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THE EUROBOND MARKET FOR CONVERTIBLE BONDS AND SOLUTIONS TO SELECTED VALUATION PROBLEMS

by

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ABSTRACT

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The Eurobond Market for corporate debt is estimated to exceed $2,000bn worth of corporate and mortgage-backed bonds, of which there is approximately one eighth with complex equity-linked features, more commonly known as convertible bonds. This thesis is directed towards analysing the nature of these securities and proposing solutions for selected valuation problems associated with them. The work leans heavily on option pricing theory and spans subjects associated with both equity derivatives and risky debt.

The thesis starts with a detailed analysis of the features of convertible bonds. The ever-increasing complexity of these financial contracts makes them one of the most demanding valuation problems in finance.

Valuation challenge associated with complex securities are addressed by developing the decomposition approach that offers benefits in understanding the nature of the security as well as offering a simplified valuation approach. I take callable and puttable zero-coupon convertible bonds, known as *Liquid Yield Option Note*, and present a methodology for stripping it into a portfolio of a callable zero coupon bond, callable option-to-convert, and callable option-to-put. In addition, the issuer's requirement to give a notice prior to exercising its option-to-call is specifically added to the decomposition. Its positive value to the bondholder is also a reason for an issuer to delay exercising its call in order to minimise agency costs associated with refinancing and reduce the underwriting risk.

The overwhelming majority of stocks underlying the convertible bonds that trade in the Eurobond Market pay regular dividends. The inertia towards using simplified dividend modelling by assuming stocks are paying continuous dividend yields (or a constant yield on particular dates) can result in significant pricing errors even for short dated call options, and especially when the cumulative amount of distributed dividend is a significant proportion of the initial stock price. For long-dated and out-of-the-money call options, such as those embedded

in convertible bonds, the pricing error is even more amplified. I explain the differences arising from various assumptions used to model dividend payments; particularly focussing on the influence that the timing of a dividend has on the value of an option. An exact numerical valuation framework is developed that takes into account the constant nature of the forecasted dividend but still allows for individual payments to be affected by sufficiently low or high stock prices. For simple European options that can be valued using the Black-Scholes formula, I propose an adjustment for known dividends based on the dividend duration concept and demonstrate the improvement in valuation accuracy.

In recent years, and especially following the Russian triggered financial crisis in 1998, the research has been focused on understanding and incorporating the risk of default into the pricing of securities and risk management. The centre of attention was afforded to corporate zero-coupon bonds with slight consideration given to more complex instruments like convertible bonds. To re-address the balance, a substantial part of this thesis addresses the issue of the default risk in convertible bonds. I present an innovative yet compatible structural type approach to default risk modelling that allows for flexible capital structure and various recovery scenarios. I analyse the change in convertible bond behaviour in the presence of default risk and apply the proposed methodology on a real life example spanning the period of the Russian financial crisis, showing that it has significant additional explanatory power compared to the standard approach.

To summarise, this thesis offers original research in several selected areas of value to both academicians and practitioners alike. The decomposition approach of Chapter 4 is a valuable teaching tool in explaining complex securities as well as modelling their value. The effect of dividend timing is a rarely addressed subject and its neglect leads to potentially significant mispricing of securities as demonstrated in Chapter 5. The most valuable contributions are perhaps the works presented in Chapter 3 'Analysis of the Convertible Bonds Trading in Eurobond Market' where for the first time the totality of features embedded in present-day convertible bonds is uniquely summed up and concisely presented; and particularly Chapter 6 'Valuation of Convertible Bonds Subject to Default Risk', which is to the best of my knowledge, at the time this thesis is written, the only research to address the issue of default risk in convertible bonds in a structural context without the explicit dependence on the firm value process.

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Acknowledgements

I would like to thank my former colleagues Mr. Sid Amin and especially Mr. Kostas Kaliakatsos for indoctrinating me with this wonderful subject and for many hours of valuable discussions. I also acknowledge that Mr. Kaliakatsos[1] had in part contributed towards my work on 'Valuation of a Liquid Yield Option Note: A Decomposition Approach', presented in Chapter 4.

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I dedicate this work to my father, Dr. Zivojin Jevtić, whose love of science and relentless pursuit of knowledge has forever been a great source of inspiration and admiration. I also dedicate this work to the memory of my mother Danica who always supported and encouraged me to do better.

1 Mr. Kaliakatsos letter acknowledging his partial contribution is lodged with the Faculty Office.

Chapter 1: Introduction

Several years ago, while working at one of the last remaining independent London based brokers, Smith New Court PLC, I was fortunate enough to start a convertible bonds project with two very talented colleagues, which has since inspired my work in this field, and has led to this thesis.

For more than 25 years convertible bonds have been researched with ever increasing frequency in both the academic and professional environments. Many of the most noted contributions will be mentioned in this thesis. My main attention is directed towards those features and the behaviour of convertible bonds commonly traded in the Eurobond Market[2].

My aim is to provide thorough analyses of the features that are present in currently traded bonds within the Eurobond market, and to offer a more in-depth analysis of selected technical issues surrounding convertible bond investments where there has been a scarcity of published research. In particular, the following issues will be covered in greater detail:

- Valuation of Liquid Yield Option Notes via the decomposition approach.
- The effect of dividend timing on the price of conversion options embedded in convertible bonds.
- Valuation of convertible bonds subject to default risk.

This thesis represents a contribution to the current academic work not only by analysing the features of currently traded bonds in great detail, but also by proposing answers to the specific valuation problems that have escaped much of the academic focus, which can be assessed from the literature review presented in Chapter 2.

2 The Eurobond Market (for the purposes of this thesis) is assumed to cover convertible bonds traded over the counter through mainly London based brokers and settling via Euroclear or CEDEL. This generally includes bonds issued by entities all over the world, but is dominated by European companies.

What does it mean when a bond is called convertible?

Conversion refers to bondholder's choice of redemption alternatives. Redeeming in cash is a typical feature of all bonds, but accepting a set quantity of common stock, thus converting cash into stock, can also satisfy redemption. In reality, the market's innovation and development has expanded the feature set of those bonds, and although they are still classed under the generic term of convertible bonds, the complexity of a modern day convertible is way beyond the simple choice of redemption. The full analysis of the convertible bond contracts that currently trade in the Eurobond Market is presented in Chapter 3.

A decomposition approach allows for better understanding of less obvious features embedded within a complex security as well as greater appreciation of how the interaction and inter-dependence of various components are contributing towards the whole. In this sense the approach presented here has both a pedagogical potential and offers an alternative modelling approach when complex securities are valued. The decomposition methodology involving convertible bonds is presented in Chapter 4.

In the face of 94% of stocks underlying convertibles that have regular dividend payments, the modelling of known dividends seems to be an almost forgotten subject, as if all answers were provided a long time ago. This thesis shows that the almost universally used method for incorporating dividend payment into the valuation equation is, even at its best, only a good approximation, and that dividends' true effect on call option value can differ significantly from common methodology, especially on long dated and out-of-the-money options, such as those embedded in convertible bonds. The thesis explains the reasons for valuation error and develops an algorithm for exact dividend modelling when numerical procedures are employed. It also proposes an alternative adjustment for constant dividend payments to the Black-Scholes valuation formula that improves its accuracy in pricing and recovering the implied volatility parameter. An approach that can be used to model known dividend payments exactly is presented in Chapter 5.

Over the recent years, and especially since the Russian triggered financial crisis of 1998, default risk has become a prime focus for academicians and practitioners alike. This thesis contributes to the rarely addressed problem of the valuation of convertible bonds in the presence of default risk. To the best of my knowledge, it appears to be the only work thus far using a structural approach to modelling default applied to convertible bonds with the flexibility to capture the default risk for varying capital structures (a convertible bond can be valued in presence of both junior and senior securities), as well as for varying recovery assumptions including the deviation from the absolute priority rules, without the explicit reference to the firm value process, and thus bringing the benefits of structural approach while retaining the tractability and compatibility with standard models for equity options. Chapter 6 shows that a valuation of this type has an improved explanatory power over the standard model.

The thesis continues with the overview of the current literature and research publications addressing convertible bonds and other subjects mentioned in the preceding paragraphs. The thesis concludes with a summary and pointers for further research.

Chapter 2: Literature Review

2.1 Introduction

There is a substantial body of literature on convertible bonds ranging from the market research perspective, to very technical analysis of the specific features for enlightened academic researchers. The objective of this chapter is to provide an overview of the research that is relevant to the selected subjects of this thesis as well as to provide a more general insight into the state of academic thinking on this subject to date.

Being a much-specialised subject, convertible bonds are mainly addressed via research papers or as a section of a larger work. However, several books dedicated to convertible bonds are worthy of citation as they provide a good balance of both technical and practical insights into the convertibles market. These are:

- "*Convertible Bond Markets*", by George A. Philips (1997).
- "*Pricing Convertible Bonds*", by Kevin B. Connolly (1998).
- "*Convertible Bonds: The Low-Risk, High-Profit Alternative To Buying Stocks*", by Thomas C. Noddings (1991).

Modern theory of finance has been applied to convertible bond analysis since shortly after the seminal work of Black and Scholes (1973) and Merton (1973). Ingersoll (1977a) and Brennan and Schwartz (1977a, 1980) published the two major works, specifically building upon the option-pricing framework to value convertible bonds. They methodically set down the rules for rational convertible bond pricing under the idealised market conditions of Black-Scholes. The underlying risk variable, affecting both the default risk and the conversion value, is taken to be the value of the firm.

Default risk affects convertible bonds via the uncertain redemption amount, which is taken to be either the promised payment at maturity or the value of the firm whichever is lower. On default the whole of remaining value immediately and entirely passes to bondholders.

Brennan and Schwartz (1980) further extend the firm value approach to model convertible bonds allowing for the existence of both senior and junior debt. Brennan and Schwartz introduce the concept of (guaranteed) partial redemption upon default for both senior and convertible debt. Their work was equally important for interest rate modelling, as it was the first to introduce the mean-reverting lognormal process for the short rates. They were also the first to describe the use of explicit finite difference method in the finance context, to solve the resulting partial differential equation.

Subsequent academic research was more targeted towards specific aspects of convertibles. Broadly it can be grouped as:

- Optimal call and conversion strategies
- Default risk
- Convertibles as an asset class
- Specific type of convertibles
- Numerical methods and specific valuation approaches
- Market behaviour

2.2 Optimal Call and Conversion Strategies

Rational call and conversion strategy was formulated in the early works of Ingersoll (1977b): "A convertible security should be called as soon as its conversion value[3] rises to equal the prevailing effective call price (i.e., the stated call price plus accrued interest)". Ingersoll further qualifies the optimal call strategy "will also be that policy which maximizes the market value of the common stock", and therefore "minimizes the market value of the convertible".

In the above definition of the optimal call, the phrase "as soon as" implies that it may be optimal to call the convertible even earlier. Circumstances when an earlier call may be optimal

[3] Adjusted for any special conversion conditions, e.g. entitlement of next/previous dividend payment, delay in receiving the registered stock, etc…

are similar to those applicable to callable straight bonds, namely decreasing interest rate environment and positive change in the credit quality of the issuer, circumstances conducive to obtaining more favourable financing rates.

Putting both conditions together yields a slightly modified formulation of call strategy as being optimal as soon as the convertible price exceeds the effective call price[4]. This is very similar to the formulation used in Chapter 4 of this work dealing with the valuations of Liquid Yield Option Notes (LYONs). I have further relaxed the zero-length call notice period assumption in order to arrive at more precise and realistic valuations.

Ingersoll (1977b) and others presented research evidencing a substantial departure from the optimal policy. Nowhere is this effect more pronounced than in the Japanese domestic CB markets, where only a handful of issues have been called for early redemption despite call features being routinely present[5]. Ingersoll points towards the call notice as a potential reason for delayed call, but finds call notice to be insufficient in explaining the effect. Subsequent research gives several reasons for this:

- Cash Flow Advantage

  Mikkelson (1981, 1983) and Asquith and Mullins (1991) attribute the delayed call to tax advantages of paying interest versus paying dividends, Dunn and Eades (1989) use the sub-optimal behaviour of *passive investors* as a potential explanation for firms delaying calling the bonds. They conclude that firms do behave optimally by not calling the bonds 'as soon as the conversion value exceeds the effective call price' if issuing the call would result in higher total cash payout, due to lower yield of convertible than its underlying stock.

- Relationship Value

  Aldred (1989) points towards the importance of maintaining good investor relationship. He notes: 'If a borrower avoided calling his issue until three years had elapsed or until its conversion privilege had substantial value (substantial

[4] This definition holds assuming no call notice period as in Ingersoll (1977a, 1977b).

meaning a conversion value of at least 140% above the call price) he should be safe from charges of exploiting the investor'. This particular reason is frequently used to explain reluctance of Japanese companies to call their bonds even when they are very deep-in-the-money[6].

- Underwriting Risk

This explanation is most commonly cited in practice and was first examined in detail by Jaffee and Shleifer (1990). It is based on the cost of financial distress, i.e. the risk that a forced conversion initiated by the firm calling the bonds, may fail during the call notice period. The firm may then be forced into (expensive) cash raising transactions to pay the redeeming bondholders. Desire to be protected against this risk (with what would frequently be a disproportionately expensive underwriting agreement) increases the effective call price to the firm and results in delayed call. I would add here that an underwriting agreement is another name for a put option that the bond issuer needs to buy if it wishes to ensure that calling the bond will result in full conversion (and not into a large scale cash redemption). If such insurance is deemed disproportionately high it is signalling that the forward volatility estimate of the underwriter is high, i.e. probability of stock price ending bellow the call price is high.

- Asymmetric Information

Constantinides and Grundy (1986) argue that the firm's decision to call a convertible bond will be influenced by its expectation of future dividend growth. The firm's managers expecting an increase in dividend payments (based on firm's private information) are likely to delay calling an in-the-money convertible bond[7]. Harris and Raviv (1985) present a theoretical case where firms shape their call policies based on privately held information about the future stock performance. Therefore, a long delay in exercising the early

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[5] By the end of December 2001 only one Japanese domestic convertible bond was called, Chugai Pharmaceutical #5, and that was due to the planned merger with Roche AG, see Chadwick and Otsuka (2002).

[6] At the time of this thesis submission, no Japanese domestic convertible bond has ever been called for reasons that may be associated with optimal call policies. The single issue that was called had a corporate merger as the rationale.

redemption option is due to private knowledge of positive news. Ofer and Natarajan (1987) and Tang, Kadapakkam and Singer (1994) provide empirical evidence that supports Harris and Raviv's theory.

2.3 Default Risk

Modelling the effect of the default risk[8] on corporate securities and associated derivative contracts has been a subject of a number of papers with increased frequency over recent years. Researchers have, however, concentrated on the areas of risky straight corporate bonds or the effect that counterparty risk has on the pricing of options or swap contracts. More complex corporate securities like convertible bonds have been occasionally mentioned in passing with a few noted exceptions.

Research into pricing of corporate debt subject to default risk can be broadly split into two types. *Structural models* use the firm value approach and specify the default trigger levels, e.g. Merton (1974), Black and Cox (1976), Ingersoll (1977a), Brennan and Schwartz (1977a, 1980), Kim, Ramaswamy, and Sundaresan, (1993). *Reduced form models* specify the residual value of the bond exogenously, e.g. Jarrow and Turnbull (1995), Duffie and Singleton (1997, 1999), Schönbucher (1997). Reduced form models attempt to value the evolution of the risk-adjusted term structure of interest rates where the initial term-structure is fitted to the current market data, e.g. to term structure of credit adjusted discount factors or risky yield curve.

The earliest published research specifically targeting convertible bonds subject to default risk were Ingersoll (1977a) and Brennan and Schwartz (1977a). They build on the ideas of Merton's (1974) valuation of risky corporate bonds and extend it to convertible bonds.

Shortly after Merton's (1974) work, researchers noted that the credit spreads[9] implied by the firm-value model are too narrow to explain the spreads observable in the market. One of the earliest attempts to introduce more realistic spreads was done by Black and Cox (1976), who studied the bond covenants and introduced the concept of default threshold, a minimum value a

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7 Calling the bond immediately would have the adverse effect of increasing the future total payout expense (total new dividend expense less coupon expense).

8 Frequently the term credit risk is used instead. I will use both terms interchangeably, although it a more rigorous sense it can be argued that reduced form models are modelling the credit risk and structural models are modelling the default risk that in turn determines the credit risk, i.e. the change in credit spreads.

9 Credit spread is defined as the difference in yield-to-maturity of a risky bond compared to an otherwise equivalent risk-free bond.

firm can reach before triggering a default. Defining the default threshold higher than the redemption value of the bonds generated credit spread values much closer to the market levels.[10]

Kim, Ramaswamy, and Sundaresan, (1993) start from Merton's (1974) model but define the net cash outflow as the basis for determining the timing of default thus incorporating the default risk in coupons. They report significantly more realistic values for the credit spreads than those obtainable using the original Merton model.

However, as the value of the firm is empirically difficult to observe coupled with the difficulties of handling more complex capital structures of real-life firms, the models of Ingersoll (1977a) or Brennan and Schwartz (1977a) have limited application in practice.

Within the class of reduced form models of default risk a notable contribution is the work of Duffie and Singleton (1997, 1999). They use the models for stochastic interest rates, but instead of the risk-free rate as the state variable, they use a default-adjusted short-rate process, defined as the risk-free rate plus mean-loss rate[11] (credit spread). Default prone securities are then valued in the usual way using the default-adjusted rate instead of the risk-free rate. Duffie and Singleton (1997, 1999) suggest an extension to their model for cases where the mean-loss rate may depend on a stock price, and suggest that this may be used to value convertible bonds.

Davis and Lischka (1999) took the route suggested by Duffie and Singleton (1997, 1999). They describe a two-and-a-half dimensional valuation approach for convertible bonds. They extend the usual lognormal process for the stock price by addition of a hazard rate, a Poisson process, which in a case of default would force the stock price to zero. Davis and Lischka (1999) then add an extended Vasicek process for the interest rate[12]. The last half-dimension is an explicit inverse relationship between the hazard rate and the stock process. This is to date, apart from the modelling approach proposed in Chapter 6 of this thesis, one of the most complete models of convertible bonds subject to default risk that can be applied in practice and

10 Most popular industry sponsored providers of credit analysis generally adopt this approach, most notably KMV (now owned by Moody's credit rating agency). Other comparable methodologies in commercial use are CreditMetrics proposed by JP Morgan based on Black and Cox (1976).

11 In this context pure stochastic process for interest rate is lost as only risky rate is available within the model that incorporate part the risk-free rate, part the credit spread, but the exact amount of each cannot be determined. This potential short coming is addressed by Schönbucher (1999), who constructs two factor model where the dynamics of risk-free rate and the credit spreads are separated, but given a degree of correlation.

12 For Extended Vasicek interest rate model see Hull and White (1994a, 1994b).

calibrated to market data for interest rates as well as term structure of credit spreads. The model is deficient in the sense that the default prone stock process as constructed for a particular issuer is incompatible with the generally accepted Black-Scholes valuation of options and warrants, and would thus create inconsistency in pricing of options embedded in convertible bonds with options and warrants issued on the same underlying stock.

The idea that exogenously supplied credit spread should be more closely linked to the value of the convertible bond itself was first mentioned in the research note by Derman and Kani (1993) who, in the context of binomial tree valuation, calculate the 'probability-of-conversion' and then scale down the amount of credit spread added to the risk-free rate based on how likely it is for the bond to be converted[13]. The 'probability-of-conversion' is closely associated with convertible bond *parity delta*, i.e. its sensitivity to changes in parity (i.e. underlying stock price).

The approach of Derman and Kani (1993) has at least resolved the issue of numerous discussions between academicians and practitioners on what is the correct discount rate to be applied when valuing convertible bonds. Using the assumption that any convertible bond may be viewed as the sum of the straight bond and an equity warrant[14], where the bond should be valued using the default-adjusted rate while the warrant should use the risk free rate.

Hull and White (1995) in their work on the effect of default on corporate securities conclude that the contracts, whereby a company issues an option/warrant on their own stock, should be treated as default-free securities as the default risk is already contained in the price of the company's stock[15]. If, however, the option is on an other company's stock, the contract should be valued using the default-adjusted rate[16].

Tsiveriotis and Fernandes (1998) introduce a concept of separate valuation of cash only and equity components of convertible bonds. For the cash component valuation is performed using risk adjusted discount rate, while the equity component is discounted using risk-free rate. The full value of convertible bond becomes the sum of the two components.

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13 Strictly speaking, the probability that the bond might be converted or called by the issuer should be taken as a weight when adjusting the discount rate. This finer point was made by Hull (1997), page 529.

14 For detailed description of the decomposition approach see Chapter 4.

15 Additionally, companies can always issue more shares if necessary.

A recent working paper by Takahashi, Kobayashi and Nakagawa (2001) uses the Duffie and Singleton (1997, 1999) approach to value risky convertible bonds in a single factor binomial model with the stock price as the main source of risk. They specify parametric form for, what Duffie and Singleton (1997, 1999) specify as the hazard rate (i.e. an instantaneous probability of default) and perform a comparison test between their model and models of Longstaff and Schwartz (1995) (adapted for convertible bonds), Derman and Kani (1993), Tsiveriotis and Fernandes (1998), and Cheung and Nelken (1994). They conclude, based on the sample of Japanese convertible bonds, that their model, along with Longstaff and Schwartz's (1995) variant, has more relative explanatory value than the other models.

Both approaches of Derman and Kani (1993) and Tsiveriotis and Fernandes (1998), while correctly apportioning credit risk to each component of the convertible bond, take the actual credit risk as a static and exogenously supplied constant. The models assume that regardless of the quality or amount of assets, the firm will actually never default and the bonds, if not converted, will be redeemed at full value. In this sense, models of this type would become particularly inaccurate when used on bonds that are highly risky or in distress. For deep in-the-money convertible bonds these models perform well as the actual amount of default risk is very small. However, their relative simplicity makes them widely used in practice.

2.4 Convertibles as an Asset Class

Research papers in this group concentrate on the reasons for companies and investors to issue/buy convertible bonds and their risk/return profile.

Melicher and Hoffmeister (1997) produce a survey of 118 chief financial officers to identify their reasons for issuing convertible debt. The most frequently cited reasons were to reduce interest expense, to enhance marketability, and to sell equity at a premium over the current market price.

Brennan and Schwartz (1988) analyse the reasons behind a corporation's decision to issue convertible debt rather than straight bonds or pure equity. They conclude that the traditionally offered argument that convertible bonds offer cheaper funding through lower coupons and

[16] In contrast Jarrow and Turnbull (1995) provide a solution to the problem of valuing options on equity with positive probability of default, which uses the default-adjusted discount rate. The additional credit spread is equal to marginal probability of default of an associated risky

enable companies to issue stock at a premium to current market price does not lend itself to rational explanation. Other suggestions that certain investors have restrictions in holding equities does not sufficiently account for the large amount of convertible bond issuance. Brennan and Schwartz offer, as the most likely reason, the relative insensitivity of the convertible bond price to varying and unknown perception of risk associated with the issuer[17]. This, they argue, is particularly attractive to high risk, fast growing companies that account for the majority of convertible issuance.

Lummer and Riepe (1993) examine the long-term risk-return profile of convertible bonds. They conclude that convertibles are a unique asset class that allows investors to experience the benefits of both fixed-income and equity investment. Convertibles are ideally suited for an investment in firms that have higher risk or where the future risk is difficult to assess. Over the long period from 1957 to 1992, the return and risk of holding convertible bonds is between that of stocks and straight corporate bonds.

Alexander and Stover (1977) further show that convertible bond returns are additionally slightly increased by the tendency of the issuers to under-price convertibles when originally issued.

Warren *et al.* (1998a, 1998b) perform a follow-up analysis from Lummer and Riepe (1993) on US and Japanese convertible bonds and report persistent long-term over performance, with convertibles delivering strongly positive returns in falling equity market and equity like returns with but much lower volatility in rising markets. They also conclude that the convertibles are useful in optimising performance in both fixed income and equity portfolios.

2.5 Specific Types of Convertibles

One of the first published practical applications of a contingent claims pricing technique on a real convertible bond was McConnell and Schwartz (1986). They price an asset (which was at the time considered cutting edge in corporate finance engineering) called the *Liquid Yield Option Note*, a zero coupon deeply discounted convertible, callable and puttable bond. McConnell and Schwartz (1986) employ a numerical technique to solve for the bond price with

bond under the augmented jump-diffusion process.

17 This is due to the offsetting effect an increased perception of risk would have on the bond part and warrant part of the convertible.

the stock price being the only source of uncertainty while assuming the interest rate to be constant throughout the life of the bond. Chapter 4 contains further analysis of the McConnell and Schwartz (1986) approach and offers an alternative approach to pricing of LYONs.

In a subsequent paper McConnell and Schwartz (1992) offer a retrospective look at the financial innovation and particular success of the Liquid Yield Option Note structure. They note that although a set of features packed into a LYON is not unusual or particularly innovative, in the preceding seven years Merrill Lynch & Co alone had issued 43 LYONs. They explain the success of LYONs at Merrill Lynch with correctly identified demand of individual retail customers, who were frequently observed buying small quantities of out-of-the-money call options that largely expired unexercised, i.e. investors were willing to expose small amounts of capital for riskier but larger potential gains. LYONs proved to be a good alternative for those clients, who in turn helped make it a success (in the case of LYONs retail investors showed a four fold increase in demand).

Starting from mid 1990 a number of Japanese banks issued approximately US$ 10bn of (subordinated) resettable convertible bonds[18] aimed at the international markets (and most of them with multiple resets and mandatory conversion at maturity). The conversion ratio could be adjusted either up or down (with caps and floors enforced) during the life of the bond and only up at maturity. Further complexity is introduced with callable and sinking fund features. Due to the amount of issuance, liquidity and the huge initial profit potential[19] these bonds were very popular for a while with some investors rumoured to have accumulated recklessly large exposures[20]. Despite the significant size of this sub-class there was no specific academic research published. The commercial researchers equally ignored them until the fall in Japanese equity markets revealed the true extent of misunderstanding and the investors started demanding answers from their brokers.

One of the first to publish about this sub-class was Howard (1997a) who points to the inadequacy of the usual binomial tree models to value the reset convertibles and suggests either

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[18] See Chapter 3 for further explanation of reset feature in section 'Conversion Ratio Contingent on Future Stock Price'. Berger, Klein and Levitan (2000) report, that 40% of Japanese domestic convertibles also have annual reset features.

[19] At the time of issue these bonds were, mainly due to lack of understanding and capable valuation tools, perceived as very cheap. In number of cases price would jump between 5-15% as soon as they started trading in the secondary market.

using a decomposition approach and valuing each embedded option separately, or using a Monte Carlo simulation. In a subsequent research note Howard (1997b) reflects on the previously proposed decomposition valuation approach and notes that is difficult to determine a meaningful delta number (hedge ratio), preferring instead to calculate a better estimate of the delta hedge by using a historic regression of observed market prices.

Other notable research comes from Davenport (1997) who concentrates on detailed analysis of the exact nature of the reset mechanism and its consequence for the investor. Davenport (1997) makes vague reference to possibility of modifying the binomial tree model to value reset convertibles[21].

Berger, Klein and Levitan (2000) analysed the modelling implications of resets and concluded that downward only reset feature combined with American optionality introduces path-dependency into the final payoff and makes the modelling problem notoriously difficult. They also make reference to Hull and White (1993) and Hull (1997) for an example of a possible solution.

A recent breakthrough in the valuation of reset convertibles was made by Hoogland, Neumann and Bloch (2001), who by likening the reset convertibles to look-back options conclude that they only have a *soft* path-dependency that can be solved as an advanced jump-condition on the reset date rather than by introduction extra dimension to the problem as in Hull and White (1993). Hoogland *et al.* present an elegant solution that can be easily incorporated into existing valuation approaches.

An another interesting sub-class of mandatory convertible securities that go under various acronyms such as ACES, CHIPS, DECS, ELKS, PEPS, PERCS, PIES, PRIDES, YEELDS, SAILS, STRYPES, TRACES[22] and number of other mandatory convertible structures, were analysed by Arzac (1997). For the rationale behind these convertible securities he points

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[20] The extent of over-exposure became evident once the degree of the losses arising from these investments become apparent and forced number of investment banks to liquidate their entire books, further exacerbating the negative momentum. See for example "UBS to upgrade its derivatives losses to $421m", Financial Times, January 31, 1998

[21] In a personal communication, Davenport revealed that a binomial tree model based on Hull and White (1993) has been implemented within Salomon Smith Barney in order to value Japanese reset convertibles.

[22] Acronyms, some of them registered trademarks, stand for: ACES – Automatically Convertible Enhanced Securities; CHIPS – Common Higher Incomes Participation Securities; DECS – Debt Exchangeable for Common Stock; ELKS – Equity-Linked Securities; PEPS – Premium Equity Participating Securities; PERCS – Preferred Equity-Redemption Cumulative Stock; PIES – Premium Income Equity Securities; PRIDES – Preferred Redeemable Increased Dividend Securities; YEELDS – Yield Enhanced Equity Linked Securities; SAILS – Stock Appreciation Income Linked Securities; STRYPES – Structured Yield Product Exchangeable for Stock; TRACES – Trust Automatic Common Exchange Securities.

towards information asymmetry that can make the equity offering particularly expensive for smaller, high-growth and highly leveraged or financially troubled companies[23]. In order to fulfil the demand for higher yielding investments, investment banks occasionally issue synthetic securities with similar features that would convert into stocks from large known companies that traditionally pay nil or very little dividend.

All of the above mandatory convertible securities can be valued via the decomposition approach as a portfolio consisting of the underlying common stock, plus the present value of net dividend (receivable less foregone dividend), plus/minus the final equity option (for example in the case of PERCS it is minus call option, etc). A good overview of various terminal payoffs for this class of convertible securities is (graphically) presented and analysed with reference to the usual convertible bond payoff in Sheriff, Berger and Klein (1999).

Hillion and Vermaelen (2000) analysed so called *Death Spiral Convertibles*.[24] These are a special type of resettable convertible preference shares or bonds. The conversion price is continuously adjusted in a fashion of a look-back put, i.e. conversion price is set at the discount to the lowest observed price of the underlying stock over the look-back period (usually two weeks) preceding the conversion request. Hillion and Vermaelen (2000) analyse 487 such bonds issued between 1995 and 1998 issued almost exclusively by high-risk companies as the last resort, and report a significant negative stock performance following the issue.

Hillion and Vermaelen (2000) attribute part of the explanation for such negative performance to the company's own business prospects, while the other part can be linked to the flawed nature of the convertible contract that, in order to create a risk-free security, encourages the holder to make riskless profit by selling the stock short (accelerating further downwards momentum), submitting for a conversion and frequently ending up receiving even more shares

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23 A very recent example of this particular rationale was the 2002 issue of France Telecom EUR 6.75% 2005 subordinated bond exchangeable into stock of STMicroelectronics that is structured as SAILS (France Telecom was a minority shareholder of STMicroelectronics). France Telecom was, during the period of offering, going through severe financial hardship that would make straight equity offering particularly painful. Bonds with such features are specific to the US market and the France Telecom is a rare example of such structure being used elsewhere.

24 This type of convertible is sometimes referred to as "floorless convertibles", "future priced convertibles", "discount convertibles", "toxic convertibles", or "junk equity".

that were needed to satisfy the original short sale[25] causing the stock price to continue down the 'death spiral'.

Death spiral convertibles can cause huge dilution, as stock price falls result in even greater numbers of shares being issued to satisfy the conversion requests. In this respect the wealth of existing shareholders is transferred to convertible bondholder. Following shareholder complaints and negative media coverage[26], Nasdaq Stock Exchange imposed certain restrictions on the terms of such convertible bonds, mainly requiring the existing shareholders approval if the transaction can create dilution in excess of 20%. Hillion and Vermaelen (2000) note that as of May 2000, 61 new issues were announced, suggesting that this type of convertible (with some reasonable restrictions) is still a 'potentially useful financial innovation that, in theory, should be an ideal financing instrument for small risky firms, where agency cost of debt and asymmetric information are large'.

2.6 Numerical Methods and Specific Valuation Approaches

The complex nature of the convertible bond contract meant that an analytic solution to the valuation problem was possible only in very special circumstances and was therefore of limited practical use. The main reason for the lack of analytic solution is similar in nature to the valuation of American style call options, i.e. the existence of a free boundary due to continuous possibility of exercise of the conversion option. If, in addition to the continuous conversion right, I add discrete fixed dividend payments, coupons, holders put option and issuer right to call the bonds for an early redemption (conditional or unconditional), the complexity of the problem is way beyond an analytical solution. Subsequently, the convertible valuation problem has not received overwhelming research attention, researchers preferring instead to address specific numerical techniques in related subjects of exotic options.

Apart from the complexity of the initial and free boundary conditions that form part of the convertible PDE, from the mathematical standpoint convertible bond solution is quite well

[25] Assume an investor short sells a number of shares, based on the conversion ratio at the time of the trade. Owing to the company's bad business performance and downwards momentum created by short selling, the share price declines further, thus resulting in a higher conversion ratio to be set. Conversion now yields more shares than originally anticipated which the investor needs to sell.

behaved in large part (by that I mean that the solution and its first and second derivatives are all smooth and continuous functions). That has enabled many researchers, practitioners and commercial solutions providers to apply relatively simple numerical solvers.

However, the problem quickly worsens once call features are added (first derivative is continuous but not smooth any more and second derivative becomes non-continuous, this is similar to barrier and digital exotic options valuation problems); and worsens further once default conditions are added, as due to the loss-of-value on default event, the solution itself becomes a non-continuous function. In such cases the numerical instability (and non-convergence) problems may become significant unless much stronger numerical solvers are employed.

Ingersoll (1977a) offers several elegant closed form solutions for specific convertible bonds in continuous time settings with the underlying variable being the value of the firm. Notably, zero coupon non-callable/non-puttable convertibles on non-dividend paying stock can be neatly valued as the sum of an ordinary zero coupon bond and a call option on the firm; and for the case of perpetual callable convertible bonds where an additional term is introduced representing the discount due to the call feature. The analytical solutions have been of limited use in practice due to difficulties of estimating the parameters of the firm value process and the restricted set of features that can be taken into account.

Brennan and Schwartz (1977a) were the first to describe the use of what they called a 'numerical algorithm' that in fact was an explicit finite difference method to solve convertible bond partial differential equations under a set of fairly general assumptions. The procedure shown uses value of the firm as the only source of uncertainty.

In their subsequent work, Brennan and Schwartz (1980) extend the valuation problem significantly to include both the capital structure of the firm as well as introduce interest rate dynamics as the second source of uncertainty. The two dimensional PDE was solved using a suitably expanded explicit finite difference method as described in their previous paper.

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[26] Ward and Cahill (2001) report, that at least three lawsuits have been filed by the issuing companies and against the lenders or financial intermediaries that organised the loan. The issuing companies alleged that they were not properly warned about the negative side effects of these loans and that the lenders were engaged in illegal short selling and stock price manipulation. Ward and Cahill (2001) note that several companies have repaid their death spiral loans (including paying the early repayment penalty fees) within weeks of receiving them after the management realised the full impact they had on the stock price performance and dilution.

Cheung and Nelken (1994) describe their implementation of a two-factor convertible bond model based on a binomial technique. They combine the Cox, Ross and Rubinstein (1979) stock price tree with the Kalotay, Williams and Fabozzi (1993) interest rate tree. Their approach takes the current yield curve into account, volatility of interest rates is assumed to be constant over all maturities with no correlation between the stock price returns and interest rate movements. From a practical standpoint, Cheung and Nelken's (1994) paper shows the state of the art in real-life implementation of convertible bond valuation techniques, which has since then formed the basis for several commercially available models[27].

Ferguson, Butman, Erickson and Rossiello (1995) apply bond plus warrant method using the stock price as underlying variable but explicitly adjust for the dilution effect. As there is no closed form solution in this case, they use a binomial tree to solve for the warrant price. Their approach uses the benefits of decomposition and applies a computationally fast closed form solutions for bond and European warrant prices modified to incorporate the effect of dilution using the method described in Hull and White (1997). It has a very limited applicability due to relatively small number of real-life convertible bonds that would have a suitably restricted set of features[28].

Zvan, Forsyth and Vetzal (1998a) describe a finite element method for solving general PDE option pricing. Conditions of stability and convergence are explicitly discussed. They show a two-dimensional[29] Crank-Nicolson[30] valuation example for a continuously callable convertible bond.

In a subsequent paper Zvan, Forsyth and Vetzal (1998b) describe the valuation of delayed barrier and Asian/Parisian options. Although, the paper does not explicitly mention convertible bonds, this type of valuation problem arises with almost every callable convertible bond (see the section 'Analysis of Issuer Call Option').

[27] Nelken was a financial technology consultant to a number of Wall Street equity derivative desks that have initiated this work. Convertible bond models commercially offered by TrueRisk Inc bear many similarities with Nelken's implementation. At approximately the same time a two-factor model was implemented at Lehman Brothers Inc that combined Hull and White's extended Vasicek interest rate model and Cox, Ross and Rubinstein's stock tree.

[28] See Chapter 3 for full description of features in convertible bonds.

[29] Underlying variables are stock price and interest rate. Interest rate process is similar to Brennan and Schwartz (1977b).

[30] For Crank-Nicolson finite difference numerical PDE solver in finance applications context see Wilmott, Dewynne and Howison (1993).

Recent paper from Yigitbasiouglu (2001) combines several sources of risk within the two dimensional valuation problem. Using a change of numeraire stock price and foreign exchange rate risks are folded into one dimension. Second dimension is given to Cox, Ingersoll and Ross (1985) mean reverting square root interest rate process. Credit risk is introduced in a fashion of Tsiveriotis and Fernandes (1998). Yigitbasiouglu (2001) employs a Crank-Nicholson finite difference method with adaptive grid to force time steps to concide with jump events (coupon payments, dividends, etc.). The free boundary condition (the American feature) is posed as a linear complimentary problem[31] and is solved using projective successive over-relaxation method[32].

2.7 Market Behaviour

The first comparison between the Ingersoll (1977a) and Brennan and Schwartz (1977a) model for convertible bonds and the market observed prices came from King (1986). King (1986) studies a sample of 103 convertible bonds between 1977 and 1980 and concludes that the model performs 'reasonably well', with 90% of all model predictions within the 10% of the market values[33] with out-of-the-money bonds being slightly overpriced and in-the-money-bonds slightly under priced[34] by the model. Additionally, King (1986) performs a cheap/dear analysis and concludes that the returns on bonds, identified at the beginning of the sample period as undervalued, significantly exceed the returns for the overvalued bonds.

Janjigian (1987) studies the effect of a firm's decision to call the bond for an early redemption and finds the evidence for abnormal common stock negative returns following the announcement of such action that he attributes to the de-leveraging. Janjigian (1987) also reports the evidence for the opposite effect, following an announcement of convertible issuance, but only for the industrial sectors firms.

31 See Wilmott, Dewynne and Howison (1993).

32 See Zhu and Sun (1999).

33 King (1986) uses the firm value as the main source of risk with initial value and volatility parameters estimated from the stock prices after adjusting for the leverage.

34 Effectively, the firm value convertible models of Ingersoll (1977a) and Brennan and Schwartz (1977a) that are based on Merton (1974), both underestimated the default risk. The result also suggest a positive volatility skew, i.e. positive relationship between the stock price and its volatility, that goes contrary to what can be the observed in practice, where lower stock prices generally relate to higher volatility. For example, as early as 1978 Schmalensee and Trippi (1978) report strong negative relationship between stock price changes and changes in implied volatility of traded option prices.

Following up from King (1986), Carayannopoulos (1996) performs a similar empirical investigation on the sample of 30 corporate convertible bonds over a period of one year starting last quarter 1989. Carayannopoulos (1996) uses a similar model to King (1996) extended to include the stochastic interest rate process as the second source of risk. The interest rate model used was Cox, Ingersoll and Ross (1985), a so-called square root model. The results of this study were similar to previous findings, i.e. model significantly overpriced out-of-the-money bonds, with overpricing being more pronounced for the firms of lower credit quality. After investigating several possibilities for this bias, including the volatility estimate for the firm value process, dividend policy, assumption of continuous coupon payments, etc, Carayannopoulos (1996) lays blame at the market inefficiency due to convertible bonds being treated by most investors as equity instruments who attached more value to equity than to the fixed income part of the bond[35]. Carayannopoulos (1996) performs the same empirical test using the simplified model without the stochastic interest rate element, and reports that so derived prices on average exceeded complex's model prices by 1%, and thus confirming the similar theoretical observation of Brennan and Schwartz (1980).

An interesting research note was presented by Lange, Sommers and Seidle (2001), who analyse the impact on convertible re-hedging activity on pre and post issuance realised volatility of the underlying stock (scaled down by the overall market volatility). They found that overall convertible issuance did not have significant dampening effect on realised volatility. However, in cases of convertible bonds with high positive gamma[36], re-hedging activity had reduced the realised volatility in nine out of ten cases. Although, Lange, Sommers and Seidle (2001) analysed data from the US convertible market, the similar effect can be observed in Europe especially for issuers that previously had lower trading volumes or had no traded options[37].

In a recent study of the French domestic convertible market Ammann, Kind and Wilde (2001) compare market prices with prices derived from three different models of varying complexity (bond + European call option, bond + European option to exchange one risk asset

[35] The same comment applies here as to King (1986). This is further evidence of the firm value model underpricing the default risk.

[36] Convertible bonds with the positive gamma have convex price profile that requires the short stock hedge position to be increased as the underlying stock price moves up and vice versa. This activity, if sufficiently large, creates narrow trading bands as in both directions large re-hedging orders will be placed that would limit the highest and the lowest realised stock prices.

for another, and binomial model as the most complete). Ammann *et al.* (2001) study a sample of 21 liquid convertible bonds over the period of 18 months starting February 1999. They report that market prices are systematically lower than any of the three models tested, with the binomial model being the most precise. Similarly to King (1986) and Carayannopoulos (1996), Ammann *et al.* (2001) find the pricing error to be the highest for out-of-the-money convertible bonds. This is further evidence of default risk underestimation within the models used in the study (their most precise model, the binomial component model, used the Tsiveriotis and Fernandes (1998) technique of incorporating and apportioning the credit risk between the fixed income part and equity components of a convertible bond).

2.8 Conclusion

As is evident, convertible bonds have frequently featured as a research subject. However, most of the research attempts to limit the complexity of the convertible bond in order to simplify the direction of the specific researched topic or to make the valuation problem more tractable.

Addressing a complex subject with many interrelating parts spanning the fields of equity derivatives, yield curve and default risk problems, can be overwhelming. Attempting to strip a large problem into constituent parts is how the work on LYON in Chapter 4 came about. The work had not only filled a void in the literature, but was useful to me to fully understand the nature of the structure being studied and also led into a fully functional valuation tool that has been successfully used in my subsequent research.

Extending from the same premises that have influenced the work on decomposition, the complexity of real life convertible bonds has not previously been analysed in great detail. That prompted me to take a critical look into the actual structure of convertible bonds and after studying numerous prospectuses reported my findings in Chapter 3.

In many cases bonds are assumed to be non-callable or convertible only at maturity. Sometimes the continuous nature of the conversion option is circumvented by assuming that the underlying stock pays no dividend, or at best that it pays a continuous dividend yield. While many of the above assumptions were successfully relaxed in related subjects in option pricing theory, on the

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[37] An example is Infineon Technologies AG stock that had 69% volatility before the issue of the first convertible bond (79 days from the

issue of exact modelling of known dividend payments I have found a distinct lack of published research. It was after I had the opportunity to examine one of the well-known commercially available solutions for option pricing and noted that the dividend-timing problem is incorrectly handled, the idea to perform a detailed research of this subject appealed quite naturally to me. It was, and still is, surprising to me that such a seemingly omnipresent issue has escaped rigorous research attention. The work on dividend timing is presented in Chapter 5.

Credit risks and the default issues have come into prominence during and after the Russian financial crisis of 1997-1998. While observing the price behaviour of convertible bonds, it was apparent that traditional valuation models were systematically underestimating the credit risk and providing incorrect sensitivity parameters and subsequently wrong hedges were applied. After studying the published and un-published research, it became evident that there was a gap, especially with regards to convertible bonds. Most frequently credit risk was, if considered at all, taken into account but only as a deterministic variable. Various other approaches that have been developed concentrated on straight bonds, while early works of Ingersoll (1977a) and Brennan and Schwartz (1977a, 1980) that specifically addressed convertible bonds depended on data and variables that would be very cumbersome or impossible to estimate if a practical application were considered. My work as presented in Chapter 6 'Valuation of Convertible Bonds Subject to Default Risk' is unique in providing a structural approach to default risk modelling that can be applied without reference to firm value process and is dependent on data that is generally available or easy to estimate.

Trying to find the answers on how to value real life convertible bonds have guided my own research effort and the structure of this thesis. In this sense I hope that a number of gaps in the available research will be at least partially filled with suitable and easily workable solutions that I humbly present in this thesis.

flotation). Subsequent 79 day volatility has dropped 60%.

Chapter 3: Analysis of the Convertible Bonds Trading in Eurobond Market

3.1 Introduction

The convertible bond market has grown in size and popularity over the past decade. Approximately half of the global issuance originates from Europe. Currently the total sizes of outstanding issues are well in excess of $100bn representing approximately 1.5% of total European equity market capitalisation[38].

This chapter provides an overview of the Eurobond Market for convertible bonds and gives detailed analysis of the features present in a variety of the convertible bond contracts that actively trade. While the previous chapter reviews the state of the current research, the aim here is to identify the extent of financial innovation presented in this product. Understanding the nature of particular provisions is a prerequisite for discovering and implementing methodology for the valuation of convertible bonds, as well as identifying potential pitfalls and explaining pricing differentials.

3.2 Overview of the Market

My analysis of the convertible bonds issues concentrates largely (but not exclusively) on the sample of convertible bonds in issue as of beginning of November 1999. In total 661 convertible bonds were identified as trading in the Eurobond market at that time. To make the sample more manageable while remaining representative, selection criteria based on a minimum rating requirement, A3/A- or better, issue size of more than EUR 50mm, and more than 6 months to maturity are employed, thus ensuring reasonably liquid secondary market activity and more reliable historic data. The sample consists of 183 bonds with the total issue size in excess of US$ 73bn. Please refer to Appendix 3-A for the full list of qualifying issues.

[38] See EuroWeek February 2001.

To get a better insight into the composition of the most liquid part of the market, I'll present some statistical information[39]. The regional split of the issuers is show in the figure below.

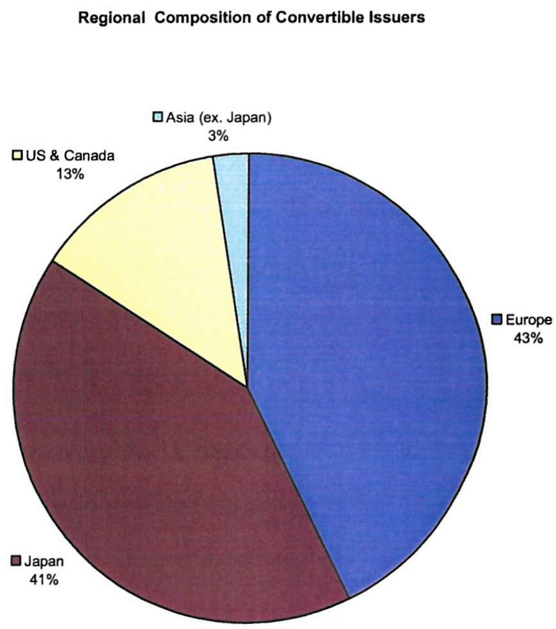

Figure 3-1. Regional composition of convertible issuers.

The European and the Japanese issuers, who in total account for 84% of the sample, dominate the market. The US and the Canadian issuers account for 13% while Asian companies account for the remaining 3%.

The high participation of the Japanese issuers is not surprising considering that as of 1999 Japan had globally the highest amount of convertible bonds outstanding. They were subsequently overtaken by the US as the majority of new issues came from there during the TMT[40] boom.

The relatively low percentage of North American companies can, to a degree, be explained by the traditional preference of the highly rated North American companies to issue straight

[39] It is fair to assume that the sample is representative of the market as a whole as the most activity would be among the higher rated issues.

[40] TMT is an abbreviation frequently used over the last few years. It refers to companies belonging to the technology-media-telecommunication sectors.

bonds, while riskier companies would favour convertible bonds in order to reduce funding costs[41]. Additionally, riskier North American issuers concentrate their marketing efforts to the US and Canadian domestic markets and would rarely feature in the Eurobond Market.

The low participation of Asian (ex. Japan) issuers were mainly due to the generally small number of highly rated companies in this geographical region.

Figure 3-2 illustrates the high level of issuance over the last few years. According to figures published by Morgan Stanley Dean Witter from the combined equity and equity-linked offerings between October 1997 and October 2000, approximately 18% was raised via convertibles.

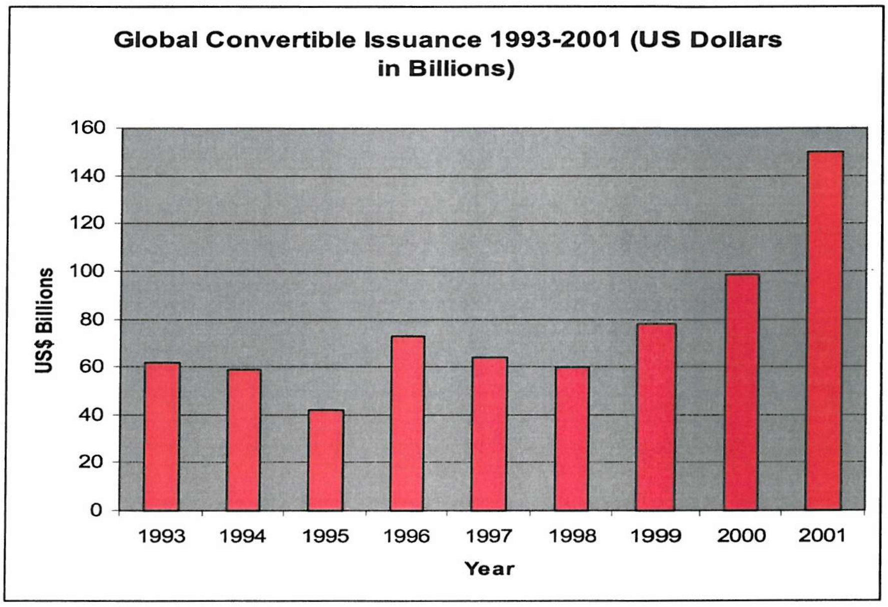

Figure 3-2. Global convertible issuance 1993-2001.

A clear jump in overall issuance over the last three years is associated with the Internet boom. The issuance from the US contributed more than all other regions put together. The second largest amount came from Europe. Year 2001 marks a historic turning point for convertible bonds: for the first time in history, the amount of new capital raised in the US markets via convertible bond issues outstrips the straight equity offerings.

The next graph shows the average number of years until maturity for the selected sample.

41 See Section 2.4 for further explanation.

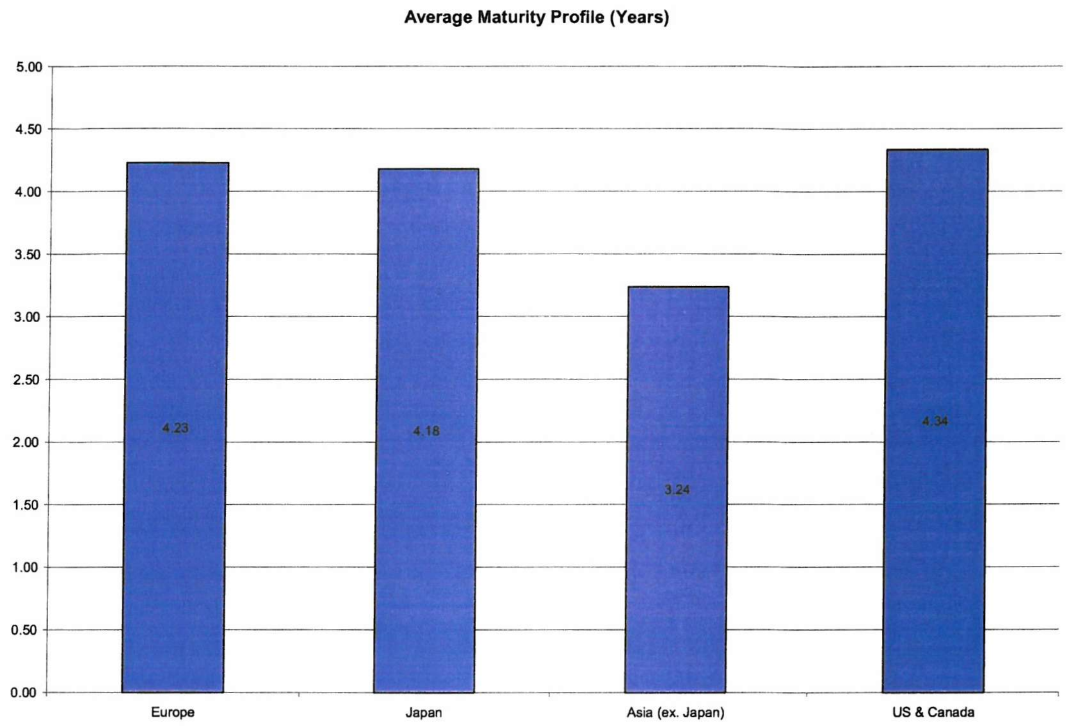

Figure 3-3. Average maturity profile

The average maturity across the developed markets (Europe, Japan and US) is very similar, at approximately 4 years and 3 months. The Asian (ex. Japan) market subsection is about one year shorter, again pointing to the higher perception of the investors about the risk in the region and subsequent lack of the demand for longer dated paper.

Figure 4-4 presents the issuers' industrial sector dispersion of the sample. The largest sector is the electronics that mainly consists of computer and semiconductor manufacturers. Japanese and other Asian issuers make up a large proportion of this category. The second most frequent issuance comes from the financial sector, i.e. banks closely followed by the insurance sector. This is the largest sector for the European issuers. Other prominent sectors are telecommunications, utilities (electricity, gas, water) and transport (mainly railways and airlines). Interestingly sovereign issuers are present, mainly due to the efforts of governments to affect delayed privatisation programmes by issuing convertible bonds that can be exchanged into state owned companies[42].

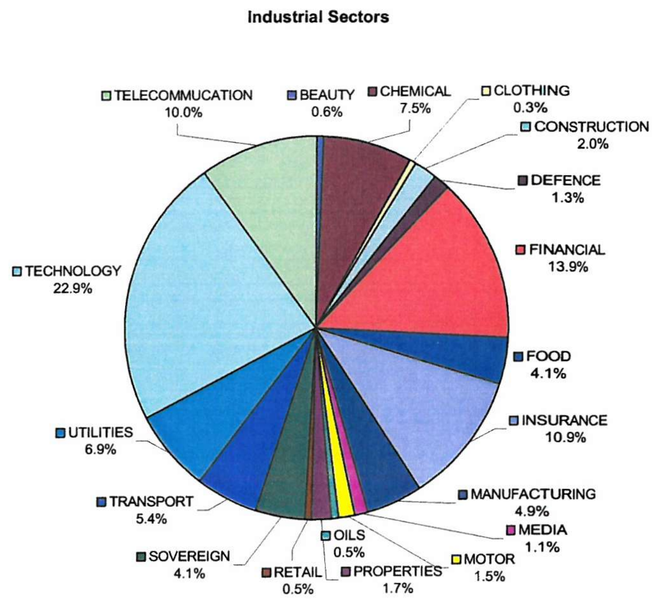

Figure 3-4. Industrial sectors

The following figure shows the regional issuance ratio between bonds issued in domestic currency (domestic to the issuer of the convertible bond) and foreign currency[43].

[42] This type of bond became very popular among the hedge fund investors as they merged good liquidity with a AAA credit rating. The most popular issues were from Republic of Italy USD 5% 2001 that can be exchanged into Istituto Nazionale Delle Assicurazioni, Singapore USD 0% 2004 exchangeable into DBS Bank and Hellenic Republic EUR 2% 2003 exchangeable into National Bank of Greece.

[43] Foreign currency in this context is most frequently US Dollar and then EURO and rarely Japanese Yen.

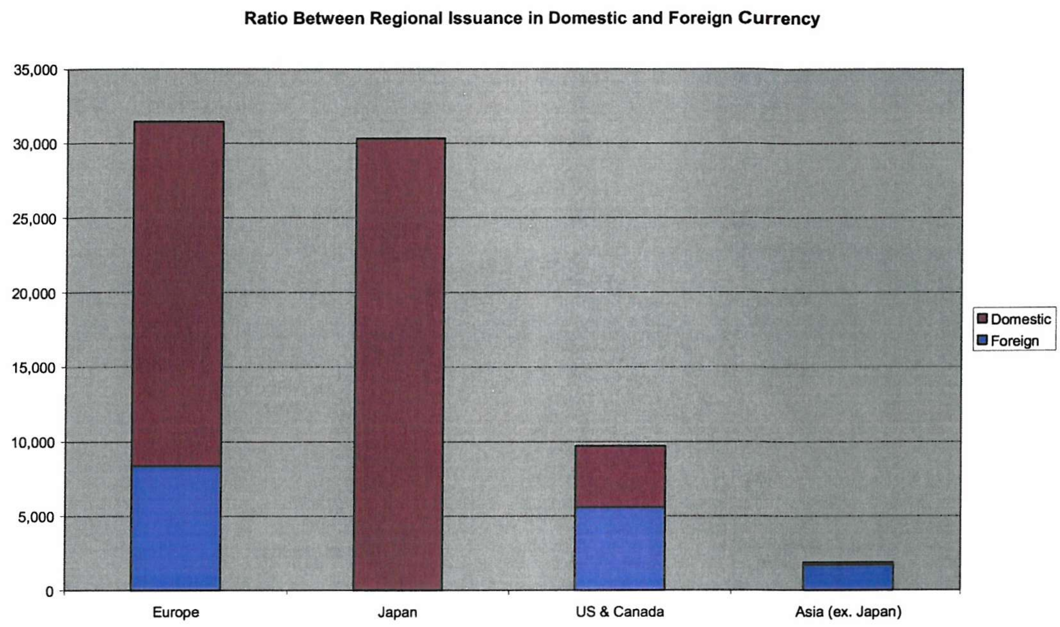

Figure 3-5. Ratio Between Regional Issuance in Domestic and Foreign Currency

The first feature to notice is that the sample contains no Japanese bonds issued in any currency other than Yen. In practice this is not the case as there is for example a sub-market in Swiss Franc and US Dollar Japanese convertible bonds. The main reason why none of this appears in my sample is due to the rating restrictions (Swiss Franc Japanese convertibles are used mainly by smaller companies that would rarely have an investment grade rating). Also, the US Dollar issues are particular resettable structures that do not count as ordinary convertibles (for example USD 2bn Mitsubishi Bank USD 3% 2002 mandatory resettable convertible).

The second feature is the almost complete absence of domestic currency convertible bond issue in Asia (ex. Japan). This can be partially explained by the relatively small demand for convertible bonds in domestic markets and general export orientation of the issuing companies that actually need foreign currency to further finance their operation.

The apparent high level of foreign currency issuance in US and Canada is mainly due to two large issues from Verizon Inc (formerly Bell Atlantic) that can be exchanged into stocks of New Zealand Telecom (domestic currency New Zealand Dollar) and Cable and Wireless Communications (domestic currency British Pound). Apart from those two bonds, which should be regarded as exceptions, the US and Canadian market is dominated by domestic currency bonds.

European issues, although mainly in domestic currency, include a fair amount of US Dollar denominated bonds, mainly by companies with significant exposures to US markets.

Other common features of importance for efficient valuation that are present in all bonds are:

- Fixed Maturity
- Fixed Coupon
- Bonds have exchange property (bonds can be exchanged/converted for a holding in a common stock on or prior to maturity).
- Average term is less than 5 years.
- Approximately 42% of the qualifying convertible universe has no other features in addition to the above.

Other frequent features are:

- Bonds are subject to an early redemption via the issuer call option, which would usually become effective 2 years after the issue date. This is present in approximately 31% of the sample.
- If the bonds are callable, a further, *stock trigger*, condition may need to be satisfied, before the issuer is allowed to call the bond. This is present in approximately 22% of the sampled issues.

Less frequent feature

- Currency of the underlying stock is different from the currency of the convertible bond. ~ 18.1%.
- Bonds may have an early redemption clause at the option of the bondholder, i.e. a put option. ~3%.

- Upon exercise of the exchange option the issuer may have a choice of delivering a cash equivalent of the underlying asset, rather than the asset itself. The exact amount of cash may be subject to small adjustments, from the observable underlying asset price. ~13%.
- A quantity of the underlying asset may be altered at certain pre-specified dates in the future in the bond holders favour if the level at which the underlying asset trades is below the initially set level, i.e. downward reset clause. ~5%.
- Maximum redemption value is subject to an overall upper limit, i.e. capped bonds. ~1%.

General properties of the stock into which bonds can be converted/exchanged:

- Stocks are liquid and available for borrowing/lending.
- Stocks are quoted on one or more exchanges.
- Stocks pay regular dividend.

3.3 Analysis of Bond Component

3.3.1 General Characteristics

A **bond** is a contract (as specified in the bond prospectus) according to which two parties enter into a loan agreement. The party lending is referred to as an investor or a bondholder (holder for short) and the party borrowing the funds is known as bond issuer or issuer for short. The contract specifies the schedule for repayment of the borrowed funds (*redemption*), payment of periodic interest (*coupon payments*), events of default and resulting procedure for the recovery of the funds[44].

A **convertible bond** commonly refers to bonds that in addition to the above have an *exchange property* feature.

[44] Holders would have a more senior claim on the residual assets in the case of liquidation. Unless the issuing entity is in default, holders would have no voting rights.

Coupon payments are usually fixed in advance and payable in regular intervals. Failure to pay any of the coupons or the redemption amount in full, or adhere to other specific conditions of the bond contract, triggers the default event, which may force the issuing entity into liquidation[45].

Par amount is the face value of a single unit of bond contract. It is the smallest tradable amount (trading unit). A trade size must be a whole number multiple of the par amount. Bonds are quoted in the currency of the par amount. Exceptionally, bonds may have dual currency par amount[46].

Bonds are initially sold at their par value. More rarely the initial price is less than par, a so called *discount issue* or above par, i.e. *a premium issue.*

Although the par amount (face value) can be arbitrarily set, the most popular values are USD 1,000 or EUR 1,000. The exception is the French market where the face value is set to equal the *conversion price* of the bond, making the conversion simple and intuitive, as one bond unit would exchangeable into one unit of the underlying stock.

Holders are entitled to periodic cash payments known as *coupons*. The amount receivable is known as a *coupon rate* and is expressed as a percentage of the bond's par amount. *Zero coupon* bonds pay no coupons.

Coupon payment periods are initially set in regular time intervals, e.g. quarterly coupon, semi-annual coupon, annual coupon, etc. However, in practice the actual payment date may need to be adjusted for non-working days to the following or preceding business day with reference to domicile of the issuer, or the clearing agent. The amount of coupon generally does not increase/reduce for a business day adjustment. The business day adjustment method and the potential coupon payment adjustment would be defined in the bond prospectus.

Another 'administrative' issue is the calculation of the coupon amount for partial payment periods, also known as *accrued interest calculation*. This situation arises when bonds have long or short first/last coupon, when bonds are called for an early redemption, when bonds are put

[45] Usually a bond trustee would be appointed to observe bond issuers' compliance with the contract provisions.

[46] See the section (3.3.3.6) on redemption in currency other then the par amount currency.

back to the issuer between the coupon days[47], and most often when bonds are traded *clean of accrued.* As with the business day adjustment, the method for calculating the accrued interest is known as *day count* method and is defined in the bond prospectus. Across the sample of bonds used in this chapter, approximately 50% of (all Asian ex. Japan, majority of European and all US & Canadian) bonds use 30/360 method, 41% (mainly Japanese) use Actual/365, and the remaining 9% (mainly French) use Actual/Actual day count. The day count conventions are listed in the Appendix 3-B.

3.3.2 Trading and Settlement Conventions

An overwhelming majority of the bonds are quoted and trade according to the Eurobond convention. That effectively means:

- Bonds are quoted as a percentage of their par value, clean of the accrued interest.
- The quantity of the bonds traded is specified as cumulative notional amount, e.g. buy £250,000 of bond X. If bond X par value is £5,000, this is equivalent to buying 50 bond units.
- Bonds are settled on a T+3 basis, i.e. three business days after the trade date, for trades completed before 5pm London time. Trades after 5pm are next-day trades[48].
- Consideration payable on settlement is calculated with reference to the quantity traded, price, and adjusted for the amount of accrued interest.
- The accrued interest is calculated from last payment date (not business-day adjusted) until the settlement date, using Euro 30/360 convention.

In the case of French domestic bonds the trading and settlement convention is that of the underlying shares:

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47 In some rare cases, the accrued interest is payable on conversions as well.

48 Involved parties may in practice agree different settlement convention.

- Bonds are exchange quoted, as price for one nominal unit (cum-accrued interest).
- The quantity of the bonds is specified as number of bond units.
- Bonds are settled on a T+3 basis (this is opposite to French equities, which are settled on a settlement account basis)[49].
- Consideration payable on settlement is calculated with reference to the quantity traded, price (no adjustment for the accrued interest as it is already included in the price).

3.3.3 Coupons and Redemption

3.3.3.1 No Coupon

This is the simplest case to analyse, and quite popular among the convertible issuers. Bond is set to have only one cash flow, the redemption payment at maturity. These bonds are frequently referred to as *zeros*.

3.3.3.2 Fixed Coupon

The overwhelming majority of convertible bonds fall into this category. The coupon frequency usually follows the convention of the fixed income market of the bond's currency, i.e. USD semi-annual, EUR annual, CHF annual, JPY semi-annual, etc.

First and last coupons may have a non-standard term, i.e. long or short coupon, in which case the amount payable is adjusted in accordance with the adopted day-count convention.

All coupons other than the first and the last are usually paid in identical amounts. Rarely bonds may be specified in advance to have several different coupon rates, each valid during a certain

[49] With the introduction of the EURO currency, the Paris Stock Exchange has adopted a T+3 settlement convention.

date period. For example a nine-year bond may pay 4.1% coupon in the first two years, and then move to 5.55% for last seven years[50].

Some bonds pay fixed rate coupons at first and then change to floating rate[51].

3.3.3.3 Floating Coupon

There are not many examples of floating coupon convertible bonds. The exact coupon rate for these bonds is not known in advance and is set with reference to an *index fixing*, usually 3 or 6 month LIBOR. The actual coupon rate is set as the value of the fixing plus a constant *spread*. The spread is set at the time of the bond's issue.

A small number of convertible bonds use the dividend rate on the underlying stock as an index for determining the coupon rate. For example the coupon is set up to give a yield equivalent to current year dividend yield increased by a given spread and often subject to a minimum coupon rate. Examples of convertible bonds with such features are

Ivaco Inc	CAD div+2.50s	2010
Thornmark Equities Inc	CAD div+1.00s	1998
Inmet Mining Corp	CAD div+1.00s	2007
Noranda Inc	CAD div+1.00s	2007
KBC Bancassurance	BEF div+2.00a	2003
Wuert Ag	DEM div+1.00a	2004

In the case of the Axa S.A. 2017 convertible bond, coupon payment can be suspended if dividends are not paid on any other Axa S.A. security that is subordinated to the bond (which itself is subordinated).

[50] For example: Banque Colbert FRF 5.55 2002 (France).

[51] For example: Credit National FRF 5.625% 2003 (France). This bond was issued in 1996 with the fixed coupon of FRF 5.625% until January 2001, thereafter the coupon becomes floating 6 month LIBOR+2.5%.

3.3.3.4 Coupon Payments Contingent on Stock Performance[52]

This type of feature started to appear in recent US convertible bonds. Bonds were officially issued as zero coupons but that might change during certain future (quarterly) periods based on the extent of stock underperformance at that time. For example, the following is an extract from Verizon Communication Inc. USD 0% 2021 prospectus, page 13, where coupon rate adjustment (referred as per annum Reset Rate) is specified as:

> "Beginning on May 15, 2004, if the closing sales price of the common stock of Verizon Communications is equal to or less than 60% of the Accreted Conversion Price of the notes for any 20 trading days out of the last 30 consecutive trading days ending three business days prior to such date or three business days prior to May 15 or November 15 thereafter, then the accretion rate on the notes for the semi-annual period commencing on such date will be subject to an increased accretion rate equal to the applicable per annum Reset Rate in effect at that time."

As of the time this thesis is written, bonds with such features have not appeared for any non-US issuers.

3.3.3.5 Coupon Payments Contingent on an External Event

I am mentioning this type of coupon payment contingency more for completeness and its novelty value; as such bonds are very rare. In 1997 Winterthur Insurance[53] (Switzerland) issued a convertible bond maturing in 2000 with each annual CHF 2.25% coupon payment being contingent on there being less than 6,000 successful car damage claims made related to icy rainfall for the year. I have no information on how many coupons were actually paid, but when this bond was issued, the average number of claims quoted by the lead manager was in the region of 4,500.

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[52] Bonds with such features started to appear in the US only during 2000 are not the part of the representative sample. They are discussed here for completeness only.

[53] Winterthur Insurance has subsequently merged with Credit Suisse Group.

3.3.3.5.1 Premium/Discount/Zero Redemption Value

Approximately 17% of the qualifying issues redeem with an amount different than its par value. In recent years it has become quite popular to issue bonds with a lower coupon rate and shorter maturity, and have them redeem at a premium to par. Structuring bonds in this way allows an issuer to achieve a yield-to-maturity as required by the market while keeping the annual coupon expenditure to a minimum. Issuers, from their perspective, believe that the strong stock performance over the coming years, would lead to certain conversion so that the high cash redemption price will never be actually paid.

A small number of issues, approx. 1%, are defined as mandatory conversion issues. At maturity, bondholders automatically become stockholders. This effectively implies a zero redemption value. Bonds with this feature are commonly referred to as *mandatory convertible.*

Mandatory convertible bonds are frequently further spiced-up by having the final conversion ratio dependent on the stock price at expiry. This feature is analysed in the section dealing with *time varying conversion ratio* features.

3.3.3.6 Redemption in Currency Other Than Par Amount Currency

Some issues, mainly from Thailand, are structured to have redemption specified as a fixed local currency amount, which the issuer will convert into the bond's issuing currency at the exchange rate prevailing at the maturity time. Out of 50 Euro-bond issues from Thailand, 18 have this feature.

Handling of such features requires much more precise estimation of the local currency yield curve and forward foreign exchange rate. Given the nature of the emerging markets, this is difficult and sometimes an impossible task. None of the bonds in the qualifying sample contain this feature and are included here for the sake of completeness.

3.3.3.7 Phased Redemption – Sinking Fund Features

A bond contract may specify that a fixed percentage of the issue must be redeemed over a number of years. This feature is known as *sinking fund.*

In some cases, the phased redemption is being specified as optional blurring the sinking fund feature. Defined in such a way the sinking fund appears very similar to the standard issuer's call option, especially when partial calls are allowed.

3.3.3.8 Default and Recovery

Events leading into a declaration of default are set in the bond prospectus, and relate to the non-payment of coupons or the redemption on the pre-specified date, and other specific provisions[54].

The first notification of the potential breach of contract would come from the clearing agent, e.g. Euroclear or CEDEL, which would notify the holders on the register as to the failure of the issuer or its agent to deliver the funds on time.

The bond's custodian would then issue a notice to the issuer giving them 1 week to comply with the contract requirements, before declaring that a default has occurred. Backed by this information the bond's trustee would initiate the proceedings for the recovery of the funds.

At this point the exact route and the time taken depend on the jurisdiction of the issuer. Needless to say, it may take years until the situation is fully resolved.

Following the declaration of default[55], bonds still continue to trade, albeit with reduced liquidity and with a larger bid-offer spread. Bonds are quoted as an all-in-price, i.e. no accrued interest is added.

The amount ultimately recovered from the issuer is difficult to predict. However, historical analysis of the past default situations[56] suggest a strong link between the rating of the issuer and the percentage of the par value recovered. Moody's Investor Service reports one-year default rate for A3 or better issuer at a rate of 0.01%. The long-term volatility (over the period of 66 years) of the default rate for this rating grade is also very low at 0.3%.

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[54] This can include cross-default provision, dividend payment restrictions, restriction on entering into another loan agreement of the same or higher seniority claims, restrictions on entering into mergers and acquisitions, restrictions on large capital expenditures, imposition of the minimum net asset worth, etc.

[55] The presence of the official declaration is important, as it would trigger the default protection options.

[56] See "Historical Default Rates of Corporate Bond Issuers, 1920-1996", Moody's Investor Service, January 1997.

The default risk is most commonly expressed as a *yield spread* over a default free bond (government issued bonds). The additional yield provided is interpreted as compensation to holders of such securities, for taking the additional risk.

3.3.4 Analysis of Exchange Property

In this thesis I am concentrating on bonds that can be exchanged into equity type assets. These are stock issued by either the same entity issuing the bonds themselves or stocks of some other companies.

To further specify the exchange option it is necessary to look closely at what is often referred in the bond prospectus as *the exchange property.*

> Exchange Property is the right but not the obligation of the bondholder to exchange his/her bond holding for a holding in another asset, known as an underlying security.
>
> The exchange option may be defined as exercisable continuously (American style), on certain dates only (Bermudan style), or only at maturity (European style).
>
> The exercise of the exchange option terminates the bond contract.

A holder may opt to exercise his/her conversion right voluntarily or be forced into conversion by following rational investor preferences. The section *Early Bond Contract Termination Summary* describes circumstances when voluntary conversion may be optimal.

Upon the exercise of the exchange option the following needs to be considered:

- Holder surrenders all the remaining bond cash flows, including the interest accrued up to the exercise date[57].
- As from the exercise date a bondholder becomes a stockholder and is entitled to future dividend payments[58].

[57] Certain convertible bonds specifically allowed for the holders to keep the accrued interest on conversion. For example Espanola de Tubos por Extrusion 2009 (Spain), Dixons 2002 (UK).

- The equity asset(s) receivable upon conversion, known as the underlying basket may contain one or more securities. Baskets may be composed of:

  1) Single stock, most common;

  2) Combination of a stock and cash;

  3) Small baskets containing two or more stocks. The exact method of constructing the basket is defined in the prospectus or subsequent notices, and can take many forms, from simple linear combination, to the choice of best performing/least performing, etc.

  4) A cash equivalent of any of the above.

- In practice there is a delay between the exercise and the delivery of the underlying shares or its cash equivalent. This is termed *conversion delivery delay*. This may be from 3 days to 2 months, with 2 weeks being the most common delay.

- In some cases bondholders would initially receive so called unregistered stock that cannot be efficiently, if at all, traded away. Holders are faced with the prospect of having to hold the stock for months until the registration process completes. Frequently such unregistered stock does not have the same dividend rights as the ordinary share[59].

- Some convertible bonds have conversion rights suspended between the coupon payment date and the next ex-dividend date[60].

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58 There are cases in the UK and France where the holders on conversion would receive a restricted dividend stock, which would be entitled to receive the dividend pertaining to the financial year in which the bonds are converted. This may in some cases exclude the holders from the next dividend payment.

59 This is the usual case with French domestic convertible bonds, when conversion is effected between the coupon date and the next ex-dividend date. This is in order to prevent a convertible bondholder from receiving both coupon and dividend payment pertaining to the same fiscal period. Most Taiwanese convertible bonds would initially convert to so called entitlement certificates, that would then be exchanged for the actual stock after approximately 60 to 90 days. During this period companies would apply to the courts to have the increase of share capital approved and registered.

60 The intention of the issuer is to prevent a convertible bondholder from receiving both coupon and dividend payment pertaining to the same fiscal period.

- Small number of UK convertible bonds allowed bonds to be converted into a cum-dividend stock, even if the stock already trading ex-dividend, providing the conversion notice is submitted before the record day.

- The quantity of each asset in the basket is fixed in advance however, if the exercise is cash settled, the amount payable would be generally determined with the reference to an average closing price stretching several trading days following the conversion request[61]. The average price would be lower than the closing spot in a rising market and vice versa in a falling market. If the issuer retains the choice of cash or stock delivery, without committing itself in advance of the averaging period, it is prudent to assume that the cheaper alternative would be chosen[62].

- Cases where an issuer has retained the option to satisfy the exercise request by either delivering physical stock or cash equivalent[63], place an additional risk to the holders of the convertible. The order of events is as follows:

  1) Holder exercises the conversion option;

  2) Issuer notifies the holder within the set period (usually 3 to 5 business days) of their intention to deliver cash, and states when the averaging period starts (the length is known in advance). The averaging period usually starts 1 to 2 days following the cash delivery notification.

  3) One or two days following the end of the averaging period, the company delivers the cash to the holder.

  This type of stock or cash delivery presents a minimal additional risk for a hedged bondholder, namely the costs of closing the short stock position in the market and potential inability to trade exactly at prices and times used to calculate the average.

---

61 Most frequently it is a 5 day closing average.

62 This is a safe assumption from the modelling perspective. In reality this is not the dominant factor which determines what is being delivered especially for issuers from non-financial sectors. If the issuer is a financial institution, the 'rational' behaviour is more frequently observable.

- Infrequently the cash delivery is based on the averaging prior to the conversion, with the issuer simply delivering either stock or cash without the prior notice. To fully describe the potential risk in such cases let's consider the following scenarios:

  1) A bondholder enters a bond into conversion (and unwinds the short stock position, if hedged), expecting to receive a delivery of cash. The company subsequently decides to deliver the stock. The holder is left with the long stock position, which he/she would need to liquidate at his/her risk.

     The holder can remove this risk by buying a (European) put option; with strike price set at the cash equivalent price, and maturing at the time of cash/stock delivery date.

  1) A bondholder enters into conversion expecting to receive a delivery of stock. He/She keeps the stock hedge. The company subsequently delivers cash equivalent. The holder is left with the short stock position, which he/she would need to cover at his/her risk.

     The holder can remove this risk by buying a call option with the strike price equal to the cash equivalent price, and maturing at cash/stock delivery date.

- The final value of the exchange property may be reduced/increased by the amount of cash payable/receivable on conversion.

- A maximum value an investor may receive by exercising the exchange option may be restricted, so called *capped bonds*[64].

Exchange rights usually start one month after the initial settlement date and last until one week before maturity and may be shortened if the bond is called for an early redemption.

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[63] At the time of the exercise of exchange option the amount of cash which company might deliver would not be known exactly.

[64] Bonds of this type are usually issued by large financial institutions and specifically structured to cover predetermined investment needs.

3.3.4.1 Bonds with Time Varying Conversion Ratio

The presence of a non-zero conversion ratio is what separates convertibles from the straight bonds. The conversion ratio is defined as:

> The conversion ratio is the number of underlying baskets receivable in exchange for one face-value unit of bond.
>
> Conversion price is the effective price of the underlying basket at which the bondholders are acquiring ownership by surrendering one face-value unit.

With the help of the above definition straight bonds can be viewed as special case of convertible bonds with conversion ratio set to zero (or alternatively conversion price is infinitely high, i.e. straight bond are 'convertible bonds' issued at *very* high conversion premium that makes the conversion probability almost surely zero).

3.3.4.1.1 Conversion Ratio as a Known Function of Time

Rarely bonds are issued with the conversion ratio specified as changing to pre-specified value at some time in the future. The intention of the issuer is to make the conversion in certain periods more likely by increasing the conversion value. Two issues in Europe have such a feature, namely[65]:

Parmalat Capital	XEU	1.00a	2005	Euro
Rexel	FRF	0.00a	2002	

The first issue initially converts into 25,706 underlying shares, but as from 31st December 2003, the holder will receive 26,668 shares. Rexel converted into 1.11 shares from issue time until 31st January 1998, and into 1 share thereafter, i.e. there was an incentive to convert early.

3.3.4.1.2 Conversion Ratio Contingent on Future Stock Price

This class of convertible bonds is one of the most complex to value. In addition to the other features, time varying conversion ratio (resets) introduces path-dependency as the conversion value of this bonds changes over their life.

Convertible bonds with reset features have so far escaped the attention of academic research. Financial instruments with similar features have been subject to research mainly under the related subjects of path-dependent options[66] such as Average Strike Options, Look-back Options, Bear Market Warrants[67], etc. Recently, Hoogland, Neumann and Bloch (2001) presented an elegant approach to valuing the reset convertibles by introducing an advanced jump condition at the reset dates rather then increasing the dimensionality of the problem.

After the initial conversion ratio has been set[68] (at issue time), the bond contract specifies one or more future dates, known as the reset dates, at which time conversion ratio may be increased (decreased) depending on the stock performance during the period immediately preceding the reset date[69]. At the first reset date the conversion ratio may only be increased, i.e. conversion price may decrease, to a pre-specified maximum adjustment level. At each subsequent reset date the ratio may be increased, if not already at the maximum, or in some cases decreased, but not below the initial value.

Bonds with such features are very popular among Japanese issuers, who appear to favour them as a way of increasing the probability of never having to redeem the bonds, as the reset of the conversion ratio would help keep the exchange option in-the-money[70].

At present the qualifying sample contains only 9 issues (less than 1% of the sample), where the reset feature can only enhance the conversion terms, i.e. add value to the convertible.

1)	77 Bank	JPY	.45s	2002
2)	First International	USD	.00a	2002
3)	G V C	USD	.00a	2002
4)	Gunma Bank	JPY	.45s	2001
5)	Lite-On Technology	USD	.00a	2002

65 Please note that due to the lack of an explicit rating, this issue is not currently part of the qualifying sample. A separate analysis would probably assign an implied A2 rating.

66 See Conze and Viswanathan (1991), Hull and White (1993), Dewynne and Wilmott (1993), Heynen and Kat (1994).

67 For description and valuation of S&P 500 Bear Market Warrants see Gray and Whaley (1997).

68 Conversion ratio is usually defined with reference to the conversion price.

69 Common method is to use the 20day closing average, or 20day minimum price.

6)	Mitsubishi	USD	3.00s	2002
7)	Shin-Etsu Chemical	JPY	.40s	2005
8)	Teco Electric & Machine	USD	.00a	2003
9)	Yamanouchi Pharmaceutical	JPY	1.25s	2014

A special class of reset convertibles are so called "death spiral convertibles"[71]. This type of convertibles is issued with continuously resettable conversion price that is defined as the lower of the current conversion price or the discounted lowest observed price over a trailing period of last several trading days. This is construed to ensure that the conversion option is always kept in-the-money.

3.3.4.1.3 Conversion Ratio as a Known Function of Final Stock Price

Certain mandatory convertible structures that are frequently issued by the US[72] companies fall into this category. They have the final conversion ratio calculated with reference to the stock price at maturity time. They are referred to as convertibles, but are generally considered to be higher yielding equity substitutes rather than convertible bonds. Conversion ratio is usually a decreasing function of stock price with upper and lower limits imposed.

A small number of convertible bonds were issued in Europe that had their maximum conversion value capped. This feature is exclusive to so called *Index Convertible Bonds* issued by French companies (see the section on Index Convertible bonds below). Capped convertible bonds can be decomposed to a portfolio consisting of long straight bond, long at-the-money call option and short out-out-the-money call option.

3.3.4.2 Bonds with Conversion Contingent on Stock Performance[73]

In recent years this type of feature started to appear in a number of zero-coupon issues from US. Most commonly these bonds allow voluntary conversion, prior to maturity, only if the

---

70 The only reset convertible to come by the US issuer was Amazon Inc USD 6.875% 2010, issued in 2000 with downward only reset in 2001 and 2002. This particular structure is called PEACS – Premium Adjustable Convertible Security. Subsequently, the Amazon's price stock has come down so much that despite the maximum reset (from $104.85 to $84.88) the bond's conversion option is still deep out-of-the-money (at the time this thesis is written Amazon's stock is trading around $18).

71 See Hillion and Vermaelen (2000).

72 PRIDES, ACES, DECS, etc. See Arzac (1997) and Sheriff, Berger and Klein (1999).

stock price remains above a certain level during the pre-specified period. A good example of the wording of such contingent conversion can be found in Verizon Communications Inc. USD 0% 2021 CB prospectus, which on page 18 states:

> "During any quarterly conversion period if the closing sales price of the common stock of Verizon Communications for at least 20 trading days in the 30 consecutive trading days ending on the first day of such quarterly conversion period is more than the Applicable Percentage of the Accreted Conversion Price on the first day of such conversion period, then holders may surrender their noted for conversion into common stock of Verizon Communications prior to maturity."

The bond prospectus further specifies other circumstances when conversion prior to maturity would also be allowed. A particularly convoluted example is the definition of '*Conversion Upon Satisfaction of Trading Price Condition*' that reads:

> "During the five business day period following the ten business days after any nine consecutive trading day period in which the trading price for each day of such period was less than 95% of the product of the closing sales price of the common stock of Verizon Communications multiplied by the number of shares into which such note is convertible for that period, then holders may surrender their notes or conversion into common stock of Verizon Communications prior to maturity."

Translated into plain English, this means if the convertible trades at more than 5% discount to its conversion value for at least two weeks, conversion is allowed. Other situations when conversion will be allowed in a case of issuer call or a number of specific corporate events, e.g. mergers, acquisitions, takeover (when holder may need to own the stock in order to exercise their voting rights or profit from favourable price action).

In practice the contingent conversion does not adversely affect bonds theoretical valuation, as the optimal conversion strategy for a rational investor would never be an early voluntary conversion unless faced with large enough dividend payments. In many cases this feature can be safely ignored. The actual reason for including this provision into the bond prospectus has

[73] Bond with such features started to appear in the US only during 2000 and are not the part of the representative sample. They are discussed here for completeness only. Thus far no issuers outside US have issued bonds with such feature.

more to do with the reporting practice in the US, as they allow companies to report higher earnings on the fully diluted basis as they would treat convertible bonds as non-convertible (unless the stock has performed well).

3.3.4.3 Exchangeable Bonds

The term 'Exchangeable Bonds' is generally used to describe convertible bonds issued by one company that are convertible into stock of a different company, where the motivation for the issuance was a possible disposal or monetisation of the cross holding.

Bonds that would fall into this category are very common in the Eurobond market. Within the sample of bonds presented in Appendix 3-A, the exchangeable bonds represent 23.7%. The following figure shows the regional issuance composition.

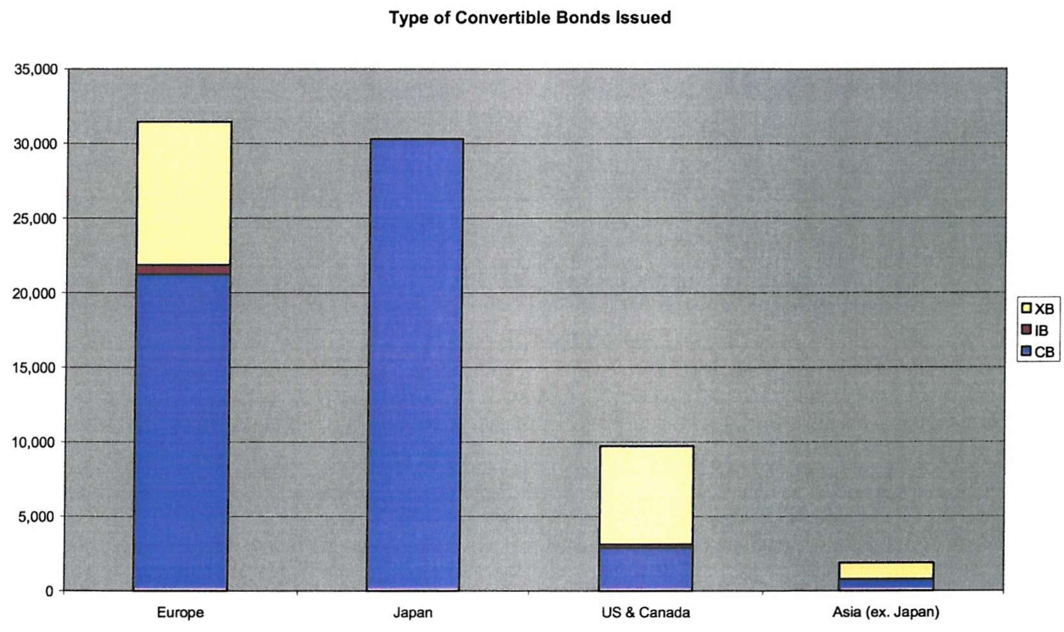

Figure 3-6. Regional Composition of Issuance by Type of Convertible Bond

Bonds in this group usually have a delayed start of conversion right. The issuer of the bonds would frequently retain the right to deliver cash equivalent value upon conversion. Both of these features are connected with cross holding disposal. For example, delayed conversion may be connected to specific agreements between the companies as to the earliest date a cross-holding can be disposed. The cash option is linked to the uncertain tax status that a disposal of cross holding may have in any particular fiscal year up to maturity. Depending on the overall tax situation, the issuing company will wish to retain the choice of tax treatment.

Another reason for the cash option is a possibility of a favourable bid for the cross holding from a market competitor. The company may decide to sell at such opportunity knowing that any future conversion requests coming from exchangeable bondholders can be satisfied by a cash payment.

3.3.4.4 Index Convertible Bonds

A number of bonds have been issued that fall into this category. They all share one common feature: the bonds convert into the cash equivalent of the underlying security (i.e. bond's redemption is indexed). The underlying security can be a single stock or a stock index. Within the sample of bonds presented in the Appendix 3-A, the index bonds represent 1.2%.

Other unusual features can also be present such as:

- Floating rate coupon (as in case of Aceites Y Proteinas ESP 6.52 1999);
- The underlying security is defined as a basket of shares (as in European Bank for Reconstruction and Development NLG 0% 2003, where the underlying basket consists of Elsevier, Walter Kluwer, VNU, and Polygram shares);
- Cash payable on conversion is capped to some value generally defined to be less than 230% of issue price (There are 19 French index convertible bonds that have a feature of this type). For example Carrefour FRF 1% 2005 has its redemption linked to the performance of its own stock (more precisely its 12 month average before maturity), with maximum redemption capped to 187% (bond redeems at 111%).

Depending on the type of issuer, the rationale for issuing convertible bonds of this type was two fold. If the issuer was a non-financial sector company, the governing logic was the desire to prevent any further stock dilution, by paying cash equivalent amount in case of conversion.

Issuance from the financial sector was mainly driven by customer demand for bonds with solid credit quality, that would have some upside linked to a particular stock or in most cases general market index performance (the bonds are marketed to the public as guaranteed capital bonds with stock market participation).

3.3.4.5 Reverse Convertible Bonds

Over the last few years continental European banks have been active in offering specific structures commonly referred to as 'Reverse Convertibles'. These bonds were generally small in size with short maturity (from 6 months, up to 3 years), and with unusually high coupon that in some cases was as high as 20% (Commerzbank EUR 20% 2001). The issuing banks were targeting wealthy private investors. Marshall (2000) reports that bonds of this type were also popular among Japanese investors, as they appeared to offer much higher coupon rates than traditional investments that were largely due to mature shortly in that year. In the US market a number of preferred convertible stock was issued over last 10 years that had similar final payoff, where these structures were known as PERCS, MCPDPS, PEPS, PIPS, CHIPS, EYES and STEP units, see Arzac (1997) or Sheriff, Berger and Klein (1999).

The reverse convertible bonds all share one common feature: the issuer retains the option to deliver either cash as redemption amount or a pre-specified number of shares of particular companies (in some cases the underlying security is a market index in which case a cash equivalent would be delivered, as in Mediobanca USD 6.1% 2010). Comparing to the normal convertible bond, where the bondholders have upside participation in the case of positive stock performance, a holder of a reverse convertible bond is exposed to negative stock performance and hence the name. The maximum redemption of a bond is the par value (i.e. 100%), however if the conversion value of the stock at maturity time is less than par, the issuing bank will deliver stock to the holder. The terminal payoff is therefore the lesser of par or stock.

To value these bonds, they can be decomposed into a portfolio of long straight bond and a short position in a European put option on the reference stock[74]. The increased coupon size reflected the value of the sold-off put option.

---

[74] On the other hand, using the put-call parity the reverse convertible can be thought of as a portfolio of one long stock and one short call option.

Alternatively, reverse convertible bonds can be valued as mandatory cash convertible with maximum conversion value capped at par.

Some variations were also present. In the case of Deutsche Shiffbank DEM 11.15 1999, the issuer had the option to deliver a basket of four stocks: Hoechst AG, SAP AG Preferred, Commercial Bankshares Inc and Daimpler-Benz AG), while in the case of LB Baden-Wuerttemberg DEM 11% 2000, the issuer had the option to deliver the worst performing stock from the basket of two: Deutsche Bank AG or Daimler-Benz AG.

Some issuers have structured a so-called knock-in reverse convertible bond. They provide for redemption in stock only if, at pre-specified monitoring periods, the stock price is lower than a knock-in barrier that is set at a price lower than the reference price (see for example Rabo Securities EUR 10% 2004).

Most of the bonds of this type were issued during the period of the bull market and the Internet boom (from 1998 to the first half of 2000). Small investors appeared to have happily bought these bonds as their positive perception of stocks used to reference the redemption value, made them believe sub-par redemptions were very unlikely, while the coupon payment made the bonds look highly attractive.

With the significant negative market movements over the last year to 18 months, the issuance of this type of bond has completely dried out, reflecting the changed perception of the targeted investor.

3.3.4.6 Bond with Warrants Attached

A small number of straight bonds was issued with warrants attached. Japanese issuers have during 1990s created a whole mini-market in Swiss Franc denominated Japanese warrants that were detached from the original bond/warrant package and traded on the Zurich exchange. The bonds, spiced up with very attractively structured warrants, enabled the issuers to offer very low coupon (usually between CHF 0.125% to 0.5%).

Once the bond with warrants is issued, the warrants can be detached and trade separately from the associated bond. The warrant would generally have the same maturity as the bond and its strike price is set to equal the face value of the bond (actually, the attached warrant was defined as an option to exchange the bond for a set number of shares).

A small number of European issuers (mainly driven by tax rationale) have issued bonds with non-detachable warrants. Examples are Generali Holdings CHF 1% 2003, Carrefour FRF 3.5% 2003, Club Mediterrane FRF 4.5% 2003, however by far largest issue was from Daimler-Benz DEM 4.125% 2003. This DEM 1.2bn issue in 1996 effectively started the German convertible bond market.

The existence of bond-warrant packages was a practical decomposition of the convertible bond. It filled the need of the investors to assume exposure to the underlying stock without necessarily assuming the credit risk, and enabled issuers to receive funding on what for them were very attractive terms and in some cases offered tax efficiencies over the comparable convertible bond alternative.

3.3.5 Analysis of Holders Put Option

> Put option is the right but not the obligation of the bondholder to request early (cash) redemption. The price at which holders can redeem the bonds early is known as the put price.
>
> The exercise of the put option terminates the bond contract.

The put option can be thought of as an additional exchange option, whereby the holders can either exchange the bond for the underlying basket or fixed cash amount at some point prior to maturity.

Common characteristics of the holder's right to an early redemption are:

- Holders are required to give an issuer a notice of their intention to put the bonds. This will usually be between 30-90 calendar days.
- Giving a notice to put cancels all other options and any entitlement to cash flows beyond the payment of the put price.

- Occasionally, the put price may be paid in shares, in which case the analysis used for situations when issuer may deliver stock or cash on conversion applies here.
- Holder is entitled to receive interest accrued up until the payment of the put price.
- In case of many US domestic convertible bonds, notice to put can be retracted by the investor up until the actual put date.

3.3.5.1 Put Options Contingent on Stock Performance

This type of put option was introduced in the case of Bangkok Bank USD 1.50% 2006. The put option gets extinguished if parity ever exceeds 131.26%. The governing logic of the issuer was to encourage bondholders to opt for an early conversion as soon as the conversion value exceeds the threshold level (for given number of days). The only reason why a (risk-neutral) holder may decide to keep the convertible bond alive is if the forecasted volatility is sufficiently high to offset the loss of an early put at the premium price.

Fortunately, such complicated formulations are very rare, and not a part of the qualifying sample.

3.3.5.2 Put Options Contingent on Corporate Event

Occasionally, a term *poison put* is used to describe a contingent put option given to the bondholders that is activated prior to the company being allowed to merge, acquire another entity, de-merge a substantial portion of its business or other large corporate or tax events. The timing of such a put option is not pre-specified in advance but is determined by the unfolding events. These events are discussed in detail in the section dealing with credit deterioration protection.

3.3.6 Analysis of Issuer Early Redemption Right - Call Option

Issuers Call Option is the right but not the obligation of the issuer of the bonds to redeem the bonds prior to maturity. The price at which the issuer can redeem the bond early is known as the call price.

The issuer's call option may be defined as exercisable continuously (American style), or on certain dates only (Bermudan style).

The exercise of the call option terminates the bond contract.

Issuers are required to give a notice to the bondholders of their intention to exercise the call option[75]. Holders use the call notice period to decide whether to convert or do-nothing and receive the call price. The conversion right (usually) ends one week before the payment of the call price.

The exercise period for the call option would usually start two to three years into the life of the bond. In many cases the call option is also made conditional on the price level of the underlying basket (see next sub-section).

- By exercising the call option (i.e. issuing the call notice), the issuer reduces the life of the exchange right to a few days (usually 7) before the call price becomes payable.
- Occasionally, the call price may be paid in shares, in which case the analysis used for situations when issuers may deliver stock or cash on conversion applies here.
- Call price is usually defined as a deterministic function of time. In many cases it is a simple constant, but most zero-coupon structures would have call price accreting at the set yield.
- An issuer decision to call the bonds conveys the message of its ability to pay the eminent call price. A rational issuer will not call the bonds likely to be redeemed in cash (rather than exchanged for stock) unless it has

[75] Call notice period varies between 15 to 90 calendar days, with the 30 (calendar) day notice being the most common.

sufficient funds available. In this sense, bonds that have been called for early redemption can be regarded (almost) risk-free.

- Issuers may call the bond for full or partial redemption. Most of the issues are callable in part or in full at the option of the issuer. However, there are exceptions in certain issues such as Lyonnnaise Des Eaux XEU 0.00% 2004. Here only 50% of the issued quantity can be called from 31st December 2001, with the rest becoming callable a year later. Call features of this type reduce the negative impact of the call provision on the bond's value.

- Almost all issues have a so-called *clean-up call*, which enables the issuer to call the bonds if the outstanding quantity is severely reduced due to exercise of voluntary conversion or holders exercising put option. The clean-up call threshold is usually set at less than 10% of the original size.

- If a holder opts to accept the call price, he/she is also entitled to the interest accrued from the last coupon payment date up to the call payment date.

- Starting in early 1999 a number of convertible issues[76] within high growth industries i.e. technology (internet), media, telecommunication, biotechnology, included so-called *make-whole provisions*. This provision enabled issuers to structure convertible bonds that would become immediately callable, subject to a stock based trigger level being satisfied. In the case of an early call (usually within the first three years from the issue date); the additional payment compensated bondholders for the premature termination of their investment.

[76] The first convertible bond to have make-whole call provisions was Amazon.com Inc USD 4.75% 2009. The bond was issued on 29th January 1999, and became callable one week later with the stock trigger level set at 150% above the conversion price for minimum of 20 out of 30 trading days immediately preceding the mailing date of the call notice. The bond would become unconditionally callable three years later. If the call condition was to have been satisfied and the issuer had decided to call the bond, the holders that opted to convert would have been entitled to receive an additional cash payment of USD 212.60 less the interest (coupons) already paid. The amount of USD 212.60 was chosen to match the premium at which the bonds were initially issued. In practice, Amazon stock never traded above the trigger level giving the company no opportunity to exercise the call.

Within the following 15 months, a total of 23 convertible bonds were issued containing make-whole provisions but only one was ever called[77]. Our sample of bonds does not contain issues with make-whole call provisions and it is included in this section for the sake of completeness.

A number of research papers have analysed how efficient are the issuers in exercising their call options. The common conclusion is that the call options are not exercised optimally, conveying extra gains to the holders. The explanation for such behaviour is varied and presented in the literature review section. In Chapter 4 I present a valuation approach that specifically takes into account the value of the call notice period put option, i.e. the cost of underwriting the issuer call, as a possible way of accounting for observed behaviour.

3.3.6.1 Call Option Contingent on Stock Performance

A number of convertible bonds have their call option impact dampened by imposing an additional condition on the level at which the underlying security has to trade before the call is allowed. The condition usually takes several forms:

1) A minimum closing price over a number of trading days (usually any 20 trading days out of preceding 30 continuous trading days) leading to and during the call period has to be above a pre-specified trigger level[78];

2) An average closing price over a number of days leading into and during the call period has to be above certain trigger level.

3) The trigger level may be specified in terms of the stock price (in the currency of the stock) or conversion value (in the currency of the bond). If the trigger level is specified in the currency of the stock, then the future development of the currency exchange rate needs to be taken into account as it may increase or decrease the effective trigger level.

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77 See Seidel (2000). The issue called was Human Genome Sciences Inc USD 5% 2006. The bond was issued on 14th December 1999 and the company issued a call notice on 2nd March 2000, giving an extra USD 150 make-whole payment per nominal bond to every holder that opted to convert.

78 Options like this are also known as Parisian barrier options. The option is triggered if the asset price stays above a barrier for a number of consecutive days. See Chesney, Cornwall, Jeanblanc-Picque, Kentwell and Yor (1997), Chesney, Jeanblanc-Picque and Yor (1997), Vetzal and Forsyth (1998).

4) In cases of convertible bonds redeeming at a premium, the trigger level may be defined as a set percentage above the accreted call price.

3.3.7 Other Features

A number of other circumstances need to be considered when convertible bonds are analysed compared to straight bonds.

3.3.7.1 Change of Control Protection

The value of a convertible bond is heavily dependent on the properties of the stock(s) into which it converts (the underlying basket). Along with the credit quality of the issuing company, these are the major parameters affecting convertibles prices.

Volatility of the underlying stock, its current price, and the dividend yield are the three most prevalent properties that largely determine the premium at which a convertible bond would trade over the price of an otherwise equivalent straight bond. What happens to the issuing company and the underlying stock during the life of the convertible bond is of great importance to the investors. The risk to convertible holders arises from ending up with a new issuer that is of lower credit quality or with underlying stock that is less volatile or pays a higher dividend, all of which could reduce the value of the convertible bond. Apart from the general bond protection clauses, holders of convertible bonds have demanded additional specific protection[79] to cover specific merger/de-merger/takeover events and depending on the form of the new exchange property:

- Straight share swap. This is the most common form of merger or takeover.
- Basket of several shares.
- Basket of shares and cash.

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[79] This is particularly important to holders of exchangeable bonds as in many cases the action required by the firm issuing an exchangeable bond was not clearly specified, that may lead to the firm opting for action that is more favourable or convenient to them, rather than making a choice based on the best interest of holders of exchangeable bonds. One example is when stock and cash is offered on take-over. If the issuing firm simply accepts such an offer, the resulting exchange property would have significantly reduced volatility. Exchangeable holders would benefit from provisions in the prospectus requiring the issuing firm to at least reinvest the cash portion into a government security or even better use the cash to buy the extra stock, and giving the holders the full benefit of underlying volatility.

- All cash offers.
- Conversion remains into shares of the legacy company.
- Convertibility is lost.

3.3.7.1.1 Premium Protection

In any of the above scenarios the convertible holder is concerned with the 'premium protection' i.e. seeing the value of their holding decrease. Over the years several methods were devised that are commonly referred to in prospectuses as 'premium protection clauses':

- Right to immediate conversion at improved conversion ratio. This would usually apply for the period of 60 days after the change of control event. The extent of conversion ratio improvement is fixed in advance and is usually linked to the remaining life of the convertible, with higher increases afforded to early years of convertible life.
- Tender offer at favourable terms. Acquiring company is obliged to make a tender offer to convertible holders at the advantageous terms.
- Cash compensation. An additional amount of cash is added to the conversion property. This usually applies to part or all cash offers. As in the previous case, the calculated cash compensation or the price of the tender offer may vary significantly and usually leave the compensation to be determined solely by the acquiring company.

In some cases, the tender offer or the cash compensation is more precisely defined as

- Compensation calculated based on market price.
- Compensation calculated based on theoretical formula.
- Compensation rate fixed in prospectus.

After all of the above is determined, an investor may still be exposed to the cash component of the compensation especially when it becomes a significant part of the exchange property. To remove and clarify this last point some prospectuses specify how the cash is held.

- Cash is re-invested. This is applicable in cases where part or full consideration is paid in cash. The cash is used to buy the shares in the acquiring company on best effort basis.
- Cash is used to buy government securities and all received interest is re-invested likewise.

3.3.7.1.2 Credit Deterioration Protection

This type of protection is common for straight bonds and has been extended to convertibles. Most common features are:

- Change of Control Put/Redemption (Poison Pill)

  In the case of takeover, merger or substantial disposal of assets, bondholders have the right (subject to majority voting or by decision of bond trustee) to an immediate redemption.

- Coupon Rate Change or Put on Credit rating Downgrade.

  In the case of a credit rating downgrade, the issuer has to increase the coupon payment by a specified amount for each notch on the credit rating scale. If the credit rating falls below a certain level, holders may request immediate redemption.

3.3.7.2 Corporate Actions

- Special Dividends

  The protection is usually defined with reference to the average dividend payment over past few years. If payment is made in significant excess over the average it qualifies as 'special' and the conversion ratio, i.e. conversion price, is adjusted to compensate the bondholders. Earlier prospectuses have relied on the dividend being declared by the issuer as special in order to trigger the conversion ratio adjustment[80]. Such definition has been subsequently updated to the above, as some issuers started using various creative names for what was effectively a special dividend in order to avoid compensating the holders of convertible bonds.

- Stock dividends

Bondholders are usually not protected against normal stock dividends. Therefore the forecasting of future dividend payments and their precise handling within the valuation algorithm becomes an important issue. Chapter 5 analyses the effect of dividend timing upon the price of options and develops a procedure for exact dividend modelling.

Very rarely, certain specially structured convertible bonds would have a coupon payment linked to stock dividend in such a way to insure that the coupon remains higher than dividend payments, effectively offering bondholders dividend protection and ensuring conversion does not happen too early, or alternatively compensating the holders for not being able to convert prior to maturity[81].

- Rights and Warrants Issues

Convertible bondholders are protected in this case by either allowing them to participate in these issues as if they have converted their bonds, or by making a subsequent adjustment to the conversion ratio. In either case the logic is to preserve the continuity of the bond price over the ex-rights/ex-warrant date.

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80 A well known 'incident' involving special dividend happened in May 1998 when Daimler-Benz declared DEM 28 payment (average dividend at the time was approximately DEM 2). Daimler-Benz's own convertible bond (or more precisely bonds with non-detachable warrants) as well as an exchangeable bond from Deutsche Bank to Daimler-Benz did not have any specific special dividend protection provided within the prospectuses. Bondholders of the both bonds suffered significant losses. Interestingly, the Deutsche Terminborse have equally failed to see any need to adjust the option strike prices for this event, resulting in the holders of the Daimler-Benz call options suffering unexpected losses while the holders of put options were rejoicing. The adviser to Daimler-Benz at the time was Deutsche Bank (who was also the main shareholder as well as the issuers of the exchangeable bond and the lead manager for Daimler-Benz bonds with non-detachable warrants). In the aftermath of this event, the special dividend protection has been included in all German prospectuses with some investors demanding the special protection clauses against companies being advised by Deutsche Bank as well.

81 For example see KBC Bancassurance 2003 (Belgium) convertible bond. Initially coupon was set to 5.75% until December 1995, then it becomes linked to annual dividend on KBC Bancassurace stock. Similar issues are Dofasco 1996 and Dofasco 2010 (Canada), Inmet 2007 (Canada), Noranda 2007 (Canada), Wuert AG 2004 (Germany), Axa 2017 (France).

- Spin-offs and other capital distributions

  In this case a number of provisions may be triggered. If the spin-off is small, convertible bondholders would be compensated by a conversion ratio adjustment.

  Alternatively, the exchange property may become a basket of two stocks: the original one and the one being spun-off.

  If the disposal is large enough to cause credit quality concerns, bondholders may request an early redemption prior to allowing the company to proceed with disposal. This works well for low coupon straight and out-of-the-money convertible bondholders, as they are likely to enjoy windfall gains. Holders of high coupon straight bonds and at/in-the-money convertible bondholders would lose their premium if redeemed early and would seek compensation differently.

- Stock Splits

  Making an adjustment to the conversion ratio protects convertible bondholders. It is worth noting that stock splits may sometimes lead to improved visibility of the stock and result in an increase in subsequent realised volatility[82].

3.3.8 Early Bond Contract Termination Summary

General convertible bond contracts can be terminated prematurely in a number of ways:

- Voluntary conversion is optimal in the following two cases:

[82] Ohlson and Penman (1985), Dubovsky (1991) and Desai *et al.* (1994) all find evidence of an increase in the post-split volatility of a stock's returns. The volatility increase was particularly evident for Swiss stocks, where the post-split price was reduced 100 times that brought additional liquidity to the market.

1) Prior to a large dividend payment[83] when expected dividend yield outstrips the expected coupon yield; and

2) Prior to an adverse change in the conversion ratio[84] when the new ratio is known to be lower then the current one. Conversion would be optimal if and only if the conversion value before the change in the conversion ratio is greater than the expected (discounted) convertible bond value following the change in the conversion ratio.

- Forced early conversion due to potential adverse change resulting from announced corporate action.
- Early redemption at issuer's option, i.e. exercise of call option;
- Early redemption at investor's option (i.e. exercise of a put option);
- Issuer's default.

3.3.9 Common Features of the Underlying Security

As stated earlier, an underlying basket of assets would generally comprise of one or more ordinary shares. These are perpetual securities with occasional dividend payments to their holders.

As noted in the introductory section of this document, approximately 18.1% of the qualifying convertible issues are denominated in a currency other that the underlying security currency. In such a case, the following effects are worth noting[85]:

1) The current conversion value becomes dependent on both the stock price and the foreign exchange spot rate;

83 See Ingersoll (1977a), page 316.

84 See Ingersoll (1977a), page 314. This can also be expressed as an adverse increase in the exercise price. Viewed in this way, circumstances when the voluntary conversion is optimal are the same as an ordinary American style call option, as established by Merton (1973).

85 Yigitbasiouglu (2001) studies convertible bonds with foreign exchange rate risk and propose a change of numeraire solution in order to avoid increasing the dimensionality of the valuation problem.

2) If stock price movements are modelled in the stock currency, then the forward conversion value is dependent on forward foreign exchange rate;

3) For dividend paying stocks, future based cash flows must be converted using the forward foreign exchange rate;

4) If the issuer's call option is conditioned on the stock price based trigger, the actual parity level at which bonds may become callable is dependent on the forward foreign exchange rate.

Ordinary shares are from time-to-time subject to corporate actions, e.g. stock splits, rights issues, special dividends. A properly defined bond contract would allow for an adjustment to the exchange property in order to protect the bondholders from any adverse effect these actions may have on the value of the convertible.

The bond contract also makes provisions for dealing with take-over and merger situations. The most common protection offered to the holders is:

1) Company may be prevented from merging or acquiring another company without the bondholders consent.

2) Company may be required to redeem the bonds at the premium before it can proceed with the merger/acquisition.

3) If a bond is callable at the time of anticipated merger/acquisition, the company may be forced to issue a call notice, when it is not optimal to do so, giving the bondholder a windfall gain.

4) Holders of bonds are allowed to elect for an early redemption prior to the merger/acquisition becoming final. This feature is known as *a poison put*.

5) The company is free to merge/acquire without the specific bondholders' approval. Bonds become convertible into the stock of a merged entity. The conversion ratio is adjusted in a way that preserves the conversion value.

Although, this adjustment may appear fair, the holders are still exposed to risk of newly merged company receiving a lower credit rating, its stock becoming less volatile, or a change in dividend yield policy

In certain take-over/merger cases, where the original stock is exchanged for a quantity of stock of the merging company with some cash payment, the adjustment to the exchange property may specify that the bond holders can convert into like package, i.e. new stock + cash. If the amount of cash is substantial in proportion to the stock, the reduction in the underlying asset volatility may reduce the value of the bond.

Dividend payments are one of the most regular features of ordinary shares. A convertible bond value has negative sensitivity to dividend yield, i.e. the higher the dividend yield, the lower is the price of convertible bond[86]. To that end the following needs to be taken into account:

1) Bond values are affected by the future dividend yield. This places a requirement for dividend payments and the associated ex-dividend/payment dates to be forecasted;

2) In most cases, a high confidence, dividend forecast is possible for at most one or two years in advance. As the average life of convertible bonds is far longer than that, long-term forecast is usually extrapolated from the currently available data and long-term analyst forecasts.

3) Stock prices are adjusted for dividend drops as of the ex-dividend date. However, the dividend payment may actually be paid months later. If this timing difference is not taken into account the model may under estimate the true value of the bonds.

4) As with ordinary equity call and put options, convertible bonds are sensitive to the timing of the dividend payments, e.g. a large dividend initially

[86] This relationship comes from two sources: a) general property of call options with reference to the dividend yield on the underlying stock; b) the default risk is positively correlated to increases in dividend payouts, that in turns reduces the value of the bond component of the convertible bond.

forecasted just after the bond's maturity and subsequently revised to fall before the maturity, would adversely affect its perceived value.

5) A large and unexpected hike in dividend or a special dividend would significantly affect the valuation, unless the bond prospectus offers a specific protection.

6) Many stocks report both gross and net dividend amounts. It is important to note that, if an investor is long stock, the dividend received would be closer to the net amount, but if short, the dividend payable would be closer to the gross amount.

Finally, borrowing shares in order to maintain the short position is subject to a fee. The more difficult it is to borrow a stock, the higher is the fee. The size of the fee is counter-proportional to the perceived value of the bond.

One of the most important components of the stock prices is their volatility. Most of the convertible bond premium is attributed to the volatility of the underlying asset. Thus, good and consistent volatility estimation and forecasting is essential. In practice traders would most commonly use 3 to 6 months historic volatility as an initial estimate[87]. This would be further refined by comparison with implied volatility of medium term traded options and subjective forward-looking volatility view. The selected valuation problems that are included in this thesis do not explicitly address the issue of volatility estimation. This is recognized in the concluding remarks where a number of pointers for further research in this area are mentioned[88].

3.4 Conclusion

Commonly, when convertible bonds are mentioned it is the conversion feature that is being emphasised. In my own presentations I would frequently simply explain a convertible bond as a straight bond with an extra choice given to the holder at maturity: an investor can either do

[87] Recent research note Conway and Weir (2001) and Mackie *et al.* (2001), both provide good overviews of prevalent practice as well as empirical test of alternative volatility estimates and re-hedging strategies designed to maximise the realised volatility.

[88] Recent paper by Yigitbasiouglu (2001) looks at ways to incorporate implied volatility information contained in single stock options and foreign exchange options in convertible bond valuation problem.

nothing and receive the final payment, or opt for conversion to a given quantity of the issuer's stock; with a rational investor choosing the more valuable alternative.

Although, for the large part such explanations are true, this chapter gives plenty of evidence that they are far from the whole truth. Modern convertible bonds have a plethora of features of significant complexity that are far beyond presently developed valuation techniques. Some of the valuation problems will be addressed in subsequent chapters, while many others I can only afford to mention in the concluding remarks to this thesis, and hope to find the solutions in future research.

Considering the ever increasing complexity and diversity of the businesses of issuing convertible bonds, competitiveness and huge resources available to investment banks, researchers would be chasing an elusive target.

Appendix 3-A. Qualifying Sample of Convertible Bonds

Region	Country	ISSUE	Same CCY	Rating	Size (USD' mm)	Reset	Put	Call
Asia	TW	First Internatio USD .00a 2002 RegS	No	AA1	200	Yes		Soft
Asia	TW	G V C USD .00a 2002 RegS	No	AA2	130	Yes		Soft
Asia	IN	Indian Petrochem USD 2.50s 2002 RegS	Yes	AA3	175			
Asia	AU	Lend Lease USD 4.75s 2003 Euro	No	A1	200			Soft
Asia	TW	Lite-On Technolo USD .00a 2002 RegS	No	AA3	70	Yes		Soft
Asia	SG	Singapore USD .00s 2004 Euro	No	AA1	438			Soft
Asia	SG	Singapore XEU .00s 2004 Euro	No	AA1	453			Soft
Asia	TW	Teco Electric & USD .00a 2003 RegS	No	AA1	200	Yes		Soft
Europe	CH	Abb USD 2.75a 2004 Euro	No	AA3	150			Hard
Europe	NL	Abn Amro NLG 2.00a 2004 Euro	Yes	AA2	94			Hard
Europe	NL	Aegon USD 4.75a 2004 Euro	No	A1	600			Hard
Europe	DE	Allianz DEM 3.00a 2003 RegS	Yes	AA1	1,060			Soft
Europe	NL	Asm Lithography NLG 2.50a 2005 RegS	Yes	AA3	282			Hard
Europe	NL	Asr Verzekerings NLG 3.00a 2005 Euro	Yes	AA3	188			Soft
Europe	NL	Asr Verzekerings NLG 5.00a 2001	Yes	AA3	71			Soft
Europe	FR	Axa XEU 2.50a 2014	Yes	A3	1,368			Soft
Europe	DE	Bahn DEM 1.13a 2003 Euro	Yes	AAA	45			Soft
Europe	IT	Banca Milano (Po ITL 2.50a 2008 RegS	Yes	A3	327			Hard
Europe	ES	Banco Bilbao Viz USD 3.50s 2006 RegS	No	A1	250			Hard
Europe	ES	Banco Santander XEU 2.00a 2003 RegS	Yes	A2	311			
Europe	AT	Bank Austria XEU .00a 2004 RegS	Yes	AA2	181		Yes	Hard
Europe	GB	Barclays Capital USD .00s 2001 Euro	No	AA2	65			Hard
Europe	GB	Barclays Capital USD 1.00a 2005 Euro	No	AA3	50			Soft
Europe	DE	Bgb Finance DEM 3.25a 2001 Euro	Yes	AA2	185			Soft
Europe	GB	British Aerospac GBP 3.75s 2006 Euro	Yes	A3	983			Soft
Europe	GB	British Airport GBP 4.88s 2004 Euro	Yes	A1	283			Soft
Europe	GB	British Airport GBP 5.75s 2006 Euro	Yes	A1	404			Hard
Europe	GB	Capital Shopping GBP 6.25s 2006 Euro	Yes	A3	329			Hard
Europe	FR	Carrefour FRF 1.00a 2005	Yes	AA3	95			
Europe	CH	Ciba Specialty C USD 1.25s 2003 Euro	No	A2	605			
Europe	GB	Commercial Gener FRF 1.50a 2003 Euro	Yes	AA2	414			Soft
Europe	DE	Commerzbank DEM .75a 2004 Euro	Yes	AA3	132			Hard
Europe	DE	Commerzbank DEM 7.50a 2000	Yes	AA3	265			
Europe	FR	Credit Communal FRF 3.25a 2005 RegS	Yes	AA1	149			Soft
Europe	FR	Credit National FRF 3.25a 2001 Euro	Yes	A1	191			Soft
Europe	FR	Credit National FRF 5.63a 2003 Euro	Yes	A3	166			Soft

Region	Country	ISSUE	Same CCY	Rating	Size (USD' mm)	Reset	Put	Call
Europe	CH	Credit Suisse USD 2.00a 2005 Euro	No	AA3	100			Hard
Europe	CH	Credit Suisse USD 2.00a 2008 Euro	No	AA3	250			Hard
Europe	CH	Credit Suisse USD 4.88a 2002 Euro	No	A2	500			
Europe	GB	Credit Suisse Fi USD 2.00a 2000 Euro	Yes	AA3	51			
Europe	NL	Cregem Finance USD 2.75s 2004 Euro	No	AA1	150			Hard
Europe	NL	Csm XEU 1.50a 2004 Euro	Yes	AA3	104			Soft
Europe	DE	Daimlerchrysler DEM 4.13q 2003 Euro	Yes	A1	392			Soft
Europe	FR	Danone FRF 3.00a 2002 Euro	Yes	A1	632			Soft
Europe	FR	Danone FRF 6.60a 2000 Euro	Yes	A1	522			Soft
Europe	DE	Deutsche Bank CHF 1.00a 2001 Euro	No	AA3	65			
Europe	DE	Deutsche Bank DEM 1.00a 2001 Euro	Yes	AA3	459			Hard
Europe	DE	Deutsche Bank USD .00s 2017 RegS	No	AA3	534		Yes	Hard
Europe	DE	Deutsche Bank XEU 2.00a 2003 RegS	Yes	AA3	1,606			Soft
Europe	XE	Ebrd USD .75s 2001 Euro	Yes	AAA	100			Soft
Europe	GB	Ebrd XEU .00a 2000 Euro	Yes	AAA	104			
Europe	SE	Ericsson SEK 4.25a 2000	Yes	A2	260			
Europe	XE	European Bank Fo NLG .00a 2003 Euro	Yes	AAA	72			
Europe	GB	Foreign & Coloni JPY 3.00a 2000 Euro	No	AA3	95			Soft
Europe	NL	Fortis NLG 2.63a 2003 RegS	Yes	AA3	705			Hard
Europe	FR	France Telecom FRF 2.00a 2004	Yes	AA1	182			Soft
Europe	DE	Hamburgische Lan USD 3.25a 2002 RegS	No	AA1	100			Hard
Europe	DE	Henkel DEM 2.00a 2003 Euro	Yes	A1	79			Hard
Europe	SE	Investor SEK 8.00a 2001	Yes	A1	394			
Europe	IT	Italy ITL 6.50s 2001 Euro	Yes	AA3	760			Hard
Europe	IT	Italy USD 5.00s 2001 Euro	No	AA3	925			Hard
Europe	BE	Kbc Bancassuranc DEM 2.50a 2005 RegS	Yes	A1	450			Soft
Europe	GB	Land Securities GBP 6.00a 2007 Euro	Yes	A1	339			Hard
Europe	GB	Land Securities GBP 7.00s 2008 Euro	Yes	A1	242			Hard
Europe	GB	Legal & General GBP 6.75s 2008 Euro	Yes	A1	194			Hard
Europe	FR	Lyonnaise Des Ea FRF 4.00a 2006	Yes	A1	474			Soft
Europe	FR	Lyonnaise Des Ea XEU .00a 2004 RegS	Yes	A1	815			Hard
Europe	DE	Mannesmann XEU 1.00a 2004 Euro	Yes	A2	2,383			Soft
Europe	CH	Merrill Lynch USD 1.00a 2003 Euro	No	AA3	150			Soft
Europe	GB	Merryll Lynch USD 2.00a 2004 Euro	No	AA3	120			Hard
Europe	GB	National Grid GBP 4.25s 2008 RegS	Yes	AA2	646			Hard
Europe	CH	Nestle USD 3.00s 2002 Euro	No	AAA	300			Hard
Europe	CH	Nestle Australia USD 1.25s 2005 Euro	No	AA1	250			Hard
Europe	DE	Norddeutsche Lb USD 3.00a 2003 RegS	No	AA1	150			Hard
Europe	DE	Nordrhein-Westfa DEM 2.13a 2002 RegS	Yes	AA1	140			

Region	Country	ISSUE	Same CCY	Rating	Size (USD' mm)	Reset	Put	Call
Europe	CH	Novartis CHF 1.25a 2002	Yes	AAA	483		Yes	
Europe	CH	Novartis USD 2.00s 2002	No	AAA	609			
Europe	GB	P & O USD 6.00s 2004	No	A3	175			Soft
Europe	FR	Peugeot FRF 2.00a 2001	Yes	A3	626			Soft
Europe	PT	Portugal Telecom XEU 1.50a 2004 Euro	Yes	A2	466			Soft
Europe	NL	Rabobank NLG 4.25a 2006 Euro	Yes	AAA	96			
Europe	NL	Rabobank USD 1.88s 2004 Euro	No	AAA	150			Hard
Europe	GB	Railtrack GBP 3.50s 2009 RegS	Yes	A1	646			Hard
Europe	GB	Royal & Sun Alli GBP 7.25s 2008 Euro	Yes	A2	250			Hard
Europe	FR	Schneider USD 2.00a 2003 Euro	No	A3	150			Hard
Europe	FR	Sophia FRF 3.75a 2004	Yes	A3	126			Soft
Europe	CH	Swiss Re NLG 1.25s 2003 Euro	Yes	AAA	435			Soft
Europe	CH	Swiss Re USD 2.00s 2000 Euro	No	AAA	439			
Europe	ES	Telefonica USD 2.00s 2002 RegS	No	A2	525			Hard
Europe	CH	U B S CHF 1.50a 2003 Euro	Yes	AA1	114			
Europe	CH	U B S DEM 1.00a 2001 Euro	Yes	AA1	63			Soft
Europe	CH	U B S USD 2.75a 2002 Euro	No	AA1	325			Soft
Europe	CH	U B S XEU 1.00a 2003 Euro	Yes	AA1	104			Hard
Europe	DE	Volkswagen DEM 1.75a 2003	Yes	A1	53			Hard
Europe	CH	Zurich Insurance USD 1.00s 2003 RegS	No	AA3	582			Hard
Japan	JP	77 Bank JPY .45s 2002 #3	Yes	A1	189	Yes		
Japan	JP	Asahi Chemical JPY 1.70s 2003 #7	Yes	A3	464			Hard
Japan	JP	Asahi Glass JPY 1.90s 2008 #5	Yes	A2	935			
Japan	JP	Canon Inc JPY 1.00s 2002 #5	Yes	A1	134			
Japan	JP	Canon Inc JPY 1.20s 2005 #4	Yes	A1	162			
Japan	JP	Canon Inc JPY 1.30s 2008 #3	Yes	A1	288			
Japan	JP	Chiyoda F&M Ins JPY .70a 2001 #4	Yes	AA3	95			
Japan	JP	Chiyoda F&M Ins JPY .80a 2003 #3	Yes	AA3	142			
Japan	JP	Chubu Ele Pwr JPY 1.00s 2006 #2	Yes	AA2	1,421			
Japan	JP	Chugai Pharmaceu JPY 1.05s 2008 #6	Yes	A3	237			
Japan	JP	Chugai Pharmaceu JPY 1.10s 2006 #5	Yes	A3	284			Hard
Japan	JP	Dai Nippon Print JPY 1.80s 2000 #9	Yes	AA1	188			
Japan	JP	Dai Nippon Print JPY 1.80s 2003 #8	Yes	AA1	281			
Japan	JP	Daiichi Pharm JPY 1.80s 2000 #4	Yes	A1	284			
Japan	JP	Dai-Tyo F&M Ins JPY 1.60a 2000 #4	Yes	AA2	41			
Japan	JP	Dai-Tyo F&M Ins JPY 1.60a 2003 #3	Yes	AA2	36			
Japan	JP	Ebara Corp JPY .90s 2003 #2	Yes	A3	189			
Japan	JP	Eisai JPY .60s 2003 #4	Yes	A1	94			
Japan	JP	Fujisawa Pharm JPY 1.70s 2001 #8	Yes	A2	103			

Region	Country	ISSUE	Same CCY	Rating	Size (USD' mm)	Reset	Put	Call
Japan	JP	Fujisawa Pharm JPY 1.70s 2004 #7	Yes	A2	109			
Japan	JP	Fujitsu Ltd JPY 1.90s 2002 #8	Yes	A2	378			
Japan	JP	Fujitsu Ltd JPY 1.95s 2003 #9	Yes	A2	379			
Japan	JP	Fujitsu Ltd JPY 2.00s 2004 #10	Yes	A2	189			
Japan	JP	Gunma Bank JPY .45s 2001 #4	Yes	A2	474	Yes		
Japan	JP	Hitachi JPY 1.30s 2003 #6	Yes	A2	907			
Japan	JP	Hitachi JPY 1.40s 2004 #7	Yes	A2	193			
Japan	JP	Hitachi JPY 1.70s 2002 #5	Yes	A2	274			
Japan	JP	Hitachi Credit JPY 1.80s 2004 #1	Yes	A2	117			
Japan	JP	House Foods Corp JPY 1.80a 2002 #3	Yes	A2	141			
Japan	JP	Kaneka Corp JPY 1.80s 2001 #7	Yes	A3	95			
Japan	JP	Kaneka Corp JPY 1.80s 2004 #8	Yes	A3	189			
Japan	JP	Kao Corporation JPY .95s 2006 #7	Yes	AA3	474			
Japan	JP	Keihin E Expr Ry JPY 1.15s 2009 #19	Yes	A3	379			
Japan	JP	Keihin E Expr Ry JPY 1.50s 2002 #18	Yes	A3	284			
Japan	JP	Kikkoman Corp JPY 1.60a 2000 #4	Yes	A3	95			
Japan	JP	Kikkoman Corp JPY 1.70a 2002 #5	Yes	A3	95			
Japan	JP	Kinki Nip Ry JPY .70s 2003 #5	Yes	A2	283			
Japan	JP	Kinki Nip Ry JPY 1.00s 2008 #6	Yes	A2	284			
Japan	JP	Kuraray JPY 1.00s 2003 #4	Yes	A3	94			
Japan	JP	Kuraray JPY 2.20s 2002 #5	Yes	A3	142			
Japan	JP	Makita Corp JPY 1.50s 2005 #3	Yes	A2	125			
Japan	JP	Marui JPY 1.15a 2012 #9	Yes	A1	379			
Japan	JP	Matsushita E Wks JPY 1.00s 2005 #9	Yes	A1	757			
Japan	JP	Matsushita Elect JPY 1.30s 2002 #5	Yes	AA2	946			
Japan	JP	Matsushita Elect JPY 1.40s 2004 #6	Yes	AA2	947			
Japan	JP	Mitsubishi USD 3.00s 2002	Yes	A3	2,000	Yes		
Japan	JP	Mitsui M&F Ins JPY .70a 2003 #3	Yes	AA1	474			
Japan	JP	Murata Mfg Co JPY 1.80s 2002 #4	Yes	A1	106			
Japan	JP	Nip Meat Packers JPY 1.90a 2003 #4	Yes	A3	84			
Japan	JP	Nishimatsu Const JPY .50a 2005 #7	Yes	A3	284			
Japan	JP	Q.P. Corp JPY .80a 2001 #5	Yes	A3	95			
Japan	JP	Q.P. Corp JPY 1.00a 2005 #4	Yes	A3	189			
Japan	JP	Ricoh Co Ltd JPY .35s 2003 #9	Yes	A3	379			Hard
Japan	JP	Ricoh Co Ltd JPY 1.50s 2002 #8	Yes	A3	373			Hard
Japan	JP	Rohm Co Ltd JPY 1.10s 2003 #2	Yes	A2	53			
Japan	JP	Sankyo Co Ltd JPY .70s 2001 #3	Yes	AA3	468			
Japan	JP	Shin-Etsu Chem JPY .40s 2005 #6	Yes	A2	474	Yes		
Japan	JP	Snow Brand Milk JPY 1.60a 2000 #5	Yes	A3	95			

Region	Country	ISSUE	Same CCY	Rating	Size (USD' mm)	Reset	Put	Call
Japan	JP	Snow Brand Milk JPY 1.70a 2000 #3	Yes	A3	94			
Japan	JP	Snow Brand Milk JPY 1.70a 2002 #6	Yes	A3	95			
Japan	JP	Snow Brand Milk JPY 1.70a 2003 #4	Yes	A3	95			
Japan	JP	Sony Corp JPY .15s 2001 #5	Yes	A1	2,841			Hard
Japan	JP	Sony Corp JPY 1.40s 2003 #3	Yes	A1	301			
Japan	JP	Sony Corp JPY 1.40s 2005 #4	Yes	AA3	2,828			Hard
Japan	JP	Sumitomo Electri JPY .25a 2008 #6	Yes	A2	474			
Japan	JP	Sumitomo M&F Ins JPY 1.10a 2002 #3	Yes	AA2	165			
Japan	JP	Sumitomo M&F Ins JPY 1.20a 2004 #4	Yes	AA2	215			
Japan	JP	Sumitomo M&F Ins JPY 1.60a 2003 #2	Yes	AA2	122			
Japan	JP	Tokyo Elec Pwr JPY 1.70s 2004 #1	Yes	AA2	743			
Japan	JP	Tokyo Gas JPY 1.10s 2007 #6	Yes	AA1	474			
Japan	JP	Tokyo Gas JPY 1.20s 2009 #5	Yes	AA1	474			
Japan	JP	Toppan Printing JPY 1.40s 2005 #7	Yes	AA3	331			
Japan	JP	Yamanouchi Pharm JPY 1.25s 2014 Euro	Yes	AA3	256	Yes		Hard
Japan	JP	Yamanouchi Pharm JPY 1.63s 2000 Euro	Yes	AA3	284			Hard
Japan	JP	Yamato Transport JPY 1.20s 2009 #7	Yes	A3	379			
Japan	JP	Yamato Transport JPY 1.70s 2002 #6	Yes	A3	441			
Japan	JP	Yamato Transport JPY 3.90s 2001 #5	Yes	A3	331			
US	US	Bell Atlantic USD 4.25s 2005 Euro	No	A1	3,180			Hard
US	US	Bell Atlantic USD 5.75s 2003 Euro	Yes	A1	2,400			Hard
US	US	Berkshire Hathaw USD 1.00s 2001	Yes	AAA	399			Hard
US	US	Centocor USD 4.75s 2005	Yes	AAA	400			Hard
US	US	Dsc Comminicatio USD 7.00s 2004	Yes	A2	300			Hard
US	US	Hewlett-Packard USD .00s 2017	Yes	AA3	1,076		Yes	Hard
US	US	I B M USD 2.00a 2003 Euro	Yes	A1	100			Soft
US	US	Loews USD 3.13s 2007	Yes	A2	1,000			Hard
US	US	Texaco USD 3.50a 2004 Euro	Yes	A1	200			Hard
US	US	Xerox USD .57s 2018	Yes	A3	500		Yes	Hard
US	US	Xerox USD 1.30s 2006	Yes	A2	150			Hard

Appendix 3-B. Day Count Conventions

The calculation of yields and accrued interest are influenced by day count conventions that differ from one bond to the other. Day count conventions vary in their assumptions on the number of days in a year as well as the number of days in a month. The most common conventions, in order of their commonness, are:

- 30 / 360 days basis

  In this convention, all months (even February) are supposed to have 30 days. For European bonds that follow *International Securities Market Association* (ISMA) rules if a month has 31 days it is adjusted to 30. Last day of February counts as 30 days. American bonds follow *Securities Industry Association* (SIA) rules where the 31^{st} day is the same as the 1^{st} day of the following month. One calendar year is taken to have 360 days.

- Actual / 365 days basis

  Here any year has 365 days for the denominator even the leap one. The nominator is generally the actual number of days (counting February 29 if leap year), except for Japanese bonds where the leap year's February is ignored.

- Actual / Actual days basis

  Both the numerator and the denominator is the number of days between the two dates. The denominator is the actual number of days in the coupon period multiplied by the coupon frequency. This normally results in day count factors of 1.0 for annual coupons, 0.5 for semi annual coupons and 0.25 for quarterly coupons.

- Actual / 360 days basis

  The number of days used in the numerator is the real number of days (as you can find in all calendars). The denominator assumes that you have 360 days (12 times 30 days) in a year.

Chapter 4:
Valuation of a Liquid Yield Option Note: A Decomposition Approach

4.1 Introduction

This chapter addresses the valuation of a Liquid Yield Option Note (LYON), a complex and interesting convertible bond structure. I employ an option valuation framework that treats LYONs as a portfolio of long-term options plus corporate discount bonds. It provides an innovative and intuitive way of valuing complex securities and compares well with the results published by McConnell and Schwartz (1986).

A LYON is a zero coupon bond with number of embedded features: the convertible feature enables the holder to exchange the LYON for the stock of the issuing company; the put feature gives the holder a right to redeem the LYON early; the call feature is the right of the issuing company to bring forward the maturity of the LYON. The literature concerning LYONs and the rationale for their issuance has been discussed in Chapter 3, section 2.5. LYONs are predominantly issued in the US domestic market, but two of them have found their way into the sample of convertible bonds that has been analysed in Chapter 3. These bonds are:

Deutsche Bank AG	USD	0.00s	2017
Hewlett-Packard Co	USD	0.00s	2017

Both bonds were issued in 1997. The Deutsche Bank LYON was called on the first opportunity, five years after issue[89]. The LYON was exchangeable into Daimler-Benz AG shares (and as from December 1998 into shares of Daimler-Chrysler[90]). The Hewlett-Packard LYON has its first put and call scheduled for three years after issue[91]. This bond is still alive.

The LYON chosen for this chapter was initially issued at a price of $250 in 1985 under the name Waste Management Inc USD 0% 2001, with full terms listed in Appendix 4-A. It was

89 The conversion option had always been deep out-of-the-money and remained there at the time of call leading to all bondholders opting for cash.

90 Just before the merger Daimler-Benz paid a surprise special dividend that left holders of this LYON nursing huge losses due to lack of suitable protection. This incident is discussed in the previous chapter under the subject of Special Dividends.

one of the first issued and is also analysed by McConnell and Schwartz (1986) which provides good ground for comparison. In 1993 Waste Management Inc changed its name to WMX Technologies Inc, and in 1998 it was acquired by USA Waste Services Inc. The enlarged company then changed the name back to Waste Management Inc. As a part of acquisition process, WMX Technologies Inc called the LYON on 30th June 1998 at a price of USD 798.34. At that time the conversion value was just above $1,220, which resulted in full conversion.

Unlike most of the subsequently issued LYONs, Waste Management Inc's LYON was characterised by an initial 'provisional call protection period' where the issuer cannot call the issue unless a special condition is met (e.g. the share price has to "hit" a predetermined level). After the end of the provisional call protection period the issuer can call the notes at any time. However, the issuer is required to give LYON holders a call notice providing them with sufficient time to make up their minds whether to accept the cash or convert. In the subsequent analysis I will show that the length of the call notice period can have a material effect on the LYON price.

This chapter proposes an innovative way of pricing this complex bond structure by decomposition. Rather than solving for the LYON value as a whole, each decomposed component becomes a part of a system of linear partial differential equations, resulting in explicit solution for each individual component. Emphasis is given to the definition of the boundary conditions and their interpretation rather than the actual modelling details.

The following sections define the optimal conversion, call and put rules, the payoff conditions, and the interrelationships between them. The valuation approach is then compared with that of McConnell and Schwartz (1986) of valuing the same LYON using the same data set.

I follow a pedagogical approach to this complex valuation problem by first defining a special type of LYON which carries only convertible features. Then I extend this simple, convertible-only LYON to incorporate the put characteristics and finally I value the actual LYON.

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[91] The opportunity to request an early redemption was utilised by approximately 75% of all holders. The conversion option of this bond has always been out-of-the-money since the issue.

There is a double advantage in adopting this approach. Firstly, it lays down a methodology of decomposing complex securities. Secondly, it helps to examine how these securities are structured. Finally, the transparency and accuracy of this approach is demonstrated by comparing it with the results listed in McConnell and Schwarz (1986).

4.1.1 Notation

$L(t)$	LYON value;
$CL(t)$	Convertible-only LYON value;
$CPL(t)$	Convertible-Puttable-only LYON value;
$S(t)$	Stock price;
α	Conversion ratio;
$P(t)$	Put price of the LYON at time t;
$C(t)$	Call price of the LYON at time t;
η	Call notice period;
$p(t)$	Call notice period put option;
$B(t)$	Current value of the discount bond that expires at time T;
$B(T)$	Face value of the bond;
$W(t)$	Value of the option to convert;
$WP(t)$	Value of the option to put;
$C^{B}(t)$	Value of the option to call the discount bond;
$C^{W}(t)$	Value of the option to call the option to convert;
$C^{WP}(t)$	Value of the option to call the option to put.

4.2 The Optimal Conversion, Call and Put Rules

The rational conversion, call and put rules have been first analysed by Brennan and Schwartz (1977a, 1980) and Ingersoll (1977a). I will go through each of these conditions as they apply to the LYON.

4.2.1 Rational Conversion Rule under Ideal Market Conditions

It will never be optimal to convert an uncalled LYON except if the conversion value (parity) is greater than the current price of the LYON. In other words, a rational LYON holder will *voluntarily* convert the LYON into stock at time t, providing that at that time the LYON can be converted, if

$$\alpha S(t) \geq L(t) \qquad [4\text{-}1]$$

where α is the conversion ratio, $S(t)$ is the stock price and $L(t)$ is the price of the LYON.

4.2.2 Rational Put (Redemption) Rule under Ideal Market Conditions

On any put date (other than maturity), if the issue has not been converted or called, the LYON holder will put the LYON if its current price is less than the respective put price. Therefore, the LYON will be put if

$$P(t) \geq L(t) \qquad [4\text{-}2]$$

where $P(t)$ is the put price at time t.

Since the put prices escalate through time, LYON holders have to make up their minds if they are going to put the security at that put date or hold it and put at the next available put date[92].

4.2.3 Rational Call Rule under Ideal Market Conditions

The issuer's rational call policy is to maximise shareholders' wealth. By invoking the Modigliani-Miller theorem, i.e. the value of the firm is independent of its capital structure; this is achieved by minimising the value of the LYON. Therefore, a rational issuer should call the LYON, subject to the call option being exercisable and the trigger condition satisfied, at times when,

$$L(t) \geq C(t) \qquad [4\text{-}3]$$

Where $C(t)$ is the call price at time t. Upon call, LYON holders must choose between the most attractive of the two: either converting and receiving $\alpha S(t)$ or accepting the redemption payment given by the time t call price, $C(t)$. Thus, by calling the LYON the issuer will either redeem the bonds or force conversion.

92 In situations where interest rates or credit spread decline it may be worth keeping the security rather than putting it. This suggests that a model taking into account the stochastic nature of interest rates and credit spread should produce an improved valuation.

Fortunately, LYON cannot be called for immediate redemption. The issuer is required to give a notice of its intention to exercise the early redemption option, thus allowing time for investors to make a choice to either convert and receive $\alpha S(t)$ shares or do nothing and receive a guaranteed cash amount equal to the call price, $C(t)$. So the value of the LYON at the time the call is announced is given by

$$L(t) = e^{-r\eta} E\left\{\max\left[\alpha S(t+\eta), C(t+\eta)\right]\right\} \qquad [4\text{-}4]$$

Where η is the call notice period and E is the expectation operator. The value of the LYON at the time of the call announcement can be interpreted as a package consisting of the conversion value and a European option to put the conversion value against the call price[93]. This option is granted to the investor at the time the call is announced. Symbolically [4-4] is equivalent to,

$$L(t) = \alpha S(t) + p(t) \qquad [4\text{-}5]$$

Where

$$p(t) = e^{-r\eta} E\left\{\max\left[C(t+\eta) - \alpha S(t+\eta), 0\right]\right\} \qquad [4\text{-}6]$$

And $p(t)$ is the value of call notice period put option[94]. The time value of the call notice period put would usually be small especially if the call is conditional on stock hitting the trigger, i.e. the put would be deep out-of-the-money. However, its value should not be completely ignored especially in cases were stock price volatility is high and the call notice period is long. Additionally, the value afforded to the call notice period put may be substantially different from the issuer's perspective due to the asymmetric information, which may influence the issuer to post-pone calling the bond until this option is deep-out-of-the-money[95]. In section 4.5 I examine the effect the length of the call notice period has on the overall value of the LYON.

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93 Alternatively, using the put-call parity, a called LYON can be seen as a package of a short (risk-free) discount bond and a European call option with the call price as a strike.

94 McConnel and Schwartz (1986) in the footnote 4 make a passing note of the LYON's 15 day call notice period. However, in subsequent analysis and valuation this is ignored. In such cases the value of the call notice period put option reduces to its intrinsic value.

95 An investor may in expectation of such circumstances attach a higher volatility estimate to the call notice period put option and fully account for this extra value.

The issuer's rationale for calling the LYON is established by the following propositions.

PROPOSITION 4.1. Assuming that the LYON is not already converted or redeemed, and the stock price is above the *trigger* level[96], a necessary and sufficient condition for the issuer to announce his intention to call the LYON in order to force conversion, is

$$E[\alpha S(t+\eta)] \geq C(t+\eta) \geq B(t+\eta). \qquad [4\text{-}7]$$

PROPOSITION 4.2. Assuming that the LYON is not already converted or redeemed, and the stock price is above the trigger level, a necessary and sufficient condition for the issuer to announce his intention to call the issue to refinance it is

$$B(t+\eta) \geq C(t+\eta) \geq E[\alpha S(t+\eta)]. \qquad [4\text{-}8]$$

4.2.4 Maturity Conditions

At maturity, the holder is entitled to receive the face value of the bond or the conversion value. Therefore, the value of the LYON at maturity time, T, is given by

$$L(T) = \max[\alpha S(T), B(T)] \qquad [4\text{-}9]$$

4.3 Decomposition of the LYON

Our approach of valuing the LYON, using contingent claims analysis, begins with the specification of the process that governs stock prices and interest rates. Following McConnell and Schwartz (1986) I also assume that the value of the LYON depends upon the issuer's common stock rather than the total market value of the company[97]. Since the lognormal process does not allow for the possibility of default I use a risk-adjusted discount rate that

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[96] Conditions about callability and the trigger are added for the sake of generality. Because the LYON under consideration is callable throughout its life span and the trigger, when it exists, is well above the call price these conditions are trivially satisfied.

[97] The effect of dilution is assumed to be negligible and is subsequently ignored. For a discussion of the dilution effect, see for example Veld (1994).

incorporates the default premium. This adjustment corrects for possible overvaluation of the LYON.

In particular, the following assumptions underlie the modelling environment:

ASSUMPTION 4.1. An environment where trading takes place continuously, transaction costs and taxes are zero, there are no borrowing or lending restrictions and investors prefer more wealth than less.

ASSUMPTION 4.2. The model assumes only one stochastic variable, the stock price, S(t). Movements in the stock price are generated by the following diffusion[98], known as Geometric Brownian motion,

$$dS = (\mu - q)Sdt + \sigma SdZ \quad [4\text{-}10]$$

where the instantaneous growth rate, μ, and the instantaneous variance, σ^2, of the process are constant, q is the continuous dividend yield, and dZ is a standard Wiener process.

ASSUMPTION 4.3. The risk-adjusted discount rate (which incorporates a risk premium), *r*, is assumed constant.

4.3.1 Valuation of Convertible-only LYON

From the conversion and maturity conditions the arbitrage-free value of the convertible-only LYON (*CL*), at time *T*, is equal,

$$CL(T) = \max\left[\alpha S(T), B(T)\right] \quad [4\text{-}11]$$

where $B(T)$ is the face value of the discount bond, \$1000, and $S(T)$ the stock price at maturity. A more careful look at relation [4-11] shows that *CL*'s payoff can be transformed into one that reveals the hybrid nature of *CL*'s more explicitly. Making some elementary transformations [4-11] can be written as

$$CL(T) = B(T) + \max[\alpha S(T) - B(T),\ 0] \qquad [4\text{-}12]$$

This shows that the value of the LYON at expiry is equal to the face value of its discount bond component increased by any excess value obtainable from conversion into stock. Thus the value of the non-callable, non-puttable, convertible-only LYON at maturity is

$$CL(T) = B(T) + W(T) \qquad [4\text{-}13]$$

Where $W(T)$ is the value of the *option to convert* (warrant), or to be more precise this is the value of an *option to exchange* bond for stock.

Since the option to convert can be exercised at any time (American style exercise), in order to prevent risk-less arbitrage, the value of the LYON should never fall below its conversion value,

$$CL(t) = \max\left[\alpha S(t), B(t), e^{-r\Delta t} E[CL(t + \Delta t)]\right] \qquad [4\text{-}14]$$

This is the greater of the conversion value, bond value or the discounted expected value of the LYON if it is held for another instant. Substituting [4-13] into [4-14] I am left to solve the following recursive maximisation problem

$$CL(t) = B(t) + W(t) \qquad [4\text{-}15]$$

Where the option to convert, $W(t)$, at time t, is defined as the maximum gain one can realise by converting immediately or holding the LYON and considering conversion at the next conversion instant, i.e.,

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[98] I could implement any "acceptable" process for stock price movements. However, the idea is to test our approach against market data and the "*all in one*" approach of McConnell and Schwartz (1986) without changing any of their assumptions.

$$W(t) = \max[W_{IV}(t), W_{TV}(t)] \qquad [4\text{-}16]$$

Where the first term in the max[·] is the intrinsic value of the option to convert

$$W_{IV}(t) = \max[\alpha S(t) - B(t), 0] \qquad [4\text{-}17]$$

And the second term is the expected value of the option to convert at the next successive conversion opportunity, conditional on the level of the stock price at time t, that is, the time value

$$W_{TV}(t) = e^{-r\Delta t} E[W(t + \Delta t)] \qquad [4\text{-}18]$$

Where Δt denotes the time until the next exercise opportunity.

The value of the corporate discount bond element at time t, under the constant interest rates assumption, is given by

$$B(t) = e^{-r\Delta t} B(t + \Delta t) = e^{-r(T-t)} B(T) \qquad [4\text{-}19]$$

Where $B(T)$ is the face value of the bond, and r, is the continuously compounded discount rate applied to the particular LYON.

Closer inspection of relation [4-11] shows that a convertible-only LYON can be valued as a call option on the maximum of two assets; the discount bond, $B(t)$, and the conversion value, $\alpha S(t)$ (i.e., the conversion ratio multiplied by the stock price). Stulz (1982) has derived closed form solutions for European type options with similar payoffs. Relation [4-12] offers alternative interpretation showing that convertible-only LYONs can be valued as a package consisting of a discount bond plus an exchange option. According to Margrabe's (1978) original paper exchange options can be viewed as either a call on the conversion value with strike price equal to the discount bond value or a put on the discount bond with strike price equal to the share price. Therefore, the alternative way to view [4-12] is,

$$CL(T) = \alpha S(T) + \max[B(T) - \alpha S(T),\ 0] \qquad [4\text{-}20]$$

Equation [4-20] simply suggests another representation of the convertible-only LYON. Expressions [4-11], [4-12], and [4-20] are, of course, all equivalent.

4.3.2 Valuation of Convertible-Puttable-only LYON

The put feature gives the LYON holder the right to put the bond back to the issuer at pre-specified prices. In a sense, the put option granted to the LYON holder reduces the overall exposure to the losses the investor can suffer should the stock performs poorly. Put prices would normally accrete over time at the rate specified in the offering circular[99].

To facilitate decomposition of the convertible-puttable-only LYON, I assume that the last put coincides with the LYON's redemption value and time. The value at maturity of the convertible-puttable (but non-callable) LYON (*CPL*) is equal to[100] (noting that at maturity $P(T) \equiv B(T)$),

$$CPL(T) = \max[\alpha S(T), P(T), B(T)] \qquad [4\text{-}21]$$

or

$$CPL(T) = B(T) + \max[\max[\alpha S(T), P(T)] - B(T), 0]$$

which can be rewritten as[101]

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[99] In the case of Waste Management LYON the accretion rate is set at 9% with semi-annual compounding.

[100] Rubinstein (1991) and Johnson (1987) obtained closed form solutions for European options on similar (and more general) payoffs. Rubinstein (1991) named them "*options delivering the best of two risky assets and cash*". Since I assume constant interest rates the above case is a special case of his valuation problem.

[101] The convertible-puttable LYON payoff can be also defined as a portfolio of cash plus a *call option on the maximum of two assets*, that is,

$$CPL(T) = P(T) + \max[\max[\alpha S(T), B(T)] - P(T), 0]$$

$$CPL(T) = B(T) + \max[W(T), WP(T)] \qquad [4\text{-}22]$$

Where the value at maturity of the *option-to-put*, *WP*(*T*), is defined as the excess value LYON holder can realise by putting the bond[102],

$$WP(T) = WP_{IV} = \max[P(T) - B(T), 0] \qquad [4\text{-}23]$$

Where $WP_{IV}(T)$ is the intrinsic value of the option to put .

Payoff [4-22] shows that at maturity the value of the convertible, puttable only LYON is equal to the sum of the discount bond and the greater of the option to convert or option to put. Choosing between the greater of the values is necessary as the holder can either convert or put, but not both at the same time.

Using the same argument as for the option to convert, the value of the LYON at any time *t* prior to maturity, is equal to a portfolio consisting of a discount bond and either the option to convert or put, whichever is greater,

$$CPL(t) = B(t) + \max[W(t), WP(t)] \qquad [4\text{-}24]$$

Where *B*(*t*) and *W*(*t*) are defined by [4-19] and [4-16] respectively, while *WP*(*t*) takes a form similar to the definition of option to convert,

$$WP(t) = \max[WP_{IV}(t), WP_{TV}(t)] \qquad [4\text{-}25]$$

Where the first term is given by [4-23], and the second term, which is the value of the put option at the next successive put opportunity, is given by[103]

---

102 Considering the definition of put price at maturity, the value of the option to put at maturity is zero.

103 The expectation operator is not strictly necessary as in this case interest rates (and default risk) are assumed deterministic.

$$WP_{TV}(t) = e^{-r\Delta t} E[WP(t + \Delta t)]. \quad [4\text{-}26]$$

The option to put is not similar to the option to convert in the sense that early exercise is restricted on certain pre-specified dates during the life of the LYON.

4.3.3 Valuation of Convertible-Puttable-Callable LYON

The option to call has important consequences for the valuation, risk management and structuring of LYONs. The payoff of the LYON at any time before maturity is given by

$$L(t) = CPL(t) - \max[CPL(t) - C(t), 0] \quad [4\text{-}27]$$

By writing a call on the convertible-puttable LYON the investor actually limits his upside potential to the exercise price of the call. However, obtaining an extra value of the LYON above the call price is still possible due to the call notice period option.

To fully decompose the convertible-puttable-callable LYON I have to examine the effect of the call on each component of the decomposed portfolio [4-24]. To achieve this I start by examining the effect of the call on the corporate discount bond.

From the definition of rational call policy, and under the ideal market conditions, the issuer should call the bond as soon as the price of the discount bond becomes equal or higher than the call price[104]. However, the issuer is restricted from exercising this right throughout the provisional call protection period. During that time the stock price has to be on or above the trigger level (in this example, $86.01) before the issuer can exercise his call option[105].

The value of the option to call the discount bond, C^B, can be written as

[104] Examination of the structure of the call prices for the LYON considered in this paper, this should happen as soon as risk adjusted discount rate falls below 9%.

[105] If the stock price reaches the initial trigger level and the issuer calls, the holders are most certain to opt for conversion as the call price during that period would have been significantly less than the conversion value. The conversion is not absolutely certain as the issuer would have to give at least 15 days notice within which the stock price could drop below the call price. Upon call the issuer grants the holders an additional put option offering the downside protection in case of a sudden stock price fall. In order to maximise the value of the stock the issuer would try to minimise the value of this implicit put. This is achieved by making the put out of the money, i.e. calling the bond when the stock price is higher than the call price or by reducing the call notice period to minimum allowed.

$$C^B(t) = \max\left[C_{IV}^B(t), C_{TV}^B(t)\right] \qquad [4\text{-}28]$$

where

$$C_{IV}^B(t) = \max\left[B(t) - C(t), 0\right] \qquad [4\text{-}29]$$

and

$$C_{TV}^B(t) = e^{-r\Delta t} E\left[C^B(t + \Delta t)\right]. \qquad [4\text{-}30]$$

Having defined the value of the option to call the discount bond I can now define the value of a callable discount bond at time t as,

$$\textit{Callable Discount Bond} = B(t) - C^B(t)\,. \qquad [4\text{-}31]$$

From the definition of the callable discount bond it is obvious that, in ideal market conditions, its value can never grow above the call price.

The effect that the call feature has on the values of the option to convert and option to put is two fold. First, both options now incorporate the callable discount bond. Second, the (convert and put) options' lifetime may be shortened as a result of an early call, thus potentially preventing them from realising their full values. However, the provisional call protection period offers some protection to LYON holders.

In the case of our particular LYON, the issuer's call is continuously exercisable (subject to trigger condition being satisfied during the first three years). In practice this means that if the issuer calls, the LYON holder may choose to exercise his/her own option to convert, therefore realising its intrinsic value. This value, regardless of the stock price level or the time of call, can never be taken away from the LYON holder.

Once the issuer calls, the intrinsic value of its call on the option to convert at time t must be equal to a) the time premium of that option[106] reduced by the call notice period put that the

[106] The time premium is defined as the difference between an option's full value and its intrinsic value at a given point in time.

issuer grants to the LYON holder and b) the time premium of the call on the discount bond that will equally become extinguished. The intrinsic value of the issuer's call on the option to convert can be written as (subject to LYON being callable at this time),

$$C_{IV}^{W}(t) = \max\{[W(t)+B(t)]-[\alpha S(t)+C^{B}(t)+p(t)],0\} \qquad [4\text{-}32]$$

On the other hand, if the LYON cannot be called at time t, the value of issuer's call on the option to convert becomes the discounted expected value for the next successive call opportunity,

$$C_{TV}^{W}(t) = e^{-r\Delta t}E\{C^{W}[t+\Delta t]\} \qquad [4\text{-}33]$$

Using the adopted notation, the value of the issuer's call on the option to convert at time t is the greater of its intrinsic and discounted expected value,

$$C^{W}(t) = \max[C_{IV}^{W}(t), C_{TV}^{W}(t)] \qquad [4\text{-}34]$$

The LYON holder is effectively given a callable option to convert into a stock of the company, a callable discount bond, where any potential future premium is conditional on the option to call not being exercised.

The value of the callable option to convert can be written as the value of the (normal) option to convert reduced by the value of the issuer's right to call it, i.e.

$$\textit{Callable Option to Convert} = W(t) - C^{W}(t) \qquad [4\text{-}35]$$

Combining the two option payoffs [4-16] and [4-34], the value of the callable option to convert can be expressed as,

$$\textit{Callable Option to Convert} = \max[W_{IV}(t) - C_{IV}^{W}(t), W_{TV}(t) - C_{TV}^{W}(t)] \qquad [4\text{-}36]$$

In a similar fashion, the value of the callable option to put is defined as a put on the callable discount bond where any later premiums are conditional on the bond not being called. A reasonable assumption, which should be satisfied by a correctly structured LYON, would be that the call price is always greater than or equal to the put price.

$$C^{WP}(t) = \max\left[C_{IV}^{WP}(t),\, C_{TV}^{WP}(t)\right] \qquad [4\text{-}37]$$

$$C_{IV}^{WP}(t) = \max\left\{\left[WP(t) + B(t)\right] - \left[P(t) + C^{B}(t)\right], 0\right\} \qquad [4\text{-}38]$$

$$C_{TV}^{WP}(t) = e^{-r\Delta t} E\left\{C^{WP}\left[t + \Delta t\right]\right\} \qquad [4\text{-}39]$$

The value of a callable option to put can be written as the value of the ordinary option to put [4-25] reduced by the value of the issuer's right to call it, i.e.

$$Callable\ Option\ to\ Put = WP(t) - C^{WP}(t) \qquad [4\text{-}40]$$

Combining the two values [4-25] and [4-37] I arrive at a direct expression for the value of the callable option to put as,

$$Callable\ Option\ to\ Put = \max\left[WP_{IV}(t) - C_{IV}^{WP}(t), WP_{TV}(t) - C_{TV}^{WP}(t)\right] \qquad [4\text{-}41]$$

Taking the call feature into account the value of LYON decomposes in to a portfolio of a callable discount bond and the greater of the callable option to convert or callable option to put. The value of such a portfolio at any time $0 \leq t \leq T$, is,

$$\begin{aligned} L(t) &= CallableDiscountBond + \\ &\max\left[CallableConvertionOption, CallablePutOption\right] \end{aligned} \qquad [4\text{-}42]$$

or fully decomposed, the LYON's value can be written as,

$$L(t) = B(t) - C^{B}(t) + \max\left[W(t) - C^{W}(t), WP(t) - C^{WP}(t)\right] \qquad [4\text{-}43]$$

Removing various LYON features can conveniently test the consistence of the current approach as the contribution of each decomposed element is clearly visible, see table 4-2. Taking away conversion and put features, the LYON reduces to a callable corporate discount bond. Removing the conversion aspect, the resulting portfolio equals a callable-puttable corporate discount bond, etc.

4.4 Empirical Results

The LYON was issued at a price of $250.00. The two parameters that are not directly observable are the discount rate (which incorporates the risk premium) and the stock price volatility. In order to make meaningful comparisons I use McConnell and Schwartz's (1986) estimates of volatility (30%), discount rates (11.21%), and dividends (1.6%).

The model is implemented as an explicit difference method with the spacing between the nodes chosen in a way that reduces it to a binomial tree[107].

Table 4-1 shows the closing market price and theoretical prices of the new "*decomposition*" approach and the McConnell and Schwartz (1986) models.

Table 4-1. LYON Market Prices Comparison
Comparison of market price, McConnell and Schwartz valuation and the new valuation approach.

Date	Stock Price	Market Price	McConnell & Schwartz	New Approach
12-Apr-85	52.25	258.75	262.70	257.52
15-Apr-85	53.00	258.75	264.60	259.74
16-Apr-85	52.63	257.50	263.70	258.70
17-Apr-85	52.00	257.50	262.10	256.87
18-Apr-85	52.38	257.50	263.00	258.01
19-Apr-85	52.75	257.50	264.00	259.08
22-Apr-85	52.50	257.50	263.30	258.46
23-Apr-85	53.25	260.00	265.30	260.63
24-Apr-85	54.25	265.00	267.90	263.45
25-Apr-85	54.25	265.00	267.90	263.47
26-Apr-85	54.00	265.00	267.20	262.79
29-Apr-85	53.75	260.00	266.60	262.14
30-Apr-85	52.13	260.00	262.40	257.55
01-May-85	49.75	252.50	256.70	251.09
02-May-85	50.50	250.00	258.40	253.12
03-May-85	50.75	252.50	259.00	253.84
06-May-85	50.50	252.50	258.40	253.20
07-May-85	50.88	255.00	259.30	254.27
08-May-85	50.75	253.75	259.00	253.94
09-May-85	51.25	255.00	260.30	255.37

[107] For discussion of explicit difference method used in this chapter and its relationship with the binomial tree techniques see Brennan and Schwartz (1978).

10-May-85	53.13	260.00	265.00	260.61

Table 4-1 clearly shows that our model provides results consistent with those from McConnell and Schwartz (1986) model. During the period under observation it also tracks the market better than the McConnell and Schwartz (1986) model and in some cases replicates the market prices exactly.

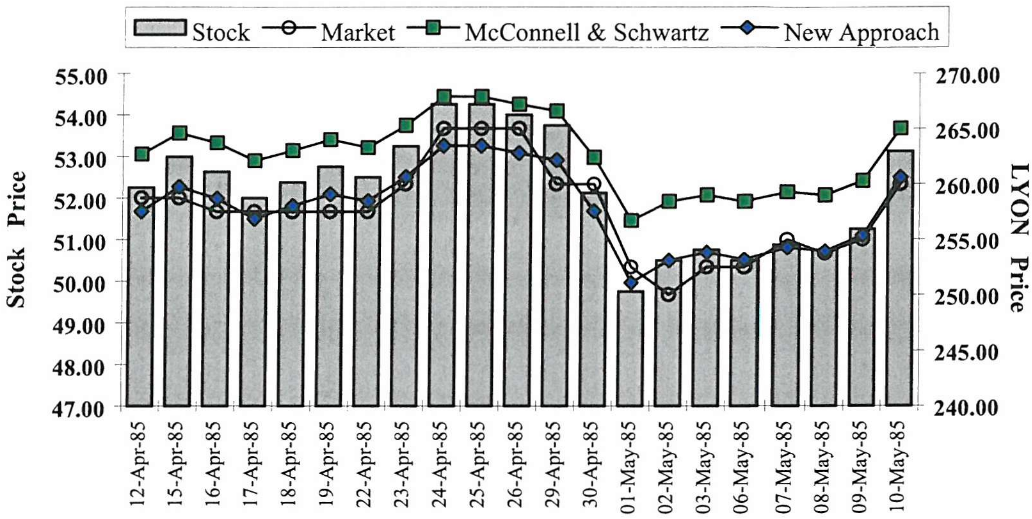

Figure 4-1. LYON Market Prices Comparison.
Comparison of market prices, McConnell and Schwartz (1986) valuation and the new valuation approach.

From the theoretical background as discussed in the previous section the two approaches should yield equivalent results. McConnell and Schwartz (1986) treat LYON as a single financial instrument and impose one set of initial and (free) boundary conditions. Their solution yields a single value for the LYON as a whole. The approach described in this chapter decomposes the LYON into a portfolio of several financial instruments and solves the problem as a system of linear partial differential equations. As a consequence my solution yields values for each individual component of the LYON and only after by summing them all together the result can be compared to McConnell and Schwartz (1986).

I attribute the difference to the different numerical convergence[108], inclusion of the call notice period put option, and possibly different treatment on interest rate discounting within the models.

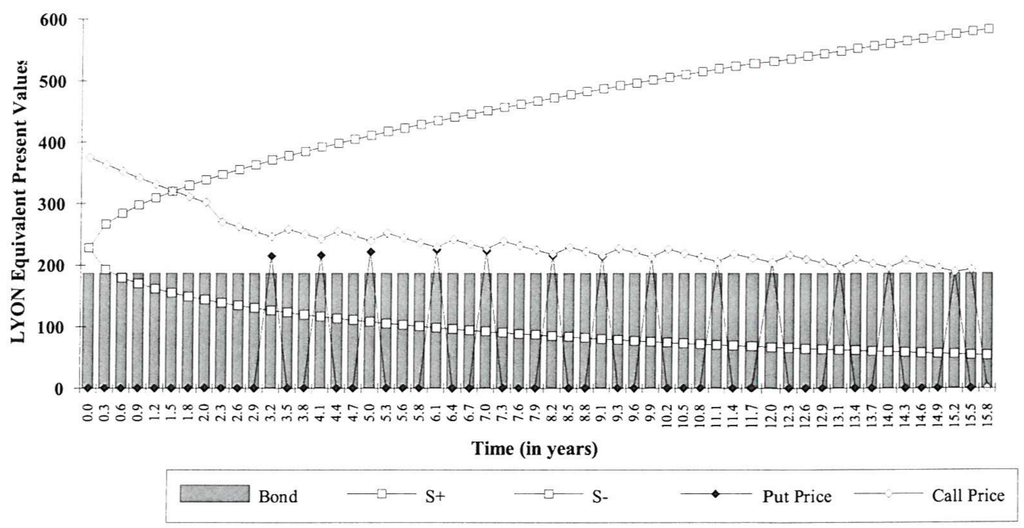

Figure 4-2. Components of LYON Price.

The above figure shows the present values of the various components of the LYON. It reveals explicitly and clearly the way that the LYON was structured.

The elements for the figure 4-2 are as composed as follows,

- S+ is the present value of the upper bound that the stock price can reach given time and volatility assumptions. This effectively presents the upper boundary a LYON can take based on the distributional assumptions for the stock price.

108 Our own testing of the speed of convergence suggests a slower rate of convergence when valuing LYONs with callable features compared to the non-callable LYONs. Similar results were reported by Ritchken (1995) in the case of the related problem of pricing barrier options. Ritchken (1995) improves the convergence by repositioning the surrounding tree nodes to coincide with the barrier and suitably adjusting the jump probabilities so the equation [5-23] is satisfied.

- S- is the present value of the lower bound that the stock price can reach given time and volatility assumptions.
- Put Price is the present value of the put price and represents the lower boundary for the value of the LYON.
- Call Price is the present value of the higher of the call price and the trigger level and this acts as a cap on the upper boundary, effectively lowering the maximum value a LYON can achieve (before being called). Note that after 8 years the effective put and call prices are the same, suggesting that it is either optimal for the investor to put or convert the LYON, or it is optimal for the company to call it.
- Bond is the price of the discount bond given a discount rate of 11.21%. It can be thought of as simply an extra and final put opportunity.

The following comments are in order,

- From the above the value of the LYON is bounded by the lesser of the upper stock price bound and the call price.
- From the below the value of the LYON is bounded by the higher of the put price and the bond price.
- With those two bounds the actual value of the LYON is calculated as the probability weighted average price.

Table 4-2 summarises the theoretical values of the LYON at the time of issue for the range of initial stock prices (in the first column). The table offers an insight in to the effect the various features embedded in the LYON have on its value. Some of our results contradict those reported by McConnell and Schwartz (1986). I summarise them as follows,

1) The effect of the put option on the value of the LYON becomes visible only when the stock price sinks below $32. This contradicts the findings of McConnell and Schwartz (1986, page 572) who argue that it is very valuable to LYON holders. The rationale is that the option to put is always

overtaken by the value of the option to convert (see relation [4-22]). The immediate conversion value for the base stock price of $52.125 is $227.27. This effectively represents the minimum value of the LYON at issue.

2) A callable-only LYON and a discount bond have the same value[109]. Since the value of the callable-only LYON is equal to $187.04, the addition of the convertible feature can only increase the value of the LYON rather than decrease it. Table III of McConnell and Schwartz (1986, page 573) presents a value of $181.94 for the convertible-callable-only LYON that violates this limit.

3) As can be seen from figure 4-2 the call price is higher than the put price till year 10. Subsequent call and put prices become identical. The figure 4-2 also shows that, given a volatility of 30%, there is almost negligible probability for the trigger condition to be satisfied that would result in the issuer calling the bond.

4) McConnell and Schwartz (1986, page 574) argue that "... as the redemption prices of the LYON increase through time, the value of the downside risk protection for holding the LYON increases." Figure 4-2 clearly shows that this is not the case. The redemption prices may escalate through time but the present values are not monotonically increasing function of time. The present values of the put prices are initially increasing but then decreasing function time. This suggests that, further into the future, the downside protection offered by the put decreases rather than increases, the peak being reached in year 6.

109 This is effectively the value of callable discount bond. For the given constant interest rate assumption of 11.21% and the call schedule, call will never be exercised and therefore has no value.

Table 4-2. New Approach Results for Various Types of Convertible Bonds.

Convertibl	✓	✓	✓	✓	✗	✗	✗
Puttabl	✓	✓	✗	✗	✓	✗	✓
Callabl	✓	✗	✗	✓	✓	✓	✗
Stock							
30	223.06	223.06	219.72	208.33	223.06	187.04	223.06
31	223.06	223.06	221.84	209.91	223.06	187.04	223.06
32	223.06	224.00	224.00	211.55	223.06	187.04	223.06
33	223.06	226.20	226.20	213.28	223.06	187.04	223.06
34	223.06	228.46	228.46	215.07	223.06	187.04	223.06
35	223.06	230.74	230.74	216.92	223.06	187.04	223.06
36	223.06	233.10	233.10	218.80	223.06	187.04	223.06
37	223.06	235.48	235.48	220.72	223.06	187.04	223.06
38	223.06	237.89	237.89	222.77	223.06	187.04	223.06
39	224.83	240.37	240.37	224.83	223.06	187.04	223.06
40	226.92	242.87	242.87	226.92	223.06	187.04	223.06
41	229.16	245.40	245.40	229.16	223.06	187.04	223.06
42	231.38	247.99	247.99	231.38	223.06	187.04	223.06
43	233.64	250.60	250.60	233.64	223.06	187.04	223.06
44	236.07	253.24	253.24	236.07	223.06	187.04	223.06
45	238.44	255.94	255.94	238.44	223.06	187.04	223.06
46	240.91	258.66	258.66	240.91	223.06	187.04	223.06
47	243.42	261.40	261.40	243.42	223.06	187.04	223.06
48	245.99	264.19	264.19	245.99	223.06	187.04	223.06
49	248.64	267.01	267.01	248.64	223.06	187.04	223.06
50	251.27	269.86	269.86	251.27	223.06	187.04	223.06
51	254.05	272.72	272.72	254.05	223.06	187.04	223.06
52	256.81	275.65	275.65	256.81	223.06	187.04	223.06
53	259.67	278.59	278.59	259.67	223.06	187.04	223.06
54	262.46	281.56	281.56	262.46	223.06	187.04	223.06
55	265.47	284.54	284.54	265.47	223.06	187.04	223.06
56	268.40	287.58	287.58	268.40	223.06	187.04	223.06
57	271.45	290.63	290.63	271.45	223.06	187.04	223.06
58	274.47	293.71	293.71	274.47	223.06	187.04	223.06
59	277.63	296.81	296.81	277.63	223.06	187.04	223.06
60	280.75	299.95	299.95	280.75	223.06	187.04	223.06
61	283.98	303.11	303.11	283.98	223.06	187.04	223.06
62	287.20	306.30	306.30	287.20	223.06	187.04	223.06
63	290.43	309.50	309.50	290.43	223.06	187.04	223.06
64	293.80	312.72	312.72	293.80	223.06	187.04	223.06
65	297.11	315.99	315.99	297.11	223.06	187.04	223.06
66	300.54	319.27	319.27	300.54	223.06	187.04	223.06
67	303.98	322.58	322.58	303.98	223.06	187.04	223.06
68	307.40	325.90	325.90	307.40	223.06	187.04	223.06
69	310.96	329.23	329.23	310.96	223.06	187.04	223.06
70	314.47	332.62	332.62	314.47	223.06	187.04	223.06
71	318.04	336.02	336.02	318.04	223.06	187.04	223.06
72	321.71	339.43	339.43	321.71	223.06	187.04	223.06
73	325.27	342.87	342.87	325.27	223.06	187.04	223.06
74	328.98	346.31	346.31	328.98	223.06	187.04	223.06
75	332.73	349.79	349.79	332.73	223.06	187.04	223.06
76	336.44	353.30	353.30	336.44	223.06	187.04	223.06
77	340.18	356.81	356.81	340.18	223.06	187.04	223.06
78	344.05	360.36	360.36	344.05	223.06	187.04	223.06
79	347.87	363.91	363.91	347.87	223.06	187.04	223.06
80	351.60	367.47	367.47	351.60	223.06	187.04	223.06
81	355.65	371.08	371.08	355.65	223.06	187.04	223.06
82	359.64	374.70	374.70	359.64	223.06	187.04	223.06
83	363.22	378.33	378.33	363.22	223.06	187.04	223.06
84	367.42	381.99	381.99	367.42	223.06	187.04	223.06
85	371.59	385.66	385.66	371.59	223.06	187.04	223.06
86	375.55	389.33	389.33	375.55	223.06	187.04	223.06
87	379.32	393.04	393.04	379.32	223.06	187.04	223.06
88	383.68	396.78	396.78	383.68	223.06	187.04	223.06
89	388.04	400.52	400.52	388.04	223.06	187.04	223.06
90	392.40	404.27	404.27	392.40	223.06	187.04	223.06

4.5 Effect of the Call Notice

Finally, I examine the effect of the call notice period length on the overall value of the LYON. The length of call notice (as would an increase in volatility assumption) has a direct effect on the value of the call notice period put option. During the unprotected call periods (periods not conditioned on the trigger), according to the proposition 4.1, the LYON should be called as soon as the (expected) conversion price exceeds the LYON price. At that time the call notice period put option will be either at-the-money or only slightly out-of-the-money. The length of the call notice period combined with the high volatility would significantly increase the value of the call notice period put option[110].

Figure 4-3 shows the effect of the call notice period on the value of the LYON at the time of issue. This effect can be as large as $2 when the call notice period is increased from 0 to 45 days.

[110] Due to the asymmetric information the volatility estimate used to price this option may differ significantly between the firm and the market. This will frequently result in the firm postponing calling the bonds in order to minimise potential underwriting cost. See Chapter 3 for further discussion on optimal call and conversion strategies.

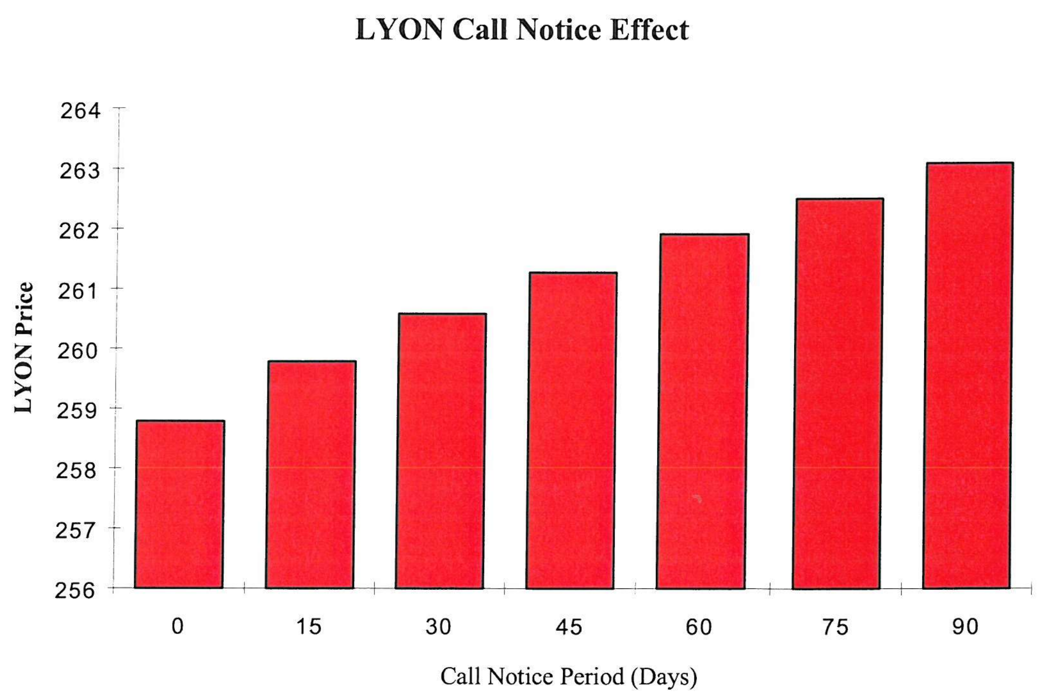

Figure 4-3. Call notice effect.

4.6 Conclusion

In this chapter I demonstrate the application of decomposition to a Liquid Yield Option Note, a complex discount bond with embedded conversion, put and call features. The decomposed LYON yields a portfolio consisting of callable-discount-bond and the higher of a callable-option-to-convert or a callable-option-to-put. I also show that even 'non-important' features such as the call notice period can affect the value of the LYON significantly and that it can be used to explain the reasons for the delayed call decisions. Decomposition also aids understanding how complex securities are structured, and to uncover the intention of the issuer under different scenarios.

The decomposed LYON is valued as a system of linear partial differential equations and the results were analysed and compared to those published by McConnell and Schwarz (1986), uncovering some discrepancies that I attribute to differences in the numerical algorithm used, the effect of a call notice period put option, and potentially different handling of the discount factor.

Appendix 4-A. Term Sheet for Waste Management LYON

Security Name: Waste Management Inc. LYON due to 21st January, 2001

Issue Date: 12th April, 1985

Maturity Date: 21st January, 2001

Issue Price $250

Stock Price at issue date: $52.125

Face Value of the LYON: $1,000

Call and Put Schedule:

Date	Call Price	Trigger	Put Price
At Issuance	$272.50	$86.01	
30 June 1986	$297.83	$86.01	
30 June 1987	$321.13	$86.01	
30 June 1988	$346.77	n/a	$301.87
30 June 1989	$374.99	n/a	$333.51
30 June 1990	$406.00	n/a	$375.58
30 June 1991	$440.08	n/a	$431.08
30 June 1992	$477.50	n/a	$470.75
30 June 1993	$518.57	n/a	$514.07
30 June 1994	$563.63	n/a	$561.38
30 June 1995	$613.04	n/a	$613.04
30 June 1996	$669.45	n/a	$669.45
30 June 1997	$731.06	n/a	$731.06
30 June 1998	$798.34	n/a	$798.34
30 June 1999	$871.81	n/a	$871.81
30 June 2000	$952.03	n/a	$952.03
At maturity	$1000.00	n/a	$1000.00

Conversion: Into 4.36 shares of Waste Management Inc common stock.

Trigger: Prior to 30th June 1987 the issue cannot be called unless the stock price rises above the $86.01 level. Thereafter, the issue can be called irrespective of the stock price level (I assume the trigger is zero).

Call Notice: The issuer must give to the LYON holders at least 15 days' notice prior to exercising the call.

Chapter 5:
The Effect of Dividend Timing on Call Option Prices

5.1 Introduction

Dividend payments are an almost universal feature of the stocks underlying the convertible bonds in the Eurobond market. Looking back at the sample of convertible bonds presented in Chapter 3 (as listed in Appendix 3-A), only a small fraction (2.73%) of bonds have underlying stocks that paid no dividend, i.e. 5 out of 183 bonds. At the other extreme, 3.28% or 6 out of 183 bonds had, as the underlying security, a market index or a basket of stocks, which can be looked at as a continuous dividend example. From the valuation perspective, these two cases are particularly simple to incorporate into any modelling framework. However, for the remaining 94% of the sample, correct dividend modelling in the context of convertible bonds, and more generally, call options becomes an important issue.

To illustrate the omnipresence of dividend paying stocks in the Eurobond sample, I have included a graph of the regional distribution of dividend payment frequency. It is immediately obvious that almost all underlying stocks are paying dividends either annually (mainly European stocks) or semi-annually (predominantly Japanese stocks), with a small number of quarterly payments (US & Canadian stocks). Asian (ex. Japan) stocks pay dividend mostly in annual or semi-annual intervals.

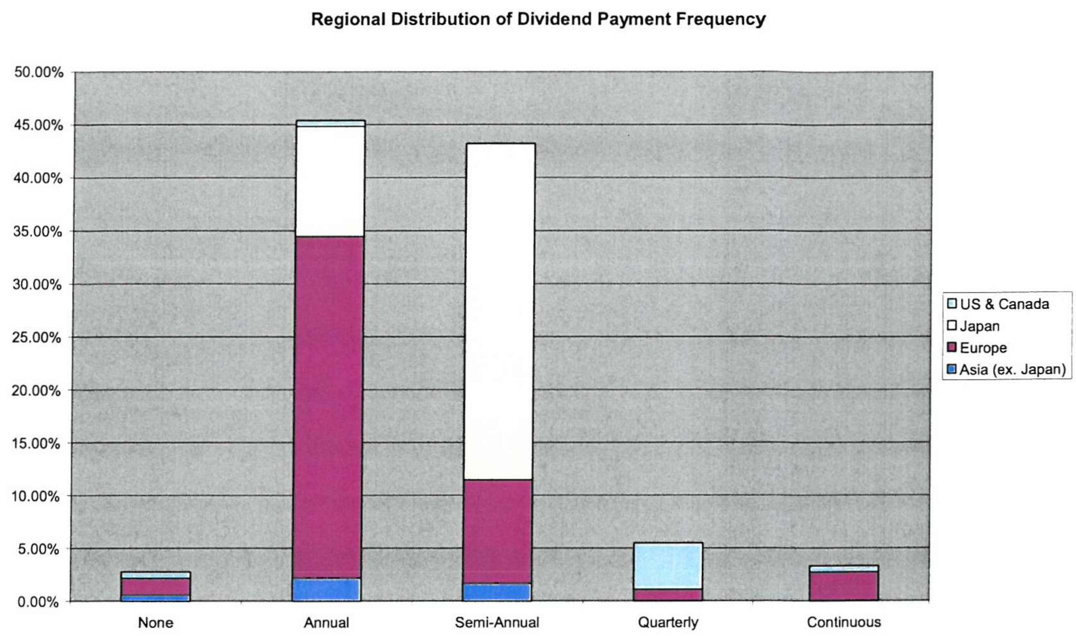

Figure 5-1. Regional Distribution of Dividend Payment Frequency.

In practice, stocks that pay no dividend are frequent in the US markets, and as mentioned in the introduction, stock indices and basket of stocks can be assumed to pay continuous dividends. For most stocks the actual dividend amounts paid over the longer period of time are generally linked to the overall success of the companies, i.e. historic dividend yield is relatively stable. However, over shorter periods, dividends are assumed to be unconditionally known constant payments. This is especially evident during the period between the dividend announcement and the actual ex-dividend date.

The exact modelling of discrete dividend payments thus becomes an issue of interest especially for derivative instruments with the shorter maturity facing a potentially large payment (for example a distribution of special dividend), or for the longer maturity derivatives, like convertible bonds, where the cumulative amount of forecasted constant dividends is large relative to the current stock price.

The first part of this chapter examines the theoretical rationale that makes the exact modelling of known discrete dividend payments produce results that are significantly different to proportional discrete/continuous dividend assumptions. I examine the effect of ex-dividend timing on prices of American and European call options when the underlying stock pays known discrete dividends. I show that the standard adjustment for known dividends, whereby the initial stock price is reduced by the sum of the present values of all dividends falling due during the option's lifetime, produces incorrect valuations, with the error increasing with the ex-dividend time and number of dividends, a common case when valuing the conversion option embedded in convertible bonds.

In the second part, I develop a numerical valuation technique, which explicitly models the stock price drops over an ex-dividend date and sets some rational limitations on the pricing of options in the presence of known discrete dividends, pointing out that in the case of long dated options some combination of constant and proportional dividend payments might be more appropriate.

I propose a more appropriate adjustment to the Black-Scholes formula for pricing European options on stock with known discrete dividends where the dividend dollar duration[111] is introduced to correct for the pricing errors. Finally, using the concepts developed in this chapter I show a similarity between risky (coupon) bonds and known dividend paying stocks and define the risk structure of dividends.

5.2 Literature Overview

The original Merton (1973) and Black and Scholes (1973) option pricing formula made a simplifying assumption that the underlying stock pays no dividend. An extension of the formula to include the continuous dividend yields was added soon after.

Most convertible bond research has either presented the cases where the underlying stock is assumed to pay no dividends of any form until the maturity of the option or at best to pay a continuous constant dividend yield. For example market behaviour research papers presented in Chapter 2 by King (1986), Carayannopulos (1996) and Ammann, Kind and Wilde (2001) all assume underlying stocks pay none or a continuous dividend. In practice neither of these two assumptions is realistic except for some very special cases.

The 'standard approach' is to assume that the stock price consists of two parts: a present value of all known dividends occurring up to the option's expiration and the residual stochastic component that becomes the new underlying variable[112]. This approach was used as long as 25 years ago by Roll (1977) to derive an analytical formula for American style call options on stock that pays a known dividend during the option's lifetime, by Geske (1979) who uses a compound option approach to derive an alternative analytical valuation for (dividend payments) unprotected American call options[113], and by Whaley (1981) who starts from Black and Scholes' (1973) formula and calculates an adjustment for the American style exercise.

All the above approaches effectively defined the meaning of a stock's volatility used to price an option as the volatility of the forward price resulting in the effective dividend timing to be at or very close to the option's valuation time point.

The first researchers to publish a paper pointing to problems arising from using the standard approach to incorporating known dividend payments into option pricing, were Berger and Klein (1998), who show that mispricing of long-dated out-of-the-money options, such as those found in a convertible bond, can be as much as 15% for a three and a half year option with reasonable parameter choice. They also point to potential problems associated with known dividend modelling that may lead to negative stock prices, but point to benefits of seeing the volatility input parameters as being on the volatility the full spot price as this is consistent with both observable volatility and the traders understanding of it. Frishling (2002) provides similar analysis of the source and the nature of the dividend timing pricing error and concludes that the numerical methods that impose an ex-dividend stock continuity condition must be used to correctly price options. More recently Bos and Vandermark (2002) published a note describing an adjustment to the Black and Scholes (1973) formula for known dividends that is in its nature similar to the one proposed in this chapter. Bos and Vandermark (2002) test the quality of the adjustment against numerically derived results and find that even 7 year options are priced within 0.05 volatility points from numerically derived prices.

[111] Duration is a concept borrowed from the world of fixed income securities. It is the time-weighted average of all payment streams or alternatively, first derivate of the payment stream w.r.t. (constant) interest rate.

[112] See for example Hull (1997, pages 249 and 354). As the residual stock value is less than the spot, Hull (1997) suggests a volatility increase by factor equal to the spot-to-residual ratio.

[113] Geske's (1979) approach has practical use in the case of one known dividend. For a higher number of dividends the solution theoretically exists but it involves multi-variate normal distributions of order one higher than the number of known dividend payments, that require complex numerical approximations.

5.3 Rational Option Pricing When Stock Pays Known Dividend

This section concentrates on the effects that the discrete known dividend payments have on option prices and the issues of modelling them. The usual Black-Scholes assumptions apply throughout: trading takes place continuously, there are no restrictions on lending and borrowing, interest rate is known and constant, stock price follows geometric Brownian motion with constant drift and variance, the holders of options are not protected against dividend payments (all these hold for convertible bonds as well).

A call option is a derivative security taken to be dependent on the stochastic price process, S, followed by its underlying stock. The stochastic process for S is assumed to be continuous with constant drift and standard deviation. The risk neutralised process for S can be written as,

$$dS_t = [rS_t - D_t(S_t)]dt + \sigma S_t dW \qquad [5\text{-}1]$$

Where r is the risk-free rate, σ is the volatility of stock price returns and the dW is the Wiener process, i.e. $E[dW] = 0$ and $E[dW^2] = dt$. The underlying security's dividend function $D_t(S_t)$ has a general format of

$$D_t(S_t) = q_t S_t + p_x \qquad [5\text{-}2]$$

A constant proportional dividend yield is introduced by setting q_t greater than zero[114], while a known dividend payment, p_x, at time x, is defined as a product of a non-negative constant, P_x, associated with the ex-dividend time, x, via the Dirac delta function

$$p_x = P_x \delta(t - x) \qquad [5\text{-}3]$$

Where the Dirac delta function, also known as the point function, is used to mathematically define actions highly localised in time or space. The defining properties of this function are $\delta(t-x) = 0,\ t \neq x$ and $\int_{t_0}^{T} \delta(t-x)dt = 1, t_0 \leq x \leq T$, and so consequently $\int_{t_0}^{T} p_x dt = P_x$.

[114] The restriction to a constant dividend yield can be easily generalised for a time dependant but deterministic proportional dividend yield.

A combination of the two dividend payment methods is also possible. Alternatively, the process given by [5-1] can be written as (assuming constant dividend yield, q),

$$S_t = S_0 e^{\left(r-q-\frac{\sigma^2}{2}\right)t+\sigma W_t\sqrt{t}} - \phi(t-x)P_x e^{r(t-x)} \quad [5\text{-}4]$$

Where $\phi(z)$ is a unit step (heavy side) function defined as 0 when $z \leq 0$ and 1 otherwise. The Dirac delta function and the unit step function are linked through the integral relationship $\phi(z) = \int_{-\infty}^{z} \delta(\zeta)d\zeta$.

I use the symbol $c(S_0,K,D)$ to denote a *European call option* on a stock with a current price of S_0, dividend policy $D_t(S_t)$ and strike K[115]. An *American call option* is denoted by uppercase C. Black and Scholes (1973) provided formula for European call options, struck at time $t_0 = 0$ and maturity at time T, on stocks following [5-1] that for the continuous dividend yield policy $D_t(S_t)=qS$ takes the form as,

$$c(S_0,K,q) = e^{-qT}S_0N(z_+) - e^{-rT}KN(z_-) \quad [5\text{-}5]$$

$$z = \frac{\ln\frac{S_0}{K} + (r-q)T}{\sigma\sqrt{T}}, \quad z_+ = z + \frac{1}{2}\sigma\sqrt{T}, \quad z_- = z - \frac{1}{2}\sigma\sqrt{T} \quad [5\text{-}6]$$

The most frequently suggested adjustment to Black-Scholes formula when the stock pays known discrete dividend[116], p_x, is to first subtract the present value of the known dividend from the initial stock price and then use the adjusted stock price as the new initial price,

$$c(S_0,K,p_x) = c(S_0 - e^{-rx}p_x,K,0) \quad [5\text{-}7]$$

However, such an adjustment to Black-Scholes is incorrect. The difference between the true price of the option and the price given by [5-7] is an increasing function of the ex-dividend date. Note that [5-7] introduces the dependency on the ex-dividend date only through the

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[115] Other parameters entering the option price equation are assumed to be constant and are not explicitly listed.

calculation of the present value of the dividend. In a special case of zero interest rates, [5-7] would become completely invariant to the timing of the dividend.

To fully understand the nature of the flaws in [5-7] and to formulate a better adjustment I first need to establish some properties of Black-Scholes call option prices[117].

Let's consider the limiting cases as the ex-dividend time, x, approached current time, t_0, and the time of expiry, T, I arrive at the following conclusions:

THEOREM 5.1. The price of a European call option on a stock that pays a known dividend amount p_x at time x, $t_0 = 0 \le x \le T$, converges, as x approaches t_0, to an otherwise equivalent call option on a non-dividend paying stock with its initial stock price reduced by the amount of known dividend,

$$\lim_{x \to t_0} c(S_0, K, p_x) = c(S_0 - p_0, K) \qquad [5\text{-}8]$$

Proof. By taking the limit of the expected payoff as the ex-dividend time approaches the current time $t_0 = 0$, (please note the relationship $p_0 = e^{-rx} p_x \; and \; p_T = e^{r(T-x)} p_x$),
$\lim_{x \to t_0} e^{-rT} \mathrm{E}[\max(S_T - p_x - K, 0)] = e^{-rT} \mathrm{E}[\max((S_T - p_0 e^{r(T-t_0)}) - K, 0)] = c(S_0 - p_0, K)$. ■

THEOREM 5.2. The price of a European call option on a stock which pays a known dividend amount p_x at time x, $t_0 = 0 \le x \le T$, converges, as x approaches T, to an otherwise equivalent call option on a non-dividend paying stock with its strike price increased by the amount of known dividend,

$$\lim_{x \to T} c(S_0, K, p_x) = c(S_0, K + p_T) \qquad [5\text{-}9]$$

Proof. By taking the limit of the expected payoff as the ex-dividend date approaches maturity,
$\lim_{x \to T} e^{-rT} \mathrm{E}[\max(S_T - p_x - K, 0)] = e^{-rT} \mathrm{E}[\max(S_T - (p_T + K), 0)] = c(S_0, K + p_T)$. ■

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116 For example see Hull (1997, page 249).

117 Through this chapter I adopt Black and Scholes (1973) assertions about the economy and the stock price process.

***COROLLARY* 5.2.1.** The present value of an ex-dividend stock price, $S_{0,x}$, equals the value of a European call option maturing on the ex-dividend date with the strike price equal to the known dividend amount, p_x,

$$S_{0,x} = c(S_0, p_x, 0) \qquad [5\text{-}10]$$

Proof. From theorem 2 by setting the option's expiry time to equal the ex-dividend time and choosing the strike price $K = 0$. ■

Note that in the majority of cases the dividend amount is small in proportion to the stock price, i.e. the option given by [5-10] is deep-in-the-money. The present value of ex-dividend stock price can be safely approximated by,

$$S_{0,x} \cong S_0 - e^{-rx} p_x \qquad [5\text{-}11]$$

***THEOREM* 5.3.** The price of an American call option on a stock which pays a known dividend amount p_x at time x, $t_0 = 0 \leq x \leq T$, converges, as x approaches T, to an otherwise equivalent American call option on a non-dividend paying stock with the same initial stock and strike price,

$$\lim_{x \to T} C(S_0, K, p_x) = C(S_0, K) \qquad [5\text{-}12]$$

Proof. By allowing for the early exercise of the American option. I can first cite the well known result that the early exercise of an American option on stock with known dividends is optimal, if at all, just prior to the ex-dividend time. The original option can be therefore approximated by an option C_x maturing an instant prior to ex-dividend time, thus allowing the holder to participate in the dividend payment. As the ex-dividend approaches maturity, so does the maturity of the approximating option. In the limit C_x becomes equal to the original option C. ■

***THEOREM* 5.4.** The price of a European call option on a stock paying a known dividend at the start of an option contract cannot be more valuable than an otherwise equivalent option when the dividend is paid at the option's expiration,

$$c(S_0 - p_0, K) \le c(S_0, K + p_T) \qquad [5\text{-}13]$$

Proof. In order to compare the two options I'll first transform them into options with the same initial stock price. This can be done, by applying a transformation to the left hand side of the inequality as, $c(S_0, K + p_T) = S_0/(S_0 - p_0)c(S_0 - p_0, (K + p_T)(S_0 - p_0)/S_0)$. Formula [5-13] then becomes,

$$c(S_0 - p_0, K) \le \frac{S_0}{S_0 - p_0} c\left(S_0 - p_0, (K + p_T)\frac{S_0 - p_0}{S_0}\right) \qquad [5\text{-}14]$$

As, $S_0 / (S_0 - p_0) \ge 1$ for every $p_0 \ge 0$, the proposition certainly holds when

$$K \ge (K + p_T)\frac{S_0 - p_0}{S_0} \Rightarrow K \ge e^{+rT}(S_0 - p_0) \qquad [5\text{-}15]$$

For lower K then in [5-15] $c(S_0 - p_0, K) \ge c(S_0 - p_0, (K + p_T)(S_0 - p_0) / S_0)$. The higher the difference, the lower the strike price. In the limit as $K = 0$, inequality in [5-15] becomes $S_0 - p_0 \le c(S_0, p_T)$, which holds for any non-negative interest rate. ■

> ***THEOREM 5.5.*** The price of an American call option on a stock paying a discrete dividend at the start of an option contract cannot be more valuable than an otherwise equivalent option when the dividend is paid at the option expiration,

$$C(S_0 - p_0, K) \le \lim_{x \to T} C(S_0, K, p_x) = C(S_0, K) \qquad [5\text{-}16]$$

Proof. From theorem 3 and from the fact that the call option on the lower initial stock price cannot have a higher value than the option with the higher initial stock price. ■

> ***THEOREM 5.6.*** A European call option on a stock paying a known discrete dividend at an earlier time, x_1, cannot be more valuable than the European call option paying the same known dividend at a later time, x_2, $t_0 = 0 \le x_1 \le x_2 \le T$,

$$c\left(S_0, K, p_{x_1}\right) \leq c\left(S_0, K, p_{x_2}\right) \qquad [5\text{-}17]$$

Proof. To prove Theorem 6 it is sufficient to show that option price is non-decreasing function in ex-dividend time or alternatively that $\partial c/\partial t_x$ is non-negative. ■

5.4 Avoiding Negative Stock Prices

Although simple in its formulation, the model in [5-1] has a positive probability of negative stock prices, effectively suggesting that companies will pay the promised/forecasted dividends even if it forces them into bankruptcy which is unsustainable from the rational and financial viewpoint.

A more realistic approach, albeit more complicated, would be to suggest that companies would endeavour to pay the fixed expected dividends as far as its stock price remains within some low bound L_x , and a high bound H_x, where $0 \leq p_x < L_x \leq S \leq H_x$. Outside this band the company pays a modified dividend equivalent to the yields associated with the two boundaries. I will refer to such a generalisation to the constant dividend method as a *Flexible Dividend Policy*. The resulting dividend payments for the range of possible stock prices (log-spaced) are plotted on the graph below.

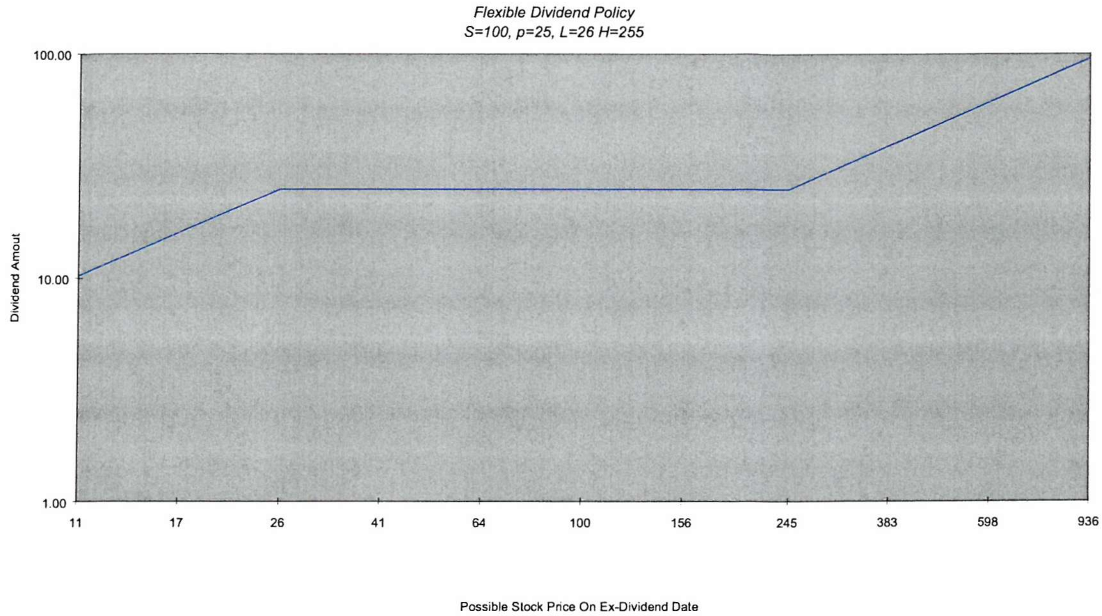

Figure 5-2. Dividend payments under *Flexible Dividend Policy.*

The determination of the limiting minimum and maximum stock prices for which a dividend is kept constant can be obtained from historic dividend yield analysis (ratio of the dividend paid over the stock price on the day before ex-dividend) with the limitation that the lower level has to be at least as high as the dividend payment, thus preventing the bankruptcy state[118].

The model in [5-1] then becomes *flexible dividend model,*

$$dS_t = \left\{ rS_t - p_x \min\left[\frac{S_t}{L_x}, \max\left(\frac{S_t}{H_x}, 1 \right) \right] \right\} dt + \sigma S_t dW \qquad [5\text{-}18]$$

Assuming that [5-18] is the stock price process I can calculate[119] the present value of the expected ex-dividend stock as a three part sum each weighted by the ratio of the known

118 In the subsequent Chapter 6 'Valuation of Convertible Bonds Subject to Default Risk' I will show that when a firms capital structure includes securities senior to common stock, the lower dividend threshold needs to be further increased to protect senior security holders.

119 For the complete derivation of this result see the appendix at the end of this chapter.

dividend and current stock price, lower relaxation limit, and reduced by the upper relaxation limit respectively[120],

$$S_{0,x} = \frac{p_x}{S_0}\left(\frac{S_0^2}{p_x} - \frac{S_0^2}{L_x}\right) + \frac{p_x}{L_x}c(S_0, L_x) - \frac{p_x}{H_x}c(S_0, H_x) \qquad [5\text{-}19]$$

Note that by setting $L_x = p_x$ and $H_x = +\infty$, expression [5-19] collapses to [5-10].

Combining the minimum and maximum dividend yields with the historic dividend growth rate can be particularly suitable for the valuation of long term options and convertible bonds. Assuming dividend growth rate is estimated as g and minimum and maximum dividend yield as q_L and q_H, [5-18] can be rewritten as

$$\mathrm{d}\,S_t = \left\{rS_t - \min\left[q_H S_t, \max\left(q_L S_t, p_{x_0}(1+g)^{t-x_0}\right)\right]\right\}dt + \sigma S_t dW \qquad [5\text{-}20]$$

This effectively means, that the current dividend grows at the (historically) estimated annualised rate g and at any ex-dividend date cannot be made to yield more than (historically) estimated maximum yield q_H or yield less than (historically) estimated yield q_L.

5.5 Modelling Of Known Dividends Numerically

In order to solve for the option price the numerical procedure must be flexible enough to deal with stock price jumps over the ex-dividend period. The explicit difference method of Hall and White (1990) and binomial approach of Nelson and Ramaswamy (1990) can both be employed. Both methods use constant mesh but allow for the possibility of multi-jumps ensuring that the local expectation and variance of the stock price process is preserved as the stock goes ex-dividend.

The numerical method used in this paper is of the explicit finite difference variety where the stock price mesh is created in such a way as to always ensure that the local expectation of the logarithmically transformed stock price is by definition on the next mesh node. If $S_{i,j}$ represents the value of the stock price at node i,j ($0 < i \leq N$, $-N \leq j \leq +N$) then the node i+1,j is defined as $\ln S_{i+1,j} = \mathrm{E}[\ln S_{i,j}]$. Initial nodes are set to, $S_{0,j} = S_0 \Delta S^j$, and $\Delta S = \exp[\sigma(\eta\Delta t)^{1/2}]$, where time step

[120] The equation [5-19] can be simplified further but than the relationship between the known dividend, the current stock price and lower and

$\Delta t = T/N$, and $1 \le \eta \le 3$. The local expectation during the ex-dividend period for the stock price process assumed in this paper is defined as[121]

$$\exp\left(\mathrm{E}\left[\ln S_{i,j}\right]\right) = S_{i,j} e^{\left(r-q-\frac{1}{2}\sigma^2\right)\Delta t} - p_i \min\left[\frac{S_{i,j}}{L_i}, \max\left(\frac{S_{i,j}}{H_i}, 1\right)\right] \qquad [5\text{-}21]$$

To illustrate the mesh construction I used the example with two dividend payments occurring after 3 and 9 months. The low dividend relaxation point L_τ is chosen as the lowest cum-dividend node still higher that the dividend payment. The high dividend relaxation point is not imposed, i.e. $H_\tau = +\infty$.

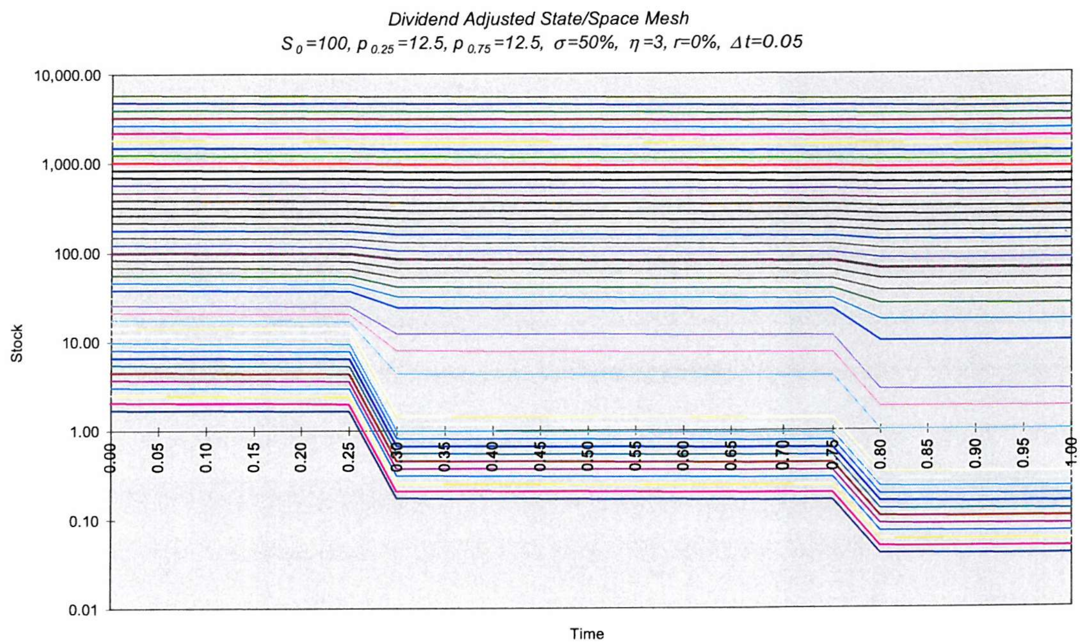

Figure 5-3. State/Space Mesh Adjusted for Dividend Payments.

Within such a mesh the probabilities which uphold the underlying stock price distributional properties are defined as,

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upper relaxation limit is somewhat obstructed.

121 When moving from continuous time to discrete time all dividends falling due within the right inclusive interval $i\Delta t$ .. $(i+1)\Delta t$ are amalgamated and assumed to occur at the end of the period.

$$\pi_{i,j-1} = \begin{cases} \left\{\sigma\sqrt{\Delta t}\left[\left(\ln S_{i,k} - \ln S_{i,k-1}\right)\left(\ln S_{i,k+1} - \ln S_{i,k-1}\right)\right]\right\}^{-1}, & \textit{if } S_{i,j+1} > S_{i,j} > S_{i,j-1} > 0 \\ 0, & \textit{otherwise} \end{cases}$$

$$\pi_{i,j+1} = \begin{cases} \left\{\sigma\sqrt{\Delta t}\left[\left(\ln S_{i,k+1} - \ln S_{i,k}\right)\left(\ln S_{i,k+1} - \ln S_{i,k-1}\right)\right]\right\}^{-1}, & \textit{if } S_{i,j+1} > S_{i,j} > S_{i,j-1} > 0 \\ 0 & \textit{otherwise} \end{cases}$$

$$\pi_{i,j} = 1 - \pi_{i,j-1} - \pi_{i,j+1}$$

[5-22]

Those are the solutions to the linear system of equations,

$$\begin{bmatrix} 1 & 1 & 1 \\ \ln S_{i,j-1} & \ln S_{i,j} & \ln S_{i,j+1} \\ \ln^2 S_{i,j-1} & \ln^2 S_{i,j} & \ln^2 S_{i,j+1} \end{bmatrix} \begin{bmatrix} \pi_{i,j-1} \\ \pi_{i,j} \\ \pi_{i,j+1} \end{bmatrix} = \begin{bmatrix} 1 \\ \mathrm{E}\left[\ln S_{i,j}\right] \\ \mathrm{E}\left[\ln^2 S_{i,j}\right] \end{bmatrix} \qquad [5\text{-}23]$$

Subject to condition,

$$0 \le \pi_{i,j} \le 1 \qquad [5\text{-}24]$$

The evolution of probabilities for the above example is tabled in Appendixes5-B and 5-C. The figure 5-4 below plots the prices of European and American call options against the ex-dividend time. The input parameters for this example are chosen to amplify the dividend timing effect. The difference in price calculated for the same example between the standard approach and the method described in this chapter can be as much as 58% in case of a ex-dividend date being close to expiry.

Bos and Vandermark (2002) report that the prices for realistic examples of 4 year call options can differ by as much as 12% for an option that is initially 25% out-of-the-money, going up to a 16% price difference for a 50% out-of-the-money call option (this is usually the case with most convertible bonds)

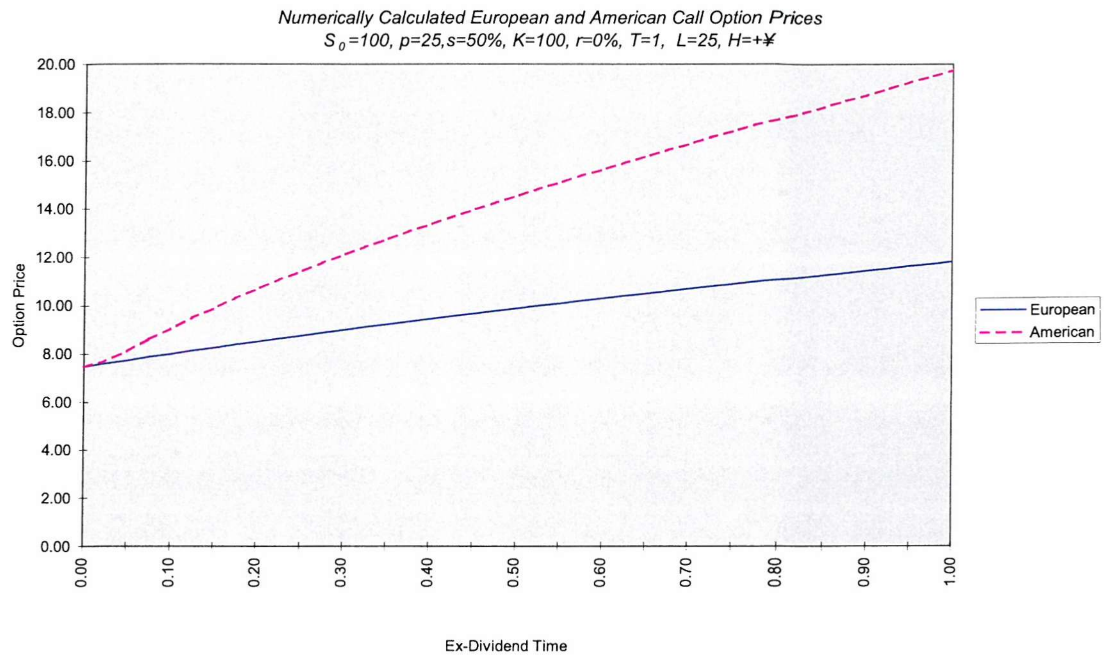

Figure 5-4. Call Option Prices as a Function of Ex-Dividend Time.

5.6 Adjustment to Black-Scholes Valuation Formula

From theorems 1 and 2 and the knowledge that the European call option price is a smooth function of the underlying stock price, for any ex-dividend time, x, that is between $t_0 = 0 < x < T$, the relationship holds,

$$c(S_0 - p_0, K) \le c(S_0, K, p_x) \le c(S_0, K + p_T) \qquad [5\text{-}25]$$

To achieve the smooth transition between the two extremes I proposed the following linear ex-dividend date weighted adjustment to initial stock price and the strike price,

$$c(S_0, K, p_x) \cong c(S_0 - p + d, K + e^{rT} d) \qquad [5\text{-}26]$$

Where p (without index) is the present value of the dividend payment, $p = e^{-rx} p_x$, and the adjustment factor[122] d is defined as $d = e^{-rx} p_x x / T = -(1/T)\, \partial p_x / \partial r$.

In a case when there is more than one dividend payment, $n \geq 1$, during the option's lifetime the above can be generalised by defining p as the sum of the present values of all dividend payments,

$$p = \sum_{i=1}^{n} e^{-rx_i} p_{x_i} \qquad [5\text{-}27]$$

The adjustment factor d then becomes the dividend dollar duration (using a term borrowed from the fixed income world[123]) counted in units of option's maturity,

$$d = \frac{1}{T} \sum_{i=1}^{n} x_i e^{-rx_i} p_{x_i} = -\frac{1}{T} \frac{\partial p}{\partial r} \qquad [5\text{-}28]$$

The graph below compares the prices of European options calculated using the usual dividend payment adjustment for Black-Scholes as in [5-7] (marked B-S/p), with prices given by Black-Scholes adjusted for both the dividend payment and the ex-dividend time as in [5-28] (marked B-S/pd), and numerically calculated prices as described in the previous section.

122 Here I have assumed that the initial settlement date is equal to t_0 and the dividend payment and ex-dividend dates are the same.

123 See for example Fabozzi and Fabozzi (1989, page 61).

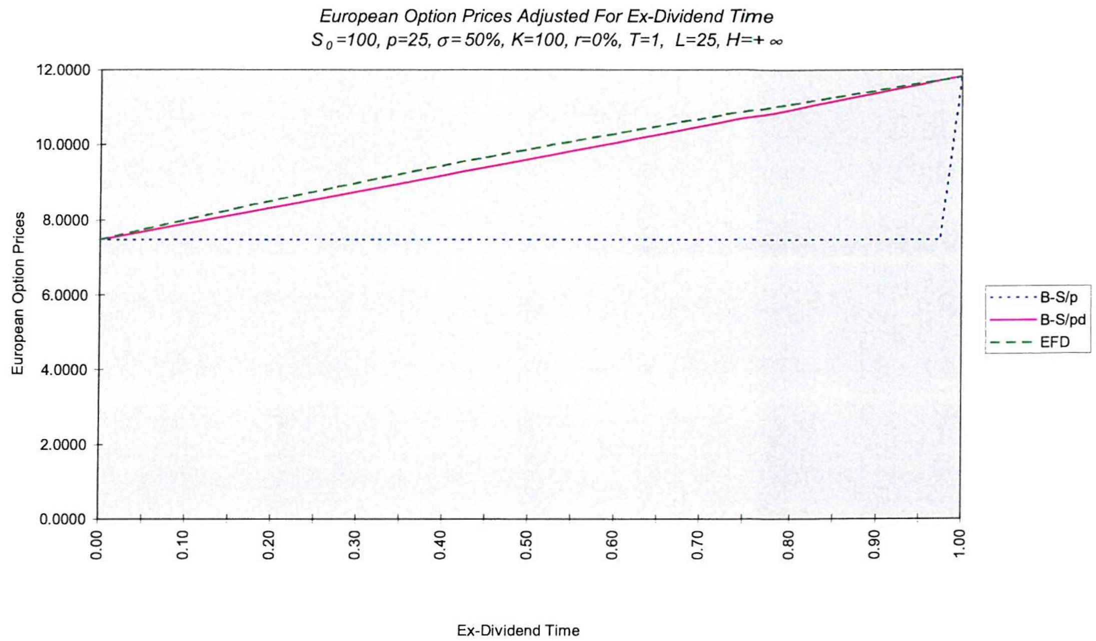

Figure 5-5. Improved Black-Scholes European call option price.

Finally I show the comparison between deltas calculated using the payment/duration adjusted Black-Scholes model and deltas returned by the numerical algorithm.

The nature of the adjustment for the Black-Scholes formula presented in this chapter ensures that the prices agree exactly for cases when ex-dividend is very close to option's inception and at expiration date. Intermediary dates are diverging slightly suggesting that further precision can be achieved by the addition of a second order component. However, the difference is small in comparison with the option price and figure 5-5 illustrates the high convergence of option's delta to its true value.

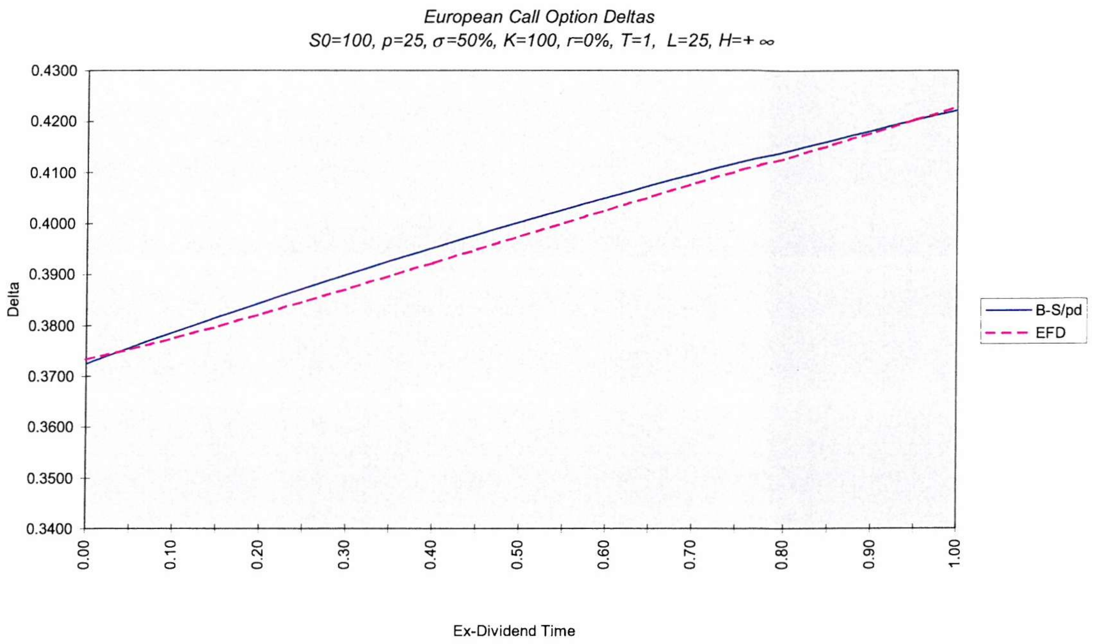

Figure 5-6. Improved Black-Scholes stock sensitivity calculation.

5.7 Risk Structure of Dividend Payments

Having defined the present value of an ex-dividend stock, which is now probabilistically conditional on the level of stock price I can introduce the dividend risk premium and term structure of dividend risk premiums.

> ***DEFINITION 5.1.*** The dividend risk premium, $k_x \geq 0$, is the additional rate of return required by stock holders for accepting the risk that the company may default on the forecasted dividend payment p_x. The dividend risk premium is an adjustment to the risk-free rate which makes the difference between the initial stock price and the risk-adjusted present value of the forecasted dividend payment equal to the present value of expected ex-dividend stock price, $S_0 - \exp[-(r + k_x)x]p_x = S_{0,x}$. Thus the dividend risk premium is,

$$k_x = -r - \frac{1}{x} \ln \frac{e^{rx} S_0 - S_x}{p_x} \qquad [5\text{-}29]$$

Definition 5.2. Term structure of dividend risk premiums is the relationship that exists between the dividend risk premiums and ex-dividend time.

Theorem 5.7. Dividend risk premium on a stock that pays a continuous dividend yield is always zero.

Proof. The total amount of dividends received until some time x, from the stock which pays (constant) continuous dividend yield is equal to $d = S_0 e^{rx} - S_0 e^{(r-q)x}$. The present value of the ex-dividend stock price calculated in the spirit of [5-10] is $S_{0,x} = c(S_0,0,q) = S_0 e^{-qx}$. Substituting these two values into equation [5-29] yields zero dividend risk premium. ■

Corollary 5.7.1. The dividend risk premium on a stock that pays dividends more frequently is lower that the dividend risk premium on a stock with less frequent payments (assuming the cumulative dividend payments being the same).

The figure below illustrates the evolution of dividend risk premium when the single ex-dividend date varies between $0 \le x \le 1$. To make the pure premium more visible I have chosen a slightly artificial case of zero interest rate, high annual volatility and high dividend payment.

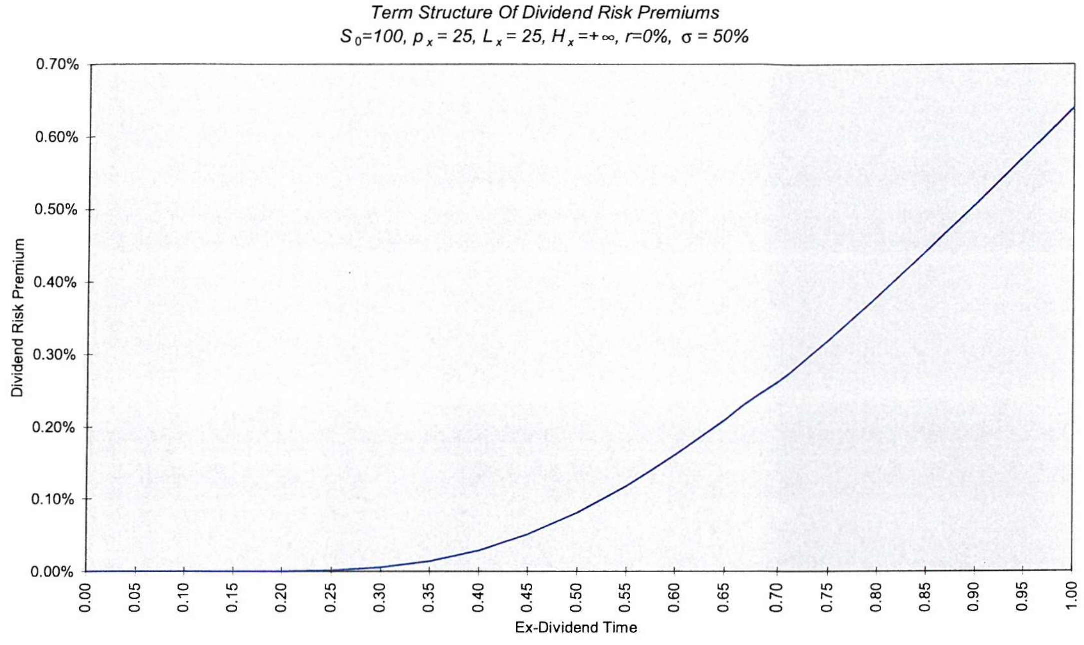

Figure 5-7. Term structure of dividend risk premiums.

5.8 Conclusion

This chapter examines in detail the side effects of the constant dividend payments assumption on the valuation of call options, and by extension on convertible bonds and other such like equity derivative securities. The standard adjustment for known dividends can produce significant valuation errors even for reasonable choice of valuation parameters, especially for longer dated and out-of-the-money options as frequently found in convertible bonds. I have developed a flexible dividend policy modelling algorithm for known and forecasted dividend payments and shown how it can be incorporated within a general numerical solver with direct applicability to the valuation of embedded conversion options. An adjustment to the Black and Scholes (1973) formula for European call option prices on stock with known dividends is also devised that produces results closely in line with those obtained by using the exact numerical solver.

Appendix 5-A. Derivation of Flexible Dividend Formula for Call Options

Assuming that [5-18] is the stock price process the expected ex-dividend value of the stock can be calculated as

$$E[S_x|S_0]=\left(1-\frac{p_x}{L_x}\right)\int_0^{L_x} S_x f(S_x)\,\mathrm{d}S_x+\int_{L_x}^{H_x}(S_x-p_x)f(S_x)\,\mathrm{d}S_x+\left(1-\frac{p_x}{H_x}\right)\int_{H_x}^{+\infty} S_x f(S_x)\,\mathrm{d}S_x$$

[5-30]

Where $f(S_x)$ is the probability density function of S_x, assumed to be lognormal, i.e. $f(S_x) = N(\ln S_x)$. and $N(z) = (2\pi)½\exp(½z^2)$ is the standard normal density function. The present value of the expected ex-dividend can be written as

$$e^{-rx}S_x \equiv A+B+C \qquad [5\text{-}31]$$

The solutions to the three integrals are

$$A=\left(1-\frac{p_x}{L_x}\right)S_0e^{rx}\left[1-N(l_+)\right] \qquad [5\text{-}32]$$

$$B=S_0e^{rx}\left[N(l_+)-N(h_+)\right]-p_x\left[N(l_-)-N(h_-)\right] \qquad [5\text{-}33]$$

$$C=\left(1-\frac{p_x}{H_x}\right)S_0e^{rx}N(h_+) \qquad [5\text{-}34]$$

$$l_+=\frac{\ln\frac{S_0}{L_x}+rx}{\sigma\sqrt{x}}+\frac{\sigma\sqrt{x}}{2},\qquad l_-=\frac{\ln\frac{S_0}{L_x}+rx}{\sigma\sqrt{x}}-\frac{\sigma\sqrt{x}}{2} \qquad [5\text{-}35]$$

$$h_+=\frac{\ln\frac{S_0}{H_x}+rx}{\sigma\sqrt{x}}+\frac{\sigma\sqrt{x}}{2},\qquad h_-=\frac{\ln\frac{S_0}{H_x}+rx}{\sigma\sqrt{x}}-\frac{\sigma\sqrt{x}}{2} \qquad [5\text{-}36]$$

That after rearranging yield [5-19].

APPENDIX 5-B. Probabilities of an Up Move

j\t	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.90	0.95	0.95	1.00
-21																					
-20	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-19	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-18	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-17	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-16	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-15	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-14	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-13	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-12	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-11	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-10	0.1667	0.1667	0.1667	0.1667	0.1667	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	0.0090	
-9	0.1667	0.1667	0.1667	0.1667	0.1667	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	0.0123	
-8	0.1667	0.1667	0.1667	0.1667	0.1667	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	0.0275	
-7	0.1667	0.1667	0.1667	0.1667	0.1667	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	0.0439	
-6	0.1667	0.1667	0.1667	0.1667	0.1667	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0600	0.0112	0.0112	0.0112	0.0112	0.0112	
-5	0.1667	0.1667	0.1667	0.1667	0.1667	0.0751	0.0751	0.0751	0.0751	0.0751	0.0751	0.0751	0.0751	0.0751	0.0751	0.0159	0.0159	0.0159	0.0159	0.0159	
-4	0.1667	0.1667	0.1667	0.1667	0.1667	0.0887	0.0887	0.0887	0.0887	0.0887	0.0887	0.0887	0.0887	0.0887	0.0887	0.0316	0.0316	0.0316	0.0316	0.0316	
-3	0.1667	0.1667	0.1667	0.1667	0.1667	0.1007	0.1007	0.1007	0.1007	0.1007	0.1007	0.1007	0.1007	0.1007	0.1007	0.0481	0.0481	0.0481	0.0481	0.0481	
-2	0.1667	0.1667	0.1667	0.1667	0.1667	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.1111	0.0640	0.0640	0.0640	0.0640	0.0640	
-1	0.1667	0.1667	0.1667	0.1667	0.1667	0.1201	0.1201	0.1201	0.1201	0.1201	0.1201	0.1201	0.1201	0.1201	0.1201	0.0787	0.0787	0.0787	0.0787	0.0787	
0	0.1667	0.1667	0.1667	0.1667	0.1667	0.1278	0.1278	0.1278	0.1278	0.1278	0.1278	0.1278	0.1278	0.1278	0.1278	0.0919	0.0919	0.0919	0.0919	0.0919	
1	0.1667	0.1667	0.1667	0.1667	0.1667	0.1342	0.1342	0.1342	0.1342	0.1342	0.1342	0.1342	0.1342	0.1342	0.1342	0.1035	0.1035	0.1035	0.1035	0.1035	
2	0.1667	0.1667	0.1667	0.1667	0.1667	0.1397	0.1397	0.1397	0.1397	0.1397	0.1397	0.1397	0.1397	0.1397	0.1397	0.1136	0.1136	0.1136	0.1136	0.1136	
3	0.1667	0.1667	0.1667	0.1667	0.1667	0.1443	0.1443	0.1443	0.1443	0.1443	0.1443	0.1443	0.1443	0.1443	0.1443	0.1222	0.1222	0.1222	0.1222	0.1222	
4	0.1667	0.1667	0.1667	0.1667	0.1667	0.1481	0.1481	0.1481	0.1481	0.1481	0.1481	0.1481	0.1481	0.1481	0.1481	0.1295	0.1295	0.1295	0.1295	0.1295	
5	0.1667	0.1667	0.1667	0.1667	0.1667	0.1513	0.1513	0.1513	0.1513	0.1513	0.1513	0.1513	0.1513	0.1513	0.1513	0.1357	0.1357	0.1357	0.1357	0.1357	
6	0.1667	0.1667	0.1667	0.1667	0.1667	0.1540	0.1540	0.1540	0.1540	0.1540	0.1540	0.1540	0.1540	0.1540	0.1540	0.1410	0.1410	0.1410	0.1410	0.1410	
7	0.1667	0.1667	0.1667	0.1667	0.1667	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562	0.1562	0.1453	0.1453	0.1453	0.1453	0.1453	
8	0.1667	0.1667	0.1667	0.1667	0.1667	0.1580	0.1580	0.1580	0.1580	0.1580	0.1580	0.1580	0.1580	0.1580	0.1580	0.1490	0.1490	0.1490	0.1490	0.1490	
9	0.1667	0.1667	0.1667	0.1667	0.1667	0.1595	0.1595	0.1595	0.1595	0.1595	0.1595	0.1595	0.1595	0.1595	0.1595	0.1520	0.1520	0.1520	0.1520	0.1520	
10	0.1667	0.1667	0.1667	0.1667	0.1667	0.1607	0.1607	0.1607	0.1607	0.1607	0.1607	0.1607	0.1607	0.1607	0.1607	0.1546	0.1546	0.1546	0.1546	0.1546	
11	0.1667	0.1667	0.1667	0.1667	0.1667	0.1618	0.1618	0.1618	0.1618	0.1618	0.1618	0.1618	0.1618	0.1618	0.1618	0.1567	0.1567	0.1567	0.1567	0.1567	
12	0.1667	0.1667	0.1667	0.1667	0.1667	0.1626	0.1626	0.1626	0.1626	0.1626	0.1626	0.1626	0.1626	0.1626	0.1626	0.1584	0.1584	0.1584	0.1584	0.1584	
13	0.1667	0.1667	0.1667	0.1667	0.1667	0.1633	0.1633	0.1633	0.1633	0.1633	0.1633	0.1633	0.1633	0.1633	0.1633	0.1598	0.1598	0.1598	0.1598	0.1598	
14	0.1667	0.1667	0.1667	0.1667	0.1667	0.1639	0.1639	0.1639	0.1639	0.1639	0.1639	0.1639	0.1639	0.1639	0.1639	0.1610	0.1610	0.1610	0.1610	0.1610	
15	0.1667	0.1667	0.1667	0.1667	0.1667	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1644	0.1620	0.1620	0.1620	0.1620	0.1620	
16	0.1667	0.1667	0.1667	0.1667	0.1667	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1648	0.1628	0.1628	0.1628	0.1628	0.1628	
17	0.1667	0.1667	0.1667	0.1667	0.1667	0.1651	0.1651	0.1651	0.1651	0.1651	0.1651	0.1651	0.1651	0.1651	0.1651	0.1635	0.1635	0.1635	0.1635	0.1635	
18	0.1667	0.1667	0.1667	0.1667	0.1667	0.1654	0.1654	0.1654	0.1654	0.1654	0.1654	0.1654	0.1654	0.1654	0.1654	0.1641	0.1641	0.1641	0.1641	0.1641	
19	0.1667	0.1667	0.1667	0.1667	0.1667	0.1656	0.1656	0.1656	0.1656	0.1656	0.1656	0.1656	0.1656	0.1656	0.1656	0.1645	0.1645	0.1645	0.1645	0.1645	
20	0.1667	0.1667	0.1667	0.1667	0.1667	0.1658	0.1658	0.1658	0.1658	0.1658	0.1658	0.1658	0.1658	0.1658	0.1658	0.1649	0.1649	0.1649	0.1649	0.1649	
21																					

APPENDIX 5-C. Probabilities of an Down Move

j\t	0.00	0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50	0.55	0.60	0.65	0.70	0.75	0.80	0.90	0.95	0.95	1.00
-21																					
-20	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-19	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-18	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-17	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-16	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-15	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-14	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-13	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-12	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-11	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	0.1667	
-10	0.1667	0.1667	0.1667	0.1667	0.1667	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	0.0504	
-9	0.1667	0.1667	0.1667	0.1667	0.1667	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	0.0068	
-8	0.1667	0.1667	0.1667	0.1667	0.1667	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	0.0199	
-7	0.1667	0.1667	0.1667	0.1667	0.1667	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	0.0359	
-6	0.1667	0.1667	0.1667	0.1667	0.1667	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0522	0.0279	0.0279	0.0279	0.0279	0.0279	
-5	0.1667	0.1667	0.1667	0.1667	0.1667	0.0678	0.0678	0.0678	0.0678	0.0678	0.0678	0.0678	0.0678	0.0678	0.0678	0.0097	0.0097	0.0097	0.0097	0.0097	
-4	0.1667	0.1667	0.1667	0.1667	0.1667	0.0821	0.0821	0.0821	0.0821	0.0821	0.0821	0.0821	0.0821	0.0821	0.0821	0.0239	0.0239	0.0239	0.0239	0.0239	
-3	0.1667	0.1667	0.1667	0.1667	0.1667	0.0949	0.0949	0.0949	0.0949	0.0949	0.0949	0.0949	0.0949	0.0949	0.0949	0.0401	0.0401	0.0401	0.0401	0.0401	
-2	0.1667	0.1667	0.1667	0.1667	0.1667	0.1061	0.1061	0.1061	0.1061	0.1061	0.1061	0.1061	0.1061	0.1061	0.1061	0.0563	0.0563	0.0563	0.0563	0.0563	
-1	0.1667	0.1667	0.1667	0.1667	0.1667	0.1158	0.1158	0.1158	0.1158	0.1158	0.1158	0.1158	0.1158	0.1158	0.1158	0.0716	0.0716	0.0716	0.0716	0.0716	
0	0.1667	0.1667	0.1667	0.1667	0.1667	0.1241	0.1241	0.1241	0.1241	0.1241	0.1241	0.1241	0.1241	0.1241	0.1241	0.0856	0.0856	0.0856	0.0856	0.0856	
1	0.1667	0.1667	0.1667	0.1667	0.1667	0.1312	0.1312	0.1312	0.1312	0.1312	0.1312	0.1312	0.1312	0.1312	0.1312	0.0979	0.0979	0.0979	0.0979	0.0979	
2	0.1667	0.1667	0.1667	0.1667	0.1667	0.1371	0.1371	0.1371	0.1371	0.1371	0.1371	0.1371	0.1371	0.1371	0.1371	0.1087	0.1087	0.1087	0.1087	0.1087	
3	0.1667	0.1667	0.1667	0.1667	0.1667	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421	0.1421	0.1181	0.1181	0.1181	0.1181	0.1181	
4	0.1667	0.1667	0.1667	0.1667	0.1667	0.1463	0.1463	0.1463	0.1463	0.1463	0.1463	0.1463	0.1463	0.1463	0.1463	0.1260	0.1260	0.1260	0.1260	0.1260	
5	0.1667	0.1667	0.1667	0.1667	0.1667	0.1498	0.1498	0.1498	0.1498	0.1498	0.1498	0.1498	0.1498	0.1498	0.1498	0.1328	0.1328	0.1328	0.1328	0.1328	
6	0.1667	0.1667	0.1667	0.1667	0.1667	0.1527	0.1527	0.1527	0.1527	0.1527	0.1527	0.1527	0.1527	0.1527	0.1527	0.1385	0.1385	0.1385	0.1385	0.1385	
7	0.1667	0.1667	0.1667	0.1667	0.1667	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551	0.1551	0.1433	0.1433	0.1433	0.1433	0.1433	
8	0.1667	0.1667	0.1667	0.1667	0.1667	0.1571	0.1571	0.1571	0.1571	0.1571	0.1571	0.1571	0.1571	0.1571	0.1571	0.1472	0.1472	0.1472	0.1472	0.1472	
9	0.1667	0.1667	0.1667	0.1667	0.1667	0.1588	0.1588	0.1588	0.1588	0.1588	0.1588	0.1588	0.1588	0.1588	0.1588	0.1506	0.1506	0.1506	0.1506	0.1506	
10	0.1667	0.1667	0.1667	0.1667	0.1667	0.1602	0.1602	0.1602	0.1602	0.1602	0.1602	0.1602	0.1602	0.1602	0.1602	0.1534	0.1534	0.1534	0.1534	0.1534	
11	0.1667	0.1667	0.1667	0.1667	0.1667	0.1613	0.1613	0.1613	0.1613	0.1613	0.1613	0.1613	0.1613	0.1613	0.1613	0.1557	0.1557	0.1557	0.1557	0.1557	
12	0.1667	0.1667	0.1667	0.1667	0.1667	0.1622	0.1622	0.1622	0.1622	0.1622	0.1622	0.1622	0.1622	0.1622	0.1622	0.1576	0.1576	0.1576	0.1576	0.1576	
13	0.1667	0.1667	0.1667	0.1667	0.1667	0.1630	0.1630	0.1630	0.1630	0.1630	0.1630	0.1630	0.1630	0.1630	0.1630	0.1592	0.1592	0.1592	0.1592	0.1592	
14	0.1667	0.1667	0.1667	0.1667	0.1667	0.1636	0.1636	0.1636	0.1636	0.1636	0.1636	0.1636	0.1636	0.1636	0.1636	0.1605	0.1605	0.1605	0.1605	0.1605	
15	0.1667	0.1667	0.1667	0.1667	0.1667	0.1642	0.1642	0.1642	0.1642	0.1642	0.1642	0.1642	0.1642	0.1642	0.1642	0.1615	0.1615	0.1615	0.1615	0.1615	
16	0.1667	0.1667	0.1667	0.1667	0.1667	0.1646	0.1646	0.1646	0.1646	0.1646	0.1646	0.1646	0.1646	0.1646	0.1646	0.1624	0.1624	0.1624	0.1624	0.1624	
17	0.1667	0.1667	0.1667	0.1667	0.1667	0.1650	0.1650	0.1650	0.1650	0.1650	0.1650	0.1650	0.1650	0.1650	0.1650	0.1632	0.1632	0.1632	0.1632	0.1632	
18	0.1667	0.1667	0.1667	0.1667	0.1667	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1653	0.1638	0.1638	0.1638	0.1638	0.1638	
19	0.1667	0.1667	0.1667	0.1667	0.1667	0.1655	0.1655	0.1655	0.1655	0.1655	0.1655	0.1655	0.1655	0.1655	0.1655	0.1643	0.1643	0.1643	0.1643	0.1643	
20	0.1667	0.1667	0.1667	0.1667	0.1667	0.1657	0.1657	0.1657	0.1657	0.1657	0.1657	0.1657	0.1657	0.1657	0.1657	0.1647	0.1647	0.1647	0.1647	0.1647	
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Chapter 6:
Valuation of Convertible Bonds Subject to Default Risk

6.1 Introduction

Corporate convertible bonds are inherently subject to default risk. The valuation of risky convertible bonds has been a rarely addressed subject except for the notable works of Ingersoll (1977a), Brennan and Schwartz (1980), and more recently by Davis and Lischka (1999) and Takahashi, Kobayashi and Nakagawa (2001). This chapter proposes a completely new structural type approach to the valuation of risky debt (both convertible and straight) via the introduction of the asset cover process. The model in its basic form requires no additional inputs beyond those already supplied in standard valuation approaches, thus offering significant practical potential over the traditional structural models and becomes a real structural valuation alternative to recently proposed reduced form models.

The chapter proceeds with a short literature overview, followed by a description of the model, numerical implementation and price sensitivity analysis. In the final part, the model is applied to a real life convertible bond over the period of the Russian triggered financial crisis in order to demonstrate an improved explanatory power.

6.2 Literature Overview

Much of the literature on the subject of default risk has been previously covered in Chapter 2. In this chapter, I will concentrate on recent research papers specifically focusing on default risk in convertible bonds.

The underlying theory of default risk is split into two branches. The structural class of models take the firm value as the main underlying source of risk. Once the capital structure is specified, a default timing and its intensity are determined internally. The initiating work came from Merton (1974), who assumed that firm value, the capital structure of which consists of a discount bond and an equity, follows a lognormal diffusion process. At the maturity of the bond two outcomes were possible: a) if the value of the firm is insufficient to pay the redemption, the bond defaults and bondholders acquire the firm; or b) the bond is redeemed in full and equity holders keep the residual value. In other words, equity is seen as a call option on the firm with the strike price equal to the face value of the bond, i.e. equity holders pay off bondholders and keep the firm.

The structural approach is an elegant and theoretically well founded idea but has a major drawback: parameters of the firm value process i.e. initial price, its volatility, as well as the exact capital structure and total cash outgoings, are mostly unobservable variables that in many cases are impossible to estimate with any degree of confidence.

Merton's (1974) model, when it can be applied, was found to forecast credit spreads that are significantly tighter than those observed in the market. To correct the pricing bias, Black and Cox (1976) introduced the notion of a default trigger and the possibility of default before maturity, as the default can occur as soon as firm's value crosses the default trigger. Many other authors as mentioned in Chapter 2 have since made further advances to the structural approach culminating with a very respectable commercial implementation of this concept that goes under the name of KMV[124].

The earliest published research specifically targeting convertible bonds subject to default risk were Ingersoll (1977a) and Brennan and Schwartz (1977a). They built on the ideas of Merton (1974) for the valuation of risky corporate bonds and extended it to convertible bonds. The underlying risk variable, affecting both the default risk and the conversion value, is taken to be the value of the firm[125]. The capital structure consisted of one convertible bond and equity. At maturity in case of default, the whole of remaining value immediately and entirely passes to bondholders. Liquidation costs and default before maturity were not considered.

In their subsequent work, Brennan and Schwartz (1980) expand the earlier work[126] to allow for the existence of both senior and junior debt and for partial redemption upon default for both senior and convertible debt (while junior debt and equity holders suffer a total loss).

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[124] KMV was founded in 1989 by, Stephen Kealhofer, John Andrew "Mac" McQuown, and Dr. Oldrich Alfons Vasicek with a single focus on credit risk, which in their implementation is termed a distance to default (difference between the current firm value and the default threshold level, divided by the volatility of the firm value). Their quantitative approach to credit risk management has gained such a wide recognition that in February 2002, Moody's Corporation had acquired the company for $210m. For a good comparative study of current commercial leaders in credit risk modelling see Crouhy, Galai and Mark (2000).

[125] As in Merton (1974) the value of the firm follows a lognormal process with constant volatility.

[126] Their work was equally important for the modelling of interest rates, as this paper was the first to introduce the mean-reverting lognormal process for the short rates.

In practice, as in subsequent theoretical studies[127] that used Ingersoll or Brennan and Schwartz models, confident estimation of firm value parameters has severely limited this model's applicability.

It took 21 years before Takahashi, Kobayashi and Nakagawa (2001) mention in passing an implementation of another structural class model[128] for default prone convertible bonds based on Longstaff and Schwartz (1995)[129], but with firm value replaced by the stock price. The default trigger was backed out by replicating market observed prices of straight bonds.

The class of reduced form models of default risk was initiated by the works of Jarrow and Turnbull (1995), who used a foreign currency analogy and define uncertain payoff at maturity as being paid in full but in currency with a volatile foreign exchange rate. Therefore, the risk of having sub par redemption is translated into foreign currency profits repatriation risk with the upside capped.

The current shape of reduced form models was formulated by the notable contributions of Duffie and Singleton (1997, 1999) and Schönbucher (1997). They use the models for stochastic interest rates, but instead of the risk-free rate as the state variable the default-adjusted short-rate process, defined as the risk-free rate plus a mean-loss rate[130] (credit spread) is used. Default prone securities are then valued as if the risk free rate is used. Reduced form models attempt to value the evolution of the risk-adjusted term structure with the initial term-structure fitted to the current market data. Duffie and Singleton (1997, 1999) suggest an extension to their model for cases where the mean-loss rate may depend on the security being valued or on some other variable such as stock price, and suggest that this may be used to value convertible bonds.

Davis and Lischka (1999) implemented a model for convertible bonds in a fashion suggested by Duffie and Singleton (1997, 1999). They introduced a default prone stock price process that at the instance of default jumps to zero and remains there, while the value of the convertible drops to a predetermined fraction of notional. The final shape of the model is two-and-a-half dimensional case: the lognormal process for the stock price, an extended Vasicek

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127 See King (1986) and Carayannopulos (1996)

128 Takahashi, Kobayashi and Nakagawa (2001) introduced this model for purely comparative purposes so they can test their main model based on the reduced form approach.

129 Longstaff and Schwartz (1995) handling of default is based on the assumption of the default threshold value as in Black and Cox (1976).

130 In this context the pure stochastic process for interest rate is lost.

process for the interest rate[131], and the last half-dimension is an explicit inverse relationship between the hazard rate[132] and the stock process, with the stock price diffusion process driving the hazard rate as well. Takahashi, Kobayashi and Nakagawa (2001) specify a very similar model with hazard rate linked to the stock price via an inverse power function[133], the parameters of which can be estimated from historic regression of credit spreads and stock prices. Ayache, Forsith and Vetzal (2002) extend the reduced approach further by considering a non-zero default assumption for the stock and number of assumptions for convertible bond recovery values. They also draw attention to the 'unclear' definition of default in Tsiveriotis and Fernandes (1998) model that, in their framework, corresponds to assuming that on default the stock price does not move, but the cash-only part of the convertible bond suffers a total loss, while the holder continues carrying the equity component of the defaulted convertible bond. Mao (2001) addresses some of the shortcomings of Tsiveriotis and Fernandes (1998) cash-only plus stock-only decomposition of convertible bond, by adding a third component: defaulted part, However, Mao (2001) still does not account for the stock's default risk nor does he imposes the (inverse) relationship between the hazard rate and the stock price process as Davis and Lischka (1999) or Takahashi, Kobayashi and Nakagawa (2001) have done[134].

These are to date the most complete models for convertible bonds subject to default risk. Some can be applied in practice and calibrated to market data for interest rates as well as the term structure of credit spreads. All models assume perfect correlation between changes in stock prices or firm value and credit spreads. Implied volatility parameters derived from any of the other models are incompatible with Black-Scholes valuation of equity options and warrants.

The valuation approach proposed in this chapter falls into the class of structural models of default. However, instead of assuming a single stochastic process for the firm's value, the equity and the remaining/other assets (the asset cover) of higher seniority that ultimately add up to the value of the firm, are each given their own correlated diffusion processes (for stock it is

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131 For Extended Vasicek interest rate model see Hull and White (1994a, 1994b)

132 Hazard rate is the probability of default over infinitesimally small periods. It can be thought as the instantaneous credit spread and is the jump frequency or the Poisson jump process that on default in Davis and Lischka (1999) model will cause stock to jump with intensity 1 to zero. Davis and Lischka (1999) avoid introducing a completely independent stochastic process for the hazard rate by letting the stock price diffusion act as hazard's rate diffusion process as well, but with its own volatility. The implied stochastic process can theoretically produce negative hazard rates.

133 In their model the hazard rate is deterministically linked to the stock price. The hazard rate, λ, is linked to the stock price via the relationship $\lambda = \theta + c\,S_t^{-b}$, where θ, c, and b are non-negative constants estimated by regressing the implied credit spreads from the non-convertible bonds of the same issuers and respective stock prices.

lognormal while for the asset cover it is normal). The asset cover in combination with the stock establishes 'collateral' for the bond. A default occurs once the collateral values falls below the bond value, i.e. there is not enough collateral to guarantee the bond's redemption.

The benefits from such a specification are multiple: the default timing and its intensity is internal to the model and depends on variables with real economic meaning. The probability of default is a function of the volatilities of both processes and the correlation between them. Provision for non-perfect correlation allows for the information asymmetry and the risk associated with the evolution of unobservable variables to be accounted for, producing more realistic hedging parameters. Similarly to other models, the proposed model can be fitted to the market observed credit spreads.

In this respect my approach is a new contribution applicable not only to the valuation of convertibles but to corporate bonds in general. The use of a standard log-normal process for the stock gives a further unique benefit, as its implied volatility outputs are fully compatible with the Black-Scholes valuation of equity options and warrants[135].

The chapter continues with a description of the model, followed by its sensitivity analysis. In the final section I examine the pricing and hedging potential of the proposed model using real life data spanning over the 1998 crash, and show that the proposed model has a much closer fit to the observable convertible bond data than the standard model even when the credit spread is adjusted daily.

6.3 Notation

$\mathbf{K} = \{\kappa_1, \ldots, \kappa_N\}$ Vector of bond payments. Market convention is to refer to the periodic payments as coupons and payment at maturity, κ_N, as redemption value. For us κ_i is generalised payment consisting of all cash flows occurring at time *i*.

L_t Value of the collateral process at time t.

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[134] I am grateful to Frank Mao for alerting me to this work.

[135] This is something that can not be achieved with other reduced form models as they all use a default prone stock price process, i.e. a lognormal diffusion with superimposed jump process, which is different from Black and Scholes (1973). Equally, structural models that assume a lognormal process for the firm value are incompatible with Black-Scholes. In this sense, the current model is unique.

ζ Expected default time.

α Conversion Ratio is the number of units of the underlying asset received upon the exercise of one bond unit.

V_t Convertible bond value.

$V_t|_{\alpha=0}$ The value of an otherwise equivalent non-convertible bond.

X_t Conversion value at time t. This is dependent on the value of the underlying stock and the specific provisions of the contract. In this case $X_t = \alpha S_t$.

6.4 Definitions and General Description of Default Prone Convertible Bonds

The subject of this chapter are the convertible bonds prone to risk of default. I define them as follows,

DEFINITION 6.1. A convertible bond is a corporate liability (of a fixed maturity) that at the option of the bondholder can be exchanged for the stock of the corporate. So long as the bond is not exchanged, the firm promises a stream of scheduled cash payments. If the firm fail on this promise, the bond defaults and the holders acquire all assets in pursuit of their claim.

The economy where these convertible (and by extension straight) bonds and the underlying stock are traded is further described as

ASSUMPTION 6.1. *Trading takes place continuously, transaction costs and taxes are zero, there are no borrowing and lending restrictions and investors prefer more than less.*

I.e. market is complete with no arbitrage, risk-neutral valuation technique can be applied.

In order to keep the problem tractable, a convertible bond under consideration has further restrictions imposed:

ASSUMPTION 6.2. Conversion terms are constant, conversion is at the option of the holder at any time prior to maturity and remain the bondholder right even in default[136]. Bonds are non-callable and non-puttable for life.

The imposition of non-callability and non-puttability is not essential. However, adding this feature would increase the complexity of the solution making it more difficult to concentrate on the default risk. Furthermore, to precisely value callable bonds, I would need to introduce a stochastic interest rate process, which I leave for future research. Note that the call event itself carries no additional default risk, while the put would attract significant risk.

Allowing for post-default conversion is not an essential requirement, as the post default stock price is expected to be zero or close to it. However, as will be seen later, the model presented here assumes post-default negotiation between bond and equity holders that might lead to positive stock recovery. It makes sense to assume that the same negotiation benefits convertible holders especially if their conversion option is kept alive.

ASSUMPTION 6.3. A convertible bond and its underlying stock are denominated in the same currency units.

This is frequently not the case in practice, however to keep the focus on the default risk this restriction is needed.

The convertible bond can be converted into stock with the following properties:

ASSUMPTION 6.4. The stock price follows the lognormal diffusion process with constant[137] instantaneous variance σ^2 and deterministic drift.

[136] Retaining the conversion right beyond the default event could be potentially a valuable option if stock price is assumed to recover at some positive value.

[137] The constant variance assumption can be easily removed to accommodate time-varying, stock price dependant, or even stochastic variance. However, the basic characteristics of the proposed model would remain the same and introducing an additional level of complexity would greatly reduce tractability.

$$dS = \mu S dt + \sigma S dW \quad [6\text{-}1]$$

Where dW is the Weiner process with properties $E[dW]=0$, $E[dW^2]=dt$. μ is drift that includes dividend payments the extent of which is restricted by the need to keep sufficient collateral for the bond to survive. The shape of the dividend payments function is discussed in Chapter 5.

Based on the above assumptions, the convertible bond contract can be prematurely terminated in two ways:

- *Voluntary conversion;*
- *Issuer's default.*

The redemption choices coupled with early termination leads to a definition of initial and free boundary conditions for the valuation problem.

From the above it can be seen that the value of a convertible bond depends on the two sources of uncertainty: stock price and asset cover. The two risks can assume perfect correlation, as in most of the other models, but, as I pointed in the introduction, there is a good case to assume lesser correlation.

6.5 Components of the Model

6.5.1 Collateral Process

Before defining the collateral process let's first examine the capital structure of a generic firm. Suppose a firm issuing a bond, V, convertible into its own stock, S, has also issued senior bonds, U, and junior bonds, J. The total value of the firm, F, is therefore,

$$F = n_S S + n_J J + n_V V + n_U U, \quad [6\text{-}2]$$

Where, n_J, n_V, and n_U are quantities of bond units issued and J, V, and U are the respective prices. Using those components of the firm value, I construct the collateral available to a holder of one unit of senior bond, L_U, as the sum of equity value (easily observable) and the remaining part (imperceptible). The sum of the easily observable part and the imperceptible parts together make up the whole firm.

$$L_U = \frac{n_S}{n_U} S + \frac{1}{n_U} (F - n_S S), \qquad [6\text{-}3]$$

Similarly, the collateral available to a convertible investor, L_V, is

$$L_V = \frac{n_S}{n_V} S + \frac{1}{n_V} (F - n_S S - n_U U). \qquad [6\text{-}4]$$

With this introduction,

DEFINITION 6.2. *The collateral, L, is the part of the firm's value upon which the promised schedule of payments to the given bondholder is made.*

The collateral can be alternatively thought as the asset backing for the given bond. In this context, the collateral is specific to the given bond under consideration[138], i.e. the collateral associated with the higher seniority claims is removed.

Structural models of default as in Merton (1974), Brennan and Schwartz (1980), Black and Cox (1976) and Longstaff and Schwartz (1994) can be seen as special cases of our model when the bond being valued is a senior bond, as in that case the collateral equals the firm. To achieve total equivalency the distributional properties of the stock price and the asset cover would need to be changed so that the collateral process becomes lognormal.

6.5.2 Asset Cover Process

Rearranging [6-4], time t collateral applicable to a unit of given bond has a general form of

[138] If the given bond is a senior debt, then the collateral are all the assets of the firm, but in general the collateral would consist of all assets available to claims seniority of which is up and equal to the bond being valued. It is all the assets less the assets covering higher seniority claims.

$$L_t = \beta S_t + A_t, \qquad [6\text{-}5]$$

Where $\beta = n_S/n_V$ is the equity-to-(convertible) debt multiplier[139], and $A \in \Re$ is the *asset cover* of the convertible bond[140]. Note that in an idealised case as [6-4], the asset cover is the sum of values of junior and convertible debt.

Equation [6-5] leads to a definition of the asset cover in the context of this model as:

> ***DEFINITION 6.3.** The asset cover is the firm's assets/liabilities account that, together with the current equity, forms the collateral upon which the promise of the given bond's repayment is based.*

In the general case, the asset cover is the residual value of the firm less the value of the equity. For mid-ranking bonds the asset cover, A, may become negative, in which case the collateral value, L, can, for sufficiently low stock price, also become negative[141] (i.e. total liabilities are greater than total assets, implying that, seen from the given bondholders point of view, the firm is insolvent). If the model is used to value senior debt, in which case the collateral consists of all assets, this implies that the firm value process can take both positive and negative values (insolvent firm). Here, I make a significant leap suggesting that although individual asset classes that together make up the firm can take only non-negative values, the firm as a whole can attain both positive and negative[142] values. The fact that the sum of the parts can be (significantly) different from the (imperceptible) whole, not only in magnitude but also in its sign, is a very real phenomenon[143] that can be readily explained with the presence of asymmetric information flow.

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[139] Correspondingly, the product of equity-to-(convertible) debt multiplier with the stock price, βS, is the equity-to-(convertible) debt ratio. This ratio is readily available from companies accounting statements and is relatively stable over time.

[140] When positive and deterministic, A, can be interpreted as a guaranteed recovery of the convertible bond. However, in this model A is explicitly allowed to take both positive and negative values.

[141] This is generally not the case for the initial value of the collateral. However, the future evolution of stock price and the asset cover would lead to the states when this would be the case.

[142] This corresponds to real life defaults where bonds are total loss, i.e. the nil recovery case.

[143] Take the Enron example in 2002. Only six months prior to collapse, stock and bond of the company traded as if everything was in order. However, it is now known that for several years this was effectively an insolvent company. So, the sum of the parts would at that time be significantly different to the unobservable true value of the firm.

Assumption 6.5. The asset cover, A, follows the normal diffusion process with constant instantaneous variance υ^2 and deterministic drift ν.

$$\mathrm{d}A = \left[\nu_t A - \kappa_{t_i}\delta(t-t_i)\right]dt + \upsilon dZ, \qquad t_i \geq t, \qquad [6\text{-}6]$$

where $E[dZ]=0$, $E[dZ^2]=dt$, $E[dWdZ]=\rho$, ν_t is drift which may include additional payouts[144], κ is a vector of bond cash flows, t_i is the date of the next coupon payment and δ is Dirac delta function[145]. The correlation between the stock price process and the asset cover process signify the extent of asymmetric information and the uncertain value of unobservable components of firm value. This is an important difference from all other structural models that assume perfect correlation. The reduced class of models, depending on implementation, would generally either assume perfect or no correlation.

The model as described has two sources of risk: stock price S and asset cover A. According to assumption 6.1 both stock and bonds issued by the firm are assumed to be tradable securities. It is less obvious that asset cover, A, is also a tradable security, but we can evoke an argument that the risk associated with the stochastic nature of the asset cover can be diversified via other tradable securities, namely straight bonds. Under this, somewhat weaker, assumption A can be also assumed to be a tradable security and we can therefore assume that A follows risk-neutral arithmetic Brownian motion as per [6-6].

The terminal distribution of the asset cover process, subject to no default is therefore

$$A_T = A_0 e^{\int_0^T \nu_t dt} - \mathbf{K}_{0,T} + \upsilon\sqrt{T}Z, \qquad [6\text{-}7]$$

The initial value of asset cover, A_0, can be estimated from the accounts or implied from the market by fitting the quoted bond prices. The coupons accumulation account, $\mathbf{K}_{0,T}$, is the sum of all coupon payments each deposited into a money-market account at their payment time, i.e.,

144 Although the drift is assumed to be equal to the risk free rate in general an additional component can be added to allow calibration of the model to exogenously supplied term structure of credit spreads.

145 See Chapter 5 for the definition of the Dirac delta function.

$$\mathbf{K}_{0,T} = \sum_{0<t_i\leq T} \kappa_{t_i} P_{t_i,T}^{-1}, \qquad [6\text{-}8]$$

Where $P_{t,T}$ is the value at time t of a risk free zero coupon bond maturing at time T. This formulation corresponds to the situation when the coupon payments are paid from current operating income and not financed by further equity issuance[146]. As a consequence of such a formulation, default at maturity can be avoided only if the initial asset cover and its drift rate were high enough to guarantee the payment of all coupons.

The coupon accumulation account may if necessary include payment/receipts not directly linked with the bond under consideration. This would be used to calibrate the model to the observed term structure of credit spreads.

A number of other basic assumptions are needed in order to specify the model fully:

> ***ASSUMPTION 6.6.*** The management of the firm acts to maximise the value of the firm's equity.

The implication of this assumption is that the management also acts to minimise the value of other asset classes within the firm, and that the management pursue the optimal dividend policy.

> ***ASSUMPTION 6.7.*** The management of the firm are prevented from selling assets to pay the dividends.

This assumption safeguards collateral and prevents situations where the management of the company may pay a dividend even if it forces the default. Dividend payments are restricted or suspended[147] if they may trigger the default. If A_{x-} and S_{x-} are the values of the asset cover and

[146] See Leland (1994). Alternatively I can omit the coupons accumulation account and make the assumption that the coupons are paid by issuing more equity, but at the expense of having to deal with stock dilution.

[147] This would also be the optimal dividend policy in those circumstances.

stock processes just before the dividend payment, then the payment must be small enough to ensure firm's survival and keep the stock price positive[148],

$$\beta S_{x^-} - dividend_x > V_{x^-}\big|_{\alpha=0} - A_{x^-} > 0 \qquad [6\text{-}9]$$

In other words, post dividend collateral has to remain above the equivalent straight bond value. If the collateral process is already below the default trigger, further dividends are suspended.

6.5.2.1 Default Event

The default event occurs whenever the current value of the collateral process, L, falls below the current value of the straight bond equivalent, $V_t|_{\alpha=0}$. Upon default the convertible bond process terminates with the residual value of the collateral process, adjusted for the bankruptcy costs if present.

> ***DEFINITION 6.4.*** The default event is the earliest stopping time ζ such that the value of the straight bond equivalent exceeds the value of the collateral, i.e.

$$\zeta = \inf_{t<\tau\leq T}\left(V_\tau\big|_{\alpha=0} > L_\tau\right), \qquad [6\text{-}10]$$

Defining the default event in such a way I am taking the default risk in coupons as well as the redemption payment. Risk free bonds would by definition have $\zeta = +\infty$.

The approach in this paper is similar to Leland and Toft's (1996) case with exogenously determined default level. However, the intensity of default (i.e. liquidation costs) is determined differently. Leland gives the post default value of a bond as a percentage of the pre-default value. In this model the post default value is given with the reference to the value of the collateral process, which can assume a whole range of values.

Although the default is allowed to occur at any time, within the present modelling proposal and with the usual parameter settings, the drop in the asset cover process on ex-coupon dates is

[148] This formulation is an extension of the dividend rules established in chapter 5.

likely to cause the default event on those dates[149] ([6-9] explicitly prevents dividend payments from triggering the default).

6.5.2.2 Recovery

Once the default has occurred I assume that the entire pre-default collateral value, $L_{\zeta-}$, adjusted for liquidation losses, is available to bond holders as recovered collateral. A number of authors have offered various scenarios as to what form of security would be received as recovery. Schönbucher (1997) proposes the most general solution where the recovered security itself may be subject to further default at some future time point. What happens in default in this model is determined by the following,

> ***ASSUMPTION 6.8.*** Upon default the bond value changes (drops down) to the recovered collateral value which the holders are then free to immediately convert or resell in the market thus avoiding any future uncertainty.

Determining the recovery levels is a very difficult task in its own right. Duffie and Singleton (1997, 1999) provide the analysis of "Corporate Bond Defaults and Default Rates" published by Moody's Investor Services, a regular historical statistical summary of the default rates within various rating categories. They conclude that the recovery rates vary greatly even for highly rated issuers and show a pronounced cyclical component.

Schönbucher (1997) quotes the research done by Franks and Torous (1994) who calculate the average recovery rate of 40% to 80% with significant variance.

I approach this problem by splitting the liquidation loss into two parts: proportional loss of the pre-default value, l_P, and an absolute loss of value, l_A, i.e. fixed cost liquidation. Those two parameters are exogenous to the model. The total loss, l, due to cost of liquidation is defined as,

$$l = \max\{0, \min[L_{\zeta-}, l_A + \max(0, L_{\zeta-} - l_A) l_P]\}, \quad 0 \le l_A \le \max(\mathbf{K}), 0 \le l_P \le 1 \qquad [6\text{-}11]$$

The formulation ensures that the total liquidation loss is locally bounded, $0 \le l \le L_{\zeta-}$, i.e. its value is non-negative and cannot be higher than the available collateral[150].

[149] Bonds with early redemption options, i.e. puttable bonds, would also have those dates as likely default dates.

What happens next to the liquidation costs is motivated by research of Fan and Sundaresan (1997). They argue that bond and equity holders (as per definition 6.1 and assumption 6.6), in trying to maximise the residual value of their claims, may be both better off by accepting a mutually agreed split of the remaining assets and avoid paying the cost of liquidation to an outside agency, which would otherwise leave them both worse off. Bondholders are better off as they bear only a half of the costs; equity holders take the other half for themselves[151]. The side effect of this assumption is that the co-operation between bondholders and equity holders leads to reorganisation and, except in the case of a total loss, $L_{\zeta-} \leq 0$, the firm survives, with post default stock price now being above zero. As noted in Fan and Sundaresan (1997) this form of the recovery has a flavour of deviating from the absolute priority rules, a phenomena frequently occurring in real-life bankruptcies.

> ***ASSUMPTION 6.9.*** Upon default bondholders and equity holders accept a mutually agreed split of liquidation costs to mutual benefit.

From the equity holder's point of view, upon default the value of their holdings initially drops to zero, but then it recovers by one half of (saved) total liquidation losses. So the default is a two-stage process:

Stage 1. As default occurs, stockholders wealth transfers to bond holders via an increase in asset cover value. The stock price jump to zero; the collateral remains the same.

$$\begin{aligned} A_{\zeta-} &\rightarrow A'_{\zeta} = L_{\zeta-} = \beta S_{\zeta-} + A_{\zeta-} \\ S_{\zeta-} &\rightarrow S'_{\zeta} = 0 \\ L_{\zeta-} &\rightarrow L'_{\zeta-} = L_{\zeta-} \end{aligned} \qquad [6\text{-}12]$$

Stage 2. Liquidation costs are applied and negotiation takes place. The liquidation costs are split between bondholders and stockholders (stockholders

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150 If the post-default collateral is negative, than the both liquidation costs are set to zero.

151 There is a transfer of wealth from debt-holders to equity-holders that is mutually beneficial.

received negotiated part of the costs, $0 \leq \omega \leq 0.5$[152]), i.e. wealth transfer goes in opposite direction, the final recovery values for the asset cover, stock and collateral are,

$$\begin{aligned} A'_{\zeta} &\rightarrow A''_{\zeta} = A'_{\zeta} - (1-\omega)l \\ S'_{\zeta} &\rightarrow S''_{\zeta} = S'_{\zeta} + \omega l \\ L'_{\zeta} &\rightarrow L''_{\zeta} = L'_{\zeta-} - (0.5-\omega)l \end{aligned} \qquad [6\text{-}13]$$

Note that the optimal negotiating process, $\omega = 0.5$, helps preserve the collateral. A net loss of wealth will occur whenever the negotiated costs split, ω, is less then optimal, $\omega < 0.5$.

Following the negotiation process, the convertible bondholders end up with choice between accepting the value of post-default asset cover (i.e. pre-default collateral reduced by their share of negotiated liquidation costs)

$$A''_{\zeta} = L_{\zeta-} - (1-\omega)l \qquad [6\text{-}14]$$

or the post-default conversion value

$$X''_{\zeta} = \alpha\omega l \qquad [6\text{-}15]$$

Ayache *et al.* (2002) have also proposed non-zero recovery for the defaulted equity, however, they do not provide any guidance as to how this is motivated and what value it should have. In my model, non-zero recovery of stock is a structurally imposed relationship that is unique to this model in its class.

Having decided what the liquidation cost is, how it is apportioned and what are the recovered values of asset cover and stock, the recovered value of a defaulted convertible bond is the greater of the post-default asset cover or the post-default conversion value.

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[152] ω is the share of liquidation costs that bondholders have negotiated to give to stockholders to ensure their cooperation. Setting ω=0 indicates no negotiation takes place, stockholders suffer a total loss, bondholder bare the full cost of liquidation and recover whatever is left. Setting it to 0.5 suggests full negotiation and optimal split of liquidation costs between bondholders and stockholders.

$$V_{\zeta} = \max\left(A''_{\zeta}, X''_{\zeta}\right). \quad [6\text{-}16]$$

The second argument in the above equation represents the additional choice bond holders have, as holders of a convertible security[153].

It is worth noting that in two particular cases the negotiation process is marginalized. The case when pre-default collateral is fully exhausted and the case when there are no liquidation costs.

6.5.2.3 Special Cases

Two special cases are of immediate importance for the analysis that will follow. Those are the cases of pure equity and risk-free debt.

6.5.2.3.1 Valuation of Equity as a Risky Convertible Bond

The model can be applied to value the equity of the firm treating it as a special type of convertible bond.

Let's assume that a convertible bond is specified with the following characteristics: bond matures in the next instant; the collateral consists only of equity, i.e. $L_t = S_t$, so that $\alpha \equiv 1$ and $\beta \equiv 1$.

The next instant the 'bond' matures. If the redemption value is greater than the available collateral, the bond defaults, but bond holders opt to 'optimally negotiate' fully preserving the value of collateral (which consists only of equity). On the other hand, if the redemption value is lower then the collateral, implying the conversion value is also above the redemption value, then the conversion takes place again.

Whatever the outcome the holders of this special convertible bond i.e. the stockholders are indifferent to default risk.

[153] This shows that the convertible holders have an extra choice compared to straight bondholders. However, the conversion value is most likely to be lower than the recovery.

6.5.2.3.2. Valuation of a Risk-Free Bond

A risk-free bond can be seen as a special case of a convertible bond where the collateral value, in all states of the world, is always greater than the value of the convertible bond. Taking our definition of the default time this implies $\zeta > T$. Note that in the generalisation of the collateral process where the asset cover, A, is allowed to follow the arithmetic random walk process, a risk-free bond is achieved (almost surely) only in the limit as $A_t \to +\infty$ and $\beta \to +\infty$, a default-risk free bond is a highly collateralised debt obligation.

6.5.2.4. Incorporating Default Risk into Pricing Equation

Having defined the collateral process, default event and the recovery; I can now proceed to define equations for valuation of convertible bonds subject to default risk.

The world where convertibles are traded is set in assumption 6.1. These are standard assumptions that enable us to use the arguments of Black and Scholes (1973) and apply a risk neutral valuation technique.

Using the standard arguments for creating an instantaneously risk-less portfolio, I arrive to the fundamental (parabolic, backward) partial differential equation that has to be satisfied by the convertible bond function:

$$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + \frac{1}{2}\upsilon^2 \frac{\partial^2 V}{\partial A^2} + \rho\upsilon\sigma S \frac{\partial^2 V}{\partial S \partial A} + (r-q)S\frac{\partial V}{\partial S} + \left(v_t - \lambda\upsilon - \frac{\kappa}{A}\right)A\frac{\partial V}{\partial A} + \frac{\partial V}{\partial t} + \kappa = rV$$

[6-17]

Where r is the risk free rate, q is the dividend yield and λ is the market price of asset cover risk.

In order to solve the above equation I need to specify terminal (initial) and boundary conditions. The terminal values are provided from the payoff definition of a convertible bond at maturity

$$V_T = \begin{cases} (\kappa_T, X_T)^+ & \kappa_T < L_T \\ (A''_T, X''_T)^+ & \kappa_T \geq L_T \end{cases} \quad [6\text{-}18]$$

Where A'' is the post-default recovery value and X'' is the post-default conversion value as defined in [6-14] and [6-15] respectively.

Boundary conditions are:

$S \to +\infty$ $$V_t - \max\left(Xe^{-(r-q)(T-t)}, \mathrm{K}_{t,T}P_{t,T}\right) = 0 \quad [6\text{-}19]$$

$S \to 0$ Same as equation [6-17] [6-20]

$|A| \to \infty$ $$\frac{1}{2}\sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV + \kappa = 0 \quad [6\text{-}21]$$

In addition to terminal and boundary conditions the continuous possibility of default and the continuous conversion option introduces two additional free boundary conditions[154]. Those conditions impose an effective range of permissible bond values. From below the bond price is bounded by the conversion value. At any time t prior to maturity when,

$V_t \leq X_t$ $$V_t = X_t \quad [6\text{-}22]$$

This is a standard condition for the American option exercise. If convertible bond price is less than or equal to the conversion value, bond is converted and contract terminates.

The second boundary condition specifies the value in case of a default event. The convertible bond value is reduced to the higher of the value of the recovered asset cover[155], A'' or, assuming conversion in default is allowed, recovered equity, X''. Therefore, at any time prior to maturity ξ when,

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154 For callable and puttable convertible bonds further free boundary conditions would be needed.

155 Depending on the choice of the proportional and absolute loss-of-value parameters, there may be a discontinuity in V_t at the time of default.

$V_\xi|_{\alpha=0} \geq L_\xi$ $\qquad V_\xi = \max\left(A''_\xi, X''_\xi\right).$ [6-23]

Bondholders accept the higher of the recovered asset cover or the recovered conversion value and terminate the contract.

Finally, a consistency condition $0 \leq \alpha \leq \beta \leq +\infty$, needs to be imposed so that the model can never produce situations where the conversion value grows faster than the collateral, potentially triggering the default. The extreme case when $\alpha=\beta$ corresponds to the example of valuing equity as a special case of convertible bond. The other extreme case when $\beta=+\infty$ leads to default free convertible bond. Case when $\alpha = 0$ corresponds to a straight bond.

6.6 Solving the PDE

Parabolic partial differential equations of the type I have described in the preceding section are common in financial literature. However, there are some specific points about the current one.

Due to the existence of the free boundary there is no analytical solution, which leaves us in search of an efficient numerical technique.

Analysing the nature of solution suggests that an implicit method would be most suitable. Implicit methods are unconditionally stable in a sense that the length of the time step is not limited by the choice of space step. Convertible bonds, in contrast to option contracts, have much longer maturity. An overly short time step would significantly increase computational effort.

Problems of this type, in financial applications, were recently studied by Zvan, Forsyth, and Vetzal (1996) and numerous authors in computational fluid dynamics. As Zvan *et al.* note, when the drift of the stochastic process is large relative to the variance, numerical solutions would contain an unwanted oscillatory component in the regions of the steep gradient unless the discretisation is very fine[156] (notably the part of the solution surrounding the default event).

[156] The space step must satisfy the *Peclet* condition, which requires the step to be smaller than the ratio between variance and drift of the process. There is also a second part of the Peclet condition that is trivially satisfied in the case of the implicit method.

To obtain an oscillation free solution, researchers in computational fluid dynamics have developed the technique called flux limiter. Flux limiter adds an artificial numerical variance in the areas of steep gradient while preserving the original variance elsewhere. This in turn produces an oscillation free solution, which has a *Total Variance Diminishing* characteristic (see Zvan *et al*). However, the solution is slightly more diffusive compared with the exact solution[157].

Several flux limiters can be used and I have found that both van Leer and van Albada work well in our case as described in LeVeque (1992).

Before presenting the results, let us mention that the free boundary conditions are introduced to the solution in a fully implicit way via the addition of a penalty term[158]. The penalty function is defined, with reference to [6-22] and [6-23], as

$$Penalty = c_1 1_{V_t \geq L_t} \max[0, V_t - \max(A_t'', X_t'')] + c_2 1_{V_t < L_t} \min[0, V_t - X_t] \qquad [6\text{-}24]$$

Where c_1 and c_1 is suitably defined large constant (in my case c_1=1E+6 and c_1=1E+1).

The first part of the penalty is forcing the bond price towards the recovery level once the default condition has been satisfied. The second part is ensuring that the bond price is never less than the conversion value.

Free boundary conditions can also be treated explicit fashion in which case equations [6-22] and [6-23] would be used to adjust the value after each time step.

To solve the equation [6-17] subject to initial condition [6-18], boundary conditions [6-19], [6-20] and [6-21], and free boundary conditions [6-22] and [6-23], I have chosen the implicit finite difference 2 dimensional solver with locally uniform adaptive grid called VLUGR2, described in Blom, Trompert and Verwer (1996), Blom and Verwer (1994) and Blom and Verwer (1996). The solver was originally used in computational fluid dynamics and intended to handle

[157] Jackson and Süli (1998) provide an example for alternative handling of areas in the solution with sharp changes or jumps. They *mollify* the solution over the nodes where the continuity and boundedness conditions are violated. *Mollification* is implemented via piecewise Hermite cubic interpolation.

[158] See Zvan, Forsyth and Vetzal (1998).

functions that have continuous solutions. I have extended it to cope with discontinuous solutions as discussed above.

6.7 Test Data and Analysis

In order to examine the properties of the model I have chosen a test case from the universe of LYON type zero coupon convertible bonds[159] as studied in Chapter 4.

Initial parameters are set up based on Pride International USD 0% 2018, subordinated convertible debt, issued 20th April 1998, with a put 5 years later @ 49.452% (non-callable during that period). The bond was issued @ 39.106% of nominal unit ($1,000). In total 511,430 nominal units of the bond were issued. One nominal unit of bond can be continuously converted into 13.794 units of common stock (conversion price $28.35). The common stock paid no dividend. At the time of issue the stock was trading around $24.3125 with 50,068,000 units of shares outstanding, and three other senior bonds of total nominal size of $613mm. The senior issues were rated BB-, convertible was rated B being subordinated debt[160].

In this example, I have set the correlation between the movements in asset cover and equity to be 1, i.e. $E[dWdZ]=1$. This also means that the original two sources of risk have collapsed into one. Based on assumption 6.1, stock is a tradable security, the asset cover risk can also be fully diversified (via stock). Using the standard risk-neutral valuation arguments, the price of asset cover market risk, λ, can be set to

$$\lambda = \frac{\nu - r}{\upsilon} \qquad [6\text{-}25]$$

Making the drift of A in equation [6-17] equal to the risk free rate.

In this model it is possible to relax these constraints, however to facilitate easier comparison with other published results I assume perfect correlation as well.

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159 Many of these bonds are not strictly non-callable, however their structure allows for a put option within the first third of their life during which period they are non-callable.

160 At the time of the convertible issue senior bonds were trading at a yield spread of less than 200bp. Consequently the initial yield spread for the convertible subordinated debt was conservatively estimated to 500bp.

Other parameters were set to:

Conversion ratio[161]	$\alpha = 0.013794$
Equity-to-Debt ratio[162]	$\beta = 0.210751$
Proportional loss-of-value on default	$l_1 = 0.0$
Absolute loss-of-value on default	$l_2 = 0.0$
Risk-free interest rate[163]	$r = 6\%$
Initial Parity[164]	$X_0 = 0.335367$
Stock volatility[165]	$\sigma = 0.56$
Initial asset cover value[166]	$A_0 = 0.11$
Stock/Asset cover correlation[167]	$\rho = 1$
Asset cover volatility[168]	$\upsilon = 0.277$

The chosen bond parameters were directly related to bond particulars and market observable data except for the initial asset cover is reverse-calculated to generate bond's initial credit spread of 500bp.

6.7.1 Price Profile at Issue Time and Maturity

The following two graphs show a number of important characteristics of the proposed model.

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[161] This is calculated as ratio between the issue price and the conversion price, i.e. 39.106%/$28.35.

[162] This is the ratio between the number of shares in issue and the US$ value of the debt at the first put date, i.e. 50,068,000 / (511,430 * $1,000 * 49.452%).

[163] Continuously compounded, approximately equal to the yield of US$ 5 year Treasury Strips at the time this convertible was issued.

[164] Based on the stock closing price of 24.3125 on 20th April 1998 and multiplied by the conversion ratio 0.013794.

[165] Estimated as 252 trading days (one calendar year) historic volatility.

[166] Chosen so that the convertible bond initial credit spread is 500bp.

[167] By definition of the problem.

[168] In this example, as perfect correlation between stock and asset cover is assumed, I have linked the asset cover volatility to the underlying stock's volatility (adjusted for the redemption value), i.e. 56%*49.452%.

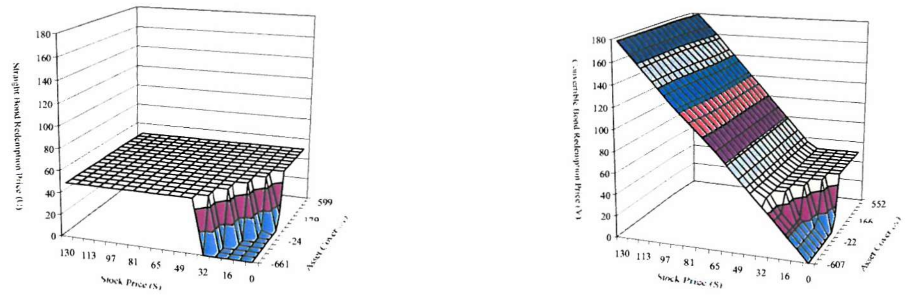

Figure 6-1. Price Profile at Maturity.
Straight bond (left) and convertible bond (right) redemption price (at $t=T$) as the function of stock price and asset cover.

First, the sharp decline of the payoff function is obvious. In general, other input parameters may produce even more severe jump characteristics, especially when the loss-of-value is greater than zero. As described in the previous section, the initial and free boundary conditions of such nature would produce numerical solutions containing spurious oscillations in the direction of the drift. Even a small amount of oscillations would cause huge swings in the estimates of delta and gamma making precise hedging impossible. However, in my case the chosen solver with adaptive grid coupled with the use of flux limiter produced remarkably stable results.

Second, the dominant[169] payoff profile of the convertible investment can be seen in the areas affected by the default. While the straight bond value diminishes faster and earlier towards zero, the convertible investor benefits from the conversion right, avoiding the total loss of value. The convertible price reaches zero value only when stock becomes totally worthless.

Later I will show how this difference has an important effect on the hedging.

[169] Dominance is defined in following sense: A is a dominant security to B if the two have equal payoff in most states of the world, and A has a higher payoff than B in some states of the world.

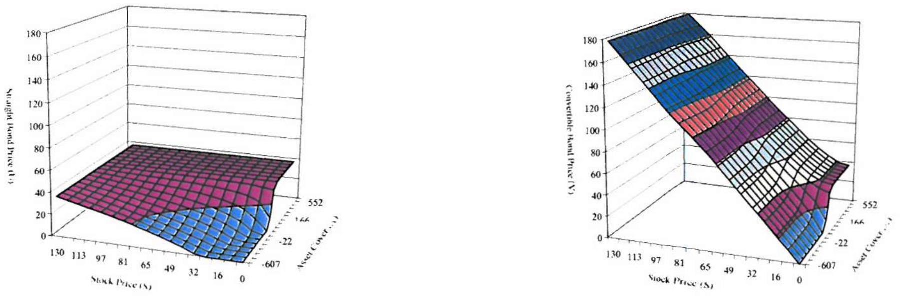

Figure 6-2. Price Profile at Issue Time.
Straight bond (left) and convertible bond (right) price at the time of issue ($t=t_0$) as the function of stock price and asset cover.

Lastly, I note the (linear) equity like payoff profile of the convertible bond for low values of asset cover. In this example, all payoffs for asset cover values of less then –660 have the shape of straight line, the same as an equity investment would have. There are numerous example in practice where bonds of low credit quality issuers trade as if they are equity.

6.7.2 Stock Price Sensitivity

Change in the underlying stock price is the major contributor to the changes in convertible bond price.

The models of Brennan and Schwartz (1980) and Ingersoll (1977a) have the stock embedded in the value of the firm, which for them is the main stochastic variable. As the firm value changes, stock and convertible bond changes with correlation 1 (this also implies for the asset cover/collateral).

Derman and Kani (1994) also use stock as the main driver. They as well have perfect correlation between stock moves and convertible bond moves.

In our model stock changes affect both the default risk, through its contribution to the collateral value, and the conversion value. Therefore, this model can also have a perfect correlation between the movements in stock and convertible bond.

However, there is room for a different assumption for the correlation between changes in the stock and the asset cover. For simplicity, and to facilitate easier comparison with the other models of default, I have assumed perfect correlation. However, a firm may put some other firm's stock[170] or even cash as an asset cover for the bonds being issued. In such cases a different correlation assumption (as well as asset cover volatility and distributional properties) may become more appropriate.

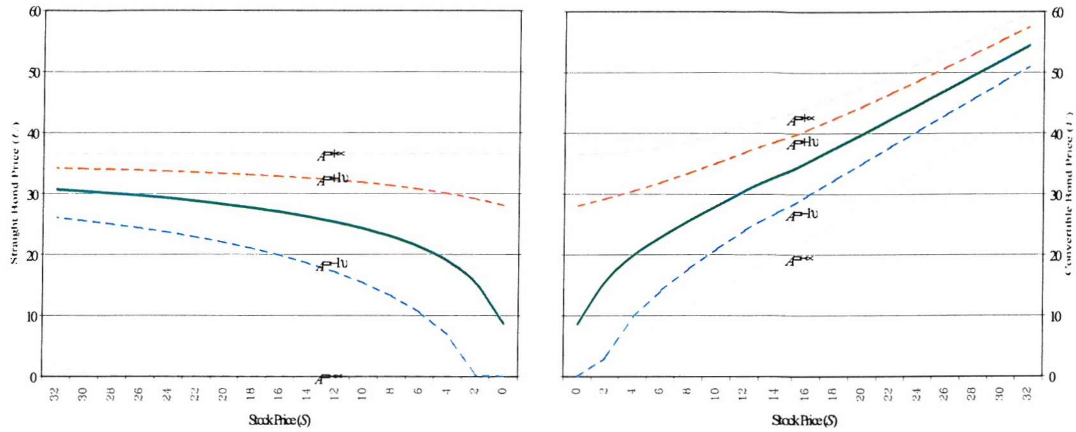

Figure 6-3. Bond Price as a Function of Stock Price.
Straight bond (left) and convertible bond (right) price as a function of stock price for the various levels of asset cover. The central line plots the values of the convertible bond for the base parameters. Lines above and below are the values for 1 standard deviation move in the value of the asset cover estimate. Top and bottom lines are the value in the two extreme case when asset cover is infinitely high, i.e. risk-free convertible bond, and for infinitely low asset cover, i.e. bond is certain to default and its value equals parity (bond has equity value only).

The above figure confirms the convexity of the convertible bond price for high asset cover values (high credit quality bonds). For lower credit quality convertible bonds, the price function is actually concave (except for the very high stock prices), which is completely opposite from the usual perception. In this example, an asset cover value of –0.50 (less than ½ standard deviation from the initial value) creates a convertible price profile that is concave in most of its normal trading region. Note that for low stock prices the bond price falls as far as its parity level.

Extremely low credit quality bonds are equivalent to stock. Their price is solely derived from the conversion property. In comparison, a straight bond would have no value at all.

Next I examine convertible bond delta (figure 6-4). The delta in the default-free case ($A=+\infty$), is the same shape as in the standard models, starting from 0 for low stock prices and

[170] Convertible bonds of this type are known as exchangeable.

monotonically approaching 1 for high stock prices. In the other limiting case when bonds are almost certain to default ($A = -\infty$), the bond price moves one-for-one with the parity and its delta is constantly 1 for all stock prices.

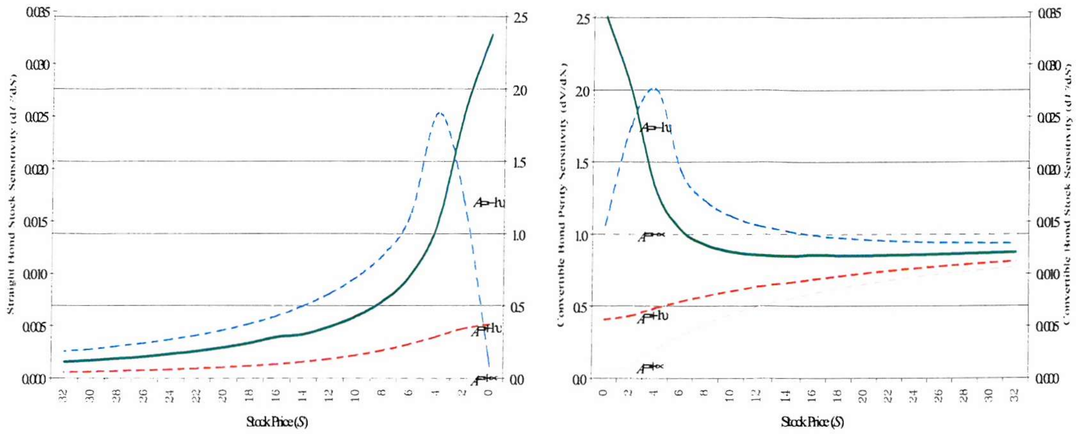

Figure 6-4. Bond Delta as a Function of Stock Price
Straight bond (left) and convertible bond (right) stock sensitivities a function of stock price for the various levels of asset cover. Middle scale is the equivalent parity sensitivity. Central lines plot the values of the convertible bond for the base parameters. Lines above and below are the values for 1 standard deviation move in the value of asset cover. Top and bottom lines are the value in the two extreme case when the asset cover is infinitely high, risk-free convertible bond, and for infinitely low asset cover, the bond is certain to default and its value equals parity (bond has equity value only).

Delta for the midrange asset cover values can be both above and below 1.

It is worth noting that the value of bond's delta (its sensitivity to stock price movements) is limited to

$$0 \le \Delta_t \le \frac{V_t}{X_t} \text{ and } \lim_{X_t \to 0} \Delta_t = +\infty, \quad \lim_{X_t \to +\infty} \Delta_t = 1 \qquad [6\text{-}26]$$

The second limit is due to the fact that, as stock price rises the conversion option gains in value and the value of the convertible itself approached the value of the stock to which it converts. This limit is the same for the standard model. On the other hand, the straight bond's sensitivity to changes in stock price goes to 0 as the stock price rises.

As the stock price reduces and the bond is moving from deep-in-the-money to the at-the-money area, delta would start to fall (except for almost-certain to default bonds where delta stays at 1).

Thereafter the behaviour of delta as the stock drops in price further is dependent on the asset cover level. High asset cover causes the delta to reduce further, while low asset cover rapidly increases delta to values well above 1.

To complete this part of the analysis, I add a gamma graph (bond's delta sensitivity to changes in stock price), figure 6-5. Except for the default-free case (A=+∞) and high asset cover case, all other gamma exhibit significant regions with negative values.

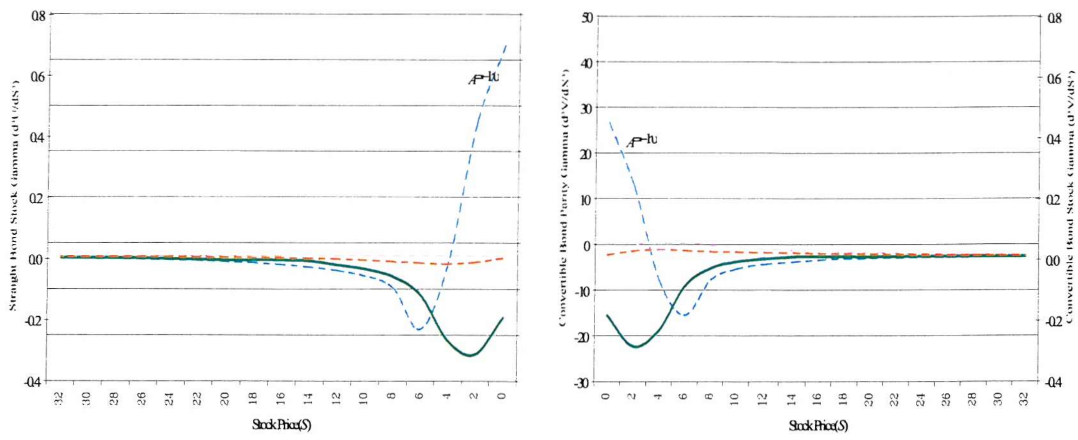

Figure 6-5. Bond Gamma as a Function of Stock Price
Straight bond (left) and convertible bond (right) stock gamma as a function of stock price for the various levels of asset cover. Central lines plot the values of convertible bond for the base parameters. Lines above and below are the values for 1 standard deviation move in the value of asset cover. Top and bottom lines are the value in the two extreme cases when the asset cover is infinitely high, risk-free convertible bond, and for infinitely low asset cover, the bond is certain to default and its value equals parity (bond has equity value only).

For our chosen parameters gamma value is estimated at 0.033, redemption in the case of default is estimated at 4.25. For the comparison, standard model estimates gamma at 0.007.

6.7.3 Asset Cover Sensitivity

Introduction of the asset cover process distinguishes this model from all thus far proposed models both for risky debt and convertible bonds. The forthcoming analysis is based on the chosen real life parameters, the asset cover process volatility and correlation with the stock price is linked. Asset cover volatility is defined as stock volatility adjusted for the non-standard redemption, $\upsilon=\sigma\kappa_T$, and correlation is set to 1. This assumption removes the need for establishing extra parameters while preserving the integrity of the model.

The sensitivity of the convertible price to changes in asset cover value in our model is strictly positive. High values of asset cover reduce the default risk and in the limit the bond is behaving as a risk-free investment, realising its maximum value. Low asset cover increases the possibility of default and in the limit as the bond becomes almost certain to default its value is derived entirely from its conversion property, i.e. its value is the same as the value of the stock it converts into. For all asset cover values in between, the bond price changes smoothly[171].

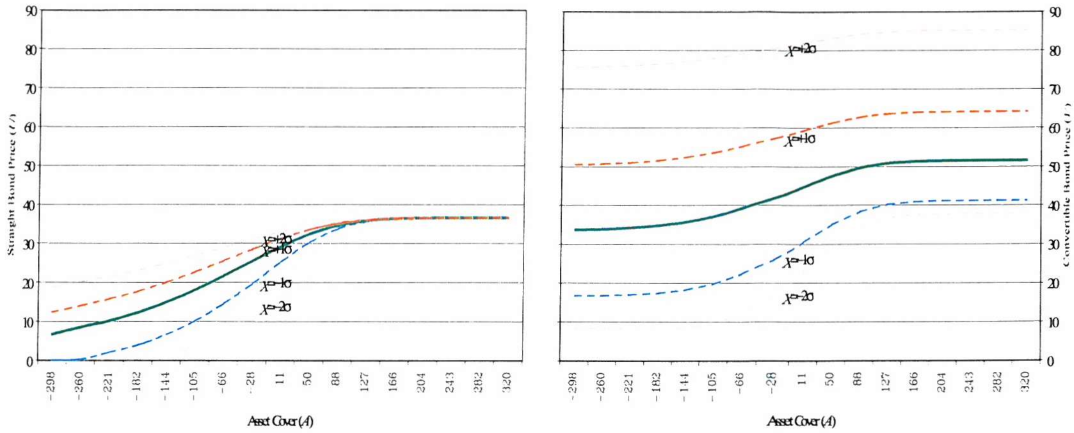

Figure 6-6. Bond Price as a Function of Asset Cover
Left straight bond and right convertible bond price as a function of asset cover for various parities. Central lines plot the values of convertible bond for the base parameters. Lines above and below are the values for 1 and 2 standard deviation moves in parity.

Even more revealing is the price graph for the straight bond. The absence of the conversion property introduces a natural upper limit to their value (equivalent to the value of the equivalent default-free bond). For sufficiently high asset cover, regardless of the level of stock price, they will all approach the same limit. As the asset cover is reduced, bonds value drops at a speed related to the stock price. Lower stock prices cause a faster drop. In our example the values associated with the lowest parity is almost certain to default for all values of asset cover below –125.

To complete the analysis in this section I look at the graph of associated implied credit spread. For clarity spreads are shown in a logarithmically transformed scale, as due to the stochastic

[171] The degree of smoothness is linked to amount of cumulative volatility, $\sigma(T-t)^{1/2}$. The smaller its value is, the sharper the fall. In the limit as the cumulative volatility approaches 0, the bond value becomes discontinuous at the point of default. The Extent of discontinuity is linked to the cost of default parameters l_1 and l_2. See the following section on effects of stock volatility on default risk.

nature of this model, bonds default with certainty only in the limit as recovery reached negative infinity. Although, the numerical solution to the PDE imposes the practical limits of the asset cover range to a finite area, the spreads still achieves very high values.

The graph below reaffirms figure 6-6 above. The model is perfectly capable of generating a wide range of realistic credit spreads, which was the critical point raised for some other models for risky bonds[172].

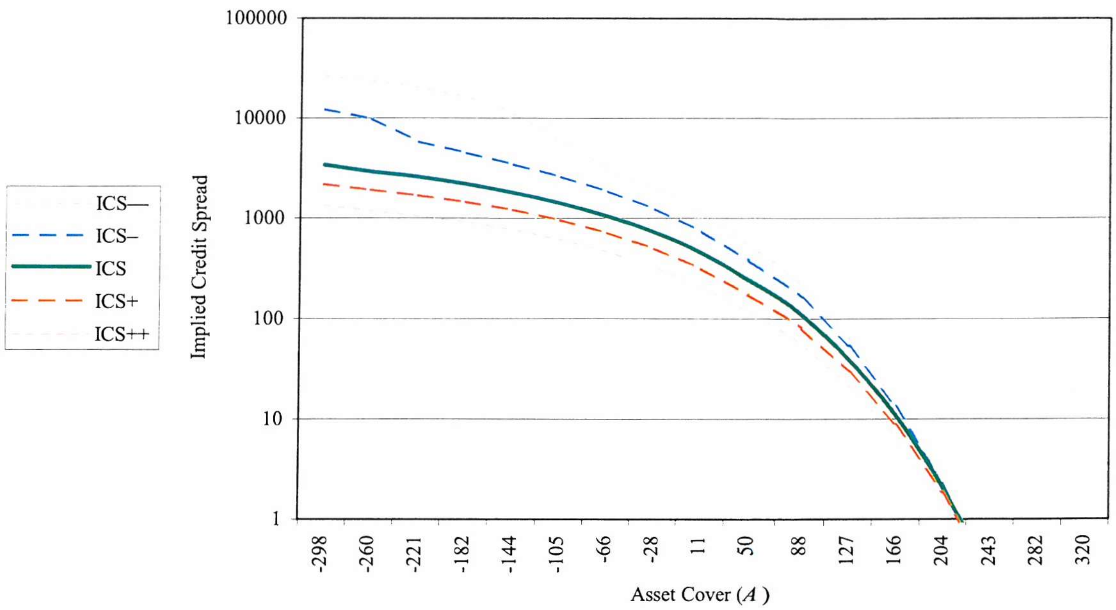

Figure 6-7. Implied Credit Spread as a Function of Asset Cover
Straight bond implied credit spread as a function of the level of asset cover for the various parities. The central line plots the implied credit spread for the base parameters. Lines above and below are the values for 1 and 2 standard deviation moves in the stock value. Spreads are shown in a logarithmically transformed scale.

I now turn my attention to the sensitivity of a convertible and a straight bond price to changes in asset cover, the proposed model's second stochastic variable. I name it *asset cover delta* and use the symbol Δ_A. Thus,

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[172] See Kim, Ramaswamy, and Sundaresan (1993).

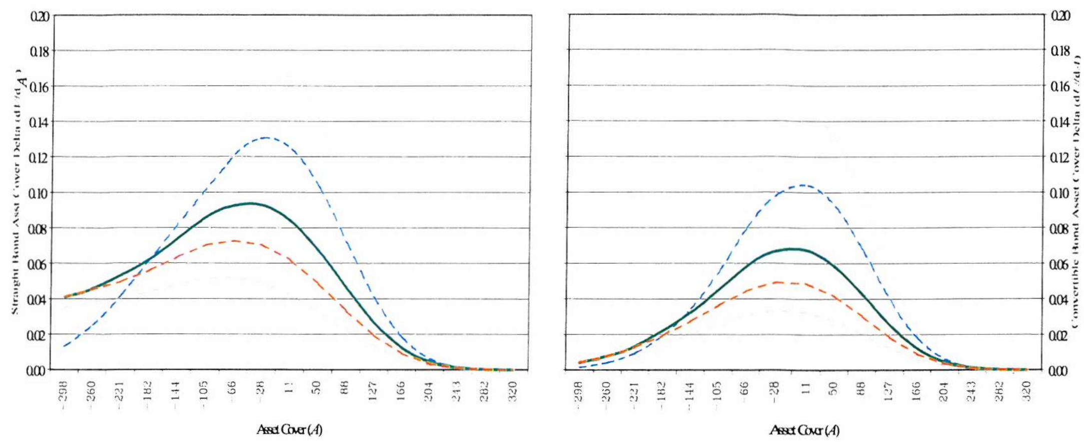

Figure 6-8. Bond Asset Cover Delta as a Function of Asset Cover
Straight bond (left) and convertible bond (right) asset cover delta (bond's sensitivity to changes in asset cover) as a function of asset cover level for the various parities. Central lines plot the values of convertible bond for the base parameters. Lines above and below are the values for 1 and 2 standard deviation moves in the conversion value.

The sensitivity of the convertible bond and the straight bond to the asset cover changes for low conversion value is very similar. However, for higher conversion values, the convertible bond is less sensitive to the lower asset cover due to the extra value contributed by presence of the conversion option.

6.7.4 Effect of Stock Volatility

When examining the effect of changes in volatility the option pricing model of Black and Scholes (1973) as well as standard convertible bond models would show that the increased volatility leads to increased option / convertible bond price[173].

However, when valuing risky debt, as noted by Leland (1994) and others, increases in volatility has the opposite effect on the price of the debt.

Combining the two effects in a model of risky convertible debt creates a more complex sensitivity curve. The effect of change in stock volatility may be either positive or negative.

For low volatility values, depending of the chosen value of the asset cover parameter, A, the price of risky convertible debt would either become risk-free, in which case reducing volatility

[173] This is true for my model with regards to the embedded conversion warrant.

leads to reduction of convertible price to either the conversion value or the recovery, depending on the moneyness of the conversion option.

For small volatility increases, the convertible bond's value responds positively. However, as volatility increases further bonds are subjected to ever increased amount of default risk and in the limit default becomes certain. The bond price then converges towards the recovery level, as its lower limit.

The net effect is the existence of price maximising volatility. Due to the complexity of the underlying PDE there is no closed form solution for the maximising volatility. From the graph 6-9, the peak is related to the level of recovery. Low recovery causes the bond to peak in price at lower levels of volatility. Higher recovery levels are shifting the peak bond price towards the higher volatility.

In the limit as asset cover is infinitely high[174] the bond is default-free and achieves its peak value for infinitely high volatility, which is the same as for the standard models as they value bonds as default-free securities.

The value of convertible bond at maximising volatility is also positively related to the asset cover, i.e. higher the asset cover the higher maximum value the bond can achieve.

An important implication for practical use of the models of this type is the need to precisely estimate input volatility. When using the standard convertible models the usual reaction is to limit the input volatility in order to get prices in line with the quoted market. This often gives a misleading impression that the bonds are trading cheaply in an implied volatility sense[175] (when in fact they are trading rich). The recent plethora of convertible bond issuance involving Internet related companies highlighted this problem even more, when much higher stock price volatility in this sector is taken into account.

The figures below show the changes in price for a range of volatility choices. As can be seen, for the low volatility values, price converges towards the higher of the (otherwise equivalent)

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174 Infinitely high asset cover translates into infinitely small default probability.

175 An other equally dangerous consequence is the low delta given by those models (although, reduced volatility may in some cases produce a higher delta estimates).

risk-free bond or parity. On the other side, high volatility levels, slowly drag the bond price towards the recovery level for the straight bond and towards conversion value for the convertible bond (in the case of our chosen parameters, parity is 33.537).

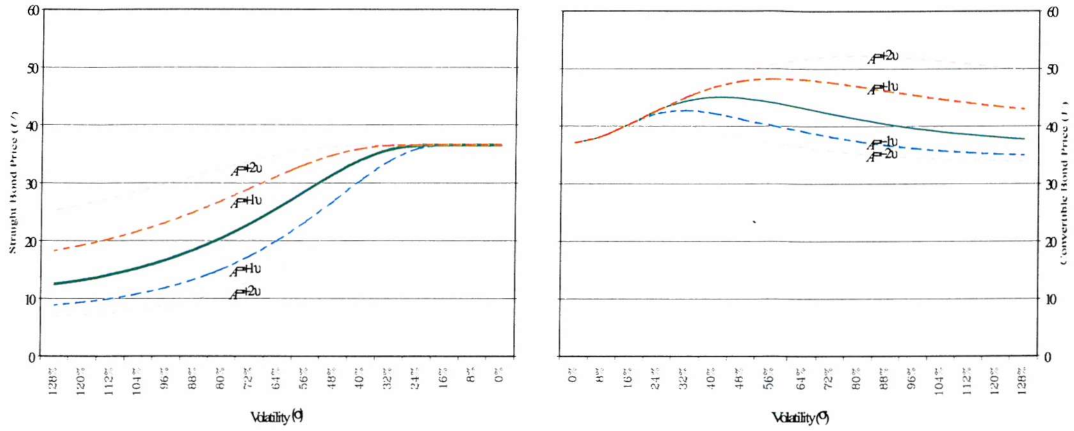

Figure 6-9. Bond Price as a Function of Volatility.
Straight bond (left) and convertible bond (right) price as a function of stock price volatility for the various levels of asset cover. Central lines plot the values of straight/convertible bond for the base parameters. Lines above and below are the values for 1 and 2 standard deviation moves in the value of the asset cover.

The model shows a strictly negative relationship between the value of risky straight debt and volatility. This is the same behaviour as in the other structural models. A similar effect can be observed in the corporate bonds market, where higher uncertainty (i.e. volatility) leads to wider credit spreads[176].

To complete the analysis of the effect of volatility on the risky convertible debt I also take a look at the values of the embedded equity warrant (option to convert). These are obtained as a difference between the value of the convertible and the straight debt. The volatility / warrant-price relationship is as in ordinary warrants, i.e. increase in volatility is positively related to warrant prices regardless of the default risk.

176 See Boldt-Christmas (2002).

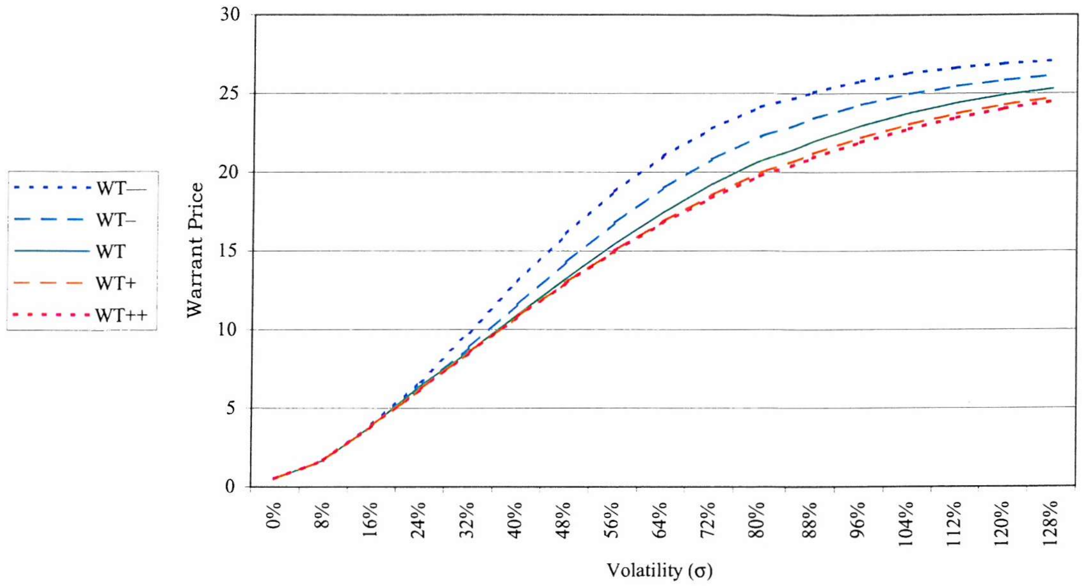

Figure 6-10. Embedded Warrant Price as a Function of Stock Price volatility.
Central line plots the values of the embedded warrant for the base parameters. Lines above and below are the values for 1 and 2 standard deviation moves in the value of the asset cover. Lower asset cover produces higher warrant values.

The above observations lead to an alternative interpretation. As first pointed out by Merton (1974), holders of risky convertible bonds can be seen as holding two embedded options: a long call option on the firm's equity, and a short put option on the collateral associated with the bond. The riskyness of the bond is proportional to the moneyness of the short put option.

6.8 Empirical Results

Table 6-1 below shows the theoretical value, delta and the implied credit spread from April 1998 to March 1999 (month-end readings). For comparison I also show the theoretical value and the delta as calculated by the industry standard model[177].

[177] In this study I used the TrueCalc® Convertible model version 3 developed by TrueRisk® Inc.

Table 6-1. Pride International Bond: Comparison of Market Prices
Parity levels and the corresponding convertible bond price as reported on each month-end for Pride International 0% 2018. Columns V_1 and dV_1/dX refer to the theoretical value and delta as reported by the model proposed in this chapter. Column ICS is the implied credit spread by the model. Columns V_2 and dV_2/dX are theoretical values and deltas obtained using the standard model with the credit spread as in column ICS.

Month-End	Parity	Market V	V_1	dV_1/dX	dV_1/dA	d^2V_1/dX^2	ICS	V_2	dV_2/dX
April-1998	33.537	41.75	44.736	0.857	0.067	0.0013	500	45.538	0.768
May	30.950	40.125	42.629	0.851	0.071	0.0014	532	43.528	0.750
June	23.445	35.0	35.707	0.874			655	36.836	0.676
July	16.553	31.25	29.900	0.932			834	30.782	0.576
August	10.949	24.438	25.006	1.380			1082	25.739	0.459
September	11.078	25.125	25.342	1.370			1078	26.085	0.455
October	15.906	26.875	29.214	0.988			892	30.554	0.556
November	10.475	25.813	25.187	1.431			1127	25.839	0.418
December	9.742	24.938	24.688	1.488			1183	25.276	0.383
Jan-1999	8.751	23.5	23.921	1.563			1260	24.530	0.344
February	6.940	22.063	22.049	1.777			1452	22.640	0.276
March	11.380	25.125	26.745	1.369			1367	25.224	0.425

The input parameters for the proposed model were kept constant (except for the parity level), while for the standard model credit spread was varied to match the implied spread given by the proposed model. From the results it is evident that the proposed model is capable of explaining the behaviour of market prices much more realistically than the standard model[178]. In other words, the proposed model, after being initially calibrated with the Pride International Inc convertible bond data at the time of issue in April 1998, was able to forecast its out-of-sample prices over the period of next 12 months with much greater precision than the standard model was, even though the latter was recalibrated on a monthly basis.

178 RMS error estimate vs. the market price of the bond for the proposed model is 4.56, while the standard model with variations in credit spread achieves RMS 6.87

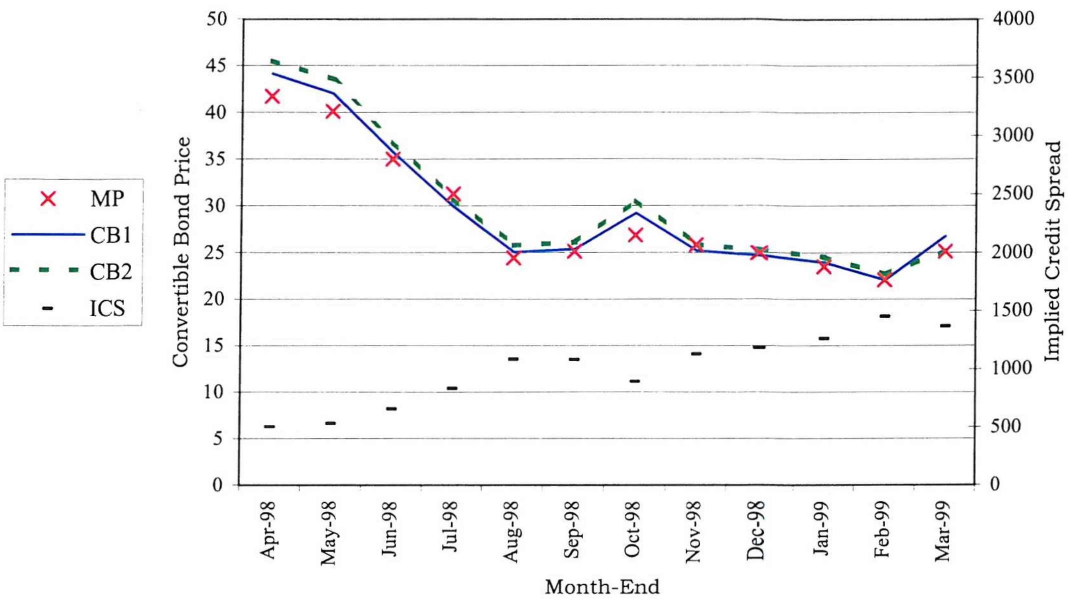

Figure 6-11. Pride International Bond: Comparison of Market Prices
CB1 line plots the proposed model values. CB2 line is the standard model. Parameters for the proposed model as chosen at the onset and then kept constant throughout the period. The standard model uses the same parameters except that the value of credit spread is chosen to match the implied credit spread as returned by the proposed model.

Particularly important is the difference in the hedging parameter delta. For relatively high levels of parity, deltas for both models are converging. However, during the period of low parity levels, with the increase in default risk, the proposed model produces significantly higher deltas. On the other hand, deltas returned by the standard models are completely inapplicable[179] (see for example the December 1998 delta estimate).

6.9 Conclusion

This chapter introduces a new type of structural model for the valuation of risky debt, applicable to both straight and convertible bonds. The model, even in its most simplified form, shows improved predictive power in valuing real-life convertible bonds. The standard convertible bond model, as in McConnell and Schwartz (1986), as well as the structural models

[179] Between the end of November and December of 1998 the bond's actual movements imply a delta of approx 1.20. The standard model suggests hedge of 0.418 while the model proposed here suggested 1.43. Although, the proposed model over-hedges the bond's movement the absolute hedging errors would be significantly smaller for the model proposed here.

of default as in Merton (1974), Brennan and Schwartz (1980), Black and Cox (1974) and Longstaff and Schwartz (1994) can be seen as special cases of our model. Implied volatility parameters obtained from this model are directly compatible with those obtained from the prices of traded options and warrants using the Black and Scholes (1973) formula, as the diffusion process assumed in this model for the stock price have the same log-normal characteristics.

Separation of firm value components to equity (observable, log-normally distributed) and asset cover (imperceptible, normally distributed) processes and allowing for correlation between them, takes this model beyond any structural class model to-date. The extent of non-perfect correlation has the real economic interpretation associated with the uncertainty associated with the estimation of unobservable data (for example firm's financial performance is available in quarterly intervals) and asymmetric information (frequently seen in practice as surprise announcements, restatement of profits, profit warnings, etc).

The proposed model offers important benefits of compatibility and simplicity that coupled with the positive aspects of its innovative structural approach to modelling of default risk make it a unique contribution to both practical and theoretical fields.

The next monthly period is even more revealing. The actual bond movements implied required the hedge to be 1.45, with the standard model suggesting 0.38 and the proposed model 1.49, an almost perfect hedge.

Chapter 7:
So What Does It Mean When a Bond is Called Convertible?

I have put forward this question in the introduction part of this thesis. It appeared as a simple question that on a face of it had a very short and simple answer: a bond that can be converted into stock! However, having taken a deeper look into how this is proposed and achieved in real life, we can find a huge variety of flavours and ways of converting a bond. It is a 'can of worms'.

My aim was to provide a detailed insight into the world of convertible bonds that are currently trading in the Eurobond market, to address selected technical issues relating to features where there had been a scarcity of written research, and hopefully to provide a good starting point for further research in this area.

This work shows the diversity of research subjects that have to be referenced in order to methodically understand convertible bonds. The complexity of convertible bonds currently found in the Eurobond market is exposed in great detail in Chapter 3. Even as I am writing, innovative features are being added to newly issued convertible bonds, and they have found their place in this work. Certain interesting convertibles found in Japanese or American domestic markets are also examined, as it is probably just a matter of time before similarly structured convertible bonds arrive in Europe. The market is also analysed as to the regional composition of the issuers, industrial sectors, preference for certain features, etc. Providing detailed yet concise analysis of a variety of the embedded features, the rationale behind them and the ways to evaluate their contribution to the overall value of a convertible bond gives particular strength to this thesis, as I have systematically failed to find such summarised research published by either an academic or a practitioner's source. The list of features remains complete up to this day and in this respect, it is a valuable contribution in its own right.

The thesis incorporates several other areas of particular importance to enable further understanding and valuation of convertible bonds.

Valuation of convertible structures, using the decomposition approach, is of benefit to both pedagogical and practical modelling purposes. Decomposition helps to understand how convertible securities are structured and what would be the intention of the issuers under different scenarios. The decomposition process is explained and used in this thesis to value a complex convertible bond called a Liquid Yield Option Note (LYON). Through the decomposition of the LYON, the importance of the call notice period really becomes prominent. Sub optimal call decisions are well documented and explained in the literature as being caused by asymmetric information and high underwriting costs. I show that the call notice period put option is a proper tool to incorporate a delayed call decision into a valuation process in a consistent and efficient way. Work presented in Chapter 4 demonstrates how to systematically apply the decomposition approach to a complex convertible bond structure and also contributes towards further understanding of the role played by the call notice and how it can be used to account for the delayed call decision.

Dividends are paid by almost every stock underlying convertible bonds in the Eurobond market. In Chapter 5, I explain in great detail the difficulties of incorporating this ubiquitous feature and point to an inherent error in the standard valuation approach that can be particularly pronounced when embedded conversion options are assessed. To correct this error, I develop a new type of dividend modelling algorithm that values the known and forecasted dividend payments but allows for rational bounds to be imposed. An alternative and more precise adjustment for the Black and Scholes (1973) formula for European call options in the presence of a known dividend is also developed and tested.

However, in my opinion, the most notable contribution of this thesis is the structural model for convertible bonds subject to default risk. This is currently an area where intensive and broad research activity is under way, but there is still lack of fusion between the generic research and its applicability to convertible bonds. The valuation approach presented in Chapter 6 has a number of unique features not present in any other models for convertible bonds prone to default. It is a structural modelling approach but with the firm value decomposed into two parts: observable (stock) and imperceptible (the rest). The stock is assumed to follow a lognormal diffusion process, while the rest of the firm's value follows a normal diffusion. This is an innovative approach that, while achieving complete compatibility of its inputs and results with the Black and Scholes (1973) model for equity options, allows for independent volatility assumptions for each part of the firm value. The correlation between the processes has a real economic meaning as a risk that is associated with information asymmetry and the uncertainty in assessing infrequently published information. The model is tested against a real life example and is shown to have significant explanatory potential. I believe this is an innovative and potentially highly valuable contribution to this subject.

Henceforth, I can offer my recommendations for future research both as an academic and a practitioner. A number of papers have been written recently, which specifically address the issue of default risk in convertible bonds, including the work presented in this thesis. All these models have an improved capacity to explain the market prices of convertible bonds. However, to gain further insight, comprehensive market testing research is required, which should provide a better understanding of the relative strength of various approaches as well as the overall improvement that these new models achieve over the standard valuation approach. It would be particularly interesting to see how the assumption of a default prone stock process, as used in reduced models for convertible bonds subject to default risk, performs when compared with the market observed data.

However, market tests have their particular difficulties. To start, parameterising convertible bonds is much more demanding than either stock options or even straight bonds. In order to make this data manageable a researcher may need to reduce the size of the sample to a much smaller number of convertible bonds than I have considered in Appendix 3-A. Additionally, time series of convertible bond prices, underlying stock prices, risk free yield curves and dividend data would be required. Some sort of automated processing would need to be put in place that would then run the test under various assumptions. Taking into account that the general form of the model of default risk, as presented in Chapter 6, is a two-factor model the processing power of the computing equipment used may also become an issue[180], requiring potentially further research into efficient numerical solvers and database management software.

One of the valuation problems that I have not explicitly addressed in this thesis and that has so far not been explored in conjunction with convertible bonds is the issue of implied volatility surfaces, i.e. implied probability density function of stock returns. This issue is extensively addressed in connection with ordinary call and put options on market indices with notable works of Rubinstein (1994), Dupire (1994) and Derman and Kani (1994b), who all derive the implied probability distribution of the underlying stock market index based on the prices of

[180] These were mainly my concerns in extending the scope of this thesis to include a market test.

traded options. Recently, Yigitbasiouglu (2001) describes a convertible bond valuation approach where the volatility surface is created consistent with information derived simultaneously from prices of stock options and options on foreign exchange rates.

However, it is rarely possible to find a Euro convertible bond that would have an underlying stock with a sufficient range of liquid traded options for this procedure to yield a reliable result. In many cases, especially involving smaller stocks or issuers from less developed markets, the conversion right embedded in a convertible bond is the only source of implied volatility[181]. Canina and Figlewski (1993) voiced further doubt on the usefulness of an option's implied volatility surface. They have found that the volatilities implied by option prices have much more to do with current demand and supply than with the realisation of future volatility. I concur with this finding from my own experience, which leads me to believe that an approach, based on the option's implied volatility, would be rarely either practical or correct especially if the main hedging instrument is the stock itself. However, the techniques developed for fitting the option's implied volatility can be used to create a convertible bond model consistent with the information contained in the historic time series of underlying prices. Such a model would have much greater chance of yielding a more accurate valuation as well as being a metric for determining the bonds true value. Stutzer (1996) proposes an approach that recovers risk-neutral distribution from historic data, and is subsequently extended by Zou and Derman (1999) for European options, and Cakici and Foster (2002) for American style options. Although they primarily address the pricing of simple call and put options, the methodology can be extended to more complex securities including convertible bonds. The efficiency of this approach can be improved further by using high-low or open-high-low-close volatility estimators, as shown in Conway and Weir (2001) and Mackie *et al.* (2001).

Another area where convertible bonds would benefit from further research is the effect of transaction costs and optimal hedging strategies. A number of potential approaches to this problem were suggested and recently reviewed by Whalley and Wilmott (1999). However, hedging convertible bonds[182] involves not only stock, but also interest rates, credit risk and

[181] Many stocks have no traded options or other derivatives written on them. If a convertible bond is issued on such a stock it will be practically impossible to create an implied volatility surface that would become an independent base for valuing that bond. An even greater problem is posed with cross currency convertible bonds, in which case the volatility surface of quanto options would be necessary, see Reiner (1992). In such cases, the convertible bond itself becomes the only quoted derivative and it would not make sense to use the convertible bond to create an implied volatility surface in order to value the same bond.

[182] Assuming that as in most common practical cases, hedging is supposed to remove all but the volatility risk.

sometimes foreign exchange, therefore extending the proposed approaches to convertible bonds offers a challenging yet useful research project.

A sub-problem in convertible valuation is the modelling of call triggers. All written research and most of the commercially implemented valuation tools ignore the effect of n-days moving average or n-days minimum price call trigger conditions. When the call is a few years away, the error this omission introduces is fairly small, but as the callability approaches, or once the bond is in the call territory, the error becomes significant. The behaviour of call triggers is related to the valuation problems for so called Parisian and Delayed Barrier options that have been studied recently by Chesney, Jeanblanc-Picque and Yor (1997) and by Forsyth and Vetzal (1998), but so far no research has applied the mentioned valuation techniques to the problem of call triggers in convertible bonds.

A partially related problem is to incorporate a more flexible conversion assumption, for example, taking into account the issuer's option to deliver stock or cash equivalent (based on n-days average price, the so called *Asian tail*), and/or the issuer's option to deliver stock (with the quantity of stock determined with reference to n-days average price) in lieu of cash on put dates or at maturity. With some evidence of increased volatility of the periods encompassing call and put dates, such future research may even consider a different volatility assumption over the critical periods.

These are some of key areas that I believe require particularly detailed examination. The plethora of features already present, the diverse needs of companies issuing convertible bonds and the creativity of investment banks, ensures that the currently achieved solutions that can be provided will always lag behind and thus provide plenty of opportunity for future research.

So What Does It Mean When a Bond is Called Convertible?

Having progressed through this thesis, I hope that one is left reasonably content with the answer to this question, but if my answers have generated even more questions, then to a degree this had been an intentional outcome.

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