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Non-Supersymmetric Deformations Of The AdS/CFT Correspondence

by

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ABSTRACT

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Non-Supersymmetric Deformations of The AdS/CFT Correspondence

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We study non-supersmmetric deformations of the AdS/CFT Correspondence. We begin with an unbounded scalar mass deformation of the $\mathcal{N} = 4$ theory. We discuss the behaviour of the dual 5 dimensional supergravity field then lift the full solution to 10 dimensions. Brane probing the resulting background reveals a potential consistent with the operator we wished to insert. We then study non-supersymmetric fermion mass and condensate deformations of the AdS/CFT Correspondence. The 5 dimensional supergravity flows are lifted to a complete and remarkably simple 10 dimensional background. A brane probe analysis shows that when all the fermions have an equal mass a positive mass is generated for all six scalar fields leaving pure Yang Mills in the deep infra-red. We call this theory Yang-Mills*. For a condensate deformation the geometry describes an unstable supergravity background around an fuzzy 5-brane. We proceed to investigate various aspects of the Yang-Mills* field theory from the perspective of the supergravity dual. A Wilson loop calculation shows there is a maximum separation of quark-antiquark pairs. This behaviour suggests string breaking, which has not been seen in other dualities. We calculate the glueball mass spectrum in Yang Mills* and $\mathcal{N} = 1^*$ gravity duals. The results agree at the 10% level with previous computations in AdS-Schwarzchild and Klebanov-Strassler

backgrounds respectively. We also calculate the spectrums of bound states of massive fermions.

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Preface

The work that appears in this thesis was carried out in collaboration with Nick Evans, James Babington, Riccardo Apreda, and Michela Petrini. Original work appears in chapters 3, 4, 5 and 6.

The work in chapter 3 has been previously published in

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J. Babington, D.E. Crooks, N. Evans, " A Stable Supergravity Dual Of Non Supersymmetric Glue" ,Phys.Rev. **D67**, 066007, (2003), hep-th/0210068.

The work in chapters 5 and 6 has appeared in

Riccardo Apreda, David E. Crooks, Nick Evans, Michela Petrini. "Confinement, Glueballs And Strings From Deformed ADS," hep-th/0308006.

Chapter 5 also contains some review material, which is clearly indicated as such in the text.

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‘As far as we can discern, the sole purpose of human existence is to kindle a light of meaning in the darkness of mere being.’

- *C. G. Jung*

Chapter 1

Introduction

Quantum Chromodynamics (QCD) describes the strong force that binds quarks together to form mesons and baryons, and binds baryons together to form nuclei. It is a classic example of a non abelian gauge theory. The most important features of QCD are asymptotic freedom and confinement. At high energies, and short distances, the strong force becomes weak, and QCD tends towards a free theory of quarks. At low energies, on the other hand, the QCD coupling becomes increasingly large. Free quarks are not seen as the theory 'confines' quarks together into bound states [4]. A quark-antiquark pair are tied together by a flux tube that acts much like a string. As the quarks are pulled apart, the force between them increases, like the tension in a stretched string. Eventually the string will break, creating another quark-aniquark pair out of the vacuum. Perturbation theory relies on the smallness of the coupling constant, and so breaks down at strong coupling. Consequently, our understanding of strong coupling phenomena, such as confinement, is quite limited and relies greatly on large scale numerical simulations. It would be useful to have some analytic control over the non-perturbative regime of QCD and related non-abelian gauge theories, and a lot of effort from the particle physics community has been directed towards this goal.

Before QCD was born, String theory had its origins as an attempt to understand the phenomenology of the strong force. In the early 1960's many new hadrons where being discovered with very high spins. The mass squared of the lightest hadronic resonance with spin J was found to be roughly $m^2 = J/\alpha'$, where α' is constant known as the Regge slope. It seemed unlikely that all these resonance's could be fundamental particles, particularly as there are no known consistent field theories for high spins. An alternative view was that the different resonant states could be harmonics of a relativistic string. This was the early motivation for studying the quantum theory of strings and there interactions [5, 6, 7, 8, 9]. A string sweeps out a world-sheet as it propagates through space time, and interacting strings produce branching world sheets. String perturbation theory sums over world sheets with different topologies. This approach to the strong interaction had some limited success, but turned out not to be in complete agreement with experiment. Whilst string scattering agreed well with experiment in some kinematic regions, it failed in other high energy regions that where little explored in the early sixties. In the end, string theory was superseded by QCD in the 1970's as the theory of the strong force. This was not the end of string theory. It was realized that closed strings could describe gravity, and that open strings gave rise to a U(N) gauge theory. String theory took on a new role as a potential theory of everything. Not only did it hold the possibility of a consistent quantum theory of gravity, but also the hope of unifying gravity with particle physics in a coherent whole. Of course, if string theory is to describe everything, it must eventually come back to its origins, and describe QCD. Perhaps string theory could help understand confinement. Are flux tubes fundamental strings?

The idea of a relation between non abelian gauge theories and strings was developed by t'Hooft [50]. He showed that if a non-abelian gauge theory is expanded in the number of colours, N, then the feynman diagrams arrange themselves into an expansion labeled by the Euler number. The Euler number describes the topology of surfaces, and this reminds us of the perturbative expansion of string theory in terms of the genus of the string world sheet.

The link between non-abelian gauge theory and string theory was given a concrete realization by Maldacena [1, 59]. He conjectured that a certain supersymmetric gauge theory was equivalent (or dual) to string theory on a Anti de Sitter space. The gauge theory, N=4 super Yang Mills, is very different to QCD. Unlike QCD, it is a conformal theory with no running coupling. It is possible to deform this correspondence and study the gravity duals of confining theories with less or no supersymmetry. The conjecture is best understood in the large N limit of the field theory, where string theory reduces to classical supergravity on AdS_5. The radial coordinate of the on-shell AdS_5 corresponds to the renormalisation group scale in the off-shell field theory [68, 69, 60, 59]. The 5d supergravity geometry can be thought of as an infinite stack of the same 4d field theory at different energy scales. This is referred to as a holographic correspondence, by analogy to a hologram, which is a two dimensional surface encoding a three dimensional image. The correspondence is motivated by observing that the global symmetries on both sides of the duality match up. For example the isometry group of AdS_5 is the same as the conformal group of the 4d gauge theory. The conjecture has passed numerous non-trivial tests, and few people now doubt its validity, at least in the supergravity limit. Scalar fields in AdS can be identified with operators in the field theory [3, 59, 60, 9]. Field theory observables, such as correlation functions and wilson loops, have been calculated from supergravity with great success. This will all be explained in much greater detail in chapter 2.

In this thesis we will study non-supersymmetric deformations of the AdS/CFT correspondence. Our motivation for studying non-supersymmetric deformations is two-fold. Firstly, we are interested in developing non-perturbative tools for exploring the infra red physics of QCD like theories. QCD is non-supersymmetric, so it is necessary to break supersymmetric. Ideally, we would like to have a string dual of QCD. In practice, we attempt to construct gravity duals of theories whose infra red properties are as close as possible to large N QCD. A second motivation is to explore the bounds of the correspondence itself. The AdS/CFT correspondence is a very well tested conjecture, but most of the tests of the correspondence have been carried out in a supersymmetric context. It is an interesting question to ask how robust the correspondence is to deformations that break all supersymmetry or introduce instabilities.

We will begin our investigations into non supersymmetric dualities with an unbounded scalar mass deformation of the $\mathcal{N} = 4$ field theory [57]. This provides a sufficiently simple example too enable us to develop the techniques needed to construct complete supergravity duals of non-supersymmetric deformations. This theory is unstable, but is interesting in its own right from the perspective of our second motivation. We will then go on to a more physically interesting example, which we call Yang Mills* [54, 55]. The majority of this thesis is devoted to constructing the complete supergravity dual to the Yang Mills* field theory and exploring its physics. $\mathcal{N} = 4$ Super Yang Mills is deformed by giving an equal mass to each of the four adjoint fermions. We find a supergravity dual to this field theory and then proceed to investigate various aspects of the physics of this field theory from the point of view of the dual supergravity description [66].

An overview of the thesis is as follows. In the remainder of the introduction we give a quick review of string theory and supergravity.

In chapter 2 we review various aspects of the AdS CFT correspondence. We introduce Anti de Sitter spaces, and $\mathcal{N} = 4$ Super Yang-Mills before explaining the conjectured correspondence between them. We describe how scalars in supergravity are related to operators in the field theory, and how deformations of the correspondence can be studied with 5d gauged supergravity.

In chapter 3 we study an unbounded scalar mass deformation as our first example of a non-supersymmetric deformation. We find the supergravity dual description and this echoes the runaway behavior of the field theory.

Chapter 4 is the heart of the thesis. In this chapter we develop Yang Mills*, a non-supersymmetric deformation which resembles pure Yang Mills in the infra-red. We give an equal mass to each of the four adjoint fermions in the field theory. We study the 5d gravity dual and find exact solutions in the infra red. We use the technology developed in the previous chapter to lift this to a complete 10d type IIB solution. A brane probe reveals that the geometry is stable and that the 6 scalars have mass terms radiatively induced as expected from the field theory.

In chapter 5 we consider Wilson loops in the Yang Mills* theory developed in the previous chapter. We review Wilson loops, in AdS and deformed geometries before examining the rich structure of Wilson loops in Yang Mills*.

In chapter 6 we consider bound states in Yang Mills* and a related supersymmetric deformation called $\mathcal{N} = 1^*$. We look at fluctuations of the dilaton and calculate the glueball mass spectrum in both theories. The results in Yang Mills* are compared to the lattice, with good agreement. We also compute the spectrum of bound states of the adjoint fermions.

We then draw our conclusions in a final chapter, followed by a number of appendices.

1.1 Strings and Branes

The standard model of particle physics consists of a bewildering array of different fundamental particles. These particles are viewed as point-like objects described by quantum field theory. The standard model has been phenomenally successful experimentally, but it leaves some unanswered questions. Why are there so many different kinds of particle? How does gravity fit into this picture? String theory attempts to address this kind of question. The fundamental particles are viewed not as point like objects, but extended string-like objects. Different particles are viewed as different modes of oscillation of the string. In this way, string theorists hope to unify the disparate particles and forces of nature. Any consistent theory of strings must also include gravity. As String theory provides a quantum theory of gravity, it unifies gravity with particle physics [5, 7, 9].

A string is a 1-dimensional extended object that sweeps out a surface, called the world sheet, as it propagates through space-time. We will also be interested in higher-dimensional extended objects, so we will consider the general case. This is illuminating because, by considering extended objects in general, we see that strings are a special case. Before we generalize to extended objects of arbitrary dimension, let us look at the familiar special case of a point particle propagating in space time. The action for a point particle propagating in space time is

$$S = -m \int d\tau \sqrt{\dot{x}^{\mu} \dot{x}_{\mu}} \tag{1.1}$$

where x^{μ} are the coordinates of the particle in the target space-time and τ is the particles proper time. This action measures the length of the worldline traced by the particle as it moves through space-time. The principle of least action is just the statement that classically, the particle goes from A to B by the shortest route. We can generalize this to objects moving in p spatial dimensions. Such an object is called a p-brane. A point particle is a 0-brane, a string is a 1-brane, a surface is a 2-brane[1] and so on. The action for a p-brane moving in D dimensional curved space-time is [5, 7, 9]

$$S = -T \int d^{p+1}\xi \sqrt{-\text{det} G_{ab}} \tag{1.2}$$

[1] 2-branes are also referred to as membranes, which is the origin of the term 'brane'

T is the brane tension, ξ the coordinates on the brane, and G_{ab} is the metric on the brane induced by the target space-time metric. It is given by the pullback of the metric

$$G_{ab} = \frac{\partial x^\mu}{\partial \xi^a}\frac{\partial x^\nu}{\partial \xi^b} G_{\mu\nu} \tag{1.3}$$

$G_{\mu\nu}$ is the metric of the target space. The action (1.2) was first written down by Dirac in the case $p = 2$. In the context of strings, with $p = 1$ it is known as the Nambo-Goto action. It is a straight forward generalization of the action (1.1) and measures the volume swept out by the p-brane in space-time. The principle of least action states that a classical brane will minimize its volume in space-time. We will see later that there are other terms that can be added, but this will suffice for now.

We will find it useful to rewrite (1.2) in terms of an auxiliary world volume metric γ_{ab}, $a, b = 0, ...p$. The action

$$S' = -\frac{T}{2}\int d^{p+1}\xi[\sqrt{-\gamma}\gamma^{ab}G_{ab} - (p-1)\sqrt{-\gamma}] \tag{1.4}$$

is called the Howe-Tucker action, and is classically equivalent to (1.2). To see this, consider the equations of motion. It is not difficult to show that

$$\frac{\delta S'}{\delta \gamma_{ab}} = 0 \Longleftrightarrow \gamma_{ab} = G_{ab} \tag{1.5}$$

Substituting back into S' we find that $S = S'$ on the equations of motion. Now that we have established that the two actions are equivalent at the classical level, we will drop the prime in (1.4).

The action for the brane is invariant under world sheet reparameterisations and target space diffeomorphisms. There is an important additional symmetry in the case of strings. It is clear from the action (1.4) that there is something special about $p = 1$. Consider the transformation

$$\gamma^{ab} \longrightarrow e^{2\omega}\gamma^{ab} \tag{1.6}$$

The action (1.4) transforms under (1.6) as

$$S \longrightarrow -\frac{T}{2}\int d^{p+1}\xi[\sqrt{-\gamma}\gamma^{ab}G_{ab}e^{-(p-1)\omega} - (p-1)\sqrt{-\gamma}e^{-(p+1)\omega}] \tag{1.7}$$

The action is invariant only in the case $p = 1$. This symmetry is called Weyl symmetry and is of vital importance for the following reason. γ^{ab} has $\frac{1}{2}(p+1)(p+2)$ independent components as it is symmetric. Reparametrisations of the worldsheet can account for $p+1$ components of the metric, leaving $\frac{1}{2}p(p+1)$ remaining components. In the special case of strings this one remaining degree of freedom can be absorbed by a Weyl transformation. Only in the case of strings can we use the symmetries of the action to bring the metric into any desired form, such as the flat unit metric. This freedom proves invaluable in quantising the string.

A second reason for considering strings rather that higher dimensional branes is that the the action (1.4) is non-renormalisable for $p \geq 2$. One of the main motivations for studying strings is to obtain a sensible perturbative description of quantum gravity. Trying to make sense of quantising the non-renormalisable action of a higher dimension brane may by just as difficult as quantising gravity itself.

Higher dimensional branes will play a very important role in this thesis, as they emerge naturally from the framework of string theory. In order to develop this framework we will focus, for now, on strings. Strings can be open, with two free ends, or closed loops. Open strings sweep out surfaces with boundaries in space time, whereas closed strings form tubes. We write the action for a string as

$$S = -\frac{T}{2} \int d^2\sigma \sqrt{-\gamma} \gamma^{ab} \partial_a X^\mu \partial_b X^\nu G_{\mu\nu} \tag{1.8}$$

This is the Polyakov action. We can use the gauge invariance (2 reparametrisations and 1 Weyl) to put the world sheet metric into 2d Minkowski form, so that the action becomes

$$S = -\frac{T}{2} \int d^2\sigma \eta^{ab} \partial_a X^\mu \partial_b X_\mu \tag{1.9}$$

The string tension T is related to the Regge slope α' by

$$T = \frac{1}{2\pi\alpha'} \tag{1.10}$$

For the time being we will work in a flat Minkowski target space, $G_{\mu\nu} = \eta_{\mu\nu}$. The variation of the action gives use the wave equation

$$\left(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2}\right) X^\mu = 0 \tag{1.11}$$

The general solution of the wave equation is

$$X^\mu(\sigma, \tau) = X^\mu_L(\sigma_+) + X^\mu_R(\sigma_-) \tag{1.12}$$

where $\sigma_\pm = \sigma \pm \tau$, corresponding to left moving and right moving modes on the string. The variation of the action also gives a surface term that must vanish, which gives the open string boundary condtions,

$$\partial_\sigma X^\mu(\tau,0) = \partial_\sigma X^\mu(\tau,\pi) = 0 \tag{1.13}$$

These boundary conditions lead to standing waves on the open string.

To discuss in detail the quantisation of the string would take us too far afield. However, it is not too difficult to provide a brief sketch, highlighting the points that will come into the present work. The target space coordinates X^μ of the string can be expanded in fourier modes, α^μ_n. To guarantee that X^μ are real the oscillators must satisfy $\alpha^\mu_{-n} = (\alpha^\mu_n)^*$. In quantising the string, the α^μ_n are promoted to operators which obey the harmonic operator algebra,

$$[\alpha^\mu_m, \alpha^\nu_n] = m\delta_{m+n}\eta^{\mu\nu} \tag{1.14}$$

The α^μ_n act as raising and lowering operators. We introduce a ground state, that is annihilated by all the lowering operators,

$$\alpha^\mu_m|0,k^\nu\rangle, \qquad m>0 \tag{1.15}$$

Excited states are obtained by acting on the ground state with α^μ_{-m} for $m>0$. This is just a 2d quantum field theory, but the interpretation is different. We are not thinking of creating and annihilating particles in a 2d space, but of different states of a single string living in a larger target space. Shortly, we will think about interacting strings. The mass of a state is given by

$$M^2 = \frac{1}{\alpha'}(\sum_{n=1}^{\infty}\alpha_{-n}.\alpha_n - 1) \tag{1.16}$$

We see that the ground state has a negative mass squared. The presence of such tachyonic states is perhaps the most unattractive feature of the theory. In the next section we will consider supersymmetric strings, and find that the tachyon can be removed from the spectrum. The first excited state, $\alpha^{\mu}_{-1}|0,k\rangle$, offers a little more hope. It is a massless vector, giving us the $U(1)$ gauge theory that we advertised in the previous section. Let us now take a quick look at closed strings. For closed strings we have right and left moving modes, so we need an extra set of oscillators, $\tilde{\alpha}^{\mu}_{m}$. These oscillators obey identical commutation relations to α^{μ}_{m}. The mass squared for closed string states is given by

$$M^2 = \frac{2}{\alpha'}(\sum_{n=1}^{\infty}(\alpha_{-n}.\alpha_n + \tilde{\alpha}_{-n}.\tilde{\alpha}_n) - 2) \tag{1.17}$$

for closed strings we must excite a left moving mode for every right moving mode. Again the ground state is a tachyon, and the first excited state is massless. In this case the the massless state $\tilde{\alpha}^{\mu}_{-1}\alpha^{\nu}_{-1}|0,0,k\rangle$ is a rank 2 tensor. This can be decomposed into a symmetric, an anti-symmetric and a scalar part that do not mix. We call the anti-symmetric part $B_{\mu\nu}$, and the scalar Φ is called the dilaton. We identify the symmetric, massless rank 2 tensor with the graviton $G_{\mu\nu}$.

We began by considering strings propagating in a flat space, but we have found gravitons in the closed string spectrum. We can think of a coherent state of gravitons as strings propagating in a curved background. When considering strings propagating in background fields, the simplest way to include the dilaton is to couple it to the Ricci Scalar. In 2d the Einstein Hilbert action is just the Euler characteristic χ of the worldsheet, which depends only on the topology. When we consider worldsheets of different topologies, as we do in string perturbation theory, closed string amplitudes will be weighted by $e^{\phi\chi}$. This means that the closed string coupling is given by the expectation value of the dilaton, $g_s = e^{\phi}$, a fact which will play an important role in the next chapter.

We have already indicated that the conformal invariance of the string is crucial. It is important this invariance is retained at the quantum level. In general in a quantum field theory conformal invariance will usually be broken by quantum effects even in a theory that is classically scale invariant. To retain scale invariance at the quantum level the beta function of the world sheet theory should vanish. This translates in string theory to conditions on the target space geometry. Conformal invariance requires that the target space that the bosonic string propagates in must have 26 dimensions. For a string propagating in a curved space-time, the 2d beta function is related to the 26 dimensional curvature tensor. Setting the two loop beta function to zero yields Einstein's equation plus stringy corrections [2]

$$R_{\mu\nu} + \frac{\alpha'}{2} R_{\mu\alpha\beta\gamma} R_{\nu}^{\alpha\beta\gamma} + \mathcal{O}(\alpha'^2) = 0 \tag{1.18}$$

This is one of the most beautiful and surprising results of string theory: Conformal invariance in two dimension implies sensible space time field equations in 26 dimension. Of course, the stringy corrections are insignificant at low energies, but may be crucial in the early universe and in resolving singularities.

String theory can be viewed as a 2d quantum field theory in which the target spacetime appears as internal degrees of freedom. The vacuum of this field theory does not correspond to empty space, but to a single unexcited string. We would like to consider more than one string, and to introduce interactions between strings. We cannot achieve this by simply adding interaction terms to the string action. This would just make a more complicated worldsheet theory of a single string. We need to allow the world sheet to split, and to include loop corrections, we need to consider different topologies of the world-sheet. String perturbation theory involves summing

[2] we have set $B_{\mu\nu}$ and Φ to zero for simplicity

over these topologies. We will see later that this ties in with an expansion in $1/N$ of a non-abelian gauge theory.

Notice that string pertubation theory can be viewed as a third quantised system. A quantum field theory, such as the 2d conformal field theory of the string world sheet, is considered to be a second quantised system. When we include higher genus topologies, we are looking at perturbation theory of a weakly coupled third quantised system. This formalism gives us no information about strongly coupled strings, as we do not know what the underlying theory is that we are doing perturbation theory on.

1.1.1 Superstrings

We have found a theory of strings that contains gauge fields and gravitons in its spectrum, but that also contains tachyons. Target space supersymmetry will remove the tachyon from the spectrum. In our quest to find a space time supersymmetric string theory we will begin by introducing supersymmetry on the world sheet. The spectrum will include states that are not space-time supersymmetric, including tachyons, but we will be able to project out the non-supersymmetric states. The gauge fixed action (1.9) can be generalized to include world sheet spinors [5, 8, 9]

$$S = \frac{T}{2}\int d^2\sigma[\partial^a X^\mu \partial_a X_\mu - i\psi^\mu \rho^a \partial_a \psi_\mu] \tag{1.19}$$

ρ satisfies the 2d dirac algebra

$$\{\rho^a, \rho^b\} = -2\eta^{ab} \tag{1.20}$$

We have suppressed the world sheet spinor indices , which take the values $A = +, -$.

This action is invariant under world sheet supersymmetry transformations

$$\delta X^{\mu} = \epsilon\psi^{\mu}, \qquad \delta\psi^{\mu} = -i\rho^{a}\partial_{a}X^{\mu}\epsilon \tag{1.21}$$

where ϵ is a constant anti-commuting spinor. We have introduced global world sheet supersymmetry because we started with the gauge fixed action. It is possible to recover a locally supersymmetric form of the action by the Noether procedure, but the gauge fixed form will suffice for our purposes.

To ensure the action is stationary on the equations of motion we need boundary conditions for the fermions such that the surface term vanishes. The surface term will vanish if $\psi_+ = \pm\psi_-$ on the endpoints of the open string. We choose

$$\psi_+(0,\tau) = \psi_-(0,\tau) \tag{1.22}$$

by convention. We then have two choices at the other end of an open string. Ramond (R) boundary conditions

$$\psi_+(\pi,\tau) = \psi_-(\pi,\tau) \tag{1.23}$$

lead to space time-fermions. On the other hand Neveu-Schwarze (NS) boundary conditions

$$\psi_+(\pi,\tau) = -\psi_-(\pi,\tau) \tag{1.24}$$

lead to space-time bosons. For closed strings we can pair up right and left moving modes leading to four distinct sectors. The R-R sector and the NS-NS sector contain bosonic states, whereas the R-NS and NS-R sectors contain fermionic states.

In addition to α_n^μ of the previous section we will have additional oscillators, b_r and d_n, corresponding to NS and R sectors respectively. r takes half integer values and n takes integer values. In the quantum theory these are operators that obey anti-commutation relations, and contribute to the mass operator. The mass operator for bosonic open strings is

$$M^2 = \frac{1}{\alpha'}(\sum_{m=1}^{\infty} \alpha_{-m}.\alpha_m + \sum_{r=1/2}^{\infty} r b_{-r} b_r - \frac{1}{2}) \tag{1.25}$$

The ground state is still a tachyon. We have included supersymmetry on the world sheet, but as it stands this does not lead to a consistent theory. The GSO projection removes the unphysical states from the spectrum by restricting to states that are space-time supersymmetric. This leads to a much more natural and satisfying theory. The GSO projection ensures there is an equal number of space time bosons and fermions at each mass level. There is an alternative approach, called the Green-Schwarz formalism, in which space-time supersymmetry is manifest from the start. This leads to the same theory as we have obtained here.

There are still a number of options leading to five distinct superstring theories. If we consider a theory with open strings, then we must included closed strings too. This is because a loop correction to the open string is topologically equivalent to a closed string propagator, so we must allow the ends of open strings to join to form closed strings. The theory with both open and closed strings is called Type I superstring theory and has $\mathcal{N} = 1$ supersymmetry.

Although it is not possible to have open strings without closed strings, it is consistent to have closed strings only. Closed strings have $\mathcal{N} = 2$ supersymmetry. If there are two supersymmetries of opposite chirality, we have type IIA string theory. If they have the same chirality, then we have type IIB.

There are two more possibilities, known as heterotic string theories. In closed string theories left and right moving modes are decoupled. In heterotic string theories the left and right modes are treated very differently. The left moving modes are supersymmetric, but the right moving modes are not. There are two heterotic theories, one with gauge group $SO(32)$ (denoted Ho) and one with gauge group $E_8 \times E_8$ (denoted He). We will not discuss these theories further as they will not enter into this work.

In the same manner as for the bosonic strings, conformal invariance leads to space time field equations, and a condition on the number of space time dimensions. For superstrings, the number of space time dimensions is ten. The field equation can be obtained from an action. The low energy effective action for a superstrings are supergravities. The bosonic part of the action for type IIA supergravity, the low energy effective action of type IIA string theory, is

$$\begin{aligned} S_{IIA} = \; & \tfrac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}e^{-2\Phi}(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H|^2) \\ & -\tfrac{1}{4\kappa_{10}^2} \int d^{10}x\sqrt{-G}(|F_2|^2 + |F_4|^2) \\ & -\tfrac{1}{4\kappa_{10}^2} \int B_2 \wedge F_4 \wedge F_4 \end{aligned} \tag{1.26}$$

where $F_4 = dA_3 - A_1 \wedge F_3$. The Ricci scalar, R, the dilaton Φ and the field H_3 are in the NS-NS sector and the fields F_2 and F_4 are in the R sector. There are, of course, stringy corrections of order α' to the action.

The bosonic part of the action for type IIB supergravity, the low energy effective action of type IIB string theory, is

$$S_{IIB} = \frac{1}{2\kappa_{10}^2} \int d^{10}x\sqrt{-G}e^{-2\Phi}(R + 4\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}|H|^2) \qquad (1.27)$$
$$-\frac{1}{4\kappa_{10}^2} \int d^{10}x\sqrt{-G}(|F_1|^2 + |F_3|^2 + \frac{1}{2}|F_5|^2)$$
$$-\frac{1}{4\kappa_{10}^2} \int C_4 \wedge H_3 \wedge F_3$$

where

$$F_3 = dC_2 - C_0 \wedge H_3, \qquad (1.28)$$
$$F_5 = dC_4 - \frac{1}{2}C_2 \wedge H_3 + \frac{1}{2}B_2 \wedge F_3 \qquad (1.29)$$

There is an additional self duality condition on the field F_5

$$F_5 = \star F_5 \qquad (1.30)$$

where $\star$ represents the hodge dual that takes p forms into $d - p$ forms. The field equations derived from the action are consistent with this condition, but they do not imply it. It must be imposed by hand as an additional constraint.

Notice the dilaton factor $e^{-2\Phi}$ multiplying the Ricci scalar in the string effective actions. This differs from the conventional Einstein-Hilbert action, and leads to slightly different field equations. A metric that is the solution of the string effective action is said to be in the String frame. The Einstein frame metric can be recovered from the string frame metric by scaling with the dilaton,

$$G_{\mu\nu}^{(E)} = e^{-\frac{\Phi}{2}} G_{\mu\nu}^{(S)} \tag{1.31}$$

We will work (mostly) in the Einstein frame.

1.1.2 T-Duality and D-Branes

Before introducing T-duality it will help to review Kaluza Klein reduction of dimensions. As string theory has far more dimensions than we see in the physical world, it is natural to consider how we might remove or hide the extra dimensions. Indeed, extra dimensions were originally introduced by hand to unify different fields. Consider gravity in $D = d + 1$ dimensions, with metric G_{MN}. We compactify a dimension on a circle by the identification [7]

$$x^d \sim x^d + 2\pi R \tag{1.32}$$

The metric decomposes to a d-dimensional metric, $G_{\mu\nu}$, a vector $G_{\mu d}$ and a scalar G_{dd}. Coordinate invariance in D-dimensions induces a gauge invariance for the vector in d-dimensions. We have effortlessly unified electromagnetism and gravity! Of course, this is a bit of a simplification- the extra dimension does not just disappear. Consider a massless scalar field propagating in a space with a compact extra dimension. We can expand the scalar field in the compact dimension as

$$\Phi(x^M) = \sum_n \Phi_n(x^\mu) e^{i\frac{n^2}{R^2}x^d} \tag{1.33}$$

The wave equation $\partial_M \partial^M \Phi = 0$ gives us a set of equations in d-dimensions labeled by n

$$\partial_\mu \partial^\mu \Phi_n = \frac{n^2}{R^2}\Phi_n \tag{1.34}$$

We have an infinite tower of fields in d-dimensions with masses $\frac{n}{R}$. At low energy scales, $E < R^{-1}$, only the massless mode can be excited, and the extra dimension is indeed invisible. But at high energies there is an infinite tower of massive states.

Now consider a similar compactification in string theory. As above, there will be a Kaluza Klein tower of massive states labeled by n. But there is also another distinctly stringy effect. A closed string can wind around the compact direction any number of times,

$$X(\sigma + 2\pi) = X(\sigma) + 2\pi R w \tag{1.35}$$

w is an integer known as the winding number. There will be an extra term contributing to the mass of a string state. The mass formula for an unexcited bosonic string becomes,

$$M^2 = \frac{n^2}{R^2} + \frac{w^2 R^2}{\alpha'^2} - \frac{4}{\alpha'} \tag{1.36}$$

This formula has an obvious symmetry

$$n \leftrightarrow w, \qquad R \leftrightarrow \frac{\alpha'}{R} \tag{1.37}$$

This symmetry is called T-duality, and it is not just a symmetry of the mass formula, but a symmetry of the whole closed string theory. It is not possible to completely remove a dimension from string theory - if we try to reduce the radius to zero, a new dimension opens up in the dual theory to take its place.

The situation is a little different for open strings. As they have free ends it is impossible for them to get caught on a compact dimension. They can always unwind. The ends of the open string behave just like particles when we try to reduce a dimension to zero. For the ends of the open string the dimension just disappears and they are confined to move in a lower dimensional space. Except for the end points, an open string is locally indistinguishable from a closed string. The middle of the string can locally wind around the extra dimension, so that it sees a dual dimension. Whilst the ends of the string are confined to a lower dimensional space, the middle is free to oscillate in the bulk space.

The lower dimensional space that the open strings end on is called a D-Brane. Of course, the D-branes are not rigid objects. They inherit dynamics from the target space in which they move. By compactifying several dimensions we can construct D-branes of different dimensions. The action for the D-brane will be a generalization of the action (1.2) for a p-brane. As open strings end on the D-branes we expect a U(1) field theory to live on the surface of the brane. So we include an anti-symmetric U(1) field strength F_{ab}. For the same reason we include a factor of the dilaton associated with the string coupling. The brane can also couple to the anti-symmetric tensor $B_{\mu\nu}$. A general $p+1$ form,

$$A_{p+1} = \frac{1}{(p+1)!} C_{\mu_1 \ldots \mu_{p+1}} dx^1 \wedge \ldots \wedge dx^{p=1} \tag{1.38}$$

couples naturally to a $p+1$ dimensional object because we can form an action of the form

$$S_C = T \int_{p+1} C_{p+1} \tag{1.39}$$

This action is diffeomorphism invariant and invariant under the gauge transformation,

$$C_{p+1} \rightarrow C_{p+1} + d\rho_p \tag{1.40}$$

We include a term of the form (1.39) coupling the Dp-brane to a RR $p+1$ form potential. Putting all this together, the action for a Dp-Brane is, in the string frame [10],

$$S_p = -\tau_p \int dx^{p+1} e^{-\Phi} \det[G_{ab} + 2\pi\alpha' F_{ab} + B_{ab}]]^{1/2} + \mu_p \int C_{p+1}, \tag{1.41}$$

Type IIB string theory has only even p form potentials, and so it only has odd p-branes. On the other hand, type IIA has only even p-branes. T-Duality interchanges odd and even forms and branes, and hence relates IIA strings on radius R to IIB strings on radius $1/R$. IIA string theory is said to be T-Dual to type IIB. The two heterotic string theories are also related by T-duality.

We have so far considered strings ending on a single brane with a U(1) gauge theory. We can consider, however, a theory with N different charges attached to the ends of the open strings. These are called Chan Paton factors. There is a global U(N) symmetry on the world sheet that rotates the N charges. In the target space-time this is a gauge symmetry because we can make different rotations at different points. The U(1) gauge group of the open string is promoted to U(N). The different charges correspond to strings ending on different branes. Hence, we have a stack of N branes that are nearly coincident on the string scale.

This gives us a nice geometric description of the the Higgs mechanism. The displacement of the branes in the transverse space appears as scalar fields in the field theory living on the branes. If we separate one of the branes from the stack, we give a vev to the relevant scalar, and break the gauge group $U(N) \rightarrow U(N-1) \times U(1)$ (fig. 1.1). Strings stretching from the brane to the stack acquire a mass corresponding to

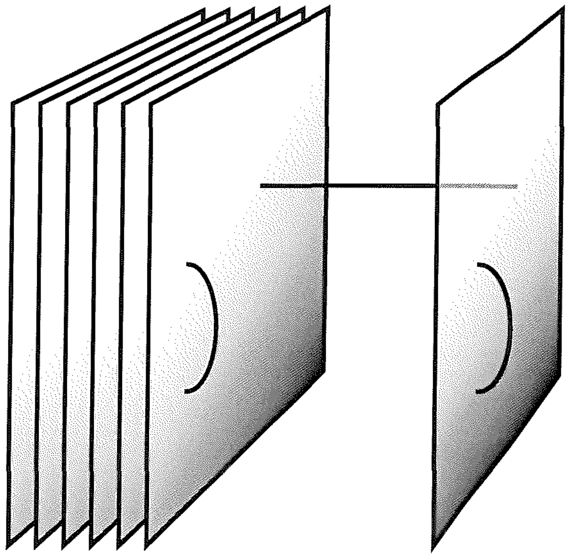

Figure 1.1: A D-brane is removed from the stack, breaking the symmetry gauge group $U(N) \rightarrow U(N-1) \times U(1)$.

W-bosons.

1.1.3 S-Duality and M Theory

Type IIB string theory is also conjectured to be self dual under the exchange of strong and weak coupling [8]. This is called S duality. Type IIB string theory contains a D-String (or D1-brane) and S duality exchanges D strings for fundamental strings. This implies that D-branes are every bit as fundamental as the fundamental strings. To understand this better we need to understand the underlying non-perturabitve theory.

It is also conjectured that the heterotic string Ho is S dual to type I strings, and that type I strings are S dual to type IIA strings. In this way, all five string theories are related by a web of S and T dualities. String theory appears to be unique. Its five different manifestations are just different perturbative expansions of an underlying

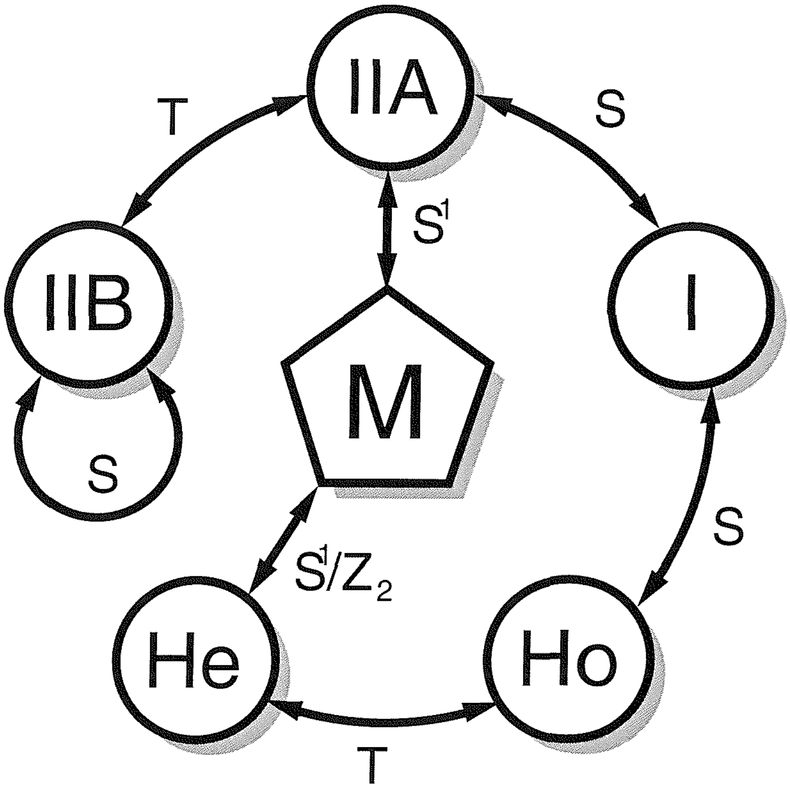

Figure 1.2: M theory, the five string theories and the dualities between them.

theory. Although not a lot is known about this theory, it already has a name: M Theory. The web of dualities is shown in fig. 1.2, with M theory in the centre. We believe M theory to be an eleven dimensional theory, with 11d supergravity as its low energy effective action. The effective action of type IIA supergravity can be obtain from 11d supergravity by dimensional reduction. IIA string theory is M theory compactified on a circle. M-theory compactified on an orbifold[3] is believed to give the heterotic string He.

We have come to the end of our short review of string theory. We have tried to provide a brief but coherent account of the relevant concepts. In a subject as complex as string theory, the result is of necessity a compromise. In the next chapter we will draw on

[3]An orbifold is a circle S^1 with points identified by reflection in a line through the center.

many of the concepts discussed here to develop the Maldecena Conjecture relating $\mathcal{N} = 4$ Super Yang-Mills to Anti-de Sitter Space.

Chapter 2

The AdS/CFT Correspondence

The idea that QCD maybe a string theory in disguise has a long history. Indeed, string theory's first incarnation was as an attempt (albeit a failed attempt) to understand the strong force. These early attempts were superseded by QCD, but this did not put an end to a stringy interpretation. The phenomena of confinement and asymptotic freedom are suggestive of strings. The flux tubes between quarks are very string like: as a pair of quarks are pulled apart the 'string' is stretched and the its tension leads to a stronger force pulling the quarks back together, until the string breaks, pulling another quark-antiquark pair out of the vacuum. Some meat was put on the bones of these ideas by t'Hooft, who showed that a large N expansion in QCD led to a genus expansion, which could be interpreted as the world sheet of a string. The AdS/CFT correspondence is the first concrete realization of these ideas. It is a duality between string theory on Anti de Sitter space, and $\mathcal{N} = 4$ Super-conformal Yang-Mills. This is not a confining theory, but we will see that it can be deformed to a confining theory. In the next sections we will describe Anti de Sitter space and $\mathcal{N} = 4$ Super Yang-Mills. We will then be in a position to introduce the correspondence in some detail. We will begin, however, with t'Hoofts $1/N$ expansion.

2.1 t'Hooft expansion

Following t'Hooft we would like to look at expanding a U(N) gauge theory in 1/N for large N [50]. We will see that this gives a tantalizing hint of an underlying stringy description of Non Abelian Gauge theory. We will consider a general non-abelian gauge theory with gauge group U(N). We can write the action as

$$S = \int dx^d [\mathrm{Tr}\,(\partial A_m \partial A_m) + g f_{mnp} \mathrm{Tr}\,(A_m A_n A_p) + g^2 h_{mnpr} \mathrm{Tr}\,(A_m A_n A_p A_r)] \quad (2.1)$$

Where A_m are gauge fields, and f_{mnp} and h_{mnpr} are arbitary couplings that do not depend on g or N. We introduce the t'Hooft coupling,

$$\lambda = g^2 N \quad (2.2)$$

The t'Hooft limit is $N \to \infty$, $g^2 \to 0$, whilst keeping the t'Hooft coupling fixed. It is convenient to rescale the gauge potential $A_m \to \frac{A_m}{g}$. The action becomes

$$S = \frac{N}{\lambda} \int dx^d [\mathrm{Tr}\,(\partial A_m \partial A_m) + f_{mnp} \mathrm{Tr}\,(A_m A_n A_p) + h_{mnpr} \mathrm{Tr}\,(A_m A_n A_p A_r)] \quad (2.3)$$

Now consider a Feynman diagram for a vacuum amplitude. From (2.3) we can see that for each vertex we will pick up a factor of $\frac{N}{\lambda}$ and each propagator will gain a factor of $\frac{\lambda}{N}$. For each loop of group indices we will pick up a factor of N since each index has N possible values. So each Feynman diagram will be waited by a factor of

$$N^{V-E+F} = N^{\chi} \tag{2.4}$$

where V is the number of vertices, E the number of propagators and F the number of loops [1]. χ is the famous Euler characteristic . It is a topological invariant of a surface [2], related to the genus, h, by $\chi = 2 - 2h$. The genus is the number of handles on the surface. There will also be a factor ofλ^{E-V}, but λ is held fixed whilst N is large. This reminds us of string perturbation theory, where we expand in the topologies of surfaces. It is an indication that non abelian gauge theory admits a string theory description. For large N only diagrams with the topology of a plane or sphere (with boundaries) will survive. Later, we will find a concrete realization of this idea in Maldacena's conjecture.

2.2 Anti de-Sitter Space

Maldacena's conjecture is a duality between a supersymmetric field theory at large t'Hooft coupling and supergravity on Anti de Sitter space. Before we can talk in detail about the conjecture, we need to introduce the theories on either side of the duality. We will begin by describing Anti de Sitter space (AdS) [3, 61, 59].

AdS_{p+2} is the hyperboloid (fig. 2.1)

$$x_0^2 + x_{p+2}^2 - \sum_{i=1}^{p+1} x_i^2 = R^2 \tag{2.5}$$

[1]Historically, Euler studied the topology of polyhedra and discovered the topological invariant χ. Hence V, E, and F stand for Vertices , Edges and Faces, respectively.

[2]The topology of a surface is completely determined by specifying the Euler characteristic, the number of boundaries, and whether or not the surface is orientable. The embedding of the surface into a target space, however may be highly non-trivial. For example, boundaries may be knotted or linked.

embedded in a flat $p+3$ dimensional space with metric

$$ds^2 = -dx_0^2 - dx_{p+2}^2 + \sum_{i=1}^{p+1} dx_i^2 \tag{2.6}$$

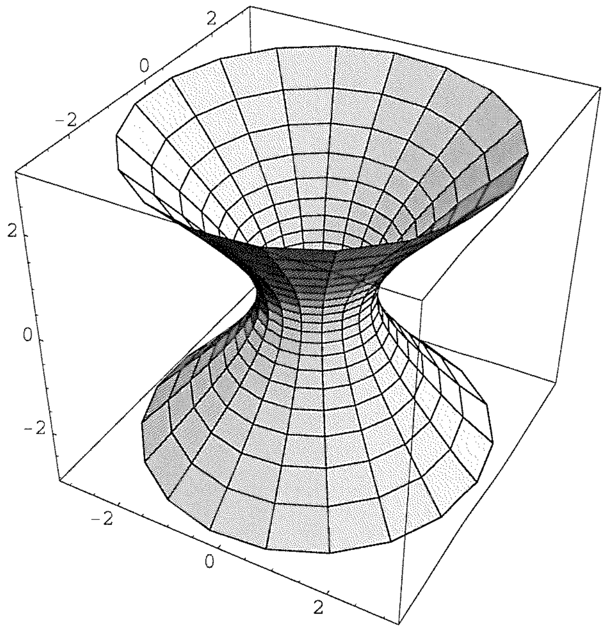

Figure 2.1: A Hyperbola

This space has an $SO(p+1,2)$ symmetry by construction. AdS space has a compactification of Minkowski space at its boundary. For large x, $x \to \infty$, R is negligible and we can write

$$x_0^2 + x_{p+2}^2 = \sum_{i=1}^{p+1} x_i^2 \tag{2.7}$$

It is easy to that this has topology $[S^1 \times S^p]/\mathbf{Z}^2$. (The $\mathbf{Z}^2$ identification comes from invariance under $x \to -x$). To see the relationship to Minkowski space we change coordinates to

$$x_{p+2} = \frac{1}{2}[a + b] \qquad (2.8)$$
$$x_{p+1} = \frac{1}{2}[a - b] \qquad (2.9)$$

So that

$$ab = \sum \eta_{ij} x_i x_j \qquad (2.10)$$

In the absence of R we have scale invariance. If $b \neq 0$ we can use this to set b=1 leaving just Minkowski space coordinates. This differs from Minkowski space because it contains additional points when $b = 0$ which compactifies the space.

We can satisfy the constraint (2.5) by changing variables to

$$x_0 = R \cosh \rho \cos \tau \qquad (2.11)$$
$$x_{p+2} = R \cosh \rho \sin \tau \qquad (2.12)$$
$$x_i = R \sinh \rho \Omega_i \qquad (2.13)$$

where

$$\sum_i \Omega^2 = 1 \qquad (2.14)$$

The metric for AdS space, in these coordinates, is

$$ds^2 = R^2(-\cosh^2\rho d\tau^2 + d\rho^2 + \sinh^2\rho d\Omega^2) \tag{2.15}$$

This covers the hyperboloid once if $\rho \geq 0$ and $0 \leq \tau < 2\pi$. There are closed time-like curves in the τ direction, so the topology is $S^1 \times R^{p+1}$. To obtain a causal space time we can unwrap the circle by allowing τ in the range $-\infty < \tau < +\infty$ without identifications.

Putting $\tan\theta = \sinh\rho$ for $0 \leq \theta \leq \frac{\pi}{2}$ we have

$$ds^2 = \frac{R^2}{\cos\theta}[-d\tau^2 + d\theta^2 + \sin^2\theta d\Omega^2] \tag{2.16}$$

which is conformal to the Einstein static universe. A conformal rescaling does not effect the causal structure. As θ takes values in the range $0 \leq \theta \leq \frac{\pi}{2}$ rather than $0 \leq \theta \leq \pi$ AdS can be mapped to one half of the Einstein static universe.

Another coordinate system on AdS that is often used is defined by

$$x^0 = \frac{1}{2u}[1 + u^2(R^2 + \sum_i X^2 - t^2)] \tag{2.17}$$

$$x^i = Rux_i \tag{2.18}$$

$$x^{p+1} = \frac{1}{2u}[1 - u^2(R^2 - \sum_i X^2 + t^2)] \tag{2.19}$$

$$x^{p+2} = Rut \tag{2.20}$$

In these coordinates the metric takes the form

$$ds^2 = R^2[\frac{du^2}{u^2} + u^2\eta_{\mu\nu}dx^\mu dx^\nu] \tag{2.21}$$

where $\mu, \nu = 0, ..., p$. These coordinates cover one half of the hyperboloid.

We will make one final change of coordinates. Putting $u = e^{\frac{2r}{R}}$ we have

$$ds^2 = dr^2 + e^{\frac{2r}{R}} \eta_{\mu\nu} dx^\mu dx^\nu \tag{2.22}$$

This is the form of the metric on AdS that we will usually use. For notational simplicity we will often set the AdS radius to unity. As we will be interested in deformations of the correspondence, we will be interested in metrics that return asymptotically to the form (2.22) when $r \to \infty$. Note that, despite the notation, r is not a true radial coordinate as it takes values in the range $-\infty < r < \infty$.

2.3 $\mathcal{N} = 4$ Super Yang-Mills

We now turn to the other side of the duality. $\mathcal{N} = 4$ Super Yang-Mills is a conformal field theory in 4d with conformal group $SO(4, 2)$ [59, 60, 67]. This symmetry survives at the quantum level: The Beta function vanishes to all orders in perturbation theory so there is no running of the coupling. The field content of $\mathcal{N} = 4$ Super Yang-Mills is one gauge boson, 4 fermions and 6 scalars all in the adjoint of the gauge group. The theory also has a global $SU(4)$ R-symmetry. The four fermions are in the fundamental representation of the $SU(4)$ R-symmetry and the 6 scalars transform under $S0(6) \sim SU(4)$. The lagrangian for the $\mathcal{N} = 4$ theory can be obtained by dimensional reduction of $\mathcal{N} = 1$ Super Yang Mills from 10 dimensions down to 4 dimensions. This is the low energy theory of open strings in 10d, which we can think of as a 9-brane in type IIB string theory. Thus when we T-dualise to a 3-brane we obtain the $\mathcal{N} = 4$ theory. The lagrangian for $\mathcal{N} = 1$ super Yang Mills is

$$\mathcal{L} = \frac{1}{g_{YM}^2}\mathrm{Tr}\,[F_{MN}F^{MN}] - \frac{i}{g_{YM}^2}\mathrm{Tr}\,[\overline{\lambda}\Gamma^M D_M \lambda] \tag{2.23}$$

λ is a Majarona-Weyl 16 spinor of $SO(1,9)$. Under the dimensional reduction the 10d lorentz group decomposes as

$$SO(1,9) \longrightarrow SO(1,3) \times SO(6) \tag{2.24}$$

The spinor decomposes as

$$\mathbf{16} \longrightarrow (\mathbf{2},\mathbf{4}) \oplus (\overline{\mathbf{2}},\overline{\mathbf{4}}) \tag{2.25}$$

The 10d vector A_M breaks down to a 4d gauge vector A_μ and 6 scalars X_i. Hence the lagrangian for $\mathcal{N}=4$ Super Yang-Mills is

$$\mathcal{L} = \frac{1}{g_{YM}^2}\mathrm{Tr}\,[F_{\mu\nu}F^{\mu\nu} + D_\mu X_i D^\mu X_i - [X_i, X_j]^2] - \frac{i}{g_{YM}^2}\mathrm{Tr}\,[\overline{\lambda}\Gamma^\mu D_\mu \lambda + i\overline{\lambda}\Gamma_i[X_i,\lambda]] \tag{2.26}$$

There is a scalar potential of the form

$$V = \frac{1}{g_{YM}^2}\mathrm{Tr}\,[X_i, X_j]^2 \tag{2.27}$$

As each term is positive, when the potential is zero we have a minimum corresponding to the $\mathcal{N}=4$ supersymmetric ground state. The moduli space is given by $[X_i, X_j] = 0$. There are two classes of solution. $\langle X^i \rangle = 0$ for all $i = 1$ to 6 is called the super conformal phase as superconformal symmetry is unbroken. $\langle X^i \rangle \neq 0$ for at least one i is called the coulomb branch, since an adjoint vev generically breaks $SU(N) \to U(1)^{N-1}$

2.4 The Maldacena Conjecture

Now that we have the basic ingredients we are ready to introduce the Maldacena conjecture connecting IIB supergravity on $AdS_5 \times S^5$ with $\mathcal{N} = 4$ super Yang Mills. We present a heuristic argument to motivate the correspondence [59, 1]. In the next section we will make a more precise statement of the correspondence and gather some evidence that it is correct.

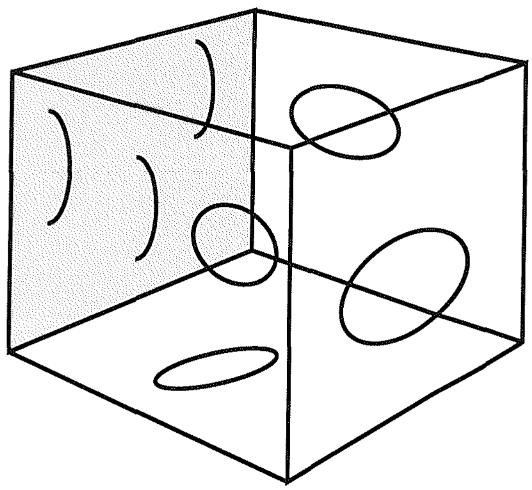

Figure 2.2: Open strings live on the branes (grey) and closed strings propagate in the bulk.

Consider a stack of N nearly coincident D3-Branes in type IIB string theory. Open strings live on the branes and closed strings propagate in the bulk space (fig. 2.2). We will begin by looking at this configuration from the perspective of the D3-Branes. The low energy effective action for the branes is $\mathcal{N} = 4$ Super Yang Mills plus higher derivative corrections. Taking the low energy limit, $\alpha' \to 0$ with N and g_s held fixed the derivative corrections and the interaction between the branes and the bulk is suppressed. We have $\mathcal{N} = 4$ Super Yang Mills living on the branes, and this is decoupled from the supergravity background.

We will now look at the supergravity description from the perspective of an observer at infinity. D-branes are massive charged objects that act as sources for supergravity fields. The stack of branes warps the space-time around them. The D3-branes couple to the four form potential, so that that the self dual five form is switched on. The solution of IIB supergravity for the geometry around the branes is given by

$$ds^2 = Z^{-\frac{1}{2}}\eta^{ij}dx_i^2 dx_j^2 + Z^{\frac{1}{2}}dy^2 + Z^{\frac{1}{2}}y^2 d\Omega_5^2 \tag{2.28}$$

where $Z = 1 + \frac{R^4}{y^4}$. Consistency of the field equations requires a non zero self dual five form

$$F_5 = dC_4 + \star dC_4, \qquad C_4 = Z^{-1}dx_0 dx_1 dx_2 dx_3 \tag{2.29}$$

We also have

$$R^4 = 4\pi g_s \alpha'^2 N \tag{2.30}$$

For $y \gg R$ we see there is an asymptotically flat region. At the other extreme, $y \ll R$, the geometry becomes

$$ds^2 = \frac{y^2}{R^2}\eta^{ij}dx_i^2 dx_j^2 + \frac{R^2}{y^2}dy^2 + R^2 d\Omega_5^2 \tag{2.31}$$

This is the metric of the product space $AdS_5 \times S^5$. The time component of the metric,

G_{00}, is not constant so there will be a red shift. An observer at infinity will measure the energy of an object at y as

$$E_{\infty} = Z^{-\frac{1}{4}} E \tag{2.32}$$

As an object is brought closer to $y = 0$ it will appear to the observer at infinity to have lower and lower energy. Even in the low energy limit, we can have any kind of excitation if it is sufficently close to $y = 0$. In this limit, the massless fields propagating in the bulk decouples from the near horizon region, as viewed by an observer at infinity.

We have looked at our configuration of branes from two different points of view, and found two disconnected pictures. The low energy effective action of the brane gave us $\mathcal{N} = 4$ Super Yang Mills, whereas the geometry seen by an observer at infinity gave us string excitations on $AdS_5 \times S^5$. We now note that both of these theories share the same global symmetries [3]. The $SU(4)$ R-symmetry of $\mathcal{N} = 4$ Super Yang Mills maps to the $SO(6)$ isometry group of the five sphere. The $\mathcal{N} = 4$ theory is a conformal theory in 4 dimensions. Hence it has the conformal symmetry group $SO(2,4)$. This is the same group as the isometry group of AdS_5. Both theories contain the same amount of supersymmetry. D-branes preserve half the supersymmetries so the 10d theory has $2^5 = 32$ supercharges. Conformal invariance doubles the number of supercharges, so $\mathcal{N} = 4$ theory has $4 \times 2 \times 4 = 32$ supersymmetries. This leads us to the tentative conclusion that these two theories may be dual, in the sense that they both describe the same physics in different ways. This is the Maldacena conjecture: $\mathcal{N} = 4$ Super Yang Mills is dual to strings on $AdS_5 \times S^5$. The couplings are related by

$$g_{YM}^2 = g_s \tag{2.33}$$

[3]We do not need to match the gauge symmetry as all observables are gauge invariant

In its strong form, the conjecture holds for all values of N and $g_{YM}^2 = g_s$. However, the full quantum string theory on the product space $AdS_5 \times S^5$ is intractable, so it is useful to consider limits that are easier to study. The t'Hooft limit involves keeping $\lambda = g_{YM}^2 N$ fixed while letting $N \to \infty$. The string coupling may be re-expressed in terms of the t'Hooft coupling as $g_s = \lambda/N$, so the t'Hooft limit corresponds to classical string theory. After taking the t'Hooft limit, the remaining parameter is λ. From 2.30, we can see that $\lambda^{-1/2}$ is proportional to α', so taking λ to be large corresponds to classical supergravity on $AdS_5 \times S^5$. This is the limit of the conjecture we will work in. In the next section we will make this correspondence a little more precise, and gather some more evidence.

2.4.1 The Operator-Field Correspondence

So far, we have suggested that there is some sort of duality between $\mathcal{N} = 4$ super Yang Mills and anti de sitter space, but we have not been very clear about precisely how this correspondence will work. We have matched the global symmetries on both sides of the correspondence, but we would like to go further. If this is a true duality, we should be able to calculate field theory observables from supergravity. We get a clue as to how to proceed from the dilaton. In $\mathcal{N} = 4$ Super Yang-Mills we can make a marginal operator deformation that changes the value of the coupling constant. The Yang-Mills coupling is related by eq.2.33 to the string coupling. But the string coupling is given by the boundary value of the dilaton at infinity. Changing the boundary conditions of the dilaton on the gravity side of the duality is related to an operator deformation on the Yang-Mills side of the duality. This suggests we might be able to identify operator deformations of the field theory with scalar fields propagating in AdS [3]. We will see below that we can indeed make this identification, and that the mass of the scalar field is related to the dimension of the operator.

AdS_5 has one more extended spatial direction than its field theory dual. The correspondence must be holographic, in the sense that the 4d field theory encodes all the information of 5d theory of gravity (for the time being we neglect the S^5). We would like to know what role the extra radial coordinate ofAdS_5 has in the field theory. Consider the kinetic term in the $\mathcal{N} = 4$ SYM action for a scalar field

$$\int d^4x \eta^{\mu\nu} \partial_\mu \phi \partial_\nu \phi \tag{2.34}$$

A conformal rescaling of the metric leaves the action invariant if

$$x \longrightarrow e^\alpha x, \qquad \phi \longrightarrow e^{-\alpha} \phi \tag{2.35}$$

The conformal group in $\mathcal{N} = 4$ SYM maps to the isometry group of AdS_5 and so the 5d metric must be invariant. Hence

$$r \longrightarrow r - \alpha \tag{2.36}$$

We see that e^r transforms in the same way as ϕ under a conformal transformation and has conformal dimension 1. A different radius in AdS_5 corresponds to a different scale in the field theory. We interpret r as a renormalisation group scale. We can think of AdS as an infinite stack of the same field theory living at different RG scales.

To pin down the operator field correspondence, we would like to take a closer look at scalar fields in AdS. Consider a free scalar field propagating in AdS with field equation

$$\Box_{AdS} \lambda = m^2 \lambda \tag{2.37}$$

At large radius r this reduces in AdS_5 to

$$\lambda'' + 4\lambda' = m^2\lambda \tag{2.38}$$

With solution

$$ae^{r(\Delta-4)} + be^{-\Delta r} \tag{2.39}$$

where

$$m^2 = \Delta(\Delta - 4) \tag{2.40}$$

We advertised above a relation between the mass of scalar fields and the dimension of operators, and this relation is given by (2.40). We conjecture that a scalar field in AdS with mass m is dual to an operator with dimension Δ. We would now like to see how the boundary conditions of the scalar field, determined by a and b, are related to the operator in the field theory. As we know e^r has conformal dimension 1 we can read off the dimensions of a and b. The scalar field is invariant under the radial isometry of AdS, so dim $a = 4 - \Delta$ and dim $b = \Delta$. Consider a field theory operator with dimension Δ. The lagrangian has dimension 4 so a source for the operator will have dimension $4 - \Delta$. This leads us to propose that a is a source for the operator. Hence, a supergravity scalar which at large r obeys (2.39) is dual to the field theory given by

$$\mathcal{L}_{CFT} + a\mathcal{O} \tag{2.41}$$

where $\mathcal{O}$ is a local operator with dimension Δ

b has the same dimension as the operator. The operator $\mathcal{O}$ can be given a vacuum expectation value (VEV) by allowing a non zero b

$$\langle 0|\mathcal{O}|0\rangle = b \tag{2.42}$$

Given our conjectured relation between operators and fields, a sensible way to imagine the correspondence is to relate the path integral of the field theory with an operator insertion to the supergravity action evaluated for an appropriate solution of the dual scalar field. The correspondence can be stated as [3, 59]

$$\langle \exp \int a\mathcal{O}\rangle = \exp(-S[\lambda]) \tag{2.43}$$

Here the left hand side is the generating functional for correlation functions in the field theory and the right hand side is the supergravity action evaluated on a solution of the scalars equation of motion with boundary condition (2.39). We can calculate correlation functions by taking functional derivatives with respect to the supergravity scalar. In the next section we will gather evidence that this prescription for relating operators to fields is correct.

2.5 5d Gauged Supergravity

We have compactified our 10d space on S^5, so much can be learnt about the correspondence at the 5d level. 5d $\mathcal{N} = 8$ gauged supergravity with gauge group SO(6) is the low energy effective action of type IIB supergravity on $AdS_5 \times S^5$. It is believed to be a consistent truncation in the sense that every 5d solution can be uniquely lifted to a 10d type IIB solution. The field content of 5d $\mathcal{N} = 8$ supergravity is a graviton, 8 spin $\frac{3}{2}$ gravitinos, 27 vectors, 48 spin $\frac{1}{2}$ fermions and 42 scalars. The 42 scalar fields in 5d gauged SUGRA transform in the $\mathbf{1}$,$\mathbf{10}$,$\overline{\mathbf{10}}$ and $\mathbf{20}$ of $SO(6)$ [16, 17, 18].

Restricting to the metric and the scalar fields the lagrangian is, schematically,

$$\mathcal{L} = -\frac{1}{4}R + \frac{1}{2}(\partial\lambda)^2 - V(\lambda) \tag{2.44}$$

The S0(6) gauge group comes from the isometry of S^5 and corresponds to the $SU(4)$ R-symmetry of $\mathcal{N} = 4$ Super Yang Mills. For the operator-field correspondence conjectured in the previous section to work, we must be able to identify scalars from a particular multiplet with field theory operators with the same transformation properties. The scalars must have the correct mass corresponding to the dimension of the operator according to (2.40). Scalars transforming in the **20** have $m^2 = -4$. From (2.40) we require an operator with dimension 2 transforming in the **20**. There is such an operator,

$$\mathrm{Tr}\,\phi_i\phi_j - \frac{1}{6}\delta_{ij}\mathrm{Tr}\,\phi_i\phi_j \tag{2.45}$$

Scalars transforming in the **10** have $m^2 = -3$. This must correspond to a dimension 3 operator in the field theory transforming in the **10**. Again, we find an operator in the field theory that satisfies these requirements,

$$\mathrm{Tr}\,\lambda_a\lambda_b \tag{2.46}$$

The $\overline{\mathbf{10}}$ scalars correspondence to the same operator only with λ replaced by $\overline{\lambda}$. The singlets are the massless dilaton and axion. They correspond to the dimension 4 operators

$$\mathrm{Tr}\,F^2, \qquad \mathrm{Tr}\,F\wedge F \tag{2.47}$$

Hence the dilaton corresponds to the gauge coupling, and the axion to the θ-angle. We find, remarkably, that the identification works and the relation (2.40) is satisfied for all the scalar fields.

2.5.1 Deformations in 5d Gauged Supergravity

$\mathcal{N} = 4$ Super Yang-Mills is a conformal theory, so does not have a characteristic mass scale like QCD. We are interested in breaking this conformal symmetry in the AdS/CFT correspondence, so that we can study mass gaps and confinement in a dual theory. As the conformal group of the field theory is dual to the isometry of AdS, breaking conformal symmetry means deforming AdS. If we introduce a scale with a mass, for example, we expect that at very high energy scales the mass will be negligible, so that at very large r the deformed geometry should return asymptotically to AdS.

The AdS/CFT correspondence [1, 3] maps Supergravity fields to operators in the field theory. Switching on 5d Supergravity fields deforms the geometry of AdS. We allow the scalar fields to vary only in the radial direction, and we look for solutions that return asymptotically to AdS:

$$ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \tag{2.48}$$

where $\mu = 0..3$, r is the radial direction in AdS_5, and $A(r) \to r$ as $r \to \infty$.

There are two independent, non-zero, elements of the Einstein tensor (G_{00} and G_{rr}) and the Ricci scalar is

$$R = 20A'^2 + 8A'' \tag{2.49}$$

We find the following equations of motion [64, 68, 11]

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda} \tag{2.50}$$

$$6A'^2 = \lambda'^2 - 2V \tag{2.51}$$

$$-3A'' - 6A'^2 = \lambda'^2 + 2V \tag{2.52}$$

In fact only two of these equations are independent. Subtracting (2.52) from (2.51) we obtain the following useful relation between the potential and derivatives of A

$$V = -3A'^2 - \frac{3}{4}A'' \tag{2.53}$$

In the large r limit, where the solution will return to AdS_5 at first order and $\lambda \to 0$ and $V \to \frac{m^2}{2}\lambda^2$, only the first equation survives. In this limit eq. 2.50 become eq. 2.38. Hence, we can still identify the scalar with operators in the field theory according to (2.39) and (2.40) as we did in the undeformed case.

If the solution retains some supersymmetry then the potential can be written in terms of a superpotential

$$V = \frac{1}{8}\left|\frac{\partial W}{\partial \lambda}\right|^2 - \frac{1}{3}|W|^2 \tag{2.54}$$

and the second order equations reduce to first order

$$\lambda' = \frac{1}{2}\frac{\partial W}{\partial \lambda}, \qquad A' = -\frac{1}{3}W \tag{2.55}$$

There are many examples of deformations in the literature. The $\mathcal{N} = 1^*$ theories [28, 29, 20] are of particular relevance to this work. The $\mathcal{N} = 1^*$ flows are obtained by giving masses to three of the four adjoint fermions, and a vev to the remaining fermion. This is closely related to the non supersymmetric Yang Mills* deformation that we develop in chapter 4. We will also calculate glueball masses in $\mathcal{N} = 1^*$ in chapter 6. The $\mathcal{N} = 2^*$ [22, 23, 25, 27, 26] deformation is related to the work in chapter 3, which involves the same scalar operator. In $\mathcal{N} = 2^*$ an equal mass is given to two of the $\mathcal{N} = 1$ multiplets of $\mathcal{N} = 4$ super Yang Mills. Another gravity dual with relevance to this work is the $\mathcal{N} = 1$ Klebanov Strassler [43] background. The geometries we consider are singular, whereas the Klebanov Strassler background is completely smooth. We will use glueball mass spectrums calculated from the Klebanov Strassler dual for comparison with the $\mathcal{N} = 1^*$ results.

Another construction relevant to the work in this thesis is Witten's finite temperature model [49]. This approach to QCD begins with M-theory 5-branes, and compactifies on two circles. Supersymmetry is broken by taking the fermions to be anti-periodic on one of the circles, leaving 4d pure Yang-Mills in the infra-red.

2.5.2 Fluctuations

We will be interested in studying bound states in the field theory using its gravity dual. Each supergravity field plays the role of a source in the dual field theory. Suppose we have an operator $\mathcal{O}$ dual to a scalar Φ and we are interested in the correlators $\langle \mathcal{O}\mathcal{O} \rangle$. From the prescription (2.43) we see that we need to take two functional derivatives of the supergravity action with respect to Φ. So we are interested in the linearised field equation for fluctuations of the scalar $\Phi = \Phi_0 + \delta\Phi$ [15, 60]

$$\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)\delta\Phi = \frac{\partial^2 V}{\partial\Phi\partial\Phi}\delta\Phi \tag{2.56}$$

To study bound states we consider plane wave fluctuations of the scalar of the form

$$\delta\Phi = \psi(r)e^{-ikx}, \qquad k^2 = -M^2 \tag{2.57}$$

where the eigenvalues M give the masses of the bound states corresponding to the operator $\mathcal{O}$.

If we make the change of coordinates $(r \to z)$ such that

$$\frac{dz}{dr} = e^A \tag{2.58}$$

and rescale

$$\psi \to e^{-3A/2}\psi \tag{2.59}$$

Then the dilaton field equation takes a Schroedinger form

$$(-\partial_z^2 + U(z))\psi(z) = M^2\psi(z) \tag{2.60}$$

where

$$U = \frac{3}{2}A'' + \frac{9}{4}(A')^2 + e^{2A}\frac{\partial^2 V}{\partial\Phi\partial\Phi} \tag{2.61}$$

Primes now denote differentiation with respect to z. Now that the equation has been cast in Schroedinger form calculating the spectrum of bound states is reduced to a familiar problem in one dimensional quantum mechanics. Note that in these coordinates the Ricci scalar is

$$R(z) = 4e^{-2A(z)}[3A'(z)^2 + 2A''(z)] \tag{2.62}$$

so that the potential can be written

$$U = \frac{3}{16}e^{2A}(R + \frac{\partial^2 V}{\partial\Phi\partial\Phi}) \tag{2.63}$$

In these coordinates the second order equations of motion become

$$\lambda'' + 3\lambda' A' = e^{2A}\frac{\partial V}{\partial \lambda} \tag{2.64}$$

$$6A'^2 = \lambda'^2 - 2e^{2A}V \tag{2.65}$$

In the ultra violet limit

$$z = -e^{-r}, \qquad \lambda = a(-z)^{4-\Delta} + b(-z)^{\Delta} \tag{2.66}$$

Again, a is interpreted as a source for an operator and b as the vev of that operator since z has conformal dimension -1. If some supersymmetry is preserved then the first order equations are

$$\lambda' = \frac{1}{2}e^A\frac{\partial W}{\partial \lambda}, \qquad A' = -e^A\frac{1}{3}W \tag{2.67}$$

We will be particularly interested in Glueballs. Gluons carry colour charge, so that in a confining non abelian gauge theory there will be bound states of gluons known as glueballs. Glueball states are labeled by their quantum numbers J^{PC} where J is the spin, P and C are parity and charge conjugation quantum numbers. We will study the scalar glueball O^{++}, corresponding to the operator $\mathcal{O} = \mathrm{Tr}\, F^2$. The dual scalar is the dilaton, which does not contribute to the supergravity potential. Hence the glueball potential is simply

$$U_{glue} = \frac{3}{2}A'' + \frac{9}{4}(A')^2 \tag{2.68}$$

In chapter 6 we will calculate the mass spectrum for glueballs in the Yang Mills* deformation developed in chapter 4 and make comparisons with lattice calculations for glueballs in QCD. We will also study glueballs in $N = 1^*$ and other bound states in both theories.

2.6 The Brane Probe

It is often useful in physics to use a test particle to probe a particular solution of a theory. For example using light rays to explore black holes in general relativity or test charges in electro-statics. We will find it extremely useful in gauge-gravity dualities to use D-branes to probe the geometries that arise as deformations of the AdS/CFT correspondence [9]. The D-brane is a probe in the sense that we neglect the back reaction of the brane on the geometry. This is reasonable in the large N limit as the effects of a single brane will be small in comparison to the large stack of branes that warps space time. The probe should be thought of a single brane removed form the stack. This breaks the symmetry from $U(N) \to U(N-1) \times U(1)$ so that the $U(1)$ theory living on the probe sheds light on the field theory dual to the geometry it is probing. The D-brane probe is described by the action eq. 4.67 and so feels the geometry through the pullback of 10d fields onto the probe surface. The $U(1)$ field theory living on the brane yields gauge theory quantities such as the gauge coupling.

2.7 Consistent Truncation And The Lift to 10 Dimensions

Five dimensional $\mathcal{N} = 8$ gauged supergravity has been conjectured to be a consistent truncation of ten dimensional type IIB supergravity [22, 20]. This means that every five dimensional solution can be uniquely lifted to a ten dimensional solution. The five dimensional fields will be embedded in the ten dimensional solution. The ten dimensional fields satisfy their field equations only when the five dimensional field embedded in the solution satisfy there respective field equations. Whilst unproved, this conjecture has been well tested. In what follows, we will work to the assumption that it is correct.

Whilst the five dimensional theory is an extremely valuable tool in studying the correspondence, it is often useful for a physical interpretation to know the full ten dimensional geometry. The 5d flows are generally singular in the infra red, and these singularities often appear to be pathological. In lifting to ten dimensions, however, the 5d metric is multiplied by a warp factor. The asymptotics of the warp factor modify the asymptotic behavior of the 5d metric, and the 10d metric is typically less singular.

Another important aspect of the lift to 10 dimensions concerns the dilaton and axion.

In general, the 5d dilaton and axion do not correspond directly to the IIB dilaton and axion. Even if the 5d dilaton and axion are set to zero it is possible for the ten dimensional dilaton and axion to be highly non-trivial. As the IIB dilaton describes the running of the gauge coupling, this is crucial for a physical interpretation of the flow.

Finally, it is nessecary to know the complete lift to ten dimensions to brane probe the geometry. The brane probe has become an indispensable tool in exploring the physics of gauge-gravity dualities. Brane probes are used to identify the fields dual to particular operator, to calculate wilson loops, and reveal qualitative features of the gauge theory.

2.8 Quarks

We would like to include quarks in the fundamental representation. An open string has two ends where charges can be placed, so it will usually appear in the adjoint representation. To obtain the fundamental representation we require only one charge, and hence only one end of a string. This can be done by placing a probe D7-brane such that it is parallel to the 3-brane stack but extends into the bulk [58]. Strings stretching from the 7-brane to the 3-branes will have only one end on living in the field theory on the 3-branes and so will appear in the fundamental representation (fig. 2.3). We would like to have a different flavor of quark, so that we will have a stack of N_f D7-branes. We work in the probe limit where $N_f \ll N$.

The metric for $AdS_5 \times S^5$ can be written

$$ds^2 = e^{2r} dx_{||} + dr^2 + d\psi^2 + \cos^2 \psi d\theta^2 + \sin^2 \psi d\Omega_3^2 \tag{2.69}$$

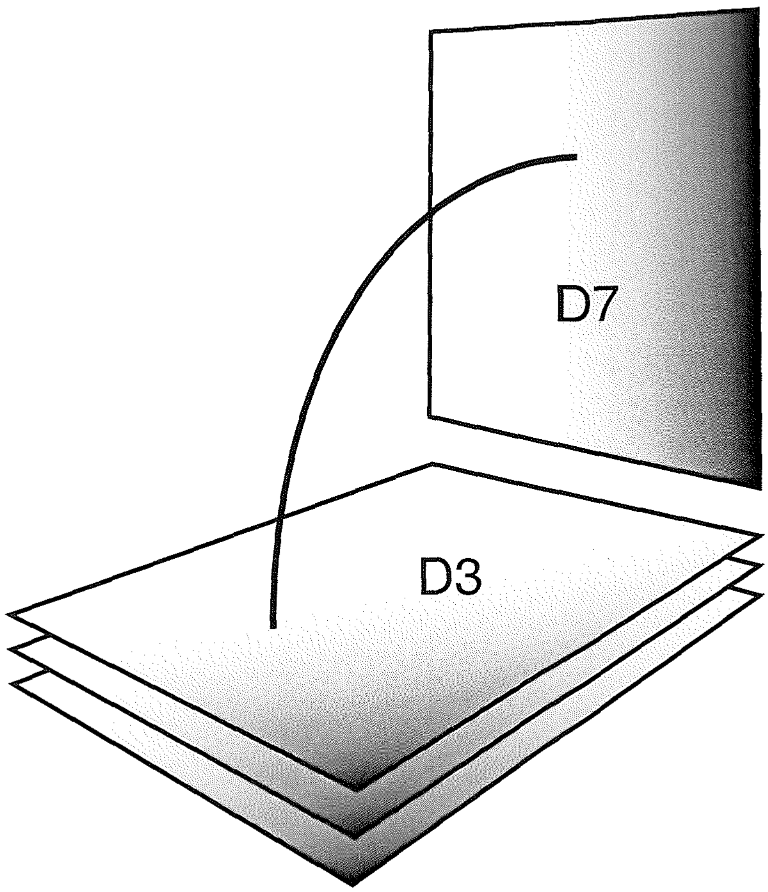

Figure 2.3: Strings stretching from the 7-brane to the 3-branes appear in the fundamental representation.

We wrap the probe 7-brane on a three sphere S_3 at $\psi = \frac{\pi}{2}$. Consider fluctuations of of the brane of the form

$$\psi = \frac{\pi}{2} + \phi(r) \tag{2.70}$$

We claim that the scalar ϕ is dual to fundamental fermions in the field theory living on the 3-branes. The probe action is

$$S_{\text{probe}} = \int dx^8 \sqrt{\det G_{ab}} \tag{2.71}$$

$$= \text{Vol}S_3 \int dx^4 dr e^{4r} \sin^3\psi \sqrt{1+\phi' r^2} \tag{2.72}$$

Hence the lagrangion for this scalar in AdS_5 is

$$\mathcal{L} = e^{4r} \sin^3\psi \sqrt{1+\phi'(r)^2} \tag{2.73}$$

In the ultra violet limit we have

$$\mathcal{L} \longrightarrow e^{4r}[1 - \frac{3}{2}\phi(r)^2 + \frac{1}{2}\phi'(r)] \tag{2.74}$$

The equations of motion at large r are

$$\phi'' + 4\phi' - 3\phi = 0 \tag{2.75}$$

with solution

$$\phi = ae^{-r} + be^{-3r} \tag{2.76}$$

From (2.40) we see that ϕ is dual to a dimension 3 operator. This is consistent with the interpretation of ϕ being dual to fermionic flavor operators in the field theory.

The motivation for including fermions in the fundamental representation is, of course, comparison with QCD. QCD has an $SU(N_f) \times SU(N_f)$ flavor symmetry, so two stacks of D7-branes are needed. This approach is useful for studying Chiral symmtry breaking and the mass spectrum of mesons. In [62] Chiral symmetry breaking has been studied in several non supersymmetric backgrounds. Although we will not use this method in this thesis, one of the attractive features of the Yang Mills* duality developed in Chapter 4 is that it is open to this approach. The finite tempterature model is less suitable for the introduction of quarks as the thermalization would induces masses for the matter fields. This is a direction for future work on Yang Mills*.

Chapter 3

A Non-Supersymetric Scalar Mass Deformation of AdS/CFT

Our main interest in this thesis is non-supersymmetric deformations of the AdS/CFT correspondence. We are motivated by the idea of developing tools for studying QCD like theories. In this chapter we will consider a very simple non supersymmetric deformation. It will be a toy model, with not much in common with QCD, so it will not in itself take us very far towards our stated goal. Our purpose in this chapter is to develop in a relatively simple setting the technology for finding non supersymmetric supergravity duals. In the next chapter we will apply these techniques to a much more physically interesting example.

One might think that the constraints of supersymmetry are an essential ingredient of the correspondence, but we will argue that this is not the case. In the AdS/CFT Correspondence supergravity fields behave as sources in the dual gauge theory. Expectation values of field theory operators are obtained from functional derivatives on the supergravity partition function with respect to the boundary values of the supergravity fields. It is therefore a crucial aspect of the correspondence that the supergravity partition function must be calculable in the presence of all infinitesimal sources in order that derivatives with respect to those sources are well defined. In fact, for sources which break the $\mathcal{N} = 4$ theory's conformal symmetry, infinitesimal has no meaning since they become the only mass scale in the problem. It should therefore be possible to find gravity duals of deformed versions of the $\mathcal{N} = 4$ super Yang-Mills theory, including non-supersymmetric theories.

In this chapter we will deform the original AdS/CFT correspondence by introducing a mass term of the form Tr $(\phi_1^2+\phi_2^2+\phi_3^2+\phi_4^2-2\phi_5^2-2\phi_6^2)$ which is naively unbounded. As our interest is in developing the technology to find and lift these solutions to 10d we will not be so concerned by the runaway behaviour (although the 10d solution we provide correctly reproduces the expected behaviour). One might hope that there would be such backgrounds that are really stable since an $SO(6)_R$ singlet scalar mass term Tr $\sum_i \phi_i^2$ is not visible in the supergravity solution. It's presence could stabilize the solution. Note that the supersymmetric deformations [12, 22, 20] already mentioned require this operator to be present. In fact our brane probe potential reveals the operator not to be present in our 10d lifts. Our solution is also of interest since it is probably the simplest example of a non-supersymmetric deformation; only the metric and four potential fields are non-zero.

In the next section we will discuss the introduction of our deformation at the 5d supergravity level. In section 3 we then lift the full solution to 10d, although one function in the four form is only found numerically. In section 4 we brane probe the background with a D3 brane and show that asymptotically the background indeed includes the operator we hoped to introduce showing the consistency of the techniques. Finally we plot the potential seen by the probe for the full solution.

3.0.1 A Scalar Operator

Let us now make a particular choice for the scalar field we will consider. We take a scalar from the multiplet in the 20 of SO(6). These operators have been identified [3] as playing the role of source and vev for the scalar operator $\mathrm{Tr}\,\phi_i\phi_j$ in the field theory. In particular we will choose the scalar corresponding to the operator

$$\mathcal{O} = \mathrm{Tr}\,(\phi_1^2 + \phi_2^2 + \phi_3^2 + \phi_4^2 - 2\phi_5^2 - 2\phi_6^2) \tag{3.1}$$

This scalar has been studied in the literature [19, 35] already in its role of describing an $\mathcal{N} = 4$ preserving scalar vev and as a mixture of a mass term and a vev in the $\mathcal{N} = 2^*$ gauge theory [22, 23]. The potential for the scalar, which we will write as $\rho = e^{\lambda/\sqrt{6}}$ is given by

$$V = -\frac{1}{\rho^4} - 2\rho^2 \tag{3.2}$$

and the three equations of motion become

$$\frac{\rho''}{\rho} - \left(\frac{\rho'}{\rho}\right)^2 + 4\frac{\rho'}{\rho}A' = \frac{\rho}{6}\frac{\partial V}{\partial \rho} \tag{3.3}$$

$$6A'^2 - 6\left(\frac{\rho'}{\rho}\right)^2 = -2V \tag{3.4}$$

$$A'' = -4\left(\frac{\rho'}{\rho}\right)^2 \tag{3.5}$$

The last of these is the sum of (2.51) and (2.52). The asymptotic ($r \to \infty$) solutions take the form

$$\lambda = \mathcal{A}e^{-2r} + \mathcal{B}re^{-2r} \tag{3.6}$$

with $\mathcal{A}$ the scalar vev and $\mathcal{B}$ a mass term for the operator $\mathcal{O}$.

In the special case where only the first part of the solution is present the deformation preserves $\mathcal{N} = 4$ supersymmetry. The superpotential is

$$W = -\frac{1}{\rho^2} - \frac{1}{2}\rho^4 \tag{3.7}$$

and the second order equations reduce to the first order equations

$$\frac{\partial \rho}{\partial r} = \frac{1}{3}\left(\frac{1}{\rho} - \rho^5\right), \qquad \frac{\partial A}{\partial r} = \frac{2}{3}\left(\frac{1}{\rho^2} + \frac{1}{2}\rho^4\right) \tag{3.8}$$

with solution [19]

$$e^{2A} = l^2 \frac{\rho^4}{\rho^6 - 1} \tag{3.9}$$

with l^2 a constant of integration.

3.0.2 Non-supersymmetric First Order Equations

In [44] it was pointed out that using Hamilton Jacobi theory the second order equations could be replaced by a system of first order equations. They further stated that a "superpotential", W, could be found which resulted in the equations (2.55) even for the non supersymmetric solution with only $\mathcal{B}$ switched on. A similar result was obtained in [45, 46] but as a requirement for the RG flow solution to be stable. Further analysis along these lines can be found in [47, 48]. Reducing the equations to first order would be very helpful, but the system we discuss here can not be.

Consider the UV of the theory where, expanding (6.22)

$$V = -3 - 2\lambda^2 + \sqrt{\frac{8}{27}}\lambda^3 + ... \qquad (3.10)$$

we can attempt to find a superpotential W that reproduces this potential via the trial form

$$W = a + b\lambda^2 + c\lambda^3 + ... \qquad (3.11)$$

Working to quadratic order one finds

$$a = -3, \qquad b = -2 \qquad (3.12)$$

The solution for b comes from a quadratic equation with degenerate roots hinting at the two forms of the solution. However, it is then easy to show that at higher orders there is a unique series (eg $c = \sqrt{2/27}$) and it is simply the supersymmetric solution. We have therefore not been able to find a superpotential that describes the non-supersymmetric solution and are forced to numerically solve the second order equations. Of course our geometry is intrinsically unstable since we have introduced an unbounded operator in the field theory. Apparently the stability of the flow is essential for the system to reduce to first order.

3.0.3 Numerical Solutions

The second order equations of motion are easily solved. In figure 3.1 we show the numerical behaviour of ρ. For this plot we fix $\rho(r = \Lambda_{UV})$ and vary the derivative. The purely vev supersymmetric solution ($\mathcal{B} = 0$) and purely masslike case ($\mathcal{A} = 0$) are labelled. The three regions (bounded by the $\mathcal{A} = 0$ and $\mathcal{B} = 0$ curves) correspond to

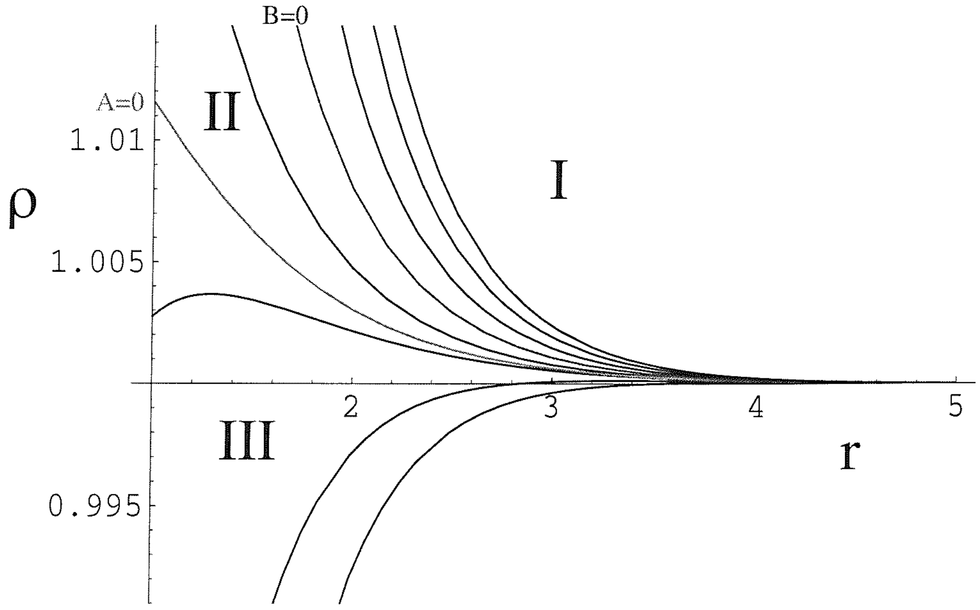

Figure 3.1: Plots of ρ vs r for a variety of initial conditions on ρ'. The vev only initial condition solution is marked with $\mathcal{B} = 0$ and mass only initial condition with $\mathcal{A} = 0$. The marked regions are explained in (3.13).

	$\mathcal{A}$	$\mathcal{B}$
I	$+ve$	$-ve$
II	$+ve$	$+ve$
III	$-ve$	$+ve$

(3.13)

In all these cases the function $A(r)$ only deviate from each other and $A(r) \sim r$ by a small amount so a plot is unrevealing. Note that most of these solutions become singular before $r = 0$. Close to the singularity, there will be stringy corrections and the supergravity approximation will not be valid. When lifted to 10d this singular point is expected [19] to correspond to the position of the D3 brane sources in the background. For most of these solutions there is a scalar vev and so the D3 branes are expected to have moved away from the origin. The mass only solution ($\mathcal{A} = 0$)

on the other hand can be extended to $r = 0$ which is consistent with the D3 branes being pinned at the origin.

It has proven difficult to extract aspects of the field theory from the 5d supergravity backgrounds. More success has been had at the 10d level where techniques such as brane probing can be used to connect to the field theory. We shall therefore move to discussing the lift of these solutions to 10d in the next section.

3.1 The 10d Background

To lift the 5d solution to 10d requires the procedure outlined in [30]. Finding the metric is complicated but we will be able to short cut the process since the lift of the 5d solution where the $\mathcal{N} = 4$ theory is on moduli space has already been written down. In particular the solution where our scalar corresponds to a vev has been studied in [19, 35] (it is also the limit of the metrics in [22, 23, 20] with some of the fields switched off). That solution is given by

$$ds^2 = \frac{X^{1/2}}{\rho} e^{2A(r)} dx_{//}^2 + \frac{X^{1/2}}{\rho}\left(dr^2 + \frac{L^2}{\rho^2}\left[d\theta^2 + \frac{\sin^2\theta}{X}d\phi^2 + \frac{\rho^6\cos^2\theta}{X}d\Omega_3^2\right]\right), \quad (3.14)$$

where $d\Omega_3^2$ is the metric on a 3-sphere and

$$X \equiv \cos^2\theta + \rho^6 \sin^2\theta \quad (3.15)$$

For consistency there must also be a non-zero C_4 potential of the form

$$C_4 = \frac{e^{4A}X}{g_s\rho^2} dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \quad (3.16)$$

Note that the solution has the same SO(2) $\times$ SO(4) symmetry as our operator (4.1).

Clearly the lift of the full solution of the second order equations has this as a limit. In fact the procedure for finding the form of the metric does not depend on the supersymmetric solution and we may take it over directly to our case. The C_4 potential though will change since the supersymmetric first order equations of motion were used in its derivation [19, 22].

In fact the 10d supergravity equations of motion we must concern ourselves with are relatively few [22] since only the metric and C_4 are non-zero. There are the Einstein equations

$$R_{MN} = T_{MN} = \frac{1}{6} F_N^{PQRS} F_{PQRSM} \tag{3.17}$$

and

$$F_{(5)} =^* F_{(5)}, \qquad dF_{(5)} = 0 \tag{3.18}$$

The self duality condition can be imposed by using the ansatz

$$F_{(5)} = \mathcal{F} +^* \mathcal{F}, \qquad \mathcal{F} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge dw \tag{3.19}$$

where $w(r,\theta)$ is an arbitrary function.

There are three independent non-zero elements of R_{MN} which factorize into the useful equations

$$R_0^0 - R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{rr} \left(\frac{\partial w}{\partial r} \right)^2 \tag{3.20}$$

$$R_0^0 + R_r^r = \frac{1}{2} g^{00} g^{11} g^{22} g^{33} g^{\theta\theta} \left(\frac{\partial w}{\partial \theta} \right)^2 \tag{3.21}$$

$$R^r_\theta = \frac{1}{2} g^{11} g^{22} g^{33} g^{44} g^{rr} \left(\frac{\partial w}{\partial \theta} \frac{\partial w}{\partial r} \right) \tag{3.22}$$

The right hand side of these equations are straightforward but laborious to explicitly calculate. We use mathematica to calculate and simplify the elements of the Ricci tensor[1] . The resulting output is lengthy but can be simplified by using the second order equations of motion to eliminate ρ'', A'' and A'^2. The resulting background will therefore reproduce the full second order equations of motion. We find

$$R^0_0 - R^r_r = -\frac{18 \sin^2\theta \cos^2\theta \rho^5 \rho'^2}{X^{5/2}} \tag{3.23}$$

$$R^0_0 + R^r_r = -\frac{(2\cos^2\theta - (\cos 2\theta - 3)\rho^6)^2}{2\rho^3 X^{5/2}} \tag{3.24}$$

$$R^r_\theta = -\frac{3 \sin^2\theta \rho' ((\cos 2\theta - 3)\rho^6 - 2\cos^2\theta)}{2X^{5/2}} \tag{3.25}$$

(3.20) thus reduces to

$$\left(\frac{\partial w}{\partial \theta} \right) = \frac{6 e^{4A} \cos\theta \sin\theta \rho'}{\rho} \tag{3.26}$$

which can be directly integrated and w put in the form

$$w(r, \theta) = \frac{e^{4A}}{\rho^2} - \frac{3 \sin^2\theta \rho' e^{4A}}{\rho} - e^{4A} F(r) \tag{3.27}$$

where $F(r)$ is as yet undetermined.

[1] 'It is unworthy of excellent men to lose hours, like slaves, in the labors of calculation' - *Leibnitz*

Note that the supersymmetric limit corresponds to $F(r) = 0$ and ρ' replaced using the supersymmetric first order equation of motion (3.8). We should not be surprised that derivatives of ρ enter directly into the solution since introducing a mass term corresponds explicitly to introducing an extra degree of freedom via precisely this derivative.

F can then be found using either of the other two equations (the third equation providing a check on the consistency of the solution). It is the solution of

$$-2 - 2\rho^6 = -4\rho^2 A' + 4\rho^4 F A' + \rho^4 F' + 2\rho\rho' \tag{3.28}$$

We have not been able to solve this equation explicitly but in the UV limit the solution takes the form

$$F = \frac{1}{3}\left(\frac{1}{\rho} - \rho^5\right) - \rho' + ... \tag{3.29}$$

which clearly vanishes in the supersymmetric limit given (3.8). For a general numerical solution of the second order equations of motion we can set the boundary conditions on F using this asymptotic form and hence find F numerically for all r.

The solution then faces its strongest test since $F_{(5)}$ must also satisfy its bianchi identity (3.18). At first sight this appears to be a challenge; since w contains a derivative of ρ the bianchi identity is a third order equation.

The Bianchi identity is

$$dF = [\partial_r[\sqrt{g}g^{00}g^{11}g^{22}g^{33}g^{rr}\partial_r\omega] + \partial_\theta[\sqrt{g}g^{00}g^{11}g^{22}g^{33}g^{\theta\theta}\partial_\theta\omega]]d\Omega_5 \wedge dr = 0 \tag{3.30}$$

Hence we must check that

$$\partial_r[X^2 e^{-4A}\rho^4 \sin\theta \cos^3\theta \sqrt{\det S_3}\partial_r\omega] + \partial_\theta[X^2 e^{-4A}\rho^6 \sin\theta \cos^3\theta \sqrt{\det S_3}\partial_\theta\omega] = 0 \quad (3.31)$$

In fact explicit computation, using the second order equations of motion and (3.28), shows that this third order equation is satisfied and the solution survives.

Given the complete numerical 10d lift of our non-supersymmetric solutions we can study the background for signals that it correctly encodes the field theory dynamics.

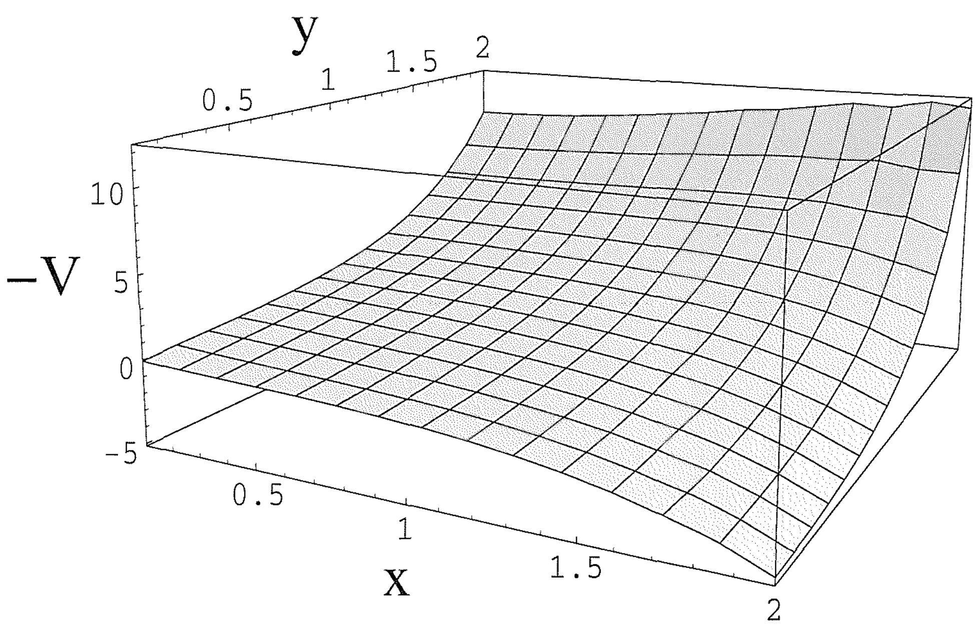

Figure 3.2: The probe potential plotted over the $r - \theta$ plane for the mass only case ($\mathcal{A} = 0$).

3.2 Brane Probe Potential

The most succesful technique for connecting backgrounds and their dual field theories has been brane probing [1, 26, 27, 21, 35, 36, 37] which converts the background to the U(1) theory on the probe's surface. We thus substitute the background into the Einstein frame Born-Infeld action

$$S_{probe} = -\tau_3 \int_{\mathcal{M}_4} d^4x \det[G^{(E)}_{ab} + 2\pi\alpha' e^{-\Phi/2} F_{ab}]^{1/2} + \mu_3 \int_{\mathcal{M}_4} C_4, \tag{3.32}$$

The resulting scalar potential is given by

$$V_{probe} = -e^{4A} \left[\frac{X}{\rho^2} + \frac{3\sin^2\theta\rho'}{\rho} - \frac{1}{\rho^2} + F \right] \tag{3.33}$$

It is illuminating to evaluate this potential at leading order in the UV with

$$\rho = 1 + ve^{-2r} + m^2 re^{-2r} + \ldots \tag{3.34}$$

We find

$$V = m^2 e^{2r}(2 - 6\sin^2\theta) + \ldots \tag{3.35}$$

The scalar vev vanishes from the potential at this order consistent with the existence of the $\mathcal{N} = 4$ moduli space. The mass term reproduces precisely the mass operator we expected in (4.1) remembering that e^r plays the role of a scalar field. We conclude that the 10d background shows all the correct behaviour to be dual to the non-supersymmetric gauge theory with scalar masses.

Finally we numerically find the full solution for $A(r)$, $\rho(r)$ and $F(r)$ for the mass only boundary conditions ($\mathcal{A} = 0$) using (2.50,2.51, 2.52) and (3.28,3.29). We then plot the full solution for the probe potential in the $r - \theta$ plane for the mass only solution ($\mathcal{A} = 0$) in figure 3.2. The plot fits well with the claim that the mass operator (4.1) is present. The supersymmetric solutions ($\mathcal{B} = 0$) give a flat probe potential.

Other non-supersymmetric solutions reproduce the form of figure 2 upto a sign change dependent on the sign of $\mathcal{B}$.

We conclude that we have successfully found the 10d gravity dual of this simple non-supersymmetric deformation of the AdS/CFT Correspondence. This deformation is unstable: The resulting field theory and supergravity background shared an instability in the scalar potential. This highlights one of the most challenging problems in constructing non-supersymmetric solutions, the need to find a stable deformation. In the next chapter we will find a stable non-supersymmetric deformation of the AdS/CFT Correspondence. We will apply the methods used in this chapter to construct a gravity dual of a field theory that resembles pure Yang Mills in the infra red. The deformation studied in this chapter is interesting in its own right for another reason. We mentioned in the introduction a second motivation for studying non supersymmetric deformations: to see how robust the correspondence itself is to the loss of supersymmetry and stability. We find here that the 10d lift correctly encodes the runaway behaviour of the field theory. This is at least an indication that the correspondence survives even under the most extreme circumstances.

Chapter 4

The Supergravity Dual of Non-supersymetric Glue

In this chapter we will deform the AdS/CFT Correspondence by including a supergravity scalar that is a source for an equal mass term for each of the four adjoint fermions of $\mathcal{N} = 4$ Super Yang-Mills. We find exact solutions of the 5d flow equations in the infra-red, and solve numerically for the whole flow. We find the complete 10d supergravity solution in terms of the 5d scalar fields. The resulting background is remarkably simple. The stability of the solution is then tested using a brane probe. The D3 brane probe indicates that the background is stable and the field theory scalars acquire equal masses. The field theory the background describes is $\mathcal{N} = 4$ Super Yang-Mills with masses for all the matter fields leaving pure non-supersymmetric Yang Mills in the infra-red. We call this theory Yang Mills* following the nomenclature used for supersymmetric deformations of $\mathcal{N} = 4$ Super Yang-Mills.

Our 10d lift can also describe a bilinear condensate for the four adjoint fermions by giving appropriate boundary conditions to the 5d scalar. The D3 branes in the core expand non commutatively into the bulk on the surface of a fuzzy sphere. We show that the geometry is unstable with a brane probe, as there is no force to support the non commutative expansion.

The Yang Mills* theory is hopefully of real use as an approximation to non-supersymmetric, pure Yang Mills theory. In subsequent chapters we will further explore the physics of this duality by calculating wilson loops and the mass spectra of bound states. The Yang Mills* is analogous to the thermalized 5d background of Witten [49] which also describes 4d non-supersymmetric Yang Mills in the infra-red. That theory has been used to compute glueball masses [51] with some success, supporting the use of these geometries. In chapter 6 we will compare the predictions of these two variants to begin to determine the size of systematic errors induced by the massive matter in each. The Yang Mills* deformation is a more systematic approach to obtaining non-supersymmetric Yang Mills and is more open to the introduction of quarks. The thermal geometry would make the quarks massive. The analysis [58] described at the end of chapter 2 of probe D7 branes in anti de-Sitter (AdS) space appears a particularly fruitful approach.

On the next page there is a flow chart (4.1) showing how the some of the steps taken in the remainder of this thesis fit together. It shows that $\mathcal{N} = 4$ super Yang Mills is deformed by the inclusion of a mass term for the fermion, which is dual to a scalar in 5d Supergravity. The 5d gravity dual is used (in chapter 6) to calculate glueball masses in Yang Mills*. The 5d geometry is lifted to 10d. The 10d lift is brane probed to reveal that the field theory scalars also acquire masses in Yang Mills*. The 10d background is also used (in chapter 5) to compute the Wilson loop in Yang Mills*. The flow chart is intended to show in a simple way how the steps taken in supergravity lead back to the field theory.

In the next section we describe the Yang Mills* deformation in 5d supergravity. In section 3 we describe the oxidation process to 10d and then in section 4 we brane probe the solution. The full background is gathered together in the appendix for ease

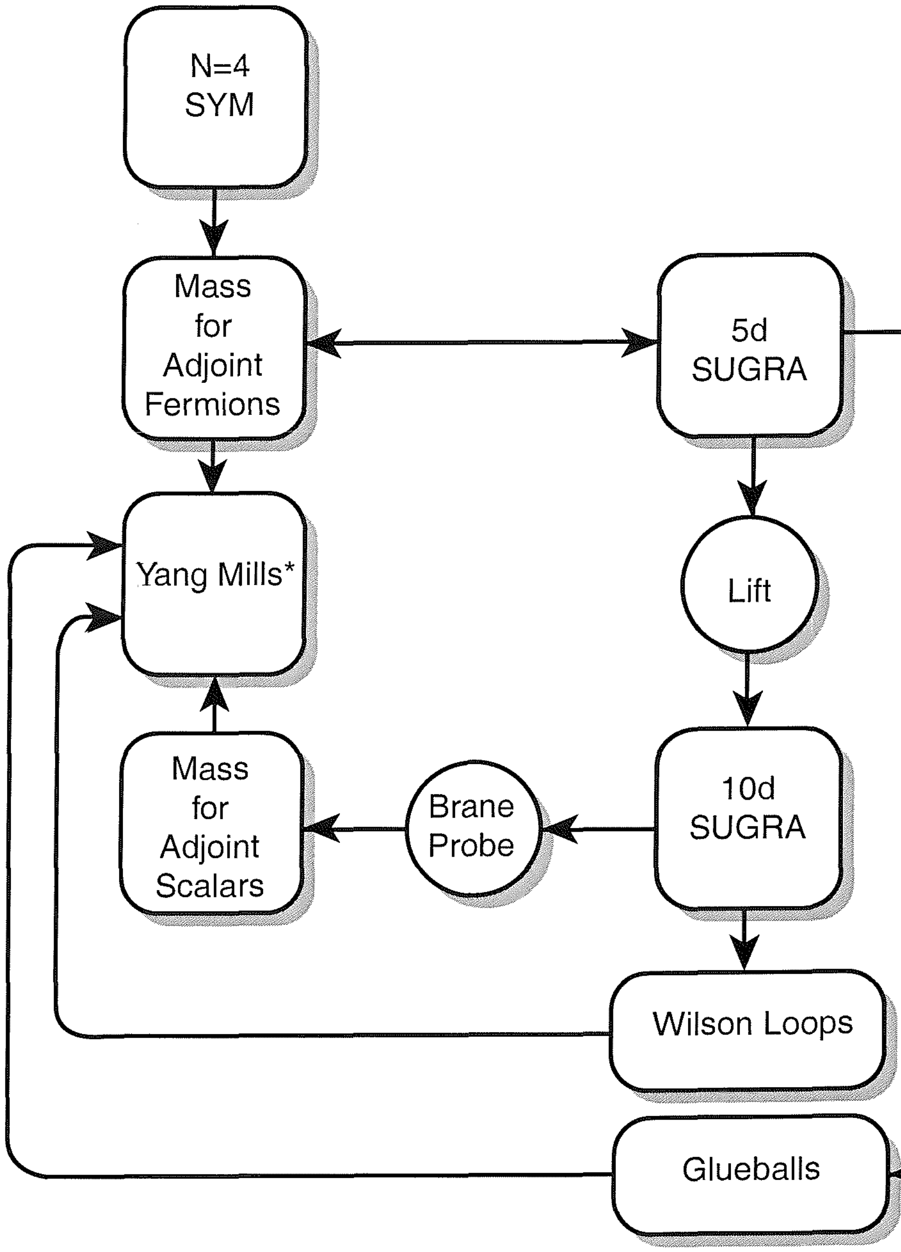

Figure 4.1: Flow chart of the Yang Mills* lift showing how the steps taken in the remainder of the thesis fit together.

of reference.

4.1 A Fermionic Operator

Let us now make a particular choice for the scalar field we will consider. We take a scalar from the multiplet in the 10 of SO(6). These operators have been identified [28] as playing the role of source and vev for the fermionic operator $\psi_i\psi_j$ in the field theory. In particular we will chose the scalar corresponding to the operator

$$\mathcal{O} = \sum_{i=1}^{4} \psi_i\psi_i \tag{4.1}$$

The potential for the scalar can be obtained from the $N = 1^*$ solution of [28] by setting their two scalars equal (to be precise one must set their $m = \sqrt{3/4}\lambda$ and $\sigma = \sqrt{1/4}\lambda$ to maintain a canonically normalized kinetic term)

$$V = -\frac{3}{2}\left(1 + \cosh^2\lambda\right) \tag{4.2}$$

In this case $m^2 = -3$ and the ultra-violet solutions are

$$\lambda = \mathcal{M}e^{-r} + \mathcal{K}e^{-3r} \tag{4.3}$$

The field theory operator has dimension 3. Thus in what follows $\mathcal{M} = 0$ corresponds to a solution with just bi-fermion vevs while $\mathcal{K} = 0$ corresponds to the purely massive case.

4.1.1 Numerical Solutions

We are not able to write down first order equations as there is no superpotential, so we are forced to solve the second order equations numericaly. The evolution of λ as a function of r for a variety of different initial conditions on λ' is shown in Fig 4.2. The mass only flow is unique. If there is even a small condensate $\lambda(z)$ diverges very rapidly. It is necessary to fine tune the initial conditions to a very high precision in order to isolate the mass only flow. The mass only and condensate only cases are highlighted. The function $A(r)$ evolves as $A(r) \sim r$ except in the very deep infra-red. Note that λ typical diverges at finite r with the $\mathcal{K} = 0$ solution lying on the boundary between solutions that blow up positively and negatively. It appears that even in the mass only case that the flow is singular. When the curvature blows up, stringy corrections become important, and supergravity cannot be trusted. The presence of a condensate leads to an unstable geometry, and an unbounded glueball potential as shown in figure 4.3. For the mass only solution, however, the glueball potential is a bounded well and the spectrum is calculable.

4.1.2 Asymptotic Solutions in the Infra Red

In the infra-red $r \to -\infty$, $\lambda(r) \to \infty$ and $A(r) \to -\infty$. The flow equations become

$$\lambda'' + 4A'\lambda' = -\frac{3}{4}e^{2\lambda} \tag{4.4}$$

$$6A'^2 = \lambda'^2 + \frac{3}{4}e^{2\lambda} \tag{4.5}$$

These equations have solutions

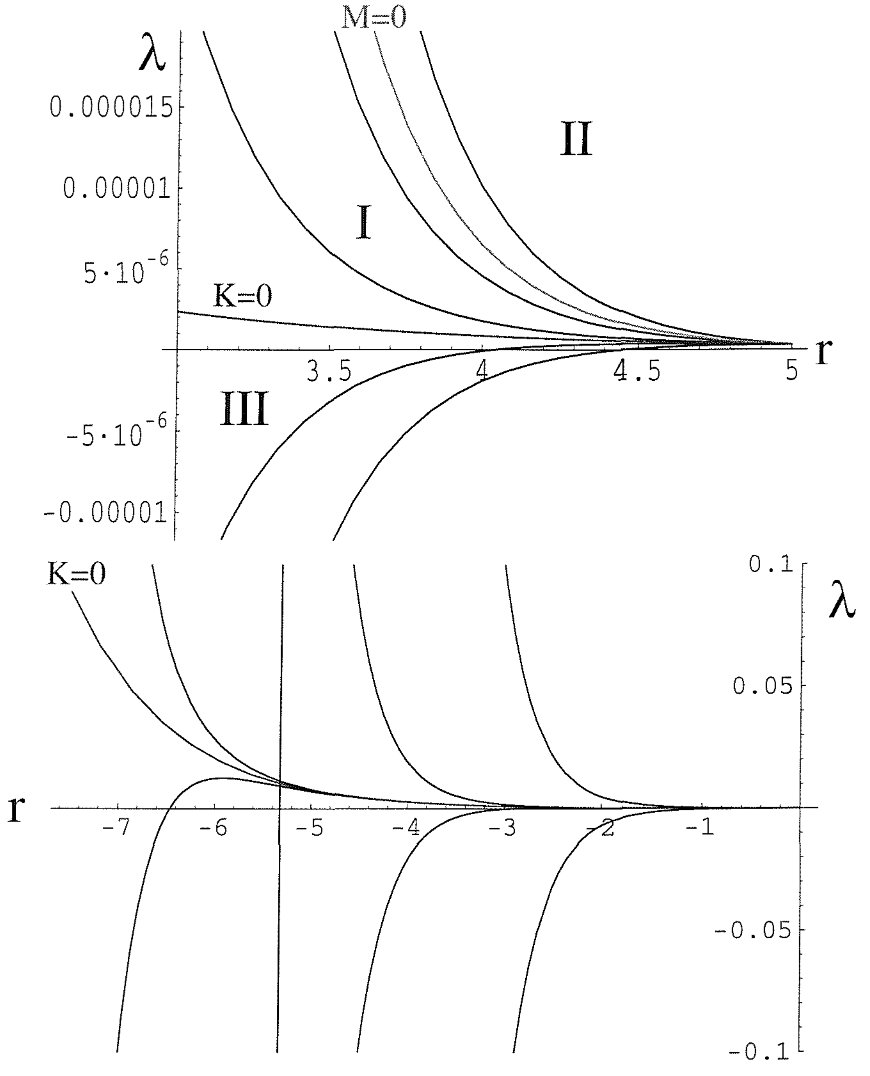

Figure 4.2: Plots of λ vs r for a variety of initial conditions on λ'. In the top figure the marked regions correspond to initial conditions: I positive mass and condensate; II negative mass, positive condensate; III positive mass, negative condensate. The lower figure shows a close up of initial conditions close to the mass only solution in the IR ($r < 0$)

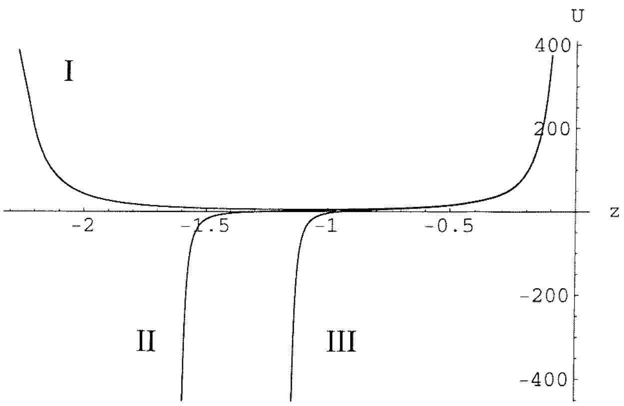

Figure 4.3: The Glueball potential showing: I mass only; II mass and condensate; III condensate only.

$$\lambda = \log \frac{|r - r_0|}{c}, \qquad c = \sqrt{\frac{20}{9}}, \qquad A = \frac{2}{3} \log |r - r_0| \tag{4.6}$$

This is clearly not a complete set of solutions, so they could represent a gluino condensate or the mass deformation corresponding to Yang-Mills*. We can easily check by changing variables $dz = e^{A} dr$ and computing the glueball potential $U(z)$

$$\begin{aligned} z &= \int e^{-A} dr \\ z - z_0 &= 3(r - r_0)^{1/3} \end{aligned} \tag{4.7}$$

Hence

$$A(z) = 2 \log |\frac{z - z_0}{3}| \tag{4.8}$$

We can now calculate the glueball potential in the infra-red,

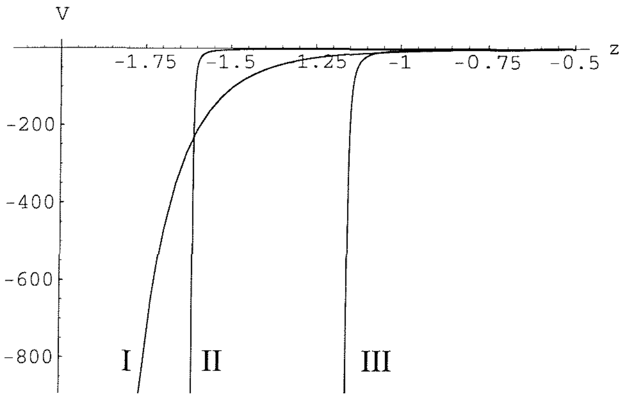

Figure 4.4: The Supergravity potential for a range of flows: I mass only; II mass and condensate; III condensate only. All cases are bounded above.

$$
\begin{aligned}
U(z) &= \frac{3}{2}A'' + \frac{9}{4}(A')^2 \\
&= \frac{6}{(z-z_0)^2}
\end{aligned}
\tag{4.9}
$$

We see that $U(z) \to \infty$ in the infra-red. Comparison with numerical study of the full second order equations implies that these solutions are the IR limit of the mass only Yang Mills * geometry. If there had been a condensate present, we would have found $U(z) \to -\infty$.

Deformed geometries of the form (2.48) have naked singularities in the infra-red. In [15] a necessary condition to distinguish healthy singularities from pathological ones is proposed. Large curvatures in geometries of the form (2.48) are allowed only if the scalar potential of 5d gauged supergravity evaluated on the solution is bounded above. This condition is the origin of the bound (6.12). Figure 4.4 shows the Supergravity potential evaluated for different asymptotic boundary conditions. All the Yang Mills* flows satisfy this condition, including the unphysical flows with a condensate. It is also a necessary condition for a confining gauge theory that the glueball potential be a bounded well. In this case this appears to be a stricter condition that successfully distinguishes the physical from the unphysical flows.

Consider a generic solution that behaves in the IR like

$$A_{IR} = \gamma \log |r - r_0| \tag{4.10}$$

The condition that the scalar potential is bounded above translates, using (2.53), to a bound on α

$$\gamma > \frac{1}{4} \tag{4.11}$$

If the geometry is dual to a confining gauge theory, we expect to be able to compute from supergravity the glueball mass spectrum. This implies a stricter bound on γ. To be able to compute a discrete spectrum the glueball potential (2.61) must be a bounded well. This implies that in the infra-red the glueball potential goes to positive infinity. Then

$$\gamma > \frac{2}{5} \tag{4.12}$$

The Yang Mills* infra-red solution for A_{IR} is of the form (4.10)with $\gamma = \frac{2}{3}$. This satisfies the above inequlities, so we hope that the singularity is not pathological.

4.2 The 10d Background

To lift the solution to a 10d background requires us to find a solution of the full set of IIB supergravity equations of motion. As was found in [20, 22] where a fermion mass term was introduced in a supersymmetric context all the supergravity fields will be non-zero. We first summarize the field equations taken from [22]

- The Einstein equations:

$$R_{MN} = T^{(1)}_{MN} + T^{(3)}_{MN} + T^{(5)}_{MN} \tag{4.13}$$

where the energy momentum tensor contributions from the dilaton, 2-form potential and 4-form potential are given by

$$T^{(1)}_{MN} = P_M P_N{}^* + P_N P_M{}^* \tag{4.14}$$

$$T^{(3)}_{MN} = \frac{1}{8}(G^{PQ}{}_M G^*_{PQN} + G^{*PQ}{}_M G_{PQN} - \frac{1}{6} g_{MN} G^{PQR} G^*_{PQR}) \tag{4.15}$$

$$T^{(5)}_{MN} = \frac{1}{6} F^{PQRS}{}_M F_{PQRSN} \tag{4.16}$$

The dilaton is written in unitary gauge where

$$P_M = f^2 \partial_M B, \qquad Q_M = f^2 \mathrm{Im}(B \partial_M B^*), \qquad f = \frac{1}{(1 - BB^*)^{1/2}} \tag{4.17}$$

The more familiar dilaton-axion field is given by

$$a + ie^{\Phi} = i\frac{(1 - B)}{(1 + B)} \tag{4.18}$$

and the 3-form field strength is defined by

$$G_{(3)} = f(F_{(3)} - BF^*_{(3)}) \tag{4.19}$$

- The Maxwell equations:

$$(\nabla^P - iQ^P)G_{MNP} = P^P G^*_{MNP} - \frac{2}{3}\, i\, F_{MNPQR}G^{PQR} \tag{4.20}$$

$$(\nabla^M - 2iQ^M)P_M = -\frac{1}{24}G^{PQR}G_{PQR} \tag{4.21}$$

- The self-dual equation:

$$F_{(5)} = \star F_{(5)} \tag{4.22}$$

- Bianchi identities:

$$F_{(3)} = dA_{(2)}, \quad dF_{(5)} = -\frac{1}{8}\,\mathrm{Im}(F_{(3)} \wedge F^*_{(3)}) \tag{4.23}$$

4.2.1 The UV limit

Let us first concentrate on lifting the ultra-violet ($r \to \infty$) limit of the 5d flow. The supergravity scalar lifts to the 2-form potential in 10d [34]. To determine its form we can use the group theory technique in [29, 31].

Parametrize the 6d space perpendicular to the D3 branes of the construction as

$$z_1 = \frac{w^1 + iy^1}{\sqrt{2}}, \quad z^2 = \frac{w^2 + iy^2}{\sqrt{2}}, \quad z_3 = \frac{w^3 + iy^3}{\sqrt{2}} \tag{4.24}$$

Under the SO(2) rotation subgroups of the 6 dimensional representation of SU(4), $z^i \to e^{i\phi_i} z_i$, the 4 dimensional representation transforms as

$$\lambda_1 \to e^{i(\phi_1 - \phi_2 - \phi_3)/2}\lambda_1, \quad \lambda_2 \to e^{i(-\phi_1 + \phi_2 - \phi_3)/2}\lambda_2,$$
$$\lambda_3 \to e^{i(-\phi_1 - \phi_2 + \phi_3)/2}\lambda_3, \quad \lambda_4 \to e^{i(\phi_1 + \phi_2 + \phi_3)/2}\lambda_4. \tag{4.25}$$

We can thus construct a 3-form field strength with the symmetry properties of a fermion mass or condensate

$$\langle\lambda_1\lambda_1\rangle dz^1\wedge d\bar{z}^2\wedge d\bar{z}^3 + \langle\lambda_2\lambda_2\rangle d\bar{z}^1\wedge dz^2\wedge d\bar{z}^3 + \langle\lambda_3\lambda_3\rangle d\bar{z}^1\wedge d\bar{z}^2\wedge dz^3 + \langle\lambda_4\lambda_4\rangle dz^1\wedge dz^2\wedge dz^3 \tag{4.26}$$

setting all the operators equal gives

$$F_{(3)} = dw^1 \wedge dw^2 \wedge dw^3 + i dy^1 \wedge dy^2 \wedge dy^3 \tag{4.27}$$

It will be useful to write the 6d space in terms of two S^2 (with metrics $d\Omega_\pm^2 = d\theta_\pm^2 + \cos^2\theta_\pm d\phi_\pm^2$) corresponding to the w and y spaces, a radial direction r, and an angular coordinate between the spheres, α. The appropriate 2-form is then

$$A_{(2)} = \cos^3\alpha \cos\theta_+ d\theta_+ d\wedge\phi_+ + i\sin^3\alpha\cos\theta_- d\theta_- \wedge d\phi_- \tag{4.28}$$

A survey of the field equations reveals that only the 2-form's Maxwell equation is of leading order in the perturbing field λ. We find that

$$A_{(2)} = 2\lambda(i\cos^3\alpha\cos\theta_+ d\theta_+ \wedge d\phi_+ - \sin^3\alpha\cos\theta_- d\theta_- \wedge d\phi_-) \tag{4.29}$$

indeed reproduces the asymptotic form of the 5d field equation

$$\lambda'' + 4\lambda' = -3 \tag{4.30}$$

when substituted into that Maxwell equation.

The ultra-violet solution for $A_{(2)}$ provides a useful check at each stage of the computation of the full lift which we come to next.

4.2.2 The Metric

Pilch and Warner [30, 20] have provided an ansatz for the lift of a 5d supergravity flow to 10d (note that, although they study supersymmetric flows, their ansatz is not restricted to the supersymmetric solution of the second order equations of motion). In particular the ansatz provides us with the metric and dilaton. We can find the lifts we want as a limit of the metrics in [20]; that lift is of the $\mathcal{N} = 1^*$ GPPZ flows [28] in which three of the fermions are given a mass and the fourth develops a bilinear condensate. We review this lift in Appendix A. Setting the scalars equal (again to be precise one must set their $m = \sqrt{3/4}\lambda$ and $\sigma = \sqrt{1/4}\lambda$ to maintain a canonically normalized kinetic term) gives the metric we require (A.20)

$$ds_{10}^2 = \xi^{\frac{1}{2}} ds_{1,4}^2 + \xi^{-\frac{3}{2}} ds_5^2 \tag{4.31}$$

The metric of the deformed five sphere in their coordinates $(u^i, v^i, i = 1..3)$ is given by

$$ds_5^2 = c^2 du^i du_i - 4s^2 u.v du^i dv_i + c^2 dv^i dv_i + c^2 s^2 d(u.v)^2 \tag{4.32}$$

where

$$c = \cosh\lambda, \qquad s = \sinh\lambda \tag{4.33}$$

This metric is subject to the constraint

$$u^2 + v^2 = 1 \tag{4.34}$$

The warp factor is given by

$$\xi^2 = c^4 - 4s^4(u.v)^2 \tag{4.35}$$

We must move to more appropriate coordinates for our problem. The metric can be diagonalised by the change of coordinates

$$U^i_\pm = \frac{1}{\sqrt{2}}[u^i \pm v^i] \tag{4.36}$$

$$ds_5^2 = (c^2 - s^2[U_+^2 - U_-^2])dU_+^i dU_+^i + (c^2 + s^2[U_+^2 - U_-^2])dU_-^i dU_-^i + 4c^2 s^2 U_+^i U_+^j dU_+^i dU_+^j \tag{4.37}$$

The constraint can now be applied using the coordinates used in the UV limit above (r, α and two S^2 parametrized by $\theta_\pm, \phi_\pm$)

$$U_+^1 = \cos\theta_+ \cos\phi_+ \cos\alpha, \quad U_+^2 = \sin\theta_+ \cos\phi_+ \cos\alpha, \quad U_+^3 = \sin\theta_+ \cos\alpha$$
$$U_-^1 = \cos\theta_- \cos\phi_- \sin\alpha, \quad U_-^2 = \sin\theta_- \cos\phi_- \sin\alpha, \quad U_-^3 = \sin\theta_- \sin\alpha \tag{4.38}$$

The metric then takes the form

$$ds_5^2 = \xi_- \cos^2\alpha \; d\Omega_+^2 + \xi_+ \sin^2\alpha \; d\Omega_-^2 + \xi_+ \xi_- d\alpha^2 \tag{4.39}$$

where the $\xi_\pm$ are given by

$$\xi_\pm = c^2 \pm s^2 \cos 2\alpha \tag{4.40}$$

and the warp factor is

$$\xi^2 = \xi_+ \xi - \tag{4.41}$$

4.2.3 The Ricci Tensor

The calculation of the Ricci tensor is carried out by computer and the second order 5d flow equations for the scalar field are used throughout to simplify the expressions. The results are lengthy but we note that the non-zero components of the Ricci tensor are

$$R_{00} = R_{11} = R_{22} = R_{33}, \qquad R_{rr}, \qquad R_{\alpha\alpha}, \qquad R_{r\alpha} = R_{\alpha r}, \qquad R_{66} = R_{77}, \qquad R_{88} = R_{99} \tag{4.42}$$

4.2.4 The Dilaton

The dilaton can again be extracted from [20] where they provide

$$\mathcal{M} \ = \ \mathcal{S}\mathcal{S}^T \ = \ \frac{1}{\xi}\begin{pmatrix} \cosh^2\lambda & \sinh^2\lambda\cos 2\alpha \\ \sinh^2\lambda\cos 2\alpha & \cosh^2\lambda \end{pmatrix} \tag{4.43}$$

where (in unitary gauge)

$$\mathcal{S} = f\begin{pmatrix} 1 & B \\ B^* & 1 \end{pmatrix}, \qquad f = \frac{1}{(1-|B|^2)^{1/2}} \tag{4.44}$$

we thus find

$$f = \frac{1}{\xi^{1/2}}\sqrt{\frac{\cosh^2\lambda + \xi}{2}}, \qquad B = \frac{\sinh^2\lambda\cos 2\alpha}{\cosh^2\lambda + \xi} \tag{4.45}$$

Note that B is a real function and therefore from (4.18) the axion is zero for this flow, and the dilaton can be writen

$$e^{-\Phi} = \sqrt{\frac{\xi_-}{\xi_+}} \tag{4.46}$$

The r dependence implies the gauge coupling runs although finding the correct coordinate system to match it to the gauge theory will be difficult.

4.2.5 The 2-form and 4-form Potentials

We now move on to determining the potentials in the solution. Motivated by the UV limit we make an ansatz for the 2-form potential of the form

$$A_{(2)} = iA_+(\lambda(r),\alpha)\cos^3\alpha\,\cos\theta_+ d\theta_+\wedge d\phi_+ - A_-(\lambda(r),\alpha)\sin^3\alpha\,\cos\theta_- d\theta_-\wedge d\phi_- \tag{4.47}$$

where A_+ and A_- are arbitrary functions that become $\lambda(r)$ in the UV.

The non-vanishing components of the three form energy momentum tensor are then

$$T_{(3)0}^{\ 0} = T_{(3)1}^{\ 1} = T_{(3)2}^{\ 2} = T_{(3)3}^{\ 3} = -\frac{1}{8}[\mathcal{A}+\mathcal{B}+\mathcal{C}+\mathcal{D}] \tag{4.48}$$

$$T_{(3)r}^{\ r} = \frac{1}{2}[\mathcal{A}+\mathcal{C}] + T_{(3)0}^{\ 0} \tag{4.49}$$

$$T_{(3)\alpha}^{\ \alpha} = \frac{1}{2}[\mathcal{B}+\mathcal{D}] + T_{(3)0}^{\ 0} \tag{4.50}$$

$$T_{(3)6}^{\ 6} = T_{(3)7}^{\ 7} = \frac{1}{2}[\mathcal{A}+\mathcal{B}] + T_{(3)0}^{\ 0} \tag{4.51}$$

$$T_{(3)8}^{\ 8} = T_{(3)9}^{\ 9} = \frac{1}{2}[\mathcal{C}+\mathcal{D}] + T_{(3)0}^{\ 0} \tag{4.52}$$

$$T_{(3)\alpha}^{\ r} = \frac{1}{2}[G^{r67}G_{\alpha 67} + G^{r89}G_{\alpha 89}] \tag{4.53}$$

where the functions $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ and $\mathcal{D}$ are given by

$$\mathcal{A} = g^{rr}g^{66}g^{77}|G_{r67}|^2, \quad \mathcal{B} = g^{\alpha\alpha}g^{66}g^{77}|G_{\alpha 67}|^2,$$
$$\mathcal{C} = g^{rr}g^{88}g^{99}|G_{r89}|^2, \quad \mathcal{D} = g^{\alpha\alpha}g^{88}g^{99}|G_{\alpha 89}|^2 \tag{4.54}$$

We now turn to the 4-potential. The self duality condition of the five form field strenth is satisfied by construction using the ansatz

$$F_{(5)} = \mathcal{F} + \star\mathcal{F}, \qquad \mathcal{F} = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\omega(r,\alpha) \tag{4.55}$$

The non-vanishing components of the five form energy momentum tensor are

$$T_{(5)0}^{\ 0} = T_{(5)1}^{\ 1} = T_{(5)2}^{\ 2} = T_{(5)3}^{\ 3} = -T_{(5)6}^{\ 6} = -T_{(5)7}^{\ 7} = -T_{(5)8}^{\ 8} = -T_{(5)9}^{\ 9} = \mathcal{X} + \mathcal{Y} \tag{4.56}$$

$$T_{(5)r}^{\ r} = -T_{(5)\alpha}^{\ \alpha} = \mathcal{X} - \mathcal{Y} \tag{4.57}$$

where the functions $\mathcal{X}$ and $\mathcal{Y}$ are given by

$$\mathcal{X} = \frac{1}{2}g^{00}g^{11}g^{22}g^{33}g^{rr}(\frac{\partial\omega}{\partial r})^2, \qquad \mathcal{Y} = \frac{1}{2}g^{00}g^{11}g^{22}g^{33}g^{\alpha\alpha}(\frac{\partial\omega}{\partial\alpha})^2 \tag{4.58}$$

and

$$T_{(5)\alpha}^{\ r} = \frac{1}{2}g^{00}g^{11}g^{22}g^{33}g^{rr}(\frac{\partial\omega}{\partial\alpha}\frac{\partial\omega}{\partial r}) \tag{4.59}$$

To solve the supergravity equations we need to dissentangle the contributions from the 2-form potential and the 4-form. Furthermore, we need to seperate the function A_+ from A_-. We achieve this with the following combinations in which the 4-form cancels

$$R^7{}_7 - R^9{}_9 + 2R^r{}_r + 2R^\alpha{}_\alpha - 2T_{(1)r}^{\ r} - 2T_{(5)\alpha}^{\ \alpha} = \mathcal{A} + \mathcal{B} \tag{4.60}$$

$$R^9{}_9 - R^7{}_7 + 2R^r{}_r + 2R^\alpha{}_\alpha - 2T_{(1)r}^{\ r} - 2T_{(5)\alpha}^{\ \alpha} = \mathcal{C} + \mathcal{D} \tag{4.61}$$

Since λ is the only function of r in the solution we can separate out the pieces proportional to λ'^2 and those not, to distinguish between, for example, $\mathcal{A}$ and $\mathcal{B}$. A lengthy calculation, in which the 5d field equations are used repeatedly to simplify expressions, yields the simple result

$$A_\pm = \frac{\sinh 2\lambda}{\xi_\pm} \tag{4.62}$$

The remaining function in the 4-form potential $\omega(r,\alpha)$ can now be found using the equations

$$R^0{}_0 + R^r{}_r - T_{(1)r}^{\ r} - T_{(3)r}^{\ r} - T_{(3)0}^{\ 0} = \mathcal{X} \tag{4.63}$$

$$R^0{}_0 - R^r{}_r + T_{(1)r}^{\ r} + T_{(3)r}^{\ r} - T_{(3)0}^{\ 0} = \mathcal{Y} \tag{4.64}$$

In fact there is no angular dependence in ω and we find again the simple result

$$\frac{\partial \omega}{\partial \alpha} = 0, \qquad \frac{\partial \omega}{\partial r} = -\frac{4}{3} e^{4A(r)} V(r) \tag{4.65}$$

And hence

$$\omega(r) = e^{4A(r)} A'(r) \tag{4.66}$$

This completes the solution. The remaining equations of motion act as a check of the solution.

4.3 Brane Probing

The most succesful technique for connecting backgrounds and their dual field theories has been brane probing [1, 27, 26, 21, 35, 36, 37, 32] which converts the background to the U(1) theory on the probe's surface. We thus substitute the background into the Born-Infeld action which, since the 2-form field is entirely orthogonal to the probe directions, takes the form [27, 26]

$$S_{probe} = -\tau_3 \int_{\mathcal{M}_4} d^4x \det[G_{ab}^{(E)} + 2\pi\alpha' e^{-\Phi/2} F_{ab}]^{1/2} + \mu_3 \int_{\mathcal{M}_4} C_4, \tag{4.67}$$

where $C_{(4)}$ is the pull back of the 4-form potential on to the brane which corresponds here to the function w above. The resulting scalar potential is given by

$$V_{probe} = e^{4A} \left[\xi - A' \right] \tag{4.68}$$

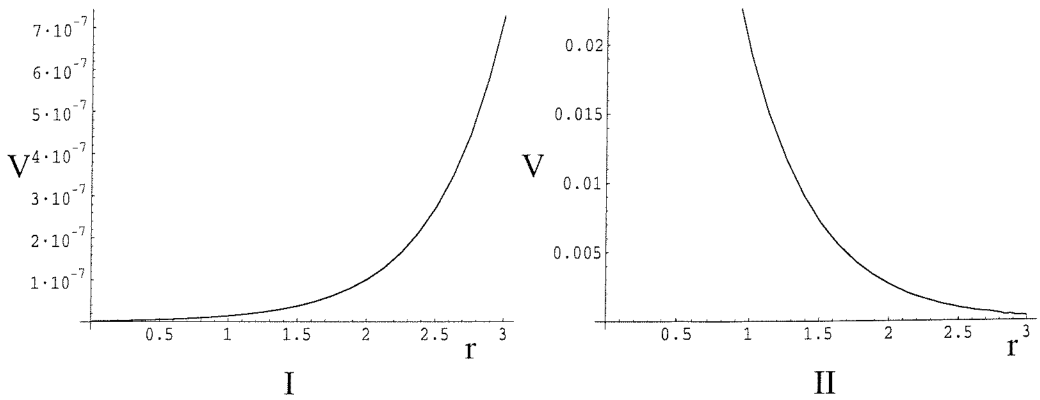

Figure 4.5: Plots of the probe potential in the infra-red for the mass only solution showing the stability of the solution (I) and the condensate only solution which is unstable (II).

4.3.1 Yang Mills* Boundary Conditions

It is illuminating to evaluate this potential at leading order in the ultra-violet with

$$\lambda = \mathcal{M}e^{-r} + ..., \qquad A = r + ... \tag{4.69}$$

We find

$$V = \mathcal{M}^2 e^{2r} + ... \tag{4.70}$$

Remembering that e^r has conformal dimension of mass this is an equal mass term for each of the 6 scalar fields. The field theory at large scalar vevs is bounded suggesting the set up is stable. Note that the potential's dependence on the angle α in ξ is subleading in the ultra-violet. The infra-red behaviour can be found numerically by solving (2.50) and (2.51) and a sample plot is shown in Fig 4.5. The potential is largely independent of α in the infra-red too. The plot supports the hypthesis that the scalar potential pins the probe at the origin of the space.

This background therefore appears to be the dual of a stable non-supersymmetric gauge theory in which all the adjoint matter fields are massive. The deep infra-red physics is pure Yang Mills. Both the curvature and

4.3.2 Fuzzy Sphere Boundary Conditions

Alternatively if we look at the other possible asymptotic solution

$$\lambda = \mathcal{K} e^{-3r} + ..., \qquad A = r + ... \tag{4.71}$$

We find

$$V = \mathcal{K}^2 e^{-2r} + ... \tag{4.72}$$

a condensate leaves a runaway potential. Again the infra-red behaviour can be found numerically (Fig 4.5) and shows the same behaviour as the asymptotic solution. This configuration, which asymptotically looks like the $F_{(3)}$ field we would expect a D5 in AdS to generate, is unstable to the emission of probe like D3 branes. This is not surprising since there is no force supporting the expansion of the D3s into a fuzzy D5 brane.

The other possible solutions with both a mass and a condensate present interpolate between the two forms of solution we've seen. In the infra-red they are unstable whilst in the ultra-violet the mass term dominates. In between there is a minimum of the probe potential. However, given the instability of the core structure there is probably little physics associated with this minimum.

4.4 Summary

In this chapter we have studied deformations of the AdS/CFT Correspondence which are bi-fermion masses or condensates in the field theory. The major challenge has been to lift the solutions to a complete 10d IIB supergravity background. The resulting background, summarized in the appendix, is surprisingly simple. In the last chapter, although there where fewer fields switched on, the 4-form potential involved a function defined by a complicated differential equation. In the case studied in this chapter all fields could be written down as simple functions of the 5d scalars A and λ. We have brane probed the solution in order to study the field theory scalar potential. For the mass only solution the probe potential is stable and the scalars massive. This theory is non-supersymmetric Yang Mills theory in the deep infra-red. We hope that this geometry will provide a new tool for studying Yang Mills theory. It should also be possible in the future to include probe D7 branes in the geometry and study the fermionic quark potential for chiral symmetry breaking. In the next few chapters we will explore the physics of Yang Mills* in more detail and, where possible, make comparisons with pure Yang Mills.

In the infra-red the dilaton and curvature blow up. Close to this singularity, stringy corrections will be important and the supergravity approximation is not valid. However, the singlularity in the interior of the space appear to play little role in the physics we study in latter chapters.

Any solution with a fermion condensate present generates an unstable probe potential. The asymptotic form of the solution suggests there is a D5 brane in the core of the geometry. We have interpreted these solutions as the geometries around a fuzzy D5 brane with no force supporting the non-commutative expansion.

Chapter 5

Wilson Loops

The Wilson loop is a well known gauge theory observable. We integrate the gauge connection around some closed loop in space-time and take the trace in some representation. If we consider the fundamental representation, then we can think of the Wilson loop as a quark-antiquark pair being created at some time, separated, and then brought back together to annihilate. The Wilson loop calculates the quark antiquark potential. We will see in the next section that if the Wilson loop goes like the area contained within the loop, then the quark-antiquark potential goes like the separation. This is a signal of confinement, so the Wilson loop is a natural object for us to study. The Wilson loop fits neatly into the idea of a stringy description of non abelian gauge theory. If quarks live on the ends of stings, then the Wilson loop is the boundary of an open string. In this chapter we will use our supergravity description of Yang Mills* to calculate Wilson loops. This will be done by placing a probe string in the supergravity background. Before we study Wilson loops in Yang Mills* it will be helpful to review Wilson loops in the AdS/CFT correspondence.

5.1 Wilson Loops in Supergravity

Consider a field theory in 4-d and a loop $\mathcal{C}$ embedded in 4 dimensional space. The Wilson loop is the path ordered integral of the gauge connection around this contour. It is defined by the operator

$$W_R(\mathcal{C}) = \mathrm{Tr}_R \exp[iP \oint_{\mathcal{C}} A] \tag{5.1}$$

The trace is taken over some representation R of the gauge group, but we will only be concerned with the fundamental. We can compute the quark-antiquark potential from the expectation value of the Wilson loop. Consider a rectangular loop of length L in the x-direction and length T in the time direction. This can be thought of as the creation of a quark-antiquark pair at a separation L propagating for a time T before annihilating. The expectation value of the Wilson loop goes like

$$\langle W \rangle \sim e^{-TE(L)} \tag{5.2}$$

$E(L)$ is the quark-antiquark potential. For a conformal field theory we expect $E(L) \propto \frac{1}{L}$ whereas for a confining theory we expect $E(L) \sim L$ leading to an area law behavior for the Wilson loop.

In the AdS/CFT correspondence we associate the flux tube of a confining theory with the fundamental string. Even in a non-confining theory, we would like to think of the end points of the open string as quarks. So we expect a Wilson loop to correspond to the boundary of an open string. The quark anti-quark interaction potential may be studied in AdS duals [24] by introducing a probe D3 brane into the geometry at some radius z_{max}. Fundamental strings between the probe and the central stack of D3 branes would represent W bosons which transform in the (N,1) of SU(N)×U(1) gauge group - we may equally think of these states as quarks since they are in the fundamental representation of SU(N). Thus a string attached to the probe with well separated ends play the role of a quark anti-quark pair with mass of order the energy scale determined by z_{max}. The action of the string corresponds to the interaction energy between the pair. To study such a configuration we necessarily require the 10d lifts of our deformed geometries since the string lives in 10d. The string will choose a minimal area configuration. Hence we compute a Wilson loop in supergravity by minimising the Nambo Goto Action for a String in AdS (or a deformed geometry).

$$\langle W \rangle \sim e^{-S(minimum)} \tag{5.3}$$

where in the Einstein frame the string action is

$$S = \int d^2\sigma e^{\frac{\phi}{2}} \sqrt{-\det \partial_a x^\mu \partial_b x^\nu G_{\mu\nu}} \tag{5.4}$$

If the supergravity solution was flat the string would just lie flat giving the confining result $S = LT \Rightarrow E(L) = L$. However in AdS the worldsheet of the string enters the interior because the metric diverges on the boundary. We can write the AdS metric as

$$ds^2 = u^2 dx^\mu dx_\nu + \frac{du^2}{u^2} \tag{5.5}$$

Again, consider a rectangular loop of length L in the x-direction and T and the time direction. We choose world sheet coordinates such that $\sigma = x$ and $\tau = t$. As the string will extend into AdS we give it a profile $u = u(x)$. For a time invariant configuration we have

$$S = T \int dx \sqrt{(\partial_x u)^2 + u^4} \tag{5.6}$$

As the action does not depend explicitly on x the hamiltonian is conserved. The hamiltonian is

$$\frac{u^4}{\sqrt{u^4 - \dot{u}^2}} = u_0^2 \tag{5.7}$$

We can rearrange for $\dot{u}$, and hence fix the constant to be u_0^2. We obtain the separation as a function of u_0 by integrating with respect to u,

$$L(u_0) = 2 \int_{u_0}^{u_{max}} du \frac{u_0^2}{u^2 \sqrt{u^4 - u - 0^4}} \tag{5.8}$$

We can also substitute for $\dot{u}$ in the action to obtain

$$E(u_0) = \int_{u_0}^{u_{max}} du \frac{u^2}{\sqrt{u^4 - U_0^2}} \tag{5.9}$$

A parametric plot of E against L (fig. 5.1) reveals the expected behaviour.

This plot is consistent with the the calculation in [59], where in is shown that

$$E(L) \propto const. - \frac{1}{L} \tag{5.10}$$

The constant is due to the mass of the W-boson comming from strings stretching from the probe to the stack, and should be subtracted. This leaves the $\frac{1}{L}$ behaviuor expected from a conformal field theory.

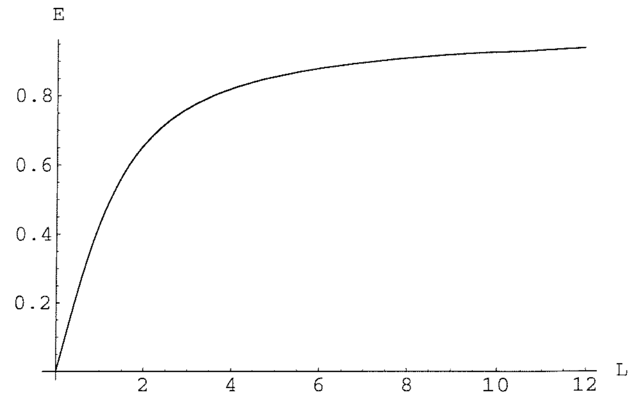

Figure 5.1: Plot of energy against separation for Wilson loops in AdS.

We can consider this calculation in the more general context of a deformed geometry. The 10d metric is

$$ds_{10}^2 = \xi^{\frac{1}{2}} ds_{1,4}^2 + ds_5^2 \tag{5.11}$$

$ds_{1,4}^2$ is a solution of 5d supergravity of the form (2.48). ξ is a warp factor necessary for the lift to 10d which may depend on the scalars of 5d supegravity. in this case the action for the string is

$$S = T \int dx e^{\frac{\phi}{2}} \xi^{\frac{1}{2}} e^A \sqrt{(\partial_x r)^2 + e^{2A}} \tag{5.12}$$

The factor $\xi^{\frac{1}{2}}$ plays the role of the string tension. We can bring this into the same form as (5.6) with the coordinate change

$$\frac{\partial u}{\partial r} = \xi^{\frac{1}{2}} e^A \tag{5.13}$$

Then the action takes the form

$$S = T \int dx \sqrt{(\partial_x u)^2 + \Omega} \tag{5.14}$$

where

$$\Omega = \xi e^{\phi} e^{4A} \tag{5.15}$$

The Wilson loop is determined by the behaviour at small u of the function Ω. Suppose $\Omega \sim u^{\gamma}$ in the infra-red. Then $\gamma = 4$ is the conformal case. If, however, $\gamma \leq 0$ then there will be area law behaviour and confinement.

We can also consider, in a similar way, t'Hooft loops. These are obtained from Wilson loops by electro-magnetic duality. The electro magnetic duality of $\mathcal{N} = 4$ super Yang Mills is dual to the S duality of Type IIB. Thus, we can study t'Hooft loops and magnetic monoploes by exchanging fundamental strings with D-strings in the above.

5.2 Wilson Loops in Yang Mills*

We will now employ the ideas developed in the previous section to explore the behaviour of Wilson loops in Yang Mills*. We can calculate the Wilson loop behaviour by lying a string in an $x_{//}$ direction and letting it move in r and α. The action for the string in Einstein frame is given by

$$S = \int d^2\sigma e^{\Phi/2} \sqrt{\det G} \tag{5.16}$$

In the z coordinates ($ds^2 = e^{2A(z)}(dx^2 + dz^2)$) we obtain

$$S = T \int dx\, e^{2A} \sqrt{\xi_+} \sqrt{1 + \left(\frac{dz}{dx}\right)^2 + e^{-2A} \left(\frac{d\alpha}{dx}\right)^2} \tag{5.17}$$

The resulting equations of motion for z and α are then

$$\frac{d}{dx}\left[\frac{e^{2A}\sqrt{\xi_+} z'}{\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}}\right] - \frac{d}{dz}\left[e^{2A}\sqrt{\xi_+}\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}\right] = 0 \tag{5.18}$$

$$\frac{d}{dx}\left[\frac{e^{2A}\sqrt{\xi_+} \alpha'}{\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}}\right] - \frac{d}{d\alpha}\left[e^{2A}\sqrt{\xi_+}\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}\right] = 0 \tag{5.19}$$

These can be solved numerically but there are an array of solutions. The new feature relative to pure AdS is that the strings can have a non-trivial α profile. The reason for this is that the fermion masses we introduced broke the SO(6) symmetry to $SO(3) \times SO(3)$ so there are potentially different strings connecting particles in different subgroups of the SO(6). To begin with lets concentrate on strings that are between two identical particles and hence have no α variation. There are such strings - in the α equation of motion the last potential term is given by

$$-\frac{d\sqrt{\xi_+}}{d\alpha}\left[e^{2A}\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}\right] = -\frac{\sinh^2\lambda \sin 2\alpha}{\sqrt{\xi_+}}\left[e^{2A}\sqrt{1 + z'^2 + e^{-2A}\alpha'^2}\right] \tag{5.20}$$

so vanishes if $\alpha = n\pi/2$. The kinetic term also vanishes if α =constant so these are solutions of the equations of motion.

Initially we study the case $\alpha = \pi/2$ which implies $\xi_+ = 1$. The z equation of motion then becomes

$$\frac{d}{dx}\left[\frac{e^{2A}z'}{\sqrt{1+z'^2}}\right] - \frac{d(e^{2A})}{dz}\sqrt{1+z'^2} = 0 \tag{5.21}$$

This is straight forward to solve in the background $A(z)$ appropriate to YM*.

Consider initial conditions where we start the string at $z = -0.5$, which is in the AdS like region. We then vary the derivative of z' and shoot off strings to more negative z corresponding to the interior of the deformed AdS space - we show numerical results of this type in Fig 5.2. Initially as z' increases the strings penetrate the geometry more and return to $z = -0.5$ further out indicating that they describe a quark anti-quark pair that are more widely separated. This is standard AdS behaviour. However when the strings begin to enter the deformed space the behaviour changes. At a critical value of z' the string, although still penetrating deeper into the space, returns to $z = -0.5$ at a *shorter* quark separation. Thus there is a maximum quark separation allowed by the geometry.

To get a better handle on this behaviour we can use the x independence of the Lagrangian which implies the Hamiltonian is conserved

$$\frac{\sqrt{\xi_+}e^{2A}}{\sqrt{1+z'^2+\alpha'^2e^{-2A}}} = \text{constant} \tag{5.22}$$

Thus, for α constant,

$$z' = \sqrt{\frac{e^{4A}\xi_+}{c^2} - 1} \tag{5.23}$$

Positioning a string so that at $x = 0$ $z = z_0$ and $z' = 0$, we find:

$$c^2 = \xi_+(z_0)e^{4A(z_0)} \tag{5.24}$$

The quark anti-quark separation is then given by

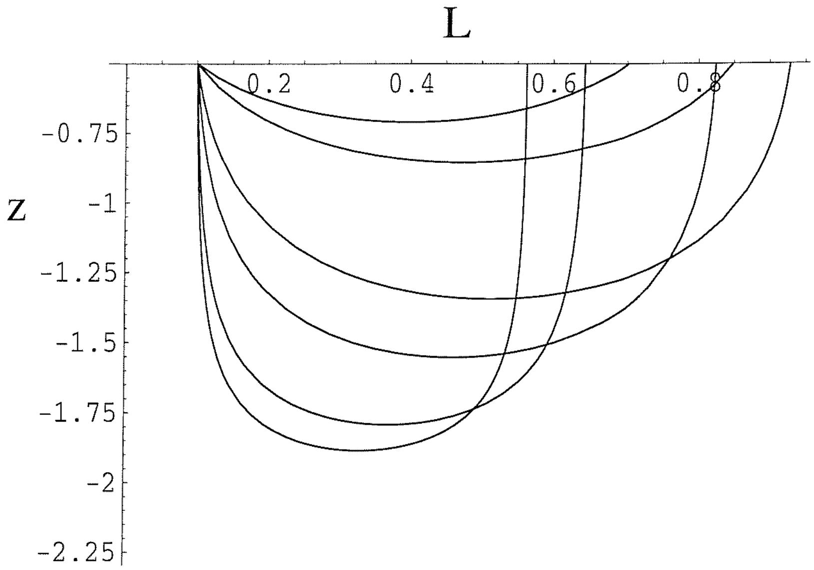

Figure 5.2: Various Wilson loops in YM* showing how the depth the probe string penetrates into the deformed space depends on the quark separation (or equivalently the initial condition of z').

$$\frac{L}{2} = \int_{z_0}^{z_{max}} dz \frac{1}{\sqrt{\frac{e^{4A}\xi_+}{c^2} - 1}} \tag{5.25}$$

The energy of the string is given by S/T so we find

$$E = \frac{1}{\pi c} \int_{z_0}^{z_{max}} dz \frac{e^{4A}\xi_+}{\sqrt{\frac{e^{4A}\xi_+}{c^2} - 1}} \tag{5.26}$$

These equations are again straightforward to solve numerically in the YM* background. In Fig 5.3 we show plots of the quark anti-quark separation vs z_0, the energy of the string vs z_0 and finally the energy of the string vs quark anti-quark separation. We see again the maximum separation but now also that the strings that penetrate into the interior of the deformed space are energetically disfavoured. The physical solutions are those that match to the AdS like solutions for small quark separation. The energy of the string connecting the quarks grows until some critical value above which there are no longer strings.

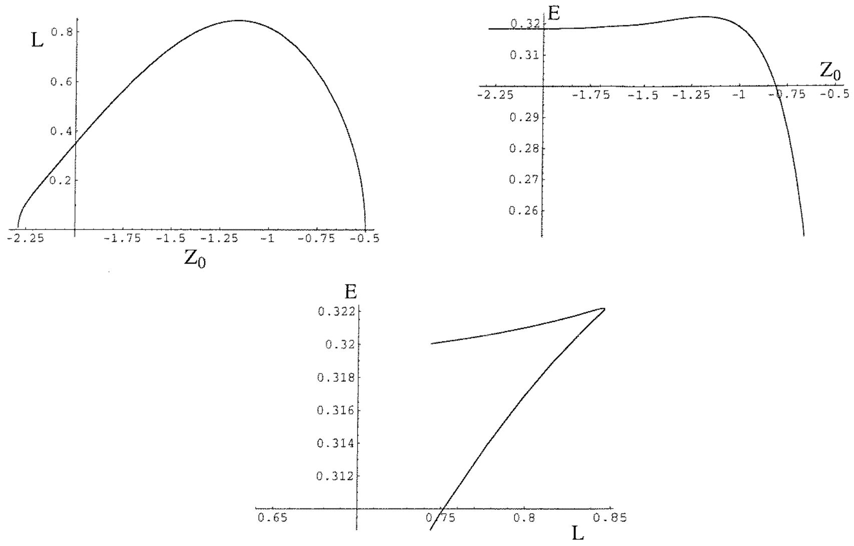

Figure 5.3: YM* Wilson loop results at $\alpha = \pi/2$; we plot the quark separation L vs the maximum depth z_0 into the space that the string reaches; the energy of the string vs the maximum depth; and finally the energy vs the quark separation.

This behaviour strongly suggests string breaking by quark anti-quark pair production. We certainly have finite mass quarks in our theory so might expect string breaking to occur. On the other hand the gauge background is at large N where finite quark flavour number effects should be suppressed. Presumably by studying the string probe we have moved away from infinite N. The situation is similar to studying a fermion gap equation in a large N background. Such an equation produces a dynamical quark mass reflecting the presence of a quark condensate which can only be present if pair production by the vacuum is allowed. Thus finite N_f effects are present in the gap equation. The success of quenched QCD on the lattice is another example of such an effect - a valence quark in a quenched background nevertheless acts as if there are quark condensates present. The same must be true here, that studying a single quark

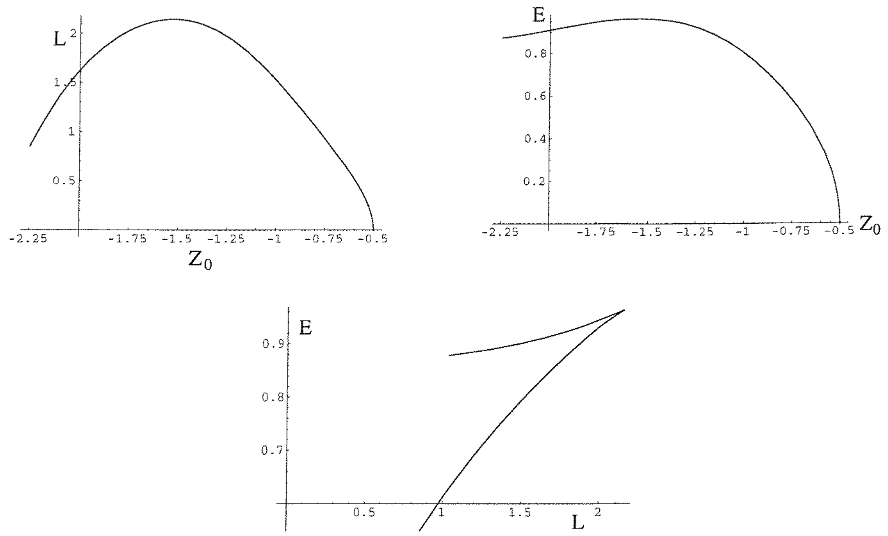

Figure 5.4: YM* Wilson loop results at $\alpha = 0$; we plot the quark separation L vs the maximum depth z_0 into the space that the string reaches; the energy of the string vs the maximum depth; and finally the energy vs the quark separation.

anti-quark pair introduces vacuum effects of those quarks.

The second constant α solution ($\alpha = 0$) has the same qualitative behaviour, as shown in Figure 5.4, although the maximum length and energy of the string differ. This presumably reflects the SO(6) symmetry breaking which means that quarks in different subgroups of SO(6) have different energies in the IR of the theory.

We have also looked at strings with varying α, corresponding to interactions between quarks in different parts of the broken SO(6), by solving the full equations of motion numerically. The qualitative behaviour is similar with a maximum string length being found. If the starting values for α and its derivative are close enough to one of the constant solutions, that is we choose the endpoint to be a quark of definite flavour, the α variation is not relevant, and the shapes for string loops are the same as the constant case. However in the general case ($\alpha_0 \neq n\pi/2$) the infra-red behaviour of the string can be quite complicated. We show a sample behaviour in Fig 5.5. The z

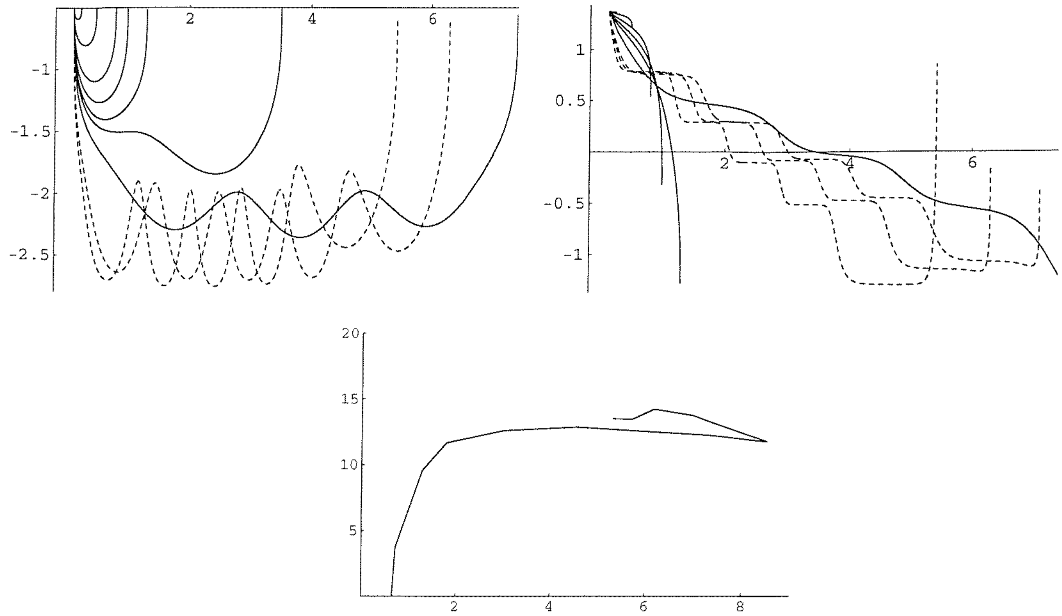

Figure 5.5: YM* Wilson loop results with initial conditions $\alpha = \pi/2 - 0.2$ and $\alpha' = 0.5$, for varying z'; we plot z and α vs x and energy vs quark separation for these cases. In the first two plots, the solid lines correspond to solutions in the regime where separation between quarks increase with z_0'; the dashed lines correspond to decreasing separation.

and α dependence of a set of solutions are shown as a function of z'. We also plot the energy of the string vs quark separation. As α moves away from zero it performs a series of jumps with an associated oscillation in the z position of the string. Only when α returns to $n\pi/2$ does the string re-emerge from the interior. The interpretation of this behaviour is unclear but the existence of a maximum length string is again apparent. Strings that penetrate deep into the deformed region are also again numerically disfavoured. We could interpretate the step-like behaviour for α in terms of the broken $SO(6)$; if we take as the initial condition a superposition of quarks of different types at an endpoint, and give enough energy to the system, the interior of the flux tube starts exploring the creation of quark pairs of different kinds.

An important part of this analysis is that the singular part of the geometry at small radius manages to hide itself from the physics. Strings that reach down into the deformed geometry towards the singularity are not physical states. This ties in well with the interpretation that the singularity of the geometry is dual to the coupling of the gauge theory becoming infinite (as happens in the $\mathcal{N} = 2^*$ theory). The string contains enough energy to pair create quarks and break before we reach IR scales where the coupling actually diverges. The precise details of the very strong coupling regime are not relevant to the physics.

We also note that the case of a D-string (that is having magnetic monopoles in place of quarks) just exchanges ξ_+ with ξ_- in (5.17). This is just a shift in α of $\pi/2$, so all the results can be carried over to the magnetic monopole case.

Chapter 6

Glueballs and Other Bound States

In Chapter 4 we deformed the AdS-CFT correspondence by including a Supergravity scalar corresponding to an equal mass term for all four adjoint fermions. We call the mass deformation Yang Mills*. As well as describing a mass deformation, the second order equations of motion for the scalar can also describe a condensate for the fermion operator. The mass only solution is a unique flow that requires the boundary conditions to be arbitrarily fine tuned. Even in the mass only case, the background appears to be singular in the infra-red. The precise interpretation of the singularity remains unclear. For example the backgrounds describing $\mathcal{N} = 4$ Super Yang Mills on moduli space [19] are singular but those singularities are understood to correspond to the presence of D3 branes in the solution. In the $\mathcal{N} = 2^*$ theory [22, 23, 25, 27, 26] the singularities correspond to the divergence of the running gauge coupling. On the other hand the backgrounds of Klebanov Strassler [43] and Maldacena Nunez [42] are championed for their smooth behavior. In this chapter we will further explore the IR behavior of Yang Mills*.

The AdS/CFT correspondence can also be deformed to $\mathcal{N} = 1^*$ [28] by giving an equal mass term to three of the four adjoint fermions. A condensate for the massless gluino results in a range of flows. This geometry is also singular (at least at the 5d level) and indeed not all flows are physical. However provided the condensate is not too large, these backgrounds are physical and the spectra of bound states can be calculated. In fact, we will find that the results of the smooth Klebanov Strassler background for the glueball mass spectrum are reproduced accurately.

In this chapter we will study Yang Mills* and $\mathcal{N} = 1^*$ theory's in the IR. We will compute the mass spectrums of various bound states by looking at fluctuations of the dual Supergravity scalars. In particular, we will calculate the mass spectrum of glueballs and of bound states of the adjoint fermions in both theories. We begin in the next section with a review of the relevant formalism. In section 3 we will study $\mathcal{N} = 1^*$, and in section 4 we will study Yang Mills*.

6.1 N=1*

The $N = 1^*$ theory is $\mathcal{N} = 4$ super Yang Mills with equal mass terms for the three adjoint chiral superfields leaving just the vector multiplet massless, with a gaugino condensate. The field theory in the large N limit has been studied in [33] and it has been shown to have a set of discrete vacua differentiated by the magnitude of the gaugino condensate which is real. The $N = 1^*$ geometry is obtained by taking the scalars m and σ from the 10 of SO(6) dual to the dimension 3 operators

$$\mathcal{O}_m = \sum_{i=2}^{4} \psi_i \psi_i, \qquad \mathcal{O}_\sigma = \psi_1 \psi_1 \tag{6.1}$$

The Potential for these two scalars is [28]

$$V = -\frac{3}{8}[\cosh^2(\frac{2m}{\sqrt{3}}) + 4\cosh(\frac{2m}{\sqrt{3}})\cosh(2\sigma) - \cosh^2(2\sigma) + 4] \tag{6.2}$$

This potential can be obtained from a superpotential

$$W = -\frac{3}{4}[\cosh(\frac{2m}{\sqrt{3}}) + \cosh(2\sigma)] \tag{6.3}$$

This ensures that N=1 supersymmetry is preserved and that the flow equations reduce to first order

$$\begin{aligned}
\frac{\partial \sigma}{\partial z} &= -\frac{3}{2}e^{A}\sinh(2\sigma) && (6.4)\\
\frac{\partial m}{\partial z} &= -\frac{\sqrt{3}}{2}e^{A}\sinh(\frac{2m}{\sqrt{3}}) && (6.5)\\
\frac{\partial A}{\partial z} &= \frac{1}{2}e^{A}[\cosh(\frac{2m}{\sqrt{3}}) + \cosh(2\sigma)] && (6.6)\\
& && (6.7)
\end{aligned}$$

In the ultra violet $z \to 0$ we find

$$\begin{aligned}
m &\to -az && (6.8)\\
\sigma &\to -bz^3 && (6.9)\\
A &\to -\log|z| && (6.10)\\
& && (6.11)
\end{aligned}$$

From (2.66), m is a mass for three of the adjoint fermions and σ is a gaugino condensate. The geometry is singular and not all flows are physical. The allowed flows, with 'good' singularities, satisfy [15]

$$\begin{aligned} b &\leq 3^{-3/2}a \\ &\approx 0.19a \end{aligned} \tag{6.12}$$

In what follows we will set a=1.

6.1.1 Glueballs

The mass spectrum of bound states can be obtained from fluctuations of the dual scalar field by finding the eigenvalues of (2.60). We will calculate the mass spectrum of glueballs with quantum numbers $J^{PC} = \mathcal{O}^{++}$ and correlators

$$\langle \mathrm{Tr}\, F^2(x) \mathrm{Tr}\, F^2(y) \rangle \tag{6.13}$$

The operator $\mathcal{O} = \mathrm{Tr}\, F^2$ is dual to the dilaton. The dilaton does not contribute to the supergravity potential, so that the potential for glueballs is just (2.68)

$$U = \frac{3}{2}A'' + \frac{9}{4}(A')^2 \tag{6.14}$$

In order to obtain a discrete spectrum with a mass gap U must be bounded below. This is indeed the case for all physical flows satisfying the bound (6.12). Unphysical flows, which violate this bound, have an unbounded glueball potential.

The eigenvalues of (2.60), and hence the glueball mass spectrum, can be obtained using the numerical shooting method. The results for b=0 and b=0.19 are shown in table 6.1. The glueball mass spectrum obtained from the $\mathcal{N} = 1$ Klebanov Strassler model [43, 53, 56] is included for comparison, and agrees very well the $\mathcal{N} = 1^*$ results.

State	N=1* (b=0)	N=1*(b=0.19)	N=1 KS
0^{++}	1.0(input)	1.0(input)	1.0(input)
0^{++*}	1.5	1.5	1.5
0^{++**}	2.0	1.9	2.0
0^{++***}	2.5	2.3	2.5
0^{++****}	3.0	2.7	3.0

Table 6.1: Spectrum of $\mathcal{N} = 1^*$ Glueball masses from supergravity with b=0 and b=0.19. In the last column the N=1 glueball spectrum obtained from Klebanov Strassler background is shown for comparison. In all cases the 0^{++} mass has been scaled to 1.

The 0^{++} mass is not a prediction, but just sets the scale , so we normalise the lowest state to 1 in all cases.

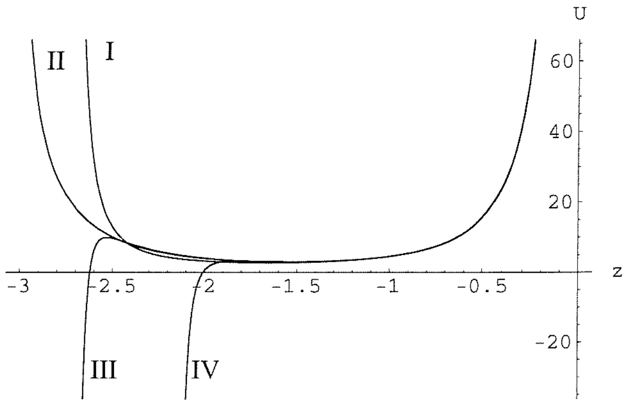

Figure 6.1: The Glueball potential in $\mathcal{N} = 1^*$ with $a = 1$ and: I b=0.1 ; II b=0.19 ; III b=0.2 ; IV b=0.25

6.1.2 Gluino Bound States

We now move on to investigate the masses of bound states of the fermions of the $\mathcal{N} = 1^*$ theory. For simplicity we will set $\sigma = 0$ ($b = 0$) in what follows. To study bound states of the massless gluino we look at fluctuations of the scalar σ. The corresponding potential (2.61) is

$$U_\sigma = \frac{3}{2}A'' + \frac{9}{4}(A')^2 + 3e^{2A}[1 - 2\cosh(\frac{2m}{\sqrt{3}})] \tag{6.15}$$

Numerically plotting this potential reveals it to be bounded giving a discrete spectrum with a mass gap. As in the case of the glueballs, we can find the eigenvalues of (2.60) with the shooting method. The results are shown in table 6.2 below. The gravity dual describes a moduli space of vacua corresponding to different background values of σ so we would expect to find a Goldstone boson. In fact we do and the smallest eigenvalue is zero.

It's interesting to note that if we study fluctuations of the supergravity scalar m which corresponds to a composite operator of all three fermions (6.1) we obtain the potential

$$U_m = \frac{3}{2}A'' + \frac{9}{4}(A')^2 + e^{2A}[-2\cosh(\frac{2m}{\sqrt{3}}) - \cosh(\frac{4m}{\sqrt{3}})] \tag{6.16}$$

This potential is unbounded. This may correspond to some instability in the field theory.

To examine bound states of a single species of massive fermion we need to subdivide the operator $\mathcal{O}_m$ further. We introduce the scalars μ and ν corresponding to

$$\mathcal{O}_\mu = \psi_2\psi_2, \qquad \mathcal{O}_\nu = \sum_{i=3}^{4} \psi_i\psi_i \tag{6.17}$$

This enables us to study bound states of the massive fermion ψ_2 by looking at fluctuations of ν. In terms of these scalars the potential is [25]

State	ψ_1	ψ_2
1	0.0	1.2
2	1.0	1.7
3	1.6	2.2
4	2.1	2.8
5	2.7	3.3

Table 6.2: The first five bound states of the massless gluino ψ_1 and the massive fermion ψ_2 in $\mathcal{N} = 1^*$ form supergravity.

$$\begin{aligned} V &= \tfrac{1}{8}[-5 + \cosh(4\mu) - 4\cosh(2\mu)] - \cosh(\sqrt{2}\nu)[\cosh(2\mu) + 1] \\ &+ \tfrac{1}{16}\left[-3 + 2\cosh(2\sqrt{2}\nu) + \cosh(4\mu)\right] \end{aligned} \tag{6.18}$$

Of course, we are still interested in preserving N=1 supersymmetry. If we set

$$\nu = \sqrt{\frac{2}{3}}m, \qquad \mu = \frac{m}{\sqrt{3}} \tag{6.19}$$

then we recover the $\mathcal{N} = 1^*$ potential with $\sigma = 0$. The Schroedinger potential we are interested in for fluctuations in μ about this background is

$$U_\mu = \frac{3}{2}A'' + \frac{9}{4}(A')^2 + e^{2A}\left[-2 - 2\cosh(\frac{2m}{\sqrt{3}}) + \cosh(\frac{4m}{\sqrt{3}})\right] \tag{6.20}$$

This is indeed a bounded potential. The mass spectrum is found by shooting and the results are displayed in table 2, normalized to the lightest glueball mass above. We find that the fermion bound states are a little heavier than the lightest glueball but far from completely decoupled.

6.2 Yang Mills*

The Yang Mills* geometry is obtained by taking a scalar λ from the 10 of SO(6) dual to the operator

$$\mathcal{O} = \sum_{i=1}^{4} \psi_i \psi_i \tag{6.21}$$

The potential for the scalar can be obtained from the $N = 1^*$ solution of [28] by setting their two scalars equal (to be precise one must set their $m = \sqrt{3/4}\lambda$ and $\sigma = \sqrt{1/4}\lambda$ to maintain a canonically normalized kinetic term)

$$V = -\frac{3}{2}\left(1 + \cosh^2\lambda\right) \tag{6.22}$$

In this case $m^2 = -3$ and the ultra-violet solutions are

$$\lambda = \mathcal{M}e^{-r} + \mathcal{K}e^{-3r} \tag{6.23}$$

The field theory operator has dimension 3. Thus in what follows $\mathcal{M} = 0$ corresponds to a solution with just bi-fermion vevs while $\mathcal{K} = 0$ corresponds to the purely massive case. Giving a mass to all four fermions breaks supersymmetrey completely and we expect that that the deep infra-red should be pure Yang Mills.

6.2.1 Glueballs

As mentioned in chapter 4, the glueball potential for the mass only case is a bounded well, fig. 4.3. We calculate the glueball mass spectrum by shooting, and the results are shown in table 6.3. The lightest glueball state is not a prediction, but can be

State	Lattice	Yang Mills*	AdS-Schwarz
0^{++}	1.00 ± 0.09(input)	1.0(input)	1.0(input)
0^{++*}	1.54 ± 0.14	1.5	1.6
0^{++**}	-	1.9	2.1
0^{++***}	-	2.3	2.7
0^{++****}	-	2.7	-

Table 6.3: Spectrum of glueball masses from the lattice, from Yang Mills* and Witten's AdS-Schwarz dual. Again, the lowest glueball mass has been scaled to one in all cases.

used to fix the scale Λ_{QCD}. As in the $\mathcal{N} = 1^*$ case we normalize the lowest state to 1. Unfortunately only the first excited state has been calculated on the lattice [52], so there is little data to compare. However the agreement for this state is very good. Witten [3] found a high temperature deformation of the gravity dual of the field theory on the surface of an M5 brane which is expected at low energies to describe a 4 dimensional non-supersymmetric Yang Mills theory (but in the UV there are extra adjoint matter fields that live in 6d). The glueball mass spectrum has also been computed in this model [51] and the results are shown in table 6.3 for comparison.

6.2.2 Fermion Bound States

We will calculate the mass spectrum of bound states of the massive adjoint fermions. a single species of fermion is dual to the scalar σ. The potential (2.61) for this scalar in terms of the Yang Mills* scalar λ is

$$U_\sigma = \frac{3}{2}A'' + \frac{9}{4}(A')^2 - 3e^{2A} \tag{6.24}$$

As in the $N = 1^*$ case, this is a well potential and the spectrum of bound states is calculable. The results are shown in table . The potentials (2.61) for m and λ are

State	mass
1	0.9
2	1.3
3	1.7
4	2.1
5	2.6

Table 6.4: First five bound states of a massive adjoint fermion in Yang Mills*.

$$U_m = \frac{3}{2}A'' + \frac{9}{4}(A')^2 - 4e^{2A}\cosh^2\lambda \tag{6.25}$$

$$U_\lambda = \frac{3}{2}A'' + \frac{9}{4}(A')^2 - e^{2A}\cosh 2\lambda \tag{6.26}$$

These are both unbounded and it is not possible to calculate a mass spectrum in these cases.

6.3 Summary

In this chapter we have studied the mass spectrum of bound states in Yang Mills* and $\mathcal{N} = 1^*$ deformations of the AdS/CFT correspondence. In the case of $\mathcal{N} = 1^*$ we have computed the first five glueball masses for the two extremes of the allowed flows($b = 0$ and $b = 0.19$). The flow without a gluino condensate agrees to 2 significant figures with results obtained from the Klebanov Strassler background. We also computed the mass spectrum of bound states of the adjoint fermions. We found that bound states of the massless fermion where heavier than bound states of the massive fermion. We also compute the spectrum of glueball masses from Yang Mills*. There is only one excited state calculated on the lattice for comparison, but the agreement is good. We also compare with AdS-Schwarzchild background, and find reasonable agreement. However, the higher excited states are more massive in this case, so it would be interesting to see lattice results for these states. We observe that all the glueball mass spectrums we have considered are similar. This suggests that the glueball mass spectrum is not very sensitive to the details of a particular theory. In both $\mathcal{N} = 1^*$ and Yang Mills* we found that Schrodinger potential for bound states of three or four species of adjoint fermion led to an unbounded potential. This presumably corresponds to some instability in the field theory. Encouragingly the Yang Mills* gravity dual appears to encode much of the physics we would expect of non-supersymmetric Yang-Mills theory.

Chapter 7

Conclusions

‘ Back off, Man. I'm a scientist.’

-Dr. Peter Venkman, ‘Ghostbusters’.

The majority of this thesis has been devoted to studying Yang-Mills*, a stable non-supersymmetric deformation of $\mathcal{N} = 4$ Super Yang-Mills. The $\mathcal{N} = 4$ theory is deformed by giving masses to all four adjoint fermions. The dual scalar has been identified and the geometry studied at the level of 5d gauged supergravity. The 5d background was used to compute glueball masses in the dual field theory. We have lifted the 5d geometry to 10d and found a complete type IIB supergravity solution. Brane probing this solution has demonstrated that the geometry is stable and that the six adjoint scalars in the duel field theory acquire masses. The IIB solution was also used to compute Wilson loops. The supergravity solution appears to encode much of the infra-red physics we would expect from non-supersymmetric Yang Mills. We have seen evidence for confinement, string breaking and glueballs.

The glueball mass spectrum is in excellent agreement with the limited lattice data, and with results from finite temperature duals. We have also calculated glueball masses in $\mathcal{N} = 1^*$ and compared with results from $\mathcal{N} = 1$ Klebanov Strassler background. These are in good agreement with each other, and are also very similar to the non-supersymmetric results. This suggests that the glueball mass spectrum is not very sensitive to the details of a particular theory.

Our Wilson loop results suggest string breaking. When quarks are separated in QCD, the flux tube between them will eventually break, producing another quark-antiquark pair out of the vacuum. It is very interesting to see this phenomena in our model, especially as it has not been seen in other confining gravity duals. It is not clear at this stage why we see string breaking in Yang-Mills* but not in other gravity duals.

A direction for future work on Yang-Mills* is the inclusion of fundamental fermions (quarks). The construction in [58], reviewed in chapter 2, should be a fruitful approach. This would enable us to study chiral symmetry breaking and meson mass spectrums in a 4d non-supersymmetric model from the perspective of supergravity.

Appendix A

The Lift to Ten Dimensions for the Metric and Dilaton

We will give a brief sketch of the machinery for lifting from five to ten dimensions, and the complete metric for $\mathcal{N} = 1^*$. The following is a summary of the relavant parts of [22]. The lift to ten dimensions is achieved by multiplying the 5d metric by a warp factor, Ω^2, and adding the metric of a deformed five sphere $ds^5 = g_{mn}dy^m dy^n$,

$$ds_{10}^2 = \Omega^2 ds_{1,4}^2 + ds_5^2 \tag{A.1}$$

where $ds_{1,4}^2$ is the metric of of the $\mathcal{N} = 8$ supergravity in five dimensions. The inverse metric, g^{pq}, on the deformed five sphere is given by

$$\Delta^{-\frac{2}{3}} g^{pq} = \frac{1}{a^2} K^{IJp} K^{KLq} \mathcal{V}_{IJab} \mathcal{V}_{KLcd} \Omega^{ac} \Omega^{bd} \tag{A.2}$$

where $\mathcal{V}$ is the scalar matrix of the $E_{6(6)}/USp(8)$ coset of 5d gauged supergravity, K are killing vectors on S^5, and Ω^{ab} is the $USp(8)$ symplectic form, and $\Delta =$

$\det^{\frac{1}{2}}(g_{mp}h^{pq})$, where H^{pq} is the inverse of the metric on the "round" S^2. The quantity Δ can be found by taking the determinant of both sides of (A.2), and is related to the warp factor by $\Omega^2 = \Delta^{-\frac{2}{3}}$.

The IIB dilton axion matrix S can be obtained, up to an $SO(2)$ gauge choice, from

$$\Delta^{-\frac{4}{3}}(SS^T)^{\alpha\beta} = \text{const} \times \epsilon^{\alpha\gamma}\epsilon^{\beta\delta}\mathcal{V}_{I\gamma}^{ab}\mathcal{V}_{J\delta}^{cd}\Omega_{ac}\Omega_{bd} \tag{A.3}$$

The remaining fields, the three forms and self dual five form, are not provided by this ansatz. However, they can be found by solving the IIB equation of motion for a given ten dimensional metric, dilaton and axion.

We will now present the lift of the metric and dilaton with the scalars m and σ turned on. These scalars are dual to the field theory operators

$$\mathcal{O}_m = \sum_{i=2}^{4} \psi_i\psi_i, \qquad \mathcal{O}_\sigma = \psi_1\psi_1 \tag{A.4}$$

The Potential for these two scalars is

$$V = -\frac{3}{8}[\cosh^2(\frac{2m}{\sqrt{3}}) + 4\cosh(\frac{2m}{\sqrt{3}})\cosh(2\sigma) - \cosh^2(2\sigma) + 4] \tag{A.5}$$

The lift of the metric is given in terms of $d\hat{s}_5^2$ where

$$ds_5^2 = \xi^{-\frac{3}{2}}\, d\hat{s}_5^2 \tag{A.6}$$

The metric for the five sphere is written in terms of the six coordinates u_i, v_i, $i = 1..3$ with a constraint $u^2 + v^2 = 1$.

$$d\hat{s}_5^2 = a_1\, du^i du^i + 2\, a_2\, du^i dv^i + a_3\, dv^i dv^i + a_4\, (d(u\cdot v))^2 + 2\, a_5\, (v^i du^i)(v^j du^j) + 2\, a_6\, (u^i du^i)(v^j dv^j) \tag{A.7}$$

The coefficients of the metric are

$$a_1 = \frac{1}{4\,\mu^2\,\nu^4}\,(1+\mu^2\,\nu^2)\,((1+\mu^2\,\nu^2)\,\nu^2\,u^2 + (\mu^2+\nu^6)\,v^2) \tag{A.8}$$

$$a_2 = -\frac{1}{4\,\mu^2\,\nu^4}\,(1-\nu^4)\,(1-\mu^2\,\nu^2)\,(\mu^2+\nu^2)\,u\cdot v \tag{A.9}$$

$$a_3 = \frac{1}{4\,\mu^2\,\nu^4}\,(1+\mu^2\,\nu^2)\,((\mu^2+\nu^6)\,u^2 + (1+\mu^2\,\nu^2)\,\nu^2\,v^2) \tag{A.10}$$

$$a_4 = \frac{1}{16\,\mu^4\,\nu^6}\,(1-\mu^2\,\nu^2)^2\,(1+\mu^2\nu^2)\,(\mu^2+\nu^6) \tag{A.11}$$

$$a_5 = \frac{1}{8\,\mu^4\,\nu^4}\,(1-\mu^4\,\nu^4)\,(\mu^4-\nu^4) \tag{A.12}$$

$$a_6 = -\frac{1}{8\,\mu^2\,\nu^6}\,(1-\nu^8)\,(\mu^4-\nu^4) \tag{A.13}$$

where

$$\mu = e^{\sigma}\ ,\nu = e^{\frac{m}{\sqrt{3}}} \tag{A.14}$$

The warp-factor, ξ, is given by:

$$\xi^2 = \frac{1}{16\,\mu^4\,\nu^8}\Big[\,\nu^2\,(1+\mu^2\,\nu^2)^3\,(\mu^2+\nu^6) + (1-\nu^4)^2\,(\mu^2-\nu^2)^2\,(1+\mu^2\,\nu^2)^2\,u^2\,v^2 - (1-\mu^2\,\nu^2)^2\,(1-\nu^4)^2\,(\mu^2+\nu^2)^2\,(u\cdot v)^2\Big] \tag{A.15}$$

and

$$\xi \equiv \Delta^{-\frac{4}{3}} \tag{A.16}$$

The dilaton-axion matrix is

$$\mathcal{M}_{11} = \frac{1}{4\,\xi\,\mu^2\,\nu^4}(1+\mu^2\,\nu^2)\,((\mu^2+\nu^6)\,\cos^2\theta + \nu^2(1+\mu^2\nu^2)\,\sin^2\theta) \tag{A.17}$$

$$\mathcal{M}_{12} = \mathcal{M}_{21} = \frac{1}{4\,\xi\,\mu^2\,\nu^4}(1-\nu^4)\,(1-\mu^2\,\nu^2)\,(\mu^2+\nu^2)\sin\theta\,\cos\theta\,\cos\phi \tag{A.18}$$

$$\mathcal{M}_{22} = \frac{1}{4\,\xi\,\mu^2\,\nu^4}(1+\mu^2\,\nu^2)\,(\nu^2(1+\mu^2\nu^2)\,\cos^2\theta + (\mu^2+\nu^6)\,\sin^2\theta) \tag{A.19}$$

The Yang Mills* metric and dilaton is obtained by setting $\mu = \nu$, which results is considerable simplification of the metric.

$$ds_5^2 = c^2 du^i du_i - 4s^2 u.v du^i dv_i + c^2 dv^i dv_i + c^2 s^2 d(u.v)^2 \tag{A.20}$$

where

$$c = \cosh\lambda, \qquad s = \sinh\lambda \tag{A.21}$$

Appendix B

Summary of the Yang Mills* Geometry

The 10d geometry is

$$ds_{10}^2 = \xi^{\frac{1}{2}} ds_{1,4}^2 + \xi^{-\frac{3}{2}} ds_5^2 \tag{B.1}$$

$$ds^2 = e^{2A(r)} dx^\mu dx_\mu + dr^2 \tag{B.2}$$

$$ds_5^2 = \xi_- \cos^2\alpha \; d\Omega_+^2 + \xi_+ \sin^2\alpha \; d\Omega_-^2 + \xi_+ \xi_- d\alpha^2 \tag{B.3}$$

where $\mu = 0..3$ and the $\xi_\pm$ are given by

$$\xi_\pm = c^2 \pm s^2 \cos 2\alpha, \quad c = \cosh\lambda, \quad s = \sinh\lambda \tag{B.4}$$

The warp factor is

$$\xi^2 = \xi_+ \xi_- \tag{B.5}$$

The dilaton is given, in unitary gauge, by the functions

$$f = \frac{1}{\xi^{1/2}} \sqrt{\frac{\cosh^2 \lambda + (\xi_+ \xi_-)^{1/2}}{2}}, \quad B = \frac{\sinh^2 \lambda \cos 2\alpha}{\cosh^2 \lambda + (\xi_+ \xi_-)^{1/2}} \tag{B.6}$$

In the more usual language the axion-dilaton field is given by

$$C + ie^{-\Phi} = i\frac{(1-B)}{(1+B)} = i\sqrt{\frac{\xi_-}{\xi_+}} \tag{B.7}$$

The two-form potential is given by

$$A_{(2)} = iA_+ \cos^3 \alpha \, \cos \theta_+ d\theta_+ \wedge d\phi_+ - A_- \sin^3 \alpha \, \cos \theta_- d\theta_- \wedge d\phi_- \tag{B.8}$$

with

Finally the four-form potential lifts to

$$F_{(4)} = F + \star F, \qquad F = dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3 \wedge d\omega \tag{B.9}$$

where

$$\omega(r) = e^{4A(r)} A'(r) \tag{B.10}$$

The functions A and λ are solutions of

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial \lambda} \tag{B.11}$$

$$6A'^2 = \lambda'^2 - 2V \tag{B.12}$$

with

$$V = -\frac{3}{2}\left(1 + \cosh^2 \lambda\right) \tag{B.13}$$

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