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Phenomenological aspects of supersymmetric family symmetries

by

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Dedicated to Charlie and my parents

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ABSTRACT

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PHENOMENOLOGICAL ASPECTS OF

SUPERSYMMETRIC FAMILY SYMMETRIES

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We study the phenomenological consequences of models with family symmetries which are broken close to the GUT scale. First, we consider a string-inspired model which becomes a supersymmetric Pati-Salam model with an abelian family symmetry at energy scales between the string scale and the GUT scale. We use the predicted rates of BR($\mu \rightarrow e\gamma$) to gauge the relative importance of a number of contributions to flavour violation, including from auxiliary fields associated with the spontaneous breaking of the family symmetry close to the GUT scale. Secondly, we consider the effect of an operator expansion in the Kähler sector similar to the operator expansions in the superpotentials of family symmetry models. When the family symmetry is broken, the fields in the effective theory become non-canonical, and have to be rescaled and mixed. This process can change the Yukawa textures, but we show that the effects on physical observables are sub-dominant provided that the textures are hierarchical.

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Preface

The work described in this thesis was carried out in collaboration with Prof. S. F. King, Prof. G. G. Ross, Dr. O. Vives and Dr. L. Velasco-Sevilla. The following list details our original work and gives the references for the material.

- Chapter 3: S. F. King and I. N. R. Peddie, Nucl. Phys. B. **678**(2004) 339 [arXiv:hep-ph/0307091]
- Chapter 4: S. F. King and I. N. R. Peddie, Phys. Lett. B. **586** (2004) 83 [arXiv:hep-ph/0312237]

  S. F. King, I. N. R. Peddie, G. G. Ross, L. Velasco-Sevilla and O. Vives, arXiv:hep-ph/0407012

No claims to originality is made for the content of Chapters 1 and 2 which were compiled using a variety of other sources.

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Chapter 1

General Introduction

1.1 Preliminaries

1.1.1 Motivation

The work in this thesis is involved with the analysis of models which extend the Minimal Supersymmetric Standard Model (MSSM) with a gauge symmetry which relates the three generations of matter, but is spontaneously broken at a high scale. Such symmetries tend to be called family symmetries. The motivation for studying these models is that they can, in principle, successfully explain in a natural way the hierarchy of quark and lepton masses, the smallness of the quark mixings and the largeness of the neutrino mixings. If these models could not be tested phenomenologically, they would be of no interest. There are several aims for studying these models:

- To understand where the hierarchy of quark and lepton masses originates from
- To understand why the mixing between generations of quarks are small, but the mixing between neutrinos is large
- To make predictions that can be tested experimentally in the near future, at the

LHC and the proposed LC. Also, to find ways of making predictions in the model which could, in principle, rule them out to allow theorists to study only those models which are consistent with experiment. Such avenues are:

- Bounds on the rates of rare decays, especially flavour violating decays such as $\mu \to e\gamma$.
- Predictions for the spectrum of supersymmetric particles, which can be tested if supersymmetry is seen at the LHC, and which can be compared with the lower bounds attained from the running of LEP-II.
- Finding that the model requires fine-tuning to be consistent with all experimental data. [1]

In general, these models are appealing because they can explain the entire fermion sector of the Standard Model, without the need for fine tuning, sometimes with fewer free parameters than there are observables, where one would not in general expect to find the fermion spectrum and mixings of the Standard Model.

1.1.2 Thesis structure

This thesis is organised as follows: in Chapter 1, we review the Standard Model (SM), and discuss the motivations for its extension. In particular, we focus on low energy $\mathcal{N} = 1$ supersymmetry, and the Minimal Supersymmetric Standard Model (MSSM). We also talk about two mechanisms which work at extremely high energy scales which can be indirectly detected at low energies, the see-saw model for explaining neutrino masses, and the Froggatt-Nielsen mechanism which can explain the Yukawa couplings in the MSSM and SM.

[1] While this would not rule a model out *per se*, if alternative models exist that do not require any fine-tuning, these would become the priority for further investigation.

In Chapter 2, we review the framework for considering models at extremely high energy. We start by considering field-theory unification by introducing the SUSY Pati-Salam model. Then we introduce some relevant aspects of string theory for model building, concentrating on brane-world setups in Type I string theory. Then we unite the two parts of the chapter by considering a Pati-Salam setup coming from a Type I brane scenario.

In Chapter 3, we consider a non-universal contribution to the trilinear scalar interactions in the soft supersymmetry breaking Lagrangian in supergravity models which contain a family symmetry. These contributions were known to be dangerous with respect to flavour violation, and we introduce a specific model to study this numerically. The model considered is a string inspired Pati-Salam model, with an added abelian family symmetry. We then calculated the branching ratios BR($\mu \to e\gamma$) and BR($\tau \to \mu\gamma$) for four points in the parameter space of the model, which correspond to different flavour violating contributions dominating.

In Chapter 4, we consider the effect of having to canonically normalise fields in the effective field theory once flavour symmetry breaking Higgs fields gain a VEV. This corresponds to a field redefinition which is generation dependent, and can in principle change the Yukawa textures. We then utilise a residual freedom in the field redefinition to demonstrate that the effect on the masses and mixing angles are always small, even if some choices of normalisation transformations do change the textures.

The overall conclusions to this thesis are presented in Chapter 5, which is followed by a number of Appendices.

1.2 The Standard Model

The Standard Model (SM) of particle physics, is a renormalisable gauge quantum field theory, [2] based on the gauge group $G_{\rm SM} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Being a quantum field theory (QFT), every *fundamental* particle observed experimentally corresponds to a field in the QFT. There are three generations of fermionic matter; each member of a generation has a corresponding member in the other two generations which are identical in every way except mass and mixing to the other generations. Each member of a single generation has a different representation under $G_{\rm SM}$; see table 1.1.

Field	Spin	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Left-handed quarks, $Q_{iL} \equiv (u_{iL}, d_{iL})$	1/2	**3**	**2**	1/6
Right-handed up quarks u_{iR}	1/2	**3**	**1**	2/3
Right-handed down quarks d_{iR}	1/2	**3**	**1**	−1/3
Left-handed leptons $L_{iL} \equiv (\nu_{iL}, e_{iL})$	1/2	**1**	**2**	−1/2
Right-handed electrons e_{iR}	1/2	**1**	**1**	−1
Higgs boson, $\phi \equiv (\phi^+, \phi_0)$	0	**1**	**2**	1/2
Gluons, g^α, $(\alpha = 1-8)$	1	**8**	**1**	0
Weak bosons, W^a, $(a = 1-3)$	1	**1**	**3**	0
Hypercharge boson, B	1	**1**	**1**	0

Table 1.1: Gauge representations of the Standard Model fields. Note that left-handed (right-handed) fields transform as fundamental (trivial) representations of $SU(2)_L$. The fermion index i is a generation index, so $u = (u, c, t)$ etc.

The gauge symmetry doesn't allow mass terms for any of the fermions or gauge bosons. Adding mass terms for the gauge bosons which explicitly break the gauge

[2]There are many excellent references to the Standard Model, such as Refs. [1, 2].

symmetry doesn't work, since this makes the theory non-unitary. Instead we use the Higgs mechanism, which itself utilises *spontaneous symmetry breaking* of the electroweak symmetry (Electroweak Symmetry Breaking (EWSB)):

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_{\text{em}} \tag{1.1}$$

In order to explain how breaking the electroweak symmetry $SU(2)_L \otimes U(1)_Y$ to electromagnetism generates gauge-invariant masses for the gauge bosons and all SM fermions, we need to start by writing down the Higgs-sector Lagrangian:

$$\mathcal{L}_{\text{Higgs}} = (D^\mu \phi)^\dagger (D_\mu \phi) - \left[-m_\phi^2 \phi^\dagger \phi + \frac{\lambda}{2} (\phi^\dagger \phi)^2 \right] \tag{1.2}$$

Where the electroweak covariant derivative is:

$$D_\mu = \partial_\mu - igW_\mu^a T^a - ig'YB_\mu \tag{1.3}$$

If $m_\phi^2 = +|\mu|^2$ and $\lambda > 0$, then this is just a standard scalar field coupled to two gauge fields. But if $m_\phi^2 = -|\mu|^2$, then $\phi = 0$ is no longer a minimum of the potential. Were we able to do exact field theoretical calculations, this wouldn't be a problem, but in order to do calculations in perturbation theory, we must expand around the minimum of the potential. The minimum of the potential is no longer at $\phi = 0$, and we say that the Higgs field has developed a Vacuum Expectation Value (VEV).

In this case we can use the gauge freedom to write the Higgs field in full generality as:

$$\phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + h(x) \end{pmatrix} \tag{1.4}$$

If we substitute Eq. (1.4) into the Higgs Lagrangian, Eq. (1.2) then the non-derivative terms in the covariant derivative multiplying the VEV will lead to mass terms for some of the gauge bosons:

$$\Delta\mathcal{L}_{\text{Higgs}} = \frac{1}{2}\frac{v^2}{4} \left[g^2 W_\mu^1 W^{1\,\mu} + g^2 W_\mu^2 W^{2\,\mu} + (-gW_\mu^3 + g'B_\mu)(-gW^{3\,\mu} + g'B^\mu) \right] \tag{1.5}$$

In terms of physical states, we form linear combinations of W^i and B. As electromagnetism survives as a gauge symmetry, we form combinations of definite electric charge. The charged states are:

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \pm iW_\mu^2) \tag{1.6}$$

With masses of $m_W = gv/2$. Then the electrically neutral states W^3 and B are mixed together by EWSB

$$\begin{pmatrix} Z_\mu^0 \\ A_\mu \end{pmatrix} = \begin{pmatrix} \cos\theta_W & -\sin\theta_W \\ \sin\theta_W & \cos\theta_W \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix} \tag{1.7}$$

The massless field is the photon of electromagnetism, and the Z^0 boson has a mass $M_Z = \sqrt{g^2 + g'^2}$. θ_W is defined by: $\tan\theta_W = g'/g$. The electromagnetic charge e is related to the $SU(2)_L$ and $U(1)_Y$ couplings via:

$$e = g' \cos\theta_W \tag{1.8}$$

This has explained how we can get masses for the gauge bosons in the theory [3]. Now we consider massive fermions. By examining table 1.1, we can see that the Higgs field has exactly the right representation to allow couplings that link left and right handed fields. Thus we can generate fermion masses if we allow the model to contain a Yukawa sector:

$$\mathcal{L}_{\text{Yukawa}} = \overline{Q}_{iL}(-i\tau_2\phi^*)u_{jR}Y_{ij}^u + \overline{Q}_{iL}\ \phi\ d_{jR}Y_{ij}^d + \overline{L}_{iL}\ \phi\ e_{jR}Y_{ij}^e + \text{h.c.} \tag{1.9}$$

When we introduce the Higgs VEV, this will generate masses and mixings for the quarks and charged leptons. So that the quark and charged lepton mass matrices will be:

$$m_{ij}^{u,d,e} = \frac{v}{\sqrt{2}} Y_{ij}^{u,d,e} \tag{1.10}$$

[3]Remarkably, the theory is now renormalizable with these mass terms [3]

Note that in the Standard Model we cannot have any neutrino masses, because there is not a right-handed neutrino field to form a Yukawa coupling with. In general these mass matrices are not diagonal, and so fermions from different generations can mix together. In order to extract the physical masses, we make a unitary transformation on the weak eigenstates to get the mass eigenstates. This is especially useful for perturbative calculations, where it is much better to have diagonal propagators and non-diagonal vertices than the other way around. In doing so, we redefine:

$$\begin{aligned} u_{iL} \rightarrow u'_{iL} = (V_L^u)_{ij} u_{jL} \quad u_{iR} \rightarrow u'_{iR} = (V_R^u)_{ij} u_{jR} \\ d_{iL} \rightarrow d'_{iL} = (V_L^d)_{ij} d_{jL} \quad d_{iR} \rightarrow d'_{iR} = (V_R^d)_{ij} d_{jR} \end{aligned} \tag{1.11}$$

Where $V_{L,R}^{u,d}$ are unitary and diagonalise the quark mass matrices:

$$m_{\text{diag}}^{u,d} = V_L^{u,d} m^{u,d} V_R^{u,d\dagger} \tag{1.12}$$

We don't make a distinction for the leptons; the absence of a RH neutrino field in the SM has the effect that we can simultaneously diagonalise the mass matrices and the couplings to the electroweak bosons for the charged leptons, and diagonalise the neutrino couplings to the electroweak bosons.

The combination $V_{CKM} = V_L^u V_L^{d\dagger}$ is the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix [4]. The standard parameterisation involves three mixing angles and one CP-violating phase which cannot be removed. The elements of V_{CKM} are observable in weak charged-current processes, and the CKM matrix is observed to be highly diagonal.

1.2.1 Successes of the Standard Model

The Standard Model has been subject to experimental testing since its inception. This testing includes the large data collected by various high energy accelerators, as well as precision electroweak data. The latest data is collated and released regularly by the Particle Data Group [5]. This includes the measurement, from the Z^0 boson decay width, that there are only three left-handed (non-sterile) neutrinos with mass $m_\nu \leq m_Z/2$. Furthermore, in order to avoid chiral anomalies, where quantum corrections break symmetries of the classical theory, there must exist only complete generations [4]. Taken together, this tells us that there must be three complete generations of matter in the SM.

Furthermore, the quark model tells us that there must exist a number of approximate flavour symmetries, from which we can successfully predict the spectra of light mesons and baryons. The unitarity of the CKM matrix leads to the Glashow-Iliopoulis-Maiani (GIM) [6] mechanism which suppresses flavour changing processes, and was used to predict the existence of the charm quark before it was observed.

The Higgs mechanism allows an understanding of where the electroweak boson masses come from; this in turn allows us to understand why the range of the electroweak interaction is so low. Since most of the couplings in the Standard Model are so small, it has been possible to perform a large number of calculations, which compare very well with experiment. The Renormalisation Group Equations (RGEs) are one such calculation, which allows us to understand the variation of physical parameters with energy scale. This allows us to accept that the QCD group $SU(3)_c$ confines in the infrared, which has the effect that quarks *confine*; quarks always form into $SU(3)_c$ singlet bound states of three quarks (a baryon) or a quark and an antiquark (a meson).

[4] This statement strongly constrains any new matter that is added, which makes going beyond the Standard Model more difficult. We will return to this when we discuss supersymmetry and the MSSM

1.2.2 Issues with the Standard Model

While the Standard Model has been very successful as far as it goes, it is becoming increasingly clear from both experimental and theoretical viewpoints that it is a low-energy effective field theory. As such, it then becomes a reasonable question to ask what lies beyond the Standard Model. Any such theory would have to replicate the successes of the Standard Model, as well as removing one or (hopefully) more of the problems that the SM has. The main issues that the Standard Model has trouble with dealing with are:

- **Neutrino Masses**

It has long been known that there was a discrepancy between the Standard Solar Model (SSM) and the earth-based experiments measuring solar neutrinos. The SSM predicted that the sun generated energy by fusion reactions, which would generate electron neutrinos. The first neutrino astrophysics experiments in the 1960s tried, and succeeded, to measure solar neutrinos. Unfortunately, the observed flux of solar neutrinos was about three times smaller than that predicted by the SSM. This discrepancy could be explained by electron neutrinos oscillating into muon and tau neutrinos, which would be undetectable by the experiments of the time. Any oscillation would require the neutrinos to be massive. However, the discrepancy between the SSM prediction and the observed flux was not conclusive because there was no way of distinguishing between the SSM prediction being wrong with no oscillation and the SSM prediction being correct with massive neutrinos oscillating into each other.

More recently, super-Kamiokande [7] has observed a deficit of atmospheric muon-neutrinos reaching the surface after being generated in the upper atmosphere. The production involved a decay chain of $\pi^- \rightarrow \mu^- + \overline{\nu}_\mu$ followed by $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$.

With no neutrino oscillation, one would expect to observe twice as many atmospheric muon neutrinos as electron neutrinos.

The issue of solar neutrino oscillations was settled when the Sudbury Neutrino Observatory (SNO) reported their results [61]. SNO was able to measure separately the flux of electron neutrinos and the total flux of all three neutrino species. The results demonstrated that the SSM prediction of neutrino flux was correct.

Combining results from the various experiments, it was possible to extract the differences between the squared masses of the three neutrinos and two of the three leptonic mixing angles in the PMNS matrix [5]. The mass differences are incompatible with zero, thus requiring at least two neutrinos to be massive. This is a problem for the Standard Model, where the lack of a right-handed neutrino field implies a zero neutrino Dirac mass, and gauge-invariance won't allow a renormalisable Majorana neutrino mass.

- **The fermion hierarchy**

In the SM, the fermion masses and mixings come from 27 parameters in Yukawa coupling matrices. However, these are all input parameters, and the SM has no way of explaining the fact that the top quark is 6 orders of magnitude heavier than the electron, other than that the top quark Yukawa is 6 orders of magnitude larger than the electron Yukawa. The fact that there is a similar hierarchy in masses in the three generations is suggestive of some organising principle, but there is no way of understanding this as anything other than a coincidence within the SM. In order to understand why such a strong hierarchy exists, one has to start looking beyond the Standard Model.

- **The Gauge Hierarchy Problem**

[5] The PMNS matrix is the leptonic equivalent of the CKM matrix

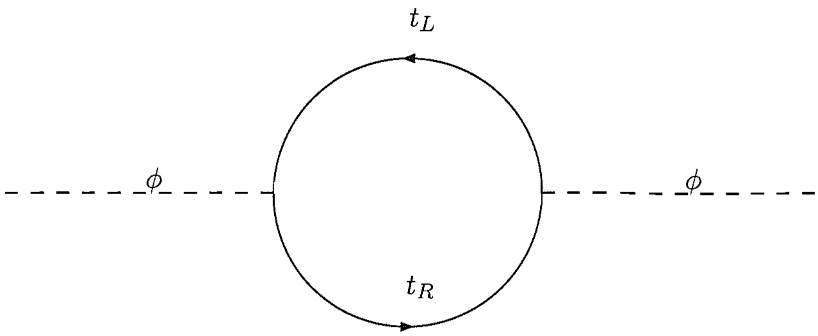


Figure 1.1: The dominant correction to the Higgs mass^2 in the Standard Model comes from the top quark bubble diagram

The Gauge Hierarchy problem amounts to the question 'why is the electroweak scale so much lower than the Planck scale?'. There are two parts to the question, neither of which the Standard Model can answer in a natural manner. The first is the question why the electroweak symmetry breaks at the order 100 GeV. There is no reason to expect the Higgs mass parameter m_ϕ^2 to become negative at this energy scale in particular. The fact that it does has to be put in by hand to make the Standard Model work. The second question is more technical. Consider the mass renormalisation of the Higgs mass, δm_ϕ^2; the dominant term comes from the top-quark loop, as in fig. 1.1. The dominant term is then:

$$\delta m_h^2 = \frac{|\lambda_t|}{16\pi^2}\left[-2\Lambda_{UV}^2 + 6m_t^2 \ln(\Lambda_{UV}/m_t) + \cdots\right] \tag{1.13}$$

The problem is that the Higgs is a scalar, so the finite part of the renormalisation is sensitive to the largest mass in the theory that couples directly or indirectly to the Higgs. As well as this, we have a naturalness problem. If there is nothing beyond the Standard Model, then the appropriate ultraviolet cutoff, Λ_{UV} is the Planck scale, where quantum gravity effects start to become large. But then, in order to have a renormalised Higgs mass of the order of 100 GeV we need to set the tree level mass to be of the order of Λ_{UV}^2, in order to cancel the quadratic divergence in the one-loop correction:

$$m_{h\ \text{physical}}^2 = (m_h^2)_0 + \delta m_h^2 = \Lambda_{UV}^2 - \Lambda_{UV}^2 + \mathcal{O}(100\ \text{GeV}^2) \tag{1.14}$$

However, doing this involves the fine tuning of the tree level mass such that the two contributions to the physical mass cancel down to a number 20 orders of magnitude smaller. Unfortunately, there is no way of getting around this in the SM.

- **Problems with gauge unification**

It is possible in the Standard Model to calculate the energy dependence of the parameters within it; when this calculation is performed for the gauge couplings it turns out that they appear to converge at a very high energy scale. This led some theorists to predict that the reason that the couplings appear to get close together was because the Standard Model gauge group, G_{SM}, is a subgroup of a larger group, such as $SO(10)$. Such models are called Grand Unified Theories (GUTs). They have the benefit of naturally explaining why each generation of matter is anomaly-free, because each generation forms a complete representation of the 'Grand Unified' group. There are many groups that could contain G_{SM} as a subgroup, and most of them require the existence of a right-handed neutrino field, which means that they predict neutrino masses.

However, improved calculations made it clear that although the gauge couplings did appear to be heading towards each other, they didn't all meet at a single point, as would be required for a GUT.

There are many well motivated solutions which each solve some of these problems, such as supersymmetry, string theory, family symmetries and the see-saw mechanism, which we will review in the following sections.

1.3 Supersymmetry

Supersymmetry (SUSY) is a symmetry which transforms between fermions and bosons. Unlike gauge symmetries, SUSY is not an internal symmetry of the Poincaré group. There are many very good introductions to the formalism of supersymmetry available [8, 9, 10, 11]. In order to construct actions which are supersymmetric, it is useful to use the *superfield* formalism. We introduce this notation in the next subsection. Then, we use it to define the Minimal Supersymmetric Standard Model (MSSM), which is the minimal consistent supersymmetric extension of the Standard Model. Then we briefly mention the problems that the MSSM solves, and some unanswered questions that it has.

1.3.1 Superfield basics

The generators of SUSY are fermionic, and satisfy an anti-commutation relation:

$$\left\{Q_\alpha, \overline{Q}_{\dot{\beta}}\right\} = 2\sigma^\mu_{\alpha\dot{\beta}} P_\mu \tag{1.15}$$

Thus supersymmetry extends the Poincaré group, called the super-Poincaré group or the super-algebra. The SUSY generators commute with the momentum operator P_μ:

$$[Q_\alpha, P_\mu] = \left[\overline{Q}_{\dot{\alpha}}, P_\mu\right] = 0 \tag{1.16}$$

In the same way that x_μ is the coordinate corresponding to the momentum operator P_μ, we introduce Weyl spinors containing Grassman variables θ_α, $\overline{\theta}_{\dot{\alpha}}$ corresponding to the SUSY charge operators Q_α, $\overline{Q}_{\dot{\alpha}}$. The properties of anti-commuting Grassman variables will help us greatly.

We can write a superfield $S(x, \theta, \overline{\theta})$ as a function of all three coordinate types. Since θ, $\overline{\theta}$ are Grassman variables, the Taylor series in θ,$\overline{\theta}$ terminates and can be written exactly in terms of component fields. The most general superfield doesn't turn out to be useful. One useful superfield is the *chiral superfield*, $\Phi(y, \theta)$, where $y^\mu = x^\mu + i\theta\sigma^\mu\overline{\theta}$:

$$\Phi(y,\theta) = \phi(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{1.17}$$

Chiral superfields have the useful property that the product of two chiral superfields is itself a chiral superfield. Another useful superfield is the vector superfield. This is the most general real superfield, i.e. $V(y) = V^\dagger(y)$.

In both superfields there are auxiliary fields, F for chiral superfields and D for vector superfields. Both have the property that under SUSY transformations they only change by a derivative term, so they leave the action invariant. This gives us a convenient way for constructing supersymmetric actions. If we extract the F term from a chiral superfield of mass dimension 3, and the D term from a mass dimension 2 vector superfield, we will have an action which is supersymmetric. Thus there are two ways of constructing SUSY invariant actions:

$$\int d^4x \left[\int d^2\theta \Phi(y,\theta) + \int d^2\overline{\theta}\Phi^\dagger(y^\dagger, \overline{\theta}) \right] \tag{1.18}$$

This works for any chiral superfield, Φ. The other way works for any vector superfield V:

$$\int d^4x d^4\theta V(x, \theta, \overline{\theta}) \tag{1.19}$$

Both are needed to generate an interesting theory. We define the *superpotential* W as a mass dimension 3 chiral superfield. In order to satisfy this constraint, the superpotential has to be a holomorphic polynomial of chiral superfields. This will contain the interactions. We also define the *Kähler potential*, K as a mass dimension 2 vector superfield.

We now demonstrate how this all pulls together in the simplest case, of a chiral fermion and its scalar partner field. $K = \Phi\Phi^\dagger$. We can write the expansion of Φ as

$$\begin{aligned}\Phi &= \phi(x) + \sqrt{2}(\theta\psi_\phi(x)) + (\theta\theta)F_\phi(x) \\ &\quad + i(\theta\sigma^\mu\overline{\theta})\partial_\mu\phi(x) - \frac{i}{\sqrt{2}}(\theta\theta)\partial_\mu\psi_\phi(x)\sigma^\mu\overline{\theta} - \frac{1}{4}(\theta\theta)(\overline{\theta\theta})\Box\phi(x) \qquad (1.20)\end{aligned}$$

In order to extract the SUSY-invariant part of the Kähler potential (which is a vector superfield), we need to extract the auxillary field D. However, in the definition of a vector superfield, there is a term:

$$V = C(x) + ... + (\theta\theta)(\overline{\theta\theta})(D(x) - \frac{1}{2}\Box C(x)) \qquad (1.21)$$

Thus in order to extract the D term, we need to add $1/2\Box$ operating on the θ independent term.

$$K|_{\theta\theta\overline{\theta\theta}} + \frac{1}{2}\Box\, K|_{\text{no }\theta} = -\frac{1}{4}\Box\phi^*\phi - \frac{1}{4}\phi^*\Box\phi + F^*_\phi(x)F_\phi(x) + \frac{1}{2}\partial^\mu\phi^*\partial_\mu\phi + \frac{1}{2}\Box(\phi^*\phi) \qquad (1.22)$$

This leads to the following contribution to the action:

$$\begin{aligned}S_K &= \int d^4x\,[\partial^\mu\phi^*\partial_\mu\phi + \frac{i}{2}\Big\{([\partial_\mu\psi^\dagger_\phi(x)]\sigma^\mu\psi_\phi(x)) \\ &\quad -\psi^\dagger_\phi(x)\sigma^\mu\partial_\mu\psi_\phi(x)\Big\} + F^*_\phi(x)F_\phi(x)\Big] \qquad (1.23)\end{aligned}$$

Thus the Kähler potential has generated the kinetic terms for the scalar field ϕ and fermionic field ψ_ϕ, with a non-propagating scalar field F_ϕ of mass dimensions 2, which is an auxiliary field. Note that these kinetic terms are canonically normalised. This is not an accident, it is because we wrote down a canonically normalised Kähler potential. Had we not done so, we would have had to rescale the fields in the Kähler potential in order to canonically normalise it. [6]

[6]Note that in an effective field theory, the effective Kähler potential, K_{eff} could become non-canonical after at least one scalar field gains a VEV. In this case, a previously canonical Kähler potential will

We use the superpotential

$$W = \frac{1}{2}m\Phi^2 + \frac{1}{3}g\Phi^3. \tag{1.24}$$

This leads to the following contribution to the action:

$$\begin{aligned} S_W &= \int d^4x \left[m(\psi_\phi(x)\psi_\phi(x)) + mF_\phi(x)\phi(x) \right. \\ &\quad \left. + g\phi(x)(\psi_\phi(x)\psi_\phi(x)) + g\phi(x)\phi(x) \right] \end{aligned} \tag{1.25}$$

The full action of the theory is gained by adding the contributions from the Kähler potential, the superpotential and the hermitian conjugate of the superpotential:

$$S_{\text{full}} = S_K + S_W + S_{W^*} \tag{1.26}$$

It should be noted that the F can be eliminated by solving its Euler-Lagrange equation, and doing the same for F^*:

$$\frac{\delta \mathcal{L}}{\delta F} = \partial_\mu \frac{\delta \mathcal{L}}{\delta \partial_\mu F} \tag{1.27}$$

This leads to the solutions:

$$-F^* = m\phi + g\phi\phi \qquad -F = m\phi^* + g\phi^*\phi^* \tag{1.28}$$

And substituting it into the action:

$$\begin{aligned} S &= \int d^4x \left[\partial^\mu \phi^*(x)\partial_\mu\phi(x) + \frac{i}{2}\left\{ \left(\left[\partial_\mu \psi_\phi^\dagger(x)\right] \sigma^\mu \psi_\phi(x) \right) - \left(\psi_\phi^\dagger(x)\sigma^\mu\partial_\mu\psi_\phi(x) \right) \right\} \right. \\ &\quad \left. -m\left[\left(\psi^\phi(x)\psi^\phi(x) \right) + \left(\psi_\phi^\dagger(x)\psi_\phi^\dagger(x) \right) \right] - \left| m\phi + g\phi^2 \right|^2 \right] \end{aligned} \tag{1.29}$$

This is a theory of a self-coupling complex scalar coupled to a fermion with an identical (Majorana) mass term.

become non-canonical, and the fields in the effective theory will have to be rescaled to correct this, but this shift has to be applied consistently throughout the theory. This point is developed in detail in Chapter 4.

None of this involves gauge fields, or chiral superfields which transform non-trivially under any gauge symmetry. It is possible to use the superfield formalism to construct superfields which contain field strength tensors, and use these to construct the gauge sector of a supersymmetric gauge field theory. However, the non-gauge sector can be constructed in the same way as already outlined, providing that each term in the superpotential is a gauge singlet. We now move on to defining the Minimal Supersymmetric Standard Model (MSSM).

1.3.2 The Minimal Supersymmetric Standard Model

The MSSM is the supersymmetric extension of the Standard Model with the minimal gauge group and minimal particle content. The minimal gauge group turns out to be the same gauge group as the standard model, $G_{\text{SM}} = SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. The minimal field content is not just a case of promoting all Standard Model fields to become superfields. This is because the fermionic partners to the Higgs fields can contribute to chiral anomalies, since it has hypercharge 1/2. This is cancelled by introducing a new Higgs doublet with hypercharge $-1/2$. This turns out to be important to generate the up-type fermion masses, due to the holomorphicity of the superpotential.

We introduce the standard tilde notation here. For every field in the SM, the partner field to the same field in the MSSM has a tilde on top. When naming the field, we prepend an 's-' if the new field is a scalar, and append an '-ino' if the new field is fermionic. There are no new vector fields introduced. We refer to the partner field of a SM field in general as its 'superpartner', and the particles corresponding to the new fields as 'sparticles'. Thus the superpartner field of a quark is a squark, and the superpartner of a Higgs is a Higgsino. The fields of the MSSM and their gauge representations are laid out in table 1.2.

Superfield	Spin 0	Spin-1/2	Spin-1	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$
Q_{iL}	$\widetilde{q}_{iL}$	q_{iL}	–	**3**	**2**	1/6
$\overline{U}_{iR}$	$\widetilde{\overline{u}}_{iR}$	$\overline{u}_{iR}$	–	**3**	**1**	−2/3
$\overline{D}_{iR}$	$\widetilde{\overline{d}}_{iR}$	$\overline{d}_{iR}$	–	**3**	**1**	1/3
L_{iL}	$\widetilde{l}_{iL}$	l_{iL}	–	**1**	**2**	−1/2
$\overline{E}_{iR}$	$\widetilde{\overline{e}}_{iR}$	$\overline{e}_{iR}$	–	**1**	**1**	1
H_u	h_u	$\widetilde{H}_u$	–	**1**	**2**	1/2
H_d	h_d	$\widetilde{H}_d$	–	**1**	**2**	−1/2
G^α	–	$\widetilde{g}^\alpha$	g^α	**8**	**1**	0
W^a	–	$\widetilde{W}^a$	W^a	**1**	**3**	0
B	–	$\widetilde{B}$	B	**1**	**1**	0

Table 1.2: The $G_{\rm SM}$ representations of the field content of the MSSM. Note that right-handed field degrees of freedom have been CP-conjugated to make them transform as left-handed fields.

The MSSM superpotential is:

$$W = Q_i H_u \overline{U}_j Y^u_{ij} + Q_i H_d \overline{D}_j Y^d_{ij} + L_i H_d \overline{E}_j Y^e_{ij} + \mu H_u H_d \tag{1.30}$$

The Kähler potential has to be modified slightly to ensure gauge invariance. To be consistent with gauge invariance and supersymmetry, we must introduce a set of superfields which contain the gauge fields g^μ_α, W^μ_a and B^μ, and their superpartner fields. Then we need to ensure that the gauge and gaugino fields have kinetic terms.

Since SUSY is broken in nature (we have not discovered any light scalar fields that could partner the charged leptons), we must break it in the MSSM. Since we do not know how it is broken, we break SUSY by introducing a set of terms which explicitly break the supersymmetry in the Lagrangian, but break it 'softly'. We define

$$\begin{aligned} -\mathcal{L}_{\text{soft}} &= \frac{1}{2}\left(M_3 \widetilde{g}\widetilde{g} + M_2 \widetilde{W}\widetilde{W} + M_1 \widetilde{B}\widetilde{B}\right) + h.c \\ &+ \widetilde{\overline{u}}_{iR} \widetilde{q}_{jL} h_u \widetilde{A}^u_{ij} + \widetilde{\overline{d}}_{iR} \widetilde{q}_{jL} h_d \widetilde{A}^d_{ij} + \widetilde{\overline{e}}_{iR} \widetilde{l}_{jL} h_d \widetilde{A}^e_{ij} \\ &+ \widetilde{q}^\dagger_{iL} m^2_{QL} \widetilde{q}_{jL} + \widetilde{\overline{u}}^\dagger_{iR} m^2_{UR} \widetilde{\overline{u}}_{jR} + \widetilde{\overline{d}}^\dagger_{iR} m^2_{DR} \widetilde{\overline{d}}_{jR} + \widetilde{l}^\dagger_{iL} m^2_{LL} l_{jL} + \widetilde{\overline{e}}^\dagger_{iR} m^2_{ER} \widetilde{\overline{e}}_{jR} \\ &+ m^2_{h_u} h^\dagger_u h_u + m^2_{h_d} h^\dagger_d h_d - (B\mu h_u h_d + h.c.) \end{aligned} \tag{1.31}$$

The M_i are gaugino mass terms, the $\widetilde{A}^f_{ij}$ are trilinear scalar interactions and the m^2_{ij} are scalar mass terms. After phase redefinitions, there are 105 additional free independent parameters in the MSSM.

We can write down the full MSSM Lagrangian $\mathcal{L}_{\text{MSSM}}$. Having done so, one can diagonalise the mass terms to get the physical masses of all of the superpartner fields. Note that there are 8 degrees of freedom in the Higgs sector, of which 3 get Goldstone modes are eaten by the $W^\pm$,Z^0 bosons. There are then 5 physical degrees of freedom in the MSSM Higgs sector:

- H^0, h^0 - two CP-even neutral Higgs bosons,

- A^0 - a CP-odd Higgs boson,

- $H^{\pm}$ - a charged Higgs boson pair

After EWSB, states with the same representation under $SU(3)_c \otimes U(1)_{em}$ mix, and the mass eigenstates are given by linear combinations of the (unbroken) gauge eigenstates. In this, the charged Higgsinos and charged gauginos mix into chargino states:

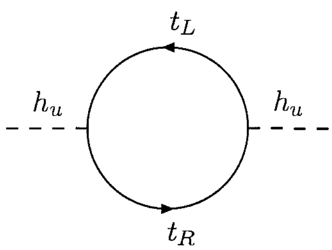
$$\widetilde{h}_u^+ \ , \ \widetilde{h}_d^- \ , \ \widetilde{W}^1 \ , \ \widetilde{W}^2 \rightarrow \widetilde{\chi}_1^{\pm} \ , \ \widetilde{\chi}_2^{\pm} \tag{1.32}$$

The neutral Higgsinos and gauginos mix into neturalino states:

$$\widetilde{h}_u^0 \ , \ \widetilde{h}_d^0 \ , \ \widetilde{W}^3 \ , \ \widetilde{B} \rightarrow \widetilde{\chi}_1^0 \ , \ \widetilde{\chi}_2^0 \ , \ \widetilde{\chi}_3^0 \ , \ \widetilde{\chi}_4^0 \tag{1.33}$$

The lightest neutralino $\widetilde{\chi}_1^0$ is often the lightest supersymmetric partner (LSP).

Successes and motivations for the MSSM



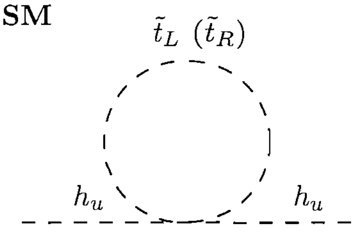


Figure 1.2: The correction to the Higgs mass^2 comes from both the top quark bubble and the top squark loop

- **Hierarchy Problem**

The MSSM helps the Hierarchy Problem in two ways. It helps answer the naturalness problem by being a softly broken SUSY theory; the corrections to the Higgs self-energy now have superpartners in the loops which exactly cancel the quadratic divergence, as in fig. 1.2. With softly broken SUSY there is an extra correction due to the parameters in $\mathcal{L}_{\text{soft}}$. The correction to the Higgs mass^2 is then:

$$\delta m_h^2 = -\frac{6|\lambda_t|}{16\pi^2} m_{\tilde{t}}^2 \ln\left(\frac{\Lambda_{UV}}{m_{\tilde{t}}^2}\right) + \cdots \tag{1.34}$$

The hierarchy is then stabilised if the soft parameters are at most of the TeV scale.

The other part of the hierarchy problem is to understand why the Electroweak breaking scale is so far below the Planck scale. This is actually addressed in the MSSM, by Radiative Electroweak Symmetry Breaking.

- **Radiative Electroweak Symmetry Breaking**

In the SM, we have to insert by hand the fact that the Higgs mass^2 becomes negative at something close to the electroweak scale to trigger Electroweak Symmetry Breaking. In the MSSM, the Renormalisation Group Equations for $m_{h_u}^2$ are such that if the top quark Yukawa is $O(1)$ then the RG running will drive $m_{h_u}^2$ negative at around the electroweak scale. This can then trigger EWSB, and helps explain why the EW scale is so much lower than the Planck scale, which of course is the second part of the hierarchy problem alluded to above.

- **Gauge Coupling Unification**

As the RGEs are changed, this is going to change the predictions for the evolution of the three gauge couplings to high energy scales. The MSSM RGEs for the gauge couplings are smaller in magnitude, leading to approximate unification at a higher scale. The correct approach is to use the SM RGEs up to the scale that the sparticle fields enter and then switch to the MSSM RGEs. As the lines are approaching each other, two lines will always meet and by changing the scale that we switch to the MSSM RGEs we can make all three points meet. However, it turns out that to get gauge coupling unification, the appropriate point to make this change is at the TeV scale, which is the scale we need SUSY to be at to solve the Hierarchy Problem. This is promising for both SUSY GUTs and string phenomenology, since a lot of string-inspired models have some amount of unification close to the Planck scale.

Unanswered Questions in the MSSM

- **The SUSY Flavour Problem**

  Although there are 100 free parameters in $\mathcal{L}_{\text{soft}}$, experimental constraints on, amongst other things, rare flavour violating decays tells us there are hints of an organising structure. Consider the lepton flavour violating process $\tau \to \mu\gamma$. If we move to the basis where the Yukawa matrices are diagonal, then the couplings of charged leptons to neutralinos and charginos are flavour universal. Then the contribution to these flavour violating decays come from the off-diagonal elements in the slepton mass matrices m_L^2, m_E^2, and are suppressed by the diagonal elements. A typical Feynman graph contributing to $\tau \to \mu\gamma$ is shown in fig. 1.3.

  This brings us to the SUSY flavour problem; in order to accommodate the very small experimental limits on $\text{BR}(\tau \to \mu\gamma)$, $\text{BR}(\tau \to e\gamma)$ and $\text{BR}(\mu \to e\gamma)$, we know that the slepton mass matrices $m_{L,E}^2$ must be nearly diagonal in the basis where the Yukawa matrices are diagonal. However, we do not expect the soft

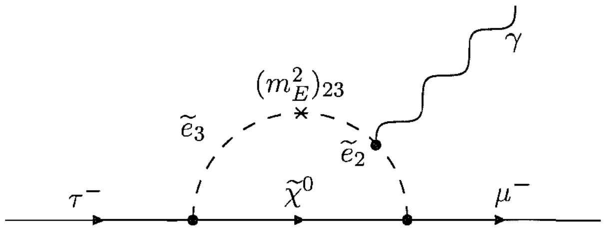


Figure 1.3: Feynman graph contributing to the process $\tau \longrightarrow \mu\gamma$ at one loop in the MSSM

couplings to originate from the same (unknown) physics as the Yukawa coupling. We therefore either have to explain why the slepton mass matrices are aligned to the Yukawa matrices, or explain why the slepton mass matrices are close to the identity. There are similar arguments for the quark sector, where we need alignment or universality in the squark mass matrices to avoid predicting rates for $b \longrightarrow s\gamma$ etc. which are far above the experimentally measured limits.

- **Free Parameters**

The other problem with the MSSM is the rather large number of independent, free parameters, most of which come from $\mathcal{L}_{\text{soft}}$. At the moment, there are no strong direct constraints on most of these, other than by limits due to rare decays, which can be ameliorated by increasing the scale of all of the soft parameters. There currently isn't a standard accepted model for SUSY breaking, and there isn't an accepted view on how SUSY breaking is transmitted to the visible sector. Once these parameters are measured, it may be possible to probe these questions. At the moment, though, the sheer number of parameters makes general predictions using the MSSM very difficult.

- **Neutrino Masses**

The MSSM, like the Standard Model doesn't have any right-handed neutrino fields, and therefore predicts massless neutrinos. This is a problem for the MSSM in the light of the neutrino oscillation observations [7, 12] which demonstrate that at least two neutrinos must have non-zero mass. There isn't a standard model for the generation of neutrino masses, so a 'new MSSM' with massive neutrinos isn't currently a viable option.

1.4 Low energy footprints of high energy physics

It turns out that in the context of supersymmetry, it is possible to consistently have fields with mass terms close to the Planck scale, which can then be integrated out of the effective field theory (the MSSM, or one of its extensions). We can't do this in the SM because radiative corrections to the Higgs mass would be of the order of the new energy scale. In a SUSY theory, the supersymmetry protects the Higgs mass, so the large scale doesn't become a technical problem.

In fact, in some circumstances it is possible to see the low energy effects of such high energy physics.

1.4.1 The see-saw mechanism

The see-saw mechanism [13] is a rather general way of producing Majorana mass terms for the left-handed neutrino fields, which are naturally of the correct order. We introduce right-handed neutrino field(s), which have masses just below the SUSY GUT scale of about $2.2 \cdot 10^{16}$GeV. While gauge invariance forbids a left-handed Majorana mass terms, right-handed neutrino fields are allowed, and Yukawa terms are also allowed. Taking there to be three right-handed neutrino generations [7], the full theory has a see-saw part of the superpotential:

$$W_{\text{see-saw}} = L_i H_u \overline{N}_j Y^{\nu}_{ij} + \frac{1}{2}\overline{N}_i \overline{N}_j \left(M_{RR}\right)_{ij} \tag{1.35}$$

Then, despite the fact that a tree-level left-handed Majorana mass matrix is not allowed by gauge invariance, one will be generated once we break $SU(2)_L \otimes U(1)_Y \to U(1)_{em}$ and integrate out the right handed neutrino fields. This then gives a term in the effective Lagrangian:

$$\mathcal{L}_{\text{eff}} = -\nu_i \nu_j \langle h_u \rangle^2 \left(Y^{\nu} \cdot M_{RR}^{-1} \cdot Y^{\nu\,\dagger}\right)_{ij} \tag{1.36}$$

Thus we have generated an effective Majorana mass matrix for the left-handed neutrinos as:

$$m_{LL} = Y^{\nu} \cdot M_{RR}^{-1} Y^{\nu\,\dagger} \langle h_u \rangle^2 \tag{1.37}$$

Now, if we set the Dirac-Yukawa mass term to be of the same order as the charged lepton mass terms, then we get Majorana masses of the right order if the right handed masses are just below the GUT scale. As there is no symmetry in the MSSM which protects the right-handed neutrino fields from attaining a mass,[8] they are expected to get masses around the high-energy scale, so thinking in terms of a string-inspired theory of a GUT, then the see-saw mechanism naturally generates neutrino masses of the right order.

It is possible to attain the neutrino mixing angles coming from the see-saw mechanism in a natural way as well, [14] which means that the neutrino measurements

[7] This works well phenomenologically, and is a requirement of most GUTs and string-inspired models

[8] While there is no symmetry in the MSSM which protects the right handed neutrino fields N from having a mass, there are in most GUTs, and in the Pati-Salam model. When these symmetries are broken, the N fields are expected to gain a mass close to the energy scale where the symmetry is broken. Thus in unified theories, we expect right handed neutrinos to acquire masses just below the unification scale $M_X \approx 10^{16}$GeV.

can actually act as a window into the very high energy physics, and it tells us that if the see-saw mechanism is correct, there is an energy scale very close to the GUT scale for SUSY theories.

1.4.2 The Froggatt-Nielsen mechanism

It is possible to understand the hierarchical form of the Yukawa matrices in a similar manner to how we understand the neutrino masses and mixings as coming from the see-saw mechanism. Following Froggatt and Nielsen, [15] we take a similar idea, of there being a high energy theory for which the MSSM is the appropriate low-energy effective field theory.

In the full theory, we introduce a new $U(1)_F$ gauge symmetry, under which there is a generation dependent charge. We can allow the Higgs fields to have charges under the new symmetry. We also introduce a new superfield, Φ_{FN}, which has charge -1 under $U(1)_F$. In the full theory, there are also messenger fields χ. Then we can generate the Yukawa interactions in the MSSM as being effective operators, which are generated when Φ_{FN} get a VEV. For example, we get a (non-renormalisable) term in the effective superpotential:

$$\mathcal{O}^u_{ij} = a^u_{ij} Q_i \overline{U}_j H_u \left(\frac{\Phi_{FN}}{M_\chi} \right)^{x_{q_i} + x_{u_j} + x_{h_u}} \tag{1.38}$$

a^u_{ij} is a coupling that we expect to be $O(1)$, and x_f is the $U(1)_F$ charge of the field f. When Φ_{FN} develops a VEV, this leads to a Yukawa interaction, in the same way that in the MSSM and SM, when the Higgs develops a VEV we get a Dirac mass term. In this case, defining $\epsilon =< \phi_{FN} > / M_\chi$ we can read off:

$$Y^u_{ij} = a^u_{ij} \epsilon^{x_{q_i} + x_{u_j} + x_{h_u}} \tag{1.39}$$

Then we can understand the hierarchical form of the Yukawa matrices, and the small quark mixing angles by having the the charges under $U(1)_F$ being different between each generation. In doing so, it is possible (at least in principle) to understand the hierarchical Yukawa matrices in terms of a number of $O(1)$ parameters, within the context of a symmetry which becomes broken at a very high energy scale.

This idea is very powerful when combined with the idea of a Grand Unified Theory, where in order to be consistent with the larger multiplets of Grand Unification, we impose the constraints that the charges of some (or all) of the fields have to be the same. For example, in a Pati-Salam unified theory enhanced with a $U(1)_F$, we must have that $x_{q_i} = x_{l_i}$, and $x_{u_i} = x_{d_i} = x_{e_i} = x_{n_i}$. This is useful in terms of model-building, because there will only be a small number of symmetries which can lead to the correct Standard Model particle spectrum which are consistent with a unified group such as Pati-Salam or $SO(10)$.

Chapter 2

Introduction to Pati-Salam and Strings

2.1 Introduction

In this chapter, we introduce two formalisms which could be appropriate for the understanding of particle physics at extremely high energies, approaching the Planck scale, at which it is believed that gravitation becomes comparable in strength to the gauge interactions, and quantum gravitational effects have to be taken into account. These two formalisms are those of the unification, in the context of the Pati-Salam model, and string theory. We then bring these threads together by describing a string-inspired Pati-Salam model.

2.2 The Pati-Salam model

Unified theories are the next logical step from the electroweak theory, which unify the left-handed quarks and leptons into left handed 'doublets' above the scale of electroweak symmetry breaking. Historically, the first such model proposed was the Pati-Salam model [16], where leptons are considered a 'fourth colour' [1], under a larger colour gauge group $SU(4)_c$, which undergoes spontaneous symmetry breaking at some very high energy scale. The Pati-Salam model has a gauge group of $G_{PS} = SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. Under G_{PS}, the left-handed matter F transforms non-trivially under $SU(4)_c$ and $SU(2)_L$ and the right-handed matter $\overline{F}$ transforms non-trivially under $SU(4)_c$ and $SU(2)_R$:

$$\begin{aligned} F^i_{\alpha,a} &= (\mathbf{4},\mathbf{2},\mathbf{1}) = \begin{pmatrix} u_{L,R} & u_{L,G} & u_{L,B} & \nu \\ d_{L,R} & d_{L,G} & d_{L,B} & e_L \end{pmatrix}^i \\ \overline{F}^{i\,\alpha,x} &= (\overline{\mathbf{4}},\mathbf{1},\mathbf{2}) = \begin{pmatrix} \overline{d}_{R,R} & \overline{d}_{R,G} & \overline{d}_{R,B} & N \\ \overline{u}_{R,R} & \overline{u}_{R,G} & \overline{u}_{R,B} & \overline{e}_R \end{pmatrix}^i \end{aligned} \tag{2.1}$$

Note that with the intention of eventually looking at the supersymmetric version of this model, the right handed field degreed of freedom have been CP-conjugated to transform as left-handed fields in order to facilitate constructing a superpotential.

In order to form a full right-handed representation, we have to introduce a right-handed neutrino field, so this model predicts neutrino masses, and we do not have to introduce right handed neutrino fields by hand. Also, in order to allow Yukawa terms that are gauge singlets, it is clear that the SM (MSSM) Higgs fields have to transform as $(\mathbf{1},\mathbf{2},\mathbf{2})$ under G_{PS} for the non-SUSY (SUSY) version of the model:

$$h = (\mathbf{1},\mathbf{2},\mathbf{2}) = \begin{pmatrix} h_u^+ & h_d^0 \\ h_u^0 & h_d^- \end{pmatrix} \tag{2.2}$$

This gauge group is clearly broken, and it must be at a very high energy scale in order to suppress operators which break baryon and lepton number converting quarks into leptons. In order to break this spontaneously, we need to introduce a new set of

[1]The original Pati-Salam model was non-supersymmetric; we are more interested in the supersymmetric variant [17]

'heavy' Higgs fields which get a VEV which breaks $G_{PS} \to G_{SM}$. These are $H, \overline{H}$:

$$H = (\mathbf{4}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} H_{u_R} & H_{u_G} & H_{u_B} & H_\nu \\ H_{d_R} & H_{d_G} & H_{d_B} & H_e \end{pmatrix}$$
$$\overline{H} = (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = \begin{pmatrix} \overline{H}_{d_R} & \overline{H}_{d_G} & \overline{H}_{d_B} & \overline{H}_e \\ \overline{H}_{u_R} & \overline{H}_{u_G} & \overline{H}_{u_B} & \overline{H}_N \end{pmatrix} \tag{2.3}$$

We have the unfortunate notation that for H, 2 represents the first row and 1 represents the second row. These attain VEVs in the 'neutrino' directions so:

$$\langle H_{\alpha,a} \rangle = v_H \delta^4_\alpha \delta^2_a \quad \langle \overline{H}_{\alpha,x} \rangle = v_{\overline{H}} \delta^4_\alpha \delta^2_x \tag{2.4}$$

These VEVs break the Pati-Salam group $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$ to the Standard Model group $SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$. Note that $B - L$ is actually a generator of $SU(4)_c$, which is broken when G_{PS} is broken. While the residual $U(1)$ groups in $SU(4) \otimes SU(2)_R$ are both broken, hypercharge can be made from a linear combination of them, and remains unbroken:

$$\frac{Y}{2} = I_{3R} + \frac{1}{2}(B - L) \tag{2.5}$$

The left-handed matter representations are then broken to the left-handed quark doublet and the left-handed lepton doublets. The right handed representations are broken apart into the up quark, down quark, electron and neutrino fields. This also breaks the Higgs bi-doublet apart into the two MSSM Higgs doublets.

Just as in the SM, the gauge bosons corresponding to broken generators become massive, while the gauge bosons corresponding to unbroken generators remain massless. After diagonalisation, the massive gauge bosons are:

- 1 $U(1)_{B-L}$ boson with mass squared $(v_H^2 + v_{\overline{H}}^2)(g_{2R}^2/4 + 3g_4^2/8)$

- 2 $SU(2)_R$ bosons with mass squared $(v_H^2 + v_{\overline{H}}^2)g_{2R}^2/4$

- 6 $SU(4)$ bosons with mass squared $(v_H^2 + v_{\overline{H}}^2)g_4^2/4$

g_4 and g_{2R} are the gauge couplings for $SU(4)_c$ and $SU(2)_{2R}$. The hypercharge coupling g' is related to these couplings via:

$$\frac{1}{g'^2} = \frac{1}{g_{2R}^2} + \frac{2}{3g_4^2} \tag{2.6}$$

Furthermore, right handed Majorana mass terms can be generated, at non-renormalisable order:

$$O_N = \overline{FF}\frac{\overline{HH}}{M_X} \tag{2.7}$$

The structure of the VEVs are such that these only generate such mass terms for the right-handed neutrinos. In SUSY Pati-Salam, such right-handed masses are generated at almost exactly the right order of magnitude to make the see-saw mechanism work without any need for fine tuning. So not only does SUSY Pati-Salam predict neutrino masses, but it can predict the masses at the correct order of magnitude.

There is no need to stop the unification here; it is possible to unify everything into a single multiplet under a larger group, such as $SO(10)$, with a single gauge coupling. This does have certain technical problems. The two main problems are predicting proton decay at rates which are competitive with the current experimental limits, and the 'doublet-triplet' splitting problem. The doublet-triplet splitting problem comes from the fact that the Higgs representations become unified. In order to sufficiently suppress Higgs-mediated proton decay, the colour triplet part has to become super-heavy, whereas the electroweak doublet part must have a mass at the electroweak scale. These problems don't occur in a fundamental Pati-Salam model.

2.3 Strings and phenomenology

In this section, we review the formalism of string theory and the current status of string phenomenology. We will ignore the more technical details, but they are covered in detail in some excellent introductory texts [9, 18, 19]. String theory is attractive since it is the only known theory that unifies the four fundamental forces (electromagnetism, strong, weak and gravity) within a *consistent* framework.

Everything that a QFT would call a field, a string theory would call a vibrational mode of a fundamental string. These fundamental strings are one-dimensional objects of length $1/M_*$, where M_* is the string scale. So scalars, fermions, vector bosons and gravitons would be seen as different vibrational modes of strings. Strings can be open, in which case they have two free ends, or they can be closed, so that the two ends have joined to form a loop [2]. The closed strings have a spin-2 massless excitation, which behaves in the way that a mediating particle of gravity would. The most successful string theories are those which the strings are supersymmetric, which tends to lead to a supersymmetric spectrum. String phenomenology is the ongoing attempt to find a way of embedding either the MSSM or the Standard Model into string theory, without too much low energy exotic matter. This task is made extremely challenging by a number of factors, which include the difficulty of performing detailed calculations in string theory, the large number of vacua that string theory has and the large number of experimental constraints that any string model has to be consistent with.

2.3.1 String dualities and M-theory

Since its inception, progress in string theory has been characterised by sudden progress after a key breakthrough or *revolution*, followed by longer periods of slow progress.

[2]A closed string can be thought of as the bound state of two open strings

The 'first string revolution' happened in 1984, when the first 10-dimensional, anomaly free type I string theory was constructed by Green and Schwarz [20]. The theory had both open and closed strings, and a gauge group of $SO(32)$. This was followed by the construction of two heterotic string theories, that combined bosonic strings in 26 dimensions with the 10d supersymmetric Green-Schwarz theory to give a 10d theory with a gauge group of $SO(32)$ and $E_8 \times E_8$ respectively [21]. All of these theories have $\mathcal{N} = 1$ spacetime supersymmetry, and large gauge groups which can easily accommodate the standard model gauge group. These string theories look promising for finding the MSSM, since the both $SO(32)$ and E_8 can break to $SU(3) \times SU(2) \times U(1)$, and there is the right amount of spacetime supersymmetry. Two type II string theories were later developed, with $\mathcal{N} = 2$ spacetime supersymmetry. These look less promising, since $\mathcal{N} = 2$ SUSY automatically preserves parity, and the weak interaction of the SM is parity-violating. Therefore, a mechanism of breaking $\mathcal{N} = 2$ to $\mathcal{N} = 1$ must be incorporated in the type II string theories in order to get acceptable low energy phenomenology. It should be noted that there is a lot to do in all five theories, as they predict large amounts of exotic states not in the MSSM, and the extra dimensions must be dealt with, in order to have only four dimensions evident at low energy.

In 1995, there was another key series of breakthroughs, the 'second string revolution', which started with the discovery of the strong/weak coupling duality [22] . This is a duality between the strong (weak) coupling phase of the heterotic $SO(32)$ theory with the weak (strong) coupling phases of the type I theory. A number of dualities were discovered linking the five string theories. The revolution culminated with the realisation that all of the five superstring theories were different limits of an 11-dimensional theory, *M-theory* . The full set of dualities is laid out in figure 2.1. Another important discovery was the discovery of extended solitonic objects, Dirichlet branes (D-branes)

[23, 24, 25], in type I and type II string theory.

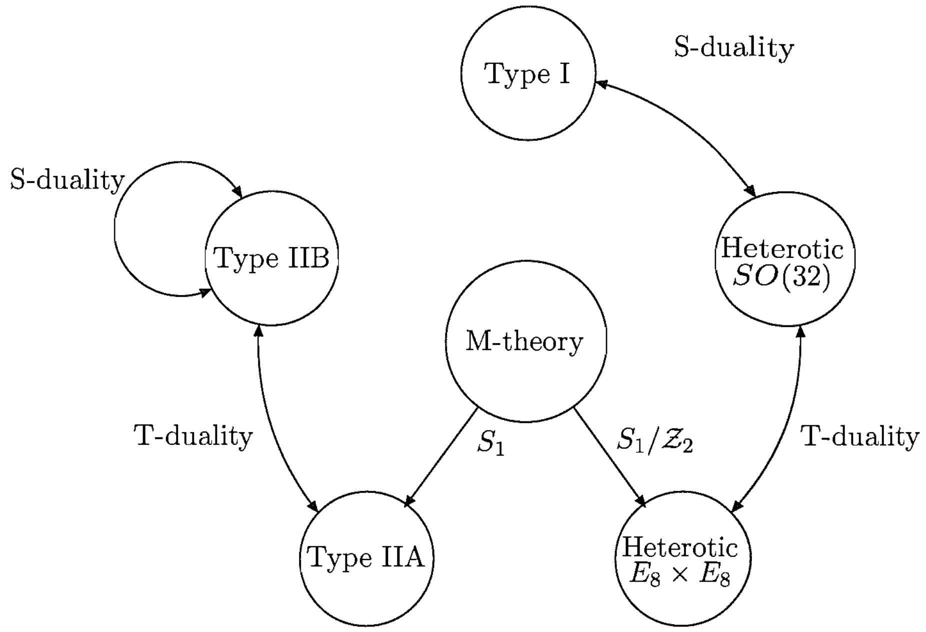


Figure 2.1: The dualities linking M-theory with the five string theories. The dualities linking the five string theories are shown as double headed arrows. The compactifications leading from M-theory to a 10d string theory is denoted by a radial arrow.

A DN-brane is a $(N + 1)$-dimensional manifold of the full 10d spacetime; the ends of open strings in the theory are constrained to lie within one of the D-branes. The endpoints of the string then have Neumann boundary conditions within the D-brane, and Dirichlet boundary conditions in the directions transverse to the D-brane. A stack of coincident D-branes leads to a symmetry under which the open strings transform. Usually, a stack of N coincident branes leads to a $U(N)$ symmetry group. This symmetry group behaves like a global symmetry when viewed from the string 'worldsheet' perspective, but behaves like a gauge symmetry when viewed from the target-space perspective. Gravity fields then arise as closed strings. Thus the study of models constraining D-branes is interesting, since it gives a natural way of considering small gauge groups. Thus scenarios in Type I and type II string theory with branes can lead to models with less exotic matter to be dealt with.

2.3.2 Aspects of Type I strings

We now move on to consider some aspects of type I supergravity[3] relevant for attempts to construct MSSM like models [4]. For a more complete discussion, see ref. [26].

Type I string theory in 10d can be obtained by taking an 'orientifold' [27, 28, 29, 30, 31] of the 10d Type IIB string theory. The orientifold takes a parity operation, Ω on the IIB string worldsheet; this operation transforms left-moving and right moving vibrations into each other. The result of this projection leads to a closed, unoriented string with $\mathcal{N} = 1$ SUSY in 10d. Furthermore, for consistency open strings whose ends are attached to D-branes have to be added to the theory. The consistency conditions (tadpole anomaly cancellation) require 32 D9-branes in the vacuum, and the open strings will transform in the adjoint representation of $SO(32)$. This leads to a $D = 10$, $\mathcal{N} = 1$ target-space string theory with open and closed strings.

One problem with this orientifold is that it doesn't break enough supersymmetry; although Type I string theory is a $\mathcal{N} = 1$ theory in 10d, compactification tends to lead to extended supersymmetry in the lower dimensional theory. This is because the fermionic supersymmetry generators get 'split' by compactification; The supersymmetry generators have to be in the fundamental spinor representation of the Poincaré group; for a 10d theory, this is $SO(9,1)$, and for a 4d theory this is $SO(3,1)$. The extra degrees of freedom don't vanish. A 10d $\mathcal{N} = 1$ theory compactified on a 6-Torus would become a $\mathcal{N} = 4$ theory in 4d. [5]. As we wish to have a $D = 4, \mathcal{N} = 1$ theory, we must

[3]Type I SUGRA is the SUGRA corresponding to type I string theory

[4]By MSSM like models, we mean models that contain either the MSSM or one of its simple extensions as its appropriate low-energy description. This can include string-inspired GUTs at energies close to the string scale.

[5]We can count the number of supersymmetries by looking at the spectrum, since the number of

find a way of breaking some (but not all) of the supersymmetry.

We will consider Type IIB 4d orientifolds, obtained by compactifying six dimensions on a six-torus $T^6 = T^2_{(1)} \times T^2_{(2)} \times T^2_{(3)}$. [32, 33, 34, 35, 36, 37, 38, 39, 40] Each pair of extra dimensions is wrapped around a symmetric two-torus $T^2_{(i)}$ with radius R_i; the two-torus will then have volume $v_i = (2\pi R_i)^2$. We label a spacetime co-ordinates as x_i, where $x_0 - x_3$ are the usual 4d Minkowski spacetime coordinates and the remaining six correspond to the compactified dimension. Since the compactification is so simple, it is convenient to treat each pair of compact dimensions as a complex number, z_i:

$$z_1 = (x_4, x_5) \quad , \quad z_2 = (x_6, x_7) \quad , \quad z_3 = (x_8, x_9) \tag{2.8}$$

With z_i spanning the two-torus $T^2_{(i)}$.

Imposing an orientifold group $\{\Omega \times G\}$, where Ω is the world-sheet parity, and G is a discrete Abelian group $G = \prod_{i=1}^{n} Z_i$ is said to 'twist' the theory. Twisting the theory in this way leads to an $\mathcal{N} = 1$ SUSY in 4d with the presence of fixed points which are invariant under the action of the orientifold group. There are only a finite number of Type IIB orientifolds which lead to $\mathcal{N} = 1$ in 4d; these have already been classified, in the context of toroidal heterotic compactifications [41]. The action of orientifolding leads to tadpole divergences; in order to cancel these, Dp-branes must be introduced into the vacuum of the theory. In order to preserve $\mathcal{N} = 1$ SUSY, p must be 5 and/or 9. Thus the string constructions can contain D9-branes and D5_i-branes. The D5_i-branes span the non-compact Minkowski space plus the two-torus $T^2_{(i)}$; the compactification radius of $T^2_{(i)}$ is often labelled as R_{5_i}. The tadpole cancellation conditions strongly constrain the massless spectrum and the gauge structure by projecting out states which are not singlets under the orientifold group [26, 42, 37, 38]. [6].

We now consider the open and closed string states which appear in generic construc-

supersymmetries must be equal to the number of gravitinos. There can be only one graviton, but there

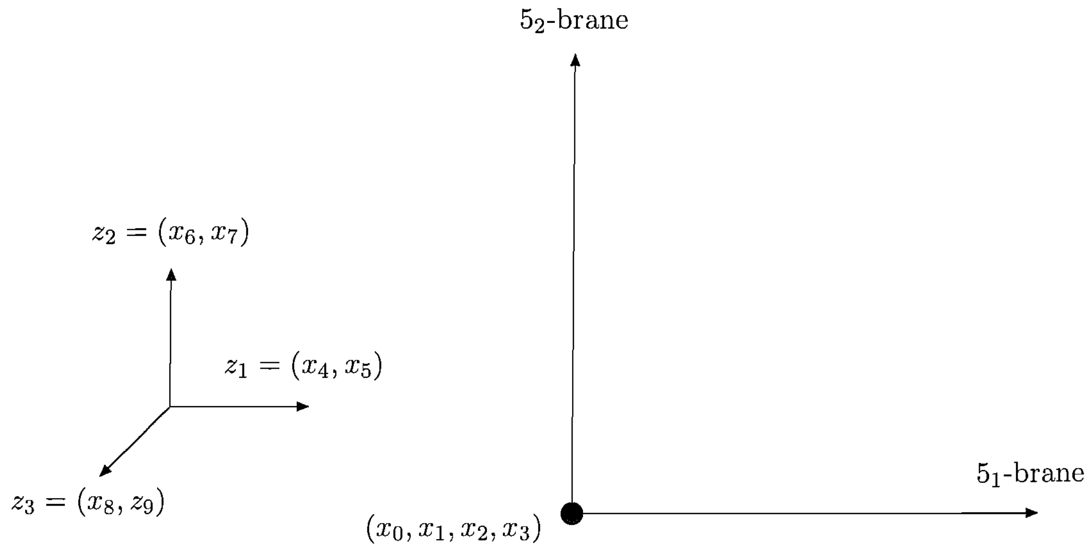


Figure 2.2: We represent the six-dimensional compact space using a complex coordinate system (*left*), where D5_i-branes are shown as straight line along the z_i direction. A D5_1-brane and a D5_2 brane will overlap in Minkowski space (which is the origin of the coordinate system) but extend out in perpendicular directions (*right*)

tions involving stacks of D9-branes and (up to) three stacks of D5_i-branes. A D9-brane fills the entire 10d spacetime, $(x_0 - x_9)$, whereas a D5_i brane spans the Minkowski spacetime, plus two extra compact dimensions which wrap the two-torus $T^2_{(i)}$. We can represent the 6-dimensional compact space on the six-torus T^6 using three complex coordinates z_i, where each coordinate corresponds to a pair of compact dimensions, as in Eq. (2.8). In this system, a stack of coincident D5_i-branes are represented by a single line along the i^{th} coordinate. See figure 2.2.

Two types of massless $\mathcal{N} = 1$ chiral fields arise in Type I string models:

- Closed string chiral singlets

---

will be $\mathcal{N}$ generators Q_α, generating $\mathcal{N}$ gravitinos when acting upon a graviton state.

[6]The presence of a background field $B_{\mu\nu}$ or non-trivial Wilson lines can modify the tadpole cancellation conditions, and lead to a reduced rank gauge group since fewer D-branes are required to cancel the tadpoles

Chiral singlets arise from the scalar excitation of closed string states. They include the 4d dilaton S and the untwisted moduli fields $T_i, (i = 1, 2, 3)$, which are all free to move in the full 10d spacetime. The VEVs of the untwisted moduli parametrise the size of the compactified dimensions, and the compactification radius of the i^{th} two torus $T^2_{(i)}$ is given by [26]:

$$R_i = \frac{\sqrt{T_i + \overline{T_i}}}{2M_*} \tag{2.9}$$

In Type I models, the gauge coupling corresponding to a stack of branes is related to the corresponding modulus, S for the 9-branes and T_i for the 5_i-branes. There are also closed string states that are trapped at the fixed point singularities of the orientifold group, the *twisted* moduli fields Y_k. The twisted moduli parametrise the size of the fixed points. The twisted states can contribute to SUSY breaking and also modify the relation of the brane gauge couplings.

- Charged open string states

Chiral superfields arise as open strings attached to D-branes. Since quarks, leptons and Higgs fields are all members of chiral superfields in the MSSM, they should correspond to open string states in the string theory. The open strings can either end on *different* D-branes (denoted as $C^{\alpha\beta}$ e.g. $C^{5_i5_j}$, or attached to the same D-brane and (denoted by the D-brane they attach to and a winding direction C^{α}_j e.g. $C^{5_1}_2$).

The most general setup consists of a D9-brane and three D5_i-branes. In this scenario, string selection rules constraining the allowed interactions which appear in the superpotential at renormalisable level [26]:

$$\begin{aligned} W &= g_{5_1}\left(C^{5_1}_1 C^{5_1}_2 C^{5_1}_3 + C^{5_1}_3 C^{5_1 5_2} C^{5_1 5_2} + C^{5_1}_1 C^{95_1} C^{95_1}\right) \\ &\quad + g_{5_2}\left(C^{5_2}_1 C^{5_2}_2 C^{5_2}_3 + C^{5_2}_3 C^{5_1 5_2} C^{5_1 5_2} + C^{5_2}_2 C^{95_2} C^{95_2}\right) \\ &\quad + g_{5_3} C^{5_1 5_2} C^{95_1} C^{95_2} + g_9\left(C^9_1 C^9_2 C^9_3 + C^9_1 C^{95_1} C^{95_1} + C^{9_2} C^{95_2} C^{95_2}\right) \end{aligned} \tag{2.10}$$

The coupling constants are given by:

$$g_{5_i}^2 = \frac{4\pi}{\Re T_i} \quad , \quad g_9^2 = \frac{4\pi}{\Re S} \tag{2.11}$$

In Appendix A, we discus the supergravity formalism that can be used for the low-energy description of Type I models in terms of the Kähler potential, superpotential and gauge kinetic functions [43]. We also provide general expressions for the terms of the soft supersymmetry breaking Lagrangian in terms of the SUSY breaking F-terms, assuming that only closed string states acquire non-zero VEVs.

2.4 A String Pati-Salam model

We now discuss a string model for which the supersymmetric Pati-Salam model discussed in section 2.2 becomes the appropriate effective field theory when the superheavy exotic states are integrated out. Both the string theoretical [44] and the model building [45] aspects have been considered in detail. We summarise the relevant parts of the model here.

The model is a Type I model with two stacks of 5-branes. The two stacks are taken to be the 5_1 and 5_2 branes, although any other combination would be equivalent. In the full string model, there is a gauge group of $U(4) \otimes U(2)_a \otimes U(2)_b$ on each brane. We take the gauge group on the 5_2 brane to have been broken to $U(4)^{(2)}$. Thus the gauge group on the 5_1 brane is $U(4)^{(1)} \otimes U(2)_L \otimes U(2)_R$, and the gauge group on the 5_2 brane is $U(4)^{(2)}$. The setup is shown schematically in fig. 2.3.

The group representations of the field content is laid out in table 2.1. In the table, h represents the MSSM Higgs, F^i the i^{th} generation of left handed matter, $\overline{F}^j$ the j^{th} generation of right handed matter, H and $\overline{H}$ are the Higgs fields responsible for breaking $SU(4) \otimes SU(2) \otimes SU(2) \to SU(3) \otimes SU(2) \otimes U(1)$. The model therefore has a

Field	$SU(4)^{(1)}$	$SU(2)_L$	$SU(2)_R$	$SU(4)^{(2)}$	$Q_4^{(1)}$	Q_{2L}	Q_{2R}	$Q_4^{(2)}$	Brane assignment
h	1	2	2	1	0	1	−1	0	$C_1^{5_1}$
F_3	4	2	1	1	1	−1	0	0	$C_2^{5_1}$
$\overline{F}_3$	$\overline{4}$	1	2	1	−1	0	1	0	$C_3^{5_1}$
$F_{1,2}$	1	2	1	4	0	−1	0	1	$C^{5_1 5_2}$
$\overline{F}_{1,2}$	1	1	2	$\overline{4}$	0	0	1	−1	$C^{5_1 5_2}$
H	4	1	2	1	1	0	−1	0	$C_1^{5_1}$
$\overline{H}$	$\overline{4}$	1	1	4	−1	0	1	0	$C_2^{5_1}$
φ_1	4	1	1	$\overline{4}$	1	0	0	−1	$C^{5_1 5_2}$
φ_2	$\overline{4}$	1	1	4	−1	0	0	1	$C^{5_1 5_2}$
$D_6^{(+)}$	6	1	1	1	2	0	0	0	$C_1^{5_1}$
$D_6^{(-)}$	6	1	1	1	−2	0	0	0	$C_2^{5_1}$

Table 2.1: The particle content of the '4224' string Pati-Salam model. We have used the isomorphism $U(N_a) = U(1)_a \otimes SU(N_a)$ to write a $U(N_a)$ representation as a $U(1)_a$ charge and a $SU(N_a)$ representation.

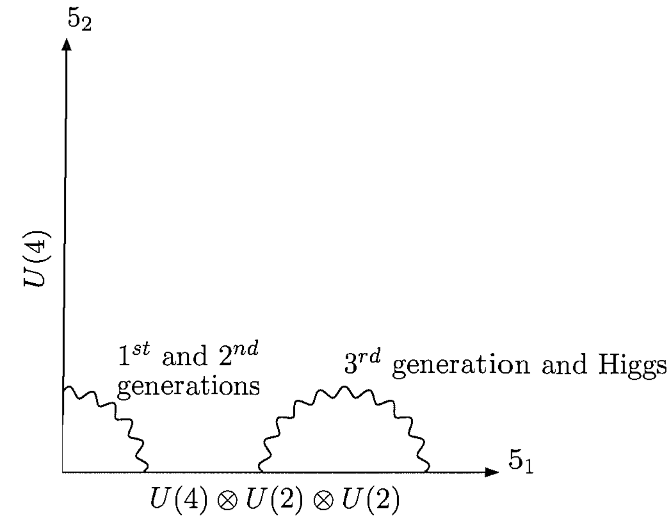


Figure 2.3: Setup of the '4224' model. The first and second generations of matter arise as $C^{5_1 5_2}$ intersections states; the third generation, Pati-Salam and Electroweak breaking Higgs fields arise as $C_i^{5_1}$ states.

gauge symmetry $SU(4)^{(1)} \otimes SU(4)^{(2)} \otimes SU(2)_L \otimes SU(2)_R \otimes U(1)^4$. The third generation arises from the 5_1 sector, and the first two generations are intersection states.

The gauge symmetry breaking pattern follows the three-family approach of Shiu and Tye [44]. There are two stages to the symmetry breaking, which are assumed to occur close to the unification scale, M_U. The first stage is the breaking of $U(4)^{(1)} \otimes U(4)^{(2)}$ to the diagonal $U(4)$ subgroup, which is identified as the $U(4)_c$ part of the Pati-Salam group. This breaking is done by the diagonal VEVs of $\varphi_{1,2}$; the resulting theory has a $U(1)^3$ enhanced Pati-Salam group. The $U(1)$s are anomalous, and are expected to decouple by the scale that the Pati-Salam group breaks to the MSSM group. The breaking of the Pati-Salam is by the PS Higgs fields, $H, \overline{H}$. The breaking should occur along flat directions in order to preserve supersymmetry; D flatness should be spoilt only by terms of the order of the soft parameters.

The symmetry breaking pattern occurs as follows:

$$U(4)^{(1)} \otimes U(4)^{(2)} \otimes U(2)_L \otimes U(2)_R \overset{<\varphi_{1,2}>}{\longrightarrow} U(4)_c \otimes U(2)_L \otimes U(2)_R \qquad (2.12)$$
$$\overset{<H,\overline{H}>}{\longrightarrow} SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \otimes U(1)^3$$

The simultaneous requirements of (approximate) D-flatness and diagonal symmetry breaking gives us the VEVs of the φ fields:

$$\langle(\varphi_1)_{a\alpha}\rangle = \delta_{a\alpha}v \ ; \ \langle(\varphi_2)_{a\alpha}\rangle = \delta_{a\alpha}v. \qquad (2.13)$$

At this point, we note that there are a number of combinations that are overall colour triplets which would lead to high rates of proton decay. The colour sextet fields $D_6^{(\pm)}$ exist to give these super-heavy mass terms in order to reduce the rate to an acceptable level. More details are beyond the present scope, but are presented in ref.[45].

2.4.1 MSSM couplings

As the string model is predictive, we can make quantitative statements about the couplings in the MSSM, which is the appropriate effective field theory below the GUT scale of $M_U \approx 10^{16}$GeV. On the string side, there are two gauge couplings, one associated with each 5_i-brane. In the MSSM, there are three, and they are related to the brane couplings g_{5_i} by:

$$\begin{aligned} g_3 &= \frac{g_{5_1} g_{5_2}}{\sqrt{g_{5_1}^2 + g_{5_2}^2}} \\ g_2 &= g_{5_1} \\ g_Y &= \frac{\sqrt{3} g_{5_1} g_{5_2}}{\sqrt{5g_{5_2}^2 + 2g_{5_1}^2}} \end{aligned} \qquad (2.14)$$

We now turn to the Yukawa couplings. The string assignments have been chosen to allow a third family-third family-higgs Yukawa coupling, but no others. We see that in eq. (2.10) the term $C_1^{5_1} C_2^{5_1} C_3^{5_1}$ is allowed, thus giving an $O(1)$ Yukawa coupling

$F_3\overline{F}_3h$. However, the terms that would give Y_{ij}, Y_{i3} and Y_{3j} $(i,j = \{1,2\})$ do not appear so only the $(3,3)$ element is allowed at renormalisable order due to the form of the superpotential, eq. (2.10) and the string assignments as given in table 2.1. The model predicts third family Yukawa unification, and therefore requires large $\tan\beta \approx 50$ to ensure the correct m_t/m_b ratio.

The small (non-zero) values of the Yukawa couplings of the first and second generations as well as the Majorana right-handed mass matrix are generated by non-renormalisable operators. These could be generated by operators of the form $F_i\overline{F}_jh(H\overline{H}^n)$ [46], by Froggatt-Nielsen operators for models extended with a family symmetry or by combining the two approaches [47].

We can use the phenomenological approach of [48] to write down the soft couplings in terms of a few goldstino parameters. Here, we make the usual assumption that the auxiliary fields of the moduli contribute dominantly over the auxiliary members of the Higgs superfields $\varphi_i, H, \overline{H}$ to all of the soft terms. This is not necessarily true for the trilinear soft parameters A_{ijk} [49], an issue that will be returned to in Chapter 3. We start off by parameterising the auxiliary fields. Then, through the expressions in Appendix A, we will obtain the gaugino soft masses, the scalar soft masses, and the trilinear soft couplings for the MSSM in terms of a few goldstino parameters, X_α and the gravitino mass $m_{3/2}$:

$$F^S = \sqrt{3}(S+S^*)m_{3/2}X_S e^{i\alpha_S} \tag{2.15}$$

$$F^{T_i} = \sqrt{3}(T_i+T_i^*)m_{3/2}X_i e^{i\alpha_i} \tag{2.16}$$

The goldstino parameters sum square to one: $\sum_i X_i^2 = 1$, and in general the auxiliary fields can have arbitrary phases α_S, α_i. The masses of the 4224 gauginos are:

$$m_4^{(1)} = m_{2L} = m_{2R} = \sqrt{3} m_{3/2} X_1 e^{-i\alpha_1} \tag{2.17}$$

$$m_4^{(2)} = \sqrt{3} m_{3/2} X_2 e^{-i\alpha_2} \tag{2.18}$$

The MSSM gaugino masses are then:

$$m_3 = \frac{\sqrt{3} m_{3/2}}{(T_1 + T_1^*) + (T_2 + T_2^*)} \left[(T_1 + T_1^*) X_1 e^{-i\alpha_1} + (T_2 + T_2^*) X_2 e^{-i\alpha_2} \right] \tag{2.19}$$

$$m_2 = \sqrt{3} m_{3/2} X_1 e^{-i\alpha_1} \tag{2.20}$$

$$m_1 = \frac{\sqrt{3} m_{3/2}}{\frac{5}{3}(T_1 + T_1^*) + \frac{2}{3}(T_2 + T_2^*)} \left[\frac{5}{3}(T_1 + T_1^*) X_1 e^{-i\alpha_1} + \frac{2}{3}(T_2 + T_2^*) X_2 e^{-i\alpha_2} \right] \tag{2.21}$$

Each scalar mass will depend only on which string assignment it is; they are:

$$m_h^2 = m_{H_u}^2 = m_{H_d}^2 = m_{3/2}^2 (1 - 3X_S^2) \tag{2.22}$$

$$m_{Q_3}^2 = m_{L_3}^2 = m_{3/2}^2 (1 - 3X_3^2) \tag{2.23}$$

$$m_{U_3}^2 = m_{D_3}^2 = m_{E_3}^2 = m_{N_3}^2 = m_{3/2}^2 (1 - 3X_2^2) \tag{2.24}$$

$$m_{Q_{1,2}}^2 = m_{L_{1,2}}^2 = m_{3/2}^2 (1 - \frac{3}{2}(X_S^2 + X_3^2)) \tag{2.25}$$

$$m_{U_{1,2}}^2 = m_{D_{1,2}}^2 = m_{E_{1,2}}^2 = m_{N_{1,2}}^2 = m_{3/2}^2 (1 - \frac{3}{2}(X_S^2 + X_3^2)) \tag{2.26}$$

The trilinear parameters $\tilde{A}_{ij}$ can then be written down in terms of A_{ij} and the Yukawa couplings Y_{ij}:

$$\tilde{A}_{ij} = \begin{bmatrix} d_1 Y_{11} & d_1 Y_{12} & d_2 Y_{13} \\ d_1 Y_{21} & d_1 Y_{22} & d_2 Y_{23} \\ d_3 Y_{31} & d_3 Y_{32} & d_4 Y_{33} \end{bmatrix} \tag{2.27}$$

where the d_i are defined as:

$$d_1 = \sqrt{3}m_{3/2}\left(X_S e^{-i\alpha_S} - X_1 e^{-i\alpha_1} - X_2 e^{-i\alpha_2}\right) \tag{2.28}$$

$$d_2 = \sqrt{3}m_{3/2}\left(\frac{1}{2}X_S e^{-i\alpha_S} - X_1 e^{-i\alpha_1} - \frac{1}{2}X_3 e^{-i\alpha_3}\right) \tag{2.29}$$

$$d_3 = \sqrt{3}m_{3/2}\left(\frac{1}{2}X_S e^{-i\alpha_S} - X_1 e^{-i\alpha_1} - X_2 e^{-i\alpha 2} + \frac{1}{2}X_3 e^{-i\alpha_3}\right) \tag{2.30}$$

$$d_4 = -\sqrt{3}m_{3/2}X_1 e^{-i\alpha_1} \tag{2.31}$$

The string-derived superpotential doesn't allow the MSSM to have a μ term. The μ and $B\mu$ terms must be generated in the effective theory, (such as by the Giudice-Masiero mechanism [50]). This is model dependent, and we will not consider the issue further. However, their magnitudes will be set by the requirements of radiative electroweak symmetry breaking.

Chapter 3

Lepton Flavour Violation from Yukawa operators, Supergravity and the See-Saw mechanism

We investigate the phenomenological impact of different sources of lepton flavour violation arising from realistic models based on supergravity mediated supersymmetry breaking with Yukawa operators. We discuss four distinct sources of lepton flavour violation in such models: minimum flavour violation, arising from neutrino masses and the see-saw mechanism with RG running; supergravity flavour violation due to the non-universal structure of the supergravity model; flavour violation due to Froggatt-Nielsen (FN) fields appearing in Yukawa operators developing supersymmetry breaking F-terms and contributing in a non-universal way to soft trilinear terms; and finally heavy Higgs flavour violation arising from the heavy Higgs fields used to break the unified gauge symmetry which also appear in Yukawa operators and behave analogously to the FN fields. In order to quantify the relative effects, we study a particular type I string inspired model based on a supersymmetric Pati-Salam model arising from intersecting D-branes, supplemented by a U(1) family symmetry.

3.1 Introduction

Lepton flavour violation (LFV) has been long known to be a sensitive probe of new physics in supersymmetric (SUSY) models [51, 52]. LFV arises in SUSY models due to off-diagonal slepton masses in the super-CKM basis in which the Yukawa matrices are diagonal. Such flavour violation could arise either directly at the high energy scale due to primordial string or SUGRA effects, or be generated radiatively by the renormalisation group equations, for example in running a grand unified theory (GUT) from the Planck scale to the GUT scale (due to the presence of Higgs triplets) [53] or in running the minimal supersymmetric standard model (MSSM) with right-handed neutrinos from the Planck scale down to low energies, through the scales at which the right-handed neutrinos decouple.

Even in minimal supergravity (mSUGRA), where there is no flavour violation at the high energy scale, the presence of heavy right-handed neutrinos as required by the see-saw mechanism explanation of small neutrino masses will lead inevitably to LFV [54, 55]. The recent neutrino experiments which confirm the matter enhanced Large Mixing Angle (LMA) solution to the solar neutrino problem [12], together with the atmospheric data [7], show that neutrino masses are inevitable, and, assuming SUSY and the see-saw mechanism, hence show that LFV must be present. For example this has recently been studied in mSUGRA models with a natural neutrino mass hierarchy [56]. There is in fact a large literature on this subject [57].

Recently it has been realised that in realistic string inspired models based on supergravity mediated supersymmetry breaking, in which the origin of Yukawa matrices

Y_{ij} is due to Froggatt-Nielsen (FN) operators [15] of the form $Y_{ij} \sim \theta^{n_{ij}}$, where n_{ij} is an integer power, there may be a new and dangerous source of LFV which arises when the FN fields θ develop supersymmetry breaking F-terms $F_\theta \sim m_{3/2}\theta$ leading to non-universal soft trilinear terms $\Delta A_{ij} = F_\theta \partial_\theta \ln Y_{ij}$ [58] which implies $\Delta A_{ij} \sim n_{ij} m_{3/2}$ [59, 60]. The effect is independent of the vacuum expectation value (vev) of the FN field θ, and is present even in minimum flavour violation scenarios such as mSUGRA.

In this chapter we shall explore the phenomenological impact of the new source of LFV arising from FN fields discussed above, and compare it to the more usual sources of LFV arising from right-handed neutrinos, and non-universal SUGRA models in order to gauge its relative importance. A phenomenological analysis is necessarily model dependent, and so we shall study a particular type I string inspired model based on a SUSY Pati-Salam model arising from intersecting D-branes, which was introduced in [45]. However in order to explore the effects of interest, it is necessary to supplement this model by a $U(1)$ family symmetry, and introduce FN fields so as to provide a realistic description of quark and lepton masses and mixing angles, including those of the neutrino sector. Recently a global χ^2 analysis of a realistic SUSY Pati-Salam model was performed [47], and a good fit to the quark and lepton mass spectrum was obtained based on a FN operator analysis with a $U(1)$ family symmetry. It is therefore natural to combine the models in [45] and [47] in order to provide a realistic framework for studying the new LFV effects arising from the FN fields, and to compare this to the effects on non-universal SUGRA and also right-handed neutrinos in a model that gives a good fit to the neutrino data.

Of course in combining the two models we are taking some liberties with string theory. In particular we assume that the combined model corresponds to the low energy limit of a string model as in [45], but with the addition of an extra state, which is a Froggatt-Nielsen [15] family field, θ. We assume that since the model without θ can be extracted from a string model, then so can the model with θ, but we make no attempt to derive it. We emphasise that the main motivation for combining the two approaches is to explore the phenomenology of LFV in a 'realistic' framework. One by-product of doing this is that we identify a genuinely new source of LFV that has not been considered at all in the literature, namely the heavy Higgs fields H that break the unified gauge symmetry at high energies. These heavy Higgs fields also appear in the operators which describe the Yukawa couplings, and they can be expected to behave in a similar way to the FN fields θ, and give rise to LFV analogously. The combined model has a number of attractive features: it includes approximate third family Yukawa unification, the number of free parameters is restricted to eight undetermined free parameters related to supergravity, and the model gives a good fit to all quark and lepton masses and mixing angles.

In order to study the phenomenological effect of the different sources of LFV, we generalise the Goldstino Angle parametrisation of the dilaton and moduli fields S, T_i to include a parametrisation of the SUSY breaking F-terms for the FN fields θ and heavy Higgs fields H. There are four distinct sources of lepton flavour violation in this model: minimum flavour violation, arising from neutrino masses and the see-saw mechanism with renormalisation group (RG) running; supergravity flavour violation due to the non-universal structure of the supergravity model; FN flavour violation due to the FN fields developing supersymmetry breaking F-terms and contributing in an non-universal way to soft trilinear terms; and finally heavy Higgs flavour violation arising from the Higgs fields used to break the unified gauge symmetry which may behave analogously to the FN fields. We propose four benchmark points at which each of these four sources separately dominate. We then perform a detailed numerical analysis of LFV arising from the four benchmark points. The numerical results show that LFV due to FN fields is the most sensitive source in the sense of leading to larger limits of $m_{3/2}$, however we find that the gluino mass is relatively light in these cases which tends to reduce fine-tuning. We also find that in some cases the LFV effects from Yukawa operators in the presence of the seesaw mechanism can be less than without the seesaw mechanism.

The outline of this chapter is as follows. In section 3.2 we introduce the specific model that we shall study, discuss the symmetries of the model, and the Yukawa and Majorana operators, and for particular choices of the order unity coefficients, show that this leads to a good fit to the neutrino data, with a prediction for the unmeasured θ_{13}. In section 3.3 we discuss the soft SUSY breaking aspects of the model. We parametrise the SUSY breaking F-terms, give the soft scalar masses, including the D-term contributions, give the soft gaugino masses and soft trilinear couplings, and explain why these are expected to lead to large flavour violation. In section 3.4 we give the results of a numerical analysis of the model, focusing on four benchmark points designed to highlight the four different sources of LFV. Finally we present our concluding remarks in section 3.5

3.2 The Model

3.2.1 Symmetries and Symmetry Breaking

The model defined in Table 3.1 is an extension of the string inspired Supersymmetric Pati-Salam model discussed in ref.[45], and summarised in Chapter 2 , based on two $D5$ branes which intersect at 90 degrees and preserve SUSY down to the TeV energy scale. The string scale is taken to be equal to the GUT scale, about 3×10^{16} GeV.

The extension is to include an additional $U(1)_F$ family symmetry and the FN op-

Field	$SU(4)^{(1)}$	$SU(2)_L$	$SU(2)_R$	$SU(4)^{(2)}$	Ends	$U(1)_F$ charge	$U(1)_{\overline{F}}$ charge
h	1	2	2	1	$C_1^{5_1}$	0	0
F_3	4	2	1	1	$C_2^{5_1}$	$\frac{5}{6}$	0
$\overline{F}_3$	$\overline{4}$	1	2	1	$C_3^{5_1}$	$-\frac{5}{6}$	0
F_2	1	2	1	4	$C^{5_1 5_2}$	$\frac{5}{6}$	2
$\overline{F}_2$	1	1	2	$\overline{4}$	$C^{5_1 5_2}$	$\frac{7}{6}$	0
F_1	1	2	1	4	$C^{5_1 5_2}$	$\frac{11}{6}$	1
$\overline{F}_1$	1	1	2	$\overline{4}$	$C^{5_1 5_2}$	$\frac{19}{6}$	4
H	4	1	2	1	$C_1^{5_1}$	$\frac{5}{6}$	0
$\overline{H}$	$\overline{4}$	1	2	1	$C_2^{5_1}$	$-\frac{5}{6}$	0
φ_1	4	1	1	$\overline{4}$	$C^{5_1 5_2}$	–	–
φ_2	$\overline{4}$	1	1	4	$C^{5_1 5_2}$	–	–
$D_6^{(+)}$	6	1	1	1	$C_1^{5_1}$	–	–
$D_6^{(-)}$	6	1	1	1	$C_2^{5_2}$	–	–
θ	1	1	1	1	$C^{5_1 5_2}$	-1	-1
$\overline{\theta}$	1	1	1	1	$C^{5_1 5_2}$	1	1

Table 3.1: The particle content of the 42241 model, and the brane assignments of the corresponding string

erators as in [47] (see also [46]). The present 42241 Model is then just the 4224 Model of [45] augmented by a $\mathrm{U}(1)_F$ family symmetry. The purpose of this extension is to allow a more realistic texture in the Yukawa trilinears Y_{abc}, along the lines of the recent operator analysis in [47].

The quark and lepton fields are contained in the representations $F, \overline{F}$ which are assigned charges X_F under $\mathrm{U}(1)_F$. In Table 3.1 we list two equivalent sets of charges $\mathrm{U}(1)_F$ and $\mathrm{U}(1)_{\overline{F}}$, where $\mathrm{U}(1)_F$ is anomaly free, but $\mathrm{U}(1)_{\overline{F}}$ is equivalent for all practical purposes and has much simpler charge assignments. The field h represents both Electroweak Higgs doublets that we are familiar with from the MSSM. The fields H and $\overline{H}$ are the Pati-Salam Higgs scalars; [1] the bar on the second is used to note that it is in the conjugate representation compared to the unbarred field.

The extra Abelian $U(1)_F$ gauge group is a family symmetry, and is broken at the high energy scale by the VEVs of the FN fields [15] $\theta, \overline{\theta}$, which have charges -1 and $+1$ under $U(1)_F$, respectively. We assume that the singlet fields $\theta, \overline{\theta}$ arise as intersection states between the two $D5-$branes, transforming under the remnant U(1)s in the 4224 gauge structure. In general they are expected to have non-zero F-term VEVs.

The two $SU(4)$ gauge groups are broken to their diagonal subgroup at a high scale due to the assumed VEVs of the fields φ_1, φ_2 [45]. The symmetry breaking at the scale M_X

$$\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R \rightarrow \mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y \tag{3.1}$$

is achieved by the heavy Higgs fields H, $\overline{H}$ which are assumed to gain VEVs [46])

$$\left\langle H^{\alpha b}\right\rangle = \langle \nu_H \rangle = V\delta_4^{\alpha}\delta_2^{b} \sim M_X \; ; \; \left\langle \overline{H}_{\alpha x}\right\rangle = \langle \overline{\nu}_H \rangle = \overline{V}\delta_{\alpha}^{4}\delta_{x}^{2} \sim M_X \tag{3.2}$$

This symmetry breaking splits the Higgs field h into two Higgs doublets, h_1, h_2. Their

[1] We will also refer to these as 'Heavy Higgs'; this has nothing to do with the MSSM heavy neutral Higgs state H^0

neutral components then gain weak-scale VEVs

$$\langle h_1^0 \rangle = v_1 \; ; \; \langle h_2^0 \rangle = v_2 \; ; \; \tan\beta = v_2/v_1. \tag{3.3}$$

The low energy limit of this model contains the MSSM with right-handed neutrinos. We will return to the right handed neutrinos when we consider operators including the heavy Higgs fields H, $\overline{H}$ which lead to effective Yukawa contributions and effective Majorana mass matrices when the heavy Higgs fields gain VEVs.

3.2.2 Yukawa Operators

The Yukawa operators, responsible for generating effective Yukawa couplings, have the following structure [2] [46]:

$$\mathcal{O} = F_I \overline{F}_J h \left(\frac{H\overline{H}}{M_X^2} \right)^n \left(\frac{\theta}{M_X} \right)^{p(i,j)} \tag{3.4}$$

where the integer $p(i,j)$ is the total $U(1)_F$ charge of $F_I + \overline{F}_J + h$ and $H\overline{H}$ has a $U(1)_F$ charge of zero. The tensor structure of the operators in Eq.3.4 is

$$(\mathcal{O})^{\alpha\rho y w}_{\beta\gamma x z} = F^{\alpha a} \overline{F}_{\beta x} h^y_a \overline{H}_{\gamma z} H^{\rho w} \theta^{p(i,j)} \tag{3.5}$$

One constructs [46] SU(4)$_{PS}$ invariant tensors $C^{\beta\gamma}_{\alpha\rho}$ that combine 4 and $\overline{4}$, 4 and 4 or $\overline{4}$ and $\overline{4}$ representations of SU(4)$_{PS}$ into **1**, **6**, **10**, $\overline{\mathbf{10}}$ and **15** representations. Similarly we construct SU(2)$_R$ tensors R^{xz}_{yw} that combine **2** representations of SU(2) into singlet and triplet representations. These tensors are contracted together and into $\mathcal{O}^{\alpha\rho y w}_{\beta\gamma x z}$ to create singlets of SU(4)$_{PS}$, SU(2)$_L$ and SU(2)$_R$. Depending on which operators are used, different Clebsch-Gordan coefficients (CGCs) will emerge.

We look at two different models for the Yukawa sector, which we refer to as model I and model II. The models represent different $\mathcal{O}(1)$ parameters a, a', a'' in the following

[2] We note that due to the allocation of charges, and since the effective Yukawa operators include the fields $F_I \overline{F}_J h$ with overall charge positive, the field $\overline{\theta}$ does not enter the Yukawa operators.

operator texture [47]:

$$\mathcal{O} = \begin{bmatrix} (a_{11}\mathcal{O}^{Fc} + a''_{11}\mathcal{O}''^{Ae})\epsilon^5 & (a_{12}\mathcal{O}^{Ee} + a'_{12}\mathcal{O}'^{Cb})\epsilon^3 & (a'_{13}\mathcal{O}'^{Cf} + a''_{13}\mathcal{O}''^{Ee})\epsilon \\ (a_{21}\mathcal{O}^{Dc})\epsilon^4 & (a_{22}\mathcal{O}^{Bc} + a'_{22}\mathcal{O}'^{Ff})\epsilon^2 & (a_{23}\mathcal{O}^{Ee} + a'_{23}\mathcal{O}'^{Bc}) \\ (a_{31}\mathcal{O}^{Fc})\epsilon^4 & (a_{32}\mathcal{O}^{Ac} + a'_{23}\mathcal{O}'^{Fe})\epsilon^2 & a_{33} \end{bmatrix} \tag{3.6}$$

where the operator nomenclature is defined in Appendix C. For convenience, from this point on, we define:

$$\delta = \frac{H\overline{H}}{M_X^2} \tag{3.7}$$

and

$$\epsilon = \frac{\theta}{M_X} \tag{3.8}$$

Eq.(3.6) then yields the effective Yukawa matrices

$$Y^u(M_X) = \begin{bmatrix} a''_{11}\sqrt{2}\delta^3\epsilon^5 & a'_{12}\sqrt{2}\delta^2\epsilon^3 & a'_{13}\frac{2}{\sqrt{5}}\delta^2\epsilon \\ 0 & a'_{22}\frac{8}{5\sqrt{5}}\delta^2\epsilon^2 & 0 \\ 0 & a'_{32}\frac{8}{5}\delta^2\epsilon^2 & a_{33} \end{bmatrix} \tag{3.9}$$

$$Y^d(M_X) = \begin{bmatrix} a_{11}\frac{8}{5}\delta\epsilon^5 & -a'_{12}\sqrt{2}\delta^2\epsilon^3 & a'_{13}\frac{4}{\sqrt{5}} \\ a_{21}\frac{2}{\sqrt{5}}\delta\epsilon^4 & (a_{22}\sqrt{\frac{2}{5}}\delta + a'_{22}\frac{16}{5\sqrt{5}}\delta^2)\epsilon^2 & a'_{23}\sqrt{\frac{2}{5}}\delta^2 \\ a_{31}\frac{8}{5}\delta\epsilon^4 & a_{32}\sqrt{2}\delta\epsilon^2 & a_{33} \end{bmatrix} \tag{3.10}$$

$$Y^e(M_X) = \begin{bmatrix} a_{11}\frac{6}{5}\delta\epsilon^5 & 0 & 0 \\ a_{21}\frac{4}{\sqrt{5}}\delta\epsilon^4 & (-a_{22}3\sqrt{\frac{2}{5}}\sqrt{\frac{2}{5}} + a'_{22}\delta\frac{12}{5\sqrt{5}})\delta\epsilon^2 & -a'_{23}\sqrt{\frac{2}{5}}\delta^2 \\ -a_{31}\frac{6}{5}\delta\epsilon^4 & a_{32}\sqrt{2}\delta\epsilon^2 & a_{33} \end{bmatrix} \tag{3.11}$$

$$Y^\nu(M_X) = \begin{bmatrix} a''_{22}\sqrt{2}\delta^3\epsilon^5 & a_{12}2\delta\epsilon^3 & a''_{13}\delta^3\epsilon \\ 0 & a'_{22}\frac{6}{5\sqrt{5}}\delta^2\epsilon^2 & a_{23}2\delta \\ 0 & a'_{32}\frac{6}{5}\delta^2\epsilon^2 & a_{33} \end{bmatrix} \tag{3.12}$$

The order unity coefficients a_{ij}, a'_{ij} of the operators are adjusted to give a good fit to the quark and lepton masses and mixing angles, and take the values given in Table 3.2.

	Model I	Model II
a_{33}	0.55	0.55
a_{11}	-0.92	-0.92
a_{12}	0.33	0.33
a_{21}	1.67	1.67
a_{22}	1.12	1.12
a_{23}	0.89	0.89
a_{31}	-0.21	-0.21
a_{32}	2.08	2.08
a'_{12}	0.77	0.77
a'_{13}	0.53	0.53
a'_{22}	0.66	0.66
a'_{23}	0.40	0.40
a'_{32}	1.80	1.80
a''_{11}	0.278	0.278
a''_{13}	0.000	1.000
A_{11}	0.94	0.94
A_{12}	0.48	0.48
A_{13}	2.10	2.10
A_{22}	0.52	0.52
A_{23}	1.29	1.79
A_{33}	1.88	1.88

Table 3.2: The a, a' and a'' parameters for model I and model II

Note that the two models differ only in the choice of a''_{13}, which is taken to be zero in model I. Model I consequently has a lower rate for $\mu \to e\gamma$, and model II has a higher $\mu \to e\gamma$ rate due to the non-zero 13 element of the neutrino Yukawa matrix, as can be understood from the analytic results in [56]. The fits assume $\delta = \epsilon = 0.22$.

3.2.3 Majorana Operators

We are interested in Majorana fermions because they can contribute neutrino masses of the correct order of magnitude via the see-saw effect. The operators for Majorana fermions are of the form

$$\mathcal{O}_{IJ} = \overline{F}_I \overline{F}_J \left(\frac{HH}{M_X}\right) \left(\frac{H\overline{H}}{M_X^2}\right)^{n-1} \left(\frac{\theta}{M_X}\right)^{q_{IJ}} \tag{3.13}$$

There do not exist renormalisable elements of this infinite series of operators, so $n < 1$ Majorana operators are not defined [3]. A similar analysis goes through as for the Dirac fermions; however the structures only ever give masses to the neutrinos, not to the electrons or to the quarks. [4]

It should be noted that the Majorana neutrinos will not affect the A-terms, as these operators do not contribute to the Yukawas. The RH Majorana neutrino mass matrix is:

$$\frac{M_{RR}(M_X)}{M_{33}} = \begin{bmatrix} A_{11}\delta\epsilon^8 & A_{12}\delta\epsilon^6 & A_{13}\delta\epsilon^4 \\ A_{12}\delta\epsilon^6 & A_{22}\delta\epsilon^4 & A_{23}\delta\epsilon^2 \\ A_{13}\delta\epsilon^4 & A_{23}\delta\epsilon^2 & A_{33} \end{bmatrix} \tag{3.14}$$

[3]Except for the 33 neutrino mass term; this is allowed because of string theoretic effects

[4]To see this note that the form of the two H VEVs is symmetric, and proportional to $\delta_4^\alpha \delta_2^x$. Symmetric structures will then contract to give neutrino mass terms. Antisymmetric structures will contract to give zero. As any structure can be written as a sum of a symmetric and an antisymmetric part, we see immediately that the only mass terms can be given to the neutrinos because of the form of the VEVs in Eq. (3.2)

3.2.4 Neutrino sector results

The neutrino Yukawa matrix in Eq.(3.12) and the heavy Majorana mass matrix in Eq.(3.14) imply that the see-saw mechanism satisfies the condition of sequential dominance [14], leading to a natural neutrino mass hierarchy $m_1 \ll m_2 \ll m_3$ with no fine-tuning. The dominant contribution to the atmospheric neutrino mass m_3 comes from the third (heaviest) right-handed neutrino, with the leading subdominant contribution to the solar neutrino mass m_2 coming from the second right-handed neutrino. In such a natural scenario, the large atmospheric angle is due to the large ratio of dominant neutrino Yukawa couplings $\tan\theta_{23} \approx Y^{\nu}_{23}/Y^{\nu}_{33}$, and the large solar angle is due to the large ratio of leading subdominant Yukawa couplings $\tan\theta_{12} \approx \sqrt{2}Y^{\nu}_{12}/(Y^{\nu}_{22} - Y^{\nu}_{32})$.

Model I for the Yukawa sector is taken from a global analysis of a SUSY Pati-Salam model enhanced with an Abelian flavour symmetry [47]. At one-loop order the Yukawa runnings only depend on the other Yukawas and the gauge couplings. Since Model II only differs from Model I in the neutrino Yukawa, we do not expect the quark masses or mixing angles to be different. We also do not expect the charged lepton masses to differ by much.

The possibility remains open, however, that the new operator in the 13 Yukawa elements could predict either a mass difference or a neutrino mixing angle in violation of the results from the various neutrino experiments [12, 7, 61, 62]. As such, we checked our predictions for the mass-differences and the mixing angles for both models, in comparison to experiment. The results of this are summarised in Table 3.3.

We note that in both model I and model II, we are within the constraints on the neutrino sector. In fact Model II is slightly closer to the central values of three observable parameters (the solar and atmospheric neutrino mixing angles, and the atmospheric mass difference). In both cases we predict values of θ_{13} below the current

Observable	Model I Prediction	Model II Prediction	Experimental Values
$\sin^2\theta_{12}$	0.316	0.308	0.28 ± 0.05
$\sin^2\theta_{23}$	0.553	0.552	0.50 ± 0.15
$\sin^2\theta_{13}$	$5.18 \cdot 10^{-3}$	$5.20 \cdot 10^{-3}$	< 0.03
Δm^2_{atm}	$1.32 \cdot 10^{-3}$	$1.33 \cdot 10^{-3}$	$(2.5 \pm 0.8)10^{-3}$
Δm^2_{sol}	$6.05 \cdot 10^{-5}$	$5.91 \cdot 10^{-5}$	$(7.0 \pm 3.0)10^{-5}$

Table 3.3: The neutrino mass differences and mixing angles in model I, model II and the experimental limits

limit.

3.3 Soft Supersymmetry Breaking Masses

3.3.1 Supersymmetry Breaking F-terms

In [45] it was assumed that the Yukawas were field-independent, and hence the only F-vevs of importance were that of the dilaton (S), and the untwisted moduli (T^i). Here we set out the parameterisation for the F-term VEVs, including the contributions from the FN field θ and the heavy Higgs fields $H, \overline{H}$. Note that the field dependent part follows from the assumption that the family symmetry field, θ is an intersection state.

$$F_S = \sqrt{3} m_{3/2} \left(S + \overline{S}\right) X_S \tag{3.15}$$

$$F_{T_i} = \sqrt{3} m_{3/2} \left(T_i + \overline{T}_i\right) X_{T_i} \tag{3.16}$$

$$F_{H^{\alpha b}} = \sqrt{3} m_{3/2} H^{\alpha b} \left(S + \overline{S}\right)^{\frac{1}{2}} X_H \tag{3.17}$$

$$F_{\overline{H}_{\alpha x}} = \sqrt{3} m_{3/2} \overline{H}_{\alpha x} \left(T_3 + \overline{T}_3\right)^{\frac{1}{2}} X_{\overline{H}} \tag{3.18}$$

$$F_\theta = \sqrt{3} m_{3/2} \theta \left(S + \overline{S}\right)^{\frac{1}{4}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{4}} X_\theta \tag{3.19}$$

We introduce a shorthand notation:

$$F_H H = \sum_{\alpha b} F_{H^{\alpha b}} H^{\alpha b} \; ; \; F_{\overline{H}} \overline{H} = \sum_{\alpha x} F_{\overline{H}_{\alpha x}} \overline{H}_{\alpha x}. \tag{3.20}$$

3.3.2 Soft Scalar Masses

There are two contributions to scalar mass squared matrices, coming from SUGRA and from D-terms. In this subsection we calculate the SUGRA predictions for the matrices at the GUT scale, and in the next subsection we add on the D-term contributions.

The SUGRA contributions to soft masses are detailed in Appendix A. From Eq. (A.7) we can get the family independent form for all scalars:

$$m_L^2 = m_{3/2}^2 \begin{bmatrix} a & & \\ & a & \\ & & b_L \end{bmatrix} \tag{3.21}$$

$$m_R^2 = m_{3/2}^2 \begin{bmatrix} a & & \\ & a & \\ & & b_R \end{bmatrix} \tag{3.22}$$

$$m_h^2 = m_{3/2}^2 \; (1 - 3X_S^2) \tag{3.23}$$

$$m_H^2 = m_{3/2}^2 \; (1 - 3X_S^2) \tag{3.24}$$

$$m_{\overline{H}}^2 = m_{3/2}^2 \; (1 - 3X_{T_3}^2) \tag{3.25}$$

where

$$a = 1 - \frac{3}{2}(X_S^2 + X_{T_3}^2) \tag{3.26}$$

$$b_L = 1 - 3X_{T_3}^2 \tag{3.27}$$

$$b_R = 1 - 3X_{T_2}^2 \tag{3.28}$$

Here m_L^2 represents the left handed scalar mass squared matrices m_{QL}^2 and m_{LL}^2. m_R^2 represents the right handed scalar mass squared matrices m_{UR}^2, m_{DR}^2, m_{ER}^2 and m_{NR}^2.

3.3.3 D-term Contributions

We now consider the D-terms from breaking the Pati-Salam group $\mathrm{SU}(4) \otimes \mathrm{SU}(2)_L \otimes \mathrm{SU}(2)_R$ down to the MSSM group $\mathrm{SU}(3) \otimes \mathrm{SU}(2)_L \otimes \mathrm{U}(1)_Y$. These will be family independent, but charge dependent, and will pull the six matrices that appear in the RGE equations apart. We shall neglect D-term contributions from the broken family symmetry which would lead to additional sources of flavour violation.

The addition to the D-terms have been written down before [63]. The corrections are, in matrix notation:

$$m_{QL}^2 = m_L^2 + g_4^2 D^2 \tag{3.29}$$

$$m_{UR}^2 = m_R^2 - (g_4^2 - 2g_{2R}^2) D^2 \tag{3.30}$$

$$m_{DR}^2 = m_R^2 - (g_4^2 + 2g_{2R}^2) D^2 \tag{3.31}$$

$$m_{LL}^2 = m_L^2 - 3g_4^2 D^2 \tag{3.32}$$

$$m_{ER}^2 = m_R^2 + (3g_4^2 - 2g_{2R}^2) D^2 \tag{3.33}$$

$$m_{NR}^2 = m_R^2 + (3g_4^2 + 2g_{2R}^2) D^2 \tag{3.34}$$

$$m_{h_u}^2 = m_{h_2}^2 - 2g_{2R}^2 D^2 \tag{3.35}$$

$$m_{h_d}^2 = m_{h_1}^2 + 2g_{2R}^2 D^2 \tag{3.36}$$

where in the appendix of Ref.[63], an expression for D^2 in terms of the soft parameters m_H^2 and $m_{\overline{H}}^2$ is derived,

$$D^2 = \frac{m_H^2 - m_{\overline{H}}^2}{4\lambda_S^2 + 2g_{2R}^2 + 3g_4^2}. \tag{3.37}$$

The gauge couplings and mass parameters in Eq.(3.37) are predicted from the model. The only free parameter is the coupling λ_S is a dimensionless coupling constant which enters the potential [63] and should be perturbative. Furthermore, we see that the largest that D^2 can be is when λ_S is zero, so not only is the order of magnitude of D^2

predicted in this model, but we also have an exact upper bound on the value.

3.3.4 Soft Gaugino Masses

The soft gaugino masses are the same as in [45], which we quote here for completeness. The results follow from Eq. (A.8) applied to the $SU(4) \otimes SU(2)_L \otimes SU(2)_R$ gauginos, which then mix into the $SU(3) \otimes SU(2)_L \otimes U(1)_Y$ gauginos whose masses are given by

$$M_3 = \frac{\sqrt{3}m_{3/2}}{(T_1+\overline{T}_1)+(T_2+\overline{T}_2)}\left[(T_1+\overline{T}_1)X_{T_1}+(T_2+\overline{T}_2)X_{T_2}\right] \quad (3.38)$$

$$M_2 = \sqrt{3}m_{3/2}X_{T_1} \quad (3.39)$$

$$M_1 = \frac{\sqrt{3}m_{3/2}}{\frac{5}{3}(T_1+\overline{T}_1)+\frac{2}{3}(T_2+\overline{T}_2)}\left[\frac{5}{3}(T_1+\overline{T}_1)X_{T_1}+\frac{2}{3}(T_2+\overline{T}_2)X_{T_2}\right] \quad (3.40)$$

The values of $T_1+\overline{T}_1$ and $T_2+\overline{T}_2$ are proportional to the brane gauge couplings g_{5_1} and g_{5_2}, which are related in a simple way to the MSSM couplings at the unification scale. This is discussed in [45].

When we run the MSSM gauge couplings up and solve for g_{5_1} and g_{5_2} we find that approximate gauge coupling unification is achieved by $T_1+\overline{T_1} \gg T_2+\overline{T}_2$. Then we find the simple approximate result

$$M_1 \approx M_3 \approx M_2 = \sqrt{3}m_{3/2}X_{T_1}. \quad (3.41)$$

3.3.5 Soft Trilinear Couplings

So far the soft masses are as in [45], with the FN fields and heavy Higgs contributions being completely negligible due to the smallness of their F-terms. However for the soft trilinear couplings these contributions are of order $O(m_{3/2})$ despite having small F-terms, so FN and Higgs contributions will give very important additional contributions beyond those considered in [45].

From Appendix A we see that the canonically normalised equation for the trilinear is:

$$A_{abc} = F_I \left[\overline{K}_I - \partial_I \ln \left(\tilde{K}_a \tilde{K}_b \tilde{K}_c \right) \right] + F_m \partial_m \ln Y_{abc} \tag{3.42}$$

This general form for the trilinear accounts for contributions from non-moduli F-terms. These contributions are in general expected to be of the same magnitude as the moduli contributions despite the fact that the non-moduli F-terms are much smaller [49]. Specifically, if the Yukawa hierarchy is taken to be generated by a FN field, θ such that $Y_{ij} \sim \theta^{p_{ij}}$, then we expect $F_\theta \sim m_{3/2}\theta$, and then $\Delta A_{ij} = F_\theta \partial_\theta \ln Y_{ij} \sim p_{ij} m_{3/2}$ and so even though these fields are expected to have heavily sub-dominant F-terms [5] they contribute to the trilinears at the same order $O(m_{3/2})$ as the moduli, but in a flavour off-diagonal way.

In the specific D-brane model of interest here the general results for soft trilinear masses, including the contributions for general effective Yukawa couplings are given in Appendix B. From Eqs.(3.4,3.5) we can read off the effective Yukawa couplings,

$$Y_{hF\overline{F}} hF\overline{F} \equiv \underbrace{(c)^{\beta\gamma}_{\alpha\rho}(r)^{xz}_{yw}\overline{H}_{\gamma z} H^{\rho w} \theta^p}_{Y_{hF\overline{F}}{}^{\beta x}_{\alpha y}} h^y_a F^{\alpha a} \overline{F}_{\beta x}. \tag{3.43}$$

Note the extra group indices that the effective Yukawa coupling $Y_{hF\overline{F}}{}^{\beta x}_{\alpha y}$ has, and proper care must be taken of the tensor structure when deriving trilinears from a given operator. For Model I and II defined earlier, we can write down the trilinear soft masses, A, by substituting the operators in Eq.(3.6) into the results in Appendix B. Having done this we find the result:

$$A = \sqrt{3} m_{3/2} \begin{bmatrix} d_1 + d_H + 5d_\theta & d_1 + d_H + 3d_\theta & d_2 + d_H + d_\theta \\ d_1 + d_H + 4d_\theta & d_1 + d_H + 2d_\theta & d_2 + d_H \\ d_3 + d_H + 4d_\theta & d_3 + d_H + 2d_\theta & d_4 \end{bmatrix} \tag{3.44}$$

[5] In our model the FN and heavy Higgs vevs are of order the unification scale, compared to the moduli vevs which are of order the Planck scale.

where

$$d_1 = X_S - X_{T_1} - X_{T_2} \tag{3.45}$$

$$d_2 = \frac{1}{2}X_S - X_{T_1} - \frac{1}{2}X_{T_2} \tag{3.46}$$

$$d_3 = \frac{1}{2}X_S - X_{T_1} - X_{T_2} + \frac{1}{2}X_{T_3} \tag{3.47}$$

$$d_4 = -X_{T_1} \tag{3.48}$$

$$d_H = (S+\overline{S})^{\frac{1}{2}}X_H + (T_3+\overline{T}_3)^{\frac{1}{2}}X_{\overline{H}} \tag{3.49}$$

$$d_\theta = (S+\overline{S})^{\frac{1}{4}}(T_3+\overline{T}_3)^{\frac{1}{4}}X_\theta \tag{3.50}$$

3.3.6 Why we expect large flavour violation

According to ref.[64], if the trilinears can be written in a certain manner, then flavour violation is expected to be small. However, it is not possible to write the trilinears in this manner if either $X_H \neq 0$ or $X_{\overline{H}} \neq 0$ or $X_\theta \neq 0$. It is possible in general to write:

$$A_{ij} = A_i^L + A_j^R + \delta_{ij} \tag{3.51}$$

If $\delta_{ij} = 0$, then the trilinears factorise.

$$\tilde{A} = \begin{bmatrix} & & \\ & Y_{ij} & \\ & & \end{bmatrix} \begin{bmatrix} A_1^R & & \\ & A_2^R & \\ & & A_3^R \end{bmatrix} + \begin{bmatrix} A_1^L & & \\ & A_2^L & \\ & & A_3^L \end{bmatrix} \begin{bmatrix} & & \\ & Y_{ij} & \\ & & \end{bmatrix} \tag{3.52}$$

If this is true at the SUSY breaking scale, then the FCNC effects are small, and the leading order contributions are proportional to $A_i^R - A_j^R$ or $A_i^L - A_j^L$. If there are any contributions which are universal, A^0, then we can add them in any linear combination to A_i^L and A_j^R provided that $A_i'^L + A_j'^R = A_i^L + A_j^R + A^0$. From the SUGRA formula for A_{ij}, it is clear which terms contribute to the universal part, the left part and the right part:

$$A_{ij} = A_{hij} = \underbrace{F^a \partial_a (\tilde{K} - \ln K_h^h)}_{A^0} - \underbrace{F^a \partial_a \ln K_i^i}_{A_i^L} - \underbrace{F^a \partial_a \ln K_j^j}_{A_j^R} + \delta_{ij} \tag{3.53}$$

We see that δ_{ij} is the terms due to the derivative of the Yukawa. If this is either zero or universal, then the A matrix can be written in the restricted form.

Unfortunately, neither the Higgs contribution or the Froggatt-Nielsen contribution can be written in this form. The Higgs contribution to Eq.(3.44) is:

$$A_H \propto \begin{bmatrix} a & a & a \\ a & a & a \\ a & a & b \end{bmatrix} = a + (b-a) \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{3.54}$$

The Froggatt-Nielsen contribution is maximally non-universal, and the elements have $A_{ij} \not\approx A_{kl}$ for $i \neq k$; $j \neq l$. In this case we expect there to be the largest contribution to flavour violation, assuming that it is not tuned down. (We could do this either by selecting a very small value for the F-term VEV by setting $X_\theta \approx 0$, or by setting the operator texture in the Yukawas to have very small off-diagonal elements, as the A-contribution multiplies Yukawa elements). Hence we see that the new sources of flavour violation will not only contribute to the trilinear terms on at least an equal footing as the moduli, but that also they cannot be written in a form where the contribution to flavour violation is expected to be small.

3.4 Results

3.4.1 Benchmark points

Since the parameter space for this model is reasonably expansive, and the intention is to compare different sources of LFV, it is convenient to consider four benchmark points, as follows. It should be noted that for all these points, we have taken all X_{T_i} to be the same, $X_{T_i} = X_T$, and also $X_H = X_{\overline{H}}$.

Point	X_S	X_T	X_H	$X_{\overline{H}}$	X_θ
A	0.500	0.500	0.000	0.000	0.000
B	0.536	0.488	0.000	0.000	0.000
C	0.270	0.270	0.000	0.000	0.841
D	0.270	0.270	0.578	0.578	0.000

Table 3.4: The four benchmark points, A-D

- Point A is referred to as 'minimum flavour violation'. At the point $X_S = X_{T_i}$ the scalar mass matrices m^2 are proportional to the identity, and the trilinears $\tilde{A}$ are aligned with the Yukawas. Also, if we look back to Eq. (4.6), Eq. (3.24) and Eq. (3.25) we see that for $X_S = X_T$, which is the case for point A (and point C, and point D) we see that the upper limit on the magnitude of the D-term contribution is zero. As such both m^2 and $\tilde{A}$ will be diagonal in the SCKM basis in the absence of the RH neutrino field.

- Point B is referred to as 'SUGRA'. With $X_S \neq X_{T_i}$ it represents typical flavour violation from the moduli fields; this is the amount of flavour violation that would traditionally have been expected with no contribution from the F_H or F_θ fields.

- Point C is referred to as 'FN flavour violation'. It represents flavour violation from the Froggatt-Nielsen sector by itself, without any contribution to flavour violation from traditional SUGRA effects since $X_S = X_{T_i}$ as in point A.

- Point D is referred to as 'Heavy Higgs flavour violation'. It represents flavour violation from the heavy Higgs sector, without any contribution from either traditional SUGRA effects since $X_S = X_{T_i}$, or from FN fields since $F_\theta = 0$. As will become apparent, at this point the seesaw mechanism actually helps reduce the LFV for $\mu \to e\gamma$ in model I and $\tau \to \mu\gamma$ in both models.

3.4.2 Numerical Results

From the benchmark points defined in Table 3.4, the F-term VEVs were determined, and from these the soft parameters at the high energy scale $M_X = 3.10^{16}$GeV were calculated. The soft parameters were then run down using the 1-loop RGEs of the MSSM + ν^c model. For our numerical results we use a modified version of SOFTSUSY [65]. The modifications were made to add the effect of the right-handed neutrino field to the RGEs and to decouple them in a manner that allows the neutrino masses and mixing angles to be calculated at the low energy scale. As a result of the RGEs having to be recoded, all of them are to one loop only in the version that was used here.

Flavour violation is proportional to non-zero off diagonal elements in the scalar mass squared matrices m^2 in the SCKM basis and to non-zero off diagonal elements in the trilinears $\tilde{A}$ in the SCKM basis. [6]. Hence, there are two ways to generate flavour violation. The first is to have non-zero off-diagonal elements in m^2 of the scalars and $\tilde{A}$ at the unification scale. The second is to have non-zero off diagonal elements radiatively generated by the β-function running down to the electroweak scale. It is possible to remove the second source by removing the RH neutrino field from the model; this allows a disentangling of the see-saw mechanism from the particular source of interest, but is unphysical since we know that the neutrinos have to be massive.

Figure 3.1 shows numerical results for BR($\mu \to e\gamma$) for Model I, plotted against the gravitino mass $m_{3/2}$, where each of the four panels corresponds to each of the four

[6] The SCKM basis is the basis where the Yukawas are diagonal at the electroweak scale

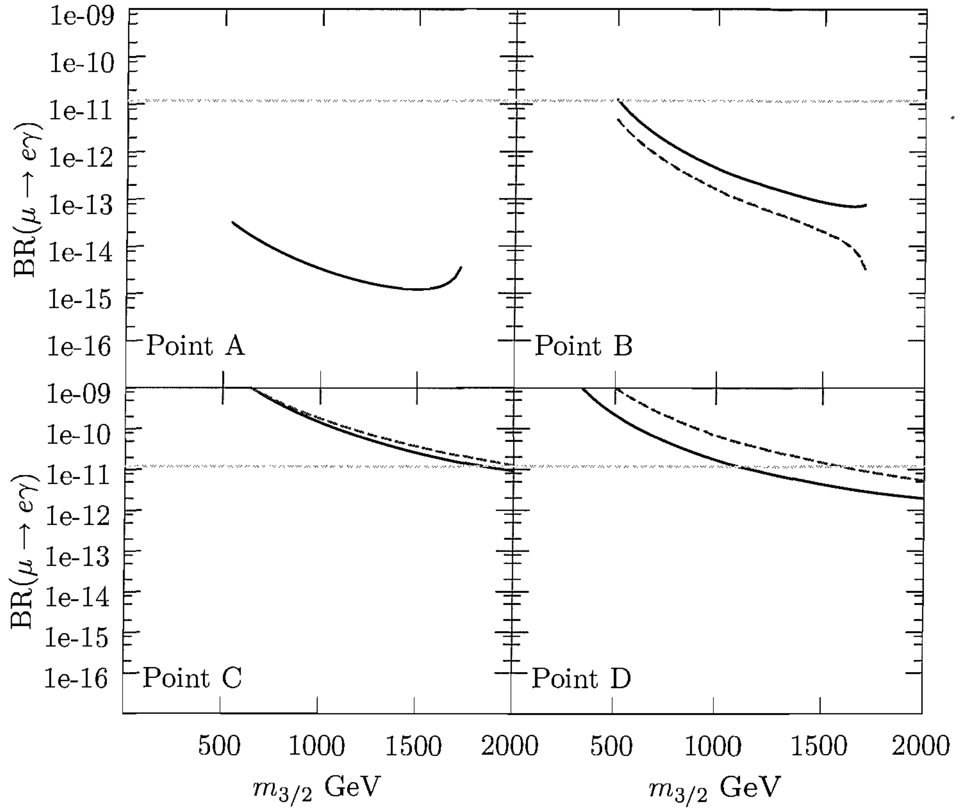


Figure 3.1: BR($\mu \to e\gamma$) for points A-D in model I (low $\mu \to e\gamma$). The solid line represents model I. The dashed line represents an unphysical model with no right-handed neutrino field whose purpose is only comparison. The horizontal line is the 2002 experimental limit from ref.[5]

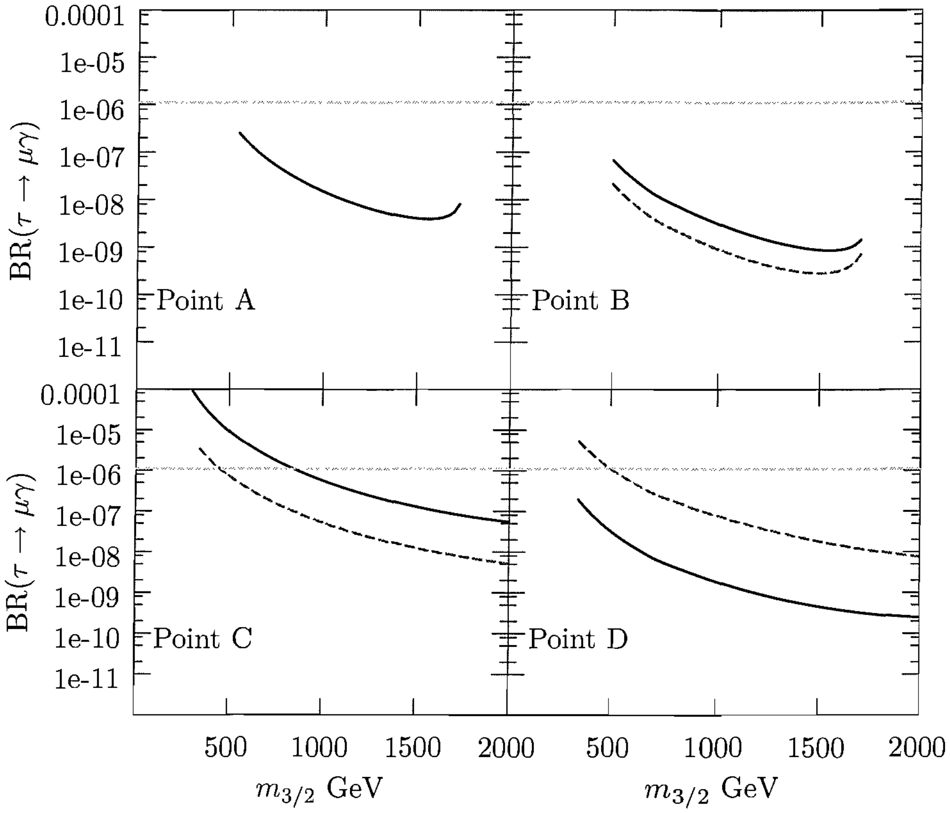


Figure 3.2: BR($\tau \to \mu\gamma$) for points A-D. The lines coincide in both model I and model II. The solid line represents models I and II (which predict very similar rates for $\tau \to \mu\gamma$). The dashed line represents an unphysical model with no right-handed neutrino field whose purpose is only comparison. The horizontal line is the 2002 experimental limit from ref.[5]

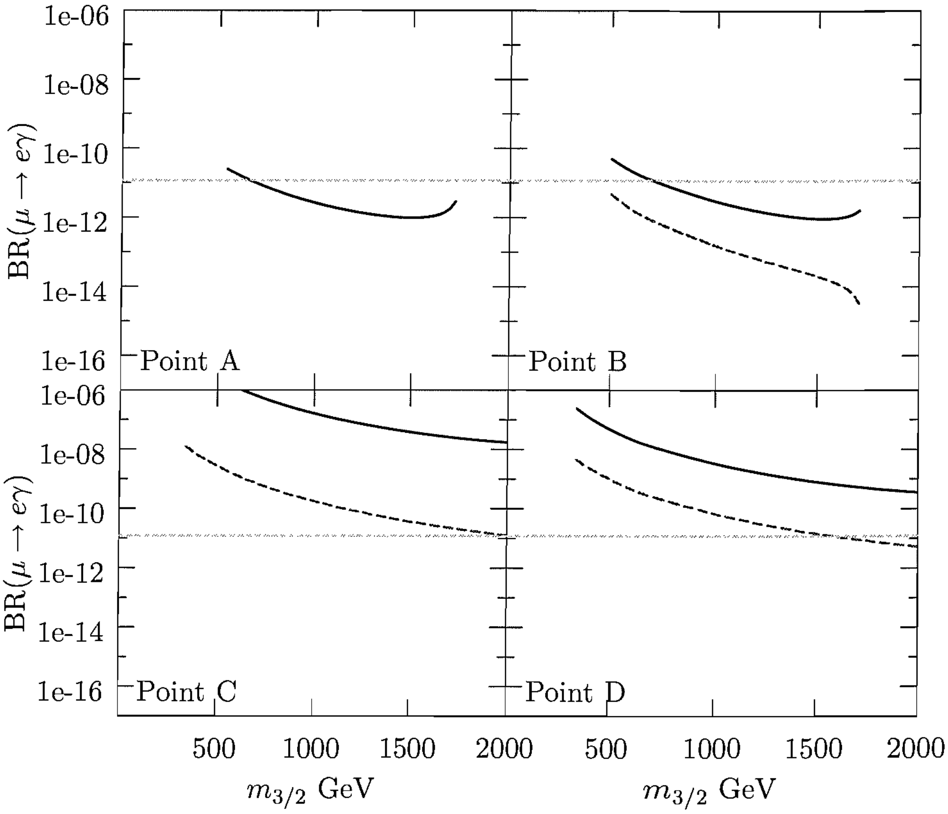


Figure 3.3: BR($\mu \to e\gamma$) for points A-D in model II (high $\mu \to e\gamma$). Note that because the rate is much higher the scale is different to that in fig. 3.1. The solid line represents model II. The dashed line represents an unphysical model with no right-handed neutrino field whose purpose is only comparison. The horizontal line is the 2002 experimental limit from ref.[5]

benchmark points A-D. As $m_{3/2}$ increases the sparticle spectrum becomes heavier. This will look different at each parameter point, but the physical masses are expected to be of the same order of magnitude as the gravitino mass. As such, high gravitino masses will start to reintroduce the fine-tuning problem of the gluino mass being too high. Point A corresponds to minimum flavour violation, where the only source of LFV is from the see-saw mechanism, which for Model I is well below the experimental limit, shown as the faint horizontal dashed line. Point B has LFV arising from SUGRA, with the FN and heavy Higgs sources of LFV switched off, and in this case we also show the results with the see-saw mechanism switched off (dashed curve) as well as with the see-saw mechanism with SUGRA contributions to LFV (solid curve). In both cases the results are below the experimental limit for $m_{3/2}$ above 500 GeV. Point C is the FN benchmark point, and for this case we see that the experimental limit is violated over the entire range of $m_{3/2}$ shown, with the see-saw mechanism making very little difference. Point D shows the heavy Higgs point, for which the experimental limit is violated for $m_{3/2}$ below 1000 GeV. Interestingly, the effect of switching off the see-saw mechanism in this case (dashed curve) is to increase the rate for BR($\mu \to e\gamma$).

Figure 3.2 shows results for BR($\tau \to \mu\gamma$) for Model I, plotted against the gravitino mass $m_{3/2}$. Point A for minimum flavour violation is below the experimental limit, as is point B corresponding to SUGRA, with the see-saw mechanism switched off corresponding as before to the dashed curve. Point C corresponding to FN violates the experimental limit for lower $m_{3/2}$, with a rather large effect coming from the see-saw mechanism. Point D shows the heavy Higgs point, with the effect of the see-saw mechanism being to reduce BR($\tau \to \mu\gamma$) in conjunction with the LFV coming from heavy Higgs, similar to the analogous effect observed previously.

Figure 3.3 shows the analogous results for BR($\mu \to e\gamma$) for Model II. As expected model II, which is supposed to give a high rate for $\mu \to e\gamma$, does give results close to the experimental limit for points A and B, and the limit is now well exceeded for points C and D. By increasing the gravitino mass sufficiently (which increases all the sparticle masses) it is possible to respect the current experimental limit, but at the expense of a very heavy superpartner spectrum. However it is worth noting that for benchmark points C,D the value of X_T is almost half its value corresponding to points A,B. According to Eq.(3.41) this implies that for points C,D the gaugino masses are almost half their values corresponding to points A,B, leading to reduced fine-tuning for a given $m_{3/2}$.

The results for BR($\tau \to \mu\gamma$) for Model II are almost identical to those shown for Model I in Figure 3.2, which is as expected since the only difference between the two models is in the 13 element of the neutrino Yukawa matrix.

3.5 Conclusions

We have investigated the phenomenological impact of different sources of lepton flavour violation arising from realistic D-brane inspired models based on supergravity mediated supersymmetry breaking, where the origin of flavour is due to Froggatt-Nielsen (FN) operators. We have discussed four distinct sources of lepton flavour violation in such models: minimum flavour violation, arising from neutrino masses and the see-saw mechanism with renormalisation group (RG) running; supergravity flavour violation due to the non-universal structure of the supergravity model; FN flavour violation due to the FN fields developing supersymmetry breaking F-terms and contributing in an non-universal way to soft trilinear terms; and finally heavy Higgs flavour violation arising from the Higgs fields used to break the unified gauge symmetry which may behave analogously to the FN fields.

In order to quantify the relative effects, we studied a particular type I string inspired model based on a supersymmetric Pati-Salam model arising from intersecting D-branes as proposed in [45], but here supplemented by a $U(1)$ family symmetry with the quarks and leptons described by the set of FN operators as in [47]. We have derived the soft supersymmetry breaking masses for the model, including the new flavour violating contributions to the soft trilinear masses arising from the FN and heavy Higgs fields. We then performed a numerical analysis of LFV for four benchmark points, each chosen to highlight a particular source of flavour violation, with the benchmark points C and D corresponding to LFV arising from the FN and heavy Higgs fields giving by far the largest effects. Since the new contributions are dominantly from the trilinears $\tilde{A}$, the amount of flavour violation is therefore strongly dependent on the choice of Yukawa matrices at the unification scale. For example the huge difference in the rate of $\mu \longrightarrow e\gamma$ between Model I and Model II is simply generated by changing the (1,3) element of Y^{ν}. Also we find that $\mu \longrightarrow e\gamma$ is more constraining than $\tau \longrightarrow \mu\gamma$.

The numerical results show that the contributions to LFV from Yukawa operators with the heavy Higgs sector and the Froggatt-Nielsen sector can give the dominant contributions to LFV processes, greatly exceeding contributions from SUGRA and the see-saw mechanism, and should be taken into account when performing phenomenological analyses of supergravity models.

Chapter 4

Canonical Normalisation and Kähler operators

We highlight the important role that canonical normalisation of kinetic terms in flavour models based on family symmetries can play in determining the Yukawa matrices. Even though the kinetic terms may be correctly canonically normalised to begin with, they will inevitably be driven into a non-canonical form by a similar operator expansion to that which determines the Yukawa operators. Therefore in models based on family symmetry canonical re-normalisation is mandatory before the physical Yukawa matrices can be extracted. In nearly all examples in the literature this is not done. As an example we perform an explicit calculation of such mixing associated with canonical normalisation of the Kähler metric in a supersymmetric model based on a SU(3) family symmetry, where we show that such effects can significantly change the form of the Yukawa matrix. We then develop techniques to simplify the analysis of such effects on the Yukawa sector. Using them we show that in the class of theories with an hierarchical structure for the Yukawa couplings the Kähler corrections to both the masses and mixing angles are subdominant. This is true even in cases that texture zeros are filled in by the terms coming from the Kähler potential.

4.1 Introduction

There is great interest in the literature in trying to understand the hierarchical pattern of Standard Model fermion masses, the smallness of the quark mixing angles and the two large and one small neutrino mixing angles. One popular way of doing this is to extend either the Standard Model, or one of its more common supersymmetric extensions, by adding a gauge or global family symmetry, G_F which is subsequently broken [15].

In such models based on family symmetry G_F, Yukawa couplings arise from Yukawa operators which are typically non-renormalisable and involve extra heavy scalar fields, ϕ, coupling to the usual three fields, for example:

$$\mathcal{O}_Y = F\overline{F}H\left(\frac{\phi}{M}\right)^n \tag{4.1}$$

where F represents left-handed fermion fields, $\overline{F}$ represents the CP-conjugate of right-handed fermion fields, H represents the Higgs field, and M is a heavy mass scale which acts as an ultraviolet (UV) cutoff. In the context of supersymmetric (SUSY) field theories, all the fields become superfields. The operators in Eq.(4.1) are invariant under G_F, but when the scalar fields ϕ develop vacuum expectation values (vevs) the family symmetry is thereby broken and the Yukawa couplings are generated. The resulting Yukawa couplings are therefore effective couplings expressed in terms of an expansion parameter, ϵ, which is the ratio of the VEV of the heavy scalar field to the UV cut-off, $\epsilon = \frac{\langle\phi\rangle}{M}$. Explaining the hierarchical form of the Yukawa matrices then reduces to finding an appropriate symmetry G_F and field content which leads to acceptable forms of Yukawa matrices, and hence fermion masses and mixing angles, at the high energy scale.

Over recent years there has been a huge activity in this family symmetry and operator approach to understanding the fermion masses and mixing angles [66], including neutrino masses and mixing angles [67]. However, as we shall show in this chapter, in analysing such models it is important to also consider the corresponding operator expansion of the kinetic terms. The point is that, even though the kinetic terms may be correctly canonically normalised to begin with, they will inevitably be driven to a non-canonical form by a similar operator expansion to that which determines the Yukawa operator. In order to extract reliable predictions of Yukawa matrices, it is mandatory to canonically re-normalise the kinetic terms once again before proceeding. *In nearly all examples in the literature this is not done.* The main point of this chapter is thus to highlight this effect and to argue that it is sufficiently important that it must be taken into account before reliable predictions can be obtained. However, even after this field redefinition, we can still perform further arbitrary unitary rotations of the chiral superfields which will preserve the canonical form of the Kähler potential. Clearly any superfield field redefinitions in the Kähler potential must be performed consistently for all the superfields in the theory and this will result in a transformation of the superpotential couplings when written in terms of the new chiral superfields. This transformation of the Yukawa couplings is the main subject of this work and we are especially interested in the observable effects of this transformation on the physical masses and mixing angles. In fact, in the literature it is usually believed that these field redefinitions can have very important observable effects in quark and squark mixings [68, 69, 70], although it has been noted that this is not always the case [71].

Many approaches combine the family symmetry and operator approach with supersymmetric grand unified theories (SUSY GUTs) [66, 67]. Such models tend to be more constraining, because the Yukawa matrices at the high scale should have the same form, up to small corrections from the breaking of the unified symmetry. The same comments we made above also apply in the framework of SUSY GUTs. In the SUSY case the Yukawa operators arise from the superpotential W, and the kinetic terms and scalar masses, as well as gauge interaction terms come from the Kähler potential, K. In nearly all examples in the literature the superpotential W has been analysed independently of the Kähler potential, K, leading to most of the published results being inconsistent. The correct procedure which should be followed is as follows.

To be consistent, the Kähler potential, K, should also be written down to the same order M^{-n} as the superpotential W. Having done this, one should proceed to calculate the elements of the Kähler metric, $\tilde{K}_{ij}$, which are second derivatives with respect to fields of the Kähler potential $\tilde{K}_{i\bar{j}} = \frac{\partial^2 K}{\partial\phi_i \partial\phi_j^\dagger}$. However, in order to have canonically normalised kinetic terms, the Kähler metric has to itself be canonically normalised $\tilde{K}_{i\bar{j}} = \delta_{i\bar{j}}$. In making this transformation, the superfields in the Kähler potential are first being mixed and then rescaled. Once this has been done, the superfields in the superpotential must be replaced by the canonically normalised fields.

Canonical normalisation is not of course a new invention, it has been known since the early days of supergravity [72]. However, as we have mentioned, for some reason this effect has been largely ignored in the model building community. A notable exception is the observation some time ago by Dudas, Pokorski and Savoy [58], that the act of canonical normalisation will change the Yukawa couplings, and could serve to cover up 'texture zeros', which are due to an Abelian family symmetry which does not allow a specific entry in the Yukawa matrix and is therefore manifested as a zero at high energies. This issue has been resurrected for abelian family models recently [73]. However, as we have already noted, this observation has not been pursued or developed in the literature, but instead has been largely ignored.

In this chapter we consider the issue of canonical normalisation in the framework of non-Abelian symmetries, in which the Yukawa matrices are approximately symmetric. In such a framework we show that the effects of canonical normalisation extend beyond the filling in of 'texture zeros', and can also change the expansion order of the leading non-zero entries in the Yukawa matrix. As an example we perform an explicit calculation of such mixing associated with canonical normalisation of the Kähler metric in a recent supersymmetric model based on $SU(3)$ family symmetry where we show that such effects can significantly change the form of the Yukawa matrix. The $SU(3)$ model we consider is a grossly simplified version of the realistic model in [74], where we only consider the case of a single expansion parameter and perform our calculations in the 23 sector of the theory for simplicity, although we indicate how the results can straightforwardly be extended to the entire theory. We then use the fact that, for supersymmetric models, once the Kähler metric is canonical, it remains canonical under unitary transformations since $U\mathbf{1}U^\dagger = \mathbf{1}$. We can therefore pick a basis for convenience; doing so, we can show that for textures with certain properties, the corrections coming from canonical normalisation are small corrections for each element. Following this up, we demonstrate that the corrections to the CKM and PMNS matrices and the quark and lepton masses are at higher powers of the expansion parameter ϵ than the non-canonical values.

The outline of the rest of this chapter is as follows. In section 4.2 we discuss the issues surrounding canonical normalisation in the Standard Model supplemented by a family symmetry, first without then with SUSY. In the SUSY case we discuss the scalar mass squared and Yukawa matrices for two types of Kähler potential where only one superfield contributes to supersymmetry breaking. In section 4.3 we discuss a particular model in some detail as a concrete example, namely the simplified $SU(3)$

family symmetry model, focusing on the second and third generations of matter, later indicating how the results can be extended to all three families. In section 4.4 we point out that there is a freedom in choosing a canonical basis, and we use this freedom to pick a more convenient basis for calculating the effect of the canonical normalisation. In section 4.5 we will show that, at least for the case of an hierarchical pattern of masses and mixing angles for the up and down sectors, the effect of the Kähler potential, is always sub-dominant and cannot change the structure coming from the superpotential. Note that this disagrees with published results claiming that the effects of Kähler terms can change the order in ϵ of the CKM mixing angles [68]. We conclude in section 4.7

4.2 Canonical normalisation

4.2.1 Standard Model with a Family Symmetry

In this section we first consider extending the Standard Model gauge group with a family symmetry, under which each generation has a different charge (for abelian family symmetries) or representation (for non-abelian family symmetries). The family symmetry typically prohibits renormalisable Yukawa couplings (except possibly for the third family) but allows non-renormalisable operators, for example:

$$\mathcal{O}_Y = F^i H \overline{F}^j \frac{\phi_i \phi_j}{M^2} \tag{4.2}$$

where i, j are generation indices, M is some appropriate UV cutoff, F represents left-handed fermion fields, and $\overline{F}$ represents CP-conjugates of right-handed fermion fields, and H is a Higgs field. When the flavon scalar field ϕ gets a VEV, which breaks the family symmetry, effective Yukawa couplings are generated:

$$Y_{ij} = \frac{\langle \phi_i \rangle \langle \phi_j \rangle}{M^2} \tag{4.3}$$

The effective Yukawa matrices are determined by the operators allowed by the symmetries of the model, $G_F \otimes SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$, as well as the form that the vev of ϕ takes.

Even though the kinetic terms are correctly canonically normalised to begin with, they will receive non-renormalisable corrections arising from operators allowed by the family symmetry, which will cast them into non-canonical form. For example,

$$F_i^\dagger \not{\partial} F^j \left(\delta_j^i + \frac{\phi^i \phi_j^\dagger}{M^2} \right) \tag{4.4}$$

This leads to a non-canonical kinetic term when ϕ is replaced by its VEV. It is therefore mandatory to perform a further canonical re-normalisation of the kinetic terms, before analysing the physical Yukawa couplings. The canonical normalisation amounts to a transformation which is not unitary but which gives all the fields canonical kinetic terms. The kinetic part of a theory with a Higgs scalar field H, a fermionic field F^i and the field strength tensor $F^{\mu\nu}$ corresponding to a gauge field A^μ when canonical will look like:

$$\mathcal{L}_{\text{canonical}} = \partial_\mu H \partial^\mu H^* + F_i^\dagger \not{\partial} F^i - \frac{1}{4} F^{\mu\nu} F_{\mu\nu} \tag{4.5}$$

Once we have done this normalisation, we have to rewrite all of our interactions in terms of the canonical fields with the shifted fields.

The important point we wish to emphasise is that all the interaction terms should be expressed in terms of canonical fields, before making any physical interpretation. If this is not done, as is often the case in the literature, then the results will not be reliable.

4.2.2 SUSY Standard Model with Family Symmetry

In the context of supersymmetric theories, it turns out to be possible to automatically canonically normalise all the fields in the theory at once. However these transformations are not always simple, and in practice calculating the relevant transformations may well turn out to be intractable for any given model.

The aim of SUSY model builders with respect to flavour is two-fold. The primary wish is to generate a set of effective Yukawa matrices which successfully predict the quark and lepton masses and mixing angles as measured by experiment. However, because of the parameters associated with softly broken SUSY models, there exist dangerous one-loop diagrams which lead to processes such as $b \to s\gamma$ and $\mu \to e\gamma$ at rates much greater than predicted by the Standard Model and also much greater than measured by experiment. A successful SUSY theory of flavour will therefore successfully describe fermion masses and mixing angles, while simultaneously controlling such flavour changing processes induced by loop diagrams involving sfermion masses which are off-diagonal in the basis where the quarks and leptons are diagonal.

In a SUSY or supergravity (SUGRA) model, very often the starting point in addressing the flavour problem is to propose a set of symmetries that will give rise to non-renormalisable superpotential operators which will lead to a hierarchical form for our Yukawa matrices, arising from some effective Yukawa operators as discussed previously. Extra fields, ϕ are introduced that spontaneously break the extra family symmetries. The general form of the superpotential is :

$$W = F^i \overline{F}^j H w_{ij} \left(\phi/M\right) \tag{4.6}$$

Here $w_{ij}(\phi/M)$ is a general function of the extra fields, ϕ, which has mass dimension zero and contracts with $F^i\overline{F}^j$ to make W a singlet of the extended symmetry group.

In models of this type, the amount of flavour violation is proportional to the size of the off-diagonal elements in the scalar mass matrices at the electroweak (EW) scale when the scalar mass matrices have been rotated to the basis where the Yukawas are diagonal (the super-CKM basis). Since the quark mixing angles are small, this suggests that any large scalar mixings at the electroweak scale would remain large when in the super-CKM basis. Since we would generally not expect the RG running of the scalar mass matrices to tune large off-diagonal values to zero, we would expect to be in trouble if there are large off diagonal scalar mass mixings predicted at the high energy scale. This scale might be, for example, the unification scale in a SUSY GUT.

We now proceed to demonstrate that this will not be a problem in for two classes of Kähler potential. The two classes of Kähler potential are: The first form is:

$$K_1 = \ln(S + \overline{S}) + F^i F^\dagger_j k^j_i\,(\phi/M) + \overline{F}^i \overline{F}^\dagger_j \overline{k}^j_i\,(\phi/M) \tag{4.7}$$

The second form we consider is:

$$K_2 = \frac{S\overline{S}}{M^2}\left(F^i F^\dagger_j k^j_i\,(\phi/M) + \overline{F}^i \overline{F}^\dagger_j \overline{k}^j_i\,(\phi/M)\right) \tag{4.8}$$

Here $k(\phi)$ and $\overline{k}(\phi)$ represent functions of the various ϕ fields that can be contracted with the matter fields to make the Kähler potential a singlet and of the correct mass dimension.

Since we are looking at gravity-mediated SUSY breaking, we may use the SUGRA equations which relate the *non-canonically normalised* soft scalar mass squared matrices $m^2_{\overline{a}b}$ in the soft SUSY breaking Lagrangian to the Kähler metric $\tilde{K}_{\overline{a}b} = \frac{\partial^2 K}{\partial\phi^\dagger_a \partial\phi^b}$, and the vevs of the auxiliary fields which are associated with the supersymmetry breaking, F_m [72]:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} - F_{\overline{m}}\left(\partial_{\overline{m}}\partial_n \tilde{K}_{\overline{a}b} - \partial_{\overline{m}}\tilde{K}_{\overline{a}c}(\tilde{K}^{-1})_{c\overline{d}}\partial_n \tilde{K}_{\overline{d}b}\right) F_n \tag{4.9}$$

where we have assumed a negligibly small cosmological constant. Roman indices from the middle of the alphabet are taken to be over the hidden sector fields, which in our case can only be the singlet field S associated with SUSY breakdown. As it happens, for both K_1 and K_2, the *non-canonically normalised* mass matrix reduces to:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} \tag{4.10}$$

This is obvious for K_1, since the Kähler metric doesn't involve S, so partial derivatives with respect to S will give zero. To see that eq. (4.9) reduces to eq. (4.10) for K_2, is less obvious. We first write:

$$\tilde{K}_{\overline{a}b} = \frac{S\overline{S}}{M^2}\mathcal{M}_{\overline{a}b} \tag{4.11}$$

Substituting this into eq. (4.9) gives a non-canonically normalised scalar mass squared matrix:

$$m^2_{\overline{a}b} = m^2_{3/2}\tilde{K}_{\overline{a}b} - F_{\overline{S}}\left(\frac{1}{M^2}\mathcal{M} - \frac{S}{M^2}\mathcal{M}\frac{M^2}{S\overline{S}}\mathcal{M}^{-1}\mathcal{M}\frac{\overline{S}}{M^2}\right)F_S \tag{4.12}$$

It is clear that eq. (4.12) reduces to eq. (4.10). However, the physical states are those for which the Kähler metric is canonically normalised, $\tilde{K} = 1$. This is attained by $\tilde{P}^\dagger\tilde{K}\tilde{P} = 1$. In order to canonically normalise the mass matrix, we apply the same transformation, and find that the *canonically normalised* squark mass squared matrix then takes the universal form:

$$m^2_{\text{c.n.}} = m^2_{3/2}\mathbf{1} \tag{4.13}$$

We conclude that models with Kähler potentials like K_1 or K_2 will result in universal sfermion masses at the high-energy scale. Of course all this is well known, and it has long been appreciated that this would tame the second part of the flavour problem, flavour violating decays. However, what is less well appreciated at least amongst the model building community, is that canonical normalisation corresponds to redefining the fields in the Kähler potential, and one must therefore also redefine these fields in the same way in the superpotential. Unless this is done consistently it could lead to a problem with the first part of the flavour problem, because the shifted fields may well no longer lead to a phenomenologically successful prediction of the masses and mixing angles for the quarks and leptons.

The 'standard' method

After the flavour symmetry is spontaneously broken we obtain a certain Yukawa texture given by non-renormalisable operators which are functions of the flavon vevs as in Eq. (4.1). In the same way the effective Kähler potential will be a general non-renormalisable real function invariant under all the symmetries of the theory coupling the superfield combinations $F_i^\dagger F_j$ to the flavon fields, and similarly for $\overline{F}_i^\dagger \overline{F}_j$, where i, j are flavour indices. The terms $F_i^\dagger F_i$, $\overline{F}_i^\dagger \overline{F}_i$ without flavon superfields are clearly invariant under gauge, flavour and global symmetries and hence give rise to a family universal contribution. However, family symmetry breaking terms involving flavon superfields give rise to important corrections [75, 76, 77]. In fact, it is interesting to notice that, due to the non-holomorphicity of the Kähler potential, new terms are allowed with different structure from the terms that appear in the Yukawa couplings of the superpotential.

In general the matter fields do not have canonical wave functions (kinetic terms) in the symmetry eigenstate basis $\hat{\overline{F}}_i$, $\hat{F}_j$. Rather, flavon field vevs contribute to the diagonal terms and also generate new flavour off-diagonal entries. Thus, we now have non-canonical kinetic terms and we must redefine the fields to obtain canonical kinetic terms. The effect of these redefinitions, which can be regarded as wave function corrections, on the Yukawa couplings and other couplings in the theory may be determined after this field redefinition, $\hat{F} = PF$.

To obtain canonical kinetic terms we have to redefine the fields to go to the canonical basis by the inverse of the square root of the Kähler metric K given by

$$\hat{F}^\dagger K \hat{F} = (PF)^\dagger (P^{-1})^\dagger P^{-1} PF \tag{4.14}$$

Thus $K = (P^{-1})^\dagger P^{-1}$. Using Supergravity (SUGRA) equations, the Kähler metric is obtained as $K_{\bar{a}b} = \partial^2 G/(\partial\Phi_a^\dagger \partial\Phi^b)$ with G the Kähler function and it determines both the kinetic terms and the non-canonically normalised soft scalar mass squared matrices $\tilde{m}^2_{\bar{a}b}$. In SUGRA, where $K_{\bar{a}b}$ represents a metric, P^{-1} is also a Hermitian matrix, such that $P^{-1} = (P^{-1})^\dagger$ and hence it can be conventionally written as [69]

$$K = (P^{-1})^\dagger P^{-1} = V^\dagger X^2 V \Leftrightarrow P^{-1} = V^\dagger X V$$

with V a unitary matrix diagonalising the Hermitian matrix K and X the square root of the eigenvalues of K. We call this solution the 'standard' form of P^{-1}.

At this point, if we had a specific model, we would then need to check that the canonically normalised Yukawas are viable. This is then the procedure which must be followed in analysing a general model. We now turn to a particular example which illustrates the effects described above, in the framework on a non-Abelian family symmetry.

4.3 A SUSY Model based on $SU(3)$ family symmetry

4.3.1 The quark sector

As an example of the general considerations above, and in order to determine the quantitative effects of canonical normalisation, we now turn to a particular example based on $SU(3)_F$ family symmetry. As mentioned the model we consider is a simplified version of the realistic model by King and Ross [74] in which we assume only a single expansion parameter. For simplicity, we shall also ignore the presence of the first generation, although later we shall indicate how the results may be extended to the three family case. This model is based on a SUSY Pati-Salam model with a gauged SU(3) family symmetry extended by a $Z_2 \otimes U(1)$ global symmetry. As shown in Table 1, the left-handed matter is contained in F^i, the right-handed matter is contained in a left-handed field $\overline{F}^i$. The MSSM Higgs doublets are contained in H; Σ is a field which has broken $SO(10)$ to $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R$. There are two $SU(3)_F$-breaking fields, ϕ_3 and ϕ_{23}.

Field	$SU(3)_F$	$SU(4)_c$	$SU(2)_L$	$SU(2)_R$	Z_2	$U(1)$
F	3	4	2	1	+	0
$\overline{F}$	3	$\overline{4}$	1	2	+	0
H	1	1	2	2	+	8
Σ	1	15	1	1	+	2
ϕ_3	$\overline{3}$	1	1	1	-	-4
ϕ_{23}	$\overline{3}$	1	1	1	+	-5

Table 4.1: The field content of the toy model

The superpotential has to be a singlet under the combined gauge group $SU(4)_c \otimes SU(2)_L \otimes SU(2)_R \otimes SU(3)_F$ and also neutral under $Z_2 \otimes U(1)$. Because of this, the standard Yukawa superpotential:

$$W = F^i \overline{F}^j H Y_{ij} \tag{4.15}$$

is not allowed because of the $Z_2 \otimes U(1)$. As such, we have to move to a superpotential containing non-renormalisable terms. We view this as being the superpotential corresponding to a supersymmetric effective field theory, where some heavy messenger fields and their superpartners have been integrated out. Then, assuming that the messenger fields have the same approximate mass scale, we write:

$$W = \frac{1}{M^2} a_1 F^i \overline{F}^j H \phi_{3,i} \phi_{3,j} + \frac{1}{M^3} a_2 F^i \overline{F}^j H \Sigma \phi_{23,i} \phi_{23,j} \tag{4.16}$$

The a_i are parameters that are expected to be of the order of unity, M is the appropriate UV cutoff of the effective field theory. This will clearly lead to a set of effective Yukawa terms when the fields ϕ_3 and ϕ_{23} gain VEVs which break the family symmetry.

We choose the vacuum structure after King and Ross [74]:

$$\langle\phi_3\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} a \; ; \; \langle\phi_{23}\rangle = \begin{pmatrix} 1 \\ 1 \end{pmatrix} b \; ; \; \langle\Sigma\rangle = \sigma \tag{4.17}$$

And we then trade these for a single expansion parameter, $\epsilon \approx \frac{1}{10}$.

$$\frac{a}{M} = \sqrt{\epsilon} \; ; \; \frac{b}{M} = \epsilon \; ; \; \frac{\sigma}{M} = \epsilon \tag{4.18}$$

Substituting eqs. (4.17, 4.18) into eq. (4.16), we can write down our high-energy Yukawa matrix:

$$Y_{\text{n.c.}} = \begin{pmatrix} a_2\epsilon^3 & a_2\epsilon^3 \\ a_2\epsilon^3 & a_1\epsilon + a_2\epsilon^3 \end{pmatrix} \tag{4.19}$$

We write it as $Y_{\text{n.c.}}$ to represent the fact that it is the Yukawa matrix corresponding to the non-canonical Kähler metric.

4.3.2 The squark sector

In order to write down the squark mass matrices, the first step is to write down our Kähler potential. This should be the most general Kähler potential consistent with the symmetries of our model up to the same order in inverse powers of the UV cutoff as the superpotential is taken to. In our case, this is M^{-3}. However, from the general arguments of section 4.2, we know that if we pick our Kähler potential, K to be of the form as K_1 (eq. (4.7)) or K_2 (eq. (4.8)) then we will have universal scalars.

The non-canonical form of the scalar mass-squared matrix is:

$$m^2_{\text{n.c.}} \sim \begin{pmatrix} 1+\epsilon & \epsilon^2 \\ \epsilon^2 & 1+\epsilon \end{pmatrix} \tag{4.20}$$

However, we already know exactly what the canonical form of this matrix will look like:

$$m^2 = m^2_{3/2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{4.21}$$

This universal form is a direct result of the simple supersymmetry breaking mechanism that we have and canonical normalisation, and is independent of other details about the model.

4.3.3 Kähler potential for the model

We saw in the previous subsection that we will not end up with dangerous off-diagonal elements in the scalar mass matrices for general Kähler potentials of the type we are going to look at. We must now write down our Kähler potential. We choose this to be of the same form K_1. There will be no M^{-3} terms, so it will suffice to write this down up to $\mathcal{O}(M^{-2})$.

The matrices which diagonalise the Kähler metric will in general be large and intractable. In order to proceed, we will have to make some simplifying assumptions. We first assume that the Kähler metric $\tilde{K}_{\overline{a}b} = \frac{\partial^2 K}{\partial \phi_a^\dagger \partial \phi^b}$ is block diagonal, of the form:

$$\tilde{K}_{\overline{a}b} = \begin{pmatrix} \tilde{K}_{LH} & & & & \\ & \tilde{K}_{RH} & & & \\ & & \tilde{K}_\phi & & \\ & & & \tilde{K}_\Sigma & \\ & & & & \tilde{K}_H \end{pmatrix}_{\overline{a}b} \tag{4.22}$$

In this, $\tilde{K}_{LH}$ represents the block for chiral superfields, F, containing left-handed matter; $\tilde{K}_{RH}$ represents chiral superfields, $\overline{F}$, containing right-handed matter; $\tilde{K}_\phi$ represents the $SU(3)_F$ breaking Higgs fields, ϕ_{23} and ϕ_3; $\tilde{K}_\Sigma$ represents the block for the Higgs field that break the GUT symmetry down to the MSSM gauge group, Σ; finally, the block $\tilde{K}_H$ represents the block corresponding to the MSSM Higgs fields, H. The block diagonal assumption is equivalent to switching off some terms in the Kähler potential. The remaining terms in the Kähler potential are listed below:

$$\begin{aligned}
K \quad &= \ln(S+\overline{S}) + b_0 F^i F_i^\dagger + \frac{1}{M^2} F^i F_j^\dagger \left\{ \phi_3^k \phi_{3,l}^\dagger (b_1 \delta_i^l \delta_k^j + b_2 \delta_i^j \delta_k^l) \right. \\
&\left. + \phi_{23}^k \phi_{23,l}^\dagger (b_3 \delta_i^l \delta_k^j + b_4 \delta_i^j \delta_k^l) + b_5 H H^\dagger \delta_i^j + b_6 \Sigma \Sigma^\dagger \delta_i^j \right\} \\
&+ c_0 \overline{F}^i \overline{F}_i^\dagger + \frac{1}{M^2} \overline{F}^i \overline{F}_j^\dagger \left\{ \phi_3^k \phi_{3,l}^\dagger (c_1 \delta_i^l \delta_k^j + c_2 \delta_i^j \delta_k^l) \right. \\
&\left. + \phi_{23}^k \phi_{23,l}^\dagger (c_3 \delta_k^l \delta_k^j + c_4 \delta_i^j \delta_k^l) + c_5 H H^\dagger \delta_i^j + c_6 \Sigma \Sigma^\dagger \delta_i^j \right\} \\
&+ d_1 \phi_3^i \phi_{3,i}^\dagger + d_2 \phi_{23}^i \phi_{23,i}^\dagger + d_3 H H^\dagger + d_4 \Sigma \Sigma^\dagger \\
&+ \frac{1}{M^2} \left\{ \phi_3^i \phi_{3,j}^\dagger \phi_3^k \phi_{3,l}^\dagger d_5 \delta_i^j \delta_k^l + \phi_3^i \phi_{3,j}^\dagger \phi_{23}^k \phi_{23,l}^\dagger (d_6 \delta_i^j \delta_k^l + d_7 \delta_k^j \delta_i^l) \right. \\
&\left. + \phi_{23}^i \phi_{23,j}^\dagger \phi_{23}^k \phi_{23,l}^\dagger d_8 \delta_i^j \delta_k^l + d_9 H H^\dagger H H^\dagger + d_{10} \Sigma \Sigma^\dagger \Sigma \Sigma^\dagger \right\} \qquad (4.23)
\end{aligned}$$

Having done this, we now need to calculate the Kähler metric $\tilde{K}$. But since we have set K up specifically such that it is block diagonal, we can instead work out the non-zero blocks, $\tilde{K}_{LH}$, $\tilde{K}_{RH}$, $\tilde{K}_\phi$, $\tilde{K}_\Sigma$ and $\tilde{K}_H$. Once we have done so, we need to canonically normalise them. This is done in two stages. The first is a unitary transformation to diagonalise each block $\tilde{K}_i$:

$$\mathcal{L} \supset F^\dagger \tilde{K} F \to (F^\dagger U)(U^\dagger \tilde{K} U)(U^\dagger F) = F' \tilde{K}' F^{\dagger\prime} \qquad (4.24)$$

The mixed Kähler metric, $\tilde{K}'$, is now diagonal. Then we rescale the fields by a diagonal matrix R such that $R_i = (\tilde{K}_i')^{-1/2}$. These new superfields are then canonically normalised.

Then:

$$\mathcal{L} \supset (F^\dagger U R^{-1}) \underbrace{(R U^\dagger \tilde{K} U R)}_{\mathbf{1}} (R^{-1} U^\dagger F) \qquad (4.25)$$

So, with this notation $P = R^{-1} U^\dagger$. As before, the Kähler metric is equal to $P^\dagger P$.

The important point to note is that in canonically normalising, we have redefined our superfields, so we must also redefine them in our superpotential. This is discussed in the next section.

4.3.4 Yukawa sector after canonical normalisation

In this section we return to the important question of the form of the Yukawa matrices in the correct canonically normalised basis. In order to do this we would have to calculate the shifting in all of the fields in the superpotential. Unfortunately, algebraically diagonalising the sub-block $\tilde{K}_\phi$ is intractable, even for such a simple model. We therefore make a second assumption and neglect the effects of canonical normalisation arising from this sector, although we shall correctly consider the effects of canonical normalisation arising from all the other sectors.

Even making this assumption, the expressions we get are not especially pleasant. We then substitute in the form of the VEVs (eq. (4.17) and eq. (4.18)). Having done this, we then expand the cofactors of $F^i\overline{F}^j H$ as a power series in ϵ around the point $\epsilon = 0$. The cofactors of ϵ^n are quite complicated, so we only write out here the expression for the effective Yukawa for the 23 element. The full expressions for all four elements are listed in Appendix E.

$$Y_{23} = -a_1 \frac{b_3}{\sqrt{b_0 c_0} b_1}\epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0} d_4}\epsilon^3 + a_1 \frac{b_3(b_2 c_0 + b_0(c_1 + c_2))}{2 b_0^{3/2} c_0^{3/2} b_1}\epsilon^3 + \mathcal{O}(\epsilon^4) \qquad (4.26)$$

The important point to note is that, compared to the 23 element of Eq.(4.19), the leading order expansion in ϵ has changed. No longer is it at ϵ^3, it is now ϵ^2.

Note that we can write the expressions for the canonically normalised off-diagonal Yukawa matrix elements Y_{23} and Y_{32} in such a way that they would transform into each other if we interchange $b_i \leftrightarrow c_i$, as would be expected. We also note that the diagonal matrix elements would transform into themselves under the same substitution, $b_i \leftrightarrow c_i$. This has been checked explicitly to the order in the Taylor expansion shown in the Appendix.

Setting the $\mathcal{O}(1)$ parameters b_i,c_i and d_i to unity, the Yukawa matrix then takes the canonical form:

$$Y_c \sim \begin{pmatrix} (a_1+a_2)\epsilon^3 & -a_1\epsilon^2+(1.5a_1+a_2)\epsilon^3 \\ -a_1\epsilon^2+(1.5a_1+a_2)\epsilon^3 & a_1\epsilon-2a_1\epsilon^2+a_2\epsilon^3 \end{pmatrix} + \mathcal{O}(\epsilon^4) \tag{4.27}$$

We emphasise again that Eq.(4.27) has a different power structure in ϵ to the original, non-canonically normalised Yukawa in eq. (4.19).

What has happened is that the unitary matrix which redefines our fields has mixed them amongst themselves. This leads to a similar (but different) high energy Yukawa texture. This certainly could be a sufficiently different set-up to ruin any predictions that the non-canonical model was designed to make. However we emphasise that this result applies to the simplified $SU(3)_F$ model with a single expansion parameter, and not the realistic $SU(3)_F$ model of King and Ross [74] with two different expansion parameters.

By comparing the non-canonical Yukawa matrix in eq. (4.19) to the canonical Yukawa matrix in eq. (4.27), we can see that the Kähler mixing angles are large, of $\mathcal{O}(\epsilon)$. In the appendix, we have an expression for the inverse P-matrix, P^{-1}. The large mixing effect can come only from the mixing part of the transformation. Schematically, the appearance of the ϵ^2 leading order terms in the off-diagonal elements can then be understood by neglecting all the coefficients of $\mathcal{O}(1)$, as follows:

$$Y_c \sim \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \begin{pmatrix} \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon \end{pmatrix} \begin{pmatrix} 1 & \epsilon \\ \epsilon & 1 \end{pmatrix} \sim \begin{pmatrix} \epsilon^3 & \epsilon^2 \\ \epsilon^2 & \epsilon \end{pmatrix} \tag{4.28}$$

which accounts for the appearance of the ϵ^2 leading order terms in the off-diagonal elements.

4.3.5 Three generations of matter

The procedure we have discussed for the second and third families can straightforwardly be generalised to include also the first family or indeed to any number of generations. The first thing to do is to write down all of the symmetries of the model. Having done this, write down all of the non-renormalisable operators up to the the chosen order in the UV cutoff, M. In the two generation case, this was to $\mathcal{O}(M^{-3})$. The next step is to write down the Kähler potential consistent with all the symmetries of the model, up to the same order in the UV cutoff M as the superpotential was expanded to. For tractability, some terms may have to be switched off to make the Kähler metric block diagonal as in eq. (4.22). At this point, the fields which break the family symmetry are replaced by their VEVs.

Then one must find the matrices which canonically normalise each sub-block of the Kähler metric. These will take the form of a unitary matrix which diagonalises the sub-block, and then a rescaling which takes it to the identity matrix of the appropriate size. Having done this, the unnormalised fields can be written in terms of the canonically normalised fields. If $\tilde{P}_S$ is the matrix which diagonalises the sub-block $\tilde{K}_S$, and ψ_S and ψ'_S are respectively the unnormalised and canonically normalised fields in the sub-block, then:

$$\tilde{P}_S \tilde{K}_S \tilde{P}_S^\dagger = \mathbf{1} \tag{4.29}$$

$$\psi_S = \tilde{P}_S \psi'_S \tag{4.30}$$

We then substitute eq. (4.30) into the superpotential. Once we have done this, the canonically normalised Yukawa matrix will be the coefficient of $F'\overline{F}'H'$. At this point, the Yukawa matrix elements may well be of the form of one polynomial in expansion parameters, (ϵ in the example model) divided by another. In this case, to understand the power structure in the expansion parameter, it is necessary to use a Taylor expansion to get a power series in the expansion parameters (we may do this because the expansion parameters must be small in order for the whole technique of non-renormalisable operators to work in the first place).

Having completed this, the end result is canonically normalised three-generation Yukawa matrices, as required. Note that any step of this calculation could in principle be intractable, and therefore some simplifying assumptions may have to be made.

4.4 An improved method for calculating P

Note that if P^{-1} is a solution of Eq. (4.14) then also $V.P^{-1}$ is a solution of Eq. (4.14), with V any unitary matrix. Of course physical quantities will not depend on V and for any choice we must always obtain the same physical result. This is due to the invariance of the Lagrangian under the so-called Weak Basis Transformations (WBT) [78, 79]. The theory is invariant if we transform the fields as,

$$q_L = V_q q'_L \quad ; \quad u_R = V_u u'_R \quad ; \quad d_R = V_d d'_R$$

where V_q, V_u and V_d are transformations from the global unitary groups $U(3)_L$, $U(3)_{u_R}$ and $U(3)_{d_R}$ respectively, while simultaneously the Yukawa couplings are transformed as,

$$Y'_u = V_q^\dagger Y_u V_u \qquad Y'_d = V_q^\dagger Y_d V_d \tag{4.31}$$

Therefore when we choose the different V_a all we are doing is to choose a particular weak basis where we write our theory and the physical results are absolutely independent of this choice. However, it is very useful to choose the unitary transformation V in the definition of $P = K^{-1/2}$ to get a simpler form for this transformation. The form that proves to be useful is the Cholesky decomposition of an Hermitian matrix. It is always possible to write an Hermitian matrix as $K = U^\dagger U$ in terms of an upper U triangular matrix,

$$K = \begin{pmatrix} K_{11} & K_{12} & K_{13} \\ K_{12}^* & K_{22} & K_{23} \\ K_{13}^* & K_{23}^* & K_{33} \end{pmatrix} = U^\dagger U = \begin{pmatrix} u_{11} & 0 & 0 \\ u_{12}^* & u_{22} & 0 \\ u_{13}^* & u_{23}^* & u_{33} \end{pmatrix} \begin{pmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{pmatrix} \tag{4.32}$$

This equation is very easy to solve,

$$u_{11} = \sqrt{K_{11}} \qquad u_{12} = \frac{K_{12}}{\sqrt{K_{11}}} \qquad u_{13} = \frac{K_{13}}{\sqrt{K_{11}}} \tag{4.33}$$

$$u_{22} = \sqrt{K_{22} - \frac{|K_{12}|^2}{K_{11}}} \qquad u_{23} = \frac{K_{23}K_{11} - K_{13}K_{12}^*}{\sqrt{K_{22}K_{11}^2 - K_{11}|K_{12}|^2}} \qquad u_{33} = \sqrt{K_{33} - |u_{23}|^2 - |u_{13}|^2}$$

The inverse of this upper triangular matrix is also upper triangular, and it is also easily obtained. Obviously we could have chosen to use lower triangular matrices L instead of the upper triangular matrices U and the explicit form of the L would then have been obtained in a similar way in terms of K.

This form for the square root of the Kähler matrix is different from the 'standard' form used in the literature [69, 70]. Clearly the 'standard' form is related to our triangular form by an unobservable WBT and therefore the two forms are physically indistinguishable. However it is evident from Eq. (4.33) that from the point of view of calculability it is much simpler to obtain the triangular form than the 'standard' form.

4.5 The Kähler corrections to Yukawa couplings

4.5.1 The form of the Yukawa coupling matrix

To proceed we need to know the form of the Yukawa couplings coming from the superpotential. A fit to the data using a form for the Yukawa matrices where the smallness of CKM mixing angles is due to the smallness of the off-diagonal entries with respect to the relevant diagonal entry yields the structure [80],

$$Y_d \propto \begin{pmatrix} 0 & \bar{\varepsilon}^3 & \bar{\varepsilon}^3 \\ . & \bar{\varepsilon}^2 & \bar{\varepsilon}^2 \\ . & . & 1 \end{pmatrix}, \qquad Y_u \propto \begin{pmatrix} 0 & \varepsilon^3 & \varepsilon^3 \\ . & \varepsilon^2 & \varepsilon^2 \\ . & . & 1 \end{pmatrix}$$

with the expansion parameters $\bar{\varepsilon} = 0.15$ and $\varepsilon = 0.05$. Some non-Abelian family symmetry models can provide such a structure quite naturally [74, 81]. Here we have suppressed coefficients of $\mathcal{O}(1)$. This structure has $Y_{kj} < Y_{ij}$ for $i > k$ and $j \geq i$ and is unique if the contribution to the left-handed mixing angles from the elements below the diagonal are negligible. If one relaxes this constraint then it is possible for some of the entries to be smaller or zero (texture zeros). We will discuss both these possibilities. To do so let us first note that, although there are no direct bounds on the Yukawa couplings below the diagonal from (right-handed) mixing angles, we can obtain some upper bounds on these entries from their contributions to the mass eigenvalues. Just requiring that the determinant of the down Yukawa matrix is $\bar{\varepsilon}^6 = 1 \times \frac{m_s}{m_b} \times \frac{m_d}{m_b}$ we arrive to the conclusion that $Y^d_{21} \leq \bar{\varepsilon}^3$, $Y^d_{31} \leq \bar{\varepsilon}$ and $Y^d_{32} \leq 1$, assuming no cancellation between different contributions to the determinant. The same bounds are valid for the up Yukawa matrix in terms of ε. With this the most general hierarchical Yukawa structure consistent with the masses and mixing angles is

$$Y \propto \begin{pmatrix} \leq \epsilon^4 & a\,\epsilon^3 & b\,\epsilon^3 \\ \leq \epsilon^3 & c\,\epsilon^2 & d\,\epsilon^2 \\ \leq \epsilon & \leq 1 & 1 \end{pmatrix} \tag{4.34}$$

where $\epsilon = \bar{\varepsilon}, \varepsilon$ for $Y = Y_d, Y_u$. Not all of the four coefficients a, b, c, d must be $\mathcal{O}(1)$ allowing for the possibility of additional texture zeros.

In the rest of this chapter we consider the case that this structure is reproduced by the terms of the superpotential in the symmetry basis and we show that the effect of the Kähler potential is then always subdominant in its effects on the masses and mixing angles.

4.5.2 The Kähler corrections

It proves to be useful in most realistic models to go to the canonically normalised basis by redefining the fields by a wave function normalisation matrix chosen to have the upper triangular form, as discussed above. Using this form the correction to the Yukawa coupling matrix in the Standard Model (SM) is of the form

$$H\hat{\overline{\psi}}_L \, Y \, \hat{\psi}_R \;\equiv\; H \, \hat{\psi}^*_{L\,i} \, Y_{ij} \, \hat{\psi}_{R\,j} \;=\; H \, \psi^*_{L\,k} P^*_{L\,ik} \, Y_{ij} \, P_{Rjm} \psi_{R\,m} \;=\; H \, \psi^*_{L\,k} \, Y^t_{km} \, \psi_{R,m}$$

If we consider, for the moment, only the transformation on the left handed fields using our triangular matrices, with $P = U$, the total (t) Yukawa is,

$$\begin{aligned} Y^t_{ij} &= \sum_{i \geq k} P^*_{ki} Y_{kj} \\ &\simeq P^*_{ii} Y_{ij} + \sum_{i>k} P^*_{ki} Y_{kj} \end{aligned} \qquad (4.35)$$

As may be seen in Eq. (4.1) the expansion parameters are given by terms of the form $< \theta > /M$ where M is the messenger mass. In the superpotential the expansion parameters come from both the LH and RH sectors. The expansion parameters, ε and $\bar{\varepsilon}$, for the up and down sectors[1] in the superpotential may differ as the SM gauge group does not relate the up and down right handed quark sectors. However the contribution from the LH sector to the mass matrix structure must be equal in the up and down

[1]Here we have implicitly assumed that $\epsilon =< \theta > /M$ is the fundamental expansion parameter. If this is not true and the true expansion parameter is larger (e.g. θ is itself generated by a higher dimension term $\phi.\phi/M$) one should allow for the possibility that the expansion parameter in the *Kähler* sector is the larger one (e.g. $< \phi > /M$).

sectors due to the $SU(2)_L$ gauge symmetry. Thus its contribution cannot be larger than ε, the smaller of the two (right handed) expansion parameters. This implies that the Kähler rescaling matrix in the LH sector, P^L_{ik}, has a strong hierarchy controlled by the small parameter ε with $P^L_{ii} \simeq 1$ and $P^L_{ik} \leq \epsilon$ for the non-zero entries of the upper triangular form. Notice that an $\mathcal{O}(1)$ value for $P^L_{ik,i\neq k}$ would destroy the hierarchy in Eq. (4.34) and therefore is not phenomenologically allowed. A similar argument applies to the up quark RH sector, $P^{R,u}_{ii} \simeq 1$ and $P^{R,u}_{ik} \leq \varepsilon$ but in the down quark RH sector the expansion parameter must be the larger one, $\bar{\varepsilon}$, so $P^{R,d}_{ii} \simeq 1$ and $P^{R,d}_{ik} \leq \bar{\varepsilon}$.

In fact it is easy to prove that for the hierarchical textures of interest here the leading correction to a given Yukawa element is suppressed by at least $\mathcal{O}(\epsilon^2)$. With the underlying family symmetry ordering the correction we know that, before symmetry breaking, the operator giving rise to the correction to a given element must transform in the same way under the family symmetry as the leading term. We have just proved that the difference of the Kähler transformations from the identity is at least of $O(\epsilon)$. Furthermore corrections to Y_{ij} after transformations to canonical Kähler with upper triangular matrices come only from $\hat{Y}_{kj}$ with $k < i$ and $\hat{Y}_{kj} < \hat{Y}_{ij}$. This implies that a new contribution to Y_{ij} is subdominant relative to $\hat{Y}_{ij}$ at least by $O(\epsilon)$ where $\epsilon =< \theta > /M$. As θ transforms non-trivially under the family symmetry, to maintain the symmetry property of the leading term, this relative correction must be given by a combination of fields which transforms as a singlet, that is at least of the form $\theta\theta^\dagger$ and hence of $\mathcal{O}(\epsilon^2)$. This result applies to hierarchical Yukawa structures. For the case that the $(2,3)$ element saturates the bound of Eq. (4.34) it violates the condition of hierarchical Yukawa couplings and our conclusions above do not apply. In what follows we consider this possibility separately.

Using this we will now calculate the canonical Yukawa through Eq. (4.35). Although we have started with the superpotential generating the form of Eq. (4.34) in the symmetry basis we have the freedom to use any basis when calculating the effects on physical quantities. It is convenient to go to the Cholesky form when determining the effects of the Kähler potential and we use an upper triangular form for the Kähler rescaling matrix in the LH sector with $P^L_{ki} = 0$ for $i < k$. The corrections to a given element of the Yukawa matrix induced by the transformation to canonical Kähler are given by $P^*_{ki} Y_{kj}$.

No additional texture zeros

We first consider the case without additional texture zeros so that all of a, b, c, d are of $\mathcal{O}(1)$. Taking into account that $Y_{kj} < Y_{ij}$ for $k < i$ and $j \geq i$ we conclude that $P_{ki} Y_{kj} < Y_{ij}$. Therefore, these corrections are always sub-dominant in ϵ. This is not yet sufficient to prove that the transformation to the canonical left handed Kähler basis does not change the observable mixings and masses because they could be sensitive to elements of Y below the diagonal. Given the bounds of Eq. (4.34) the only dangerous term is the $(3,1)$ term because for $Y_{3,1} < Y_{2,1}$ the Kähler correction can dominate the $(3,1)$ element. However in this case, from the structure in Eq. (4.34), $Y^t_{3,1} \leq \epsilon^5$. Clearly this is too small to affect masses or LH mixing angles at leading order. As we have discussed, for the hierarchical textures of interest here, the leading correction to a given Yukawa element is suppressed by at least $\mathcal{O}(\epsilon^2)$.

One might worry that the condition $Y_{kj} < Y_{ij}$ for $k < i$ and $j \geq i$ is too strong and that what are constrained are the elements after Kähler mixing, i.e. $Y^t_{kj} < Y^t_{ij}$ for $k < i$ and $j \geq i$ and the condition on Y_{kj} is not satisfied. However this is inconsistent. To see this note that the phenomenological structure of Y^t_{kj} in Eq. (4.34) would correspond both to the basis of canonical Kähler with upper triangular transformations or to the basis of "standard" canonical transformations. This is due to the fact that both basis are related by a small rotation which does not change the order of the elements if the departure of the original Kähler metric from the identity is also hierarchical as expected in models with a spontaneously broken family symmetry. Thus we still have $P_{ik} \leq \epsilon$ for $i \neq k$. Therefore, we would need $Y_{kj} > Y_{ij}$ for $k < i$, or more exactly the power in ϵ of Y_{kj} is smaller than the power in ϵ of Y_{ij} for $k < i$ so that $P^*_{ki} Y_{kj} > Y_{ij}$ is possible. However in this case we necessarily have $Y^t_{kj} = Y_{kj} > Y_{ij} + P_{ik} Y_{kj} = Y^t_{ij}$ for $k < i$ and $j \geq i$ (neglecting smaller contributions from Y_{mj} with $m < k$ if present) and we arrive to an inconsistency with the initial statement $Y^t_{kj} < Y^t_{ij}$. thus even with the weaker condition we need $Y_{kj} < Y_{ij}$ for $k < i$ and $j \geq i$.

So far we have discussed the transformations to canonical Kähler for the left handed fields. Now, we have to proceed exactly in the same way for the right-handed transformation. Clearly, if the Yukawa structures are also hierarchical we can perform the same analysis using upper triangular matrices and we would again arrive to the conclusion that corrections from the Kähler to any Yukawa element are always sub-dominant at least by ϵ^2 ($\epsilon = \bar{\varepsilon}, \varepsilon$ for $Y = Y_d, Y_u$). There is an exception to this conclusion if Y_{23} does not preserve the hierarchical structure and is of $\mathcal{O}(1)$ saturating the bound in Eq. (4.34). In this case it is possible that $P^R_{23} = \mathcal{O}(1)$ and therefore corrections $\mathcal{O}(1)$ to Y_{i3} are still possible. Even in this case, it is clear that we can never modify the order in ϵ of the different elements of the Yukawa matrix, all it can do is to change the $\mathcal{O}(1)$ coefficients of the Y_{i3} elements. To determine whether this special case is possible one needs to know Y_{32} and this can be done through measurement of flavour changing neutral currents [82, 83] or lepton flavour violation [84].

Thus, using the triangular form, we have shown that the Kähler corrections to the Yukawa matrix are sub-dominant for hierarchical Yukawa matrices. In the next section we prove that this is also true for the observable mixing angles and mass eigenstates.

Additional texture zeros

A special situation occurs when one of a, b, c, d is $< \mathcal{O}(1)$ giving rise to an approximate texture zero. This can spoil the hierarchical structure of our Yukawa textures, $Y_{kj} < Y_{ij}$ for $k < i$ and $j \geq i$ and therefore must be analysed separately. An example of the origin of such zeros occurs in spontaneously broken Abelian theories through the so-called holomorphic zeros [68]. In this case the symmetry breaking is through flavon field(s) carrying only one sign of charge (say negative) and then a net negative charge of the fermionic fields cannot be compensated with insertions of the flavon field because, due to the holomorphicity of the superpotential, the charged conjugated flavon can not be used. However the Kähler potential is non holomorphic and therefore these zeros can be filled after the transformation to the canonical basis.

As before, if we are only interested in the physical effects of this texture zero filling we can choose a convenient basis [68]. Once more our choice of upper triangular matrices is especially simple. In a hierarchical texture we can have a texture zero in any position of the matrix except in Y_{33} which is necessarily $\mathcal{O}(1)$. Although it is clear that the texture zeros can be filled in by the Kähler corrections we can immediately use the analysis presented above to show that physical observables will not be affected by these corrections. The point, as is explicitly demonstrated in the next section, is that the form of Eq. (4.34) gives the value of each entry of the Yukawa matrix that has a leading effect on a mass or a mixing angle. If the entry is larger than the value shown it will give a mass or mixing angle in conflict with the measured value. If the entry is smaller it will only contribute to measurable quantities at subleading order.

In the previous section we showed that, for the case of hierarchical textures, the

Kähler corrections only contribute to the Yukawa matrix elements suppressed relative to the order shown in Eq. (4.34) by at least $\mathcal{O}(\epsilon^2)$. For example we can see that a zero in Y_{11} is never filled by any other element. In the same way a zero in Y_{12} or Y_{21} is only filled by a non-zero entry in Y_{11}. Taking into account the constraints from the determinant of the Yukawa matrix, $Y_{11} \leq \epsilon^4$ and in the hierarchical case with $P^{L(R)}_{i \neq j} \leq \epsilon$ this implies that they can only be filled at $\mathcal{O}(\epsilon^5)$. In the same way Y_{13} Y_{31} and Y_{22} can only be filled at $\mathcal{O}(\epsilon^4)$ $(Y_{12}, Y_{21} \leq \epsilon^3)$. Finally a zero in Y_{23} or Y_{32} implies that $Y_{22} = \epsilon^2$ and hence these zeros can be filled at most at $\mathcal{O}(\epsilon^3)$. As we will now show, these subleading terms only contribute to physical quantities at subleading order even though the texture zero may be filled in. The only exception to this is when the hierarchical structure is spoilt through an $\mathcal{O}(1)$ term in Y_{23}. In this case, following the discussion given above, the Kähler corrections can contribute at $\mathcal{O}(1)$ to physical quantities.

4.6 Kähler corrections to the mass matrix eigenvalues and mixing angles

To complete our proof we need to demonstrate that the entries of Eq. (4.34) are the smallest that can affect masses and mixing angles and thus the Kähler corrections, which we have shown are smaller than those of Eq. (4.34), are necessarily subdominant in determining physical quantities.

4.6.1 Quark and charged lepton masses and mixing angles.

Since the Kähler corrections are wave function corrections which cannot change the rank of the mass matrix we know that they lead to multiplicative normalisations of the masses. For hierarchical Yukawa matrices the wave function normalisation has the form

$P_{ik} = \delta_{ik} + \mathcal{O}(\leq \varepsilon)$ and this means the Kähler corrections to masses are necessarily sub-dominant. To see this explicitly, consider only the left handed canonical normalisation P_{ik} with P upper triangular. Now using Eq. (4.35) the canonical Yukawa and the fact that the Yukawa and Kähler matrices are hierarchical in the left handed sector, the determinant of Y^t is,

$$\mathrm{Det}(Y^t) = \mathrm{Det}(P)\mathrm{Det}(\hat{Y}) \simeq (1 + \mathcal{O}(\leq \epsilon))\mathrm{Det}(\hat{Y})$$

Moreover, from the hierarchical structure in Eq. (4.34) we know that any element of the matrix is corrected only at $\mathcal{O}(\leq \epsilon^2)$ under the transformations to canonical left-handed Kähler. In particular, the heaviest eigenvalue in Y^t will be still be $1 + \mathcal{O}(\leq \epsilon^2)$. Therefore this implies that the product of the two lightest eigenvalues can only be changed at $\mathcal{O}(\leq \epsilon^2)$. Finally the second eigenvalue is basically obtained from the lightest eigenvalue of the $(2,3)$ submatrix and thus we obtain again that any change to this eigenvalue will be sub-dominant in ϵ and therefore the same is true for the first generation eigenvalue.

In the case of a non-hierarchical structure in the $(2,3)$ entry with P^R_{23} of $\mathcal{O}(1)$ we expect $\mathrm{Det}(P^R)$ to be $\mathcal{O}(1)$ barring accidental cancellations. In this case the corrections to the eigenvalues, while still not changing their order in ϵ, could be $\mathcal{O}(1)$.

Concerning the mixing angles, with the use of triangular matrices we have not changed the hierarchical structure of the Yukawa matrices. Hence, we can still use the usual perturbative expansion. In this way, after the transformations to left handed canonical Kähler we have,

$$\begin{aligned}\theta_{23} &= \theta^d_{23} - \theta^u_{23} = \frac{(Y^d_{23})^t}{(Y^d_{33})^t} - \frac{(Y^u_{23})^t}{(Y^u_{33})^t} = \frac{\hat{Y}^d_{23}(1 + \mathcal{O}(\varepsilon^2))}{\hat{Y}^d_{33}(1 + \mathcal{O}(\varepsilon^2))} - \frac{\hat{Y}^u_{23}(1 + \mathcal{O}(\varepsilon^2))}{\hat{Y}^u_{33}(1 + \mathcal{O}(\varepsilon^2))} \\ &= \hat{\theta}_{23}(1 + \mathcal{O}(\varepsilon^2)) \end{aligned} \tag{4.36}$$

the discussion is identical for the θ_{13} mixing angle. The case of θ_{12} is slightly more complicated, now we have,

$$\theta_{12}^d = \frac{(Y_{12}^d)^t}{(Y_{22}^d)^t - (Y_{23}^d)^t (Y_{32}^d)^t} \tag{4.37}$$

where the denominator is really the Y_{22}^d element in the basis where we have already diagonalised the $2,3$ sector, and it is approximately equal to $m_s/m_b = \bar{\varepsilon}^2$. However, we know that both $(Y_{22}^d)^t \leq \bar{\varepsilon}^2(1+\mathcal{O}(\varepsilon^2))$ and $(Y_{23}^d)^t(Y_{32}^d)^t \leq \bar{\varepsilon}^2(1+\mathcal{O}(\varepsilon^2))$. This means that the denominator can also be corrected only at $\mathcal{O}(\varepsilon^2)$, then we have,

$$\theta_{12}^d = \frac{\hat{Y}_{12}^d(1+\mathcal{O}(\varepsilon^2))}{(\hat{Y}_{22}^d - \hat{Y}_{23}^d \hat{Y}_{32}^d)(1+\mathcal{O}(\varepsilon^2))} = \hat{\theta}_{12}^d(1+\mathcal{O}(\varepsilon^2)) \tag{4.38}$$

doing the same for θ_{12}^u we arrive immediately to $\theta_{12} = \theta_{12}^d - \theta_{12}^u = \hat{\theta}_{12}(1+\mathcal{O}(\varepsilon^2))$.

Moreover, it is easy to check that the effect of the transformation to canonical Kähler for the right handed fields on the left handed mixings is usually negligible. To see this, we consider the limit of trivial left handed Kähler and nontrivial right-handed Kähler. Then, we consider the diagonalisation of the Hermitian matrix H^t,

$$H^t = Y^t (Y^t)^\dagger = \hat{Y} P_R P_R^\dagger \hat{Y}^\dagger = \hat{Y} K^{-1} \hat{Y}^\dagger = \hat{V}_L^\dagger \hat{M}_f \hat{V}_R K^{-1} \hat{V}_R^\dagger \hat{M}_f V_L \equiv \hat{V}_L^\dagger \hat{M}_f \tilde{K}^{-1} \hat{M}_f \hat{V}_L$$

where we have written $\hat{Y} = \hat{V}_L^\dagger \hat{M}_f \hat{V}_R$ and reabsorbed the right-handed rotation in $\tilde{K}^{-1}$, i.e. we have written the inverse of the Kähler in the basis of right handed mass eigenstates. Now it is trivial to see that the matrix diagonalising H^t will be the product of V_L with the matrix diagonalising $\hat{M}_f \tilde{K}^{-1} \hat{M}_f$. As we have seen $\hat{M}_f$ are approximately equal to the eigenvalues of the total Yukawa matrix, this implies that $\hat{M}_f \tilde{K}^{-1} \hat{M}_f$ is strongly hierarchical and then the mixing angles diagonalising this matrix will be,

$$\tilde{\theta}_{i3} \simeq \frac{m_i m_3 (\tilde{K}^{-1})_{i3}}{m_3^2 (\tilde{K}^{-1})_{33}} \qquad \tilde{\theta}_{12} \simeq \frac{m_1 m_2 (\tilde{K}^{-1})_{12}}{m_2^2 \left((\tilde{K}^{-1})_{22} - \dfrac{|(\tilde{K}^{-1})_{23}|^2}{(\tilde{K}^{-1})_{33}} \right)}$$

therefore these contributions are suppressed both by the smallness of off-diagonal entries in the Kähler with respect to diagonal ones and by ratios of fermion masses. This last suppression is usually enough to make $\tilde{\theta}_{ij} \ll \theta_{ij}$ and then we can safely neglect the effect of right handed transformation in left handed mixings.

The exception to this rule arises when the right handed Kähler in the basis of right handed mass eigenstates is not hierarchical and has $\mathcal{O}(1)$ entries in K_{23}, K_{22} and K_{33}. In this case the correction to the angle θ_{23} from the down quark right handed Kähler could be of leading order as both θ_{23} and m_s/m_b are $\mathcal{O}(\bar{\varepsilon}^2)$. Still this situation can be understood as an exception to the main rule we formulated above. The correction from the right handed Kähler in the left handed mixing angles would still be of the same order as the contribution from the non-canonical Yukawa matrix and therefore would only modify the unknown $\mathcal{O}(1)$ coefficients. Usually, we find this structure in $U(1)$ models with lopsided Yukawa textures [85]. These models depend precisely on the existence of different $\mathcal{O}(1)$ coefficients in the elements of the Yukawa texture to obtain the correct masses and mixing angles. However, the $U(1)$ symmetry has no control on these $\mathcal{O}(1)$ coefficients and so this means that we do not need to worry about these effects. Only in a theory where we can control these unknown coefficients we should worry about the effects of this right-handed field redefinition.

4.6.2 Neutrino masses and mixing angles

The case of neutrino masses can be analysed with similar techniques. In this case, we obtain the effective Majorana mass matrix for the left handed neutrinos through the seesaw mechanism. The neutrino mass matrix structure has the form

$$L_\nu = -\nu_{Li} Y^\nu_{ij} \nu^c_{Rj} - \frac{1}{2} \nu_{Ri} M_{Rij} \nu^c_{Rj} + h.c.$$

giving the effective Majorana mass matrix of the effective low energy neutrinos, M_ν of the form

$$M_\nu = \chi_\nu \ (v \sin\beta)^2 = Y^\nu (M_R)^{-1} Y^{\nu T} \quad (v \sin\beta)^2$$

The transformation properties of the effective neutrino mass matrix under the transformations to canonical Kähler for both left handed and right handed fields is given by

$$\begin{aligned}\chi_\nu^t &= Y^{\nu\,t}(M_R^t)^{-1}(Y^{\nu\,t})^T = P_L^T\,\hat{Y}^\nu\,P_R(P_R)^{-1}\,M_R^{-1}\,(P_R^T)^{-1}P_R^T\,\hat{Y}^{\nu T}\,P_L \\ &= P_L^T\,\hat{Y}^\nu\,\hat{M}_R^{-1}\,\hat{Y}^{\nu T}\,P_L \;=\; P_L^T\,\hat{\chi}_\nu\,P_L \qquad (4.39)\end{aligned}$$

Hence, we see that the effective neutrino coupling χ_ν is transformed only by the left handed canonical transformations and the right-handed transformations cancel exactly.

However the neutrino sector can be special because in this case, we do not know much about the hierarchy of the leptonic Yukawa couplings Y^ν and Y^e. In fact we can find two different situations:

1. Y^ν and Y^e are hierarchical and $Y_{kj} < Y_{ij}$ for $k < i$ and $j \geq i$. This is this situation in realistic non-Abelian flavour theories explored to date [74].

2. Y^ν or Y^e have two rows of similar size. We can find this situation in some $U(1)$ models [86].

In case 1 the Kähler metric is also very close to the identity with small off-diagonal entries. Therefore we can choose P_L to be upper triangular with $(P_L)_{ii} \simeq 1$ and $(P_L)_{ij} \leq \epsilon$. Then both Y_ν and Y_e are only changed at higher order in ϵ and neutrino masses and mixings are only changed at sub-dominant order. In the case of non-Abelian symmetries χ_ν^t and Y^e are changed at most at order ε^2. Then we can immediately use the standard formulae for the neutrino mixings compiled in Ref. [87]. For all the different cases compatible with hierarchical rows in the lepton Yukawa matrix, we can immediately see that neutrino mixings will only be changed at sub-leading order. Although small, this might still be relevant for the difference of the solar mixing angle from maximality.

Case 2 arises if two left handed fields have identical flavour symmetry charges. As a result the Kähler metric will have large mixing between these two fields and therefore $\mathcal{O}(1)$ off-diagonal entries. In this case, it is possible to modify the $\mathcal{O}(1)$ coefficients in the different elements of the canonical Yukawa matrices, but the order in ϵ of these entries is not changed. Therefore, in this case, it is possible to generate changes at leading order in neutrino masses and mixings. This corresponds again to the case where right-handed mixing angles can modify left-handed mixings in the quark sector. Since only the $\mathcal{O}(1)$ coefficients are modified these corrections do not change the predicted structure if the family symmetry does not predict the value of these coefficients.

4.6.3 Soft SUSY breaking masses and mixing angles

Finally, we would also like to comment on the effects of the Kähler transformations on the soft breaking masses which may give rise to dangerous flavour changing neutral current processes [82]. Notice that the F-term contributions to soft breaking masses in supergravity are closely related to the Kähler potential [72]. In fact the non canonical soft breaking masses are,

$$\hat{m}^2_{\overline{a}b} = m^2_{3/2} K_{\overline{a}b} - F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K_{\overline{a}b} - \partial_{\overline{m}} K_{\overline{a}c} (K^{-1})_{c\overline{d}} \partial_n K_{\overline{d}b} \right) F_n$$

To obtain the canonical soft breaking masses we have to multiply this matrix by the inverse of the square root of K, $m^2 = (K^{-1/2})^\dagger \hat{m}^2 K^{-1/2}$. Then we obtain,

$$\begin{aligned} m^2 &= m^2_{3/2} \mathbf{1} - (K^{-1/2})^\dagger F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K - \partial_{\overline{m}} K (K^{-1}) \partial_n K \right) F_n K^{-1/2} \\ &\equiv m^2_{3/2} \mathbf{1} - P^\dagger F_{\overline{m}} \left(\partial_{\overline{m}} \partial_n K - \partial_{\overline{m}} K (K^{-1}) \partial_n K \right) F_n P \end{aligned}$$

Therefore we see that we have a universal contribution proportional to $m^2_{3/2}$ plus other terms which in principle will depend on flavour. These terms depend on the derivatives of the Kähler potential with respect to fields with non vanishing F-terms.

If the field with non-vanishing F-term is a hidden sector field it must be neutral under the flavour symmetry and therefore the structure in powers of ϵ of $\partial_{\overline{m}}\partial_n K$ or $\partial_{\overline{m}}K$ will be the same as the structure of K. However, factors $\mathcal{O}(1)$ can be different and indeed can sometimes be zero. The important point is that no terms larger in powers of ϵ are generated than are in K itself. Due to this difference in the $\mathcal{O}(1)$ coefficients the product $(K^{-1/2})^\dagger \partial_{\overline{m}} K K^{-1/2}$ will be different from the identity, but will be bounded by the same power in ϵ as the original K matrix [71].

Another possibility is that the field with non-vanishing F-term is a flavon field with non-trivial quantum numbers under the flavour symmetry. As shown in [88], the natural size for F_θ for θ a flavon field is $m_{3/2}\langle\theta\rangle$, although it can be smaller depending on the characteristics of the scalar potential. In this case, we also have that $F_{\overline{m}}\partial_{\overline{m}}K$ cannot generate terms larger in powers of ϵ than the terms initially present in K itself and the conclusion above still applies.

We have also to consider the possibility of a non-vanishing flavour D-term contributing to the soft masses. Although this possibility is extremely dangerous for the phenomenology of flavour changing neutral currents (FCNCs) it can be realised for heavy sfermion masses in some Abelian flavour models. In this case we obtain a new contribution to the soft masses,

$$(\hat{m}^2_{\overline{a}b})^D \;=\; g\; q_b\; K_{\overline{a}b}\; \langle D\rangle$$

with q_b the charge of the field ϕ_b under the $U(1)_{fl}$ symmetry. Notice that due to the dependence on the charges of the different fields this contribution to the soft masses is not diagonalised when we make the transformation to the basis of canonical Kähler and therefore it gives rise to new FCNC effects.

To analyse these FCNC effects it is convenient to work in the SCKM basis where the corresponding Yukawa matrix is diagonal. Therefore, to obtain the sfermion mass matrix in the SCKM basis we have to do two transformations. First we go to the basis of canonical Kähler with our triangular matrices and second we diagonalise the corresponding Yukawa matrix with a rotation of the full superfield. Now, we can compare the effects of the transformations to the basis of canonical Kähler with the effects of the second transformation to the SCKM basis. First it is easy to see that in $U(1)$ models the structure in ϵ of our triangular Kähler transformations are always smaller or equal that the corresponding rotation diagonalising the Yukawa matrix. For instance, the left handed Kähler transformation is usually of the same order as the left handed rotation diagonalising the up quark Yukawa matrix and smaller than the left handed rotation diagonalising the down quark Yukawa. If the diagonal elements of the Kähler metric are $\mathcal{O}(1)$, this means that the corrections to off-diagonal elements that we obtain from the transformations to the SCKM basis are larger or equal than the corrections obtained in the transformation to the canonical basis. As before, if we are not interested in coefficients $\mathcal{O}(1)$, we can also ignore the effects of transformation to canonical Kähler in the soft breaking masses.

4.7 Conclusions

In this chapter we have studied the effects of the transformations to the canonical Kähler basis on the Yukawa textures for quarks and leptons and their contributions to physical masses and mixing angles. We demonstrated using a simple example model that the texture in the Yukawa matrices seems to change after canonical normalisation using the 'standard' method for canonically normalising the matter fields. We have developed a simple formalism that allows a straightforward calculation of the necessary Kähler transformations and simplifies enormously the phenomenological analysis. Using this formalism we have proved that, in the case of models with a hierarchical structure of the Yukawa matrices, the corrections obtained through the transformations to canonical Kähler are always suppressed by a factor $\leq \epsilon^2$ with ϵ the expansion parameter in the Yukawa matrix. This implies that, in this case, fermionic masses and mixing angles receive only corrections at ϵ^2 from the Kähler transformations. We have seen that although texture zeros can be filled by transformations to canonical Kähler the physical effects of this texture zero filling are only subdominant corrections in ϵ to observable masses and mixing angles. We have also discussed some exceptions to the case of completely hierarchical Yukawa matrices where some corrections at leading order are possible. In any case, we have seen that in these models only unknown $\mathcal{O}(1)$ coefficients are modified. We have also shown that the corrections to the scalar soft breaking mass matrices can only change the unknown $\mathcal{O}(1)$ coefficients. We conclude that in the large class of models considered here the leading order superpotential couplings in the non-canonical Kähler basis are essentially unchanged when transformed to the canonical Kähler basis. Agreement on this issue was arrived at independently by Refs. [89, 90]

Chapter 5

Conclusions

In this thesis we have studied the connections between family symmetries and low-energy observable physics in the context of supersymmetric unified theories. The emphasis has been on trying to find low energy constraints since the energy scale where the new physics arises is far too high for direct observation. Such effects may be considered the low energy footprints of models which otherwise could not be distinguished, as they would all have the same low energy limit.

In Chapter 3, we numerically investigated a contribution to flavour violation that had previously been neglected, coming from the auxiliary fields to the Higgs fields that break a gauge family symmetry. Since the investigation was numerical, it had to be done in the context of a specific model; the model chosen was a string-inspired Pati-Salam model with a $U(1)$ family symmetry. The most constraining flavour violation turned out to be $\mathrm{BR}(\mu \to e\gamma)$. In the model chosen, we picked four benchmark points which demonstrate different dominant contributions to flavour violation. These were running effects from having a see-saw mechanism, non-universal scalar masses from the model itself, auxiliary fields associated with breaking the family symmetry and auxiliary fields associated with the breaking of the Pati-Salam group down to the MSSM group. We found that when the flavour violation is dominated by the auxiliary fields, the decay rates are close to being ruled out by the current experimental limits, but when other forms dominate, the rate can be well below the experimental limit.

In Chapter 4, we considered the effects of non-renormalisable flavon operators in the Kähler potential. These were considered to be consistent with non-renormalisable flavon operators in the superpotential. When the flavons are replaced by their VEVs, the effective Kähler potential becomes non-canonical. Canonically normalising is a simultaneous field rescaling and mixing effect, which will change the superpotential couplings. We then use the freedom to apply a further unitary transformation on the fields post-canonical normalisation. Using this freedom, we show that the effects of canonical normalisation will not change the leading order predictions of masses, CKM and PMNS matrix elements, provided that the original Yukawa matrices have a hierarchical form. Since such a hierarchical form is common in family symmetry models, we concluded that when building such models the effects of canonical normalisation can usually be neglected.

The ultimate aim of family symmetry models is to explain the structure of the Yukawa matrices in the MSSM in a natural way. This would involve a number of non-renormalisable operators with numerical coefficients which are all order 1, preferably with less operators than constraints. However, there is no way of directly telling the difference between such a model, and the MSSM with the same Yukawa matrices. Therefore, we would like to find indirect ways of distinguishing the two. The work in this thesis has looked at a couple of ways that the GUT-scale family symmetry model might leave traces in the low energy physics accessible at current and near-future experiments. However, there is still a large amount of work to do in this field.

Appendix A

Soft terms from supergravity

We summarise here the standard way of getting soft SUSY breaking terms from supergravity. Supergravity is defined in terms of a Kähler function, G, of chiral superfields ($\phi = h, C_a$). Taking the view that the supergravity is the low energy effective field theory limit of a string theory, the hidden sector fields h are taken to correspond to closed string moduli states ($h = S, T_i$), and the matter states C_a are taken to correspond to open string states. In string theory, the ends of the open string states are believed to be constrained to lie on extended solitonic objects called $Dp-$branes.

Using natrual units:

$$G(\phi, \overline{\phi}) = \frac{K(\phi, \overline{\phi})}{\tilde{M}_{Pl}^2} + \ln\left(\frac{W(\phi)}{\tilde{M}_{Pl}^3}\right) + \ln\left(\frac{W^*(\overline{\phi})}{\tilde{M}_{Pl}^3}\right) \tag{A.1}$$

$K(\phi, \overline{\phi})$ is the Kähler potential, a real function of chiral superfields. This may be expanded in powers of C_a:

$$K = \overline{K}(h, \overline{h}) + \tilde{K}_{\overline{a}b}(h, \overline{h})\overline{C}_{\overline{a}}C_b + \left[\frac{1}{2}Z_{ab}(h, \overline{h})C_aC_b + h.c.\right] + ... \tag{A.2}$$

$\tilde{K}_{\overline{a}b}$ is the Kähler metric. $W(\phi)$ is the superpotential, a holomorphic function of chiral superfields:

$$W = \hat{W}(h) + \frac{1}{2}\mu_{ab}(h)C_aC_b + \frac{1}{6}Y_{abc}C_aC_bC_c + ... \tag{A.3}$$

We expect the supersymmetry to be broken; if it is broken, then the auxiliary fields $F_\phi \neq 0$ for some ϕ. Lacking a model of SUSY breaking we can proceed no further without parameterising our ignorance. We do this using goldstino angles. We introduce a matrix, P that canonically normalises the Kähler metric, $P^\dagger K_{\overline{J}I} P = 1$ [1] [91]. We also introduce a column vector Θ which satisfies $\Theta^\dagger\Theta = 1$. We are completely free to parameterise Θ in any way which satisfies this constraint.

Then the un-normalised soft terms and trilinears appear in the soft SUGRA breaking potential [43]:

$$V_{\text{soft}} = m^2_{\overline{a}b}\overline{C}_{\overline{a}}C_b + \left(\frac{1}{6}A_{abc}Y_{abc}C_aC_bC_c + h.c.\right) + ... \tag{A.4}$$

The non-canonically normalised soft trilinears are then:

$$\begin{aligned} A_{abc}Y_{abc} &= \frac{\hat{W}^*}{|\hat{W}|}e^{\overline{K}/2}F_m\left[\overline{K}_m Y_{abc} + \partial_m Y_{abc} - \left(\left(\tilde{K}^{-1}\right)\partial_m\tilde{K}_{\overline{e}a}Y_{dbc}\right.\right. \\ &\qquad \left.\left. + (a\leftrightarrow b) + (a\leftrightarrow c)\right)\right] \end{aligned} \tag{A.5}$$

In this equation, it should be noted that the index m runs over h, C. However, by definition, the hidden sector part of the Kähler potential and the Kähler metrics are independent of the matter fields.

Assuming that the terms $\partial_C Y_{abc} \neq 0$, the canonically normalised equation for the trilinear is:

$$A_{abc} = F_I\left[\overline{K}_I - \partial_I \ln\left(\tilde{K}_a\tilde{K}_b\tilde{K}_c\right)\right] + F_m\partial_m \ln Y_{abc} \tag{A.6}$$

If the Yukawa hierarchy is taken to be generated by a Froggatt-Nielsen field, ϕ such that $Y \propto \phi^p$, then we expect $F_\phi \propto m_{3/2}\phi$, and then $F_\phi\partial_\phi \ln Y \propto m_{3/2}$ and so even though these fields are expected to have heavily sub-dominant F-terms, they contribute to the trilinears on an equal footing as the moduli.

[1] The subscripts on the Kähler potential K_I means $\partial_I K$. However, the subscripts on the F-terms are just labels.

If the Kähler metric is diagonal and non-canonical, then the canonically normalised scalar mass-squareds are given by

$$m_a^2 = m_{3/2}^2 - F_{\overline{J}} F_I \partial_{\overline{J}} \partial_I \left(\ln \tilde{K}_a \right) \tag{A.7}$$

And the gaugino masses are given by

$$M_\alpha = \frac{1}{2\mathrm{Re} f_\alpha} F_I \partial_I f_\alpha \tag{A.8}$$

Where f_α is the 'gauge kinetic function'. α enumerates D-branes in the model. In type I string models without twisted moduli these have the form $f_9 = S$; $f_{5^i} = T^i$.

For the models considered in this thesis, we use a Kähler potential that doesn't have any twisted-moduli [26]:

$$\begin{aligned}
K \;=\; & -\ln\left(S+\overline{S}-\left|C_1^{5_1}\right|^2-\left|C_2^{5_2}\right|^2\right)-\ln\left(T_1+\overline{T}_1-\left|C_1^9\right|^2-\left|C_3^{5_3}\right|^2\right) \\
& -\ln\left(T_2-\overline{T}_2-\left|C_2^9\right|^2-\left|C_3^{5_1}\right|^2\right)-\ln\left(T_3-\overline{T}_3-\left|C_3^9\right|^2-\left|C_2^{5_1}\right|^2-\left|C_1^{5_1}\right|^2\right) \\
& +\frac{\left|C^{5_1 5_2}\right|^2}{\left(S+\overline{S}\right)^{1/2}\left(T_3+\overline{T}_3\right)^{1/2}}+\frac{\left|C^{95_1}\right|^2}{\left(T_2+\overline{T}_2\right)^{1/2}\left(T_3+\overline{T}_3\right)^{1/2}} \\
& +\frac{\left|C^{95_2}\right|^2}{\left(T_1+\overline{T}_1\right)^{1/2}\left(T_3+\overline{T}_3\right)^{1/2}}
\end{aligned} \tag{A.9}$$

The notation is that the field theory scalars, the dilaton S and the untwisted moduli T_i originate from closed strings. Open string states C_i^b are required to have their ends localised onto $D-$branes. The upper index then specifies which brane(s) their ends are located on, and if both ends are on the same brane, the lower index specifies which pair of compacitified extra dimensions the string is free to vibrate in.

Appendix B

Parameterised trilinears for the 42241 Model

We here write the general form of the trilinear parameters A_{ijk} assuming nothing about the form of the Yukawa matrices.

$$
\begin{aligned}
A_{C_1^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = \; & \sqrt{3} m_{3/2} \left\{ X_S \left[1 + \left(S + \overline{S}\right) \partial_S \ln Y_{abc}\right] \right. \\
& + X_{T_1} \left[-1 + \left(T_1 + \overline{T}_1\right) \partial_{T_1} \ln Y_{abc}\right] \\
& + X_{T_2} \left[-1 + \left(T_2 + \overline{T}_2\right) \partial_{T_2} \ln Y_{abc}\right] \\
& + X_{T_3} \left(T_3 + \overline{T}_3\right) \partial_{T_3} \ln Y_{abc} \\
& + X_H \left(S + \overline{S}\right)^{\frac{1}{2}} H \partial_H \ln Y_{abc} \\
& + X_{\overline{H}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{2}} \overline{H} \partial_{\overline{H}} \ln Y_{abc} \\
& \left. + X_\theta \left(S + \overline{S}\right)^{\frac{1}{4}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{4}} \theta \partial_\theta \ln Y_{abc} \right\} \qquad \text{(B.1)}
\end{aligned}
$$

$$
\begin{aligned}
A_{C_1^{5_1} C_3^{5_1} C^{5_1 5_2}} &= \sqrt{3} m_{3/2} \Big\{ X_S \left[\frac{1}{2} + \left(S + \overline{S}\right) \partial_S \ln Y_{abc} \right] \\
&+ X_{T_1} \left[-1 + \left(T_1 + \overline{T}_1\right) \partial_{T_1} \ln Y_{abc} \right] \\
&+ X_{T_2} \left(T_2 + \overline{T}_2\right) \partial_{T_2} \ln Y_{abc} \\
&+ X_{T_3} \left[-\frac{1}{2} \left(T_3 + \overline{T}_3\right) \partial_{T_3} \ln Y_{abc} \right] \\
&+ X_H \left(S + \overline{S}\right)^{\frac{1}{2}} H \partial_H \ln Y_{abc} \\
&+ X_{\overline{H}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{2}} \overline{H} \partial_{\overline{H}} \ln Y_{abc} \\
&+ X_\theta \left(S + \overline{S}\right)^{\frac{1}{4}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{4}} \theta \partial_\theta \ln Y_{abc} \Big\}
\end{aligned}
\tag{B.2}
$$

$$
\begin{aligned}
A_{C_1^{5_1} C_2^{5_1} C^{5_1 5_2}} &= \sqrt{3} m_{3/2} \Big\{ X_S \left[\frac{1}{2} + \left(S + \overline{S}\right) \partial_S \ln Y_{abc} \right] \\
&+ X_{T_1} \left[-1 + \left(T_1 + \overline{T}_1\right) \partial_{T_1} \ln Y_{abc} \right] \\
&+ X_{T_2} \left[-1 + \left(T_2 + \overline{T}_2\right) \partial_{T_2} \ln Y_{abc} \right] \\
&+ X_{T_3} \left[\frac{1}{2} \left(T_3 + \overline{T}_3\right) \partial_{T_3} \ln Y_{abc} \right] \\
&+ X_H \left(S + \overline{S}\right)^{\frac{1}{2}} H \partial_H \ln Y_{abc} \\
&+ X_{\overline{H}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{2}} \overline{H} \partial_{\overline{H}} \ln Y_{abc} \\
&+ X_\theta \left(S + \overline{S}\right)^{\frac{1}{4}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{4}} \theta \partial_\theta \ln Y_{abc} \Big\}
\end{aligned}
\tag{B.3}
$$

$$
\begin{aligned}
A_{C_1^{5_1} C_2^{5_1} C_3^{5_1}} \quad = \quad & \sqrt{3} m_{3/2} \left\{ X_S \left(S + \overline{S}\right) \partial_S \ln Y_{abc} \right. \\
& + X_{T_1} \left[-1 + \left(T_1 + \overline{T}_1\right) \partial_{T_1} \ln Y_{abc}\right] \\
& + X_{T_2} \left(T_2 + \overline{T}_2\right) \partial_{T_2} \ln Y_{abc} \\
& + X_{T_3} \left(T_3 + \overline{T}_3\right) \partial_{T_3} \ln Y_{abc} \\
& + X_H \left(S + \overline{S}\right)^{\frac{1}{2}} H \partial_H \ln Y_{abc} \\
& + X_{\overline{H}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{2}} \overline{H} \partial_{\overline{H}} \ln Y_{abc} \\
& \left. + X_\theta \left(S + \overline{S}\right)^{\frac{1}{4}} \left(T_3 + \overline{T}_3\right)^{\frac{1}{4}} \theta \partial_\theta \ln Y_{abc} \right\}
\end{aligned}
\qquad \text{(B.4)}
$$

Appendix C

$n = 1$ operators

The $n = 1$ Dirac operators are the complete set of all operators that can be constructed from the quintilinear $F\overline{F}h\overline{H}H$ by all possible group theoretical contractions of the indices in

$$\mathcal{O}^{\alpha\rho y w}_{\beta\gamma x z} = F^{\alpha a}\overline{F}_{\beta x} h^{y}_{a} \overline{H}_{\gamma z} H^{\rho w} \tag{C.1}$$

We define some SU(4) invariant tensors C and some SU(2) invariant tensors R as follows [1]:

$$\begin{aligned}
(C_1)^{\alpha}_{\beta} &= \delta^{\alpha}_{\beta} \\
(C_6)^{\rho\gamma}_{\alpha\beta} &= \epsilon^{\rho\gamma\omega\chi}_{\alpha\beta\omega\chi} \\
(C_{10})^{\alpha\beta}_{\rho\gamma} &= \delta^{\alpha}_{\rho}\delta^{\beta}_{\gamma} + \delta^{\alpha}_{\gamma}\delta^{\beta}_{\rho} \\
(C_{15})^{\beta\gamma}_{\alpha\rho} &= \delta^{\beta}_{\rho}\delta^{\gamma}_{\alpha} - \frac{1}{4}\delta^{\beta}_{\alpha}\delta^{\gamma}_{\rho} \\
(R_1)^{x}_{y} &= \delta^{x}_{y} \\
(R_3)^{wx}_{yz} &= \delta^{x}_{y}\delta^{w}_{z} - \frac{1}{2}\delta^{x}_{z}\delta^{w}_{y}
\end{aligned} \tag{C.2}$$

[1] The subscript denotes the dimension of the representation they can create from multiplying $\mathbf{4}$ or $\overline{\mathbf{4}}$ with $\mathbf{4}$ or $\overline{\mathbf{4}}$. For example $(C_{15})^{\beta\gamma}_{\alpha\rho}\overline{\mathbf{4}}_{\gamma}\mathbf{4}^{\rho} = \mathbf{15}^{\beta}_{\alpha}$

Operator Name	Operator Name in [46]	$Q\bar{U}h_2$	$Q\bar{D}h_1$	$L\bar{E}h_1$	$L\bar{N}h_2$
O^{Aa}	O^A	1	1	1	1
O^{Ab}	O^B	1	-1	-1	1
O^{Ac}	O^M	0	$\sqrt{2}$	$\sqrt{2}$	0
O^{Ad}	O^T	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$
O^{Ae}	O^V	$\sqrt{2}$	0	0	$\sqrt{2}$
O^{Af}	O^U	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$
O^{Ba}	O^C	$\frac{1}{\sqrt{5}}$	$\frac{1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$
O^{Bb}	O^D	$\frac{1}{\sqrt{5}}$	$\frac{-1}{\sqrt{5}}$	$\frac{-3}{\sqrt{5}}$	$\frac{3}{\sqrt{5}}$
O^{Bc}	O^W	0	$\sqrt{\frac{2}{5}}$	$-3\sqrt{\frac{2}{5}}$	0
O^{Bd}	O^X	$\frac{2\sqrt{2}}{5}$	$\frac{\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$
O^{Be}	O^Z	$\sqrt{\frac{2}{5}}$	0	0	$-3\sqrt{\frac{2}{5}}$
O^{Bf}	O^Y	$\frac{\sqrt{2}}{5}$	$\frac{2\sqrt{2}}{5}$	$\frac{-6\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$
O^{Ca}	O^a	$\sqrt{2}$	$\sqrt{2}$	0	0
O^{Cb}	O^F	$\sqrt{2}$	$-\sqrt{2}$	0	0
O^{Cc}	O^E	0	2	0	0
O^{Cd}	O^b	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$	0	0
O^{Ce}	O^N	2	0	0	0
O^{Cf}	O^c	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0	0
O^{Da}	O^d	$\sqrt{\frac{2}{5}}$	$\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^{Db}	O^e	$\sqrt{\frac{2}{5}}$	$-\sqrt{\frac{2}{5}}$	$-2\sqrt{\frac{2}{5}}$	$2\sqrt{\frac{2}{5}}$
O^{Dc}	O^G	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$	0
O^{Dd}	O^H	$\frac{4}{5}$	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$
O^{De}	O^O	$\frac{2}{\sqrt{5}}$	0	0	$\frac{4}{\sqrt{5}}$
O^{Df}	O^f	$\frac{2}{5}$	$\frac{4}{5}$	$\frac{8}{5}$	$\frac{4}{5}$
O^{Ea}	O^g	0	0	$\sqrt{2}$	$\sqrt{2}$
O^{Eb}	O^h	0	0	$-\sqrt{2}$	$\sqrt{2}$
O^{Ec}	O^i	0	0	2	0
O^{Ed}	O^j	0	0	$\frac{2}{\sqrt{5}}$	$\frac{4}{\sqrt{5}}$
O^{Ee}	O^I	0	0	0	2
O^{Ef}	O^J	0	0	$\frac{4}{\sqrt{5}}$	$\frac{2}{\sqrt{5}}$
O^{Fa}	O^P	$\frac{4\sqrt{2}}{5}$	$\frac{4\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^{Fb}	O^Q	$\frac{4\sqrt{2}}{5}$	$\frac{-4\sqrt{2}}{5}$	$\frac{-3\sqrt{2}}{5}$	$\frac{3\sqrt{2}}{5}$
O^{Fc}	O^R	0	$\frac{8}{5}$	$\frac{6}{5}$	0
O^{Fd}	O^L	$\frac{16}{5\sqrt{5}}$	$\frac{8}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$
O^{Fe}	O^K	$\frac{8}{5}$	0	0	$\frac{6}{5}$
O^{Ff}	O^S	$\frac{8}{5\sqrt{5}}$	$\frac{16}{5\sqrt{5}}$	$\frac{12}{5\sqrt{5}}$	$\frac{6}{5\sqrt{5}}$

Table C.1: Operator names, CGCs and names in [46]

Then the six independent SU(4) structures are:

$$\begin{array}{llcl}
\text{A.} & (C_1)^{\beta}_{\alpha}\,(C_1)^{\gamma}_{\rho} & = & \delta^{\beta}_{\alpha}\delta^{\gamma}_{\rho} \\
\text{B.} & (C_{15})^{\beta\chi}_{\alpha\sigma}\,(C_{15})^{\gamma\sigma}_{\rho\chi} & = & \delta^{\beta}_{\rho}\delta^{\gamma}_{\alpha} - \frac{1}{4}\delta^{\beta}_{\alpha}\delta^{\gamma}_{\rho} \\
\text{C.} & (C_6)^{\omega\chi}_{\alpha\rho}\,(C_6)^{\beta\gamma}_{\omega\chi} & = & 8(\delta^{\beta}_{\alpha}\delta^{\gamma}_{\alpha} - \delta^{\gamma}_{\alpha}\delta^{\beta}_{\rho}) \\
\text{D.} & (C_{10})^{\omega\chi}_{\alpha\rho}\,(C_{10})^{\beta\gamma}_{\omega\chi} & = & 2(\delta^{\beta}_{\alpha}\delta^{\gamma}_{\rho} + \delta^{\gamma}_{\alpha}\delta^{\beta}_{\rho}) \\
\text{E.} & (C_1)^{\beta}_{\rho}\,(C_1)^{\gamma}_{\alpha} & = & \delta^{\beta}_{\alpha}\delta^{\gamma}_{\alpha} \\
\text{F.} & (C_{15})^{\gamma\chi}_{\alpha\sigma}\,(C_{15})^{\beta\sigma}_{\rho\chi} & = & \delta^{\gamma}_{\rho}\delta^{\alpha}_{\beta} - \frac{1}{4}\delta^{\gamma}_{\alpha}\delta^{\beta}_{\rho}
\end{array} \tag{C.3}$$

And the six SU(2) structures are:

$$\begin{array}{llcl}
\text{a.} & (R_1)^{z}_{w}\,(R_1)^{x}_{y} & = & \delta^{z}_{w}\delta^{x}_{y} \\
\text{b.} & (R_3)^{zq}_{wr}\,(R_3)^{xr}_{yq} & = & \delta^{x}_{w}\delta^{z}_{y} - \frac{1}{2}\delta^{x}_{y}\delta^{z}_{w} \\
\text{c.} & \epsilon^{xz}\epsilon_{yw} & = & \epsilon^{xz}\epsilon_{yw} \\
\text{d.} & \epsilon_{ws}\epsilon^{xt}\,(R_3)^{sq}_{yr}\,(R_3)^{zr}_{tq} & = & \delta^{x}_{w}\delta^{z}_{y} - \frac{1}{2}\epsilon_{wy}\epsilon^{xz} \\
\text{e.} & (R_1)^{z}_{y}\,(R_1)^{x}_{w} & = & \delta^{z}_{y}\delta^{x}_{w} \\
\text{f.} & (R_3)^{zq}_{yr}\,(R_3)^{xr}_{wq} & = & \delta^{x}_{y}\delta^{z}_{w} - \frac{1}{2}\delta^{x}_{w}\delta^{z}_{y}
\end{array} \tag{C.4}$$

All possible $n = 1$ operators were then named $O^A ... O^Z O^a ... O^j$ in [46]. We rename them here in a manner consistent with the $n > 1$ operators $O^{(n')}$, so that the names are $O^{\Pi\pi}$ where Π is the SU(4) structure and π is the SU2 structure. See Table C.1 for the translation into the names of ref.[46] and the CGCs.

All of these operators are operators for the case without a U(1) family symmetry. In the case when there is, we follow the prescription:

$$\mathcal{O}_{IJ} \rightarrow \mathcal{O}_{IJ}\left(\frac{\theta}{M_X}\right)^{p_{IJ}} \tag{C.5}$$

Where $p_{IJ} = |X_{\mathcal{O}_{IJ}}|$ is the modulus of the charge of the operator. If the charge of the operator is negative, then the field θ should be replaced by the field $\overline{\theta}$. The prescription makes the operator chargeless under the $U(1)_F$ while simultaneously not changing the dimension.

Appendix D

$n > 1$ operators

In the case that $n > 1$, there will be more indices to contract, which allows more representations, and hence more Clebsch coefficients. To generalise the notation, it is necessary only to construct the new tensors which create the new structures. However, it will always be possible to contract the new indices between the H and $\overline{H}$ fields to create a singlet $H\overline{H}$ which has a Clebsch of 1 in each sector u, d, e, ν. In this case, the first structures are the same as the old structures, but with extra δ symbols which construct the $H\overline{H}$ singlet.

Thus taking a $n = 2$ operator, say $\mathcal{O}'^{Fb}$, which forms a representation that could have been attained by a $n = 1$ operator, the Clebsch coefficients are the same. This is what we mean by $\mathcal{O}^{n' \, \Pi\pi}$, as we have only used $n > 1$ coefficients which are in the subset that have $n = 1$ analogues.

Appendix E

Expressions for the canonically normalised Yukawa elements, and P_{LH}^{-1}

We write here the full expressions for the four Yukawa elements.

$$Y_{22} = a_2 \frac{1}{\sqrt{b_0 c_0 d_4}} \epsilon^3 + a_1 \frac{b_3 c_3}{\sqrt{b_0 c_0} b_1 c_1} \epsilon^3 + \mathcal{O}(\epsilon^4) \quad \text{(E.1)}$$

$$Y_{23} = -a_1 \frac{b_3}{\sqrt{b_0 c_0} b_1} \epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}} \epsilon^3 + a_1 \frac{b_3(b_2 c_0 + b_0(c_1 + c_2))}{2 b_0^{3/2} c_0^{3/2} b_1} \epsilon^3 + \mathcal{O}(\epsilon^4) \quad \text{(E.2)}$$

$$Y_{32} = -a_1 \frac{c_3}{\sqrt{b_0 c_0} c_1} \epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}} \epsilon^3 + a_1 \frac{c_3(c_2 b_0 + c_0(b_1 + b_2))}{2 b_0^{3/2} c_0^{3/2} c_1 \sqrt{d_4}} \epsilon^3 + \mathcal{O}(\epsilon^4) \quad \text{(E.3)}$$

$$Y_{33} = a_1 \frac{1}{\sqrt{b_0 c_0}} \epsilon + -a_1 \frac{c_0(b_1 + b_2) + b_0(c_1 + c_2)}{2 b_0^{3/2} c_0^{3/2}} \epsilon^2 + a_2 \frac{1}{\sqrt{b_0 c_0 d_4}} \epsilon^3$$
$$+a_1 \frac{1}{8 b_0^{5/2} c_0^{5/2}} \Big(\frac{c_0^2 (3 b_1^4 + 6 b_1^3 b_2 - 4 b_0^2 b_3^2 + b_1^2 (3 b_2^2 - 4 b_0 (b_3 + b_4 + b_6)))}{b_1^2} \quad \text{(E.4)}$$
$$+ \frac{b_0^2 (3 c_1^4 + 6 c_1^3 c_2 - 4 c_0^2 c_3^2 + c_1^2 (3 c_2^2 - 4 c_0 (c_3 + c_4 + c_6)))}{c_1^2}$$
$$+2 b_0 c_0 (b_1 + b_2)(c_1 + c_2) \Big) \epsilon^3 + \mathcal{O}(\epsilon^4) \quad \text{(E.5)}$$

These follow from the expressions for the inverse P-matrix after it has been Taylor expanded in ϵ to order ϵ^3 around the point $\epsilon = 0$. The full expression for the left-handed P-matrix is then, to sub-leading order in ϵ:

$$P_{LH}^{-1} = \begin{pmatrix} \frac{1}{\sqrt{b_0}} - \frac{b_2\epsilon}{2b_0^{3/2}} & \frac{b_3\epsilon}{\sqrt{b_0}b_1} - \frac{(b_1+b_2)b_3\epsilon^2}{2b_0^{3/2}b_1} \\ -\frac{b_3\epsilon}{\sqrt{b_0}b_1} + \frac{b_2b_3\epsilon^2}{2b_0^{3/2}b_1} & \frac{1}{\sqrt{b_0}} - \frac{(b_1+b_2)\epsilon}{2b_0^{3/2}} \end{pmatrix} \tag{E.6}$$

The structure of the right-handed equivalent is exactly the same, but with every b_i replaced with a c_i.

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