UNIVERSITY OF SOUTHAMPTON

FACULTY OF LAW, ARTS AND SOCIAL SCIENCES

Economics Division

School of Social Sciences

OPTIMAL TAXATION IN DYNAMIC GENERAL EQUILIBRIUM

by

Sheikh Tareq Selim

Thesis for the degree of
Doctor of Philosophy

September 2005

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF LAW, ARTS AND SOCIAL SCIENCES

Economics Division, School of Social Sciences

Doctor of Philosophy

OPTIMAL TAXATION IN DYNAMIC GENERAL EQUILIBRIUM

by Sheikh Tareq Selim

The thesis addresses the issue of optimal choice of income tax rates for a benevolent government (the *Ramsey* problem) in variants of multi-sector dynamic general equilibrium models. Using the primal approach to taxation, the thesis recasts the optimal taxation problem as one in which the choice variables for the government are allocations rather than tax rates. Permissible allocations are those that satisfy resource constraints and implementability constraints, where the latter are budget constraints in which the consumer and firm first order conditions are used to substitute out for prices and tax rates.

There are five chapters in this thesis. Chapter one sets the tone of the thesis by introducing the Ramsey problem and the current state of the art in dynamic Ramsey taxation. This chapter is a detailed review of existing literature, ongoing trends, and established models and results of dynamic Ramsey taxation in representative agent economies.

Chapter two solves the Ramsey problem in a simple infinite-horizon two-sector neoclassical production economy where the two sectors produce consumption goods and new capital goods, both tradable in competitive markets. The startling finding of this chapter is that the celebrated result of long run zero capital income tax does not hold unconditionally for a wider class of neoclassical production economy models. The chapter prescribes that capital income can be taxed at a nonzero rate in the consumption goods sector as long as the other capital income tax is set at zero. It also shows that an ex ante restriction of identical income tax rates across sectors results in nonzero capital income tax.

Chapter three and four solve the Ramsey problem in environments characterized by monopoly power in private markets. Chapter three addresses the issue of optimal labor income taxation in an economy without capital, and establishes that the optimal policy involves a lower labor income tax which offsets the distortions created by monopoly pricing. An extension with monopolistic wage setting is also presented. Chapter four develops a model with private market distortions and a richer set of income taxes which include sector-specific labor income taxes, profit tax and capital income tax. The main finding of this chapter is that the optimal levels of sector-specific labor income tax rates are not equal (with lower tax in sector with monopoly pricing), and optimal steady state capital income tax is nonzero. The sign of the optimal capital income tax depends on the relative strength of two opposing effects, namely, the welfare effect of investment, and the monopoly distortion effect. Both these chapters present numerical results based on calibration of the models to fit stylized facts of the post war US economy.

Chapter five is devoted to examining the policy relevance of Ramsey tax rules. It defends the established Ramsey tax rules and the key results derived in this thesis against the commonly held criticisms, which are typically based on practicality, efficiency, administrative costs and fairness.

The thesis belongs to the stream of literature that addresses optimal fiscal policy issue in a class of dynamic general equilibrium models. It is therefore particularly intended to contribute to macroeconomic and public economic theory research.

Contents

Detailed Contents

List of Tables

List of Figures

List of Definitions

Preface

Much of this thesis is about optimal taxation in multi-sector dynamic general equilibrium frameworks. Optimal Taxation, ever since its orientation in the influential paper by Ramsey (1927), has continued to be a rapidly expanding research area. In its original form, optimal taxation problem is the government's quest for average levels and compositions of welfare maximizing second best taxes that finance a predetermined amount of revenue and are consistent with equilibrium behaviour of taxpayers. At its current state, optimal taxation has become an immense field with diverse applications. The current thesis does not pretend to survey the field, but attempts only to sample, complement, and enrich it.

The inevitability of distorting taxes has both a long history and a strong underlying intuition. Governments have resorted to all sorts of taxes in history, all the while summoning reasons that went from simple pragmatism to progressive paternalism. Traditionally, the public sector played the key role in most economic activities. In order to finance heavy public expenditures the pursuit of taxing transaction was relentless, which in turns made the predicament of taxes and their disincentive effects a major source of anxiety amongst policymakers and economists. With regard to defining uncertainty, Benjamin Franklin once quoted, rather metaphorically, that "....in this world nothing is certain but death and taxes." Franklin's view as a policymaker may seem cynical to many, but the follow up of this thought was even less compassionate for the advocates of taxation. For instance, in connection to Franklin's quote, Erwin Griswold stated, "....we have long had death and taxes as the two standards of inevitability. But there are those who believe that death is the preferable of the two. 'At least', as one man said, 'there's one advantage about death; it doesn't get worse every time Congress meets'."

Quotes of such kind actually apply to all forms of taxes --- be it poll tax which is nearly independent of any disincentive effects, or be it taxes on transactions, which are inconsistent with pareto optimality since they distort welfare through their effect on allocation decisions. The economic analysis of taxation generally rules out the plausibility of poll taxes on grounds of information asymmetry. Since evidence on income distributions very rarely supports the assumption of identically endowed economic agents, and since it is extremely costly for the government to collect accurate information on the ability to pay taxes, poll taxes or lump sum taxes are not practical. This implies that the government can raise its required revenue by using taxes that distort allocation decisions through their effect on the relative prices. Hence, if taxes are as certain as death, the available taxes are distorting as well. Optimal taxation theory finds the optimal levels and composition of such distorting taxes, in the sense that the government's choice of these taxes is based purely on welfare maximizing motive. Optimal taxation, in its basic form, is therefore the quest for the second best optimal taxes.

The government's problem of welfare maximization with distorting taxes is associated with second best optimality relative to the benevolent social planner's problem mainly due to an additional constraint, known popularly in the relevant literature as the *implementability* constraint. This constraint ensures that the resulting taxes and the associated allocations and prices are consistent with equilibrium behaviour of taxpayers. This is the central idea of Ramsey (1927), but his investigation was for a system of excise taxes in a static framework. The current thesis follows the same methodology, but it embarks on recursive representations of competitive (and imperfectly competitive) equilibria distorted by income taxes in multi-sector general equilibrium models, and examines a simple mechanism design problem, namely, one that seeks to find the optimal temporal pattern of these distorting income taxes.

The main agenda of the optimal taxation theory has enormous importance both in macroeconomic theory and fiscal policy. Chapter one of this thesis is a documentation that represents the theoretical strength, analytical depth and historical contributions of the optimal taxation theory in providing normative benchmarks for fiscal policy design. It presents an introduction to the classic optimal taxation problem and a review of its practice in literature. It inaugurates the Ramsey problem, and following the state of the art practice in macroeconomic and public economic theory, presents the primal approach to optimal taxation in simple static and dynamic general equilibrium environments. The first model is one of optimal commodity taxation which recovers the Ramsey rule of taxing necessities heavily than luxuries. The discussion follows with a section devoted to non-stochastic models of dynamic optimal income taxation. The section presents three core variants of dynamic models, namely, the neoclassical growth model, the endogenous growth model, and the overlapping generations model, to highlight the celebrated result of long run zero tax on capital income. A brief introduction to the *time consistency* and *credibility* issues associated with optimal capital income taxation, a review of the current tax systems and trends, and current state of the art of optimal taxation literature are also included. The main contribution of chapter one is that it presents an extended review of the existing literature on optimal taxation and documents the stylized established models and results, which facilitate the discussions to follow in the remainder of the thesis.

Chapter two examines dynamic optimal income taxation problem in a two-sector neoclassical general equilibrium model where the government is able to commit to a sequence of tax plans for future. The main finding of chapter two is that while it is optimal to set a zero long run capital tax for the capital goods sector, steady state optimal capital tax is in general nonzero in the consumption goods sector. The distortion created by the nonzero capital tax in consumption goods sector, given that the other capital tax is set at zero, is in no way compounding in nature. This is because along the transition to steady state economic agents can avoid the compounding tax liabilities simply by shifting depreciated capital. The chapter examines the optimal steady state capital tax in consumption goods sector with three popular classes of utility functions and finds that the set of conditions under which this tax is zero is in no way inferred by the model. It is also shown that if the government faces an *ex ante* constraint of setting equal factor income taxes across sectors, the optimal level of capital tax rate is nonzero.

The key finding of chapter two, therefore, is that the celebrated result of zero capital taxation in the long run cannot be generalized for neoclassical growth models. This result is path-breaking and striking, but it does not necessarily make chapter two and its arguments part of a somewhat inclined political campaign for taxing capital income. The result is more sensibly interpreted in connection to its point of departure, i.e. the zero capital tax result. Proponents of the zero capital tax policy argue that capital income should not be taxed in the long run because distortions of a nonzero capital tax compound over time that creates exploding distortions in intertemporal allocations. Chapter two of the current thesis establishes that this result is not as strong as it is generally treated, and the optimal policy may involve nonzero taxes on capital income as long as the distortions created by such a tax are smooth in nature. Put more elaborately, in an economy with demarcated features where capital owners can shift old capital to a sector where capital income is untaxed, the optimal policy may involve nonzero taxation of capital income in the sector where the tax distortions are uniform over time.

Chapter three and chapter four both contribute to the emerging literature concerning fiscal policy with imperfectly competitive private markets, or more precisely, optimal corrective taxation. The Pigovian function of distorting taxes, which is a borrowed theme from environmental taxation, acts as the key intuition behind the general set of results from these two chapters. Chapter three studies the Ramsey problem of optimal labor income taxation in a simple model economy which deviates from a first best representative agent economy in three important aspects, namely, flat rate second best tax, monopoly power in intermediate product market, and monopolistic wage setting. The first model in chapter three is one that assumes perfectly competitive labor market but monopoly distortion in pricing of intermediate goods. The extended model introduces monopolistic wage setting. There are three key findings, which hold for both models: *(a)* In order to correct for monopoly distortion the Ramsey tax prescription is to set the labor income tax rate lower than its competitive market analogue; *(b)* At the Ramsey equilibrium, the government's choice of income tax policy is independent of the government's fiscal treatment of distributed profits; and *(c)* For higher levels of monopoly distortions Ramsey policy is more desirable than the first best policy. The key analytical results are verified by a calibration which fits the model to the stylized facts of the US economy.

Evidence of declining trend in OECD economies' income tax rates and the emerging policy concern of enhancing competition in the US and the EU product markets mutually motivate chapter four, which examines optimal labor and capital income tax policy in a multi-sector general equilibrium model with monopoly distortion. Chapter four is the ideal extension of chapter three, though by all means these two chapters can be treated independently. One of the main findings of chapter four is that the welfare-maximizing income tax policy is distortion-neutralizing. A complete characterization of the set of tax instruments which accomplishes the distortion-neutralizing function is presented. More specifically, it is shown that with monopoly power in pricing of intermediate goods, the optimal policy may involve capital income taxes or subsidies depending on the relative strength of two opposing effects, namely, the monopoly distortion effect, and the relative effect of investment on tax distorted equilibrium welfare. For remarkably high degree of monopoly distortion, the optimal capital tax is negative which compensates a remarkably low level of output. For low degrees of monopoly distortion, economic agents invest in search of pure profits since the perceived relative effect of investment over-rules the realized monopoly induced distortions. A positive tax on capital income in such a case discourages investment and thus neutralizes the monopoly induced distortions. The chapter also presents a calibration of the model to fit the long run characteristics of the US economy.

Over the last three decades, there has been a marked enthusiasm amongst the critics of optimal taxation theory in attempts to establish the limits of optimal tax formulas and prescriptions in designing tax policy. Some attempts have stimulated interesting debates and consequently have been archived as important parts of the relevant literature. Recently, in many presentations of the main findings of chapters two, three and four, I have encountered some criticisms along these lines from the seemingly less convinced parts of my audience. In pursuit of investigating the importance and policy relevance of Ramsey tax rules, chapter five, which is the concluding chapter of this thesis, establishes that most of the common grounds of such criticisms, be it realistic, such as administrative and compliance costs, or be it rather abstract, such as fairness, are either unimportant or irrelevant for Ramsey taxation in a representative agent economy. The more important inadequacy of the traditional Ramsey tax models is their limited applicability in designing tax policy in developing countries. The main discussion of chapter five evolves around dynamic Ramsey taxation and the Ramsey rule for capital taxation.

Acknowledgements

My greatest and most sincere thanks go to my supervisor, Akos Valentinyi, for his advice, support, seemingly endless patience and warm care. Akos made me believe in myself and rely on my abilities to conduct independent research in Macroeconomic Theory, and he literally tailored me with the intellectual vision of an academic and the professional etiquettes of an economist. He was always willing to listen to my new (and at times naïve) ideas. He created an environment for me which was conducive to self motivated innovative research. By keeping the protocol formalities to a minimum, we established an effective custom of mutual esteem for opinions and exchange of ideas, all of which benefited this particular research and my other endeavours during the PhD program to an immense extent.

I thank Robin Mason, Xavier Mateos-Planas, and Berthold Herrendorf (Arizona State), especially, for opening up new spheres of economic theory research to me. Words seem insufficient to thank Robin for his connoisseur advice, patience, time, friendship and mentoring companionship during my tenure as a doctoral student. I thank all other members of the Economics Division, in particular, Patricia Rice, Maurice Kugler, Jan Podivinsky, Peter Smith, Jackie Wahba, Yves Pitarakis, Anurag Banerjee, John Aldrich, Ray O Brien, Thibaud Verge, Antonella Ianni, David Benjamin, Max Kwiek, Christian Schluter, Maozu Lu, Giovanni Pica, Mathias Hoffmann, Jusso Valimaki, and Giulio Seccia, for their help, suggestions, encouragement and friendship. I take this opportunity to thank Fatima Fauzel, Claire Caffrey, Gill Crumplin, Glenda Stevens, Jing Perny, Anne Edwards and all other administrative staffs of Economics Division and the School of Social Science, for being always supportive, that too with a smile.

I take this opportunity to thank the Commonwealth Scholarship Commission and the Foreign and Commonwealth Office, not only for funding my PhD program in University of Southampton, but also for their administrative support and monitoring of my research progress.

During the progress of this research, I have had the privilege of presenting different versions of the related papers in seminars, conferences and small group discussions (including job interviews). The comments, suggestions and encouragements that I have accumulated from these intellectual interactions have helped me in revising, rethinking and rearranging the main results. I thank all participants of these seminars, conferences and small group discussions who have implicitly contributed to the grooming of this thesis.

Very special thanks to all my friends in the Economics Division who have made my last three years a remarkable experience. This list includes, but may not be limited to, Emanuella Lotti, Ioannis Kingdom, Szabolcs Deak, Yun Zhu (my office mate), Federico Martellosio, Xinyi Li, Johnson Asiama, Souraya El Yaman, Abdol Samad Nawi, Wilson Correa, Rosita Chong, Vandana Bhattacharya, Jie Ma and Hairong Mu.

I thank my family for having confidence in my abilities and complementing my efforts with their patience. I always believe my family is my most prized endowment, and this is a good opportunity to publicise it.

I dedicate this thesis to my mother, Begum Rashida Shahed, and to my son, Aymaan Roshan Sheikh --- representatives of two generations, for whom I am what I am today.

Chapter 1

The Optimal Taxation Problem: A Review

1.1 An Introduction to the Ramsey Problem.

The classic taxonomy of optimal taxation problem is due to the seminal work of Ramsey (1927). It restates the government's problem of choosing tax rates to maximize welfare when taxing commodity transactions and/or income is the sole fiscal instrument to finance a predetermined stream of expenditures, and each subset of taxes are subject to equilibrium reactions of consumers and producers. In addressing the issue of optimal choice of tax rates, the seminal work of Ramsey (1927) characterized the optimal levels for a system of taxes on consumption goods assuming that the government's objective was to choose these taxes to maximize social welfare subject to a set of constraints it faced. The set of constraints generally comprises of preset revenue target of the government and the economic agents' competitive equilibrium reaction to the tax policy. Each optimal tax plan from a standard Ramsey model therefore implies a feasible allocation of factor services and goods along with prices that fully reflects the equilibrium feedback of consumers and firms.

Ramsey's methodology of addressing the optimal taxation problem and the results were rediscovered by Samuelson (in a note to the US Secretary of Treasury 1951, published in 1986), and later extended to an economy with several consumers by Diamond & Mirrlees (1971). Led mainly by Mirrlees (1971), economists in the 1970s developed a series of models that approaches the issue of optimal taxation by insisting on the trade off between equity and efficiency criteria for taxation[1]. The basic idea of this stream of literature is that in a perfectly competitive economy, the right lump sum distribution is capable of attaining any pareto efficient outcome. This stems from the widely known second fundamental welfare theorem. From this point of view, in principle, lump sum taxes obviously are the optimal form of taxation since they can accomplish any redistributive objectives at minimum social cost. Lump sum taxes, however, are impractical as a policy option. This is because in order to implement and administer these taxes the government must collect accurate information on individual abilities to pay taxes --- a task which is likely to be overwhelmingly costly. This argument, coupled with the emerging role of inevitable public expenditure to be raised through taxation of transactions, acts as the most important theme in the optimal taxation literature. If taxation is the sole instrument to finance inevitable public expenditure and lump sum taxes are not feasible, the government can only tax economic transactions (e.g. consumption of goods and factor income). But by taxing economic transactions the government influences the economic decisions of private agents which lead to inefficiencies. The optimal taxation problem therefore can be stated simply as given the tax revenue that the government has decided to collect, and the perfectly foreseen reactions of consumers and producers for any chosen tax instrument, how the government should choose the rates of the various taxes to maximize social welfare.

The general equilibrium tradition of optimal taxation is due to the work of, among others, Cass (1965), Koopmans (1965), Kydland & Prescott (1982), Lucas & Stokey (1983), Judd (1985), Chamley (1986), Jones, Manuelli & Rossi (1993 & 1997) and Chari & Kehoe (1990, 1994, 1999). The Ramsey problem in a general equilibrium, as

[1] The trade off between equity and efficiency as an argument is usually put forward because optimal taxation literature, allegedly, is concerned with efficiency and abstracts from equity. These arguments in some detail are presented as critique to optimal taxation theme in chapter 5.

popularly known in this literature, is the one in which for a given welfare criterion which the government uses to evaluate different allocations, the problem for the government to pick the fiscal policy (or one of them if there are many) that generates the competitive equilibrium allocation in a decentralized regime giving the highest value of the welfare criterion (e.g. Chamley (1986)).

An equivalent (and alternative) way of characterizing and formulating the Ramsey problem, widely known as the primal approach, is primarily due to Atkinson & Stiglitz (1980, ch. 12). In the primal approach, the government is allowed to pick an allocation directly (rather than a set of taxes). However, the set of allocations from which the government is allowed to choose is restricted by the implementability constraints, which is generally the consumer's budget constraint in which the prices and policies are substituted out using the consumer's and firm's necessary (and sufficient) optimizing conditions. Under any arbitrarily chosen fiscal policy, the optimal behaviour of consumers and firms generates a competitive equilibrium allocation which is one element in the set of allocations from which the government can choose. Such an allocation is referred to as an implementable allocation. To implement this particular allocation as a competitive equilibrium, the government needs to choose the fiscal policy that generated it. The Ramsey problem with implementability constraints therefore consists of choosing among all implementable allocations the one that maximizes a welfare criterion.

More compactly, the primal approach to optimal taxation characterizes the set of allocations that can be implemented as a competitive equilibrium with distorting taxes by two simple conditions: a (set of) resource constraint(s) and an implementability constraint. This characterization implies that optimal allocations are solutions to a simple programming problem. In the current stream of literature concerning optimal taxation, this is generally referred to as the Ramsey problem, and the associated solutions are referred to as the Ramsey allocations and Ramsey plan (see for instance, Chari & Kehoe (1999) and Erosa & Gervais (2001) for details).

The general approach to characterizing equilibria with distorting taxes portrayed in this chapter and in the remainder of the thesis is the primal approach. This approach involves finding the optimal (second best) wedges between marginal rates of substitution and marginal rates of transformation. The government chooses the policy that implements those wedges. Typically, many tax policies can decentralize the Ramsey allocations. This implies that even if the allocation is unique, there may be more than one policy that implements the same wedges. The prescriptions for optimal taxes therefore depend on the details of the particular tax system. The current thesis has particular focus on income taxes, and hence mostly considers a tax system that comprises labor income tax and capital income tax. However, in representative agent frameworks consumption taxes and labor income taxes are equivalent, in the sense that they distort exactly the same margin and both their distortions are uniform over time. This in turn implies that the optimal policy for labor income taxation equivalently applies to consumption taxation.

The remainder of this chapter is organized as follows. Section 1.2 presents a simple static model of optimal commodity taxation, and deduces the optimal levels of taxes for a number of commodities traded in a competitive market. This section is intended to demonstrate the benchmark results of optimal commodity taxation and establish the general methodology of addressing the optimal taxation problem in general equilibrium in the remainder of the thesis. Section 1.3 in three subsections presents the three standard variants of dynamic general equilibrium models of optimal taxation, namely, neoclassical growth model, endogenous growth model and the model with overlapping generations. The key and celebrated result of established infinite horizon models is that it is optimal to set the long run capital income tax equal to zero. The overlapping generations' model infers that only under special conditions the tax rate on capital income is zero in the steady state. This, often termed as the celebrated Chamley-Judd result of dynamic optimal taxation (due to Chamley (1986) and Judd (1985)), is derived, highlighted and explained intuitively for all three models in this section. Section 1.4 introduces, rather non-technically, the time consistency problem associated with optimal tax rules in the absence of an effective commitment technology at the society's disposal. Section 1.5 presents a brief review of the current tax systems of the world and the current state of the art of dynamic optimal taxation research. Section 1.6 concludes.

1.2 Optimal Commodity Taxation.

Following Atkinson & Stiglitz (1980, ch. 12), consider a static general equilibrium framework of a simple model economy, in which j types of consumption goods are produced using labor as the only input. All economic activity takes place in a single period. The resource constraint is given by:

$$f(x_1, x_2, \ldots\ldots, x_j, n) = 0 \tag{1.1}$$

where x_i denotes output of consumption good i, with $i = 1, 2, \ldots\ldots, j$; n denotes the labor input, and $f(.)$ denote the constant returns to scale technology with $f_i > 0, f_{ii} < 0, f_n < 0, f_{nn} > 0$, where f_i and f_n denote the partial derivatives of the production function with respect to the i th good and labor, respectively. The consumption goods are bought by consumers for private consumption (c_i) and purchased by the government for exogenously determined government consumption (g_i).

The consumers are endowed with one unit of time which can be allocated to work and leisure. Normalizing wage to unity, the utility maximization problem of the representative consumer is given by:

$$\max_{\{c_i\}, n} \quad u(c_1, c_2, \ldots\ldots, c_j, n) \tag{1.2}$$

$$s.t. \quad \sum_{i=1}^{j} p_i (1 + \tau_i) c_i = n \tag{1.3}$$

where p_i is the price of good i, and τ_i is the consumption tax on good i. Thus the tax system for this model economy comprises of j linear commodity taxes. The utility function $u : \mathbf{R}_+^{j+1} \to \mathbf{R}$ is continuously differentiable, strictly increasing in consumption (c_i), decreasing in labor (n), and strictly concave.

The government uses the tax revenue to finance a given amount of public consumption of each good (g_i). The government's budget constraint is given by:

$$\sum_{i=1}^{j} p_i g_i = \sum_{i=1}^{j} p_i \tau_i c_i \qquad (1.4)$$

The goods market clearing condition is simply:

$$c_i + g_i = x_i \qquad \text{for } i = 1,2,\ldots\ldots,j \qquad (1.5)$$

Following the tradition of optimal taxation, the Ramsey problem for the government is how the government should set tax rates to finance the given level of public consumption (g_i) such that it maximizes consumer's welfare given by the utility function in *(1.2)* and the resulting allocation constitutes a competitive equilibrium.

The solution to the representative consumer's problem yields $j+2$ first order conditions. The combined conditions which characterize the consumer's decisions are:

$$\frac{u_i}{u_n} = -p_i(1+\tau_i) \qquad \text{for } i = 1,2,\ldots\ldots,j \qquad (2.1)$$

which is standard and fairly intuitive. It states that the representative consumer maximizes utility where her marginal rate of substitution between consumption of good i and labor is equal to the relative price of consumption good i and labor, for all $i = 1,2,\ldots\ldots,j$.

The representative firm chooses output levels $\{x_i\}$ and labor input (n) to maximize profits defined by $\left[\sum_{i=1}^{j} p_i x_i - n\right]$ subject to technology constraint defined by *(1.1)*. The first order conditions from this maximization problem can be summarized as:

$$p_i = -\frac{f_i(x_1, ..., x_j, n)}{f_n(x_1, ..., x_j, n)} \qquad \text{for } i = 1,2,....., j \qquad (2.2)$$

The Competitive Equilibrium for this model economy is defined as follows:

Definition 1.2.1a (Competitive Equilibrium). A competitive equilibrium is a policy $\pi = \{\tau_i\}_{i=1}^{j}$, allocations $(\{c_i, x_i\}_{i=1}^{j}, n)$ and a price system $p = \{p_i\}_{i=1}^{j}$ such that

(1) Allocations c and n solve the consumer's problem;

(2) Allocations x and n solve the firm's problem;

(3) The government budget constraint *(1.4)* holds;

(4) Under the allocations c and x, the markets clear, thus,

$$c_i + g_i = x_i \qquad \text{for } i = 1,2,....., j \qquad (2.3)$$

□

The following proposition presents a necessary condition that ensures that an allocation, generated by a particular set of tax rates chosen by the government that maximizes welfare, constitutes a competitive equilibrium.

Proposition 1.2.1a. The allocations $(\{c_i, x_i\}_{i=1}^{j}, n)$ in a competitive equilibrium satisfy the resource constraint

$$f(c_1 + g_1, ..., c_j + g_j, n) = 0 \qquad (2.4)$$

and the implementability constraint

$$\sum_{i=1}^{j} u_i c_i + u_n n = 0 \qquad (2.5)$$

Furthermore, if an allocation satisfies *(2.4)* and *(2.5)*, one can construct policies and prices such that the allocation constitutes a competitive equilibrium.

Proof: It is obvious that any feasible allocation must satisfy the resource constraint defined by *(2.4)*. Substituting *(2.1)* in *(1.3)* yields $\sum_{i=1}^{j}(-1)\frac{u_i}{u_n}c_i = n$ which can be rearranged to derive the implementability constraint *(2.5)*. The implementability constraint, *(2.5)*, therefore is a constraint that incorporates, only in terms of allocations, the consumer's optimizing conditions. It is a constraint on the set of allocations that can be implemented as a competitive equilibrium with distorting taxes.

Note that allocations c, g and n satisfy *(2.4)*and *(2.5)*. Also, *(2.4)* implies that markets clear, i.e. $c_i + g_i = x_i$ for $i = 1,2,.....,j$. The first order condition from firm's optimization problem, *(2.2)*, determines the prices according to:

$$p_i = -\frac{f_i(c+g,n)}{f_n(c+g,n)} \qquad (2.6)$$

Given the allocations and the prices, one can construct a policy π such that the consumer's first order conditions are also satisfied. In particular,

$$1+\tau_i = (-1)\frac{u_i(c+g,n)}{u_n(c+g,n)} p_i^{-1} \qquad (2.7)$$

Substituting *(2.7)* into the implementability constraint *(2.5)*, it is straightforward to show that the consumer's budget constraint (the remaining first order condition from consumer's optimization problem) is also satisfied. The government's budget constraint is satisfied by Walras' law. ■

It is obvious that different tax rates are consistent with different competitive equilibrium. With exogenous government expenditure, the Ramsey problem for the government is to choose the tax rates that generate a competitive equilibrium with the maximum welfare.

Definition 1.2.1b (Ramsey Equilibrium). A Ramsey equilibrium is a policy π, an allocation $(c(\pi), x(\pi), n(\pi))$ and a price function $p = p(\pi)$ such that

1. The policy π solves

$$\max_{\pi} \quad u(c(\pi), n(\pi)) \qquad (3.1)$$

$$s.t. \quad \sum_{i=1}^{j} p_i(\pi) g_i = \sum_{i=1}^{j} p_i(\pi)\tau_i c_i(\pi) \qquad (3.2)$$

2. For every π, the allocations, the price function and the policy constitute a competitive equilibrium. □

The resulting allocations and prices from Ramsey equilibrium are known as Ramsey allocations and Ramsey prices, respectively. If a competitive equilibrium associated with each policy is unique, the Ramsey equilibrium is also unique.

Optimal Taxation

The Ramsey allocations solve the Ramsey problem, which according to the primal approach is to choose c and n to maximize utility defined in *(1.2)* subject to the resource constraint *(2.4)* and the implementability constraint *(2.5)*. The Lagrangian of the Ramsey problem is given by:

$$L = u(c,n) + \mu_1 f(c+g,n) + \mu_2\left(\sum_{i=1}^{j} u_i c_i + u_n n\right) \qquad (3.3)$$

where μ_1 and μ_2 are the two associated Lagrange multipliers on the resource constraint and the implementability constraint, respectively.

Proposition 1.2.1b. If preferences are additively separable, i.e. $u_{ik} = 0 \, \forall i \neq k$, with $i,k = 1,....j$, and $u_{in} = 0$, then Ramsey taxes are higher for the goods with lower income elasticity.

Proof: The first order conditions from by *(3.3)* can be rearranged to characterize the welfare maximizing allocation as:

$$\frac{u_i + \mu_2(u_i + u_{ii}c_i)}{u_n + \mu_2(u_n + u_{nn}n)} = \frac{f_i}{f_n} \qquad \text{for } i = 1,2,.....,j \qquad (4.1)$$

In order to determine the tax rate that is consistent with *(4.1)*, combine *(2.7)* with *(4.1)* to derive:

$$\frac{1}{1+\tau_i} = \frac{[u_i + \mu_2(u_i + u_{ii}c_i)]u_i^{-1}}{[u_n + \mu_2(u_n + u_{nn}n)]u_n^{-1}} \tag{4.2}$$

which can be rewritten as:

$$\frac{1}{1+\tau_i} = \frac{1 + \mu_2[1 - \{-u_i^{-1}(u_{ii}c_i)\}]}{1 + \mu_2[1 + u_n^{-1}(u_{nn}n)]} \tag{4.3}$$

The term $\{-u_i^{-1}(u_{ii}c_i)\} > 0$ in *(4.3)* is the elasticity of marginal utility with respect to good i, and the right hand side and left hand side of *(4.3)* are decreasing in this elasticity and the tax rate, respectively. Consequently, optimal taxes are higher for goods with higher elasticity of the marginal utility with respect to consumption.

The higher the elasticity of the marginal utility with respect to consumption, the lower is the elasticity of demand with respect to prices. This is because the marginal utility of consumption of good i is closely related to the relative price of good i. So, high elasticity of marginal utility with respect to consumption means a high elasticity of the price with respect to quantity, implying that one percentage point change in the quantity leads to a large change in the price. But this also means that a one percentage point change in the price induces only a small change in consumption implying the demand is inelastic. Thus, necessities are taxed more than luxuries. ■

Proposition 1.2.1c. If utility is weakly separable across consumption goods and is homothetic in consumption, optimal commodity taxation is uniform, in the sense that the Ramsey taxes satisfy $\tau_i = \tau_k$ for $i, k = 1, \ldots\ldots, j$.

Proof: Consider a utility function of the form $u(c,n) = V(\zeta(c), n)$ where $\zeta(.)$ is homothetic, and c is a vector of consumption of j commodities. The utility is weakly separable across consumption goods and homothetic in consumption, which implies:

$$\sum_{k=1}^{j}\frac{c_k u_{ik}}{u_i}=\sum_{k=1}^{j}\frac{c_k u_{lk}}{u_l} \qquad \text{for all } i,l=1,......,j \qquad (4.4)$$

Substituting the firm's first order condition in *(2.7)* yields:

$$1+\tau_i=\frac{u_i f_n}{u_n f_i} \qquad (4.5)$$

which implies $\tau_i=\tau_k$ for $i,k=1,......,j$ if and only if $\frac{u_i}{f_i}=\frac{u_k}{f_k}$.

Now consider the first order condition for c_i implied by *(3.3)*:

$$u_i+\mu_2\left(u_i+u_{ni}n+\sum_{k=1}^{j}c_k u_{ik}\right)=-\mu_1 f_i \qquad (4.6)$$

Imposing $u(c,n)=V(\zeta(c),n)$ and *(4.4)* in *(4.6)*, yields:

$$(1+\mu_2)V_1\zeta_i+\mu_2[AV_1\zeta_i+nV_{12}\zeta_i]=-\mu_1 f_i \qquad (4.7)$$

where A is some constant for which $\sum_{k=1}^{j}c_k u_{ik}=Au_i$ for all i. Since *(4.7)* holds for all i,k, it is straightforward to show that $\frac{u_i}{f_i}=\frac{u_k}{f_k}$. ■

Commodity Taxation with Intermediate Goods

Extensions of this simple static model include, among others, addressing the issue of optimal taxation of intermediate goods. An important contribution in this issue is the one by Diamond & Mirrlees (1971). The extension is typically accomplished by introducing an intermediate input good, z, in the production technology of the final good, and introducing another technology to produce the intermediate input good.

More specifically, consider a composite consumption good, c, produced using labor and intermediate goods by a constant returns to scale technology $f(c,z,n_c)=0$ where n_c is labor input in consumption goods producing sector. The intermediate input good and government consumption good is produced using labor as the only input by constant returns to scale production technology $h(g,z,n_z)=0$. The households pay the ad velorem tax on composite consumption at the rate τ_c. The government also taxes the expenditure on intermediate input good at the rate τ_z, which is levied on producers of the final consumption good. Markets for all goods are competitive, and standard assumptions ensuring existence of economy-wide competitive equilibria apply.

Let p_i denote the price of goods where $i=c,z,g$, and let w denote the wage rate in both sectors. One of the first order conditions from the maximization problem of representative firm producing final consumption goods is $w^{-1}p_z(1+\tau_z)=f_{nc}{}^{-1}f_z$. The representative firm producing intermediate input good and government consumption good has a first order condition $w^{-1}p_z=-h_{nz}{}^{-1}h_z$. Now consider the optimal taxation problem for this environment. The optimal taxation problem for the government is to maximize utility of households defined by $u(c,1-n_c-n_z)$ subject to implementability constraint $u_c c-u_n(n_c+n_z)=0$, and the two technologies. One of the necessary conditions for the Ramsey equilibrium is $f_{nc}{}^{-1}f_z=-h_{nz}{}^{-1}h_z$, which, combined with the firms' first order conditions stated above imply $\tau_z=0$ at an optimum. Hence optimal tax on intermediate input good is zero, or in other words, the government can implement the Ramsey allocations keeping the tax rate on intermediate input good at zero and generating all revenue by taxing consumption.

The main intuition behind the undesirability of a tax on an intermediate input good is that such a tax creates distortions on two margins: one on the allocation of intermediate input good which in turns distorts productive efficiency margin, and the other on labor supply to the intermediate input goods firm. The optimal policy must be the one that minimizes distortions, which in such a setting is the one that taxes consumption and sets no tax on intermediate input good.

1.3 Dynamic Optimal Taxation.

Taxation of capital income deserves its own analysis. This is mainly because taxation of capital income influences the intertemporal allocation decisions of taxpayers and thereby affects the intertemporal incentive structure underlying the tax policy. This section presents optimal capital income taxation in deterministic dynamic general equilibrium frameworks where the government can commit to a plan of tax rates. The advancement and sophistication of this trend in optimal taxation literature is primarily due to Judd (1985) and Chamley (1986)[2]. The main research issue in this trend is to find the optimal rule for taxing capital income where current period decisions of agents depend crucially on next period's accumulation of capital stock, and there is a commitment device at the society's disposal with which it can restrict the government from changing initially announced tax plans.

Capital taxation, however, is a vast sub-discipline since many things go under the name of capital. Taxation of capital in fact involves two sorts of taxes, (1) taxes on the stock of capital, e.g. wealth tax, the tax on bequests, property taxes, and (2) taxes on the income from savings, e.g. corporate income tax, taxation of interest and dividends, taxation of capital gains, etc. The economic analysis of these two cases in a general equilibrium model is very similar since all capital stocks stem from accumulated savings.

The following three subsections present three most commonly used dynamic general equilibrium frameworks of optimal capital income taxation. The models presented here are otherwise unanimously rated as *representative* models of this stream of literature. The first model deals with optimal capital income taxation in a competitive equilibrium version of simple infinite horizon one-sector neoclassical growth model. The framework presented here closely follows Ljungqvist & Sargent (2000, ch.12). Next, the issue of taxing both physical and human capital is addressed in a simple infinite horizon endogenous growth model following Jones, Manuelli &

[2] The literature on optimal income taxation goes back to Mirrlees (1971), who addresses redistributive progressive income taxation in a timeless economy without capital, and ignores, as a means to simplification, intertemporal problems of taxing savings and property income.

Rossi (1993 & 1997). Finally, capital taxation in a simple overlapping generations model is presented in the spirit of Atkeson, Chari & Kehoe (1999) and Chari & Kehoe (1999).

1.3.1 Neoclassical Growth Model.

Consider a simple production economy where the private sector comprises of firms and households. The economy is populated by a continua of measure one of identical infinitely-lived households. There is no population growth. The time subscript t for a variable is used to denote the level of that particular variable at time t. The final good (y_t), which is produced using labor (n_t) and capital (k_t) as inputs, is traded in a competitive market and can be used for private consumption (c_t), exogenously determined government consumption (g_t), or used to augment capital stock for investment (i_t). The resource constraint of the economy is defined as:

$$c_t + g_t + i_t = F(k_t, n_t) \qquad (5.1)$$

where the technology $F(k,n)$ exhibits Constant Returns to Scale, with $F:\mathbf{R}^2_+ \to \mathbf{R}_+$ continuously differentiable, strictly increasing, strictly concave in both k and n, and satisfies standard *Inada* conditions such as $\mathbf{lim}_{k_t \to 0}\mathbf{F}_k(t) = \infty$ and $\mathbf{lim}_{k_t \to \infty}\mathbf{F}_k(t) = 0$ for any $n > 0$. Throughout this section (and in most parts of the remainder of this thesis), I will use the notation of type $\mathrm{Z}_x(t)$ to denote the partial derivative of the function $\mathrm{Z}(x_t, ...)$ with respect to x evaluated at time t. Hence for instance, the notation $\mathbf{F}_k(t)$ represents the marginal product of capital evaluated at time t.

Capital depreciates at a constant rate $\delta \in (0,1)$. The equation $i_t = k_{t+1} - (1-\delta)k_t$ characterizes accumulation of capital stock[3].

[3] Although the linear capital accumulation specification is a useful benchmark and a standard practice, there are potential variants of this specification. In the literature concerning fiscal policy in the presence of adjustment costs of capital, alternative formulation of the law of motion for capital is often considered. Cassou & Lansing (1998), for instance, assume that the law of motion is

Firms

There is a continua of measure one of identical firms who own nothing except the technology. These firms simply hire labor and capital from households, produce the final output using technology $F(k,n)$, sell the output to the households in a competitive market and return profits to households. With competitive market for the final good and constant returns to scale technology, equilibrium pure profits are zero, and hence ignored hereafter. The representative firm faces the following sequence of static maximization problems:

$$\max_{k,n} \quad [F(k_t, n_t) - r_t k_t - w_t n_t]$$

where r_t and w_t are the rental price of capital and labor, respectively, both in terms of the numeraire. Competitive pricing ensures that factor prices equate their marginal products. The first order conditions for this problem are:

$$r_t = F_k(t) \tag{5.2a}$$

$$w_t = F_n(t) \tag{5.2b}$$

$k_{t+1} = A_1 k_t^{1-\delta} i_t^{\delta}$ with parameters $A_1 > 0$ and $\delta \in (0,1]$ governing the relationship between new investment and next period's capital stock. This formulation was primarily proposed by Lucas & Prescott (1971). Note that with $\delta \in (0,1)$ in this formulation capital is long lasting, and when both parameters are equal to one, capital depreciates fully after one period. By forward iteration and substituting out corresponding next period capital stock, this specification for any finite $T > 0$ yields $k_{T+1} = \prod_{s=0}^{T} \left[A^{(1-\delta)^s} i_{T-s}{}^{\delta(1-\delta)^s} \right] k_0{}^{(1-\delta)^{T+1}}$. The parameter δ therefore is no longer the constant average rate of depreciation and better viewed as associated with the relative quality of old capital relative to investment in each period.

Kim & Kim (2003) use the specification $k_{t+1} = \left[\delta(\delta^{-1} i_t)^{1-\phi} + (1-\delta) k_t^{1-\phi} \right]^{(1-\phi)^{-1}}$ with $\phi \geq 0$. With this specification $\phi > 0$ implies presence of adjustment cost in investment (no adjustment cost otherwise).

Hassler, Krusell, Storesletten & Zilibotti (2004) address the issue of optimal timing of capital taxes assuming quasi-geometric depreciation structure. They specify a parameter $\rho \geq 0$ such that a unit of investment at time t leads to one unit of productive capital at time $t+1$, $1-\rho\delta$ units in period $t+2$, and $(1-\rho\delta)(1-\delta)^S$ units in period $t+2+S$. The capital accumulation equation with this specification is simply $k_{t+1} = i_t + (1-\delta)k_t + \delta(1-\rho)i_{t-1}$. Note that with this specification $\rho < (>)1$ captures lower (higher) initial depreciation than in the standard geometric case ($\rho = 1$).

Since the production function satisfies constant returns to scale, one can define $\hat{k}$ as the capital-labor ratio, and $\hat{y}$ as the output-labor ratio, such that the production function in intensive form can be written as $\hat{y} = f(\hat{k})$. The corresponding Inada conditions are $\mathbf{lim}_{\hat{k}_t \to 0} f_{\hat{k}}(t) = \infty$ and $\mathbf{lim}_{\hat{k}_t \to \infty} f_{\hat{k}}(t) = 0$. The first order conditions of the firm's maximization problem can be rewritten as:

$$r_t = f_{\hat{k}}(t) \tag{5.2c}$$

$$w_t = f(\hat{k}) - \hat{k}[f_{\hat{k}}(t)] \tag{5.2d}$$

Thus in the current setting both factor prices depend solely on the capital-labor ratio.

Government

The government has a preset revenue target $\{g_t\}_{t=0}^{\infty}$ which it finances by levying flat rate taxes on earnings from capital at rate θ_t (gross of depreciation) and from labor at rate τ_t. The government also trades one period real bonds, and let b_t denote the government's indebtness to the private sector, denominated in time t goods, maturing at the beginning of period t. The government's time t budget constraint is defined as:

$$g_t = \theta_t r_t k_t + \tau_t w_t n_t + R_t^{-1} b_{t+1} - b_t \tag{5.3}$$

where R_t is the gross rate of return on one-period bonds held from t to $t+1$, denominated in units of time t goods. The government is benevolent. Interest earnings on bonds are assumed to be tax exempt. The government can effectively commit to the sequence of tax plans announced in the initial period.

Households

Each Household is endowed with one unit of time at each instant that can be devoted to work (n_t) or leisure, a given level of initial capital stock (k_0) that is rented out to firms for production, and property rights of the firms. The representative household derives utility from consumption and *disutility* from work, and its preferences for consumption and labor service streams $\{c_t, n_t\}_{t=0}^{\infty}$ is defined by the utility function:

$$\mathbf{U} = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \qquad (5.4)$$

where the subjective discount rate is $\beta \in (0,1)$ which varies inversely with the rate of time preference, and $u : \mathbf{R}_+^2 \to \mathbf{R}$ is continuously differentiable, strictly increasing in consumption (c), decreasing in labor (n), strictly concave, and satisfies standard Inada conditions, namely $\mathbf{lim}_{c_t \to 0} [u_n(t)]^{-1} u_c(t) = \infty$, and $\mathbf{lim}_{c_t \to \infty} [u_n(t)]^{-1} u_c(t) = 0$ for any n.

The representative household's problem is summarized as follows[4]:

$$\max_{c_t, n_t, k_{t+1}, b_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t u(c_t, n_t)$$

$$s.t. \quad c_t + k_{t+1} + R_t^{-1} b_{t+1} = (1-\tau_t) w_t n_t + (1-\theta_t) r_t k_t + (1-\delta) k_t + b_t$$

[4] One may assume that the government's capital income tax rate is a tax *net* of depreciation, implying that the after tax return on capital is $1 + (1-\theta_t)(r_t - \delta)$. With the current set up this implies the household's budget constraint is:

$$c_t + i_t + R_t^{-1} b_{t+1} = (1-\tau_t) w_t n_t + (1-\theta_t) r_t k_t + \theta_t \delta k_t + b_t$$

The key benefit of this formulation is that it allows a richer description of the tax code by introducing the notion of depreciation allowance, possible investment tax credit, etc. However, for a macro assessment of average effective tax rates on factor incomes, both formulations provide analytically similar results. See subsection 1.3.3 for such a formulation in an overlapping generations model.

where k_0, b_0 given, and standard non-negativity restrictions apply. In addition to the household's budget constraint, the consolidated first order conditions of the household's maximization problem are[5]:

$$u_n(t) = -u_c(t)(1-\tau_t)w_t \tag{5.5a}$$

$$u_c(t) = \beta u_c(t+1)[(1-\theta_{t+1})r_{t+1} + (1-\delta)] \tag{5.5b}$$

$$R_t = [(1-\theta_{t+1})r_{t+1} + (1-\delta)] \tag{5.5c}$$

$$\lim_{T\to\infty} \frac{k_{T+1}}{\Pi_{i=0}^{T-1} R_i} = 0 \tag{5.5d}$$

$$\lim_{T\to\infty} \frac{R_T^{-1} b_{T+1}}{\Pi_{i=0}^{T-1} R_i} = 0 \tag{5.5e}$$

The first two conditions are standard optimality conditions. The first one equates marginal rate of substitution of work and consumption to their relative price. The second condition is the standard Euler equation which states that at the optimum the household is indifferent between consuming today and saving for a later time.

Condition *(5.5c)* constitutes a no-arbitrage condition for trades in capital and bonds that ensures that these two assets have the same rate of return. The remaining two are Transversality conditions which restrict household's choice of the size of k_{t+1} and

[5] An alternative way to model bonds in the household's budget constraint *may* be to assume that b_{t+1} denotes real government bonds carried into period $t+1$, and these bonds pay interest at the rate r_{bt}, say. The household's budget constraint can be written as:

$$c_t + k_{t+1} + b_{t+1} = (1-\tau_t)w_t n_t + (1-\theta_t)r_t k_t + (1-\delta)k_t + (1+r_{bt})b_t, \quad k_0, b_0 \text{ given}$$

The government's budget constraint becomes

$$g_t = \theta_t r_t k_t + \tau_t w_t n_t + b_{t+1} - b_t(1+r_{bt})$$

and bond interest $r_{bt}b_t$ is not explicitly taxed. Note that since households have no control over r_{bt}, the government can choose the interest rate on bonds that yields a target after-tax interest income. Such a formulation is presented in subsection 1.3.3 in an overlapping generations model. With this formulation under the current set up, the corresponding arbitrage condition (as in *(5.5c)*) is simply

$$(1+r_{bt+1}) = [(1-\theta_{t+1})r_{t+1} + (1-\delta)].$$

A note on modelling government bonds in these environments deserves attention, however. Government bonds modelled in these frameworks in any standard way will yield similar analytical insights for the optimal tax rates. In particular, the key result of long run zero capital tax which appears later as proposition 1.3.1a is completely insensitive to which way (or *at all*) bond financing is modelled. Hence a bond financed economy or an economy where the government maintains a period by period balanced budget yields same analytical result for optimal capital income tax rate in the steady state.

b_{t+1}, and state that for an optimal consumption allocation the present discounted value of the household's capital and bond holdings must be zero as time goes to infinity.

For the following definition, symbols without time subscripts denote the one-sided infinite sequence for the corresponding variables, e.g. $n \equiv \{n_t\}_{t=0}^{\infty}$.

Definition 1.3.1a (Competitive Equilibrium). A competitive equilibrium is a 3-tuple of price sequences (w, r, R) that depends on the government policy (τ, θ, b) and supports an allocation (k, c, n), such that

(1) given the price system and government policy, the allocation solves both the firm's problem and the household's problem;

(2) given the price systems and allocation, the government policy satisfies the sequence of government budget constraints *(5.3)*; and

(3) all markets clear. □

It is understandable that each government policy *potentially* generates a competitive equilibrium. The multiplicity of competitive equilibria prompts the Ramsey problem.

Definition 1.3.1b (Ramsey Problem). Given (k_0, b_0, g), the Ramsey problem for the government is to choose a tax policy that maximizes expression *(5.4)* such that the resulting allocations and prices are consistent with competitive equilibrium defined by 1.3.1a. □

The Ramsey Problem

In order to characterize the Ramsey problem according to the primal approach, first note that the household's present-value budget constraint is:

$$\sum_{t=0}^{\infty} q_t^o c_t = \sum_{t=0}^{\infty} q_t^o (1-\tau_t) w_t n_t + [(1-\theta_0) r_0 + (1-\delta)] k_0 + b_0 \qquad (5.6)$$

where $q_t^o = \prod_{i=1}^{t} R_i^{-1}$, and $q_0^o = 1$ is held as the numeraire. One can also interpret q_0^o as the relative price of the good in period zero in terms of the good in period zero. Reconsider the problem of the representative household with this budget constraint. Maximizing expression *(5.4)* subject to *(5.6)* yields the household's optimality conditions and arbitrage condition with the *Arrow-Debreu* price q_t^o. The firm's problem and its solution are same as before. The constraint that puts a restriction on Ramsey allocations to constitute a competitive equilibrium is derived then by solving the household's and firm's optimality conditions for $\{q_t^o, r_t, w_t, \tau_t, \theta_t\}_{t=0}^{\infty}$ as functions of allocations only, and substituting these expressions for taxes and prices in *(5.6)*. The resulting intertemporal constraint that involves only allocations, initial capital tax (held fixed), initial capital endowment and initial bond holdings, is the implementability constraint, which is given by:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t + u_n(t)n_t] - \Omega(c_0, n_0, \theta_0) = 0 \qquad (5.7)$$

where $\Omega(c_0, n_0, \theta_0) \equiv u_c(0)\{[(1-\theta_0)F_k(0) + (1-\delta)]k_0 + b_0\}$

The primal approach recasts the Ramsey problem as one in which the government chooses allocations to maximize expression *(5.4)* subject to the resource constraint *(5.1)* and the implementability constraint *(5.7)*. The problem can be solved as a *Pseudo* planner problem. Define the function

$$V(c_t, n_t, \Phi) \equiv u(c_t, n_t) + \Phi[u_c(t)c_t + u_n(t)n_t]$$

where $\Phi \geq 0$ is the Lagrange multiplier on *(5.7)*. Intuitively, Φ is a measure of the utility cost of raising government revenues through distorting taxes. Maximizing the corresponding Lagrangian with respect to $\{c_t, n_t, k_{t+1}\}_{t=0}^{\infty}$ yields the set of Ramsey equilibrium conditions. The consolidated Ramsey equilibrium conditions are:

$$V_c(t) = \beta V_c(t+1)[F_k(t+1) + (1-\delta)] \qquad \forall t \geq 1 \qquad (5.8a)$$

$$V_n(t) = -V_c(t)F_n(t) \qquad \forall t \geq 1 \qquad (5.8b)$$

$$V_n(0) = [\Phi\Omega_c - V_c(0)]F_n(0) + \Phi\Omega_n \qquad (5.8c)$$

and constraints *(5.1)* and *(5.7)*.

Assume there is a $T \geq 0$ for which $g_t = g$ for all $t \geq T$, and solution to the Ramsey problem converges to a time-invariant allocation. In other words assume that after some point fluctuations in government spending become arbitrarily small. The following proposition establishes the celebrated Chamley-Judd result of long run zero capital taxation for the Ramsey problem.

Proposition 1.3.1a. For the Ramsey equilibrium characterized by *(5.8), (5.7)* and *(5.1)*, the steady state capital income tax is zero.

Proof: The stationary version of *(5.8a)* implies $1 = \beta(F_k + 1 - \delta)$.

With $q_t^o = \prod_{i=1}^{t} R_i^{-1}$, as $t \to \infty$, $\frac{q_t^o}{q_{t+1}^o} \to \beta^{-1}$. This implies, from household's optimality conditions (in particular the arbitrage condition), $1 = \beta[(1-\theta)F_k + 1 - \delta]$, which together with $1 = \beta(F_k + 1 - \delta)$ imply $\theta = 0$. ■

The government's long run tax policy would then comprise of a labor income tax and a zero tax on capital income. The government could raise all revenues through a time 0 capital levy, and then lend the proceeds to the private sector and finance government expenditure by using the interest from the loan.

The key argument put forward to support this striking result is that nonzero capital taxes (e.g. a savings tax) serve neither efficiency nor redistributive purposes in the long run. Unlike period by period labor income tax or consumption tax that creates smooth distortions over the long run, capital taxes compound over time. The capital tax-induced wedge between marginal rate of substitution between consumption at two different dates and their corresponding marginal rate of transformation grows exponentially over time (see Judd (1999) and section 5.3.1 of chapter 5 of this thesis for details). Such exponentially growing tax distortions are inconsistent with commodity tax principle. A long run policy involving nonzero capital tax therefore cannot be optimal. A simple non-numeric example explains the intuition. Consider two individuals denoted by *X* and *Y*, both receiving the same discounted labor income over their life-cycles. Individual *X* spends her income within each period, but individual *Y* saves for her retirement. If capital taxes are zero, both *X* and *Y* surely pay the same tax on labor income. But if the tax structure is such that savings are taxed (a positive capital income tax), *Y* pays more income tax since she is taxed at retirement time on the income she draws from her accumulated savings. A current period savings tax in this example would mean that each period the tax burden on income from savings compounds with accumulation of interest payments.

Steady State and Calibration

In order to provide a sample characterization of the steady state associated with Ramsey equilibrium *(5.8)*, *(5.7)* and *(5.1)*, consider specifications $u(c_t, n_t) = \ln c_t + [1 - An_t]$ and $F(k_t, n_t) = Bk_t^{\alpha} n_t^{1-\alpha}$ for the utility function and the production function, respectively, where $A, B > 0$, $\alpha \in (0,1)$ ensure these functions satisfy standard properties. The time-invariant version of the Ramsey equilibrium with the specifications is defined by the following system of equations:

$$1 = \beta[\alpha Bk^{\alpha-1} n^{1-\alpha} + 1 - \delta] \quad (5.9a)$$

$$A(1+\Phi) = c^{-1}(1-\alpha)Bk^{\alpha} n^{-\alpha} \quad (5.9b)$$

$$A(1+\Phi) = [c_0^{-1} - \Phi\Omega_{c0}](\alpha - 1)Bk_0^{\alpha} n_0^{-\alpha} - \Phi\Omega_{n0} \quad (5.9c)$$

$$c + g = Bk^{\alpha} n^{1-\alpha} - \delta k \quad (5.9d)$$

$$(1-\beta)^{-1}(1-An) - \Omega(c_0, n_0, \theta_0) = 0 \quad (5.9e)$$

where $\Omega(c_0, n_0, \theta_0) = c_0^{-1}\{[(1-\theta_0)\alpha Bk_0^{\alpha-1} n_0^{1-\alpha} + 1 - \delta]k_0 + b_0\}$

Proposition 1.3.1b. There is a unique solution to the system *(5.9)* implying unique steady state allocations, taxes and prices associated with the Ramsey equilibrium. Put simply, there is a unique Ramsey steady state.

Proof: The process of solving the system *(5.9)* is as follows. First solve *(5.9a)* for $n^{1-\alpha}$. Substitute in *(5.9d)* to derive expression $k = [1-\beta+\beta\delta-\alpha\beta\delta]^{-1}(c+g)\alpha\beta$. Then, substitute back in the expression for $n^{1-\alpha}$ and solve for n. Substitute for both k and n in *(5.9b)* to derive $c = \frac{(1-\alpha)B}{(1+\Phi)A}\left(\frac{1-\beta+\beta\delta}{\beta\alpha B}\right)^{\frac{\alpha}{\alpha-1}}$.

Note that c is unique if the multiplier Φ is unique. One can compute a unique value for Φ using *(5.9b)* and *(5.9c)* in terms of the initial conditions. For unique c and given g, both k, n and their corresponding prices are unique. The competitive equilibrium condition $(1-\tau)(1-\alpha)Bk^{\alpha}n^{-\alpha} = Ac$ gives a unique labor income tax rate. With $\theta = 0$, the time-invariant version of the no-arbitrage condition *(5.5c)* gives R.

In order to find the steady state level of government bond, b, evaluate the household's time t budget constraint at time $t+1$, and substitute in the household's first order conditions. This gives the following recursive equation:

$$u_c(t)k_{t+1} = \beta u_c(t+1)(k_{t+2} + R_{t+1}^{-1}b_{t+2} - b_{t+1}) + \beta u_n(t+1)n_{t+1} + c_{t+1}$$

The time-invariant version of this equation, with the specifications of utility and production function yields $b = [\beta(R^{-1}-1)]^{-1}[k(1-\beta)+\beta Acn - c^2]$, which is unique. ∎

Now consider, for illustration only, a simple calibration of the steady state using post war US economy data approximately for the period 1960-2001. The set of parameters of the model are $(\beta,\delta,\alpha,B,A)$, and government expenditure is treated as exogenous. These values are pinned down so that steady state of the model matches characteristics identified from the long run US data. Consistent with the fiscal decision frequency of the government, the time period is considered as one year. The steady state observations of government consumption-output and bond-output ratios are the ones taken from the *Federal Reserve Bank of St. Louis Economic Data*-FRED II. The 1960-1996 US series for k and i include business equipment and structures, consumer durables and residential components taken from *Revised Fixed Reproducible Tangible Wealth in the United States*, US Department of Commerce, that give $k/y = 3.31$ and $i/y = 0.22$. The US series for b is gross federal debt which gives steady state government debt ratio $b/y = 0.51$. Given a time endowment normalized to one, Cooley & Prescott (1995) pin down the fraction of worked time to a range of 0.2 to 0.3. For the current study $n = 0.3$ is held as a benchmark for numerical results. The steady state government spending ratio is $g/y = 0.23$.

There are two most commonly used steady state real interest rates for the US economy: 6.5% per annum for studies dealing with quarterly data (such as in Cooley & Prescott (1995)), and 4% per annum for studies dealing with annual data (such as Guo & Lansing (1999)). For the purpose of illustration, consider the annual real interest rate of 4% consistent with one year time period consideration. Using *(5.9a)* this yields $\beta = 0.9615$. There are some established estimates of the depreciation parameter δ which may be model specific. Instead, consider the steady state version of the investment's law of motion, which with steady state observations of capital-output and investment-output ratio give $\delta = 0.0664$. Next, *(5.9a)* with estimates of δ and β gives $r = 0.1064$. Using $k/y = 3.31$ and $r = 0.1064$ in the steady state version of firm's first order condition *(5.2a)* gives $\alpha = 0.3523$. The steady state version of the resource constraint gives 0.55 as the consumption-output ratio.

Consider now the steady state version of the government budget constraint with zero capital tax, and divide both sides by y. Evaluate the resulting expression for the observed steady state government expenditure-output ratio and bond-output ratio to derive $(w\tau)/y = 0.8322$. Steady state version of the firm's first order condition *(5.2b)* gives $(w/y) = 2.159$, and hence $\tau = 0.3823$. Estimate for the Lagrange multiplier is $\Phi = 0.4921$.

The calibration of the model is thus quite useful because it does fairly well to characterize the steady state level of average effective tax rate on all economic activities for the US economy. The calibrated labor income tax here represents the period by period average effective tax rate which distorts the long run margins of consumption (and hence welfare) through its effect on labor income. The estimated labor income tax of 38% is reasonably close to calibrated estimate of 22% labor income tax in Jones *et al.* (1997). The calibrated estimate of Jones *et al.* (1997) was for an economy with a much richer tax code (see next subsection) which perhaps better identifies average effective tax rate on labor income in particular. The current estimate is also close to empirical estimate of 27% in Mendoza, Razin & Tesar (1994) and Carrey & Tchilinguirian (2000), respectively. Their estimates assume a separate consumption tax which is not modelled here.

1.3.2 Endogenous Growth Model.

In two very important papers, Jones *et al.* (1993 & 1997) address the issue of optimal taxation of physical and human capital income in differentiated versions of endogenous growth model. Their studies show that the optimality of a limiting zero tax also applies to human capital with some restrictions on the process of creating human capital. The sketch of the model presented in this subsection is a simple endogenous growth framework of optimal taxation as in Jones *et al.* (1997). This model introduces demarcation of the stock and flow components of effective labor services with human capital accumulation process postulated as an internal activity. It deduces two important results. First, with a sufficiently rich tax code and zero profit from accumulating either capital stock, both capital and labor income taxes can be chosen to be zero in the limit. Second, for some specifically featured preferences *all* taxes can be chosen to be zero in the steady state.

Time is discrete and runs forever. There is a continua of measure one of identical infinitely-lived households. The representative household derives utility from consumption (c_t) and leisure (l_t). The preference for $\{c_t, l_t\}_{t=0}^{\infty}$ is defined by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t) \qquad (6.1)$$

where the subjective discount rate is $\beta \in (0,1)$, and the utility function $u : \mathbf{R}_+^2 \to \mathbf{R}$ has standard features as discussed in the preceding subsection. Leisure is defined as the residual of one unit of time endowment and number of hours allocated to human capital formation (n_{ht}) and market activities (n_{mt}). Human capital technology is defined as:

$$h_{t+1} = (1-\delta_h)h_t + H(x_{ht}, h_t, n_{ht}) \qquad (6.2)$$

The function $H(x_{ht}, h_t, n_{ht})$ describes how new human capital is created with the input of a market good x_{ht} , the stock of human capital h_t, and working time n_{ht}. Human capital is in turn used to produce *efficiency units* of labor e_t. The technology $e_t = M(x_{mt}, h_t, n_{mt})$ describes the production of e_t which requires three inputs, namely, quantity of market goods x_{mt}, human capital stock h_t and working time n_{mt}. Both $H(.), M(.)$ functions are homogeneous of degree one in market goods and human capital stock, and twice continuously differentiable with decreasing positive marginal products of all factors. Efficiency units of labor e_t and physical capital stock k_t are used as inputs for the production of final good. The resource constraint corresponding to the final goods sector of the economy is given by[6]:

$$c_t + g_t + k_{t+1} + x_{mt} + x_{ht} = F(k_t, e_t) + (1-\delta)k_t \qquad (6.3)$$

where $e_t = M(x_{mt}, h_t, n_{mt})$. The government has four tax instruments to finance its exogenously determined revenue target g_t. It taxes capital, labor, market goods used to produce efficiency labor, and consumption, at flat rates $\theta_t, \tau_t^n, \tau_t^m$, and τ_t^c , respectively. The government also issues one period bonds which have specification

[6] The resource constraint of the economy corresponding to the human capital producing sector is the technology given by *(6.2)*, which the government considers as an independent constraint in the Ramsey problem.

similar to that used in subsection 1.3.1. Moreover, it is committed to carry on with its initially announced tax plans.

The household's present-value budget constraint is given by:

$$\sum_{t=0}^{\infty} q_t^o (1+\tau_t^c) c_t = \sum_{t=0}^{\infty} q_t^o [(1-\tau_t^n) w_t e_t - (1+\tau_t^m) x_{mt} - x_{ht}] + [(1-\theta_0) r_0 + (1-\delta)] k_0 + b_0 \tag{6.4}$$

where $q_t^o = \prod_{i=1}^{t} R_i^{-1}$, and $q_0^o = 1$ is held as the numeraire. The representative household's problem is to maximize *(6.1)* subject to constraints *(6.2)* and *(6.4)* after substituting for $e_t = M(x_{mt}, h_t, n_{mt})$. Solution to this problem yields six necessary (and sufficient) conditions for marginal changes in $\{c_t, n_{mt}, n_{ht}, x_{mt}, x_{ht}, h_{t+1}\}$. Substituting these and the firm's optimizing conditions back in *(6.4)*, and after *considerable* manipulation, one can derive the following implementability constraint[7]:

$$\sum_{t=0}^{\infty} \beta^t u_c(t) c_t - \Lambda(c_0, n_{m0}, n_{h0}, x_{m0}, x_{h0}) = 0 \tag{6.5}$$

where $\Lambda(c_0, n_{m0}, n_{h0}, x_{m0}, x_{h0}) = u_c \{[H_x^{-1}(H_h + 1 - \delta_h) + F_e M_h] h_0 + [F_k + 1 - \delta] k_0 + b_0\}$ with partial derivatives evaluated at time 0.

The labor income tax rate τ_t^n has some special features in this setting. Note that τ_t^n affects not only the household's static choice of supplying effective labor into production but also its dynamic choice of human capital. In the Ramsey problem, the government therefore faces an additional constraint that allocations are consistent with the same τ_t^n that affects the margins of two optimizing conditions of the household, namely, the condition for static choice of e_t and the condition for dynamic choice of h_t . Jones *et al.* (1997) derive the additional constraint using the household's optimizing conditions but solve the Ramsey problem without imposing it, and show

[7] The manipulation of the necessary conditions from household's problem includes invoking homogeneity properties of the functions $H(.)$ and $M(.)$.

that in the steady state of the Ramsey equilibrium the additional constraint is satisfied. Thus if the Ramsey problem without the additional constraint and the one with it converge to a unique steady state, they will converge to the same steady state.

According to the primal approach, the Ramsey problem without the additional constraint, is the government's problem of maximizing *(6.1)* subject to constraints *(6.2)*, *(6.3)* and *(6.5)*. The problem can be approached as a Pseudo planner problem as in the previous subsection[8]. The corresponding first order conditions with respect to $\{c_t, n_{mt}, n_{ht}, x_{mt}, x_{ht}, h_{t+1}, k_{t+1}\}$ evaluated at the steady state collectively deduce optimal steady state tax policy $\tau^m = \tau^n = \theta = 0$. Moreover, with $\Phi \geq 0$ denoting the Lagrange multiplier on *(6.5)*, the steady state corresponding to the Ramsey equilibrium implies:

$$1 + \tau^c = \frac{u_c u_l + \Phi u_c u_{cl} c}{u_c u_l + \Phi(u_c u_l + u_{cc} u_l c)} \qquad (6.6)$$

With the second best solution where $\Phi > 0$, $\tau^c = 0$ if and only if $u_c u_l + u_{cc} u_l c = u_c u_{cl} c$. This condition holds for a special class of utility functions and hence cannot be generalized.

Jones *et al.* (1997) show that utility functions of the type $u(c,l) = (1-\sigma)^{-1} c^{1-\sigma} + v(l)$ where $\sigma > 0$, is consistent with this condition. Therefore in the steady state of the Ramsey equilibrium, optimal solution for these preferences is eventually to set all taxes equal to zero. Note that the class of utility function for which this result holds satisfies homotheticity and separability properties. These are the essential utility function properties for the uniform commodity taxation result as in proposition 1.2.1c. The optimal plan with these preferences therefore involves collecting tax revenues in excess of expenditures in the initial periods, and when the government has amassed large enough claims to finance subsequent expenditure with interest earnings, all taxes are set equal to zero.

---

[8] The Pseudo utility function for this model is

$$V(c_t, n_{mt}, n_{ht}, \Phi) = u(c_t, 1 - n_{mt} - n_{ht}) + \Phi u_c(t) c_t .$$

1.3.3 Overlapping Generations Model.

Kotlikoff (1998) presents a useful survey of an extensive literature on optimal fiscal policy in environments with overlapping generations. In a recent paper, Yakita (2003) addresses the growth effects of income taxation in an overlapping generations model with endogenous growth. The key result from this stream of literature is that the tax rate on capital income in a steady state is zero if certain homotheticity and separability conditions in the utility function are satisfied (see for instance, Atkinson & Stiglitz (1980, ch. 3), Atkeson *et al.* (1999)). The sketch of the neoclassical exogenous growth framework presented here closely follows Atkeson *et al.* (1999) and Chari & Kehoe (1999).

Consider an economy populated by overlapping generations (without bequests) who live for two periods. Time t is discrete and zero is the initial period. The agent of generation t is young in period t and old in period $t+1$. There is an initial old generation. In each period t, a representative young and representative old agent consumes c_{1t} and c_{2t} of the final good, respectively, which is produced using corresponding labor services n_{1t} and n_{2t}, and capital stock k_t as inputs. The government consumes exogenous g of the final good. The economy's resource constraint is given by:

$$c_{1t} + c_{2t} + k_{t+1} + g = F(k_t, n_{1t}, n_{2t}) + (1-\delta)k_t \qquad (7.1)$$

The government uses flat rate taxes to finance its exogenous consumption expenditure. The *net* of depreciation tax on capital income is denoted by θ_t, while τ_{1t} and τ_{2t} denote flat rate taxes on labor incomes. The government trades one period bonds b_t which is carried into period $t+1$ with gross return R_t. The government budget constraint is:

$$g = \theta_t r_t k_t + \tau_{1t} w_{1t} n_{1t} + \tau_{2t} w_{2t} n_{2t} + b_{t+1} - R_t b_t \qquad (7.2)$$

The government assigns weight λ^t to generation t with $\lambda < 1$. Each young agent in period t solves the following problem:

$$\begin{aligned} \textbf{max} \quad & \left[u(c_{1t}, n_{1t}) + \beta u(c_{2t+1}, n_{2t+1})\right] \\ s.t. \quad & c_{1t} + k_{t+1} + b_{t+1} = (1 - \tau_{1t}) w_{1t} n_{1t} \\ and \quad & c_{2t+1} = (1 - \tau_{2t+1}) w_{2t+1} n_{2t+1} + [1 + (r_{t+1} - \delta)(1 - \theta_{t+1})] k_{t+1} + R_{t+1} b_{t+1} \end{aligned}$$

The implementability constraints associated with each generation is given by:

$$R(c_{1t}, n_{1t}) + \beta R(c_{2t+1}, n_{2t+1}) = 0 \qquad \text{for each } t \qquad (7.3)$$

where $R(c,n) \equiv c u_c(c,n) + n u_n(c,n)$. The Ramsey problem for the government is:

$$\textbf{max} \quad \left[\lambda^{-1} u(c_{20}, n_{20}) + \sum_{t=0}^{\infty} \lambda^t [u(c_{1t}, n_{1t}) + \beta u(c_{2t+1}, n_{2t+1})] \right]$$

subject to the *(7.1)* and *(7.3)*. The term $\lambda^{-1} u(c_{20}, n_{20})$ in the government's objective function is the weighted utility of the initial old generation. If the solution to the Ramsey problem converges to a steady state, the corresponding Ramsey allocations satisfy:

$$\lambda^{-1} = F_k + 1 - \delta \qquad (7.4)$$

On the other hand, the steady state version of the agent's optimality condition for capital accumulation implies:

$$u_c(c_1, n_1) = [1 + (F_k - \delta)(1 - \theta)] \beta u_c(c_2, n_2) \qquad (7.5)$$

Equations *(7.4)* and *(7.5)* jointly imply $\theta = 0 \Leftrightarrow \lambda u_c(c_1, n_1) = \beta u_c(c_2, n_2)$.

The condition $\lambda u_c(c_1, n_1) = \beta u_c(c_2, n_2)$ normally would not hold, unless one restricts the utility function to follow two properties more closely connected to uniform commodity taxation, namely, (*a*) homotheticity over consumption, and (*b*) separability of consumption and labor services (or leisure). A utility function of the type $u(c,n) = \ln c + [1-n]$, for instance, satisfies these two properties, and hence would yield a steady state zero optimal tax on capital income in this framework. Recall that these properties of the utility function are the essential ones for the long run policy choice of setting all taxes equal to zero in the endogenous growth model.

1.4 The Time Consistency Problem.

In environments where societies (and governments) have no ability to bind future policy choices, the policy design problem typically is characterized by incentive compatibility restrictions. In a path-breaking paper, Kydland & Prescott (1977) argued that in such environments the sensible way to set up the policy design problem was to formulate the decision problems of both government and the private agents sequentially, requiring that choices be optimal at each point of time. For a finite horizon model, they presented the computational technique for the optimal policy using backward induction. Their paper logically questioned the appropriateness of the optimal control technique as a device for designing and evaluating macroeconomic policy over the long run. This is because with no commitment, current decisions of economic agents depend in part upon their expectations of future policy actions, and only if these expectations were invariant to the future policy plan selected would optimal control technique be appropriate.

In addition to questioning the appropriateness of optimal control technique in designing macroeconomic policy over the long run, the paper by Kydland & Prescott (1977) is perhaps the most cited one for introducing two very important issues of macroeconomic policymaking, namely, *dynamic inconsistency* and *credibility*. The paper presents a very clear statement of the tension between *ex ante* optimal and *ex post* optimal policy that indicates the existence of dynamic inconsistency and credibility problems in once-and-for-all (non sequentially) optimally chosen policy rules. The perception of time-inconsistency therefore assists one to formalize and hence better understand, *say*, a government's incentive to first promise that accumulated capital will not be taxed and then weigh the possibilities of capital levies in the face of demanding revenue shortfalls. The government's incentive towards time-inconsistent behaviour in this case also helps explain why capital accumulation may be remarkably low in economies with relatively weaker cushions against such behaviour.

To put the matter more specifically, consider, for instance, a simple macroeconomic model where the policymaker wants to design an optimal policy rule for future policy at some arbitrary point in time *t*. One important contemplation when selecting her policy for some future date *t+s* will be how the expected policy at *t+s* affects private agents' economic decisions in the time interval from *t* to *t+s*. Now since the private economic decisions between *t* and *t+s* are already bygone at *t+s* so that policy can no longer influence them, the ex ante optimal plan, i.e. the plan that was optimal given the date *t* constraints will not be optimal ex post given the date t+s constraints. Put differently, the policymaker faces different constraints ex post than she did ex ante and this makes her prefer a different policy. Hence, the original plan, although optimally designed, is said to be dynamically inconsistent or time inconsistent. Now consider the model with a specific example of taxation. Suppose the policymaker's choice of the tax rate for time *t+s* chosen at time *t+j* is denoted by $\eta_{t+s}(t+j)$ where $0 \leq j \leq s$. A forward looking policymaker can obviously wait until *t+s* to choose the tax rate for that date, or she can choose the *t+s* tax rate at *t*. With no changes in the policymakers' preferences, state or technology between *t* and *t+s*, basic dynamic programming implies the date of choosing tax rate does not pose a serious problem. Time inconsistency arises if without any unanticipated shocks or changes, these choices are not equal, i.e. $\eta_{t+s}(t+s) \neq \eta_{t+s}(t)$.

In the presence of an effective commitment device, or a *commitment technology*, this problem is however not obvious. If for example the policymaker has access to a commitment technology with which she could make a binding commitment at date *t* to pursue a particular policy at *t+s*, time consistency would obviously be irrelevant. But policymakers involved in monetary and fiscal policy, as Persson & Tebellini (2002, ch. 11 & 12) argue in detail, rarely possess commitment technologies, unless there exists a binding constraint in the constitution, or some form of treaty, for instance. Instead they generally operate in a discretionary policy environment where sequential decision making is the norm and revisions of policy decisions are frequent. In such frameworks, private agents will anticipate the future incentive to abandon the ex ante optimal policy, and hence the optimal policy will not be credible. A policymaker who has discretion to sequentially revise her policy instruments will therefore face a set of additional incentive constraints at time *t*. Only policies that satisfy these constraints will be believed by forward-looking private agents with rational expectations. Surely, these additional constraints in the policy problem worsen the equilibrium outcome from the viewpoint of the policymaker.

1.4.1 A Simple Model of Dynamic Taxation without Commitment.

To put the matter of time-inconsistency of optimal capital taxation more formally, consider the following simple representative agent two-period model, in the spirit of Fischer (1980). The representative agent is endowed with income **y** which she allocates between first period consumption, c_1, and accumulation of capital to be used in the second period, k_2. Production and government activity occur only in the second period. The representative agent consumes during both periods but supplies labor, n_2, only during the second period. The utility of the representative agent over the two periods is defined by:

$$V = \ln c_1 + \beta[\ln c_2 + \zeta \ln(1-n_2) + \gamma \ln g_2] \qquad (8.1)$$

where g_2 is government spending that affects utility of agents, β is the psychological discount factor, and $\beta = \frac{1}{1+\rho}$, where ρ is the rate of time preference. The parameters ζ and γ govern the utility weights attached to leisure and government spending, respectively. The production function is linear in its arguments (capital and labor), so that the market clearing conditions for product market are:

$$c_1 + k_2 = \mathbf{y}$$
$$c_2 + g_2 = wn_2 + rk_2 \qquad (8.2)$$

where w and r are marginal products of labor and capital, respectively. The benevolent government's objective is to maximize the welfare of the representative agent, which necessarily is the planner solution or *command* optimum, yielding the first best allocation. This problem can be solved by maximizing *(8.1)* over quantities. Using the market clearing conditions *(8.2)*, the first best allocation is therefore the solution to the problem of choosing $\{c_1, c_2, n_2, g_2\}$ to maximize utility subject to the resource constraint $wn_2 + r\mathbf{y} - rc_1 - c_2 - g_2 = 0$.

The set of first order conditions for this problem yields unique expressions for first best allocation of consumption, labor supply and government spending[9], implying that the period two capital stock is given by:

$$k_2 = \frac{\beta(1+\zeta+\gamma) - r^{-1}w}{1+\beta(1+\zeta+\gamma)} \qquad (8.3)$$

The command optimum therefore would be achieved if the government had available sufficient non-distorting fiscal policy tools (e.g. a single lump sum tax). Consider the case where the government does not have access to such means of financing its expenditures g_2 and must use distorting taxes in the second period. Assume τ and θ are the two tax rates on labor and capital income, respectively. The representative agent's budget constraint for two periods now becomes:

$$c_1 + k_2 = \mathbf{y}$$
$$c_2 = (1-\tau)wn_2 + (1-\theta)rk_2 \qquad (8.4)$$

With the tax rates, government's budget constraint now becomes:

[9] The solution to this problem yields $c_1 = \frac{\mathbf{y}+r^{-1}w}{1+\beta(1+\zeta+\gamma)}$, $c_2 = r\beta c_1, n_2 = \frac{w - r\beta\zeta c_1}{w}$ and $g_2 = r\gamma\beta c_1$. The command optimum allocation of k_2 is then derived by substituting for c_1 in the first market clearing condition.

$g_2 = wn_2\tau + rk_2\theta$ *(8.5)*

Note that the representative agent takes the government spending g_2 as exogenous since she is atomistic and g_2 is predetermined. Hence the representative agent's decisions do not influence economy-wide aggregates. The problem of the representative agent is to choose $\{c_1, c_2, n_2\}$ to maximize utility defined by *(8.1)* subject to the budget constraint $(1-\tau)wn_2 + (1-\theta)r(\mathrm{y} - c_1) - c_2 = 0$. The solution to this problem yields demand and supply functions for consumption for two periods, labor supply and capital stock for period two, where the arguments of the functions are the expected tax rate on first period and actual tax rates on second period. More precisely, the solutions $c_2(\tau,\theta), n_2(\tau,\theta)$ will depend on first period decisions $c_1(\tau^e,\theta^e), k_2(\tau^e,\theta^e)$, and hence will depend on the expected tax rates (τ^e,θ^e).

Given the representative agent's decisions, the government chooses actual tax rates (τ,θ) in the second period to maximize welfare subject to government's budget constraint that incorporates the agent's decision, i.e. $g_2 = \tau\, wn_2(\tau,\theta) + \theta\, rk_2(\tau^e,\theta^e)$. With this set up, the key question then is how expectations of tax rates are formed, and whether the vector (τ,θ) that the government chooses at period two converges with (τ^e,θ^e). If these do not converge, there is an inherent time consistency problem.

As of period two with capital k_2 being accumulated, the government has the incentive to minimize distortions by taxing only capital and leaving labor untaxed. Hence a benevolent government has an incentive to be time inconsistent by announcing a low level of capital taxation ex ante, and once it is believed and capital has been accumulated, taxing it heavily ex post by announcing a surprise capital levy. Put differently, as Drazen (2000, ch. 4) states, "....what makes the phenomenon interesting is that it occurs in cases where time-inconsistent policy is chosen to maximize the welfare of those who are misled. Put simply, the policymaker has an incentive to mislead people for their own good!"

Suppose that there is an effective commitment device, or a commitment technology at the society's disposal, with which it can make the government to commit itself to whatever policy it announced in the first period, so that $\theta = \theta^{e}, \tau = \tau^{e}$. The government chooses (τ, θ) to maximize welfare *(8.1)*, knowing (and considering) the equilibrium reactions of the representative agent. It announces tax rates and carries on the originally announced plan, i.e. it has a mechanism not to re-optimize in the second period when capital has been accumulated. The solution to the government's optimization problem with this assumption emphasizes that the commitment to a policy existed prior to the time period in which a change in policy is the central issue. This solution is popularly known as the *precommitment* solution. The precommitment solution thus suppresses the notion of time inconsistency by assuming existence of a commitment device at the society's disposal.

1.4.2 A Note on Credibility.

The issue of credibility arises generically in dynamic optimal taxation --- a classic example of this issue in *capital-levy* problem is presented in the preceding subsection. The famous paper by Fischer (1980) brings home these points in a very clear way, by contrasting between the ex ante and ex post optimal policies. As Fischer (1980) points out, since without the additional set of constraints the policymaker was already facing a second best equilibrium outcome with the ex ante optimal policy (Fischer defines this second best tax plan as the optimal open-loop policy), an additional constraint of time consistency makes the equilibrium outcome third best. In particular, policies with long run desirable properties will often violate incentive constraints, and they are thus not *credible* and not implementable in equilibrium unless the incentive constraints can somehow be relaxed.

Extending Fischer's idea in an infinite horizon framework, Chari & Kehoe (1990) and Benhabib & Rustichini (1997), in two influential papers, addressed the optimal capital taxation problem by explicitly modelling the trade off between the cost of revising the tax plan and the benefit of the revision under the assumption that the commitment power is not perfect. Chari & Kehoe (1990) studied an infinite horizon version of Fischer's capital taxation model, and defined their time periods so that they can stack one capital accumulation problem into each of them and then assume that capital is non storable between these periods. The paper then asked what equilibria can be sustained by *reputational* forces if the government cannot commit capital taxation ex ante. Adopting the approach suggested in the seminal game theoretic work by Abreu (1988), they manage to characterize completely how the set of sustainable welfare levels depends on the parameters. Not surprisingly, as they point out, a low enough discount rate makes the second best commitment equilibrium (the Ramsey equilibrium) sustainable.

Benhabib & Rustichini (1997) show that the sequence of tax rates under the second best tax plan without commitment has a bang bang feature: capital is typically taxed maximally in the first periods, and then taxes are shifted onto labor. They, like Kydland & Prescott (1977), Fischer (1980) and Chari & Kehoe (1990), also claim that in absence of commitment power, the second best tax plan is time inconsistent. In the early periods it is optimal to announce low tax rates on capital income in the future in order to promote accumulation. When future becomes present and capital has been accumulated, it becomes convenient to do the opposite and tax capital income rather than to impose distorting taxes on labor. Unless the government has some commitment power to bind itself to implement the plan it has announced in the first period, the plan will not be credible. Like most other contributions in this tradition, this paper also assumes that such a commitment power is hardly acceptable. It therefore introduces the idea that the government is aware that a change in plans may have costs, for example, from the point of view of its own credibility. As a result, when commitment is not possible, both the limit tax rate and the steady state capital are different from their levels in the second best solution. Limit taxes on capital may be strictly positive; but it may also be the case that the only sustainable plan has subsidies to capital. The subsidies induce an over accumulation of capital which becomes a commitment device against revisions of the tax plan.

From the discussion on time consistency and credibility presented so far, it is evident that in absence of a commitment device not every tax structure promised in the initial period is credible. As argued by Persson & Tabellini (2002, ch. 12), a credible equilibrium tax structure must satisfy three requirements: (1) Individual economic decisions are optimal, given the expected policies and the decisions of all other individuals in the economy; (2) The tax structure is ex post optimal, given the allocations and individual equilibrium responses to the tax structure after a policy deviation; and (3) Individual expectations are fulfilled and markets clear in every period.

For the optimal taxation models constructed, presented and analyzed in the remaining chapters of this thesis, the assumption that the government can commit to a sequence of tax plans for future will be maintained. The solutions and the Ramsey equilibria emphasized hereafter will therefore be time consistent, and the issue of policy credibility will not be of immediate (or any) concern. This is an innocuous simplification, which in no way limits the analysis to follow. The time consistency and credibility issues of optimal policy have amassed immense popularity amongst economic policy researchers, which in turns has created a distinct family of Macroeconomic and Political Economic literature[10]. It may be acknowledged that relaxing the commitment assumption in the new variants of optimal taxation models presented in chapters two, three and four of this thesis will be an important extension. The extended versions, however, will potentially belong to another distinct rich stream of literature. This is why the current version of this thesis, within its scope, abstracts from these extensions.

1.5 The Current State of the Art.

The remaining chapters of this thesis will emphasize mainly on optimal income taxation principles in variants of neoclassical general equilibrium frameworks with infinite horizon. The review of current state of the art presented in this section therefore will highlight the facts and literature that are relevant to neoclassical dynamic general equilibrium frameworks of optimal income taxation. The models developed in chapters two, three and four actually infer the government's policy of

[10] The issue has also been popular in the fields of Game Theory, Financial Economics and other branches of modern economic theory where some form of dynamic decision making is involved. For a detailed survey of the time consistency problem of optimal policy, see Persson, Persson & Svensson (1987), Abreu (1988), Chari & Kehoe (1990), Stokey (1991), Karp & Lee (2000), Drazen (2000, ch. 4 & 5), Persson & Tabellini (2002, ch. 12) and Alvarez, Kehoe & Neumeyer (2004), among others.

choosing average effective tax rate (AETR, hereafter) on labor and capital income[11]. AETR for capital, labor and consumption is empirically estimated for OECD economies by Mendoza *et al.* (1994), and later by Carey & Tchilinguirian (2000). In deriving AETR, both these studies basically link the realised tax revenues directly to the relevant macroeconomic variables in the national accounts. Understandably, since these estimates take into account the effective overall tax burden from the major taxes, the approximations are consistent with the concept of tax rates affecting national aggregates and the assumption of a representative agent. These tax rate estimates are therefore useful approximations to the taxes that distort economic decisions in dynamic models.

The celebrated Chamley-Judd result of long run zero capital income taxation has been the central focus of optimal income taxation literature, and discussions in the remainder of the thesis will evolve around this as a benchmark result. Switching to Ramsey policy, with this finding being the core of calibrated welfare effects computation, has been high on the agenda of the US and other OECD economies' tax reform. But how much weight one should attach to reviewing capital tax rates in redesigning or reforming a tax system? If the fundamental structure of income taxes from country to country is broadly the same, is the tax bill for taxpayers in more or less the same position same? Moreover, if a calibrated model for post war US data suggest that switching to Ramsey policy is associated with remarkable welfare gains, should this result generalize tax reforms of other industrialized economies? Answering these questions from a class of purely theoretical models are more likely to be an impulsive task. It is, therefore, useful to present a summary of the current tax systems in practice followed by the current stream of relevant literature.

[11] It is possible to introduce richer tax codes and hence abstract from finding optimal average effective income tax rates in these general equilibrium environments. This is suggested by Jones *et al.* (1997), and applied in a calibrated version of a general equilibrium model by Guo & Lansing (1999). For instance, as in Guo & Lansing (1999), one may introduce accelerated depreciation allowance on capital by introducing a non-negative parameter ϕ on the tax bill of the households, such that the tax bill with usual notations becomes $\tau_t w_t n_t + \theta_t (r_t - \phi\delta) k_t$. Under such a set up, the labor income tax rate is the average effective labor income tax rate, but the capital income tax rate θ is not necessarily.

1.5.1 Current Tax Systems.

Salanie (2002, pp. 2-8) presents a detailed discussion of the historical evolution of tax systems of leading economies of the world. The survey presented here is rather moderated and more focussed towards the *current* practice of taxation.

Table 1.1 presents the breakdown of tax revenue into its main components in five major economies of the world. Some minor taxes are omitted, and hence one should not add these percentages to an aggregate of 100. Over 80% of tax revenue generally come from three sources, namely, (1) income taxes, including personal income tax (PIT) and corporate income tax (CIT); (2) taxes on goods and services, including general consumption taxes such as Value Added Tax (VAT), or Goods & Services Tax (GST), and taxes on specific goods and services (mainly excise and custom duties), and (3) social security contributions (SSC). The fourth important component of current tax systems is the tax on property. Payroll taxes and other taxes are negligible in most countries.

Table 1.1: Components of Tax revenue (in per cent) in leading economies, 1999.

	PIT	CIT	SSC	Property	Goods & Services
USA	40.7	8.3	23.9	10.7	16.4
UK	26.2	7.2	24.5	11.0	30.9
Japan	18.5	12.9	37.2	11.0	20.1
OECD	26.3	8.8	26.1	5.4	31.7
EU	25.6	8.7	28.6	4.9	30.4

Source: OECD Revenue Statistics.

The revenue generated from each category varies widely across countries (and subset of countries). Consider first the share of revenue generated from *direct* taxation, corresponding to the columns PIT, CIT, SSC and Property. Income taxation and social security contributions in these countries are quite similar because of the tax base they use. Personal income tax is the largest source of revenue from *income* taxation in OECD countries, accounting for 26.3% of total tax revenue in 1999, just ahead of

26.1% in social security contributions. While the US government receives 41% of its tax revenue in personal income tax, in Europe and Japan social security contributions account for the major share, at 29% and 38% respectively. Japan has the highest share of corporate income tax, at 13%, compared to approximately 9% in EU, the US and an average OECD level. Japan also has the highest share of social security contributions. Social security contributions were the main source of general government revenue in seven OECD countries in 1998-99, namely, Austria, the Czech Republic, France, Germany, Japan, the Netherlands and Spain. Australia and New Zealand do not collect social security contributions at all, but manage social spending from core taxation. Property taxes accounted for 10% or more of revenue in the US and Japan. Revenue share of Property taxes are generally lower in Europe, at 5%, although in the UK the figure is quite high, at 11%.

There has been an overall shift in the last four decades towards general consumption taxes like VAT or GST, more at the expense of other taxes on goods and services (like excise duties) than personal and corporate income taxes. This change reflects an acceptance that broad based consumption taxes are less distorting, more feasible and more effective in raising revenue. The OECD on an average has a relatively larger reliance on consumption tax, at 32% of the total revenue compared to 31% in the EU (and in the UK), 20% in Japan and 17% in the US. Consumption tax is the single most important tax in the UK, Hungary, Iceland, Korea, Mexico, Norway, Poland and Turkey. The average standard rate of VAT was 12.5% when introduced, as compared to the current average rate of 17.5%. The EU countries generally tend to rely more on consumption taxes and social security contributions than on personal income tax. The US collects more in personal income tax and social security contributions. Japan falls somewhere between, with a low share of consumption and personal income taxes, but more reliance on corporate tax and social security contributions.

The fundamental structure of personal income tax systems imposed by central governments is very similar in every OECD country. A certain amount of income may be exempted from tax. An alternative system is to tax all income, and give all taxpayers a reduction in their tax bill in the form of a *basic tax credit*. Despite such similarities in the structure of income tax system, the tax bill for taxpayers in more or less the same position may be quite different. One major reason is that different countries provide different tax relief to their citizens. For instance, of the average worker's gross wage, Greece exempts only 3%, Korea 7%, the Netherlands 14% and France 20%. The UK and the US offer relief to the tune of 24% of the average wage. Income above the personal exemption is generally divided into brackets, and all income belonging to one particular bracket is taxed at the same rate. The rate applied to the income in successive brackets increases resulting in a progressive income tax system. The top marginal income tax rates, i.e. the highest percentage of tax imposed on every additional dollar, sterling, yen or euro earned above standard taxable income levels varies remarkably across countries. The top marginal rate of personal income tax levied by central government ranges from 25% in Sweden and 33% in New Zealand to as much as 60% in the Netherlands. In Austria, Belgium, Canada, Finland, France, Germany, the Netherlands and the UK, workers must earn about *twice* the average before they start paying the top rate. On the other hand, Swiss and the US employees are not confronted by the top rate unless their salaries reach *ten times* the average production worker's wage.

In recent years most countries' top marginal rates of income tax have been reduced. However, while top rates of personal income tax have come down, personal income tax revenues have not moved. Across the OECD economies, share of personal tax revenues in GDP was 10.3% in 1999 compared to 10.5% in 1980. This is largely due to two reasons, namely, strong economic growth which elevated taxpayers into higher tax brackets, and many governments partly financed their rate reductions contemporaneously by reducing permissible deductions against taxable income. Apart from cutting top rates, the number of tax brackets in OECD countries has been reduced. This is in pursuit of making the tax system easier to manage and understand for both taxpayers and administrators. Trends in corporate income tax have followed personal income tax. Various incentive schemes including investment credits and property related tax shelters have been moderated or abolished in numerous countries such as Australia, Austria, Finland, Germany, Iceland, Ireland, Portugal, Spain and the US. Also, several countries have revised the allowances for depreciation of capital equipment that companies can use to cut down on taxable income, bringing them nearer to the actual reduction in the economic value of the equipment. Still, corporate profits and personal capital income (dividends, interest etc.) are generally less heavily taxed than labor income in the OECD area and the EU, mainly because of social security contributions.

Table 1.2: Average Effective Tax Rate Estimates (in per cent), 1991-97.

	Capital Tax[a]		Labor Tax		Consumption Tax	
	Mendoza *et al.* (1994)	Carrey & Tchilinguirian (2000)	Mendoza *et al.* (1994)	Carrey & Tchilinguirian (2000)	Mendoza *et al.* (1994)	Carrey & Tchilinguirian (2000)
USA	27.3	31.1	26.7	22.6	5.2	6.1
UK	31.9	38.4	23.7	21.0	16.7	16.9
Japan	24.1	32.6	28.3	24.0	6.0	6.7
OECD	22.0	26.6	36.8	33.4	16.5	17.1
EU	21.2	25.1	42.8	36.8	19.3	18.7

a: These estimates are based on gross operating surplus.
Source: Carey & Tchilinguirian (2000). 'Average Effective Tax Rates on Capital, Labor and Consumption', ***OECD Economics Department Working Papers No. 258.***

Table 1.2 presents AETR estimates using both Mendoza *et al.* (1994) and Carey & Tchilinguirian (2000) methodology, as reported in Carey & Tchilinguirian (2000). As mentioned earlier, the concept and approximations of these rates directly correspond to the tax rates considered in standard representative agent dynamic general equilibrium models.

The AETR for capital includes corporate profit taxes, taxes on household capital income and various property taxes. All income generated from labor, social security charges (excluding employers' contribution to private pension funds) and payroll taxes are allocated to AETR for labor. The approximations from both methodology reflect that the AETR for labor and capital are much higher than AETR for consumption in all five major economies. The EU on an average has the highest AETR for labor and consumption, while the UK has the highest AETR on capital. All AETR for capital estimates are far away from zero. For the US and Japan, AETR on consumption is remarkably low.

The actual tax bill of individual taxpayers also reflects the impact of various deductions such as mortgage interest and employee contributions to occupational pension plans, and exemptions such as capital gains or interest received. This means that AETR in countries with lower statutory rates but little in the way of basic relief, deductions and exemptions, could well be higher than effective rates in countries which combine higher statutory rates with much more generous exemptions and deductions. This holds for AETR for capital income and consumption also.

1.5.2 The Current Stream of Literature.

The literature concerning optimal income taxation with commitment in dynamic general equilibrium frameworks, as may be clear by now, has evolved in two main directions over the last two decades[12]. The stream of literature that deals with optimal taxation principles in standard neoclassical growth models embarks mainly on the infinite horizon framework. As highlighted previously, the most celebrated result in these studies is that an optimal income tax policy entails taxing capital at confiscatory rates in the short run and setting capital income taxes equal to zero in the long run. Put intuitively, this result implies that since capital tax compounds over time, only labor income should be taxed in the long run to support uniform pattern of long run distortion in the economy. On the other hand, most applied studies concerned with dynamic impact of fiscal policy use the life-cycle framework.

A large division of the relatively more recent stream of literature on optimal taxation with commitment presents variants of general equilibrium models with micro foundations of individual behaviours and/or market structure mainly to provide theoretical justifications for a nonzero limiting capital tax. From a somewhat different perspective, these models are useful illustrations that reinforce the robustness of the zero capital tax result in simpler standard settings. It is interesting to note the extent of variations in standard assumptions made and/or the ideas and dimensions of extensions undertaken in these general equilibrium models, which in turns make the literature diverse, stimulating and competitive.

[12] See Chari & Kehoe (1999) for an excellent technical coverage of the established models of optimal taxation with commitment in (static and) dynamic general equilibrium frameworks where the problem is addressed using the primal approach. For a rather non-technical yet comprehensive survey, see Erosa & Gervais (2001). For an earlier survey which may be useful in realizing the evolution and emergence of the literature on optimal taxation, see Sandmo (1976).

Examples of important papers of such kind appeared during the last decade, and their key assumptions underlying the particular framework include, but not limited to, (1) Aiyagari (1995), borrowing constraints, incomplete markets and constant discounting; (2) Judd (1997/2003), imperfect competition; (3) Jones *et al.* (1997), endogenous growth with richer tax code; (4) Ha & Sibert (1997), strategic capital taxation with mobile capital; (5) Diamond (1998), U-shaped optimal marginal tax pattern; (6) Cassou & Lansing (1998), public capital and productivity slowdown; (7) Guo & Lansing (1999), imperfectly competitive product market and richer capital tax code; (8) Aronsson & Sjogren (2001), unionized wage bargaining; (9) Koskela & Thadden (2002), monopolistic wage bargaining; (10) Barreto & Alm (2003), heterogeneous agents, corruption and growth; (11) Golosov, Kocherlakota & Tsyvinsky (2003), private information of agents' skills; (12) Sleet (2004), private government information; (13) Schmitt-Grohe & Uribe (2004a & b), imperfect competition & sticky prices; (14) Hassler *et al.* (2004), optimal timing of capital taxes; and (15) Abel (2005), endogenous benchmark consumption level. The remaining chapters of this thesis complement this stream of literature.

Numerical approximations of welfare losses due to distorting taxes and potential welfare gains from switching to Ramsey policy in economies with commitment has also embarked as important parts of the relevant literature. Recent contributions of such kind include, and again are not limited to, Ortigueira (1998), Coleman II (2000), Kim & Kim (2004), Turnovsky (2004) and Jonsson (2004). The current stream of literature concerning optimal taxation without commitment and the general time consistency problem associated with optimal fiscal policy include important papers such as (1) Benhabib & Rustichini (1997), without commitment; (2) Phelan & Stacchetti (2001), sequential equilibria in a Ramsey tax model; (3) Ortigueira (2003), instantaneous and non-instantaneous commitment; (4) Klein, Quadrini & Rios-Rull (2003), international mobility of capital, and (5) Alvarez *et al.* (2004), time consistency of fiscal and monetary policy.

1.6 Concluding Remarks.

This chapter has presented an asymptotically detailed review of how the primal approach to the classic optimal taxation problem is used in both static and dynamic general equilibrium frameworks to find optimal levels of tax rates. In doing so it has resorted to stylized models established in relevant literature. This approach draws a number of substantive lessons for policymaking, namely, *(a)* Taxing necessities heavily relative to taxing luxuries; *(b)* steady state zero capital taxation, and *(c)* roughly constant levels of labor income tax. A simple calibration of the one-sector neoclassical growth model using post war US economy's data is presented as an illustration of the numerical methodology most commonly used to characterize the relevant steady states of these models. The chapter also presents a non-technical introduction to the time consistency and credibility issues associated with optimal fiscal policy in the absence of an effective commitment device. Finally, a brief survey of the current tax systems and current state of the literature are presented.

Conceivably the most startling finding of this literature is that the optimal steady state tax on physical capital is equal to zero, which is robust in both government bond-financed and period by period balanced budget environments. The extension to endogenous growth with human capital formation illustrates that for a rather restricted class of preferences and technology all taxes can be set equal to zero in the long run. While this contribution by Jones *et al.* (1997) is of vital importance for many reasons, the zero tax result on human capital is established with more restrictions relative to the number of restrictions required to establish the zero tax result for physical capital. In this sense, physical capital and its taxation are special and hence deserve special attention.

The models presented and their implied results are based on a crucial assumption that the economy is characterized by competitive markets. Environments popular in literature that establish steady state nonzero capital tax necessarily resort to changing basic working assumptions, and the one commonly altered assumption is of perfect competition (see for instance, Judd (1997/2003) and Guo & Lansing (1999)).

Researchers motivated by empirical evidence on labor market characteristics have shown that monopolistic wage setting and/or unionized wage bargaining can induce the nonzero capital tax result (see for instance, Aronsson & Sjogren (2001) and Koskela & Thadden (2002)). The main theme of this extension is to introduce a larger array of distortions in the economy and hence observe the interaction between tax and non-tax distortions.

The results highlighted in this survey are derived for environments without such imperfections or any other form of distortion in private markets. In models with private market imperfections, optimal policy not only must be responsive to efficiency concerns but also must attempt to cure the imperfections. Such corrective functions of optimal taxes support the policy of nonzero tax on capital income. But even with economy-wide competitive markets assumption, chapter two of this thesis shows that capital tax rate for a particular sector can be nonzero in the steady state. Moreover, any restriction on the government's ability to independently tax factor income according to sector may lead to nonzero capital taxes.

A seemingly sensitive abstraction this survey (and the remainder of the thesis) has made is from the literature on optimal fiscal policy over the business cycles, or more technically, optimal fiscal policy in a stochastic economy (as in Chari, Christiano & Kehoe (1994), for instance). Studies of such kind highlight the indeterminacy of state-contingent debt and capital taxes and prescribe debt taxation as a shock absorber. Chari *et al.* (1994) numerically estimate the time period required to reach the steady state level of zero capital tax in the presence of stochastic business cycle effects --- an allegedly important task often beyond the scope of deterministic settings. Put more intuitively, optimal policy from deterministic general equilibrium settings typically entails taxing capital income at confiscatory rates in initial periods and then setting it equal to zero in the steady state. But the length of the initial period is often simplified and the transition is left unclear in numerical estimations. In light of the key findings of this thesis, such experiments would certainly be interesting follow ups.

Chapter 2

Optimal Taxation in a Two-sector Neoclassical Economy[13]

2.1 Introduction.

Contributions to the literature on optimal taxation of factor income in dynamic settings, ever since its advancement and sophistication, has established and endorsed a set of substantive results. In the competitive equilibrium version of the standard neoclassical growth model with infinitely-lived individuals, Judd (1985) and Chamley (1986) establish that an optimal income tax policy entails taxing capital at confiscatory rates in the short run and setting capital income taxes equal to zero in the long run. Over the last two decades, this result has been rated as one of the most important contributions of dynamic Ramsey taxation.

[13] A version of this chapter, titled "Optimal Taxation with Commitment in a Two-sector Neoclassical Economy" was presented in Royal Economic Society Annual Conference 2005 in Nottingham, UK.

An earlier version of the same paper was presented in Royal Economic Society Easter School 2004 in Birmingham, UK, and in Southampton University Staff Seminar Series in November 2003.

On a special note, I thank Akos Valentinyi for the idea of extending the one-sector simple model of optimal taxation to a multi-sector one.

The zero capital tax result has a strong underlying intuition. A positive tax on the return from current period's savings effectively makes consumption next period more expensive relative to consumption in the current period. This distortion grows exponentially over time, and such exploding distortions are not optimal. The long run optimal policy therefore should not involve revenue-raising nonzero tax on capital income. In a one-sector production economy model with infinitely-lived representative agent, Judd (1985) shows that a positive tax on capital income in the steady state implies that the implicit tax rate of consumption in future has an unbounded increasing trend. A current period nonzero tax on capital income therefore induces a compounding form of distortion on future allocations which is unlike the uniform distortion created by period by period labor income tax, for instance.

This chapter approaches the Ramsey problem in a dynamic general equilibrium framework of a two-sector neoclassical economy, and investigates the steady state optimal choice of tax rates for capital income generated from two production sectors. Problems related to time consistency of optimal plans are suppressed by assuming that there is an effective commitment device which binds the government to continue with its initially announced tax plans. More specifically, this chapter proposes a general framework of a neoclassical model which comprises of infinitely-lived agents with identical preferences, two factors of production which are labor and physical capital, two production sectors producing consumption goods and new capital goods, and a benevolent government with preset revenue target to finance its consumption, linear income tax instruments to furnish the expenditure, and an effective commitment device to restrict itself from changing initially announced policies. It follows the primal approach to optimal taxation and examines the steady state properties of the optimal capital income tax rules.

The striking result of this chapter is, while it is optimal to tax capital income from capital goods sector at a zero rate, the steady state capital income tax for consumption goods sector is in general nonzero. This nonzero capital income tax is sustainable in the Ramsey equilibrium, since it would not create distortions that compound over time. This is because along the transition to the steady state economic agents have the option of shifting depreciated capital good to the sector where its income is untaxed and thus avoid the compounding tax liabilities. Thus, the long run optimal policy may involve nonzero tax on capital income from consumption goods sector as long as the capital income tax is zero in the sector producing capital goods.

This chapter also shows that the set of optimal policies at the government's disposal for which competitive equilibrium exists implies conditional convergence of zero steady state capital income tax for the consumption goods sector. The chapter considers some specific utility functions in order to characterize the set of optimal policies, and thus identifies the set of conditions for which the steady state capital income tax for consumption goods sector converges to zero. Neither the model nor a specific economic intuition guarantees the fulfilment of these conditions. Hence, the celebrated Chamley-Judd result of zero steady state tax on capital income cannot be unconditionally generalized for a class of neoclassical models.

In addition, this chapter considers the case where the government faces an *ex ante* constraint of keeping the two labor income tax rates and the two capital income tax rates equal, i.e. the constraint that the factor income tax rates are not sector-specific. There is a strong reason why this experiment is important. Without such a constraint, the benchmark model prescribes that two different steady state capital income tax rates can be sustained in a Ramsey equilibrium, one of which should be kept zero in the long run. The key intuition is that a nonzero tax on capital income from consumption goods sector is optimal since it creates uniform distortions as long as the other capital tax instrument is set at zero. This is because since shifting capital is costless, households can shift the depreciated capital at end of each period to the sector where capital income is not taxed (capital goods sector) and avoid the compounding distortions of capital tax. The nonzero capital income tax in the consumption goods sector becomes, in terms of consequences, a tax which has uniform distortion pattern, similar to a period by period consumption tax, for example.

While this is theoretically proven to be the optimal sustainable choice of capital income tax rates for the government, it may be subject to criticism from a real world point of view. Governments in the real world often face the constraint of keeping income tax rates same irrespective of production and investment sectors. Put differently, and more practically, real world political economy considerations often restrict governments from adopting sector based taxation. It is shown that imposing such a constraint in the Ramsey problem results in a fiscal policy outcome with lower welfare, implying that restricting the government's choice of income tax rates is pareto-worsening. Restricting the government's choice of income taxes ex ante triggers an outcome with both nonzero capital income tax rates which induces lower welfare. Since capital tax compounds over time, this outcome cannot be optimal.

Chamley (1986) shows that with a steady state policy of zero capital taxation in the scheme, it is possible for the government to announce high tax on capital income in period 0. With no exogenous bounds on the magnitude of tax rates, this initial high tax rate on capital income may even be confiscatory. This chapter endorses this finding by examining the optimal policy for taxing capital income in the initial period.

The way the tax structured is modelled in this chapter implies that its key focus is on the income tax policy of the government. Consumption taxes and labor income taxes are equivalent in a representative agent framework since they distort exactly the same margin. The optimal policy for labor income taxation as implied by this chapter is reminiscent of the optimal policy for consumption taxation. Abstraction from incorporating consumption taxes in the tax structure therefore does not limit the scope of the underlying model or the analysis.

The set of policies which generates allocations that can be implemented as competitive equilibrium, as this chapter advocates, prescribes that the optimal steady state capital income tax for capital goods sector is unambiguously zero, but the steady state optimal capital income tax for consumption goods sector is only conditionally zero. The set of conditions for which the celebrated Chamley-Judd result can be established are neither inferred by the model nor justified by simple intuitions. This is verified by three experiments using variants of commonly used utility functions. The experiments suggest that the nonzero capital tax result holds for a wider class of utility functions with standard properties, irrespective of separability and marginal rate of substitution of labor across sectors, intratemporal labor adjustment costs and varying types of labor. The three examples considered in this chapter are in the spirits of Herrendorf & Valentinyi (2003), Huffman & Wynne (1999) and Jones *et al.* (1997).

The already enriched literature on optimal taxation is diverse in both the models analyzed and the types of fiscal experiments undertaken. No matter how diverse the modelling approaches and spotlights of these studies are, three major findings appear from the solution to the Ramsey problem in representative infinitely-lived agent models. The famous chapter by Chari & Kehoe (1999) presents a comprehensive survey of these findings in variants of environments. Of these, this chapter's focus is on the one that states capital income should not be taxed in the long run. The second interesting finding is that tax rates on labor income may be nonzero in the limit but should be roughly constant, and in no circumstances should be confiscatory. The third motivating finding from the literature on optimal taxation in infinitely-lived agent models stems directly from the time inconsistency of optimal policies. The fact that an optimal capital income tax plan can be time inconsistent was established seminally by Kydland & Prescott (1977). There has been a marked enthusiasm in relatively recent literature on political economics addressing such issues, which are logically relevant to fiscal policy choice of governments (see for instance, Persson *et al.* (1987), Chari & Kehoe (1990), Stokey (1991), Benhabib & Rustichini (1997), Phelan & Stacchetti (2001) and Alvarez *et al.* (2004), among others).

This particular chapter, therefore, belongs more to the tradition of Jones *et al.* (1993 & 1997), and is intended to complement the same dynasty. In the remainder of the chapter, section 2.2 presents the model and competitive equilibrium; section 2.3 addresses the Ramsey problem using the primal approach, derives the optimal capital income tax rules, and presents the constrained tax choice experiment; section 2.4 presents examples of utility functions and thereby characterizes the optimal steady state capital income tax rates; section 2.5 concludes.

2.2 The Economy.

To my knowledge, the prototype version of the two-sector neoclassical model was primarily proposed by Uzawa (1963 & 1964) and Srinivasan (1964) to examine growth process and stability properties of the balanced growth equilibria. These studies considered standard neoclassical framework with two factors of production simultaneously in operation in two production sectors that produce perishable consumption goods and new capital goods, and focused on growth and equilibrium properties with varying factor intensities. Fiscal policy under multi-sector neoclassical models of endogenous growth was examined primarily by Rebelo (1991), Jones *et al.* (1993) and Stokey & Rebelo (1995), followed by contributions such as Jones *et al.* (1997) which introduce a labor-leisure choice and an internal human capital accumulation process.

In this chapter, the following dynamic general equilibrium environment is considered. Time t is discrete, runs forever, and t belongs to the set of integers $N = \{0,1,2,.....\}$. The economy has two production sectors indexed by j, where j = *C, X* denotes the consumption goods and capital goods sector, producing perishable consumption goods and new capital goods, respectively. There is a continua of measure one of identical infinitely-lived households, of identical firms in sector *C* that own a technology with which a perishable consumption goods can be produced, and of identical firms in sector *X* that own a technology with which new capital goods can be produced. The representative household is endowed with initial capital stock, the property rights of the representative firms, and one unit of time at each period. Firms combine capital and labor, the two factors of production, for final production. At each point in time, four commodities are traded in sequential markets: the consumption good, new capital good, working time in sector *C*, and working time in sector *X*.

All households have identical preferences over intertemporal consumption and labor services. The representative household derives utility from consumption (c_t) and disutility from effort given in terms of labor units in the two sectors of production (n_{ct} and n_{xt} in sectors *C* and *X*, respectively) at all time t, such that household's preferences for consumption and labor service streams $\{c_t, n_{ct}, n_{xt}\}_{t=0}^{\infty}$ can be defined by the utility function over infinite horizon:

$$\mathrm{U}(c_0, c_1, ..., n_{c0}, n_{c1}, ..., n_{x0}, n_{x1}, ...) = \sum_{t=0}^{\infty} \beta^t u(c_t, n_{ct}, n_{xt}) \qquad (1)$$

where $\beta \in (0,1)$ is the subjective discount rate.

Assumption 1: The current period utility function $u : \mathbf{R}^3_+ \to \mathbf{R}$ is continuously differentiable, strictly increasing in consumption, decreasing in labor, strictly concave, and satisfies Inada conditions, namely:

$$\lim_{c_t \to 0} \frac{u_c(t)}{u_{n_j}(t)} = \infty, \quad \text{and} \qquad \lim_{c_t \to \infty} \frac{u_c(t)}{u_{n_j}(t)} = 0,$$

for $n_j > 0$ where j = *C*, *X*. □

The household purchases new capital goods and rents capital to the firms for one period. Capital decays at the fixed rate $\delta \in (0,1)$. Firms return the rented capital stock next period net of depreciation δ, and pay unit cost of capital employed r_c and r_x, for capital stocks employed in sector *C* and *X*, respectively. Firms own nothing except the technologies; they hire labor and capital on a rental basis, sell the output produced back to households, and return profits to shareholders. The technology for the representative firm in sector *C* producing consumption goods for (private) consumption, c_t, and exogenously determined government consumption, g_t, for all time t is:

$$c_t + g_t \leq F^c(k_{ct}, n_{ct}) \tag{2.1}$$

and the technology for the representative firm in sector *X* producing new capital goods is:

$$x_{ct} + x_{xt} \leq F^x(k_{xt}, n_{xt}) \tag{2.2}$$

where x_{ct} and x_{xt} are the investments in the two sectors.

Assumption 2: The technology $F^j(k_{jt}, n_{jt})$ exhibits Constant Returns to Scale (CRTS), with $F^j : \mathbf{R}^2_+ \to \mathbf{R}_+$ continuously differentiable, strictly increasing, strictly concave in both *k* and *n*, and satisfies Inada conditions, namely:

$\mathbf{lim}_{k_{jt}\to 0} F^{j}_{kj}(t)=\infty$ and $\mathbf{lim}_{k_{jt}\to \infty} F^{j}_{kj}(t)=0$ for all $n_j>0$, for j = C, X.

□

The level of government expenditures is considered to be given (following the optimal taxation tradition), and the expenditures program is assumed to converge to a constant level when time goes to infinity. The government finances the exogenous stream of consumption expenditures $\{g_t\}_{t=0}^{\infty}$ solely by taxing income from capital and labor employed in both sectors. Throughout the chapter, the assumption that the government has access to some commitment device, or a commitment technology that allows the government to commit itself once and for all to the sequence of tax rates announced at time 0, is maintained. In other words, the commitment technology prevents the government from revising the path of fiscal instruments over time. This assumption allows one to avoid the general time inconsistency problem of optimal policies in dynamic settings. The benevolent government therefore is assumed to seek a tax system that provides revenues to finance g_t and to maximize household's welfare defined by *(1)*.

The government taxes labor income and capital income from sector j, with j = C, X, at rates τ_t^j per unit and θ_t^j per unit, respectively. There are no government bonds in the economy, implying that the government runs a balanced budget each period. The set of analytical results this chapter focuses on are insensitive to this assumption, which can be reconfirmed if one examines a one-sector bond economy analogue (see for instance, Chamley (1986) and Ljungqvist & Sargent (2000, ch.12)). The government's budget constraint for all time t can be written as:

$$g_t=\tau_t^c w_{ct}n_{ct}+\tau_t^x w_{xt}n_{xt}+\theta_t^c r_{ct}k_{ct}+\theta_t^x r_{xt}k_{xt} \qquad (3)$$

where w_{jt} is the before tax return on per unit labor employed and r_{jt} is the before tax return on per unit capital employed in sector j, with j = C, X. Initially, the government announces the program of tax rates and its expenditures. The representative household and firms are endowed with perfect foresight and behave competitively. Under the assumption of commitment technology, the announced program of tax rates cannot be changed at a later date.

2.2.1 Household's Problem.

For the remainder of the chapter, the consumption good will be treated as the numeraire. Let p_{jt} denote the relative price of a new capital good to be used in sector j with $j = C, X$. With CRTS technology in both sectors, competitive equilibrium profits are zero (and will be ignored in household's budget constraint).The representative household's problem can be illustrated as program (4), as follows:

$$\max_{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_{ct}, n_{xt}) \tag{4.1}$$

$s.t.$

$$c_t + p_{ct}x_{ct} + p_{xt}x_{xt} \leq (1-\theta_t^c)r_{ct}k_{ct} + (1-\theta_t^x)r_{xt}k_{xt} + (1-\tau_t^c)w_{ct}n_{ct} + (1-\tau_t^x)w_{xt}n_{xt} \tag{4.2}$$

$$x_{ct} = k_{ct+1} - (1-\delta)k_{ct} \tag{4.3}$$

$$x_{xt} = k_{xt+1} - (1-\delta)k_{xt} \tag{4.4}$$

$$k_{c0} > 0\,, \qquad k_{x0} > 0 \qquad (given) \tag{4.5}$$

Using *(4.3)* and *(4.4)* to substitute for x_{ct} and x_{xt} , and defining $R_t^j \equiv [p_{jt}^{-1}(1-\theta_t^j)r_{jt} + (1-\delta)]$, the household's budget constraint can be rewritten as:

$$c_t + p_{ct}k_{ct+1} + p_{xt}k_{xt+1} \leq (1-\tau_t^c)w_{ct}n_{ct} + (1-\tau_t^x)w_{xt}n_{xt} + p_{ct}k_{ct}R_t^c + p_{xt}k_{xt}R_t^x \tag{4.2a}$$

The representative household's problem can now be illustrated as the problem of maximizing utility subject to *(4.2a)* and *(4.5)*. With $\beta^t\lambda_t$ as the Lagrange multiplier on time t budget constraint, the first order conditions for the household's maximization problem are the period budget constraints *(4.2a)* along with the followings:

$$c_t \quad : \quad u_c(t) = \lambda_t \qquad (5.1)$$
$$n_{ct} \quad : \quad u_{nc}(t) = -\lambda_t(1-\tau_t^c)w_{ct} \qquad (5.2)$$
$$n_{xt} \quad : \quad u_{nx}(t) = -\lambda_t(1-\tau_t^x)w_{xt} \qquad (5.3)$$
$$k_{ct+1} \quad : \quad \frac{\lambda_t}{\lambda_{t+1}} = \beta\frac{p_{ct+1}}{p_{ct}}R_{t+1}^c \qquad (5.4)$$
$$k_{xt+1} \quad : \quad \frac{\lambda_t}{\lambda_{t+1}} = \beta\frac{p_{xt+1}}{p_{xt}}R_{t+1}^x \qquad (5.5)$$

and the Transversality conditions that put a restriction on the terminal value of the household's capital stocks in terms of utility:

$$\lim_{t\to\infty}[\lambda_t p_{jt} k_{jt+1}] = 0 \qquad \text{for } j = C, X. \qquad (5.6)$$

The Transversality condition implies that the discounted lifetime utility is maximal when the terminal value of the capital stock in sector j, with $j = C, X$, is zero. Consolidating the first order conditions yields:

$$\frac{u_c(t)}{u_c(t+1)}\left(\frac{p_{ct}}{p_{ct+1}}\right) = \beta R_{t+1}^c \qquad (5.4a)$$
$$\frac{u_c(t)}{u_c(t+1)}\left(\frac{p_{xt}}{p_{xt+1}}\right) = \beta R_{t+1}^x \qquad (5.5a)$$

that imply

$$\frac{R_{t+1}^c}{R_{t+1}^x} = \frac{p_{ct}}{p_{xt}}\left(\frac{p_{xt+1}}{p_{ct+1}}\right) \qquad (5.7)$$

$$u_{nc}(t) = -u_c(t)(1-\tau_t^c)w_{ct} \qquad (5.2a)$$
$$u_{nx}(t) = -u_c(t)(1-\tau_t^x)w_{xt} \qquad (5.3a)$$

that imply

$$\frac{u_{nc}(t)}{u_{nx}(t)} = \frac{(1-\tau_t^c)w_{ct}}{(1-\tau_t^x)w_{xt}} \qquad (5.8)$$

These equations are fairly intuitive. Equation *(5.7)* is a no-arbitrage condition that combines the two Euler equations (*5.4a & b*) from the solutions to the household's maximization problem. The Euler equations state that household's one period ahead capital stock decision that maximizes utility is determined at the point where the household is indifferent between consuming today and saving for a later date. Equation *(5.8)* is a static optimization equation that implies that the representative household will maximize its utility at the point where its marginal rate of substitution of labor across sectors is equal to the relative after tax wage rate of the two sectors. As long as assumptions 1 and 2 are valid, these equations represent the necessary and sufficient conditions for a maximum.

2.2.2 Firms' Problems.

Since the representative household's preferences are strictly monotone and all factors have strictly positive marginal products, $p_{jt} > 0$, $r_{jt} > 0$, and $w_{jt} > 0$, for all time *t*, for sector *j* = *C, X*. In sector *C*, problem of the representative firm producing consumption goods is:

$$\max_{k_{ct}, n_{ct}} \quad [c_t + g_t - r_{ct}k_{ct} - w_{ct}n_{ct}] \tag{6.1}$$

$$s.t. \quad c_t + g_t \leq F^c(k_{ct}, n_{ct}) \tag{6.2}$$

$$0 \leq c_t, g_t, k_{ct}, n_{ct} \tag{6.3}$$

$$g_t = \overline{g}_t \tag{6.4}$$

Competitive pricing ensures that returns are equal to their marginal products. The necessary and sufficient conditions for the maximization problem are, therefore:

$$r_{ct} = F^c_{kc}(t) \tag{6.5a}$$

$$w_{ct} = F^c_{nc}(t) \tag{6.5b}$$

In sector *X*, problem of the representative firm producing new capital goods is:

$$\max_{x_{ct}, x_{xt}, n_{xt}, k_{xt}} \quad [p_{ct}x_{ct} + p_{xt}x_{xt} - r_{xt}k_{xt} - w_{xt}n_{xt}] \tag{7.1}$$

$$s.t. \quad x_{ct} + x_{xt} \leq F^{x}(k_{xt}, n_{xt}) \tag{7.2}$$

$$0 \leq x_{ct}, x_{xt}, k_{xt}, n_{xt} \tag{7.3}$$

With competitive pricing, and for ℓ_t as the Lagrange multiplier associated with the problem, necessary and sufficient conditions for the maximization problem are:

$$p_{ct} = p_{xt} = \ell_t \tag{7.4a}$$

$$r_{xt} = \ell_t F_{kx}^{x}(t) \tag{7.4b}$$

$$w_{xt} = \ell_t F_{nx}^{x}(t) \tag{7.4c}$$

Accordingly, I will simplify the model by denoting $p_{ct} = p_{xt} = p_t$ hereafter. Hence for both firms, inputs should be employed until the marginal revenue product of the last unit is equal to its rental price.

2.2.3 Competitive Equilibrium.

For definitions in this subsection, symbols without time subscripts denote the one-sided infinite sequence for the corresponding variables, e.g. $n_c \equiv \{n_{ct}\}_{t=0}^{\infty}$.

Definition 2.2.3a (Competitive Equilibrium): A competitive equilibrium is an allocation $(c, n_c, n_x, x_c, x_x, k_c, k_x)$, a price system (w_c, w_x, r_c, r_x, p), and a government policy $(\tau^c, \tau^x, \theta^c, \theta^x)$ such that

(a) Given the price system and the government policy, the allocation $(c, n_c, n_x, x_c, x_x, k_c, k_x)$ solves the problem of the representative household.

(b) Given the price system, the allocation (c, n_c, k_c) solves the problem of the representative firm in sector *C*.

(c) Given the price system, the allocation (n_x, x_c, x_x, k_x) solves the problem of the representative firm in sector *X*.

(d) The markets clear, i.e. the two resource constraint defined by *(2.1)* and *(2.2)* hold simultaneously. □

Note that the government budget constraint did not appear in the definition *2.2.3a.* Given the assumption about the utility function, the household's budget constraint is satisfied with equality in equilibrium. The government policy, the household's budget constraint and the two resource constraint defined by *(2.1)* and *(2.2)* imply that the government budget constraint *(3)* holds in equilibrium.

Given total time endowment at each instant for the household, define $\Im : \mathrm{R}^2_+ \to \mathrm{R}$ with $\Im$ (strictly) convex, such that the total time allocation constraint can be written as $\Im(n_{ct}, n_{xt}) \leq 1$. For (strict) convexity of the function $\Im : \mathrm{R}^2_+ \to \mathrm{R}$, imposing separability, the household's utility function is (non) linear in labor. Combining the first order conditions derived from the representative household's problem and the representative firms' problems, the resource constraints and time allocation constraint, it can be shown that the (competitive) equilibrium dynamics is characterized by the Transversality conditions together with the following system in the set of unknowns $\{c_t, k_{ct}, k_{xt}, n_{ct}, n_{xt}, r_{ct}, r_{xt}, w_{ct}, w_{xt}, p_t, \tau^c_t, \tau^x_t, \theta^c_t, \theta^x_t\}$:

$$\Im(n_{ct}, n_{xt}) \leq 1 \tag{7.5a}$$

$$c_t + \bar{g}_t = F^c(k_{ct}, n_{ct}) \tag{7.5b}$$

$$x_{ct} + x_{xt} = F^x(k_{xt}, n_{xt}) \tag{7.5c}$$

$$x_{ct} = k_{ct+1} - (1-\delta)k_{ct} \tag{7.5d}$$

$$x_{xt} = k_{xt+1} - (1-\delta)k_{xt} \tag{7.5e}$$

$$u_{nc}(t) = -u_c(t)(1-\tau^c_t)w_{ct} \tag{7.5f}$$

$$u_{nx}(t) = -u_c(t)(1-\tau^x_t)w_{xt} \tag{7.5g}$$

$$\frac{u_c(t)}{u_c(t+1)} = \frac{p_{t+1}}{p_t}\beta R^c_{t+1} \tag{7.5h}$$

$$\frac{u_c(t)}{u_c(t+1)} = \frac{p_{t+1}}{p_t}\beta R^x_{t+1} \tag{7.5i}$$

$$r_{ct} = F^c_{kc}(t) \tag{7.5j}$$

$$w_{ct} = F_{nc}^{c}(t) \tag{7.5k}$$

$$r_{xt} = p_t F_{kx}^{x}(t) \tag{7.5l}$$

$$w_{xt} = p_t F_{nx}^{x}(t) \tag{7.5m}$$

Equation *(7.5a)* represents the time allocation constraint. Equations *(7.5b)* and *(7.5c)* represent goods market clearing conditions. The next two are laws of motion for capital. The rest of the equations are the set of equilibrium conditions derived from household's and firms' optimization problems. A few observations deserve attention here. Note *(7.5h)* and *(7.5i)* together imply that after tax returns from capital are equal in a competitive equilibrium, which does not necessarily imply that pre-tax returns are equal. Note also that with *(7.5f)* and *(7.5g)*, a non-unitary marginal rate of substitution of labor across sectors would imply that after tax wage rates are not equal in equilibrium.

There are many competitive equilibria, indexed by different government policies. The multiplicity of competitive equilibrium motivates the Ramsey problem, defined as follows.

Definition 2.2.3b (Ramsey problem): Given the time 0 (initial) endowments of capital stocks and the preset revenue target, the Ramsey problem is to choose a policy that maximizes expression *(1)* subject to government's budget constraint such that the resulting allocations and prices are consistent with the competitive equilibrium defined by *(7.5)*. □

For a given welfare criterion, which the government uses to evaluate different allocations, the Ramsey problem for the government, therefore is to pick the fiscal policy (or one of them if there are many) that generates the competitive equilibrium allocation giving the highest value of the welfare criterion. This way of formulating the Ramsey problem was examined for a one-sector neoclassical model by Chamley (1986). Applying Chamley's (1986) methodology to the current setting yields analytical results which require interpretation of an array of Lagrange multipliers, and hence lacks tractability.

Derivation and interpretation of the key analytical results of this chapter are relatively more convenient if the primal approach to optimal taxation is adopted. According to the primal approach, one can equivalently formulate the Ramsey problem by allowing the government to pick an allocation directly (rather than a set of taxes). However, the set of allocations from which the government is allowed to choose is restricted by the implementability constraints. The Ramsey problem therefore consists of choosing among all implementable allocations (generated by arbitrary fiscal policy), the one that maximizes a welfare criterion (see for details, Atkinson & Stiglitz (1980, ch. 12), and for applications, Lucas & Stokey (1983), Jones *et al.* (1997), Benhabib & Rustichini (1997), Chari & Kehoe (1999), and Erosa & Gervais (2001)).

2.3 The Ramsey Problem.

In the primal approach to the Ramsey problem, the government can be thought of as directly choosing a feasible allocation, subject to constraints that ensure the existence of prices and taxes such that the chosen allocation is consistent with the optimization behaviour of household and firms. This approach of characterizing the Ramsey problem was primarily applied in Lucas & Stokey's (1983) analysis of an economy without capital. For a model economy with two (or more) factors comprising both physical and human capital, Jones *et al.* (1997) applies the primal approach by using a present-value household budget constraint which, after substituting for equilibrium prices and taxes, characterizes a set of implementability constraints to be incorporated in the government's optimization problem. This is the current trend in recursive formulation of the Ramsey problem.

For the primal approach to the Ramsey problem corresponding to the current setting, the first step would therefore be to introduce a present-value budget constraint for either the government or the representative household (one of them is redundant since both resource constraints are imposed). It turns out that the problem simplifies nicely if one chooses the present-value budget constraint of the household, in which future capital stocks can be (algebraically) eliminated. Note that since firms' problems are equivalent to a series of one-period maximization problems, in equilibrium $R_t \equiv R_t^c = R_t^x$. Consider, therefore, household's time T budget constraint:

$$c_T - (1-\tau_T^c) w_{cT} n_{cT} - (1-\tau_T^x) w_{xT} n_{xT} \leq p_T R_T [k_{cT} + k_{xT}] - p_T [k_{cT+1} + k_{xT+1}] \tag{8a}$$

Let $\prod_{s=1}^{0} R_s \equiv 1$ be the numeraire. Divide *(8a)* by the period T term $p_T \prod_{s=1}^{T} R_s$ and evaluate the resulting expression at time $T-1$. Then add these two and evaluate the resulting expression at time $T-2$. Iterating this procedure (and finally adding the time 0 expression) and taking the limit of both sides of the sum as $T \to \infty$ results in the following expression:

$$\sum_{t=0}^{\infty} \frac{c_t - (1-\tau_t^c) w_{ct} n_{ct} - (1-\tau_t^x) w_{xt} n_{xt}}{p_t \prod_{s=1}^{t} R_s} \leq R_0 [k_{c0} + k_{x0}] \tag{8b}$$

where $\lim_{t \to \infty} k_{jt+1} \left(\prod_{s=1}^{t} R_s \right)^{-1} = 0$ is already imposed since the present discounted value of the capital stock in sector j, $j = C, X$, in period t evaluated using period t market prices is asymptotically zero as $t \to \infty$. Expression *(8b)* is the household's present-value budget constraint, which delivers the interpretation that the present-value of consumption expenditures net of (net) labor earnings cannot exceed the value of the net initial assets. Assume that *(8b)* binds, i.e. there are no unused resources in the limit. Define the Arrow-Debreu price, $q_t^o \equiv p_t^{-1} \left(\prod_{s=1}^{t} R_s \right)^{-1}$ such that *(8b)* becomes:

$$\sum_{t=0}^{\infty} q_t^o c_t = \sum_{t=0}^{\infty} q_t^o (1-\tau_t^c) w_{ct} n_{ct} + \sum_{t=0}^{\infty} q_t^o (1-\tau_t^x) w_{xt} n_{xt} + R_0^c k_{c0} + R_0^x k_{x0} \tag{9}$$

with $q_0^o = p_0^{-1}$.

A summary of the primal approach to the Ramsey problem is as follows. In the first step, the necessary conditions from the household's maximization problem are derived by maximizing the representative household's utility subject to *(9)*. Then, the representative firms' problems and corresponding necessary conditions are reconsidered (which are necessarily same as derived before). The set of these competitive equilibrium conditions are solved for prices and taxes $\{q_t^o, r_{ct}, r_{xt}, w_{ct}, w_{xt}, \tau_t^c, \tau_t^x, \theta_t^c, \theta_t^x\}_{t=0}^{\infty}$, as functions of the allocations $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$. When these expressions are substituted into the household's present-value budget constraint *(9)*, the resulting expression becomes an intertemporal budget constraint involving only the implementable allocations. The government maximizes welfare subject to the two resource constraints and the adjusted intertemporal budget constraint, and solution to this problem characterizes the Ramsey allocation[14]. Once the Ramsey allocations are characterized, one can solve for the Ramsey equilibrium taxes and prices.

With λ^P as the Lagrange multiplier on the household's present-value budget constraint[15], solution to the household's problem yields the following first order conditions with respect to changes in consumption and labor supply for all time *t*:

$$c_t : \beta^t u_c(t) = \lambda^P q_t^o \tag{10a}$$

$$n_{ct} : \beta^t u_{nc}(t) + \lambda^P q_t^o (1 - \tau_t^c) w_{ct} = 0 \tag{10b}$$

$$n_{xt} : \beta^t u_{nx}(t) + \lambda^P q_t^o (1 - \tau_t^x) w_{xt} = 0 \tag{10c}$$

With $q_0^o = p_0^{-1}$, the time 0 version of *(10a)* implies $\lambda^P = p_0 u_c(0)$. Substituting for λ^P in *(10a)* gives the Arrow-Debreu price in terms of consumption allocations and initial relative price of capital goods, which is:

$$q_t^o = \frac{\beta^t u_c(t)}{p_0 u_c(0)} \tag{10d}$$

[14] The definition of Ramsey allocation is given in subsection 2.3.1.

[15] λ^P measures the value of additional units of resources available in the initial period evaluated in utility terms.

Using *(10d)* and $\lambda^p = p_0 u_c(0)$ in *(10b)* and *(10c)*, the after tax wage rates for sector C and X for all time t in terms of consumption and labor supply allocations is given by:

$$(1-\tau_t^j)w_{jt} = \frac{-u_{nj}(t)}{u_c(t)} \qquad \text{for sector } j = C, X \qquad (10e)$$

The formulation of the representative firms' problems is unchanged, implying that the first order conditions from firms' problem are also unchanged. With *(10d)* and *(10e)*, *(9)* may be rewritten as:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t + u_{nc}(t)n_{ct} + u_{nx}(t)n_{xt}] - p_0 u_c(0)[R_0^c k_{c0} + R_0^x k_{x0}] = 0 \qquad (10f)$$

With $R_0^c = R_0^x$, the time 0 definition of R_t^j for sector j = C, X, gives:

$$p_0 = \frac{(1-\theta_0^c)F_{kc}^c(0)}{(1-\theta_0^x)F_{kx}^x(0)} \qquad (10g)$$

such that *(10f)* may be rewritten as:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t + u_{nc}(t)n_{ct} + u_{nx}(t)n_{xt}] - \Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x) = 0 \qquad (11)$$

where $\Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x) \equiv \left[\dfrac{(1-\theta_0^c)F_{kc}^c(0)}{(1-\theta_0^x)F_{kx}^x(0)}\right] u_c(0)[R_0^c k_{c0} + R_0^x k_{x0}]$

Expression *(11)* is, therefore, the intertemporal constraint that involves only allocations and initial capital income tax rates that can be implemented in a competitive equilibrium, and is known in the literature as the implementability constraint of the corresponding Ramsey problem. The Ramsey problem for the government, therefore, is to maximize *(1)* subject to the two (binding) resource constraints *(2.1)* and *(2.2)* and the implementability constraint defined by *(11)*.

2.3.1 Solution to the Ramsey Problem.

Let $\Phi \geq 0$ be a Lagrange multiplier on *(11)*, and define[16]

$$\mathbf{V}(c_t, n_{ct}, n_{xt}, \Phi) \equiv u(c_t, n_{ct}, n_{xt}) + \Phi[u_c(t)c_t + u_{nc}(t)n_{ct} + u_{nx}(t)n_{xt}] \qquad (12.1)$$

The Lagrangian of the Ramsey problem can be written as:

$$\begin{aligned}\tilde{\mathbf{J}} = \sum_{t=0}^{\infty} \beta^t \{ & \mathbf{V}(c_t, n_{ct}, n_{xt}, \Phi) \\ & + \chi_{1t}[F^c(k_{ct}, n_{ct}) - c_t - g_t] \\ & + \chi_{2t}[F^x(k_{xt}, n_{xt}) + (1-\delta)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}] \} - \Phi\Omega(c_0, n_{c0}, n_{x0}, \theta_0^c, \theta_0^x)\end{aligned}$$

$$(12.2)$$

where $\{\chi_{1t}, \chi_{2t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the two resource constraints. For given government revenue target $\bar{g}_t$ and initial capital endowments k_{c0} and k_{x0}, the problem is therefore to fix initial capital income tax rates θ_0^c and θ_0^x and maximize *(12.2)* with respect to $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$. The necessary conditions for an optimum for this problem due to changes in allocations are:

$$c_t: \quad \mathbf{V}_c(t) = \chi_{1t}, \qquad \forall t \geq 1 \qquad (12.3a)$$

$$n_{ct}: \quad \mathbf{V}_{nc}(t) = -\chi_{1t} F_{nc}^c(t), \qquad \forall t \geq 1 \qquad (12.3b)$$

$$n_{xt}: \quad \mathbf{V}_{nx}(t) = -\chi_{2t} F_{nx}^x(t), \qquad \forall t \geq 1 \qquad (12.3c)$$

$$k_{ct+1}: \quad \chi_{2t} = \beta[\chi_{1t+1} F_{kc}^c(t+1) + \chi_{2t+1}(1-\delta)], \qquad \forall t \geq 0 \qquad (12.3d)$$

[16]The following expression *(12.1)* is commonly referred to as the *Pseudo* utility function which combines the utility function and the infinite horizon part of the implementability constraint. The detailed interpretation of the Lagrange multiplier Φ is given in subsection 2.3.4.

$$k_{xt+1}: \quad \chi_{2t} = \beta\chi_{2t+1}[F_{kx}^{x}(t+1)+(1-\delta)], \qquad \forall t \geq 0 \tag{12.3e}$$

$$c_0: \quad \mathbf{V}_c(0) = \chi_{10} + \Phi\Omega_{c0} \tag{12.3f}$$

$$n_{c0}: \quad \mathbf{V}_{nc}(0) = -\chi_{10}F_{nc}^{c}(0) + \Phi\Omega_{nc0} \tag{12.3g}$$

$$n_{x0}: \quad \mathbf{V}_{nx}(0) = -\chi_{20}F_{nx}^{x}(0) + \Phi\Omega_{nx0} \tag{12.3h}$$

Consolidating *(12.3)* yields the following five equations:

$$\mathbf{V}_c(t)\frac{F_{kc}^{c}(t)}{F_{kx}^{x}(t)} = \beta\mathbf{V}_c(t+1)\frac{F_{kc}^{c}(t+1)}{F_{kx}^{x}(t+1)}[F_{kx}^{x}(t+1)+(1-\delta)], \quad \forall t \geq 1 \tag{13.1a}$$

$$\mathbf{V}_{nc}(t) = -\mathbf{V}_c(t)F_{nc}^{c}(t), \qquad \forall t \geq 1 \tag{13.1b}$$

$$\mathbf{V}_{nx}(t) = -\mathbf{V}_c(t)\frac{F_{kc}^{c}(t)}{F_{kx}^{x}(t)}F_{nx}^{x}(t), \qquad \forall t \geq 1 \tag{13.1c}$$

$$\mathbf{V}_{nc}(0) = [\Phi\Omega_{c0} - \mathbf{V}_c(0)]F_{nc}^{c}(0) + \Phi\Omega_{nc0} \tag{13.1d}$$

$$\mathbf{V}_{nx}(0) = -\mathbf{V}_c(0)\frac{F_{kc}^{c}(0)}{F_{kx}^{x}(0)}F_{nx}^{x}(0) + \Phi\Omega_{nx0} \tag{13.1e}$$

The other three necessary conditions are the two (binding) resource constraints *(2.1)* and *(2.2)*, and the implementability constraint *(11)*, which are repeated here for convenience:

$$c_t + g_t = F^{c}(k_{ct}, n_{ct}) \tag{13.1f}$$

$$k_{ct+1} + k_{xt+1} - (1-\delta)(k_{ct} + k_{xt}) = F^{x}(k_{xt}, n_{xt}) \tag{13.1g}$$

$$\sum_{t=0}^{\infty}\beta^{t}[u_c(t)c_t + u_{nc}(t)n_{ct} + u_{nx}(t)n_{xt}] - \Omega(c_0, n_{c0}, n_{x0}, \theta_0^{c}, \theta_0^{x}) = 0 \tag{13.1h}$$

Let N denote the set of policies for which a competitive equilibrium exists.

Definition 2.3.1a (Ramsey Equilibrium): A Ramsey equilibrium is a policy η in N, an allocation rule $\Gamma(.)$, and a price system $\mathrm{P}(.) = \{w_j(.), r_j(.), p(.)\}$ for j = *C*, *X*, such that

(a) The policy η maximizes the household's utility *(1)* subject to the resource constraints *(2.1)* and *(2.2)* and implementability constraint *(11)*.

(b) For every η', the allocation $\Gamma(\eta')$, the price system $\mathrm{P}(\eta')$, and the policy η' constitute a competitive equilibrium. □

Definition 2.3.1b (Ramsey Allocation): A Ramsey allocation that corresponds to the Ramsey problem is a 5-tuple of sequences $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$ and a multiplier Φ that provides a solution to the system of difference equations *(13.1)* and characterizes the Ramsey equilibrium defined by *2.3.1a*. □

First, note that the Ramsey equilibrium requires optimality by households and firms for all policies that the government might choose. Hence for a given value of the initial price level p_0 for which the Transversality condition *(5.6a)* is satisfied, an allocation $\{c_t, n_{ct}, n_{xt}, k_{ct+1}, k_{xt+1}\}_{t=0}^{\infty}$ and a multiplier Φ that satisfy the system of difference equations presented by *(13.1)* will characterize the Ramsey equilibrium. Using the resulting Ramsey allocation, one can then compute the Ramsey equilibrium values of all endogenous variables of the system. For instance, one can obtain q_t^o from *(10d)*, r_{ct} from *(6.5a)*, w_{ct} from *(6.5b)*, and τ_t^c from *(10e)*. Condition *(10e)* for sector X gives $(1-\tau_t^x)w_{xt}$, and so on.

2.3.2 Optimal Capital Income Tax.

Consider a case in which there is a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$. Assume solution to the Ramsey problem converges to a time-invariant allocation, so that c, n_c, n_x, k_c and k_x are constant after some time. Then because $\mathbf{V}_c(t)$ converges to a constant, the time-invariant version of *(13.1a)* implies:

$$1 = \beta[F_{kx}^x + (1-\delta)] \qquad (14.1a)$$

Proposition 1: For a steady state solution to the Ramsey problem and a corresponding steady state Ramsey allocation, the limiting tax rate on capital income from the capital goods sector X is zero.

Proof: With $\dfrac{q_{t+1}^o}{q_t^o} = \beta \dfrac{u_c(t+1)}{u_c(t)}$, as $t \to \infty$, $\dfrac{q_t^o}{q_{t+1}^o} \to \dfrac{1}{\beta}$

Also by definition, $\dfrac{q_t^o}{q_{t+1}^o} = \dfrac{p_{t+1}}{p_t}[(1-\theta_{t+1}^x)F_{kx}^x(t+1) + (1-\delta)]$

that implies as $t \to \infty$, $\dfrac{q_t^o}{q_{t+1}^o} \to [(1-\theta^x)F_{kx}^x + (1-\delta)]$.

Hence for $t \to \infty$

$$1 = \beta[(1-\theta^x)F_{kx}^x + (1-\delta)] \tag{14.1b}$$

(14.1a) and *(14.1b)* together imply $\theta^x = 0$. ∎

Proposition 1 has analogy to the celebrated Chamley-Judd result of dynamic optimal taxation literature --- the optimal steady state tax on capital income is zero. The finding for the case of capital goods sector is similar to what Judd (1985) and Chamley (1986) find using a one-sector model. This result is intuitive, since a nonzero tax on capital income in steady state would mean that distortions created by the tax evolve explosively, contrary to a uniform distortion that might be created by simple labor or consumption taxes (see Judd (1999) for details). One way the current modelling approach differs from a conventional one-sector competitive model is how savings and capital accumulation occurs across sectors. Note that households pay a strictly positive relative price for the new capital goods and rent it out to firms in anticipation of income from investment. Firms return the rented capital stock net of depreciation. Of these two installed capital stocks, only k_x is required to produce future capital goods. Hence if income from k_x is taxed in a steady state, this will induce compounding nature of distortions. The zero limiting tax rate of capital income from capital goods sector holds irrespective of specifications, as long as specified functions satisfy assumptions 1 and 2.

The result does not necessarily hold as robust for capital income tax in consumption goods sector, however. For the consumption goods sector, as $t \to \infty$, $\frac{q_t^o}{q_{t+1}^o} \to \left[(1-\theta^c)\frac{(1-\tau^x)F_{nx}^x u_{nc}}{(1-\tau^c)F_{nc}^c u_{nx}} F_{kc}^c + (1-\delta)\right]$. This in turns implies that $1 = \beta\left[(1-\theta^c)\frac{(1-\tau^x)F_{nx}^x u_{nc}}{(1-\tau^c)F_{nc}^c u_{nx}} F_{kc}^c + (1-\delta)\right]$ holds for $t \to \infty$. Together with *(14.1a)*, this implies $\theta^c = 1 - \frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x}\left[\frac{(1-\tau^c)u_{nx}}{(1-\tau^x)u_{nc}}\right]$. The government's set of policies N for which a competitive equilibrium exists is therefore:

$$\mathrm{N} = \left\{(\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, \frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x}\left[\frac{(1-\tau^c)u_{nx}}{(1-\tau^x)u_{nc}}\right] = 1 - \theta^c\right\} \tag{14.1c}$$

Proposition 2: If the utility function is separable in consumption and labor and linear in labor, and if the government sets the labor income tax rates equal across sectors, the limiting tax rate on capital income from consumption goods sector C is zero. Otherwise, it is not zero.

Proof: Consider $\theta^c = 1 - \frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x}\left[\frac{(1-\tau^c)u_{nx}}{(1-\tau^x)u_{nc}}\right]$, and recall $\frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x} = \frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}$ which is derived from the Ramsey equilibrium system defined by *(13.1)*,.

Since $\mathbf{V}_{nc} = u_{nc} + \Phi[u_{cnc}c + u_{nc} + n_c u_{ncnc} + n_x u_{nxnc}]$

and, $\mathbf{V}_{nx} = u_{nx} + \Phi[u_{cnx}c + u_{nx} + n_c u_{ncnx} + n_x u_{nxnx}]$,

the term $\frac{F_{kx}^x F_{nc}^c}{F_{kc}^c F_{nx}^x}\left[\frac{(1-\tau^c)u_{nx}}{(1-\tau^x)u_{nc}}\right] = 1$ if and only if *(1)* the utility function separable in consumption and labor and linear in labor, for which $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{u_{nc}}{u_{nx}}$, and *(2)* the government sets labor income tax rates equal across sectors. Unless both conditions are satisfied simultaneously, $\theta^c \neq 0$. ■

A few clarifications deserve attention in the proof of proposition 2. First, for utility function defined by *(1)*, it is not explicitly assumed that utility is linear in labor, and that the marginal rate of substitution of labor across sectors is unitary. The first simplification is common in literature that deals with similar models, which (together with separability of utility function in consumption and labor) dramatically simplifies the expressions of $\mathbf{V}_{nj}$ by ruling out the second and cross derivatives of labor services. The second simplification (unitary marginal rate of substitution of labor) would imply that after tax wages are equal across sectors. The fact that workers may receive varying disutility from working in different sectors is empirically supported, evidence of which will be presented in the next section. Such simplifications are not obvious where there exists some intratemporal adjustment cost of labor across sectors (see for instance, Huffman & Wynne (1999)). For such a class of utility functions where $\Im : \mathrm{R}_{+}^{2} \to \mathrm{R}$ is strictly convex, $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{u_{nc}}{u_{nx}}$ does not necessarily hold. Moreover, the additional condition for which the government taxes capital income from consumption goods sector at a zero rate is that government's *ex post* choice of labor income tax rates are equal across sectors, which is not inferred by the model. Unless both conditions, $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{u_{nc}}{u_{nx}}$ and $\tau^{c} = \tau^{x}$, hold simultaneously, $\theta^{c} \neq 0$.

This particular analytical result has a very sharp intuition. Since capital is produced in a different sector than the consumption goods sector, nonzero capital income tax in the consumption goods sector is similar, in terms of consequences, to a simple consumption tax which has uniform distortion pattern. The subscript *j* to capital stock and to labor denotes the level of capital and labor employed in a particular sector, and in no way restricts factors to be sector-specific. Since capital is freely movable between sectors, and following proposition 1, it is feasible for the household to purchase two new capital goods and invest the new capital k_x and both forms of the depreciated old capital goods in the capital goods sector. The next period capital to produce consumption goods is available through production of new capital goods. Hence, along the transition the depreciated capital good from consumption goods sector is transferred to capital goods sector for production. In this respect, a nonzero tax on capital income from capital goods sector would definitely have a chaotic distorting effect, which cannot be optimal and duly recognized by the government. The household earns capital income from consumption goods sector in each period, gets taxed at a nonzero rate, and can avoid the compounding tax liabilities by shifting depreciated capital to the other sector.

2.3.3 Constrained Tax Choice.

The previous analysis concluded that the government's optimal choice of steady state capital tax rates in general varies across sectors. Consider, for instance, a class of utility functions for which $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}} = \frac{u_{nc}}{u_{nx}}$ holds[17]. The government's set of policies for which a competitive equilibrium exists would then be:

$$\tilde{\mathrm{N}} = \left\{ (\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, 1 - \theta^c = \frac{(1-\tau^c)}{(1-\tau^x)} \right\}$$

implying that the government sets a limiting zero tax on capital income from consumption goods sector if and only if it sets labor income tax rates equal across sectors. Hence, given that particular class of utility functions, for any subset of Ramsey policy that prescribes varying labor income tax rates across sectors, the optimal steady state tax on capital income from consumption goods sector is nonzero and non-explosive in nature.

While this may be a robust theoretical proposal for fiscal policy choice, it may be subject to criticisms from a realistic point of view. Real world governments often face a constraint of keeping same tax rates for the same factor across different sectors. Considering the proposed model, it becomes interesting to test what happens to government's optimal capital tax choice in the event when the government, ex ante,

[17] One may consider the utility function $u(.) = \ln c_t + [1 - n_{ct} - n_{xt}]$ which is supported by the lottery argument of Hansen (1985). This functional form is popular in real business cycle literature, as may be found in Herrendorf & Valentinyi (2003), among others. This specification, however, endorses very limited empirical justification. I will introduce it more formally in the next section.

faces an additional constraint of keeping all factor-specific tax rates same, i.e. same labor income tax rates and same capital income tax rates across sectors. Same labor income tax rates across sectors can be empirically justified, since it is observed in most tax plans laid out by governments. In the present context, equal labor income tax rates across sectors would imply (in general) that in a competitive equilibrium the marginal rate of substitution of labor across sectors equals the before-tax wage ratio.

In principle, it is predictable that such additional constraints in the Ramsey problem *(12.2)* would necessarily yield an outcome with lower welfare. While the celebrated Chamley-Judd result of zero steady state capital tax is typically claimed to be the most efficient outcome in a tax distorted one-sector economy, the two-sector analogue of this result would suggest that any nonzero tax on capital income from capital goods sector would induce lower welfare than the unconstrained Ramsey equilibrium corresponding to *(13.1)*. The prescription of a nonzero tax on capital income from consumption sector is backed up by a clear intuition that such a capital tax will not have compounding distortion effects as long as the government keeps the other capital tax zero. If the government's choice of capital tax rates is constrained to be same ex ante, the only pareto improving rule for the government would be that both capital tax rates are zero. Hence, in a Ramsey problem with constrained capital tax choice, any nonzero optimal tax on capital income would be a pareto-worsening outcome for the government.

To test it formally, note that since the after tax returns to capital are equal across sectors in a competitive equilibrium, constraining capital income taxes to be same is tantamount to constraining pre-tax returns to capital across sectors to be same. In other words, one can test the restriction of equal capital income taxes across sectors by incorporating the additional constraint $F_{kc}^{c}(t) = p_t F_{kx}^{x}(t), \forall t$ in the Ramsey problem *(12.2)*. Substituting for the equilibrium relative price of new capital goods, and imposing the constraint that government keeps the labor income tax rates same across sectors, the additional constraint becomes $\frac{F_{kc}^{c}}{F_{kx}^{x}} \cdot \frac{F_{nx}^{x}}{F_{nc}^{c}} = \frac{u_{nx}}{u_{nc}}$.

Consider the Lagrangian form of the Ramsey problem with constrained tax choice for the government,

$$\mathfrak{J} = \sum_{t=0}^{\infty} \beta^t \{ \mathbf{V}(c_t, n_{ct}, n_{xt}, \Phi)$$
$$+ \chi_{1t}[F^c(k_{ct}, n_{ct}) - c_t - g_t]$$
$$+ \chi_{2t}[F^x(k_{xt}, n_{xt}) + (1-\delta)(k_{ct} + k_{xt}) - k_{ct+1} - k_{xt+1}]$$
$$+ \chi_{3t}\left[\frac{F^c_{kc}}{F^x_{kx}} \cdot \frac{F^x_{nx}}{F^c_{nc}} - \frac{u_{nx}}{u_{nc}}\right] \} - \Phi\Omega(c_0, n_{c0}, n_{x0}, \theta^c_0, \theta^x_0)$$

(14.2)

where $\{\chi_{1t}, \chi_{2t}, \chi_{3t}\}_{t=0}^{\infty}$ is a sequence of Lagrange multipliers on the two resource constraints and the additional ex ante tax choice constraint. The necessary conditions for an optimum for this problem for changes in consumption, labor supply and one period ahead capital stocks are:

$$c_t : \mathrm{V}_c(t) = \chi_{1t}, \qquad \forall t \geq 1 \qquad (14.3a)$$

$$n_{ct} : \mathbf{V}_{nc}(t) = -\chi_{1t} F^c_{nc}(t) - \chi_{3t}\left[\frac{F^x_{nx}(t)}{F^x_{kx}(t)}\left\{\frac{F^c_{kcnc}(t)}{F^c_{nc}(t)} - \frac{F^c_{kc}(t)F^c_{ncnc}(t)}{[F^c_{nc}(t)]^2}\right\} - \left\{\frac{u_{nxnc}(t)}{u_{nc}(t)} - \frac{u_{nx}(t)u_{ncnc}(t)}{[u_{nc}(t)]^2}\right\}\right], \quad \forall t \geq 1$$

(14.3b)

$$n_{xt} : \mathbf{V}_{nx}(t) = -\chi_{2t} F^x_{nx}(t) - \chi_{3t}\left[\frac{F^c_{kc}(t)}{F^c_{nc}(t)}\left\{\frac{F^x_{nxnx}(t)}{F^x_{kx}(t)} - \frac{F^x_{nx}(t)F^x_{kxnx}(t)}{[F^x_{kx}(t)]^2}\right\} - \left\{\frac{u_{nxnx}(t)}{u_{nc}(t)} - \frac{u_{nx}(t)u_{ncnx}(t)}{[u_{nc}(t)]^2}\right\}\right], \quad \forall t \geq 1$$

(14.3c)

$$k_{ct+1} : \chi_{2t} = \beta\left\{\chi_{1t+1}F^c_{kc}(t+1) + \chi_{2t+1}(1-\delta) + \chi_{3t+1}\left[\frac{F^x_{nx}(t+1)}{F^x_{kx}(t+1)}\left\{\frac{F^c_{kckc}(t+1)}{F^c_{nc}(t+1)} - \frac{F^c_{kc}(t+1)F^c_{nckc}(t+1)}{[F^c_{nc}(t+1)]^2}\right\}\right]\right\}$$

(14.3d)

$$k_{xt+1}: \chi_{2t} = \beta\left\{\chi_{2t+1}[F_{kx}^{x}(t+1)+(1-\delta)] + \chi_{3t+1}\left[\frac{F_{kc}^{c}(t+1)}{F_{nc}^{c}(t+1)}\left\{\frac{F_{nxkx}^{x}(t+1)}{F_{kx}^{x}(t+1)} - \frac{F_{nx}^{x}(t+1)F_{kxkx}^{x}(t+1)}{[F_{kx}^{x}(t+1)]^{2}}\right\}\right]\right\}$$

(14.3e)

Consolidating *(14.3)* yields three necessary conditions for Ramsey equilibrium, and the one of interest is:

$$\mathbf{V}_{c}(t)\frac{F_{kc}^{c}(t)}{F_{kx}^{x}(t)} = \beta\left[\mathbf{V}_{c}(t+1)\frac{F_{kc}^{c}(t+1)}{F_{kx}^{x}(t+1)}[F_{kx}^{x}(t+1)+(1-\delta)] + \chi_{3t+1}[\Theta_{t+1}]\right] - \chi_{3t}[\Lambda_{t}]$$

(14.3f)

where Θ_{t+1} and Λ_t are terms comprising derivatives of $F^{c}(.)$ and $F^{x}(.)$, evaluated at time *t+1* and *t*, respectively, defined as[18]:

$$\Theta_{t+1} \equiv \left\{\frac{F_{nx}^{x}}{F_{kx}^{x}}\left[\frac{F_{kckc}^{c}}{F_{nc}^{c}} - \frac{F_{kc}^{c}F_{nckc}^{c}}{[F_{nc}^{c}]^{2}}\right]\left[\frac{F_{kx}^{x}+(1-\delta)}{F_{kx}^{x}}\right] + \frac{F_{kc}^{c}}{F_{nc}^{c}}\left[\frac{F_{nxkx}^{x}}{F_{kx}^{x}} - \frac{F_{nx}^{x}F_{kxkx}^{x}}{[F_{kx}^{x}]^{2}}\right]\left[\frac{\delta-1}{F_{kx}^{x}}\right]\right\}$$

$$\Lambda_{t} \equiv \left\{\frac{F_{nx}^{x}}{F_{kx}^{x}}\left[\frac{F_{kckc}^{c}}{F_{nc}^{c}} - \frac{F_{kc}^{c}F_{nckc}^{c}}{[F_{nc}^{c}]^{2}}\right] - \frac{F_{kc}^{c}}{F_{nc}^{c}}\left[\frac{F_{nxkx}^{x}}{F_{kx}^{x}} - \frac{F_{nx}^{x}F_{kxkx}^{x}}{[F_{kx}^{x}]^{2}}\right]\right\}\frac{1}{F_{kx}^{x}}$$

Recall the otherwise equivalent condition derived from the Ramsey problem *(12.2)* where factor income tax choices were not constrained for the government. For a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$, and assuming convergence of the solution to the Ramsey problem to a time-invariant allocation, the time-invariant version of *(13.1a)* implied $1 = \beta[F_{kx}^{x} + (1-\delta)]$, which acted instrumentally for the proof of proposition 1. In the current Ramsey problem, for $t \to \infty$, $1 = \beta[(1-\theta^{x})F_{kx}^{x} + (1-\delta)]$ still holds in the Ramsey equilibrium. Unless $1 = \beta[F_{kx}^{x} + (1-\delta)]$ holds from the time-invariant version of *(14.3f)* corresponding to the Ramsey problem *(14.2)*, it is trivial that $\theta^{x} \neq 0$ *vis a vis* $\theta^{c} \neq 0$. In proposition 3, it is formally proved that $1 = \beta[F_{kx}^{x} + (1-\delta)]$ does

[18] The time notations attached to the derivatives are omitted in defining Θ_{t+1} and Λ_t, without loss of generality, just to avoid notational clutter.

not hold in the Ramsey equilibrium with constrained factor income tax, resulting in a Ramsey equilibrium outcome with nonzero steady state capital income tax.

Consider a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$, and assume that the solution to the Ramsey problem *(14.2)* converges to a time-invariant allocation. The time-invariant version of *(14.3f)* is:

$$\mathbf{V}_c \frac{F_{kc}^c}{F_{kx}^x} = \beta \mathbf{V}_c \frac{F_{kc}^c}{F_{kx}^x}[F_{kx}^x + (1-\delta)] + \chi_3 \Sigma \qquad (14.4a)$$

where

$$\Sigma \equiv \left\{ \frac{F_{nx}^x}{F_{kx}^x}\left[\frac{F_{kckc}^c}{F_{nc}^c} - \frac{F_{kc}^c F_{nckc}^c}{[F_{nc}^c]^2}\right]\left[\frac{\beta[F_{kx}^x + (1-\delta)] - 1}{F_{kx}^x}\right] + \frac{F_{kc}^c}{F_{nc}^c}\left[\frac{F_{nxkx}^x}{F_{kx}^x} - \frac{F_{nx}^x F_{kxkx}^x}{[F_{kx}^x]^2}\right]\left[\frac{1-\beta(1-\delta)}{F_{kx}^x}\right]\right\}$$

In order to prove that both capital tax rates are nonzero, it is sufficient to prove that $\Sigma \neq 0$, which in turn implies $1 \neq \beta[F_{kx}^x + (1-\delta)]$ in the Ramsey equilibrium with constrained tax choice.

Proposition 3: For a steady state solution to the Ramsey problem *(14.2)* and a corresponding steady state Ramsey allocation, the limiting tax rate on capital income is nonzero.

Proof: Suppose not, and hence $\Sigma = 0$ such that *(14.4a)* implies $1 = \beta[F_{kx}^x + (1-\delta)]$.

Given the underlying parameter restrictions and assumption 2, $\frac{F_{nx}^x}{F_{kx}^x}\left[\frac{F_{kckc}^c}{F_{nc}^c} - \frac{F_{kc}^c F_{nckc}^c}{[F_{nc}^c]^2}\right] < 0$, $\frac{F_{kc}^c}{F_{nc}^c}\left[\frac{F_{nxkx}^x}{F_{kx}^x} - \frac{F_{nx}^x F_{kxkx}^x}{[F_{kx}^x]^2}\right] > 0$ and $[1-\beta(1-\delta)] > 0$. Hence for $\Sigma = 0$, it must be that $\beta[F_{kx}^x + (1-\delta)] - 1 > 0$, which is a contradiction. ■

Thus if the government faces an *ex ante* constraint of keeping factor income tax rates same across sectors, the Ramsey equilibrium outcome comprises taxing capital income from both sectors at a strictly nonzero rate. This is an outcome with lower welfare since with nonzero capital income tax in both sectors, the distortions created by the taxes would be compounding in nature. With this tax plan in the scheme, the household will not be able to avoid the compounding tax liabilities by simply shifting depreciated capital.

2.3.4 Taxation of Initial Capital.

Given the Ramsey problem *(12.2)*, if the government is free to choose θ_0^j for $j = C, X,$ how does the government tax income from initial capital?[19] Since steady state tax on capital income is 0 for capital goods sector, there exists a strong impetus for the government to set a scheme involving high taxation of initial capital income, likely due to high values of government consumption expenditure. A few observations deserve attention in this context. Since household's preferences are strictly monotone, a high tax on income from initial capital employed in consumption goods sector must accompany high tax on income from initial capital employed in capital goods sector. Without exogenous bounds on the tax rates, the high initial tax rates may even be confiscatory, i.e. either $\theta_0^j > 1$ for both sectors holds in equilibrium, or both initial capital income tax rates are fractions. In either case, of course, these two tax rates are not *necessarily* equal, and *(10g)* implies $\theta_0^j \neq 1$ for $j = C, X,$ since p_0 is finite and strictly positive.

Denote the *maximum value* Lagrangian associated with the Ramsey problem as $\tilde{J}^*$. The derivative of $\tilde{J}^*$ with respect to θ_0^x is:

$$\tilde{J}^*_{\theta_0^x} = \Phi(1-\theta_0^c)F_{kc}^c(0)u_c(0)(k_{c0}+k_{x0})\left[\frac{(\delta-1)}{(1-\theta_0^x)^2 F_{kx}^x(0)}\right] \tag{14.5}$$

[19] Hereafter, I will consider the Ramsey problem *(12.2)* as the benchmark, and Ramsey problem with constrained tax choice defined by *(14.2)* as a special experimental case. I will extend all further analysis on the basis of the benchmark Ramsey problem defined by *(12.2)*.

which is strictly positive for $\theta_0^c > 1$ (and consequently $\theta_0^x > 1$) and $\Phi > 0$. Ljungqvist & Sargent (2000, ch.12) define the non-negative Lagrange multiplier Φ as a measure of the utility costs of raising government revenues through distorting taxes. Without distorting taxes, a competitive equilibrium would attain first best outcome for the representative household, and Φ would be zero, so that the household's present-value budget constraint would not exert any additional constraining effect on welfare maximization beyond what is present in the economy's set of technologies. Contrary to the first best outcome, when the government has to use some tax device, the multiplier Φ is strictly positive and can be interpreted as the welfare cost of the distorted margin, implicit in the implementability constraint *(11)*.

With the presence of distorting taxes and $\Phi > 0$, $\tilde{J}^*_{\theta_0^x} > 0$ if $\theta_0^c > 1$ (and consequently $\theta_0^x > 1$). This result is fairly intuitive. By raising θ_0^j for $j = C, X$, and thereby increasing the revenues from taxation of the initial capital stocks employed in the two sectors, the government reduces its future need to rely on distorting taxation. With this policy, the value of the utility cost of raising government revenue through distorting taxation falls. Hence the implication of *(14.5)* is that the government should set θ_0^j for $j = C, X$, high enough to drive down Φ quickly to zero. With a zero optimal limiting capital income tax for capital goods sector, the government should raise *bulk* of revenue through a time 0 capital levy and for time onwards should only use tax instruments which do not induce compounding distortions.

2.4 Specific Utility Functions.

In this section I will characterize the optimal steady state capital tax for consumption goods sector corresponding to the Ramsey equilibrium *(13.1)* with a variant of commonly used utility functions. A few observations from established literature deserve special attention in this context. The utility function defined in (*1*) is standard in literature that adopts models with consumption-labor choice. There exist a handful of studies that consider a class of utility functions which are separable, logarithmic in consumption and linear in labor (see for instance, Jones *et al.* (1993) and Herrendorf & Valentinyi (2003)). Although such utility functions aid theoretical tractability of models, their empirical justification can be contentious. Huffman & Wynne (1999) propose a class of utility functions that captures the idea of intratemporal labor adjustment cost assuming that shifting labor across sectors is costly. Their proposed functional form characterizes strict convexity of the function $\Im : \mathrm{R}_+^2 \rightarrow \mathrm{R}$. Jones *et al.* (1997) present a useful specification of a utility function where the planner is unable to distinguish between income from two types of labor. I will consider these utility function specifications as experimental cases in order to verify the key analytical results, acknowledging that there may be many other interesting cases to consider.

2.4.1 Equal Marginal Disutility of Labor.

Consider the broader class of utility functions:

$$U(c_t, n_{ct}, n_{xt}) = \frac{[c_t \exp(1 - n_{ct} - n_{xt})]^{1-\sigma} - 1}{1-\sigma} \qquad (15.1a)$$

where $\sigma \geq 0$ is the inverse of elasticity of intertemporal substitution. Consider $u(.)$ as a special case of $U(.)$ where $\sigma \rightarrow 1$. As $\sigma \rightarrow 1$, using l'Hôpital's rule, it is possible to show that

$$u(c_t, n_{ct}, n_{xt}) = \ln c_t + (1 - n_{ct} - n_{xt}) \qquad (15.1b)$$

Specification *(15.1b)* thus characterizes utility linear in labor services, which can be justified by the lottery argument of Hansen (1985). In the context of the current chapter's analytical tractability, such utility functions simplify the expressions of $\mathbf{V}_{nj}$ by ruling out the second and cross derivatives of labor services. The specific form *(15.1b)* also exhibit unitary marginal rate of substitution of labor across sectors. While this simple assumption (that workers receive equal marginal disutility of effort from different sectors) is typically held in a subset of multi-sector general equilibrium models established in literature, empirically, there is strong evidence against it. In the US, for instance, the Bureau of Labor Statistics (BLS) 2002 survey reports suggest that injury related incidence per 100 worker varies greatly across different industrial sectors, and incidence rates are relatively higher in goods producing sector as compared to the service producing sector. Hence, one can argue that such utility functions are increasingly stylized and ignores the empirically supported evidence of varying disliking for jobs across sectors.

The set of policies for the government which can be implemented in a competitive equilibrium, given *(15.1b)*, is presented by:

$$\tilde{\mathrm{N}} = \left\{ (\tau^c, \tau^x, \theta^c, \theta^x) \middle| \theta^x = 0, 1 - \theta^c = \frac{(1-\tau^c)}{(1-\tau^x)} \right\}$$

which states that the optimal steady state capital income tax for consumption goods sector is zero if and only if the government keeps the two labor income tax rates equal across sectors. Now consider competitive equilibrium condition which states that the marginal rate of substitution of labor must equal the relative after tax wage rates. Given specification *(15.1b)*, the marginal rate of substitution of labor across sectors is one. This implies that the after tax wage rates across sectors are equal (and not the tax rates). Hence for $\tilde{\mathrm{N}}$, the government's optimal choice of labor income tax rates may or may not be equal across sectors, although both choices will generate allocations which can be implemented as a competitive equilibrium. In the particular policy choice where labor income tax rates vary, the government taxes capital income from consumption goods sector at a nonzero rate.

A possible extension to this specification may be to consider varying marginal disutility of labor across sectors maintaining the assumption that utility is linear in labor services. The simplest form that specifies this idea is perhaps $u(c_t, n_{ct}, n_{xt}) = \ln(c_t) + [1 - \upsilon(n_{ct}, n_{xt})]$ where $\upsilon : \mathbf{R}_+^2 \to \mathbf{R}$ is a convex function and linear in its two arguments, such that $\upsilon_{n_j n_j} = 0$ for j = C, X and $\upsilon_{n_c n_x} = \upsilon_{n_x n_c} = 0$. In order to incorporate the non-unitary marginal rate of substitution of labor in this functional form, one can define a parameter $\mu > 0$ such that $\upsilon_{n_c} = \mu \upsilon_{n_x}$. Due to the empirical evidence from US industrial sector, it is sensible to assume that $\mu \neq 1$. Invoking this specification yields the same policy set for the government as given by $\tilde{\mathrm{N}}$, and same conclusion holds.

2.4.2 Intratemporal Labor Adjustment.

This functional form, as mentioned earlier, is in the spirit of Huffman & Wynne (1999). Assume there exists some intratemporal adjustment cost of labor across sectors, and consider the following utility function:

$$u(c_t, n_{ct}, n_{xt}) = \mathbf{ln}(c_t) + \{1 - \zeta\,[\psi\, n_{ct}^{-\omega} + (1-\psi) n_{xt}^{-\omega}]^{-\frac{1}{\omega}}\} \qquad (15.1c)$$

where $\omega \leq -1, \zeta > 0$ and $1 \geq \psi \geq 0$. This specification of the utility function allows for the idea that it is costly to reallocate labor from one-sector to the other. Note that with $\omega = -1, \zeta = 2$ and $\psi = \frac{1}{2}$, *(15.1c)* reduces to $\mathbf{ln}(c_t) + \{1 - n_{ct} - n_{xt}\}$, which exhibits unitary marginal rate of substitution of labor across sectors, and is tantamount to saying that the household receives equal disutility from labor services from the two sectors. There is an issue, of course, that how the adjustment costs should be interpreted here, which I will not focus in detail. Given the main purpose of this experiment, such details which may be important otherwise, are relatively less important here.

The marginal rate of substitution of labor across sectors for this specification, for all permissible values of ω, is:

$$\frac{u_{nc}}{u_{nx}} = \frac{\psi\,\zeta\, n_c^{-\omega-1}[\psi\, n_c^{-\omega} + (1-\psi) n_x^{-\omega}]^{-\frac{1}{\omega}-1}}{(1-\psi)\zeta\, n_x^{-\omega-1}[\psi\, n_c^{-\omega} + (1-\psi) n_x^{-\omega}]^{-\frac{1}{\omega}-1}}$$

For any $\omega < -1$, which can be interpreted as the adjustment cost parameter, the optimal steady state tax rate for capital income from consumption goods sector is:

$$\theta^c = 1 - \left[\frac{\psi \zeta n_c^{-\omega-1} [\psi n_c^{-\omega} + (1-\psi) n_x^{-\omega}]^{-\frac{1}{\omega}-1} (1+\Phi) - \Phi[n_c u_{ncnc} + n_x u_{nxnc}]}{(1-\psi) \zeta n_x^{-\omega-1} [\psi n_c^{-\omega} + (1-\psi) n_x^{-\omega}]^{-\frac{1}{\omega}-1} (1+\Phi) - \Phi[n_c u_{ncnx} + n_x u_{nxnx}]} \right] \left[\frac{(1-\tau^c) u_{nx}}{(1-\tau^x) u_{nc}} \right]$$

with $u_{ncnc} \neq 0$, $u_{nxnx} \neq 0$, $u_{ncnx} \neq 0$ and $u_{nxnc} \neq 0$. This implies that the set of policies at the government's choice which can be implemented in competitive equilibrium comprises of θ^c which is nonzero, even in the case when the government sets labor income tax rates equal across sectors.

2.4.3 Two Types of Labor.

The particular functional form of utility where labor services are of two specific types is taken from the work of Jones *et al.* (1997), and here it is intended to represent the case where the planner is unable to distinguish between income from two types of labor. A probable *rationale* for this utility function may be the often realized and empirically supported fact that producing capital goods is typically more skill-intensive than producing consumption goods. The example considered therefore features one household that sells two types of labor in the market. Jones *et al.* (1997) invoke this specification with an ex ante restriction on the choice of labor income tax rates. I will consider the unconstrained version, and examine the optimal choice of capital tax for consumption goods sector.

Consider the following utility function:

$$u(c_t, 1-n_{ct}, 1-n_{xt}) = \frac{c_t^{1-\sigma}(1-n_{ct})^{\gamma_c}(1-n_{xt})^{\gamma_x}}{1-\sigma} \qquad (15.1d)$$

with $\sigma \geq 0$, and $\gamma_j < 0$ for $j = C, X$.

This utility function can be interpreted as that of a household with two members, each of which is able to supply one unit of leisure to the market each period. The marginal rate of substitution of labor across sectors with specification *(15.1d)* is:

$$\frac{u_{nc}}{u_{nx}}=\frac{\gamma_c(1-n_x)}{\gamma_x(1-n_c)}$$

Since now the utility function has cross derivatives of consumption and labor supply, it is useful to state the following expression:

$$\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}=\frac{\gamma_c}{\gamma_x}\left[\frac{\Phi\{\mathbf{Y}\}-(1+\Phi)(1-n_x)^{\gamma_x}(1-n_c)^{\gamma_c-1}}{\Phi\{\mathbf{Z}\}-(1+\Phi)(1-n_c)^{\gamma_c}(1-n_x)^{\gamma_x-1}}\right]$$

where

$$\mathbf{Y}\equiv n_c(1-n_x)^{\gamma_x}(\gamma_c-1)(1-n_c)^{\gamma_c-2}+n_x(1-n_x)^{\gamma_x-1}\gamma_x(1-n_c)^{\gamma_c-1}-(1-\sigma)(1-n_x)^{\gamma_x}(1-n_c)^{\gamma_c-1}$$

$$\mathbf{Z}\equiv n_c(1-n_x)^{\gamma_x-1}\gamma_c(1-n_c)^{\gamma_c-1}+n_x(1-n_x)^{\gamma_x-2}(\gamma_x-1)(1-n_c)^{\gamma_c}-(1-\sigma)(1-n_x)^{\gamma_x-1}(1-n_c)^{\gamma_c}$$

It is straightforward to notice that for all permissible values of the parameter γ_j, the condition $\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}=\frac{u_{nc}}{u_{nx}}$ does not hold. This implies the set of policies at the government's disposal for which a competitive equilibrium exists, i.e. $\mathrm{N}=\left\{(\tau^c,\tau^x,\theta^c,\theta^x)\middle|\theta^x=0,\frac{\mathbf{V}_{nc}}{\mathbf{V}_{nx}}\left[\frac{(1-\tau^c)u_{nx}}{(1-\tau^x)u_{nc}}\right]=1-\theta^c\right\}$, prescribes that an ex post choice of equal labor income tax rates is not sufficient to guarantee zero steady state tax on capital income from consumption goods sector. The Ramsey equilibrium consequences due to an ex ante restriction of equal factor-specific tax rates have already been discussed in subsection 3.3. Since the proof of proposition 3 holds for any utility function specification as long as it satisfies assumption 1, it holds for *(15.1d)*.

2.5 Conclusion.

The chapter formulated a two-sector neoclassical production model with infinitely-lived agents in order to analyze the optimal income taxation problem (the Ramsey problem) and to examine the celebrated optimal capital taxation principles derived from one-sector and endogenous growth analogues. The extension of one-sector model to a two-sector one with endogenous capital good's price makes it convenient to scrutinize sector-specific optimal capital income taxes in the steady state. The analysis reached a startling conclusion. While it is optimal to set a long run zero tax on capital income from capital goods sector, the optimal steady state capital income tax for consumption goods sector can be nonzero. For a standard class of utility functions that has desirable properties, this result holds, and the set of conditions for which this tax rate is zero is in no way inferred by the model. The set of feasible policies which generate competitive equilibrium allocation (from which the government can choose) prescribes that both optimal steady state capital taxes are zero if and only if the utility function is separable in consumption and labor services and linear in labor services, and the government sets the *ex post* optimal labor income tax rates equal across sectors. Unless both conditions hold simultaneously, it is not zero, i.e. for all other cases, the optimal steady state capital tax from consumption goods sector is nonzero and creates uniform nature of distortion.

An experiment of adding a constraint that restricts the government to keep factor-specific income tax rates same across sectors was conducted, which resulted in an outcome with two nonzero long run capital income taxes. Hence restricting the government's choice of income tax rates ex ante eventually forces the government to choose two nonzero capital income taxes, which cannot be the optimal policy. Initial tax rates on capital income and their inherent properties are also analyzed. The celebrated result of confiscatory taxation of initial capital in the absence of exogenous tax bounds is re-established. The optimal steady state capital tax in consumption goods sector is characterized using three popular classes of utility functions, all of which cohere the key analytical finding of nonzero capital taxation in consumption goods sector. Since there is no explicit inference from the model that the government chooses equal labor income taxes across sectors optimally, the optimal steady state capital tax from consumption goods sector is nonzero in general.

This chapter advocates that the government's long run tax policy may comprise of three income tax instruments --- the two labor income tax rates and nonzero tax on capital income in the consumption goods sector --- all of which have uniform distortion pattern. Capital income from consumption goods sector can be taxed at a nonzero rate optimally without creating compounding distortions in the long run as long as the other capital tax is set at zero. This allows economic agents to shift depreciated capital to the untaxed sector and avoid the compounding capital tax liabilities. Although a one-sector analogue of this model with similar assumptions would possibly provide consensus to the celebrated result of zero limiting capital income taxation, the current chapter suggests that simple observant arguments claiming to clarify the result are less likely to be very useful. The result of limiting zero capital income tax cannot be unconditionally generalized for a wider class of neoclassical production models. A more functional path to understanding the basic forces that drive the properties of optimally chosen tax rates is to demarcate features of the economy which are often conjectured to account for the result.

The deterministic convex model presented in this chapter incorporated two factors and corresponding two tax instruments at the disposal of the government to finance its exogenously determined consumption expenditure. This set up is probably the simplest form of two-sector dynamic general equilibrium model of optimal income taxation. It is predictable that introducing any further complexity in the framework is not likely to yield pareto improving taxes. This is proved by the experiment of constrained tax choice.

The model, as may be argued, is sensitive to the simplifying assumption of commitment technology, which enables avoidance of potential problems of time inconsistency of optimal policies typical in a dynamic general equilibrium setting with distortions. In the case where commitment power is not perfect, both the limit tax rate and the steady state allocation of capital are different from their levels found in the second best outcome. The assumption of a commitment technology is hardly acceptable in extreme form. Within the context of the current chapter the commitment device is not explicitly modelled, although one might simply consider that the government can commit to its future actions by a restriction on its constitution. It is therefore acknowledged that relaxing the commitment assumption or modelling it formally in the current setting may be important for another stream of literature. In addition, an interesting extension may be to compute the entire time path of capital taxes and hence characterize the optimal policy during the transition to the steady state. With such an extension the model can be effectively used to provide complete policy advice to governments.

Chapter 3

Monopoly Power and Optimal Taxation of Labor Income[20]

3.1 Introduction.

Until very recently, the dynamic general equilibrium tradition of optimal taxation seemed more or less silent about the departure from the simplifying assumption of economy-wide competitive markets. To my knowledge, an attempt to formally address the issue of dynamic optimal taxation with imperfect competition in private markets appeared with Judd (1997). The follow up of this literature includes Guo & Lansing (1999), Judd (2002), Koskela & Thadden (2002) and the recent paper by Schmitt-Grohe & Uribe (2004a). Except these, most general equilibrium models of Ramsey taxation with representative agent established in literature that deal specifically with optimal income taxation typically consider environments without imperfections in private markets.

[20] An earlier version of this chapter was presented in Southampton University Economics Division staff seminar series in September 2004.

Some key results as part of a paper titled "Optimal Taxation with Entry Barriers" were presented in the XII Annual Congress of Public Economics, Palma de Mallorca, Spain in February 2005.

The imperative findings of dynamic optimal taxation are therefore in most parts based on the simplifying assumption that the private sector of the economy is characterized by perfect competition in all markets. But modern economies are characterized by distortions from imperfect competition in private market, which implies that economic welfare is lower than what it could have been if markets were fully competitive[21]. In practice, therefore, the economy-wide competitive markets assumption is too restrictive, and does not always seem to be a realistic description of the incentive structure underlying policy.

This chapter first develops a model of a two-sector neoclassical production economy with tax distortions and distortions arising from monopoly power in pricing of intermediate goods. In the relevant literature, it is a well-known finding that with private market distortions optimal taxes perform a corrective function that assists in minimizing productive inefficiency. The main focus of this chapter is optimal labor income tax policy and its corrective role in the presence of private market imperfections. In the basic model of this chapter, the sector producing final goods is characterized by perfectly competitive market, but producers of intermediate goods possess a degree of monopoly power which is characterized by a single parameter. More specifically, firms in the intermediate goods sector create distortions by manipulating prices through the exploitation of a downward sloping demand curve for their output. This formulation of monopoly power is drawn primarily from the work of Dixit & Stiglitz (1977). In the model economy, government bonds are the only traded assets which yield tax free returns. The benevolent government taxes labor income and distributed profits to finance an exogenous stream of consumption expenditure. The basic model is then extended through the introduction of monopolistic wage setting. With imperfectly competitive labor market, the source of non-tax distortions diversifies and a natural intuition is that the Ramsey policies tend to be more corrective in nature.

The framework developed in this chapter is simple but useful and insightful since its economy-wide perfect competition analogue is embedded for a particular value of

[21] Jonsson (2004) presents the recent empirical evidence of this fact for the US economy.

the parameter that indexes the degree of monopoly power. The optimal taxation problem in this setting is simple in the sense that the emphasis is on the optimal choice of a single tax instrument. The government's quest is therefore to find the average level of a single tax, which in this setting is the labor income tax. This simple setting allows one to exclusively examine the temporal pattern of a corrective tax and the particular characteristics of its period by period effects. With the basic framework and its extension to imperfectly competitive labor market, the chapter derives the first best tax rules and the Ramsey tax rules, and discusses, both analytically and quantitatively, how these are designed to offset the distortions due to monopoly power.

Three main results emerge from this chapter --- (1) government's optimal choice of labor income tax rate with monopoly distortions is completely independent of how government treats taxes on distributed profits; (2) the optimal tax rate with monopoly distortions is lower than its competitive market analogue, which holds irrespective of how the government treats distributed profits fiscally; and (3) for remarkably high degrees of monopoly power, economic agents prefer distorting Ramsey taxes than first best taxes. All three results hold for both models.

The corrective function of optimal taxes in economies with private market distortions has been through an exciting process of intellectual investigations. The main concentration however has been the optimal capital income tax policy, which may be due to its political sensitivity. Judd (1999) in a competitive market model shows that a positive tax on asset income generates exponentially growing MRS/MRT distortions among goods over time. Since such explosive distortions are inconsistent with commodity taxation, the long run tax on capital income must be zero. This is however not the long run optimal policy when private market distortions violate the productive efficiency condition, as may be found in Guo & Lansing (1999), Koskela & Thadden (2002), Golosov *et al.* (2003) and Judd (2002 & 2003). In the presence of private market distortions where the efficiency condition is already violated, optimal fiscal policy can be designed to alleviate the distortion, or more precisely, as a corrective device for the distortions.

The corrective function of optimal *labor* income taxes has been partly emphasized in the literature by using models that involve both labor and capital taxes. But as mentioned earlier, the capital income tax policy has dominated the intellectual discussions. The paper by Koskela & Thadden (2002) is an exception, which for instance shows that with imperfectly competitive labor market the wage tax policy faces two conflicting demands when capital tax is set at zero. Due to such conflict, Koskela & Thadden (2002) argue that both instruments should be used, which in turn invalidates the zero capital income tax result. In referring to optimal labor income tax policy, Guo & Lansing (1999) argue that when distributed profits can be fully taxed, the entire revenue raising tax burden falls on labor, while capital income receives a subsidy as a corrective device. This result is also one of the key findings in Judd (2002).

The economic policy issue and the key results presented in this chapter are of extreme importance to policymakers. Many macroeconomic policies aimed towards outlawing monopolies and price agreements are actually targeted to enhance competition. There is a popular debate about the choice of an appropriate policy that effectively enhances competition, between the proponents of direct regulations and advocates of fiscal policy. This chapter does not pretend to examine the details of this debate, but does attempt to establish the usefulness of labor income tax policy in compensating the distorted margins of allocations due to private market imperfection. It is often argued in the literature that taxation is relatively more effective as a policy device in enhancing competition[22]. The basic idea behind this argument is that since imperfect competition creates a marginal distortion in the productive efficiency condition of an economy, tax policy must be designed in a manner such that it minimizes the inequality between marginal rate of substitution and marginal rate of transformation among goods. With no concerns of redistribution, the Ramsey taxes in such settings become more of a corrective nature rather than revenue-raising nature.

[22] Auerbach & Hines Jr. (2001b) argue that other policy instruments, such as enforcement of antitrust, may be more cost-effective at correcting the distortions of private market imperfections. In line with Judd (2003), I agree that this view has limited scope both intuitively and empirically, since there is no (or insignificant) evidence that pricing above marginal cost is related to violations of antitrust law. It is therefore difficult to think of any policy instrument other than taxation which could counterbalance the distortions due to imperfect competition, when *say*, a firm is pricing its innovated output above marginal cost since it owns a copyright that legally entitles it to do so. In a separate paper, Auerbach & Hines Jr. (2001a) however admit that when it is possible to identify imperfectly competitive market structure, an appropriate set of taxes and subsidies as a curative device is more attractive to policymakers than regulatory devices.

The theoretical proposition that monopoly power and pure profits are important in determining the function and optimal choice of tax rates actually goes back to the 1970s stream of relevant research. In a well known paper, Stiglitz & Dasgupta (1971) show that the optimal commodity tax policy for a monopolistic industry with a bound on profit taxation generally includes both differential taxes and subsidies. In a dynamic general equilibrium, differential commodity taxation is accomplished by introducing a distortion of the savings decision so that present and future consumption goods are taxed at different rates. This intuition is most commonly held for the interpretation of an optimal nonzero capital tax in models where firms in a particular sector practices monopoly power (see for instance, Guo & Lansing (1999), Judd (2002 & 2003)). Moreover, Diamond & Mirrlees (1971) argue that the existence of pure profits may require a deviation from the productive efficiency condition, implying that taxes should generally be levied on final and not on intermediate goods. This important finding is ignored in Myles (1989), who examine Ramsey taxation with imperfect competition but abstracts from general equilibrium with both intermediate and final goods.

With a much greater emphasis on optimal capital taxation, the relevant literature allows some scope to contribute in resolving the specific concerns related to optimal labor income taxation with private market imperfections. This is exactly where the current chapter is intended to contribute. The three main results of this chapter are based on strong intuitions and therefore provide some very useful insights into these policy issues. The first result, which is derived analytically for the first best tax rule and numerically for the Ramsey tax rule, is not surprising but its underlying intuition is strong. In the model economy considered here, profits actually represent the income to a fixed factor, namely, monopoly power. It is trivial that with this formulation the Ramsey planner would like to tax profits at a rate of 100% and reduce other distorting taxes. In reality, however, governments cannot implement a complete confiscation of this type of income. This may be due to the situation where the government is unable to distinguish profits from other income (or firms *somehow* hiding profits). The political viability and the consequential practicality of such a policy are also of considerable reservation. If the government cannot tax distributed profits separately, in an economy without capital a single tax rate applied to labor income must also function as a profit tax.

The models developed here consider a non-negative parameter that linearly characterizes the government's fiscal treatment of profits. Different values of this parameter characterize the different relative weight attached to profit taxation --- from no profit tax to 100% profit tax --- which enables one to consider a range of non-confiscatory profit tax solutions. It is found that for all permissible values of this parameter, both the first best labor income tax rule and the Ramsey tax rule remain unaffected. This is because distributed pure economic profit is not one of the choice variables for the households' optimization problem (unless otherwise specified) implying that profits and profit taxes do not influence households' allocation decisions at the margin. The household's equilibrium allocation decisions are sensitive to labor income tax rate which has both income and welfare effects at the margin. In the Ramsey equilibrium, the government's optimal choice of labor income tax rate is therefore independent of how the government treats profit for taxation.

The second result is the normative benchmark of optimal taxation with monopoly distortions. This is proved both analytically and numerically. The popular intuition of making a welfare maximizing distorting tax a curative device for monopoly distortions can effectively be attributed to this result. This result is driven by the fact that distortions interact, and cost of one distortion depends on the level of another. Since monopoly distortions drive a wedge between social and private returns to factors, setting the optimal tax rate lower than its competitive market analogue can compensate for resulting loss in output in the economy. Put differently, a relatively lower labor income tax than its competitive market analogue is optimal since it compensates for the monopoly induced distorted margin between social and private returns to labor. The first best tax policy in the presence of monopoly power is to subsidize labor income and impose a heavy lump sum tax which finances both the inevitable government expenditure and the subsidy. In the Ramsey equilibrium, for some degrees of monopoly power there is an optimal labor income tax, and after a threshold level of monopoly power it becomes optimal to subsidize labor income. The threshold level depends on the number of non-tax distortions. Hence starting from the competitive market analogue, higher degrees of monopoly power are associated with lower levels of Ramsey taxes, and after the threshold level higher levels of Ramsey subsidies. This result holds irrespective of how the government treats profit taxes.

For higher degree of monopoly power, there are more than proportionate increments in the wedge between social and private marginal returns to factors. This is because an elastically demanded good (or factor) sold with a price mark up possess a multiplier like demand shock effect. Since this effect induces more than proportionate increase in the wedge, its curative device must also be equivalently responsive. For remarkably high levels of price mark up (or wage mark up), the first best subsidy is higher but so is the lump sum tax. Economic agents facing such a situation will therefore be less willing to replace distorting Ramsey taxes with lump sum taxes. For high degrees of monopoly power, the utility cost of Ramsey taxes are therefore relatively lower, which explains the third important result of this chapter.

In the next section, a model of a simple economy where firms in the intermediate goods sector possess some degree of monopoly power and government taxes labor income and distributed profits to finance preset revenue target, is developed. The optimal taxation problem is presented in section 3.3. Section 3.4 introduces monopolistic wage setting in the model economy. Section 3.5 calibrates the model for the post war US economy and presents some intuitive quantitative results. Section 3.6 concludes.

3.2 The Model Economy.

In order to focus exclusively on optimal choice of labor income tax policy in the presence of monopoly power in intermediate product market, this section builds a dynamic model of an economy without physical capital where monopoly power of firms in intermediate goods sector is indexed by a single parameter. The framework developed here is simple but useful for the purpose. Its technology, as mentioned earlier, is that of Dixit & Stiglitz (1977), and its closest (and perhaps wealthier) relatives are the ones used in Benhabib & Farmer (1994), Guo & Lansing (1999) and Benassy (2002, ch. 4). This model is later extended in section 3.4 where the labor market is subject to imperfect competition due to monopolistic wage setting.

The framework of an economy without capital has amassed popularity in the literature concerning optimal fiscal and monetary policy, as may be found in Lucas & Stokey (1983), Correia, Nicolini & Teles (2002) and Schmitt-Grohe & Uribe (2004a & b). In terms of heritage, the model developed and presented here belongs to the tradition of two-sector deterministic dynamic general equilibrium framework of Ramsey taxation with representative agents, intermediate goods, and seemingly the simplest notion of market power in intermediate goods sector.

3.2.1 The Environment.

Consider a simple model economy that consists of households, firms and the government. Time t is discrete, runs forever, and t belongs to the set of integers $N = \{0,1,2,.....\}$. The production environment has two sectors: one producing intermediate goods and the other producing final goods. In the remainder of this chapter, I will hold the final good as the numeraire. The final goods sector of the economy is characterized by perfectly competitive markets. Producers of intermediate goods may possess a degree of monopoly power and hence can earn positive economic profits. All firms are owned by households who receive profits in the form of dividends.

More specifically, there is a continua of measure one of identical infinitely-lived households, each of whom are endowed with one unit of time at each instant and ownership of firms. The one unit of time can be allocated to a combination of work and leisure. In the final goods sector there is a continua of measure one of identical firms that own a technology with which a perishable final good, y_t, can be produced combining a continuum of intermediate goods z_{jt}, with $j \in [0,1]$. The final good can be used for private consumption (c_t) and exogenously determined government consumption (g_t). The final good is produced using the following constant returns to scale technology:

$$y_t = \left(\int_0^1 z_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \qquad (1.1)$$

where $\sigma \in [0,1)$ indexes the degree of monopoly power exercised by suppliers of the intermediate good z_{jt}. This is because with this specification, σ^{-1} is the elasticity of substitution between intermediate goods, and for $\sigma = 0$ intermediate goods are perfect substitutes in the production of final goods making the intermediate goods market perfectly competitive. On the other hand, $\sigma \to 1$ represents very low elasticity of substitution between intermediate goods giving higher market power to firms in the intermediate goods sector.

The intermediate goods sector comprises of j firms who own a technology with which a continuum of intermediate goods (z_{jt}) can be produced using labor service (n_{jt}) as the only input. The technology is defined as:

$$z_{jt} = n_{jt}^{\alpha} \qquad (1.2)$$

where $\alpha \in (0,1]$.

The representative household supplies labor service to firms in the intermediate goods sector. Since all households are identical, they have identical preferences over consumption of final good and labor supply. At each period, the representative household derives utility from consumption (c_t) and disutility from labor service (n_t). Preferences for the representative household are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \qquad (2)$$

where $\beta \in (0,1)$ is the subjective discount rate which varies inversely with the rate of time preference. The utility function $u : \mathbf{R}_+^2 \to \mathbf{R}$ is continuously differentiable, strictly increasing in consumption, decreasing in labor, strictly concave, and satisfies standard Inada conditions, namely $\mathbf{lim}_{c_t \to 0} [u_n(t)]^{-1} u_c(t) = \infty$, and $\mathbf{lim}_{c_t \to \infty} [u_n(t)]^{-1} u_c(t) = 0$ for any $n > 0$.

The government consumes exogenous g_t of the final good each period and has, at its disposal, taxation of labor income and pure distributed profits as the *fiscal* instruments to finance the predetermined revenue target. The proportional tax rate is denoted by τ_t. Since profits influence household's decisions only through an income effect, a trivial solution for the government would be to confiscate profits by taxing it at a rate of 100% and reduce other distorting taxes. In order to consider other optimal solutions, consider the situation where government taxes profits at a rate $\kappa\tau_t$, where κ is a parameter and $\tau_t^{-1} \geq \kappa \geq 0$. In the Ramsey equilibrium, different values of the parameter κ will illustrate the government's fiscal treatment of distributed profits. For instance, $\kappa = 0$ implies profits escape all direct taxation, and $\kappa = 1$ implies profits and labor income are taxed at the same rate[23].

The government also trades one period bonds to households, and b_{t+1} denotes real government bonds carried into period $t+1$, which pay interest at the rate r_{bt}. Interest earning from bonds are assumed to be tax-exempt. The government's period t budget constraint is given by:

$$g_t + b_t(1 + r_{bt}) = \tau_t \left[\int_0^1 w_{jt} n_{jt} dj + \kappa \int_0^1 \pi_{jt} dj \right] + b_{t+1} \qquad (3)$$

where w_{jt} denotes real wage, and π_{jt} denotes pure profits. The government has access to an effective commitment technology with which it can sustain all initially announced tax plans. The government is benevolent, i.e. it maximizes welfare of the economy.

[23] For $\kappa = \tau_t^{-1}$ profits are taxed at the rate of 100%, although in most parts of the analysis this obvious case is ignored.

3.2.2 Firms' Problems.

Let p_j denote the relative price of intermediate good z_j. The representative firm in the final goods sector competitively maximizes profits. It faces the following sequence of problems:

$$\max_{z_{jt}} \quad \left[\left(\int_0^1 z_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} - \int_0^1 p_{jt} z_{jt} dj \right] \qquad (4.1)$$

The first order condition with respect to a change in z_{jt} yields the inverse demand function of the j th intermediate good:

$$p_{jt} = y_t^{\sigma} z_{jt}^{-\sigma} \qquad (4.2)$$

The demand for the j th intermediate good is therefore $z_{jt} = (p_{jt})^{-\frac{1}{\sigma}} y_t$. The corresponding price elasticity of demand, $\eta_z = -\sigma^{-1}$, is strictly negative for $\sigma \in (0,1)$.

Firms in the intermediate goods sector possess monopoly power in pricing and face the demand function *(4.2)* for j th intermediate good. The firms take the wage rate and prices of other firms as given when choosing price and labor to maximize profits. The decision problem of the representative firm in the intermediate goods sectors is:

$$\max_{p_{jt}, n_{jt}} \quad [p_{jt} z_{jt} - w_{jt} n_{jt}] \qquad (4.3)$$

$$s.t. \quad z_{jt} = n_{jt}^{\alpha}$$

$$p_{jt} = y_t^{\sigma} z_{jt}^{-\sigma}$$

Substituting both constraints in *(4.3)* yields the following sequence of unconstrained problems for the representative firm of the intermediate goods sector:

$$\max_{n_{jt}} \quad [y_t^{\sigma} n_{jt}^{\alpha(1-\sigma)} - w_{jt} n_{jt}] \qquad (4.4)$$

The necessary condition for maximum profits is:

$$w_{jt} n_{jt} = \alpha(1-\sigma) p_{jt} z_{jt} \qquad (4.5)$$

I will restrict my attention to a symmetric equilibrium where all firms in the intermediate goods sector produce at the same level, employ the same labor and charge the same relative price. It is important to make this assumption here, although a much detailed illustration of the equilibrium is presented later. The symmetry assumption simplifies $n_{jt} = n_t$ and $p_{jt} = p_t$ for all j. Moreover, *(1.1)* and *(1.2)* imply that the aggregate production technology is given by:

$$y_t = n_t^{\alpha} \qquad (4.6)$$

Since the final goods sector is characterized by perfectly competitive markets, firms producing final goods earn zero profits in equilibrium, i.e. $\left[y_t - \int_0^1 p_{jt} z_{jt} \right] = 0$. Using *(4.2)* in the zero profit condition and imposing symmetry yields $p_{jt} = p_t = 1$ for all j. Moreover, the symmetric equilibrium imposed on *(4.5)* together with $p_{jt} = p_t = 1$ gives the equilibrium wage rate:

$$w_t = \alpha(1-\sigma) z_t (n_t)^{-1} \qquad (4.7)$$

Using *(4.7)*, the equilibrium profits for the intermediate goods sector is given by:

$$\pi_t = z_t [1 - \alpha(1-\sigma)] \qquad (4.8)$$

Since the parameter σ controls the degree of monopoly power, it is also associated with the equilibrium profit to output ratio. The equilibrium profit to output ratio for this model is linked to the degree of returns to scale in intermediate goods sector and the parameter σ. It is convenient to express the relationship between equilibrium profit to output ratio and the price mark up ratio (price over marginal cost) as:

$$n_t^{1-\alpha} = \mu\left[1 - \frac{\pi_t}{y_y}\right] \tag{4.9}$$

where μ denotes the price mark up ratio. Equation *(4.9)* is derived using *(4.7)* and *(4.6)*. For instance, if the profit ratio is 5% and degree of returns to scale in the intermediate goods sector is 1, equation *(4.9)* gives $\mu = 1.05$. With $\alpha = 1$, the profit ratio is simply equal to σ. According to Basu & Fernald's (1997) estimates on typical US industry profit ratio, the value of the price mark up ratio assuming constant returns to scale technology in manufacturing industry is 1.03. More recent empirical estimates of price mark up ratio from a study by Bayoumi, Laxton & Pesenti (2004) are equal to 1.23 for the overall US economy and 1.35 for the Euro area. The estimate for the US for instance, assuming that $\alpha = 1$ in the current setting amounts to an estimate of σ equal to 0.186 (for the Euro area it is 0.259). The other estimates established in literature also indicate lack of competition in the Euro area as compared to the US economy (see Martins, Scarpetta & Pilat (1996) for details).

To get an idea of how the distortion created by monopoly power affects factor return, consider the social marginal product of labor given by $\alpha[z_t(n_t)^{-1}]$. For $\sigma > 0$ implying practice of monopoly power, the equilibrium wage rate given by *(4.7)* is less than its social marginal product by an amount $\alpha\sigma[z_t(n_t)^{-1}]$. This is the equilibrium distortion margin in factor return due the monopoly power of firms in intermediate goods sector.

3.2.3 Household's Problem.

Each of the continua of measure one of infinitely-lived households intertemporally chooses allocations to maximize a stream of discounted utilities over consumption and labor. The decision problem of the representative household is defined by the following program:

$$\max_{c_t, n_t, b_{t+1}} \sum_{t=0}^{\infty} \beta^t u(c_t, n_t) \qquad (5.1)$$

$s.t.$

$$c_t + b_{t+1} \leq (1-\tau_t) w_t n_t + (1+r_{bt}) b_t + (1-\kappa\tau_t)\pi_t \qquad (5.2)$$

where b_0 is given, and standard non-negativity restrictions apply. Equation *(5.2)* is the time t budget constraint of the representative household. The households view w_t, r_{bt}, π_t and the government's tax policy as determined outside of their control. In addition, it is also assumed that there is no intra-household trading of bonds. This is assumed simply to avoid the complexities of having a private market for bonds. It is, however, acknowledged that relaxing this assumption may be interesting for future research. Given the main purpose of this chapter, holding this assumption is fairly understandable.

With $\beta^t \lambda_t$ as the Lagrange multiplier on the time t budget constraint, the first order conditions for this problem are the period budget constraint *(5.2)* itself and the followings:

$$c_t: \quad \lambda_t = u_c(t) \qquad (5.3a)$$

$$n_t: \quad u_n(t) = -\lambda_t (1-\tau_t) w_t \qquad (5.3b)$$

$$b_{t+1}: \quad \lambda_t = \lambda_{t+1} \beta (1 + r_{bt+1}) \qquad (5.3c)$$

and the Transversality condition (TVC)

$$TVC: \; \lim_{t\to\infty} \beta^t \lambda_t b_{t+1} = 0 \qquad (5.3d)$$

Consolidating *(5.3)*, and defining $R_{bt} \equiv (1 + r_{bt})$, the set of the household's optimality conditions that carries more convenient intuitions is:

$$-u_n(t) = u_c(t)(1-\tau_t) w_t \qquad (5.3e)$$

$$u_c(t) = u_c(t+1)\beta R_{bt+1} \qquad (5.3f)$$

$$\lim_{t\to\infty} \beta^t u_c(t) b_{t+1} = 0 \qquad (5.3g)$$

Equation *(5.3e)* states that the representative household's utility is at its maximum when the marginal rate of substitution between labor and consumption is equal to the price ratio of labor to consumption. Equation *(5.3f)* is the standard Euler equation which makes the household indifferent between consuming today and saving for a later date at the optimum. Equation *(5.3g)* states that the discounted utility is maximum when the present discounted value of government bonds in terms of consumption is zero as time goes to infinity. Thus the Transversality condition *(5.3g)* ensure that the household's within period budget constraint *(5.2)* can be transformed into an infinite-horizon present-value budget constraint.

3.2.4 Equilibrium.

As mentioned earlier, when solving for equilibrium my attention is restricted to symmetric equilibrium. Since all households are identical, they make exactly the same decisions. In a symmetric equilibrium all firms in the intermediate goods sector produce at the same level, employ the same labor and charge the same relative price. For the following definition symbols without time subscripts represent one-sided infinite sequence of the corresponding variable. The definition to a symmetric equilibrium of the model economy is as follows.

Definition 3.2.4 (Equilibrium). An equilibrium is an allocation (c, n, z, y), a price system (w, p, r_b), and a government policy (τ, b), such that

(1) given the price system and government policy, the allocation solves the firms' problems and the household's problem;

(2) given the price system and allocation, the government policy satisfies the sequence of government budget constraints *(3)*; and

(3) all markets clear in the long run. □

The equilibrium as defined above is characterized by the following system *(6.1)* for the set of endogenous variables $\{c_t, n_t, b_t, w_t, p_t, \pi_t, z_t, y_t, \tau_t\}$:

$$0 < n_t \leq 1 \tag{a}$$

$$y_t = c_t + g_t \tag{b}$$

$$y_t = z_t \tag{c}$$

$$z_t = n_t^{\alpha} \tag{d}$$

$$-u_n(t) = u_c(t)(1-\tau_t)w_t \tag{e}$$

$$u_c(t) = u_c(t+1)\beta R_{bt+1} \tag{f}$$

$$\lim_{t\to\infty} \beta^t u_c(t) b_{t+1} = 0 \tag{g}$$

$$w_t = \alpha(1-\sigma)z_t(n_t)^{-1} \tag{h}$$

$$\pi_t = z_t[1-\alpha(1-\sigma)] \tag{i}$$

$$p_t = y_t^{\sigma} z_t^{-\sigma} \tag{j}$$

Since interpretations of equations *(6.1a-j)* have already been presented, interpretation of the equilibrium system *(6.1)* is straightforward. With *(6.1b, c & d)*, the model economy's aggregate resource constraint in terms of allocations is simply:

$$c_t + g_t = n_t^{\alpha} \tag{6.2}$$

3.3 Optimal Taxation.

With r_{b0} and g_t specified exogenously, the optimal taxation problem for the government is to choose an implementable allocation $\{c_t, n_t\}_{t=0}^{\infty}$ to maximize household's utility defined by *(2)*. The notion of implementable allocations deserves further explanation in the current context. For each arbitrarily chosen fiscal policy of the government, there is a unique equilibrium allocation and prices from system *(6.1)*. This can be verified by solving *(6.1)* for any fixed policy. Thus the set of allocations that are consistent with *(6.1)* is implementable as equilibrium. If a particular tax policy that maximizes welfare is consistent with the implementable allocations, it is consistent with equilibrium feedback of the private sector. Given the preset revenue target of the government, the optimal taxation problem for the government is to choose from the set of implementable allocations an allocation that maximizes welfare, such that the resulting taxes and prices along with allocations are consistent with the equilibrium. This approach is the primal approach to optimal taxation problem, which has been introduced previously in chapter one and chapter two.

Put more technically, the optimal taxation problem for the government in this model economy is simply a programming problem of choosing $\{c_t, n_t\}_{t=0}^{\infty}$ to maximize household's utility defined by *(2)* subject to *(a)* the resource constraint defined by *(6.2)*, and *(b)* an implementability constraint that ensures the resulting taxes and prices along with allocations are consistent with the equilibrium system *(6.1)*. Since g_t is specified exogenously, this approach to the optimal taxation problem is in fact one characterization of the Ramsey problem. The implementability constraint is an intertemporal constraint involving only allocations and initial conditions, and is typically derived by using equilibrium conditions to recursively substitute out prices and taxes in the household's present-value budget constraint.

The process to derive the implementability constraint for the current model is as follows. First, I consider the household's time t budget constraint *(5.2)* and multiply both sides by $u_c(t)$. Then I use *(5.3e)* to substitute out $u_c(t)(1-\tau_t)w_t$. The resulting equation is:

$$u_c(t)c_t + u_c(t)b_{t+1} = -u_n(t)n_t + u_c(t)R_{bt}b_t + u_c(t)(1-\kappa\tau_t)\pi_t \tag{7.3a}$$

Now I evaluate *(7.3a)* for $t=1$, multiply both sides by β, and substitute for βR_{b1} using the $t=0$ version of *(5.3f)*. The resulting equation is:

$$u_c(0)b_1 = \beta[u_c(1)c_1 + u_c(1)b_2 + u_n(1)n_1 - u_c(1)(1-\kappa\tau_1)\pi_1] \tag{7.3b}$$

I evaluate *(7.3b)* for $t = 2$, and substitute back the resulting expression for $u_c(1)b_2$ in *(7.3b)*. I repeat the similar process for $t = 3,4,.........$, and as $t \to \infty$ impose the condition $\lim_{t\to\infty} \beta^t u_c(t)b_{t+1} = 0$. Finally I use the $t = 0$ version of *(7.3a)* to substitute for $u_c(0)b_1$. This gives the following intertemporal equation:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t)c_t + u_n(t)n_t - u_c(t)(1-\kappa\tau_t)\pi_t] - u_c(0)R_{b0}b_0 = 0 \qquad (7.3c)$$

Equation *(7.3c)* is still not the implementability constraint since it is not yet expressed in terms of allocations only. In order to derive an expression for $(1-\kappa\tau_t)\pi_t$ in terms of allocations, consider first *(6.1d & i)* which gives:

$$\pi_t = n_t^{\alpha}[1-\alpha(1-\sigma)] \qquad (7.3d)$$

Now consider *(6.1e, h & d)* to find:

$$\tau_t = 1 + \frac{u_n(t)}{u_c(t)\alpha(1-\sigma)n_t^{\alpha-1}} \qquad (7.3e)$$

With *(7.3d & e)*, the implementability constraint is *(7.3c)*, where

$$(1-\kappa\tau_t)\pi_t = \left[\frac{1-\alpha(1-\sigma)}{\alpha(1-\sigma)}\right]\left[\alpha(1-\sigma)(1-\kappa)n_t^{\alpha} - \kappa\frac{u_n(t)n_t}{u_c(t)}\right] \qquad (7.3f)$$

3.3.1 The Ramsey Problem.

The Ramsey problem for the government is to choose a policy $\{\tau_t\}_{t=0}^{\infty}$ that maximizes welfare defined by *(2)* subject to the government budget constraint defined by *(3)* such that the resulting policy and the associated allocations and prices are consistent with equilibrium feedback of taxpayers. According to the primal approach, this problem can be characterized as one where the government chooses $\{c_t, n_t\}_{t=0}^{\infty}$ to maximize household's utility defined by *(2)* subject to *(6.2)*, *(7.3c)* and *(7.3f)*. Let $\Phi \geq 0$ be the Lagrange multiplier associated with the implementability constraint, and define the Pseudo objective function as:

$$V(c_t, n_t, \Phi) \equiv u(c_t, n_t) + \Phi[u_c(t)c_t + u_n(t)n_t - u_c(t)(1-\kappa\tau_t)\pi_t] \qquad (8.1)$$

where $(1-\kappa\tau_t)\pi_t = \left[\dfrac{1-\alpha(1-\sigma)}{\alpha(1-\sigma)}\right]\left[\alpha(1-\sigma)(1-\kappa)n_t^{\alpha} - \kappa\dfrac{u_n(t)n_t}{u_c(t)}\right]$.

With $\{\chi_t\}_{t=0}^{\infty}$ as the sequence of Lagrange multipliers on the resource constraint *(6.2)*, the Ramsey problem's Lagrangian is:

$$J = \sum_{t=0}^{\infty} \beta^t \{V(c_t, n_t, \Phi) + \chi_t(n_t^{\alpha} - c_t - g_t)\} - \Phi u_c(0) R_{b0} b_0 \qquad (8.2)$$

For exogenously determined g_t, R_{b0} and b_0, the Ramsey problem amounts to maximizing *(8.2)* with respect to $\{c_t, n_t\}_{t=0}^{\infty}$. The necessary conditions for an optimum for this problem due to changes in allocations are:

$$c_t: \quad V_c(t) = \chi_t \qquad \forall t \geq 1 \qquad (8.3a)$$

$$n_t: \quad V_n(t) = -\alpha\chi_t n_t^{\alpha-1} \qquad \forall t \geq 1 \qquad (8.3b)$$

$$c_0: \quad V_c(0) = \chi_0 + u_{cc}(0)\Phi R_{b0} b_0 \qquad (8.3c)$$

$$n_0: \quad V_n(0) = \Phi R_{b0} b_0 u_{cn}(0) - \alpha\chi_0 n_0^{\alpha-1} \qquad (8.3d)$$

Consolidating *(8.3)* yields the following two conditions:

$$V_n(t) = -\alpha V_c(t) n_t^{\alpha-1} \qquad \forall t \geq 1 \qquad (8.4a)$$

$$V_n(0) = [u_{cc}(0)\Phi R_{b0} b_0 - V_c(0)]\alpha n_0^{\alpha-1} + \Phi R_{b0} b_0 u_{cn}(0) \qquad (8.4b)$$

The Ramsey equilibrium is therefore characterized by a system of equations comprising *(8.4a)*, *(8.4b)*, *(7.3c)*, *(7.3f)* and *(6.2)*.

Note first that the Lagrange multiplier Φ represents the utility cost of raising revenue through distorting taxes. In other words, Φ is the amount in units of time 0 consumption that households would be willing to pay in order to replace one unit of distorting tax revenue by one unit of lump sum tax revenue. To solve the system for Ramsey allocations and Ramsey taxes, one can fix Φ and solve *(8.4)* and *(6.2)* for an allocation. Then one can substitute these allocations in the implementability constraint *(7.3c & f)*, and depending on whether the implementability constraint is binding or slack, one can increase or decrease the value of Φ. Once the resulting allocations satisfy the implementability constraint, a unique value of Φ is obtained, and allocations and prices constitute equilibrium as defined in 3.2.4.

In the next subsection, it is shown that a unique steady state Ramsey tax rule exists for a unique value of the multiplier Φ. Furthermore, in section 3.5 it is formally demonstrated that for a unique steady state allocation there exists a unique value of the multiplier Φ. In general, for a $T \geq 0$ for which fluctuations in government expenditure is arbitrarily small for all $t \geq T$, the solution to *(8.4)* can be characterized by a set of stationary allocation rules $c_t(c_{t-1}, n_{t-1}, \Phi)$ and $n_t(c_{t-1}, n_{t-1}, \Phi)$. Given these allocation rules, one can use *(4.7)*, *(5.3a)*, *(5.3b)*, *(6.1d)*, *(7.3d)* and *(7.3f)* to compute a set of stationary rules for the factor price and tax rate: $w_t(c_{t-1}, n_{t-1}, \Phi)$ and $\tau_t(c_{t-1}, n_{t-1}, \Phi)$ for $t \geq T$. A stationary allocation rule for government bonds can be computed by recursively solving the household's budget constraint *(5.2)* by substituting out prices and taxes for allocations. The optimal allocations for $t \leq T$ can be computed by solving *(8.4)* backwards in time, starting from $t = T$ and by imposing the stationary allocation rules for $t \geq T$ as the boundary conditions. The entire sequence of allocations, together with the initial conditions, determines the multiplier Φ such that the implementability constraint *(7.3c)* is satisfied.

3.3.2 Fiscal Policy.

I will start the analysis of optimal fiscal policy by considering the first best tax policy. If the government had an access to a lump sum tax ($\equiv \ell_t$) and could take up the first best tax policy, the equilibrium allocations would coincide with those chosen by a benevolent social planner who maximizes utility as defined by *(2)* subject to the resource constraint *(6.2)*. In this problem government bond do not affect the equilibrium allocations, and hence it is convenient to set $b_t = 0$ for all t.

Proposition 1: The first best fiscal policy corresponding to equilibrium *(6.1)* is to subsidize labor income, and generate all revenues by a lump sum tax. In particular, the first best fiscal policy implies:

$$\tau_t^1 = \frac{-\sigma}{1-\sigma} \quad \text{and} \quad \ell_t = g_t + \frac{\sigma}{1-\sigma}[y_t\{\alpha(1-\sigma)(1-\kappa)+\kappa\}]$$

Proof: To account for lump sum taxes, the term ℓ_t is added to the right hand side of the symmetric version of the government's budget constraint *(3)*. Now, consider $\beta^t \lambda_t^1$ as the Lagrange multiplier associated with the resource constraint *(6.2)*. The necessary conditions for the optimum of the planner's problem for changes in allocations are:

$$c_t: \quad u_c(t) = \lambda_t^1 \tag{8.5a}$$

$$n_t: \quad u_n(t) = -\alpha\lambda_t^1 y_t (n_t)^{-1} \tag{8.5b}$$

Recall household's optimizing conditions *(5.3)*. Comparing *(8.5a)* with *(5.3a)* gives $\lambda_t = \lambda_t^1$. Hence *(8.5b)* and *(5.3b)* yield $\tau_t^1 = \frac{-\sigma}{1-\sigma}$ which is strictly negative for $\sigma \in (0,1)$. The government's budget constraint with lump sum tax then yields $\ell_t = g_t + \frac{\sigma}{1-\sigma}[y_t\{\alpha(1-\sigma)(1-\kappa)+\kappa\}]$ after substituting for $w_t n_t, \tau_t^1$ and π_t. ■

A welfare maximizing social planner would seek to implement an allocation which is characterized by the optimality condition *(8.5)*. To replicate these conditions in an (imperfectly) competitive equilibrium, as is inferred from the first order conditions of the representative household's maximization problem, taxes have to be set according to $\tau_t^1 = \frac{-\sigma}{1-\sigma}$. The first best policy therefore involves subsidizing labor income for inefficiency due to the monopoly power and generating all revenues by a heavy lump sum tax. The monopoly power creates a wedge between social and private marginal returns to labor which is corrected by the subsidy.

Note that the competitive market analogue of this result, which is derived by setting $\sigma = 0$, is zero distorting tax and $\ell_t = g_t$. Moreover, for $\sigma \in (0,1)$ the lump sum tax is strictly greater than government's planned expenditures when profits are taxed at the same rate as labor income (i.e. $\kappa = 1$), and when profits are not taxed at all (i.e. $\kappa = 0$). Understandably, the case of 100% profit tax for the first best fiscal policy is ignored. What is important to notice here is that for higher degrees of monopoly power, both the amount of subsidy and lump sum tax increases, and the rate of increase in lump sum tax is higher than that of the first best labor income subsidy.

Now consider the Ramsey policy where government do not have an access to lump sum tax. At this point, consider some standard simplifications in the utility function only for the sake of analytical tractability. Let $u : \mathbf{R}_+^2 \to \mathbf{R}$ be separable in consumption and labor, and linear in labor, as supported by Hansen (1985), among others. Imposing these restrictions is tantamount to assuming $u_{cn}(t) = u_{nc}(t) = u_{nn}(t) = 0$. Furthermore, following the literature it is assumed that there exists a solution to the Ramsey problem which converges to a time-invariant allocation, giving rise to a unique steady state income tax. From *(8.4a)* and *(5.3b)*, the steady state level of the optimal tax rate is given by the following equation:

$$1 - \tau = \frac{u_c\{1 + \Phi[\kappa + \alpha - \alpha\sigma - \alpha\kappa + \alpha\sigma\kappa]\} + \Phi u_{cc}[c - (1-\kappa)\pi]}{u_c\{1 - \sigma + \Phi[1 - \sigma + \kappa(\alpha)^{-1} - \kappa + \sigma\kappa]\}} \tag{8.6}$$

Consider the competitive equilibrium version of *(8.6)* and denote the corresponding steady state tax rate by τ^{p}. This is obtained simply by setting $\sigma = 0$ in *(8.6)*, which results in the following equation:

$$1-\tau^{p} = \frac{u_c\{1+\Phi[\kappa+\alpha(1-\kappa)]\}+\Phi u_{cc}[c-(1-\kappa)\pi]}{u_c\{1+\Phi[1+\kappa(\alpha)^{-1}-\kappa]\}} \qquad (8.7)$$

Since the sign and relative magnitudes of τ and τ^{p} are rather inconclusive from *(8.6)* and *(8.7)*, I will resort to calibration and numerical results to analyze the key findings. Nevertheless, one analytical result is quite insightful and comes right out of the above two expressions. It is formalized as follows.

Proposition 2: If profits are taxed at the same rate as labor income is taxed (i.e. $\kappa = 1$), equation *(8.6)* from the Ramsey equilibrium implies that optimal tax rate is lower than its competitive market analogue.

Proof: If profits and labor income are taxed at the same rate, the government cannot set $\tau \geq 1$, since it violates Transversality condition *(5.3g)*. Hence *(8.6)* with $\kappa = 1$ implies $[u_c(1+\Phi)+\Phi u_{cc}c] > 0$. Comparing *(8.6)* with $\kappa = 1$ and *(8.7)* with $\kappa = 1$, it is straightforward to show that $(\tau - \tau^{p}) < 0$. ■

The intuition for this result is clear. In the presence of monopoly power, a lower tax rate relative to its competitive market analogue is optimal since it offsets the distortions created by the monopoly power. As will be shown later in section 3.5, proposition 2 actually holds for all permissible values of the parameter κ.

3.4 Monopolistic Wage Setting.

In this section I will introduce the simplest form of monopolistic wage setting behaviour of workers in the model economy. This is in the spirit of Koskela & Thadden (2002). The optimal income taxation problem now deviates from a first best representative agent economy in three aspects: first, to raise revenue the government must use distorting second best tax; second, the intermediate product market is imperfectly competitive; and third, the labor market is imperfectly competitive and subject to monopolistic wage setting by workers, i.e. wages are set with a mark up compared to a fully competitive outcome leading to a socially suboptimal level of working hours.

Consider specialization of the environment presented in section 3.2 to introduce monopolistic wage setting. Assume that households collectively organize in a trade union which acts as a monopolistic wage setter. Wages are set for one period, and the wage setting behaviour takes into account the static constraint imposed by the labor demand schedule $n_{jt} = n(w_{jt})$. Since firms are small relative to the economy, they are unable to behave in a strategic manner towards the wage setting behaviour. This assumption abstracts the model from the hold-up problem which typically arises under firm specific bargaining. Assume that the behaviour of the union is myopic in the sense that intertemporal feedback effects of wage setting are not taken into account. The union is also assumed not to influence profits which are distributed back to its members. Assume further that the institutional set up which generates the market inefficiency is taken as given by the government when designing the tax policy, implying that corrective taxes or subsidies are the only channel to address the labor and intermediate product market distortion. The proportional tax rate on wage is denoted by τ_t^m.

Recall the profit maximization problem of the representative firm in intermediate goods sector, given by *(4.4)*. Imposing symmetry, the first order condition to this problem yields the following wage function which is the wage setting constraint for the trade union's maximization problem:

$$w_t = \alpha(1-\sigma)n_t^{\alpha-1} \qquad (9.1a)$$

The wage elasticity of labor demand therefore is

$$\eta_w = (-1)\frac{1}{[1-\alpha(1-\sigma)]} \qquad (9.1b)$$

Acting on behalf of its members, the trade union maximizes utility defined by *(2)* subject to constraints *(5.2)* and *(9.1a)*. The first order condition for variation in labor supply is:

$$u_n(t) = -u_c(t)\alpha(1-\sigma)(1-\tau_t^m)w_t \qquad (9.1c)$$

The mark up of net wages over the marginal rate of substitution between labor and consumption is therefore $\frac{1}{\alpha(1-\sigma)}$, which is equal to $\frac{|\eta_w|}{|\eta_w|-1}$. Comparing *(9.1c)* with social planner's equilibrium *(8.5)*, the first best policy (in steady state) for this model is $\tau^{m1} = 1 - \frac{1}{\alpha(1-\sigma)^2} < 0$.

Following the procedure presented in section 3.3, it is straightforward to derive the implementability constraint for the corresponding Ramsey problem, which is:

$$\sum_{t=0}^{\infty}\beta^t\left[u_c(t)c_t + \frac{u_n(t)n_t}{\alpha(1-\sigma)} - u_c(t)(1-\kappa\tau_t^m)\pi_t\right] - u_c(0)R_{b0}b_0 = 0 \qquad (9.2)$$

where $(1-\kappa\tau_t^m)\pi_t = \left[\frac{1-\alpha(1-\sigma)}{\alpha(1-\sigma)}\right]\left[\alpha(1-\sigma)(1-\kappa)n_t^{\alpha} - \kappa\frac{u_n(t)n_t}{\alpha(1-\sigma)u_c(t)}\right]$.

Define the Pseudo utility function associated with the Ramsey problem as:

$$V^m(c_t,n_t,\Phi^m) \equiv u(c_t,n_t) + \Phi^m\left[u_c(t)c_t + \frac{u_n(t)n_t}{\alpha(1-\sigma)} - u_c(t)(1-\kappa\tau_t^m)\pi_t\right] \qquad (9.3)$$

where $\Phi^m \geq 0$ is the multiplier associated with the implementability constraint, and represents the utility cost of raising revenue by distorting taxes. The Lagrangian for the Ramsey problem is:

$$J^m = \sum_{t=0}^{\infty} \beta^t \left\{V^m(c_t, n_t, \Phi^m) + \chi_t^m(n_t^{\alpha} - c_t - g_t)\right\} - \Phi^m u_c(0) R_{b0} b_0 \qquad (9.4)$$

The first order condition with respect to variation in labor supply for $t \geq 1$ is:

$$V_n^m(t) = -\alpha V_c^m(t) n_t^{\alpha-1} \qquad \forall t \geq 1 \qquad (9.5)$$

where, imposing $u_{cn}(t) = u_{nc}(t) = u_{nn}(t) = 0$,

$$V_n^m(t) = u_n(t)\left\{1 + \frac{\Phi^m}{\alpha(1-\sigma)} + \frac{\Phi^m[1-\alpha(1-\sigma)]\kappa}{[\alpha(1-\sigma)]^2}\right\} - \Phi^m u_c(t)(1-\kappa)[1-\alpha(1-\sigma)]\alpha n_t^{\alpha-1} \qquad (9.6a)$$

$$V_c^m(t) = u_c(t)(1+\Phi^m) + \Phi^m u_{cc}(t)\left\{c_t - (1-\kappa)n_t^{\alpha}[1-\alpha(1-\sigma)]\right\} \qquad (9.6b)$$

As in the previous section, assume solution to the Ramsey equilibrium converges to a time-invariant allocation. Equation *(9.5)* is one of the Ramsey equilibrium conditions, which generates allocations and prices which are consistent with the (imperfectly) competitive equilibrium. Combining steady state versions of *(9.5)* with *(9.1c & a)*, one can derive the following expression for the steady state optimal tax rule:

$$1-\tau^m = \frac{u_c\{1+\Phi^m[\kappa+\alpha-\alpha\sigma-\alpha\kappa+\alpha\sigma\kappa]\} + \Phi^m u_{cc}[c-(1-\kappa)\pi]}{u_c\{\alpha(1-\sigma)^2 + \Phi^m[1-\sigma+\kappa(\alpha)^{-1}-\kappa+\sigma\kappa]\}} \qquad (9.7)$$

Note first that the denominator of the right hand side of expression *(9.7)* has the term $\alpha(1-\sigma)^2$, which in expression *(8.6)* is $(1-\sigma)$. Although both *(9.7)* and *(8.6)* include mostly the same structural parameters of the model, analytical comparison of these two is not conclusive since the multipliers associated with the implementability constraints of these two problems are not same. Since the main function of the optimal distorting taxes are corrective, and since the sources and levels of market distortions are different in the two models, it is reasonable to conjecture that Ramsey taxes will have different social costs.

3.5 Calibration and Numerical Results.

I will use US economy's data to calibrate the model in order to focus on a subset of interesting numerical results, which in turns will highlight the key policy findings of this chapter. The model is calibrated for both versions, namely, the perfectly competitive labor market version and monopolistic wage setting version. In line with the basic assumptions underlying the period utility function given in *(2)* and the assumption $u_{cn}(t) = u_{nc}(t) = u_{nn}(t) = 0$, consider the following specification:

$$u(c_t, n_t) = \ln(c_t) + [1 - \Lambda n_t] \tag{10.1}$$

where $\Lambda > 0$ is a constant associated with marginal disutility of work. For the model with monopolistic wage setting, I will denote this parameter by Λ^m. For the same set of long run observations, the calibrated Λ and Λ^m will be two different values since these are pinned down using *(5.3e)* and *(9.1c)*, respectively.

First, consider the model with perfectly competitive labor market. With *(10.1)*, and dropping time subscripts, the Ramsey equilibrium condition *(8.4a)* can be written as:

$$\frac{c}{y}\left\{\frac{\Lambda n[1 + \Phi + \frac{\Phi}{\alpha(1-\sigma)}\kappa\{1 - \alpha(1-\sigma)\}]}{\alpha[1 + \Phi(1-\kappa)\frac{\pi}{c}]}\right\} + \frac{\Phi(1-\kappa)\{1 - \alpha(1-\sigma)\}}{[1 + \Phi(1-\kappa)\frac{\pi}{c}]} = 1 \tag{10.2a}$$

Now consider the model with monopolistic wage setting. The analogous condition is:

$$\frac{c}{y}\left\{\frac{\Lambda^{m}\, n[1+\frac{\Phi^{m}}{\alpha(1-\sigma)}+\frac{\Phi^{m}}{\{\alpha(1-\sigma)\}^{2}}\kappa\{1-\alpha(1-\sigma)\}]}{\alpha\,[1+\Phi^{m}\,(1-\kappa)\frac{\pi}{c}]}\right\}+\frac{\Phi^{m}\,(1-\kappa)\{1-\alpha(1-\sigma)\}}{[1+\Phi^{m}\,(1-\kappa)\frac{\pi}{c}]}=1$$

(10.2b)

The idea of the calibration is as follows. The set of parameters for the model is $(\beta,\alpha,\sigma,\kappa,\Lambda,\Lambda^{m})$. First, these parameters are pinned down to fit the stylized facts of the US economy for data period 1960-2001. The time period considered is one year which is consistent with the frequency of fiscal policy revision. In particular, I will parameterize the model for $(\beta,\alpha,\sigma,\kappa,\Lambda,\Lambda^{m})$ to fit the facts of the US economy for the approximate data period of 1960-2001. Some estimates are also used from the literature. Using these pinned down values in *(10.2)* will give estimates of the multipliers Φ and Φ^{m}. Then, the set of calibrated parameter values and the calibrated multiplier values are used to derive an estimate of the optimal tax rate using *(8.6)*, *(9.7)*, *(7.3d)* and *(10.1)*.

All numerical results are therefore based on the assumption that the instantaneous utility function is separable in its arguments and linear in labor. The two key parameters of the model for which variations may be of interest are the profit tax treatment parameter, κ, and the parameter associated with market power, σ. Once the model has been calibrated, I will vary these two parameters within reasonable range to derive insights regarding the sensitivity of the key results with respect to these.

3.5.1 Data and Parameterization.

Annual data of the US economy for the period 1960-2001 are taken from the *Federal Reserve Bank of St. Louis Economic Data*-FRED II. According to this data, in seasonally adjusted real terms average government consumption to output ratio is equal to 0.23, profit to output ratio is equal to 0.11, and government bond to output ratio is equal to 0.51. Since the model is without capital, the only interest rate is the interest rate on government bonds. I use an interest rate value of 6% which is a reasonable approximation of the series of interest rate on US government securities[24]. This is consistent with an estimate of $\beta = 0.9434$. Working hours estimate is set at 0.3 which implies that the average time an individual spends in employment is about $\frac{1}{3}$ of total time. This approximation is frequently used as a benchmark that reflects the average time people between 18-64 years in the US spend in employment. The calibration, however, was verified for working hours range of 0.2 to 0.3, following Cooley & Prescott (1995), and it was found that the key findings are consistent within this range. The target statistics are summarized in table 3.1.

Table 3.1: Steady state ratios for the US economy, 1960-2001.

Ratio	Description	Value
g/y	Government consumption to output ratio.	0.23
π/y	Profit to output ratio.	0.11
b/y	Government bond to output ratio.	0.51

Source: Federal Reserve Bank of St. Louis Economic Data-FRED II.

According to its specification, the parameter κ stands for the fiscal treatment of profits and is the ratio between profit tax and labor tax. The profit tax in this model is the tax that households pay on distributed profits. McGrattan & Prescott (2005) estimate a tax rate on corporate distributions for the US and the UK economy, which is the personal income tax rate on dividend income if corporations make distributions to households by paying dividends. I use their period average estimate of 17.4% for 1990-2000 for the US economy. Note that McGrattan & Prescott's (2005) period average estimate for this tax rate for 1960-1969 is 41.1%, and the dramatic decline in this rate is due to three tax reforms between 1964-1986. For the average effective tax rate on labor income for the US economy, I use a value of 22.6% from Carey & Tchilinguirian (2000). This pins down $\kappa = 0.76991$.

[24] Interest rate sensitivity of the key numerical results is not noteworthy. The model was calibrated with interest rate values of 4%, 5% and 6%, which yielded minor changes in the third and fourth decimal digits of the main numerical results.

There are two convenient ways one can pin down the parameter σ. First, one can simply *assume* $\alpha = 1$, which pins down σ equal to the profit to output ratio. The second way is to use price mark up estimates from the literature and derive an estimate of σ that is consistent with the mark up value. This in turn will pin down α which is consistent with both profit ratio and the mark up value. Here I follow the latter. There is, however, some difficulty associated with choosing the appropriate value for price mark up. From the literature on empirical estimation and evaluation of monopoly power and price mark up ratio, an interesting observation is the range of estimates for the price mark up ratio (denoted μ for the current setting). The estimates for price mark up ratio for the US economy ranges from as low as 1.03 in Basu & Fernald (1997) to as high as 1.23 in Bayoumi *et al.* (2004). There are even higher estimates of this ratio for particular industries of the US, as may be found in detail in Martins *et al.* (1996). For the current model, I choose $\mu = 1.12$ as the price mark up ratio, which is the Martins *et al.* (1996)'s 1970-1992 average estimate for US industries producing differentiated goods. Given the range of available estimates, this is a reasonable approximation. From the steady state version of *(4.9)*, this pins down $\alpha = 0.99734$. Steady state versions of *(4.8)* and *(6.1c)* are therefore consistent with $\sigma = 0.10763$. For the model with monopolistic wage setting, the baseline wage mark up estimate is therefore equal to 1.12, which is very close to the recent estimate of 1.16 for the US economy, as in Bayoumi *et al.* (2004).

With average government consumption to output ratio equal to 0.23, the steady state version of *(6.1b)* gives private consumption to output ratio equal to 0.77. The baseline estimate for Λ and Λ''' are then 2.8075 and 2.4987, respectively. Using these parameter values in *(10.2)* gives $\Phi = 0.4963$ and $\Phi''' = 0.5978$. The parameters, their brief description and their baseline values are presented in table 3.2.

Table 3.2: **Baseline parameter values.**

Parameter	Description	Value
β	Subjective discount rate.	0.9434
α	Degree of returns to scale in intermediate goods sector.	0.9973
σ	Inverse of the elasticity of substitution between intermediate goods.	0.1076
κ	Fiscal treatment of profits.	0.7699
Λ	Value of marginal disutility of labor (competitive labor market).	2.8075
Λ^m	Value of marginal disutility of labor (monopolistic wage setting).	2.4987

For comparison and sensitivity of the calibrated optimal tax rate for changes in κ and σ, a range of values such that $\kappa \in [0,1]$ with 0.1 difference and $\sigma \in [0,0.4]$ with 0.05 difference between two consecutive values, is considered. Note that varying the value for κ and σ requires recalibration of the multipliers Φ and Φ^m. This implies that the utility cost of raising revenue through distorting taxes varies for changes in fiscal treatment of profits and the parameter controlling the degree of monopoly power, which is fairly intuitive.

3.5.2 Quantitative Findings.

The main quantitative findings are summarized in table 3.3 and figures 3a-e. In constructing the figures, a single parameter was varied while simultaneously recalibrating the other parameters and the multipliers to match the long run characteristics of US data. Consider first calibration of the model with perfectly competitive labor market. The calibrated optimal tax rate is equal to 27.13%, which is reasonably close to the estimated average effective labor income tax rate of 26.7% and 22.6% for the US economy for data period 1991-1997, as reported in Carey & Tchilinguirian (2000), using Mendoza *et al.* (1994) and Carey & Tchilinguirian (2000)'s methodology, respectively. Even without capital, the model therefore presents is a sensible imitation of the US economy. For the model with monopolistic wage setting, the baseline parameter values gives optimal tax rate equal to 28.21% ---

a slightly higher estimate than the one for competitive labor market model. The calibrated tax estimates for both models are preserved for all permissible values of κ, implying that the government's optimal choice of tax rate is completely insensitive to its fiscal treatment of profits. This is not surprising, since profit tax as modelled here distorts the welfare margin only through an income effect. More intuitively, household's allocation decisions are not affected at the margin by κ which enables the government to choose optimal tax rate without any concern of its fiscal treatment of profits.

Table 3.3: Calibrated optimal tax rates.

	τ^P (Ramsey, $\sigma = 0$)	τ (Ramsey, $\sigma = 0.1076$)	τ^1 (First Best, $\sigma = 0.1076$)
Competitive Labor Market	0.3497	0.2713	-0.1206
Monopolistic Wage Setting	0.4011	0.2821	-0.2591

Table 3.3 presents the competitive market analogue of optimal tax rate (τ^P) with recalibrated parameters and multipliers, the baseline calibrated Ramsey tax rate (τ), and the first best tax rate (τ^1) with baseline parameters, for both competitive labor market and monopolistic wage setting specialization of the model. For $\sigma = 0$, the optimal tax rates for the model with competitive labor market and monopolistic wage setting are equal to 34.97% and 40.11%, respectively. Not surprisingly, these estimates (for $\sigma = 0$) are also insensitive to changes in the parameter κ. Combining these findings imply that proposition 2 holds for all permissible values of κ; more generally, the optimal tax rate with monopoly distortions is lower than its competitive market analogue irrespective of how the government treats taxes on distributed profits.

Figure 3a and 3b present how the utility cost of distorting taxes varies with different values of the parameters σ and κ. Figure 3a shows that a higher degree of monopoly power is associated with a relatively lower utility cost of distorting taxes, which holds for both models. Higher σ is associated with households' willingness to pay lesser amount of time 0 consumption goods to replace a unit of distorting tax by a unit of lump sum tax, implying that households facing higher monopoly distortions would be more willing to accept a distorting tax as a corrective device. Note that in figures 3a and 3b, the rate of decline in Φ^m is much sharper than that of Φ, implying that introducing an additional distortion in the model makes corrective Ramsey taxes relatively more desirable from social cost of taxation point of view.

Fig 3a: Utility cost of taxation vs. sigma.

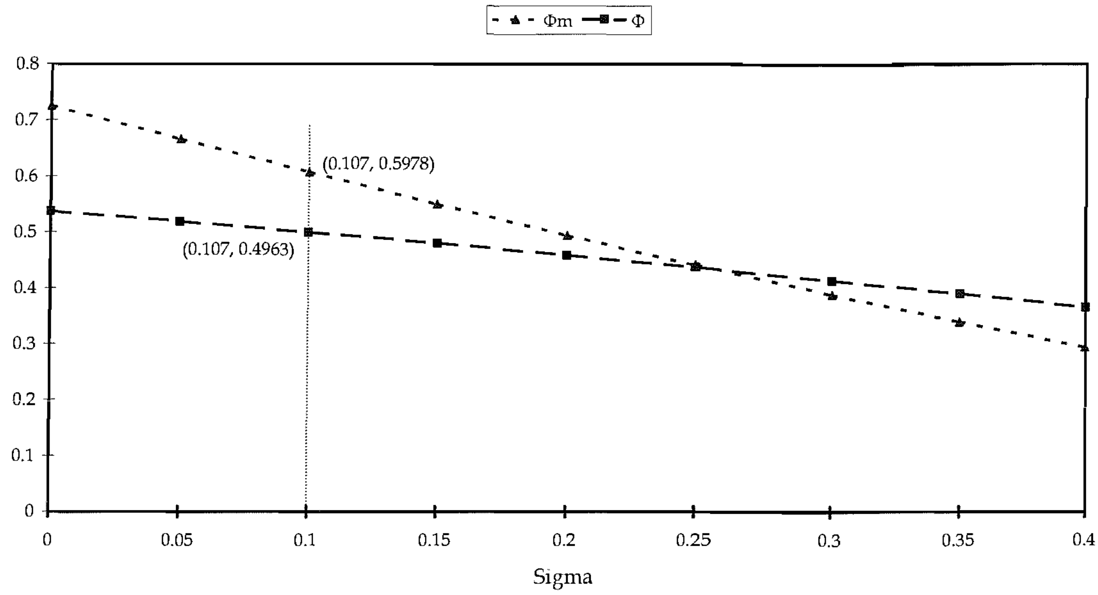


Fig 3b: Utility cost of taxation vs. kappa.

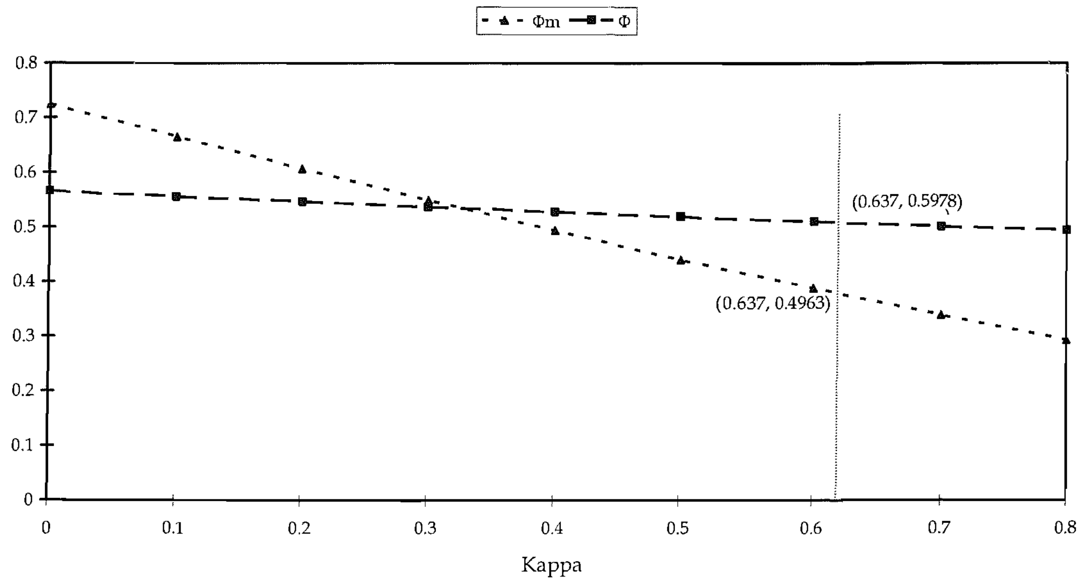


Figure 3c presents the Ramsey tax rates for both models for a range of values of the parameter σ. Figures 3d and 3e compare the Ramsey taxes with the first best taxes for the competitive labor market model and the monopolistic wage setting model, respectively, for a range of values of the parameter σ. First consider figure 3c. For higher values of the parameter σ, the optimal tax rate continues to be lower. For the competitive labor market model, it reaches the zero level at approximately $\sigma = 0.34$, and for any σ higher than this level it becomes optimal to subsidize labor income. For the monopolistic wage setting model, the optimal tax reaches zero level for $\sigma = 0.24$ and continues to be subsidy thereafter. Since the optimal choice of tax rate is influenced by both the wedge between social and private marginal returns to labor and the diminishing utility cost of distorting taxes for higher values of the parameter σ, the decline in τ^m is much sharper than the decline in τ.

Fig 3c: Ramsey tax rates vs. sigma.

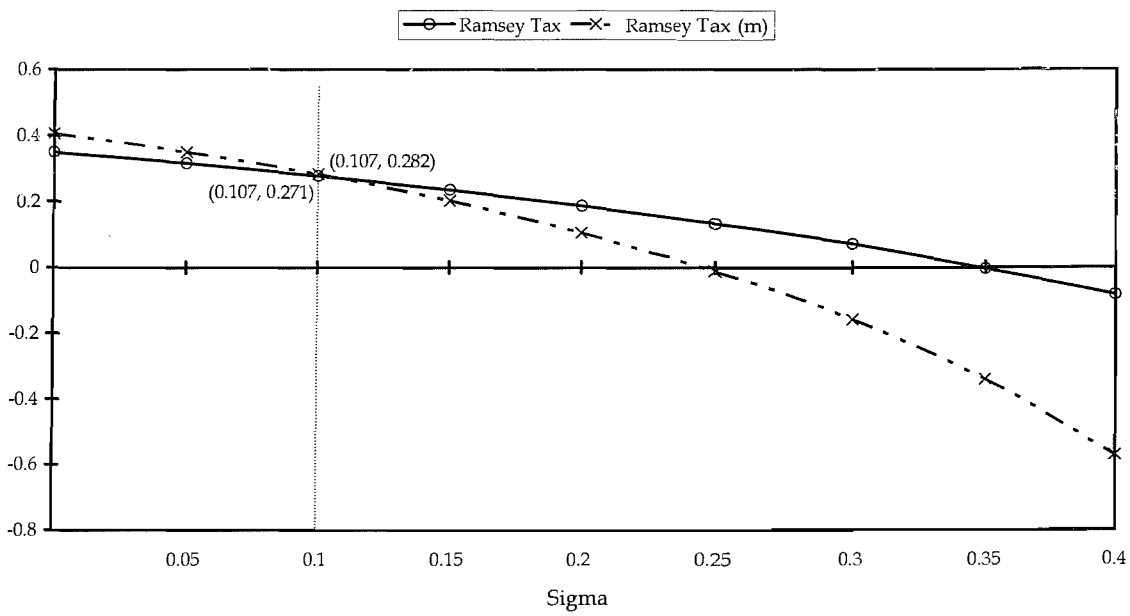


Fig 3d: Ramsey and first best tax (competitive labor market) vs. sigma.

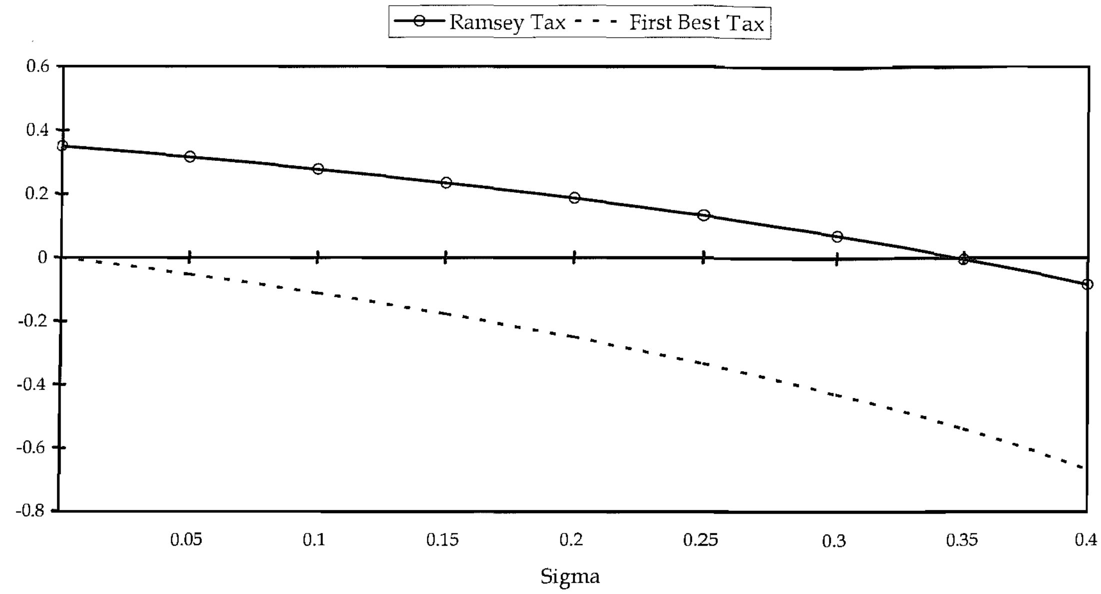
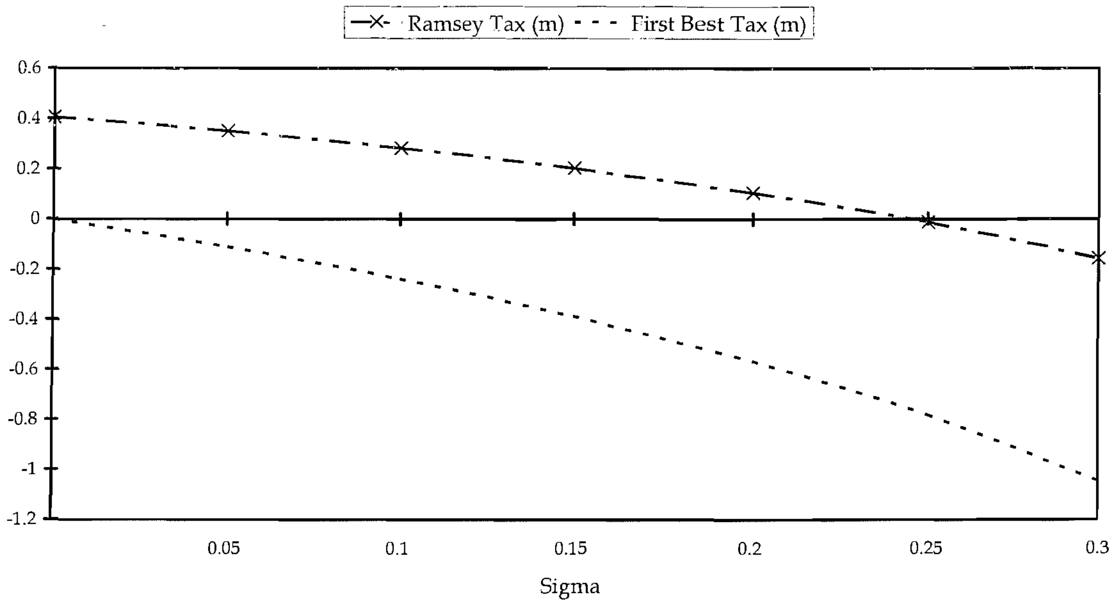


Fig 3e: Ramsey and first best tax (monopolistic wage setting) vs. sigma.



The sharp decline in optimal tax rate for extremely high values of σ indicates that with elastic demand for intermediate goods (and elastic demand for labor in the wage setting model), monopoly distortions create compounding effect in the wedge between social and private returns to labor, and it becomes optimal to cure its more than proportionate distortions with more than proportionate decrease in tax rates. For the monopolistic wage setting model, the multiplier effect is much larger, since there are multiple sources of market distortions.

3.6 Conclusion.

In order to address the issue of optimal choice of labor income tax in the presence of monopoly power in private market, this chapter presents a simple dynamic optimal taxation model of an economy without capital. In the model with competitive labor market, firms in the intermediate goods sector exert monopoly power in pricing and hence distort the productive efficiency condition of the economy. In the model with monopolistic wage setting, monopoly power distorts productive efficiency from two sources: intermediate goods market and labor market. The main purpose of this study is to derive the optimal policy for labor income taxation, and to examine whether and how these optimal choices act as corrective policy. Both analytical and quantitative investigations are undertaken, which cohere to the same set of findings.

The study finds that optimal choice of labor income tax rate is independent of how the government treats distributed profits fiscally. This holds for both models. This is primarily because as long as households treat distributed profits as exogenous, profits and profit taxes do not affect their equilibrium allocation decisions. The only tax that affects household's decisions both through an income and incentive effect is the labor income tax. Optimal choice of this tax is independent of how profits are taxed. Stiglitz & Dasgupta (1971) in this regard argue that with an exogenous upper bound on profit taxes (i.e. no confiscation), productive efficiency is no longer desirable. The current analysis is consistent with an extended version of this interpretation. More precisely, since the optimal choice of labor income tax rate is insensitive to how the government treats distributed profits fiscally, any level of profit taxation (including zero taxation) *may* indicate violation of the productive efficiency. This finding motivates the second result of the current study, i.e. optimal tax rate with monopoly distortions is lower than its competitive market analogue.

The first best intuition of a relatively lower optimal labor income tax due to monopoly distortion is obvious: a lower optimal tax rate at least partly compensates for the loss of output due to mark up pricing (or wages). In particular, for all levels of monopoly distortion the Ramsey tax rate is lower than its competitive market analogue and higher than its first best counter part. The first best policy with any nonzero monopoly distortion is a subsidy, but the Ramsey policy for certain levels of monopoly power is a tax, and after a threshold it is a subsidy. Another important finding is that for remarkably high levels of monopoly distortions, economic agents are less willing to replace Ramsey taxes with lump sum taxes. This is a striking result, since in a sense it establishes that with monopoly distortion second best taxes are more desirable as curative devices than first best taxes. This finding also establishes that the Ramsey taxes are more desirable as corrective policy rather than revenue-raising policy.

A relevant intuition behind these two results is presented in Solow (1998, ch. 2 & 3). In the presence of some degree of monopoly power in private market, a demand shock typically has multiplier like effect. Since the intermediate good's demand (and the labor demand in monopolistic wage setting model) is elastic in addition, a small increase in its price will reduce its demand more than proportionately, which in turn will reduce the production of final good. Since the only factor of production of intermediate goods is labor, employment demand in next period will decrease making intermediate sector firms increase wages in offer. But with a relatively low labor input, production of intermediate goods will fall further, which makes the intermediate goods firms increase its price further. Hence the distorted margins of social and private returns to labor will continue to grow more than proportionately. The only way the government can compensate for this effect is to introduce a lower income tax, which for remarkably high levels of monopoly distortion can be a subsidy. The more than proportionate or compounding wedge between social and private returns to labor makes economic agents prefer distorting taxes rather than lump sum tax, since high degrees of monopoly power in the pricing of an elastically demanded good is associated with high equilibrium profits making the first best lump sum tax heavier. In the model with monopolistic wage setting, the source of private market distortion diversifies that induces a sharper decline in Ramsey tax rate for higher degrees of monopoly power.

Obviously, high level of market power is not a desirable situation, and a long run optimal steady state subsidy to both labor income and profits is also not consistent with the Transversality condition. However, for high degrees of monopoly power there is no need to tax or subsidize profits, since the optimal policy is insensitive to fiscal treatment of distributed profits. The optimal subsidy in the steady state therefore can be financed by bond earnings, which is mainly why tax exempt real government bonds play an essentially important role in the model.

The lower optimal tax result may well be empirically (and policy wise) disputable when one considers the aggregate levels of competition and labor income tax rates in the Euro zone and in the US. The average effective tax rates on labor income in the Euro zone is much higher than in the US, although level of competition in the US is higher than that in the Euro zone. But deciding the equivalence of this result from mere statistics alarmingly ignores the inherent features of tax rules and tax administration systems. What this normative study shows is how Ramsey taxes should be designed to offset monopoly distortion. But how fiscal policy can be effectively used to increase competition and completely remove monopoly power is to a certain extent a distinct research.

Chapter 4

Monopoly Power and Optimal Taxation[25]

4.1 Introduction.

What is the optimal capital income tax policy in an economy with monopoly distortion? As the relevant literature suggests, there is an agreement on one principle --- with monopoly distortion, since output is lower than its optimal level, there must be some form of *Pigovian* element in optimal taxes such that it offsets the distortion created by monopoly power. In other words, with monopoly distortion in private markets, optimal taxes need to be welfare-maximizing as well as distortion-neutralizing. In chapter three this idea acted as the key intuition behind the result that optimal labor income tax under imperfectly competitive private market is lower than what it would have been if markets were perfectly competitive. The questions then remain, does the government's optimal policy for capital income taxation follow the same principle, and with capital income tax, does the optimal labor income tax principle hold.

[25] A version of this chapter has been accepted for presentation at the 2005 Australian Conference of Economists to be held at the University of Melbourne in 2005.

An earlier version was presented in the PhD workshop of Southampton University Economics Division in November 2004.

This chapter attempts to answer these questions in a simple two-sector dynamic general equilibrium model of optimal income taxation with imperfectly competitive intermediate product market. The simple notion of monopoly distortion through mark up pricing of intermediate goods is introduced in the spirit of Dixit & Stiglitz (1977). The optimal taxation problem is solved using the primal approach to the Ramsey problem. The analytical results suggest that the steady state optimal capital income tax can be positive or negative depending on the relative strength of two opposing effects, namely, the distortion effect of monopoly power, and the relative effect of investment on tax distorted equilibrium welfare. It is also established that with the limiting capital tax rule depending on two effects, the optimal tax on labor income from imperfectly competitive sector is lower than the optimal tax on labor income from perfectly competitive sector. The quantitative importance of the results is established by calibrating the model to fit the stylized facts of the post war US economy.

The policy problem addressed in this chapter is one of central importance. The *OECD Revenue Statistics* and various issues of *OECD Observer* suggest that there has been a general tendency amongst the OECD countries to cut the top marginal rates of income taxes and shift the revenue reliance more towards general consumption taxes. While the 1999 OECD average revenue share of consumption taxes was 32%, revenue share of corporate income tax and property tax in the same year were only 9% and 5.5%, respectively[26]. Individual studies and the statistics of the major industrialized economies also indicate the same trend of declining reliance on capital taxation. On the other hand, empirical estimates of price mark ups, such as the Bayoumi *et al.* (2004) estimates of 1.23 for the US economy and 1.35 for the Euro area, motivate the research towards designing competition enhancing policy tools. The important question that stems from these facts therefore is: are low income tax rates the optimal policy choice for economies with high price mark ups?

To my understanding, contributions to the literature on optimal taxation with private market distortions that are of immediate relevance to this chapter are the ones

[26] The OECD average of the revenue share of personal income tax in 1999 was 26.3%, which of course is a high proportion. Personal income tax revenue involves some revenue from taxing capital at the household level, although it is a minor proportion. The major source of capital tax revenue is the corporate income tax and property tax.

by Stiglitz & Dasgupta (1971), Diamond & Mirrlees (1971), Judd (1997/2003), Guo & Lansing (1999), Auerbach & Hines Jr. (2001a), Judd (2002) and Golosov *et al.* (2003). One of the main results of Stiglitz & Dasgupta (1971) is that the optimal commodity tax policy for a monopolistic industry with a bound on profit taxation generally includes both differential taxes and subsidies. Diamond & Mirrlees (1971) argue that the existence of pure profits may require a deviation from the productive efficiency condition implying that taxes should generally be levied on final and not on intermediate goods. Of the various important results stemming from the relevant literature on capital taxation, it is however to some extent difficult to establish a general principle of optimal capital taxation. Nevertheless, there is one result which is common in Judd (1997 & 2002), Guo & Lansing (1999) and Golosov *et al.* (2003) --- the celebrated Chamley-Judd result of zero limiting tax on capital income, due to Judd (1985) and Chamley (1986), which stands more or less robust for models with economy-wide competitive market, does not hold in models with imperfectly competitive markets.

Consider Judd (1997 & 2002) who establishes the result that in an economy with imperfectly competitive private market, the optimal tax on capital income is negative, but it is not optimal to subsidize pure profits. Put more elaborately, Judd (1997) finds that if all intermediate goods are affected symmetrically by market power, a homogeneous subsidy of capital goods' purchase would be an appropriate curative policy tool. This result can be generalized to a subsidy of capital income if one distinguishes between income to capital goods and pure profits[27]. Judd (1997) argues that private market distortions act like a privately imposed tax on purchase of intermediate goods and that for a sufficiently flexible set of tax instruments the optimal tax policy will offset the privately created distortions. The key result highlighted in Judd (1997) is that capital formation should be subsidized but not pure

[27] The intuition put forward by Judd (2003) is that if profits are exhausted by fixed costs, the free-entry zero profit oligopoly equilibrium is equivalent to a competitive market where tax revenues finance fixed costs. This is quite insightful in interpreting the equilibrium, but the equivalence concept is oversimplified from welfare cost point of view. Consider Jonsson (2004) who uses a model with private market distortions arising from both product and labor market. In a calibrated version of the model Jonsson (2004) finds that the welfare cost of imperfect competition with distorting taxes with zero steady state profit (but strictly positive fixed cost) is significantly higher than welfare cost of distorting taxes in a perfectly competitive economy.

profits, and this can be used as a corrective policy device to offset the distortions created by monopoly power[28].

Judd's (1997) general principle of optimal capital subsidy in the presence of monopoly distortions stems primarily from the idea that tax policy may involve subsidies to bring buyer price down to social marginal cost. There is, however, a problem associated with the general principle of using subsidies to neutralize mark ups. These subsidies would require substantial revenues, and the optimal policy would have to tax some goods in order to provide mark up reducing subsidies for other goods. Hence, this general principle would further necessitate identifying which goods to tax and which to subsidize. Judd (2002) provides a clear answer to this question. Since mark up on capital goods distort investment just as a capital income tax does, a positive tax on capital income and a mark up on capital good combine to produce exploding distortion which is inconsistent with commodity tax principle. Taxing labor income (and consumption) is not associated with such compounding distortions, even if these elevate the more uniform monopoly distortions in labor and consumption decisions. Therefore, the exploding distortions due to mark up in the capital market should be reduced with subsidies even if the necessary revenues are generated by taxing labor income and consumption.

Judd's (1997) analysis and the key result of optimal capital subsidy are appealing for further verification because with monopoly distortion the welfare effect of investment as perceived by the planner and the private sector are very likely to be different. This in turns imply that agents either over-invest or under-invest. A general principle of optimal capital subsidy to encourage investment in such a case may not be appropriate. This chapter contributes by specifying Judd's (2002 & 2003) finding for a particular range of monopoly distortions. More specifically, this chapter shows that with monopoly distortion in private markets, the government's optimal policy may involve a capital income tax or a capital income subsidy depending on the relative strengths of two effects which are of opposite signs. These effects are the monopoly

[28] In a relatively more recent paper, Golosov *et al.* (2003) model taxation in an environment where agents' skills are private information and show that if source of distortion is not only confined to taxation, a positive tax on capital income may be sustainable as a pareto efficient outcome. With a rather different source of private market distortion, the paper's key findings are necessarily banked on the same set of intuitions.

distortion effect and the relative effect of investment on equilibrium welfare. Depending on the level of monopoly distortion, economic agents may under-invest or over-invest which makes one of these effects stronger than the other, and consequently motivates the government to use an optimal policy involving a capital income subsidy or a capital income tax. Through numerical investigation, the chapter establishes that for low degrees of monopoly power agents over-invest in search of profits which reduces welfare. This motivates the government to use an optimal capital income tax to discourage investment. For high degrees of monopoly power agents under-invest, which implies the optimal policy involves a capital income subsidy that encourages investment.

This chapter, however, is not the first attempt to establish such a 'two effect' result. A somewhat similar result can be found in Guo & Lansing (1999), but unfortunately the intuition behind their result is not particularly convincing. The underlying explanations that they provide are also rather incomplete. Guo & Lansing (1999) develop a simple two-sector neoclassical model of optimal income taxation and introduce monopoly distortions in pricing of intermediate goods. Their paper introduces a rich capital tax code that involves, in addition to simple flat rate income taxes, tax on distributed pure profits and accelerated depreciation. The key finding of Guo & Lansing (1999) is that the optimal capital tax rate balances two opposing forces, namely, an underinvestment effect and a profit effect, and depending on their relative strength, can either be negative or positive. First, agents invest less than socially optimal level since return to investment is less than social marginal product of capital. A negative tax on capital income assists to correct this underinvestment effect since it encourages investment. On the other hand, since monopoly power earn pure profit the relative strength of a profit effect motivates the use of a positive tax on capital income.

Guo & Lansing's (1999) interpretation of the two effects leaves room for *two* important questions: for what degrees of monopoly distortion the underinvestment (profit) effect dominates the profit (underinvestment) effect, or more specifically, for what levels of monopoly power the government should tax/subsidize capital income? Is there a case where these two effects completely offset each other which recovers the limiting zero capital tax result with imperfectly competitive market? The analytical findings of Guo & Lansing (1999) do not provide clear-cut answers to these very important questions. Nevertheless, in the quantitative section, Guo & Lansing (1999, p. 987) show that when profits escape all taxation, the underinvestment (profit) effect dominates for very low (high) degrees of monopoly power. Based on their set of intuitions, this implies that government attempts to discourage investment when monopoly distortions and profits are high, which I find rather vague. Investment at a high level of monopoly distortion is not an attractive decision since the private return to capital is much lower than its socially optimal level. For relatively low degrees of monopoly power, agents invest in search of profits since the perceived wedge between social and private returns to investment is not remarkably high. A more sensible policy option would therefore be to use a tax on capital income for low degrees of monopoly power.

This chapter answers these two important questions both analytically and numerically and with strong underlying intuitions. It argues that a more useful and sensible track to understand and interpret the two effects that govern the sign of steady state optimal capital income tax is to demarcate them into monopoly distortion effect and the relative effect of investment on tax distorted equilibrium welfare. Unlike the findings of Guo & Lansing (1999), in the quantitative exercise the current chapter finds that the optimal *tax* on capital income is high if monopoly power is low, and it is optimal to subsidize capital income if monopoly power is high. This chapter therefore specifies Judd's (2002 & 2003) prescription of optimal capital income subsidy for a range of high degrees of monopoly power. The magnitude of the optimal Ramsey subsidy is larger for higher levels of monopoly distortion, but is always smaller than the first best subsidy.

These important findings are banked on strong intuitions. With monopoly distortions, investment decision of agents depends on both the discounted real return to capital and the wedge between social and private return to capital. Since investment generates profits, returns to investment in physical capital as perceived by the private sector and the Ramsey planner do not coincide, which would in an otherwise competitive economy. This is because with profits distributed back to households, capital accumulation becomes an argument in the implementability constraint, and hence investment decisions directly affect welfare. For low degrees of monopoly power, agents over-invest in anticipation of pure profits. This investment is associated with welfare loss, but the relative effect of this investment as perceived by agents is stronger than the wedge. This motivates the government to use a positive tax on capital income in order to discourage welfare distorting investment. On the contrary, agents under-invest when monopoly power is high, since for higher monopoly distortion the rate of increase in the wedge is higher than the rate of increase in realized relative effect of investment. This is the case where the distortion effect dominates, and the optimal policy involves a capital income subsidy. The monopoly distortion effect and the relative effect of investment on equilibrium welfare do not completely offset each other for any plausible set of parameter values. This is because the allocation for which this may hold is not supported by equilibrium prices and policy. Thus with private market imperfection, the optimal policy never involves the celebrated Chamley-Judd prescription of zero limiting tax on capital income.

The chapter also establishes that whether a tax or a subsidy, the optimal capital income tax policy is distortion-neutralizing. With monopoly distortion, the marginal product of capital is equal to private return to capital grossed up by the price mark up factor. If one considers the equilibrium cost of capital with two sources of distortions, monopoly power actually acts as a second tax rate on capital, which is neutralized by an optimal choice of capital income tax/subsidy. For low degrees of monopoly power, profit-seeking high investment distorts the equilibrium welfare which necessitates a distortion-neutralizing tax on capital income. For high degrees of monopoly power, underinvestment drives the wedge between social and private return to capital at a high level, which necessitates a distortion-neutralizing subsidy on capital income.

The next section presents the model economy and maximization problems of producers and consumers. Section 4.3 formulates and solves the optimal taxation problem. It derives the analytical set of solutions to both the first best and the Ramsey policy problems. It also presents the key propositions based on the analytical results. Section 4.4 explains the intuitions underlying the key propositions. Section 4.5 calibrates the model to fit the stylized facts of the US economy and presents insightful quantitative results. Section 4.6 concludes.

4.2 The Model Economy.

In this section I will consider a specialization of the model presented in the previous chapter. The current model economy has two sectors of production indexed by y and z, producing final goods and intermediate goods, respectively. With private and government consumption, an additional use of the final good is private investment which can be accomplished by accumulating capital stocks. The intermediate goods sector uses capital and labor as inputs, and the final goods sector uses intermediate goods and labor as inputs. It will be convenient hereafter to index household's labor supply with subscripts y and z to denote the working time in the final goods sector and the intermediate goods sector, respectively.

These specifications introduce an additional labor argument in the household's utility function, and allow the government to tax capital income and impose two sector-specific taxes on labor income. Since government bonds' role in determining the key results are insignificant, assume the only asset at the household's possession are capital stocks. Hence, the household's intertemporal allocation decision involves consumption, labor supply to two sectors and a period ahead capital stock, which are the choice variables in their optimization problem. A close match of this setting may be the one presented in Guo & Lansing (1999), but the current model claims to be analytically stronger. The current model introduces sector-specific labor supply choice for the household and its key focus is on the average effective tax rates on labor, capital and distributed profits. Since monopoly power is exercised in only one sector, the sector-specific labor income tax rates allow one to compare optimal labor income tax policy for competitive market structure and imperfectly competitive market structure within the same model.

4.2.1 The Environment.

Time t is discrete, runs forever, and t belongs to the set of integers $N = \{0,1,2,.....\}$. There is a continua of measure one of firms in sector y, which produce the final good, y_t, using labor, n_{yt}, and a continuum of intermediate goods, z_{jt} where $j \in [0,1]$, as inputs. A continuum of $j \in [0,1]$ firms in sector z combine capital, k_t, and labor, n_{zt}, to produce a continuum of intermediate goods, z_{jt}. Market for final goods is characterized by perfect competition, but producers of intermediate good may possess some degree of monopoly power. The final good can be used for private consumption, government consumption and private investment, denoted by c_t, g_t and i_t, respectively. Initial endowment of capital, one unit of time at each period and property rights of firms are owned by each of a continua of measure one of identical infinitely-lived households.

The constant returns to scale technology used to produce the final good is:

$$y_t = \left\{ \left(\int_0^1 z_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \right\}^{\nu} n_{yt}^{1-\nu} \tag{1.1}$$

where $\nu \in (0,1)$ is a share parameter, and $\sigma \in [0,1)$ indexes the degree of monopoly power exercised by suppliers of the intermediate good. With this specification, σ^{-1} is the elasticity of substitution between any two intermediate goods, and for $\sigma \to 0$ ($\sigma \to 1$) the intermediate goods sector possesses low (high) monopoly power.

The technology for intermediate goods sector is defined as:

$$z_{jt} = k_{jt}^{\alpha} n_{zjt}^{1-\alpha} \tag{1.2}$$

where $\alpha \in (0,1)$ is the share parameter for capital.

Households have identical preferences over consumption and labor supply. The representative household derives utility from consumption sequences $\{c_t\}_{t=0}^{\infty}$ and disutility from labor service sequences $\{n_{yt}, n_{zt}\}_{t=0}^{\infty}$. Preferences for the representative household are given by:

$$\sum_{t=0}^{\infty} \beta^t u(c_t, n_{yt}, n_{zt}) \tag{2}$$

where $\beta \in (0,1)$ is the subjective discount rate which varies inversely with the rate of time preference. The utility function $u : \mathbf{R}_+^3 \to \mathbf{R}$ is continuously differentiable, strictly increasing in consumption, decreasing in labor and strictly concave. It is also assumed that the utility function satisfies standard Inada conditions, namely $\mathbf{lim}_{c_t \to 0}[u_{ns}(t)]^{-1} u_c(t) = \infty$, and $\mathbf{lim}_{c_t \to \infty}[u_{ns}(t)]^{-1} u_c(t) = 0$ for $s = y, z$.

The government consumes exogenous g_t each period and raises the required revenue by taxing capital income, distributed profits, and labor income at rates θ_t, $\kappa\theta_t$, and τ_{st} for $s = y, z$, respectively. The government's period t budget constraint is given by:

$$g_t = \tau_{yt} w_{yt} n_{yt} + \tau_{zt} \int_0^1 w_{zjt} n_{zjt} dj + \theta_t \left[\int_0^1 r_{jt} k_{jt} dj + \kappa \int_0^1 \pi_{jt} dj \right] \tag{3}$$

where w_{yt} and w_{zjt} denote wage rates in the final goods and intermediate goods sector, respectively, r_{jt} denotes rental price of capital, and π_{jt} denotes pure profits from intermediate goods sector. The benevolent government has access to an effective commitment technology with which it can sustain all initially announced tax plans. All optimal plans are therefore dynamically consistent.

Unlike the model presented in the previous chapter, tax rate on distributed profits is now linked to the capital tax rate, an assumption more thoughtful from fiscal policy point of view. The parameter $\kappa \geq 0$ represents the tax treatment of distributed corporate profits. For instance, the restriction $\kappa \in [0,1]$ in the current setting implies the government's set of tax treatments $[\textit{no tax}, \textit{at par with capital tax}]$ for distributed corporate profits. In principle, ignoring the rather obscure possibility of more than 100% tax on distributed corporate profits, the restriction $\theta_t^{-1} \geq \kappa \geq 0$ would be more appropriate, since $\kappa = \theta_t^{-1}$ then would represent the case where distributed profits are taxed at the rate of 100%. But for most parts of the analysis to follow, I will consider $\kappa \in [0,1]$. This is because although a 100% tax on profits is optimal, it is an impractical policy option. The assumption $\kappa \in [0,1]$ is also empirically supported. For instance, using McGrattan & Prescott (2005)'s estimates of tax rate on corporate distributions and Carey & Tchilinguirian (2000)'s estimates of the average effective tax rates on capital income, one can compute $\kappa = 0.6373$ and $\kappa = 0.1222$ for the US and the UK economy, respectively.

4.2.2 Firms' Problems.

The final good is held as the numeraire. The representative firm in the final goods sector competitively maximizes profits. It faces the following sequence of problems:

$$\max_{z_{jt}, n_{yt}} \left[\left\{ \left(\int_0^1 z_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}} \right\}^{\nu} n_{yt}^{1-\nu} - \int_0^1 p_{jt} z_{jt} dj - w_{yt} n_{yt} \right] \tag{4.1}$$

where p_j denotes the relative price of intermediate good z_j. The first order conditions associated with this problem are:

$$z_{jt}: \quad p_{jt} = \nu (y_t)^{1-\frac{1-\sigma}{\nu}} z_{jt}^{-\sigma} (n_{yt})^{\frac{(1-\nu)(1-\sigma)}{\nu}} \tag{4.2}$$

$$n_{yt}: \quad w_{yt} n_{yt} = (1-\nu) y_t \tag{4.3}$$

From *(4.2)*, the price elasticity of demand for the j th intermediate good is equal to $(-1)\sigma^{-1}$, which is strictly negative for $\sigma \in (0,1)$.

Firms in the intermediate goods sector possess monopoly power in pricing and face the demand function *(4.2)* for j th intermediate good. The profit maximization problem of the representative firm in the intermediate goods sectors is:

$$\max_{p_{jt}, n_{zjt}, k_{jt}} [p_{jt} z_{jt} - r_{jt} k_{jt} - w_{zjt} n_{zjt}] \qquad (5.1)$$

$$s.t. \qquad z_{jt} = k_{jt}^{\alpha} n_{zjt}^{1-\alpha}$$

$$p_{jt} = \nu (y_t)^{1-\frac{1-\sigma}{\nu}} z_{jt}^{-\sigma} (n_{yt})^{\frac{(1-\nu)(1-\sigma)}{\nu}}$$

Substituting both constraints in *(5.1)* yields a sequence of unconstrained problems for the representative firm which can be maximized with respect to k_{jt} and n_{zjt}. The first order conditions associated with this problem are:

$$n_{zjt}: \quad w_{zjt} n_{zjt} = (1-\alpha)(1-\sigma) p_{jt} z_{jt} \qquad (5.2)$$

$$k_{jt}: \quad r_{jt} k_{jt} = \alpha (1-\sigma) p_{jt} z_{jt} \qquad (5.3)$$

Consider a symmetric equilibrium where all firms in the intermediate goods sector produce at the same level, employ the same levels of factors and charge the same relative price, such that $n_{zjt} = n_{zt}$, $k_{jt} = k_t$ and $p_{jt} = p_t$ for all j. The model economy's aggregate resource constraint is then given by:

$$c_t + g_t + i_t = k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} \qquad (6.1)$$

where the aggregate production technology exhibit constant returns to scale. Equilibrium profits for the intermediate goods sector is given by:

$$\pi_t = (\nu\sigma) k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} \qquad (6.2)$$

The symmetric equilibrium factor prices can be expressed in terms of the final good. These are:

$$w_{yt} = (1-\nu)(n_{yt})^{-1} y_t \tag{6.3a}$$

$$w_{zt} = (1-\alpha)\nu(1-\sigma)(n_{zt})^{-1} y_t \tag{6.3b}$$

$$r_t = \alpha(1-\sigma)\nu(k_t)^{-1} y_t \tag{6.3c}$$

From *(6.2)*, the profit to output ratio for this model economy is equal to $\nu\sigma$, which implies that apart from indexing the degree of monopoly power the magnitude of the parameter σ also governs the equilibrium profit to output ratio.

In order to derive the price mark up ratio, it is convenient to redefine the problem of the representative firm in the intermediate goods sector as one of choosing output to maximize profits. Let $TC_t(z_t, w_{zt}, r_t)$ denote the total cost function for the representative firm, and redefine the profit maximization problem as:

$$\max_{z_t} \left[\nu y_t^{1-\frac{1-\sigma}{\nu}} z_t^{1-\sigma} n_{yt}^{\frac{(1-\nu)(1-\sigma)}{\nu}} - TC_t(z_t, r_t, w_{zt}) \right] \tag{6.4}$$

The first order condition associated with this problem is:

$$z_t: \quad p_t = \frac{1}{(1-\sigma)} MC_t(z_t, r_t, w_{zt}) \tag{6.5}$$

From *(6.5)*, the price mark up ratio is simply $(1-\sigma)^{-1}$.

4.2.3 Household's Problem.

The infinitely-lived representative household chooses $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$ to maximize *(2)* subject to the following constraints:

$$c_t + i_t \leq (1-\tau_{yt})w_{yt}n_{yt} + (1-\tau_{zt})w_{zt}n_{zt} + (1-\theta_t)r_t k_t + (1-\kappa\theta_t)\pi_t \qquad (7.1)$$

$$i_t = k_{t+1} - (1-\delta)k_t \qquad (7.2)$$

with $k_0 > 0$ given, and where $\delta \in (0,1)$ is the capital depreciation rate. Equations *(7.1)* and *(7.2)* can be combined to derive the household's budget constraint:

$$c_t + k_{t+1} \leq (1-\tau_{yt})w_{yt}n_{yt} + (1-\tau_{zt})w_{zt}n_{zt} + [(1-\theta_t)r_t + (1-\delta)]k_t + (1-\kappa\theta_t)\pi_t \qquad (7.3)$$

Maximizing *(2)* subject to *(7.3)* and consolidating the resulting first order conditions yield the following system of equations, which along with *(7.3)* characterizes the household's equilibrium decisions:

$$-u_{ns}(t) = u_c(t)(1-\tau_{st})w_{st} \qquad \text{for } s = y, z \qquad (7.4a)$$

$$\beta R_{t+1} = \frac{u_c(t)}{u_c(t+1)} \qquad (7.4b)$$

$$\lim_{t\to\infty} \beta^t u_c(t) k_{t+1} = 0 \qquad (7.4c)$$

where $R_t \equiv [(1-\theta_t)r_t + (1-\delta)]$. Equation *(7.4a)* states that the representative household's utility maximizing choice of labor supply is the one that equates the marginal rate of substitution of labor across sectors to the after tax wage ratio of labor. Equation *(7.4b)* is the Euler equation stating that the marginal utility of consuming an additional unit today and the discounted marginal utility of saving that unit for a later date are equal at the optimum. In other words the Euler equation in this setting is the intertemporal consumption and savings arbitrage condition. Equation *(7.4c)* is the standard Transversality condition that puts a restriction on the terminal value of capital stock.

4.2.4 Equilibrium.

As mentioned earlier, I will restrict my attention to a symmetric equilibrium. The symbols without time subscripts used in the definition denote the one-sided infinite sequence for the corresponding variables, e.g. $n_z \equiv \{n_{zt}\}_{t=0}^{\infty}$.

Definition 4.2.4 (Equilibrium). A symmetric equilibrium is an allocation (c, n_y, n_z, k, z, y), a price system (w_y, w_z, p, r), and a government policy (τ_y, τ_z, θ), such that

(1) given the price system and government policy, the allocation solves the firms' problems and the household's problem;

(2) given the price system and allocation, the government policy satisfies the sequence of government budget constraints *(3)*; and

(3) all markets clear in the long run. □

The symmetric equilibrium is characterized by the following system *(8)* in the set of unknowns $\{c_t, n_{yt}, n_{zt}, k_t, w_{yt}, w_{zt}, r_t, p_t, \pi_t, z_t, y_t, \tau_{yt}, \tau_{zt}, \theta_t\}$.

$$0 < n_{yt} + n_{zt} \le 1 \qquad (a)$$

$$y_t = c_t + g_t + i_t \qquad (b)$$

$$y_t = z_t^{\nu} n_{yt}^{1-\nu} \qquad (c)$$

$$i_t = k_{t+1} - (1-\delta)k_t \qquad (d)$$

$$z_t = k_t^{\alpha} n_{zt}^{1-\alpha} \qquad (e)$$

$$p_t = \nu(y_t)^{1-\frac{1-\sigma}{\nu}} z_t^{-\sigma} (n_{yt})^{\frac{(1-\nu)(1-\sigma)}{\nu}} \qquad (f)$$

$$w_{yt} = (1-\nu)(n_{yt})^{-1} y_t \qquad (g)$$

$$w_{zt} = (1-\alpha)\nu(1-\sigma)(n_{zt})^{-1} y_t \qquad (h)$$

$$r_t = \alpha(1-\sigma)\nu(k_t)^{-1} y_t \qquad (i)$$

$$\pi_t = (\nu\sigma)k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} \qquad (j)$$

$$-u_{ny}(t) = u_c(t)(1-\tau_{yt})w_{yt} \qquad (k)$$

$$-u_{nz}(t) = u_c(t)(1-\tau_{zt})w_{zt} \qquad (l)$$

$$\beta R_{t+1} = \frac{u_c(t)}{u_c(t+1)} \qquad (m)$$

$$\lim_{t\to\infty} \beta^t u_c(t)k_{t+1} = 0 \qquad (n)$$

The system *(8)* characterizes the symmetric equilibrium allocations and prices for the government's choice of tax instruments. Each arbitrarily chosen tax policy generates a symmetric equilibrium for the model economy.

4.3 Optimal Taxation.

Following the primal approach, the optimal taxation problem for the government is to choose allocations $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$ to maximize welfare defined by *(2)* subject to the aggregate resource constraint *(6.1)* where investment is defined by *(7.2)*, and an implementability constraint that ensures that the resulting taxes, prices and allocations are consistent with equilibrium system *(8)*. This is a characterization of the underlying Ramsey problem. Once the optimal taxation problem is solved, the resulting Ramsey allocations, given the initial conditions $\{R_0, k_0\}$, can be used to recover a sequence of prices $\{w_{zt}, w_{yt}, r_t, p_t\}_{t=0}^{\infty}$ and policy variables $\{\tau_{zt}, \tau_{yt}, \theta_t\}_{t=0}^{\infty}$ that will support the Ramsey allocations as a decentralized equilibrium.

In order to formulate the Ramsey problem, it is convenient to first solve the household's problem using a present-value budget constraint. The present-value budget constraint of the household is[29]:

[29] The detailed procedure of backward iteration of household's budget constraint in order to derive its present-value version is presented in section 2.3 of chapter 2 of this thesis. This way of formulating the implementability constraint (and the Ramsey problem) is proposed earlier in Jones *et al.* (1997) and Ljungqvist & Sargent (2000, ch. 12).

$$\sum_{t=0}^{\infty} q_t^o c_t = \sum_{t=0}^{\infty} q_t^o (1-\tau_{yt}) w_{yt} n_{yt} + \sum_{t=0}^{\infty} q_t^o (1-\tau_{zt}) w_{zt} n_{zt} + \sum_{t=0}^{\infty} q_t^o (1-\kappa\theta_t)\pi_t + R_0 k_0 \tag{9.1}$$

where the Arrow-Debreu price is given by $q_t^o = \left(\prod_{s=1}^{t} R_s\right)^{-1}$ with $R_t \equiv [(1-\theta_t) r_t + (1-\delta)]$, and $\prod_{s=1}^{0} R_s \equiv 1$ is the numeraire which makes $q_0^o = 1$.

Consider the sequential market version of the household's optimization problem. More specifically, consider the problem of the household of choosing $\{c_t, n_{yt}, n_{zt}\}_{t=0}^{\infty}$ to maximize utility defined by *(2)* subject to *(9.1)*. Let λ^p be the associated Lagrange multiplier of this problem. The first order conditions with respect to allocations are:

$$c_t : \beta^t u_c(t) = \lambda^p q_t^o \tag{9.2a}$$

$$n_{yt} : \beta^t u_{ny}(t) + \lambda^p q_t^o (1-\tau_{yt}) w_{yt} = 0 \tag{9.2b}$$

$$n_{zt} : \beta^t u_{nz}(t) + \lambda^p q_t^o (1-\tau_{zt}) w_{zt} = 0 \tag{9.2c}$$

Consolidating *(9.2)* gives:

$$q_t^o u_c(0) = \beta^t u_c(t) \tag{9.3a}$$

$$u_{ny}(t) = -u_c(t)(1-\tau_{yt}) w_{yt} n_{yt} \tag{9.3b}$$

$$u_{nz}(t) = -u_c(t)(1-\tau_{zt}) w_{zt} n_{zt} \tag{9.3c}$$

The implementability constraint is the intertemporal constraint involving only allocations and initial values, which is derived by substituting out taxes, factor prices and Arrow-Debreu price in *(9.1)* using *(9.3)*, *(6.2)*, *(7.4b)* and *(6.3c)*. The implementability constraint is therefore:

$$\sum_{t=0}^{\infty} \beta^t [u_c(t) c_t + u_{ny}(t) n_{yt} + u_{nz}(t) n_{zt} - u_c(t)(1-\kappa\theta_t)\pi_t] - u_c(0) R_0 k_0 = 0 \tag{9.4a}$$

where

$$(1-\kappa\theta_t)\pi_t = \begin{cases} v\sigma(1-\kappa)k_t^{\alpha v} n_{zt}^{(1-\alpha)v} n_{yt}^{1-v} + k_t \dfrac{\kappa\sigma}{\alpha(1-\sigma)}\left[\dfrac{u_c(t-1)}{\beta u_c(t)} - (1-\delta)\right] & for\, t \geq 1 \\ (1-\kappa\theta_0)(v\sigma)k_0^{\alpha v} n_{z0}^{v(1-\alpha)} n_{y0}^{1-v} & for\, t = 0 \end{cases}$$

(9.4b)

4.3.1 The Ramsey Problem.

The Ramsey problem for the government is to choose a policy $\{\theta_t, \tau_{yt}, \tau_{zt}\}_{t=0}^{\infty}$ that maximizes welfare defined by *(2)* subject to the government budget constraint defined by *(3)* such that the resulting policy and the associated allocations and prices are consistent with equilibrium defined by *(8)*. Following the primal approach, this problem can be characterized as one in which the government chooses allocations $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$ to maximize *(2)* subject to constraints *(6.1)*, *(7.2)* and *(9.4)*. Note that with *(9.4b)* the Pseudo utility function corresponding to the Ramsey problem now incorporates current period capital stocks as one of the arguments. This implies in the tax distorted equilibrium, investment induces both a direct and an indirect welfare effect.

With $\Phi \geq 0$ representing the utility cost of raising revenue through distorting taxes, the Pseudo utility function for the Ramsey problem is defined as:

$$V(c_t, n_{yt}, n_{zt}, k_t, \Phi) \equiv u(c_t, n_{yt}, n_{zt}) + \Phi[u_c(t)c_t + u_{ny}(t)n_{yt} + u_{nz}(t)n_{zt} - u_c(t)(1-\kappa\theta_t)\pi_t]$$

(9.5)

where $(1-\kappa\theta_t)\pi_t$ is defined by *(9.4b)*. The economy's aggregate resource constraint after substituting for investment is:

$$c_t + g_t + k_{t+1} = k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} + (1-\delta)k_t \tag{9.6}$$

Let $\{\chi_t\}_{t=0}^{\infty}$ be the sequence of Lagrange multiplier on the resource constraint *(9.6)*. The Lagrangian of the Ramsey problem is defined as:

$$J = \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} V(c_t, n_{yt}, n_{zt}, k_t, \Phi) \\ + \chi_t [k_t^{\alpha\nu} n_{zt}^{\nu(1-\alpha)} n_{yt}^{1-\nu} + (1-\delta)k_t - c_t - g_t - k_{t+1}] \end{array} \right\} - \Phi u_c(0) R_0 k_0 \tag{9.7}$$

where $(1-\kappa\theta_t)\pi_t$ in $V(.)$ is defined by *(9.4b)*. For exogenously given g_t, R_0 and k_0, the Ramsey problem is to maximize *(9.7)* with respect to $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$. The first order conditions due to changes in $t \geq 1$ allocations are:

$$c_t: \quad V_c(t) = \chi_t, \qquad \forall t \geq 1 \tag{9.8a}$$

$$n_{yt}: \quad V_{ny}(t) = -\chi_t (1-\nu) k_t^{\alpha\nu} n_{zt}^{(1-\alpha)\nu} n_{yt}^{-\nu}, \qquad \forall t \geq 1 \tag{9.8b}$$

$$n_{zt}: \quad V_{nz}(t) = -\chi_t \nu(1-\alpha) k_t^{\alpha\nu} n_{zt}^{(1-\alpha)\nu-1} n_{yt}^{1-\nu}, \qquad \forall t \geq 1 \tag{9.8c}$$

$$k_{t+1}: \quad \chi_t = \beta \{ V_k(t+1) + \chi_{t+1} [\nu\alpha\, k_{t+1}^{\alpha\nu-1} n_{zt+1}^{(1-\alpha)\nu} n_{yt+1}^{1-\nu} + (1-\delta)] \}, \qquad \forall t \geq 1 \tag{9.8d}$$

Consolidating *(9.8)* and using *(6.3)*, the Ramsey equilibrium conditions for $t \geq 1$ are:

$$V_{ny}(t) = -V_c(t) w_{yt} \tag{9.9a}$$

$$V_{nz}(t) = -V_c(t) \frac{w_{zt}}{(1-\sigma)} \tag{9.9b}$$

$$V_c(t) = \beta \left\{ V_k(t+1) + V_c(t+1) \left[\frac{r_{t+1}}{(1-\sigma)} + (1-\delta) \right] \right\} \tag{9.9c}$$

together with the implementability constraint defined by *(9.4)* and aggregate resource constraint defined by *(9.6)* for $t \geq 1$. Note that with $\sigma = 0$ *vis a vis* $V_k(t+1) = 0$, equation *(9.9c)* captures the standard intertemporal trade off described by the Euler equation from the household's optimization problem. By contrast, $V_k(t+1) \neq 0$ indicates that returns to investment in physical capital as perceived by the private sector and the Ramsey planner no longer coincide.

4.3.2 The First Best Policy.

Consider the first best tax policy as a useful scale. If there is an access to lump sum tax ($\equiv \ell_t$), the government can implement the first best tax policy which generates the equilibrium that coincides with the equilibrium derived by solving the benevolent social planner's problem. The benevolent social planner's problem in this setting is to choose allocations $\{c_t, n_{yt}, n_{zt}, k_{t+1}\}_{t=0}^{\infty}$ to maximize utility defined by *(2)* subject to the economy-wide resource constraint *(9.6)*. With $\beta^t \lambda_t^1$ as the period t Lagrange multiplier, the first order conditions with respect to allocations for the social planner's problem can be summarized as:

$$c_t: \quad u_c(t) = \lambda_t^1 \tag{10.1a}$$

$$n_{yt}: \quad u_{ny}(t) = -u_c(t)(1-\nu)(n_{yt})^{-1} y_t \tag{10.1b}$$

$$n_{zt}: \quad u_{nz}(t) = -u_c(t)\nu(1-\alpha)(n_{zt})^{-1} y_t \tag{10.1c}$$

$$k_{t+1}: \quad \beta\left[\nu\alpha\, k_{t+1}^{\alpha\nu-1} n_{zt+1}^{\nu(1-\alpha)} n_{yt+1}^{1-\nu} + (1-\delta)\right] = \frac{u_c(t)}{u_c(t+1)} \tag{10.1d}$$

The social planner's allocations also satisfy resource constraints *(9.6)* and the Transversality condition *(7.4c)*.

Proposition 1: With social planner's equilibrium implied by *(10.1)*, the first best fiscal policy is to *(a)* set zero tax on labor income from final goods sector (competitive markets); *(b)* set a uniform subsidy on labor income and capital income from intermediate goods sector (imperfectly competitive market), and *(c)* impose

$$\ell_t = g_t + \frac{\nu\sigma}{(1-\sigma)} y_t [1-\sigma(1-\kappa)]$$

as a lump sum tax that is strictly greater than planned government consumption expenditure.

Proof: The term ℓ_t is added to the right hand side of the symmetric version of government budget constraint *(3)*. Comparing *(10.1)* with *(7.4)*, it is straightforward to show that $\tau_{yt} = 0$, $\tau_{zt} = \theta_t = \frac{-\sigma}{(1-\sigma)} < 0$. Substituting for these taxes and using equilibrium conditions *(8)* in the government's budget constraint with lump sum taxes yields:

$$\ell_t = g_t + \frac{\nu\sigma}{(1-\sigma)} y_t [1-\sigma(1-\kappa)]$$

which is strictly positive, and strictly greater than g_t. ■

The intuition behind proposition 1 is clear. Since final goods sector is perfectly competitive, the associated first best labor income tax rate is zero. For the sector where monopoly power is exercised, factor returns are less than their social marginal products, which is compensated by a first best uniform subsidy. In order to finance the subsidies, the lump sum tax is charged at a higher level than government's planned consumption expenditure. For instance, if $\kappa = 0$ (no profit tax), $\ell_t = g_t + \pi_t$, and if $\kappa = 1$, $\ell_t = g_t + \frac{\pi_t}{(1-\sigma)}$.

4.3.3 The Ramsey Policy.

Now consider Ramsey tax policy when lump sum tax is not a policy option. I will introduce a rather innocuous simplification to the model only for the sake of tractability of analytical results. Following Hansen (1985), assume that $u : \mathbf{R}_+^3 \to \mathbf{R}$ is separable in consumption and labor, and linear in labor. Imposing these restrictions is tantamount to assuming that the derivatives of *(a)* the marginal utility of consumption and marginal disutility of labor supplies with respect to labor supplies, and *(b)* the marginal disutility of labor supply with respect to consumption, are zero [30]. Furthermore, I will restrict my attention to steady state Ramsey tax policy. This is accomplished by assuming that there exists a $T \geq 0$ for which $g_t = \bar{g}$ for all $t \geq T$, and solution to the Ramsey problem converges to a time-invariant allocation.

In order to analyze the underlying intuitions of the Ramsey policy in a detailed manner, I will focus only on the technical results in this subsection and defer the intuitions and explanations of these results to the next section. Recall the Pseudo utility function for this problem, as defined by *(9.5)*. The derivatives of the Pseudo utility function with respect to allocations, evaluated at steady state, and after some algebra, are:

$$V_c = u_c + u_c\Phi\left[1 + \frac{u_{cc}c}{u_c}\left\{1 - \sigma\nu\frac{y}{c}\left[(1-\kappa) - (1-\delta-\beta^{-1})r^{-1}\kappa\right]\right\}\right] \qquad (10.2a)$$

$$V_k = u_c\Phi\left[\frac{\kappa\sigma}{\alpha(1-\sigma)}(1-\delta-\beta^{-1}) - \nu\sigma(1-\kappa)\frac{r}{(1-\sigma)}\right] \qquad (10.2b)$$

$$V_{ny} = u_{ny}(1+\Phi) - \Phi u_c\nu\sigma(1-\kappa)w_y \qquad (10.2c)$$

$$V_{nz} = u_{nz}(1+\Phi) - \Phi u_c\nu\sigma(1-\kappa)\frac{w_z}{(1-\sigma)} \qquad (10.2d)$$

The derivatives in *(10.2)* represent the steady state marginal effect of change in allocations on tax distorted equilibrium welfare. In other words, for a particular tax policy, these derivatives represent the long run effect on equilibrium welfare for small changes in allocation decisions. The sector-specific Ramsey tax rules for labor income is analytically computed by comparing the steady state versions of the Ramsey equilibrium conditions *(9.9a & b)*, and equilibrium conditions *(8k & l)*, or *(9.3b & c)*, which are primarily derived from representative household's optimization problem. This gives:

[30] Put technically, this simplification imply $u_{cns}(t) = u_{nsc}(t) = u_{nsns}(t) = u_{nsnl}(t) = 0$ for $s, l = y, z$, and $l \neq s$.

$$(1-\tau_y)=\frac{\left[1+\Phi\left(1+\frac{u_{cc}c}{u_c}\left\{1-\sigma\nu\frac{y}{c}\left[(1-\kappa)-(1-\delta-\beta^{-1})r^{-1}\kappa\right]\right\}\right)\right]-\Phi\nu\sigma(1-\kappa)}{(1+\Phi)}$$

(10.3a)

$$(1-\tau_z)=\frac{\left[1+\Phi\left(1+\frac{u_{cc}c}{u_c}\left\{1-\sigma\nu\frac{y}{c}\left[(1-\kappa)-(1-\delta-\beta^{-1})r^{-1}\kappa\right]\right\}\right)\right]-\Phi\nu\sigma(1-\kappa)}{(1+\Phi)(1-\sigma)}$$

(10.3b)

It is convenient for the following two propositions (2 and 3) to combine *(10.3a & b)* to derive:

$$\frac{(1-\tau_z)}{(1-\tau_y)}=\frac{1}{(1-\sigma)} \qquad (10.4)$$

Proposition 2: If all markets are perfectly competitive, it is optimal for the government to tax labor income from the two sectors at the same rate.

Proof: The competitive market analogue of the model economy corresponds to the case where there is no monopoly power in pricing of the intermediate goods. This is tantamount to saying that all markets are competitive if $\sigma = 0$. Equation *(10.4)* with $\sigma = 0$ implies $\tau_z = \tau_y$. ∎

Proposition 3: For any $\sigma \in (0,1)$, it is optimal for the government to set the labor income tax in the intermediate goods sector (where monopoly power is exercised) lower than the labor income tax in the final goods sector (competitive market), i.e. $\tau_z < \tau_y$ is optimal policy as long as monopoly power is exercised in the intermediate goods sector.

Proof: For any $\sigma \in (0,1)$, $(1-\sigma)^{-1} > 1$ holds. Equation *(10.4)* then implies $\tau_z < \tau_y$. ■

In order to compute the steady state optimal capital income tax rate, consider first the time-invariant version of *(9.9c)*:

$$\beta\left\{V_k + V_c\left[\frac{r}{(1-\sigma)} + (1-\delta)\right]\right\} = V_c \qquad (11.1a)$$

The steady state condition *(11.1a)* can be compared to the steady state version of equilibrium condition *(8m)*, which is:

$$\beta\{(1-\theta)r + (1-\delta)\} = 1 \qquad (11.1b)$$

Proposition 4: If all markets are perfectly competitive, the steady state level of optimal capital tax is zero, i.e. for $\sigma = 0$, the model recovers the celebrated Chamley-Judd result of zero capital tax in the long run.

Proof: Consider *(10.2b)* with $\sigma = 0$ which gives $V_k = 0$. Equations *(11.1a & b)* with $\sigma = 0$ and $V_k = 0$ gives $\theta = 0$. ■

Proposition 5: For $\sigma \in (0,1)$, there are two opposing effects that determine the sign and magnitude of the steady state optimal capital tax rate, namely, the distortion effect of monopoly power, and the relative effect of investment on tax distorted equilibrium welfare. For any $\kappa \in [0,1]$, and depending on the relative strength of these two effects, the government's long run optimal policy may involve a capital income subsidy that is smaller in magnitude than the first best subsidy, or a capital income tax.

Proof: Equations *(11.1a & b)* together yield:

$$\theta = \frac{-\sigma}{(1-\sigma)} + \left(-\frac{V_k}{rV_c}\right) \qquad (11.2)$$

For $\sigma \in (0,1)$, the two effects which determine the sign and magnitude of θ are therefore $\frac{-\sigma}{(1-\sigma)}$ which represents the monopoly distortion effect, and $\left(-\frac{V_k}{rV_c}\right)$, which is a measure of the relative effect of investment on equilibrium welfare.

The first effect is the effect due to monopoly power, which is equal to the first best subsidy. The second effect comprises of derivatives of the Pseudo utility function with respect to capital and consumption (evaluated at steady state). Consider first *(10.2b)* with $\kappa \in [0,1]$. Since $(1-\delta-\beta^{-1})<0$, this implies $V_k < 0$. Moreover, with $\kappa \in [0,1]$, *(10.2c)* implies $V_{ny} < 0$, which together with *(9.9a)* implies $V_c > 0$. Hence the term $\left(-\frac{V_k}{rV_c}\right)$ is strictly positive.

The sign and magnitude of the steady state optimal capital tax therefore depends on the relative strengths of these two effects. If the relative effect of investment on tax distorted equilibrium welfare is stronger (weaker) than the monopoly power effect, the long run optimal policy is a tax (subsidy) on capital income. In any case, the optimal subsidy is smaller than the first best subsidy. ∎

4.4 Intuitions and Explanations.

Proposition 2 states that if all markets are competitive, the government's optimal policy involves equal labor income taxes across sectors. The intuition behind this result is straightforward, but its implication is strong since the result does not depend on the marginal disutility of effort, or more generally, preference specification. Since all households are identical, in equilibrium they make exactly the same set of decisions. In other words, in equilibrium it must be optimal for the representative household to behave the way everyone else does. Moreover, the government's Ramsey policy must be consistent with equilibrium reactions of the households. The optimal choice of labor income tax rates therefore must induce households to react with equilibrium labor supply to the two sectors, such that the wage rates across sectors are exactly the equilibrium wage rates. Since with economy-wide competitive markets the only source of distortion is the tax policy, it is optimal for the government to set equal labor income tax rates across sectors.

In a competitive setting, there is also a concern of intra and intertemporal smoothing of labor income taxes. Since the government is benevolent, each period it wants to minimize the total disutility of effort for the household, both over two subsequent periods and across the two sectors. Since the disutility is convex, it is best to induce the representative household to supply same number of hours in each period, which suggests that the optimal labor income tax should be smooth over time. Likewise, as long as the marginal rate of substitutions of labor across sectors is unitary, it is best to induce the representative household to supply same number of hours in the two sectors within a period, than making them work different hours in the two sectors with the same total. This implies that the optimal policy with economy-wide competitive markets should be one which smoothes the labor income tax rate across sectors.

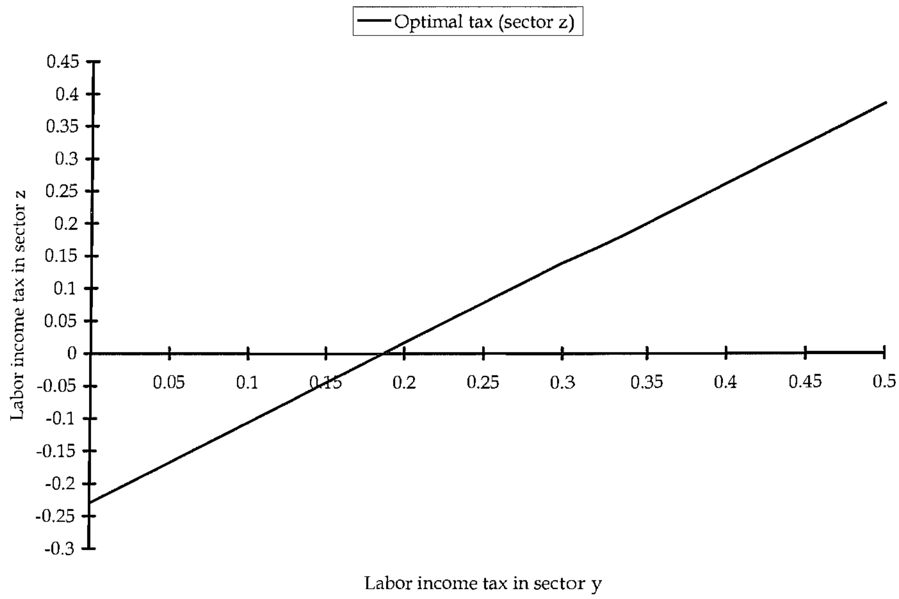
Proposition 3 makes the intuition behind proposition 2 even clearer, and the result from proposition 3 is the normative benchmark of optimal labor income taxation where firms in a particular sector practice monopoly power in pricing. Rearranging *(10.4)* one can derive:

$$\tau_z = \frac{\tau_y - \sigma}{1-\sigma} \qquad (11.3)$$

which implies that the optimal labor income tax for the intermediate goods sector is the sum of two elements, namely, the first best subsidy, and the price mark up adjusted optimal labor income tax for the competitive sector. Due to monopoly distortions, the private marginal return to labor in the intermediate goods sector is lower than the social marginal return. It is therefore optimal to set the labor income tax for this sector lower than a competitive sector's labor income tax such that the distorted efficiency margins are corrected. This is the simple Pigovian consideration while optimally designing labor income tax in the presence of monopoly power.

Since the sign of optimal tax rate on labor income is inconclusive from *(10.3)*, a simple numerical example using *(11.3)* and assuming $\sigma \in (0,1)$ makes the intuition clearer. Figure 4a is presented to illustrate the function $\tau_z = 1.23(\tau_y) - 0.23$, which is derived by setting $\sigma = 0.186$ in *(11.3)*. The choice for σ is not arbitrary. Recall from *(6.5)* that the price mark up ratio for the current setting is $(1-\sigma)^{-1}$. For this illustration, I have chosen 1.23 as the price mark up ratio, which is Bayoumi *et al.* (2004)'s average estimate of price mark up ratio for the US economy. For figure 4a, I have considered $\tau_y \in [0, 0.5]$ which resulted in $\tau_z \in [-0.23, 0.38]$. Consider, for instance, the case where government sets $\tau_z = 0$. This implies the optimal policy is simply to set the labor income tax in final goods sector equal to the parameter σ. Next, consider the case where the government sets $\tau_y = 0$, which simply converge to the first best labor income tax policy without lump sum taxes. In principle, an optimal subsidy for labor income in the final goods sector therefore accompanies a larger optimal subsidy for labor income in the intermediate goods sector.

Fig 4a: Optimal tax on labor income for sigma = 0.186



Proposition 4 recovers the Chamley-Judd result of zero limiting capital tax, and for the current model this is established by setting $\sigma = 0$. It is more convenient to think of this result in connection with proposition 2. With no monopoly power, the steady state optimal tax policy for the government should be one that minimizes tax distortions, which can be accomplished by taxing labor income only and leaving capital income untaxed. With $\sigma = 0$ there is no Pigovian consideration in designing tax policy, i.e. the only consideration for the government in this case is to minimize the *tax induced* wedge between social and private marginal returns, and not to allow the tax distortions to compound over time.

In a competitive setting with no corrective functions of taxes, a nonzero capital income tax between period t and $t+1$ creates tax distortions that compound over time. Judd (1999) provides a clear intuition behind this result. A positive tax on capital income at period t, for instance, implicitly taxes consumption at period $t+1$. With no Pigovian considerations, this implies that the ratio of the marginal rate of substitution between period t consumption and period $t+1$ consumption and their corresponding marginal rate of transformation compounds over time, inducing explosive distortions in the commodity tax equivalent. A long run nonzero capital tax in a competitive setting therefore is not consistent with commodity tax principle and not sustainable as a Ramsey policy. Combining proposition 2 and 4, one can recover the normative benchmark of optimal income taxation in a competitive setting --- smooth labor income taxes with uniform distortions, and zero limiting capital tax.

Proposition 5 is one of the main results of the current chapter. What it implies is that a more useful track to explain the optimal capital income tax policy in the presence of monopoly power is to demarcate the effects that motivate the policy into a distortion effect and an investment effect. The first effect, which is due to monopoly distortion, is simply equal to the first best subsidy, or in other words, one minus the price mark up. The second effect is due to Ramsey taxation, since it is not observed in the first best policy. Note first that from *(11.1a)*, one can derive:

$$\frac{V_k}{V_c} = \frac{1}{\beta} - \left[\frac{r}{(1-\sigma)} + (1-\delta) \right] \qquad (11.4)$$

The terms V_k and V_c in *(11.4)* are simply the steady state marginal effect of investment and consumption in tax distorted equilibrium welfare. In a zero profit competitive market setting $V_k = 0$, and the Ramsey equilibrium implies $\beta^{-1} = [r + 1 - \delta]$. With monopoly distortion and nonzero equilibrium profits in the implementability constraint, equilibrium welfare is adversely affected by profit-seeking investment. For $\kappa \in [0,1]$, $V_k < 0$ and $V_c > 0$, which implies that the relative effect of investment is negative, i.e. investment in the tax and monopoly distorted equilibrium reduces welfare.

The government's optimal capital tax policy, as in *(11.2)*, therefore is determined by the relative strengths of two terms, one representing the distortion effect, and the other representing the welfare effect of investment. If the latter dominates the former, agents invest in search of profits and it is optimal to tax capital income to discourage such investment. More intuitively, investment in the presence of monopoly distortion is associated with a much lower private return to capital, but opportunity for more profits. For low degrees of monopoly power, agents invest to increase profits since the realized increase in profits over-rules the realized wedge between social and private returns to capital. Although higher profits increase agents' income, lower returns to factors induce welfare loss. The optimal policy in such case should be one to discourage investment by imposing a capital income tax. For remarkably high degrees of monopoly power, the wedge between social and private returns to factors increases proportionately more than the realized profit gain from investment. This is the situation where agents under-invest, and the optimal policy response should be one that encourages investment through a capital income subsidy.

Is the optimal capital income tax policy distortion-neutralizing? Note that the equilibrium cost of capital in this setting is determined by total distortion created by the interaction of taxation and monopoly power. More specifically, denoting the social marginal product of capital by MP_k, equilibrium condition *(8i)* implies:

$$r = (1 - \sigma) MP_k \qquad (11.5a)$$

Equation *(11.5a)* shows that in equilibrium, the marginal product of capital equals the private return to capital grossed up by the price mark up factor $(1-\sigma)^{-1}$. In an efficient outcome $r = MP_k$; which means the distortion in the demand for capital is created by monopoly power. With $\tilde{r} \equiv (1-\theta)r$, which is the after tax return to capital, equation *(11.5a)* implies that the equilibrium cost of capital is expressed by the following equation:

$$\tilde{r} = (1-\sigma)(1-\theta)MP_k \qquad (11.5b)$$

Equation *(11.5b)* shows that monopoly power is equivalent to a privately imposed tax rate of σ, and the optimal choice of θ neutralizes this distortion.

Finally, there remains a question if proposition 5 implies a corollary of a limiting zero capital tax result when the two effects completely offset each other. In principle the answer is no, since the case where these two effects completely offset each other is inconsistent with equilibrium conditions. To show it formally, note that *(11.4)* and *(11.2)* together imply:

$$\theta = 1 - \frac{1}{r}\left[\frac{1-\beta(1-\delta)}{\beta}\right] \qquad (11.6)$$

and $\theta = 0$ if and only if $\beta^{-1} = [r+1-\delta]$. Given the current setting, $\beta^{-1} = [r+1-\delta]$ cannot hold in equilibrium for $\sigma \in (0,1)$, implying that $\theta \neq 0$.

4.5 Calibration and Computation.

In order to verify and reinforce the analytical findings in propositions 2, 3, 4 and 5, this section calibrates the steady state of the model and presents some useful insights from the numerical results. The steady state of the model is calibrated to fit the stylized facts of the US economy for data period of 1960-2001. The time period is considered to be one year which is consistent with frequency of revision of fiscal decision. In calibrating, the parameters of the model are pinned down so that the steady state of the model matches characteristics identified from the US data for time period 1960-2001.

Consistent with the standard assumptions of the utility function and the simplifying assumption of $u_{cns}(t) = u_{nsc}(t) = u_{nsns}(t) = u_{nsnl}(t) = 0$ for $s,l = y,z,$ and $l \neq s$, consider the following specification:

$$u(c_t, n_{yt}, n_{zt}) = \ln(c_t) + [1 - n_{yt} - n_{zt}] \qquad (12.1)$$

which implies $\frac{u_{cc}}{u_c} c = -1$[31].

The set of parameters for the model is $(\alpha, \nu, \sigma, \kappa, \beta, \delta)$. The multiplier Φ (the utility cost of raising revenue through distorting taxes) can be computed once the set of parameters are calibrated. The parameters are pinned down to match the steady state characteristics identified from the US data for time period 1960-2001. This gives the baseline values for the set of parameters.

The baseline values are used to calibrate the multiplier and then the Ramsey tax rates and the first best tax rates. Given the main purpose of the chapter, the parameter indexing the degree of monopoly power, σ, is of prime interest. Once the model has been calibrated, sensitivity of the key results is tested by varying σ within a reasonable range. Note that the parameters $\beta, \alpha, \nu, \delta$ and κ are the structural parameters which are calibrated directly from data, and hence do not require

[31] Specification *(12.1)* ignores the possibility of having different marginal disutility of work across sectors --- an abstraction which may be empirically questionable. The *Bureau of Labor Statistics* survey reports suggest that injury related incidence per 100 workers varies greatly across different industrial sectors of the US economy, and incidence rates are relatively higher in goods producing sector as compared to the service producing sector. Considering this observation, one can specify $u(c_t, n_{yt}, n_{zt}) = \ln(c_t) + [1 - n_{yt} - \Lambda n_{zt}]$ with $\Lambda > 0$. For the current purpose, however, the implicit assumption of unitary marginal rate of substitutions of labor across sectors is innocuous because the optimal labor income tax rates as in *(10.3)* are not at all sensitive to this assumption. Moreover, the calibration was verified by adding another parameter in *(12.1)* to account for the relative marginal disutility of work, which did not produce any remarkable changes in the key results.

recalibration. Varying σ for the current setting is tantamount to varying the profit ratio for given values of the structural parameters, and requires recalibration of the multiplier indexing for utility cost of distorting taxes, Φ. A reasonable range of values for the parameter κ, that describes government's fiscal treatment of distributed profits, is also considered. Varying κ requires recalibration of the multiplier Φ.

4.5.1 Parameterization.

Annual data of the US economy's real output, government consumption and corporate profits for the period 1960-2001 are taken from the *Federal Reserve Bank of St. Louis Economic Data*-FRED II. This data gives average government consumption to output ratio equal to 0.23, and profit to output ratio equal to 0.11. Annual data for the US economy's capital stock and investment for the period 1960-1996 are collected from the US Department of Commerce's *Revised Fixed Reproducible Tangible Wealth in the United States*. The series for capital and investment include business equipment and structures, residential components and consumer durables, and give steady state capital to output ratio equal to 3.31, and investment to output ratio equal to 0.22. Table 4.1 summarizes the target statistics.

Table 4.1: Steady state ratios for the US economy, 1960-2001.

Ratio	Description	Value
g/y	Government consumption to output ratio.	0.23
π/y	Profit to output ratio.	0.11
k/y	Capital to output ratio.	3.31[a]
i/y	Investment to output ratio.	0.22[a]

a: These estimates are for 1960-1996, collected from Revised Fixed Reproducible Tangible Wealth in the United States, US Department of Commerce.
Source: Federal Reserve Bank of St. Louis Economic Data-FRED II, and Revised Fixed Reproducible Tangible Wealth in the United States, US Department of Commerce.

The baseline parameter values are presented in table 4.2. The parameter β is chosen such that it is consistent with annual real interest rate of 4%. This pins down $\beta = 0.9615$. The value of the parameter κ stands for the fiscal treatment of profits and is the ratio between tax on distributed profit and tax on capital income. The tax on distributed profits for the US economy, from McGrattan & Prescott (2005)'s period average estimate for 1990-2000, is 17.4%. For the average effective tax rate on capital income for the US economy, I use 27.3% as in Carrey & Tchilinguirian (2000). This pins down $\kappa = 0.6373$.

Table 4.2: Baseline parameter values.

Parameter	Description	Value
β	Subjective discount rate.	0.9615
δ	Capital depreciation rate.	0.0664
α	Share parameter for capital in intermediate goods sector.	0.5759
ν	Share parameter for intermediate goods in final goods sector.	0.7351
σ	Inverse of the elasticity of substitution between intermediate goods.	0.1496
κ	Fiscal treatment of distributed profits.	0.6373

Capital's share of final output is set equal to 0.36, an approximation consistent with long run US data, and also frequently used in relevant literature[32]. This is consistent with $\nu = 0.7351$, $\sigma = 0.1496$, and $\alpha = 0.5759$. The calibrated value for the parameter σ yields the price mark up ratio equal to 1.175, which is a reasonable approximation of the range of values typically used in established literature, such as the ones presented in Martins *et al.* (1996), Basu & Fernald (1997) and Bayoumi *et al.* (2004). With $k/y = 3.31$ and $i/y = 0.22$, the steady state version of *(8d)* pins down $\delta = 0.0664$, and consequently $c/y = 0.55$. In order to calibrate the utility cost of distorting taxes, Φ, note first that with baseline parameters $r = 0.1087$, which from *(11.4)* yields $\frac{V_k}{V_c} = -0.0213$. Using this in *(10.2)* pins down $\Phi = 0.9890$.

[32] In the current setting, the three income shares add up to $1 - \nu\sigma$, which is simply one minus the profit ratio.

4.5.2 Quantitative Findings.

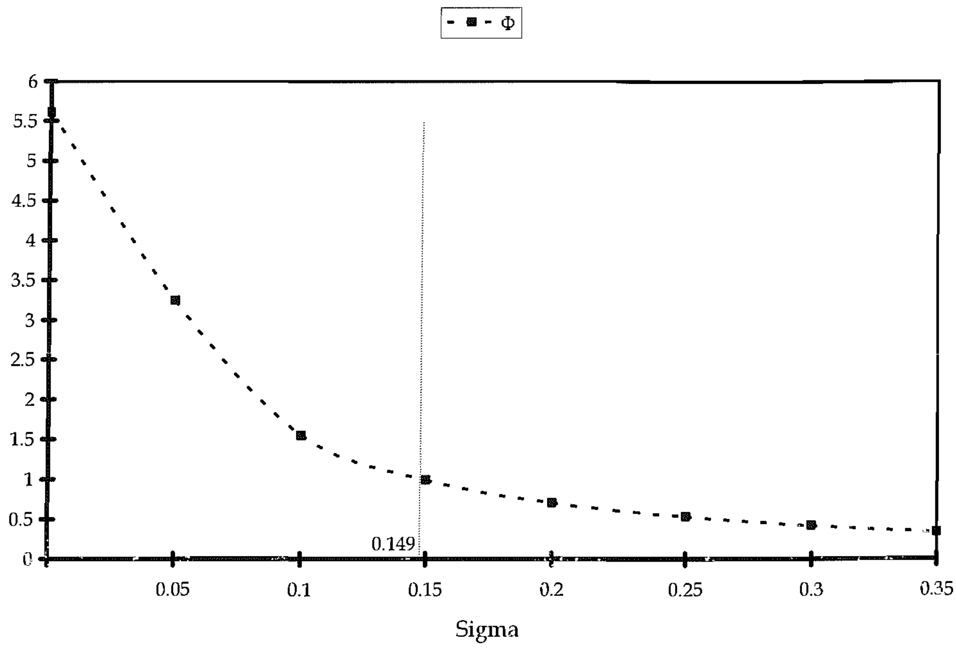
The quantitative findings are summarized in table 4.3 and figures 4b-g. Table 4.3 summarizes the calibrated Ramsey policy and the first best policy for baseline parameter values. The calibration with baseline parameter values suggests that the long run optimal policy for the government involves tax on all income and no subsidy. The calibrated Ramsey tax rates are approximately equal to 2%, 32% and 42% for capital income, labor income from intermediate goods sector, and labor income from final goods sector, respectively. Given the baseline value of the parameter κ, the calibrated tax rate on distributed corporate profits is 1.3%. The first best policy would involve 0% tax on labor income from final goods sector, a uniform subsidy of approximately 18% on capital income and labor income from intermediate goods sector, and a lump sum tax such that the ratio of lump sum tax to output is equal to 0.3522.

Table 4.3: Calibrated optimal tax rates.

	Capital income tax (θ)	Sector z labor income tax (τ_z)	Sector y labor income tax (τ_y)
Ramsey Policy	0.0206	0.3167	0.4189
First Best Policy	-0.1759	-0.1759	0

The figures are constructed to show the sensitivity of the key results for changes in parameters σ and κ. Figure 4b and 4c examine the efficiency of the Ramsey policy for a range of values for the parameters σ and κ. Figure 4b suggests that economic agents prefer the Ramsey policy than the first best policy for high price mark up ratio. This is because the Ramsey policy compensates for monopoly distortion and induces lesser welfare cost than a heavy lump sum tax. Higher degrees of monopoly power results in higher losses of output and drives a larger wedge between social and private returns to factors, which in turn distorts the work and investment incentives. Although a first best subsidy can be used to compensate the wedge, a heavy lump sum tax in addition reduces disposable income. The Ramsey policy for high degrees of monopoly power diversifies the tax burdens through different tax instruments, which imply that the social cost of distorting taxes becomes relatively lower. With remarkably high degrees of monopoly power, the utility cost of distorting taxes are lower, implying that the households are willing to pay lesser amount in terms of consumption goods to replace one unit of distorting tax by one unit of lump sum tax.

Fig 4b: Utility cost of taxation (Φ) vs. sigma.



Not surprisingly, this is also true for higher values of the parameter κ, as in figure 4c. The more the tax (or subsidy) on distributed profits, the less is the government's reliance on factor income tax instruments. Consequently, for high values of the parameter κ, the welfare cost of Ramsey taxes is low, and Ramsey taxes are preferred over lump sum tax. It will be shown later in figure 4g that different fiscal treatments of distributed corporate profits such that $\kappa \in [0,1]$ does not affect the optimal choice of capital income tax at all, and affect only the optimal labor income tax policy. More specifically, higher values of the parameter κ for a given monopoly distortion level are associated with lower values of labor income tax rates, implying that the Ramsey policy becomes more desirable for higher taxes (or subsidies) on distributed profits.

Fig 4c: Utility cost of taxation (Φ) vs. kappa

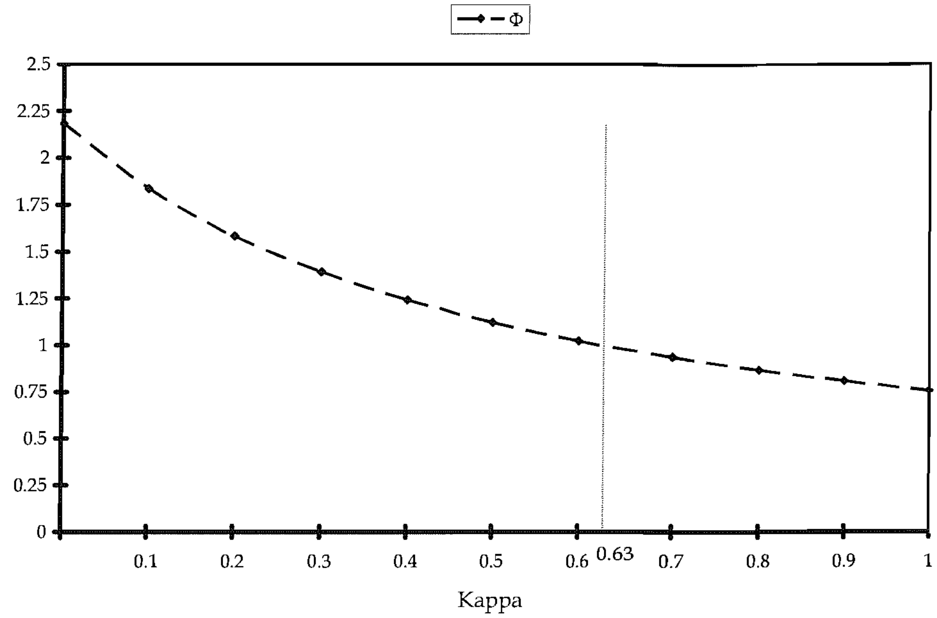
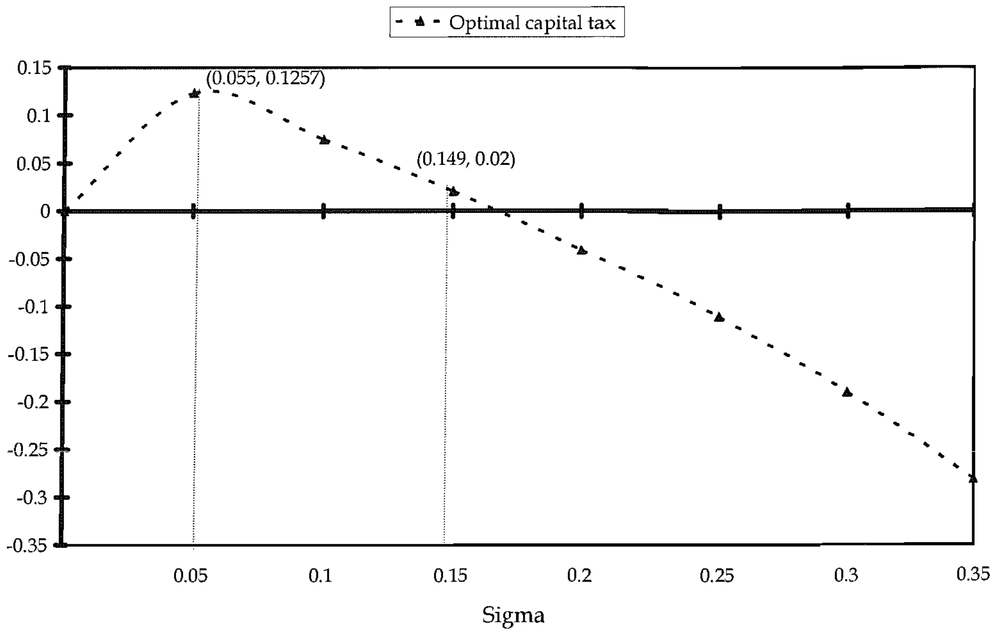


Figure 4d, 4e and 4f present the calibrated Ramsey and the first best taxes for $\sigma \in [0, 0.35]$. Consider first the calibrated optimal capital income tax presented in figure 4d. For $\sigma \in (0, 0.17)$, the relative effect of investment dominates the distortion effect of monopoly power which motivates an optimal tax on capital income. The peak of capital income tax is 13% which is for $\sigma = 0.055$. For any $\sigma \in (0.17, 0.35)$, the converse happens, and the optimal policy involves a capital income subsidy. For different values of σ, the sensitivity of the relative effect of investment is much less than the sensitivity of the distortion effect of monopoly power, implying that the relative effect of investment dominates the monopoly distortion effect over a much smaller range of σ [33]. Although high degrees of monopoly power are associated with high profits, they are also associated with larger wedges between social and private returns to factors and consequent larger loss in output. For high degrees of monopoly power, the rate of increase in the wedge between social and private marginal return to capital is much larger than the rate of increase in welfare effect of investment. For remarkably high degrees of monopoly distortions economic agents therefore set

[33] The optimal capital subsidy result actually holds for $\sigma \in (0.17, 1)$. The only permissible range of values for σ for which it is optimal to tax capital income is therefore $(0, 0.17)$.

investment at a very low level. Investment in the range of $\sigma \in (0.17, 0.35)$ can be encouraged only by setting a capital income subsidy.

Fig 4d: Optimal capital tax vs. sigma



Does this imply for $\sigma \in (0.17, 0.35)$ the optimal policy involves heavy subsidies to both capital income and profits? For remarkably high degrees of monopoly power the profits are high, and subsidizing both capital income and high profits requires raising heavy revenues which may be infeasible, especially in an economy without government bonds and low levels of labor income tax rates. In fact, for $\sigma \in (0,1)$ the optimal tax/subsidy on capital income can be accompanied by little or no tax/subsidy on distributed profits. As will be shown in figure 4g, the optimal tax rate on capital income is completely insensitive for different values of the parameter κ within a reasonable range. What the parameter κ affects are the optimal labor income taxes. This implies that the government can pursue the optimal policy of subsidizing capital income with little or no subsidy on distributed profits, and hence make the revenue requirement feasible. This is also clear from figure 4g, since for low values of κ both optimal labor income taxes are higher than their baseline calibrated estimates. Hence, for high degrees of monopoly power, the government's optimal policy may be one that attaches a low weight on profit taxation and finances the capital subsidy by heavily taxing labor income.

Fig 4e: Optimal labor income tax vs. sigma

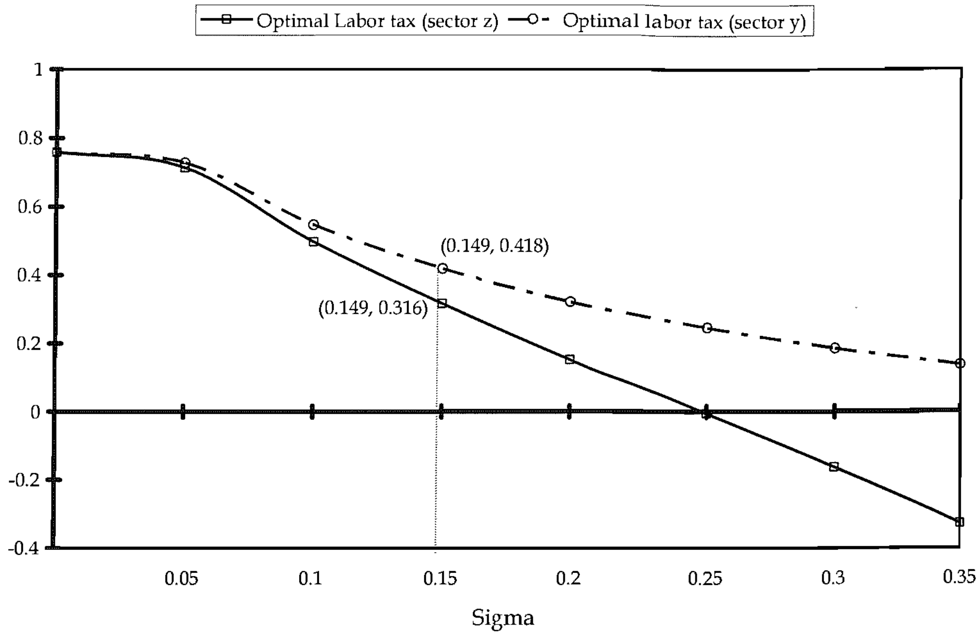


Figure 4e presents the calibrated optimal labor income taxes for $\sigma \in [0, 0.35]$. Figure 4f presents the calibrated Ramsey taxes and the first best taxes for the same range of σ. For any $\sigma \in (0, 0.35]$, the optimal labor income tax for intermediate goods sector is lower than the optimal labor income tax in competitive sector. Higher degrees of monopoly power are associated with lower optimal taxes on labor income in both sectors. For $\sigma = 0.25$ the optimal labor income tax in intermediate goods sector becomes zero, implying that it is optimal to subsidize labor income in the intermediate goods sector for any $\sigma > 0.25$. Moreover, for any $\sigma \in (0, 0.35]$, neither the optimal capital income subsidy nor the optimal labor income subsidy for the intermediate goods sector converges to the first best subsidy, i.e. both Ramsey subsidies are smaller in magnitude than the first best subsidy. These results were verified for $\sigma \in (0,1)$ which suggests that the optimal labor income tax for the final goods sector never converges to zero.

Fig 4f: Ramsey and first best taxes vs. sigma

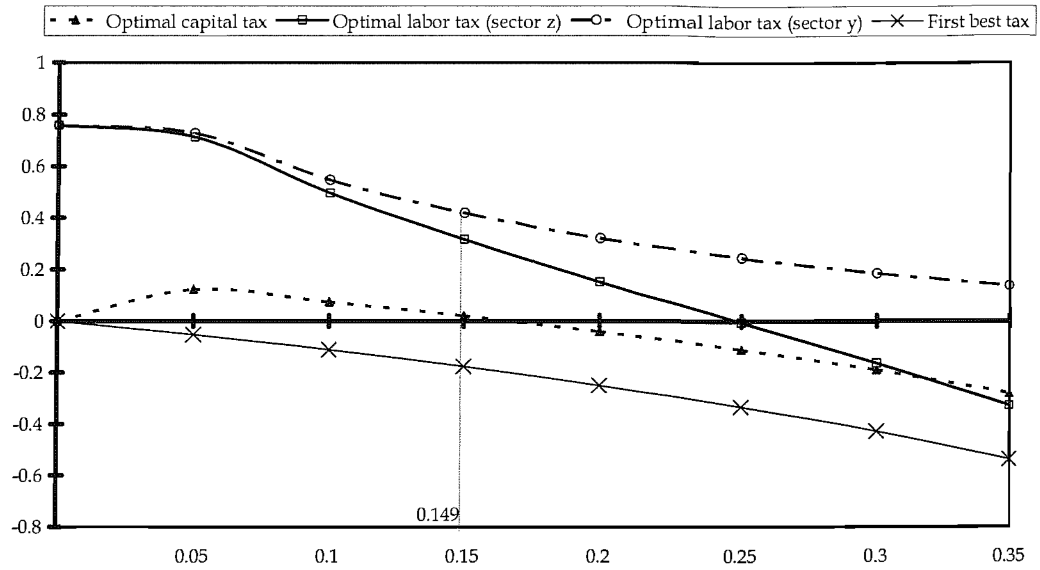


Fig 4g: Ramsey taxes vs. kappa

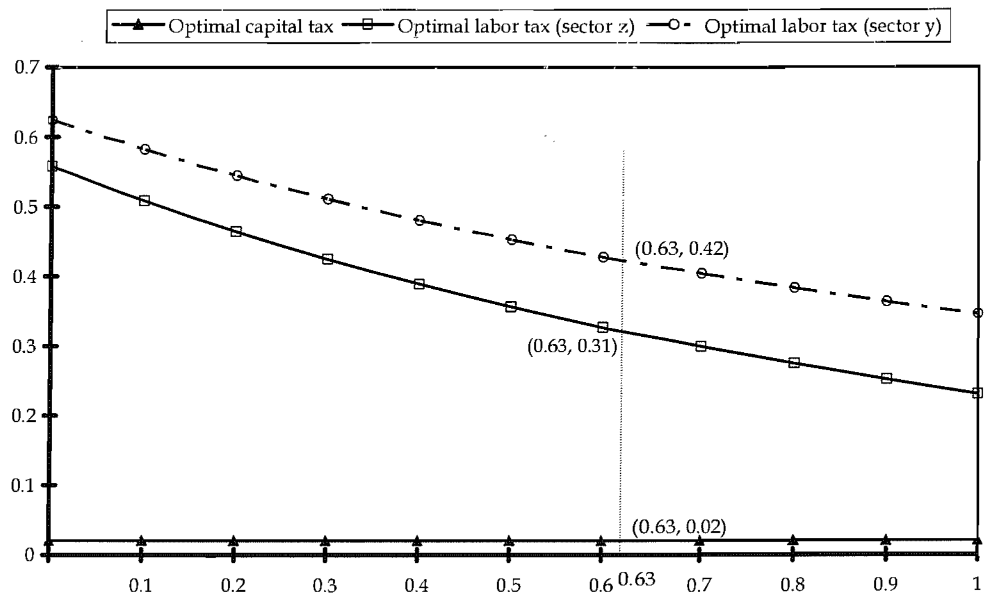


Figure 4g presents the Ramsey taxes for different values of the parameter describing fiscal treatment of distributed profits, i.e. for $\kappa \in [0,1]$. As has been mentioned before, varying the parameter κ within the specified range does not affect the optimal capital income tax at all, and affects only the optimal labor income taxes. As the government increases tax on distributed profits, it relies less on taxing factor income. Moreover, since both the profit taxes and the labor income taxes create uniform distortion, an increase in profit tax rate is accompanied by a decrease in labor income tax rates. Hence, higher values of the parameter κ are associated with lower optimal tax rates on labor income from both sectors.

4.6 Conclusion.

This chapter examines the optimal income taxation problem in a simple two-sector general equilibrium model where firms in the intermediate goods sector practice monopoly power in pricing of differentiated intermediate goods. It shows that with the introduction of monopoly power in a standard neoclassical optimal taxation model, the choice of optimal capital income tax policy becomes somewhat analytically ambiguous. Resolving the ambiguity requires correct assessment of the degree of monopoly distortion. The sign of the optimal capital income tax is determined by two opposing effects, which are the distortion effect of monopoly power and the welfare effect of investment. The distortion effect motivates the use of a capital income subsidy, while the relative effect of investment on welfare supports the use of a positive tax on capital income. For an empirically plausible set of parameters which are consistent with long run characteristics of the US economy, the chapter finds that the welfare effect of investment dominates the distortion effect for very low degrees of monopoly power which supports the optimal policy of a capital income tax. For high degrees of monopoly power, it is optimal to subsidize capital income. This result therefore specifies Judd's (2002) prescription of optimal capital income subsidy for a range of high degrees of monopoly power.

The chapter also shows that optimal tax on labor income in the imperfectly competitive sector is lower than the optimal tax on labor income in the perfectly competitive sector, and for all permissible degrees of monopoly power Ramsey taxes are higher than the first best income taxes. However for high degrees of monopoly power Ramsey taxes induce lesser welfare cost since they neutralize the distortions created by monopoly pricing. The optimal labor income tax policy is sensitive to government's fiscal treatment of distributed profits, but the optimal capital income tax policy is not. This implies that for excessively high degrees of monopoly power, the optimal policy *may* involve capital income subsidy, no subsidy for distributed profits, and taxes on labor income.

Both monopoly power and income taxes induce distortions in allocations and are associated with significant welfare costs. In a recent paper, Jonsson (2004) presents a quantitative analysis of the US economy's welfare costs due to monopoly power and taxation. One of the major findings of his work is that the welfare cost of taxation depends on the level of competition. As reported in Jonsson (2004), the long run welfare cost of imperfect competition in product market and distorting taxes are 48.26% and 12.79%, respectively. Moreover, based on the computed welfare cost approximations, Jonsson (2004) establishes that in an economy with imperfectly (perfectly) competitive markets labor income taxes are more (less) distorting than capital income taxes. From this point of view, the current chapter's key finding of a nonzero limiting tax/subsidy on capital income in principle is less distorting than what it would have been if markets were perfectly competitive.

The chapter abstracts from formal analysis of taxation of initial capital income, or more generally, capital income taxation during transition. Since the initial endowment of capital is fixed, government has a strong incentive to tax capital income in the initial period at a confiscatory rate. The practicality and political feasibility of confiscatory taxation of capital income are however of natural suspicion. Coleman II (2000) shows that different assumptions regarding the pattern of tax rates allowed during transition can dramatically influence the welfare gains from adopting long run optimal policy, and some specific set of restrictions on transitional tax rates may completely remove the welfare gains of policy reform. This is because agents change consumption plans during transition to achieve the higher capital stock associated with the long run policy. The current chapter argues that the optimal policy itself is highly sensitive to particular modelling assumptions, and therefore its tractability with transition consideration becomes more complex. This, however, may be a potential extension to this research. Another important extension may also be to introduce endogenous monopoly power, which may be accomplished by modelling some form of interaction between fiscal policy and anti trust policy.

Chapter 5

On Policy Relevance of Ramsey Tax Rules[34]

5.1 Introduction.

Mapping pure theory into policy is always a demanding task for economists. While principles and normative benchmarks typically dictate intellectual forums, establishing their policy relevance and political realism remains a different and at times an appealing challenge. Over the last three decades, there has been a marked enthusiasm amongst the critics of optimal taxation theory in attempts to establish the limits of optimal tax formulas and prescriptions in designing tax policy. Important contributions in this spirit include Slemrod (1990), Heady (1993), and Alm (1996). In most parts of this particular literature, it is often argued that optimal taxation is in fact largely irrelevant to realistic tax design, because it typically abstracts from a range of considerations associated with fiscal and societal institutions that are crucial elements in the normative and positive analysis of taxation.

[34] I would like to thank James Hines Jr. (Michigan & NBER), Patrick Minford (Cardiff) and Olivier Bargain (IZA) for their important comments on policy relevance of Ramsey tax rules during my presentations of chapters 2 and 3, which in turns motivated this chapter.

The seminal approach to optimal taxation is of Ramsey's (1927). The well known optimal taxation principles which are derived from general equilibrium settings that follow Ramsey's (1927) approach are known as, and will be referred to as, Ramsey tax rules. In the standard set up of this approach, it is typically assumed that the government's commitment power is perfect such that it cannot deviate from the announced policy rules. This chapter is devoted to examining the mapping of 'Ramsey tax rules under commitment' into fiscal policy design. Throughout this analysis, I clearly maintain an activist role for Ramsey tax rules, mainly through analyses of their strength and importance in designing fiscal policy, and instantaneous attempts to refute their policy-related criticisms. I begin with a general but strong proposition that the aim of optimal tax analysis is to describe the taxes that governments should set, and not necessarily to explain the taxes that governments do set. A close relationship between the optimal tax prescriptions and the taxes that are actually implemented should not be expected, since there are a number of reasons for believing that governments do not follow normative approach to policy design and policy implementation.

Before proceeding to details, it becomes, to some extent, necessary to introduce the two stylized traditions in the optimal taxation theory, although to my understanding there is barely any reason why this classification is strict and important to many[35]. As the literature suggests, a research on optimal taxation is typically based on any one of the two traditions, namely, the *Ramsey* tradition due to Ramsey (1927), and the *Mirrlees* tradition due to Mirrlees (1971). Much of the thoughts and practice of the Ramsey tradition has already been introduced in the previous chapters. The Mirrlees tradition is more concerned on redistribution issues and utilitarian arguments of taxation, which is why its primary focus is on marginal tax rates in an economy where agents have heterogeneous types and endowments. In this tradition, the key underlying assumption is that the optimal level of tax depends on the consumer's ability to earn money. If the government had perfect knowledge of this ability, it could levy an

[35] I think the classification is a mere stylization. Contributions to the literature that follow either of these two traditions are of similar importance in designing tax policy. A combined set of results accumulated from the two streams is more helpful in understanding the policy relevance of optimal tax rules. Given the current chapter's key focus, results from the *non-Ramsey* tradition are regarded as important mainly for a complete assessment of policy relevance of optimal tax rules.

ability-dependent lump sum tax that would not distort the consumer's allocation decisions. In the presence of incomplete information about ability, the government can only base the income tax policy on realized income. The income tax schedule can be seen as an incentive scheme eliciting information about the consumer's ability. Most of this tradition therefore highlights the importance and policy relevance of non linear taxation of income.

The substantive lessons of taxation derived from the two stylized traditions are, from a broad perspective, nearly similar. The important difference is perhaps their methodology and focus[36]. The current chapter's objective is to establish the policy relevance of Ramsey tax rules, which is accomplished mainly by emphasizing their importance and defending them against their criticisms. Most of the Mirrlees literature and results are held as supporting arguments, which will be highlighted within the line of discussion to follow. A summary of the key results and modelling techniques of the Mirrlees tradition, apart from the pioneering paper by Mirrlees, can be found in Renstrom (1999).

A review of the critiques of optimal taxation theory suggests that critics typically target the underlying assumptions and features of optimal taxation models, and practicality and political acceptability of the optimal tax rules. A summary of the three common features which all standard Ramsey taxation models (with commitment) possess is useful before analyzing their criticisms. First, each model specifies a given revenue requirement for the government and a set of proportionate taxes on transactions. The models typically rule out lump sum taxes due to its impracticality. Second, each model specifies how consumers and producers react to a particular tax policy. Third, the government has an objective function (typically the social welfare function) for evaluating different configurations of taxes. The theory, in general, does not consider the details of the political process that generates tax policy, and does not deal with the possibility that policymakers' objectives may be something other than

[36] Some authors, such as Renstrom (1999) and Golosov *et al.* (2003) are explicitly in favour of one over the other. The reason for such a bias, as implied by their papers, is specific to the particular purpose of their papers. The *general* principles of taxation drawn from these two traditions are indisputably equivalent. It is, however, important to mention that the recent campaigners of progressive taxation and fair taxation in the US are following the line of discussions from the Mirrlees tradition, where flat tax plans are severely criticized. The current chapter assumes that details of this campaign are irrelevant, but for those who are interested, a useful survey may be found in Shapiro (1996).

benevolent. The desirability of any tax policy is evaluated solely by its consequences for taxpayers, and is not judged independently on how closely it meets abstract principles such as fairness.

Most critics claim that the simplifying assumptions and some underlying features (e.g. ruling out lump sum taxes, abstraction from formal modelling of administrative costs) of standard Ramsey tax models are limitations of the theory. In this chapter, I argue that the simplifying assumptions of optimal taxation models in general (and Ramsey tax models in particular) are innocuous, since elaborate attention to such details is relatively less important than the broader set of goals of the theory, which their abstraction facilitates to achieve. The features of Ramsey tax models that are often under scrutiny are the ones which allegedly fail to simultaneously justify fairness and efficiency of a particular Ramsey tax rule. From a political perspective, fairness of tax policy is an attractive feature (or campaign cliché). In a way, such an issue is actually addressed in most standard representative agent Ramsey taxation models, although with much less emphasis than the critics would like to see. For instance, a concern for fairness may be embedded in the concavity of the social welfare function. Efficiency of tax policy is one of the main focuses of optimal taxation theory, which is reflected in the welfare maximization process of finding tax rules that reduce disincentive effects in allocation. Critics argue that greater emphasis of efficiency is associated with trading off fairness. This trade off problem is conceivable and perhaps inevitable, but as will be detailed later, it is not such a serious problem.

Critics also question the implementability and practicality of Ramsey tax rules. Such arguments are generally irrelevant, and this can be verified by simple intuitions. A theoretical general equilibrium model of Ramsey taxation is particularly intended to imitate the fiscal policy design process and specify the normative benchmark levels and composition of taxes. Without further specialization and extension of focus, one cannot expect the model to yield instrumental and applicable results that specify how such policies can be practically implemented. Moreover, Ramsey tax rules in dynamic settings in particular directly infer to the optimal average effective tax rates on the taxable transactions, which, in practical policy designing process, can be attained with a combination of different tax instruments. Finding the right combination of taxes that achieves the optimal policy, or identifying the problems associated with doing so is a different issue, and hence should be addressed separately. Ramsey tax rules are therefore more useful and insightful from the macroeconomic perspective (the level and composition of tax revenue) than the microeconomic perspective (design aspects of specific taxes), implying that criticizing Ramsey tax rules on grounds of practicality of tax systems is in fact far from relevant.

One of the most important contributions of optimal taxation theory is its lessons for capital income tax policy, which is largely due to three influential papers by Kenneth Judd (1985), Christophe Chamley (1986) and Larry Jones, Rodolfo Manuelli & Peter Rossi (1997). The key result of these papers is that the welfare maximizing second best policy involves zero tax on capital income in the long run, and smooth and roughly constant labor income tax and consumption tax in order to finance revenues. The strong underlying intuitions of these principles make them more or less irrefutable and laudable in modern tax reform proposals, which is perhaps why there has been an observed tendency of cutting down capital income taxes in recent OECD economies' tax reforms. Moreover, Judd (1997) establishes that with imperfect competition, since output is lower than its optimal level, the optimal policy should involve corrective subsidies to capital income. The current thesis, in chapters two, three and four, adds some new insights to these results. This chapter discusses the underlying intuitions and the policy relevance of these popular results and their most recent extensions.

If there is a criticism that truly characterizes a limitation of the Ramsey tradition to optimal taxation, it is the one which nullifies the usefulness of standard Ramsey tax rules in designing tax policy in developing countries. Optimal taxation in developing countries is essentially a futuristic research. It has been attempted very recently in Penalosa & Turnovsky (2004). An economic model that explores fiscal policy in developing countries requires particular attention to some *special* features, which are not typically included in standard economic models. These may be predominance of informal sector activities which makes it difficult to identify taxable income and calculate taxable base, commercial integration which makes the feasible margin of deviating tax policy from other countries' tax policy very low, and issues related to tax administration and collection. Given such unconventional features and their potential impacts, tax policy in developing countries is the art of the possible rather than the pursuit of the optimal. It is not surprising that optimal taxation theory, as it has been practiced, will have relatively little impact on the design of tax systems in these countries. This chapter includes a particular section that highlights the findings of Penalosa & Turnovsky (2004) and discusses the extent to which the Ramsey tax rules can be mapped into such an unconventional tax policy design problem.

In the remainder of this chapter, section 5.2 presents the optimality criterion of Ramsey tax rules and examines the second best tax policy's compatibility with the *apparently* essential features of a tax system. Section 5.3 discusses the policy relevance of optimal capital income tax rules. Section 5.4 discusses the policy relevance of Ramsey tax rules in developing economies. Section 5.5 presents concluding remarks.

5.2 Optimality.

The optimality criterion of Ramsey tax rules under commitment has been through rigorous investigations, mostly on grounds of fairness, feasibility of collection and compliance, and disincentive effects. Important papers that belong to this practice are Shavell (1981), Slemrod (1990), Mayshar (1991), Heady (1993), Alm (1996) and Shapiro (1996). All these authors agree that the abstraction from formally modelling the costs associated with tax collection and tax compliance significantly weakens the usefulness and policy relevance of optimal tax rules. More precisely, these authors argue that the second best tax policy ceases to be consistent with the theory of second best because of its abstraction from the cost of administering and implementing such a policy. Their views, however, differ in characterizing the fairness of a tax system, and hence the criticism of optimal tax rules on fairness ground is more or less unclear.

The technical evaluation of Ramsey tax rules' optimality is rather simple and clear. Due to the impracticality associated with implementing first best lump sum taxes --- which would otherwise keep optimal allocation decisions unaffected --- a welfare maximizing Ramsey tax must be the second best policy and the associated outcome must be the second best outcome. This section first presents the general representation of this proposition. Mapping the second best taxes into practical tax policy design, allegedly, is far from simple. According to Heady's (1993) proposition, for a tax policy to be politically desirable and implementable, the set of criteria other than simple utility maximization that must be satisfied includes *(a)* fairness; *(b)* economy in collection and compliance; and *(c)* minimum disincentive effects. This section also analyzes to what extent the standard second best Ramsey plans fulfil these additional considerations.

5.2.1 General Representation of the Model.

That the Ramsey tax rules under commitment are in general the second best outcomes can be verified by considering a general representation of the Ramsey problem with commitment in an economy with competitive markets. Specialized versions of this model are the one or multi-sector neoclassical growth models presented in the previous chapters.

Recall that in environments where the commitment technology is assumed to be effective, the additional implicit assumption is that the government has a technology that permits it to choose an action first, i.e. ahead of the private sector[37]. Consider a simple one period economy where these assumptions are operative. There is a continuum of households, each of whom chooses an action $\varepsilon \in X$, in response to the government's choice of an action $\gamma \in Y$. Both X and Y are assumed to be sequentially compact sets, i.e. the sequence of elements of X and Y have convergent subsequence whose limits lie in X and Y, respectively.

The average level of ε across households is denoted by $\bar{\varepsilon} \in X$. When the government chooses γ, and given that the average level of households' action is $\bar{\varepsilon}$, a particular household chooses ε which gives utility $\mathbf{u}(\varepsilon,\bar{\varepsilon},\gamma)$. Assume that the preferences are strictly monotone in ε, and the utility function is strictly concave and

[37] This assumption is effective in models developed and presented in most parts of this thesis. However, for the general representation of the Ramsey problem presented here, the focus is restricted to an economy which deviates from the first best representative agent economy only due to distorting revenue-raising taxes. Most of the models discussed in chapter 1 and the model developed in chapter 2 are the specialized versions of this general representation.

continuously differentiable. For realized levels of γ and $\bar{\varepsilon}$, the representative household faces the following problem:

$$\mathbf{max}_{\varepsilon\in X} \qquad \mathbf{u}(\varepsilon,\bar{\varepsilon},\gamma) \tag{1}$$

The solution to *(1)* is a function denoted by $\varepsilon=\varepsilon(\bar{\varepsilon},\gamma)$. With the commitment assumption, the representative household acts to set its equilibrium response $\varepsilon=\varepsilon(\bar{\varepsilon},\gamma)$ for the government's action γ, and for the belief that the average level of other households' actions is set at $\bar{\varepsilon}$. Furthermore, if one assumes that all households are identical, then actual level of $\bar{\varepsilon}$ is $\varepsilon(\bar{\varepsilon},\gamma)$. For expectations about the average to be consistent with the average outcome, one would require $\varepsilon=\bar{\varepsilon}$, or simply, $\bar{\varepsilon}=\varepsilon(\bar{\varepsilon},\gamma)$. The following equilibrium definition is therefore required.

Definition 5.2.1a (Competitive Equilibrium). A Competitive *(Rational Expectations)* Equilibrium is an action $\bar{\varepsilon}\in X$ that is consistent with $\bar{\varepsilon}=\varepsilon(\bar{\varepsilon},\gamma)$. □

A competitive equilibrium therefore satisfies $\mathbf{u}(\bar{\varepsilon},\bar{\varepsilon},\gamma)=\mathbf{max}_{\varepsilon\in X}\mathbf{u}(\varepsilon,\bar{\varepsilon},\gamma)$, and each chosen action of the government, $\gamma\in Y$, is consistent with a competitive equilibrium. More specifically, for each chosen action of the government, $\gamma\in Y$, let $\bar{\varepsilon}=\xi(\gamma)$ denote the corresponding rational expectations equilibrium. The set of competitive equilibria is therefore defined as $E=\{(\bar{\varepsilon},\gamma)|\ \bar{\varepsilon}=\xi(\gamma)\}$.

The Ramsey plan in this setting is a derived result of the following sequence of actions. First, the government chooses $\gamma\in Y$. Knowing the setting of γ, the households respond with $\bar{\varepsilon}\in X$, such that $\bar{\varepsilon}=\varepsilon(\bar{\varepsilon},\gamma)$. The government is benevolent, i.e. it evaluates its set of policies $\gamma\in Y$ on the basis of welfare maximizing motive. More specifically, the government chooses a particular policy $\gamma\in Y$ that *(a)* maximizes $\mathbf{u}(\varepsilon,\bar{\varepsilon},\gamma)$, and *(b)* is consistent with the government's *correctly foreseen* equilibrium reaction of households, $\bar{\varepsilon}=\xi(\gamma)$. The following two definitions complete the model.

Definition 5.2.1b (Ramsey Problem). The Ramsey problem for the government is $\mathbf{max}_{\gamma\in Y}\ \mathbf{u}[\xi(\gamma),\xi(\gamma),\gamma]$, or equivalently, $\mathbf{max}_{(\bar{\varepsilon},\gamma)\in E}\ \mathbf{u}[\bar{\varepsilon},\bar{\varepsilon},\gamma]$. □

Definition 5.2.1c (Ramsey Plan). Let $\gamma^R \in Y$ denote the policy that attains the maximum of the Ramsey problem. The Ramsey plan is (γ^R,ε^R) where $\varepsilon^R = \xi(\gamma^R)$. □

Consider the problem of a benevolent *dictator*. A benevolent dictator would simply choose a pair of actions that solves the problem $\mathbf{max}_{\varepsilon\in X,\gamma\in Y}\ \mathbf{u}(\varepsilon,\varepsilon,\gamma)$. Any such pair (γ^F,ε^F) is known as the first best outcome. In general such outcomes cannot be reached under a decentralized regime. Consider, however, the outcome where the government's action $\gamma \in Y$ is dictatorial, i.e. without any consideration of the equilibrium feedback. Surely the Ramsey plan $(\gamma^R,\xi(\gamma^R))$ is inferior to the dictatorial outcome (γ^F,ε^F), because the restriction $(\gamma,\varepsilon)\in E$ is in general binding. Moreover, in general $\varepsilon^F \neq \xi(\gamma^F)$, so first best outcomes are not Ramsey plans.

More intuitively, the dictatorial outcome in such a setting would imply that the government *may* attain a first best outcome by choosing $\gamma^F \in Y$, if for any $\varepsilon \in X$, $\varepsilon \le \varepsilon^F$, and the government does not take into account the competitive equilibrium reaction $\bar{\varepsilon}^F = \varepsilon(\bar{\varepsilon}^F,\gamma^F)$. As long as the policy $\gamma^F \in Y$ is plausible, it can be implemented without any consideration of $\bar{\varepsilon}^F = \varepsilon(\bar{\varepsilon}^F,\gamma^F)$. Since preferences are strictly monotone and $\varepsilon \le \varepsilon^F$ for any $\varepsilon \in X$, the policy $\gamma^F \in Y$ does not distort welfare through its effect on households' equilibrium decision, implying that it would attain first best optimality, i.e. $\mathbf{u}(\bar{\varepsilon}^F,\bar{\varepsilon}^F,\gamma^F) \ge \mathbf{u}(\bar{\varepsilon},\bar{\varepsilon},\gamma)$.

On the other hand, if the policy $\gamma^F \in Y$ is not plausible, the government must resort to the Ramsey plan $(\gamma^R,\xi(\gamma^R))$, which solves $\mathbf{max}_{(\bar{\varepsilon},\gamma)\in E}\ \mathbf{u}[\bar{\varepsilon},\bar{\varepsilon},\gamma]$. The corresponding policy $\gamma^R \in Y$ is welfare maximizing, but its effect on the households' competitive equilibrium decisions distorts welfare from the $\mathbf{u}(\bar{\varepsilon}^F,\bar{\varepsilon}^F,\gamma^F)$ margin.

Put in terms of notations, since $\gamma^R \in Y$ induces $\xi(\gamma^R) < \varepsilon^F$, $\mathbf{u}(\bar{\varepsilon}^F, \bar{\varepsilon}^F, \gamma^F) > \mathbf{u}(\bar{\varepsilon}, \bar{\varepsilon}, \gamma^R)$, for $\gamma^F \in Y$. Hence the Ramsey plan $(\gamma^R, \xi(\gamma^R))$ attains second best optimality.

5.2.2 Fairness.

What actually is a *fair* tax policy? Heady (1993, p. 17) wisely asserts that "fairness of a tax policy means different things to different people". Such disagreement prevails amongst the critiques of the optimal taxation theory. The critiques, in general, differ in ideological characterization of fairness of tax policy. This is not surprising, since fairness itself is an obscure feature and its characterization requires adhering to proxy features of tax systems such as progressiveness, equity and compliance. A fair tax system is typically characterized by attaching different weights to horizontal and vertical equity (see for instance, Shapiro (1996) for details), minimization of inequality (see for instance, Shavell (1981) for details), and tax compliance (see for instance, Alm (1996) for details). The current analysis of fairness is limited to the first two concepts, while the issue of tax compliance is deferred to the subsection analyzing administrative costs of taxation.

First of all, there is a trade off between efficiency and equity of a tax system, which perhaps is an inherent feature of any tax policy[38]. The broad objective of Ramsey taxation is minimizing inefficiency of taxes, and from a macroeconomic perspective establishing the importance of equity of a second best policy is somewhat obscure. In the literature involving Ramsey taxation, except for Judd (1985) and Ljungqvist & Sargent (2000, ch.12) who use models with heterogeneous agents and lump sum transfers, the issue of equity is typically simplified by assuming that all taxpayers are identical in tastes and endowment. Renstrom (1999) argues that such simplifications of Ramsey models induce a trade off between efficiency and both vertical and horizontal equity, and greater emphasis of efficiency and abstraction from equity limits its policy practicality. As is clear by now, the trade off argument is somewhat undeniable. The

[38] A first best policy which involves zero taxes on transactions and a lump sum tax, for instance, is the most efficient tax policy since it is associated with minimum disincentive effects. But such a first best policy is the least fair tax policy unless taxpayers are identical and have identical endowments.

issue is therefore the cost of this trade off, or more precisely, the opportunity cost of emphasizing efficiency.

The efficiency-equity trade off debate can be partly resolved if one considers the relative importance of these two rather abstract principles. Consider for instance the issue of vertical equity. A tax policy is fair in terms of vertical equity if the tax burden is consistent across taxpayers of different means. This is the typical focus of the Mirrlees (1971) tradition of optimal taxation[39]. The Mirrlees tradition develops models with heterogeneous agents who differ in endowments, and argues that the resulting optimal non-linear tax rules are fair since they are consistent with vertical equity. This intuition is conceivable, but commodity taxes which vary with the circumstances of the buyer are in general impractical.

The Ramsey rule for commodity taxation is that the efficiency cost minimizing commodity taxes will in general differ by commodity, such that more inelastically demanded goods tend to attract higher tax rates. This result is path-breaking, since it rules out the commonly held perception that uniform commodity taxation is optimal[40]. Ramsey's (1927) intuition was that a uniform tax on all commodities (other than leisure) reduces the relative price of leisure with respect to each commodity, causing an inefficiently large consumption of leisure. The optimal tax pattern should take advantage of commodities' relative substitutability and complementarity with leisure. A complement to leisure should be taxed relatively heavily, and a substitute for leisure should be taxed relatively lightly.

In light of this strong intuition, and with no redistribution considerations, the issue of equity ceases to be of major importance. Moreover, in a dynamic Ramsey tax model where the focus is on average consumption and income taxes, the relative importance

[39] In fact one of the main motivations behind the evolution of the Mirrlees (1971) tradition of optimal taxation was based on the argument that Ramsey taxation oversimplifies equity by assuming identically endowed taxpayers.

[40] The perception that uniform commodity taxation is optimal is quite natural. At a first instinct, it is sensible to assume that the lowest efficiency cost will be achieved with the fewest distortions in relative prices. Since uniform commodity taxation alters none of the relative prices of goods, it is most likely to be the optimal policy. But as Ramsey (1927) shows, uniform commodity taxation is optimal only under very restrictive conditions on preferences (see proposition 1.2.1c in chapter 1 of this thesis for details). Uniform commodity tax principle has not been ruled out in policy reforms; rather it has been revived through the introduction of commodity taxes like VAT or GST. This reform is based on minimizing administrative costs of taxation, which I will focus in some detail later.

of equity is much lower than the broader set of objectives of the model. The concern of efficiency is one of major importance, since Ramsey taxes are designed to weigh advantages of incentive effects more than the advantages of fairness. Too much attention to equity may be associated with allowing for too much inefficiency, resulting in too much distortion in intertemporal allocations. A simple example, although not particularly reminiscent of dynamic Ramsey taxation, makes this proposition clearer.

Consider a hypothetical situation where the taxation authority seeks to raise a given amount of revenue to finance local government expenditure, and has the option to implement a flat community charge (e.g. a poll tax) or a proportionate local income tax. In choosing between the two, most taxpayers would regard the local income tax as fairer. But a local income tax would have a greater disincentive effect on labor supply than the community charge. In order to choose a policy, it is therefore necessary for the authority to weigh the fairness advantage of the local income tax against its disadvantage of discouraging work. The main theme of optimal taxation theory is to create welfare maximizing tax policy which has minimum disincentive effect on allocations. In this sense, the fairness advantage of local income tax is likely to be out-weighed.

An alternative and perhaps a more sensible view of fairness is associated with inequality, which in turns is related to social welfare. In Ramsey tax models, social welfare is seen as an indicator of well being of society and is taken to depend on the utilities of individuals. In its simplest setting, and as in the previous chapters, social welfare is defined by the utility of the measure one of households. Social welfare can also depend on how equally these utilities are distributed as long as agents differ in endowments[41]. In the utilitarian school of thought it is typically assumed that social welfare decreases as inequality increases. In this way, the concept of social welfare captures one idea of fairness of a tax system. Taxes are fair if they reduce the degree of inequality, implying that attempt to maximize welfare will involve an instantaneous attempt to achieve fairness. Given this idea, the social welfare function must place more weight on utility gains of poor people than those of rich people, which is one of

[41] Heady (1993) devotes a section of his paper defining social welfare functions that are typically used in the optimal taxation literature.

the main motivations of the heterogeneous agent redistributive taxation models typically used in the Mirrlees tradition.

This does not imply that it is strictly necessary to follow the Mirrlees methodology to derive such insights. Judd (1985) and Ljungqvist & Sargent (2000, ch. 12) show that a Ramsey model with heterogeneous agents and optimal redistributive taxation can actually recover the key findings of the optimal redistributive taxation that follow the Mirrlees tradition, and that too at a relatively lower cost of methodology.

5.2.3 Administrative Costs.

The most severe criticism of the optimal taxation theory stems from its moderate attention to the details of administrative and compliance costs of taxation. A compelling survey of such criticisms can be found in Alm (1996). Administrative and compliance costs are actually important issues from the perspective of design and implementation of specific taxes. However, they are not so important if one is only concerned about the average level and composition of taxes. In dynamic models of Ramsey taxation that deal specifically with the average level of taxes, minimizing administrative costs will be reflected in social welfare, because higher administrative costs will require a greater amount of gross revenue to be collected that reduces individual utilities. Modelling administration costs formally in Ramsey taxation is therefore not really appealing.

Nevertheless, there have been attempts to include administrative costs formally in optimal taxation models. To my knowledge the seminal attempt is of Yitzhaki's (1979), which presents a simple static model of optimal commodity taxation with administrative costs where the aim of the taxation authority is to minimize the social cost of taxation[42]. According to his specification, the social cost of taxation is the sum of the administrative cost and the deadweight loss caused by the tax system. By

[42] A relatively more recent attempt to model administrative costs in optimal taxation model can be found in Mayshar (1991), which derives the conditions that characterize the optimal use of the tax assuming that there are costs to both the taxpayer and the government from collecting a generic form of taxes.

varying the number of feasible tax rates, Yitzhaki (1979) finds that the relative effect of administrative costs worsens the optimality of the second best policy. This is why one of his conclusions was that if one allows the number of feasible taxes to vary, the optimal taxation problem with cost of collecting taxes ceases to be a problem in the theory of the second best. While his assumption of varying tax instruments is tempting, it is not consistent with Ramsey taxation theory. It is intuitively clear that if one allows the number of taxes to vary to account for variable government expenditures, the second best optimality of Ramsey policy is in question, even without explicit modelling of administrative costs.

Even if one relies on Yitzhaki's (1979) interpretation that administrative costs of taxation is a proportion of the social cost of taxation, there is hardly any reason to model it formally. In standard dynamic Ramsey taxation models, the non-negative Lagrange multiplier associated with the implementability constraint provides a *shadow* measure of the social cost of distorting taxes. This multiplier is typically known as the utility cost of distorting taxes, and its value represents the amount taxpayers are willing pay to replace a unit of distorting tax with a unit of lump sum tax. If one assumes that lump sum taxes are less costly to administer, the utility cost of distorting taxes actually represents a broader measure of administrative costs of taxation. For instance, for high values of this parameter, the social cost of distorting taxes is high, but that of lump sum taxes are low, implying that administering the second best tax policy is relatively more costly.

In the real world, however, administering and collecting taxes is overwhelmingly costly. Interesting evidence of such costs is provided by Slemrod (1990) and Alm (1996). For instance in the US, operating the tax system requires the participation of over 100 million taxpayers, hundreds of thousands of tax professionals, and a multi-billion dollar budget for the Internal Revenue Service and its state subsidiaries. Apart from such direct costs, there are hidden costs of tax compliance, tax evasion and creating the ease of administering taxes. Alm (1996) reports that for the US economy the budget cost of collecting individual income, business income, and sales tax is generally in excess of 1% of the revenues from these taxes. The approximate compliance cost of personal and corporate income taxes for the US economy range from 3% to 7% of their revenue, while for UK and Australia these figures range from 2% to 24% of revenues for selected taxes.

There is little information on how these costs vary with various tax instruments and tax bases. It may be that administrative costs vary in large and discrete amounts with the scale of collections, a hypothesis that is roughly similar to Yitzhaki's (1979) assumption of discontinuous administrative costs for changes in tax base. This is more likely to be the main reason why most parts of optimal taxation literature abstracts from modelling these costs formally. Administrative and compliance costs of taxation do not vary continuously with taxes, but they tend to vary with such things as the number of different rates of tax or the number of tax allowances. This makes them difficult to include in the mathematical analysis of aggregative models, and if somehow incorporated, these additional details are likely to inhibit the smooth tractability of the relatively more important results.

5.2.4 (Dis) incentive Effects.

It is simply impossible for an individual to pay a higher tax bill without reducing consumption, increasing income, reducing savings or increasing borrowing. Tax reforms, such as changing marginal tax rates can affect a number of relative prices, which in turns affect behavioural choice, resource allocation, and real economic activity. In particular, tax-induced relative price changes affect choices between work and leisure, consumption and future consumption, and taxable and non-taxable activity. Optimal taxation theory formalizes these responses to taxation in a manner that is consistent with the specification of utility and intertemporal allocation decisions.

Modelling disincentive effects in a standard optimal taxation framework is likely to be selective, however. This is the standard practice, and there are strong reasons, such as tractability, for doing so. For instance, income taxation can have significant effects on decisions other than labor supply, which may be savings decision, consumption plans and human capital formation (e.g. educational choice). Most standard Ramsey models look at these disincentive effects either in conjunction with labor supply or separately, but to my knowledge there is no model that attempts to combine them all. This is because the imminent complexity associated with such models would be too substantial to yield any useful insights. Does this imply selective modelling of disincentive effects limits the usefulness of Ramsey tax rules in explaining the incentive structure underlying an optimal policy? Surely not, and yet again the intuition stems from characterizing the mapping of aggregate levels of optimal taxes into specific tax instruments.

To explain this intuition, consider first that the Ramsey tradition of optimal taxation assumes an exogenous level of government expenditure and a fixed set of feasible tax instruments. The assumption of a preset (nonzero) revenue target in obviously essential, for otherwise distortion minimizing taxes could just be reduced to zero. This implies that solution to the optimal taxation problem will depend on the size of the revenue requirement, and more importantly, any changes in taxes should be revenue-neutral. Now consider a hypothetical tax reform of a labor income tax cut. A first instinct from microeconomics is demarcating the effects of a tax cut into income and substitution effects. The income effect of the tax cut is that it increases after-tax income which in turns increases the taxpayers' time allocation to leisure in pursuit of enjoying increased consumption. On the other hand, the substitution effect of the tax cut is that marginal return to work becomes high which encourages more work. The net incentive effect of the tax cut, in principle, could go either way, depending on the relative strengths of the income and substitution effects. With revenue-neutral taxation, however, the average taxpayer's income effect is embedded in the loss or gain of welfare through fall or rise in consumption. Only the substitution effect will operate in factor allocations, implying that the tax cut will increase total labor supply.

If the labor income tax is the only tax instrument, modelling disincentive effect is therefore simple. With multiple taxes, being selective in disincentive effects is actually necessary for tractability. To illustrate it further, consider the same example, now with a broader set of taxes that include capital income tax and consumption tax. With revenue-neutral taxation, a labor income tax cut financed by an instantaneous increase in the capital income tax rate, for instance, will induce increased labor supply due to the substitution effect. But in this case the effect on consumption *vis a vis* welfare becomes ambiguous. Higher capital income tax reduces savings, which adversely affects intertemporal consumption decision. A cut in labor income tax at tandem on the other hand provides higher disposable income for consumption. Unless one is able to numerically characterize the welfare effect, it is analytically inconclusive which effect dominates.

Nevertheless, one point is clear from the above discussion --- a tax reform with significant net *disincentive* effects will necessarily be welfare-worsening. Wynne (1997) presents a calibrated version of growth, welfare, and disincentive effects of hypothetical tax reforms in the US economy using a simple endogenous growth model. The calibration, for instance, suggests that halving the labor income tax rate and financing it by an increase in the capital income tax induces a 17% loss of initial consumption (a welfare loss) and slows the economic growth rate from 1.7% to 1.5% . In contrast, a same cut in labor income tax financed by an increased consumption tax boosts economic growth from 1.7% to 2.8%, and increases welfare by increasing initial consumption by a massive 39%. In terms of incentive effects, both policies increase labor supply but by different amounts. The capital income tax increase causes labor supply to increase by 8%, but the consumption tax increase results in a 14% increase in labor supply.

Such results are interesting for policymakers, politicians, and proponents of optimal taxation theory, since they necessarily establish that determining growth and welfare effects of taxes is far from simple, and that growth and welfare effects of taxes depends crucially on the incentive effects of tax reforms. A significant part of Stokey & Rebelo's (1995) paper is devoted to documenting this proposition in a purely technical manner, and like many others I rate their paper to be one of extreme significance in understanding the incentive effects of taxes and how such effects should guide tax reform proposals. According to their findings, growth effects of a particular tax policy is highly sensitive to, among others, elasticity of intertemporal substitution and long run elasticity of labor supply, both of which are closely related to incentive effects of tax reforms through their effect on beliefs about changes in the interest rate. Since interest rate governs intertemporal allocation decisions, a tax reform that affects the interest rate will have long run incentive effects, and hence long run effects in growth and welfare.

5.3 Capital Tax Policy.

Economists in general accept the hypothesis that capital income taxes drive a wedge between pre-tax capital rental rates and the intertemporal marginal rate of substitution between consumption at different dates. The wedge grows at a compounding rate over time which is inconsistent with commodity tax principles (see for instance, Judd (1999 & 2002) and Mulligan (2003) for details). Due to such a strong intuition, the literature (and perhaps policymakers, as OECD tax reform trends suggest) unanimously hold the principle that capital income should receive tax-favoured treatment.

If private markets are imperfectly competitive, this result is stronger. Monopoly pricing in private markets induce a loss in output and drives a wedge between private returns and socially optimal returns to capital and other factors. In this regard, tax policy may use subsidies to bring buyer price down to social marginal cost. The subsidy result cannot be generalized for all transactions, since there is a concern of raising enough revenue to use corrective subsidies. The optimal policy therefore must choose some transactions to tax in order to subsidize other transactions. Since capital income tax induces explosive distortions in intertemporal allocation decisions, and consumption tax and labor income taxes induce uniform distortions, the optimal policy is the one that subsidizes capital income and taxes consumption and labor income.

The next two subsections analyze the policy relevance of these two important results, namely, the zero tax result, and the optimal subsidy result. Optimal taxation of capital income in open economy framework has emerged as an important subdivision of the literature and has established some substantive policy lessons. The particular policy issues addressed in this field of research are of extreme importance. Important contributions in this particular field include Sibert (1985 & 1990), Klien *et al.* (2003), Palomba (2004) and Kim & Kim (2005). The third subsection discusses the policy relevance of their key findings.

5.3.1 The Zero Tax Result.

Perhaps the most celebrated finding of the dynamic optimal taxation literature is that with competitive markets the long run optimal tax on capital income is equal to zero. Judd (1985) and Chamley (1986) are the promoters of this idea, who seminally established that in a standard neoclassical competitive growth model where the commitment power is effective, the Ramsey rule is consistent with a long run zero tax on capital income. A significant part of the current thesis has focused mainly on the implications and justification of this result. With a limiting zero tax, the optimal capital income tax policy may be frontloaded. Put differently, the Ramsey taxation models' prescription in general is that the optimal policy may involve high taxes on initial capital income that raise more than the required revenue, and zero taxes thereafter that avoids explosive distortions.

With competitive settings and demarcated production sectors, chapter two of the current thesis establishes that the steady state optimal policy may involve nonzero tax on capital income from consumption goods sector as long as the tax on capital income from capital goods sector is set at zero. This result is striking --- since on one hand it complements the Chamley-Judd result, but on the other hand proves that the Chamley-Judd result cannot be generalized for neoclassical competitive growth models.

Following the work of Judd (2002 & 2003), consider first a simple example that explains the underlying intuition of the zero tax result. An agent saves some money at time 0 for consumption $(\equiv c)$ at a date $t > 0$. The taxation authority taxes investment income between time 0 and t, which implicitly implies that consumption at time t is taxed. Denote the before-tax interest rate and the tax rate on interest by r and θ, respectively. The social cost of one unit of consumption at time t in units of time 0 good is $(1+r)^{-t}$, and the after tax price is $(1+(1-\theta)r)^{-t}$. The ratio of marginal rate of substitution between the two dated consumption, $MRS(c_0, c_t)$, and the marginal rate of transformation between the two dated consumption, $MRT(c_0, c_t)$, is therefore:

$$\frac{MRS(c_0,c_t)}{MRT(c_0,c_t)}=\left[\frac{1+r}{1+(1-\theta)r}\right]^t \qquad (2)$$

which represents the tax distortions. Now suppose that the time t consumption is taxed at rate τ_c. The tax distortion in *(2)* is equivalent to the tax distortion induced by τ_c, i.e.

$$1+\tau_c=\left[\frac{1+r}{1+(1-\theta)r}\right]^t \qquad (3)$$

Equation *(3)* illustrates that the commodity tax equivalent of interest tax θ is exploding exponentially in time, which is inconsistent with commodity tax principle. If utility is separable across time and between consumption and leisure, and the elasticity of demand for consumption does not change over time, the best tax system would have a constant commodity tax equivalent. Since a nonzero capital income tax violates this principle, it cannot be optimal[43]. This result is robust in settings with heterogeneous agents (see for instance, Judd (1985) for details).

Ever since its induction, this result has had the privilege of being one of the most popular and powerful policy lessons of Ramsey taxation. Analytical robustness of this result and strength of the underlying intuition leave little scope for critics to question the associated policy proposal on any grounds. A tax reform that reduces the average tax rate on capital income is convenient (i.e. administratively less costly), more desirable (i.e. fairer), efficient (i.e. little or no disincentive effects) and politically acceptable (i.e. implementable). In fact this has been the trend in OECD tax reforms over the last decade or more. A long run zero tax on capital income not only rules out explosive tax distortion, but also boosts investment. However, policy relevance of this result is weaker when government's commitment power is less than perfect, or when government policies (and government itself) lack credibility. In a society where the

[43] Limiting the choice of preferences aids analytical tractability but is not binding for this particular result to hold. Judd (1985 & 1999) show that the optimal tax on asset income is zero in the long run even when preferences are far more general than the ones typically used in dynamic models with taxes.

government frequently changes announced policies, a zero tax on capital income in future is not a credible policy and is less likely to boost investment.

With two production sectors producing consumption goods and capital goods, this result cannot be generalized for all income from capital. Chapter two of the current thesis shows that if the government can use sector-specific taxes, the long run optimal policy involves zero tax on capital income from capital goods sector, but nonzero tax on capital income from consumption goods sector. If the government's choice of factor specific tax rates are restricted to be sector-indifferent, optimal tax on capital income is nonzero. Consider first the unrestricted case, where the government is allowed to use sector-specific taxes on capital income. The long run policy of setting nonzero tax on capital income from consumption goods sector is optimal and sustainable, since along the transition to steady state economic agents can shift depreciated capital to the sector for which the government has announced zero long run tax. This shifting allows agents to avoid compounding tax liabilities associated with nonzero capital income tax. Since there is no accumulated old capital stock in consumption goods sector, the distortion created by a nonzero capital income tax is analogous to distortion created by a tax on any period by period transaction. Put differently, due to the transitional shifting of depreciated capital, distortion created by the optimal nonzero capital income tax does not violate the commodity tax principle, implying that the policy is sustainable.

Practicality of sector-specific taxation *may* seem unrealistic at the first instant, because on top of information cost associated with identifying capital income by sector, there is a potential monitoring cost associated with the implementation and enforcement of this policy. On such grounds one may claim that this result has limited scope in positive policy design process. I disagree to such thoughts. Given the structure of the problem, I think a more intuitive track to measure the value of this result is to evaluate its normative implications. From a normative point of view this result is both sound and useful, since the worse outcome of two nonzero capital income taxes is embedded in the test where one restricts government's choice of tax rates to be sector indifferent. Issues of administrative and information costs may be important, but in light of the gains associated with sector-specific taxation such problems can be out-weighed. In attaching relative weights to the efficiency gains from sector-specific taxation and the potential costs associated with it, one should consider the outcome that *could* arise if the second best policy would have been ruled out only on grounds of implementation costs.

Put more elaborately, chapter two of the current thesis suggests that the second best Ramsey policy may actually involve nonzero tax on capital income from consumption goods sector as long as the capital goods sector's capital income tax is zero, and taxing capital income in consumption goods sector is optimal since it creates a uniform pattern of tax distortions. In simple terms, if capital is used to produce capital, the optimal policy is not to tax it, but if capital is used otherwise, the optimal policy is to tax it. Due to any considerations, be it abstract such as political acceptability, or be it *realistic* such as information and administrative cost, both of which may be associated with the proposal of sector-specific taxation, if one restricts the government's choice of tax rates to be sector indifferent, the outcome is one of third best where optimal policy is to tax capital income from both sectors. The effects are chaotic --- since with third best optimality, there is an efficiency loss due to disincentive effects on both investment and consumption. Tax distortions would be compounding, which would induce more than proportionate loss of social welfare. This conclusion holds for a wider class of utility functions. This implies that an overwhelming concern of abstract or secondary principles such as practicality or administrative costs of implementing a policy trades off both its optimality and efficiency.

5.3.2 Optimal Capital Subsidy.

The presence of imperfect competition supports the scheme against capital income taxation, and substantially strengthens the case for moving more towards consumption taxation and away from income taxation. With imperfect competition in private markets, Judd's (1997 & 2002) key finding is that the optimal policy involves corrective capital income subsidy and revenue-raising consumption tax and labor income tax. Judd's (1997 & 2002) findings are scrutinized and further specialized in Guo & Lansing (1999), who establish that the optimal policy may involve capital income tax or subsidy depending on the deviation of equilibrium investment from socially optimal level of investment. The key finding of chapter four of the current thesis is similar, although the underlying intuition and insights are quite different and much stronger.

The intuition behind the optimal capital subsidy result is as follows. With imperfectly competitive markets, optimal tax policy must perform a corrective function in addition to their usual revenue-raising function. The corrective function of taxes can be accomplished by subsidizing some transactions while taxing others. Since mark up on capital goods distort investment just as an asset income tax does, a combination of capital income tax and capital goods mark up create exploding distortions in intertemporal allocations. Hence, the optimal policy should include taxes on consumption and labor income for raising enough revenue to finance both government expenditure and corrective subsidies to capital income. This is the central argument in Judd (1997, 2002), and an important one in Guo & Lansing (1999) and chapter four of the current thesis.

Corrective capital income subsidies are often deemed as a costly alternative of other competition enhancing policy options. Auerbach & Hines Jr. (2001b), for instance, argue that antitrust policy can be more cost effective as a policy for enhancing competition and correcting monopoly induced distortions. Antitrust policy has intrinsic restrictions of application. The presence of monopoly power in pricing may be attributable to many circumstances, one of which is product differentiation. Distortions of such various forms actually limit the general implications of competition enhancing antitrust policy. For instance, if there are fixed costs of production, competition cannot push price down to marginal cost, and having firms specialize in differentiated goods is desirable. Also, extending the models of chapters three and four to include innovation requires a richer analysis. But even then antitrust policy would be of dubious significance since the point of a patent is to give incentives for innovations. With no fixed costs and innovation, Judd (1997) shows that product differentiation induced monopoly distortions in capital goods market are more damaging than those in consumer goods market, implying that antitrust policy should give priority to intermediate goods market.

One of Judd's (1997) conclusions was that capital income subsidies could be paid directly to the investors, or to the firms in the form of investment tax credits or accelerated depreciation schedules. Since equipment markets are more distorted by market power, the capital income subsidy should look similar to the investment tax credit (ITC) for new equipment, which has been occasionally part of the US tax code[44]. This is because equipment makers generally engage in substantial R&D effort and the resulting innovation often exemplifies new technology protected by patents and trade secrecy. Equipment can also be substantially differentiated that enhances market power. A new type of equipment is likely to incorporate the newest technology. It is also likely to be differentiated with respect to both used equipment and to other new equipment. If one rules out the possibility of imitation, the producer of new equipment is better able to set a price over marginal cost. This argument is in favour of the original ITC proposal that involved corrective subsidies only to the purchase of new equipment.

Guo & Lansing (1999) consider depreciation allowance as a means to subsidize capital income. Depreciation allowances in excess of economic depreciation are another form of investment subsidy which is in practice, in a rather *generous* fashion, in both the US and the UK tax codes. For instance in the UK, starting from 1972 the initial allowance received by industrial buildings ranged between 40% and 75%. Inventories received tax relief due to high inflation in the 1970s. According to the US corporate tax structure, physical rents from capital are taxed at a constant rate after the allowance of a deduction for depreciation.

Nevertheless, the practicality or policy relevance of an optimal capital subsidy to offset monopoly distortions is difficult to establish if one considers the imprecise knowledge of mark ups. Calibration of such models, as presented in chapter three and four of this thesis, are typically based on mark up estimates which are available in the

[44] The investment tax credit (ITC) was an on-and-off policy device in the US. It was introduced in 1962, repealed in 1969, reintroduced in 1971, and finally eliminated in 1986. According to this policy firms receive a tax credit proportional to their purchase of new equipment but not structures. The ITC fluctuated between 0% and 10% until 1986 when it was completely eliminated. The US code currently includes a 20% tax credit for qualifying expenditures on research and development activities. In the UK, a similar subsidy to capital investment is paid through corporate grants for the purchase of new capital goods. This was first introduced in 1967. For a rather detailed documentation of this and other relevant policy such as depreciation allowances, and associated policy reforms in the US and the UK, see McGrattan & Prescott (2005).

literature. The results from these calibrations suggest that corrective capital income subsidies, as compared to other options such as antitrust policy, is welfare maximizing and a relatively more effective policy tool in an economy which is experiencing monopoly pricing and monopoly induced distortions. But the implementability and desired outcomes of this policy is subject to collecting perfect measures of mark ups.

5.3.3 Capital Taxation in an Open Economy.

Capital taxation in an open economy has emerged as a distinct and important research theme, mainly due to interdependencies of taxes and their induced cross-border incentive effects. In an integrated world where capital is relatively more mobile across borders than labor, there are a number of channels through which domestic taxation of capital exerts international effects. For instance, domestic taxes affect the international allocation of the existing stock of world capital. These taxes also affect international growth and the process of capital accumulation over time. A country's capital tax reforms can influence the level of savings both at home and abroad, which in turn affects international rate of capital accumulation and economic growth. Moreover, domestic taxation of capital can be associated with different effects on economic growth and welfare. This is because in an open economy tax reforms result in two distinct effects: one on domestic product, and the other on national income. Changes in welfare due to capital tax reforms induce important distributional effects, since such reforms have different effects on welfare of individuals at home and abroad.

Two country models with overlapping generations has been a popular workhorse to investigate this broad range of issues related to international capital taxation. An important contribution to this trend is Sibert (1985), which examines foreign investment taxation as a means to restricting capital mobility in a two country overlapping generations model. The main idea of Sibert (1985) is that since the degree of capital mobility affects gains from trade, incentive effects of capital accumulation, and intergenerational welfare, restricting capital mobility through foreign investment taxation affects all three. More specifically, Sibert (1985) shows that at least one country's welfare is improved by taxation, and since investment taxation adversely affects savings rate, generational preference for a smaller or larger tax on foreign investment depends crucially on generational location which may be a capital-exporting or capital-importing country. The steady state incentive effects across borders also vary considerably for location-specific choice of small or large taxes. For instance, if the home country implements smaller taxes for home investors on the after-tax earnings from exported capital, the long run levels of foreign rental price of capital rises and home rental price of capital falls. Converse happens if the foreign country imposes a smaller tax on the earning of imported capital.

The welfare effects of capital taxation in a large open economy have been examined and analyzed in a number of important papers, and Sibert (1990) is one of them. Extending Sibert's (1990) idea into a two country growth model, Palomba (2004) examines both the welfare and growth effects of international capital taxation. Findings of these papers are interesting, although they deliver much less robust policy prescriptions as compared to what closed economy models generally do. Nevertheless, there is one finding which is common in most of these papers: in an open economy, there is a distinction between the effect of taxes on domestic product and the effect of taxes on residents' claims on that product (national product). For instance, a country can increase domestic productivity and the growth rate of its product by lowering its taxes, but this may lower the level of domestic saving, which in turns reduces the claims of its citizens on future product and their welfare. Moreover, international tax interdependencies pose subtle problems of policy design to national governments. Governments may use taxes on capital income both to compete for the existing stock of world capital and to affect the rate of capital accumulation over time. But a policy that increases the domestic share of current capital may not increase the growth rate of that capital in future.

Furthermore, there are important issues related to cyclical properties of tax reforms, much of which is the main agenda in stochastic versions of two country models. Kim & Kim (2005), for instance, develop an infinite horizon stochastic general equilibrium model of optimal taxation in two countries, and examine the possibility of welfare-improving active, contingent tax policies. They find that the cyclical properties of optimal tax rules can be significantly different in a closed and an open economy setting. More precisely, in a closed economy setting, optimal tax policy is countercyclical in capital income taxes, implying that optimal tax response to an increase in productivity is to increase capital tax rate. However in the open economy setting where capital moves across borders, optimal tax policy becomes procyclical in capital income taxes. The procyclical tax policy generates efficiency gains by correcting market incompleteness.

5.4 Optimal Tax Policy in Developing Countries.

The leap from the doctrines to the real world is a large one when it comes to taxation, and a larger one when it comes to taxation in developing countries. Implementing an optimal tax policy in developing countries is subject to many impediments, some of which have not been highlighted or analyzed so far in this chapter. For instance, there is a predominantly active informal sector in these countries which cannot be taxed by the government. This amounts to incomplete taxation of factors, which in turns is likely to change the standard Ramsey tax principles and composition of revenues[45]. In designing tax policy, developing countries must also consider the margin of deviation from tax system in other countries. This is because with commercial integration there is an issue of designing tax policy that is conducive to foreign investors and expatriate workers. Such integration also raises concerns of raising revenue with much less reliance on foreign trade taxation.

Most workers in developing countries are typically employed in agriculture or in small informal enterprises. According to the International Labor Organization (ILO) 2002 report, on an average more than half of the total workforce of South Asian developing countries is employed in informal sectors. For India, Pakistan, Nepal and Bangladesh, this figure stands at 56%, 65%, 74% and 59%, respectively. The predominance of informal sector employment is also observed in other developing countries, such as 75% in Ethiopia, 72% in Lithuania, 37% in Kenya, 32% in

[45] From a broader perspective, incomplete taxation of factors may be attributable to an industrialized economy as well, where for instance there is an observed tax evasion tendency in the relatively more labor intensive service sector. Having no tax in informal sector may be tantamount to saying that there is complete tax evasion in the service sector.

Mexico, and 43% in Fiji. As workers in the informal sector are seldom paid a regular, fixed wage, their earnings fluctuate. Because of surplus labor, in some cases their marginal wage is zero, or some form of payment which is *off the books*. The base for an income tax in such economies is therefore difficult to calculate. Moreover, workers in these countries generally do not spend their earnings in large stores that keep accurate records of sales and inventories. As a result, modern means of raising revenue, such as income taxes and consumer taxes, play a rather vague role in these economies.

Informal production sector has been formally modelled in the recent optimal taxation literature, albeit with simple technology and very selective focus on its consequences in fiscal policy. Penalosa & Turnovsky (2004) develop a two-sector model of Ramsey taxation where they assume that economic activities in one sector are informal, i.e. non-taxable by the government. Their model is in the spirit of Jones *et al.* (1993), but due to private factor allocation in informal sector their main attention is on optimal incomplete taxation. The main motivation of Penalosa & Turnovsky (2004) is to establish the Ramsey tax principles for developing countries. Saying that their simple model provides some very useful insights in pursuit of a rather obscure policy design problem will not be an overstatement. For instance, one of their findings is that the optimal capital income tax in such a setting is nonzero irrespective of how the revenues are used. Moreover, the welfare maximizing labor income tax and capital income tax rates depend crucially on how the government uses the tax revenue, which may be simple redistribution, or investment in infrastructure.

Implementing an optimal tax policy in developing countries is also subject to problems related to tax administration, some of which are of peculiar nature. It is difficult to create an efficient tax administration without a well educated and well trained staff, when money is lacking to pay good wages to tax officials and to computerize the operation, and when taxpayers have limited ability to keep accounts. There are concerns of corruption in tax administration and tax collection, a high tendency of tax evasion, and strong and influential corporate lobbies which, through campaign contribution, almost determine the policy to be implemented. Moreover, because of the informal structure of the economy and financial limitations, in many developing countries statistical and tax offices have inflexibility in generating and documenting reliable statistics. This lack of data prevents policymakers from assessing the potential impact of major changes to the tax system. As a consequence, marginal changes are often preferred over major structural changes, even when the latter are clearly preferable.

A relatively more globally integrated developing country faces, on top of what has been discussed so far, another subset of problems in designing tax policy. The world price of an imported capital good is the social cost of capital for a small developing country. In such a case the optimal capital subsidy result does not apply, since the country should not subsidize imported capital goods as long as its internal price equals the world price. This implies that policies like investment tax credit have little scope in neutralizing monopoly distortions. This situation is further complicated if there are foreigners who own a domestic firm in a developing country which produces a monopolized capital service. Since the rent goes to the foreigners, the true social cost to the developing country is the monopoly price, which cannot justify the optimal capital subsidy principle.

Finally, income tends to be disproportionately distributed within developing countries. Although raising high tax revenues in this situation ideally calls for the rich to be taxed more heavily than the poor, this is rarely reflected in their fiscal policy designs and reforms. The economic and political power of rich taxpayers often allows them to prevent fiscal reforms that would increase their own tax burdens. This problem is analogous to the influential corporate lobbying problem. This explains, albeit in part, why many developing countries have not fully exploited personal income and property taxes, and why their tax systems rarely achieve reasonable progressiveness.

5.5 Concluding Remarks.

This chapter has attempted to establish the policy relevance of optimal tax rules drawn from the Ramsey tradition. To accomplish its objective, the chapter has analyzed the technical importance of Ramsey tax rules in practical policy design, and has argued in favour of Ramsey tax rules by refuting its common criticisms. The main finding of this chapter is that the Ramsey tax rules that summarize the optimal average taxes are important and relevant in designing fiscal policy, but they serve moderately to provide guidelines and insights into specific design of taxes. Criticisms of Ramsey tax rules on grounds of practicality is irrelevant, since the correct interpretation of Ramsey tax rules is linked to average levels and compositions of taxes rather than specific design of taxes. A Ramsey tax rule illustrates the macroeconomic tax rate on a taxable transaction which in turn reflects the optimal proportion of that particular transaction to be taxed. Given this formulation, and given the welfare maximizing objective, Ramsey tax rules are the normative benchmarks which are not subject to criticism from positive policy design perspective. A Ramsey tax rule can be practically implemented with a combination of different tax instruments, and finding the right combination of specific tax instruments is not a problem of the underlying Ramsey tax model.

The chapter attaches a relatively high weight on analyzing the policy relevance of Ramsey rule for capital income taxation. The underlying intuitions of the two most popular results (the optimal zero tax result and the optimal subsidy result) are strong enough for their widespread acceptability. Abstraction from a detailed analysis of labor income tax rules' policy relevance in a way limits the scope of this analysis. The Ramsey rule for labor income taxation is simple, which states that optimal labor income taxes with competitive markets should be smooth and roughly constant. This result stands robust for standard Ramsey tax models, and proponents of flat rate taxes also support this rule on grounds of intra and intertemporal smoothing of consumption and labor allocation decisions. With imperfect competition, the optimal levels of labor income tax rates are lower than what it would have been under perfectly competitive markets. This result reflects the corrective functions labor income taxes perform in presence of monopoly power in pricing and/or wage setting.

The optimal policy for labor income taxation is much debated on its progressivism, an issue which is much better handled in the Mirrlees tradition of optimal taxation. The degree of abstraction often embedded in Mirrlees tradition's models of optimal nonlinear taxation, however, has limited the policy relevance of their results. For instance, one often cited result is that the marginal tax rate on the highest income person, who presumably has the highest ability, is zero. The intuition behind this result is that a nonzero marginal tax rate distorts the labor supply of the highest ability person. If this tax rate were changed to zero, the highest ability person might work more, which would make that person better off. However, government revenue would not change, because with a positive tax rate this labor is not provided, and with a zero tax rate the extra labor supply is not taxed. The logic of this argument applies only at the top of the income distribution, because changes in marginal tax rates below this level affect the taxes paid by people with higher incomes. Unfortunately, this result does not give any information about how high marginal tax rates should be just below the top of the income distribution. Also, from a practical standpoint, it is almost impossible to determine the top of the ability or income distribution.

Abstract issues like fairness and more generic issues like administrative and compliance costs are typically embedded in the social welfare function and the welfare maximizing process of a Ramsey tax model. Such simplifications aid analytical tractability of the policies that are welfare maximizing and distortion minimizing. The efficiency-equity trade off argument, which stands more or less robust against any tax policy, is a relatively less important issue for Ramsey tax rules. This is because the equity issue is weighed away by emphasizing the minimization of disincentive effects of taxes. Put differently, since Ramsey tax rules weighs efficiency of taxes more than fairness of taxes, they are in principle associated with minimum disincentive effects. But Ramsey tax rules have very little scope in fiscal policy design process of developing countries, largely due to a set of non-conventional factors that inhibit the scope of optimal policy analysis.

Bibliography

[1] Abel, A. B. (2005). 'Optimal Taxation when Consumers Have Endogenous Benchmark Levels of Consumption', *The Review of Economic Studies,* Vol. 72, pp. 21-42.

[2] Abreu, D. (1988). 'On the Theory of Infinitely Repeated Games with Discounting', *Econometrica*, Vol. 56, pp. 383-396.

[3] Aiyagari, R. (1995). 'Optimal Capital Income Taxation with Incomplete Markets, Borrowing Constraints, and Constant Discounting', *The Journal of Political Economy,* Vol. 103, pp. 1158-1175.

[4] Alm, J. (1996). 'What is an "Optimal" Tax System?', *National Tax Journal,* Vol. 49, pp. 117-133.

[5] Alvarez, F., Kehoe, P. & Neumeyer, P. A. (2004). 'The Time Consistency of Optimal Monetary and Fiscal Policies', *Econometrica,* Vol. 72, pp. 541-567.

[6] Aronsson, T. & Sjogren, T. (2001). 'Optimal Taxation and Provision of Public Goods in a Unionized Economy', University of Umea, *Umea Economic Studies 560.*

[7] Atkeson, A., Chari, V. V. & Kehoe, P. (1999). 'Taxing Capital Income: A Bad Idea', Federal Reserve Bank of Minneapolis *Quarterly Review*, Vol. 23, pp. 3-17.

[8] Atkinson, A. B. & Stiglitz, J. E. (1980). *Lectures on Public Economics,* McGraw-Hill, New York, 1980.

[9] Auerbach, A. J. & Hines, J. R. Jr. (2001a). 'Perfect Taxation with Imperfect Competition', *NBER Working Paper, No. 8138.*

[10] Auerbach, A. J. & Hines, J. R. Jr. (2001b). 'Taxation and Economic Efficiency', *NBER Working Paper, No. 8181.*

[11] Barreto, R. A. & Alm, J. (2003). 'Corruption, Optimal Taxation and Growth', *Public Finance Review*, Vol. 31, No. X.

[12] Basu, S. & Fernald, J. G. (1997). 'Returns to Scale in U.S. Production: Estimates and Implications', *The Journal of Political Economy*, Vol. 105, pp. 249-283.

[13] Bayoumi, T., Laxton, D., & Pesenti, P. (2004). 'Benefits and Spillovers of Greater Competition in Europe: A Macroeconomic Assessment', *NBER Working Paper, No. 10416.*

[14] Benassy, J. P. (2002). *The Macroeconomics of Imperfect Competition and Non Clearing Markets: A Dynamic General Equilibrium Approach*, MIT Press, USA, 2002.

[15] Benhabib, J. & Farmer, R. E. A. (1994). 'Indeterminacy and Increasing Returns', *Journal of Economic Theory*, Vol. 63, pp. 19-41.

[16] Benhabib, J. & Rustichini, A. (1997). 'Optimal Taxes Without Commitment', *Journal of Economic Theory*, Vol. 77, pp. 231-259.

[17] Carey, D. & Tchilinguirian, H. (2000). 'Average Effective Tax Rates on Capital, Labor and Consumption', *OECD Economics Department Working Papers No. 258.*

[18] Cass, D. (1965). 'Optimum Growth in an Aggregative Model of Capital Accumulation', *The Review of Economic Studies*, Vol. 32, pp. 233-240.

[19] Cassou, S. P. & Lansing, K. J. (1998). 'Optimal Fiscal Policy, Public Capital, and the Productivity Slowdown', *Journal of Economic Dynamics and Control,* Vol. 22, pp. 911-935.

[20] Chamley, C. (1986). 'Optimal Taxation of Capital Income in general Equilibrium with Infinite Lives', *Econometrica*, Vol. 54, pp. 607-622.

[21] Chari, V. V. & Kehoe, P. J. (1990). 'Sustainable Plans', *The Journal of Political Economy,* Vol. 98, pp. 783-802.

[22] Chari, V. V. & Kehoe, P. J. (1999). 'Optimal Fiscal and Monetary Policy', in J. B. Taylor and M. Woodford, eds., *Handbook of Macroeconomics, Vol. 1.* Amsterdam: North-Holland, 1999, pp.1671-1745.

[23] Chari, V. V., Christiano, L. J. & Kehoe, P. J. (1994). 'Optimal Fiscal Policy in a Business Cycle Model', *The Journal of Political Economy*, Vol. 102, pp. 617-652.

[24] Coleman, II, W. J. (2000). 'Welfare and Optimum Dynamic Taxation of Consumption and Income', *Journal of Public Economics*, Vol. 76, pp. 1-39.

[25] Cooley, T. F. & Prescott, E. C. (1995). 'Economic Growth and Business Cycles', in T. F. Cooley and E. C. Prescott, eds., *Frontiers of Business Cycle Research,* Princeton University Press.

[26] Correia, I. H. (1996). 'Should Capital Income be Taxed in the Steady State?', *Journal of Public Economics*, Vol. 60, pp. 147-151.

[27] Correia, I. H., Nicolini, J. P. & Teles, P. (2002). 'Optimal Fiscal and Monetary Policy: Equivalence Results', *CEPR Discussion Papers 3730.*

[28] Diamond, P. & Mirrlees, J. (1971). 'Optimal Taxation and Public Production, II: Tax Rules', *The American Economic Review*, Vol. 61, pp. 261-278.

[29] Diamond, P. (1998). 'Optimal Income Taxation: An Example with a U-shaped Pattern of Optimal Marginal Tax Rates', *The American Economic Review*, Vol. 88, pp. 83-95.

[30] Dixit, A. K. & Stiglitz, J. (1977). 'Monopolistic Competition and Optimum Product Diversity', *The American Economic Review*, Vol. 67, pp. 297-308.

[31] Drazen, A. (2000). *Political Economy in Macroeconomics*, Princeton: Princeton University Press, 2000.

[32] Erosa, A. & Gervais, M. (2001). 'Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review', Federal Reserve Bank of Richmond *Economic Quarterly*, Vol. 87/2, pp. 23-44.

[33] Fischer, S. (1980). 'Dynamic Inconsistency, Cooperation, and the Benevolent Dissembling Government', *Journal of Economic Dynamics and Control*, Vol. 2, pp. 93-107.

[34] Golosov, M., Kocherlakota, N. & Tsyvinski, A. (2003). 'Optimal Indirect and Capital Taxation', *The Review of Economic Studies*, Vol. 70, pp 569-587.

[35] Guo, J-T. & Lansing, K. J. (1999). 'Optimal Taxation of Capital Income with Imperfectly Competitive Product Markets', *Journal of Economic Dynamics and Control,* Vol. 23, pp. 967-995.

[36] Ha, J. & Sibert, A. C. (1997). 'Strategic Capital Taxation in Large Open Economies with Mobile Capital', *International Tax and Public Finance*, Vol. 39, pp. 243-262.

[37] Hansen, G. D. (1985). 'Indivisible Labor and the Business Cycle', *Journal of Monetary Economics*, Vol. 16, pp. 309-328.

[38] Hassler, J., Krusell, P., Storesletten, K. & Zilibotti, F. (2004). 'On the Optimal Timing of Capital Taxes', University of Rochester, *Working paper*.

[39] Heady, C. (1993). 'Optimal Taxation as a Guide to Tax Policy: A Survey', *Fiscal Studies*, Vol. 14, pp. 15-41.

[40] Herrendorf, B. & Valentinyi, A. (2003). 'Determinacy Through Intertemporal Capital Adjustment Costs', *Review of Economic Dynamics*, Vol. 6, pp. 483-497.

[41] Huffman, G. W. & Wynne, M. A. (1999). 'The Role of Intratemporal Adjustment Costs in a Multisector Economy', *Journal of Monetary Economics*, Vol. 43, pp. 317-350.

[42] Jones, L. E. & Manuelli, R. E. (1992). 'Finite Lifetimes and Growth', *Journal of Economic Theory*, Vol. 58, pp. 171-197.

[43] Jones, L. E., Manuelli, R. E. & Rossi, P. E. (1993). 'Optimal Taxation in Models of Endogenous Growth', *The Journal of Political Economy*, Vol. 101, pp. 485-517.

[44] Jones, L. E., Manuelli, R. E. & Rossi, P. E. (1997). 'On the Optimal Taxation of Capital Income', *Journal of Economic Theory*, Vol. 73, pp. 93-117.

[45] Jonsson, M. (2004). 'The Welfare Cost of Imperfect Competition and Distortionary Taxation', *Sveriges Riksbank Working Paper Series 170*.

[46] Judd, K. L. (1985). 'Redistributive Taxation in a Simple Perfect Foresight Model', *Journal of Public Economics*, Vol. 28, pp. 59-83.

[47] Judd, K. L. (1999). 'Optimal Taxation and Spending in General Competitive Growth Models', *Journal of Public Economics*, Vol. 71, pp. 1-26.

[48] Judd, K. L. (2002). 'Capital Income Taxation with Imperfect Competition', *The American Economic Review*, Vol. 92, pp. 417-421.

[49] Judd, K. L. (2003). 'The Optimal Tax Rate for Capital Income is Negative', *NBER Working Paper W6004 (1997)*, Hoover Institution Version 2003.

[50] Karp, L. and Lee, I. H. (2000). 'Time-Consistent Policies', *Discussion Paper in Economics*, University of Southampton.

[51] Kim, J. & Kim, S. H. (2004). 'Welfare Effects of Tax Policy in Open Economies: Stabilization and Cooperation', *Working paper in Economics*, Department of Economics, Tufts University.

[52] Klein, P., Quadrini, V. and Rios-Rull, J-V. (2003). Optimal and Time-consistent fiscal policy with international mobility of capital: Why does the US tax capital more than Europe?, *mimeo*, University of Pennsylvania.

[53] Koopmans, T. C. (1965). 'On the Concept of Economic Growth', in: *The Econometric Approach to Development Planning*, RAND McNally, Chicago, IL.

[54] Koskela, E. & Thadden, L. (2002). 'Optimal Factor Taxation Under Wage Bargaining - A Dynamic Perspective', *Discussion Paper 31/02, Economic Research Centre of the Deutsche Bundesbank.*

[55] Kotlikoff, L. J. (1998). 'The A-K Model: Its Past, Present and Future', *NBER Working Paper, No. 6684.*

[56] Kydland, F. E. & Prescott, E. C. (1977). 'Rules Rather than Discretion: The Inconsistency of Optimal Plans', *The Journal of Political Economy*, Vol. 85, pp. 473-491.

[57] Kydland, F. E. & Prescott, E. C. (1982). 'Time to Build and Aggregate Fluctutations', *Econometrica*, Vol. 50, pp. 1345-1370.

[58] Ljungqvist, L. & Sargent, T. J. (2000). *Recursive Macroeconomic Theory.* London: The MIT Press, 2000.

[59] Lucas, R. E. Jr., & Prescott, E. (1971). 'Investment Under Uncertainty', *Econometrica,* Vol. 39, pp. 659-681.

[60] Lucas, R. E. Jr., & Stokey, N. (1983). 'Optimal Fiscal and Monetary Policy in an Economy without Capital', *Journal of Monetary Economics*, Vol. 12, pp. 55-93.

[61] Martins, J. O., Scarpetta, S. & Pilat, D. (1996). 'Mark-up Pricing, Market Structure, and the Business Cycle', *OECD Economic Studies, No. 27*, 1996/11.

[62] Mayshar, J. (1991). 'Taxation with Costly Administration', *Scandinavian Journal of Economics*, Vol. 93, pp. 75-88.

[63] McGrattan, E. R. & Prescott, E. C. (2005). 'Taxes, Regulations, and the Value of U.S. and U.K. Corporations', *The Review of Economic Studies*, Vol. 72, pp. 767-796.

[64] Mendoza, E. G., Razin, A. & Tesar, L. L. (1994). 'Effective Tax Rates in Macroeconomics: Cross-country Estimates of Tax Rates on Factor Incomes and Consumption', *NBER Working Paper, No. 4864.*

[65] Mirrlees, J. A. (1971). 'An Exploration in the Theory of Optimal Income Taxation', *The Review of Economic Studies*, Vol. 38, pp. 175-208.

[66] Mirrlees, J. A. (1982). 'Migration and Optimal Income Taxes', *Journal of Public Economics*, Vol. 18, pp 319-341.

[67] Mulligan, C. B. (2003). 'Capital Tax Incidence: Fisherian Impression from the Time Series', *NBER Working Paper, No. 9916.*

[68] Myles, G. D. (1989). 'Ramsey Tax Rules for Economies with Imperfect Competition', *Journal of Public Economics*, Vol. 38, pp. 95-115.

[69] Ortigueira, S. (1998). 'Fiscal Policy in an Endogenous Growth Model with Human Capital Accumulation', *Journal of Monetary Economics*, Vol. 42, pp. 323-355.

[70] Ortigueira, S. (2003). 'Markovian Optimal Taxation: The Macroeconomic Implications of Two Notions of Time Consistency', European University Institute, *Working paper.*

[71] Palomba, G. (2004). 'Capital Income Taxation and Economic Growth in Open Economies', *IMF Working Paper, WP/04/91.*

[72] Penalosa, C. G. & Turnovsky, S. J. (2004). 'Second Best Optimal Taxation of Capital and Labor in a Developing Economy', University of Washington, *Working paper.*

[73] Persson, M., Persson, T. & Svensson, L. E. (1987). 'Time Consistency of Fiscal and Monetary Policy', *Econometrica*, Vol. 5, 1419-1431.

[74] Persson, T. & Tabellini, G. (2002). *Political Economics: Explaining Economic Policy*, Cambridge: The MIT Press, 2002.

[75] Phelan, C. & Stacchetti, E. (2001). 'Sequential Equilibria in a Ramsey Tax Model', *Econometrica*, Vol. 69, pp. 1491-1518.

[76] Ramsey, F. P. (1927). 'A Contribution to the Theory of Taxation', *The Economic Journal*, Vol. 37, pp. 47-61.

[77] Rebelo, S. (1991). 'Long-Run Policy Analysis and Long-Run Growth', *The Journal of Political Economy*, Vol. 99, pp. 500-521.

[78] Renstrom, T. I. (1999). 'Optimal Dynamic Taxation' in S. B. Dahiya, eds., *The Current State of Economic Science,* Spellbound Publications, 1999.

[79] Salanie, B. (2002). *The Economics of Taxation*, London, The MIT Press, Cambridge, 2002.

[80] Samuelson, P. (1986). 'Theory of Optimal Taxation', *Journal of Public Economics*, Vol. 30, pp. 137-143.

[81] Sandmo, A. (1976). 'Optimal Taxation: An Introduction to the Literature', *Journal of Public Economics*, Vol. 6, pp. 37-54.

[82] Schmitt-Grohe, S. & Uribe, M. (2004a). 'Optimal Fiscal and Monetary Policy under Imperfect Competition', *Journal of Macroeconomics*, Vol. 26, pp. 183-209.

[83] Schmitt-Grohe, S. & Uribe, M. (2004b). 'Optimal Fiscal and Monetary Policy under Sticky Prices', *Journal of Economic Theory*, Vol. 114, pp. 198-230.

[84] Shapiro, R. J. (1996). 'Why Fairness Matters: Progressive Versus Flat Taxes', in *Taxes & The New Economy,* Progressive Foundation, 1996.

[85] Shavell, S. (1981). 'A Note of Efficiency vs. Distributional Equity in Legals Rulemaking: Should Distributional Equity Matter Given Optimal Income Taxation?', *The American Economic Review*, Vol. 71, pp. 414-418.

[86] Sibert, A. C. (1985). 'Capital Accumulation and Foreign Investment Taxation', *The Review of Economic Studies*, Vol. 52, pp. 331-345.

[87] Sibert, A. C. (1990). 'Taxing Capital in a Large, Open Economy', *Journal of Public Economics*, Vol. 41, pp. 297-317.

[88] Sleet, C. (2004). 'Optimal Taxation with Private Government Information', *The Review of Economic Studies,* Vol. 71, pp. 1217-1239.

[89] Slemrod, J. (1990). 'Optimal Taxation and Optimal Tax Systems', *The Journal of Economic Perspectives*, Vol. 4, pp. 157-178.

[90] Solow, R. M. (1998). *Monopolistic Competition and Macroeconomic Theory,* Federico Caffe Lectures, Cambridge: Cambridge University Press, 1998.

[91] Srinivasan, T. N. (1964). 'Optimal Savings in a Two-sector Model of Growth', *Econometrica*, Vol. 32, pp. 358-373.

[92] Stiglitz, J. E. & Dasgupta, P. (1971). 'Differential Taxation, Public Goods, and Economic Efficieny', *The Review of Economic Studies*, Vol. 38, pp. 151-174.

[93] Stokey, N. L. & Rebelo, S. (1995). 'Growth Effects of Flat-Rate Taxes', *The Journal of Political Economy*, Vol. 103, pp. 519-550.

[94] Stokey, N. L. (1991). 'Credible Public Policy', *Journal of Economic Dynamics and Control*, Vol. 15, pp. 627-656.

[95] Turnovsky, S. (2004). 'The Transitional Dynamics of Fiscal Policy: Long-run Capital Accumulation and Growth', *Journal of Money, Credit, and Banking*, Vol. 36, pp. 883-910.

[96] Uzawa, H. (1963). 'On a Two-sector Model of Economic Growth II', *The Review of Economic Studies*, Vol. 30, pp. 105-118.

[97] Uzawa, H. (1964). 'Optimal Growth in a Two-sector Model of Capital Accumulation', *The Review of Economic Studies*, Vol. 31, pp. 1-24.

[98] Wynne, M. A. (1997). 'Taxation, Growth and Welfare: A Framework for Analysis and Some Preliminary Results', *Economic Review*, First Quarter, pp. 1-13, Federal Reserve Bank of Dallas.

[99] Yakita, A. (2003). 'Taxation and Growth with Overlapping Generations', *Journal of Public Economics,* Vol. 87, pp. 467-487.

[100] Yitzhaki, S. (1979). 'A Note on Optimal Taxation and Administrative Costs', *The American Economic Review*, Vol. 69, pp. 475-480.