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Essays on Pricing Games with Asymmetric Players

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<u>ABSTRACT</u>

FACULTY OF SOCIAL SCIENCES
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ESSAYS ON PRICING GAMES WITH ASYMMETRIC PLAYERS

by Grazia Rapisarda

In this thesis we study pricing games where asymmetric players compete for the right to sell their products or services to a common buyer. The sellers' asymmetries may be informational and be a consequence of a pre-existing contract relationship with the buyer. They may also derive from either specialisation or cost efficiency in the provision of the good or service they sell. The focus is on the impact that these asymmetries have on the nature of the competition among the sellers and on the equilibrium price paid by the buyer.

In Chapter 2, two banks compete on interest rates for the provision of a loan to a firm in which one of the banks holds an equity stake. As opposed to his competitor, the equity-holding bank not only has more precise information on the creditworthiness of the firm but, regardless of her winning the loan contract, she receives dividend payments. As a result, the competition is biased in her favour and the equilibrium expected interest rate on the loan increases not only with the degree of informational differential between banks - as traditional theory would predict - but also with the size of the equity stake held by the informed bank.

Chapter 3 contains some evidence on the effectiveness of interest subsidies in relieving credit-rationing constraints. In the underlying pricing game, a firm is applying for a loan and interest payments on a fraction of it can be subsidised by a government agency. Bank competition is distorted by informational frictions and by market power (banks control unequal shares of the market or specialise in the provision of different forms of subsidies). If extended to existing clients, subsidies do not reach the intended beneficiaries and they end up generating a pure windfall for the bank-firm coalition. The bank managing the subsidy appropriates of at least part of this surplus by charging higher interest rates on the recipient's non-subsidised loans, and the extent of appropriation increases with the bank's monopoly power.

In Chapter 4 we consider a procurement game for the supply of a divisible requirement to a government agency who can affect the outcome of the bidding game by designing ex-ante the set of possible organisations of productions suppliers can bid upon. The agency is never worse off restricting the maximum number of sub-contracts to exclude at least one supplier from getting any award, even at the cost of inducing an efficiency loss. Further restrictions are revenue enhancing only if they reduce the distance, in terms of cost efficiency, between bidders who are pivotal to the equilibrium organisation and those who are not, to an extent which compensates for the efficiency loss the restriction induces. Removing *selected* allocations involving exclusively advantaged bidders may also reduce the expected price paid by the agency in equilibrium.

Chapter 1 introduces the thesis and summarises the main results. Chapter 5 concludes by suggesting directions for future research.

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Chapter 1

Introduction

In many real competitive situations some agents are known to have a competitive advantage over the others. In takeover battles, for instance, buyers who have toeholds in the target firm are believed to be favoured, and usually manage to win the auction at a low price. In affirmative procurement, the auctioneer can bias the competition in favour of certain bidders who belong to disadvantaged categories like minority owned firms, for instance. In procurement, a frequently encountered situation is one where there is an asymmetry in the initial cost position of potential suppliers, resulting from differences in their previous production experiences. When banks compete on loan pricing, they are believed to have an informational advantage to competitors over their old clients, as they learn about their creditworthiness during the lending relationship.

This work consists of three essays in which we study economic situations similar to those just mentioned. The strategic interaction among agents can be modeled as a pricing game where bidders compete for the right to sell their products or services to a common buyer, and market participants acknowledge the existence of asymmetries among the bidders. Asymmetries may be informational, and be a consequence of pre-existing contract relationships with the buyer, or represent different degrees of specialisation or cost efficiency across sellers in the provision of the good they sell. Despite the diversity of the specific economic environments we analyse, the common denominator is the emphasis on the impact that these asymmetries have on the nature of the strategic interaction among the sellers and on the resulting equilibrium price paid by the buyer.

In bidding contests like auctions, ex-ante asymmetries among sellers are well known to reduce the expected revenue of the buyer. One of the first to point this out was Myerson (1981), in his seminal paper on optimal auction design, where he suggests that a non-discriminating auction may be less profitable than one which discriminates in favour of the bidders with known lower valuations. Myerson (1981) investigates the issue by taking a mechanism design approach and invoking the "revelation principle", which has the advantage of making the analysis simple by abstracting from the strategic interaction among the sellers. The approach adopted in this work is instead more in line with that part of the literature (Cantillon (1999), Corns and Schotter (1999), Klemperer (1998), Maskin and Riley (1998) and McAfee and McMillan (1989) among the others) that finds in the analysis of the strategic interaction among sellers the key to understand the means by which asymmetries affect competition. It is indeed the acknowledgement of some aspect of the asymmetries by bidders that induce them to formulate asymmetric beliefs about their competitors' preferences. The possibility to assess their own relative competitive position lessens the competitive pressure and therefore also the buyer's expected revenue.

The objective of this thesis is not to provide a general framework to study the role played by asymmetries in pricing games. We actually consider alternative economic settings where the nature of the asymmetries arises from the specific problem analysed, and our interest is mainly descriptive of the channels through which asymmetries affect the outcome of the competition in each context.

In what follows we briefly summarise the content of each chapter, and we discuss the different approaches adopted. Given the diversity of topics considered, the survey of the relevant literature is provided separately in the introductory section of each chapter.

In Chapter 2 we examine the problem of a firm who needs to raise funds to finance her investment project. The only investors she can approach are two universal banks competing on prices (interest rates on loans), one of which holds an equity claim in the firm itself. As opposed to his competitor, the equity-holding bank not only holds more precise information on the creditworthiness of the firm but, regardless of whether she wins the competition on loan pricing or not, she is entitled to dividend payments. These are defined as residual rights to the firm's profits once interest rates are paid out to debt-holders.

We model competition on loan pricing as a sealed-bid common-value auction. Even though, in practise, firms do not formally set up auctions when they need a bank loan, there are nevertheless a few aspects of banking competition that are remindful of some relevant features of sealed-bid common-value auctions. The most important similarities, in relation to the objective of this paper, are the following. The entrepreneur can be seen as the "seller" of rights on the future cash flow of his project in the form of a loan contract. These rights can be seen as the common "object" bidders compete for and whose value depends on the value of the project's return the banks have imperfect information about. The rights will be sold to the bank who bids the *lowest* interest rate, i.e. the bank who advances the lowest claim on the project's cash flow thus leaving the firm with the highest residual claim. In a sealed-bid auction, bidders submit their offers *simultaneously*, and also this feature suits our bidding framework as banks usually cannot observe the offers made by their competitors.

We show that as a consequence of pay-off asymmetries between the two banks - due to the information structure and to the different nature of their claims - the competition is biased in favour of the equity-holding bank, and the equilibrium expected interest rate on the loan increases not only with the degree of information differential between banks - as traditional theory would predict - but also with the size of the equity stake held by the informed bank. In other words, equity stakes reinforce the competitive advantage of the informed bank by resulting in a higher expected interest rate on loans for the borrower. Correspondingly, the informed shareholding bank expects to lose the borrower to the outside competitors with a probability which is decreasing in her informational advantage and in the size of her equity share. This result is consistent with the empirical evidence on the phenomenon of bank equity stakes acquisition available for the Italian case as provided by Bianco and Chiri (1997). Indeed, there seems to be a positive correlation between the presence of a bank in the ownership structure of a firm and the tendency of the firm to concentrate its credit relationships at the shareholding bank over time.

Chapter 3 contains empirical evidence on the effectiveness of interest subsidies in relieving credit rationing constraints[1]. In spite of the importance assigned to credit programs by policy makers, as revealed by the significant amount of resources that they absorb worldwide, credit subsidies have received little attention among applied economists. Our analysis is made possible by a unique dataset containing information on a panel of firms operating in Southern Italy during the period 1995-1998. The information includes details of the terms and conditions on all contractual relationship between the firms and the anonymous bank that provided the data. We find empirical evidence that, even though interest subsidies may be used to start new firm-bank relationships, they seem not too have considerable impact on investment when granted to the existing clients of the bank. Subsidised loans tend to substitute one-to-one existing ones with no substantial impact on total quantities loaned. This is taken as evidence that subsidies reach firms that would have received funding even without subsidies and that the latter represent a pure windfall for the bank-firm coalition. We find that the bank managing the subsidy may exploit her monopoly power and appropriate at least part of this surplus. Our results show that subsidised borrowers are charged interest rates on their non-subsidised loans which are higher than those charged on generic borrowers with similar characteristics. Moreover, we find that a marginal increment in the size of the subsidy generates a significant increment in the price of subsidised borrowers' non-subsidised loans.

The pricing framework that we consider to justify this "price spillover" as evidence

[1]The study was started while I was visiting the "Ente 'Luigi Einaudi' per gli Studi Bancari, Monetari e Finanziari" in Rome. Prof. Luigi Guiso and Prof. Luigi Zingales supervised my work and contributed in the specification of the empirical models.

of appropriation is still one in which banks' asymmetries play a crucial role. We consider a spatial competition model with adverse selection, where two banks differ in that they specialise in the provision of certain financial products or services, including different forms of subsidised finance. The location of borrowers represents their preference for the services provided by each bank. Each firm can apply for a loan to any of the banks but she can receive an interest subsidy only on a portion of it. This models the fact that subsidies can only be extended on loans of a limited size, and borrowers usually need additional non-subsidised loans to cover their financial needs. The subsidised interest rate and the fraction of it paid by the subsidising body are fixed by the program, but banks can still compete in the pricing of the non-subsidised part of the loan. We assume that banks Bertrand compete on new borrowers, but that they charge their old borrowers with an *average* interest rate that just prevents them from switching to the competition. They hold proprietory information on the quality of their old clients, and these cannot be distinguished from new ones when they apply at outside banks. In the absence of frictions, one dollar subsidy should translate in one dollar savings in interest payments for these borrowers. Nevertheless, informational frictions, switching costs, and asymmetries in the provision of subsidised finance all represent means by which the bank can limit the appropriation of these savings. This is consistent with the evidence on the presence of a "price spillover" which is larger whenever the bank's monopoly power is more severe.

In Chapter 4 we depart from a banking environment and we consider a procurement problem where a government agency needs to award the supply of a given quantity of a divisible requirement to one or *more* of the potential producers operating in the market. Moreover, the buyer plays here an active role and can affect the nature of the competition among sellers by designing the bid solicitation format so as to maximise her expected revenue. We do not take a mechanism design approach, as the agency is only allowed to choose the bid solicitation format out of a limited set of design variations. We model the procurement game as a menu (split-award) auction and the agency can only decide on which of the feasible allocations of the requirement the sellers can bid upon. The set of feasible allocations is given by all the possible organisations of production that can be attained combining the participating sellers. The agency can constrain this set by setting a cap on the number of sub-contractors she can commit to deal with, or by deciding on the relative size of the shares to be awarded to each of them.

The sellers can observe each others' (private) valuations - the production cost functions - and they can thus exactly assess the extent of their reciprocal "distance". The agent is aware of this, but he only knows the random process that generates the bidders' valuations, so that he can only formulate expectations on how distant the bidders might be. In some cases, we allow the agency to be able to screen out, from the pool of bidders, those who have a cost advantage over the others, and to constrain the set of feasible allocations using this piece of information as well. We want to emphasise that, even though we assume incomplete information between the agency and the bidders, we are only interested in results that do not depend on the specific probability distribution over the bidders' types that form the agency's beliefs.

The menu-auction game that we consider falls into the more general class of common agency games with complete information among principals. These games typically admit a multiplicity of equilibria some of which, known in the literature as *truthful equilibria*, have the desirable property of resulting in an *efficient* equilibrium allocation. We restrict our attention to games that admit a *unique* truthful equilibrium (or *marginal contribution equilibrium*). In equilibrium, the price paid to suppliers is a function of the actual total cost associated with the equilibrium organisation, plus a mark-up which depends on the *marginal contributions* of suppliers to this total cost. Asymmetries in the suppliers' cost efficiency play a role in that total marginal contributions depend on the "distance" or degree of substitutability between suppliers who take part to the equilibrium organisation (pivotal bidders) and the most efficient of those who do not (non-pivotal bidders). Any restriction on the feasibility set can only be preferred by the agency if the reduction in this distance is large enough to compensate for the efficiency loss that may result from the restriction itself.

We find that the agency is never worse off restricting the maximum number of sub-contracts to exclude at least one supplier from getting any award. Nevertheless, if bidders are identical any reduction of the maximum number of sub-contractors below $n - 1$ is never strictly profitable to the agency. If instead bidders are not identical, the optimal restriction depends on the relationship between the marginal properties of the cost functions and the distance among suppliers. We concentrate on environments characterised by diseconomies of scale in production and find that the more severe the extent of asymmetries in cost efficiency is, the more likely restrictions on the number of sub-contracts are revenue enhancing for the agency. We finally analyse alternative types of restrictions on the allocation possibilities set. We first consider an example that shows that the agency can improve her revenue by altering the relative size of the shares awarded to each supplier in a split-award outcome. We also find that the agency might increase her revenue by biasing the competition in favour of disadvantaged players by excluding *selected* allocations involving advantaged players only. Other contributions have shown how preferential treatment of disadvantaged bidders can increase the auctioneer's revenue in procurement auctions. In our example, we investigate this issue in a multi-unit setting, where the bias can be tailored to favour certain categories of bidders in *some* of the possible allocations and not in others.

Chapter 5 contains concluding remarks and outlines directions for future research.

Chapter 2

Bank Equity Stakes in Borrowing Firms and Credit Market Competition

2.1 Introduction

The range of services that can be offered by banking institutions differs widely across the globe. Two general models can be distinguished: universal banking and functionally separated banking. In a universal banking system, banks perform both investment and commercial banking functions while, in a functionally separated system, they are allocated to different institutions. One of the most controversial issues is whether universal banks should engage in securities underwriting and be allowed to acquire equity stakes in non-financial firms. The general argument in favor of universal banking is that artificial limitations on bank activities could potentially constrain optimal configurations. On the other hand, a commonly raised concern is the fear that the affiliation of a bank with a commercial firm could increase the risks of bank failure and thereby impose greater costs on the safety net. This paper considers the impact the presence of banks holding mixed debt-equity claims in a firm might have on the degree of competition in credit markets.

We model the problem of a wealth-constrained owner-manager who needs to finance its project. The only investors firms can approach are two universal banks competing on prices (interest rates on loans) and operating in the form of conglomerates, one of which holds an equity claim in the firm itself. Equity claims are defined as rights to receive dividend payments as the residual right to the firm's profits once interest rates are paid off to debt-holders. We also assume that the equity-holding bank can have access to private information about the future prospects of the firm. Since the shareholding bank is universal, information synergies between banking activities can result. The information collected by the investment banking department (or subsidiary) is also used by the commercial banking department to assess the creditworthiness of the firm. As an "insider", the shareholding bank turns out to have an informational advantage which generates a "lemons" problem for outside potential lenders and reduces the degree of competition in the market for loans, leading to a limited informational capture of the borrower. The original contribution of this paper as compared to this classical paradigm is to show that the temporary monopoly power enjoyed by the shareholding bank is a function not only of her informational advantage (*information effect*), but also of the size of the equity stake she holds (*dividend effect*).

Many authors (Cable (1985), Flath (1993), Hoshi,Kashyap and Sharfstein (1991), Krosner and Strahan (1999), Prowse (1990,1992), Sheard (1989)) have emphasised that, by owning equity in a firm as well as debt, the bank becomes even more of an insider than if it remained just a privileged creditor. This argument is justified not only by the fact that being a shareholder implies additional informational rights compared to other stakeholders, but also by the fact that, in general, shareholding banks do have their representatives in the firm's supervisory board or in the board of directors. Through board representation the bank acquires a *full* insider status, which internalises and perfects the information flow from the firm to the bank.

Other theoretical studies (Shaffer (1997), Rajan (1992), Broecker (1990),von Thadden (1998)) have identified a "lemons" problem in bank lending, resulting from the ability of rejected applicants to apply at additional banks. As a consequence of this ability, it becomes difficult for one less informed bank to draw off another bank's good customers without attracting the less desirable ones as well. In order to capture this aspect of banking competition, we model the loan pricing game between banks as a sealed-bid, first-price common-value auction with asymmetric bidders. The bidding banks differ in two respects. Firstly, one of the bank (the insider) holds an equity stake in the firm applying for the loan. Secondly, they are asymmetrically informed about the value of the contract. Both banks have a prior belief on the distribution of good and bad firms in the industry, but only the equity-holding bank receives an informative signal about the value of the project. Since we assume that the signal space is discrete, no pure strategy equilibrium exists (Wang (1991)), and competing banks optimally choose to bid randomly over a range of feasible interest rates. As in von Thadden (1998), because of a "winner's curse" effect caused by incomplete information, the outside uninformed bank decides not to take part to the competition with a positive probability and, correspondingly, the insider decides to "squeeze" the borrower by bidding the highest possible interest rate. Our setting differs in that the informed bank holds an equity stake in the firm demanding the loan. We show that if the informed bank acquires a positive share of residual rights on the firm's net surplus, her optimal bidding strategy remains unaffected - compared to the case of no equity - while, ceteris paribus, the outsider would refuse finance with a higher probability the larger is the equity share. In other words, equity stakes reinforce the competitive advantage of the informed bank by resulting in a higher expected interest rate on loans for the good borrower and a lower probability of the informed bank to lose good borrowers to competitors. Indeed, the "switching" probability turns out to be decreasing in the equity-holding bank's informational advantage and in the size of her equity share. This result is consistent with the empirical evidence on the phenomenon of bank equity stakes acquisition available for the Italian case provided by Bianco and Chiri (1997). In fact, there seems to be a positive correlation between the presence of a bank in the ownership structure of a firm and the tendency of the firm to concentrate its credit relationships at the shareholding bank over time.

The intuition behind these results is the following. Dividends represent an additional source of profit of which the bank benefits regardless of her being also a debt financier. By increasing the shareholding bank's expected profits from the loan pricing competition, dividends reinforce her competitive advantage by worsening the "winner's curse" effect for the outside uninformed bank. As the size of the equity stake increases, the outsider optimally chooses not to take part to the competition with a higher probability, thus reducing the chance of the firm switching to outside sources of finance.

These observations are remindful of the literature on almost common-value auctions (Klemperer (1998)) and of the related literature on *toeholds* in takeover battles (Bulow, Huang and Klemperer (1999), for instance). Small asymmetries between bidders, like a small value advantage or a small ownership of the object by one bidder, can have dramatic effects on equilibrium prices in common-value auctions because of an increased winner's curse faced by disadvantaged competitors. This result is obtained in a setting where competition is modelled as an ascending auction with common values, rather than as a sealed-bid auction, which, we believe, is a more realistic way to describe credit market competition. In Bulow, Huang and Klemperer (1999), the authors do suggest that a sealed-bid auction would soften the winner's curse problem (bidders have no "incentive to bid up the price purely in order to sell high"), but they do not formally model this intuition.

Our results are also similar to those independently achieved by Tröge (2000), who explores the relationship between bank industry ownership and credit market competition by considering a more general setting in which two *symmetrically* informed banks (they receive independent but equally informative signals) hold equity stakes in the applicant firm. Asymmetries in the size of the shares increase the expected cost of bank finance for the firm. This result is not driven by a winner's curse problem (banks bid with perfect information) but by the banks being uncertain about the number of their competitors.

The contribution of this paper to the above literature lies in the joint analysis of information effect and dividend (or toehold) effect, which have been separately studied in previous work. By analysing both effects in the same model, we provide a more complete characterisation of the nature of the competition between asymmetric banks. If we take the "switching probability" as a proxy for the monopoly power of the equity-holding bank, we find that, for any given degree of informational asymmetry, an increment in the size of the equity stake increases the informed bank competitive advantage over the uninformed bank, and viceversa. Nevertheless, the way dividend and information effect interact with each other depends on the the relative size of the information advantage (p), the size of the equity stake (α), and the degree of adverse selection in the market. The reduction in the switching probability resulting from a marginal increment in p is larger the larger is α (*information* effect and *dividend* effect reinforce each other) only for a subset of combinations (α, p). Otherwise, the decrement in the switching probability is larger the smaller is α (*information* effect and *dividend* effect are substitutes). As the degree of adverse selection in the market gets more severe, the set of values (α, p) for which the second effect holds gets larger.

Equity affects the insider's expected pay-off in mainly two ways. First of all, as compared to the case of no equity stakes, the insider can now extract the same value from good borrowers by bidding a lower interest rate. What is lost by the credit division in terms of lower interest payments can indeed be recovered by the investment division through higher dividends. Equity may therefore play a disciplinary role on the insider's bank and induce her to bid more aggressively (i.e. bid lower rates); correspondingly, the response of the outsider would be to bid less aggressively. From this point of view, the effect of a larger equity share is similar to that of information acquisition, as they both keep the outsider out of the competition with a higher probability. On the other hand, equity secures the insider a minimum amount of income even in case she loses the loan contract, and this may induce her to bid less aggressively even at the expense of a higher probability of losing the game. This "insurance" effect is typical of equity shares in our model. If α is very large (close to one), the outsider stays out of the competition with a high probability as he anticipates that the insider is mainly concerned about winning and will try to bid low interest rates. As α decreases, the insider becomes more and more concerned about winning with a high interest rate. The "insurance" effect of equity plays a more relevant role here, and this is why acquiring information may be more effective in keeping the outsider out of the competition when α is relatively low. As the degree of adverse selection worsens, the return from acquiring information may be high enough to make the "insurance" effect of equity irrelevant, and this would explain why for a larger set of values (α, p) the equity and the dividend effects become substitutes.

The present paper is organised as follows. In section 2.2 we present the model. In section 2.3 we characterise the equilibrium bidding strategies of players. In section 2.4 we compute and analyse the equilibrium interest rate and the probability with which the firm can switch to outside bank credit in a comparative statics framework. Final remarks in section 2.5 conclude the Chapter.

2.2 The model

Assume a risk neutral world where there is an entrepreneur who intends to undertake a one-period investment project. The project can be of a good or bad quality; its quality determines the random stream of returns to the investment at the end of the period. The good project requires an initial investment I at t=0 and pays out $S \geq I$ at the end of the period (t=1). The bad project, on the contrary, is doomed to fail at t=1 with probability one. The output produced by each project is observable and verifiable.

The are only two universal banks operating in the credit market which the entrepreneur can approach for finance. One of the banks holds an equity stake in the firm itself, which implies that she is entitled to receive a fraction α of the firm's net surplus at t=1.

At t=0, both banks know that the probability of the entrepreneur being a good type is $\theta \in (0, 1)$. Nevertheless, before competition starts, the shareholding bank observes an informative private signal ξ of the project's quality. The signal delivers a message $\xi = 1$ with probability one if the borrower is a good type. If the borrower is a bad one, the signal delivers $\xi = 0$ with probability $(1 - q)$, and $\xi = 1$ with the complementary probability. The parameter q measures the precision of the signal received by the insider. The smaller is q the more informative is the signal. In particular, if $q = 0$, the insider can perfectly screen out bad from good types at t=1. The outsider receives no signal. Implicit in the assumption of asymmetric information between the insider shareholding bank and outsider is the idea that banks do not readily divulge information concerning the profitability of their borrowers. Clearly, such an information sharing would help competing banks to bid away their best customers.

Let the risk-free interest rate as well as the discounting rate be zero.

In summary, the sequence of events is the following.

t=0

1. The shareholding bank observes the signal ξ.
2. The entrepreneur approaches the two banks operating in the market and apply for a loan of size I.
3. The shareholding insider bank and the outside bank respond by simultaneously quoting an interest rate (i^{in} and i^{out}, respectively) that gives them an expected return greater than or equal to their cost of funds (normalised to 1).
4. The entrepreneur accepts to write a contract with the bank that quotes the lowest interest rate, borrows and invest I. If indifferent, he chooses the inside bank's offer.

t=1 At the end of the first period the output is realised and the borrower will repay the face value of debt only if the output $S \neq 0$. All cash flow from the project is paid out in the form of dividends or debt service.

2.3 The equilibrium strategies

The competition on loan pricing is described as a sealed bid, first price common value auction with asymmetrically informed bidders. The claims attached to the loan contract can be seen as the common "object" bidders compete for. The "seller" is the entrepreneur and he sells rights on the future cash flow of his project, which represent the common value the banks have imperfect information about. The rights will be sold to the bank who offers the *lowest* interest rate, i.e. the bank who advances the lowest claim on the project's cash flow thus leaving the firm the highest residual claim. In addition, as in a sealed-bid auction, banks submit their offers *simultaneously*. This feature of sealed-bid auctions suits our bidding framework as banks usually compete on loans without being able to observe the offers made by their competitors.

The bidding banks differ in two respects. Firstly, one of the bank (the insider) holds an equity stake in the firm offering the loan contract. Secondly, they have different information about the value of the contract. The information structure is determined at the beginning of period one, by Nature's random choice of the borrowers' types, and by the observation of the signal ξ by the insider. The outsider bank only knows the prior θ, which represents the probability of the entrepreneur being a good one and the distribution of the signal received by the insider at t=1. The insider can observe a signal ξ of the borrower's type. After receiving the signal, the inside bank updates her beliefs on the type of borrower being financed. If the signal reports $\xi = 0$, the insider believes that the borrower is a bad one, and that the project is to fail with probability one. If $\xi = 1$, the insider expects the borrower to be a good type with a probability $\beta(\theta, q) = \theta\left[\theta + (1-\theta)\, q\right]^{-1}$, derived using Bayes' rule, where q measures the precision of the signal received by the insider.

Before deriving the equilibrium strategies, we need to define important benchmark loan rates:

- $i_0^{out} \equiv (1-\theta)/\theta$.

- $u \equiv (S/I) - 1$.
- $i_0^{in}(q,\alpha) \equiv \left(\beta(q)^{-1} - \alpha u\right)/(1-\alpha)$, $\alpha \neq 1$.[1]

The interest rate i_0^{out} ensures the outside, uninformed bank with zero expected profits in case only the prior θ affects his beliefs, where θ represents the public information available in the market. The interest rate u leaves the good entrepreneur with zero profits and allows the lending bank to extract all of the project's surplus through interest rate payments. The rate $i_0^{in}(q,\alpha)$ represents the zero-profit interest rate[2] for the insider conditional on receiving a good signal, in case the posterior $\beta(q)$ summarises her beliefs. If she observes $\xi = 0$, then the zero-profit interest rate is obviously $i = +\infty$, which we conventionally identify with the choice of "not bidding at all". Note that, in computing these rates, we abstract from any strategic consideration.

We can easily derive the following relation between the insider's and the outsider's "zero-profit" interest rates when $\alpha = 0$, that is in the case the insider holds no equity claims:

$$i_0^{in}(q,0) = \frac{1}{\beta(q)} - 1 = \frac{(1-\theta)q}{\theta} = q i_0^{out} \tag{2.3.1}$$

The above relation implies that the value the insider attaches to borrowers that are screened out as "good" is higher than the value an outsider would attach to it. This implies:

$$i_0^{in}(q,\alpha) \leq i_0^{in}(q,0) \leq i_0^{out}$$

[1]If $\alpha = 1$ the shareholding bank always gets an expected profit equal to $\beta(\theta,q)S - I$ regardless of the interest rate charged.

[2]We have assumed that universal banks operate in the form of a conglomerate, so that the investment and the commercial banking department operate as part of a unique legal entity. Expression $i_0^{in}(q,\alpha)$ represents the zero-profit interest rate for the entire conglomerate.

for any given level of adverse selection $(1-\theta)$. The profit that the insider obtains by charging an interest rate $i \geq i_0^{out}$ to good borrowers is higher than what an outsider would get by charging the same interest rate. And this is not only because the insider is more informed, but also because she can receive dividend payments as a residual claim once interests are paid out.

Banks' bidding decisions at $t = 0$ depend on various factors. Firstly, the characteristics of the information structure, which is represented by the parameters q and θ and by the discrete random process generating the signal ξ. Secondly, the *size* of the equity share α, which determines the entity of profits the investor earns in addition to interest payments. According to the value taken by these parameters, we might have equilibria in pure strategies or in mixed strategies.

We rule out the trivial case in which $u < i_0^{in}$, as in this case not even the informed inside bank would find it worthwhile to invest in the project. Both the insider and the outsider refuse credit to the entrepreneur with probability one. Also the case $i_0^{in} \leq u \leq i_0^{out}$ is not an interesting one, as the pricing game has a unique Bayesian Nash equilibrium in *pure strategies* where the outsider refuses finance (i.e. plays $i^{out} = +\infty$), and the insider "squeezes" the firm by bidding the highest possible interest rate $i = u$. the outsider would not undercut this rate as she would not break even in expectation. In addition, any finite interest rate above u would only attract bad borrowers.

In the rest of this paper we will therefore consider values of the parameters such that $i_0^{out} < u$. The following Lemma gives a first characterisation of the possible equilibria of the game:

Lemma 1 *If $i_0^{out} < u$, the pricing game at $t = 0$ might have a pure or mixed strategy equilibrium according to the value taken by the parameter α and q :*

[a] If $q = 1$, for every α the game has a unique Nash equilibrium in pure strategies in which the outsider and the insider bids the interest rate $i^{in} = i_0^{out}$.

[b] If $q < 1$ and $\alpha = 1$, the game has a unique Bayesian Nash equilibrium in pure strategies in which the outsider refuses finance with probability one, the insider refuses finance if

the signal received is $\xi = 0$*, and bids the interest rate* $i^{in} = i_0^{out}$ *if the signal is* $\xi = 1$.
[c] *If* $q < 1$ *and* $0 \leq \alpha < 1$ *the game has no Nash equilibrium in pure strategies.*

Proof. [a] If $q = 1$, the insider's signal is not informative and both banks are symmetrically uninformed. Given the outsider bids i_0^{out}, the insider's best response would be to bid i_0^{out}: a higher rate would make her lose the contract and a lower rate would make her win the game at a lower net profit. Given the insider bids i_0^{out}, the outsider is indifferent between i_0^{out} and any rate above this as she loses the competition in all cases; i_0^{out} is nevertheless strictly preferred to any rate below i_0^{out} as this would give him negative expected profits.
[b] If the shareholding bank holds an equity stake $\alpha = 1$, then, in case she wins the loan contract, she will appropriate of all the surplus produced by the project, regardless of the size of the interest rate she bids. It will be an optimal strategy for her to bid i_0^{out} with probability one as, if she bids an interest rate $i^{in} > i_0^{out}$ with a positive probability, she might lose with a positive probability and get a lower expected payoff.
[c] If $\alpha < 1$ and $q < 1$, the outsider will never play a pure strategy in equilibrium. Let us take the outsider's point of view. If he bids $i^{out} \leq i_0^{out}$, he bears an expected loss even if she manages to attract, with this interest rate, both good and bad borrowers. If instead $i_0^{out} \leq i^{out} < +\infty$, the outsider can only attract bad borrowers as the insider can always keep good ones by charging $i^{in} = i^{out}$ ($i_0^{in} \leq i^{out}$ and the borrower will choose the insider in case of ties). This is because the outsider believes that the insider is always better off taking part to the bidding game whatever the size of the equity stake α. Indeed, if the insider does not participate, she expects to get dividend payments $\alpha\beta(q)\left[S - (1 + i^{out})I\right]$, which is less than what she gets by bidding i^{out} and getting interest payments $(\beta(q)(1 + i^{out}) - 1)I$ on top of those dividends. Now, the only alternative left to the outsider is to refuse finance with probability one, which implies bidding an interest rate $i^{out} = +\infty$. Given the outsider's strategy, though, the insider's best response will be to refuse finance to bad type borrowers and to extract all the surplus from good ones by bidding $i^{in} = u$. Given $i^{in} = u$, to "stay out" is no longer a best response for the outsider. The latter

could indeed undercut and bid any $i^{out} = u - \varepsilon$, $\varepsilon > 0$ and get positive expected profits. ■

From now on we restrict our attention to the case in which $i_0^{out} < u$ and $0 \leq \alpha < 1$, and the bidding game admits a unique Bayesian Nash Equilibrium in mixed strategies. Under these restrictions of the parameters, the mixed strategies are a direct consequence of two facts. First of all, the outsider acknowledges the fact that the set of the insider's possible information types is discrete (see Wang (1991). Secondly, she also knows that the dividends the insider gets as pure equity-holder do not make her participation to the bidding game unprofitable. This means that the outsider knows that the insider has always an incentive to undercut any of her pure strategies.

We can now start to characterise the players' bidding strategies. In Lemma 2 we derive lower and upper bounds to the equilibrium strategies' support.

Lemma 2 *In equilibrium, the following statements are true:*
[*a*] $i^{in} \geq i_0^{out}$ *and* $i^{out} \geq i_0^{out}$ *with probability one.*
[*b*] *For* $k \longrightarrow +\infty$, $i^{in} \in (u, k)$ *and* $i^{out} \in (u, k)$ *with probability zero.*
[*c*] *Given* $\xi = 1$, $i^{in} = +\infty$ *with probability zero.*

Proof. [*a*] The outsider will never bid below i_0^{out}, as she would make negative expected profits. Therefore $i^{out} \geq i_0^{out}$ and the insider expects to win the auction with probability one for every $i \leq i_0^{out}$. If we assume that the insider bids an interest rate $i < i_0^{out}$ with positive probability, her expected pay-off for any given i will be:

$$(\beta(q)(1+i)-1)I + \beta(q)\alpha[S-(1+i)I] =$$
$$= \beta(q)[(1-\alpha)(1+i)I + \alpha(S-I)]$$

which is strictly increasing in i as $0 \leq \alpha < 1$. The insider could bid $i + \varepsilon$, $\varepsilon > 0$ and be strictly better off without reducing the probability of winning the auction. Therefore $i^{in} \geq i_0^{out}$.

[b] The insider refuses finance with probability one to bad borrowers, i.e. $i^{in} = +\infty$ if $\xi = 0$. If $\xi = 1$, assume that the insider bids, in equilibrium, an interest rate $\infty > k > i^{in} > u$ with positive probability. In case she wins the auction, she gets no higher pay-off than bidding u. Therefore, by bidding a lower interest rate $i^{in} - \varepsilon$, $\varepsilon > 0$, she could reduce the probability of losing the auction without affecting the pay-off in case of win. The outsider can anticipate that the insider will never bid over (u, k) with positive probability and that any bid above u will only attract bad borrowers. Therefore the only interest rate above u that both bidders might bid in equilibrium is $i = +\infty$.

[c] Assume the insider plays $i^{in} = +\infty$ if $\xi = 1$ with probability L_ε. Under this circumstance, with a probability L_ε she will lose the competition on loan pricing and receive dividend payments only if the outsider plays $i^{out} \leq u$. If also the outsider refuses finance with positive probability, the insider will not even receive dividend payments. She will not be worse off if she moves this probability mass on u, for instance, and get not only the same dividend payments, but also interest payments when $i^{out} = +\infty$. ■

In order to proceed with the characterisation of the equilibrium strategies, we need to formally derive the players' expected pay-off functions. The outsider's and the insider's expected profits are given by expressions (3.2) and (3.3) respectively:

$$P^{out}(i;q) = [\theta + (1-\theta)q]\left(1 - G^{in}(i)\right)[\beta(q)(1+i) - 1]I + \\ - (1-\theta)(1-q)I \quad (2.3.2)$$

$$P^{in}(i;\alpha,q) = \left(1 - G^{out}\left(i^{-}\right)\right)[(\beta(q)(1+i) - 1)I + \beta(q)\alpha[S - (1+i)I]] + \\ + G^{out}\left(i^{-}\right)\beta(q)\alpha\left[S - \left(1 + E\left(i^{out}|\ell \leq i^{out} < i\right)\right)I\right] \quad (2.3.3)$$

where $G^{in}(i)$ and $G^{out}(i)$ represent the insider's and the outsider's bidding strategies, $G^{out}(i^-) \equiv \lim_{x \to i^-} G^{out}(x)$, ℓ represents the lower bound of the players' bidding strategy support, and $E(i^{out} | \ell \leq i^{out} < i)$ denotes the expected interest rate bid by the outsider conditional on her winning the auction. The first additional term in (2.3.2) represents the profits (interest rate payments plus dividends) the insider gets in case she wins the auction. The second term represents the profits (dividends) she gets in case she loses, and they are a function of the average interest rate bid by the outsider, given that the outsider wins the auction.

In order to derive the actual functional form of the players' bidding strategies we need to verify continuity of $G^{in}(i)$ and $G^{out}(i)$ - and of their derivatives - over the interval $[\ell, u)$.

Lemma 3 *In equilibrium, the following statements are true:*
[a] Both the insider and the outsider bid atomlessly over the interval $[\ell, u)$.
[b] The outsider's bidding strategy $G^{out}(i)$ *is continuous on* $i = u$.
[c] Both $G^{in}(i)$ *and* $G^{out}(i)$ *are strictly increasing on* $[\ell, u]$.

Proof. See Appendix A.1. ■

Lemma 3 enables us to reformulate the expected profits of the insider as follows:

$$P^{in}(i; \alpha, q) =$$
$$\left(1 - G^{out}(i)\right) \left[(\beta(q)(1+i) - 1) I + \beta(q) \alpha [S - (1+i) I]\right] +$$
$$+ G^{out}(i) \beta(q) \alpha \left[S - \left(1 + \frac{1}{G^{out}(i)} \int_{-\infty}^{i} t dG^{out}(t)\right) I\right] \quad (2.3.2')$$

The insider's and the outsider's pay-offs differ in many respects. First of all, the insider's expected pay-off is discontinuous as a result of the discrete nature of the signal she receives. The outsider's expected payoff is continuous, as he is not informed, but her beliefs reflect the discrete nature of the insider's signal. Indeed, she anticipates that the insider can be of two types: with a probability $[\theta + (1-\theta) q]$ the insider will take part to the bidding game on good borrowers' loans, and with the complementary probability $(1-\theta)(1-q)$ she will refuse finance to bad borrowers. In addition, the outsider's pay-off is not affected by the insider holding equity stakes. The size of the equity share α affects only the insider's expected pay-off and in essentially two ways. First of all, as compared to the case of no equity stakes, the insider can now extract the same value from good borrowers by bidding a lower interest rate. From the borrower's point of view, equity may therefore play a disciplinary role on the insider's bank and induce her to bid more aggressively (i.e. bid lower rates): what is lost by the credit division in terms of lower interest payments can indeed be recovered by the investment division through higher dividend payments. Nevertheless, the possibility to get dividend payments even without winning the competition on loan pricing may induce a less aggressive behaviour on the part of the insider's, as dividends guarantee a sort of minimum income in any event. Which one of these effects prevail will be reflected in the outsider's equilibrium response.

The equilibrium mixed strategies for the insider and the outsider are described in Proposition 1:

Proposition 1 *The bidding game at t =1 has a unique Bayesian Nash Equilibrium in mixed strategies in which:*

[a] The insider refuses finance with probability one if the borrower is detected as a bad one. If the signal received at $t = 0$ is good, then the insider bids u with probability $\mu = (\ell - i_0^{in}(0))/(u - i_0^{in}(0))$ and atomlessly over the interval $[\ell, u)$ according to the cumulative probability distribution function: $G^{in}(i) = 1 - \ell(1-q)/(i - \ell q)$.

[b] The outsider refuses finance with probability $\mu^{(1-\alpha)}$ and with the complementary probability she bids atomlessly over the whole range $[\ell, u]$ according to the cumulative distribution function: $G^{out}(i) = 1 - (\ell(1-q)/(i-\ell q))^{1-\alpha}$.

[c] The lower bound of the bidding strategies' support is $\ell = i_0^{out}$ and the outsider earns zero-expected profits in equilibrium.

Proof. See Appendix A.2. ■

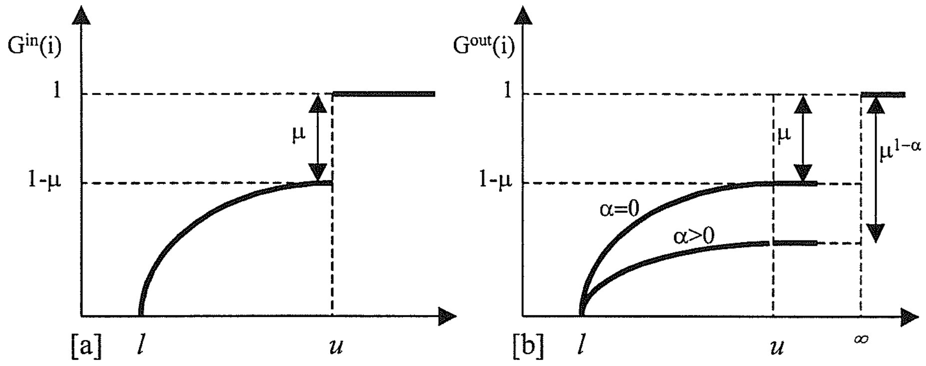

Figure 2-1: The insider's and the outsider's bidding strategies.

The equilibrium strategies are represented in Fig.2.1. Proposition 1 shows that the equilibrium bidding strategy of the insider depends on q but not on α. In a mixed strategies equilibrium, the insider's strategy is only determined by the outsider's expected pay-off, which is in fact only affected by changes in q. The outsider's strategy is instead a function of both α and q. If $\alpha = 0$, we have $G^{in}(i) = G^{out}(i)$ over the interval $[\ell, u)$. Therefore, in this particular case, the insider's and the outsider's cumulative distribution functions differ only in that, in equilibrium, the insider puts probability mass μ on the point $i = u$, extracting the entire surplus from the firm, whilst with the same probability the outsider refuses finance by bidding $i = +\infty$. Nevertheless, as α gets larger, the probability distributions $G^{in}(i)$ and $G^{out}(i)$ get more and more distant over the interval $[\ell, u)$ as the outsider's strategy turns into a less aggressive one. Indeed, the outsider optimally decides to refuse finance with a probability $\mu^{1-\alpha}$ which is increasing in α.

The fact that the outsider earns zero-expected profit in equilibrium is a consequence of the fact that the probability with which he does not take part to the competition is positive for any $\ell \geq i_0^{out}$. Indifference between bids in equilibrium then implies that bidding $i = +\infty$ or any $i \in [\ell, u]$ should give him the same expected pay-off. This finding is consistent with results by Engebrelcht-Wiggans, Milgrom and Weber (1983) and Milgrom and Weber (1982), and originates from the assumption of the insider's information partition being finer than the outsider's.

2.4 The switching probability and the equilibrium interest rate

In this section we will complete the characterisation of the equilibrium by formally deriving the interest rate the good borrower expects to pay in equilibrium and the probability with which he expects to switch from the inside informed shareholding bank to the outside uninformed bank. Both can be taken as proxies of the monopoly power of the insider.

The switching probability is given by (see intermediate steps in Appendix A.3.):

$$\sigma(\alpha, q) \equiv \Pr\left(i^{out} < i^{in} | i^{out} \text{ or } i^{in} \leq u\right) = \frac{1-\alpha}{2-\alpha}\left[1 - \left(\frac{\ell(1-q)}{u-\ell q}\right)^{2-\alpha}\right] \tag{2.4.1}$$

The term in square brackets in (2.4.1) represents the probability with which the insider and the outsider actually compete with one another. Indeed, the term $\left(\frac{\ell(1-q)}{u-\ell q}\right)^{2-\alpha}$ is the probability with which the outsider refuses finance and the insider squeezes the firm, by bidding the highest possible interest rate u. The switching probability is therefore increasing in the intensity of competition provided by the outsider.

The expected interest rate for the good borrower is given by:

$$\begin{aligned} i^e(\alpha, q) &= E\left[\min\left(i^{in}, i^{out}\right)\right] = \\ &= \int_{\ell}^{u} \left(i\left(g^{in}(i)\left(1 - G^{out}(i)\right) + g^{out}(i)\left(1 - G^{in}(i)\right)\right) di + \right. \\ &\qquad + u\left(1 - G^{in}(u^-)\right)\left(1 - G^{out}(u)\right) \end{aligned}$$

where $G^{out}(i)$ and $G^{in}(i)$ are described in Proposition 1, $g^{in}(i)$ and $g^{out}(i)$ are their first derivatives. By rearranging the above expression we get:

$$i^e(\alpha,q)=\ell\left\{1+\frac{1-q}{1-\alpha}\left[1-\left(\frac{\ell(1-q)}{u-\ell q}\right)^{1-\alpha}\right]\right\} \tag{2.4.2}$$

2.4.1 The information effect and the dividend effect

We will now conduct a few exercises of comparative statics and examine the effect of variations in the parameters q and α on the outcome of the pricing game between banks. In what follows we will only consider variations of q and α in [0,1) as a mixed strategy equilibrium occurs only if $q \neq 1$ and $\alpha \neq 1$ (Lemma 1).

The parameter q measures the precision of information acquired by the inside bank and, given that the outside bank is uninformed, it also represents the degree of informational asymmetry between them. Changes in q are captured by variations in the posterior probability $\beta(q)$ of the project being successful conditional on the signal observed by the insider being a good one. This probability is monotonically decreasing in q, which means that, as q decreases, the insider bank's signal becomes more informative about the borrower's type. From the outsider's point of view, this means that the probability to be facing a bad borrower rejected by the insider increases. It follows that, as the precision of information increases, the outsider moves probability mass towards the upper tail of his distribution, which translates into a monotonically increasing probability with which she refuses finance to the applicant. As a result, we have:

$$G^{out}(i)\underset{q\to 0}{\to}1-\left(\frac{\ell}{i}\right)^{1-\alpha} \quad \text{and} \quad \mu^{1-\alpha}\underset{q\to 0}{\to}\left(\frac{\ell}{u}\right)^{1-\alpha}$$

The insider exploits this to his advantage and, as q reduces, he progressively moves probability mass onto u, the upper bound of the interval of feasible bids. Formally:

$$G^{in}(i) \underset{q\to 0}{\to} 1-\frac{\ell}{i} \quad \text{and} \quad \mu \underset{q\to 0}{\to} \frac{\ell}{u}$$

The shareholding bank's informational advantage gives her the status of incumbent in the market for loans; incumbents have always the most to lose from an unsuccessful bid, and so place a higher value on winning. New entrants - in our setting, the outside bank - therefore calculate that, if they win the auction, they must have overpaid. Accordingly, they back off, allowing incumbents to win cheaply. In auction theory, this is what is called the "winner's curse" effect for common-value auctions. We will now show that changes in α can increase the insider's competitive advantage in a similar way.

Changes in α do not increase the probability of a borrower being detected as a bad one by the insider. Nevertheless, an increase in α increases the value that the insider can extract from a good borrower for any given level of interest rate bid. She can extract the same surplus in case she wins by bidding a lower interest rate. The outsider anticipates this increased competitive advantage and, in order to guarantee indifference of the insider between his possible bids, she needs to bid less aggressively. As already mentioned, equity stakes also guarantee the insider dividends even in case she loses the competition, and this might induce her to bid less aggressively and the outsider to bid more aggressively. Nevertheless, in equilibrium the disciplinary role of equity prevails and as α increases, the probability with which the outsider refuses finance increases and the the distance between $G^{out}(i)$ and $G^{in}(i)$ gets larger for any given $i \in [\ell, u)$. On the contrary, as α tends to zero we have:

$$G^{out}(i) \underset{\alpha\to 0}{\to} G^{in}(i)$$

In other words, both reductions in q and in α tend to have similar effects on the behaviour of the outsider in equilibrium. If either the informational advantage or the size of the equity stake held by the insider reduces, the outsider tends to participate to the bidding game with a higher probability. On the contrary, since changes in α do not affect the outsider's expected pay-off function, they do not affect the insider's optimal bidding strategy.

The effect of this behaviour on the equilibrium switching probability is that, for a given size of the equity share α, as the informational advantage of the insider gets more significant, the probability of a switch to outside sources of finance decreases. In fact, it can be easily shown that $\partial\sigma(q,\alpha)/\partial(1-q) < 0$ for any α. Similarly, for any given level of information differential q, the switching probability decreases with α, i.e. $\partial\sigma(q,\alpha)/\partial\alpha < 0$ for any q.

The impact of changes in α and q on expected interest rates for good borrowers just mimicks those occurring in the switching probability but with opposite sign. So, we can easily verify that $\partial i^e(q,\alpha)/\partial(1-q) > 0$ for any α, and $\partial i^e(\alpha,q)/\partial\alpha > 0$ for any q. In other words, the expected cost of a loan for good borrowers is increasing in the informational advantage of the insider and in the size of his equity stake.

These results are consistent with the empirical evidence on the phenomenon of bank equity stakes acquisition available for the Italian case (see Bianco and Chiri (1997)). Indeed, there seems to be a positive correlation between the presence of a bank in the ownership structure of a firm and the tendency of the firm to concentrate its credit relationships around the shareholding bank and to pay higher interest rates on loans.

In summary, in our model, both an increment in the precision of information and an increment in the entity of dividend payments would increase the equilibrium price paid by the borrower by increasing the equity-holding bank's competitive advantage.

We can now look at how α and q interact with each other in affecting the players' equilibrium behaviour. For simplicity, we look at the impact of changes in α and q on the switching probability $\sigma(\alpha,p)$, where $p = 1-q$.

The mixed derivative of $\sigma(\alpha,p)$ with respect to α and p is given by the following expression:

$$\frac{\partial\sigma(\alpha,p)}{\partial\alpha\partial p} = \mu^{1-\alpha}\left[1+(1-\alpha)\ln\mu\right]\frac{d\mu}{dp} \tag{2.4.3}$$

where $\mu = (\ell p)/(u-\ell(1-p))$ is the probability with which the insider bids the highest possible interest rate u, and $\mu^{1-\alpha}$ is the probability with which the outsider refuses finance in equilibrium for given levels of α and p. The mixed derivative is non-negative as far as the term in square brackets in (2.4.3) is non-negative, and this occurs for values of α and p such that:

$$\alpha \geq 1 + (\ln\mu)^{-1} \equiv f(p;\ell)$$

where $f(p;\ell)$ is a monotonically decreasing function of p such $\lim_{p\to 0} f(p;\ell) = 1$. Moreover, as the degree of adverse selection in the industry gets larger (ℓ increases), the locus of points $\alpha = f(p;\ell)$ rotates downwards. The way dividend and information effect interact with each other depends on the the relative size of the information advantage (p), the size of the equity stake (α), and the degree of adverse selection in the market (ℓ). The reduction in the switching probability resulting from a marginal increase in p is larger the larger is α (the *information effect* and the *dividend effect* complement and reinforce each other) only for values of α below $f(p;\ell)$. For values of α above this function, the decrement in the switching probability is larger the smaller is α (the two effects are somewhat substitutes). If the degree of adverse selection in the industry gets worse or, equivalently, the interest rate ℓ increases, then the locus $\alpha = f(p;\ell)$ rotates downward, so that the set of values (α, p) for which the mixed derivative (2.4.3) is positive gets larger.

The interpretation of the above result could be the following. For a fixed level of adverse selection - which determines the level of ℓ - if α is relatively high, the equity stake allows the insider to extract a large part of the surplus of good firms even when she charges small interest rates on loans, and this occurs regardless of how precise her information technology is. In other words, when bidding on the loan contract, she is mainly interested in maximising the probability of winning the loan contract, rather than securing the highest possible income through interest payments. The return (in terms of lower switching probability) from acquiring information is therefore relatively small, as the insider does not need to increase the outsider's "winner's curse" to keep him out of the competition. As α gets smaller, though, the outsider anticipates that the insider becomes more concerned about securing a sufficient level of return on loans as well, and that she might try to bid less aggressively. Acquiring information might therefore work as an effective entry deterrent for the insider when α is relatively smaller. Nevertheless, if α gets too small, dividends play more the role of a safe source of income the insider receives regardless of her winning the competition on loan pricing. This may induce a less aggressive behaviour on the part of the insider, who may try to bid high to maximise the revenue in case of win even at the expense of a higher probability of losing the game. Under this circumstance, the return from acquiring information might need a larger stake to be more effective in making the outsider less aggressive. As the degree of adverse selection worsens, the return from acquiring information may be high enough to make the "insurance" effect of equity irrelevant, and this would explain why for a larger set of values (α, p) we would have returns to information declining with α.

2.5 Concluding remarks

The analysis carried out in this paper showed that differences in the nature of banks' financial claims, if combined with incomplete information, may distort the loan pricing competition in credit markets. When a bank holds equity stakes in a firm who is applying for a loan, the bank's competitive advantage derives not only from her superior knowledge of the firm's future prospects but also from the possibility to extract surplus from the firm via dividend payments. The firm ends up being partially "captured" by the shareholding bank so that she expects to pay interest rates on new loans which are increasing not only in the shareholding bank's *informational advantage*, but also in the *size* of the equity stake she holds. Correspondingly, the equity-holding bank expects to lose good borrowers to outside competitors with a probability which is decreasing in her informational advantage and in the size of her equity share.

We also provided a few preliminary insights on the nature of the interaction between the "information" and the "dividend" effect in affecting the switching probability. For instance, we found that these effects reinforce each other only if the equity stake and the precision of information held by the inside bank are not too high. Otherwise, the two effects can be seen as substitutes in affecting the insider monopoly power, especially when the degree of adverse selection in the industry is particularly severe.

The model we have examined can be easily extended to allow for alternative forms of pay-off asymmetries between competing banks. For example, the shareholding bank might want to sell its equity share on the capital market at some point in the future and cash capital gains rather than dividends. Winning the competition on loan pricing in early stages could be important, as being a debt-financier at the time of the sale could deliver a good signal to the market and yield higher sale revenues. Alternatively, one may think of any future cash flow accruing to the informed bank as a consequence of its relationship with the firm, and whose value could be affected by the outcome of the competition on loan pricing today. For example, the banks might be competing for the sale of an "information-intensive" service to the firm that could make the monitoring or screening activity on the borrower either cheaper or more precise. The informed bank's expected value of any existing contract with the firm might therefore be increased by the possibility of winning loan-pricing competition today. As far as the outside uninformed bank perceives that the value that the insider attaches to "winning" is somewhat increased by this "link", the competitive advantage of the inside bank will increase.

The analysis carried out in this paper therefore suggests that the monopoly power of an informed bank in credit markets can be reinforced by the possibility of extracting surplus from the firm through alternative channels. The firm might be captured not only as a consequence of information asymmetries, but also as a consequence of differences in the claims the competing banks hold on the surplus produced by its activities.

The limits of this analysis are mainly related to the fact that we have abstracted from the circumstances that might have induced the firm to offer an equity contract to a bank (or the bank to acquire an equity stake in the firm). An interesting extension would be to allow a pre-bidding stage in which the borrower invites the banks to compete for the acquisition of an equity stake (and associated informational advantage) in the firm. Banks anticipate that the stake can give a competitive advantage in the sub-sequent loan-pricing competition and may be willing to pay a higher price for it. As a result, the overall cost of capital may not increase. If the "price" paid for the equity claims on the firm's future surplus is expressed in terms of the size of the equity share required by banks in equilibrium, the higher interest rates paid by the borrower on loans would be rebated in the form of a smaller share sold ex-ante to the banks. And this could be particularly advantageous for small, family-owned businesses, for instance, that find particularly costly to give up control rights on their business.

Finally, a proper analysis of the interaction between the dividend and the information effect could provide useful insights in explaining how firms decide on the optimal allocation of financial claims (equity and/or debt) and related information rights among their investors. Firms usually have preferences over different allocations of equity and information rights, simply because, as already mentioned, giving up equity rights might be more expensive (in terms of loss of control) than revealing information on the firm's prospects. If dividends and information may have a similar effect on lenders' monopoly power, a firm operating in a highly adverse selected industry and in need of equity to survive, the combination of a small share and substantial information could represent a convenient way to sell banks a competitive advantage in credit markets.

Chapter 3

Who Benefits from Credit Subsidies?

3.1 Introduction

Credit subsidies are widespread worldwide. In the United States the federal government runs over 300 programs meant to provide credit aid to various groups - including households and firms, particularly small businesses - under various forms (provision of loan guarantees, direct lending, interest rate subsidies). Over the 1980-1987 period federal lending programs extended over $1,200 billion of net subsidised credit and accounts for about a third of total credit market debt (Gale, (1991)). Between 1995 and 1999 the European Community budget for credit programs was as large as 12,7 billion Euro and financed as many as 55,000 small businesses[1]. The present paper studies the case of southern Italy, where credit subsidies have been the main instrument of choice for industrial policy since the 1950s, even though their incidence reduced progressively over the years[2]. A large and increasing proportion (35% in 1996, 41% in 2000[3]) of total subsidised credit has been extended by banks under a variety of regional and national schemes to sustain growth in distressed regions like Basilicata, Molise, Sardinia and Sicily. Sub-

[1]Source: European Commission Bulletin 18-10-2000 Com (2000) 653.

[2]The percentage of subsidised loans on total loans extended by banks was 14% in 1974 and shrank to 4% in 2000 (De Bonis, Piazza and Tedeschi (2001)).

[3]De Bonis, Piazza and Tedeschi (2001).

sidised credit has taken here mainly two forms, direct loans and interest subsidies. This paper focuses on the latter form of intervention, which has been used as an instrument of industrial policy also by the European Commission in the aftermath of the deep recession that characterised the early 1990s[4].

Credit subsidies find their theoretical justification in the presence of credit market imperfections. In loan markets, potential borrowers may retain private information on the riskiness of their project after banks have evaluated their loan applications. When banks set interest rates on loans they anticipate that the interest rate itself might affect the riskiness of the pool of loans because of adverse selection or moral hazard problems. Borrowers who are willing to pay higher interest rates may be, on average, more risky: they may be willing to pay higher rates just because they know they have a low probability of repaying the loan. In addition, a higher interest rate may induce a borrower to undertake projects with lower probability of success but higher return when successful. The existence of a negative correlation between the interest rate and the likelihood of repayment implies that, in equilibrium, the bank may set an interest rate that does not clear the market and credit rationing may occur (Stiglitz and Weiss (1981)).

The target of lending programs are indeed usually businesses, households or individuals who belong to highly adverse selected categories of borrowers (e.g. small businesses, students) that are more likely to face difficulties in obtaining bank credit to finance their activities. Subsidised credit is generally provided under easier terms, such as reduced interest, reduced collateral or longer maturity compared to non-subsidised credit. Interest subsidies, in particular, are designed in such a way that the fraction of interests actually paid by the borrower on a subsidised loan is lower than that computed at market rates. In a frictionless and perfectly competitive capital market, the effect of a dollar interest

[4]During the period 1994-1998, the European Investment Bank (EIB) - under its temporary lending facility (SME Facility) - extended about 92.3 million ECU to small and medium-sized enterprises in the form of interest subsidies on loans. The general feeling of uncertainty of the time obliged credit institutions to be more selective in the granting of loans and to provide themselves with high interest spreads. The measure was motivated by the fear that firms could be "forced to give up a number of investment projects which would offer insufficient returns in view of the [high] interest rate" (Commission of the European Community, Brussels 22/06/2000).

subsidy translates into a dollar saving in interest payments for borrowers. In other words, the surplus generated by the subsidy should in principle be entirely rebated to the borrower in the form of lower average interest rates. By allowing targeted borrowers to pay lower interest rates on loans, interest subsidies should attract those good borrowers that were discouraged to demand loans by the high rates, thereby raising the probability of repayment of the target group. Consequently, if a target group is credit rationed, easier terms on interest payments should exert their effect by releasing the rationing constraint.

Concerns have been raised about the effectiveness of credit subsidies as a way to promote investment and growth (De Meza and Webb (1987), Gale (1989), Gale (1990), Hoff and Stiglitz (1997)). The general line of argument in most of this theoretical work is that the very same informational frictions that justify public intervention in the form of subsidies can compromise their effectiveness. Gale (1990), for instance, considers a model where all agents take prices as given and borrowers retain private information concerning their ability to repay loans. He examines the impact of alternative credit policies in different market regimes of the target group (market clearing, rationing or redlining) and shows that interest subsidies can be ineffective because they do not alter banks' return on lending to the target group and therefore their willingness to lend. In a market-clearing equilibrium, interest subsidies raise the targeted group's probability of repayment and loan demand, crowd out general borrowers - as subsidies require funding which reduces the available supply of funds - and increase targeted investment. If the target group is credit-rationed or red-lined, subsidies fail to increase the bank's return to lending, and no increase in the bank's effective demand for loans to the targeted group occurs.

In spite of the importance assigned to credit programs by policy makers, as revealed by the documented significant amount of resources that they absorb, credit subsidies have received little attention among applied economists. The only exception is Gale (1991) who studies the economic and welfare effects of the US federal government credit programs. He develops a simple theoretical set up extending the Stiglitz and Weiss (1981) framework to a situation where lenders (including the government) can sort borrowers into various risk groups - some of which are targeted by the credit programs. He parameterizes the model and simulates the effects of various credit policies. He concludes that "most direct welfare gains appear to accrue to borrowers who would have received credit without subsidies. These subsidies represent pure windfall gains for the recipient, with no obvious social benefit" (p.135). While interesting, these simulations can only be suggestive and cannot be taken as evidence.

In this paper we provide some empirical evidence on the effectiveness of interest subsidies on a panel of firms operating in southern Italy during the period 1995-1998, when subsidies were a pervasive phenomenon. Our analysis is made possible by a unique dataset containing information on all loans extended by a bank to its corporate customers. The dataset provides details of the terms of the loan contracts - interest rates, size of loans, contractual forms - and allows us to distinguish between subsidised and non-subsidised loans.

We first analyse whether interest rates subsidies stimulate investment, as, from a policy perspective, this should be the ultimate goal of these programs. We look at the impact of subsidies on the total amount of credit granted to subsidised firms. Credit subsidies foster corporate investments if an overall increase in the amount of loans received by the subsidised firm can be observed. If instead subsidised loans replace one-to-one pre-existing loans, the effect of the subsidy is simply to increase the profit of the coalition bank-borrower, with very limited effect on investment. We find evidence in line with the latter argument: recipient firms tend, on average, to substitute non-subsidised loans with subsidised loans, with no significant difference in the overall amount of loans. One dollar increase in subsidised loans translates into an increase in total loans of only 30-40%. What we observe is a substantial substitution between subsidised and non-subsidised loans for those borrowers who already receive finance for their projects, and for which subsidies are a relatively cheaper way to finance their current activities. We also observe considerable substitution for those borrowers who use their loans in excess of what they have been granted. These are probably riskier borrowers, who indeed resort to overdrafts to finance their activities despite their higher cost. Moreover, persistent overdrafts eventually lead to default. Subsidies, for these borrowers, are a convenient way to normalise the use of their accounts and avoid default on loan repayments. This has the worrying implication that subsidies may increase the riskiness of the *healthy* portfolio of the bank. This is consistent with the results by De Bonis, Piazza and Tedeschi (2000), who provide evidence on the existence of a positive correlation between credit subsidies and riskiness of Italian banks' assets over the period 1984-1996.

The evidence on the impact of subsidies on quantities lent shows that subsidies are costly as they fail to address the intended beneficiaries of the programs, and they represent a pure windfall for the coalition formed by the bank and the recipient firms. In order to assess who is appropriating the surplus generated by subsidies, we study whether they reduce the overall cost of loans for recipient firms. Interest rates actually paid by borrowers on subsidised loans are lower than those computed at market rates. Nevertheless, subsidies can only be extended on loans of limited size, and borrowers might therefore need additional non-subsidised finance to cover all their financial needs. Interest rates on these loans are freely negotiated on the market and, if the bank managing the subsidy has considerable monopoly power on her clients, she can seize great part of the surplus generated by the subsidy by charging higher interest rates on non-subsidised loans.

Interestingly, we find that firms with subsidies pay higher rates on non-subsidised loans compared to non-subsidised firms once individual effects have been controlled for. Moreover, a dynamic analysis confirms that just after a firm obtains a subsidised loan, the interest charged on non subsidised loans increases while it goes down as a subsidised loan is paid off. The presence of price spillovers implies that the bank managing the subsidies is probably appropriating of that part of the surplus generated by the subsidy aimed at benefiting the borrower. The extent of appropriation is limited by the market interest rate, which reflects the imperfect information on the creditworthiness of the firm, the monopoly power of the bank, switching costs the firm bears when moving away from her preferred lender. We indeed find evidence that the "spillover effect" worsens for borrowers that have most of their contractual relationships in local markets where the bank is a monopolist and for those who are more informationally captured by the bank. In other words, the spillover effect reflects market imperfections and imperfect competition in the market for loans.

The empirical results show that, if granted to the existing clients of a bank, subsidies may not promote growth and investment as firms tend to substitute existing non-subsidised loans with subsidised ones. To test for this hypothesis we were forced to limit our analysis to firms with at least one non-subsidised contractual relationship. This runs the risk of underestimating the impact of credit subsidies, since one of their potential benefits is to promote the initiation of new credit relationships to customers that were unable to borrow otherwise. Therefore, we separately study whether subsidies give rise to new bank-firm relationships. We find that 35% of new relationships are born with a subsidized loan, and that more than 90% of these relationships involve relatively small and young firms. Moreover, only 1% of these borrowers appear to have non-subsidised finance in the quarter following the first subsidy. For these firms, the spillover effect does not take place, and subsidies are indeed associated to savings in interest rates payments. This suggests that credit subsidies may have a nontrivial role in starting new relationship among those firms who probably find it more difficult to obtain external finance.

The paper is organized as follows. In section 3.2 we describe the allocation mechanism of interest subsidies. In section 3.3 we describe the dataset used. In section 3.4 we present our empirical results on the impact of subsidies on quantities loaned and on the rise of new relationships. In section 3.5 we first provide a description of the spillover mechanism using a simple model, and we then provide evidence which suggests that appropriation of surplus by the bank managing the subsidy takes place. Section 3.6 concludes the paper and discusses the policy implications of our results.

3.2 The allocation mechanism of interest subsidies

Beneficiaries of interest subsidies receive loans at basic interest rates which are paid partly by the subsidised borrower and partly by the subsidising agency according to split ratios that may vary across the various programs. The interest subsidy on loans extended by the European Investment Bank (EIB) under its temporary lending facility in the 1990s took the form of an interest rebate fixed at 200 base points (2%), and which was paid to firms which were already borrowers of EIB loans under easier terms that would otherwise be obtained in credit markets. In Italy, subsidised loans have been (and currently are) in great part granted at a conventional fixed interest rate, which is the same for all eligible borrowers, and which equals the bank's cost of funds plus a modest mark-up. The actual subsidy is the fraction of interest paid by the subsidising government agency, and this may change across programs. Under the terms specified by "Legge Sabatini" (n.1329, 28/11/65), for instance, the share of the conventional rate paid by the government agency is usually higher for recipients located in more distressed geographical areas. In order to address specific target groups, the law regulating the program defines eligibility requirements in terms of objective and verifiable characteristics of the beneficiary, like belonging to certain categories (small young businesses, households, students) or industries (agriculture, mining...).

The government body that sponsors the program relies on banks as intermediaries in the allocation of subsidised funds. Banks are endowed with screening technologies that may sort out borrowers who are most in need of financial support among all those belonging to a certain targeted group. In order to obtain a given form of subsidy the applicant firm needs to provide the subsidising government agency with evidence that they have successfully passed the screening process of a bank authorised to provide that form of subsidy. To limit collusion between banks and bad firms, the default risk on the repayment of the principal and the borrower's portion of interests is entirely born by the bank managing the subsidy.

Authorisation to supply subsidised funds is granted through different mechanisms according to whether the subsidising body is the central national government or local government authorities. All banks are automatically entitled to provide subsidised finance under national schemes. Nevertheless, banks need to apply to local authorities to obtain an official authorisation to intermediate in the allocation subsidies regulated by these authorities. The public body that provides the majority of subsidies extended to the clients of the bank that we analyse in this work is the Regional Authority. Great part of the funds are provided by the European Community, and what the Regional Authority does is to decide which categories of individuals or firms to target and to which banks to grant the licences. The criterion generally adopted is market share. The rationale behind this criterion might be that banks with a larger market share can reach a larger fraction of targeted firms and, presumably, allocate funds more efficiently.

The mechanism used to allocate authorisations to supply subsidies can give certain banks market power in the provision of certain forms of subsidised finance. In equilibrium, banks may even find it profitable to specialise in certain forms of services and products by applying for certain licences and not for others. This means that subsidies provided by competing banks may not be perfect substitutes for borrowers. The latter might therefore face switching costs when moving away from their preferred bank and banks may turn this into their advantage.

3.3 The data

We base our analysis on an extremely rich and, in several respects, unique database obtained from a medium sized bank operating in Italy with a strong geographical concentration. For obvious confidentiality reasons we do not report its name; for brevity we will refer to it as "the Bank". The Bank has over 2,000 employees and extends loans to about 50,000 clients. A large fraction of its customers are small, as inferred by the fact that only 20 percent of them receive loans in excess of the threshold for reporting information the Credit Register (Sample of CR firms in Table B.1). The Bank operates some

400 branches, mainly in local markets (over 330 municipalities), where - at least over the span of time under analysis - she has considerable market power. She appears to be a monopolist in 70% of the municipalities in which she operates. The Bank has considerable market power in the provision of subsidised finance, too. In the geographical area under analysis, between 1994 and 1998, the group the Bank belongs to extended more than 47% (38% was the Bank's share) of loans subsidised by local authorities against 50% and 1.5% extended by its main competitors. The share of total interest subsidies distributed by the group on those loans was just over 40% (33% by the Bank) against 57% and 1% by the main competitors. By looking at the range of subsidised forms of finance the Bank and her main competitor provide, we can say that the former is basically monopolist in the provision of subsidised finance to agricultural firms while the latter specialises in the provision of subsidised finance to manufacturing firms. The banks compete on commercial firms and artisans, but the Bank provides a much wider range of products to these categories of firms.

Data are quarterly snapshots of the population of firms that are funded by the Bank and cover the period 1995, third quarter, 1999, second quarter[5]. For each firm we know a set of demographic characteristics (legal form, location, location of the loan etc.); for the subset of incorporated firms we also have information on various items in their balance sheet. For each loan the firm obtains from the Bank, the information includes the amount granted and actually used (for credit lines), the type of the loan (whether it is a mortgage, a credit line, a discount window etc.), the interest rate charged and the interest rate actually paid. We also know whether collateral has been pledged and the type of the collateral, the date the loan was first issued, when it was revised and, who, within the Bank approved the loan and who is in charge of it. Furthermore, for each customer the Bank computes a score obtained weighting various pieces of information relating to the firm performance as a client of the Bank. The score - which we term

[5]Needless to say, the data we have access to do not enable identification of the borrowers, who remain anonymous.

"internal score" - is a summary measure of the information on the firm that is specific to the Bank and provides a useful control for firm's quality as perceived by the Bank. Finally, and most importantly for the purpose of this paper, for each loan, we know whether it is a subsidised loan and the extent of the subsidy, measured as the share of interest paid by the subsidising body.

As all other banks, the Bank reports information to and receives some from the Credit Register (CR form now on). The Register is a device administered by the Bank of Italy whose purpose is to allow information-sharing among lenders on the credit quality of clients who borrow more than 150 million Liras ($ 75,000). Regardless of the amount borrowed, all debtors in default are reported to the CR. Each bank obtains from the CR information on the overall financial exposure of its clients with the banking system, the number of lending institutions and indicators of distressed loans. We supplement these data with information on the market share of the Bank in its local markets, identified with the individual municipalities in which the Bank operates.

Table B.1 in Appendix B reports summary information about the sample, separately for the whole sample and for the sample of firms that are reported to the Credit Register. Computations use the cross-section 1999/II. Over 50% percent of the Bank's customers have at least one subsidised loan showing that credit subsidies are pervasive. This partly reflects the geographical location of the Bank, which operates in the South of Italy, where subsidies are particularly widespread, and partly the fact that the Bank enjoys monopoly power in the provision of subsidised finance. Subsidised borrowers have, on average, a number of 1.4 subsidised loans each. Firms reported to the CR have on average 2.07 subsidised loans. The average nominal interest rate on subsidised loans is on average 1.5 (3% for CR sample) percentage points higher than the average rate on non subsidised loans. Among subsidised loans the entity of the subsidy, as measured by the fraction of interest rate paid by the subsidising body, is on average equal to 70% and ranges from a minimum of 38% to a maximum of 1%. Differences may reflect variation in the generosity of the various subsidy schemes as well as heterogeneity across firms in the eligibility criteria to the various programs.

The table also reports measures of borrower quality. The first is an internal score computed combining all relevant information on the client's relation with the bank. It weights several indicators of borrower's behavior such as regularity in repayments, the frequency and persistence of overdrafts on loans. The score varies between 0 and 100, with values of the score closer to 100 signalling firms in situation of dramatic distress and values closer to 0 identifying essentially healthy firms. More than 50% of firms are overdrafting, signalling that a great part of the Bank's clients are using their loans abnormally. Nevertheless, a smaller percentage (40%) of the clients reported to the CR are overdrafting.

3.4 The welfare impact of subsidies

3.4.1 Effect of subsidies on quantities

We first look at the impact of subsidies on total quantities loaned to a target firm. If subsidies relieve credit constraints, we should observe that the total quantities loaned to a firm increase as a result of the subsidy. If instead we find that subsidies hardly affect total quantities we take this as evidence of a substitution effect between subsidised and non-subsidised quantities. From a welfaristic point of view, subsidies would be ineffective, as they would tend to finance existing investment rather than new ones.

Total loans for each firm are scaled by total sales. The null hypothesis of "no effect on quantities" is that scaled total quantities remain unaffected by a firm being beneficiary of a subsidy. This test has an important methodological advantage since it is robust to the criticism of reverse causality, which applies to the tests based on the correlation between the (existence and size of the) subsidy and the subsequent performance of the firm.

Table B.2 reports results on estimations computed using annual data over the period 1995/III-1999/II. We first use the whole sample of firms (Total sample) and we then exclude those borrowers receiving only subsidised loans (Reduced sample), as these would bias the results towards finding "no substitution". In panel A, column 1 and 3, the dependent variable is the total amount of outstanding loans in each quarter (divided by annual sales) received by all banks if the borrower is reported to the Register, and by the Bank if he is not. The explanatory variables are the value of quarterly outstanding subsidised loans (normalised by annual sales), time dummies and firm fixed effects. We find that a unit increment in total normalised subsidy increases total normalised quantities by 0.52 units. Under the null hypothesis of "no effect on quantities", this coefficient is zero. As expected, we find a lower impact on quantities when we exclude borrowers with only subsidised loans (0.45). Column 2 and 4 report results on the same regression but using as dependent variable the total amount of quarterly outstanding loans (scaled by annual sales) obtained by the Bank. A higher impact on quantities as revealed by the results suggests that subsidised borrowers tend, on average, to substitute subsidised funds provided by the Bank with funds provided by *other* banks. Nevertheless, these results might underestimate this substitution effect as, for firms not reported to the CR, the dependent variable might not represent their total indebtedness to the banking system. In panel B, we therefore run the same regression restricting the attention to the sub-sample of firms reported to the Register and for which we have the information on the *total* indebtedness of each firm. In panel B, the dependent variable is now the total amount of quarterly outstanding loans received by the borrower from *all* banks. We find that, in this case, the impact on total quantities reduces to a coefficient of 0.44. When we exclude borrowers with only subsidised loans, we find that a unit increment in total normalised subsidies increases total normalised quantities by a mere 0.36 units. Note that the substitution effect on the Bank's loan is slightly larger for firms reported to the CR.

In table B.3 we try to decompose the impact on quantities by taking a dynamic perspective. The first column refers to the whole sample, the last consider the sub-sample of firms reported to the Register. The dependent variable is the change in total loans between time t and $t-1$ over time $t-1$ sales. The explanatory variables are calendar year dummies and firm random effects. We also include two "dummies": *dummy* 1 equals the change in subsidised loans between t and $t-1$ (scaled by annual sales at $t-1$) when positive and zero otherwise; *dummy* 2 equals the change in subsidised loans between t and $t-1$ (normalized by annual sales at $t-1$) when negative and zero otherwise. We find that a unit increment (decrement) in total subsidies from time t and $t-1$ accounts for 0.14 (0.57) units of the increment (decrement) in total quantities between the same interval of time. When restricting attention to the sub-sample of firms reported to the Register, the impact of subsidies reduces to 0.05 and -0.52 respectively. These results provide additional evidence that subsidies tend to replace existing non-subsidised loans and have a very small impact on total quantities. Table B.3 also shows that the substitution effect when a new subsidy is granted is much larger than the one associated to a subsidy being paid off. This is probably due to the fact that subsidies are often used by firms with overdrafts to normalise their accounts. We therefore verify this conjecture by running the same static regression as in table B.2 separately on firms with overdrafts and those with no overdrafts. If we look at the sub-sample of firms reported to the CR we indeed find that firms with overdrafts substitute more than firms with no overdrafts (.30 against .39).

3.4.2 New relationships

In order to estimate the extent of the substitution effect, we had to concentrate on subsidised borrowers with at least another non-subsidised contractual relationship. This runs the risk of underestimating the impact of credit subsidies, since one of their potential benefits should be to foster the initiation of new credit relationships to customers that were unable to borrow otherwise. We document the rise of new relationships year by year, and we find out how many of the new relations are born thanks to a subsidised loan. We also investigate who are the firms that start a new relation with a subsidy and compare their characteristics with those of the firms who started without. If subsidies are meant to relieve credit constraints we should find that these firms are younger, smaller, minority-owned.

In table B.5 and B.6 we report the number of new bank-firm relationships born quarter by quarter from 1995/III to 1996/II. We compute how many of these new relationships start with a subsidised loan[6] and how many started with non-subsidised loans only. On average, 35% of new relationships are born with a subsidy. At least 94% of these relationships involve firms not reported to the Credit Register, and which are supposedly relatively small, as they have a very modest indebtedness towards the Bank. This is probably the reason why information on their characteristics as reported in table B.7 is available only for few individuals. We look at the age of the firm (or owner), their legal form and some balance sheet information (sales, leverage). The data confirms that subsidies might indeed relieve credit constraints: firms born with a subsidy are in great part either individual firms or partnerships. Even though they are not on average younger (13.04 years of age compared to 11.52 years for the control sample), they are nevertheless smaller (average sales of 455.33 million Liras compared to 9,132.17 of the control sample) and they have a slightly higher leverage (0.33 compared to 0.30). Also, we find that more than 56% of female-owned firms start their relationship with the Bank with a subsidy. Interestingly, we also find that only 1% of borrowers who start a relationship with the bank with a subsidy at time t have non-subsidised loans at $t+1$. In other words, these firms can easily cover their financial needs with only subsidised finance and they probably expect to fully benefit of interest rates savings as a result of the subsidy.

3.5 The distributional impact of subsidies

3.5.1 The pricing of non-subsidised loans

In what follows we consider a simplified spatial competition framework to model the pricing behaviour of banks in the presence of interest subsidies. A spatial model is a

[6] A client is defined as "new" if it was not in the dataset for at least four quarters.

convenient and simple way to model market power and imperfect competition among the intermediaries and it allows us to view banks as competing on interest rates rather than on quantities.

The objective is to describe the spillover mechanism and identify the relevant variables that might affect its size. We only consider the case in which subsidies are granted in markets where equilibrium prices clear the market even without subsidies, as we think this is the category of firms subsidies reach. We are aware that a proper analysis of the problem would require investigation of this effect also when the market is not cleared, but we leave this as the object of future research.

We adapt the model developed in Dell'Ariccia (2001) and, for simplicity, we consider a risk-neutral world with only two banks, which we assume to be located at the two extremes of a linear city of length 1 rather than on a circle, as we are not concerned with entry issues. We assume that banks specialise in the provision of different forms of finance, including subsidised one, and that a population of firms is uniformely distributed along the line. The location of each firm measures their preference for the different services the banks provide.

Each firm needs a loan of size D to finance its investment activities and she can approach any of the banks operating in the market. Each bank has unlimited access to the money market at the constant interest rate C. When choosing with which bank to contract a loan, firms take into account the interest rate they charge and the unit transportation cost τ of moving away from their preferred bank.

Each bank i can provide a proportion σ_i of D at a nominal interest rate $r_s > C$. All firms in the population are eligible to receive some form of subsidised finance, in that they possess the objective characteristics that define eligibility to interest subsidy programs. Subsidised borrowers will pay only a fraction ϖ of r_s, the residual fraction being paid by the relevant government agency sponsoring the program. The parameters σ_i, ϖ and r_s are exogenously given and specified by the credit subsidy program so that banks can only compete on interest rates to be charged on the non-subsidised portion of each loan. We also make the crucial assumption that borrowers cannot split their contractual relationships with more than one bank as this would be too costly. This means that firms cannot borrow the subsidised portion of the loan from one bank and the non-subsidised one from another. This is consistent with the characteristics of the firms in our dataset, which are relatively small - especially if subsidised - and tend to concentrate their banking relations at one bank.

A fraction λ of the population of firms consists of "new" borrowers, that is firms who are applying for credit for the first time. The remaining fraction consists of the "old" clients of the two banks, equally split between them. Each group of borrowers (new and old) consists of a share θ of good types and a share $(1-\theta)$ of bad types. Bad types invest in projects which are doomed to fail with probability one, while good ones invest in projects which yield a return of G with probability $p \in (0,1)$. In order to capture the sorting role played by interest rates in the presence of adverse selection, we assume that good borrowers and borrowers who do not know their type will demand the amount D only if they expect to make non-negative profits. Bad borrowers, instead, will accept to borrow at any interest rate. All firms know their location along the line but they only get to know their type when they are old. We also assume that banks hold *proprietory* information about their old clients. They know type and location of their old borrowers, but they can't distinguish bad from good among new borrowers or their competitors' old customers from new borrowers.

The pricing game between banks takes place in two stages. In stage one, banks compete in a Nash fashion over the interest rate for new borrowers and old bad borrowers rejected by competitor banks. In stage two, banks observe the interest rate charged by competitors to new borrowers and offer their old good clients a rate to keep them from switching to the competition. Entrepreneurs move last and choose to sign the best contract on offer.

Our objective is to determine under what conditions the nominal interest rate charged on the non-subsidised part of subsidised borrowers' loan $(1-\sigma)D$ is higher than the interest rate that non-subsidised firms with similar characteristics would pay on the entire loan D.

We start by considering the benchmark case in which banks do not provide subsidised finance ($\sigma_i = 0$, $i = 1, 2$). Consider stage two and the contracting between bank i and her old clients. We denote with r_i^o and r_i the interest rate charged by bank i on old and new borrowers, respectively. Since rates on new borrowers are decided in stage one, bank i takes r_{-i} as given when maximising his expected profits from his known borrowers. From now on we take Bank 1's point of view. Bank 1 refuses finance to her old bad borrowers with probability one. An old good borrower located at x earns zero expected profits when charged the average interest rate:

$$r_{\max}^o(x) = [(pG - \tau x)/pD] - 1$$

Therefore, Bank 1 will charge old borrowers an interest rate:

$$r_1^o(x) = \min(r_2 + (\tau/pD)(1 - 2x), r_{\max}^o(x))$$

We can now compute the symmetric Nash Equilibrium interest rate r^* charged by banks on new borrowers in stage 1. We make the simplifying assumption that each bank wants to break even on each market segment, so that bank i's best response to the opponent bidding an interest rate r_{-i} would be given by the maximisation of expected profits made on new borrowers only. Again, let us take Bank 1's point of view. On the free market there are $(\lambda/2)(1-\theta)$ old bad borrowers rejected by bank 2, on which bank 1 is going to make a loss of $D(1 + C)$ whatever the interest rate charged. There are also $(1 - \lambda)$ new borrowers, who don't know their type but know their location. We assume that the good project cash flow, G, and the proportion θ of good borrowers in the industry are high enough to guarantee the existence, even without subsidies, of a symmetric equilibrium in which banks break even on new borrowers by charging an interest rate r^* which covers the market:

$$r^* \leq \frac{p\theta G - \tau}{p\theta D} - 1 = r_{\max}$$

A new borrower located at x will accept finance from bank 1 at an interest rate r_1 only if:

$$p\theta(G - D(1 + r_1)) - \tau x \leq p\theta(G - D(1 + r_2)) - \tau(1 - x)$$

Each bank's demand for loans at the interest rate r_i given r_{-i} would therefore be given by:

$$s_i(r_i, r_{-i}) = (1 - \lambda)[\frac{1}{2} + \frac{p\theta D}{2\tau}(r_{-i} - r_i)]$$

It follows that the equilibrium interest that maximises each bank's profit on the free market is given by:

$$r^* = \min(\frac{\tau - D(1 + C)}{p\theta D} - 1, r_{\max}) \tag{3.3.1}$$

By using the above expression we can compute the interest rate paid by bank 1's old customers:

$$r_1^o(x) = \min(r^* + \frac{\tau(1 - 2x)}{pD}, r_{\max}^o(x)). \tag{3.3.2}$$

The rate $r_1^o(x)$ paid by non-subsidised borrowers is an increasing function of their idiosyncratic characteristics, captured by their location x (or $1-x$), and of the degree of adverse selection in the market, as reflected by the presence of θ at the denominator of r^*. In case we had considered a market with more than two banks, $r_1^o(x)$ would also be a decreasing function of the number of competing banks. This means that the banks extract surplus from their old borrowers exploiting their market power and their inside information on the quality and location of the borrower.

We now consider the case in which each bank provides an interest subsidy on a portion σ of subsidised finance on the total loan D. The effective (average) interest rate paid by an old or a new borrower on D is now given by $(1-\sigma)r_i + \sigma\varpi r_s$. In order to keep her old customers, bank 1 charges an interest rate:

$$r_1^{o,ns}(x) = \min(r_2 + \frac{\tau(1-2x)}{pD(1-\sigma)}, \frac{r_{\max}^o(x) - \sigma\varpi r^s}{1-\sigma})$$

Whether this interest rate is going to be larger than the interest rate (3.3.2) depends on the nature of the competition between banks in stage 1 and on the corresponding value taken by r_2 in equilibrium.

The possibility for banks to provide subsidised finance on new borrowers affects their expected profit function $\Pi_i(r_i, r_{-i})$ by changing the structure of both the gross return from good borrowers and the market share $s_i = (1/2 + p\theta D\,(1-\sigma)\,(r_{-i} - r_i)/2\tau)$ that they cover. We have:

$$\Pi_i(r_i, r_{-i}) = D\{[p\theta(1 + (1-\sigma)\,r_i + \sigma r^s) - (1+C)](1-\lambda)s_i - \frac{\lambda}{2}(1-\theta)(1+C)\}$$

The equilibrium interest rate on the free market for the non-subsidised part of the loan is:

$$r^{ns} = (r^* - \sigma r^s)/(1-\sigma) \tag{3.3.3}$$

and banks equally share the market. What expression (3.3.3) says is that, due to the symmetric competition between banks in the provision of subsidised finance, new borrowers totally appropriate the subsidy by paying an *average* interest rate $r^* - \sigma(1 - \varpi)r^s$ rather than r^*, as given by (3.3.1). This implies that also old firms will save an amount $\sigma(1-\varpi)r^s$ on interest payments compared to old non-subsidised borrowers:

$$r_1^{o,ns}(x) = \min((r^* + \frac{\tau(1-2x)}{pD} - \sigma r^s)/(1-\sigma), \frac{r_{\max}^o(x) - \sigma\varpi r^s}{1-\sigma}) \tag{3.3.4}$$

It follows that, if $r^s \geq r^* + \tau(1-2x)/pD$, i.e. if subsidies increase the bank's gross return on each unit of finance, we would observe $r_1^{o,ns}(x) \leq r_1^o(x)$. If instead $r^s < r^* + \tau(1-2x)/pD$, we observe a spillover on interest rates of non-subsidised loans which is *not* due to the appropriation of the subsidy by the bank - the whole subsidy is indeed rebated to the borrower - but to that fact that in equilibrium the bank needs to cash a total amount equal to $r^* + \tau(1-2x)/pD$ in order to break even.

The analysis carried out sofar shows that, if banks compete on the provision of subsidised finance to both new and old borrowers, then borrowers should appropriate all the surplus generated by the subsidy. We now examine the case in which banks compete on the provision of interest rates subsidies on old borrowers but not on new ones. This may occur, for instance, if the programs allow subsidies to be granted on pre-existing loans only, which is usually the case for subsidies sponsored by the European Union, for instance. In stage two, bank 1 and the representative old borrower renegotiate the terms of a loan of size D of which a portion σ can be subsidised. The interest rate charged on the non-subsidised portion of the loan would be:

$$r_1^{o,ns}(x) = (\min(r_2 + \frac{\tau(1-2x)}{pD}; r^o_{\max}(x)) - \sigma\varpi r^s)/(1-\sigma) \tag{3.3.5}$$

where $r_2 = r^*$, which implies that the average interest rate paid by old borrowers is equal to the interest rate that non-subsidised firms would pay. Old borrowers do not benefit from any savings in interest rate payments and the surplus $\sigma\,(1-\varpi)\,r^s$ is entirely seized by the bank. As a consequence of the appropriation, we would observe a spillover on $r_1^{o,ns}(x)$ of size $\sigma\min(r_1^o(x); r^o_{\max}(x)) - \varpi r^s)/(1-\sigma)$ as far as $\varpi r^s \leq r_1^o(x)$, i.e. as far as interest subsidies can potentially generate interest savings for the firm.

Another circumstance in which a spillover on non-subsidised loans may occur, as a result of surplus appropriation by the bank, is when only one of the two banks can provide interest subsidies to borrowers belonging to a certain industry. And this may be due, for instance, to the banks specialising in the provision of subsidies to some categories of borrowers and not to others. The interest rate $r_1^{o,ns}(x)$ would still be given by (3.3.5) but r_2 would be the outcome of an asymmetric competition between bank 1 and bank 2 in stage 1. If bank 2 cannot provide subsidies, bank 2's expected profits can only be affected by subsidies through changes in her market share. The equilibrium interest rates would be given by:

$$\begin{aligned}\tilde{r}_1^{ns} &= (r^* - \frac{2}{3}\sigma(1-\varpi)r^s)/(1-\sigma) \\ \tilde{r}_2 &= r^* - \frac{1}{3}\sigma(1-\varpi)r^s\end{aligned} \tag{3.3.6}$$

As a result of the asymmetric competition between banks on the provision of subsidised finance, only part of the subsidy is returned to new borrowers. This has obviously repercussion on the cost of finance for the captive borrowers of the two banks. In particular, bank 1's old borrowers will appropriate only one third of the surplus $\sigma(1-\varpi)r^s$. As a result, a spillover can be observed on interest rate $r_1^{o,ns}(x)$ depending on the relative size of r^s, ϖ and r^*. In particular, a spillover takes place if r^s is not too much larger than r^*, i.e. $r^s \simeq r^*$, or if ϖ - the portion of interest paid by the borrower - is relatively small.

In summary, if we can reasonably assume that $r^s \geq r^*$, which means that subsidies are granted only if they guarantee the bank a return at least as large as the market return, then a spillover on the price of subsidised borrowers' non-subsidised loans can only indicate that the bank is appropriating of part or all of the surplus generated by the subsidy. Therefore, in what follow, we take the existence of price spillovers as evidence that appropriation is actually taking place.

3.5.2 The effect of subsidies on interest rates

In order to verify whether appropriation of surplus by the Bank actually takes place, we look at the average interest rate subsidised firms pay on non-subsidised loans. In line with the above considerations, we should find that a positive variation in the size of the subsidy should translate in a positive variation in the cost of non-subsidised loans and that this "spillover effect" worsens when controlling for measures of the bank's monopoly power.

This part of the analysis is again conducted on the sample of all subsidised firms excluding those that receive only subsidised loans (Reduced sample). The variable to be explained is the average interest rate (or its variations) charged on the non-subsidised loans of a borrower. One of the variables that significantly affect interest rates is the firm's internal score, which controls for possible variations in the firm's creditworthiness during its relationship with the Bank. Since this information is not available for the entire sample, we are forced to restrict our attention to the quarterly snapshots of interest rates on all outstanding loans between 1998.III and 1999.II only. To control for unobservable individual characteristics, we introduce in all regressions firm specific effects. Quarterly dummies are also included. In order to test for significance of subsidies in affecting interest rates, we include proxies of the *size* of subsidies among the explanatory variables. If a redistribution of the surplus generated by the subsidy actually takes place, we should observe that the size of the subsidy affects the entity of the price spillover.

In table B.8, we compare the quarterly average contractual interest rate charged on non-subsidised loans of firms with no subsidies and that of firms with at least a subsidised loan. In column one, the variable used as a proxy for the size of the subsidy is the number of outstanding subsidised loans (contracts) of a firm. The estimated coefficient suggests that, on average, an additional subsidised loan determines an increment of 0.26 basis points in the interest rate of non-subsidised loans. This effect is highly statistically significant. In column two, we repeat the same exercise by replacing the indicator variable with the total amount of subsidised loans of a firm. Again, we find that a unit increment in the total amount of the subsidy increases interest rates by 0.0016 points.

The above results are indicative of a spillover effect of subsidies on the price of non-subsidised loans. In order to decompose this effect, in Table B.9 we consider the same sample and we study how variations in the number (or in the total amount) of subsidies granted between quarter t and quarter $t-1$ to a firm affects variations in the average interest rate on its non-subsidised loans. We run a random effects estimation on the following model specification: we consider variation of the firm's internal score between quarter t and quarter $t-1$, quarterly dummies and two indicator dummies. *Dummy* 1 takes a value equal to one if at least one new subsidy was granted between time t and time $t-1$ and zero otherwise. *Dummy* 2 takes a value equal to one if at least one new subsidy was paid off between time t and time $t-1$ and zero otherwise. We find that an increase in the number of subsidies of at least one unit determines an increment of 0.05 points (10% significance level) in the average non-subsidised interest rate while, whenever a subsidy is paid off, the interest rate decreases of 3.54 basis points (5% significance level). Again, this confirms that the surplus generated by the subsidy is at least partially appropriated by the Bank through higher interest rates on non-subsidised loans.

3.5.3 Sample splits by market concentration and by local market share

In table B.10 and B.11 we consider sample splits to verify whether the extent of the spillover gets larger whenever the Bank enjoys more relevant monopoly power.

In table B.10 we consider traditional measures of market power. Namely, the number of bank branches operating in the market where most of the contractual relationships of the borrower take place (first two columns), and the Bank's share of total credit granted in the market (last two columns). We distinguish between the case in which the Bank is the only bank in the market from the case in which she is not. These two measures split the sample in an asymmetric way, as we have information on the number of bank branches operating in a market but no information on whether they belong to the same bank or not.

We find that the extent of the spillover is much higher for borrowers that hold most of their contractual relationships in markets where the Bank is a monopolist. Under this circumstance, an additional subsidised loan increases the price of non-subsidised loans of 0.87 basis points more than in the case in which more than one bank operates in the market. The qualitative conclusions remain unaffected when considering the alternative proxy for market power, the Bank's market share (last two columns).

Unfortunately, at the time in which the regressions were run, no precise information was available on where the Bank was a monopolist as a provider of subsidised finance, so that we cannot provide evidence on whether specialisation can lead to appropriation.

3.5.4 Informational capture based sample splits

In order to verify whether informational monopoly may play a role too, we split the sample by using informational capture based proxies of market power. In the first two columns of table B.11, panel A, we compare the extent of the spillover for firms not reported to the CR and that for firms reported. The Bank shares information with competing banks on the latter category, and therefore we expect the Bank to benefit of a reduced informational monopoly on these borrowers. Indeed, the spillover effect for these borrowers is almost 1 point basis lower than for firms not reported to the CR (0.02 versus 0.95 respectively).

We obtain similar results when we isolate firms for whom the Bank is the only lender, or main bank (second two columns), and which should be more informationally captured. Computations have been done on the sub-sample of firms reported to the CR, as the information on the number of lenders of a borrower is available only for these firms. Results confirm that, when the Bank is the only lender of a borrower, the spillover effect is of 0.03 points while it becomes negative (-0.81) when the borrower has multiple banking credit relationships. This might be evidence that, if the Bank has less monopoly power, the surplus generated by the subsidy is actually at least partially rebated to the borrower. This is confirmed also by the results reported on the last two columns of panel B, where we split borrowers by size (measured by annual sales): larger firms obtain on average lower interest rates on private funds compared to those charged on non-subsidised borrowers.

When splitting by length of the bank-firm relationship we find that the extent of appropriation by the Bank is smaller for firms that have been clients for more than 8.5 years (the median). This result seems to contradict our conjecture that old customers are more informationally captured than new ones. In truth, the length of a bank-firm relationship is not only a signal of a borrower's good quality - he has successfully been borrowing money from the bank for many years - but it is also positively correlated with the age of the firm. If the length of a relationship can be observed by other banks, older customers should be less informationally captured than new ones. In other words, what the results may suggest, is that the relationship between length of lending relationship and informational capture may not be linear. A proper test of our conjecture should therefore consider a finer partition of the total sample.

3.6 Concluding Remarks

By using an exclusive dataset of bank-firm relationships, we were able to provide preliminary evidence on the effectiveness of interest-rate subsidy programs. We have shown that they can promote the rise of new bank-firm relationships with small, young firms, which are usually thought to suffer from credit rationing constraints. Nevertheless, we find that, on average, subsidies do not finance new investments when granted to the existing clients of the Bank. Subsidised loans replace almost one-to-one pre-existing loans, suggesting very limited effect on total quantities loaned and, consequently, on new investment.

We also provided evidence in support of our conjecture that the bank managing the subsidy appropriates at least part of the surplus generated by the subsidy, with supposedly very little benefit for the intended beneficiaries of the programs. Interest subsidies can only be extended on loans of a limited size, as specified by each program, so that often borrowers need to raise non-subsidised finance as well. The surplus generated by the subsidy in terms of lower interest rates on subsidised loans can therefore be appropriated by the Bank in the form of higher interest rates on non-subsidised loans. Our results indeed show that a subsidy generates a "spillover effect" on the price of a targeted borrower's non-subsidised loans, and that this spillover tends to be more severe whenever the bank has more market power or has informational monopoly on the target borrower.

In this paper we have provided evidence that appropriation by the bank managing the subsidy actually takes place. It would be interesting to extend the analysis and not only quantify the extent of this appropriation, but also identify its relevant determinants.

Given the pervasiveness of credit programs and their cost for governments, we believe that more research is needed to evaluate the effectiveness of this form of intervention in promoting economic efficiency. For example, there seems to be no empirical evidence on the relative performance of alternative forms of intervention (interest subsidies versus loan guarantees, for instance), and it would be appealing to test whether the way credit programs are designed can impact their effectiveness. Some indeed believe that certain forms of intervention may indeed be more effective than others (Gale (1989), (1990)). However, as this study showed, a proper analsysis of the problem should not neglect issues related to the structure of the credit market which is supposed to deliver the subsidies.

Chapter 4

Split-Award Auctions in Procurement

4.1 Introduction

Split-award procurement auctions fall into the more general category of multi-unit auctions. They are typically used to allocate a procurement contract to more than one supplier in circumstances in which the requirement is severable into two or more economic purchase lots. In the case of construction contracting, for example, one may think of different portions of a project which may be undertaken by different contractors.

Split-awards are common practise in a variety of real-world contexts. The Metropolitan Washington Airports Authority (MWAA) currently makes multiple-split awards to procure cleaning, outside audit support and blueprint reproduction services, for instance. Many public and private institutions in the U.S. (State of Indiana, University of Virginia, University of California Lawrence Livermore National Laboratory among the others) also procure the supply of services or goods reserving the right to split the requirement between two or more vendors. Split-award auctions have also been employed in the past to procure complex technologies, including missiles by the U.S. government and computer chips by IBM (as reported in Anton and Yao (1989)).

A possible economic motivation for multiple awards in procurement is efficiency. If suppliers' cost functions are likely to exhibit increasing marginal cost of production, for instance, then it is cost-minimising to break down the requirement into a certain number of sub-units and let each sub-unit be produced by a different supplier. In the "Contract Pricing Reference Guide" by the U.S. Department of Defence, local government agencies are invited to opt for multiple awards "whenever prospective offerors are likely to perceive no significant economies of scale/scope from an aggregate award"[1]. In a dynamic context, split-awards can be the optimal choice of a revenue-maximising agency to the extent they maintain competitive sources for a product to work as a safeguard against monopolistic pricing (Anton and Yao (1989)). Split awards may also provide incentives (Anton and Yao (1989) and McGuire and Riordan (1991)) or be used by benevolent agencies to implement affirmative action and support disadvantaged enterprises. For instance, partial "set-aside" award auctions specify that a portion of the requirement be *set aside* for small businesses, minority-owned firms or hystorically disadvantaged firms.

In split-award auctions, the bid solicitation format may take in practise a variety of forms, depending on the nature of the good to be procured and of the expectations that the buyer has on potential suppliers' characteristics. In a discriminatory pay-your-bid (or menu) auction bidders are invited to submit a price schedule over alternative portions of the requirement (often defined on a percentage share basis, if the good is homogeneous) and the auctioneer chooses the allocation that minimises the total procurement costs. As opposed to winner-take-all auctions, they explicitly allow for divided as well as sole-source production awards. Progressive awards are also used whenever some of the potential competitors are believed not to have the capability to supply the entire quantity required by the agency but might be in a position to offer the lowest price for some of the needed units. Bidders are requested to bid unit prices and state their maximum quantity allocations, and if the low responsive offeror offers a quantity allocation which is smaller than the Government's requirement, the Government makes progressive awards to the

[1] See www.acq.osd.mil/dp/cpt/pgv1_0/.

other offerors to meet its total requirement.

The existing literature seems to suggest that a government agency that needs to choose whether to set up a "winner-take-all" allocation rule or a "split-award" one faces a conflict between efficiency and revenue maximisation objectives. Anton and Yao (1989) analyse a static context where two suppliers compete for the award of a procurement contract in a discriminatory pay-your-bid auction setting. They conclude that a buyer who chooses a split-award setting would be mainly motivated by allocative efficiency objectives rather than by revenue-maximisation ones. A split-award auction would result in efficient equilibrium outcomes but, due to implicit coordination by bidders, the efficiency gain generated by the split is totally appropriated by the bidders. On the other hand, a winner-take-all rule, even when resulting in an inefficient outcome, would guarantee a lower procurement price for the auctioneer. Split-award contracting results in a higher expected price than sole-source contracting also in Perry and Sàkovics (2001), where the buyer uses a sequential second-price auction to award a larger primary contract and a smaller secondary contract. The higher expected price resulting from a split-award setting - as opposed to a sole-source one - is due to the fact that the premium paid to the winner of the secondary contract must also be paid by the winner of the primary contract as an opportunity cost of not winning the secondary contract.

Under some circumstances revenue and efficiency objectives can be reconciled. In Anton and Yao (1989) a setting is considered in which the less efficient producer can engage in a costly cost-reducing innovation activity before the construction phase. The positive profits earned ex-post in correspondence of a split-award outcome provide incentives for ex-ante investment. The improved competitiveness of the less efficient producer allows the government agency to share in the efficiency gain associated with a split and yields higher revenue than a sole-source setting, where no investment takes place. Perry and Sàkovics (2001), instead, examine how an optimal design of the "secondary contract can induce entry by a new supplier and result in a lower expected price paid by the buyer", thus making a split-award setting preferred to a sole-source one in a sequential context as well.

The above contributions mainly focus on the analysis of split awards among at most two contractors, neglecting not only the fact that, in practise, contracts are often awarded to more than two suppliers (as in the case of MWAA); but also overlooking the importance of the relationship between the number of the sub-contracts and the number of suppliers competing for them when revenue maximisation is the agency's objective. One exception is Seshadri, Chatterjee and Lilien (1991), where the optimal number of sub-contracts is derived in a context where also a pre-bid entry decision is considered. Their static model differs from the one adopted in this paper as shares of equal size are auctioned using a non-discriminatory allocation rule. Perry and Sàkovics (2001) also allow for multiple investors, but only two sub-contracts are procured sequentially and the emphasis is on the relative size of the sub-contracts in affecting pre-bid entry decision and thereby the auctioneer's revenue.

The novelty of the present paper is to consider a general n suppliers-r contractors setting where the revenue-efficiency conflict is analysed in the light of recent developments in common agency theory (Bernheim and Whinston (1986), Bergemann and Välimäki (2001), Laussel and Le Breton (2001)). We study the decision problem of a revenue-maximising government agency who needs to procure a given requirement and has the possibility to split the contract among a given number of potential suppliers.

We model the procurement auction as a common agency game with complete information among the principals (the suppliers). We analyse the agent's (the government agency's) problem as the optimal choice of the set of *possible* allocations the principals face out of the set of *feasible* allocations[2]. For instance, we allow the agency to choose either an upper bound or a lower bound (or both) on the *number* of sub-contractors the agency intends to deal with. She might instead decide on the relative size of the shares to be awarded to winning contractors in case of a split-production outcome. Alternatively,

[2] We do not adopt a mechanism design approch here, as the agency can restrict the feasibility set by choosing one of a limited set of design variations.

we consider the possibility of introducing a bias in favour of a certain category of bidders by allowing the agency to exclude certain types of allocations.

We concentrate on games that admit *marginal contribution equilibria*, as defined in Bergemann and Välimäki (2001). If a common agency game admits a marginal contribution equilibrium, this is the unique equilibrium of the game. Uniqueness allows us to easily compare the revenue properties of games resulting from alternative restrictions imposed on the allocation possibilities set. In equilibrium, the price paid by the agency to suppliers is a function of the actual cost associated with the equilibrium organisation of production, plus a mark-up which depends on the *marginal contributions* of suppliers that are *pivotal* to the equilibrium organisation. Restrictions on the possibility set may induce an efficiency loss, thus increasing the actual cost of production; nevertheless, they might turn out to be profitable to the agency if they alter the competition among suppliers in such a way to reduce the total marginal contributions of pivotal bidders. A restriction is in other words preferred by the agency if the total reduction in marginal contributions of pivotal bidders more than compensates the efficiency loss caused by the restriction.

This argument is in line with Bergemann and Välimäki (2001), where the conflict between efficiency and revenue is solved by allowing the principals to lobby the agency in the choice of the possibilities set in a pre-bid "agenda setting" game. The focus is on the existence of a marginal contribution equilibrium of the agenda game, which coincides with a situation in which the principals don't need to lobby the agent to induce the efficient outcome. In the context that we are investigating, their analysis would answer the question of whether the *feasibility* set is the agenda that maximises the agent's revenue.

In our paper, we address the question of *what* is the agenda that maximises the agency's pay-off so that the agency has no incentive to impose further restrictions on this agenda. We are therefore close in spirit to the work by Laussel and le Breton (2001), where they characterise games where the agency gets either no-rent or positive rent in equilibrium. In their terminology our procurement games are "private" common agency games, in which the agency usually gets a rent in equilibrium. They suggest that the agency's rent is higher the lower is the degree of congruence among the principals' interests. In our procurement setting, we try to investigate how the agency can constrain the feasibility set to increase the degree of conflict or competition among the principals'. This is going to depend on the relationship between the number of bidders who are pivotal to the equilibrium allocation and those who are not, the marginal properties of individual cost functions and the extent of asymmetries between bidders (i.e. their "distance").

We first consider the agency's choice of an upper-bound to the number of sub-contractors among which to split the requirement. We find that, if there are n suppliers taking part in the competition, the agency is never worse off by restricting the number of possible sub-contracts to be $n - 1$. In other words, the agency never loses in making at least one bidder *non-pivotal* to the equilibrium organisation of production. This is a general result in that it does not depend on the specific properties of the cost functions, once conditions for the existence of a marginal contribution equilibrium are satisfied. This also generalises the result by Anton and Yao (1989) for the case of two bidders.

We obtain insights for the particular case in which participating bidders are characterised by identical cost functions. Under this special circumstance, and for generic cost functions, the agency cannot strictly increase her revenue by reducing the number of sub-contractors below $n - 1$. In the presence of asymmetries among the bidders' cost functions, the agency's preferences are going to depend on the relationship between the degree of substituitability among bidders (or their distance) in each alternative setting and the marginal properties of the cost functions. The assumptions on the cost functions, as well as the information available to the agency on such properties, become crucial in the analysis. We provide a few illustrative examples which show how the investigation can, in general, get very complicated even in a complete information setting. We consider an environment in which bidders have convex cost functions and show that marginal reductions in the number of sub-contracts are convenient whenever the degree of substitutability among bidders is sufficiently low compared to the degree of diseconomies of scale in production.

We finally analyse alternative types of restrictions on the allocation possibilities set. We first consider an example that shows that the agency can improve her revenue by altering the relative size of the shares awarded to each supplier in a split-award outcome. We also find that the agency might increase her revenue by biasing the competition in favour of disadvantaged players by excluding *selected* allocations involving advantaged players only. This is in line with the well known result in optimal auctions theory that a non-discriminating auction may be less profitable than one which discriminates in favour of the bidders with known lower valuations (Myerson (1981)). Other contributions (Corns and Schotter (1999), among others) showed how preferential treatment of disadvantaged bidders can increase the auctioneer's revenue in procurement auctions. In our example we investigate this issue in a multi-unit setting, where the bias can be designed in such a way to favour certain categories of bidders in some of the possible allocations and not in others.

The chapter is organised as follows. In section 4.2 we introduce the model; in section 4.3 we first summarise the main results on marginal contribution equilibria in common agency theory, and then apply these findings to characterise environments that admit marginal contribution equilibria. In section 4.4 we analyse the agency's choice of an upper bound to the number of sub-contractors. In section 4.5 we analyse alternative restrictions on the set of feasible allocations. Final remarks conclude the paper in section 4.6.

4.2 The model

A governmental agency needs to procure a unit of a good/service and there are $n \geq 2$ potential suppliers she can approach. Each producer i can supply a fraction a_i in $[0, 1]$ of the good at a cost $c(a_i, \sigma_i)$, where σ_i represents the idiosyncratic cost parameter for supplier i. Let $\boldsymbol{\sigma} = (\sigma_1, \sigma_2, ..., \sigma_n)$. We assume that firms are ordered in terms of their relative cost efficiency so that $\sigma_i \leq \sigma_j$ for $i \leq j$. For instance, $i = 1$ identifies the most efficient supplier and $i = n$ the least efficient one. We also make the following assumptions:

Assumption 1. The cost functions $c(a_i, \sigma_i)$ are monotonically non decreasing in a_i over the interval [0,1] and in the cost-parameter σ_i.

Assumption 2. The functional form of the cost functions $c(\cdot, \cdot)$ is common knowledge among all market participants (agency and suppliers); the suppliers also observe the realisation of the cost parameters $\boldsymbol{\sigma}$, while the agency has only knowledge of the random process generating them.

The assumption of complete information among bidders is made for simplicity, but it also allows us to isolate the impact of the degree of competition among bidders from the impact that informational frictions might have on the revenue of the agency.

The set of all feasible organisations of production is denoted by A. Each element $a \in A$ is represented by an $n-$vector $(a_1, a_2..., a_n)$ where $a_i \in [0, 1]$ for every $i \in \Im$. and $\sum_{i=1}^{n} a_i \leq 1$.

The agency's preferences over the possible organisation of production are denoted by a function $v\left(\sum_{i=1}^{n} a_i\right)$ on which we make the following assumption:

Assumption 3

$$\begin{aligned} v\left(\sum_{i=1}^{n} a_i\right) &= 0 \text{ if } \sum_{i=1}^{n} a_i = 1 \\ v\left(\sum_{i=1}^{n} a_i\right) &= -Q \text{ if } \sum_{i=1}^{n} a_i \neq 1 \end{aligned}$$

where Q tends to infinity. Assumption 3 means that the agency needs to procure no more and no less than one unit of the requirement and that she can only obtain this amount from the competing bidders.

Before competition takes place, the government agency selects an element $A^{'}$ out of the power set of A, which we denote as $\wp(A)$. We interpret this action as the choice of a specific bid solicitation format out of a set of given alternatives. For instance, the agency might select a subset A^r, $r \leq n$, whose elements are all vectors $a \in A$ with at least $n - r$ components equal to zero. In other words, by selecting the bid solicitation format A^r, the agency is restricting the set of feasible allocations so as to have *at most* r sub-contractors supplying the requirement. In particular, if the agency chooses A^1 the requirement is treated as indivisible (*sole-source* setting); if instead she chooses A^r with $r > 1$, the contract is treated as divisible into *at most* a number r of sub-contracts (*multiple-source* setting).

After the agency announces the competition format A', bidders are requested to simultaneously submit price schedules of the form $t_i^{'}(a)$[3], $a \in A^{'}$. Note that any choice of A' by the agency completely identifies the competition game among bidders at the procurement stage. Therefore, from now we will use A' to define the solicitation format as well as the corresponding pricing game.

The agency determines the award by selecting the combination of bids that provides the requirement at the lowest total price, i.e.:

$$\alpha' \in \arg\max_{a \in A'}(v(a) - \sum_{i \in \Im} t_i'(a)) \tag{4.2.1}$$

We assume that bidders have quasi-linear preferences, so that, for a given game A', if the equilibrium allocation turns out to be α', bidder i receives a transfer $t_i'(\alpha')$ and his net payoff is $n_i'(\alpha') = t_i'(\alpha') - c(\alpha_i', \sigma_i)$.

Let us denote with a' the efficient organisation of production when the allocation possibility set is given by $A^{'}$. Formally:

[3] In the absence of allocative externalities, we will see that in equilibrium we have $t_i^{'}(a) = t_i^{'}(a_i)$ and so this formulation of the game is equivalent to one in which bidders are requested to submit price schedules $t_i(a_i)$ over the interval $[0, 1]$.

$$a' \in \arg\max_{a\in A'} \left(v(a) - \sum_{i\in\Im} c(a_i, \sigma_i) \right) \tag{4.2.2}$$

Let us also denote as a'_{-S} the efficient organisation if the possibility set is A' and the sub-set of bidders $S \subseteq \Im$ is not competing for the contract:

$$a'_{-S} \in \arg\max_{a\in A'} \left(v(a) - \sum_{i\in\Im\backslash S} c(a_i, \sigma_i) \right) \tag{4.2.2'}$$

Let C' (C'_{-S}) be the value of the cost-minimising organisation a' (a'_{-S}).

The *marginal contribution* of the subset of bidders $S \subseteq \Im$ in game A' is denoted with M'_S and is defined by:

$$M'_S = C' - C'_{-S} \tag{4.2.3}$$

Note that, since the value functions C' and C'_{-S} have values in $\Re^-$, the marginal contributions M'_S are non-negative numbers. In particular, a supplier $i \in \Im$ is said to be *non-pivotal* if $M_i^r = 0$ and *pivotal* otherwise.

Note also that a', a'_{-S}, C', C'_{-S} and therefore M'_S are all function of the signal realisations $\boldsymbol{\sigma}$, which we omit for ease of notation.

4.3 The procurement game

4.3.1 Marginal contribution equilibria

In this section we solve the competition game at the procurement stage resulting from the agency's choice of a bid solicitation format $A' \subseteq A$ and from a given realisation

$\tilde{\boldsymbol{\sigma}} = (\tilde{\sigma}_1, ..., \tilde{\sigma}_n)$ of the bidders' signals. In what follows, and just for ease of notation, we will omit the argument $\tilde{\boldsymbol{\sigma}}$ unless specifically required by the analysis.

A Nash Equilibrium for any pricing game $A' \subseteq A$ is given by a strategy profile $\left\{\{t_i'(\cdot)\}_{i\in\Im}, \alpha'\right\}$ such that α' solves problem (4.2.2) and for every i, the price schedule $t_i'(\cdot)$ over the set of possible allocations in A' maximises bidder $i's$ net payoff $n_i'(\cdot)$.

As already mentioned, we see the competition game as an n-principal static common agency game with complete information, where the principals are the bidders. This class of games generally admits a multiplicity of equilibria. Some of these equilibria, known in the literature as *truthful equilibria*, have a series of desirable properties and they are defined as follows (Bernheim and Whinston (1986)):

Definition 1 (Truthful Equilibrium)

1. *Consider a game A'. A price schedule $t_i'(\cdot)$ is said to be truthful relative to a certain allocation a if for all $\hat{a} \in A'$, either*
   (a) $n_i'(\hat{a}) = n_i'(a)$, or,
   (b) $n_i'(\hat{a}) < n_i'(a)$, *and* $t_i'(\hat{a}) = 0$.
2. *The strategies $\left\{\{t_i'(\cdot)\}_{i\in\Im}, a\right\}$ are said to be a Truthful Nash Equilibrium if they form a Nash Equilibrium and $\{t_i'(\cdot)\}_{i\in\Im}$ are truthful relative to a.*

Truthful Nash Equilibria are characterised by the fact that they are sustained by transfers which reflect the relative willingness to pay of each principal for the various alternatives. It follows that, since gross pay-offs do not exhibit allocative externalities, in equilibrium we have $t_i'(a) = t_i'(a_i)$. In other words, the procurement games under analysis can be treated as "private" common agency games, where each principal communicates to the agency a monetary transfer contingent on the quantities he will receive. Since we are interested in assessing how the size of the total price paid by the agency changes across alternative settings $A' \subseteq A$, we restrict our attention to games that admit a *unique* truthful equilibrium, as this would make comparisons easier.

A common agency game admits a unique (truthful) equilibrium if and only if it is a *marginal contribution equilibrium.* We adapt the definition of *marginal contribution equilibrium* as given in Bergemann-Välimäki (2000) to our framework:

Definition 2 (Marginal Contribution Equilibrium)

A marginal contribution equilibrium of the common agency game A' is a truthful Nash Equilibrium with $n_i'(a') = M_i'$.

If the game A' admits a MCE, the bidders equilibrium net payoffs coincide with their marginal contribution M_i' to the social value C', which is the cost of the efficient organisation of production. These transfers coincide with the Vickrey-Groves-Clarke mechanism transfers: each supplier receives a payment equal to the externality that he generates on competitors by taking part in the competition.

The agency's payoff (the negative value of total transfers to the winning bidders) is given by the residual value:

$$C' - \sum_{i=1}^{n} M_i' \tag{4.3.1}$$

Marginal contribution equilibria, as all truthful equilibria, have the desirable property of resulting in an *efficient* equilibrium allocation. It follows that the equilibrium allocation α' as defined in (4.2.1) will coincide with the efficient allocation a' as given by (4.2.2). The outcome of the game will therefore be determined by the characteristics of the cost function $c(\cdot)$ and be contingent on the bidders' signals $\tilde{\boldsymbol{\sigma}}$.

The following theorem by Bergemann and Välimäki (2001) on the existence of marginal contribution equilibria will be used throughout the paper:

Theorem 1 ***(Bergemann and Valimaki (2001))***

1. The game $A^{'} \subseteq A$ admits a marginal contribution if and only if

$$\forall S \subseteq \Im, \sum_{i \in S} M_i^{'} \leq M_S^{'} \tag{4.3.2}$$

2. If $M_S^{'}$ is superadditive:

$$\forall S, T \subseteq \Im, S \cap T = \emptyset, M_S^{'} + M_T^{'} \leq M_{S \cup T}^{r} \tag{4.3.2'}$$

then the truthful equilibrium is unique.

Superadditivity of the marginal contributions can equivalently be expressed as follows:

$$C^{'} - C_{-S}^{'} \leq C_{-T}^{'} - C_{-T \cup S}^{'} \tag{4.3.2"}$$

In other words, the marginal contributions are superadditive if for any subsets S and T satisfying the assumptions of Theoreom 1, the marginal contribution of the subset of principals S when the subset T of principals is included in the computation of the social surplus is not larger than in the case in which T is excluded.

In a game $A' \subseteq A$ with $n \leq 2$ players, a marginal contribution equilibrium exists for any characterisation of the cost functions (see Bergemann-Välimäki (2000)). If $n = 1$ relation (4.3.2) is trivially satisfied. If $n = 2$, the possible sub-sets S of $\Im = \{1, 2\}$ are $\{1\}, \{2\}$ and the set $\Im$ itself. Relation (4.3.2) is an identity for the singleton sets $\{1\}$ and $\{2\}$, and it is satisfied for $S = \Im$ as $M_{\Im} = \infty$ (bidders 1 and 2 are the only suppliers in the market, and the agency has no outside option). If $n > 2$, superadditivity of the marginal contributions in the class of games considered depends, in general, on the properties of the cost functions $c(\cdot)$. Since we are interested in procurement games that admit a (unique) marginal contribution equilibrium, we need to consider cost structures such that the marginal contributions of players satisfy the properties expressed in Theorem 1 for *any* game $A' \subseteq A$.

4.3.2 Games with an upper bound on the number of contractors

In this section we will verify existence of MCE for procurement games resulting from a particular restriction on the set of feasible allocations, namely the imposition of an upper bound on the number of sub-contractors the agency can commit to deal with in equilibrium. These games can be identified with subsets A^r of A, where the index $r \leq n$ denotes the maximum number of sub-contractors among which the requirement can be split. In this auction format the actual number of contracts awarded in equilibrium and the size of each contract depends on the bidders' behaviour. In other words, the agency just imposes an upper bound to the number of contracts, but the actual split is going to be endogenously determined by the bid structure in equilibrium.

Given the partial ordering among the suppliers that can be established for any given signal $\tilde{\sigma}$, in each game A^r the efficient (equilibrium) organisation of production is never going to involve the last $n-r$ producers[4]. And this is true for any of the specific forms the cost function $c(\cdot)$ can take. Therefore, the efficient allocation can be represented by some $n-$dimension vector $a^r = [a_1, a_2..., a_r, 0..0]$, where $a_i \in [0,1]$ for $1 \leq i \leq r$.

By using assumption 1 to 4 and definitions (4.2.1) to (4.2.3) we can easily verify a series of properties of the values C^r and C^r_{-S} for a given game A^r :

Property 1 $C^r \geq C^{r'}$ and $C^r \geq C^r_{-S} \geq C^r_{-\Im} = -\infty$, $S \subseteq \Im$, $r' \leq r \leq n$.

Property 2 $C^r_{-S} = C^r$ if $S = \{z \in \Im | n \geq z \geq r+1\}$, $r \leq n$.

[4]In case of symmetric cost functions, any r out of the n producers will be involved. We can then assume that they can be ranked randomly and that the first r producers get the award. This would basically work as a tie-breaking rule in the presence of symmetric bidders.

Property 3 $C^{n}_{-S} = C^{n-1}_{-S} = ... = C^{n-|S|}_{-S} \geq C^{n-|S|-1}_{-S} \geq ... \geq C^{1}_{-S}$, $S \subseteq \Im$.

Property 1 simply states that any constraint imposed on the maximum number of subcontractors is never going to induce an efficiency gain. This just reflects the fact that restrictions on the set of feasible allocations may result in a Pareto inferior outcome compared to the case in which all feasible allocations are possible. In particular, we have $C^{r}_{-\Im} = -\infty$ as a consequence of the fact that we have excluded the possibility that the agency might have an outside option. Property 2 says that, given the ordering between bidders (assumption (2)), in any game A^r the efficient allocation is never going to involve suppliers $r+1, ..., n$. Therefore any sub-set of suppliers made out of them is *non-pivotal* to the efficient allocation. Property 3 expresses the fact that restrictions on the set of feasible allocations constrain the optimisation problem (4.2.2') only if they impose the maximum number of sub-contractors to be strictly less than $n - |S|$, where $|S|$ denotes the cardinality of S. Indeed, if A^r is the set of possible allocations, a^{r}_{-S} will be *at most* a split among the remaining $n - |S|$ suppliers. Therefore, as far as $r \geq n - |S|$ the value of a^{r}_{-S} will remain unaffected. In particular, when restricting the maximum number of sub-contractors to be $n-1$, we have $a^{n}_{-S} = a^{n-1}_{-S}$. This follows from the fact that the efficient allocation without S is going to involve a number of sub-contractors less than or equal to $n-1$ even under the allocation set A^r[5].

We will now use the above properties to verify the existence of MCE in certain environments relevant to our analysis. For instance, if we assume existence of MCE for a given game A', we might be interested to know whether this would imply existence of MCE also for games which are "contained" in A'. We now formulate the following result:

Lemma 1 *If A^r admits a marginal contribution equilibrium, then a sufficient condition for A^z, $z \leq r$, to admit a marginal contribution equilibrium is:*

[5] The same conclusion can be derived from the fact that $n - |S| - 1 = n - 1$ only when $|S| = 0$ which is never going to be the case as far as S includes at least one element.

$$\sum_{i\in S} C^r_{-i} - C^r_{-S} \leq \sum_{i\in S} C^z_{-i} - C^z_{-S}, \forall S \subseteq \Im \qquad (4.3.3)$$

Proof. Let us consider the games A^r and A^z, $z \leq r$. If A^r admits a marginal contribution equilibrium condition 5 in Theorem 1, necessary condition, implies $(|S|-1)C^r \leq \sum_{i\in S} C^r_{-i} - C^r_{-S}$ $\forall S \subseteq \Im$. Now assume relation (4.3.3) holds: then condition (4.3.2) in Theorem 1 (sufficient condition) guarantees the existence of a marginal contribution equilibrium for A^z too. Indeed, since $C^r \geq C^z$ (Property 1), for every $S \subseteq \Im$ we have:

$$(|S|-1)C^z \leq (|S|-1)C^r \leq \sum_{i\in S} C^r_{-i} - C^r_{-S} \leq \sum_{i\in S} C^z_{-i} - C^z_{-S}.$$

■

Whether condition (4.3.3) holds for all r and z, probably depends on the characteristics of the cost functions $c(\cdot, \tilde{\sigma}_i)$. Nevertheless, it is certainly satisfied if $r = n$ and $z = n-1$. Indeed, Property 2 ensures that $\forall S \subseteq \Im$:

$$\sum_{i\in S} C^n_{-i} - C^n_{-S} = \sum_{i\in S} C^{n-1}_{-i} - C^{n-1}_{-S}$$

We can therefore formulate the following proposition:

Proposition 1 *If the game A^n admits a marginal contribution equilibrium, then also A^{n-1} admits a marginal contribution equilibrium.*

Another particular environment that will be considered in the following paragraphs is the case of identical bidders. In our model, this would occur under the circumstance in which bidders receive perfectly correlated signals and $\sigma_i = \sigma$ for every $i \in \Im$. The analysis of this particular case will work as a useful benchmark for the analysis of more complex environments. We therefore formulate the following proposition on the existence of MCE in this case:

Proposition 2 *Consider a game A^r, $r \leq n-1$. If potential suppliers have identical cost functions, then A^r admits a MCE for any cost function $c(\cdot,\cdot)$.*

Proof. A marginal contribution equilibrium exists for a game A^r if the following relation holds for every sub-set of principals $S \subseteq \Im$ (we just reformulate the relation in Theorem 1):

$$|S|C^r - \sum_{i \in S} C^r_{-i} \leq C^r - C^r_{-S}$$

If $r \leq n-1$ and principals are symmetric, none of them is pivotal to the efficient allocation: if principal $i \leq r$ does not compete, it can be replaced - without any loss in welfare - by any of the identical principals $j \geq r+1$. Formally, $C^r_{-i} = C^r$ for every $i \in \Im$. Therefore the above relation is trivially satisfied as $C^r_{-S} \leq C^r$ (property 1). ∎

Existence of MCE under the assumptions of Proposition 2 is a consequence of the fact that, if the maximum number of sub-contractors is $n-1$, then the supplier that is not involved in production is a *perfect substitute* of any of the suppliers involved in production, and these are consequently characterised by a marginal contribution equal to zero. In other words, none of the suppliers involved in production are *pivotal* to the equilibrium allocation.

The existence of a marginal contribution equilibrium for games A^n with symmetric bidders, when $n \geq 3$[6] relies on the properties of $c(\cdot,\cdot)$.

Insofar, we have assumed $c(\cdot,\cdot)$ be monotonically non-decreasing in both its arguments. If we also assume differentiability of $c(\cdot,\cdot)$, then cost functions can only be either

[6] If $n \leq 2$, a marginal equilibrium exists regardless of the properties of the cost functions (see Anton and Yao [1989], Laussel and Le Breton [1996]).

convex or concave in quantity. Existence of MCE for this case is stated in Proposition 3.

Proposition 3 *Consider a game A^n. If potential suppliers have identical cost functions $c(a_i, \cdot)$ which exhibit either increasing or decreasing marginal costs over the entire support [0,1], then A^n admits a MCE.*

Proof. If $c(a_i, \cdot)$ exhibits *decreasing* marginal costs, a^n is a sole source to *any* of the participating suppliers. Each supplier is characterised by a marginal contribution to the social value $c(1, \cdot)$ equal to zero and relation (4.3.2) in Theorem 1 holds. If $c(a_i, \cdot)$ exhibits *increasing* marginal costs, differentiability ensures that a^n is a $t-$ *split* with equal shares with $t \leq n$.[7] If $t < n$, each supplier has a marginal contribution equal to zero, as it can be replaced by any out of the $n - t$ suppliers who do not take part to the production process, and relation (4.3.2) of Theorem 1 is therefore satisfied. If $t = n$, all bidders are pivotal to the equilibrium allocation. Since the game is symmetric, only the cardinality of each subset $S \subseteq \Im$ matters in computing marginal contributions M_S^n. Therefore, we have $M_S^n = C^n - C^{n-|S|}$ for any subset $S \subseteq \Im$ of cardinality $|S|$. Condition (4.3.2) therefore reduces to:

$$[(n - |S|)c(\frac{1}{n - |S|}) - nc(\frac{1}{n})]/|S| \leq (n - 1)c(\frac{1}{n - 1}) - nc(\frac{1}{n})$$

Where $|S| < n$, as when $|S| = n$ inequality (4.3.2) is trivially satisfied. Rearranging the above expression we get:

$$\frac{n - |S|}{|S|(n - 1)}c(\frac{1}{n - |S|}) + \frac{n(|S| - 1)}{|S|(n - 1)}c(\frac{1}{n}) \geq c(\frac{1}{n - 1}) \tag{4.3.4}$$

[7] For example, consider $c(\cdot) = v(\cdot) + \sigma$, where σ represents a fixed cost component. Let us define $\Gamma^t \equiv tv(1/t) - (t + 1)v(1/(t + 1))$, where $1 \leq t \leq n$ and set $\Gamma^n = 0$. The efficient allocation is a $t -$ *split* if $\Gamma^t \leq \sigma \leq \Gamma^{t-1}$.

Since $c(a_i, \cdot)$ is a convex function, then $c(\lambda x + (1-\lambda)y) \leq \lambda c(x) + (1-\lambda)c(y)$ for any $\lambda \geq 0$ and quantities x and y. The above inequality is therefore satisfied if we choose

$$x = \frac{1}{n - |S|}, \quad y = \frac{1}{n} \text{ and } \lambda = \frac{n - S|}{|S|(n-1)}$$

■

In general, if cost functions are convex, Proposition 3 says that, if efficency requires n producers, a marginal equilibrium exists if the average efficiency loss of excluding $|S|$ suppliers from production is larger than the efficiency loss of excluding one producer (*marginal efficiency loss*). Equivalently, a MC equilibrium exists if the marginal efficiency loss is, in absolute terms, a convex function of the number of suppliers excluded from production.

In order to illustrate the result expressed in Proposition 3, consider the simple case $n = 3$. Relation (4.3.2) reduces to:

$$3c(\frac{1}{3}, \cdot) - 2c(\frac{1}{2}, \cdot) \geq 2c(\frac{1}{2}, \cdot) - c(1, \cdot)$$

which means that a MCE exists if the efficiency gain of adding one additional producer decreases with the number of current producers. This coincide with the definition of convex cost function.

We now restrict our attention to the case in which bidders have convex but asymmetric cost functions.

Example 1

Let us first consider the case $c(a_i) = (\sigma_i a_i^2)/2$. For a given game A^r, the efficient organisation of production is an $r-$ *split* among the first r most efficient suppliers, represented by an $n-$vector $a^r = [a_1^r, ..., a_r^r, 0...0]$ where $a_j^r = \left(\sigma_j \sum_{i=1}^r \frac{1}{\sigma_i}\right)^{-1}$. Let us denote with H_x the first x most efficient suppliers in $H \subseteq \Im$.

We can now compute the value functions C^r, C^r_{-j} and C^r_{-S} and M^r_{-S} as follows:

$$C^r = -\left(2\sum_{i\in\Im_r}\frac{1}{\sigma_i}\right)^{-1} \tag{4.3.5}$$

$$C^r_{-S} = -\left(2(\sum_{i\in(\Im\backslash S)_r}\frac{1}{\sigma_i}\right)^{-1} \tag{4.3.5'}$$

Where the number of terms in (4.3.5') is given by $\min(r, |\Im\backslash S|)$. The following proposition states existence of MCE for this example.

Proposition 4 *If cost functions are given by* $c(a_i) = (\sigma_i a_i^2)/2$, $i \in \Im$, *then a MCE exists for every game* A^r, $1 \leq r \leq n$.

Proof. See Appendix C. ■

Example 2

Assume suppliers are characterised by cost functions $c(\cdot, \sigma_i) = v(\cdot) + \sigma_i$, where $\sigma_1 \leq ... \leq \sigma_n$, the component $v(\cdot)$ satisfies the usual convexity assumptions, and $c(0, \sigma_i) = 0$. This formulation of the cost function is the simplest way to parameterise separately the marginal properties of $c(\cdot)$ and the degree of asymmetries among players. The limit of this specification is that asymmetries are invariant across allocations. A more general environment would let the σ_i be a function of the quantity produced.

Let us define the following quantity:

$$1 \leq y \leq n, \qquad \Gamma^y = yc(\frac{1}{y}, \cdot) - (y+1)c(\frac{1}{y+1}, \cdot)$$

Γ^y represents the efficiency gain in total costs from having the requirement produced by $(y+1)$ rather than y producers. In any game A^r, $r \leq n$, the efficient allocation is a $p-$ *split* among the first $p \leq r \leq n$ most efficient suppliers, where p turns out to be a function of the relationship between the extent of asymmetries among suppliers and the extent of economies that can be generated by splitting production:

$$\begin{aligned} p &= 1 \text{ if } \sigma_2 > \Gamma^1; \\ 1 &< p < r \text{ if } \sigma_p \leq \Gamma^{p-1} \text{and } \sigma_{p+1} > \Gamma^p; \\ p &= r \text{ if } \sigma_r \leq \Gamma^{r-1}; \end{aligned}$$

Note that, in order to verify the condition (4.3.2) in Theorem 1, we only need to consider the marginal contributions of sub-sets of pivotal bidders. For any sub-set $S \subseteq \Im$, let us define $P = \Im_r \cap S$. In words, P is the set of pivotal bidders contained in S. We have $M_P^r \leq M_S^r$, and if $\sum_{i \in P} M_i^r \leq M_P^r$, we also have $\sum_{i \in S} M_i^r = \sum_{i \in P} M_i^r \leq M_P^r \leq M_S^r$, and superadditivity of the marginal contributions of S is automatically satisfied.

Let us denote with Z a generic subset of P, and let also $|P| = p$ and $|Z| = z$. If $x \equiv |\Im \backslash Z|$, then this is also the number of pivotal bidders in Z that can be collectively be substituted by non-pivotal bidders in $\Im \backslash Z$. We have:

$$M_Z^r = \sum_{j=1}^{x} \min(\sigma_{p+j}, \Gamma^{p-1-z+j}) + \sum_{j=x+1}^{z} \Gamma^{p-1-z+j} - \sum_{j \in Z} \sigma_j$$

where we have set $\min(\sigma_{p+1}, \Gamma^0) = \sigma_{p+1}$, and $\min(\sigma_{n+1}, \Gamma^{n-1}) = \Gamma^{n-1}$ to have a unique formula for the case $z < p$ and $z = p$.

Proposition 5 *Consider a procurement game A^r, $r \leq n$. If the bidders' cost functions are given by $c(\cdot) = v(\cdot) + \sigma_i$, where $\sigma_1 \leq ... \leq \sigma_n$ and $v(\cdot)$ is a convex function, then a MCE exists.*

Proof. See Appendix C. ■

For illustrative purposes, let us consider the simple case in which $n = 3$. If $r = 3$, the following table indicates what the efficient allocation is according to the values taken by the signal $\boldsymbol{\sigma}$:

	$\sigma_2 \geq \Gamma^1$	$\sigma_2 \leq \Gamma^1$ and $\sigma_3 \geq \Gamma^2$	$\sigma_3 \leq \Gamma^2$
a^3	$(1,0,0)$	$(1/2,1/2,0)$	$(1/3,1/3,1/3)$
p	1	2	3

For every bidder in any set $Z \subseteq P$ of pivotal players we have:

$$M_i^r = \min(\sigma_{p+1}, \Gamma^{p-1}) - \sigma_i$$

If $p = 1$, we can only have $Z = \{1\}$, and $M_1^r = \min(\sigma_2, \Gamma^0) - \sigma_1 = \sigma_2 - \sigma_1$. Under this circumstance relation (4.3.2) in Theorem 1 is trivially satisfied. If $p = 2$ we can have at most $x = 1$, which implies $M_Z^r = \min(\sigma_3, \Gamma^{2-z}) + \sum_{j=2}^{z} \Gamma^{1-z+j} - \sum_{j \in Z} \sigma_j$. If the number of pivotal bidder is $z = 2$ (and consequently $Z = \{1, 2\}$), we have:

$$M_Z^r = \min(\sigma_3, \Gamma^0) + \Gamma^1 - \sigma_1 - \sigma_2 = \sigma_3 + \Gamma^1 - \sigma_1 - \sigma_2$$

Also in this case condition (4.3.2) is satisfied as:

$$2\min(\sigma_3, \Gamma^1) - \sigma_1 - \sigma_2 \leq \sigma_3 + \Gamma^1 - \sigma_1 - \sigma_2$$

Finally, if $p = 3$ we have $x = 0$ and $M_Z^r = \sum_{j=1}^{z} \Gamma^{2-z+j} - \sum_{j \in Z} \sigma_j$. So, if $z = 2$ we have $M_Z^r = \sum_{j=1}^{2} \Gamma^j - \sum_{j \in Z} \sigma_j$ and condition (4.3.2) is satisfied:

$$2\Gamma^2 - \sum_{i \in Z} \sigma_i \leq \sum_{j=1}^{2} \Gamma^j - \sum_{j \in Z} \sigma_j$$

A marginal contribution equilibrium therefore exists for the game A^3. Corollary 1 ensures the existence of a marginal contribution equilibrium for $r = 2$ as well. If $r = 1$ a marginal contribution equilibrium exists for all cost specifications.

4.4 The agency's choice of contract specifications

4.4.1 The conflict between efficiency and revenue maximisation objectives

In Laussel and Le Breton (2001) they show that, in private common agency games, the supermodularity/submodularity properties of the agent's and the principals' utility functions over the set of possible actions determine whether the agency gets a rent in equilibrium or not. The existence of a rent would reflect the presence of a minimum degree of conflict among the bidders' interests. Rather than assessing the existence of a rent, our objective is to explore whether this conflict, and the agency's rent with it, can be increased by appropriately tailoring the set of possible allocations the principals can bid upon. The assumptions made on the agency's preferences make our setting a very simple one in which, if a MCE exists, the agency always gets a rent in equilibrium[8]. In order to assess the relative convenience of alternative restrictions A' of the set of feasible allocations A, we therefore just need to compare the equilibrium net transfers from the agency to the contractor(s) in alternative settings A'.

At the procurement stage, the agency's equilibrium pay-off (total transfers to the (winning) contractors with a negative sign) for a given game A' and for a given realisation

[8]The only exception is represented by a very special case that will be analysed in section 4.4.2 (Example 2).

of the bidders' signals $\boldsymbol{\sigma}$ is given by (4.3.1). Since the agency cannot observe $\boldsymbol{\sigma}$, at the pre-bid stage she chooses the bid solicitation format $A^{'}$ so as to maximise the following expected profit:

$$E_{\sigma}[C^{'}(\sigma) - \sum_{i \in \Im} M_{i}^{'}(\sigma)] \tag{4.4.1}$$

Using the definition of marginal contribution as given in (4.2.3), the argument in (4.4.1) becomes:

$$(1-n)C^{'}(\sigma) + \sum_{i \in \Im} C_{-i}^{'}(\sigma) \tag{4.4.1'}$$

If we assume that the choice is between setting A' and setting A'', where $A'' \subseteq A'$, the agency prefers setting A'' if (we omit the argument $\boldsymbol{\sigma}$ for convenience):

$$E_{\sigma}\left[\sum_{i \in \Im}(C_{-i}^{'} - C_{-i}^{''})\right] \leq (n-1)E_{\sigma}\left(C^{'} - C^{''}\right) \tag{4.4.2}$$

Since $A'' \subseteq A'$, we have $C' \geq C''$ and $C'_{-i} \geq C''_{-i}$ for every $i \in \Im$ and for any given signal $\boldsymbol{\sigma}$. It follows that a necessary condition for a restriction to be *strictly* preferred by the agency is that it induces an efficiency loss in expectation. Indeed, if $E_{\sigma}\left(C^{'} - C^{''}\right) = 0$, (4.4.2) implies $E_{\sigma}\left[\sum_{i \in \Im}(C_{-i}^{'} - C_{-i}^{''})\right] \leq 0$ which can at most be satisfied with equality. In other words, the agency is never better off imposing a restriction on a given possibilities set $A' \subseteq A$ unless the restriction alters the set of pivotal bidders in such a way to induce an efficiency loss for at least some $\boldsymbol{\sigma}$.

Let us now concentrate on the left-hand-side of (4.4.2). We can say that the agency is never worse off imposing a restriction A'' on the possibility set A' if the restriction is such that, for every realisation of $\boldsymbol{\sigma}$, the values C_{-i} remain unaffected or - equivalently -

$C'_{-i} = C''_{-i}$ for all bidders.

When a restriction induces a non-zero variation in *both* sides of (4.4.2), the agency's preferences are probably going to depend on both the properties of the value functions C and C_{-i} and on the agency's beliefs over types $\boldsymbol{\sigma}$.

4.4.2 The number of subcontracts

The restrictions that the agency might want to impose on the set of feasible organisations A can be diverse. In this section we will investigate whether it could be revenue enhancing for the agency to commit on splitting the contract among at *most* a certain number of sub-contractors r^*. In the common agency (menu-auction) setting that we are considering, this is equivalent to set a maximum number of principals that would be *pivotal* to the equilibrium (efficient) allocation. The sort of restrictions that we are considering are therefore represented by the possibility sets A^r examined in section 4.3.2 and for which properties 1,2 and 3 hold.

The considerations made in the preceding paragraph on the conflict between revenue maximisation objectives and efficiency allow us to formulate the following general result:

Proposition 6 *If the procurement game A^n admits a marginal contribution equilibrium, then the agency is never worse off by restricting the maximum number of sub-contractors to $n-1$.*

Proof. Corollary 1 ensures the existence of a marginal contribution equilibrium for A^{n-1}. Since $C^n_{-i} = C^{n-1}_{-i}$ for every i (Property 3) condition (4.4.2) reduces to $(n-1)E_\sigma(C^n - C^{n-1}) \geq 0$, which always holds (Property 1). ■

In essence, Proposition 6 says that the agency is never worse off if she restricts the set of feasible allocations A^n so as to make one of the participant bidders non-pivotal. This is a consequence of the fact that when restricting the number of bidders to $n-1$ the values C_{-i} remain unaffected. This result is general in that it depends neither on the specific form taken by the cost function nor on the agency's beliefs over the bidders' types. It generalises a result by Anton-Yao (1992) for the case $n = 2$. If the agency's choice is between a sole-source setting (A^1) and a 2-split award one (A^2) there is a total conflict of interest between efficiency and revenue maximisation. If the efficient allocation is a sole-source to the most efficient producer, then the total price paid under A^1 and A^2 is the same. If the efficient allocation is a split between the two producers, then - whatever the assumptions on the cost functions are - the total price paid under A^2 is larger than the one paid under A^1. Any efficiency gain associated with a split compared to a sole-source production is appropriated by the producers.

Given that a reduction of the number of sub-contracts to $n-1$ is never going to make the agency worse off, we can now investigate under what characteristics of the suppliers' cost functions restrictions below $n-1$ are profitable or not.

Proposition 7 *If the procurement games A^z and A^{z-1}, $z \leq n$, admit a marginal contribution equilibrium, a necessary condition for the agency to prefer a reduction in the maximum number of sub-contractors from z to $z-1$ is:*

$$\sum_{i=1}^{z-1} M_i^z > \sum_{i=1}^{z-1} M_i^{z-1} \tag{4.4.3}$$

for at least some signal realisations $\boldsymbol{\sigma}$.

Proof. In A^z (A^{z-1}) the maximum number of sub-contractors is z ($z-1$) and, therefore, the efficient allocation is going to be at most a $z - split$ ($(z-1) - split$) among the $z\,(z-1)$ most efficient suppliers. It follows that, for any given signal realisation $\boldsymbol{\sigma}$, $M_i^z = 0$ ($M_i^{z-1} = 0$) for $i \geq z+1$ ($i \geq z$). The setting A^{z-1} is not strictly preferred to A^z if, for any given $\boldsymbol{\sigma}$, the following inequality (obtained from (4.4.2)) holds:

$$\sum_{i=1}^{z-1} (C_{-i}^z - C_{-i}^{z-1}) + C_{-z}^z - C^{z-1} \geq (z-1)(C^z - C^{z-1})$$

Since $C^{z-1} = C^{z-1}_{-z}$ and $C^{z}_{-i} \geq C^{z-1}_{-i}$ for every $i \in \Im$ (Property 2), we have z non-negative terms on the left-hand side of the above inequality and $z-1$ non-negative terms on the right-hand side. The quantity $C^{z}_{-z} - C^{z-1}$ is less than or equal to any of the terms on the right hand side as $C^{z}_{-z} \leq C^{z}$, but it is still a non negative quantity. Therefore, for the above inequality to hold, it suffices that:

$$\sum_{i=1}^{z-1}(C^{z}_{-i} - C^{z-1}_{-i}) \geq (z-1)(C^{z} - C^{z-1})$$

which implies (4.4.3), once the definition of marginal contribution of principal i is applied. ■

The agency is never better off imposing marginal reductions in the number of contractors if, for any signal realisation $\boldsymbol{\sigma}$, the sum of the marginal contributions of the bidders who are pivotal in both the *restricted* scenario A^{z-1} and the original one A^{z} does not reduce as a result of the restriction. Alternatively, as far as the efficiency loss resulting from the restriction is smaller than the average increase in the absolute value of the terms C_{-i} for the set of bidders who remain pivotal after the restriction.

A special case: symmetric bidders

An immediate application of Proposition 4 is the one in which the procurement games A^{z} and A^{z-1}, $z \leq n$, admit a marginal contribution equilibrium and $M^{z}_{i} = M^{z-1}_{i}$ for all $i \leq z-1$, i.e. the marginal contributions of the bidders who remain pivotal do not change as a result of the restriction. This occurs if bidders have identical cost functions, for instance, which, in our setting, is going to occur if bidders's signals are perfectly correlated. Namely, $\sigma_i = \sigma$ for every $i \in \Im$.

We have already shown that a MCE exists under this circumstance (Proposition 2 and 3). Moreover, $M_i^r = M_i^{r-1} = 0$ for every $i \leq r$ and for any arbitrary $r \leq n-1$. Relation (4.4.3) is therefore satisfied with equality for any realisation σ of the signals. We can therefore formulate the following:

Corollary 2 *If bidders have identical cost functions, reducing the maximum number of subcontractors below $n-1$ is never going to make the agency strictly better off.*

Symmetric principals are perfect substitutes as any given quantity can be produced by any supplier at the same cost. If at least one supplier is non-pivotal ($r = n-1$), each producer's marginal contribution to the social surplus is zero. The conflict among bidders is already maximum at $r = n-1$ as it is the agency who appropriates of all the surplus. By reducing further the maximum number of sub-contractors the agency just incurs in an efficiency loss without increasing the conflict among bidders. Since the agency's revenue-maximisation objective does not clash with efficiency for $r \leq n-1$, she will never find it profitable to set the maximum number of subcontractors below $n-1$.

If the game A^r admits a marginal contribution equilibrium also for $r = n$, then the "optimal" upper bound to the number of sub-contractors would be exactly $n-1$ (Proposition 6). If instead A^n admits a multiplicity of truthful equilibria, then in any such equilibrium there is at least one bidder who is receiveing *less* than its marginal contribution to the social surplus (Bernheim and Whinston (1989)). This means that an $n-$split could still be preferred to an $(n-1)-$split. If n is sufficiently large, the choice between A^n and A^{n-1} becomes irrelevant, and we summarise the above result by saying that, in case of identical bidders, the agency has essentially no incentive to set a cap on the maximum number of subcontractors.

The case of asymmetric bidders: some examples

In a setting in which bidders are not identical, the agency's decision problem is going to depend on the specific relationship between the characteristics of the cost functions and the extent of the asymmetries among principals. As already discussed, the convenience of setting an upper bound to the maximum number of suppliers depends on the impact on the suppliers' marginal contributions relative to the efficiency loss that such a restriction might induce. For a given game A^r, the marginal contribution of each *pivotal* supplier depends on his "distance" from the most efficient *non-pivotal* supplier in $\Im$. This bidder is going to play the role of an "inside option" or "substitute" for the agency in relation to the supply of the portion of requirement supplied by each pivotal bidder. If bidders are asymmetric, reducing the upper bound on the maximum number of subcontractors can be convenient because it makes the best non-pivotal bidder more efficient and reduces its distance from the pivotal bidders.

In general, if $r \leq n-1$, the agency's preferences over alternative competition settings A^r are going to depend on the prior beliefs the agency has over the suppliers' types. In this paragraph, we will show, by means of a few examples, how the presence of asymmetries can complicate the analysis even in the case of complete information between the agency and the suppliers. In what follows we therefore assume that the suppliers' cost functions are common knowledge among all players.

Example 1

In the case of cost functions taking the form $c(a_i) = (\sigma_i a_i^2)/2$, we have already shown that a marginal contribution equilibrium exists for each $r \leq n$. We know that the agency would always restrict the maximum number of subcontractors to $n-1$, and we need to verify whether reductions of the number of sub-contractors below $n-1$ is profitable to the agency or not.

For the specific functional form under consideration, combining expressions (4.3.5) and (4.3.5'), we obtain:

$$
\begin{aligned}
C^r - C^{r-1} &= 2\sigma_r |C^r||C^{r-1}| \\
C^r_{-j} - C^{r-1}_{-j} &= 2\sigma_{r+1} |C^r_{-j}||C^{r-1}_{-j}| \\
M^r_j = C^r - C^r_{-j} &= 2\left(\frac{1}{\sigma_j} - \frac{1}{\sigma_{r+1}}\right)|C^r||C^r_{-j}|
\end{aligned}
\tag{4.4.4}
$$

From (4.4.2) we know that a restriction of the number of pivotal bidders from r to $r-1$ $(r \leq n-1)$ is not profitable to the agency if the following inequality holds:

$$
(r-1)\sigma_r |C^r||C^{r-1}| \leq \sigma_{r+1} \sum_{i=1}^{r} |C^r_{-j}||C^{r-1}_{-j}| \tag{4.4.5}
$$

Notice that, regardless of the values taken by the parameters, we have:

$$
(r-1)|C^r||C^{r-1}| \leq \sum_{i=1}^{r} |C^r_{-j}||C^{r-1}_{-j}|
$$

Therefore a crucial role is going to be played by the "distance" between the parameters σ_r and σ_{r+1}. These are the cost parameters for the suppliers who are potential substitutes for each pivotal player in A^r and A^{r-1} respectively. The choice of the agency is driven by the relationship between the extent of the economies associated with production splitting and the degree of substitutability between players in A^r and A^{r-1}. As an illustration, let us assume that the agency's choice is between $r = 2$ and $r = 1$. Inequality (4.4.5) becomes:

$$
\sigma_1^2/(\sigma_1 + \sigma_2) \leq \sigma_1^2/(\sigma_1 + \sigma_3) + \sigma_2^2/(\sigma_2 + \sigma_3)
$$

The parameter σ_3 appears only on the right hand side of the inequality and, holding fixed the parameters σ_1 and σ_2, sufficiently high values for σ_3 can be found such that the above inequality does not hold. For instance assume for simplicity that parameters σ_i only assume integer values. If $\sigma_1 = 1$ and $\sigma_2 = 2$, then reducing the number of pivotal bidders from two to one is not profitable as far as $\sigma_3 \leq 14$, i.e. as far as the "substitute" bidder in A^2 is sufficiently close to the pivotal bidders 1 and 2. For higher values of σ_3, the agency would choose to induce an efficiency loss by imposing a sole source setting.

The choice of the agency is therefore driven by the relationship between the extent of the economies associated with production splitting and the relative degree of substitutability between players 1 and 2 in A^1, and players 1,2 and 3 in A^2.

Let us now consider a particular specification of the parameters σ_i that will allow us to focus on the role played by asymmetries.

Assume that $\sigma_i = \sigma_l$ for $i \leq z$ and $\sigma_i = \sigma_h$ for $i > z$, where $z < n$. In order to examine the agency's optimal choice of r^*, we distinguish two cases: $r = z$ and $r \neq z$, for any $r \leq n - 1$.

If $r \neq z$, a marginal reduction of the number of sub-contractors is never convenient to the agency because each pivotal principal $j \leq r$ can be substituted by a supplier with the same cost parameter in the two settings A^r and A^{r-1}. Indeed, it is either $\sigma_r = \sigma_{r+1} = \sigma_l$ $(r < z)$ or $\sigma_r = \sigma_{r+1} = \sigma_h$ $(r > z)$. It follows $M_i^r = M_i^{r-1} = 0$ for every $i \leq r - 1$ and Proposition 4 applies. The agency's problem can, as a consequence, be reduced to a choice among the settings A^{n-1}, A^z and A^{z-1}.

Let us compare A^{n-1} and A^z. Since the marginal contributions of the last $z+1$ (high cost) producers are zero in A^{n+1}, we have $A^{n-1} \succeq A^z$ if and only if:

$$C^z - zM^z \leq C^{n-1} - zM^{n-1}$$

where $M^{n-1}(M^z)$ is the marginal contribution of each *low-cost* producer in $A^{n-1}(A^z)$. In order to show that $A^{n-1} \succeq A^z$, it is enough to verify that $M^z \geq M^{n-1}$ (Proposition 7) for the first z low-cost producers, who are pivotal both in A^{n-1} and A^z. Indeed, for each of them we have (from (4.4.4)):

$$M^z = 2\left(\frac{1}{\sigma_l} - \frac{1}{\sigma_h}\right)|C^z||C^z_{-i}| \geq 2\left(\frac{1}{\sigma_l} - \frac{1}{\sigma_h}\right)|C^{n-1}||C^{n-1}_{-i}| = M^{n-1}$$

For each $i \leq z$, the "substitute" or "inside option" for the agency is going to be a high-cost producer. It follows that a reduction in the number of subcontractors from $n-1$ to z is just going to induce an efficiency loss without reducing the distance between pivotal and non-pivotal bidders. The agency's decision therefore boils down to the comparison between A^{n-1} and A^{z-1}.

The necessary condition in proposition 4 is instead satisfied when we compare A^{n-1} and A^{z-1}. In this case, low-cost pivotal bidders have $M_i^{z-1} = 0$ in A^{z-1} and $M^{n-1} \geq 0$ in A^{n-1} and reducing the number of subcontractors below $n-1$ might be profitable. We have $A^{n-1} \succeq A^{z-1}$ if and only if:

$$C^{z-1} \leq C^{n-1} - zM^{n-1}$$

It turns out that a restriction to a number $z-1$ of subcontractors is never profitable if and only if:

$$\frac{\sigma_h}{\sigma_l} \leq \delta(z,n)$$

where $\delta(z,n) \geq 1$, $\frac{d\delta(z,n)}{dz} < 0$ and $\lim_{z \to n} \delta(z,n) = 1$. In other words, reducing the number of subcontractors below $n-1$ is not convenient as far as the "distance" between low-cost and high-cost suppliers is not too large. Moreover, the larger the number z of symmetric low-cost producers, the less distant low and high types should be for the agency to accomodate efficiency and choose A^{n-1} rather than A^{z-1}. As z increases, the game degenerates to one with identical bidders where, as we have shown in the preceding paragraph, there is no conflict between revenue and efficiency objectives as far as $r \leq n-1$.

Example 2

The analysis of the case in Example 1 is complicated by the fact that both economies and asymmetries are parameterised by the same coefficient σ_i in each cost function. Further insights can be obtained if we simplify the setting by parameterising asymmetries with a fixed cost component.

Let the cost functions be given by $c_i(\cdot) = v(\cdot) + \sigma_i$, where $v(\cdot)$ is a convex function and $\sigma_1 \leq ... \leq \sigma_n$. Let $A^x - A^y$, where $x, y \leq n$ and $x \geq y$, represent a bid solicitation format prescribing not only a maximum (x) but also a minimum (y) number of sub-contractors. We denote with $M_S^{r-r'}$ the marginal contribution of a subset $S \subseteq \Im$ of bidders in game $A^r - A^{r'}$.

We consider the simple case in which the agency faces three suppliers only and she can choose one out of the following competition settings $A^3 \equiv A$, A^2, A^1, $A^3 - A^1$, $A^3 - A^2$ and $A^2 - A^1$.

The case $A^3 - A^2$ is never going to be preferred by the agency, as it is one in which the only possible allocation is a split among all potential suppliers, each of them characterised by a marginal contribution $M_i^{3-2} = \infty$. In other words, this game has the "no-rent" property.

The agency's preferences over the remaining settings depend on what is the efficient allocation a^3 in A^3 contingent on the values taken by the parameters σ_i, $i \leq 3$. In Table 1 we report the suppliers' marginal contributions in each contingency. The agency's preferences in each sub-case are summarised in Table 2, where in case (a) we have $\sigma_3 \geq \Gamma^1$, and in case (b1) and (b2) we have $\sigma_3 \leq (\Gamma^1 + \sigma_2)/2$ and $(\Gamma^1 + \sigma_2)/2 < \sigma_3 < \Gamma^1$ respectively.

First of all, we can notice that the agency is going to prefer restrictions on A which do not involve an efficiency loss. For example, if the efficient organisation of production is $a^3 = (1, 0, 0)$, the agency is never going to be strictly better off by imposing any sort of restriction on the set of feasible allocations A^3. She is indifferent between whether to impose an upper bound on the number of sub-contractors or not. Secondly, the agency is never going to impose a restriction for which the necessary condition expressed in Proposition 7 is not verified. Indeed, she can worsen her situation by setting a lower bound on the number of sub-contractors in A. For instance, if the efficient organisation of production is a sole-source to supplier $i = 1$, increasing the number of pivotal bidders to at least 2 is going to increase not only the cost of production, but also the marginal contributions of the participating suppliers. In A^3 only supplier 1 is pivotal, with a marginal contribution equal to $\sigma_2 - \sigma_1$. In A^{3-1}, instead, not only also supplier 2 is pivotal (and therefore has a positive marginal contribution) but also supplier 1's marginal contribution increases up to $\sigma_3 - \sigma_1$. The extent of coordination among suppliers is therefore enhanced by the imposition of a lower bound on the number of pivotal bidders.

Table 1. The suppliers' marginal contributions.

	$a_1^3 = 1$	$a_i^3 = 1/2$ $\sigma_3 \geq \Gamma^1$	$a_i^3 = 1/2$ $\sigma_3 < \Gamma^1$	$a_i^3 = 1/3$
M_i^3	$\sigma_2 - \sigma_1*$	$\Gamma^1 - \sigma_i$	$\sigma_3 - \sigma_i$	$\Gamma^2 - \sigma_i$
M_i^2	$\sigma_2 - \sigma_1*$	$\Gamma^1 - \sigma_i$	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$
M_i^1	$\sigma_2 - \sigma_1*$	$\sigma_2 - \sigma_1*$	$\sigma_2 - \sigma_1*$	$\sigma_2 - \sigma_1*$
M_i^{3-1}	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$	$\Gamma^2 - \sigma_i$
M_i^{2-1}	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$	$\sigma_3 - \sigma_i$

*Marginal contribution of bidder 1

Table 2. The agency's preferences.

$a_i^3 = 1$	$A^3 \sim A^2 \sim A^1 \succsim A^3 - A^1 \sim A^2 - A^1$
$a_i^3 = 1/2$, case(a)	$A^3 \sim A^2 \precsim A^1 \succsim A^3 - A^1 \sim A^2 - A$
$a_i^3 = 1/2$, case (b1)	$A^3 \sim A^2 \sim A^3 - A^1 \sim A^2 - A^1 \succsim A^1$
$a_i^3 = 1/2$, case (b2)	$A^3 \sim A^2 \sim A^3 - A^1 \sim A^2 - A^1 \precsim A^1$
$a_i^3 = 1/3$, case (b1)	$A^3 \sim A^3 - A^1 \precsim A^2 - A^1 \sim A^2 \succsim A^1$
$a_i^3 = 1/3$, case (b2)	$A^3 \sim A^3 - A^1 \precsim A^2 - A^1 \sim A^2 \precsim A^1$

On the other hand we have the case in which all market participants are pivotal to the efficient allocation, i.e. $a^3 = (1/3, 1/3, 1/3)$. Under this circumstance, the agency is strictly better off reducing the number of sub-contractors to at most two and make at least one supplier non-pivotal to the equilibrium allocation. Depending on the relative size of σ_3, σ_2 and Γ^1, the agency might even find it revenue increasing to induce a further efficiency loss and have at most one pivotal supplier. Indeed, in A^2 the agency pays a price which is twice the production cost of the non-pivotal supplier in A^2 (supplier 3). In A^1 she pays a price equal to the cost of the most efficient non-pivotal bidder in A^1. If $\Gamma^1 - \sigma_2 \leq 2(\sigma_3 - \sigma_2)$ - (case (b2)) - imposing a sole-source setting A^1 is profitable because the resulting efficiency loss $\Gamma^1 - \sigma_2$ is smaller than the total increase $2(\sigma_3 - \sigma_2)$ in the suppliers' marginal contributions. Therefore the choice between A^1 and A^2 is going to be determined by the distance between suppliers 2 and 3, which play as substitutes in A^1 and A^2 respectively; the number of suppliers involved in production in each setting; and the extent of the economies of scale generated by production splitting $(\Gamma^1 - \sigma_2)$. A similar reasoning applies to the case in which the efficient allocation is $a^3 = (1/2, 1/2, 0)$ where, again, the agency prefers to have at most one pivotal bidder rather than two if $\Gamma - \sigma_2 \leq 2(\sigma_3 - \sigma_2)$.

This example has shown that the distribution of surplus between the agency and the suppliers is affected by the specific relationship between the distance between pivotal suppliers and the non-pivotal ones (actually, the most efficient among them) and the entity of the efficiency gains deriving from production splitting in each setting.

4.5 Comparison with alternative restrictions

Besides setting an upper (or lower) bound to the maximum number of subcontractors, the agency might attempt to increase her revenue by imposing other types of restrictions on the set of feasible allocations. The agency might have some information on the nature of the asymmetries among potential suppliers and might want to use this information to tailor the competition setting accordingly in order to increase her expected revenue.

In this section we will consider a couple of examples to illustrate, firstly, that the agency can increase her revenue by deciding on the relative size of the shares awarded to each sub-contractor. Secondly, it could also be profitable to alter the set of feasible allocations so as to create a bias in favour of a certain category of suppliers.

The Size of the Shares

The choice of the optimal number of subcontractors by the agency is just a special case of the more general problem of deciding on the relative size of the shares to be awarded to each of the participating suppliers.

We consider the very simple case in which $n = 2$, so that existence of MCE is guaranteed whatever is the specific form taken by the cost functions.

Rather than asking the suppliers to submit continuous price schedules on the possibility set $A = \{(\alpha, 1-\alpha) | 0 \leq \alpha \leq 1\}$, the agency can restrict the set of possible allocations to be the finite set $B = \{[1, 0], [\beta, 1-\beta]\}$ with β a given number in the interval $[0, 1]$. The particular restriction considered is such that the values C_{-i}, $i \leq 2$, remain unaltered because the allocations $[1, 0]$ and $[0, 1]$ are part of the restricted set B. Therefore, if the unconstrained efficient allocation $a^2 = [a^*, 1-a^*]$ is an element of B, i.e. the restriction does not induce an efficiency loss, then the agency would be indifferent between A and B. If instead the restriction induces an efficiency loss (i.e. $\alpha^* < 1$ and $\beta \neq \alpha^*$), then the agency would rather set β to minimise the marginal contributions of the sellers.

In A, the agency's total transfers to suppliers are given by:

$$c_1(\alpha^*) + c_2(1-\alpha^*) - c_1(1) - c_2(0)$$

If instead the game is B, then the agency's payoff is:

$$\min[c_1(1), c_1(\beta) + c_2(1-\beta)] - c_1(1) - c_2(0)$$

Since $\min[c_1(1), c_1(\beta)+c_2(1-\beta)] \geq c_1(\alpha^*)+c_2(1-\alpha^*)$, the agency strictly prefers scenario B, and the optimal β^* will be such that:

$$\beta^* \in \arg\max[\min(c_1(1), c_1(\beta) + c_2(1-\beta))]$$

For example, if cost functions are given by $c(\alpha) = \sigma_i(\alpha)^2/2$, the set of optimal β^* is $\{[0, \beta']; 1\}$ where β' is a function of the parameters σ_1 and σ_2. When $\beta^* = 0$ or $\beta^* = 1$, the optimal setting coincides with a sole-award setting. Otherwise, it coincides with a split-award setting which is revenue equivalent to a sole-source one.

Partial Set-Aside Awards

Split-award procurement auctions often allow for *set-aside awards,* taking the following form[9]. All participants can bid on a certain specified fraction of the contract, while the remaining fraction is *set aside* for suppliers that are believed to be disadvantaged (small businesses, minority-owned firms, entrants...). Disadvantaged bidders can bid on both portions while the remaining bidders can only bid on the portion which has not been *set-aside*[10]. In our setting, this could be represented by the set of feasible allocations being constrained differently for different categories of bidders.

It is a well known result in optimal auctions theory that a non-discriminating auction may be less profitable than one which discriminates in favour of the bidders with known

[9] In the "Contract Pricing Reference Guide" by the U.S. Department of Defence, partial set-aside awards are recommended whenever one or more small business concerns is expected to have the technical competence and productive capacity to satisfy the set-aside portion of the requirement at a fair market price.

[10] Also in the British third-generation (3G) mobile telephony licence auction, the UK designers considered an auction with 5 licenses of which the largest had been reserved for a new entrant (Jehiel and Moldovanou, (2000), p.3).

lower valuations (Myerson (1981)). Other contributions (Corns and Schotter (1999), among the others) showed how preferential treatment of disadvantaged bidders can increase the auctioneer's revenue in single-unit procurement auctions. In our example we investigate this issue in a multi-unit setting, where the bias can be tailored in two dimensions so as to favour some bidders in some allocations and not in others.

Assume that the agency faces four suppliers, characterised by cost functions $v(\cdot)+\sigma_i$. Let $\sigma_i = 0$ for $i \leq 2$ ("advantaged" bidders or "incumbents") and $\sigma_i = \sigma > 0$ for $i > 2$. The parameter σ can either be common knowledge among market participants or be a random draw from a given support, $[\sigma_l, \sigma^u]$ and be the private information of the suppliers only. We will indeed consider a class of restrictions on which the agency's preferences can be derived regardless of her beliefs on the value taken by σ.

We take it as given that the agency wants to split production between at most two suppliers. She can choose to set up one out of a set of six possible settings. Three of them are "unbiased", in that they do not favour one category of bidders to the other:

- A 2-split award setting in which sole-source outcomes are possible (A^2).
- A 2-split award setting in which sole-source outcomes are not possible (A^{2-1});
- A sole-source award setting (A^1).

The remaining three settings are *biased* split-award settings:

- The first is represented by set B^0, which includes all possible allocation possibilities but those involving *exclusively* incumbent suppliers (suppliers 1 and 2), so that allocations (1,0,0,0) and (1/2,1/2,0,0) are excluded.
- The second biased setting, B^1, excludes just allocation (1,0,0,0), which means that incumbents cannot play as a sole-source for the whole requirement.
- Finally, we consider a setting represented by B^2, which only excludes allocation (1/2,1/2,0,0).

Let us denote with a^r and b^r the efficient allocation under setting A^r and B^r, $r \leq 2$, respectively. Entries in Table 3 represent the efficient allocations in each setting. If the set of efficient allocations is not a singleton, entries represent the allocation that assigns a non-zero quantity to the first bidder in each category. For example, between the equivalent allocations $(1,0,0,0)$ and $(0,1,0,0)$ we would select $(1,0,0,0)$.

Table 3. The efficient allocation.

	$\sigma \leq \Gamma^1$	$\sigma > \Gamma^1$
a^2	$(\frac{1}{2},\frac{1}{2},0,0)$	$(\frac{1}{2},\frac{1}{2},0,0)$
a^{2-1}	$(\frac{1}{2},\frac{1}{2},0,0)$	$(\frac{1}{2},\frac{1}{2},0,0)$
a^1	$(1,0,0,0)$	$(1,0,0,0)$
b^0	$(\frac{1}{2},0,\frac{1}{2},0)$	$(\frac{1}{2},0,\frac{1}{2},0)$
b^1	$(\frac{1}{2},\frac{1}{2},0,0)$	$(\frac{1}{2},\frac{1}{2},0,0)$
b^2	$(\frac{1}{2},0,\frac{1}{2},0)$	$(1,0,0,0)$

where $\Gamma^1 = c(1) - 2c(1/2)$. The only circumstance in which the efficient allocation depends on the relative size of σ and Γ^1 is in B^2. In all other settings the agency would know what types of bidders the efficient allocation involves. Entries in Table 4 represent the marginal contributions of bidders. The only circumstance in which they depend on the relative size of σ and Γ^1 is in A^2.

By comparing the alternative specifications, we will now show that the agency is never worse off by setting up a biased procurement competition of type B^2, in which incumbents can either be sole-source producers or split production with an entrant firm.

First of all we should stress the fact that a restricted set which does not induce a strictly positive efficiency loss is never going to be preferred to the set A^2. Therefore we can already say that $A^{2-1} \precsim A^2$. For the same reason $B^1 \precsim A^2$. Moreover, the agency would be indifferent between settings A^{2-1} and B^1: the players' marginal contributions are identical in both settings and we have $a^{2-1} = b^1 = (1/2, 1/2, 0, 0)$.

Table 4. The suppliers marginal contributions.

	$\sigma \leq \Gamma^1$	$\sigma > \Gamma^1$
A^2	$(\sigma, \sigma, 0, 0)$	$(\Gamma^1, \Gamma^1, 0, 0)$
A^{2-1}	$(\sigma, \sigma, 0, 0)$	$(\sigma, \sigma, 0, 0)$
A^1	$(\sigma, \sigma, 0, 0)$	$(\sigma, \sigma, 0, 0)$
B^0	$(0, 0, 0, 0)$	$(0, 0, 0, 0)$
B^1	$(\sigma, \sigma, 0, 0)$	$(\sigma, \sigma, 0, 0)$
B^2	$(0, 0, 0, 0)$	$(0, 0, 0, 0)$

Compared to A^2, A^1 implies not only an efficiency loss but also an increment equal to $\sigma - \Gamma$ in total marginal contributions if $\sigma > \Gamma^1$. Therefore, among the unbiased settings, A^2 is the one that would never make the agency worse off in any contingency.

Moving to the comparison of the biased settings, we can say that $B^2 \succsim B^0$, as B^0 would imply an efficiency loss of $\sigma - \Gamma^1$ if $\sigma > \Gamma^1$ and no reduction in total marginal contributions. When comparing B^1 and B^2, B^2 involves a reduction of 2σ in the total marginal contribution of (incumbent) suppliers in any contingency, and an efficiency loss equal to σ if $\sigma \leq \Gamma^1$ and equal to Γ^1 if $\sigma > \Gamma^1$. In both cases the efficiency loss is more than compensated by the reduction in the total marginal contributions of pivotal suppliers so that $B^2 \succsim B^1$. Therefore B^2 gives the agency the highest pay-off among the biased settings.

We can now verify that the biased setting B^2 is preferred to the unbiased setting A^2. If $\sigma \leq \Gamma^1$, the efficency loss of σ associated to a mixed incumbent-entrant split as compared to a pure incumbent split is more than compensated by the reduction of 2σ in total marginal contributions, flattened to zero in B^2. If $\sigma > \Gamma^1$, total marginal contribution reduce of an amount 2Γ which is larger than the efficiency loss of Γ^1. The reason why the partially biased setting B^2 is preferred to A^2 and to any other constrained setting is that the efficency loss induced by the bias is minimised in each contingency, i.e. it is given by $\min[\sigma, \Gamma^1]$ as σ varies, and the marginal contributions of pivotal bidders (1 incumbent and 1 entrant) are zero. Indeed, each of the pivotal suppliers has a perfect substitute in the market with respect to the mixed two-split allocation.

4.6 Concluding remarks

In this Chapter we have analysed the decision problem of a revenue-maximising government agency who needs to procure a fixed quantity of a given divisible good and can choose the bid solicitation format out of a limited set of design variations. We modelled the pricing game as a menu auction (a common agency game) with complete information among bidders, where the agency chooses the bid solicitation format by selecting the set of allocations bidders can bid upon out of a set of feasible altenatives. The agency has an incentive to constrain the set of feasible allocations only if this reduces the extent of coordination among bidders, measured by their marginal contribution to the equilibrium allocation, so as to compensates for the efficiency loss that may result from constraining the feasibility set.

We mainly considered environments with a cost structure characterised by increasing marginal costs - implying that split production generates cost economies - and investigated under what circumstances the agency's revenue-maximisation objective does conflict with efficiency. When analysing the choice of the upper bound to impose on the number of possible sub-contractors, we found that, even though the agency is always better off setting the maximum number of sub-contracts so as to make at least one bidder non-pivotal to the equilibrium allocation, further reductions of the number of sub-contracts may not be profitable at all. A crucial role is played by the the agency's beliefs about the degree of complementarity among production units (measured by the extent of economies generated by split production) and the degree of substitutability of potential suppliers (measured by the extent of asymmetries between them, or the distance among their idiosyncratic cost parameters). The more distant the suppliers are between each other, the lower the degree of substitutability and the higher the extent of their coordination. A marginal reduction in the maximum number of sub-contractors may induce an efficiency loss, with a resulting increase in the cost of procuring the given requirement; nevertheless, the extent of coordination among bidders - as measured by their total marginal contribution to the social cost of production - might reduce. Firstly, at least the marginal contribution of one supplier reduces to zero. Secondly, each of the pivotal bidders in the restricted scenario can be substituted by a "closer" bidder, the one the restriction has excluded. Nevertheless, if bidders are expected to be quite far apart, the total reduction in marginal contributions may not compensate for the expected efficiency loss thereby making the restriction not profitable.

Another case considered was one in which the agency could decide whether to bias the competition in favour of a category of bidders so as to increase her revenue. We considered a four-bidders/two-split setting with increasing marginal costs, where two of the participating bidders have a cost disadvantage. We found that the agency gets a higher pay-off when the possibility of the advantaged players splitting production between them is removed. It is not necessary to exclude *all* allocations involving advantaged bidders, but only those in which they would be the only pivotal bidders. This is the circumstance in which their "coordination power" is maximised.

In this paper we have restricted our attention to the case in which bidders do not engage in any pre-bid investment or decision that might alter the total surplus to be shared at the procurement stage. Therefore, in the context analysed, any restriction on the feasibility set can reduce the extent of coordination among bidders only at the expense of an efficiency loss. It would be interesting to analyse the conflict between revenue maximisation and efficiency if bidders make pre-bid decisions (participation to the bidding contest, entry in the industry, investment in cost-saving technology) which are contingent on the agency's choice of the bid solicitation format and that might increase the total surplus produced, or reduce the extent of coordination among bidders. For instance, the agency might decide to allow for allocations that would make investment in a cost-reducing technology profitable to disadvantaged bidders. The latter may therefore gain competitiveness and reduce their "distance" to their competitors and the agency might benefit from this. Another interesting possible extension of the model could be the analysis of the choice of the bid solicitation format in a dynamic context where the agency interacts repeatedly with the same set of suppliers, who may engage in pre-bid investment/entry decisions once the contract specifications have been announced by the agency in each period.

The main limits of the analysis carried out in this paper are the following. First of all, we have confined our attention to games admitting a unique marginal contribution equilibrium so as to make comparisons easier. It would be interesting to analyse the case in which the pricing games admit a multiplicity of truthful equilibria and compare sets of equilibrium pay-offs for the agency in each game. Secondly, we have not provided any insights on the role played by the agency's beliefs on the extent of asymmetries among bidders, as we have confined the attention to situations that did not require specific assumption on the probability distributions over the bidders' types. Moreover, we have assumed complete information among suppliers. The latter is a particularly strong assumption, especially because in multi-unit auctions information asymmetries may not only reduce the agency's payoff, but also induce allocative inefficiencies.

Chapter 5

Conclusions

Asymmetries among sellers can have dramatic effects on the extent of competition in pricing games, and, consequently, on the expected price paid by the buyer. The reciprocal acknowledgement of asymmetries by market participants affects their perception of their relative competitive position and, as a result, the competitive pressure may be lessened and the buyer's expected revenue reduced.

We have examined three examples in which we described the effects of bidders' asymmetries on competition by taking the nature and the extent of those asymmetries as given by the particular competitive situation considered. In other words, we have abstracted from any consideration of how those asymmetries originated. All the economic problems we have studied can nevertheless be extended to allow for a pre-bid stage in which either the buyer or the sellers (or both) can take decisions that affect their asymmetries and, therefore, their competitive position in future stages.

In Chapter 2, we have examined the competition on loan pricing between two banks one of which is linked to the borrowing firm by a pre-existing equity contract that not only gives her an informational advantage over the competitor, but also generates asymmetries in the return that each bank gets from the loan contract. The possibility to extracts surplus via dividend payments reinforces the informed bank's competitive advantage so that the borrower expects to pay higher interest rates the larger is her informational advantage and the larger is the size of the equity stake she holds. It would be interesting to extend this setting by allowing a pre-bid stage in which the firm invites the banks to compete for the acquisition of the equity stake (and associated informational advantage) in the firm. Banks anticipate that the stake can give a competitive advantage in the subsequent loan pricing competition and may be willing to pay a higher price for it. As a result, the overall cost of capital may not increase. Alternatively, the firm could be allowed to decide on the optimal allocation of financial claims (equity and/or debt) and information rights among its investors. Small, family-owned firms, for instance, usually have preferences over different combinations of equity and information rights, simply because giving up equity rights might be more expensive (in terms of loss of control) than revealing information on their business prospects. If dividends and information have a similar effect on lenders' monopoly power in credit markets, the combination of small shares and substantial information rights could represent a convenient way to raise equity finance for this category of firms.

The analysis on the effectiveness of credit subsidies contained in Chapter 3 could be completed and extended in several directions. We have shown that interest subsidies can promote the rise of new bank-firm relationships with small, young firms, which are usually thought to suffer from credit constraints. Nevertheless, when granted to the existing clients of a bank, they may be ineffective because they tend to reach those firms who would have received finance even without a subsidy. These firms tend to substitute existing non-subsidised funds with subsidised ones, so that the surplus generated by the subsidy represents a pure windfall for the firm-bank coalition. We found evidence that the bank managing the programs appropriates of at least part of this surplus by charging higher rates on non-subsidised loans, and that appropriation is more substantial when she benefits of more monopoly power. From an empirical point of view, a natural extension of the work would be to quantify the extent of this appropriation and to identify its relevant determinants. In addition, it would be interesting to provide evidence on the relative performance of alternative forms of intervention (interest subsidies versus loan guarantees, for example) to test whether the way subsidy programs are designed can impact their effectiveness. From a theoretical point of view, we have identified the sources of appropriation in the informational asymmetries between competing banks, or in the fact that banks specialise in different forms of subsidised programs. We have restricted attention to the case in which subsidies are granted to firms who are not credit rationed, as these seem to be those to be reached by the programs. Nevertheless, a proper analysis of appropriation would consider the pricing behaviour of banks when firms are credit rationed without subsidies. Moreover, it would be interesting to consider different categories of borrowers belonging to non-targeted or different targeted sectors, and investigate whether specialisation equilibria can be endogenously derived, where banks develop expertise and specific informational advantages in certain targeted sectors and not in others.

The example analysed in Chapter 4 is the only one in which the the buyer, a government agency, engages in pre-bid decisions that affect the nature of the competition in the subsequent procurement game. Procurement competition is modelled as a menu auction with complete information among sellers, and the agency can decide on which of the feasible organisations of production to include in the "menu". In equilibrium, the price paid to suppliers is a function of the actual total cost associated with the equilibrium organisation, plus a mark-up which depends on the *marginal contributions* of suppliers to this total cost. Asymmetries in the suppliers' cost efficiency play a role in that total marginal contributions depend on the "distance" or degree of substitutability, in terms of cost efficiency, between suppliers who take part to the equilibrium organisation and those who do not. We have restricted our attention to the case in which bidders do not engage in any pre-bid investment or decision that might alter the total surplus to be shared at the procurement stage. Therefore, any restriction on the feasibility set by the agency can reduce the marginal contributions of bidders only at the expense of an efficiency loss. It would be interesting to analyse the conflict between revenue maximisation and efficiency if bidders engage in pre-bid decisions (participation to the bidding contest, entry in the industry, investment in cost-saving technologies) which are contingent on the agency's choice of the bid solicitation format and that might either increase the total surplus produced or reduce the extent of coordination among bidders. Another possible extension is to consider the choice of the bid solicitation format in a dynamic context where the agency interacts repeatedly with the same set of suppliers, who may in turn engage in pre-bid investment or entry decisions once the contract specifications have been announced by the agency in each period.

Appendix A

Proofs for Chapter 1

A.1 Proof of Lemma 3

[a] The continuity of the functions $G^{in}(i)$ and $G^{out}(i)$ are a consequence of the fact that they are *right*−continuous by definition and that the players' pay-off functions are strictly increasing in i for given values taken by $G^{in}(\cdot)$ and $G^{out}(\cdot)$. Consider a point $x \in [\ell, u)$. Given right continuity of $G^{in}(i)$ on x we have $G^{in}(x^{+}) = G^{in}(x)$ and therefore there exists an ε positive and sufficiently small such that $G^{in}(i) = G^{in}(x)$ for any $i \in (x, x+\varepsilon]$. Therefore, $P^{out}(x) < P^{out}(i)$ for $i \in (x, x+\varepsilon]$ and the outsider will put no probability mass on x. Therefore, G^{out} (as well as the expectation $E(i^{out}|\ell \leq i^{out} \leq i)$ is continuous on x. Similarly, we can show continuity of $G^{in}(i)$.

[b] The outsider puts no probability mass, L_{ε} say, on $i = u$ because we have $G^{in}(u) = 1$ from Lemma 2 point [c] and the outsider's expected profits would be negative at u. The outsider will be strictly better off by concentrating the mass L_{ε} on $i = +\infty$ and expecting zero profits.

[c] Given Lemma 1 (point [c]), in equilibrium, the outsider chooses $G^{out}(i)$ so that the insider's expected profits are constant on $[\ell, u]$. Now, suppose $G^{in}(i)$ is constant on some interval $[a, b] \subseteq [\ell, u]$. Let $[a', b] \supseteq [a, b]$ be the maximal of such interval with respect to $G^{in}(i)$. By definition of ℓ, $\ell \equiv \inf\{i; G^{in}(i) > 0\}$, and continuity

of $G^{in}(i)$, we have $a > \ell$. If $G^{in}(i)$ is constant, it follows that $P^{out}(i)$ is strictly increasing on $[a', b]$ and, consequently, $G^{out}(i)$ is constant on $[a', b)$. Since $G^{out}(i)$ is continuous, $G^{out}(i)$ is constant on the whole interval $[a', b]$. It follows that $P^{in}(i)$ is strictly increasing on $[a', b]$, a contradiction to Lemma 1.

A.2 Proof of Proposition 1

Lemma 1 point [c] ensures the non existence of pure strategies equilibria. Lemma 2 ensures that the bidding support of the insider's optimal strategy is $[\ell, u]$. Lemma 3 proves continuity over $[\ell, u)$ for both the insider and the outsider bidding strategies,and their derivatives, over the relevant support. Given Lemma 1 (point [c]) the insider chooses $G^{in}(i)$ so that the outsider's expected pay-off is constant on $[\ell, u)$. Hence, the following equation must hold in equilibrium, where the LHS of the expression is given by (3.1):

$$\left\{ [\theta + (1-\theta) q] \left(1 - G^{in}(i)\right) [\beta(\theta)(1+i) - 1] - (1-\theta)(1-q) \right\} I = c$$

The value of the constant c can be obtained by evaluating the value of the expression at any interest rate in the bids' support. If we set $i = \ell$, then $G^{in}(\ell) = 0$ by Lemma 2. Therefore the constant c is given by the following expression:

$$I\left\{\theta(1+\ell) - \theta - (1-q) - (1-\theta)(1-q)\right\} = c$$
$$I(\theta\ell - (1-\theta)) = c$$

Hence, the insider's optimal bidding strategy $G^{in}(i)$ is given by the solution of the following equation:

$$[\theta + (1-\theta) q] \left(1 - G^{in}(i)\right) [\beta(\theta)(1+i) - 1] - (1-\theta)(1-q) = I(\theta\ell - (1-\theta))$$

Rearranging the above expression we get the following:

$$G^{in}(i) = 1 - \frac{\ell - q i_0^{out}}{i - q i_0^{out}}$$

It can be easily shown that $G^{in}(i^-) = \frac{1-\frac{\ell}{u}}{1-\frac{q\ell}{u}} \equiv 1 - \mu < 1$. Given continuity of $G^{in}(i)$ on $[\ell, u)$ and Lemma 2, which implies $G^{in}(u) = 1$, we can conclude that the distribution has a point mass of $\mu \equiv \frac{\ell - q i_0^{out}}{u - q i_0^{out}}$ at $i = u$.

By maximising $P^{in}(i, \alpha)$ (3.2') with respect to i and solving the associated differential equation.

$$\begin{aligned}
\frac{\partial P^{in}(i;\alpha,q)}{\partial i} &= (1-\alpha)\left[1 - G^{out}(i)\right] + \frac{dG^{out}(i)}{di}\left(i_0^{in} - i\right) = 0 \\
\int \frac{di}{i_0^{in} - i} &= \int \frac{dG^{out}(i)}{(1-\alpha)(1 - G^{out}(i))} + c \\
\ln\left(q i_0^{out} - i\right) &= \ln\left(1 - G^{out}(i)\right)^{-\frac{1}{1-\alpha}} + k
\end{aligned}$$

and setting $i = \ell$ we get $k = (1-\alpha)\ln(\ell - q i_0^{out})$ and therefore:

$$G^{out}(i) = 1 - \left(\frac{\ell - q i_0^{out}}{i - q i_0^{out}}\right)^{1-\alpha}$$

Given continuity of $G^{out}(i)$ on $i = u$, the outsider will put no positive probability mass on u. As a result, since $\lim_{i \to u^-} G^{out}(i) = G^{out}(i) = 1 - \left(\frac{\ell - qi_0^{out}}{u - qi_0^{out}}\right)^{1-\alpha} \equiv 1 - \mu^{1-\alpha} < 1$, the outsider refuses finance with probability $\mu^{1-\alpha} > 0$. It follows that, in order to guarantee indifference between bids in equilibrium, the outsider must earn zero expected profits. And this implies $I(\theta\ell - (1-\theta)) = c = 0$ or, equivalently, $\ell = i_0^{out}$.

A.3 Computation of expression (2.4.1)

We need to show that $\sigma(\alpha, q) = \frac{1-\alpha}{2-\alpha}\left(1 - \mu^{2-\alpha}\right)$.

$$\sigma(\alpha,q) = \int_\ell^u \int_\ell^{i^{in}} g^{in}(i^{in})\, g^{out}(i^{out})\, di^{out} di^{in} + \left[1 - \left(\frac{\ell - i_0^{in}}{u - i_0^{in}}\right)^{1-\alpha}\right] \frac{\ell - i_0^{in}}{u - i_0^{in}}$$
$$= \int_\ell^u g^{in}(i)\left[G^{out}(i) - G^{out}(\ell)\right] di + \left(1 - \mu^{1-\alpha}\right)\mu$$
$$= \int_\ell^u g^{in}(i)\, G^{out}(i)\, di + \left(1 - \mu^{1-\alpha}\right)\mu$$
$$= \int_\ell^u G^{out}(i)\, dG^{in}(i) + \left(1 - \mu^{1-\alpha}\right)\mu$$
$$= \left[G^{out}(i)\, G^{in}(i)\right]_\ell^u - \int_\ell^u G^{in}(i)\, dG^{out}(i) + \left(1 - \mu^{1-\alpha}\right)\mu$$
$$= 1 - \mu^{1-\alpha} - \int_\ell^u \left(1 - \frac{\ell - i_0^{in}}{i - i_0^{in}}\right)(1-\alpha)\frac{\left(\ell - i_0^{in}\right)^{1-\alpha}}{\left(i - i_0^{in}\right)^{2-\alpha}} di$$
$$= (1-\alpha)\left(\ell - i_0^{in}\right)^{2-\alpha} \int_\ell^u \frac{1}{\left(i - i_0^{in}\right)^{2-\alpha}} di$$
$$= \frac{(1-\alpha)}{2-\alpha}\left[1 - \left(\frac{\ell - i_0^{in}}{u - i_0^{in}}\right)^{2-\alpha}\right] = \frac{(1-\alpha)}{2-\alpha}\left[1 - (\mu)^{2-\alpha}\right]$$

Appendix B

Tables for Chapter 2

Table B.1: Sample Statistics

Computations use the cross-section 1999/II.

	Total Sample			Sample of CR firms		
	Mean	S.D.	No. obs.	Mean	S.D.	No. obs.
No. firms	44,047	-	-	6,664	-	-
No. of loans	71,527	-	-	18,181	-	-
No. subsidised firms	22,939	-	-	2,323	-	-
No. subsidised loans	31,506	-	-	4,808	-	-
interest rate	9.63%	3.75%	71,527	8.63%	3.82%	18,181
interest rate on non-subsidised loans	8.87%	3.71%	40,021	7.75%	3.5%	13,373
interest rate on subsidised loans	10.59%	3.57%	31,506	11.09%	3.61%	4,808
interest rate paid by public body	7.59%	3.47%	5,100*	6.61%	3.12%	817*
interest rate paid by firm	3.59	2.82%	5,100*	4.48%	2.25%	817*
firm's internal score	14.94	29.32	42,786	23.30	32.86	6,606
firms with overdrafts	28,179	-	-	2,705	-	-
No.firms for which the bank is main bank	-	-	-	5,371	-	-

*Figures on interests paid arecomputed over quarters where interest payments are actually liquidated.

Table B.2: Effect of subsidies on quantities

Computations use panel 1995/III-1999/II. Quarterly quantities scaled by annual sales.

A	Total sample	Total sample	Reduced sample*	Reduced sample*
Dependent variable:	Total loans	Loans from Bank	Total loans	Loans from Bank
Subsidised	.52	.65	.45	.65
loans	(.00)	(.00)	(.00)	(0.00)
R^2	.07	.03	.04	.02
F-test	30.95	12.49	30.06	11.92
fixed effects	(.00)	(.00)	(.00)	(.00)
No. observations	74,933	74,933	70,057	70,057
No. of groups	6,848	6,848	6,702	6,702

B	CR sample	CR sample	Reduced CR sample*	Reduced CR sample*
Dependent variable:	Total loans	Loans from Bank	Total loans	Loans from Bank
Subsidised	.44	.62	.36	.61
loans	(.00)	(.00)	(.00)	(.00)
R^2	.07	.03	.04	.02
F-test	37.09	11.86	35.77	11.92
fixed effects	(.00)	(.00)	(.00)	(.00)
No. observations	50,051	50,051	46,284	46,284
No. of groups	4,774	4,774	4,624	4,624

*Debtors with only subsidised loans are excluded

Table B.3: Effect of subsidies on quantities: dynamics

Computations use panel 1995/III-1999/II. Variations of average annual quantities scaled by annual sales.

A	Total sample	Total sample	CR sample	CR sample
Dependent variable:	Change in total loans	Change in total loans	Change in total loans	Change in total loans
Change in subsidised loans	.32	-	.23	-
	(.00)	-	(.00)	-
dummy 1	-	.15	-	.05
	-	(.00)	-	(.00)
dummy 2	-	-.57	-	-.53
	-	(.00)	-	(.00)
R^2	.19	.14	.11	.17
F-test fixed effects	1.55	1.59	1.70	1.67
	(.00)	(.00)	(.00)	(.00)
No. observations	1,637	1,637	1,135	1,135
No. of groups	737	737	541	541

Table B.4: Effect of subsidies on quantities: sample splits

Computations use panel 1995/III-1999/II. Quarterly quantities scaled by annual sales.

	Splits by degree of exposure to liquidity constraints			
	Reduced sample*		Reduced CR sample*	
	No overdrafts	With overdrafts	No overdrafts	With overdrafts
Dependent variable:	Total loans (quarterly data)	Total loans (quarterly data)	Total loans (quarterly data)	Total loans (quarterly data)
Subsidised	.50	.37	.39	.30
loans	(.00)	(.00)	(.00)	(0.00)
R^2	.04	.03	.04	.03
F-test	27.99	16.89	30.36	18.91
fixed effects	(.00)	(.00)	(.00)	(.00)
No. observations	51,240	18,814	34,134	12,148
No. of groups	5,410	4,046	4,127	2,496

Table B.5: Effect of subsidies on new relationships

Computations use panel 1995/III-1999/II.

	Sub-sample of firms not registered to CR		
year/quarter (t)	Relations born with a subsidy at time t	Total new relations at time t	Existing relations at time t
95/09	205	1,128	31,908
95/12	944	2,201	32,307
96/03	853	2,931	37,975
96/06	544	1,532	37,051
96/09	781	1,462	36,165
96/12	532	2,200	37,675
97/03	458	1,310	37,408
97/06	705	1,557	37,374
97/09	565	1,264	37,482
97/12	544	1,289	37,404
98/03	551	1,317	37,652
98/06	421	1,233	37,728
98/09	356	1,059	37,773
98/12	426	1,307	37,600
99/03	444	1,296	37,427
99/06	821	1,215	37,733

Table B.6: Effect of subsidies on new relationships

Computations use panel 1995/III-1999/II.

	Sub-sample of firms registered to CR		
year/quarter (t)	Relations born with a subsidy at time t	Total new relations at time t	Existing relations at time t
95/09	58	143	10,440
95/12	98	192	10,516
96/03	92	226	6,680
96/06	19	58	6,799
96/09	15	47	6,572
96/12	14	1,817	9,503
97/03	14	67	9,458
97/06	22	78	9,449
97/09	18	75	9,538
97/12	19	78	9,421
98/03	13	72	9,430
98/06	25	99	9,456
98/09	13	66	9,553
98/12	20	98	9,669
99/03	16	75	6,634
99/06	22	108	10,112

Table B.7: Effect of subsidies on new relationships: summary statistics

Computations use sub-sample of new clients at time t over period 1995/III-1999/II.

	Firms not registered to CR				Firms registered to CR			
	Born without subsidies		Born with subsidies		Born without subsidies		Born with subsidies	
A	mean	obs.	mean	obs.	mean	obs.	mean	obs.
Firm age (years)	9.77	494	7.31	17	13.60	444	14.97	51
Entrepreneur age (years)	45.13	10,632	51.01	22,097	48.45	1,478	49.61	1,577
Sales (ml Liras)	4,812.67	2,215	382.95	82	19,211	962	492.62	162
Leverage	.26	2,215	.27	82	.41	962	.36	162

B	obs.		obs.		obs.		obs.	
Public or no-profit	1,416		45		99		13	
Individual	10,579	2,157*	22,088	2,951*	1,475	235*	1,552	137*
Partnership	4,045		2,655		681		674	
Limited Co.	2,041		1,305		829		200	
Totals	18,081		24,923		18,081		2,439	

*Individual firms with female owners; **quarter following the subsidy

Table B.8: Effect of subsidies on interest rates:price spillovers

Computations use panel 1998/III-1999/II. Quarterly interest rates.

	Reduced sample*	
Dependent variable:	Quarterly average interest rates of non-subsidised loans.	
No. of subsidised loans	0.26 (0.00)	-
Total subsidised loans	-	0.0016 (0.0000)
Firm's internal score	0.01 (0.00)	0.01 (0.00)
R^2	0.03	0.02
F-test fixed effects	18.52 (0.00)	18.55 (0.00)
No. observations	89,742	89,742
No. of groups	24,762	24,762

*Debtors with only subsidised loans are excluded

Table B.9: Effect of subsidies on contractual interest rates: dynamics

Computations use panel 1998/III-1999/II. Quarterly interest rates.

	Reduced sample*
Dependent variable:	Variations of quarterly average interest rates of non-subsidised loans.
dummy 1	.05
	(.12)
dummy 2	-3.54
	(.00)
Variation of firm's internal score	.11
	(.00)
R^2	.12
Fraction of variance due to fixed effects	0
No. observations	67,854
No. of groups	23,670

*Debtors with only subsidised loans are excluded

Table B.10: Effect of subsidies on interest rates: sample splits

Computations use panel 1998/III-1999/II. Quarterly interest rates.

	Traditional measures of market power.			
	Reduced sample*			
Dependent variable:	Average interest rate on non-subsidised loans			
	No. banks = 1	No. banks > 1	% Bank assets =1	% Bank assets >1
No. of subsidised loans	1.11	.24	.50	.19
	(.00)	(.00)	(.00)	(0.00)
Firm's internal score	.02	.01	.01	.01
	(.00)	(.00)	(.00)	(.00)
R^2	.01	.03	.02	.03
F-test fixed effects	18.06	18.58	17.66	19.18
	(.00)	(.00)	(.00)	(.00)
No. observations	9,030	80,814	41,849	47,893
No. of groups	2,451	22,337	11,640	13,266

*Debtors with only subsidised loans are excluded

Table B.11: Effect of subsidies on interest rates: sample splits

Computations use panel 1998/III-1999/II. Quarterly interest rates.

	Informational capture measures of market power.			
	Reduced sample*			
Dependent variable:	Average interest rate on non-subsidised loans			
A	Firms not :registered to CR	Firms :registered to CR	Bank is main bank	Bank is not main bank
No. of subsidised loans	.96	.02	.02	−.81
	(.00)	(.00)	(.00)	(0.00)
Firm's internal score	.01	.02	.02	.03
	(.00)	(.00)	(.00)	(.00)
R^2	.01	.03	.02	.03
F-test fixed effects	17.88	15.21	15.19	8.26
	(.00)	(.00)	(.00)	(.00)
No. observations	80,625	9,117	8,796	325
No. of groups	22,550	2,780	2,721	109

B	Relationship length < 8.5 years	Relationship length ≥ 8.5 years	Sales ≥ 23 ml Liras	Sales ≥ 23 ml Liras
No. of subsidised loans	.03	.0013	.0058	-.08
	(.00)	(.0000)	(.0000)	(.00)
Firm's internal score	.01	.02	.22	.03
	(.00)	(.00)	(.00)	(.00)
R^2	0.05	0.09	0.04	0.05
F-test fixed effects	16.38	13.21	22.66	14.90
	(.00)	(.00)	(.00)	(.00)
No. observations	2,406	3,224	1,286	870
No. of groups	763	1,005	353	236

*Debtors with only subsidised loans are excluded

Appendix C

Proofs for Chapter 4

C.1 Proof of Proposition 4

In order to prove the existence of MCE, we verify that condition (4.3.2') in Theorem 1 is satisfied. We therefore need to compute the values $C^r_{-T} - C^r_{-T\cup S}$, where $S, T \subseteq \Im$ and $S \cap T = \emptyset$:

$$\frac{1}{2}|C^r_{-T}||C^r_{-T\cup S}|\left(\sum_{i\in(\Im\setminus T)_r}\frac{1}{\sigma_i} - \sum_{i\in(\Im\setminus T\cup S)_r}\frac{1}{\sigma_i}\right) \qquad (4.3.5")$$

Note that expression (4.3.5") represents $C^r - C^r_{-S}$ if $T = \phi$.

Inequality (4.3.2') of Theorem 1 is satisfied for every $S, T \subseteq \Im$ such that $S \cap T = \emptyset$ and $S \cup T = \Im$ because $M^r_{\Im} = \infty$. It is also satisfied for all $S, T \subseteq \Im$ such that $S \cap T = \emptyset$ and either S or T is equal to the empty set or contains non-pivotal principals. Let us consider the case $r = n$. We have $C^n - C^n_{-S} = \frac{1}{2}|C^n||C^n_{-S}|\sum_{i\in S}\frac{1}{k_i}$ and $C^r_{-T} - C^r_{-T\cup S} = \frac{1}{2}|C^n_{-T}||C^n_{-T\cup S}|\sum_{i\in S}\frac{1}{k_i}$. Therefore we have that inequality (4.3.1') is satisfied since $|C^n| \leq |C^n_{-T}|$ and $|C^n_{-S}| \leq |C^n_{-T\cup S}|$. Let us now consider the case $r < n$. We can rewrite the term in brackets in (4.3.5") as follows:

$$\sum_{i \in (\Im \backslash T)_r \cap S} \frac{1}{\sigma_i} - \sum_{i \in (\Im \backslash (\Im \backslash T)_r \cup S)_{|(\Im \backslash T)_r \cap S|}} \frac{1}{\sigma_i}$$

The first term in the above expression can be further decomposed into the following:

$$\sum_{i \in \Im_r \cap S} \frac{1}{\sigma_i} + \sum_{i \in \left\{(\Im \backslash \Im_r \cup T)_{|(\Im_r \cap T|} \cap S\right\}} \frac{1}{\sigma_i}$$

We now need to verify that $C^r - C^r_{-S} \leq C^r_{-T} - C^r_{-T \cup S}$. Since we have $|C^r| \leq |C^r_{-T}|$ and $|C^r_{-S}| \leq |C^r_{-T \cup S}|$ and each term in brackets in (4.3.5") is positive we just need to check that:

$\sum_{i \in \Im_r \cap S} \frac{1}{\sigma_i} - \sum_{i \in (\Im \backslash \Im_r \cup S)_{|\Im_r \cap S|}} \frac{1}{\sigma_i} \leq$
$\sum_{i \in \Im_r \cap S} \frac{1}{\sigma_i} + \sum\limits_{i \in \left\{(\Im \backslash \Im_r \cup T)_{|(\Im_r \cap T|} \cap S\right\}} \frac{1}{\sigma_i} - \sum\limits_{i \in (\Im \backslash (\Im \backslash T)_r \cup S)_{|(\Im \backslash T)_r \cap S|}} \frac{1}{\sigma_i}.$

which holds because:

$\sum\limits_{i \in (\Im \backslash (\Im \backslash T)_r \cup S)_{|(\Im \backslash T)_r \cap S|}} \frac{1}{\sigma_i} \leq \sum_{i \in (\Im \backslash \Im_r \cup S)_{|\Im_r \cap S|}} \frac{1}{\sigma_i}$

The LHS sums over the terms $\frac{1}{\sigma_i}$ of bidders in $\Im \backslash T$ who substitute the bidders in S who are pivotal in $\Im \backslash T$; the RHS sums over the terms $\frac{1}{\sigma_i}$ of bidders in $\Im$ who substitute the bidders in S who are pivotal in $\Im$. Therefore, the number of terms on the LHS is less than or equal to the number of terms on the RHS and each term on the LHS is large than or equal to each term on the RHS.

C.2 Proof of proposition 5

Using the expression for M_Z^r computed above, condition (4.3.2) reduces to:

$$z \cdot min(\sigma_{p+1}, \Gamma^{p-1}) \leq \sum_{j=1}^{x} \min(\sigma_{p+j}, \Gamma^{p-1-z+j}) + \sum_{j=x+1}^{z} \Gamma^{p-1-z+j}$$

Each term on the right hand side is less than or equal to any of the identical terms on the left hand side. Indeed consider a generic j :

(a) If $min(\sigma_{p+1}, \Gamma^{p-1}) = \sigma_{p+1}$, then if $j \leq x$ and $\min(\sigma_{p+j}, \Gamma^{p-1-z+j}) = \sigma_{p+j}$ the inequality is obviously satisfied. If $j \leq x$ and $\min(\sigma_{p+j}, \Gamma^{p-1-z+j}) = \Gamma^{p-1-z+j}$ or if $j \geq x+1$, then we have $\sigma_{p+1} \leq \Gamma^{p-1} \leq \Gamma^{p-1-z+j}$. Indeed, $j \leq z$ and $p-1 \geq p-1-z+j$.

(b) If $min(\sigma_{p+1}, \Gamma^{p-1}) = \Gamma^{p-1}$, then if $j \leq x$ and $\min(\sigma_{p+j}, \Gamma^{p-1-z+j}) = \sigma_{p+j}$ we have $\Gamma^{p-1} \leq \sigma_{p+1} \leq \sigma_{p+j}$. If $j \leq x$ and $\min(\sigma_{p+j}, \Gamma^{p-1-z+j}) = \Gamma^{p-1-z+j}$ or if $j \geq x+1$, then we have $\Gamma^{p-1} \leq \Gamma^{p-1-z+j}$.

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