital of the family. Human capital can be built by investing time in education. Since this time has to be taken out of current labour supply, there is a trade-off between current and future income. This is the approach followed by many authors, i.e., Uzawa (1965), Mulligan and Sala-i-Martin (1993), Caballe and Santos (1993), etc. In particular, these authors assumed a human capital production function of the type (in its simplest form)

$$\dot{H}_t = BH_t u_t^a - \theta H_t \tag{3.7}$$

where H_t is the aggregate human capital, u_t is the portion of time devoted to education, and θ is the constant depreciation rate. This functional form is in fact the same used— with variations— by the authors of the 1970s (i.e., Ben-Porath [1967], Ryder, Stafford and Stephan [1976], etc). The difference though is that these authors studied the problem of human capital formation at the micro-economic level, that is, the optimal education and human capital formation of a single individual. It is questionable whether or not this production function can be imported to the aggregate level as, contrary to the physical, human capital is not inheritable. Perhaps Lucas (1988) was the first that noticed this problem, but argued that the human capital production function of a single individual can be aggregated, if one only assume that the newly born start with a human capital level that is proportional to the average of the economy. Since however the aim of this chapter is the effects of demographic factors on the aggregate human capital, I disaggregate human capital to the individual level. That is, I assume an *individual's* human capital production function of the type of (3.7), similarly to the authors of the 1970s.

The human capital therefore of the individuals born at time s is at time t equal to

$$h_{st} = h_{ss} e^{\int_s^t (Bu_{sv}^a - \theta)dv} \tag{3.8}$$

where h_{ss} is the initial human capital of the individuals and u_{sv} is the portion of their time that they invested in education, at time v. Finally, θ is the depreciation rate of their human capital, and it may be due to various reasons, i.e., obsolescence of skills or deterioration of the ability to work productively— due perhaps to aging. It is also assumed that $a < 1$. I next adopt the assumption of Lucas, that the initial human capital is a portion of the average human capital in the economy. If therefore the later is denoted by H_t it is $h_{ss} = \zeta H_s$, where ζ is exactly the portion of the average human capital that the newly born start from. If finally ω_{ht} stands for the growth rate of the average human capital H_t, it is $H_s = H_t e^{\int_t^s \omega_{hv} dv}$. Substituting all the above into (3.8) we get the final expression for an individual's human capital:

$$h_{st} = \zeta H_t e^{\int_s^t (Bu_{sv}^a - \theta - \omega_{hv})dv} \tag{3.9}$$

3.2.4 Utility maximisation

The extended family maximises (3.1) under the budget constraint (3.2). In the budget constraint also appears the term y_t, which stands as said for the per capita labour income of the family. This is given by

$$y_t = w_t \int_{t-T}^{t} n_{st} h_{st} (1 - u_{st}) ds \tag{3.10}$$

where n_{st}, h_{st} and u_{st} stand respectively for the relative size, human capital, and time devoted to education, for the generation born at time s, while w_t is the wage rate.

In other words, y_t is not exogenous to the family; in fact it is an instrument the family uses for its maximisation problem, and the optimal solution also requires optimal allocation of the members' time between work and education. The two decisions can however be separated; the family can maximise the present value of the path of the labour income (y_t) and then import the optimal solution in its utility maximisation problem.

This second problem is written as

$$\begin{aligned} \max_{\{c_t\}} U &= \int_0^\infty e^{-\rho t} \frac{c_t^{1-\sigma} - 1}{1-\sigma} dt \\ \text{s.t.} \qquad \dot{q}_t &= (r_t - n)\, q_t + c_t - y_t = 0 \end{aligned}$$

This problem is well known and its optimal solution is given by

$$\dot{c}_t = \frac{1}{\sigma} (r_t - n - \rho)\, c_t \tag{3.11}$$

$$\dot{q}_t = (r_t - n)\, q_t + c_t - y_t \tag{3.12}$$

$$\lim_{t \to \infty} e^{-\int_0^t (r_v - n) dv} q_t = 0 \tag{3.13}$$

Equation (3.13) is the transversality condition, and states that eventually the family consumes all its real wealth.

3.2.5 Optimal human capital investment

As said, the family can separate the two decisions, that is, the decision on the optimal paths of its consumption and wealth, and the decision on its human capital investment and the resulting labour income. This subsection deals with the second decision.

Investment in human capital involves as said an externality from the average human capital to the initial human capital of the new generations. Yet one family alone has negligible influence on the average human capital of the economy, therefore the externality is not internalised[2]. Since there are no other externalities in the production of human capital, the optimal allocation of the time of the family members between work and education is disaggregated to the level of each individual family member. That is, the family maximises the present value of its labour income by maximising the present value of the labour income of each one member. This is a problem of life-cycle choice, and has been well analysed in the literature (i.e., Ben-Porath [1967]). It is therefore only briefly presented here:

As argued by Ben-Porath, the individuals at any time t equate the marginal returns to education and work. The latter are equal to $w_t h_{st}$, where w_t is the market wage rate and h_{st} is as said earlier the human capital at time t of an individual born at time s. The marginal returns to education are equal to $h_{st} aBu_{st}^{a-1} \int_t^{T+s} e^{-\int_t^v (r_\mu+\theta)d\mu} w_v dv$. The term $h_{st} aBu_{st}^{a-1}$ corresponds to the human capital generated by the marginal unit of time that is invested in education, while the integral gives the present value of a unit of human capital. This is equal to the discounted stream of future wages. The discount rate is equal to the interest rate, plus the depreciation rate θ of human capital.

Assuming therefore that there are no corner solutions where the optimal education time exceeds unity— the individuals' time endowment— the optimal education choice is

[2] Alternatively it can be assumed that the externality is from the average human capital of the family rather than the economy. Yet although this assumption internalises the externality, it also makes human capital a partly non-rival good, as the amount transfered to the young generations is not affected by the size of these generations. In other words, even if the externality of human capital is internalised, the later still does not resemble physical capital, which per capita value is always affected by the size of the population.

given by

$$u_{st}^{1-a} = aB \int_t^{T+s} e^{\int_v^t (r_\mu - g_\mu + \theta) d\mu} dv \tag{3.14}$$

where by g_t the growth rate of the wage w_t is denoted.

3.2.6 The aggregate human capital

The average human capital is given as

$$H_t = \int_{t-T}^{t} n_{st} h_{st} ds$$

that is, the human capital h_{st} of each generation, weighted by the relative size n_{st} of this generation. Substituting (3.3) and (3.9) in the above expression one has the following law of motion for the average human capital of the economy:

$$1 = \zeta \int_{t-T}^{t} \beta_s e^{\int_s^t (B u_{sv}^a - n_v - \omega_{hv} - \theta) dv} ds \tag{3.15}$$

In (3.15) n_v and ω_{hv} are the population and average human capital growth rates at time v, as previously explained, while the time u_{sv} invested in education is given by (3.14).

3.2.7 Physical output and capital markets

As the interest of this chapter is human capital, the physical capital is assumed away. The physical output is therefore produced by labour (in effective units) alone, with a linear production technology. This also implies a constant wage rate per unit of effective labour[3]. Further, the absence of physical capital means that all of the physical output is consumed.

The extended family has two means for saving and investment: The first is by taking time out of labour and investing it in education, as previously described. The second

[3]With a linear production technology the marginal productivity of effective labour is constant.

is by lending to or borrowing from other families, at the market interest rate r_t. The stock of net lending of the family consists its real assets, which are given— in per capita units— by q_t. The supply of new loans for the family is equal to the change in its real assets, which in per capita terms is given by $\dot{q}_t + nq_t$, which from the intertemporal budget constraint (3.2) is equal to $r_t q_t + y_t - c_t$, that is, the income from labour and assets, net of consumption.

Since there are no other real assets in the economy and lending has to add to zero among families, we have for the representative family that $q_t = \dot{q}_t = 0$[4]. The capital markets are assumed perfect, and the interest rate continuously adjusts to achieve equilibrium between the supply and demand of loans.

3.3 General equilibrium

Having described the model economy, the next task it's to derive its general equilibrium:

Definition 1 *A general equilibrium is a set of variables c_t, r_t, q_t, u_{st}, h_{st}, H_t, ω_{ht} such as:*

1. c_t and q_t are the per capita consumption and real assets of the family that maximise its intertemporal utility, given its expectations for the future interest rates and its future labour income.

2. r_t is the interest rate that achieves equilibrium in the loans' market, given the demand and supply of loans by the families.

3. u_{st} is the time spent in education by each member of generation s, which maximise the present value of their intertemporal labour income, given their expectations for the future interest rates.

[4]Yet the main results of the chapter are equally robust for non zero real wealth, consisting i.e., on physical capital or government bonds. For this reason this property will be ignored in what follows, and all results will be proved under general assumptions for the real wealth.

4. h_{st} is the human capital of each member of generation s, and it is a function of the previous education decisions of these members, as well as of their initial human capital, which depends on the average human capital of the economy at the time (s) that they were born.

5. H_t is the average human capital of the representative family and the economy, and it is a weighted average of the human capital of its members.

6. ω_{ht} is the growth rate of human capital.

Thus the general equilibrium is given by the first order condition (3.11), the intertemporal budget constraint (3.2), the transversality condition (3.13), the expression (3.9) for the human capital of an individual, the optimal education choice (3.14), the law of motion of the average human capital (3.15), and the equilibrium value of the real wealth. These equations are repeated below (slightly modified), for convenience:

$$\dot{q}_t = (r_t - n)\, q_t + c_t - y_t \tag{3.16}$$

$$y_t = \beta \int_{t-T}^{t} (1 - u_{st}) h_{ss} e^{\int_s^t (B u_{sv}^a - n - \theta) dv} ds \tag{3.17}$$

$$h_{tt} = \zeta H_t \tag{3.18}$$

$$\dot{c}_t = \frac{1}{\sigma} (r_t - n - \rho)\, c_t \tag{3.19}$$

$$u_{st}^{1-a} = aB \int_t^{T+s} e^{\int_v^t (r_\mu + \theta) d\mu} dv \tag{3.20}$$

$$1 = \zeta \int_{t-T}^{t} \beta_s e^{\int_s^t (B u_{sv}^a - n_v - \omega_{hv} - \theta) dv} ds \tag{3.21}$$

$$H_t = H_0 e^{\int_0^t \omega_{hv} dv} \tag{3.22}$$

$$q_t = 0 \tag{3.23}$$

Equations (3.16) and (3.19) are repetitions of (3.2) and (3.11) respectively. (3.17) emerges from substitution of (3.8) into (3.10), and use of the property that the wage rate is constant— which is further normalised to unity. In other words, (3.17) gives the per capita labour income, which is equal to the per capita efficient labour. The later is given by the integral of the efficient labour of all family members, which in turn depends on the portion of time they allocate to work, their initial human capital, and the cumulative time they invested in their education. (3.18) states formally the assumption that the newly born start from a level of human capital that is proportional to the average in the economy, while (3.20) is a repetition of (3.14), with substitution again of the property of constant wage rate. Finally, (3.21) is repetition of (3.15), (3.22) is a definition of ω_{ht}, while equation (3.23) gives the property discussed above, that the aggregate real wealth is equal to zero. It must be stressed once again though that this is not necessary for the results that follow; it would equally well do if the real wealth was i.e., equal to physical capital or to public debt; what is needed is to define what the real wealth consists of.

Before defining the steady state of the model economy, the following propositions will be proved:

Proposition 1 *In the long run the growth rates of average consumption and human capital are equal.*

Proof: It will be shown first that the growth rates of consumption and effective labour are equal, and then that the second is equal to the growth rate of human capital:

With the wage rate normalised to unity, the per capita effective labour is equal to the labour income y_t. Since this is equal to the per capita consumption c_t, the two grow at the same rate[5].

The effective labour supply— or the labour income y_t— is given by (3.17).

Since u_{st} is bounded between 0 and 1, it is only through the initial human capital h_{ss} that y_t can grow in the long run. Yet h_{ss} is proportional to the average human capital of the economy, which ties the growth rate of y_t to ω_{ht}.

Q.E.D.

[5]One however does not need the two to be equal; even if there were real assets in the economy, that is, if it was $q_t > 0$, then it can be easily shown that the transversality condition (3.13) would eventually tie c_t and y_t together.

Proposition 2 *At any point of time t the older a generation is, the less education they acquire.*

Proof: Derivation of (3.20) by s gives

$$(1-a)\, u_{st}^{-a} \frac{du_{st}}{ds} = aBe^{\int_{T+s}^{t}(r_\mu+\theta)d\mu}$$

Since all terms in the above expression are positive, positive has to be $\frac{du_{st}}{ds}$ as well, that is, the optimal education is higher for the younger generations (higher s).

Q.E.D.

This result is due to the finite horizon of the individuals, and it is a generalisation of the result of Ben-Porath, who found that an individual's education is declining with the process of aging. Ben-Porath however proved this result under the assumption of constant prices and interest rate. Without this assumption his result does not necessarily hold; although the shorter horizon still discourages education, heavy enough swings of the interest rate may well offset or even reverse this "horizon effect", at least for some period of the individual's life. What however proposition 2 states is that whatever the education swings in the lifetime of an individual, at any given point of time, the economic agents take more education than their contemporaneous older and less than their contemporaneous younger.

3.3.1 Steady state growth

I prefer to define the steady state with as less properties as possible, and derive the remaining:

Definition 2 *Steady state is an equilibrium path where the interest rate r_t and the common growth rate ω_t of per capita human capital and consumption are constant.*

Next the following is established for the steady state:

Proposition 3 *In the steady state the time allocation u_{st} and human capital h_{st} of an individual can be given as functions of the age $t-s$ alone, with the real time t and the date of birth s being individually irrelevant.*

Proof: The time devoted to education, u_{st}, is given from (3.20) as

$$u_{st}^{1-a} = aB \int_t^{T+s} e^{\int_v^t (r_\mu + \theta) d\mu} dv$$

In the steady state the interest rate is constant, which simplifies the above expression to

$$u_{st}^{1-a} = aB \int_t^{T+s} e^{(t-v)(r+\theta)} dv$$

which in turn gives

$$u_{st}^{1-a} = aB \int_{t-s-T}^{0} e^{(r+\theta)v} dv$$

The r.h.s. of the above expression depends on the difference $t-s$ but not on t and s individually.

Q.E.D.

The steady state can therefore be summarised by the following system of equations:

$$r - n - \rho - \sigma\omega = 0 \tag{3.24}$$

$$1 = \zeta\beta \int_0^T e^{\int_0^x (Bu_v^a - n - \omega - \theta) dv} dx \tag{3.25}$$

$$u_x = \left(aB \int_{x-T}^{0} e^{(r+\theta)v} dv \right)^{\frac{1}{1-a}} \tag{3.26}$$

where r and ω stand for the constant interest and growth rates, ζ is the portion of the average human capital of the economy that is passed on to the new generations, β and n are the fertility and population growth rates, θ is the depreciation rate of human capital, which has been assumed independent of age, and u_x is the time devoted to education by the agents of age x. Next (3.26) is substituted into (3.25) to yield[6]

$$1 = \zeta\beta \int_0^T \exp\left\{\int_0^x \left[B\left(aB\int_{v-T}^0 e^{(r+\theta)\mu} d\mu\right)^{\frac{a}{1-a}} - n - \omega - \theta\right] dv\right\} dx \qquad (3.27)$$

This equation states a relationship between the interest and growth rates alone, and it can be seen that this relationship is negative and concave. The steady state system is therefore block recursive, with equations (3.24) and (3.27) consisting the first block, and (3.26) consisting the second. The first block is presented graphically on graph 3.1, on the next page. In this graph, the locus HH gives the growth rate of the average human capital given the interest rate, while the locus CC gives the growth rate of consumption, given again the interest rate. The steady state is given by point A, where the two loci intercept.

Next the steady state effects of parameter changes are studied:

3.4 Comparative steady state

The effects on the optimal education u_x will be studied first:

Proposition 4 *The optimal education is decreasing with respect to r and θ.*

Proof: Differentiation of (3.26) yields

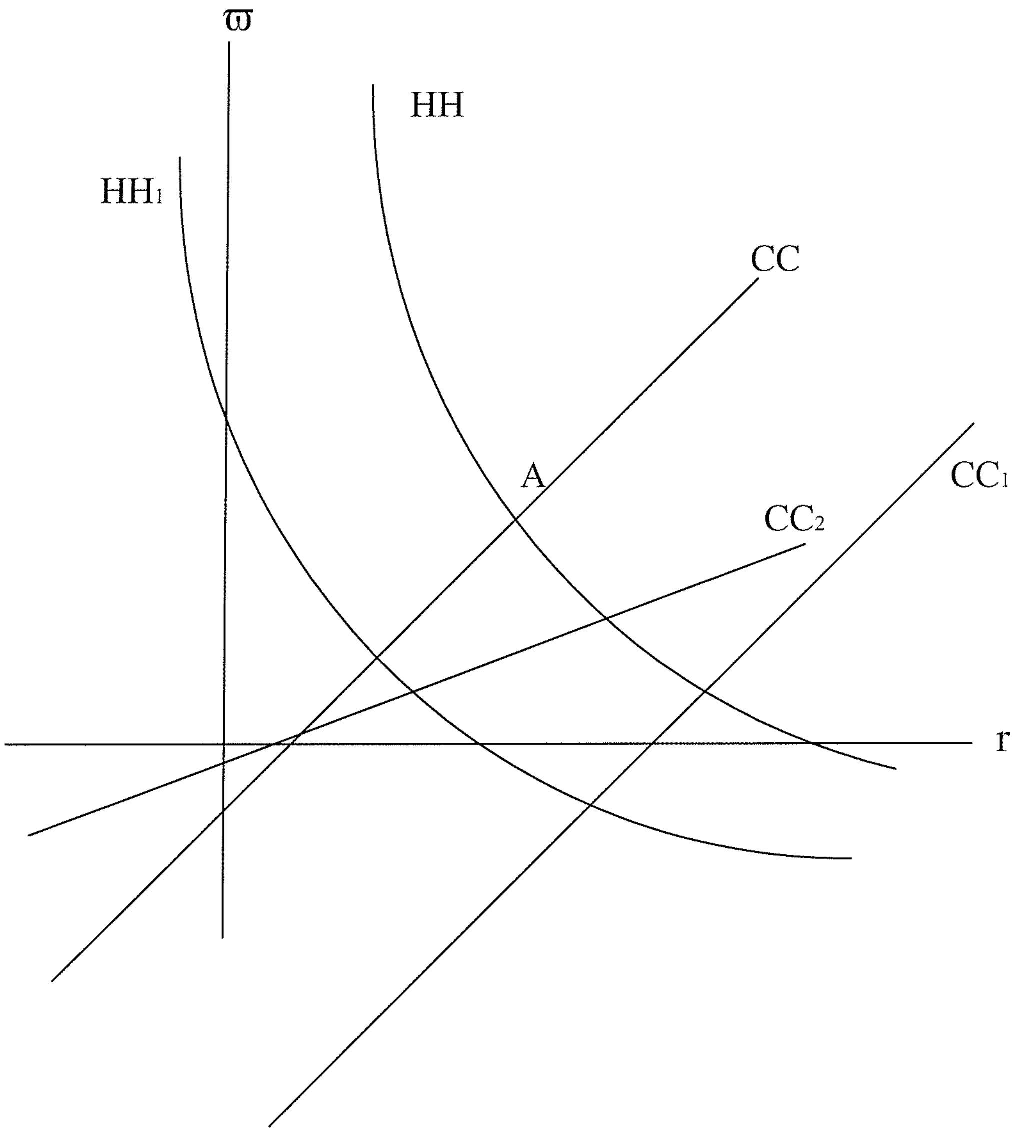
$$\frac{du_x}{dr} = \frac{du_x}{d\theta} = (1-a)^{-1} (aB)^{\frac{1}{1-a}} \left(\int_{x-T}^0 e^{(r+\theta)v} dv\right)^{\frac{a}{1-a}} \int_{x-T}^0 v e^{(r+\theta)v} dv$$

This expression is negative because v is always negative in the integral

Q.E.D.

Next the effects of the human capital depreciation θ and birth rate β on the per capita growth ω and the interest rate r are studied. This is done with the aid of graph 3.1:

[6] Although the integral in (3.26) can be evaluated, it is more concise algebrically to leave it as an integral.



Graph 3.1. Comparative Steady State

Proposition 5 *An increase in θ shifts the HH locus to the left while it does not affect the CC locus. An increase in σ on the other hand leaves the HH locus unaffected, while it rotates the CC locus clockwise from the point it intercepts with the r-axis. An increase finally in β shifts the CC locus to the right, while it has an ambiguous effect on the HH locus.*

The proposition is proved with implicit differentiation of (3.24) and (3.27), as shown in appendix A. Its general equilibrium implications are studied next:

As can be seen from graph 3.1, an increase in the human capital depreciation rate θ has a negative effect on both economic growth and the interest rate, by shifting the HH locus to HH_1: θ has a direct negative effect on the growth rate of the average human capital, but also an indirect one, again negative, which consists on reducing the returns to human capital, and consequently education effort, as stated by proposition 4. As all families now prefer to invest in real assets, the supply of loans increases, with negative effects on the interest rate. Although this mitigates the original effect of θ on education, this effect cannot be reversed, and eventually both ω and r are lower.

σ on the other hand is the inverse of the intertemporal elasticity of substitution. A higher value of σ— lower intertemporal elasticity of substitution— makes the households smooth their consumption further. To achieve that, they increase their borrowing, at the expense of their future consumption[7]. Since everyone wants to borrow the interest rate goes up, which reduces education, with negative effects on economic growth.

These results are also shown algebraically in appendix A. Less straightforward are the effects of the birth rate β though, which are next analysed:

[7]This is so for the case the growth rate is positive. A negative growth rate implies otherwise, as consumption smoothing now means increasing the *future* rather than the current consumption. In what follows it is assumed that the growth rate is positive, keeping in mind that if the growth rate is negative everything is the exact opposite.

3.4.1 Effects of the birth rate

The birth rate affects the steady state equilibrium through two streams, that is, the population growth rate and age structure. The first stream is captured by the CC locus, while the second by the HH one.

Starting from the first effect, an increase in β has from (3.6) a positive effect on the growth rate of population, which is realised, as stated in proposition 5, with a shift of the CC locus to the right (CC_1): In order to maintain the growth of its average consumption, the family wants to increase its borrowing; its faster growing population will more easily repay the debt. As all families want to borrow the interest rate goes up, which exactly means that the CC locus shifts to the right.

This is an important result by itself, as it states that other things equal a higher birth rate leads to less education for all, by increasing the interest rate, which in turn reduces education. In other words, a high birth rate increases the amount of resources required for education, which ceteris paribus reduces the education effort. This is the effect mentioned by authors like Tu (1969).

The age structure effect of β on the other hand, captured by the curve HH, is the sum of two sub-effects: The first, consists on the entry of new generations, which start with a human capital level that is proportional to the average of the economy. Yet as long as this initial human capital level is lower than the average, the entry of the new generation has a human capital dilution effect which, similarly to the physical capital dilution effect of the literature, is increasing with the size of the new generation.

The second effect of the population age structure is also the one mostly neglected by the aggregate human capital literature, and depends on the human capital investment by age. In particular, as was shown by proposition 2, younger generations invest more in education. Consequently, the younger an economy's population, the more the investment in human capital and the faster the growth rate of the average human capital of that economy.

As the first effect of the population age structure on human capital growth is negative and the second is positive, the total cannot be signed without knowledge of the exact values of the parameters. In other words, the effect of the birth rate β on the HH locus is ambiguous, as stated in proposition 5.

For a better understanding of the properties of these two effects, the second will be assumed away for the time being. In particular, it is assumed that all generations invest the same amount of time to their education, which exactly eliminates the life cycle effect of the age structure of the population on the average human capital. To achieve constant education, infinite horizon is assumed that is, T$=\infty$. In that case, the optimal education equation (3.14) is modified as

$$u_{st}^{1-a} = aB \int_t^{\infty} e^{\int_v^t (r_\mu+\theta)d\mu} dv$$

which yields for the steady state

$$u_x = \left(aB \int_{-\infty}^{0} e^{(r+\theta)v} dv\right)^{\frac{1}{1-a}} = \left(\frac{aB}{r+\theta}\right)^{\frac{1}{1-a}} = u\text{— constant.}$$

This simplifies the human capital equation (3.27) to[8]

$$\begin{aligned} 1 &= \zeta\beta \int_0^{\infty} e^{x\left(Bu^{\frac{a}{1-a}}-\beta-\omega-\theta\right)} dx \Rightarrow \\ 1 &= \zeta\beta\left(\beta+\omega+\theta-Bu^{\frac{a}{1-a}}\right)^{-1} \Rightarrow \\ \omega &= (\zeta-1)\beta + Bu^{\frac{a}{1-a}} - \theta \end{aligned}$$

Differentiation of ω with respect to β yields

$$\frac{d\omega}{d\beta} = \zeta - 1$$

That is, even under the most simplistic assumptions, the growth rate of human capital is still dependent on the demographic variables of the economy, even if one adopt the

[8]With infinite horizon the population growth rate n is equal to β.

argument of Lucas that the new generations inherit a portion of the average human capital of the economy; even if one both concentrate to the steady state and assume away the life-cycle theory of human capital investment, the birth rate still dilutes the human capital of the economy, in per capita terms. This is often accounted for in the literature with a depreciation rate of the aggregate human capital. Yet this approach neglects the fact that this depreciation rate is not constant, but depends on the demographic variables of the economy.

The above finding also stresses the significance of the second effect, that of the age dependent education: With the capital dilution effect negative, it is the more human capital investment of the young that makes the total effect of the population age structure on human capital investment ambiguous.

A general equilibrium analysis must of course take into account both the population growth and population age structure effects of the birth rate, or to put it in a more technical way, the effects of β on both CC and HH loci. These effects are studied algebraically in appendix A. Perhaps the most important result is that the total effect of β on per capita growth remains uncertain and dependent on the values of all parameters of the model. Surprising however is that the effect of β on the interest rate is uncertain as well. In particular, if the intertemporal elasticity of substitution is higher or equal to unity, that is, if σ is lower or equal to 1, then the effect of β on the interest rate is always positive. With inelastic intertemporal elasticity of substitution though, that is, for $\sigma > 1$, this is not certain anymore; all one can say is that whenever β increases economic growth, it also increases the interest rate. If however the growth effect of β is negative, then its effect on the interest rate can go either way.

The above results with respect to r can be explained as follows: An increase of the birth rate β shifts as said the CC locus to the right. Yet when σ is low this locus is more vertical, therefore shifts of the HH locus affect more the growth rather than the interest rate. Consequently, for low values of σ the shift of the HH locus cannot offset the increase of the interest rate, generated by the shift of the CC locus. Yet when the

CC locus is more horizontal ($\sigma > 1$) and the birth rate shifts the HH locus to the left, the total effect to the interest rate may well be negative. This is more probable to occur if the intertemporal elasticity of substitution is very low— if σ is very high— as shown in appendix A.

The effect of the birth to the interest rate is reflected to the education decisions of the economic agents, as these decisions depend on r. In particular, if r goes up, the education of all agents goes down, as shown by proposition 4. If on the other hand the interest rate falls despite the upward pressure of the CC shift to the right, the human capital investment of all agents goes up. This result may also explain some counter-intuitive findings of some studies (i.e., Jeon and Berger [1996]), which found that the size of a generation has a positive effect on its education effort.

This later result implies that whenever the birth rate affects r negatively, it increases ω. Yet it was just shown that the birth rate can reduce the interest rate only if it also reduces economic growth. The answer to this puzzle is the human capital dilution effect of population: Although education does go up, this is outweighted by the lower initial human capital of the new generations.

3.5 Conclusions

In this chapter the effects of demographic factors on the aggregate human capital formation and through it on economic growth were studied. Using a simple model of intertemporal maximisation where perfect altruism ensures the efficient contemporaneous and intertemporal allocation of resources, I first derived the optimal human capital investment of an altruistic individual. The result of Ben-Porath, that education attainment is declining at the process of aging, was generalised for the case where the interest rate is not constant.

Three effects of the birth rate on the aggregate human capital were then identified: The first, has to do with the effect of the birth rate on the population growth, which in turn pushes the interest rate upwards, with negative effects on education. The second effect consists on dilution of the average human capital by the newly born generations, which start from a lower than average level. The last and most neglected in the literature effect is related to the life cycle nature of human capital investment. In particular, as the young agents invest in education more than the old, the higher the birth rate, the higher the portion of the young and the more human capital investment on the average. As this last effect is opposite to the other two, the overall effect of the birth rate is ambiguous. Ambiguous also is the overall effect of the birth to the interest rate. Since the latter affects education attainment, the size of a generation may affect its education either positively or negatively. This can possibly explain the positive such effect found in some empirical studies.

Chapter 4

Effects of demographic factors on technological change

4.1 Introduction

The endogenous technological change literature implies a scale effect of the population size on the per capita growth rate of the economy. Yet this is not supported by the evidence: As Young (1998) argues, after the second world war not only the scale of the economy but also other growth promoting variables (such as trade liberalisation and increased education) were very favourable, yet without the growth rate increasing. Increasing indeed was the growth rate during the industrial and pre-industrial ages, yet this increase was much more modest than what the scale effects argument would imply.

Subsequent authors tried to fix the scale effects problem, while maintaining the endogeneity of technological progress. Two are the alternative approaches they followed. The first (i.e., Kortum [1997] and Segerstrom [1998]) argues that the more advanced a technology is the more difficult it is to improve it further, which exactly implies that more and more resources are required for same amounts of improvement. This assumption is sufficient to eliminate the scale effects of population, albeit it yields the undesirable result that without population growth there is no output growth either.

The second approach (i.e., Young [1998], Dinopoulos and Tompson [1998] and Peretto [1998]) stresses the argument of 2-dimensional R&D, that is, both quality improving and variety expanding. According to these authors, as population increases the variety of products expands, which has a *dispersion effect* on the amount of resources that are allocated to quality-improving R&D. That is, although the total resources allocated to R&D increase, they are also divided to an increasing number of products. The result is that it is the population growth rate rather than size that affects the output growth.

Both approaches however maintained the "tradition" of the original papers on endogenous technological change, of ignoring the question of human capital formation: Although all authors[1] recognise human capital as the engine of innovation and technological improvement, they take it as exogenous and proportional to the size of the population[2]. The objective of this chapter is to restore the role of human capital investment for innovation and long run growth and, to the extend that education decisions are affected by demographic factors, to study the role of these factors for R&D and growth. In particular, it is shown that although population affects economic growth in the way described by the 2-dimensional R&D approach, it also affects human capital investment, which feedbacks to economic growth. In this way, demographic changes may provide a better than scale effects explanation for the increasing growth rates of the last two centuries, as well as for the more recent growth stationarity.

Following the 2-dimension approach, I assume that the physical output is produced on a continuum of intermediate products which are not perfect substitutes for each other. The engine of growth is technological progress, which similarly to the 2-dimensional R&D models can be either variety expanding or quality improving. Input for both R&D activities is labour, yet measured in effective units, which exactly takes human capital into account.

In this framework three types of economic agents are assumed: First, the firms that

[1]Including the "founders" of the endogenous technology theory, i.e., Romer (1990), Aghion and Howitt (1992), etc.

[2]Zeng (1997) is among the few exceptions.

produce the intermediate products, which enjoy perpetual patent rights of their inventions and consequently maximise their profits under conditions of monopolistic competition. Competitive however is the market in the final output sector. The third type of agents are extended families, which consist of members of all generations. An extended family— which can be seen as all the descendants of an individual born far in the past— seeks to maximise an intertemporal utility function with respect to the members' average consumption. Decision variables of the family are on the one hand the allocation of its members' time between labour and education and on the other the allocation of its total income between consumption and saving.

The main result is that population has a direct and an indirect— through human capital— effect on economic growth. Further, it is shown that it is not only the growth rate of population but its age structure as well that matter, even if the effects analysed in the previous chapter are assumed away. A theoretical explanation is also offered for the finding of Bils and Klenow (2000) that "growth causes schooling rather than the other way round", as well as the finding of Jeon and Berger (1996) and other authors, that the size of a generation affects their schooling positively rather than negatively.

The structure of this chapter is as follows: The model is presented in the next section and in section 3 the general equilibrium and steady state are derived. The comparative statics of demographic changes are studied in section 4, while section 5 summarises the main results.

4.2 Economic environment

A closed economy is assumed, consisting of three different types of economic agents: families, final output firms, and firms that produce the intermediate products that are used as inputs in the final output sector.

4.2.1 Families and population

It is assumed that the economy consists of many extended families, identical in terms of preferences, real wealth, age structure and population dynamics. This assumption allows one to speak about a "representative family". This representative family is assumed to maximise an intertemporal utility function with respect to the average consumption of its members, which is given by

$$U = \int_0^\infty e^{-\rho t} \ln c_t dt \tag{4.1}$$

where c_t is per capita consumption. The fact that it is only the per capita consumption that matters implies perfect altruism among the family members, which in turn implies that the individual members of the family would take by themselves exactly the same decisions with a family planner.

The intertemporal budget constraint of the family is given as

$$\dot{q}_t = (r_t - n)\, q_t + P_t c_t - \ell_t \tag{4.2}$$

where r_t is the interest rate at time t, n is the constant growth rate of population, q_t, c_t and ℓ_t stand for the per capita real assets, consumption, and effective labour supply, and P_t is the price of the single consumption good, with the wage rate set as numeraire[3].

The real wealth consists on shares of the firms that produce the intermediate products. By "effective labour supply" the hours supplied to the labour market is meant, weighted by the human capital of the workers. The family therefore has two means of investment: shares of the "intermediate" firms, and human capital.

Regarding the population of the representative family, constant birth (ε) and death (λ) rates are assumed for simplicity. That is, at any point of time εN_t new members are born to the family and λN_t members die. It is also assumed that the probability of

[3]It will become apparent later on that it's more convenient to set the wage rate rather than the output price as numeraire.

death (λ) is the same for all age groups. Thus the population growth rate (n) is given as $n = \varepsilon - \lambda$, while by following the same steps as in the previous chapter we get that the relative size n_{st} of a generation born at time s is at time t given by

$$n_{st} = \varepsilon e^{\varepsilon(s-t)} \tag{4.3}$$

Utility maximisation

The extended family maximises its utility (4.1) subject to the intertemporal budget constraint (4.2). Of course the per capita efficient labour ℓ_t is not exogenous to the family, but depends on its human capital investment. Yet as argued in the previous chapter, the family can separate the two decisions of optimal human capital investment and consumption paths. This subsection therefore studies the second decision while the first is studied in the next.

The current value Hamiltonian of the second problem is given as

$$\mathcal{H} = \ln c_t + \xi_t \left[(r_t - n)\, q_t + P_t c_t - \ell_t \right]$$

and the first order conditions are

$$c_t^{-1} = -\xi_t P_t \tag{4.4}$$

$$\dot{\xi}_t = \xi_t \left(r_t - n - \rho \right) \tag{4.5}$$

which yield the following optimal paths for the per capita consumption (c_t) and real assets (q_t) of the family:

$$\dot{c}_t = \left(r_t - n - \rho - \hat{P}_t \right) c_t \tag{4.6}$$

$$\dot{q}_t = (r_t - n)\, q_t + P_t c_t - \ell_t \tag{4.7}$$

$$\lim_{t \to \infty} e^{-\int_0^t (r_v - n) dv} q_t = 0 \tag{4.8}$$

where $\hat{P}_t$ is the growth rate of P_t[4]. Equation (4.7) is a repetition of the budget constraint (4.2), while (4.8) is the transversality condition, and states that the family eventually consumes all its real wealth.

Human capital and labour supply

The effective labour supply of the family, mentioned previously, depends on the working hours supplied and on the human capital of the individuals that supply them. Human capital can be built by investing time in education, which has to be taken out of current labour supply. In particular, it is assumed that each individual is endowed with one unit of non-leisure time, which they allocate between work and education. The latter adds to the individuals' human capital, according to an accumulation function that is similar to that of the previous chapter, that is[5],

$$\dot{h}_{st} = Bh_{st}u_{st}^{\delta} - \varphi h_{st} \tag{4.9}$$

where h_{st} and u_{st} are the human capital and portion of time devoted to education, for an individual born at time s. φ is a constant human capital depreciation rate, and can also be attributable to deterioration of skills, due to ageing. Finally, it is assumed that the returns to education are diminishing, that is, $\delta < 1$.

As said, the family can separate the two decisions, that is, the decision on the optimal paths of its consumption and wealth, and the decision on its human capital investment and the resulting labour income. The first decision has already been studied; regarding the second, the family maximises the present value of its labour income by maximising the present value of the labour income of each one member, as explained in the previous chapter. As was also said, the condition for optimal education is that the marginal returns to education and work are equal. These returns are given by "what can labour buy", which depends on the price P_t of the final output, since the wage rate has been

[4] In what follows the growth rate of a variable y_t will be denoted by $\hat{y}_t$.

[5] Providing of course that the individual will be alive in the next moment.

taken as numeraire. Thus the marginal returns to work are equal to $h_{st}P_t^{-1}$, while the marginal returns to education are equal to $h_{st}\delta Bu_{st}^{\delta-1}\int_t^\infty e^{-\int_t^v(r_\mu+\varphi+\lambda)d\mu}P_v^{-1}dv$. The term $h_{st}\delta Bu_{st}^{\delta-1}$ corresponds to the human capital generated by the marginal unit of time that is invested in education, while the integral gives the present value of a unit of human capital. This is equal to the discounted stream of future wages in terms of the final good, which with the wage rate taken as numeraire are equal to the reciprocal of the price of the final good. The discount rate is equal to the interest rate, plus the depreciation rate φ of human capital, plus the probability λ that the individual will die in the next moment. The horizon is infinite, as a constant probability of death was assumed.

Assuming therefore that there are no corner solutions where the optimal education time exceeds unity— the individuals' time endowment— the optimal education choice is given by

$$u_{st}^{1-\delta} = \delta B \int_t^\infty e^{\int_v^t(r_\mu+\hat{P}_\mu+\varphi+\lambda)d\mu}dv \tag{4.10}$$

Noteworthy is that the optimal education is the same for all age groups. This is due to the assumption of constant probability of death, and eliminates the life-cycle effect of the population age structure on human capital accumulation— discussed in the previous chapter. The other effect of the age structure, that of human capital dilution, has already been eliminated by the assumption that the new generations start at a human capital level that is equal to the average of the economy rather than a portion of this average. These two assumptions together imply that all agents have the same stock of human capital and consequently the law of motion of the average human capital is the same as that of the human capital of the individual, that is,

$$g_t = Bu_t^\delta - \varphi \tag{4.11}$$

where g_t is the growth rate of the average human capital and u_t is the equal among generations time invested in education, as given by (4.10).

4.2.2 The final output sector

A single good is produced in the economy, which is used entirely for consumption purposes. This good is assumed to be produced under conditions of perfect competition and with a C.E.S. production technology that demonstrates constant returns to scale. The inputs used are a variety of intermediate products which completely depreciate in the procedure. What is important for these intermediate products is that they are not perfect substitutes for each other.

The production function of the representative final output firm is therefore given by

$$Y = \left(\int_0^A x_i^a di \right)^{\frac{1}{a}} \tag{4.12}$$

where x_i are the intermediate products used and A is the number of the available different types of intermediate products. It is also assumed that $a < 1$. The firms of the physical output sector decide on the quantities of the inputs they use in order to maximise their profits, which because of the assumption of perfect competition are given by

$$\Pi = PY - \int_0^A p_i x_i di \tag{4.13}$$

where p_i and P are the prices of the intermediate and final products respectively. In their maximisation problem the firms take the variety of the intermediate products (A) as given, and because of the assumption of perfect competition so they do for the prices p_i and P. Solving this maximisation problem yields the demand function for the intermediate products:

$$x_i = \left(\frac{P}{p_i} \right)^{\frac{1}{1-a}} Y \tag{4.14}$$

The next task is to derive an expression for the price of the final output. For that, equations (4.14) and (4.12) are substituted into (4.13) to yield

$$\Pi = P \left[\int_0^A \left(\frac{P}{p_i} \right)^{\frac{a}{1-a}} Y^a di \right]^{\frac{1}{a}} - \int_0^A p_i \left(\frac{P}{p_i} \right)^{\frac{1}{1-a}} Y di$$

which by arranging terms and using the property that under perfect competition the profits are zero, yields the following expression for price of the final output:

$$P = \left(\int_0^A p_i^{\frac{a}{a-1}} di \right)^{\frac{a-1}{a}} \tag{4.15}$$

4.2.3 The intermediate products' firms

The production of the intermediate products is assumed to be restricted by perpetual patent rights of the firms that first introduced them[6]. This implies monopolistic competition in the intermediate products' market. It is also assumed that no one but the initial patent holder can improve the quality of an intermediate product: Although it would be more realistic to allow for R&D races and business stealing, this would only complicate the analysis without adding anything to it.

The production of the intermediate products requires (effective) labour alone, and their "quality" is defined as the reciprocal of the labour input required for the production of one unit. In particular, it is assumed that the production function of the intermediates is given by

$$x_{it} = z_{it}^{\theta} \ell_{x_i t} \tag{4.16}$$

where θ is a constant, ℓ_{x_i} is labour input, and the labour productivity z_i evolves according to[7]

$$\dot{z}_{it} = \beta z_{it} \ell_{z_i t} \tag{4.17}$$

The intermediate firms maximise at any time t the present value of their expected profits, which is given by[8]

$$V_{it} = \int_t^{\infty} e^{\int_{\mu}^{t} r_v dv} \left[\left(p_{i\mu} - z_{i\mu}^{-\theta} \right) x_{i\mu} - \ell_{z_i \mu} \right] d\mu$$

[6]This subsection, as well as the next, draws from Peretto (1998).

[7]This quality improvement function is different from the one used in the literature, in the sense that in the literature it is the average rather than individual quality that matters.

[8]Recall that the wage rate is set as numeraire.

which by substitution of x_i from its demand function (4.14) yields

$$V_{it} = \int_t^{\infty} e^{\int_{\mu}^{t} r_v dv} \left(p_{i\mu} - z_{i\mu}^{-\theta}\right) p_{i\mu}^{\frac{1}{a-1}} P_{\mu}^{\frac{1}{a-1}} Y_{\mu} - \ell_{z_i\mu}\Big] d\mu \tag{4.18}$$

The intermediate firms therefore maximise (4.18) under the constraint (4.17). The current value Hamiltonian is given as

$$\mathcal{H}_i = \left(p_{it} - z_{it}^{-\theta}\right) p_{it}^{\frac{1}{a-1}} P_t^{\frac{1}{a-1}} Y_t - \ell_{z_it} + \xi_t \beta z_{it} \ell_{z_it}$$

and the first order conditions are

$$p_{it} = \frac{1}{a} z_{it}^{-\theta} \tag{4.19}$$

$$\xi_t = \frac{1}{\beta} z_{it}^{-1} \tag{4.20}$$

$$\dot{\xi}_t = \xi_t \left(r_t - \beta \ell_{z_it}\right) - \theta a^{\frac{1}{1-a}} P^{\frac{1}{1-a}} Y z_i^{\frac{\theta}{1-a} - \theta - 1} \tag{4.21}$$

Taking next the time derivative of (4.20) and substituting into (4.21) we get after arranging terms the following expression:

$$r_t = \beta \theta a^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} Y_t z_{it}^{\frac{a\theta}{1-a}} \tag{4.22}$$

Equations (4.17), (4.19) and (4.22) give the paths of quality (z_{it}), quality improving R&D effort ($\ell_{z_{it}}$), and output price (p_{it}), for the intermediate product industry i.

4.2.4 Variety expansion

Although the intermediate products are protected with patent rights, there are no restrictions in inventing a new product. This implies perfect competition in the variety expanding R&D sector. The variety expanding technology is assumed of the type

$$\dot{A}_t = \gamma L_{at} \tag{4.23}$$

where L_{at} stands for (effective) labour input in the expansion R&D sector and γ is a constant. For simplicity it is also assumed that the quality level of all new products is equal to the average quality of the existing ones. This is sufficient to achieve same quality for all intermediate products.

In order to introduce a new variety, an R&D firm compares the cost of invention with the present value of the expected profits of this invention. From (4.23) the cost of invention is equal to $\frac{1}{\gamma}$. The present value of the expected profits on the other hand is given by (4.18). In other words, positive R&D in the expansion sector implies that

$$V_t = \frac{1}{\gamma}$$

4.3 General equilibrium

Having described the model, the next task is to derive its general equilibrium. First though, the symmetry among the firms of the intermediate products' sector must be stressed: The assumption that the quality of new products is equal to the average quality makes all firms identical and therefore make the same decisions. This allows one to talk about a "representative intermediate firm", which simplifies the notation and derivation of the general equilibrium.

Definition 3 *A general equilibrium is a set of variables* c_t, r_t, q_t, u_t, ℓ_t, H_t, P_t, V_t, g_t, A_t, x_t, p_t, Y_t, z_t, ℓ_{xt}, ℓ_{zt} *and* L_{at} *such as:*

1. c_t *and* q_t *are the per capita consumption and real assets of the family that maximise its intertemporal utility, given its expectations for the future interest rates, price level, and its own effective labour supply.*

2. u_t *is the time spent in education by each individual, which maximises the present value of their intertemporal labour income, given their expectations for the future prices and interest rates.*

3. H_t is the average human capital of the economy, which is also equal to the human capital of each individual agent and it is a function of their previous education decisions.

4. ℓ_t is the average efficient labour supply of the representative family, and depends on the average human capital H_t and the time 1-u_t that is devoted to labour activities.

5. Y_t and x_t are the output produced and the inputs used by the final output firms, which maximise their profits given the prices P_t and p_t of the final output and intermediate products respectively.

6. V_t is the present value of the expected profits of an intermediate firm, and depends on the demand for their product, their current technology level z_t, and the expected interest rates as well as the future decisions of the firm.

7. ℓ_{xt}, ℓ_{zt} and p_t are respectively the labour inputs in production and quality improving R&D of the intermediate firms and the price of their output, that maximise their value V_t just described.

8. L_{at} is the amount of labour employed in variety expanding R&D, given the value V_t of the intermediate firms.

9. z_t and A_t are the quality level and variety of the intermediate products respectively, and depend on the cumulative labour investment in quality improving (ℓ_{zt}) and variety expanding (L_{at}) R&D.

10. P_t is the price level of the final product, which clears its market.

11. r_t is the interest rate that achieves equilibrium between supply and demand for savings, the first given by the desired assets (q_t) of the families and the second by the investment plans of the R&D firms.

12. g_t is the growth rate of the average human capital.

The general equilibrium is given by the following system of equations:

$$\dot{c}_t = \left(r_t - n - \rho - \hat{P}_t\right) c_t \tag{4.24}$$

$$\dot{q}_t = (r_t - n)\, q_t + P_t c_t - \ell_t \tag{4.25}$$

$$u_{st}^{1-\delta} = \delta B \int_t^{\infty} e^{\int_v^t (r_\mu + \hat{P}_\mu + \varphi + \lambda) d\mu} dv \tag{4.26}$$

$$g_t = B u_t^{\delta} - \varphi \tag{4.27}$$

$$Y = \left(\int_0^A x_i^a di \right)^{\frac{1}{a}} \tag{4.28}$$

$$x_t = z_t^{\theta} \ell_{xt} \tag{4.29}$$

$$\dot{z}_t = \beta z_t \ell_{zt} \tag{4.30}$$

$$\dot{A}_t = \gamma L_{at} \tag{4.31}$$

$$P_t = A_t^{\frac{a-1}{a}} p_t \tag{4.32}$$

$$V_t = \int_t^{\infty} e^{\int_\mu^t r_v dv} \left[\left(p_\mu - z_\mu^{-\theta} \right) x_\mu - \ell_{z\mu} \right] d\mu \tag{4.33}$$

$$p_t = \frac{1}{a} z_t^{-\theta} \tag{4.34}$$

$$r_t = \beta \theta a^{\frac{1}{1-a}} P_t^{\frac{1}{1-a}} Y_t z_t^{\frac{a\theta}{1-a}} \tag{4.35}$$

$$V_t = \frac{1}{\gamma} \tag{4.36}$$

$$A_t \left(\ell_{xt} + \ell_{zt} \right) + L_{at} = N_t \ell_t \tag{4.37}$$

$$\ell_t = (1 - u_t) H_t \tag{4.38}$$

$$N_t q_t = A_t V_t \tag{4.39}$$

Equations (4.24) and (4.25) are repetition of (4.6) and (4.7) respectively, and express the optimal consumption and saving decisions of the family. Equation (4.26) is repetition of the optimal education expression (4.10), while (4.27) repeats the expression for the growth rate of the average (and individual) human capital. (4.28), (4.29), (4.30) and (4.31) restate the production technologies of the final output, intermediate products, quality improvement of the intermediates, and variety expanding respectively. Equation (4.32) emerges by using in (4.15) the symmetry property of the intermediate firms, and gives the price of the final output in terms of the price of the intermediate products. As can be seen, the price P_t is decreasing with respect to the variety A_t of the intermediates because higher variety allows higher production without reducing the marginal product of the intermediates used. (4.33) gives the value of intermediate firms, while (4.34) and (4.35) are the optimality conditions (4.19) and (4.22) of the intermediate firms. Finally, (4.36) gives the condition for positive (and finite) variety expansion R&D, (4.37) is the equilibrium condition in the labour market[9], (4.38) gives the aggregate labour supply in per capita terms, and (4.39) states the equilibrium between supply and demand for assets, that is, the value of all stocks of all intermediate firms must equal the real wealth of the extended family.

The next task is to reduce the number of equations and variables to those of our interest, that is, r_t, u_t, g_t, ℓ_{xt}, ℓ_{zt}, L_{at}, ω_t and s_t. By ω_t the growth rate of the per capita final output is meant, while s_t stands for the aggregate labour supply per intermediate firm and is defined by $s_t = \frac{N_t \ell_t}{A_t}$. The simplified general equilibrium system is as follows[10]:

$$\omega_t = r_t - \varepsilon + \lambda - \rho + \frac{1-a}{a}\hat{A}_t + \beta\theta\ell_{zt} \tag{4.40}$$

$$\omega_t = \frac{1}{a}\hat{A}_t + \beta\theta\ell_{zt} + \hat{\ell}_{xt} - \varepsilon + \lambda \tag{4.41}$$

$$u_t^{1-\delta} = \delta B \int_t^{\infty} e^{\int_v^t \left(r_\mu + \lambda + \varphi + \frac{a-1}{a}\hat{A}_\mu - \beta\theta\ell_{z\mu}\right) d\mu} dv \tag{4.42}$$

$$\ell_{xt} = \frac{1}{\beta\theta} r_t \tag{4.43}$$

$$\ell_{zt} = \left(\frac{1-a}{a\beta\theta} - \frac{1}{\gamma}\right) r_t \tag{4.44}$$

$$s_t = \ell_{xt} + \ell_{zt} + \frac{1}{\gamma}\hat{A}_t \tag{4.45}$$

plus equations (4.27), (4.31) and (4.37). I prefer to keep the growth rate of varieties (A_t) as it is, for reasons that will become obvious. The steady state growth path is defined next:

[9] N_t is the population size.

[10] The derivation of the system is described in appendix B1.

4.3.1 Steady state growth

Definition 4 *Steady state is an equilibrium path that is characterised by the following properties:*

1. The interest rate r_t, firm size s_t and the portion of time allocated to education u_t are all constant.

2. The total labour inputs in the production of intermediate products, quality improving R&D, and variety expanding R&D are proportional to the variety A_t of the intermediate products.

3. The per capita final output and human capital grow at the constant rates of ω and g respectively.

Paragraph 2 is another way of saying that in the steady state growth path, ℓ_{xt}, ℓ_{zt} and $\frac{L_{at}}{A_t}$ are constant. This can only be the case if labour supply and product variety grow at the same rate. The first is given as $N_t \ell_t = N_t H_t (1 - u_t)$, which by the steady state property of constant u_t implies that the growth rate of total labour supply is the sum of the growth rates of total population $(\varepsilon - \lambda)$ and average human capital (g). This in turn implies that the steady state growth rate of product variety is given as

$$\hat{A}_t = \varepsilon - \lambda + g \tag{4.46}$$

The steady state system is next given by

$$r - g = \varepsilon + \rho - \lambda \tag{4.47}$$

$$Bu^{\delta} - g = \varphi \tag{4.48}$$

$$(1-a)\, g + a\delta B u^{\delta-1} + \left(1 - 2a - \frac{a\beta\theta}{\gamma}\right) r = a\varphi + (a-1)\,\varepsilon + \lambda \tag{4.49}$$

$$\ell_x = \frac{1}{\beta\theta} r \tag{4.50}$$

$$\ell_z = \left(\frac{1-a}{a\beta\theta} - \frac{1}{\gamma}\right) r \tag{4.51}$$

$$\omega = \frac{1-a}{a}(\varepsilon - \lambda) + \frac{1}{a}g + \beta\theta\ell_z \tag{4.52}$$

$$s = \ell_x + \ell_z + \frac{1}{\gamma}(\varepsilon - \lambda + g) \tag{4.53}$$

To make things as simple as possible, the steady state system (4.47)-(4.53) was made block recursive, with the first three equations forming the first block and the remaining equations being one block each. Equation (4.47) is the steady state expression of (4.40), after subtraction of (4.41) and substitution of $\hat{A}_t$ from (4.46). (4.48) emerges from (4.27) while (4.49) emerges by substitution of (4.44) and (4.46) in (4.42), and solving the integral. (4.52) is the steady state expression of (4.41), after substitution of $\hat{A}_t$. Finally, (4.50), (4.51) and (4.53) are repetitions of (4.43), (4.44) and (4.45) without the time index, while (4.46) was also used in (4.53).

4.4 Steady state effects of demographic changes

This section studies the steady state effects of the two demographic parameters, fertility (ε) and mortality (λ), on the growth rates of per capita human capital (g) and output (ω), the interest rate (r), the time allocated to education (u), the labour allocation variables ℓ_x and ℓ_z, and the firm size s. Before proceeding, it is useful to recall that the demographic parameters give the population growth rate and age structure; in particular, it is $n = \varepsilon - \lambda$ and $n_x = \varepsilon e^{-\varepsilon x}$ where n is the population growth rate and n_x is the relative size of the generation of age x. Changes in λ reflect therefore opposite changes in the population growth rate while changes in ε reflect changes in both population growth rate and age structure. With this in mind I next establish the following proposition:

Proposition 6 *Assuming that the C.E.S. parameter a is "high", an increase of the death rate λ ceteris paribus reduces all of r, g, u, ω, ℓ_x, ℓ_z, s and variety expansion $\hat{A}_t$, while an increase in the birth rate ε affects positively r, ℓ_x, ℓ_z, s and $\hat{A}_t$, negatively u and g,* *and has an ambiguous effect on the per capita growth rate ω.*

The proposition is proved in appendix B.2. It is also shown that a value for a higher or equal to $\frac{2}{3}$ is sufficient for proposition 6 to hold, but not necessary; smaller values for a may still produce the same results. Noteworthy also is that the assumption of high a implies elastic substitution between intermediate inputs in the final output production function. Whether we interpret the intermediates as consumption goods[11] or as production inputs (i.e., Young [1998]), this assumption of elastic substitution is realistic; in modern economies there is a huge variety of both final products and skills, with often very minor differences between them.

Some of the results stressed in proposition 6 were anticipated: Starting from the interest rate, although it was shown in the previous chapter that the population growth rate may affect it negatively, it was also shown that this can only occur if the elasticity of intertemporal substitution is strictly lower than unity. In the model of the present chapter though, this elasticity is exactly unity which means that the positive effect of ε and the negative effect of λ on r are in line with the findings of the previous chapter.

As Peretto argued, higher population growth increases both the firm size s and the growth rate $\hat{A}_t$ of product variety. The reason why is that higher population growth implies higher expected demand— and therefore profits— for the intermediate firms. As a consequence they increase both their production and quality improving R&D, or in other words, they increase their size. Yet this increases the firm value, which results in more resources allocated to variety expanding R&D. It is exactly for this reason that the birth rate ε is found to affect s and $\hat{A}_t$ positively, while the effects of the death rate λ are negative.

As the demographic variables affect ℓ_x and r in the same way, a positive correlation between the two is implied. This does not come as a surprise; A higher interest rate reduces the returns to future quality improvements, and the intermediate firms concentrate

[11]This is the assumption of i.e., Peretto (1998), who studied the growth of utility rather than output, with an expanding variety of *consumption* goods. Both approaches give the same results, providing that one always remembers which of the two it is about.

to production rather than R&D. However, the labour (ℓ_z) devoted to quality improving R&D is too positively related with the interest rate! The answer must be sought at the firm size: by affecting as said s, the birth and death rates affect (positively the first, negatively the second), the returns to R&D as well. This effect proves stronger than the one through the interest rate, with the result of positive relation between r and ℓ_z.

Negative is the effect of the birth rate on the amount u of time spent in education. This is due to the positive effect of ε on r, which as can be seen from (4.42) affects u negatively. This is due to the fact that a high interest rate reduces the expected returns to education. Yet equation (4.42) also reveals a second stream through which the birth rate affects education, that is, through variety expanding ($\hat{A}_t$) and quality improving (ℓ_{zt}) R&D. Both these factors are positively affected by ε while positive also is their own effect on education effort: As it is shown in appendix B.1, the growth rate of the price P_t of the final product is inversely affected by the amount of resources that are devoted to R&D, of either type. This is the same as saying that the growth rate of the purchasing power of effective labour is positively affected by R&D. Consequently, the higher the R&D effort, of either type, the higher the future returns to effective labour and the more it pays to invest in education.

The above effect is reminiscent of the argument that not only schooling causes growth, but there also is an opposite causality between the two[12]. Under the assumption of unit elasticity of intertemporal substitution though this effect of population growth is dominated by that of the interest rate, with the result of lower education. Yet education does not fall now as much as it would in the absence of R&D. Further, with elastic intertemporal substitution in the utility function the birth rate may in fact have a positive effect on education; as is well known, with high elasticity of intertemporal substitution changes in exogenous variables (such as population growth) tend to influence more the consumption growth rather than the interest rate. With sufficiently elastic intertemporal

[12]Noteworthy is the finding of Bils and Klenow (2000), that the "growth causes schooling" stream yields higher relationship between education and growth than the opposite stream.

substitution therefore the total effect of the birth rate on education can be positive, with its effect through the R&D stream dominating over the effect of the moderate increase of the interest rate. This result provides a theoretical explanation for the finding of authors like Jeon and Berger (1996) —that a size of a generation affects its education positively— without any need of the interest rate to be affected negatively.

However, education is negatively affected by the death rate λ as well. This may come as a surprise, as λ affects the interest rate and both types of R&D effort in a way opposite to that of ε. The answer is that λ also has a direct negative effect on education, as the probability of death raises the discount factor of future labour income. This effect dominates, resulting to a negative total effect of the death rate on education. Finally, from (4.48) it is straightforward that the way ε and λ affect education, the same way they affect the growth of the per capita human capital. That is, under the assumptions of the present model they both reduce g.

According to the 2-dimensional R&D literature, population growth boosts the per capita output growth as well, through the above mentioned stream of higher R&D effort in both quality and variety dimensions. The result of proposition 6 therefore, that λ reduces growth, is consistent with the literature. Puzzling however is the ambiguity of the sign of the effect of ε, which according to the literature should have been unambiguously positive. As can be seen from equation (4.52) the per capita output growth (ω) is a weighted sum of three factors: population growth, quality improvement, and per capita human capital growth. The first affects ω both directly— by increasing the number of shares the final output will be divided to— and indirectly, by increasing the growth rate of product variety. On the aggregate though the indirect effect dominates. Positive also is the effect of population growth on the quality improvement of the intermediate products, as shown previously. Yet the last factor, the per capita human capital, was found to be affected negatively by both fertility ε and mortality λ. Although this leads to an unambiguously negative effect of λ on ω, it makes the total effect of ε ambiguous. This ambiguity is entirely attributable to human capital investment, which is exactly the factor the literature assumed away: Although what matters for per capita growth is the growth rate of the total human capital, by assuming its formation away, the literature regarded the growth rates of human capital and population as one and the same thing. This is wrong, because human capital investment is anything but unaffected by population growth.

Interesting also is to study the effects of an equal increase of the birth and death rates. This will increase the portion of the young without altering the growth rate of the population. The results are summarised in the following proposition:

Proposition 7 *A demographic change that increases the portion of the young but leaves the population growth rate unaffected has a negative effect on all of r, g, u, ω and s.*

The proposition is proved in appendix B.2. The explanation of these results must be sought again in the area the 2-dimensional R&D literature assumed away, that is, human capital investment; by assuming it away, as long as the population growth rate does not change nothing else does. However, the population age structure is very important for human capital investment: A higher death rate λ increases the discount factor of future labour income, which reduces an individual's education and through it the growth rate of the average human capital. This effect is additional to the effect of λ throught the stream of the population growth and because of that it is not offset by the equal increase of the birth rate ε. In short, an equal increase of ε and λ, or in other words a younger but not faster growing population, implies less human capital investment, lower growth rate of the average human capital, and consequently lower growth of the total human capital.

In fact the effects of the population age structure on human capital investment and through it technological progress are much richer; in the present model very simplifying assumptions were made with respect to the death rate and the human capital of the newly born. In particular, the assumption of age independent death rate results in same education effort for all age groups which is not the case under the more realistic assumption of finite horizons, as was seen in the previous chapter. It's also more reasonable to assume that the new generations start at a human capital level that is proportional to the average of the economy, but less than that. The first result implies a positive effect of the portion of the young on human capital growth, while the second implies the opposite. What was however shown by proposition 7 is that even under very simplifying assumptions the population age structure still affects innovation and technological progress.

The next task is to see how the above described effects of the demographic variables on economic growth explain the data. Similarly to Romer (1986), I study the annual per capita growth rate of the technology leader[13], which I compare to various demographic variables. This is done on table 4.1.

Table 4.1. Population and growth since early industrialisation

Period and leader	Per capita growth (%)	Initial popul. (000)	Median age	Median age (initial)	Population growth (%) (20 yr. lag)
UK 1785-20	0.50	8,664	25.8	26.5	0.99
UK 1820-90	1.40	16,736	25.6	24.9	1.39
US 1840-80	1.44	17,120	23.1	22.0	2.90
US 1880-20	1.78	50,262	26.1	24.5	2.23
US 1920-60	1.68	106,461	30.4	27.7	1.39
US 1960-90	1.97	179,979	33.0	31.6	1.46
US 1960-70	2.54	179,979	31.8	31.6	1.37
US 1970-80	1.61	203,810	32.7	32.0	1.76
US 1980-90	1.76	226,546	34.2	33.5	1.25

Note: UK population data refers to England & Wales only.

Sources of the raw data: (1) Romer (1986), Tables 1 and 2.

[13]Although it would be more accurate to take the OECD rather than the US as "technology leader" for the recent years, this simplification should not have a considerable effect on the results.

(2) Wrigley and Schofield (1981).

(3) Censuses of England & Wales.

(4) US Bureau of census.

(5) OECD- National accounts.

Perhaps the very first thing one can see from the table is that the scale effects never were the case, at least since the industrial revolution; although the per capita growth rate has been increasing, this increase was much more moderate than the one implied by the scale effects argument. Further, plenty is the evidence of non increasing growth in the recent years. Neither however the growth rate of population appears to keep pace with per capita growth[14] as the 2-dimensional R&D literature implies, although authors like Dinopoulos and Tompson (1998) argued that the observed growth patterns may be due to a long adjustment period towards the steady state growth. Yet by stressing the multiplicity of the effects of the demographic factors on technological progress and economic growth, this chapter offers an alternative explanation for the growth patterns of the last two centuries: As can be seen from table 4.1, the population growth rate has been steadily declining (with the exception of the decade of 1950-60), at least during the period of US leadership. Steadily on the other hand had increased the median age of the population, which reflects its age structure. In addition, although it is not shown on the table, both fertility and mortality declined in the last two centuries. The fall in mortality had an unambiguously positive effect on growth, while the fall of fertility although implied slower population growth it also resulted in more education and growth of the per capita human capital. On the overall, the combination of slower population growth and higher population age and life expectancy resulted in faster output growth, in spite of the recent literature that would expect the output growth rate to follow that of the population. However, it is doubtful that these growth pattern will not be reversed if

[14]A 20 years lag was used for the population growth, because it was assumed that it takes approximately that time for population to affect technological change and through it economic growth. The result however is exactly the same if contemporaneous population growth is used instead.

the demographic trends that generated it continue; as said, the population age structure affects human capital investment in various ways and consequently if the birth rates in developed countries fall further we may well end up with an older population, less human capital investment, and slower economic growth.

4.5 Conclusions

In this chapter the effects of population on technological progress and through it economic growth were studied. This was done by introducing human capital investment in the framework of a model with both quality improving and variety expanding R&D. According to this rather recent approach, it is the population growth rate, not size, that matters for per capita economic growth. It was found in this chapter that when education decisions are also taken into account many results of this 2-dimensional R&D literature are reduced in size or even reversed. This is so because what actually matters is the growth rate of human capital rather than population, which depends not only on population as has been assumed in the literature, but on education and per capita human capital as well.

In particular, it was found that R&D is positively affected by the population growth rate, a finding which is in line with the 2-dimensional R&D literature. Yet population growth also exercises an upward pressure to the interest rate, which reduces education and human capital growth. This effect is mitigated by the expected productivity growth which increases the returns to education. Further, with high elasticity of intertemporal substitution it may well be the case that a high birth rate leads to higher rather than lower education, as found by some empirical studies. Always negative however is the effect of the death rate, as it also has a direct negative effect to the returns to education.

For all the above reasons, negative is the effect of the death rate to per capita output growth. What comes as a surprise though is the ambiguous effect of the birth rate; as the later increases population growth, it would be expected by the 2-dimensional R&D theory to have an indisputably positive effect on growth. The ambiguity comes from the effects of the birth rate to investment in human capital. Although very simple assumptions were made with respect to human capital formation, the population age structure still was found to affect it and through it economic growth, albeit only through the death rate. Interesting would be to include the more realistic assumptions of chapter 3, which yielded on the one hand declining with age education effort, and on the other hand human capital dilution from the entry of new generations.

Yet even under very simplifying assumptions, the present chapter shed more light into the forces that drove economic growth in the last two centuries: although the decline of population growth should according to the 2-dimensional R&D theory have reduced economic growth as well, the lower mortality and more balanced population age structure resulted in more education and through it increasing rather than decreasing growth rates. This however may be reversed if the current demographic trends in the developed countries continue, as education may not increase any more to make up for further reductions in the population growth rate.

Chapter 5

Effects of borrowing constraints when the population is endogenous

5.1 Introduction

Earlier literature on financial development[1] showed that it has a positive effect on aggregate investment, by facilitating the firms in their investment plans. Yet it has also been well established in the literature (Zeldes [1989] is just one of the many studies), that households too face borrowing constraints. It is on these constraints that the recent literature on borrowing constraints (i.e., Jappelli and Pagano [1994] and [1999]) concentrates at. The conclusion of this literature is that borrowing constraints on households have in fact a positive effect on capital accumulation and growth, as they force the households to borrow less, that is to save more.

To this framework authors like De Gregorio (1996), Buiter and Kletzer (1995), and Barro et. al. (1995) added the dimension of human capital. These authors argued the one way or the other that borrowing on human wealth is much more difficult than borrowing to build physical capital. Therefore human capital formation has to be financed out of current income. The conclusion of these authors is similar to that of the early literature

[1]Pagano (1993) gives a survey of this literature.

on financial development; more borrowing opportunities for households increase their opportunities for human capital formation, exactly the way more borrowing opportunities for the firms increase investment in physical capital. Interesting also is that although borrowing constraints still promote physical capital accumulation, their negative effect on the accumulation of human capital dominates, and the growth rate is lower for a constrained economy.

Perhaps the only thing common to all the above authors— apart from the borrowing constraints— is that they take population as not only exogenous, but most times even constant. However population is very important for both physical and human capital formation; life cycle saving and the capital dilution effect of population growth link physical capital to population, while the literature on endogenous fertility choice has stressed the trade-off between population and human capital. Borrowing constraints on the other hand are very likely to affect fertility choice as well. There is therefore a good reason to introduce endogenous fertility in a liquidity— or borrowing— constraints' framework. This is the object of this chapter.

In particular, it is assumed that the economic agents maximise a utility function with respect to their old age consumption and the number of their children, which they have in their youth. It is also assumed that the parents are selfish. This assumption is necessary, as the opposite would nullify the borrowing constraint through borrowing from the altruistic parents. Old age consumption depends on life time income, which can be increased by acquiring education earlier in life. Since both education and children are costly as will be explained, there is a trade off between children and investment in education— which in turn implies a trade off between children and old age consumption.

Following the mainstream assumption, child rearing requires time. Yet "adult" rather than "parental" time is required in the model of the present chapter. This amendment has no effect other than allowing the economic agents to effectively relocate their lifetime time endowment among periods, by hiring someone to look after their children. Time is also important for education, both directly and indirectly— working to buy material inputs implied by the latter. Yet it is again assumed that own and educators' time are perfect substitutes in the production of human capital. The reason is again to permit the relocation of one's lifetime time endowment.

These two amendments have no effect other than replacing the time endowment constraints of each separate period with one lifetime constraint. This leaves the borrowing constraint as the only period-related one, which allows to concentrate on it. The borrowing constraint is usually introduced in the literature with the assumption that the economic agents can borrow up to a certain fraction of their future or current income. Then marginal changes of this fraction are studied, which correspond to marginal tightening or relaxing of the constraint. However this approach is inapplicable in the present framework, because the endogeneity of population makes the derivation of an analytical solution impossible. Thus an alternative approach is followed. In particular, the two extreme scenarios of zero and unlimited borrowing are simulated for an ample range of parameter values, and their results are compared. For a better understanding of how the endogeneity of population interacts with borrowing constraints, the scenario of zero borrowing but with exogenous population is also simulated.

The results are striking: Comparison between the scenarios of unlimited and zero borrowing with exogenous population gives exactly the same results with the previous literature: the borrowing constraint increases physical and reduces human capital and economic growth. Yet when comparing the scenarios of unlimited borrowing and of zero borrowing but endogenous population, the results are different. In particular, in the constrained scenario the investment in physical capital is still higher while that of human capital is still lower. Yet the endogeneity of population mitigates the second result while it also affects the magnitude of the first. But the most important result is that when the population is endogenous the borrowing constraint may now have a positive rather than negative effect on per capita growth, exactly because it reduces fertility.

The structure of this chapter is as follows: The next section presents the economic environment. Section 3 derives the competitive equilibrium of the model and the steady state. Section 4 presents the simulations' results and compares the three scenarios of unlimited borrowing and no borrowing with either endogenous or exogenous population. Finally section 5 summarises the main results.

5.2 Description of the model

5.2.1 Individuals

The economy is assumed to consist on four overlapping generations, each of them living for four periods: Childhood, young adulthood, mature adulthood, and old age. In the first period of their lives (childhood) the individuals make no decisions. In the second and third periods the individuals are endowed with a unit of non-leisure time. This time they allocate between education, work and childrearing. In the fourth period they retire and consume their savings. To keep things as simple as possible no consumption is assumed in periods 1,2 and 3. It is further assumed that child-bearing takes place only in the second period.

The intertemporal utility function the individuals maximise is given by

$$U = \ln c + a\frac{n^{1-e}}{1-e} \tag{5.1}$$

where c is old age consumption and n is number of children. This utility function is a generalisation of that of Iyigun (2000) or Zhang (1997)[2]. Similarly to them, the arguments of the utility function are consumption in period 4 (old age), and number of children (n). The parameter e can take any value between zero and infinity and corresponds to the relative elasticity of demand for consumption and children. In particular, the higher e is the higher the elasticity of consumption comparing to that of children[3]. The difference of

[2]Zhang also included children's utility, but in the present framework it would only complicate matters even further without adding anything.

[3]One can think of the two extreme values, zero and infinity. As e goes to infinity, the marginal utility of children becomes more sensitive to the number of children itself, which exactly implies inelastic

this utility function comparing to those of the above authors consists in its more general form. In particular, both Iyigun and Zhang assumed that the second term of the utility function is logarithmic, that is, $e = 1$ in the framework of the present model. Yet the elasticity of preferences is very important for the problem studied in this chapter, as will be seen, therefore a CRRA utility with respect to children is deemed necessary[4]. The parameter a on the other hand is a scaling parameter and shows the desirability of n comparing to consumption— which coefficient has been normalised to unity.

Important also is that the children's utility does not appear as argument in the utility function, either directly or indirectly. The reason why is that perfect altruism is not compatible with the assumption of borrowing constraints: the constrained young would borrow from the altruistic parents, nullifying the constraint. Although some form of imperfect altruism may escape this problem, it only complicates the analysis without adding anything to it.

5.2.2 Production of children and human capital

Children require a fixed amount v of *adult* time. This assumption is more general than the mainstream one that children require *parental* time, and allows parents to hire someone to look after their children.

Human capital is assumed to be accumulated according to the accumulation function

$$h_{t+1} = \varphi h_t^{1-s} u_t^b q_t^s \tag{5.2}$$

where h_t is the individual's human capital at time t, u_t is time spent in education, and q_t is physical capital input in the production of human capital. Following standard assumption of the literature, the new entrants in the labour market (period 2 agents) have the same

demand. With $e = 0$ on the other hand, the marginal utility of children is constant which implies constant marginal utility of consumption. This in turn implies constant consumption, that is, inelastic demand for consumption.

[4] Yet a logarithmic form is necessary for consumption, otherwise there is no steady state growth path.

level of human capital with the previous generation. Since only two generations are in labour force at a time, this is equivalent to saying that all workers have the same amount of human capital. It is further assumed that $b+s<1$, while own and educators' time are perfect substitutes in education. Since this assumption is rather strong, one can think that there is an optimal mix of own and educators' time. The reason for this assumption and the previous one, that children don't necessarily need their parents to look after them, is to allow the individuals to effectively "borrow time" by exactly hiring someone to educate them or look after their children.

Since at time t an individual's human capital is fixed, the above human capital production function exhibits diminishing returns. For the society though the production of human capital exhibits constant returns with respect to the two capital inputs, due to the assumption that human capital is effectively inheritable[5].

5.2.3 Utility maximisation

The individuals therefore maximise at time t their utility (5.1) under the constraint (5.2). To this we should add the intertemporal budget constraint

$$w_t h_t(1-vn_t-u_t)-R_t^{-1}q_t+R_{t+1}\left[\varphi h_t^{1-s}u_t^b q_t^s w_{t+1}-R_{t+2}c_{t+2}\right]=0 \qquad (5.3)$$

where w_i is the wage per unit of human capital on period i, R_i the discount factor of future income, h_t is the human capital of the individuals when they enter the labour force (period 2)[6], u_t is the time invested in education on period 2, and q_t is the physical capital used by the individual for education purposes in that period[7]. Both wages w_i and

[5] The above human capital production function may at a first glance look rather unusual. Yet it can be rewritten as $h_{t+1}=\varphi h_t^{1-s-b}\left(u_t h_t\right)^b q_t^s$. Now the human capital of the next period is a function of the human ($u_t h_t$) and physical (q_t) capital inputs, while there is also an externality from the average human capital of the economy— which is as said equal to the initial human capital of the individual. This is similar to the formulation of i.e. Mulligan and Sala-i-Martin (1993), if we only adjust the notation.

[6] The human capital of the third period of an individual's life has been substituted in the budget constraint from the human capital accumulation function (5.2).

[7] As in the fourth period the individuals retire there is no point in taking education on the third, since it'll never yield any returns.

discount factors R_i are taken by the maximising agents as given.

The budget constraint needs to be explained further: In period 2 (t) the economic agents have an earning potential of $w_t h_t$. Their expenditure in this period consists on time spent on child rearing and education, as well as the rent paid for the physical capital used for education purposes. The rent for this physical capital is equal to R_t^{-1} that is the reciprocal of the discount factor[8], or in other words the "gross" interest rate, i.e., $R_t = \frac{1}{1+r_t}$, where r_t is the interest rate. The rearing of each child costs $vw_t h_t$, that is, the adult time v required, times the compensation for this time. Since all economic agents have the same amount of human capital it doesn't matter whether it's the parent's or a "carer's" time. The same argument applies for the education time u_t as well; due to same level of human capital among generations own and educators' time have the same cost. In period 3 the economic agents work and receive an amount equal to their human capital, times the current wage. Finally in period 4 they retire and consume their previous savings. Period 3 is discounted by R_{t+1} and period 4 by $R_{t+1}R_{t+2}$.

The economic agents are also assumed to face a borrowing constraint in period 2. This constraint has been specified in the literature as a maximum fraction of their current (De Gregorio) or future (Jappelli and Pagano) income the individuals can borrow. Then the effects of marginal changes of this fraction were studied. This approach is inapplicable though in the present framework, because the endogeneity of population makes an analytical solution impossible to derive. Instead, a strict constraint of zero borrowing is introduced, and its results are compared to those when there is no borrowing constraint at all. This strict constraint is formally given as

$$w_t h_t(1 - vn_t - u_t) - R_t^{-1} q_t \geq 0 \tag{5.4}$$

The economic agents therefore maximise (5.1) under the constraints (5.3) and (5.4).

[8] Standard economic theory says that the rent (user cost) of physical capital equals the sum of the interest (r_t) and depreciation (δ) rates. Yet, as will be said, it is assumed that the physical capital depreciates fully in one period, or in other words $\delta = 1$.

The Lagrangian of the problem is

$$\mathcal{L} = \ln c_{t+2} + a\frac{n_t^{1-\varepsilon}}{1-\varepsilon} + \lambda\left\{w_t h_t(1 - vn_t - u_t) - R_t^{-1}q_t\right\} +$$
$$\mu\left\{w_t h_t(1 - vn_t - u_t) - R_t^{-1}q_t + R_{t+1}\left[\varphi h_t^{1-s}u_t^b q_t^s w_{t+1} - R_{t+2}c_{t+2}\right]\right\} \Leftrightarrow$$
$$\mathcal{L} = \ln c_{t+2} + a\frac{n_t^{1-\varepsilon}}{1-\varepsilon} + (\mu+\lambda)\left\{w_t h_t(1 - vn_t - u_t) - R_t^{-1}q_t\right\} +$$
$$\mu R_{t+1}\left[\varphi h_t^{1-s}u_t^b q_t^s w_{t+1} - R_{t+2}c_{t+2}\right]$$

This specification is general and allows one to study the problem under the alternative assumption of no borrowing constraint, by just setting $\lambda = 0$. The first order conditions are

$$\frac{1}{c_{t+2}} = \mu R_{t+1}R_{t+2} \tag{5.5}$$
$$an_t^{-\varepsilon} = (\lambda+\mu)w_t h_t v \tag{5.6}$$
$$\mu b\varphi h_t^{1-s}u_t^{b-1}q_t^s w_{t+1}R_{t+1} = (\lambda+\mu)w_t h_t \tag{5.7}$$
$$\mu s\varphi h_t^{1-s}u_t^b q_t^{s-1}w_{t+1}R_{t+1} = (\lambda+\mu)R_t^{-1} \tag{5.8}$$

The first order conditions along with the budget (5.3) and borrowing (5.4) constraints give the optimal solution to the individuals' maximisation problem. Of course in the unrestricted borrowing scenario the later constraint drops out. Instead, we have the condition $\lambda = 0$. Next the optimal solution is expressed in terms of the four variables c_{t+2}, n_t, u_t, q_t alone:

Division of (5.7) by (5.8) yields

$$q_t = \frac{s}{b}w_t R_t h_t u_t \tag{5.9}$$

while solving (5.5) with respect to μ and (5.6) w.r.t. $\lambda+\mu$ and substituting the solution in (5.7) gives

$$\frac{1}{c_{t+2}R_{t+2}}b\varphi h_t^{1-s}u_t^{b-1}q_t^s w_{t+1} = \frac{a}{v}n_t^{-\varepsilon}$$

which with substitution of q_t yields

$$c_{t+2} = \frac{vb\varphi}{a}\left(\frac{s}{b}\right)^s w_{t+1}R_{t+2}^{-1}\left(w_tR_t\right)^s u_t^{b+s-1}n_t^{\varepsilon}h_t \tag{5.10}$$

The optimal solution to the individuals' problem is given by equations (5.9) and (5.10) alone with the budget (5.3) and borrowing (5.4) constraints. For the unrestricted borrowing scenario though, the later is replaced as said with the condition that λ, the Lagrange multiplier of the borrowing constraint, is zero. Using this property into the first order condition (5.7) we have

$$\varphi bh_t^{1-s}u_t^{b-1}q_t^s w_{t+1}R_{t+1} = w_th_t \tag{5.11}$$

This equation states a familiar property, that is, that in the optimal solution the marginal return to education time $\left(\varphi bh_t^{-s}u_t^{b-1}q_t^s w_{t+1}R_{t+1}\right)$ equals its marginal cost w_t.

Next the physical output sector of the model economy is introduced:

5.2.4 The physical output sector

The economy produces a single good which can be either consumed or added to the physical capital of the next period. This good is produced under conditions of perfect competition and according to the Cobb-Douglas production function

$$Y_t = K_t^{\gamma}H_t^{1-\gamma} \tag{5.12}$$

where K and H stand for physical and human capital respectively. The demand for the production factors is given by

$$w_t = (1-\gamma)K_t^{\gamma}H_t^{-\gamma} \tag{5.13}$$

$$\frac{1}{R_t} = \gamma K_t^{\gamma-1}H_t^{1-\gamma} \tag{5.14}$$

where $\frac{1}{R_t}$ is the reciprocal of the discount factor, that is, the gross interest rate.

Regarding the inputs employed in the physical output sector, it is assumed for simplicity that the physical capital is totally depreciated in one period. As for the human capital employed in the sector it is given by

$$H_t = N_{1t}h_t(1 - vn_t - u_t) + N_{2t}h_t$$

where N_{1t} and N_{2t} are the numbers of individuals of age 1 and 2 (that is in periods 2 and 3 of their lives) respectively, and h_t is their (common) human capital stock. That is, the human capital employed in the final output sector equals the total human capital stock of the economy minus the part of it that is used for the education of period 2 agents, or the rearing of their children. Recalling now that $N_{1t} = n_{t-1}N_{2t}$ that is, the individuals of age 1 are in fact the offsprings of those of age 2, the above expression simplifies to

$$H_t = N_{2t}h_t\left[1 + n_{t-1}(1 - vn_t - u_t)\right] \tag{5.15}$$

The demand next for the physical output sector is given by

$$Y_t = K_{t+1} + N_{1t+1}q_{t+1} + N_{3t}c_t$$

that is, the consumption of old (age 3) individuals, plus the physical capital of the next period, which is given as the sum of its amounts used in the two sectors. The physical capital used in the human capital sector in particular, is equal to the size of the generation that takes education (N_1), times the physical capital (q) used for the education of one member of this generation.

Finally, we have the market clearing condition of the physical output sector:

$$K_t^{\gamma}H_t^{1-\gamma} = K_{t+1} + N_{1t+1}q_{t+1} + N_{3t}c_t \tag{5.16}$$

Having concluded the model the next task is to derive its general equilibrium:

5.3 Competitive equilibrium

Definition 5 *A general equilibrium is a set of variables* $c_t, n_t, u_t, q_t, K_t, H_t, w_t, R_t$ *such as:*

1. n_t, u_t *and* q_t *are the number of children, education time and physical capital used for the education of individuals of age 1 (period 2) that maximise their lifetime utility, given the paths of wages and interest rates during their lifetime.*

2. c_t *is the consumption of individuals of period 4 that maximise their lifetime utility, given the paths of wages* (w) *and interest rates* (R^{-1}) *during this lifetime, as well as their earlier decisions on the above mentioned variables* n, u and q.

3. The physical capital employed in the physical output sector (K_t) *equals the savings of the active population (aged 1 and 2) of the previous period, minus the total amount of physical capital that is used in the human capital sector.*

4. The human capital H_t *available for the physical output sector is given by equation (5.15) and equals the total human capital of the two active generations— which is for any time t given— minus the part of it that is devoted to the child-rearing and education sectors.*

5. The wage level w_t *and capital rent* R_t^{-1} *achieve equilibrium between supply and demand of human and physical capital respectively. The demand for the two capital variables is given by the profit maximising behaviour of the physical output firms, while their supply has been defined in paragraphs 3 and 4 above.*

5.3.1 The equilibrium system

The task of this subsection is to derive the general equilibrium of the model economy under the alternative assumptions of unrestricted borrowing, constrained borrowing, and constrained borrowing with exogenous fertility (n).

The "unrestricted" scenario

From the physical output sector we have

$$w_t = (1-\gamma)K_t^{\gamma}H_t^{-\gamma}$$
$$\frac{1}{R_t} = \gamma K_t^{\gamma-1}H_t^{1-\gamma}$$
$$K_t^{\gamma}H_t^{1-\gamma} = K_{t+1} + N_{1t+1}q_{t+1} + N_{3t}c_t$$

Setting now $x_t = \dfrac{K_t}{H_t}$ that is, the physical/human capital ratio in the physical output sector, we have

$$x_t^{\gamma}H_t = x_{t+1}H_{t+1} + N_{1t+1}q_{t+1} + N_{3t}c_t \tag{5.17}$$
$$w_t = (1-\gamma)x_t^{\gamma} \tag{5.18}$$
$$\frac{1}{R_t} = \gamma x_t^{\gamma-1} \tag{5.19}$$

Substituting next (5.18) and (5.19) into the agents' optimal equations (5.3) and (5.9)-(5.11) we get after some algebraic manipulations

$$q_t = Bx_t h_t u_t \tag{5.20}$$
$$c_t = \frac{vb\varphi\,(1-\gamma)\,\gamma}{a}B^s x_{t-2}^s x_{t-1}^{\gamma} x_t^{\gamma-1} u_{t-2}^{b+s-1} n_{t-2}^e h_{t-2} \tag{5.21}$$
$$1 - vn_t - \frac{b+s}{b}u_t + \frac{\varphi}{\gamma}B^s x_{t+1} x_t^{s-\gamma} u_t^{b+s-1}\left(u_t - \frac{vb}{a}n_t^e\right) = 0 \tag{5.22}$$
$$\frac{\varphi b}{\gamma}B^s u_t^{b+s-1} = x_t^{\gamma-s}x_{t+1}^{-1} \tag{5.23}$$

where $B = \dfrac{s\,(1-\gamma)}{\gamma b}$

To complete the system the population dynamics must be included as well. These are given by the law of motion

$$N_{0t} = n_t N_{1t} \tag{5.24}$$

and the definition

$$N_{it} = N_{i+1,t+1} \tag{5.25}$$

Thus the general equilibrium under the assumption of unrestricted borrowing is given by the system of the equations (5.20)-(5.23) that describe the individuals' optimisation, the equations (5.15) and (5.17)-(5.19) that give the equilibrium in the physical output sector, and equations (5.24) and (5.25) that give the population dynamics.

We now want to reduce this system to one in the four variables of interest, that is, fertility n_t, education time u_t, physical capital employed in the education of one individual (q_t) and physical/human capital ratio in the physical output sector (x_t).

Substituting the variables x, q, c and H into the market clearing condition (5.17) and with some algebraic manipulations of the population variables we get the following expression:

$$\begin{aligned} &\gamma u_t n_{t-1} \left[B u_{t+1} n_t + 1 + n_t \left(1 - v n_{t+1} - u_{t+1}\right)\right] + \\ &\frac{v b^3 \left(1-\gamma\right)}{a} u_{t-2}^{-1} u_{t-1}^{-1} n_{t-2}^{e-1} - b \left[1 + n_{t-1} \left(1 - v n_t - u_t\right)\right] = 0 \end{aligned} \tag{5.26}$$

Substitution of (5.23) into (5.22) yields

$$1 - v n_t + \frac{1-b-s}{b} u_t - \frac{v}{a} n_t^e = 0 \tag{5.27}$$

Equations (5.26) and (5.27) alone with (5.20) and (5.23) fully give the "unrestricted" system in terms of the variables n, u, q, x.

The "constrained" scenario

The constrained general equilibrium system is but one equation same as the unrestricted. In particular, it is described by equations (5.17)-(5.22) and (5.15), while equation (5.23) is replaced with the borrowing constraint (5.4), which after substitution of w_t, R_t and q_t simplifies to[9]

$$1 - vn_t - \frac{b+s}{b}u_t = 0 \tag{5.28}$$

Substituting now the borrowing constraint (5.28) into (5.22) one gets a much simplified expression of the budget constrained:

$$u_t - \frac{vb}{a}n_t^e = 0 \tag{5.29}$$

These two equations alone determine the fertility (n_t) and the education time (u_t). Yet important also is that they are both atemporal! Therefore under the constrained scenario the two variables are constant (providing of course that the constraint is binding). This is not surprising, as both variables are determined by the borrowing constrained young adults (period 2 individuals), and a borrowing (or "liquidity") constraint operates exactly by breaking the link between time periods.

The third equation of the constrained system is (5.20), as before. The system is completed with the market clearing condition in the physical output sector. This is derived in the same way as it was derived under the unrestricted scenario, with the only exception that the capital ratio x cannot be substituted now. Instead, the atemporal property of n and u is used. In addition, we substitute into (5.17) the borrowing constraint (5.28), and after all these manipulations we get

$$\varphi B^s u^{b+s} n x_t^{s-\gamma} x_{t+1}\left(Bun + 1 + \frac{s}{b}un\right) - \left(1 + \frac{s}{b}un\right) +$$

[9]The implicit assumption is that the constraint is binding, since the opposite is of no interest.

$$\frac{vb\,(1-\gamma)\,\gamma}{a\varphi}B^{-s}u^{-1-b-s}n^{e-1}x_{t-1}^{\gamma-s}x_t^{-1}=0 \tag{5.30}$$

Yet the object of this chapter is how the effects of borrowing constraints vary with the endogeneity of population. This will be studied more effectively if a third scenario is introduced, in which the economic agents will again face a borrowing constraint but now their fertility is exogenous (fixed for comparability purposes at the level of the unrestricted scenario):

The constrained with exogenous fertility scenario

With exogenous fertility we don't have for the individuals' maximisation problem the first order condition (5.6) with respect to n. Consequently, any general equilibrium equation in which (5.6) has been used is not valid anymore. This leaves only equations (5.17)-(5.20), (5.15) and (5.28). The exogeneity of fertility raises the number of equations to 7, which is just one short of the required 8 for a finite number of solutions of the general equilibrium system. This eighth equation is the budget constraint, which after substitution of the borrowing constraint is simplified to

$$c_{t+2}=\varphi h_t^{1-s}u_t^b q_t^s\frac{w_{t+1}}{R_{t+2}}$$

By substituting w_{t+1}, R_{t+2} and q_t and taking two periods' lag we have the final expression

$$c_t=\frac{(1-\gamma)\,\gamma}{\varphi}B^{-s}u_{t-1}^{-b-s}x_{t-1}^{\gamma-s}x_t^{\gamma-1}n^{-1}h_{t-2} \tag{5.31}$$

The next task is to reduce this system to one in the three variables u_t, q_t and x_t, as before. For this purpose we substitute (5.18)-(5.19), (5.28), (5.31) and (5.24)-(5.25) into (5.17), and after some algebraic manipulations we get the expression

$$\varphi B^s u^{b+s}nx_t^{s-\gamma}x_{t+1}\left(Bun+1+\frac{s}{b}un\right)-\left(1+\frac{s}{b}un\right)+$$

$$\frac{(1-\gamma)\gamma}{\varphi}B^{-s}u^{-b-s}n^{-1}x_{t-1}^{\gamma-s}x_t^{-1}=0 \tag{5.32}$$

Thus the general equilibrium system in u_t, q_t and x_t, is given for the constrained with exogenous n scenario by the three equations (5.32), (5.20) and (5.28). The two later are repeated for convenience[10]:

$$1-vn-\frac{b+s}{b}u=0 \tag{5.33}$$

$$q_t=Bx_th_tu \tag{5.34}$$

Having derived the general equilibrium the next task is to define a steady state growth and derive it for each of the alternative 3 scenarios. It will however simplify matters if we can drop one variable. This variable is the physical capital input (q_t) in the human capital sector: in all scenarios it is given by the same expression,

$$q_t=Bx_th_tu_t \tag{5.35}$$

which can be rewritten as

$$\frac{q_t}{h_tu_t}=Bx_t \tag{5.36}$$

That is, the physical/human capital ratio in the human capital sector is a constant fraction of the respective capital ratio in the physical output sector. This is a well-known result[11], and enables us to concentrate on x and ignore q:

5.3.2 Steady State Growth

I prefer to define the steady state with as less properties as possible, and prove the remaining:

Definition 6 *Steady state is a path where the physical/human capital ratio x_t, the fer-*

[10] From (5.28) it is made obvious that the exogeneity of n implies that u is constant.
[11] I.e., Barro and Sala-i-Martin (1995).

tility n_t and the time u_t devoted to an individual's education are all constant.

Next the following is established for the steady state:

Proposition 8 *In the steady state the following are also the case:*

a. The population grows at rate $n - 1$, where n is the steady state fertility rate.

b. The wages w_t and interest rate R_t^{-1} are constant.

c. The human capital h_t of the individuals grows at a constant rate $\omega = \varphi B^s u^{b+s} x^s - 1$.

d. The aggregate human and physical capital employed in the physical output sector as well as this sector's output grow an a constant rate of $n(1+\omega) - 1$.

Proof: (a): From (5.24) we have that in the steady state the size of each generation is equal to the size of the previous one, times n. Aggregating this property for all generations and using (5.25) we have that the population grows at a rate $n - 1$.

(b): Proved immediately by applying the definition of the steady state in (5.18)-(5.19).

(c): Simple substitution of (5.36) into (5.2) proves the proposition.

(d): In the steady state (5.15) is written as $H_t = N_{2t} h_t \left[1 + n(1 - vn - u)\right]$. The term inside the brackets is constant, while $N_{2t} = nN_{2t-1}$ and $h_t = (1+\omega) h_{t-1}$. This yields $H_t = n(1+\omega) H_{t-1}$, that is, H_t grows at a rate $n(1+\omega) - 1$. The constant x then implies that K_t grows at the same rate as well, and because of constant returns to scale so does the physical output.

Q.E.D.

The next task is to derive the steady state and per capita growth (ω) for each of the three alternative scenarios. All of them share a common expression for per capita growth, which is given by

$$\omega = \varphi B^s u^{b+s} x^s - 1 \tag{5.37}$$

Yet this does not imply that the growth rate is equal among scenarios, because the variables u and x are not:

A. The unrestricted steady state:

By eliminating the time dimension in equations (5.23), (5.26) and (5.27) we have

$$\frac{\varphi b}{\gamma} B^s u^{b+s-1} - x^{\gamma-s-1} = 0 \tag{5.38}$$

$$\gamma un\left(Bun + 1 + n - vn^2 - nu\right) - b\left(1 + n - vn^2 - nu\right) + \frac{vb^3(1-\gamma)}{a} u^{-2} n^{e-1} = 0 \tag{5.39}$$

$$1 - vn + \frac{1-b-s}{b}u - \frac{v}{a}n^e = 0 \tag{5.40}$$

B. The constrained steady state:

Time elimination from (5.28)-(5.30) yields

$$1 - vn - \frac{b+s}{b}u = 0 \tag{5.41}$$

$$u - \frac{vb}{a}n^e = 0 \tag{5.42}$$

$$\varphi B^s u^{b+s} n x^{s-\gamma+1}\left(Bun + 1 + \frac{s}{b}un\right) - \left(1 + \frac{s}{b}un\right) + \frac{vb(1-\gamma)\gamma}{a\varphi} B^{-s} u^{-1-b-s} n^{e-1} x^{\gamma-s-1} = 0 \tag{5.43}$$

C. The constrained with exogenous n steady state:

By the same means of time elimination we take from (5.33) and (5.32)

$$1 - vn - \frac{b+s}{b}u = 0 \tag{5.44}$$

$$\varphi B^s u^{b+s} n x^{s-\gamma+1}\left(Bun + 1 + \frac{s}{b}un\right) - \left(1 + \frac{s}{b}un\right) + \frac{(1-\gamma)\gamma}{\varphi} B^{-s} u^{-b-s} n^{-1} x^{\gamma-s-1} = 0 \tag{5.45}$$

In the next section the three alternative steady states are compared:

5.4 The effects of the borrowing constraint

In this section the effects of the constraint on the steady state are studied. This is done by simulating and comparing for a wide range of parameter values the steady states of the three alternative scenarios of unrestricted borrowing, borrowing constrained agents, and borrowing constrained with exogenous fertility. The third scenario is useful in revealing how the effects of the borrowing constraint vary with the assumptions about population. For comparability purposes, the exogenous fertility in this third scenario is assumed equal to its level in the unrestricted scenario.

The variables of concern are the physical/human capital ratio (x), the time allocation (u) of agents on age 1 between work and education, the fertility n, and the per capita growth rate ω.

5.4.1 Simulation method

The model contains seven parameters: The two utility parameters e and a, the three parameters of the production function of human capital φ, b and s, the time cost v of one child, and finally γ, the Cobb-Douglas coefficient in the production of physical output. Obviously, the more the parameters the less their alternative values that can be studied. Some amount of sacrifices was therefore essential in order to concentrate to the effects of the most important parameters.

In particular, the Cobb-Douglas coefficient γ was set equal to 0.3, a value well established empirically. Sample simulations also found that the scaling coefficient φ in the human capital industry as well as the time cost v of children are both very important for the steady state values of all variables, yet they have rather limited effects in the relations of these variables among the three alternative scenarios. It was therefore decided to keep them fixed, at the (arbitrary) values of 5 for φ and 0.5 for v. Although the chosen value for v may by too high[12], it has the advantage that it makes the borrowing constraint

[12]Even if basic education is included, which in the present model is implicitly assumed exogenous.

binding in most cases, a requirement necessary if one wants to study the effects of that constraint.

Of the remaining 4 parameters, the scaling coefficient (a) of fertility in the utility function is less important and because of that its effects are only briefly studied. Next a is fixed to the value of unity, that is, equal to the corresponding scaling coefficient of consumption. Having thus fixed the 4 of the 7 parameters I concentrate on the three most important, that is, the inverse utility elasticity of children (e), and the Cobb-Douglas coefficients b and s in the human capital industry. Starting from e, it corresponds as said earlier to the relative demand elasticity of fertility and consumption. Relevant for this "relative demand elasticity" also is the CRRA coefficient of consumption, which is equal to unity. Four values of e were therefore studied, that is, two at each side of unity with the one being very near to it and the other being rather extreme. In particular, the values studied were $0.1, 0.8, 1.5$ and 6.

Regarding the Cobb-Douglas coefficients b and s of the human capital production function, instead of studying alternative values for each of them, alternative values for the one (b) and for their sum ($b + s$) were studied. This allows one to study the effects of the returns to scale for the individual ($b + s$) as well as the effects of changes in the significance of the two production factors, for given returns to scale. The values studied for the returns to scale were $0.5, 0.7, 0.8$ and 0.9, a range that covers all reasonable values. Regarding b, the values studied were $0.1, 0.4, 0.6,$ and 0.8. Table 5.1 summarises the values studied for the three variables e, b and $b + s$:

Table 5.1. Parameter values

e	0.1	0.8	1.5	6
b	0.1	0.4	0.5	0.6
b+s	0.5	0.7	0.8	0.9

All possible combinations of the above values were studied, with the only restriction that the value of b is smaller than that of $b + s$. This enables one to study the effects of each parameter under various conditions with respect to the others. Of the resulting

48 parameters' combinations one is dropped out, because in the constrained scenario the borrowing constraint is not binding. For the remaining 47 combinations the steady states among the three alternative scenarios were compared. An additional restriction is that these steady states are stable, as an unstable steady state will never be reached and it is therefore pointless to compare it with anything else. From the simulations it emerged that all three scenarios have one stable and one unstable steady state, at least for the parameter values studied. In what follows it is the stable steady states that are compared. These are presented in Table 5.2. The number of the available combinations implies that each value of each of the parameters e, b and $b+s$ is combined with approximately 12 different combinations of the other parameters. This number is sufficient for the effects of parameter changes to be studied. In this respect, it was deemed better to study all possible combinations of values of the important parameters, instead of studying deviations from a benchmark case: The results will be more robust if derived under a variety of conditions with respect to the other parameters.

5.4.2 Simulation results

Before studying each variable in detail a few general points are deemed necessary. To start with, comparison between the unrestricted and the constrained-exogenous-fertility scenarios re-establishes the results of the existing literature; less time (u) is invested in education in the constrained scenario, while in this scenario the physical/human capital ratio (x) is higher and the growth rate smaller. This is an important result, as it implies that the results that follow are indeed due to the endogeneity of population and not to some other assumption of the model.

Comparing next the unrestricted scenario with the constrained-endogenous-fertility one reveals that the above results with respect to u and x remain. Yet two new results emerge: first, in the constrained scenario the fertility is lower. Second, with endogenous fertility the unrestricted growth is not necessarily higher than the constrained. These two results will be analysed in more detail later. Meanwhile a comparison between the

Table 5.2. Simulation results

	Parameters			Unrestricted scenario				Constrained scenario				Constrained-fixed n scenari		
no.	e	b	b+s	n	u	x	growth	n	u	x	growth	u	x	growth
1	0.1	0.1	0.5	1.5842	0.0631	0.0795	0.1147	1.4800	0.0520	0.1050	0.1309	0.0416	0.1170	0.0562
2	0.1	0.1	0.7	1.3959	0.0716	0.1087	0.0161	1.2824	0.0513	0.1398	-0.0651	0.0431	0.1492	-0.1384
3	0.1	0.1	0.8	1.2857	0.0778	0.1193	0.0337	1.1862	0.0509	0.1610	-0.0920	0.0446	0.1638	-0.1721
4	0.1	0.1	0.9	1.1579	0.0864	0.1268	0.0993	1.0920	0.0504	0.1821	-0.0950	0.0468	0.1832	-0.1502
5	0.1	0.4	0.5	1.6269	0.2707	0.0441	0.8042	1.4800	0.2080	0.1125	0.7367	0.1492	0.1300	0.4924
6	0.1	0.4	0.7	1.3959	0.2866	0.0872	0.1858	1.2824	0.2050	0.1689	0.1440	0.1726	0.1810	0.0354
7	0.1	0.4	0.8	1.2857	0.3111	0.1059	0.1233	1.1862	0.2034	0.1961	0.0230	0.1786	0.2044	-0.0628
8	0.1	0.4	0.9	1.1579	0.3454	0.1206	0.1391	1.0920	0.2018	0.2231	-0.0449	0.1871	0.2263	-0.1011
9	0.1	0.6	0.7	1.3959	0.4298	0.0461	0.8520	1.2824	0.3076	0.1547	0.6536	0.2589	0.1648	0.4752
10	0.1	0.6	0.8	1.2857	0.4667	0.0691	0.5145	1.1862	0.3052	0.1942	0.3256	0.2679	0.2025	0.2044
11	0.1	0.6	0.9	1.1579	0.5182	0.0894	0.4044	1.0920	0.3027	0.2314	0.1514	0.2807	0.2349	0.0808
12	0.1	0.8	0.9	1.1579	0.6909	0.0437	1.3173	1.0920	0.4035	0.2012	0.6639	0.3742	0.2042	0.5571
13	0.8	0.1	0.5	1.4024	0.0713	0.0840	0.2113	1.3604	0.0640	0.0973	0.2168	0.0598	0.0994	0.1863
14	0.8	0.1	0.7	1.2661	0.0790	0.1112	0.1028	1.1936	0.0576	0.1379	0.0061	0.0524	0.1389	-0.0540
15	0.8	0.1	0.8	1.1885	0.0841	0.1206	0.1095	1.1225	0.0548	0.1605	-0.0380	0.0507	0.1611	-0.0938
16	0.8	0.1	0.9	1.1013	0.0908	0.1273	0.1532	1.0583	0.0523	0.1819	-0.0655	0.0499	0.1822	-0.1030
17	0.8	0.4	0.5	1.4650	0.3289	0.0498	1.0130	1.3604	0.2558	0.1063	0.9152	0.2140	0.1115	0.7599
18	0.8	0.4	0.7	1.2661	0.3159	0.0898	0.2809	1.1936	0.2304	0.1642	0.2310	0.2097	0.1677	0.1597
19	0.8	0.4	0.8	1.1885	0.3366	0.1074	0.2031	1.1225	0.2194	0.1936	0.0811	0.2029	0.1962	0.0210
20	0.8	0.4	0.9	1.1013	0.3632	0.1211	0.1942	1.0583	0.2093	0.2224	-0.0146	0.1997	0.2234	-0.0531
21	0.8	0.6	0.7	1.2745	0.4886	0.0484	1.0357	1.1936	0.3456	0.1512	0.7903	0.3109	0.1542	0.6658
22	0.8	0.6	0.8	1.1885	0.5049	0.0703	0.6185	1.1225	0.3291	0.1918	0.4046	0.3043	0.1943	0.3228
23	0.8	0.6	0.9	1.1013	0.5448	0.0899	0.4714	1.0583	0.3139	0.2307	0.1887	0.2996	0.2317	0.1413

Table 5.2. Simulation results (continued)

	Parameters			Unrestricted scenario				Constrained scenario				Constrained-fixed n scenari		
no.	e	b	b+s	n	u	x	growth	n	u	x	growth	u	x	growth
24	0.8	0.8	0.9	1.1013	0.7264	0.0440	1.4258	1.0583	0.4185	0.2006	0.7190	0.3994	0.2015	0.6488
25	1.5	0.1	0.5	1.2961	0.0772	0.0871	0.2782	1.2778	0.0722	0.0947	0.2789	0.0704	0.0951	0.2649
26	1.5	0.1	0.7	1.1953	0.0837	0.1126	0.1573	1.1438	0.0612	0.1379	0.0495	0.0575	0.1379	0.0047
27	1.5	0.1	0.8	1.1378	0.0879	0.1214	0.1539	1.0898	0.0569	0.1605	-0.0093	0.0539	0.1605	-0.0512
28	1.5	0.1	0.9	1.0737	0.0931	0.1275	0.1814	1.0423	0.0532	0.1820	-0.0512	0.0515	0.1820	-0.0791
29	1.5	0.4	0.5	1.3529	0.3706	0.0537	1.1529	1.2778	0.2889	0.1048	1.0321	0.2588	0.1061	0.9259
30	1.5	0.4	0.7	1.1953	0.3347	0.0913	0.3405	1.1438	0.2446	0.1631	0.2812	0.2299	0.1643	0.2293
31	1.5	0.4	0.8	1.1378	0.3515	0.1083	0.2497	1.0898	0.2275	0.1931	0.1120	0.2155	0.1940	0.0668
32	1.5	0.4	0.9	1.0737	0.3725	0.1213	0.2231	1.0423	0.2128	0.2223	0.0001	0.2058	0.2226	-0.0286
33	1.5	0.6	0.7	1.2043	0.5259	0.0497	1.1491	1.1438	0.3670	0.1507	0.8664	0.3410	0.1515	0.7739
34	1.5	0.6	0.8	1.1378	0.5273	0.0710	0.6791	1.0898	0.3413	0.1914	0.4457	0.3233	0.1922	0.3854
35	1.5	0.6	0.9	1.0737	0.5588	0.0901	0.5066	1.0423	0.3192	0.2305	0.2066	0.3088	0.2309	0.1715
36	1.5	0.8	0.9	1.0737	0.7451	0.0441	1.4827	1.0423	0.4256	0.2006	0.7451	0.4117	0.2008	0.6937
37	6.0	0.1	0.7	1.0690	0.0935	0.1156	0.2709	1.0519	0.0677	0.1393	0.1336	0.0665	0.1389	0.1175
38	6.0	0.1	0.8	1.0490	0.0953	0.1228	0.2415	1.0322	0.0605	0.1611	0.0433	0.0594	0.1608	0.0277
39	6.0	0.1	0.9	1.0263	0.0974	0.1279	0.2334	1.0151	0.0547	0.1821	-0.0264	0.0541	0.1820	-0.0367
40	6.0	0.4	0.5	1.1258	0.4648	0.0619	1.4452	1.1024	0.3590	0.1077	1.2717	0.3497	0.1069	1.2401
41	6.0	0.4	0.7	1.0690	0.3742	0.0944	0.4641	1.0519	0.2709	0.1639	0.3780	0.2660	0.1635	0.3595
42	6.0	0.4	0.8	1.0490	0.3813	0.1099	0.3416	1.0322	0.2419	0.1934	0.1688	0.2378	0.1932	0.1519
43	6.0	0.4	0.9	1.0263	0.3897	0.1218	0.2762	1.0151	0.2189	0.2223	0.0258	0.2164	0.2223	0.0151
44	6.0	0.6	0.7	1.0732	0.6007	0.0523	1.3706	1.0519	0.4063	0.1524	1.0067	0.3972	0.1517	0.9741
45	6.0	0.6	0.8	1.0490	0.5720	0.0723	0.7984	1.0322	0.3629	0.1921	0.5195	0.3566	0.1917	0.4978
46	6.0	0.6	0.9	1.0263	0.5846	0.0905	0.5712	1.0151	0.3283	0.2307	0.2376	0.3246	0.2306	0.2248
47	6.0	0.8	0.9	1.0263	0.7795	0.0444	1.5871	1.0151	0.4377	0.2008	0.7899	0.4328	0.2007	0.7714

two constrained scenarios reveals that both education time and per capita growth are always higher when the population is endogenous, while there is no general result for the capital ratio x.

After these general points a detailed look to the simulation results comes next. Of interest is not only under which scenario the four variables n, u, x and ω are higher or lower, but also how their variations among scenarios are affected by the parameters of the model. In particular, of interest is how the model parameters affect the following quantities:

$$\begin{aligned}
N &= \frac{n(\text{unrestricted}) - n(\text{constrained})}{n(\text{unrestricted})} \\
U &= \frac{u(\text{constrained}) - u(\text{constrained-fixed-n})}{u(\text{unrestricted}) - u(\text{constrained-fixed-n})} \\
X &= \frac{x(\text{constrained}) - x(\text{constrained-fixed-n})}{x(\text{unrestricted}) - x(\text{constrained-fixed-n})} \\
G_1 &= \frac{\omega(\text{unrestricted}) - \omega(\text{constrained})}{1 + \omega(\text{unrestricted})} \\
G_2 &= \frac{\omega(\text{constrained}) - \omega(\text{constrained-fixed-n})}{\omega(\text{unrestricted}) - \omega(\text{constrained-fixed-n})}
\end{aligned}$$

N gives the difference between the unconstrained and the constrained fertility as a portion of the former, or in other words, it gives the effect of the borrowing constraint on fertility. Regarding U, its denominator gives the education difference between the unrestricted and constrained-exog-n scenarios, that is, the education effect of the borrowing constraint when the fertility is assumed exogenous. The numerator of U on the other hand gives the education that is generated when— under conditions of borrowing constraints— the fertility is endogenised. The ratio U therefore gives the portion of the education gap, generated by the borrowing constraint, that the endogeneity of population fills. In what follows this ratio will be referred to by just "U" while the term "u-gap"— or "education gap"— will refer to the denominator of U that is, to the education difference under exogenous fertility assumptions. Obviously a high value of U means that the endogeneity of population closes a high part of the "u-gap".

Similar to U are the new variables X and G_2. The denominators of these variables give the difference in x and ω generated by the borrowing constraint under exogenous population assumptions, while the numerators give the part of this difference that the endogeneity of population closes. Finally, G_1 gives the growth difference between the unrestricted and constrained-endog-n scenarios, as a portion of the "gross" growth— the growth generated if depreciation is assumed away— in the unrestricted scenario. The reason for using the gross rather than the net growth is that the later can be either positive, negative, or (near to) zero, therefore dividing by it may distort the results.

Next the effects of parameter changes are studied, starting as said from the scaling parameter a, and then follows the more detailed analysis of the effects of e, b and $b + s$.

5.4.3 Effects of a

The effects of a were studied by means of numerical simulations. In particular, three alternative values of a were tried: 0.2, 1 and 5. These three values were combined with a range of combinations of values of e, b and $b + s$, and in each case I studied how a change in a would affect the variables of the model, in the three alternative scenarios.

The first result is that the higher the value of a, the smaller the gap between constrained and unrestricted fertility, as captured by the variable N defined above. The reason why is that the higher a is the higher the fertility, at the expense of education. When the borrowing constraint is lifted, the economic agents increase their fertility further, by borrowing out of their future income. Yet this income is low, due to the low education effort. Thus the fertility cannot increase much.

Puzzling however is the positive effect of a on U; as it reduces the fertility gap N it should lead to convergence of the education u between the two constrained scenarios. The reason this is not the case is that although a reduces the fertility gap it also leads to high fertility, in all alternative scenarios. This keeps the residual u low, thus easy to fill the u-gap— the denominator of U— even with a low *relative* fertility gap.

The above result implies that with a higher value for a the constrained-endog-n human capital is relatively higher. Yet a higher a was also seen to imply lower fertility gap, which in turn implies a higher dilution of the per capita physical capital, for the constrained scenario. As the physical capital is the numerator of the capital ratio x while the human capital is its denominator, the obvious consequence is that a suppresses the constrained-endog-n value of x, comparing to the same value in the constrained-exog-n scenario. This is exactly what the simulations found.

From equation (5.37) the per capita growth rate is given as a function of u and x. Consequently, the effects of all parameters on ω as well as on the growth indices G_1 and G_2 reflect their effects on u and x. Starting from G_2— which compares the two constrained scenarios— the simulations show a small and with ambiguous sign effect of a on it. This is due to the positive effect of a on both U and X; the first implies that a increases the endogenous-n education time while the later implies that it increases the exogenous-n capital ratio[13]. The simulations results show that the two effects more or less offset each other, which means that a does not affect the portion of the "growth gap" that the endogeneity of fertility closes. Negative finally was found the effect of a on G_1, that is, the higher a is the higher the constrained growth comparing to the unrestricted. The reason why, is that a affects the fertility gap negatively in relative terms (the index N), but also affects positively the total fertility and the fertility gap in absolute terms. That is, when the preference for children is high the education can be only low. Imposing the borrowing constraint under these circumstances reduces education considerably in relative terms, but only marginally in absolute terms, because there is no further room to reduce it. In addition, when fertility is high, both physical and human capital are low; the second because of low education as just described, the first because of low income and the capital dilution effect of population. Imposing the borrowing constraint under these circumstances reduces drastically the denominator of the physical/human capital

[13]As said earlier, the unrestricted value of x is lower than that in the two constrained scenarios. The denominator of X is therefore negative, which exactly means that the positive effect of a on X reflects its negative effect on the numerator.

ratio x, but not as much the numerator; physical capital investment is not as much liable to the borrowing constraint, because it is undertaken by both active generations. The result is that the x-ratio is increased. In short, the higher a is the nearer— in absolute terms— the constrained u is to the unrestricted, while the constrained x increases its distance from the unrestricted. Both effects boost the constrained growth.

5.4.4 Effects of e, b and $b + s$

As said, the effects of these parameters were studied by simulating the steady states of the three scenarios— unrestricted borrowing, no borrowing with endogenous fertility and no borrowing with exogenous fertility— with alternative values of the three parameters. It was also said that 4 values were considered for each of them, and all possible combinations between these values were simulated and compared, to achieve more robust results. The simulations results were presented in Table 5.2. In this section, these results are studied:

Effects on fertility

As said, the fertility is lower in the constrained scenario. This result was anticipated. What one may like to study though is which parameters are more important for the difference between the constrained and the unrestricted fertility, or in other words, how the effects of the borrowing constraint on fertility vary with the values of the model parameters.

The simulations show that the higher the values of e, b and returns $(b + s)$ in the human capital sector, the smaller the gap between constrained and unrestricted fertility. Starting from e, its negative relation with the gap index N implies that the higher e is, the more the constrained fertility converges to the unrestricted. The reason why is that e is negatively related to the utility— therefore demand— elasticity for children. In other words, a high value of e implies inelastic demand for children, which in turn implies that small is the effect of the borrowing (or any other!) constraint on this demand.

The sum $b + s$ refers to the returns to education. The higher this sum is the more it pays to invest in education and consequently the economic agents take more education at the expense of their fertility. Due to the concavity of the utility function with respect to fertility, when the agents face a borrowing constraint they are more reluctant to reduce their already low fertility. In addition, the higher value of $b+s$ implies that the production function of human capital is less concave, that is, the economic agents can more easily reduce their human capital investment. Both effects lead to a smaller fertility gap between the unrestricted and constrained scenarios when the returns $b+s$ are high. This is exactly the finding of the simulations.

Regarding b, for given returns $b+s$ it gives the relative significance of human capital in the human capital industry; a high value of b implies that more human rather than physical capital is used, and vise verca. The prices of these inputs depend on their relative abundance, which is given by the capital ratio x. This ratio is as said lowest in the unrestricted scenario, that is, human capital is relatively more expensive in the unrestricted scenario. When therefore a borrowing constraint is imposed, the physical capital and consequently education become cheaper. This has a substitution effect from children to education. The higher b is the less the cost of education is reduced, that is, the lower the substitution from children to education. This is exactly why b has a negative effect on the fertility gap N.

Effects on the time devoted to education (u)

As said earlier, the education time (u) is highest in the unrestricted scenario and lowest in the constrained-exogenous-n, with the constrained-endogenous-n lying in between. This subsection studies where exactly this "between" lies, and how it is affected by the values of the model parameters.

Of relevance to this question is the quantity

$$U = \frac{u(\text{constrained}) - u(\text{constrained-fixed-n})}{u(\text{unrestricted}) - u(\text{constrained-fixed-n})}$$

above defined, which exactly gives the portion of the education difference between the two extreme scenarios that the endogeneity of population fills. The simulation results show that U is affected negatively by e and $b + s$, with the effect of the latter being stronger. The effect of b on the other hand is negligible. In both constrained scenarios, education is the residual of the second period income after the cost of fertility has been subtracted. This helps explaining the effects of the model's parameters on the education portion U :

High scale returns to education $b + s$ imply as said low fertility and high education. It was also said that the fertility gap is low when $b + s$ is high. A low value of $b + s$ on the other hand implies low education thus low difference between the unrestricted and the constrained-exog-n education, which can be easily filled by the high fertility gap. In other words, the higher the returns $b + s$ the smaller the portion of the education gap that is filled by reducing fertility and the higher the portion that is accounted for by the borrowing constraint.

Similar arguments apply as well for the negative effects of e; this parameter reduces as said the fertility gap and consequently reduces the portion of the education difference (between the two extreme scenarios) that the endogeneity of population closes.

The insignificance on the other hand of b, the Cobb-Douglas coefficient in the human capital production function, can be explained as follows: for given returns $b + s$, what b gives is as said the relative weight of human and physical capital in the human capital industry. As was also explained, the higher b is, the more the constrained cost of education converges to the unrestricted, or equivalently the more the constrained education diverges from the unrestricted. Yet this is the case for both constrained-exog-n and constrained-endog-n scenarios, that is, b increases both the numerator and the denominator of U. It appears from the simulations that neither of the two effects dominates over the other, which exactly renders b insignificant for U.

Effects on the physical/human capital ratio

The physical/human capital ratios between the two sectors are different as shown by equation (5.36) in only the scaling coefficient B, which depends only on the parameters of the model. Thus the physical/human capital ratio can be studied from the physical output sector (x) alone.

In this subsection I study how the model parameters affect the quantity

$$X = \frac{x(\text{constrained}) - x(\text{constrained-fixed-n})}{x(\text{unrestricted}) - x(\text{constrained-fixed-n})}$$

as well as the difference of x between the unrestricted and the two constrained scenarios.

Starting from X, its denominator gives the capital ratio difference generated by the borrowing constraint under exogenous population assumptions, while the numerator gives the part of this difference that the endogeneity of population closes. As said in the beginning of this section, both the constrained and constrained-exog-n capital ratios are higher than the unrestricted, but when compared to each other either of them can be higher. This may be surprising, because the lower fertility is the higher the per capita physical capital, due to the capital dilution effect. However x is not an index of physical capital, but rather the physical/human capital ratio. Since the endogeneity of population increases as said the investment in human capital, its total effect on the capital ratio is ambiguous.

Obviously, the fertility gap is very important for X. Yet now its effect is not as straightforward as it was for U; fertility does suppress education and through it human capital, but it also suppresses physical capital, through the capital dilution effect of population. However, it is reasonable to expect that X is positively related with U[14], while opposite is its relation with N, once the relation of the latter with education choice has been accounted for. With this in mind, it is easy to interpret the parameters' effects on X by means of their effects on N and U:

[14] Recall that the denominator of X is negative.

By affecting both N and U in the same (negative) way, only moderate are the effects of e and $b+s$ on X, despite their strong effects on N and U. Yet the simulation results show that it is the effects through U that dominate, implying that the higher e and $b+s$ are the more the constrained-endog-n value of x is increased, comparing to the constrained-exog-n value. In order next to explain the U-shaped effect of b on X, its effects on the capital ratio x of all three scenarios are studied.

Starting by comparing the unrestricted with the two constrained scenarios, the simulations show that b increases the constrained x's more than the unrestricted. The reason why, is that b increases as said before the constrained cost of human capital investment, which has a negative effect on the denominator of x. The difference finally x(constrained)$-x$(constrained-fixed-n), that is, the numerator of X, is declining with b, at an increasing rate. The cost of human capital investment is the reason for this effect of b: Although the cost of human capital investment increases with b, it is easier to meet it if the fertility in endogenous. At high values of b this cost is relatively high, which without endogeneity of fertility has a stronger negative effect on human capital investment. This is exactly why the effect of b on the difference x(constrained)$-x$(constrained-fixed-n) is more and more negative as b goes up.

In the light of the above results, it is easy to explain the U-shaped effect of b on X: For low values of b the numerator of X— the difference x(constrained)$-x$(constrained-fixed-n)— declines less rapidly than the denominator. For higher b though it is the other way round.

Effects on economic growth

Perhaps the most important finding of this chapter though is that when endogenous population is assumed the established in the literature negative growth effect of the borrowing constraint is not necessarily the case. These opposite cases are relatively few (only 3 out of the 47 parameter combinations give higher growth for the constrained scenario), but enough to manifest themselves. What one would like to know though is which parameter values are more likely to generate this "inverse growth effect" of the borrowing constraint. Although the number of such cases is very small, it is interesting that they all occur for the lowest values of both b and $b + s$. For a better understanding though of how the model parameters affect the growth differences among the three alternative scenarios, all 47 simulated parameter combinations are studied.

This is done by the use of the quantities G_1 and G_2, previously defined. The first of them gives the growth difference between the unrestricted and constrained-endog-n scenarios, while the second gives the portion of the "exogenous population growth gap"— the growth difference between the two extreme scenarios— that the endogeneity of population fills.

Starting from the returns to education $b + s$, the simulations show that they have a strong positive effect on the difference between the unrestricted and the constrained-endog-n growth. In other words, as the returns $b + s$ increase the constrained-endog.-n growth converges to that when the fertility is exogenous. This finding is also backed by the negative effect of $b + s$ on G_2. This result was anticipated; as previously argued, the lower $b + s$ is the more concave the production function of human capital becomes, which implies that when facing the borrowing constraint the economic agents prefer to reduce the number of their children rather than their education. This has obviously a positive effect on human capital accumulation, but another positive effect on physical capital as well, through the capital dilution effect of population and the higher income— therefore savings— of the period 3 agents. Consequently lower returns in the human capital industry increase the constrained growth.

Human capital investment is also the explanation for the effects of b. As said earlier, for given returns $b + s$, what b stands for is the relative weight of human capital in the human capital industry. The cost of one unit of investment in human capital depends on the one hand on this weight of the two capital inputs and on the other on the relative cost of these inputs. The latter is a function of their ratio x, which is higher for the constrained scenario, that is, the user cost of physical capital is in the constrained scenario lower than in the unrestricted. As was shown in the previous subsection, a low value of b implies lower difference between the unrestricted and the constrained x's, thus lower difference in the rental cost of physical capital. On the other hand though, the lower b is, the higher the weight of the physical capital is in the human capital industry. If the human capital industry uses physical capital very intensively the "unit cost effect" of b is very strong and dominates its total effect on human capital investment. If on the other hand b is relatively high the physical capital is not used very intensively in the production of the human and it is the "weight effect" that prevails. It is exactly for this reason that the simulations found a positive effect of b on G_1 in all cases except for when b is as low as 0.1. Interesting also is the fact that the effect of b on G_2 is the exact opposite of its effect on G_1. The reason why is that the constrained-endog-n growth rate enters the two quantities with opposite signs.

Very important for the growth effects of the constraint also is the inverse utility elasticity of children (e). In particular, the simulations show that the higher e is the higher G_1 and the lower G_2 are. As said, a high e implies inelastic demand for children, therefore limited effects of the borrowing constraint on fertility. In other words the higher e is, the more the model behaves as an exogenous population one— that is, the more the constrained-endog-n growth converges to the constrained-exog-n. This is exactly what the simulation results show.

To conclude this discussion of the growth effects of the endogeneity of population in an environment with borrowing constraints, one last word must be said for the index G_2: The mean of G_2 in the simulations' sample is 0.315, while the respective means of N,U and X are 0.044, 0.137 and 0.037. In addition, the average values of the fractions $\frac{U}{N}$, $\frac{G_2}{N}$ and $\frac{G_2}{U}$ are 3.08, 7.89 and 2.36 respectively. This clearly shows how the effect of the borrowing constraint on fertility is magnified when transmitted to human capital investment, and then magnified again when transmitted to per capita growth. Even if all the previous results are assumed away, this alone is quite eloquent for the importance of the assumptions regarding population in studies of the growth effects of borrowing constraints.

5.5 Conclusions

It has been well established in the literature that borrowing constraints on households amplify physical and suppress human capital accumulation, while the total effect on per capita growth is negative. Yet with endogenous population borrowing constraints are also expected to affect fertility choice as well. Given that the latter interacts with the accumulation of both types of capital, introducing endogenous population is the obvious step forward in the theory of borrowing constraints.

The purpose of the present chapter was exactly to take this step. This was done by studying a more general model, which pools together elements of models with endogenous population and elements of models with borrowing constraints. Since it was impossible to derive analytical solutions even for the steady state, simulations were used instead.

The results showed that the fertility is indeed affected by the borrowing constraint, this effect depending positively on the elasticity of the demand for children $\frac{1}{e}$, and negatively on the intensity a of the utility of children, the human capital intensity b in the human capital sector, and the scale returns $b + s$ to the production of human capital.

Regarding the other variables of interest, the time u devoted to education is negatively affected by the borrowing constraint, yet this effect is mitigated when the population is endogenised. How much it is mitigated depends on the parameter values. In particular, a high value of $b+s$ clearly reduces the portion of the education gap that the endogeneity of population closes. Opposite however are the effects of the elasticity of demand and the intensity of the utility of children.

The positive effect of the borrowing constraint on the physical/human capital ratio x is maintained when the population is endogenised, although it can be higher for either of the two constrained scenarios. This is not invariant to the parameters of the model; the intensity a of the utility of children boosts the x ratio for both constrained scenarios, but this effect is stronger for the exogenous population one. On the other hand, the coefficient b tends to increase the difference of both constrained x -values from the unrestricted.

Perhaps the most important of the results of this chapter though is that when the population is endogenised the borrowing constraint may have a positive rather than negative effect on economic growth. Even when this is not the case, the endogeneity of population still mitigates the negative growth effect of the constraint. In addition, this effect is an "enlargement" of the mitigating effect of the endogeneity of population on education.

How exactly the endogeneity of population affects the constrained growth depends on the parameters of the model. From the reduced form equation (5.37), growth is given as a weighted product of u and x, the first roughly standing for the total resources allocated to human capital investment and the second, again roughly, standing for the opposite of the unit cost of this investment. The parameters' effects on growth obviously reflect their effects on these two variables. Indeed, strong and negative are the effects of e and $b+s$ on the constrained-endog-n growth while the effect of a is positive. Finally, b boosts the unrestricted growth, comparing to the constrained-endog-n, unless it is very low.

Chapter 6

Conclusion

This thesis is composed of three essays on the effects of demographic variables on human capital investment and through it on economic growth. Each essay discusses a different aspect of this relation. Chapter 3 studies how the age structure of population affects the aggregate human capital of an economy. The human capital is different to the physical, in the sense that it can only exist embodied in people and consequently it is not transferable, at least in the sense the physical capital is. Further, since the human capital investment of the economic agents depends on their age, the aggregate human capital of an economy depends in turn on the demographic features of the economy.

Lucas (1988) partly identified this special nature of human capital, but argued that one only has to assume the existence of inter-generational externalities in the production of human capital, to have it behaving on the aggregate level in the same manner with the physical. In chapter 3 I show that this is not the case. In particular, I develop a simple model were human capital investment is disaggregated to the level of the individuals. As the life-cycle theory argues, when they make their investment decisions they also take into account their expected life. Due to its embodiment nature, the investment in human capital always follows the pattern implied by the life-cycle theory, regardless of altruism assumptions. The assumption of Lucas is also included, that is, the new generations are assumed to start from a human capital level that is proportional to the average.

It is shown that there are three effects of the birth rate to the average human capital investment and its growth rate: The first, has to do with the effect of the birth rate on population growth and the consequent pressure on the education facilities of an economy. Second, as long as the new generations start at a human capital level that is proportional to the average but lower than that, they imply a human capital dilution effect, which is higher the larger the new generation. The third effect of population consists on the life cycle property of human capital investment: Since the young generations invest in human capital more than the old, the higher the portion of the young in the economy the higher the average investment. This final effect depends on the birth rate positively rather than negatively, contrary to the other two. As a consequence, the total growth effect of the birth rate is ambiguous.

In short, chapter 3 shows that because of its embodiment nature, the aggregate human capital depends on the demographic factors of an economy and cannot be assumed to behave in the same way as the physical, as is usually assumed in the literature. In addition, this was shown by assuming away all other streams through which the economy can be affected by demographic factors.

This finding of chapter 3 is assumed away in the next chapter, which studies the effects of demographic factors on technological change, and through it economic growth. In particular, I include human capital investment in the framework of a model with endogenous technological change. Although the endogenous technology literature recognises human capital as the engine of innovation and technological improvement, it usually takes it as exogenous and proportional to the size of the population, implying probably that human capital investment and technological improvement do not interfere with each other. In chapter 4 I show that this is not the case, even under very simple assumptions with respect to human capital formation.

In particular, I include human capital investment in a framework of endogenous technology without scale effects. The engine of growth is technological progress, which can be either variety expanding or quality improving. This 2-dimensional nature of R&D is shown in the literature to eliminate the scale effect of population. Input for both R&D activities is labour, yet measured in effective units, which exactly takes human capital into account. Regarding human capital investment, although it is still disaggregated to the level of the individuals, certain simplifying assumptions eliminate the dependence of its aggregate level on the demographic features of the economy.

The main result of the literature of 2-dimensional R&D is that the population growth rate— not size— has a positive effect on economic growth. In chapter 4 I show that when human capital investment is also taken into account the relations between population, innovation, and human capital become less straightforward: Population growth has a direct positive effect on innovation, as shown in the literature. This increases the returns to human capital, which in turn boosts education. On the other hand though, a high population growth rate pushes the interest rate upwards, with an opposite to the above effect on education. On the overall, the birth rate can affect education either way, a result that offers theoretical grounding to some empirical studies that found that the size of a generation affects their education positively. Finally, human capital investment affects innovation in the same manner the latter is affected by population growth. The reason why is that what in fact matters is the growth rate of human capital rather than population, which depends on both population and per capita human capital. The effects of the death rate on the other hand are negative, because the latter is also associated with the depreciation rate of human capital.

The chapter finally argues that these multiple effects of population on R&D and technological progress provide a better explanation for the growth patterns of the last two centuries, than those attempted in the literature: The increasing education effort— partly fuelled by the lower mortality rates— outweighted the negative growth effect of the declining population growth. Further, education and human capital growth may have also been promoted by the lower birth rate and more balanced population age structure, through the mechanisms analysed in chapter 3.

Chapter 5 studies the effects of borrowing constraints when the population is endogenous. Under the opposite assumption— exogenous population— borrowing constraints have been found to affect negatively the accumulation of human capital and positively the accumulation of the physical, with the total effect on economic growth being negative. However, population is very important for both physical and human capital formation; life cycle saving and the capital dilution effect of population growth link physical capital to population, while the literature on endogenous fertility choice has stressed the trade-off between population and human capital. Borrowing constraints on the other hand are very likely to affect fertility choice as well. Introducing endogenous fertility in a borrowing constraints framework is therefore the obvious way to carry forward the theory of borrowing constraints.

An overlapping generations model was developed, where the economic agents work for two periods of their lives, in the first of which they are borrowing constrained. In that period they also bear their children and take education, which pays its returns in the next period. The trade-off between these two decisions is studied, by comparing the three alternative scenarios of unlimited borrowing, no borrowing, and no borrowing with exogenous population. The third scenario was included for a better understanding of how the endogeneity of population interacts with borrowing constraints.

Simulations showed that under borrowing constraints, the expenditure for education (measured in time units) is lower, albeit this effect is mitigated when the economic agents can also reduce their fertility. The physical/human capital ratio on the other hand was found to be higher when the economic agents face borrowing constraints. This is due to the fact that inability to borrow implies more saving and consequently higher investment, as emphasised in the literature. In addition, when fertility is endogenous the per capita physical capital is increased further due to the lower fertility, although this result is not as much reflected to the capital ratio because with endogenous fertility the human capital is higher as well.

But the most important result of chapter 5 is that with endogenous population the borrowing constraint may have a positive rather than negative effect on economic growth.

Even when this is not the case, the endogeneity of population still mitigates the negative growth effect of the constraint. This is due to the reduced fertility, which as said has a positive effect on the per capita levels of both capital inputs. The size and sign of the growth effect of the borrowing constraint depend on whether the economic agents reduce their fertility or their human capital investment, as well as how the borrowing constraint affects the cost of this investment.

Overally, the thesis addresses some of the questions on the interaction between human capital and population, and their effects on economic growth. Its contribution is related to different aspects of the problem. Of course it is impossible to answer all questions in such a vast subject. In this respect, I'm glad to have had the opportunity to look at three related but separate questions, instead of having concentrated on just one.

Appendix A

Derive the steady state effects of parameter changes of chapter 3

The steady state variables r and ω are given from equations (3.24) and (3.27), which are graphically shown in graph 3.1 by the CC and HH loci respectively. These equations are repeated below for convenience:

$$r - n - \rho - \sigma\omega = 0 \tag{A.1}$$
$$1 = \zeta\beta \int_0^T \exp\left\{\int_0^x \left[B\left(aB\int_{v-T}^0 e^{(r+\theta)\mu}d\mu\right)^{\frac{a}{1-a}} - n - \omega - \theta\right] dv\right\} dx \tag{A.2}$$

where (A.1) is same as (3.24) and stands for the CC locus, while (A.2) is same as (3.27) and stands for the HH locus. In order to study how the two loci shift when the parameters change, I take the implicit derivatives of r:

It is easy to show that

$$\left.\frac{\partial r}{\partial(\beta, \theta, \sigma)}\right|_{\mathrm{CC}} = (A, 0, \omega)$$

where $A = \dfrac{dn}{d\beta} > 0$.

Thus, an increase in β shifts the CC locus to the right, while an increase in σ rotates it at the point it crosses the horizontal axis ($\omega = 0$), and θ has no effect on it.

Regarding the HH locus, it is

$$\begin{aligned}\frac{\partial(A.2)}{\partial r} &= \zeta\beta\int_0^T \exp\left\{\int_0^x \left[B\left(aB\int_{v-T}^0 e^{(r+\theta)\mu}d\mu\right)^{\frac{a}{1-a}} - n - \omega - \theta\right]dv\right\}F(x)dx \\ \text{where } F(x) &= (1-a)^{-1}(aB)^{\frac{1}{1-a}}\int_0^x \left(\int_{v-T}^0 e^{(r+\theta)\mu}d\mu\right)^{\frac{2a-1}{1-a}}\int_{v-T}^0 \mu e^{(r+\theta)\mu}d\mu dv\end{aligned}$$

$F(x)$ is always negative, therefore $\frac{\partial(A.2)}{\partial r} < 0$.

Further, it is

$$\frac{\partial(A.2)}{\partial\omega} = -\zeta\beta\int_0^T x\exp\left\{\int_0^x \left[B\left(aB\int_{v-T}^0 e^{(r+\theta)\mu}d\mu\right)^{\frac{a}{1-a}} - n - \omega - \theta\right]dv\right\}dx < 0$$

and

$$\begin{aligned}\frac{\partial(A.2)}{\partial(\beta,\theta,\sigma)} &= \left(\frac{1}{\beta} + LA, K + L, 0\right) \\ \text{where } L &= \frac{\partial(A.2)}{\partial\omega}, \ K = \frac{\partial(A.2)}{\partial r} \text{ and } A = \frac{dn}{d\beta}\end{aligned}$$

Using the principles of implicit differentiation one has that

$$\left.\frac{\partial r}{\partial(\beta,\theta,\sigma)}\right|_{\mathrm{HH}} = \left(\frac{\frac{1}{\beta} + LA}{-K}, \frac{K+L}{-K}, 0\right)$$

Substituting the signs of K, L and A from above one has that σ does not affect the HH locus, θ shifts it to the left, and the effect of β is ambiguous.

Next the general equilibrium results of the parameter changes are studied: It is

$$\frac{\partial\begin{pmatrix}\omega\\ r\end{pmatrix}}{\partial(\beta,\theta,\sigma)} = -\left[\frac{\partial\begin{pmatrix}A.1\\ A.2\end{pmatrix}}{\partial(\omega,r)}\right]^{-1}\cdot\frac{\partial\begin{pmatrix}A.1\\ A.2\end{pmatrix}}{\partial(\beta,\theta,\sigma)} \Rightarrow$$

$$\frac{\partial\begin{pmatrix}\omega\\ r\end{pmatrix}}{\partial(\beta,\theta,\sigma)} = -\begin{bmatrix}-\sigma & 1\\ L & K\end{bmatrix}^{-1}\begin{bmatrix}-A & 0 & -\omega\\ AL+\frac{1}{\beta} & K+L & 0\end{bmatrix} \Rightarrow$$

$$\frac{\partial\begin{pmatrix}\omega\\ r\end{pmatrix}}{\partial(\beta,\theta,\sigma)} = \frac{1}{D}\begin{bmatrix}A(K+L)+\frac{1}{\beta} & K+L & \omega K\\ AL(\sigma-1)+\frac{\sigma}{\beta} & \sigma(K+L) & -\omega L\end{bmatrix}$$

where $D = -\sigma K - L > 0$ and K, L and A are as above.

The above results therefore show that θ reduces both growth and interest rates. σ on the other hand reduces ω and increases r if the former is positive, with the opposite being the case if $\omega < 0$. Regarding finally β, its total effect on economic growth is ambiguous. As for its effect on the interest rate, it can be seen that it is always positive if σ is lower or equal to unity. For $\sigma > 1$ though, this effect is rewritten as $\frac{\partial r}{\partial \beta} = \frac{\sigma\left(AL+\frac{1}{\beta}\right) - AL}{D} = \sigma\frac{\partial \omega}{\partial \beta} - \frac{AK+AL}{D}$. With the second part of this expression negative, a non-negative effect of β on ω guarantees that it affects the interest rate positively. If however $\frac{\partial \omega}{\partial \beta} < 0$, the sign of $\frac{\partial r}{\partial \beta}$ can be negative as well, especially if σ is very high.

Appendix B

Appendices of chapter 4

B.1 Derive the reduced general equilibrium

Substituting first (4.29) into (4.28) and using the symmetry property one has

$$Y_t = A_t^{\frac{1}{a}} z_t \ell_{xt} \tag{B.1}$$

while substitution of (4.34) into (4.32) gives for the price of the final output

$$P_t = a^{-1} A_t^{\frac{a-1}{a}} z_t^{-\theta} \tag{B.2}$$

The value of the intermediate firms is next given as

$$V_t = \int_t^{\infty} e^{\int_\mu^t r_v dv} \Pi_\mu d\mu$$

where by Π_t the net cash influx of the intermediate firms is meant. This is given by $\Pi_t = \left(p_t - z_t^{-\theta}\right) x_t - \ell_{zt}$, which by substitution of (4.29) and (4.34) yields

$$\Pi_t = \frac{1-a}{a} \ell_{xt} - \ell_{zt} \tag{B.3}$$

The time derivative of V_t is given by $\dot{V}_t = r_t V_t - \Pi_t$. By equation (4.36) it is $V_t = \frac{1}{\gamma}$ which yields $r_t = \gamma \Pi_t$ that is,

$$r_t = \gamma \left(\frac{1-a}{a} \ell_{xt} - \ell_{zt} \right) \tag{B.4}$$

Substitution next of (B.2) and (B.1) in (4.35) yields

$$r_t = \beta \theta \ell_{xt} \tag{B.5}$$

while log differentiation of (B.2) and use of equation (4.30) gives the growth rate of the price P_t of the final product:

$$\hat{P}_t = \frac{a-1}{a} \hat{A}_t - \beta \theta \ell_{zt} \tag{B.6}$$

where by $\hat{A}_t$ the growth rate of the variety of intermediate products is meant. What equation (B.6) states is that the more the resources that are devoted to R&D (of either type), the more rapidly the price P_t of the final product declines. Substituting next (B.6) into (4.26) and (4.24) one has

$$u_t^{1-\delta} = \delta B \int_t^{\infty} e^{\int_v^t \left(r_\mu + \lambda + \varphi + \frac{a-1}{a} \hat{A}_\mu - \beta \theta \ell_{z\mu} \right) d\mu} dv \tag{B.7}$$

$$\omega_t = r_t - \varepsilon + \lambda - \rho + \frac{1-a}{a} \hat{A}_t + \beta \theta \ell_{zt} \tag{B.8}$$

Equation (B.7) gives the optimal education decision while (B.8) gives the growth rate of per capita consumption, which in equilibrium equals that of per capita final product. The supply-side expression for per capita output growth is given by log-differentiation of (B.1) and— taking into account that Y_t is the total rather than per capita output— it is

$$\omega_t = \frac{1}{a} \hat{A}_t + \beta \theta \ell_{zt} + \hat{\ell}_{xt} - \varepsilon + \lambda \tag{B.9}$$

Dividing next (4.37) by A_t and using (4.31) we get for the firm size s_t

$$s_t = \ell_{xt} + \ell_{zt} + \frac{1}{\gamma}\hat{A}_t \tag{B.10}$$

Finally, by solving the system of (B.4) and (B.5) we get the labour inputs in the production (ℓ_{xt}) and quality improving R&D (ℓ_{zt}) of the intermediate sector as functions of the interest rate alone:

$$\ell_{xt} = \frac{1}{\beta\theta} r_t \tag{B.11}$$

$$\ell_{zt} = \left(\frac{1-a}{a\beta\theta} - \frac{1}{\gamma}\right) r_t \tag{B.12}$$

B.2 Proof of the propositions

B.2.1 Proof of proposition 1

By total differentiation of equations (4.47) to (4.49) one gets

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & -1 & \delta B u^{\delta-1} \\ K & 1-a & L \end{bmatrix} \begin{bmatrix} dr \\ dg \\ du \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 0 \\ a-1 & 1 \end{bmatrix} \begin{bmatrix} d\varepsilon \\ d\lambda \end{bmatrix}$$

where $K = 1 - 2a - \frac{a\beta\theta}{\gamma}$ and $L = a\delta\left(\delta - 1\right) Bu^{\delta-2}$. The solution of the above system is

$$\begin{bmatrix} dr \\ dg \\ du \end{bmatrix} = \frac{1}{D} \begin{bmatrix} -L & L - a\delta Bu^{\delta-1} \\ \delta Bu^{\delta-1}\left(K - a + 1\right) & -\delta Bu^{\delta-1}\left(K+1\right) \\ K - a + 1 & -\left(K+1\right) \end{bmatrix} \begin{bmatrix} d\varepsilon \\ d\lambda \end{bmatrix} \tag{B.13}$$

where

$$D = -L - \delta Bu^{\delta-1}K - \left(1-a\right)\delta Bu^{\delta-1} = -L - \delta Bu^{\delta-1}\left(K + 1 - a\right) \tag{B.14}$$

It is assumed that $K+1-a<0$. A sufficient condition for this to hold is that $a \geq \frac{2}{3}$. Under this assumption it is $D>0$, since L is negative. It also is $K+1=2(1-a)-\frac{a\beta\theta}{\gamma}>(1-a)-\frac{a\beta\theta}{\gamma}>0$. The last inequality stems from equations (B.11) and (B.12) and the assumption that ℓ_x and ℓ_z are both strictly positive.

Thus, g and u are negatively affected by both ε and λ, which effects on r are positive and negative respectively.

Next we have that

$$\frac{dg}{d\varepsilon}=\frac{\delta Bu^{\delta-1}(K-a+1)}{-L-\delta Bu^{\delta-1}(K-a+1)} \Rightarrow \frac{dg}{d\varepsilon} \in (-1,0) \tag{B.15}$$

It is from (4.46) $\hat{A}_t=\varepsilon-\lambda+g$. From the above the negative effect of λ on $\hat{A}_t$ is obvious, while $\frac{d\hat{A}_t}{d\varepsilon}=1+\frac{dg}{d\varepsilon} \Rightarrow \frac{d\hat{A}_t}{d\varepsilon} \in (0,1)$.

Regarding next the firm size s, equation (4.53) is first written as

$$s=\left(\frac{1}{a\beta\theta}-\frac{1}{\gamma}\right)r+\frac{1}{\gamma}\hat{A}_t$$

Since ε affects both r and $\hat{A}_t$ positively and λ affects them negatively, it is obvious that positive is the effect of ε on s while the effect of λ is negative.

From equations (4.51) and (4.52) we have for the per capita growth rate

$$\omega=\frac{1-a}{a}(\varepsilon-\lambda)+\frac{1}{a}g+\left(\frac{1-a}{a}-\frac{\beta\theta}{\gamma}\right)r \tag{B.16}$$

Since λ affects both g and r negatively and $\frac{1-a}{a}-\frac{\beta\theta}{\gamma}$ is positive as was shown above, the negative total effect of λ on ω is straightforward. Regarding the effect of ε, we have

$$\frac{d\omega}{d\varepsilon}=\frac{1-a}{a}+\frac{1}{a}\frac{\partial g}{\partial\varepsilon}+\left(\frac{1-a}{a}-\frac{\beta\theta}{\gamma}\right)\frac{\partial r}{\partial\varepsilon}$$

Substituting $\frac{\partial g}{\partial \varepsilon}$ and $\frac{\partial r}{\partial \varepsilon}$ from (B.13) we have next

$$\frac{d\omega}{d\varepsilon} = \frac{1}{D}\left\{\frac{1-a}{a}D + \frac{\delta}{a}Bu^{\delta-1}\left(K-a+1\right) - L\left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)\right\}$$

where D, K and L were defined above. After a considerable amount of calculations we get the final expression

$$\frac{d\omega}{d\varepsilon} = \frac{1}{D}\delta Bu^{\delta-2}\left\{\left(K+1\right)\left(1-\delta\right) + u\left(K+1-a\right)\right\}$$

which sign can go either way and depends on the values of all parameters of the model.

Finally, it is obvious from (4.50) and (4.51) that the labour allocation variables ℓ_x and ℓ_z are affected by changes in the exogenous variables in the same way the interest rate is affected.

Q.E.D.

B.2.2 Proof of proposition 2

The effects on r, g and u of an equal change of ε and λ are given by adding the two columns of the matrix in (B.13), that is

$$\begin{aligned} dr &= \frac{-a\delta Bu^{\delta-1}}{D} < 0 \\ dg &= \frac{-a\delta Bu^{\delta-1}}{D} < 0 \\ du &= \frac{-a}{D} < 0 \end{aligned}$$

The changes in s and ω are given by

$$\begin{aligned} ds &= \frac{1}{\gamma}dg + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)dr \\ d\omega &= \frac{1}{a}dg + \left(\frac{1-a}{a} - \frac{\beta\theta}{\gamma}\right)dr \end{aligned}$$

and they are negative because both dg and dr are.

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