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"Three Essays on Personal Bankruptcy"

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Abstract

This thesis presents three essays on the field of personal bankruptcy. The research carried out is mostly, but not fully, theoretical in nature. The first chapter develops a model of general equilibrium with default and asks, among other things, how the level of default penalties can affect the welfare of different agents in an economy, when do agents face binding borrowing limits, and why some agents are denied credit upon application. In answer to these questions, it shows that there exists an intermediate range of Pareto optimal punishment levels where changes in the punishment level benefit one type of agent but harm another; outside this range, punishment levels can be changed to benefit all agents. It also proves that the existence of default is a sufficient condition for the emergence of borrowing limits *which are binding*; asymmetric information is not necessary for this to occur. Finally, is shows that some agents can be denied credit if default penalties are too low *or* too high. In these instances, it shows that even though these agents would like to borrow at the market interest rates, they are unable to do so. The chapter then also asks whether a competitive market will select the socially optimal punishment level, and shows that this is not, in general, the case - welfare can be improved if default penalties are set by a social planner. Finally, a numerical experiment is performed to evaluate the welfare effects of a change in the law which allows only low-income households to file for default on their debts. The result shows the welfare effects of this will depend on the degree of agent heterogeneity; if the gap between rich agents and poor agents is not large, then this policy benefits all agents. If however the gap between the rich and the poor is substantially large, then this policy will benefit those who save and rich agents who borrow, while harming poor agents who borrow. This illustrates the importance of agent heterogeneity when evaluating different policies in bankruptcy legislation under a general equilibrium setting. Following this, we argue that the welfare evaluations from various studies of bankruptcy may be biased upwards if they do not allow for enough heterogeneity among agents.

The second chapter extends this model to a world consisting of many open economies from between which funds can be transferred without cost, and examines how default penalties are set endogenously by the governments of each region. It then explores the implications of such endogenous punishment formation. From a normative perspective, it asks whether the resulting non-cooperative Nash equilibrium is welfare-maximising. We find that this is not

in general the case, and that welfare can increase if governments set their punishment levels cooperatively. From a positive point of view, it compares the effect of an exogenous shock in income distributions to the equilibrium allocations in the case of fixed punishment levels and in the case of variable punishment levels (i.e. the case when governments are able to react to a change in the economic environment by altering bankruptcy legislation). The analysis shows that allowing for endogenous punishment levels can reverse the results of such exogenous shocks. For example, one of the puzzles in the literature is that default rates have been increasing, despite rising incomes. We show that, keeping punishments fixed, higher incomes result in more borrowing but less default. However, when we allow punishments to vary endogenously, these fall in response to higher incomes and the result is more borrowing and *more* default.

Finally, the third chapter develops the model to include bankruptcy exemptions which resemble legislation in the US and examines how these should be set optimally in a partial equilibrium environment. It then studies how optimal exemption levels depend on various economic variables. It derives a simple rule in a completely general environment which states that any exogenous shock which increases the level of borrowing implies a higher optimal exemption level. For example, all else equal, higher (future) incomes imply higher levels of borrowing. Therefore, economies with higher incomes should have higher exemption levels. It then tests this empirically using data from the US, where exemption levels vary between states. The main innovation from the empirical part with respect to similar studies is that the analysis controls for neighbourhood effects in the sense that states may take into account the exemption levels in neighbouring states when setting their own exemption level. The results are two-fold: first, the evidence is consistent with the theoretical prediction made above; secondly, there is some evidence of the existence of spatial effects, so that, along with economic variables such as the level of incomes and unemployment, states do take into account the level of exemptions in neighbouring states when setting their own exemption levels. However, we find that such effects have steadily been getting weaker over the years and are only present in earlier years of our sample.

Contents

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Introduction

The field of personal bankruptcy has been one of increasing volume and depth since the early 1990s, and also of growing importance. Over this period of time, there has been a vast increase in unsecured borrowing, and with that a large rise in the number of households declaring bankruptcy and defaulting on their debts[1], which has attracted a lot of interest from academic researchers in recent years. Theoretical research has concentrated on the implications of bankruptcy law on social welfare, with papers written of a pure abstract theory and of a more quantitative nature using calibration methods. Empirical papers have tried to identify, among other things, the determinants of households' decision to declare bankruptcy and the effects of bankruptcy legislation on variables such as the availability of credit, the aggregate bankruptcy filing rate and the volatility of consumption. In this section, we describe the research which has been carried out over the years in the field of personal bankruptcy. We will then conclude with a full description of the research carried out in the remaining part of this thesis.

Past contributions

Two seminal papers, Dubey et al (2005)[2] and Zame (1994), have led the theoretical research in this field. Zame (1994) outlined three ideas on why it may be efficient to allow for default in an economy where markets are incomplete. The first of these originated from Dubey et al (1988): with incomplete markets, allowing for default may expand the set of possible contingent contracting and improve the efficiency of equilibrium allocations. To illustrate this, consider the following example. Suppose that an efficient allocation is one which allows trade between two states, say from s_1 to s_2. To achieve this, an investor will have to buy a security which promises a delivery in state s_1 and sell one which promises a security in state s_2. One way in which market incompleteness will not allow for trade is if there are no such securities promising a return in state s_2. However, another way in which this may occur is if such securities do exist,

[1] It is a well-documented fact that the proportion of households declaring bankruptcy in the US has risen five-fold since 1984, from 0.3% then to more than 1.5% in 2003. In the UK too, figures from the DTI (Department of Trade and Industry) reveal that the number of insolvent households has increased sharply. For example, in 1997, a total of 24,400 households were insovlent in England and Wales, while in 2005 this had increased to almost 70,000 households.

[2] This may be a recent publication but working versions of this paper have been circulating as far back as 1988.

but that they also promise a delivery in some other state of nature, perhaps one which occurs with a low probability. Suppose that the investor's endowment in one of these is insufficient to honour that promise[3]. Then, if investors are only able to enter into agreements that they can honour in all possible contingencies, this investor will not be able to sell such securities, so the opportunities for risk sharing will be limited and the equilibrium allocation will be far from efficient. In such an environment, allowing for default can improve welfare by allowing agents to enter into such agreements which they can honour with a high probability but fail to honour with a low probability. The second idea is the following. Opening new markets may be a less costly way of shrinking the gap between equilibrium and efficiency, but this is not necessarily true. Zame shows, via an illustrative example, that even as the set of markets expands to approximately cover a full set of contingencies, the gap does not necessarily shrink much further. Finally, in such a case, one has to allow for default to achieve this; therefore, default and opening new markets can be complements rather than substitutes.

Dubey, Geanakoplos and Shubik[4] were the first authors to model default in a general equilibrium setting. They allow assets to be defined by their return, the penalty for defaulting on any promises made by such assets and by a quantity restriction on those who sell it. A central feature of their model is that assets are thought of as pools. This is done in an attempt to replicate the features of a modern financial market with securitised debt. Different sellers of each security choose the quantity they want to sell up to some exogenously fixed constraint, and typically default in different proportions and on different occasions. The overall return on each asset is the average return from all these trades. Furthermore, banks are not explicitly modelled; they only play a minor, administrative role, collecting debt from some agents and passing on the risk to the shareholders of the pool. This eliminates any game-theoretic considerations in the case where the banks cannot distinguish between agent types and makes the analysis far simpler and, in the view of the authors, more akin to the reality of today's financial markets. The authors prove the existence of equilibrium with default in exactly the same conditions necessary to prove the existence of equilibrium in a standard model of general equilibrium with incomplete markets (GEI) without default, as in Radner (1972). These conditions are either

[3] And there is a sufficient absence of other securities.
[4] Henceforth, DGS.

that all assets promise payoffs in the same good, or that some arbitrary, non-binding limit Q be placed on the sale of any asset[5]. One important finding by DGS is that the subset of actively traded assets is usually much smaller than the set of available assets. Thus, unlike GEI where all available (non-redundant) assets are typically traded, their model can endogenously determine which markets, although available, are not active. The authors then ask how harsh default penalties *should* be. They show, via a simple example, that penalties should be lenient enough so that agents sometimes default even when they are able to deliver their promises, but should not be so lenient so that agents always default. They find a unique such punishment level which Pareto dominates all others. In other words, there exists an intermediate level of punishment which is optimal[6]. DGS then proceed by asking how harsh penalties *will* be if they are selected by the market. They find that the market does select a unique penalty, which in their example turns out to be the optimal penalty.

Subsequent theoretical work, spurred on by these two papers, has tended to be of a more dynamic nature, with the most well-known studies employing infinite horizon representative agent models. The advantage of such models is that can be calibrated to target specific policy questions. A prominent researcher on the topic of personal bankruptcy is Kartik Athreya, who has asked a range of questions using such calibrated dynamic stochastic models of general equilibrium. In Athreya (2001), he analyses two long-run equilibrium scenarios, one with low credit availability and another with high credit availability, in order to assess the welfare impacts of an increase in the availability of credit which has been observed over the years. The paper attributes this to a court ruling in 1978, which allowed banks from other states to issue loans to residents in Minnesota at interest rates higher than the ceiling imposed by that state, and instead issue loans according to the legislation of the state which they were based at. He does this by calibrating a model with exogenous borrowing limits twice: once with low limits and once with high limits. The main result of his analysis is that this expansion in credit availability may have had a detrimental effect on the welfare of poor households via the higher interest rates

[5]This condition is necessary because, as Radner (1972) has pointed out, sometimes two assets may promise different commodities, but may become nearly equivalent at some spot prices because they promise nearly the same money. At such a case, the existence of equilibrium is destroyed as agents try to go infinitely long in one asset and infinitely short in the other. Placing some arbitrarily large borrowing limit eliminates this problem.

[6]Zame (1994) provides an interesting analogy to this, stating that there is an optimal intermediate level of default, just like there exists intermediate level of pollution which is optimal.

and more frequent punishments which result from an increase in bankruptcy filing rates.

Athreya (2002) uses a similar setting to examine the impact of allowing for means-tested bankruptcy, as was proposed in the Bankruptcy Reform Act of 1999 in the US. To do this, he only allows households whose wealth is below a certain threshold level to declare bankruptcy - all other households are not allowed to do so - and finds a welfare gain of an estimated \$80 per household annually. In addition, he also examines the welfare effect of removing bankruptcy option to all households, and finds a much larger welfare gain, estimated at \$280 per household annually.

In Athreya (2003), the author examines the interactions between unemployment insurance and personal bankruptcy by extending the model in Athreya (2002) to include unemployment in the form of a drop in their endowments to some low level with an exogenous probability of occurrence. Unemployment insurance is modelled as a payment made from the state to the unemployed for one time period, where the payment is equal to some predetermined fraction θ of the mean labour income in the economy - a high θ represents an economy with generous unemployment insurance and vice versa for a low θ. After this one period, if a household remains unemployed it does not receive any more unemployment benefits but instead its endowment falls to an even lower subsistence level $Y_{\min}$, which can be thought of as the combination of all social insurance programs beyond unemployment insurance. He then compares the welfare in two benchmark economies with different levels of unemployment insurance with and without bankruptcy, and finds that allowing bankruptcy lowers welfare in both cases, although it makes the distribution of wealth more equal. His conclusion from this experiment is that if a society has to choose between any combination of unemployment insurance and bankruptcy, it should choose unemployment insurance alone. However, this result is subsequently reversed as the subsistence level of income $Y_{\min}$ is lowered even further; in this case, allowing for bankruptcy raises welfare regardless of the level of unemployment insurance.

These papers do however suffer from certain fallbacks. First, in these models, when a household files for bankruptcy, all its debt is fully discharged[7]; this is a rather extreme event since creditors cannot recover any portion of the defaulting household's debt. In practice, a

[7]The household is punished by having restricted access to credit markets (i.e. being able to save but not able to borrow) for a stochastic period of time and further by receiving a fixed disutility for one time period. Without such punishment, there would be no commitment to repaying one's debts and so there could be no borrowing.

defaulting household is not fully exempt from repayment and is obliged to transfer some of its wealth to its creditors. The extent of this transfer is determined by state legislation in the form of a specified bankruptcy exemption level. As Zha (2001) has shown, there may be scope for improving welfare by allowing for bankruptcy with a limited exemption level. His paper extends the results of Zame and DGS to a world with infinite horizons. Unlike Zame and DGS, who simplify default penalties in the form of disutility inflicted upon a defaulting agent, he specifically models US legislation by including bankruptcy exemption levels which specify the minimum level of wealth which a defaulting agent can retain in bankruptcy - creditors can seize a defaulting agent's assets if their total wealth exceeds the exemption level, but cannot claim anything if their wealth is below these levels. Zha builds a dynamic stochastic model of general equilibrium with capital accumulation and finds that intermediate exemption levels can improve social welfare and distributive equity. Therefore, with reference to Athreya (2002, 2003), it may be a little premature to conclude that removing the bankruptcy option altogether improves welfare, as the only point of comparison in those papers is a world where bankruptcy is allowed but with essentially unlimited exemptions.

The second restriction of these papers is that borrowing constraints are fixed exogenously. The existence of such borrowing constraints has been empirically identified by several authors[8], so their endogenous derivation could affect any calculations of such calibrated models. Mateos-Planas and Seccia (2006) investigate this by building a stochastic dynamic model of general equilibrium with infinite horizons and assessing the impact of, first, a reduction in T, the period of exclusion from credit markets for defaulting households, and second, a reduction in the volatility of income, perhaps the result of some social insurance program. These experiments are performed under two settings: one with fixed borrowing constraints, and one where they are endogenously determined. The main result of their analysis is that allowing for an endogenous borrowing constraint may reverse the welfare evaluations of such experiments in models of bankruptcy. For example, in a partial equilibrium setting (i.e. a small and open economy), a reduction in T results in an increase in welfare with endogenous borrowing limits, but in a fall in welfare with exogenous constraints. A similar pattern is observed with a mean-preserving reduction in income fluctuations meant to represent a social insurance program - with endoge-

[8] A review of such papers, and others, is presented in the section below this.

nous constraints, welfare increases, but under exogenous constraints the opposite is true again and welfare declines.

Thirdly, these papers only model shocks to income over time and abstract from other sources of uncertainty in the form of expense shocks such as medical bills, divorce and child care. On the other hand, Livshits et al (2003) develop a life cycle model with both income uncertainty and expense shocks which are more permanent in nature. The main conclusion of that study is that, in the absence of such expense shocks, it may be optimal to remove a Chapter 7-type bankruptcy option whereby a creditor can seize all non-exempt assets from a bankrupted agent[9] and all remaining debt is completely discharged. However, once such expense shocks are included in the model, this is no longer optimal and it is preferable from a welfare point of view to include such an option. Therefore, in the absence of expense shocks, any policy recommendations which suggest removing a Chapter 7 bankruptcy option may be premature and require careful consideration.

Chatterjee et al (2002) develop a model of partial equilibrium with bankruptcy and endogenous constraints, so that the model represents a small, open economy. The characteristics of each household are fully observable to banks so that debt contracts are individually tailored for each household type[10]. Households that declare bankruptcy are punished by exclusion from the credit market for some finite period of time. From a theoretical point of view, the authors show that under such a setting, for each level of debt and each household type, the set of earnings that trigger default lies in a closed interval. In other words, contrary to intuition, households with very low earnings do not file for default[11] - for low incomes, the exclusion from the credit market following bankruptcy is too high a punishment. The authors are also able to derive credit limits endogenously, defined as the amount above which banks will refuse to lend. The paper then examines, via a numerical experiment, the impact of two policies: first, a reduction in the amount of time that households are denied credit following bankruptcy, and second, as in Athreya (2002), a change in the law which does not allow households with above-median

[9] Up to and including the amount owed, of course.

[10] This, according to the empirical evidence presented in Edelberg (2003), is more akin to our present reality, which finds that during the 1990s interest rates charged to consumers showed a greater dependence on agent characteristics than in previous years.

[11] More conventionally, neither do very high-income households - just households withing an intermediate range of earnings.

earnings to declare bankruptcy. With respect to the first experiment, the paper finds a very modest quantitative effect. The second experiment however, unlike Athreya (2002), has a much larger impact and leads to a three-fold increase in debt but without a significant increase in the aggregate bankruptcy rate. The authors also find significant welfare effects with this experiment, with households on average willing to pay a one-off lump sum equivalent to a quarter of their yearly earnings.

Further studies have sought to emphasise the interactions, through bankruptcy, between the market for unsecured credit and other markets by adding further dimensions of choice. Two such studies are those of Athreya and Simpson (2006), and Li and Sarte (2006).

Athreya and Simpson (2006) extend the model provided by Athreya (2003) by endogenising the job search effort by unemployed households, whereas in Athreya (2003), an unemployed household had an exogenously given probability of being employed in the next time period. Secondly, they also derive credit limits endogenously and allow borrowing rates to depend on borrowers' observed characteristics. The main conclusion of this paper is that current bankruptcy law hinders the ability of unemployment insurance to improve welfare. For example, when bankruptcy is allowed, generous unemployment provision leads to a loss in welfare, whereas when bankruptcy is not allowed, generous unemployment can improve welfare substantially. Intuitively, the authors find that introducing bankruptcy into the model distorts job search effort amongst unemployed households as households can receive a temporary boost in wealth by filing for bankruptcy. Then, increasing unemployment insurance encourages households to declare bankruptcy all else equal, since one of the consequences of bankruptcy is exclusion from the credit market for a certain period of time. An insured household will have less need to borrow to smooth consumption, and so the threat of exclusion is not as serious. This in turn encourages households to borrow more, as defaulting is an easier option. Perhaps counter-intuitively, higher unemployment insurance leads to *more*, not less bankruptcy, and as a result, households suffer greater penalties in the form of default costs and higher interest rates. In other words, bankruptcy strengthens the adverse effects of unemployment on job search effort, and unemployment insurance strengthens the adverse effects of bankruptcy costs and higher interest rates that come with bankruptcy, so that the policies counteract each other. As a result, the authors conclude that if unemployment insurance exists, the state should not allow for bankruptcy. If on the other hand bankruptcy exists, there should be little or no unemployment insurance. This conclusion however is derived by using an 'all or nothing' bankruptcy procedure, where a defaulting household is discharged from all its debt; in reality, we might expect there to be a case for allowing bankruptcy with strict exemption levels instead.

Li and Sarte include not only production and labour supply in their model, but also model agents' choice between Chapter 7 and Chapter 13 bankruptcy. Chapter 7 is the bankruptcy procedure studied in most other papers. However, under Chapter 13 bankruptcy, a debtor has to propose a new debt payment plan which has to be approved by a bankruptcy court. In order for this to be approved, it must satisfy either a 'full repayment' or a 'disposable income' criterion. That is, a debtor must either propose to repay his debt in full, or alternatively propose to pay his creditors his entire disposable income which remains after all essential expenses[12] over the following five years. The authors distinguish the two bankruptcy procedures as an asset tax for Chapter 7 and a wage tax for Chapter 13. Furthermore, they allow firms to borrow at the risk-free rate of interest and invest in their optimal capital stock. They then conduct three numerical experiments. First, they examine the effect of completely removing the Chapter 7 bankruptcy option[13], and find a substantial loss in welfare as a result, contradicting Athreya (2002). Briefly, in their model, removing the bankruptcy option lowers the consumer borrowing rate and as a result borrowing increases. This causes the risk-free interest rate to increase in order to attract the extra deposits required, which leads to a fall in the capital stock as firms now find it more expensive to borrow. This in turn causes the demand for labour by firms to decrease, and as a result labour input falls. Relative to their benchmark model with bankruptcy, production falls by 4.4%, and this fall more than offsets the benefits of eliminating Chapter 7 bankruptcy as reported in Athreya (2002). The result is an overall decrease in welfare of 3.3%.

The second experiment of that paper examines the effect of a means-tested approach to bankruptcy, allowing only households with below-median income to file under Chapter 7. The authors find that if the tests are made strict enough, welfare falls by up to 1%[14]. With a restricted choice, there is an increase in Chapter 13 filings and a fall in Chapter 7 filings. Via

[12]These expenses cover essential purchases such as rent and food for the debtor and his dependants.

[13]Agents can still however file for bankruptcy under a Chapter 13 procedure.

[14]At best, welfare remains unchanged if the means-test is reduced so that no households are affected.

the same effects as the first experiment, the capital stock and labour input fall[15]. As a result, production falls and this leads to a fall in welfare as well.

Finally, the authors examine the impact of a reduction in Chapter 7 bankruptcy exemption levels and find that such a move can increase economic efficiency by reducing borrowing and hence Chapter 13 bankruptcies. One of the effects is an increase in labour input and this can result in an increase in welfare. It has to be said however, that these results, interesting as they are, have been derived with an exogenously fixed borrowing limit. A suggestion therefore for future research would be to extend this model to include endogenous constraints. Furthermore, it would be an interesting extension to examine the effects of removing Chapter 13 bankruptcy with respect to a benchmark model which includes both Chapter 7 and Chapter 13 bankruptcy procedures. Moreover, the paper includes transaction costs which increase monotonically with the amount of borrowing and with the number of bankruptcies - it would be interesting to find out how robust these results are to the reduction or elimination of such costs.

Thesis summary

The research carried out in this thesis is mostly theoretical in nature, although some applied work is carried out in Chapter 3. The theoretical work most closely resembles the work of DGS (2005) by working under a two-period general equilibrium setting. Using a two-period environment gives the model a certain tractability and enables work to be done which would otherwise be too complex to carry out.

Chapter 1 of the thesis builds a model of general equilibrium which incorporates default, with the aim of understanding certain positive and normative features of the market for unsecured consumer credit. The difference with the model of DGS (2005) is that we explicitly model the banking sector; rather than modelling assets as pools with exogenous credit constraints which agents can trade, banks offer debt contracts which are type-dependent and take into account *individual* and not aggregate expected default rates. Whereas more recent papers model borrowing in a similar way[16], the majority of studies that exist tend to use exogenous borrowing limits.

[15]In addition, labour input is also affected by incentives; with a greater proportion of households under Chapter 13 bankruptcy, the incentive to work or look for work is reduced, since a potentially significant fraction of income is garnished by creditors.

[16]For example, Chatterjee et al (2003); Athreya and Simpson (2006).

Households are punished for defaulting on their debts in the form of a disutility received for default. This disutility depends on the extent of the default, and is used as a proxy for losses incurred during bankruptcy such as seizure of assets owned, exclusion from the credit market and social stigma.

We first proceed by examining the impact that changes in the punishment level have on different agent types. The main conclusion from the analysis is that, contrary to the findings of DGS (2005), there exists an intermediate *range* of punishment levels which are Pareto efficient. At first, increasing penalties from a very low level results in higher equilibrium utilities for savers and borrowers. Hence, low punishment levels are Pareto inefficient. After a certain point, within an intermediate range, increasing the punishment level benefits savers but harms borrowers. Therefore, this intermediate range consists of Pareto efficient punishment levels. Finally, after a certain point, further increases in the punishment level result in lower equilibrium utilities for both savers and borrowers, which shows that, as with low punishment levels, high punishment levels can be inefficient as well. This differs from DGS (2005), who find a *unique* punishment level which Pareto dominates all others. One implication of this, as we show further on, is that, when we allow banks to specify the punishment level in the contracts they offer consumers, the result, unlike DGS (2005), is not welfare-maximising. The reason for this is that, as we show, banks always select an efficient punishment level. In DGS (2005), as there is a unique efficient punishment level, this is necessarily welfare-maximising. However, in our model, since there is no such unique punishment, this does not have to be welfare-maximising; the welfare-maximising punishment level will depend on how welfare is evaluated. Since the punishment level offered by banks at equilibrium is invariant to how welfare is evaluated, it follows that this will not, generically, be welfare-maximising.

The first chapter then discusses borrowing constraints. Plenty of evidence exists that borrowing constraints exist which affect the ability of households to borrow. For example, Gross and Souleles (2002) show that households increase borrowing in response to an increase in their credit card limit, indicating that they are credit-constrained and would like to borrow in excess of their credit limit at the given interest rate. We prove, under general conditions, that the existence of default is a sufficient condition for the existence of such *binding* borrowing limits - asymmetric information is not necessary for these to occur. Furthermore, evidence exists of households being denied credit after submitting applications for loans to banks. According to conventional logic, these households are being denied credit because they are too high a risk - in other words, there is not enough commitment to repay their debts, so the punishment for default is too low. However, according to our model, this can also occur if the punishment for default is too high - at too high or too low a punishment, households are constrained to zero borrowing, even though they would like to borrow at the given interest rates. This can have important policy implications, since a large percentage of households are constrained in this way - for example, up to 15% of households in the US are denied credit upon application, according to evidence in Jappelli (1990).

Finally, in a numerical exercise, we examine the effect of allowing only agents with low incomes to default on their debt. We find, first of all, that in an economy with just one type of borrower, this policy is unambiguously welfare-improving as it results in higher equilibrium utilities for both savers and borrowers. However, when we introduce some agent heterogeneity in the model so that there are two types of borrowers, this is no longer the case. While introducing the policy makes savers better off and one type of borrowers better off, the second type of borrowers are made worse off through general equilibrium effects, so that the policy is no longer Pareto improving. Through this, we illustrate the importance of agent heterogeneity in general equilibrium models of this kind. We argue that this implies that welfare calculations in other studies could be biased upwards by not including enough agent heterogeneity in their experiments.

The second chapter extends the model above to a world with many open economies, where funds can be transferred between regions without cost. In this model, default penalties are set endogenously by a benevolent government in each economy whose aim is to maximise welfare in its region. Under this setting, governments are the players of a game where the action of each player is a profile of type-dependent punishment levels. The game is solved using the Nash equilibrium concept, so that each government sets the punishments which maximise welfare in its region given the punishments set by all other governments.

The first theoretical contribution of this chapter is to prove, in a completely general setting, the existence of a competitive general equilibrium with default and type-dependent contracts. The chapter then examines some positive and normative implications of such endogenous punishment formation. From a normative point of view, we show, via a numerical example, that the non-cooperative equilibrium is not welfare-maximising - by cooperating, welfare can increase in every region. This occurs even when, as in our numerical example, regions are so small that their actions do not affect world interest rates and therefore do not affect the actions of other governments. Intuitively, as we show, this occurs because when an individual government sets its punishment level, it does not take into account the general equilibrium effects of its actions to the extent that a central planner for the whole world economy would. This is made clear if we view the world as a single closed economy. Then, changes in the punishment level may benefit certain types of borrowers but may harm other agents via general equilibrium effects on the risk-free rate of interest. If setting punishment levels in the world collectively, one would take into account these general equilibrium effects. However, when setting punishment levels individually in each region, governments do not take these effects into account, at least not to the same extent, and the resulting equilibrium yields a lower welfare. This can have important implications for policy, since bankruptcy law is set individually by different countries. Even within the US, states are allowed to set their own bankruptcy legislation; our model suggests that there may be scope for welfare to improve with closer cooperation between states. It must be stressed however that this is meant to be a qualitative, not quantitative exercise. As such, the analysis does not make any inferences about the extent of the welfare loss arising from a non-cooperative equilibrium. This would be more appropriate for a more dynamic and complex model to address the issue with calibration methods, and is a suggested topic for future research.

Furthermore, past papers[17] have claimed that falling default penalties are to blame for an increase in the bankruptcy filing rate over the years. To examine this claim, we *exogenously* vary the punishment level in a single economy, holding all else constant, and observe the effects. The results show that a fall in default penalties may explain the trends observed over recent years since they result in higher default rates and may result in higher borrowing too[18]. However, this does not explain what triggered a fall in default penalties in the first place; to answer this question, we argue that a model needs to take this into account by deriving default penalties

[17] For example, Gross and Souleles (2002); Fay et al (2002).

[18] On the other hand, a fall in default penalties may result in a fall in borrowing, in contrast to recent trends.

endogenously, as we do in our model.

The chapter then proceeds to examine some positive implications of such endogenous punishment formation by conducting a series of numerical experiments, where several parameters are altered in a single economy. The experiments are conducted under two settings: under the first setting, we keep default penalties fixed while under the second setting, we allow governments to react to a change in economic conditions and set default penalties accordingly so that they vary endogenously. We then conduct the following five experiments: an increase in future endowments, a mean-preserving spread in future endowments, an increase in the probability of an expense shock occurring, a fall in intermediation costs, and a fall in interest rate spreads as a proxy to an increase in banking sector competition. The main conclusion from these experiments is that keeping penalties fixed and allowing them to vary endogenously can lead to contrasting observations. For example, one of the main puzzles in the literature has been the simultaneous increase in default rates and real incomes. We show that, keeping penalties fixed, an increase in incomes results in an increase in borrowing and a *fall* in the default rate. However, when we allow penalties to vary endogenously, the reverse is true and higher incomes result in more borrowing and an *increase* in the default rate. The reason for this is that, following a rise in incomes, the government finds it optimal to decrease default penalties. In turn, this puts upwards pressure on the default rate, which ends up higher at equilibrium. Therefore, higher incomes can result in more borrowing, more default and lower penalties, which is what is observed in the data. In other words, bankruptcy rates may have increased not despite of, but because of higher incomes; rather than a cause for concern, higher bankruptcy rates may be the result of governments' optimising behaviour.

The third and final chapter examines whether bankruptcy legislation is influenced by economic fundamentals by looking at the level of bankruptcy exemptions across the US. We first develop a two period, partial equilibrium model of with a bankruptcy option resembling Chapter 7 bankruptcy in the US, and derive, under a completely general setting, a simple rule to specify bankruptcy exemptions. This states that, all else equal, any macroeconomic shock which increases borrowing implies a higher optimal exemption level. For example, we would expect agents in regions with higher incomes to borrow more, as they require more borrowing in absolute terms to smooth their consumption over time. Following this, our rule would imply that, if set optimally, regions with higher incomes will have higher exemption levels.

We then proceed to test this empirically by looking at data from the US. The US provides an ideal source of data, since states are effectively allowed to set their own exemption levels and this has produced great variation across regions. The main innovation of the analysis is to incorporate spatial effects in our empirical modelling. This methodology is employed in response to casual observations that neighbouring states tend to have similar exemption levels and that, over the years, states have tended to increase or lower their exemptions (in real terms) if their neighbouring states have done so too. We use spatial error autoregressive (SER) and spatial lag autoregressive (SAR) frameworks to examine whether such spatial dependencies do indeed exist, or whether they can be explained by economic fundamentals, given that neighbouring states may have similar macroeconomic conditions. Our empirical results suggest that a significant relationship exists between macroeconomic variables and the level of bankruptcy exemptions, one that is broadly in line with the simple rule which we derive[19]. This suggests that governments do take into account economic data and change their exemption levels accordingly, which may have important implications for predicting future long-term rates of default, as chapter 2 illustrates. Furthermore, we detect the presence of positive spatial effects, so that states are likely to have exceptionally high or exceptionally low exemption levels if their neighbours do, and are likely to increase or decrease their exemption levels after neighbouring states have done so. However, we find that these spatial effects were more prevalent in previous years and have steadily been getting weaker since the mid-nineties.

To conclude, the aim of the research presented in this thesis has been to increase our understanding of the market for unsecured credit and personal bankruptcy. Among other things, it has provided insights into how default penalties influence the welfare of different agents in an economy, how they affect credit rationing and what implications their endogenous derivation can have for overall welfare and bankruptcy trends. Suggestions for future research lie on further developing such models and applying them in different contexts. For example, to the best of our knowledge, such models of bankruptcy have not been applied in the context of student loans and higher education. In the US for example, student debts cannot be discharged

[19]For example, it was found that states with higher incomes per capita have significantly higher home exemption levels.

with bankruptcy, and there may be scope for exploring this further. Furthermore, the topic of student loans and fees is a heated debate in the UK, with the government reducing its financial support for students in recent times. The motivation for doing so centres on the argument that, if higher education raises one's lifetime income, students should be able to invest in their futures themselves. However, this is countered by the argument that students are unable to borrow for such purposes due to credit rationing. Bankruptcy models may be helpful in explaining the reasons for this and could provide further insights on this subject.

Another topic of interest would be to examine changes in the current bankruptcy law with a view to making exemption levels more dynamic. For instance, there may be significant scope for improvements in welfare by linking exemption levels to aggregate income or economic growth so that they fluctuate with an economy's business cycle. For instance, it may be welfare-improving for exemptions to fall in boom times and to increase in recessions. More research could also be conducted into making exemption levels type-dependent - it may be beneficial for example to link these to education, income or age. For example, our third chapter states that any increases in borrowing imply a higher optimal exemption level. Therefore, given that young agents are likely to borrow more than older agents, it may be welfare-enhancing for type-dependent exemption levels to fall with a debtor's age.

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Chapter 1

General equilibrium and default: modelling the market for unsecured consumer credit

Abstract

We build a model of the market for unsecured consumer credit which encompasses endogenous borrowing limits, interest rates and rates of default. We first show that default punishments that are too harsh or too lenient can be Pareto inefficient and can lead to certain agents being excluded from the credit market. We also show that there exists an intermediate range of Pareto efficient punishments; there is no unique punishment level which Pareto dominates all others. We then show that when positive default exists, borrowing constraints must necessarily exist and be binding. In other words, default is sufficient for the existence of binding borrowing constraints, even in the absence of asymmetric information. Next, we ask what happens if the punishment level is selected endogenously by the market and show that this will not, in general, be socially optimal. Finally, we illustrate, by a numerical example, the importance of agent heterogeneity when evaluating the welfare effects of a certain policy; in an economy consisting of just one type of borrower, a policy may be unambiguously welfare improving, but this may not be the case in a mixed economy with two different types of borrowers. We argue that this result may imply an upwards bias in the welfare calculations of related studies.

1.1 Introduction

Until recently, models of general equilibrium (GE) and their extensions to incorporate incomplete markets (GEI) have not made any room for default by explicitly assuming that all agents always keep their promises. However, there has been a growing presence of default in our society, whether personal, corporate or sovereign. The present chapter builds a GEI model which incorporates default, having in mind the market for unsecured consumer credit. The motivation has been the recently observed upwards trends in credit levels and default rates. For example, unsecured credit in the US accounted for \$1.56 trillion, up from \$0.8 trillion in 1990, while personal bankruptcy filing have risen from 0.3% of households in 1984 (Fay et. al. (2000)) to more than 1.5% in 2003 (White, 2003). In the UK, figures from the DTI[1] show that unsecured lending has increased from £67bn to £168bn in real terms over the past decade, while the number of personal bankruptcies has been on the rise, with an estimated 20,400 personal bankruptcies filed in the UK between October and December 2005, compared to 7,700 in the same quarter of 2002. Furthermore, this has been at a period of historically high growth and low unemployment.

Default occurs because borrowers are unable to commit to fully repaying their debts in the future. Borrowers must therefore be induced into keeping their promises with the imposition of a penalty for default, otherwise they will not be able to commit to repaying their debt ex-post; recognising this, no agent will be willing to lend to borrowers, and so no intertemporal trade will occur. There is clearly an incentive therefore from society's point of view for a third party such as a government to impose a credible and enforceable punishment on those who default upon their debts.

The costs to default are as such. First, creditors are less inclined to lend since they anticipate that they will not be repaid in full and second, the imposition of a punishment is a deadweight loss to society. However, having some default can actually be welfare-enhancing. As Zame (1994) argues, default can be welfare enhancing in incomplete markets by allowing agents to enter into agreements which they can honour with a high probability and fail to honour with a low probability; without the option to default, borrowers would only enter into agreements which

[1]Department of Trade and Industry.

they knew that they would be able to honour in all conceivable situations, and this could greatly reduce the amount of intertemporal trade. Hence, infinite (or very high) punishment levels which completely eliminate default are not optimal. Dubey, Geanakoplos and Shubik (2005)[2] demonstrate how welfare is maximised with an intermediate punishment level in a general equilibrium model with default - they provide an example where the optimal punishment level is unique and Pareto dominates all other punishment levels - every other punishment results in an equilibrium where all agents are worse off. At high punishment levels, agents are deterred from borrowing, while at low punishment levels, the incentive to default is too high and lenders are deterred from lending.

The present chapter develops a two-period model of general equilibrium which encompasses such default and punishment as in DGS, with a punishment function which enters directly into the utility function of the agent. Unlike DGS, we explicitly model the banking sector, which collects deposits from one subset of consumers ('savers') and issues loans to another ('borrowers'). Retail banks act as financial intermediaries between savers and borrowers, who cannot trade with each other due to indivisibility and enforceability constraints. In our model, interest rates, default rates and quantity constraints are all determined endogenously, which, to the best of our knowledge, other papers do not do. For example, DGS derive two out of these three variables endogenously while keeping one fixed. Chatterjee et. al (2002) on the other hand build a model of a small economy with a fixed rate at which banks can obtain funds. In our model, this rate is determined endogenously by the forces of demand and supply - this allows us to evaluate the welfare of savers as well as borrowers, since changes in variables such as punishment levels affect the savings rate as well as the borrowing rate and credit constraints. Other models, such as Athreya (2002) and Li and Sarte (2002) assume exogenous quantity constraints. We then extend this in our last section by allowing default penalties to be set by a competitive market.

We obtain the following results. First, we show that default punishments which are too harsh or too lenient can lead to zero levels of borrowing and saving within an economy. We then show that, in an equilibrium with positive saving and borrowing, too high or too low a punishment level can be Pareto inefficient, which is similar to the findings of DGS - at very

[2]Henceforth, DGS (2005).

low punishment levels, increasing the punishment results in higher equilibrium utilities for both savers and borrowers, while at very high levels, savers and borrowers both benefit by decreasing the punishment. Unlike that paper however, we find that there is no unique Pareto efficient punishment level - instead, there is an intermediate *range* of Pareto efficient punishment levels. If the punishment level is within this range, changing it makes one type of agent better of but the other worse off. Therefore, the socially optimum punishment level is not unique and will then depend upon how welfare is evaluated. For example, if welfare is evaluated by the weighted sum of the utilities of all agent types, then the optimum punishment level will depend on the weights attached to each agent type.

Thirdly, evidence exists that certain agents are excluded from the market from borrowing (see, for example, Jappelli (1990)). We can derive such an equilibrium, and show that this can happen if default punishments are too lenient, or too harsh.

Next, we prove the existence of binding borrowing limits in an economy with default, which is consistent with the empirical evidence on the subject. For example, Gross and Souleles (2002) find that households increase credit card borrowing in response to an increase in their credit limit, while Grant (2003) finds that, on average, households would borrow up to $4,000 more if unrestricted. We prove that the existence of strategic default is a sufficient condition for the existence of such constraints. This is in contrast to previous papers, which derive credit constraints with some other form of market imperfection. For example, Stiglitz and Weiss (1981) derive credit constraints in a model with asymmetric information. In our model however, there are no informational asymmetries and the borrowing rate varies to reflect the risk of each individual borrower. Kehoe and Levine (1993) on the other hand derive credit constraints endogenously but these are set at the point where there is no default in the economy. In our equilibrium, credit constraints are derived endogenously but default does exist.

We then ask what happens if default penalties are chosen endogenously by the market. DGS (2005) ask the same question, and show via an example that the market selects the socially optimal punishment. We contradict this, and show that the punishment selected by the market is *not* socially optimal. To see this, we show that a competitive market always selects a Pareto efficient penalty. However, as mentioned above, DGS find a unique such penalty, and so this results in a socially optimum outcome. In contrast, in our paper there is a range of

Pareto efficient default penalties, so the socially optimum penalty will depend on how welfare is evaluated. Our endogenously determined penalty is invariant to this - no matter how welfare is evaluated, the chosen penalty is always the same. Therefore, in general, the outcome is not socially optimal.

Finally, we illustrate the importance of including different types of borrowers when evaluating the welfare implications of certain policy changes. We examine the effect of a policy which does not allow high-income households to default upon their debt. We find that in our simple setting, this is Pareto improving in two economies with different types of borrowers. However, in an economy which consists of both types of borrowers, such a policy may benefit one type but harm the other. We argue that such an omission could upwardly bias welfare calculations in other relevant studies.

We now proceed with the chapter as follows. Section 1.2 describes the model economy, as well as the maximisation problems of consumers and banks, and the solutions to them. Section 1.3 then defines and characterises an equilibrium, and gives a proof for the existence of credit constraints when default is present. We then offer a numerical example where punishment levels are varied for a given set of parameters to illustrate the workings of the model, and show how there exists an intermediate range of efficient punishment levels. Default penalties are then determined by the market in Section 1.4. In Section 1.5, the importance of including more than one type of borrower on a welfare analysis is illustrated. The paper concludes with a brief summary in Section 1.6. All proofs can be found in the appendix.

1.2 The model economy

Our model economy consists of two time periods, $t = 0, 1$ and idiosyncratic uncertainty in the second time period which is represented by S states of nature. The probability of each state $s = 1, ..., S$ occurring is given by π_s. There is one consumption good in each state, which we call 'money', and financial markets are incomplete. The purpose of this study is not to explore the *reasons* for this incompleteness of markets, but rather to assume this as a given and explore the implications of this for welfare in a world with bankruptcy. This is in accordance with most papers in the field, which simply take the incomplete markets as a given. It is also representative of what occurs in reality, with debt contracts specifying an interest rate which is constant and does not vary. However, we can justify this assumption if only for reasons of complexity. We assume it is too costly to include all contingencies in a debt contract - in reality, this would just about be impossible to achieve and we aim to replicate credit car and other borrowing arrangements which only specify a single interest rate. Secondly, we assume this is so because of costly state verification - borrowers will always pretend to be worse off than they really are, while banks cannot verify the true state they are located in. As banks are not able to accurately determine an agent's level of endowment, it is thus not able to make the interest rates fully state-contingent.

We denote the set of all agent types by Θ. Each agent of type $\theta \in \Theta$ is endowed with ω_θ^s of the consumption good in state s. Preferences over consumption in each state s are represented by a utility function $u : \mathbb{R}_+ \rightarrow \mathbb{R}$, which is strictly increasing and strictly concave[3]. Agents have rational expectations concerning future prices and rates of return. Agents can smooth consumption over time through the use of a bank; they can either save by depositing money at a bank and earn interest, or they can borrow at a certain interest rate and repay the bank in the second time period. For their services, banks can charge borrowers a higher interest rate than they offer savers. There is perfect competition in the banking sector, and banks can observe agent types.

1.2.1 The trading process

The trading process then takes place as follows. The first time period is divided into two stages. In the first stage, each bank offers a single contract $(y_\theta, r_\theta) \in \mathbb{R}^2$ to consumers of type $\theta \in \Theta$. An agent's type is common knowledge and hence contractible; banks will not trade a contract (y_θ, r_θ) with agents of type $\theta' \neq \theta$. In the second stage, consumers, who are perfectly informed about all the contracts on offer in the market, then face a binary choice - they can either accept or reject each bank's offer[4]. Each consumer of type θ will then trade the contract available to them which offers them the highest utility, subject to that being higher than their reservation utility; for instance, this could be the utility from not trading any contracts and consuming

[3]We could instead assume that u is type- and state- dependent. However, for ease of notation, we assume that utility is the same across agents and states.

[4]Without loss of generality, we can assume that consumers only trade one contract.

their endowments in each state, or it could be the utility received from using some available storage technology.

We now make the simplifying assumption that there are just two types of representative agents in the economy i.e. $\Theta = \{d, b\}$ and a single bank. We call agents of type $\theta = b$ 'borrowers' while agents of type $\theta = d$ 'savers' or 'depositors'[5]. The bank acts as if it were in a perfectly competitive environment by assuming zero barriers to entry.

In the second time period, all uncertainty is resolved and the state of nature s of each agent is revealed. If contracts were traded, the bank will have to honour its liabilities towards depositors and pay them $r_d y_d$, while borrowers can choose to partially default a proportion λ_b^s upon their debt and bear a punishment function P. This punishment function takes the form $P\left(\lambda_\theta^s, y_\theta, r_\theta\right) = p\, f\left(\lambda_\theta^s, y_\theta, r_\theta\right)$, where f (the 'punishment technology') is convex and increasing in λ_θ^s and increasing in the amount owed, while p is a constant which we call the 'punishment level'. It enters directly into the utility of agents and is fixed exogenously by the law. Default penalties are therefore increasing with λ_s^θ and $|r_\theta y_\theta|$ - for any given liability $|r_\theta y_\theta|$, a higher default rate will yield a higher penalty, whereas the penalty on a given default rate is higher if the amount owed is higher.

If the default rate chosen by borrowers in state s is λ_b^s then the bank will receive $-(1-\lambda_b^s) r_b y_b$ from each borrower it trades a contract with. Any ex-post profits Π (whether positive or negative) are kept by the bank[6].

The consumer's 'payoff function' and budget set

Finally, we define a consumer's 'payoff function' and their budget set. The payoff function U_θ is the ex-ante expected utility of a consumer once the punishment for default in each state is taken into account. This is given as:

$$U_\theta\left(x_\theta, \lambda_\theta\right) = u\left(x_\theta^0\right) + \sum_{s=1}^{S} \pi^s \left(u\left(x_\theta^s\right) - p\, f\left(\lambda_\theta^s, y_\theta, r_\theta\right)\right) \tag{1.1}$$

[5]We can think of these two agents types as 'young' and 'old' agents. We then fix their endowments in such a way so that, at equilibrium, young agents are borrowers and old agents are savers.

[6]We could just assume that profits are redistributed across the population which consists of the bank's shareholders. However, we avoid this for ease of notation - it does not affect our conclusions and does not affect the first order necessary conditions of our succeeding optimisation problems. Furthermore, due to perfect competition, this will be zero anyway.

where π_s is the probability of the agent being located in state s.

For any given set of parameters $\{\omega_\theta, y_\theta, r_\theta\}$, the budget set $B_\theta(\omega_\theta, y_\theta, r_\theta)$ of consumers of type θ is given as:

$$B_\theta(\omega_\theta, y_\theta, r_\theta) = \quad (x_\theta, \lambda_\theta) \in \mathbb{R}_+^{n+1} \times [0,1]^n \mid \tag{1.2}$$

$$x_\theta^0 \leq \omega_\theta^0 - y_\theta; \tag{1.3}$$

$$x_\theta^s \leq \omega_\theta^s + (1-\lambda_\theta^s) r_\theta y_\theta, \forall s \in \{1, ..., S\}; \tag{1.4}$$

$$\lambda_\theta^s \in [0,1], \forall s \in \{1, ..., S\} \tag{1.5}$$

We now solve for equilibrium by backwards induction.

1.2.2 The second time period

In the second time period, each agent of type θ holds a contract (y_θ, r_θ). Each individual's state of nature s is then revealed, and agents choose their optimal consumption-default vector $(x_\theta^s, \lambda_\theta^s)$ in that state. Due to monotonicity in preferences, for each state $s = 1, ..., n$, the optimal consumption for any given λ_θ^s is given by $x_\theta^s = \omega_\theta^s + (1-\lambda_\theta^s) r_\theta y_\theta$. Their optimal default decision $\lambda_\theta^s : \mathbb{R}^2 \to [0,1]$ is then given by:

$$\lambda_\theta^s(y_\theta, r_\theta; p) \in \arg \max_{\lambda_\theta^s \in [0,1]} \left(u(x_\theta^s) - p\, f(\lambda_\theta^s, y_\theta, r_\theta)\right) \tag{1.6}$$

Obviously, savers do not default and so $\lambda_d^s = 0$ in all states s.

Due to the law of large numbers, the proportion of agents of type θ who are located in state s is given by the probability π_s of that state occurring. The bank then has an income stream of:

$$\Pi = -\sum_{\theta \in \Theta} \sum_{s=1}^{S} \pi_s \xi_\theta (1-\lambda_\theta^s) r_\theta y_\theta \tag{1.7}$$

where ξ_θ is the fraction of θ-type agents the bank trades contracts with.

1.2.3 The first time period

Stage 2 - Consumers

In the second stage of the first time period, each consumer of type θ has a choice of available contracts (y_θ, r_θ) on offer to trade with each bank. We now define the function $V_\theta(y_\theta, r_\theta; p)$ as the value function to the problem:

$$\max_{(x_\theta,\lambda_\theta)\in B_\theta(\omega_\theta,y_\theta,r_\theta)} U_\theta(x_\theta,\lambda_\theta) \tag{1.8}$$

In other words, $V_\theta(y_\theta, r_\theta)$ is the maximum expected utility achievable by agent θ if he trades the contract (y_θ, r_θ) and chooses his optimal consumption and default rate in each state, given the punishment level p. Furthermore, each consumer has a reservation utility of $\underline{U}_\theta$. Therefore, at equilibrium, they will trade the contract which offers them the highest utility, subject to that being higher than $\underline{U}_\theta$.

Stage 1 - The bank

Due to the assumption of perfect competition and zero barriers to entry, we can assume without loss of generality that there is just one bank which acts like a competitive firm. Each bank's ex-ante expected profits (from here on, profits) are given by:

$$\Pi = -\sum_{\theta\in\Theta}\sum_{s=1}^{S}\pi_s\xi_\theta\left(1-\lambda_\theta^s\right)r_\theta y_\theta \tag{1.9}$$

if its contracts are traded, zero otherwise. The bank's maximisation problem is then given by:

$$\max_{\{(y_\theta,r_\theta),\xi_\theta\}_{\theta\in\Theta}} \quad \Pi = -\sum_{\theta\in\Theta}\sum_{s=1}^{S}\pi_s\xi_\theta\left(1-\lambda_\theta^s\right)r_\theta y_\theta \tag{1.10}$$

$$\text{s.t.} \quad \sum_{\theta\in\Theta}\xi_\theta y_\theta \geq 0 \tag{1.11}$$

$$V_\theta(y_\theta, r_\theta) \geq \widetilde{U}_\theta,\ \forall\theta\in\Theta \tag{1.12}$$

The first constraint is a feasibility constraint, and the second is an individual rationality one, where $\widetilde{U}_\theta$ is the utility of the next best option available to each consumer - this could be their reservation utility $\underline{U}_\theta$ or it could be the utility on offer from a contract available by another bank.

The equilibrium contract We now solve for the above maximisation problem. At equilibrium, every contract offered to each consumer type must be the solution to the problem given below. If this is not the case, it will be possible for the bank to enter the industry and capture the entire market share. This states that the contract offered to each agent type θ must be such that, given the contracts offered to all other agents:

$$\max_{(y_\theta, r_\theta), \xi_\theta} \quad V_\theta (y_\theta, r_\theta) \tag{1.13}$$

$$\text{s.t.} \quad \Pi = -\sum_{\theta \in \Theta} \sum_{s=1}^{S} \pi_s \xi_\theta (1 - \lambda_\theta^s) r_\theta y_\theta = 0 \tag{1.14}$$

$$\sum_{\theta \in \Theta} \xi_\theta y_\theta = 0 \tag{1.15}$$

Essentially, this solution is in the spirit of Allen and Gale (2004). A bank has to offer contracts which satisfy the participation constraint given in (1.12). Competition and free entry will result in banks undercutting one another by offering more attractive contracts to consumers. The zero-profit condition in (1.14) arises because a contract which yields a positive profit does not maximise a consumer's utility; a new entrant can steal customers by offering a more attractive contract which still yields a positive profit. Therefore, profits must equal zero[7]. Secondly, (1.15) arises because the bank will always 'clear its books'; it cannot lend more than it has in deposits and it will never hold more deposits than it needs to as doing so will reduce its profits ceteris paribus.

Finally, if the contracts offered to agents of type θ yield a utility which is strictly higher than their reservation utility, then the bank will not be able to exclude any of these agents from trade; they will have to set $\xi_\theta = 1$ or there will be an opportunity for a new bank to enter the

[7]Obviously, a bank will not offer any contracts which yield negative profits.

market and capture the entire market share.

To see this, suppose that there are some borrowers who are excluded from trading the contract (y_b, r_b) on offer from the incumbent bank when they could benefit from doing so (i.e. $V_b(y_b, r_b) > \underline{U}_b$). Another bank could then enter the market and offer those agents who are excluded a contract $(y_b, r_b + \varepsilon)$ with the same quantity y_b but with a slightly higher interest rate (i.e. $\varepsilon > 0$). Since preferences are continuous, for some small enough ε, the contract it offers will still yield a utility higher than $\underline{U}_b$, and so the excluded borrowers will trade the contract. This bank could then offer savers a contract $(y_d, r_d + \delta)$ with the same quantity they are offered by the incumbent bank, but with a slightly higher interest rate ($\delta > 0$), in order to lure them away from the incumbent bank. Then, if δ is sufficiently small, the resulting spread in interest rates will be large enough for the new bank to make a positive profit. The same logic applies in the case of depositors being excluded. Therefore, for all $\theta \in \Theta$:

$$V_\theta(y_\theta, r_\theta) \geq \underline{U}_\theta \Rightarrow \xi_\theta = 1 \tag{1.16}$$

$$V_\theta(y_\theta, r_\theta) = \underline{U}_\theta \Rightarrow \xi_\theta \leq 1 \tag{1.17}$$

or, more compactly :

$$V_\theta(y_\theta, r_\theta) \geq \underline{U}_\theta,\ \xi_\theta \leq 1, \text{ with complementary slackness} \tag{1.18}$$

By complementary slackness, we mean that when one constraint is loose, the other is binding and vice versa.

1.3 Equilibrium

To summarise, an equilibrium consists of a vector $\left(\{(x_\theta^e, \lambda_\theta^e), ((y_\theta^e, r_\theta^e), \xi_\theta^e)\}_{\theta\in\Theta}\right)$ such that:

1. The consumption-default vector $(x_\theta^e, \lambda_\theta^e)$ for each consumer $\theta \in \Theta$ is the solution to the maximisation problem given in (1.8).

2. The vector $\left(\{(y_\theta^e, r_\theta^e), \xi_\theta^e\}_{\theta\in\Theta}\right)$ is the solution to the bank's maximisation problem given in (1.10)-(1.12).

These two conditions guarantee that the goods and asset markets clear. Since there is only one consumption good in each state, we set this as the numeraire good and markets will clear (due to monotonicity of preferences), while the solution to the bank's maximisation problem ensures that the asset market clears through condition (1.15). In addition, it must be the case that, at equilibrium, no cross-subsidisation exists between contracts - in other words, every contract traded yields a marginal profit of zero to a firm's portfolio:

Proposition 1 *At equilibrium, there can be no cross-subidisation between contracts; in other words,* $\sum_{s=1}^{S} \pi_s r_\theta \left(1 - \lambda^*_{s\theta}\right) = \sum_{s=1}^{S} \pi_s r_{\theta'} \left(1 - \lambda^*_{s\theta'}\right)$ *for all* $\theta, \theta' \in \Theta$.

1.3.1 Characterisation

Borrowing limits

We begin our characterisation of this equilibrium with a discussion on borrowing limits. These are an everyday phenomenon, as most people who own a credit card or have asked for a loan would testify - in many cases, the amount we are allowed to borrow is less than what we would like to borrow at the given interest rates. We therefore define a borrowing constraint as binding if the amount borrowed is less than the optimum *at the given borrowing rate* i.e. if at the given borrowing rate r_b, $\frac{\partial V_b[y_b;r_b]}{\partial y_b} < 0$ - keeping the interest rate fixed, utility can increase by lowering y_b or in other words by increasing borrowing. Such borrowing constraints have been empirically identified by several authors. An interesting application is that of Gross and Souleles (2002), who find that borrowing increases in response to an increase in the credit limit, which suggests that these limits are in most cases binding. Other authors, amongst them Grant (2003), and Zeldes (1989) have also identified the existence of such limits.

In this section, we prove that the existence of strategic default is sufficient for the existence of such borrowing constraints. Our model therefore is capable of deriving such credit constraints endogenously. This is important, since models with endogenous borrowing limits may reverse results obtained using models with exogenous borrowing limits. An example of such a case is in Lochner and Monge-Naranjo (2002); they find that student subsidies for university education are substitutes to personal debt levels with exogenous borrowing limits, but become complements with endogenous borrowing limits - higher student subsidies increase investment in human capital which results in a higher expected future expected income of the individual and hence a higher borrowing limit. Mateos-Planas and Seccia (2006) also show how the endogenous determination of borrowing limits can have important positive and normative implications. Furthermore, borrowing constraints have been identified as a main reason for the observed failure of Friedman's permanent income hypothesis (see, for example Feigenbaum (2004)). It seems therefore that the inclusion of endogenous borrowing limits can only be a welcome feature of any model.

Proposition 2 *At equilibrium, in the presence of voluntary default, the partial derivative (keeping interest rates fixed)* $\frac{\partial V_b(y_b^e, r_b)}{\partial y_b} < 0$*; therefore, borrowing limits exist and are binding.*

The equilibrium in this economy is therefore equivalent to the competitive equilibrium in an economy with an exogenous, binding borrowing limit $|y_b^e|$ (which is derived endogenously in our model). In such an economy, interest rates for savers and borrowers will adjust so that at a competitive equilibrium, all agents trade their optimum quantity, given the exogenously imposed borrowing limits for each agent type, and banks make a profit of zero.

This result is significant for it proves that borrowing limits must necessarily arise in the presence of positive default and are an equilibrium, rather than a disequilibrium, phenomenon. Furthermore, we have derived these borrowing limits in perfect competition with no informational asymmetries. Therefore, in the presence of default, no other market imperfections are necessary for the existence of borrowing limits. This differs from Stiglitz and Weiss (1981), where borrowing constraints (or "credit rationing") are the result of imperfect information about the probability of default. This also differs from Kehoe and Levine (1993), where contracts are complete and no default occurs at equilibrium, whereas in our model borrowing constraints exist alongside default.

Excluded agents

There is ample empirical evidence of certain agents being excluded from the credit market and not being able to obtain credit. For example, Jappelli (1990) uses survey evidence to find that such households accounted for up to 15% of the US population. Our model can derive such an equilibrium.

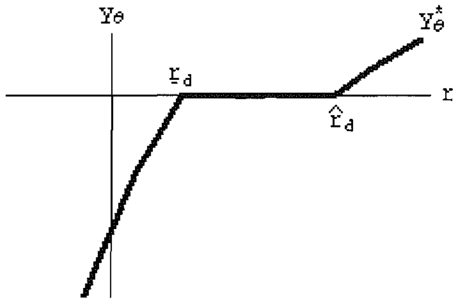

Figure 1-1: Equilibrium amount of trading given the risk-free rate of interest r_d.

Proposition 3 *There exists an interval of risk-free interest rates $(\underline{r}_d, \widehat{r}_d)$ at which the agent does not trade the asset. Furthermore, at every such interest rate, the agent would like to borrow but is unable to do so. This is illustrated in Figure 1-1 (y_θ^* is the quantity offered to agent θ as part of the equilibrium contract for any given risk-free interest rate r_d).*

With just two agent types, there can be two equilibria; one where the asset is traded, and one where no asset is traded - either both agents are offered zero quantities as part of the equilibrium contract, or one agent is a saver and the other a borrower. Figure 1-2 depicts the first situation and Figure 1-3 the second. They each plot the *absolute* quantity of the asset offered by the bank given the risk-free interest rate, $|y_b^*|$ and $|y_d^*|$. Equilibrium occurs where these two curves intersect. In Figure 1-3, these intersect at the flat parts of the curves, so none of the asset is traded by either agent.

With three types of agents, we can get an equilibrium where trade occurs between two agents, but the third agent is in autarky. This is illustrated in Figure 1-4. At the equilibrium risk-free interest rate, agents of type b and d trade, while agents of type z do not. This equilibrium, where agents of type z are rationed occurs when the punishment faced by those agents is either too high or too low. At intermediate punishment levels, all three agents trade contracts - agents of type b and z borrow while agents of type d save. This occurs because the interval of interest rates at which an agent is not offered any credit is smallest for intermediate punishment levels. We are unable to provide a general proof but numerical experiments confirm this.

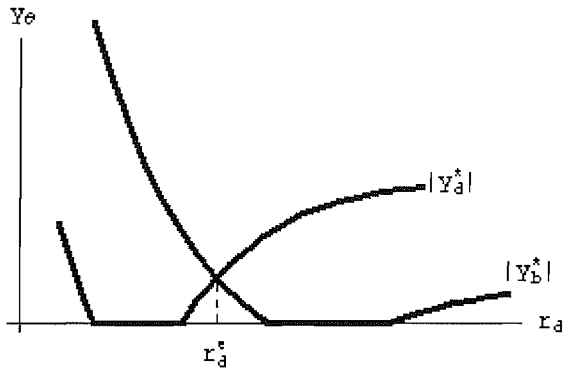

Figure 1-2: Asset is traded at equilibrium.

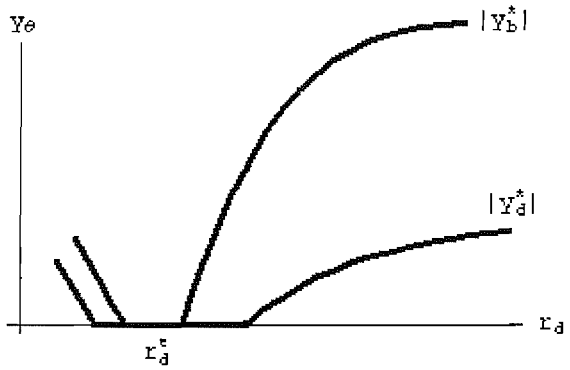

Figure 1-3: Asset is not traded at equilibrium.

1.3.2 A numerical example

In this section, we provide a brief numerical example to illustrate the workings of our model. There are three states of nature $s = 1, 2, 3$[8] in the second time period. The probabilities of each state occurring are given by $\alpha_1 = 0.45$, $\alpha_2 = 0.45$ and $\alpha_3 = 0.10$. Agents are given quadratic utility functions, and so the utility of agent of type θ from consuming $x_{s\theta} \in [0, 1]$

[8]The number of states is not crucial, as long as it is greater than one in order to include a 'bad state'. The reason we use three states instead of two is because we are plotting discrete points, since we vary the punishment level discretely - with three states, we get much smoother graphs than we do with two at the given accuracy levels.

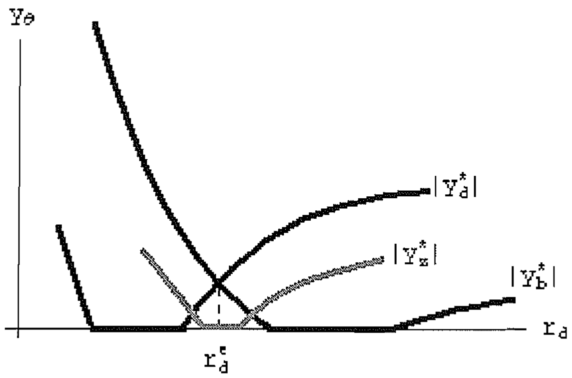

Figure 1-4: Equilibrium with three types of agents; only two agents trade contracts.

in state s is given by $u_{s\theta}(x_{s\theta}) = x_{s\theta} - \frac{1}{2}x_{s\theta}^2$. The punishment function P is specified as $P(\lambda_{s\theta}, y_\theta, r_\theta) = -pr_\theta y_\theta \lambda_{s\theta}^2$[9], where p is the punishment level and is varied. Each depositor's endowments $\omega_d = (\omega_{0d}, \omega_{1d}, \omega_{2d}, \omega_{3d})$ are given as $\left(\frac{6}{10}, \frac{4}{10}, \frac{4}{10}, \frac{4}{10}\right)$, while those of borrowers are $\left(\frac{4}{10}, \frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right)$. The masses of depositors and borrowers, M_d and M_b, are both equal to 1. We make the additional assumption that the borrower's budget set in state 3 is 'empty'. In other words, the borrower's endowment in that state, ω_{3b}, is equal to the minimum consumption level required for survival, and so the borrower will therefore always default on 100% of their debt. Default in such a state is therefore entirely involuntary; this is similar to the notion of an empty budget set in Chatterjee et al. (2003), and DGS (2005) do this too in a numerical example. We call such a state a 'bad' state. We let p vary discretely from $\frac{1}{50}$ to $\frac{210}{50}$ (at intervals of $\frac{1}{50}$). The results are plotted below.

Welfare and (sub)optimal punishment levels

We now plot the utilities at equilibrium of borrowers and depositors (U_b^* and U_d^* respectively) as the punishment is varied in Figure (1-5). Notice how setting the punishment at too high or too low a level results in a Pareto inefficient outcome; for values between $\frac{1}{50}$ and $\frac{41}{50}$, increasing

[9] If we do not scale the punishment by $r_\theta y_\theta$, then the punishment is the same for someone who defaults on 100% of two different amounts - e.g. someone who defaults 100% on a loan of \$1 is punished as harshly as someone who defaults 100% on a loan of \$1,000,000. This is obviously not representative of reality. However this is not necessary and does not qualitatively change the results.

p results in higher utilities for both borrowers and savers, while for values of p greater than $\frac{50}{100}$, decreasing p does so. On the other hand, there is a Pareto efficient intermediate range (which has been shaded in grey) from $\frac{41}{50}$ to $\frac{50}{50}$, where changes in the punishment level result in an increase in one agent's utility but a decrease in the other's. This interval corresponds to the peaks of each agent's equilibrium utility; the equilibrium utility of borrowers peaks at $\frac{41}{50}$, and that of savers at $\frac{50}{50}$.

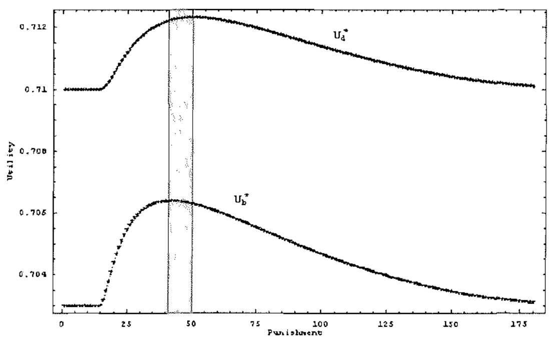

Figure 1-5: Equilibrium utilities at given punishment levels.

The intuition behind this observation is as follows. At very low punishment levels ($p < \frac{15}{50}$), there is too high an incentive to default and there is no trade in the credit market and no agent borrows or saves. Therefore, their equilibrium utilities are just their reservation utilities. This equilibrium corresponds to that depicted in Figure 1-3. As the punishment level is then increased, the incentive to default falls and banks are willing to make small loans. The banks' demand for deposits is therefore low too, and so the rate offered on deposits is low.

As the punishment level increases further, banks are willing to make larger loans and at lower interest rates (given that r_d remains constant), and this results in better consumption smoothing for borrowers. Since the banks are lending more, they require a larger amount of deposits from the market. The interest rate for deposits must therefore increase to attract the extra funds, and this makes savers better off. The effect on borrowers is two-fold. On the one hand, borrowers benefit from better consumption smoothing, while on the other hand the higher punishment level and higher rate on deposits (which raises the rate on loans to keep profits at zero) harms them. Initially, these gains outweigh the losses, and so the utility of borrowers increases. Therefore, a higher punishment level results in higher equilibrium utilities for both savers and borrowers. This occurs for punishment levels between $\frac{15}{50}$ and $\frac{41}{50}$, to the left of the shaded region.

After a certain point, increasing the punishment level harms borrowers, as the benefits from better consumption smoothing are outweighed by the harmful effects of a higher punishment and higher rates on deposits. Higher punishment levels reduce the incentive to default, and so credit is extended. Therefore, banks still demand more deposits from the market, which raises the interest rate on deposits and benefits savers. Hence, increasing the punishment level harms borrowers but benefits savers. This is the Pareto efficient intermediate range, which is the shaded grey area from $\frac{41}{50}$ to $\frac{50}{50}$.

At high punishment levels ($p > \frac{50}{50}$), borrowing is too unattractive so borrowing levels per agent start falling while the utility of borrowers keeps on decreasing. The lower levels of borrowing lead to a lower demand for deposits by banks, so the interest rate on deposits decreases, which leads to lower saving and less utility for savers. Therefore, increasing the punishment level results in lower equilibrium utilities for both borrowers and savers (or in other words decreasing the punishment leads to higher equilibrium utilities for both agents). This occurs to the right of the shaded area. Eventually, at some high punishment level, borrowing is too unattractive, and credit in the economy reduces to zero.

Credit levels

In this section, we plot the equilibrium credit levels for every given punishment level. These are plotted in Figure 1-6:

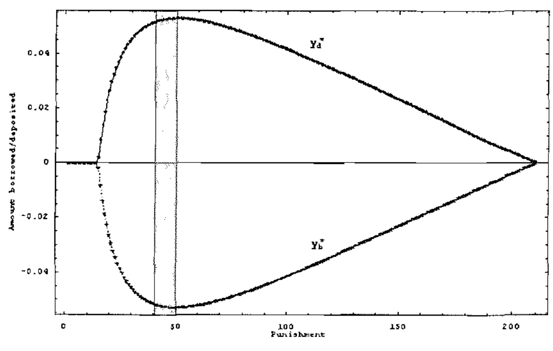

Figure 1-6: Borrowing and saving per agent at given punishment levels.

The top line plots the amount traded per saver and the bottom line plots the amount traded per borrower. Notice how for low punishment levels ($p < \frac{15}{50}$) there no contracts are traded hence credit levels are zero. After that, deposits per saver (the top line) and hence the total amount of credit in the economy initially expands, but then starts to fall. This occurs at after the end of the Pareto inefficient range of $\frac{50}{50}$ onwards, which agrees with the intuition given above. Credit levels eventually fall to zero at $p = \frac{210}{50}$, and trade seizes at higher punishment levels; at such high punishment levels, agents are discouraged from borrowing, and so they will not want to borrow unless the interest rate is sufficiently low. However, if interest rates are too low then savers will not want to save - the end result will be that no trade will occur, and credit levels in the economy will be zero, as Figure (1-6) illustrates.

Borrowing limits

We now illustrate the presence of binding borrowing limits by plotting the partial derivative $\frac{\partial V(y_b^e, r_b)}{\partial y_b}$ in Figure 1-7. As the figure shows, this is negative at all punishment levels, which implies that borrowing limits exist and are binding, and is a visual representation of Proposition 2:

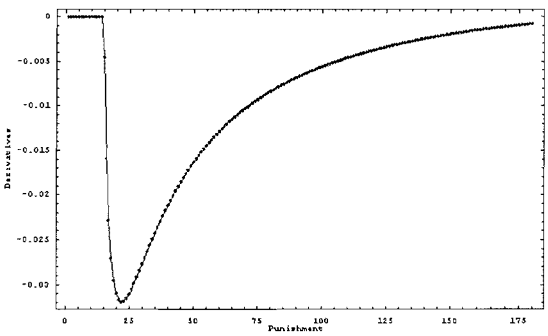

Figure 1-7: Illustration of borrowing constraints at equilibrium.

1.4 Market selection of punishment levels

In the previous section, we asked how high punishment levels *should* be and showed that setting them at too high or too low a level can be inefficient - outside a certain range, the punishment level can be increased or decreased to make everyone better off. The government should then choose a punishment level inside that range which maximises social welfare according to its goals. For example, if it places a higher value on the welfare of borrowers than that of savers then it should set a relatively low punishment level whereas if it places a greater value on the welfare of savers then it should set a relatively high punishment level.

We now ask what happens if the punishment level is set by the market - does this result in an optimal punishment which maximises welfare? Just as in DGS (2005), the market selects a Pareto efficient punishment level. However, in their paper they obtain a single punishment level which Pareto dominates all others and is therefore welfare maximising, whereas in this paper we obtain a range of Pareto efficient punishment levels, so the punishment level selected need not be socially optimal. In fact, the level selected by the market is invariant to how welfare is measured, and so the punishment level selected by the market will not generically be socially optimal.

We now describe how the punishment level is endogenously selected by the market. The consumer's optimisation problem is unchanged; consumers will still trade the contract which yields them the highest utility subject to that being higher than their reservation utility. On the other hand, the bank's optimisation problem is slightly altered since the bank must now choose the punishment level p_θ to specify in its contracts in addition to choosing the other variables in (1.10). Contracts specified by the bank now consist of a quantity y_θ, an interest rate r_θ and a punishment level p_θ. Specifically, the bank's optimisation problem is now given by:

$$\max_{\{(y_\theta,r_\theta,p_\theta),\xi_\theta\}_{\theta\in\Theta}} \quad \Pi = -\sum_{\theta\in\Theta}\sum_{s=1}^{S}\pi_s\xi_\theta\left(1-\lambda_\theta^s\right)r_\theta y_\theta \tag{1.19}$$

$$\text{s.t.} \quad \sum_{\theta\in\Theta}\xi_\theta y_\theta \geq 0 \tag{1.20}$$

$$V_\theta\left(y_\theta,r_\theta,p_\theta\right) \geq \widetilde{U}_\theta,\ \forall\theta\in\Theta \tag{1.21}$$

As in the case where the punishment is set by the government, competition and free entry, will force the bank to offer the contract which maximises each consumer's utility subject a zero-profit condition and subject to its net asset position being zero. In other words, it will be the solution to the following maximisation problem:

$$\max_{(y_\theta, r_\theta, p_\theta)} \quad V_\theta\left(y_\theta, r_\theta; p_\theta\right) \tag{1.22}$$

$$\text{s.t.} \quad \Pi = -\sum_{\theta\in\Theta}\sum_{s=1}^{S}\pi_s\xi_\theta\left(1-\lambda_\theta^s\right)r_\theta y_\theta = 0 \tag{1.23}$$

$$\sum_{\theta\in\Theta}\xi_\theta y_\theta = 0 \tag{1.24}$$

In addition, as before, the bank will have to offer all agents contracts if the utility on offer is strictly higher than their reservation utility:

$$V_\theta\left(y_\theta, r_\theta, p_\theta\right) \geq \underline{U}_\theta,\ \xi_\theta \leq 1, \text{ with complementary slackness} \tag{1.25}$$

1.4.1 A numerical example

We now solve our previous numerical example with an endogenous punishment level - we leave all variables unchanged. An equilibrium vector is denoted as:

$$\left(\left\{\left(x_\theta^e, \lambda_\theta^e\right), \left(\left(y_\theta^e, r_\theta^e, p_\theta\right), \xi_\theta^e\right)\right\}_{\theta\in\Theta}\right)$$

This is a point which satisfies the consumer's optimisation problem given in (1.8) and the bank's new optimisation problem given in (1.22). In our example, we get a unique such point, at $p_b^e \approx 0.9165$ (or in other words, a point close to $\frac{46}{50}$). At this point, the equilibrium allocations are $y_d = 0.053$, $y_b = -0.053$, $r_d = 0.812$, $r_b = 1.194$, $\lambda_b = (0.191, 0.297, 1)$ and every agent trades a contract, so $\xi_d = \xi_b = 1$.

An initial observation is that this point must lie within the Pareto efficient range of punishment levels; if the bank specifies a punishment level outside this range, another bank can capture the entire market by offering contracts with a different punishment level which make all agents better off. However, this point need not be welfare-maximising. Since there is a range of Pareto optimal punishment levels, the one which maximises welfare will depend on how welfare is evaluated. The punishment selected by the market however is invariant to this and so will not in general be welfare-maximising. To see this, consider the following example. Suppose the government chooses the socially optimal punishment level by maximising a social welfare function W which is the weighted sum of the two equilibrium utilities U_d^e and U_b^e:

$$W = \beta_d U_d^e + \beta_b U_b^e \tag{1.26}$$

where $\beta_d + \beta_b = 1$. Then, if the government places a high value on borrowers, such as $\beta_b = \frac{8}{10}$ (so $\beta_d = \frac{2}{10}$), the socially optimal punishment level p_b^* will be (approximately) 0.86, as shown in Fig. (1-8). Therefore, the punishment selected by the market is too high. On the other hand,

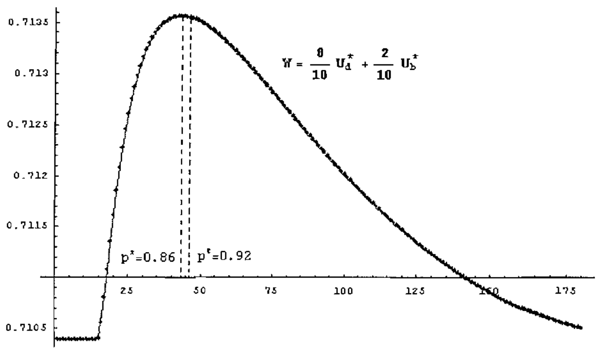

Figure 1-8: Endogenously selected punishment level is too high.

if the government places a high value on savers, such as $\beta_d = 0.8$ (so $\beta_b = 0.2$), the socially optimal punishment level p^* will be (approximately) 0.98, as shown in Fig. (1-9). In this case therefore, the punishment selected by the market is too low.

1.5 The importance of agent heterogeneity

Up to this point, we have assumed that the punishment level in each state is the same. However, if we allow for state-dependent punishment levels, then it is clear that, if chosen optimally, welfare can be improved; constant punishment levels are just a restricted case of state-dependent punishment levels. Athreya (2002) calibrates a dynamic model of general equilibrium and examines state-dependent punishments by evaluating a law that prohibits bankruptcy procedures for US households with above-median income ('stringent means-tests'). In our model, this is similar to setting the punishment level in the good state ($s = 1$) to infinity or at very high levels,

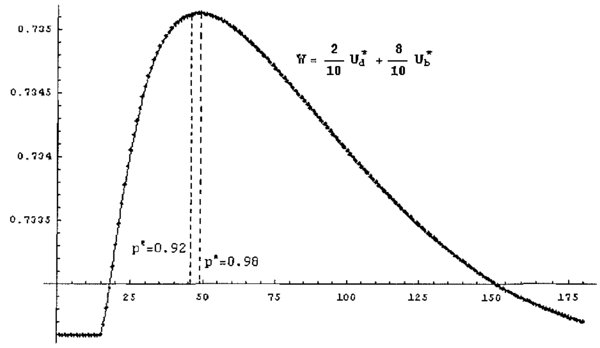

Figure 1-9: Endogenously selected punishment level is too low.

thus not allowing for any default in that state. The author finds that such a reform improves welfare by a small margin. In that paper however, borrowing constraints are set exogenously. Chatterjee *et al.* (2002) on the other hand evaluate such a proposal using a calibrated model with endogenous credit constraints and find that the welfare gains are substantial; however, in their model, banks can obtain funds at a fixed interest rate.

In this section we examine the effects of a certain state-dependent punishment using the more abstract model already developed with two different types of borrowers, and show how increasing the punishment level to a very high (or infinite) value in the good state to rule out default in that state is Pareto improving in two separate economies. Both consist of the same type of savers, but of different types of borrowers; these borrowers have the same endowments in each state, but different probabilities of each state occurring; the borrowers in one economy have a higher probability of the good state ($s = 1$) occurring and a lower probability of the medium state ($s = 2$). We call these agents 'skilled', and the other type 'unskilled'.

This policy is Pareto improving in both economies - both borrowers and savers are better off as a result. However, when we repeat the same experiment on a mixed economy which consists of both types of borrowers, the policy is no longer Pareto improving; the equilibrium utility of savers and skilled borrowers increases, while that of unskilled borrowers falls.

1.5.1 The effect on different agents

Suppose that an economic environment consists of two time periods, with one state of nature in the first time period, and three states nature in the second time period, where endowments in states $s = 0, 1, 2, 3$ are given as $\frac{40}{100}, \frac{70}{100}, \frac{50}{100}, \frac{10}{100}$ respectively for borrowers (where $s = 0$ is the state in the first time period, $s = 1$ is the good state, $s = 2$ is the medium state and $s = 3$ is the bad state with the empty budget), and the endowments of savers are constant in each time period and are given as $\frac{60}{100}, \frac{40}{100}, \frac{40}{100}, \frac{40}{100}$. As in the example above, all agents have quadratic utility functions and the punishment function is given as $P(\lambda_{sb}, y_b, r_b) = -pr_b y_b \lambda_{sb}^2$. Now consider two different economies with the same economic environment, but with different idiosyncratic uncertainty. In the first economy, borrowers are more educated ('skilled') and therefore have a higher chance of the good state occurring, while in the second economy borrowers are less educated ('unskilled') and therefore have a lower chance of the good state occurring but a higher chance of the 'medium' state ($s = 2$) occurring. More specifically, in the first economy, the probabilities of each state occurring are $\alpha_1 = 0.20$, $\alpha_2 = 0.65$ and $\alpha_3 = 0.15$ while in the second economy they are given as $\alpha_1' = 0.05$, $\alpha_2' = 0.80$ and $\alpha_3' = 0.15$. We start with a punishment level which is constant and equal to $\frac{75}{50}$ across all states.

Skilled agents

We now experiment on the economy with skilled borrowers, first setting the punishment level at $\frac{75}{50}$ across all states and then disallowing default in the good state. The first column shows the equilibrium quantities and utilities when the punishment level is constant across states while the second column shows the equilibrium quantities and utilities when no default is allowed in the good state.

Table 1-1

	Constant p	No default at $s = 1$
y_d^e	0.0247	0.0282
y_b^e	−0.0247	−0.0282
U_d^e	0.740467	0.740616
U_b^e	0.68505	0.68520

As the table shows, the change is a good one; disallowing default in the good state has resulted in more saving and borrowing, and as a result, both savers and borrowers are better off so we have a Pareto improvement.

Unskilled agents

We now repeat the same experiment on the economy with unskilled borrowers and show the results in Table 1-2 below.

Table 1-2

	Constant p	No default at $s=1$
y_d^e	0.0186	0.0195
y_b^e	−0.0186	−0.0195
U_d^e	0.740262	0.740288
U_b^e	0.676343	0.676372

Once again, it would seem that disallowing default in the good state is beneficial; the end result is more saving and borrowing, and a higher equilibrium utility for both borrowers and savers. However, as we now show in the following section, even though such a policy is Pareto improving in both economies, this may no longer be the case in a mixed economy consisting of both types of borrowers.

The effects on a mixed economy

The experiment is now carried out on an economy with the same savers but with a mixed population of borrowers consisting of 50% skilled borrowers (type $b1$) and 50% unskilled borrowers (type $b2$). The results are shown in Table 1-3 below.

Table 1-3

	Constant p	No default at $s=1$
y_d^e	0.0217	0.0239
y_{b1}^e	−0.0281	−0.0332
y_{b2}^e	−0.0153	−0.0146
U_d^e	0.740325	0.74037
U_{b1}^e	0.685221	0.685483
U_{b2}^e	0.676233	0.67621

As Table 1-3 shows, despite being Pareto improving in both separate economies, disallowing default in the good state is no longer Pareto improving in this economy; the end result is an increase in saving, an increase in borrowing by skilled agents and a decrease in borrowing by unskilled agents, while the utilities of savers and skilled borrowers rise and that of unskilled borrowers falls. The reason for this is simple - disallowing default in the good state leads to more borrowing from skilled agents and therefore more deposits are required from the market, which raises the interest rate on savings. This then discourages borrowing by all agents and makes them unambiguously worse off *ceteris paribus*. In our case, this general equilibrium effect outweighs the commitment benefits to unskilled agents due to the high punishment level, and the end result is that they borrow less and are worse off after the change.

This example has just illustrated the importance of general equilibrium effects with different agent types on the quantitative evaluation of welfare when examining policy changes in bankruptcy law. It has direct relevance to the studies in this field of literature such as Athreya (2002), Chatterjee *et at.* (2002), and Lehnert and Maki (2000). As this example shows, by not allowing for enough agent heterogeneity or having an exogenous rate on savings, the welfare evaluations of these models could be biased upwards; it may be necessary to repeat such experiments by allowing for greater heterogeneity.

1.6 Conclusion

The objective of this paper has been to develop a model with the aim of achieving a better understanding of the market for unsecured consumer credit. We build a two-period model of general equilibrium, where credit constraints, interest rates, default rates and later on punishment levels are all determined endogenously within the model.

We first asked whether there exist Pareto sub-optimal punishments for default. To this end, we found that, just as in DGS (2005), the punishment can be sub-optimal if set at too high or too low a level and that by decreasing or increasing it, both borrowers and savers can be made better off at equilibrium. However, unlike DGS, we found that there was not a unique Pareto efficient punishment level. Rather, there exists a range such punishments - changing the punishment in that range makes one set of consumers better off at equilibrium but another worse off. This range was such that for any punishment level within that range, lower punishment levels benefited borrowers and higher punishment levels benefited savers. We also show that if the punishment level is too high or too low, no borrowing or saving will occur. This finding is very similar to that of DGS. We then ask why certain agents are excluded from the credit market; in our model, this can happen if punishment levels are too high or too low.

Next, we examined the existence of credit constraints. We defined a household as being constrained if they would borrow a larger amount at the existing market rates than the limit imposed upon them by banks. The existence of such constraints has been well documented in papers such as Grant (2003) and Gross and Souleles (2003). We proved that, given that the average default rate varies continuously with the amount borrowed, the existence of default is sufficient for the existence of borrowing constraints. This condition is sufficient even in otherwise perfect markets with no informational asymmetries and where the borrowing rate reflects the risk of each individual borrower. This differs from previous studies such as Stiglitz and Weiss (1981), who attribute the existence of credit constraints to informational asymmetries where every agent borrows at the same interest rate.

We then asked what happens if default penalties are set by the market. We derive an equilibrium where the penalty is determined endogenously, along with quantity constraints, interest rates and default rates. We find that the chosen penalty will be Pareto efficient, but will not be welfare-maximising (unless this occurs by chance). This is in contrast to DGS (2005), the endogenously selected punishment level selected is the socially optimal one. The difference between our model and theirs is that we also derive quantity constraints endogenously, whereas they fix them exogenously. In their model, they obtain a unique punishment level which Pareto dominates all others and therefore is welfare-maximising. In our model, we obtain a range of Pareto efficient punishment levels, so a punishment which is Pareto efficient need not be welfare-maximising - that will depend on how social welfare is evaluated. Therefore, even though the punishment level selected by the market is Pareto efficient, it will not, in general, be socially optimal. This difference with DGS has a fundamental policy implication; whereas their result would seem to suggest that a government can achieve the social optimum by allowing the market to determine default penalties, in our model this is no longer the case.

Finally, we end the paper with a brief illustration of the importance of including different agent types on the welfare analysis of policies; we show how a certain policy (ruling out default for agents with high endowments) is Pareto improving in two separate economies, each with a different type of borrower, but is no longer so in an mixed economy with both types of borrowers. We argue that this may imply that the welfare evaluations in certain related papers might be biased upwards.

1.7 Appendix

1.7.1 Comparative statics - Optimum default levels

In this section, we show how the optimal default rate changes with the interest rate and the level of borrowing. The consumer's optimal bankruptcy decision is the solution to the problem in Eq. (1.6) on p. 33:

$$\max_{\lambda_b^s} \quad U_b^s = \left(u_b^s\left(\omega_b^s + \left(1-\lambda_b^s\right) r_b y_b\right) - pf\left(\lambda_b^s\right)\right)$$

We now examine the cross-partial derivative of U_b^s with respect to λ_b^s and either r_b or y_b. By the Conjugate Pairs Theorem (p. 81, Currier, (2000)), $\frac{\partial U_b^s}{\partial \lambda_b^s \partial r_b} \frac{\partial \lambda_b^s}{\partial r_b} > 0$, and $\frac{\partial U_b^s}{\partial \lambda_b^s \partial y_b} \frac{\partial \lambda_b^s}{\partial y_b} > 0$. The partial derivative of U_b^s with respect to λ_b^s is given as $\frac{\partial U_b^s}{\partial \lambda_b^s} = -r_b y_b u'(\cdot) - pf'(\bullet)$. Therefore, the cross-partials are:

$$\begin{aligned} \frac{\partial U_b^s}{\partial \lambda_b^s \partial r_b} &= \overbrace{-y_b u'(\cdot)}^{<0} - \overbrace{\left(1-\lambda_b^s\right) r_b y_b^2 u''(\cdot)}^{<0} > 0 \\ \Rightarrow \frac{\partial \lambda_b^s}{\partial r_b} &> 0 \end{aligned}$$

$$\begin{aligned} \frac{\partial U_b^s}{\partial \lambda_b^s \partial y_b} &= \overbrace{-r_b u'(\cdot)}^{>0} - \overbrace{\left(1-\lambda_b^s\right) r_b^2 y_b u''(\cdot)}^{>0} < 0 \\ \Rightarrow \frac{\partial \lambda_b^s}{\partial y_b} &< 0 \end{aligned}$$

since $u_b^{s\prime}(\cdot) > 0$, $u_b^{s\prime\prime}(\cdot) < 0$ and $y_b < 0$. These results agree with intuition; the higher the borrowing rate or the higher the quantity borrowed, the larger an individual's liability and therefore the more likely it is that they will default.

1.7.2 Borrowing limits

Proof of Proposition 1. To prove this proposition, we first argue that all agents who save, or depositor types, must necessarily save at the same interest rate, which we will call r_f. Suppose this is not so and the bank is offering a lower interest rate to some depositors (type d') than to others. Then this cannot be an equilibrium, since there will be an opportunity for another bank to enter the market and make a profit. It can do this by offering those agents of type d' a slightly higher interest rate. It can then attract some borrowers by offering them a slightly lower interest rate than that offered by the incumbent bank. If the increase in the rate offered to depositors d' and the decrease in the rate offered to borrowers are small enough then this bank will make a positive profit - this is because the cost of funds of the incumbent bank was a combination of the low and high interest rates offered to depositors, whereas the new entrant obtains funds at just the lower interest rate (or, more accurately, at a rate slightly above that). Therefore, at equilibrium, all depositors must save at the same rate r_f. Furthermore, the contracts offered to borrowers must be such that $\sum_{s=1}^{S} \pi_s (1 - \lambda_\theta^s) r_\theta = r_f$ for all borrowers of type θ. If this is not the case then cross-subsidisation exists, given that a bank's profits must equal zero at equilibrium. That is, contracts offered to some borrowers (e.g. type b) will yield positive profits, or $\sum_{s=1}^{S} \pi_s (1 - \lambda_b^s) r_b < r_f$, while contracts offered to other borrowers (e.g. type b') will yield negative profits so that $\sum_{s=1}^{S} \pi_s (1 - \lambda_{b'}^s) r_{b'} < r_f$. In this case, another bank can offer borrowers of type b (whose contracts yield a positive profit) a slightly lower interest rate while at the same time offering savers a slightly higher interest rate. If these differences are not too large, the bank will capture that part of the market and make a positive profit, while the incumbent bank will be left with the loss-making borrower contracts. Therefore, cross-subsidisation cannot exist at equilibrium and the following condition holds:

$$\sum_{s=1}^{S} \pi_s (1 - \lambda_\theta^s) r_\theta = r_f \quad \forall \theta \in \Theta \tag{1.27}$$

■

Proof of Proposition 2. The proof is straightforward and arises as part of the solution to the bank's maximisation problem in (1.10)-(1.12). This states that the contract offered to *each consumer type* θ must be such that, *taking as fixed the contracts offered to all other agents*:

$$\max_{(y_\theta, r_\theta), \xi_\theta} \qquad V_\theta (y_\theta, r_\theta) \tag{1.28}$$

$$\text{s.t.} \qquad \Pi = -\sum_{\theta' \in \Theta} \sum_{s=1}^{S} \pi_s \xi_{\theta'} \left(1 - \lambda_{\theta'}^{s}\right) r_{\theta'} y_{\theta'} = 0 \tag{1.29}$$

$$\sum_{\theta' \in \Theta} \xi_{\theta'} y_{\theta'} = 0 \tag{1.30}$$

Applying the equilibrium condition given in Proposition 1 ($\sum_{s=1}^{S} \pi_s \left(1 - \lambda_{\theta'}^{s}\right) r_{\theta'} = r_f$ for all $\theta' \in \Theta$) to the contracts offered to all other agents $\theta' \neq \theta$, we obtain:

$$\max_{(y_\theta, r_\theta), \xi_\theta} \qquad V_\theta (y_\theta, r_\theta) \tag{1.31}$$

$$\text{s.t.} \qquad \sum_{\theta' \in \Theta} \sum_{s=1}^{S} \pi_s \left(1 - \lambda_{\theta'}^{s}\right) r_{\theta'} = 0 \tag{1.32}$$

We now solve this problem using the Lagrangian method. We first set up the Lagrangian function:

$$\mathbb{L}_\theta = V_\theta (y_\theta, r_\theta) - \gamma \sum_{\theta \in \Theta} \sum_{s=1}^{S} \pi_s \left(1 - \lambda_{\theta}^{s}\right) r_\theta \tag{1.33}$$

where γ is the Lagrangian multiplier for the constraint of the problem. The solution must then satisfy the first-order conditions:

$$\frac{\partial \mathbb{L}_\theta}{\partial y_\theta} = 0 \tag{1.34}$$

$$\frac{\partial \mathbb{L}_\theta}{\partial r_\theta} = 0 \tag{1.35}$$

$$\frac{\partial \mathbb{L}_\theta}{\partial \gamma} = 0 \tag{1.36}$$

From equation (1.34), we get the following:

$$\frac{\partial \mathbb{L}_\theta}{\partial y_\theta} = \frac{\partial V_\theta\left(y_\theta^e, r_\theta\right)}{\partial y_\theta} + \gamma \sum_{s=1}^{S} \pi_s r_\theta \frac{\partial \lambda_\theta^s}{\partial y_\theta} = 0 \tag{1.37}$$

$$\Rightarrow \quad \frac{\partial V_\theta\left(y_\theta^e, r_\theta\right)}{\partial y_\theta} = -\gamma \sum_{s=1}^{S} \pi_s r_\theta \frac{\partial \lambda_\theta^s}{\partial y_\theta} \leq 0 \tag{1.38}$$

The concluding inequality follows because $\gamma < 0$[10] and $\frac{\partial \lambda_\theta^s}{\partial y_\theta} \leq 0$. When y_θ is positive and the agent is a saver, there is no default and so $\frac{\partial \lambda_\theta^s}{\partial y_\theta} = 0$, which implies that $\frac{\partial V\left(y_\theta^e, r_\theta\right)}{\partial y_\theta} = 0$ and savers trade the optimum amount at the given interest rates. However, when y_θ is negative and the agent is a borrower, $\frac{\partial \lambda_\theta^s}{\partial y_\theta} < 0$[11]. Overall therefore, $\frac{\partial V\left(y_\theta^e, r_\theta\right)}{\partial y_\theta} < 0$ and borrowing limits which are binding arise. ■

1.7.3 Excluded agents

Proof of Proposition 3. Consider the quantity $y_\theta\left(r_d; p\right)$ offered to agent θ as part of the equilibrium contract, which is an increasing function of r_d, the risk-free interest rate. For low enough values of r_d, $y_\theta\left(r_d\right)$ will be negative and will approach zero as the interest rate is raised to $\underline{r}_d$:

$$y_\theta\left(\underline{r}_d\right) = 0 \tag{1.39}$$

At this point, default in each state is zero, since borrowing is equal to zero.

Now consider a world without default. In this world, quantity traded by agent θ at any given interest rate r_d is given by $Y_\theta\left(r_d\right)$. For high enough values of r_d, $Y_\theta\left(r_d\right)$ will be positive and will approach zero as the interest rate is lowered to $\widehat{r}_d$:

$$Y_\theta\left(\widehat{r}_d\right) = 0 \tag{1.40}$$

[10] Maximising the consumer's utility subject to achieving a strictly positive profit will yield a utility which is less than maximising subject to a profit of zero.

[11] In other words, more borrowing (y_θ more negative) will result in more default. See the appendix section above for the proof.

We now make the claim that the interest rate $\underline{r}_d$ is less than $\widehat{r}_d$:

$$\underline{r}_d < \widehat{r}_d \tag{1.41}$$

To see this, note that, by Proposition 2:

$$\frac{\partial V_\theta \left(y_\theta \left(\underline{r}_d\right), \underline{r}_d\right)}{\partial y_\theta} < 0 \tag{1.42}$$

By the envelope theorem, keeping default fixed at zero, this implies:

$$\frac{\partial V_\theta \left(y_\theta \left(\underline{r}_d\right), \underline{r}_d\right)}{\partial y_\theta}|_{\lambda_\theta = 0} < 0 \tag{1.43}$$

In other words, if default is zero, the agent would like to borrow at an interest rate of $\underline{r}_d$:

$$Y_\theta \left(\underline{r}_d\right) < 0 = y_\theta \left(\underline{r}_d\right) = Y_\theta \left(\widehat{r}_d\right) \tag{1.44}$$

Since $Y_\theta \left(r_d\right)$ is increasing in r_d, this implies that $\underline{r}_d < \widehat{r}_d$.

Next, for all interest rates $r_d \in \left(\underline{r}_d, \widehat{r}_d\right)$, the agent prefers not to trade the asset than to save, and this follows from the strict concavity of U_θ:

$$y_\theta > 0 \Rightarrow V_\theta \left(0, r_d\right) > V_\theta \left(y_\theta, r_d\right) \quad \forall r_d \in \left(\underline{r}_d, \widehat{r}_d\right) \tag{1.45}$$

Since $y_\theta \left(r_d\right)$ is increasing in r_d, combining Equations (1.44)-(1.45) implies that the quantity offered to agents for all interest rates between $\underline{r}_d$ and $\widehat{r}_d$ will be zero:

$$y_\theta \left(r_d\right) = 0 \quad \forall r_d \in \left(\underline{r}_d, \widehat{r}_d\right) \tag{1.46}$$

Furthermore, keeping the interest rate fixed, the agent would like to borrow but is unable to. This follows from the fact that $Y_\theta \left(r_d\right) < 0$ for all such interest rates; keeping default at zero, the agent would like to borrow, and would therefore do so if we allow default to vary optimally.

■

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Chapter 2

Endogenous default penalties and their implications for welfare and borrowing trends

Abstract

We develop a model of unsecured borrowing with default in a world with many open economies, and investigate how default penalties are set by each region's government at equilibrium. We first show that the resulting equilibrium is not welfare-maximising; by cooperating, welfare can increase in every region. Numerical experiments are then conducted to examine reasons behind upwards trends in borrowing and default over the years, coupled with falling default penalties. These include the level of incomes, the degree of income uncertainty, and costs to financial intermediation. The experiments are at first conducted by keeping default penalties fixed, and are then repeated by allowing penalties to vary endogenously. We find that these sets of results may contradict each other. That is, when default penalties are allowed to vary endogenously, the effects on borrowing and default may be reversed.

2.1 Introduction

Default can play an important role as a form of insurance against uninsurable risks in the market for unsecured consumer credit. As Zame (1994) has shown, allowing for some default may actually enhance welfare by allowing agents to enter into agreements which they can honour with a high probability but fail to honour with a low probability; without the option to default, agents would only be able to enter into agreements which they could honour in any conceivable situation, reducing the scope for borrowing and consumption smoothing over time.

Over recent years, default rates and credit levels in the US and the UK have been on the rise. For example, the number of personal bankruptcy filings per year in the US have increased five-fold between 1980 and 2003, from 300,000 to more than 1,500,000; this accounts for nearly 1.5% of US households and an annual rate of 7% - $120 billion of unsecured debt (White, 2005)[1]. Trends are similar in the UK, where unsecured debt has increased from £67 billion to £168 billion in real terms over the last decade (DTI[2], 2003), and personal insolvencies have increased from around 2,300 per quarter in 1988 to over 14,000 today (BoE[3], 2004). Other European countries have experienced similar trends in credit levels, but due to the lack of formal personal insolvency procedures, data on default is scarce[4].

There has been some empirical work into why these trends have been observed, but that is fairly limited and mostly related to bankruptcy legislation. For example, Agarwal et al. (2003) examine the effect of exemption laws on default rates and find that more lenient laws increase the probability of default. Gross and Souleles (2002) estimate that, ceteris paribus, a household in 1997 was nearly 1% more likely to declare bankruptcy than in 1995 and attribute this to a fall in the 'social stigma' of declaring bankruptcy.

Default penalties have indeed been falling, at least in the US. For example, Table 2-1 records the mean level of exemptions under Chapter 7 bankruptcy filings in the US, as documented by Grant (2003). These specify the value of assets that may be retained in bankruptcy; all assets worth less than the specified exemption level can be kept by the filer, while all assets worth

[1] In fact, this number is misleadingly low. Since households who file for bankruptcy are not allowed to file again for 5 years, this translates into more thant 1.5% *distinct* households filing each year over a 5-year period. Summing up, this means that over 6% of US households will file for bankruptcy in any 5-year period.

[2] Department of Trade and Industry, UK.

[3] Bank of England.

[4] For example, Germany only introduced such legislation in 2001.

more than the exemption level are sold and the difference is paid to creditors. All else equal, the higher the level of these exemptions, the more lenient the bankruptcy law. As the table shows, these exemptions levels have, on average, increased.

Table 2-1: Exemption levels under Chapter 7 bankruptcy

Year	Mean
Homestead	
1988	22,576
1993	31,262
1999	42,588
Other assets	
1988	9,521
1993	11,424
1999	15,873
'Tools of trade'	
1988	2,225
1993	2,438
1999	3,128

Source: Grant (2003)

However, while it might be true to say that default penalties have become more lenient, causing higher default rates, this then leads to the question of why this is happening - why have penalties been cut even though bankruptcies are on the rise?

The aim of this paper is to take a step towards answering this question by providing a framework where default penalties are determined endogenously. We develop a model of general equilibrium which consists of many open economies, where funds can be transferred from one region to another without cost and where each government is free to set its own default penalties. We examine how these penalties arise endogenously at equilibrium, and show that the result does not maximise welfare - by cooperating in setting default penalties, welfare can be increased.

This suggests there may be scope for improving welfare in today's economies, where default penalties are set non-cooperatively.

A novel feature of this model is that, along with default punishments, borrowing limits are derived endogenously, as are interest rates and default rates. Other papers in the related literature, such as Dubey, Geanakoplos and Shubik (2005) and Athreya (2002) have exogenously fixed borrowing constraints. Some papers on the other hand have endogenous borrowing constraints but fix some other variable. For example, Chatterjee et al (2002) have a fixed rate of interest at which banks can obtain funds, whereas other papers such as Kehoe and Levine (1993) or Zhang (1997) have an endogenous borrowing limit, but that is set at a point where no default occurs, so such models could not be used for the purposes of this paper.

The paper then proceeds with a series of numerical experiments on the model. We assess the impact of several types of exogenous shocks in an economy:

1. a rise in incomes;
2. an increase in income uncertainty;
3. an increase in the frequency of expenditure shocks;
4. a fall in the costs of financial intermediation;
5. a fall in interest rate spreads, perhaps arising from increasing competition.

In each case, two types of experiments are performed. Initially, the effects of each shock are examined when the punishment level remains constant. We might expect this to apply in the short run, given that new legislation is costly and time consuming for governments to implement. In the long run however, we would expect legislation to be changed to better reflect the new economic environment[5]. Therefore, to capture the long-run effects of such shocks, we

[5]Default penalties vary widely across different economies. For example, default punishments are generally perceived to be much more lenient in the US than in some European countries such as Germany or Italy (see, for example, Livshits et al. (2001)). In addition, default penalties can vary widely within the US itself, as a result of individual states being allowed to set their own bankruptcy laws. Table 2 in the Appendix documents the home exemption levels (HE) and the personal exemption levels (PE) which are set across states; the exemption levels are low in Delaware (DE) at \$0 and \$5,000 respectively, slightly higher in New Jersey(NJ) at \$15,000 and \$10,700 and much higher in Texas (TX) at \$1,000,000 and \$30,000. Whether this variation can be explained by economic factors is a subject of further research.

repeat the experiments, but allow the punishment level to vary endogenously.

We find the results quite striking - by allowing governments to react to exogenous shocks, we get conflicting results to when punishments are held fixed. This occurs because the change in the punishment for default which occurs in the long run often puts opposing pressures on the default rate.

For example, one of the puzzles in the literature is that default rates have been increasing, despite rising incomes. We find that higher incomes lead to more borrowing and less default when penalties are held fixed, but when penalties are determined endogenously so that they vary within the model as its economic parameters change, the opposite is true and default rates go up. We also find that increasing the probability of expense shocks occurring leads to lower borrowing and lower default rates when penalties are fixed, but in the long run they lead to less borrowing and *higher* default rates. A similar pattern is observed with other experiments such as lower intermediation costs and lower interest rate spreads, although in the case of higher income uncertainty, both cases of exogenous and endogenous penalties lead to qualitatively similar results.

The chapter proceeds as follows. Section 2.2 presents the model (a more detailed description can be found in chapter 1 of this thesis), and shows how a non-cooperative equilibrium is not welfare-maximising. In section 2.3, the numerical experiments are carried out, and section 2.4 concludes.

2.2 Description of the model

2.2.1 The world

Our world consists of a number of open economies indexed by $\kappa \in K = \{1, ..., k\}$, where by open we mean that a bank can obtain funds from one economy and lend them to agents in another without incurring any costs. Each region's population consists of different types of agents indexed by $\theta \in \Theta$. There are two time periods in this world, $t = 0, 1$, and idiosyncratic uncertainty in the second time period which is represented by n states of nature. The probability of each state $s = 1, ..., n$ occurring is given by $\alpha_{s\kappa}$. There is one consumption good in each state, and financial markets are incomplete[6].

2.2.2 Agent types

Agents live for two time periods. An agent's type is fully described by a vector $\eta = (\theta, \kappa) \in \Theta \times K$; θ denotes preferences over consumption and endowments in each state, represented by a utility function $u_\theta : \mathbb{R}_+ \to \mathbb{R}$ and endowment vector $\omega_\theta \in \mathbb{R}_+^{n+1}$, while κ denotes the region they are located in. The utility function u_θ has the following properties:

Assumption 1 The function $u_\theta : \mathbb{R}_+ \to \mathbb{R}$ is continuous, strictly concave and strictly increasing on $\mathbb{R}_+$.

Furthermore, agents have rational expectations concerning future prices and rates of return. They can smooth consumption over time through the use of a bank; they can either save by depositing money at a bank and earn interest, or they can borrow at a certain interest rate and repay the bank in the second time period. For their services, banks can charge borrowers a higher interest rate than they offer savers. There is perfect competition in the banking sector, and banks can observe agent types. Finally, the population mass of type η agents is given by M_η.

2.2.3 The sequence of events

The sequence of events takes place as follows. The first time period is divided into three stages; stages zero, one and two. In stage zero, the government of every region κ sets the 'punishment level' for default, p_η, faced by consumer of type $\eta = (\theta, \kappa)$[7]. In stage one, each bank offers a contract $(y_\eta, r_\eta) \in \mathbb{R}^2$ to each consumer of type η. In stage two, consumers, who are perfectly informed about all the contracts available to them, then face a binary choice; they can either accept or reject each bank's offer[8]. Each consumer will then trade the contract which offers them the highest utility, subject to that being higher than their reservation utility; for example,

[6] See Chapter 1 for a justification of this assumption.

[7] Note that this is type dependent, implicitly assuming that governments can observe agent types. However this need not be the case; punishment levels could instead be indexed by κ so that each region had a unique punishment level which applied to all its agents - this would not have any significant impact on the analysis. A full description of the punishment level is given below.

[8] Without loss of generality, we can assume that each consumer only trades one contract.

this could be consuming just their endowments in each state, or it could be the utility achieved from using some available storage technology. For simplicity of notation, we assume that there is just one bank which acts competitively in every economy by assuming zero barriers to entry in every region.

In the second time period, all uncertainty is resolved, and the endowment of each agent is revealed to them. If they have traded a short contract (i.e. borrowed money - $y_\eta < 0$), then they decide on their optimal default rate $\lambda_\eta^s \in [0,1]$ in that state. By defaulting, they can increase their consumption, but default bears a punishment $P_\eta\left(\lambda_\eta^s, y_\eta, r_\eta\right)$; this punishment is the disutility caused by the seizing of assets and future income, exclusion from the credit market for defaulting households[9], social stigma[10] and legal fees[11]. This punishment function is of the form $P_\eta\left(\lambda_\eta^s, y_\eta, r_\eta\right) = p_\eta f\left(\lambda_\eta^s, y_\eta, r_\eta\right)$, where p_η is the constant which we call the 'punishment level', and f (the 'punishment technology') has the following properties:

Assumption 2 The function $f : [0 \times 1] \times \mathbb{R}^2 \to \mathbb{R}$ is continuous and convex on $[0,1] \times \mathbb{R}^2$.

Assumption 3 f is increasing in $\lambda_\eta^s : \frac{\partial f}{\partial \lambda_\eta^s} > 0$. In other words, the higher the default rate on a given loan, the higher the punishment.

Assumption 4 f is decreasing in $y_\eta : \frac{\partial f}{\partial y_\eta} < 0$. For example, a default rate of 20% on a liability of \$1,000 is punished more than defaulting 20% on a liability of \$1.

Assumption 5 f is increasing in $r_\eta : \frac{\partial f}{\partial r_\eta} > 0$ (see example above).

In other words, f is convex and strictly increasing in λ_η^s and in the amount owed. Finally, we assume that the bank cannot default, so if agents have traded a long contract, they receive the return promised to them.

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[9] Musto (1999).

[10] Fay, Hurst and White (2002).

[11] Athreya and Simpson (2005).

2.2.4 The consumer's 'payoff function' and budget set

A consumer's payoff function $U : \mathbb{R}_{+}^{n+1} \times [0,1]^{n} \rightarrow \mathbb{R}$ ranks any consumption-default vector $(x_{\eta}, \lambda_{\eta})$ in terms of preference, where $x_{\eta} = \left(x_{\eta}^{0}, ..., x_{\eta}^{n}\right)$ is a vector of consumption in each state $s = 0, ..., n$ over both time periods and $\lambda_{\eta} = \left(\lambda_{\eta}^{1}, ..., \lambda_{\eta}^{n}\right)$ is a vector of default rates in each state $s = 1, ..., n$ in the second time period. In other words, the payoff function is the expected utility of a consumer once default has been taken into account, and is given as:

$$U_{\eta}\left(x_{\eta}, \lambda_{\eta}; p_{\eta}, y_{\eta}, r_{\eta}\right) = u_{\theta}\left(x_{\eta}^{0}\right) + \sum_{s=1}^{n} \alpha_{s}\left(u_{\theta}\left(x_{\eta}^{s}\right) - p_{\eta} f\left(\lambda_{\eta}^{s}, y_{\eta}, r_{\eta}\right)\right) \tag{2.1}$$

Given assumptions on u_{θ} and f, it is straightforward to prove the following proposition:

Proposition 4 *$U_{\eta}\left(x_{\eta}, \lambda_{\eta}\right)$ is a strictly concave function.*

Proof.

$$\begin{aligned} & U_{\eta}\left[\gamma x + (1-\gamma)\, x_{\eta}', \gamma \lambda_{\eta} + (1-\gamma)\, \lambda_{h}'\right] & (2.2) \\ = \; & u_{\theta}\left[\gamma x_{\eta}^{0} + (1-\gamma)\, x_{\eta}^{\prime 0}\right] & (2.3) \\ & + \sum_{s=1}^{n} \alpha_{s}\left(u_{\theta}\left[\gamma x_{\eta}^{s} + (1-\gamma)\, x_{\eta}^{\prime s}\right] - p_{\eta} f\left[\gamma \lambda_{\eta}^{s} + (1-\gamma)\, \lambda_{\eta}^{\prime s}, y_{\eta}, r_{\eta}\right]\right) & (2.4) \end{aligned}$$

By Assumption 1, u_{θ} is a strictly concave function, so:

$$u_{\theta}\left[\gamma x_{\eta}^{s} + (1-\gamma)\, x_{\eta}^{\prime s}\right] > \gamma u_{\theta}\left(x_{\eta}^{s}\right) + (1-\gamma)\; u_{\theta}\left(x_{\eta}^{\prime s}\right) \;\; \forall s \in \{0, 1, ..., n\} \tag{2.5}$$

Furthermore, by condition 2, f is convex, so:

$$f\left[\gamma \lambda_{\eta}^{s} + (1-\gamma)\, \lambda_{\eta}^{\prime s}, y_{\eta}, r_{\eta}\right] \leq \gamma f\left(\lambda_{\eta}^{s}, y_{\eta}, r_{\eta}\right) + (1-\gamma)\; f\left(\lambda_{\eta}^{\prime s}, y_{\eta}, r_{\eta}\right) \tag{2.6}$$

Combining (2.5) and (2.6) together implies that:

$$U_{\eta}\left[\gamma x + (1-\gamma)\, x_{\eta}', \gamma \lambda_{\eta} + (1-\gamma)\, \lambda_{h}'\right] < \gamma U_{\eta}\left(x_{\eta}, \lambda_{\eta}\right) + (1-\gamma)\, U_{\eta}\left(x_{\eta}', \lambda_{\eta}'\right) \tag{2.7}$$

and this completes the proof. ■

It is also clear that U_η is continuous, as it is the sum of two continuous functions. Next, a consumer's budget correspondence $B_\eta : \mathbb{R}^{n+1} \times \mathbb{R}^2 :\longrightarrow P\left(\mathbb{R}^{n+1} \times \mathbb{R}^n\right)$[12] maps any given parameters $\{\omega_\eta, y_\eta, r_\eta\}$ onto the set of all feasible consumption-default vectors and is given as:

$$\begin{aligned} B_\eta(\omega_\eta, y_\eta, r_\eta) &= (x_\eta, \lambda_\eta) \in \mathbb{R}_+^{n+1} \times \mathbb{R}^n : && (2.8)\\ & x_\eta^0 \leq \omega_\eta^0 - y_\eta; && (2.9)\\ & x_\eta^s \leq \omega_\eta^s + \left(1 - \lambda_\eta^s\right) r_\eta y_\eta, \ \forall s \in \{1, ..., n\}; && (2.10)\\ & 0 \leq \lambda_\eta^s \leq 1, \ \forall s \in \{1, ..., n\} && (2.11) \end{aligned}$$

Proposition 5 *The budget correspondence B_η is convex-valued. In other words, $B_\eta(\omega_\eta, y_\eta, r_\eta)$ is a convex set for all $(\omega_\eta, y_\eta, r_\eta)$.*

Proof. Assume that $(x_\eta, \lambda_\eta) \in B_\eta$ and $\left(x'_\eta, \lambda'_\eta\right) \in B_\eta$. Then $\gamma(x_\eta, \lambda_\eta) + (1-\gamma)\left(x'_\eta, \lambda'_\eta\right) \in B_\eta \forall \gamma \in (0, 1)$, as we will now show. By definition, this implies that B_η is a convex set. For any $\gamma \in (0, 1)$:

$$x_\eta^0 \leq \omega_\eta^0 - y_\eta \quad (2.12)$$

$$x_\eta'^0 \leq \omega_\eta^0 - y_\eta \quad (2.13)$$

$$\Rightarrow \gamma x_\eta^0 + (1-\gamma) x_\eta'^0 \leq \omega_\eta^0 - y_\eta \quad (2.14)$$

$$x_\eta^s \leq \omega_\eta^s + \left(1 - \lambda_\eta^s\right) r_\eta y_\eta \quad (2.15)$$

$$x_\eta'^s \leq \omega_\eta^s + \left(1 - \lambda_\eta'^s\right) r_\eta y_\eta \quad (2.16)$$

$$\begin{aligned} \Rightarrow \gamma x_\eta^s + (1-\gamma) x_\eta'^s \leq & \ \omega_\eta^s + \left[\gamma\left(1 - \lambda_\eta^s\right) + (1-\gamma)\left(1 - \lambda_\eta^{s\prime}\right)\right] r_\eta y_\eta && (2.17)\\ = & \ \omega_\eta^s + \left[\gamma - \gamma\lambda_\eta^s + 1 - \lambda_\eta'^s - \gamma + \gamma\lambda_\eta'^s\right] r_\eta y_\eta && (2.18)\\ = & \ \omega_\eta^s + \left(1 - \left[\gamma\lambda_\eta^s + (1-\gamma)\lambda_\eta'^s\right]\right) r_\eta y_\eta && (2.19) \end{aligned}$$

[12] Here, the operator P is the power set. In other words, $P\left(\mathbb{R}^{n+1} \times \mathbb{R}^n\right)$ is the set of all subsets in $\mathbb{R}^{n+1} \times \mathbb{R}^n$.

$$0 \leq \lambda_\eta^s \leq 1 \tag{2.20}$$

$$0 \leq \lambda_\eta'^s \leq 1 \tag{2.21}$$

$$\Rightarrow 0 \leq \gamma\lambda_\eta^s + (1-\gamma)\,\lambda_\eta'^s \leq 1 \tag{2.22}$$

By equations (2.14), (2.19) and (2.22), it follows that $\gamma\,(x_\eta, \lambda_\eta) + (1-\gamma)\,(x'_\eta, \lambda'_\eta) \in B_\eta \forall \gamma \in (0,1)$. This completes the proof. ■

Proposition 6 *The budget correspondence B_η is compact valued. In other words, $B_\eta\,(\omega_\eta, y_\eta, r_\eta)$ is a compact set for all $(\omega_\eta, y_\eta, r_\eta)$.*

Proof. By definition, any given set $B_\eta\,(\omega_\eta, y_\eta, r_\eta)$ is the intersection of the following three sets:

$$(x_\eta, \lambda_\eta) \in \mathbb{R}_+^{n+1} \times \mathbb{R}^n : x_\eta^0 \leq \omega_\eta^0 - y_\eta \tag{2.23}$$

$$(x_\eta, \lambda_\eta) \in \mathbb{R}_+^{n+1} \times \mathbb{R}^n : x_\eta^s \leq \omega_\eta^s + \left(1-\lambda_\eta^s\right) r_\eta y_\eta, \ \forall s \in \{1, ..., n\} \tag{2.24}$$

$$(x_\eta, \lambda_\eta) \in \mathbb{R}_+^{n+1} \times \mathbb{R}^n : 0 \leq \lambda_\eta^s \leq 1 \tag{2.25}$$

It is straightforward to see that, for any given y_η and r_η, each of these sets is closed and bounded and therefore compact. From equation (2.23), x_η^0 is bounded below by zero and bounded above by $\omega_\eta^0 - y_\eta$. From equation (2.24), each x_η^s is bounded below by zero and bounded above by $\omega_\eta^s + \left(1-\lambda_\eta^s\right) r_\eta y_\eta$, which is finite for any given λ_η^s. Finally, from equation (2.25), each λ_η^s is clearly bounded by zero and one. Secondly, consider the complements of each of the thee sets. We will show that these are open, so that each of the three sets is closed. The complement equation (2.23), is the set $x_\eta^0 \in (-\infty, 0) \cup \left(\omega_\eta^0 - y_\eta, \infty\right)$. This is the union of two open intervals, or two open sets, and hence is open itself. It therefore follows that the set in (2.23) is itself closed. Applying similar logic, it follows that the remaining two sets (2.24) and (2.25) are closed. Any real-valued set which is bounded and closed is compact, so each of the three sets is compact. Finally, the intersection of compact sets is itself compact, from which it follows that $B_\eta\,(\omega_\eta, y_\eta, r_\eta)$ is a compact set for all $(\omega_\eta, y_\eta, r_\eta)$, hence B_η is compact-valued. ■

2.2.5 The second time period

In the second time period, each agent of type η holds a contract (y_η, r_η). Each individual's state of nature s is then revealed, and agents choose their optimal consumption-default vector $(x_\eta^s, \lambda_\eta^s)$ in that state. Due to monotonicity in preferences, for each state $s = 1, ..., n$, the optimal consumption for any given λ_η^s is given by $x_\eta^s = \omega_\eta^s + (1 - \lambda_\eta^s) r_\eta y_\eta$. Their optimal default decision $\lambda_\eta^s : \mathbb{R}^2 \to [0,1]$ is then given by:

$$\lambda_\eta^s (y_\eta, r_\eta; p_\eta) \in \arg \max_{\lambda_\eta^s \in [0,1]} \left(u_\eta^s (x_\eta^s) - p_\eta f(\lambda_\eta^s)\right) \tag{2.26}$$

The decisions of any individual have no effect on bank profits.

Due to the law of large numbers, the proportion of agents of type η who are located in state s is given by the probability α_s of that state occurring. The bank then has an income stream of:

$$\Pi = - \sum_{\eta \in \Theta \times K} \sum_{s=1}^{n} \alpha_s \xi_\eta M_\eta (1 - \lambda_\eta^s) r_\eta y_\eta \tag{2.27}$$

where M_η is the population mass of s-type agents, and ξ_η is the fraction of η-type agents the bank trades contracts with.

2.2.6 The first time period

Stage 2 - Consumers

In the second stage, every consumer of type η has a choice of available contracts to trade with each bank[13]. We define the function $V(y_\eta, r_\eta; p_\eta)$ as the solution to the problem:

$$V(y_\eta, r_\eta; p_\eta) = \max_{(x_\eta, \lambda_\eta) \in B(\omega_\eta, y_\eta, r_\eta)} U(x_\eta, \lambda_\eta; p_\eta) \tag{2.28}$$

In other words, $V(y_\eta, r_\eta; p_\eta)$ is the maximum expected utility achievable by agent η if he trades the contract (y_η, r_η) and chooses his optimal consumption and default rate in each state.

Each agent has a reservation utility of $\underline{U}_\eta$, which may be the utility derived from not saving or borrowing and consuming his endowments, or it could for instance be the utility derived

[13] As we have already stated, in theory there are many banks offering contracts, although in our model, for reasons of notation and simplicity, there is only one bank acting as if it were in perfect competition with others.

from borrowing money from a relative. Then, each consumer will trade the contract which offers them the highest utility, subject to that being higher than (or equal to) the reservation utility $\underline{U}_\eta$.

Proposition 7 *The value function* $V(y_\eta, r_\eta; p_\eta)$ *is a continuous function.*

Proof. This follows from the Maximum Theorem and from the fact that U is a continuous and strictly concave function maximised over B_η, a continuous and convex-valued correspondence. ■

Stage 1 - The bank

The bank has to choose the contract (y_η, r_η) it offers consumers of type η, as well as the proportion ξ_η of such agents it offers a contract to, in order to maximise its profit Π, subject to a feasibility and a participation constraint. More specifically, the maximisation problem it faces is:

$$\max_{\{(y_\eta, r_\eta), \xi_\eta\}_{\eta\in\Theta\times K}} \quad \Pi = -\sum_{\eta\in\Theta\times K}\sum_{s=1}^{n} \alpha_s \xi_\eta M_\eta \left(1-\lambda_\eta^s\right) r_\eta y_\eta \tag{2.29}$$

$$\text{s.t.} \quad \sum_{\eta\in\Theta\times K} \xi_\eta M_\eta y_\eta \geq 0 \tag{2.30}$$

$$V(y_\eta, r_\eta, p_\eta) - \widetilde{U}_\eta \geq 0, \forall \eta \in \Theta\times K \tag{2.31}$$

The first constraint is the feasibility constraint, and says that the bank cannot lend out more money than it has collected in deposits. The second constraint is the participation constraint; it says that the contract which the bank offers consumers must yield a utility which is at least as high as that of the next best available option, $\widetilde{U}_\eta$. This could be their reservation utility $\underline{U}_\eta$, or it could be the utility on offer from a contract available by another bank. The solution to this problem, which we call z_η, is such that the contract (y_η, r_η) offered to each consumer will be the one that maximises their utility subject to the bank's profit from that contract being non-negative:

$$z_\eta(r_f; p_\eta) \in \arg\max_{(y_\eta, r_\eta)} V(y_\eta, r_\eta; r_f, p_\eta) \quad \forall \eta \in \Theta \times K \tag{2.32}$$

subject to:

$$(y_\eta, r_\eta) \in C_\eta(r_f) \tag{2.33}$$

where r_f is the risk-free interest rate. This is the interest rate offered to all savers[14]. Furthermore:

$$\begin{aligned} C_\eta(r_f) &= (y_\eta, r_\eta) \in \mathbb{R} \times \mathbb{R}_+ | & (2.34) \\ & \pi(y_\eta, r_\eta) = y_\eta r_f - \sum_{s=1}^{n} \alpha_s \left[1 - \lambda^*_{s\eta}(y_\eta, r_\eta)\right] r_\eta y_\eta \geq 0 & (2.35) \\ & y_\eta \leq \omega_{0\eta} & (2.36) \\ & V(y_\eta, r_\eta) \geq \underline{U}_\eta & (2.37) \end{aligned}$$

Briefly, given the risk-free rate of interest, the contracts offered to consumers must be feasible and must yield the highest possible utility, driving the bank's profits on each contract to zero. This will guarantee that there will be no incentive for another bank to enter the industry and capture the incumbent bank's market share[15]. The second constraint, $y_\eta \leq \omega^0_\eta$, is a feasibility constraint which simply states that an agent cannot lend more than their endowment in the first time period. The final constraint $V(y_\eta, r_\eta) \geq \underline{U}_\eta$ does not affect the solution to the maximisation problem but is useful in proving the existence of a solution to this problem, as we will see below. Finally, we assume for convention that $r_\eta = r_f$ if $y_\eta = 0$; without this, r_η would in theory be unbounded since any $r_\eta \in \mathbb{R}$ would satisfy $\pi(0, r_\eta) \geq 0$. We now prove that a solution to this problem exists for any $r_f > 0$.

Proposition 8 *The correspondence $C_\eta : \mathbb{R} \rightarrow P(\mathbb{R}^2)$ given in (2.34)-(2.37) is compact-valued (i.e. $C_\eta(r_f)$ is a compact set for each r_f).*

[14] By Proposition 1 in Chapter 1, all savers must save at the same risk-free rate of interest, and this we call r_f.
[15] A full description of the solution to the bank's optimisation problem is provided in chapter 1 of this thesis.

Proof. We will first show that y_η and r_η are both bounded. Consider r_η - this is bounded below by zero by definition. However, without the constraint in (2.37), r_η is not bounded from above[16]. This problem is eliminated by the constraint $V(y_\eta, r_\eta) \geq \underline{U}_\eta$. This constraint is in a sense redundant as the solution to the maximisation problem does not change with its inclusion[17]. Hence, if a solution exists with this constraint, it must exist and is identical without this constraint. However, it does place a bound on r_η which is required when proving that $C(r_f)$ has compact values. This occurs because as r_η is increased, there will come a point where each agent η will want to save and not borrow - in fact, borrowing will yield a utility which is strictly less than not trading and achieving $\underline{U}_\eta$. We call such an interest rate $\overline{r}_\eta$:

$$\exists \overline{r}_\eta \; V(y_\eta, r_\eta) < \underline{U}_\eta \; \forall r_\eta > \overline{r}_\eta \wedge \forall y_\eta < 0 \tag{2.38}$$

Furthermore, for $y_\eta = 0$, $r_\eta = 0$ by convention, while for positive y_η (i.e. for savers), the interest rate offered on savings, r_η, is bounded above by r_f, since $\pi(y_\eta, r_\eta)$ is negative for any higher r_η. Hence, r_η is bounded above and below:

$$r_\eta \in [0, \max\{r_f, \overline{r}_\eta\}] \tag{2.39}$$

Now consider y_η. For all $y_\eta < 0$:

$$\pi(y_\eta, r_\eta) = r_f y_\eta - \sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta y_\eta \geq 0 \tag{2.40}$$

$$\Leftrightarrow \quad \overline{\pi}(y_\eta, r_\eta) = \sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta - r_f \geq 0 \tag{2.41}$$

Hence, the set of points $\pi(y_\eta, r_\eta) \geq 0$ is equivalent to the set of points:

$$(y_\eta, r_\eta) \in \mathbb{R}^2 : \overline{\pi}(y_\eta, r_\eta) = \sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta - r_f \geq 0 \tag{2.42}$$

[16] To see this, consider any $r_\eta \geq r_f$. The bank's profit is given as $y_\eta r_f - \sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta y_\eta$, which, for negative y_η, is positive if $\sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta - r_f \geq 0$. For $y_\eta = 0$, default is zero in each state, so this term reduces to $r_\eta - r_f \geq 0$. Since λ_s is continuous in y_η, it follows that there exists a y_η close to zero for which this term is still positive. In other words, for any $r_\eta \geq r_f$, there exists a $y_\eta < 0$ for which the positive profit constraint holds. Hence, without the constraint $V(y_\eta, r_\eta) \geq \underline{U}_\eta$, r_η is unbounded.

[17] This is because the contract $(0, r_\eta)$ yields a utility $V(0, r_\eta) = \underline{U}_\eta \; \forall r_\eta \geq 0$.

Given that $\lambda_\eta^s(y_\eta, r_\eta)$ is decreasing in y_η (see chapter 1), $\overline{\pi}(y_\eta, r_\eta)$ is increasing in y_η. Furthermore, as $-y_\eta$ becomes arbitrarily large, $\lambda_\eta^s(y_\eta, r_\eta)$ approaches 1, so that $\overline{\pi}(y_\eta, r_\eta)$ becomes negative. Hence, for every r_η there exists a $y_\eta < 0$ such that $\overline{\pi}(y, r_\eta) < 0$ for all y smaller that y_η:

$$\begin{aligned} \forall \quad & r_\eta \exists y_\eta < 0 \, \overline{\pi}(y, r_\eta) < 0 \, \forall y < y_\eta & (2.43) \\ \Rightarrow \exists \quad & \overline{y}_\eta < 0 \, \overline{\pi}(y_\eta, r) \leq 0 \, \forall y_\eta \leq \overline{y}_\eta \wedge \forall r_\eta \in [0, \overline{r}_\eta] & (2.44) \end{aligned}$$

Hence, following this, y_η is bounded above and below:

$$y_\eta \in \left[\overline{y}_\eta, \omega_\eta^0\right] \tag{2.45}$$

Given that y_η and r_η are both bounded, it follows that $C_\eta(r_f)$ is a bounded set for any r_f[18].

We now prove that $C_\eta(r_f)$ is a closed set by showing that its complement $C_\eta(r_f)^c$ is open. The set $C_\eta(r_f)^c$ is given as:

$$\begin{aligned} C_\eta(r_f)^c \quad = \quad & (y_\eta, r_\eta) \in \mathbb{R} \times \mathbb{R}_+ : & (2.46) \\ & \pi(y_\eta, r_\eta) = y_\eta r_f - \sum_{s=1}^{n} \alpha_s \left[1 - \lambda_\eta^s(y_\eta, r_\eta)\right] r_\eta y_\eta < 0 & (2.47) \\ \text{and/or:} \quad & y_\eta > \omega_\eta^0 & (2.48) \\ \text{and/or:} \quad & V(y_\eta, r_\eta) < \underline{U}_\eta & (2.49) \end{aligned}$$

Consider any such point $(y_\eta, r_\eta) \in C_\eta(r_f)^c$. Then, either $\pi(y_\eta, r_\eta) < 0$ holds, or $y_\eta > \omega_\eta^0$, or $V(y_\eta, r_\eta) < \underline{U}_\eta$, or any combination of the three. By continuity, for any such point, there will exist an $\varepsilon > 0$ such that at least one of these constraints still holds for all points in the ε-neighbourhood of (y_η, r_η). Therefore, $C_\eta(r_f)^c$ is an open set, so $C_\eta(r_f)$ is closed for all r_f.

We have shown that $C_\eta(r_f)$ is a closed and bounded set in Euclidean space, and therefore it follows that it is compact. ■

Proposition 9 *$z_\eta(r_f; p_\eta)$ is an upper hemicontinuous correspondence.*

[18]This is easy to see - for any r_f there exists an open ball with an ε-neighbourhood of $\varepsilon > \max\left\{\overline{r}_\eta, \left|\overline{y}_\eta\right|, \omega_{0\eta}\right\}$ which contains $C_\eta(r_f)$.

Proof. This follows from the fact that $V(y, r)$ is a continuous function maximised over a compact-valued continuous correspondence. ■

Proposition 10 *The graph of z_η is closed.*

Proof. This follows from the fact that z_η is upper hemicontinuous. ■

Now, let an agent's demand correspondence for the asset, $\zeta_\eta(r_f)$, be the quantity of the asset traded by agent η. In other words, ζ_η is a projection of z_η onto y_η; while $z(r_f)$ returns the optimal contract correspondence, ζ_η just returns the optimal quantities. Finally, we define $\zeta(r_f)$ as the excess demand correspondence, or the difference between long and and short contracts (saving and borrowing):

$$\zeta(r_f) = \sum_\eta M_\eta \, \zeta_\eta(r_f; p_\eta) \tag{2.50}$$

Then ζ is well defined and the graph of ζ is closed, and the range of ζ is compact (Allen and Gale (2004)).

Equilibrium in the credit markets

To summarise, for any given punishment levels $\{p_\eta\}_{\eta\in\Theta\times K}$, an equilibrium in the credit markets is a vector $\mathcal{E} = \left(\left\{\left(x_\eta^e, \lambda_\eta^e\right), \left(\left(y_\eta^e, r_\eta^e\right), \xi_\eta^e\right)\right\}_{\eta\in\Theta\times K}\right)$ such that:

- The consumption-default vector $\left(x_\eta^e, \lambda_\eta^e\right)$ is a solution to the consumer's maximisation problem:

$$\max_{(x_\eta,\lambda_\eta)\in B(\omega_\eta, y_\eta, r_\eta)} U(x_\eta, \lambda_\eta; p_\eta) \tag{2.51}$$

- The vector $\left(\left(y_\eta^e, r_\eta^e\right), \xi_\eta^e\right)_{\eta\in\theta\times K}$ is the solution to the bank's maximisation problem:

$$\max_{\left\{(y_\eta, r_\eta), \xi_\eta\right\}_{\eta\in\Theta\times K}} \Pi = -\sum_{\eta\in\Theta\times K}\sum_{s=1}^{n}\alpha_s\xi_\eta M_\eta\left(1-\lambda_\eta^s\right)r_\eta y_\eta \tag{2.52}$$

subject to:

$$\sum_{\eta\in\Theta\times K}\xi_\eta M_\eta y_\eta \geq 0 \tag{2.53}$$

$$V\left(y_\eta, r_\eta, p_\eta\right) - \widetilde{U}_\eta \geq 0, \forall\eta\in\Theta\times K \tag{2.54}$$

Proposition 11 *An equilibrium exists for any given profile of punishment levels* $\{p_\eta\}_{\eta\in\Theta\times K}$.

Proof. Let Z denote a compact convex set containing the range of ζ and let z be any element of Z. Furthermore, let $R = [\underline{r}, \overline{r}]$, where $\underline{r}$ is an interest rate at which no agent wants to save, and $\overline{r}$ an interest rate at which all agents want to save. Let $\rho : Z \rightarrow P(R)$ be a correspondence which minimises the product of the excess demand correspondence and r_f:

$$\rho(z) = \arg\min_{r_f\in R} z\ r_f \tag{2.55}$$

By the Berge Theorem, $\rho(z)$ minimises $z\ r_f$, a continuous function concave function in r_f for each z, over R, a convex set, so $\rho(z)$ is an upper hemicontinuous, convex-valued correspondence.

We now define a grand correspondence $\Phi : Z\times R \rightarrow P(Z\times R)$ as:

$$\Phi(z, r_f) = co\ \zeta(r_f)\times\rho(z) \tag{2.56}$$

where $co\ \zeta(r_f)$ denotes the convex hull of $\zeta(r_f)$. Then, Φ is an upper hemicontinuous correspondence with nonempty, compact, and convex values, and maps $Z\times R$, a compact and convex set, onto itself. It therefore satisfies the conditions of the Kakutani fixed point theorem and thus has a fixed point $\left(z^*, r_f^*\right)\in\Phi\left(z^*, r_f^*\right)$.

Next, we claim that any fixed point is necessarily an equilibrium. An equilibrium occurs when, at a given risk-free rate of interest r_f, $z = 0$, or borrowing equals saving. If $z > 0$, then $\rho(z) = \underline{r}$. But $\underline{r}$ is the interest rate at which no agent saves, so any $z' \in \zeta(\underline{r}) \leq 0$. Hence,

(z, r_f) cannot be a fixed point for any $z > 0$. By the same reasoning, (z, r_f) cannot be a fixed point for any $z < 0$. If $z < 0$, then $\rho(z) = \overline{r}$. However, $\overline{r}$ is the interest rate at which no agent borrows, so any $z' \in \zeta(\overline{r}) \geq 0$, so (z, r_f) cannot be a fixed point for any $z < 0$. Therefore, any fixed point must have $z = 0$ and must therefore be an equilibrium. ■

Stage zero - Governments

The question now arises of how governments reach decisions about their chosen punishment level at stage zero of the first time period and what the equilibrium punishment levels are. To answer this, we model stage zero as a game $\{\kappa, A_\kappa, W_\kappa\}_{\kappa \in K}$ of simultaneous moves, where κ denotes the government of each region κ (the players of this game), A_κ denotes their action set, and $W_\kappa : \{A_\kappa\}_{\kappa \in K} \to \mathbb{R}$ is the welfare arising in economy κ given the action of each government in every economy - this is each government's payoff.

Each government's action a_κ is the punishment level p_η it sets for each possible consumer type $\eta = (\theta, \kappa)$ in that economy:

$$a_\kappa = \left\{p_{(\theta,\kappa)}\right\}_{\theta \in \Theta} \tag{2.57}$$

Hence the action set A_κ is given as $A_\kappa = \mathbb{R}^{\#\Theta}$.[19] The welfare function W_κ is a function of the equilibrium utilities of each type of agent in the region, which in turn fully depend on the punishments chosen in each economy[20] (or in other words the on action profile $(a_\kappa)_{\kappa \in K}$), hence W_κ is itself a function of $(a_\kappa)_{\kappa \in K}$.

We now define a government's best response correspondence, $b_\kappa(a_{-\kappa})$ as the set of actions α_κ which maximise its welfare W_κ given the action profile of all other governments excluding κ, $\alpha_{-\kappa}$:

$$b_\kappa(a_{-\kappa}) = a_\kappa \in A_\kappa | \; \alpha_\kappa \in \arg\max_{\alpha_\kappa} \quad W_\kappa(\alpha_\kappa, \alpha_{-\kappa}) \tag{2.58}$$

Due to our assumption that every region is small, any changes in a region's punishment levels which affect borrowing in that region have a negligent effect on the total amount of borrowing in the world economy. Hence, the world interest rate on savings will not be affected.

[19] $\#\Theta$ denotes the cardinality of (or number of elements in) the set Θ.

[20] This is because the penalties chosen in other economies affect the equilibrium amount of borrowing and saving and hence affect the equilibrium r_f. This in turn affects the optimal punishment level chosen in the economy under consideration.

As an implication, the government in each region can only affect the welfare of borrowers in that region by setting default penalties; the welfare of savers will remain fixed, since changes in the punishment level in one region do not affect world interest rates and therefore do not affect the interest rate at which agents can save. Therefore, a government maximises its welfare by choosing the punishment level which maximises the equilibrium utility of borrowers in that region at the given world interest rates[21].

Given the action profile $(\alpha)_{\kappa \in K}$, the equilibrium utility offered to agents of type η is $V\left(y_{\eta}^{*}, r_{\eta}^{*}; p_{\eta}\right)$[22]. Therefore, a government's best response correspondence in 2.58 is equivalent to:

$$b_{\kappa}\left(a_{-\kappa}\right) = a_{\kappa} \in A_{\kappa} | \; p_{\eta} \in \arg\max_{p_{\eta}} \quad V_{\eta}\left(y_{\eta}^{*}, r_{\eta}^{*}; p_{\eta}\right), \quad \text{given } \alpha_{-\kappa} \tag{2.59}$$

where $\eta = (\theta, \kappa)$, $\forall \theta \in \Theta$.

Equilibrium punishment levels We use the Nash equilibrium concept to solve the game at stage zero. Equilibrium is therefore an action profile $(a_{\kappa})_{\kappa \in K} = (a_1, ..., a_k)$, such that:

$$a_{\kappa} \in b_{\kappa}\left(a_{-\kappa}\right), \quad \forall \kappa \in K \tag{2.60}$$

In other words, given the punishment in every other economy, no government has an incentive to deviate from its chosen action p_{κ}.

2.2.7 Welfare implications of a non-cooperative equilibrium

In this section, we focus on the welfare implications of such a non-cooperative equilibrium. This is representative of what we observe in reality, where different countries, and even individual states within the US, set their own bankruptcy laws. We ask whether this results in a first-best outcome, and show that the resulting equilibrium is not welfare-maximising; under full cooperation and commitment, welfare can increase in every region.

[21] This is not necessary but simplifies numerical calculations.

[22] Where y_{η}^{*} and r_{η}^{*} are the solution to the bank's maximisation problem, which is: $\max_{(y_{\eta}, r_{\eta})} V\left(y_{\eta}, r_{\eta}; p_{\eta}\right)$, subject to $\Pi = 0$ and $\Sigma_{\eta \in \Theta \times K}(\xi_{\eta} M_{\eta} y_{\eta}) = 0$, given above.

Proposition 12 *Let the government of each region set default penalties non-cooperatively. Then, the resulting Nash equilibrium is not, generically, welfare maximising.*

Proof. We prove this by a numerical example. Consider our benchmark world, which we will now outline. The benchmark model consists of a large number κ of identical economies. Idiosyncratic uncertainty is represented by three states of nature, $s = 1, 2, 3$, and the probabilities of each state occurring are $\alpha_1 = 0.45$, $\alpha_2 = 0.45$ and $\alpha_3 = 0.10$. Agents are given quadratic utility functions, and so the utility of agent of type η from consuming $x_\eta^s \in [0, 1]$ in state s is given by $u_\eta^s\left(x_\eta^s\right) = x_\eta^s - \frac{1}{2}\left(x_\eta^s\right)^2$. The punishment function P is specified as $P\left(\lambda_\eta^s, y_\eta, r_\eta\right) = -pr_\eta y_\eta \left(\lambda_\eta^s\right)^2$[23], where p is the punishment level and is varied. Therefore, the payoff function of each agent is given as:

$$U_\eta\left(x_\eta, \lambda_\eta\right) = x_\eta^0 - \frac{1}{2}\left(x_\eta^0\right)^2 + \sum_{s=1}^{n} \alpha_s \left(x_\eta^s - \frac{1}{2}\left(x_\eta^s\right)^2 + pr_\eta y_\eta \left(\lambda_\eta^s\right)^2\right)$$

Endowment vectors $\omega_\eta = \left(\omega_{0\eta}, \omega_{1\eta}, \omega_{2\eta}, \omega_{3\eta}\right)$ are given as $\left(\frac{6}{10}, \frac{4}{10}, \frac{4}{10}, \frac{4}{10}\right)$ for savers, while those of borrowers are $\left(\frac{4}{10}, \frac{7}{10}, \frac{5}{10}, \frac{1}{10}\right)$. State $s = 3$ is the borrower's 'bad' state, where default is always equal to 100%. The population size of depositors and borrowers in each economy are both equal to 1.

The equilibrium punishment level faced by borrowers in each region is $p^e = 0.9165$. Keeping the interest rate on savings fixed, this punishment level maximises the equilibrium utility of borrowers in every region. Given this punishment level then in every economy, the amount borrowed and saved is 0.0527 per agent, the average default rate in the economy is 32%, and the equilibrium utilities are 0.70637 for borrowers and 0.742306 for savers. However, this is not necessarily a welfare-maximising equilibrium, as we will now show.

Suppose that the welfare in each region is evaluated in the same manner, as the weighted sum of the equilibrium utilities of savers and borrowers in each economy:

$$W_\kappa = w_d U_d^e + w_b U_b^e \qquad (2.61)$$

[23] If we do not scale the punishment by $r_\theta y_\theta$, then the punishment is the same for someone who defaults on 100% of two different amounts - e.g. someone who defaults 100% on a loan of \$1 is punished as harshly as someone who defaults 100% on a loan of \$1,000,000. This is obviously not representative of reality. However this is not necessary and does not qualitatively change the results.

for every region κ, where w_d and w_b are constants, such that $w_d + w_b = 1$.

Suppose now that all regions set the punishment level cooperatively, and can commit to their agreement. Let us assume that $w_d = 0.2$ and $w_b = 0.8$, so that the government of each region places a high weight on the welfare of borrowers[24]. Suppose too that all there is one punishment level that applies to all regions. Then, welfare in each region is maximised at a punishment level of $p^* = 0.8585$, which yields a welfare of 0.713569, as illustrated in Figure (2-1) below. In a non-cooperative Nash equilibrium however, the punishment level in each economy

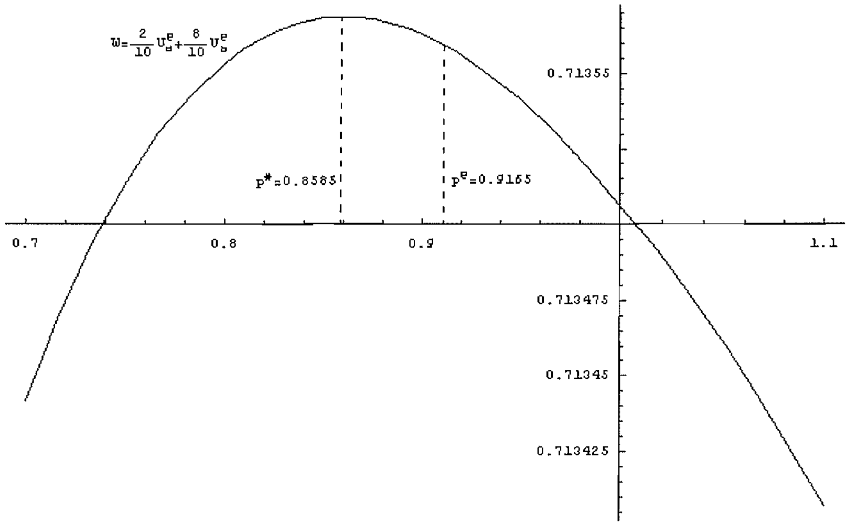

Figure 2-1: Welfare-maximising punishment level in a cooperative equilibrium.

will be $p^e = 0.9165$, which yields a welfare of 0.713557 in each economy. Clearly then, in a non-cooperative setting, the punishment level set by each government will not be the one that collectively maximises welfare. This completes the proof of Proposition 1. ■

The problem arises because ex-post, a government will have no incentive to cooperate - with full cooperation, the punishment level is the one that collectively maximises welfare. However, the resulting punishment level selected endogenously is different to this.

The reason for this is that when an individual region sets its punishment level, there are no general equilibrium effects on its agents - interest rates on savings are not affected. On the

[24] Of course, the analysis is not affected by these weights.

other hand, when acting cooperatively, regions have to take into account how changes in the punishment faced by borrowers affects the welfare of savers through general equilibrium effects on interest rates. This lack of welfare maximisation is for the same reason Chapter 1 - the fact that governments do not use a 'global' social welfare function toset punishment levels[25]. However, there is an element of a lack of coordination since, as we show below, any agreements made between governments cannot be sustained without a certain form of punishment - there is always an incentive to deviate and the resulting equilibrium is not the one which maximises social welfare.

We will now show this. Suppose that a single punishment level, p, is set in the world to maximise welfare in each region. Specifically, if $U_\theta(p, r_d(p))$ is the equilibrium utility of agents of type θ (irrespective of the region they are located in) given the punishment level p and the resulting equilibrium risk-free rate of interest $r_d(p)$, the welfare maximisation problem is:

$$\max_{p} \quad w_d U_d[p, r_d(p)] + w_b U_b[p, r_d(p)] \tag{2.62}$$

which yields the necessary first-order condition:

$$w_d\left[\frac{\partial U_d[p^*, r_d(p^*)]}{\partial r_d}\frac{dr_d(p^*)}{dp}\right] + w_b\left[\frac{\partial U_b[p^*, r_d(p^*)]}{\partial p} + \frac{\partial U_b[p^*, r_d(p^*)]}{\partial r_d}\frac{dr_d(p^*)}{dp}\right] = 0 \tag{2.63}$$

Suppose now that this first-order condition is satisfied, so that the punishment level in every other region is p^*. Then, a region κ can choose not to cooperate and set the punishment level p_κ which maximises welfare in that economy, given the punishment level p^* in every other economy - given that this will have no effect on the risk free rate of interest r_d[26], the maximisation problem of this region becomes:

$$\max_{p_k} \quad w_d U_d[p_k, r_d(p^*)] + w_b U_b[p_k, r_d(p^*)] \tag{2.64}$$

[25] As inChapter 1, banks chose the punishment level by maximising the utility of borrowers *ceteris paribus* and not by maximising the social welfare function in the economy, while the same occurs in this case. However, this is only because we assume that changes in the level of borrowing in a single economy do not affect the world risk-free rate of interest. If we assume however that changes in the level of borrowing in one economy do affect world interest rates, the solution will not be the same as that in Chapter 1.

[26] That is, $\frac{dr_d(p^*)}{dp_k} = 0$.

while the first-order necessary condition is:

$$w_b \frac{\partial U_b \left[p_k^*, r_d\left(p^*\right)\right]}{\partial p_k} = 0 \tag{2.65}$$

so that p_k^*, the optimum punishment level in this economy[27], does not necessarily coincide with p^*. Hence cooperating, is a weakly dominated strategy - an economy can achieve at least as high a welfare by defecting[28].

To see this, consider again the numerical example provided above from the point of view of an individual government. Suppose that every other region (the "rest of the world") cooperates and sets $p^* = 0.8585$, resulting in an equilibrium risk-free interest rate of $r_d\left(p^*\right) = 0.811$. Then, given that r_d remains fixed at 0.811, the government of a typical region κ can maximise its welfare by setting its punishment level at $p_\kappa = 0.917$, which yields higher equilibrium utility for its borrowers, and so a higher social welfare of 0.713589. Hence, each government will have an incentive to defect. If on the other hand the rest of the world defects and sets $p = 0.9156$, then co-operating and setting $p = 0.8585$ yields a government a social welfare of 0.713539, while defecting and setting $p = 0.9165$ yields a higher social welfare of 0.713557. Hence, whether or not the rest of the world cooperates, an individual government will have an incentive to defect.

The payoffs of this game are similar to that of a prisoner's dilemma, and are illustrated in the table below[29]. The top row gives the action of a typical region, while the first column gives the action of the rest of the world[30]. If every government cooperates the welfare in every region is 1 in every region. However, there is always an incentive to defect so that at equilibrium, every government defects and this results in a lower welfare of 0.9998 in every region.

[27] Taking as fixed the punishment level in all other economies.

[28] They only time it is not a dominated strategy is when the Nash equilibrium of the game at stage zero coincides with the welfare-maximising punishment level. When the two do not coincide then co-operating is a strongly dominated strategy.

[29] For ease of notation, we normalise the payoffs by dividing them by 0.713569, which is the welfare achieved if every region cooperates.

[30] The bold font indicates the highest-paying action for the top player.

Table 2-2 - government payoffs

	Cooperate	Defect
Cooperate	1	**1.00003**
Defect	0.99996	**0.99998**

This example has just illustrated that the punishment level in a non-cooperative equilibrium is not welfare maximising. The welfare implications of this result are obvious - since default penalties such as exemption levels, garnishment of future income and regulations on credit scores[31] are set without cooperation across different economies, and even within the US itself, there may be scope for improvements in welfare in these regions with some form of cooperation.

2.3 Numerical experiments

The paper now proceeds with the numerical experiments on the benchmark economy outlined above. We first investigate the effects of falling punishment levels.

2.3.1 Default penalties

As well as penalties imposed by the state, individuals who default on their debts are subject to further penalties. An obvious cost are the legal fees incurred upon filing for bankruptcy. Furthermore, there may be social disapproval or 'stigma' associated with filing for bankruptcy. For example, U.S legislation allows credit agencies to report past bankruptcies of up to ten years old; as a result, landlords may be less willing to let their accommodation to bankruptcy filers, and perhaps more significantly, banks may restrict lending[32]. These extra costs can act as a significant deterrent - for example, White (1998) finds that while 15 percent of households would find it financially beneficial to declare bankruptcy, only about 1 percent do.

[31] For example, a government can state for how long an individual's credit score is allowed to show a bankruptcy finding.

[32] Musto (1999) finds that bankrupt individuals are indeed restricted to credit - upon losing their 'bankruptcy flag' after the ten year period, such agents enjoy significantly higher access to credit. On the other hand, a survey by Staten finds that among bankrupt individuals, three quarters are able to obtain new credit within a year of filing for bankruptcy, but can only do so at higher interest rates. Either way, consumers who go bankrupt are penalised.

Various authors have argued that in addition to more lenient exemption levels, there has been a reduction in such other forms of punishment. For instance, in an empirical analysis, Gross and Souleles (2002) find that an increase in bankruptcy filing rates between 1995 and 1997 could not be explained by risk-related factors such as income so attributed this to a fall in stigma. Furthermore, Fay et al (2002) find that, after controlling for risk-related factors, an increase in bankruptcy filings in one year leads to a further increase in the following year. This could be interpreted as a fall in social stigma (going bankrupt may become more socially acceptable as more people do so), or, as the authors argue, a fall in the informational costs of bankruptcy - as more people go bankrupt, their friends and relatives learn from their experience and find it easier to file themselves as they are more informed.

In this section, we briefly examine the impact that decreasing penalties can have on borrowing and default levels. Consider the benchmark world economy outlined above. Equilibrium in this world yields a risk-free interest rate of $r_f = 0.812$, and each government sets the punishment level at $p = 0.9165$ in their economies. Given these parameters, we exogenously vary the punishment level in a single economy - since the size of each economy is negligible, this will have no impact on the rest of the world and the risk-free rate will remain constant[33]. We let the punishment level in this economy vary from 0.35 to 1.6. The effects on borrowing and default are depicted in Figure 2-2.

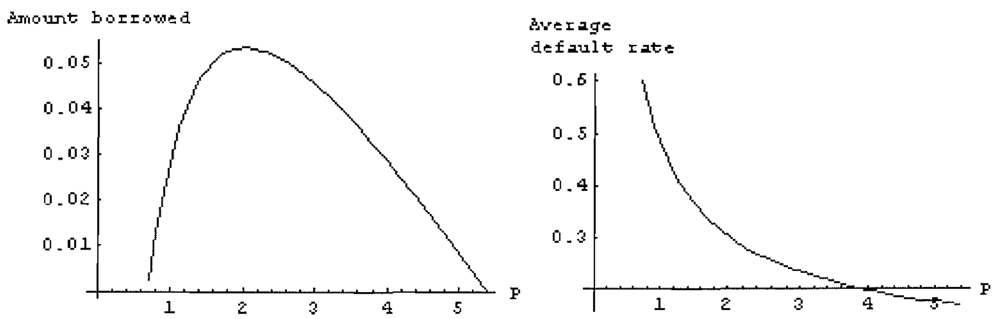

Figure 2-2: The effects of default penalties on borrowing and default rates.

The results show that the average default rate varies monotonically with the punishment

[33]However, qualitatively similar results will apply if we let r_d vary in general equilibrium.

level - lower punishments lead to more default at equilibrium. On the other hand, the effect on borrowing is not monotonic; at too low a punishment level, there is too high an incentive to default, so banks are deterred from lending, while at high punishment levels consumers are deterred from borrowing for fear of excessive punishment in the case of 'bad luck' leading to default.

Clearly then, decreasing penalties *may* lead to more borrowing and more default. However, if the punishment level is at its equilibrium value of 0.9165, then decreasing it further leads to more default but less borrowing[34]. Nevertheless, one could still conclude that decreasing penalties may be behind upwards trends in borrowing and default.

In our opinion however, the above view is somewhat simplistic. It is as if, analysing the market for new cars, one concludes that the demand for cars has increased because their price has fallen; while this may be true, it would be more informative to ask what may have caused this fall in prices instead. By endogenising the punishment level, this is the aim of the upcoming sections.

2.3.2 Income levels

One of the puzzles in the literature is that default rates have been on the rise even though incomes have steadily been increasing. To illustrate this, Figure 2-3 plots the number of personal insolvencies in the UK against the level of personal income, along with a line of best fit. As the figure shows, high incomes coincide with a high number of personal insolvencies.

This section examines whether this could be a causal relationship rather than just a correlation. For instance, it could be that higher incomes lead to more borrowing and as a result greater default. To answer this, we perform the following experiment on a single economy in the world. The short-run effects are first analysed, by assuming that default penalties remain static. The punishment level is held fixed at its benchmark value of 0.9165, and the risk-free rate is fixed at $r_d = 0.812$. The endowment of borrowers in this economy in states 1 and 2 is then uniformly varied from 0.6 and 0.4 respectively, to 0.9 and 0.7, so that average future incomes steadily rise[35].

[34]This is consistent with the findings of Athreya (2004), who calibrates a model to US data and finds that decreasing the punishment for default leads to less borrowing.

[35]To be more specific, endowments are initially set at $\frac{6+\varepsilon}{10}$ and $\frac{4+\varepsilon}{10}$. The parameter ε is then varied from 0 to

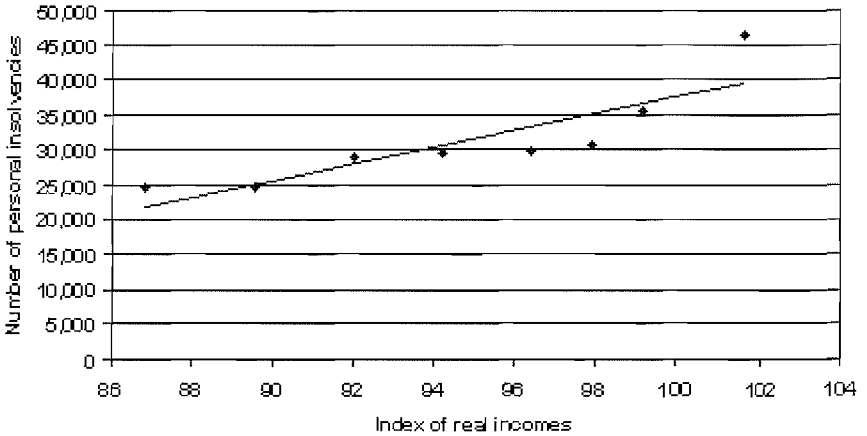

Figure 2-3: Correlation between incomes and personal insolvencies (1997-2004)

On the basis of our results, we find no evidence to support this hypothesis in the short run. Figure 2-4 plots the results of this experiment. As this shows, as incomes rise, borrowing increases but the default rate falls.

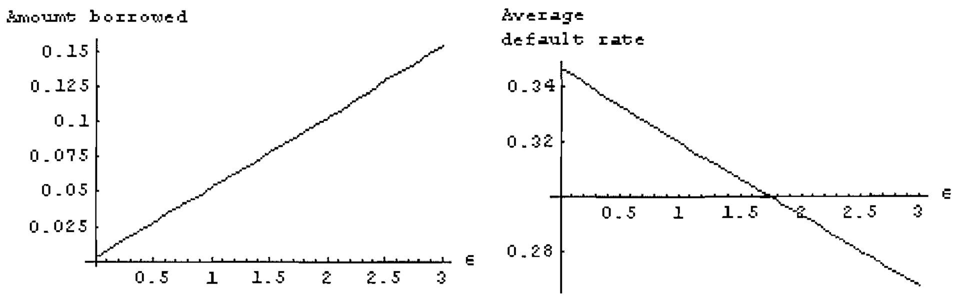

Figure 2-4: The short-run effect of higer incomes on borrowing and default levels.

Long-run effects

In this example, keeping default penalties fixed, higher incomes do not cause more borrowing and higher default rates. In the long run however, we would expect the government to react to

3.

react to any changes in incomes and set the punishment level accordingly. This section examines how the equilibrium punishment level varies with incomes, and what the overall effects are on borrowing and default.

We now repeat the experiment on incomes outlined above, but allow the punishment level to be set endogenously at its equilibrium value. The effects on the equilibrium punishment level are illustrated in Figure 2-5.

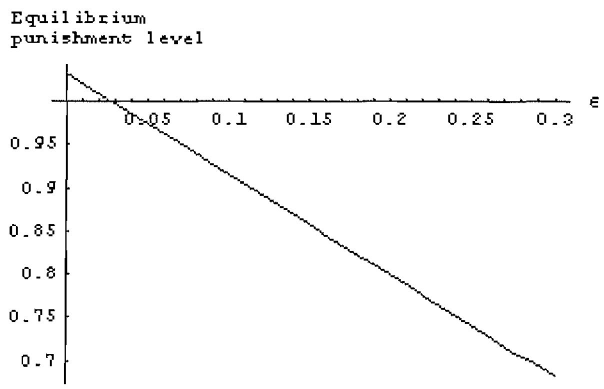

Figure 2-5: Effect of higher incomes on the equilibrium punishment level.

However, default penalties can act as a commitment device towards the repayment of debt, lowering interest rates and increasing the supply of credit. As incomes rise, agents are more able to repay a given amount of debt, and so there is less need for strict penalties as a commitment device. Overall, this effect dominates, and the equilibrium punishment level falls as incomes rise. In the long run therefore, higher incomes result in lower default penalties[36]. In turn, lower default penalties put upwards pressure on default rates, and default rates rise, *despite* the increase in incomes. Borrowing still increases, albeit by not as much.

As this example has illustrated, the effects of rising incomes in the long run can be quite different to those in the short run. While in the short run, higher incomes can lead to more borrowing and less default, in the long run they can lead to lower default penalties and *more* default. The evidence from the US shows that borrowing and default have been increasing,

[36]This is consistent with what is observed in reality, at least between Europe and the US, where incomes are higher in the latter and default penalties more lenient.

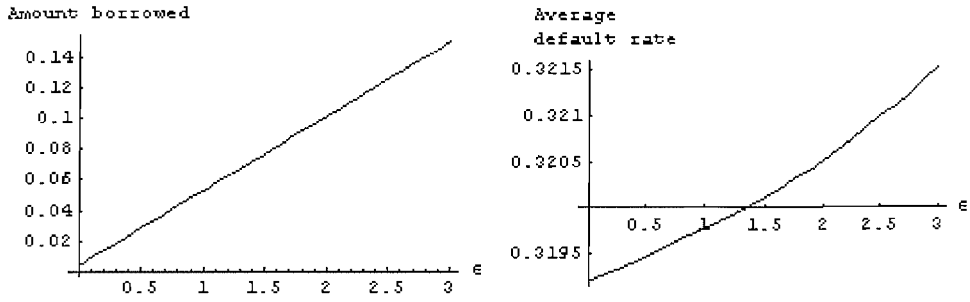

Figure 2-6: Effect of higher incomes on borrowing and default with endogenous default penalties.

while default penalties have been falling. According to our results, rising incomes may be one of the factors underlying these trends.

2.3.3 Income uncertainty

While economies have enjoyed increased prosperity through economic growth and technological progress, the degree of uncertainty faced by individuals today may also have been increasing. There may be several reasons why this could have occurred. For example, rapid innovation may replace jobs which were once carried out manually, forcing previously skilled people to take lower paid jobs. Moreover, increased trade liberalisation may lead to the closure of some domestic industries, resulting in job losses and lower incomes for those involved.

Evidence of an increase in income uncertainty is the sharp rise in the degree of wage inequality in various developed economies, particularly in the US and the UK (Acemoglu, 2002); greater inequality may imply greater uncertainty by increasing the range of an agent's attainable income in the future.

In this section we examine the effects of such uncertainty on the market for unsecured consumer credit by applying a mean-preserving spread on the future endowments of borrowers. We initially set endowments in states 1 and 2 to $\frac{6}{10}$, which represents the lowest degree of uncertainty, and apply a mean-preserving spread so that endowments in these states gradually reach $\frac{9}{10}$ and $\frac{3}{10}$. The experiment is carried out in the short run, keeping p fixed at 0.9165. The results on borrowing and default from this experiment are plotted in Figure 2-7. As these results illustrate, greater uncertainty may lead to more borrowing and more default.

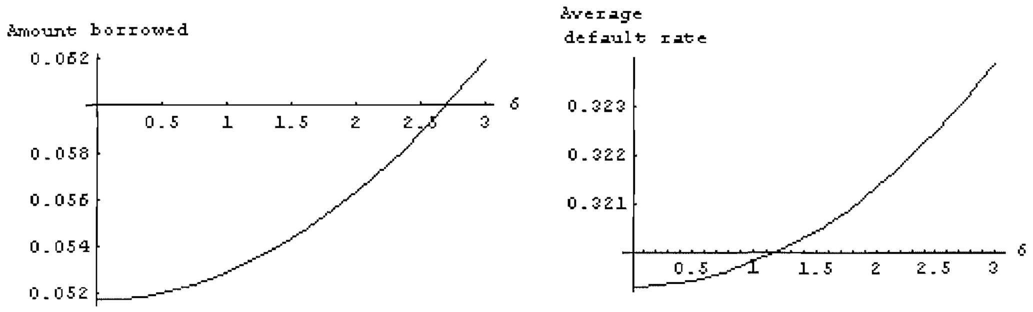

Figure 2-7: Short-term effects of income uncertainty on borrowing and default.

The long run

The general consensus in the literature is that economies with more uncertainty should have more lenient bankruptcy laws to allow for better consumption smoothing. For example, this view is expressed by Grant and Koeniger (2005), Livshits et al (2001), and by Athreya and Simpson (2005). We examine the long-term effects of a mean-preserving spread in future endowments by repeating the above experiment and allowing the punishment level to vary endogenously. Our results are consistent with this view; as Figure 2-8 shows, the equilibrium punishment level falls as uncertainty increases. Once again, this result is consistent with the evidence in Europe which has less income inequality, a more generous social net, and harsher bankruptcy legislation than the US.

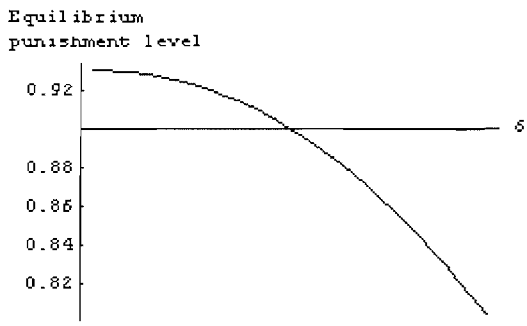

Figure 2-8: Effects of income uncertaity on the equilibrium punishment level.

2.3.4 Expense shocks

Besides income uncertainty, individuals face uncertainty over possible adverse expense shocks such as a large medical expense or a divorce. These can be especially relevant, as noted by Fay et al (2002), who find that such shocks can be the main force behind decisions to go bankrupt. Himmelstein et al (2005) find that, in 2001, nearly half (46%) of bankrupt debtors report 'major' medical expenses, and the average out-of-pocket expenses among those for whom illness led to bankruptcy were $11,854. They also find that bankruptcy isn't restricted to just those without medical insurance, since 76% of such filers were actually insured.

Evidence suggests that such shocks have been on the rise. Consider divorce for example - although divorce rates as a percentage of marriage have remained stable at around 50% in the US over the past two decades, they have generally been increasing heavily in the UK, where they have been increasing from 38% in 1980 to 51% in 2001, and in most of Europe too (UNECE[37], 2005). On the other hand, the US has seen an increase in the medical expenses of its population; a study by Kashihara and Karper (2005) finds that the average medical expense in the US increased by 21% in real terms between 1997 and 2002.

We investigate the effects of more frequent expense shocks in the following manner. As outlined above, state $s = 3$ represents such circumstances, by assuming that endowments in that state are so low that an agent will always default. We thus simulate more frequent expense shocks by increasing α_3, the probability of that state occurring. At the same time, we decrease α_1, the probability of state 1 taking place, and leave α_2 unchanged. More specifically, we set these values as $\alpha_1 = 0.5 - \gamma$, $\alpha_2 = 0.45$ and $\alpha_3 = 0.05 + \gamma$, and vary γ from 0 to 0.1. The effects in the short run are depicted in Figure 2-9. As this reveals, more frequent expense shocks lead to lower borrowing and lower default rates.

When faced with a higher probability of an expense shock, borrowing becomes less attractive for consumers for two reasons: first, agents now face a higher chance of defaulting and receiving a punishment; second, the interest rate on loans must rise to take into account the higher probability of an expense shock.

In turn, lower equilibrium borrowing leads to lower default rates in states 1 and 2. Alto-

[37] United Nations Economic Commision for Europe.

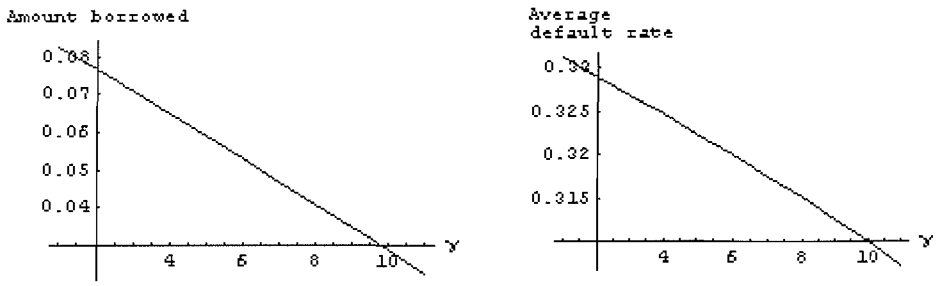

Figure 2-9: Short run effects of expenditure shocks on borrowing and default.

gether, this effect dominates, and the average default rate falls, despite a higher probability of complete default. Consequently, the short run effects of more frequent expense shocks do not explain the upwards trends observed in borrowing and default.

The long run

The long run effects of this experiment once again contradict the short run effects above. We first investigate the effect on the equilibrium punishment level; as Figure 2-10 illustrates, increasing the probability of an expense shock results in a lower equilibrium punishment level. Intuitively, we would expect this to be the case, as the need for default as a form of insurance increases.

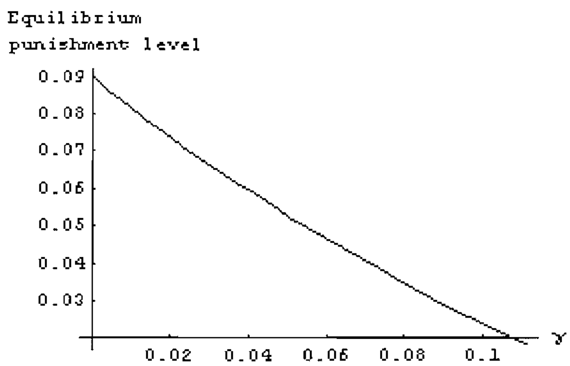

Figure 2-10: Effects of expense shocks on the equilibrium punishment level.

As equilibrium punishment levels fall, this puts upwards pressure on default rates, but downwards pressure on borrowing levels. As a result, borrowing still falls, but default rates increase, once again contradicting the short run effects.

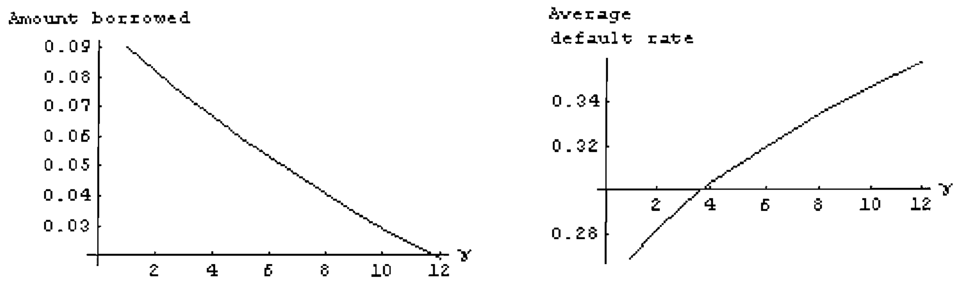

Figure 2-11: Long run effects of expenditure shocks on borrowing and default.

To summarise, more likely expense shocks *can* explain falling default penalties and rising levels of default in the long run. However, they can *not*, on their own, account for recent trends as they lead to less, and not more, borrowing.

2.3.5 Intermediation costs

Up to this point, we have assumed that there are no costs to financial intermediation, a simplifying but unrealistic assumption. Costs to financial intermediation do exist, and can consist of operating costs, monitoring costs, and general administration costs such as data keeping.

It is reasonable to assume that, over the years, such costs have been falling due to advances in information technology and perhaps the wider availability of finance courses, increasing the availability of, and hence providing cheaper access to, a specialised workforce[38].

We extend the model by assuming that the bank faces a fixed cost c per agent it trades a contract with. After including this cost, the bank's profit function becomes:

$$\Pi = -\sum_{\eta \in \Theta \times K} \sum_{s=1}^{n} \left[\alpha_s \xi_\eta M_\eta \left(1 - \lambda_\eta^s\right) r_\eta y_\eta - \xi_\eta M_\eta c \right]$$

[38] An example of falling intermediation costs, offered by Athreya (2004), is the introduction of the PS2000 payments authorisation system introduced by VISA.

This obviously does not affect the solution to the bank's maximisation problem and the equilibrium conditions. The cost c is then varied from 0 to 0.07.

Figure 2-12 plots the short-term effects on borrowing and default, assuming the punishment level remains fixed. As these reveal, a fall in these costs results in more borrowing but a lower default rate.

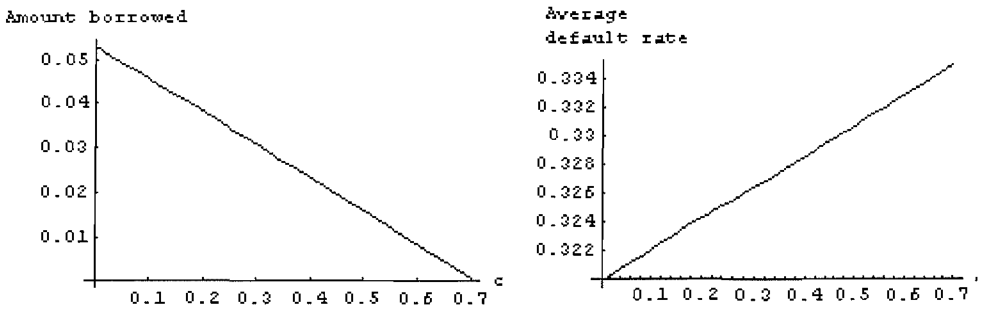

Figure 2-12: Short run effect of higher intermediation costs on borrowing and default.

The explanation is fairly intuitive. As costs fall, interest rates decline. Therefore, borrowing increases as loans are more affordable for consumers to pay off. On the other hand, there are two effects on default rates: lower interest rates put downwards pressure on default, while more borrowing exerts upwards pressure. On the whole however, the effect of lower interest rates prevails and default rates decrease. Therefore, in the short run, falling transaction costs are only partly consistent with recent trends: while they can offer a reason for an increase in borrowing, they cannot explain trends in default rates, as they lead to less, and not more default.

The long run effects

Once again, the long run effects contradict those obtained in the short run. In the long run, lower intermediary costs lead to a lower equilibrium punishment level, as illustrated in Figure 2-13. As costs fall, and with them interest rates, loans become more affordable to pay off. On the one hand, this reduces the need for lenient default penalties as a form of insurance; on the other hand, however, there is less need for strict punishments as a form of commitment. Overall, the second effect dominates, and the equilibrium punishment level falls.

Bos and Kolari (2003) find that banks in the US tend to be more cost-efficient than banks

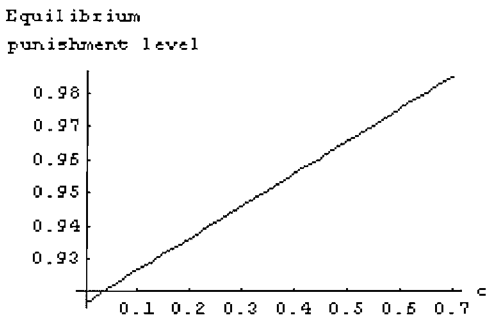

Figure 2-13: Effect of transaction costs on the equilibrium punishment level.

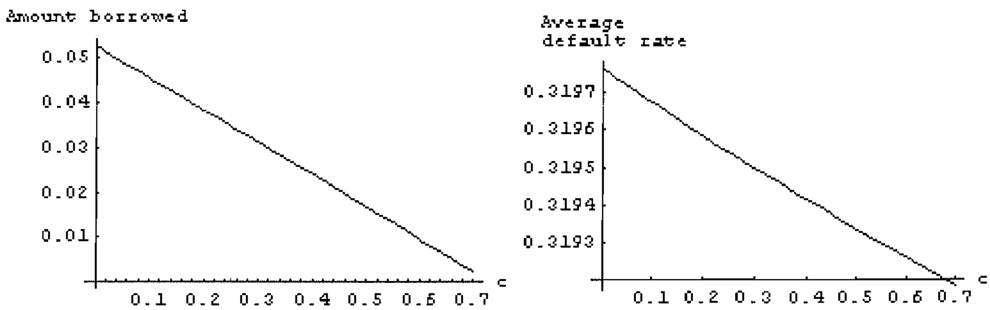

Figure 2-14: Long run effects of transaction costs on borrowing and default.

in Europe. Our results suggest that this may be another reason for the US to have more lenient default penalties.

The long-run effects of a fall in intermediation costs are illustrated in Figure (2-14). The fall in the punishment level puts upwards pressure on equilibrium default rates, and as a result, the default rate *increases*, reversing the short-run effects, while borrowing also increases.

To conclude therefore, a fall in intermediation costs can only partially explain the data in the short run, as they lead to more borrowing but less default. In the long run however the results are fully consistent with the evidence on falling default penalties, an increase in borrowing, and rising default rates. Lower intermediation costs may therefore be another reason for observing such trends.

2.3.6 Falling interest rate spreads

We now examine the impact of a fall in the interest rate at which agents can borrow by inserting an additional parameter as a proxy for the degree of competition in the model. A fall in the cost of borrowing for instance may occur if the real interest rate on savings falls. Evidence by Orr et al (1995) supports this, who reveal that real interest rates in many developed countries were significantly lower in the mid 1990s than their peak values in the early 1980s[39]. A fall in the borrowing rate may also occur if for some reason the spread between saving and borrowing rates declines. Evidence of this can be found in Del-Rio and Young (2005), who report a fall in the interest rate spread between unsecured debt interest rates and the retail bank base rate in the UK between 1995 and 2000, despite the fact that default has been increasing.

Athreya (2004) models this phenomenon by inserting an exogenous parameter τ in the interest rate spread, where a higher τ implies a larger spread, over and above the spread required for zero profits. Athreya interprets this as a proxy to the degree of competition in the credit market; the larger τ is, the more market power banks have over consumers. We will proceed with the same experiment, which we now outline.

Without the parameter τ, the interest rate r_b^* charged to borrowers at equilibrium is given by:

$$r_b^* = \frac{r_d}{\sum_{s=1}^{n} \alpha_s \left(1 - \lambda_b^s\right)}$$

where λ_b^s is the optimal, ex-post default rate of borrowers in state s given the amount borrowed and the interest rate charged. We now insert the parameter τ in this equation, so that r_b^* is given as:

$$r_b^* = \frac{r_d + \tau}{\sum_{s=1}^{n} \alpha_s \left(1 - \lambda_b^s\right)}$$

We set r_d to its benchmark value of 0.812, and the punishment level to 0.9165. The parameter τ is then varied from 0 to 0.2, and the effects on borrowing and default are depicted in Figure 2-15. As this reveals, a fall in the parameter τ leads to more borrowing and a higher default rate.

As interest rates decrease, borrowing increases. This increase in borrowing puts upwards

[39] One reason for this may be demographic trends - as the population of developed economies ages, there are more savers relative to borrowers, and this drives interest rates down.

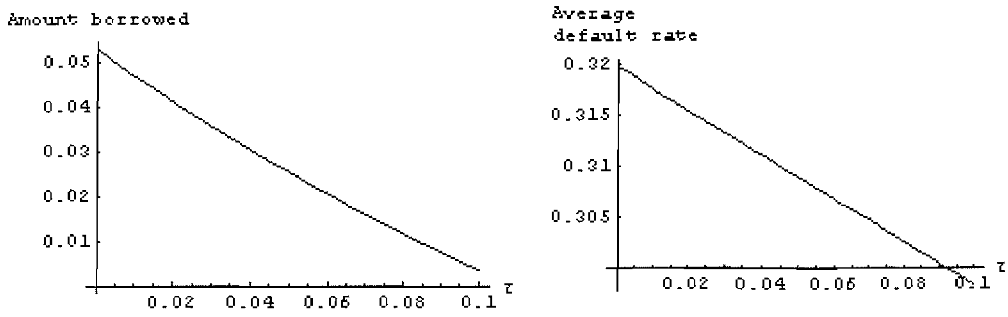

Figure 2-15: Short run effects of a fall in interest rates on borrowing and default.

pressure on the default rate and as a result, default rates rise, despite the initial fall in the interest rate. Figure 2-16 plots the equilibrium interest rate - as this illustrates, the interest rate on loans does indeed fall as τ is lowered, even though default rates increase.

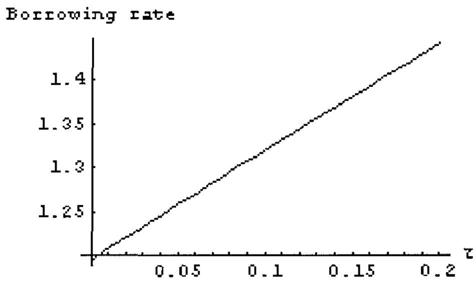

Figure 2-16: Short run effects of a fall in τ on the equilibrium borrowing rate.

Hence, in the short run, a fall in τ leads to higher borrowing, more default and lower interest rates. This result is on par with Athreya (2004), who obtains the same results in a similar experiment.

The long run

Athreya (2004) reaches the conclusion that falling intermediation costs may be responsible for the increases in borrowing and default observed in the US, and our short run results are consistent with this view. In the long run however, our results are once again reversed. Unlike the case of lower transaction costs, a lower interest rate spread leads to a higher equilibrium punishment level (see Figure 2-17). On the one hand, a lower interest rate spread makes loans more affordable and so reduces the need for lenient penalties and default as a form of insurance. On the other hand, there is less need for strict penalties as a form of commitment. Overall, the first effect dominates, and the equilibrium punishment level increases.

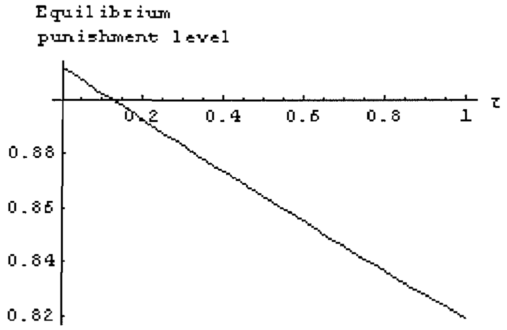

Figure 2-17: Effects of a fall in τ on the equilibrium punishment level.

In the long run, the effects on borrowing and default are once more reversed. The increase observed in the punishment level puts downwards pressure on the equilibrium default rate, and is enough to reverse the increase in default which occurs in the short run. Overall then, in the long run, a fall in the parameter τ still results in more borrowing, but the default rate *falls*, as depicted in Figure 2-18. Hence, while lower interest rate spreads may explain higher borrowing and higher default rates in the short run, they cannot do so in the long-run as they lead to lower default rates.

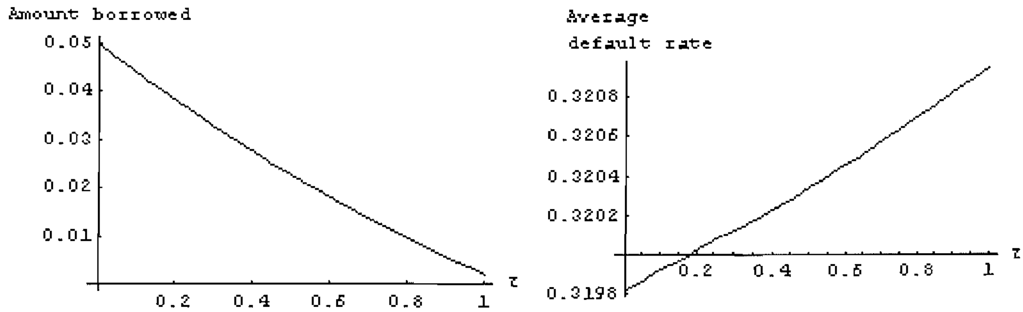

Figure 2-18: Long run effects of a fall in τ on borrowing and defaut rates.

2.4 Conclusion

The aim of this paper has been to examine the implications of endogenous default penalties in the market for unsecured credit, both from a normative perspective and with a view to understanding the observed trends in most developed economies, and particularly the US and UK. More specifically, the main questions it seeks to answer are: what can account for the increase in borrowing *and* default over the years? Why have banks been extending credit, even though default is becoming more common? Finally, why have governments, in the face of increasing default, lowered default penalties?

To this end, we extend the model which was developed in chapter 1 of this thesis to a world which consists of many open economies and has the following features. First, it is assumed that funds can be transferred from one region to another without cost; second, agents who borrow can choose to default on their debt, subject to default penalties they face in their region and third, the government of each region is free to set its own default penalties. The novel feature of this model is that, along with credit constraints, interest rates and default rates, default penalties are determined endogenously, which, to the best of our knowledge, has not been attempted by any other study. Default penalties are derived in a non-cooperative game, where each government sets its default penalties individually. We show that this does not coincide with a cooperative equilibrium where there is full commitment, so the outcome under no cooperation yields a lower welfare collectively. This may point to a greater need for cooperation in setting default penalties between different economies. For example, within the US, states are allowed to set their own default penalties individually, as is the case for member states within the EU. This suggests that there is scope to increase welfare across these regions with increased cooperation, so further research may be necessary into this matter.

We then proceed with the main theme of the paper, which focuses on the possible reasons behind the higher levels of borrowing and default, and the relaxation of default penalties. Numerical experiments are then conducted to examine the impact on borrowing and default of:

1. rising incomes;

2. greater income uncertainty (or inequality);

3. more likely expense shocks;
4. lower intermediation costs; and
5. cheaper credit availability.

We distinguish between the effects in the short run, and those in the long run. In the short run, we assume that default penalties remain fixed, given that it is both time-consuming and costly to introduce new legislation. In the long run however, governments can react to the new set of economic circumstances, so default penalties are allowed to vary endogenously along their equilibrium path.

The main message from these experiments is that allowing for endogenous punishment formation can often yield an opposing set of conclusions to the case where penalties are assumed to be fixed. For example, in the experiment involving incomes, we find that, keeping penalties fixed, higher incomes lead to more borrowing but less default. This perhaps is one of the biggest puzzles in this field - if greater incomes lead to more borrowing but less default, why is default becoming more common as incomes have risen? The answer may lie in the long-run consequences of rising incomes. In the long run, higher incomes lead to lower default penalties - the optimal default penalty in a region is a balance between allowing for default as a form of insurance against states with lower endowments, and punishing default as a form of commitment device, which benefits agents through lower interest rates. In the case of higher incomes, there is less need for default as a form of insurance, which implies a higher punishment level, but there is also less of a need for punishments to act as a commitment device, given that the incentive to default is lower. The latter effect dominates, and the result is a fall in the equilibrium punishment level. In turn, lower punishment levels put upwards pressure on default rates. This effect is too strong, and the end result is higher levels of borrowing, and a *higher* default rate, along with more lenient punishments. In the long run therefore, contrary to the short run, higher incomes are consistent with the data: default rates may be rising, and penalties falling, not despite higher incomes, but *because* of higher incomes.

Next, we evaluate the effects of a mean-preserving spread in the future endowments of agents. This is meant to represent a greater extent of income uncertainty, either due to more inequality, or perhaps a lower provision of social security such as unemployment insurance. In the short run, more uncertainty led to more borrowing and more default. In the standard case with no default, we would expect a mean-preserving spread like that to lead to lead to less borrowing, as agents reduce seek to increase consumption in the low-income state. However, when default is allowed, borrowing may increase as agents borrow against their higher income in one state and use default to increase their consumption in the low-income state. In this case, our long run results are consistent with our short run results, as more uncertainty leads to more borrowing and more default in both cases. It also results in lower punishment levels, since there is a greater need for default to facilitate intra-temporal consumption smoothing. Therefore, along with higher incomes, greater uncertainty may also explain why borrowing and default have increased, and default punishments fallen.

The chapter then proceeds by investigating the effects of an increase in the likelihood of a large expense shock, having in mind mainly medical expenses and divorces. Over the years there has been an increase in such expenditure shocks, both in Europe (mainly through higher divorce rates), and in the US (mainly through higher medical expenses). We find that the results are not entirely consistent with the data. In the short run, more likely expense shocks lead to less borrowing and less default, which completely contradicts the evidence. In the long run, they lead to a lower equilibrium punishment level due to the greater need for default as a form of insurance; this puts upwards pressure on the default rate, and as the result, default rates increase, reversing the short-term result. The amount of borrowing however still falls in the long run; therefore, while more likely expenditure shocks may have contributed to the increase in default rates and the fall in default penalties over the years, this does not, on its own, account for these trends as it leads to less borrowing.

The effects of lower costs to financial intermediation are analysed next. We model intermediation costs, such as monitoring and general administration costs, by assuming that the bank pays a fixed cost c for every agent it trades a contract with. We then vary this cost while keeping the punishment level fixed. In the short run therefore, as costs decrease, interest rates fall, leading to an increase in borrowing and a decrease in the equilibrium default rate. In the long run, the equilibrium punishment falls as costs are reduced - lower interest rates reduce the incentive to default, so there is less need for strict penalties to act as a commitment device.

This fall in the punishment level is then enough to reverse the short-term reduction in default rates - in the long run, borrowing *and* default rates increase. Hence, while a fall in transaction costs cannot explain the data in the short run, in the long run it leads to more borrowing, more default, and lower punishment levels. Thus, along with higher incomes and greater income uncertainty, lower intermediation costs may be another reason for the trends observed over the years.

Finally, this chapter ends with an experiment involving a reduction in the cost which consumers face in obtaining credit. In a similar experiment to Athreya (2004), we introduce a parameter τ as the the extent of which the interest rate spread is over and above that required for zero profits, which we interpret as a proxy to the degree of competition in the banking sector - the lower τ is, the greater the degree of competition. We find that, keeping default penalties fixed, a lower interest rate spread leads to more borrowing and more default, a result which is consistent with that of Athreya (2004); it could therefore explain increases in borrowing and default over a short period of time. However, in the long run, the results are yet again reversed. As a result of a reduction in the interest rate spread, the equilibrium punishment level increases since there is less need for default as a form of insurance. This then puts downwards pressure on default rates; the end result is an increase in borrowing but a fall in default rates. Therefore, in the long run, lower interest rate spreads cannot fully explain the trends in the market.

To conclude the analysis, the main message that comes out of this chapter is that allowing for endogenous punishment formation can have important positive and normative implications. We have shown that the resulting non-cooperative equilibrium with many open economies is not welfare maximising - this suggests there may be real scope for welfare improvements in reality, where default penalties are set without cooperation amongst various economies, and even within the US itself, where bankruptcy legislation is determined at state level. We have also found that under this setting, higher incomes, more income uncertainty and lower intermediation costs can, in the long run, lead to more borrowing, more default, and lower punishments, thus explaining such empirical evidence. This result might go some way towards easing confusion over why default is becoming more common and why default penalties are being relaxed - rather than a cause for concern, it could just be the result of optimising behaviour on the part of governments.

2.5 Appendix

Table 2-3 - Bankruptcy exemption levels across states in the US

State	Home	Personal	State	Home	Personal
AL	5,000	6,925	MT	60,000	5,700
AK	54,000	8,000	NE	12,500	2,400
AZ	100,000	9,250	NV	12,5000	4,500
AR	1,000,000	1,400	NH	30,000	11,350
CA	50,000	5,000	NJ	15,000	10,700
CO	30,000	4,800	NM	30,000	8,050
CT	75,000	7,100	NY	10,000	7,400
DE	0	5,000	NC	10,000	5,000
FL	1,000,000	2,000	ND	80,000	7,425
GA	5,000	5,400	OH	5,000	2,900
HI	20,000	2,000	OK	1,000,000	10,925
ID	50,000	5,750	OR	25,000	9,150
IL	7,500	7,125	PA	15,000	10,700
IN	7,500	4,000	RI	15,000	10,700
IA	1,000,000	10,600	SC	15,000	10,700
KS	1,000,000	24,650	SD	1,000,000	3,250
KY	5,000	6,500	TN	5,000	7,925
LA	15,000	15,125	TX	1,000,000	30,000
ME	12,500	2,900	UT	10,000	2,500
MD	0	6,000	VT	75,000	9,400
MA	15,000	12,200	VA	5,000	14,750
MI	15,000	10,700	WA	30,000	12,675
MN	200,000	13,000	WV	15,000	3,200
MS	75,000	10,000	WI	40,000	7,200
MO	8,000	3,000	WY	15,000	2,400

Source: Agarwal et al (2003)

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Chapter 3

Optimal bankruptcy exemptions: theory and evidence

Abstract

We study a simple two-period economy with bankruptcy and a law which resembles the current system of legislation in the US, namely one which specifies bankruptcy exemptions. We then devise a simple rule for setting the optimal exemption level which states that, if exemption levels are set optimally, any exogenous shock which increases borrowing implies a higher exemption level. This is then tested using data from the US under a spatial panel regression framework. We employ this methodology to test for spatial interactions in exemption levels between neighbouring states, and, to our best knowledge, this is the first study to do so. We find the empirical results to be broadly in line with the theoretical predictions of our model and also detect the presence of some limited spatial effects, so that exemption levels in a state are influenced by those of its neighbours. However, we find evidence that these spatial effects have steadily become weaker in more recent years.

3.1 Introduction

Default can play an important role as a form of insurance against uninsurable risks in the market for unsecured consumer credit. As Zame (1994) has shown, allowing for some default may be beneficial by allowing agents to enter into agreements which they can honour with a high probability but fail to honour with a low probability; without the option to default, agents would only be able to enter into agreements which they could honour in any conceivable situation, reducing the scope for borrowing and smoothing consumption over time.

Government legislation can influence an individual's decision to default on their debts, and such legislation can vary widely between regions[1]. In the US, state legislation determines the bankruptcy exemption levels. These specify the maximum value of certain types of assets that are protected from seizure by a bankrupt consumer's creditors. Such exemptions take the form of homestead exemption (HE) levels, which protect individuals' homes, and personal property exemption (PPE) levels, which cover other assets such as clothes and jewellery, and vary greatly between states and over time.

This chapter examines the reasons behind the high heterogeneity in the level of bankruptcy exemptions across US states under a theoretical framework and via an empirical analysis. This is an issue which has not been addressed very often - the existing literature has instead focused on the effects of bankruptcy legislation on other variables such as aggregate default rates and the level of borrowing.

We apply a similar framework to Posner et al (2004) but add to their contributions in a variety of ways. First, their paper, as with similar studies, concentrates mainly on political factors to explain the variation in US exemption levels. On the other hand, the emphasis of this paper is on economic explanations, focussing instead on macroeconomic variables. Second, we extend their data set to a larger time frame of 1975-2004, adding 7 years of more recent observations. Finally, we control for 'neighbourhood' effects in our estimation to account for the fact that exemption levels in states may be influenced by the level of exemptions in their neighbouring states. For this, we use a panel spatial error autoregressive (SER) model and a panel spatial lag autoregressive (SAR) model, both with fixed effects, which to the best of our

[1] For example, such legislation is generally perceived to be much more lenient in the US than in some European countries such as Germany or Italy (see Livshits et al. (2003)).

knowledge, has not been carried out by other studies in a similar context.

We believe it is important to understand how economic fundamentals can influence the setting of exemption levels, as this can have important implications for making inferences about future rates of default, as, for example, is illustrated by chapter two of this thesis. In that chapter, we build a model of general equilibrium with unsecured borrowing and default, where default penalties are determined endogenously by a welfare-maximising government. The study conducts numerical experiments under two settings: under the first setting, default penalties are held constant, while under the second setting they are determined endogenously by the model, so that the government is allowed to react to the change in the macroeconomic environment by setting the punishment level which maximises welfare under the new economic settings. The experiments are then carried out to assess the impact of various macroeconomic shocks on aggregate default rates.

The main result of the previous chapter is that allowing for such endogenous default penalty determination can yield contrasting results - following a macroeconomic shock such as a rise in incomes or a fall in uncertainty, it is possible for equilibrium default rates to fall under the first setting (fixed penalties) but to rise under the second setting (endogenous penalties). For example, one of the puzzles in the literature is that aggregate default rates have been increasing despite a continuing rise in real incomes over the years. In chapter 2 we show that, keeping default penalties fixed, an increase in incomes results in a lower equilibrium default rate, but under endogenous penalty formation, higher incomes can result in *higher* default rates. The reason for this is that, when penalties are set endogenously, the government reacts optimally to higher incomes by lowering default penalties. With lower penalties, agents are more likely to default, and this results in *higher* equilibrium default rates, despite the rise in incomes. Therefore, if we can show that the setting of exemption levels is influenced by economic variables, this can have implications for estimating future rates of default - if, for example, exemption levels are raised as incomes rise, this may result in higher bankruptcy rates.

This paper consists of a theoretical part and an empirical part. For the theoretical part, we develop a simple, two-period model where interest rates and borrowing constraints are determined endogenously. We then derive a simple rule for setting exemptions optimally, which forms the central hypothesis of the paper. This rule states that: *ceteris paribus, any exogenous shock which increases the amount of borrowing implies a higher optimal exemption level.* For instance, higher incomes should, all else equal, lead to more borrowing. Therefore, regions with higher incomes should have higher exemption levels. The same applies for other variables such as unemployment, income inequality and divorce rates. For example, a higher probability of divorce occurring may lead banks to restrict credit. In such a case, we may expect regions with higher divorce rates to have lower exemption levels.

In the empirical part, we then set to test whether this rule applies by using a panel of data from the US, where individual states are effectively allowed to set their own exemption levels[2]. The result has been a wide dispersion in exemption levels across states; for example, in 2004, the HE and PPE levels in Michigan were \$7,500 and \$2,000 while in Arizona they were a much more lenient \$150,000 and \$20,200. This pattern is similar across all states[3]. At the same time, economic and sociological variables can vary widely as well. For instance, while the median income across the US in 2004 was \$44,346, this ranged from \$33,465 in West Virginia to \$56,973 in New Hampshire[4]. Unemployment rates are equally varied, and in July 2004 ranged from 3.1 per cent in Hawaii to 7.4 per cent in Oregon[5].

Of course, repossession of personal belongings is not the only form of punishment for default; other costs to bankruptcy include legal fees, being denied credit post-bankruptcy[6], or perhaps even social stigma. For example, White (1998) estimates that over 15 per cent of households would benefit financially by filing for bankruptcy, yet only 1.1 percent did so in 1996. The author suggests this may be because certain agents default on their loans without formally declaring bankruptcy and are not chased up by their creditors, or that by not declaring bankruptcy, agents can preserve the option to declare bankruptcy at some point in the future, which can be

[2]To be more precise, bankruptcy exemptions are set at a federal level, but individual states are allowed to opt out and set their own exemptions. The majority of states have chosen to do so, but in the few remaining states that have not, bankruptcy filers are alowed to choose the state or federal exemptions, so naturally choose the highest of the two.

[3]The level of exemptions, along with our data sources, is provided further on.

[4]Source: US Cencus Bureau.

[5]Source: US Bureau of Labour Statistics.

[6]When an individual files for bankruptcy, a record of this is created on their credit history, and government legislation can determine how long the record remains. Evidence points strongly to this having an adverse effect on the individual's capacity to borrow - Musto (2004) for instance finds that upon losing this 'bankruptcy flag', individuals enjoy significantly higher access to credit.

valuable if they face substantial income uncertainty. Nonetheless, this could also be interpreted as the existence of other costs, preventing agents from declaring bankruptcy even though it may be financially beneficial for them to do so. However, these are not likely to vary significantly across US states and are also captured by fixed effects in our estimation, so we do not expect their inclusion to affect our results.

The chapter proceeds as follows. In section 3.2, we provide a review of the relevant literature. In section 3.3, we discuss trends in exemption levels over time and across US states. In section 3.4, we develop our model and derive our simple rule for setting exemption levels optimally. In section 3.5, we test this empirically and present the results. Section 3.6 concludes.

3.2 Literature review

3.2.1 Bankruptcy exemptions as exogenous variables

Plenty of research exists involving bankruptcy exemptions in the US, but its focus has been mainly on how exemption levels influence other variables - in other words, exemption levels have been treated as exogenous. In this section, we provide an overview of such studies.

One line of research examines the effect of exemptions on individual and aggregate bankruptcy rates. An early such study by White (1987) regresses the aggregate bankruptcy filing rate across counties in the US against the exemption levels applicable in those counties and finds a significant positive relationship - the higher the exemption levels, the higher is the aggregate bankruptcy rate. On the other hand, Weiss, Bhandari and Robbins (1996), found no such relationship. Furthermore, Peterson and Aoki (1984) and Domowitz and Eovaldi (1993) found that changes in the bankruptcy law in 1978 did not lead to higher bankruptcy filing rates.

Another study by Domowitz and Sartain (1999) examines whether the level of exemptions influences borrowers' choice of bankruptcy procedure, and finds that a decrease in exemption levels makes consumers more likely to file under Chapter 13 rather than Chapter 7 of the bankruptcy code. Furthermore, Fay et al (2002) look at individual data from the Panel Study of Income Dynamics (PSID) for the years 1984-1985 to analyse the decision to file for bankruptcy, and find that increases in the exemption levels would lead to a higher bankruptcy rate. More specifically, they estimate that a proposed adoption of uniform national exemption levels for personal property[7] would result in a rise of 16 per cent in the number of filings per year. In a similar study, Agarwal et al (2003) also find that loose exemption laws encourage default.

Another topic examined has been the effect of bankruptcy exemptions on the supply of credit and consumer spending. For example, Gropp, Scholz and White (1997) use data from the 1983 Survey of Consumer Finance to examine the effects of exemption levels on credit supply and the level of debt held, and find that borrowers are more likely to be denied credit if they live in a state with high exemption levels. They also find that the level of debt held by individual borrowers is lower in such states. Another study by Grant (2003) uses a larger data set by analysing a panel from the Consumer Expenditure Survey for the years 1980-1999. He examines the effect of bankruptcy exemption levels on variables such as the level of borrowing, the interest rate charged on loans, and the effect on consumption volatility and finds that, once state fixed effects are included in the regression, higher exemption levels lead to significantly lower levels of debt as banks reduce credit. Furthermore, he finds that higher exemption levels reduce the volatility of consumption across short time intervals, at the cost of making it more difficult to smooth consumption over longer periods of time; this supports the idea that bankruptcy exemptions provide insurance against low income levels. Grant and Koeniger (2004) argue that if this is the case, then states with high bankruptcy exemptions should have less need for redistributive taxation as a means of consumption smoothing. They investigate this empirically using data across time from 18 US states, and find that states with higher exemption levels have significantly lower tax redistribution policies.

A further field of study is the effect of bankruptcy exemption levels on the level of *secured* debt in an economy. This is the subject of two separate papers by Berkowitz and Hynes (1999) and Lin and White (2001), which present opposing theories on the effect of exemption levels on the level of secured borrowing. On the one hand, Lin and White (L-W) argue that higher exemption levels should reduce the supply of secured credit, arguing that secured creditors may restrict lending if they face some transaction cost whenever a borrower defaults on their unsecured debt, perhaps arising from a delay in payment. On the other hand, Berkowitz and Hynes (B-H) argue that this should not be the case; they reason that higher exemption levels preserve the wealth of bankruptcy filers, so they are more able to repay their secured creditors,

[7] This would be $20,000 for homeowners and $35,000 for non-homeowners, doubled for married couples.

and this should not have any adverse effects on the supply of secured credit. The empirical evidence is not clear-cut; while L-W find that applicants in states with higher exemption levels face a significantly higher probability of being denied mortgage credit, B-H do not find this to be the case.

Yet another line of research focuses on the effects of bankruptcy exemption levels on small businesses. This may be particularly important for unlisted private companies, since their owners are directly liable for any company debts. Fan and White (2003) look at the effects of exemption levels on the probability of starting a new business and find that households in states with higher exemption levels are more likely to own a business - more specifically, they find that households in states with unlimited exemptions are 35 per cent more likely to own a business than households in states with low exemptions, and this is statistically significant. The intuition behind this is that high exemption levels encourage risk-taking by households, which increases their willingness to start a business - in the case of business failure, they can declare bankruptcy. On the other hand, Berkowitz and White (2003) examine the effects of exemptions on access to small business credit, and find that small private businesses in states with unlimited exemptions face a 30 per cent higher chance of being denied credit than similar businesses in states with low exemptions. This follows earlier work by Scott and Smith (1986), who found that the relaxation of bankruptcy laws with the introduction of the new US bankruptcy code in 1978 caused interest rates charged to small businesses to rise.

Such research was extended by Mathur (2005), who examined the impact of bankruptcy exemptions on entrepreneurship using a model with spatial interactions to capture the effect of bankruptcy exemption levels in neighbouring states. Like Berkowitz and White (2001), Mathur found that high exemptions encourage entrepreneurship in a state by making it easier to start and shut down a business. In addition, the author finds that high exemptions in neighbouring states have an effect towards entrepreneurship. More specifically, entrepreneurs are significantly less likely to start a business in a state whose neighbouring states have high exemption levels while at the same time existing business owners are more likely to shut down their business in such states. A possible explanation for this is that high exemptions in neighbouring states encourage entrepreneurs to set up businesses across the border, and therefore less likely to start a business in that state. The author finds evidence of entrepreneurs shutting down businesses in a particular state, and restarted them in neighbouring states, which supports this argument.

3.2.2 Bankruptcy exemptions as endogenous variables

As mentioned above, all these studies have treated bankruptcy exemptions as exogenous variables. In comparison, the empirical literature on endogenous exemption levels is somewhat limited. A related study, that of Nunez and Rosenthal (2002), examines whether campaign contributions and 'ideology' influence voting on bankruptcy legislation in the House of Representatives and in the Senate by examining voting tendencies on the "Bankruptcy Reform" bill in 2001 which proposed removing the bankruptcy option for households with above-median income. This was backed by major lending and credit card institutions such as VISA USA and MBNA America Bank following fears of strategic bankruptcy decisions by American households. The study found that while ideology had a strong effect on voting[8], after controlling for this they found that contributions by major credit institutions did influence voting, estimating that at least 11-19 Democrats would have voted against the bill had they not received any contributions; since 290 votes are required to override a presidential veto, subtracting this from the 306 votes in favour of the bill would have put the veto-proof majority in question[9].

Probably the most cited study is that of Posner et al (2004), which examines whether changes in exemption levels can be explained by political factors based on the introduction of new federal bankruptcy laws in 1978. In 1978, the federal government introduced the Bankruptcy Reform Act, which introduced uniform exemption levels across states. Bankruptcy filers could then choose which set of exemptions to file under, those of the state they were filing in, and those set by the federal government. Naturally, a filer would choose the highest of the two, so in effect the new legislation raised the exemption levels in states where they were lower while states with higher exemptions were unaffected. The authors assumed that the states which were affected would opt out of the new legislation keeping their initial exemption levels. This was confirmed empirically as would be expected; states with exemption levels lower than the ones set by the federal government were significantly more likely to opt out than states with higher

[8] For example, all republicans in the House vote voted for the bill, while more liberal Democrats were more inclined to vote against it.

[9] A bill achieves a veto-proof majority with 306 votes or more. This makes it immune to any presidential vetoes. If majorities total less than 306 votes, the US president may veto against any bill.

exemptions. The authors also hypothesised that once such states opted out, they would raise their exemption levels[10] in order to 'bribe' those who preferred the higher federal exemptions and would otherwise try and block any attempts by the state government to opt out. The evidence supports this, since the decision by state governments to raise their exemptions is correlated with the decision to opt out at a 1% significance level. The paper then tests whether the level of exemptions is related to the presence of various groups within the population, such as migrants, lawyers or doctors[11], but finds no significant relationship.

Other than Posner et al (2004), there is a wide gap in this field, and this study seeks to address this. However, unlike their study, which focuses on political economy explanations, our focus is on whether exemption levels are influenced by economic fundamentals. Furthermore, this paper introduces spatial effects in the estimation of exemption levels to account for the fact that exemption levels are influenced by exemptions in other states. As mentioned above, research by Mathur (2005) examined the relationship between bankruptcy exemptions and entrepreneurship with a spatial effects model. However, that paper treated bankruptcy exemptions as the independent variable. To the best of our knowledge, this study is the first to estimate spacial effects in the determination of bankruptcy exemptions.

3.3 A first look at exemptions

Prior to 1979, there were no federal laws concerning personal bankruptcy. In 1979, new federal exemption levels were enacted which introduced sharply higher exemption levels than average. States were allowed to keep their own exemption levels, in which case bankruptcy filers could choose between the state and the federal exemption levels, or they were given the option to opt out of the new scheme. Opting out meant introducing new legislation in each state and so it took some time before most states had chosen to do so. In 1979, just 4 out of 50 states

[10] Obviously, to a level below the federal exemptions.

[11] The explanation given is the following. States may attempt to attract migrant workers with higher levels of bankruptcy exemptions, so the presence of a large migrant population may coincide with high exemption levels. Lawyers may pressure the government for higher exemption levels, since that would raise the number of bankruptcy filings, earning them more in fees. Doctors may also pressure for higher exemption levels as protection against lawsuits that they may face.

had opted out of the scheme[12]; by 1980, a further 11 did so[13]; by 1982, an additional 17 states opted out[14]; finally, most of the states which opted out had done so by 1983, when a further 3 did so[15].

This third chapter concentrates on the variation in home exemption levels in the United States rather than personal property exemptions. The reason for this is that it is very difficult to measure the true extent of personal property exemptions, as they specify exemptions on a variety of items rather than one overall exemption level. For example, they specify exemption levels on clothing, types of household furniture and appliances, types of jewellery, cars, pension schemes and other assets, which makes their measurement problematic and difficult[16]. A further problem with personal property exemptions is that one could quite easily conceal such property from creditors, whereas the same cannot be done with home exemptions.

3.3.1 Cross-sectional variation

As was mentioned above, exemption levels vary significantly between states. Table 3-6 in the appendix documents the exemption levels for a family of four in 2004. As we can see, home exemptions range from $0 in Delaware to $150,000 in Arizona and are unlimited in some states such as Kansas. To measure the spread in exemption levels, we assigned to all states with unlimited exemptions the largest (finite) value from all other states. Using this method, the average home exemption in 2004 was $191,000 while the standard deviation of the sample was substantial at $298,000.

[12] Florida, Louisiana, Ohio and Virginia.

[13] Alabama, Arizona, Georgia, Indiana, Kansas, Kentucky, Nebraska, Oklahoma, South Dakota, Tennessee and Wyoming.

[14] Arkansas, Colorado, Delaware, Iowa, Idaho, Illinois, Maryland, Maine, Minnesota, North Carolina, North Dakota, New Hampshire, Nevada, Oregon, South Carolina, Utah and West Virginia.

[15] Alaska, Missouri and New York.

[16] To see this, consider two states which specify exemptions on cars and fridges. State A specifies an exemption of $600 on a fridge and $900 on cars. State B specifies an exemption of $1300 on cars and no exemption on fridges. By just adding the two, one would conclude that state A has a more generous exemption level, since the total exemption in each state is $1500 for state A and $1300 for state B. However, this depends on the distribution of assets for the household. For example, if an average household owned a fridge whose second-hand value was $200 and a car with a second-hand value of $1500, it would prefer to file for bankruptcy in state B instead. Filing for bankruptcy in state A obliges it to pay $600 to its creditors, which is the excess value of its car (the fridge is competely exempt). By filing instead in state B, the household would have to pay its creditors a total of $400 ($200 excess on the car and $200 excess on the fridge). Therefore, by just adding up the different exemptions one would conclude that state A had the more lenient laws, when in fact an average household like the above would prefer to file in state B.

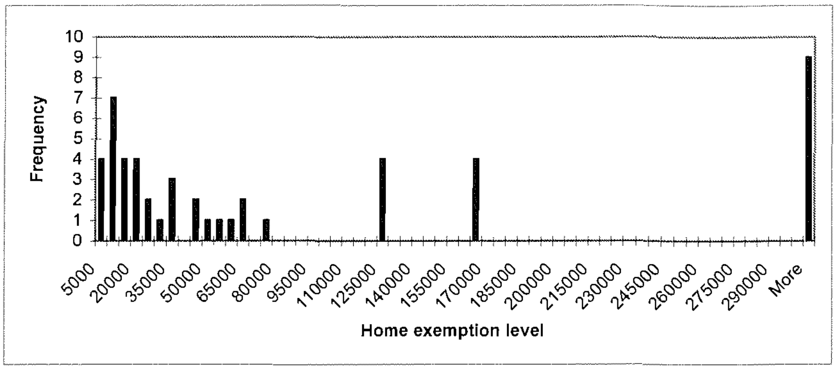

Figure 3-1: Histogram of home exemptions, 2004

Figure 3-1 plots the histogram of home exemptions across states in 2004, and reveals two interesting features. First, there is indeed a wide dispersion of exemption levels across states. Secondly, there is a cluster of states with very high exemptions in the tail end of each distribution, and this is mainly due to states with unlimited exemptions.

3.3.2 Variation across time

While there is significant variation in exemptions across states, there has also been significant variation over the last 30 years. As many other authors have noted, there has been an increase in average exemption levels across the US over time. This is illustrated in Figure 3-2 below, which plots the average home exemption across US states in real prices[17], from 1975 to 2004.

As the Figure 3-2 reveals, average home exemptions across the US have steadily been rising, more than doubling from $61,500 in 1975 to $155,700 in 2004. However, a closer inspection reveals a more complicated picture; out of the 50 US states, home exemptions (in real terms) have actually fallen in 15 cases[18]. The average home exemption across these states has nearly halved from $68,400 in 1975 to $35,700 in 2004, with a large drop coming after 1990, and this is plotted in Figure 3-3. Exemptions have remained at zero in 2 states throughout this period[19] and have increased in the remaining 33, from an average of $62,100 in 1975 to $222,400 in 2004, as Figure 3-4 illustrates.

[17] In 1996 US$.

[18] These are Alabama, Alaska, Hawaii, Illinois, Louisiana, Minnesota, Nebraska, New Mexico, North Dakota,

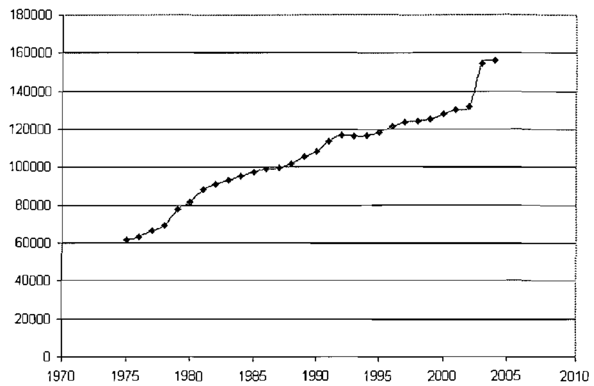

Figure 3-2: Average home exemptions, 1975-2004.

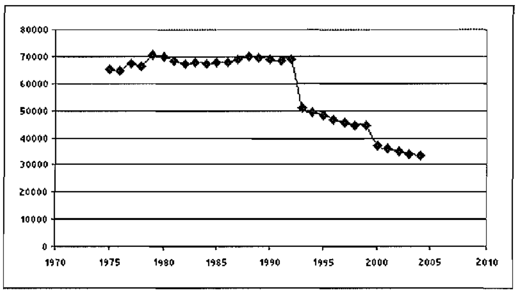

Figure 3-3: Average of states with falling home exemptions, 1975-2004.

3.3.3 Geographical links

Casual observation of the data suggests a possible correlation in exemption levels between neighbouring states. For example, Posner et al (2004) provide visual representations of the level of bankruptcy exemptions across US states and find that states with high exemptions and states with low exemptions can be found in clusters. This is further illustrated in the map in Figure 3-5. States which are unshaded (white) are the ones which have unlimited exemption levels and they form a spine down the middle. States with the darker shading are those which have lowered exemption levels, and states with the lighter shading have increased them. As

Oregon, Tennessee, Virginia, Washington, Wisconsin and Wyoming.

[19]These are Delaware and Maryland.

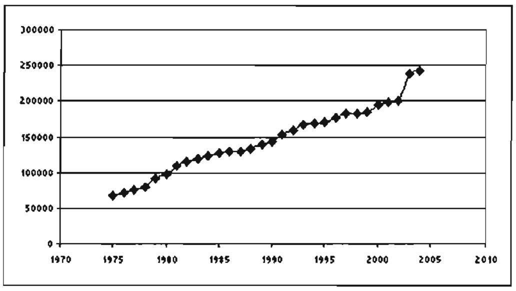

Figure 3-4: Average of states with rising home exemptions, 1975-2004.

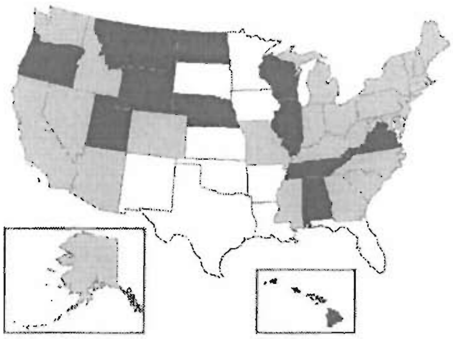

Figure 3-5: Changes in home exemption levels, 1975-2004.

the map shows, there is a greater proportion of states in the West which have lowered their exemption levels than there is in the East, and such states are broadly clustered together.

To summarise, there are two aims of this study. The first aim is to help understand the reasons for such wide heterogeneity in bankruptcy exemptions across states and over time. The second aim is to evaluate whether the perceived spatial interactions between neighbouring states really have a role in the setting of exemption levels, or whether they can be explained by economic fundamentals[20].

The remaining part of this chapter focuses on providing answers to these questions. In section 3.4, we develop a simple, two-period model, where we prove that any exogenous shock which causes borrowing to increase implies a higher exemption level if these are set optimally.

[20] Under the assumption that neighbouring states have similar economic environments, we might expect them to have similar exemption levels.

Section 3.5 continues with the empirical analysis, which tests, first, whether this rule can explain the variation observed in bankruptcy exemptions, and second, whether neighbouring effects exist. Finally, section 3.6 concludes.

3.4 Theory

The model presented follows similar lines as that of the previous two chapters, but has been simplified for ease of notation. As we are examining the optimal policy in a small economy, we assume an exogenous risk-free rate of interest r_f at which banks can obtain funds[21]. For this reason, government cannot affect the welfare of savers in the economy; in a competitive market, such agents will save at the risk-free rate of interest, which is exogenously fixed and is not affected by changes in the exemption level. Therefore, for simplicity, we assume that the economy consists of a continuum of just one type of agent whose endowment is such that they end up borrowing at equilibrium. There are two time periods denoted by $t = 0, 1$ with one good in each time period. The endowment of each agent at time 0 is given by e_0, while at time $t = 1$ we drop the subscript for simplicity and denote endowments as e. In the first time period, all agents start off with a constant e_0, whereas in the second time period there is idiosyncratic uncertainty and each agent's endowment $\widetilde{e}$ is a continuous random variable ranging from $\underline{e} > 0$ to $\overline{e} > \underline{e}$ with a probability density function $f(e)$. Agents have standard concave, time-separable utility functions.

3.4.1 The trading process

In the first time period, banks (from any region) offer a contract (y, r) to each consumer, where y specifies the quantity traded and r the interest rate. Agents then trade the contract which yields them the highest utility. In the second time period, all uncertainty is resolved, and each agent observes their true endowment. Borrowers then choose whether or not to declare bankruptcy. Bankruptcy legislation is set by the state and resembles the current system in the

[21] In the US, each state sets its own exemption levels. Given that the size of each state is small compared to the world economy, it is not unrealistic to assume that governments take the risk-free interest rate as given when doing so.

US[22]. That is, the state sets an exemption level X, which specifies how much of a bankruptcy filer's endowment is exempt from repossession. If an agent declares bankruptcy, they have to repay their outstanding debts with any non-exempt assets, and this is the value of their endowment over and above the specified exemption level X - if their total endowment is less than this exemption level, then it is all exempt and they do not have to repay their creditors anything[23]. We now solve the model by backwards induction.

3.4.2 The second time period

Agents

In the second time period, agents hold the contract (y, r). The true endowment of each agent, e, is then revealed. If they have saved from the first time period (i.e. $y > 0$), they receive $r\ y$ from the bank, and consume $e + r\ y$. If instead they have borrowed (i.e. $y < 0$), they must choose whether or not to declare bankruptcy. If they do not declare bankruptcy, they pay their creditors in full and consume $e + r\ y$. If they do declare bankruptcy, then the amount they pay their creditors will depend on their endowment. If this is below the exemption level X, then they are fully exempt from making any payments. If on the other hand their endowment is above X, then they have to pay their creditors an amount up to and including their liability, using their non-exempt assets. Hence, if agents declare bankruptcy, the amount they have to pay to their creditors is given as $\min\{\max\{0,\ e - X\}, -r\ y\}$[24].

Proposition 13 *Agents will declare bankruptcy for all endowment levels e such that:*

$$e < X - r\ y \tag{3.1}$$

[22] This is chosen optimally by the government to maximise the welfare arising from subsequent equilibrium allocations in the economy. We present this maximisation problem formally in the sections below.

[23] For example, suppose that the exemption level is \$10, and an agent owes the bank \$5. If the total endowment of the agent is less than \$10, then his entire endowment will be exempt, so the agent will not pay his creditors anything in bankruptcy. If the endowment is \$12, then the agent can go bankrupt and only pay his creditors \$2. If the endowment is \$15 or more, then there is no use in going bankrupt as the agent will have to pay back the entire amount owed to the bank.

[24] If $e < X$, the agent is fully exempt and this is equal to zero. If $e \geq X$, the agent is not fully exempt and so has to pay his creditor from his non-exempt assets. If these are enough to cover his debts then the agent must repay the full amount, $-r\ y$. If however these are not enough then the agent just pays back with all his non-exempt assets, an amount equal to $e - X$. See note above for an example.

The proof can be found in the appendix. This is fairly intuitive; for any income levels below $X - r\ y$, the amount paid in bankruptcy is less than the amount owed and hence it is beneficial to go bankrupt. For all endowment levels above that, the agent is indifferent between declaring bankruptcy or not since their consumption is the same in both cases, so we assume that he does not. Taking into account this optimal bankruptcy decisions, whether the agent borrows or saves, consumption $x(e)$ as a function of the agent's endowment in the second time period is given as:

$$x(e; y, r, X) = e - \min\{\max\{0,\ e - X\}, -r\ y\} \tag{3.2}$$

We now plot this function for agents who borrow in Figure 3-6 below.

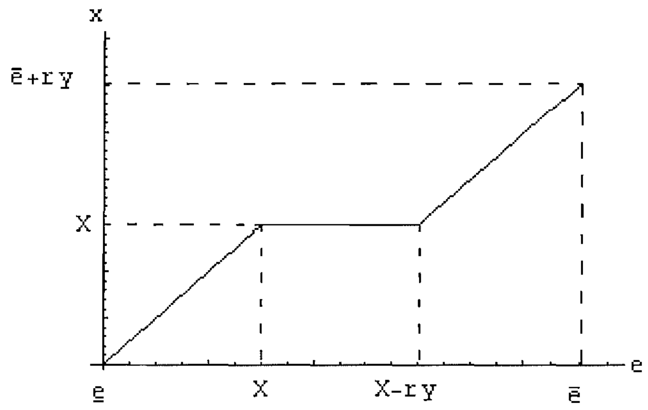

Figure 3-6: Second-period consumption of borrowers as a function of their endowment.

The bank

In the second time period, the bank has to honour its liabilities towards savers and pay them the amount promised, while from borrowers it collects the full amount, $-r\ y$, if they have not declared bankruptcy, while if instead they have declared bankruptcy, the bank collects $\min\{\max\{0,\ e - X\}, -r\ y\}$. In general, the bank receives $\min\{\max\{0,\ e - X\}, -r\ y\}$ from all agents it trades a contract with. Furthermore, a bank can invest every deposit it collects at the risk-free rate of interest r_f, while any loans it makes it has to fund with that rate of interest. Therefore, the profit Π from each contract offered to agents is given as:

$$\Pi = y\ r_f + \int_{\underline{e}}^{\bar{e}} f(e)\ \min\left\{\max\left\{0,\ e - X\right\}, -r\ y\right\} de \tag{3.3}$$

3.4.3 The first time period

In the first time period, banks offer contracts (y, r) to each consumer. Consumers then trade the contract which yields them the highest utility. We call this the value of each contract and this is given as:

$$V(y,r) = u(e_0 - y) + \int_{\underline{e}}^{\bar{e}} f(e)\ u(x(e; y, r, X))\ de \tag{3.4}$$

If $y > 0$, this is just given as:

$$V(y,r) = u(e_0 - y) + \int_{\underline{e}}^{\bar{e}} f(e)\ u(e + r\ y)\ de \tag{3.5}$$

However, for borrowers (i.e. $y < 0$), this is given as:

$$\begin{aligned} V(y,r) &= u(e_0 - y) + \int_{\underline{e}}^{X} f(e)\ u(e)\ de + \int_{X}^{X - r\ y} f(e)\ u(X)\ de \\ &\quad + \int_{X - r\ y}^{\bar{e}} f(e)\ u(e + r\ y)\ de \end{aligned} \tag{3.6}$$

which takes into account the optimal bankruptcy decision in (3.1)[25].

The bank has to offer the contracts which maximise its profit in a perfectly competitive environment. Due to free entry and perfect competition, the contract offered to each consumer will be the one which offers the highest value, subject to a zero-profit constraint (see Allen and Gale (2005)). In other words, each contract which is offered must be the solution to the

[25] For all endowment levels below X, the agent declares bankruptcy and is fully exempt from repaying his creditors. Therefore, for all such endowment levels, he consumes e. In addition, for all endowment levels between X and $X - r\ y$, the agent goes bankrupt but has to repay his creditors with any non-exempt assets. Therefore, the agent just consumes his exempt assets, whose value is given by X. Finally, for all endowment levels above $X - r\ y$, the agent does not go bankrupt and fully repays his creditors. There, for all such endowment levels, his consumption is given as $e + r\ y$.

following maximisation problem:

$$\max_{y,r} V(y,r) \tag{3.7}$$
$$\text{s.t.} \quad \Pi = 0 \tag{3.8}$$

3.4.4 Optimal exemption levels

We now examine the problem from the point of view of an authority, or government, which decides on the exemption level X. The aim of this government is to choose the exemption level which maximises the value of the equilibrium contract offered to consumers. In the proposition that follows, we devise a simple rule for setting exemption levels optimally.

Proposition 14 *An exogenous shock which increases borrowing at every given exemption level implies a higher exemption level.*

Proof. To see this, consider the government's maximisation problem. This is to choose an exemption level X which maximises the borrower's utility at the equilibrium allocations:

$$\max_{X} \left\{ \begin{array}{cc} \max\limits_{y,r} & V(y,r;X) \\ \text{s.t.} & \Pi = 0 \end{array} \right\} \tag{3.9}$$

We can restate this problem in the following way:

$$\max_{X,y,r} \quad V(y,r;X) \tag{3.10}$$
$$\text{s.t.} \quad \Pi = 0 \tag{3.11}$$

Using the Lagrange maximisation method, we first set up a Lagrangian L:

$$L = V(y,r;X) + \lambda\Pi$$

The necessary first-order conditions for an optimum are given as[26]:

$$\frac{\partial L}{\partial y} = -u'(e_0 - r\,y) + r\int_{X-r\,y}^{\bar{e}} f(e)\;u'(e+r\,y)\;de + \lambda\left[r_f - r\int_{X-r\,y}^{\bar{e}} f(e)\;de\right] = 0 \quad (3.12)$$

$$\frac{\partial L}{\partial r} = y\left[\int_{X-r\,y}^{\bar{e}} f(e)\;u'(e+r\,y)\;de - \lambda\int_{X-r\,y}^{\bar{e}} f(e)\;de\right] = 0 \quad (3.13)$$

$$\frac{\partial L}{\partial X} = \left[u'(X) - \lambda\right]\int_{X}^{X-r\,y} f(e)\,de = 0 \quad (3.14)$$

$$\frac{\partial L}{\partial \lambda} = r_f\;y + \int_{X}^{X-r\,y} f(e)\;\left[e - X\right]\;de - \int_{X-r\,y}^{\bar{e}} f(e)\;r\;y\;de = 0 \quad (3.15)$$

Now consider the third condition, equation (3.14). Since $f(e) > 0$ for all realisable endowment levels, this implies that for any $y < 0$ and $r > 0$, $\int_X^{X-r\,y} f(e)\;de > 0$, so this condition is only satisfied if $\lambda = u'(X)$. Substituting this into the second Lagrange condition and assuming $y < 0$, we obtain:

$$\int_{X-r\,y}^{\bar{e}} f(e)\;u'(e+r\,y)\;de - \int_{X-r\,y}^{\bar{e}} f(e)\;u'(X)\;de = 0 \quad (3.16)$$

Finally, this equation can only hold if $r = \frac{X-\bar{e}}{y}$, or $X - r\,y = \bar{e}$. This is because for all endowment levels greater than $X - r\,y$, consumption is greater than X:

$$e > X - r\,y \Rightarrow e + r\,y > X \quad (3.17)$$

$$\Rightarrow \quad u'(e+r\,y) < u'(X) \quad (3.18)$$

$$\Rightarrow \quad \int_{X-r\,y}^{\bar{e}} f(e)\;u'(e+r\,y)\;de - \int_{X-r\,y}^{\bar{e}} f(e)\;u'(X)\;de < 0 \quad (3.19)$$

The only case where we get equality is when $X - r\,y = \bar{e}$:

$$X - r\,y = \bar{e} \quad (3.20)$$

$$\Rightarrow \quad \overbrace{\int_{X-r\,y}^{\bar{e}} f(e)\;u'(e+r\,y)\;de}^{=0} - \overbrace{\int_{X-r\,y}^{\bar{e}} f(e)\;u'(X)\;de}^{=0} = 0 \quad (3.21)$$

[26]These have been simplified for the sake of brevity, but their full derivation is available in the appendix.

Furthermore, it is easy to show that the solution to the government's maximisation problem is unique. After solving the second and third Lagrangian conditions ($\frac{\partial L}{\partial r} \equiv 0$ and $\frac{\partial L}{\partial X} \equiv 0$) for $\lambda = u'(X)$ and $r = \frac{X-\bar{e}}{y}$, the remaining two conditions simplify to:

$$\frac{\partial L}{\partial y} = r_f\, u'(X) - u'(e_0 - y) \equiv 0 \quad (3.22)$$

$$\frac{\partial L}{\partial \lambda} = r_f\, y + \int_X^{\bar{e}} f(e)\, [e - X]\, de \equiv 0 \quad (3.23)$$

Solving for y from $\frac{\partial L}{\partial \lambda}\# = 0$, we obtain

$$y = -\frac{\int_X^{\bar{e}} f(e)\, [e - X]\, de}{r_f} \quad (3.24)$$

Substituting this into $\frac{\partial L}{\partial y}$ yields:

$$\frac{\partial L}{\partial y} = r_f\, u'(X) - u'\left(e_0 + \frac{\int_X^{\bar{e}} f(e)\, [e - X]\, de}{r_f}\right) \equiv 0 \quad (3.25)$$

We now take the derivative of this with respect to X, which is strictly negative for all X:

$$\frac{\partial}{\partial X}\left(\frac{\partial L}{\partial y}\right) = r_f\, u''(X) + \frac{\left[\int_X^{\bar{e}} f(e)\, de\right] u''\left(e_0 + \frac{\int_X^{\bar{e}} f(e)\, [e-X] de}{r_f}\right)}{r_f} < 0 \quad (3.26)$$

which follows due to the strict concavity of u ($u'' < 0$). Therefore, there can only be one X for which Eq. (3.25) holds. Hence, there can only be one solution to the government's maximisation problem.

Finally, consider the last part of the proof. We have shown in Eq. (3.25) that the optimal exemption level X^* must be such that:

$$r_f\; u'(X^*) - u'(e_0 - y(X^*)) = 0 \quad (3.27)$$

where, by Eq. (3.24):

$$y(X) = -\frac{\int_X^{\bar{e}} f(e)\, [e - X]\, de}{r_f} \quad (3.28)$$

Let equilibrium borrowing for any given exemption level X and economic parameter σ be denoted by $y(X;\sigma)$. Suppose that after an exogenous shock which moves the parameter to σ', borrowing increases for any given exemption level:

$$y(X;\sigma') < y(X;\sigma)\ \forall X \in [0,\bar{e}] \tag{3.29}$$

Then, keeping X fixed, $u'(e_0 - y(X))$ decreases, due to the concavity of u. Since (3.27) is strictly decreasing in X (see Eq. (3.26)), this implies that X has to increase; let the new optimum be denoted by X'. Then, $X' > X^*$:

$$u'\left(e_0 - y\left(X^*,\sigma'\right)\right) < u'\left(e_0 - y\left(X^*,\sigma\right)\right) \tag{3.30}$$

$$\Rightarrow\ r_f\, u'(X^*) - u'\left(e_0 - y\left(X^*,\sigma'\right)\right) >$$

$$r_f\, u'(X^*) - u'\left(e_0 - y\left(X^*,\sigma\right)\right) = 0 \tag{3.31}$$

$$\therefore\ r_f\, u'\left(X'\right) - u'\left(e_0 - y\left(X',\sigma'\right)\right) = 0 \Leftrightarrow X' > X^* \tag{3.32}$$

■

Thus, we have derived a simple rule for setting exemption levels: anything that causes credit to expand implies a higher exemption level, and vice versa. What is remarkable about this derivation is that it departs from conventional thinking on exemption levels and shocks to endowments such as unemployment. This relates the level of exemptions to degree of income volatility in an economy, stating that regions with higher income volatility should have higher exemptions to enable consumers to use bankruptcy as a form of insurance against a drop in income. However, in our model, income volatility does not have a *direct* effect on the optimal exemption level, but rather an indirect one through its effect on the level of borrowing.

3.5 Data and estimation

We now proceed to the empirical part of our analysis. The main aims are: (i) to test the above proposition, that any shock which increases borrowing implies higher exemption levels; and (ii) to examine whether neighbourhood effects exist in the setting of exemption levels. Such spatial effects have not been included in the theoretical analysis above. However, we include them in our empirical modelling following arguments for, and casual evidence of, their existence. This can also be seen essentially as a specification issue: if such effects exist which we do not control for, this may distort our empirical results.

The data for exemption levels has been obtained from two sources. For the years 1975-1996, the data has been obtained from Eric Posner and Richard Hynes, who used this in their (2004) coauthored paper with Anup Malani. From 1997 to 2004, data on exemption levels has been obtained from a series of legal books (Elias et al).

An initial problem with the data is that several states specify unlimited home exemptions, so bankruptcy filers can keep their entire property regardless of its value. To deal with this, we follow the method used by Posner et al (2004) and assign the average of the two largest exemption levels (in real terms) across all time periods to states which specify unlimited exemptions. This makes our results directly comparable with the most similar study to ours. For robustness however, we also deal with this in other ways, but find no significant impact to our results[27].

3.5.1 Hypotheses

We test the effect of the following variables on the level of exemptions in each state.

Income Higher incomes should, all else equal, lead to higher levels of borrowing, as agents borrow more in absolute terms to smooth consumption. Therefore, by Proposition 2, we would expect states with higher incomes to have higher bankruptcy exemptions.

Unemployment Unemployment has two potential effects on borrowing: (i) on the one hand, more unemployment may reduce the amount of borrowing and hence imply a lower optimal exemption level, since this increases the risk of default and banks may respond by restricting credit; (ii) on the other hand, if unemployment spells are brief, and if it they occur mainly at earlier stages of an agent's life, then they may increase the need to borrow against future income to smooth consumption over time, in which case they imply a higher exemption level.

[27] First, we use Grant and Koeniger's (2004) method of assigning just the largest exemption. We also ran regressions which included just those states without unlimited exemptions. Neither method altered our results in any significant way.

The evidence suggests the latter is true; for example, in 2005, the unemployment rate for age groups 20-24 and 25-34 was 8.8% and 5.1% respectively, while that for age groups 35-44 and 45-54 was only 3.9% and 3.5% in 2004[28]. It seems then that unemployment affects younger agents more than it does older agents, so we would expect states with higher unemployment to have higher exemption levels.

Divorce Much evidence points to divorce being a major influence on the decision to declare bankruptcy[29]. If banks take this into account, this could reduce the supply of credit to consumers. Consumers themselves may demand less credit as a result of the future uncertainty brought about by a higher chance of divorce. Hence, if this holds, states with higher divorce rates should have lower exemption levels.

Inequality We use the Gini coefficient to measure the degree of inequality in the US[30]. As with unemployment, this could have two effects - if inequality is mainly between people of different age groups, with older members of the population on much higher incomes than younger members of the population, then more inequality may lead to an increase in the amount borrowed as agents smooth their consumption over time. If this is the case, then higher inequality will imply higher optimal bankruptcy exemptions. However, if it mainly takes the form of high inequality between members of the same age group, it could have the opposite effect and reduce the amount borrowed as it makes future incomes uncertain for young agents, which could induce them to borrow less. In this case, higher inequality will imply lower bankruptcy exemptions.

Tax We use the average tax rate as a proxy to the size of the welfare state in each region. Grant and Koeniger (2004) found that states with high bankruptcy exemptions were associated with lower welfare provision as both are designed to help smooth intra-temporal consumption. Under our model, a larger welfare state could have two effects on exemptions. On the one hand, all else equal, it may make individuals less likely to declare bankruptcy as they receive

[28] Source: Bureau of Labor Statistics.

[29] For example, Fay et al (2002) find that such shocks can be the main force behind decisions to go bankrupt.

[30] The Gini coefficient is a number between zero and one. A reading of zero implies full equality, where everyone has the same income, whole a reading of one implies perfect inequality, where one person has all the income and everyone else has none.

more support from the state at times of trouble. This should lower interest rates and therefore lead to more borrowing, which implies a higher bankruptcy exemption. On the other hand, it could reduce borrowing by agents who experience a temporary drop in their earnings as this is counteracted by state benefits. It could also lead to less borrowing by young agents with families by providing them with greater help in the form of, for example, tax relief or child support benefits. If this is so, then a larger welfare state should lead to less borrowing and hence will imply lower bankruptcy exemptions.

House prices With high house prices come higher levels of secured debt, as larger mortgages are required by agents. This has a similar effect to reducing an agent's disposable income, as it costs agents more in terms of mortgage repayments for younger agents who have mortgages, or higher rent payments for agents who do not own a property. Lower lifetime disposable incomes should reduce borrowing, as agents need less in absolute terms to smooth their consumption over time. This is backed empirically by Del Rio and Young (2005), who find evidence that agents with higher mortgages have lower levels of unsecured debt. Furthermore, higher mortgage repayments should, all else equal, make agents more likely to declare bankruptcy[31], which may lead banks to restrict credit. If this is the case, higher house prices lead to less unsecured borrowing and therefore imply a lower optimal exemption level.

Politics We adjust for political ideology of each state by including a dummy variable in our regressions which takes into account the electoral college of each state. How political affiliation should influence exemption levels in a state is unclear. Some may argue that Republican politicians tend to favour 'pro-creditor' or strict laws whereas more liberal Democratic politicians tend to favour 'pro-debtor' or lenient legislation[32]. According to this view, we might expect Republican-controlled states to have lower exemption levels than Democrat-controlled states. Another view might be that Republican politicians tend to represent the middle classes whereas Democrats gain more support from working class sections of the population. Given that the middle classes are likely to borrow more in absolute terms, our model suggests that they would benefit by higher exemption levels, so if Republican politicians are biased towards that section

[31] This has been documented empirically by Domowitz and Sartain (1999).
[32] See, for example, Nunez and Rosenthal (2002).

of the population then they would favour higher exemption levels. In this case, we might expect Republican-controlled states to have higher exemption levels than Democrat-controlled states.

Geographical links Finally, we test whether bankruptcy exemptions in each state are affected by those in neighbouring states. As discussed above, preliminary evidence certainly suggests this, as illustrated in Figure 3-5 and in figures by Posner et al (2004), so ignoring these may distort inferences. The reason for this is unclear and we have not provided a model to illustrate this. One plausible reason could be that states are competing with each other to attract high income households with large amounts of debt accumulated. For example, evidence of extremely wealthy individuals moving to Florida before filing for bankruptcy in order to take advantage of the unlimited homestead exemptions, saving them millions of dollars, can be found in Rochter (1993)[33]. Further evidence of bankruptcy-induced state migration is presented by Elul and Subramanian (2001), who find that had bankruptcy not been a factor in migration decisions, there would be 3.1% less moves to neighbouring higher-exemption states (p. 18).

3.5.2 Data sources

The data used in our regressions was obtained from several sources. These are summarised in Table 3-1 below. All variables are state-specific, apart from the Gini coefficient which is US-wide. We provide some descriptive statistics in the Appendix, under Section 3.7.6..

[33]In this article, a Florida judge was quoted as saying "*You could shelter the Taj Mahal in this state and nobody could do anything about it*".

Table 3-1. Data description and sources

Variable	Description	Source
HE	Home exemption	1975-1996: Posner et al (2004) Elias et al (1998-2005)
ipc	Real income per capita	Bureau of Economic Analysis
tax	Income tax rate $\left(= 1 - \frac{\text{Real disposable income per capita}}{\text{Real income per capita}}\right)$	Bureau of Economic Analysis
unem	Unemployment rate	Bureau of Labor Statistics
div	Divorce rate (Number of divorces per 1,000 people)	National Centre for Health Statistics
hprice	Real house prices (index)	Federal Housing Finance Board
gini	US Gini coefficient (nationwide)	US Census Bureau
politics	Dummy variable for political inclination Republican = 0; Democrat = 1.	www.wikipedia.org

The analysis consists of two types of spatial model specifications. Pooled regressions with fixed effects are run under a spatial autoregressive lag (SAR) model and a spatial autoregressive error model (SER). The former includes exemption levels in neighbouring states as a regressor, so that states take into account exemption levels in neighbouring states when setting their own exemptions. On the other hand, the SER takes into account spatial correlation in the error terms of the regression. This may be present if observations are interdependent through unmeasured variables that are correlated through space. However, it could also occur if there is co-movement in exemption levels. For example, if a state increased its exemption level, this should show up in the model as a positive error term. If, as a result, neighbouring states increase their exemption levels as well, then this could also show up as a positive error term. In this case, if neighbouring states tend to change their exemption levels at around the same time, the data would exhibit positive spatial correlation in the error terms.

3.5.3 Regression results

We begin our analysis with four statistical tests to detect the presence of spatial autocorrelation in the error term for each individual year[34]. The test is what is known as Moran's I test. This is based on the residuals obtained from performing a simple cross-sectional OLS regression, as such:

$$Y = X\beta + \rho WY + \varepsilon \quad (3.33)$$

$$\varepsilon \sim N\left(0, \sigma^2 I\right) \quad (3.34)$$

Moran's I statistic is then given as:

$$I = \frac{\varepsilon' W \varepsilon}{\varepsilon' \varepsilon} \quad (3.35)$$

where W denotes the spatial weights matrix, a 50×50 matrix whose $(i,j)^{th}$ term is equal to $\frac{1}{n_i}$ if states i and j are neighbours, where n_i is the number of neighbours of state i, and zero otherwise[35]. The mean and variance of this statistic are given as:

$$E(I) = \operatorname{tr}(MW)/(N-K) \quad (3.36)$$

$$V(I) = \left[MWMW' + \operatorname{tr}(MW)^2 + \left(\operatorname{tr}(MW)^2\right)\right]/d - E(I)^2 \quad (3.37)$$

$$M = I - X\left(X'X\right)^{-1}X' \quad (3.38)$$

where N is the number of observations and K is the number of regressors. The I statistic can then be standardised by subtracting its mean and dividing by its standard deviation, so that

[34]Full details of all these tests are given in Anselin (1988) and LeSage (1999).
[35]This matrix is standardised, so that the rows of matrix W add up to 1.

its asymptotic distribution follows a standard normal distribution (Cliff and Ord, 1972).

$$Z_I = [I - E(I)] / V(I)^{1/2} \tag{3.39}$$

The second test is a likelihood ratio test based on the difference between the log likelihood from a SER regression and the log likelihood from a least-squares regression, and this statistic follows a $\chi^2(1)$ distribution. The SER model is estimated by maximum likelihood estimation[36] and takes the form:

$$\begin{aligned} Y &= X\beta + u && (3.40) \\ u &= \lambda W u + \varepsilon && (3.41) \\ \varepsilon &\sim N\left(0, \sigma^2 I\right) && (3.42) \end{aligned}$$

where λ denotes the spatial error autoregressive parameter. The explanatory variables (matrix X) consist of real income per capita, unemployment, the divorce rate, the tax rate, house prices and the political ideology dummy.

The third test is known as the Wald test for spatial error autocorrelation and this statistic also follows a $\chi^2(1)$ distribution, which is given as:

$$\begin{aligned} w &= \lambda^2 \left(t_2 + t_3 - t_1^2/n\right) \sim \chi^2(1) && (3.43) \\ t_1 &= \operatorname{tr}\left(W.*B^{-1}\right) && (3.44) \\ t_2 &= \operatorname{tr}\left(WB^{-1}\right)^2 && (3.45) \\ t_3 &= \operatorname{tr}\left(WB^{-1}\right)'\left(WB^{-1}\right) && (3.46) \end{aligned}$$

where λ is the maximum likelihood estimation of λ in Eq. (3.41), $B = (I_n - \lambda W)$, and $.*$ denotes element-by-element matrix multiplication.

The fourth and final test is known as the Lagrange Multiplier test. This statistic too follows

[36]The Matlab programs required for this estimation and all other tools employed are freely available from James P. LeSage's website at www.spatial-econometrics.com.

a $\chi^2(1)$ distribution and is given as:

$$LM = (1/T)\left[\left(e'We/\sigma^2\right)\right]^2 \sim \chi^2(1) \tag{3.47}$$

$$T = \left[\text{tr}\left(W + W'\right)\right] W \tag{3.48}$$

where e denotes the least-square residuals of Eq. (3.41).

The full results of each of these tests are provided Table 3-7 in the appendix, and are summarised in the graphs contained within Figure (3-7). These reveal an interesting pattern and seem to suggest that such spatial effects have been declining over the years; while the test statistics are initially high and significant, they are constantly falling and become insignificant from around 1990 onwards. The dotted line in each case indicates the 10% significance level for each statistic.

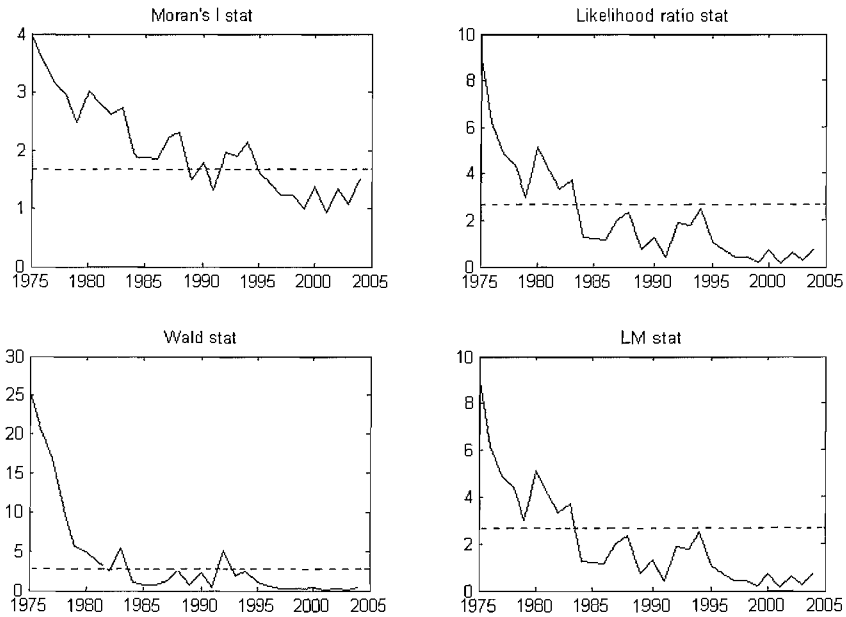

Figure 3-7: Test results for the presence of spatial autoregressive error terms, 1975-2004.

We also run the following SAR regression, estimated by maximum likelihood estimation, for

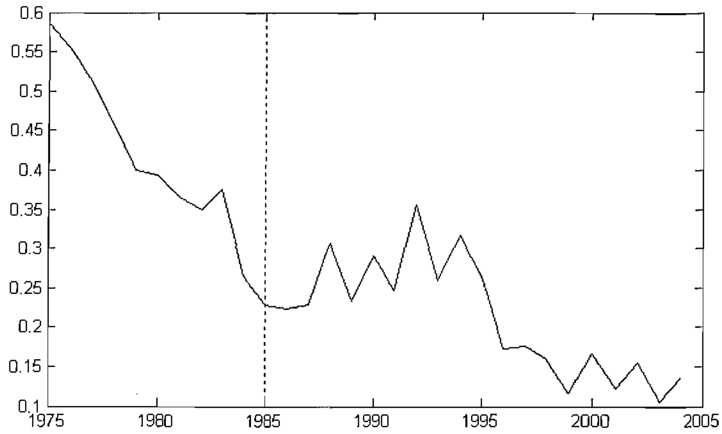

Figure 3-8: Coefficient of spatial autocorrelation, 1975-2004.

each individual year:

$$Y = X\beta + \rho WY + \varepsilon \tag{3.49}$$

$$\varepsilon \sim N\left(0, \sigma^2 I\right) \tag{3.50}$$

where ρ is the coefficient of spatial autocorrelation and is plotted in Figure (3-8). This reveals a similar pattern as above - the results seem to suggest the presence of some spatial effects, particularly in earlier years. The coefficient ρ was found to be positive in every year, but has been declining steadily and becomes insignificant from 1986 onwards[37]. Overall therefore, these preliminary results suggest the presence of positive but declining spatial effects.

We proceed with the analysis through the use of spatial panel regressions. We run two separate panel regressions - a spatial error autoregressive (SER) model and a spatial lag au-

[37] With the exception of the years 1988, 1990, 1993 and 1995.

toregressive (SAR) regression. The SER model is given as:

$$Y_{it} = \alpha_0 + \sum_i \alpha_i state_i + X_{it}\beta + u_{it} \quad (3.51)$$

$$u_{it} = \lambda W_i u_t + \varepsilon_{it} \quad (3.52)$$

$$\varepsilon_{it} \sim N\left(0, \sigma^2\right) \quad (3.53)$$

where $state_i$ is the dummy variable capturing the fixed effects in each state, W_i is a 1×50 spatial weights vector of state i whose j^{th} entry is equal to $\frac{1}{n_i}$ if states i and j are neighbours, where n_i is the number of neighbours of state i, and zero otherwise. The explanatory vector X_{it} consists of real income per capita, unemployment, the divorce rate, the tax rate, house prices, the US gini coefficient of inequality, and political ideology. Using the same explanatory vector, we also run the following SAR panel regression:

$$Y_{it} = \alpha_0 + \rho W_i Y_t + \sum_i \alpha_i state_i + X_{it}\beta + \varepsilon_{it} \quad (3.54)$$

$$\varepsilon_{it} \sim N\left(0, \sigma^2\right) \quad (3.55)$$

The results are illustrated in Tables 3-2 and 3-3 below[38]. These results are very interesting, since they are broadly in line with the predictions of our simple model in both regressions.

Table 3-2. Panel SER regression results with fixed effects, 1975-2004

ipc	*tax*	*unem*	*gini*	*div*	*hprice*	*politics*
6.48 (0.00)	$-264,068$ (0.11)	**4,540** (0.00)	**473,006** (0.00)	**−9,945** (0.00)	**−318.9** (0.00)	**−7,608** (0.08)
λ	*Adjusted* R^2					
0.054 (0.11)	0.86					

[38] In addition, the fixed effects coefficients for the SER panel regression are given in Table A3-8 in the appendix. The terms in brackets are the p-values of each coefficient.

Table 3-3. **Panel SAR regression results with fixed effects, 1975-2004**

ipc	*tax*	*unem*	*gini*	*div*	*hprice*	*politics*
6.55 (0.00)	**−277,932** (0.08)	**4,444** (0.00)	**471,073** (0.00)	**−9,744** (0.00)	**−323.0** (0.00)	**−7,432** (0.08)
ρ	*Adjusted* R^2					
0.005 (0.89)	0.86					

Consider first the coefficient of real income per capita. This was found to be significantly positive in both cases, and is consistent with the predictions of our model, since a rise in lifetime incomes is likely to lead to more borrowing and therefore implies a higher optimal exemption level.

Furthermore, the tax rate coefficient was found to be negative in both cases, but was with a p-value of 0.11 and 0.08. We can rationalise a negative tax coefficient through our model. We argued that, if seen as a proxy to the welfare state and degree of redistribution, a higher tax rate may imply less borrowing by smoothing the income of agents over time - greater support for the unemployed and for young family households could reduce the need for borrowing to finance their expenses. Less borrowing in turn implies a lower optimal exemption level, and hence a negative relationship between the tax rate and the home exemption level.

In addition, the unemployment rate was found to be significantly positive in both regressions. In the context of our model, we argued that if unemployment affects mostly younger agents, which evidence supports, then it is likely to lead to more borrowing, as such agents borrow more against their future incomes to finance their current expenditures and smooth consumption. In turn, more borrowing implies imply a higher optimal exemption level, so that a positive relationship exists between the unemployment rate and the home exemption level, as was found in the data. Note that this is in effect the opposite effect to higher tax rates; higher tax rates reduce the need for such consumption smoothing, implying a lower optimal exemption level, while higher unemployment increases the need for borrowing to smooth consumption and hence implies a higher optimal exemption level.

Similarly, the gini coefficient was found to be significantly positive for both regressions. We argued that this may be the case if income inequality occurred mainly between younger and older agents, since this would increase borrowing and hence by our model imply a higher exemption level.

We also argued that a higher divorce rate may imply lower exemption levels; higher divorce rates may lead banks restricting credit since divorce is one of the main factors behind bankruptcy, and this in turn would imply a lower exemption level. Our findings are consistent with this hypothesis, with the divorce rate coefficient negative and strongly significant in both regressions.

In addition, we argued that higher house prices may reduce borrowing by lowering agents' disposable income, since they raise the cost of living through higher rent and mortgage payments, and this is confirmed with empirical evidence in Del Rio and Young (2005). Higher mortgage repayments may also increase the probability of filing for bankruptcy, as documented empirically by Domowitz and Sartain (1999), and lead to banks restricting credit. Therefore, higher prices should imply lower bankruptcy exemptions. This is consistent with our results, which find a significantly negative coefficient for house prices so that, *all else equal*, states with higher house prices have lower bankruptcy exemptions.

The *politics* dummy coefficient was found to be significantly negative in both regressions, so that democrat-controlled states have, on average, lower home exemption levels. This may occur if Democrat politicians tend to support lower-income households, are likely to borrow less in absolute terms than households with higher incomes, and so would benefit from lower exemption levels.

Finally, we did not detect any significant spatial effects in either regression. Although both were positive, the λ coefficient of spatial error autocorrelation was only just insignificant, while the ρ coefficient of spatial lag autocorrelation was highly insignificant. Our preliminary cross-sectional tests seem to suggest that such spatial effects have declined over time, and this could be why these were found to be insignificant over the whole time period 1975-2004. We investigate this further in the section below by running the same panel regressions on restricted sub-periods.

A point worth noting is the high adjusted R^2 of each model, at 0.86 for both panel regressions. This could be the result of a spurious regression between dependent and independent variables in our regression. For example, augmented Dicky-Fuller (ADF) tests on individual state exemption levels reveal an apparent unit root process behind many of these[39]. As would be expected, these tests suggest that income per capita follows a unit root process in all states. Furthermore, according to the ADF tests, tax rates follow a unit root process in all states, unemployment rates follow a unit root process in all states apart from Arizona, and so do the Gini coefficient of inequality, the divorce rate[40] and house prices[41]. We deal with this issue by using a statistic developed by Pedroni (2004), dubbed the "panel-t" statistic, and tests for the stationarity of the pooled error terms. Our results for this test strongly reject the hypothesis no panel cointegration and leads us to conclude that our results are apparently cointegrated. The results and full details of this test are provided in the appendix.

Interactions over time In order to test whether spatial effects are in fact becoming weaker over time, we perform the same panel regressions but restrict the sample period by one year at a time. For example, we first run the panel regression over the whole period, 1975-2004, then repeat that but restricting the time period by a year to 1975-2003, then 1975-2002 and so on. The λ and ρ coefficients for each panel regression are given below in Table 3-4.

[39] In fact, home exemptions in all states apart from Alabama, Connecticut, Georgia, Hawaii, Kentucky, Louisiana, North Dakota, Tennessee, Virginia and Wyoming follow a unit root process.

[40] In all states apart from Delaware, Indiana, Louisiana, Maine, Minnesota, New Hampshire, New Mexico, North Carolina, Vermont and West Virginia.

[41] In all states apart from Arkansas, Iowa, Mississippi, Montana, New Mexico, North Dakota and Wyoming.

Table 3-4. Coefficient of spatial autoregression at given sample sizes

	λ	ρ
75-04	0.054 (0.11)	0.005 (0.89)
75-03	0.056 (0.11)	0.005 (0.89)
75-02	0.036 (0.31)	0.004 (0.91)
75-01	0.036 (0.32)	0.004 (0.91)
75-00	0.042 (0.25)	0.005 (0.90)
75-99	0.051 (0.17)	0.005 (0.90)
75-98	0.056 (0.14)	0.007 (0.86)
75-97	**0.068** (0.08)	0.009 (0.83)
75-96	**0.092** (0.02)	0.011 (0.79)
75-95	**0.112** (0.00)	0.011 (0.79)
75-94	**0.121** (0.00)	0.016 (0.71)
75-93	**0.136** (0.00)	0.017 (0.70)

These results reveal two things. First, the results confirm what was found in the cross-sectional analysis on λ, the coefficient of spatial error autocorrelation. This steadily increases as the sample size is decreased to exclude more recent years - it has increased from 0.054 in the 1975-2004 sample to 0.136 in the 1975-1993 sample, implying that such spatial effects have steadily become weaker over the years. On the other hand, there are no spatial effects found in any of the SAR panel regressions; while ρ increases as the sample size is decreased, it remains highly insignificant in all the regressions. This contradicts the cross sectional results on ρ, which find significant spatial effects in earlier years. However, this is entirely due to the inclusion of state fixed effects - if fixed effects are not included in the regressions, the coefficient for ρ increases and is found to be significant when the sample is restricted to the years 1975-1986 or less.

We also report the estimates of the remaining coefficients for each sub-sample in the table below[42].

Table 3-5. SER coefficients at given sample sizes

	ipc	*tax*	*unem*	*gini*	*div*	*hpirce*	*politics*
75-04	**6.48** (0.00)	−264,068 (0.11)	4,540 (0.00)	**473,006** (0.00)	**−9,945** (0.00)	**−318.9** (0.00)	**−7,608** (0.08)
75-03	**6.01** (0.00)	−213,725 (0.19)	**3,897** (0.00)	**551,951** (0.00)	**−7,956** (0.00)	**−419.4** (0.00)	−6,761 (0.10)
75-02	**5.03** (0.00)	−71,455 (0.67)	**3,167** (0.01)	**633,947** (0.00)	**−6,070** (0.00)	**−505.3** (0.00)	−6,441 (0.10)
75-01	**5.04** (0.00)	−79,780 (0.64)	**3,075** (0.01)	**629,836** (0.00)	**−5,951** (0.00)	**−503.7** (0.00)	−6,287 (0.11)
75-00	**4.97** (0.00)	−54,155 (0.75)	**3,112** (0.01)	**641,977** (0.00)	**−5,877** (0.01)	−490.9 (0.00)	−6,389 (0.11)
75-99	**5.22** (0.00)	−28,390 (0.87)	**3,233** (0.00)	**642,327** (0.00)	**−5,152** (0.01)	−489.1 (0.00)	**−6,315** (0.10)
75-98	**5.35** (0.00)	−68.49 (1.00)	**3,140** (0.00)	**648,887** (0.00)	**−4,193** (0.05)	**−482.9** (0.00)	**−6,452** (0.07)
75-97	**5.43** (0.00)	22,634 (0.90)	**3,152** (0.00)	**647,154** (0.00)	**−3,689** (0.09)	**−474.7** (0.00)	**−6,850** (0.05)
75-96	**5.27** (0.00)	46,735 (0.79)	**3,259** (0.00)	**669,511** (0.00)	−3,063 (0.15)	**−455.5** (0.00)	**−7,375** (0.07)
75-95	**5.05** (0.00)	119,802 (0.52)	**3,152** (0.00)	**728,143** (0.00)	−1,724 (0.41)	**−430.2** (0.00)	**−8,116** (0.03)
75-94	**5.18** (0.00)	124,831 (0.49)	**3,259** (0.00)	**719,264** (0.00)	−863 (0.69)	−404.9 (0.00)	**−6,409** (0.09)
75-93	**4.54** (0.00)	180,295 (0.31)	**3,078** (0.00)	**826,465** (0.00)	−348 (0.87)	**−366.0** (0.00)	−4,294 (0.25)

As these results reveal, the tax rate coefficient cannot be considered to have an important effect on the level of exemptions in each state, since its *p*-value quickly loses its significance as the sample is restricted, while the coefficient itself eventually becomes positive as the sample is restricted by seven years (although it remains insignificant). As we show in the Appendix (Section 3.7.7), this is mainly because there exists little variation in tax rates, both cross-sectionally and over time. Another possible cause for this is that, as we argue, a higher tax rate

[42] As the estimates for the SEM and SAR panel regressions are qualitatively similar, we only report those for the SEM panel regressions for brevity. The SAR results however are available upon request.

can have an ambiguous effect on the level of borrowing. On the one hand, it may reduce the need for borrowing by providing young households with financial assistance and by helping households smooth temporary income shocks[43]. However, on the other hand, it may increase borrowing if banks extend credit as a result of a larger social 'safety net' which reduces consumers' tendency to declare bankruptcy. The results also reveal that while the effects of income per capita, the unemployment rate and the divorce rate on exemption levels have become stronger over the years, the effects of inequality[44] and house prices have become weaker while and the politics coefficient has broadly remained steady.

3.6 Conclusion

This paper has provided evidence on the effect of macroeconomic variables on the setting of exemption levels. It has derived a rule to set exemption levels optimally which relates the optimal bankruptcy exemption to the level of borrowing, and tests this empirically. This rule states that any exogenous shock which, all things equal, leads to more borrowing, implies a higher optimal exemption level. This departs from conventional thinking on this subject, which, broadly speaking, relates exemption levels to the level of risk[45] faced by consumers. This states that higher risk implies higher exemption levels, since bankruptcy offers a form of insurance against a drop in incomes. Rather, in our model, risk affects the optimal exemption level *indirectly*, through its effect on borrowing. For instance, higher risk may actually reduce borrowing and so may actually imply a lower optimal exemption level. The empirical results from our panel regressions suggest that macroeconomic variables do indeed influence the setting of exemption levels, and in a way which our theoretical model would predict. We believe that this can be important for making inferences about future long-term rates of default. For example, all things equal, the aggregate bankruptcy rate should fall as average incomes in an economy increase. However, if, as our theory predicts, higher incomes result in higher exemption levels[46], then it is likely that the aggregate bankruptcy rate does not fall by as much, or indeed it is possible that bankruptcy rates may actually increase. Our empirical evidence supports

[43]For example, via the greater provision of unemployment insurance.
[44]As measured by the gini coefficient.
[45]Or in other words income volatility.
[46]This, because higher incomes should increase the level of borrowing.

this as we obtain positive coefficients for income per capita, so on average, states with higher incomes per capita have higher exemption levels. Another example is the divorce rate. All things equal, higher divorce rates should increase the aggregate bankruptcy rate as divorce is major determinant in the decision to declare bankruptcy. However, if banks restrict credit following an increase in divorce rates, exemption levels may eventually fall. As a result, bankruptcy rates may not increase by as much as otherwise expected. This too is supported by the empirical analysis, which finds that, on average, states with higher divorce rates have lower exemption levels.

An original contribution of this paper has also been the addition of spatial interactions in the estimation of exemption levels. Casual evidence suggests that there exists some correlation between bankruptcy exemptions in neighbouring states - states with high exemptions or low exemptions can generally be found in clusters, as can be found states which have increased or decreased their (real) exemption levels. We model this empirically by use of a spatial error and a spatial lag autoregressive specification, so that each model captures any spatial correlation in the regression error terms or in the dependent variable. Positive spatial lag autocorrelation would imply that states are likely to have higher exemption levels if neighbouring states do too, while positive spatial error autocorrelation would imply either that exemption levels are influenced by unobserved variables which are correlated through space, or that there is a degree of co-movement in exemptions. Our empirical results detect only a limited presence of such spatial dependence. In the SAR models, cross sectional regressions suggest the existence spatial lag autocorrelation in the earlier years of our sample of 1975-2004. However, once state fixed effects are included in our panel analysis, such spatial effects are no longer detected in the data. On the other hand, spatial error autocorrelation is detected in the SER analysis, but we also find that these spatial effects have not been as strong in more recent years and have been declining steadily over time. To conclude therefore, our study finds that while economic fundamentals do appear to influence the level of exemptions across the US, and in a way which is consistent with our basic model, the importance of spatial effects is, at best, limited.

3.7 Appendix

This appendix contains tables and proofs that have not been provided in the main text, starting with Table 3-6 on the following page, which depicts HE levels and PPE levels across US states in 2004. It also provides a derivation of the Lagrangian optimisation conditions used in Proposition 2.

3.7.1 Exemption levels

Table 3-6. Home exemptions (HE) across US states, 2004

	HE			HE
Alabama	10,000		Montana	200,000
Alaska	67,500		Nebraska	12,500
Arizona	150,000		Nevada	400,000
Arkansas	U		New Hampshire	200,000
California	75,000		New Jersey	36,900
Colorado	90,000		New Mexico	60,000
Connecticut	150,000		New York	20,000
Delaware	0		North Carolina	20,000
Florida	U		North Dakota	80,000
Georgia	20,000		Ohio	10,000
Hawaii	36,900		Oklahoma	U
Idaho	50,000		Oregon	33,000
Illinois	15,000		Pennsylvania	36,900
Indiana	15,000		Rhode Island	200,000
Iowa	U		South Carolina	10,000
Kansas	U		South Dakota	U
Kentucky	10,000		Tennessee	7,500
Louisiana	25,000		Texas	U
Maine	70,000		Utah	40,000
Maryland	0		Vermont	150,000
Massachusetts	1,000,000		Virginia	12,000
Michigan	36,900		Washington	40,000
Minnesota	200,000		West Virginia	50,000
Mississippi	150,000		Wisconsin	40,000
Missouri	15,000		Wyoming	20,000

3.7.2 Lagrange maximisation conditions

The Lagrangian is given as:

$$
\begin{aligned}
L &= V(y, r, X) + \lambda \Pi \qquad (3.56) \\
&= \overbrace{u(e_0 - y)}^{a} + \overbrace{\int_{\underline{e}}^{X} f(e)\, u(e)\, de}^{b} + \\
&\quad \overbrace{\int_{X}^{X - r\,y} f(e)\, u(X)\, de}^{c} + \overbrace{\int_{X - r\,y}^{\bar{e}} f(e)\, u(e + r\,y)\, de}^{d} + \\
&\quad \overbrace{\lambda \left[r_f\, y + \int_{X}^{X - r\,y} f(e)\, [e - X]\, de - \int_{X - r\,y}^{\bar{e}} f(e)\, r\, y\, de \right]}^{e} \qquad (3.57)
\end{aligned}
$$

We now derive the four first-order conditions.

$$
\begin{aligned}
\frac{\partial L}{\partial y} &= \overbrace{-u'(e_0 - y)}^{\frac{\partial a}{\partial y}} \overbrace{- r\, f(X - r\, y)\;\; u(X)}^{\frac{\partial c}{\partial y}} + \\
&\quad \overbrace{r \left[\int_{X - r\, y}^{\bar{e}} f(e)\;\; u'(e + r\, y)\;\; de + f(X - r\, y)\;\; u(X) \right]}^{\frac{\partial d}{\partial y}} + \\
&\quad \overbrace{\lambda \left[r_f - r \int_{X - r\, y}^{\bar{e}} f(e)\;\; de \right]}^{\frac{\partial e}{\partial y}} \qquad (3.58) \\
&= -u'(e_0 - r\, y) + r \int_{X - r\, y}^{\bar{e}} f(e)\;\; u'(e + r\, y)\;\; de + \\
&\quad \lambda \left[r_f - r \int_{X - r\, y}^{\bar{e}} f(e)\;\; de \right] \qquad (3.59)
\end{aligned}
$$

$$\frac{\partial L}{\partial r} = \overbrace{-y\, f(X - r\, y)\ u(X)}^{\frac{\partial c}{\partial r}} +$$

$$\overbrace{y\left[\int_{X-r\,y}^{\overline{e}} f(e)\ u'(e + r\,y)\ de + f(X - r\,y)\ u(X)\right]}^{\frac{\partial d}{\partial r}} -$$

$$\overbrace{\lambda\, y \int_{X-r\,y}^{\overline{e}} f(e)\ de}^{\frac{\partial e}{\partial r}} \tag{3.60}$$

$$= y\left[\int_{X-r\,y}^{\overline{e}} f(e)\ u'(e + r\,y)\ de - \lambda \int_{X-r\,y}^{\overline{e}} f(e)\ de\right] \tag{3.61}$$

$$\frac{\partial L}{\partial X} = \overbrace{f(X)\ u(X)}^{\frac{\partial b}{\partial X}} +$$

$$\overbrace{[f(X - r\,y) - f(X)]\ u(X) + \int_{X}^{X-r\,y} f(e)\ u'(X)\ de}^{\frac{\partial c}{\partial X}} -$$

$$\overbrace{f(X - r\,y)\ u(X)}^{\frac{\partial d}{\partial X}} +$$

$$\overbrace{\lambda\left[r\,y\,f(X - r\,y) - r\,y\,f(X - r\,y) - \int_{X}^{X-r\,y} f(e)\ de\right]}^{\frac{\partial e}{\partial X}} \tag{3.62}$$

$$= \int_{X}^{X-r\,y} f(e)\ u'(X)\ de -$$

$$\lambda \int_{X}^{X-r\,y} f(e)\ de = [u'(X) - \lambda] \int_{X}^{X-r\,y} f(e)\ de \tag{3.63}$$

$$\frac{\partial L}{\partial \lambda} = r_f\ y + \int_{X}^{X-r\,y} f(e)\ [e - X]\ de - \int_{X-r\,y}^{\overline{e}} f(e)\ r\ y\ de \tag{3.64}$$

3.7.3 Proofs

We now provide the proofs which have not been provided in the main text.

Proof of Proposition 1. Under bankruptcy, the agent consumes $e_1 - \min\{\max\{0,\ e_1 - X\}, -r\ y\}$ whereas if the agent does not declare bankruptcy he consumes $e_1 + r\ y$. The agent will therefore declare bankruptcy if consumption under bankruptcy is higher than that under non-bankruptcy, or in other words if:

$$e_1 - \min\{\max\{0,\ e_1 - X\}, -r\ y\} \ > \ e_1 + r\ y \tag{3.65}$$

$$\Rightarrow \min\{\max\{0,\ e_1 - X\}, -r\ y\} \ < \ -r\ y \tag{3.66}$$

$$\Rightarrow \max\{0,\ e_1 - X\} \ < \ -r\ y \tag{3.67}$$

The inequality in the final line is satisfied either if $e_1 - X < 0 \Rightarrow e_1 < X$, or $e_1 - X < -r\ y \Rightarrow e_1 < X - r\ y$. Therefore, since $X - r\ y > X$, the agent will always declare bankruptcy if $e_1 < X - r\ y$. ■

3.7.4 SER test results

Table 3-7. Tests for spatial error autocorrelation, 1975-2004

	Moran's I	Likelihood ratio	Wald test	Lagrange multiplier
1975	**3.96** (0.00)	**12.67** (0.00)	**25.43** (0.00)	**8.96** (0.00)
1976	**3.53** (0.00)	**9.68** (0.00)	**20.72** (0.00)	**6.20** (0.01)
1977	**3.16** (0.00)	**7.82** (0.01)	**16.92** (0.00)	**4.87** (0.03)
1978	**2.95** (0.00)	**6.14** (0.01)	**10.29** (0.00)	**4.37** (0.04)
1979	**2.49** (0.01)	**4.22** (0.04)	**5.61** (0.02)	**2.97** (0.08)
1980	**3.02** (0.00)	**5.01** (0.03)	**4.91** (0.03)	**5.12** (0.02)
1981	**2.80** (0.01)	**4.07** (0.04)	**3.53** (0.06)	**4.13** (0.04)
1982	**2.62** (0.01)	**3.27** (0.07)	2.54 (0.11)	**3.31** (0.07)
1983	**2.73** (0.01)	**4.34** (0.04)	**5.39** (0.02)	**3.73** (0.05)
1984	**1.87** (0.06)	1.45 (0.23)	1.11 (0.29)	1.27 (0.26)

1985	**1.87** (0.06)	1.24 (0.27)	0.78 (0.38)	1.18 (0.28)
1986	**1.86** (0.06)	1.18 (0.28)	0.76 (0.39)	1.16 (0.28)
1987	**2.25** (0.02)	1.99 (0.16)	1.32 (0.25)	1.98 (0.16)
1988	**2.30** (0.02)	**2.81** (0.09)	2.57 (0.11)	2.32 (0.13)
1989	1.50 (0.13)	1.02 (0.31)	0.82 (0.37)	0.74 (0.39)
1990	**1.80** (0.07)	2.03 (0.15)	2.26 (0.13)	1.29 (0.26)
1991	1.31 (0.19)	0.56 (0.45)	0.38 (0.54)	0.44 (0.51)
1992	**1.96** (0.50)	**3.16** (0.08)	**5.12** (0.02)	1.88 (0.17)
1993	**1.90** (0.57)	2.11 (0.15)	1.86 (0.17)	1.75 (0.19)
1994	**2.15** (0.32)	**2.86** (0.09)	2.47 (0.12)	2.48 (0.12)
1995	1.61 (0.11)	1.34 (0.25)	1.03 (0.31)	1.11 (0.29)
1996	1.42 (0.16)	0.82 (0.37)	0.53 (0.47)	0.70 (0.40)
1997	1.22 (0.22)	0.46 (0.50)	0.25 (0.62)	0.43 (0.51)
1998	1.23 (0.22)	0.48 (0.49)	0.25 (0.62)	0.44 (0.51)
1999	1.00 (0.32)	0.30 (0.58)	0.20 (0.65)	0.21 (0.65)
2000	1.39 (0.16)	0.78 (0.38)	0.46 (0.50)	0.72 (0.40)
2001	0.93 (0.35)	0.17 (0.68)	0.08 (0.78)	0.17 (0.68)
2002	1.34 (0.18)	0.60 (0.44)	0.33 (0.57)	0.59 (0.44)
2003	1.07 (0.28)	0.25 (0.62)	0.11 (0.74)	0.26 (0.61)
2004	1.51 (0.13)	0.75 (0.39)	0.41 (0.52)	0.74 (0.39)

3.7.5 SER panel fixed effects

Table 3-8. Fixed effects coefficients for SER panel regression

Alabama	−211,697 (0.205)	Louisiana	−214,931 (0.188)	Ohio	−243,804 (0.134)
Alaska	−218,053 (0.198)	Maine	−210,433 (0.193)	Oklahoma	240,023 (0.155)
Arizona	−141,107 (0.397)	Maryland	−257,459 (0.099)	Oregon	−207,495 (0.206)
Arkansas	248,124 (0.143)	Massachusetts	−130,238 (0.4)	Pennsylvania	−250,900 (0.114)
California	−162,888 (0.312)	Michigan	−242,952 (0.136)	Rhode Island	−212,907 (0.173)
Colorado	−184,764 (0.258)	Minnesota	35,420 (0.819)	South Carolina	−223,767 (0.164)
Connecticut	−213,964 (0.173)	Mississippi	−119,926 (0.47)	South Dakota	205,070 (0.199)
Delaware	−246,287 (0.124)	Missouri	−234,131 (0.15)	Tennessee	−219,284 (0.187)
Florida	219,423 (0.188)	Montana	−123,060 (0.451)	Texas	218,642 (0.184)
Georgia	−217,384 (0.18)	Nebraska	−234,029 (0.142)	Utah	−199,258 (0.218)
Hawaii	−154,057 (0.323)	Nevada	−62,632 (0.734)	Vermont	−150,358 (0.348)
Idaho	−158,763 (0.341)	New Hampshire	−209,786 (0.191)	Virginia	−227,275 (0.155)
Illinois	−250,095 (0.118)	New Jersey	−256,947 (0.103)	Washington	−182,131 (0.267)
Indiana	−228,435 (0.169)	New Mexico	−167,653 (0.314)	West Virginia	−216,033 (0.191)
Iowa	203,352 (0.199)	New York	−247,396 (0.117)	Wisconsin	−213,089 (0.177)
Kansas	211,122 (0.195)	North Carolina	−216,214 (0.182)	Wyoming	−209,487 (0.215)
Kentucky	−221,999 (0.176)	North Dakota	−74,141 (0.641)		

3.7.6 Panel cointegration test

Pedroni's panel t-statistic for a panel of N equations and T time periods, is given as:

$$Z_{\widehat{t}_{NT}} = \left(\sum_{i=1}^{N} \widetilde{\sigma}_{NT}^{2} \sum A_{22i}\right)^{-1} \sum_{i=1}^{N} \left(A_{21i} - T\widehat{\lambda}_i\right) \tag{3.68}$$

where:

$$\widetilde{e}_{it} = (\triangle\varepsilon_{it}, \varepsilon_{i,t-1})' \tag{3.69}$$

$$A_i = \sum_{t=1}^{T} \widetilde{e}_{it}\widetilde{e}_{it}' \tag{3.70}$$

A_{11} and A_{22} are the upper- and lower-diagonal blocks of A, and the $\widehat{\varepsilon}_{it}$ terms are the error terms estimated from a simple panel regression without spatial effects, while:

$$\widehat{\mu}_{it} = \widehat{\varepsilon}_{it} - \widehat{\rho}\,\widehat{\varepsilon}_{i,t-1} \tag{3.71}$$

$$\widetilde{\sigma}_{NT}^{2} = (NT)^{-1}\sum_{i=1}^{N}\sum_{t=2}^{T}\widehat{\mu}_{it}^{2} + 2T^{-1}\sum_{s=1}^{K} w_{sK} \sum_{t=s+1}^{T} \widehat{\mu}_{it}\,\widehat{\mu}_{i,t-s} \tag{3.72}$$

$$w_{sK} = 1 - \frac{s}{1-K} \tag{3.73}$$

where $\widehat{\rho}$ is the regression coefficient of $\widehat{\varepsilon}_{it}$ on its lag. Pedroni shows that $Z_{\widehat{t}_{NT}}$ asymptotically follows a normal distribution under the null hypothesis of no cointegration, whose mean and variance depend on the data-generating processes[47]. We obtain a $Z_{\widehat{t}_{NT}}$ statistic of -7.49, which we standardise by subtracting it from assuming a mean of -2.29 (case 3) and a variance of 1.50 (case 1), so that we bias the result towards accepting the null of no cointegration. However, we obtain a standardised statistic of -6.40 - this is significantly different from zero, which leads us to reject the null of no cointegration and conclude that our results are apparently cointegrated.

[47] If the data is constructed from variables following a standard Wiener process (case 1), $Z_{\widehat{t}_{NT}} \sim N(-1.01,\ 1.50)$. If the variables follow a demeaned Wiener process (case 2), $Z_{\widehat{t}_{NT}} \sim N(-1.73,\ 0.93)$, while if the variables follow a demeaned and detrended Wiener process (case 3), $Z_{\widehat{t}_{NT}} \sim N(-2.29,\ 0.66)$.

3.7.7 An general outline of trends

We now provide an outline of some trends in home exemption levels and in the independent variables. We show that considerable variation exists in the independent variables (real income per capita, unemployment rates etc.), both cross-sectionally and over time. However, we find that, relative to the means, cross-sectional variation has generally remained fairly stable or increased slightly over the years. This is reflected in exemption levels, whose cross-sectional variation, while significant, has not increased substantially over the time period analysed (1975-2004).

Home exemption levels

Average exemption levels have increased by 156% over the period analysed. Furthermore, the cross-sectional variation in exemption levels has vastly increased in absolute terms. However, relative to the mean, the standard deviation does not seem to have increased by as much; while it has increased when compared to 1975, it has been relatively stable since 1990:

Table 3-9. Cross-sectional variation in home exemption levels across US states, 1975-2004

	1975	1980	1985	1990	1995	2000	2004
σ^2 (millions)	4292	9974	18,675	27,623	34,542	43,743	59,492
Mean	61,492	87,600	99,604	109,781	120,776	130,414	157,506
$\frac{\sigma}{mean}$	1.14	1.14	1.37	1.51	1.54	1.60	1.55

Real income per capita

Not surprisingly, real income per capita has seen steady overall rises in all states between 1975 and 2004, while the average across all states has risen by 51% over this time period, as Figure (3-9) illustrates[48].The average increase in real incomes per capita among the top half of gainers

[48]The big exception is Alaska, whose real income per capita has actually fallen by 10% - however, having started at an exceptionally high level in 1975 (possibly due to its oil reserves), it still remained among the higher-income states in 2004. Furthermore, the real HE level in Alaska remained steady with a fall of just $32 over this period. This is consistent with our regression estimates, which predicts that states with lower-than-average gains in income will have lower-than-average rises in exemption levels.

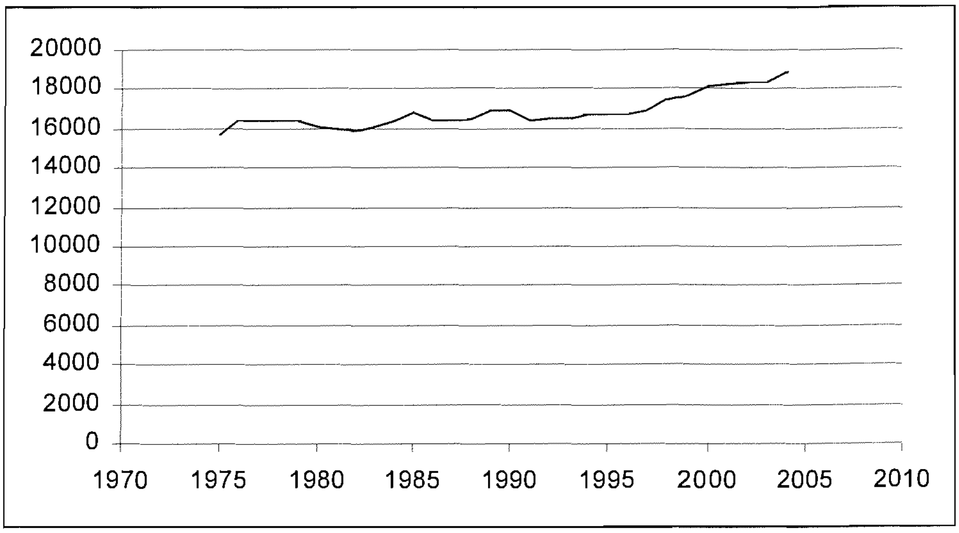

Figure 3-9: Average real incomes across US states, 1975-2004.

was 63%, compared with a 39% increase among the bottom half of states. Incidentally, the average increase in real HE levels between these two groups was approximately $109,000 for states with the highest gains in incomes compared with $83,000 for the states with the lower gains in incomes. This is consistent with the findings of our model, which predicts that states with higher gains in income will, on average, experience higher increases in exemption levels.

In addition, we observe the cross-sectional variance in real income per capita among states - as Table 3-10 illustrates, variance in absolute terms has increased. However, relative to the mean, the standard deviation has fallen slightly.

Table 3-10. Cross-sectional variation in real income per capita across US states, 1975-2004

	1975	1980	1985	1990	1995	2000	2004
σ^2 (000s)	886	843	916	1,177	942	1,544	1,358
Mean	10,477	11,434	11,911	12,897	13,671	14,931	15,877
$\frac{\sigma}{mean}$	0.28	0.25	0.25	0.27	0.22	0.26	0.23

Income tax rates

As Figure (3-10) illustrates, income tax rates have remained relatively stable in individual states throughout 1975 and 2004.On the other hand, some cross-sectional variation does exist. This

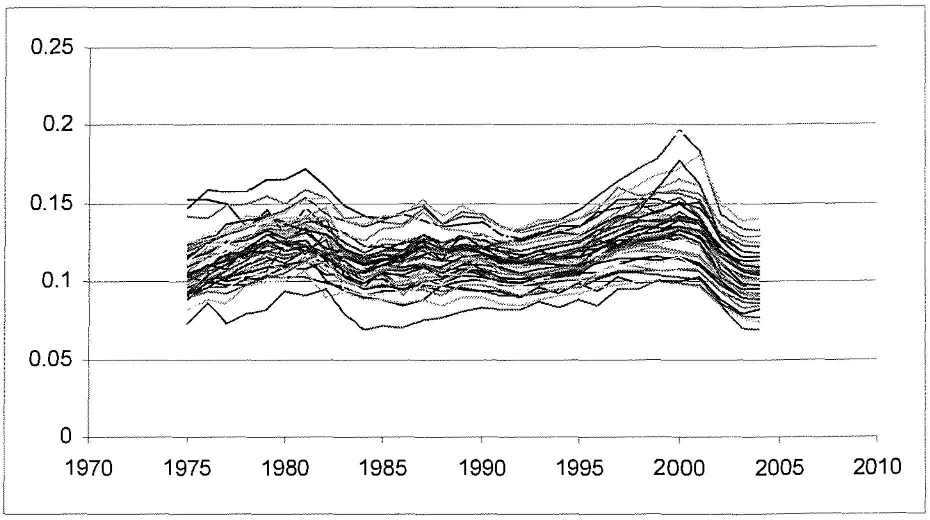

Figure 3-10: Income tax rates across 50 US states, 1975-2004

however, As Table 3-11 illustrates, is relatively small compared to the mean and has remained fairly stable too, both in absolute terms and relative terms. This small degree of variation, both cross-sectionally and over time, is the reason for not finding a weak statistical relationship between the tax rate real HE levels.

Table 3-11. Cross-sectional variance in income tax rates amongst US states, 1960-2004

	1975	1980	1985	1990	1995	2000	2004
$\sigma^2 \times 1000$	0.24	0.19	0.20	0.17	0.20	0.29	0.26
Mean	0.11	0.12	0.11	0.12	0.12	0.13	0.10
$\frac{\sigma}{mean}$	0.15	0.11	0.13	0.11	0.12	0.16	0.16

Unemployment rates

Unemployment rates have generally fallen across states, with the average unemployment rate across all US states 27% lower in 2004 than in 1975[49]:Furthermore, cross-sectional variation

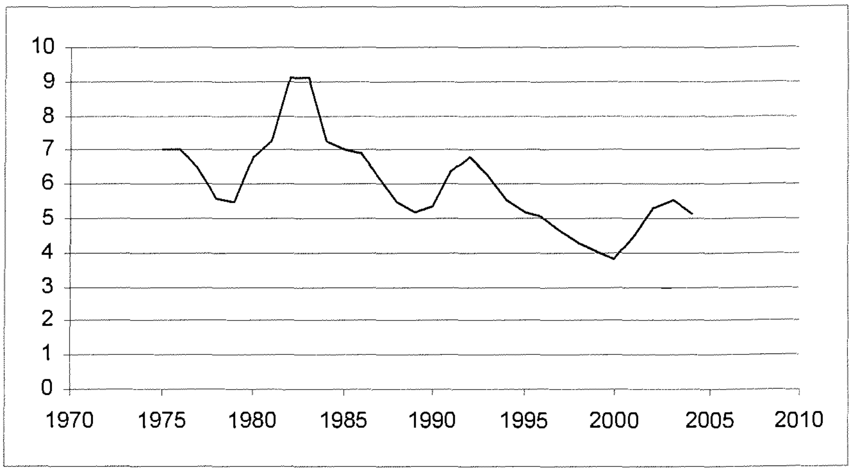

Figure 3-11: Average unemployment rate across US states, 1975-2004

in unemployment has fallen over the period analysed, both in absolute and relative terms, suggesting that unemployment rates have converged between states. This is shown in the table below:

Table 3-12. Cross-sectional variance in divorce rates amongst US states, 1960-2004

	1975	1980	1985	1990	1995	2000	2004
σ^2	3.53	2.50	4.06	1.37	1.35	0.83	1.00
Mean (%)	7.05	6.79	7.05	5.38	5.16	3.84	5.14
$\frac{\sigma}{mean}$	0.27	0.23	0.26	0.22	0.22	0.24	0.19

Finally, we compare the change in exemption levels between the 25 states which have experienced the greatest fall in unemployment against those 25 states which have experienced

[49]There are five exceptions, where the unemployment rate has in fact increased. These are: Iowa (+17%), Kansas (+31%), Nebraska (+19%), South Dakota (+3%) and Texas (+5%).

the lowest falls. We find the respective changes in real HE levels to be $111,000 and $81,000. This is consistent with out regression results which find a positive relationship between the unemployment rates and real HE levels, so that, all things equal, states with greater falls in unemployment will experience greater falls, or lower increases (since exemption levels have risen due to other factors such as income), in exemption levels.

Divorce rates

Divorce rates have generally fallen under the period considered (1075-2004), with the average divorce rate 27% lower in 2004 than in 1975. This is illustrated in Figure (3-12) below.Out

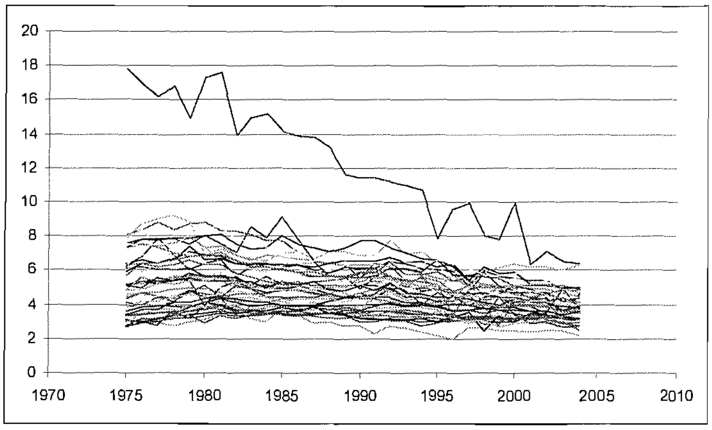

Figure 3-12: Divorce rates (per 1000) across 50 US states, 1975-2004.

of the 50 US states, divorce rates increased in just 7 states, by an average of just 7.5%[50]. These states saw an average increase in their real HE levels of just $2,293, compared to an average increase of $96,014 across the 50 states. This is in line with our regression model, which estimates a negative relationship between divorce rates and real HE levels. Furthermore, one state has clearly experienced a much larger fall in divorce rates than other states - this is Nevada, which saw its divorce rate fall by 64% between 1975 and 2004, from 17.8 per mil to 6.4 per mil. Incidentally, this state has seen one of the largest increase in exemption levels, having experienced an increase of $252,501 in real terms. Once again, this is in line with the findings

[50]These states were Virginia, Rhode Island, North Dakota, North Carolina, Wisconsyn, New Jersey and Kentucky.

of our regression model, so that, all else equal, states with above-average falls in divorce rates will experience an above-average rise in real HE levels.

We also examine the cross-sectional variance of divorce rates between states in given years. As Table 3-13 illustrates, this variance has fallen significantly, both in absolute terms and relative to the mean, suggesting that divorce rates have converged between states over the years.

Table 3-13. Cross-sectional variance in divorce rates amongst US states, 1960-2004

	1975	1980	1985	1990	1995	2000	2004
σ^2	5.96	5.20	3.49	2.24	1.50	1.53	0.90
Mean	5.55	5.67	5.31	4.97	4.64	4.30	3.90
$\frac{\sigma}{mean}$	0.46	0.40	0.35	0.30	0.26	0.29	0.24

Real house prices (index)

House prices have on average risen by 36% across US states in real terms between 1965 and 2004. Furthermore, cross-sectional variation this time period, doubling relative to the mean. This may have contributed to the slight increase in the cross-sectional variation present in HE levels:

Table 3-13. Cross-sectional variance in index of real house prices amongst US states, 1960-2004

	1975	1980	1985	1990	1995	2000	2004
σ^2	490	1,007	1,077	1,685	1,261	1,484	3,672
Mean	132.4	131.2	121.6	140.9	135.0	156.0	182.3
$\frac{\sigma}{mean}$	0.17	0.24	0.27	0.29	0.26	0.25	0.33

US Gini coefficient

As Figure (3-13) illustrates, the US Gini coefficient has risen by 17% between 1965 and 2004, an increase of about 0.5% per annum:

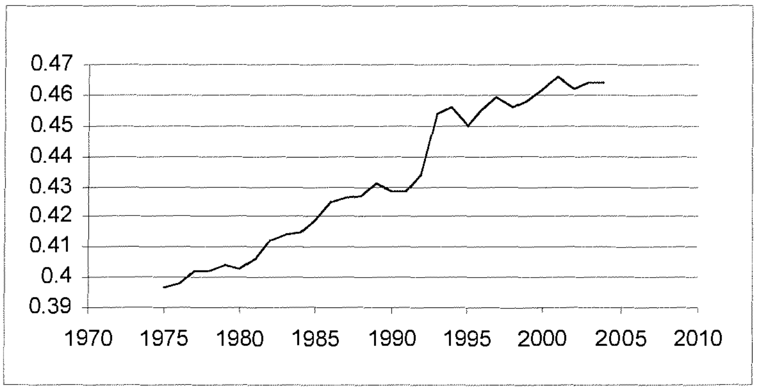

Figure 3-13: US Gini coefficient of inequality, 1965-2004

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