University of Southampton

Hecke groups and their subgroups of low index

Catherine Campbell -Grant

June 2008

School of mathematics

M.Phil.

Contents

Chapter 1

Introduction

We first look at Fuchsian groups. To understand what these are we look at hyperbolic geometry. One model of the hyperbolic plane is the upper half-plane model. The underlying space of this model is
$\mathbb{H} = \{z \in \mathbb{C} \mid Im(z) > 0\}$

The group of conformal automorphisms of the upper half-plane is isomorphic to
PSL(2,$\mathbb{R}$)=SL(2,$\mathbb{R}$)/$\{\pm I\}$
where
SL(2,$\mathbb{R}$)= $\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{R}, ad - bc = 1\}$
The isomorphism is obtained by letting A $\in$ SL(2,$\mathbb{R}$) act as a Mobius transformation
z $\rightarrow \frac{az+b}{cz+d}$.

A Fuchsian group is a discrete subgroup of PSL(2,$\mathbb{R}$).

To begin with we study the modular group and its subgroups, particularly concentrating on congruence subgroups of low index. We then extend this work to some Hecke groups in chapter 5.

The modular group is PSL(2,$\mathbb{Z}$). It is isomorphic to the triangle group $(0; 2, 3, \infty)$ which has a presentation
$\{x, y, z \mid x^2 = y^3 = xyz = 1\}$.The quotient space $\mathbb{H}$ / PSL(2,$\mathbb{Z}$) is referred to as a modular triangle.
The problem which we concentrated on was finding all the subgroups of low index (≤ 10) in the modular group and determining which were *congruence* subgroups.
Any subgroup G of PSL(2,$\mathbb{Z}$) containing a principal congruence subgroup $\Gamma(N)$ is called a congruence subgroup. This is when a principal congruence

subgroup is defined as

$\Gamma(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \epsilon PSL(2,\mathbb{Z}) \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{modN}\}$

The least N such that $G \geq \Gamma(N)$ is called the level of G.

This idea is generalised to Hecke groups in chapter 5, where we show that H^5 contains non-congruence subgroups.

With the idea of cusp widths we can generalise the concept of level as below.

Definition A A cusp is a fixed point of a parabolic element.

Definition B *The cusp width of a particular cusp (i.e. a point at ∞) in* the quotient space $\mathbb{H} \, / \, \Gamma_1$ (where Γ_1 is an arbitrary modular subgroup) is the number of modular triangles within the quotient space that touches the cusp.

Definition C Let Γ_1 be an arbitrary subgroup of finite index of the modular group. The geometric level of Γ_1 can be defined to be the l.c.m. of the cusp widths of the quotient of the upper half-plane by Γ_1.

Wohlfahrt brought these two definitions together by proving the following theorem.

Theorem(Wohlfahrt) Let N be the geometric level of a modular subgroup Γ_1. Then Γ_1 is a congruence subgroup if and only if it contains $\Gamma(N)$.

We also study the dessin d'enfants of the coset permutation representations of these subgroups of the modular group. These pictorially show the permutation of the cosets. Conjugate subgroups give isomorphic dessins so non-isomorphic dessins give non-conjugate subgroups. Dessins with different structures therefore show the quotient groups of our Hecke group by different non-conjugate non-isomorphic subgroups.

We then look at Hecke groups. We know that the modular group is discrete and is generated by $R(z) = z + 1$ and $S = \frac{-1}{z}$. Hecke extended this by going on to look at when groups generated by $R_1(z) = z + \lambda$ and $S_1(z) = \frac{-1}{z}$ are discrete.

He found in a paper published in 1936 [3] that they were discrete if and only if

(i) $\lambda = 2cos\frac{\pi}{q}$ for some integer $q \geq 3$, or

(ii) $\lambda \geq 2$.

In the first case we find that ${S_1}^2 = (S_1R_1)^q = 1$. We denote this group by H^q and it is isomorphic to the free product $C_2 * C_q$. H^3 is the modular

group.
In this thesis we concentrated on H^5. We once again looked at the subgroups of low index. During our studies we used two main methods. These being:
(i) The permutation method
(ii) Arithmetic methods
The description of these techniques follows in chapters four and five.

The main result is in chapter six where we find a non-congruence subgroup of finite index in H^5. We do this by extending work done in the modular group.

In the modular group we can use three lemmas to show that there exists a normal subgroup N of the modular group Γ which is of finite index and contains none of the principal congruence subgroups $\Gamma(N)$. The three lemmas are listed below:

Lemma One Let $\mathbb{Z}_N$ be the ring of integers *mod* N, and let $PSL(2, \mathbb{Z}_N)$ be the group of Mobius transformations with $a, b, c, d \in \mathbb{Z}_N$. Then the quotient group $\Gamma/\Gamma(N)$ of the Nth principal congruence subgroup in the modular group is isomorphic with $PSL(2, \mathbb{Z}_N)$.

Lemma Two The only nonabelian quotient groups that can appear in a composition series of $PSL(2, \mathbb{Z}_N)$ are the groups $PSL(2, \mathbb{Z}_p)$, where p is a prime number.[This implies $p \geq 5$, in which case $PSL(2, \mathbb{Z}_p)$ is simple.]

Lemma Three The alternating group A_{11} on 11 symbols is a quotient group of the modular group, and A_{11} is not isomorphic with any group $PSL(2, \mathbb{Z}_p) = \Gamma/\Gamma(p)$.

We go on to extend this proof into H^5 and from this process we prove our main result which is as follows.

Main Result There is a non-congruence subgroup of index 11 in H^5.

In the appendix we look at subgroups of low index in H^5 and their corresponding transitive permutation representations. In doing so we find epimorphisms from H^5 onto A_{11} which are of great interest in our work.

Chapter 2

Hyperbolic geometry and Fuchsian groups

The results in this chapter can be seen in chapters 4 and 5, Jones and Singerman, Complex functions [9].

2.1 Hyperbolic geometry

2.1.1 Models of the hyperbolic plane

By a model, we mean a choice of underlying space, together with a choice of how to represent basic geometric objects, such as points and lines, in this underlying space.

2.1.2 The upper half-plane model

The underlying space of this model is the upper-half plane in the complex plane, defined to be

$$\mathbb{H} = \{z \in \mathbb{C} | Im(z) > 0\}$$

We use the usual notion of point and of angle that $\mathbb{H}$ inherits from $\mathbb{C}$ and we define hyperbolic lines to be the intersection of $\mathbb{H}$ with a Euclidean line in $\mathbb{C}$ perpendicular to the real axis or the intersection of $\mathbb{H}$ with a Euclidean circle centred on the real axis.

The metric ds of hyperbolic geometry is defined on $\mathscr{U}$ and is given by the formula

$$ds^2 = \frac{|dz|^2}{y^2} \qquad \text{with} \qquad z = x + yi$$

We also have the formula

$$dA = \frac{dxdy}{y^2}$$

for infinitesimal hyperbolic area.

2.2 The group of automorphisms of the upper half plane

Let SL(2,$\mathbb{R}$) be the group

$$SL(2,\mathbb{R}) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} |a,b,c,d\epsilon\mathbb{R}, ad-bc=1\}$$

Dividing this by its centre $\{\pm I\}$ we get the group

$$PSL(2,\mathbb{R}) = SL(2,\mathbb{R})/\{\pm I\}$$

PSL(2,$\mathbb{R}$) is the group of automorphisms of the upper half-plane. It acts on the upper half-plane as the transformations

$$SL(2,\mathbb{R}) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} : z \rightarrow \frac{az+b}{cz+d}\}$$

They are orientation preserving hyperbolic isometries.

Theorem 1 (i) PSL(2, $\mathbb{R}$) is transitive on $\mathscr{U}$.
(ii) PSL(2,$\mathbb{R}$) is doubly transitive on $\mathbb{R} \cup \{\infty\}$.

Proof (i) Let $ai+b \in \mathscr{U}$, so that $a > 0$. Then if $T(z) = az+b$, $T \in PSL(2,\mathbb{R})$ and $T(i) = ai+b$. Thus the orbit of i under the action of $PSL(2,\mathbb{R})$ is $\mathscr{U}$ and so $PSL(2,\mathbb{R})$ is transitive on $\mathscr{U}$.
(ii) If $a,b \in \mathbb{R}$, $a > b$ then if $S(z) = \frac{(z-a)}{(z-b)}$, $S \in PSL(2,\mathbb{R})$ maps the ordered pair (a,b) to $(0,\infty)$. Also $z \rightarrow \frac{-1}{z}$ maps $(0,\infty)$ to $(\infty,0)$ and $z \rightarrow z+b$ maps $(0,\infty)$ to (b,∞). It follows that the orbit of $(0,\infty)$ under the action of $PSL(2,\mathbb{R})$ consists of all ordered pairs (a,b), $(a,b \in \mathbb{R}\cup\{\infty\}$, $a \neq b)$, so that $PSL(2,\mathbb{R})$ is doubly transitive on $\mathbb{R}\cup\{\infty\}$.

2.2.1 The elements of this group

Let $T(z) = \frac{az+b}{cz+d}$, $\quad a,b,c,d \in \mathbb{R}$, $\quad \triangle = ad-bc > 0$

The elements of this group can be classified according to the value of their trace as follows: If $|a+d| = 2$ then we say that T is parabolic, if $|a+d| > 2$ then we say that T is hyperbolic, and if $|a+d| < 2$ then we say that T is elliptic.

These have different geometric properties. A parabolic element acts with a single fixed-point on the boundary of $\mathscr{U}$, a hyperbolic element acts with two fixed-points on the boundary of $\mathscr{U}$, and an elliptic element acts with one fixed point on $\mathscr{U}$.

2.3 Riemann surfaces

We will now go on to look at Fuchsian groups; these are discrete subgroups of PSL(2,$\mathbb{R}$). A reason for our interest in these subgroups is that every connected Riemann surface may be obtained from one of the three simply connected Riemann surfaces which are the sphere, the plane, and the upper half plane. They are obtained by factoring out by a discontinuous group of one of the automorphism groups of these surfaces. In the following theorem we show that with the exception of four simple cases all Riemann surfaces are obtained by factoring out a Fuchsian group from the upper half plane.

Theorem 2 (see reference[9], p.213) If S is a connected Riemann surface not conformally equivalent to the sphere Σ, the plane $\mathbb{C}$, the punctured plane $\mathbb{C}\setminus\{0\}$, or a torus $\mathbb{C}\setminus\Omega$, then S has universal covering space $\widehat{S} = \mathscr{U}$,the upper half-plane, and S is conformally equivalent to $\mathscr{U}/$G for some subgroup G of PSL(2,$\mathbb{R}$) acting discontinuously on $\mathscr{U}$.

2.4 Fuchsian groups

Identifying the matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ of PSL(2,$\mathbb{R}$) with the point (a,b,c,d) of $\mathbb{R}^4$ we can view PSL(2,$\mathbb{R}$) as a topological space with topology inherited from the topology of $\mathbb{R}^4$. In this way PSL(2,$\mathbb{R}$) becomes a topological group and we define a discrete subgroup Ω of it to be a subgroup with the property that there is a neighbourhood U in PSL(2,$\mathbb{R}$) of I the identity matrix such that $U \cap \Omega = \{I\}$.

Definition A Fuchsian group is a discrete subgroup of PSL(2,$\mathbb{R}$).

The modular group The modular group is an example of a Fuchsian group. It consists of the elements defined below.

$$SL(2,\mathbb{Z}) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} | a,b,c,d \in \mathbb{Z}, ad-bc=1\}$$

$$PSL(2,\mathbb{Z}) = SL(2,\mathbb{Z})/\{\pm I\}$$

2.4.1 Properly discontinuous

Let G be a group of homeomorphisms of a topological space Y. Then G acts properly discontinuous on Y if each point $y \in Y$ has a neighbourhood V such that if $g(V) \cap V \neq \emptyset$ for $g \in G$, then $g(y) = y$.

Theorem 3 (i) Let Γ be a subgroup of PSL(2,$\mathbb{R}$). Then Γ is a Fuchsian group if and only if Γ acts properly discontinuously on $\mathscr{U}$.

(ii) Let Γ be a Fuchsian group and let $p \in \mathscr{U}$ be fixed by some element of Γ. Then there is a neighbourhood W of p such that no other point of W is fixed by an element of Γ other than the identity.

Theorem 4 Let Γ be a subgroup of PSL(2,$\mathbb{R}$). Then Γ is a Fuchsian group if and only if for all $z \in \mathscr{U}$, Γ_z, the Γ - orbit of z, is a discrete subset of $\mathscr{U}$.

Chapter 3

Fundamental regions

The results in this chapter can be seen in chapter 5, Jones and Singerman, Complex functions [9].

3.1 Fundamental regions

F is a fundamental region for Γ if F is a closed set such that

(i) $\cup_{T\epsilon\Gamma} T(F) = \mathscr{U}$,
(ii) $\overset{\circ}{F} \cap T(\overset{\circ}{F}) = \emptyset$, for all $T \in \Gamma \setminus \{I\}$, where $\overset{\circ}{F}$ is the interior of F.

The Dirichlet region Let Γ be an arbitrary Fuchsian group and let $p \in \mathscr{U}$ be not fixed by any element of $\Gamma \setminus \{I\}$. Such points exist by theorem 3 (ii). We define the Dirichlet region for Γ centred at p to be the set

$$D_p(\Gamma) = \{z \in \mathscr{U} \mid \rho(z,p) \leq \rho(z, T(p)) \qquad \text{for all} \qquad T \in \Gamma\}$$

By the invariance of the hyperbolic metric under PSL(2,$\mathbb{R}$) this region can also be defined as

$$D_p(\Gamma) = \{z \in \mathscr{U} \mid \rho(z,p) \leq \rho(T(z), p) \qquad \text{for all} \qquad T \in \Gamma\}$$

Theorem 5 If p is not fixed by any element of $\Gamma \setminus \{I\}$, then $D_p(\Gamma)$ is a connected fundamental region for Γ.

Example If we take Γ to be the modular group then we find that $D_{ki} = F$ $(k > 1)$ where

$$F = \{z \in \mathscr{U} \mid |z| \geq 1, |Re(z)| \leq \frac{1}{2}\}$$

See the following picture.

3.1.1 Locally finite

Definition A fundamental region F for a Fuchsian group Γ is called locally finite if every point $a \in F$ has a neighbourhood $V(a)$ such that $V(a) \cap T(F) \neq \emptyset$ for only finitely many $T \in \Gamma$.

Theorem 6 A Dirichlet region is locally finite.

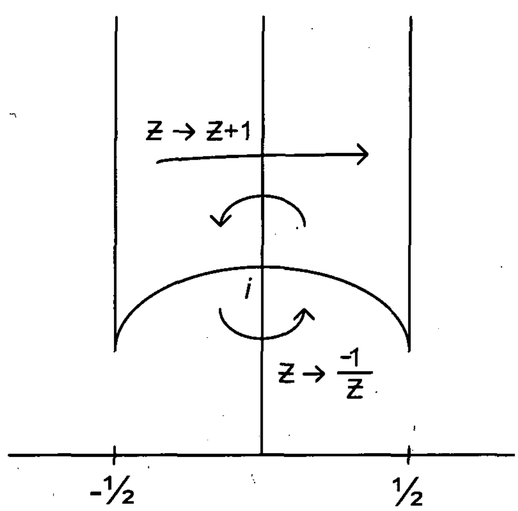
3.1.2 Congruence of sides

Let s be a side of a Dirichlet region F for a Fuchsian group Γ. If $T \in \Gamma \setminus \{I\}$ and $T(s)$ is a side of F then s and $T(s)$ are called congruent sides.

There cannot be more than two sides in a congruent set.

Theorem 7 Let $\{T_i\}$ be the subset of Γ consisting of those elements which pair the sides of some fixed Dirichlet region F. Then $\{T_i\}$ is a set of generators for Γ.

Example (The modular group) The two vertical sides of the Dirichlet region found above can be paired using the transformation $z \to z + 1$.
The part of the boundary consisting of a segment of the unit circle consists of two sides split by the fixed point (namely i). These can be paired using the transformation $z \to \frac{-1}{z}$. Therefore Theorem 7 implies that the modular group is generated by $z \to z + 1$ and $z \to \frac{-1}{z}$.



3.1.3 Presentations of Fuchsian groups

The most general presentation of a Fuchsian group Γ with a fundamental region of finite area is

$$\langle x_1, \ldots, x_r, a_1, b_1, \ldots, a_g, b_g, p_1, \ldots, p_s \mid x_1^{m_1} = x_2^{m_2} = \ldots = x_r^{m_r} = \Pi_{i=1}^{g}[a_i, b_i]\Pi_{j=1}^{r}x_j\Pi_{k=1}^{s}p_k = 1\rangle$$

where

$x_1, \ldots, x_r$ are elliptic elements, $a_1, b_1, \ldots, a_g, b_g$ are hyperbolic elements, and $p_1, \ldots, p_s$ are parabolic elements.

We then say Γ has signature $(g; m_1, m_2, \ldots, m_r; s)$

3.1.4 A triangle group

A triangle group is denoted by (l, m, n) where l, m, n are positive integers or ∞. It is a Fuchsian group with signature $(0; l, m, n; -)$ and it has a presentation

$$\{x, y, z \mid x^l = y^m = z^n = xyz = 1\}$$

When an elliptic element has infinite period it is a parabolic element. A parabolic element generates an infinite cyclic group. For instance $z \to z + 1$ would be an example of a parabolic element in the modular group. These are easily recognised when looking at the quotient space of a Fuchsian group by the generators pairing the sides joined by cusps (or vertices of zero angle that lie on the boundary of $\mathbb{H}$). A triangle group (l, m, ∞) has the presentation

$$\{x, y, z \mid x^l = y^m = xyz = 1\}$$

For a geometric interpretation of the triangle group (l, m, n) we consider a triangle T with angles $\frac{\pi}{l}, \frac{\pi}{m}, \frac{\pi}{n}$ in a space X where X is either the sphere, the Euclidean plane, or the hyperbolic plane.

The group generated by the reflections of X in the sides of T has a subgroup of index two, consisting of all the orientation preserving transformations, isomorphic to (l, m, n).

The integers (l, m, n) determine completely the underlying space X, namely X is:
(i) The sphere iff $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} > 1$
(ii) The Euclidean plane iff $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} = 1$
(iii) The hyperbolic plane iff $\frac{1}{l} + \frac{1}{m} + \frac{1}{n} < 1$

Seeing the modular group as a triangle group we get an isomorphism $\Gamma \cong (2, 3, \infty)$ (where Γ is the modular group). We can see this from the earlier fundamental region.

3.1.5 The hyperbolic area of a fundamental region

Theorem 8 Let F_1, F_2, be two fundamental regions for a Fuchsian group Γ. Suppose that the boundaries of F_1, F_2 have zero hyperbolic area. Then $\mu(F_1) = \mu(F_2)$.

Now looking at the area of the Riemann surface produced by a Fuchsian group and using the Gauss-Bonnet theorem we prove the following.

Theorem 9 Let Γ have signature $(g; m_1, \ldots, m_r)$.

If F is a fundamental region for Γ whose boundary has zero hyperbolic area then

$$\mu(F) = 2\pi\Big\{(2g-2) + \sum_{i=1}^{r}(1 - \frac{1}{m_i})\Big\}.$$

Theorem 10 If $g \geq 0$, $m_i \geq 2$ are integers and if

$$2g - 2 + \sum_{i=1}^{r}(1 - \frac{1}{m_i}) > 0,$$

then there exists a Fuchsian group with signature $(g; m_1, \ldots, m_r)$.

Theorem 11 If F is a Dirichlet region of a Fuchsian group Γ with $\mathscr{U} \setminus \Gamma$ compact then $\mu(F) \geq \frac{\pi}{21}$. If $\mu(F) = \frac{\pi}{21}$ then Γ is a triangle group with signature $(0; 2, 3, 7)$.

Theorem 12 Let Γ be a Fuchsian group and Λ a subgroup of index n. If

$$\Gamma = \Lambda T_1 \cup \Lambda T_2 \cup \ldots \cup \Lambda T_n$$

is a decomposition of Γ into Λ-cosets and if F is a fundamental region for Γ then

(i) $F_1 = T_1(F) \cup T_2(F) \cup \ldots \cup T_n(F)$ is a fundamental region for Λ,

(ii) if $\mu(F)$ is finite and the H-area of the boundary of F is zero then $\frac{\mu(F_1)}{\mu(F)} = n$.

Chapter 4

The permutation method and arithmetic methods

4.1 Subgroups of Fuchsian groups and finite permutation groups

The following theorems are obtained from a paper written by D. Singerman entitled *Subgroups of Fuchsian groups and finite permutation groups.* This can be seen as number 15 in my list of references.

Theorem 13 (Singerman [15]) Let Γ have the signature $(g; m_1, m_2, \ldots, m_r; s)$. Then Γ contains a subgroup Γ_1 of index N with signature
$(g'; n_{11}, n_{12}, \ldots, n_{1\rho_1}, \ldots, n_{r1}, n_{r2}, \ldots, n_{r\rho_r}; s')$
if and only if
(a) There exists a finite permutation group G transitive on N points, and an epimorphism $\theta : \Gamma \to G$ satisfying the following conditions:
(i) The permutation $\theta(x_j)$ has precisely ρ_j cycles of lengths less than m_j, the lengths of these cycles being $\frac{m_j}{n_{j1}}, \ldots, \frac{m_j}{n_{j\rho_j}}$,
(ii)If we denote the number of cycles in the permutation $\theta(\gamma)$ by $\delta(\gamma)$ then $s' = \sum_{k=1}^{s} \delta(p_k)$.
(b) $\frac{\mu(F_1)}{\mu(F)} = N$ (where F,F_1 are fundamental regions of Γ and Γ_1 respectively).

Let G be a finite permutation group which acts transitively on N points and which is a homomorphic image of a triangle group $(0; l, m, n)$. Then G has generators a, b, c obeying the relations

$$a^l = b^m = c^n = abc = 1$$

Let the permutation a have λ_u u-cycles ($u = 1, 2, \ldots, l-1$), b have μ_v v-cycles ($v = 1, 2, \ldots, m-1$),and c have ν_w w-cycles ($w = 1, 2, \ldots, n-1$).Then we have

Theorem 14 (Singerman [15]) There exists an integer $g \geq 0$ such that

$$2g-2+\sum_{u=1}^{l-1}\lambda_u(1-\frac{u}{l})+\sum_{v=1}^{m-1}\mu_v(1-\frac{v}{m})+\sum_{w=1}^{n-1}\nu_w(1-\frac{w}{n}) = N(1-\frac{1}{l}-\frac{1}{m}-\frac{1}{n})$$

4.2 *Homogeneous* and *inhomogeneous groups*

Let Γ_1 be a subgroup of the *homogeneous* group SL(2,$\mathbb{Z}$). The map
$\varphi : A \mapsto \bar{A}, \qquad A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma_1, \qquad \bar{A} : z \to \frac{az+b}{cz+d},$
induces a homomorphism of Γ_1 onto a subgroup
$\bar{\Gamma_1} = \varphi(\Gamma_1)$
of the *inhomogeneous* modular group PSL(2,$\mathbb{Z}$). This homomorphism has the kernel $\pm I$ if $-I \in \Gamma_1$, and is an isomorphism if $-I \notin \Gamma_1$

4.3 Congruence subgroups of the modular group

The principal congruence group is defined to be

$$\Gamma(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid \begin{pmatrix} a & b \\ c & d \end{pmatrix} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \text{modN}\}$$

Any subgroup G of the modular group Γ containing a principal congruence subgroup $\Gamma(N)$ is called a congruence subgroup, and the least N such that $G \geq \Gamma(N)$ is called the level of G.

Some examples of congruence subgroups are:

$$\Gamma_1(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid a \equiv d \equiv 1\text{modN}, c \equiv 0\text{modN}\}$$

$$\Gamma_0(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid c \equiv 0\text{modN}\}$$

$$\Gamma^0(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid b \equiv 0\text{modN}\}$$

$$\Gamma^0_0(N) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma \mid b \equiv c \equiv 0\text{modN}\}$$

defined for any positive integer N.

We can now see that the following inclusion holds for any positive integer:

$\Gamma(N) \leq \Gamma_1(N) \leq \Gamma_0(N) \leq \Gamma$

If N > 2, for the indices (in the inhomogeneous modular group) of the above inclusions we have the formulae:

$| \Gamma : \Gamma_0(N) |= N\Pi_{p|N}(1 + \frac{1}{p})$

$| \Gamma : \Gamma_1(N) |= \frac{N^2}{2}\Pi_{p|N}(1 - \frac{1}{p^2})$

$| \Gamma : \Gamma(N) |= \frac{N^3}{2}\Pi_{p|N}(1 - \frac{1}{p^2})$

$| \Gamma : \Gamma_0^0(N) |= \frac{\phi(N)}{2} = \frac{N}{2}\Pi_{p|N}(1 - \frac{1}{p})$

and

$| \Gamma : \Gamma_0(2) |= 3$, $| \Gamma : \Gamma_1(2) |= 3$, $| \Gamma : \Gamma(2) |= 6$, and $| \Gamma : \Gamma_0^0(2) |= 6$.

The products in these equations run over the distinct prime divisors of N.

A well known isomorphism is:

$\mathrm{PSL}(2,\mathbb{Z}_N) \cong \Gamma/\Gamma(N)$

Specific examples of these groups are:

$\mathrm{PSL}(2,\mathbb{Z}_2) \cong S_3$, $\mathrm{PSL}(2,\mathbb{Z}_3) \cong A_4$,

$\mathrm{PSL}(2,\mathbb{Z}_4) \cong S_4$,$\mathrm{PSL}(2,\mathbb{Z}_5) \cong A_5$

where S_n, A_n are the symmetric and alternating groups on n points.

Example We can prove that $\mathrm{PSL}(2,\mathbb{Z}_2)$ is isomorphic to S_3 by calculating the multiplication table of $\mathrm{PSL}(2,\mathbb{Z}_2)$. By identifying specific elements of $\mathrm{PSL}(2,\mathbb{Z}_2)$ with specific elements of S_3 it is easy to see that the structure of $\mathrm{PSL}(2,\mathbb{Z}_2)$ is the same as that of S_3.
In the table below we use the following notation for the matrices of $\mathrm{PSL}(2,\mathbb{Z}_2)$.
$e = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $a_1 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$, $a_2 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$,
$a_3 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $a_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$, $a_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

	e	a_1	a_2	a_3	a_4	a_5
e	e	a_1	a_2	a_3	a_4	a_5
a_1	a_1	e	a_3	a_2	a_5	a_4
a_2	a_2	a_5	a_4	a_1	e	a_3
a_3	a_3	a_4	a_5	e	a_1	a_2
a_4	a_4	a_3	e	a_5	a_2	a_1
a_5	a_5	a_2	a_1	a_4	a_3	e

The multiplication table of PSL(2,$\mathbb{Z}_2$)

By reordering these elements we can clearly see the isomorphism between the two groups.

	e	a_2	a_4	a_3	a_1	a_5
e	e	a_2	a_4	a_3	a_1	a_5
a_2	a_2	a_4	e	a_1	a_5	a_3
a_4	a_4	e	a_2	a_5	a_3	a_1
a_3	a_3	a_5	a_1	e	a_4	a_2
a_1	a_1	a_3	a_5	a_2	e	a_4
a_5	a_5	a_1	a_3	a_4	a_2	e

This table is identical to the multiplication table of S_3. Therefore knowing that $S_3 = < a, b | a^2 = b^2 = (ab)^3 = 1 >$ we can find the isomorphism $a \longrightarrow a_2$, $b \longrightarrow a_3$.

Example: Looking at Theorem 13 As shown in Singerman's theorem we can describe (conjugacy classes of) subgroups Γ_1 of the modular group in terms of transitive permutation representations of the modular group.

If we look at the fundamental region of $\Gamma(2)$ (which is later illustrated more thoroughly)and we number the modular triangles we can see this in action. The permutations are as follows:

$$X = (1,3)(2,6)(4,5)$$

$$Y = (1,4,6)(3,2,5)$$

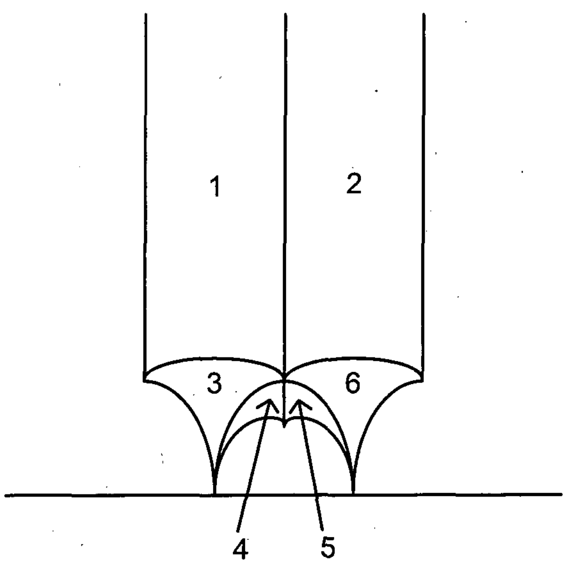
The permutation X being induced by $E = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ and the permutation Y being induced by $V = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$.

By calculating

$$XY = (1,3)(2,6)(4,5)(1,4,6)(3,2,5) = (1,2)(3,4)(5,6)$$

we can determine that this modular subgroup has three conjugacy classes of parabolic cyclic subgroups.

We can also see that its presentation is $< a, b| -- >\cong F_2$, the free group on two elements.

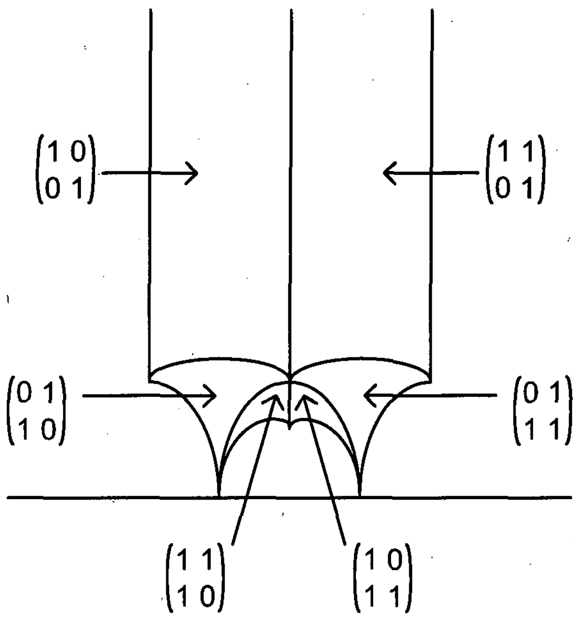


4.3.1 The cosets of some congruence subgroups

Let A,B be two elements of the modular group Γ as shown below
$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$,$B = \begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix}$
then I.Ivrissimtzis proved the following theorem in his thesis (see reference [7]).

Theorem 15 (Ivrissimtzis [7]) Let N be a positive integer. The matrices A, B belong in the same coset of
(i) $\Gamma_0(N)$ if and only if $ac' - ca' \equiv 0\text{modN}$
(ii) $\Gamma_1(N)$ if and only if $a \equiv a'\text{modN}$, $c \equiv c'\text{modN}$ or $a \equiv -a'\text{modN}$, $c \equiv -c'\text{modN}$
(iii)$\Gamma(N)$ if and only if $a \equiv a'\text{modN}$, $b \equiv b'\text{modN}, c \equiv c'\text{modN}$, $d \equiv d'\text{modN}$ or $a \equiv -a'\text{modN}$, $b \equiv -b'\text{modN}, c \equiv -c'\text{modN}$, $d \equiv -d'\text{modN}$

An application Using both Theorem 13 and Theorem 15 we can look at the fundamental region for $\Gamma(2)$. Seeing how different matrices relate to different sections of the region as illustrated below.



An interesting theorem related to the fundamental regions of subgroups Γ_1 of the modular group Γ which illustrates the above example is

Theorem 16 (Hoare and Singerman [4]) There exists a fundamental region F_1 for Γ_1 which is constructed from modular triangles in such a way that every pair of them can be connected by a chain of modular triangles with a common side. This fundamental region F_1 is simply connected.

The system of inequivalent cusps for the homogeneous principal congruence subgroup $\Gamma(N)$ (This result can be seen as theorem 8, in chapter 4, Elliptic modular functions [14].)

Theorem 17 If $N > 2$, the number $\sigma_\infty(N)$ of inequivalent rational cusps for $\Gamma(N)$ is equal to one half the number of pairs (a, b) incongruent $modN$ with $(a, b, N) = 1$. If $N = 2$, $\sigma_\infty(N)$ is equal to the number of these pairs.

Example When we apply theorem 17 to $N = 12$ we find the following incongruent pairs.
(0, 1),(0, 5),(0, 7),(0, 11),
(1, 0),(1, 1),(1, 2),(1, 3),(1, 4),(1, 5),(1, 6),(1, 7),(1, 8),(1, 9),(1, 10),(1, 11),
(2, 1),(2, 3),(2, 5),(2, 7),(2, 9),(2, 11),
(3, 1),(3, 2),(3, 4),(3, 5),(3, 7),(3, 8),(3, 10),(3, 11),
(4, 1),(4, 3),(4, 5),(4, 7),(4, 9),(4, 11),
(5, 0),(5, 1),(5, 2),(5, 3),(5, 4),(5, 5),(5, 6),(5, 7),(5, 8),(5, 9),(5, 10),(5, 11),
(6, 1),(6, 5),(6, 7),(6, 11),
(7, 0),(7, 1),(7, 2),(7, 3),(7, 4),(7, 5),(7, 6),(7, 7),(7, 8),(7, 9),(7, 10),(7, 11),
(8, 1),(8, 3),(8, 5),(8, 7),(8, 9),(8, 11),
(9, 1),(9, 2),(9, 4),(9, 5),(9, 7),(9, 8),(9, 10),(9, 11),
(10, 1),(10, 3),(10, 5),(10, 7),(10, 9),(10, 11),
(11, 0),(11, 1),(11, 2),(11, 3),(11, 4),(11, 5),(11, 6),(11, 7),(11, 8),(11, 9),
(11, 10),(11, 11).
There are 96 pairs and so $\Gamma(12)$ has 48 inequivalent cusps by the above theorem.

4.4 Wohlfahrt's result

A parabolic element L of the modular group can always be written in the form $\pm A^{-1}\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^m A$ where A is a matrix within the modular group and $A^{-1}\infty$ is a parabolic fixed point of the induced transformation. In this form, m is a rational integer, not zero and uniquely determined by L. The modulus $\mid m \mid$ of m will be called the amplitude of the parabolic matrix L. If a subgroup Γ_1 of the homogeneous modular group contains parabolic elements P their fixed points are called cusps. If we take ξ to be a cusp of a modular subgroup Γ_1 then the subgroup of Γ_1, generated by $-I$ and a certain parabolic element P, contains all the matrices in Γ_1 with fixed point ξ. The element P cannot be uniquely determined by Γ_1 and ξ but the choice is limited to $\pm P$,$\pm P^{-1}$ of a common amplitude m. This common amplitude is equivalent to the width of the cusp in the quotient of the upper half plane by this modular subgroup. The width of a cusp is the number of modular triangles in the quotient space that touch this cusp.
All equivalent cusps have equal cusp widths.

Definition Let Γ_1 be any subgroup of the modular group and denote $C(\Gamma_1)$ to be the subset of all cusp widths of Γ_1 in the set $\mathbb{Z}$. If $C(\Gamma_1)$ is nonempty and bounded in $\mathbb{Z}$, the lowest common multiple of all the numbers in $C(\Gamma_1)$ is a number $m \in \mathbb{Z}$ and will be called the level of Γ_1. If $C(\Gamma_1)$ is empty or unbounded, the level of Γ_1 is defined to be the number zero.

Example A normal subgroup Γ_1 of finite index of PSL(2,$\mathbb{Z}$) has just one class of cusps and so $C(\Gamma_1)$ has just one element. This is illustrated by looking back at the diagram of the fundamental region of $\Gamma(2)$. There we see that all of the cusps have width two and therefore the level of this group is two.

Looking at subgroups We can now begin to see how theorem 13 works in terms of parabolic elements. When we look at subgroups of the modular group we see that the cusp widths of the subgroups are equivalent to the length of the cycles in the coset permutation representation of the parabolic element. Therefore the lowest common multiple of these cycle lengths is the level of the subgroup.

Theorem 18(Wohlfahrt, An extension of F.Klein's level concept [17]) Let m be the level of a modular subgroup Γ_1. Then Γ_1 is a congruence subgroup if and only if it contains $\Gamma(m)$.

Theorem 19(Wohlfahrt, An extension of F.Klein's level concept [17]) Let Γ_1 be a subgroup of the modular group of index $\mu \leq 6$. Then Γ_1 is a congruence subgroup.

4.5 Dessins d'enfants

If we let G be a triangle group then dessin d'enfants can be thought of as:

1.Drawings (structured in a manner consistent with G) on connected orientated surfaces,

In these terms a map or dessin is an embedding of a graph G' in a surface S such that the components of S-G' are simply connected. As the graphs become more complex we often require a surface with genus greater than zero to achieve this. From this we acquire the genus of our subgroup.

2.Subgroups of G,

The stabilizer of a dessin is a subgroup of G. It consists of group elements that leave the dessin unchanged.

3.Transitive permutation representations of G, along with a choice of basepoint.

The transitive permutation representation of G corresponding to a subgroup $G_1 \leq G$ is the permutation of the cosets of G_1 in G. We will describe the coset representations of subgroups of the modular group using the following notation.
Let

$$\gamma_0 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \gamma_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \gamma_\infty = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix},$$

where the following relation is satisfied

$$\gamma_0^3 = \gamma_1^2 = \gamma_0\gamma_1\gamma_\infty = 1$$

in G. This is the complete set of defining relations in our group and so to obtain a transitive permutation representation of G, we need only choose transitive permutations $\sigma_0, \sigma_1, \sigma_\infty$ that satisfy the above relation. Dessin d' enfants then go onto represent these permutations pictorially.

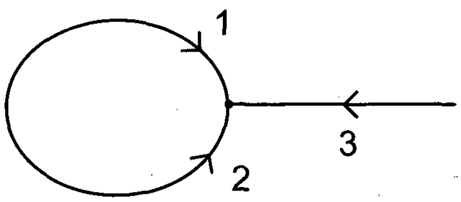
In the dessin d'enfants we number each of the darts in our diagram. Then:

1. The edges relate to σ_1

By transposing any two numbered darts which lie on the same edge we obtain our permutation σ_1 (the other darts remain constant).

2. The vertices relate to σ_0
By rotating our numbered darts anticlockwise around the vertices (i.e. where the edges intersect) we obtain our permutation σ_0.
3. The faces relate to σ_∞
By the combined effect of the two actions in 1 and 2 we obtain the permutation σ_∞.

Example As an example we look at the permutations and dessin of the coset representation of $\Gamma_0(2)$ as a subgroup of the modular group.
The permutations are $\sigma_0 = (1,2,3)$, $\sigma_1 = (1,2)(3)$, and $\sigma_\infty = (1)(2,3)$. They have the dessin below:



Using theorem 13 we can deduce from the permutations in this example that $\Gamma_0(2)$ (conjugate to the stabiliser of a point) has the signature $(0; 2, \infty, \infty)$.

4.5.1 Imprimitivity

Let Ω be a set. A partition of Ω is a set P of nonempty disjoint subsets of Ω whose union is Ω. We use $\mathrm{Part}(\Omega)$ to denote the set of partitions of Ω. Now suppose that G is a group which acts transitively on Ω. Then a system of imprimitivity for G on Ω is a partition B whose members are permuted by G. We denote the set of systems of imprimitivity for G on Ω by $Imp_G(\Omega)$.

Definition We define a map Φ_1 from $Imp_G(\Omega)$ to subgroups of G by saying that, for $B \in Imp_G(\Omega)$, $\Phi_1(B)$ is the stabiliser of the member of B containing 1.

We also note the following definition.

Definition A permutation group G acting on a set Ω is called primitive if G preserves no nontrivial partition of Ω.

Using this definition we can use the following theorem on the above example.

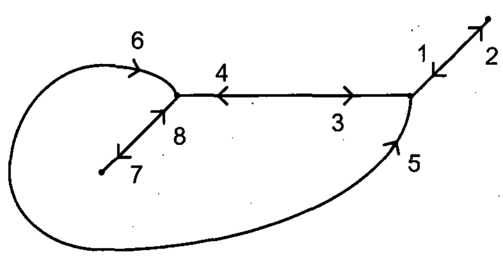
Theorem 20 If G is transitive on the set Ω, then G is primitive on Ω if and only if for each $\alpha \in \Omega$, G_α is a maximal subgroup of G. Here $G_\alpha = Stab_G(\alpha)$ is the stabiliser of $\alpha \in \Omega$.

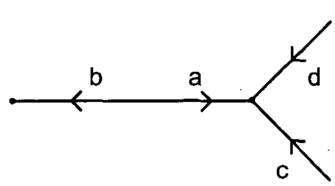
We can therefore see that in the above example we must be looking at a maximal subgroup. This theorem also comes in very handy when we are building our table of subgroups at the end of this chapter.

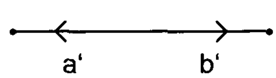
4.6 Calculations

We will use some of the above results to study subgroups of low index in the modular group.

Example 1 (applying the theory of imprimitivity) A subgroup of index eight in the modular group has a coset permutation representation generated by:
$E = (1,2)(3,4)(5,6)(7,8)$,
$V = (1,3,5)(4,6,8)(2)(7)$.
Two systems of imprimitivity of these permutations are the partitions:
(i) $a = \{1,8\}$, $b = \{2,7\}$, $c = \{3,4\}$, $d = \{5,6\}$.
These blocks are permuted by the generators as follows:
E=(a, b)(c)(d),
V=(a, c, d)(b).
(ii) $a' = \{1,3,5,7\}$, $b' = \{2,4,6,8\}$.
These blocks are permuted by the generators as follows:
E=(a', b'),
V=(a')(b').
Therefore our original group of index 8 in the modular group (represented by the permutations above) lies in a subgroup of index four in the modular group (represented by the permutations found in (i)) and a subgroup of index two in the modular group (represented by the permutations found in (ii)). The dessins are as below:





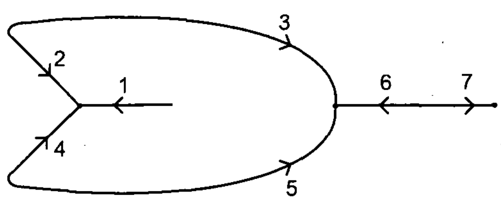


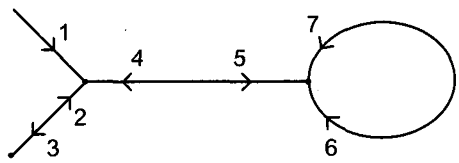
Example 2 (finding non-congruence subgroups of index 7 in the modular group) As we know, seven is the lowest index that a non-congruence subgroup can have in the modular group (Wohlfahrt, An extension of F.Klein's level concept [17]).

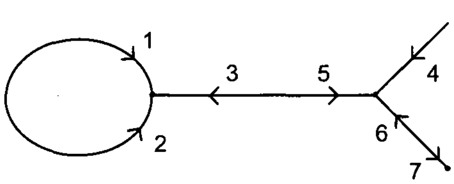
For index seven in the modular group we have the following permutation representations for the cosets of four non-conjugate subgroups:

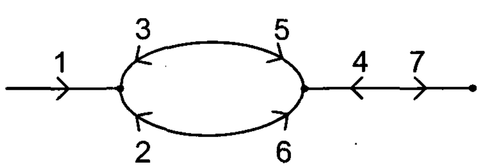
$X = (1)(2,3)(4,5)(6,7), Y = (1,2,4)(3,5,6)(7), XY = (1,2,5)(3,4,6,7)$
$X = (1)(2,3)(4,5)(6,7), Y = (1,2,4)(3)(5,6,7), XY = (1,2,3,4,6,5)(7)$
$X = (1,2)(3,5)(6,7)(4), Y = (1,2,3)(4,5,6), XY = (1,3,6,7,4,5)(2)$
$X = (1)(3,5)(2,6)(4,7), Y = (1,2,3)(4,5,6), XY = (1,2,4,7,5)(3,6)$

These subgroups have geometric levels 12, 6, 6, and 10 respectively. Therefore to be congruence subgroups (by Wohlfahrt's Theorem) they must contain $\Gamma(12)$, $\Gamma(6)$, $\Gamma(6)$, $\Gamma(10)$ respectively. The indexes of $\Gamma(12)$, $\Gamma(6)$, $\Gamma(6)$, $\Gamma(10)$ within the modular group are 576, 72, 72, and 360 respectively (by the before mentioned formula). For these to lie in our four subgroups the indexes must be divisible by seven. These numbers are not divisible by seven and so these subgroups(the stabilisers of one in these four permutation representations) must be non-congruence.

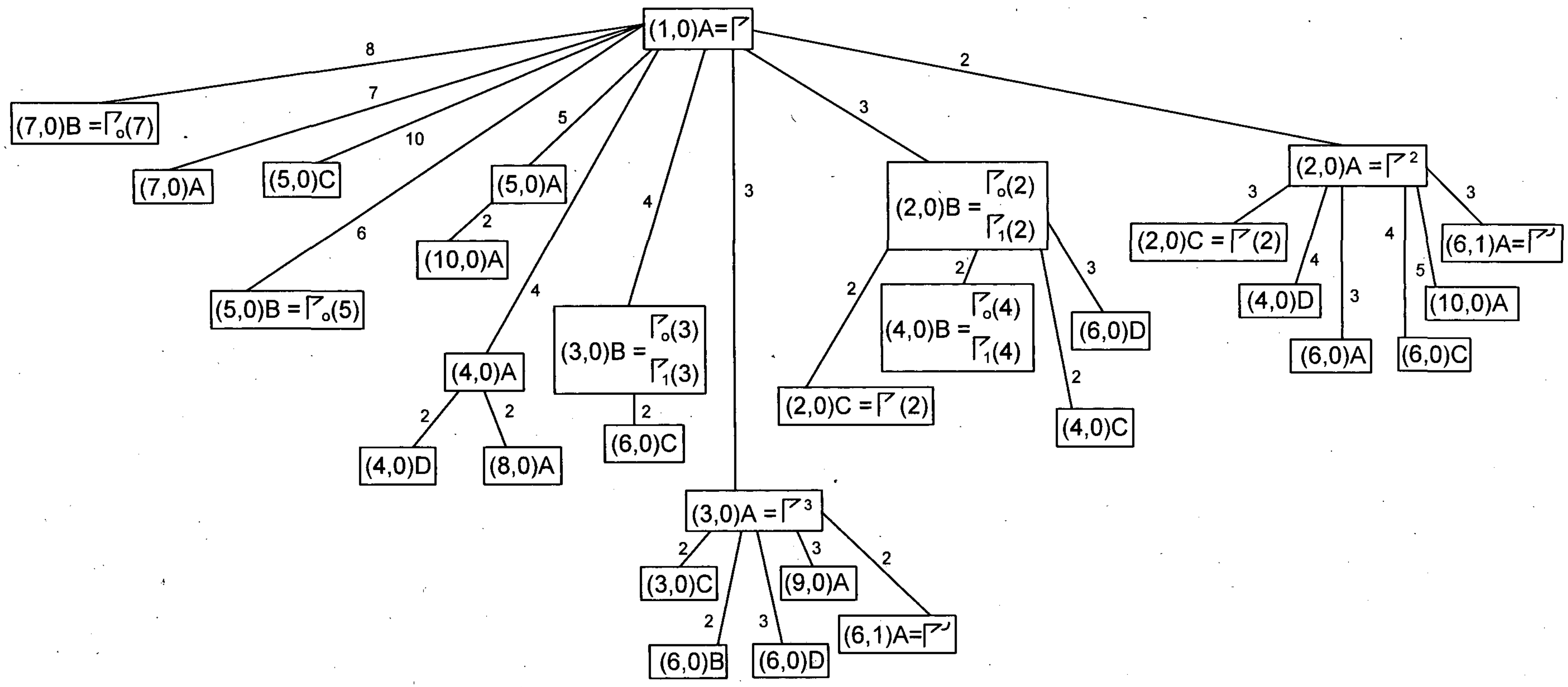








A table of all the conjugacy classes of congruence subgroups of index ≤ 10



In this table the first number before the letters distinguishing the groups tells you their level and the second number indicates the genus of the group. The numbers which lie on the lines that join the group boxes are the indices (i.e. the index of the lower group to the group it is joined to above). Using calculations we have been able to identify some of the groups. Many of these groups have been defined previously in this thesis. The groups which are as yet undefined are the following:

(a)Γ'

This is the commutator subgroup of the modular group, i.e. the subgroup generated by the commutators of its elements, that is

$\Gamma' = < g^{-1}h^{-1}gh|g, h \in \Gamma >$.

(b)Γ^2

This is the power subgroup generated by the squares of the elements of Γ.

(c)Γ^3

This is the power subgroup generated by the cubes of the elements of Γ.

4.7 Some sample calculations performed to obtain the table

Calculation 1 There are only two possible coset permutation representations for a subgroup of index 3 in the modular group. These being:
1. X = (1)(2)(3), Y = (1, 2, 3) where XY = (1, 2, 3)
2. X = (1, 2)(3), Y = (1, 2, 3) where XY = (1, 3)(2)

As you can see only one of these has level 2 and so this must be the coset permutation representation for $\Gamma_0(2)$ (or equivalently $\Gamma_1(2)$).

Calculation 2 There is only one possible coset permutation representation for a subgroup of index 5 in the modular group. This is:
X = (1)(2, 3)(4, 5), Y = (1, 2, 4)(3)(5) where XY=(1, 2, 3, 4, 5).
This must be a congruence subgroup due to Wohlfahrt's result.

Calculation 3 We know that $\Gamma(2)$ lies in $\Gamma_0(2)$ (or equivalently $\Gamma_1(2)$) therefore the coset permutation representation in calculation one must be a system of imprimitivity of the coset permutation representation of $\Gamma(2)$. By calculations (finding all the possible dessins on six points) we find that there is only one subgroup of index six in the modular group which is of level two. This is:
$X = (1,4)(2,6)(3,5), Y = (1,2,3)(4,5,6)$ where $XY = (1,5)(2,4)(3,6)$.
Splitting this into the following partition:

$$A = \{2,6\}, B = \{3,4\}, C = \{1,5\}$$

we find that these blocks are permutated as follows by the permutations above.
X = (A)(B, C), Y = (A, B, C)
We can easily see that these are the same permutations as in calculation one. Therefore X and Y must be our coset permutation representation for $\Gamma(2)$.

Calculation 4 There is only one coset permutation representation of index six in the modular group which has level five. This is:
X = (1)(2)(3, 4)(5, 6), Y = (1, 2, 3)(4, 5, 6) where XY = (1, 2, 3, 5, 4)(6).
Therefore this must be the coset permutation representation of $\Gamma_0(5)$.

A lot of my work was finding all the possible dessin d'enfants which have either one or three lines touching each vertex (i.e. which represent subgroups of the modular group).

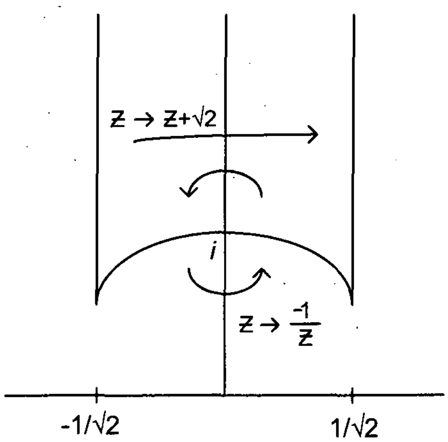
Chapter 5

Hecke groups

We will now move onto to look at Hecke groups and try to see how we can generalise the above ideas in these groups.
Consider the group generated by $z \to \frac{-1}{z}$ and $z \to z + \lambda$.
Hecke (see reference [3]) showed that these groups are discrete if either:
1. $\lambda \geq 2$
or
2. $\lambda = 2\cos\frac{\pi}{q}$ where $q \in \mathbb{N}$, $q \geq 3$.

The groups of type two are isomorphic to the triangle groups $(2, q, \infty)$. We will be concentrating of the groups of type two. These are Hecke groups of the first kind, where the limit set is dense in the reals. The second kind occurs when $\lambda > 2$. We let $H(\lambda_q)$ or H^q denote the Hecke group of type two generated by $z \to \frac{-1}{z}$ and $z \to z + \lambda_q$ where $\lambda_q = 2\cos\frac{\pi}{q}$.

An example of a fundamental region of one of these Hecke groups (i.e. $H(\sqrt{2})$) is shown below. This can be easily generalised.



The fundamental domain of H(√2)

5.1 Examples of Hecke groups

If we take $q = 3$ in the second case then we have $H(1)$ known to us as the modular group.

If $q = 4$ we have $H(\sqrt{2})$. In this group there are two types of matrices. The two types are
$\begin{pmatrix} a & b\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}$ and $\begin{pmatrix} a\sqrt{2} & b \\ c & d\sqrt{2} \end{pmatrix}$ where $a, b, c, d \in \mathbb{Z}$ and the determinants of all of these matrices equals one (i.e. ad - 2bc = 1 in the first case and 2ad - bc = 1 in the second case).

We can prove that all of the elements of $H(\sqrt{2})$ lie in these two sets of matrices by first observing that both of our generators are of one of these two forms. For example $\begin{pmatrix} 1 & \sqrt{2} \\ 0 & 1 \end{pmatrix}$ is in the first set and $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ is in the second. We will now look at what effect each of our generators has on an arbitrary member of both of these sets.

$\begin{pmatrix} a & b\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a & (a+b)\sqrt{2} \\ c\sqrt{2} & 2c+d \end{pmatrix}$,

$\begin{pmatrix} a & b\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} b\sqrt{2} & a \\ d & c\sqrt{2} \end{pmatrix}$,

$\begin{pmatrix} a\sqrt{2} & b \\ c & d\sqrt{2} \end{pmatrix}\begin{pmatrix} 1 & \sqrt{2} \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} a\sqrt{2} & 2a+b \\ c & (c+d)\sqrt{2} \end{pmatrix}$,

$\begin{pmatrix} a\sqrt{2} & b \\ c & d\sqrt{2} \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} b & a\sqrt{2} \\ d\sqrt{2} & c \end{pmatrix}$.

As the generators of the Hecke group have determinant one and as det A det B = det AB, all the matrices of the Hecke group have determinant one. Therefore the generators do not move the matrices outside these two sets and so our proof is complete.
To be specific what we have just proved is

$$H(\sqrt{2}) \subseteq H_e(\sqrt{2}) \cup H_e(\sqrt{2})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

where $H_e(\sqrt{2})$ consists of the matrices $\begin{pmatrix} a & b\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}$ with a determinant of one. The reverse inclusion can be proved. Therefore $H_e(\sqrt{2})$ and $H_e(\sqrt{2})\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ are two cosets of this Hecke group.
If $q = 6$ we have $H(\sqrt{3})$. In this group we see a similar pattern to $H(\sqrt{2})$. There are two types of matrices.
The two types are
$\begin{pmatrix} a & b\sqrt{3} \\ c\sqrt{3} & d \end{pmatrix}$ and $\begin{pmatrix} a\sqrt{3} & b \\ c & d\sqrt{3} \end{pmatrix}$ where $a, b, c, d \in \mathbb{Z}$ and the determi-

nants of all of these matrices equals one (i.e. ad - 3bc = 1 in the first case and 3ad - bc = 1 in the second case).

$H(\sqrt{2})$ and $H(\sqrt{3})$ are commensurable in the wide sense with the modular group (see 5.2) and so once we have information about subgroups in the modular group it is relatively easy to extend it to these two Hecke groups. It is however more difficult to find out information about the subgroups of the Hecke group where $q = 5$.

5.2 Working with commensurable Hecke groups

Definition A pair of subgroups H_1 and H_2 of a group G are commensurable in the wide sense if there exists $g \in G$ such that $(g^{-1}H_1g) \cap H_2$ is a finite index subgroup of both $g^{-1}H_1g$ and H_2 (when g is trivial, we say H_1 and H_2 are commensurable).

The following examples show how we can use information we know about subgroups of the modular group to obtain information about subgroups of the Hecke groups where $q = 4$ and $q = 6$.

Example 1 If we let $m = 2$ or $m = 3$ then we can calculate
$$\begin{pmatrix} m^{\frac{-1}{4}} & 0 \\ 0 & m^{\frac{1}{4}} \end{pmatrix} \begin{pmatrix} a & b(xm) \\ c & d \end{pmatrix} \begin{pmatrix} m^{\frac{1}{4}} & 0 \\ 0 & m^{\frac{-1}{4}} \end{pmatrix} = \begin{pmatrix} a & xb\sqrt{m} \\ c\sqrt{m} & d \end{pmatrix}$$
where $x \in \mathbb{Z}$.

Therefore the modular subgroups $\Gamma^0(mx)$ produce the subgroups $\begin{pmatrix} a & xb\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}$, $\begin{pmatrix} a & xb\sqrt{3} \\ c\sqrt{3} & d \end{pmatrix}$ in H^4 and H^6 respectively (where $x \in \mathbb{Z}$).

The even subgroups of the Hecke groups H^4 and H^6 are all the matrices of the form $\begin{pmatrix} a & b\sqrt{2} \\ c\sqrt{2} & d \end{pmatrix}$ and $\begin{pmatrix} a & b\sqrt{3} \\ c\sqrt{3} & d \end{pmatrix}$ respectively. Therefore as you can see above by conjugating $\Gamma^0(2)$ we can obtain the even subgroup of H^4 and by conjugating $\Gamma^0(3)$ we can obtain the even subgroup of H^6.

The even subgroups have index two in the Hecke groups. $\Gamma^0(2)$ is of index 3 and $\Gamma^0(3)$ is of index 4 in the modular group.

Example 2 Looking at principal congruence subgroups within the modular group we conjugate specific examples of them to obtain the corresponding subgroups in the Hecke groups as below.

$$\begin{pmatrix} m^{\frac{-1}{4}} & 0 \\ 0 & m^{\frac{1}{4}} \end{pmatrix} \begin{pmatrix} a(mx)+1 & b(mx) \\ c(mx) & d(mx)+1 \end{pmatrix} \begin{pmatrix} m^{\frac{1}{4}} & 0 \\ 0 & m^{\frac{-1}{4}} \end{pmatrix} = \begin{pmatrix} a(mx)+1 & bx\sqrt{m} \\ cmx\sqrt{m} & dmx+1 \end{pmatrix}$$

where $x \in \mathbb{Z}$.

However $H(\lambda_5)$ is not commensurable with the modular group (S.Katok, reference[10]) and its elements are much more difficult to describe.

Some information we do know about $\mathrm{H}(\lambda_5)$ is:
(i) $\mathbb{Z}[\lambda_5]$ is a principal ideal domain.
(ii) The set of cusps of $\mathrm{H}(\lambda_5)$ is $\mathbb{Q}(\lambda_5) \cup \{\infty\}$.

The two methods for working out the subgroups of the $\mathrm{H}(\lambda_5)$ which we have used extensively are:
(a) The permutation method
(b) Arithmetic methods

5.3 The permutation method

We looked at this method in chapter 4 (in particular 4.1, 4.5, 4.6, 4.7, and the examples in 4.3). We will now go on to use this method in H^5. Using the permutation method we obtain the results shown in the appendix.
We arrived at these results by using dessin d'enfants and coset permutation representations.

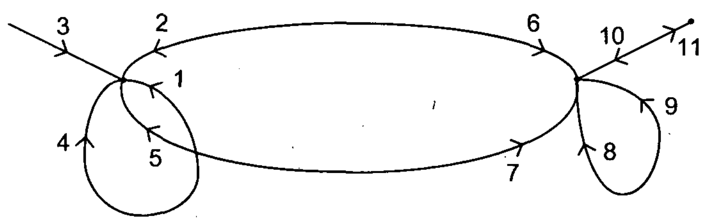
The level of a subgroup of a Hecke group As part of our results we obtain levels of our subgroups of H^5. We should therefore take the time here to explain exactly what level means in these cases. Here instead of looking at modular triangles to obtain our level we use instead fundamental regions of the specific Hecke group we are studying (see the opening page to chapter five). In our case this will be the fundamental region of H^5. Once we find a subgroup of H^5 we then go onto to observe how many of the fundamental regions of H^5 touch each cusp in our fundamental region of the subgroup we have found. The numbers we obtain here are very similar to the cusp widths we defined in the case of a modular subgroup. Similarly to the modular group we now take the lowest common multiple of all of these numbers to obtain our geometric level.

We will now go onto to look at an example of the calculations carried out.

Example The coset permutation representation of one of these subgroups is:

$X = (1,4)(5,7)(2,6)(8,9)(10,11)$
$Y = (1,2,3,4,5)(6,7,8,9,10)$
$XY = (1,5,8,10,11,6,3,4,2,7)(9)$

We can therefore deduce that the signature is $(1;2,5,\infty,\infty)$. We can also see that
$\alpha = 10$
$\beta = 11$
where α is the level of our subgroup and β is the index of our subgroup in H^5
The dessin d'enfant for this subgroup is as below:



5.4 Arithmetic methods

When we talk about using arithmetic methods we mean looking at congruence subgroups, etc. We discussed this method previously in chapter 4 (in particular 4.3 and 4.4). We have to approach this slightly differently in H^5. To implement this method we first need to define what we mean by a congruence subgroup (i.e. how principal congruence subgroups translate across to Hecke groups).

Congruence subgroups in Hecke groups If we let I be an ideal of $\mathbb{Z}[\lambda_q]$ then we can define
$PSL(2,\mathbb{Z}[\lambda_q],I) = \{\begin{pmatrix} a & b \\ c & d \end{pmatrix} \in PSL(2,\mathbb{Z}[\lambda_q])|a-1,b,c,d-1 \in I\}$
We define $PSL_1(2,\mathbb{Z}[\lambda_q],I)$ and $PSL_0(2,\mathbb{Z}[\lambda_q],I)$ similarly. For these we require $a-1,c,d-1 \in I$ and $c \in I$ respectively.

Our principal congruence subgroup is as below:
$H^q(I) = PSL(2,\mathbb{Z}[\lambda_q],I) \cap H^q$
$H^q_1(I)$ and $H^q_0(I)$ are similar.
Now, clearly $H^q(I) \leq H^q_1(I) \leq H^q_0(I) \leq H^q$.

I.Ivrissimtzis proved the following theorem in his thesis (see reference [7]).

Theorem 21 If p is a prime then any proper principal congruence subgroup of H^p is torsion free.

Proof H^p is generated by R(z) = $\frac{-1}{z}$, S(z) = z + λ_p, obeying only the relations $R^2 = (RS)^p = 1$ (where p is a prime). We write T = RS. Then, as H^p is a Fuchsian group, the only elements of finite order are conjugates of R, or conjugates of powers of T. If a principal congruence subgroup H^p(I) contains an element of finite order, then as H^p(I) is normal, it must contain R or a power of T. If it contains R = $\pm\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, then R $\equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ mod I. Thus 1 $\in$ I, and so I = Z[λ_p], and H^p(I) = H^p. If the principal congruence subgroup contains a power of T = $\pm\begin{pmatrix} 0 & -1 \\ 1 & \lambda_p \end{pmatrix}$ then as p is a prime it must contain T itself. Therefore as above, T $\in$ H^p(I) implies that 1 $\in$ I, so that I = Z[λ_p] and H^p(I) = H^p. The theorem above has therefore been proved true.

From this theorem we can observe using theorem 13 (shown earlier) that the index of any proper principal congruence subgroup within H^5 must be divisible by ten. This is because for our subgroup to be torsion free neither of the permutations X or Y can have a fixed point and so our index must be divisible by both 2 and 5. Therefore the index must be divisible by ten. This is where X and Y are permutations induced by right multiplication of the cosets of the generating elements of orders 2 and 5.

Chapter 6

Looking for a non-congruence subgroup

6.1 Proof with the modular group

In the modular group we can use three lemmas to show that there exists a normal subgroup N of the modular group Γ which is of finite index and contains none of the principal congruence subgroups $\Gamma(N)$. (These lemmas can be seen in chapter 3, Noneuclidean tesselations and their groups, Magnus [13].) The three lemmas are listed below:

Lemma One Let $\mathbb{Z}_N$ be the ring of integers *mod* N, and let $PSL(2, \mathbb{Z}_N)$ be the group of Mobius transformations with $a, b, c, d \in \mathbb{Z}_N$. Then the quotient group $\Gamma/\Gamma(N)$ of the Nth principal congruence subgroup in the modular group is isomorphic with $PSL(2, \mathbb{Z}_N)$ (This can be seen as theorem 6.9.3 and corollary 6.9.4 in Jones and Singerman, Complex functions [9]).

Proof Looking at SL(2,$\mathbb{Z}$)= $\{\begin{pmatrix} a & b \\ c & d \end{pmatrix} | a, b, c, d \in \mathbb{Z}, ad - bc = 1\}$. Let a,b,c,d be integers representing residue classes *mod* N such that $ad - bc \equiv 1 modN$. Then there exist integers a', b', c', d' such that
$a \equiv a' modN$, $b \equiv b' modN$, $c \equiv c' modN$, $d \equiv d' modN$, $a'd' - b'c' = 1$.
From this it follows that the natural mapping of SL(2,$\mathbb{Z}$) into SL(2,$\mathbb{Z}_N$) is a surjective mapping.
Let g be the greatest common divisor of a and b, and let $a^* = \frac{a}{g}$ and $b^* = \frac{b}{g}$.

Then there exist integers c^*, d^* such that
$a^*d^* - b^*c^* = 1$.
We have

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix}^{-1} = \begin{pmatrix} a'' & 0 \\ c'' & d'' \end{pmatrix} \; (a''d'' \equiv 1 mod N).$$

Now let $a''d'' = 1 + kN$ and $(c'' - k)d'' = t$. Then

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix}^{-1}\begin{pmatrix} a'' & N \\ k & d'' \end{pmatrix}^{-1}\begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}^{-1} \equiv \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} mod N,$$

and the matrix

$$\begin{pmatrix} a' & b' \\ c' & d' \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ t & 1 \end{pmatrix}\begin{pmatrix} a'' & N \\ k & d'' \end{pmatrix}\begin{pmatrix} a^* & b^* \\ c^* & d^* \end{pmatrix}$$

contains the elements a', b', c', d' which we wanted to find. Therefore our mapping has been proved to be surjective and so by using the well known formula
$Im\phi \cong \frac{G}{Ker\phi}$
(where $Im\phi = PSL(2, \mathbb{Z}_N)$, $G = \Gamma$, $Ker\phi = \Gamma(N)$)
we complete our proof.

(See exercise 6I, Jones and Singerman, Complex functions [9])

Lemma Two The only nonabelian quotient groups that can appear in a composition series (see section 6.1.1) of $PSL(2, \mathbb{Z}_N)$ are the groups $PSL(2, \mathbb{Z}_p)$, where p is a prime number.[This implies $p \geq 5$, in which case $PSL(2, \mathbb{Z}_p)$ is simple.]

Proof We show that $SL(2, \mathbb{Z}_N)$ is the direct product of the groups $SL(2, \mathbb{Z}_q)$, where q denotes any one of the distinct prime powers $q_\nu = p_\nu^{e_\nu}$ the product of which equals N. Let again a, b, c, d be integers representing residue classes mod N, such that $ad - bc \equiv 1$ mod N, and let a_ν, b_ν, c_ν, d_ν be integers such that
$a \equiv a_\nu$ mod q_ν, $b \equiv b_\nu$ mod q_ν, $c \equiv c_\nu$ mod q_ν, $d \equiv d_\nu$ mod q_ν,
$a_\nu \equiv 1$ mod q_μ, $b_\nu \equiv 0$ mod q_μ, $c_\nu \equiv 0$ mod q_μ, $d_\nu \equiv 1$ mod q_μ for $\mu \neq \nu$.
According to the Chinese remainder theorem, the a_ν, b_ν, c_ν, d_ν exist and are uniquely determined mod N. Let

$$M_\nu = \begin{pmatrix} a_\nu & b_\nu \\ c_\nu & d_\nu \end{pmatrix}$$

Then the product M of the M_ν has the property to be congruent M_ν mod q_ν for all ν, regardless of the arrangement of the factors. Therefore,

$$M \equiv \begin{pmatrix} a & b \\ c & d \end{pmatrix} \text{ mod } N.$$

Since $ad - bc \equiv a_\nu d_\nu - b_\nu c_\nu \equiv 1$ mod q_ν, we have a unique decomposition of the elements of $SL(2, \mathbb{Z}_N)$ into a product of pairwise commuting matrices M_ν which define elements of $SL(2, \mathbb{Z}_{q_\nu})$. Therefore $SL(2, \mathbb{Z}_N)$ is indeed the direct product of the $SL(2, \mathbb{Z}_{q_\nu})$, and a quotient group that appears in the composition series of $PSL(2, \mathbb{Z}_N)$ must also appear as a quotient group in a composition series of an $SL(2, \mathbb{Z}_q)$. Next we show that the quotient groups of a composition series of $SL(2, \mathbb{Z}_q)$ are all abelian, with the possible exception of $PSL(2, \mathbb{Z}_p)$, which is the quotient group of $SL(2, \mathbb{Z}_p)$ with respect to its centre. The proof is based on the remark that $SL(2, \mathbb{Z}_{p^{\nu+1}})$ contains a normal abelian subgroup N_ν whose quotient group is $SL(2, \mathbb{Z}_{p^\nu})$, where N_ν consists of the matrices P_ν defined by

$$P_\nu \equiv \begin{pmatrix} 1 + \lambda p^\nu & \rho p^\nu \\ \sigma p^\nu & 1 - \lambda p^\nu \end{pmatrix} \bmod p^{\nu+1},$$

in which λ, ρ, σ are representatives of residue classes *mod* p. From direct calculations we see that the P_ν form an abelian group of exponent p and rank 3 which is normal in $SL(2, \mathbb{Z}_{p^{\nu+1}})$, and that $M^{-1}M^*$ is a matrix of the type of P_ν if $M, M^* \in SL(2, \mathbb{Z}_{p^{\nu+1}})$ and $M \equiv M^* mod p^\nu$. This completes the proof.

To prove lemma three we require the following theorem.

Theorem 22 (see reference [16], p.34) A primitive group which contains a transposition is isomorphic to S_n. A primitive group which contains a 3-cycle is isomorphic to A_n or S_n.

Lemma Three (see reference [13]) The alternating group A_{11} on 11 symbols is a quotient group of the modular group, and A_{11} is not isomorphic with any group $PSL(2, \mathbb{Z}_p) = \Gamma/\Gamma(p)$.

Proof We observe first that the simple group A_{11} cannot be isomorphic with any group $PSL(2, \mathbb{Z}_p)$, because the latter group is known to be of order p(p^2-1)/2, and p is the greatest prime number dividing its order. Since 11 is the greatest prime number dividing the order of A_{11}, we would have p = 11. But A_{11} has order 11!/2 which is larger than 11(11^2-1)/2. Now we show that A_{11} is a quotient group of the modular group $PSL(2, \mathbb{Z})$. To prove this, we merely have to show that A_{11} can be generated by an element X of order two and an element Y of order 3. We choose

X = (1, 5)(6, 7)(8, 10)(9, 11),

Y = (1, 2, 3)(4, 5, 6)(7, 8, 9). Then

YX = (1, 2, 3, 5, 7, 10, 8, 11, 9, 6, 4).

Since YX is an eleven-cycle, the group generated by X,Y is certainly transitive. Since 11 is a prime number, the group is also primitive. If we can show it contains a three- cycle, it must be A_{11} or S_{11}, according to theorem 22. Since X,Y $\in A_{11}$ we merely have to compose a three-cycle out of these

permutations to complete the proof of lemma three. Now
$Y^{-1}XY = (2, 6)(4, 8)(7, 11)(9, 10)$,
$YXY^{-1}X = (1, 5, 11, 10, 6, 7, 8, 9)(3, 4)$,
$(YXY^{-1}X)^4 = (1, 6)(5, 7)(8, 11)(9, 10)$,
$Z = Y^{-1}XY(YXY^{-1}X)^4 = (1, 6, 2)(4, 11, 5, 7, 8)$.
Therefore $Z^5 = (1, 2, 6)$ is a three cycle and our proof is complete.

6.1.1 The Jordan-Hölder Theorem

An important result that is used in the proof above is called the Jordan-Hölder Theorem. To understand this theorem we must first define what we mean by a composition series.

Definition Every finite group G of order greater than one possesses a finite series of subgroups, called a composition series, such that
$e \lhd T_s \lhd \cdots \lhd T_2 \lhd T_1 \lhd G$,
where T_{i+1} is a maximal normal subgroup of T_i and $T \lhd G$ means that T is a normal subgroup of G. A composition series is therefore a normal series without repetition whose factors are all simple. The quotient groups G/T_1, T_1/T_2, ..., T_{s-1}/T_s, and T_s are called composition factors of G.

The Jordan-Hölder Theorem The composition quotient groups belonging to two composition series of a finite group G are, apart from their sequence, isomorphic in pairs. In other words, if
$e \subset T_s \subset \cdots \subset T_2 \subset T_1 \subset G$
is one composition series and
$e \subset K_t \subset \cdots \subset K_2 \subset K_1 \subset G$
is another, then $t = s$, and corresponding to any composition quotient group K_j/K_{j+1}, there is a composition quotient group T_i/T_{i+1} such that
$K_j/K_{j+1} \cong T_i/T_{i+1}$.

The kernel of the homomorphic mapping from the modular group Γ to A_{11} provides us with a non congruence subgroup because the simple group A_{11} cannot appear as a quotient in a composition series of any group $\Gamma/\Gamma(N)$.

We can generalise this proof to H^5.

6.2 Proof extended to H^5

We now begin to think about how to extend this proof to H^5.
For any ideal I of $\mathbb{Z}[\lambda_q]$, $H^q(I)$ is the kernel of a natural homomorphism $\rho : H^q \to PSL(2, \mathbb{Z}[\lambda_q]/I)$. Thus each principal congruence subgroup of H^q is normal and of finite index.
This is equivalent to lemma one in the previous proof therefore lemma one holds. When we look at lemma two and how its proof works we see that lemma two follows through similarly to before as we are working with a Euclidean domain and so primes factorization works in a similar way. We are therefore left with lemma three to look at.

In a paper by Lang, Lim, and Tan (see reference [11]) they prove the following result.

Corollary (Lang, Lim, and Tan [11]) Let p be the positive rational prime that lies below the prime ideal (τ) of $\mathbb{Z}[\lambda_5]$. The indices of the congruence subgroups of H^5 of level (τ) are given by

p	(τ)	$[H^5 : H^5(\tau)]$	$[H^5 : {H_1}^5(\tau)]$	$[H^5 : {H_0}^5(\tau)]$
2	$(\tau) = (2)$	10	5	5
3	$(\tau) = (3)$	60	20	10
5	$(\tau) = (2 + \lambda_5)$	60	12	6
$\equiv \pm 3 (mod 10), \neq 3$	$(\tau) = (p)$	a	$(p^4 - 1)/2$	$p^2 + 1$
$\equiv \pm 1 (mod 10)$	$(\tau) \neq (p)$	b	$(p^2 - 1)/2$	$p + 1$

where $a = \frac{1}{2}(p^2 - 1)p^2(p^2 + 1)$ and $b = \frac{1}{2}(p - 1)p(p + 1)$

Looking at the results in the appendix (at the end of my thesis)
In the appendix we look at subgroups of low index in H^5 and their corresponding transistive permutation representations. We see epimorphisms from H^5 onto various finite permutation groups. By the coset permutation representations we find we can see that A_{11} is a quotient group of H^5. In fact using GAP we are able to deduce that there are up to automorphisms of A_{11}, 160 epimorphisms from H^5 onto A_{11} in the appendix. The representatives of these are indicated by stars in their first column.
One specific example of these epimorphisms is:
X =(1, 10)(5, 7)(2, 8)(9, 11)
Y =(1, 2, 3, 4, 5)(6, 7, 8, 9, 10)

We can see that this system is primitive by observing that it cannot have any systems of imprimitivity as 11 is prime(see section 4.5.1). Using theorem 22 and calculating the equation below we can see that these per-

mutations do indeed generate A_{11}.
By calculation we get
$(XY)^4 = (1, 6, 7)$
for the above permutations. Therefore since both X and Y lie in A_{11} they must generate this group.

We can easily see that A_{11} does not have the same index as any of the principal congruence subgroups mentioned in Lang, Lim, and Tan's corollary. This can be seen because 11 is the greatest prime that divides the order of A_{11}. However neither 10 or 60 are divisible by 11. As for the other indexes the p used to calculate these is the greatest prime integer that divides the indexes and if we substitute $p = 11$ into these formulae we get $(10 \times 11 \times 12)/2 = 660$ and $(120 \times 121 \times 122)/2 = 885720$. The order however of A_{11} is $11!/2 = 19958400$ and this is not the same as either of the above so we can see that A_{11} is none of the groups mentioned in the theorem by Lim, Lang, and Tan. Therefore the kernel of the epimorphism from H^5 onto A_{11} gives us a non-congruence subgroup (in the same way as before). By combining all our work we can deduce our main result which is as follows.

Main Result There is a non-congruence subgroup of index 11 in H^5.

Chapter 7

Acknowledgements

I would like to thank Professor David Singerman for all his help and guidance during my studies. I would also like to thank the School of Mathematics at Southampton University for all their support during my time there. I am grateful for the financial support from EPSRC.

Chapter 8

References

[1]J.W.Anderson, *Hyperbolic Geometry*, Springer-Verlag, 2005
[2]C.J.Cummins and S.Pauli, *Congruence subgroups of PSL(2,$\mathbb{Z}$) of genus less than or equal to 24*, Experimental Mathematics, Vol.12, Issue 2, 243-255, 2003
[3]E.Hecke, *Über die Bestimmung Dirichletscher Reihen durch ihre Funktionalgleichung*, Mathematische Annalen, Vol.112, 664-699, 1936
[4]Hoare and Singerman, *Groups St. Andrews*, LMS lecture notes, Vol.71, 221-227, 1982
[5]T.Hsu, *Permutation techniques for coset representations of modular subgroups*, Geometric Galois Actions II: Dessins d'enfants, Mapping Class Groups and Moduli, Vol.243 of L.M.S. lecture notes, 67-77, Cambridge University Press, 1991
[6]T.Hsu, *Identifying congruence subgroups of the modular group*, Proceedings of the American mathematical society, Vol.124, No.5, 1351-1359, May 1996
[7]I.P.Ivrissimtzis, *Congruence subgroups of Hecke groups and regular dessins*, PhD Thesis, University of Southampton, 1998
[8]I.P.Ivrissimtzis and D.Singerman, *Regular maps and principal congruence subgroups of Hecke groups*, European Journal of Combinatorics, Vol.26, 437-456, 2005
[9]G.A.Jones and D.Singerman, *Complex functions: An algebraic and geometric viewpoint*, Cambridge University Press, 1997
[10]S.Katok, Fuchsian Groups, University of Chicago Press, Section 5.6, 1992
[11]Mong-Lung Lang, Chong-Hai Lim, and Ser-Peow Tan, *Principal Congruence Subgroups of the Hecke Groups*, Journal of Number Theory, Vol.85, 220-230, 2000
[12]C.Maclachlan, *Topics on Riemann surfaces and Fuchsian groups*, L.M.S. lecture series 287, Cambridge university press, 2001
[13]W.Magnus, *Noneuclidean tessellations and their groups*, Academic press,

New York and London, Chapter 3, 1974
[14]B.Schoeneberg, *Elliptic modular functions*, Springer-Verlag, Chapter 4, p.71-103, 1974
[15]D.Singerman, *Subgroups of Fuchsian groups and finite permutation groups*, Bull.London Math.Soc, 2(1970), 319-323
[16]Helmut Wielandt, *Finite Permutation Groups*, Academic press, New York and London, 1964
[17]K.Wohlfahrt, *An extension of F.Klein's level concept*, Illinois J. Math., 8 (1964), 529-535

Chapter 9

Appendix

In the following table we look at subgroups of low index in H^5 in the form of their corresponding transitive permutation representations. We see epimorphisms from H^5 onto various finite permutation groups.The representative 160 epimorphisms from H^5 onto A_{11} that we discussed in section 6.2 are indicated by stars in their first column (i.e. next to the index figures).

Index	Permutations	Signature	Level	Para. cycle lengths
2	$X = (1,2)$ $Y = (1)(2)$	$g = (0; 5, 5, \infty)$	2	2
5	$X = (1,2)$ $Y = (1,2,3,4,5)$	$g = (0; 2, 2, 2, \infty, \infty)$	4	4, 1
5	$X = (1,3)$ $Y = (1,2,3,4,5)$	$g = (0; 2, 2, 2, \infty, \infty)$	6	3, 2
5	$X = (1)(2)(3)(4)(5)$ $Y = (1,2,3,4,5)$	$g = (0; 2, 2, 2, 2, 2, \infty)$	5	5
5	$X = (1,2)(3,4)$ $Y = (1,2,3,4,5)$	$g = (0; 2, \infty, \infty, \infty)$	3	3, 1, 1
5	$X = (1,3)(4,5)$ $Y = (1,2,3,4,5)$	$g = (0; 2, \infty, \infty, \infty)$	2	2, 2, 1
5	$X = (1,3)(2,4)$ $Y = (1,2,3,4,5)$	$g = (1; 2, \infty)$	5	5

6	$X=(1,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,2,2,5,\infty)$	6	6
6	$X=(1,2)(3,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	5	$5,1$
6	$X=(1,2)(4,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	5	$5,1$
6	$X=(1,2)(5,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	5	$5,1$
6	$X=(1,3)(2,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	3	$3,3$
6	$X=(1,3)(4,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	4	$4,2$
6	$X=(1,3)(5,6)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,\infty,\infty)$	4	$4,2$
6	$X=(1,2)(3,4)(5,6)$ $Y=(1,2,3,4,5)$	$g=(0;5,\infty,\infty,\infty)$	4	$4,1,1$
6	$X=(1,3)(4,5)(2,6)$ $Y=(1,2,3,4,5)$	$g=(0;5,\infty,\infty,\infty)$	6	$3,2,1$
6	$X=(1,3)(2,4)(5,6)$ $Y=(1,2,3,4,5)$	$g=(1;5,\infty)$	6	6
7	$X=(4,6)(5,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,2,5,5,\infty)$	7	7
7	$X=(5,6)(3,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,2,5,5,\infty)$	7	7
7	$X=(1,2)(4,6)(5,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	6	$6,1$
7	$X=(1,3)(4,6)(5,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	10	$5,2$
7	$X=(2,3)(4,6)(5,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	6	$6,1$
7	$X=(1,2)(5,6)(3,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	6	$6,1$
7	$X=(1,4)(5,6)(3,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	12	$4,3$
7	$X=(2,4)(5,6)(3,7)$ $Y=(1,2,3,4,5)$	$g=(0;2,5,5,\infty,\infty)$	12	$4,3$
8	$X=(1,6)(2,7)(3,8)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,5,5,\infty)$	8	8
8	$X=(1,6)(2,7)(4,8)$ $Y=(1,2,3,4,5)$	$g=(0;2,2,5,5,5,\infty)$	8	8

8	$X = (1,2)(3,6)(4,7)(5,8)$ $Y = (1,2,3,4,5)$	$g = (0; 5,5,5,\infty,\infty)$	7	7, 1
8	$X = (1,3)(2,6)(4,7)(5,8)$ $Y = (1,2,3,4,5)$	$g = (0; 5,5,5,\infty,\infty)$	15	5, 3
9	$X = (1,6)(2,7)(3,8)(4,9)$ $Y = (1,2,3,4,5)$	$g = (0; 2,5,5,5,5,\infty)$	9	9
10	$X = (1,6)(2,7)(3,8)(4,9)(5,10)$ $Y = (1,2,3,4,5)$	$g = (0; 5,5,5,5,5,\infty)$	10	10
10	$X = (1,6)(2,7)(3,8)(4,9)(5,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (2; \infty)$	10	10
10	$X = (1,6)(2,8)(3,7)(4,9)(5,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; \infty,\infty,\infty)$	4	4, 4, 2
10	$X = (1,6)(2,9)(3,7)(4,8)(5,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (2; \infty)$	10	10
10	$X = (1,6)(2,7)(3,10)(4,9)(5,8)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; \infty,\infty,\infty)$	6	6, 2, 2
10	$X = (1,6)(2,8)(3,7)(4,10)(5,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; \infty,\infty,\infty)$	6	6, 2, 2
10	$X = (1,6)(2,8)(3,10)(4,7)(5,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (2; \infty)$	10	10
10	$X = (1,6)(2,10)(3,9)(4,8)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; \infty,\infty,\infty,\infty,\infty)$	2	2, 2, 2, 2, 2
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; \infty,\infty,\infty,\infty,\infty)$	6	6, 1, 1, 1, 1
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,3,2,4,5)(6,7,8,9,10)$	$g = (1; \infty,\infty,\infty)$	8	8, 1, 1
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,3,4,2,5)(6,7,8,9,10)$	$g = (0; \infty,\infty,\infty,\infty,\infty)$	10	5, 2, 1, 1, 1
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,3,2,4,5)(7,9,8,10,6)$	$g = (2; \infty)$	10	10
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,3,4,2,5)(7,9,10,8,6)$	$g = (0; \infty,\infty,\infty,\infty,\infty)$	4	4, 2, 2, 1, 1
10	$X = (1,2)(3,4)(5,6)(7,8)(9,10)$ $Y = (1,3,2,4,5)(7,9,10,8,6)$	$g = (1; \infty,\infty,\infty)$	14	7, 2, 1
10	$X = (1,2)(3,8)(4,9)(5,10)(6,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; \infty,\infty,\infty)$	8	8, 1, 1
10	$X = (1,2)(3,8)(4,9)(5,10)(6,7)$ $Y = (1,3,2,4,5)(6,8,7,9,10)$	$g = (1; \infty,\infty,\infty)$	12	4, 3, 3

10	$X=(1,2)(3,8)(4,9)(5,10)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,2)(3,10)(4,8)(5,9)(6,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	8	8, 1, 1
10	$X=(1,2)(3,10)(4,8)(5,9)(6,7)$ $Y=(1,3,2,4,5)(6,8,7,9,10)$	$g=(2;\infty)$	10	10
10	$X=(1,2)(3,10)(4,8)(5,9)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,2)(3,10)(4,9)(5,8)(6,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;\infty,\infty,\infty,\infty,\infty)$	4	4, 2, 2, 1, 1
10	$X=(1,2)(3,10)(4,9)(5,8)(6,7)$ $Y=(1,3,2,4,5)(6,8,7,9,10)$	$g=(2;\infty)$	10	10
10	$X=(1,2)(3,10)(4,9)(5,8)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(1,2)(3,9)(4,8)(5,10)(6,7)$ $Y=(1,3,2,4,5)(6,8,7,9,10)$	$g=(2;\infty)$	10	10
10	$X=(1,2)(3,9)(4,8)(5,10)(6,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;\infty,\infty,\infty,\infty,\infty)$	6	3, 3, 2, 1, 1
10	$X=(1,2)(3,9)(4,8)(5,10)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,2)(3,9)(4,10)(5,8)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	20	5, 4, 1
10	$X=(1,2)(3,8)(4,10)(5,9)(6,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;\infty,\infty,\infty,\infty,\infty)$	6	3, 3, 2, 1, 1
10	$X=(1,2)(3,8)(4,10)(5,9)(6,7)$ $Y=(1,3,2,4,5)(6,8,7,9,10)$	$g=(1;\infty,\infty,\infty)$	4	4, 4, 2
10	$X=(1,2)(3,8)(4,10)(5,9)(6,7)$ $Y=(1,3,2,4,5)(6,7,8,9,10)$	$g=(1;\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,2,2,\infty)$	10	10
10	$X=(1,6)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	8	8, 2
10	$X=(2,6)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	21	7, 3
10	$X=(2,7)(5,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	12	6, 4
10	$X=(1,7)(5,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	5	5, 5
10	$X=(1,6)(5,7)(2,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 2, 2

10	$X=(1,6)(5,7)(2,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	30	$5,3,2$
10	$X=(1,6)(5,7)(3,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	30	$5,3,2$
10	$X=(1,6)(5,7)(3,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	4	$4,4,2$
10	$X=(2,10)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	12	$6,4$
10	$X=(2,7)(5,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	5	$5,5$
10	$X=(2,8)(1,10)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	12	$4,3,3$
10	$X=(3,8)(1,10)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	12	$4,3,3$
10	$X=(3,8)(2,10)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	12	$4,3,3$
10	$X=(4,8)(3,6)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	4	$4,4,2$
10	$X=(4,8)(1,9)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	4	$4,4,2$
10	$X=(2,6)(4,9)(1,7)(5,8)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,\infty,\infty,\infty,\infty)$	4	$4,2,2,2$
10	$X=(2,10)(4,7)(1,6)(5,8)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	8	$8,2$
10	$X=(2,10)(1,7)(4,6)(5,8)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	8	$8,2$
10	$X=(2,10)(1,7)(5,6)(4,8)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	12	$6,4$
10	$X=(2,10)(1,8)(5,6)(4,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	8	$8,2$
10	$X=(2,10)(1,8)(5,7)(4,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	12	$6,4$
10	$X=(4,10)(5,8)(1,7)(2,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,\infty,\infty,\infty,\infty)$	6	$3,3,2,2$
10	$X=(1,10)(5,8)(4,7)(2,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	5	$5,5$
10	$X=(5,10)(1,8)(4,7)(2,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	21	$7,3$
10	$X=(4,10)(1,8)(5,7)(2,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	21	$7,3$

10	$X = (5,10)(1,8)(2,7)(4,6)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	6	$3, 3, 2, 2$
10	$X = (5,6)(1,8)(2,7)(4,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	8	$8, 2$
10	$X = (5,6)(2,8)(4,7)(1,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	6	$3, 3, 2, 2$
10	$X = (1,6)(2,8)(5,7)(4,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	8	$8, 2$
10	$X = (1,6)(2,8)(5,10)(4,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	21	$7, 3$
10	$X = (1,7)(2,8)(5,10)(4,6)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	8	$8, 2$
10	$X = (1,7)(2,8)(4,10)(5,6)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	5	$5, 5$
10	$X = (1,2)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (3,4)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (1,2)(5,7)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (1,2)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (3,4)(5,7)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (2,3)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (1,2)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (3,4)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	8	$8, 1, 1$
10	$X = (2,3)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (1,2)(3,4)(5,7)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	7	$7, 1, 1, 1$
10	$X = (1,2)(3,4)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	7	$7, 1, 1, 1$
10	$X = (1,2)(3,4)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	7	$7, 1, 1, 1$
10	$X = (1,6)(2,3)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	14	$7, 2, 1$

10	$X = (1,6)(3,4)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X = (1,6)(2,3)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	6	6, 2, 1, 1
10	$X = (1,6)(2,4)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	10	5, 2, 2, 1
10	$X = (1,6)(2,4)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	10	5, 2, 2, 1
10	$X = (1,6)(2,4)(5,7)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	4	4, 2, 2, 2
10	$X = (1,4)(2,6)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	9	9, 1
10	$X = (1,4)(2,6)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	9	9, 1
10	$X = (1,4)(2,7)(5,6)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	9	9, 1
10	$X = (1,4)(2,7)(5,6)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	9	9, 1
10	$X = (1,4)(2,7)(5,6)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,2,2,\infty)$	10	10
10	$X = (1,4)(2,6)(5,7)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	8	8, 2
10	$X = (1,4)(2,7)(5,6)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	8	8, 2
10	$X = (1,6)(2,4)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,2,\infty,\infty,\infty)$	6	6, 2, 2
10	$X = (2,6)(5,7)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,2,\infty,\infty,\infty)$	30	5, 3, 2
10	$X = (1,4)(2,6)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,2,2,\infty)$	10	10
10	$X = (1,3)(2,7)(5,6)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,\infty,\infty)$	8	8, 2
10	$X = (2,7)(3,4)(5,6)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	12	4, 3, 2, 1
10	$X = (1,6)(2,3)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	6	6, 2, 1, 1
10	$X = (1,6)(3,4)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,\infty,\infty,\infty,\infty)$	6	6, 2, 1, 1
10	$X = (1,3)(2,6)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,2,2,\infty)$	10	10

10	$X=(2,6)(3,4)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(2,6)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(2,6)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(2,6)(1,3)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	9	9, 1
10	$X=(1,3)(2,6)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,\infty,\infty)$	9	9, 1
10	$X=(2,7)(3,4)(5,8)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,\infty,\infty,\infty,\infty)$	15	5, 3, 1, 1
10	$X=(2,6)(3,4)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,\infty,\infty,\infty,\infty)$	15	5, 3, 1, 1
10	$X=(1,3)(2,7)(5,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,2,\infty)$	10	10
10	$X=(2,7)(3,4)(5,6)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,3)(5,7)(6,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(1,3)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(1,3)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(2,4)(5,7)(6,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(2,4)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(2,4)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	14	7, 2, 1
10	$X=(1,4)(5,7)(6,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,4)(5,7)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,4)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,\infty,\infty,\infty)$	6	6, 3, 1
10	$X=(1,3)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	8	8, 2
10	$X=(2,4)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,\infty,\infty)$	8	8, 2

10	$X = (1,4)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,2,2,\infty,\infty)$	21	7, 3
10	$X = (1,3)(2,4)(5,7)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2,2,\infty,\infty)$	9	9, 1
10	$X = (1,3)(5,7)(6,10)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	6	6, 2, 1, 1
10	$X = (2,7)(5,6)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,\infty,\infty,\infty)$	20	5, 4, 1
10	$X = (2,7)(5,6)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,\infty,\infty,\infty)$	20	5, 4, 1
10	$X = (1,3)(2,7)(5,6)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2,2,\infty,\infty)$	9	9, 1
10	$X = (1,3)(2,7)(5,6)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2,2,\infty,\infty)$	9	9, 1
10	$X = (2,7)(5,6)(3,4)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	15	5, 3, 1, 1
10	$X = (2,7)(5,6)(3,4)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	15	5, 3, 1, 1
10	$X = (1,7)(5,6)(2,3)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,\infty,\infty,\infty)$	20	5, 4, 1
10	$X = (1,7)(5,6)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,\infty,\infty,\infty)$	20	5, 4, 1
10	$X = (1,7)(5,6)(2,3)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	4	4, 4, 1, 1
10	$X = (1,7)(2,3)(5,6)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	4	4, 4, 1, 1
10	$X = (1,7)(5,6)(3,4)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	4	4, 4, 1, 1
10	$X = (1,7)(5,6)(2,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,2,\infty,\infty,\infty)$	30	5, 3, 2
10	$X = (1,7)(5,6)(2,4)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	12	4, 3, 2, 1
10	$X = (1,7)(5,6)(2,4)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	12	4, 3, 2, 1
10	$X = (2,10)(1,6)(5,7)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	10	5, 2, 2, 1
10	$X = (2,9)(1,6)(5,7)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,\infty,\infty,\infty,\infty)$	12	4, 3, 2, 1
10	$X = (2,9)(1,6)(5,7)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2,2,\infty,\infty)$	8	8, 2

10	$X = (3,10)(1,6)(5,7)(2,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	8	$8, 2$
10	$X = (3,10)(1,6)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	12	$4, 3, 2, 1$
10	$X = (1,6)(5,7)(2,4)(3,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	8	$8, 2$
10	$X = (1,3)(2,4)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (1,3)(5,7)(6,8)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	10	$5, 2, 2, 1$
10	$X = (1,3)(2,4)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (2,4)(5,7)(6,10)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	6	$6, 2, 1, 1$
10	$X = (2,4)(5,7)(6,8)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	10	$5, 2, 2, 1$
10	$X = (1,4)(2,3)(5,7)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	6	$6, 2, 1, 1$
10	$X = (1,4)(5,7)(6,10)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	15	$5, 3, 1, 1$
10	$X = (1,4)(5,7)(6,8)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	12	$4, 3, 2, 1$
10	$X = (1,4)(2,3)(5,7)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	6	$6, 2, 1, 1$
10	$X = (1,3)(2,4)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, 2, 2, \infty)$	10	10
10	$X = (1,4)(2,3)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	14	$7, 2, 1$
10	$X = (2,7)(5,10)(1,3)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, 2, 2, \infty)$	10	10
10	$X = (1,4)(2,7)(5,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, 2, 2, \infty)$	10	10
10	$X = (2,7)(5,10)(3,4)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	4	$4, 4, 1, 1$
10	$X = (1,3)(2,7)(5,10)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (1,4)(2,7)(5,10)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	$9, 1$
10	$X = (1,3)(2,7)(5,10)(6,8)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	12	$6, 4$

10	$X = (1,3)(2,7)(5,10)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	5	5, 5
10	$X = (2,7)(5,10)(1,4)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	12	6, 4
10	$X = (1,10)(5,7)(2,8)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	3	3, 3, 3, 1
10	$X = (1,10)(5,7)(2,8)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	12	6, 4
10	$X = (2,10)(1,3)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, 2, 2, \infty)$	10	10
10	$X = (2,10)(5,7)(1,3)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	9, 1
10	$X = (2,10)(5,7)(1,3)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	12	6, 4
10	$X = (2,10)(5,7)(1,3)(6,8)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	5	5, 5
10	$X = (2,10)(5,7)(3,4)(8,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	4	4, 4, 1, 1
10	$X = (2,10)(5,7)(3,4)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	9	9, 1
10	$X = (2,10)(5,7)(1,4)(6,9)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	5	5, 5
10	$X = (2,10)(5,7)(1,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, 2, 2, \infty)$	10	10
10	$X = (2,10)(5,7)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	20	5, 4, 1
10	$X = (2,7)(5,10)(3,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, \infty, \infty, \infty)$	20	5, 4, 1
10	$X = (1,10)(5,7)(3,8)(2,4)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	21	7, 3
10	$X = (2,6)(5,7)(4,9)(8,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 2, \infty, \infty)$	21	7, 3
10	$X = (1,2)(3,6)(5,7)(4,8)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	12	4, 3, 2, 1
10	$X = (1,9)(5,7)(4,8)(6,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, \infty, \infty, \infty, \infty)$	12	4, 3, 2, 1
11*	$X = (1,11)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, 2, 2, 2, 5, \infty)$	11	11
11*	$X = (2,11)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 2, 2, 2, 2, 2, 2, 5, \infty)$	11	11

11*	$X=(3,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,2,5,\infty)$	11	11
11*	$X=(4,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,2,2,5,\infty)$	11	11
11	$X=(1,6)(5,7)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,6)(5,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,6)(5,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(5,7)(2,6)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,7)(2,6)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(5,7)(2,6)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(5,7)(2,6)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(5,7)(2,6)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(5,7)(2,6)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(5,6)(2,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,6)(2,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,6)(2,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,6)(2,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,6)(2,7)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,6)(2,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,6)(1,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,6)(1,7)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,6)(1,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5

11∗	$X=(2,10)(1,6)(5,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	14	7, 2, 2
11∗	$X=(2,10)(1,6)(5,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	14	7, 2, 2
11∗	$X=(1,6)(5,7)(2,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11	$X=(1,6)(5,7)(2,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(1,6)(5,7)(2,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11∗	$X=(1,6)(5,7)(2,9)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(1,6)(5,7)(3,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(1,6)(5,7)(3,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11	$X=(1,6)(5,7)(3,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(1,6)(5,7)(3,10)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11∗	$X=(1,6)(5,7)(3,9)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11∗	$X=(1,6)(5,7)(3,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11	$X=(5,7)(1,11)(2,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,7)(2,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,7)(2,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2;2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(5,10)(2,7)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(5,10)(2,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11	$X=(2,7)(5,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	30	6, 5
11∗	$X=(1,10)(5,7)(2,8)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	4, 4, 3
11∗	$X=(1,10)(5,7)(2,8)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	4, 4, 3

11*	$X = (1,10)(5,7)(2,8)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11*	$X = (1,10)(5,7)(2,8)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11*	$X = (1,10)(5,7)(3,8)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	12	4, 4, 3
11*	$X = (1,10)(5,7)(3,8)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11*	$X = (2,10)(5,7)(3,8)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	12	4, 4, 3
11*	$X = (2,10)(5,7)(3,8)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11*	$X = (3,6)(5,7)(4,8)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11*	$X = (3,6)(5,7)(4,8)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11*	$X = (1,9)(5,7)(4,8)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11*	$X = (1,9)(5,7)(4,8)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11	$X = (1,6)(5,7)(4,8)(3,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	10	5, 2, 2, 2
11	$X = (1,6)(5,7)(3,8)(4,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (1,6)(5,7)(3,8)(4,9)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (2,10)(1,7)(5,8)(4,6)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (2,10)(1,7)(5,8)(4,6)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (1,6)(5,8)(4,7)(3,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	30	6, 5
11	$X = (2,10)(4,7)(1,8)(5,6)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (2,10)(1,8)(5,6)(4,7)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	9, 2
11	$X = (2,10)(1,8)(4,6)(5,7)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	28	7, 4
11	$X = (2,10)(1,8)(4,6)(5,7)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	28	7, 4

11	$X=(1,6)(5,7)(4,8)(3,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	$4,3,2,2$
11	$X=(1,7)(5,6)(4,9)(3,8)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	$6,5$
11	$X=(2,6)(5,10)(1,8)(4,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	$8,3$
11	$X=(2,6)(5,10)(1,8)(4,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	$7,4$
11	$X=(2,6)(4,10)(5,7)(1,8)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	$8,3$
11	$X=(2,6)(1,8)(4,10)(5,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	$7,4$
11	$X=(2,7)(1,8)(5,10)(4,6)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	$4,3,2,2$
11	$X=(2,7)(1,8)(4,10)(5,6)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	$9,2$
11	$X=(2,8)(4,7)(5,6)(1,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	$4,3,2,2$
11	$X=(1,9)(4,8)(5,7)(3,6)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	$9,2$
11	$X=(1,6)(2,8)(5,10)(4,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	$7,4$
11	$X=(1,6)(5,10)(2,8)(4,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	$8,3$
11	$X=(1,8)(5,7)(4,10)(3,6)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	$9,2$
11	$X=(2,9)(1,8)(5,7)(4,6)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	$6,5$
11	$X=(1,2)(3,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$
11	$X=(1,2)(4,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$
11	$X=(1,2)(6,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$
11	$X=(1,2)(10,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$
11	$X=(1,2)(9,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$
11	$X=(1,2)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	$10,1$

11	$X=(3,4)(2,11)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(1,11)(3,4)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(3,4)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(3,4)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(3,4)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(3,4)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11*	$X=(1,2)(6,10)(5,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(1,2)(3,11)(5,7)(6,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(1,2)(5,7)(6,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(1,2)(5,7)(6,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(1,2)(3,11)(5,7)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(1,2)(5,7)(4,11)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(3,4)(5,7)(6,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X=(3,4)(5,7)(6,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11	$X=(2,3)(5,7)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(2,3)(5,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(2,3)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(2,3)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(2,3)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1
11	$X=(2,3)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	10	10, 1

11*	$X = (2,3)(1,11)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (2,3)(4,11)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (1,2)(3,11)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (1,2)(4,11)(5,7)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (1,2)(6,11)(9,10)(5,7)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (1,2)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (1,11)(5,7)(3,4)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (3,4)(5,7)(2,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (3,4)(5,7)(6,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11*	$X = (3,4)(5,7)(8,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	9	9, 1, 1
11	$X = (1,2)(3,4)(5,7)(6,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11	$X = (1,2)(3,4)(5,7)(6,10)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11	$X = (1,2)(3,4)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11	$X = (1,2)(3,4)(5,7)(6,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11	$X = (1,2)(3,4)(5,7)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11	$X = (1,2)(3,4)(5,7)(8,9)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	8	8, 1, 1, 1
11*	$X = (1,6)(5,7)(2,3)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,6)(5,7)(2,3)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,6)(5,7)(2,3)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11	$X = (1,6)(5,7)(2,3)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1

11	$X = (1,6)(5,7)(3,4)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11∗	$X = (1,6)(5,7)(3,4)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11∗	$X = (1,6)(5,7)(3,4)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11∗	$X = (1,6)(5,7)(3,4)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11	$X = (1,6)(5,7)(2,3)(4,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11	$X = (1,6)(5,7)(2,3)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11	$X = (1,6)(5,7)(2,4)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	6	6, 2, 2, 1
11	$X = (1,6)(5,7)(2,4)(3,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X = (1,6)(5,7)(2,4)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	6	6, 2, 2, 1
11	$X = (1,6)(5,7)(2,4)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X = (1,6)(5,7)(2,4)(8,10)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 2, 2
11	$X = (1,4)(5,7)(2,6)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,7)(2,6)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(2,6)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,7)(2,6)(9,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,6)(2,7)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,6)(2,7)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,6)(2,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11	$X = (1,4)(5,6)(2,7)(3,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	10, 1
11∗	$X = (1,4)(5,6)(2,7)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,2,2,5,\infty)$	11	11

11∗	$X=(1,4)(5,6)(2,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11∗	$X=(1,4)(5,6)(2,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11∗	$X=(1,4)(5,6)(2,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(1,4)(5,7)(2,6)(8,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(1,4)(5,7)(2,6)(8,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	8, 3
11	$X=(1,4)(5,6)(2,7)(8,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(1,4)(5,6)(2,7)(8,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	8, 3
11∗	$X=(1,6)(5,7)(2,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	14	7, 2, 2
11	$X=(1,6)(5,7)(2,4)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(1,6)(5,7)(2,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	14	7, 2, 2
11∗	$X=(1,6)(5,7)(2,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	14	7, 2, 2
11∗	$X=(2,6)(5,7)(8,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11∗	$X=(2,6)(5,7)(1,11)(8,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	20	5, 4, 2
11∗	$X=(2,6)(5,7)(8,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(2,6)(5,7)(8,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11∗	$X=(2,6)(5,7)(1,4)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11∗	$X=(2,6)(5,7)(1,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11∗	$X=(2,6)(5,7)(1,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(2,6)(5,7)(1,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(5,6)(2,7)(1,3)(8,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	8, 3

11	$X=(2,7)(5,6)(1,3)(4,11)(8,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(2,7)(5,6)(1,11)(3,4)(8,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X=(2,7)(5,6)(3,4)(8,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 3, 1
11	$X=(1,6)(5,7)(3,4)(2,11)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11	$X=(1,6)(5,7)(2,3)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11	$X=(2,6)(5,7)(1,3)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,6)(5,7)(1,3)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,6)(5,7)(1,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,6)(5,7)(1,3)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,6)(5,7)(3,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(5,7)(2,6)(1,11)(3,4)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,6)(5,7)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(1,11)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,6)(5,7)(3,11)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(9,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(4,11)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,6)(5,7)(3,11)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1

11*	$X=(2,6)(5,7)(8,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(2,6)(5,7)(1,3)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,6)(5,7)(1,3)(4,11)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,6)(5,7)(1,3)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,6)(5,7)(1,3)(9,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,6)(5,7)(1,11)(3,4)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11	$X=(2,6)(5,7)(3,4)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11	$X=(2,6)(5,7)(3,4)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11	$X=(2,6)(5,7)(1,11)(3,4)(9,10)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11*	$X=(2,7)(5,6)(1,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(5,6)(2,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(5,6)(2,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(5,6)(2,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,6)(1,11)(3,4)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,7)(5,6)(3,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,7)(5,6)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(2,7)(5,6)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,3)(5,7)(6,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(1,3)(5,7)(6,10)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,3)(5,7)(6,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1

11	$X = (1,3)(5,7)(6,10)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,3)(5,7)(6,11)(9,10)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,3)(5,7)(9,10)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,3)(5,7)(9,10)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X = (1,3)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11	$X = (1,3)(5,7)(8,9)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,3)(5,7)(8,9)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X = (1,3)(5,7)(8,9)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (1,3)(5,7)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11	$X = (2,4)(5,7)(6,10)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(6,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X = (2,4)(5,7)(6,10)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(6,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(9,10)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(9,10)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(9,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X = (2,4)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X = (2,4)(5,7)(8,9)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X = (2,4)(5,7)(8,9)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1

11	$X=(2,4)(5,7)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(1,4)(5,7)(6,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,4)(5,7)(6,10)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,4)(5,7)(6,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,4)(5,7)(6,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,4)(5,7)(9,10)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,4)(5,7)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11*	$X=(1,4)(5,7)(9,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,4)(5,7)(9,10)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(1,4)(5,7)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11	$X=(1,4)(5,7)(8,9)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	21	7, 3, 1
11	$X=(1,4)(5,7)(8,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(1,4)(5,7)(8,9)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(1,3)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,3)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,3)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,3)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,3)(5,7)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,3)(5,7)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(1,11)(2,4)(5,7)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2

11	$X=(2,4)(5,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(2,4)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(2,4)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(2,4)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(2,4)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	18	9, 2
11	$X=(1,4)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(1,4)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(1,4)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(1,4)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	24	8, 3
11	$X=(1,4)(5,7)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(1,4)(5,7)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,2,2,5,\infty,\infty)$	28	7, 4
11	$X=(1,3)(2,4)(5,7)(6,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(1,3)(2,4)(5,7)(6,10)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(1,3)(4,11)(5,7)(6,10)(8,9)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11	$X=(1,3)(5,7)(6,10)(8,9)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11*	$X=(2,7)(5,6)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,7)(5,6)(8,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,7)(5,6)(8,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11	$X=(2,7)(5,6)(8,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,7)(5,6)(9,10)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1

11	$X=(2,7)(5,6)(9,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,7)(5,6)(9,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,7)(5,6)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(2,7)(5,6)(8,9)(1,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,6)(1,3)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,6)(1,3)(9,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,6)(1,3)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,6)(3,4)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11	$X=(2,7)(5,6)(3,4)(8,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11	$X=(2,7)(5,6)(3,4)(9,10)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11	$X=(2,7)(5,6)(3,4)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 3, 1, 1
11*	$X=(1,7)(5,6)(2,3)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,7)(5,6)(2,3)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,7)(5,6)(2,3)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,7)(5,6)(2,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(1,7)(5,6)(3,4)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(1,7)(5,6)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,7)(5,6)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(1,7)(5,6)(3,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(1,7)(5,6)(2,3)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1

11	$X=(1,7)(5,6)(2,3)(9,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11	$X=(1,7)(5,6)(2,3)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11	$X=(1,7)(5,6)(3,4)(9,10)(2,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11*	$X=(1,7)(5,6)(2,4)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	15	5, 3, 3
11*	$X=(1,7)(5,6)(2,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11*	$X=(1,7)(5,6)(2,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11	$X=(1,7)(5,6)(2,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	6	6, 3, 2
11	$X=(1,7)(5,6)(2,4)(8,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 3, 1
11	$X=(1,7)(5,6)(2,4)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X=(1,7)(5,6)(2,4)(9,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 3, 1
11	$X=(1,7)(5,6)(2,4)(9,10)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X=(2,10)(1,6)(5,7)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 2, 2, 1
11	$X=(2,10)(1,6)(5,7)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	6	6, 2, 2, 1
11	$X=(2,9)(1,6)(5,7)(3,4)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	4	4, 4, 2, 1
11	$X=(2,9)(1,6)(5,7)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1
11	$X=(2,9)(1,6)(5,7)(8,10)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(2,9)(1,6)(5,7)(8,10)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(1,6)(5,7)(3,10)(2,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(1,6)(5,7)(3,10)(2,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	18	9, 2
11	$X=(1,6)(5,7)(3,10)(8,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	30	5, 3, 2, 1

11	$X = (1,6)(5,7)(3,10)(8,9)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	4	$4,4,2,1$
11	$X = (1,6)(5,7)(3,9)(2,4)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	$9,2$
11	$X = (1,6)(5,7)(3,9)(2,4)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	18	$9,2$
11	$X = (1,3)(2,4)(5,7)(9,10)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	$10,1$
11	$X = (1,3)(2,4)(5,7)(9,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	$10,1$
11	$X = (1,3)(5,7)(6,8)(9,10)(4,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	6	$6,2,2,1$
11	$X = (1,3)(5,7)(6,8)(9,10)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	30	$5,3,2,1$
11	$X = (1,3)(2,4)(5,7)(8,9)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	$10,1$
11	$X = (1,3)(2,4)(5,7)(8,9)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1;2,5,\infty,\infty)$	10	$10,1$
11	$X = (2,4)(5,7)(6,10)(8,9)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	$7,2,1,1$
11	$X = (2,4)(5,7)(6,10)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	6	$6,3,1,1$
11	$X = (2,4)(5,7)(6,8)(9,10)(1,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	6	$6,2,2,1$
11	$X = (2,4)(5,7)(6,8)(9,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	30	$5,3,2,1$
11	$X = (1,4)(2,3)(5,7)(6,10)(8,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	$7,2,1,1$
11	$X = (1,4)(2,3)(5,7)(6,10)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	$7,2,1,1$
11	$X = (1,4)(5,7)(6,10)(8,9)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	20	$5,4,1,1$
11	$X = (1,4)(5,7)(6,10)(8,9)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	20	$5,4,1,1$
11	$X = (1,4)(5,7)(6,8)(9,10)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	4	$4,4,2,1$
11	$X = (1,4)(5,7)(6,8)(9,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	4	$4,4,2,1$
11	$X = (1,4)(2,3)(5,7)(8,9)(6,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0;2,5,\infty,\infty,\infty,\infty)$	14	$7,2,1,1$

11	$X=(1,4)(2,3)(5,7)(8,9)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	14	7, 2, 1, 1
11*	$X=(1,3)(2,4)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(2,4)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(2,4)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,3)(2,4)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(1,4)(2,3)(5,7)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(1,4)(2,3)(5,7)(10,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(1,4)(2,3)(5,7)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(1,4)(2,3)(5,7)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	8	8, 2, 1
11*	$X=(2,7)(5,10)(1,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,3)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,3)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,3)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,4)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,7)(5,10)(1,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(2,7)(5,10)(3,4)(8,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11	$X=(2,7)(5,10)(1,3)(8,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,10)(1,3)(8,9)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1

11	$X=(2,7)(5,10)(1,4)(8,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,10)(1,4)(8,9)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,7)(5,10)(1,3)(6,8)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4
11	$X=(2,7)(5,10)(1,3)(6,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5
11	$X=(2,7)(5,10)(1,3)(6,9)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5
11	$X=(2,7)(5,10)(1,4)(6,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4
11	$X=(1,10)(5,7)(2,8)(3,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 3, 1
11	$X=(1,10)(5,7)(2,8)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	12	4, 3, 3, 1
11	$X=(1,10)(5,7)(2,8)(6,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5
11	$X=(1,10)(5,7)(2,8)(6,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5
11*	$X=(2,10)(5,7)(1,3)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(2,10)(5,7)(1,3)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,10)(5,7)(1,3)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,10)(5,7)(1,3)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(2,10)(5,7)(1,3)(8,9)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,10)(5,7)(1,3)(8,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,10)(5,7)(1,3)(6,9)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4
11	$X=(2,10)(5,7)(1,3)(6,9)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4
11	$X=(2,10)(5,7)(1,3)(6,8)(4,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5

11	$X=(2,10)(5,7)(3,4)(8,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,5,\infty,\infty,\infty,\infty)$	20	5, 4, 1, 1
11	$X=(2,10)(5,7)(3,4)(6,9)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,10)(5,7)(3,4)(6,9)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	10	10, 1
11	$X=(2,10)(5,7)(1,4)(6,9)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	30	6, 5
11*	$X=(2,10)(5,7)(1,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,10)(5,7)(1,4)(3,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,10)(5,7)(1,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11	$X=(2,10)(5,7)(1,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,2,2,5,\infty)$	11	11
11*	$X=(2,10)(5,7)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(2,10)(5,7)(3,4)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,10)(5,7)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,10)(5,7)(3,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,7)(5,10)(3,4)(8,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(2,7)(5,10)(3,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	5	5, 5, 1
11*	$X=(2,7)(5,10)(3,4)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11*	$X=(2,7)(5,10)(3,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(0;2,2,2,5,\infty,\infty,\infty)$	12	6, 4, 1
11	$X=(1,10)(5,7)(3,8)(2,4)(9,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	24	8, 3
11	$X=(1,10)(5,7)(3,8)(2,4)(6,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4
11	$X=(2,6)(5,7)(4,9)(8,10)(1,11)$ $Y=(1,2,3,4,5)(6,7,8,9,10)$	$g=(1;2,5,\infty,\infty)$	28	7, 4

11	$X = (2,6)(5,7)(4,9)(8,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (1; 2, 5, \infty, \infty)$	24	$8, 3$
11	$X = (1,2)(3,6)(5,7)(4,8)(9,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 5, \infty, \infty, \infty, \infty)$	30	$5, 3, 2, 1$
11	$X = (1,2)(3,6)(5,7)(4,8)(10,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 5, \infty, \infty, \infty, \infty)$	30	$5, 3, 2, 1$
11	$X = (1,9)(5,7)(4,8)(6,10)(3,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 5, \infty, \infty, \infty, \infty)$	30	$5, 3, 2, 1$
11	$X = (1,9)(5,7)(4,8)(6,10)(2,11)$ $Y = (1,2,3,4,5)(6,7,8,9,10)$	$g = (0; 2, 5, \infty, \infty, \infty, \infty)$	30	$5, 3, 2, 1$

Table 9.1: Subgroups of H_5 up to and including index eleven