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Type I String Phenomenology and Extra Dimensional Models

by

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Dedicated to Claudia and my family

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ABSTRACT

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Type I String Phenomenology and Extra Dimensional Models

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We study the phenomenological consequences of type I string constructions and higher-dimensional effective field theories involving Dirichlet-branes with the aim of forging a connection between the underlying string theory and observable low-energy physics. First, we propose a mechanism for mediating supersymmetry (SUSY) breaking in type I string constructions with intersecting D-branes. We consider an explicit example with a Pati-Salam-like gauge symmetry, where only the third family and Higgs scalars acquire large soft masses. We compare the low-energy sparticle spectrum with gaugino mediation and no-scale supergravity models. Second, we use a model-independent parametrisation to study the localised twisted moduli contributions to supersymmetry breaking in the effective low-energy supergravity description of type I models. We derive general expressions for soft masses and trilinears in terms of Goldstino angles that control the relative contributions to supersymmetry breaking from the closed string sector. Finally, we study electroweak symmetry breaking in a five-dimensional effective field theory where supersymmetry is broken on a spatially-separated brane. We evaluate the dominant Kaluza-Klein (KK) contributions to the 1-loop effective potential, and calculate the physical Higgs mass spectrum as a function of $\tan\beta$ and the compactification scale M_C.

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Preface

The work described in this thesis was carried out in collaboration with Prof. S. F. King and Dr. V. Di Clemente. The following list details our original work and gives the references for the material.

- Chapter 2: S. F. King and D. A. J. Rayner, Nucl. Phys. B **607** (2001) 77 [arXiv:hep-ph/0012076].
- Chapter 3: S. F. King and D. A. J. Rayner, arXiv:hep-ph/0111333.
- Chapter 4: V. Di Clemente, S. F. King and D. A. J. Rayner, Nucl. Phys. B **617** (2001) 71 [arXiv:hep-ph/0107290]; *ibid* arXiv:hep-ph/0205010.

No claims to originality is made for the content of Chapter 1 which was compiled using a variety of other sources.

Acknowledgements

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Chapter 1

Introduction

1.1 Preliminaries

1.1.1 Motivation

The work in this thesis explores the connections between superstring theory and low-energy observable physics. There are strong motivations for supersymmetric extensions of the Standard Model that arise naturally in string theory, which in turn, provides the only consistent framework for uniting the four fundamental forces of Nature. There are several general aims to this approach:

- to understand how the Standard Model can be embedded into a higher-dimensional string theory
- to identify the mechanism(s) responsible for supersymmetry breaking
- to make predictions that can be tested at the Tevatron or LHC, which in turn provide constraints on viable models e.g.

– the hierarchy of fermion masses

– the mass spectra of supersymmetric partners

In particular, type I string theory provides an appealing framework to consider low-energy phenomenology due to the flexibility (or rather uncertainty) in defining the fundamental string scale M_*. This is in stark contrast to heterotic theory, where M_* is around an order of magnitide below the Planck scale independently of the details of compactification. The presence of extended Dirichlet-branes in type I (and II) vacua offer many important consequences, most notably, the realisation that gauge and gravity fields can live in different numbers of dimensions. D-brane constructions also have a very rich gauge structure in which to embed the MSSM or some other well-motivated gauge extension. These provide us with strong motivations for studying the phenomenological consequences of type I models.

1.1.2 Thesis Structure

This thesis is organised as follows: in chapter 1 we review the Standard Model (SM) and discuss the motivations for its extension. In particular, we focus on low-energy $\mathcal{N} = 1$ supersymmetry and the Minimal Supersymmetric Standard Model (MSSM). Supersymmetry (SUSY) is an essential ingredient in superstring theories that offer the only consistent framework for combining all four fundamental forces together within a single theory. We review the main features of type I string theory and discuss how developments in string theory have stimulated interest in higher-dimensional effective field theories involving Dirichlet-branes.

In chapter 2, we propose a mechanism for mediating SUSY breaking in type I string models with intersecting D-branes. We consider an explicit construction involving intersecting D5-branes, where only the third family and Higgs scalars couple directly to the SUSY breaking sector to receive large soft masses. We compare the sparticle spectrum of our model with the predictions from gaugino mediation and no-scale supergravity models.

We use a model-independent parametrisation to study the localised twisted moduli contributions to SUSY breaking in chapter 3. We propose a phenomenologically-motivated Kähler potential for states that are sequestered away from the twisted moduli and derive general expressions for the soft masses and trilinears.

In chapter 4, we study a higher-dimensional effective field theory that is motivated by the model discussed in chapter 2. We consider electroweak symmetry breaking (EWSB) in the presence of a single large extra dimension, and use a matrix method to extract the top and stop Kaluza-Klein spectra that yield the dominant contribution to the 1-loop effective potential. We calculate the Higgs mass spectrum as a function of $\tan\beta$ and the compactification scale of the extra dimension M_C. We find that the standard MSSM bounds on the lightest Higgs scalar mass can be violated when the third family lives in the extra dimension.

The overall conclusions to this thesis are presented in chapter 5, which is followed by a number of Appendices.

1.2 The Standard Model

The Standard Model [1] (SM) of particle physics combines the strong and electroweak forces within the framework of a renormalisable gauge field theory with a gauge group $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$. There are three generations of elementary fermions, where each generation contains a family of quarks and leptons. Table 1.1 lists the gauge quantum numbers of these fields under the gauge group G_{SM}. Demanding that the lagrangian is gauge-invariant generates interactions between fermions

[1]There are many excellent introductions to the SM, see Refs. [1, 2] for example.

that are mediated by the exchange of intermediate vector bosons.

Particles	Spin	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Left-handed quarks, $Q_{iL} \equiv (u_{iL}\ d_{iL})$	1/2	**3**	**2**	1/6
Right-handed up quarks, u_{iR}	1/2	**3**	**1**	2/3
Right-handed down quarks, d_{iR}	1/2	**3**	**1**	-1/3
Left-handed leptons, $L_{iL} \equiv (\nu_{iL}\ e_{iL})$	1/2	**1**	**2**	-1/2
Right-handed electrons, e_{iR}	1/2	**1**	**1**	-1
Higgs boson, $\Phi \equiv (\Phi^+\ \Phi^0)$	0	**1**	**2**	1/2
Gluons, g^α $(\alpha = 1-8)$	1	**8**	**1**	0
Weak bosons, W^a $(a = 1-3)$	1	**1**	**3**	0
Hypercharge boson, B	1	**1**	**1**	0

Table 1.1: G_{SM} gauge quantum numbers of the fields in the SM. Notice that the left (right) handed components transform as doublets (singlets) under $SU(2)_L$. The quark and lepton fields carry an additional generation (or family) index $i = 1, 2, 3$ e.g. $u_i = (u, c, t)$, $d_i = (d, s, b)$, $e_i = (e, \mu, \tau)$ and $\nu_i = (\nu_e, \nu_\mu, \nu_\tau)$.

Gauge symmetry forbids gauge boson and fermion (Dirac) mass terms in the lagrangian [2] since left and right-handed fields transform in different representations of $SU(2)_L$. However, we are free to construct the following gauge-invariant Yukawa interaction term by adding a fundamental Higgs scalar doublet Φ to the SM spectrum:

$$-\mathcal{L}_{yuk} = Y^u_{ij}\, Q^\dagger_{iL} \Phi^c u_{jR} + Y^d_{ij}\, Q^\dagger_{iL} \Phi d_{jR} + Y^e_{ij}\, L^\dagger_{iL} \Phi e_{jR} + h.c. \tag{1.1}$$

where $\Phi^c = i\,\sigma_2\,\Phi^*$. Note that gauge indices have been suppressed, and Y^{ude}_{ij} are 3×3 Yukawa matrices in generation space. We propose that the Higgs scalar has a potential that develops a non-trivial vacuum expectation value (VEV) which spontaneously

[2]However, neutrinos are massless in the SM since there are no right-handed neutrinos ν_R to form Dirac masses, $m_D \sim \nu^\dagger_L \nu_R + h.c.$. Also, Majorana mass terms violate $SU(2)_L \times U(1)_Y$ gauge-symmetry and are forbidden by conservation of lepton number.

breaks the electroweak gauge symmetry down to electromagnetism

$$SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em} \tag{1.2}$$

and generates Dirac mass terms from the Yukawa interactions in Eq.(1.1).

Consider the following lagrangian for the complex Higgs doublet:

$$\mathcal{L}_{higgs} = (D^\mu \Phi)^\dagger \, (D_\mu \Phi) - m_\Phi^2 \, \Phi^\dagger \Phi - \lambda_\Phi \left(\Phi^\dagger \Phi\right)^2 \tag{1.3}$$

and the covariant derivative is given by:

$$D_\mu \Phi = \left(\partial_\mu - i\frac{g'}{2} B_\mu - igW_\mu^a \tau^a\right) \Phi \tag{1.4}$$

where g' and g are the $U(1)_Y$ and $SU(2)_L$ gauge couplings, and $\tau^a = \sigma^a/2$ are the generators of $SU(2)_L$. The Higgs potential in Eq.(1.3) has a symmetry breaking minimum away from the origin at $\langle \Phi^\dagger \Phi \rangle = -m_\Phi^2/2\lambda_\Phi$. Using our freedom to make a global $SU(2)_L$ rotation, we can choose to locate the real VEV in the lower, neutral component of the Higgs doublet

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \qquad where \quad v = \sqrt{\frac{-m_\Phi^2}{\lambda_\Phi}} \tag{1.5}$$

Expanding around this vacuum, we make the redefinition:

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.6}$$

where $h(x)$ is the physical Higgs field. The remaining three degrees of freedom from the original complex Higgs scalar doublet have been "eaten" by the gauge fields associated with the broken electroweak generators to give them mass. We can see these mass terms explicitly by expanding Eq.(1.3) around the symmetry breaking minimum:

$$(D^\mu\Phi)^\dagger\,(D_\mu\Phi) \supset \frac{g^2v^2}{8}\left(W^{1\mu}W_{1\mu} + W^{2\mu}W_{2\mu}\right) + \frac{v^2}{8}\left(-gW^{3\mu} + g'B^\mu\right)^2 + \ldots \quad (1.7)$$

where $W_\mu^{1,2}$ combine to form a charged mass eigenstate $W_\mu^\pm = (W_\mu^1 \mp iW_\mu^2)/\sqrt{2}$ with a mass $M_W = gv/2$. Notice that W_μ^3 and B_μ are mixed together by EWSB, and the mass eigenstates are given by:

$$\begin{pmatrix} Z_\mu^0 \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \quad (1.8)$$

where $\tan\theta_W = g'/g$ is the weak mixing angle. The photon linear combination A_μ is still massless since electromagnetism is unbroken, while the neutral weak boson Z_μ^0 has obtained a mass $M_Z = \sqrt{g^2 + g'^2}\,v/2$. We find that the observable electromagnetic charge is related to the weak and hypercharge gauge couplings by the relation:

$$e = g'\,\cos\theta_W \quad (1.9)$$

Substituting the Higgs scalar VEV $\langle\Phi\rangle$ into Eq.(1.1), we can generate the following fermion mass matrices:

$$m_{ij}^{u,d,e} = \frac{1}{\sqrt{2}} Y_{ij}^{u,d,e}\,v \quad (1.10)$$

In general, $m^{u,d}$ will not be diagonal and so quarks from different generations can mix together. We can extract the physical masses (in the mass basis) by making a unitary transformation on the weak eigenstates $(u_{iL}, d_{iL}, u_{iR}, d_{iR})$ that will diagonalise the mass matrices. For example, consider the transformations between the weak and mass bases:

$$\begin{aligned} u_{iL} &\longrightarrow u'_{iL} = (V_L^u)_{ij}\,u_{jL} \qquad & u_{iR} &\longrightarrow u'_{iR} = (V_R^u)_{ij}\,u_{jR} \\ d_{iL} &\longrightarrow d'_{iL} = (V_L^d)_{ij}\,d_{jL} \qquad & d_{iR} &\longrightarrow d'_{iR} = (V_R^d)_{ij}\,d_{jR} \end{aligned} \quad (1.11)$$

where $V^{u,d}_{L,R}$ are unitary and diagonalise the quark mass matrices:

$$\begin{aligned} m^u_{diag} &= V^u_L\, m^u\, V^{u\dagger}_R \\ m^d_{diag} &= V^d_L\, m^d\, V^{d\dagger}_R \end{aligned} \tag{1.12}$$

The combination $V_{CKM} = V^u_L V^{d\dagger}_L$ is the unitary Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix [3] and the standard parametrisation involves three physical mixing angles and a single phase that can lead to CP violation. The CKM matrix elements are observable in weak charged current processes, and V_{CKM} is found to be highly diagonal.

1.2.1 Successes of the Standard Model

The SM has been rigorously tested at high-energy accelerators as illustrated by the amount of high precision data collected by the Particle Data Group [4]. Measurements of the Z^0 width constrain the number of (active) neutrinos with masses $m_\nu \leq M_Z/2$ to be three. Hypercharge gauge anomalies arising from triangle graphs involving internal loops of fermions are found to be exactly cancelled within each complete generation of quarks and leptons [3]. Taken together, we conclude that there must be three complete generations in the SM to maintain consistency.

The quark model of mesons and baryons invokes approximate flavour symmetries to successfully predict the light hadronic spectra of bound quark states. The unitarity of the CKM matrix offers a method called the Glashow-Illiopoulos-Maiani (GIM) mechanism [5] to suppress rare flavour-changing processes. In fact, the GIM mechanism predicted the charm quark prior to its experimental discovery.

The Higgs mechanism for EWSB explains how the weak bosons acquire mass, and

[3] This criteria will prove important when we decide to supplement the matter content of the SM.

therefore the short-range nature of the weak force. We can make perturbative calculations of physical observables to test against experiment, and we understand the renormalisation group equation (RGE) evolution of physical parameters with energy scale. Indeed, we can understand the confinement of quarks inside hadrons from the infra-red behaviour of the $SU(3)_C$ gauge group.

1.2.2 Unanswered questions in the SM

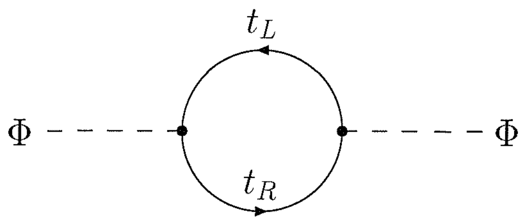
Despite the vast amount of experimental data, there is increasing observational and theoretical motivation for extending the SM which must be incomplete since gravity is not included. Quantum gravity effects are anticipated to appear around the Planck scale $M_{Pl} \sim 10^{19}$ GeV, but this leaves a vast "energy desert" above the electroweak scale where "new physics" can appear.

• Neutrino masses and the fermion hierarchy

The most significant experimental evidence for new physics *beyond the Standard Model* is the observation of neutrino oscillations [6] which imply that neutrinos must have non-zero (albeit very small) masses. However, there are no right-handed neutrino states in the SM and so we cannot form Dirac masses or Majorana masses since the SM also conserves lepton number. An obvious extension is to trivially include right-handed neutrinos ν_R. Unfortunately, this offers no explanation why neutrino masses are so small (and similarly the observed hierarchy in the other fermion masses) and thus a more appealing approach is to embed the SM in a deeper theory where masses and couplings could be predicted *a priori*.

• Hierarchy problem

Prior to EWSB, fermions and gauge bosons are massless since mass terms explicitly violate gauge-invariance. However, there is no symmetry that protects the Higgs boson mass m_Φ from receiving large radiative corrections of order the cutoff scale Λ_{UV}, e.g. $\delta m_\Phi \sim \Lambda_{UV}$. Figure 1.1 shows the dominant top-quark radiative correction to the Higgs mass.



$$\delta m_\Phi^2 \approx \frac{y_t^2}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_t^2 \ln\left(\frac{\Lambda_{UV}}{m_t}\right)\right] + \ldots$$

Figure 1.1: The dominant top quark 1-loop correction to the Higgs mass is found to have a leading quadratic divergence.

Suppose that this cutoff is associated with the Planck scale M_{Pl}, then the Higgs boson mass correction would involve Planck-scale particles in virtual loops that would push the Higgs mass towards M_{Pl}. However, we know that EWSB occurs at the weak scale $\mathcal{O}(M_W)$ which begs the question how the hierarchy between these two different scales is stabilised. A possible solution is to fine-tune the tree-level mass and 1-loop corrections such that the sum is around the weak scale, but this seems highly unnatural. There is also no explanation of how the Higgs potential develops an instability (i.e. $m_\Phi^2 < 0$) in the SM.

- **Gauge unification problems**

Most theorists would agree that it would be desirable if the SM could be embedded into a deeper theory, perhaps uniting the three SM gauge groups within a grand unified theory (GUT) with a single gauge coupling e.g. $SU(5)$ or $SO(10)$. GUTs are inherently more predictive and right-handed neutrino states arise naturally [4].

[4] In Appendix A, we will discuss a supersymmetric Pati-Salam GUT where lepton number is like

However, when the observed low-energy gauge coupling constants $\alpha_i(M_Z)$ are extrapolated up to high energies through RGE running, we find that they do not unify at a single value (using the matter content of the SM) as shown in Figure 1.2.

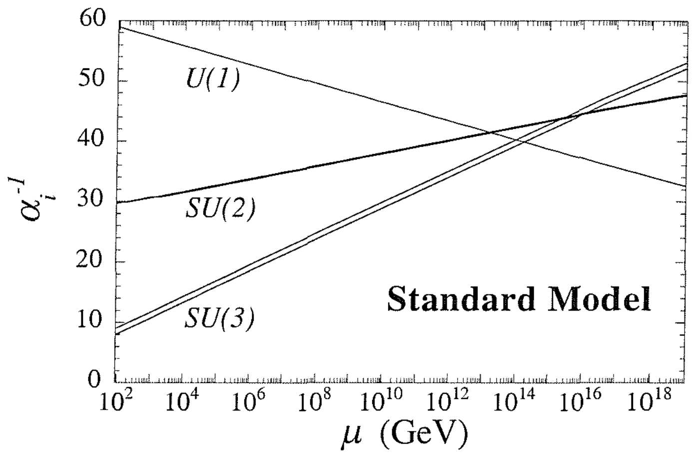


Figure 1.2: The renormalisation group equation (RGE) running of gauge coupling constants α_i as a function of the renormalisation scale μ in the SM. Notice that the gauge couplings do not meet at a single point.

There are many well-motivated solutions to these problems including supersymmetry, grand unification, superstring theory and extra-dimensions that we will review in subsequent sections.

1.3 Supersymmetry

In this section, we aim to highlight the important features of phenomenologically-viable low-energy supersymmetric models. However, there are plenty of excellent reviews that provide further technical details [8, 9, 10, 11].

Supersymmetry (SUSY) unites bosons and fermions through some underlying symmetry. It non-trivially combines spacetime Poincare symmetries with internal sym-

a "fourth colour" [7] and right-handed neutrinos ν_{iR} arise in the same multiplets as u_{iR}, d_{iR} and e_{iR}.

metries, and extends ordinary spacetime into *superspace* by including anti-commuting coordinates $(\theta, \bar{\theta})$ along with the usual (commuting) spacetime coordinates (x_μ). Indeed, a local (or gauged) theory of SUSY is called *supergravity* since it includes general coordinate transformations to offer a theory of gravity. SUSY is also an essential ingredient in superstring models that (so far) provide the only consistent framework for combining all four fundamental forces.

1.3.1 Minimal Supersymmetric Standard Model

The minimal supersymmetric extension of the SM (MSSM) adds a fermion (boson) superpartner for each boson (fermion) particle in Table 1.1. This immediately offers a potential solution to the hierarchy problem shown in Figure 1.1 since an additional diagram involving the scalar partner of the top quark contributes to δm_Φ^2 to soften the quadratic divergence into a logarithm, provided the mass-splitting is around $\mathcal{O}(\text{TeV})$. However, the fermionic Higgsino partner of the Higgs scalar will couple to the hypercharge boson and re-introduces a triangle anomaly. Hence, a second Higgsino (and Higgs boson) with opposite hypercharge is required to cancel this anomaly. We label these two Higgs doublets H_u and H_d since we will find that they couple to fields with different weak isospin component I_3. The extended matter content of the MSSM is given in Table 1.2.

It is convenient to work in terms of chiral (and vector) superfields which unite scalars with fermions (and fermions with vector bosons) within a single field. For instance, a chiral superfield can be expanded in superspace notation into component fields [5]:

$$\begin{aligned}\Psi(x_\mu, \theta, \bar{\theta}) = \phi(x) + \sqrt{2}\,\theta\psi(x) + \theta\theta\, F(x) + i\partial_\mu\phi(x)\,\theta\sigma^\mu\bar{\theta} \\ -\frac{i}{\sqrt{2}}\theta\theta\,\partial_\mu\psi(x)\,\sigma^\mu\bar{\theta} - \frac{1}{4}\partial_\mu\partial^\mu\phi(x)\,\theta\theta\,\bar{\theta}\bar{\theta}\end{aligned} \tag{1.13}$$

[5] We will consistently suppress the spinorial indices on ψ and θ. See for example, Refs. [10, 11] for technical details about spinor algebra.

Particles		Spin 0	Spin 1/2	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
squarks, quarks	Q_{iL}	$(\tilde{u}_{iL}\ \tilde{d}_{iL})$	$(u_{iL}\ d_{iL})$	$\mathbf{3}$	$\mathbf{2}$	1/6
	U^c_{iR}	$\tilde{u}^*_{iR}$	$u^\dagger_{iR}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	-2/3
	D^c_{iR}	$\tilde{d}^*_{iR}$	$d^\dagger_{iR}$	$\overline{\mathbf{3}}$	$\mathbf{1}$	1/3
sleptons, leptons	L_{iL}	$(\tilde{\nu}_{iL}\ \tilde{e}_{iL})$	$(\nu_{iL}\ e_{iL})$	$\mathbf{1}$	$\mathbf{2}$	-1/2
	E^c_{iR}	$\tilde{e}^*_{iR}$	$e^\dagger_{iR}$	$\mathbf{1}$	$\mathbf{1}$	1
Higgs, Higgsinos	H_u	$(H_u^+\ H_u^0)$	$(\tilde{H}_u^+\ \tilde{H}_u^0)$	$\mathbf{1}$	$\mathbf{2}$	1/2
	H_d	$(H_d^0\ H_d^-)$	$(\tilde{H}_d^0\ \tilde{H}_d^-)$	$\mathbf{1}$	$\mathbf{2}$	-1/2
Particles		Spin 1/2	Spin 1	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
gluino, gluon	(α=1-8)	$\tilde{g}^\alpha$	g^α	$\mathbf{8}$	$\mathbf{1}$	0
winos, W bosons	(a=1-3)	$\tilde{W}^a$	W^a	$\mathbf{1}$	$\mathbf{3}$	0
bino, B boson		$\tilde{B}$	B	$\mathbf{1}$	$\mathbf{1}$	0

Table 1.2: G_{SM} gauge quantum numbers of the matter content of the MSSM. We have taken the CP-conjugate of all right-handed singlet fields to change them into left-handed fields.

where ϕ is a complex scalar (e.g. squark, slepton or Higgs boson); ψ is a left-handed 2-component Weyl fermion (e.g. quark, lepton or Higgsino) and $\sigma^\mu \equiv (\mathbf{1}_2\,,\,\sigma^i)$ with the Pauli matrices σ^i . Notice that we have included a complex scalar *auxiliary* field F that will allow us to close the SUSY algebra off-shell and helps us keep track of degrees of freedom. However F is a non-dynamical field which can be eliminated using its classical equation of motion. The conjugate superfield $\Psi^\dagger$ expansion is found by taking the hermitian conjugate of Eq.(1.13) where the Weyl fermion is right-handed. We will refer to Ψ ($\Psi^\dagger$) as a left (right) superfield since they involve left-handed (right-handed) Weyl spinors respectively. Superfields are usually distinguishable from their components as shown in Table 1.2, but in the case of $SU(2)_L$ doublets, it is convenient to use the same symbol for the superfield and its SM component, e.g. Q_{iL} represents both the quark doublet superfield and the quark doublet $(u_{iL}\ d_{iL})$, whereas the squark doublet is denoted by $\tilde{Q}_{iL} = (\tilde{u}_{iL}\ \tilde{d}_{iL})$.

Similarly, vector superfields V can be decomposed [6] into gauge bosons A_μ, 2-component Weyl fermion gauginos λ and another auxiliary field D:

$$V(x_\mu, \theta, \bar{\theta}) = \theta\sigma^\mu\bar{\theta}\, A_\mu(x) + i\theta\theta\,\bar{\theta}\lambda^\dagger(x) - i\bar{\theta}\bar{\theta}\,\theta\lambda(x) + \frac{1}{2}\theta\theta\,\bar{\theta}\bar{\theta}\,D(x) \tag{1.14}$$

Under infinitessimal (global) SUSY transformations, we find that fermions and bosons transform into each other as expected, while the auxiliary fields transform into total-derivatives that vanish in the action. This provides a simple method of constructing SUSY-invariant lagrangians by extracting the coefficients of $\theta\theta$ (F-terms) and $\theta\theta\,\bar{\theta}\bar{\theta}$ (D-terms) from a (gauge-invariant) combination of superfields.

In the MSSM, we generalise the Yukawa couplings of Eq.(1.1) so that supersymmetric interactions are determined by a gauge-invariant, holomorphic superpotential of chiral superfields [7]:

$$W_{MSSM} = Y^u_{ij}\, Q_{iL} H_u U^c_{jR} + Y^d_{ij}\, Q_{iL} H_d D^c_{jR} + Y^e_{ij}\, L_{iL} H_d E^c_{jR} - \mu H_u H_d \tag{1.15}$$

where masses are generated after EWSB when $H_{u,d}$ acquire VEVs as given in Eq.(1.17). The supersymmetric Higgs mass parameter μ plays an important rôle in obtaining the correct EWSB VEVs and must be stabilised around $\mu \sim \mathcal{O}(M_Z) \ll M_{Pl}$. Models have been proposed that generate the small μ-term - these include the next-to-minimal supersymmetric Standard Model (NMSSM) which introduces an additional gauge singlet superfield [12, 13] and also the Giudice-Masiero mechanism [14]. There are other renormalisable, gauge-invariant combinations of chiral superfields from Table 1.2 that

[6] We are working in the so-called Wess-Zumino gauge, where non-physical degrees of freedom have been gauged away.

[7] Renormalisable terms involve up to three chiral superfields, although non-renormalisable interactions can be constructed with higher-dimensional operators suppressed by powers of some mass scale.

can be included in the MSSM superpotential

$$W_{RPV} = \frac{1}{2}\lambda_{ijk}\, U^c_{iR} D^c_{jR} D^c_{kR} + \frac{1}{2}\lambda'_{ijk}\, L_{iL} L_{jL} E^c_{kR} \\ + \lambda''_{ijk}\, L_{iL} Q_{jL} D^c_{kR} + \mu'_i\, L_{iL} H_u \tag{1.16}$$

where gauge indices have been suppressed. However, the first term leads to baryon number violation, while the last three terms violate total lepton number. Proton lifetime measurements severely constrain these interactions since tree-level diagrams can be constructed where squarks mediate very rapid proton decay. In the MSSM, we often invoke a new discrete symmetry called "R-parity" that forbid the superpotential terms in Eq.(1.16). This symmetry has important phenomenological implications since it predicts that the lightest supersymmetric particle (LSP) is stable against decay into ordinary SM particles and, if electrically neutral, it offers a potential Dark Matter candidate.

Electroweak symmetry is broken when the neutral components of the two Higgs scalar doublets acquire VEVs

$$\langle H_u \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_u \end{pmatrix} \quad , \quad \langle H_d \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v_d \\ 0 \end{pmatrix} \tag{1.17}$$

where we know $v_u^2 + v_d^2 = v^2 \approx (246\,\mathrm{Ge}V)^2$ from measurements of the Fermi constant G_F. Notice that we do not know the ratio between VEVs given by $\tan\beta = v_u/v_d$, and look to experiment to constrain the parameter space.

SUSY cannot be an exact symmetry in Nature since we have not observed degenerate-mass superpartners at high-energy accelerators. However, we do not know the mechanism(s) responsible for breaking SUSY, although many models have been proposed. The allowed parameter space is only weakly constrained at present, and we must surely wait for the observation of sparticles at the Tevatron or LHC to identify viable models. As discussed earlier, SUSY offers a solution to the hierarchy problem provided the sparticle masses lie around the TeV scale. In the absence of experimental data, we assume that the SUSY breaking parameters in the *soft* lagrangian [8] $\mathcal{L}_{soft}$ are generated from some underlying theory (e.g. supergravity or string theory):

$$\begin{aligned}
-\mathcal{L}_{soft}^{MSSM} &= \frac{1}{2}\left(M_1\tilde{B}\tilde{B} + M_2\tilde{W}\tilde{W} + M_3\tilde{g}\tilde{g}\right) + h.c. \\
&\quad + A_{ij}^u\,\tilde{u}_{iR}^*\tilde{Q}_{jL}H_u - A_{ij}^d\,\tilde{d}_{iR}^*\tilde{Q}_{jL}H_d - A_{ij}^e\,\tilde{e}_{iR}^*\tilde{L}_{jL}H_d + h.c. \\
&+\tilde{Q}_{iL}^\dagger m_{Q_L}^2\tilde{Q}_{jL} + \tilde{L}_{iL}^\dagger m_{L_L}^2\tilde{L}_{jL} + \tilde{u}_{iR}^* m_{U_R^c}^2\tilde{u}_{jR} + \tilde{d}_{iR}^* m_{D_R^c}^2\tilde{d}_{jR} + \tilde{e}_{iR}^* m_{E_R^c}^2\tilde{e}_{jR} \\
&\qquad + m_{H_u}^2 H_u^* H_u + m_{H_d}^2 H_d^* H_d - (B\mu H_u H_d + h.c.)
\end{aligned} \tag{1.18}$$

where M_i are gaugino masses, and the trilinears A_{ij} and squark/slepton masses m^2 are 3×3 matrices in generation space. However, these parameters can have complex entries (although the mass matrices must be hermitian) which introduces 105 additional masses, phases and mixing angles that cannot be rotated away.

Using this soft lagrangian and the superpotential, we can calculate the physical mass spectra of the superpartners by diagonalising the lagrangian to extract the squark and slepton masses. The two complex Higgs scalars contain 8 real degrees of freedom, of which 3 Goldstone modes are eaten by the $W^\pm, Z^0$ weak bosons to give them mass, and the remaining 5 physical degrees comprise the physical Higgs sector of the MSSM:

- $H^\pm$ - a charged Higgs boson pair
- A^0 - a CP-odd, neutral Higgs boson
- H^0, h^0 - two CP-even, neutral Higgs bosons

After EWSB, the (observable) mass eigenstates are given by linear combinations of gauge eigenstates since fields with identical quantum numbers can mix together. For

[8] Notice that terms in $\mathcal{L}_{soft}$ must not re-introduce quadratic divergences [15] and spoil the solution of the hierarchy problem.

instance, the charged Higgsinos and gauginos mix to form charginos:

$$\tilde{H}_u^+ \, , \, \tilde{H}_d^- \, , \, \tilde{W}^1 \, , \, \tilde{W}^2 \longrightarrow \tilde{\chi}_1^\pm \, , \, \tilde{\chi}_2^\pm \tag{1.19}$$

and the neutral Higgsinos mix with the neutral gauginos to form neutralinos:

$$\tilde{H}_u^0 \, , \, \tilde{H}_d^0 \, , \, \tilde{W}^3 \, , \, \tilde{B} \longrightarrow \tilde{\chi}_1^0 \, , \, \tilde{\chi}_2^0 \, , \, \tilde{\chi}_3^0 \, , \, \tilde{\chi}_4^0 \tag{1.20}$$

where the lightest neutralino $\tilde{\chi}_1^0$ is often the LSP.

1.3.2 Successes and motivations for low-energy SUSY

Supersymmetry offers solutions to many of the problems in the SM and there is strong theoretical motivation for pursuing supersymmetric extensions of the SM.

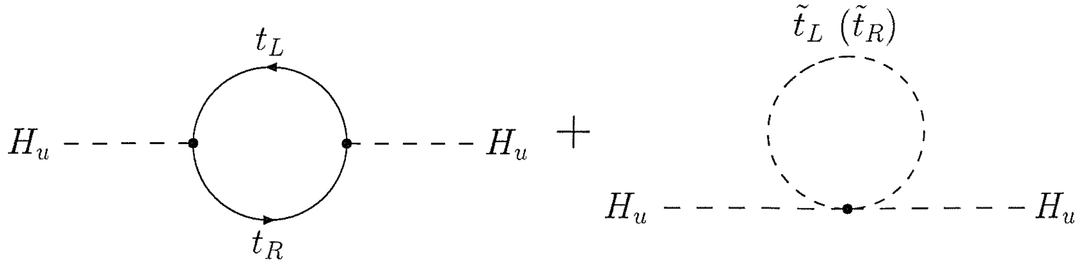
- **Hierarchy problem**

As shown in Figure 1.1, the Higgs scalars are not protected in the SM from acquiring quadratically-divergent radiative corrections from virtual loops of Planck-scale particles. However, SUSY stabilises the hierarchy between the electroweak and Planck scales by contributing sparticle loops for every particle loop that *softens* the quadratic divergence into a logarithmic divergence as shown in Figure 1.3.

This supersymmetric cancellation avoids the problem of fine-tuning provided that SUSY is broken around the TeV scale.

- **Radiative electroweak symmetry breaking**

Supersymmetry can also provide an explanation for the mysterious Higgs mechanism and the origin of its tachyonic mass. For example, in supergravity models with universal soft parameters at the GUT scale, we find that $m_{H_u}^2$ receives large radiative



$$\delta m_{H_u}^2 \approx -\frac{6y_t^2}{16\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\Lambda_{UV}}{m_{\tilde{t}}}\right) + \ldots$$

Figure 1.3: The dominant top and stop 1-loop corrections to the Higgs mass, where the additional stop contribution cancels the quadratic divergence. In the limit that SUSY is preserved ($m_{\tilde{t}} = m_t$) the Higgs mass would be 1-loop finite.

corrections from top and stop loops that do not exactly cancel. Extrapolating to low energies using RGEs, we see that $m_{H_u}^2$ is driven negative at the weak scale which triggers EWSB [16].

- **Gauge coupling unification**

We have already shown that we do not obtain gauge coupling unification in the SM. However, the additional sparticles with masses around the TeV scale carry gauge quantum numbers and therefore modify the RGE running to yield approximate unification at a scale $M_X \approx 2 \times 10^{16}$ GeV as shown in Figure 1.4.

This unification is consistent with embedding the MSSM within a supersymmetric GUT group that can push the potentially dangerous proton decay rate *above* experimental lower bounds.

1.3.3 Unanswered questions in the MSSM

However, our understanding of low-energy SUSY is not complete since there are still many unanswered questions. One of the main concerns is the huge number of

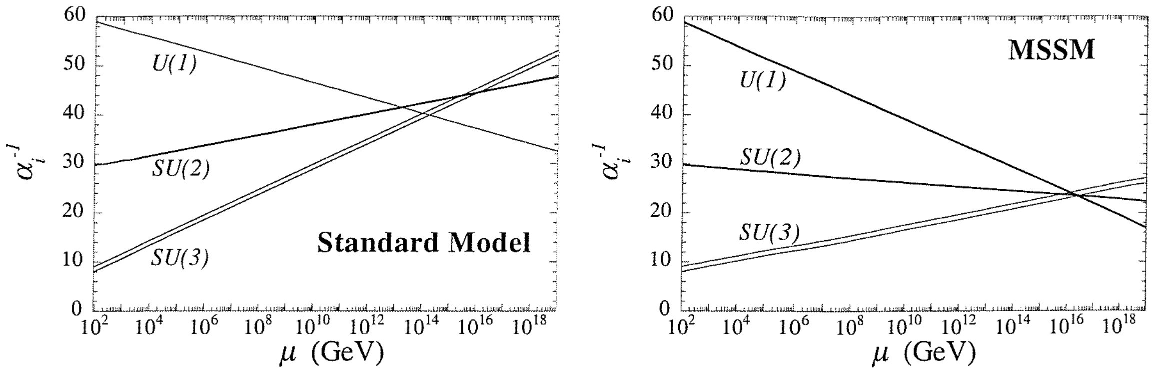


Figure 1.4: The RGE extrapolation of gauge coupling constants α_i to high energies in the SM and MSSM. Notice how additional sparticle loops in the MSSM modify the evolution of the gauge couplings which now meet meet at a unification scale of $M_{GUT} \simeq 2 \times 10^{16}$ GeV.

(arbitrary) parameters in the MSSM - there are 124 masses, phases and mixing angles (excluding neutrino masses) compared to the 19 parameters in the SM. In the absence of sparticle data, we cannot identify the mechanism(s) responsible for SUSY breaking.

In order to make progress, we assume that the MSSM arises from an underlying theory where these free parameters are inter-related by some (unknown) symmetries, e.g. supergravity, SUSY GUTs or string theory. For example, there has been extensive study of the phenomenology of minimal supergravity (mSUGRA) models where SUSY is broken in a "hidden sector" that is coupled through gravitational interactions to SM particles [17]. In these models, the soft parameters in Eq.(1.18) are unified at the Planck scale, i.e. universal gaugino masses $M_{1/2}$; squark, slepton and Higgs scalar masses M_0^2; and trilinears A_0. These high-scale parameters provide upper boundary conditions for running RGEs down to the weak scale to extract the physical low-energy sparticle spectra. Another variant is called "no-scale supergravity" [18] which assumes that $M_{1/2} \gg M_0$, A_0 and leads to characteristic experimental signatures such as very light right-handed sleptons. However, SUSY breaking can also be communicated to the visible SM fields via an intermediate non-SM gauge sector [19, 20].

Developments in superstring theory involving Dirichlet-branes and large extra dimensions offer exciting new possibilities for SUSY breaking since the hidden sector can now be physically separated or sequestered from the SM fields along an extra dimension(s). Parallel brane constructions also form the basis of the Horava-Witten model in strongly-coupled heterotic string theory [21] which in turn motivated the formulation of anomaly [22, 23] and gaugino [24, 25] mediated SUSY breaking effective field theory models.

We expect to observe sparticles at the Tevatron and/or LHC within the next ten years. However, experimental sensitivity is at the level where mSUGRA effects are (in principle) accessible in current experiments. For example, off-diagonal squark and slepton mass matrix elements cause flavour violation at 1-loop that is measurable in $K^0 - \overline{K}^0$ mixing, $b \to s\gamma$ and $\mu \to e\gamma$ processes. Flavour-changing neutral-current (FCNC) data imposes very strong constraints on these off-diagonal mass matrix elements [26], and electric dipole moment observations constrain CP-violating phases in soft parameters [27].

We have shown that supersymmetry is a very powerful framework and offers many solutions to SM problems. However, we still need to consider how the MSSM can be embedded within a deeper theory such as superstring theory which is the focus of the next section.

1.4 Superstring Theory and Extra Dimensions

In this section, we aim to review the status of superstring phenomenology. There are many excellent introductions and reviews of superstring theory that can be consulted for technical details [10, 28, 29].

String theories offer the only consistent framework for unifying all four fundamental forces of Nature together. Matter fermions, Higgs scalars, gauge bosons and gravity fields are all represented as different vibrational modes of microscopic strings of length $\sim 1/M_*$, where M_* is the fundamental string scale. Strings can have their ends free (open) or tied together to form a loop (closed), where a closed string is often regarded as a bound state of two open strings. Closed strings are found to possess a spin 2 resonance representing a graviton which is the proposed mediator of gravitational interactions. Supersymmetric strings (or superstrings) offer the most promising models and string phenomenology [30] is the study of how to embed the MSSM within string theory.

1.4.1 String Revolutions and Dualities

String theory has undergone a series of important developments or *revolutions*. The first revolution occurred in 1984 when Green and Schwarz [31] constructed the 10d anomaly-free type I string theory involving open and closed strings with a gauge group $SO(32)$. This was followed by the formulation of the two 10d heterotic closed string theories that combined bosonic strings in 26d with the 10d supersymmetric Green-Schwarz theory to give a gauge group $SO(32)$ or $E_8 \times E_8$ [32]. Heterotic and type I both exhibit $\mathcal{N} = 1$ spacetime supersymmetry and contain very large gauge groups which can easily accommodate the MSSM. Type II models with $\mathcal{N} = 2$ SUSY in 10d were later developed, giving a total of five consistent, perturbative superstring theories that are all candidates for containing the MSSM. However, these models contain a plethora of exotic matter outside the MSSM states and the extra dimensions must be compactified from 10d to 4d in order to forge a connection with observable low-energy physics.

The second string revolution occurred in 1995 with the discovery of weak/strong coupling dualities [34]. As shown in Figure 1.5, dualities are found to unite the five superstring theories and 11d supergravity as different perturbative limits of an underlying 11d *M-theory* [35]. Another important development was the discovery

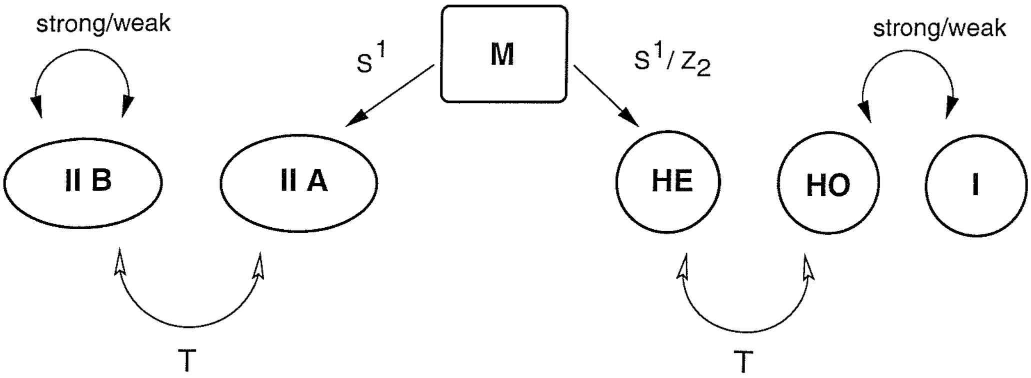


Figure 1.5: The web of dualities that inter-relate the five string theories with 11d M-theory. HE and HO refer to the two heterotic theories with the gauge groups $E_8 \times E_8$ and $SO(32)$. This figure is taken from Ref. [33].

of extended, solitonic objects called Dirichlet-branes [33, 36, 37] in type I and II vacua. Dp-branes span a $(p+1)$ sub-manifold of the full 10d spacetime and they are identified as the hyper-surfaces where open strings end. These open strings have Dirichlet boundary conditions in the $(9-p)$ coordinates transverse to the Dp-brane, and Neumann conditions inside the world-volume of the brane. Open strings attached to a given Dp-brane have Kaluza-Klein (winding) states along the compact dimensions with Neumann (Dirichlet) boundary conditions. A coincident stack of N D-branes generates a $U(N)$ gauge group [9] and open strings carry Chan-Paton gauge quantum numbers for the gauge groups of the branes to which they are attached. Gravity fields arise as closed strings, so confining gauge fields to D-branes raises the (hitherto) unexplored possibility that gauge and gravity fields can live in different numbers of dimensions in type I and II theory. These discoveries ended the heterotic monopoly in string phenomenology and made other corners of the M-theory moduli space accessible to study.

[9] Imposing a mirror symmetry Ω (which is called "orientifolding" [38]), or groups of discrete symmetries Ωg, can generate alternative SO and Sp gauge groups.

1.4.2 Type I Strings

In this section we will review type I string theory which has been the focus of intense study recently [39, 40] and many semi-realistic models have already been constructed [41]-[55]. Perturbative type I models are particularly interesting since they are dual to strongly-coupled heterotic constructions.

We can understand 10d $SO(32)$ Type I string theory as an "orientifold" [38],[56]-[61] - or Z_2 projection - of type IIB theory in 10d. This projection is identified with the parity operation Ω on the surface swept out by the string (world-sheet) that exchanges left and right-moving vibrations. It projects out states that are not invariant under Ω and breaks $\mathcal{N} = 2$ SUSY down to $\mathcal{N} = 1$. This results in an (unoriented) type I closed string theory in 10d [10]. However, orientifolding leads to string amplitudes containing tadpole divergences that must be cancelled for a consistent theory. The solution is to add states which are "twisted" with respect to Ω - and these are simply type I open strings with their ends attached to D-branes [11]. We find that we require 32 D9-branes (with world-volumes filling the whole 10d spacetime) in the vacuum, where the attached open strings give rise to massless 10d gauge fields transforming in the adjoint of $SO(32)$.

We need to generalise this orientifolding technique to compactify six dimensions while leaving a single unbroken supersymmetry in 4d [12]. We will consider type IIB 4d orientifolds [41]-[50] obtained by compactifying six dimensions on a six-torus $T^6 \equiv T^1 \times T^2 \times T^3$, where each pair of compactified dimensions is wrapped around a symmetric two-torus T^i of radius R_i and volume $v_i = (2\pi R_i)^2$. We will label the 10d spacetime coordinates as $(x_0 - x_9)$, where $(x_0 - x_3)$ span the usual 4d Minkowski

[10]Compare with type IIB theory where the closed strings are said to be *oriented*.

[11]D-branes and orientifold planes both carry Ramond-Ramond charges, and so tadpole cancellation is equivalent to a net Ramond-Ramond charge of zero.

[12]Notice that $\mathcal{N} = 1$ SUSY in 10d is equivalent to $\mathcal{N} = 4$ in 4d, and so the compactification must simultaneously break three of the 4d supersymmetries.

spacetime. It is convenient to pair up the remaining six compact coordinates ($x_4 - x_9$) within three complexified dimensions as shown in Figure 1.6

$$z_1 = (x_4, x_5) \quad , \quad z_2 = (x_6, x_7) \quad , \quad z_3 = (x_8, x_9) \tag{1.21}$$

where z_i spans the two-torus T^i.

Twisting the compactified theory by an orientifold group $\{\Omega \times G\}$, where Ω is the world-sheet parity and G is a discrete Abelian group Z_N, $Z_N \times Z_M$ and $Z_{N_1} \times \ldots \times Z_{N_n}$, leads to $\mathcal{N} = 1$ SUSY in 4d and the presence of singular fixed points that are invariant under the action of the orientifold group [13]. However, there are only a finite subset of type IIB orientifolds (type I orbifolds) that give rise to $\mathcal{N} = 1$ in 4d and these have already been classified in the context of toroidal heterotic compactifications [62]. The action of orientifolding leads to tadpole divergences that must be cancelled by introducing Dp-branes into the vacuum, where $p = 5$ and/or 9 to preserve $\mathcal{N} = 1$ SUSY [14]. The tadpole cancellation conditions strongly constrain the gauge structure and massless spectrum by projecting out states that are not invariant under the orientifold group action [39, 40, 47, 48]. Notice that the presence of a background field $B_{\mu\nu}$ field, or non-trivial Wilson lines, can modify the tadpole cancellation conditions and reduce the rank of the gauge groups [51, 53, 63] since fewer D-branes are required for consistency.

Instead of using the world-sheet parity Ω, we are free to use different Z_2-parities to orientifold the type IIB theory, and other choices lead to vacua where D3 and/or D7-branes must be added to cancel tadpoles. This scenario is related to the previous

[13]The origin of the compact space is a trivial example of a fixed point.

[14]The D5_i-branes span Minkowski space plus the two-torus T^i, and the compactification radius of T^i is often labelled $R_i \equiv R_{5_i}$.

one involving $D5_i$ and/or D9-branes via *T-duality* transformations:

$$R_i \longleftrightarrow \frac{1}{M_*^2\, R_i} \qquad\qquad \lambda_I \longleftrightarrow \frac{\lambda_I}{M_*^2\, R_i} \tag{1.22}$$

where R_i is the radius of compactification of the two-torus T^i, M_* is the string scale and λ_I is the 10d dilaton that controls the type I string coupling. Therefore, T-dualising a pair of compact dimension will exchange Neumann with Dirichlet boundary conditions, and Dp-branes become D($p \pm 2$)-branes.

Type I models are phenomenologically appealing since the fundamental string scale M_* can take a range of values. This is in contrast to heterotic models where the string scale is fixed close to the Planck scale (independently of the compactification radii) by the relation $M_* = \sqrt{\alpha_{GUT}/8}\, M_{Pl}$ with $\alpha_{GUT} \approx 1/24$. Dimensionally reducing the effective 10d lagrangian, we can extract expressions for the Planck mass and D-brane gauge couplings in terms of the compactification scales $M_i = 1/R_i$ and M_* [39, 40]:

$$G_N \;=\; \frac{1}{M_{Pl}^2} = \frac{\lambda_I^2\, M_1^2 M_2^2 M_3^2}{8 M_*^8} \tag{1.23}$$

$$\alpha_{5_i}^2 \;=\; \frac{\lambda_I\, M_i^2}{2M_*^2} \quad , \qquad \alpha_9^2 = \frac{\lambda_I\, M_1^2 M_2^2 M_3^2}{2M_*^6} \tag{1.24}$$

where $\alpha_{5_i(9)} = g_{5_i(9)}^2/4\pi$ and $\lambda_I \leq \mathcal{O}(1)$ to remain perturbative. This explicit dependence on the size and shape of the compactification manifold gives the string scale its freedom $1\,\mathrm{Te}V \lesssim M_* \lesssim M_{Pl}$. In particular, type I string theory offers the exciting possibility of large extra dimensions, where quantum gravity effects could (in principle) be accessible to the next generation of accelerators [64].

We will now consider the open and closed string states that arise in generic constructions involving stacks of coincident D9-branes and three types of $D5_i$-branes. Recall that D9-branes fill the whole 10d spacetime ($x_0 - x_9$), while a $D5_i$-brane spans the usual 4d Minkowski space ($x_0 - x_3$) plus two extra compact dimensions that wrap around the two-torus T^i.

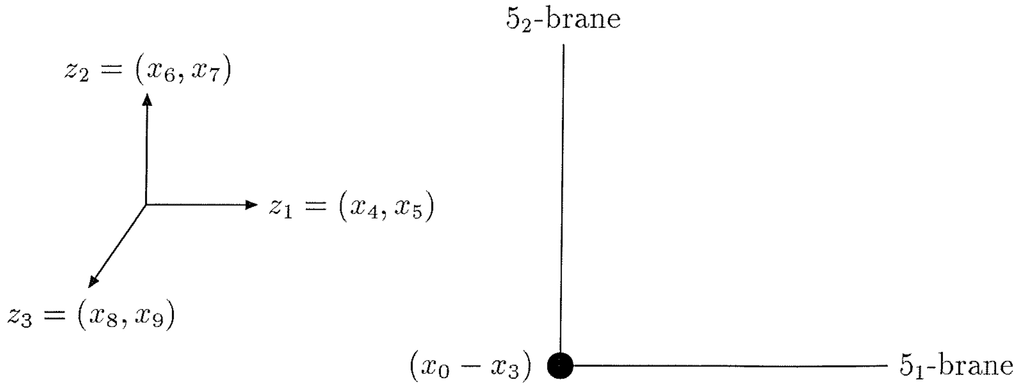


Figure 1.6: We can represent the six-dimensional compact space using a complexified coordinate system, where $D5_i$-branes are shown as straight lines along the z_i direction. Therefore, a $D5_1$ and $D5_2$-brane will overlap at the origin of the coordinate system $(x_0 - x_3)$, but extend out into "perpendicular" compact dimensions.

We can represent the 6d compact space on the six-torus T^6 using a three-dimensional coordinate system z_i where each coordinate corresponds to a pair of compact dimensions as given in Eq.(1.21). Therefore, stacks of coincident D5_i-branes are represented by a single line along the i^{th} coordinate. This is shown in Figure 1.6, where we have two different stacks of 5-branes that overlap at the origin $(x_0 - x_3)$, but have worldvolumes that extend into different perpendicular compact dimensions.

There are two types of massless $\mathcal{N} = 1$ chiral fields that arise in type I models:

- Closed strings chiral singlets

Chiral singlets arise from the closed string sector and include a complex dilaton S and untwisted moduli fields T_i $(i = 1, 2, 3)$ that freely move in the whole 10d spacetime. The untwisted moduli fields parametrise the size of the compactified dimensions where the compactification radius is given by:

$$R_i = \frac{\sqrt{T_i + \bar{T}_i}}{2M_*} \tag{1.25}$$

In heterotic models, the 4d dilaton is uniquely defined by the string coupling (10d dilaton) and volume-modulus. However in type I models, couplings are related to either the dilaton or untwisted moduli from T-duality. Compactifying the 10d theory using an orientifold leads to the presence of fixed points that are invariant under the orientifold group. Twisted moduli Y_k are closed strings that have been *trapped* at these fixed point singularities and parametrise the size of the fixed points. As we will discuss in chapter 3, these twisted states can contribute to SUSY breaking and also modify gauge couplings. Twisted moduli play an important rôle in cancelling anomalous $U(1)$'s through a 4d generalisation of the Green-Schwarz mechanism [47, 48].

- Charged open string states

Chiral matter, gauge bosons and Higgs fields arise as open strings attached to D-branes. They can either end on *different* D-branes (e.g. $C^{5_i5_j}$, C^{95_i}) and carry charges under both gauge groups, or have both ends attached to the *same* D-brane (e.g. $C_j^{5_i}$, C_j^9). Notice that these latter states carry an additional index that can be exploited to constrain the form of the renormalisable superpotential.

In the most general setups involving three types of $D5_i$-branes and $D9$-branes, string selection rules constrain the allowed combinations of open string states that appear in the superpotential (subject to gauge invariance) [39]:

$$\begin{aligned} W_{ren} = g_{5_1} & \left(C_1^{5_1}C_2^{5_1}C_3^{5_1} + C_3^{5_1}C^{5_15_2}C^{5_15_2} + C_1^{5_1}C^{95_1}C^{95_1}\right) \\ & + g_{5_2}\left(C_1^{5_2}C_2^{5_2}C_3^{5_2} + C_3^{5_2}C^{5_15_2}C^{5_15_2} + C_2^{5_2}C^{95_2}C^{95_2}\right) + g_{5_3}C^{5_15_2}C^{95_1}C^{95_2} \\ & + g_9\left(C_1^9C_2^9C_3^9 + C_1^9C^{95_1}C^{95_1} + C_2^9C^{95_2}C^{95_2}\right) \end{aligned} \quad (1.26)$$

where the Yukawa coupling constants (associated with fields arising from each 5-and 9-brane) are given by:

$$g_{5_i}^2 = \frac{4\pi}{Re\,T_i} \quad , \quad g_9^2 = \frac{4\pi}{Re\,S} \quad (1.27)$$

In Appendix B, we discuss the supergravity formalism that is used for the low-energy description of type I models in terms of the Kähler potential, superpotential and gauge kinetic functions [65]. We also provide general expressions for soft masses and trilinears in terms of SUSY breaking F-terms, where we will assume that only closed string states acquire non-zero VEVs.

1.4.3 Extra Dimensional Models

The recent developments in string theory involving D-branes, dualities and potentially low string scales [66, 67] has stimulated interest in higher-dimensional effective field theories [15] and their phenomenological implications [64]. In particular, the realisation that gravity and gauge fields can live in different numbers of dimensions has inspired the construction of "brane-world" models where SM fields are confined to D-branes embedded in a higher-dimensional space felt by gravity [70]. These models offer new (non-supersymmetric) solutions to the hierarchy problem [71] and can explain the relative weakness of the gravitational force due to a volume suppression from the additional dimensions felt by gravity. Brane-world models also offer a realisation of hidden sector SUSY breaking, where the hidden sector is spatially separated from the SM fields on a distant brane [22, 23, 24, 25].

Historically, the idea of (microscopic) extra dimensions pre-dates string theory and first appeared in the context of unification, when Kaluza and Klein attempted to combine Maxwell's theory of electromagnetism with Einstein's theory of general relativity by embedding both theories in a generally covariant 5d spacetime [72]. They assumed that quantum corrections caused the extra dimension to curl up into a circle of radius $R \sim \mathcal{O}(M_{Pl}^{-1}) \approx 10^{33}$ cm which is too small to observe. The fifth dimension y has the topology of a circle S^1, so that a scalar field $\phi(x, y)$ propagating in the extra

[15]See Refs.[68, 69] for recent reviews.

dimension satisfies the boundary conditions:

$$\phi(x,y) = \phi(x, y + 2\pi R) \tag{1.28}$$

Hence, $\phi(x,y)$ can be expanded as an infinite tower of 4d *Kaluza-Klein* modes $\phi_n(x)$:

$$\phi(x,y) = \sum_n \phi_n(x)\, e^{i\,p_n\,y} \tag{1.29}$$

where the momentum (mass) is quantised $p_n = n/R$. The non-zero modes correspond to particles with masses around the Planck scale, which can be neglected in the low-energy limit. Decomposing the Einstein-Hilbert action in 5d, they recovered the usual Maxwell theory coupled to general relativity where the off-diagonal fifth component of the metric tensor can be identified with the Maxwell field $A_\mu = g_{5\mu}$ and the electric charge is given by $e \sim \sqrt{8\pi\, G_N}/R$.

Instead of circular compactifications, recent higher-dimensional models have been studied where the extra dimension(s) are compactified on *orbifolds* such as S^1/Z_2. These are constructed by dividing a continuous manifold (S^1) with a discrete symmetry (Z_2) as shown in Figure 1.7 which leads to the appearance of discontinuous singularities, or fixed points, that are invariant under the orbifold group action [16].

However, we can still decompose higher-dimensional fields as infinite towers of Kaluza-Klein excitations, but now the orbifold symmetry provides a mechanism to project out unwanted states [17].

In chapter 4, we will study a 5d supersymmetric effective field theory [18], where the

[16] We have already seen in section 1.4.2 how closed strings (twisted moduli) can become trapped at such fixed points.

[17] Recall how the additional supersymmetries that remain when the 10d type IIB theory was dimensionally reduced to 4d, are projected out by the orientifold group.

[18] Notice that $\mathcal{N} = 1$ SUSY in 5d is equivalent to $\mathcal{N} = 2$ SUSY in 4d, and it is convenient to construct $\mathcal{N} = 2$ hypermultiplets by combining an $\mathcal{N} = 1$ chiral supermultiplet with its CP-mirror partner as discussed in Appendix C.

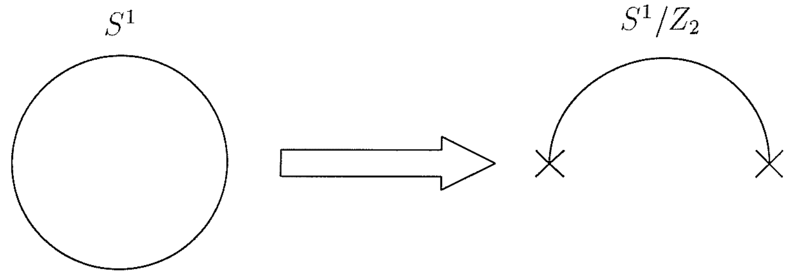


Figure 1.7: We can construct an S^1/Z_2 orbifold by dividing a circle S^1 by a discrete Z_2 group. This identifies coordinates on opposite sides of the circle, and leads to the presence of two fixed points that are invariant under the Z_2 symmetry.

extra dimension y is compactified on an S^1/Z_2 orbifold. In analogy to Eq.(1.29), we can decompose 5d fields into infinite Kaluza-Klein towers:

$$\begin{aligned} E(x,y) &= \sum_n E_n(x)\cos\left(\frac{n\,y}{R}\right) \\ O(x,y) &= \sum_n O_n(x)\sin\left(\frac{n\,y}{R}\right) \end{aligned} \tag{1.30}$$

where the orbifold leads to a classification of fields into even (E) and odd (O) Z_2-parity. Notice that we can project out non-MSSM states at $y=0$ by assigning them odd Z_2-parity.

Chapter 2

Brane Mediated SUSY Breaking

We propose a mechanism for mediating supersymmetry breaking in Type I string constructions. The basic set-up consists of a system of three D-branes: two parallel D-branes, a matter D-brane and a source D-brane, with supersymmetry breaking communicated via a third D-brane, the mediating D-brane, which intersects both of the parallel D-branes. We discuss an example in which the first and second family matter fields correspond to open strings living on the intersection of the matter D-brane and mediating D-brane, while the gauge fields, Higgs doublets and third family matter fields correspond to open strings living on the mediating D-brane. As in gaugino mediated models, the gauginos and Higgs doublets receive direct soft masses from the source brane, and flavour-changing neutral currents are naturally suppressed since the first and second family squarks and sleptons receive suppressed soft masses. However, unlike the gaugino mediated model, the third family squarks and sleptons receive unsuppressed soft masses, resulting in a very distinctive spectrum with heavier stops, sbottoms and staus.

2.1 Preamble

The process of SUSY breaking continues to be an active area of research as discussed in section 1.3.3. Over the years there have been various mechanisms proposed, including gravity [17], gauge [19, 20] and anomaly mediation [22, 23]. An alternative mechanism has been been put forward [24, 25] called *gaugino* mediated supersymmetry breaking ($\tilde{g}$MSB) which has the attractive property of solving the flavour problem, since scalars masses effectively vanish at the GUT scale and are generated through radiative corrections for which a GIM-like mechanism prevents flavour-changing neutral current (FCNC) problems. This is rather like the no-scale supergravity mechanism [18], but is implemented within a Horava-Witten [21] type set-up [1] consisting of two parallel but spatially separated D3-branes with SUSY broken on one brane, the supersymmetric matter fields living on the other brane and the gauge sector living in the bulk and communicating the SUSY breaking from one brane to the other. The Higgs doublets may also be in the bulk providing a solution to the μ problem via the Giudice-Masiero mechanism [14]. The advantage of this set-up is that the contact terms arising from integrating out states with mass M are suppressed by a Yukawa factor e^{-ML} if $M \geq L$, and so a modest separation L between the two branes can lead to negligible direct communication between the SUSY breaking brane and the matter brane. This is the starting point of both the anomaly mediated and the gaugino mediated models, and underpins the solution to the FCNC problem in both cases.

In this chapter we shall propose a mechanism for mediating SUSY breaking in Type I string models based on open strings starting and ending on D-branes. Type I string theories can provide an attractive setting for ideas such as gaugino mediated SUSY breaking ($\tilde{g}MSB$), and we shall explore this possibility in this paper. In place of the Horava-Witten set-up we shall consider a Type I toy model consisting of two

[1] Note that in the Horava-Witten model the branes are dynamical and appear at strong coupling. Also, they are not Dirichlet-branes and the gauge fields do not live in the bulk.

parallel D-branes with a third D-brane intersecting with both of the parallel D-branes. Instead of having the gauge fields in the bulk we shall put the gauge fields onto the third mediating D-brane, which allows SUSY breaking to be communicated between the SUSY breaking brane and the matter brane. Thus the role of the bulk is played by the third mediating D-brane, and it is the gauge fields which live on this brane that communicates the SUSY breaking. However in Type I models it is natural for a matter family to also live on the mediating D-brane, and this provides a characteristic signature of the brane mediated SUSY breaking mechanism.

To illustrate these ideas we consider a toy model inspired by the work of Shiu and Tye [52] using intersecting D5-branes, where the intersection regions are effectively parallel D3-branes within a higher-dimensional spacetime. In this model two chiral families occur in the 4d intersection region at the origin fixed point ($5_1 5_2$ sector), with a third family on the $D5_2$-brane ($5_2 5_2$ sector). However our model differs from Shiu-Tye since we include a further $D5_1'$-brane which intersects with the $D5_2$-brane at a point located away from the origin fixed point, and suppose that SUSY gets broken on that brane and is communicated via the states on the $D5_2$-brane which intersect with both $D5_1$-branes at the two fixed points – brane mediated SUSY breaking (BMSB). In this example gauge fields, Higgs fields and the third family all live in the mediating D-brane which plays the role of the bulk in the original $\tilde{g}$MSB scenario. This separation of the third family[2] provides an explanation for the large mass of the third family of quarks and leptons, without perturbing the solution to the flavour problem since the first and second families remain almost degenerate.

[2]Remember that the first two families are localised within an effective 4d overlapping region, while the third family feels two extra dimensions

2.2 Gaugino Mediated Supersymmetry Breaking

In this section we review the $\tilde{g}MSB$ mechanism [24, 25]. This toy model involves D3-branes embedded in a higher-dimensional space. Two parallel D3-branes are spatially separated along (at least) one extra dimension as shown in Figure 2.1. Standard Model quark and lepton fields are localised on the matter brane at $y = 0$ as open strings, while the gauge and (possibly) Higgs fields propagate in the bulk[3]. Supersymmetry is broken on the displaced source D3-brane at $y = L$. SUSY breaking is communicated to the bulk fields by direct higher-dimensional interactions[4], and mediated to the quark/lepton fields by Standard Model loops[5].

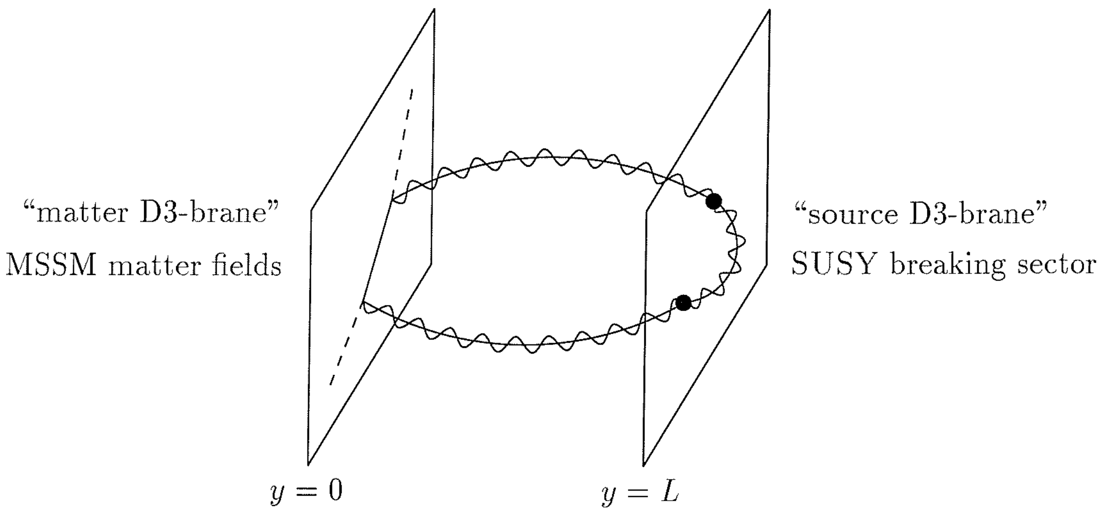


Figure 2.1: An extra dimensional loop diagram that contributes to SUSY breaking scalar masses. It is similar to a self-energy diagram, but with the virtual gaugino not confined to either 4-dimensional brane. This figure is taken from Ref. [24].

The full D-dimensional lagrangian is split into two distinct pieces - a bulk term

[3]Thus feeling all 5-dimensions.

[4]Higher-dimensional operators are assumed to arise from the underlying string theory, although this is not clear at present.

[5]Gauginos in the bulk couple directly to chiral fermions on the matter brane. They also couple to the hidden sector directly through mass-insertions on the source brane.

involving only bulk fields and terms localised on either D3-brane that allow direct bulk-brane field coupling.

$$\mathcal{L}_D = \mathcal{L}_{bulk}\left[\xi\left(x,y\right)\right] + \sum_j \delta^{D-4}\left(y-y_j\right)\mathcal{L}_j\left[\xi\left(x,y_j\right),\eta_j\left(x\right)\right] \tag{2.1}$$

where j runs over the branes, x are coordinates for the 4 non-compact dimensions, y are coordinates for the $D-4$ compact spatial dimensions, ξ is a generic bulk field, and η_j is a field localised on the j^{th} brane.

A Naive Dimensional Analysis (NDA) [73] allows the 5d (or higher) effective theory to be *matched* on to the observed 4d theory at the compactification scale. The 4 and D-dimensional gauge couplings can be related by the volume of the compact dimensions V_{D-4}:

$$g_4^2 = \frac{g_D^2}{V_{D-4}} \tag{2.2}$$

The D-dimensional gauge coupling g_D must be smaller than its strong-coupling limit, otherwise perturbative results become meaningless[6]

$$g_D^2 \sim \frac{\epsilon l_D}{M^{D-4}} \tag{2.3}$$

where l_D is a geometrical loop factor for D dimensions, $l_D = 2^D\pi^{D/2}\Gamma(D/2)$, M is the fundamental scale in the effective field theory which acts as a regulating cutoff, and ϵ suppresses the coupling strength. Note that $\epsilon \sim 1$ corresponds to the strong coupling limit. This places a constraint, along with FCNC suppression, that restricts the maximum size of the extra dimensions [24, 25].

Following the work of Randall and Sundrum on spatially-separated D3-branes in extra

[6] Extra dimensions (and Kaluza-Klein excitations) change the energy-dependence of couplings to power law running above the compactification scale. This allows for unification at lower scales, see [66, 74] for a review.

dimensions [22], contact terms between fields on opposite branes are exponentially suppressed by an amount e^{-ML}, where L is the separation between D3-branes along the extra dimension(s). Eq. (2.4) is an example of an exponentially suppressed 4-point operator involving superfields from the matter and source branes that generates scalar masses:

$$\delta\mathcal{L}_{brane} \sim \frac{e^{-ML}}{M^2} \int d^4\theta \left(\phi_S^\dagger \phi_S\right) \left(\phi_M^\dagger \phi_M\right) \tag{2.4}$$

where ϕ_S, ϕ_M are source and matter superfields respectively.

Compare the suppressed contact terms with the operators giving rise to gaugino masses and Higgs SUSY breaking parameters, from Higgs superfields H_u, H_d and gauge field strengths W_α living in the bulk.

$$\begin{aligned}\delta\mathcal{L}_{brane} \sim \frac{l_D}{l_4}\left(\int d^2\theta \frac{1}{M^{D-3}}\phi_S W^\alpha W_\alpha + h.c.\right) + \frac{l_D}{l_4}\int d^4\theta \Big\{ \frac{1}{M^{D-3}}\left(\phi_S^\dagger H_u H_d + h.c.\right) \\ + \frac{1}{M^{D-2}}\phi_S^\dagger \phi_S \left[H_u^\dagger H_u + H_d^\dagger H_d + (H_u H_d + h.c.)\right]\Big\} \end{aligned} \tag{2.5}$$

which leads to the following soft terms when we match to the D-dimensional theory[7]:

$$m_\lambda, \mu \sim \frac{F_S}{M}\frac{l_D/l_4}{M^{D-4}V_{D-4}} \sim \frac{1}{\epsilon l_4}\frac{F_S}{M}, \qquad B\mu, m_{H_u}^2, m_{H_d}^2 \sim \frac{F_S^2}{M^2}\frac{l_D/l_4}{M^{D-4}V_{D-4}} \sim \frac{1}{\epsilon l_4}\frac{F_S^2}{M^2} \tag{2.6}$$

where we have used Eqs.(2.2,2.3) with $g_4 \approx 1$.

Notice that we are following Ref. [25] with the Higgs fields living in the extra-dimensional bulk so that the μ-term is generated on the source brane via the Giudice-Masiero mechanism [14]. In comparison, Ref. [24] choose to localise the Higgs doublets on the matter D3-brane and include an additional gauge singlet (NMSSM [12, 13]) to generate μ with an extra superpotential term W $\sim \lambda N H_u H_d$. Hence, an effective

[7]Notice that the $B\mu$ term and Higgs mass-squared terms are enhanced by a volume factor relative to the gaugino mass m_λ and the μ-term.

μ-term can be generated if the scalar component of the singlet N acquires a non-zero VEV.

2.3 Type I String-Inspired Model

Now we turn to Type I string constructions and introduce a toy model motivated by the work of Shiu and Tye [52]. The string scale M_* is usually considered to be of the order 10^{16} GeV, but recently the gauge unification scale was suggested to be as low as 1 TeV, which could allow the string scale at a comparable value. Shiu and Tye [52] discuss the phenomenological possibilities within Type I string theory and overlapping D5-branes. They use the duality between the compactification of 10-dimensional Type IIB string theory on an orientifold, with Type I theory on an orbifold, to recover a 4-dimensional $\mathcal{N} = 1$ supersymmetric chiral string model with Pati-Salam-like gauge symmetry [8] [7, 75].

We have already discussed in section 1.4.2 that tadpole cancellation conditions and a non-zero background B-field constrain the number and type of D-branes allowed within the model to D5 and D9-branes only [53]. In a particular scenario they consider only one type of D5 brane (5_3) together with the D9 brane, and after T-dualising two pairs of compact dimensions, they obtain a scenario with two intersecting branes, namely 5_1 and 5_2 branes which intersect at the origin fixed point. A gauge group $U(4) \otimes U(2) \otimes U(2)'$ exists on each brane, and they discuss three scenarios where the SM gauge group originates from different brane combinations. Their third scenario is of particular interest since it leads to three chiral families - two families on the $5_1 5_2$ overlap and a third family on the $D5_2$-brane as shown in Figure 2.2.

Using Eq.(1.26), we can extract the renormalisable superpotential terms allowed by

[8] See Appendix A for details of the Pati-Salam gauge group.

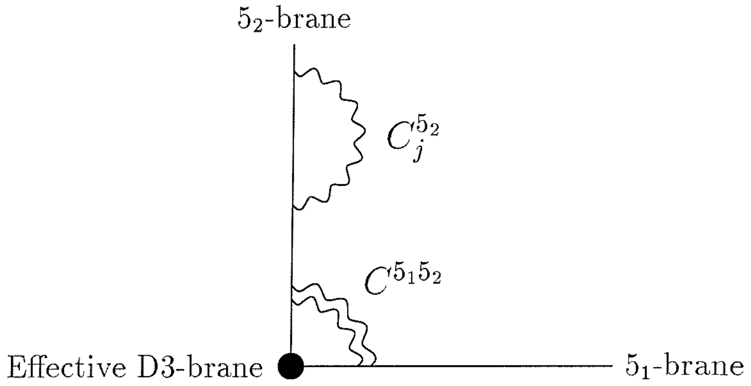


Figure 2.2: The matter fields and Higgs doublets resulting from Shiu and Tye's third scenario with intersecting D5-branes, where C_i^p is an open-string state (matter field) starting and ending on the p^{th} brane. C^{pq} is an open-string state starting on the p^{th} brane and ending on the q^{th} brane.

string selection rules:

$$W_{ren} = C_1^{5_2} C_2^{5_2} C_3^{5_2} + C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2} \tag{2.7}$$

where we have neglected all other open string states.

We now proceed to introduce a toy model based on the above construction. In order to allow the third family Yukawa couplings ($F_{3L} F_{3R} H$) consistent with the allowed terms in Eq.(2.7), we assign the Higgs doublets $H = H_u, H_d \equiv C_1^{5_2}$ or $C_2^{5_2}$. This leads to the four possible allocations of 5_2 states in Table 2.1.

5_2 states	A	B	C	D
$H = H_u\,,\, H_d$	$C_1^{5_2}$	$C_1^{5_2}$	$C_2^{5_2}$	$C_2^{5_2}$
$F_{3L} = Q_{3L}\,,\, L_{3L}$	$C_2^{5_2}$	$C_3^{5_2}$	$C_3^{5_2}$	$C_1^{5_2}$
$F_{3R} = U_{3R}^c\,,\, D_{3R}^c\,,\, E_{3R}^c\,,\, N_{3R}^c$	$C_3^{5_2}$	$C_2^{5_2}$	$C_1^{5_2}$	$C_3^{5_2}$

Table 2.1: Allocation of 5_2 states that lead to third family-only Yukawa couplings at lowest order. We use the lower index to distinguish between doublets, singlets and Higgs fields.

Notice that there are no free indices on the intersection states

$$F_{iL}\,,\, F_{iR} = Q_{iL}\,,\, L_{iL}\,,\, U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR}\,,\, N^c_{iR} \equiv C^{5_1 5_2} \qquad (i=1,2) \tag{2.8}$$

which means that we cannot distinguish between the first two families.

In our Type I string-inspired model, we shall assign the gauge groups and matter fields as in Table 2.2, where we have ignored the custodial $SU(4)_{5_1} \otimes U(1)^6$ symmetry. The states $\phi, \phi', \mathcal{H}, \mathcal{H}'$ are used to break the gauge groups down to the G_{SM} as discussed in Appendix D.1.

States	Sector	$SU(4)_{5_2}$	$SU(2)_{5_{2R}}$	$SU(2)_{5_{2L}}$	$SU(2)_{5_{1R}}$	$SU(2)_{5_{1L}}$
$F_{iL} = Q_{iL}\,,\, L_{iL}$	$5_1 5_2$	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$
$F_{iR} = U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR}\,,\, N^c_{iR}$	$5_1 5_2$	$\bar{\mathbf{4}}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$F_{3L} = Q_{3L}\,,\, L_{3L}$	5_2	$\mathbf{4}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$
$F_{3R} = U^c_{3R}\,,\, D^c_{3R}\,,\, E^c_{3R}\,,\, N^c_{3R}$	5_2	$\bar{\mathbf{4}}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
ϕ	$5_1 5_2$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$
ϕ'	$5_1 5_2$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
$\mathcal{H}$	5_2	$\mathbf{4}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$\mathcal{H}'$	5_2	$\bar{\mathbf{4}}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{1}$
$H = H_u\,,\, H_d$	5_2	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{1}$

Table 2.2: $SU(4)_{5_2} \otimes SU(2)_{5_{2R}} \otimes SU(2)_{5_{2L}} \otimes SU(2)_{5_{1R}} \otimes SU(2)_{5_{1L}}$ quantum numbers for left and right-handed chiral fermion states and symmetry breaking Higgs fields.

Gauge invariance with respect to the initial gauge group $SU(4)_{5_2} \otimes SU(2)_{5_{2R}} \otimes SU(2)_{5_{2L}} \otimes SU(2)_{5_{1R}} \otimes SU(2)_{5_{1L}}$ provides a mechanism to forbid both first and second family Yukawa couplings ($F_{iL}F_{jR}H$) and R-parity violating superpotential terms in Eq.(1.16) without any other assumptions[9]. Note that the μ-term is forbidden by string selection rules, which also forbid a superpotential term involving a matter brane

[9]Notice that the third family right-handed neutrinos and sneutrinos receive large Majorana masses from the operators $F_{3R}F_{3R}\mathcal{H}\mathcal{H}$ resulting in a see-saw mechanism. This is discussed in Ref. [76], along with a discussion of higher-dimensional operators suitable for first and second family fermion masses.

singlet[10] where W $\sim \lambda N H_u H_d$. Therefore, the Giudice-Masiero mechanism offers the best opportunity for producing a μ-term from the soft potential as discussed later.

2.4 Brane Mediated Supersymmetry Breaking

We now augment the model in section 2.3, by including an additional $D5'_1$-brane located at an orbifold fixed point away from the origin as shown in Figure 2.3. The idea of including the extra $5'_1$-brane is that SUSY is broken on this brane and communicated by the MSSM states that live on the 5_2-brane which intersects it [11]. Thus, the gauge fields on the 5_2-brane play the role of the gauge fields in the bulk in Figure 2.1. In Appendix E, we will discuss the various mass scales in this model.

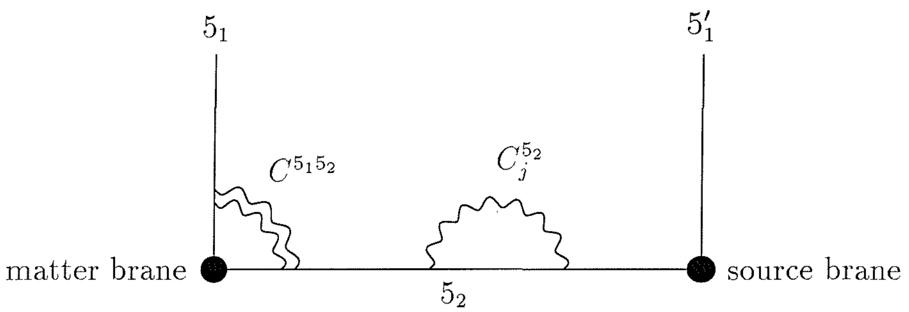


Figure 2.3: A brane-construction using overlapping D5-branes, with effective D3-branes at the intersection points spatially separated along the $D5_2$-brane. The first two chiral families ($C^{5_1 5_2}$) live on the first intersection region. The third family and Higgs doublets ($C_j^{5_2}$) live on the $D5_2$-brane in the "bulk" between the source and matter branes. The gauge-singlet source field in principle can either live on the $D5'_1$-brane or be localised on the $5'_1 5_2$ intersection, but for definiteness we assume the latter possibility.

We now consider a limiting case in which the model in Figure 2.3 reduces to the

[10] A non-renormalisable higher-dimensional 4-point superpotential term may be generated by two additional gauge singlet fields, eg. $W \sim N_1 N_2 H_u H_d$. This can become the 3-point term when one of the singlet fields acquire a VEV.

[11] We will assume an arbitrary gauge group on the $5'_1$-brane, and ignore tadpole cancellation conditions at this fixed point.

$\tilde{g}MSB$ model discussed in section 2.2, namely that the D5_2 compactification radius is very much larger than the $D5_1$ radius[12]

$$R_{5_2} \gg R_{5_1} \gg M_*^{-1} \tag{2.9}$$

In this limit, the model reduces to that shown in Figure 2.1, where the matter and source D3-branes correspond to the intersection regions of the D5-branes, and the bulk corresponds to the mediating 5_2 brane, as shown in Figure 2.4. Note that the first two families are located on the matter brane, while the third family and Higgs doublets live on the mediating 5_2 brane.

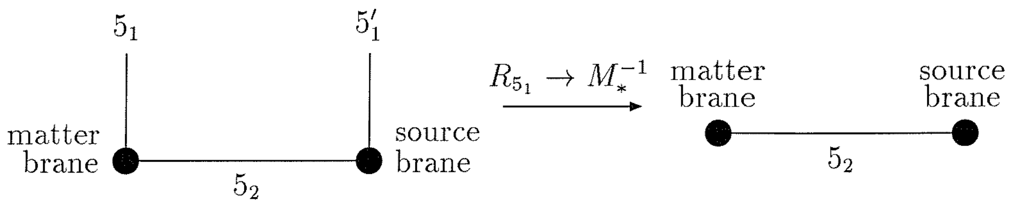


Figure 2.4: The intersecting D5-brane construction in the limit of small $D5_1$ compactification radius. The $D5_1$-branes reduce to effective D3-branes, separated in two orthogonal dimensions along the $D5_2$-brane (*bulk*). The allocation of Higgs and chiral matter fields are the same as in Figure 2.3 and Table 2.2.

From Eq.(1.24), we know that the gauge couplings on the branes are given by:

$$g_{5_2}^{-2} = \frac{M_*^2 v_{5_2}}{(2\pi)^3 \lambda_I} \quad , \quad g_{5_1}^{-2} = \frac{M_*^2 v_{5_1}}{(2\pi)^3 \lambda_I} \tag{2.10}$$

where λ_I is the 10d dilaton in type I string theory and the compact volume spanned by the D5_i-brane is $v_{5_i} = (2\pi\, R_{5_i})^2$. We can see that the coupling-squared is inversely proportional to the compactification volume v_{5_i}, which implies that $g_{5_1} \gg g_{5_2}$ from Eq.(2.9). This limiting case of the symmetry breaking is discussed in Appendix D.2, but the important results are that the dominant components of the gauge fields live

[12]Both radii must be larger than the inverse string scale M_*^{-1} otherwise the original T-dual description in terms of 9 and 5_3-branes from Ref. [52] is more appropriate, and we lose the intersecting brane structure.

on the $D5_2$-brane which is consistent with $\tilde{g}MSB$ with two extra bulk dimensions. After the gauge symmetry is broken down to the Standard Model, we recover the relationship between gauge couplings:

$$g'_Y \sim \sqrt{\frac{3}{5}} g_3 \tag{2.11}$$

where $g_3 \equiv g_{5_2}$, and this is consistent with gauge coupling unification if $g_{5_2} \equiv g_{GUT}$ at the GUT scale.

It is also interesting to note that the restrictions we place on the radii do not restrict the radius of the third complexified dimension too strongly. This could allow a large extra dimension felt by gravity alone (with a size of the order of 1mm) as considered recently [70], but we will not discuss that possibility here.

In this limiting case, we can use the results of Ref. [25], where we identify $L \equiv R_{5_2}$, to extend the analysis for the size of the extra dimensions and exponential suppression factors. Ref. [25] considers the maximum dimension size in the strong coupling limit $\epsilon \sim 1$, but for a small number of extra dimensions the theory does not need to be strongly coupled at the string scale, ie. $\epsilon \neq 1$.

Consider our symmetric toroidal compactification where the volume of the compact dimensions is

$$v_{5_2} \sim L^2 \equiv R_{5_2}^2 \tag{2.12}$$

Using Eqs. (2.2,2.3) with $D = 6$, we can relate dimension size to the parameter ϵ for $g_4 \sim \mathcal{O}(1)$ (as observed for SM couplings).

$$\begin{aligned} g_{5_2}^2 &\sim \frac{\epsilon l_6}{M_*^2} \sim L^2 \\ L M_* &\sim (\epsilon l_6)^{\frac{1}{2}} \end{aligned} \tag{2.13}$$

Note that from Eqs. (2.12, 2.13), we have:

$$v_{5_2} M_*^2 \sim \epsilon l_6 \tag{2.14}$$

ϵ	LM_*	$e^{-LM_*/2}$
1	63	2×10^{-14}
0.8	56	6×10^{-13}
0.6	49	3×10^{-11}
0.4	40	2×10^{-9}
0.2	28	8×10^{-7}
0.1	20	5×10^{-5}
0.05	14	9×10^{-4}
0.01	6	4×10^{-2}

Table 2.3: Estimates for the toroidal compactification length L and exponential suppression factor for $D = 6$, where $L \equiv R_{5_2}$.

We have just seen how to recover the $\tilde{g}MSB$ model, but with two extra dimensions and the third family in the bulk. We can use the $\tilde{g}MSB$ operators that generate scalar and Higgs masses, A and $B\mu$-terms and even a μ-term via the Giudice-Masiero mechanism[13]. However, in our model with $M \equiv M_*$ and $R_{5_2} \gg R_{5_1} > M_*^{-1}$, there are only two extra dimensions in the bulk between D3-branes[14].

We use Eqs. (2.4-2.6) with the following identifications:

$$\begin{aligned}
\phi_M &\equiv \left(C^{5_1 5_2}\right) \quad Q_{iL}\,,\, L_{iL}\,,\, U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR}\,,\, N^c_{iR} \qquad (i = 1, 2) \\
W^\alpha &\equiv W_{SM} \\
H_u, H_d &\equiv \left(C^{5_2}_j\right) \quad H_u\,,\, H_d\,,\, Q_{3L}\,,\, L_{3L}\,,\, U^c_{3R}\,,\, D^c_{3R}\,,\, E^c_{3R}\,,\, N^c_{3R}
\end{aligned} \tag{2.15}$$

[13]Remember that a superpotential μ-term is forbidden by string selection rules for our choice of states.

[14]This allows us to use Table 2.3 to get restrictions on the size of R_{5_2}.

$$\phi_S \equiv \left(C^{5_1' 5_2}\right) \quad \phi_S$$

to generate higher-dimensional operators, subject to the full 42222 gauge invariance. We assume that the F-component of the gauge-singlet field ϕ_S, which we assume to be a string state localised at the intersection between the source brane and the mediating brane, acquires a non-zero VEV and breaks supersymmetry. We now proceed to discuss the different types of masses in the limiting case of the BMSB model.

2.4.1 Gaugino masses

In the limit of $R_{5_2} \gg R_{5_1}$, the Standard Model gauge fields are dominated by their components on the $D5_2$-brane (bulk). In agreement with $\tilde{g}MSB$, we generate gaugino masses of the same order of magnitude from Eqs. (2.5,2.6)

$$m_\lambda \sim \frac{F_S}{M_*} \frac{l_6/l_4}{M_*^2 v_{5_2}} \sim \frac{1}{\epsilon l_4} \frac{F_S}{M_*} \tag{2.16}$$

(where v_{5_2} is the volume of the compact dimensions inside the $D5_2$-brane world-volume.) Notice that gauginos on the $D5_1$-brane will have exponentially-small masses due to their separation from the SUSY breaking, and we would expect to observe them at colliders unless they acquire large masses through some (unspecified) mechanism.

2.4.2 First and second family scalar masses

This is the generic 4-point contact term between fields on opposite branes that leads to exponentially suppressed first and second family squark and slepton masses[15], using

[15]This operator also leads to first and second family mixing and off-diagonal mass matrix elements. There may be another operator leading to first and third family mixing, eg. $\delta\mathcal{L} \sim \int d^4\theta C^{\dagger 5_1 5_2} C_j^{5_2} \phi_S^\dagger \phi_S$.

Eq.(2.4).

$$\begin{aligned} \delta\mathcal{L}_{soft} &\sim \frac{e^{-M_* R_{5_2}}}{M_*^2}\int d^4\theta C^{\dagger 5_1 5_2} C^{5_1 5_2}\phi_S^\dagger \phi_S \\ V_{soft} &\sim \frac{e^{-M_* R_{5_2}} F_S^2}{M_*^2}\left(\tilde{Q}_{iL}^\dagger \tilde{Q}_{jL} + \tilde{u}_{iR}^* \tilde{u}_{jR} + \tilde{d}_{iR}^* \tilde{d}_{jR} + \tilde{L}_{iL}^\dagger \tilde{L}_{jL} + \tilde{e}_{iR}^* \tilde{e}_{jR} + \tilde{\nu}_{iR}^* \tilde{\nu}_{jR}\right) \end{aligned} \tag{2.17}$$

Table (2.3) shows that the exponential suppression factor is strong for two extra dimensions. Therefore, contact term contributions to the first and second family scalar masses are negligible at high energies, and they are generated by RGE effects instead.

Loop contributions to first and second family scalar masses as shown in Figure 2.1 are much larger than contact terms and anomaly mediated contributions. So, although the first and second family squark/slepton masses are not zero at high-energies, they are suppressed by a loop factor relative to third family scalar masses.

2.4.3 Higgs mass terms and third family scalar masses

Extending Eq. (2.5) to include third family scalars, we have the following higher-dimensional operators:

$$\begin{aligned} \delta\mathcal{L}_{soft} \sim \frac{l_6}{l_4}\int d^4\theta \Bigg\{ &\frac{1}{M_*^3}\left(\phi_S^\dagger H_u H_d + h.c.\right) + \frac{1}{M_*^4}\phi_S^\dagger \phi_S \Big[H_u^\dagger H_u + H_d^\dagger H_d + (H_u H_d + h.c.) \\ &+ Q_{3L}^\dagger Q_{3L} + U_{3R}^{c\dagger} U_{3R}^c + D_{3R}^{c\dagger} D_{3R}^c + L_{3L}^\dagger L_{3L} + E_{3R}^{c\dagger} E_{3R}^c + N_{3R}^{c\dagger} N_{3R}^c\Big]\Bigg\} \end{aligned} \tag{2.18}$$

From Eqs. (2.6, 2.18), we obtain the μ-term,

$$\mu \sim \frac{F_S}{M_*}\frac{l_6/l_4}{M_*^2 v_{5_2}} \sim \frac{1}{\epsilon l_4}\frac{F_S}{M_*} \tag{2.19}$$

Higgs and third family scalar masses.

$$B\mu, m_{H_u}^2, m_{H_d}^2, m_{F_3}^2 \sim \frac{F_S^2}{M_*^2}\frac{l_6/l_4}{M_*^2 v_{5_2}} \sim \frac{1}{\epsilon l_4}\frac{F_S^2}{M_*^2} \tag{2.20}$$

2.4.4 Scalar mass matrix

We have generated a scalar mass matrix with an explicit third family mass hierarchy at lowest order:

$$m_{scalar}^2 \sim \frac{1}{\epsilon l_4}\frac{F_S^2}{M_*^2}\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.21}$$

The first and second family mass matrix elements are dominated by loop corrections since the contact term contributions are exponentially suppressed. However these contributions are still smaller than the third family masses due to the location of the third family in the bulk and its direct coupling to the SUSY breaking hidden sector.

2.4.5 Trilinear A-terms

Gauge invariant operators can be constructed for third family A-terms as follows:

$$\delta\mathcal{L}_{soft} \sim \frac{l_6}{l_4}\int d^2\theta \frac{1}{M_*^4}\phi_S\left[H_d D_{3R}^c Q_{3L} + H_u U_{3R}^c Q_{3L} + H_d E_{3R}^c L_{3L} + H_u N_{3R}^c L_{3L} + h.c.\right] \tag{2.22}$$

These operators lead to trilinear A-terms that are proportional to the Yukawa couplings:

$$A_{ij} \sim \frac{F_S}{M_*}\frac{l_6/l_4}{M_*^3 v_{5_2}^{3/2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \sim \frac{F_S}{M_*}\frac{1}{\epsilon l_4 \left(\epsilon l_6\right)^{1/2}} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{2.23}$$

where we have used Eq.(2.14).

The first and second family A-terms are negligible in comparison to the third family term, and receive loop-suppressed contributions instead.

2.4.6 Yukawa textures

Using our choice of states and Eq. (2.7), we obtain a third family hierarchical Yukawa texture for the quark and lepton sectors at lowest-order. This texture reflects the observation that $\mathrm{m}_t \gg m_c, m_u$; $\mathrm{m}_b \gg m_s, m_d$ and $\mathrm{m}_\tau > m_\mu, m_e$.

$$Y_{ij}^a \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad where\ a \equiv u, d, e, \nu \tag{2.24}$$

Smaller next-to-lowest order Yukawa couplings (and associated trilinear A-terms) are generated by higher-dimensional operators. Notice that an interesting operator is allowed by 42222 gauge invariance, and appears to be such a small Yukawa term:

$$\delta\mathcal{L} \sim F_{iL} F_{iR} H \phi \phi' \tag{2.25}$$

The scalar components of the superfields H, ϕ and ϕ' (Higgs) acquire VEVs and spontaneously break the gauge symmetry. When each field is replaced by its VEV, we can generate a first and second family mass term. This operator will be suppressed by powers of the string scale such that the first and second family have much smaller masses relative to the third family in the bulk.

2.4.7 Mass ratios and FCNC constraints

Consider the ratio of Higgs and third family scalar masses $B\mu, m_{H_u}^2, m_{H_d}^2, m_{F_3}^2$ to gaugino masses m_λ^2:

$$\frac{m_\phi^2}{m_\lambda^2} \sim \frac{l_4}{l_6} M_*^2 v_{5_2} \sim \epsilon l_4 \tag{2.26}$$

using Eqs. (2.2,2.3) and $g_4 \sim 1$, where $\mathrm{m}_\phi^2 \equiv B\mu, m_{H_u}^2, m_{H_d}^2, m_{F_3}^2$.

Also consider the ratio of trilinear soft masses A_{33} to gaugino masses m_λ using Eqs. (2.16,2.23):

$$\frac{A_{33}}{m_\lambda} \sim \frac{1}{(\epsilon l_6)^{1/2}} \tag{2.27}$$

ϵ	$e^{-LM_*/2}$	m_ϕ^2/m_λ^2	m_ϕ/m_λ	A_{33}/m_λ
1.0	2×10^{-14}	158	12.6	0.016
0.8	6×10^{-13}	126	11.2	0.018
0.6	3×10^{-11}	95	9.7	0.020
0.4	2×10^{-9}	63	7.9	0.025
0.2	8×10^{-7}	32	5.6	0.035
0.1	5×10^{-5}	16	4.0	0.050
0.05	9×10^{-4}	8	2.8	0.071
0.01	4×10^{-2}	1.6	1.3	0.159

Table 2.4: Estimates for the ratio of scalar masses and third family A-terms to gaugino masses for different ϵ and the exponential suppression factor (for masses-squared) arising from toroidal compactification.

Experimental constraints on FCNC [26] from mass-squared matrix elements require a suppression of $\sim 10^{-3} - 10^{-4}$ at the high-scale for first and second family scalar masses in Eq. (2.17). Using Table 2.4, we get a *lower* limit of say $\epsilon \sim 0.01$. However, phenomenological considerations restrict the ratio of m_ϕ and m_λ, and places an *upper* limit of say $\epsilon \sim 0.1$. This amount of suppression requires that the effective D3-branes are separated by a distance of order $R_{5_2} \sim 10/M_*$.

2.4.8 Phenomenology

As in [25] we shall consider the phenomenology based on an inverse compactification scale ($R_{5_2}^{-1}$ in our case) close to the unification scale $M_{GUT} \sim 2 \times 10^{16}$ GeV. It is natural to assume a high energy unification scale in the limiting case $g_{5_1} \gg g_{5_2}$ since in this limit the light physical gauge fields all arise from the mediating 5_2 brane, and so are all subject to a single gauge coupling constant, $g_{5_2} \equiv g_{GUT}$.

We have seen that in the BMSB model (at M_{GUT}) the trilinear and first and second family soft masses are negligible, while the third family soft masses, and the Higgs mass parameters are larger than the gaugino masses. In Table 2.5 we compare a sample spectrum in the BMSB model to that in both the $\tilde{g}MSB$ model [24, 25] and the no-scale supergravity model [18], where the ratio of Higgs VEVs $\tan\beta = 20$ and a universal gaugino mass of $M_{1/2} = 300$ GeV, are chosen to give a lightest Higgs boson mass of about 115 GeV, consistent with the recent LEP signal [77, 78]. [16]

In the no-scale model the only non-zero soft mass is $M_{1/2}$, which results in a very characteristic spectrum where the right-handed slepton is very light and is in danger of becoming lighter than the lightest neutralino $\tilde{\chi}_1^0$. The $\tilde{g}MSB$ model differs from the no-scale model only by the inclusion of soft Higgs masses which we have taken to be degenerate and somewhat higher than the gaugino masses. The main effect is

[16]Note that $\tan\beta = 20$ is sufficiently small that we may neglect all Yukawa couplings except the top Yukawa coupling in the RGEs.

	BMSB	$\tilde{g}MSB$	no-scale
$M_{1/2}$	300	300	300
A_0	0	0	0
$m_{F_{1,2}}$	0	0	0
m_{F_3}	500	0	0
m_{H_u}	500	500	0
m_{H_d}	500	500	0
$\tilde{g}$	830	830	830
$\tilde{\chi}_1^0$	124	119	124
$\tilde{\chi}_2^0$	239	200	237
$\tilde{\chi}_3^0$	506	258	472
$\tilde{\chi}_4^0$	517	314	485
$\tilde{\chi}_1^\pm$	238	195	237
$\tilde{\chi}_2^\pm$	518	314	486
$\tilde{E}_{1L}$, $\tilde{E}_{2L}$	220	220	220
$\tilde{E}_{3L}$	546	220	220
$\tilde{E}_{1R}^c$, $\tilde{E}_{2R}^c$	124	124	124
$\tilde{E}_{3R}^c$	515	124	124
$\tilde{N}_{1L}$, $\tilde{N}_{2L}$	205	205	205
$\tilde{N}_{3L}$	540	205	205
$\tilde{U}_{1L}$, $\tilde{U}_{2L}$	740	740	740
$\tilde{U}_{3L}$	783	653	676
$\tilde{U}_{1R}^c$, $\tilde{U}_{2R}^c$	715	715	715
$\tilde{U}_{3R}^c$	628	520	577
$\tilde{D}_{1L}$, $\tilde{D}_{2L}$	744	744	744
$\tilde{D}_{3L}$	787	658	681
$\tilde{D}_{1R}^c$, $\tilde{D}_{2R}^c$	713	713	713
$\tilde{D}_{3R}^c$	871	713	713
$\tilde{t}_1$	613	492	544
$\tilde{t}_2$	832	718	745
$\tan\beta$	20	20	20
m_{h^0}	115	114	115
m_{H^0}	738	596	511
m_{A^0}	738	596	511
$m_{H^\pm}$	742	602	517
$\mu(M_Z)$	500	250	467

Table 2.5: Comparison of spectra (in GeV) for the three models BMSB, $\tilde{g}MSB$ and no-scale supergravity. The common parameters are $\tan\beta = 20$, universal gaugino mass $M_{1/2} = 300$ GeV, trilinear soft mass $A_0 = 0$, first and second family squark and slepton masses $m_{F_{1,2}}^2 = 0$. The parameters are chosen to give a lightest Higgs boson mass consistent with the LEP signal [77, 78]. The μ parameter (assumed positive) and B are determined from the low-energy EWSB conditions.

to reduce the μ parameter, which is determined here from the EWSB condition, and taken to be positive, which results in lighter charginos and neutralinos. Also in the $\tilde{g}MSB$ model the heavy Higgs and third family squark spectrum is also noticeably different from the no-scale model. [17] Turning to the BMSB model, we see that the effect of having both the Higgs and third family soft masses is to raise the μ parameter, and of course to significantly increase the third family squark and slepton masses, providing an unmistakable spectrum and a characteristic smoking gun signature of the model.

2.5 Conclusions

We have proposed a mechanism for mediating SUSY breaking in Type I string theories - BMSB. Rather similar to the $\tilde{g}MSB$ set-up in Figure 2.1 we have proposed a Type I string-inspired set-up consisting of three intersecting D5-branes as shown in Figure 2.3 in which the gauge fields, Higgs doublets and third family matter fields all live on the third mediating 5_2-brane which plays the role of the bulk in the $\tilde{g}MSB$ scenario. The presence of the third matter family on the mediating D-brane is characteristic of Type I string constructions and provides the main experimentally testable difference between the BMSB and $\tilde{g}MSB$ models.

We have considered a limiting case in which $R_{5_2} \gg R_{5_1}$, and shown that in this case the model reduces to the original $\tilde{g}MSB$ model with the role of the bulk being played by the mediating 5_2-brane. In this limiting case, the model naturally leads to approximately universal gaugino masses and a single unified gauge coupling constant, which motivates the identification of the string scale with the usual GUT scale. In this case the phenomenology of the BMSB model is rather interesting, and it may be

[17]As noted in [25], if we had taken non-degenerate Higgs soft masses then the lightest right-handed slepton mass could have been significantly increased relative to the no-scale model due to the hypercharge Fayet-Illiopoulos D-term.

compared to the predictions of the no-scale supergravity and the $\tilde{g}MSB$ model. As in the $\tilde{g}MSB$ model, the first two families naturally receive very small masses at the high energy scale leading to flavour-changing neutral currents being naturally suppressed. The presence of third family soft masses will not alter this conclusion very much since FCNC limits involving the third family are much weaker. However the third family soft masses will lead to a characteristic squark and slepton mass spectrum which may be easily distinguished from that of both no-scale supergravity and the $\tilde{g}MSB$ model as shown in Table 5. The μ-problem is solved by the Giudice-Masiero mechanism as in the original $\tilde{g}MSB$ model.

In this limiting case the BMSB model bears a close resemblance to both the no-scale supergravity and the $\tilde{g}MSB$ models. The fact that the third family receives a non-zero soft SUSY breaking mass is strictly not an unambiguous signal of the underlying Type I string model, since it is possible for this to happen in both the other cases also. For example in the old heterotic models based on orbifolds, matter may be localised in the fixed points of the orbifold (the twisted sector) or not (the untwisted sector) so it is possible to have the third family playing a different role from that of the first two families. What is different in the model presented here is that the gauge group is localised on two different branes, but in the limiting case (above) the physical gauge group arises essentially from one brane, and in this limit we return to a situation similar to that of the old heterotic string theories. There are however three points worth noting here. Firstly, the presence of two families at the intersection points of two branes, and one family on a single brane, seems to be typical of Type I string constructions [52]. Secondly in Type I string constructions we have the possiblity of full unification of both gravity and gauge forces, precisely because gravity exists in 10 dimensions whereas the gauge groups live in a 6 dimensional sub-manifold, which is not possible in old heterotic string theories. Thirdly, the limiting case of $R_{5_2} \gg R_{5_1}$ would be expected to apply only approximately, and therefore in practice there will be corrections, for example, to gauge coupling unification which may be observable.

In the more general non-limiting case, the model will have an even richer structure. In this limit (ie. $R_{5_2} > R_{5_1} \neq M_*^{-1}$), we must use the full gauge state expressions listed in Appendix D.1. The light gauge states are no longer dominated by their $D5_2$-brane components, but are instead mixtures of fields from either brane, with the exception of the gluon/gluino states that *only* arise from the $D5_2$-brane. The result is that the high energy gluino mass will be larger than the high energy wino and bino masses. In this more general case the gauge couplings are no longer equal, so there is less motivation to identify the string scale with the GUT scale. Generally the string scale can take any value from a few TeV to 10^{16} GeV, and we have the possibility of a mm scale large extra dimension.

The toy model has other interesting features such as the fact that the gauge symmetry forbids first and second family Yukawa couplings at lowest order, and naturally forbids R-parity violating operators that cannot be forbidden by string selection rules alone, while allowing the third family Yukawa coupling. Most importantly, however, the toy model demonstrates the BMSB mechanism, which is based on having at least three branes with two different intersection points. This minimum requirement implies that constructions with all the branes at the origin fixed point are inadequate for our purpose. Although there are examples in the literature of intersecting branes at different fixed points [54], such models are generally more complicated than the simple set-up considered here. Nevertheless our BMSB mechanism could provide a useful alternative starting point from which to address the problem of SUSY breaking in more general Type I string theories.

Finally note that it has been been suggested, in the context of Type I theories, that singlet twisted moduli, which appear in the tree-level gauge kinetic function, might be responsible for generating gaugino masses if they acquire non-vanishing F-terms, and that this might provide a brane realisation of $\tilde{g}MSB$ if the standard model gauge symmetry originates from 9-branes providing that there are in addition two sets of

D5-branes located at two different fixed points [79]. This suggestion shares some of the features with the work in the present chapter, although model building issues were not discussed, and the characteristic possibility of the third family on the mediating brane was not considered. Also additional contributions from the F-terms of dilaton S and moduli fields T_i were also generically allowed, whereas in this chapter we have implicitly assumed them to be absent. In the next chapter, we will explore the rôle of twisted moduli as sources of SUSY breaking in the context of general D-brane models involving intersecting D-branes.

Chapter 3

Twisted Moduli and Supersymmetry Breaking

We consider twisted moduli contributions to supersymmetry breaking in effective type I string constructions involving intersecting $D5_i$ and $D9$-branes using Goldstino angles to parametrise the supersymmetry breaking. It is well known that twisted moduli enter at tree-level into the gauge kinetic functions, and can provide new sources of gaugino mass if they develop F-term vacuum expectation values. It is generally assumed that string states which are sequestered from the twisted moduli receive a zero soft mass in the twisted modulus domination limit, however the standard form of Kähler potential does not reproduce this expectation. We therefore propose a new form of the Kähler potential which is consistent at leading order with the sequestered form proposed by Randall and Sundrum, and show that it leads to exponentially suppressed sequestered soft masses. Including the effects of Green-Schwarz mixing, we write down the soft scalar and trilinear masses arising from a type I string construction involving intersecting $D5_i$ and $D9$-branes in the presence of untwisted and twisted moduli. If the squarks and sleptons are identified with sequestered states then in the twisted moduli dominated limit this corresponds to gaugino mediated supersymmetry breaking, and we discuss two different scenarios for this. The general results will be useful for phenomenological studies involving a combination of gravity and gaugino mediated SUSY breaking due to the dilaton, untwisted and twisted moduli contributions, and enable the soft masses to be studied as a function of the different compactification radii.

3.1 Preliminaries

Superstring theories offer the only consistent method for unifying the four fundamental forces of Nature within a single framework, as discussed in section 1.4, Prior to 1995, heterotic strings with a gauge group of $SO(32)$ (heterotic-O) or $E_8 \times E_8$ (heterotic-E) offered the most promising possibility of constructing superstring models containing the MSSM. However, the discovery of Dirichlet-branes and string dualities ended this heterotic monopoly on string phenomenology, and semi-realistic 4d $\mathcal{N} = 1$ type I models have been constructed from type IIB orientifolds, as reviewed in section 1.4.2.

The heterotic-O and type I models share common features, but differ in phenomenologically important ways. For instance, both scenarios contain dilaton and (twisted and untwisted) moduli fields - closed string states that are related to the geometry of the compactified space and appear in the low-energy four-dimensional effective SUGRA theory [65]. However, in contrast to heterotic models, type I theories include extended, solitonic Dirichlet-branes which are needed to cancel tadpole anomalies. Stacks of coincident D-branes are found to generate $U(N)$ gauge groups within their world-volume (which can be smaller than the full 10d space), and the attached open strings are identified with chiral matter and gauge fields. Hence, type I models necessarily include open *and* closed strings for consistency. This raises the hitherto unexplored possibility that gravity (closed) and gauge/chiral (open) fields can live in different numbers of dimensions. Another important diference is that in type I models the fundamental string scale M_* is no longer fixed at the grand unification M_{GUT} or Planck scales M_{Pl}, and in principle can be as low as the TeV scale [66, 67] with the lower bound determined by phenomenology. The low energy gauge structure is controlled by the number and type of D-branes present in the vacuum, and background B-fields can also induce further symmetry breaking [51, 53].

An important difference between heterotic and type I is the rôle played by twisted moduli fields - closed string states that are *trapped* at fixed points in the underlying manifold due to the action of orbifold compactification. Consider the gauge kinetic function f_α that appears in the SUGRA lagrangian [65]. In weakly coupled heterotic string theory, the string coupling constant is uniquely determined by the dilaton $g_s^{-2} \sim Re(S)$, and:

$$f_\alpha = k_\alpha S + \Delta_{1-loop}(T_i) \tag{3.1}$$

where k_α is the Kac-Moody level of the gauge factor[1] and the dependence on the untwisted moduli $\Delta_{1-loop}(T_i)$ only arises from 1-loop string threshold corrections [80] [2]. In contrast, type I models have gauge kinetic functions that depend on the dilaton S for 9-branes *and* the moduli fields T_i for 5_i-branes at tree-level, giving rise to different gauge couplings on different branes. In addition the gauge kinetic functions have a tree-level dependence on the twisted moduli, and this gives rise to different gauge couplings even within a particular D-brane sector. The tree-level dependence on the twisted moduli fields Y^k from the k^{th} twisted sector (within the world-volume of a

[1]In the MSSM, $k_{SU(3)_C} = k_{SU(2)_L} = \frac{3}{5}k_{U(1)_Y} = 1$.

[2]In contrast, there is a very different situation for the strongly coupled case (from M-theory) where f_α receives comparable contributions at tree-level from the dilaton and untwisted moduli fields $f \sim S + T$ [81].

given D-brane sector) are given by:

$$f_\alpha^9 = S + \sum_k \frac{s_{\alpha,k}}{4\pi} \sum_q Y^{k,q} \tag{3.2}$$

$$f_\beta^{5_i} = T_i + \sum_k \frac{s'_{\beta,k}}{4\pi} \sum_{p_i} Y^{k,p_i} \tag{3.3}$$

where the gauge coupling is found by the relation:

$$Ref_\alpha = \frac{4\pi}{g_\alpha^2} \tag{3.4}$$

and $q(p_i)$ label the fixed points within the $9(5_i)$-brane and $s_{\alpha,k}, s'_{\beta,k}$ are calculable model-dependent coefficients involving Chan-Paton matrices [39]. Thus twisted moduli tend to induce non-universal gauge couplings even for gauge groups living on a common brane sector. The twisted moduli also play an important rôle in the cancellation of gauge and gravitational anomalies in type I models - like the dilaton in heterotic string theory - through a generalised four-dimensional Green-Schwarz mechanism [48] that mixes twisted and untwisted moduli together.

The Kähler potential and superpotential can also receive non-perturbative contributions, and in the absence of a complete model one may adopt a phenomenologically-motivated parametrisation in order to make progress [39, 82]. The relative contributions to the overall SUSY breaking F-term vacuum expectation value from different fields can be parametrised in terms of *Goldstino* angles [3]. In such an approach one can derive the soft parameters in terms of these Goldstino angles, and examine various limits in which the dilaton or moduli fields dominate. As envisaged by the originators of the approach, it may also be used to investigate the contributions to SUSY breaking from twisted moduli in effective type I theories, in addition to the usual dilaton and untwisted moduli fields [83]. However the analyses that have been done so far have only considered the explicit situation where the gauge group and matter

[3]See the discussion in Appendix B.2 for further details.

fields arise from a stack of D9-branes, and thus share the same world-volume as all of the twisted moduli fields. It is one of the purposes of this chapter to extend the scope of such analyses to include more general set-ups involving intersecting $D5_i$ and $D9$-branes. In so doing we encounter a difficulty that is not present in the case of a single D9-brane set-up, namely the problem of sequestered states which do not share the same world volume as the twisted moduli, and we show how this problem may be successfully resolved.

In this chapter, then, we shall consider twisted moduli contributions to SUSY breaking in effective type I string constructions based on a general set-up involving intersecting $D5_i$ and $D9$-branes, using Goldstino angles to parametrise the SUSY breaking. It is well known that the F-term VEVs of the twisted moduli fields provide a new source of gaugino masses [79]. It is also generally assumed that states that do not live in the same world-volume should receive zero soft mass contributions in the twisted moduli dominated limit, which offers a possible string realisation of gaugino mediated SUSY breaking ($\tilde{g}MSB$) [24, 25, 79]. However we show that the standard form of Kähler potential is not consistent with this physical requirement. We therefore propose a new form of the Kähler potential which is consistent at leading order with the sequestered form proposed by Randall and Sundrum [22], and which leads to exponentially suppressed sequestered soft masses, in agreement with physical expectations. Including the effects of Green-Schwarz mixing we then write down soft scalar and trilinear masses arising from a general string construction involving intersecting $D5_i$ and $D9$-branes in the presence of untwisted and twisted moduli. We show how the results may be applied to $\tilde{g}MSB$ and discuss two explicit scenarios for this. The general results will be useful for phenomenological studies involving a combination of gravity and gaugino mediated SUSY breaking due to the dilaton, untwisted and twisted moduli contributions, and enable the soft masses to be studied as a function of the finite compactification radii.

3.2 Effective Type I String Theory and Twisted Moduli

3.2.1 Kähler Potentials

In this section we will introduce a generic type I string construction involving intersecting $D5_i$-branes embedded within $D9$-branes, where coincident D-branes give rise to gauge groups localised within the world-volume of the corresponding D-brane. In section 1.4.2, we discussed how chiral charged matter fields appear as open-strings with their ends attached to D-branes. Chan-Paton factors at the string ends carry the gauge quantum numbers under the attached gauge group. This type of construction leads to two distinct types of matter field - $C_j^{5_i}$ and C_j^9 are open strings with both ends attached to the same $D5(9)$-brane, while $C^{5_i5_j}$ and C^{95_i} have their ends attached to different D-branes and the string tension forces the inverse length of the strings to become of order the string scale M_*. The $C^{5_i5_j}$ states become localised at the 4d intersection point between the two D5-branes, while the C^{95_i} states have one end attached anywhere along the 5_i-brane world-volume. The spectrum also contains closed strings that correspond to the gravity multiplet and dilaton (S) and moduli fields (T_i). Notice that this construction is entirely general and is T-dual to alternative scenarios involving D7- and D3-branes. A construction involving two sets of intersecting branes within a D9-brane is shown in Figure 3.1, but our analysis can be extended for a full set of three perpendicular intersecting branes and the open/closed string states that result.

We can now exploit the string duality between 10d $SO(32)$ heterotic theory and 10d type I theory to derive the 4d Kähler potential $K(S, T_i, C_a)$ for the dilaton S, (untwisted) moduli T_i and charged chiral superfields C_a that arise in the low energy supergravity description of the model with two sets of intersecting D5-branes embed-

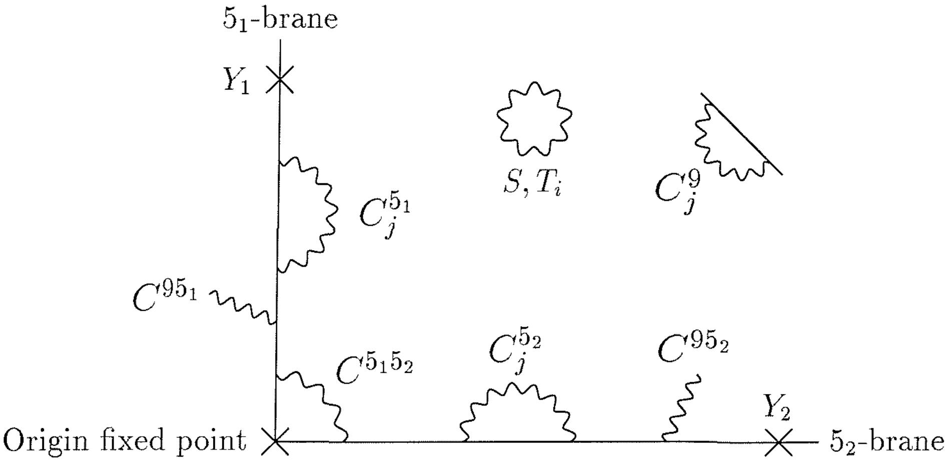


Figure 3.1: A generic type I string construction involving two sets of perpendicular D5-branes embedded within a D9-brane, where the D5-brane world-volumes intersect at the origin. Charged chiral fields appear as open strings with both ends attached to the same D-brane $C_j^{5_i}$ and C_j^9, or different branes $C^{5_1 5_2}$ and C^{95_i}. Closed strings (S, T_i) can live in the full 10d space, although orbifolding leads to closed strings (twisted moduli Y_k) localised at 4d fixed points within the $D5_i$-brane world-volume.

ded within a D9-brane as shown in Figure 3.1. Ignoring the twisted moduli for the moment, the result is [39]:

$$
\begin{aligned}
K = &-\ln\left(S+\bar{S}-|C_1^{5_1}|^2-|C_2^{5_2}|^2\right)-\ln\left(T_1+\bar{T}_1-|C_1^9|^2-|C_3^{5_2}|^2\right)\\
&-\ln\left(T_2+\bar{T}_2-|C_2^9|^2-|C_3^{5_1}|^2\right)-\ln\left(T_3+\bar{T}_3-|C_3^9|^2-|C_2^{5_1}|^2-|C_1^{5_2}|^2\right)\\
&+\frac{|C^{5_1 5_2}|^2}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}}+\frac{|C^{95_1}|^2}{(T_2+\bar{T}_2)^{1/2}(T_3+\bar{T}_3)^{1/2}}+\frac{|C^{95_2}|^2}{(T_1+\bar{T}_1)^{1/2}(T_3+\bar{T}_3)^{1/2}}
\end{aligned}
\tag{3.5}
$$

where the results can easily be extended to include a third 5_3-brane.

Expanding the Kähler potential in the lowest order in the matter fields (i.e. $(S+\bar{S}) \gg |C_1^{5_1}|^2+|C_2^{5_2}|^2$) yields:

$$
K=-\ln\left(S+\bar{S}\right)-\sum_{i=1}^{3}\ln\left(T_i+\bar{T}_i\right)+\sum_{i=1}^{2}\frac{|C_i^{5_i}|^2}{(S+\bar{S})}+\frac{|C_3^{5_2}|^2}{(T_1+\bar{T}_1)}+\frac{|C_1^9|^2}{(T_1+\bar{T}_1)} \tag{3.6}
$$

$$+\frac{|C_3^{5_1}|^2}{(T_2+\bar{T}_2)}+\frac{|C_2^9|^2}{(T_2+\bar{T}_2)}+\frac{|C_2^{5_1}|^2}{(T_3+\bar{T}_3)}+\frac{|C_1^{5_2}|^2}{(T_3+\bar{T}_3)}+\frac{|C_3^9|^2}{(T_3+\bar{T}_3)}$$
$$+\frac{|C^{5_15_2}|^2}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}}+\frac{|C^{95_1}|^2}{(T_2+\bar{T}_2)^{1/2}(T_3+\bar{T}_3)^{1/2}}+\frac{|C^{95_2}|^2}{(T_1+\bar{T}_1)^{1/2}(T_3+\bar{T}_3)^{1/2}}$$

Using Appendix B, we can identify the individual Kähler metrics (which are diagonal $\tilde{K}_a = \tilde{K}_{\bar{a}b}\delta_{\bar{a}b}$) for each type of charged chiral field:

$$\begin{aligned}
\tilde{K}_{C_i^{5_i}} &= \frac{1}{(S+\bar{S})} \\
\tilde{K}_{C_j^{5_i}} &= \frac{1}{(T_k+\bar{T}_k)} && (i \neq j \neq k \neq i) \\
\tilde{K}_{C_i^9} &= \frac{1}{(T_i+\bar{T}_i)} \\
\tilde{K}_{C^{5_15_2}} &= \frac{1}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}} \\
\tilde{K}_{C^{95_i}} &= \frac{1}{(T_j+\bar{T}_j)^{1/2}(T_k+\bar{T}_k)^{1/2}} && (i \neq j \neq k \neq i)
\end{aligned} \tag{3.7}$$

The twisted moduli $Y^{k,q}$ also contribute to the Kähler potential, but the precise form of the contribution is strongly model-dependent. For simplicity we shall consider a single twisted modulus within each of the three D5-brane sectors, which we denote by Y_k where $k = 1, 2, 3$ labels the $D5_k$ branes. Each of the Y_k may be regarded as a linear combination of all the twisted moduli within that $D5_k$ brane, so that the simplified gauge kinetic function is from Eq.(3.3),

$$f_\alpha^{5_k} = T_k + \frac{s_\alpha}{4\pi} Y_k \tag{3.8}$$

Modular anomaly cancellation via the Green-Schwarz mechanism suggests that this contribution mixes twisted and untwisted moduli together while preserving modular invariance. We will work in terms of a general even function $\hat{K}$ with an argument:

$$(Y_k + \bar{Y}_k) - \delta_{GS}^{kj} \ln(T_j + \bar{T}_j) \tag{3.9}$$

For simplicity, we will initially drop the Green-Schwarz term ($\delta_{GS} = 0$) and assume a very simple form [84]

$$\hat{K}(Y_k, \bar{Y}_k) = \frac{1}{2}(Y_k + \bar{Y}_k)^2 \tag{3.10}$$

which is valid for small values of $Y_k + \bar{Y}_k$ close to the fixed point. Hence the tree-level Kähler potential for the closed string states is:

$$\bar{K} = -\ln\left(S + \bar{S}\right) - \sum_{i=1}^{3} \ln\left(T_i + \bar{T}_i\right) + \sum_{k=1}^{3} \hat{K}(Y_k, \bar{Y}_k) \tag{3.11}$$

We will repeat our analysis in the presence of a Green-Schwarz mixing term ($\delta_{GS} \neq 0$) in section 3.3.

Recall from Eq.(1.26), that the perturbative superpotential is given by:

$$\begin{aligned} W_{ren} = g_{5_1} &\left(C_1^{5_1} C_2^{5_1} C_3^{5_1} + C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2} + C_1^{5_1} C^{95_1} C^{95_1}\right) \\ +g_{5_2} &\left(C_1^{5_2} C_2^{5_2} C_3^{5_2} + C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2} + C_2^{5_2} C^{95_2} C^{95_2}\right) + g_{5_3} C^{5_1 5_2} C^{95_1} C^{95_2} \\ &+ g_9 \left(C_1^9 C_2^9 C_3^9 + C_1^9 C^{95_1} C^{95_1} + C_2^9 C^{95_2} C^{95_2}\right) \end{aligned} \tag{3.12}$$

where the Yukawa coupling constants (associated with fields arising from each 5-and 9-brane) are given by:

$$g_{5_i}^2 = \frac{4\pi}{ReT_i} \quad , \quad g_9^2 = \frac{4\pi}{ReS} \tag{3.13}$$

However, the superpotential can also receive (unknown) non-perturbative contributions, e.g. from gaugino condensation[4], that require the F-terms to be parametrised in terms of Goldstino angles.

In this general setup, we are assuming that SUSY breaking originates from the closed string sector. In the absence of a Green-Schwarz anomaly cancelling term

[4]See [85] for a recent discussion in the context of stabilising the dilaton potential in type I string theory.

in Eq.(3.10), the Kähler metric is diagonal at leading order since $ReS, ReT_i \gg |C_a|^2$. Using Eqs.(3.6,3.10,B.8) we can write down the SUSY breaking F-term VEV in terms of two Goldstino angles (θ, ϕ), where $\theta(\phi)$ describes the relative contributions from the dilaton and moduli (twisted and untwisted moduli) F-terms respectively, and we are assuming a vanishing cosmological constant V_0.

$$F_S = \sqrt{3}m_{3/2}\sin\theta e^{i\alpha_S}(K_{\bar{S}S})^{-1/2} = \sqrt{3}m_{3/2}\sin\theta e^{i\alpha_S}(S+\bar{S}) \tag{3.14}$$

$$F_{T_i} = \sqrt{3}m_{3/2}\cos\theta\sin\phi\,\Theta_i\, e^{i\alpha_i}(K_{\bar{T}_iT_i})^{-1/2} = \sqrt{3}m_{3/2}\cos\theta\sin\phi\,\Theta_i\, e^{i\alpha_i}(T_i+\bar{T}_i) \tag{3.15}$$

$$F_{Y_k} = \sqrt{3}m_{3/2}\cos\theta\cos\phi\,\Phi_k\, e^{i\alpha_{Y_k}}(K_{\bar{Y}_kY_k})^{-1/2} = \sqrt{3}m_{3/2}\cos\theta\cos\phi\,\Phi_k\, e^{i\alpha_{Y_k}} \tag{3.16}$$

where Θ_i (Φ_k) parametrise the relative contributions from different untwisted (twisted) moduli respectively, and satisfy $\sum_{i=1}^{3}\Theta_i^2 = 1$ and $\sum_{k=1}^{3}\Phi_k^2 = 1$.

We can study three limits of phenomenological interest where different sources of SUSY breaking dominate: dilaton (S) domination where $\sin\theta = 1$; untwisted moduli (T_i) domination where $\cos\theta = \sin\phi = 1$; and twisted moduli ($Y_k$) domination where $\cos\theta = \cos\phi = 1$. In the next sub-section we shall see that there is a problem with the sequestered masses in the twisted moduli dominated limit, and then we show how this problem may be resolved.

3.2.2 Problems with the Standard Kähler Potential

In order to illustrate the problem let us consider the case of a *single* linear combination of twisted moduli located inside the 5_2-brane, which we denote by Y_2, corresponding to the simplified gauge kinetic function $f_\alpha^{5_2} = T_2 + s_\alpha Y_2/4\pi$. Thus we take the SUSY breaking parameter $\Phi_2 = 1$ in Eq.(3.16). We regard this linear combination Y_2 to be located in the world-volume of the 5_2-brane at a distance $\mathcal{O}(R_{5_2})$ from the intersection states $C^{5_15_2}$. Figure 3.1 shows that only $C_j^{5_2}, C_j^9$ and C^{95_2} states can couple directly to the Y_2 twisted moduli, while $C_j^{5_1}, C^{95_1}$ are confined on the 5_1-brane, and $C^{5_15_2}$ is confined to the origin fixed point. We refer to the states $C_j^{5_1}, C^{95_1}$ and $C^{5_15_2}$ which are spatially separated from Y_2 as being sequestered from it.

Using Eqs.(3.14-3.16,B.17) with the standard Kähler metric for the intersection and 5_1-brane states of Eq.(3.7), the sequestered state scalar masses are found to be (still ignoring the Green-Schwarz mixing term $\delta_{GS} = 0$):

$$\begin{aligned} m^2_{C^{5_15_2}} &= m^2_{3/2}\left[1 - \frac{3}{2}\left(\sin^2\theta + \Theta_3^2 \cos^2\theta \sin^2\phi\right)\right] \\ m^2_{C_1^{5_1}} &= m^2_{3/2}\left[1 - 3\sin^2\theta\right] \\ m^2_{C_2^{5_1}} &= m^2_{3/2}\left[1 - 3\Theta_3^2 \cos^2\theta \sin^2\phi\right] \\ m^2_{C_3^{5_1}} &= m^2_{3/2}\left[1 - 3\Theta_2^2 \cos^2\theta \sin^2\phi\right] \\ m^2_{C^{95_1}} &= m^2_{3/2}\left[1 - \frac{3}{2}\cos^2\theta \sin^2\phi \left(\Theta_2^2 + \Theta_3^2\right)\right] \end{aligned} \tag{3.17}$$

In the twisted moduli dominated limit where the F-term F_{Y_2} is the only contribution to the SUSY breaking ($\cos\theta = \cos\phi = 1$) the intersection state masses from Eq.(3.17) are:

$$m^2_{C^{5_15_2}} = m^2_{C_j^{5_1}} = m^2_{C^{95_1}} = m^2_{3/2} \qquad (j = 1, 2, 3) \tag{3.18}$$

These soft masses are found to be independent of the separation between the origin and the fixed point at which the twisted moduli live. Physically, this is not what we expect for fields trapped at the origin fixed point. Since these states are sequestered from the twisted moduli we would expect that their soft masses be exponentially suppressed by the spatial separation between the two fixed points, as the following argument explains. In the twisted moduli dominated limit, the situation regarding the sequestered states is physically equivalent to the $\tilde{g}$MSB scenario [24, 25] as shown in Figure 3.2.

In section 2.2 we discussed that, in gaugino mediation, SUSY is broken on a 4d hidden

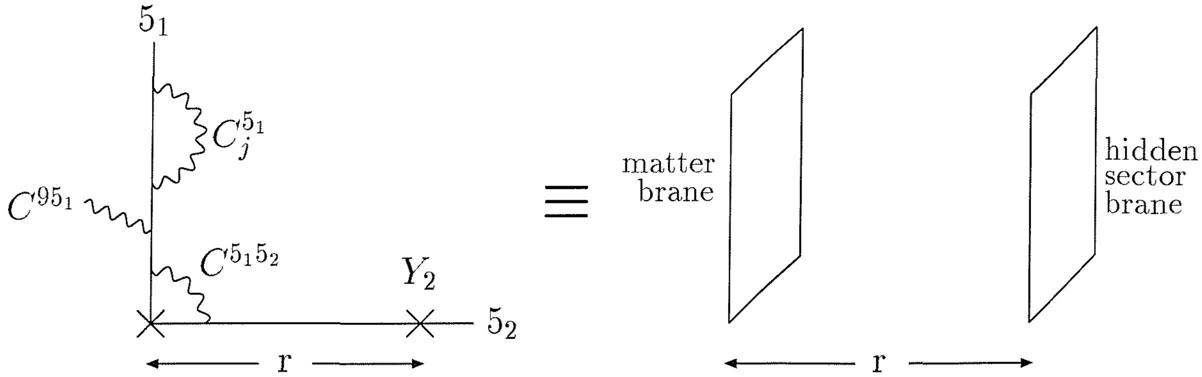


Figure 3.2: The intersecting D5-brane construction shares similar features with the gaugino mediated SUSY breaking model in the limit of a small compactification radius R_{5_1}. In this limit, the $C_j^{5_1}$ and C^{95_1} states are effectively localised at the origin, and these intersection states are equivalent to the matter brane while the localised twisted modulus is equivalent to the hidden sector brane where SUSY is broken. The spatial separation between the two fixed points (matter and hidden sector brane) is $r \sim \mathcal{O}(R_{5_2})$.

sector brane which is spatially separated along one (or more) extra dimensions from another parallel 4d matter brane where matter fields are localised. Scalar masses on the matter brane are exponentially suppressed at tree-level by the distance between the branes but are radiatively generated at one-loop via gaugino mediation. In the string theory realisation, the rôle of the hidden sector brane is played by the twisted moduli localised at a non-trivial fixed point separated from the origin fixed point which corresponds to the matter brane.

Clearly in the limit of large spatial separation between the two fixed points $r \sim \mathcal{O}(R_{5_2})$ the sequestered soft masses should be exponentially suppressed [5]:

$$m^2_{C^{5_1 5_2}}\,,\; m^2_{C_j^{5_1}}\,,\; m^2_{C^{95_1}} \sim e^{-M_* r} m^2_{3/2} \tag{3.19}$$

[5]Physically, this suppression is due to integrating out the heavier modes (with masses above the cutoff M_*) that propagate between sectors with a Yukawa-like propagator.

where M_* is the ultraviolet cutoff for the effective theory, which we will associate with the string scale. Obviously, a sufficiently large separation will lead to a negligibly small mass as in $\tilde{g}MSB$ which offers a solution to the flavour problems and suppression of flavour-changing neutral-currents. In the next sub-section we propose a new form of the Kähler potential which give rise to the correct exponentially suppressed soft sequestered masses in Eq.(3.19) rather than the result in Eq.(3.18). In Appendix G we consider an alternative exponential suppression factor $e^{-(M_* r)^2}$ that is attributed to non-perturbative world-sheet instanton corrections [86].

3.2.3 A New Kähler Potential

We need to modify the intersection state Kähler potentials $\tilde{K}_{C^{5_1 5_2}}, \tilde{K}_{C_j^{5_1}}$ and $\tilde{K}_{C^{95_1}}$ to give the desired exponentially suppressed mass prediction in Eq.(3.17) with an explicit dependence on the separation. Notice that in the limit of very small separation, we should be able to recover the previous (standard) form of Eq.(3.7). To begin with we will only consider the Kähler potential for $C^{5_1 5_2}$ states and later generalise to the $C_j^{5_1}, C^{95_1}$ states as well.

Consider the scalar mass relation of Eq.(B.17) for the intersection states $C^{5_1 5_2}$ in the limit of twisted moduli domination and let us determine the Kähler potential from the requirement that it be exponentially suppressed:

$$\begin{aligned} m^2_{C^{5_1 5_2}} &= m^2_{3/2} - F_{\bar{Y}_2} F_{Y_2} \partial_{\bar{Y}_2} \partial_{Y_2} \left(\ln \tilde{K}_{C^{5_1 5_2}} \right) \\ &= m^2_{3/2} \left[1 - \frac{3}{\tilde{K}_{C^{5_1 5_2}}} \partial_{\bar{Y}_2} \partial_{Y_2} \tilde{K}_{C^{5_1 5_2}} + \frac{3}{(\tilde{K}_{C^{5_1 5_2}})^2} \left(\partial_{\bar{Y}_2} \tilde{K}_{C^{5_1 5_2}} \right) \partial_{Y_2} \tilde{K}_{C^{5_1 5_2}} \right] \\ &\equiv e^{-R_{5_2} M_*} m^2_{3/2} \end{aligned} \tag{3.20}$$

where $F_{Y_2} = \sqrt{3} m_{3/2} \cos\theta \cos\phi \, e^{i\alpha_{Y_2}}$; twisted moduli domination corresponds to $\cos\theta = \cos\phi = 1$, and R_{5_2} is the compactification radius of the second complex dimension and is of order the separation between the hidden sector Y_2 moduli and the intersection states. The 4d untwisted moduli field T_i can be decomposed into real and imaginary parts:

$$T_i = \frac{2R_i^2 M_*^2}{\lambda_I} + i\eta_i \tag{3.21}$$

where $R_i^2 = R_{5_i}^2$ is the compactification radius on the i^{th} torus, M_* is the string scale, η_i is an untwisted closed string from the Ramond-Ramond sector, and λ_I is the 10d dilaton which is related to the fundamental (perturbative) string coupling and can be set equal to unity. We can find a relationship between the real part of the untwisted T-modulus field T_2 and the compactification radius R_{5_2}:

$$R_{5_2} = \frac{\sqrt{T_2 + \bar{T}_2}}{2M_*} \tag{3.22}$$

We solve for the Kähler potential that leads to the equivalence of the last two lines of Eq.(3.20):

$$\tilde{K}_{C^{5_1 5_2}} = exp\left[\left(1 - e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\frac{(Y_2+\bar{Y}_2)^2}{6}\right]\zeta[S,T_1,T_3] \tag{3.23}$$

where ζ is some arbitrary function of S, T_1 and/or T_3. The condition that the previous expression for the Kähler potential of Eq.(3.6) is reproduced in the limit of a small compactification radius, i.e. $R_{5_2} \sim \sqrt{T_2+\bar{T}_2} \longrightarrow 0$ fixes the function $\zeta[S,T_1,T_3]$:

$$\tilde{K}_{C^{5_1 5_2}} \longrightarrow \zeta[S,T_1,T_3] \equiv \frac{1}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}} \tag{3.24}$$

Then in the limit of very large separation $R_{5_2} \sim \sqrt{T_2+\bar{T}_2} \longrightarrow \infty$:

$$\tilde{K}_{C^{5_1 5_2}} \longrightarrow \frac{e^{(Y_2+\bar{Y}_2)^2/6}}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}} \tag{3.25}$$

In this limit, using Eqs.(3.14-3.16,B.17) we obtain:

$$m^2_{C^{5_1 5_2}} = m^2_{3/2}\left[1 - \frac{3}{2}\left(\sin^2\theta + \Theta_3^2\cos^2\theta\sin^2\phi\right) - \cos^2\theta\cos^2\phi\right] \tag{3.26}$$

which replaces the form in Eq.(3.17) in the large separation limit, and vanishes in the limit of twisted moduli domination ($\cos\theta = \cos\phi = 1$) due to the strong exponential suppression factor.

As shown in Appendix F, Randall and Sundrum [22] have discussed the conditions under which a visible matter sector may be sequestered from a SUSY breaking hidden sector. The Kähler potential in Eq.(F.7) leads to scalar masses that vanish exactly:

$$K_{RS} = -3\tilde{M}^2_{Pl}\ln\left[1 + e^{-K_{vis}/3\tilde{M}^2_{Pl}} + e^{-K_{hid}/3\tilde{M}^2_{Pl}}\right] \tag{3.27}$$

where $K_{vis}(K_{hid})$ is the separate Kähler potential for the visible (hidden) sectors, and $\tilde{M}_{Pl}$ is the reduced Planck mass. If we write the visible sector (C) and hidden sector (Y_2) Kähler potentials as:

$$K_{vis} = 3\,|C|^2 \quad , \quad K_{hid} = \frac{3}{2}(Y_2 + \bar{Y_2})^2 \tag{3.28}$$

then the combined Kähler potential may be expanded for small $|C|^2$ and $(Y_2+\bar{Y_2})^2$ as:

$$\begin{aligned} K_{RS} &= -3\tilde{M}^2_{Pl}\ln\left[1 + e^{-|C|^2/\tilde{M}^2_{Pl}} + e^{-(Y_2+\bar{Y_2})^2/2\tilde{M}^2_{Pl}}\right] \\ &= \frac{1}{2}(Y_2+\bar{Y_2})^2 + \mathcal{O}\left[(Y_2+\bar{Y_2})^4\right] + |C|^2\left[1 + \frac{(Y_2+\bar{Y_2})^2}{6\tilde{M}^2_{Pl}} - \frac{(Y_2+\bar{Y_2})^4}{72\tilde{M}^4_{Pl}}\right] + \ldots \end{aligned} \tag{3.29}$$

The expansion of the coefficient of $|C|^2$ in Eq.(3.29) is equivalent to the expansion of $e^{(Y_2+\bar{Y_2})^2/6}$ in Eq.(3.25) up to $\mathcal{O}\left[(Y_2+\bar{Y_2})^4\right]$, where we have adopted "natural" units and set $\tilde{M}_{Pl} = 1$. Therefore the numerator in Eq.(3.25) is equivalent to the Randall-Sundrum sequestered form of the Kähler potential in Eq.(3.29) up to $\mathcal{O}\left[(Y_2+\bar{Y_2})^4\right]$, which is sufficient for all practical purposes.

We can now write down the modified form of the tree-level Kähler potential that yields the "correct" mass for the intersection states $C^{5_1 5_2}$ (and similarly the 5_1-brane states $C_j^{5_1}$ and C^{95_1}) - in the limit of Y_2-domination - with an explicit dependence on the separation between the intersection point and the twisted moduli hidden sector:

$$\begin{aligned} K(S,\bar{S},T_i,\bar{T}_i,Y_2,\bar{Y}_2) &= -\ln\left(S+\bar{S}\right) - \sum_{i=1}^{3}\ln\left(T_i+\bar{T}_i\right) + \frac{1}{2}(Y_2+\bar{Y}_2)^2 \\ &+ \sum_{C_1^{5_1}} \frac{\xi(T_2,Y_2)}{(S+\bar{S})}|C_1^{5_1}|^2 + \sum_{C_2^{5_1}} \frac{\xi(T_2,Y_2)}{(T_3+\bar{T}_3)}|C_2^{5_1}|^2 + \sum_{C_3^{5_1}} \frac{\xi(T_2,Y_2)}{(T_2+\bar{T}_2)}|C_3^{5_1}|^2 \\ &+ \sum_{C_1^{5_2}} \frac{|C_1^{5_2}|^2}{(T_3+\bar{T}_3)} + \sum_{C_2^{5_2}} \frac{|C_2^{5_2}|^2}{(S+\bar{S})} + \sum_{C_3^{5_2}} \frac{|C_3^{5_2}|^2}{(T_1+\bar{T}_1)} + \sum_{C^{5_1 5_2}} \frac{\xi(T_2,Y_2)}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}}|C^{5_1 5_2}|^2 \\ &+ \sum_{C_1^{9}} \frac{|C_1^{9}|^2}{(T_1+\bar{T}_1)} + \sum_{C_2^{9}} \frac{|C_2^{9}|^2}{(T_2+\bar{T}_2)} + \sum_{C_3^{9}} \frac{|C_3^{9}|^2}{(T_3+\bar{T}_3)} \\ &+ \sum_{C^{95_1}} \frac{\xi(T_2,Y_2)}{(T_2+\bar{T}_2)^{1/2}(T_3+\bar{T}_3)^{1/2}}|C^{95_1}|^2 + \sum_{C^{95_2}} \frac{|C^{95_2}|^2}{(T_1+\bar{T}_1)^{1/2}(T_3+\bar{T}_3)^{1/2}} \end{aligned} \tag{3.30}$$

where

$$\xi(T_2,Y_2) = exp\left[\left(1-e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\frac{(Y_2+\bar{Y}_2)^2}{6}\right] \tag{3.31}$$

Before we repeat this analysis with the Green-Schwarz mechanism to cancel anomalies, we will show that our result can be generalised to a construction involving three perpendicular D5-branes that all intersect at the origin. We will assume that there are three separate linear combinations of twisted moduli Y_i - one combination within each $D5_i$-brane world-volume - each at a distance $\mathcal{O}(R_{5_i})$ from the origin intersection states, where

$$R_{5_i} = \frac{\sqrt{T_i+\bar{T}_i}}{2M_*} \tag{3.32}$$

We can immediately write down the form of the Kähler potential that will give the correct prediction for the masses in the Y-dominated SUSY breaking limit:

$$
\begin{aligned}
K \supset \frac{\xi(T_2,Y_2)\,\xi(T_3,Y_3)}{(S+\bar{S})}|C_1^{5_1}|^2 + \frac{\xi(T_1,Y_1)\,\xi(T_3,Y_3)}{(T_3+\bar{T}_3)}|C_1^{5_2}|^2 \\
+\frac{\xi(T_1,Y_1)\,\xi(T_2,Y_2)}{(T_2+\bar{T}_2)}|C_1^{5_3}|^2 + \frac{\xi(T_1,Y_1)\,\xi(T_2,Y_2)\,\xi(T_3,Y_3)}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}}|C^{5_1 5_2}|^2 \\
+\frac{\xi(T_2,Y_2)\,\xi(T_3,Y_3)}{(T_2+\bar{T}_2)^{1/2}(T_3+\bar{T}_3)^{1/2}}|C^{95_1}|^2 + \frac{\xi(T_1,Y_1)\,\xi(T_3,Y_3)}{(T_1+\bar{T}_1)^{1/2}(T_3+\bar{T}_3)^{1/2}}|C^{95_2}|^2 + \ldots
\end{aligned}
\tag{3.33}
$$

Notice that the $C_j^{5_i}$ and C^{95_i} states will couple directly to the twisted moduli within the same brane (Y_i), but will receive exponentially suppressed SUSY breaking contributions from the twisted moduli on different branes ($Y_{k\neq i}$). The C_j^9 states live in the full 10d space and therefore can couple to all twisted moduli.

3.3 Green-Schwarz Mixing

In this section we will repeat our previous analysis, but with the inclusion of an anomaly cancelling Green-Schwarz term that requires mixing between the twisted and untwisted moduli fields as discussed in section 1.4.2. This mixing leads to a non-diagonal Kähler metric (at leading order) and we use a canonically normalising P-matrix in our parametrisation to define the SUSY breaking F-terms [6].

For simplicity, we will again assume that only a single linear combination of twisted moduli fields Y_2 (within the $D5_2$-brane world-volume at a distance $\mathcal{O}(R_{5_2})$ from the intersection point) contributes to the SUSY breaking. Since only the twisted moduli from the 5_2-brane contribute, it is not too unreasonable to suppose that only the T_2 untwisted modulus field participates in the anomaly cancellation. Using Eqs.(3.9,3.10)

[6] See Appendix B.2 for details.

we propose that Y_2 has the following Kähler potential [84]:

$$\hat{K}(Y_2, \bar{Y_2}) = \frac{1}{2}\left[Y_2 + \bar{Y_2} - \delta_{GS}\ln(T_2 + \bar{T_2})\right]^2 \tag{3.34}$$

We can calculate the Kähler metric by using Eq.(3.30) with the modified twisted moduli potential:

$$K_{J\bar{I}} = \begin{pmatrix} \frac{1}{(S+\bar{S})^2} & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{(T_1+\bar{T}_1)^2} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{(T_2+\bar{T}_2)^2}\left(k + \delta_{GS}^2\right) & 0 & -\frac{\delta_{GS}}{T_2+\bar{T}_2} \\ 0 & 0 & 0 & \frac{1}{(T_3+\bar{T}_3)^2} & 0 \\ 0 & 0 & -\frac{\delta_{GS}}{T_2+\bar{T}_2} & 0 & 1 \end{pmatrix} \tag{3.35}$$

where $I = S, T_i, Y_2$ and $k = 1 + \delta_{GS}\left[Y_2 + \bar{Y_2} - \delta_{GS}\ln(T_2 + \bar{T_2})\right]$. For simplicity, we assume that $|C|^2 \ll (S + \bar{S}), (T_i + \bar{T}_i)$ [7].

The Kähler metric is block-diagonal, and we can canonically normalise the metric using a (5×5) P-matrix [83] as discussed in Appendix B.2. Using Eq.(B.8), the F-terms are defined as:

$$F_I \equiv \begin{pmatrix} F_S \\ F_{T_1} \\ F_{T_2} \\ F_{T_3} \\ F_{Y_2} \end{pmatrix} = \sqrt{3}m_{3/2}\,P \begin{pmatrix} \sin\theta\, e^{i\alpha_S} \\ \cos\theta\, \sin\phi\, \Theta_1\, e^{i\alpha_1} \\ \cos\theta\, \sin\phi\, \Theta_2\, e^{i\alpha_2} \\ \cos\theta\, \sin\phi\, \Theta_3\, e^{i\alpha_3} \\ \cos\theta\, \cos\phi\, e^{i\alpha_{Y_2}} \end{pmatrix} \tag{3.36}$$

where θ and ϕ are Goldstino angles, $\sum_i \Theta_i^\dagger \Theta_i = 1$ and we have included CP-phases α_i.

---

[7]Notice that in this limit the exact form of the intersection state Kähler potential $\tilde{K}_{C^{5_1 5_2}}$ is not important.

Using Eq.(3.35) and imposing the condition $P^\dagger K_{\bar{J}I} P = 1$ we obtain a very complicated expression for the P-matrix that can be expanded for large values of $(T_2 + \bar{T}_2)$ to give:

$$P = \begin{pmatrix} S+\bar{S} & 0 & 0 & 0 & 0 \\ 0 & T_1+\bar{T}_1 & 0 & 0 & 0 \\ 0 & 0 & \frac{T_2+\bar{T}_2}{\sqrt{k}} & 0 & -\frac{\delta_{GS}}{T_2+\bar{T}_2} \\ 0 & 0 & 0 & T_3+\bar{T}_3 & 0 \\ 0 & 0 & \frac{\delta_{GS}}{\sqrt{k}} + \frac{\sqrt{k}\delta_{GS}}{(T_2+\bar{T}_2)^2} & 0 & 1 - \frac{\delta_{GS}^2}{(T_2+\bar{T}_2)^2} \end{pmatrix} \tag{3.37}$$

where $k = 1 + \delta_{GS}\left[Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2 + \bar{T}_2)\right]$.

From Eqs.(3.36,3.37) we find that the SUSY breaking F-terms up to $\mathcal{O}\left[1/(T_2+\bar{T}_2)^3\right]$ are:

$$\begin{aligned} F_S &= \sqrt{3}m_{3/2}\sin\theta\, e^{i\alpha_S}\,(S+\bar{S}) \\ F_{T_1} &= \sqrt{3}m_{3/2}\cos\theta\sin\phi\,\Theta_1\, e^{i\alpha_1}\,(T_1+\bar{T}_1) \\ F_{T_2} &= \sqrt{3}m_{3/2}\cos\theta\left[\sin\phi\frac{(T_2+\bar{T}_2)}{\sqrt{k}}\,\Theta_2\, e^{i\alpha_2} - \cos\phi\, e^{i\alpha_{Y_2}}\,\frac{\delta_{GS}}{T_2+\bar{T}_2}\right] \\ F_{T_3} &= \sqrt{3}m_{3/2}\cos\theta\sin\phi\,\Theta_3\; e^{i\alpha_3}(T_3+\bar{T}_3) \\ F_{Y_2} &= \sqrt{3}m_{3/2}\cos\theta\left[\sin\phi\left(\frac{\delta_{GS}}{\sqrt{k}} + \frac{\sqrt{k}\delta_{GS}}{(T_2+\bar{T}_2)^2}\right)\Theta_2\, e^{i\alpha_2}\right. \\ &\qquad\qquad\qquad\qquad \left. + \cos\phi\, e^{i\alpha_{Y_2}}\left(1 - \frac{\delta_{GS}^2}{(T_2+\bar{T}_2)^2}\right)\right] \end{aligned} \tag{3.38}$$

Notice that in the limit of T_2 (or Y_2) modulus domination, *both* F_{T_2} and F_{Y_2} are nonzero. However, setting $\cos\theta = \cos\phi = 1$ still corresponds to the Y_2-domination limit, even in the presence of Green-Schwarz mixing, and we expect the intersection state masses to depend on the separation from the Y_2-fields as before.

Our previous analysis, in the absence of a Green-Schwarz mixing term, leads us to propose the following generalisation of the Kähler potential $\tilde{K}_{C^{5_1 5_2}}$ in Eq.(3.23):

$$\tilde{K}_{C^{5_1 5_2}} = \frac{exp\left[\frac{1}{6}\left(1 - e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\left\{Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2\right]}{(S+\bar{S})^{1/2}(T_3+\bar{T}_3)^{1/2}} \tag{3.39}$$

which leads to an exponentially suppressed intersection state mass in the limit of Y_2-domination:

$$m^2_{C^{5_1 5_2}} = e^{-\sqrt{T_2+\bar{T}_2}/2}\, m^2_{3/2} + \mathcal{O}\left[\frac{1}{(T_2+\bar{T}_2)^{3/2}}\right] \tag{3.40}$$

Similar results apply to the $C^{5_1}_j$ and C^{95_1} states and we obtain a Kähler potential as in Eq.(3.30), but with Eq.(3.31) generalised to

$$\xi(T_2, Y_2) = exp\left[\frac{1}{6}\left(1 - e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\left\{Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2\right] \tag{3.41}$$

In section 3.5 we will consider an explicit example and analyse the soft parameters in various limits.

Notice that our comment at the end of section 3.2.3 about including the effects of multiple SUSY breaking twisted moduli still holds. The previous expression of Eq.(3.33) is easily generalised by replacing the arguments as follows:

$$(Y_k + \bar{Y}_k) \longrightarrow (Y_k + \bar{Y}_k) - \delta_{GS}\ln(T_k + \bar{T}_k). \tag{3.42}$$

3.4 Soft SUSY Breaking Parameters

We can now write down the complete list of soft scalar masses and trilinears that arise in a general string construction involving two intersecting D5-branes embedded within a D9-brane, where a single linear combination of twisted moduli Y_2 is located at a fixed point within the world-volume of one of the branes (5_2). It is straightforward to generalise these results to more twisted moduli fields, and we have explicitly discussed the case of three twisted moduli Y_i in section 3.2. The results presented in this section will be useful for performing more general phenomenological analyses of sparticle spectra in string theory than have been done so far in the literature [8].

Note that the gaugino masses require knowledge of the gauge group embedding, and therefore gaugino masses are more model-dependent. We will consider a simple example in section 3.5.

3.4.1 Scalar Masses

Using Eqs.(3.30,3.38,3.41,B.17) we can write down the scalar masses for the non-sequestered states $C_j^{5_2}, C_j^9$ and C^{95_2} which couple directly to the twisted moduli Y_2:

$$\begin{aligned}
m^2_{C_1^{5_2}} = m^2_{C_3^9} &= m^2_{3/2}\left[1 - 3\Theta_3^2 \cos^2\theta \sin^2\phi\right] \\
m^2_{C_2^{5_2}} &= m^2_{3/2}\left[1 - 3\sin^2\theta\right] \\
m^2_{C_3^{5_2}} = m^2_{C_1^9} &= m^2_{3/2}\left[1 - 3\Theta_1^2 \cos^2\theta \sin^2\phi\right] \\
m^2_{C_2^9} &= m^2_{3/2}\left[1 - 3\Theta_2^2 \cos^2\theta \sin^2\phi\right] \\
m^2_{C^{95_2}} &= m^2_{3/2}\left[1 - \frac{3}{2}\cos^2\theta \sin^2\phi \left(\Theta_1^2 + \Theta_3^2\right)\right]
\end{aligned} \tag{3.43}$$

The sequestered states $C_j^{5_2}, C^{5_15_2}$ and C^{95_1} are spatially separated from the twisted modulus field Y_2 and have masses of the form:

$$\begin{aligned}
m^2_{C^{5_15_2}} &= \tilde{m}^2 - \frac{3}{2}m^2_{3/2}\left(\sin^2\theta + \Theta_3^2 \cos^2\theta \sin^2\phi\right) \\
m^2_{C^{95_1}} &= \tilde{m}^2 - \frac{3}{2}m^2_{3/2} \cos^2\theta \sin^2\phi \left(\frac{\Theta_2^2}{k} + \Theta_3^2\right)
\end{aligned}$$

[8]In appendix G we consider an alternative exponential suppression factor $e^{-(M_* r)^2}$ that can be attributed to non-perturbative world-sheet instanton corrections [86], and we calculate the modified expressions for scalar masses and trilinears.

$$
\begin{aligned}
m^2_{C^{5_1}_1} &= \tilde{m}^2 - 3m^2_{3/2}\sin^2\theta \\
m^2_{C^{5_1}_2} &= \tilde{m}^2 - 3m^2_{3/2}\Theta_3^2\cos^2\theta\sin^2\phi \\
m^2_{C^{5_1}_3} &= \tilde{m}^2 - \frac{3}{k}m^2_{3/2}\Theta_2^2\cos^2\theta\sin^2\phi
\end{aligned}
\tag{3.44}
$$

where

$$
\begin{aligned}
\tilde{m}^2 = m^2_{3/2}\Bigg[1 &- \cos^2\theta\cos^2\phi\left(1-e^{-\sqrt{T_2+\bar{T}_2}/2}\right) \\
&-\frac{\cos^2\theta\sin^2\phi\,\Theta_2^2\,\delta_{GS}}{k}\left(1-e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\left\{Y_2+\bar{Y}_2-\delta_{GS}\ln(T_2+\bar{T}_2)\right\} \\
&+\frac{\cos^2\theta\sin^2\phi\,\Theta_2^2\,e^{-\sqrt{T_2+\bar{T}_2}/2}}{32k}\sqrt{T_2+\bar{T}_2}\left\{Y_2+\bar{Y}_2-\delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2\left(2+\sqrt{T_2+\bar{T}_2}\right) \\
&-\frac{\cos^2\theta\cos\phi\sin\phi\left(\Theta_2\,e^{i(\alpha_2-\alpha_{Y_2})}+\Theta_2^\dagger\,e^{-i(\alpha_2-\alpha_{Y_2})}\right)e^{-\sqrt{T_2+\bar{T}_2}/2}}{32\sqrt{k}\,(T_2+\bar{T}_2)}\left\{Y_2+\bar{Y}_2-\delta_{GS}\ln(T_2+\bar{T}_2)\right\} \\
&\times\left(8(T_2+\bar{T}_2)^{3/2}+\delta_{GS}\left\{Y_2+\bar{Y}_2-\delta_{GS}\ln(T_2+\bar{T}_2)\right\}\right)\Bigg]+\mathcal{O}\left[\frac{\delta_{GS}}{(T_2+\bar{T}_2)^{3/2}}\right]
\end{aligned}
\tag{3.45}
$$

In the limit of a large separation, $R_{5_2}\sim\sqrt{T_2+\bar{T}_2}\to\infty$

$$
\begin{aligned}
\tilde{m}^2 \to m^2_{3/2}\Bigg[1 &- \cos^2\theta\cos^2\phi \\
&-\frac{\cos^2\theta\sin^2\phi\,\Theta_2^2\,\delta_{GS}}{k}\left\{Y_2+\bar{Y}_2-\delta_{GS}\ln(T_2+\bar{T}_2)\right\}\Bigg]
\end{aligned}
\tag{3.46}
$$

and for a small separation $R_{5_2}\sim\sqrt{T_2+\bar{T}_2}\to 0$

$$
\tilde{m}^2 \to m^2_{3/2} \tag{3.47}
$$

Now we will consider the different limits of SUSY breaking:

- Dilaton domination ($\sin\theta = 1$):

$$
m^2_{C^{5_2}_{1,3}} = m^2_{C^{5_1}_{2,3}} = m^2_{C^9_{1,2,3}} = m^2_{C^{95_{1,2}}} = m^2_{3/2}
$$

$$
\begin{aligned}
m^2_{C^{5_2}_2} &= m^2_{C^{5_1}_1} = -2m^2_{3/2} \\
m^2_{C^{5_1 5_2}} &= -\frac{1}{2}m^2_{3/2}
\end{aligned}
\tag{3.48}
$$

Notice that this limit generally gives rise to tachyonic states.

- T-moduli domination ($\cos\theta = \sin\phi = 1$):

$$
\begin{aligned}
m^2_{C^{5_2}_1} &= m^2_{C^9_3} = m^2_{3/2}\left(1 - 3\Theta_3^2\right) \quad , \quad m^2_{C^{5_2}_2} = m^2_{3/2} \\
m^2_{C^{5_2}_3} &= m^2_{C^9_1} = m^2_{3/2}\left(1 - 3\Theta_1^2\right) \quad , \quad m^2_{C^9_2} = m^2_{3/2}\left(1 - 3\Theta_2^2\right) \\
m^2_{C^{5_1 5_2}} &= \tilde{m}^2_T - \frac{3}{2}m^2_{3/2}\Theta_3^2 \quad , \quad m^2_{C^{95_2}} = m^2_{3/2}\left[1 - \frac{3}{2}\left(\Theta_1^2 + \Theta_3^2\right)\right] \\
m^2_{C^{95_1}} &= \tilde{m}^2_T - \frac{3}{2}m^2_{3/2}\left(\frac{\Theta_2^2}{k} + \Theta_3^2\right) \quad , \quad m^2_{C^{5_1}_1} = \tilde{m}^2_T \\
m^2_{C^{5_1}_2} &= \tilde{m}^2_T - 3m^2_{3/2}\Theta_3^2 \quad , \quad m^2_{C^{5_1}_3} = \tilde{m}^2_T - \frac{3}{k}m^2_{3/2}\Theta_2^2
\end{aligned}
\tag{3.49}
$$

where

$$
\begin{aligned}
\tilde{m}^2_T = m^2_{3/2}\Bigg[1 &- \frac{\Theta_2^2\,\delta_{GS}}{k}\left(1 - e^{-\sqrt{T_2+\bar{T}_2}/2}\right)\left\{Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2 + \bar{T}_2)\right\} \\
&+ \frac{\Theta_2^2\, e^{-\sqrt{T_2+\bar{T}_2}/2}}{32k}\sqrt{T_2 + \bar{T}_2}\left\{Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2 + \bar{T}_2)\right\}^2\left(2 + \sqrt{T_2 + \bar{T}_2}\right)\Bigg]
\end{aligned}
\tag{3.50}
$$

- Y_2-moduli domination ($\cos\theta = \cos\phi = 1$):

$$
\begin{aligned}
m^2_{C^{5_2}_{1,2,3}} &= m^2_{C^9_{1,2,3}} = m^2_{C^{95_2}} = m^2_{3/2} \\
m^2_{C^{5_1 5_2}} &= m^2_{C^{5_1}_{1,2,3}} = m^2_{C^{95_1}} = e^{-\sqrt{T_2+\bar{T}_2}/2}\, m^2_{3/2}
\end{aligned}
\tag{3.51}
$$

where the 5_2-and 9-brane states couple directly to the SUSY breaking twisted moduli and are not exponentially suppressed.

Physically the twisted Y_2 moduli dominated limit, corresponds to gaugino mediated

SUSY breaking, if the standard model quark and lepton states are identified with the sequestered states (see later example). The dilaton and T-moduli domination limits correspond to different examples of gravity mediated SUSY breaking. In the general case where we are not in any particular limit, SUSY breaking will have contributions from the F-terms of the dilaton and untwisted moduli as well as the twisted moduli, and then we must use the general formulae for the scalar masses in Eqs.(3.43,3.44).

3.4.2 Trilinears

The trilinear and Yukawa couplings arise from the superpotential, where the dominant tree-level contributions are shown in Eq.(3.12) in terms of open string states. The precise structure of the Yukawa and trilinear matrices depend on the identification of these string states with MSSM fields. As shown in Eq.(3.12), the leading terms are constrained by string selection rules and gauge invariance. Higher order corrections can be generated by higher-dimensional operators where powers of the model cutoff (e.g. the string or Planck scales) lead to a large suppression. We will illustrate how different identifications lead to alternative Yukawa structures in section 3.5, as recently discussed in Ref. [87].

We can list the dominant trilinear couplings that arise from the perturbative superpotential of Eq.(3.12). Using Eqs.(3.30,3.38,3.41,B.19) and making the standard assumption that the Yukawa couplings Y_{abc} have no dependence on the dilaton and moduli fields ($\partial_I \ln Y_{abc} = 0$) [9] we find:

$$\begin{aligned} A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} &= -\sqrt{3} m_{3/2} \cos\theta \left[\sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}} \right. \\ & \left. + \sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{12\sqrt{k}} \, e^{-\sqrt{T_2+\bar{T}_2}/2} \sqrt{T_2+\bar{T}_2} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2) \right\}^2 \right. \end{aligned} \qquad (3.52)$$

[9]Notice that by definition, the Yukawa couplings are related to the moduli fields through Eq.(3.13) so this assumption is not really valid, but we make it for illustrative purposes so that our results may be compared to others in the literature.

$$- \cos\phi \, \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-\sqrt{T_2+\bar{T}_2}/2}\right) \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\Big]$$

$$\begin{aligned} A_{C_1^9 C^{95_1} C^{95_1}} &= -\sqrt{3} m_{3/2} \Big[\sin\theta \, e^{i\alpha_S} \\ &+ \cos\theta \sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{12\sqrt{k}} e^{-\sqrt{T_2+\bar{T}_2}/2} \sqrt{T_2+\bar{T}_2} \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}^2 \\ &- \cos\theta \cos\phi \, \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-\sqrt{T_2+\bar{T}_2}/2}\right) \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\Big] \end{aligned} \tag{3.53}$$

$$\begin{aligned} A_{C^{5_1 5_2} C^{95_1} C^{95_2}} &= -\sqrt{3} m_{3/2} \left[\frac{1}{2} \sin\theta \, e^{i\alpha_S} + \frac{1}{2} \cos\theta \sin\phi \left(\Theta_1 \, e^{i\alpha_1} + \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}} - \Theta_3 \, e^{i\alpha_3}\right)\right. \\ &+ \cos\theta \sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{12\sqrt{k}} \, e^{-\sqrt{T_2+\bar{T}_2}/2} \sqrt{T_2+\bar{T}_2} \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}^2 \\ &\left. - \cos\theta \cos\phi \, \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-\sqrt{T_2+\bar{T}_2}/2}\right) \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\right] \end{aligned} \tag{3.54}$$

$$\begin{aligned} A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} = A_{C_1^{5_1} C^{95_1} C^{95_1}} &= -\sqrt{3} m_{3/2} \cos\theta \Big[\sin\phi \, \Theta_1 \, e^{i\alpha_1} \\ + \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{8\sqrt{k}} \, e^{-\sqrt{T_2+\bar{T}_2}/2} &\sqrt{T_2+\bar{T}_2} \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}^2 \\ - \cos\phi \, e^{i\alpha_{Y_2}} \, e^{-\sqrt{T_2+\bar{T}_2}/2} &\left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\Big] \end{aligned} \tag{3.55}$$

$$\begin{aligned} A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}} = A_{C_2^{5_2} C^{95_2} C^{95_2}} &= -\sqrt{3} m_{3/2} \cos\theta \left[\sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}}\right. \\ &\left. - \cos\phi \, e^{i\alpha_{Y_2}} \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\right] \end{aligned} \tag{3.56}$$

$$\begin{aligned} A_{C_1^9 C_2^9 C_3^9} = A_{C_2^9 C^{95_2} C^{95_2}} &= -\sqrt{3} m_{3/2} \Big[\sin\theta \, e^{i\alpha_S} \\ &- \cos\theta \cos\phi \, e^{i\alpha_{Y_2}} \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}\Big] \end{aligned} \tag{3.57}$$

Now we will consider the different limits of SUSY breaking:

- Dilaton domination ($\sin\theta = 1$):

$$\begin{aligned} A_{C_1^9 C^{95_1} C^{95_1}} = A_{C_1^9 C_2^9 C_3^9} = A_{C_2^9 C^{95_2} C^{95_2}} = 2\, A_{C^{5_1 5_2} C^{95_1} C^{95_2}} &= -\sqrt{3} m_{3/2} \, e^{i\alpha_S} \\ A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} = A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} &= 0 \\ A_{C_1^{5_1} C^{95_1} C^{95_1}} = A_{C_2^{5_2} C^{95_2} C^{95_2}} = A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}} &= 0 \end{aligned} \tag{3.58}$$

- T-moduli domination ($\cos\theta = \sin\phi = 1$):

$$\begin{aligned}
A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} &= -\sqrt{\frac{3}{k}}\, m_{3/2}\, \Theta_2\, e^{i\alpha_2} - \hat{A} \\
A_{C_1^9 C^{95_1} C^{95_1}} &= -\hat{A} \\
A_{C^{5_1 5_2} C^{95_1} C^{95_2}} &= -\frac{\sqrt{3}}{2} m_{3/2} \left(\Theta_1\, e^{i\alpha_1} + \frac{\Theta_2\, e^{i\alpha_2}}{\sqrt{k}} - \Theta_3\, e^{i\alpha_3} \right) - \hat{A} \\
A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} &= A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} = A_{C_1^{5_1} C^{95_1} C^{95_1}} = -\sqrt{3} m_{3/2}\, \Theta_1\, e^{i\alpha_1} - \frac{3}{2}\hat{A} \quad (3.59) \\
A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}} &= A_{C_2^{5_2} C^{95_2} C^{95_2}} = -\sqrt{\frac{3}{k}}\, m_{3/2}\, \Theta_2\, e^{i\alpha_2} \\
A_{C_1^9 C_2^9 C_3^9} &= A_{C_2^9 C^{95_2} C^{95_2}} = 0
\end{aligned}$$

where

$$\hat{A} = m_{3/2} \frac{\Theta_2\, e^{i\alpha_2}}{4\sqrt{3k}}\, e^{-\sqrt{T_2+\bar{T}_2}/2} \sqrt{T_2 + \bar{T}_2} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2 + \bar{T}_2) \right\}^2 \qquad (3.60)$$

- Y_2-moduli domination ($\cos\theta = \cos\phi = 1$):

$$\begin{aligned}
A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} &= A_{C_1^9 C^{95_1} C^{95_1}} = A_{C^{5_1 5_2} C^{95_1} C^{95_2}} = \left(1 + 2\, e^{-\sqrt{T_2+\bar{T}_2}/2} \right) \tilde{A} \\
A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} &= A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} = A_{C_1^{5_1} C^{95_1} C^{95_1}} = 3e^{-\sqrt{T_2+\bar{T}_2}/2}\, \tilde{A} \qquad (3.61) \\
A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}} &= A_{C_2^{5_2} C^{95_2} C^{95_2}} = A_{C_1^9 C_2^9 C_3^9} = A_{C_2^9 C^{95_2} C^{95_2}} = 3\tilde{A}
\end{aligned}$$

where

$$\tilde{A} = m_{3/2} \frac{e^{i\alpha_{Y_2}}}{\sqrt{3}} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2 + \bar{T}_2) \right\} \qquad (3.62)$$

3.5 Gaugino Mediated SUSY Breaking Revisited

The preceeding results are based on a general set-up of intersecting $D5_i$ and $D9$ branes. In order to discuss $\tilde{g}MSB$ it is sufficient to specialise to the case of just two intersecting sets of $D5$ branes, 5_1 and 5_2. This set-up arises for example in the explicit string constructions of Ref. [52]. We shall assume that the MSSM gauge group arises from the 5_2-brane only. This enables approximate gauge coupling unification to be achieved. The MSSM matter fields are identified as either $C^{5_15_2}$ or $C_j^{5_2}$ states. We assume that any $C^{5_15_2}$ states are gauge singlets with respect to any gauge groups on the 5_1-brane. Such a set-up may be achieved in practice from constructions involving severely asymmetric compactifications (for example $R_{5_2} \gg R_{5_1}$ as shown in section 2.4), where the combined gauge groups generally arise from linear combinations of groups on each set of branes. The asymmetry ensures that the dominant contributions live on the 5_2-brane, a limit we refer to as "single brane dominance".

Although the perpendicular 5_1-brane seems to be irrelevant in this scenario, in fact it plays an important rôle since the $C^{5_15_2}$ states are sequestered at a distance $r \sim \mathcal{O}(R_{5_2})$ from the fixed point associated with the twisted modulus Y_2. From Eq.(3.44) we find the soft mass for the sequestered state to be

$$m^2_{C^{5_15_2}} \;\; = \;\; \tilde{m}^2 - \frac{3}{2}m^2_{3/2}\left(\sin^2\theta + \Theta_3^2\,\cos^2\theta\,\sin^2\phi\right) \qquad (3.63)$$

where $\tilde{m}^2$ is given in Eq.(3.45). In the twisted moduli dominated limit ($\cos\theta = \cos\phi = 1$)

$$m^2_{C^{5_15_2}} = e^{-\sqrt{T_2+\bar{T}_2}/2}\, m^2_{3/2}. \qquad (3.64)$$

If the standard model states are all identified as intersection states $C^{5_15_2}$ then, for a large compactification radius, this setup corresponds to gaugino mediated SUSY breaking. However the soft mass in Eq.(3.63) is valid away from the twisted modulus dominated limit, and is also valid for a small compactification radius. It therefore allows more general and detailed studies of gaugino mediation to be performed, including the effects of finite compactification radii, and the contributions from gravity mediation effects, which in type I theories correspond to the dilaton and untwisted moduli F-term VEVs.

The non-sequestered soft masses are given by Eq.(3.43),

$$\begin{aligned} m^2_{C_1^{5_2}} = m^2_{C_3^9} &= m^2_{3/2}\left[1 - 3\Theta_3^2 \cos^2\theta \, \sin^2\phi\right] \\ m^2_{C_2^{5_2}} &= m^2_{3/2}\left[1 - 3\sin^2\theta\right] \\ m^2_{C_3^{5_2}} &= m^2_{3/2}\left[1 - 3\Theta_1^2 \cos^2\theta \, \sin^2\phi\right] \end{aligned} \tag{3.65}$$

The MSSM gauge groups all arise from the 5_2-brane, and using Eq.(3.3) with a single linear combination of twisted moduli fields within the 5_2-brane world-volume, we find:

$$f_\alpha^{5_2} = T_2 + \frac{s_\alpha}{4\pi} Y_2 \qquad (\alpha = SU(3)_C, SU(2)_L, U(1)_Y) \tag{3.66}$$

where s_α are model-dependent coefficients that depend on the details of the orbifold compactification [10].

We can find the gaugino masses using Eqs.(3.4,3.38,B.18):

$$\begin{aligned} M_\alpha = \frac{\sqrt{3} m_{3/2} \, g_\alpha^2}{8\pi} \cos\theta & \left[\sin\phi \, \Theta_2 \, e^{i\alpha_2} \left\{ \frac{T_2 + \bar{T}_2}{\sqrt{k}} + \frac{s_\alpha}{4\pi}\left(\frac{\delta_{GS}}{\sqrt{k}} + \frac{\sqrt{k}\,\delta_{GS}}{(T_2 + \bar{T}_2)^2} \right) \right\} \right. \\ & \left. - \cos\phi \, e^{i\alpha_{Y_2}} \left\{ \frac{\delta_{GS}}{T_2 + \bar{T}_2} - \frac{s_\alpha}{4\pi}\left(1 - \frac{\delta_{GS}^2}{(T_2 + \bar{T}_2)^2} \right) \right\} \right] \end{aligned} \tag{3.67}$$

Now consider different limits of SUSY breaking:

[10] Notice that for Z_3 and Z_7 orbifolds, these coefficents are proportional to the MSSM 1-loop beta-function coefficients b_α.

• Dilaton domination ($\sin\theta = 1$):

$$M_\alpha = 0 \qquad\qquad (\alpha = SU(3)_C, SU(2)_L, U(1)_Y) \tag{3.68}$$

• T-modulus domination ($\cos\theta = \sin\phi = 1$):

$$M_\alpha = \frac{\sqrt{3}m_{3/2}\,g_\alpha^2}{8\pi}\Theta_2\, e^{i\alpha_2}\left\{\frac{T_2+\bar{T}_2}{\sqrt{k}} + \frac{s_\alpha}{4\pi}\left(\frac{\delta_{GS}}{\sqrt{k}} + \frac{\sqrt{k}\,\delta_{GS}}{(T_2+\bar{T}_2)^2}\right)\right\} \tag{3.69}$$

• Y_2-modulus domination ($\cos\theta = \cos\phi = 1$):

$$M_\alpha = -\frac{\sqrt{3}m_{3/2}\,g_\alpha^2}{8\pi}e^{i\alpha_{Y_2}}\left\{\frac{\delta_{GS}}{T_2+\bar{T}_2} - \frac{s_\alpha}{4\pi}\left(1 - \frac{\delta_{GS}^2}{(T_2+\bar{T}_2)^2}\right)\right\} \tag{3.70}$$

3.5.1 Scenario A - Gaugino Mediated SUSY Breaking For All Three Families

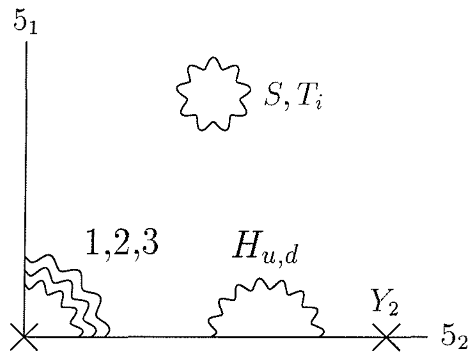


Figure 3.3: The allocation of charged chiral fields in scenario A which is similar to the gaugino mediated SUSY breaking model [24, 25]. The MSSM gauge group arises from the 5_2-brane, and all three MSSM chiral families (1,2,3) are localised at the origin, while the Higgs (H_u, H_d) and MSSM gauge fields live on the 5_2-brane. The dilaton and moduli fields S, T_i live in the full 10d space and a single twisted moduli Y_2 is localised at a fixed point inside the 5_2-brane world-volume.

In scenario A, depicted in Figure 3.3, the three chiral families are open string states localised at the origin fixed point, and the Higgs fields feel two extra dimensions as open string states with both ends attached to the 5_2-brane.

$$\begin{aligned} Q_{iL}\,,\, L_{iL}\,,\, U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR} &\equiv C^{5_15_2} \qquad (i=1,2,3) \\ H_u\,,\, H_d &\equiv C^{5_2}_j \end{aligned} \tag{3.71}$$

The Higgs states carry an extra index that plays an important rôle in constructing the perturbative superpotential from open string states. The tree-level superpotential from Eq.(3.12) contains the terms:

$$W_{ren} = g_{5_2}\left[C^{5_2}_1 C^{5_2}_2 C^{5_2}_3 + C^{5_2}_3 C^{5_15_2} C^{5_15_2}\right] \tag{3.72}$$

In order to obtain non-zero third family Yukawa couplings (at tree-level), we can immediately see that the Higgs fields must be $C^{5_2}_3$ states. This leads to a "democratic" Yukawa texture (and trilinear matrix) where all entries are equal:

$$Y^{ude}_{ij} \sim \mathcal{O}(g_{5_2}) \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{3.73}$$

The democratic structure arises due to the presence of three (indistinguishable) chiral families, localised at the origin fixed point ($C^{5_15_2}$). However, type I compactifications do not generally lead to low-energy spectra with this property as one (or more) families generally arise with both ends attached to the same D5-brane (C^{5_i}) which is the situation we will discuss in scenario B.

The squark, slepton and Higgs soft masses are given by Eqs.(3.63,3.65) with the identifications in Eq.(3.71). In general the squark and slepton ($C^{5_15_2}$) soft masses receive unsuppressed contributions from the dilaton and untwisted moduli F-term VEVs, which corresponds to the string version of conventional gravity mediation. In the limit of twisted moduli domination, we see that the quarks and lepton states acquire exponentially small soft scalar masses:

$$m^2_{Q_iL} = m^2_{U^c_{iR}} = m^2_{D^c_{iR}} = m^2_{L_iL} = m^2_{E^c_{iR}} = e^{-\sqrt{T_2+\bar{T}_2}/4}\, m^2_{3/2} \qquad (i = 1, 2, 3) \quad (3.74)$$

and the Higgs scalars obtain much larger masses due to their direct coupling with the SUSY breaking sector (twisted moduli):

$$m^2_{H_u} = m^2_{H_d} = m^2_{3/2} \qquad (3.75)$$

This yields the same spectrum as the gaugino mediated SUSY breaking scenario [24, 25], where vanishingly small scalar masses at the high-scale (due to the separation between sectors) offers an attractive (and natural) solution to the SUSY flavour problem [11]. However, unlike $\tilde{g}MSB$, the third family trilinear $A_{33} \equiv A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}}$ from Eq.(3.61) is not loop suppressed and depends on the explicit function of twisted moduli $\hat{K}(Y_2, \bar{Y}_2, T_2, \bar{T}_2)$.

The general results in Eqs.(3.63,3.65) enable us to smoothly move away from the twisted moduli dominated limit (corresponding to $\tilde{g}$MSB) and also consider the contributions of the dilaton and untwisted moduli to the soft masses (corresponding to the gravity contributions to SUSY breaking). We can also consider the effect of smoothly changing the compactification radius R_{5_2} (corresponding to varying the distance r in Figure 3.2.)

Notice that if we assign the Higgs fields as different 5_2-brane states - for example $H_u \equiv C_3^{5_2}$ and $H_d \equiv C_{1,2}^{5_2}$ - then it is possible to generate a μ-term in the tree-level

[11] As discussed in section 2.4.7, flavour-changing neutral-current suppression places a lower limit on the size of the separation.

superpotential

$$W_{ren} \supset \lambda\, N H_u H_d \longrightarrow \mu \sim \lambda\langle N\rangle \tag{3.76}$$

if we add a gauge singlet that acquires a non-zero VEV, e.g. $N \equiv C_1^{5_2}$.

3.5.2 Scenario B - Gaugino Mediated SUSY Breaking For the First and Second Families Only

In scenario B, depicted in Figure 3.4, the third family is moved on to the 5_2-brane along with the Higgs and gauge fields.

$$\begin{aligned} Q_{iL}\,,\, L_{iL}\,,\, U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR} \;&\equiv\; C^{5_1 5_2} \qquad (i=1,2) \\ Q_{3L}\,,\, L_{3L}\,,\, U^c_{3R}\,,\, D^c_{3R}\,,\, E^c_{3R}\,,\, H_u\,,\, H_d \;&\equiv\; C^{5_2}_j \end{aligned} \tag{3.77}$$

This setup is a simplification of the BMSB model of Chapter 2 in the limit of a vanishing 5_1-brane compactification radius, where the gauge fields are dominated by their components on the 5_2-brane - “single brane dominance”.

In order to generate a third family Yukawa coupling at tree-level, the Higgs and third family singlets and doublets must carry different indices. However, we are still free to choose whether the allocation of Higgs fields allows first and second family Yukawa couplings at tree-level. For example, suppose that we choose the following allocations:

$$Q_{3L}\,,\, L_{3L} \equiv C_1^{5_2} \qquad U^c_{3R}\,,\, D^c_{3R}\,,\, E^c_{3R} \equiv C_2^{5_2} \qquad H_u\,,\, H_d \equiv C_3^{5_2} \tag{3.78}$$

We will generate block-diagonal Yukawa textures that are not consistent with exper-

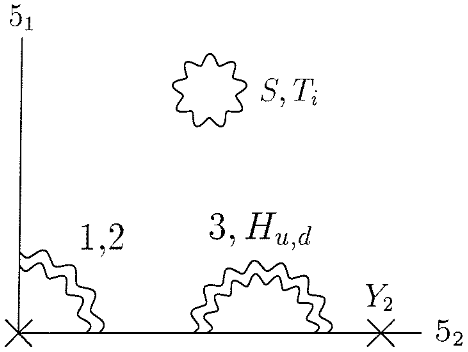


Figure 3.4: The allocation of charged chiral fields in scenario B which is a simplification of the brane mediated SUSY breaking model in Chapter 2. The MSSM gauge group arises from the 5_2-brane, and only the first two chiral families (1,2) live at the origin while the third family (3), Higgs (H_u, H_d) and gauge fields live on the 5_2-brane. The dilaton and moduli fields S, T_i live in the full 10d space and a single twisted moduli Y_2 is localised at a fixed point inside the 5_2-brane world-volume.

imental data:

$$Y_{ij}^{ude} \sim \mathcal{O}(g_{5_2}) \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.79}$$

However, if we choose that $H_{u,d} \neq C_3^{5_2}$, then we generate a Yukawa texture with only a single non-zero value in the (33) entry:

$$Y_{ij}^{ude} \sim \mathcal{O}(g_{5_2}) \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3.80}$$

which is more compatible with data, as higher order corrections can generate the required structure. We also obtain a trilinear matrix with a single (33) entry:

$$\tilde{A}_{ij}^{ude} = A_{ij} Y_{ij}^{ude} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & A_{33} \end{pmatrix} \tag{3.81}$$

where $A_{33} = A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}}$ from Eq.(3.56).

The soft masses for this scenario are very similar to scenario A, except that the third family is a $C_2^{5_2}$ state, so it is now a non-sequestered state. In the twisted moduli domination limit of Eq.(3.51), the first two families receive exponentially suppressed masses as for scenario A. However, the third family and Higgs acquire large soft masses $\sim \mathcal{O}(m_{3/2})$ which may be read off from Eq.(3.65).

As discussed earlier, the experimental constraints from FCNC data is only sensitive to the first two families [26], and this scenario (with a hierarchically larger third family) may not violate these constraints, thereby providing an interesting alternative solution to the flavour-changing problem.

3.6 Conclusions

We have considered twisted moduli contributions to supersymmetry breaking in effective type I string constructions based on intersecting $D5_i$ and $D9$-branes, using the formalism of Goldstino angles and extending the scope of previous analyses which were based on a single $D9$-brane sector. The more general set-up allows the possibility of states which are sequestered from twisted moduli states which are located at fixed points and cannot move freely. The sequestered states should have suppressed soft mass contributions from distant twisted moduli, and this observation has been used to suggest how $\tilde{g}MSB$ might be implemented in type I string theory [79]. However, contrary to this expectation, we find that the standard form of the Kähler potential leads to non-zero soft masses for the sequestered states in the twisted moduli dominated limit. This motivated us to look for a new form of Kähler potential for the sequestered states. We have proposed a new Kähler potential which is consistent at leading order with the sequestered form proposed by Randall and Sundrum [22], and which leads to exponentially suppressed sequestered soft masses. Including the effect of Green-Schwarz mixing we have derived trilinears and soft scalar masses arising from a general string construction involving intersecting $D5_i$ and $D9$-branes in the presence of untwisted and twisted moduli. We have shown how the results may be applied to $\tilde{g}MSB$, and discussed two explicit scenarios for this based on two intersecting 5_1 and 5_2 brane sectors, in which the MSSM gauge group is placed on the 5_2 sector. The second scenario in which $\tilde{g}MSB$ only applies to the first two families, while the third family receives an unsuppressed soft mass, is a simplification of the model discussed in Chapter 2, but we have extended it to incorporate both gravity and gaugino mediation effects.

These general results will be useful in phenomenological studies involving a combination of gravity and gaugino mediated SUSY breaking due to the dilaton, untwisted and twisted moduli contributions, which enable the soft masses to be studied as a function of the compactification radii. Previous analyses [83] have only considered the effect of twisted moduli in the case where the gauge group and matter fields live on the D9-brane, and share the same world-volume with all twisted moduli fields. However such a scenario does not give rise to localised matter fields (confined at intersection points) and in general one does not encounter states which are sequestered from twisted moduli. Hence the standard Kähler potentials used in those analyses are perfectly acceptable. By contrast our analysis opens the door for more general type I string constructions involving $D9$ *and* $D5_i$-branes, where potentially more realistic phenomenology and hierarchies between observables can be obtained with some or all of the matter fields sequestered from twisted moduli SUSY breaking sectors.

Chapter 4

Supersymmetric Higgs Bosons in a 5d Orbifold Model

We consider a 5d simplification of the BMSB model from Chapter 2, where the extra dimension is compactified on an S_1/Z_2 orbifold. We analyse the phenomenology of the Higgs sector in this effective field theory for a compactification scale $M_C = 1/R$ around the TeV scale. We show that the conventional MSSM Higgs boson mass bounds in 4d can be violated when we allow the gauge sector, Higgs, top and stop fields to live in the fifth extra dimension. Supersymmetry is broken at an orbifold fixed point which is spatially separated from the Yukawa brane where two chiral families are localised. When the brane-localised supersymmetry breaking term for the stop sector is arbitrarily large, we find that the stop Kaluza-Klein mass spectrum is completely independent of the Higgs fields. Hence, the Higgs masses only receive radiative contributions from the top Kaluza-Klein modes. We find that the 1-loop effective potential is insensitive to the cutoff scale of the theory. This yields a negative correction to the Higgs scalar squared-mass that triggers electroweak symmetry breaking in the range $1.5 \lesssim \tan\beta \lesssim 20$, where bottom/sbottom loop effects can be ignored. The recent LEP Higgs bound at $m_{h^0} > 114.1$ GeV, in conjunction with naturalness arguments, allows us to bracket the compactification scale $1.5 \lesssim M_C \lesssim 4$ TeV. Within this parameter space, we find that the lightest Higgs boson mass has an upper bound $m_{h^0} \lesssim 160$ GeV with the magnitude of the μ-parameter restricted to the range $33 \lesssim |\mu| \lesssim 347$ GeV.

4.1 Introduction

Extra-dimensional supersymmetric models with a TeV compactification scale [66, 67] offer an exciting new environment for investigating electroweak symmetry breaking (EWSB). Various models have been proposed to study EWSB in extra dimensions and their phenomenological implications [88]-[93] and the majority of models break SUSY through Scherk-Schwarz (SS) boundary conditions [1] [95]. However we will concentrate only on those models which recover the MSSM below the compactification scale and are anomaly-free after the orbifold compactification [96]. An important feature of these types of models are that the contribution of quark/squark Kaluza-Klein (KK) modes to the Higgs mass is negative which triggers EWSB at the TeV scale. Also the 1-loop radiative correction to the effective potential is found to be free of ultraviolet divergences, which implies that the Higgs physics is completely independent of the high-energy physics above some cutoff scale. This is due to the requirement of $\mathcal{N} = 1$ SUSY in 5d (which is equivalent to $\mathcal{N} = 2$ SUSY in 4d after compactification) which leads to a finite 1-loop effective potential because even though SUSY is globally softly broken, it is still preserved locally [91, 97] [2].

In this chapter we show that the conventional MSSM lightest Higgs boson mass bound $m_{h^0} \lesssim 130$ GeV [99] can be violated by allowing some of the fields to live in a fifth

[1] Notice that Ref.[94] demonstrate the equivalence of SS boundary conditions with Wilson line symmetry breaking.

[2] Recently, the finiteness of this kind of theory has been showed explicitly at 2-loops [98].

extra dimension [3]. Motivated by fine-tuning arguments, we are led to a (conservative) upper bound on the compactification scale $M_C \lesssim 4$ TeV. At this compactification scale, the lightest Higgs boson mass can be pushed as high as $m_{h^0} \sim 160\,GeV$. We find that in order to have EWSB through radiative corrections, $\tan\beta$ can have a wide range $1.5 \lesssim \tan\beta \lesssim 20$ rather than the small range allowed in the model of Ref. [92] $35 \lesssim \tan\beta \lesssim 40$, where SUSY is broken through the SS mechanism. We only limit $\tan\beta \lesssim 20$ since we neglect bottom-sbottom loop effects. Note that, unlike the MSSM, $\tan\beta \approx 1.5$ is not ruled out by experiment in our extra dimensional model.

4.2 Our Model

4.2.1 Outline

In this section we introduce our string-inspired model which is a simplification of the BMSB scenario discussed in Chapter 2, but with only a *single* extra dimension compactified on an S^1/Z_2 orbifold as shown in Figure 4.1. We regard this model as an effective field theory which is defined below a cutoff that we identify with the string scale M_*. Compactification of the extra dimension on the orbifold leads to a description of 5d bulk fields (ξ) as infinite towers of KK resonances. The Z_2 orbifold lets us classify bulk fields into even and odd parities which satisfy the following transformation rules under ($y \to -y$):

$$\begin{array}{cc} \mathbf{E}\mathit{ven} & \mathbf{O}\mathit{dd} \\ \xi_{even}(x,-y) = \xi_{even}(x,y)\,, & \xi_{odd}(x,-y) = -\xi_{odd}(x,y) \end{array} \tag{4.1}$$

[3]Higher upper bounds on the lightest boson mass have been recently calculated in the context of SUSY in warped extra dimensions [100] and a (de)constructed model [101].

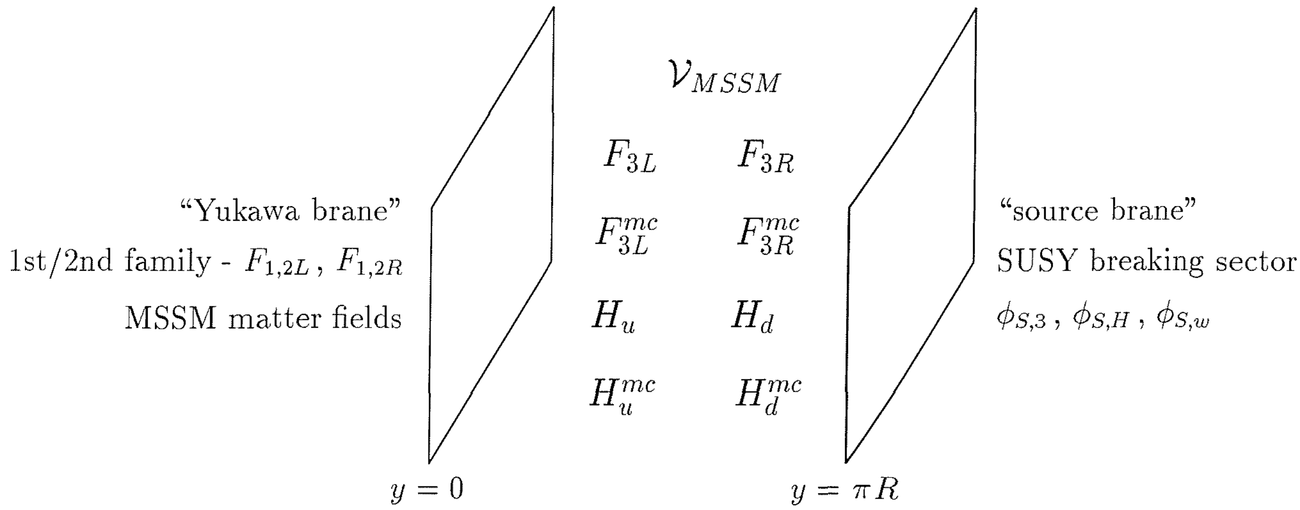


Figure 4.1: Our model showing the parallel 3-branes spatially separated along the extra dimension y. This extra dimension is compactified on the orbifold S^1/Z_2 that leads to two fixed points at $y = 0, \pi R$, where the two 3-branes are located. The first two chiral families ($F_{1,2}$) live on the Yukawa brane at $y = 0$, while SUSY is broken by F-terms of gauge singlet fields (ϕ_S) on the source brane at $y = \pi R$. The third family (F_3), gauge sector ($\mathcal{V}_{MSSM}$) and Higgs superfields ($H_{u,d}$) live in the extra dimensional bulk along with their $\mathcal{N} = 2$ SUSY partners which are required for consistency. The fields present in the model are summarised in Table 4.1.

The odd fields have KK expansions involving $\sin(ky/R)$ or $\sin(m_k y)$ where k is the KK number and m_k is the k^{th} KK-mode mass[4]. Notice that odd fields do not have zero modes which can be associated with MSSM fields since they have wavefunction profiles that vanish at the fixed points. In contrast, the even fields have $\cos(ky/R)$ or $\cos(m_k y)$ expansions and therefore do not vanish at the orbifold fixed points[5]. These Z_2-parity transformation properties are important when we come to couple bulk fields to boundary fields at either fixed point [102], for example when we localise the third family Yukawa couplings at $y = 0$ in section 4.3.2.

From a 4d perspective, $\mathcal{N} = 1$ SUSY in 5d is equivalent to $\mathcal{N} = 2$ in 4d, since the

[4]Usually the KK modes have masses of the form $m_k = k/R$.

[5]We can choose that the familiar MSSM fields are even with respect to the Z_2-symmetry, and so have massless zero modes before SUSY breaking.

Kaluza-Klein modes can combine in pairs to form $\mathcal{N}=2$ invariant states. However CP-conjugate "mirror" fields also need to be added to the theory to respect the $\mathcal{N}=2$ SUSY and form hypermultiplets as discussed in Appendices C.1 and C.3. For instance, each 5d vector supermultiplet $\mathcal{V}_{MSSM}$ contains a five-component gauge field $A_{M=\mu,5}$, a real scalar σ and two Weyl fermions $\lambda_{1,2}$ that all transform in the adjoint representation of the gauge group [93, 103]. Each $\mathcal{V}_{MSSM}$ can be decomposed into an $\mathcal{N}=1$ vector supermultiplet (containing a gauge boson A_μ and a gaugino λ_1) and an $\mathcal{N}=1$ chiral supermultiplet (containing a scalar $\Sigma \sim \sigma + iA_5$ and a fermion λ_2) where the fifth component of the 5d gauge boson becomes the longitudinal component of the scalar. Similarly each 5d matter hypermultiplet can be decomposed into an $\mathcal{N}=1$ chiral supermultiplet and its CP-"mirror" chiral supermultiplet, e.g. the up-like Higgs hypermultiplet $\overline{\overline{H}}_u$ contains the familiar MSSM Higgs chiral superfield H_u and its CP-"mirror" H_u^{mc} partner. The location of the fields present in our model are shown in Table 4.1.

States	Location	Z_2-parity
$F_{iL} = Q_{iL}\,,\, L_{iL} \quad (i=1,2)$	$y=0$	
$F_{iR} = U^c_{iR}\,,\, D^c_{iR}\,,\, E^c_{iR} \quad (i=1,2)$	$y=0$	
$F_{3L} = Q_{3L}\,,\, L_{3L}$	bulk	even
$F_{3R} = U^c_{3R}\,,\, D^c_{3R}\,,\, E^c_{3R}$	bulk	even
$F^{mc}_{3L} = Q^{mc}_{3L}\,,\, L^{mc}_{3L}$	bulk	odd
$F^{mc}_{3R} = U^{mc}_{3R}\,,\, D^{mc}_{3R}\,,\, E^{mc}_{3R}$	bulk	odd
$V = A_\mu\,,\, \lambda_1$	bulk	even
$\Sigma = \sigma + iA_5\,,\, \lambda_2$	bulk	odd
$H_u\,,\, H_d$	bulk	even
$H^{mc}_u\,,\, H^{mc}_d$	bulk	odd
$\phi_{S,3}\,,\, \phi_{S,H}\,,\, \phi_{S,w}$	$y=\pi R$	

Table 4.1: The location of the states present in our model. Bulk fields are also classified by their transformation with respect to Z_2-parity. Notice that the superfields Q, U, D, L, E implicitly include the scalar and fermion components, e.g. $Q_{iL} \sim Q_{iL}\,,\, \tilde{Q}_{iL}$.

4.2.2 Lagrangian

From section 2.2, we know that the 5d lagrangian can be split into an $\mathcal{N}=2$ invariant bulk term [103] consisting of 5d bulk fields, and separate 4d $\mathcal{N}=1$ invariant brane terms localised at either fixed point. Notice that the 4d brane terms are formed from the boundary fields and the even projections of the bulk fields on to the boundary branes using an off-shell formalism of $\mathcal{N}=2$ SUSY in 5d [102].

$$\mathcal{L} = \mathcal{L}_5 \left[\, \xi\,(x,y)\,\right] + \sum_j \delta\,(y-y_j)\,\mathcal{L}_j\left[\,\xi\,(x,y_j)\,,\eta_j\,(x)\,\right] \tag{4.2}$$

where j runs over the two branes at the orbifold fixed points, x are coordinates for the 4 non-compact dimensions, y is the coordinate for the extra compact spatial dimension, ξ is a bulk field, and η_j is a field localised on the j^{th} brane.

The 5d lagrangian for vector (A_M,σ,λ_i) and matter hypermultiplets ($\Phi_i^a = H_{u,d}, \tilde{F}_3$ and $\Psi_a = \tilde{H}_{u,d}, F_3$) given below [93, 103] includes the standard kinetic energy terms and supersymmetric Yukawa interaction terms:

$$\begin{aligned}
\mathcal{L}_5 = Tr\frac{1}{g^2}&\left\{-\frac{1}{2}F_{MN}^2 + |D_M\sigma|^2 + i\lambda_i\gamma^M D_M\lambda^i - \overline{\lambda}_i\left[\sigma,\lambda^i\right]\right\} \\
&+|D_M\Phi_i^a|^2 + i\overline{\Psi}_a\gamma^M D_M\Psi^a - \left(i\sqrt{2}\Phi_a^{\dagger i}\overline{\lambda}_i\Psi^a + h.c.\right) - \overline{\Psi}_a\sigma\Psi^a \\
&\qquad\qquad -\Phi_a^{\dagger i}\sigma^2\Phi_i^a - g^2\sum_{m,\alpha}\left[\Phi_a^{\dagger i}\left(\tau^m\right)_i^j T^\alpha\Phi_j^a\right]^2
\end{aligned} \tag{4.3}$$

where a labels the bulk matter fields (including both Higgs doublets and the third family superfields); $i,j=1,2$ are $SU(2)_R$ (R-symmetry) indices and $M,N=0-3,5$. D_M is a covariant derivative and τ^m are $SU(2)$ generators where m=1,2,3. $\Phi_i^a(\Psi_a)$ are the scalar (Dirac fermion) components of the Higgs and third family superfields. The 5d Dirac matrices are given in Appendix C.2.

Unlike previous models, we assume that supersymmetry is broken by non-zero F-terms of 4d gauge-singlet fields $\phi_{S,3}$, $\phi_{S,H}$, $\phi_{S,w}$ localised on the source brane at the fixed point $y = \pi R$ and mediated across the extra dimension by bulk fields as discussed in Chapter 2. Notice that the SUSY breaking fields ϕ_S carry additional indices that allow for non-universal couplings, e.g. $F_{S,3}$ and $F_{S,H}$ are not necessarily equal. However, we assume that the same gauge-singlet $\phi_{S,3}$ ($\phi_{S,H}$) couples to the whole third family (Higgs) fields respectively. Generalising the D-dimensional operators from Eq.(2.18), but for one extra dimension, we can generate third family scalar and gaugino masses:

$$\delta\mathcal{L}^{(1)}_{\pi R} = \delta\left(y - \pi R\right)\left[-\int d^4\theta\, \frac{c_3}{M_*^3} F_3^\dagger F_3\, \phi_{S,3}^\dagger \phi_{S,3} + \int d^2\theta\, \frac{c_w}{M_*^2}\phi_{S,w}\, W^\alpha W_\alpha + h.c.\right] \quad (4.4)$$

where F_3 represent the third family superfields $Q_{3L}, L_{3L}, U^c_{3R}, D^c_{3R}, E^c_{3R}$; W_α is the 5d gauge field-strength superfield that contains the gaugino as its lowest component, and c_3 (c_w) are the couplings of the third family (gauge fields) to the SUSY breaking sector.

Similarly we can also generate soft Higgs masses, $B\mu$ and μ-terms through the following operators:

$$\begin{aligned}\delta\mathcal{L}^{(2)}_{\pi R} = -\delta\left(y - \pi R\right)\Bigg[\int d^4\theta \frac{1}{M_*^3}\left[c_{H_u} H_u^\dagger H_u + c_{H_d} H_d^\dagger H_d + c_{B\mu} H_u H_d + h.c.\right]\phi_{S,H}^\dagger \phi_{S,H} \\ + \int d^4\theta\left(\frac{c_\mu}{M_*^2} H_u H_d\, \phi_{S,H}^\dagger + h.c.\right)\Bigg] \quad (4.5)\end{aligned}$$

Terms with even-parity fields replaced by their mirror pairs are forbidden by Z_2-parity, as only even fields couple directly to the 3-brane boundaries at the orbifold fixed points. However, the y-derivative of an odd field is actually even with respect to Z_2-parity, so terms like:

$$\delta\left(y - \pi R\right)\int d^4\theta \frac{1}{M_*^5}\left(\partial_y Q_{3L}^{mc\dagger}\right)\left(\partial_y Q_{3L}^{mc}\right)\phi_{S,3}^\dagger \phi_{S,3} \quad (4.6)$$

are allowed by the Z_2 symmetry, but can be neglected since they are so heavily suppressed by high powers of the cutoff scale M_*.

We choose to localise the Yukawa couplings on the "Yukawa" brane at $y = 0$ since we are unable to produce Yukawas in the bulk from higher-dimensional operators because such operators would explicitly break the $\mathcal{N} = 2$ SUSY[6]. Therefore couplings between bulk and brane fields can only be consistently defined on the boundaries of the 5d space. In order to discuss EWSB, we will consider the dominant top/stop sector Yukawa couplings and ignore the rest [7]. The lagrangian term that generates Yukawa couplings is given by:

$$\delta\mathcal{L}_0^{Yuk} = -\delta(y)\,\frac{f_t}{M_*^{3/2}}\int d^2\theta\,(Q_{3L}\,H_u\,U_{3R}^c + h.c.) \tag{4.7}$$

where $f_t = (\pi R M_*)^{3/2}\,y_t$ and $f_t\,(y_t)$ is the 5d (4d) Yukawa coupling. We will make an additional assumption that only the zero KK-modes of the Higgs scalars acquire non-zero VEVs and participate in EWSB. It is convenient to expand the zero modes of the neutral components of each complex (scalar) Higgs doublet in terms of real and imaginary parts:

$$H^0_{u,\,k=0} = \frac{1}{\sqrt{2}}\,(h_u + i\chi_u) \qquad , \qquad H^0_{d,\,k=0} = \frac{1}{\sqrt{2}}\,(h_d + i\chi_d) \tag{4.8}$$

where electroweak symmetry is spontaneously broken when the real parts acquire non-zero VEVs $< h_u >, < h_d > \neq 0$ and $< \chi_u > = < \chi_d > = 0$.

4.3 Kaluza-Klein mass spectra

In this section we will calculate the top and stop KK mass spectra in the presence of Yukawa couplings using a variant of the matrix method developed in Refs. [104, 105]. This method allows us to diagonalise the infinite KK mass matrices, where non-trivial

[6]Notice that it is possible to construct higher dimensional operators in the bulk that respect the weaker constraint of $SU(2)_R$ (R-parity) invariance, but $\mathcal{N} = 2$ SUSY is still explicitly broken.

[7]We will also neglect bottom/sbottom effects by taking $\tan\beta \lesssim 20$.

mixing between different KK modes is induced by the delta-functions in Eqs.(4.4) and (4.7). We find that for realistic phenomenology, we require a large mixing parameter ($\alpha_3 \gg 1$) between stop KK modes and small mixing ($\alpha_H \ll 1$) between Higgs KK modes. Finally, we will calculate the stop/top sector KK contributions to the 1-loop effective potential, and we find that it is finite due to supersymmetry in 5d.

4.3.1 Top KK mass spectrum

We will begin by diagonalising the top KK mass matrix which is independent of the SUSY breaking sector. Using Appendix C.3 and Eq.(C.7), we can combine Eqs.(4.3,4.7) to find the terms in the 5d lagrangian that contribute to the top mass matrix:

$$\mathcal{L}_{5,t}^{mass} = t_L^\dagger \partial_5 t_L^{mc} - t_L^{mc\dagger} \partial_5 t_L + t_R^\dagger \partial_5 t_R^{mc} - t_R^{mc\dagger} \partial_5 t_R - \frac{f_t}{M_*^{3/2}} H_u^0 \, \delta(y) \left(t_L^\dagger t_R + t_R^\dagger t_L \right) \quad (4.9)$$

where $f_t = (\pi R M_*)^{3/2} \, y_t$. The 5d top fields have the following KK-expansions:

$$t_{L(R)}(x,y) = \frac{1}{\sqrt{\pi R}} t_{L(R),0}(x) + \sum_{k=1}^{\infty} \sqrt{\frac{2}{\pi R}} \cos\left(\frac{ky}{R}\right) \tilde{t}_{L(R),k}(x) \quad (4.10)$$

$$t_{L(R)}^{mc}(x,y) = \sum_{k=1}^{\infty} \sqrt{\frac{2}{\pi R}} \sin\left(\frac{ky}{R}\right) \tilde{t}_{L(R),k}^{mc}(x) \quad (4.11)$$

and since we assume that only the real part of the zero-mode of H_u^0 acquires a non-zero VEV, we can neglect all other KK-modes in the Higgs expansion:

$$H_u^0 = \frac{1}{\sqrt{\pi R}} H_{u,k=0}^0 = \frac{1}{\sqrt{2\pi R}} h_u \quad (4.12)$$

where we have used Eq.(4.8) and dropped the imaginary part.

Substituting these KK expansions into Eq.(4.9) and integrating out the extra dimension, we obtain the 4d lagrangian:

$$
\begin{aligned}
-\mathcal{L}_{4,t}^{mass} &= -\sum_{k=1}^{\infty}\frac{k}{R}\left(t_{L,k}^{\dagger}t_{L,k}^{mc}+t_{R,k}^{mc\dagger}t_{R,k}\right) \\
&+\frac{y_t h_u}{\sqrt{2}}\left[t_{L,0}^{\dagger}t_{R,0}+\sqrt{2}\sum_{k=1}^{\infty}\left(t_{L,0}^{\dagger}t_{R,k}+t_{L,k}^{\dagger}t_{R,0}\right)+2\sum_{k,l=1}^{\infty}t_{L,k}^{\dagger}t_{R,l}\right]+(L\leftrightarrow R)
\end{aligned}
\tag{4.13}
$$

where we have used the following identity

$$
\int_0^{\pi R} dy\,\cos\left(\frac{ky}{R}\right)\,\cos\left(\frac{ly}{R}\right)=\frac{\pi R}{2}\delta_{kl}
\tag{4.14}
$$

This infinite matrix can be rewritten in block-diagonal form:

$$
\begin{aligned}
-\mathcal{L}_{4,t}^{mass} &= \Psi_{t,L}^{\dagger}\,\mathcal{M}_t\,\Psi_{t,R}+h.c.= \\
&\frac{1}{R}\sum_{k,l}\left(\begin{array}{ccc} t_{L,0}^{\dagger} & t_{L,k}^{\dagger} & t_{R,k}^{mc\dagger}\end{array}\right)\left(\begin{array}{ccc}\frac{1}{\sqrt{2}}y_t h_u R & y_t h_u R\mathcal{I} & 0\\ y_t h_u R\mathcal{I}^T & \sqrt{2}y_t h_u R\,(\mathcal{I}^T\mathcal{I}) & -M\\ 0 & -M & 0\end{array}\right)\left(\begin{array}{c} t_{R,0}\\ t_{R,l}\\ t_{L,l}^{mc}\end{array}\right)+h.c.
\end{aligned}
\tag{4.15}
$$

where $\mathcal{I}=(1,1,1,1,\ldots)$ and $M_{kl}=k\delta_{kl}$.

Following the method discussed in Ref. [105], we consider the eigenvalue equation:

$$
\mathcal{M}_t\left(\begin{array}{c} a\\ B_k\\ C_k\end{array}\right)=m_{t,k}[h_u]\left(\begin{array}{c} a\\ B_k\\ C_k\end{array}\right)
\tag{4.16}
$$

where $m_{t,k}[h_u]$ is the field-dependent mass of the k^{th} top KK excitation; a is a number, while B_k and C_k are infinite-dimensional vectors. After multiplying out Eq.(4.16) and some algebra, we obtain three independent relations:

$$
a \;=\; \frac{\sqrt{2}\,y_t h_u}{\left(\sqrt{2}\,m_{t,k}[h_u]-y_t h_u\right)}\sum_{k'=1}^{\infty}B_{k'}
\tag{4.17}
$$

$$C_k = -\frac{k}{R}\frac{B_k}{m_{t,k}[h_u]} \tag{4.18}$$

$$B_k = \frac{2\, y_t h_u\, m_{t,k}^2[h_u]}{\left(\sqrt{2}\, m_{t,k}[h_u] - y_t h_u\right)} \frac{R^2}{\left(m_{t,k}^2[h_u]\, R^2 - k^2\right)} \sum_{k'=1}^{\infty} B_{k'} \tag{4.19}$$

where we can sum Eq.(4.19) using the identity

$$\sum_{k=1}^{\infty} \frac{1}{\left(m_{t,k}^2[h_u]\, R^2 - k^2\right)} = \frac{-1 + m_{t,k}[h_u]\, \pi R\, \cot\left(m_{t,k}[h_u]\, \pi R\right)}{2 m_{t,k}^2[h_u]\, R^2} \tag{4.20}$$

to derive an eigenvalue condition for each KK-mode:

$$\tan\left[m_{t,k}[h_u] R\right] = \frac{y_t h_u\, \pi R}{\sqrt{2}} \tag{4.21}$$

This yields the following field-dependent KK mass eigenvalues

$$m_{t,k}[h_u] = \left| k M_C + \frac{M_C}{\pi} \arctan\left(\frac{y_t\, h_u\, \pi}{\sqrt{2} M_C}\right)\right| \qquad (k = -\infty, \ldots, \infty) \tag{4.22}$$

where $M_C = 1/R$. We can identify the observable top mass with the $k = 0$ KK mode,

$$m_t[h_u] = \frac{M_C}{\pi} \arctan\left(\frac{y_t\, h_u\, \pi}{\sqrt{2} M_C}\right) \tag{4.23}$$

Notice that we recover the usual MSSM relation $m_t[h_u] = y_t h_u/\sqrt{2}$ in the limit that $\mathrm{M}_C \to \infty$ as expected since the theory becomes four-dimensional. However, for general M_C, Eq.(4.23) is different due to the non-trivial mixing between KK-modes on the Yukawa brane from Eq.(4.7).

4.3.2 Stop KK mass spectrum

We will now repeat the diagonalisation procedure for the stop fields. However, the stop analysis is complicated by an additional source of KK mixing from the SUSY breaking sector in Eq.(4.4), where the mixing parameter α_3 is related to the F-term VEV of the SUSY breaking singlet $\phi_{S,3}$:

$$\alpha_3 = c_3 \pi \left(\frac{F_{S,3}^2}{M_*^4} \right) (M_* R) \tag{4.24}$$

Using Eqs.(4.3,4.4,4.7), we can extract the stop mass terms in the 5d lagrangian:

$$\begin{aligned} -\mathcal{L}_{5,\tilde{t}}^{mass} = \partial_5 \tilde{t}_L^* \partial_5 \tilde{t}_L + \partial_5 \tilde{t}_R^* \partial_5 \tilde{t}_R + \partial_5 \tilde{t}_L^{mc*} \partial_5 \tilde{t}_L^{mc} + \partial_5 \tilde{t}_R^{mc*} \partial_5 \tilde{t}_R^{mc} & \\ + \delta(y) \frac{f_t}{M_*^{3/2}} H_u^0 \left[\tilde{t}_L F_{U_{3R}^c}^\dagger + F_{Q_{3L}} \tilde{t}_R^* + \tilde{t}_L^* F_{U_{3R}^c} + F_{Q_{3L}}^\dagger \tilde{t}_R \right] & \\ + \delta(y - \pi R) \frac{c_3}{M_*^3} F_{S,3}^2 \left(\tilde{t}_L^* \tilde{t}_L + \tilde{t}_R^* \tilde{t}_R \right) & \end{aligned} \tag{4.25}$$

where $f_t = (\pi R M_*)^{3/2} y_t$. Notice that there is an important difference from the previous top analysis since expansion of the Yukawa coupling operator Eq.(4.7) into component fields leads to terms in Eq.(4.25) that involve 5d stop fields, Higgs scalars and quark superfield auxiliary F-terms that need to be integrated out.

The standard definition of auxiliary F-terms are modified when 5d bulk fields are coupled to 4d boundary fields due to the requirement that the localised coupling preserves $\mathcal{N} = 1$ SUSY at the boundary. We use the method developed in Ref. [102] that exploits an off-shell formulation of SUSY to define the auxiliary fields in terms of the even-parity projections of bulk fields on to the boundary [8]:

$$F_{Q_{3L}}^\dagger = \delta(y) \frac{f_t}{M_*^{3/2}} \tilde{t}_R^* H_u^0 - \partial_5 \tilde{t}_L^{mc*} \tag{4.26}$$

$$F_{U_{3R}^c}^\dagger = \delta(y) \frac{f_t}{M_*^{3/2}} \tilde{t}_L^* H_u^0 - \partial_5 \tilde{t}_R^{mc*} \tag{4.27}$$

Substituting $F_{Q_{3L}}$ and $F_{U_{3R}^c}$ into Eq.(4.25) will lead to the appearance of squared

[8]Notice that this expression also involves odd-parity mirror fields ($\tilde{t}_L^{mc}$) since the extra-dimensional derivative of an odd field is even.

delta-functions $\delta^2(y)$. These can be re-expressed as $\delta^2(y) = \delta(0)\delta(y)$, where we recast $\delta(0)$ as

$$\delta(0) = \frac{1}{\pi R}\sum_{n=-\infty}^{\infty} 1 = \frac{1}{\pi R} + \frac{2}{\pi R}\sum_{n=1}^{\infty} 1 = \frac{2}{\pi R}\sum_{n=0}^{\infty}\left(\frac{1}{2}\right)^{\delta_{n0}} = \frac{2}{\pi R}\mathcal{D} \tag{4.28}$$

and $\mathcal{D}$ is an infinite quantity. However, we find that $\mathcal{D}$ factors out and will not appear in the KK mass eigenvalues.

The 5d stop fields have the standard KK expansion

$$\tilde{t}_{L(R)}(x,y) = \frac{1}{\sqrt{2\pi R}}\tilde{t}_{L(R),0}(x) + \sum_{k=1}^{\infty}\frac{1}{\sqrt{\pi R}}\cos\left(\frac{ky}{R}\right)\tilde{t}_{L(R),k}(x) \tag{4.29}$$

$$\tilde{t}^{mc}_{L(R)}(x,y) = \sum_{k=1}^{\infty}\frac{1}{\sqrt{\pi R}}\sin\left(\frac{ky}{R}\right)\tilde{t}^{mc}_{L(R),k}(x) \tag{4.30}$$

and can be substituted with Eqs.(4.12,4.26,4.27) into Eq.(4.25) to obtain the lagrangian terms that contribute to the 4d KK mass matrix:

$$\begin{aligned}
-\mathcal{L}^{mass}_{4,\tilde{t}} = \frac{1}{2R^2}\Bigg[&\sum_{k=1}^{\infty} k^2\left(\tilde{t}^*_{L,k}\tilde{t}_{L,k} + \tilde{t}^{mc*}_{R,k}\tilde{t}^{mc}_{R,k}\right) + \left(\frac{\alpha_3}{\pi^2} + 2y_t^2\, h_u^2\, R^2\mathcal{D}\right)\tilde{t}^*_{L,0}\tilde{t}_{L,0} \\
&+ \sum_{k=1}^{\infty}\left(\frac{\alpha_3\sqrt{2}}{\pi^2}(-1)^k + 2\sqrt{2}y_t^2\, h_u^2\, R^2\mathcal{D}\right)\left(\tilde{t}^*_{L,k}\tilde{t}_{L,0} + \tilde{t}^*_{L,0}\tilde{t}_{L,k}\right) \\
+ \sum_{k,l=1}^{\infty}&\left(\frac{2\alpha_3}{\pi^2}(-1)^{k+l} + 4y_t^2\, h_u^2\, R^2\mathcal{D}\right)\tilde{t}^*_{L,k}\tilde{t}_{L,l} - \sum_{k=1}^{\infty} y_t\, h_u\, k\, R\left(\tilde{t}^{mc*}_{R,k}\tilde{t}_{L,0} + \tilde{t}^*_{L,0}\tilde{t}^{mc}_{R,k}\right) \\
&- \sum_{k,l=1}^{\infty}\sqrt{2}y_t\, h_u\, k\, R\left(\tilde{t}^{mc*}_{R,k}\tilde{t}_{L,l} + \tilde{t}^*_{L,k}\tilde{t}^{mc}_{R,l}\right)\Bigg] + (L \leftrightarrow R)
\end{aligned} \tag{4.31}$$

where we have integrated out the extra dimension and used the identity in Eq.(4.14).

This infinite mass matrix can be re-written in the basis:

$$\Psi_{\tilde{t},L} = \left(\tilde{t}_{L,0} \quad \tilde{t}_{L,k} \quad \tilde{t}^{mc}_{R,l}\right)^T \qquad k,l = 1,\ldots,\infty \tag{4.32}$$

so that Eq.(4.31) becomes:

$$
\begin{aligned}
-\mathcal{L}_{4,\tilde{t}}^{mass} &= \frac{1}{2}\Psi_{\tilde{t},L}^{\dagger}\,\mathcal{M}_{\tilde{t}}^{2}\,\Psi_{\tilde{t},L} + h.c. \\
&= \frac{1}{2R^2}\sum_{k,l=1}^{\infty}\left(\tilde{t}_{L,0}^{*}\ \ \tilde{t}_{L,k}^{*}\ \ \tilde{t}_{R,l}^{mc*}\right) \times \qquad (4.33)
\end{aligned}
$$

$$
\begin{pmatrix}
\frac{1}{\pi^2}\alpha_3 + 2\Gamma^2\,\mathcal{D} & \frac{\sqrt{2}}{\pi^2}\alpha_3\,\tilde{\mathcal{I}} + 2\sqrt{2}\Gamma^2\,\mathcal{D}\,\mathcal{I} & -\Gamma\,(\mathcal{I}\cdot M) \\
\frac{\sqrt{2}}{\pi^2}\alpha_3\,\tilde{I}^T + 2\sqrt{2}\Gamma^2\,\mathcal{D}\,\mathcal{I}^T & M^2 + \frac{2}{\pi^2}\alpha_3\,\tilde{\mathcal{J}} + 4\Gamma^2\,\mathcal{D}\,\mathcal{J} & -\sqrt{2}\Gamma\,(\mathcal{J}\cdot M) \\
-\Gamma\left(M\cdot\mathcal{I}^T\right) & -\sqrt{2}\Gamma\,(M\cdot\mathcal{J}) & M^2
\end{pmatrix}
\begin{pmatrix}
\tilde{t}_{L,0} \\ \tilde{t}_{L,k} \\ \tilde{t}_{R,l}^{mc}
\end{pmatrix} + h.c.
$$

where $\Gamma = (y_t\,h_u\,R)$, $\mathcal{I} = (1,1,1,1,\ldots)$, $\tilde{\mathcal{I}} = (-1,1,-1,1,\ldots)$, $M_{kl} = k\,\delta_{kl}$ and

$$
\mathcal{J} = \mathcal{I}^T\mathcal{I} = \begin{pmatrix} 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & \cdots \\ 1 & 1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad \tilde{\mathcal{J}} = \tilde{\mathcal{I}}^T\tilde{\mathcal{I}} = \begin{pmatrix} 1 & -1 & 1 & \cdots \\ -1 & 1 & -1 & \cdots \\ 1 & -1 & 1 & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \qquad (4.34)
$$

Suppose that the eigenvector $\Phi_{\tilde{t},L} = \left(\phi_{L,0}\ \phi_{L,1}\ \phi_{L,2}\ \ldots\ \phi_{R,1}^{mc}\ \phi_{R,2}^{mc}\ \ldots\right)^T$ satisfies the mass eigenvalue equation:

$$
\mathcal{M}_{\tilde{t}}^2\,\Phi_{\tilde{t},L} = m_{\tilde{t},k}^2[h_u]\Phi_{\tilde{t},L} \qquad (4.35)
$$

Then we obtain the following relations:

$$
\begin{aligned}
\lambda^2\,\phi_{L,0} &= \mathcal{P}_+\phi_{L,0} + \sqrt{2}\,(S_E\mathcal{P}_+ + S_O\mathcal{P}_-) - \Gamma\sum_{l=1}^{\infty} l\,\phi_{R,l}^{mc} && (k=0) \qquad (4.36)\\
\lambda^2\,\phi_{L,k} &= \sqrt{2}\mathcal{P}_-\phi_{L,0} + k^2\phi_{L,k} + 2\,(S_O\mathcal{P}_+ + S_E\mathcal{P}_-) - \sqrt{2}\,\Gamma\sum_{l=1}^{\infty} l\,\phi_{R,l}^{mc} && (k\in odd)\\
\lambda^2\,\phi_{L,k} &= \sqrt{2}\mathcal{P}_+\phi_{L,0} + k^2\phi_{L,k} + 2\,(S_E\mathcal{P}_+ + S_O\mathcal{P}_-) - \sqrt{2}\,\Gamma\sum_{l=1}^{\infty} l\,\phi_{R,l}^{mc} && (k\in even)\\
\lambda^2\,\phi_{R,k}^{mc} &= -\Gamma\,k\,\phi_{L,0} - \sqrt{2}\,\Gamma\,k\,(S_O + S_E) + k^2\,\phi_{R,k}^{mc} && (k=1,2,3,\ldots)
\end{aligned}
$$

where $\lambda = m_{\tilde{t},k}[h_u]\,R$ and

$$\mathcal{P}_\pm = \pm\frac{\alpha_3}{\pi^2} + 2\,\Gamma^2\,\mathcal{D} \qquad , \qquad S_O = \sum_{l\in odd} \phi_{L,l} \qquad , \qquad S_E = \sum_{l\in even} \phi_{L,l} \quad (4.37)$$

Following a series of manipulations, we can eliminate $\mathcal{D}$ and solve Eq.(4.36) to obtain a transcendental equation for the stop KK mass $m_{\tilde{t},k}[h_u] = \lambda/R$:

$$m_{\tilde{t},k}[h_u]\,R\left[\tan\left(\pi\, m_{\tilde{t},k}[h_u]\,R\right) - \frac{\pi^2}{2}\,\Gamma^2\,\cot\left(\pi\, m_{\tilde{t},k}[h_u]\,R\right)\right] = \frac{\alpha_3}{\pi}\left[1 + \frac{\pi^2}{2}\,\Gamma^2\right] \quad (4.38)$$

which can be solved iteratively [9] by considering different limits of the KK mixing parameter α_3.

- No SUSY breaking ($\alpha_3 \to 0$)

In the limit that SUSY is unbroken ($\alpha_3 = 0$), Eq.(4.38) becomes

$$m_{\tilde{t},k}[h_u]\,R\left[\tan^2\left(\pi\, m_{\tilde{t},k}[h_u]\,R\right) - \frac{(\pi\, y_t\, h_u\, R)^2}{2}\right] = 0 \quad (4.39)$$

where the non-trivial solution is identical to the top KK mass eigenvalue in Eq.(4.22) as expected since supersymmetry is preserved.

- Weak SUSY breaking ($\alpha_3 \ll 1$)

In the limit of weak SUSY breaking, there is minimal mixing between KK modes ($\alpha_3 \ll 1$), and the stop mass eigenvalues [10] are given by:

$$m^2_{\tilde{t},k}[h_u] = \left[\left(kM_C + m_t[h_u]\right)^2 + \frac{\alpha_3}{\pi^2}\,M_C^2 + Z_k^2[h_u,\alpha_3]\right]\left(1 + \mathcal{O}(\alpha_3^2)\right) \quad (4.40)$$

[9]Notice that we are unable to solve Eq.(4.38) for $\alpha_3 \sim 1$ since the $\mathcal{O}(\alpha_3^{-1})$ corrections can no longer be neglected.

[10]Notice that this eigenvalue also applies for the mirror stop fields $\tilde{t}^{mc}_{L/R}$ after EWSB when the Yukawa couplings in Eq.(4.7) induce mixing between even and odd-parity stop fields. However, in the absence of EWSB ($< h_u >= 0$), the mirror stop fields will have the usual KK masses k/R since they do not couple directly to the SUSY breaking sector.

where we have used the expression for the field-dependent top mass $m_t[h_u]$ from Eq.(4.23), and $Z_k^2[h_u, \alpha_3]$ is given by:

$$
\begin{aligned}
Z^2_{k\neq 0}[h_u, \alpha_3] &= \begin{cases} \alpha_3\, M_C^2/\pi^2 & \text{if } < h_u >= 0 \\ 0 & \text{otherwise} \end{cases} \\
Z^2_{k=0}[h_u, \alpha_3] &= \quad 0
\end{aligned}
\tag{4.41}
$$

From Eq.(4.40), we see that the mass of the stop $k = 0$ mode is given by:

$$
m^2_{\tilde{t},k=0}[h_u] \;=\; m_t^2[h_u] + \frac{\alpha_3}{\pi^2}\, M_C^2 \tag{4.42}
$$

where the stop mass is equal to the top mass and SUSY breaking soft mass added in quadrature [11]. Hence, we can generate soft masses around the TeV scale with minimal mixing ($\alpha_3 \ll 1$) if the compactification scale M_C lies at very high energies..

$$
\frac{\alpha_3}{\pi^2}\, M_C^2 \sim \mathcal{O}(TeV) \;\Rightarrow\; M_C \gg 1\,\mathrm{TeV} \tag{4.43}
$$

Therefore, below the compactification scale we recover the usual 4d MSSM phenomenology.

- Strong SUSY breaking ($\alpha_3 \gg 1$)

Finally we will consider the strong SUSY breaking limit ($\alpha_3 \gg 1$) which leads to maximal mixing between KK-modes. This scenario is equivalent to a large extra dimension and the solution of Eq. (4.38) is given by:

$$
m^2_{\tilde{t},k} = \left(k + \frac{1}{2}\right)^2 M_C^2 \left[1 + \mathcal{O}\left(\alpha_3^{-1}\right)\right] \qquad (k = -\infty, \ldots, \infty) \tag{4.44}
$$

Notice that to leading order the stop mass eigenvalues are independent of the Higgs

[11] For simplicity, we have neglected the trilinear mixing term A_t between the stop left and right fields.

background field (h_u). This independence can be explained by the arbitrarily large mixing term on the source brane that makes the Yukawa brane become transparent which washes out any field dependence, and so the stop mass is identified by its SUSY breaking contribution. We find that the compactification scale should be $M_C \approx \mathcal{O}(\mathrm{T}eV)$ to provide soft terms at the electroweak scale.

4.3.3 Stop/Top KK contributions to the effective potential

We will now calculate the dominant stop and top Kaluza-Klein contributions to the effective potential using the KK mass spectra:

$$\begin{aligned} m_{t,k}[h_u] &= \left| kM_C + \frac{M_C}{\pi} \arctan\left(\frac{y_t\, h_u\, \pi}{\sqrt{2}M_C}\right)\right| \\ m^2_{\tilde{t},k} &= \left(k + \frac{1}{2}\right)^2 M_C^2 \end{aligned} \tag{4.45}$$

The KK contributions to the 1-loop effective potential are given by the relation:

$$V_{1-loop} = \frac{1}{2} Tr \sum_{k=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \ln\left[\frac{p^2 + m^2_{\tilde{t},k}}{p^2 + m^2_{t,k}[h_u]}\right] \tag{4.46}$$

where the trace is over all degrees of freedom [12]. Since the stop KK mass eigenvalue is independent of the Higgs field h_u, we find that the stop contribution is constant and can be absorbed into the cosmological constant.

We perform the momentum integral in Eq.(4.46) using dimensional regularisation, and find that the top and stop contributions can each be separated into an infinite pole part and a finite piece. For instance, the infinite part from the top sector is given

[12] Recall that each top/stop (and mirror) field has three colours and four degrees of freedom, i.e. a four-component fermion or two complex scalars, so the trace gives an overall factor of $N_c = 12$.

by:

$$V^{\infty}_{top}[h_u] = \frac{3}{8\pi^2\epsilon}\left[m_t^4[h_u] + \sum_{k=1}^{\infty}\left(\frac{k}{R} + m_t[h_u]\right)^4 + \sum_{k=1}^{\infty}\left(-\frac{k}{R} + m_t[h_u]\right)^4\right] \tag{4.47}$$

where the second term arises from $k \neq 0$ top KK modes, and the third term from the CP-"mirror" fields. Using zeta-function regularisation to perform the infinite summation [106, 107], we find that the three terms in Eq.(4.47) exactly cancel each other. Therefore, V^{∞}_{top} (and similarly V^{∞}_{stop}) vanishes due to the explicit $\mathcal{N} = 2$ SUSY present in the bulk, and the top (stop) contributions to the 1-loop effective potential are found to be finite. Notice that in a non-supersymmetric model, the third term in Eq.(4.47) involving CP-"mirror" fields does not appear and so the top contribution is no longer finite. Also note that in the case of small extra dimensions, the non-zero KK-modes are decoupled from the low-energy physics, and so only the first term in Eq.(4.47) arises and we recover the usual MSSM effective potential.

We find that the finite top contribution is:

$$V_{top}\,[h_u] \;=\; \frac{9M_C^4}{16\pi^6}\sum_{n=1}^{\infty}\frac{1}{n^5}\cos\left[\frac{2\pi\, n\, m_t[h_u]}{M_C}\right] \tag{4.48}$$

and the constant field-independent stop contribution is given by:

$$V_{stop} \;=\; -\frac{9M_C^4}{16\pi^6}\sum_{n=1}^{\infty}\frac{(-1)^n}{n^5} = \frac{135\,\zeta(5)\,M_C^4}{256\pi^6} \tag{4.49}$$

which can be absorbed into the cosmological constant and dropped from our subsequent analysis.

4.3.4 SUSY Breaking Higgs Parameters

The soft Higgs mass parameters are generated on the source brane at $y = \pi R$ through the higher-dimensional operators in Eq.(4.5). Substituting the standard KK expansions for the 5d Higgs doublet fields will lead to mixing between KK modes, where the mixing parameter is given by:

$$\alpha_H = c_H \pi \left(\frac{F_{S,H}^2}{M_*^4} \right) \left(\frac{M_*}{M_C} \right) \tag{4.50}$$

and $F_{S,H}$ is the F-term VEV associated with the singlet $\phi_{S,H}$ field, and we have assumed that $c_{H_u} = c_{H_d} = c_H$.

In section 4.3.2, we were led to maximal mixing ($\alpha_3 \gg 1$) in the stop sector to generate soft masses around the TeV scale. We also know that for EWSB via top/stop radiative corrections, we require a negative mass-squared to trigger spontaneous symmetry breaking. However, this is harder to achieve when $\alpha_3 \approx \alpha_H$ since the tree-level and (negative) 1-loop contributions have comparable magnitude. We conclude that the KK-modes in the Higgs sector must be minimally mixed ($\alpha_H \ll 1$) for viable radiative EWSB.

Comparing Eqs.(4.24) and (4.50) shows that we have two options, either (i) the couplings in the higher-dimensional operators are hierarchical ($c_3 \gg c_H, c_{B\mu}, c_\mu$) which appears unattractive, or else (ii) there is a non-univerality in the hidden sector. We have assumed the latter option where different SUSY breaking fields couple to the stop and Higgs sectors separately which allows for non-universal F-terms ($F_{S,H} \neq F_{S,3}$) with comparable hidden sector couplings $c \sim \mathcal{O}(1)$.

In the case of minimal mixing, the Higgs KK mass matrix is dominated by the diagonal elements, so that we can decouple the non-zero KK excitations. Instead we will impose the EWSB conditions on the lightest ($k = 0$) KK-modes where the soft parameters can be taken directly from Eq.(4.5):

$$m_H^2 = \frac{c_H F_{S,H}^2}{M_*^3 \pi} M_C = \frac{\alpha_H}{\pi^2} M_C^2 \tag{4.51}$$
$$B\mu = \frac{c_{B\mu}}{c_H} m_H^2 \quad , \quad \mu = \frac{c_\mu}{c_H} m_H^2$$

where $m_{H_u}^2 = m_{H_d}^2 = m_H^2$.

4.4 Reliability and Perturbativity

We have not yet imposed any constraint on the relationship between the compactification scale M_C and the cutoff scale M_*. The requirement of perturbativity (where our perturbative analysis is valid) allows us to find an upper bound on the ratio (M_*/M_C). We know that the gauge and Yukawa couplings exhibit power law running behaviour in extra-dimensional models [74]. Indeed the beta functions of these couplings depend on powers of the renormalisation scale μ due to the inclusion of the KK-modes that makes the physics highly sensitive to the renormalisation scale. This implies that gauge coupling unification and the emergence of the Landau pole in the Yukawa couplings are accelerated with respect to the usual (logarithmically-running) 4d theory. The top Yukawa coupling y_t is found to develop a singularity at energies close to the compactification scale M_C. In our model, the presence of the third family in the 5d bulk gives the Yukawa coupling beta function (β_{y_t}) a quadratic dependence on the ratio between the renormalisation scale μ and the compactification scale M_C:

$$\beta_{y_t} \sim y_t^3 \left(\frac{\mu}{M_C}\right)^2 + \dots \tag{4.52}$$

Note that this dependence on (μ/M_C) is stronger than the corresponding beta function for the gauge coupling (β_{y_t}) which is only linearly-dependent

$$\beta_g \sim g^3 \left(\frac{\mu}{M_C}\right) + \ldots \tag{4.53}$$

Suppose μ_{NP} is the scale where the top Yukawa coupling becomes non-perturbative, which was found by the authors of Ref. [92] to be $\mu_{NP} \approx 5M_C$. Therefore, we can maintain a (reliable) perturbative regime by imposing the following constraint on the cutoff scale of our theory M_*

$$M_* \lesssim \mu_{NP} \qquad \longrightarrow \qquad M_* \lesssim 5M_C \tag{4.54}$$

Similarly the cutoff is bounded from below by the compactification scale M_C, and we find that the string scale M_* is restricted to the range:

$$M_C \lesssim M_* \lesssim 5M_C \tag{4.55}$$

4.5 Higgs Mass Spectrum

In this section we will calculate the mass eigenvalues in the Higgs sector, where the light and heavy CP-even Higgs mass eigenstates (h^0, H^0) are linear combinations of the real fields h_u and h_d. We can use the standard MSSM relations to find the masses of the charged $(H^\pm)$ and CP-odd (A^0) Higgs fields. The tree-level potential in terms of the neutral components of the $k = 0$ Higgs doublets $(H^0_{u,0}, H^0_{d,0})$ is:

$$\begin{aligned} V_{tree} = m_1^2 \left|H^0_{d,0}\right|^2 + m_2^2 \left|H^0_{u,0}\right|^2 - B\mu\left(H^0_{u,0}H^0_{d,0} + h.c.\right) \\ + \frac{M_Z^2}{2v^2}\left[\left|H^0_{u,0}\right|^2 - \left|H^0_{d,0}\right|^2\right]^2 \end{aligned} \tag{4.56}$$

where we are free to define $B\mu$ as real and positive by absorbing any phase into $H^0_{u,0}$ and $H^0_{d,0}$; and we have traded the $U(1)_Y$ and $SU(2)_L$ gauge couplings (g', g) for the physical Z^0 mass M_Z^2 and the VEV $v = 246$ GeV. Using Eq.(4.51), we find that the Higgs doublets share the same soft mass since we assume $c_{H_u} = c_{H_d} = c_H$:

$$m_{1,2}^2 = m_{soft}^2 = |\mu|^2 + m_H^2 \tag{4.57}$$

and we regard m_{soft}^2 and $B\mu$ as input parameters.

Combining Eqs.(4.8,4.23,4.48,4.56), we find an expression for the total 1-loop effective potential in terms of the real Higgs fields h_u, h_d:

$$\begin{aligned} V = \frac{1}{2} m_{soft}^2 \left(h_u^2 + h_d^2\right) - B\mu\, h_u h_d + \frac{M_Z^2}{8v^2}\left[h_u^2 - h_d^2\right]^2 \\ + \frac{9M_C^4}{16\pi^6}\sum_{n=1}^{\infty}\frac{1}{n^5}\cos\left[2n\ \arctan\left(\frac{y_t\, h_u\, \pi}{\sqrt{2}\, M_C}\right)\right] \end{aligned} \tag{4.58}$$

where we can neglect the imaginary parts $\chi_{u,d}$ since they acquire vanishing VEVs and we have also dropped the constant stop contribution.

Applying the EWSB conditions at the usual minimum $\langle h_u \rangle = v \sin\beta$, $\langle h_d \rangle = v\cos\beta$ with $v = 246$ GeV.

$$\left.\frac{\partial V}{\partial h_u}\right|_{\langle h_u\rangle, \langle h_d\rangle} = \left.\frac{\partial V}{\partial h_d}\right|_{\langle h_u\rangle, \langle h_d\rangle} = 0 \tag{4.59}$$

allows us to eliminate $B\mu$ and M_Z^2 in terms of the other parameters. By imposing the correct observable Z^0 mass ($M_Z = 91.18$ GeV), we can also eliminate m_{soft} so that the compactification scale M_C and $\tan\beta$ (or equivalently m_{A^0}) are regarded as the two free parameters. In Figure 4.2, we plot m_{soft} as a function of the compactification scale M_C for two different values of $\tan\beta$.

We construct the CP-even mass matrix from the second derivatives of the effective

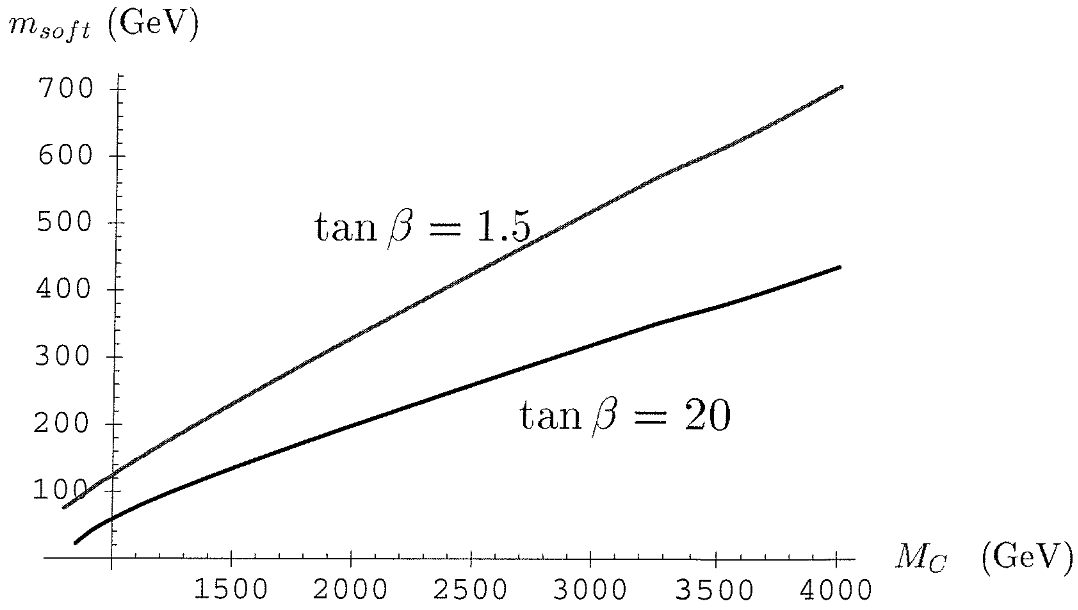


Figure 4.2: The universal Higgs soft mass $m_{soft}^2 = |\mu|^2 + m_H^2$ against the compactification scale M_C for $\tan\beta = 1.5$ and 20.

potential at the minimum, which can be diagonalised to find the mass eigenvalues (m_{h^0}, m_{H^0}):

$$\mathcal{M}_{even}^2 = \begin{pmatrix} \dfrac{\partial^2 V}{\partial h_u^2} & \dfrac{\partial^2 V}{\partial h_u \partial h_d} \\ \dfrac{\partial^2 V}{\partial h_d \partial h_u} & \dfrac{\partial^2 V}{\partial h_d^2} \end{pmatrix} \tag{4.60}$$

Notice that these CP-even eigenvalues include the 1-loop effects, but only m_2^2 is 1-loop improved since we are neglecting bottom-sbottom loops for $\tan\beta \leq 20$:

$$m_{2,imp}^2 = m_2^2 + \frac{1}{2} \left. \frac{\partial^2 V_{top}}{\partial h_u^2} \right|_{\langle h_u \rangle, \langle h_d \rangle} \tag{4.61}$$

where $m_2^2 = m_{soft}^2$ from Eq.(4.57). However $B\mu$ (and m_1^2) are not 1-loop improved, so we can use the standard 4d tree-level MSSM expressions to find the CP-odd (A^0)

and charged Higgs ($H^{\pm}$) masses [11]

$$m_{A^0}^2 = \frac{2B\mu}{\sin 2\beta} \tag{4.62}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + M_W^2 \tag{4.63}$$

where M_W^2 is the physical $W^{\pm}$ mass. We solve for M_Z^2 as a function of the free parameters $(M_C, \tan\beta)$, and use the standard definition of fine-tuning Δ [108, 109] (but neglecting the variation of $\tan\beta$ with respect to changes in M_C) to find an upper limit on the compactification scale M_C.

$$\Delta = \left|\frac{M_C}{m_{Z^0}^2}\frac{\partial m_{Z^0}^2}{\partial M_C}\right| \tag{4.64}$$

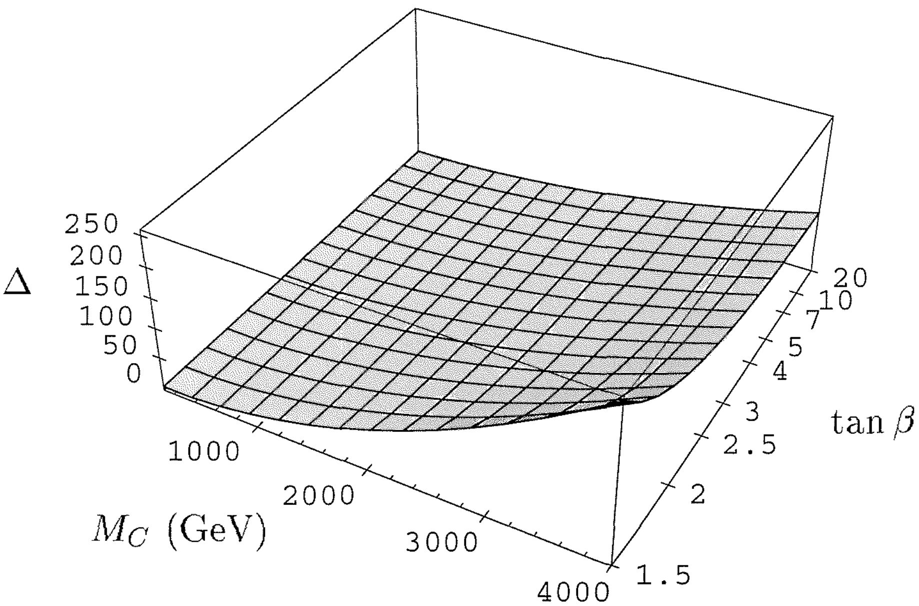


Figure 4.3: The fine-tuning parameter Δ as a function of $\tan\beta$ and the compactification scale M_C in GeV. Δ is shown to have a very weak dependence on $\tan\beta$ in the range $2.5 \lesssim \tan\beta \lesssim 20$ for which we can neglect bottom-sbottom effects. However, the fine-tuning rapidly becomes singular as $\tan\beta \to 1$.

Motivated by the fine-tuning as shown in Figure 4.3, we choose to investigate the parameter space $M_C \leq 4$ TeV and $1.5 \leq \tan\beta \leq 20$ where the fine-tuning $\Delta \sim \mathcal{O}(10^2)$.

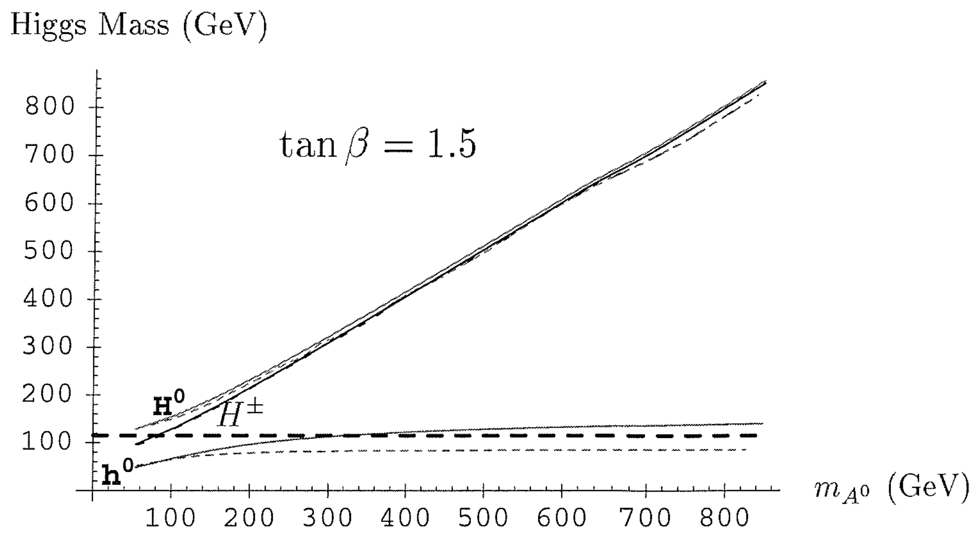


Figure 4.4: The Higgs masses ($m_{h^0}, m_{H^\pm}, m_{H^0}$) against m_{A^0} for $\tan\beta = 1.5$. We have run $1 \leq M_C \leq 4$ TeV parametrically along each curve. The LEP candidate at 115 GeV (*bold dotted-line*) [77] is shown for comparison with m_{h^0}. We have also included the MSSM results (*dotted-lines*) taken from Ref.[4] to compare against our model (*bold-lines*).

In Figures 4.4 and 4.5, we plot the eigenvalues (m_{h^0}, m_{H^0}) of the CP-even mass matrix and the charged Higgs mass ($m_{H^\pm}$) against the CP-odd mass (m_{A^0}) for two fixed values of $\tan\beta = 1.5$ and 20. In Figure 4.4 we also include the MSSM predictions for $\tan\beta = 1.5$ taken from [4] for comparison. Notice that, unlike the MSSM predictions from [4], our model is *not* excluded by the LEP signal for $\tan\beta = 1.5$. There are additional experimental lower limits for the other Higgs masses

$$m_{A^0} \geq 92\,GeV \qquad and \qquad m_{H^\pm} \geq 69\,GeV \tag{4.65}$$

but these provide a much weaker constraint on our model.

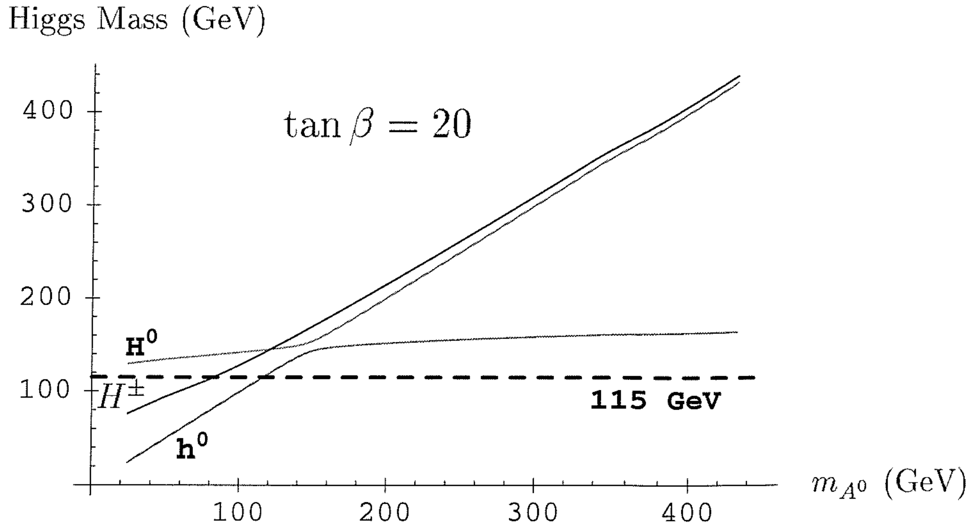


Figure 4.5: The Higgs masses $(m_{h^0}, m_{H^\pm}, m_{H^0})$ against m_{A^0} for $\tan\beta = 20$. We have run $1 \leq M_C \leq 4$ TeV parametrically along each curve. The LEP candidate at 115 GeV (*dotted-line*) [77] is shown for comparison with m_{h^0}.

In Figure 4.6, we plot the lightest Higgs mass (m_{h^0}) as a function of the compactification scale M_C and $\tan\beta$. The LEP data excludes the parameter space below the first contour at $m_{h^0} = 115$ GeV [77] which corresponds to a compactification scale $M_C \approx 1.5 - 1.7$ TeV over the whole range of $\tan\beta$. Combining Figures 4.3 and 4.6 we find a very conservative estimate for the allowed window of compactification scales in our model:

$$1.5\,\mathrm{TeV} \lesssim M_C \lesssim 4\,\mathrm{TeV} \tag{4.66}$$

Our model can easily accommodate the conventional 4d MSSM upper bound on the lightest Higgs boson mass $m_{h^0} \sim 130$ GeV, and can be pushed as high as $m_{h^0} \sim 160$ GeV with $M_C \approx 4$ TeV and $5 \lesssim \tan\beta \lesssim 20$. Recall that including additional matter (e.g. gauge singlets in the NMSSM [12, 13]) can also raise the upper bound on the lightest Higgs mass, but our "minimal" extension of the MSSM in 5d achieves higher

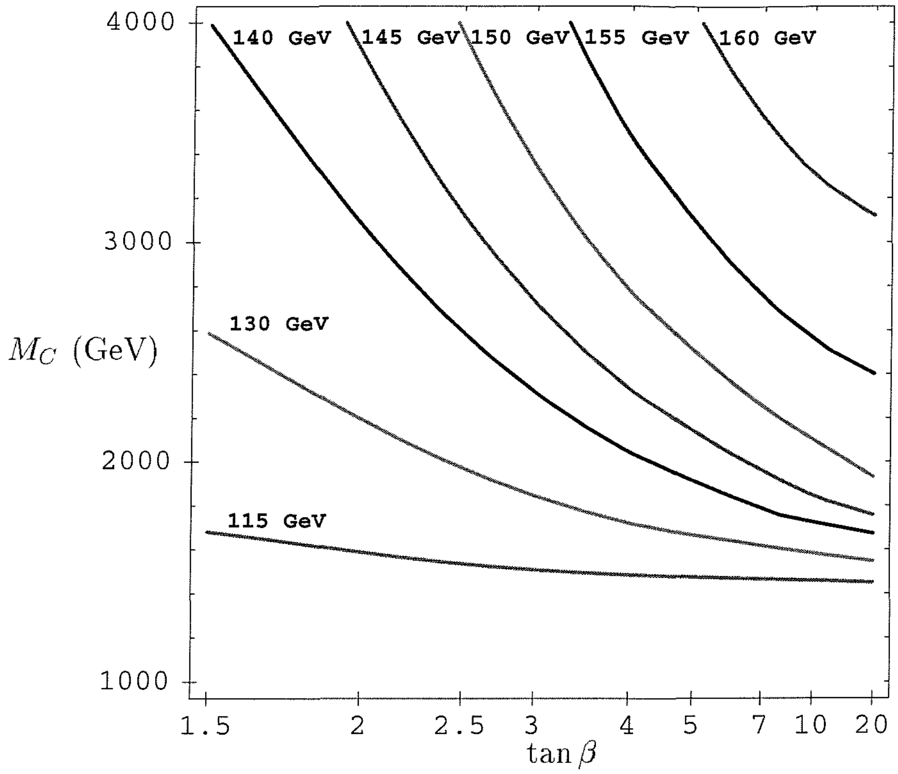


Figure 4.6: Mass contour plot for the lightest Higgs mass m_{h^0} as a function of the compactification scale M_C (GeV) and $\tan\beta$ in the absence of bottom sector effects (i.e. $\tan\beta \leq 20$). The LEP candidate at 115 GeV [77] is easily accommodated over the range of $\tan\beta$ with a compactification scale $M_C \approx 1.5 - 1.7$ TeV. At large $\tan\beta \to 20$ and $M_C \to 4$ TeV, the lightest Higgs mass can be as large as $m_{h^0} \sim 164$ GeV.

mass bounds without adding extra matter content.

We can use Figure 4.2 to find limits on the μ-parameter from the universal soft Higgs mass m_{soft} which is constrained when we impose EWSB at the electroweak minimum. From Eq.(4.51) we find the ratio between $m_{H_u}^2 = m_{H_d}^2 = m_H^2$ and $|\mu|^2$ in terms of the compactification and cutoff scales:

$$\frac{m_H^2}{|\mu|^2} \sim \frac{M_* \pi}{M_C} \tag{4.67}$$

where we have assumed that $c_H, c_\mu \sim 1$.

Recall from section 4.3.4 that we assume non-universality in the SUSY breaking sector ($F_{S,H} \ll F_{S,3}$) to obtain maximal (minimal) mixing in the stop (Higgs) sectors for viable radiative EWSB while maintaining similar couplings $c_3 \sim c_H$. We can use the constraints from Eq.(4.55) to find limits on the ratio between the soft Higgs mass and the μ-parameter from Eq.(4.67)

$$\pi \lesssim \frac{m_H^2}{|\mu|^2} \lesssim 5\pi \tag{4.68}$$

and therefore limits on m_{soft} in terms of μ:

$$m_{soft}^2 = m_H^2 + |\mu|^2 \qquad \longrightarrow \qquad (\pi + 1)\,|\mu|^2 \lesssim m_{soft}^2 \lesssim (5\pi + 1)\,|\mu|^2 \tag{4.69}$$

The constraints on the μ-parameter for different compactification scales and values of $\tan\beta$ are shown in Table 4.2:

	$M_C = 1.5\,\mathrm{TeV}$	$M_C = 4\,\mathrm{TeV}$
$\tan\beta = 1.5$	$114 \gtrsim \|\mu\| \gtrsim 57$	$347 \gtrsim \|\mu\| \gtrsim 173$
$\tan\beta = 20$	$66 \gtrsim \|\mu\| \gtrsim 33$	$215 \gtrsim \|\mu\| \gtrsim 107$

Table 4.2: Upper and lower limits on the size of the μ-parameter for two different values of $\tan\beta = 1.5$ and 20, and compactification scales $M_C = 1.5$ and 4 TeV.

Therefore, the magnitude of the μ-parameter is constrained to the range $33 \lesssim |\mu| \lesssim 347$ GeV in our model.

4.6 Conclusions

In conclusion we have considered the Higgs sector of an $\mathcal{N}=1$ supersymmetric 5d theory compactified on an orbifold S^1/Z_2 where the compactification scale $M_C \sim \mathcal{O}(TeV)$. This effective field theory is inspired by the explicit type I string model in chapter 2. Orbifolding leads to fixed points at either end of the extra dimension (y) where 4d branes can be localised. Supersymmetry is broken by the F-term VEV of a gauge singlet on the brane at $y = \pi R$ that is spatially separated from another Yukawa brane at $y = 0$ where the first two MSSM families and Yukawa couplings are localised. Direct coupling between the two sectors (and therefore soft squark masses at the high-scale) is suppressed by the separation between the branes which alleviates the flavour-changing neutral-current problem since the first and second family squark masses are only generated through flavour-blind loops. The third family, gauge sector and Higgs fields live in the extra dimensional bulk with their $\mathcal{N}=2$ supersymmetric partners (which are required for consistency) and therefore receive unsuppressed soft masses due to their direct coupling to the SUSY breaking sector.

We assume a non-universality in the SUSY breaking sector, where different gauge singlets couple separately to the stop (Higgs) fields and the associated F-term VEVs are hierarchical ($F_{S,H} \ll F_{S,3}$) which induces maximal (minimal) mixing between different KK-modes. The maximal mixing between stop modes requires the use of a matrix method to diagonalise the infinite mass matrix. In contrast the Higgs KK-modes are minimally mixed so that the mass matrix is dominated by the diagonal components and we can effectively decouple the non-zero KK-modes from the analysis. We find that the soft Higgs parameters are generated by non-renormalisable operators. The presence of the third family in the bulk is particularly important for its dominant 1-loop contribution to the Higgs effective potential. The full tower of top and stop Kaluza-Klein modes contribute to the potential and trigger radiative electroweak symmetry breaking. Following dimensional regularisation and zeta-function regularisation techniques, we find that the 1-loop contributions to the effective potential are separately finite and therefore insensitive to the high-energy cutoff M_*. However in the maximal mixing limit we find that the top contribution has a non-trivial dependence on the Higgs background fields, whereas the stop sector will only contribute to the cosmological constant.

We minimise the 1-loop effective potential and impose the conditions for electroweak symmetry breaking to find the physical Higgs mass eigenvalues. Requiring the correct physical Z^0-mass allows us to eliminate parameters in terms of the compactification scale M_C and $\tan\beta$ (or equivalently m_{A^0}). We use fine-tuning arguments to constrain the parameter space $M_C \lesssim 4$ TeV, and choose to study the region $1.5 \lesssim \tan\beta \lesssim 20$ where bottom sector effects can be neglected. We obtain physical Higgs mass eigenvalues for different values of $\tan\beta$ and find that the LEP signal [77] imposes a lower limit on the compactification scale $M_C \gtrsim 1.5$ TeV. We also find that, unlike the MSSM, $\tan\beta = 1.5$ is not excluded by experiment and our model can easily accommodate the LEP signal over the full parameter space. In fact the usual MSSM upper bound ($m_{h^0} \lesssim 130$ GeV) and the NMSSM bound ($m_{h^0} \lesssim 150$ GeV) can be trivially exceeded and raised to $m_{h^0} \lesssim 164$ GeV for a compactification scale $M_C \sim 4$ TeV and $5 \lesssim \tan\beta \lesssim 20$.

Note that radiative electroweak symmetry breaking is viable over a large range $1.5 \lesssim \tan\beta \lesssim 20$ in comparison to an alternative model [92] that is severely constrained to the smaller range $35 \lesssim \tan\beta \lesssim 40$. The requirement of perturbativity in our model imposes a constraint on the relationship between the compactification scale M_C and the cutoff M_* scale ($M_C \lesssim M_* \lesssim 5M_C$). We use this constraint in combination with the assumption of a universal soft Higgs mass ($m^2_{H_u} = m^2_{H_d}$) to deduce limits on the μ-parameter, and we find that the magnitude of μ is inside the range $33 \lesssim |\mu| \lesssim 347$ GeV.

Chapter 5

Conclusions

In this thesis we have studied connections between superstring theory and low-energy observable physics in the context of type I string models and higher-dimensional effective field theories involving intersecting Dirichlet-branes. In chapters 2 and 3, we studied how generic models with localised supersymmetry breaking can be realised in intersecting D-brane constructions. We considered an explicit three-family example involving three intersecting D-branes that, in the limit of asymmetric compactification radii, reduces to the gaugino mediated SUSY breaking model, but with the Pati-Salam gauge group arising from a linear combination of gauge groups on different branes. Type I string selection rules constrained the form of the renormalisable superpotential which, for our assignment of open string states, forbids R-parity violating terms and leads to a third family hierarchical Yukawa texture. We compared the low-energy sparticle spectra with gaugino mediation and no-scale supergravity predictions, and found that our model has a characteristic experimental signature with much heavier stops, sbottoms and staus due to the separation of the third family states which is a common feature of type I constructions.

We extend these ideas, in chapter 3, by identifying the localised SUSY breaking fields with twisted moduli - closed string states that become trapped at orbifold fixed points. Intersecting D-brane constructions provide a mechanism for confining open string states (corresponding to MSSM fields) at the four-dimensional intersection point between branes. This offers a string realisation of gaugino and anomaly mediated SUSY breaking when the twisted moduli, that acquire SUSY breaking VEVs, are localised at another spatially-separated fixed point in the compact space. However, in the absence of a complete understanding of non-perturbative corrections to the Kähler potential and superpotential, we used a model-independent parametrisation to derive general expressions for soft masses and trilinears in models with D5 and D9-branes. This framework allows us to perform more general investigations of SUSY breaking that involve both conventional gravity mediation and gaugino mediated SUSY breaking.

In chapter 4, we considered electroweak symmetry breaking in an effective field theory motivated by the model in chapter 2. The equivalence of $\mathcal{N} = 1$ SUSY in 5d with $\mathcal{N} = 2$ in 4d necessitates the inclusion of additional exotic CP-"mirror" states into the spectrum. We are forced to localise Yukawa couplings on a 4d boundary which induces mixing between different Kaluza-Klein modes. We utilise a matrix method to extract the Kaluza-Klein mass spectrum of the top and stop fields which provide the dominant contributions to the 1-loop effective potential. After minimization, we derived expressions for the physical Higgs boson spectra as a function of $\tan\beta$ and the compactification scale M_C. We found that the conventional 4d MSSM bounds on the lightest Higgs boson can be exceeded for a conservative choice of parameters by allowing the third family and Higgs fields to live in the extra dimension.

The "holy grail" of string phenomenology is to understand how the MSSM can be derived from a higher-dimensional string theory that unites all four fundamental forces together in a consistent framework. We would like to understand the observed pattern of fermion masses and the mechanism(s) responsible for supersymmetry breaking. However, we must not underestimate the importance of experiment which provides invaluable constraints on constructing realistic string-inspired models. The work presented in this thesis goes some way to address these issues and bridge the gap between low-energy observations and the underlying string theory.

Appendix A

Supersymmetric Pati-Salam Model

In this Appendix we will review the supersymmetric Pati-Salam model [7] which arises in heterotic [110] and type I string constructions [1] [52, 55]. The original (non-supersymmetric) Pati-Salam model [75] was an attempt to embed the SM in a larger gauge group. It treated lepton number as the fourth quark colour so that leptons and quarks are united within the same multiplets, and right-handed neutrinos arise naturally.

The supersymmetric Pati-Salam model has the same gauge group:

$$G_{PS} = SU(4) \otimes SU(2)_L \otimes SU(2)_R \tag{A.1}$$

and each generation i of quarks and leptons transform in the following representations under G_{PS}:

$$F_{iL} \quad = \quad (\mathbf{4}, \mathbf{2}, \mathbf{1}) = \begin{pmatrix} u_{iL}^R & u_{iL}^B & u_{iL}^G & \nu_{iL} \\ d_{iL}^R & d_{iL}^B & d_{iL}^G & e_{iL} \end{pmatrix} \tag{A.2}$$

[1]However, this type I model has multiple $SU(2)_L$ and $SU(2)_R$ factors. This model will be studied in more detail in chapter 2.

$$F_{iR} = (\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}) = \begin{pmatrix} d_{iR}^{\dagger R} & d_{iR}^{\dagger B} & d_{iR}^{\dagger G} & e_{iR}^{\dagger} \\ u_{iR}^{\dagger R} & u_{iR}^{\dagger B} & u_{iR}^{\dagger G} & \nu_{iR}^{\dagger} \end{pmatrix} \tag{A.3}$$

where we have suppressed gauge indices, and superfields are represented by their fermionic components from Table 1.2. Notice that right-handed superfields have been CP-conjugated to obtain left-handed superfields to be able to construct a holomorphic superpotential.

The two MSSM Higgs doublet superfields are contained in the representation:

$$H = (\mathbf{1}, \mathbf{2}, \mathbf{2}) = \begin{pmatrix} H_u^+ & H_d^0 \\ H_u^0 & H_d^- \end{pmatrix} \tag{A.4}$$

where this embedding of the MSSM Higgs fields avoids the phenomenological Higgs doublet-triplet splitting problems of minimal $SU(5)$ models [111].

There are additional heavy Higgs fields $\mathcal{H}, \mathcal{H}'$ in the representations:

$$\mathcal{H} = (\mathbf{4}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} u_{\mathcal{H}}^R & u_{\mathcal{H}}^B & u_{\mathcal{H}}^G & \nu_{\mathcal{H}} \\ d_{\mathcal{H}}^R & d_{\mathcal{H}}^B & d_{\mathcal{H}}^G & e_{\mathcal{H}} \end{pmatrix} \tag{A.5}$$

$$\mathcal{H}' = (\overline{\mathbf{4}}, \mathbf{1}, \overline{\mathbf{2}}) = \begin{pmatrix} d_{\mathcal{H}'}^R & d_{\mathcal{H}'}^B & d_{\mathcal{H}'}^G & e_{\mathcal{H}'} \\ u_{\mathcal{H}'}^R & u_{\mathcal{H}'}^B & u_{\mathcal{H}'}^G & \nu_{\mathcal{H}'} \end{pmatrix} \tag{A.6}$$

These heavy Higgses will break the Pati-Salam gauge symmetry down to the SM when their "neutrino" components develop VEVs around the GUT scale M_{GUT}:

$$\langle \mathcal{H} \rangle = \langle \nu_{\mathcal{H}} \rangle = v_{\mathcal{H}} \sim M_{GUT} \quad , \quad \langle \mathcal{H}' \rangle = \langle \nu_{\mathcal{H}'} \rangle = v_{\mathcal{H}'} \sim M_{GUT} \tag{A.7}$$

where $SU(3)_C$ is contained in $SU(4)$, and the hypercharge $U(1)_Y$ is a linear combination of the residual $U(1)$ subgroups in $SU(4)$ and $SU(2)_R$. This symmetry breaking separates the two MSSM Higgs doublets in Eq.(A.4), and we find that the hypercharge (and electromagnetism) assignments are naturally quantised.

In analogy to the discussion of the Higgs mechanism in section 1.2, the broken generators of the Pati-Salam group have corresponding gauge bosons that become massive after symmetry breaking, while the SM gauge bosons remain massless. Diagonalising the mass matrices, we find the following heavy gauge bosons:

- 6 $SU(4)$ bosons with mass-squared $(v_{\mathcal{H}}^2 + v_{\mathcal{H}'}^2)g_4^2/4$
- 2 $SU(2)_R$ bosons with mass-squared $(v_{\mathcal{H}}^2 + v_{\mathcal{H}'}^2)g_{2R}^2/4$
- 1 $SU(2)_{B-L}$ bosons with mass-squared $(v_{\mathcal{H}}^2 + v_{\mathcal{H}'}^2)(g_{2R}^2/4 + 3g_4^2/8)$

where g_4 and g_{2R} are the gauge couplings for $SU(4)$ and $SU(2)_R$ respectively. They are related to the hypercharge coupling g' by the relation:

$$\frac{1}{g'^2} = \frac{1}{g_{2R}^2} + \frac{2}{3g_4^2} \tag{A.8}$$

Appendix B

Supergravity Basics

We will use conventional supergravity formalism to describe the 4d effective theory that describes the low energy limit of type I string models. Supergravity (local SUSY) is defined in terms of a Kähler function (G) of generic chiral superfields ($\phi = h, C_a$) including the hidden sector closed strings ($h = S, T_i, Y_k$) and open string matter states ($C_a = C_j^{5_i}, C^{5_i 5_j}, C^{5_i 9}, C_i^9$) [1]:

$$G(\phi, \bar{\phi}) = \frac{K(\phi, \bar{\phi})}{\tilde{M}_{Pl}^2} + \ln\left(\frac{W(\phi)}{\tilde{M}_{Pl}^3}\right) + \ln\left(\frac{W^*(\bar{\phi})}{\tilde{M}_{Pl}^3}\right) \tag{B.1}$$

The Kähler potential $K(\phi, \bar{\phi})$ is a real function of chiral superfields and may be expanded in powers of matter states C_a [65] (including non-perturbative contributions):

$$K = \bar{K}(h, \bar{h}) + \tilde{K}_{\bar{a}b}(h, \bar{h})\bar{C}_{\bar{a}}C_b + \left[\frac{1}{2}Z_{ab}(h, \bar{h})C_aC_b + h.c.\right] + \dots \tag{B.2}$$

where $\tilde{K}_{a\bar{b}}$ is the (generally non-diagonal) matter metric and a non-zero bilinear term Z_{ab} can generate the μ-term through the Guidice-Masiero mechanism [14] subject

[1]Notice that we have included powers of the reduced Planck mass ($\tilde{M}_{Pl}$) that appear in the Kähler function to obtain the correct dimensions, although it is conventional to adopt natural units and set $\tilde{M}_{Pl} = 1$.

to gauge-invariance. The superpotential $W(\phi)$ is a holomorphic function of chiral superfields that can also be expanded:

$$W = \hat{W}(h) + \frac{1}{2}\mu_{ab}(h)C_a C_b + \frac{1}{6}Y_{abc}C_a C_b C_c + \dots \tag{B.3}$$

Notice that it includes a trilinear Yukawa term (that will generate fermion masses) and a bilinear μ-term. However, the Kähler potential and superpotential also receive non-perturbative contributions that are often difficult to predict. To make progess we will utilise a simple parametrisation of our ignorance of the non-perturbative sector in terms of Goldstino angles and CP-phases.

B.1 Supergravity Potential

We know that SUSY must be broken in Nature, but the precise mechanism responsible is not known at present. It is convenient to analyse the SUSY breaking by considering the F-part of the SUGRA scalar potential [2]. It can be expressed in terms of derivatives of the Kähler function $G(\phi, \bar{\phi})$, or equivalently in terms of the F-term auxiliary fields that can acquire non-zero VEVs and trigger the SUSY breaking [65]. Using Eq.(B.1) we obtain:

$$V(\phi, \bar{\phi}) = e^G \left[G_I (K^{-1})_{I\bar{J}} G_{\bar{J}} - 3\right] = F_{\bar{J}} K_{\bar{J}I} F_I - 3e^K |W|^2 \tag{B.4}$$

where $I, J \equiv \phi_I, \phi_J \in S, T_i, Y_k, C_a$ and

$$G_I \equiv \frac{\partial G}{\partial \phi_I} = \frac{W_I}{W} + K_I \tag{B.5}$$

$$F_I = e^{G/2} (K^{-1})_{I\bar{J}} G_{\bar{J}} \tag{B.6}$$

[2]We will ignore the D-term contributions arising from the gauge sector.

where $(K^{-1})_{I\bar{J}}$ is the inverse of the metric $K_{\bar{J}I}$, and satisfies the relation $(K^{-1})_{I\bar{J}}K_{\bar{J}L} = \delta_{IL}$. A subscript on G denotes partial differentiation, while the same subscript on F is just a label. A barred subscript on an F-term denotes its conjugate field $F_{\bar{I}} \equiv (F_I)^\dagger$. We make no distinction between upper and lower indices.

After SUSY breaking, the supersymmetric partner of the Goldstone boson (Goldstino) is *eaten* by the massless gravitino through the super-Higgs mechanism. The gravitino now has a mass given by

$$m_{3/2}^2 = e^{\langle G\rangle} = e^{\langle K\rangle}\,|\langle W\rangle|^2 = \frac{1}{3}\langle F_{\bar{J}}\,K_{\bar{J}I}\,F_I\rangle \tag{B.7}$$

and sets the overall scale of the soft parameters.

In the absence of F-term vacuum expectation values $(\langle F_I\rangle = 0\ \forall\,\phi_I)$, the locally supersymmetric vacuum is negative $V_{SUSY} = -3e^G$. However if one (or more) of the auxiliary F-terms acquires a non-zero VEV, the negative vacuum energy can be (partially) cancelled. This raises the exciting possibility that the vacuum energy, or rather the cosmological constant V_0, can be made vanishingly small in agreement with experimental limits. Notice that such a possibility cannot arise in *global* SUSY.

B.2 SUSY breaking F-terms

As previously mentioned, (unknown) non-perturbative contributions to the Kähler function require a parametrisation of our ignorance in terms of Goldstino angles and CP-phases that *control* the relative contributions to SUSY breaking from the various F-terms VEVs. We can define a column vector of F-term VEVs F in terms of a matrix P and column vector Θ (which also includes a CP-phase), where Θ has unit length and satisfies $\Theta^\dagger\Theta = 1$, and P canonically normalises the Kähler metric $P^\dagger K_{\bar{J}I} P = 1$:

$$\begin{aligned} F &= \sqrt{3}\, C\, m_{3/2}\, (P\Theta) \\ F^\dagger &= \sqrt{3}\, C\, m_{3/2} \left(\Theta^\dagger P^\dagger\right) \end{aligned} \tag{B.8}$$

Replacing the fields by their VEVs, we can rewrite Eq.(B.4) as a matrix equation:

$$\begin{aligned} \langle V \rangle \equiv V_0 &= F^\dagger\, K_{\bar{J}I}\, F - 3m_{3/2}^2 \\ &= 3C^2 m_{3/2}^2\, \Theta^\dagger\Theta \left(P^\dagger K_{\bar{J}I} P\right) - 3m_{3/2}^2 \\ &= 3m_{3/2}^2 \left(C^2 - 1\right) \end{aligned} \tag{B.9}$$

where V_0 is the cosmological constant and hence $C^2 = 1 + \frac{V_0}{3m_{3/2}^2}$. Therefore, choosing a vanishingly small cosmological constant sets $C = 1$.

As an example consider the case of the dilaton S and an overall moduli field T with diagonal Kähler metric. The SUGRA potential would be a "sum of squares" $V_F \sim |F_S|^2 + |F_T|^2 + \ldots - 3e^G$ and hence the P-matrix is a diagonal normalising matrix:

$$P_{I\bar{J}} = (K_{I\bar{I}})^{-1/2} \delta_{I\bar{J}} \tag{B.10}$$

In this special case we would recover the expressions of Refs.[39, 65, 82]:

$$F \equiv \begin{pmatrix} F_S \\ F_T \end{pmatrix} = \sqrt{3} C\, m_{3/2} \begin{pmatrix} (K_{S\bar{S}})^{-1/2} \sin\theta\, e^{i\alpha_S} \\ (K_{T\bar{T}})^{-1/2} \cos\theta\, e^{i\alpha_T} \end{pmatrix} \tag{B.11}$$

so that dilaton(moduli) dominated SUSY breaking corresponds to $\sin\theta(\cos\theta) = 1$ respectively. However in the more general case, the potential includes terms that mix different F-terms. The action of the P-matrix is to canonically normalise the Kähler metric and maintain the validity of the parametrisation [3].

B.3 Soft Masses and trilinears

Using Eqs.(B.2,B.3) we can write down the *un-normalised* SUSY breaking masses and trilinears that arise in the soft SUGRA potential [65]:

$$V_{soft} = m^2_{\bar{a}b}\bar{C}_{\bar{a}}C_b + \left(\frac{1}{6}A_{abc}Y_{abc}C_aC_bC_c + h.c.\right) + \ldots \tag{B.12}$$

where the Kähler metrics are in general not diagonal leading to the non-canonically normalised soft masses

$$\begin{aligned} m^2_{\bar{a}b} &= \left(m^2_{3/2} + V_0\right)\tilde{K}_{\bar{a}b} - F_{\bar{m}}\left(\partial_{\bar{m}}\partial_n\tilde{K}_{\bar{a}b} - \partial_{\bar{m}}\tilde{K}_{\bar{a}c}(\tilde{K}^{-1})_{c\bar{d}}\partial_n\tilde{K}_{\bar{d}b}\right)F_n & \text{(B.13)} \\ A_{abc}Y_{abc} &= \frac{\hat{W}^*}{|\hat{W}|}e^{\bar{K}/2}F_m\left[\bar{K}_mY_{abc} + \partial_mY_{abc} - \left((\tilde{K}^{-1})_{d\bar{e}}\partial_m\tilde{K}_{\bar{e}a}Y_{dbc}\right.\right. & \text{(B.14)} \\ &\qquad\qquad \left.\left.+(a \leftrightarrow b) + (a \leftrightarrow c)\right)\right] \end{aligned}$$

where the subscript $m = h, C_a$. Notice that a non-diagonal Kähler metric for the matter states will generate a mass matrix between different fields. The physical masses and states are obtained by transforming to the canonically normalised Kähler metric,

$$\tilde{K}_{\bar{a}b}\bar{C}_{\bar{a}}C_b \longrightarrow \bar{C}'_{\bar{a}}C'_a. \tag{B.15}$$

The Kähler metric is canonically normalised by a transformation $\tilde{P}^\dagger\tilde{K}\tilde{P} = 1$, so that the physical canonically normalised masses m^2_a are related to the previous non-

[3]The Kähler metric always receives off-diagonal components from the matter fields, but these are conventionally assumed to be small in comparison to the diagonal entries. However, the anomaly cancelling Green-Schwarz term in string theory mixes different fields at the same level to introduce off-diagonal components of comparable size.

canonical mass matrix $m^2_{\bar{a}b}$ by the relation

$$m^2_a = \tilde{P}^\dagger m^2_{\bar{a}b} \tilde{P}. \tag{B.16}$$

If the Kähler matter metric is diagonal (but not canonical) $\tilde{K}_a = \tilde{K}_{\bar{a}b}\delta_{\bar{a}b}$ then the canonically normalised scalar masses m^2_a are simply given by

$$m^2_a = m^2_{3/2} - F_{\bar{J}} F_I \partial_{\bar{J}} \partial_I \left(\ln \tilde{K}_a\right) \qquad (I, J = h, C_a). \tag{B.17}$$

The soft gaugino mass associated with the gauge group G_α is:

$$M_\alpha = \frac{1}{2\,Re f_\alpha} F_I \partial_I f_\alpha \qquad (I = S, T_i, Y_k) \tag{B.18}$$

and the canonically normalised SUSY breaking trilinear term for the scalar fields $A_{abc} Y_{abc} C_a C_b C_c$ is

$$A_{abc} = F_I \left[\overline{K}_I + \partial_I \ln Y_{abc} - \partial_I \ln \left(\tilde{K}_a \tilde{K}_b \tilde{K}_c\right)\right]. \tag{B.19}$$

Appendix C

$\mathcal{N}=2$ supersymmetry formalism

C.1 Mirror fields in $\mathcal{N}=2$ supersymmetry

As discussed in section 1.4.3, we can construct $\mathcal{N}=2$ SUSY hypermultiplets from an $\mathcal{N}=1$ chiral supermultiplet in combination with its CP-"mirror" superfield. We will remove any ambiguity by specifying what we mean by a "mirror" partner. Consider the left-handed quark MSSM doublet, Q_{iL}, as an explicit example. Under the MSSM gauge symmetry and Lorentz symmetry, Q_{iL} has the following quantum numbers respectively:

$$Q_{iL}: \quad \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) \qquad \left(\frac{1}{2}, 0\right) \tag{C.1}$$

Now the mirror Q^m_{iL} has the opposite gauge quantum numbers, but still transforms like a left-handed field:

$$Q^m_{iL}: \quad \left(\overline{\mathbf{3}}, \overline{\mathbf{2}}, -\frac{1}{6}\right) \qquad \left(\frac{1}{2}, 0\right) \tag{C.2}$$

However, the CP-operation recovers a right-handed "CP-mirror" with the same gauge quantum numbers:

$$CP\{Q^m_{iL}\} \mapsto Q^{mc}_L : \quad \left(\mathbf{3}, \mathbf{2}, \frac{1}{6}\right) \qquad \left(0, \frac{1}{2}\right) \tag{C.3}$$

When we consider the Higgsinos and top quark, it will be useful to form 4-component Dirac spinors from the $\mathcal{N} = 1$ MSSM fields and the CP-conjugates of their mirror fields. This leads to mixing in the classical equations of motion as discussed in Appendix C.3. Similary for gauginos we associate the usual $\mathcal{N} = 1$ gaugino and its $\mathcal{N} = 2$ superpartner together in a 4-component spinor. Note that use of the term "mirror" will implicitly include CP-conjugation.

C.2 5d Dirac matrices

In this Appendix we will review the Dirac matrices that appear in the fermion terms of the 5d Lagrangian. We will use the notation that the indices M, N run over 0,1,2,3,5; and μ runs over 0,1,2,3 as usual. We use a timelike metric $\eta_{MN} = diag\,(1, -1, -1, -1, -1)$, and take the following basis for the 5d Dirac matrices:

$$\gamma^M = \left[\begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0 \end{pmatrix}, \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \right] \tag{C.4}$$

where $\sigma^\mu = (1, \underline{\sigma})$ and $\overline{\sigma}^\mu = (1, -\underline{\sigma})$.

C.3 $\mathcal{N} = 2$ spinors and 5d kinetic terms

It is convenient to work in terms of $\mathcal{N} = 2$ hypermultiplets, which we have seen are formed from conventional $\mathcal{N} = 1$ supermultiplets by adding the CP-conjugates of their mirror superfields with opposite quantum numbers.

We will consider an explicit example where the third family lives in the extra-dimensional bulk. The third family scalars and their mirrors are uncoupled, and so only the even parity (MSSM) scalars couple directly to the SUSY breaking sector to acquire a soft mass. However, the form of the 5d Dirac matrices causes mixing between fermion fields of even and odd Z_2-parity.

Consider the top fields charged with respect to the (unbroken) $SU(2)_L$ gauge group in the MSSM - the left-handed top is contained within the left-handed quark doublet Q_{3L} along with the left-handed bottom quark. The right-handed top is a singlet with respect to $SU(2)_L$, and so a Dirac mass term $\sim m_t\left(t_L t_R^\dagger + t_R^\dagger t_L\right)$ is forbidden by gauge invariance[1].

In the $\mathcal{N} = 2$ generalisation, we must include additional mirror fields to construct the full 5d hypermultiplet. The left-handed top t_L and the CP-conjugate of its mirror, t_L^{mc}, can be combined into a 4-component Dirac spinor, since the charge-conjugated left-handed mirror is equivalent to a right-handed fermion. Similarly for the right-handed top t_R and its mirror t_R^m. Notice that $SU(2)_L$ singlets and doublets appear in different Dirac spinors, and therefore do not break the gauge symmetry. We have two 4-component Dirac spinors for the top sector, where the index labels the handedness of the MSSM fermion:

$$T_L = \begin{pmatrix} t_L \\ t_L^{mc} \end{pmatrix} \qquad\qquad T_R = \begin{pmatrix} t_R^{mc} \\ t_R \end{pmatrix} \tag{C.5}$$

and similarly $\bar{T} = T^\dagger \gamma^0$

$$\bar{T}_L = \begin{pmatrix} t_L^{mc\dagger} & t_L^\dagger \end{pmatrix} \qquad\qquad \bar{T}_R = \begin{pmatrix} t_R^\dagger & t_R^{mc\dagger} \end{pmatrix} \tag{C.6}$$

[1]A Dirac mass may be formed *after* the $SU(2)_L$ gauge symmetry is broken which is what happens in the (MS)SM through the Higgs mechanism and EWSB.

We can now construct the kinetic terms in the 5d lagrangian in the absence of interactions.

$$\begin{aligned}\mathcal{L}_{KE} &= i\bar{T}_L\gamma^M\partial_M T_L + i\bar{T}_R\gamma^M\partial_M T_R \\ &= it_L^\dagger\bar{\sigma}^\mu\partial_\mu t_L + it_R^\dagger\sigma^\mu\partial_\mu t_R + it_L^{mc\dagger}\sigma^\mu\partial_\mu t_L^{mc} + it_R^{mc\dagger}\bar{\sigma}^\mu\partial_\mu t_R^{mc} \\ &\qquad -t_L^{mc\dagger}\partial_5 t_L + t_L^\dagger\partial_5 t_L^{mc} + t_R^\dagger\partial_5 t_R^{mc} - t_R^{mc\dagger}\partial_5 t_R\end{aligned} \tag{C.7}$$

Notice that the 4d kinetic terms do not mix fields, while γ^5 leads to mixing between fields and their mirror states. This leads to non-trivial classical equations of motion.

Appendix D

Spectrum of Gauge bosons in BMSB

D.1 General case

In this Appendix we consider the effect of symmetry breaking on massless gauge field states and gauge couplings in the BMSB model of chaper 2. We begin with the gauge group $SU(4)_{5_2} \otimes SU(2)_{5_{1L}} \otimes SU(2)_{5_{2L}} \otimes SU(2)_{5_{1R}} \otimes SU(2)_{5_{2R}}$, where the scale-dependence of the gauge couplings is governed by RGEs. Conventionally, the symmetry breaking all occurs at high energies $(10^{15} - 10^{16} GeV)$ except for $SU(2)_L \otimes U(1)_Y \longrightarrow U(1)_{EM}$ which happens at the electroweak scale ($v \approx 246$ GeV). In the tables that follow, gauge couplings are assumed to be at high energies unless otherwise stated. Notice that i, a and m are adjoint indices for SU(2), SU(3) and SU(4) respectively.

(a) First combine the chiral SU(2) groups from either brane via diagonal symmetry

Gauge group	$SU(4)_{5_2}$	$SU(2)_{5_{1L}}$	$SU(2)_{5_{2L}}$	$SU(2)_{5_{1R}}$	$SU(2)_{5_{2R}}$
Coupling	g_{5_2}	g_{5_1}	g_{5_2}	g_{5_1}	g_{5_2}
States	$G^m_{5_2}$	$W^i_{5_1L}$	$W^i_{5_2L}$	$W^i_{5_1R}$	$W^i_{5_2R}$

Table D.1: The initial gauge groups, gauge couplings and states in our model.

breaking to recover the Pati-Salam gauge group.

$$\begin{aligned} SU(2)_{5_{1L/R}} \otimes SU(2)_{5_{2L/R}} &\overset{\substack{diagonal \\ v_\phi, v_{\phi'}}}{\longrightarrow} SU(2)_{L/R} \\ &\Rightarrow SU(4)_{5_2} \otimes SU(2)_L \otimes SU(2)_R \equiv G_{PS} \end{aligned} \tag{D.1}$$

Spontaneous symmetry breaking (SSB) induces a change of basis, parametrised by

$$\cos\theta_\phi = \frac{g_{5_2}}{\sqrt{g_{5_1}^2 + g_{5_2}^2}} \tag{D.2}$$

We can express the new massless states and gauge couplings in terms of the original parameters. The Higgs mechanism generates massive gauge bosons with masses of the order of the symmetry breaking scale.

Gauge group	$SU(4)_{5_2}$	$SU(2)_L$ $\qquad$ $SU(2)_R$
Coupling	g_{5_2}	$g_L = \frac{g_{5_1} g_{5_2}}{\sqrt{g_{5_1}^2 + g_{5_2}^2}} = g_R$
States	$G^m_{5_2}$	$W^i_{L/R} = \frac{1}{\sqrt{g_{5_1}^2 + g_{5_2}^2}} \left(g_{5_1} W^i_{5_2L/R} + g_{5_2} W^i_{5_1L/R} \right)$

Table D.2: The new massless states and couplings after the original gauge symmetry is broken down to the Pati-Salam gauge group.

plus 3 massive $SU(2)_L$ $\left(\bar{W}_L\right)$ and 3 massive $SU(2)_R$ $\left(\bar{W}_R\right)$ bosons, of mass
$M^2_{\bar{W}_{L/R}} = \frac{1}{2} v_\phi^2 \left(g_{5_1}^2 + g_{5_2}^2\right)$

(b) QCD $SU(3)_C$ is contained within $SU(4)_{5_2}$. The U(1)s combine to give the hypercharge U(1) using the relationship $Y = (B - L) + 2I_R$.

$$\begin{aligned} SU(4)_{5_2} &\supset SU(3)_C \otimes U(1)_{B-L} \\ SU(2)_R &\supset U(1)_{I_R} \end{aligned} \tag{D.3}$$

The Pati-Salam gauge group is broken down to the Standard Model by giving VEVs to the Higgs fields $\mathcal{H}, \mathcal{H}'$ as discussed in Appendix A.

$$\begin{aligned} & U(1)_{B-L} \otimes U(1)_{I_R} \xrightarrow{v_{\mathcal{H}}} U(1)_Y \\ & \Rightarrow SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \end{aligned} \tag{D.4}$$

The change of basis is parametrised by

$$\cos\theta_H = \frac{\sqrt{\frac{3}{2}}g_{5_2}}{\sqrt{g_R^2 + \frac{3}{2}g_{5_2}^2}} = \sqrt{\frac{3\left(g_{5_1}^2 + g_{5_2}^2\right)}{5g_{5_1}^2 + 3g_{5_2}^2}} \tag{D.5}$$

Gauge group	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Coupling	g_{5_2}	$g_L = \frac{g_{5_1}g_{5_2}}{\sqrt{g_{5_1}^2 + g_{5_2}^2}}$	$g_Y' = \frac{g_{5_1}g_{5_2}\sqrt{3}}{\sqrt{5g_{5_1}^2 + 3g_{5_2}^2}}$
States	$G_{5_2}^a$	$W_L^i = \frac{g_{5_1}W_{5_2L}^i + g_{5_2}W_{5_1L}^i}{\sqrt{g_{5_1}^2 + g_{5_2}^2}}$	$B_Y = \frac{\sqrt{3}\left(g_{5_1}W_{5_2R}^3 + g_{5_2}W_{5_1R}^3\right) + \sqrt{2}g_{5_1}G_{5_2}^{15}}{\sqrt{5g_{5_1}^2 + 3g_{5_2}^2}}$

Table D.3: The Standard Model massless states and gauge couplings expressed in terms of the original parameters.

plus 6 massive $SU(4)_{5_2}$ bosons $\left(G_{5_2}^9 - G_{5_2}^{14}\right)$, mass $M_G^2 = \frac{1}{4}v_{\mathcal{H}}^2 g_{5_2}^2$,
2 massive $SU(2)_R$ bosons $\left(W_R^{\pm}\right)$, mass $M_{W_R^\pm}^2 = \frac{1}{4}v_{\mathcal{H}}^2 g_R^2$,
and 1 massive $SU(2)_{B-L}$ boson (X_{B-L}), mass $M_{X_{B-L}}^2 = \frac{1}{4}v_{\mathcal{H}}^2\left(g_R^2 + \frac{3}{2}g_{5_2}^2\right)$

(c) Finally, we can recover the QCD and EM Standard Model gauge group via the familiar low-energy Higgs mechanism, parametrised by

$$\cos\theta_W = \frac{g_L(v)}{\sqrt{g_L^2(v)+g_Y'^2(v)}} = \sqrt{\frac{5g_{5_1}^2(v)+3g_{5_2}^2(v)}{8g_{5_1}^2(v)+6g_{5_2}^2(v)}}.$$

Electroweak symmetry breaking occurs when the Higgs fields H_u, H_d acquire non-zero VEVs. Notice that the gauge couplings are evaluated at the electroweak scale $g = g(v)$.

$$\begin{aligned} SU(2)_L \otimes U(1)_Y &\xrightarrow{v} U(1)_{EM} \\ &\Rightarrow SU(3)_C \otimes U(1)_{EM} \end{aligned} \tag{D.6}$$

Gauge group	$SU(3)_C$	$U(1)_{EM}$
Coupling	$g_{5_2}(v)$	$e = \dfrac{g_{5_1}(v)g_{5_2}(v)\sqrt{3}}{\sqrt{8g_{5_1}^2(v)+6g_{5_2}^2(v)}}$
States	$G^a_{5_2}$	$A = \dfrac{\sqrt{3}g_{5_1}(v)\left(W^3_{5_2L}+W^3_{5_2R}\right)+\sqrt{3}g_{5_2}(v)\left(W^3_{5_1L}+W^3_{5_1R}\right)+\sqrt{2}g_{5_1}(v)G^{15}_{5_2}}{\sqrt{8g_{5_1}^2(v)+6g_{5_2}^2(v)}}$

Table D.4: The massless gauge states and couplings after electroweak symmetry breaking.

plus 3 massive $SU(2)_L$ bosons $(W_L^\pm, Z_L^0)$ with masses:

$$M_{W_L^\pm} = \frac{1}{2}vg_L(v) = \frac{g_{5_1}(v)g_{5_2}(v)v}{2\sqrt{g_{5_1}^2(v)+g_{5_2}^2(v)}} \text{ and}$$

$$M_{Z_L^0} = g_{5_1}(v)g_{5_2}(v)v\sqrt{\frac{4g_{5_1}^2(v)+3g_{5_2}^2(v)}{2\left(g_{5_1}^2(v)+g_{5_2}^2(v)\right)\left(5g_{5_1}^2(v)+3g_{5_2}^2(v)\right)}}$$

D.2 Limiting case $R_{5_2} \gg R_{5_1}$

In this Appendix we repeat the symmetry breaking analysis for the limiting case

$$R_{5_2} \gg R_{5_1} \Leftrightarrow g_{5_2} \ll g_{5_1} \qquad \text{(D.7)}$$

We find that the dominant components of the massless gauge fields live on the 5_2-brane ("bulk") which is consistent with $\tilde{g}MSB$ - "single brane dominance".

(a) After diagonal symmetry breaking we recover the Pati-Salam gauge group

$SU(4)_{5_2} \otimes SU(2)_L \otimes SU(2)_R$

Gauge group	$SU(4)_{5_2}$	$SU(2)_L$ $\quad$ $SU(2)_R$
Coupling	g_{5_2}	$g_{L/R} \sim g_{5_2}$
States	$G^m_{5_2}$	$W^i_{L/R} \sim W^i_{5_2 L/R}$

Table D.5: The dominant components of massless states and couplings after symmetry has been broken down to the Pati-Salam group.

$$\text{plus 3 massive } SU(2)_L \text{ and 3 massive } SU(2)_R \text{ bosons } \left(\bar{W}_{L/R} \sim W^i_{5_1 L/R}\right),$$
$$M^2_{\bar{W}_{L/R}} \approx \tfrac{1}{2} v_\phi^2 g_{5_1}^2$$

(b) We break the Pati-Salam group down to the Standard Model. Notice the relationship between the hypercharge gauge coupling and the other gauge couplings, which is consistent with gauge coupling unification. This will happen if the 5_2 gauge coupling equals g_{GUT} at the GUT scale.

$$\text{plus 6 massive } SU(4)_{5_2} \text{ bosons } \left(G^9_{5_2} - G^{14}_{5_2}\right),\ M^2_G \approx \tfrac{1}{4} v_{\mathcal{H}}^2 g_{5_2}^2,$$
$$\text{2 massive } SU(2)_R \text{ bosons } \left(W^\pm_R \sim \tfrac{1}{\sqrt{2}}\left(W^1_{5_2 R} \mp i W^2_{5_2 R}\right)\right),\ M^2_{W^\pm_R} \approx \tfrac{1}{4} v_{\mathcal{H}}^2 g_{5_2}^2$$
$$\text{and 1 massive } SU(2)_{B-L} \text{ boson } \left(X_{B-L} \sim \sqrt{\tfrac{3}{5}} G^{15}_{5_2} - \sqrt{\tfrac{2}{5}} W^3_{5_2 R}\right),\ M^2_{X_{B-L}} \approx \tfrac{5}{8} g_{5_2}^2 v_{\mathcal{H}}^2$$

Gauge group	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Coupling	g_{5_2}	$g_L \sim g_{5_2}$	$g_Y' \sim \sqrt{\frac{3}{5}} g_{5_2}$
States	$G^a_{5_2}$	$W^i_L \sim W^i_{5_2 L}$	$B_Y \sim \sqrt{\frac{3}{5}} W^3_{5_2 R} + \sqrt{\frac{2}{5}} G^{15}_{5_2}$

Table D.6: The dominant components of the massless states and couplings after the Pati-Salam group is broken down to the Standard Model.

(c) Finally the Higgs mechanism induces electroweak symmetry breaking, and generates the massive W and Z bosons.

Gauge group	$SU(3)_C$	$U(1)_{EM}$
Coupling	$g_{5_2}(v)$	$e \sim \sqrt{\frac{3}{8}} g_{5_2}(v)$
States	$G^a_{5_2}$	$A \sim \sqrt{\frac{3}{8}} \left(W^3_{5_2 L} + W^3_{5_2 R} \right) + \frac{1}{2} G^{15}_{5_2}$

Table D.7: The dominant components of the familiar massless gauge states after electroweak symmetry.

plus 3 massive $SU(2)_L$ bosons:

$$\left(W_L^{\pm} \sim \frac{1}{\sqrt{2}} \left(W^1_{5_2 L} \mp i W^2_{5_2 L} \right) \right), \; M_{W_L^{\pm}} \approx \frac{1}{2} v g_{5_2}(v)$$

and $\left(Z_L^0 \sim \sqrt{\frac{5}{8}} W^3_{5_2 L} - \frac{3}{2\sqrt{10}} W^3_{5_2 R} - \frac{1}{2}\sqrt{\frac{3}{5}} G^{15}_{5_2} \right), \; M_{Z_L^0} \approx \sqrt{\frac{2}{5}} v g_{5_2}(v)$

Appendix E

Mass scales in BMSB

In this Appendix we consider the different mass scales present in the BMSB model. Each time the gauge symmetry is spontaneously broken down towards the Standard Model, the gauge bosons associated with the broken generators acquire masses via the Higgs mechanism, where the masses are around the symmetry breaking VEVs. Our model already assumes an order for symmetry breaking, which creates a VEV hierarchy $v_\phi \geq v_\mathcal{H} \gg v \sim O(M_W)$. For instance, we know that $v_\phi, v_\mathcal{H} \gg O(M_W)$ since these broken symmetry bosons have not been observed.

We must also consider the (inverse) compactification radii of the D5-branes. Their relative sizes are arbitrary, but we choose to start with the relationship $R_{5_2} > R_{5_1}$ or equivalently $R_{5_1}^{-1} > R_{5_2}^{-1}$ as shown in Figure E.1. Notice that we have not specified how R_{5_3} is related to the other two compactification radii, suffice to say that a large third dimension (felt by gravity alone) is not forbidden, i.e. $R_{5_1}^{-1} > R_{5_2}^{-1} \gg R_{5_3}^{-1}$.

In this work, we have adopted the standard scenario with symmetry breaking occurring at a scale comparable to the first two compactification radii and string scale. Soft masses are also generated at around the same scale. We have deliberately not specified these scales, but we claim that the formalism applies for GUT/string scales

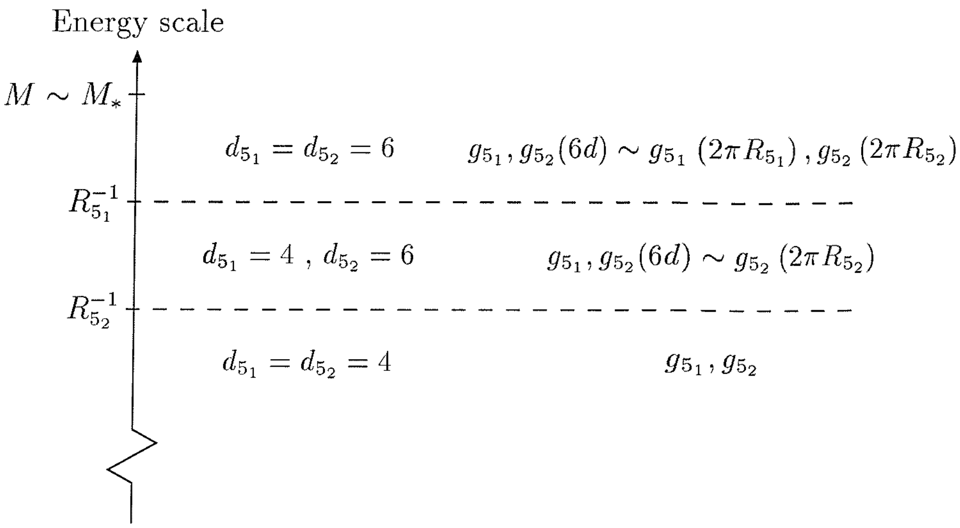


Figure E.1: At energy scales below an inverse compactification radii, the dimension appears too small to observe. The coupling in a higher-dimension is related to the same coupling in a lower dimension via Eq. (2.2).

in the region 1 TeV to 10^{16} GeV.

We impose the following restrictions:

$$\begin{aligned} R_{5_1}^{-1} &> R_{5_2}^{-1} \\ v_\phi &\geq v_{\mathcal{H}} \gg v \\ M_* &\geq R_{5_1}^{-1}, R_{5_2}^{-1} \sim v_\phi, v_{\mathcal{H}} \gg v \sim O(M_W) \end{aligned} \tag{E.1}$$

These constraints provide six ways of ordering the inverse radii and VEVs. The supersymmetry breaking scale (where soft masses are generated) also needs to be assigned, thus giving a total of 30 possibilities.

In Table 13 we list the various possibilities for the relative ordering of mass scales, VEVs and inverse compactification radii within the constraints of Eq.(E.1).

A	v	$v_{\mathcal{H}}$	v_ϕ	$R_{5_2}^{-1}$	$R_{5_1}^{-1}$	M_*
B	v	$v_{\mathcal{H}}$	$R_{5_2}^{-1}$	v_ϕ	$R_{5_1}^{-1}$	M_*
C	v	$R_{5_2}^{-1}$	$v_{\mathcal{H}}$	v_ϕ	$R_{5_1}^{-1}$	M_*
D	v	$v_{\mathcal{H}}$	$R_{5_2}^{-1}$	$R_{5_1}^{-1}$	v_ϕ	M_*
E	v	$R_{5_2}^{-1}$	$v_{\mathcal{H}}$	$R_{5_1}^{-1}$	v_ϕ	M_*
F	v	$R_{5_2}^{-1}$	$R_{5_1}^{-1}$	$v_{\mathcal{H}}$	v_ϕ	M_*

Table E.1: Possible ordering of symmetry breaking VEVs and inverse compactification radii within the constraints of Eq. (E.1).

Appendix F

Sequestered Hidden Sectors á la Randall-Sundrum

Randall and Sundrum [22] constructed a model with parallel 3-branes separated along a fifth extra dimension y. Matter fields are localised on one brane at $y = 0$ while SUSY was broken on the other brane at $y = L$. SUSY breaking is communicated between sectors via bulk fields living in the extra dimension including gravity and the conformal anomaly multiplet. The hidden sector is truly "sequestered", or hidden, from the visible sector due to the very weak gravitational coupling strength.

Consider the supergravity lagrangian kinetic term for chiral matter superfields $Q = (\tilde{q}, q)$ coupled to generic bulk superfields Φ,

$$\mathcal{L} = \sqrt{-g}\int d^4\theta \left[f\left(Q^\dagger, Q\right)\Phi^\dagger\Phi - \frac{1}{6} f\left(\tilde{q}^\dagger, \tilde{q}\right)\mathcal{R} + \dots \right] \tag{F.1}$$

where $g = det(g_{\mu\nu})$, $\mathcal{R}$ is the Ricci scalar, and we adopt the "compensator formalism" of Wess and Bagger [112] for the gravity fields:

$$\Phi \equiv 1 + F_\Phi \theta^2 \tag{F.2}$$

such that flat space corresponds to $\Phi = 1$, $g_{\mu\nu} = \eta_{\mu\nu}$ and Eq.(F.1) recovers the familiar globally supersymmetric lagrangian.

The kinetic lagrangian function f is related to the supergravity Kähler potential K by:

$$f \equiv -3\tilde{M}_{Pl}^2\, e^{-K/3\tilde{M}_{Pl}^2} \tag{F.3}$$

where $\tilde{M}_{Pl}^2 = M_{Pl}^2/8\pi$ is the reduced Planck mass and is conventionally set equal to unity in cosmological units. Notice that for a canonically normalised Kähler potential $K(\tilde{q}^\dagger, \tilde{q}) = \tilde{q}^\dagger\tilde{q}$ then

$$f(\tilde{q}^\dagger, \tilde{q}) \approx -3\tilde{M}_{Pl}^2 + \tilde{q}^\dagger\tilde{q} + \ldots \tag{F.4}$$

which gives the standard Ricci scalar term from the Einstein-Hilbert action

$$\mathcal{L} \approx \sqrt{-g}\frac{\tilde{M}_{Pl}^2}{2}\mathcal{R} + \ldots \tag{F.5}$$

Now we will include a hidden sector field Σ which acquires a non-zero VEV and breaks SUSY. The effect of SUSY breaking is communicated to the visible sector via the bulk Φ fields, and any direct coupling between the two sectors arises from a non-renormalisable operator with a coefficient suppressed by powers of $\tilde{M}_{Pl}$.

Previous attempts to incorporate a hidden sector field have simply added the visible and hidden sector matter Kähler potentials together ($K = K_{vis} + K_{hid}$) but from Eq.(F.3) this will clearly lead to a non-zero tree-level scalar mass, albeit suppressed by powers of the Planck mass. Randall and Sundrum realised that in order to obtain a truly sequestered hidden sector, the hidden and visible sector Kähler potentials must be added non-linearly:

$$
\begin{aligned}
f &= -3\tilde{M}_{Pl}^2 + f_{vis} + f_{hid} \\
&= -3\tilde{M}_{Pl}^2 \left[1 + e^{-K_{vis}/3\tilde{M}_{Pl}^2} + e^{-K_{hid}/3\tilde{M}_{Pl}^2}\right] \qquad \text{(F.6)}
\end{aligned}
$$

which gives a combined Kähler potential:

$$
K_{RS} = -3\tilde{M}_{Pl}^2 \ln \left[1 + e^{-K_{vis}/3\tilde{M}_{Pl}^2} + e^{-K_{hid}/3\tilde{M}_{Pl}^2}\right] \qquad \text{(F.7)}
$$

Notice that there is no problem adding the individual hidden and visible sector superpotentials together to form a combined superpotential, since each superpotential is radiatively stable due to supersymmetric non-renormalisation theorems. The Randall-Sundrum Kähler potential possesses the property that - due to a "magical" cancellation - the scalar mass vanishes exactly at tree-level.

Appendix G

Alternative exponential suppression factor

In this Appendix we consider an alternative suppression factor $e^{-(M_* r)^2}$ that is attributed to non-perturbative world-sheet instanton corrections [86], and differs from the (previous) field theory interpretation of the suppression due to propagating massive modes.

We summarise the modified scalar masses and trilinears found by repeating the earlier calculations in section 3.4, but with the alternative suppression factor. The non-sequestered scalar masses of Eq.(3.43) remain unchanged, but Eq.(3.44) becomes:

$$\begin{aligned}
m^2_{C^{5_1 5_2}} &= \tilde{m}^2 - \frac{3}{2} m^2_{3/2} \left(\sin^2\theta + \Theta_3^2 \cos^2\theta \sin^2\phi \right) \\
m^2_{C^{9 5_1}} &= \tilde{m}^2 - \frac{3}{2} m^2_{3/2} \cos^2\theta \sin^2\phi \left(\frac{\Theta_2^2}{k} + \Theta_3^2 \right) \\
m^2_{C_1^{5_1}} &= \tilde{m}^2 - 3 m^2_{3/2} \sin^2\theta \\
m^2_{C_2^{5_1}} &= \tilde{m}^2 - 3 m^2_{3/2} \Theta_3^2 \cos^2\theta \sin^2\phi \\
m^2_{C_3^{5_1}} &= \tilde{m}^2 - \frac{3}{k} m^2_{3/2} \Theta_2^2 \cos^2\theta \sin^2\phi
\end{aligned} \tag{G.1}$$

where

$$\begin{aligned}\tilde{m}^2 = m_{3/2}^2 \Bigg[& 1 - \cos^2\theta \cos^2\phi \left(1 - e^{-(T_2+\bar{T}_2)/4}\right) \\ & - \frac{\cos^2\theta \sin^2\phi\, \Theta_2^2\, \delta_{GS}}{k} \left(1 - e^{-(T_2+\bar{T}_2)/4}\right) \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\} \\ & + \frac{\cos^2\theta \sin^2\phi\, \Theta_2^2\, e^{-(T_2+\bar{T}_2)/4}}{32k} (T_2+\bar{T}_2)^2 \left\{Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2+\bar{T}_2)\right\}^2 \\ & - \frac{\cos^2\theta \cos\phi \sin\phi \left(\Theta_2\, e^{i(\alpha_2 - \alpha_{Y_2})} + \Theta_2^\dagger\, e^{-i(\alpha_2-\alpha_{Y_2})}\right) e^{-(T_2+\bar{T}_2)/4}}{32\sqrt{k}} \\ \times & \left\{Y_2 + \bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\} \left(8(T_2+\bar{T}_2) + \delta_{GS}\left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}\right)\Bigg]\end{aligned} \tag{G.2}$$

which replaces Eq.(3.45); and the masses are expanded up to $\mathcal{O}\left[\frac{\delta_{GS}}{T_2+\bar{T}_2}\right]$.

Similarly we can find modified expressions for the trilinears of Eqs.(3.52 - 3.57).

$$\begin{aligned}A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} = & -\sqrt{3} m_{3/2} \cos\theta \Bigg[\sin\phi \frac{\Theta_2\, e^{i\alpha_2}}{\sqrt{k}} \\ & + \sin\phi \frac{\Theta_2\, e^{i\alpha_2}}{12\sqrt{k}} e^{-(T_2+\bar{T}_2)/4} (T_2+\bar{T}_2) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2 \\ & - \cos\phi \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-(T_2+\bar{T}_2)/4}\right) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}\Bigg]\end{aligned} \tag{G.3}$$

$$\begin{aligned}A_{C_1^9 C^{95_1} C^{95_1}} = & -\sqrt{3} m_{3/2} \Bigg[\sin\theta\, e^{i\alpha_S} \\ & + \cos\theta \sin\phi \frac{\Theta_2\, e^{i\alpha_2}}{12\sqrt{k}} e^{-(T_2+\bar{T}_2)/4} (T_2+\bar{T}_2) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2 \\ & - \cos\theta \cos\phi \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-(T_2+\bar{T}_2)/4}\right) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}\Bigg]\end{aligned} \tag{G.4}$$

$$\begin{aligned}A_{C^{5_1 5_2} C^{95_1} C^{95_2}} = & -\sqrt{3} m_{3/2} \Bigg[\frac{1}{2} \sin\theta\, e^{i\alpha_S} + \frac{1}{2}\cos\theta\sin\phi \left(\Theta_1\, e^{i\alpha_1} + \frac{\Theta_2\, e^{i\alpha_2}}{\sqrt{k}} - \Theta_3\, e^{i\alpha_3}\right) \\ & + \cos\theta \sin\phi \frac{\Theta_2\, e^{i\alpha_2}}{12\sqrt{k}} e^{-(T_2+\bar{T}_2)/4} (T_2+\bar{T}_2) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2 \\ & - \cos\theta \cos\phi \frac{e^{i\alpha_{Y_2}}}{3} \left(1 + 2\, e^{-(T_2+\bar{T}_2)/4}\right) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}\Bigg]\end{aligned} \tag{G.5}$$

$$\begin{aligned}A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} = A_{C_1^{5_1} C^{95_1} C^{95_1}} = & -\sqrt{3} m_{3/2} \cos\theta \Big[\sin\phi\, \Theta_1\, e^{i\alpha_1} \\ & + \sin\phi \frac{\Theta_2\, e^{i\alpha_2}}{8\sqrt{k}} e^{-(T_2+\bar{T}_2)/4} (T_2+\bar{T}_2) \left\{Y_2+\bar{Y}_2 - \delta_{GS}\ln(T_2+\bar{T}_2)\right\}^2\end{aligned} \tag{G.6}$$

$$- \cos\phi \, e^{i\alpha_{Y_2}} \, e^{-(T_2+\bar{T}_2)/4} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2 + \bar{T}_2) \right\} \Big]$$

$$A_{C_1^{5_2} C_2^{5_2} C_3^{5_2}} = A_{C_2^{5_2} C^{95_2} C^{95_2}} = -\sqrt{3} m_{3/2} \cos\theta \left[\sin\phi \, \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}} \right.$$

$$\left. - \cos\phi \, e^{i\alpha_{Y_2}} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2 + \bar{T}_2) \right\} \right] \qquad \text{(G.7)}$$

$$A_{C_1^9 C_2^9 C_3^9} = A_{C_2^9 C^{95_2} C^{95_2}} = -\sqrt{3} m_{3/2} \left[\sin\theta \, e^{i\alpha_S} \right.$$

$$\left. - \cos\theta \cos\phi \, e^{i\alpha_{Y_2}} \left\{ Y_2 + \bar{Y}_2 - \delta_{GS} \ln(T_2 + \bar{T}_2) \right\} \right] \qquad \text{(G.8)}$$

where all trilinears have been expanded up to $\mathcal{O}\left(\frac{1}{T_2+\bar{T}_2}\right)$.

It is now straightforward to consider these modified expressions in different limits of SUSY breaking domination as before.

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