UNIVERSITY OF SOUTHAMPTON

# Small Yukawa Couplings in Particle Physics and Cosmology from Type I String Theory

by

Oliver John Eyton-Williams

A thesis submitted for the degree of

Doctor of Philosophy

School of Physics and Astronomy

April 2006

*To Nicola, obviously.*

This thesis was submitted for examination in 2006. It does not necessarily represent the final form of the thesis as deposited in the University after examination.

UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF SCIENCE

SCHOOL OF PHYSICS AND ASTRONOMY

Doctor of Philosophy

# SMALL YUKAWA COUPLINGS IN PARTICLE PHYSICS AND COSMOLOGY FROM TYPE I STRING THEORY

Oliver John Eyton-Williams

Within the framework of Type I string theory we construct phenomenologically interesting models. The string theory is formulated in a 10 dimensional space of which 6 are compact and the anisotropies in the compact dimensions lead to a hierarchy of gauge and Yukawa couplings. We make use of this hierarchy to construct a model of inflationary particle physics and a consistent model for Dirac neutrino masses. The inflation model solves the strong CP and $\mu$ problems of the MSSM and predicts a range of allowed ratios for $\mu$ and the soft masses for the Higgs doublets. We demonstrate that it is possible to obtain Dirac masses in agreement with current experimental data.

# Contents

# List of Figures

# List of Tables

# Preface

The work described in this thesis was carried out in collaboration with Prof. S. F. King and Dr S. Antusch. The following list details of and references for our original work.

- Chapter 3 - O. J. Eyton-Williams and S. F. King *Phys. Lett.* **B610** (2005) 87 hep-ph/0411170
- Chapter 4 - O. J. Eyton-Williams and S. F. King, JHEP **0506** (2005) 040 [arXiv:hep-ph/0502156].
- Chapter 5 - S. Antusch, O. J. Eyton-Williams and S. F. King, JHEP **0508** (2005) 103 [arXiv:hep-ph/0505140].

The majority of the original work in this thesis is to be found in chapter 3 and all following chapters. The material in chapters 1 and 2 was collected from a number of different sources and the original presentations are referenced therein.

# Acknowledgements

I would like to take this opportunity to thank all the people who have made my life better/easier/more productive for the duration of my studies. In no particular order:

My officemates. Most obviously for their friendship, but also the subsidiary details of sharing of cartoons, games, blathering about nothing, blathering about physics, the list goes on. So, thanks Jon (L, M and B), Martin and Martin, Phil, Iain, Tom and Olly. It's been fun.

Steve, my supervisor for providing me with enough encouragement to keep me going and for having the determination one needs when building models. My collaborator, Stefan, for his insightful input and inhuman tolerance of stupid questions.

My family for supporting me when things go slightly awry. That and dragging me to family functions.

And Nicola for being unreasonably lovely.

# Chapter 1

# Introduction

## 1.1 Thesis Structure

The main body of the thesis is organised as follows. The current chapter consists solely of reviews of known physics. First we discuss one of the most well tested physical theories: the Standard Model (SM) of particle physics. After discussing some of the successes and shortcomings of this model we introduce low energy supersymmetry (SUSY), first covering the basics of the formalism, then moving on to the well studied Minimal Supersymmetric Standard Model (MSSM). The Next to Minimal Supersymmetric Standard Model (NMSSM) is also discussed as a mechanism for generating the supersymmetric Higgs mass $\mu$ of the MSSM. After this we consider the implications of making supersymmetry local, namely we discuss supergravity (SUGRA) and how this provides a useful framework for breaking supersymmetry. We then discuss neutrino physics in the SM and beyond. Our penultimate review is a brief review of the salient points of supersymmetric string theory, focusing on the elements that will prove important in the main body of the thesis. Finally we introduce inflationary cosmology paying particular attention to models of hybrid inflation.

In chapter 2 we expand on the parts of string theory most relevant to our model building efforts, namely the low energy effective superpotential of Type I strings in the presence of intersecting stacks of D-branes. In addition we discuss the soft spectrum expected in this class of models and how the geometry of the underlying space is of great importance to the models we later build. Specifically we show that it is possible to obtain very small Yukawa couplings without invoking particularly small extra dimensions.

Chapter 3 is concerned with a field theory model of hybrid inflation. The requirements of inflation are imposed and the phenomenology of the model investigated. A prediction for the ratio of soft and supersymmetric masses for the Higgs bosons will be obtained, coming directly from the inflationary requirements. From the field theory perspective a number of seemingly arbitrary assumptions are required for the model to work. These assumptions are justified in chapter 4, making use of the framework laid out in chapter 2 to demonstrate that it is possible to build the model of hybrid inflation in this framework. The small Yukawa couplings obtained in chapter 2 will be put to work connecting the Peccei-Quinn and electroweak (EW) scales. It will be demonstrated how the soft spectrum can be made to accommodate our inflation model.

In chapter 5 we provide another application of the string framework. In this case the small Yukawa coupling allows for the generation of Dirac mass matrices. This requires consideration of non-renormalisable operators, which was not the case for the inflation model, and they are generated using the supersymmetric generalisation of the Froggatt-Nielsen (FN) mechanism.

The thesis is rounded off by a general discussion of the findings in chapter 6. This is followed by appendices A and B and concludes with the bibliography.

## 1.2 Underlying Physics

The rest of this chapter is set aside for discussions of the physical theories that underpin the model building efforts that appear in chapters 3 to 5. Due to the range of different theories that have relevance to the model building it is impossible to do justice to all, or indeed any of the subjects. Instead we attempt to provide sufficient information for the reader to better understand the following chapters and do not attempt to make these discussions self-contained. It should also again be stressed that the author makes no claim as to the originality of the work presented in this or the succeeding chapter. Finally to keep the bibliography under control we generally only cite reviews, texts and illustrative examples intending no slight to the original authors in the first two chapters of this thesis. With these disclaimers in place we start our discussion with the Standard Model of particle physics.

## 1.3 Standard Model

For textbook treatments of the SM see for example [1, 2] The SM is a renormalisable field theory containing fields that transform under the (spontaneously broken) gauge symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$ and can be thought of as the union of electroweak theory [3, 4] and quantum chromodynamics [5, 6]. To completely define the model one must write down the Lagrangian for all of the fields. Strictly speaking, if all the indices, gamma matrices, couplings and group generators are included in full this completely specifies the model. Nonetheless it is vastly more convenient and comprehensible if the fields are grouped into representations of the symmetries governing the SM. Firstly and most fundamentally the fields can be classified by their representation under the Lorentz group, of which two representations, spin-1/2 (quarks and leptons) and spin-1 (gauge bosons) have been observed in nature. It has long been expected that spin-0 fields play a crucial role in breaking the gauge symmetry of the SM from $SU(3)_c \times SU(2)_L \times U(1)_Y \to SU(3)_c \times U(1)_{EM}$, this role being played by the famous Higgs boson. However, while it is to be expected that the Higgs boson will appear at the Large Hadron Collider (LHC), until that moment the existence of the Higgs can only be inferred. As yet we have not properly defined what we mean by mean by quarks, leptons and so on, to do so we must require that they transform under representations of the gauge symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y$. In other words a field can be thought of as a vector in the space upon which the representation matrices act. This allows us to specify the standard model fields by their gauge and Lorentz transformation properties, summarised in table 1.1.

| Field | Spin | $SU(3)_c$ | $SU(2)_L$ | $U(1)_Y$ |
|---|---|---|---|---|
| Higgs boson, $H \equiv (H^+, H_0)$ | 0 | **1** | **2** | 1/2 |
| Left-handed quarks, $Q_i \equiv (u_{Li}, d_{Li})$ | 1/2 | **3** | **2** | 1/6 |
| Right-handed up quarks, $u_{Ri}$ | 1/2 | **3** | **1** | 2/3 |
| Right-handed down quarks, $d_{Ri}$ | 1/2 | **3** | **1** | −1/3 |
| Left-handed leptons, $L_i \equiv (\nu_{Li}, e_{Li})$ | 1/2 | **1** | **2** | −1/2 |
| Right-handed electrons, $e_{Ri}$ | 1/2 | **1** | **1** | −1 |
| Gluons, $g^\alpha$, $(\alpha = 1-8)$ | 1 | **8** | **1** | 0 |
| Weak bosons, $A^a$, $(a = 1-3)$ | 1 | **1** | **3** | 0 |
| Hypercharge boson, $B$ | 1 | **1** | **1** | 0 |

Table 1.1: Gauge and Lorentz representations of the Standard Model fields. The $SU(2)_L$ doublets are decomposed into their representations under $U(1)_{EM}$ and the index $i$ labels generations.

The fields in different generations are distinguished with either the index $i = 1, 2, 3$ or as follows: $u_i = (u, c, t)$, $d_i = (d, s, b)$, $\nu_i = (\nu_e, \nu_\mu, \nu_\tau)$ and $e_i = (e, \mu, \tau)$. To be more precise the notation with the numerical index should be reserved for the weak eigenstate basis and the letters for the mass eigenstates, however we will use these two interchangeably. The choice of $H$ to represent the Higgs field, instead of the traditional $\phi$, is to avoid any confusion with the inflaton, which is denoted $\phi$.

The most important consequence of table 1.1 is that gauge invariant fermion mass terms cannot be written down with the fields transforming as shown. Any two Weyl spinors that have been suggestively labelled to imply that they are left and right-handed components of a Dirac spinor in fact have different gauge transformation properties, hence cannot form an invariant bi-linear. This implies that, if gauge symmetry is a good symmetry of the Lagrangian, i.e. intact after quantum corrections, then the fermions must be exactly massless in the unbroken phase.

However the gauge symmetry of the SM is spontaneously broken and this allows mass terms to be included for both fermions and vector bosons. For the vector boson masses we need to consider a generalisation of the kinetic term for the Higgs field that is symmetric under local symmetry transformations. First the derivative must be covariantised, such that $\partial_\mu H \to D_\mu H$ where $D_\mu H$ transforms like $H$, where the covariant derivative is given by

$$D_\mu H = \left(\partial_\mu - igA^a_\mu \tau^a - i\frac{1}{2}g' B_\mu\right) H \tag{1.1}$$

where $\tau^a = \sigma^a/2$ and $\sigma^a$ are the Pauli sigma matrices. The kinetic term is now

$$|D_\mu H|^2 \tag{1.2}$$

where it is clear from expanding out Eq. (1.2) that it will contain terms that are bilinear in the gauge bosons and quadratic in the Higgs boson. When spontaneous symmetry breaking occurs the Higgs boson is replaced by a vacuum expectation value (vev), since the Higgs potential does not minimise at zero, but at finite, non-zero values for the field $H$. Three of the degrees of freedom of the Higgs field will be re-interpreted as longitudinal modes for massive gauge bosons, but one remains as a perturbation about this new minimum; a massive scalar field.

The potential for the Higgs must be of the following form

$$V(H) = -\mu^2 H^\dagger H + \lambda \left(H^\dagger H\right)^2 = \lambda \left(H^\dagger H - \frac{\mu^2}{2\lambda}\right)^2 - \frac{\mu^4}{4\lambda} \tag{1.3}$$

which has a minimum at $|H|^2 = \frac{\mu^2}{2\lambda}$ where residual gauge freedom allows the Higgs vev to be expressed as

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{1.4}$$

To put it another way this freedom comes from the degeneracy of the different vacua means that all choices are equally good and we are allowed to make a global transformation to choose a convenient vacua[1].

Eq. (1.4) is not invariant under the full $\mathrm{SU}(2)_L\times$ $\mathrm{U}(1)_Y$ rotations under which $H \longrightarrow e^{i(\alpha^a\tau^a+\beta/2)}H$ as we see from table 1.1. However it is invariant under a subset of $\mathrm{SU}(2)_L\times$ $\mathrm{U}(1)_Y$ rotations in which $\alpha^3 = \beta$ and $\alpha^1 = \alpha^2 = 0$. In the rest of the fields we see that the remaining transformation distinguishes between elements of $\mathrm{SU}(2)_L$ doublets and rotates each element by a local phase that depends on both the hypercharge and its $\tau^3$ eigenvalue. Hence we can say that the gauge symmetry of the model has undergone the following transformation, termed electroweak (EW) symmetry breaking

$$\mathrm{SU}(3)_c \times \mathrm{SU}(2)_L \times \mathrm{U}(1)_Y \rightarrow \mathrm{SU}(3)_c \times \mathrm{U}(1)_{EM}. \tag{1.5}$$

[1]This is not true if different vacua are reached in spatially separated regions since we can only make a global redefinition not a local (gauge) transformation

In the broken phase the gauge bosons acquire masses from their coupling to $H$ in Eq. (1.2) and, after we change basis to

$$W_\mu^\pm = \frac{1}{\sqrt{2}}\left(A_\mu^1 \mp iA_\mu^2\right) \tag{1.6}$$

$$Z_\mu^0 = \frac{1}{\sqrt{g^2+g'^2}}\left(gA_\mu^3 - g'B_\mu\right) \tag{1.7}$$

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g'A_\mu^3 + gB_\mu\right) \tag{1.8}$$

we obtain the following masses

$$m_W = g\frac{v}{2},\ m_Z = \sqrt{g^2+g'^2}\frac{v}{2} \quad \text{and} \quad m_A = 0 \tag{1.9}$$

where $m_W$ is the mass for both $W^+$ and $W^-$.

To find the masses for the fermions we must make further use of the symmetry breaking mechanism. We note that it is possible to write down couplings between two fermions and one Higgs boson:

$$\mathcal{L}_{Yuk.} = -\lambda_{ij}^e \overline{L}_i H e_{Rj} - \lambda_{ij}^d \overline{Q}_i H d_{Rj} - \lambda_{ij}^u \overline{Q}_i H^\dagger u_{Rj} + h.c. \tag{1.10}$$

where all of the indices, except for generational indices, are suppressed. It is clear that when $H$ is replaced with its vev Eq. (1.10) provides Dirac masses for the quarks and charged leptons. Note that the SM cannot generate Dirac masses for the neutrinos since no singlet fermion exists to provide an analogous Yukawa coupling; since non-renormalisable operators are excluded it is impossible to include any masses for the neutrinos.

### 1.3.1 Standard Model Successes

The first and perhaps the most obvious success of the SM is its remarkable agreement with experiment: at the time of writing there are no predictions of the SM out of line with experimental tests. Notable successes of the SM include the prediction of the existence of the $c$ quark, given the existence of the $u$, $d$ and $s$ quarks. Without the inclusion of the $c$ quark loop effects enhance flavour changing processes, but Glashow, Iliopoulos and Maiani (GIM) observed [7] that including a quark with the same quantum numbers as the $u$ quark brought these effects under control.

The decays of the $K^o$ meson were shown to violate CP and hence, to introduce CP violation into particle physics, Kobayashi and Maskawa [8] introduced a third generation of matter. This was done so that there would be one irreducible phase left in the couplings between $W$ bosons and quarks, which is not the case in two generations as all the phases can be rotated away. The subsequent discovery of the $b$ quark required the existence of the $t$ quark which, when subsequently discovered, further confirmed the standard model. In addition the mixing matrix induced in the $W$ boson couplings by diagonalising the quark mass matrix, the Cabibbo-Kobayashi-Maskawa (CKM) matrix, has been subject to extensive experimental test. As yet all experimental data is consistent with the CKM description of the quark sector.

### 1.3.2 Standard Model Problems

There are a number of unresolved issues with the SM, both on a conceptual level and with regard to experiment. Firstly how is a period of inflation possible within the SM? It seems that this is not possible since there are a lack of scalars within the SM, only the Higgs. Secondly, while the gauge couplings do not unify in the SM, they do come close providing a strong hint that there is something between the electroweak scale and the scale of the Grand Unified Theories (GUTs). That is, if we take the near miss as being more than fluke.

In addition there is the hierarchy problem. This stems from the observation that the mass of the Higgs boson, $m_H$, is extremely sensitive to unknown high energy physics.

It can be shown [9] that, if there are any couplings between massive particles and the Higgs, one loop effects introduce an additive, quadratic dependence on the momentum cut-off, $\delta m_H^2 \propto \Lambda^2$. This would have to be cancelled by the bare mass squared of the order of $10^{36}$ GeV$^2$, if we take the SM to be valid up to the Planck scale, to give a remainder of the order of $10^4 - 10^6$ GeV$^2$. This would have to be done at every order in perturbation theory. While there is nothing intrinsically wrong with such fine-tuning it essentially seems arbitrary. Let us instead phrase the problem as a question: why should the Higgs vev be down at the electroweak scale when the natural scale for the Higgs mass is the highest scale in the theory? One possible answer to this question is supersymmetry, which we discuss in section 1.4.

Also there is the strong CP problem, namely why is pure Quantum ChromoDynamics (QCD) (without electroweak effects) CP conserving? If all of the renormalisable, gauge invariant terms in QCD Lagrangian are allowed the following term must be included [2]

$$\mathcal{L}_\theta = \theta_{QCD} \frac{g^2}{16\pi^2} Tr(G_{\mu\nu}\tilde{G}_{\mu\nu}) \tag{1.11}$$

where $G_{\mu\nu}$ is the gluon field strength and $\tilde{G}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}F^{\rho\sigma}$. Eq. (1.11) cannot just be set to zero, despite the fact this can be expressed as a total derivative, since it can be shown that the integral of the total derivative is not identically zero due to instantons contributions[2]. The physical coupling can be obtained when the most general quark mass matrix is subjected to a chiral rotation $q_L \to U_L q_L$ and $q_R \to U_R q_R$ such that all the quark Yukawa couplings are rendered real and diagonal. If this is performed the Lagrangian is modified (since the chiral symmetry is anomalous) and the effective

[2] Instantons are field configurations that interpolate between different, degenerate QCD vacua

$\theta$ parameter becomes

$$\theta = \theta_{QCD} - \theta_{EW} \tag{1.12}$$

where $\theta_{EW}$ arises from the diagonalisation of the quark mass matrices and is given by

$$\theta_{EW} = \arg(\det(M^u)\det(M^d)) \tag{1.13}$$

where $M^u$ and $M^d$ are the quark mass matrices for the up-like and down-like quarks, respectively. Since a non-zero $\theta$ can be shown to [10] give rise to a electric dipole moment for the neutron, $d_n$, experimental limits of $|d_n| < 3 \times 10^{-26} e\,cm$ [11] place an upper bound on $\theta \leq 10^{-10}$ since $d_n < 0.63 \times 10^{-25} e\,cm$ [10]. So the strong CP problem is essentially, why should these two separate areas of physics, the QCD and EW sectors, conspire to give $\theta = 0$?

### Peccei-Quinn Mechanism

The Peccei-Quinn (PQ) mechanism [12, 13] provides a possible solution to the strong CP problem. Essentially the PQ mechanism introduces an additional field, $a$, that has a linear coupling to the CP-violating gluon field strength term, $G_{\mu\nu}\tilde{G}_{\mu\nu}$. Now the strong CP violating Lagrangian is

$$\mathcal{L}_{axion} = \left(\theta - \frac{a}{f_a}\right)\frac{g^2}{16\pi^2}Tr(G_{\mu\nu}\tilde{G}_{\mu\nu}). \tag{1.14}$$

Now it is clear that, if $a$ has no other potential, then it will be energetically favourable to take on a value such that Eq. (1.14) is minimised, i.e. $\frac{\langle a \rangle}{f_a} = \theta$. With an effective $\theta$ coupling of zero the strong CP problem is solved.

For the PQ mechanism to be interesting it needs to be explained how this particular potential might be arrived at. The idea is to introduce additional, anomalous global $U(1)_{PQ}$ symmetry under which the quarks are charged and an additional scalar field, which must carry a charge under the symmetry. The axion, $a(x)$ is identified with the dynamical phase of the scalar field, $\sigma$, and if the global symmetry were exact would have a shift symmetry $a(x) \to a(x) + \alpha$, where $\alpha$ is a c-number, forbidding any non-derivative interactions. However, as noted, $U(1)_{PQ}$ is anomalous and hence the shift symmetry is broken by additional terms induced by the anomaly. This conspires to give axion a mass and introduce Eq. (1.14). Finally $f_a$ can be shown [10] to be given by

$$f_a = Q_\sigma \langle \sigma \rangle \tag{1.15}$$

where $Q_\sigma$ is the PQ charge of $\sigma$.

The axion is subject to experimental bounds coming from cosmology, astrophysics and conventional particle physics [10, 14]:

$$10^{10}\ \text{GeV} \leq f_a \leq 10^{13}\ \text{GeV}. \tag{1.16}$$

These bounds will play important roles in the models built in the main body of the thesis.

## 1.4 Supersymmetry

One of the problems with supersymmetry is the multitude of differing notation. For this thesis we maintain that Greek indices $\mu$, $\nu$, $\rho$ and $\sigma$ will be retained for Minkowski space vector and tensor indices, while spinorial indices will be taken from the beginning of the Greek alphabet. With all other indices we will try to avoid Greek if at all possible. Unfortunately this is not the convention used in [15] (in which the tensorial indices are Latin), but does agree with [2] and [9] and the rest of the thesis.

Historically, one[3] of the origins of supersymmetry was in the development of string

[3] SUSY has several origins: for an overview of the history see [16].

theory, in which supersymmetry is needed to introduce fermions. Since this thesis makes use of string theory the connection between the two theories is helpful. From a particle physics perspective low energy supersymmetry [4] is helpful for a number of reasons, including the following. The hierarchy problem is solved since [9] the non-renormalisation theorem of SUSY [17] states that there is no infinite renormalisation required, beyond, logarithmic, wavefunction renormalisation and cancelling this requires a much milder tuning. This still does not explain why the electroweak scale and the Planck scale are different, in that this separation is not an *a priori* prediction of SUSY. It can however provide a mechanism to split these scales given appropriate parameters within the right class of SUSY models, hence it is very much a model dependent prediction. We will return to this question in section 1.4.1.

Supersymmetry enlarges the Poincaré symmetry to include generators, $Q_\alpha$, that mix fermions and bosons. Heuristically

$$Q|boson>=|fermion> \tag{1.17}$$

$$Q|fermion>=|boson> \tag{1.18}$$

and we can immediately deduce two things. One, $Q_\alpha$ is fermionic (as was implied by the spinor index) and two, $Q$ must have a non-zero mass dimension. In principle there can be more than one SUSY generator, $Q^i_\alpha$, but we restrict our attention to $\mathcal{N}=1$ SUSY. In this case the anticommutators of the spinor generators are given by

$$\{Q_\alpha, \overline{Q}_{\dot\beta}\} = 2\sigma^\mu_{\alpha\dot\beta}P_\mu \tag{1.19}$$

$$\{Q_\alpha, Q_\beta\} = 0 = \{\overline{Q}_{\dot\alpha}, \overline{Q}_{\dot\beta}\} \tag{1.20}$$

where $\sigma^0 = \mathbb{I}_{2\times2}$ and $\sigma^i$ are the Pauli matrices. The undotted indices denote left-

[4] By low energy we mean that supersymmetry is manifest at scales not much in excess of the electroweak scale.

handed two component Weyl spinors whereas a dot over the index implies that this field is a right-handed two component Weyl spinor. Eq. (1.19) shows that the mass dimensions of $Q$ and $\overline{Q}$ are given by $[Q] = 1/2 = [\overline{Q}]$. The rest of $Q$'s algebraic properties can be inferred from the Poincaré transformations of a spinor operator as discussed in [18]. For a more through discussion of notation and conventions see [15], bearing in mind the the differing convention for Lorentz 4-vectors. To simplify the process of constructing supersymmetric models we utilise superspace formalism and sketch out the details necessary for model building.

Our first step will be to introduce Grassmannian spinor parameters, $\theta^{\alpha}$ and $\overline{\theta}_{\dot{\alpha}}$ which obey the following anti-commutation relations

$$\{\theta^{\alpha}, \theta^{\beta}\} = \{\overline{\theta}_{\dot{\alpha}}, \overline{\theta}_{\dot{\beta}}\} = \{\theta^{\alpha}, \overline{\theta}_{\dot{\beta}}\} = 0. \tag{1.21}$$

This allows us to re-formulate the SUSY algebra as a lie algebra, hence making it possible to exponentiate the generators and obtain a unitary operator corresponding to a finite SUSY rotation. The only non-zero commutator is given by

$$[\theta Q, \overline{\theta Q}] = 2\theta\sigma^{\mu}\overline{\theta}P_{\mu} \tag{1.22}$$

where the spinor summation convention $\psi\chi = \psi^{\alpha}\chi_{\alpha} = \chi\psi$ and $\overline{\psi}\overline{\chi} = \overline{\psi}_{\dot{\alpha}}\overline{\chi}^{\dot{\alpha}} = \overline{\chi}\overline{\psi}$ has been employed. This leads to

$$G(\theta, \overline{\theta}) = \exp\left[i\left(\theta Q + \overline{\theta Q}\right)\right]. \tag{1.23}$$

To prevent the action of $\theta Q$ from changing the mass dimension of any field it operates on $\theta$ must have a mass dimension of -1/2, in addition Eq. (1.23) would make little sense if $\theta Q$ were dimensionful.

### Superfields

We now need an object onto which the SUSY transformations can act. Let us consider a general scalar function living in superPoincaré space i.e. a representation of the Poincaré group augmented by the SUSY generators.

$$\Phi(x,\theta,\overline{\theta}) = f(x) + \theta\phi(x) + \overline{\theta}\overline{\chi}(x) + \theta\theta m(x) + \overline{\theta}\overline{\theta}n(x)$$
$$+\theta\sigma^{\mu}\overline{\theta}v_{\mu}(x) + (\theta\theta)\overline{\theta}\overline{\lambda}(x) + (\overline{\theta}\overline{\theta})\theta\psi(x) + (\theta\theta)(\overline{\theta}\overline{\theta})d(x) \tag{1.24}$$

where this is a series expansion in $\theta$ and $\overline{\theta}$, which naturally terminates due to the anticommutation relations given in Eq. (1.21). Fierz identities have been applied to ensure that this includes all possible, independent, non-zero combinations of $\theta$ and $\overline{\theta}$.

The action of Eq. (1.23) on $\Phi$ allows one to derive the SUSY transformation laws of the components. We do not show all the transformations, but let us consider the infinitesimal transformation of $d$

$$\delta_{\xi}d = \frac{i}{2}\partial_{\mu}\left[\psi\sigma^{\mu}\overline{\xi} + \xi\sigma^{\mu}\overline{\lambda}\right] \tag{1.25}$$

where $\xi$ is a constant, Grassmann spinor analogous to $\theta$.

There are two important features of this equation. One, the SUSY transformation is doing what one expects: it is mixing fermions, $\chi$ and $\phi$, into a boson, $d$. Two, this field transforms as a total derivative of the ordinary space-time co-ordinates, and so we expect that a quantum field theory (QFT) built entirely of this, and similar components, will be automatically supersymmetric. It is exactly this approach which we will now pursue.

We wish to consider field theories constructed out of irreducible representations of the SUSY transformations. To find chiral representations we require the general superfield, $\Phi$, be subject to the covariant constraint

$$\overline{D}_{\dot{\alpha}}\Phi = 0 \tag{1.26}$$

where $\overline{D}_{\dot{\alpha}}$ is a covariant derivative in superspace

$$\overline{D}_{\dot{\alpha}} = \partial_{\alpha} + i\theta^{\beta}\sigma^{\mu}_{\beta\dot{\alpha}}\partial_{\mu}. \tag{1.27}$$

The most general function that satisfies this constraint is $\Phi(y, \theta)$ where

$$y^{\mu} = x^{\mu} + i\theta\sigma^{\mu}\overline{\theta} \tag{1.28}$$

hence

$$\Phi(y, \theta) = A(y) + \sqrt{2}\theta\psi(y) + \theta\theta F(y) \tag{1.29}$$

is the most general, left-handed, chiral superfield (also referred to as chiral superfields), where $A$ and $F$ are complex scalar fields and $\psi^{\alpha}$ is a complex left-handed Weyl spinor, note the undotted index[5]. This justifies our assertion that this representation is chiral. Eq. (1.29) can be expanded out in terms of $x, \theta$ and $\overline{\theta}$ which introduce higher derivative terms, crucial in the derivation of the kinetic terms:

$$\begin{aligned}\Phi(y, \theta) =& A(x) + \sqrt{2}\theta\psi(x) + \theta\theta F(x) + i\theta\sigma^{\mu}\overline{\theta}\partial_{\mu}A(x) \\ &+ \frac{i}{\sqrt{2}}(\theta\theta)\partial_{\mu}\psi(x)\sigma^{\mu}\overline{\theta} - \frac{1}{4}(\theta\theta)(\overline{\theta}\overline{\theta})\Box A(x).\end{aligned} \tag{1.30}$$

An analogous construction can be carried out for the other covariant derivative

$$D_{\alpha} = \partial_{\alpha} + i\sigma^{\mu}_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_{\mu} \tag{1.31}$$

imposing

$$D_{\alpha}\Phi = 0 \tag{1.32}$$

[5]The undotted index of $\psi$ can be inferred from the spinorial summation conventions

on a general superfield. We note that superfields satisfying this constraint are right-handed, chiral (also denoted anti-chiral superfields) superfields and that the hermitian adjoint of a chiral superfield provides an important example of such a superfield.

Applying either covariant derivative to a function of chiral and anti-chiral fields shows, via the chain rule, that any function of only chiral superfields is chiral. Also a function of anti-chiral superfields remains anti-chiral whereas, via the product rule, it is possible to show that functions of both chiral and anti-chiral fields are neither chiral nor anti-chiral. We also note that the $\theta\theta$ ($\overline{\theta\theta}$) component of $\Phi$ ($\Phi^\dagger$) transforms as a total derivative and is a candidate for building a SUSY theory. This is not the case for the more general superfield, Eq. (1.24), which contains additional fields that contribute to the transformation of $\theta\theta$ and $\overline{\theta\theta}$ components of Eq. (1.24). See [15] for a complete discussion of the transformation laws.

Since the chiral superfields do not contain any fields with vector indices it is clear that we must look elsewhere for gauge bosons. The general scalar superfield, Eq. (1.24), does contain the vector components, $v_\mu$, and we impose $V(x,\theta,\overline{\theta})^\dagger = V(x,\theta,\overline{\theta})$, where $V$ is initially general. This ensures that $v_\mu^* = v_\mu$ is a candidate for an Abelian vector boson. Implementing gauge transformations in a supersymmetric way requires

$$V \rightarrow V + \Phi + \Phi^\dagger \tag{1.33}$$

where $\Phi$ is a chiral superfield. This leads to

$$v_\mu \rightarrow v_\mu + \partial_\mu\big(i(A - A^*)\big) = v_\mu + \partial_\mu \Lambda \tag{1.34}$$

where this is clearly a gauge transformation in the usual sense. We note that the general scalar superfield, Eq. (1.24), would clearly not have been a suitable replacement for the sum of chiral superfields since it would introduce $v_\mu \rightarrow v_\mu + v'_\mu$ which clearly is not a gauge transformation.

To bring our notation closer in line with [15] and [19] we redefine the components of Eq. (1.24) as follows

$$v_\mu \to -v_\mu, \ \overline{\lambda} \to i\overline{\lambda} \text{ and } d \to \frac{1}{2}D. \tag{1.35}$$

This has no bearing on any physics since the components in Eq. (1.24) are arbitrary, up to their Lorentz transformation properties which are unchanged. As a final note many of the fields in $V$ can be gauged away under Eq. (1.33). To see this consider the first component of $\Phi + \Phi^\dagger = A + A^*$: since $\Phi$ is an arbitrary chiral field $A + A^*$ can be chosen to exactly cancel the first component of $V$ and thus it cannot be a physical degree of freedom. We chose the Wess-Zumino gauge in which

$$V = V_{WZ} = -\theta\sigma^\mu\overline{\theta}v_\mu + i(\theta\theta)\overline{\theta}\overline{\lambda} - i(\overline{\theta}\overline{\theta})\theta\lambda + \frac{1}{2}(\theta\theta)(\overline{\theta}\overline{\theta})D. \tag{1.36}$$

The final superfield we will need before we can write down a general $\mathcal{N} = 1$ global SUSY Lagrangian is a spinor superfield, containing the gauge field strength as a component. We do not attempt to derive the form of the spinor superfields, $W^\alpha$, here; instead we quote [15]

$$W^\alpha = -i\lambda^\alpha(y) + \theta^\alpha D(y) + \theta^\beta\sigma_\beta^{\mu\nu\alpha}F_{\mu\nu}(y) - (\theta\theta)\overline{\sigma}^{\mu\dot{\beta}\alpha}\nabla_\mu\overline{\lambda}_{\dot{\beta}}(y) \tag{1.37}$$

where $\nabla_\mu$ is the gauge covariant derivative.

If we wish to consider the supersymmetric theory of gravity (SUGRA) this extends the theory to include the spin-2 graviton and the spin-3/2 gravitino. For completeness we should consider the transformation of superfields with these new fields as components, but since we do no physics with these new fields we omit this discussion, but instead see, for example [16, 19].

**SUSY Lagrangian**

Since any product of chiral superfields is also chiral and the transformations of the components are determined by (1.22) it can be shown that the $\theta\theta$ component of such a product transforms as a total derivative. To extract this component we introduce integration over Grassmannian parameters.

$$\int da\ a = 1 \tag{1.38}$$

$$\int da\ 1 = 0 \tag{1.39}$$

where $a$ is a single Grassmann variable and since $aa = 0$ we do not need to consider integrals that are any more complicated. Returning to Grassmann spinors we have

$$\int d^2\theta\ \theta\theta = 1 \tag{1.40}$$

$$\int d^2\overline{\theta}\ \overline{\theta\theta} = 1 \tag{1.41}$$

$$\int d^4\theta\ (\theta\theta)(\overline{\theta\theta}) = 1 \tag{1.42}$$

and all other integrals over different products of $\theta$ and $\overline{\theta}$ are zero. The exact form of the measures is irrelevant for this thesis since we are using them for book-keeping purposes, but they may be found in [15].

We define the superpotential, $W(\Phi)$, as a general function of chiral superfields and the Kähler potential, $K(\Phi, \Phi^\dagger)$ as a general, real function of both chiral and anti-chiral superfields. Coupled with Eqs. (1.40)- (1.42) this allows us to now write down a general

$\mathcal{N} = 1$ Lagrangian [15] [6]

$$\mathcal{L} = \int d^4\theta K\left(\Phi^i, \Phi^{j\dagger} e^{2V}\right) - \left[\int d^2\theta W(\Phi^i) + h.c.\right]$$
$$+\frac{1}{4}\left[\int d^4x \int d^2\theta \; \mathrm{Tr}(f(\Phi^i)^{ab} W_a^\alpha W_{\alpha b}) + h.c.\right] \tag{1.43}$$

where $f(\Phi^i)^{ab}$ is the gauge kinetic function, whose expectation value will set the gauge couplings for the theory. In general the fields in $f(\Phi_i)^{ab}$ may be drawn from all of the fields in the theory, but $f(\Phi^i)^{ab}$ must be holomorphic in said fields and can only have gauge transformation properties such that it is still possible to make the trace invariance. The indices $a$ and $b$ denote the fact that, in general, there will be more than one gauge group in a given theory. While, in general, the gauge kinetic function can be very complicated, in all the models we will consider it is taken to be diagonal. Eq. (1.43) also shows that the renormalisable contributions to the superpotential must have mass dimension 3 or less and likewise the contributions to the Kähler potential must be dimension 2 or less.

Obtaining the Lagrangian from Eq. (1.43) for general $K$ and $W$ is reasonably involved, but the essential principle is as follows. First perform the integration over the Grassmannian spinors and then integrate out the $F$ and $D$ fields. This last step is valid because $F$ and $D$ have no kinetic terms allowing one to perform the path-integral, hence these fields are termed auxiliary. Equivalently the equations of motion for the auxiliary fields can be solved and the solutions used to replace them with propagating fields. If we temporarily ignore the gauge fields the result of this process is the following

[6] The form of the $W^\alpha$ part of the potential has been modified such that it agrees with the expression commonly used in effective theories arising from string theory. Specifically, that the gauge couplings depend on the real part of $f(\Phi_i)^{ab}$, see for example [20, 21].

Lagrangian[19]

$$\mathcal{L} = -g_{ij^*}\partial_\mu A^i \partial^\mu A^{*j} - i g_{ij^*}\overline{\psi}^j \overline{\sigma}^\mu D_\mu \psi^i - g^{ij^*}\frac{\partial W}{\partial A^i}\frac{\partial W^*}{\partial A^{*j}} + \frac{1}{4}R_{ij^*kl^*}\psi^i\psi^k\overline{\psi}^j\overline{\psi}^l$$
$$-\frac{1}{2}\left(\frac{\partial^2 W}{\partial A^i \partial A^j} - \Gamma^k_{ij}\frac{\partial W}{\partial A^k}\right)\psi^i\psi^j - \frac{1}{2}\left(\frac{\partial^2 W^*}{\partial A^{*i}\partial A^{*j}} - \Gamma^{k*}_{i^*j^*}\frac{\partial W^*}{\partial A^{*k}}\right)\overline{\psi}^i\overline{\psi}^j \quad (1.44)$$

where $D_\mu\psi^i = \partial_\mu\psi^i + \Gamma^i_{ij}\partial_\mu A^j \chi^k$, not to be confused with $D_\alpha$, which acts on superfields. To simplify the form of Eq. (1.44) we have introduced the following notation

$$g_{ij^*} = K_{ij^*} = \frac{\partial}{\partial A^i}\frac{\partial}{\partial A^{*j}}K|_{\theta=\bar{\theta}=0} \quad (1.45)$$

$$g_{ij^*,k} = \frac{\partial}{\partial A^k} g_{ij^*} = g_{mj^*}\Gamma^m_{ik} \quad (1.46)$$

$$g_{ij^*,k^*} = \frac{\partial}{\partial A^{*k}} g_{ij^*} = g_{im^*}\Gamma^{m^*}_{j^*k^*} \quad (1.47)$$

and hence we see that to obtain canonical kinetic terms we require that $g_{ij^*} = \delta_{ij^*}$. This is clearly satisfied if

$$K = \sum_i \Phi^i \Phi^{i\dagger} \quad (1.48)$$

and hence this is termed the canonical form for the Kähler potential.

Returning to the gauge fields we find that the pure gauge part of the Lagrangian is given by

$$\mathcal{L}_{gauge} = \mathrm{Tr}\left[-\frac{1}{4}\mathrm{Re}(f(\Phi^i)^a)\left(F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^a\sigma^\mu\nabla_\mu\overline{\lambda}^a + \frac{1}{2}(D^a)^2\right)\right.$$
$$\left.+\frac{1}{4}\mathrm{Im}(f(\Phi^i)^a)F^a_{\mu\nu}\tilde{F}^{\mu\nu a}\right] \quad (1.49)$$

where we have imposed $f(\Phi^i)^{ab} = \delta^{ab}f(\Phi^i)^a$ as will be the case for all the models we consider and we have defined $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$. The important point to note here, is that the gauge coupling is given by the real part of the gauge kinetic function. $D$-term contributions are not considered in this thesis so we go no further in this analysis.

### SUSY Breaking Lagrangian

It is manifest that SUSY is not a good symmetry of nature therefore it must be broken to give agreement with nature. Since we need SUSY to stabilise the electroweak scale any breaking must not disturb the ultraviolet behaviour of the theory: such breaking is termed "soft". In other words we cannot allow corrections to the Higgs mass that depend quadratically on the cut-off. By power counting arguments ([22, 16] and references therein) it can be decided which terms can be included and we now summarise those terms relevant to our model building efforts. First, trilinear scalar interactions[7],

$$\frac{1}{3!}\tilde{A}_{ijk}\phi_i\phi_j\phi_k + h.c. = \frac{1}{3!}\lambda_{ijk}A_{ijk}\phi_i\phi_j\phi_k + h.c. \tag{1.50}$$

and Soft scalar mass-squares,

$$m^2_{ij}\phi^\dagger_i\phi_j. \tag{1.51}$$

In general one has bilinear scalar interactions, $\frac{1}{2}b_{ij}\phi_i\phi_j + h.c.$, and gaugino masses, $(\frac{1}{2}M_a\lambda^a\lambda^a + h.c.)$. However, in our models the bilinears will be zero as discussed in section 1.4.3 and the gauginos are not considered.

### R-Symmetry

The anti-commutator of the SUSY generators, Eq. (1.19), can be seen to be symmetric under $Q_\alpha \to e^{i\phi}Q_\alpha$. Given that $Q$ acting on a given state either kills the state or moves to a different part of the SUSY multiplet we see that the fermionic and bosonic members of a given multiplet have different charges under the symmetry. These are known as R-symmetries and may well be realised in nature, though it is not a requirement of supersymmetry that they do so. From Eq. (1.22) it is possible to see how $\theta$ and $\overline{\theta}$

[7] It is convenient in string constructions to extract a factor of the Yukawa coupling since this appears to be the natural form for $\tilde{A}_{ijk}$.

transform. If $Q_\alpha$ is given a charge $-1$ and defined in a way consistent with Eq. (1.22):

$$Q_\alpha = \frac{\partial}{\partial \theta^\alpha} - i\sigma^\mu_{\alpha\dot{\beta}}\overline{\theta}^{\dot{\beta}}\partial_\mu \tag{1.52}$$

then it is clear that $\theta$ must have charge $+1$ and $\overline{\theta}$ charge $-1$. Since the superpotential contribution to the Lagrangian is constructed from its $\theta\theta$ component it follows that the superpotential must carry an R-charge of $+2$ to yield an invariant Lagrangian. A similar argument shows that the Kähler potential must have zero R-charge.

### 1.4.1 Minimal Supersymmetric Standard Model

The Minimal Supersymmetric Standard Model (MSSM) is the simplest possible extension of the SM to include supersymmetry. This is achieved by promoting all the fields in the SM to superfields and adding an extra Higgs field. The additional Higgs is required because the superpotential can only contain fields and not their conjugates. Since the conjugate Higgs field was required to generate down-like quarks it is clear that a field with the same quantum numbers as the conjugate Higgs field must be introduced to act in the same capacity.

To define the MSSM we must write down the superpotential, Kähler potential and soft breaking terms. First we assume canonical kinetic terms and so $K$ is given by Eq. (1.48) where this runs over all quarks, leptons and Higgs superfields in the theory[8] Then we define MSSM superpotential, with gauge indices suppressed, as

$$W_{MSSM} = Y_u^{ij} Q_i H_U U_j + Y_d^{ij} Q_i H_d D_j + Y_e^{ij} L_i H_d E_j + \mu H_u H_d \tag{1.53}$$

[8]For this to be gauge invariant $e^{2V_i}$ must be sandwiched between the two superfields, where $V_i$ is the sum of the vector superfields associated with $\Phi^i$[15]. For example, if $\Phi^1$ is a bifundamental of SU(2) and SU(3) then $V_1 = V_{SU(2)} + V_{SU(3)}$.

with the following soft terms

$$\begin{aligned}
\mathcal{L}_{Soft} = & \frac{1}{2}\left[M_3\tilde{g}\tilde{g} + M_2\tilde{W}^a\tilde{W}^a + M_1\tilde{B}\tilde{B} + h.c.\right] \\
& + \left[bH_uH_d + \lambda_{u_{ij}}A_{u_{ij}}Q_iH_uU_i + \lambda_{d_{ij}}A_{d_{ij}}Q_iH_dD_j + \lambda_{e_{ij}}A_{e_{ij}}L_iH_dE_j + h.c.\right] \\
& + m^2_{H_d}|H_d|^2 + m^2_{H_u}|H_u|^2 + m^2_{Q_{ij}}Q_iQ_j^* \\
& + m^2_{L_{ij}}L_iL_i^* + m^2_{U_{ij}}U_iU_j^* + m^2_{D_{ij}}D_iD_j^* + m^2_{E_{ij}}E_i^*E_j.
\end{aligned} \tag{1.54}$$

In a slight abuse of notation we have denoted the scalar components of the superfields by the same symbols as the superfields themselves. The fermionic degrees of freedom are differentiated by the inclusion of tildes.

It is possible to address the hierarchy problem within the MSSM since this is a concrete SUSY model. One can show (see [22] and references therein) that the renormalisation group running can, with a large enough top Yukawa, drive the soft mass for the up-like Higgs negative. Since the soft mass can be shown to go negative around the electroweak scale and give rise to phenomenologically viable Higgs vevs it can be said that SUSY incorporates electroweak symmetry breaking (EWSB) in a natural way. We state without derivation[9] the minimisation conditions for the Higgs potential given in [22]

$$|\mu|^2 + m^2_{H_d} = b\tan\beta - \frac{m_Z^2}{2}\cos 2\beta \tag{1.55}$$

$$|\mu|^2 + m^2_{H_u} = b\cot\beta + \frac{m_Z^2}{2}\cos 2\beta \tag{1.56}$$

and in terms of $\mu$

$$\mu^2 = \frac{m^2_{H_d} - m^2_{H_u}\tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m_Z^2. \tag{1.57}$$

In the MSSM, the term $\mu$ is considered to be an explicit mass parameter and is not tied to the EW or soft breaking scales, yet Eqs. (1.55)- (1.57) suggest that it should be

[9] To obtain these expressions one analyses the Higgs potential under the requirement that the Higgs vevs are non-zero and finite, in the minimum

of the same order as the soft terms and the EW scale. In other words why should $\mu$ be around the EW scale rather than, say, the Planck scale? This is the $\mu$ problem of the MSSM and while similar to the hierarchy problem it is not as severe since, although it is not predicted, the $\mu$ scale is stable under radiative corrections. We propose a possible solution to the $\mu$ problem in chapter 3.1.1.

We expect, by a similar argument, that the soft terms should also be not much above the EW scale. In our models we will make use of the supergravity framework to generate the soft terms and make use of the Planck scale to suppress the soft parameters to the TeV scale. Hence the soft terms will be discussed further after supergravity has been introduced.

Finally we define our R-charge conventions, taken from [22]. The Higgs superfields have R-charge of +1 while the remaining chiral fields have charge +1/2 and the gauge vector superfields have charge zero.

### 1.4.2 NMSSM

The NMSSM [23] removes the $\mu$ term from the MSSM superpotential and replaces it with two new terms, as follows:

$$\mu H_u H_d \rightarrow W \lambda N H_u H_d + \frac{\kappa}{3} N^3 \tag{1.58}$$

hence the soft breaking sector is given by

$$V_{soft}^{\text{NMSSM}} = V_{soft}^{\text{MSSM}} + \lambda A_\lambda N H_u H_d + \frac{1}{3}\kappa A_\kappa - b H_u H_d + h.c. + m_N^2 |N|^2. \tag{1.59}$$

Since the potential for $N$ is entirely determined by renormalisable couplings and soft parameters it follows that $\langle S \rangle$ will likely be around the soft breaking scale. Hence the NMSSM dynamically generates a $\mu$ term of $\lambda \langle N \rangle$ when $N$ is minimised. For discussions of NMSSM phenomenology see, for example, [24].

The NMSSM possesses a discrete $\mathbb{Z}_3$ symmetry and as such can fall into three distinct vacua post inflation. The minimum energy field configuration that interpolates between these vacua is know as a domain wall and is analogous to the behaviour of ferro-magnets below the Curie temperature. These domain walls present a serious problem for the NMSSM since they generate an unacceptably large contribution to the CMB anisotropies. For a discussion of the domain wall problem within the NMSSM see [25].

### 1.4.3 SUGRA

Now that we have given the formalism for global SUSY and considered its simplest phenomenologically viable incarnation, the MSSM, we address the issue of making gravity supersymmetric and hence consider a theory invariant under local supersymmetry transformations. There are numerous reasons for considering local SUSY, but in this thesis we only wish to consider the following two reasons. Firstly it appears naturally as the low energy limit of string theory and secondly it provides a natural framework for SUSY breaking to be realised and communicated to low energies.

Supergravity (SUGRA) arises when the parameter of SUSY transformation is promoted from a constant to having a spacetime dependence: $\epsilon \to \epsilon(x)$. Since $P_\mu$ is an element of the SUSY algebra this requires our theory to be locally translation invariant, and we see that we have obtained general co-ordinate invariance. This leads us to suspect that the theory should include General Relativity (GR). More directly we can apply the Noether method and, starting from a free theory invariant under global SUSY, allow local transformations. To correct for the variation in the Lagrangian new terms must be added and the transformation laws modified. This process is iterated and leads to a locally invariant Lagrangian, for an example of this see [18]. When this a similar method is applied to supersymmetrise GR we obtain SUGRA. A more involved procedure will lead to the SUGRA Lagrangian in the presence of a general matter sector and it is this Lagrangian that will prove important in this thesis. In this thesis we only consider the potential for the scalars [22]

$$V = \exp(\kappa^2 K)[K^{ij^*}(D_i W)(D_j W)^* - 3\kappa^2 |W|^2] \tag{1.60}$$

$$= \kappa^{-6} \exp(G)[K^{ij^*}(G_i)(G_{j^*}) - 3\kappa^2] \tag{1.61}$$

and the two point component of the gravitino's, $\tilde{G}_\mu$, superpotential

$$V_{3/2} \supset \exp(\kappa^2 K/2)\kappa^2 \left( W^* \tilde{G}_\mu \sigma^{\mu\nu} \tilde{G}_\nu + W \overline{\tilde{G}}_\mu \overline{\sigma}^{\mu\nu} \overline{\tilde{G}}_\nu \right) \tag{1.62}$$

where $G = k^2 K + \ln(k^3 W)$, $D_i W = W_i + \kappa^2 K_i W$ and $\kappa^2 = 8\pi G_N$ and $K$ and $W$ are reduced to functions of the scalar components of the superfields.

Given this potential we now want to consider the possible applications of this to phenomenology. Specifically we will show how one can obtain a low energy effective field theory after SUSY is broken. To achieve this we introduce a hidden sector containing fields that do not transform under the MSSM gauge groups with a superpotential separated from the MSSM's:

$$W = W_{\text{MSSM}} + W_{\text{h}}. \tag{1.63}$$

For the sake of this discussion we first consider a canonical Kähler potential $K = \sum_I |\Phi^I|^2$, but note that we will have to go beyond this assumption in later chapters and revisit this issue there. Also, we only consider one hidden sector field $z$ though this can be generalised.

So doing we obtain

$$V = \exp\left(\kappa^2(|z|^2 + |\Phi^i|^2)\right) \left[ \left| \frac{\partial W_{\text{h}}}{\partial z} + \kappa^2 z^* W \right|^2 + \left| \frac{\partial W_{\text{MSSM}}}{\partial \Phi^i} + \kappa^2 \Phi_i^* W \right|^2 - 3\kappa^2 |W|^2 \right]. \tag{1.64}$$

One can show [2] that the following vevs minimise Eq. (1.64)

$$\langle z \rangle = a\kappa^{-1}, \ \langle W_{\rm h} \rangle = bm^2\kappa^{-1}, \ \left\langle \frac{\partial W_{\rm h}}{\partial z} \right\rangle = m^2 \tag{1.65}$$

where $m$ is an intermediate mass scale, and $a$ and $b$ are arbitrary real parameters.

From Eq. (1.62) we see that after the hidden sector fields get their vevs

$$m_{3/2} = \kappa^2 \exp(\kappa^2 K/2) W^* \approx \exp(a^2/2) bm^2 \kappa. \tag{1.66}$$

We will see that this sets the scale of the soft terms, as it will appear as a common mass for all the MSSM scalars. In order to simplify matters for low energy phenomenology we wish to discard any non-renormalisable operator, decouple gravity and consider a renormalisable effective field theory. Formally this is achieved by taking the flat limit by sending $\kappa \to 0$, while keeping $m_{3/2}$ constant. Since $\kappa^2 = 8\pi G_N$ we see that gravity becomes non-interacting. It is necessary to keep $m_{3/2}$ constant to keep the soft breaking scale unchanged.

This leads us to the following expression for the scalar potential

$$\begin{aligned} V_{Eff} = e^{a^2}\left(1+\kappa^2|\Phi^i|^2\right)\Bigg[ & m^4\left((1+ab)^2-3b^2\right) + \left|\frac{\partial W_{\rm MSSM}}{\partial \Phi^i}\right|^2 + \kappa^2|\Phi^i|^2 b^2 m^4 \\ & + m^2\kappa(a^2b+a-3b)(W_{\rm MSSM}+W^*_{\rm MSSM}) + \kappa\left(\frac{\partial W_{\rm MSSM}}{\partial \Phi^i}\Phi^{i*}bm^2 + h.c\right)\Bigg]. \end{aligned} \tag{1.67}$$

The series expansion in the Kähler potential of the matter fields, $\exp(\kappa^2 K) = 1 + \kappa^2 K + \mathcal{O}(K^2)$, has been truncated at first order since all higher orders vanish in the flat limit. However the first order picks out the constant inside the square brackets and hence provides an important contribution to the soft mass. Re-expressing this in terms of $m_{3/2}$ we obtain

$$\begin{aligned} V_{Eff} = & \left|\frac{\partial \hat{W}}{\partial \Phi^i}\right|^2 + V_0 + (m_{3/2}^2 + \kappa^2 V_0)|\Phi^i|^2 \\ & + m_{3/2}\left(a^2 + \frac{a}{b} - 3\right)(\hat{W} + \hat{W}^*) + \left(m_{3/2}\frac{\partial \hat{W}}{\partial \Phi^i}\Phi^{i*} + h.c.\right) \end{aligned} \tag{1.68}$$

where we have let $V_0 = e^{a^2} m^4 \big((1+ab)^2 - 3b^2\big)$ and defined $\hat{W} = e^{a^2/2} W_{\text{MSSM}}$.

From this expression we can see that we have obtained an expression for global SUSY plus soft breaking terms. A notable feature of this potential is that there are two universal contributions to the soft masses: one from $m_{3/2}$ and the other from $\kappa^2 V_0$. However, when non-canonical Kähler potentials for the hidden and observable sectors are considered there are additional non-universal contributions to the scalar soft masses. In general this is the case for string theories and we will consider these effects in chapter 2.

We now give a quite general expression for the soft-masses in a SUGRA model, but without explicit masses in the superpotential before or after SUSY breaking[10]. The original presentation of these results is to be found in [26] and in [27] the effects of a non-zero cosmological constant were considered. This is defined in terms of the following series expansions for the super and Kähler potentials:

$$W = \hat{W}(h_m) + \frac{1}{6} Y_{\alpha\beta\gamma}(h_m) \Phi^\alpha \Phi^\beta \Phi^\gamma + \cdots , \tag{1.69}$$

$$K = \hat{K}(h_m, h_m^\dagger) + \tilde{K}_{\alpha^*\beta}(h_m, h_m^\dagger) \Phi^{\alpha *} \Phi^\beta + \cdots \tag{1.70}$$

where higher powers of the visible sector fields have been neglected since they disappear in the flat limit and $h_m$ represent the hidden sector fields, with $m$ being an index associated with the hidden sector.

With the F-terms for the hidden sector fields given by $F^m = e^{G/2} \hat{K}^{mn^*} G_{n^*}$ we invert the Kähler metric[11], take the flat limit and the couplings in Eq. (1.69) are replaced by

[10]This simplification foreshadows the string models, in which this will always be the case.

[11]By no means a trivial step, but one that is made possible by the fact that the inverting matrix can be expressed in terms of power series in $m_P^{-1}$[26] and hence solved iteratively.

effective Yukawa couplings[12] given by

$$Y'_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{K}/2} Y_{\alpha\beta\gamma}. \tag{1.71}$$

In addition a soft potential is generated with the following form

$$V_{soft} = m'^2_{\alpha^*\beta} \Phi^{\alpha^*} \Phi^\beta + \left( \frac{1}{6} A'_{\alpha\beta\gamma} \Phi^\alpha \Phi^\beta \Phi^\gamma + \frac{1}{2} B'_{\alpha\beta} \Phi^\alpha \Phi^\beta + h.c. \right) \tag{1.72}$$

where $B'_{\alpha\beta} = 0$ for all low energy fields in our models, since they have no explicit supersymmetric masses and the bilinear coupling is zero. The remaining non-zero soft parameters can be expressed as

$$m'^2_{\alpha^*\beta} = \left( m^2_{3/2} + V_0 \right) \tilde{K}_{\alpha^*\beta} - F^{m^*} \left( \partial_{m^*} \partial_n \tilde{K}_{\alpha^*\beta} - \partial_{m^*} \tilde{K}_{\alpha^*\gamma} \tilde{K}^{\gamma\delta^*} \partial_n \tilde{K}_{\delta*\beta} \right) F^n \tag{1.73}$$

$$A'_{\alpha\beta\gamma} = \frac{\hat{W}^*}{|\hat{W}|} e^{\hat{W}/2} F^m \left[ \hat{K}_m Y_{\alpha\beta\gamma} + \partial_m Y_{\alpha\beta\gamma} - \left( \tilde{K}^{\delta\rho^*} \partial_m \tilde{K}_{\rho^*\alpha} Y_{\delta\beta\gamma} \right.\right.$$
$$\left.\left. (\alpha \leftrightarrow \beta) + (\alpha \leftrightarrow \gamma) \right) \right] \tag{1.74}$$

where $(\alpha \leftrightarrow \beta)$ implies that one should repeat the preceding expression with the indices exchanged.

Note that these expressions, Eqs. (1.71) - (1.74), are un-canonically normalised and the normalisation of the matter fields will modify these expressions. In all the models we consider the Kähler metric for the matter fields will be diagonal, making the canonical normalisation a simple case of rescaling. Later in the thesis, whenever explicit values or expressions are quoted for the parameters they will be in canonical form. This concludes our discussion of SUGRA.

[12] Effective supersymmetric masses can also be obtained if they are contained in the original superpotential, or if the bilinear coupling Kähler potential term has non-zero coupling. Neither of these cases will be realised in any of the models we wish to consider so these terms are omitted.

## 1.5 Neutrino Physics

The majority of this section was compiled from [28, 29, 30]. In the original formulation the SM contains no right-handed neutrinos. This forbids Dirac type masses and Majorana masses can only be generated from non-renormalisable operators, so if we take the SM to be valid up to the Planck scale[13] then we can estimate the size of possible Majorana operators:

$$\frac{LLHH}{M_P} \to \nu_L \nu_L \left(\frac{200^2}{10^{19}}\ \mathrm{GeV}\right) \sim \nu_L \nu_L \left(10^{-6}\ \mathrm{eV}\right) \tag{1.75}$$

which is clearly far too small to be the only contribution to the neutrino's mass. From this we conclude that it is likely that there is some additional physics that generates the observed masses and mixings.

The majority of neutrino model building is focused around the see-saw mechanism [30, 28] essentially due to the observed smallness of neutrinos. Without introducing a very small Yukawa coupling, $\mathcal{O}(10^{-13})$, the Higgs mechanism cannot explain why neutrino masses appear with an upper bound, from tritium $\beta$ decay, of $< 3$ eV [31]. However we will demonstrate that sufficiently small Yukawa couplings can be found within a string theory framework, hence the see-saw mechanism is unnecessary. In this model we will generate neutrinos with pure Dirac masses $m^{\nu}_{\mathrm{LR}} = Y_\nu v_{\mathrm{u}}$ in a SUSY framework, with the following term in the superpotential

$$\lambda_{\nu_{ij}} L_i H_{\mathrm{u}} \nu^{\mathrm{c}}_{\mathrm{R}j}\ . \tag{1.76}$$

We suppose that the neutrino spectrum is hierarchical with the lowest mass eigenvalue being zero. Hence the neutrino mass eigenstates are Dirac spinors with masses determined by their splittings so $m_2 \simeq \Delta_{sol.} \simeq 0.01$ eV and $m_3 = \Delta_{atm.} \simeq 0.05$ eV.

[13] Of course this moves beyond the SM since it requires there to be new physics around the Planck scale

In appendix A we define our mixing conventions, showing how one goes from a theory with general Yukawa matrices to one with diagonal Yukawas, but off-diagonal couplings to the $W$ boson.

Finally a few remarks on the experimental signatures of Dirac neutrinos. If there are no Majorana masses at all then we have pure Dirac neutrinos and we expect to find no neutrinoless double $\beta$ decay (a $\Delta L = 2$ process)[31]. If, instead there are small Majorana masses in the model then we have Pseudo-Dirac neutrinos and the mass spectrum has additional small splittings generated when the eigenstates are rotated into the Majorana basis. This can be seen when considering the mass matrix for one generation

$$\begin{pmatrix} m_L & m_D \\ m_D & m_R \end{pmatrix} \rightarrow \begin{pmatrix} m_a & 0 \\ 0 & m_a \end{pmatrix} \tag{1.77}$$

where

$$m_a = \frac{1}{2}\left(m_L + m_R - \sqrt{4m_D^2 - (m_L - m_R)^2}\right) \tag{1.78}$$

and

$$m_b = \frac{1}{2}\left(m_L + m_R + \sqrt{4m_D^2 - (m_L - m_R)^2}\right) \tag{1.79}$$

hence

$$\delta_m^2 = m_b^2 - m_a^2 \simeq 2(m_L + m_R)m_D, \tag{1.80}$$

when $m_L$, $m_R \ll m_D$. Such a splitting would be hard to detect with terrestrial baselines, but studies have been conducted on the possibility of utilising astrophysical scale baselines to enhance the effects of such a splitting (see [32] and references therein). With a hierarchical neutrino mass spectrum, $m_3 > m_2 > m_1$, the most stringent current bounds on $\delta_m^2$ are [32]

$$\delta_{m_{1,2}}^2 < 10^{-12}\ \mathrm{eV}^2 \tag{1.81}$$

and

$$\delta^2_{m_3} < 10^{-4}\ \text{eV}^2. \tag{1.82}$$

Neutrinoless double $\beta$ decay is in principle possible in this framework. However this will be heavily suppressed since, by assumption, the dominant contribution to the mass eigenstates are the Dirac masses. This coupled with Eqs. (1.82) and (1.80) implies that $(m_L + m_R) < 10^{-3}$ eV. Since neutrinoless double $\beta$ decay is suppressed by the effective mass of the Majorana neutrino this will be very hard to detect[32]

$$\langle m \rangle_{eff} = \frac{1}{2}\sum_j U^2_{ej}\frac{\delta m^2_j}{2m_j} \leq 10^{-4} \tag{1.83}$$

and current limits are $\langle m \rangle_{eff} < \mathcal{O}(1)$ eV.

## 1.6 String Theory

In this section we try to elucidate a few general features of string theory, without delving too deeply into the formalism. Our model building efforts start with four dimensional effective actions consistent with string theory. Therefore, while the full machinery of string theory is of great importance when deriving the low energy theory, it is of less importance to physics below the string scale. This being the case we focus our attention on what string theory has to say about the form of the effective field theory.

Nonetheless a few remarks about the underlying theory are in order, to put the effective field theory in context[14]. Firstly the fundamental object is a one-dimensional relativistic string as opposed to the zero-dimensional point particles found in quantum field theories. Strings come two distinct varieties: closed and open. At the most basic level closed strings are topologically circular whereas open strings are topologically equivalent to a line. A number of remarkable properties spring from considering strings

[14]For textbook accounts of the subject see [18, 33, 34, 35, 36]

instead of particles, perhaps the most startling being that the number of spacetime dimensions is predicted, on grounds of mathematical consistency. The exact prediction depends on the type of string theory considered: if the theory only contains bosons, then the prediction is 26, but if fermions are included, then the prediction is 10. To be more precise we quote the Polyakov [37] action for the superstring [38]

$$S = -\frac{1}{4\pi\alpha'}\int d^2\xi \left(\partial_a x^\mu \partial^a x_\mu - i\overline{\psi}^\mu \rho^a \partial_a \psi_\mu\right) \tag{1.84}$$

where $\xi$ are the two dimensional co-ordinates describing the string's proper time and the position along its length. $x^\mu$ are the "target" space co-ordinates of the string, namely the four spacetime dimensions with which we are familiar and six additional spatial co-ordinates, with which we are not. $\mu$ runs from 0 to 9 where the first four will denote Minkowski space and the remaining six a compact space, discussed below. $\psi^\mu$ are spinorial fields and it can be shown that this action is invariant under world-sheet[15] supersymmetry and $\rho^a$ are two dimensional "gamma" matrices. Finally $\alpha'$ is a dimensionful constant that sets the overall scale of string theory, since it is related to the tension, $T$, of the string by $T = \frac{1}{2\pi\alpha'}$. In addition $\alpha'$ is used to define the string scale $M_* = (\alpha')^{-1/2}$.

There are two good reasons for including fermions into the worldsheet: one, the spectrum of the bosonic string has a tachyonic ground state and two, there are no fermions in the spectrum of purely bosonic strings. The first reason implies that there is no stable vacuum about which the theory can be formulated and the second reason means that the bosonic string cannot describe the SM.

[15]The path of the string through spacetime. Compare with a worldline for a particle.

### Compactified Space

Since strings exist in 10 dimensional spacetime it must be explained why the extra six dimensions do not have obvious observable features. In an effective field theory sense, why is the four dimensional SM a good description of physics up to, at least, the EW scale? Perhaps the simplest possibility is that the extra dimensions are not extended dimensions like the first four, but instead are small, compact spaces, for example a 6-torus. To see how this could work, and clarify what is meant by small, we consider field theory in $4+1$ dimensions where the extra dimension will be compactified on a circle[16]. For simplicity we consider a real scalar field in this framework given by [41]

$$S = \int d^4x \int_0^{2\pi R} dy \mathcal{L} = \int d^4x \int_0^{2\pi R} dy \frac{1}{2} \partial_A \Phi \partial^A \Phi \tag{1.85}$$

where $A = (x^\mu, y)$ and $\Phi$ is a function of $A$. The integration over the $y$ co-ordinate spans the circumference of the circle, $2\pi R$, and we require that $\Phi$ be invariant under $y \to y + 2\pi R$, therefore $\Phi(x,y) = \Phi(x, y+2\pi R)$. This implies that $\Phi$ can be decomposed into Fourier components

$$\Phi(x,y) = \sum_{n=-\infty}^{+\infty} \phi_n(x) e^{iny/R} \tag{1.86}$$

where $\phi(x)_n$ are complex fields subject to $\phi_n^* = \phi_{-n}$ stemming from $\Phi^* = \Phi$. Note this implies that $\phi_0$ is real.

Performing the integral over $y$ in Eq. (1.85) and rescaling $\phi_n \to \frac{\phi_n}{\sqrt{2\pi R}}$ yields

$$S = \int d^4x \left[\frac{1}{2}\partial_\mu \phi_0 \partial^\mu \phi_0\right] + \int d^4x \sum_{n=1}^{+\infty} \left(\partial_\mu \phi_n \partial^\mu \phi_n^* + \frac{n^2}{R^2}\phi_n \phi_n^*\right) \tag{1.87}$$

where the $\phi_n$ fields are termed Kaluza-Klein (KK) states and their masses are proportional to the inverse of the size of the extra dimension.

[16] This idea was originally put forward as a fascinating attempt to unify gravity and electromagnetism, by Kaluza and Klein (KK) [39, 40]

Now it is possible to address the question of how small the extra-dimensions must be, since they govern the masses of the $\phi_n$ fields. If interaction energies are significantly below the mass of the lowest excitation then there will be no KK states in the outgoing particles. While KK states can appear in loops these effects decouple for sufficiently large KK masses, for SM bounds from precision EW tests see, for example [42, 43] with the strongest upper bound being $R^{-1} > 700$ GeV if the SM fields are allowed to propagate in one extra dimension. Since our model includes six extra dimensions this bound can only be taken as a guide, not a hard prediction.

To obtain a phenomenologically interesting low energy spectrum one must consider more complicated spaces than tori since it can be shown [2, 18] that compactification on a 6-torus leads to $\mathcal{N} = 4$ SUSY in four dimensions. $\mathcal{N} \geq 2$ SUSY theories are not chiral since the extra SUSY generators place the left and right-handed fields in the same SUSY multiplet, hence cannot describe the SM. However more complicated spaces, such as orbifolds, allow one to break the extended supersymmetry such that one is left with $D = 4$, $\mathcal{N} = 1$ SUSY, below the compactification scale. In addition to obtaining $\mathcal{N} = 1$ SUSY orbifolds reduce the (very large) gauge symmetry [17] of the theory. These both arise from the fact that not all of the massless four-dimensional states, analogous to $\phi_0$ in the circle case, that arise when one compactifies on a torus appear in the spectrum of an appropriate orbifold. In other words only subsets of the components of gauge and SUSY multiplets are present in the effective four dimensional theory. A phenomenologically interesting example where this is discussed and the low energy spectrum derived is [44].

Finally we note that the size and shape of the extra dimensional space are given by moduli fields. In this thesis we will be concerned with both twisted and untwisted

[17] The origin of this will be discussed in the following subsection.

moduli where twisted moduli are associated with orbifold fixed points and untwisted moduli determine the size of the extra dimensions. A field similar to the moduli is the dilaton which determines the strength of the string couplings, in that it is analogous to a coupling constant in QFT.

### D-Branes and Gauge Symmetries

Besides one dimensional strings there are additional extended objects, known as D-branes [45], generically present in string theory. These objects are associated with the boundary conditions of a string which can be one of two forms, Neumann or Dirichlet. Neumann boundary conditions in a given co-ordinate allow the string end to move along that co-ordinate. In contrast a string is fixed with respect to co-ordinates which obey Dirichlet boundary conditions. Therefore one can define a D$p$ brane as a $p+1$ dimensional hypersurface in which $p+1$ dimensions are Neumann and the string end is fixed by $9-p$ Dirichlet conditions. The number and dimensionality of the branes present in the theory are determined in part by the type of string theory one is working in and in part by the particular compact space utilised to reach four dimensions; see [21] for a discussion of this point.

From a phenomenological perspective D-branes are important because they are subspaces upon which open strings can end. This is vital because open strings can give rise to chiral superfields and hence, potentially, the MSSM (or a viable extension thereof). In addition a stack of coincident D-branes can give rise to a non-Abelian gauge symmetry. One introduces non-dynamical degrees of freedom at the end of a string, known as Chan-Paton factors: one for each brane in the stack. It is possible to show that [38], for $N$ branes, there are gauge fields transforming in the adjoint of U(N) present in the low energy spectrum. In addition open string amplitudes, which must be approximated by interactions in the effective field theory, contain traces over the Chan-Paton factors and will be invariant under U(N) rotations. Thus we can say that the effective field theory possesses a U(N) symmetry.

The last important feature we will mention is T-duality. This is purely string theoretic symmetry whereby the physics is invariant under the exchange $R \leftrightarrow \frac{1}{R}$. This results from the fact that strings are extended objects and hence can wrap around compact dimensions. Since strings are tensionful objects energy is required to stretch a string a distance $L$ given by $E = LT$. Hence to wrap $n$ times around a circle of radius $R$ increases the energy of the string by $nR/\alpha' = nRM_*^2$. Since we expect a tower of KK states in the same spectrum we can see that if $R \leftrightarrow \frac{1}{R}$ we have retained exactly the same mass spectrum: it is simply that KK and winding states have swapped roles. Since a particle theory would only have KK states it could not be symmetric under this exchange. This duality acts non-trivially on any D$p$-branes in the theory since it can be shown [33] that this swaps Dirichlet and Neumann boundary conditions in the direction that the duality acts. We will return to this issue when we have more precisely defined our framework, in chapter 2.

## 1.7 Inflation

The majority of this section was drawn from [46, 47, 48, 49]

In order to discuss inflation [50] we must first sketch the elements of standard big bang cosmology. Let us begin with the cosmological principle: simply stated this is the idea that, on large scales, the universe is homogeneous and isotropic. This principle can be used to derive equations of motion for the evolution of the universe as a whole, but first we must introduce the Robertson-Walker (RW) line element, which is implied by the cosmological principle [2, 51][18]

$$ds^2 = dt^2 - a(t)^2 \left[\frac{dr^2}{1-kr^2} + r^2\left(d\theta^2 + sin^2\theta d\phi^2\right)\right] \tag{1.88}$$

where $a(t)$ is a scale factor that describes the evolution of the universe and $t$ is referred to as the cosmic time, a common time that independent observers could measure irrespective of their position in the universe. $r, \theta$ and $\phi$ are co-moving coordinates, in that they do not change with the evolution of the universe and $k$ is a measure of spatial curvature. If $k = 0$ the universe is flat and if $k = 1$ it is closed whereas $k = -1$ implies that it is open.

The dynamics of matter and energy can be described, on large scales, by the equations of General Relativity, the Einstein equations

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + \Lambda g_{\mu\nu} = -8\pi G_N T_{\mu\nu} \tag{1.89}$$

where loosely put the left hand side describes curvature and the right hand side, matter and energy, with a certain ambiguity in the placement of $\Lambda$. The factor of $g_{\mu\nu}$ suggests that $\Lambda$ belongs with gravity, but from a particle physics point of view it could have a dynamical origin[19]. To put these equations in a more tractable form we must impose the symmetries inherent in the cosmological principle on both the metric and the stress-energy tensor. Utilising the cosmological principle and assuming that the universe can be treated as a perfect fluid (no frictional forces) leads to a stress-energy tensor with the following non-zero elements

$$T_{00} = \rho \quad \text{and} \quad T_{ii} = p g_{ii} \tag{1.90}$$

[18]As we are dealing with the evolution of matter and energy on the largest of scales we make use of General Relativity.

[19]The conflict between possible physical origins and the expected size of $\Lambda$ is known as the cosmological constant problem [52]. We make no attempt to address it in this thesis.

where $\rho$ is the energy density of the fluid, $p$ defines the pressure and $i$ runs over $r, \theta$ and $\phi$.

The metric elements in Eq. (1.90) are those giving rise to the RW line element, Eq. (1.88) and are expressed as

$$g_{00} = 1, \quad g_{rr} = -\frac{a^2}{1 - kr^2}, \quad g_{\theta\theta} = -a^2 r^2, \quad g_{\phi\phi} = -a^2 r^2 \sin^2\theta. \tag{1.91}$$

It can be shown that Eq. (1.90) coupled with Eq. (1.91), when inserted into the Einstein equations, give the Friedmann equations in the presence of a non-zero $\Lambda$:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G_N}{3}(\rho + 3p) + \frac{\Lambda}{3} \tag{1.92}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3}\rho - \frac{k}{a^2} + \frac{\Lambda}{3}. \tag{1.93}$$

The fact that there are now only two Einstein equations is a result of the symmetries implied by the cosmological principle.

We now turn to the issue of inflation in this framework. First let us consider a simple example with one scalar field, the so called inflaton, $\phi$,[20] and the following Lagrangian

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - V(\phi) \tag{1.94}$$

where we do not specify the exact form of $V(\phi)$ at this stage. For the moment, we require that for a particular trajectory of $\phi$ the dominant contribution to the energy density is the potential, i.e.

$$V(\phi) \gg (\partial_\mu \phi)^2 \tag{1.95}$$

for a range of values of $\phi$ along its trajectory. If this is the case then Eq. (1.93) reduces to

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G_N}{3} V(\phi) \tag{1.96}$$

[20]Not to be confused with the angle $\phi$ in the RW metric.

and it is easy to show that

$$a(t) \propto e^{Ht} \tag{1.97}$$

solves Eq. (1.96) where Eqs. (1.96) and (1.93) show that $H$ is a constant while Eq. (1.95) holds. Thus we see that a universe dominated by a slowly varying potential will exponentially expand, i.e. undergo a period of inflationary expansion.

Next we will consider the conditions under which the potential can give rise to inflation, namely the slow-roll conditions. The action is given by

$$\int \sqrt{-\det(g)}d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right) \tag{1.98}$$

where, since we are interested in gravitational interactions, we have included the metric in $\sqrt{-\det(g)}$.

The equation of motion of the scalar field is

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0 \tag{1.99}$$

where it is clear that, for positive $H$, the $3H\dot{\phi}$ term acts analogously to a frictional force, counteracting the effects of $V'(\phi)$.

For inflation to last $\ddot{\phi}$ must provide a negligible contribution [21] to Eq. (1.99) hence

$$3H\dot{\phi} \simeq -V'(\phi). \tag{1.100}$$

Re-deriving the acceleration term from this equation and requiring consistency with the assumption that $\ddot{\phi} \ll 3H\dot{\phi}$ in Eq. (1.99) leads to the famous slow roll conditions

$$\eta \equiv \frac{M_P^2}{8\pi}\left|\frac{V''(\phi)}{V(\phi)}\right| = m_P^2\left|\frac{V''(\phi)}{V(\phi)}\right| \ll 1 \tag{1.101}$$

$$\epsilon \equiv \frac{M_P^2}{16\pi}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 = \frac{m_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 \tag{1.102}$$

[21] Strictly speaking this is not necessary for all inflation models. In [53] so called "fast-roll" inflation is discussed whereby the slow-roll conditions are violated, but inflationary expansion is still obtained. The model presented in this thesis gives rise to slow-roll inflation

where the reduced Planck mass is given by $m_P = M_P/\sqrt{8\pi}$ and if these conditions are satisfied then inflation can occur.

One final issue is the generation of structure in the universe. At some point the cosmological principle must break down since when one considers small (from a cosmological viewpoint) scales, such as the level of galaxies and below inhomogeneities and anisotropies are manifest. For inflation models it is shown in [54] and references therein that

$$\delta_H^2(k) = \frac{32}{75}\frac{V_N}{m_P^4}\epsilon_N^{-1} \tag{1.103}$$

where in this instance the subscript $N$ denotes that this quantity is evaluated $N$ e-folds before the end of inflation, when the scale $k$ left the horizon.

### Hybrid Inflation

Since the model we construct in chapter 3 is one of hybrid inflation we will use a hybrid inflation model to illustrate some of this section's points. This class of models was introduced in [55, 56] and made use of the following scalar potential

$$V(\sigma, \phi) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 + \frac{m^2}{2}\phi^2 + \frac{g^2}{2}\phi^2\sigma^2 \tag{1.104}$$

where $\lambda$ and $g$ are dimensionless couplings while $M$ is coupling of mass dimension one. For simplicity we assume that $\phi$ and $\sigma$ are real scalar fields.

First let us consider the potential when $\phi = 0$,

$$V(\sigma) = \frac{1}{4\lambda}(M^2 - \lambda\sigma^2)^2 \tag{1.105}$$

this shows that, at the origin, the $\sigma$ field is unstable in exactly the same way as the Higgs field in Eq. (1.3) and is minimised away from the origin. If, instead, $\sigma = 0$, then Eq. (1.104) assumes the particularly simple form

$$V(\phi) = \frac{m^2}{2}\phi^2 \tag{1.106}$$

which evidently minimises at the origin.

Finally, to see the whole picture, we consider the coupling $\frac{g^2}{2}\phi^2\sigma^2$ that contributes to the effective mass of $\phi$ or $\sigma$, respectively, if $\sigma$ or $\phi$ is non-zero. Since this is always a positive contribution it can give $\sigma$ a positive effective mass squared if the inflaton is above a critical point, $\phi^2 > \phi_c^2 = \frac{M^2}{g^2}$. If $|\phi| > |\phi_c|$ then Eq. (1.104) is positive semi-definite and is minimised for $\sigma = 0$. The critical value of the inflaton marks the transition between a region in which $\sigma$ is stable and an unstable region where its potential takes a form similar to Eq. (1.105). Since $\frac{g^2}{2}\phi^2\sigma^2$ adds to the inflaton's mass squared its potential steepens as $\sigma$ increases, hastening its arrival at $\phi = 0$. This signals the end of inflation.

It is now possible to postulate an inflationary trajectory for $\phi$ and $\sigma$ and see if this has a period of slow roll. The smallest possible effective mass squared for $\phi$ is obtained when $\sigma = 0$ hence the inflaton must start its evolution above $\phi_c$, and we require the energetically favoured position of $\sigma = 0$. Hence the slow roll conditions can be expressed as

$$\epsilon = \frac{m_P^2 m^4 \phi^2}{2\left(M^4/(4\lambda)\right)^2} = \frac{m_P^2 m^4 \phi^2}{2V(0)^2} \ll 1 \tag{1.107}$$

$$\eta = \frac{m_P^2 m^2}{M^4/(4\lambda)} = \frac{m_P^2 m^2}{V(0)} \ll 1 \tag{1.108}$$

where we have made the replacement $\frac{M^4}{4\lambda} = V(0)$. The $\eta$ condition is independent of the inflaton's trajectory, depending only on the ratio of the massive parameters in the model. As $\epsilon$ depends on $\phi$ we can impose

$$\epsilon \geq \frac{m_P^2 m^4 \phi_c^2}{2V(0)^2} = \frac{2\sqrt{\lambda}}{g^2} \frac{m_P^2 m^4}{V(0)^{3/2}} \tag{1.109}$$

Now it is clear that satisfying inflationary requirements is a matter of choosing appropriate parameters for the model, of which some interesting choices are discussed in [56].

Strictly speaking $\eta$ and $\epsilon$ must also be evaluated a number of e-folds[22] before the end of inflation. While this has no impact on $\eta$ it can modify the $\epsilon$ condition. We omit this here, but will return to this matter in chapter 3.

[22]The number of e-folds, $N$, in a given expansion is defined by $N = \ln(a_{end}/a_{start})$.

# Chapter 2

# String Model Building

In this chapter we lay out the string framework in detail. We focus on the parts of the framework most relevant to the models we will construct in the following chapters, but include more general information wherever it is relevant and does not draw the reader too far from the essential points.

## 2.1 D-Brane Framework

This thesis is concerned primarily with utilising string theory to solve particle physics problems and attempts to do this study at a reasonably high level. Specifically we tried not to delve deep into one explicit string construction, but rather keep our results as general as possible. Obviously some choices of string model had to be made and the first of these was to work in type I string theory. This necessarily leads to the inclusion of D-Branes within the spectrum [45]. As mentioned in section 1.6 the underlying orbifold determines the D-Brane setup needed for consistent vacua. However, in this thesis we consider the most general setup of D5 and D9 branes, without specifying a specific orbifold. To present an intuitive picture for branes and states in the theory we refer to fig. 2.1 where the locations of the various string states are shown. The

| | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|---|
| $M^4$ | – | – | – | – | . | . | . | . | . | . |
| $D5_1$ | – | – | – | – | – | – | . | . | . | . |
| $D5_2$ | – | – | – | – | . | . | – | – | . | . |
| $D5_3$ | – | – | – | – | . | . | . | . | – | – |
| D9 | – | – | – | – | – | – | – | – | – | – |

Table 2.1: D-Brane notation: columns are labelled by the dimension they correspond to in ten dimensional spacetime.

two orthogonal directions in fig. 2.1 each represent two compact spatial dimensions with identical radii, around which the D-branes wrap. There are two more compact dimensions orthogonal to the first four, which are not presented in the figure since they do not directly affect the models in this thesis. Finally we note that a useful notation for dealing with D-branes is the following. The spacetime dimensions in which a string end is free to move are denoted by dashes and the dimensions in which they are fixed are given by dots. This allows us to represent Minkowski space, the D5 branes and the D9 branes in table 2.1.

Given this starting point the effective low energy Lagrangian can be derived for the D-Brane setup. This can be done because in string theory, unlike $\mathcal{N} = 1$ SUGRA, the superpotential, Kähler potential and gauge kinetic functions are not arbitrary, but instead are predicted. The Kähler potential for the untwisted moduli and the matter

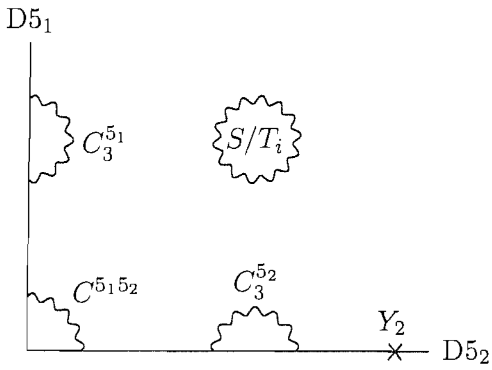


Figure 2.1: Schematic representation of two stacks of D5-branes. The stacks of branes overlap in Minkowski space, but are orthogonal in the compactified dimensions. The $C$ states correspond to chiral matter fields, $S$ the dilaton, $T_i$ the untwisted moduli and $Y_2$ is a twisted modulus (introduced in section 4.2) localised within the extra dimensions, but free to move in Minkowski space. We have only presented the string states involved in our construction: for a more complete picture see Figure 1 in [57].

fields can be shown to be [21]

$$K = -\ln\left(S + S^* - \sum_{i=1}^{3} |C_i^{5_i}|^2\right) - \sum_{i=1}^{3} \ln\left(T_i + T_i^* - |C_i^9|^2 - \sum_{j,k=1}^{3} d_{ijk}|C_j^{5_k}|^2\right) \tag{2.1}$$
$$+ \frac{1}{2}\sum_{i,j,k=1}^{3} d_{ijk}\frac{|C^{5_j 5_k}|^2}{(S+S^*)^{1/2}(T_i+T_i^*)^{1/2}} + \frac{1}{2}\sum_{i,j,k=1}^{3} d_{ijk}\frac{|C^{95_i}|^2}{(T_j+T_j^*)^{1/2}(T_k+T_k^*)^{1/2}}$$

and, to lowest order in the matter fields[1],

$$K = -\ln(S+S^*) - \sum_{i=1}^{3}\ln(T_i+T_i^*) + \sum_{i=1}^{3}\frac{|C_i^9|^2}{(T_i+T_i^*)} + \sum_{i=1}^{3}\frac{|C_i^{5_i}|^2}{(S+S^*)} \tag{2.2}$$
$$+ \sum_{j,k=1}^{3} d_{ijk}\frac{|C_j^{5_k}|^2}{(T_i+T_i^*)} + \frac{1}{2}\sum_{i,j,k=1}^{3} d_{ijk}\frac{|C^{5_j 5_k}|^2}{(S+S^*)^{1/2}(T_i+T_i^*)^{1/2}}$$
$$+ \frac{1}{2}\sum_{i,j,k=1}^{3} d_{ijk}\frac{|C^{95_i}|^2}{(T_j+T_j^*)^{1/2}(T_k+T_k^*)^{1/2}}$$

where the $C$-terms represent the low energy excitations of strings starting and ending on D-Branes and $d_{ijk} = |\epsilon_{ijk}|$ and $S$ and $T_i$ are, respectively, the dilaton and the untwisted moduli. The dilaton and moduli are $D = 4$ closed string singlets of the following form

[1] A valid approximation in our case since the dilaton/moduli vevs are at least $M_P$.

[21]

$$S = \frac{2R_1^2 R_2^2 R_3^2}{\lambda_I (\alpha')^3} + i\theta \tag{2.3}$$

and

$$T_i = \frac{2R_i^2}{\lambda_I \alpha'} + i\eta_i, \tag{2.4}$$

where $\lambda_I$ is the $D = 10$ dilaton, $\theta$ and $\eta_i$ are untwisted Ramond-Ramond closed string states that are included for completeness; only the real components of $S$ and $T_i$ appear in the rest of the thesis.

As such both their gauge interactions and the form of the superpotential will be constrained. First we consider the gauge interactions. As shown in chapter 1.6 the gauge groups of the string arise from the stacks of branes the open strings end on. We note that, as demonstrated in [44], the particular groups living on a given brane are given by the underlying compactification. In this thesis we do not try to find the specific compactification that can lead to our model. Instead we motivate its existence by appealing to similar models in the literature[2] and simply require the correct gauge groups and particle spectrum. For the gauge groups we assume that, in the low energy regime, the unbroken symmetry is that of the MSSM: $\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3)$. How this is arrived at is beyond the scope of the thesis: we only impose that, in the effective theory, there are no exotic remnants. T-Duality requires that a copy of this group appears on each of the stacks of branes. As a result, in full generality, we have four gauge groups and, in principle, four different gauge couplings: $(\mathrm{U}(1)_Y \times \mathrm{SU}(2)_L \times \mathrm{SU}(3))^4$ with couplings $g_{5_1}$, $g_{5_2}$, $g_{5_3}$ and $g_9$. Anticipating the models constructed in chapters 3 and 5 we restrict our focus to just two of the possible gauge groups, those associated with

[2]There are numerous examples of Standard Model and MSSM-like models [44, 58] and we propose a high level study, the successes of which should motivate more explicit constructions.

the $D5_1$ and $D5_2$ branes. We denote the location of the group with a superscript, so an $SU(3)^{5_2}$ is the SU(3) associated with the $D5_2$ brane.

The superscripts of a given $C$-term denote the location of the ends of the string and hence which representations are allowed for any fields associated with that term. For example a $C^{5_1 5_2}$ field could transform as $(\mathbf{1}, \mathbf{3}; \mathbf{2}, \mathbf{1})$, [3] a bi-fundamental representation of $SU(3)^{5_1} \times SU(2)^{5_2}$, whereas a field like $C_3^{5_2}$ could transform like $(\mathbf{1}, \mathbf{1}; \mathbf{2}, \mathbf{3})$ in the same way as a quark doublet. Note that the subscript denotes fields with different gauge transformation properties and, as shown in Eq. (2.5), different Yukawa couplings. While it is possible to consider models where the gauge group is drawn from the groups of several different branes we will be considering models where the MSSM gauge group is entirely contained within one stack of branes. We will return to this question in chapter 4.

The superpotential can be derived by considering three point amplitudes of open string states [59] and can be shown to be, before canonical normalisation:

$$W = \left( C_1^9 C_2^9 C_3^9 + C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1} + \sum_{i=1}^{3} C_i^9 C^{95_i} C^{95_i} \right) + \sum_{i,j,k=1}^{3} \left( C_1^{5_i} C_2^{5_i} C_3^{5_i} \right.$$
$$\left. + C_i^{5_i} C^{95_i} C^{95_i} + d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} + \frac{1}{2} d_{ijk} C^{5_j 5_k} C^{95_j} C^{95_k} \right) \tag{2.5}$$

where all the Yukawa couplings are order one. This is in contrast to more complicated setups in which the branes do not intersect at one point (for example see [60]) resulting in a geometric suppression: $e^{-A/\alpha'}$ where $A$ is the area spanned by the intersection. In our case the branes are all assumed to intersect at one point in the extra-dimensional space and do not feel this suppression.

[3] Our convention is to quote the representations in ascending order of both their associated groups and branes. The example in the text has its $SU(2)^{5_1}$ representation followed by its $SU(3)^{5_1}$ rep. and after the semi-colon the sequence repeats for the $D5_2$ brane.

The final pieces of the puzzle are the gauge kinetic functions which T-duality invariance requires to have the following forms [21]:

$$f_9 = S \text{ and } f_{5_i} = T_i. \tag{2.6}$$

In fact, in the presence of twisted moduli these expressions are modified as follows [61, 62, 63].

$$f_{9\alpha} = f_9 + \sum_k c_\alpha^k Y_k \text{ and } f_{5_i\beta} = f_{5_i} + \sum_k c_\beta^k Y_k \tag{2.7}$$

where $c^k_{\alpha,\beta}$ are model dependent coefficients, $\alpha$ and $\beta$ run over the different gauge groups and $k$ runs over all possible twisted moduli. For the moment we neglect the contributions of the twisted moduli, making the assumption that the dilaton and untwisted moduli provide the dominant contributions to the gauge kinetic functions. This assumption will be re-evaluated in section 4.2.1.

From Eq. (2.6) it can be shown [15] that the gauge couplings on the stacks of branes are given by

$$g_9^2 = \frac{4\pi}{\mathrm{Re}(S)} \text{ and } g_{5_i}^2 = \frac{4\pi}{\mathrm{Re}(T_i)}. \tag{2.8}$$

It is possible to show [21] how the superpotential changes after canonically normalising the Kähler potential and taking the flat limit, in which $M_P \to \infty$. We quote this here:

$$\begin{aligned} W =& g_9 \left( C_1^9 C_2^9 C_3^9 + C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1} + \sum_{i=1}^{3} C_i^9 C^{95_i} C^{95_i} \right) + \sum_{i,j,k=1}^{3} g_{5_i} \Big( C_1^{5_i} C_2^{5_i} C_3^{5_i} \\ &+ C_i^{5_i} C^{95_i} C^{95_i} + d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} + \frac{1}{2} d_{ijk} C^{5_j 5_k} C^{95_j} C^{95_k} \Big). \end{aligned} \tag{2.9}$$

It is illustrative to briefly detour, before returning to Eqs. (2.1) and (2.5), to canonically normalise more general Kähler potentials and superpotentials [27] given by Eq. (1.70) and Eq. (1.69). To obtain canonical kinetic terms we must redefine our fields such that $K_{\alpha^*\beta} \rightarrow \delta_{\alpha^*\beta}$. Let us simplify matters slightly by assuming that, as is the case in (2.1), the Kähler metric for the matter fields is diagonal, but different from the identity. The relevant part of the Kähler potential can be re-expressed as

$$K_{\alpha^*\beta}\Phi^{\alpha^*}\Phi^{\beta} = K_{\alpha}|\Phi^{\alpha}|^2. \tag{2.10}$$

With this assumption we can make a simple re-scaling

$$\Phi^{\alpha} \rightarrow \frac{1}{(K_{\alpha})^{1/2}}\Phi^{\alpha} \tag{2.11}$$

and obtain canonical kinetic terms.

This change manifests itself in the superpotential and the soft terms with the final Yukawa couplings being given by

$$\tilde{Y}_{\alpha\beta\gamma} = Y'_{\alpha\beta\gamma}(K_{\alpha}K_{\beta}K_{\gamma})^{-1/2} = e^{\hat{K}/2}(K_{\alpha}K_{\beta}K_{\gamma})^{-1/2}Y_{\alpha\beta\gamma}. \tag{2.12}$$

where $Y'_{\alpha\beta\gamma}$ is the effective Yukawa coupling after symmetry breaking given by Eq. (1.71).

At the end of our detour we have an expression for the physical, low energy couplings in terms of the high energy couplings and elements of the Kähler potential. We can now make use of this expression to obtain the canonical form for the superpotential given in [21].

By way of demonstration we consider two couplings, the only two couplings we make use of in this model: $g_{5_1}C_3^{5_1}C^{5_15_2}C^{5_15_2}$ and $g_{5_2}C_3^{5_2}C^{5_15_2}C^{5_15_2}$. Taking the values for $\hat{K}$, $K_{C_3^{5_1}}$ and $K_{C^{5_15_2}}$ from Eq. (2.2) we obtain

$$Y_{C_3^{5_1}C^{5_15_2}C^{5_15_2}} = e^{\hat{K}/2}(K_{C_3^{5_1}}K^2_{C^{5_15_2}})^{-1/2} \tag{2.13}$$

$$= e^{\left(-\left(\ln(S+S^*)+\sum_{i=1}^3\ln(T_i+T_i^*)\right)/2\right)}((T_2+T_2^*)(S+S^*)(T_3+T_3^*))^{-1/2}$$

$$= (T_1+T_1^*)^{-1/2} = g_{5_1}(8\pi)^{-1/2} \tag{2.14}$$

which is, up to a factor of $(8\pi)^{-1/2}$, same result as presented in [21], given in Eq. (2.5). This factor is likely due to an inconsistent definition of the gauge kinetic function in

[21], the definition in [64] of $\mathrm{Re}(f) = 1/g^2$ improves matters by $\sqrt{4\pi}$. Also [64] perform an analogous calculation for the heterotic string obtaining, as we do, a dependence on $S+S^*$ in the effective superpotential. Since it is $S+S^*$ that appears rather than Re(S) this introduces an additional factor of $\sqrt{2}$ not present in the [21] superpotential. So the end result is that the superpotential, Eq. (2.9), should be divided by $\sqrt{2}$. While this discrepancy is odd it will not materially affect any of the conclusions of the thesis due to our freedom to set the value of the radii. As a result we use the results of [21], aware that the values of the $S/T_i$ moduli may be subject to order one corrections. Note, by symmetry the other analogous terms $C_k^{5_i} C^{5_i 5_j} C^{5_i 5_j} \rightarrow g_{5_i}(2)^{-1/2} C_k^{5_i} C^{5_i 5_j} C^{5_i 5_j}$

With this $\sqrt{8\pi}$ uncertainty we have the following expression

$$W \supset \sum_{i,j,k=1}^{3} \left( g_{5_i} d_{ijk} C_j^{5_i} C^{5_i 5_k} C^{5_i 5_k} \right) \tag{2.15}$$

where we have only presented the class of terms that will be relevant for our model building. The complete set can be derived using the rules laid out above.

### 2.1.1 Soft Terms

Before we discuss the soft terms in this framework we need to introduce a convenient parametrisation for the F-terms, in vector form, given by [57]

$$F = \sqrt{3} C m_{3/2} (P\Theta) \tag{2.16}$$

$$F^\dagger = \sqrt{3} C m_{3/2} (\Theta^\dagger P^\dagger) \tag{2.17}$$

where $C^2 = 1 + \frac{V(0)}{3m_{3/2}^2}$ [21] and $\kappa = 1$ has been imposed with $V(0)$ the vev of the potential when all of the hidden sector fields obtain their vevs and the visible sector fields are set to zero. Also $\Theta$ is a vector, defined shortly, and $P$ is defined as the canonically normalising matrix, $P^\dagger K_{\alpha^* \beta} P = 1$ where $\alpha$ and $\beta$ run over the hidden sector fields, but not the low energy fields. For this parametrisation to be of use it must yield the correct vacuum expectation value for the scalar field potential and be possible to freely choose how much each field contributes to the SUSY breaking. Since $\Theta$ is a vector with trigonometric components, $\Theta_i$, such that $\sum_i \Theta_i^2 = 1$ and $|\Theta_i| \leq 1 \; \forall i$ this parametrisation is valid with $\langle V \rangle = V(0)$ shown in [57].

We now discuss the soft terms in Type I string theory under the assumption that the SUSY contributions come solely from the F-terms of the $S/T_i$ fields given by

$$F \equiv \begin{pmatrix} F^S \\ \\ F^i \end{pmatrix} = \sqrt{3} C m_{3/2} \begin{pmatrix} (K_{SS^*})^{-1/2} \sin\theta e^{-i\gamma_S} \\ \\ (K_{T_i T_i^*})^{-1/2} \cos\theta \Theta_i e^{-i\gamma_{T_i}} \end{pmatrix} \tag{2.18}$$

where since $K_{\alpha^*\beta} = K_{\alpha^*}\delta_{\alpha^*\beta}$, $P$ is a diagonal, rescaling matrix and we obtain

$$F^S = \sqrt{3} C m_{3/2} (S + S^*) \sin\theta e^{-i\gamma_S} \tag{2.19}$$

$$F^i = \sqrt{3} C m_{3/2} (T_i + T_i^*) \cos\theta \Theta_i e^{-i\gamma_i} \tag{2.20}$$

Now all that remains to be done is to substitute these F-terms in to the soft-term expressions given in section 1.4.3, Eqs. (1.73) and (1.74) to obtain the soft masses in this framework. The soft masses that will prove relevant to our model building efforts are [21]

$$m^2_{C^{5_i}_j} = m^2_{3/2} + V(0) - 3C^2 m^2_{3/2} \Theta_k^2 \cos^2\theta \tag{2.21}$$

$$m^2_{C^{5_i 5_k}} = m^2_{3/2} + V(0) - \frac{3}{2} C^2 m^2_{3/2} \left(\sin^2\theta + \Theta_j^2 \cos^2\theta\right) \tag{2.22}$$

and we will make use of the following trilinear couplings

$$A_{C^{5_i}_j C^{5_i 5_k} C^{5_i 5_k}} = -\sqrt{3} C m_{3/2} \Theta_i \cos\theta e^{-i\gamma_i}. \tag{2.23}$$

Given this parametrisation it is easy to show that the following sum rule must be obeyed

$$m^2_{C^{5_i}_j} + 2 m^2_{C^{5_i 5_k}} = |A_{C^{5_i}_j C^{5_i 5_k} C^{5_i 5_k}}|^2 + 2V(0) \tag{2.24}$$

For a similar analysis using the heterotic string see [27].

## 2.2 Asymmetric Compactifications

For our purposes we need to consider a highly anisotropic compactification[4]. This is necessary for the models to be constructed in chapters 3 and 5, since they both require Yukawa couplings of order $10^{-10}$. Since gauge and Yukawa couplings are very closely linked the requirement of $\mathcal{O}(1)$ gauge couplings for the MSSM gauge group means there is a ratio between Yukawa couplings of approximately $10^{10}$. In this thesis we propose that this ratio arises due to an asymmetric compactification in which two radii become large and four remain small. From Eqs. (2.3) and (2.4) with Eq. (2.8) we see that

$$g_{5_i}^2 = \frac{2\pi\lambda_I\alpha'}{R_i^2} \text{ and } g_9^2 = \frac{2\pi\lambda_I(\alpha')^3}{R_1^2R_2^2R_3^2}. \tag{2.25}$$

As is generally the case in extra-dimensional theories the effective four dimensional Planck mass depends on the higher dimensional Planck mass and the volume of the extra dimensions:

$$M_P^2 = \frac{8M_*^8R_1^2R_2^2R_3^2}{\lambda_I^2} \tag{2.26}$$

and this relation will prove important when deriving the gauge couplings.

At this stage, having specified none of the parameters, we are free to choose the couplings as desired. Selecting the small coupling to be $g_{5_1} \sim 10^{-10}$ and the MSSM gauge coupling to be $g_{5_2} \sim 1$ requires $\frac{R_1}{R_2} \sim 10^{10}$. While these two couplings do not fix the third ratio we see, via Eq. (2.26), that

$$R_3^2 = \frac{M_P^2 g_{5_1}^2 g_{5_2}^2}{32\pi^2 M_*^4} \tag{2.27}$$

so we see the question of what size is $R_3$ is equivalent to asking what is the string scale. This expression is only true when there are no twisted moduli in the spectrum, or their

[4]For a recent example of a explicit string construction which naturally incorporates anisotropic extra dimension see [65] which is analogous to the KKLT construction [66]

expectation values are zero. In section 4.2 we will need to go beyond this assumption and, while it will not turn out to a have far reaching consequences, we quote the precise expression

$$R_3^2 = \frac{M_P^2}{2M_*^4 \mathrm{Re}(T_1)\mathrm{Re}(T_2)}. \tag{2.28}$$

The final parameter we can choose is $\lambda_I$, the $D = 10$ dilaton, that governs the strength of the of string loop corrections. This can be expressed as a function of the couplings using Eq. (2.8) and also in terms of the $S/T_i$ fields

$$\lambda_I = \frac{g_{5_1} g_{5_2} g_{5_3}}{2\pi g_9} = \left( \frac{2\mathrm{Re}(S)}{\mathrm{Re}(T_1)\mathrm{Re}(T_2)\mathrm{Re}(T_3)} \right)^{1/2} \tag{2.29}$$

We also note that we can re-express the relationship between $M_*$ and $M_P$ in terms of the gauge couplings, and equivalently in terms of the dilaton and moduli, as follows:

$$32\pi^2 \left( \frac{M_*}{M_P} \right)^2 = g_{5_1} g_{5_2} g_{5_3} g_9 = \frac{16\pi^2}{\left(\mathrm{Re}(T_1)\mathrm{Re}(T_2)\mathrm{Re}(T_3)\mathrm{Re}(S)\right)^{1/2}} \tag{2.30}$$

Fixing the $D = 4$ Planck mass and $g_{5_1}$ and $g_{5_2}$ means that $M_*$ is determined by the remaining two couplings $g_{5_3}$ and $g_9$ which in turn fix the value of $\lambda_I$. From this we see that if we input all four couplings, and $M_P$, then we completely determine the string scale, string coupling constant and hence the radii. Of course the converse is true, and perhaps is a more physical perspective. However, we neither claim to know the origin of the particular compact space nor address the question of how the radii we require are reached. Instead we input the couplings we require and select order 1 parameters for the remaining two couplings[5].

[5]These couplings only play a very minor role in our model building, beyond the fixing of the radii discussed here. They can be used to fix $\lambda_I$ to be very small, $\sim 10^{-10}$, in order to ensure that stringy effects are dominated by the tree level contribution. This improves the validity of our effective field theory approach.

Selecting $g_{5_1} = 10^{-10}$, $g_{5_2} = \sqrt{\frac{4\pi}{24}}$ and $g_{5_3} = g_9 = 2$ fixes, via Eq. (2.30), $M_* \approx 10^{13}$ GeV. Since this choice and Eq. (2.29) fix $\lambda_I \approx 10^{-11}$ we see that, from Eq. (2.25), the inverse radii are fixed to be

$$R_1^{-1} \approx 10^8 \text{ GeV}, \; R_2^{-1} \approx 10^{18} \text{ GeV and } R_3^{-1} \approx 10^{18}. \text{ GeV} \tag{2.31}$$

These values will receive very minor corrections when we consider the impact of twisted moduli, in section 4.2, but the order of magnitude estimates remain intact. We see that there are two radii at just above the Planck length and one a factor of $10^{10}$ larger, and that it is this anisotropy that, as expected, generates the necessary hierarchy in Yukawa couplings. Anisotropic compactifications are not without precedent in string theory, see for example [65].

These radii are all too small to have Kaluza-Klein (KK) or winding modes that will be readily excitable at collider energies. The winding modes of $R_1$ are $\approx n10^{18}$ GeV and $R_2$ and $R_3$ have winding modes of $\approx n10^8$ GeV. The KK modes for $R_1$ are $\approx n10^8$ GeV and $R_2$ and $R_3$ are $\approx n10^{18}$ GeV.

For the majority of this thesis the exact values of the coupling will not prove to be important so, unless specifically stated, we will use the following approximate expressions for the couplings:

$$g_{5_1} \sim 10^{-10} \text{ and } g_{5_2} \sim g_{5_3} \sim g_9 \sim 1. \tag{2.32}$$

# Chapter 3

# Inflationary Solution to the Strong CP and $\mu$ Problems

This chapter concerns an inflationary particle physics model, inspired by Type I string theory. The model uses the vev of the inflaton post inflation to both solve the strong CP problem via the Peccei-Quinn mechanism and generate the supersymmetric Higgs mass term, $\mu$. We will show that this gives a high-scale prediction for the $\mu$ mass, expressed in terms of the soft SUSY breaking parameters. Some of the values of parameters for this model are taken as assumptions, but many of these will be shown, in chapter 4, to have a natural origin within Type I string theory.

Our goal when we embarked on this study was to find a simple model, consistent with Type I string theory, and study its predictions. Our approach was to first of all start with an interesting and phenomenologically viable model and then, in Chapter 4, see if this model is compatible with Type I string theory.

## 3.1 The Model

The model we considered was inspired by the $\phi$NMSSM model of Bastero-Gil and King [67]:

$$W_{\phi\text{NMSSM}} = \lambda N H_u H_d - \kappa \phi N^2 + \left(W_{\text{MSSM}} - \mu H_u H_d\right) \tag{3.1}$$

where $\lambda$ and $\kappa$ are Yukawa couplings of order $10^{-10}$. $W_{\text{MSSM}}$ is the superpotential for the MSSM as given in section 1.4.1. In this model there is no necessity for these couplings to be equal, only that they be small. This will turn out not to be the case for the model we build in this chapter. Briefly summarised, the $\phi$NMSSM is consistent with the MSSM after inflation, solves the strong CP and $\mu$ problems, provides an inflation model with a spectral index very close to unity and has the curvature perturbations being generated by the inflaton, $\phi$. Clearly this summary does not do justice to the model, but more details can be found in [67].

Our model uses the following superpotential

$$W = \lambda \phi H_u H_d + \kappa \phi N^2 + y_t Q_3 H_u t_R^c + y_b Q_3 H_d b_R^c \tag{3.2}$$

leading to a SUSY potential of

$$V_{SUSY} = |\lambda H_u H_d + \kappa N^2|^2 + |\lambda \phi H_u + y_b Q_3 b_R^c|^2 + |\lambda \phi H_d + y_t Q_3 t_R^c|^2 + 4\kappa^2 |\phi N|^2 \tag{3.3}$$

and a soft potential of

$$\begin{aligned} V_{soft} = {} & V(0) + \lambda A_\lambda \phi H_u H_d + \kappa A_\kappa \phi N^2 + y_t A_{y_t} Q_3 H_u t_R^c + y_b A_{y_b} Q_3 H_d b_R^c + h.c. \\ & + m_0^2 \left(|H_u|^2 + |H_d|^2 + |N|^2 + |t_R^c|^2 + |b_R^c|^2\right) + m_Q^2 |Q_3|^2 - m_\phi^2 |\phi|^2 \end{aligned} \tag{3.4}$$

where $\phi$ and $N$ are, respectively, the inflaton and waterfall fields and are singlets of the MSSM gauge group. It is these two fields that provide both the mechanism for inflation and the means to end it since $\phi$ experiences a period of slow roll and $N$ becomes unstable at a given point along $\phi$'s trajectory, ending inflation. The Higgs fields $H_u$ and $H_d$ have standard MSSM quantum numbers and are not involved in inflation. The dimensionless couplings $\lambda$ and $\kappa$ are both $\mathcal{O}(10^{-10})$ parameters, and while there is no field theoretic reason for them to be equal, the string construction will require $\lambda = \kappa$, which we now enforce. The remaining dimensionless couplings $y_t$ and $y_b$ are both taken to be order one and again they will identified in the string construction. The scalar soft mass, $m_0$, is common to $N$, $H_u$, $H_d$, $t_R^c$ and $b_R^c$ at the string scale and is taken to be approximately a TeV. The quark doublets' soft masses $m_Q$ are also approximately a TeV, but can differ from $m_0$. We allow a lighter, negative soft mass squared for the inflaton in order to satisfy the slow roll conditions and yield an acceptable inflationary trajectory. We require that the magnitude of the soft mass of order an MeV or less, as will be demonstrated in section 3.1.2. Again these are assumptions to be justified in the string construction in section 4. It should be noted that we do not explicitly re-create the entirety of the rest of the MSSM in this thesis, instead we concentrate on the model of inflation and, in chapter 5, the Dirac neutrino model. The quark Yukawas included are not an attempt to provide a realistic quark sector, they merely show that quark masses can be realised in this framework.

The model is one of inverted hybrid inflation, since the negative mass squared for $\phi$ will result in an inflationary trajectory rolling away from the origin. It solves the $\mu$ problem in a way similar to that of the NMSSM as shown in section 1.4.2, i.e. the $\phi$ field obtains a vev post inflation and hence generates a $\mu = \lambda \langle \phi \rangle$. This model also replaces the discrete $\mathbb{Z}_3$ symmetry of the NMSSM with a continuous $\mathrm{U}(1)_{PQ}$ symmetry and the invariance of the two terms in Eq. (3.2) clearly leads to the constraint that

$$Q^{PQ}(\phi) + Q^{PQ}(H_u) + Q^{PQ}(H_d) = 0 \text{ and } Q^{PQ}(\phi) + 2Q^{PQ}(N) = 0 \tag{3.5}$$

where the usual charges of the quarks, leptons and Higgs [22] lead to the following charges for $\phi$ and $N$: $Q^{PQ}(\phi) = -2$ and $Q^{PQ}(N) = 1$. The entire spectrum will be laid out in table 4.1 at the end of the supersymmetric construction in section 4.1. The fact that the inflaton is charged under the $U(1)_{PQ}$ symmetry means that it can spontaneously break the symmetry after inflation, if it obtains a vev. We will demonstrate that it does so at a scale consistent with current bounds on the axion decay constant, $f_a$.

We now consider the minimisation of the potential post inflation and defer the consideration of the inflationary period as it relies on the results of the minimisation procedure.

### 3.1.1 The Potential

In this section we construct and minimise the potential in order to calculate the vevs relevant to our model. We initially search the potential under the assumption that the Higgs obtain no vevs immediately post inflation. This assumption will be justified by analysing the turning points we discover and demonstrating they are minima, under certain important constraints. The requirement of zero Higgs vev is crucial, because the order of magnitude estimates for any vevs post inflation are either zero or $A_\lambda/\lambda$. The latter of which is clearly in conflict with the experimental value determined by $\left(\langle H_u\rangle^2 + \langle H_d\rangle^2\right)^{1/2} = (\sqrt{2}G_F)^{-1/2} = 246$ GeV [31].

In order to map on to the MSSM at low energies both Higgs must be minimised at zero at the high scale to allow radiative electroweak symmetry breaking (EWSB) to occur in the usual way, as discussed in section 1.4.1. This results in non-zero Higgs vevs at low energy consistent with experimental data. We shall now demonstrate that the Higgs can keep their vevs at zero while the inflaton's vev generates an effective TeV scale $\mu$ term, as required for an effective MSSM theory valid below the Peccei-Quinn scale.

For the first stage of the analysis we re-parametrise the complex scalars. We rewrite the Lagrangian in polar co-ordinates, eg. $\phi(x) = \frac{|\phi(x)|}{\sqrt{2}} \exp\left(\frac{i\alpha(x)}{|\langle\phi\rangle|}\right)$[1]. We denote the dynamical phases of $\phi$, $N$, $H_u$ and $H_d$ as, respectively, $\alpha$, $\beta$, $\gamma$ and $\delta$. There is also one constant phase, in $A_\lambda = |A_\lambda|e^{i\sigma}$. So doing we find the following SUSY potential:

$$V_{SUSY} = \lambda^2 \left(\frac{1}{4}|H_d|^2|H_u|^2 + \frac{1}{2}|H_u||H_d||N|^2 \cos(\gamma + \delta - 2\beta)\right. \tag{3.6}$$

$$\left. + \frac{1}{4}|\phi|^2 \left(|H_u|^2 + |H_d|^2\right) + \frac{1}{4}|N|^4 + |\phi|^2|N|^2\right) \tag{3.7}$$

and the soft potential:

$$\begin{aligned} V_{soft} = & V(0) + \frac{\lambda}{\sqrt{2}}|A_\lambda||\phi||H_u||H_d| \cos(\alpha + \gamma + \delta + \sigma) \\ & + \frac{\lambda}{\sqrt{2}}|A_\lambda||\phi||N|^2 \cos(\alpha + 2\beta + \sigma) + \frac{1}{2}m_0^2 \left(|H_u|^2 + |H_d|^2 + |N|^2\right) - \frac{1}{2}m_\phi^2|\phi|^2. \end{aligned} \tag{3.8}$$

We now attempt the minimisation of $V_{SUSY} + V_{soft}$ under the assumption that $\langle H_u\rangle = \langle H_d\rangle = 0$ [2]. Under this assumption it is clear that the potential is minimised for $\cos(\alpha + 2\beta + \sigma) = -1$ since the trilinear is the only contribution to the potential that can be negative. We note that we can consistently set $\cos(\alpha + \gamma + \delta + \sigma) = -1$, but that this requires $\cos(\gamma + \delta - 2\beta) = 1$. To make the notation legible we henceforth

[1]Canonical normalisation of the dynamical phases only makes sense when the modulus obtains a vev. We wish to consider the possibility of non-zero Higgs vevs and so parametrise them in this fashion, even though we will show that their vevs are zero at the end.

[2]More precisely we consider the potential at $\langle H_u\rangle = \langle H_d\rangle = 0$, and show that it can be minimised there.

drop the modulus signs, but it should be remembered that any negative or complex vevs are clearly not allowed.

Taking partial derivatives with respect to $H_u$ and $H_d$ we obtain the following two equations

$$\frac{\partial V}{\partial H_u} = \frac{\lambda^2}{2} H_d^2 H_u + \frac{\lambda^2}{2}\phi^2 H_u + \frac{\lambda^2}{2} H_d N^2 \cos(\gamma + \delta - 2\beta) + \frac{\lambda}{\sqrt{2}} A_\lambda \phi H_d + m_0^2 H_u = 0 \tag{3.9}$$

$$\frac{\partial V}{\partial H_d} = \frac{\lambda^2}{2} H_u^2 H_d + \frac{\lambda^2}{2}\phi^2 H_d + \frac{\lambda^2}{2} H_u N^2 \cos(\gamma + \delta - 2\beta) + \frac{\lambda}{\sqrt{2}} A_\lambda \phi H_u + m_0^2 H_d = 0 \tag{3.10}$$

and we clearly see that $\langle H_u \rangle = \langle H_d \rangle = 0$ solves these equations irrespective of the phases. Turning to $\phi$ and $N$ with the phases set as discussed above and $\langle H_u \rangle = \langle H_d \rangle = 0$ we find that

$$\frac{\partial V}{\partial N} = \lambda^2 N^3 + 2\lambda^2 \phi^2 N - \sqrt{2}\lambda A_\lambda \phi N + m_0^2 N = 0 \tag{3.11}$$

$$\frac{\partial V}{\partial \phi} = 2\lambda^2 \phi N^2 - \frac{\lambda}{\sqrt{2}} A_\lambda N^2 = 0. \tag{3.12}$$

We now see that the minimisation conditions alone cannot fix the phases since all that has been enforced so far is that $\alpha + 2\beta + \sigma = 2n\pi + \pi$. Hence this does not determine the phase of $\mu$.

We first note that there is a trivial solution to Eqs. (3.11 - 3.10) when all the fields are set to zero. This solution is of little interest, but we note it for completeness. To find non-trivial solutions we simply solve Eqs. (3.11) and (3.12) algebraically. Since $m_\phi$ is very small when compared with $m_0$ we discount it in the following as including it provides a negligible correction. So doing we see that we have two equations and two variables with the following solutions:

$$\langle\phi\rangle = \frac{A_\lambda}{2\sqrt{2}\lambda} \tag{3.13}$$

$$\langle N\rangle = \frac{A_\lambda}{2\lambda}\sqrt{1-\frac{4m_0^2}{A_\lambda^2}} \tag{3.14}$$

and we had already found that

$$\langle H_u\rangle = \langle H_d\rangle = 0. \tag{3.15}$$

Considering Eqs. (3.9) and (3.10) we see that they go into each other under the exchange of $H_u$ and $H_d$ (which clearly must be the case as $V$ is symmetric under the exchange) hence we can just consider $H_u = H_d = H$ and search for other solutions. Repeating the analysis under the assumption that $N = 0$ and $H \neq 0$ requires different phases to minimise the potential. Specifically we require that $\cos(\alpha+\gamma+\delta+\sigma) = -1$. Eqs. (3.9) and (3.10) become:

$$\frac{\partial V}{\partial H} = \frac{\lambda^2}{2}H^3 + \frac{\lambda^2}{2}\phi^2 H - \frac{\lambda}{\sqrt{2}}A_\lambda \phi H + m_0^2 H = 0 \tag{3.16}$$

and we obtain the following expression for

$$\frac{\partial V}{\partial \phi} = \lambda^2 \phi H^2 - \frac{\lambda}{\sqrt{2}}A_\lambda H^2 = 0 \tag{3.17}$$

This yields another set of solutions at

$$\langle H\rangle = \frac{A_\lambda}{\sqrt{2}\lambda}\sqrt{1-\frac{4m_0^2}{A_\lambda^2}} \tag{3.18}$$

$$\langle\phi\rangle = \frac{A_\lambda}{\sqrt{2}\lambda} \tag{3.19}$$

$$\langle N\rangle = 0 \tag{3.20}$$

Substituting these expressions back into $V$ we find the following expressions for the vacuum energy post inflation

$$V_{H=0} = V(0) - \frac{A_\kappa^4}{64\kappa^2}\left(1 - \frac{4m_0^2}{A_\kappa^2}\right)^2 \tag{3.21}$$

$$V_{N=0} = V(0) - \frac{A_\lambda^4}{16\lambda^2}\left(1 - \frac{4m_0^2}{A_\lambda^2}\right)^2 \tag{3.22}$$

where we have relaxed the assumption that $A_\lambda = A_\kappa$ and $\lambda = \kappa$. This allows us to see that if our assumptions are in place then the $N = 0$ minimum is energetically preferred. Also we see that, in principle, we could escape this problem if we relaxed our assumptions. For example, requiring that $\frac{4m_0^2}{A_\lambda^2} > 1$ would remove $H \neq 0$ as a solution and setting $A_\kappa = A_\lambda$ and $\lambda^2 > 4\kappa^2$ would make $V_{H=0}$ the global minimum. However, removing either assumption takes us away from the string construction, though they remain acceptable field theoretic models in their own rights. To demonstrate that this situation was physically viable would require a calculation of the tunnelling probability. If the half-life is significantly longer than the measured age of the universe then it seems likely that the universe will survive long enough to be observed.

As we wish to remain in contact with the string theoretic origins we need to demonstrate that the local minimum, with zero Higgs vevs, can be reached after inflation. To do so, in section 3.1.2 we will consider a possible trajectory of the inflaton, $\phi$, that will destabilise to the desired minimum. Before this we must review the consequences of arriving in the minimum described by Eqs. (3.13-3.15).

Firstly we see that, since $\phi$ and $N$ have obtained vevs, $U(1)_{PQ}$ has been spontaneously broken and hence the axion can relax to the CP conserving minimum, with

$$f_a \sim \langle\phi\rangle \sim \langle N\rangle \sim 10^{13}\ \text{GeV}. \tag{3.23}$$

Then, from Eq. (3.2) it is clear that, when $\phi$ obtains its vev the first term, $\lambda\phi H_u H_d$, becomes a supersymmetric mass term for $H_u$ and $H_d$, a $\mu$ term:

$$\mu = \lambda \langle \phi \rangle = -\frac{A_\lambda}{4} \tag{3.24}$$

where the phases have been neglected. We see that our assumption that there is only one Yukawa coupling, $\lambda$, means that $\mu$ is automatically at the soft breaking scale. This, with the requirement of low energy supersymmetry, leads us to a $\mu$ term at the electroweak scale, solving the $\mu$ problem of the MSSM.

There are further constraints on $\mu$ coming from the requirements that inflation end and that there be a phenomenologically viable minimum to be reached post inflation. In order for inflation to end the $N$ field must become unstable at a point along $\phi$'s trajectory and rapidly roll to its minimum. When $N$ starts to roll it destabilises $\phi$ through their couplings in $V$; specifically the trilinear coupling $\lambda A_\lambda \phi N^2$ causes $\phi$ to accelerate and violate the slow roll conditions.

Since this means $N$ must obtain a non-zero vev post inflation, Eq. (3.14) implies the following constraint

$$A_\lambda^2 > 4m_0^2. \tag{3.25}$$

So far we have not proved that the stationary point described by Eqs. (3.13)- (3.15) is in fact a minimum of the potential. To prove this we need to show that the Hessian is positive definite. If

$$V_{ij} = \begin{pmatrix} V_{H_uH_u} & V_{H_uH_d} & V_{H_u\phi} & V_{H_uN} \\ V_{H_dH_u} & V_{H_dH_d} & V_{H_d\phi} & V_{H_dN} \\ V_{\phi H_u} & V_{\phi H_d} & V_{\phi\phi} & V_{\phi N} \\ V_{NH_u} & V_{NH_d} & V_{N\phi} & V_{NN} \end{pmatrix} \tag{3.26}$$

is a positive definite matrix at a given stationary point, then that point is a local minimum. If instead the Hessian is negative definite, then the point is a local maximum and finally if there are both positive and negative eigenvalues, then it is a saddle point. Re-expressing Eq. (3.26) in terms of $m_0$ and the ratio $x = \frac{A_\lambda}{m_0}$ we find that the ratio must fall in the following range

$$8 > x^2 > 4 \text{ and hence } 8m_0^2 > A_\lambda^2 > 4m_0^2. \tag{3.27}$$

So we finally arrive at a prediction for the supersymmetric Higgs mass squared, $\mu^2$, in terms of its soft mass, $m_0$:

$$0.25m_0^2 < \mu^2 < 0.5m_0^2 \tag{3.28}$$

where we have used Eq. (3.24) to re-express Eq. (3.27) in terms of $\mu$.

Clearly this prediction is valid at the string scale, but to make contact with experiment the couplings would need to be run down to the electroweak scale. However, a full study of collider phenomenology was not undertaken in this thesis.

We note that we have not included the quarks in this analysis. The reason for this is that they are held at zero throughout inflation, do not modify the critical point analysis and remain at zero in the minimum reached after inflation. This can be demonstrated simply by including the quarks in the above analysis, but since doing so does not modify our findings we omit it here.

Incidentally we can now obtain upper and lower bounds on $V(0)$, expressed in terms of $\mu$. Using Eqs. (3.21) and (3.24) we find that to have an effective cosmological constant of zero after inflation we need the following

$$V(0) = \frac{A_\lambda^4}{64\lambda^2}\left(1 - \frac{4m_0^2}{A_\lambda^2}\right)^2 = \frac{4\mu^4}{\lambda^2}\left(1 - \frac{4m_0^2}{A_\lambda^2}\right)^2 \tag{3.29}$$

and from Eq. (3.27) we see that

$$0 < V(0) < \frac{2\mu^4}{\lambda^2}. \tag{3.30}$$

As a rough guide we use $\mu \sim 1$ TeV motivated by the standard expression for $\mu^2$[22]:

$$\mu^2 = \frac{m_{H_d}^2 - m_{H_u}^2 \tan^2\beta}{\tan^2\beta - 1} - \frac{1}{2}m_Z^2 \tag{3.31}$$

where we take $M_Z \sim 91$ GeV[31] leading to a reasonable estimate for $V(0)$ being

$$V(0)^{1/4} \sim 10^8 \text{ GeV}. \tag{3.32}$$

### 3.1.2 Inflation

For this model to describe inflation it must satisfy some basic requirements: it must have a field that slowly rolls for a sufficient period of expansion, it must generate curvature perturbations in line with CMB data and it must predict a spectral index consistent with current observations [3]

To meet the first requirement it must satisfy the slow roll conditions, $\epsilon_N \ll 1$ and $|\eta_N| \ll 1$ where they are defined as:

$$\epsilon_N = \frac{1}{2}m_P^2\left(\frac{V'}{V}\right)^2 \ll 1 \tag{3.33}$$

$$|\eta_N| = \left|m_P^2\frac{V''}{V}\right| \ll 1 \tag{3.34}$$

where the subscript, $N$, implies that $\epsilon$ and $\eta$ were evaluated $N$ e-folds before the end of inflation[4]. Specifically this must be at the time of horizon exit of the scales that are currently re-entering the horizon. For our model, we have a relatively small vacuum energy during inflation, $V(0) \sim 10^{32}$ GeV$^4$, which we imposed to give an effective cosmological constant of zero post inflation, as was shown in section 3.1.1. This leads

[3] For a survey of similar particle physics models and a discussion of how they satisfy the inflationary requirements see for example [48].

[4] As a consistency check $\epsilon$ and $\eta$ should be evaluated at this point and at a point just before the end of inflation to ensure that the slow roll conditions hold for the entire process. We will see that the strongest bounds come at the end of inflation and we impose those bounds on our model building.

to a small number of e-folds between horizon exit and the end of inflation. To prove this we need to know the vacuum energy at the end of inflation and the temperature of reheating so we will consider the number of e-folds again, once these have been determined.

We now discuss the hybrid inflation mechanism of which the model described here is a slightly different realisation than the one presented in [56]. While the analysis is similar to that of section 1.7 it is not quite as simple and necessitates re-examination. First we will address the issue of whether or not the inflaton's trajectory naturally falls into the phenomenologically desirable minimum after inflation. This requires us to consider the dynamics towards the end of inflation. During inflation a second "waterfall" field, $N$ in our model, has a positive mass squared and hence is held at zero. It is subsequently destabilised, i.e. obtains a negative effective mass squared, when the inflaton passes a critical value. Geometrically speaking the critical point marks the transition of the inflaton from a region where all other fields are locally minimised to a saddle region in which the $N$ field is unstable.

To see this we consider the behaviour of the Hessian, Eq. (3.26), along possible inflaton trajectories. We only consider possibilities in which all other fields are set to zero, essentially for simplicity. We make no claim that this the only possible inflationary trajectory, merely that it is a valid option. With this assumption we are entitled to set both the cosines multiplying the trilinears equal to minus one. Then the Eq. (3.26) takes the following form:

$$V_{ij} = \begin{pmatrix} m_0^2 + \frac{\lambda^2}{2}\phi^2 & -\frac{\lambda}{\sqrt{2}}A_\lambda\phi & 0 & 0 \\ -\frac{\lambda}{\sqrt{2}}A_\lambda\phi & m_0^2 + \frac{\lambda^2}{2}\phi^2 & 0 & 0 \\ 0 & 0 & -m_\phi^2 & 0 \\ 0 & 0 & 0 & m_0^2 - \sqrt{2}\lambda A_\lambda\phi + 4\lambda^2\phi^2 \end{pmatrix} \tag{3.35}$$

The critical values are the roots of the eigenvalue equations in the Higgs and N sectors. From Eq. (3.35) we see that the roots are expressible in terms of the soft parameters: the critical points at which $N$ becomes unstable are

$$\phi_{\text{crit.(N)}} = \frac{A_\lambda}{2\sqrt{2}\lambda}\left(1 \pm \sqrt{1 - \frac{4m_0^2}{A_\lambda^2}}\right) \tag{3.36}$$

and the Higgs fields destabilise at

$$\phi_{\text{crit.(H)}} = \frac{A_\lambda}{\sqrt{2}\lambda}\left(1 \pm \sqrt{1 - \frac{4m_0^2}{A_\lambda^2}}\right). \tag{3.37}$$

Within the ranges of $\phi$ bounded by these critical values the associated field is unstable. This result explains the necessity that our model be one of inverted hybrid inflation rather than the standard. The trajectory starts with $\phi$ at a point with a small value and all other fields set to zero. This is a stable point for all but $\phi$ which slowly rolls away from the origin, driven by its negative effective mass squared.

As $\phi$ rolls it will reach $\phi_{\text{crit.(N)}}$ before $\phi_{\text{crit.(H)}}$, assuming that $m_0$ is non-zero. Therefore it is the phenomenologically preferred minimum with $N \neq 0$ and $H_u = H_d = 0$ that is reached on this trajectory. If the soft mass squared for the inflaton were positive, and the initial value for $\phi$ were large and stable, then it would slowly roll towards the origin and would always destabilise in the Higgs direction rather than $N$. It is possible to allow for $m_\phi^2$ to be positive since there is a stable region in between the two unstable ranges bounded by Eqs. (3.36) and (3.37), which can be non-zero. If $\sqrt{1 - \frac{4m_0^2}{A_\lambda^2}} < \frac{1}{3}$, then this region opens up. It is interesting that this gives much tighter constraints on the soft masses than the inverted case,

$$\frac{9}{2}m_0^2 > A_\lambda^2 > 4m_0^2 \tag{3.38}$$

and hence a stronger prediction for $\mu$:

$$\frac{9}{32}m_0^2 > \mu^2 > \frac{1}{4}m_0^2. \tag{3.39}$$

However the required initial conditions for the standard hybrid case are difficult to imagine being satisfied, so for the rest of this thesis we focus our attention on the inverted case.

We shall now discuss the slow roll period that occurs, for the inverted hybrid inflation scenario.

For this trajectory, the potential effectively simplifies to

$$V = V(0) - \frac{1}{2}m_\phi^2\phi^2. \tag{3.40}$$

In this case the slow roll conditions become

$$\epsilon_N = \frac{1}{2}\frac{m_P^2 m_\phi^4 \phi_N^2}{V(0)^2} \ll 1 \tag{3.41}$$

$$|\eta_N| = m_P^2 \frac{|m_\phi|^2}{V(0)} \ll 1. \tag{3.42}$$

To ensure that these conditions are satisfied we first derive an upper limit on $|m_\phi|$ from Eq. (3.42):

$$|m_\phi| \ll 4 \text{ MeV}. \tag{3.43}$$

To find how to satisfy Eq. (3.41) we must first calculate $\phi_N$. From the standard [46, 49, 68] equations describing the evolution of $a$ in the slow roll approximation we can show that, for a slowly varying Hubble constant, $H$, we obtain [48]

$$N(\phi) = \int_{\phi_{\text{crit.}}}^{\phi} M_P^{-2}\frac{V}{V'}d\phi \tag{3.44}$$

which gives the number of e-folds the universe will undergo between the input value of $\phi$ and the critical point at which inflation ends.

For $V$ given by Eq. (3.40) this obtains the simple form

$$N(\phi) = -\frac{V(0)}{M_P^2 m_\phi^2}\ln(\phi/\phi_{\text{crit.}}) = -\frac{\ln(\phi/\phi_{\text{crit.}})}{|\eta|} \tag{3.45}$$

and $\phi_N$ is obtained from Eq. (3.45):

$$\phi_N = \phi_{\text{crit.(N)}} e^{-N|\eta|} \tag{3.46}$$

where $N$ is the number of e-folds between the time at which the largest measured scales leave the horizon and the end of inflation. We now calculate $N$ using the standard expression from [48]:

$$N = 62 - \ln\left(10^{16}\text{GeV}/V_{\text{end}}^{1/4}\right) - \frac{1}{3}\ln\left(V_{\text{end}}^{1/4}/\rho_{\text{reh}}^{1/4}\right) \tag{3.47}$$

where $V_{\text{end}}$ is the value of the potential at the end of inflation and $\rho_{\text{reh}}^{1/4}$ is the reheat temperature. We must make a brief detour to calculate these quantities and then obtain $N$.

First we note that [47]:

$$\rho_{\text{reh}} \sim g_* T_{\text{reh}}^4 \simeq g_* \left(1.2 g_*^{-1/4}\sqrt{M_P \Gamma_\phi}\right)^4 = 2.1 M_P^2 \Gamma_\phi^2 \tag{3.48}$$

with the inflaton's decay rate [69]:

$$\Gamma_\phi \sim \frac{M_\phi^3}{64\pi f_a^2} \sim \frac{\lambda^2}{8\pi}\left(1 - \frac{4m_0^2}{A_\lambda^2}\right) M_\phi \tag{3.49}$$

where post inflation $M_\phi^2 \approx 2\lambda^2 \langle N \rangle^2 = \frac{A_\lambda^2}{2}\left(1 - \frac{4m_0^2}{A_\lambda^2}\right) \sim 1\ \text{TeV}^2$.

Now with $g_* \sim 80$ [47] we are in a position to calculate all the desired quantities:

$$\Gamma_\phi \sim 10^{-10}\ \text{eV} \quad T_{\text{reh}} \sim 0.2\ \text{GeV} \quad \rho_{\text{reh}} \sim 0.1\ \text{GeV}^4. \tag{3.50}$$

This low reheat temperature slightly relaxes the upper bound on the axion decay constant, allowing $f_a \sim 10^{13}$ GeV [67, 69] and arrive at

$$N \approx 37. \tag{3.51}$$

With $N$ known we can set about satisfying $\epsilon_N$. From Eq. (3.41) we obtain[5]

[5]While the details of the standard hybrid inflation case are not included it differs slightly from the inverted case. The difference being that there are be slightly tighter constraints on the soft parameters to satisfy $\epsilon_N \ll 1$, since $\phi_N$ is larger.

$$m_\phi \ll \left(\frac{2V(0)e^{2N|\eta|}}{M_P^2\phi^2_{\mathrm{crit.(N)}}}\right)^{1/4}. \tag{3.52}$$

In the limiting case when $\eta \ll 1$ and

$$\phi_{\mathrm{crit.(N)}} = \frac{A_\lambda}{2\sqrt{2}\lambda} \tag{3.53}$$

where this is the largest value $\phi_{\mathrm{crit.(N)}}$ can have to provide the strongest bound,

$$m_\phi \ll 5 \text{ MeV}, \tag{3.54}$$

where this bound and the corresponding bound on $\eta$ depend on the exact details of SUSY breaking so, lacking an specific SUSY breaking mechanism, these are left as order of magnitude constraints.

However it turns out the most stringent constraint on $m_\phi^2$ comes from the density perturbation data. From [54] we see that

$$\delta_H = \frac{32}{75}\frac{V(0)}{m_P^4}\epsilon_N^{-1} = 1.92 \times 10^{-5}. \tag{3.55}$$

Satisfying this requirement with the inflaton would drive its mass down to below the eV scale. This would require a high degree of fine-tuning[6]. If the mass of the inflaton $\phi$ during inflation is in the MeV range this satisfies the slow roll constraints, but precludes the possibility that the density fluctuations are provided by the inflaton itself. Thus extreme fine-tuning is alleviated [70] if we use a different field, a curvaton [71, 72, 73], to generate the curvature perturbations. There are numerous examples of this mechanism in the literature, but we do not speculate as to which one might be compatible with our model.

[6]For a discussion of fine-tuning and the expected radiative corrections in the closely related $\phi$NMSSM model see [67].

We now consider the spectral index given by[48]

$$n = 1 + 2\eta - 6\epsilon \tag{3.56}$$

it is perhaps safest to say that we expect the spectral index to be very close to unity. Although it is impossible to make an exact prediction without a better understanding of the details of SUSY breaking and hence predictions for the soft parameters. It is however clear that if the inflaton is responsible for the curvature perturbations $n-1$ will be negligible. This is in agreement with the WMAP one year data, $n = 0.99 \pm 0.04$ [74].

Tied into inflation is the issue of domain walls. Since this model does not possess the $\mathbb{Z}_3$ symmetry of the NMSSM it sidesteps the domain wall problem discussed in section 1.4.2. However, domain walls can still be created when the PQ symmetry breaks [75, 76]. However it is possible [77] to avoid this problem if, as is the case in our model, a field charged under $U(1)_{PQ}$ already has a non-zero vev by the end of inflation and retains a non-zero vev in the minimum post inflation. In our model this role is taken by the inflaton which has a vev of $\phi_{crit.(N)}$ at the end of inflation and obtains $\langle\phi\rangle = \frac{A_\lambda}{2\sqrt{2}\lambda}$ in the minimum reached after inflation.

### 3.1.3 Inflation summary

We close this section with a brief review of the main features of the field theory model before embarking upon its string construction in the following section. This model puts forward the $\phi$ field as the field responsible both for inflation and for generating the supersymmetric Higgs mass term, $\mu$, of the MSSM. In addition it has been demonstrated that with the vacuum expectation value of the inflaton at the Peccei-Quinn axion scale the $\mu$ term automatically appears at the soft breaking scale, given as a simple function of the soft parameters. The very small Yukawa coupling, $\lambda$, provides the link between these scales, giving rise to $f_a$ once the soft parameters are determined. This means the physics of inflation simultaneously solves the strong CP and $\mu$ problems and give rise to a testable prediction for the $\mu$ parameter in terms of the Higgs' soft scalar mass: $0.25m_0^2 < \mu^2 < 0.5m_0^2$. This implies deep connections between supersymmetric Higgs phenomenology, inflation and the absence of CP violation in QCD.

This analysis has rested on a number of assumptions, namely that the soft masses of the Higgs field and of $N$ are equal, that small Yukawa couplings can be obtained and these small Yukawa couplings can be equal: $\lambda = \kappa$ in our model. In the next chapter we will construct the supersymmetric and soft sectors of the model and in so doing provide justifications for these assumptions.

# Chapter 4

# String Construction of Inflation and Small Yukawa Couplings

In this chapter we investigate the application of the D-brane framework discussed in chapter 2, making particular use of the small Yukawa couplings uncovered in section 2.2. We demonstrate how string selection rules, arising from the D-brane setup, impose non-trivial constraints on the set of allowed superpotentials. The applications of the framework are made apparent when the inflationary model of chapter 3 is constructed and many of the model's assumptions are seen to be consequences of the underlying string theory.

First we construct the supersymmetric side of the model, consistent with the string selection rules, before moving on to consider SUSY breaking. Initially only the dilaton and untwisted moduli are considered, but it will be necessary to expand the analysis to include the effects of twisted moduli.

## 4.1 Supersymmetric Sector

This section draws heavily on the information contained within chapter 2 and attempts to use this to justify a number of assumptions of the previous section. Chiefly we wish to address the origin of the small couplings required by the model and consider the main result of the model, namely the high scale relationship between the Higgs' soft and supersymmetric masses in Eq. (3.28).

The superpotential, Eq. (2.15), the relationship between the Yukawa couplings, Eq. (2.30), and the expression for $\lambda_I$, Eq. (2.29), are all the tools we need for the string construction. Figure 1 displays the two branes that feature in our construction. The $D5_2$ brane is assumed to have an order one gauge coupling and possess a twisted modulus at a fixed point of the orbifold. This fixed point is taken to lie on the $D5_2$ brane, but to be spatially separated from the intersection point of the branes. However, because the radius of the $D5_2$ brane is very small, as given by Eq. (2.31), the separation is similarly limited. Intuitively we expect there to be very little by way of geometric effects arising from the small separation and this will be borne out when the calculations are performed. All the MSSM fields will be required to transform under representations of the $D5_2$ brane's gauge group and not the $D5_1$ brane which has an order $10^{-10}$ coupling.

The goal of the following subsection is to elucidate the string selection rules and demonstrating how they can be applied. This will show how one obtains a given renormalisable superpotential within string theory and how some simple examples are not compatible, at the renormalisable level.

### 4.1.1 Methodology

It should perhaps be re-emphasised that our approach in this paper is one of string inspired phenomenology. We make use of a number of the generic properties of low energy effective string theory so as to keep our analysis as general as possible and avoid specialising to a particular model. Some of the obvious strengths and weaknesses of this approach are as follows. We can consider a large class of models in one fell swoop and, if it proves impossible to embed our model, be reasonably confident that there is little reason to undertake more involved, specific constructions. However without the explicit realisation of our model we cannot know that there exists an appropriate compactification leading to the right sets of branes and the desired low energy spectrum. However the number of possible $D = 4$, $\mathcal{N} = 1$, low energy string models is vast and a complete survey is far beyond the scope of this thesis. With this caveat in mind we may still undertake the construction.

The rules that we enforce are as follows:

- All supersymmetric terms must found within the low energy effective superpotential, Eq. (2.9).
- The string states, $C^{5_1 5_2}$ etc., can represent more than one low energy field.
- Each low energy field can only be assigned to one string state.
- The gauge transformation properties of a string state are determined by the stacks of branes to which its ends attach.

As previously mentioned we only make use of a small subset of these terms. That is not a rule that we require, but it is the case for all models in this thesis. Also, when we come to consider neutrinos it will be necessary to make use of the supersymmetric generalisation of the Froggatt-Nielsen mechanism to generate non-renormalisable operators. However this only makes use of renormalisable operators from the canonically normalised Eq. (2.15) so does not violate our first rule. The final point summarises the discussion in section 1.6.

The approach in this thesis is to take a purely field theoretic superpotential and see if it can be realised in the string superpotential, using the rules discussed above. Let us consider a toy field theory with just three, gauge singlet superfields, $A$, $B$ and $C$ with the following superpotential:

$$W = \lambda_a A^3 + \lambda_b A^2 B + \lambda_c ABC \tag{4.1}$$

where the $\lambda$'s are constants.

We now consider the assignment of each term in Eq. (4.1) individually.

$\lambda_a A^3$ cannot appear in the string superpotential for the simple reason that Eq. (2.9) contains no terms that are cubic in a single superfield. Were we to assign $A$ to any string state we would be then forced to assign it to another string state, which would violate our third rule. Incidentally this means that the NMSSM cannot be realised using purely renormalisable operators as it includes just such a term. However, the same problem does not afflict the $\lambda_b A^2 B$ term since there are quadratic terms in Eq. (2.9). For example it could be assigned to $C^9_{\bar{1}} C^{95_1} C^{95_1}$ and other similar terms. To be concrete we will have to assign $A$ and $B$ to $C^{95_1}$ and $C^9_{\bar{1}}$, respectively, if we wish to use $C^9_{\bar{1}} C^{95_1} C^{95_1}$. However, terms like $C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1}$ are not acceptable candidates. Finally $\lambda_c ABC$ can be assigned to any of the terms in the string superpotential, Eq. (2.9). In addition we are completely free to choose the string states to which the fields are assigned, within a particular coupling. In other words, we can freely permute the superfield assignments once we have chosen a string term. This is similar for the $\lambda_b A^2 B$ term, except that only a subset of the string superpotential terms are available.

We should now note that all the normal rules of model building apply, except where they contradict the string selection rules. By way of example, gauge invariance must be insisted upon for all terms. Since our toy model only consists of singlets, this is not very restrictive, but remains instructive. Ordinarily one would write down all possible terms that have the appropriate mass dimension, are holomorphic and are gauge invariant. For our toy model this would include terms like $\Lambda A^2$, $\Lambda^2 B$, $AB^2$ and so on, where we might guess that the high scale couplings are order one where $\Lambda$ is the high scale. Clearly, since there are only mass dimension 3 terms in Eq. (2.9) $\Lambda A^2$ and $\Lambda^2 B$ are ruled out, but we note that $AB^2$ is analogous to $BA^2$, which we have already shown is acceptable. However if we attempt to realise them both simultaneously we see that this is impossible, since $B$ must be assigned to $C_1^9$ and this only appears linearly within Eq. (2.9). We stress that this is not an artifact of our choice of $C_1^9 C^{95_1} C^{95_1}$ for $A^2 B$, as all viable terms have the same form.

In summary we have seen that the string selection rules can forbid interactions that are otherwise allowed by all the gauge symmetries of the theory. The non-renormalisation theorem of supersymmetry will keep these couplings at zero. Naturally this will not hold when SUSY is broken, but it does remain valid above the soft scale.

The model building generally proceeds as follows: first we choose a set of radii and hence the determine the non-canonical Kähler potential. Then we must canonically normalise the Kähler potential and work out the effective superpotential. Having done so we assign fields in accordance with the string selection rules and then write down all terms that are allowed by both gauge invariance and the aforementioned rules. This determines the supersymmetric side of the construction and we will now apply this procedure to our model.

### 4.1.2 String Assignments

Now and hereafter we focus our attention on the specific anisotropic compactification considered in section 2.2. Having fixed the Yukawa couplings there is only one small coupling, $g_{5_1} = 10^{-10}$, which can serve as the $\lambda$ in the model. The remaining couplings are taken to be $g_{5_2} \sim g_{5_3} \sim g_9 \sim 1$.

We start our assignments by considering $\lambda\phi N^2$. The reason for this is that it has exactly the same form as $\lambda_b A^2 B$ in section 4.1.1. It was shown there that this class of term can only be assigned to a restricted subset of the complete superpotential. This is in contrast to the $\lambda\phi H_u H_d$ term which could be assigned to any term in Eq. (2.9) with $g_{5_1}$ as its Yukawa coupling.

So doing we see that there are three terms that have the correct form and right coupling constant:

$$(i): \; g_{5_1} C_1^{5_1} C^{95_1} C^{95_1}, (j): \; g_{5_1} C_2^{5_1} C^{5_1 5_3} C^{5_1 5_3} \text{ and } (k): \; g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2} \qquad (4.2)$$

where $(j)$ and $(k)$ are symmetric under relabelling of 2 and 3. Notice also that Eq. (2.9) is symmetric under permutations of the 1, 2 and 3 labels if the radii are allowed to vary (hence altering the size of the coupling constants). Since the size of the radii is a free choice by assumption we see that $(j)$ and $(k)$ are equally good choices. Considering the fact that T-Duality is a symmetry of the theory, this and the freedom to relabel links all possible permutations of the branes, hence $(i)$, $(j)$ and $(k)$ are effectively equivalent. Due to this fact we only consider the $C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}$ and hence the assignments of $\phi$ to $C_3^{5_1}$ and $N$ to $C^{5_1 5_2}$. As $N$ does not appear again we must look for the $\lambda\phi H_u H_d$ term.

There are only two terms with $g_{5_1}$ coupling that include $C_3^{5_1}$, $(a)$: $g_{5_1} C_3^{5_1} C_2^{5_1} C_3^{5_1}$ and $(b)$: $g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}$. Notice they are inequivalent under T-Duality and relabelling.

Now the question of gauge assignments must be addressed. See section 2.1 for a discussion of the possible transformation properties of the string states. These rules imply that $C_i^{5_1}$ states can only transform under the D$5_1$ brane's gauge groups, with $g_{5_1}$ as their gauge coupling. On the other hand $C^{5_15_2}$ states can have quantum numbers from both the D$5_1$ and the D$5_2$ branes.

We must now ensure that our fields can transform appropriately under the MSSM gauge group and that each term can be made invariant. Term $(a)$ requires that we assign $H_u$ to $C_1^{5_1}$ and $H_d$ to $C_2^{5_1}$ or vice-versa. This means that both Higgs fields must transform with a gauge coupling of $10^{-10}$. Since we expect the MSSM gauge couplings to be order one at the string scale this is clearly unacceptable.

Since $\phi$ obtains a large vev it must not couple to any MSSM gauge bosons or their masses will be pushed up to $\sim 10^{13}$ GeV. It is easy to see that $\phi$ does not couple to any MSSM gauge bosons because $\phi$ belongs to the $C_3^{5_1}$ string state and hence cannot transform under any gauge groups with order one couplings. However the $N$ field also obtains a large vev, comparable to $\langle\phi\rangle$, unless $4m_0^2$ is tuned to be very close to $A_\lambda^2$. As such we must require that it does not transform under any MSSM gauge groups. Since $N$ is an intersection state, $C^{5_15_2}$, it must transform under one of the gauge groups on the D$5_2$ brane. This would seem to present us with a problem, as it would seem to imply that the $N$ field must transform non-trivially under one of the MSSM groups as discussed in section 2.1. However this is only true if string theory gives us exactly SU(3)$\times$SU(2)$_L\times$U(1)$_Y$. Consider as a possibility the following gauge group, U(1)$_Y\times$SU(2)$_L\times$SU(3)$\times$G$_X$, where G$_X$ is an unspecified non-Abelian group with an $X$ dimensional fundamental representation. This could allow $N$ to transform as follows $(\mathbf{1},\mathbf{1},\overline{\mathbf{X}};\mathbf{1},\mathbf{1},\mathbf{X})$. Now in principle we could use this in our model, assuming we found an appropriate G$_X$ and could form the correct invariant with $\phi$. However we feel this is more suited to an explicit construction since we cannot know the exact spectrum in our approach and the exact choice of this additional group lacks the physical motivation that has been guiding us so far. We do note that the $H_u$ and $H_d$ field are also intersection states and must transform under one of the $D5_1$ brane's groups. Because of this there has to be at least one spontaneously broken group on the $D5_1$ brane so that the Higgs fields can appear simply as $SU(2)_L^{5_2}$ doublets, not as bi-fundamentals of both branes. In addition to the group being spontaneously broken we require that a effect analogous to doublet-triplet splitting[1] in an SU(5) GUT [80] so that in low energies we only see one copy of each Higgs doublet. It is possible to imagine a phenomenologically viable models with multiple Higgs doublets (see [81] and references therein), but for simplicities sake we just want to consider a two Higgs doublet model. Again this is a question that could only be properly addressed in a more complete model, however we anticipate that any such model could have similar properties to those discussed here.

We now see that the previously *ad hoc* assumption of $\kappa = \lambda$ that was made in Chapter 3 has been justified in the string construction.

To complete the superpotential as defined in Eq. (3.2) we need to find order one quark Yukawa couplings consistent with these assignments. The specific terms we wish to assign are $y_t Q_3 H_u t_R^c$ and $y_b Q_3 H_d b_R^c$.

Since both $H_u$ and $H_d$ are assigned to $C^{5_1 5_2}$, the only order one terms allowed are

$$(\alpha)\ g_{5_2} C_1^{5_2} C^{5_2 5_1} C^{5_2 5_1},\ (\beta)\ g_9 C^{5_2 5_3} C^{5_1 5_2} C^{5_3 5_1} \text{ and } (\gamma)\ g_{5_3} C^{95_2} C^{5_1 5_2} C^{95_1} \tag{4.3}$$

where we must allow the radii to vary slightly to accommodate ($\beta$) and ($\gamma$). Again the requirement of gauge invariance must be satisfied. To do so the quark doublet, which transforms as a $(\mathbf{2}, \mathbf{3})$ under $SU(2)_L$ and SU(3), must be assigned to a string state with both ends on branes with $\mathcal{O}(1)$ gauge couplings. Of the three possibilities ($\alpha$) is the simplest choice since it has all the standard model gauge factors coming from the

[1] For some examples where this is realised see [78, 79]

| | $SU(3)$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_{PQ}$ | String State |
|---|---|---|---|---|---|
| $Q_3$ | $\mathbf{3}$ | $\mathbf{2}$ | 1/6 | −1/2 | $C_3^{5_2}$ |
| $t^c_R$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | −2/3 | −1/2 | $C^{5_1 5_2}$ |
| $b^c_R$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | 1/3 | −1/2 | $C^{5_1 5_2}$ |
| $H_u$ | $\mathbf{1}$ | $\mathbf{2}$ | 1/2 | 1 | $C^{5_1 5_2}$ |
| $H_d$ | $\mathbf{1}$ | $\mathbf{2}$ | −1/2 | 1 | $C^{5_1 5_2}$ |
| $\phi$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | −2 | $C_3^{5_1}$ |
| $N$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 1 | $C^{5_1 5_2}$ |

Table 4.1: Inflation Fields

same stack of branes. Since it has already been shown that the Higgs fields transform under a gauge group coming from the $D5_2$ branes, assigning $Q_3$ to $C_1^{5_2}$ means that the entire MSSM gauge group can be found on the same stack of branes. In contrast ($\beta$) and ($\gamma$) entail diagonal symmetry breaking from $\left(SU(3) \times SU(2)_L \times U(1)_Y\right)^2$ to $SU(3) \times SU(2)_L \times U(1)_Y$. Of course this does not rule out ($\beta$) and ($\gamma$), but in this thesis we concentrate on the simplest model, ($\alpha$), for the rest of the analysis.

A summary of the model as it stands is presented in table 4.1. This contains all the information about the spectrum that we have constructed so far, namely the string assignments and symmetry representations.

## 4.2 Supersymmetry Breaking Sector

In this section we start with the assumption that the SUSY breaking is dominated by the dilaton/moduli sector, as discussed in section 2.1.1, as this is the simplest possible case in Type I string theory. Having made this assumption we consider the soft mass constraints, Eq. (3.27) derived in chapter 3.

In the dilaton/moduli dominated case we found that the following sum rule, first presented in Eq. (2.24), must be enforced.

$$m^2_{C_j^{5_i}} + 2m^2_{C^{5_i5_k}} = |A_{C_j^{5_i}C^{5_i5_k}C^{5_i5_k}}|^2 + 2V(0). \tag{4.4}$$

It will soon become clear that this sum rule will have important consequences for our model. If we wish to satisfy the slow roll conditions Eq. (3.41) and (3.42) we require that the inflaton's soft mass must be essentially zero, when compared with the other soft masses which are at the TeV scale. Putting aside, for a moment, the mechanism for obtaining this we will consider its implications.

From table 4.1 we see that $\phi$ is assigned to $C_3^{5_1}$ and $N$, $H_u$ and $H_d$ to $C^{5_15_2}$. In section 2.1.1 it was demonstrated that the fields belonging to a particular string state have a common soft mass, so we see that the assumption of a common mass, $m_0$, for $H_u$, $H_d$ and $N$ is justified in the string construction. In addition we see that, while there are similarities between their expressions, $m_\phi^2$ and $m_0^2$ generically have different values. Even so it must be demonstrated that the Higgs fields can obtain TeV scale masses and, at the same time, an MeV scale mass for the inflaton is allowed.

First of all it can be seen from Eq. (2.21) and (2.22) that, in the limit where $V(0) \to 0$, it is possible to set $m_\phi^2 = m^2_{C_3^{5_1}} = 0$ while $m_0^2 = m^2_{C^{5_15_2}}$ remains non-zero. For example, setting $\Theta_2^2 = \Theta_3^2 = \frac{1}{3}$ and $\cos\theta = 1$ gives $m^2_{C_3^{5_1}} = 0$ and $m^2_{C^{5_15_2}} = \frac{1}{2}m^2_{3/2}$. From Eq. (2.23) $A_{C_3^{5_1}C^{5_15_2}C^{5_15_2}} = -m_{3/2}e^{-i\gamma_i}$ and clearly Eq. (4.4) is satisfied. Relaxing the $V(0) = 0$ assumption, as we must do for our model, will modify the exact values of the parameters so they are no longer neat rational numbers, but angles can still be chosen to make $m^2_{C_3^{5_1}}$ arbitrarily close to zero, or indeed negative (with an arbitrarily small magnitude), while retaining a non-zero $m^2_{C^{5_15_2}}$. Reinstating the powers of $M_P$

we see that $V(0)$ contributes to the mass squares as

$$\frac{V(0)}{M_P^2} \approx \frac{10^{32}}{(10^{18})^2} \text{ GeV}^2 = 10^2 \text{ MeV}^2 \tag{4.5}$$

which is negligible when compared with the Higgs' soft masses which are of the order of a TeV. Therefore Eq. (4.4) takes the much simpler form of

$$2m_0^2 \approx |A_\lambda|^2 \tag{4.6}$$

where $A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_3}} = A_\lambda$ is required by string theory. Since this trilinear, by virtue of the chosen string assignments, is common to the $\lambda A_\lambda \phi N^2$ and $\kappa A_\kappa \phi H_u H_d$ terms we see that the assumption of $A_\lambda = A_\kappa$ is also required[2].

Unfortunately it is clear that Eq. (4.6) does not satisfy the lower bound on the ratio of the trilinears and soft masses shown in Eq. (3.27) so these soft terms are inconsistent with the model of inflation. To overcome this problem we must violate the sum rule in Eq. (4.4), which clearly necessitates modifications to the soft parameters. However since this sum rule is independent of the Goldstino angles it is clear that we cannot simply change the F-terms to avoid this problem: all dilaton or moduli dominated models of SUSY breaking will give rise to the same sum rule. So it is clear that we must look for sources of SUSY breaking other than just the $S$ and $T_i$ moduli, as they are currently formulated. The fields we put forward as additional sources are the twisted moduli discussed in section 2.1. We do not introduce these fields, as such, since they are already present in the spectrum; instead we allow them to take part in SUSY breaking and hence obtain F-terms. In the following section we consider the effect of twisted moduli and address the issue of why $m_\phi$ is so small when compared with the rest of the soft masses.

[2] $\lambda = \kappa$ has already been demonstrated since $\kappa\phi N^2$ and $\lambda\phi H_u H_d$ have been shown to be contained in the same string superpotential term in section 4.1.2.

### 4.2.1 Twisted Moduli

In spaces with singular points, in our case orbifolds, twisted moduli must be present in the spectrum as discussed in section 2.1. For simplicity's sake we only consider the effect of one twisted modulus, $Y_2$ located at a fixed point in the orbifold spanned by $D5_2$, i.e. in the 2-torus with dimension $R_2$. $Y_2$ is taken to be at a fixed point spatially separated by a distance $R_2$ from the origin, as shown in fig. 2.1.

In the following analysis we draw heavily on the analysis of [57], but generalise it to allow $\lambda_I \neq 1$ in accordance with our $\lambda_I \sim 10^{-11}$. In so doing we find that $\lambda_I$ has a significant effect on the soft spectrum and that it simplifies considerably in the $\lambda_I \to 0$ limit.

The presence of twisted moduli must be represented by modifications to the Kähler potential and gauge kinetic functions. Since we desire additional contributions to the soft parameters we must also include include new F-terms parametrised by additional Goldstino angles and phases.

First we consider the Kähler potential for the twisted modulus, $K(Y_2)$. The exact form is not known, but $K(Y_2)$ must be an even function, see [62] and references therein, of $Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)$. Where $\delta_{GS}$ is a model dependent coefficient connected with anomaly cancellation via Green-Schwarz mixing [61]. The simplest non-trivial possibility is to have $K$ as follows

$$\hat{K}(Y_2, \overline{Y}_2) = \frac{1}{2} \left[ Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2) \right]^2 . \tag{4.7}$$

This is not the only modification that the Kähler potential must undergo. Since the twisted modulus is spatially separated from the origin we expect the effects of SUSY breaking coming from that brane to be suppressed, as a function of that distance. Intuitively this can be thought of as arising from the fact that the twisted moduli have their wavefunctions localised away from the origin and hence their overlap with fields at the origin are geometrically suppressed. With this suppression in place the contribution of the F-terms corresponding the $Y_2$ superfield will be reduced. The soft spectrum should reflect this and fields sequestered from $Y_2$ should have lighter soft masses, assuming that $Y_2$ is the dominant source of SUSY breaking. For a more rigorous argument and the original derivation see [57].

The states considered to be sequestered are those that are localised away from the fixed point where $Y_2$ resides. The most obvious example of a sequestered state is $C_3^{5_1}$: since both ends of the string are only free to move on the $D5_1$ brane it is held apart from $Y_2$. Another example is $C^{5_1 5_2}$, despite the fact that it has one end on the same brane as the $Y_2$. This is because the string tension localises the intersection states at the origin, away from $Y_2$. For an example of an unsequestered state we consider $C_2^{5_2}$. This state comes from a string which has both ends attached to the $D5_2$ branes, which means that it is free to move throughout the space containing $Y_2$ field and feels no suppression.

To impose this sequestration we require that a multiplicative factor, $\xi$, be introduced to the Kähler potential, where $\xi$ is given by

$$\xi(T_2, Y_2) = exp\left[\frac{1}{6}\left(1 - e^{-(T_2+\overline{T}_2)\lambda_I/4}\right)\left\{Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right\}^2\right]. \tag{4.8}$$

This gives rise to a new Kähler potential $K = K_{seq.} + K_{unseq.}$ where

$$\begin{aligned} K(S,\overline{S},T_i,\overline{T}_i,Y_2,\overline{Y}_2)_{seq.} =& \frac{1}{2}\left[Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right]^2 \\ &+ \sum_{i\neq 2}\frac{\xi(T_2,Y_2)}{(S+\overline{S})}|C_i^{5_i}|^2 + \frac{1}{2}\sum_{i\neq 2}\frac{\xi(T_2,Y_2)}{(T_k+\overline{T}_k)}|C_j^{5_i}|^2 d_{ijk} \\ &+ \frac{1}{2}\sum_{i}\frac{\xi(T_2,Y_2)}{(S+\overline{S})^{1/2}(T_k+\overline{T}_k)^{1/2}}|C^{5_i 5_j}|^2 d_{ijk} \\ &+ \frac{1}{2}\sum_{i\neq 2}\frac{\xi(T_2,Y_2)}{(T_j+\overline{T}_j)^{1/2}(T_k+\overline{T}_k)^{1/2}}|C^{95_i}|^2 d_{ijk} \end{aligned} \tag{4.9}$$

and

$$K(S,\overline{S},T_i,\overline{T}_i)_{unseq.} = -\ln(S+\overline{S}) - \sum_{i=1}^{3}\ln(T_i+\overline{T}_i) + \sum_{i=1}^{3}\frac{|C_i^9|^2}{(T_i+\overline{T}_i)}$$
$$+\frac{|C_2^{5_2}|^2}{(S+\overline{S})} + \frac{1}{2}\sum_i^3 \frac{|C_k^{5_2}|^2}{(T_i+\overline{T}_i)}d_{ik} + \frac{|C^{95_2}|^2}{(T_1+\overline{T}_1)^{1/2}(T_3+\overline{T}_3)^{1/2}} \quad (4.10)$$

where $d_{ij} = |\epsilon_{ij}|$.

For our model $\lambda_I \sim 10^{-11}$ and $T_2 + \overline{T}_2 \sim 40$ as can be seen from Eqs. (2.29), (2.8) and (2.32). Hence $\xi \approx 1$ for our model. The exact value for $\text{Re}(T_2)$ depends on $\text{Re}(Y_2)$ so we only give an approximate value here. However, over the range of values of $\text{Re}(Y_2)$ considered in this thesis $\text{Re}(T_2)$ only varies by a factor of two and the result $\xi \approx 1$ is dominated by $\lambda_I$'s contribution. We note that if this were not the case and $\xi$ differed from one then canonical normalisation would create substantial modifications of the superpotential given in Eq. (2.9).

Having now specified our Kähler potential, we parametrise the F-terms as follows [57]

$$F_S = \sqrt{3}m_{3/2}\sin\theta\, e^{i\alpha_S}\,(S+\overline{S})$$
$$F_{T_1} = \sqrt{3}m_{3/2}\Theta_1\cos\theta\sin\phi\, e^{i\alpha_1}\,(T_1+\overline{T}_1)$$
$$F_{T_2} = \sqrt{3}m_{3/2}\cos\theta\left[\sin\phi\frac{(T_2+\overline{T}_2)}{\sqrt{k}}\,\Theta_2\, e^{i\alpha_2} - \cos\phi\, e^{i\alpha_{Y_2}}\frac{\delta_{GS}}{T_2+\overline{T}_2}\right]$$
$$F_{T_3} = \sqrt{3}m_{3/2}\Theta_3\cos\theta\sin\phi\, e^{i\alpha_3}(T_3+\overline{T}_3)$$
$$F_{Y_2} = \sqrt{3}m_{3/2}\cos\theta\left[\sin\phi\left(\frac{\delta_{GS}}{\sqrt{k}} + \frac{\sqrt{k}\delta_{GS}}{(T_2+\overline{T}_2)^2}\right)\Theta_2\, e^{i\alpha_2} + \cos\phi\, e^{i\alpha_{Y_2}}\left(1 - \frac{\delta_{GS}^2}{(T_2+\overline{T}_2)^2}\right)\right]. \quad (4.11)$$

where these expressions are valid up to $\mathcal{O}\left[\frac{1}{(T_2+\overline{T}_2)^2}\right]$ and the Goldstino parameters, $\Theta_i$, satisfy the following conditions $\sum_{i=1}^{3}\Theta_i^2 = 1$ and $\Theta_i^2 \leq 1\ \forall i$. Also

$$k = 1 + \delta_{GS}\left(Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2+\overline{T}_2)\right) \quad (4.12)$$

is introduced to simplify the expressions.

Given the Kähler potential and a parametrisation for the F-terms we can re-derive our soft spectrum using the techniques discussed in section 2.1.1.

We now present the full expressions for the relevant soft masses, $\tilde{m}$, $A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}}$ and $A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}}$, up to $\mathcal{O}\left(\frac{1}{T_2+\overline{T}_2}\right)$

$$m_Q^2 = m_{C_3^{5_2}}^2 = m_{3/2}^2 - 3m_{3/2}^2 \Theta_1^2 \cos^2\theta \sin^2\phi \tag{4.13}$$

$$m_0^2 = m_{C^{5_1 5_2}}^2 = \tilde{m}^2 - \frac{3}{2} m_{3/2}^2 \left(\sin^2\theta + \Theta_3^2 \cos^2\theta \sin^2\phi\right) \tag{4.14}$$

$$m_\phi^2 = m_{C_3^{5_1}}^2 = \tilde{m}^2 - \frac{3}{k} m_{3/2}^2 \Theta_2^2 \cos^2\theta \sin^2\phi \tag{4.15}$$

where all of the dependence on $\lambda_I$ is contained within $\tilde{m}$ given by

$$\begin{aligned}
\tilde{m}^2 = m_{3/2}^2 \Bigg[ & 1 - \cos^2\theta \cos^2\phi \left(1 - e^{-\lambda_I (T_2+\overline{T}_2)/4}\right) \\
& - \frac{\cos^2\theta \sin^2\phi \, \Theta_2^2 \, \delta_{GS}}{k} \left(1 - e^{-\lambda_I (T_2+\overline{T}_2)/4}\right) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \\
& + \frac{\cos^2\theta \sin^2\phi \, \Theta_2^2 \, e^{-\lambda_I (T_2+\overline{T}_2)/4}}{32k} \lambda_I^2 (T_2 + \overline{T}_2)^2 \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}^2 \\
& - \frac{\lambda_I \cos^2\theta \cos\phi \sin\phi \left(\Theta_2 \, e^{i(\alpha_2 - \alpha_{Y_2})} + \Theta_2^\dagger \, e^{-i(\alpha_2 - \alpha_{Y_2})}\right) e^{-\lambda_I (T_2+\overline{T}_2)/4}}{32\sqrt{k}} \\
& \times \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\} \left(8(T_2 + \overline{T}_2) + \lambda_I \delta_{GS} \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}\right) \Bigg] .
\end{aligned} \tag{4.16}$$

$$\begin{aligned}
A_\lambda = A_{C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}} = & -\sqrt{3} m_{3/2} \cos\theta \Big[\sin\phi \, \Theta_1 \, e^{i\alpha_1} \\
& + \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{8\sqrt{k}} e^{-\lambda_I (T_2+\overline{T}_2)/4} \lambda_I (T_2 + \overline{T}_2) \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}^2 \\
& - \cos\phi \, e^{i\alpha_{Y_2}} \, e^{-\lambda_I (T_2+\overline{T}_2)/4} \left\{Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2)\right\}\Big]
\end{aligned} \tag{4.17}$$

$$A_Q = A_{C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}} = -\sqrt{3} m_{3/2} \cos\theta \left[ \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{\sqrt{k}} \right.$$

$$+ \sin\phi \frac{\Theta_2 \, e^{i\alpha_2}}{12\sqrt{k}} \, e^{-\lambda_I (T_2 + \overline{T}_2)/4} \, \lambda_I (T_2 + \overline{T}_2) \left\{ Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2) \right\}^2$$

$$\left. - \cos\phi \, \frac{e^{i\alpha_{Y_2}}}{3} \left( 1 + 2 e^{-\lambda_I (T_2 + \overline{T}_2)/4} \right) \left\{ Y_2 + \overline{Y}_2 - \delta_{GS} \ln(T_2 + \overline{T}_2) \right\} \right] \tag{4.18}$$

For our value of $\lambda_I$ it is interesting to note that, to a very good approximation, $\tilde{m} = m_{3/2}$, consistent with [21], so the effects of the sequestering are not felt by the soft masses. To clarify, the soft masses are clearly different from those presented in [21], but this difference is not due to the spatial separation. This is unsurprising since the separation between $Y_2$ and the origin, $R_2 \approx 10^{-18}$ GeV$^2$ is below $L_{string} \approx 10^{-13}$ GeV so in this sense they are "close". Also the exponentials vanish from $A_\lambda$ so it is not the sequestering that breaks the sum rule in Eq. (4.6). The sum rule is broken by the Kähler potential for the twisted moduli in Eq. (4.7). If it was logarithmic, as all the other moduli's potentials are, then the sum rule would hold, but the fact it is quadratic breaks the sum rule.

### 4.2.2 Allowed soft terms

In this subsection the soft parameters given by Eqs. (4.14)- (4.18) are examined to see how the inflationary constraints of Eq. (3.27) can be satisfied. First we write out the soft masses, working in the limit that $\lambda_I \to 0$, which is essentially true, to exceptionally good precision.

$$m_Q^2 = m_{3/2}^2 - 3 m_{3/2}^2 \Theta_1^2 \cos^2\theta \sin^2\phi \tag{4.19}$$

$$m_0^2 = m_{3/2}^2 - \frac{3}{2} m_{3/2}^2 \left( \sin^2\theta + \Theta_3^2 \cos^2\theta \, \sin^2\phi \right) \tag{4.20}$$

$$m_\phi^2 = m_{3/2}^2 - \frac{3}{k} m_{3/2}^2 \Theta_2^2 \cos^2\theta \sin^2\phi \tag{4.21}$$

and the expressions for the trilinears simplify in this limit to

$$A_\lambda = -\sqrt{3}m_{3/2}\,\cos\theta\left[\Theta_1 \sin\phi\, e^{i\alpha_1} - \cos\phi\, e^{i\alpha_{Y_2}}\left\{Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right\}\right] \tag{4.22}$$

$$A_Q = -\sqrt{3}m_{3/2}\,\cos\theta\left[\sin\phi\,\frac{\Theta_2\, e^{i\alpha_2}}{\sqrt{k}} - \cos\phi\, e^{i\alpha_{Y_2}}\left\{Y_2 + \overline{Y}_2 - \delta_{GS}\ln(T_2 + \overline{T}_2)\right\}\right]. \tag{4.23}$$

Our aim is to find values for the Goldstino parameters that satisfy the soft term ratio, Eq. (3.27), thereby providing an existence proof. In order to find parameters that satisfy all of our constraints, in particular $m_\phi^2 = 0$, to a given precision, we note that unless

$$0 < k \leq 3 \tag{4.24}$$

we see from Eq. (4.21) that it is impossible to obtain $m_\phi^2 = 0$. This in turn places constraints on the allowed values of $Y_2+\overline{Y}_2$ and $T_2+\overline{T}_2$ since they appear in Eq. (4.12). While we do not specify any particular GUT into which our model could be embedded we would like to see if unification is, in principle, possible. To do so we consider Eq. (2.7) when there is only one non-zero twisted modulus, $Y_2$

$$\frac{4\pi}{g^2_{5_2,\beta}} = Re(f_{5_2,\beta}) + c^2_\beta Re(Y_2) \approx 24 \tag{4.25}$$

where we have imposed that $\frac{g^2_{5_2,\beta}}{4\pi} = \alpha_{GUT}$ where this is the standard MSSM value. As discussed in [21] the coefficients, $c^k_\beta$, can be of the same order as the beta functions for some orientifolds. However, not wanting to specify a particular compactification, we simply set $c^2_\beta = 1\ \forall\beta$. In addition we assume that all the phases are set to zero, since we are attempting an existence proof, it is enough to prove it for one choice of phase.

The parameters were generated numerically, using the following method. A value is chosen for $\delta_{GS}$, then a random set of Goldstino parameters and moduli are generated within the range of values known to satisfy $m_\phi^2 = 0$ and Eq. (4.25), to a precision specified at the outset. It is then easy to calculate the soft parameters in units where $m_{3/2} = 1$. These parameters are then compared with Eq. (3.27) and we also ensure that $m_Q^2 > 0$. If all this goes through then these parameters are accepted.

A sample of points that meet all of our requirements are presented in table 4.2.

| $\delta_{GS}$ | $\theta$ | $\phi$ | $\Theta_1$ | $\Theta_2$ | $\Theta_3$ | $m_0^2$ | $m_Q^2$ | $A_\lambda$ | $A_Q$ | $Re(T_2)$ | $Re(Y_2)$ |
|---|---|---|---|---|---|---|---|---|---|---|---|
| −2 | 5.69 | 5.33 | 0.807 | 0.173 | 0.565 | 0.312 | 0.113 | 1.34 | 1.40 | 27.9 | −3.92 |
| −2 | 5.54 | 5.25 | 0.810 | 0.497 | 0.311 | 0.248 | 0.215 | 1.11 | 1.23 | 27.8 | −3.84 |
| −4 | 6.14 | 5.20 | 0.463 | 0.176 | 0.869 | 0.100 | 0.507 | 0.887 | 1.19 | 32.2 | −8.22 |
| −4 | 5.57 | 5.26 | 0.777 | 0.407 | 0.479 | 0.219 | 0.237 | 1.01 | 1.13 | 32.2 | −8.23 |
| −6 | 5.54 | 4.74 | 0.705 | 0.529 | 0.472 | 0.13 | 0.195 | 0.901 | 1.00 | 36.9 | −12.9 |
| −6 | 5.39 | 3.88 | 0.937 | 0.163 | 0.31 | 0.0679 | 0.532 | 0.551 | 0.867 | 36.8 | −12.8 |
| −8 | 6.68 | 5.19 | 0.693 | 0.156 | 0.704 | 0.279 | 0.0313 | 1.07 | 1.09 | 41.6 | −17.6 |
| −8 | 7.18 | 4.57 | 0.696 | 0.707 | 0.127 | 0.0724 | 0.448 | 0.735 | 0.992 | 41.7 | −17.7 |
| −10 | 6.4 | 4.94 | 0.379 | 0.451 | 0.808 | 0.0625 | 0.596 | 0.652 | 1.02 | 46.7 | −22.7 |
| −10 | 5.43 | 4.34 | 0.916 | 0.316 | 0.248 | 0.117 | 0.0505 | 0.938 | 0.963 | 46.6 | −22.6 |

Table 4.2: Goldstino parameters and soft terms satisfying all constraints

Note that the values of the twisted modulus presented in table 4.2 have a negligible effect on the $g_{5_1}$ coupling since its vev is order 1 and Re($T_1$)$\sim 10^{20}$. Also the values of $m_\phi^2$ have not been displayed since they can be rendered arbitrarily small with sufficient numerical precision.

The data in table 4.2 shows that we have managed to achieve our goal of obtaining soft masses that agree with Eq. (3.27) and also allow an arbitrarily small $m_\phi$. It is clear, from a brief analysis of the allowed sets of parameters in table 4.2, that the sum rule is no longer independent of the Goldstino angles. Obtaining a small $m_\phi$ and satisfying all constraints now requires careful choice of the Goldstino angles. The Goldstino angle dependence of the sum rule is just a result of the more complex Kähler potential and, while it was pleasing to be able to put forward reasonably model independent rules, they rested upon the assumption of dilaton/moduli domination. It is unsurprising that more complicated models do not share these rules. Acquiring a small $m_\phi^2$ requires some justification: why should we expect the angles to fall into such a pattern? Ultimately the answer must lie with the method of SUSY breaking. We do not attempt a detailed answer in this thesis, but instead motivate it with an explicit example [62] that yields similar results to those required here. In [62] the stabilisation of the dilaton is addressed in a Type I string framework, including the effects of twisted moduli, but assuming an isotropic compactification. The soft masses are calculated and shown to have a similar hierarchy to the model presented here, resulting in the following soft masses $m^2_{C^{5_i}_j} = \frac{V(0)}{M_P^2}$ and $m^2_{C^{5_i 5_j}} \approx \frac{1}{2} m^2_{3/2}$. However they still observe the sum-rule and the soft mass squared of $\frac{V(0)}{M_P^2}$ gives

$$\eta = 1 \tag{4.26}$$

which clearly does not allow slow roll. For these reasons it is clear that the example of [62] cannot provide our model's SUSY breaking mechanism. Nonetheless the fact that it incorporates twisted moduli and generates a clear hierarchy in its soft spectrum makes this class of SUSY breaking mechanisms attractive candidates for explicit realisations of our model.

### 4.2.3 String Construction Summary

The chapter has made use of the framework discussed in chapter 2 and demonstrated that it is possible to make use of it to construct the model of inflation presented in chapter 3. The assumptions of small Yukawa couplings, identification of said couplings and the particular soft spectra put forward in Eq. (3.4) have all been justified within the construction. This was all achieved without requiring an exceptionally low string scale or especially large extra-dimensions. For the inflation model considered we utilised Yukawa couplings of order $10^{-10}$ with a string scale of order $10^{13}$ GeV, the largest extra dimensions having a compactification scale of order $10^{8}$ GeV. Therefore we have achieved the very large hierarchy between the Yukawa couplings without needing the exceptionally small string scales put forward by [82].

In the analysis we showed the importance of the contribution of the twisted moduli to the soft spectrum. This required the extension of the previous analysis away from the $\lambda_I = 1$ limit which is all that had been previously considered. Through this it was shown that moving beyond the assumption that SUSY breaking is provided solely by the $S/T_i$ fields removed the Goldstino angle independence of the sum rules and allowed the inflationary requirements of Eq. (3.27) to be met.

It is also clear that this model does not exhaust the possibilities of this framework. In chapter 5 we further extend the construction, making use of the small Yukawa couplings to include a model of Dirac neutrinos.

# Chapter 5

# Dirac Neutrinos and Hybrid Inflation from String Theory

We now turn to another application of the D-brane framework: Dirac masses for neutrinos. The anisotropies in the radii give rise to small Yukawa couplings that are utilised for the neutrino masses. Not only is the generation of Dirac masses formulated in the same framework as the inflationary particle physics model of chapter 3, but they can both be realised at the same time, as different aspects of the same model.

We write down a minimal neutrino model that reproduces bi-large mixing. This model is not minimal in the sense that it has the smallest particle content, but in the sense that it is the easiest to construct, while remaining phenomenologically viable.

## 5.1 Particle Physics Model

As discussed in section 1.5 the main objection to Dirac neutrinos is the relative smallness of their Yukawa couplings, when compared with other leptons in the Standard Model. In the introduction the desired Yukawa couplings were shown to be $10^{-12} - 10^{-13}$ and we will discuss how these couplings might arise in a string theory context. Our starting point will be to utilise the small coupling, $g_{5_2}$, and we will construct non-renormalisable contributions to the neutrino mass matrix. We will demonstrate how this is consistent with the Type I string framework laid out in chapter 2.

In addition to providing a model of neutrino masses we will also demonstrate that it is possible to link this model with the inflationary scenario presented in chapter 3.

### 5.1.1 The Model

In this chapter we again take the approach that the phenomenology comes first then we demonstrate that it can be constructed within the string framework. To be more precise we lay out the mass matrix we wish to construct, find operators suitable for generating said matrix and then see how this might be accommodated within the string superpotential. The Dirac mass matrices we wish to find are:

$$m^{\nu}_{LR} \sim \begin{pmatrix} 0 & a & 0 \\ 0 & b & e \\ 0 & c & f \end{pmatrix} \cdot \langle H_u \rangle \ , \quad m^{E}_{LR} \sim \begin{pmatrix} 0 & 0 & 0 \\ 0 & A & 0 \\ 0 & 0 & \mathcal{O}(1) \end{pmatrix} \cdot \langle H_d \rangle \tag{5.1}$$

where for the moment we take the elements of the matrices to be numbers, determined by phenomenology, but they will be shown to arise from the vevs of fields within non-renormalisable operators.

To analyse the matrices in Eq. (5.1) we first make the assumption that the couplings are sequentially dominant[1]

$$e, f \gg a, b, c \tag{5.2}$$

where much larger means, in this case, that they are greater by a factor of approximately 5. Clearly, since $m^{E}_{LR}$ is diagonal, the lepton mixing matrix $U_{MNS} = R_{23}U_{13}R_{12}$ (see

[1]For a review of sequential dominance in neutrinos and charged leptons see [83].

appendix A for details) will entirely be given by the diagonalisation matrix for the neutrino sector, i.e. $U_{MNS} = U^{\dagger}_{\nu L}$ with $\mathrm{diag}(m_1, m_2, m_3) = U_{\nu_L}\, m^{\nu}_{LR}\, U^{\dagger}_{\nu R}$. Thus we find for the MNS mixings:

$$\begin{aligned} \tan(\theta_{23}) &\approx \frac{e}{f}\,, && (5.3)\\ \tan(\theta_{12}) &\approx \frac{a}{c_{23}\, b - s_{23}\, c}\,, && (5.4)\\ \theta_{13} &\approx 0\,, && (5.5) \end{aligned}$$

with $m_3$ and $m_2$ given by

$$\begin{aligned} m_3 &\approx \sqrt{e^2+f^2}\, v_u = \frac{e}{s_{23}}\, v_u\,, && (5.6)\\ m_2 &\approx \sqrt{a^2+(c_{23}\, b - s_{23}\, c)^2}\, v_u = \frac{a}{s_{12}}\, v_u\,, && (5.7)\\ m_1 &\approx 0 && (5.8) \end{aligned}$$

where the measured values for the masses and mixings are, assuming the lowest mass is zero [31]

$$\theta_{23} \approx 45^{\circ},\ \theta_{12} \approx 34^{\circ} \text{ and } \theta_{13} \approx 0 \tag{5.9}$$

$$m_3 \approx 0.05 \text{ eV},\ m_2 \approx 0.01 \text{ eV and } m_1 \approx 0. \tag{5.10}$$

It is clear from these expressions that we expect that $e$ and $a$ can be, at most, of the order of $10^{-13}$, given the hierarchical neutrinos with $m_3 \simeq 0.05$ eV and $m_2 \simeq 0.01$ eV. It is clear that the Yukawa coupling of the inflation sector, $\lambda = 10^{-10}$, is too large and cannot be identified with any of these couplings. The choice of $\lambda = 10^{-10}$ was motivated by the desire to connect the Peccei-Quinn and electroweak scales and so it is worth considering how tight these bounds are.

From Eqs. (3.13) and (3.23) we see that

$$f_a \sim \frac{A_\lambda}{2\sqrt{2}\lambda} \tag{5.11}$$

and from [14]

$$10^{10}\ \text{GeV} \leq f_a \leq 10^{13}\ \text{GeV} \tag{5.12}$$

where the upper bound is pushing at the bounds considered in [14], but [69] argue that much higher $f_a$ is possible. In principle we would need to push $f_a$ to $10^{16}$ GeV to allow $\lambda \simeq 10^{-13}$; however, the full structure of Eq. (5.1) cannot be obtained by the string selection rules alone, suggesting that we can utilise the mechanism of structure formation to increase the suppression. Instead of using $f_a = 10^{16}$ GeV we choose to be moderately conservative, making use of the upper bound on $f_a$ in Eq. (5.12) requiring that we set $\lambda \sim 10^{-10}$. Therefore the axion physics motivates that $\lambda$ lie in the range $10^{-7} > \lambda > 10^{-10}$ (with the aforementioned uncertainty in the lower bound), but it is the neutrino physics that motivates the choice of the smallest possible coupling.

The mechanism we introduce to generate Eq. (5.1) is a minimal FN (see appendix B) construction, minimal in the sense that we only go up to dimension four superpotential terms. Since $\lambda \sim 10^{-10}$ and all of the elements in Eq. (5.1) are $\leq 10^{-13}$, dimension three operators cannot contribute directly to Eq. (5.1). As such we restrict our attention to dimension four operators. The price we pay for keeping the dimensions of the operators low is that large ($\mathcal{O}(10^{-3})$) ratios between the flavon vevs and the messenger fields are required. To generate the structure observed in Eqs. (5.3-5.5) we must require that, in general, the flavon vevs are not equal and hence we require multiple flavons. In this sense our model is not minimal. We do not believe that higher dimensional models with fewer flavons are necessarily inconsistent with the string framework, but they are more troublesome to construct.

Before we embark on the FN construction we will consider obtaining Eq. (5.1) via the string selection rules with only one flavon. The motivation for this study would be to provide a geometric origin for the structure of Eq. (5.1) in addition to setting the scale. The exact values of the order one Yukawas, stemming from the small extra dimensions, could give rise to Eq. (5.1). However, it is difficult to obtain the measured masses and mixings presented in Eqs. (5.9) and (5.10) just from geometric arguments for the following reasons. Firstly there are only three independent order one Yukawa couplings, $g_{5_2}$, $g_{5_3}$ and $g_9$ and, as stated, Eq. (5.1) has five parameters. However it is possible to obtain realistic masses and mixings with the following assumptions: $e = f$ and $a = c$. The main difficulty lies in differentiating the three generations: the right-handed neutrinos and left handed leptons must be assigned differently, between the generations. This leads to differing messenger assignments and, since we choose not to impose any additional symmetries, the flavon will necessarily be a gauge singlet. Care must be taken to ensure that it does not couple with the inflaton and invalidate the inflation model. For the above reasons we instead choose the following approach: we impose additional flavour symmetries to differentiate the generations and use the same assignments in all three generations.

### 5.1.2 Froggatt-Nielsen Construction

To build up the FN sector we need to write down the renormalisable operators that make up the dimension four operators. Let us consider the elements of Eq. (5.1) which will share a common structure in the FN construction. In full, the operators we wish to assemble are

$$\lambda \frac{\langle F_{ij} \rangle}{\langle \psi \rangle} L_i H_u \nu_{Rj}^c \, , \tag{5.13}$$

where $\langle \psi \rangle$ is the messenger vev[2] and $F_{ij}$ represents all the possible flavons. $\lambda$ is a small Yukawa coupling that will be identified with $g_{5_1}$. To reproduce the required order of

[2] The mass being given by the coupling and this vev

magnitude imposed by Eq. (5.6) we need

$$\frac{\langle F_{ij}\rangle}{\langle\psi\rangle} \sim 10^{-2} - 10^{-3}. \tag{5.14}$$

and to restrict the allowed operators and hence introduce the zero entries in the mass matrix, Eq. (5.1), we will impose additional flavour symmetries. This will be discussed in section 5.1.4.

Eq. (5.13) is built up out of the following three operators:

$$\lambda H_u \nu^c_{Rj}\chi_{F_{ij}},\ \chi_{F_{ij}}\overline{\chi}_{F_{ij}}\psi \text{ and } \overline{\chi}_{F_{ij}} L_i F_{ij} \tag{5.15}$$

where $\overline{\chi}_{F_{ij}}$ and $\chi_{F_{ij}}$ are messenger fields. Schematically the generation of Eq. (5.13) can be represented as

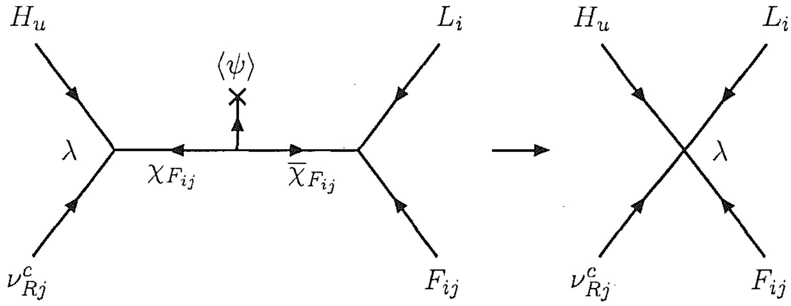


Figure 5.1: Froggatt Nielsen supergraphs generating the neutrino mass matrix.

We now consider how this can be embedded in the string framework presented in chapter 2 using the methodology discussed in section 4.1.1.

### 5.1.3 String Construction

The starting point of the string construction is the inflation model, as presented in table 4.1. In order to simultaneously accommodate both models the assignments of the inflation model are taken as fixed and then consistent neutrino assignments are made. The only fields that appear in both sectors are the $H_u$ and $H_d$ fields, which are assigned to $C^{5_1 5_2}$. For $\lambda H_u \nu^c_{R_j} \chi_{F_{ij}}$ we look in Eq. (2.5) for any terms that contain both $C^{5_1 5_2}$ and $g_{5_1}$. The only term that contains both is

$$g_{5_1} C_3^{5_1} C^{5_1 5_2} C^{5_1 5_2}. \tag{5.16}$$

We now see that there are now two reasons to assign $\nu^c_{R_j}$ to $C_3^{5_1}$. Firstly $C_3^{5_1}$ always appears with the small coupling, $g_{5_1}$, and hence it is impossible to write down right-handed neutrinos that do not couple with the small coupling[3]. Secondly if we instead tried to assign $\chi_{F_{ij}}$ to $C_3^{5_1}$ it would be impossible to write down an unsuppressed mass term for the messenger fields. Since we want the messenger masses to be integrated out before we reach the electroweak scale we require that their masses be as high as possible. Before we quantify "as high as possible" we must complete the FN construction.

With $\nu^c_{R_j}$ assigned to $C_3^{5_1}$ Eq. (5.16) requires that $\chi_{F_{ij}}$ be assigned to $C^{5_1 5_2}$. In turn this leads us to consider all possible terms that can contain $C^{5_1 5_2}$, but with a large coupling:

$$g_9 C^{5_1 5_2} C^{5_2 5_3} C^{5_3 5_1}, \; g_{5_2} C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2} \text{ and } g_{5_3} C^{5_1 5_2} C^{9 5_1} C^{9 5_2}. \tag{5.17}$$

It can be shown that each of these choices are equally good candidates for $\chi_{F_{ij}} \overline{\chi}_{F_{ij}} \psi$ in the sense that they all satisfy gauge invariance and give rise to the same field theory operators. Essentially the analysis in all three cases is identical, so we focus on the term with coupling $g_{5_2}$. In our choice of assignment for $\overline{\chi}_{F_{ij}}$ it is again possible to show that both choices are equally good and again we focus on one case, with $\overline{\chi}_{F_{ij}}$ assigned to $C^{5_1 5_2}$. This leads us to the final term, $\overline{\chi}_{F_{ij}} L_i F_{ij}$. Once again we are presented with choices: it is possible to assign $\overline{\chi}_{F_{ij}} L_i F_{ij}$ any of the (many) terms in Eq. (2.9) with order one couplings. We select perhaps the simplest of possibilities in which the term is found within $g_{5_2} C_3^{5_2} C^{5_1 5_2} C^{5_1 5_2}$ and let $L_i$ be assigned to $C_3^{5_2}$ and hence $F_{ij}$ is assigned

[3]As can be seen on close inspection of Eq. (2.9): $C_3^{5_1}$ always appears with the coupling $g_{5_1}$

to $C^{5_15_2}$. It is now possible to re-draw fig. 5.1 including the string couplings.

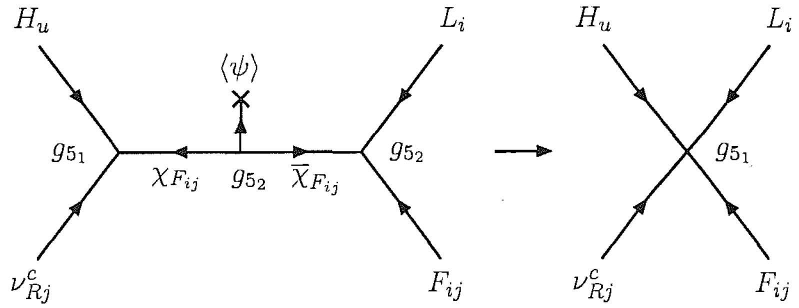


Figure 5.2: Froggatt Nielsen supergraphs leading to neutrino Yukawa couplings which are suppressed compared with the already small tree-level value $g_{5_1} = 10^{-10}$.

As in 4.1 we note that our approach does not allow us to fully specify the manner in which these particular representations are arrived at, but we can confidently state the following: if we found that MSSM-like fields were forced to have both ends on the $D5_1$ brane stack it would be impossible to give them order one gauge transformations. Hence it would be inconsistent to assign them this way. However we can see that gauge invariance should always be satisfied, since all the string states that have MSSM charges ($\chi_{F_{ij}}$, $\overline{\chi}_{F_{ij}}$ and the normal MSSM states) have at least one end attached to the $D5_2$ brane. In section 4.1, in the paragraph following Eq. (4.3), it was decided that the $D5_2$ brane would contain all the MSSM gauge groups. Since the fields we are considering are, at most, representations of one non-abelian gauge group, $SU(2)_L$, this can be satisfied by states with one string end on the "MSSM" brane, $D5_2$.

The set of assignments and corresponding string terms are best summarised by table 5.1.

| $g_{5_1}$ | $C_3^{5_1}$ | $C^{5_15_2}$ | $C^{5_15_2}$ | | $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_15_2}$ | $C^{5_15_2}$ | | $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_15_2}$ | $C^{5_15_2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\lambda$ | $\nu^c_{R_j}$ | $H_u$ | $\chi_{F_{ij}}$ | | 1 | $\psi$ | $\chi_{F_{ij}}$ | $\overline{\chi}_{F_{ij}}$ | | 1 | $L_i$ | $\overline{\chi}_{F_{ij}}$ | $F_{ij}$ |

Table 5.1: Neutrino FN String assignments

Now all the elements making up Eq. (5.13) have been assembled we can address the question of exactly how large is "as high as possible". If the framework were purely four dimensional effective field theory then the messenger mass could be sent as high as the cut-off and no problems would arise. However the underlying theory is $D = 10$ string theory and the size of the extra dimensions proves relevant here. For simplicity, we just consider the effect of a flat torus and its corresponding KK and winding modes. If we allowed the messenger mass to be comparable to the KK or winding modes of its string state then it would be necessary to consider the effects of exchanging higher KK/winding states in an FN diagram. It is easy to see that[4], if the a level $i$ KK state is exchanged, it will give the same contribution to the superpotential as the zero mode with the following coefficient

$$\frac{M_\chi}{M_\chi + iM_{KK}}. \tag{5.18}$$

This becomes troublesome when $M_\chi \geq M_{KK}$ as the higher $M_{KK}$ modes become increasingly relevant as $M_\chi$ grows. An approximate value for the sum of all the KK/winding contributions can be obtained by integrating Eq. (5.18) over $i$ from 1 to $n$. We obtain

$$\frac{M_\chi}{M_{KK}}\left(\ln\left(1+n\frac{M_{KK}}{M_\chi}\right)-\ln\left(1+\frac{M_{KK}}{M_\chi}\right)\right) \tag{5.19}$$

which diverges as $n \to \infty$. Clearly, as we are working in an effective field theory, the existence of a cut-off prevents $n$ from going towards infinity. Nonetheless a finite contribution remains, which we may estimate using Eq. (5.19) with the lowest KK/winding modes being approximately $10^8$ GeV as discussed after Eq. (2.31). From Eq. (5.19) it can be seen that the coefficient varies linearly with the ratio of $M_\chi$ and $M_{KK}$, with a slowly varying logarithmic correction. If we require that the theory be

[4]This is because the FN diagram exchanging KK/winding modes is identical to the zero mode diagram, but with a higher mass propagator.

cut off at $M_* \approx 10^{13}$ GeV and then impose $M_\chi = 10^8$ GeV we find that $n \approx 10^5$ and the correction is $\mathcal{O}(10)$, whereas if $M_\chi = 10^7$ GeV the correction is $\mathcal{O}(1)$.

Since the we are interested in the overall contribution to the mass matrix, Eq. (5.1), we see that increasing the messenger mass above the compactification scale introduces very little additional suppression. Put another way, the maximum suppression achievable is determined by the compactification scale. As such we set the messenger mass, $g_{5_2} \langle \phi \rangle$, to be $10^7$ GeV requiring the product of the flavon vevs with $g_{5_2}$ to be $\sim 10^4 - 10^5$ GeV. This leaves Eq. (5.14) intact.

Majorana masses for the right-handed neutrinos are not allowed at renormalisable level due to the string selection rules. $C_3^{5_1}$ terms never appear quadratically in the string superpotential, Eq. (2.9). Higher-dimensional operators for Majorana masses are suppressed by $(g_{5_1})^2 \sim (10^{-10})^n$ with $n \geq 2$. As a result the see-saw mechanism cannot appear in our model, but small Majorana masses could be included. This is not allowed in the model as it stands since the FN sector does not generate such masses. As such we have several possibilities. If we wish to forbid these masses then we can either impose an additional symmetry, for example $\mathrm{U}(1)_{B-L}$, or we simply leave the model as it stands and postulate that the necessary messenger fields simply are not present in the spectrum. On the other hand, to include these masses we can extend the FN sector to generate them. So doing the model would include pseudo-Dirac neutrinos with a correspondingly rich phenomenology [84, 32]. Unfortunately, due to the freedom inherent in the FN approach, it is difficult to make hard predictions for the Majorana masses, beyond the upper limit of the Yukawa coupling, $10^{-20}$. However if we restrict all flavon vevs to be of the same orders as found thus far and impose the same restriction on the messenger masses we expect dimension 4 operators of the following form.

$$\frac{\lambda^2 f f \nu_R^c \nu_R^c}{\langle \psi \rangle} \approx \nu_R^c \nu_R^c \left(10^{-10} - 10^{-12}\right) \text{eV} = m_M \nu_R^c \nu_R^c. \tag{5.20}$$

Given a Dirac mass of $m_2 = 0.01$ eV this results in a pseudo-Dirac splitting of

$$\delta m_2^2 \simeq 2m_2(m_M) \simeq 2 \times \left(10^{-12} - 10^{-14}\right) \tag{5.21}$$

which is intriguing when considering the current bound of $\delta m_2^2 < 10^{-12}$ eV$^2$ [32]. Since our expected splitting is somewhere between just above and just below the current bounds the results of the next generation of neutrino telescopes, eg. IceCube [85], could be important to our model. The splitting of $m_3$ is expected to be larger by a factor of $m_1/m_2 \sim 5$, but the bounds are much weaker, $\delta m_1^2 < 10^{-4}$, so this is of less interest. Since we can also explain why there might be no splitting, Eq. (5.21) is by no means a prediction of our model, but would be an interesting confirmation.

The analysis for the charged lepton FN contribution mass matrix is largely the same as for the neutrinos, with the obvious exception that only order 1 Yukawa couplings can be utilised. As such we omit the derivation and present the results here in table 5.2 and in fig. 5.3.

| $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_1 5_2}$ | $C^{5_1 5_2}$ | | $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_1 5_2}$ | $C^{5_1 5_2}$ | | $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_1 5_2}$ | $C^{5_1 5_2}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | $L_2$ | $H_d$ | $\chi_A$ | | 1 | $\psi$ | $\chi_A$ | $\overline{\chi}_A$ | | 1 | $A$ | $\mu_R^c$ | $\overline{\chi}_A$ |

Table 5.2: FN String assignments for the muon

In one respect the charged leptons' mass matrix differs from the neutrinos: one element is renormalisable. Since we do not require any FN suppression to get a realistic tau mass and so that the flavon vevs can all be order $10^4 - 10^5$ GeV we generate this mass from a renormalisable operator. The operator and its assignments are given in table 5.3.

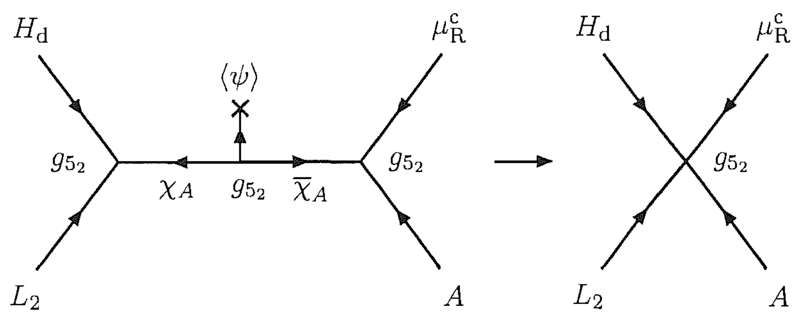


Figure 5.3: Froggatt-Nielsen supergraphs leading to the muon mass. Higher-dimensional FN diagrams can generate NNLO Yukawa couplings, e.g. for realising the electron mass.

| $g_{5_2}$ | $C_3^{5_2}$ | $C^{5_1 5_2}$ | $C^{5_1 5_2}$ |
|---|---|---|---|
| 1 | $L_3$ | $H_d$ | $t^c_R$ |

Table 5.3: Tau Lepton String assignments

To restrict the set of allowed operators we must impose additional flavour symmetries under which the flavons and matter fields both have charges. This will be the subject of the next section.

### 5.1.4 Flavour Symmetries

The approach of this section is to impose additional symmetries to disallow certain couplings between matter and flavons. This is necessary because the string assignments chosen in section 5.1.3 allow all intergenerational Yukawa couplings. In fact we would expect the mass matrix to be proportional to

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \tag{5.22}$$

since all the flavons would couple equally since they would have the same quantum numbers.

Manifestly this is not the case in nature, so we impose a flavour symmetry, $U(1)_R \times \mathbb{Z}_3$, under which the different generations of neutrinos and charged leptons are distinguished. We note that a $U(1)_R$ symmetry is to be expected (if not explicitly required) in a supersymmetric theory and that discrete symmetries are common in orbifolds [86]. The $U(1)_R$ symmetry will be broken to its $\mathbb{Z}_2$ subgroup, R-Parity as described in [22]. The full set of assignments and charges are given in tables 5.4 and 5.5.

For completeness we include the full renormalisable superpotential consistent with these assignments and symmetries

$$\begin{aligned} W_{ren.} = \; & g_{5_2} L_3 H_d \tau_R^c + g_{5_2} L_2 H_d \chi_A + g_{5_2} \chi_A \psi \bar{\chi}_A + g_{5_2} \bar{\chi}_A \mu_R^c A \\ & + g_{5_1} \chi_a H_u \nu_{R2}^c + g_{5_2} \chi_a \psi \bar{\chi}_a + g_{5_2} L_1 \bar{\chi}_a a + g_{5_2} L_2 \bar{\chi}_a b + g_{5_2} L_3 \bar{\chi}_a c \\ & + g_{5_1} \chi_e H_u \nu_{R3}^c + g_{5_2} \chi_e \psi \bar{\chi}_e + g_{5_2} L_2 \bar{\chi}_e e + g_{5_2} L_3 \bar{\chi}_e f \\ & + g_{5_1} \phi H_u H_d + g_{5_1} \phi N^2 + g_{5_2} Q_3 H_u t_R^c + g_{5_2} Q_3 H_d b_R^c. \end{aligned} \tag{5.23}$$

As discussed in [87] the soft terms can be seen to explicitly break $U(1)_R$. This can be seen by considering the gaugino mass terms,

$$m\lambda\lambda, \tag{5.24}$$

which have R-charges of 2. If we parametrise the $U(1)_R$ rotation by $e^{i\omega Q}$, where $Q$ is the R-charge and $\omega$ a real parameter, then $\omega = n\pi$, where $n$ is an integer, leaves Eq. (5.24) invariant. Since the smallest charge possessed by any field in the model is 1/2 then, under an $\omega = n\pi$ rotation such a field will pick up a phase of $e^{\frac{i\pi}{2}} = i$, hence the remaining symmetry is $\mathbb{Z}_4$. This is in turn broken when $N$ or $A$ obtains a vev. If we require that $A$ be stabilised at its minimum before inflation then the potential domain wall problem is avoided since they will be inflated away. It does not seem unreasonable that this should take place since the inflation and flavon sectors are decoupled.

| | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_R$ | $\mathbb{Z}_3$ | String State |
|---|---|---|---|---|---|---|
| $Q_3$ | $\mathbf{3}$ | $\mathbf{2}$ | 1/6 | -1/2 | 1 | $C_3^{5_2}$ |
| $t_R^c$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | -2/3 | -1/2 | 1 | $C^{5_1 5_2}$ |
| $b_R^c$ | $\bar{\mathbf{3}}$ | $\mathbf{1}$ | 1/3 | -1/2 | 1 | $C^{5_1 5_2}$ |
| $H_u$ | $\mathbf{1}$ | $\mathbf{2}$ | 1/2 | 1 | 1 | $C^{5_1 5_2}$ |
| $H_d$ | $\mathbf{1}$ | $\mathbf{2}$ | -1/2 | 1 | 1 | $C^{5_1 5_2}$ |
| $\nu_{R2}^c$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | -3/2 | 0 | $C_3^{5_1}$ |
| $\nu_{R3}^c$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | -7/2 | 1 | $C_3^{5_1}$ |
| $L_1$ | $\mathbf{1}$ | $\mathbf{2}$ | -1/2 | -1/2 | 2 | $C_3^{5_2}$ |
| $L_2$ | $\mathbf{1}$ | $\mathbf{2}$ | -1/2 | -3/2 | 1 | $C_3^{5_2}$ |
| $L_3$ | $\mathbf{1}$ | $\mathbf{2}$ | -1/2 | -5/2 | 1 | $C_3^{5_2}$ |
| $\mu_R^c$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 | 3/2 | 1 | $C^{5_1 5_2}$ |
| $\tau_R^c$ | $\mathbf{1}$ | $\mathbf{1}$ | 1 | 7/2 | 1 | $C^{5_1 5_2}$ |
| $\phi$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 1 | $C_3^{5_1}$ |
| $N$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 1 | 1 | $C^{5_1 5_2}$ |
| $A$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 1 | 0 | $C_3^{5_2}$ |
| $a$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 3 | 0 | $C^{5_1 5_2}$ |
| $b$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 4 | 1 | $C^{5_1 5_2}$ |
| $c$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 5 | 1 | $C^{5_1 5_2}$ |
| $e$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 6 | 0 | $C^{5_1 5_2}$ |
| $f$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 7 | 0 | $C^{5_1 5_2}$ |
| $\psi$ | $\mathbf{1}$ | $\mathbf{1}$ | 0 | 0 | 0 | $C_3^{5_2}$ |

Table 5.4: Matter fields and flavons

| | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_R$ | $\mathbb{Z}_3$ | String State |
|---|---|---|---|---|---|---|
| $\chi_A$ | **1** | **1** | 1 | 5/2 | 1 | $C^{5_1 5_2}$ |
| $\overline{\chi}_A$ | **1** | **1** | -1 | -1/2 | 2 | $C^{5_1 5_2}$ |
| $\chi_e$ | **1** | **2** | -1/2 | 9/2 | 1 | $C^{5_1 5_2}$ |
| $\overline{\chi}_e$ | **1** | **2** | 1/2 | -5/2 | 2 | $C^{5_1 5_2}$ |
| $\chi_a$ | **1** | **2** | -1/2 | 5/2 | 2 | $C^{5_1 5_2}$ |
| $\overline{\chi}_a$ | **1** | **2** | 1/2 | -1/2 | 1 | $C^{5_1 5_2}$ |

Table 5.5: Messenger fields

Finally we wish to address the following technical point, that has been touched on earlier in the thesis. In string theory we expect that all matter fields should transform, at some stage, under a gauge symmetry, yet we have numerous examples of gauge singlets within our model. For these two statements to agree it must be possible to assign charges to all the matter fields that are currently singlets.

To demonstrate this, let us consider two additional U(1) symmetries, $U(1)_{5_1}$ on $D5_1$ and $U(1)_{5_2}$ on $D5_2$, and assign charges to the SM-singlet fields $\phi, \nu^c_{Ri}, N$ and $\psi$. First, giving $H_u$ and $H_d$ $U(1)_{5_1}$ charges of 1, we see that $\phi$ has charge $-2$ and thus $N$ has charge 1. From the FN diagram in Fig. 5.1 we can determine the charges of the messenger fields if we assign a $U(1)_{5_1}$-charge $q$ to the right-handed neutrinos $\nu^c_{Ri}$ and finally the flavons $F_{ij}$, which are intersection states $C^{5_1 5_2}$, end up with charge $-(q+1)$. Note that only fields which are assigned to string states $C_3^{5_1}$ and $C^{5_1 5_2}$ can be charged under $U(1)_{5_1}$ and only fields $C_3^{5_2}$ and $C^{5_1 5_2}$ can be charged under $U(1)_{5_2}$. Similarly for the $C_3^{5_2}$ state $\psi$, from the FN diagram in Fig. 5.1 we see how giving it a $U(1)_{5_2}$ charge $p$ determines e.g. the charge of $\bar{\chi}_{F_{ij}}$ to be $-p$ and the charge of the flavons $F_{ij}$ to be $p$. It is easy to see that this charge assignment can be extended consistently to all the fields of the model.

This completes the supersymmetric side of the string construction. For the soft spectra we see that it has already been calculated to the same extent as in the inflation sector. Because of our particular choice of assignments only three string states are made use of: $C_3^{5_2}$, $C^{5_1 5_2}$ and $C_3^{5_1}$. In section 4.2.1 analytic expressions for $m^2_{C_3^{5_2}}$, $m^2_{C^{5_1 5_2}}$ and $m^2_{C_3^{5_1}}$ were given by Eqs. (4.13-4.15) respectively and table 4.2 summarises some example points satisfying all of the constraints on the inflation model. Imposing more constraints on the model, coming from a more rigorous phenomenological study of the combined inflation and neutrino model, is likely to modify table 4.2.

### Dirac Neutrino Conclusions

It is clear from this long string of assignments that there are many ways in which the neutrino sector could be realised within the string superpotential. This is in contrast to the case of the inflation model, discussed in chapter 4, in which there are very few choices to be made. Nonetheless we believe it is sufficient to show that there exists at least one realisation, the hope being that a full non-perturbative string calculation would select the correct model if such a model exists. Be that as it may our goal was to simultaneously construct both models within the same framework and it has been demonstrated that this is possible with the set of assignments given in tables 5.4 and 5.5.

From a phenomenological viewpoint it is interesting that, using a small Yukawa coupling, we are able to relate physics at very different scales: namely the neutrino mass, electroweak and Peccei-Quinn scales. Of course this required us to introduce the flavon vevs and messenger masses into the theory, more scales that need justifying. Though we did not attempt to construct the flavon and messenger sectors in full, not to the extent that we achieved for the inflaton sector, we can conceive of possible origins for those scales. For the messenger masses we noted that the compactification scale sets a upper limit on the amount of suppression it is possible to obtain, regardless of the messenger mass. With this in mind we could instead use the lowest KK excitation as a messenger mass, if the orbifold symmetry forbids the zero mode. As noted in the text following Eq. (5.19) the sum will give an order of magnitude increase with respect to the lowest mode's contribution. However, since the lowest mode is an order of magnitude larger than the messenger mass given in the model the vevs of the flavons remain at the same order of magnitude. Since the flavon vevs are within two orders of magnitude of the soft scale it is not unreasonable that the soft terms govern the size of the vevs. This could be analogous to, but not as extreme as, the soft terms setting the scale for the $\phi$ and $N$ vevs.

In this model we restricted the set of allowed operators by the inclusion of a $U(1)_R \times \mathbb{Z}_3$ symmetry leading to the neutrino mass matrix, Eq. (5.1). It would be interesting to utilise the string framework to include the quark sector. While it is clear that an analogous construction could be made, with one flavon per entry in the Yukawa matrix, it would be more satisfying to use fewer flavons and higher orders. It may be possible to relate the Cabibbo angle, $\theta_C$, to the neutrino mass hierarchy, $m_2/m_3$, in terms of an expansion parameter $\lambda = \theta_C$. However, this would require a substantial re-working of the neutrino sector as well as careful construction of the quark sector.

Finally we note that the Dirac nature of the neutrinos is difficult to determine since it would chiefly be confirmed by the non-observation of neutrinoless double $\beta$ decay. Clearly, since one can only set limits by this approach it is not possible to use it to prove that neutrinos are Dirac. However it may be possible to obtain pseudo-Dirac neutrinos within the string framework and hence produce measurable effects. We expect a splitting given by $2 \times 10^{-12}$ eV$^2 \geq \delta m^2 \geq 2 \times 10^{-14}$ eV$^2$ from Eq. (5.21), but unfortunately cannot turn this expectation into a hard prediction. Nonetheless detection of this level of splitting in forthcoming neutrino telescopes would be an exciting confirmation of our model.

# Chapter 6

# Conclusions

To sum up we have presented a Type I string construction for a model of Dirac neutrinos and hybrid inflation. This model is consistent with experimental observations of neutrino masses and the MNS matrix and provides a viable candidate for early universe inflation. Our string construction was performed in a very general framework and it was interesting to see that even this placed strong restrictions on the models we could build. Requiring all interactions to be renormalisable proved too strong a constraint for realistic flavour physics: this results from the small number of free parameters and the form of the Type I superpotential. While we do not provide a no-go theorem to this effect it is a very strong conjecture that flavour physics cannot result from the renormalisable superpotential. This is further strengthened if we wish to include the quark sector. However going beyond renormalisable level considerably relaxes these constraints and it should be possible to accommodate realistic quark models within this framework.

It was also interesting to discover that phenomenologically viable soft terms can necessitate moving beyond the assumption of dilaton and untwisted moduli dominance. While the soft spectrum is appealingly simple, in terms of the sum-rule relationships, it proved too restrictive for the inflation model. In this thesis we put forward the twisted moduli sector as an additional source of SUSY breaking and thereby violating the sum-rules to obtain acceptable soft parameters. It is pleasing to note that it was possible to achieve the desired SUSY breaking simply by including the effect of fields already present in the theory whose effects had been turned off in the original analysis.

# Appendix A

# Lepton mixing conventions

For the mass matrix of the charged leptons $m_{LR}^E = Y_e v_d$ defined by $\mathcal{L}_e = -m_{LR}^E \overline{e}_L^f e_R^{cf}$ + h.c. and for the Dirac neutrino mass matrix $m_{\mathrm{LR}}^\nu = Y_\nu v_u$ defined by $\mathcal{L}_\nu = -m_{LR}^\nu \overline{\nu}_L^f \nu_R^{cf}$ + h.c., where $v_u = \langle H_u^0 \rangle$ and $v_d = \langle H_d^0 \rangle$, the change from flavour basis to mass eigenbasis can be performed with the unitary diagonalisation matrices $U_{e_L}, U_{e_R}$ and $U_{\nu_L}, U_{\nu_R}$ by

$$U_{e_L}\, m_{LR}^E\, U_{e_R}^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_{\nu_L}\, m_{LR}^\nu\, U_{\nu_R}^\dagger = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}. \tag{A.1}$$

This rotation manifests itself in the interactions with the $W$ bosons. The $W^+$ couples to the lepton current

$$J_W^{\mu+} = \frac{1}{\sqrt{2}} \left( \overline{\nu}_{Li}^f \gamma^\mu e_{Li}^f \right) \tag{A.2}$$

which is not invariant under the diagonalising rotations. In the mass basis $J_W^{\mu+}$ becomes

$$J_W^{\mu+} = \frac{1}{\sqrt{2}} \left( \overline{\nu}_{Li} \gamma^\mu U_{ij}^{MNS} e_{Lj} \right) \tag{A.3}$$

with the mixing matrix in the lepton sector, the MNS matrix, given by

$$U^{MNS} = U_{e_L} U_{\nu_L}^\dagger \,. \tag{A.4}$$

We use the parameterisation $U_{MNS} = R_{23} U_{13} R_{12}$ with $R_{23}, U_{13}, R_{12}$ defined as

$$R_{12} := \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{13} := \begin{pmatrix} c_{13} & 0 & \widetilde{s}_{13} \\ 0 & 1 & 0 \\ -\widetilde{s}_{13}^{*} & 0 & c_{13} \end{pmatrix}, \quad R_{23} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix},$$

and where $s_{ij}$ and $c_{ij}$ stand for $\sin(\theta_{ij})$ and $\cos(\theta_{ij})$, respectively. $\delta$ is the Dirac CP phase relevant for neutrino oscillations and we have defined $\widetilde{s}_{13} := s_{13} e^{-i\delta}$.

# Appendix B

# Froggatt-Nielsen Mechanism

The Froggatt-Nielsen mechanism [88] utilises higher dimensional effective operators to generate realistic Yukawa couplings. The original approach allowed the left and right-handed quarks to obtain charges under a new symmetry $\mathrm{U}(1)_F$, which required extra, "flavon" fields, $F_{ij}$, to be used to make up an effective operator, invariant under the entire symmetry group of the model. Since these operators are non-renormalisable they must be suppressed by a large mass scale, $M_\chi$, in the effective field theory. It was assumed that $\mathrm{U}(1)_F$ was broken around $M_\chi$ both giving rise to a vev for the flavons and providing a mass $M_{\chi_{ij}}$ to "messenger" fields, $\chi_{ij}$, that, when integrated out, generated the effective operators.

In the original work only one flavon and one messenger were utilised and so effective operators were of the following form

$$-\overline{Q}_i H d_{Rj} \left(\frac{F}{M_\chi}\right)^{(a_i+b_j)} + h.c. \tag{B.1}$$

where $F$ is assumed to have charge $-1$, $Q_i$ charge $-a_i$ and $d_{Rj}$ charge $b_j$. $H$ is assumed to have zero charge, but this is a choice, not a requirement. If $F$ obtains a vev such that $\langle F \rangle / M_\chi = \epsilon < 1$ then Eq. (B.1) becomes

$$-\epsilon^{a_i+b_j}\overline{Q}_i H d_{Rj} + h.c. \tag{B.2}$$

and, for appropriate choices of $\epsilon$ and charges for the quark, this can be made to generate the SM quark Yukawas in Eq. (1.10). This can be generalised in a straightforward manner to include the neutrinos and charged leptons.

In our work we differ from the original paper in several important ways: our model is supersymmetric, we allow more than one flavon and we restrict ourselves to dimension four superpotential terms.

## B.1 Supergraph Formalism

Starting with the following superpotential, which is an abstracted version of those found in chapter 5

$$W = A_i C \chi_{ij} + \langle \psi \rangle \chi_{ij} \overline{\chi}_{ij} + \overline{\chi}_{ij} B_j F_{ij} \tag{B.3}$$

one can see intuitively that this represents the generation of effective operators by writing diagrams in which a heavy superfield is exchanged, see fig. B.1.

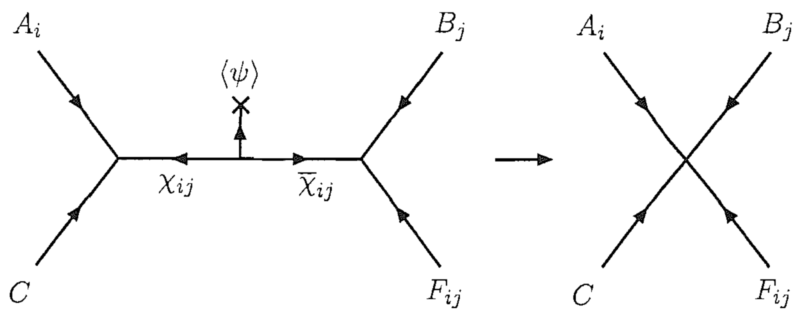


Figure B.1: Generic Froggatt-Nielsen supergraphs diagram

This is analogous to the exchange of heavy $W$-bosons generating effective, four-fermion operators suppressed by $M_W^2$. However in a supersymmetric theory the effective operators generated must be supersymmetric and hence it is possible to represent them as superfields. To find the coupling of the effective superpotential term it suffices to find the component field Lagrangian, and calculate the coupling constant for an effective operator, say a two fermion, two boson interaction. This operator will be calculable from an effective non-renormalisable superpotential term with the same form, but all fields upgraded to superfields and the same coupling constant. Hence, by supersymmetry, we expect all other terms corresponding to the superpotential to be present, which can be checked at the level of components. Alternatively this may be done at the level of supergraphs and manifest SUSY is maintained at all times. We note that corrections to the Kähler potential appear suppressed by one more power of the large mass scale [89] and we neglect them in this analysis.

Finally we note that one must be cautious when applying this proceedure since one can give large masses to, supposed, low energy fields. When all flavons and $\psi$ are replaced by their vevs it must be demonstrated that it is possible to find a zero mass state that can be identified with the low energy field. In our notation this will be a mixture of $C$ and $\chi_{ij}$ which, after symmetry breaking have the same quantum numbers.

# Bibliography


[1] M. E. Peskin and D. V. Schroeder, *An Introduction to Quantum Field Theory,* Reading, USA: Westview Press (1995)

[2] P. D. B. Collins, A. D. Martin and E. J. Squires *Particle Physics and Cosmology,* New York, USA: John Wiley & Sons, Inc. (1989)

[3] A. Salam, *Weak And Electromagnetic Interactions,*

[4] S. Weinberg, *A Model Of Leptons,* Phys. Rev. Lett. **19** (1967) 1264.

[5] M. Gell-Mann, *A Schematic Model Of Baryons And Mesons,* Phys. Lett. **8** (1964) 214.

[6] H. Fritzsch, M. Gell-Mann and H. Leutwyler, *Advantages Of The Color Octet Gluon Picture,* Phys. Lett. B **47** (1973) 365.

[7] S. L. Glashow, J. Iliopoulos and L. Maiani, *Weak Interactions With Lepton - Hadron Symmetry,* Phys. Rev. D **2** (1970) 1285.

[8] M. Kobayashi and T. Maskawa, *CP Violation In The Renormalizable Theory Of Weak Interaction,* Prog. Theor. Phys. **49** (1973) 652.

[9] S. P. Martin, *A supersymmetry primer,* arXiv:hep-ph/9709356.

[10] J. E. Kim, *Light Pseudoscalars, Particle Physics And Cosmology,* Phys. Rept. **150** (1987) 1.

[11] C. A. Baker *et al.*, *An improved experimental limit on the electric dipole moment of the neutron,* arXiv:hep-ex/0602020.

[12] R. D. Peccei and H. R. Quinn, *CP Conservation In The Presence Of Instantons,* Phys. Rev. Lett. **38** (1977) 1440.

[13] R. D. Peccei and H. R. Quinn, *Constraints Imposed By CP Conservation In The Presence Of Instantons,* Phys. Rev. D **16** (1977) 1791.

[14] G. G. Raffelt, *Particle physics from stars,* Ann. Rev. Nucl. Part. Sci. **49** (1999) 163 [arXiv:hep-ph/9903472].

[15] J. D. Lykken, *Introduction to supersymmetry*, arXiv:hep-th/9612114.

[16] S. Weinberg, *The Quantum Theory of Fields, Volume III: Supersymmetry*, Cambridge, UK: Cambridge University Press, (2000)

[17] N. Seiberg, *Naturalness versus supersymmetric nonrenormalization theorems,* Phys. Lett. B **318** (1993) 469 [arXiv:hep-ph/9309335].

[18] D. Bailin and A. Love *Supersymmetric Gauge Field Theory and String Theory*, Bristol, UK: Institute of Physics Publishing (1996)

[19] J. Bagger and J. Wess *Supersymmetry and Supergravity*, Princeton, New Jersey, USA: Princeton University Press (1992).

[20] V. Kaplunovsky and J. Louis, *On Gauge couplings in string theory,* Nucl. Phys. B **444** (1995) 191 [arXiv:hep-th/9502077].

[21] L. E. Ibanez, C. Munoz and S. Rigolin, *Aspects of type I string phenomenology,* Nucl. Phys. B **553** (1999) 43 [arXiv:hep-ph/9812397].

[22] D. J. H. Chung, L. L. Everett, G. L. Kane, S. F. King, J. D. Lykken and L. T. Wang, Phys. Rept. **407** (2005) 1 [arXiv:hep-ph/0312378].

[23] P. Fayet, *Supergauge Invariant Extension Of The Higgs Mechanism And A Model For The Electron And Its Neutrino,* Nucl. Phys. B **90** (1975) 104.

[24] J. R. Ellis, J. F. Gunion, H. E. Haber, L. Roszkowski and F. Zwirner, *Higgs Bosons In A Nonminimal Supersymmetric Model,* Phys. Rev. D **39** (1989) 844.

[25] S. A. Abel, S. Sarkar and P. L. White, *On the cosmological domain wall problem for the minimally extended supersymmetric standard model,* Nucl. Phys. B **454** (1995) 663 [arXiv:hep-ph/9506359].

[26] S. K. Soni and H. A. Weldon, *Analysis Of The Supersymmetry Breaking Induced By N=1 Supergravity Theories,* Phys. Lett. B **126** (1983) 215.

[27] A. Brignole, L. E. Ibanez and C. Munoz, *Soft supersymmetry-breaking terms from supergravity and superstring models*, arXiv:hep-ph/9707209.

[28] S. F. King, *Neutrino mass models,* Rept. Prog. Phys. **67** (2004) 107 [arXiv:hep-ph/0310204].

[29] Y. Grossman, *TASI 2002 Lectures on Neutrinos,* arXiv:hep-ph/0305245.

[30] R. N. Mohapatra *et al.*, *Theory of neutrinos: A white paper,* arXiv:hep-ph/0510213.

[31] S. Eidelman *et al.* [Particle Data Group], *Review of particle physics,* Phys. Lett. B **592** (2004) 1.

[32] J. F. Beacom, N. F. Bell, D. Hooper, J. G. Learned, S. Pakvasa and T. J. Weiler, *Pseudo-Dirac neutrinos, a challenge for neutrino telescopes,* Phys. Rev. Lett. **92** (2004) 011101 [arXiv:hep-ph/0307151].

[33] J. Polchinski, *String Theory: Volume I, An Introduction to the Bosonic String,* Cambridge, UK: Cambridge University Press (1998).

[34] J. Polchinski, *String Theory: Volume II, Superstring Theory and Beyond,* Cambridge, UK: Cambridge University Press (1998).

[35] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory, Volume I: Introduction,* Cambridge, UK: Cambridge University Press, (1987)

[36] M. B. Green, J. H. Schwarz and E. Witten, *Superstring Theory, Volume II: Loop Amplitudes, Anomalies & Phenomenology,* Cambridge, UK: Cambridge University Press, (1987)

[37] A. M. Polyakov, *Quantum Geometry Of Fermionic Strings,* Phys. Lett. B **103** (1981) 211.

[38] R. J. Szabo, *BUSSTEPP lectures on string theory: An introduction to string theory and D-brane dynamics,* arXiv:hep-th/0207142.

[39] T. Kaluza, *On The Problem Of Unity In Physics,* Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys. ) **1921** (1921) 966.

[40] O. Klein, *Quantum Theory And Five-Dimensional Theory Of Relativity,* Z. Phys. **37** (1926) 895 [Surveys High Energ. Phys. **5** (1986) 241].

[41] G. Gabadadze, *ICTP lectures on large extra dimensions,* arXiv:hep-ph/0308112.

[42] T. Appelquist and H. U. Yee, *Universal extra dimensions and the Higgs boson mass,* Phys. Rev. D **67** (2003) 055002 arXiv:hep-ph/0211023.

[43] T. Flacke, D. Hooper and J. March-Russell, *Improved bounds on universal extra dimensions and consequences for LKP dark matter,* arXiv:hep-ph/0509352.

[44] G. Shiu and S. H. H. Tye, *TeV scale superstring and extra dimensions*, Phys. Rev. D **58** (1998) 106007 [arXiv:hep-th/9805157].

[45] J. Polchinski, *Dirichlet-Branes and Ramond-Ramond Charges*, Phys. Rev. Lett. **75** (1995) 4724 [arXiv:hep-th/9510017].

[46] G. Lazarides, *Inflationary cosmology,* Lect. Notes Phys. **592** (2002) 351 [arXiv:hep-ph/0111328].

[47] E. W. Kolb and M. S. Turner, *The Early Universe* Cambridge, Massachusetts, USA: Perseus Publishing (1994)

[48] D. H. Lyth and A. Riotto, *Particle physics models of inflation and the cosmological density perturbation,* Phys. Rept. **314** (1999) 1 [arXiv:hep-ph/9807278].

[49] P. Coles and F. Lucchin *Cosmology: The Origin and Evolution of Cosmic Structure* Chichester, UK: John Wiley & Sons (2002)

[50] A. H. Guth, *The Inflationary Universe: A Possible Solution To The Horizon And Flatness Problems,* Phys. Rev. D **23** (1981) 347.

[51] S. Weinberg *Gravitation and Cosmology* John Wiley & Sons (1972)

[52] S. Weinberg, *The Cosmological Constant Problem,* Rev. Mod. Phys. **61** (1989) 1.

[53] A. Linde, *Fast-roll inflation,* JHEP **0111** (2001) 052 [arXiv:hep-th/0110195].

[54] A. R. Liddle and D. H. Lyth, *The Cold dark matter density perturbation,* Phys. Rept. **231** (1993) 1 [arXiv:astro-ph/9303019].

[55] A. D. Linde, *Axions in inflationary cosmology,* Phys. Lett. B **259** (1991) 38.

[56] A. D. Linde, *Hybrid inflation,* Phys. Rev. D **49** (1994) 748 [arXiv:astro-ph/9307002].

[57] S. F. King and D. A. J. Rayner, *Twisted moduli and supersymmetry breaking,* JHEP **0207** (2002) 047 [arXiv:hep-ph/0111333].

[58] D. Cremades, L. E. Ibanez and F. Marchesano, *Standard model at intersecting D5-branes: Lowering the string scale,* Nucl. Phys. B **643** (2002) 93 [arXiv:hep-th/0205074].

[59] M. Berkooz and R. G. Leigh, *A D = 4 N = 1 orbifold of type I strings*, Nucl. Phys. B **483** (1997) 187 [arXiv:hep-th/9605049].

[60] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, *Intersecting brane worlds*, JHEP **0102** (2001) 047 [arXiv:hep-ph/0011132].

[61] Z. Lalak, S. Lavignac and H. P. Nilles, *String dualities in the presence of anomalous U(1) symmetries,* Nucl. Phys. B **559** (1999) 48 [arXiv:hep-th/9903160].

[62] S. A. Abel and G. Servant, *Dilaton stabilization in effective type I string models,* Nucl. Phys. B **597** (2001) 3 [arXiv:hep-th/0009089].

[63] G. Aldazabal, A. Font, L. E. Ibanez and G. Violero, *D = 4, N = 1, type IIB orientifolds,* Nucl. Phys. B **536** (1998) 29 [arXiv:hep-th/9804026].

[64] L. J. Dixon, V. Kaplunovsky and J. Louis, *On Effective Field Theories Describing (2,2) Vacua Of The Heterotic String,* Nucl. Phys. B **329** (1990) 27.

[65] J. P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: Moduli spectrum and D3/D7 soft supersymmetry breaking,* JHEP **0508** (2005) 007 [arXiv:hep-th/0505076].

[66] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, *De Sitter vacua in string theory,* Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240].

[67] M. Bastero-Gil and S. F. King, *A next-to-minimal supersymmetric model of hybrid inflation,* Phys. Lett. B **423** (1998) 27 [arXiv:hep-ph/9709502].

[68] J. Foster and J. D. Nightingale *A Short Course in General Relativity* London, UK: Springer (1994)

[69] K. Choi, E. J. Chun and J. E. Kim, *Cosmological implications of radiatively generated axion scale,* Phys. Lett. B **403** (1997) 209 [arXiv:hep-ph/9608222].

[70] K. Dimopoulos and D. H. Lyth, *Models of inflation liberated by the curvaton hypothesis,* Phys. Rev. D **69** (2004) 123509 [arXiv:hep-ph/0209180].

[71] D. H. Lyth and D. Wands, *Generating the curvature perturbation without an inflaton,* Phys. Lett. B **524** (2002) 5 [arXiv:hep-ph/0110002].

[72] T. Moroi and T. Takahashi, *Effects of cosmological moduli fields on cosmic microwave background,* Phys. Lett. B **522** (2001) 215 [Erratum-ibid. B **539** (2002) 303] [arXiv:hep-ph/0110096].

[73] T. Moroi and T. Takahashi, *Cosmic density perturbations from late-decaying scalar condensations,* Phys. Rev. D **66** (2002) 063501 [arXiv:hep-ph/0206026].

[74] D. N. Spergel *et al.* [WMAP Collaboration], *First Year Wilkinson Microwave Anisotropy Probe (WMAP) Observations: Determination of Cosmological Parameters,* Astrophys. J. Suppl. **148** (2003) 175 [arXiv:astro-ph/0302209].

[75] J. A. Casas and G. G. Ross, *Domain Walls In Superstring Models,* Phys. Lett. B **198** (1987) 461.

[76] J. A. Casas and G. G. Ross, *A Solution To The Strong CP Problem In Superstring Models,* Phys. Lett. B **192** (1987) 119.

[77] K. Dimopoulos, G. Lazarides, D. Lyth and R. Ruiz de Austri, *The Peccei-Quinn field as curvaton,* JHEP **0305** (2003) 057 [arXiv:hep-ph/0303154].

[78] C. Kounnas, D. V. Nanopoulos, M. Quiros and M. Srednicki, *Natural Triplet - Doublet Splitting From Supergravity,* Phys. Lett. B **127** (1983) 82.

[79] A. Masiero, D. V. Nanopoulos, K. Tamvakis and T. Yanagida, *Naturally Massless Higgs Doublets In Supersymmetric SU(5),* Phys. Lett. B **115** (1982) 380.

[80] H. Georgi and S. L. Glashow, *Unity Of All Elementary Particle Forces,* Phys. Rev. Lett. **32** (1974) 438.

[81] G. C. Branco, M. N. Rebelo and J. I. Silva-Marcos, *CP-odd invariants in models with several Higgs doublets,* Phys. Lett. B **614** (2005) 187 [arXiv:hep-ph/0502118].

[82] J. D. Lykken, *Weak Scale Superstrings,* Phys. Rev. D **54** (1996) 3693 [arXiv:hep-th/9603133].

[83] S. Antusch and S. F. King, *Sequential dominance,* New J. Phys. **6** (2004) 110 [arXiv:hep-ph/0405272].

[84] M. Kobayashi and C. S. Lim, *Pseudo-Dirac scenario for neutrino oscillations,* Phys. Rev. D **64** (2001) 013003 [arXiv:hep-ph/0012266].

[85] J. Ahrens *et al.* [The IceCube Collaboration], *IceCube: The next generation neutrino telescope at the South Pole,* Nucl. Phys. Proc. Suppl. **118** (2003) 388 [arXiv:astro-ph/0209556].

[86] F. Quevedo, *Lectures on superstring phenomenology,* arXiv:hep-th/9603074.

[87] J. Bagger, E. Poppitz and L. Randall, *The R axion from dynamical supersymmetry breaking,* Nucl. Phys. B **426** (1994) 3 [arXiv:hep-ph/9405345].

[88] C. D. Froggatt and H. B. Nielsen, *Hierarchy Of Quark Masses, Cabibbo Angles And CP Violation,* Nucl. Phys. B **147** (1979) 277.

[89] M. Cvetic, L. L. Everett and J. Wang, *Effects of heavy states on the effective $N = 1$ supersymmetric action,* Nucl. Phys. B **538** (1999) 52 [arXiv:hep-ph/9807321].