

UNIVERSITY OF SOUTHAMPTON

Thoughts, Propositions, and Unities: A Historical and Critical Examination

by

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Doctor of Philosophy

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ABSTRACT

FACULTY OF ARTS

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This thesis examines the impact of early analytical philosophy on issues that, in turn, impact on work in current philosophy of language and mind. The contribution made by the thesis is therefore twofold: firstly it addresses exegetical issues regarding the origins of the analytical tradition and, secondly, extracts from this study insights into the current state of the philosophies of language and mind. I begin by discussing Frege's revolutionary work in logic and isolate three doctrines which I hold to be essential to the philosophical study of content: the independence of thoughts from the psychological process of thinking, the distinction between sense and reference, and the recognition that thoughts are structured unities. The thesis explores, defends, and develops these three notions and argues, ultimately, that any attempt to provide a philosophical account of thought and meaning must make provision for these concerns.

The thesis divides into five chapters, following a historical and thematic course. Chapter one introduces the central doctrines of Frege's philosophy and those of Russell's Principles of Mathematics. The problem of the unity of the proposition is introduced and the differing approaches to the problem offered by Frege and Russell are compared and evaluated. Chapter two examines in detail Russell's attempts to overcome the paradoxes of set theory in his development of the logical theory of Principia Mathematica. Special attention is given to the role of the theory of descriptions and the substitutional theory of classes and relations, as well as the later ramified theory of types and multiple relation theory of judgement, exploring how the problem of the unity of the proposition plays a pivotal role in determining the successes and failures of these approaches. These issues are further developed in chapter three by examining Wittgenstein's criticisms of Russell's logical theory and the subsequent development of Wittgenstein's own solution to these problems in his Tractatus Logico-Philosophicus.

The fourth chapter charts Russell's course after Wittgenstein's damning attack, examining how those criticisms influenced Russell's adoption of a causal theory of meaning and his propulsion into psychologism. The issues are developed through comparisons of Russell's position and that of more recent proponents of causal theories of content, such as Fodor, and the naturalistic project of Quine. This discussion of Quine is developed in the last chapter by examining how Quine's project stands in relation to the issues raised in the earlier parts of the thesis. The Quinean/Davidsonian attempt to reduce the theory of meaning to a theory of reference is criticized. The chapter concludes with a discussion of the relation between thought and language, arguing that any successful philosophical study of thought must be conducted through the philosophy of language.

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## Preface & Acknowledgements

This thesis was originally intended to be a critique of cognitive science. After completing a Master's degree in the philosophy of mind, I was eager to pursue a similar line of inquiry in my doctoral thesis and set myself the task of inquiring into the nature of thought, hoping that in doing so I would be able to offer some worthwhile criticisms of the various positions occupied by current philosophers of mind. As my research progressed, however, several influences conspired to drive the trajectory of the project away from the philosophy of mind and into the philosophy of language. Perhaps the most influential reason for this shift was my study of Frege and Russell who, at first, I had read only in the hope of casting further light on the gloom that had descended over my attempts to make sense of Wittgenstein's *Tractatus*. Such illumination was, indeed, readily forthcoming and, furthermore, light was cast on wider issues that troubled me and I thus commenced a lengthy investigation into the foundations of analytical philosophy that reshaped my thinking and my thesis.

Although the idea of writing this thesis first took shape during my time as an MA student, many of the thoughts expressed in it are more sophisticated versions of thoughts that were first given a chance to come to the fore as an undergraduate student. I would like to take this opportunity to pay a debt of gratitude to my teacher at that time, Roy Birch, for providing me with my first introduction to the ideas which have occupied my thoughts for much of the subsequent years. It was he who first encouraged me to devote my studies to philosophy, for which I am immensely grateful.

I have received much support and assistance during the completion of this project. The inevitable difficulties faced in writing a doctoral thesis have been greatly

eased by my friends among the staff and postgraduate students in the philosophy department of the University of Southampton. I am especially indebted to my supervisor, Ray Monk, whose guidance and insightful criticisms have constantly improved my thinking and writing. In addition, his vast knowledge of the works of Russell and Wittgenstein has provided me with an invaluable source material in its own right. The many long conversations I have shared with him, his published work, and the unpublished work of his which he has kindly allowed me to read and discuss with him, have been highly influential on the development of my thoughts on all areas of philosophy. He is not, of course, responsible for any shortcomings this thesis may contain. I would also like to thank Maria Alvarez who has been a constant and generous source of encouragement and invaluable advice during the completion of this project. She very kindly translated an article from Spanish into English for me and has offered many insightful comments on earlier drafts of parts of this thesis. I am also grateful to her for the guidance she offered during two enjoyable semesters when I worked as an assistant on an Applied Ethics course that she taught in the department. Other members of the philosophy department at Southampton have offered helpful criticism and advice on many occasions, especially when earlier drafts of some of this material have been presented during research seminars. I have benefited from numerous conversations from Denis McManus who has also offered much support. In the first year of my research on this project, I had the opportunity to discuss my work on the *Tractatus* with Tony Palmer, who has since retired. I would like to thank him for those conversations and for kindly allowing me to read some of his unpublished work on Wittgenstein. I would also like to thank the research students in the philosophy department whose time here has overlapped with mine for their friendship and for many lively philosophical discussions of my own ideas and theirs: Martin Ladbury, Geoff Eavy, Vincent Iyengar, James Cuthbert, Melenia Arouh, and Øystein Hide. I am grateful to Martin Ladbury for furnishing me with a copy of his PhD thesis and for reading and commenting on parts of this thesis. I also received very helpful comments from the panel at my upgrade viva: Ray Monk, Maria Alvarez, and Aaron Ridley.

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Southampton

G.S.

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## Introduction

The structure of this thesis is largely determined by historical considerations. In one sense this is, of course, inevitable; one can hardly examine the development of a line of thought in a given intellectual tradition without seeking to remain sensitive to the evolutionary course taken by that idea. One cannot, for example, understand many of the ideas running through the *Tractatus*, unless one sees them in the light of their origins in the works of those (mainly Frege and Russell) to whom Wittgenstein was responding. Hence the extraordinary elegance and ingenuity of the picture theory is likely to be lost on the reader who is unaware of the problems faced by Russell's many attempts to capture the essence of the proposition and its logical form prior to Wittgenstein's work. There is another sense, however, in which the link between philosophy and its past is essential to philosophical inquiry that does not have the same suspicious ring of triviality to it. Philosophical ideas do not, generally, survive a separation from their place in the history of philosophy. Many philosophical problems do not even appear immediately problematic unless they are seen in the context of the developments of preceding theories and the concerns of theorists out of which they have grown. The philosophical significance of Gödel's incompleteness theorems, to give a particularly vivid example, will only be recognized by someone sufficiently aware of the philosophical drives which motivated the logicist project in mathematics in the late nineteenth and early twentieth centuries, and such concerns, in turn, require an understanding of the philosophical perplexities which arose out of reflection on the infinite and the infinitesimal in previous centuries. Before long, it becomes apparent that the problems which provide the context in which the incompleteness results are deemed significant stretch back to the very beginnings of mathematical reasoning and grew out of paradoxes that troubled Plato and Euclid long before the completeness of higher-order logics could ever have been dreamt of.

This is, no doubt, one of the main reasons why philosophy often presents such a formidable face to those outside of it. To even understand a philosophical problem is, very often, to understand a substantial portion of philosophy itself. Nowhere is this more noticeable than in the kind of philosophy now called “analytical philosophy”. The three founding figures of that tradition scrutinized here, Frege, Russell, and Wittgenstein, are all very obviously “difficult” philosophers. The *Grundgesetze der Arithmetik*, *Principia Mathematica*, and *Tractatus Logico-Philosophicus*, are all quite fearsomely technical works that are likely to strike the unwary reader as virtually impenetrable on her first introduction to them. To even understand the problems these works are concerned with is already to achieve a significant amount in philosophy. But no progress can be made in the study of those works without such an understanding. For this reason, it is futile to try and approach them without some sensitivity to the history of ideas into which they were born.

Of course, a line must be drawn somewhere. A full account of the origins of the philosophical problems that these figures wrestled with would take almost as long as that history itself. The limitations of time and space imposed on a PhD thesis have meant that the line has here been drawn earlier than I should have otherwise preferred. A more elaborate account of the problems I have discussed in this thesis would have included at least two more lengthy chapters; one on the fascinating mathematical climate into which Frege’s *Begriffsschrift* was (at the time, unspectacularly) introduced, and one on the place of Carnap and the Vienna Circle in the development of the analytical tradition. As it is, I have had to content myself with no more than gestures towards the kinds of issues that those chapters would have explored in parts of chapters one, four, and five. The thesis is not, however, deformed by their omission as it is an omission of supporting detail rather than essence of argument, and the thesis is complete without their presence.

Analytical philosophy is not renowned for historical sensitivity; if anything, it is notorious for its *ahistorical* approach. The same is not true of philosophy labelled as “continental”. Heidegger’s *Basic Problems of Phenomenology*, for example, is given over in tremendous detail to the historical excavation of the origins of the problems Heidegger seeks to tackle, on the grounds that such investigation is essential to the investigation of those problems. It is almost unthinkable that his contemporaries in the analytical fold would have ventured a similar approach. The last decade or so has witnessed a change on this front, however. The publication of Michael Dummett’s

*Origins of Analytical Philosophy* in 1993 signalled the arrival of a new interest in the historical setting and development of the analytical tradition which has been eagerly taken up by others.<sup>1</sup> It should not be thought that no-one had turned their attention to these issues before this book, of course,<sup>2</sup> but what was noticeably rare (and welcome) about Dummett's work was that a prominent figure in the analytical tradition felt it necessary to examine the history of the ideas of analytical philosophy as a work of analytical philosophy, on the basis that: 'it is important to analytical philosophy that it understand its own history' (Dummett, 1993a, p. 1). The list of books instantiating this healthy self-reflective turn of the analytical movement has since grown steadily and continues to do so.<sup>3</sup> One aim of this thesis is to make a contribution to that movement.

The aim of this thesis is not, however, just to produce something of interest to the historian of philosophy. The history of philosophy is a worthwhile subject for study for its own sake, but a *philosophical* history must seek to do something more than simply reveal aspects of the past development of the subject. It must draw out from such explorations questions for further philosophical study and undertake that study. Such is the task of this thesis. The project commences with a discussion of Frege's revolutionary work in logic and isolates three doctrines which I hold to be essential to the philosophical study of content: the independence of thoughts from the psychology of thinking, the separation of sense and reference, and the recognition that thoughts are structured as unities. The thesis explores, defends, and develops these three notions and argues, ultimately, that any attempt to provide a philosophical account of thought and meaning must make provision for these concerns. The examination of these issues divides into five chapters, as follows.

Chapter One provides an exposition of Frege's work and critically examines the salient features of his philosophy. In particular I argue that Frege's analysis of propositional content on the model of mathematical functionality is well-suited to accommodate the unity of the proposition. I invoke his context-principle in defence of my claim that Frege should not be understood as putting forward a straightforwardly

<sup>1</sup> A particularly fruitful collection of essays, largely written in response to Dummett's book, is that edited by Glock (1997).

<sup>2</sup> The most notable, and still one of the best, works in this tradition prior to Dummett's book was J. Alberto Coffa's, *The Semantic Tradition from Kant to Carnap*, published posthumously in 1991. Others such as Hylton (1990) and Griffin (1991) had studied Russell's position in relation to the genesis of analytical philosophy.

<sup>3</sup> See, for example, Monk & Palmer (1996), Stroll (2001), Reck (2002).

combinatorial semantics (in Chapter Three I also argue, however, that it is wrong to perceive Frege's philosophy of language as making sentences the smallest unit of meaning and attribute to him, instead, an intermediate position). I also examine his recognition of the need to grant that linguistic items (and their referents) divide into logical types or semantic categories and offer a defence of his distinction between concept and object. A failing of Frege's project, I argue, is his dependence on an unsatisfactory degree of platonism as a way of achieving the benefits of anti-psychologism. I then go on to give an account of the philosophical basis for Russell's *Principles of Mathematics*, showing how it is derived from two sources: the extreme pluralism jointly developed with Moore in response to Hegelian idealism, and the mathematical achievements of Peano. I examine Russell's doctrine of denoting concepts and argue that the traditional interpretation of the doctrine as countenancing Meinongian non-actual objects is inaccurate. The chapter concludes with a comparison of the relative merits of Russell and Frege with regard to their treatment of the unity of the proposition. I argue that, at this time, Russell lacks a satisfactory account of propositional unity.

Chapter Two gives a detailed analysis of Russell's development of ramified type-theory in response to his discovery of the paradox. In particular I seek to elucidate Russell's desire to construct a system of logic which remains faithful to the philosophical position taken up in the *Principles*. In order to do so, I draw heavily on both Russell's published work and unpublished manuscript material. The first advance made against the contradictions is Russell's theory of descriptions which allowed denoting phrases and classes (and, eventually, propositions themselves) to be treated as logical subjects without any commitment to them ontologically. The advantage of this theory over the earlier theory of denoting is specifically in the ability of the 1905 theory to apply this treatment to classes. The theory of descriptions led Russell on to his substitutional theory of classes and relations which allowed the construction of a system which is equivalent to a simple theory of types without any commitment to different types of entities. This theory, I suggest, has much to commend to it; most notably, it provides a solution to the logical paradoxes which plagued Russell and, furthermore, the theory offers Russell's best response to the problem of the unity of the proposition in a way which is philosophically palatable to the Russellian pluralist. However, the substitutional theory depends on the admission of propositions as entities. Once Russell had discovered that this assumption also led

to paradoxes, he was driven towards ramification of the substitutional theory in (1908). Eventually, he rejected propositions (and, therefore, substitution) altogether and hence needed new foundations for the type-theory of *Principia*. Those foundations were to be provided by the multiple-relation theory of judgement which was expanded in the 1913 *Theory of Knowledge* manuscript. With the multiple-relation theory, Russell's problems with unities returned. The fiercest critic of the theory was Wittgenstein, who temporarily brought Russell's work on logic to a direct halt.

Chapter Three looks in detail at Wittgenstein's criticisms of the multiple-relation theory, providing a new interpretation of those criticisms which draws on the account given in the previous chapter of Russell's development of the ramified theory of types and on Wittgenstein's own remarks on the theory of types and Russell's paradox in the *Tractatus*. Contrary to the now popular view that Wittgenstein showed the multiple-relation theory of judgement to be incompatible with ramified type-theory, I argue that Wittgenstein convinced Russell that the theory of judgement could not be made to fit with Russell's desire to avoid projecting type-theory onto his ontology if he was to avoid the problem of the unity of the proposition. In the second half of the chapter, I develop these arguments by showing how they bear on Wittgenstein's picture theory. I argue that the theory not only accounts for the unity of the proposition, but also retains (and develops) Frege's form of anti-psychologism in such a way as to avoid the platonic excesses of Frege and Russell. I conclude the chapter with a discussion of the recent debate over the method of the *Tractatus*. I argue against Conant and Diamond's argument (from the context-principle) that the *Tractatus* does not contain a theory of meaning. On the contrary I show that the theory of meaning is perfectly compatible with the presence of the context-principle. The picture theory is a theory of meaning (and thought) which critically develops the Fregean model in crucially important ways.

Chapter Four follows Russell's path, subsequent to Wittgenstein's attack on the multiple-relation theory of judgement and the influence over Russell of the *Tractatus*, from platonism back to a form of psychologism coupled with naturalism. Again, a central motivation for Russell is his refusal to accept that linguistic considerations are relevant to the study of meaning and thought. Having been convinced by Wittgenstein that logic is linguistic, and maintaining a degree of anti-psychologism with regard to logic, he decided that logic (being linguistic) was

irrelevant to the study of thought and meaning and turned to psychology in explaining these. Meaning (and propositions) were held to be private contents of the mind which exert a causal power. The inadequacy of this approach is made explicit in the chapter, firstly by looking at Russell's theory and, then, by comparing it with a modern alternative—Fodor's causal theory of content. Though a causal theory of content may be made to comport with the structural limitations on thought revealed in the previous chapters, it cannot, I argue, explain it. Such criticism is avoided by Quine (who is the subject of the remainder of the chapter) by his decision to simply abandon the attempt to give a systematic theory of meaning altogether in favour of a theory of reference. The remainder of the chapter is devoted to an exposition and critical study of the relevant aspects of Quine's philosophy in preparation for the criticisms to be directed at it in the final chapter. In particular, I isolate Quine's holism as the salient feature of his philosophy from which his doctrines of the indeterminacy of translation, ontological relativity, and naturalized epistemology are all derived.

Chapter Five is an extended discussion of Quinean naturalism in relation to the philosophy of language. Criticisms are levelled at Quine's holism (which, it is argued, is the most compelling argument for his naturalistic project). Davidson's theory of meaning is criticised extensively for the same reasons and I argue that Quine's rejection of epistemology as 'first philosophy' relies on his implicitly treating the philosophy of language as primary. I argue that a theory of meaning (reference) in the Quinean/Davidsonian tradition fails on two main counts to address the issues which have been shown in the thesis to be of central importance to the philosophy of language concerning the unity of the proposition. Firstly I argue that the expulsion of sense in favour of just reference in the theory of meaning makes impossible an account of what it is that a speaker knows when he knows a language and, therefore, is unable to explain how linguistic competence is systematic; how a language user constructs novel sentences out of parts of which he already grasps the content. I also suggest that the holism contained in Quine and Davidson cannot accommodate for the context-principle, as it seeks to extend it to the level of sentences within the context of languages and thereby loses an account of how a word is meaningful only in the context of a proposition. I conclude the chapter by examining how this and the conclusions of previous chapters bear on the question of the relation of thought and language. Drawing on arguments from Wittgenstein and Frege which have been developed throughout the thesis, I argue that thought is inseparable from its linguistic

vehicle, denying the view that thoughts are prior in the order of explanation to their linguistic expressions. While I do not deny that some grasp of content can be attributed to non-language users, I resist the view that these contents can be justifiably called ‘thoughts’, suggesting Dummett’s notion of ‘proto-thought’ as a way of distinguishing between the reflective intelligence of non-language users and fully-fledged thinkers. What emerges from the thesis is not a direct repudiation of naturalistic attempts to explain thought and propositional content, but a demonstration that such accounts, whether or not they are convincing accounts of how content is generated, will fail to provide answers to the insights regarding the nature and structure of thoughts which has been excavated from those figures studied throughout the thesis. The philosophy of thought, I conclude, can only be successfully approached through the philosophy of language.

## 1.

## Thoughts and Propositions in Frege and Early Russell

In a posthumously published fragment, written around 1915, Frege reflects on his ‘basic logical insights’, remarking that ‘the following may be of some use as a key to the understanding of my results’:

Whenever anyone recognizes something to be true, he makes a judgement. What he recognizes to be true is a thought. It is impossible to recognize a thought as true before it has been grasped. A true thought was true before it was grasped by anyone. A thought does not have to be owned by anyone. A thought can be grasped by several people. Making a judgement does not alter the thought that is recognized to be true.

(Frege, 1915, p. 251)

Frege recognised that his greatest contribution to philosophical logic was his treatment of the thought. His many other contributions to the subject, independently important though they unquestionably are, can all be viewed as tributaries and subsidiaries of that central insight which has helped to define a substantial portion of subsequent analytical philosophy. The de-psychologising of logic meant that the thought, in the hands of Frege, was brought out of the sphere of psychology and into that of logic, thereby achieving, for the first time, a genuinely public status.<sup>1</sup> Essential to this achievement is Frege’s notion of propositional content.

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<sup>1</sup> The admission of thoughts into the public realm did not come without a price for Frege, admittedly. From the independence of thoughts from psychology, Frege inferred that thoughts were abstract logical objects inhabiting a platonian third-realm. Dummett (1973a, pp. 364-370) argues forcefully, that an

### 1.1 Frege on Judgeable Contents

Frege's *Begriffsschrift* (concept-script)<sup>2</sup> is often taken to embody a straightforwardly combinatorial approach to sentences—judgements are built up out of independently meaningful elements in accordance with syntactical rules.<sup>3</sup> As we shall see, however, this idea is a gross oversimplification of Frege's position. From his earliest writings on logic, to his mature writings, Frege maintained that the proposition had, as it were, semantic priority over its parts. In his early period, this idea is presented through the notion of *Judgeable content*. A judgeable content is, as its name implies, a content of possible judgement; that is, a truth-evaluatable semantic item. Truth-evaluatable items are, of course, propositions and Frege makes it clear that the proposition is divisible *into* its parts, rather than being constructed out of independently subsisting elements:<sup>4</sup>

I do not believe that concept-formation can precede judgement, because this would presuppose the independent existence of concepts, but I think of a concept as having arisen by decomposition from a judgeable content.  
(Letter to Marty, 29/8/1882, in Frege, 1980, p. 101)

Frege's notation distinguishes content from assertion. The content-stroke is used to signify a 'complex of ideas' (Frege, 1879, § 2), the assertion of which is expressed by the addition of a vertical 'judgement' or 'assertion' stroke (see *Ibid.*). Wittgenstein rather hastily dismisses the judgement stroke as 'logically quite meaningless' (Wittgenstein, 1922, 4.442) but the distinction actually serves to illustrate clearly Frege's idea of a proposition. The unasserted content is a *possible* content of

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acceptance of thoughts as de-psychologised truth-bearers does not commit us to sharing Frege's platonism.

<sup>2</sup> I italicise 'Begriffsschrift' when referring to Frege's (1879) work by that name; when not italicised the word refers to Frege's formal calculus.

<sup>3</sup> C.f., Fodor, 1987, p. 150.

<sup>4</sup> Some care is needed in interpreting Frege on this point, however. As Dummett has pointed out, Frege holds to two seemingly incompatible theses, namely that (1) a thought is not constructed out of concepts but concepts are arrived from analysis of a thought, and (2) that a thought is built up out of constituents which correspond (roughly) to the part of sentences (see Dummett, 1981, p. 261). Dummett's suggested resolution to the difficulty requires the introduction of Frege's distinction between sense and reference, hence discussion of it will be postponed until that distinction has been introduced below.

judgement—a Fregean thought. In 1879, prior to availing himself of his later sense/reference distinction, Frege clearly saw this unasserted content to be a proposition: ‘In this case we *qualify* the expression with the words “*the circumstance that*” or “*the proposition that*”’ (Frege, 1879, § 2). Far from being ‘logically quite meaningless’ the separation of content from assertion was an attempt to fulfil that most Wittgensteinian of objectives—the independence of a thought or proposition from its truth or falsity. A thought, on Frege’s model, is not *just* a true or false assertion, it is a truth-evaluative content and *as such* a possible content of judgement or assertion; in other words, a proposition’s truth or falsity is not what makes it a judgeable content, what distinguishes a proposition and, indeed, a thought, is that it is capable of *being judged* true or false.<sup>5</sup>

Both Frege and Russell, at various stages in their philosophical developments, had a recurring concern with the problem of the unity of the proposition. If a proposition is, as Frege suggests, a ‘certain complex of ideas’,<sup>6</sup> then any theory of the proposition should furnish us with an account of why certain complexes are propositions, and certain others are not. The relative successes of Frege and Russell as regards this project can be traced, in part, to their differing ontological commitments (as we shall see more clearly in later discussion of the evolution of Russell’s thought). Aside from this, however, Frege had a distinct advantage over Russell from the start, resulting from his awareness of the primacy of the judgeable content over its parts. Frege’s awareness of the need to give an account of the unity of the proposition shows itself in his view of the kinds of parts into which a judgement can be decomposed.

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<sup>5</sup> This difference between Frege and Wittgenstein on the relation between sense and force has resurfaced in more recent philosophy of language through the work of Dummett, who is profoundly influenced by both Frege and Wittgenstein. Dummett holds the later Wittgenstein’s rejection of a distinction between the sense and force of an assertoric sentence to be, in part, what stands between the possibility of reconciling Wittgenstein’s conception of meaning as determined by use with an attempt at providing a systematic theory of meaning. Dummett, rejecting Wittgenstein’s opposition to the distinction, sets about seeking such a reconciliation.

## 1.2 Frege on Concepts

A concept for Frege is the meaning (after 1891, the reference or *Bedeutung*) of a predicate. Frege's account, however, should not be re-interpreted as a revised form of the account given of logical form by classical logicians in terms of subject and predicate. Wittgenstein remarked, somewhat bewilderingly, that 'Frege's "Concept and Object" is the same as subject and predicate' (Wittgenstein, 1974, p. 205); a theme that Wittgenstein returns to on more than one occasion (see, for example, Wittgenstein, 1975, §§ 93 & 115; & 1974, p. 202).<sup>7</sup> In fact Frege was insistent throughout his career that the distinction between grammatical subject and predicate is superfluous in logic. The important distinction in logic is that between concept and object.

It is often remarked that logic had not significantly progressed beyond the syllogism before Frege's "logical revolution". Boolean algebra, for example, has its roots in the calculus of classes and so, in this sense, does not make a radical departure from the analysis of the various forms of the proposition offered by classical logicians. 'All whales are mammals', contains, in the eyes of the classical logician, three distinct terms: a subject term (whales), a predicate term (mammals), and the copula (always a derivative of the verb 'to be'; in this case 'are'). Hence the proposition amounts to an assertion regarding two classes, namely that the class of whales is contained in the class of mammals. Frege's responses to this analysis are truly revolutionary. For one thing, the various classical forms of the proposition (categorical, hypothetical, etc.) are dismissed as having merely a 'grammatical significance' which has no bearing on the

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<sup>6</sup> Frege's use of the term 'idea' here is, perhaps, unfortunate, and should not be confused with its usage as a technical term in his later work where it signifies a private mental item. Frege is adamantly opposed to the idea that a proposition's meaning is composed of a complex of mental items.

<sup>7</sup> Coffa has an interesting interpretation of Wittgenstein's remarks on Frege's concept-object distinction which draws on, and helps to elucidate, Wittgenstein's flirtation with phenomenology in the *Philosophical Remarks*. Coffa points to Wittgenstein's discussion with the Vienna Circle, where he talks of a phenomenal language which need not respect the logical structure of ordinary language, from which Wittgenstein concludes that: 'the logical structure of elementary propositions need not have the slightest similarity with the logical structure of propositions' (Wittgenstein, 1979, p. 42). In other words, says Coffa, the concept-object distinction need not be given the special status granted to it by Frege. This conclusion is somewhat threatened, however, by Coffa's interpretation of Wittgenstein's critique of Russell's theory of judgement: 'Wittgenstein was telling Russell that he no longer agreed with the arguments in *Principles* that concepts can occur in subject and in predicate position' (Coffa, 1991, p. 152; see also pp. 148-150).

logical (conceptual) content of the assertion. Likewise the distinction between assertoric and apodeictic judgements is of no consequence to Frege's system.<sup>8</sup>

The key to Frege's advances lies in his replacement of the subject-predicate form with the mathematical notion of function and argument. According to a classical analysis of the sentence 'Socrates is mortal', the logical form would be that of a universal affirmative proposition stating that a class with one member (Socrates) is contained in the class of mortals. This curious analysis results from the observation that the subject term is distributed. Clearly such analysis is undesirable and it is quickly disposed of by Frege's method. The sentence is now broken up into just two parts: 'Socrates' and '...is mortal'; where '...is mortal' is viewed as a *function* of 'Socrates'. Having thus decomposed the sentence or judgeable content, we can now represent the incomplete part of the sentence through the substitution of a variable for the name which has been removed; hence, we can derive the expression 'x is mortal' from the sentence 'Socrates is mortal'.<sup>9</sup> The doctrine is not fully developed in the *Begriffsschrift* but, as his views developed, Frege extended the analysis by assimilating concepts to functions and maintaining that the value of this kind of function (a propositional function) is always a truth-value (c.f., Frege, 1891 & 1892a).<sup>10</sup> A concept, for the mature Frege, therefore becomes 'a function whose value is always a truth-value' (Frege, 1891, p. 30).<sup>11</sup>

The benefits of such analysis are enormous. An immediate result which would re-shape our whole conception of logical form was Frege's subsequent treatment of generality, the development of which led him to invent modern quantification theory.

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<sup>8</sup> See Frege, 1879, § 4.

<sup>9</sup> It is important to realise that, for Frege, the variable part of a function does not stand for any substantial entity but merely serves to hold the argument spaces open ready for saturation; 'x is mortal' could equally be written as '( ) is mortal' (c.f., Frege, 1891, p. 24).

<sup>10</sup> Baker and Hacker (1984) attribute to the early Frege a position not dissimilar to his mature doctrine, namely one where functions are seen as mapping their non-linguistic arguments to non-linguistic values (in this case, propositions or judgeable contents). As Dummett (1984, pp. 163-165) points out, Baker and Hacker are compelled to interpret Frege thus by their insistence that Frege held sentences to be proper names (of judgeable contents) in the *Begriffsschrift*. However, such a claim is unsupportable in the absence of Frege's later distinction between sense and reference. Only once Frege has adopted this doctrine, can he identify sentences as proper names of truth-values; Baker and Hacker's claim that Frege previously held sentences to be proper names of judgeable contents is made without adequate textual support and does not fit comfortably with the development of Frege's thought. Baker's (2001) defence of the non-linguistic interpretation of functions in the *Begriffsschrift* does not address the difficulties raised by interpreting the early Frege retrospectively from his later point of view where *Sinn* and *Bedeutung* were firmly established elements of his philosophy of language.

Having observed that ‘Socrates is mortal’ does not represent a relation between two distinct classes, it now becomes clear that the proposition is of a very different sort to ‘all men are mortal’, which, to the classical logician, is also a universal affirmative proposition. On the function and argument analysis, we can see that ‘all men’ is not a straightforward substitution of the argument ‘Socrates’ in the way that a subject-predicate analysis suggests. A belief in the logical primacy of the subject-predicate form will again mislead us here. ‘All men’ and ‘Socrates’ can both play grammatical subject to the predicate term ‘mortal’, but this is merely to conceal their difference. In Frege’s *Begriffsschrift*, the difference is pivotal. Whereas ‘Socrates is mortal’ gives us one value of the variable in the function ‘ $x$  is mortal’, ‘All men are mortal’ provides information about a collection of possible arguments for the function. It says that for all values of  $x$ , if  $x$  is a man, then  $x$  is mortal. Hence ‘all men’ is not a simple subject term like a proper name, but a complex concept containing a higher order concept (the quantifier) within whose scope falls the first-order concept (men). Whereas the classical analysis of the two propositions results in the same form for each, namely: ‘ $x \in y$ ’; the difference, upon Frege’s analysis, is: ‘Socrates is mortal’ = ‘ $Ms$ ’; ‘All men are mortal’ = ‘ $(\forall x)(Hx \rightarrow Mx)$ ’. The analysis in the functional calculus shows the apparent similarity in form to be illusory and hence provides the justification for Frege’s dismissal of the distinction between subject and predicate as indicative of logical form.<sup>12</sup> The generalisation of a subject-predicate sentence like ‘Socrates is mortal’ cannot be of the same form ‘ $Fx$ ’, for it essentially amounts to an assertion that ‘ $Fa, Fb, Fc, \dots Fn$ ’; in other words, ‘Socrates is mortal’ is already contained in the expansion of ‘All men are mortal’, therefore they cannot share the same logical form.

Once again, Frege was drawing on his mathematical background in using quantifiers to bind variables,<sup>13</sup> a method which was necessitated by the kinds of multiple generality present in the mathematical reasoning he sought to express. Syllogistic logic was ill-equipped to handle inferences resting on propositions with nested expressions of generality, such as ‘Every number has a successor’; in order to

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<sup>11</sup> It should be noted that concept and function are not synonymous terms for Frege. A concept is a certain species of function; other functions may yield, for example, numerical values as in ‘ $2 \cdot x^2$ ’, which is also a function of  $x$ .

<sup>12</sup> Frege, of course, did not utilise this notation, but used his own elegant (though cumbersome) *Begriffsschrift*.

carry out his project of providing arithmetic with purely logical foundations, Frege had to find a way of expressing multiple generality. The binding of variables by quantifiers gives an immediate solution, for now the scope of the quantifier can be clearly demarcated, and quantifiers can thus be placed within the different parts of the judgement as desired. Hence, 'Every number has a successor' may be (roughly) symbolised as:

$$(\forall x) [Nx \rightarrow (\exists y) (Ny \& Syx)]^{14}$$

Frege's system allows for such use of quantifiers to continue to an indefinite level of complexity, a tool which he would eventually put to use, in the elegant proofs of his *Grundgesetze*, to set out his logicist programme in full.

The primacy of the judgement is explicitly reaffirmed in Frege's introduction of generality into his *Begriffsschrift*. The following passage indeed might be viewed as the earliest appearance in Frege's work of an appeal to some embryonic form of his *Grundlagen* context-principle:<sup>15</sup>

What is asserted of the number 20 cannot be asserted in the same sense of [the concept] 'every positive integer'; of course it may in certain circumstances be assertible of every positive integer. The expression 'every positive integer' just by itself, unlike 'the number 20,' gives no complete idea; it gets a sense only through the context of the sentence.

(Frege, 1879, § 9)

In the *Grundlagen*, Frege went further still. Explicitly citing the context-principle as fundamental to his enquiry into the concept of number he now states that 'it is *only* in

<sup>13</sup> Variable binding operators had already been utilised in arithmetical analysis. Frege was the first to apply the idea to purely logical analysis (See Editor's Introduction to Bynum's translation of Frege, 1879, p. 65).

<sup>14</sup> This is not, of course, the rendering of this sentence that Frege would offer as its correct analysis, as it treats 'the successor of  $x$ ' as a relation rather than as a definite description. This need not concern us at present, however. It is convenient to use the existential quantifier here, although Frege only had one sign for generality, and used negation to derive existential claims from universal ones, ' $\exists x Fx$ ' being of course equivalent to ' $\neg\forall x \neg Fx$ '. The use of a single quantifier with negation is much less arduous in Frege's notation than it would be in modern notation.

the context of a proposition that words have any meaning' (Frege, 1884, § 62; my italics). "The number 20" is now precisely the kind of expression which Frege invokes the context-principle in order to define, apparently now conceiving of it in a similar way to his earlier view of expressions like "every positive integer":<sup>15</sup>

How, then, are number words to be given to us, if we cannot have any ideas or intuitions of them? Since it is only in the context of a proposition that words have any meaning, our problem becomes this: To define the sense of a proposition in which a number word occurs.

(Ibid.)

This remark is, undoubtedly, a pivotal moment in the philosophy of mathematics. Michael Dummett, eager to nail Frege's banner to the mast of analytical philosophy, locates in this passage *the* linguistic turn, leading him to proclaim it 'arguably the most pregnant philosophical paragraph ever written' (Dummett, 1991a, p. 111). This rather generous claim is arguably true insofar as the context-principle has borne philosophical fruit in the form of unequalled controversy amongst Frege scholars as well as exerting a significant influence on major figures such as Wittgenstein, Davidson and Quine. A thorough immersion in those issues is to be postponed until later chapters of this thesis, however; the immediate concern of this discussion is to examine how Frege utilises the context-principle in his treatment of thought. To fully appreciate that, one must appreciate Frege's reason's for invoking the principle in the first place.

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<sup>15</sup> I.e. 'never to ask for the meaning of a word in isolation, but only in the context of a proposition' (Frege, 1884, p. x). The suggestion that the following quote marks the principle's first appearance in Frege's work is made by Beaney in his translation of Frege (1879). See Frege, 1997, p. 67, fn. 31.

<sup>16</sup> The similarity should not blind us to the difference however. 'Every positive integer' is essentially incomplete, or unsaturated, being an expression which involves a second-order concept of generality (a quantifier); 'the number 20', by contrast, names an object and is therefore complete (it has no empty argument space). Frege clearly states, however, that the status of numbers as objects does not undermine the need for the context-principle in fixing the meaning of a number word: 'The self-subsistence which I am claiming for number is not to be taken to mean that a number word signifies something when removed from the context of a proposition, but only to preclude the use of such words as predicates or attributes, which appreciably alters their meaning' (Frege, 1884, § 60).

### 1.3 The De-Psychologising of Number

The context-principle is introduced as one of three such ‘fundamental principles’ in the *Grundlagen*:

Always to separate sharply the psychological from the logical, the subjective from the objective:

never to ask for the meaning of a word in isolation, but only in the context of a proposition;

never to lose sight of the distinction between concept and object.

(Frege. 1884, p. x)

All three principles, as we shall see at various stages in this thesis, are inextricably combined. Clearly, however, it was the connection between the first two that most impressed their author. The context-principle is not used in the work to *refute* psychologism—Frege’s critique of psychologism is hardly lacking without it—but, rather, plays a *prohibitionary* role. As such it acts as a warning against, and explanation of, the temptation to slip into either psychologism or empiricism. Frege’s suggestion is that, if we ignore the context-principle and try to define the meaning of a word in isolation, we will be forced to import some kind of entity, either physical or psychological, in order to do so: ‘only by adhering to [the context-principle] can we, as I believe, avoid a physical view of number without slipping into a psychological view of it’ (Ibid., § 106). In a way which is strikingly reminiscent of the philosophical method later utilised by Wittgenstein (and, no doubt, a prominent influence on it), Frege is suggesting that philosophical mistakes are the result of a failure to fully grasp the subtleties of language. Just as an unthinking acceptance of subject-predicate analysis prejudiced our conception of logical form, so also our failure to recognise the primacy of the judgement over its parts leads us to nonsensical metaphysical and epistemological positions. What are these positions, and why are they nonsensical? Frege’s answers to these questions make the *Grundlagen* one of the most penetrating examples of philosophical critique ever composed.

The clearest example of what Frege means by a ‘physical view of number’ is to be found in Mill’s *System of Logic*. Firmly committed to a stubborn empiricism,

which persuaded him that anything known *a priori* was trivial and uninformative. Mill reasoned that anything which *was* informative thereby had to be known *a posteriori*. With the added premise that both logic and mathematics are genuinely informative, he drew the (presumably informative) conclusion that logic and mathematics were empirical sciences. In Mill's terminology, they assert 'real propositions' as opposed to merely 'verbal' ones: 'It is this class of [real] propositions only which are in themselves instructive, or from which any instructive propositions can be inferred' (Mill, 1843, Bk. I, Ch. VI, § 4).<sup>17</sup> This devout adherence to the empiricist ethic that necessary truths are necessarily trivial is apparent in Mill's conception of number as an abstraction from physical objects which are given in experience. Mill advances the thesis that our knowledge that  $3 = 2 + 1$  is not a truth known *a priori*, but 'a truth known to us by early and constant experience—an inductive truth'. This inductive truth is inferred, he suggests, from observations of such 'matters of fact' as that a collection of three pebbles can be broken up and rearranged to form a single pebble and a collection of two pebbles:

Such truths are the foundation of the science of Numbers. The fundamental truths of that science all rest on the evidence of sense; they are proved by showing to our eyes and our fingers that any given number of objects, ten balls, for example, may by separation and rearrangement exhibit to our senses all the different sets of numbers the sum of which is equal to ten.

(Ibid., Bk. II, Ch. VI, § 2)

Mill's account, in Frege's view, confuses the application of the science of number with the pure science itself. The example that Mill gives may well be an *application* of arithmetical concepts, but to claim that the 'inductive truth' that this application is taken to prove gives a foundation to arithmetic is tantamount to denying that arithmetic has any foundations at all. In response to Mill's attempt to define our grasp of the number 3 in the matter of fact that 3 pebbles can be pieced together out of 1 pebble + 2 pebbles, Frege remarks: 'What a mercy, then, that not everything in the

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<sup>17</sup> In a footnote to the passage quoted, Mill observes that his distinction between verbal and real propositions matches Kant's distinction between analytic and synthetic judgements. Verbal (analytic)

world is nailed down; for if it were, we should not be able to bring off this separation, and  $2 + 1$  would not be  $3$ ! What a pity that Mill did not also illustrate the physical facts underlying the numbers  $0$  and  $1$ ! (Frege, 1884, § 7). He goes on to offer a *reductio ad absurdum* of Mill's position by pointing out that it makes talk of "three chimes of the clock", or "three methods of solving an equation" strictly incorrect, for neither of these apparent occurrences of the number three impress themselves on our senses in quite the fashion required by Mill's definition.

The strongest argument, perhaps, against the idea that numbers and other mathematical objects are derived through abstraction from our perceptions of physical objects comes from the difficulty faced by such an account in explaining the sense of those mathematical objects which we do not have any apparent acquaintance with in our immediate sensory experiences. Numbers like  $\pi$  or  $\aleph_0$ , and calculations utilising the  $\sqrt{}$  operator, for example, all feature in our mathematical vocabulary, but it is not apparent that we can make sense of the idea of someone having any sense-data corresponding to square-roots in the physical world, nor is it conceivable that someone has ever observed  $\aleph_0$  things or  $3.1415926535\dots$  things. Frege fixes on an example necessary for even the most elementary arithmetic—the number  $0$ :

Up to now no one, I take it, has ever seen or touched  $0$  pebbles. Mill, of course, would explain  $0$  as something that has no sense, a mere manner of speaking; calculations with  $0$  would be a mere game, played with empty symbols, and the only wonder would be that anything rational could come of it. If, however, these calculations have a serious meaning, then the symbol  $0$  cannot be entirely without sense either.

(Ibid., § 8)

The temptation to deny any meaning to the expression ' $0$ ' on the grounds that there is no object (either physical or psychological) which corresponds to it is precisely the danger which the context-principle is intended to guard against. The number word ' $0$ ' has no meaning in isolation, but only in the context of a proposition: 'That we can form no idea of its content is therefore no reason for denying all meaning to a word, or for excluding it from our vocabulary. We are indeed only imposed on by the opposite

view because we will, when asking for the meaning of a word, consider it in isolation, which leads us to accept an idea as the meaning' (Ibid., § 60). Once we recognise that a word has meaning only in the context of a proposition, it is clear that propositions like ' $12 - (6 + 6) = 0$ ' or 'the number of living organisms on Mars is 0' are perfectly intelligible judgements. Hence '0' is not a meaningless sign, but must have a sense, despite its absence from the realm of our sensory experience (and, hence, from the realm of Mill's numerical definitions).<sup>18</sup>

Having shown the inadequacy of Mill's physical, *a posteriori* definition of number, Frege now examines the options for arguing that arithmetical knowledge is known *a priori*. The most sophisticated version of this doctrine, at the time, was Kant's claim that arithmetical propositions are synthetic *a priori* judgements. Kant argued that mathematical judgements are *a priori* on the grounds that the necessity implied by them is not to be found in experience. He rejected, however, the Leibnizian view that arithmetical propositions are analytic. Numbers, on Kant's account, are known through intuition, hence the number flanking one side of a numerical identity statement is not to be known purely from the meanings of the numbers flanking the other:

We might, indeed, at first suppose that the proposition  $7 + 5 = 12$  is a merely analytical proposition, following (according to the principle of contradiction) from the conception of a sum of seven and five. But if we regard it more narrowly, we find that our conception of the sum of seven and five contains nothing more than the uniting of both sums into one, whereby it cannot at all be cogitated what this single number is which embraces both. The conception of twelve is by no means obtained by merely cogitating the union of seven and five; and we may analyse our conception of such a possible sum as long as we will, still we shall never discover in it the notion of twelve ... Arithmetical propositions are therefore always synthetical, of which we may become more clearly convinced by trying large numbers. For it will thus become quite evident, that turn and twist our conceptions as we may, it is impossible, without

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<sup>18</sup> Frege, of course, defines the number 0 as the class of concepts under which no objects fall: '0 is the Number which belongs to the concept "not identical with itself"' (Frege, 1884, § 74).

having recourse to intuition, to arrive at the sum total or product by means of the mere analysis of our conceptions.

(Kant. 1787, p. 33)

Clearly, this account runs into similar problems to Mill's, however. Just as there is no clear sense in which we can have any acquaintance in experience with, say, the number 7,798, there is also no clear sense of what an intuition of this number would be. The difference between 7,798 and 7,799 is not a difference which is experienced by comparing an observation of a crowd consisting of precisely 7,798 people with an observation of a crowd of 7,799 people. Even allowing that I do have an intuition of 7,798, there is no reason to suppose a qualitative difference between this intuition and my intuition of 7,799.

Frege is charitable with Kant, interpreting him as 'thinking only of small numbers. So that for large numbers the formulae would be provable, though for small numbers they are immediately self-evident through intuition' (Frege. 1884, § 5). But now we must ask where this distinction between large and small numbers is to be drawn. Perhaps one might be tempted to say that it is the numbers corresponding to single-digit numerals which are known through intuition, the larger numbers being constructed out of the smaller in the same way that the corresponding symbols are in the decimal system. Surely, however, this will be an uncertain foundation for a synthetic *a priori* science; arithmetic, on such an account, would be based on nothing more than a symbolic convention. There does not, then, appear to be any clear distinction between the so-called large and small numbers: 'If the numerical formulae were provable from, say, 10 on, we should ask with justice "Why not from 5 on? Or from 2 on? Or from 1 on?"' (Ibid.). The path is, therefore, left clear for Frege to argue that numerical propositions are provable, analytic judgements.

The inadequacy of both Mill's and Kant's definition of number can be traced back to their failure to address the points of philosophical logic which underlie Frege's three *Grundlagen* principles. In broad terms, they do so by failing to comprehend the logical structure and status of arithmetical propositions. To avoid Mill's empiricism and Kant's psychologism, Frege seeks a definition of number which is neither physical nor psychological, and draws on the context-principle and the distinction between concept and object to do so.

#### 1.4 Numbers as Logical Objects

Frege's simultaneous rejection of psychologistic and empiricist accounts of number can be summarised as an argument to the effect that numbers are both objective and non-physical. The inadequacy of psychologism shows itself through consideration of the objective status of arithmetical propositions. Arithmetical propositions—arithmetical thoughts—are public, not private, property. In other words, the truth of an arithmetical proposition is not to be determined by psychological criteria, but requires an objective criterion. In his *Grundgesetze*, Frege maintains his scathing attitude towards the idea that the laws of thought (the laws of logic) could be dependent on the psychological processes of the judging subject:

But what if beings were even found whose laws of thought flatly contradicted ours and therefore frequently led to contrary results even in practice? The psychological logician could only acknowledge the fact and say simply: those laws hold for them, these laws hold for us. I should say: we have here a hitherto unknown kind of madness ... One could scarcely falsify the sense of the word "true" more mischievously than by including in it a reference to the subjects who judge.

(Frege, 1893, p. 14)

The laws of logic, for Frege, must be mind-independent. A true thought is true before, and regardless of whether it ever actually is, grasped by a thinking subject.<sup>19</sup> The laws of logic are, therefore, prescriptive laws; they prescribe how one *should* think, rather than describe how one *does* think. Mathematical truths, it follows, are objective truths which are discovered, not invented. Frege's de-psychologising of logic thereby leads directly into his platonism. Having shown that numbers cannot be physical entities in the world, nor psychological objects in the mind, Frege concludes that they are 'logical objects' which inhabit a platonic realm. The basic foundation of this view is entirely logical. An object, Frege insists, must remain the same throughout spatial, or

temporal, alterations. In other words, an object must always be identical with itself. Numbers clearly fulfil such a requirement, as we have seen from Frege's critique of psychologism. To say that the number 1 is different for one to person to another, or from one time to another, is incoherent. Frege's critique of Mill shows that it is equally incoherent, however, to treat numbers as physical objects or properties of physical objects. Hence Frege reaches the conclusion that numbers must be objects which have no physical status, yet remain unalterable throughout time. Platonism, for Frege, is not the incentive to logicism (as it was for Russell); it is the result of logicism. Just as it was for Leibniz, metaphysics, for Frege, is justified by purely logical considerations.

Frege's definition of number is, essentially, equivalent to Russell's definition of cardinal numbers as classes of similar classes. For Frege, however, there is a difference in the means of arriving at such a definition. A class (or, in Frege's terminology, value-range) is the extension of a concept. Hence an appreciation of the nature of concepts and their relation to numbers is essential to understanding Frege's philosophy of mathematics. Let us reiterate Frege's position thus far. Numbers, we have good reason to believe, are (or at least behave like) objects. Having also shown that these objects are not physical, Frege now poses what might seem an obvious question: 'When we make a statement of number, what is that of which we assert something?' (Frege, 1884, § 45). Frege has given us examples of what will *not* be a satisfactory answer to this question: the most celebrated achievement of the *Grundlagen*, however, is his account of what will. Frege's reply to the question of what is involved in an assertion of number is well-known: 'the content of a statement of number is an assertion about a concept' (Ibid., § 46).<sup>19</sup> Clearly, Frege is concerned with cardinal numbers here; a statement of number is a statement which can be given in response to a question of the form "how many *F*s are there?".<sup>20</sup> The system of logic we have been examining in this chapter, it should be evident, is tailor-made for the definition of number that Frege is here suggesting. With Frege's rigorous distinction

<sup>19</sup> See Frege, 1897, p. 133, for a particularly clear statement of this view.

<sup>20</sup> 'Die *Zahlangabe* enthält eine Aussage von einem Begriffe'. Dummett draws attention to the difficulty of translating this concise statement, and offers, as an alternative to Austin's translation: 'The content of an ascription of number consists in predicating something of a concept' (Dummett, 1991a, p. 88). By translating 'Aussage' as 'predication', Dummett is obviously not suggesting that a concept can, contrary to Frege's view, play the role of grammatical subject. To predicate something of concept, on Frege's view, is to state that the concept falls within a second-order concept.

between concept and object in place, we can (roughly) characterise a concept as a property of an object or set of objects. A property of a concept, or set of concepts, will therefore be a higher-order concept. Numbers, on Frege's definition, are the extensions of certain higher-order concepts. We may wish, therefore, to distinguish Frege's definition of cardinal number from Russell's; rather than the Russellian classes of similar classes, Frege's cardinals are classes of similar (or *equinumerous*) concepts.<sup>22</sup>

The situation can be elucidated by taking an example of Frege's: 'If I say "the King's carriage is drawn by four horses", then I assign the number four to the concept "horse that draws the King's carriage"' (Ibid., § 46). In making the statement, I predicate of the concept "horse that draws the King's carriage" the property of having four things in its extension. That is to say, I predicate the property of membership of the class of concepts which have four objects in their extensions, to the concept. The cardinal number 4 is just this class. The superiority of this account over the alternatives we have considered is evident from a further example of Frege's: 'Venus has 0 moons'. If the proposition is true, then it surely is not a property of the moons of Venus that we are predicing when we make the assertion as, by our own admission, Venus *has* no moons. It must, Frege insists, therefore be the concept "moon of Venus" which is the number-bearing entity. The extension of this concept will, of course, have no members, but it is a perfectly intelligible concept nonetheless. The class of all such concepts is the object 0.<sup>23</sup>

As definitions fit for Frege's logicist program, these preliminary accounts will, of course, not do. For one thing, there is no *a priori* case for assuming that the concept

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<sup>21</sup> See Dummett, 1991a, p. 88.

<sup>22</sup> The subtlety of Frege's definition unfortunately evades many accounts of it. Richard Jeffrey, for example, states that cardinal numbers were defined by Frege as 'properties of properties' (Jeffrey, 1989, p. 157). This is incorrect. As we have seen, Frege insists that numbers *must* be objects, not concepts: hence an understanding of his philosophy requires an understanding of the important, and inviolable, difference between a second-order concept and its extension. Jeffrey's proclaimed Fregean analysis of what is required for a concept to have the number 0: ' $\forall A (0A \leftrightarrow \exists x Ax)$ ' (Ibid.), is consistent with Frege's account, but misleading in that it presents '0' as a predicate, rather than (more accurately) making clear that what is being predicated is the property of being equinumerate with the extension of a concept under which no object falls. For further discussion of the possibility of defining numbers as concepts, see Wright, 1983, pp. 36-40 & Bostock, 1974 & 1979.

<sup>23</sup> This 'positive' definition of '0' shows, once more, how Frege's ontological commitments are driven by logical considerations. '0' is assumed to be an object on the grounds that it behaves as the referent of an object expression. By contrast, Husserl sees '0' as a negative quality ('none', as it were), something given as a negative answer to the question 'how many?'. This was one of the aspects of Husserl's

“moon of Venus” will define a memberless class. Furthermore, defining the number 4 in terms of predication of a class the property of having 4 members is unacceptably circular.

In tightening the definition to adequately fulfil the aims of logicism, Frege invokes the context-principle in perhaps its most famous form. Having decided that numbers are ‘self-subsistent objects’ which are accessible neither through sense-data nor intuition, he points out that, since a word has meaning only in the context of a proposition, our problem is here being misconstrued. What is required for its solution is not a way of *perceiving* numbers, but a definition of the sense of a proposition in which a number *word* occurs.<sup>24</sup> In other words, the route to a definition of number is through the propositions in which their linguistic expressions feature. This use of the context-principle is clearly a pivotal moment in the history of philosophy. Its influence over Wittgensteinian remarks such as that ‘there is no more direct way of reading thought than through language. Thought is not something hidden; it lies open to us’ (Wittgenstein, 1980, p. 26) is unmistakable. The importance of the context-principle in driving philosophical analysis towards the twentieth-century preoccupation (amongst the analytically-minded, at least) with philosophy of language, has not gone unnoticed. Dummett has grandly pronounced its occurrence in Frege’s definition of number as the point at which Frege directs philosophy into the linguistic turn. In short, by invoking the context-principle in such a way, Frege literally ‘makes the linguistic turn’:

He offers no justification for making it, considers no objection to it and essays no defence of it: he simply executes the manoeuvre as if there were no novelty to it, and does it so skilfully that the reader scarcely perceives the novelty. Yet it was in fact unprecedented in the history of philosophy.

Plenty of philosophers - Aristotle, for example - had asked linguistic questions, and returned linguistic answers: Frege was the first to ask a *non-linguistic* question and return a linguistic answer. If it were on the strength of *Grundlagen* § 62 and its sequel alone, he would still deserve to

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philosophy of mathematics vigorously attacked by Frege in his review of Husserl (see Frege, 1894, pp. 83-84).

be rated the grandfather of analytical philosophy.

(Dummett. 1991a. p. 112)

Taking Frege's remark on board, we can now make sense of his approach to defining number. Rather than taking the (non-linguistic) Kantian or empiricist routes, we obtain a fixed definition of number by obtaining a firm understanding of the operation of number words within the context of a proposition. Frege does so by examining what Dummett calls the 'cardinality operator', i.e., the operator: 'the number of  $\phi$ s' (Ibid., p. 113). The concept of number is to be grasped by fixing the sense of an identity statement of the form: 'the number which belongs to the concept  $F$  is the same as that which belongs to the concept  $G$ ' (Frege, 1884, § 62). In other words we must fix the sense of the identity statement:

the number of  $F$ s = the number of  $G$ s

in a non-circular fashion (i.e., avoiding repetition of the cardinality operator in the definiens). This will furnish us with the objective criterion of identity we are seeking, allowing us to pick out the object under consideration (in this case, a number) irrespective of our spatial or temporal perspective.

Frege, of course, does so by reducing numbers to the relation of equinumerosity (*Gleichzahligkeit*). Two concepts are equinumerous if the objects falling under each can be placed in one-one correlation with those falling under the other. Hence Frege's definition of equinumerous concepts is equivalent to Russell's definition of similar classes. Two classes are similar, in Russell's sense, if their members can be placed in one-one correspondence; thus the class of husbands is similar to the class of wives (in a monogamous society), the two classes being correlated by the marriage-relation.<sup>24</sup> In Frege's terminology, the *concepts* denoted by the expressions 'x is a husband' and 'x is a wife' are equinumerous, as their extensions can be correlated one-one by the marriage-relation.

Having established the relation of equinumerosity between concepts as an objective criterion of numerical identity, Frege is now in a position to give a non-

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<sup>24</sup> See Frege, 1884, § 62.

<sup>25</sup> See Russell, 1903, § 109.

circular definition of number. Beginning with 0, Frege gives the following set of definitions:

The number 0 = the extension of the concept 'equinumerous with the concept *not identical with oneself*';

The number 1 = the extension of the concept 'equinumerous with the concept *identical with 0*';

The number 2 = the extension of the concept 'equinumerous with the concept *identical with 0 or 1*'.

In this way, we can obtain the general definition:

The number  $n + 1$  = the extension of the concept 'equinumerous with the concept *member of the natural number series ending with n*'.

Clearly, there is nothing circular about such definitions providing we have given a non-circular definition of the cardinality operator. This, we have seen is easily obtained:

The number of  $\Phi$ s = the extension of the concept 'equinumerous to the concept  $\Phi$ '.

The cardinality operator can be defined in purely logical terms, as follows (so as to avoid Frege's unwieldy notation, this demonstration will follow that given by Russell and Whitehead in *Principia Mathematica*):<sup>26</sup>

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<sup>26</sup> See Whitehead & Russell, 1912, \* 100, and Russell, 1908, p. 96. Frege's formal definition of the cardinality operator is given in Frege, 1893, § 40, and is given in terms of extensions of functions, rather than the earlier definition in terms of classes of concepts. The cardinal number of a concept ' $U$ ' is the class of value ranges equinumerous to the extension of the concept ' $U$ '. This class, considered as an object, is the cardinal number in question. Hence our definition of the cardinality operator in Russell's terms is justified. Russell noted that Frege had pre-empted his own definition of cardinal number in his

First, we introduce a relation  $R$  which is a one-one relation (a member of the class  $1 \rightarrow 1$ ):

$$R \in 1 \rightarrow 1 \equiv \{[(Rxz \cdot Ryz) \supset_{x,y,z} (x = y)] \cdot [(Rxy \cdot Rxz) \supset_{x,y,z} y = z]\}$$

We now define the relation of equinumerosity (or similarity) in terms of the domain and converse domain of  $R$ :

The domain of  $R$  is defined as

$$D 'R = \hat{x} \{(\exists y) Rxy\} \quad \text{Df.}$$

The converse domain of  $R$  is defined as

$$D 'R = \hat{y} \{(\exists x) Rxy\} \quad \text{Df.}$$

Two classes are, therefore, equinumerate (or similar) if they fulfil the following requirement:

$$\alpha \text{ sm } \beta \equiv (\exists R) [(R \in 1 \rightarrow 1) \cdot (\alpha = D 'R) \cdot (\beta = D 'R)]$$

The cardinality operator, which (again, adopting the notation of *Principia*) we will symbolise by ‘Nc’, is therefore defined as the relation of equinumerosity (or similarity):

$$\text{Nc} = \text{sm} \quad \text{Df.}$$

and, hence, the cardinal number of a class ‘ $\alpha$ ’ will be the class of all classes equinumerous to  $\alpha$ :

$$\text{Nc } \alpha = \text{sm } \alpha \quad \text{Df.}$$

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notes made during his reading of the *Grundgesetze*: the relevant page is reproduced in Russell, 1994a.

Hence, for example

$$\mathrm{Nc} \cdot \alpha = \hat{\beta}(\beta \mathrm{sm} \alpha).$$

The only difference between this definition of the cardinality operator, which is equivalent to that given by Frege in the *Grundgesetze*, and that given in the *Grundlagen*, is that Frege now opts for a value-range of value-ranges rather than a value-range of concepts. As a value-range is simply the extension of a concept for Frege, this adoption of value-ranges leads to a neater, yet equivalent definition.

It should be evident that, despite this shift towards treating functions in extension (i.e., in terms of their value-ranges), functions, in particular that species of function Frege calls a concept, remain central to Frege's philosophy. A re-examination of Fregean concepts is thus called for. In order to undertake such a re-examination, however, we must first consider a further thesis of Frege's, introduced in his middle-period, which has exerted an influence over twentieth-century philosophy as strong as anything else instigated by him; namely, the distinction between sense (*Sinn*) and reference (*Bedeutung*).

### 1.5 Frege on Sense and Reference

The distinction between sense (*Sinn*) and reference (*Bedeutung*) first appears in Frege's 1891 article 'Function and Concept', and is established in detail in two papers of the following year: 'On Sense and Reference' and 'Comments on *Sinn* and *Bedeutung*' (published posthumously). The thesis begins by outlining the situation for singular expressions: 'We must distinguish between sense and *Bedeutung*. "2<sup>4</sup>" and "4.4" certainly have the same *Bedeutung*, i.e. are proper names of the same number; but they have not the same sense' (Frege, 1891, p. 29). It follows that '2<sup>4</sup> = 4.4' or 'the evening star = the morning star' are informative identity statements in a way that '2<sup>4</sup> = 2<sup>4</sup>' or 'Venus = Venus' clearly are not. Identity statements can be informative because

two expressions sharing the same *Bedeutung* may differ in sense. In other words, the same referent can be presented to us in different ways; the sense of an expression will therefore be the 'mode of presentation' of its referent.<sup>27</sup> Hence the sense of an expression may be characterised as the manner in which a referent is picked out: '2+' and '4.4' pick out the same object in different ways. Furthermore, problems of intensionality can now be, to some extent, explained by the fact that my knowing that the morning star is the planet Venus does not imply my knowing that the evening star is the planet Venus. A grasp of the sense of an expression does not guarantee a grasp of the senses of all other expressions sharing the same *Bedeutung*.

Whilst few would disagree that there is at least some degree of explanatory force to the thesis at the level of singular expressions like 'the morning star' or '2+'. Frege's subsequent application of the thesis to sentences seems decidedly counter-intuitive. Frege's claim is that the sense of a sentence is the thought it expresses; the *Bedeutung* of the sentence, however, is a truth-value. Central to Frege's philosophical logic is his insistence that there are only two truth-values:

For brevity I call the one the True, the other the False. Every assertoric sentence concerned with the *Bedeutung* of its words is therefore to be regarded as a proper name, and its *Bedeutung*, if it has one, is either the True or the False.

(Frege. 1892a, p. 63)<sup>28</sup>

If a proper name is to have a *Bedeutung* at all, Frege insists, that *Bedeutung* must be an object. Hence the two truth-values are objects, in Frege's view.<sup>29</sup> Admittedly they, like numbers, are not objects in the usual sense of the word as used by the physicist, but *logical* objects: 'Places, instants, stretches of time, logically considered, are objects; hence the linguistic designation of a definite place, a definite instant, or a

<sup>27</sup> See Frege. 1892a, p. 57.

<sup>28</sup> Although Frege adheres to a strict two-valued logic, he does not assume that every assertoric sentence must have a truth-value. Sentences concerning fictional situations or characters, for example, may have a sense but not a *Bedeutung* (See Frege, 1892c, p. 122). Dummett suggests that Frege's description of sentences containing names which lack a bearer as neither true nor false may be interpretable as alluding to a third 'undesignated' truth-value (See Dummett, 1973a, pp. 421-426). As Evans (1982, p. 11) points out, however, it is clear that while such a course was indeed open to Frege, he did not choose to take it and retained instead a paradigm of the classical interpretation of logic as 2-valued.

<sup>29</sup> See also Frege, 1891, p. 32.

stretch of time is to be regarded as a proper name' (Frege, 1892a, p. 71). A sentence is a name of a truth-value and, being the sense of a sentence, a *thought* is a mode of presentation of a truth-value.<sup>30</sup> Again, Frege's ontology is being determined by logical considerations; an object is that which behaves like an object when considered as subject matter for the logician.<sup>31</sup> Furthermore, this lends support to Dummett's insistence that it was Frege who ushered in the linguistic turn, as Frege's basic point here seems to be that an object, from the point of view of the logician, is simply the referent of a singular term or definite description, whether it be 'the hour before dawn', 'Nine', or 'the True'.<sup>32</sup> The somewhat disconcerting consequence of this view is that, considered as a class of complex names, the class of all assertoric sentences divides into just three types of sentences: names of the True, names of the False, and sentences which fail to name any bearer (the two-valued status of Frege's logic ensuring that there are only two possible bearers of a complex name). Therefore '2 + 3 = 5' and 'Caesar conquered Gaul' are names of the same thing.

The recognition of some relation between an assertoric sentence and a truth-value is, of course, unobjectionable: indeed it is impossible to generate any chain of inference and, hence, any *logic*, without some such relation. Furthermore, as Dummett

<sup>30</sup> We are now in a position to consider Dummett's resolution to the problem, mentioned above, of how to interpret Frege's commitment to the apparently contradictory claims that (1) thoughts are built up out of parts corresponding to the parts of sentences and (2) thoughts are not constructed out of independent concepts but concepts can only be obtained by analysis of thoughts. Concepts are in the realm of reference, thoughts are senses; with this distinction in mind, Dummett distinguishes between *analysis* and *decomposition*: every thought has an analysis into ultimate constituents out of which it is constructed; therefore thoughts are constructed out of their component senses, and the sense of a thought can only be grasped by grasping the sense of its constituents. Concepts, however, are reached through decomposition which is undertaken in order to show the validity of inferences in which a sentence features. Decomposition, directed at sentences, removes one or more complete expression, leaving the unsaturated expression that stands for the concept essential to the inference. These concepts, accordingly, can only be reached through decomposition, hence a concept is only to be understood from seeing it as removed from a sentential context of which we have a prior understanding. Analysis applies to (1) and decomposition applies to (2), hence (1) and (2) happily co-exist. See Dummett, 1981, ch. 15. Dummett's view, first expressed in his (1973a) has been criticized by Sluga (1976), Geach (1975), Currie (1985), and more recently by Levine (2002) who, interestingly sees an awareness of the distinction in Russell, but not in Frege. Landini (1996c) defends Dummett. For further discussion, see Beaney, 1996, pp. 234-245.

<sup>31</sup> This interpretation has also been argued for by Wright: 'The basis of Frege's platonism is the thesis that objects are what singular terms, in the ordinary intuitive sense of "singular term", refer to. From the point of view of the correct order of explanation, the converse claim, that singular terms are those expressions which refer to objects, is true but back to front. For Frege it is the syntactic category which is primary, the ontological one derivative' (Wright, 1983, p. 13; see also p. 25).

<sup>32</sup> This becomes strikingly evident in Frege's discussion of the concept-object distinction, where he maintains that to speak of 'the concept *horse*' is, unavoidably, to turn the expression into an object expression which, as such, must refer to an object, leading to the apparently paradoxical claim that 'the concept *horse* is not a concept' (Frege, 1892b, p. 46). We will return to this point shortly.

points out, perceiving this relation as a relation of *reference* does not commit one to viewing the relation as a naming relation. Reference could, for example, be construed as a matter of semantic equivalence, in which case calling the truth-value of a sentence its reference would amount merely to an observation of the semantic role of that sentence within the context of a language without committing us to the view that a sentence *names* its truth-value.<sup>33</sup> Frege's stronger claim, however, does commit him to this latter view. The relation between a name and its bearer is not merely a helpful analogy in elucidating that relation which obtains between an assertion and its truth-value, but is taken as the prototype of it; hence the two relations are construed by Frege as equivalent. A sentence literally names a truth-value in a way which is directly equivalent to the naming of an individual by a proper name.

Frege's justification for viewing sentences as complex names would appear to be drawn, albeit indirectly, from his distinction between concept and object. Expressions, on the Fregean model, fall into two general classes (with corresponding classes of referents): complete expressions and incomplete expressions. Making this distinction is not quite equivalent to saying that there are only concept and object expressions, as not all incomplete expressions denote concepts. That is to say, not every function is a concept; operators like '+' or '( )'<sup>2</sup> are considered to be functions on Frege's account, but not concepts, as they do not by themselves map an object to a truth-value.<sup>34</sup> The fact that a sentence is a complete expression means that sentences fall into the class of object, rather than functional, expressions and must, therefore, be regarded as a species of name. What, then, is the relation of reference for incomplete expressions? Frege maintains that 'a concept is the *Bedeutung* of a predicate' (Frege, 1892b, pp. 47-48). But, by his own admission, 'it is a mere illusion to suppose that a concept can be made an object without altering it' (Frege, 1884, p. x), so, whatever the relation is which does obtain between a predicate and a concept, it cannot be that of

<sup>33</sup> See Dummett, 1973a, Ch. 12. Dummett notes that such a view is similar to that forwarded by Tugendhat, 1970, in his interpretation of Fregean reference: If we take the notion of semantic equivalence as dividing expressions into equivalence classes (construed as abstract objects), we may take those classes as the referents of the expressions which are members of them (See *Ibid.*, p. 402). Another approach, urged by Dummett himself, would be to construe reference not as a *naming*-relation but as an assignment of semantic value, where the semantic value of an expression will simply be 'that feature of it that goes to determine the truth of any sentence in which it occurs' (Dummett, 1991b, p. 24).

<sup>34</sup> A concept, we have seen, is a function whose value is always a truth-value. See Frege, 1891, p. 30.

the usual relation of a name picking out its bearer, insofar as the referent in question is not ‘self-subsistent’ in the way that an object is.

It is, to say the least, unusual to conceive of a predicate as an expression which *refers* to anything at all. In an atomic sentence ‘Fa’, it is natural to think of the object expression referring to an object (taking ‘a’ as having a constant value), namely a particular value of the variable ‘x’ in ‘Fx’. The predicative part of the sentence, however, is more commonly thought of as having an entirely different role to that of the object-expression. That is, it is more commonly thought of as doing just what its name suggests: *predicating*. In predicating a property of some object, we are not necessarily positing concepts as entities existing independently of the objects in which, as properties, those concepts are instantiated. Thus far we have only committed ourselves to the existence of objects and observed that those objects have certain characteristics which allow us to say (predicate) things *of* them. At what point does our talk of properties or concepts shift from this, apparently innocuous, *façon de parler* to a fully-fledged ontological commitment to concepts as entities referred to by predicates? The Quinean response to this question will, of course, be that ontological commitment to entities of a given logical type comes at the point at which we are prepared to quantify over those posited entities. Our willingness to quantify over something, Quine maintains, reveals the ontological requirements of our theory, hence the famous dictum: ‘To be is to be the value of a variable’ (Quine. 1948, p. 15). By this criterion, there can be little doubt that Frege’s ontology is overflowing with concepts, as he showed no hesitation in quantifying over them (including concepts which are themselves of higher order<sup>35</sup>), and could not have developed his logicist project without doing so. If we accept, for the sake of argument, Quine’s criterion, we are in no better position to seek an answer to our original question, however. We may have provided good grounds for concluding that Frege really is committed to concepts as entities, but it is still unclear what those entities are.<sup>36</sup>

An alternative approach may be to ask a slightly different question. We have seen that the sense of an expression, for Frege, is the mode of presentation of its referent. Reference, therefore, is not something which can be grasped independently of sense: if a referent is to be presented to us, it must be presented in *some* fashion. If

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<sup>35</sup> For example, in axiom IIb of the *Grundgesetze*. See Frege. 1893. § 25.

sense is the route to reference, we may be better placed to give an account of the reference of an incomplete expression if we can provide an answer to the question of what it is to have a sound grasp of its sense. An answer is suggested in passages such as the following:

It is only in virtue of the possibility of something not being wise that it makes sense to say “Solon is wise.” The content of a concept diminishes as its extension increases; if its extension becomes all-embracing, its content must vanish altogether.

(Frege, 1884, § 29)

Although this passage was obviously written prior to Frege’s adoption of the sense/reference distinction, it is fair to conclude from it that Frege’s point here will apply to what he would later call sense. A predicate which was (logically) true of all permissible arguments (i.e., all arguments of the correct logical type) would be a predicate lacking in both sense and reference. It would lack a sense precisely because there would be nothing to be grasped and, therefore, no reference to be presented.

Whether Frege would have repudiated this view in his later work is not clear.<sup>37</sup> What is clear is that Frege is here asserting a connection between the sense of a concept-expression and its extension. The connection would seem to be that having a grasp of the sense of a predicative expression involves being capable of determining which objects fall under its extension. With the sense/reference distinction in place, Frege develops the argument further:

We have to throw aside concept words that do not have a *Bedeutung*.

These are not such as, say, contain a contradiction—for there is nothing at all wrong in a concept’s being empty—but such as have vague boundaries.

It must be determinate for every object whether it falls under a concept or

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<sup>36</sup> For a discussion of the relative merits of Quine’s and Frege’s positions on this issue, see Dummett, 1973a, pp. 223-226.

<sup>37</sup> Dummett notes that Frege would not have maintained the principle for complex predicates in his later period, as it leads to a denial of sense for analytic statements. Whether he would have repudiated the claim in regard to primitive predicates (e.g., ‘ $\xi$  is an object’) is not clear. See Dummett, 1973a, p. 222. See also Frege’s letter to Russell of 29/6/1902 in Frege, 1980, pp. 135-137.

not; a concept word which does not meet this requirement on its *Bedeutung* is *bedeutungslos*.

(Frege, 1892c, p. 122)

A concept word which does not have clearly defined boundaries, then, does not have a reference; and Frege is explicitly stating that the clearly defined boundaries in question are simply the limits of the concept's extension. The implication here, undoubtedly, is that a grasp of the sense of a predicate amounts to an ability to determine which objects the predicate is true of and which it is false of. In other words, knowing the sense of a predicate is knowing how to determine its extension: and knowing how to determine the extension of a predicate is knowing how to use it as a predicate. Can we, then, identify the extension of a predicate with its reference? Is the reference of a predicate the set of things it is true of?

An immediate problem arises here in that this would not seem to be compatible with Frege's concept/object distinction. The extension of a predicate is, on Frege's account, an object. We have already observed that treating a value-range as an object is essential to Frege's philosophical account of number, and that the inviolable distinction between objects and concepts is essential to his rejection of the view that numbers are concepts or properties. In short, if we are to take the reference of a predicate as the set of things it can be veridically applied to, then we must dispense with the model of name and bearer as the prototype for the relation between a predicate and its reference. Furthermore, the problem does not just surface if we try to identify a predicate with its extension. The problem remains when we want to identify concepts as the referents of concept-expressions. To say 'the concept  $\Phi$  = the concept  $\Gamma$ ' is to strip the concepts in question of their predicative, or incomplete, status and turn them into objects. Frege was fully aware of this and, accordingly, states that only objects can flank either side of an identity-sign. Hence 'the concept  $\Phi$  = the concept  $\Gamma$ ' is a genuine identity statement *because* it asserts an identity between objects. To talk directly of a concept, it seems, is to turn it into an object, hence justifying Frege's apparently paradoxical sentence 'the concept *horse* is not a concept' (Frege, 1892b, p. 46). Now consider the sentence "the reference of the predicate '( ) is red' is the concept *red*". Again, in trying to identify the concept which we take to be the referent of the predicate, we are left with an identity statement between objects, thereby failing

to pick out a concept. One might be tempted to view this as a *reductio ad absurdum* of the idea that the sense/reference distinction can be applied to concepts in the way it can be applied to other expressions. Accordingly, one will (with some justification if looking no further than the works published by Frege in his lifetime) conclude that Frege does not wish to make a literal extension of the doctrine to incomplete expressions, but merely to present it as a helpful analogy.

Frege's posthumously published work exposes the inadequacy of such a conclusion.<sup>38</sup> Frege makes explicit, therein, his commitment to a literal extension of the name/bearer relation of reference to predicates: 'To every concept word or proper name, there corresponds as a rule a sense and a *Bedeutung*, as I use these words ... A concept word stands for a concept, if the word is used as is appropriate for logic' (Frege, 1892c, p. 118). Furthermore, he is explicit that the reference of a predicate cannot be identified with its extension for the reason given above.<sup>39</sup> What then, of the problem of identifying a concept with the reference of a predicate?:

We should really outlaw the expression 'the *Bedeutung* of the concept word *A*', because the definite article before '*Bedeutung*' points to an object and belies the predicative nature of a concept. It would be better to confine ourselves to saying 'what the concept-word *A* stands for', for this at any rate is to be used predicatively.

(Frege, 1892c, p. 122)

Frege's solution to the problem is to deny that the reference of a predicate *can* be given in the way we have been seeking for one. The reason why, it is absolutely crucial to appreciate, is both a *logical* and a *linguistic* one. If we accept the logical distinction between concept and object, along with the linguistic aspect of that logical point; namely that the status of an expression as either complete or incomplete is determined by the logical role of that expression *within the context of a proposition*, then concepts cannot be the referents of object-expressions. As a result, they cannot be subjects of identity statements. Hence, it will never be logically articulate to give an account of what a given concept is in terms of a sentence such as 'the referent of the

<sup>38</sup> Especially in Frege, 1892c. This point is given detailed examination in Dummett, 1973a, Ch. 7.

<sup>39</sup> See Frege, 1892c, pp. 118-119.

predicate  $\phi$  is the concept  $F$ '. The presence of the definite articles here shows us that the logical roles of the parts of the sentence we are interested in are the logical roles of object expressions. We cannot give conditions for identity of concepts in the way that we can for objects. To attempt to do so is to misunderstand what logical role concept expressions play. Frege's solution, with this awareness in mind, is to treat concept expressions in a way, as it were, in keeping with their logical status. To do so, he introduces a new relation which is equivalent to the role that identity plays for objects: that of co-extensionality. In this way we can give conditions for two predicates sharing the same reference without being tempted to say what *the* reference of those predicates is. Simply put, two predicates share the same reference if, and only if, they are true of just the same objects: ' $\forall x (Fx \leftrightarrow Gx)$ ' is therefore the relation between predicates equivalent to the relation for objects, ' $x = y$ '. Roughly aiming at a logically articulate version of this relation in ordinary language, we might offer: "what the concept-expression ' $F$ ' stands for is the same as what the concept-expression ' $G$ ' stands for". Hence we have a condition *equivalent* to identity which is applicable to concepts without being obliged to actually *identify* concepts outright in the way that we would for objects. Concepts, Frege concludes, evade substantival expressions, but the evasion in no way threatens the existence of concepts as the referents of predicative expressions. Predicates are literally names of concepts. This metaphysics, in all its platonic excess, is not the indolent withholding of Ockham's Razor that Russell, we shall see, came to view it as. Concepts are maintained by Frege as a result of a rigorous consistency with his own logical doctrines. Concepts, for Frege, are logically primitive, and revealed as such by logical analysis: 'One cannot require that everything shall be defined, any more than one can require that a chemist shall decompose every substance. What is simple cannot be decomposed, and what is logically simple cannot have a proper definition' (Frege, 1892b, pp. 42-43).

In effect, Frege's point regarding the reference of incomplete expressions is an important precursor to Wittgenstein's distinction between what can be said in language and what can only be shown by language. Frege's point might be summed up by saying that it is nonsensical to attempt to say something like: ' $F = G$ '. What, in a confused way, we are trying to say by this nonsensical pseudo-proposition, can be properly shown by the logically appropriate relation of co-extensionality: ' $\forall x (Fx \leftrightarrow$

$Gx$ '.<sup>40</sup> Wittgenstein's philosophy will concern us, however, only after we have first looked in detail at the work of Russell. The remainder of this chapter will be devoted to examining the philosophical enterprise and logical doctrines of Russell's *Principles of Mathematics*; the next chapter will be devoted to studying in detail the development of Russell's thought between 1903 and 1913.

### 1.6 Russell and the Origins of Analysis

Frege's interest in philosophy was exclusively restricted to mathematical and logical issues. Throughout his career, he never wrote on any other topic. Russell, by contrast, was vastly more eclectic in both his influences and his ambitions. The young Russell was immersed in the Hegelian philosophy of McTaggart and Bradley, and his early works reflect this.<sup>41</sup> The causes of his rejection of Hegelianism were twofold—one mathematical and one philosophical—and led to the creation of what we now know as analytical philosophy. The greatest influence on Russell's mathematical philosophy came from Peano's work in logic. The philosophical influence was nearer to home, originating in a dissent from Hegelianism which Russell shared with Moore. The combined impact of these forces on Russell was due, in part, to their consistency with one another; as Francisco Rodriguez-Consuegra aptly puts it:

While Moore showed Russell *what* to do in philosophy, i.e. building definitions in terms of simples, Peano showed him *how* to carry it out in practice through his logical techniques and his theory of definition, which coincided, at least in the essentials, with Moore's.

(Rodriguez-Consuegra, 1991, p. 36)

<sup>40</sup> Dummett has pointed out another way in which it is useful to invoke the saying/showing distinction in elucidating Frege's notion of reference, by understanding that sense cannot be stated, as such, but can be shown by stating reference. Stating a referent will provide, necessarily, a mode of presentation of the referent: this mode of presentation cannot itself be referred to, but is *shown* in referring to that which it is a mode of presentation of (See Dummett, 1973a, p. 227).

<sup>41</sup> Two outstanding texts detailing Russell's early commitment to Hegelian idealism are Griffin (1991) and Hylton (1990).

The results of Russell and Moore's discussions first appeared in Moore's 1899 paper 'The Nature of Judgement',<sup>42</sup> wherein he rejected outright the monism of Bradleyian-Hegelian idealism and offered in its place an atomistic ontology of mind-independent 'concepts' as the basis of a strict realism.<sup>43</sup> Criticising Bradley for failing to maintain a distinction between ideas and the objects that those ideas are ideas of,<sup>44</sup> Moore argued that the contents of our ideas must have some status independent of their being part of that content: 'It is our object to protest against this description of a concept as an "abstraction" from ideas' (Moore, 1899, p. 2). Moore's concepts are objectively subsistent entities only some of which are 'existent' (i.e., those which stand in a certain relation to the concept of existence),<sup>45</sup> and provide the substance for his theory of propositions (or judgements, which are not distinguished from propositions in Moore's doctrine). This theory of the proposition is just as vehemently anti-psychologistic, in its own way, as that forwarded by Frege. In asserting a proposition or judgement, Moore states, one is not asserting something about one's mental states, but rather is asserting 'a specific connexion of concepts':

If the judgement is false, that is not because my *ideas* do not correspond to reality, but because such a conjunction of concepts is not to be found among existents.

With this, then, we have approached the nature of a proposition or judgement. A proposition is composed not of words, nor yet of thoughts, but of concepts.

(Ibid., p. 4)

This view, endorsed by Russell, differs significantly, despite the shared rejection of psychologism, from Frege's. Whereas Frege had sought to extract *thoughts* from the

<sup>42</sup> Russell's generous acknowledgement to Moore in the preface to the *Principles of Mathematics* suggests that Moore was the primary instigator of their mutual rejection of Hegelianism. This is certainly over-modest of Russell, as his own thoughts can be seen to be developing along similar lines in his early work on Leibniz (see Russell, 1900a & 1900b). This is enforced by Moore's remarks in his autobiographical essay: 'I do not know that Russell has ever owed to me anything except mistakes; whereas I have owed to his published works ideas which were certainly not mistakes and which I think very important' (Moore, 1942, p. 15). For detailed discussion of Moore's influence on Russell see Griffin, 1991, Ch. 7, § 2, and Hylton, 1990, Ch. 4, §§ 1-2.

<sup>43</sup> The metaphysical holism that Moore objects to in Bradley is presented at its most explicit in Bradley, 1893.

<sup>44</sup> See Moore, 1899, p. 1.

psychological processes involved in thinking by considering a thought to be the sense of a sentence (i.e., the mode of presentation of a truth-value), Moore and Russell sought to defeat psychologism by making propositions themselves independent of thought.<sup>46</sup> Russell was profoundly influenced, in a way which would remain constant for the rest of his career, by the resulting picture of the world. For, if a proposition is a complex concept composed of less complex concepts, it follows that we are now in a position to justifiably ask after the nature of those constituents concepts. As Moore realised, the complexity does not appear to continue through the analysis. Sooner or later, one assumes, we must come upon the simple constituents which combine to form the complex:

It would seem, in fact ... that a proposition is nothing other than a complex concept. The difference between a concept and a proposition, in virtue of which the latter alone can be called true or false, would seem to lie merely in the simplicity of the former. A proposition is a synthesis of concepts.

(Ibid., p. 5)

These concepts, Moore claims, are the genuine ontological furniture of the world. Our knowledge of the world, therefore, is our knowledge of its constituent concepts. Moore captures the spirit of this prototype of Russellian logical atomism perfectly when he baldly states that: 'A thing becomes intelligible first when it is analysed into its constituent concepts' (Ibid., p. 8). In place of the conception of philosophy as a process of Hegelian synthesis, we now have an explicitly stated conception of philosophy as analysis.

In 1900 Russell met Peano and, impressed by his logical acumen, set about mastering his symbolic notation which, Russell, soon realised, served to facilitate the expression of his (and Moore's) ideas and allow their application to mathematics.<sup>47</sup> Prior to his study of Peano, the best logical system at Russell's disposal was Boole's

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<sup>45</sup> See Ibid., p. 8.

<sup>46</sup> The divergence from Frege is apparent from Russell's insistence to Frege that 'in spite of all its snowfields, Mont Blanc itself is a component part of what is actually asserted in the proposition "Mont Blanc is more than 4000 metres high"' (Russell to Frege, 12/12/1904, in Frege, 1980, p. 169)

<sup>47</sup> See Russell, 1967, p. 144.

algebra of classes; under Peano's influence he quickly recognised the importance of the propositional calculus (the philosophical grounding of which he based on his and Moore's theory of the proposition) as distinct from the calculus of classes. Russell also developed an application of Peano's symbolism to his own 'logic of relations'.<sup>48</sup>

Perhaps the most important influence on Russell, as he himself acknowledged, was the distinction between *material* and *formal* implication.<sup>49</sup> This led Russell to distinguish between the *real* and the *apparent* variable. Both these (interconnected) distinctions, like much of the logical achievements of Peano and Russell, had been pre-empted by Frege.<sup>50</sup> Peano expresses implication by the (now familiar) two place relation-sign ' $\supset$ '. Thus ' $p \supset q$ ' signifies that  $p$  implies  $q$ , a relation which can be otherwise expressed as ' $p$  is not true, or  $q$  is true' (where 'or' is used in the non-exclusive sense to simply state that the disjunction cannot be true if both disjuncts are false.).<sup>51</sup> This is material implication. Formal implication serves to generalise the relation over a fixed domain, a procedure symbolised by affixing the relevant variable to the implication sign. Thus, ' $p \supset_{p,q} q$ ' can be read as ' $p$  implies  $q$ , for all values of  $p$  and  $q$ '. In other words, the subscripted variables turn ' $\supset$ ' into a variable-binding operator, effectively attaching the universal quantifier to the relation of conditionality. This provides for the distinction between real and apparent variables. In a propositional function ' $\phi x$ ' the variable ' $x$ ' is a real variable, because it can be replaced by *any* value of  $x$  in order to give a proposition. In this sense it is simply marking an open space in the same way that we have seen Frege signify by ' $\xi$  is mortal'. Hence a propositional function containing a real variable is not a truth-evaluative item, as it is essentially incomplete: it only has a truth-value when the variable is assigned a value. A formal implication, however, makes an assertion about *all values* of the variable, hence the variable is called an apparent variable and the proposition can be assigned a truth-value:

<sup>48</sup> See Russell, 1901a.

<sup>49</sup> See Russell, 1959, p. 66.

<sup>50</sup> Russell notes the superiority of Frege's treatment of both concepts in his appendix on Frege in the *Principles* (see Russell, 1903, § 493). He later attributes priority to Frege in making the distinction between real and apparent variables (see Russell, 1908, p. 66).

<sup>51</sup> Frege's superior statement of ' $p \supset q$ ' is actually given in the truth-functional terms later transformed by Wittgenstein into the, now standard, truth-table format (see Frege, 1879, § 5). See also Russell's letter to Frege of 24/5/1903, in Frege, 1980, p. 158, for an explicit statement of the equivalence.

Wherever a proposition containing an apparent variable is asserted, what is asserted is the truth, for all values of the variable or variables, of the propositional function corresponding to the whole proposition.

(Russell, 1903, § 92)

There are severe limitations on the potential applications of this quantification theory. Mostly these result from Peano's failure to separate the variable-binding operation from the ' $\supset$ ' function. Frege's quantification does not suffer from the same drawback and, consequently, can be conveniently applied to propositions of different forms. For example, ' $(x) \phi x$ ' is not easily achieved in Peano's notation, and Russell's adoption of this form surely resulted from his study of Frege and subsequent development of Peano's symbolism.<sup>52</sup> Neither did Peano furnish his system with a genuine existential quantifier. The closest Peano got to an existential quantifier was a symbol used to assert the existence (non-emptiness) of a class, which he defined as:

$$\exists \alpha =_{df} \sim (\alpha = \Lambda)$$

But here ' $\exists$ ' does not act as a variable binding operator, and cannot be defined out of the universal quantifier, in the way Frege's quantification device allows. Landini (1996b, p. 566) suggests that there are hints in the *Principles* of Russell's later definition of Peano's symbol, utilising existential quantification in the definiens, as found in \*24.03 of *Principia*:

$$\exists! \alpha = (\exists x) x \in \alpha \quad \text{Df}$$

Landini draws attention to Russell's discussion of the calculus of classes in the *Principles*, where we find the following remark:

A class is said to exist when it has at least one term. A formal definition is as follows:  $\alpha$  is an existent class when and only when any proposition is

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<sup>52</sup> Russell does not attempt a symbolic rendering of ' $\phi x$  is always true' (i.e., ' $(x) \phi x$ ') in the *Principles*. Landini suggests that the only option utilising Peano's quantification would be ' $\sim \exists x \supset, \phi x$ ' (Landini, 1996b, p. 566).

true provided “ $x$  is an  $a$ ” always implies it whatever value we may give to  $x$ .

(Russell, 1903, § 25)

This ‘formal definition’ is far from clear, but Russell’s intention can nonetheless be seen as the claim that a class exists when the corresponding class-concept has at least one term in its extension. Russell goes on to say: ‘A class  $a$  exists when the logical sum of all propositions of the form “ $x$  is an  $a$ ” is true, i.e. when not all such propositions are false’ (Ibid.). Landini reads Russell’s intention as:

$$\exists \hat{x} (x \text{ is an } a) = \sim (x) \sim (x \text{ is an } a) \quad \text{Df.}$$

and is hence justified in asserting that the passage ‘certainly foreshadows Russell’s later definition of the existential quantifier’ (Landini, 1996b, p. 566). Of course, the symbolism used in Landini’s version of the definition was not available to Russell at the time in question. Within the confines of Peano’s logic, Russell would have been forced to adopt the somewhat awkward

$$\exists a = \sim [x \text{ is an } a \supset_x \sim (x \text{ is an } a)] \quad \text{Df.}$$

where the definiens can be read as ‘it is not always false that  $x$  is an  $a$ ’.

Passages such as the one examined above serve to warn us of the dangers of dismissing out of hand what Nicholas Griffin disparagingly labels ‘the fruitless complexities of Russell’s early theory of quantification’ (Griffin, 1996, p. 46). Nonetheless, it is evident that Russell did not, at this stage, equal Frege’s clarity on the subject.<sup>53</sup>

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<sup>53</sup> Landini (1996b) raises another important point regarding the development of quantification theory in the *Principles*. There are passages in the book which suggest that Russell was concerned to establish the correct rules for quantifiers in conditional proofs in the *Principles*. As Frege did not sanction conditional proofs, there is no possibility that these ideas could have been introduced to Russell through his study of Frege. As our discussion is concerned with Frege’s quantificational *device* as a complex concept, rather than the rules for the treatment of quantifiers in proofs, this point will not be pursued in this discussion.

### 1.7 The Doctrine of the Unrestricted Variable

It is important to be clear about Russell's conception of logic in this period. Although Russell shared in Frege's platonism at this time, he did not in any way endorse the explicitly linguistic spin that Frege gave to his own conception of logic. Frege's forays into philosophy of language resulted from his conception of a thought as the sense of a sentence; a mode of presentation of a truth-value. Frege's platonic (or logical) entities, be they numbers, concepts or truth-values, were all deemed to be accessible via language. Thoughts, for Frege, have the same logical structure as the sentences that express them (at least after the proper analysis of those sentences—the *logical* structure of a sentence may, of course, differ greatly from its *grammatical* structure). For this reason Frege's analysis of language is always at pains to accommodate such structures, as we have seen in his careful treatment of the distinctions between concepts and objects and between concepts of differing 'level'. In other words, Frege always showed an awareness of the differences in *logical type* of the constituents of propositions. In the next chapter we shall examine in detail the lengthy process which led to Russell's eventual acceptance of the need to accommodate such distinctions in his own account.<sup>54</sup> At the time of writing the *Principles*, however, he vigorously maintained the principle that restrictions on the values a given variable may be assigned are to be avoided wherever possible. Contrary to what some may think, this did not reflect a lack of insight, or even mere carelessness regarding logical grammar on Russell's part. Rather, it should be seen as the reflection of what Russell took to be a strong philosophical argument concerning the nature of logic itself. For Russell, at this time, logic was a synthetic *a priori* science.<sup>55</sup> As such, it is to be viewed as the foundation for every other scientific activity and, therefore, as the science of the most general structural forms of propositions (and here it must be remembered that propositions are considered to be non-linguistic, objective complexes). In other words, it is to be applied to anything and everything. Logic must be all-encompassing.

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<sup>54</sup> It is interesting to note that, in his letter to Frege of 8/8/1902, Russell discusses the possibility of avoiding the contradiction through the introduction of type-theoretic restrictions on variables and remarks that: 'This theory is analogous to your theory about functions of the first, second, etc. levels' (Russell to Frege, in Frege, 1980, p. 144). In the *Principles*, Russell had explicitly stated that a class is of 'the same logical type as its simple parts' (Russell, 1903, § 137, ft. nt.). Frege discusses the similarities between *Principia*'s type-theory and his hierarchy of levels in a letter to Jourdain written 28/1/1914 (See Frege, 1980, pp. 78-84).

Russell's first explicit statement of the universality of logic—a principle which becomes the defining feature of his logicism in the *Principles*—was given in a popular article proclaiming the virtues of Peano's symbolic treatment of mathematics:

Now the fact is that, though there are indefinables and indemonstrables in every branch of applied mathematics, there are none in pure mathematics except such as belong to general logic. Logic, broadly speaking, is distinguished by the fact that its propositions can be put into a form in which they apply to anything whatever. All pure mathematics—Arithmetic, Analysis, and Geometry—is built up by combinations of the primitive ideas of logic, and its propositions are deduced from the general axioms of logic, such as the syllogism and other rules of inference.

(Russell, 1901b, p. 76)

Perhaps because it was written with a non-professional audience in mind, the claim made here is somewhat vague. For example, Russell must have been well aware that the syllogism was wholly inadequate for analysing mathematical propositions such as, for example, require a method of expressing multiple generality. Indeed, he makes just this point in his discussion, in the *Principles*, of Kant's failure to identify mathematics and logic. Kant may be excused, Russell suggests, on the grounds that 'the syllogism still remained the one type of formally correct reasoning; and the syllogism was certainly inadequate for mathematics' (Russell, 1903, § 434). In a draft of Part 1 of the *Principles*, dated 1901, Russell is less vague in his statement of the logicist thesis:

Pure mathematics is the class of all propositions of the form " $a$  implies  $b$ ", where  $a$  and  $b$  are propositions each containing at least one variable, and containing no constants except logical constants or such as can be defined in terms of logical constants. And logical constants are classes or relations whose extension either includes everything, or at least has as many terms as if it included everything.

(Russell, 1901c, p. 185)

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<sup>55</sup> See Russell, 1903, § 434.

The description of the logical constants given here is in need of some improvement.<sup>56</sup> Otherwise, however, the idea that logic must range over *everything* is retained without alteration in the finished version of the *Principles*. The importance that Russell placed on this principle is apparent on further comparison with Frege. As mentioned above, Frege's interests were almost wholly restricted to mathematical and logical issues. Furthermore, we also witnessed in some detail in the earlier part of this chapter how Frege was led to his metaphysical commitments as a result of his logical doctrines. In particular, Frege's platonism, I have argued, is best understood as a *result* of his logicism rather than as a precondition for it. Although it would be an oversimplification to say that the exact reverse is the case for Russell, it is fair to say that Russell was more driven by metaphysical considerations than was Frege. Having freed himself of the trappings of neo-Hegelianism, Russell was determined to replace it with a metaphysics that would overthrow its predecessor.<sup>57</sup> The atomism of this new metaphysics, as conceived by Russell, entailed an analysis of propositions into simple constituents which were all on an equal ontological footing—both in regard to each other *and* to the complex objects composed of them. It was of fundamental importance, therefore, that the variable should be able to take any entity as a value.

This requirement for complete generality is reflected formally in Russell's treatment of the logical constants. Implication, which is taken as the most primitive relation in the *Principles*, is not to be automatically read as equivalent to the modern statement connective ' $\rightarrow$ ' ('if ..., then ...'), which can only be flanked by well-formed formulas. Rather, Russell's ' $\supset$ ' is a dyadic predicate which can be flanked by any logical subjects. Of course, Russell was well aware that, among those logical subjects, only *propositions* can enter into true or false relations with one another. However, as this means that ' $p \supset q$ ' can only be *true* in the case in which  $p$  and  $q$  are propositions, Russell can ensure that this condition is met without imposing restrictions on variables by simply including as hypothesis ' $p \supset p$ ' wherever necessary.<sup>58</sup>

The doctrine of the unrestricted variable, then, maintains that anything which has an ontological status may be made into a logical subject. Russell calls such entities

<sup>56</sup> Russell might have phrased the description better by saying that the logical constants are relations which are *significant* for every term.

<sup>57</sup> See Russell, 1959, ch. 5.

<sup>58</sup> See Russell, 1903, § 16. This habit of Russell's was much to the annoyance of Wittgenstein (see Wittgenstein, 1922, 5.5351).

terms in the *Principles*, although we must be careful not to conclude from this that terms are linguistic entities. Terms are entities which have the ontological status of *being*:

Whatever may be an object of thought, or may occur in any true or false proposition, or can be counted as *one*, I call a *term*. This, then, is the widest word in the philosophical vocabulary. I shall use as synonymous with it the words unit, individual, and entity. The first two emphasize the fact that every term is *one*, while the third is derived from the fact that every term has being, *i.e.* is in some sense. A man, a moment, a number, a class, a relation, a chimaera, or anything else that can be mentioned, is sure to be a term; and to deny that such and such a thing is a term must always be false.

(Russell, 1903, § 47)

Russell's 'terms' are divided into two kinds; *things* and *concepts*: 'The former are the terms indicated by proper names, the latter those indicated by all other words' (Ibid., § 48).<sup>59</sup> This is not to be viewed as equivalent to Frege's distinction between concept and object, however. Indeed the doctrine of the unrestricted variable expressly forbids the Fregean distinction. Concepts, as entities granted an ontological status, must be terms on Russell's account. Therefore it must be permissible to make them into logical subjects. Indeed, echoing (and denying) the thinking behind Frege's remark that 'the concept *horse* is not a concept' (Frege, 1892b, p. 46), Russell points out that the very claim that adjectival terms cannot be made into logical subjects is self-contradictory, requiring as it does that the terms in question be made into logical subjects in the propositions which deny their logical subject-hood:

In short, if there were any adjectives which could not be made into substantives without change of meaning, all propositions concerning such adjectives (since they would necessarily turn them into substantives) would be false, and so would the proposition that all such propositions are

false, since this itself turns the adjectives into substantives. But this state of things is self-contradictory.

(Russell, 1903, § 49)

Concepts, Russell concludes, have an essentially *two-fold* nature. A concept can act as the logical subject of an assertion, and can also be used predicatively to assert something of a logical subject.

The division of terms into concepts and things is, undoubtedly, an attempt on Russell's part at retaining the doctrine of the unrestricted variable without foregoing an account of propositional unity. The fact that some terms can be used to assert something of other terms (their logical subjects) goes some way towards explaining the unity of the proposition as an assertion, in much the same basic way that Frege's decomposition of the thought into concept and object does. Russell clearly has this in mind when introducing the idea of a propositional function:

When a proposition is completely analysed into its simple constituents, these constituents taken together do not reconstitute it. A less complete analysis of propositions into subject and assertion ... does much less to destroy the proposition. A subject and an assertion, if simply juxtaposed, do not, it is true, constitute a proposition; but as soon as the assertion is actually asserted of the subject, the proposition reappears.

(Ibid., § 81)

The inferiority of Russell's account compared to Frege's becomes apparent, however, when we turn to propositions containing quantifiers. Russell was hampered by a quantification theory that lacked the power and sophistication of Frege's. As a result, Russell's analysis of the proposition (at this time) does not make such a radical departure from subject-predicate form as the analysis we have examined in Frege's work. In part, this may be attributed to the doctrine of the unrestricted variable which holds that any term may be the value of a real variable and hence that every concept must also be a term or logical subject. Thus the hierarchy of levels inherent in Frege's

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<sup>59</sup> Within the class of concepts, Russell makes a further distinction between predicates (or class-concepts) and relations. The former correspond to the grammatical category of the adjective, the latter

notion of quantification is not available to Russell in the *Principles*. Instead, Russell developed his infamous early theory of *denoting concepts*. Russell's reasons for abandoning the early theory in favour of the theory of descriptions in 1905 are still much debated. From a purely formal perspective, one simple but often overlooked contributory reason is that Russell's adoption of a Fregean quantification theory meant that he no longer had any *need* for denoting concepts.

### 1.8 Denoting Concepts

Russell's analysis of propositions, in the *Principles*, lacks the simplicity of the Fregean model.<sup>60</sup> Whereas Frege's concept-object distinction leads naturally into a hierarchy of levels of concepts which provides the philosophical foundation for quantification theory, Russell's theory demands that all the constituents of a proposition be terms. The qualification that only some terms are concepts ensures that not all terms can occupy the predicate position, hence avoiding such abominations as 'Mortality is Socrates'. Nonetheless, in opposition to Frege's account, predicative terms can be turned into logical subjects without loss of meaning; the term 'mortality' can feature as logical subject ('Mortality is a burden') or as 'assertion' ('Socrates is mortal') without any change in content. As a result, Russell did not (at this stage) make the same radical departure from the traditional analysis of the proposition into subject and predicate terms that Frege did.

We have seen, in our discussion of Frege, that the subject-predicate analysis breaks down when confronted by sentences containing quantifiers: 'All men are mortal' cannot be analysed on the same lines as 'Socrates is mortal'. Lacking the functional analysis found in Frege, however, Russell would appear to be faced with a problem at this point. In the proposition 'All men are mortal', 'All men' appears to be a term of which mortality is predicated. But, as Russell notes, it is not the *concept* 'all men' which is mortal, but the members of the class picked out by this concept. By contrast, in the proposition 'Socrates is mortal', the *term* 'Socrates' is mortal as the term is none other than Socrates himself. Similar problems arise with the

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to the verb (See Russell, 1903, § 48).

quantificational phrases 'any', 'every', 'a', 'some' and 'the'. Russell's theory of denoting is introduced to provide an account of the concepts expressed by these phrases, all of which (when annexed to a class-concept) are considered to be denoting concepts. The mark of a denoting concept, then, is that it can occupy the subject place of a proposition and yet not, itself, be what that proposition is about:

A concept *denotes* when, if it occurs in a proposition, the proposition is not *about* the concept, but about a term connected in a certain peculiar way with the concept. If I say "I met a man," the proposition is not about *a man*: this is a concept which does not walk the streets, but lives in the shadowy limbo of the logic-books. What I met was a thing, not a concept, an actual man with a tailor and a bank-account or a public-house and a drunken wife.

(Russell, 1903, § 56)

In a typical subject-predicate proposition containing a denoting concept, the denoting concept may be a *term* but it, in turn, denotes a further *object*. The word 'object' is used by Russell in the context of this discussion to cover those entities which are denoted by denoting concepts. As such, however, it has an application outreaching that of the word 'term'; for objects, in this case, are the denotations of terms. But this contradicts Russell's earlier claim that 'term' is 'the widest word in the philosophical vocabulary' (Ibid., § 47). Russell himself notes that 'the fact that a word can be framed with a wider meaning than *term* raises grave logical problems' (Ibid., p. 55, fn.). It is not difficult to see why. The doctrine of the unrestricted variable provides philosophical foundations for the logicist project of the *Principles* and, as we have seen, requires that all the ultimate constituents of propositions be of the same ontological category. In other words, it requires that the simple constituents be terms. Yet denoting concepts, at least on the face of it, point to objects which are not terms and, furthermore, are not among the ultimate constituents of the propositions which are about them. The requirement for denoting concepts is the requirement that we have acquaintance with all the constituents of any proposition, yet a consequence of

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<sup>20</sup> Russell admits as much himself, though without being convinced by Frege's position. See Russell, 1903, § 480.

the doctrine is that some logical subjects elude the very propositions which treat them as subjects. In the proposition ‘I met a man’, a particular (though ambiguous) man is the logical subject. But this man does not feature as a constituent of the proposition: rather he is *denoted* by a constituent. Again, it is important to recognise that this denoting relation is not a *referential* relation equivalent to Frege’s relation of *Bedeutung* which obtains between a word and its referent. Denotation, in Russell’s sense, is not a linguistic feature. Denoting concepts are not linguistic and, hence, do not *name* a referent. Rather, it is a purely ‘logical’ relation between those concepts and the objects they pick out.<sup>61</sup>

The ‘grave logical problems’ raised by the attribution of a wider extension to the word ‘object’ than to the word ‘term’ may be partly responded to by noting that Russell’s position does not exclude the possibility of all objects being terms, provided they can be made into logical subjects in propositions other than those containing denoting concepts. For example, the man ambiguously denoted in the proposition ‘I met a man’, may well feature unambiguously as a term in a proposition of the form ‘S is mortal’. Hence it does not seem that Russell’s ontology need extend beyond the extension of the word ‘term’. Nonetheless, the theory of denoting remains problematic on the grounds that it leaves us with propositions whose logical subjects evade membership of the constituency of those propositions.

Much discussion of Russell’s early theory of denoting has centred on the question of non-actual entities. This is unsurprising: recognising a concern in Russell’s later work (particularly in the 1905 theory of descriptions) with ontological pruning of one sort or another, it is usually assumed that the earlier theory of denoting committed Russell to postulating some ontological status for non-actual objects. It is common to read in the voluminous literature on the subject an interpretation of the theory of denoting which places it in much the same camp as the Meinongian theory directly criticised by Russell in ‘On Denoting’.<sup>62</sup>

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<sup>61</sup> See Russell, 1903, § 56.

<sup>62</sup> For a brief review of the foremost contributors to this interpretation, see Griffin, 1996. It should be noted that one of the most prominent of those eager to attribute this view to Russell is Quine, who has a clear and well-known agenda of his own when it comes to ontological pruning. This agenda will be discussed in a later chapter of this thesis.

This theory regards any grammatically correct denoting phrase as standing for an *object*. Thus ‘the present King of France’, ‘the round square’, etc.. are supposed to be genuine objects. It is admitted that such objects do not *subsist*, but nevertheless they are supposed to be objects.

(Russell, 1905a, p. 45)

The attribution of such a theory to the Russell of the *Principles* certainly has much textual support. At several places in the book, Russell explicitly states that all terms have *being*—even those which are non-actual:

Numbers, the Homeric gods, relations, chimeras and four-dimensional spaces all have being, for if they were not entities of a kind, we could make no propositions about them. Thus being is a general attribute of everything, and to mention anything is to show that it is ... *Existence*, on the contrary, is the prerogative of some only amongst beings.

(Russell, 1903, § 427)

Russell goes so far as to assert even that ‘in some sense nothing is something’ (Ibid.. § 73), making plain his intention of granting ‘being’ to whatever one cares to mention. This is entirely consistent with Russell’s claim that ‘every word occurring in a sentence must have *some meaning*’ (Ibid., § 46). For Russell, logical analysis is not an analysis of words, but of the *meanings* of words. As such, he takes meanings to be *entities* of some kind. This is, seemingly, a necessary requirement if those meanings are to be constituents of propositions. Of course, non-existent objects can be constituents in propositions (e.g., ‘The round square does not exist’), so Russell is obliged to grant some status to the non-existent logical subject. Russell exploits the distinction between being and existence in order to give an account of how non-existent objects can be logical subjects of propositions denying their existence without contradiction:

‘For what does not exist must be something, or it would be meaningless to deny its existence; and hence we need the concept of being, as that which

belongs even to the non-existent'.

(*Ibid.*, § 427)

Passages such as these lend powerful support to the Meinongian interpretation of Russell and, as such, provide ample justification for reading 'On Denoting' as an exercise in culling the populous which results from such ontological excess. This interpretation has recently been challenged, however.

Griffin (1996) suggests that the Meinongian interpretation of Russell rests on a misunderstanding of the early theory of denoting.<sup>63</sup> In particular, it fails to recognise that Russell was not obliged to extend his ontology to non-actual objects in the *Principles*. This obligation could, in fact, be avoided through the theory of denoting. As Griffin rightly points out, a closer reading of the *Principles* reveals that Russell was committed only to the existence of denoting concepts, *not* their denotations. This is entirely consistent with Russell's claim that 'every word occurring in a sentence must have *some* meaning' (Russell, 1903, § 46). To see this, we have only to recognise the equivocation in Russell's use of the word 'meaning' in the *Principles*. Russell uses the word in at least two senses: on the one hand there is the 'linguistic' meaning which is involved in the relation a word has to something which it *indicates*, and on the other hand there is the 'logical' kind of meaning, which Russell equates with *denoting*.<sup>64</sup> It is this second, entirely non-linguistic notion of meaning which Russell intends when discussing the relation between a denoting concept and its denotation.

With this distinction in mind we can clearly see that, far from being committed to the existence of non-actual objects in the *Principles*, Russell's early theory of denoting enables him to reconcile his claim that every word has a meaning with the view that not every denoting concept has a denotation. All that Russell is committed to is the weaker claim that every denoting *phrase indicates* a denoting *concept*; there is no reason to assume that every denoting concept in turn denotes an object. Indeed it is precisely this claim which Russell explicitly makes in order to resolve the apparently paradoxical problems raised by the admission that 'in some sense nothing

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<sup>63</sup> Griffin's rejection of the Meinongian reading is shared by Noonan (1996) and Makin (2000).

<sup>64</sup> The unravelling of this equivocation was first performed by Cassin, 1970.

is something', and thereby gives just this picture in relation to the ontological status of the null-class:

We may now reconsider the proposition "nothing is not nothing"—a proposition plainly true, and yet, unless carefully handled, a source of apparently hopeless antinomies. *Nothing* is a denoting concept, which denotes nothing. The concept which denotes is of course not nothing, *i.e.*, it is not denoted by itself. The proposition which looks so paradoxical means no more than this: *Nothing*, the denoting concept, is not nothing, *i.e.*, is not what itself denotes. But it by no means follows from this that there is an actual null-class: only the null class-concept and the null concept of a class are to be admitted.

(Russell, 1903, § 73)

In other words, the theory of denoting presented in the *Principles* allows for the case where a denoting concept fails to denote, *i.e.*, has no denotation. Griffin concludes from this that it must be misguided to interpret the later theory of descriptions as ridding Russell of a 'bloated ontology' (Griffin, 1996, p. 57), for his ontology was not bloated in the first place.

### 1.9 Denoting in 1905

Griffin is right to point out that the traditional interpretation of 'On Denoting' as an exercise in ontological pruning is flawed. Although we will return to the theory of descriptions in the following chapter, it will be instructive to take a brief look ahead at just how these issues bear on the received interpretation of the 1905 theory. An adequate interpretation of 'On Denoting' must reconcile the realisation that Russell was not pruning his ontology of any previously maintained commitment to non-actual objects with Russell's own characterisation of his earlier view as 'the belief that, if a word means something, there must be something that it means' (Russell, 1959, p. 63) and his acknowledgement that it was the 1905 theory which 'showed that this was a mistake and swept away a host of otherwise insoluble problems' (Ibid.). The answer

we shall now explore offers only a partial explanation of Russell's motives but, nevertheless, is one which comports with all of these points. The partial answer is that Russell's development of a new treatment of generality served to make denoting concepts entirely superfluous.

We have seen above that it is unfair to assume that Russell's development of quantification theory was wholly dependent on what he took from Frege.<sup>65</sup> However, it is evident that Russell's approach to general propositions altered significantly after his study of Frege. This new approach is clearly signalled in 'On Denoting'. Whereas the early theory of denoting took denoting concepts to be constituents of the propositions in which they figure, Russell now joins Frege in giving priority to the proposition itself. Furthermore, although Russell gives an informal exposition of the theory, it is plain that he has in mind the symbolic renderings of general propositions that feature in *Principia*:

$C(\text{everything})$  means 'C( $x$ ) is always true';  
 $C(\text{nothing})$  means '“C( $x$ ) is always false” is always true';  
 $C(\text{something})$  means 'It is false that “C( $x$ ) is false” is always true'.  
(Russell, 1905a, p. 42)

Russell's decision to give an informal interpretation of the structure of these propositions was, of course, dictated by the demands of his readership (who would have been largely unfamiliar with the technical symbolism of mathematical logic). Nonetheless, these interpretations clearly show themselves to be interpretations of the, now familiar, forms:

1.  $(x) Cx$
2.  $(x) \sim Cx$
3.  $\sim(x) \sim Cx$

This becomes immediately clear when Russell continues: 'Here the notion of “C( $x$ ) is always true” is taken as ultimate and indefinable, and the others are defined by means

of it' (Ibid.).<sup>65</sup> This is simply the Fregean idea of taking the universal quantifier as primitive and defining the existential quantifier from it, and negation, in order to obtain the square of opposition. Russell does just this in 'The Existential Import of Propositions', also written in 1905.

With this new approach to quantificational phrases, Russell follows Frege in seeing such phrases as significant only within the context of the propositions in which they feature: '*Everything, nothing, and something* are not assumed to have any meaning in isolation, but a meaning is assigned to every proposition in which they occur' (Russell, 1905a, p. 42). Of course Russell goes further than Frege by applying the very same principle to definite descriptions. The tools required for the disposal of the description operator (a form of which had been favoured by both Frege and Peano) are provided by the treatment of quantification outlined above (with the addition of identity). The method by which this disposal of definite descriptions is effected will be returned to in the next chapter.

Let us summarise the argument regarding Russell's treatment of denoting thus far: Russell's 1903 theory of denoting does not entail the overloaded ontology it has often been assumed to be committed to. As a consequence, it is senseless to interpret the 1905 theory as an attempt to rid Russell of ontological commitments he never in fact had. An alternative response to the question of precisely what the motivation was behind the 1905 theory can be offered by seeing the theory as a result of the development of Russell's formal treatment of quantificational propositions. The approach to quantification proffered by 'On Denoting' relieves Russell of the burden of giving an account of denoting concepts as constituents of propositions by shifting the emphasis away from the denoting concept and onto the propositions whose verbal expressions contain denoting phrases. In adopting this approach, Russell follows Frege in viewing quantificational expressions as dependent on their propositional contexts, rather than as expressions for separable entities. It remains to examine how

<sup>65</sup> This interpretation is, in fact, forwarded by Griffin: 'It was not until [Russell] studied Frege, just after completing the *Principles*, that he came upon anything approaching the modern treatment of quantification (Griffin, 1996, p. 33, ft. nt.).

<sup>66</sup> Confirmation that Russell's locution ' $C(x)$  is always true' is to be read as an informal interpretation of ' $(x) \phi x$ ' is to be found in the 1905 manuscript, 'On Fundamentals'. Russell also utilises there the new notation for quantifiers in contextual definitions which do not differ significantly from those found in *Principia* (a point made by Landini, 1998, p. 83).

this interpretation fits with Russell's own remarks about the importance of the 1905 theory of descriptions.

It should be evident how this interpretation comports with Russell's characterisation of his early theory of denoting as 'the belief that, if a word means something, there must be something that it means' (Russell, 1959, p. 63). Russell's early view held that every word in a sentence had some referent. In the case of a denoting phrase, that referent was held to be a denoting *concept* (though, as we have seen, he was not committed to the idea that this referent—the denoting concept—necessarily denotes anything). This fits with our earlier usage of 'indication' and 'denotation' as a way of distinguishing Russell's equivocation over the word 'meaning'. The referent of a denoting phrase is the denoting concept which it indicates; the denoting concept, in turn, may or may not denote an object. In 1905 this model is abandoned completely. The denoting phrase is no longer assumed to indicate anything; sentences in which denoting phrases occur are taken to be significant but, after their correct analysis, will no longer contain any constituent which corresponds to the denoting expression. Hence, on the interpretation considered here, there is no longer any requirement that there must be something which is *meant* by every word in a sentence.

It is important to notice how this supports Russell's infamous 'Gray's Elegy' argument in 'On Denoting'. The 'inextricable tangle' that Russell claims to have pointed out in that argument hinges on his insistence regarding the difficulties we are faced with in attempting to get at the *meaning* of a denoting phrase, rather than its *denotation*. It does not fall within the confines of our discussion to pass judgement on the validity of that argument, as such a judgement would lead us too far afield into well known exegetical skirmishes; but it is clear that the discovery of a technique for disposing of the purported meanings (that is, *indications*) of denoting phrases, provides a neat solution to the problem Russell perceives himself to have unearthed. Having become convinced that denoting concepts (meanings of denoting phrases) cannot themselves be successfully denoted,<sup>67</sup> it seems most plausible that Russell

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<sup>67</sup> This is precisely the point which Russell makes in the following passage: 'The difficulty in speaking of the meaning of a denoting complex may be stated thus: The moment we put the complex in a proposition, the proposition is about the denotation; and if we make a proposition in which the subject is 'the meaning of C', then the subject is the meaning (if any) of the denotation, which was not intended' (Russell, 1905a, p. 49).

would have prized a method for showing that there were no such concepts, rather than engaged in an unnecessary cull of their denotations.

The most important consequence of the new theory of denoting was, of course, Russell's subsequent treatment of classes. Just as contextual definitions could be utilised to dispose of the requirement for denoting concepts, so also could they be applied to classes. In the period immediately following 'On Denoting', Russell developed this 'no-classes' approach to mathematical logic in his (eventually aborted) substitutional theory. Although the substitutional theory was eventually abandoned (for reasons which will greatly concern us in the next chapter), *Principia* retained a 'no-classes' theory which disposed of classes contextually through a direct application of the 1905 theory of descriptions:

The symbols for classes, like those for descriptions, are, in our system, incomplete symbols: their *uses* are defined, but they themselves are not assumed to mean anything at all. That is to say, the uses of such symbols are so defined that, when the *definiens* is substituted for the *definiendum*, there no longer remains any symbol which could be supposed to represent a class. Thus classes, so far as we introduce them, are merely symbolic or linguistic conveniences, not genuine objects as their members are if they are individuals.

(Whitehead & Russell, 1910, p. 75)

In accordance with the theory of descriptions, Russell and Whitehead give the following contextual definition for the use of a class symbol in a straightforward propositional context (*Ibid.*, \*20.01):

$$f\{\hat{z}(\psi z)\} . = : (\exists \phi) : \phi!x . \equiv_x . \psi x : f\{\phi! \hat{z}\} \quad \text{Df}$$

This, of course, is ontological pruning. Russell is rejecting outright his earlier view that every 'class as many' can be counted as a 'class as one'—a view he unquestionably accepted prior to discovery of the Russell paradox in 1901. This may help to explain why so many people have viewed the treatment of denoting concepts in a similar light. But that is to miss the vital difference between a class expression

and a denoting expression: The meaning of a class expression (prior to the ‘no-classes’ theories post-1905) is the class it signifies. The meaning of a denoting expression is a denoting concept which in turn denotes (or does not denote, as we have seen) an object. Therefore, whilst it is true that the application of the theory of descriptions to the theory of classes allowed Russell to abandon any commitment to classes as entities, it does not follow that the theory of descriptions was engineered in order to purge Russell’s ontology of non-actual objects such as ‘the round square’ or ‘the present king of France’. Rather the theory ensures that the denoting concepts previously taken to be constituents of the propositions expressed by sentences containing the denoting expressions will not survive a thorough analysis. The real casualties of the theory of descriptions are not non-actual entities, but the meanings of certain phrases. As Wittgenstein says in the *Tractatus*, the service performed by Russell in 1905 was that of ‘showing that the apparent logical form of a proposition need not be its real one’ (Wittgenstein, 1922, 4.0031). The performance of that service was facilitated by logical, not ontological, developments on Russell’s part.

### 1.10 Conclusion: Frege and Russell on the Unity of the Proposition

Having examined some of the central doctrines of Frege and of Russell at one stage in his development, we are now in a position to draw some conclusions regarding the merits of each approach and the relative successes of the two regarding their analyses of propositional content. Frege and Russell, as the founders of logicism, are often forced together in a way which does not do justice to the subtleties of their thought. The differences which we have seen in the preceding discussion have a profound effect on the direction which one can choose to follow if any of the insights of logicism are to be inherited, as we shall see more clearly in the progression of this thesis.

Frege’s philosophical logic confronts the unity of the proposition as a primary concern from the outset. The three *Grundlagen* principles, as many have observed, play a central role in acknowledging propositional unity, but this should not lead us to conclude that Frege only recognised the importance of the issue in 1884; rather, those principles are to be correctly understood as developments of an awareness which is

present in the *Begriffsschrift* of 1879 and is, as it were, written into the philosophical foundations of Frege's functional analysis of the proposition. This awareness is most perspicuous in the notion of *judgeable content*. The functional analysis of the sentence is grounded on the notion of a truth-value (although, again, this is to be properly developed in later work), hence the sentence takes precedence over its constituents as it is only the asserted sentence which can lead us to a truth-value. Logical relations, for Frege, are inferential relations which can only obtain between truth-evaluable entities. The three *Grundlagen* principles are merely an extension of this basic principle.

Although the three *Grundlagen* principles are interrelated, it is clear that the distinction between concept and object serves most effectively in maintaining the unity of the proposition. It is the incompleteness of concepts which safeguards the unitary status of a thought according to Frege, for 'not all parts of a thought can be incomplete; at least one must be unsaturated or predicative; otherwise they would not hold together' (Frege, 1892b, p. 54).<sup>68</sup> This principle, leading further into the hierarchy of levels for concepts, provides the philosophical foundation for Frege's quantification theory, extending to the higher-order logic required for mathematical analysis. We have seen that, in a revolutionary move, Frege draws this principle, along with the two connected *Grundlagen* principles, from linguistic considerations. Evidently, the distinctions between concepts and objects and concepts of different level amount to distinctions in logical type. However, we should not be hasty in establishing too strong a similarity between Frege's position and Russell's later theory of types. As we shall see in the next chapter, the linguistic considerations behind Frege's distinctions are largely absent from Russell's theory and have more in common with Wittgenstein's views. In particular, these considerations entail an awareness of contextual provisions on the course of logical analysis. The logical type of a propositional constituent is determined by its logical role in the context of a proposition.

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<sup>68</sup> Sainsbury (1996) disputes this, claiming that it is an erroneous interpretation of Frege that sees the notion of unsaturatedness as an explanation, rather than merely a labelling, of the unity of the proposition. In response to this I would say that, strictly speaking, it is neither. Frege's notion of unsaturatedness is intended to reflect the primacy of the judgement; in other words, the incompleteness of concepts is not intended to explain or merely to label the unity of the proposition. It serves to preserve the unitary status of the thought after analysis. This is the feature which we have isolated as lacking in Russell's 1903 theory of the proposition.

Russell's position regarding the unity of the proposition is less convincing in the *Principles*. A major inadequacy in Russell's analysis is his lack of a functional analysis such as Frege's. The function, in Frege's logic, maps objects to truth-values and, as such, has an awareness of propositional unity built into it. Russell had only a partial awareness of this possibility in the *Principles*. More damaging to his account of the proposition, however, is the metaphysical position that Russell occupied in advance of his logical investigations. Having abandoned neo-Hegelianism, Russell was determined to replace it with a realist metaphysics which demanded that there was only one ultimate ontological category. Hence Russell arrived at an atomism of terms, reflected in the doctrine of the unrestricted variable, which analysis should reveal to be the ultimate constituents of the world. This metaphysics demanded of Russell's logic that it treat all terms the same. Importantly, universals must be reducible to terms in just the same way as particulars—everything must be treatable as a logical subject for logic, the most basic of sciences, must encompass everything as subject matter.

To some extent, Russell was fully aware in the *Principles* that, despite the philosophical importance of this doctrine, it had severe limitations when practised as a logical theory. Hence in the important chapter on 'Whole and Part' we find Russell admitting that: 'though analysis gives us the truth, and nothing but the truth, yet it can never give us the whole truth' (Russell, 1903, § 138). Russell's solution is something of a compromise. Recognising the importance of the verb in guaranteeing the unity of the proposition, Russell suggests that an analysis of the proposition into subject and assertion (where the assertion contains the verb) will serve to avoid the falsification threatened by a less cautious analysis. There are still problems with such an analysis, but they no longer seem insurmountable. Recall Russell's remark, quoted above:

This analysis does much less to destroy the proposition. A subject and an assertion, if simply juxtaposed, do not, it is true, constitute a proposition; but as soon as the assertion is actually asserted of the subject, the proposition reappears.

(Ibid., § 81)

Russell, it must be said, is very close to the Fregean analysis here. The assertion is 'everything that remains of the proposition when the subject is omitted' (Ibid.). In other words, an assertion is a propositional function. Propositional functions, as the intensional determinants of classes, are vital to the system of logic outlined in the *Principles* (as they are to Frege's system in the *Grundgesetze*). Yet there is an unsettling lack of clarity in the idea at this stage in Russell's thought. The doctrine of the unrestricted variable forbids logical type distinctions in the *Principles* and, as a result, Russell presents this analysis of the proposition into subject and assertion as something of an *ad hoc* solution to the problem of the unity of the proposition. If there is nothing to stop the predicative part of a proposition being made into a logical subject, there is no immediately apparent reason why an analysis of the proposition should retain the predicative quality of the assertion beyond the requirements of a convenient method for reconstituting the proposition after analysis. Unlike Frege, Russell has given us no convincing argument as to why we should halt our analysis at this point, other than a recognition of the difficulties which will arise if we do not. Clearly, the account is unsatisfactory and Russell himself did not remain content with it. Indeed the unity of the proposition turned out to be a recurring problem for Russell, as we shall see in the next two chapters.<sup>69</sup>

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<sup>69</sup> It is widely assumed that Russell's problems with the unity of the proposition were recurring and insoluble within the confines of his logical theory. See, for example, Griffin, 1985, 1986 & 1993; Candlish, 1996; Sainsbury, 1996, and Palmer, 1988. Perhaps the earliest critic of Russell's position regarding unities was Bradley, who raised objections in a series of articles both to Russell's views in the *Principles* and to the multiple-relation theory of judgement advocated in *Principia*. These articles are reprinted in Bradley 1914, pp. 280-309.

## 2.

## Russell on Types

2.1 Introduction

In the summer of 1902, Russell wrote to Frege and communicated to him the paradox which now bears Russell's name: 'Let  $w$  be the predicate of being a predicate which cannot be predicated of itself. Can  $w$  be predicated of itself? From either answer follows its contradictory' (Russell to Frege, 16/6/1902, in Frege, 1980, p. 130). The devastating consequences of the contradiction were immediately apparent to Frege, who replied: 'Your discovery of the contradiction has surprised me beyond words, and, I should almost like to say, left me thunderstruck, because it has rocked the ground on which I meant to build arithmetic' (22/6/1902, *Ibid.*, p. 132).

In fact, as he pointed out in his reply, Frege's system is not prone to the contradiction in quite the way Russell expresses it above; we have seen in the preceding chapter that Frege's hierarchy of levels will not allow the self-predication of a concept. Nonetheless, the paradoxical function has a more famous extensional counterpart: the class of all classes which are not members of themselves. This, the Russell-class, is adequate for constructing a contradiction unavoidable in the system of Frege's *Grundgesetze*. Axiom V of that work is as follows:

$$(\{x: Fx\} = \{y: Gy\}) \leftrightarrow \forall x (Fx \leftrightarrow Gx)$$

The axiom states that two classes (or 'value-ranges', in Frege's terminology) are identical if, and only if, their defining properties are co-extensive. The basic assumption, and the assumption which invites the formation of the Russell paradox, is one which is also pivotal to Russell's *Principles of Mathematics*; namely, that every

property determines a class. This is simply the comprehension principle of set-theory but, as Russell's paradox demonstrates, the unrestricted comprehension principle of naïve set-theory is intolerably generous, positing classes for any property we care to mention, even those that offer contradictions in return. Frege, we have seen, held that the extension of a concept is an object and, therefore, that there is no reason to deny that a concept could be predicated of its own extension. That is to say, ' $F\{x: Fx\}$ ' is a well-formed formula in Frege's logic. But if we now consider the class whose defining property is that of non-self-membership of classes, we obtain the Russell class of all classes which are not members of themselves. Call this class  $w$ , then:

$$w = \{x: x \notin x\} \quad \text{Df.}$$

If we now seek to apply the defining property of  $w$  to  $w$  itself, we arrive at Russell's paradox. The deductive steps are, indeed, very simple:

1.  $\forall x (x \in w \leftrightarrow x \notin x)$  (by Df of  $w$ )
2.  $w \in w \leftrightarrow w \notin w$  (1, UI)

The intensional version of the paradox, the so-called 'paradox of predication', is equally easy to derive. Recall that the doctrine of the unrestricted variable at the heart of Russell's philosophical logic in the *Principles* forbids the kinds of logical type distinctions forwarded in Frege's hierarchy of levels. On Russell's account, everything must be a potential logical subject, hence self-predicating concepts are not excluded in the way that they are in Frege's system. There is nothing, therefore, to outlaw the following property  $\psi$ :

$$\forall\varphi (\psi\varphi \leftrightarrow \sim\varphi\varphi)$$

Universal instantiation to  $\psi$  allows us, again, to apply this property to itself, yielding the contradictory:

$$\psi\psi \leftrightarrow \sim\psi\psi.$$
<sup>1</sup>

The contradiction left the *Principles* deeply flawed, and Russell ended the book with the admission that:

The totality of all logical objects, or of all propositions, involves, it would seem, a fundamental logical difficulty. What the complete solution of the difficulty may be, I have not succeeded in discovering; but as it affects the very foundations of reasoning, I earnestly commend the study of it to the attention of all students of logic.

(Russell, 1903, § 500)

Russell himself tentatively proposed a doctrine of logical types in an appendix to the *Principles*. This prototypical type-theory falls a long way short of the sophisticated theory at the heart of the 1910 *Principia Mathematica*, however. This is, of course, unsurprising. The very conception of logic given in the *Principles* seems, on a first consideration, to be fundamentally opposed to any theory of types, for the universality of logic as expressed by the doctrine of the unrestricted variable demands that every entity be of the same logical type. Much of the complicated developments in Russell's logic between the *Principles* and *Principia* are attempts, on Russell's part, to reconcile the need for an adequate solution of the contradictions with the requirement that any such solution be philosophically acceptable. In 1903, type-theoretic distinctions may have been successful in meeting the first condition, but Russell was at a loss to see how they could be made to conform to the second requirement without abandoning too much of the philosophical foundations of his logicist project.

Russell's eventual solution to the paradoxes involved not only a ramified version of type-theory, but also utilised the theory of descriptions in establishing a 'no-classes' theory which enabled Russell to relieve himself of the ontological burden

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<sup>1</sup> See Russell, 1903, § 101. In Russell's letter to Jourdain of 15<sup>th</sup> March, 1906 (reprinted in Grattan-Guinness, 1977, pp. 78-80), Russell states that the paradox of predication first emerged in May of 1903 only after he had attempted to reconstruct logic without classes by treating propositional functions as entities. The situation here is confusing and it may be that Russell's memory is inaccurate; as we have already seen above, the contradiction in this form was certainly known to Russell in the summer of 1902 when he communicated it to Frege. For a detailed discussion of the discrepancy see Landini, 1992.

of classes in *Principia*, without forfeiting their logical benefits. The importance of the theory of descriptions as a weapon against the contradiction is evident from Russell's comment in a letter to his first wife that he and Whitehead had 'had a happy hour yesterday, when we thought the present King of France had solved the Contradiction; but it turned out finally that the royal intellect was not quite up to that standard. However, we made a distinct advantage' (Russell to Alys Russell, 14/4/1904, cited in Monk, 1996a, p. 176). We shall see shortly that the main benefit of the theory of descriptions is the role it plays in the development of Russellian type-theory, by helping to build what may otherwise appear to be no more than an *ad hoc* solution to the paradoxes on firm philosophical foundations which do not abandon the central tenets of Russell's original statement of the logicist thesis. Before examining these issues in detail, however, it will be beneficial to state, in outline, the basic ideas behind ramified type-theory. For ease of exposition we shall do so in somewhat un-Russellian terms; we will then be in a position to better appreciate the details of Russell's system and, in particular, to see how it differs from many accepted interpretations.

## 2.2 The Theory of Types—Ramified and Simple

The ramified theory of types is best understood if separated into two separate elements: the type hierarchy of functions (or classes),<sup>2</sup> and the hierarchy of orders. Russell himself, it should be noted, chose not to make this distinction explicit. One of the first to disentangle the theory thus was Ramsey, who also lays claim to making the important distinction between the logical and epistemological (semantic) paradoxes:

These contradictions it was proposed [in *Principia*] to remove by what is called the Theory of Types, which consists really of two distinct parts directed against the two groups of contradictions. These two parts were

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<sup>2</sup> Classes, as mentioned previously, feature in *Principia* only as 'incomplete symbols'. Hence the hierarchy of types applies to functions there. Should one decide to admit classes, however, there is no reason why they should not be type-stratified in a way which is precisely equivalent to the type distinctions placed on their defining functions.

unified by being both deduced in a rather sloppy way from the 'vicious-circle principle', but it seems to me essential to consider them separately.  
 (Ramsey, 1925, p. 24)

Ramsey abstracted the hierarchy of types from the hierarchy of orders and called the abstraction the 'simple theory of types'. As the logical paradoxes do not require a theory of orders for their solution, Ramsey argued (with some influence) that the simple theory of types was all that was required by the mathematical logician who is under no obligation to resolve semantic antinomies. Gödel concurred with Ramsey's segregation of the two hierarchies:

The paradoxes are avoided by the theory of simple types, which in *Principia* is combined with the theory of orders (giving as a result the "ramified hierarchy") but is entirely independent of it and has nothing to do with the vicious circle principle.  
 (Gödel, 1944, pp. 134-135).

The solution of the logical paradoxes within the simple theory of types is effected through the construction of the following hierarchy: At the base level of the hierarchy are 'individuals'; these are of type 0. Next we have functions of individuals (or classes whose only members are individuals), which are of type 1. Functions of functions of individuals (or classes of classes) will be of type 2 and so on. so that, in general, a function of an argument of type  $n$  will be of type  $n+1$ , and a class with members of type  $n$  will be of type  $n+1$ . Furthermore, classes or functions of type  $n$  can only have members or arguments of type  $n-1$ . In this way, self membership and self-predication is blocked in a way which is more or less equivalent to that achieved by Frege's hierarchy of levels. We can see how this stratification blocks the formation of Russell's paradox in both its extensional and intensional forms by showing the logical types of functions or classes through superscript type-indices. The class-abstract for the paradoxical Russell-class will now be written:

$$w^{n+1} = \{x^n : x^n \notin x^n\}$$

which is a clear violation of logical type distinctions. Likewise, the paradoxical property of non-self-predication cannot be formed, for we would now have:

$$\forall \varphi^n (\psi^{n+1} \varphi^n \leftrightarrow \sim \varphi^n \varphi^n)$$

where the right-hand side of the biconditional fails to conform with the requirements of type-theory.

By restricting the arguments any given function may take, or placing equivalent restrictions on the conditions for class-membership, the simple theory of types blocks all of the logical paradoxes which threaten predicate logic and set-theory. The semantic paradoxes, however, are not so easily dismissed. Arguably the most pernicious of the semantic paradoxes is the ancient antinomy of the Liar, a form of which is the 'Epimenides' which arose when the Cretan Epimenides asserted that all statements made by Cretans are false. If we make the added assumption that all Cretans other than Epimenides are indeed liars, then we reach the paradoxical situation where Epimenides's own assertion can be true if, and only if, it is false.<sup>3</sup> The following demonstration shows how the Epimenides fares in a system safeguarded by simple type-theory:<sup>4</sup>

Individual variables designate propositions and  $\varphi$  means the property of being asserted by a Cretan. Superscripts are logical type indices.

1.  $\exists p^n \{ [p^n = \forall q^n (\varphi^{n+1} q^n \rightarrow \sim q^n)] \& \varphi^{n+1} p^n \} \& \forall q^n \{ [\varphi^{n+1} q^n \& \sim (q^n = p^n)] \rightarrow \sim q^n \}$
2.  $\exists p^n \{ [p^n = \forall q^n (\varphi^{n+1} q^n \rightarrow \sim q^n)] \& \varphi^{n+1} p^n \}$  (1, simp)
3.  $[p^n = \forall q^n (\varphi^{n+1} q^n \rightarrow \sim q^n)] \& \varphi^{n+1} p^n$  (2, EI)
4. Assume  $p^n$ , then
5.  $\forall q^n (\varphi^{n+1} q^n \rightarrow \sim q^n)$
6.  $\varphi^{n+1} p^n \rightarrow \sim p^n$  (5, UI)
7.  $\varphi^{n+1} p^n$  (3, simp)

<sup>3</sup> It is the requirement of this extra condition which separates the Epimenides from the original Liar paradox which is attributed to Eubulides, and takes the simpler form of a man saying 'What I am saying is false'. Countless variants circulate, but we shall focus on the Epimenides as it was particularly fascinating to Russell.

<sup>4</sup> For an alternative formulation of the Epimenides in simple type-theory, see Myhill, 1979, pp. 83-84.

8.  $\sim p^n$  (6, 7, mp)

9.  $\_\_4 \rightarrow F$  (CP)

10. Assume  $\sim p^n$  (9, reductio)

11.  $\exists q^n (\varphi^{n+1} q^n \& q^n)$

12.  $\varphi^{n+1} r^n \& r^n$  (11, EI)

13.  $\forall q^n \{[\varphi^{n+1} q^n \& \sim (q^n = p^n)] \rightarrow \sim q^n\}$  (1, simp)

14.  $[\varphi^{n+1} r^n \& \sim (r^n = p^n)] \rightarrow \sim r^n$  (13, UI)

15.  $r^n$  (12, simp)

16.  $\sim \sim r^n$  (15, dn)

17.  $\sim [\varphi^{n+1} r^n \& \sim (r^n = p^n)]$  (14, 16, mt)

18.  $\varphi^{n+1} r^n \rightarrow (r^n = p^n)$  (17, imp)

19.  $\varphi^{n+1} r^n$  (12, simp)

20.  $r^n = p^n$  (18, 19, mp)

21.  $p^n$  (15, 20)

22.  $10 \rightarrow F$

The contradiction goes through without violating type restrictions, so the simple theory of types is clearly ineffective against Epimenides.

The solution offered by the ramified theory of types lies in the realisation that the Epimenides, in common with the other semantic paradoxes, results from a certain kind of vicious circle or reflexiveness:

The vicious circles in question arise from supposing that a collection of objects may contain members which can only be defined by means of the collection as a whole. Thus, for example, the collection of *propositions* will be supposed to contain a proposition stating that “all propositions are either true or false”. It would seem, however, that such a statement could not be legitimate unless “all propositions” referred to some already definite collection, which it cannot do if new propositions are created by statements about “all propositions.” We shall, therefore, have to say that statements about “all propositions” are meaningless.

(Whitehead & Russell, 1910, p. 39)

We can easily see from this how the diagnosis of the Epimenides contradiction will be given. “All propositions asserted by Cretans are false”, if taken to be a member of the collection of propositions asserted by Cretans, would be defined in terms of its own totality; but this could not be legitimate unless “all propositions asserted by Cretans are false” already referred to a definite collection, which it cannot do if the new proposition itself adds to the totality. So the conclusion Russell arrived at was that, for any given set of objects for which the assumption of a totality of its members must include in that total some members which themselves presuppose the total, that set cannot have a total—i.e., ‘no significant statement can be made about “all its members”’ (Ibid.). Such a set (e.g., the set of all propositions, the set of all propositional functions, etc.), he came to believe, must be broken up into smaller sets of differing *order*, each of which may legitimately have a total. Illegitimate totalities are then forbidden by the *vicious-circle principle*: ‘Whatever involves *all* of a collection must not be one of the collection’ (Ibid. p. 40).

This division of sets into smaller sets can be best explained by examining the hierarchy of orders for propositional functions. The ramification does not apply to individuals but will begin with functions of individuals which are not definable by reference to any totality of functions. Such functions are called *first-order* functions. Next we have *second-order* functions: functions which are definable by reference to the totality of first-order functions; *third-order* functions are definable by reference to the totality of second-order functions. This hierarchy continues indefinitely with the functions of order  $n+1$  being definable by reference to the totality of  $n^{\text{th}}$ -order functions, without being included in the lower-order totality. Hence, for example, a second-order function can make an assertion about “all first-order functions” because it is not included in that totality itself.

Whereas the hierarchy of types requires that a function of type  $n$  must have an argument of precisely type  $n-1$ , the hierarchy of orders requires merely that a function of which the argument of highest order is of order  $n$  must be of *at least* order  $n+1$ . A function is called a *predicative* function if, and only if, it is of the lowest order compatible with the highest order function contained in it. For example, a second-order function is predicative if the highest order argument contained in it is a variable of the first-order.

If we apply order-indices to our earlier version of the Epimenides paradox, we can see that the self-referential feature of the paradoxical assertion is now neutralised by the ramified theory. Our original statement:

$$\exists p^n \{ [p^n = \forall q^n (\varphi^{n+1} q^n \rightarrow \sim q^n)] \& \varphi^{n+1} p^n \} \& \forall q^n \{ [\varphi^{n+1} q^n \& \sim (q^n = p^n)] \rightarrow \sim q^n \}$$

will now have, in addition to the superscript type indices, order indices which we will express through subscripts:<sup>5</sup>

$$\exists p_{k+1}^n \{ [p_{k+1}^n = \forall q_k^n (\varphi^{n+1} q_k^n \rightarrow \sim q_k^n)] \& \varphi^{n+1} p_{k+1}^n \} \\ \& \forall q_k^n \{ [\varphi^{n+1} q_k^n \& \sim (q_k^n = p_{k+1}^n)] \rightarrow \sim q_k^n \}$$

As  $p$  is identical with  $\forall q (\varphi q \rightarrow \sim q)$ , it follows that  $p$  must be of higher order than  $q$  in order to avoid a vicious-circle fallacy, for, if  $p$  and  $q$  were of the same order,  $p$  would be an admissible value of the variable  $q$  quantified over in  $p$ , hence violating the vicious-circle principle. As a result, Epimenides's assertion that all Cretans are liars, when brought in line with the theory of orders, becomes a harmless  $k+1^{\text{th}}$ -order proposition stating that all  $k^{\text{th}}$ -order propositions asserted by Cretans are false.<sup>6</sup>

A problem now arises in that the hierarchy outlined above makes it illegitimate to talk of, for example, “all  $a$ -functions” (all functions satisfied by the argument  $a$ ). We can legitimately talk about “all first-order properties of  $a$ ” or “all second-order properties of  $a$ ”, but not “all properties of  $a$ ”. The consequences of this are somewhat played down in *Principia*, but made more explicit in Russell's earlier paper “Mathematical Logic as Based on the Theory of Types”:

It is absolutely necessary, if mathematics is to be possible, that we should have some method of making statements which will usually be equivalent to what we have in mind when we (inaccurately) speak of ‘all properties of  $x$ .’ This necessity appears in many cases, but especially in connexion with mathematical induction. We can say, by the use of *any* instead of *all*.

<sup>5</sup> For the moment, we will postpone discussion of how closely related types and orders are in *Principia*, and follow Gödel and Ramsey in treating orders as wholly independent of types.

<sup>6</sup> See Russell, 1908, p. 79; and Whitehead & Russell, 1910, p. 65.

‘Any property possessed by 0, and by the successors of all numbers possessing it, is possessed by all finite numbers’. But we cannot go on to: ‘A finite number is one which possesses *all* properties possessed by 0 and by the successors of all numbers possessing them’.

(Russell, 1908, p. 80)

The problem arises for the following reason. If we take the principle of mathematical induction, i.e., that “any function satisfied by 0, and satisfied by  $x + 1$  if satisfied by  $x$ , is satisfied by all natural numbers”, to be confined to functions of a restricted order of natural numbers, we cannot legitimately infer that it holds of functions of *every* order. If we phrase the principle of mathematical induction thus:

$$(\forall F) \{ \{F(0) \& (\forall x) [Nx \rightarrow (Fx \rightarrow Fx + 1)]\} \rightarrow (\forall x) (Nx \rightarrow Fx) \}$$

we can see that the culprit here is the quantifier which must range over all orders of functions  $F$ —a clear violation of the vicious-circle principle. The intolerable result, when brought within the structure of the ramified theory, is that: ‘we shall be unable to prove that if  $m, n$  are finite numbers, then  $m + n$  is a finite number ... It is obvious that such a state of things renders much of elementary mathematics impossible’ (Ibid., pp. 80-81). The same problem arises with Leibniz’s law of the identity of indiscernibles, which again requires quantification over *all* functions of identical individuals:

$$(\forall x) (\forall y) (x = y) \rightarrow (\forall F) (Fx \leftrightarrow Fy)$$

Clearly, a system which seeks to provide a basis for all mathematical reasoning cannot do without both identity and mathematical induction.

A further unacceptable, and much cited, casualty of ramified type theory is the theorem of the Least Upper Bound, which states that for any bounded collection of real numbers, there is a number which is the Least Upper Bound of the collection. According to the ramified theory of types, however, the Least Upper Bound of a

collection of real numbers must be of a higher order than the collection of numbers whose Least Upper Bound it is.<sup>7</sup>

The only solution, Russell thought, was to find some way around the problem by relaxing the vicious-circle principle enough to facilitate the essential mathematical and logical reasoning currently prohibited, without opening the floodgates to the contradictions excluded by the ramified theory. The answer, he decided, was his axiom of reducibility, which states that for any function  $\phi\hat{x}$ , there is a formally equivalent predicative function which is true when  $\phi\hat{x}$  is true and false when  $\phi\hat{x}$  is false:<sup>8</sup>

$$(\exists \psi) (\phi\hat{x} \equiv \psi ! x)$$

Likewise for two-place functions:

$$(\exists \psi) \phi(x, y) \equiv \psi ! (x, y)$$

and so on.

The axiom of reducibility is the most controversial component in a controversial theory. Russell himself said of it:

That the axiom of reducibility is self-evident is a proposition which can hardly be maintained. But in fact self-evidence is never more than a part of the reason for accepting an axiom, and is never indispensable. The reason for accepting an axiom, as for accepting any other proposition, is always largely inductive, namely that many propositions which are nearly indubitable can be deduced from it, and that no equally plausible way is known by which these propositions could be true if the axiom were false.

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<sup>7</sup> See Copi, 1971, pp. 92-93; and Quine, 1963, pp. 249-250.

and nothing which is probably false can be deduced from it.

(Whitehead & Russell, 1910, p. 62)<sup>9</sup>

A suspicion lingered in the minds of some that this last sentence of Russell's was overly generous, on the grounds that the ramified theory of types, taken in conjunction with the axiom of reducibility, makes the theory effectively indistinguishable from the simple theory of types of the *Principles*, hence allowing the semantic paradoxes, if not the logical ones, to re-emerge.<sup>10</sup> This criticism was soundly refuted by Myhill (1979), however.<sup>11</sup>

The most forceful objection to the axiom of reducibility came from Wittgenstein:

Propositions like Russell's 'axiom of reducibility' are not logical propositions, and this explains our feeling that, even if they were true, their truth could only be the result of a fortunate accident.

It is possible to imagine a world in which the axiom of reducibility is not valid. It is clear, however, that logic has nothing to do with the question whether our world really is like that or not.

(Wittgenstein, 1922, 6.1232-6.1233)<sup>12</sup>

In the introduction to the second edition of *Principia*, under the influence of Wittgenstein, Russell admitted that 'one point in regard to which improvement is

<sup>3</sup> Russell places a circumflex over the variable to denote a propositional function as opposed to a proposition. The function ' $\phi\hat{x}$ ' ambiguously denotes its values; ' $\phi\hat{x}$ ' is one of the ambiguously denoted values (See Whitehead & Russell, 1910, p. 42).

<sup>9</sup> An interesting early remark on the notion of reducibility appears in Russell's 1906 manuscript 'The Paradox of the Liar'. Acknowledging the problem posed for mathematical induction by the hierarchy of orders, he remarks that a theory of types must be constructed which yields reducibility, then continues: 'Such a theory of types should, if possible, be constructed so that this could be proved without an axiom *ad hoc*; but if any very self-evident axiom *ad hoc* could be discovered, & no way could be found of getting on without it, it might be well to admit some such axiom' (Russell, 1906e, p. 9).

<sup>10</sup> Both the logical and the semantic paradoxes are to be further distinguished from the pseudo-paradoxes such as the barber paradox (the barber who shaves all, and only, those who do not shave themselves), which is resolved by the simple recognition that no such barber can exist.

<sup>11</sup> See also Linsky, 1999, Ch. 6.

<sup>12</sup> This view was shared by Ramsey who rejected the ramified theory of types as a result: 'Such an axiom has no place in mathematics, and anything which cannot be proved without using it cannot be regarded as proved at all' (Ramsey, 1925, p. 28).

obviously desirable is the axiom of reducibility. This axiom has a purely pragmatic justification: it leads to the desired results, and to no others. But clearly it is not the sort of axiom with which we can rest content' (Whitehead & Russell, 1925, p. xiv). He proceeded to sketch a version of the ramified theory without the axiom, attempting to retrieve a proof of mathematical induction, but met with only limited success.<sup>13</sup>

This completes our overview of ramified type-theory. The differences between this theory and the theory of logic located in the *Principles* which we examined in the previous chapter should be evident. How is Russell's insistence on the universality of logic and the central importance of the doctrine of the unrestricted variable to be reconciled with the type-stratified formal grammar of *Principia*? Has Russell simply abandoned the philosophical foundations of his logicist project in favour of an *ad hoc* solution to the paradoxes? To answer these questions fairly we must return the theory of types to its proper historical context.

### 2.3 The Theory of Descriptions

The publication of Russell's 1905 paper 'On Denoting' is justifiably regarded as a defining moment in the analytic tradition. As such, the primary purpose of the theory—to act as a weapon against the paradox—is easily overlooked.<sup>14</sup> The prominence that the theory is given in *Principia*, however, betrays Russell's original motivations. Much later on, he referred to the theory of descriptions as 'the first step towards overcoming the difficulties which had baffled me for so long' (Russell, 1967, p. 152).<sup>15</sup>

<sup>13</sup> See Appendix B of Russell & Whitehead, 1925. Gödel found a defect in the proof at \*89.16 (see Gödel, 1944, pp. 145-146) which is used as an important lemma in the appendix. The extent of Russell's successes in this project are debated by Myhill (1974), Cocchiarella (1989), and Landini (1996a).

<sup>14</sup> This aspect of the theory of descriptions is overlooked by recent commentators such as Noonan (1996) and Makin (2000), for example. It is recognised by Rodriguez-Consuegra (1993) and given a central position in Landini's (1996d & 1998) exegesis. Important biographical support is offered in Monk (1996a). Certain remarks of Russell's provide strong evidence in favour of this interpretation, as we shall see in what follows.

<sup>15</sup> Russell goes on to remark that: 'In 1906 I discovered the Theory of Types. After this it only remained to write the book out' (Russell, 1967, p. 152). For a detailed account of the true complexities involved in Russell's movement from the theory of descriptions to the final version of type theory, see Landini, 1998 & Monk, 1996a, esp. chs 5 & 6.

In 'On Denoting' Russell states that his earlier theory of denoting, forwarded in the *Principles*, is 'very nearly the same' as Frege's distinction between *Sinn* and *Bedeutung* (Russell, 1905a, p. 42 fn.).<sup>16</sup> Definite descriptions like 'the present King of France' or 'the round square' would be dealt with on the Fregean model by admitting that such phrases have a sense (*Sinn*) but no reference (*Bedeutung*). Russell's new approach went further, denying that the denoting phrases in isolation had any meaning at all. Making a radical departure from his earlier insistence that 'every word occurring in a sentence must have *some* meaning' (1903, § 46), Russell now defined denoting phrases as *incomplete symbols* which attain their meaning only in use.

Russell's procedure for analysing definite descriptions can be seen by examining his example of 'the present King of France' in the propositions (1) 'The present King of France is bald'; and (2) 'The present King of France is not bald'. (1) is clearly false; according to the law of excluded middle, it should follow that (2) is true: 'Yet if we enumerate the things that are bald, and then the things that are not bald, we should not find the present King of France in either list. Hegelians, who love a synthesis, will probably conclude that he wears a wig' (Russell, 1905a, p. 48). Russell's solution is to break each proposition down into three constituent propositions. Taking (1) as our example, we discover after the Russellian analysis the three propositions:

- (i) There is at least one person who is presently King of France.
- (ii) There is at most one person who is presently King of France.
- (iii) That person is bald.

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<sup>16</sup> It is interesting to note, therefore, that in his discussion of Frege in Appendix A of the *Principles*, Russell does *not* translate *Bedeutung* as "denotation" on the grounds that: 'this word has a technical meaning different from Frege's, and also because *bedeuten*, for him, is not quite the same as *denoting* for me' (Russell, 1903, p. 502 fn.). I believe that this comment of Russell's lends support to the interpretation of the 1903 theory of denoting advocated in the preceding chapter. The similarity between Russell's early theory and Frege's distinction between sense and reference lies in Russell's (1903) claim that every word has a meaning, but not every word has a denotation. In other words, as explained in the last chapter, every denoting phrase *indicates* a denoting concept, but not every denoting concept *denotes* an entity. Nonetheless, Russell's decision to acknowledge the difference between his *denotation* and Frege's *Bedeutung* is well-measured—*Bedeutung*, for Frege, is a relation which obtains between a word and an entity; Russell's *denotation*, by contrast obtains between a non-linguistic *concept* and an entity.

The first proposition is straightforward, being simply of the form:  $\lceil \exists x (\phi x) \rceil$ . The second proposition provides the uniqueness implied by the definite article:  $\lceil (\phi x . \phi y) \supset_y (x = y) \rceil$ . The third proposition, of course, is not problematic once the existence or otherwise of its grammatical subject has been specified, having the simple form:  $\lceil fx \rceil$ .

The original proposition (1) can now be rephrased as a conjunction:

$$(1') \quad (\exists x) \{(\phi x) . [(\phi x . \phi y) \supset_y (x = y)] . fx\}$$

The same procedure is applied to (2):

$$(2') \quad (\exists x) \{(\phi x) . [(\phi x . \phi y) \supset_y (x = y)] . \sim fx\}$$

So long as the first conjunct,  $'(\exists x) (\phi x)'$ , is false, both conjunctions in which it features are false without any threat to the law of excluded middle. Alternatively, (2) could be analysed as:

$$(2'') \quad \sim (\exists x) \{(\phi x) . [(\phi x . \phi y) \supset_y (x = y)] . fx\}$$

yielding a true proposition. This reflects the ambiguity of the proposition 'The present King of France is not bald' which, whilst naturally interpreted to be a false statement (namely that 'the present King of France exists and is not bald'), could also be interpreted as a true statement ('it is not the case that the present King of France exists and is bald'). These two possible interpretations display what Russell calls the *primary* (as in 2') or *secondary* (as in 2'') occurrence of the denoting phrase; whenever the subject of the definite description under scrutiny does not exist, a primary occurrence of it leads to a false proposition, and a secondary occurrence leads to a true proposition. For propositions in which the subject of the definite description does exist, primary and secondary occurrences lead to equivalent results. Either way, a truth-value can be safely established (C.f., Russell, 1905a, pp. 52-53; and Whitehead & Russell, 1910, p. 72).

A further requirement placed on a theory of denotation is that it should provide an account of *informative* identity statements. Frege had noticed that 'the evening star

is the morning star' can be informative, in a way which 'Venus is Venus' clearly is not. Hence, for Frege, 'the morning star' differs from 'the evening star' in sense, but not in reference. To tell someone that the two are identical is to inform them that two signs of which they know the sense, share the same reference. Russell took as his example, the phrase 'Scott is the author of *Waverley*', recalling an inquiry made by George IV:

Now George IV wished to know whether Scott was the author of *Waverley*; and in fact Scott *was* the author of *Waverley*. Hence we may substitute *Scott* for *the author of Waverley*, and thereby prove that George IV wished to know whether Scott was Scott. Yet an interest in the law of identity can hardly be attributed to the first gentleman of Europe.

(Russell, 1905a, pp. 47-48)

Russell and Whitehead adopt Peano's description operator ' $\iota x$ ' to express the definite article. Thus ' $(\iota x)(\phi x)$ ' is to be read: 'the  $x$  which satisfies  $\phi x$ ' or 'the  $x$  with the property  $\phi$ '. 'The present King of France is bald' may be written: ' $f[(\iota x)(\phi x)]$ ', for example. Enlisting this notation, we can see the problem posed by 'Scott is the author of *Waverley*' by observing that:

$$a = (\iota x)(\phi x)$$

is clearly not the same proposition as:

$$a = a.$$

Russell's analysis of definite descriptions used in identity statements follows the same pattern we have already witnessed. After analysis, ' $a = (\iota x)(\phi x)$ ' will be:

$$(\exists x) \{(\phi x) . [(\phi x . \phi y) \supset_y (x = y)] . (x = a)\}$$

or its more concise equivalent:

$$(\exists y) \{[\phi x \equiv_x (x = y)] \cdot (y = a)\}$$

Notice that the denoting phrase, '( $\iota x$ )( $\phi x$ )', is once again absent from the complete analysis. The conclusion that Russell draws is that '( $\iota x$ )( $\phi x$ )' is an incomplete symbol. It has no meaning in isolation but only in use; it *contributes* to the overall meaning of a proposition in which it figures. Hence, Russell does not define '( $\iota x$ )( $\phi x$ )' in itself, but rather defines the *use* it has in propositions such as the following:<sup>17</sup>

$$E ! (\iota x)(\phi x)$$

the definition of which will be

$$E ! (\iota x)(\phi x) = (\exists y) \{[\phi x \equiv_x (x = y)] \quad \text{Df.}$$

where the original proposition asserts the existence of the denoted object (as in 'The golden mountain exists').<sup>18</sup>

Russell expresses his conclusion by saying that ' $a = (\iota x)(\phi x)$  is not a value of the propositional function  $a = y$ , from which it follows that  $(\iota x)(\phi x)$  is not a value of  $y$ . But since  $y$  may be anything, it follows that  $(\iota x)(\phi x)$  is nothing. Hence, since in use it has a meaning, it must be an incomplete symbol' (Whitehead & Russell, 1910, p. 70). Denoting phrases, according to the theory of descriptions, turn out not to mean anything at all. Let us now examine how this conclusion would benefit Russell in his treatment of the paradox:

What was of importance in this theory was the discovery that, in analysing a significant sentence, one must not assume that each separate word or phrase has significance on its own account. "The golden mountain" can be part of a significant sentence, but is not significant in isolation. It soon appeared that class-symbols could be treated like descriptions, i.e., as non-

<sup>17</sup> See Whitehead & Russell, 1910, pp. 69-70.

<sup>18</sup> See Whitehead & Russell, 1910, pp. 31-32, 69-75, & \*14.

significant parts of significant sentences. This made it possible to see, in a general way, how a solution of the contradictions might be possible.  
(Russell, 1944, pp. 13-14)

We saw, in the last chapter, one sense in which the theory of descriptions is applied to classes in *Principia* through the use of contextual definitions such as \*20.01:

$$f\{\hat{z}(\psi z)\} . = : (\exists \phi) : \phi!x . \equiv_x . \psi x : f\{\phi! \hat{z}\} \quad \text{Dt}^{19}$$

Yet we should not simply assume that this is the application of the theory of descriptions to classes which Russell is referring to as the one which revealed the eventual solution of the paradoxes. Indeed, there is no immediately obvious reason why one *must* abstain from assuming the existence of classes as independent entities within a system which is adequately protected against paradoxical classes such as the Russell class. The simple theory of types is easily (and naturally) extended to classes, after all. To fully appreciate the importance of the theory of descriptions within the context of the philosophical foundations of *Principia*'s logic, we must examine Russell's immediate attempts to apply the insights heralded in 'On Denoting' to the paradoxes in 1905. The missing link in the development of Russell's thought which connects the unrestricted logic of the *Principles* with the ramified theory of types in *Principia* is Russell's substitutional, or 'no-classes', theory developed between 1905 and 1907.<sup>20</sup>

## 2.4 The Substitutional Theory of Classes and Relations

Russell's substitutional theory was largely unknown (or at least misunderstood) for many years. Two important papers, both written in 1906, were not published in

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<sup>19</sup> The sign, ' $\hat{z}(\phi z)$ ', is Russell's notation for the class-abstract sign, ' $\{z: \phi z\}$ '.

<sup>20</sup> The substitutional theory was, of course, just one of several 'no-classes' theories considered by Russell, the most famous being that of *Principia* itself. We shall see, however, that the theory of *Principia* has its roots in the substitutional theory.

English until 1973.<sup>21</sup> The importance of the theory is, therefore, easily overlooked. Russell's unpublished manuscripts from this period, however, reveal him working incessantly on the theory from the discovery of the theory of descriptions in 1905, through to the seminal 'Mathematical Logic as Based on the Theory of Types' of 1908 which retains a version of the substitutional theory.<sup>22</sup>

There can be no doubt that Russell's sole intent in developing the substitutional theory was to find a method of retaining the logicism of the *Principles* without admitting the paradoxes which had plagued him, and all practitioners of mathematical logic, since the publication of the book. Central to the achievement of this goal was the non-assumption of classes as entities. Russell admitted that this interpretation of propositions concerning classes might be better effected by a different approach: 'The method of substitution, by which I have proposed to effect this interpretation, is more or less in the nature of a technical device. to be replaced by a more convenient device if one should be discovered' (Russell, 1906b, p. 200). We should be careful how we interpret this remark, however. Russell is not suggesting that the substitutional theory is merely an *ad hoc* technical means of disposing of the kinds of propositions which lead to contradictions. On the contrary, what commends the method of substitution is its ability to provide a genuine solution to the paradoxes in a way which is philosophically justified. Indeed, it is for precisely this reason that Russell was not content to avoid the paradoxes by simply brandishing the vicious-circle principle as a grammatical proscription. The vicious-circle principle is, rather, a condition which must be met by any philosophically adequate solution to the paradoxes: 'the vicious-circle principle is not itself the solution of vicious-circle paradoxes, but merely the result which a theory must yield if it is to afford a solution

<sup>21</sup> Namely 1906b & 1906c. In 1906a, Russell had tentatively suggested the substitutional theory as one possible method of solving the paradoxes, but had not lent it his complete support. However, on publication of the paper, he had added the note: 'From further investigation I now feel hardly any doubt that the no-classes theory affords the complete solution of all the difficulties stated in the first section of this paper' (Russell, 1906a, p. 164). For varying (and, sometimes, conflicting) commentaries on Russell's substitutional theory, see Lackey, 1976; Hylton, 1980; Cocchiarella, 1980; and Landini, 1989 & 1998.

<sup>22</sup> This paper is often assumed to be forwarding the same theory of types as features in *Principia*. In fact, there is a major difference; the 1908 version of the theory retains an ontology of propositions which are divided into a hierarchy of orders (the ramification of the theory). The *type*-hierarchy of functions, however, is presented as a technical convenience which could, in principle, be replaced by the method of substitution. I take this to mean that Russell, at this time, held the substitutional approach to be the philosophically correct one (see Russell, 1908, p. 77). The crucially important presence of propositions as entities in 'Mathematical Logic' appears to have gone largely unnoticed until it was pointed out by Cocchiarella (1980).

of them' Russell, 1906c, p. 205).<sup>23</sup> Russell is merely acknowledging that, if another method can be found which is technically superior and retains the philosophical insights of the substitutional theory (which are, in effect, developed out of the theory of descriptions), then pragmatism will, of course, recommend the more convenient alternative.

The most important consequence of the substitutional theory is that it disposes of the paradoxes without recourse to artificial restrictions on variation. In other words, it remains consistent with the doctrine of the unrestricted variable. This has obvious implications for our understanding of the development of the theory of types.<sup>24</sup> It also has implications, as we shall see, for the problem of the unity of the proposition. Russell first publicly advocated the substitutional theory in 1906, in a paper read before the London Mathematical Society.<sup>25</sup> The paper concludes with the following evaluation:

Of the philosophical consequences of the theory I will say nothing, beyond pointing out that it affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many; for, while allowing that there are many entities, it adheres with drastic pedantry to the old maxim that, 'whatever is, is one'.

(Russell, 1906b, p. 189)

A class has no existence 'as one', because it is not an entity. As Russell and Whitehead would later put it, 'classes, so far as we introduce them, are merely symbolic or linguistic conveniences, not genuine objects as their members are if they are individuals' (Whitehead & Russell, 1910, p. 75). We have already noted how, in *Principia*, classes are removed through contextual definitions in accordance with the

<sup>23</sup> Similar remarks occur throughout Russell's manuscripts of this period. For example, in 'The Paradox of the Liar', Russell remarks that the Liar paradox shows the impossibility of a proposition about a set of propositions being a member of that set: 'This impossibility cannot, however, be simply decreed because of the paradox; we must find some reason in the nature of the propositions which shows that the impossibility subsists' (Russell, 1906e, p. 2). He goes on to express reservations about blocking the paradox through either rejecting propositions as entities or introducing orders of propositions because 'it is difficult to express either in a form with anything to command it except the solution of paradoxes' (Ibid., p. 7).

<sup>24</sup> These implications are examined in detail in Landini, 1993 & 1998.

<sup>25</sup> Russell, 1906b. Russell subsequently withheld the paper from publication, having developed the theory further.

theory of descriptions. To see how this disposal of classes as entities is effected in the substitutional theory, we must examine the basic grammar of substitution.

There are no distinctions in types of entities in the substitutional theory. In fact there is only one kind of entity supposed by the theory, hence there is only one type of variable: the 'entity' or 'individual' variable. The doctrine of the unrestricted variable is clearly, therefore, retained 'with drastic pedantry'. An ontology of propositions is required by the theory but propositions stand on an equal ontological footing to their constituents. There are no propositional functions in the theory but, in their place, we find *matrices* of the form ' $p/a$ '. Here  $p$  is called the *prototype* and  $a$  is called the *argument*. The argument is a constituent of the prototype. The heart of the system is the operation of substitution, whereby an entity is substituted for the argument in a matrix; for example, the substitution of  $x$  for  $a$  in  $p$ , which is written:

$$p \frac{x}{a}$$

For the sake of notational convenience, Russell often expresses this as ' $p/a|x$ '.<sup>26</sup> This is to be read as 'the  $q$  which results from the substitution of  $x$  for  $a$  in  $p$ '. The formula,

$$p \frac{x}{a}!q$$

(which can also be written as ' $p/a|x!q$ ') is to be read as, ' $q$  results from  $p$  by substituting  $x$  for  $a$  in all those places (if any) where  $a$  occurs in  $p$ ' (Russell, 1906b, p. 168). Strictly speaking, ' $p/a|x$ ' is a definite description and, in accordance with the theory of descriptions, is therefore an incomplete symbol which is to be defined contextually. That is to say, having defined ' $p/a|x$ ' as follows,

$$p/a|x = (\iota q) (p/a|x!q) \quad \text{Df.}^{27}$$

<sup>26</sup> On occasions Russell uses the alternative notation of ' $p(x|u)$ ' which means the same as ' $p/a|x$ ' (see Russell, 1906a, p. 155).

<sup>27</sup> See Russell, 1905b, p. 4 (\*12.12).

we need a contextual definition of ' $(\iota q) (p/a'x!q)$ '. Russell gives us a technique for doing so which is as follows:

Thus ' $p/a'b$  has the property  $\phi$ ' is to mean: 'There is a  $q$  such that  $p \frac{b}{a}!r$  is true when and only when  $r$  is identical with  $q$ , and  $q$  has the property  $\phi$ '.  
(Russell, 1906c, p. 201)

Following Russell's practice, we can define ' $(\iota q) (p/a'x!q)$ ' in a propositional context:

$$\phi\{(\iota q) (p/a'x!q)\} = (\exists q) [(p \frac{x}{a}!r \equiv_r r = q) \cdot \phi(q)] \quad \text{Df.}^{28}$$

In this manner, Russell ensures that there is only one type of entity involved in substitution. The variable can, therefore, be given an unrestricted range; anything which can be a logical subject, can be a value of a free variable in the context of the substitutional theory. Furthermore, the above application of the theory of descriptions ensures that the limitations placed on the range of an apparent (i.e., bound) variable are not dictated by type-distinctions as such, but merely by the constraints of meaningfulness. The significance of a proposition containing a denoting phrase is directly attached to the existence or non-existence of the denoted subject.

As stated above, there are no propositional functions as such in the theory, but equivalents of higher order functions can be derived by increasing the complexity of the matrices involved. Hence, in place of a function of a function of individuals, we perform 'dual substitutions' of complex arguments in a prototype, as in matrices of the form ' $q/(p/a)$ ' which yield substitutions such as:

$$q \frac{p'/a'}{p/a}$$

By a similar process, equivalents of  $n$ -adic relations can also be obtained. For example: ' $q/(p.a)(r.c)$ ' expresses a 'dual substitution' equivalent to a dyadic relation.

<sup>28</sup> See Russell, 1905b, p. 4. (\*12.11).

It is important to note, however, that there is no intrinsic difference between the matrix for a dyadic relation and that for a function of functions. Both are examples of 'dual substitutions' and Russell does not need to exhibit the difference (though he sometimes chose to) through the differing notations of ' $q/(p/a)(r/c)$ ' and ' $q/(p,a)(r,c)$ '. The matrix ' $q/(p,a)$ ' may be equivalent to a function of functions of individuals or a dyadic relation; it will be a function of functions of individuals only if  $p$  and  $a$  appear in  $q$  in the form ' $p/a$ '. Russell gives the following as a formal definition of the condition which must be met for this to be so:

$$(r, c, r', c') [(r/c = r'/c') \supset (q \frac{r, c}{p, a} \equiv q \frac{r', c'}{p, a})].^{29}$$

So long as this condition is met by our use of the matrix ' $q/(p,a)$ ', the truth of the proposition denoted by the substitution will be dependent on the value of the matrix ' $r/c$ ', rather than  $r$  and  $c$  separately (as would be the case if ' $q/(p,a)$ ' was equivalent to a dyadic relation). Such matrices give rise to analogues of classes of classes without any supposition of classes as singular logical subjects.<sup>30</sup>

In effect, this provides all that would be provided by a simple type-theory without the requirement for different types of entities.<sup>31</sup> The 'type' of a matrix is fixed, not by appeal to a typed hierarchy of entities, but by virtue of the formal grammar of substitution. In other words, there is only one (wholly unrestricted) type of variable, but type distinctions are generated by the complexity of a matrix in the sense that the type of a matrix is simply determined by the number of arguments it contains. Hence the second type will include both dyadic relations of individuals and functions of functions of individuals; the third type will contain triadic relations and functions of functions of functions of individuals, and so on, with, in general, the  $n^{\text{th}}$  type containing  $n$ -adic relations and  $n^{\text{th}}$ -level functions.<sup>32</sup>

Any concern which we may have harboured about the theory of types being simply an *ad hoc* formal patch for a leaking logic should, it is clear, be dispelled at

<sup>29</sup> See Russell, 1906b, p. 176. Note that Russell uses ' $(r, c, r', c')$ ' as an abbreviation of the quantifier-expressions ' $(\forall r)(\forall c)(\forall r')(\forall c')$ '.

<sup>30</sup> See Russell, 1906b, pp. 176-177.

<sup>31</sup> For a more detailed account of how the substitutional theory proxies a predicate calculus with simple type-theory, see Landini, 1998. In particular, see pp. 140-144 for a demonstration of the equivalence.

<sup>32</sup> See Russell, 1906b, pp. 176-177.

least within the context of the substitutional theory. The hierarchy of types is not a set of artificial restrictions imposed on variation here; it is just a natural consequence of the primitive operation of substitution. The simple “theory” of types simply *is* the grammar of substitution.<sup>33</sup>

To see that this is so, we just need to consider how the calculus of classes is to be proxied in the substitutional theory. Russell says: ‘The theory which I wish to advocate is that this shadowy symbol  $p/a$  represents a *class*’ (Russell, 1906b, p. 170). Just as a matrix of the form  $p/a$  is to take the place of a function, so also can it be taken (extensionally, as it were) to proxy a class:

Any two entities  $p$  and  $a$  define a class, namely  $p/a$ , and  $x$  is a member of this class if  $p/a|x$  is true. (If  $p$  does not contain  $a$ , the class contains everything if  $p$  is true, and nothing if  $p$  is false). To say that  $x$  is a member of the class  $\alpha$  is now to say that for some values of  $p$  and  $a$ ,  $\alpha$  is the matrix  $p/a$  and  $p/a|x$  is true.

(Russell, 1906b, p. 172)

A little reflection shows that the Russell paradox simply cannot be formulated in this system. Quite simply, ‘ $p/a$ ’ does not stand for an entity; and only entities can be members of “classes”, because only entities can be sensibly substituted for  $a$  in the matrix  $p/a$ . The Russell paradox would require the formation of a matrix which was akin to something like:

$$p \frac{p/a}{a}$$

which is just ungrammatical nonsense (literally saying ‘the result of replacing  $a$  in  $p$  by the result of replacing  $a$  in  $p$  by...’).<sup>34</sup> As Russell goes on to say:

<sup>33</sup> It is precisely this crucial point which Quine failed to recognise in his hugely influential interpretation of Russellian type-theory. In his brief discussion of the substitutional theory, Quine (1967) comments that the theory of types is ‘not even mentioned’ in Russell’s (1906a) published account of substitution. It is unsurprising, therefore, that Quine regarded the theory of types as unpalatable (see Quine, 1967, p. 150).

<sup>34</sup> See Russell, 1906c, p. 201.

But now ‘ $x$  is an  $x$ ’ becomes meaningless, because ‘ $x$  is an  $\alpha$ ’ requires that  $\alpha$  should be of the form  $p/a$ , and thus not an entity at all. In this way membership of a class can be defined, and at the same time the contradiction is avoided.

(Ibid.)

The result is that only individuals can be members of “classes”, only “classes” can be members of “classes of classes”, and so forth. The theory of types is built into the formal grammar of the substitutional calculus in the sense that violations of type distinctions now become impossible if one wishes to remain within the bounds of sense.

At this point, then, Russell would appear to be in a strong position. He has a solution to all of the paradoxes which can be blocked by simple type-theory within a system which remains faithful to the philosophy of logic at the heart of the *Principles*. Most notably, the doctrine of the unrestricted variable seems tailor-made for the substitutional theory. We shall see shortly that this also makes Russell better-placed than at any other time in his philosophical career to give a satisfactory account of the unity of the proposition—something which, we saw in the last chapter, he could not justifiably lay claim to in 1903. Before exploring such benefits of substitution further, however, it is worth pausing to examine some of the costs incurred by Russell’s rapidly mutating ideas about logic and mathematics.

## 2.5 A Retreat from Pythagoras ?

The immediate consequence of the theory of descriptions and its application to classes via the substitutional theory is a significant depletion in the populous of Russell’s ontology. These ontological cutbacks are not as drastic as has often been supposed; Meinongian non-existent objects, for example, were not granted access to the 1903 ontology, as we have already seen, and, hence, do not need to be expelled in 1905. Nonetheless, the expulsion of classes from Russell’s ontology is surely a drastic move. If classes are just ‘symbolic or linguistic conveniences’ (Whitehead & Russell, 1910, p. 75), and numbers are classes of similar classes, it must surely follow that

numbers are also just symbolic conveniences or 'logical fictions' as Russell sometimes calls classes.

Russell later spoke of his work during this period as the first steps taken in 'a gradual retreat from Pythagoras' (Russell, 1959, p. 208). The theory of descriptions, seen in this light, is a turning point of enormous significance in Russell's development. It should be stressed that Russell is often careful not to over-commit himself to the *non-existence* of classes, he merely relies on the theory of descriptions in order to abstain from the burden of asserting their existence. For example, in his first formulation of the substitutional 'no-classes' theory he states: 'It is not necessary to assume that no functions determine classes and relations; all that is essential to the theory is to abstain from assuming the opposite' (Russell, 1906a, p. 154). Later in the same year, he was more willing to denounce classes outright:

[T]here really are no such things as classes, and statements about a class will only be significant when they can be analysed into statements about all or some of the members of the class. Language which speaks about classes is, in fact, merely a form of short-hand, and becomes illegitimate as soon as it is incapable of translation into language which says nothing about classes.

(Russell, 1906b, p. 166)

By the time of *Principia*, however, Russell was again prepared to temper Occam's razor when it came to the question of classes:

It is not necessary for our purposes, however, to assert dogmatically that there are no such things as classes. It is only necessary for us to show that the incomplete symbols which we introduce as representatives of classes yield all the propositions for the sake of which classes might be thought essential. When this has been shown, the mere principle of economy of primitive ideas leads to the non-introduction of classes except as incomplete symbols.

(Whitehead & Russell, 1910, p. 75)

In other words, Russell is neither asserting nor denying the existence of classes; as he later phrased it, he is ‘merely agnostic as regards them’ (Russell, 1919, p. 184).

Whether it is agnosticism or outright atheism regarding classes that we attribute to Russell, there will be certain consequences of his view which mark a radical alteration in his metaphysics. The classes on which mathematics is to be built in the *Principles of Mathematics* are the eternally subsisting logical objects of an unrestrained platonist. Numbers, on such an account, are inhabitants of a platonic realm which plays host to a mathematics characterised by its elegance and eternality. With the demise of classes, however, the privilege of subsistence must now be withdrawn from numbers too:

The theory which I wish to advocate is that classes, relations, numbers, and indeed almost all the things that mathematics deals with, are ‘false abstractions’, in the sense in which ‘the present King of England’, or ‘the present King of France’ is a false abstraction. Thus e.g. the question ‘what is the number *one*?’ will have no answer; the question which has an answer is ‘what is the meaning of a statement in which the word *one* occurs?’ And even this question only has an answer when the word occurs in a proper context.’

(Russell, 1906b, p. 166)

There is a striking similarity between this passage and the famous § 62 of Frege’s *Grundlagen*, where the context-principle is invoked as a means of finding a definition of number:

How, then, is a number to be given to us, if we cannot have any ideas or intuitions of them? Since it is only in the context of a proposition that words have any meaning, our problem becomes this: to define the sense of a proposition in which a number word occurs.

(Frege, 1884, § 62)

Despite the apparent similarity, there is a sizeable gulf between the views expressed in the above two quotations. For Frege, the above passage marks a triumphant insight—

numbers (which are real, albeit platonic, objects for Frege) are neither physical nor psychological, but their essence can be grasped through the language in which we speak of them. For Russell, however, the insight is decidedly less triumphant. The reduction of number to sentences in which number words occur serves to avoid the contradictions which had plagued Russell, but only at the expense of numbers as subsistent entities. Taking a step forward in his solution of the paradoxes, meant taking two steps backwards from his platonist metaphysics. As Russell would later express it: 'The solution of the contradictions ... seemed to be only possible by adopting theories which might be true but were not beautiful. I felt about the contradictions much as an earnest Catholic must feel about wicked Popes. And the splendid certainty which I had always hoped to find in mathematics was lost in a bewildering maze' (Russell, 1959, p. 212).

Having noted this, however, we should not be too hasty in overstating Russell's 'retreat from Pythagoras'. The retreat, after all, was a gradual one and Russell was far from ready to abandon his platonism outright at this time.<sup>35</sup> Indeed, it seems that Whitehead somewhat misjudged Russell's motives in developing the substitutional theory when he complained: 'It finds the whole of mathematics on a typographical device and thus contradicts the main doctrines of Vol. I' (Whitehead to Russell, 22/2/1906, quoted in Russell, 1973, p. 131). In fact, though it is true that numbers turn out to be symbolic conveniences on the analysis proffered by the substitutional theory, it is quite wrong to interpret this as founding mathematics on a typographical device. The subjects of the primitive operation of substitution are *entities*, not symbols. And where Russell had been willing to forego a platonist interpretation of classes, he now held firmly to an 'unadulterated' platonism<sup>36</sup> when it

<sup>35</sup> Indeed, as Linsky notes, Russell *never* really abandoned platonism in one sense: 'Russell never gave up his belief in the reality of relations. Rather he shifted away from his concern with the Pythagorean universe of mathematical entities' (Linsky, 1999, p. 15).

<sup>36</sup> The phrase 'unadulterated platonist' was used, somewhat dismissively, by Russell to describe Gödel: 'Gödel turned out to be an unadulterated Platonist, and apparently believed that an eternal "not" was laid up in heaven, where virtuous logicians might hope to meet it hereafter' (Russell, 1968, p. 270). Gödel's response, contained in a letter to Kenneth Blackwell of the Russell archives (which Gödel refrained from posting), correctly notes that Russell's platonism survived *Principia* by some years (though perhaps not until the time Gödel says): 'Concerning my "unadulterated" Platonism, it is no more "unadulterated" than Russell's own in 1921 when in the *Introduction to Mathematical Philosophy* he said: "Logic is concerned with the real world just as truly as zoology, though with its more abstract and general features". At that time evidently Russell had met the "not" even in this world, but later on under the influence of Wittgenstein he chose to overlook it' (Gödel, letter to K. Blackwell, dated 1971, quoted in Monk, 2000, p. 270).

came to propositions. Indeed, as he acknowledged, the only non-simple entities he remained committed to at this time were propositions: 'and it is very hard to believe that there are no such things as propositions, or to see how, if there were no propositions, any general reasoning would be possible' (Russell, 1906b, p. 188). As we shall see, Russell would eventually be forced to step down from even this position, but, at the time of the substitutional theory, propositions are fundamental and, indeed, crucial. These propositions are precisely the propositions which were central to the logicism of the *Principles*—mind independent, extra-linguistic, subsisting complexes. Far from being built on the foundations of a typographical device, mathematical truths were to be maintained as eternal truths about eternally subsisting propositions.

The admission that there are no numbers independently inhabiting this platonic realm (i.e., the admission that numbers are 'symbolic conveniences') is not, in itself, a radical comedown from the original logicist thesis. Logicism, in the hands of Russell, is merely the claim that mathematical (arithmetical, for Frege) truths are a proper subset of the logical truths. There is no change regarding this position in the substitutional theory. The conceptual analysis of number becomes more complicated on the substitutional analysis than on the analysis which admitted classes, but this is to be counterbalanced by increased ontological simplicity. Mathematics (as a branch of logic) remains well and truly in the grip of platonism on either account.

There is, however, a significant development in Russell's analysis of the proposition in the substitutional theory, as compared to the 1903 analysis. We saw, in the last chapter, that the *Principles* lacks a satisfactory account of the unity of the proposition. One of the benefits of the substitutional theory is that Russell is now in a position to offer a far more convincing account of propositional unity.

## 2.6 Substitution and the 'hoary difficulties about the one and the many'

Let us briefly recall the nature of Russell's problem with the unity of the proposition in the *Principles*. The doctrine of the unrestricted variable at the heart of that work demands that every entity be a possible value of a free variable and, hence, that all entities be treated on an ontological par. Thus the hierarchy of levels and distinction between concepts and objects invoked by Frege to safeguard the unity of the

proposition is unavailable to Russell. Russell, following Moore, adopts a form of extreme pluralism; the world (and indeed propositions), on this account, is composed of individual atoms in certain combinations and these atoms are all that there is. So, for example, universals and particulars, if granted ontological status, must share the same atomistic form—it must be possible to make either into a logical subject without any alteration of their content. But the problem of the unity of the proposition arises when we now apply this view to the analysis of propositions. For, if a proposition is to be analysed into a series of logical subjects, it seems that we will be left with a mere list of names (or objects, on the Russellian analysis): ‘John loves Mary’, for example, will be analysed into the (unordered) set of constituents {John, Love, Mary}. There is nothing here to enable a reconstruction of the proposition from its constituents; the unity of the propositional whole is lost when it is broken into its parts. Russell’s partial answer to the problem in 1903 was to attribute a ‘twofold’ nature to concepts. Concepts can occur both predicatively and as terms. However, this does not solve the problem. If a concept can occur *either* predicatively *or* as a term, there is nothing about the nature of the constituents of a proposition in themselves (after analysis) which will show which kind of occurrence a concept is to have. The unacceptable consequence for the Russellian pluralist is that the proposition appears to be somehow more than the sum of its parts.

It is commonplace to read, in commentaries on Russell, that this problem is an insoluble one within the framework of Russell’s philosophical logic.<sup>37</sup> However, this is not so. The formal grammar of the substitutional system enables the preservation of propositional unity while, most importantly, retaining the *Principles* conception of logic as universal and all-encompassing. In other words, the substitutional theory delivers a method of analysis which does not destroy the unity of the proposition and, whilst maintaining the doctrine of the unrestricted variable, is every bit as convincing as the Fregean method of decomposing a judgeable content into its complete and incomplete parts.

The superiority of this analysis over the earlier *Principles* account is due, in no small measure, to Russell’s increased (or, perhaps we should say, *enforced*) sensitivity to the proposition in logical analysis. Whereas the earlier analysis threatened to reduce the constituents of the proposition to a mere set of elements, forming no more than a

list, the new analysis only presupposes some connection between those elements in that it treats certain elements as being constituents of others (rather than being obliged to give an account of how those parts are related to one another). Furthermore, there is no intrinsic distinction between a prototype and argument which can be irrevocably lost on analysis in the way that the predicative quality of a concept is on the *Principles* account. For both argument and prototype are simply logical subjects which do not require any predicative quality in order to be united with one another. Whereas the result of analysis in the *Principles* left us with a set of elements which resists reconstitution as a proposition, the substitutional analysis leaves us with a set of elements which can, without any mystery, stand in certain relations (such as, for example, 'being contained in') to one another. The existence of such relations is guaranteed by the substitution operation which provides the unity required.

Now it may be noted that this will still require that a universal features in our analysis; namely, the universal *substitution*.<sup>38</sup> In the proposition expressed by ' $p/a;b!q$ ', for example, the elements  $p$ ,  $a$ ,  $b$ , and  $q$ , are related by the substitution operation which, at least according to Russell, will be a 'logical universal'. It should also be noted, however, that this is not a re-emergence of the original problem as located in the *Principles* analysis. The problem there was one of destroying the unity of the proposition by analysing universals, as they occur as propositional constituents, into logical subjects and hence losing the essential predicative quality which is required of a universal in order to provide the unity of the proposition. The substitution operation, by contrast, need not be considered a constituent of the proposition; rather it is the operation performed on those constituents in order to achieve the desired unity.<sup>39</sup>

This brings us on to the decisive point. Having successfully developed an analysis of propositions which only presupposes a commitment to individual entities (i.e., logical subjects), Russell is also in the immensely fortunate position of no longer

<sup>37</sup> See, for example, Griffin, 1985, 1986 & 1993; Candlish, 1996; Sainsbury, 1996; Palmer, 1988.

<sup>38</sup> I am grateful to Gregory Landini for raising this point.

<sup>39</sup> See Russell, 1906a, p. 170. Alternatively, if one insisted on considering *substitution* to be itself a constituent of the proposition, this would merely show that the logical constants are not to be treated as equivalent to the other propositional constituents (for 'substitution' stands for a logical constant in the context of the substitutional theory). Wittgenstein would later urge Russell to accept a radical distinction between logical and non-logical constants, but Russell was already aware of the unique status of logical constants. He was aware, for example, that the notion of substitution must be taken as a primitive idea in the substitutional theory.

being obliged to provide an account of how universals and particulars can enter into relations with one another. This problem, perhaps the pivotal question regarding propositional unity, was one that Russell returned to once he had abandoned the substitutional theory.<sup>40</sup> In the substitutional theory, however, the problem simply does not arise. Everything is a logical subject on this account, and the distinction invoked in the *Principles* between things and concepts, whereby concepts were adjudged to have a two-fold nature occurring as either ‘entity’ or ‘meaning’, is no longer of any significance to the substitution operation. All that is relevant in the substitutional theory is that individuals can be constituents of propositions (which are, in turn, also individuals). Whether those individuals are universals or particulars is of no consequence; there is no call for any explicitly predicative constituents anymore.

There remains the matter of the arrangement of the constituents of the proposition. The difference between ‘John loves Mary’ and ‘Mary loves John’ will of course be a further casualty of analysis on the *Principles* model. Notice, however, that this feature of the propositions would also be lost on analysis into its Fregean parts. Likewise, in the substitutional analysis, the arrangement of the constituents in the proposition expressed by ‘ $p/a\cdot b!q$ ’ will be lost on analysis. But this should not be taken as a failure on the part of the analysis to account for the unity of the proposition. Rather, it is simply to be viewed as a direct result of the fact that, for example, the two propositions mentioned above are composed of the same elements. The important thing to notice here is that we are confronted with a choice of propositions which result from re-arrangement of the constituents. On the analysis given in the *Principles*, we do not even achieve this; all we are left with there is a set of particles, {John, Love, Mary}, for which, as Russell admits, no amount of re-arrangement will produce a proposition: ‘A proposition, in fact, is essentially a unity, and when analysis has destroyed the unity, no enumeration of constituents will restore the proposition’ (Russell, 1903, § 54). It is the unity lacking here which is preserved by substitution.<sup>41</sup>

The benefits offered by the substitutional theory, then, are remarkable. The logical paradoxes are solved without recourse to artificial restrictions on variation and

<sup>40</sup> Most notably, of course, in Russell (1912a & 1913), but also in Russell (1911a).

<sup>41</sup> Of course the problem, as we shall see shortly, is more severe when it re-emerges in the context of the later multiple-relation theory of judgement. There Russell is seeking to give an account of ‘propositional’ attitudes, hence the need to pick out the correct propositional content for such attitudes is paramount.

in such a way that the solution is philosophically, as well as formally, palatable. In addition, the problem of the unity of the proposition which troubled Russell without resolution in the *Principles* does not translate into the substitutional analysis of propositions. Russell, therefore, had good reason to be quietly confident, having apparently found a solution to the paradoxes which remains faithful to his original philosophical principles:

Of the philosophical consequences of the theory I will say nothing, beyond pointing out that it affords what at least seems to be a complete solution of all the hoary difficulties about the one and the many; for, while allowing that there are many entities, it adheres with drastic pedantry to the old maxim that, 'whatever is, is one'.

(Russell, 1906b, p. 189)

Russell's confidence, however, was to be short-lived. At the high-point of his faith in the substitutional theory, he remarked: 'The only serious danger, so far as appears, is lest some contradiction should be found to result from the assumption that *propositions* are entities; but I have not found any such contradiction' (Ibid., p. 188). Shortly after writing this passage, he discovered just such a contradiction.

## 2.7 The Failure of Substitution and the Road to Ramification

It is not clear when, exactly, Russell discovered that his substitutional theory was inconsistent. There is no mention of any contradictions in the manuscripts written in 1905, but they are mentioned regularly in 1906, most notably in the manuscript 'On Substitution', which devotes detailed attention to a paradox unique to substitution. The paradox in question is briefly sketched in another 1906 manuscript, 'The Paradox of Liar', where Russell refers to it as 'the paradox which led to the abandonment of substitution before':

$$p_0 . = : (\exists p, a) : a_0 . = . p \frac{b}{a} ! q : \sim p \frac{a_0}{a} ::$$

$$\supset : p_0/a_0 : p_0 \frac{b}{a_0} !q : \sim p_0/a_0 : p_0 \frac{b}{a_0} !q$$

(Russell, 1906e, p. 72)

This sketch, however, is a little *too* sketchy, and requires some elucidation. Russell begins with a proposition  $p_0$ , which is as follows:

$$p_0 = (\exists p, a) [(a_0 = p/a \cdot b!q) \& \sim (p/a \cdot a_0)]$$

Substituting the proposition  $p_0/b; a_0!q$  for  $a_0$  in  $p_0$  yields:

$$(p_0/a_0 \cdot \{p_0/a_0 \cdot b!q\}) = (\exists p, a) [(\{p_0/a_0 \cdot b!q\} = \{p/a \cdot b!q\}) \& \sim (p/a \cdot \{p_0/a_0 \cdot b!q\})]$$

But, noticing the identity of  $\{p_0/a_0 \cdot b!q\}$  and  $\{p/a \cdot b!q\}$ ,<sup>42</sup> we can prove that  $p_0 = p$  and  $a_0 = a$ .<sup>43</sup> From here, one can go on to deduce the contradiction:

$$p_0/a_0 \cdot \{p_0/a_0 \cdot b!q\} \equiv \sim (p_0/a_0 \cdot \{p_0/a_0 \cdot b!q\}).$$

This paradox, Russell later referred to as 'the paradox which pilled the substitution theory' and remarked that: 'In trying to avoid this paradox, I modified the substitution-theory in various ways, but the paradox always reappeared in more and more complicated forms' (Russell, 1907).<sup>44</sup> A complete formal derivation of the paradox can be given as follows:<sup>45</sup>

<sup>42</sup> Curly brackets '{}' are used as nominalizing braces in the context of the substitutional theory; that is to say, they enclose a proposition to signify its use as a term (see Appendix). They are occasionally dropped where the context prevents confusion. Outside of the substitutional calculus their use here is merely as punctuation, or to form class-abstracts in accordance with the usual convention.

<sup>43</sup> See Russell, 1907. The proof of  $(r, c, p, a) [\{r/c; b!q\} = \{p/a; b!q\}] \supset [(r = p) \& (c = a)]$  is surprisingly difficult, and somewhat tedious. It is not necessary to reproduce it here. Landini provides a proof in his version of the substitutional theory in Landini, 1998, pp. 119-125.

<sup>44</sup> This paradox of the substitutional theory was first uncovered from Russell's unpublished manuscripts by Landini, who christened it the ' $p_0/a_0$ ' paradox, after the matrix from which it is derived (See Landini, 1989, 1993 & 1998).

<sup>45</sup> This derivation of the paradox is, structurally, much the same as that offered by Landini, 1998, p. 204. I have presented it slightly differently here for reasons of brevity and continuity.

As mentioned above, we will need to rely on the theorem which states that identical propositions have identical constituents in identical positions, namely the theorem:

$$(T_{(=)}) \vdash (b, q) (r, c, p, a) ([\{r/c\} b!q] = \{p/a\} b!q) \supset [(r = p) \ \& \ (c = a)]$$

We also use a form of the contextual definition schemata, explained above, for definite descriptions of propositions (which, for convenience, we will refer to as ‘desc.’):

$$p/a\dot{x} = (\iota q) (p/a\dot{x}!q) \quad \text{Df.},$$

and

$$\varphi\{(\iota q) (p/a\dot{x}!q)\} = (\exists q) [(p/a\dot{x}!q) \ \& \ (r) (p/a\dot{x}!r \supset r = q) \ \& \ \varphi q] \quad \text{Df.}$$

We will also need

$$(T_{(=)}^*) \vdash (x) (x = x).$$

We begin, as above, by defining the proposition  $p_0$ :

$$p_0 = (\exists p, a) [(a_0 = p/a\dot{b}!q) \ \& \ \sim (p/a\dot{a}_0)] \quad \text{Df.}$$

1.  $p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}! \{(\exists p, a) [(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \ \& \ \sim (p/a\dot{\{p_0/a_0\dot{b}!q\}})]\}$
2.  $\rightarrow (\exists p, a) [(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \ \& \ \sim (p/a\dot{\{p_0/a_0\dot{b}!q\}})]$
3.  $(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \ \& \ \sim (p/a\dot{\{p_0/a_0\dot{b}!q\}}) \quad (2, \text{ EI})$
4.  $\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\} \quad (3, \text{ simp})$
5.  $(p_0 = p) \ \& \ (a_0 = a) \quad (4, T_{(=)})$
6.  $\sim (p/a\dot{\{p_0/a_0\dot{b}!q\}}) \quad (3, \text{ simp})$
7.  $\sim (p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}) \quad (5, 6)$
8.  $(\exists r) [(p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}!r) \ \& \ (s) (p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}!s \supset s = r) \ \& \ \sim r] \quad (7, \text{ desc.})$
9.  $(p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}!r) \ \& \ (s) (p_0/a_0\dot{\{p_0/a_0\dot{b}!q\}}!s \supset s = r) \ \& \ \sim r \quad (8, \text{ EI})$

10.  $p_0/a_0 \{ p_0/a_0 \cdot b!q \} !r$  (9, simp)
11.  $r = (\exists p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \& \sim (p/a \{ p_0/a_0 \cdot b!q \})]$  (10, df.  $p_0$ )
12.  $\sim r$  (9, simp)
13.  $\sim (\exists p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \& \sim (p/a \{ p_0/a_0 \cdot b!q \})]$
14.  $2 \supset F$  (CP)
15.  $\sim (\exists p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \& \sim (p/a \{ p_0/a_0 \cdot b!q \})]$  (14, red.)
16.  $(p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \supset (p/a \{ p_0/a_0 \cdot b!q \})]$  (15, QN)
17.  $(\{ p_0/a_0 \cdot b!q \} = \{ p_0/a_0 \cdot b!q \}) \supset (p_0/a_0 \{ p_0/a_0 \cdot b!q \})$  (16, UI)
18.  $(\{ p_0/a_0 \cdot b!q \} = \{ p_0/a_0 \cdot b!q \}) \supset (\exists h) [(p_0/a_0 \{ p_0/a_0 \cdot b!q \}) !h]$ 
 $\& (t) (p_0/a_0 \{ p_0/a_0 \cdot b!q \} !t \supset t = h) \& h]$  (17, desc.)
19.  $\{ p_0/a_0 \cdot b!q \} = \{ p_0/a_0 \cdot b!q \}$  ( $T_{(=)}$ \*)
20.  $(\exists h) [(p_0/a_0 \{ p_0/a_0 \cdot b!q \}) !h] \& (t) (p_0/a_0 \{ p_0/a_0 \cdot b!q \} !t \supset t = h) \& h]$  (18, 19.mp)
21.  $(p_0/a_0 \{ p_0/a_0 \cdot b!q \}) !h) \& (t) (p_0/a_0 \{ p_0/a_0 \cdot b!q \} !t \supset t = h) \& h$  (20, EI)
22.  $(t) (p_0/a_0 \{ p_0/a_0 \cdot b!q \} !t \supset t = h) \& h$  (21, simp)
23.  $h = (\exists p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \& \sim (p/a \{ p_0/a_0 \cdot b!q \})]$  (22, df.  $p_0$ )
24.  $h$  (22, simp)
25.  $(\exists p, a) [(\{ p_0/a_0 \cdot b!q \} = \{ p/a \cdot b!q \}) \& \sim (p/a \{ p_0/a_0 \cdot b!q \})]$  (23, 24)
26.  $15 \supset F$

The basic substitutional theory, despite all its promises, is thus proved inconsistent.

Predictably, Russell's first response to the problem was to once again rethink his ontology. The substitutional paradox is not a semantic paradox, it merely follows from the admission of general propositions (with, of course, the absence of restrictions on values of the bound variables of those propositions).<sup>46</sup> The conclusion Russell drew from this diagnosis was that the existence of general propositions was now in doubt.

Russell's manuscript of April-May 1906, 'On Substitution', is an extended treatment of the paradoxes which had surfaced in the substitutional theory. Russell

<sup>46</sup> This recognition is, I believe, vital if we are to understand Russell's motives in creating the ramified type-theory of *Principia*. Russell's desire to find a common source of both the logical and semantic paradoxes becomes clearer when we understand that the original invocation of the vicious-circle principle came in response to what was, in effect, a *logical* contradiction; namely, the paradox of substitution. It is not surprising that he then diagnoses, for example, the Epimenides as stemming from

considers (and rejects) several possibilities in his quest for a solution (including a hierarchy of orders of propositions), with no clear resolution. By June, however, he had (if only temporarily) decided how to proceed. In 'On "Insolubila" and their Solution by Symbolic Logic', Russell rejects an ontology of general propositions and presents a streamlined version of the substitutional theory which grants ontological status to only those propositions that contain no bound variables. To this end, Russell, for the first time, publicly adopts his own form of Poincaré's vicious-circle principle, in the form: 'Whatever involves an apparent variable must not be among the possible values of that variable' (Russell, 1906c, p. 204). From this principle (and the belief that, as it were, *genuine* propositions must remain values of unrestricted variables), it follows that any *statement* about 'all propositions' cannot itself be a proposition. In other words, to assert something of all (or some) values of a propositional variable, is not to assert another proposition in addition to these values.

It is important to note that this is not, as yet, the full-blooded ramification of the substitutional theory that some have taken it to be.<sup>47</sup> Russell is not stratifying propositions (as entities) into a hierarchy of orders, but is making a distinction between *propositions* and *statements* and then denying that statements (unlike propositions) are entities. The intention is to extend the application of the theory of descriptions, which had been so successful when applied to classes and relations, to general propositions. Furthermore, Russell explicitly states that this procedure is required in order to *preserve* the doctrine of the unrestricted variable. Russell expresses the problem by stating that what is desired is that an individual variable has its 'range of significance' given with it. But, if the doctrine of the unrestricted variable is to be retained, all individuals should be included in this range. This is clearly intolerable in the case of vicious-circle fallacies. Treating general propositions as incomplete symbols solves the problem by outlawing vicious-circle fallacies on the grounds that only individuals can be values of bound variables, and general propositions are not individuals:

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the same cause. Russell considered such paradoxes to be logical in the sense that they all appear to be paradoxes of *quantification*, rather than just resulting from semantic notions.

<sup>47</sup> This is the interpretation offered by Hylton, 1980, for example.

[W]e have to assume that a single letter, such as  $x$ , can only stand for an individual; and that can only be the case if individuals are really all entities, and classes, etc., are merely a *façon de parler*. Thus our variable  $x$  now again has an unrestricted range, since it may be any individual, and there is really nothing that is not an individual. Hence to reconcile the unrestricted range of the variable with the vicious-circle principle which might seem impossible at first sight, we have to construct a theory in which every expression which contains an apparent variable (i.e. which contains such words as *all*, *any*, *some*, *the*) is shown to be a mere *façon de parler*, a thing with no more independent reality than belongs to (say)  $\frac{d}{dx}$  or  $\int_a^b$ . For in that case, if (say)  $\phi x$  is true for *every* value of  $x$ , it will be not true, but meaningless, if we substitute for  $x$  an expression containing an apparent variable. And such expressions include all descriptive phrases (the so-and-so), all classes, all relations in extension, and all *general* propositions, i.e. all propositions of the form ‘ $\phi x$  is true for all (or some) values of  $x$ ’.

(Russell, 1906c, p. 206)

Russell goes on to say: ‘To show in detail how this is to be done would require much mathematics, and is impossible in the present article’ (Ibid.). In fact, the mathematics required for such a demonstration runs into an immediate problem. As Russell notes in ‘On Insolubilia’, the restrictions which result in the substitutional theory once general propositions have been abandoned will have an intolerably destructive impact on the ability of the theory to generate arithmetic and, hence, to provide the demonstration of logicism which is its whole *raison d’être*. Consider the (from this time onwards for Russell, increasingly problematic) case of mathematical induction. The rejection of general propositions seems to demand that the principle of mathematical induction be somehow phrased without the use of such words as ‘any’, ‘all’, ‘some’, or ‘the’. But, as Russell acknowledges:

[U]nless this restriction is mitigated by an axiom, it will render most of the usual uses of induction fallacious; and in other ways it will destroy many pieces of ordinary mathematical reasoning. Take such a proposition as: 'If  $m$  and  $n$  are finite numbers, either  $m < n$  or  $m = n$  or  $m > n$ '. If we consider this as a property of  $m, n$  is an apparent variable; thus induction does not warrant the conclusion that this holds for all finite numbers from the fact that it holds for 0, and that if it holds for  $m$  it holds for  $m + 1$ .

(Russell, 1906c, pp. 211-212)

Russell's proposed solution is to adopt an axiom to the effect that, although no general proposition can be *identified* with a quantifier-free proposition, it may still be *equivalent* to one. Whilst it should be noted that this is not a fully-fledged reducibility axiom such as we find in *Principia* (we do not have a hierarchy of orders to 'reduce' here), many of the criticisms of that axiom also hold good for this. For example, there is an enormous doubt as to the status of the axiom—there is no evidence that it could be construed as a truth of logic. To make matters worse, as Landini has pointed out, the axiom re-introduces the very paradox that the rejection of general propositions was intended to avoid.<sup>48</sup> Recall that Russell's original problem arose from the following substitution of  $\{p_0/a_0\dot{b}!q\}$  for  $a_0$  in  $p_0$ :

$$p_0/a_0\dot{b}\{p_0/a_0\dot{b}!q\}!(\exists p,a)[(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \& \sim(p/a\dot{b}\{p_0/a_0\dot{b}!q\})]$$

This will now be impossible with the rejection of general propositions, for it is now inadmissible to have the term  $\{(\exists p,a)[(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \& \sim(p/a\dot{b}\{p_0/a_0\dot{b}!q\})]\}$ . With the new axiom in place, however, we can legitimately obtain

$$p_0/a_0\dot{b}\{p_0/a_0\dot{b}!q\} \equiv (\exists p,a)[(\{p_0/a_0\dot{b}!q\} = \{p/a\dot{b}!q\}) \& \sim(p/a\dot{b}\{p_0/a_0\dot{b}!q\})]$$

where we have a quantified statement which is equivalent to a proposition and is not treated as a term. This is sufficient to allow a form of the paradox to re-emerge.<sup>49</sup> As Landini says: 'The bald fact is that with the addition of [the axiom] the system is

<sup>48</sup> See Landini, 1993, p. 374 & 1998, pp. 231-233.

<sup>49</sup> See Landini, 1998, pp. 232-233 for a full demonstration of the contradiction in this form.

inconsistent. Yet without it the system cannot recover arithmetic. Russell is trapped. ‘Les paradoxes’ was a failure’ (Landini, 1998, p. 233).<sup>50</sup>

The failure of the substitutional theory without general propositions left Russell with only one real option: ramification. In the manuscript ‘The Paradox of the Liar’ written after ‘On Insolubilia’, a hierarchy of orders gradually emerges. This hierarchy, however, is still significantly different from the hierarchy of orders embedded into the ramified type-theory of *Principia*. For one thing, Russell does not decide to abandon propositions (though he does consider it); even more significantly, he does not reject substitution.

## 2.8 Ramification and the Demise of Substitution

Russell’s main reason for retaining an ontology of propositions at this time was that he could not find any suitable philosophical grounds for a theory which lacked propositions as entities. Abandoning propositions outright was an appealing option but one which he could not, at this time, justify. This is not to say that he didn’t look for justification, however:

A possible philosophical basis for the view that a variable proposition is inadmissible will be as follows. Whatever there is in the world, not only of existence, but of being, is simple; propositions are not simple, & therefore there are none ... Now *variation* can only be applied to *entities*; hence variation of *propositions* is impossible.

(Russell, 1906e, p. 16)

The other option considered in the manuscript is full-scale ramification of propositions into a hierarchy of orders. Much of the manuscript is devoted to ways in which such a hierarchy might be constructed, the most popular being the stratification of propositions in accordance with the kinds of generality utilised in them. So a proposition which quantifies over propositions of order  $n$ , will be of order  $n + 1$ . The formal advantages of such a system, as we know, are immense. Vicious-circle

<sup>50</sup> The (original) French title of ‘On Insolubilia’ was ‘Les Paradoxes de la Logique’.

fallacies become inexpressible in a language whose formal grammar is constructed in such a way. The philosophical justifications for adopting a ramified hierarchy of propositions are, again, perhaps tenuous, but the idea at least has the benefit of being formally possible. Russell, in time, would see a way of coping without propositions in a way which could proceed formally and have at least some philosophical justification. At the time in question, however, it is hard to see how propositions could be removed from Russell's logic even from a purely formal perspective. If Russell wished to maintain the substitutional theory (which the evidence from 'The Paradox of the Liar' suggests he did), there is no way of devising a workable system without propositions. As a result, Russell kept both substitution and propositions, opting for a ramification of the substitutional theory to produce what many take to be the first public statement of the mature ramified theory of types in 'Mathematical Logic as Based on the Theory of Types'.

'Mathematical Logic' was not published until 1908, but was written in 1907, and marks the last appearance of the substitutional theory in Russell's published writings. Many commentators have failed to notice the appearance of the substitutional theory at all in the paper, and one might think its importance there is only slight. In fact its importance (as should now be clear) is fundamental. Russell speaks of the substitutional theory as underlying the hierarchy of functions (i.e., the hierarchy of types) and says: 'Functions of various orders may be *obtained* from propositions of various orders by the method of *substitution*' (Russell, 1908, p. 77. first emphasis added). In other words, a hierarchy of types is obtained from (and hence justified by) the operation of substitution. Substitution provides the philosophical foundation for the type part of the ramified hierarchy in 'Mathematical Logic'; the 1908 theory of types is just the substitutional theory with a hierarchy of orders bolted on. The order part of the hierarchy is more problematic, however.

Ramification is not justified by substitution, it is imposed on it. As a purely formal solution to the paradoxes of substitution and the semantic paradoxes, it is reassuringly effective, but it lacks philosophical foundations. To see this, recall just what a proposition is for Russell. Propositions are not considered linguistic expressions, they are held to be objectively obtaining (or non-obtaining) features of the world. Now, should one opt for a linguistic account of propositions, then the theory of orders may seem more plausible. It is quite feasible to interpret the

expression “ $\forall x Fx$ ” as an abbreviated form of the expressions “ $Fa, Fb, Fc, \dots, Fn$ ”, and hence to see some justification for viewing the general proposition as being, in some sense, of differing order to the expressions which it abbreviates and, hence, contains. If one allows propositions as *entities*, however, the situation becomes grotesque. How is one to justify the claim that entities differ in order? Substitution provides a neat explanation for types because the type of a matrix is simply determined by the number of its arguments; i.e., the number of entities contained in it. There is only one kind of entity as a result. The division of propositions into orders has no such justification. The ramified substitutional theory is formally adequate, but philosophically untenable. Russell’s problem is precisely this: the substitutional theory can only work if one maintains an ontology of propositions but, as the paradoxes of propositions show, the admission of propositions requires the ramification of the substitutional theory. But the ramification of the substitutional theory conflicts with the very philosophical foundations which recommend the theory in the first place. Something had to give. It turned out to be the ontology of propositions on which the substitutional theory depends. In the final version of Russellian ramified type-theory, there are no propositions and, consequently, there is no more substitutional theory either. With the demise of the substitutional theory, Russell made the final step in completing the formal system of *Principia* but did so at the expense of a theory whose elegance and simplicity could not be sustained in his final demonstration of logicism. And, unfortunately, many of Russell’s old problems, most notably his problem with the unity of the proposition, were waiting in the wings to re-emerge as soon as substitution was finally abandoned.

## 2.9 *Principia Mathematica* and the Multiple Relation Theory of Judgement.

Russell’s substitutional theory is central to his developing thought from 1905 to 1908. We have seen, through careful attention to the manuscripts and published articles written during this period, how substitution arose as an immediate consequence of the theory of descriptions and in turn provided the philosophical framework which validated the theory of logical types. Cut off from this history, however, the ramified theory of types in *Principia* appears both Baroque, and *ad hoc*. Commentators have

tended to follow Quine in viewing the work as confused to the point of incoherence and most subsequent versions of type theory have sought to make significant amendments to the system propounded in *Principia*.

Quine's influential interpretation of ramified type-theory is historically inaccurate, however. Missing the significance of the substitutional theory as the genuine origin of the theory of types, Quine takes the theory of types to be an *ontological* theory which divides up what there is in stark contrast to the earlier doctrine of the unrestricted variable:

Russell sees the universe as dividing into levels, or *types*. We can speak of all the things fulfilling a given condition only if they are all of the same type. The members of a class, then, must all be of the same type. So must the values of the variable of any one quantification.

(Quine, 1967, p. 151)

Having misread the basic idea behind the hierarchy of types as advancing an ontological stratification, Quine is left unable to make sense of the vicious-circle principle and decides that the whole messy business of ramified type-theory is little more than a confusion between posited entities and their linguistic expressions: 'Variables, in the easiest sense, are letters; and what contain them are notational expressions. Is Russell then assigning types to his objects or to his notations?' (Ibid.). Quine's conclusion is that Russell's equivocations over use and mention lead him to think he has reduced classes to 'symbolic conveniences' when, in fact, all he has done is reduced them to platonic attributes in intension.<sup>51</sup>

It is quite evident that Quine's criticisms are wide of the mark in regard to the substitutional theory. Substitution generates a hierarchy of "types" which is easily reconciled with the demand for unrestricted variation (and its subsequent demand for uniform ontological homogeneity). Logical types are elements of formal grammar in

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<sup>51</sup> See also Quine, 1953c, pp. 122-123, & 1966. It is, of course, true that Russell was willing to countenance attributes in intension as inhabitants of his ontology in *Principia*. Russell's retreat from Pythagoras had not backed off that far from Plato at this point. Indeed Quine is disapprovingly aware of this (see Quine, 1966b, p. 8). Note that Quine's (1967) comments are directed at 'Mathematical Logic as Based on the Theory of Types' where, as we have already seen, Russell explicitly states that the type-hierarchy of functions is properly obtained through substitution and, therefore (contrary to Quine), should *not* be viewed as an ontological hierarchy.

the substitutional theory, not ontological distinctions. Quine seems to have misunderstood the basic principles of Russell's substitutional theory and been too quick to read into them the principles of the modern idea of substitutional quantification, preferring to read Russell's motives as directed towards substitutions into sentences (Quine, 1967, p. 150). This is an important error.<sup>52</sup> Russell's propositions are not sentences, as we know, and the substitutions which lay the foundations for the calculus of the substitutional theory are properly understood as substitutions of entities into other entities, not linguistic operations. The ramified theory of 'Mathematical Logic' maintains this ontology of propositions as a basis for the hierarchy of types and hence can hardly be accused of failing for the reasons offered by Quine.

What, then, of the ontology of *Principia Mathematica*? Quine may have misunderstood the substitutional theory appealed to in 'Mathematical Logic', but that theory, along with its ontology of propositions, is explicitly rejected in *Principia*. If Quine's criticism of the 1908 form of ramification fails, perhaps it yet stands as telling against the system of Russell's 1910 *magnum opus*. Without the substitutional theory to appeal to in establishing the hierarchy of types, Russell seems to be left with just 'propositional functions' (i.e., either attributes in intension or open sentences, on Quine's reading) as the material from which the hierarchy is to be formed. It seems natural, one might think, to assume that Russell ontologized propositional functions and then divided them into a hierarchy of entities, thus dispensing with the desire for the universality of logic and accepting a universe inhabited by objects of differing logical type.<sup>53</sup> Before drawing hasty conclusions, however, we should recall that Russell did not simply abandon propositions (and hence substitution) without putting anything in their place. According to *Principia*, propositions are incomplete symbols which require the context of a judging mind in order to achieve a meaning; propositions, in other words, are to be replaced by judgements. Where 'Mathematical

<sup>52</sup> The same error seems to be suggested in Grattan-Guinness's mysterious comment that 'Russell interpreted quantification in a way which is somewhat similar to the modern substitutional interpretation of quantification, in speaking of constants rather than the individuals which the constants name' (Grattan-Guinness, 1977, p. 75).

<sup>53</sup> This, indeed, is just the interpretation that Hylton favours, arguing that, in adopting the mature version of ramified type-theory, 'Russell acknowledges the inevitable consequence of the paradox: he abandons the claim that there are no ultimate ontological distinctions among entities' (Hylton, 1990, p. 286. See also Hylton, 1993). We are now in a position to see that the paradox does not compel one to

'Logic' was based on a version of the substitutional theory. *Principia* has, in its place, a version of Russell's infamous multiple-relation theory of judgement.

Having decided to abstain from acknowledging the existence of propositions. Russell is left in need of alternative truth-bearers in *Principia*. Fortunately he had a theory ready to hand which would provide just what was needed (or so Russell at least hoped). In 1907 he had tentatively suggested a correspondence theory of truth which placed the truth-bearing load on to *judgements*, rather than propositions. These judgements were to be complexes consisting of a judging subject (or at least her judging mind), a series of objects, and the relation by which the judging subject was to be "multiply-related" to the objects of the judgement.

The multiple-relation theory has been almost universally condemned as unworkable. Most of these negative evaluations have, however, centred around the inability of the theory to provide an adequate account of the unity of judgement (a recurrence of the problem of the unity of the proposition, in effect); much less attention has been directed at the role played by the theory in generating the ramified hierarchy of *Principia*'s type-theory.<sup>54</sup> Before addressing the philosophical problems with the theory, we will examine this formal role further and specifically place it in the context of replacing the substitutional theory.

The multiple-relation theory generates orders of judgements through the construction of a hierarchy of senses of truth and falsehood. Simply put, the theory is a correspondence theory but the kind of correspondence which obtains between a judgement and a fact will vary depending on the nature of the judgement in question. For example, the kind of correspondence involved in setting out the truth-conditions for an atomic judgement will differ from the relation between a general judgement and its truth-conditions. The obviously convenient result is that truth and falsity will divide into a hierarchy of levels which reflect the kinds of quantifiers used in any judgement. The hierarchy of orders is thereby generated by the recognition of (and sensitivity to) these different notions of truth and falsity:

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abandon the view that there are no ultimate ontological categories, for the substitutional theory enables the doctrine of the unrestricted variable to co-exist with a satisfactory solution to the Russell paradox.

<sup>54</sup> Notable exceptions are Griffin, 1985 & 1986, Cocchiarella, 1980 & 1987, & Landini, 1991, 1996d & 1998.

That the words “true” and “false” have many different meanings, according to the kind of proposition to which they are applied, is not difficult to see. Let us take any function  $\phi\hat{x}$ , and let  $\phi a$  be one of its values. Let us call the sort of truth which is applicable to  $\phi a$  “*first truth*.”

... Consider now the proposition  $(x) . \phi x$ . If this has truth of the sort appropriate to it, that will mean that every value  $\phi x$  has “*first truth*.” Thus if we call the sort of truth which is appropriate to  $(x) . \phi x$  “*second truth*,” we may define “ $\{(x) . \phi x\}$  has second truth” as meaning “every value for  $\phi\hat{x}$  has first truth,” i.e. “ $(x) . (\phi x \text{ has first truth})$ .” Similarly, if we denote by “ $(\exists x) . \phi x$ ” the proposition “ $\phi x$  sometimes,” i.e. as we may less accurately express it, “ $\phi x$  with some value of  $x$ ,” we find that  $(\exists x) . \phi x$  has second truth if there is an  $x$  with which  $\phi x$  has first truth; thus we may define “ $\{(\exists x) . \phi x\}$  has second truth” as meaning “some value for  $\phi\hat{x}$  has first truth,” i.e. “ $(\exists x) . (\phi x \text{ has first truth})$ .” Similar remarks apply to falsehood.

(Whitehead & Russell, 1910, pp. 44-45)

The hierarchy thus constructed builds successive notions of truth and falsity up from the base level of truth attached to elementary propositions (or, more correctly, judgements) in accordance with the scope of the most prominent quantifier in the judgement (i.e., the quantifier whose scope is the whole judgement). So a judgement about all or some values of a second-order proposition will have third-order truth (will be a third-order proposition). Truth is defined recursively on the correspondence-relation at the base level between elementary judgements and facts.<sup>55</sup> Far from dividing up Russell’s ontology into complexes of differing order, the net result of this recursion is that orders of propositions can be generated in such a way as to retain an order-free ontology. As Griffin correctly summarises:

We see, thus, how the multiple relation theory was built in at the bottom of the theory of orders, and how the theory of orders was expected to

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<sup>55</sup> I believe Landini (1996d) was the first to explicitly state that the multiple-relation theory in *Principia* gives a recursive definition of truth in order to generate orders of judgements without orders of entities.

emerge from the theory of judgement. Without it, Russell's ramified theory of types and orders, interpreted along realist lines, as Russell was wont to interpret things, would have resulted in perhaps the most baroque ontology ever devised by the philosophical imagination. Armed with the multiple relation theory, however, the ramified theory could be built up epistemologically, on the basis of a very simple and attractive commonsense ontology.

(Griffin, 1985, p. 217)

Ontologically speaking, *Principia* thus avoids stratifying propositions into different orders and is only committed to the constituents of judgements. There are individuals and universals (qualities and relations), but as propositions are incomplete symbols and hence do not feature in Russell's ontology, there is no need (at least in regard to the order part of the order-type hierarchy) for any ontological type distinctions.

Judgements are composed of their constituents, but these constituents are of the same order, whether they feature in elementary judgements or  $n^{\text{th}}$ -order judgements. The order distinctions are provided by the type of truth-value a judgement can be assigned. Ramification of the hierarchy of types thereby achieves what it had lacked in its 1908 incarnation: a coherent philosophical justification.<sup>56</sup>

Now, however, it seems that a problem arises which is the converse of that in 'Mathematical Logic'. In that system, the hierarchy of orders was an unjustified *ad hoc* attachment, whereas the hierarchy of types was generated by the formal grammar of the substitutional calculus. In *Principia*, the multiple-relation theory takes care of orders, but substitution features nowhere in the system; indeed it *cannot*, because it requires an ontology of propositions which must be abandoned if the multiple-relation

<sup>56</sup> Weiss (1995) raises doubts over the intended role of the multiple-relation theory in both *Principia* and *Theory of Knowledge*. In reply to his complaint that it is 'unclear' how the multiple-relation theory is to provide the foundations for the ramified type-theory of *Principia*, we can observe that it is far less clear what other role could be assigned to the theory in a work on the foundations of mathematics. In regard to the 1913 version of the theory, Weiss points out a significant difference in that Russell intended to extend the theory so as to include an account of molecular judgements which had been explicitly excluded in the earlier statement of the theory (See Weiss, 1995, p. 270). Unfortunately, *Theory of Knowledge* was abandoned just at the point when Russell was ready to turn his attention to molecular judgements. Whatever further divergences from his earlier theory might have emerged had this work been completed, however, it seems wildly implausible that Russell would have been willing to sacrifice the theory of truth underpinning the hierarchy of orders needed to guarantee the consistency of his mathematical logic.



theory is to be upheld. So it now seems that, with the demise of propositions, Russell gains a justification of orders while simultaneously losing the justification for types.

To find out if this really is so, we must examine more closely just what Russell takes a 'propositional function' to be in *Principia*. Russell and Whitehead's justification for the hierarchy of types of functions is often referred to as the 'direct inspection' argument; the plausibility of types is to be revealed by a direct inspection of the nature of a propositional function.<sup>57</sup> Such direct inspection of functions will reveal, according to Russell and Whitehead, that:

not only is it impossible for a function  $\phi\hat{z}$  to have itself or anything derived from it as argument, but that, if  $\psi\hat{z}$  is another function such that there are arguments  $a$  with which both " $\phi a$ " and " $\psi a$ " are significant, then  $\psi\hat{z}$  and anything derived from it cannot significantly be argument to  $\phi\hat{z}$ .

(Whitehead and Russell, 1910, p. 50)

The reason they give in support of this claim is that it follows from consideration of the very nature of a function. A propositional function is introduced in *Principia* as 'something which contains a variable  $x$ , and expresses a *proposition* as soon as a value is assigned to  $x$ . That is to say, it differs from a proposition solely by the fact that it is ambiguous' (Ibid., p. 41). It is this intrinsic ambiguity of the function which Whitehead and Russell now invoke as justification for the construction of the hierarchy of types, arguing that 'if [a function] is to occur in a definite proposition, it must occur in such a way that the ambiguity has disappeared, and a wholly unambiguous statement has resulted' (Ibid., p. 50).

Clearly there is something similar to the Fregean hierarchy of functions (and concepts) at work here.<sup>58</sup> Frege also justifies the hierarchy of types through appeal to the incompleteness (or ambiguity) of functions. For Frege, however, this is an ontological distinction as much as a logical one, 'founded deep in the nature of things' (Frege, 1891, p. 41). Furthermore, this distinction is the very distinction which Russell had been steadfastly resisting since the *Principles* on the grounds that it seems to directly contradict the doctrine of the unrestricted variable. It should also be noted,

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<sup>57</sup> See Whitehead & Russell, 1910, p. 50.

however, that Russell's own attempt to reconcile the doctrine of the unrestricted variable with type-theory—namely, the substitutional theory—is also suggested by *Principia*'s remarks on the hierarchy of functions. Russell is no doubt attempting to find, in the absence of the substitutional theory, an account of the hierarchy of types which will track the type-distinctions generated by the formal grammar of substitution. Certainly, the resulting solution to the paradox of predication is more or less equivalent. The substitution required to form the equivalent paradox in the substitutional theory would be (as we saw above) something like  $p/a(p/a)$ , which is simply ungrammatical. The (attempted) substitution is ungrammatical because it attempts to substitute an incomplete matrix for an entity, thereby producing the nonsensical 'the result of replacing  $a$  in  $p$  by the result of replacing  $a$  in  $p$  by...'. Russell surely has this in mind when limiting the kinds of arguments a function may take by appealing to the incompleteness or ambiguity of the function. Nonetheless it is far from evident that Russell's argument here can be invoked to perform the same kind of reconciliation of type-theory with the doctrine of the unrestricted variable which is effected by the logic of substitution. To do that, Russell would need to show that all functions, if they have any ontological status at all, must have *the same* ontological status.<sup>59</sup>

Some support for the suggestion that Russell did not "ontologize" propositional functions at all comes from his and Whitehead's characterisation of a function as 'not a definite object', but 'a mere ambiguity awaiting determination' (Whitehead & Russell, 1910, p. 50). Indeed this ought to follow from the rejection of propositions as entities coupled with the claim that a propositional function 'differs from a proposition solely by the fact that it is ambiguous: it contains a variable of which the value is unassigned' (Ibid., p. 41). If we take these passages seriously, it may be plausible to understand Russell's intention as being to take functions as

<sup>58</sup> This point is also made by Landini, 1996d, p. 310.

<sup>59</sup> One option, urged by Landini (1996d & 1998), is to interpret propositional functions as schematic letters which are given assignments in a nominalistic semantics that must comply with type-distinctions (of the Fregean kind). In this way, a function may occupy a subject position in the formal calculus so long as it carries the correct type-index to represent its (type-stratified) predicative status in the semantics. On such an interpretation, propositional functions will not be entities as such; they are open formulae of the object-language whose values will be *statements*. Type-distinctions at the ontological level are thereby avoided. Obviously this relies on accepting Landini's contentious claim that object-language and meta-language can be coherently differentiated in *Principia*. However, Landini's reading receives some support from the recognition, made earlier in this chapter, that Russell distinguished between propositions and statements in 'On Insolubilia'.

linguistic items—open expressions which are themselves just incomplete parts of ‘incomplete symbols’.<sup>60</sup> As yet, however, this is not sufficient to thwart Quine’s accusation of reification. Quine holds that Russell attempted to reduce classes to symbolic conveniences but only succeeded in reducing them to type-stratified platonic attributes. That criticism is not inconsistent with maintaining that Russell mistakenly *thought* that propositional functions were purely linguistic; Quine’s criticism in fact turns on the attribution of such confusion to Russell. What is required, in order to denounce Quine’s criticism as spurious, is evidence that Russell had a way of treating propositional functions which would allow him to avoid the kinds of ontological commitments that Quine attributes to him.

Clearly equivocation occurs in Russell’s use of the phrase ‘propositional function’ in *Principia*. In passages such as those just quoted, Russell takes a propositional function to be an open formula of the formal calculus. At other times, however, propositional functions are treated as predicate variables which Russell happily quantifies over. Russell did not need Quine to point out the ontological consequences of such unrestrained quantification. As early as 1906 he had noticed that ‘whatever can be an apparent variable must have some kind of being’ (Russell, 1906e, p. 106).<sup>61</sup> In such cases, Russell is surely taking propositional functions to stand for universals. If these are to be restricted in such a way as to track the type-distinctions generated by substitution, there will be no choice but to construct a hierarchy of functions. It must be inadmissible for certain functions to take the same arguments as certain other functions if the introduction of logical types is to serve the purpose for which it is invoked. Taking functions to stand for universals, this can only be done by dividing universals into different types of entities. Viewed from this perspective, Russell is in the (somewhat ironic) position of having to abandon the doctrine of the unrestricted variable in order to proxy the grammar of the substitutional theory which grew out of that doctrine. Were we to go the other way, however, and take Russell seriously in those cases where he treats propositional functions as open formulas whose values are *statements*,<sup>62</sup> type-distinctions can once again be treated as

<sup>60</sup> It is interesting to note that Frege also complained that it was difficult to determine whether Russell intended functions to be signs or entities in *Principia*. See his letters to Jourdain in Frege, 1980, pp. 78ff.

<sup>61</sup> This was first pointed out by Lackey in Russell, 1973, p. 134.

<sup>62</sup> As Landini does: see note above.

grammatical strictures rather than ontological categories. Whether Russell intended this or not is by no means easy to determine; the least one can say here, however, is that there does seem to be a route left open in *Principia* for reconciling the hierarchy of types with the desire for unrestricted variation.

Unfortunately, of course, history was to be unkind to Russell on, as it were, another flank. The multiple-relation theory of judgement was Russell's attempt to generate orders without excessive multiplication of his ontology. Famously, the theory was a quite remarkable failure. In the next chapter, we will examine Wittgenstein's criticisms which led to Russell abandoning the theory in 1913. Before doing so, however, we will conclude this chapter by returning to the issue of how Russell's old problem of the unity of the proposition reappeared in a new guise to haunt the multiple-relation theory.

### 2.10 The Unity of the Proposition Returned to.

Having rejected propositions as ontological entities in an effort to defeat the paradoxes of propositions, Russell is left in urgent need of some account of how those things which *are* granted ontological status (i.e., the constituents of what were formerly held to be propositions) are united into truth-bearing complexes. As we have seen, Russell's solution is now that they are united in judgement:

Thus "the proposition 'Socrates is human'" uses "Socrates is human" in a way which requires a supplement of some kind before it acquires a complete meaning; but when I judge "Socrates is human," the meaning is completed by the act of judging, and we no longer have an incomplete symbol.

(Whitehead and Russell, 1910, p. 46).

Explaining just how the act of judging would achieve such completion of the meaning of our prospective proposition, however, proved to be harder than Russell had originally envisaged. In the 1912 *Problems of Philosophy* version of the theory, judgement is a many-placed relation holding between the constituents of the

judgement, where one of those constituents will be the judging subject. So, for example, Othello's belief that Desdemona loves Cassio is a complex composed of Othello, Desdemona, Cassio, and the universal *love*, all of which are to be united by the four-place relation of belief.<sup>63</sup> In Russell's preferred method of symbolism:

$$B\{o, d, L, c\}.$$

An obvious problem with this analysis (the so-called 'narrow direction problem') is that there is a clear difference between Othello's belief that Desdemona loves Cassio and Othello's belief that Cassio loves Desdemona. Yet in both cases Othello is multiply-related to precisely the same objects. In 1912, Russell thought this problem could be overcome by relying on the 'sense' (or 'direction') of the belief-relation.<sup>64</sup> In this way, the subjects of the relation would be ordered appropriately in each of Othello's beliefs regarding the love between Desdemona and Cassio. In other words, the judging relation (belief) itself places the constituents of the judgement in the desired order.

As far as the problem of the unity of the proposition as it features in the *Principles* goes, this is only a minor improvement. The problem there was that, after analysis, we are left with a series of objects that lack any apparent property which will unify them. Exploiting the notation of the multiple-relation theory, we can characterise the situation in the *Principles* by saying that a proposition after analysis will be of the form:

$$\{x_1, x_2, \dots, x_n\}.$$

We are left with an (unordered) set of atoms (where one of these atoms is a universal), but lack any convincing account of what will ensure that these atoms form something

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<sup>63</sup> Russell, 1912a, pp. 72-73.

<sup>64</sup> See Russell, 1912a, p. 73. In an earlier version of the theory, Russell attempted to resolve the problem by attaching the sense to the subordinate relation which the subject is related to—the relation between Desdemona and Cassio in our example. He retracted it in 1912, however, realising that the subordinate relation, being unasserted in the overall complex, is not a 'relating relation'. In other words, the sense of the relation does not survive once the relation is treated as a term: 'The relation "loving", as it occurs in the act of believing, is one of the objects—it is a brick in the structure, not the cement' (Russell, 1912a, p 74).

more than a list. In *The Problems of Philosophy* the hope is that this unity can be established by supplementing the set of objects with a unifying relation of judgement:

$$J\{S, x_1, x_2, \dots, x_n\}$$

where not only will one of the atoms be a universal but, in addition,  $S$  will be the judging subject. However, as Russell was soon to realise, this is also unsatisfactory. How is the act of judging to order the constituents correctly? The act of judgement is not akin to rearranging linguistic signs which have been previously disarranged. This may be possible through an observation of the grammatical categories of the signs involved, but these grammatical categories do not run deeper than the linguistic expressions of Russellian complexes.<sup>65</sup> The arrangement of the constituents of judgements is not to be determined by linguistic considerations in Russell's philosophical logic.

The *Theory of Knowledge* manuscript of 1913 shows Russell attempting to find a way around the difficulty through the introduction of 'logical forms' as elements in the judgement. Russell makes a distinction between, what he calls, *permutative* and *non-permutative* judgements; a judgement is permutative if the content of the judgement is altered by permuting its objects and is non-permutative otherwise.<sup>66</sup> So, for example,  $S$ 's judgement that John is related to Mary is non-permutative, as its truth or falsity will be unaffected by the permutation of John and Mary in the judged complex while  $S$ 's judgement that Russell wrote *Theory of Knowledge* will, for obvious reasons, count as permutative. Permutative complexes, then, are complexes prone to the narrow direction problem. Russell now characterises the general form of a non-permutative judgement complex as:

$$J\{S, F, x_1, x_2, \dots, x_n\}.$$

<sup>65</sup> See Russell, 1913, p. 87. This point is also made by Landini (1991). Griffin (1985) points out another major problem with the 1912 theory: bestowing the unifying powers on the judgement-relation suggests that it is the act of judging which actually *does* the arranging of the constituents of the judgement. But if  $S$  judges that  $aRb$  and  $aRb$  is actually the case, then the judgement has added nothing to the order of  $aRb$ ; alternatively, if  $aRb$  is not the case, then no amount of judging will provide the order we are seeking (see Griffin, 1985, p. 221; c.f., Russell, 1913, p. 116).

<sup>66</sup> See Russell, 1913, p. 144.

Here, ' $\mathcal{J}$ ' stands for the judgement-relation, ' $S$ ' for the judging subject, ' $F$ ' for the *form* of the judgement, and ' $x_1, x_2, \dots, x_n$ ' for the objects to which the subject is to be severally related in judgement.<sup>67</sup> Russell is somewhat vague as to the exact status of logical forms in *Theory of Knowledge*. At first sight, the logical form would appear to be complex; for example, the logical form of Othello's belief that Desdemona loves Cassio appears to have a structure, as we can see if we introduce it into our, by now familiar, belief-complex (where ' $xRy$ ' represents the logical form):<sup>68</sup>

$$B\{o, xRy, d, L, c\}.$$

This, of course, will be hopelessly regressive—all the problems concerning the structure of the original judgement are just redistributed onto the structure of the logical form. Russell's response is to deny that it makes sense to say of the logical form that it *has* a structure as it is, in fact, the structure (or form) *of* the judgement: 'In a sense, it is simple, since it cannot be analysed. At first sight, it seems to have a structure, and therefore to be not simple; but it is more correct to say that it *is* a structure' (Russell, 1913, p. 114).

The introduction of logical forms into the *permutative* judgement-complex will not alone be enough to secure a solution to the narrow direction problem. In addition, Russell invokes the notion of position. Hence, not only must Othello's belief that Desdemona loves Cassio conform to a certain specified logical form, but also the constituents of that belief must have a definite position within that logical form. Russell therefore introduces into a complex,  $\gamma$ , the positions  $C_1, C_2, \dots, C_n$ , corresponding to the objects  $x_1, x_2, \dots, x_n$ . Russell symbolises the complex as:

$$(1\gamma) (x_1C_1\gamma \cdot x_2C_2\gamma \cdot \dots x_nC_n\gamma).^{69}$$

<sup>67</sup> See Russell, 1913, p. 144.

<sup>68</sup> Hochberg (2000) has " $(\exists\emptyset)(\exists y)\emptyset y$ " as the logical form of an atomic complex  $Fa$ . Russell does characterise the form of dual complexes as 'something having some relation to something' (Russell, 1913, p. 114). As we shall see below, this suggests a hopeless circularity in that quantified judgements are intended to be higher order judgements whose truth is defined cumulatively in terms of quantifier-free judgements. A further confusion is apparent in Russell's equivocation over the term "form". "Something has some relation to something" may be construed as a general fact (as Hochberg chooses to construe it) or as the form *of* a fact (an interpretation which seems closer to what Russell most commonly intends).

<sup>69</sup> See Russell, 1913, p. 147. This formula is to be read as, 'the complex,  $\gamma$ , such that  $x_1$  has position  $C_1$  in  $\gamma$ ,  $x_2$  has position  $C_2$  in  $\gamma$ , and ...  $x_n$  has position  $C_n$  in  $\gamma$ '. In accordance with the 1905 theory of

No position will be required for the subordinate verb in this complex, according to Russell, because the position of that verb will be determined by positions of the entities it relates. In other words, the complex,  $\gamma$ , can be denoted either by the symbol " $R(x_1, x_2, \dots x_n)$ " or by the symbol " $(\forall \gamma) (x_1 C_1 \gamma \cdot x_2 C_2 \gamma \cdot \dots x_n C_n \gamma)$ ".<sup>70</sup>

This approach, fraught with difficulties though it is, may go some way towards countering the narrow direction objection. It illustrates clearly, however, the amount of work that the act of judgement will have to do in the multiple relation theory. Russell's problems with unities are only exacerbated by the expulsion of propositions from his ontology. Furthermore, a central problem with this account is merely a new version of the central problem with the *Principles* account. There is nothing about the nature of the constituents of the judgement themselves which ensures that they will respect the unity of the proposition (or judgement) in the way that, for example, Frege's concepts do. The unity of the proposition hinges, once again, on no more than Russell's stipulations.

The multiple-relation theory, as we shall see in detail in the next chapter, was not destined to last. Wittgenstein's criticisms of the theory left Russell reportedly 'paralysed' (Wittgenstein, 1995, p. 33) and caused Russell to permanently halt work on the *Theory of Knowledge* manuscript.<sup>71</sup> Geach (1957) directed an equally damning attack on the theory which is widely perceived as being derived from Wittgenstein's critique. Following the publication of *Theory of Knowledge* in 1984, some interest in the theory has been revived but this has been largely restricted to examinations of Russell's book as a means of either gleaning further understanding of the exact nature of Wittgenstein's objections or shedding light on the role played by the multiple-relation theory in *Principia*. The consensus view, then, is that the theory was simply a failure; resorting to the mind of the judging subject as the provider of the essential unity of propositional content amounts to not really explaining that unity at all. As Griffin puts it:

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descriptions, and Russell's 'no-propositions' theory in *Principia*, the above complex is an incomplete symbol.

<sup>70</sup> See Russell, 1913, p. 147.

<sup>71</sup> In 1914, Russell once again conceded to Bradley that his account of unities was so far inadequate but suggested that he now looked to Wittgenstein for solutions: 'Chiefly through the work of an Austrian pupil of mine, I seem now to see answers about unities; but the subject is so difficult & fundamental that I still hesitate' (Russell to Bradley, 30/1/1914, in Bradley, 1999, p. 182).

Russell, like many philosophers before him, was using the mind as a pseudo-explanatory device: processes which otherwise might seem to be entirely mysterious were referred to the mind, where mystery was normal. In this way problems were not solved so much as relegated.

(Griffin, 1993, p. 175)

It might be concluded from this that the problem of the unity of the proposition was intractable for Russell. Most commentaries on the topic would certainly seem to support such a reading. The main reason for this, however, is that they move swiftly from the *Principles* account to the multiple-relation theory, with little or no consideration of developments in between. Having eschewed this pattern in favour of a historically accurate study of Russell's logical developments, we are now in a position to see the situation more clearly. Russell lacked a genuine account of the unity of the proposition in 1903. Contrary to a widespread belief that he never adequately faced the problem, we have seen that he had a genuine solution in the form of the substitutional theory. The tragedy for Russellian logicism lies in the fact that he abandoned substitution (and propositions) in favour of the multiple-relation theory of judgement. In doing so he solved one set of problems at the expense of another: namely, 'all the hoary difficulties about the one and the many' (Russell, 1906b, p. 189). In effect, Russell had provided his mammoth demonstration of the logicist thesis that all mathematics could be captured within the realm of logic,<sup>72</sup> but had left unresolved the question of what logic itself is. In 1912, Russell set to work on addressing the issue directly in a manuscript entitled 'What is Logic?'. After two days of inconclusive struggles, however, the manuscript was aborted. Writing to his lover, Lady Ottoline Morrell, he confessed: 'I can't get on with "what is logic?", the subject is hopelessly difficult, and for the present I am stuck. I feel very much inclined to leave it to Wittgenstein' (Russell to Morrell, quoted in Russell, 1992, p. 54).

In the next chapter we shall see how, in the eyes of his student, the nature of logic was held to be vastly different to the ideas underpinning *Principia*. The theory of

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<sup>72</sup> Many would, of course, now contest the thesis in this form in the light of Gödel's (1931) incompleteness result which demonstrated the existence of undecidable arithmetical propositions within the system of *Principia*. This point will be returned to later in this thesis (see Chapter 4, below).

logic which had taken Russell the best part of a decade to construct was about to be pulled out from under his feet.

## 3.

## What is Logic?

## Wittgenstein's Critique of Russell

Russell's work on his *Theory of Knowledge* manuscript was brought to an abrupt halt in the summer of 1913. *Theory of Knowledge* was never finished and remained unpublished in Russell's lifetime. Russell aborted the project in the face of Wittgenstein's criticisms of the multiple-relation theory—criticisms which left Russell, reportedly, 'paralysed'.<sup>1</sup> The criticisms, Russell confessed to his lover, made the work he was attempting in *Theory of Knowledge* 'impossible for years to come probably' (Russell, letter to Lady Ottoline Morrell, 20/6/13, cited in Editor's introduction to Russell, 1913, p. xx).<sup>2</sup> Unfortunately, there is no precise record of the nature of Wittgenstein's objection. The only surviving clues are to be found in the correspondence between the two men, and the various often cryptic remarks located in their published and unpublished writings from the period in question. Unsurprisingly, therefore, attempts to ascertain the exact nature of Wittgenstein's attack on the theory have tended to deviate from one another quite dramatically.

A shortcoming of most accounts has been the failure to link Wittgenstein's criticisms of the multiple-relation theory with what he has to say about the theory of types in his early work. Some commentators have insisted on locating Wittgenstein's objections to the multiple-relation theory in the context of Russell's development of type-theory,<sup>3</sup> but few have taken Wittgenstein's own remarks on types seriously. By

<sup>1</sup> See Wittgenstein, 1995, p. 33.

<sup>2</sup> Russell later wrote to Morrell, discussing Wittgenstein's criticisms in even stronger terms: 'His criticism ... was an event of first-rate importance in my life, and affected everything I have done since. I saw he was right, and I saw that I could not hope ever again to do fundamental work in philosophy' (Russell, letter to Lady Ottoline Morrell, 4/3/1916, in Russell, 1968, p. 57).

<sup>3</sup> Most notably, Sommerville, 1981; Griffin, 1980, 1985 & 1986; & Landini, 1991.

contrast, those who do address Wittgenstein's remarks on the theory of types often exhibit an opposite failing—they fail to provide an adequate account of the subtleties of the ramified theory of types as it features in *Principia*.<sup>4</sup>

In the first half of this chapter, I will suggest an interpretation of Wittgenstein's criticisms of the multiple-relation theory which will also serve to elucidate the passages of the *Tractatus* which deal with the theory of types and Russell's paradox. My intention in doing so is to reveal a crucial difference between Russell and Wittgenstein regarding their understanding of the nature of logic and its relation to linguistic and ontological concerns. Furthermore, we shall see that Wittgenstein's criticisms of the theory of judgement and his remarks on the Russell paradox and the theory of types are not isolated and independent reactions to Russell's philosophical and logical doctrines but, rather, constitute a consistent and thematic philosophical critique of Russell's logical theory. As a consequence, the elucidation of Wittgenstein's criticisms of the multiple-relation theory will serve to throw into sharper relief some of the notoriously enigmatic passages in the *Tractatus*.<sup>5</sup> In the second half of the chapter, I will look in detail at the picture theory of meaning contained in the *Tractatus*, paying particular attention to the role of theory in explicating the notion of thought. In addition I will look at how a recent controversy over how to interpret the book may have implications for the issues discussed in this thesis.

### 3.1 Wittgenstein's Criticisms: Some Preliminary Considerations

The multiple-relation theory is often misinterpreted as little more than a straightforward attempt on Russell's part to avoid the problem of 'false propositions'.<sup>6</sup> Put crudely, for the moment, the multiple-relation theory provides an account of

<sup>4</sup> Classic works of Wittgenstein exegesis such as Anscombe, 1971; Kenny, 1973; & Hacker, 1986; for example, whilst all emphasising the need to recognise and understand Wittgenstein's response to Russell's logical theory in order to obtain a proper understanding of the *Tractatus*, are noticeably brief and unsympathetic in their accounts of *Principia*'s ramified type-theory. Ishiguro, 1981, takes more care with her interpretation of Russell.

<sup>5</sup> It is commonly maintained that Wittgenstein's views altered drastically between 1913 and the completion of the *Tractatus* some five years later. Nonetheless, we shall see that Wittgenstein's criticisms of the multiple-relation theory were an important precursor to many of the ideas forwarded in the *Tractatus*.

<sup>6</sup> See, for example, Linsky, 1993, pp. 198-199; Candlish, 1996, p. 109; & Hylton, 1996, pp. 210-212; Weiss, 1995, pp. 263-265; & Hochberg, 2000.

“propositional” attitudes without endorsing any ontology of actual propositions. Prior to his conversion to the multiple-relation theory, Russell had held propositions to be subsistent, non-linguistic, complex entities. Hence a propositional attitude is construed as a dyadic relation obtaining between the proposition and the subject holding the relevant attitude to the proposition in question. Taking propositions, thus reified, as the objects of the propositional attitudes, however, poses something of a problem in the case of false beliefs. Othello’s belief that Desdemona loves Cassio, if understood as a dyadic relation between Othello and the proposition ‘Desdemona loves Cassio’, leaves us requiring a *false* proposition for Othello to be related to. Hence Russell would have to extend his ontology to embrace every false proposition. The multiple-relation theory avoids the problem by replacing the dyadic relation between the holder of a propositional attitude and the proposition in question with a multiple relation between a judging subject and the elements of the judgement—i.e., the individual constituents of what was previously held to be a proposition. So now, instead of being committed to the existence of a false proposition (e.g., ‘Desdemona loves Cassio’), which a subject is to be acquainted with, Russell is only committed to the existence of the objects of the judgement-complex (e.g., *Desdemona*, *Cassio*, *love*, the judging subject—in this case *Othello*, and the relation of judgement—in this case *belief*).

Although it is undoubtedly true that one benefit of the multiple-relation theory is its ability to avoid the problem of false beliefs sketched above (indeed the argument was explicitly invoked by Russell himself in support of the theory),<sup>7</sup> it is a grave error to think that this is the full story, or even the most significant element in the story, of why Russell abandoned propositions in favour of judgement-complexes. We have seen in the preceding chapter that the multiple-relation theory plays a pivotal role in generating the hierarchy of orders which lie at the heart of *Principia*’s formal system. It seems most probable that this element of the theory would have been the one most prized by Russell. As we saw in the last chapter, Russell’s intention was not just to rid his ontology of false propositions; he also intended to expel true propositions in order to erect mathematics on foundations which were not prone to the same paradoxes which had led to the downfall of the substitutional theory. The mechanism by which

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<sup>7</sup> See, for example, Russell, 1912a, p. 72.

he sought to lay those foundations, however, we have seen to be plagued with difficulties.

The first problem was the so-called ‘narrow-direction problem’. If, for example, Othello’s belief that Desdemona loves Cassio is a complex consisting just of Othello, Desdemona, Cassio, the relation of *love*, and the relation of *belief* (in the symbolism introduced in the last chapter,  $B\{O, d, L, c\}$ ), then what distinguishes Othello’s belief that Desdemona loves Cassio from his belief that Cassio loves Desdemona? Some extra element seems to be required in order to preserve the direction of the judgement. Russell, it will be recalled, attempted to solve the problem in 1913 with the introduction of logical forms and through the provision of a determinate ‘position’ for each element within a judgement complex. The solution is insufficient, however, for dealing with the so-called ‘wide-direction problem’ raised by Wittgenstein. This problem, like the narrow direction problem, concerns the status of the subordinate verb in the judgement complex but the problem is now more severe.

The multiple relation theory requires that all constituents of a judgement-complex are to be suitable terms for the relation obtaining between them. We saw, in the last chapter, that the subordinate verb, being unasserted in the overall judgement, is not a ‘relating relation’. As Russell put it, ‘it is a brick in the structure, not the cement’ (Russell, 1912a, p. 74). But if the subordinate relation is to be treated as a term on a par with the objects it is intended to relate, there is no longer any significant difference between the relation and the objects. The resulting complex, stripped of any clues as to its content, will be of the form

$$J\{S, x_1, x_2, \dots, x_n\}.$$

With no syntactic differences between the constituents of the judgement, we are now confronted with an extension of the narrow direction problem: Not only do we have the problem of distinguishing between Othello’s belief that Desdemona loves Cassio and his belief that Cassio loves Desdemona, we also lack any reason to deny the possibility of Othello’s believing that Love desdemona Cassio.<sup>8</sup> In short, the subordinate relation is forced into playing two seemingly inconsistent roles: it must be

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<sup>8</sup> See Wittgenstein, 1913, p. 96. Wittgenstein uses the example of ‘this table penholders the book’.

both a term in the overall judgement-relation, and the relation which does the relating in the subordinate complex. But, if it fulfils the first requirement, it will lose the relational status which it needs in order to fulfil the second. Consequently, we no longer have any way of securing any structural limitations on the sense of a judgement complex. As Wittgenstein puts it in the *Tractatus*: ‘The correct explanation of the form of the proposition, “*A* makes the judgement *p*”, must show it is impossible for a judgement to be a piece of nonsense. (Russell’s theory does not satisfy this requirement)’ (Wittgenstein, 1922, 5.5422).

In one sense, this is just a recurrence of an old problem for Russell; that of the unity of the proposition. In the *Principles*, Russell had realised that a proposition has an essential unity which is not captured by simply listing its constituents as they are offered by analysis: ‘When a proposition is completely analysed into its simple constituents, these constituents taken together do not reconstitute it’ (Russell, 1903, § 81). Russell may have subsequently abandoned propositions as entities, but the possibility of nonsense being permitted by the multiple-relation theory shows that Russell still has problems in accounting for the unity of the judged content. Nonetheless, the problem does not seem to be any worse for Russell here than it was in 1903. Russell’s answer to the problem then had been to acknowledge that ‘though analysis gives us the truth, and nothing but the truth, yet it can never give us the whole truth’ (*Ibid.*, § 138). One might be tempted to think, therefore, that this later recurrence of the difficulty is not quite the problem that Russell clearly considered it to be. After all, as Griffin asks, ‘why not abandon the requirement that the judgement be significant? Why not simply allow nonsense to be believed? (If the positivists are right, it very often is.)’ (Griffin, 1986, p. 144).<sup>9</sup> In answer to this question, Griffin draws attention to the way in which Wittgenstein first presented to Russell his ‘exact’ objection to the multiple-relation theory in a letter dated June, 1913:

I can now express my objection to your theory of judgement exactly: I believe that it is obvious that, from the prop[osition] “*A* judges that (say) *a* is in the Rel[ation] *R* to *b*”, if correctly analysed, the prop[osition] “*aRb* ∨ ∼*aRb*” must

<sup>9</sup> See also Griffin, 1980, pp. 176-178; & 1985. As will become evident in what follows, I disagree with Griffin’s suggestion that it is possible to judge nonsense. To judge that *Love desdemona Cassio*, or that *the table penholders the book* is, surely, not really to judge anything at all. One *cannot* judge nonsense, because a nonsensical string of words *cannot* be a judgement. Of course, this does not effect our ability to judge that ‘*Love desdemona Cassio*’ is nonsense on these very grounds.

follow directly *without the use of any other premiss*. This condition is not fulfilled by your theory.

(Wittgenstein, 1995, p. 29)<sup>10</sup>

Following Sommerville, Griffin interprets this form of the objection as making a reference to \*13.3 of *Principia*:

$$\vdash :: \phi a \vee \sim \phi a . \supset :: \phi x \vee \sim \phi x . \equiv : x = a . \vee . x \neq a .$$

This proposition shows that if  $\phi a$  is significant, then  $\phi x$  is significant if, and only if,  $x$  is either identical with  $a$  or not identical with  $a$ .<sup>11</sup> The proposition is used in the proof of *Principia*'s \*20.8 which shows that if  $\phi a$  is significant, then the arguments to a function  $\phi \hat{z}$  will be of the same type as  $a$ .<sup>12</sup> Sommerville and Griffin point out that the following proposition will be the dyadic analogue of \*13.3:

$$\vdash :: aRb \vee \sim aRb . \supset :: xRy \vee \sim xRy . \equiv : x = a \& y = b . \vee . x \neq a \& y = b . \vee . x = a \& y \neq b . \vee . x \neq a \& y \neq b .$$

The antecedent of this proposition ( $aRb \vee \sim aRb$ ) simply provides the condition under which  $aRb$  is significant. But, according to Sommerville and Griffin, Wittgenstein's remarks show that this is problematic when analysed on the lines of the multiple-relation theory.  $S$ 's judgement that  $aRb$  will be analysed as  $J\{S, a, R, b, xRy\}$ , where "xRy" stands for the form of the judged complex. Now, Wittgenstein's point is that, from a correct analysis of this judgement, the tautological  $aRb \vee \sim aRb$  'must follow directly *without the use of any other premiss*' (Wittgenstein, 1995, p. 29).

Wittgenstein is correct to say that this requirement is not met by Russell's theory for further stipulations are indeed required; for example, it must be stipulated that  $a$  and  $b$  are individuals, that  $R$  is a relation of the correct order, etc. But, as yet, this does not appear to be any worse than the problems we have already seen in relation to the

<sup>10</sup> That the objection presented in this form delivered the fatal blow to Russell's theory is suggested by Wittgenstein's comment in a letter to Russell of the following month: 'I am very sorry to hear that my objection to your theory of judgement paralyses you. I think it can only be removed by a correct theory of propositions' (Wittgenstein, 1995, p. 33).

<sup>11</sup> See Whitehead and Russell, 1910, p. 180.

<sup>12</sup> \*20.8 is then subsequently used in the proof of \*20.81 which shows that if  $a$  is a significant argument for two functions, then the two functions are of the same type (see Whitehead & Russell, 1910, p. 209).

theory. Why, then, would this objection have been so crushing for Russell? Griffin gives the following answer:

Why won't Wittgenstein allow us these stipulations? Because to make them would require further judgements. We are trying to analyze what is supposed to be the simplest type of elementary judgement. But to do so would seem to involve us in yet further judgements. Moreover the further judgements required are of an extremely problematic character. For to judge that  $a$  and  $b$  are suitable arguments for a first-order relation is to make a judgement of higher than first-order. Yet, as Russell makes quite clear in *Principia* (pp. 44-6), higher-order judgements are to be defined cumulatively on lower-order ones. Thus we cannot presuppose second-order judgements in order to analyze elementary judgements.

(Griffin, 1986, p. 144)

In other words, we are told. Wittgenstein's criticisms expose a quite devastating inadequacy in the multiple-relation theory—its incompatibility with the ramified theory of types which it was originally invoked in order to explain. That Wittgenstein had this in mind is also suggested, according to Griffin, by an earlier letter to Russell from January, 1913, where Wittgenstein says, '[I]f I analyse the prop[osition] Socrates is mortal into Socrates. Mortality and  $(\exists x, y) \varepsilon_1(x, y)$  I want a theory of types to tell me that "Mortality is Socrates" is nonsensical, because if I treat "Mortality" as a proper name (as I did) there is nothing to prevent me to make the substitution the wrong way round' (Wittgenstein, 1995, pp. 24-25). Russell, it would appear, is stuck in something of a *cul-de-sac*: the multiple-relation theory does not seem capable of blocking nonsensical judgements which violate type distinctions: Wittgenstein's point, if we are to believe Sommerville and Griffin, is that this can only be avoided by invoking the theory of types as a means of correctly structuring judgements. But we know this to be impossible because of the aforementioned incompatibility of the multiple-relation theory with the theory of types. In short, if Russell wanted to keep the theory of types, he would have to abandon the multiple-relation theory of judgement. On Sommerville and Griffin's version of events, this is just what Wittgenstein's critique led Russell to do.<sup>13</sup>

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<sup>13</sup> It may be more accurate to say that Russell, rather than deciding to abandon the multiple-relation theory in order to keep the theory of types, simply remained in a state of uncertainty about each. For example, in the section of his 1918 lectures on 'The Philosophy of Logical Atomism' dealing with the

Ingenious though the Sommerville/Griffin interpretation of Wittgenstein's criticisms is, I do not find it convincing. The account of ramified type-theory given in the preceding chapter showed that the type part of the ramified hierarchy has no significant connection with the multiple-relation theory of judgement (which was shown to be responsible purely for the order part of the hierarchy). The kinds of type distinctions Wittgenstein suggests are called for in order to prohibit nonsensical pseudo-judgements such as Othello's purported belief that Love desdemona Cassio do not require the multiple-relation theory for their generation but, rather, are intended to be justified by the direct inspection argument in *Principia*.

Griffin holds that the direct inspection argument is insufficient, as it stands, for furnishing Russell with the kinds of type-distinctions that Wittgenstein's objection shows to be needed if the constraint of meaningfulness is to be met. Only when supplemented by the multiple-relation theory, he argues, can the notion of acquaintance provide the necessary distinctions. His argument (following Sommerville's) is that we cannot be acquainted with type *differences*: '[E]very act of acquaintance with a logical object is of a different logical type to an act of acquaintance with a different logical object' (Griffin, 1985, p. 243). Griffin concludes that no single act of acquaintance can take in more than one logical object, hence the very idea of acquaintance with type differences between two or more logical objects must be out of the question. Having decided that acquaintance will not help, Griffin takes it that the weight of providing the necessary distinctions will be forced back onto the act of judgement: 'In Russell's epistemological system of 1910-1913, the type distinctions required for judgement could *only* be obtained by means of prior judgement' (*Ibid.*).

Griffin's recognition of the distinction between types and orders, though welcome, is not fine grained enough to do justice to *Principia*'s formal grammar. 'Acts of acquaintance' do not divide into types in the sense that acts of judgement divide into orders in *Principia*'s explanation of ramified type-theory. Judgements

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problems involved in describing the logical form of belief, Russell says: 'I hope you will forgive the fact that so much of what I say to-day is tentative and consists of pointing out difficulties. The subject is not very easy and it has not been much dealt with or discussed ... one has to be content on many points at present with pointing out difficulties rather than laying down quite clear solutions' (Russell, 1918b, pp. 226-227). A similar lack of confidence pervades his discussion of types in the 1919 *Introduction to Mathematical Philosophy*: 'Now the theory of types emphatically does not belong to the finished and certain part of our subject: much of this theory is still inchoate, confused, and obscure' (Russell, 1919a, p. 135).

divide into orders corresponding to the kind of truth (or falsehood) applicable to them; acquaintance (as a mental occurrence independent of, and more primitive than, judgement) is not something to which we can attribute a truth-value, and hence the justification for its division into a hierarchy is unfounded.

With this in mind, it is clear that the kinds of distinctions Griffin's Wittgenstein demands for the avoidance of nonsense are type, rather than order, distinctions. The wide direction problem shows that some distinction in type must be made between, say, a dyadic relation and its referents and relata; the relational status of the relation must survive analysis if we are to fortify the theory of judgement against the possibility of admitting nonsense. There is no reason to share Griffin's insistence that such distinctions will rely on the very theory of judgement they have been invoked to serve, however. Types are fixed independently of orders, and there does not seem to be any reason for assuming that types cannot also be fixed prior to the generation of orders. Hence the kinds of distinctions which Wittgenstein calls for seem to be adequately provided for. Why, then, was Wittgenstein's objection so devastating for Russell? In order to adequately answer this question we must first turn our attention to Wittgenstein's remarks on the theory of types and the Russell paradox in the *Tractatus*.

### 3.2 Wittgenstein on Types

Russell spent the best part of a decade working out a satisfactory solution to the paradox that bears his name. In the *Tractatus*, Wittgenstein claims to have 'disposed' of the same contradiction in just a few short paragraphs. In addition, and in the same paragraphs, Wittgenstein claims to make short work of the theory of types. After briefly remarking that the theory must be wrong because Russell 'had to mention the meaning of signs when establishing the rules for them' (Wittgenstein, 1922, 3.331), he goes on to inform us that 'the whole of the "theory of types"' amounts to no more than the realisation that '[n]o proposition can make a statement about itself, because a propositional sign cannot be contained in itself' (Ibid., 3.332). The 'disposal' of the Russell paradox is then outlined in the following passage:

The reason why a function cannot be its own argument is that the sign for a function contains the prototype of its argument, and it cannot contain itself.

For let us suppose that the function  $F(fx)$  could be its own argument: in that case there would be a proposition ' $F(F(fx))$ ', in which the outer function  $F$  and the inner function  $F$  must have different meanings, since the inner one has the form  $\phi(fx)$  and the outer one has the form  $\psi(\phi(fx))$ . Only the letter ' $F$ ' is common to the two functions, but the letter by itself signifies nothing.

This immediately becomes clear if instead of ' $F(Fu)$ ' we write ' $(\exists\phi) : F(\phi u)$  .  
 $\phi u = Fu$ '.

That disposes of Russell's paradox.

(Wittgenstein, 1922, 3.333)

The first thing one might be inclined to say of the rather obscure argument above is that the only thing it makes immediately clear is that, as a solution to Russell's paradox, it simply fails. If  $\phi u = Fu$ , then, assuming the intersubstitutivity *salva veritate* of identicals, we can surely substitute  $Fu$  for  $\phi u$  in the inner brackets of  $F(\phi u)$ , thereby obtaining  $F(Fu)$  and inviting the formation of Russell's paradox in its intensional form.<sup>14</sup> Presumably, Wittgenstein has something else in mind.

The above passage, although explicitly referring to Russell, requires an awareness of its implicitly Fregean motives if it is to be fully understood. It is a perfect example of what Dummett means when he says that the *Tractatus* is 'virtually unintelligible without an understanding of its Fregean background' (Dummett, 1973a, p. 662). When Russell first informed Frege of his discovery of the contradiction, Frege was quick to point out that the contradiction did not arise in his system in quite the same way that it crippled Russell's.<sup>15</sup> Russell first communicated the contradiction in the form of the attribute of *non self-predicability* which is predicable of itself if, and only if, it is not predicable of itself. For Frege, however, the rigorous distinction between concept and object blocked such a formulation of the contradiction from the outset. A predicate, which is just the name of a concept for Frege, cannot be its own argument; for concepts are essentially incomplete, requiring an object for saturation (the saturation of a concept by the insertion of an object into its argument space maps

<sup>14</sup> In addition, as Ishiguro (1981, p. 46) notes, one might wish to point to the self-applicable operations of Church's lambda-calculus, and conclude that Wittgenstein is simply wrong in thinking that a function cannot be its own argument.

<sup>15</sup> See Chapter 2, above.

that object to a truth-value; an unsaturated expression, Frege insists, cannot refer to a truth-value). In Frege's *Begriffsschrift*, a hierarchy of functions is built into the very idea of what a function is, hence *no* function can take itself as argument:

Now just as functions are fundamentally different from objects, so also functions whose arguments are and must be functions are fundamentally different from functions whose arguments are objects and cannot be anything else. I call the latter first-level, the former second-level, functions. In the same way, I distinguish between first-level and second-level concepts.

(Frege, 1891, p. 38)

Frege describes the difference by saying that an object falls *under* a first-level concept, whereas a first-level concept falls *within* a second-level concept (see Frege, 1892b, pp. 50-51). It will, of course, be noted that the distinctions Frege is making here are, in effect, distinctions in logical type. There is a pivotal difference between Frege's view and Russell's, however. For Frege, such distinctions are largely driven by linguistic considerations. A proposition is the linguistic expression of a thought for Frege, and the parts of the proposition divide into types in accordance with their logical role within a propositional context.

Wittgenstein's understanding of the proposition is indebted to, and is an advancement on, Frege's. Frege realised that the distinction between concept and object meant that the same word would operate differently as a concept word from how it operates as an object word. Hence it would mean something different in each case. Wittgenstein, unlike Frege, does not speculate about any extra-linguistic reason for this, but locates the distinction in the symbolism itself. A concept-expression differs from an object-expression in its logico-syntactic employment:

In order to recognize a symbol by its sign we must observe how it is used with a sense.

A sign does not determine a logical form unless it is taken together with its logico-syntactic employment.

(Wittgenstein, 1922, 3.326-3.327)

Wittgenstein shares the view, common to both Frege and Russell, that a proposition is ‘a function of the expressions contained in it’ (Wittgenstein, 1922, 3.318).<sup>16</sup> But Wittgenstein makes a significant advance; expanding on Frege’s distinction between concept and object, he sees the meanings of the constituent parts of the proposition as being determined by their logical application. A proposition, for Wittgenstein, is a *logical application of signs*. And it is in virtue of this property that a proposition gives expression to a thought, for a thought is just a ‘logical picture of facts’ (Ibid., 3). It follows from this that a proposition coincides with a thought and, furthermore, that this coincidence is a logical one, not a psychological one. To speak of a proposition or thought as the *meaning* of a sentence will not, as a result, need to enlist the metaphysical baggage carried by Russell (or, for that matter, Frege); for the meaning of a sentence, according to the Tractarian picture theory, is nothing more than that which is expressed by the logical application of its constituent signs. That is to say, the meaning of a propositional sign is simply the putative fact which the elements of the sign are used to picture.

When this insight is brought to bear on Russell’s paradox, Wittgenstein’s position becomes clear: A proposition cannot contain itself, for that would require a function to take itself as argument; and that is not impermissible—it is *impossible*. A function cannot take itself as argument because to predicate a function of itself is to turn the predicating function into a different kind of function, which simply means that the function is given a different logical application. If a sign only determines a logical form when taken together with its ‘logico-syntactic employment’, then it follows that, in the propositional sign ‘ $F(Fu)$ ’, the inner and outer  $F$ s *must* have differing logical forms. Or, to put the same point in more explicit terms, the inner and outer  $F$ s must be of differing *logical type*. Adopting Frege’s habit of using parentheses to show the argument spaces requiring saturation in our functions, we can portray the situation thus: one has the form ‘ $F( )$ ’; the other has the form ‘ $F( ( ))$ ’.<sup>17</sup>

<sup>16</sup> Of course, this is slightly misleading in Russell’s case, as it suggests that propositions are linguistic. Russell would probably have preferred to say “a proposition is a function of the entities contained in it”.

<sup>17</sup> See, e.g., Frege, 1891, p. 24. Alternatively, we could make the use of such parentheses explicitly refer to type-indices. Our two functions now become expressible by the signs: ‘ $F^{(0)}$ ’, and ‘ $F^{(0)}( )$ ’, where the first sign stands for a function of individuals and the second is a function of functions of individuals. ‘ $F^{(0)}(F^{(0)}x^0)$ ’ is a well-formed formula in simple type-theory, for example. Wittgenstein’s point, seen from this perspective, is not that there are no logical types, but that types are properties of symbolism which arise naturally out of the application of signs; hence, the theory of types does not need to be stated.

The sign—the letter *F*—is all that the two symbols have in common. But the sign by itself, unemployed and out of context, signifies nothing.

The picture of Wittgenstein which emerges from this discussion is not of one who objects to the whole notion of logical type distinctions in any form, but of one who disputes the plausibility of constructing a *theory* of types in the sense of laying down restrictions on the values a given variable may take independently of the use that variable is put to. Wittgenstein sees type-distinctions as arising naturally out of the logical grammar of ordinary language, and considers an adequate logical calculus to be one that reflects this formal property of language by generating type distinctions naturally out of applications of signs in accordance with the formal grammar of the calculus. It is for this reason that he sees any statement of the theory of types as being superfluous, as type distinctions simply result from the application of signs. Hence the dictum: 'Logic must take care of itself' (Wittgenstein, 1922, 5.473).<sup>18</sup>

It will of course be noted that distinctions in logical type are now firmly located in symbolism. Type distinctions, being distinctions in the kinds of syntactical applications that signs are given, apply to linguistic elements on Wittgenstein's model. This is no doubt why Wittgenstein said of Russellian type theory that '[i]t can be seen that Russell must be wrong, because he had to mention the meaning of signs when establishing the rules for them' (Ibid., 3.33-3.331). On reading the *Tractatus* for the first time in 1919, Russell objected to this criticism and responded thus: 'The theory of types, in my view, is a theory of correct symbolism: (a) a simple symbol must not be used to express anything complex; (b) more generally, a symbol must have the same structure as its meaning' (Russell to Wittgenstein, 13/8/1919, in Wittgenstein, 1995, p. 122). Wittgenstein's response shows that he considered Russell to have misunderstood the full meaning of, not just this point, but also of what Wittgenstein calls the 'main point' of the book,<sup>19</sup> namely the distinction between what can be said and what can be shown: 'That's exactly what one can't say. You cannot prescribe to a symbol what it *may* be used to express. All that a symbol CAN express, it MAY express. This is a short answer but it is true!' (Ibid., p. 125). So, according to Wittgenstein, what a sign can be used to express is something which cannot be said.

<sup>18</sup> There are obvious parallels here with Russell's substitutional theory, in which type distinctions are an unstated feature of the formal grammar (see ch. 2, above).

<sup>19</sup> See Wittgenstein, 1995, p. 124.

but can only be shown. Why does he say this, and what bearing does it have on the theory of types? Some clue is offered in the following passages:

If a sign is possible, then it is also capable of signifying. Whatever is possible in logic is also permitted. (The reason why ‘Socrates is identical’ means nothing is that there is no property called ‘identical’. The proposition is nonsensical because we have failed to make an arbitrary determination, and not because the symbol, in itself, would be illegitimate.)

In a certain sense, we cannot make mistakes in logic.

(Wittgenstein, 1922, 5.473)

Elaborating on this point a few lines later, Wittgenstein adds: ‘the reason why “Socrates is identical” says nothing is that we have not given *any adjectival* meaning to the word “identical”’ (Ibid., 5.4733).

The point that Wittgenstein is urging in these passages is that nonsensical sentences such as ‘Socrates is identical’ are not nonsensical because they have violated any prescribed limits of logical syntax. They are nonsensical because we have failed to mean anything by them; in other words, they have not been given a significant application. What a sign expresses is revealed in its use, according to Wittgenstein: ‘[w]hat signs fail to express, their application shows. What signs slur over, their application says clearly’ (Ibid., 3.262). The series of signs ‘Socrates is identical’, lacks a meaning simply because it lacks a use: ‘If a sign is *useless*, it is meaningless’ (Ibid., 3.328).<sup>20</sup>

It should be evident that such a conception of the relation between a sign and its meaning is not compatible with Russell’s philosophical logic. For Russell, the meanings which are expressed by linguistic elements are non-linguistic entities (be they simple or complex) which either are, or feature in, judgement complexes.

<sup>20</sup> The question over the extent to which *use* plays a significant role in understanding Wittgenstein’s early philosophy of language is, of course, relevant to the current debate over how one is to understand the *Tractatus*. Against the standard reading of the work as containing an ineffable metaphysics, an increasingly influential alternative reading (advocated most prominently by Cora Diamond and James Conant) has been suggested which holds that Wittgenstein rejected the idea that there could be any ineffable truths or ‘substantial nonsense’. Advocates of this reading have appealed to the (commonly overlooked) role of the use of linguistic items in the *Tractatus* in support of their interpretation and have gone on to stress the resulting continuity between the early Wittgenstein which emerges from their reading and the later Wittgenstein. The merits of this approach to understanding the early Wittgenstein will be discussed below, in the meantime the current discussion can be viewed as applying the Tractarian conception of use which is hinted at in a schematic fashion by the new reading to a particular set of problems addressed by Wittgenstein in the *Tractatus*.

Wittgenstein's emphasis on the *use* of signs as pivotal in determining both their meaning *and* their logical form, is necessarily opposed to Russell's position, which demands that propositions (and, later, judgements) be entirely non-linguistic. The importance of use cannot play the same role in Russell's doctrines, requiring, as it does, the kind of linguistic turn made by Wittgenstein in the *Tractatus*—it is, after all, (orthographic or phonographic) *signs* that are to be *used* in asserting a proposition or judgement, not the entities which they purportedly stand for. With this fundamental difference between Russell and Wittgenstein in mind, we are now in a position to re-examine Wittgenstein's criticisms of the multiple-relation theory of judgement, and to place those criticisms in the broader context of a more general critique of Russell's conception of logic and logical form.

### 3.3 Wittgenstein's Criticisms of the Multiple-Relation Theory of Judgement.

Russell's lectures on 'The Philosophy of Logical Atomism', given in London in 1918 and later published in *The Monist*, are, according to Russell: 'very largely concerned with explaining certain ideas which I learnt from my friend and former pupil Ludwig Wittgenstein' (Russell, 1918b, p. 177). In the section dealing with the logical form of belief, Russell draws attention to the following situation arising from the analysis of beliefs:

[W]hen *A* believes that *B* loves *C*, you have to have a verb in the place where 'loves' occurs. You cannot put a substantive in its place. Therefore it is clear that the subordinate verb (i.e., the verb other than believing) is functioning as a verb, and seems to be relating two terms, but as a matter of fact does not when a judgement happens to be false. That is what constitutes the puzzle about the nature of belief.

(Ibid., p. 225)

Russell goes on to say (of belief and the analysis of its logical form) that: 'I have got on here to a new sort of thing, a new beast for our zoo, not another member of our former species but a new species. The discovery of this fact is due to Mr. Wittgenstein' (Ibid., p. 226). Russell's 'puzzle about belief' does not, at first, seem to

be a particularly striking or even new, problem. If the subordinate verb in the judgement '*A* believes that *B* loves *C*' is to actually relate *B* and *C*, then this is obviously problematic in the case where *A* *falsely* believes that *B* loves *C*; for now we are invoking a non-existent love for *A* to be multiply-related to. Desdemona's love for Cassio is no more admissible into the catalogue of Russellian existents than any false proposition was, and so we must conclude that the subordinate verb in a judgement-complex characterising a false belief does not relate its subjects. Why will this be such a problem? Here Wittgenstein's criticisms take effect.

Clearly the 'puzzle' outlined above is concerned with a form of the wide, rather than the narrow, direction problem; the problem is one of how (or whether) the subordinate relation can serve as a relation and actually *relate* the subjects of the subordinate complex in a judgement, rather than of how it guarantees the 'sense' or direction of the complex. Wittgenstein's objection shows that the place occupied by the subordinate verb in the judgement-complex can only be occupied by a verb (a relating relation) if nonsense is to be avoided and unity maintained. Hence Russell's analysis breaks down, as it requires that the subordinate verb be treated as an object of the same status as the other constituents of the judgement complex. The requirement is not just needed in order to avoid the problems which result from falsely believing that such and such a thing stands in a certain relation to another (or, for that matter, *correctly* believing that such a such a thing is *not* related to another);<sup>21</sup> it is also the impetus behind the multiple-relation theory as it features in *Principia*. The requirement that all the constituents of a judgement stand on an equal ontological footing is what enables the multiple-relation theory to generate orders of propositions at the epistemological rather than ontological level, thus avoiding the outlandish ontology which would otherwise ensue. Wittgenstein's objection, however, pulls the theory up short. The subordinate relation has to be treated as an object if the purposes of the theory are to be met but, if the subordinate relation is treated as an object, then the theory fails on the grounds that it lacks any safeguards against 'Love desdemona Cassio' and the like.

Russell's problems are, in the main, attributable to his insistence on the non-linguistic nature of the proposition (or judgement-complex). This is made clearer by

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<sup>21</sup> The problem will arise equally in the case of *S*'s *correct* judgement that  $\sim aRb$ . Take, for example, Emilia's belief that Desdemona does not love Cassio; what is to actually relate Desdemona and Cassio in this true belief?

comparing his position with that of Wittgenstein, who explicitly takes propositions to be composed of linguistic elements. For Wittgenstein, as we have seen, type distinctions apply to signs, not the referents of those signs;<sup>22</sup> as he phrased it in a letter to Russell: '[t]here cannot be different Types of things! ...all theory of types must be done away with by a theory of symbolism showing that what seem to be *different kinds of things* are symbolised by different kinds of symbols which *cannot* possibly be substituted in one another's places' (Wittgenstein, 1995, pp. 24-25). As a result, Russell's 'puzzle about the nature of belief' is unlikely to be particularly puzzling for Wittgenstein. For him, the necessity of *R*'s occurrence as a verb (as a relating relation) in *S*'s judgement that *aRb* will be no more problematic in the case of Othello's belief that Desdemona loves Cassio than it would be in the case of Emilia's belief that Desdemona does *not* love Cassio, or in the case of my belief that Wittgenstein wrote the *Tractatus Logico-Philosophicus*. In all these cases, the status of the subordinate verb is guaranteed by its role in the proposition; its logico-syntactic employment. There is no need, therefore, to invoke any exotic entity which "*R*" stands for (such as Desdemona's non-existent love for Cassio) and, hence, there is no puzzle to be solved.<sup>23</sup>

A belief, being a variety of thought, will be a 'logical picture of facts' for Wittgenstein (Wittgenstein, 1922, 3). That is to say, it will be a certain logical arrangement modelling an obtaining or non-obtaining fact. That the fact corresponding to Othello's belief that Desdemona loves Cassio does not obtain simply makes the picture (the belief) false. By making the proposition linguistic (or, at least, pictorial), the problem of non-existent entities need not arise. As Wittgenstein says,

<sup>22</sup> This is not to say, however, that the ramified type theory of *Principia* offers an ontological hierarchy of types of entities. That view, however standard it may have become, we have seen to be mistaken. This, I am sure, explains Russell's indignant reply to Wittgenstein, quoted earlier, that he held the theory of types to be 'a theory of correct symbolism' (Russell to Wittgenstein, 13/8/1919, in Wittgenstein, 1995, p. 122). However, Russell's accompanying remark, 'a symbol must have the same structure as its meaning' (*Ibid.*), is telling. It is just this claim which Wittgenstein thinks cannot be justifiably asserted if logical form is to be determined by the *use* of symbols, rather than by any extra-linguistic considerations.

<sup>23</sup> The point is made clearly in the notes that Wittgenstein dictated to Moore in Norway during April, 1914: 'in "aRb", "*R*" is *not* a symbol, but *that* "*R*" is between one name and another symbolizes. Here we have *not* said: this symbol is not of this type but of that, but only: *This* symbolizes and not that' (Wittgenstein, 1914, p. 108). I do not wish to suggest that the picture theory adheres to some form of nominalism. The theory remains silent on the ontological status of universals. What the theory certainly does offer a solution to, however, is Frege's difficulty (discussed in chapter one) of referring to a concept. Picturing a state of affairs will show what is unsuccessfully aimed at when using a term to try and refer to a concept: that is to say, it allows for the concept to retain its predicative or incomplete status by providing referents only for the complete parts of the sentence, but utilising them in such a way as to *show* the relations that hold between them, or the concepts that they fall under.

‘[a] picture represents its subject from a position outside it. (Its standpoint is its representational form). That is why a picture represents its subject correctly or incorrectly’ (Ibid., 2.173). This vital distinction between the proposition and the fact which it represents is also invoked by Wittgenstein in his brief discussion of the multiple-relation theory in the *Tractatus*. After denouncing what he considers to be the erroneous view that apparent ascriptions of propositional attitudes place a proposition in relation to some kind of object (an interpretation which Wittgenstein attributes to the ‘modern theory of knowledge’ of ‘Russell, Moore, etc.’ (Ibid., 5.541)), Wittgenstein gives the following account:

It is clear, however, that ‘*A* believes that *p*’, ‘*A* has the thought *p*’, and ‘*A* says *p*’ are of the form “‘*p*’ says *p*”: and this does not involve a correlation of a fact with an object, but rather the correlation of facts by means of the correlation of their objects.

(Ibid., 5.542)

The important point to draw from this remark is that the pivotal relation concerned here is not that between *A* and *p*, but that between “*p*” and *p*. To know that *A* believes that *p* is simply to know that *A* holds a certain picture or proposition to be true. It is the relation between “*p*” and the possible fact which “*p*” pictures which determines whether “*p*” is true. Wittgenstein, holding a proposition to itself be a fact (Ibid., 3.14), accounts for this correspondence between the proposition and the fact, through the correspondence of their objects. In other words, the logical picture is true if the arrangement of its constituents accurately pictures the arrangement of the objects those constituents go proxy for. With the distinction between the proposition and the fact which it pictures firmly in place, Wittgenstein is immune to the problems raised by Russell’s multiple-relation theory.

I have attempted, in this discussion, to show how Wittgenstein’s rejection of Russell’s theory of judgement is best understood, not as being aimed at a very specific element of the formal system of *Principia*, but as a manifestation of Wittgenstein’s general dissatisfaction with Russell’s conception of logic. Wittgenstein’s target was not, I believe, the alleged failure of the multiple-relation theory to generate the hierarchy of orders, as Sommerville and Griffin suggest. It was the failure of the theory to account for the division of propositional content into parts which will reflect

and preserve its unity and hence debar nonsensical pseudo-judgements such as 'this table penholders the book' or 'Love desdemona Cassio'. This failure resulted from Russell's determination to maintain that the constituents of judgements were extra-linguistic entities, all of which could be treated alike in analysis. Justification for this interpretation of the impact of Wittgenstein's criticisms on Russell is to be found in the few remarks which Russell made on the subject in later work. For example just prior to the 'puzzle about the nature of belief' discussed above, the discovery of which Russell credits to Wittgenstein, Russell remarks that, in a belief-complex, 'both verbs have got to occur as verbs, because if a thing is a verb it cannot occur otherwise than as a verb' (Russell, 1918b, p. 225). In particular, Russell informs us, it is impossible to treat the subordinate verb 'on a level with its terms as an object term in the belief' (Ibid., p. 226). This, as he goes on to acknowledge, is precisely what he had attempted to do prior to Wittgenstein's criticisms of the multiple-relation theory; Russell admits that this is 'a point in which I think that the theory of judgement which I set forth once in print some years ago was a little unduly simple, because I did then treat the object verb as if one could put it as just an object like the terms, as if one could put "loves" on a level with Desdemona and Cassio as a term for the relation "believe"' (Ibid., p. 226). By contrast there is nothing about the relation of belief complexes to the hierarchy of orders mentioned in the same lectures, as one would expect there to be if the Sommerville/Griffin interpretation was accurate, despite the fact that Russell goes on to discuss the theory of types and, specifically, the restrictions on generality required to avoid the Liar paradox, later on in the lectures. Indeed, Wittgenstein did not seem to influence Russell's thoughts on the theory of types (as a purely formal exercise) particularly drastically. He did, however, have a powerful influence on Russell's understanding of logic, convincing him that the propositions of logic consist of tautologies. Russell later characterised this view, perhaps rather bleakly, in his *History of Western Philosophy*, by saying that it reduces mathematical knowledge to being 'all of the same nature as the "great truth" that there are three feet in a yard' (Russell, 1946, p. 860).<sup>24</sup> Wittgenstein's criticisms of the multiple-relation theory of judgement, we can see from the above discussion, play an important role in driving Russell to accepting this conclusion.

<sup>24</sup> Russell's comment, of course, refers not directly to the notion of a tautology, but rather to the derivative Tractarian notion of a mathematical proposition as an "equation" or identity. See Wittgenstein, 1922, 6.2-6.241. Russell clearly states in 1940's *An Inquiry into Meaning and Truth* that he holds logical and mathematical truth to be 'a syntactical concept' (Russell, 1940, p. 140).

Russell's later receptivity towards the Tractarian conception of logic is, of course, the manifestation of a dramatic change of heart. It is no coincidence that, in the section of the *History of Western Philosophy* discussing the topic, Russell resorts to using Mill's distinction between 'real' and 'verbal' propositions. As mentioned in chapter one, above, the distinction corresponds to Kant's distinction between analytic and synthetic propositions with real propositions equating to synthetic ones and verbal propositions equating to analytic.<sup>25</sup> A defining feature of Russell's logicist project was his insistence (contrary to Frege) that logic is a synthetic *a priori* science. This point was crucial for Russell; analytic propositions, being merely verbal (that is true by virtue of their meanings), were not informative. Russell's attempt to reduce mathematics to logic was not an attempt to show that mathematics was uninformative: it was an attempt to show that it had firm foundations. Logical truths, whilst being knowable *a priori*, were not held to be trivial truths by Russell; they were held to be informative truths about an eternal platonic realm of logical objects.<sup>26</sup> The reduction of such truths to analytic propositions was equated in Russell's thought with their reduction to merely verbal propositions laying out necessarily trivial truths about linguistic factors. In holding that logic consists of synthetic (real) propositions, Russell was maintaining that logic was a body of truths about the world. Once, under the influence of the *Tractatus*, he had become convinced that logic consisted of tautologies, he also became convinced that logic (and the mathematics which he had reduced to it) was itself trivial. Such was the conclusion of Russell's 'gradual retreat from Pythagoras' (Russell, 1959, p. 208).

Russell's retreat from Pythagoras had, undoubtedly, begun with the discovery of the paradox in 1901. But Wittgenstein's criticisms of the multiple-relation theory play a more central role in accelerating this retreat than has been commonly realised. What Wittgenstein showed Russell, in a nutshell, was that the kinds of type distinctions required for the preservation of the unity of the proposition left him with two, equally unattractive, options. If Russell chose to maintain his non-linguistic conception of propositional content (i.e., judgement), then he would need to account for the unity of that content by positing type-distinctions amongst the constituents of

<sup>25</sup> See Mill, 1843, Bk. I, Ch. VI, § 4, ft. nt.

<sup>26</sup> The same is of course true of Frege, but his approach was significantly different. Frege accepted that logical truths were analytic and hence sought to show that arithmetic was analytic. In doing so, however, he hoped to show that Kant's understanding of analytic propositions as necessarily trivial was misguided.

judgements, thereby imposing the theory of types onto his ontology in just the way he had sought so hard to avoid in the development of *Principia* (and, most pertinently, in the multiple-relation theory). Were he to withhold those distinctions from the ontological sphere, however, he would have no choice but to reject the thesis that judgements were extra-linguistic complexes composed of the entities which featured in his ontology. Russell had no choice but to abandon *Theory of Knowledge* and, in 1913 at any rate, was left entirely without a solution to the problem.

### 3.4 Wittgenstein on Thought and Language: The Picture Theory

Wittgenstein's conception of a proposition as a picture first appears in his Notebooks in an entry dated 29<sup>th</sup> September, 1914: 'In a proposition a world is as it were put together experimentally. (As when in the law-court in Paris a motor-car accident is represented by means of dolls, etc.)' (Wittgenstein, 1961, p. 7). Of course, the example of representing a motoring accident with dolls, cars, and so on, is deceptive in one way. Dolls have a similarity to people and model cars look like cars prior to their use in modelling the situation in question. It is not this sense in which Wittgenstein holds a proposition to be a picture, for, in the propositional sign "a is to the right of b" it is unlikely that the bearer of the name "a" will look particularly like the first letter of the English alphabet. What Wittgenstein has in mind is that a structural similarity obtains between the picture and the state of affairs which it pictures. This structure, Wittgenstein tells us in the *Tractatus*, is the way in which the elements of a picture are related to one another in a determinate way (Wittgenstein, 1922, 2.15). It is by virtue of this structure that the picture represents, and the possibility of this structure is what Wittgenstein calls 'the pictorial form of the picture' (Ibid.).<sup>27</sup> In order to picture reality, correctly or incorrectly, the pictorial form must be common to the picture and to reality (Ibid., 2.17-2.171).

In the light of the preceding discussion of Wittgenstein's views on the theory of types and the Russell paradox, we can see that the picture theory is a crucial

<sup>27</sup> 'Form', according to Wittgenstein, 'is the possibility of structure' (Wittgenstein, 1922, 2.033). It seems plausible that remarks such as this one have Russell's views in mind; we saw above that Russell himself took "form" to be more or less synonymous with "structure". Wittgenstein's remark that 'a picture cannot, however, depict its pictorial form: it displays it' (Ibid., 2.171) highlights the situation which had caused Russell such problems in 1913 when he sought to preserve the unity of the judgement through the introduction of logical forms.

component in Wittgenstein's rejection of the Russellian conception of logic. The 'disposal' of Russell's paradox suggested at *Tractatus* 3.333 makes an explicit appeal to the use which a linguistic element has within the context of a proposition in fixing the syntactic (in addition to semantic) role of that element. This, we will now see, is quite consistent with the picture theory.

A standard interpretation of the picture theory in the *Tractatus* is that it explains the representational capacity of a sentence by appeal to a prior representational capacity of thought which, in recognition of the threat of infinite regress, is itself explained by appeal to a pre-existent harmony obtaining between language, thought and world. Hence, on this model, the proposition adequately pictures because it has a logical structure which is just a mirror of the logical structure of reality. Logical form, on this model, is very similar to Russellian logical form—it is, essentially, structure. Peter Hacker is perhaps the foremost proponent of this view, which, as can be seen from the following passage, attributes to the *Tractatus* a semantic theory burdened with definite ontological commitments in addition to a devoutly mentalistic philosophy of mind:

The meaning of a subsistent name is the object for which the name stands. The logic-syntactical form of the name, i.e. the combinatorial rules determining its employment, indicate the logico-metaphysical form of the object of which it is the name (i.e. its ontological category). Which particular object out of the totality of objects belonging to a given ontological category is signified by 'A' is not determined by logical syntax but by conventional correlation of name with object. How is the correlation done, and how is meaning (*Bedeutung*) conveyed? The short answer is—the correlation is mental (intentional) and meaning is conveyed by *Erläuterungen*.<sup>28</sup>

(Hacker, 1975, p. 606)

Hacker goes on to say that it is evident that names 'pin' the formal structure of language onto the world, and equally evident that 'the correlation is mental or psychological' (Ibid.). Central to Hacker's interpretation of the Tractarian notion of

<sup>28</sup> .."Erläuterungen" (elucidations) is, as Hacker has noted, a term that Wittgenstein takes over from Frege in the *Tractatus* (Frege's most notable use of the term occurs in his discussion of "the concept horse" in 'On Concept and Object' discussed in Chapter 1, above. A very different interpretation of Frege and Wittgenstein on the status and role of *Erläuterungen* is offered by Conant (2000). Conant's reading of the *Tractatus* is addressed below.

meaning as dependent on psychological processes is his understanding of Wittgenstein's remark that 'the method of projection is to think the sense of the proposition' (Wittgenstein, 1922, 3.11).<sup>29</sup> On Hacker's view, this remark is to be understood as a claim to the effect that sentences (or, indeed, names) have meaning only by virtue of the mental act involved in actually *thinking* the proposition. where such thinking is construed as a mysterious internal process. Hence, for Hacker's Wittgenstein, it is a psychological act which correlates the elements of a picture (i.e.. names) with the elements of what is pictured (i.e., objects), and elucidations (*Erläuterungen*) are ostensive definitions 'seen through a glass darkly' (Hacker, 1986, p. 77). The preceding discussion of Wittgenstein's remarks on the theory of types. however, suggests that Hacker's interpretation cannot be quite correct.

It has been suggested that Hacker's interpretation merely ascribes to the early Wittgenstein the very psychologism which he, following Frege and Russell, was at pains to overthrow.<sup>30</sup> Such suggestions are inaccurate, however. It is important to make a clear distinction between *psychologism* and *mentalism*: anti-psychologism can be, and very often is, agnostic on the question of what mental processes are involved in meaning. What is objected to is the idea that any *logical* features of sentences (paradigmatically, their truth-conditions) could be dependent on the mental processes of individual thinkers. This is quite compatible with at least some degree of mentalism—the view that meanings are private entities which can only be directly known by their owners. Both Frege and Russell subscribed to a vigorous anti-psychologism while retaining some degree of mentalism.<sup>31</sup> Hacker's interpretation of the early Wittgenstein places him in just the same camp; he is construed as adhering to an anti-psychologism regarding logic, and a mentalistic theory of meaning. Indeed. a key element of Hacker's interpretation is that Wittgenstein, in his later period. rejects the philosophy of the *Tractatus* largely because of his 'realization that the

<sup>29</sup> See Hacker, 1986, pp. 74-75 & 1999. The first part of *Tractatus*, 3.11 is: 'We use the perceptible sign of a proposition (spoken or written, etc.) as a projection of a possible situation'.

<sup>30</sup> For example, Conant writes: 'To think that one can fix the meanings of names by means of such an act just is to think that one can fix their meanings prior to and independently of their use in propositions; and it is just this psychologistic conception of meaning that Frege's and early Wittgenstein's respective versions of a context principle are concerned to repudiate' (Conant, forthcoming a, p. 111, end nt. 115).

<sup>31</sup> For Frege, mentalism applies to the realm of "ideas" whilst thoughts, as we examined in chapter 1, are de-psychologised truth-bearers (see Frege, 1918). Russell's mentalism is prominent in his distinction between knowledge by acquaintance and knowledge by description (see Russell, 1910a).

particular form of anti-psychologism in logic which he took over from Frege was misguided' (Hacker, 1986, p. 80).

The accusation that Hacker attributes to the early Wittgenstein an unsupportable psychologism is spurious. But Hacker's interpretation of the picture theory as a mentalistic theory of meaning is equally inaccurate. To see this we only need to observe the redundancy of any appeal to mental acts in the fixing of content once the picture theory has been established. If a propositional sign symbolises its content by virtue of its logico-pictorial form and, in addition, the syntactical status of its constituent signs are to be fixed by their logical role within the context of that propositional sign, then there is no real work left for a mental act to perform. There will, obviously, be a degree of convention involved in determining the referents of names, but this should not be understood as being determined by the mysterious correlation of a word with an object by means of an inward intention. On the contrary, such convention can only be understood as dependent on the uses of linguistic items within the public domain. Furthermore, we must be careful not to conflate the 'method of projection' of a proposition with the correlation of names with their objects. Even if we were to grant that a mental process is required to fix the correlation between names and objects, this would not justify the mentalistic interpretation of Wittgenstein's remark that 'the method of projection is to think the sense of the proposition' (Wittgenstein, 1922, 3.11).

The distinction between sense and reference which Wittgenstein takes over from Frege in the *Tractatus* is essential to the anti-psychologism which Hacker, correctly, notes as common to both Frege and Wittgenstein. The distinction, as one would expect, is not taken over without alteration in the *Tractatus*. In particular, Wittgenstein utilises the distinction between sense and reference to make a further distinction between the semantic roles of names and propositions; propositions have sense and names refer: 'Situations can be described but not *given names*. (Names are like points; propositions like arrows—they have sense)' (Ibid., 3.144). The distinction is then brought out in Wittgenstein's re-phrasing of the Fregean context principle (with the key terms returned to the original German): 'Only propositions have *Sinn*; only in the context of a proposition has a name *Bedeutung*' (Ibid., 3.3).<sup>32</sup> It follows

<sup>32</sup> Ogden translation. The wording of this translation makes the reference to Frege more explicit than the Pears/McGuinness version. McGuinness (1981) admits that the rendering of "Bedeutung" as "meaning" in the Pears/McGuinness translation of this proposition may be misleading, saying: 'I

from Wittgenstein's context-principle, with its reformulation of the sense/reference distinction, that a sentence cannot, as Frege believed, be a complex name.<sup>33</sup> Equally, it follows that we cannot conclude from the assumption of a mentalistic requirement for the fixing of reference in the case of a name that an equivalent requirement must be met for the grasping of the sense of a proposition.

In any case, it is evident that the assumption of a mental act which attaches language to the world through the correlation of names with objects is, itself, unnecessary in the context of the picture theory.<sup>34</sup> A name stands for its bearer because it is used as such. Wittgenstein was perfectly aware of this in the *Tractatus*: for example, it is quite obvious that this is what he has in mind when discussing the difference between "Green" (used as a proper name) and the same word used predicatively in the sentence "Green is green": 'these words do not merely have different meanings: they are *different symbols*' (Wittgenstein, 1922, 3.323). The method of recognizing this difference, Wittgenstein goes on to say, is by observing how the sign is used with a sense (Ibid., 3.325), which follows from his earlier assertion that 'what signs fail to express, their application shows' (Ibid., 3.262). This is merely to recognize the difference between a name and any other subsentential expression through a recognition of how they are used.

Immediately following this last proposition is Wittgenstein's enigmatic remark about elucidations (*Erläuterungen*):

The meanings of primitive signs can be explained by means of elucidations. Elucidations are propositions that contain the primitive signs. So they can only be understood if the meanings of those signs are already known.

(Ibid., 3.263)

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believe it is very clear in Wittgenstein's *Tractatus* that the *Bedeutung* of a name *is* meant to be identical with the object named. Indeed translating "*Bedeutung*" as "meaning", for which I am partly responsible, could be criticized on this ground alone' (McGuinness, 1981, p. 66). Ogden also translates "*Bedeutung*" as "meaning" in this proposition.

<sup>33</sup> See Wittgenstein, 1922, 3.143.

<sup>34</sup> It is, in fact, unnecessary in just about any theory of naming, for the notion of naming already presupposes that we have a clear idea of what names do (i.e., refer). Naming, in this trivial sense, is one of the things that we have prior to any theory of meaning. As Dummett says, 'no one ever had to *argue* that singular terms, in general, have reference' (Dummett, 1993a, p. 40). The contentious issue has always been whether to accept that singular terms have a *sense*. Wittgenstein, in opposition to Frege, held that they do not.

And immediately after this remark comes Wittgenstein's restatement of the Fregean context-principle (Ibid., 3.3). These *Erläuterungen* are what Hacker holds to be ostensive definitions 'seen through a glass darkly' (Hacker, 1986, p. 77).<sup>35</sup> In the light of the foregoing discussion, however, this interpretation no longer appears supportable. A more probable explanation of what Wittgenstein has in mind, I would suggest, is that we grasp the meanings of primitive signs through observation of their uses in elementary propositions—that is, we grasp the meanings of names by paying heed to their uses as referring terms in propositions which predicate something of the objects they are being used to stand for.<sup>36</sup> The context principle is of central importance here, for it is only in the context of a proposition that a name can be said to actually (or, as it were, *actively*) refer to an object; it is in predicating something of an object that one uses a name to refer to its bearer and hence fixes the reference of that name. This is not ostensive definition—not even ostensive definition seen through a glass darkly—it is an acknowledgement of how the semantic features of expressions are to be understood in terms of the use they have within propositions which say something.

The picture theory is the ideal vehicle for elucidating this fact. It is the fact that names are used to stand for objects in propositions which picture facts about those objects that ensures that those names refer. If this is correct, then the mentalistic theory of reference which Hacker locates in the *Tractatus* becomes entirely redundant. Likewise the interpretation of 'the method of projection' (Wittgenstein, 1922, 3.11) as an internal act of the mind will be misguided.<sup>37</sup> The method of projection, Wittgenstein tells us, is to 'think the sense of the proposition'. Just prior to this, he states that 'we use the perceptible sign of a proposition (spoken or written, etc.) as a projection of a possible situation' (Ibid.). Far from being an appeal to any mysterious property injected into language by the mind, this is an observation of how a

<sup>35</sup> Hacker draws attention to the similarity between this remark and the discussion of indefinables in\*1 of *Principia* (see Hacker, 1975, p. 605).

<sup>36</sup> Hacker, in fact, appears to discount this very idea: 'Wittgenstein was *not*, I think, suggesting that we just "pick up" the meanings of simple names from attending to their use in various elementary propositions' (Hacker, 1986, p. 76, emphasis added). If my interpretation is correct, then something along these lines is just what Wittgenstein was suggesting after all.

<sup>37</sup> Peter Winch also objected to this interpretation of the picture theory. In his (1987), he directed those objections at Norman Malcolm who had forwarded such a view in his (1977). Malcolm subsequently defended his interpretation (see Malcolm, 1986), and Hacker recently mounted a further defence of the position in his (1999). The interpretation offered in this thesis comes down firmly in favour of Winch on this issue. Hans-Johann Glock chooses to leave out what he calls 'the dark, psychologistic side of the *Tractatus*' and characterizes the work as drawing 'limits to thought by establishing the limits of the *linguistic expression of thought*' (Glock, 1997, p. 157).

propositional sign gains its meaning through its use—in particular, through its use to say something true or false, which is just what the projection of a possible situation consists in. To assert a proposition is to assert that something is the case, and to do so on the Tractarian model of meaning is to use the elements of a proposition in order to picture a state of affairs. To think the sense of a proposition is no more than to grasp the conditions under which it would be true according to the *Tractatus*, and this requires no special act of the mind other than the act of using the proposition to picture the state of affairs that would, depending on whether it obtains or not, determine its truth or falsehood. Understanding a proposition is not separable from our being able to use the proposition; each requires that we grasp the sense of the proposition and, for Wittgenstein (like Frege), a grasp of the sense of a proposition is a grasp of what would make that proposition true (see Wittgenstein, 1922, 4.024). The picture theory gives an account of sense in terms of the structural similarity between the proposition and what it pictures; were the theory to need supplementation by appeal to acts of the mind it is evident that it would not have succeeded in elucidating the representational relation it is intended to be a theory of. Hence Wittgenstein insists that propositions do not need to inhabit minds in order to be meaningful. Indeed, they do not even need to inhabit books or be composed of words at all: 'The essence of a propositional sign is very clearly seen if we imagine one composed of spatial objects (such as tables, chairs, and books) instead of written signs. Then the spatial arrangement of these things will express the sense of the proposition' (Ibid., 3.1431). No recourse to mentalism is needed for this or any other kind of proposition to be imbued with meaning if the picture theory is maintained. The 'method of projection' of a possible state of affairs is simply the use of signs to picture that state of affairs—or, in more familiar terms, to *say* something.<sup>38</sup>

The picture theory provides, then, a decisively anti-mentalistic conception of meaning. Meaning is not separable from use. Of course, the observation that the meanings of words have an intrinsic relation to their use is a fairly broad claim which, in some forms, will hardly extend beyond triviality.<sup>39</sup> It would be a wholly misplaced and inaccurate kind of revisionism which sought to suggest that Wittgenstein's later

<sup>38</sup> This point is actually made more clearly in the earlier draft of the *Tractatus* now published as the *Prototracatus*: 'A propositional sign with its mode of depiction is a proposition' (Wittgenstein, 1971, 3.2).

<sup>39</sup> Dummett, in making this same point, suggests that 'the limit of total triviality would be reached if it were permissible to describe someone's use of, e.g., the word "ought" by saying, "He uses it to mean "ought"' (Dummett, 1963, p. 188).

identification of meaning with use features in anything like the same form in the *Tractatus*. Wittgenstein's later work extends the conception of what is involved in the use of language far beyond any features which are recognizably present in the *Tractatus*. Indeed, the complexity of such linguistic practices is taken by the later Wittgenstein as evidence of the impossibility of any systematic theory of meaning such as the one he had offered in the *Tractatus*.<sup>40</sup> Nonetheless, the connection between meaning and use which is present in the *Tractatus* is not a trivial one. It is, rather, an essential component in the picture theory which serves to give an account of the representational capacity of sentences without resorting to any mysterious property of the mind as the source of that meaning. In addition, the appeal to use allows Wittgenstein to avoid the problem of the unity of the proposition which had haunted Russell even after it had driven him to appeal to just such a mysterious mental property in the multiple-relation theory of judgement. Finally, the move provides Wittgenstein with a means of extending the Fregean answer to the problem of the unity of the proposition so as to avoid the unattractive metaphysics underlying Frege's philosophy of language; by appealing to the use of linguistic items in the picture theory. Wittgenstein is able to justify his claim that 'in logical syntax the meaning of a sign should never play a rôle' (Wittgenstein, 1922, 3.33).

If the interpretation I have urged here is correct, then Hacker's interpretation of the relation between thought and language in the picture theory has things back to front. Noting Wittgenstein's remark that 'thinking is a kind of language ... a thought too is, of course, a logical picture of the proposition, and therefore it just is a kind of proposition' (Wittgenstein, 1961, p. 82),<sup>41</sup> Hacker construes Wittgenstein's position as 'a remote ancestor of much nonsense in contemporary "cognitive psychology" about "mental representations" and "the language of thought" [which] Wittgenstein was later to demolish root and branch' (Hacker, 1986, p. 75).<sup>42</sup> On my interpretation,

<sup>40</sup> Assuming, that is, that such a theory is present in the *Tractatus*. There are some who dispute this, as we shall see shortly.

<sup>41</sup> I believe that Hacker has misinterpreted the sentence immediately preceding this passage by paraphrasing it as 'thinking and language are the same' (Hacker, 1986, p. 74), a misinterpretation which he repeats in a recent paper (see Hacker, 2000, p. 355). The original sentence is: 'Now it is becoming clear why I *thought* that thinking and language *were* the same' (Wittgenstein, 1961, p. 82, emphasis added). Prior to this remark, Wittgenstein had written (in an entry written just the day before): 'The way in which language signifies is mirrored in its use' (Ibid.).

<sup>42</sup> The foremost proponent of the 'nonsense' which Hacker refers to is Jerry Fodor, whose influential book *The Language of Thought* (1975) postulates an innate grammar in order to explain natural language acquisition. In developing the theory, Fodor has supplemented this grammar with the

however, this is the precise opposite of Wittgenstein's position in the *Tractatus*. The recognition that a thought, like a proposition, is a logical picture does not shroud meaning in the mystery of the mental processes involved in thinking: it shows thought to be parasitic on its linguistic expression. For a thought, in order to express and articulate a truth evaluable content, must be logically articulated in the correct fashion if it is to picture a state of affairs. Thought is dependent on, and finds its expression in, language.

### 3.5 The Method of the *Tractatus*: A Recent Controversy

Hacker's interpretation of the *Tractatus* is, as far as is possible with such a notoriously enigmatic subject, a standard one.<sup>43</sup> Recently however an alternative, and increasingly influential, reading of the *Tractatus* has been urged which diverges from the standard interpretation not just in terms of disputing the details of isolated aspects of the work but so fundamentally as to seek to overturn the very conception of both what the book achieves and what it sets out to achieve. The end result of this interpretation, first urged by Cora Diamond and elaborated by James Conant, is the claim that the very thing which the *Tractatus* is usually understood to have asserted is, in fact, the very thing which Wittgenstein intended the book to expose and overthrow as fallacious; namely, the distinction between what can be said and what can only be shown. The grounds for this interpretation has its origins in the recognition of a tension between this apparent doctrine of the work and Wittgenstein's own evaluation of the status of the propositions of the work in its penultimate remark:

My propositions serve as elucidations in the following way:  
anyone who understands me eventually recognizes them as nonsensical.  
when he has used them—as steps—to climb up beyond them. (He must,  
so to speak, throw away the ladder after he has climbed up it.)

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postulation of an internal lexicon, the semantics of which are to be provided by a causal theory of content. Fodor's language of thought hypothesis will be returned to later in this thesis (See Ch. 4, below).

<sup>43</sup> The difficulty faced by a commentator on the *Tractatus* is reflected vividly in Max Black's frank admission in the opening paragraph of his extensive *Companion* to the book: 'There can be no question here of any "definitive" reading: since my own views have oscillated while I was writing, I cannot be confident that my final judgement has always been best' (Black, 1964, p. vii).

He must transcend these propositions, and then he will see the world aright.

(Wittgenstein, 1922, 6.54)

On the face of it, this seems utterly self-destructive. In exposing the metaphysical speculations of philosophers as nonsensical, Wittgenstein appears to have committed himself to denouncing any doctrine asserted in the *Tractatus* as likewise nonsensical and thereby haplessly landed himself in paradox. As Hacker notes, Wittgenstein, if we are to take him literally here, not just ‘deliberately saws off the branch upon which he is sitting’ but finds himself embracing the hopelessly paradoxical situation whereby the very doctrine by which such a result is achieved must also be regarded as itself nonsensical and thus ‘*a reductio ad absurdum* of the very argument that led to the claim’ (Hacker, 2000, p. 365).

On the traditional reading of the book, of which Hacker is the foremost proponent,<sup>44</sup> this internal collapse of the book is avoided by making a distinction between *kinds* of nonsense: *misleading* nonsense on the one hand, and *illuminating* nonsense on the other. The doctrines of the *Tractatus* are held to fall into the latter category. Thus, on this reading, whilst Wittgenstein is, strictly speaking, correct in saying that the propositions of the book are nonsensical, they represent a special case of nonsense which gestures at a substantial yet otherwise incommunicable body of truths.<sup>45</sup> These ineffable truths (about, for example, the nature of ethics and, most appropriately to this discussion, the logical structure of language) cannot sensibly be said, but can be revealed through the use of illuminating nonsense which is the inevitable result of trying to use language to say what is shown by its proper use in non-philosophical contexts. The *Tractatus* deliberately flouts the very rules of logical syntax which it has set out in order to convey something which, by the truth of those doctrines, it cannot successfully articulate. Yet, in this failed attempt to say

<sup>44</sup> See Hacker, 1975, 1986, 1999, & 2000. Other notable examples (though differing in detail from one another) are Anscombe, 1971, Black, 1964, Geach, 1976, Malcolm, 1986, and Pears, 1987.

<sup>45</sup> The first explicit attribution of the doctrine of illuminating nonsense to the *Tractatus* appears to be in Engelmann (1967). The distinction between saying and showing met with ‘a certain sense of intellectual discomfort’ from Russell, who remarked: ‘What causes hesitation is the fact that, after all, Mr Wittgenstein manages to say a good deal about what cannot be said’ (Russell, 1922, p. xxi). Ramsey was equally sceptical, on the grounds that: ‘what we can’t say, we can’t say, and we can’t whistle it either’ (Ramsey, 1931, p. 238). Carnap similarly complained that Wittgenstein was ‘inconsistent’ on the grounds that ‘he tells us that one cannot state philosophical propositions and that whereof one cannot speak, thereof one must be silent: and then instead of keeping silent, he writes a whole philosophical book’ (Carnap, 1935, pp. 37-38).

something, it nonetheless achieves its aim: ‘Illuminating nonsense will guide the attentive reader to apprehend what is shown by other propositions which do not purport to be philosophical; moreover it will intimate, to those who grasp what is meant, its own illegitimacy’ (Hacker, 1986, pp. 18-19).

In her influential paper “Throwing Away the Ladder: How to Read the *Tractatus*”, Diamond has accused the adherents to this standard interpretive strategy of ‘chickening out’ (Diamond, 1991a, p. 181) by not taking Wittgenstein’s request that we throw away the ladder (i.e., the nonsensical pseudo-propositions of the *Tractatus*) seriously. To take Wittgenstein seriously, she urges, will be ‘to say that it is not, not really, his view that there are features of reality which cannot be put into words but show themselves’. The view she prefers to attribute to the early Wittgenstein is, rather, the view that ‘that way of talking may be useful or even for a time essential, but it is in the end to be let go of and honestly taken to be real nonsense, plain nonsense, which we are not in the end to think of as corresponding to an ineffable truth’ (Ibid.).<sup>46</sup> Marie McGinn has characterised this interpretation as a ‘therapeutic reading’ (McGinn, 1999, p. 492) a label which fits well with Diamond’s understanding of the method of the *Tractatus*. On Diamond’s account, the elucidatory strategy of the *Tractatus* is intended to free us of the temptation to believe that there could be ineffable truths lying behind sentences which fail to directly state those truths. And that project, if successful, will leave us in some sense better off without such delusions:

We are left using ordinary sentences, and we shall genuinely have got past the attempt to represent to ourselves something in reality, the possibility of what a sentence says being so, as not sayable but shown by the sentence. We shall genuinely have thrown the ladder away.

(Diamond, 1991a, p. 184)

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<sup>46</sup> See also Diamond, 1991b, 1997, & 2000. Others have sought to develop Diamond’s interpretation, most notably: Conant, 1989, 1990, 1991, 1995, 1997, 1998, & forthcoming a; Goldfarb, 1997; Ricketts, 1996, and Floyd, 1998. Other exegetes have objected fiercely to the sharp distinction between the earlier and later periods of Wittgenstein’s philosophy which is often entailed by the traditional reading of the *Tractatus* without specifically objecting to the notion of illuminating nonsense. See, for example, Winch, 1969, 1987, & 1992; Mounce, 1981; Ishiguro, 1969, & 1981; Kenny, 1973; McGuinness, 1981; and Palmer, 1988, 1992, & 1996.

Conant concurs with this and stresses the continuity between the early and later Wittgenstein which, he claims, is located in Wittgenstein's faithfulness to this therapeutic method, summed up in the *Investigations*: 'My aim is: to teach you to pass from a piece of disguised nonsense to something that is patent nonsense' (Wittgenstein, 1953, § 464).<sup>47</sup> The *Tractatus* presents to us, Conant maintains, the 'unfolding of a therapeutic strategy' (Conant, 1989, p. 262).

This new reading is clearly a radical one. In his reply to Diamond and her followers, Hacker remarks that 'One cannot but be struck by ... the sparseness of the evidence they muster' (Hacker, 2000, p. 360).<sup>48</sup> Indeed, there is no place where Wittgenstein explicitly states the view which the therapeutic interpretation of his early philosophy seeks to ascribe to him. Both Diamond and Conant make much of the preface and the conclusion to the book, exempting them from the body of nonsensical ladder-rungs to be thrown away, on the grounds that these sentences constitute the 'frame' of the book—providing us with instructions as to how we should read what comes in between.<sup>49</sup> There is some support for this view to be found in a letter from Wittgenstein to Ludwig von Ficker wherein he remarks, of the *Tractatus*, that 'the book's point is an ethical one', and goes on to offer some advice on how to approach the work: 'I would recommend that you read the *preface* and the *conclusion*, because they contain the most direct expression of the point of the book' (cited in Engelmann, 1967, pp. 143-144). Aside from such scattered circumstantial evidence, however, there seems to be very little in the way of convincing textual support for the therapeutic reading. Unperturbed by this seemingly serious threat, however, Diamond has gone a step further and, in a recent article, suggested that the absence of any explicit mention of a given position in the *Tractatus* is not, by itself, sufficient evidence that the position is not (in some deeper sense) nonetheless *in* the book: 'When I argue that something is "in" the *Tractatus* I do not mean that it is explicitly said there ... Nor do I mean that it follows from what is explicitly said there'

(Diamond, 2000, p. 263).<sup>50</sup> Such exegetical adventurism is not to be recommended.

<sup>47</sup> See Conant, 1990, p. 346; 1997; forthcoming a, p. 82n.

<sup>48</sup> See Proops (2001) for another critique of the therapeutic reading.

<sup>49</sup> See Diamond, 1991b, p. 149. Hacker points out that the feasibility of the reading also depends on the implicit conception of many other propositions of the work; namely, those which present the arguments in favour of dismissing the apparent arguments of the book as nonsense. See Hacker, 2000, p. 360.

<sup>50</sup> Diamond is, perhaps, forced to make such a claim in order to maintain the thesis of the paper in question as it seeks to show that one of the unstated things 'in' the *Tractatus* in this sense is a version of the private language argument.

As Hacker justifiably complains, it makes it near to impossible to determine what would count as sufficient evidence against such interpretive claims: 'If the internal and external evidence mustered in this paper against the post-modernist interpretation does not suffice to undermine it, it would be instructive of Diamond and her followers to inform us what would count as sufficient or telling evidence against their account' (Hacker, 2000, p. 381).

This thesis is not the place to undergo a detailed evaluation of how best to read the *Tractatus*. There is, however, an important issue raised by the debate we have just outlined which is of relevance. A central claim made by Diamond and Conant is that Frege's context-principle, if its implications are properly thought out (as they claim they were in the *Tractatus*),<sup>51</sup> renders impossible any attempt at offering a systematic *theory of meaning*.

That languages, and our understanding of them, are *systematic* to some degree seems undeniable; to understand what is said by the sentence "Desdemona loves Cassio", one must also understand what is said by the sentence "Cassio loves Desdemona". Hence it seems encumbered on any attempted theory of meaning to provide some account of how sentences are made up of their parts, what roles these parts play in the production of meaning, and, in particular, how the semantic properties of these parts contribute to the determination of the semantic properties of

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<sup>51</sup> Diamond and Conant are not in full agreement here. Both take it that Wittgenstein realised the full implications of the context-principle as they see them; Conant holds that Wittgenstein was mounting a critique of Frege in doing so, but Diamond suggests that Frege himself, contrary to what one might think, also reached the same conclusions. See Conant, forthcoming a, pp. 12-13 & nt. 24; and Diamond, 1991a, p. 79. Both Conant and Diamond reject Geach's (1976) suggestion that the saying/showing distinction in the *Tractatus* is itself a reiteration of a thesis which is already present in Frege. Weiner, in her interpretation of Frege, sees him as committed to a paradoxical situation much like that apparently facing the *Tractatus* whereby his own remarks fail to assert truths by their own lights. She concludes from this that Frege, in all his work other than the formal proofs, is best understood as pursuing an 'elucidatory' strategy. Her argument runs as follows: 'A thought has a truth-value only if that thought can be expressed in the words of a systematic science. If what is expressed in the sentences of Frege's philosophical writings cannot be expressed in the words of a systematic science, his sentences may express something that can be communicated, but they cannot express objective truths or falsehoods' (Weiner, 1990, p. 228). Of course, some support is to be found for Weiner's view, in Frege's famous request for 'a reader who would be ready to meet me halfway—who does not begrudge a pinch of salt' (Frege, 1892b, p. 54). Nonetheless, I am not prepared to share Weiner's conclusion that all of Frege's work should be viewed as 'elucidatory' simply on the basis that it is written in prose and not in the *Begriffsschrift*. Frege's exact condition on when a sentence expresses a thought with a truth-value is simply that all parts of the sentence should have a *Bedeutung*. There is no reason why this requirement cannot be met in most of Frege's philosophical discussion. Weiner is in fact falling prey to a remarkably common fallacy in holding that Frege's conception of logic as a universal science does not allow for any metatheoretical statements to be made about it. Similar remarks have been directed at Russell (see, for example, Hylton, 1980, pp. 2-6). In neither case are the accusations valid, for the simple reason that logic cannot be identified with a calculus embodying its principles (a point discussed by Landini, 1998, Ch. 1).

sentences in which they feature (standardly, the truth-values of such sentences). This demand, vague though most characterisations of it are, is commonly taken to be the minimum requirement which anything worthy of the title “theory of meaning” ought to meet. As Davidson famously put it, ‘a satisfactory theory of meaning must give an account of how the meanings of sentences depend upon the meanings of words’ (Davidson, 1967a, p. 17). The provision of such an account, of course, is no easy task; the assignments of entities to expressions on the model of assigning bearers to names leaves us (if left unchecked) with an abundant ontology of attributes, numbers, classes, and so on, which will find the resort to platonism or psychologism largely unavoidable, as we witnessed in chapters one and two of this thesis. A more plausible approach, explored by Davidson himself, and by Dummett in a somewhat different fashion, is to assume that the parts of sentences have a meaning only in ‘the ontologically neutral sense of making a systematic contribution to the meaning of the sentences in which they occur’ (*Ibid.*, p. 22).<sup>52</sup>

According to Conant and Diamond’s understanding of the context-principle, however, the principle exposes as chimerical any attempt to offer a systematic account of how sentences are built up out of their parts because such an account must reverse the correct order of explanation. The meanings of sentences should not be understood as dependent on their constituent parts, they suggest, but rather the meanings of those constituent parts are wholly dependent on their use within a sentential context which is sufficient for the assertion of a judgeable content. The faithful adherent to the context-principle, they claim, must respect ‘the primacy of the propositional whole over its parts’ (Conant, forthcoming a, p. 20). The claim is central to the therapeutic reading of the *Tractatus*, for it is the main weapon of the reading’s proponents against the standard view of ‘substantial nonsense’ as the result of a ‘violation of logical syntax’—a clash of semantic categories. If the context-principle (as understood by Conant and Diamond) shows it to be impossible for individual words to be assigned any meaning prior to, and independently of, their featuring in significant sentences, then it seems impossible to decree that certain combinations of signs are intrinsically meaningless by virtue of their being forced into ill-fitting

<sup>52</sup> Davidson reiterates the point, making an explicit appeal to truth-conditions, in a recent paper: ‘Individual words don’t have meanings. They have a role in determining the truth conditions of sentences’ (Davidson, 1997, p. 79). Dummett’s account is given in terms of the notion of *semantic value*: ‘The semantic value of an expression is that feature of it that goes to determine the truth of any sentence in which it occurs; we thus arrive at an account of the determination of the sentence as true or otherwise in accordance with its internal structure’ (Dummett, 1991b, p. 24).

combinatorial relations to one another. Indeed Wittgenstein says in the *Tractatus* that ‘the reason why “Socrates is identical” says nothing is that we have not given *any* *adjectival* meaning to the word “identical”’ (Wittgenstein, 1922, 5.4733; see also *Ibid.*, 5.473). On the therapeutic reading of the *Tractatus*, this proposition amounts to a claim that nonsense arises when *we* fail to mean something by our words; there is no possibility of us attempting to mean something which is ineffable by virtue of its necessarily violating any pre-ordained grammatical strictures. Hence, on this account, there is only one kind of nonsense: mere nonsense. There is no halfway house between nonsense and sense to be occupied by ‘substantial nonsense’ of the kind traditionally postulated by commentators on the *Tractatus*. If a word has meaning only in the context of a sentence, then it has meaning only in the context of a *significant* sentence, for a meaningless sentence does not assert a propositional content. So, the argument runs, a nonsensical string of words is not nonsensical because those words don’t fit together adequately as, in a nonsensical string, they don’t fit together at all and do not *mean* anything at all.<sup>53</sup> Nonsense is nonsense, and does not come in kinds or degrees.

But now it seems impossible to provide anything even approaching a systematic explanation of the meanings of sentences in terms of their composition out of sub-sentential parts. There is no reason to assume that individual words will behave in a constant fashion across differing sentential contexts. Taking the context-principle to be urging the strong claim that there is nothing more to the meaning of a word than its use in a sentential context accordingly seems to make any attempt to specify the meanings of sentences in terms of their parts a misguided and doomed enterprise. It is no surprise that Dummett was eager to repudiate the interpretation of the context-principle as anything resembling such a claim long before it arose in the context of understanding the method of the *Tractatus*: ‘Quine says that Frege discovered that the unit of meaning is not the word but the sentence. Likewise grammarians debate whether the word or the sentence is the primary element in meaning. This dispute seems to me empty and Frege’s alleged discovery absurd’ (Dummett, 1956, p. 39).<sup>54</sup> Indeed, it seems perverse to attribute such a view to Frege when one considers that the

<sup>53</sup> See Diamond, 1991b, p. 150.

<sup>54</sup> Quine’s attribution of the discovery to Frege is made in ‘Two Dogmas of Empiricism’ and actually attributes to Frege (and Bentham) ‘the reorientation whereby the primary vehicle of meaning came to be seen no longer in the term but in the statement’ (Quine, 1951, p. 39). Quine, of course, has a generally ‘dim view of the notion of meaning’ (Quine, 1953a, p. viii), discussion of which will occupy the remaining chapters of this thesis.

Begriffsschrift is a language designed to decompose sentences into their constituent parts and then perform substitution-operations on those parts (both singular terms and functional expressions) in order to portray the form of valid mathematical inferences. Furthermore, the power of Frege's analysis surely lies in its ability to capture the logical structure of propositional contents (the Begriffsschrift was intended, after all, to be 'a formalized language of pure thought'), strongly suggesting that language is likewise structured. This, indeed, is something that we do not need a formal language to convince us of; as Dummett goes on to point out, there would seem to be empirical verification of it at every turn: 'Any attempt to express clearly the idea that the sentence is *the* unit of meaning, or even the idea that the meaning of sentences is primary, that of words derivative, ends in implicitly denying the obvious fact—which is of the essence of language—that we can understand new sentences which we have never heard before' (Ibid.).

Dummett's understanding of the context-principle is more conservative:

'When I know the sense of all the sentences in which a word is used, then I know the sense of that word; what is then lacking to me if I am to determine its reference is not linguistic knowledge' (Ibid.). But, as Dummett goes on to admit, immediate problems arise with this formulation also.<sup>55</sup> For example, it must be too strong a claim to hold that I can only know the sense of a word if I know the sense of any sentence in which it features, as there may be instances where that word features in a sentence along with other terms of which I have no familiarity. If, for example, I need to know the sense of every sentence in which the word "function" features in order to know the sense of that word, but do not have any familiarity with the word "recursive", then I appear to be destined to run into a problem if I encounter the sentence "algorithmically computable functions are recursive functions". But, to say that my lack of understanding of this sentence will automatically entail my failure to fully grasp the meaning of the word "function" seems absurd; on the contrary, one would probably appeal to my knowledge of what a function is in order to explain to me what it means to call a function recursive, if one wished to explain the meaning of the word "recursive" to me. In addition, there is a further, perhaps more serious, problem for Dummett's account in the form of a completely unprecedented and unexpected use of

<sup>55</sup> One such problem, of course, is Dummett's phrasing of the principle in the terms of Frege's distinction between sense and reference—a distinction unavailable to him when he wrote the *Grundlagen*. See Dummett, 1993a, p. 97, ft. nt., for further discussion of how the context-principle alters under the influence of the later distinction.

a term. Frege's example, "Trieste is no Vienna",<sup>56</sup> uses the term "Vienna" in a way which would not usually be construed as a use that one must grasp in order to know what the term "Vienna" means. The sudden use of a singular term in a predicative role is not one that must be foreseen by all users of the term prior to that use in order for them to be correctly attributed with a grasp of the sense of the term. It may well be that a plausible response here is to argue that any predicative use of a term which is usually a singular term will have to make a tacit appeal to the referent of the cognate singular term. For instance, "Trieste is no Vienna" will be understood only by someone who understands how to use the singular term "Vienna" in sentences of the form "Vienna is a bustling metropolis" or "Vienna is the capital of Austria". Nonetheless, we can see from this that Dummett's version of the context-principle needs some revision in order to account for the fact that many of our linguistic capacities are latent ones; my understanding of the meaning of the word "Vienna" consists in my ability to make sense of the sentence "Trieste is no Vienna" should I be confronted by it. Were I never to encounter that sentence, however, one could not conclude that I thereby lacked a full grasp of the meaning of the term "Vienna".

Despite these problems with Dummett's version of the context-principle, it should be noted that there are even more severe problems for the version urged by the therapeutic reading of the *Tractatus*. To conclude from the presence of the context-principle in either the *Grundlagen* or the *Tractatus* that Frege and Wittgenstein held to the belief that there is no such thing as a difference in semantic category which, if not respected, can lead into nonsense can only be justified by a wilful misreading of Wittgenstein's remark that 'we cannot give a sign the wrong sense' (Wittgenstein, 1922, 5.4732). We saw, in the first half of this chapter that Wittgenstein did not think, for example, that there were no distinctions in logical type amongst expressions: rather he insisted that there were but maintained that they can only be located through observation of the logical role of a sign in the context of a proposition. One sign differs in type from another if it is *used* differently. Hence, far from making distinctions in semantic category impossible, the appeal to use suggested by the context-principle actually serves to provide us with a method by which the semantic category of a linguistic item is to be discovered. If one accepts, as Conant and Diamond surely must, that there are different ways of using words and that, in a

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<sup>56</sup> See Frege, 1892b, p. 50.

sentence, not all of the words are used in the same way,<sup>57</sup> then one has already accepted that those words can be divided into different categories. Wittgenstein's innovation, greatly more developed in his later work, was to make the *use* of a word the means by which we could come to grips with what semantic category it fits into. Certainly, in his later work, Wittgenstein was fundamentally opposed to the idea that any systematic theory of meaning could be adequately constructed for the diverse uses to which language is put. Indeed he was adamant that no philosophical purpose would be served by attempting such a theory anyway: having made a sharp distinction between philosophical method and scientific inquiry, he concluded that the business of philosophy was to describe the uses of language, piecemeal, in an effort to dissolve apparent philosophical problems. It may well be the case that a similar spirit underlies the *Tractatus*, surfacing, for example, in his insistence that 'Philosophy is not a body of doctrine but an activity ... Philosophy does not result in "philosophical propositions", but rather in the clarification of propositions' (Wittgenstein. 1922, 4.112); but this does not mean that we can trace his rejection of the possibility of a systematic theory of meaning back to the context-principle. *A fortiori* it does not permit us to trace the doctrine back to Frege.

The question whether a systematic theory of meaning is beyond our means, or even a desirable commodity should it be constructed, is one of the abiding legacies of the contributions made by Frege, Russell and Wittgenstein to their descendants in the analytical tradition, as we shall see in the remainder of this thesis. If such a theory is indeed an impossibility, it is not because of any direct consequence of Frege's context-principle.

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<sup>57</sup> The obvious exception are those sentences consisting of just one word such as "Fire!". "Lo!". "Sorry", etc. These sentences are heavily dependent on context in a different way, of course.

## 4.

## Naturalistic Developments: From Russell to Quine

In the aftermath of what he later referred to as ‘Wittgenstein’s onslaught’ (Russell to Morrell, 4/3/1916, in Russell, 1968, p. 57), Russell was left in a somewhat awkward position. *Principia* had, to Russell’s mind at least, demonstrated the logicist thesis that the mathematical truths are a proper subset of the logical truths.<sup>1</sup> What Wittgenstein’s criticisms revealed, however, was Russell’s lack of an adequate account of what logic itself is. In the spring of 1914 he delivered the Lowell lectures at Boston and steered clear of the question, addressing instead the metaphysical question of what matter is and the epistemological question of what knowledge we have of it. Writing to Ottoline Morrell two years later, he confessed: ‘I *had* to produce lectures for America. but I took a metaphysical subject although I was and am convinced that all fundamental work in philosophy is logical. My reason was that Wittgenstein persuaded me that what wanted doing in logic was too difficult for me (*Ibid.*)’. The lectures, subsequently published as *Our Knowledge of the External World*, despite containing a lecture entitled “Logic as the essence of philosophy”, offered little of interest to the philosophy of logic but instead sought to apply, in a far less technical form, some of the methods of *Principia* to metaphysical and epistemological subjects.

<sup>1</sup> Many, including Quine, would reject the claim that all mathematics can be reduced to logic in the light of Gödel’s famous proof of the incompleteness of arithmetic and, indeed, in the light of Russell’s paradox. Most would now agree that logic proper extends only to quantification theory and does not embrace higher-order calculi. Of course, one could still maintain a version of the logicist thesis but supplement it with the recognition that higher-order “logics” are, despite their incompleteness, logics nonetheless. But, as William and Martha Kneale aptly point out, ‘it seems pointless to assert that all mathematics can be reduced to logic, if at the same time we must maintain that logic involves all the diversity we find in mathematics’ (Kneale & Kneale, 1962, p. 724). Accordingly they, like Quine, prefer to say that mathematics reduces to logic and set theory. For a dissenting view, see Wang, 1986, and for further discussion of the issue of Russell’s logicism in relation to Gödel’s incompleteness result, see Sainsbury, 1979, Ch. 8. See also Linsky, 1999, Ch. 8.

In particular, the theory of descriptions (in a general form) was extended to questions of knowledge as Russell sought to repeat the successes it had previously brought him in logic and mathematics.

#### 4.1 Knowledge by Acquaintance and Knowledge by Description

Russell's diversion of the theory of descriptions towards metaphysical ends was not unprecedented.<sup>2</sup> In a lengthy manuscript, 'On Fundamentals', written in 1904, from which the theory of descriptions gradually emerges, Russell made the following observation regarding the relation of denoting to knowledge:

This topic is very interesting in regard to theory of knowledge, because most things are only known to us by denoting concepts. Thus Jones = the person who inhabits Jones's body. We don't have *acquaintance* with Jones, but only with his sensible manifestations. Thus if we think we know propositions about Jones, this is not quite right; we only know propositional functions which he satisfies, unless indeed we *are* Jones.

Thus there can be no such thing as affection for persons other than ourselves; it must be either their sensible manifestations or the concepts denoting them that we like. It cannot be the latter, for it would be absurd to say that we loved some of these and hated others.

(Russell, 1904, p. 369)

This distinction, with the theory of descriptions fully worked out, became Russell's distinction between *knowledge by acquaintance* and *knowledge by description*. In 'On Denoting', Russell explained the distinction thus: 'in every proposition that we can apprehend ... all the constituents are really entities with which we can have immediate acquaintance. Now such things as matter ... and the minds of other people are known to us only by denoting phrases, i.e. we are not acquainted with them, but

<sup>2</sup> Dummett suggests that the theory of descriptions inevitably portrays metaphysical concerns, being an attempt to resist the pull of realism without sacrificing the principle of bivalence. Russell's analysis shows, according to Dummett, that realism cannot be characterised in purely metaphysical terms, but must involve semantic notions of, for example, denotation, truth, and falsity. Integral to realism is a commitment to bivalence and an interpretation of statements at 'face value'. The theory of descriptions departs from realism on the second count. See Dummett, 1991b, p. 325.

we know them as what has such and such properties' (Russell, 1905a, p. 56). The distinction was addressed in detail in the paper, 'Knowledge by Acquaintance and Knowledge by Description', read before the Aristotelian Society in 1910, and features in 1912's *Problems of Philosophy*.

The basic principle guiding Russell's ontological development in this period, enunciated in "The Relation of Sense-Data to Physics", was one of logical parsimony: 'The supreme maxim in scientific philosophising is this: Wherever possible, logical constructions are to be substituted for inferred entities' (Russell, 1914b, p. 149). Whereas his previous work had maintained a realism more in line with traditional rationalism, his work from now on became increasingly influenced by an empiricist ethic in the Humean mould. A substantial measure of platonism was retained in the form of a belief in the existence of universals,<sup>3</sup> but even this is given an empiricist gloss by Russell's strange insistence that they be admitted only on the grounds that we have acquaintance with them.<sup>4</sup> Of course this had been a requirement for the theory of descriptions all along; if 'the present King of France is bald' is to be restructured as ' $\exists x \{[(x \text{ is presently King of France}) \& (\forall y) (y \text{ is presently King of France} \rightarrow x = y)] \& x \text{ is bald}\}$ ', then acquaintance with the constituent universals is required if the doctrine is to pull any epistemological weight. Knowledge of the external world, Russell held, was to be admitted only if it consisted in, or could be constructed from, that which we have immediate knowledge of; i.e., those things we have 'direct acquaintance' with.

Russell's project is similar to that of the Cartesian sceptic. In order to carry out his analysis of human knowledge, he must first find some sure ground from which to begin; something we are surely acquainted with. Logical constructs must be constructed out of some raw material just as surely as inferred entities had to be inferred from some prior experience not itself inferred. Russell, predictably, suggests sense-data as suitable candidates for the role.<sup>5</sup> A sense-datum is not, as it were, a slice of our whole experience at any given moment, but a particular component of that whole. Thus a particular patch of red in my visual field will count as a sense-datum

<sup>3</sup> See Russell, 1911a. In a note added to the 1956 reprinting of this paper, Russell alludes to arguments raised in his (1948) which, he claims, cast doubt on the existence of particulars. In the face of such doubt, Russell urges their rejection—again on the grounds of 'logical parsimony' (see Russell, 1911a, p. 124).

<sup>4</sup> See Russell, 1912a, Ch. 10.

<sup>5</sup> See Russell, 1910a, p. 201.

under suitable conditions.<sup>6</sup> Russell also allows, though with some reservation,<sup>7</sup> for the existence of unsensed sense-data or *sensibilia*: 'the relation of a sensibile to a sense-datum is like that of a man to a husband: a man becomes a husband by entering into the relation of marriage, and similarly a sensibile becomes a sense-datum by entering into the relation of acquaintance' (Russell, 1914b, p. 143).

With the raw material of sense-data in place, Russell set about rebuilding the external world that his scepticism had previously balked at. Matter could now be defined as series of sense-data (or, more precisely, series of classes of sense-data; an object at any instant is defined as the class of all sense-data perceived by any possible spectator of that object from any given perspective, and an object over time will be a series of such classes). The obvious objection here, that an explanation of a physical object in terms of the sense-data it induces in observers does not give an account of what the object *is*, left Russell untroubled. Noticing the increasing tendency of physicists to move away from materialistic explanations, Russell prided himself on having shown that objects could be viewed as logical constructs out of sense-data. These considerations lay at the heart of his consideration of psychology in the 1921 *Analysis of Mind* and its 1927 companion volume, *The Analysis of Matter*.

In the Preface to the *The Analysis of Mind*, Russell set himself the task of trying to 'reconcile the materialistic tendency of psychology with the anti-materialistic tendency of physics' (Russell, 1921, p. xvii). The doctrine which he thought could effect such reconciliation was the neutral monism of William James.<sup>8</sup> The behaviourist school of psychology was, to Russell's mind, correct in viewing physical phenomena as the most fundamental, but this seems somewhat paradoxical when taken alongside Russell's equally strong conviction that physicists, under the influence of relativity theory, were correct in their replacement of the crude materialist notion of matter with the ontologically basic notion of *events*. On Russell's understanding of relativity theory, as he put in his *ABC of Relativity*, 'relativity demands the abandonment of the old conception of "matter", which is infected by the

<sup>6</sup> One such condition is that the datum be something which can be named rather than asserted. Hence, a red patch is a sense-datum but *that* a red patch is to the left of a blue patch is not. See Russell, 1914b, p. 142.

<sup>7</sup> The part played by unsensed sensibilia, Russell says, is as 'an illustrative hypothesis and as an aid in preliminary statement [rather] than as a dogmatic part of the philosophy of physics in its final form' (Russell, 1914b, p. 152).

<sup>8</sup> A clear statement of the doctrine, as James construed it, is to be found in his 1904 paper 'Does Consciousness Exist?'. The advocacy of neutral monism was a change of heart for Russell: he had previously rejected the view in the *Theory of Knowledge* manuscript (see Russell, 1913, p. 32).

metaphysics associated with “substance”, and represents a point of view not really necessary in dealing with phenomena’ (Russell, 1925, p. 142). The view Russell favoured in place of this outdated metaphysics was that ‘a piece of matter will thus be resolved into a series of events’ (*Ibid.*).<sup>9</sup>

*The ABC of Relativity* had an eventful history, undergoing several changes in various editions. These alterations were made by a physicist, Felix Pirani, with Russell’s blessing, in order to keep the book relevant to subsequent advances in relativity physics and its applications. The changes are interesting with regard to the present discussion. For example, Russell has the following discussion of what he calls ‘the indestructibility of matter’ in the first edition:

The statement that matter is indestructible ... ceases to be a proposition of physics, and becomes instead a proposition of linguistics and psychology. As a proposition of linguistics: “Matter” is the name of the mathematical expression in question. As a proposition of psychology: our senses are such that we notice what is roughly the mathematical expression in question, and we are led nearer and nearer to it as we refine upon our crude perceptions by scientific observation. This is much less than physicists used to think they knew about matter.

(Russell, 1925, 1<sup>st</sup> ed., pp. 185-186)

In later editions, however, the word “energy” is substituted for “matter” throughout this passage. While the substitution brings the passage in line with modern terminology, it also, I think, distorts Russell’s intention of giving a definition of matter in non-materialistic terms. Russellian matter, it would seem, is far less destructible than the matter of previous metaphysicians.<sup>10</sup>

With matter reduced to event-series, and armed with the set-theoretic weaponry developed in *Principia*, Russell invoked sense-data as a means of reconciling the disparity between the physicists’ rejection of matter, and the psychologists’ embracement of it. Both matter and mind were conceived of as constructions out of sense-data. Sense-data, the source material for the logical

<sup>9</sup> There are some similarities here with Quine, as we shall see later on in this chapter.

<sup>10</sup> I am grateful to Ray Monk, Kenneth Blackwell, and Omar Rumi for helping to shed light on the disparities between the various editions of the book.

constructs required to flesh out scientific theory, were neither physical nor mental, but neutral occupants of space-time. Admittedly, Russell's doctrines on sense-data underwent revision in the *Analysis of Mind*. Having rejected any ontological commitment to the subject, or self, as a basic entity, Russell decided that the previously held distinction between sensations and sense-data could no longer be maintained. Russell's reasoning on the point is confusing.<sup>11</sup> The basic argument is that without a sensing subject, there is no room for a distinction between sensations had by that subject and the sense-data that she senses: 'Accordingly, the sensation that we have when we see a patch of colour simply is that patch of colour, an actual constituent of the physical world, and part of what physics is concerned with' (Russell, 1921, p. 142). Having thus justified the materialistic tendency of behaviouristic psychology by turning the material in question into something consonant, at least to Russell's mind, with the ultimate subject matter of physics, Russell was able to embrace the behaviouristic turn of psychology and set about utilising it in offering a crude version of what would later become known as a *naturalistic* theory of meaning.

#### 4.2 Russell's Naturalism and the Re-Psychologising of Content

Naturalism, in its broadest form, seeks to provide an account of some aspect of human life in purely scientific terms. That is to say, those features are to be explained as being part of the causal order in just the same way that any feature of the world studied by the natural sciences is deemed to be. As Russell's views progressed on this naturalistic trajectory, he became more and more determined to show that *thought* could be understood in purely causal terms. The behaviourism of *The Analysis of Mind* was the first manifestation of this direction.

It has been suggested by Stace (1944) that Russell's neutral monism in *The Analysis of Mind* is not entirely 'pure', invoking as it does the non-neutral phenomena of 'images'.<sup>12</sup> Mind is reduced to sensations and images in this work and, unlike

<sup>11</sup> Jager (1972) holds that Russell's reasoning is not just confusing, but actually fallacious when taken at face value. If, as Russell claims, the functions performed by the subject can always be performed by the logical construction he favours in its place, then surely the distinction which held for the subject ought to hold for the logical construction too (see Jager, 1972, p. 329).

<sup>12</sup> See Stace, 1944, p. 362.

sensations, images are not properly dissolved into the neutral stuff out of which mind and matter are to be reconstituted. Images, on Stace's interpretation, are not features of the physical world but are entirely subjective, psychological phenomena.<sup>13</sup> For Russell, mind and matter are to be distinguished not by stuff from which they are composed but by the different causal laws they obey. Images are intended to play a role in explicating mental causation. As Monk (1996a) points out, there is a striking similarity in many respects between the accounts of thought given in *The Analysis of Mind* and in the *Tractatus*.<sup>14</sup> For Wittgenstein, a thought is 'a logical picture of facts' (Wittgenstein, 1922, 3), for Russell, at this period, thoughts are 'image-propositions' (Russell, 1921, p. 248). As Monk also notes, there is also a profoundly important difference, however. For Russell, it is of the utmost importance that the picture plays a causal role and is located within the mind. Thoughts, contrary to the anti-psychologistic movement begun by Frege and inherited by Wittgenstein, are in the minds of thinkers according to Russell. It is not this point at which Russell backtracks on his own anti-psychologism, though. Russell's rejection of psychologism, indebted to Moore, relied on the independence of propositions from thoughts.<sup>15</sup> A proposition was construed by the early Russell as an objective, mind-independent complex. It was by treating the proposition as either identified with, or derivative from, the thought as an internal content, that Russell was retreating back into psychologism of the very sort he had so vehemently (and correctly) opposed in his early philosophy.

The redundancy of the psychological component of Russellian image-propositions is just the redundancy which, as we saw in the last chapter, the mentalistic strand often attributed to the Tractarian picture-theory falls prey to. If a propositional content can genuinely picture a fact, then it will picture the fact regardless of where it is situated. No amount of internal willing on the part of a thinker will do the work required to make a picture into a picture of a fact. That can only be achieved by actually using the picture to *picture* something. And this just amounts, on the model of the proposition as a picture, to using a sentence to *say* something, to make an assertion or, in the terminology Wittgenstein later adopted, to make a move in a language-game.

<sup>13</sup> In defence of Russell, however, he does hold this dualism between images and sensations to be 'perhaps not ultimate' (Russell, 1921, p. 137).

<sup>14</sup> See Monk, 1996a, p. 560.

<sup>15</sup> See Chapter 1, above.

Russell's conception of an image-proposition was explored in a revealing paper of 1919; 'On Propositions: what they are and how they mean'. A proposition is there defined as 'what we believe when we believe truly or falsely' (Russell, 1919b, p. 285). In developing this preliminary definition, Russell (unsurprisingly) rejects the multiple-relation theory of judgement. Interestingly, his approach here is to contend that the theory is inadequate on the grounds that it requires a subject who stands in a series of multiple relations to the constituents of her belief. In abandoning the subject, therefore, the correspondence relation between a judgement-complex and the objective fact which previously determined its truth or falsity breaks down. Though this might seem to be an obvious consequence of the expulsion of the subject from the ontology of the theory of judgement, it is in fact a strange diagnosis of the failings of the multiple-relation theory. For one thing, there is no immediately obvious reason why treating the subject as a logical construction should prohibit it from entering into the kinds of relations required of the multiple-relation theory. Classes were contextually defined, eliminable constructs in *Principia*, but this did not stop Russell from thinking that these logical fictions could enter into relations with other logical or mathematical objects. The statement that ' $4 > 2$ ' asserts a relation between logical constructs according to the ontological strictures of *Principia*, so it is by no means clear that a problem will suddenly emerge if a subject's belief in the truth of this statement (i.e., if there is a belief complex of the form  $[B\{S, x, R, y\}]$  which is  $S$ 's belief that  $4 > 2$ ) is the belief of a logically constructed subject. If ' $x$ ' and ' $y$ ' are to be understood as variables that can range over logical constructions, then it is far from clear why ' $S$ ' cannot be similarly understood.

A more telling problem for the multiple-relation theory emerges from Russell's discussion of facts in 'On Propositions'. If a fact has the form  $[xRy]$ , Russell notes, then there will be possible facts of the form  $[yRx]$  or  $[\sim yRx]$ , but the relation cannot be substituted for either of the logical subjects it relates: 'Thus if there is such a fact as "Socrates loves Plato", there is either "Plato loves Socrates" or "Plato does not love Socrates", but neither Socrates nor Plato can replace *loves*.' (Russell, 1919b, p. 286). It was the failure to guarantee that judgements correspond to this limitation on what a fact can conceivably be that Wittgenstein had located in the multiple-relation theory, as we saw in the previous chapter.

Russell's response, contrary to Wittgenstein's approach, was to turn away from logic as a means of explicating thought and look instead to psychology. Those of us steeped in post-Fregean philosophy of language cannot fail to be surprised by what Russell has to say about the relation of logic to meaning: 'Logicians, so far as I know, have done very little towards explaining the nature of the relation called "meaning", nor are they to blame in this, since the problem is essentially one for psychology' (Ibid., p. 290). The implication, however, is not that logicians such as Frege had not attempted to address the issue; rather it is that they had tried and failed. Russellian anti-psychologism in logic, coupled with Russell's belief that meaning is a psychological notion, led him to conclude that logic *cannot* offer insight into the nature of meaning. This is, in fact, quite consistent with his view in the *Principles of Mathematics*; 'meaning, in the sense in which words have meaning', he said there, 'is irrelevant to logic' (Russell, 1903, § 51). Where he does depart drastically from the earlier philosophy, however, is in his decision to exclude propositions and propositional contents from the scope of logical inquiry. In the *Principles*, Russell had adamantly resisted what he held to be a deeply confused notion, namely, 'the notion that propositions are essentially mental and are to be identified with cognitions' (Ibid.). By the time of 'On Propositions' and *The Analysis of Mind*, it was precisely this notion that he had embraced.

Having gone back on his previous depychologising of the proposition, Russell retained a conception of the proposition as a psychological element for the remainder of his philosophical career.<sup>16</sup> Two decades after 'On Propositions' in 1940's *Inquiry into Meaning and Truth*, Russell defined propositions as complexes independent of, and 'expressed' by, sentences: 'They are to be defined as psychological occurrences of certain sorts—complex images, expectations, etc.' (Russell, 1940, p. 189). This conception of propositions as entities, in some sense, inhabiting the minds of thinkers was given an increasingly naturalistic slant as Russell's views developed. Russell's last real contribution to philosophy, *Human Knowledge*, contained a causal theory of content crude enough to shock the linguistic

<sup>16</sup> It should be noted, however, that it is often remarkably difficult to extract a consistent account of propositions from Russell's work. Not only did he change his mind repeatedly about just what he thought propositions were, but he also had, on occasions, a frustrating habit of failing to maintain rigorous distinctions between propositions, sentences, judgements, etc. See White (1979) for a detailed study of the remarkably varied positions held by Russell on the nature of propositions.

philosophers of the time.<sup>17</sup> On this account, not only is there a causal story behind the learning of a word's meaning, there is a causal story behind the ability of words to function at all within the everyday business of communication. A word is, of course, a fragment of a language; on Russell's causal theory of content, what one has gained upon mastering such a fragment is the ability to enter into certain causal relations with the world and with other speakers. While behaviourist psychology serves to explain the causal process involved in how the meaning of a word is first ingested in learning, the causal theory of content also features in explaining the use of words in communication. The following passage from *Human Knowledge* illustrates the extent to which Russell thought meaning could be naturalistically construed within the causal strictures of science:

Suppose that, when I am walking with a friend, he says: "There was an explosion here yesterday." ... Let us suppose that I believe him, and that I believe what his words assert, not merely that the words are true. The most important word in the sentence is "explosion". This word, when I am actively understanding it, rouses in me faint imitations of the effects of hearing an actual explosion—auditory images, images of nervous shock, etc. Owing to the word "here", these images are combined in my mental picture with the surrounding scenery. Owing to the word "yesterday", they are combined with recollections of yesterday's experiences.

(Russell, 1948, p. 117)

To utter the sentence, "there was an explosion here yesterday", in the right circumstances, on this account, is to exert a causal power over one's audience. The mental images correlated with the linguistic items are essential to this causal process.

It was exactly this kind of internalist explanation of meaning that Wittgenstein was at pains to counteract in his later philosophy. He is alleged to have dismissed the causal element of the theory by saying that, if I push you, causing you to fall over, your fall is not the *meaning* of my push.<sup>18</sup> This remark brings out an important problem, often overlooked, which faces Russell's particular brand of the causal theory

<sup>17</sup> Russell's advocacy of a causal theory of content is, perhaps, unsurprising. As Linsky (1999, p. 43) notes, most theories of "direct reference", such as Russell consistently adhered to, tend to take causal connections as paradigm cases of the word-referent relation.

<sup>18</sup> Cited in Monk. 2000, p. 211.

of content without impinging on most other versions of the thesis. Whereas, roughly speaking, causal theories standardly seek to identify the meaning of an expression with its causes, Russell also wants to grant the expression causal powers of its own—to at least partly identify the meaning of an expression with its effects.<sup>19</sup> Hence the meaning of the word “explosion” in the above passage is given in terms of the images induced in my mind by my hearing an utterance of “explosion”. Wittgenstein’s remark quite adequately shows the hopelessness of such an account. No criteria are available for distinguishing meaningful effects from non-meaningful ones. My utterance of “explosion” may have the effect of angering a librarian if enunciated with sufficient volume at a suitable location, resulting eventually in my expulsion from that location, but this does not count as a meaning of “explosion”. What is required is some way of distinguishing the semantically relevant effects and such distinctions, it would seem, are unlikely to be available in causal terms.

Unsurprisingly, this version of the causal theory has not exerted much influence on subsequent theories of content. As mentioned before, a more popular and promising, alternative is to seek to try and explain the meaning of a word in terms of that which causes an utterance of it. Such views have indeed been developed to a far greater degree of sophistication than Russell’s theory by contemporary philosophers of mind and cognitive scientists. Such theories are not without problems, however, as a brief survey of the leading candidate reveals.

#### 4.3 Content and Causality

Jerry Fodor is best known for his “Language of Thought” hypothesis. Fodor’s contention is that linguistic creativity, along with first language acquisition, is to be explained by appeal to an innate neuro-linguistic system which provides us with an internal language in which cognitive tasks are performed including the cognitive tasks that he holds to be involved in the acquirement and operation of a natural language.<sup>20</sup> In order to avoid a gauntlet of charges of infinite regress and vicious circularity, without recourse to nativism of a form that would make even the Plato of the *Meno*

<sup>19</sup> Russell states in *An Inquiry into Meaning and Truth* that: ‘It is only in certain cases that the “meaning” of a verbal utterance can be identified with the effect that it is intended to have on the hearer’ (Russell, 1940, p. 28).

<sup>20</sup> See Fodor, 1975.

blush, the language of thought enthusiast needs a non-regressive, non-circular theory of content. As Fodor holds the internal language to be compositional, some account is needed of how the semantic values of the internal lexicon are to be fixed. Fodor's attempt to meet this requirement is one of the most sophisticated brands of causal theory on the naturalist's market; his asymmetric dependence theory. To appreciate what is at stake in the theory, it is essential to consider two basic problems which must be addressed by any theory of content.

Probably the most severe blow for internalist theories of content is Putnam's well known "twin-earth" argument.<sup>21</sup> Putnam expresses his doubts about internalist semantics with a possible-world thought experiment: In some other part of the universe there is a planet, Twin Earth, which is identical to our Earth in every respect except one—wherever  $H_2O$  is found on Earth, a different compound, XYZ, is found on Twin-Earth. Despite their differences in chemical composition,  $H_2O$  and XYZ are superficially identical, that is to say that they are both transparent, odourless, tasteless, and so on. The point that Putnam wishes to make is that on Twin-Earth there is no water, and so if, on Earth, I refer to  $H_2O$  as "water" (without knowing any chemical theory) and, on Twin-Earth, my twin (who is identical to me in every way, including my ignorance of chemical theory) uses the term "water" to refer to XYZ, then "water" is not being used to refer to the same substance in each case. It would therefore seem, if we accept the twin-earth argument against meanings being "in the head", that something external must determine meaning. Without an external determinant of meaning, it would seem impossible for meaning to determine extension. The hope of the causal theorist is that causes will count as suitable external determinants.

Fodor's version of the causal theory of mental content has its roots in the "informational semantics" first propounded by Dretske, and the notion of "natural signs".<sup>22</sup> A natural sign can be defined as a sign whose meaning can be expressed solely by virtue of naturally occurring causal relations. Thus clouds "mean" rain by virtue of the causal relation between clouds and rain; likewise, shadows in the west mean that the sun is in the east, and so on.<sup>23</sup> A central problem for any theory of content of this form, however, is to show how a token can misrepresent. The

<sup>21</sup> See Putnam, 1975.

<sup>22</sup> See Dretske (1981 & 1986).

<sup>23</sup> The very same causal conception of meaning lies behind Russell's remark, made in an exchange with Braithwaite at an Aristotelian Society Symposium, that: 'I am inclined to think that, whenever I notice a horse, I have an impulse to say "horse", though the impulse may be inhibited' (Russell, 1938, p. 365).

importance of this stems from the need for a theory of content to account for the normativity of meaning—the fact that there is, in some sense, a standard of correctness against which the use of a term can be judged. If content is taken to be simply the effect of a cause, then an account must be given for why misrepresentation, the propensity for a semantic item to be wrongly applied, is possible. Why are only some effects correct representations? The basic claim that representations are caused by what they represent needs some further condition if it is to account for the possibility of misrepresentation. Without any further condition, such a basic causal theory would either have to deny that misrepresentation does occur (which is obviously not an option), or say that a representation is caused by a potentially infinite class of causes. The problem is therefore referred to as the ‘disjunction’ problem: content threatens to become simply a disjunction of all its possible causes. Say for example that a causal theory states that a “cow” token is caused by a cow. How can this theory account for the fact that a “cow” token can also be caused by a horse on a dark night, and a seemingly endless list of other ‘wild’ causes? In order to account for the disjunction problem, a causal theory of content must place constraints on the occasions when the nomic connection between a symbol’s cause and its token are meaning-forming.

Fodor’s response to the disjunction problem is to opt for a causal account of misrepresentation. Fodor’s version is what he calls the ‘asymmetric dependency theory’.<sup>24</sup> Just as the name implies, the disjunction problem is accounted for by an asymmetric relation between wild causes of tokens and meaning-forming ones. The wild tokens depend on the meaning-forming tokens, but the meaning-forming ones do not depend on the wild. Therefore we can say that any non-cow caused “cow” tokens are asymmetrically dependent on cow caused “cow” tokens. In a nearby possible world where there are no cow caused “cow” tokens, it surely follows that there cannot be any non-cow caused “cow” tokens. However, in a possible world where there were cow caused “cow” tokens but no wild tokens (say, horse caused “cow” tokens) there would still be cow caused “cow” tokens. The theory is commonly explained by the counterfactual: if there were no cow caused “cow” tokens, there would be no non-cow caused “cow” tokens.

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<sup>24</sup> See Fodor, 1987, Ch. 4, & 1990, Ch. 4.

The asymmetric dependency theory provides a resolution to the problem raised by the twin-earth argument in the following way: my token, “water”, can now refer to a different token to my twin’s token, “water”, on Twin-Earth. My token locks onto H<sub>2</sub>O, whilst my twin’s locks onto XYZ. The asymmetric dependence is reversed in each case: for me the XYZ caused “water” tokens are asymmetrically dependent on the H<sub>2</sub>O caused “water” tokens, but not vice versa. For my twin, however, the H<sub>2</sub>O caused “water” tokens are asymmetrically dependent on the XYZ caused ones, but not vice versa.<sup>25</sup>

Clearly we have here a causal theory of content far more refined than the crude theory advocated in Russell’s later philosophy. There is, however, a surprising aspect of Russell’s thought which lingers on in its modern descendants. In abandoning his anti-psychologism, Russell had also abandoned any hope of accounting for the unity of propositional content. A theory of content which appeals to causal relations in fixing the content of a fragment of language must ensure that that fragment is clearly demarcated from the rest of the language if it is to have its content directly determined by its cause. The paradigm case, of course, is that of a name being caused by its bearer and the hope, drawn from formal semantics in the model-theoretic tradition, is that a derivative relation for predicative expressions whereby they denote objects in their extensions can be provided. Such an approach, if it is to place due weight on causality, will have no option but to treat languages as atomistic. If semantic items mean their causes, then they mean those causes independently of their connections with other semantic items. This is not, of course, to say that such semantic atomism is incompatible with a satisfactory account of the unity of the proposition or propositional content; indeed, I argued in the previous chapter that Frege’s context-principle, invoked in part to preserve the unity of the proposition, does not exclude the possibility of a systematic semantic theory. But it is true that semantic atomism of the causal variety advocated by Russell and developed by Fodor leaves the subject unaddressed.

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<sup>25</sup> The alternative solution to the disjunction problem, preferred by Millikan (1984), is the teleological account. Teleological theories attempt to answer the disjunction problem by invoking the notion of ‘biological proper function’. The nomic connection between a token and its cause is meaning-forming if it conforms to the proper function of the cognitive mechanism in question. A symbol misrepresents if it fails to meet meaning-forming conditions which are, roughly, naturally selected traits. If it is the proper function of the cognitive mechanism to produce “cow” tokens in response to cows, then the production of non-cow caused “cow” tokens signals a failure of the cognitive mechanism to meet the conditions of its naturally selected function.

A systematic theory of meaning has two options in accounting for content: so-called top-down or bottom-up approaches. A top-down approach will begin with sentences or even larger fragments of language and then show how the content of subsentential expressions can be understood in the terms of their roles within the larger fragments. A bottom-up account will begin with the smallest unit of meaning, a subsentential expression, and then view sentences as compositional complexes formed out of these parts. Obviously, the top-down approach has a great advantage in accounting for the unity of propositional content.<sup>26</sup> For the atomistic account, the question remains unresolved. Fodor suggests that recognition of the productivity and systematicity of linguistic knowledge demands that the meanings of sentences be treated as dependent on the meanings of their parts,<sup>27</sup> but this is a labelling of a problem, not a solution of it. Fodor then proceeds on the assumption that such a dependence does indeed obtain, but has no account whatsoever of *how* it obtains. The problem of the unity of the proposition is simply ignored.

A theory of meaning which fails to deliver on the question of just how the meaning of a sentence depends on the meanings of its parts has failed to deliver on perhaps the most important question faced by the meaning-theorist. For a minimum requirement placed on a satisfactory theory of meaning is that it should yield some explanation of what it is that I know, what it is that I am able to do and how it is that I am able do it, when I have mastered a language. If no account of how I am able to construct sentences out of words in order to successfully say something is forthcoming, then the theory in question will be ultimately dissatisfying. An account of mental content in terms of causal relations, however, is poorly equipped to provide us with such answers. The most it can do is to give an account of how contents can be assigned to expressions. It will, however, remain silent on how those contents are to be manipulated into unified thoughts. As such, it leaves the pivotal questions at the heart of the theory of meaning unresolved.

<sup>26</sup> Frege, on the interpretation I have argued for in this thesis, is best understood as having a top-down approach. The sentence is taken as the primary vehicle of meaning and the reference of a sub-sentential expression is determined by the role it plays in a sentence. Dummett's understanding of the reference of a sub-sentential expression as its semantic value is a happier one than the basic idea that a predicate refers to a concept. Brandom's (1994) top-down *use* theory of meaning takes the starting point to be not the sentence but the normative procedures involved in being able to give and ask for reasons and thereby enter into inferential practices. Moving downwards in the direction of explanation, he then seeks to explain singular terms and predication (in terms of substitutional commitments and anaphoric reference) as derivative from the wider role played by a fragment of language in its inferential relations.

<sup>27</sup> See Fodor, 1987, pp. 147-149.

Of course it is open to Fodor to try and give an account of the unity of the proposition by appealing to the syntactical strictures of the internal language of thought, the tokens of which are to have as semantic values the causally fixed content provided by the asymmetric dependence theory. Two things should be noted in relation to this point, however: firstly, the need to fall back on the language of thought is evidence in itself of the inability of the causal theory of content to account for the unity of the proposition. The language of thought serves to compensate for the inadequacies of the causal theory. Secondly, the language of thought should not be understood as explaining *how* the unity of the proposition is guaranteed but simply as positing a biological mechanism as guarantor.

The preceding paragraphs should not be understood as making an objection to naturalism. The causal theory of content may or may not be correct. The point made here is that it remains silent on the question that has been isolated as of central importance to the philosophy of language, namely, the problem of the unity of the proposition. But naturalism in general is not prone to the same inadequacy. Indeed, as we shall now see, Quine, who has exerted perhaps the most profound influence on contemporary naturalism, is far from insensitive to the problem of the unity of the proposition.

#### 4.4 Quine on Meaning Holism and the Web of Belief

We had reason to mention in the last chapter Quine's interpretation of the Fregean context-principle as 'the reorientation whereby the primary vehicle of meaning came to be seen no longer in the term but in the statement' (Quine, 1951, p. 39). As was noted there, Dummett's concern that this amounts to something of a parody of Frege's insight is a valid one. Quine's reasoning, however, is not to be just dismissed as a wilful misreading of Frege. On the contrary, the semantic reorientation he refers to is central to Quine's own philosophical project. The exaggeration of the principle expands it to suit Quine's designs, such that it might be rephrased to read "only in the context of a language does a sentence have meaning". For, according to Quine, 'even in taking the statement as unit we have drawn our grid too finely. The unit of empirical significance is the whole of science' (Ibid., p. 42). The resulting picture of

language is Quine's holism.<sup>28</sup> This Quinean doctrine is often traced back to Pierre Duhem,<sup>29</sup> the so-called Duhem-Quine thesis being the thesis that each theory confronts experience as a whole, rather than in piecemeal, sentential chunks. As Quine famously phrased it: 'our statements about the external world face the tribunal of sense-experience not individually but only as a corporate body' (*Ibid.*, p. 41).<sup>30</sup> Quine's concern, then, is not just with the unity of the proposition, but with the unity of science. Nor should science be construed in the narrow sense that popular culture often favours. Science, for Quine, is continuous with philosophy; the scientist and the philosopher are each, to borrow the metaphor that Quine liked to borrow from Neurath, embarked on the same voyage in search of an ever clearer grasp of the world: 'Neurath has likened science to a boat which, if we are to rebuild it, we must rebuild plank by plank while staying afloat in it. The philosopher and the scientist are in the same boat' (Quine, 1960, p. 3). In thus construing the relationship between science and philosophy, Quine rejects the view, prevalent amongst Wittgensteinian and ordinary language philosophers, that philosophy serves merely to clarify our understanding of language rather than to assert truths and actively revise our understanding of the world.<sup>31</sup> Philosophy, for Quine, is taken to be an activity in the business of asserting truths and revising beliefs. This conception of the relationship between science and philosophy is the key to Quine's naturalism: 'I hold that knowledge, mind, and meaning are part of the same world that they have to do with, and that they are to be studied in the same empirical spirit that animates natural science. There is no place for a prior philosophy' (Quine, 1968a, p. 26).

The holistic system that Quine is concerned with consists, at root, of sentences. It is sentences that we end up turning to if we are embarked on the scientific philosopher's pursuit of truth: 'In sober fact the pursuit resolves into

<sup>28</sup> However, the use of contextual definitions to define symbols in the context of their sentential uses, as exemplified in the theory of descriptions, remains vital to Quine's ontological program, as we shall see shortly.

<sup>29</sup> See Duhem, 1906.

<sup>30</sup> As Esfeld (1998 & 1999) points out, the holism found in the works of both Quine and Wittgenstein raises an apparent problem for our understanding of analytical philosophy. Insofar as analytical philosophy is concerned with analysis of a whole into its parts, holism appears to be departing from a guiding principle of the analytical tradition (indeed, though Esfeld does not mention this, it appears to be returning to something strikingly reminiscent of the very same views that the atomism of Russell and Moore was intended to overthrow). Esfeld seeks to effect a reconciliation of these apparently divergent approaches. One does not have to accept his reconciliation, however, to realise that acceptance of holists into the analytical fold will have important consequences for our understanding of the nature of that tradition.

<sup>31</sup> See Quine, 1960, p. 3. Consider also Quine's remark about his way of talking of bodies as thinking: 'It sounds odd, but philosophically that's all right, outside of midcentury Oxford' (Quine, 1995, p. 85).

concern with particular sentences, ones important to us in one or another way' (Quine, 1995, p. 67). Of course, taking a sentence as true is equivalent to *believing* that sentence. Hence the web of sentences which make up the holistic system in question is, at the same time, the web of belief—the set of interconnected sentences which we (either jointly or severally) hold true. One of Quine's most significant contributions to the philosophies of logic and language is relevant here, namely his clarification of the confusion between sentence and proposition.<sup>32</sup>

In an autobiographical piece, Quine reveals that he had observed and developed a distaste for the 'confusion of sign and object' as early as 1930 when he objected to the confusion in the work of his teacher at Harvard, H. M. Sheffer.<sup>33</sup> In 1934, he began to clean up this confusion as manifested in the standard understanding of the propositional calculus. Quine objected to the notion of propositions as 'hypostatized entities, inferred denotations of given signs' (Quine, 1934, p. 266). Quine's preference was to treat the variables "*p*", "*q*", "*r*", etc., of the calculus, not as ambiguously denoting *propositions*, but just as ambiguously denoting *sentences*. The logical constants, " $\rightarrow$ ", " $\vee$ ", "&", and " $\sim$ ", thereby become sentence connectives rather than predicates or relations that take names of entities as arguments.<sup>34</sup> The variables of the calculus become, in the terminology later adopted by Quine, *schematic letters*: 'the letter "*p*" is no variable ranging over objects; it is only a schematic letter for sentences, only a dummy to mark a position appropriate to a component sentence in some logical form or grammatical construction' (Quine, 1970a, p. 12).<sup>35</sup>

<sup>32</sup> It is interesting to note, therefore, that Fodor and Lepore see in Quine's argument in favour of semantic holism a fallacy of equivocation with regard to the status of *statements*. Quine, they suggest, is 'very careful *not to say what a statement is*' (Fodor & Lepore, 1992, p. 44). They offer three potential candidates (formulas, propositions, and formulas with their conditions of semantic evaluation) and reject all of them, concluding that 'apparently, there is nothing that statements can be, consonant with the use to which the "Two Dogmas" argument for semantic holism wants to put them' (*Ibid.*, p. 53).

<sup>33</sup> See Quine, 1986a, p. 9.

<sup>34</sup> In 1940's *Mathematical Logic*, Quine was influential in bringing the attention of logicians to the difference between the formulas, expressed in a more modern notation, " $p \Rightarrow q$ " (read "*p*" implies "*q*") and " $p \rightarrow q$ " read "if *p*, then *q*". To conflate the two is to conflate use and mention; implication is a two-placed predicate whose terms are the *names* of statements, whereas the conditional is a statement connective. See Quine, 1940, p. 28.

<sup>35</sup> In an autobiographical piece in the *Library of Living Philosophers* volume dedicated to him, Quine makes the following observation on his thought in 1934: 'I was aware that propositions *could* be dispensed with, as witness my "Ontological Remarks on the Propositional Calculus." 1934; there already, terminology aside, was the doctrine of schematic letters' (Quine, 1986a, p. 14).

Quine's dismissal of propositions as entities is not the nominalist's distaste for abstract entities.<sup>36</sup> Abstract entities, though their unnecessary multiplication is not looked on favourably by him, are admitted into Quine's ontology so long as the conditions for identifying them are sufficiently clear.<sup>37</sup> Where these conditions are not sufficiently clear, he maintains, is in the case of *intensional* entities: 'Intensions are creatures of darkness, and I shall rejoice with the reader when they are exorcised' (Quine, 1956, p. 188). In particular, Quine objects to what he calls the 'referential opacity' of intensional contexts.<sup>38</sup> Likewise, his attack on essentialism and modal logics turns on his refusal to tolerate the effects of the modal operators on truth-value preservations across substitutions of *identicals*. His favourite example is that of the failure to preserve truth in the transition from " $\square(9 > 7)$ " to " $\square(\text{the number of planets} > 7)$ ", despite the fact that that  $9 = \text{the number of planets}$ .<sup>39</sup> The shadowy nature of the intensional is seldom at its most misleading, according to Quine, than in the hopelessly confused conception of the proposition as the meaning of a sentence. If propositions are the meanings of sentences, he argues, then it must follow that there is a relation of synonymy for sentences such that two sentences are synonymous when they express the same proposition. This relation, Quine argues, 'makes no objective sense at the level of sentences' (Quine, 1970a, p. 3).

<sup>36</sup> Though Quine does express some sympathy with nominalism, characterizing himself as one with 'a taste for desert landscapes' (Quine, 1948, p. 4.), he should not be understood as advocating the position. As we shall see, Quine holds that certain entities disallowed by the nominalist (most notably classes), are essential to science and hence unavoidable. Quine's sympathies with nominalism were given their fullest expression in a paper co-authored with Nelson Goodman in 1947, 'Steps toward a constructive nominalism', which led many to overestimate the extent of his nominalist sympathies (See Quine, 1960, p. 243, ft. nt.; & 1986a, p. 26). Even Carnap appears to have assumed that Quine was a nominalist at one point. In a letter to Quine, dated 21/7/49, inviting Quine to comment on a manuscript, Carnap wrote: 'I should like to hear whether you think that I did or did not do justice to your position and that of the other nominalists' (Carnap to Quine, 21/7/49, in Carnap & Quine, 1990, pp. 414-415). In marginalia on his copy, Quine notes that he replied to the letter six days later. Unfortunately, no copy of the reply appears to have survived (see *Ibid.*, p. 415).

<sup>37</sup> See Quine, 1992, p. 31.

<sup>38</sup> See Quine, 1953b, p. 142.

<sup>39</sup> See Quine, 1953b. See Føllesdal (1986) for an excellent exposition and discussion of Quine's view on modalities. See also Mondadori (1995) for a more detailed analysis of Quine's objections to quantifying into referentially opaque modal contexts.

#### 4.5 The Attack on Analyticity and the Flight From Intension

Quine's seminal 1951 paper, 'Two Dogmas of Empiricism', directs a battery of arguments against the notion of synonymy commonly appealed to in explicating the concept of analyticity. His rejection of meanings as entities, on the grounds that it marks a failure to appreciate the distinction between meaning and reference is, of course, inherited from Frege, and his dismissal of meaning as 'what essence becomes when it is divorced from the object of reference and wedded to the word' (Quine, 1951, p. 22) is strongly reminiscent of Wittgenstein's warnings against the temptation to view meanings as objects.<sup>40</sup> The two arguments against *propositions* as meanings of sentences which are of most relevance to this discussion are the following: (1) Quine denies that identity conditions of sufficient clarity to pick out the proposition expressed by synonymous sentences can be given; and (2) he takes his aforementioned holism to discount the possibility that the meaning of a statement can be given independently of the wider body of sentences in which it is embedded. The first point attacks the notion of meanings as entities, the second the reductionism contained in the verificationist theory of meaning urged by the logical positivists.

According to Quine's empiricist account of our knowledge of objects, all objects are 'reifications'; aspects of the external world which we posit as independent entities. Experience is merely a matter of the irritations on our sensory surfaces in response to which we begin to posit entities. The infant, according to Quine's behaviourist account of learning, is helped along by the instruction she gains from the rewards and punishments thrust upon her by her parents. The positing of physical bodies is, naturally enough, held to be the paradigm case of this primitive categorization of the world into distinct entities: 'Bodies are our first reifications: the first objects to be taken as objects. It is in analogy to them that all further positing of objects takes place' (Quine, 1995, p. 24). Objects are, typically, at variance of one form or another with their surrounding environment; they differ in colour, movement, shape, and exhibit a resistance to pressure if touched, for example: 'It is merely such traits, at first, that distinguish bodies from the glow of the evening sky, the feel of a

<sup>40</sup> In 'Ontological Relativity' (amongst other places) Quine dismisses the view that meanings are objects as 'the myth of a museum in which the exhibits are meanings and the words are labels' (Quine, 1968a, p. 27) and credits Wittgenstein with exposing the myth. It is unlikely that Wittgenstein exerted much influence on Quine's position, however; a more likely influence is Dewey, who Quine points out was suggesting a similar view long before it was developed by Wittgenstein (See *Ibid.*).

cool breeze, or other details of the passing show' (Ibid.). However, this is not full blown reification—only its beginnings. Full blown reification, Quine maintains, requires an additional ingredient of further sophistication: the shift from so-called “pronouns of laziness” to “essential pronouns”.<sup>41</sup>

In the sentence “If Mary wants to arrive on time, she should leave now”, the pronoun “she” is replaceable by its antecedent “Mary” without loss of sense. The *lazy* pronoun is a grammatical shortcut avoiding repetition. The same cannot be said for the sentence, “If any positive integer is even, adding it to one yields an odd number”, which clearly has a different sense to the sentence resulting from a repetition of the antecedent in place of the pronoun: “If any positive integer is even, adding any positive integer to one yields an odd number”. The former sentence is true, the latter false. Such essential pronouns mark the shift from singular reference (as in the reference to Mary in our first sentence) to *quantificational* reference. Essential pronouns, Quine concludes, are the natural language correlates of bound variables. This leads us to Quine’s ontological criterion, captured in the well-rehearsed slogan, ‘to *be* is to be the value of a variable’ (Quine, 1939, p. 199). Of course, the act of existentially quantifying over some domain does not bring the elements of that domain into being. It does, however, by this criterion, assert the existence of those elements. To assert the quantificational sentence “ $\exists x Fx$ ” is to assert that there is something which is *F*. Hence the criterion is intended to reveal the ontological commitments of a theory; that which must exist for the theory in question to be correct. If a theory contains, as a vital ingredient, the sentence “ $\exists x (x \text{ is a man} \ \& \ x \text{ is mortal})$ ”, then it hinges on the truth of that sentence. This truth, in turn, hinges on the existence of men and mortals.<sup>42</sup>

We saw in chapter two, above, that Quine objected to the contextual definition of classes in *Principia* on the grounds that classes were only eliminated at the expense of their reduction to platonic attributes in intension. This conclusion follows from Quine’s criterion, as the contextual definition in question does indeed quantify over

<sup>41</sup> See Quine, 1995, p. 27. The terms originated with Geach, 1962. Brandom’s derived terminology of pronouns serving either a ‘lazy’ or ‘quantificational’ purpose makes the distinction far clearer (See Brandom, 1994, pp. 301-303).

<sup>42</sup> Another way of phrasing the point, noted by Orenstein (2002, p. 27) is to say that Quine takes predication to be, in some sense, more primitive than naming. There are two forceful reasons for doing so: firstly, quantification (objectually interpreted) over transfinite domains allows us to refer to elements of collections whose cardinality exhausts our stock of names, as in the case of the real numbers. Secondly, Quine takes names to be eliminable from a fully regimented canonical notation. See Quine, 1966a, p. 95. & 1960, § 37.

propositional functions.<sup>43</sup> Quine objects to higher-order logic, not because it condemns one to platonism, however, but because of the kind of platonic entity it introduces. In his 1963 book *Set Theory and Its Logic*, he introduces what he calls *virtual classes*. Virtual classes, along with the two-placed predicate of class membership are defined contextually by the principle of *concretion*:

$$y \in \{x : Fx\} = Fy \quad \text{Df.}^{44}$$

The sole purpose of this device is one of convenience. It allows talk of classes without enlisting classes as features of one's ontology. The limit that Quine wishes to impose on the extent to which such contextual definitions are useful, however, ends with the paraphrasing of formulas in first-order logic. To go beyond this into the realms of higher-order quantification is to quantify over things other than objects. Though he accepts that a degree of platonism is needed for the firm foundation of mathematical truth, Quine does not countenance quantification over properties; whereas properties are shadowy, intensional entities, classes are well-defined objects which, despite their abstract nature, fit easily into the extensional language of acceptable science. far better then, in Quine's view, to quantify over classes and take our ontological commitment squarely in the form of well-defined abstract objects.<sup>45</sup>

An integral part of recognising something as an object worthy of admission into one's ontology is the establishment of identity conditions for that object. It was just this insight which we saw brilliantly utilised in our discussion of Frege's contextual definition of number. Numbers are to be treated as objects on the grounds that we can reliably establish the sense of a statement of numerical identity of the form "the number of *Fs* = the number of *Gs*". Quine's famous dictum 'no entity without identity' (Quine, 1958, p. 23) is an acknowledgement of much the same point: 'Identity is intimately bound up with the dividing of reference. For the dividing of reference consists in settling conditions of identity: how far you have the same apple and when you are getting onto another' (Quine, 1960, p. 115).<sup>46</sup>

<sup>43</sup> See Whitehead & Russell, 1910, \*20.01.

<sup>44</sup> See Quine, 1963, p. 16.

<sup>45</sup> See Quine, 1960, § 55.

<sup>46</sup> Quine, indeed, holds that quantification presupposes a determinate identity relation. See his (1964). See also Dummett, 1991d, for an extended discussion of the topic.

In rejecting propositions as the shared meanings expressed by synonymous sentences, Quine is rejecting the idea that we can give sufficiently precise conditions under which two sentences can be said to express (or mean) the *same* proposition. If there is no entity without identity, then we ought to be able to specify when any entity is identical with itself, or equivalently, when any two expressions turn out to designate the same thing. In the case of propositions, Quine thinks the attempt to meet this demand is hopeless. The proposed equivalence relation needed for two sentences to express the same proposition, he insists, ‘makes no objective sense at the level of sentences’ (Quine, 1970a, p. 3). “Two Dogmas” contains a series of attacks on the intelligibility of this equivalence relation, all based around the idea that any attempt to give a sufficient specification of the relation will fall into a circle of murky intensional concepts, such as those of analyticity, synonymy, and so forth. Having shown that any attempt to explicate such concepts must draw on other members of that group, he concludes that such explications are viciously circular and thereby spurious.<sup>47</sup>

Many have objected to the implicit notion contained in the attack that the only acceptable account of analyticity will have to result in a reduction of the concept to purely extensional terms.<sup>48</sup> Extensionality is, in Quine’s view, ‘the watchword of austere science’ (Quine, 1986b, p. 115); for those who do not share this preference, however, the appeal to other intensional concepts required for the explanation of analyticity is not necessarily consigned to vicious circularity. There is little to be gained in dwelling on this point, however. As Dummett (1973b) has pointed out, the important part of the argument against analyticity in “Two Dogmas” is the second argument enumerated a few pages back, namely, the objection to the reductionism involved in the verificationist theory of meaning.<sup>49</sup> Quine’s argument on this front is certainly more forceful.

The verificationist principle, that the meaning of a statement is the method of its verification, presupposes a certain degree of sensitivity to the unity of the proposition. By taking the statement as the unit of verification, propositional contents are granted a special status not attributable to the content of subsentential expressions. It is in response to this, however, that Quine insists on taking the whole of science as

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<sup>47</sup> See Quine, 1951, §§ 1-4.

<sup>48</sup> The first to make this criticism were Grice and Strawson (1956). More recently, Hacker (1996, pp. 211-216) and Glock (1996a, pp. 204-205) have reiterated and developed the point.

<sup>49</sup> This is also noted by Glock (1996a, pp. 205-206).

the smallest unit of empirical significance.<sup>50</sup> The Vienna Circle held the distinction between empirical and *a priori* truths to reside in the distinction between sentences about the world (sentences with a *factual* component, as Quine puts it) and sentences true or false regardless of the world (sentences true or false by virtue of their meanings alone—described by Quine as those with only a *linguistic* component). The second class of sentences, the analytic ones, were held to be true come what may. But this depends on the possibility of making such a sharp distinction between factual and linguistic content and, if one accepts Quine's holism, this seems spurious.

Quine's point can be made with the help of an example from Carnap. Carnap made a distinction between two kinds of verification: *direct* and *indirect* verification. A proposition is verified directly if it is about a present perception, in which case it is verified by my present perception. A proposition P is verified indirectly if the verification depends on direct verification of propositions deduced from P together with other already verified propositions. For example, take the proposition P<sub>1</sub>, "This key is made of iron". According to Carnap, the process of deduction proceeds as follows:

Premises: P<sub>1</sub>: "This key is made of iron;" the proposition to be examined.

P<sub>2</sub>: "If an iron thing is placed near a magnet, it is attracted;" this is a physical law, already verified.

P<sub>3</sub>: "This object—a bar—is a magnet;" proposition already verified.

P<sub>4</sub>: "The key is placed near the bar;" this is now directly verified by our observation.

From these four premises we can deduce the conclusion:

P<sub>5</sub>: "The key will now be attracted by the bar."

<sup>50</sup> As Hacker (1996, p. 202) notes, this position was refined in Quine's later work as he accepted that 'some middle-sized scrap of theory usually will embody all the connections that are likely to affect our adjudication of a given sentence' (Quine, 1960, p. 13). Later still, in answer to the question how large this scrap must be, he remarks: 'For the most part ... a testable set or conjunction of sentences has to be pretty big, and such is the burden of holism. It is a question of critical semantic mass' (Quine, 1992, p. 17).

This proposition is a prediction which can be examined by observation.  
(Carnap, 1935, pp. 11-12)<sup>51</sup>

Quine's criticism is not just a rejection of a clear distinction between direct and indirect verification, though this will indeed follow from his holism. More importantly, however, the determinateness that Carnap sees in the systematic deduction of an indirect verification is also under threat. If it is the system as a whole which is apt for empirical verification, then there is no reason why changes within the system could not affect the evaluation we make of a supposedly direct verification. The system is, as Quine puts it, 'so underdetermined by its boundary conditions. experience, that there is much latitude of choice as to what statements to reëvaluate in the light of any single contrary experience' (Quine, 1951, pp. 42-43). The distinction between factual and linguistic content accordingly collapses; no sentence is independent of the other sentences it is logically interconnected with and, by the same token, every sentence is connected, eventually, to the experiential periphery of the system by virtue of its outward connections with other sentences in the system. Seen thus, it is evident that we have the (at least theoretical) ability to make drastic enough changes to the inner sentences of the system to ensure that any sentence can be held true come what may. The traditional distinction between analytic and synthetic statements clearly cannot withstand such pressure, for now any synthetic statement could, if we so desired, be made into an analytic one and, of course, vice versa:

Any statement can be held true come what may, if we make drastic enough adjustments elsewhere in the system. Even a statement very close to the periphery can be held true in the face of recalcitrant experience by pleading hallucination or by amending certain statements of the kind called logical laws. Conversely, by the same token, no statement is immune to revision. Revision even of the logical law of the excluded middle has been proposed as a means of simplifying quantum mechanics; and what difference is there between such a shift and the shift whereby

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<sup>51</sup> Carnap goes on to point out that, if  $P_5$  is an 'instance of verification' of  $P_1$  then  $P_1$  has still not been conclusively verified. The number of instances deducible from  $P_1$  is infinite, therefore there is always the possibility, however small, of finding a negative instance at some point in the future. Thus  $P_1$  remains a *hypothesis*. See Carnap, 1935, p. 13.

Kepler superseded Ptolemy, or Einstein Newton, or Darwin Aristotle?  
(Quine, 1951, p. 43)

Are we to assume, then, that the laws of logic are merely a matter of preference, chosen on the basis of their convenience for the scientist? In the above passage Quine seems to be saying as much. It has not gone unnoticed, however, that he has seemed to retreat from this position in later work. Dummett, for example, accuses Quine of having 'totally reversed his position', and suggests that by 1970's *Philosophy of Logic* Quine held that 'it is impossible for anyone to deny a law of classical logic' (Dummett, 1976, p. 270). Haack (1978, p. 237) also draws attention to the apparent disparity between Quine's extravagant claim in 'Two Dogmas' and the increasing conservatism which first surfaces in *Word and Object*.<sup>52</sup> This conservatism appears to be, in effect, a decision on Quine's part to exempt the logical constants from his otherwise damning attack on meaning-notions. As both Dummett and Haack notice, this paves the way for a so-called 'meaning-variance' argument designed to temper the potentially threatening consequences of deviant logics for Quine's conservatism. In the following section, however, I will argue that Quine's conservative shift is not an illicit one, being in fact quite consistent with his attack on the analytic/synthetic distinction. Nonetheless, there are problems with his position, as we shall see.

#### 4.6 A Partial Defence of Quine on Logical Truth

A logic can be (roughly) defined as deviant if it is *incompatible* with classical logic: that is to say: if some theorems of classical logic are no longer valid in the alternative system. So, for example, many-valued logics, relevance logics, quantum logics, all involve a rejection of some classical theorems, principles, or rules of inference (roughly: the principle of bivalence, *modus ponens*, and the distributive laws, respectively). It is logics such as these (if logics are what they are) which Quine's conservatism seeks to exclude. His argument against the deviants proceeds as follows: Should one wish to propose a system of logic wherein the classical laws were no longer held to be valid, *and* hold that the proposed system were a genuine

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<sup>52</sup> See Quine, 1960, p. 59.

advancement on classical logic (as opposed to a mere un-interpreted mathematical curiosity, say), we would have no choice but to reinterpret this new system as either (1) a peculiar dialect which translates as essentially equivalent to our familiar classical logic, or (2) concerned with a different subject matter all together. Hence neither (1) nor (2) marks a genuine threat to classical logic. In the first example, the terms are unfamiliar but the meanings remain the same. In the second, the terms are familiar but the meanings are alien.

As an example of the first scenario, Quine imagines an aspiring deviant logician who decides to apply all and only the laws which are classically reserved for conjunction to disjunction and vice versa. The only sensible response, he suggests, would be to ‘impute our orthodox logic to him, or impose it on him, by translating his deviant dialect’ (Quine, 1970a, p. 81). Quine’s suggestion, in other words, is that we interpret this putative deviation from the norm as a straightforward codification of classical logic, with no loss (or gain) in scope or meaning.<sup>53</sup> Most of us, presumably, would be happy to side with Quine thus far. Less palatable, however, are his reasons for (2).

Turning his attention to, as he puts it, ‘a popular extravaganza’ (*Ibid.*), Quine considers the consequences of attempting to reject the law of non-contradiction so as to admit the possibility of certain sentences being true along with their negations. Aside from the very real problem of imagining just what it would *mean* for a sentence and its negation to be simultaneously true, we are also faced with the devastating consequence of making all subsequent inferences valid. Any conjunction of the form ‘ $p \ \& \ \sim p$ ’ logically implies every other sentence, hence the admission of the paradoxical conjunction collapses the distinction between good and bad inference. All inference becomes good inference in our new system which can now be used to prove anything at all. Such is the character of an inconsistent set of axioms to the classical logician but, amongst deviant logicians, this is not necessarily held to be so. Some have argued that allowing the legitimate formation of paradoxes and then seeking to isolate them so as to stop them infecting the rest of the system with the inability to differentiate good inferences from bad is an acceptable response to the appearance of

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<sup>53</sup> See Levin (1978) for a detailed account of the role played by Quine’s theories of holism and translation in his understanding of the nature of logical truth.

a contradiction in an otherwise healthy-looking system.<sup>54</sup> Faced with such a logic, there are those who see the new system as an abomination threatening to undermine the very foundations of science, and then there are those who argue that it is, on the contrary, a more accurate logic and therefore a *firmer* foundation for science. Quine has little time for either party, saying:

My view of this dialogue is that neither party knows what he is talking about. They think they are talking about negation ... but surely the notation ceased to be recognizable as negation when they took to regarding some conjunctions of the form ' $p \cdot \sim p$ ' as true, and stopped regarding such sentences as implying all others. Here, evidently, is the deviant logician's predicament: when he tries to deny the doctrine he only changes the subject.

(Quine, 1970a, p. 81)

Of course, rejection of the law of non-contradiction is a radical move which is rarely urged in all seriousness. The dubious merits of paraconsistency need not detain us further. But the problem with Quine's argument here does not depend on whether or not we sympathise with deviant logicians. The problem is: can Quine legitimately appeal to a meaning-variance argument to defend the laws of classical logic if he is to remain consistent with his own doctrines concerning the indeterminacy of meaning?

The meaning-variance argument takes the form of denying that deviant logics constitute a substantial threat to classical laws, but simply offer alternative systems which give different meanings to the logical connectives. Hence, it is often argued, the adoption of truth-values over and above the standard two must lead to different readings of the truth-functions, for the truth-functions are characterised by axioms, rules, and matrices which are unavoidably altered by the admission of a new value or set of values. Hence, the absence of the law of excluded middle from 3-valued logics simply shows that the wedge ' $\vee$ ' or the tilde ' $\sim$ ' are no longer being used to mean 'or'

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<sup>54</sup> These so-called 'paraconsistent' logics vary in their approaches, but all have the same object: the preservation of systemic consistency without automatically exiling contradictions. Often quoted contenders for the title of 'true contradiction' or 'dialetheia' are Russell's paradox of predication (see chapter 2, above), and the Liar paradox. For a helpful exposition of paraconsistent logics see Priest, 2001, especially Chs. 7-8.

or ‘not’ in the same sense that the signs for disjunction and negation are in classical two-valued logic.

Now the argument may be a good one: certainly one would hope that someone who took to claiming that ‘ $p \ \& \ \sim p$ ’ was true, had a different idea in mind of what the logical vocabulary signifies in that sentence and, certainly, such a claim would normally be accompanied by certain suggested alterations of the rules governing the use of that vocabulary. The problem for Quine, however, is that the argument is strikingly similar to the kinds of arguments which Quine takes to be inadequate responses to his own attack on the traditional notion of meaning (See Dummett, 1976, p. 270). What grounds, then, does Quine have for making an exception in the case of the logical vocabulary?

Quine defines a logical truth as ‘*a sentence from which we only get truths when we substitute sentences for its simple sentences*’ (Quine, 1970a, p. 50). So,

$$\sim \exists x (x \text{ floats} \ \& \ \sim (x \text{ floats}))$$

counts as a logical truth, but

$$\sim \exists x (x \text{ floats} \ \& \ x \text{ burns})$$

does not. The former sentence is true because any substitution instance of the simple sentence ‘ $x$  floats’ will yield a true instance of the overall schema ‘ $\sim \exists x (Fx \ \& \ \sim Fx)$ ’. The definition is, essentially the model-theoretic definition of logical truth given by Tarski in his 1936 article ‘On the Concept of Logical Consequence’. Tarski defines a logically true sentence as a sentence which is true in all models, that is to say, true in every interpretation.<sup>55</sup> But, of course, ‘ $\sim \exists x (x \text{ floats} \ \& \ \sim (x \text{ floats}))$ ’ is true in all models only because the logical constants have a determinate meaning in all such models. Were we to grant that

$$\sim \exists x (x \text{ floats} \ \vee \ \sim (x \text{ floats}))$$

<sup>55</sup> To be more accurate, Tarski defines the relation of *logical consequence* between sentences, as follows: ‘*The sentence  $X$  follows logically from the sentences of the class  $K$  if and only if every model of the class  $K$  is also a model of the sentence  $X$* ’ (Tarski, 1936, p. 417). The logically true sentences are just those sentences which follow, in accordance with this definition, from any set of sentences whatsoever (see Etchemendy, 1999, p. 11).

is a valid substitution instance for the schema ' $\sim\exists x (Fx \ \& \ \sim Fx)$ ', then our original sentence could hardly be taken as logically true. The meanings of the logical constants have to be determinate if the constants are to *be* constant. And Quine's definition of logical truth demands that they are. Before concluding that Quine has quietly turned his back on the extreme position occupied in 'Two Dogmas' and allowed himself to slide back into a reassuring confidence in the very dogma he urged us to reject, we should note that the meaning-variance argument is but one aspect of Quine's reply to the deviant logician. The argument may be an adequate reply to the paraconsistent logician's claim to have abandoned the law of non-contradiction, but there are other deviations from classical standards which are not so easily dismissed. The most well-known, of course, is the logic of Intuitionism which was inspired by Brouwer and formally developed by Heyting.

The intuitionists' rejection of the law of excluded middle (along with other rejections of the principle, such as in the case of quantum logics) is, Quine concedes, a coherent one.<sup>56</sup> It is not, however, the kind of approach he recommends. Such innovations run counter to what Quine calls 'the maxim of minimum mutilation' (Quine, 1970a, p. 85). Here is the nub of Quine's argument against deviant logics. Quine's holism shows that we can exploit the under-determination of theory by experience to provide sufficient resources to *avoid*, on purely pragmatic grounds, any alterations to the logical and mathematical laws. Sentences do not stand alone as independent items to be accepted or rejected on their own merits, but stand together as an interconnected system of compatible beliefs. Expulsion of one sentence from the overall body of sentences will damage the web of connections surrounding that sentence. And the extent of that damage will depend on the status (roughly, the number of connections with other sentences) of the sentence rejected. Sentences such as logical laws stand at the very centre of the web, impinging as they do on just about

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<sup>56</sup> The concession is, indeed, essential if Quine is to avoid collapsing into absurdity. The meaning-variance argument will be ineffectual as a response to the intuitionist's treatment of the logical constants for a variance in meaning is precisely what the intuitionists are demanding. As Dummett has repeatedly pointed out, intuitionism is no mere mathematical curiosity; its whole purpose is philosophical: 'Intuitionists are engaged in the wholesale reconstruction of mathematics, not to accord with empirical discoveries, nor to obtain more fruitful applications, but solely on the basis of philosophical views concerning what mathematical statements are about and what they mean' (Dummett, 1977, p. viii). Similar arguments apply to deviant interpretations of the quantifiers, as we shall see below.

every other sentence in the system. Hence the damage caused by their expulsion would be devastating.

Quine's maxim of minimum mutilation is a plea for calm. As such it is entirely consistent with the claim cited earlier from 'Two Dogmas'; revision of logical laws is conceivable, but it is not advisable. A statement can indeed be held true, come what may, even in the face of conflicting evidence, if we make a suitable adjustment in our logical laws but, similarly, the laws of classical logic can be preserved with all their simplicity, elegance, and utility, if we opt instead for making suitable adjustments elsewhere in the system. Quine points out that intuitionistic logic, deviating as it does not just from the classical 2-valued paradigm but even from the perspicuity of truth-functional many-valued logics, lacks 'the familiarity, the convenience, the simplicity, and the beauty of our logic' (Quine, 1970a, p. 87). Accordingly, he urges restraint. Constructivism can be practiced within the confines of orthodox logic, and hence Quine sees intuitionism as an impractical and unnecessary deviation when one remembers that 'constructivist scruples can be reconciled with the convenience and beauty of classical logic' (Ibid., p. 88).<sup>57</sup> Similar reservations drive Quine's rejection of tensed logics. Whereas some have proposed to extend classical logic so as to accommodate the introduction of tense operators which attribute differing truth-values to tensed sentences over time,<sup>58</sup> Quine advocates the semantic innovation of 'temporal quantifiers' which quantify over so-called 'epochs'. An epoch, according to Quine, is a 'slice of the four-dimensional material world, exhaustive spatially and perpendicular to the time axis' (Quine, 1960, p. 172). The use of such quantificational devices allows for each sentence to be indexed with an eternally valid four-dimensional set of co-ordinates. Ambiguities of tense are overcome by the resulting Quinean 'eternal sentences'.<sup>59</sup> The consonance of this approach with relativity theory is, Quine happily notes, an obvious further advantage: relativity theory, he insists, 'leaves no reasonable alternative to treating time as spacelike' (Ibid.).

<sup>57</sup> Quine is here referring to Weyl's constructive set theory which, as Quine puts it, 'goes constructivist only in its axioms of existence of sets' (Quine, 1970a, p.88).

<sup>58</sup> E.g., Prior, 1967 & 1968. See Smart (1986) for discussion of Quine on space-time.

<sup>59</sup> The influence of this doctrine on both Davidson's philosophy of action and his philosophy of language is obvious.

Dummett's accusation that Quine 'totally reversed his position' between 1951 and 1970 is, it would seem, unfair.<sup>60</sup> Likewise the accusation that Quine relies on the very notion of determinate meaning which he is supposed to have undermined is misguided in the sense that the meaning-variance argument is not the key argument behind Quine's logical conservatism and, certainly, is not essential to it. Having defended Quine thus far, however, I now want to draw attention to another feature of his conservatism which is less easily disentangled from the meaning-variance argument.

In the chapter on deviant logics in *Philosophy of Logic* Quine devotes a fairly substantial part to discussion of substitutional quantification.<sup>61</sup> Substitutional quantification, as Quine went to great lengths to explain in the early 60s, is not what Quine calls quantification. Quantification properly construed, he contends, is to be interpreted *objectually*. That is to say, the bound variables of quantification range over a universe of objects, things, entities. And this, of course, is simply to reiterate that 'to be is to be the value of a variable'. By contrast, the *substitutional* quantifier is interpreted on different lines. Whereas ' $\forall x Fx$ ' is true if and only if all objects are  $F$ , and ' $\exists x Fx$ ' is true if and only if some object is  $F$ , on the objectual interpretation, the substitutional interpretation of the quantifier (first urged by Marcus) holds that ' $\forall x Fx$ ' is true if and only if every substitution instance of a name for the variable ' $x$ ', on elimination of the quantifier, yields a true sentence. The avowed intention is to avoid the very ontological commitments that Quine embraces.

Earlier, I characterised Quine's objection to the deviant logician's attempt to revise classical logic as dividing all such attempts into two broad categories: in the first category the deviant logician's terms are unfamiliar but the meanings turn out the same as those of our familiar terms after translation. In the second category, the terms are familiar enough, but the meanings they are taken to have are alien to us. In the latter case, Quine concludes that the new system is not so much a rival to classical logic, as it is a different subject altogether. The substitutional interpretation of the

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<sup>60</sup> Dummett does recognise that Quine appeals to the maxim of minimum mutilation rather than a meaning-variance argument in reply to the challenge of intuitionism. However, he holds that this position still amounts to a retreat from the position of 'Two Dogmas'. See Dummett, 1976. pp. 270-271.

<sup>61</sup> Substitutional quantification is deviant in the sense given earlier: certain classical theorems are lost if the universe of quantification is taken to be a set of names, as there are (according to a famous theorem of classical logic) more objects than can be named. For example, there are more real numbers than can be named.

quantifiers, presumably, would fit into the second category and, hence, fall prey to the meaning-variance argument. Substitutional quantification, it will be said, is not a rival to objectual quantification; it is simply a different type of thing altogether. The substitutional quantification theorist and the objectual quantification theorist are simply talking about different things.

As an objection to substitutional quantification, however, this is not a very good one. It is not a very good one, precisely because it reaches the very same conclusion that the advocates of substitutional quantifiers themselves urge. The notation of predicate logic is not under threat from substitutional logic, nor (in general) are any syntactic features of predicate logic (with the exception of those theorems which fail on semantic grounds for the reason given earlier). What is at stake is how best to interpret the syntactic elements of the calculus. What is being suggested is an interpretation which may be more palatable to those of a certain philosophical persuasion (i.e., those keen to avoid Quine's ontological criterion). The variance of meaning from objectual to substitutional interpretation is hardly surprising, therefore. It is precisely what is intended. Nor is it immediately clear that adoption of substitutional quantifiers would clash too much with the maxim of minimum mutilation. One may of course cite Cantor's theorem as an innocent casualty of the shift in interpretation but, on the one hand this may well be a desirable result for those of a nominalist tendency anyway,<sup>62</sup> and, on the other, adoption of the substitutional quantifier in some contexts does not disallow the use of an objectual quantifier in others. Quine's argument is that substitutional quantification is not quantification proper. But, so far as his arguments show, we ought only to conclude that it is not *Quinean* quantification. Indeed, the only obvious casualty of substitutional quantification is Quine's claim that to be is to be a value of a variable.

In the next chapter, we will address Quine's influential doctrine of naturalized epistemology. Again this thesis relies heavily on the conclusions he draws from his holistic conception of linguistic meaning. Before doing so, however, it is helpful to examine two further, connected, components of his philosophy which likewise originate in his holism, and provide much of the justification for his naturalism; namely, the theses of ontological relativity and the indeterminacy of translation.

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<sup>62</sup> See Marcus, 1978, p. 167.

#### 4.7 Radical Translation and Ontological Relativity

Quine's indeterminacy of translation thesis is first introduced by way of a thought experiment. Quine invites us to imagine a field linguist who enters into a completely unfamiliar community and sets about constructing a manual for translating the terms of this alien language into his own. With no background of shared linguistic practices, the linguist must embark on a process of 'radical translation', determining the meanings of native utterances with nothing to go on other than the behaviour of the speakers under investigation:

The recovery of a man's current language from his currently observed responses is the task of the linguist who, unaided by an interpreter, is out to penetrate and translate a language hitherto unknown. All the objective data he has to go on are the forces that he sees impinging on the native's surfaces and the observable behaviour, vocal and otherwise, of the native.

(Quine, 1960, p. 28)

Quine introduces the notion of *stimulus meaning* as the content attributable to the speaker in the light of his observable responses to certain stimuli. The *affirmative stimulus meaning* of a sentence for a given speaker is defined as the class of all the stimulations that would prompt the speaker's assent to that sentence.<sup>63</sup> If a speaker reliably produces the utterance "Gavagai" in the presence of rabbits, then the linguist will, as a working hypothesis, take such stimulation to be the stimulus meaning of the 'occasion sentence' "Gavagai".<sup>64</sup> The linguist's equation of the native utterance "Gavagai" with the English occasion sentence "Rabbit" (where "Rabbit" is construed as elliptical for something like "Lo, a rabbit") is, according to Quine, an 'analytical hypothesis' (Quine, 1960, p. 68). The indeterminacy of translation surfaces once we realise the extent to which the choice of translation for a given Jungle term or sentence is underdetermined by its stimulus meaning.

<sup>63</sup> See Quine, 1960, p. 32.

<sup>64</sup> Quine points out that it is important to remember that it is a class of stimulations that are relevant here, not the class of rabbits. The same response could be evoked through confronting the speaker with a counterfeit rabbit so long as the stimulation was sufficiently akin to that produced by a genuine rabbit. See Quine, 1960, p. 31.

The field linguist gathering data for his translation manual purely on the empirical evidence to hand will be faced with a number of competing translations for the sentence “Gavagai”. There is no available method, however, for settling determinately on one translation alone. If the sentence “Gavagai” is uttered only in the presence of rabbits, for example, the sentence may be translated as “Rabbit”, “Rabbit stage”, or even “Un-detached rabbit part”. All are equally justified translations on the basis of the available stimulus meanings. Should, as we might expect, the linguist opt for “Rabbit” as translation, this will just reflect his own linguistic habits rather than those of the native population under investigation: ‘When, from the sameness of stimulus meanings of “Gavagai” and “Rabbit” the linguist leaps to the conclusion that a gavagai is a whole enduring rabbit, he is just taking for granted that the native is enough like us to have a brief general term for rabbits and no brief general term for rabbit stages or parts’ (Ibid., p 52). The indeterminacy of translation might manifest itself in the existence of two or more translation manuals from Jungle to English, all of which have an equal claim to correctness but yield incompatible English sentences: ‘There can be no doubt that rival systems of analytical hypotheses can fit the totality of speech behaviour to perfection, and can fit the totality of dispositions to speech behaviour as well, and still specify mutually incompatible translations of countless sentences insusceptible of independent control’ (Ibid., p. 72).

From what we have already seen of Quine’s views on reference it should be evident that two competing translation manuals will, in effect, be assigning different referents to Jungle terms. Indeterminacy of translation reflects the inscrutability of reference and, Quine insists, ‘the inscrutability of reference is not the inscrutability of a fact; there is no fact of the matter’ (Quine, 1968a, p. 47).<sup>65</sup> Of course, to interpret such a claim as amounting to the assertion that there really is no objective difference between a rabbit, a rabbit stage or an un-detached rabbit part, would be, as Quine admits, to degenerate into absurdity.<sup>66</sup> The escape from this absurd conclusion lies in his acceptance of the principle of ‘ontological relativity’; reference and ontological commitment are fixed only relative to our ‘frame of reference’:

<sup>65</sup> As Alston (1986, p. 57) notes, Quine does not intend the point to be merely an epistemological or methodological one: translation is literally indeterminate in his view.

<sup>66</sup> See Quine, 1968a, pp. 47-48.

It is meaningless to ask whether, in general, our terms “rabbit,” “rabbit part,” “number,” etc., really refer respectively to rabbits, rabbit parts, numbers, etc., rather than to some ingeniously permuted denotations. It is meaningless to ask this absolutely; we can meaningfully ask it only relative to some background language ... Querying reference in any more absolute way would be like asking absolute position, or absolute velocity, rather than position or velocity relative to a given frame of reference.

(*Ibid.*, pp. 48-49)

The indeterminacy of translation, inscrutability of reference, principle of ontological relativity, and rejection of the analytic/synthetic distinction, are all consequences of Quine's holism. If Quine's conception of language is accurate then we are in a position to exploit the under-determination of theory (for languages simply are theories on Quine's account) by experience in re-evaluating and reorganising our theoretical understanding of the world to meet our pragmatic ends. To return to Neurath's metaphor, we may indeed be unable to abandon the boat we are afloat in, but the holistic structure of the vessel allows us to continue the rebuilding of its parts while remaining afloat in the whole. In the next chapter of this thesis we will examine an example of how Quine thinks such a reconstruction of one part of the vessel should be approached by studying his proposal for a naturalized epistemology.

## 5.

## Meaning, Mind, and Language

Philosophers are often prone to an obsession with categorising the practitioners of their subject into trends, traditions, and movements. Sometimes this can be less than helpful, as is often the case when the hastily imposed divide between philosophers in the “analytical” and “continental” traditions is pressed to unnecessary lengths. At other times it can reflect genuinely important patterns in the development of the subject. One of the more important divisions occurred within the analytical tradition and came to the fore in the second half of the last century. Roughly speaking, that divide was between the practitioners of ‘ordinary language philosophy’ (associated with Wittgenstein and the Oxford philosophers of the fifties such as Ryle, Strawson, and Austin) and those (like Davidson) who took their lead from Quine. Of course, such divisions are only helpful up to a point; closer scrutiny will always reveal exceptions which suggest the division to be spurious. As a rough characterisation of the landscape of analytical philosophy, however, it serves as a useful navigational aid. Whether there were enough similarities between the so-called ‘ordinary language philosophers’ to yield an identifiable school or tradition which answers to the name of ‘Oxford’ philosophy is debatable. Whether Wittgenstein can be accurately located as involved in a similar project in anything more than a superficial way is unlikely. Nonetheless there is one common theme running through the work of the philosophers in question which is sufficiently relevant to make the division stick: the conception of their own subject held by these philosophers was largely consistent in placing philosophy distinctly away from, and separate to, the sciences.<sup>1</sup> Whereas science

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<sup>1</sup> Hacker agrees with this interpretation of the shared methodology of these philosophers but goes too far in seeing such a conception of philosophy as part of the very essence of what analytical philosophy is. The unsatisfactory consequence is that those philosophers who, like Quine, reject such a conception,

seeks to broaden our understanding of the world, offering new knowledge and seeking out new truths in such a way as to *revise* our beliefs about the world. philosophy, in Wittgenstein's words, 'leaves everything as it is' (Wittgenstein, 1953, § 124). This conception of philosophy is, of course, just the opposite of Quine's philosophy as it was characterised in the previous chapter.

In an influential attack on many of the central tenets of both Wittgensteinian and 'Oxford' philosophy, Dummett has pointed out that this conception of philosophy as discontinuous with science intends "science" to be understood, in very broad terms, as any subject which is in the business of discovering or asserting truths.<sup>2</sup> It is important to note, however, that Quine's rejection of such a conception of philosophy is a great deal further removed from Dummett's. Quine is not content with philosophy to be viewed as just a subject, *analogous* to science, which contributes to knowledge. His more revolutionary claim is that philosophy and science are largely indistinguishable. There is *no* essential difference between philosophy and science, for Quine. In other words, when he talks of philosophy being continuous with science, he intends to be taken literally. This becomes immediately evident if we turn our attention to Quine's proposed 'naturalized epistemology'.

### 5.1 Epistemology Naturalized?

Quine's attitude towards epistemology is partly a response to those traditional epistemologists who, following Descartes, held themselves to be embarked on a 'first philosophy'. This traditional conception of epistemology was not, of course, restricted

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are not analytical philosophers by Hacker's definition. See Hacker, 1996a, p. 229. Hacker does recognise the fact (stressed in the previous chapter of this thesis) that Quine is, in many respects, the natural heir to the Russellian project, but holds that analytical philosophy proper is Wittgensteinian, as opposed to Russellian: 'One cannot swim back in the stream of history. The river bed of analytic philosophy was decisively shifted by the *Tractatus*, and shifted in a direction inimical to Russell's conception of philosophy, which had no further influence upon the analytic movement. By the time Quine's major work was published in 1960, it was not continuous with mainstream analytic philosophy as it had flowed for the previous forty years' (Ibid., p. 319). Hacker's account is misleading, it seems to me, in that it ignores the role played by Carnap in fostering the ideas of Russellian analytical philosophy and subsequently passing them down to the American philosophers (including Quine) guided by his influence. The Oxford philosophers may have been opposed to Carnap's views but, across the Atlantic, analytical philosophy was beginning to head down a route primarily instigated by Carnap and the positivists. Quine, in his 'Homage to Rudolf Carnap' referred to Carnap as 'the dominant figure in philosophy from the 1930s onward, as Russell had been in the decades before' (Quine, 1970b, p. 40).

<sup>2</sup> See Dummett, 1975a, p. 438.

to those who shared Descartes' rationalist bent; traditional empiricism likewise sought to build upwards from an epistemological base, and the figure who Quine defined his position in response to more than any other was Carnap.<sup>3</sup> In *Der logische Aufbau der Welt*, Carnap sought to reconstruct the external world from sense-data, making additional use only of the resources of logic and mathematics in addition to the bare minimum of non-logical vocabulary.<sup>4</sup> It is unclear whether Carnap's ambition was to effect an ontological or purely epistemological reduction. Perhaps, like, Russell, the ambition was twofold, encompassing ontological and epistemological ends. Either way, such reductionism will not survive an adoption of Quinean holism. Taking himself to have dismissed the reductionism necessary for the provision of an epistemological base prior to, and foundational for, empirical science, Quine goes on to dismiss the very notion of epistemology (or, indeed, any branch of philosophy) as the foundation of subsequent investigations. The planks of Neurath's boat are firmly in place only relatively speaking and none are immune to revision or rejection so long as we lean on others, held firm only temporarily, while we tamper with other parts of the system. Stepping out of the metaphor: there is no place for a first philosophy.

Quine's naturalistic alternative to traditional epistemology denies the philosopher any standpoint external to that of the scientist. The extent of his naturalism is illustrated in the following passage from his 1954 paper, 'The Scope and Language of Science':

I am a physical object sitting in a physical world. Some of the forces of this physical world impinge on my surface. Light rays strike my retinas: molecules bombard my eardrums and fingertips. I strike back, emanating concentric air waves. These waves take the form of a torrent of discourse about tables, people, molecules, light rays, retinas, air waves, prime numbers, infinite classes, joy and sorrow, good and evil.

(Quine, 1954, p. 228)

<sup>3</sup> Koppelberg goes so far as to state that 'Quine himself regards his naturalistic epistemology as a consequence of the failure of Carnap's theory of knowledge' (Koppelberg, 1990, p. 200).

<sup>4</sup> In fact, just one undefined relation of *remembered similarity* is introduced, taken to range over temporal points on the continuum of our immediate experience. It is interesting to note that Quine takes the two primary influences on Carnap during his construction of the *Aufbau* to be *Principia*, and *Our Knowledge of the External World*. See Quine, 1995, p. 14.

In place of the ontology of private objects invoked by traditional empiricist epistemologists, Quine's ontology, as the above passage shows, is in line with the respectably extensional ontology of the physical sciences. In place of sense-data, impressions, or ideas, Quine has irritations of subjects' sensory surfaces manifested in their behavioural propensities and utterances of observation sentences. Epistemology has no means, on Quine's account, of stepping outside of our scientific world-view in order to give justifications for that view.<sup>5</sup> Rather, it must continue within the confines of that framework as itself a section of scientific investigation:

Epistemology, or something like it, simply falls into place as a chapter of psychology and hence of natural science. It studies a natural phenomenon, viz., a physical human subject ... The old epistemology aspired to contain, in a sense, natural science; it would construct it somehow from sense data. Epistemology in its new setting, conversely, is contained in natural science, as a chapter of psychology.

(Quine, 1968b, pp. 82-83)

The radical break with tradition urged by the naturalization of epistemology has led to frequent doubts as to whether naturalized epistemology really is epistemology at all.<sup>6</sup> By blurring the distinction between *justification for* knowledge and *causes of* knowledge, Quine appears to have subtly changed the subject in such a way that it no longer answers to what we actually *mean* by "epistemology". On the face of it, it is doubtful whether Quine should be particularly troubled by this objection, however. If naturalized epistemology is a new subject in place of the old, rather than a revamping or reduction of it, then the results would appear to remain largely equivalent. Unless the objection is supplemented with a convincing riposte to Quine's attack on the conception of epistemology as external to science, then naturalized epistemology remains on strong ground whether it is construed as a metamorphosis of the old epistemology or as a replacement of it. One objection to the extravagant conclusion Quine wishes to draw, however, may emerge if we examine just how much of the philosophy of language Quine has helped himself to in order to reach this position.

<sup>5</sup> Quine's conception of our world-view as fundamentally scientific is evident in remarks such as: 'Science is a continuation of common sense' (Quine, 1951, p. 45).

<sup>6</sup> See, for example, Rorty, 1980, p. 225; Kim, 1998; and Orenstein, 2002, pp. 185-190.

## 5.2 Quine and the Linguistic Turn

Dummett has famously argued that Frege first instigated the linguistic turn, thus issuing in the reign of analytical philosophy.<sup>7</sup> His renowned priority thesis, that ‘the only route to the analysis of thought goes through the analysis of language’, has been labelled by him ‘the fundamental axiom of analytical philosophy’ (Dummett, 1993a, p. 129). Of course, as we saw in the first chapter of this thesis, Frege was not a *linguistic* philosopher in the commonly accepted sense of that label; he frequently voiced his suspicion of natural languages as deceptive regarding their underlying logical form and, accordingly, developed an artificial fragment of language free from the misleading aspects of ordinary parlance. The fragment, though artificial, was nonetheless a fragment of a language which was intended to reveal the logical structure of the ordinary language of which it was supposed to be a refinement. Dummett’s definition of analytical philosophy as ‘post-Fregean philosophy … that which follows Frege in accepting that the philosophy of language is the foundation of the rest of the subject’ (Dummett, 1975a, p. 441) may be overly narrow,<sup>8</sup> but his contention that Frege instigated the linguistic turn is a valid one.

Locating Quine in relation to the linguistic turn is not a simple task. Above all, his desire to place philosophy within the same domain as science puts him more in line with Russell’s conception of philosophy than it does with the typical proponents of linguistic philosophy. Yet the insights of the linguistic turn are largely embraced by him; indeed, the holism which we have seen to be the central component in his philosophy is a thesis about language. It is sentences which, ultimately, stand in holistic clusters rather than any non-linguistic entities. We saw in the previous chapter that this conception of language retains a sensitivity to the unity of the proposition by taking sentences (or statements) to be the smallest unit of significance and thus extending the context principle so as to apply to these units within the context of the language or theory in which they feature. However, it is important to sharply

<sup>7</sup> See, for example, Dummett, 1991a, pp. 111-112, 1975a, pp. 441-442, 1993a, pp. 4-5.

<sup>8</sup> Dummett’s definition is unsatisfactory on account of the philosophers to whom it denies analytical status. As Dummett himself notes, an obvious casualty is Gareth Evans who reversed Dummett’s priority thesis (See Evans, 1982; Dummett, 1993a, pp. 4-5). Monk (1996b & 1996c) points out that Russell likewise fails to fit Dummett’s definition.

distinguish between a holism which takes the whole language to be required in order to achieve, in Quine's phrase, 'critical semantic mass' (Quine, 1992, p. 17), and the more restricted holism which takes a smaller set of sentences of the language. i.e.. a certain theory, to be sufficient.<sup>9</sup>

An expansion of the context-principle to the level of global holism may very well suggest, to one convinced that such an expansion were required, that the systematic account of language sought by Fregean semantics is inherently unobtainable. Such a position has been associated with the later Wittgenstein and is just the position that we saw advanced by the 'therapeutic' reading of the *Tractatus* made by Diamond and Conant in chapter three. Their interpretation was rejected as an insupportable inference from the presence of the context-principle in the *Tractatus* to the conclusion that the early Wittgenstein had provided convincing grounds for denouncing any attempt to give a systematic account of meaning. Indeed, Wittgenstein, in the *Tractatus*, *did* offer a systematic account of the meanings of sentences as functions of the meanings of their parts, as well as advocating the context-principle as an essential component in that account (as, of course, did Frege). The later Wittgenstein's arrival at a position opposed to Fregean philosophy of language is, unsurprisingly, more sophisticated than a simple denial of the systematic nature of language.

To deny that languages are systematic at the level of their semantics is clearly absurd. The difference in meaning between the sentences "John loves Mary" and "Luke loves Mary", or the between the sentences "John loves Mary" and "John hates Mary", can only be accounted for by recognising that the names "John" and "Luke", or the relational expressions "loves" and "hates" differ in their semantic values. What these semantic values are is, of course, open to discussion; but *that* the terms in question have some semantic value, and that the two sentences in each of the above pairs differ solely in virtue of the difference in the semantic values of their constituent parts is surely an obvious truth. A theory which contests this simple truth is unworthy of further consideration. Wittgenstein's objections, in his later philosophy, to a theory of meaning along the lines of the Fregean model do not deny that the

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<sup>9</sup> Dummett calls the wider form of holism 'global holism'. As he points out, the more restricted form of holism that Quine later settled for is far more credible than global holism. Treating the sentences of a given scientific theory as holistically intertwined is credible enough, but it seems less likely that these theories will always be sufficiently continuous with all other areas of discourse and thought to justify the global extension of the principle. See Dummett, 1991b, pp. 230-37.

meanings of sentences are systematic in this sense, however. Rather, he denies that an exhaustive theory of how such systematicity operates can be provided.<sup>10</sup> What is more, he rejects the idea that such a theory is desirable in the first place. There is an appearance of tension in Quine's philosophy which manifests itself in a similarity (albeit largely superficial) with Wittgenstein's attitude that sits uncomfortably with other areas of his philosophy of language.

Like Wittgenstein, Quine despairs of the project of giving a theory of meaning. Central to the thought of both is their rejection of the notion that meanings are entities. Wittgenstein famously sought to counteract the tendency to reify meanings by insisting that 'for a *large* class of cases—though not for all—in which we employ the word "meaning" it can be defined thus: the meaning of a word is its use in the language' (Wittgenstein, 1953, § 43). Quine goes perhaps further in claiming that 'there is nothing in linguistic meaning beyond what is to be gleaned from overt behavior in observable circumstances' (Quine, 1992, p. 38). However, unlike Wittgenstein, Quine does not conclude from this that there is no way of achieving a systematic account of the workings of language. Quine's abandonment of the hope of obtaining a systematic theory of meaning is coupled with his preference for a theory of reference, as we saw in the previous chapter. By sharply distinguishing between meaning and reference, Quine is able to ensure that his objections to meanings do not simultaneously damage his quest for a clear notion of reference: 'we can acknowledge a worldful of objects, and let our singular and general terms refer to those objects in their several ways to our hearts' content, without ever taking up the topic of meaning' (Quine, 1951, p. 47). The mechanisms with which Quine sets about explaining and systematising reference were, of course, discussed in the last chapter.

It is evident that the methods Quine employs in 'clearing ontological slums' (Quine, 1960, p. 275), that is, making explicit the ontological commitments of a given theory, are thoroughly indebted to the linguistic turn as taken by Frege. Dummett's characterisation of that turn as the manoeuvre whereby 'an epistemological problem, with ontological overtones, is ... converted into one about the meanings of sentences' (Dummett, 1991a, p 111) shows it to be an important precedent to Quine's ontological criterion. For Quine, question about the existence or non-existence of extra-linguistic

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<sup>10</sup> Central to Wittgenstein's view here is his rejection of the Fregean idea that some uniform fragment of language (such as a sentence) can be isolated and then called on to act as a basic unit in any such enterprise. See, especially, Wittgenstein, 1953, §§ 65 & 136.

entities are replaced by question about the values of variables; the ontological commitments of a theory are revealed by scrutinising the range of the variables of quantification, once the theory under investigation has been properly regimented into the canonical notation of formal logic. Quine refers to this process whereby questions about extra-linguistic elements are deflected onto questions about the language used to talk about them as a process of *semantic ascent*: ‘It is the shift from talk of miles to talk of “mile”. It is what leads from the material (*inhaltlich*) mode into the formal mode, to invoke an old terminology of Carnap’s. It is the shift from talking in certain terms to talking about them’ (Quine, 1960, p. 271). Quine does not, however, hold that semantic ascent is a feature unique to philosophy. Carnap had taken the shift from the material to the formal mode of speech to demarcate philosophy from other areas of discourse,<sup>11</sup> but Quine does not concur. The passage continues: ‘It is precisely the shift that Carnap thinks of as divesting philosophical questions of a deceptive guise and setting them forth in their true colors. But this tenet of Carnap’s is the part that I do not accept. Semantic ascent, as I speak of it, applies anywhere’ (Ibid., pp. 271-272). The manoeuvre is present, for example, in relativity theory and in the axiomatization of mathematical theories, according to Quine. Once again, philosophy is viewed as continuous with science as a whole but now, rather than philosophy collapsing into natural science as something removed from empirical study only by degree, it is the fact that features previously held to be unique to philosophy are drawn on by the scientist that debars any permanent sundering of philosophy from science.

The linguistic turn as taken by Quine is taken in much the same spirit that it was by Frege in that it does not mark a turning away from metaphysics, only a clarification and sharpening of the focus of metaphysical questions. Paraphrase of metaphysical questions in linguistic terms is adopted as the best method of approaching, not dismissing, those questions. The central component in this move, for Quine, is the construction of a canonical logical notation adequate for a theory of reference. From apparently similar starting points, then, Quine and Wittgenstein reach profoundly different conclusions. Whereas Wittgenstein’s holism leads him to deny the possibility of a systematic theory of meaning, and to denounce the confusions of the metaphysicians, Quine seeks to make sense of metaphysical confusions by abandoning the theory of meaning and turning instead to a theory of reference which

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<sup>11</sup> See Carnap, 1937, esp. §§ 75-79.

places formal logic at the heart of an enterprise viewed as continuous with that of the scientist:

The quest of a simplest, clearest overall pattern of canonical notation is not to be distinguished from a quest of ultimate categories, a limning of the most general traits of reality. Nor let it be retorted that such constructions are conventional affairs not dictated by reality; for may not the same be said of a physical theory? True, such is the nature of reality that one physical theory will get us around better than another; but similarly for canonical notations.

(Quine, 1960, p. 161)

Wittgenstein, by contrast, always remained adamant that philosophy and science were sharply discontinuous, and held mathematical logic responsible for having ‘completely deformed the thinking of mathematicians and philosophers’ (Wittgenstein, 1956, V § 48). Insofar as semantic ascent embodies the use of such formal techniques, it is also fundamentally at odds with Wittgenstein’s approach to philosophical problems. Wittgenstein steadfastly refused to recognise the value of a distinction between meta-languages and their object languages,<sup>12</sup> whereas Quine takes Tarski’s definition of the truth-predicate to be the paradigm case of semantic ascent.<sup>13</sup> as, of course, does Davidson. Wittgenstein would, one suspects, have dismissed such projects as exemplifying the ‘craving for generality’ (Wittgenstein, 1969a, pp. 17-18) that he diagnosed as the source of much philosophical confusion. Quine, after noting the importance of semantic ascent for generalising beyond individual examples, comments that: ‘Wittgenstein’s characteristic style, in his later period, consisted in avoiding semantic ascent by sticking to the examples’ (Quine, 1960, p. 274, ft. nt.).

The naturalism that Quine takes to be suggested by his holism is not wholly of a piece with the naturalism instigated by Russell’s program for ‘the scientific method in philosophy’. The relation that Quine sees obtaining between philosophy and natural science is a reciprocal, symbiotic one; the continuity of science and philosophy is not just a matter of philosophy being contained in some wider domain of discourse called ‘science’ but consists in areas of discourse differing only in degree and each sharing

<sup>12</sup> See, for example, Wittgenstein, 1953, § 121.

<sup>13</sup> See Quine, 1970a, p. 12.

techniques and concepts with one another. Scientific study is as much indebted to philosophy, as philosophy is to science. Nonetheless, semantic ascent is primarily an activity of the philosopher. Though the scientist may, on occasions, have recourse to semantic ascent, it is primarily philosophers who are driven to conduct their analyses through the analysis of language.<sup>14</sup>

### 5.3 Empiricism and the Philosophy of Language

Quine's replacement of the search for a theory of meaning with the enterprise of radical translation turns the philosophy of language into an empirical inquiry. If all there is to meaning is 'what is to be gleaned from overt behavior in observable circumstances' (Quine, 1992, p. 38), then the assignment of content to linguistic elements on the basis of empirical study, along with a suitable account of how the content of certain elements is dependent on the construction of those elements out of their constituent elements, is all that is required to produce a workable translation manual of the language under investigation. Deeper perplexities, such as the question of what a speaker *knows* when she understands a language, seem to fall by the wayside. All that is open to study is the presumably causal processes of stimulation and conditioned responses that constitute linguistic behaviour.

To say that languages are systematic is a shorthand way of saying that *linguistic competency* is systematic. I have mastery of the ability to use a language when I am able to produce and understand sentences, which I have not previously constructed or encountered, through combining their constituent parts according to an accepted pattern or set of rules. Understanding a sentence is, of course, just equivalent to knowing what that sentence means; hence, if one expects a philosophical account of language to reveal the nature of the systematic ability of the language user, then it does not seem an unfair demand to expect such an account to yield an answer to the question of what a speaker must know in order to understand a sentence or, indeed, a language. Evidently, however, such a demand will not be met by Quine's theory. Indeed, it is difficult to see how it could be met by any theory which embodied an empiricism as austere as Quine's. One might think that a turn away from the direct

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<sup>14</sup> Quine concedes that 'semantic ascent is more useful in philosophical connections than in most' (Quine, 1960, p. 272).

study of language and linguistic behaviour in favour of the underlying brain processes of speakers will provide sufficient resources to meet the demand. If the outward behaviour of speakers will not provide sufficiently rich data for the explanation of their inward states, then perhaps a study of their internal, neural, states will. The naturalist may opt for rejecting the linguistic turn and taking a neurological one.<sup>15</sup> Such a move will not provide the answer to the question of what knowledge of a language consists in, however. At most it might present an account of how such knowledge is to be brought into the sphere of naturalistic explanation by showing how it relates to some underlying physiology. Even such an account, should it be made available, is unlikely to contribute a great deal to the theory of meaning, however. Quine is well aware that uniformity in speaker's brain states is neither likely, nor of interest in explaining the social interaction involved in communication through language:

Different persons growing up in the same language are like different bushes trimmed and trained to take the shape of identical elephants. The anatomical details of twigs and branches will fulfil the elephantine form differently from bush to bush, but the overall outward results are alike.

(Quine, 1960, p. 8)

One may not agree with Quine's behaviourist claim that social conditioning performs such neural topiary,<sup>16</sup> but the following basic insight is correct: Language is a social phenomenon and is to be explained by the outward behaviour of those who partake in it, not by speculation about whatever internal mechanisms might underlie their activity.

The most notable attempt to give a systematic account of meaning largely harmonious with Quine's concerns is, of course, Davidson's application of Tarski's definition of truth for formalized languages to natural languages. Davidson's aim is

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<sup>15</sup> In a recent book on Dummett, Karen Green has argued that just such a turn away from what she calls 'armchair philosophy of language' towards 'empirically informed cognitive science' (Green, 2001, p. 192) has, in fact, already been taken by contemporary analytical philosophy. Determined to reconcile Dummett's philosophy of language with this recent turn, she seeks to incorporate his views into the naturalist's project by arguing that he can be construed as claiming that intentionality requires the prior evolution of language (See *Ibid.*, pp. 199-200). See my (forthcoming) for further discussion and criticism of her position.

<sup>16</sup> Referring back to this passage in *Pursuit of Truth*, Quine writes: 'The outward uniformity is imposed by society, in inculcating language and pressing for smooth communication' (Quine, 1992, p. 44).

somewhat more ambitious than Quine's. Rather than seeking to provide sufficient resources for radical translation of an alien language into one's own, Davidson seeks sufficient resources to effect the process of 'radical interpretation', thus enabling one to 'provide an interpretation of all utterances, actual and potential, of a speaker or group of speakers' in a non-circular fashion (Davidson, 1984, p. xiii).

#### 5.4 Truth and Meaning

Like Quine, Davidson is sensitive to the seemingly intractable problems that arise once meanings are taken to be objects. His proposal is therefore to simply abandon meanings in the theory of meaning altogether and to lean instead on the notion of truth as a way of explicating that which we expect a theory of meaning to give an account of. The result is a theory of meaning without *meanings*. If we take one of the central tasks of a theory of meaning for a given language to be that of providing an interpretation of any utterance of a speaker in that language, then we are faced with the task of providing, for any utterance in the language, an instance of the schema

*S* means that *P*

where "S" is to be replaced by the name of a sentence in the language in question, and "P" is to be replaced by a further sentence which provides the *meaning* of that sentence.

Clearly, work has to be done to avoid the threat of circularity. Recognising the progress made in defining truth in non-circular terms for formal languages by Tarski,<sup>17</sup> Davidson thinks that an application of the same techniques will go a long way towards answering the questions that trouble the meaning-theorist in an equivalently non-circular form.<sup>18</sup>

Tarski's definition of truth, though revolutionary, proceeds with an elegant simplicity. Only a brief account of it need be given here. Firstly, he makes a distinction between object-language and meta-language, in order to avoid the circularity that threatens to descend when we try to state the truth-conditions of a

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<sup>17</sup> See Tarski, 1931.

<sup>18</sup> An obvious benefit of paraphrasing away the locution "means that" in favour of "is true iff" is that the new locution avoids the pitfalls of intensionality.

sentence. A statement of what would make a sentence true will be of little use if it is just a restatement of the sentence, but of course, the sentence “snow is white” is true by virtue of snow’s being white. Provision of this truth-condition must be stated in a language if it is to be stated at all and, hence, circularity threatens. By distinguishing object language from meta-language, the problem is avoided. Truth-conditions for a sentence of an object language can be given by applying the truth predicate to a *name* of that sentence in a meta-language. It is just that procedure of shifting from talking *in* certain terms to talking *about* them that Quine calls semantic ascent. In the following schema,

*S* is true *iff* *P*,

“*S*” is the name of an object language sentence, and we use the meta-language to give its truth-conditions. Hence, a valid instance of the schema will be

“Snow is white” is true *iff* snow is white

where the quotation marks serve to show that what they enclose is a name of an object language sentence. The insistence that an object language cannot contain its own truth-predicate, famously, is sufficient for the avoidance of semantic paradoxes such as the Liar. Tarski then proceeds to define the truth-predicate in terms of the notion of *satisfaction*. An open sentence “*Fx*” is satisfied by an object if, and only if, the object is *F*. Hence *a* satisfies the predicate “*x* is white” if, and only if, *a* is white. So, for example, snow satisfies the predicate “*x* is white”.<sup>19</sup> A recursive procedure can be given to define satisfaction of a predicate by a sequence of objects.<sup>20</sup> Definition of the truth-predicate then proceeds from Tarski’s axiomatic definition of satisfaction so as to ensure that a sentence “*S*” is true if, and only if, it is satisfied by all sequences of objects.<sup>21</sup>

<sup>19</sup> See Tarski, 1931, p. 190. Tarski uses the term ‘sentential function’ in place of ‘predicate’.

<sup>20</sup> See Tarski, 1931, pp. 192-193.

<sup>21</sup> See Tarski, 1931, p. 195. Anxiety over the status of the notion of satisfaction in the definition, concern that the definition of satisfaction somehow presupposes the notion of truth, is common (though unwarranted). Platts (1979, pp. 21-24) devotes attention to dispelling such concerns. Field (1972) disputes Tarski’s claim that he has constructed a definition of truth without appeal to any irreducibly semantic concepts (see Tarski, 1931, pp. 152-153) and maintains that the definition of truth serves *only* to reduce one semantic notion to other semantic ones. See Field, 1972, p. 86.

Davidson's claim is that this definition of truth, suitably modified for application to natural language, will go a long way (if not all of the way) towards providing all that we demand of a theory of meaning. Davidson states that the theory of meaning on offer is 'an empirical theory' (Davidson, 1967a, p. 24) and proposes that it can be used to effect the radical interpretation of another's utterances in the language for which the theory is given.<sup>22</sup> Both Frege and Wittgenstein had previously recognised the close relationship between the concepts of meaning and truth, as we have seen in this thesis, but neither of them had maintained that truth alone is sufficient for sustaining a theory of meaning.<sup>23</sup>

It is not entirely clear, however, that Davidson's account *does* amount to a theory of meaning rather than a theory of reference (at least in so far as a theory of truth depends on reference, as we shall see below). The only assignments given to expressions are those required for the generation of T-sentences: objects are assigned to singular terms and predicates are understood as making a systematic contribution to fixing the truth-conditions of the sentences in which they feature by virtue of being satisfied by objects or sequences of objects. In this sense, Davidson's advance on Quine's position is less significant than at first it may seem. As Davidson approvingly notes, Quine had previously forwarded a similar line: 'in point of *meaning* ... a word may be said to be determined to whatever extent the truth or falsehood of its contexts is determined' (Quine, 1936, p. 89).<sup>24</sup> Whereas Quine urges the abandonment of the theory of meaning in favour of the theory of reference, Davidson holds that a theory of meaning can fall out of such an enterprise; that reference is sufficient for generating a theory of meaning.<sup>25</sup> In Frege's terms, Davidson holds that sense is

<sup>22</sup> See Davidson, 1973, p. 130.

<sup>23</sup> See Frege, 1893, § 32; and Wittgenstein, 1922, 4.024.

<sup>24</sup> See Davidson, 1967a, p. 24, fn. nt.

<sup>25</sup> There is a subtle tension at play in Davidson's thought here, which my talk of Davidson's theory of reference should not be misconstrued as overlooking. As Alvarez (1994, p. 360) points out, Davidson takes the inscrutability of reference that results from his semantic holism as grounds for eschewing the notion of reference as a basic component to a theory of meaning (see Davidson, 1977, p. 221). However, reference (or some such equally problematic semantic relation) seems to be needed for the theory of truth which Davidson does take to be basic, indeed essential, to the theory of meaning. His proposed solution to the problem is to embrace 'the distinction between explanation *within* the theory and explanation *of* the theory' (*Ibid.*). In other words, he takes the aforementioned semantic notions to be postulated concepts invoked in order to adequately specify truth-conditions (*within* the theory). Armed with this notion of truth, we can then turn to this in our explanation *of* the theory: 'When it comes to interpreting the theory as a whole, it is the notion of truth, as applied to closed sentences, which must be connected with human ends and activities ... I suggest that words, meanings of words, reference, and satisfaction are posits we need to implement a theory of truth' (*Ibid.*, p. 222). The model Davidson likens the whole process to, is that of physics where macroscopic phenomena are explained by appeal to posited microphysical ones. My talk of reference in what follows is intended to be

expendable for the theory of meaning: 'a Tarski-type truth definition tells us all we need to know about sense. Counting truth in the domain of reference, as Frege did, the study of sense thus comes down to the study of reference' (Davidson, 1979, p. 109).

It is vitally important to be clear about just what is being sacrificed by expulsion of a theory of sense as a component part of a theory of meaning. On the one hand, a theory of meaning which appeals only to reference will have the advantage of a far greater simplicity than a theory which calls on other notions in addition. On the other hand, there is a very real danger of throwing the baby out with the bathwater. Frege introduces the notion of sense, it will be recalled, in order to account for the role of *thought* in the understanding of language. The sense of a sentence is the thought which is expressed by the sentence and this thought just is the mode of presentation of a truth-value. It therefore serves a dual function. It provides a route to reference, explaining how sentences are connected to their truth-values, and equally importantly, provides an account of what it is that someone *knows* when they understand a sentence. The connection of a sentence to its truth-value is explained by invoking thoughts as intermediary truth-bearers, and the account of linguistic knowledge is hence provided by the recognition that what someone knows when they understand a sentence is the thought that it expresses (what would be the case for that sentence to be assigned a truth-value).

With the repudiation of sense, comes a host of problems. For one, there is an obvious problem involved in refusing to see no more to meaning than there is to be found in reference: what of the meanings of expressions that lack a reference? Of course, Davidson's account allows for this by appealing to the role such terms play in fixing the truth-conditions of the sentential contexts in which they feature. But more needs to be said. Evans pointed out, in *Varieties of Reference*, a significant problem for the Fregean model.<sup>26</sup> Frege took sentences to be complex names whose referents are truth values. But two theses of Frege's lead into difficulties here; firstly, he adheres strictly to a classical two-valued logic.<sup>27</sup> and, secondly, holds concepts to be functions from objects to truth-values (where truth-values are deemed to be special kinds of objects). In the case of a non-referring singular term, there is no object to be mapped onto a truth-value. Hence there cannot be a truth-value within the confines of

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sensitive to this distinction, while remaining neutral as to its correctness, as it is directed at the work that the notions in question do *within* the theory.

<sup>26</sup> See Evans, 1982, p. 32.

<sup>27</sup> See Chapter one, above for discussion of Evans's interpretation of Frege on this point.

Frege's classical logic. Consequently, there is nothing for such sentences to *name*.

Frege's solution was to decree that, in such cases, sentences do not name truth-values but refer to their sense. However, if the sense of the sentence is the mode of presentation of its truth-value, there no longer seems, in these cases, to be anything for it to be a mode of presentation of. Just as in the case of empty definite descriptions, or non-referring singular terms like "Pegasus", appealing to the sense of the expressions as a proxy for the absent referent is an unsatisfactory solution, for the simple reason that, without a referent, there is no explanation of what the sense of the expression is. Sense, for Frege is construed as the road to reference; sense without reference, surely, is a road to nowhere.<sup>28</sup>

It might be thought that this puts Davidson at something of an advantage, being unburdened by the perplexities arising from the inclusion of an account of sense in a theory of meaning. However, a correlate of the problem resurfaces and is not easily dismissed. If Davidson intends the theory of meaning to operate solely in the realm of reference, and, in particular, to operate solely in terms of truth, how is he to account for sentences which do not fall neatly into either of the two classes defined by the classical truth-values? The treatment that "Pegasus is a horse" would receive in a Fregean semantics, for all its faults, is intended to recognise that the sentence is perfectly intelligible, but does not connect sufficiently with anything in the realm of reference to merit the attribution of a truth-value. For Davidson, however, the attribution of truth-conditions is essential to recognising a sentence as meaningful, hence, denying that "Pegasus is a horse" is conducive to such an analysis is tantamount to either denying that it means anything, or exempting it from the scope of one's theory. Neither option is desirable, so some manoeuvre must be made in order to bring the sentence within the scope of a truth-conditional analysis. This can be done, of course; the analysis of the sentence can be so engineered that it turns out false. As he shares Quine's holistic conception of language, Davidson has significant tools to hand for preserving classical logic on such pragmatic grounds. But pragmatic grounds are not the only grounds for constructing a theory of meaning, and the less

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<sup>28</sup> See Dummett, 1981, for a defence of Frege's position here, and see also Makin, 2000, pp. 112-117, for further discussion.

conducive to analysis within the confines of classical logic a sentence is, the more *ad hoc* classical truth-theoretic accounts of meaning start to look.<sup>29</sup>

The rejection of sense as a potential component in meaning drastically reduces the available resources for providing a satisfactory account of thought. This is a serious worry from any perspective simply on account of the obvious fact that thoughts are expressed in language, and a large part of the role played by language in day to day living is the role of communicating thought. In all but the most unusual of circumstances, one's utterance of a sentence such as, for example, "Jones is at the door", or, "Jones is late again", counts as piece of language only because it is used to communicate its content. And I have either lost grip of language or my senses, if I utter the sentence "Jones is late again" without intending to communicate, or give expression to, my thought about Jones's recurring lateness.<sup>30</sup> Now, of course, it may well be that Davidson's account will go some way towards explaining this. It seems feasible to say that I will understand the sentence "Snow is white" if I can frame its truth-conditions and, more importantly perhaps, if I *cannot* frame its truth-conditions, then I can hardly be said to understand it. But, other problems of extending this analysis to more complex sentences aside, there is a big difference between, on the one hand, providing an adequate condition which, if met, will ensure that those who meet the condition grasp the meaning of a sentence and, on the other hand, providing an explanation of what it is that a speaker knows when he masters a language or when he understands a sentence. In other words, while the ability to recognise the truth of the T-sentence

"Snow is white" is true *iff* snow is white

is a necessary condition for understanding the sentence "Snow is white", it does not follow that recognising the truth of the T-sentence is a sufficient condition for *understanding* the embedded object-language sentence. So long as I understand the sentence-forming operator ["..." is true], and understand that "*iff*" is a sentential

<sup>29</sup> Such problems arise, of course, not just for non-referring terms, but also for statements about the distant past, the future, and (as we have already seen) certain kinds of mathematical statements.

<sup>30</sup> Of course, this is a claim which is intended to refer only to utterances made in correct contexts. There will undoubtedly be contexts in which I may utter the sentence "Jones is late again" without any intention to communicate any details of Jones's poor record of punctuality. Perhaps I may be referring to the poorly timed tackle made by a participant in a football match, or I may find myself uttering the sentence as the line of a song with no thought of any content it may have.

connective yielding truth only for equivalent sentences, then I have all the knowledge I need to recognise the truth of any T-sentence. Take, for example, the following sentence from a pharmacology textbook opened at random: ‘Acetylcholine is broken down by the enzyme acetylcholinesterase’ (Downie, Mackenzie, & Williams, 1995, p. 355). I have very little understanding of what this sentence is asserting, not knowing what the terms “acetylcholine” and “acetylcholinesterase” mean. Certainly I cannot vouch for the truth of the sentence. Nonetheless, I am perfectly happy to vouch for the truth of the T-sentence

“Acetylcholine is broken down by the enzyme acetylcholinesterase” is true *iff*  
 Acetylcholine is broken down by the enzyme acetylcholinesterase.

Perhaps to my shame, I have little hesitation in stating that there are textbooks in many other disciplines which, were I to open them at random, would yield sentences similarly baffling to me.<sup>31</sup> So long as I could recognise them as sentences, however, this would not prohibit me from asserting T-sentences containing names of them along with the sentences themselves on the right-hand side of the biconditional.

What the preceding discussion reveals is that talk of the knowledge of a sentence’s truth conditions is ambiguous. For, in a sense, I know the truth-conditions of any sentence if I am familiar with the technical methods involved in framing T-sentences. In another sense, however, my ability to provide truth-conditions in this manner does not count as knowledge of those truth conditions. In the first sense, I know what the truth-conditions of the sentence “Snow is white” are if I know that the sentence is true if, and only if, snow is white; in the second sense I know what this means because I know what it means for snow to be white. Hence, it seems that the T-schema is only helpfully extended to an account of linguistic knowledge if supplemented with the requirement that a speaker who knows a given T-sentence also knows how to work out if the truth-conditions are actually met. Knowing that “snow

<sup>31</sup> Much the same argument is presented by Dummett (1975b), who in turn derives it from Kripke’s example of ‘Horses are called “horses”’, the truth of which should be evident to anyone who understands how the phrase “is called” is used in English. What would be lacking, says Dummett, is knowledge of the meaning of that sentence: ‘a person may know that the sentence “Horses are called “horses”” is true, without knowing the propositions expressed by the sentence’ (Dummett, 1975b, p. 9). Dummett’s use of the term ‘proposition’ is not meant here to imply that propositions are entities of any sort (see *Ibid.*, p. 8).

is white" is true if, and only if, snow is white, is not equivalent to knowing whether or not snow is white. Hence it is not equivalent to knowing what "snow is white" *means*.

Unsurprisingly, perhaps, Davidson's position on the relation between knowledge of a truth-theory and knowledge of meaning has fluctuated to some degree over the years. In 'Truth and Meaning', he claimed that 'a theory of meaning for a language  $L$  shows "how the meanings of sentences depend upon the meanings of words" if it contains a (recursive) definition of truth-in- $L$ ' (Davidson, 1967a, p. 23). Six years on, in 'Radical Translation', he displays less confidence:

It is tempting ... simply to say that a T-sentence 'gives the meaning' of a sentence. Not, of course, by naming or describing an entity that is a meaning, but simply by saying under what conditions an utterance is true.

But on reflection it is clear that a T-sentence does not give the meaning of the sentence it concerns: the T-sentence does fix the truth value relative to certain conditions, but it does not say the object language sentence is true *because* the conditions hold.

(Davidson, 1973, p. 138)

The problem for a Davidsonian theory of meaning is not that it cannot make good on its claim to show how the meanings of sentences depend on the meanings of words (indeed it gives one of the most convincing accounts available of how this is so); the problem it does face is that, having rejected the notion of sense as something additional to reference, it fails to give an account of what it is that a speaker *knows* in order to effect the construction of sentences out of words. The semantic account of sentence construction, it will be recalled, is based on the notion of satisfaction of a predicate by an infinite sequence. Standardly, this is preceded by providing a model wherein objects are assigned to names, and predicates are defined over the domain of those objects as follows:<sup>32</sup>

<sup>32</sup> As noted earlier, Davidson is adamant that truth is a more basic notion than reference. However, truth (on the theory he favours) is dependent on a notion of reference, as the above discussion shows. It is worth noting that reference plays a significant role in the whole of Davidson's philosophy. For example, in his account of the logical form of action sentences he endorses Quine's ontological criterion, to the effect that the bound variables of quantification have referential powers, and maintains that a correct characterisation of the logical form of an action sentence will require quantification over a domain of events. Hence, the logical form of the sentence "Shem kicked Shaun" will come out as " $(\exists x) (Kicked(Shem, Shaun, x))$ ", where the variable  $x$  ranges over events, and "Kicked" is a three-place predicate, so that the formula can be roughly translated into English as "There is an event  $x$  such

“*a*” refers to *a*

“*b*” refers to *b*

...

An object  $\alpha$  satisfies “*F*” iff  $\alpha$  is *F*

An object  $\alpha$  satisfies “*G*” iff  $\alpha$  is *G*

...

Evidently, the problems raised regarding knowledge of T-sentences can be reapplied to the knowledge of assignments of objects to names within a structure as here presented. A theory of meaning which issues in the statement that a speaker’s knowledge of the meaning of a term “*a*” consists in his knowledge that “*a*” refers to *a* runs the same risk of vacuity as did the T-sentences in the above examples. Anyone who knows how the locution “refers to” operates in English can recognise that “*a*” refers to *a*. It seems hard to find a way, therefore, in which we could understand a Davidsonian truth-theoretic account of meaning as giving any substantial account of what it is that a speaker knows when he knows a language. Of course, one can only expect of a theory of meaning that it provide an account of the *implicit* knowledge a speaker must have to use the language effectively. The explanation demanded is an explanation of the knowing *how*, rather than the knowing *that*. But there would appear to be no convincing account of either without appeal to something more than the bare empiricism offered by Quinean and Davidsonian theories. It is surely unconvincing to claim that a speaker of a language has an implicit grasp of the technical notions employed in a recursive definition of the truth predicate for the language. The other option, and the one which Davidson prefers, is to view the truth-theory not as an account of what the speaker (either implicitly or explicitly) knows, but rather as a description of the conditions which must be met for a speaker to be attributed with a grasp of the language.<sup>33</sup> The capacity for the theory to fulfil this task has not been challenged here, but we have seen good reasons for thinking that the fulfilment of the

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that *x* is a kicking of Shaun by Shem” (See Davidson, 1967b, p. 118). It is the fact that the sentence contains a (concealed) reference to an event which justifies Davidson’s commitment to an ontology of events.

<sup>33</sup> The extent to which Davidson’s theory of meaning assimilates understanding and interpretation is debated by Glock (1993 & 1995) and Alvarez (1994).

task leaves a great deal unexplained. The desire to locate all aspects of meaning in the realm of reference, in particular, suggests that much is likely to remain unexplained on a Davidsonian account.

### 5.5 Holism and the Unity of the Proposition

What I want to suggest now is that this failure of the Quinean/Davidsonian enterprise to provide a satisfactory account of meaning results from their failure to fully appreciate the importance of the unity of propositional content. In effect, the root cause of the failure is the radical extension of the context-principle urged by linguistic holism. Of course, there is a genuinely important insight in holism. As Wittgenstein famously remarked, ‘to understand a sentence means to understand a language. To understand a language means to be master of a technique’ (Wittgenstein, 1953, § 199). Clearly, one cannot be justifiably attributed with knowledge of the meaning of a sentence if one does not know how to use it in accordance with the conventions in place for its use, and it is inconceivable that someone could have such a mastery of a sentence independently of having the same mastery of a great many other sentences. If an infant only utters one sentence correctly, and shows no understanding of any other sentences, we would not say that the infant fully understood the sentence he was uttering. When, however, Davidson extends the context-principle by saying that ‘only in the context of the language does a sentence (and therefore a word) have meaning’ (Davidson, 1967a, p. 22),<sup>34</sup> or when Quine asserts that ‘the unit of empirical significance is the whole of science’ (Quine, 1951, p. 42), we should be on guard against making the unreasonable inference from the fact that sentences are not logically independent of one another to the grand claim that only a large set of sentences can bear the weight of content.

Wittgenstein is famous for abandoning logical atomism upon discovering that sentences, contrary to what he had claimed, are not logically independent. The two sentences “this patch is red all over” and “this patch is green all over” impinge on

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<sup>34</sup> Davidson repeats the same sentiments with greater clarity (but less pithily) in a paper written ten years later: ‘The meaning (interpretation) of a sentence is given by assigning the sentence a semantic location in the pattern of sentences that comprise the language’ (Davidson, 1977, p. 225).

each other in such a way as to make the conjunction of them necessarily false.<sup>35</sup> It is unsurprising that, in subsequent investigations, he was led to embrace a holistic conception of language.<sup>36</sup> Holism, however, should be understood as coming in degrees. Some degree of holism is obviously correct and ought to result from any careful scrutiny of, and reflection on, linguistic practices. Holism taken to the degree that Quine and Davidson urge, however, loses sight of the importance of the very elements out of which it is composed. As the example just quoted from Wittgenstein shows, the connections between sentences which disable their logical independence are intrinsically bound up in the systematic way that sentences are constructed out of words; “this patch is red all over” and “this patch is green all over” fail to achieve logical independence of one another because of the meanings of the words “red” and “green” in the sentential context “this patch is … all over”. Just as the Fregean context-principle should not compel us to assume (as we saw some do in chapter three) that no account can be given of how propositional content is made up of meaningful parts, so the holistic context-principle should not compel us to think that no explanation of the meanings of sentences can be given beyond seeing them as truth-bearing supports of a wider system. Furthermore, there are very good reasons for not treating the relation between sentences and the languages to which they belong as analogous to the relation of words to sentences in the way that holism tends to do.

There are, of course, similarities between the word-sentence relation and sentence-language relation which cannot be overlooked. Words combine to form sentences, and sentences combine to form languages, for example. It is debatable how far the similarity can be stretched, however. Languages are not simply sets of sentences; they are socially instantiated phenomena embodying complex rules, conventions, and so on. Admittedly, meaningful sentences are not simply sets of words; they are words of certain types combined in accordance with certain syntactical and semantical rules. Nonetheless, there is no *more* to a sentence than its constituent words, even if their arrangement must conform to a structure in order to count as a meaningful sentence. To think that this structure is itself a constituent of

<sup>35</sup> Or, as Wittgenstein preferred to say, the possibility of their conjunction is necessarily *excluded*. See Wittgenstein, 1929, p. 35.

<sup>36</sup> Wittgenstein’s most famous advocacy of holism is to be found in *On Certainty*: ‘When we first begin to believe anything, what we believe is not a single proposition, it is a whole system of propositions. (Light dawns gradually over the whole.)’ (Wittgenstein, 1969b, § 141). For accounts of the similarities and differences between Wittgenstein’s brand of holism and Quine’s, see Gibson (1996), Glock (1996a), and Hacker (1996a & 1996b).

the sentence is wholly misguided, as we saw in examining Russell's multiple-relation theory of judgement in chapters two and three, whereas it seems likely that a language must contain rules. Indeed, the structures that sentences must conform to are, rightly understood, rules of the language. Hence it is an unwarranted simplification to assert that a language simply is the sum total of all its sentences.<sup>37</sup> In short, there is no reason to assume that languages are *objects*, in the sense that sentences are. Even if one wished, as some do, however, to insist that a language is a set of sentences, the analogy will still break down.

The grounds for accepting the Fregean context-principle are convincing. Words have meaning only in the context of sentences because only sentences can be used to *say* something, to communicate a thought. In the overwhelming majority of cases, the utterance of a word in isolation achieves very little. There are, of course, instances where the context of such an utterance will allow for communication to be effected, as in the case of one-word orders, instructions, or requests. Beyond these cases, however, it is sentences which are used to communicate a unified content that can be understood by other participants in conversation. In particular, sentences are used to say things which can be evaluated by others as true or false in a way which words (with few exceptions) cannot. However, by giving such priority to sentences, there is room for a substantial account of how the meanings of sentences depend upon the meanings of their parts by understanding sentences as composed of those parts. Sentences are complex but, as we have seen throughout this thesis, they are complex *unities*. It is this unity which accords them their priority. The same is not true of the composition of languages out of their constituent sentences. A sentence abstracted from its wider context does not cease to express a unified content, even though its content is intrinsically connected to the content of other sentences. This is evident from reflection on how those inter-sentential connections obtain. They obtain by virtue of the truth-values of sentences and the inferential links between them. These links simply could not obtain were it not for the unitary status of the sentences which are so linked. No equivalent property of words explains the principles guiding our concatenation of them to form sentences.

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<sup>37</sup> This difference between sentences and languages is also evident when one considers the unbounded cardinality of the set of sentences of a language. It is hard to imagine a sentence containing an unbounded number of words.

Languages are not *unities* in anything like the same sense that sentences are. It may be acceptable to view artificial languages as sets of sentences, or even to abstract a well defined set of sentences from a natural language in order to provide a systematic semantic theory for that fragment of the language. Indeed, it seems likely that this is the only hope of success in such a project. But this should not lead us into thinking that natural languages are unities in the same sense. Natural languages are constantly undergoing change and vary under the pressures of many influences. They vary colloquially in accordance with the geographical location they are used in, for example, and under many other social pressures. A semantic theory will unavoidably be directed at a somewhat regimented fragment of the language it is intended to apply to, but the measure of its success will hinge on its capacity to be applied beyond the fragment. If it works on the assumption that the natural language is regimented in the same fashion as the fragment, it will be of little use. In this case, the difference between Quinean/Davidsonian holism and Wittgensteinian holism comes sharply into focus, as the latter is largely founded on the realisation that natural languages are not sharply defined entities like formal languages. Wittgenstein's preferred analogy makes the point well: 'Our language can be seen as an ancient city: a maze of little streets and squares, of old and new houses, and of houses with additions from various periods; and this surrounded by a multitude of new boroughs with straight regular streets and uniform houses' (Wittgenstein, 1953, § 18). One does not have to accept Wittgenstein's rejection of the attempt give a systematic theory of meaning to appreciate the force of his analogy. One needs only to see that a language is not just a construction out of sentences in the way that a sentence is a construction out of words.

Having refused to give full credit to the status of sentences as the vehicles of thoughts as unitary meanings, the holist is left unable to recognise the unity of the proposition as central to any successful account of meaning. The systematicity of linguistic knowledge can be explained if the meanings of words are understood in terms of their roles within sentences, because they are now viewed from the perspective of their use in saying something. If sentences are not viewed as capable of such work, however, then no satisfactory account of the unity of the proposition will be forthcoming. Holism makes the isolation of manageable sized chunks of language impossible. Sentences are counted as meaningful only when combined into sets which achieve, in Quine's phrase, 'critical semantic mass' (Quine, 1992, p. 17). Hence, the holist has no account of what knowledge of the meaning of a sentence consists in

beyond knowledge of an entire language. But, as Wittgenstein noted in the passage quoted earlier, knowledge of a language is inseparable from knowledge of how to use a language. The mistake of linguistic holism is to take this insight as implying that there is no way of understanding that knowledge as founded on other knowledge, no way of understanding that mastery of a language involves understanding the meanings of sentences. For the holist, language is a single, tangled edifice that falls apart into nothing if analysed into its parts. Hence there is no intermediary stage between lacking linguistic knowledge and having a total grasp of one's language. The foregoing arguments should suggest that this picture is flawed. Whilst it may be required that one knows a language in order to know the meaning of a sentence, this does not mean that knowledge of the meaning of a sentence is knowledge of the meaning of a language in its entirety. To know the meaning of a sentence is to know the thought expressed by the sentence. This, minimally at least, is what Frege's notion of sense is invoked to explain. Davidson's rejection of it is a costly one, the cost being an account of what a speaker knows when she knows the meaning of a sentence. It is highly doubtful that something purporting to be a theory of meaning can cover this cost.<sup>38</sup>

Considerations such as these should induce suspicion of Quine's claim to have naturalized epistemology. That claim relies on his insistence that questions which were previously reserved for epistemologists collapse into the domain of the psychologist once we recognise that there is no ultimate division between philosophy and the natural sciences. Psychology (construed as behaviourist psychology) is all that we have to go on in making sense of our conception of the world we inhabit. That claim, however, is heavily reliant on Quine's holism. It is holism that ensures that theories are underdetermined by evidence and thereby disallows the reduction of sentence-meanings to sense-data or some other epistemic intermediary between words and the world. But we have seen how holism implies, ultimately, no real account of how the meanings of sentences are composed out of the meanings of words. Or, to be more accurate, it dispenses with any account of what a user of a language knows when he understands a sentence and, subsequently, any account of the systematicity of this knowledge. Naturalized epistemology, so far as it leans on holism for support,

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<sup>38</sup> If the cost cannot be covered then, of course, Davidson will have failed to meet one of the requirements that he places on an acceptable theory of meaning, namely that 'knowledge of the theory suffices for understanding the utterances of speakers of [the language in question]' (Davidson, 1977, p. 215).

is a theory of knowledge based not on knowledge, but on a philosophy of language. The correctness of the doctrine, therefore, will not be decided by any considerations of natural science but, rather, will be determined by the correctness of that underlying philosophy of language. If that philosophy of language, in turn, reveals (as Quine suggests) that the traditional questions of that discipline also turn on nothing more than the empirical study of human behaviour, then that will be so much to the good for naturalism. If, however, it fails to adequately explain the systematic nature of linguistic knowledge, then there is scope for scepticism about the validity of the enterprise. Either way, it seems that the real issue at stake is not whether Quine has shown epistemology to be undeserving of any special status as first philosophy, but whether, in doing so, he has implicitly recognised the philosophy of language as holding that status. If he has unwittingly given this privileged position to the philosophy of language, then he is in distinguished company. Indeed, as mentioned above, Dummett holds that granting such a position to language, and holding that the analysis of thought must proceed through the analysis of language, is an essential characteristic of the analytical philosopher.<sup>39</sup> I have already pointed out that many have, correctly, disputed this characterisation of analytical philosophy. In the remainder of this chapter, I will not pursue further the question to what extent adherence to the principle is a defining feature of analytical philosophers, but will examine the validity of the principle itself.

## 5.6 Thought and Language

Acceptance of Dummett's thesis that the analysis of thought can only be arrived at via the analysis of language leaves space for different positions regarding the nature of thought and its relation to language. One can accept, as a contingent matter, that thought can only be analysed through language and yet hold to one of at least three distinct positions about the nature of thought.<sup>40</sup> One may hold (1) that language is the vehicle for thought, and that thought is inseparable from its linguistic manifestation.

<sup>39</sup> See Dummett, 1993a, p. 4. Dummett takes Quine and Davidson to adhere to the principles that a philosophical account of thought can, and can only, be reached through a philosophical account of language (see *Ibid.*).

<sup>40</sup> A clear outline of the three positions is offered by Glock (2000), who defends a version of the third position.

or (2) that thought is independent of and prior to language so that language is simply the expression of an underlying non-linguistic bearer of meaning, or, finally, (3) an intermediate position whereby thought is held to be, in some sense, determined by the linguistic capacities of the thinker but is not necessarily undertaken *in* a linguistic medium. The first position is most readily associated with Davidson, Dummett, and (on some interpretations) Wittgenstein.<sup>41</sup> The second position is associated with, amongst others, Russell (as we have seen), some logical positivists, Searle, philosophers of thought such as Evans and Peacocke, and many contemporary cognitive scientists.<sup>42</sup> The third position has been urged by Malcolm and (on some other interpretations) Wittgenstein. For example, in a recent defence of (3), Glock attributes the position to 'a coalition (rare, some might say) of common sense and Wittgenstein' (Glock, 2000, p. 36).<sup>43</sup> All three positions are quite compatible with the claim that the only means of *analysing* thought is via the analysis of language, they simply differ over whether this reflects any deeper conceptual priority of language over thought.

The recognition that any philosophical analysis of thought must be undertaken through a linguistic medium seems incontrovertible. If this was all that the linguistic turn amounted to then, while it may still lay claim to originality, it is not likely to be objected to by many. Even Russell, who steadfastly maintained that the analysis of thought was not an analysis of language, was forced to undertake his analysis in language. Unless one resorts to language in explaining the content of a thought, there is simply nothing to be said about what that content amounts to. The controversy arises once we ask why this is so. If one takes it to be just the contingent factor that language is used to communicate thoughts which drives the analysis in this direction, then one can still maintain that language is derived from a prior notion of thought. Our common language, on such a view, is simply a reflection of the way we think. If, on the other hand, one takes it that this connection between language and thought is a necessary one on the grounds that thoughts are, of their very essence, linguistic in character, then it will turn out that the analysis of thought through language is to be explained by the fact that the analysis of language *is* the analysis of thought.

<sup>41</sup> See, for example, Davidson (1975, 1982, 1997), Dummett (1991e, 1993a), Wittgenstein (1969a, 1953).

<sup>42</sup> See Searle (1983), Evans (1982), Peacocke (1992), Dretske (1981), and Millikan (1984). Fodor and other language of thought enthusiasts are probably best thought of as falling in this category though they are not so easily positioned within this schema, for obvious reasons.

<sup>43</sup> See Malcolm (1972-1973), Glock (1997, & 2000).

There are very good reasons for moving in the direction of the latter claim, because there are good reasons for doubting that thought can withstand being separated from its verbal expression. This is just the point that Wittgenstein makes in passages such as the following:

Make the following experiment: say and mean a sentence, e.g.: "It will probably rain tomorrow". Now think the same thought again, mean what you just meant, but without saying anything (either aloud or to yourself). If thinking that it will rain tomorrow accompanied saying that it will rain tomorrow, then just do the first activity and leave out the second.—If thinking and speaking stood in the same relation of the words and the melody of a song, we could leave out the speaking and do the thinking just as we can sing the tune without the words.

(Wittgenstein, 1969a, p. 42)

The argument is compelling; there seems no real possibility of separating the thought that it will probably rain tomorrow from its linguistic vehicle. Of course, the argument becomes more convincing the more complicated the thought is. So, for example, we may be able to make sense of a creature that lacks a language holding simple beliefs (and therefore grasping simple thoughts) such as the belief that it is raining, but not the belief that it will probably rain tomorrow, and certainly not the belief that neutrinos lack mass.<sup>44</sup> One must be careful not to conclude that this inseparability of thought from its linguistic vehicle implies that thought can be identified with its linguistic expression, of course. The thought that snow is white is not the same as the sentence "snow is white" as it can be just as easily expressed by the sentence "Schnee ist wiess".

Nonetheless, it is impossible to deny that this content, whatever it may be, is entirely public. This was the insight we saw developed by Frege in chapter one of this thesis. It is, indeed, more important than the bare realisation that thoughts require a vehicle; for it is only once one has grasped the importance of Frege's attack on psychologism and thereby seen the need to view meaning as essentially public, that

<sup>44</sup> Wittgenstein makes the same point when he asks why it is that a dog can believe his master is at the door, but not that his master will come the day after tomorrow (see Wittgenstein, 1953, p. 174, & § 650). Glock draws attention to such passages in ascribing the intermediate position to Wittgenstein (see Glock, 2000, p. 62, nt. 1).

one realises the need for thoughts to have a vehicle which will enable them to be traded between speakers in the business of communication if they are to be meaningful at all. Frege himself was only partly aware of the full consequences of his achievement. As a result, his anti-psychologism incurred a heavy and unacceptable cost, namely the need to invoke a third realm in which to locate thoughts.

Unacceptable though such platonism is, it is evident, as we saw in chapter one, that Frege's achievement was nonetheless of remarkable importance and deeply insightful. For the first time, content had been lifted out of the subjective realm and made objective. In treating thought as objective, Frege made one of the most (if not *the* most) revolutionary observations in the history of the philosophical study of thought—he distinguished between *thoughts* and *thinking*. This move, above any other, allows for meaning to be freed of its psychological trappings.

A similar insight was an essential ingredient in Russell's momentous contributions to mathematical and philosophical logic. Russell, originally at least, shared Frege's repudiation of psychologism and, like Frege, embraced a strong form of realism in doing so which also led him to resort to platonism. Significant differences also separate the two, however. The most pronounced is Russell's steadfast refusal to grant any significance to language in the philosophical analysis of meaning. The following passage, from the *Principles*, is one which he largely adhered to throughout the development of his philosophy, even in the face of drastic alterations of his views on the nature of logic and meaning:

To have meaning, it seems to me, is a notion confusedly compounded of logical and psychological elements. *Words* all have meaning, in the simple sense that they are symbols which stand for something other than themselves. But a proposition, unless it happens to be linguistic, does not itself contain words. Thus meaning, in the sense in which words have meaning, is irrelevant to logic.

(Russell, 1903, § 51)

This marks a crucial difference between Frege and Russell. The views expressed in this passage are utterly central to Russell's anti-psychologism and yet utterly alien to Frege's. As we saw in chapter one of this thesis, the key difference between Frege and Russell on this point is the following: Frege sought to free logic from psychology by

removing thought from its psychological trappings; thoughts are distinct from and independent of the psychological process of thinking. For Russell (and Moore), however, logic was to be freed from psychology by making propositions independent of thought.

Ultimately, the insistence on this divergence from Fregean anti-psychologism was disastrous for Russell. We witnessed, in chapter two, how Russellian propositions could not survive the more pressing need to rid the foundations of mathematics of the paradoxes that had plagued Russell, and other practitioners of mathematical logic, since the discovery of the Russell paradox in 1901. Having abandoned propositions in favour of the multiple-relation theory of judgement, however, he was left with nowhere to turn when faced with Wittgenstein's withering attack on that theory in 1913. Wittgenstein convinced Russell that logic could not be separated from linguistic considerations. The story of how he did so was told in chapter three. In the face of this realisation, however, Russell did not abandon the form of anti-psychologism detailed in the passage just quoted. Instead he concluded that logic was trivial, that it is 'all of the same nature as the "great truth" that there are three feet in a yard' (Russell, 1946, p. 860). Holding to his previous anti-psychologism regarding logic now meant that Russell was unequipped to resist psychologism in his account of meaning and, indeed, of thought. Hence, as we saw in chapter four, he retreated into psychologism by treating propositions now as psychological entities and insisting that logic, insofar as it concerns meaning at all, is at least partly dependent on psychology. Insofar as it is removed from psychology, Russell maintained that it tells us nothing (as the previous quote testifies). In 1938, Russell wrote: 'The problem of meaning is one which seems to me to have been unduly neglected by logicians; it was this problem which first led me, about twenty years ago, to abandon the anti-psychological opinions in which I had previously believed' (Russell, 1938, p. 362). It is unsurprising that questions of meaning should have had such an effect on Russell. Once his retreat from Pythagoras had denied him any platonic objects to invoke in explaining meaning, he turned to thoughts as suitable objects. But Russell's conception of thought did not admit of de-psychologism in the way that Frege's did. Russell's anti-psychologism rested on making propositions independent of the minds of those who apprehended them. With propositions gone, and sentences dismissed as irrelevant, Russell looked to thoughts as the bearers of meaning and, assuming that thoughts must be contents of the minds of thinkers, concluded that meaning was to be understood psychologically.

Wittgenstein, by contrast, remained rigorously faithful to the Fregean form of anti-psychologism and, in the *Tractatus*, presented a way of approaching the problems that had confounded Russell which marked a significant development of the Fregean point of view. Wittgenstein's achievement was to retain Frege's anti-psychologism without falling into the unsatisfying platonism which both Frege and Russell had resorted to. The key to his success lay in the extent to which he was willing to rely on language as a means of supporting this project. The picture theory enabled Wittgenstein to give an account of meaning without recourse to platonism in explaining the objectivity of meaning. By drawing attention to the use of linguistic elements in expressing a thought, Wittgenstein was able to give an account of thought which avoided the twin perils that had hampered Frege and Russell respectively, namely the Scylla of platonism and the Charybdis of psychologism. Meanings do not need to be situated in postulated abstract realms, be they internal or external; meaning is determined by use. Of course, it was only much later, when Wittgenstein was working on the material eventually published as the *Philosophical Investigations* and had abandoned the picture theory that the idea came to full fruition. Nonetheless, the discussion of Wittgenstein's early theory of meaning in chapter three showed that the appeal to use was also an essential feature of his early philosophy. It is important to recognise that this is a development of Frege's position, however, not a rejection of it. Unlike Russell's philosophy, Frege's model leaves scope for development in such a way as to avoid his platonic excesses while maintaining the full force of his anti-psychologism.<sup>45</sup> The key to achieving this development lies, as Wittgenstein realised, in appealing to the *intersubjectivity* of linguistic practices in place of the objectivity of a third realm. This development of Fregean anti-psychologism is also a guiding principle for Quine and Davidson as we have seen in this chapter and the last.

Russell's failure to take language seriously as a vehicle for thought made psychologism irresistible when providing an account of thought. It seems highly probable that the same will be the case for anyone who tries to follow him in taking language as of no consequence to the analysis of thought. If thoughts are to be given their proper due as publicly communicable contents, then they must be brought into the realm of intersubjective interaction between thinkers and there is no other apparent means of doing so other than by locating them within the use of language.

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<sup>45</sup> Those recent philosophers of language who have sought to develop theories of meaning on broadly Fregean lines, such as Dummett, Davidson, and Brandom are evidence of the truth of this claim.

While there are very good reasons for not identifying thoughts and sentences, there are equally good reasons for seeing thoughts as dependent on language, to see thought as incapable of surviving separation from its linguistic vehicle. For one thing, language provides a medium into which thoughts can be translated without remainder. To see this, we only have to consider what it would mean to try and frame a thought in a non-linguistic medium or, more decisively, to attempt to frame a thought which *cannot* be put into language. As Wittgenstein points out in the quote from the *Blue Book* given earlier, once we try to strip a thought of its linguistic clothing, we are left with nothing. The thought that it will probably rain tomorrow is simply not accessible to a creature which lacks sufficient linguistic resources for representing it. If there is one lesson, above all others, to be learned from the revolution in philosophy effected by Frege, Russell and Wittgenstein then it is this.

Equally important is the recognition of thoughts as central to our understanding of meaning and the subsequent position of priority accorded to the analysis of sentences in the philosophy of language. Again, Russell's struggles are illuminating here. Russell, more than any other philosopher, was the instigator of the method of logical analysis in philosophy. Admittedly, he was preceded by Frege in this respect, but it was predominantly Russell's influence which was responsible for the dominant position taken up by that method in the development of analytical philosophy. In addition, the reasons behind Russell's analytical project were not the same as Frege's. Frege sought to analyse language in order to provide a systematic theory of meaning for a fragment of language, and this theory of meaning, in turn, was intended to make plain the logical structure of thought. For Russell, however, analysis, being an analysis of propositions, was neither an analysis of thought nor of language, but was intended to be a metaphysical analysis of the ultimate structures of reality. Russell approached the task with a preconception that propositions were composed out of 'simples' or atoms. Hence he was led to struggle, with very little success, from the *Principles* through the development of type-theory and beyond with the problem of reconciling this conception of the proposition with the need to account for its unity. Propositions, like thoughts, and indeed sentences, are not just the list of their constituents, but have a structure which arranges those constituents into a unity. This is favourable to the view that thoughts require a linguistic vehicle, for thoughts have a structure and, to be blunt, we have absolutely no idea of how to begin to

account for that structure without appealing to the structure of the linguistic vehicle by which thoughts are presented to us.

Nonetheless, the extreme “lingualism” advanced by Davidson is likely to strike many as overly austere. Davidson, as he points out, ‘imputes no priority to language, epistemological or conceptual’ (Davidson, 1975, p. 157), but he does advocate the austere thesis that ‘a creature cannot have thoughts unless it is an interpreter of the speech of another’ (*Ibid.*); a thesis which clearly boils down to the thesis that ‘a creature cannot have a thought unless it has language’ (Davidson, 1982, p. 100). As Glock points out, this is a claim which is likely to cause just as much offence among zoologists as it will among pet-owners.<sup>46</sup>

One way of softening the blow to those intent on treating animals unblessed with linguistic abilities as minded entities is offered by Dummett. Dummett distinguishes between thoughts, which are only ascribable to language users, and *proto-thoughts* which can be justifiably ascribed to other sentient beings: ‘Proto-thought is distinguished from full-fledged thought, as engaged in by human beings for whom language is its vehicle, by its incapacity for detachment from present activity and circumstances ... Our thoughts may float free of the environment ... proto-thought cannot float free, but can occur only as integrated with current activity’ (Dummett, 1993a, pp. 122-123). It seems that, unless one is to deny that non-language users have any reflective intelligence, some such allowance for the kinds of mental operations performed by them must be granted. What justifies withholding the title of thought from them is their lack of any recognisable structure and the poverty of the content we would wish to attribute to them. As soon as we attribute a content rich enough and sufficiently articulated to be expressed in language, we would be attributing a grasp of the meanings of linguistic concepts to the animals in question and this is just what we have given convincing arguments against doing. Hence, we can accept Davidson’s claim that thought is dependent on language, but soften it with the concession that there are sufficient criteria available for attributing a considerably less rich prototypical version of thought to non-language users. The recognition that some such content is available to those outside a linguistic community does not threaten the view that the analysis of thought must go through the analysis of language, however, for it

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<sup>46</sup> See Glock, 2000, p. 36.

is only the engagement in linguistic practices which provides sufficient criteria for the attribution of thought.

## Appendix: Russell's Substitutional Theory of Classes and Relations

What follows is a brief sketch of the formal system of Russell's 1905 substitutional theory, to accompany the discussion of the theory undertaken in chapter two of this thesis. It should be noted that Russell nowhere constructed a finished axiomatization of the theory, and some aspects of the system must therefore be reconstructed from study of his practices in the manuscripts, and consideration of his treatment of the relevant notions elsewhere. For example, the rules of inference given here are not explicitly stated in the manuscripts from which the rest of the system is taken. The rules are the same as those given by Landini in his presentation of the 1905 system (see Landini, 1998, p. 104) and are surely unobjectionable.<sup>1</sup> The system sketched here is mainly drawn from Russell's 1905 manuscript 'On Substitution'. I have retained the original numbering of propositions from that source. For a more detailed study of the substitutional system, the reader is directed to Landini's book.

### The Substitutional System of 1905

The following are the primitive signs: (,), [ ], {, }, /, !, ;, and  $\supset$ .<sup>2</sup> Individual variables, taken from the set  $\{x_1, x_2, x_3, \dots\}$ , are represented by lower case letters from the English alphabet. All individual variables are terms, and all well-formed formulas (wffs) are terms. There are no other terms.  $\{\dots\}$  is a term-forming operation, hence if

<sup>1</sup> See also Landini, 1998, p. 105, for a deduction theorem to validate the use of a conditional proof technique in the substitutional calculus.

<sup>2</sup> One might think that another logical constant will be required to generate a complete set of axioms. By taking the universal quantifier as a primitive idea, however, Russell can define negation as follows:  $\neg \alpha = (\mu)(\alpha \supset \mu) \text{ Df.}$

$A$  is a wff, then  $\{A\}$  is a term.  $\{A\}$  is a *nominalization* of  $A$ . The atomic wffs are of the form  $[x \supset y]$ , or  $[p/a; x!q]$ .

The notion of *substitution* is primitive and is explained as follows:

$p/a[x!q]$

means that  $q$  results from the substitution of  $x$  for  $a$  in all those places, if any, where  $a$  occurs in  $p$ . For example,

$(\text{Socrates is a man}) / \text{Socrates} [Plato] (\text{Plato is a man}).$

We have the following definitions:

- \*12.1  $x = y = (p, q, r, a) ([ (p/a[x!q] \& p/a[y!r]) \supset (q \supset r)] \supset (q \supset r))$  Df.
- \*12.11  $\phi\{(\iota x) (Ax)\} = (\exists b) [(Ax \equiv_x x = b) \& \phi b]$  Df.
- \*12.111  $\text{Ex! } (\iota x) (Ax) = (\exists b) (Ax \equiv_x x = b)$  Df.
- \*12.12  $p/a[x] = (\iota q) (p/a[x!q])$  Df.
- \*12.13  $(x) A(p/a[x]) = (x, q) (p/a[x!q] \supset Aq)$  Df.
- \*12.14  $a \text{ out } p = (x) p/a[x!p]$  Df.
- \*12.141  $a \text{ in } p = \sim(a \text{ out } p)$  Df.
- \*12.15  $a \text{ ind } b = (a \text{ out } b) \& (b \text{ out } a)$  Df.
- \*12.16  $a \text{ ex } b = \sim(\exists x) [(x \text{ in } a) \& (x \text{ in } b)]$  Df.

Axioms are as follows:

- \*12.2  $(\exists q) (p/a[x!q])$
- \*12.201  $(p/a[x!q] \& p/a[x!r]) \supset q = r$
- \*12.21  $p/p[x!x]$
- \*12.211  $p/x[x!p]$ <sup>3</sup>
- \*12.212  $[(a \neq \sim p) \& (p/a[x!q])] \supset (\sim p)/x[a!\sim q]$

<sup>3</sup> Russell's original \*12.211 is clearly a misprint: 'p/x[x!x]'.

- \*12.22  $\neg[(y) a \text{ in } (Ay)] \supset (Aa)/a^{\dot{x}}!(Ax)$
- \*12.23  $(\exists q) p \text{ ex } q$
- \*12.24  $(a \text{ in } p \ \& \ p \text{ in } q) \supset a \text{ in } q$
- \*12.241  $(a \text{ in } b \ \& \ b \text{ in } a) \supset a = b$
- \*12.25  $\neg[(x, y) a \text{ in } A(x, y)] \supset \{(y) A(a, y)\}/a^{\dot{a}}!(\{(y) A(a', y)\})$
- \*12.26  $(\neg[(c, d) a \text{ in } A(c, d)] \ \& \ (p/a^{\dot{x}}!p' \ \& \ q/a^{\dot{x}}!q'))$   
 $\supset \{A(p, q)\}/a^{\dot{x}}!\{A(p', q')\}$

The rules of inference are the more familiar

*Modus Ponens:*

From A and  $\{A\} \supset \{B\}$  infer B

*Universal Generalization:*

From A infer  $(\mu)A$  where  $\mu$  is free in A

*Definitional Substitution:*

Definiens and definiendum are interchangeable in all contexts.

This completes the system as set out in Russell's manuscripts of 1905.

### Comments

It should be noted that I have here used the notation “Ax” in place of Russell's “ $\psi!x$ ”. The shriek “!” here is a habit carried over from earlier theories and serves no purpose in the 1905 system. I have opted for the change in notation to make explicit the status of “Ax” as schematic for wffs rather than as a predicate variable (there are, of course,

no predicate variables in the logic of substitution).<sup>4</sup> The theorems relied on in chapter two, namely

$$(T_{(=)}) \quad | \vdash (b, q) (r, c, p, a) ([\{r/c; b!q\} = \{p/a; b!q\}] \supset [(r = p) \& (c = a)])$$

and

$$(T_{(=)*}) \quad | \vdash (x) (x = x),$$

vary in difficulty.  $(T_{(=)*})$  is relatively easily proved from \*12.1 and \*12.201, once we have established  $| \vdash x = y \supset x \equiv y$  (Russell's \*12.58). The proof of  $(T_{(=)})$  is, by contrast, somewhat tortuous, requiring several lemmas.<sup>5</sup> The contextual definitions of descriptive phrases referred to as 'desc' in chapter two are obtained from \*12.11,

$$\phi\{(1x) (Ax)\} = (\exists b) [(Ax \equiv_x x = b) \& \phi b] \quad \text{Df.},$$

which is equivalent to

$$*12.11a \quad \phi\{(1x) (Ax)\} = (\exists x) ([Ax \& (Ab \supset_b x = b)] \& \phi x),$$

and \*12.12.

In addition to the above axioms, we can call on the deductive system of the propositional calculus. Landini suggests the following, drawn from Russell's 1906 paper 'The Theory of Implication':<sup>6</sup>

- A<sub>1</sub>  $\alpha \supset (\beta \supset \alpha)$
- A<sub>2</sub>  $[\alpha \supset (\beta \supset \delta)] \supset [\beta \supset (\alpha \supset \delta)]$
- A<sub>3</sub>  $(\alpha \supset \beta) \supset [(\beta \supset \delta) \supset (\alpha \supset \delta)]$
- A<sub>4</sub>  $[(\alpha \supset \beta) \supset \alpha] \supset \alpha$
- A<sub>5</sub>  $(\mu)A\mu \supset A\alpha|\mu$ , where  $\alpha$  is free for  $\mu$  in A

<sup>4</sup> Again, I follow Landini here.

<sup>5</sup> The proof is not given, or even sketched, by Russell but is alluded to in his (1907). See Landini, 1998, pp. 119-125, for a full proof.

<sup>6</sup> See Landini, 1998, p. 104.

A<sub>6</sub>  $(\mu)(\alpha \supset A\mu) \supset [\alpha \supset (\mu)A\mu]$ , where  $\mu$  does not occur free in  $\alpha$  and  $\mu$  is not  $\alpha$

It is interesting to note some of Russell's comments in the 1905 manuscript where the system is sketched (these manuscripts were clearly intended to be used by Whitehead in formal work on *Principia* and often read almost like letters written to him). In regard to \*12.111, Russell says, 'Ex is short for exists.  $\text{Ex}!(\text{ix})(\psi x)$  asserts that the denoting phrase  $(\text{ix})(\psi x)$  does denote an individual' (Russell, 1905b, p. 4). Following the introduction of \*12.14, Russell remarks: 'This defines "a is not a constituent of p". If the operation  $a:x$  never affects p, that can only be because a does not occur in p' (Ibid., p. 6). After \*\*12.2 & 12.201, Russell discusses an alternative:

These two Pps jointly assure us that the operation  $\frac{x}{a}$  always has a unique result. The simple equivalent Pp

$$(\exists r) : p \frac{x}{a}!q . \equiv_q . q = r$$

is not so convenient in establishing the elementary propositions of identity.

(Ibid., p. 7)

Russell uses variations with regard to the notation. The following propositions are all identical, showing the three variations in notational form:

$$p/a:x!q = p(x/a)!q = p \frac{x}{a}!q .$$

Dual substitutions (proxies for functions of functions or classes of classes) are of the form:  $\lceil p/(a,b)(x,y) \rceil$ , and are explained as follows:

Let  $a$  be Philip and  $b$  be Alexander, and let  $p$  be 'Philip is the father of Alexander'. Then  $p/(a,b)(x,y)$  is ' $x$  is the father of  $y$ ', which is true when and only when  $x$  has to  $y$  the relation of paternity, which is the relation that  $p$  affirms as subsisting between  $a$  and  $b$ . Thus the matrix  $p/(z,b)$  may

be taken as representing the relation of paternity.

(Russell, 1906b, p. 174)

This process can then be extended, as explained in chapter two, to triadic relations and beyond. In Lackey's discussion of the substitutional theory in Russell (1973), he suggests that Russell abandoned substitution as a result of the overwhelming technical difficulty involved in incorporating substitution into the theory of types. This is wrong on two counts; firstly, the substitutional theory *was* the original theory of types, and, secondly, there is no insurmountable technical difficulty involved in ramification of the substitutional theory of types—indeed just such a system underlies 'Mathematical Logic as Based on the Theory of Types'—rather, it was the unacceptable philosophical consequences of placing *ad hoc* order-restrictions on the variables of substitution. The doctrine of the unrestricted variable, as we have witnessed in some detail throughout this thesis, was the very *raison d'être* of the substitutional theory. Without it, the theory could no longer fulfil the task for which it had been constructed and, sadly, was consigned to unpublished and unknown manuscripts for the best part of seventy years. It is to be hoped that the forthcoming publication of these manuscripts in volume five of the *Collected Papers of Bertrand Russell*, will rouse new interest in this remarkable contribution to the very foundations of logical theory.

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