UNIVERSITY OF SOUTHAMPTON

FACULTY OF LAW, ARTS & SOCIAL SCIENCES

School of Management

Forecasting the Time-Varying Beta of UK and US Firms: Evidence from GARCH and Non-GARCH Models

by

Hao Wu

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ABSTRACT

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This thesis investigates the forecasting ability of four different GARCH models and the Kalman filter method in forecasting the time-varying beta. The four GARCH models applied are bivariate GARCH, BEKK GARCH, GARCH-GJR and GARCH-X; and the Kalman filter approach is the representative of non-GARCH models. The study provides comprehensive comparison analyses on the modelling ability of alternative methods, with an emphasis on their forecasting performance. The study is accomplished by using daily data from UK and US stock market, ranging from January 1989 to December 2003.

According to estimation results, GARCH models are successful in capturing the time-varying beta. Moreover, bivariate GARCH and BEKK GARCH outperform other models in terms of out-of-sample beta forecasts. Kalman filter is found to be less competent in constructing time dependent beta. However, measures of forecast errors overwhelmingly support the Kalman filter approach in terms of out-of-sample return forecasts. Among the GARCH models, GJR model appears to provide somewhat more accurate forecasts than other GARCH model.

This study contributes to financial economics research on modelling time-varying beta by providing empirical evidence from UK and US stock markets. These empirical results are helpful for both market participators and academic researchers in their decision making or research development.

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I have too many to give thanks. I would summarise in one verse of scripture:

"*Every good and perfect gift is from above, coming down from the Father of the heavenly lights, who does not change like shifting shadows.*"

James 1:17

To my wife…

Chapter 1
Introduction

1.1 Motivation and Aim

Since its introduction by Markowitz (1952, 1959), beta has occupied the centre stage in both risk measurement and risk management. According to the Capital Asset Pricing Model (CAPM), the expected return of an asset is linearly related to the single risk factor beta, with no other variables affecting the expected return. Known as systematic risk, the concept of beta has been widely applied in finance and economics, including test of asset pricing theories, estimation of the cost of capital, evaluation of portfolio performance and calculation of hedge ratios for index derivatives and many other areas.

As noted by Brooks *et al.* (1998), systematic risk of any asset may be easily estimated in the form of market-model-generated point estimates of beta. In fact, the empirical application and test of the classic CAPM generally assumes that the beta of a risky asset or portfolio is constant over time. However, in only two decades after its introduction, empirical evidence from numerous literatures has indicated that the beta stability assumption is not true (see Fabozzi and Francis, 1978; Sunder, 1980 for example). According to Bos and Newbold (1984), the variation in the stock's beta may be due to influence of either microeconomic factors and/or macroeconomic factors. Consequently, modified versions of CAPM, which take conditional expectations into consideration, have been proposed by many studies. The logic underlying the modification is that economic agents have conditional expectations rather than homogeneous constant expectations of the first and second moments of asset returns; because agents update their estimates of the mean and covariance of returns each period using newly revealed information in last period's asset returns (Bollerslev *et al.*, 1988). As summarised by Campbell (2000) that the CAPM may hold conditionally but fail unconditionally, empirical evidence generally supports the conditional CAPM rather than the classic CAMP. Therefore, modelling the time series of conditional beta is an area of considerable research interest. In particular, examining the accuracy of beta forecasts obtained from various becomes extraordinarily important, since beta forecasts are crucial for a variety of practical applications. For instance, given accurately forecasted beta, investor can easily outperform the market; and market regulators and firm managers can make more effective risk management decisions. Consequently the study seeks to dedicate itself to such an attractive and valuable research topic.

As indicated by Brooks *et al.* (1998) several econometrical methods have been applied to estimate time-varying betas of different countries and firms in the recent literature. Two of the well methods are the multivariate GARCH model first introduced by Bollerslev (1990) and the Kalman filter approach derived from the engineering literature of the 1960s. The multivariate GARCH model utilise the conditional variance and covariance information produced by the multivariate GARCH model to construct the time-varying beta series. The Kalman filter approach recursively estimates the beta series from an initial set of priors, generating a series of conditional alphas and betas in the market model. Although both modelling techniques have been applied in a variety of contexts, they have generally been conducted in isolation. Studies comparing the modelling ability of both models have concentrated on Australian stock markets (Brooks *et al.*, 1998; Faff *et al.*, 2000). Hence, this study is designed to provide a comparison of different models when applied to data sets from the UK and US stock markets. Moreover, most previous studies applied the modelling techniques for estimation and not for forecasting purpose. This thesis provides empirical evidence of forecasting the time-varying beta in addition to estimating the time-varying beta.

This study employs four GARCH-type models and the Kalman filter method to model the time-varying beta. GARCH models applied are the standard bivariate GARCH, the BEKK GARCH, the GARCH-GJR and the GARCH-X specifications. The standard bivariate model applied is the diagonal representation suggested by Bollerslev *et al.* (1988), which restricts the matrices of ARCH and GARCH term to be diagonal in order to reduce the number of coefficients to a manageable level. The BEKK GARCH model proposed by Engle and Kroner (1995) is an improvement to the standard GARCH, as the positive definiteness of the conditional variance matrix is guaranteed. The GARCH-GJR model due to Glosten *et al.* (1993) allows for the broadly reported leverage effect of financial time series, with two additional parameters incorporated in the model. Proposed by Lee (1994), the GARCH-X model allows for the effect of short term deviations between two cointegrated series, with the lagged error correction term incorporated in conditional variance and conditional covariance equations. As the representative of non-GARCH models, the Kalman filter method can be used to incorporate unobserved variables into, and estimate them along with, the observable model to impose a time-varying structure of the CAPM beta.

Data applied in this study are daily data from UK and US stock markets, ranging from January 1989 to December 2003. In order to avoid the sample effect and the overlapping issue, three out-of-sample forecast horizons are considered, including two one-year forecast horizons (2001 and 2003) and a two-year forecast horizon (2002 to 2003). To conduct the out-of-sample forecasting, each model is employed to estimate in three shorter periods (1989 to 2000, 1989 to 2001 and 1989 to 2002) and accordingly predict the time-varying beta in three forecast samples (2001, 2003 and 2002 to 2003) with estimated parameters.

To summarise, this thesis aims to estimate and forecast the time-varying beta of UK and US firms by means of GARCH and Kalman filter approaches. The study intends to investigate the relative superiority of alternative econometric models in forecasting the time-varying beta. The ultimate goal is to find the best forecasting model for the time-varying beta among a variety of available candidates.

1.2 Research Question and Thesis Outline

In order to answer the question what is the best forecasting model for the time-varying beta, several research questions are developed to approach the task.

1. Why is forecasting the time-varying beta so important?
2. Which model can be applied to estimate and forecast the time-varying beta?
3. What is the difference among the conditional beta estimated by different models?
4. Which econometric model generates the most accurate beta forecasts?
5. What is the practical implication for the research outcomes?

To answer the questions, the remainder of the thesis proceeds as follows.

Chapter 2

To establish the theoretical foundation of the study, the chapter describes the evolution of the CAPM framework from the classical static CAPM to the condition version of the CAPM. This chapter aims to justify the research subject by basically thinking the question why the forecasting of time-varying beta is valuable. It also discusses the potential benefit of forecasting the time-varying beta from both investors' and corporate financial managers' perspectives.

Chapter 3

The methodology chapter sets the scene for this thesis by discussing relevant econometric techniques and models. It covers a wide range of econometric models, varying from simple linear regression to complicated multivariate nonlinear regression, and also possesses the main part of the thesis.

Chapter 4

This chapter reviews existing literature relevant to forecasting time-varying betas. Four categories of literature are coved in the chapter, including stock return forecasts, stock market volatility forecasts, beta forecasts and forecasting with GARCH models.

Chapter 5

This chapter describes the data applied in the study and presents some statistics of the time series. It also reports the cointegration test results between the log of firm price and the log of market index.

Chapter 6

This chapter reports empirical results of forecasting time-varying betas using UK daily data. The results of time-varying beta estimation of different econometric models are discussed in details. The chapter also compares the out-of-sample forecasting ability of alternative models by a variety of approaches.

Chapter 7

This chapter presents empirical results of forecasting time-varying betas with US daily data. The chapter discusses the performance of alternative models in both estimating and forecasting time-varying betas. In addition, this chapter compares the results of UK and US results, in both estimation and forecast aspects.

Chapter 8

The chapter concludes the main findings in the empirical tests and accordingly suggests the possible implication of research outcomes.

Chapter 2
Conditional CAPM and Time-Varying Beta

2.1 Introduction

This chapter aims to establish the theoretical foundation for the study on the conditional CAPM; and thus to justify the research subject of forecasting the time-varying beta by basically answering the question why the beta coefficient in the CAPM framework is time-varying and worth forecasting. The chapter begins with the review on the theory, implications, debates and intrinsic weaknesses of the classical static CAPM. To overcome the shortcoming of the static CAPM, many researchers propose different versions of the conditional CAPM. In particular, the conditional CAPM proposed by Bodurtha and Mark (1991) is described in details. In addition, the chapter analyses possible reasons for the time dependent feature of systematic risk. Finally, the chapter discusses potential benefits of forecasting the conditional beta from both investors' and corporate financial managers' perspectives.

2.2 The CAPM Framework

2.2.1 Background

Intuitively, the reasonable goal of any investor in the stock market is to select a portfolio of shares that will provide the best distribution of future consumption. However, until Markowitz published his classic article on portfolio selection in 1952, there was very little literature concerning any theory about the measurement of risk, the relationship between risk and return, or the selection of portfolios. This article and its subsequent works on the subject changed the foundation of investment theory[1]. Since then, an investment decision has been equivalent to whether or not a particular portfolio is dominated by other portfolios in the mean-variance space. In the mean-variance space, return and risk characteristics are irrespectively measured by the mean return and the variance of the return, or its square root, the standard deviation.

[1] Markowitz was the first to define risk in terms of the variability of returns and the first to demonstrate how the risk of a portfolio is related to the risk of the individual assets it contains.

Markowitz's theory was based on several assumptions concerning investors:

1. Investors consider the probability distribution of expected returns over a specific holding period.
2. Investors maximize expected returns and diminish marginal utility of wealth.
3. Investors estimate risk on the basis of the variability of expected returns.
4. Investors base decisions solely on expected returns and risk.
5. Higher returns are preferred to lower returns and less risk is preferred to more risk.

Given all these assumptions, Markowitz developed the concept of the efficient frontier and identified a set of efficient portfolios that recognized investors face portfolio risk rather than the risk of individual securities. The trade-off between the expected return and risk is one of the founding pillars of modern financial theory. On the basis of the Markowitz's mean-variance portfolio selection theory, researchers have produced an abundance of articles and textbooks on portfolio theory and capital market theory during the past 30 to 40 years[2]. Among them, the Capital Asset Pricing Model (CAPM) is one of the most remarkable achievements.

2.2.2 Capital Asset Pricing Model

While Markowitz's portfolio theory only focused on choosing risky assets, the dynamic of the riskless asset helped develop the portfolio theory into the Capital Asset Pricing Model in the mid-1960s. The model was pioneered and developed by Sharp (1964), Lintner (1965) and Mossin (1966)[3]. The three practitioners independently derived similar equations, which can be generalized as:

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \tag{2.1}$$

where $E(R_i)$ is the expected return on asset i; R_f is the risk-free rate of return; β_i is the beta value of asset i; and $E(R_m)$ is the expected return on the market portfolio m.

[2] In 1990, Markowitz was awarded the Alfred Nobel Memorial Prize in Economics for his contributions to the theory of portfolio choice.

[3] The work was accredited to William Sharpe who was given a Nobel Prize in 1990 along with Markowitz. John Lintner and J. Mossin also derived similar equations independently. Consequently, the model is referenced as the Sharpe-Lintner-Mossin model in this thesis.

Apparently, the risk-free rate of return and the return on the market portfolio are the same for all assets. Therefore, in an equilibrium framework, the expected return of asset i is linearly related to a single risk factor called beta (β_i), with no other variables affecting the expected return. In the CAPM, the beta coefficient represents systematic risk of the capital asset. The beta value of an individual asset is measured as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2.2}$$

where $Cov(R_i, R_m)$ is the covariance between the returns of asset i and market portfolio m; $Var(R_m)$ is the variance of the market return.

The linear relationship between betas and returns is described by the security market line (SML). As illustrated in Figure 2.1, the SML has an intercept equal to the riskless rate of return; and its slope is the expected market premium. According to the CAPM, the risk return profile of all assets should be located along the SML.

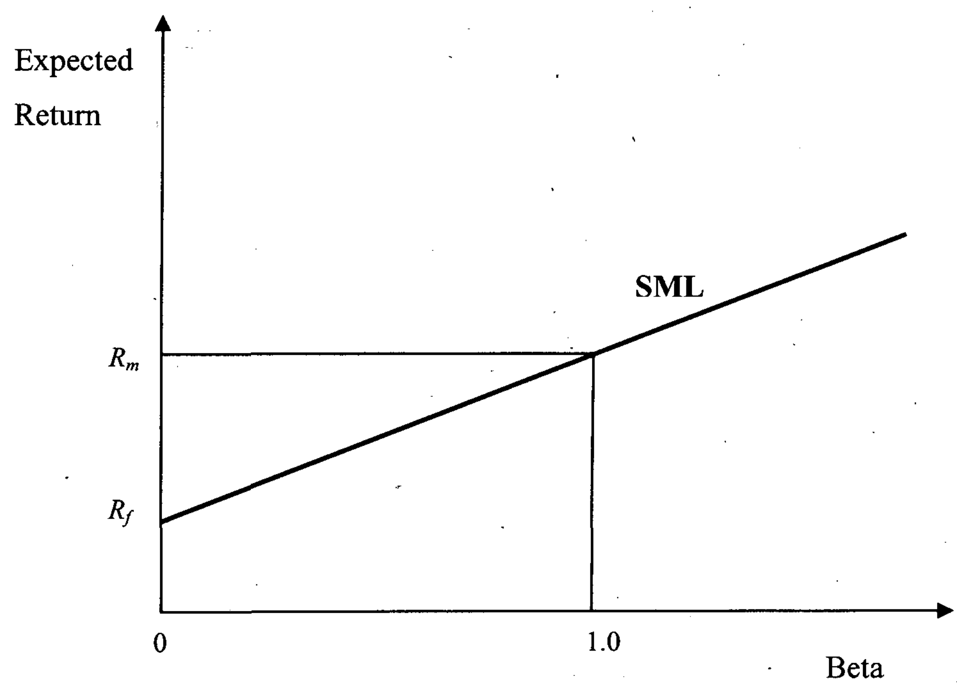

Figure 2.1: Security Market Line

The classical Sharpe-Lintner-Mossin CAPM enables the equilibrium asset pricing relationship to be explained in a simple and intuitively appealing way. As the first theory to explain the relationship between the expected return and risk of capital assets in a rigorous manner, the CAPM has been widely used for a variety of purposes, such as the cost estimation of capital and the performance measure of managed fund, due to its simple and appealing feature. Meanwhile, the usefulness and validity of the CAPM has been a hot spot of academic research ever since. As a result, the importance of the CAPM has been broadly acknowledged by both market participators and financial researchers, as Smith *et al.* (1992, p. 170) state: *"It would be difficult to overstate the impact that the capital asset pricing model (CAPM) has had on both theory and practice in the field of finance".*

The CAPM builds on the basis of portfolio theory to explore the equilibrium relationship between the expected return and risk. It makes the following additional assumptions to Markowitz portfolio theory:

6. Investors can borrow or lend any amount at the risk-free rate.
7. All investors are price takers and have homogeneous expectations or identical information about the future risk and return of each security.
8. Capital markets are in equilibrium. There are no taxes or transaction costs.
9. Assets are completely divisible and liquid.

In the world of the CAPM, where everyone behaves like a Markowitz portfolio optimizer, the only portfolio of risky assets that every investor will hold is the market portfolio. This optimal market portfolio is defined as a portfolio in which the fraction invested in any asset is equal to the market value of the asset divided by the market value of all risky assets.

2.2.3 Systematic Risk and Unsystematic Risk

Based on the assumption of perfect markets and identical investors, the CAPM implies that the only worthy of holding risk of a stock is systematic risk. Systematic risk is the risk results from exposure to general stock market movements. In the CAPM, systematic risk is represented by the beta coefficient. As mentioned above, the beta value of an individual security is obtained as:

$$\beta_i = \frac{Cov(R_i, R_m)}{Var(R_m)} \tag{2.2}$$

The beta coefficient reflects not only the relative volatility of the stock but also the degree to which its return is correlated with the market return. In addition, the concept of systematic risk simplifies the calculation of portfolio risk. The beta value of a portfolio is a simple weighted average of the betas of all assets included in the portfolio, where the weight is the proportion of the asset's value to the total value of the portfolio.

However, it is necessary to point out the beta is an index of relative systematic risk rather than a measure of total systematic risk. When the difference between a share's actual return and the expected return is noticed in the regression analysis, equation (2.1) can be rewritten as:

$$R_i = R_f + \beta_i(R_m - R_f) + \varepsilon_i \tag{2.3}$$

where R_i and R_m are the actual return of asset i and the market portfolio m, and ε_i is the stochastic disturbance or residual term. In order to measure total risk of asset, the variance of R_i can be partitioned into systematic and unsystematic risk:

$$Var(R_i) = Var(R_f) + Var[\beta_i(R_m - R_f)] + Var(\varepsilon_i) \tag{2.4}$$

The risk-free rate of return R_f, as the intercept term in the regression, is usually accepted as a constant, therefore the variance of R_f and $\beta_i R_f$ is zero. As a result, the equation can be restated as:

$$Var(R_i) = \beta_i^2 Var(R_m) + Var(\varepsilon_i) \tag{2.5}$$

or equivalently

$$\sigma_i^2 = \beta_i^2 \sigma_m^2 + \sigma_\varepsilon^2 \tag{2.6}$$

Equation (2.5) and (2.6) express the same meaning in different forms. The left side of the equation presents the total risk of the asset i. The right side explains the component of the total risk, where $\beta_i^2\sigma_m^2$ measures systematic risk and σ_ε^2 evaluates unsystematic risk. The equation demonstrates that a stock's systematic risk is a function of the variance of the market portfolio as well as the beta coefficient. Since the variance of the market is the same for all stocks, beta is thus the appropriate measure of relative systematic risk. In other words, the higher the beta, the higher systematic risk of the asset.

Systematic risk is the market-wide and pervasive influence on all security prices. Since it cannot be eliminated through diversification, it is also known as market risk or undiversifiable risk. Sources of systematic risk may include interest rate changes, changes in the rate of inflation and any other factor which impacts on the market as a whole.

In contrast, the CAPM is not concerned with unsystematic risk or idiosyncratic risk, which is specific to an individual firm, because investors can eliminate specific risk by holding diversified portfolios. Such firm-specific or diversifiable risk may generally include competition, changing preferences, lawsuit, death of a manager, or any other element that impacts on a particular company alone but not the whole market. Since investors are mean-variance efficient, they will diversify away unsystematic risk. Therefore, the CAPM only considers systematic risk in determining the required rate of return, because in equilibrium investors will maximize a utility function according to mean-variance efficiency.

2.2.4 The CAPM Debate

Since the introduction of the CAPM, there have been significant academic debates on the validity of the model. In the early life of the CAPM, most empirical tests found supportive evidence for the beta coefficient. As one of the earliest representatives, Black *et al.* (1972) find a positive relationship between average stock returns and betas, using monthly data of nearly all shares on the NYSE from 1931 to 1965. The study of Fama and MacBeth (1973) confirms that data generally support the CAPM, using return data of NYSE stocks for the period from 1926 to 1968.

Although the CAPM thus passed its first major empirical tests, the usefulness of beta as the only measure of systematic risk for a security has been challenged by a number of succeeding studies. As summarised by Pettengill *et al.* (1995), there are generally three categories of argument doubting the CAPM. First, empirical evidence has challenged the conception that the beta is the most efficient measure of systematic risk. Some researchers have argued that several macroeconomic variables are appropriate measures for systematic risk (see Chen *et al.*, 1986 for example). Second, empirical tests have found that various measures of unsystematic risk have impact on capital asset pricing. A variety of measures, such as size of the market capitalization, the book-to-market ratio, have been detected to significantly relate to stock returns (See Banz, 1981; Reinganum, 1981; Gibbons, 1982; Shanken, 1985; Fama and French, 1992 for example). Finally, some studies even detected a flat cross-sectional relationship between the beta and the rate of return, indicating the absence of the systematic relationship between betas and returns. This most challenging argument comes from the heavily cited paper of Fama and French (1992), in which the authors concluded that CAPM cannot describe the last 50 years of average returns of NYSE stocks. Their results have been widely reported in the financial press as the death knell of beta. Therefore, empirical studies have not only thrown doubts on the efficiency and sufficiency of the beta as the measure of systematic risk, but also have criticised the trade-off between risk and return implied by the CAPM.

Although negative proofs have been widely observed as anomalies against the CAPM, Fama (1991, p. 1593) asserts that "*market professionals (and academics) still think about risk in terms of market* β*.*" The preference of the beta is presumably due to the convenience of using a single factor to measure risk and the intuitive appeal of beta (Pettengill *et al.*, 1995). However, these advantages should not be sufficient to explain the prevailing use of the beta coefficient. Therefore, it seems necessary for market professionals and academic researchers to justify the use of the CAPM and beta. Such necessity is demonstrated by the fact that more studies have emerged to challenge the death knell of beta. Many explanations, both theoretical and empirical, have been proposed by researchers to provide the answer to the anomaly of the CAPM. For instance, Amihud *et al.* (1992) and Kothari *et al.* (1995) argue the data employed by Fama and French (1992) are too noisy to invalidate the CAPM. Black (1993) suggests that the size effect noted by Banz (1981) could simply be due to the sample period effect, since the size effect is observed in some periods and absent in others. Although these studies still cannot produce complete conviction for the usefulness of betas and the academic debate continues, diagnoses of anomalies provides new insights into the CAPM and beta from different angles.

2.2.5 Intrinsic Weaknesses of the CAPM

Whereas empirical tests have been found that some inconsistency of the CAPM may be due to data snooping or the sample effect, those diagnoses only provide partial explanations for anomalies. In fact, the essential season for deviations is that the CAPM is a powerful model but not a perfect description of the real world. In other words, the inconsistent evidence in stock markets has touched on some intrinsic weaknesses of the Sharpe-Lintner-Mossin CAPM, such as unrealistic assumptions and the inevitable market portfolio issues. Therefore, this section focuses on exploring the weakness of the original CAPM in terms of anomalies, which helps to understand the CAPM and beta more comprehensively and more deeply.

2.2.5.1 Unrealistic Assumptions

The CAPM is an abstraction from the reality. The model holds on the basis of the assumptions, some of which are unrealistic in the real world. Due to the assumption, the original CAPM is a robust model to describe a general equilibrium relationship in the capital market and is a fundamental contribution to understanding the manner in which capital markets function. On the other hand, the unrealistic simplification and assumption may also mislead to the rejection of the role of the beta in explaining the expected return, while the CAPM is implemented or tested with the real data.

Some assumptions of the CAPM concerning characteristics of investors, such as risk-aversion and utility maximisation, are fairly reasonable. However, many other underlying assumptions about investors and the financial market made by the model are not reasonable. As far as investors are concerned, the CAPM simply assumes:

1. Investors can borrow or lend any amount at the risk-free rate.
2. They all have homogeneous expectations or identical information about what the uncertain future holds.
3. Inversions have a single-period investment time horizon.

Furthermore, the CAPM assumes the capital market is perfect and in equilibrium. In the market:

1. There are no taxes or transaction costs.
2. Prices of capital assts are in equilibrium.

All these inappropriate assumptions may have significant influence on the reliability of the CAPM. Basically, the influence of assumptions on the validity of CAPM can be explored through two questions. Firstly, can these impractical assumptions be sufficient to explain the observed deviations from the CAPM? If the answer is positive, the CAPM is still an effective model for estimating expected rate of return with appropriate considerations incorporated. Second, if deviations are not completely explainable by assumptions, are they economically important enough to reject the validity of CAPM and beta? If the soundness of the CAPM cannot be rejected in any economically meaningful sense, it is still a valid model with some shortcomings.

In the CAPM world, there is a riskless asset with constant returns in every state of nature. In practice, the short term Treasury bill is usually used as the risk-free asset and its yield as the risk-free rate of return. Risk of the Treasury bill is extremely low, but it is not an exact riskless asset. Thus, how is the CAPM affected by the inexistence of risk-free asset? Black (1972) solves the problem by replacing the risk-free rate of return with the rate of return on the zero-beta portfolio. A zero-beta portfolio with minimum variance is constructed and acts as the riskless asset in the model. Thus, equation (2.1) of the traditional CAPM is revised as:

$$E(R_i) = E(R_z) + \beta_i [E(R_m) - E(R_z)] \qquad (2.7)$$

where $E(R_z)$ is the expected return on the unique minimum variance zero-beta portfolio. Roll (1977) mathematically confirms the correctness of the extension form of the CAPM. This extension of the CAPM indicates that the CAPM do not require the existence of a pure riskless asset. In this case, beta is still the appropriate measure of systematic risk for an individual security, and the linearity of the model is still valid. Hence, the modified model of Black (1972) provides a positive answer to the first question.

Unlike the assumption of risk-free asset, other underlying assumptions of the CAPM, such as perfect markets and homogeneous expectations of investors cannot be easily relaxed. The CAPM assumes the stock market is efficient and there is no transaction cost or tax. However, this is not true in reality. Consequently, any empirical test of the CAPM becomes a joint test of the model and market efficiency. If the empirical tests results in evidence against the CAPM, it cannot be used to conclude either the model is incorrectly specified or the market is not exactly efficient. In addition, if investors have different information about the distribution of future returns, they will perceive different opportunity sets, and thus will choose different portfolios. According to Lintner (1969), the existence of heterogeneous expectations does not alter the CAPM except that expected returns and covariances are expressed in forms of weighted averages of investor expectations. However, heterogeneous expectations may cause that the CAPM is not testable, since the market portfolio is not necessarily efficient in this situation. Hence, some assumptions may not be easily relaxed and have significant impacts on the CAPM. However, as mentioned by Mayer (2006), all these has not deterred market participants and risk managers to use CAPM as the workhorse for pricing risk, implying that the assumptions are not economically important to reject the validity of the CAPM. Furthermore Thomas Mayer, Chief European Economist at Deutsche Bank, pointed out that "*perhaps we should look at it as a theoretical framework in the background that allows market participants to think about and manage risk successfully, even if they not always behave exactly as the theory predicts*". This assertion provides a fundamental guideline to the practical application of the CAPM and beta.

In summary, underlying assumptions imply that the CAPM is not a perfect representation of the reality. It is not surprising that data show some systematic deviations from the CAPM, as long as *ex post* observations rather than *ex ante* expectations are used in empirical tests. Researchers have extensively investigated the impact of relaxing each of assumptions on the conclusion of the CAPM and have derived more complicated models, including a tax effect on dividends (Brennan, 1970), non-marketable assets (Mayers, 1972) and accounting for inflation and international assets (Stulz, 1981). Since the improved models generally describe a similar CAPM-type relationship of security prices, the CAPM identifies a major determinant of an asset's return.

2.2.5.2 One-Period Model

The CAPM is a one-period model, in which asset returns are calculated to be over the next period and investors are maximizing returns over the single period. Any length of period is considered as a unit. Hence, the CAPM is applicable in a two-date setting. However because capital assets, such as stocks, are typically with multi-date payoffs, requiring investors have the same holding period is impractical. Therefore, if investors differentiate their utilities depending on when wealth is received, they should choose different investment horizons to maximize their benefits. Accordingly a more complex model is necessary to capture the required rate of return.

Fama (1996) explains the potential issue caused by the single-period feature of the CAPM from corporate financial managers' perspective. If asset pricing is governed by the CAPM, the expected one-period simple returns on the net cash flows (NCF) of investment projects are constant through time. The constant one-period return is used as the discount rate to price the NCF. However, according to Fama (1996), this leads to the result that the distribution of NCF more than one-period are likely to be skewed right because of the compounding of returns. Therefore, expected payoffs are then larger than median payoffs, and expected payoffs are progressively more unusual outcomes for longer investment horizons. Fama (1996) shows the biased outcome with respect to the one-period weakness. Similarly, the biased outcome exists when the CAPM is applied to the stock returns. Accordingly, many extensions of the model have been proposed, which can be applied in either a discrete or continuous trading framework. Merton (1973) derives an intertemporal capital asset pricing model, which coincides with the Sharpe-Lintner-Mossin CPAM. Levy and Samuelson (1992) indicate that when portfolio rebalancing is allowed, the CAPM holds in the multi-date setting.

2.2.5.3 Efficiency of the Market Portfolio

The market portfolio is considerably important for the CAPM, because it is the benchmark to calculate the beta coefficient and its risk premium represents the price of systematic risk. However, the ideal market portfolio is unobservable, since it includes every individual asset in the world. As a result, the market portfolio is replaced by a proxy of the market index such as FT-SE 100 or S&P 500 to implement or test the CAPM. Therefore, there is always an inevitable issue regarding the benchmark, caused by difference between the proxy and the real market portfolio.

In the mean-variance space, the market portfolio is at the tangency point between the efficiency frontier and a ray from the riskless asset. Hence, the market portfolio is known as the tangent portfolio and theoretically mean-variance efficient. Fama and MacBeth (1973) approves of Black's (1972) inference that the market portfolio is ex-ante efficient given the hypothesis that investors regard as optimal those mean-variance efficient portfolios.

Roll (1977) concludes that if any *ex post* mean-variance efficient portfolio is chosen as the proxy of the market portfolio in the test of CAPM, then the equation of two-parameter CAPM must hold. When the CAPM is tested using a proxy portfolio for the market portfolio, there are two basic types of errors, "*even if the true expected returns and the covariance-variance matrix of the returns on all the assets are used*" (see Kandel, 1984).

1. The chosen proxy may be inefficient, while the true market portfolio is efficient. Difficulty encountered here is the type I error which is rejecting the correct model.
2. Since efficient portfolios exist over a period, a market proxy may be selected that satisfies all the implications of the CAPM, even when the market portfolio is inefficient. In this case, tests may result in type II error of accepting the incorrect model.

Roll (1977) also extends the notion of the CAPM by means of mathematical techniques. The CAPM holds for any efficient portfolio and its according zero-beta portfolio, instead of for the market portfolio and the riskless asset. If the benchmark portfolio is efficient, implications of the CAPM are actually tautological and independent of the way equilibrium of the capital market is set or of investor's attitude toward risk. Therefore, as stated by Kandel (1984), the validity of the CAPM is equivalent to the mean-variance efficiency of the market portfolio.

2.3 Conditional CAPM

The traditional CAPM assumes that all investors have the same subjective expectations on the means, variances and covariances of returns. In the real world, economic agents may have common expectations on the moments of future returns, but these are conditional expectations (see Bollerslev *et al.*, 1988). Agents update their estimates of the mean and covariance of returns each period using newly revealed information in last period's asset returns. Therefore these variables are time-varying rather than constant; and it seems appropriate to relax the strong assumption of homogeneous unconditional expectations underlying the CAPM to allow for homogeneous conditional expectations. This extension leads to the conditional CAPM, in which investors update their estimates each period to reflect an expectation of the information set. In order to distinguish with the conditional CAPM, the classical CAPM is also called the static CAPM.

The conditional CAPM overcomes the intrinsic one-period weakness of the static CAPM. It is virtually applicable to value capital assets with multi-date payoffs, such as stocks. Additionally, the conditional expectation implies the time variation of betas, since variances and covariances to calculate the beta are both time-varying. Although there is a consensus about time variation in market betas, it is not clear how this variation should be captured. Several researchers have proposed different versions of the conditional CAPM[4]. Among them, this study focuses on the conditional CAPM from Bodurtha and Mark (1991), which has a similar form to the static CAPM except that the parameter is measured as the mathematical expectation. Let $R_{i,t}$ denote nominal returns on asset i ($i = 1, 2, \ldots, n$) and $R_{m,t}$ denote nominal returns on the market portfolio. The risk premium of asset i and the market portfolio is given by $r_{i,t}$ and $r_{m,t}$. The conditional CAPM is expressed by the equation:

$$E(r_{i,t} | I_{t-1}) = \beta_{i I_{t-1}} E(r_{m,t} | I_{t-1}) \tag{2.8}$$

where β_{it-1} is the conditional beta of asset i defined as:

[4] Merton (1973) proposes an intertemporal CAPM that applies in continuous time. Ross (1975) provides a simple deviation of conditional CAPM, using the discrete-time first order condition and linearizing the expression for marginal utility coincides with the security market line. Jagannathan and Wang (1996) proposes a conditional CAPM with the return on human capital incorporated.

$$\beta_{i|t-1} = \frac{\text{cov}(R_{i,t}, R_{m,t} | I_{t-1})}{\text{var}(R_{m,t} | I_{t-1})} = \frac{\text{cov}(r_{i,t}, r_{m,t} | I_{t-1})}{\text{var}(r_{m,t} | I_{t-1})} \tag{2.9}$$

$E(\mid I_{t-1})$ is the mathematical expectation conditional on the information set available in the last period $(t-1)$, I_{t-1}. Expectations are rational based on the definition of Muth (1961), where the mathematical expected values are interpreted as the agent's subjective expectations. According to equation (2.8), the conditional CAPM implies that expected excess returns vary with time to reflect time variation in market risk premium in addition to the time-varying beta. Existing literature documents that the expected risk premium on the market is not constant and varies over the business cycle (See Keim and Stambaugh, 1986; Fama and French, 1989; Chen, 1991; Ferson and Harvey, 1991 for example). Therefore as noted by Bodurtha and Mark (1991), an asset's risk premium varies over time due to three time-varying variables: the conditional variance of the market return, the conditional covariance between the asset's return and the market's return and the conditional risk premium of the market portfolio.

The conditional CAPM allows investors to have common conditional instead of unconditional expectations is both theoretically and empirically attractive. As mentioned above, the CAPM holds conditionally from the theoretical perspective, as it overcomes the one-period limitation. From the empirical perspective, tests of the CAPM that treat the conditional covariance matrix of asset returns as constant are invariably inappropriate. For instance, Hansen and Richard (1987) show that the omission of relevant conditioning information as occurs with the unconditional CAPM can lead to erroneous conclusions regarding the conditional mean-variance efficiency of a portfolio. On the other hand, allowing for conditional moments leads to more powerful empirical tests of the CAPM.

2.4 Time-Varying Feature of Systematic Risk

As mentioned before, systematic risk pervasively influences all share prices. Sources of systematic risk include changes in interest rate, the rate of inflation and any other factor which impacts on the market as a whole. Such statement is true for the well-diversified portfolio. However there are some differences worthy of note between beta and systematic risk, when beta is employed as the single risk factor. First, the beta coefficient is an index of systematic risk. It measures market risk of securities or portfolios, only when it is incorporated and applied in the CAPM. Second, betas of large portfolios are relatively stable over time, but the stability deteriorates as the portfolio size decreases (see See Levy 1971; Brigham and Gapenski, 1985). Finally, asserted by Beaver *et al.* (1970), sources of systematic risk in terms of the beta of an asset may include corporate risk variables, such as financial leverage, dividend payout and earning yield instability measures. In other words, market-determined risk is relevant to corporate risk variables.

In most empirical studies on the CAPM, the parameters of the model are estimated by ordinary least square regression (OLS). The one-period hypothesis of the CAPM implicitly assumes the beta of securities or portfolios is constant through time. However in recent years, the general assumption of a stationary risk factor has come under increasing scrutiny and there now exist substantial evidences that systematic risk is unstable (See Fabozzi and Francis, 1978; Sunder, 1980; Bos and Newbold, 1984; Collins *et al.*, 1987; Faff *et al.*, 1992; and Kim, 1993). According to Bos and Newbold (1984), the variation of systematic risk may arise through the influence of either microeconomics factors or macroeconomics factors. Since the relative risk of a firm's cash flow is affected by these factors, there are reasonable economic reasons that suggest the beta may be time varying.

In addition to the factor listed above, there are many possible sources of systematic risk in common stocks[5]. Furthermore, there is a substantial body of empirical evidence that equity beta coefficients are not stable over time. The conditional CAPM make it possible to capture the time variation feature of systematic risk. Equation (2.9) from the conditional CAPM provides a convenient way to understand and capture the time-varying beta through conditional variance and covariance:

$$\beta_{it-1} = \frac{\operatorname{cov}(R_{i,t}, R_{m,t} | I_{t-1})}{\operatorname{var}(R_{m,t} | I_{t-1})} = \frac{\operatorname{cov}(r_{i,t}, r_{m,t} | I_{t-1})}{\operatorname{var}(r_{m,t} | I_{t-1})} \tag{2.9}$$

Furthermore, according to Klemkosky and Martin (1975), betas will be time-varying

[5] A detailed discussion of possible sources of systematic risk is provided by Rosenberg and Guy (1976a, 1976b).

if excess returns are conditionally heteroscedasticity. In the financial literature, the evidence of volatility clustering has been broadly found. The introduction of ARCH model by Engle (1982) and its subsequent generalization (GARCH) by Bollerslev (1986) provides powerful econometric techniques to capture the volatility clustering in financial data. Particularly, the multivariate GARCH models are useful to estimate the time-varying beta, which require the modelling of both conditional variance and covariance. Both GARCH-type models and non-GARCH methods to calculate the time-varying beta are discussed in the next chapter in details.

2.5 Benefits of Forecasting Beta

As one of the most widely used measures of risk among market practitioners and financial economists, beta has various applications in financial economics, including testing of asset pricing theories, estimation of the cost of capital, evaluation of portfolio performance and calculation of hedge ratios for index derivatives. This section discusses the potential benefit of forecasting beta from both investors' and corporate financial managers' perspectives.

2.5.1 Benefits for Investors

Systematic risk is the only risk that investors should concern about. The security market line illustrates there is a linear cross-sectional relationship between systematic risk and the expected rate of return on stocks. Therefore, forecasting beta can aid investors to fulfil their investment goal. Utility maximization is a fairly reasonable assumption for investors. In economic terms, people investing in the capital market always attempt to achieve their highest possible utility/indifferences curve. With forecasted systematic risk, the approach known as 'interior decoration' can help investors to successfully achieve the target.

$$E(R_i) = R_f + \beta_i[E(R_m) - R_f] \qquad (2.1)$$

Equation (2.1) of the CAPM helps to understand the 'interior decoration' approach. The right side of the equation indicates that the expected return on any capital asset can be divided into the compensation for time (R_f) and the compensation for risk

$(\beta_i[E(R_m)-R_f])$. When the market premium is expected to be positive, investors can choose high beta ($\beta > 1$) portfolios. In this case, the positive market premium times the beta value which is larger than one will provide an over-average risk premium for the portfolio. Such aggressive investment strategy is useful for investor to outperform the market, especially in bull markets. On the contrary, when the market return is expected to be lower than the risk-free rate, investors should choose the portfolio with a low beta value ($\beta < 1$) to reduce the potential loss. The two alternative investment strategies make forecasting of beta profitable from investors' perspective. In both cases, it is important for investors to hold the diversified portfolio. Although unsystematic risk will not pay the investor any reward, it will increase the volatility of the return and hence decrease the possibility to complete the investment goal. Furthermore, prediction of beta values enables investors not only to beat the market but also to carry out investment with a particular intention. Investment analysts can use the beta to design portfolios to match their risk preferences. Finally, forecasting time-varying betas makes it possible for investors to construct portfolios more delicately by adjusting investment decisions frequently with the latest prediction.

Another application of beta by market participants is to measure the performance of fund managers through Treynor ratio. Proposed by Treynor (1965), this ratio is also known as the reward to volatility ratio; and it is the ratio of a fund's average excess return to the fund's beta:

$$\text{Treynor ratio} = \frac{R_{i,t} - R_{f,t}}{\beta_{i,t}} \tag{2.10}$$

If the CAPM holds exactly, the ratio for any fund will be the same and equal to the market risk premium. According to the market efficiency hypothesis, all fund managers' risk-adjusted performance will have no difference in the strong-form efficiency case. However, empirical evidence shows that the UK and US stock market are quite efficient, but not perfectly so (see Fama and French, 1988; Lehmann and Modest, 1987; Gregory *et al.*, 1994). In order to find out the difference of performance, Treynor ratio measures the returns earned in excess of those that could have been earned on a riskless investment per unit of systematic risk assumed. The higher the Treynor ratio, the better performance of the fund managers. According to this risk-adjusted rate of return, it is convenient to evaluate the performance of different managers.

2.5.2 Benefits for Corporate Financial Managers

The financial managers stand between the company's operation and the financial markets. They make decisions on financing and investment, which are crucial for the business firm. The CAPM can be applied well to both categories of capital structure and investment appraisal decisions.

2.5.2.1. Capital Structure Decisions

Capital structure refers to the combination of debt and equity capital which a firm uses to finance its long-term operations. A firm's gearing ratio, which is the ratio of debt to equity finance in its capital structure, is an important measure of a firm's level of financial risk. Financial risk is the possibility that the company not be able to pay its financial commitments. Unlike business risk, the level of financial risk can be controlled by financial mangers through adjusting the capital structure, and thus the gearing ratio. In addition, the capital structure has a significant effect on the overall cost of capital. Although Modigliani and Miller (1958, 1963) argue that capital structure is irrelevant to neither the cost of capital nor the value of a firm, the propositions have received a number of academic criticisms, and observed practices in the real world tend to offer a different view. Financial managers are engaged to determine an optimal capital structure with rational proportions of various finance resource. The optimal capital structure can be defined as one that minimises a firm's cost of capital and maximises its market value.

Capital structure decision making is usually conducted on the basis of estimating the cost of individual sources of capital. Therefore, the capital structure decision is equivalent to the cost of capital estimation. Generally, there are two approaches to estimate the cost of capital. The traditional way is the weighted average cost of capital (WACC) approach. The method is based on the logic that the rate is implied by the current value of the financial asset concerned, and by future expectations of cash flows from that specific asset. The CAPM provides another means to calculate the cost of capital based on capital market information. The CAPM explicitly produce a risk premium via the beta value, and therefore the required rate of return for investors on the share. This required rate of return is also the minimum rate that financial managers seek to pay their shareholders. Thus, expected rate of return generated by the CAPM is the cost of equity from corporate financial managers' point of view.

For most listed companies, the objective of capital structure decisions is to establish and maintain an optimal balance between debt and equity. Since the cost of debt is more stable and more predictable, forecasting the beta value is crucial for the decision making. McLaney *et al.* (1998) find that 47 percent of large UK firms use the CAPM approach to derive the cost of equity. With the cost of equity predicted by the beta, managers can use the resources more efficiently by deciding the optimal debt/equity ratio. Furthermore, given the predicted beta value and the relationship between risk variables of the firm and the beta, financial managers can monitor and control the beta of the common stock. Therefore, they can influence the price and the required rate of return of stocks.

2.5.2.2. Investment Appraisal Decisions

To achieve shareholder wealth maximisation, financial managers should select real investment projects with positive net present values (NPVs). The idea using the borrowing/lending interest rate as the discount rate to calculate the NPV is not tenable in the real world, since the comparable alternative to real investment is not risk-free lending. The opportunity with which the project under consideration must logically be compared is one of equal risk to that project.

There are three arguments concerning the discount rate decision:

1. The value of a firm is the sum of the NPVs of all projects in operation. Thus, undertaking a new project with a positive NPV should increase the value of the firm by the amount of the NPV.
2. Research on stock market efficiency suggests that a firm's share price reflects events of economic significance occurring within the firm. Therefore, a real investment with a positive NPV should increase the market value of the firms' securities.

3. The CAPM states that the expected return is directly proportional to the systematic risk of each individual investment project.

The three points lead to the assertion that the logical discount rate for an individual project should be derived from the CAPM. The beta value reflects the covariance of expected returns from the project with those from the generality of risky investment. The appropriate discount rate $E(R_{project})$ can be obtained through the equation:

$$E(R_{project}) = R_f + [E(R_m) - R_f]\beta_{project} \tag{2.11}$$

According to the CAPM, the only relevant measure of a project's risk is the project's beta. With the estimated beta value, financial managers can estimate the required rate of return on particular project easily. Lumby and Jones (1999, p. 281) state *"using the CAPM to provide an NPV discount rate is certainly a considerable improvement on estimating a discount rate on the basis of management's own subjective value judgement, or not taking risk into account at all."* Due to its advantage, the CAPM is the most often used model by financial managers for assessing the risk of the cash flow from a project and for arriving at the appropriate rate to use in valuing the project.

However, there are two major difficulties to generate the discount rate using the project beta in both conceptual and practical considerations. The conceptual difficulty concerns the single-period weakness of static CAPM. Fama (1996) shows the outcome will be biased, when the constant beta is used to calculate the discount rate for capital budgeting. Fortunately, the time-varying beta provides the better choice, since it discounts future cash flows period by period using the appropriate required rate of return. The practical challenge concerns the identification of the project's beta value. Obviously, the project beta cannot be modelled in the way that a stock' beta is captured. It is inappropriate to forecast a project's beta through estimating the individual components of the beta value expression. One possible way is to use the beta value of the industry within which the project could be classified. The industry beta is simply an average of the betas of the firms within the industry. However, some adjustments might have to be made to the beta value, if the project's systematic risk characteristics differ from those of the cement industry generally. Moreover, financial managers should consider benefits of diversification implied by the CAPM. To choose projects having little or even negative correlation may help firms to take advantage of risk diversification. Such diversification can be conveniently achieved, if beta values of different projects have been forecasted.

2.6 Conclusion

This chapter discusses the evolution of the CAPM framework from the classical static CAPM to the condition version of the CAPM. As the first theory to explain the relationship between the expected return and risk of capital assets in a rigorous manner, the model has been widely used in the cost of capital estimation and the performance measure of managed fund due to its simple and appealing feature. Being one of the hotspots in finance research, there has been a considerable debate on the validity of the CAPM and the utility of beta as the only measure of systematic risk for a capital asset. Although all studies still cannot produce complete conviction for the usefulness of betas and the academic debate continues. The reality is that the CAPM is one of the major benchmarks of finance theory. As Fama (1991, p. 1593) asserts "*market professionals (and academics) still think about risk in terms of market β*".

Noticing the intrinsic weakness of the static CAPM, financial researchers have proposed various amended CAPM to overcome the shortcomings. Especially from the later 1980's, researchers have reached a consensus on time variation of betas, and accordingly proposed conditional CAPM to allow for homogeneous conditional expectations of agents in the real world. Although empirical tests of the conditional version of CAPM also produce conflicting evidence, researchers have generally realised that the CAPM may hold conditionally but fail unconditionally (Campbell, 2000). Hence improvements in the measurement and forecast of the time-varying beta would have broad applications in different areas.

Finally, the chapter discusses how the forecast of time-varying betas can benefit investors and corporate financial managers. Since systematic risk is the only risk that investors should concern about, prediction of the beta value helps investors to make their investment decisions easier. Such application is illustrated by the 'interior decoration' approach. The value of beta can also be used by market participants to measure the performance of fund managers through Treynor ratio. For corporate financial managers, forecasts of the conditional beta not only benefit them in the capital structure decision but also in investment appraisal. All these make the forecasting of time-varying valuable.

Chapter 3
Methodology

3.1 Introduction

This methodology chapter sets the scene for this thesis by discussing econometric techniques and models involved in the study. As financial econometrics can be defined as the application of statistical techniques to problems in finance (Brooks 2002, p. 1), research in this area is generally more method-driven than other financial studies, in the sense that financial modelling techniques play a vital role for achieving the research objective. Accordingly, financial econometricians usually focus on the knowledge of model building and capability of model application, rather than data collection.

A wide range of econometric models, including simple linear regression and multivariate nonlinear regression, are employed in the process of forecasting time-varying betas. This chapter presents the statistical theory, the mathematic formula and the empirical evaluation of these models. As a result, the chapter possesses a main part of the thesis.

All relevant econometric models are grouped in four categories and discussed in four sections respectively. Like many other financial econometric studies, the starting point of the chapter is some basic notations and important concepts of time series data. As an introduction to time series models, section 3.2 also describes their motivations and the characteristics of data that they can capture. Moreover, this section discusses stationary processes and tests for unit roots in time series. In section 3.3, a class of Generalised Autoregressive Conditional Heteroscedasticity (GARCH) models are presented in both univariate and bivariate contexts, including the standard GARCH, BEKK GARCH and GARCH-GJR and GARCH-X specifications. Section 3.4 presents details for the Kalman filter approach, which stands for the non-GARCH models in the competition with GARCH models for predicting the conditional beta. Finally, section 3.5 covers different measures of forecast accuracy, which are used to judge the forecasting performance of the candidate models. Measures of forecast accuracy include statistics derived from the forecast error and developed tests for equal forecast accuracy.

3.2 Time Series

3.2.1 Time Series Data

The availability of appropriate data is essential for the success of any econometric analysis. There are three types of data available for empirical analysis: time series, cross-sectional, and pooled data. In this study, data employed to forecast time-varying betas are time series data. Therefore, the chapter begins with the discussion of time series data.

A time series is a set of observations on the values that a variable takes at different times (Gujarati, 1995). In other words, it is a sequence of data in which each item is associated with a particular instant in time. Such instants in time may be at various regular time intervals, such as daily (e.g. stock closing prices), weekly (e.g. foreign exchange rates), and annually (e.g. national GDP). In addition, the data collected may be quantitative (e.g. income, prices) or qualitative (e.g. male or female, married or single). According to Manddala (1992), it is necessary to point out that qualitative data can be as important as quantitative variables in some empirical studies, since some authors may ignore the dummy variable of time series and define time series only as numerical data.

A time series process of the variable y is usually denoted as $\{ y_t \}$, where t is the time window. The process is characterised by its time ordering and its systematic correlation between observations in the sequence. The signature feature of a time series process is that empirically, the data generating mechanism produces exactly one realisation of the sequence. When the time series data are modelled formally, it is useful to regard an observed series ($y_1, y_2, ..., y_n$) as a particular realisation of a stochastic process. In general, a time series model describes a variable y_t in terms of contemporaneous (or probably lagged) factors x_t, its own past values y_{t-1}, and disturbance terms u_t (see section 12.2 in Greene, 2003 for example). This typical form of the time series model is stated as follow:

$$y_t = \alpha + \beta_1 x_t + \beta_2 y_{t-1} + u_t \tag{3.1}$$

According to Stigler (1986), time series data have been used since the dawn of empirical analysis in the mid-seventeenth century. Nowadays, a large proportion of economic studies aim to model financial and economic time series. Unfortunately, most time series used in social science studies are non-experimental. Accordingly, researchers have paid more and more attention to the quality of data. As Gujarati (1995, p.27) states, "*the researchers should always keep in mind that the results of research are only as good as the quality of the data.*"

3.2.2 Modelling of Time Series Data

Since the recorded history of the economy is often in the form of time series, time series data play a significant role in econometric analysis, especially in economic forecasting. However, economists may have limited knowledge about the economic process underlying the observed time series. When models involving such data are formulated by economic theory and then tested using econometric techniques, it is important to be recognized that economic theory in itself is not enough. For instance, Hendry *et al.* (1984) argue that theory may provide little evidence about the process of adjustment, which variables are exogenous and which are irrelevant or constant for the particular model under research. Therefore, a contrasting approach based on statistical theory is usually used to characterize the statistical process and generate the sequence of data.

A variety of univariate models can be used to model or generate time series data, which include Moving-Average (MA), Autoregressive (AR), Autoregressive Moving-Average (ARMA) and Autoregressive Integrated Moving-Average (ARIMA) processes. All these models describe the behaviour of a variable in terms of its own past values. If each time series observation is a vector of numbers, a more intricate multivariate time series model is appropriate, such as Autoregressive Moving-Average Vector (ARMAV) model. In this case, the time series is modelled on the basis of combined information of its collective past and exogenous time series, which provides insight into the dynamical interrelationships between variables.

3.2.2.1 Autoregressive Models

As the simplest statistical time series model, the first-order autoregression model, or AR(1) process, can be used to describe the data generating process, if the observation at time t depends on its past value at time $t-1$. AR(1) model is given by equation (3.2):

$$y_t = \rho y_{t-1} + u_t \tag{3.2}$$

where u_t is the white noise term. The white noise time series process $\{u_t\}$ is the essential building block for a number of econometric models, which represents the influence of all other variables excluded from the model. Consequently, each element u_t in the sequence is treated as a random variable and follows the classical assumptions:

1. It has a zero mean [$E(u_t) = 0$];
2. It has a constant variance [$E(u_t^2) = \sigma^2$];
3. It is nonautocorrelated [$E(u_t, u_s) = 0$].

AR(1) model states that current values of the variable y_t depend on the last period's value y_{t-1}, plus a white error term u_t, the latter encapsulating all other random influences. The AR parameter ρ measures the extent of impact of the past value y_{t-1} on y_t, which also determines some main underlying properties of the stochastic process. Generally, there are three categories of stochastic structure for AR(1) processes.

1. If the absolute value of ρ is larger than 1 ($|\rho| > 1$), y_{t-1} has a magnifying effect on the present value y_t. Observations will tend to become larger and larger in absolute value, while t increases. Thus, the data generating process has a tendency to drift, and the error term tends to accumulate rather than die out over time. In this case, the stochastic sequence is an explosive series, which is

nonstationary[6].

2. If $|\rho| = 1$, variable y_t can be accumulated and rearranged for n periods, beginning with an initial value of y_{t-n}:

$$y_t = y_{t-n} + \sum_{i=0}^{n-1} u_{t-i} \tag{3.3}$$

Equation (3.3) is a special case of the AR(1) model, which is also known as a random walk process. Apart from the initial value, all disturbance terms between period $t-n+1$ and period t have an accumulative influence on the current value y_t. Recall the residual u_t is identically independently distributed with the constant variance σ^2, y_t therefore has a variance equal to $t\sigma^2$. As t increases infinitely, the variance of y_t may become infinitely large. In addition, y_t does not converge to a mean value, since if at some point $y_t = c$ then the expected time for y_t returns to the same value c is infinitely. In this instance, the structure of the stochastic process is nonstationary.

3. If $|\rho| < 1$, then y_t will be a stationary process. Following the same idea as equation (3.3), the generated data can be obtained by:

$$y_t = \rho^n y_{t-n} + \sum_{i=0}^{n-1} \rho^i u_{t-i} \tag{3.4}$$

Since $|\rho| < 1$, as $n \to \infty$, the influence of the initial value on y_t will die out. The value of y_t tend to be determined solely by a moving average (MA) process $\sum_{i=0}^{n-1} \rho^i u_{t-i}$. Thus, y_t has a constant mean and variance that are independent of time. Stationarity is an important issue for time series econometrics; most empirical work assumes that the underlying time series is stationary. In order to generate stationary data, the condition $|\rho| < 1$ is compulsory for the AR(1) model.

The lag operator L is often employed by time series models for notational convenience. This operator L is also called as the backshift operator, since it shifts time one step back.

[6] The conception and condition of stationarity or nonstationarity are discussed in more details in Section 3.2.3.

$$L^n y_t = y_{t-n} \quad \text{for } n = \ldots\text{-}2, \text{-}1, 0, 1, 2, \ldots \tag{3.5}$$

The algebra of the L operator is discussed in many textbooks; and some of its properties are helpful for simplifying and summarizing complicated time series models (see Dhrymes, 1981 for example). One of algebraic characteristics is, when $|\rho| < 1$

$$1/(1-\rho L) = 1 + \rho L + \rho^2 L^2 + \rho^3 L^3 + \cdots \tag{3.6}$$

Note equation (3.2) can be rearranged as:

$$y_t = [1/(1-\rho L)]u_t \tag{3.7}$$

Assume that the autoregressive parameter $-1 < \rho < 1$, which is the stationary condition for the AR(1) model. Thus, using the property described by (3.6), the AR(1) model can be converted to an infinite order moving average of the lagged disturbance terms:

$$y_t = u_t + \rho u_{t-1} + \rho^2 u_{t-2} + \cdots \tag{3.8}$$

This property is also called invertibility, inverting a moving average process to produce an autoregressive representation. Invertibility is the counterpart to stationarity of an autoregressive process. Based on equation (3.8), it is straightforward to infer some statistical properties of a stationary series generated by the AR(1) model:

1. The mean of y_t is zero.
2. The constant variance of y_t can be calculated through $\text{var}(y_t) = \sigma^2/(1-\rho)$, where σ^2 is the variance of the residual.

An alternative popular model in time series econometrics is the AR(1) model with a constant term α:

$$y_t = \alpha + \rho y_{t-1} + u_t \tag{3.9}$$

Similarly, $|\rho| < 1$ is the restriction to generate stationary time series. Also, y_t has a variance $\text{var}(y_t) = \sigma^2/(1-\rho^2)$. However, the mean of the series is no longer equal to zero, but $E(y_t) = \alpha/(1-\rho)$.

Apart from the above two models, a more general pth-order autoregression or AR(p) process can be written:

$$y_t = \rho_1 y_{t-1} + \rho_2 y_{t-2} + \cdots + \rho_p y_{t-p} + u_t \tag{3.10}$$

where $\rho_1, \rho_2 \ldots \rho_n$ are autoregressive parameters; and u_t is the stochastic residual term. The value of y at time t is determined by its previous values up to a lag length of p. Notice that in all AR models only the current and previous y values are involved; there are no other regressions. Therefore, AR models actually are a kind of reduced form model, in which "data speak for themselves".

If $A(L)$ represents the polynomial lag operator $1-\rho_1 L-\rho_2 L^2-\cdots-\rho_p L^p$, the pth-order AR model can be abbreviated as:

$$A(L)y_t = u_t \tag{3.11}$$

The stationary condition for higher order AR(p) models is that all roots of the polynomial equation $A(L)=0$ lie outside the unit circle. Since there may be complex roots, it will be discussed in the unit root test section.

3.2.2.2 Moving Average Models

In some practical cases, it may be convenient to consider the data generating mechanism as follows:

$$y_t = u_t + \theta u_{t-1} \tag{3.12}$$

where u_t is the white noise term. Equation (3.12) states that the current value of y depends on the moving average of the current and past residual terms. This model is known as a first-order moving average, or a MA(1) process; and θ is the moving average parameter, indicating to what extent the lagged error term impact the current value of time series. Unlike the autoregressive parameter, the stationary condition does not impose restrictions upon the size of θ.

On the basis of statistical properties of the disturbance term u_t, it is straightforward to derive those of time series generated by the moving average model. For the first-order moving average model given by equation (3.12):

1. The mean of y_t is simply zero[7].
2. The variance of y_t is $\text{var}(y_t)=(1+\theta^2)\sigma^2$, where σ^2 is the variance of the

[7] Many textbooks define the MA model with an interception term, which differs from (3.12) and (3.13) with a constant α involved in the right side of equations. In this case, the mean of y_t is α.

residual.

3. The first autocorrelation coefficient is $\rho_1 = \theta/(1+\theta^2)$, but higher autocorrelation coefficient are equal to zero.

A more general q-th order moving average model allows capturing past values up to a lag length of q:

$$y_t = u_t + \theta_1 u_{t-1} + \cdots + \theta_q u_{t-q} \tag{3.13}$$

where $\theta_1, \theta_2, \ldots, \theta_q$ are moving average parameters; and u_t is the stochastic error term. If we define $B(L)$ as the polynomial lag operator $1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$, the q-th order moving average model is equivalent to:

$$y_t = B(L)u_t \tag{3.14}$$

An important feature of a MA(q) model is that lagged residual terms are unobserved and have to be estimated using the available sample data. This may cause estimation problems; and thus q is usually kept at a small value. As Franses (1998, p.39) states, the order q is generally set at 1 or 2. According to Harris and Sollis (2003, p.5), lower order MA models have been found to be more useful in econometrics than higher order in practice.

3.2.2.3 Autoregressive Moving Average Models

Occasionally, in the face of certain kinds of statistical evidence, one might conclude that the more elaborate model would be preferable. An extremely general model that encompasses AR(p) and MA(q) models is the autoregressive moving average of order (p,q), or ARMA(p,q) model:

$$y_t = \rho_1 y_{t-1} + \cdots + \rho_p y_{t-p} + u_t + \theta_1 u_{t-1} + \cdots + \vartheta_q u_{t-q} \tag{3.15}$$

where $\rho_1,\ldots,\rho_p$ are autoregressive parameters; and $\theta_1,\ldots,\theta_q$ are moving average parameters. Again u_t is the stochastic error term with the zero mean and constant variance. The current value y_t is determined by the linear combination of a p-th order autoregressive model and a q-th order moving average model, which is the most flexible data generating process for a univariate series.

For (3.15) to be useful in practice, it is usually required that $p+q$ in ARMA(p,q) model is smaller than p in (3.10) for an AR(p) model. Since the value of p in the AR(p) model is usually quite large, and thus quite a number of unknown parameter in (3.10) are needed to be estimated. Making use of the properties of the L operator, it is possible to approximate a lengthy AR polynomial $A(L)$ by a ratio of two polynomials $A(L)$ and $B(L)$, which in sum involve less parameters. The resultant univariate time series model is

$$[A(L)/B(L)]y_t = u_t \text{ or } A(L)y_t = B(L)u_t \qquad (3.16)$$

Notice that (3.16) is the ARMA model expressed in the form of lag operators, which also explains the advantage of the ARMA(p,q) over AR(p) model. In fact, researchers have found that ARMA models with relatively small values of p and q are quite effective, even for the forecasting purpose. The ARMA model has an important feature that makes itself different from other econometric models. A certain time series generated by the ARMA process can be recognized by autocorrelations or partial autocorrelations[8]. Additionally, the stationarity condition of an ARMA (p, q) model is determined by its AR component, which is the roots of $A(L)=0$ must be larger than 1.

3.2.2.4 Autoregressive Integrated Moving Average Models

Univariate time series models discussed above implicitly assume that the time series involved are stationary. However, most economic variables, such as GDP and the price level, exhibit strong trends and are not stationary. In many cases, stationarity can be simply achieved by differencing. A nonstationary series is integrated of order d, denoted I(d), if it becomes stationary after being differenced d times. When the time series y_t is replaced by $\Delta^d y_t$, a further generalization of the ARMA (p, q) model would be

$$\Delta^d y_t = \rho_1 \Delta^d y_{t-1} + \rho_2 \Delta^d y_{t-2} + \cdots + \rho_p \Delta^d y_{t-p} + u_t + \theta_1 u_{t-1} + \cdots + \theta_q u_{t-q} \qquad (3.17)$$

where $\Delta^d y_t = (1-L)^d y_t$ is the d-th difference of y_t. Model (3.17) is an autoregressive integrated moving average model of order (p, d, q), or briefly ARIMA(p, d, q). With polynomials in the lag operator, the equation (3.17) can also be written compactly as:

[8] The process of recognizing an appropriate model is called identification; see Box and Jenkins (1970).

$$A(L)[(1-L)^d y_t] = B(L)u_t \tag{3.18}$$

Therefore, if a time series has to be differenced d times to make it stationarity and then apply the ARMA(p, q) to model it, then the original time series is an ARIMA(p, d, q) time series, in which p denotes the number of autoregressive terms, d the number of times of difference to achieve stationary, and q the number of moving average terms.

ARIMA models became popular with practitioners through the seminal work of Box and Jenkins (1970). Granger and Newbold (1986) set out a number of reasons why univariate Box-Jenkins methods in particular deserve consideration. The most pertinent reason is that *"They are quick and inexpensive to apply, and may well produce forecast of sufficient quality for the purposes at hand"* (Granger and Newbold 1986, p.151).

3.2.3 Stationary and Nonstationary Time Series

3.2.3.1 Definition of Stationarity

From a theoretical point of view, any time series data can be regarded as being generated by a stochastic process[9]. A concrete set of data, either continuous data or discrete data, is a particular realization of the underlying stochastic process. The random variable $\{ y_t \}$ are generally not independent and lack replication in the particular observation period. Therefore, the available observation is usually called a single realization, because there is no way of getting another one. Consequently, the two features of dependence and lack of replication make it necessary to specify some highly restrictive models for the statistical structure of the stochastic process.

Stationarity is one way of describing a stochastic process by specifying the joint distribution of the series y_t. There are two classes of stationarity: weak or covariance stationarity and strong or strict stationarity. A stochastic process y_t is weakly stationary if it satisfies all the following requirements:

1. Mean $E(y_t) = \alpha$

[9] The word *stochastic* has a Greek origin and means "pertaining to chance".

2. Variance $Var(y_t) = \sigma_y^2$

3. Covariance between y_t and y_s is a finite function of $|t-s|$.

The first two requirements assert that a stationary stochastic process has a constant mean and variance over time. The third condition requires that the value of covariance between two observations depends only on how far they are in time, not the actual time at which they occur.

A time series is said to be strictly stationary if the joint distribution of any set of *n* observations is invariant to when the observations are made. In other words, the joint distribution of $(y_1, y_2, ..., y_n)$ is the same as the joint distribution of $(y_{1+k}, y_{2+k}, ..., y_{n+k})$ for all values of *n* and *k*. Thus, it requires not just the mean and variance are constant, but all higher order moments are independent of time *t*. In fact, this is a very strong assumption and might be too restrictive to use in practice. As Greene (2003, p. 612) points out, the statement of strong stationarity is a theoretical fine point for some practical purposes in econometrics. In most practical situations, weak stationarity suffices for application. Therefore in this paper stationarity just refers to the weak form of definition.

If a time series is not stationary as defined above, it is called a nonstationary time series. According to Nelson and Plosser (1982), the trend stationary process and difference stationary process are two main classes of nonstationary models. The trend stationary model assumes that the series y_t is generated by the mechanism of time-determining, in which time *t* is the dependant variable. Thus the movement of trend stationary series is predominantly in one direction, up or down depending on the sign of the regression coefficient of *t*. The difference stationary models are random walks, which are AR(1) processes with the AR parameter $|\rho| = 1$. In these two settings, detrending or differencing is the appropriate approach to eliminate nonstationarity.

According to Granger and Newbold (1974), the conventional hypothesis testing procedure based on *t*, *F*, chi-square tests, and many other tests may be suspect if time series data involved in regression analysis is not stationary. This is due to the spurious correlation and spurious regression, in which the dependent variable and one or more independent variables are spuriously correlated. Granger and Newbold (1974) argue that the conventional *t* an *F* tests would tend to reject the hypothesis and suggest that researchers use a larger critical value than the standard value to assess the significance level of a coefficient estimate[10]. Based on a more general model, Phillips (1986) confirms that the familiar test statistics are invalid and may lead to serious errors in inferences. Therefore, if a variable is nonstationary, and unless it combines with other nonstationary to form a stationary cointegration relationship, then regressions involving the series can falsely imply the existence of a meaningful economic relationship.

3.2.3.2 Stationary Conditions for Univariate Time Series Models

According to the requirements of stationarity, the stationary condition of various univariate models will be discussed in this section.

Autoregressive Models

The stability of mean of a AR(1) series can be quite intuitively aware of on the basis of the autoregressive parameter. However, the variance and covariances of the AR(1) time series are unfortunately more complicated, since the stochastic structure varies according to different ρ values.

1. If $|\rho| < 1$, For the AR(1) process, it is obvious that mean of the time series is zero, since we characterised it as a disturbance process (3.4). When a constant α is involved in the model (3.9), the mean of y_t is equal to α for all *t*. In this case, the AR(1) model passes the first stationary requirement automatically. The variance can be obtained through formula $\text{var}(y_t) = \sigma^2/(1-\rho^2)$; and covariances $Cov(y_t, y_s) = \rho^{|t-s|}\sigma^2/(1-\rho^2)$. When $|\rho| < 1$, the variance and covariances satisfy the second and the third requirements. Therefore, if $|\rho| < 1$, then this process is stationary.
2. If $|\rho| = 1$, the process is known as a random walk[11]. Given the initial value y_0, by successive substitution, the equation of the random walk process can be

[10] For example, a critical *t* value of 11.2 is suggested to replace the standard value of 1.96 in this case.
[11] The random walk process is often used to describe the behaviour of stock prices, although there are some dissidents who disagree with this random walk theory.

expressed as $y_t = y_0 + \sum_{i=1}^{t} u_i$. On the basis of statistical properties of the disturbance term, hence $E(y_t) = y_0$. The random walk process has a constant mean over time. Additionally, the variance is simply equal to that t multiplies the variance of the disturbance term σ^2, or $Var(y_t) = t\sigma^2$. Since the variance changes with time t, it fails to pass the second requirement. Therefore, the process is nonstationary.

3. If $|\rho| > 1$, the process will be explosive with the magnifying effect of the autoregressive parameter ρ. Thus, the series does not have a constant mean. Furthermore, the variance and covariances are undefined in this case. As a result AR(1) model is nonstationary, when $|\rho| > 1$.

For the more general case, the autoregressive process is stationary if the roots of the characteristic equation

$$A(L) = 1 - \rho_1 L - \rho_2 L^2 - \cdots - \rho_p L^p = 0 \qquad (3.19)$$

has modulus great than one. In other words, the roots of the characteristic equation may lie outside the unit circle. The AR(1) process is the simplest case with the characteristic equation $A(L) = 1 - \rho L = 0$. This equation has a single root $1/\rho$. The root lies outside the unit circle if $|\rho| < 1$, as discussed earlier.

Moving Average Models

For any MA(q) series $y_t = u_t + \theta_1 u_{t-1} + \cdots + \theta_q u_{t-q}$, we have

1. $E(y_t) = E(u_t) + \theta_1 E(u_{t-1}) + \cdots + \theta_q E(u_{t-q}) = 0$,
2. $Var(y_t) = (1 + \theta_1^2 + \cdots + \theta_q^2)\sigma^2$,
3. $Cov(y_t, y_{t-1}) = (\theta_1 + \theta_1\theta_2 + \theta_2\theta_3 + \cdots + \theta_{q-1}\vartheta_q)\sigma^2$,
4. $Cov(y_t, y_{t-(q-1)}) = (\theta_{q-1} + \theta_1\theta_q)\sigma^2$,
5. $Cov(y_t, y_{t-q}) = \theta_q\sigma^2$,

and for any lag larger than q, the autocovariances are zero. Therefore, the finite moving average processes are stationary regardless of the values of the parameters.

Although stationarity might not be an issue for a moving average process, MA time series have the counterpart to stationarity which is called invertibility[12]. Invertibility makes it possible to invert a moving average process to produce an autoregressive representation. In other words, the invertibility condition enables to calculate the residuals from $u_t = [B(L)]^{-1} y_t$, provided that $[B(L)]^{-1}$ converges. Similarly, for invertibility of the MA process, it requires that the roots of the characteristic equation $B(L) = 0$ lie outside the unit circle.

Autoregressive Moving Average Models

An ARMA(p,q) model is the combination of an AR(p) process and a MA(q) process. The stationary condition for the ARMA model is completely determined by its AR component, since all moving average processes with finite coefficients are stationary. For an ARMA(p,q) to be stationary, we require that the roots of the characteristic equation $A(L) = 0$ lie outside the unit circle.

Autoregressive Integrated Moving Average Models

A nonstationary time series may be transferred to a stationary time series by taking the difference. In the ARIMA(p,d,q) model, the time series y_t is replaced by $\Delta^d y_t$. Therefore, there are no extra stationary conditions for the ARIMA model. Time series generated or modelled by ARIMA are automatically stationary.

Difference is one of the most important ways to adjust nonstationary time series. The first difference is obtained through

$$\Delta y_t = y_t - y_{t-1} \tag{3.20}$$

where Δ is differencing operator or differencing filter; and t can be any value. The differencing operator can be more generally defined by

$$\Delta^d = (1 - L)^d \tag{3.21}$$

where $d = \ldots,-2, -1, 0, 1, 2,\ldots$

In practice, the first and second order differenced time series are of significant importance. For instance, time series of share prices are usually regarded as random

[12] It is necessary to point out that invertibility has no bearing on the stationarity of a process.

walk processes, which is not directly suitable for regression analysis. The first order differenced stock returns series, often replace the price data to be used in the regression analysis. While first order differences represent growth rates, the second order differences thus are changes in the growth rate. In the case where a time series must be differenced *d* times to be stationary, it is said to be integrated of order *d*, or abbreviated as I(*d*). Consequently, if a time series y_t is I(0), it means the series is stationary and needs no differencing. If y_t is I(2), then time series y_t is nonstationary, but after differenced twice, $\Delta^2 y_t$ is stationary.

The times that a series needs to be differenced in order to become stationary are exactly the number of unit roots in a nonstationary time series. Therefore, with the number of unit roots generated by some stationary tests, we can tell how many times a series should be differenced to achieve stationarity.

3.2.4 Unit Root Tests

Nonstationary data are not directly suitable for the conventional regression analysis. Hence, it is necessary to examine whether a time series is stationary or not before conducting regression analysis. Each nonstationary time series is characterised by the presence of a unit root, which means that the characteristic equation has a single root equal to one. Consequently, the property of stationarity of a time series is examined through testing for the presence of unit roots.

If a nonstationary time series y_t is differenced once and the differenced series is stationary, then y_t is integrated of order one, or denoted as y_t~I(1). Similarly, if series y_t is integrated of order two, y_t has to be differenced twice to achieve stationarity. In general, if a time series y_t must be differenced *d* time before it becomes stationary, y_t is integrated of order *d*, or y_t~I(*d*).

In practice, many economic time series are clearly nonstationary in the sense that the mean and variance vary time to time; and they tend to depart even further from the given value as time goes on. Therefore, it is important to test the order of integration of each variable in a model, to examine whether it is nonstationary and how many times the variable needs to be differenced to achieve stationarity.

As Maddala (2001, p. 547) states, the single topic that attracted the most attention and to which most econometricians have devoted their energies is testing for unit roots. According to a survey reported in Diebold and Nerlove (1990), hundreds of papers on this topic were published. There are many ways of testing for the presence of unit roots, such as the DF test of Dickey and Fuller (1979, 1981), the PP test of Phillips and Perron (1988), the GPH test of Geweke and Porter-Hudak (1983) and the Robinson approach of Robinson (1995). The DF test and the PP test are the standard unit root tests, which use the discrete integrated values for the difference operator. The GPH method and Robinson approach are fractional integration tests, which allow the integrated order to be any value. In this paper, the DF test is utilised to examine the presence of unit roots.

3.2.4.1 The Dickey-Fuller Test

The celebrated papers of Dicker and Fuller (1979, 1981) pioneer an approach to test the null hypothesis that a series does contain a unit root against the alternative of stationarity. Among different unit root test approaches, the DF test tends to be more popular either due to its simplicity or more general nature.

In order to discuss the DF test, consider the model

$$y_t = \rho y_{t-1} + u_t \tag{3.22}$$

where $u_t \sim$ IID(0, σ^2). Note that the residuals u_t are assumed to follow a DF distribution rather than the normal distribution. The DF method is based on testing the null hypothesis H_0: $\rho = 1$ in equation (3.22). The alternative hypothesis is H_1: $\rho < 1$. If ρ_a denotes the OLS estimate of ρ from the equation (3.22), the statistic $(\rho_a - 1)/Se(\rho_a)$ can be used to test the null hypothesis, in which $Se(\rho_a)$ is the standard error of ρ_a. The standard approach to test such a hypothesis is to construct a *t*-test. However when the regression model is applied to nonstationary data, statistics do not follow the standard *t* or *F* distribution. Thus, the *t*-test is invalid to examine the hypothesis in this case. In order to overcome the limitation, Fuller (1976) calculates critical values for the DF distribution using Monte Carlo techniques[13]. Since the absolute values of the DF τ-distribution are generally larger than those of t-distribution, failure to use the DF τ-distribution tends to over-reject the null hypothesis. Comparing the statistic $(\rho_a - 1)/Se(\rho_a)$ with critical values of the DF distribution, it is straightforward to accept or reject the null hypothesis at particular significance levels.

The simplest form of the DF test using equation (3.22) implicitly assumes that the underlying data generate process for y_t an AR(1) model with a zero mean and no trend component. In addition, it assumes that the initial observation of the series y_0 is equal to zero, so that the overall mean of the series is zero. If $y_0 \neq 0$, Nankervis and Savin (1985) find that model (3.22) can lead to the problem of over-rejection of the null. Consequently, when testing for the unit root, it sometimes may be more appropriate to allow for a constant α in the regression model:

$$y_t = \alpha + \rho y_{t-1} + u_t \tag{3.23}$$

where $u_t \sim$ IID(0, σ^2). Let ρ_b denotes the OLS estimator, then the statistic $(\rho_b - 1)/Se(\rho_b)$ can be used to test the hypothesis H_0: $\rho = 1$, and thus the existence of unit roots for the underlying data generating process (3.23). Fuller (1976) also reports appropriate critical values of the DF distribution in this case. To differentiate from previous critical values, we can label them as critical values of the DF τ_α-distribution. These critical values are invariant with respect to y_0, and thus the test can be undertaken without knowing the value of y_0. If the null hypothesis is acceptable, the series y_t follows a stochastic trend which drifts upwards or downward depending on the sign of α. Otherwise, under the alternative hypothesis $\rho_b < 1$, the variable y_t is stationary around a constant mean of $\alpha/(1-\rho_b)$.

So far we have discussed two forms of DF tests. In practice, one of the most common univariate time series models might be the trend stationary models. Both (3.22) and (3.23) cannot be used to test the unit root of trend stationary models. As a result, further medication should be undertaken to implement the DF test. Accordingly,, a

[13] The DF distribution is also known as the DF τ (tau) distribution.

time trend t should be involved in the regression model as a deterministic component:

$$y_t = \alpha + \beta t + \rho y_{t-1} + u_t \tag{3.24}$$

where $u_t \sim$ IID(0, σ^2). If ρ_c denotes the OLS estimator, statistic $(\rho_c - 1)/Se(\rho_c)$ is easily available for testing the null hypothesis H_0: $\rho = 1$. Also, Fuller (1976) provides the appropriate critical values given by the DF distribution at this case, which can be known as τ_t values. Note also that both ρ_c and τ_t are independent of y_0 and α in (3.24), so neither the initial observation nor the drift term have any impact on the test statistic τ_t.

Critical values for τ, τ_α, τ_t are all computed with Monte Carlo techniques, respectively according to underlying data generating processes (3.22), (3.23) and (3.24). It is interesting to note that $\tau_t < \tau_\alpha < \tau$, which indicates involving a constant and a trend to the model makes it more and more difficult to reject the null. In other words, unnecessary parameters in the regression model will lead to under-rejection of the null, and thus lower the power of test against stationary alternatives.

3.2.4.2 Two Unit Roots Tests

The three forms of Dickey-Fuller tests introduced above are applicable to test a unit root. Furthermore, through differencing, we can extend them for testing two unit roots by replacing y_t with its first order difference Δy_t in equations. Consequently, regression models (3.22), (3.23) and (3.24) change to:

$$\Delta y_t = \rho \Delta y_{t-1} + u_t \qquad u_t \sim \text{IID}(0, \sigma^2) \tag{3.25}$$

$$\Delta y_t = \alpha + \rho \Delta y_{t-1} + u_t \qquad u_t \sim \text{IID}(0, \sigma^2) \tag{3.26}$$

$$\Delta y_t = \alpha + \beta t + \rho \Delta y_{t-1} + u_t \qquad u_t \sim \text{IID}(0, \sigma^2) \tag{3.27}$$

As the same, the null hypothesis is H_0: $\rho = 1$ against the alternative H_1: $\rho < 1$. Follow the same hypothesis testing rules as for one unit root, we can determine to accept or reject the null. When the null is rejected, the time series may have one or no unit root. In contrast, when $\rho = 1$ is acceptable, the time series has at least two unit roots. Clearly, whenever the null of a series is acceptable in the two unit roots test, the one unit root test will produce the same result. Therefore, we usually do unit root tests for two unit roots first. Only as the null is rejected, we will undertake the one unit root test.

3.2.4.3 The Augmented Dickey-Fuller (ADF) Test

The Dickey-Fuller tests described above assume that the disturbance term u_t is white noise. If a time series y_t following an AR(p) process is modelled by a simple AR(1) DF model, then the error term will be autocorrelated to compensate for the misspecification of the dynamic structure of y_t. Autocorrelated u_t invalidate the use of the DF distribution, which assumes u_t is white noise. In this case, the DF test is invalid to investigate the existence of unit roots of y_t.

An extension which can accommodate some forms of serial correlation is the augmented Dickey-Fuller (ADF) test. Dickey and Fuller (1981), Said and Dickey (1984), Phillips (1987) and many other researchers have developed the modifications of the DF test to allow for that u_t is not white noise. The ADF test involves estimating the equation:

$$y_t = \alpha + \beta t + \rho y_{t-1} + \sum_{i=1}^{p} \vartheta_i \Delta y_{t-i} + u_t \qquad (3.28)$$

The extra terms Δy_{t-i} is added to allow for ARMA error processes. The lagged first differences of the series are included, so that the error term in (3.28) is serially independent. The number of lagged first differences is often determined empirically. The ADF test is comparable with the simple DF test, with the test statistic $(\hat{\rho}-1)/Se(\hat{\rho})$ and the same the critical values of DF τ-statistic. By subtracting y_{t-1} from both sides of the equation, we obtain an alternative formulation of (3.28):

$$\Delta y_t = \alpha + \beta t + \rho^* y_{t-1} + \sum_{i=1}^{p-1} \vartheta_i \Delta y_{t-i} + u_t \qquad (3.29)$$

The unit root test is carried out by testing the null hypothesis H_0: $\hat{\rho}^* = 0$ against the alternative hypothesis H_1: $\hat{\rho}^* < 0$.

The DF and ADF tests are so frequently used that they are known as standard unit root tests. However there are some significant problems with the methods. Schwert (1989a) first finds the size distortion problems of the unit root tests through Monte Carlo simulations. To determine the appropriate lag length for the augmented regression is the first step and thus a potential issue to use the ADF test. Schwert (1989a) suggests that the maximum lag length should be the integer part of $[12 \times (T/100)^{0.25}]$. Additionally, DeJong *et al.* (1992) complain about the low power of unit root tests. They find the unit root tests have low power against plausible trend stationary alternatives. Taylor (1997) explains the main reasons for poor power. It can be due to that the alternative hypothesis is typically close to the null or the testing approaches are sensitive to the way in which lag structure is modelled. As Blough (1992) states, there is a trade-off between size and power problems. Unit root tests must have either high probability of falsely rejecting the null of nonstationary when the true time series is nearly stationary (poor size properties) or low power against any stationary alternative.

3.3 GARCH Models

3.3.1 Univariate GARCH Models

3.3.1.1 Stock Market Volatility

Volatility measures variability, or dispersion about a central tendency. As a concept, volatility is simple and intuitive. However, this simple concept is the cause of many difficulties in finance, because unlike many other market parameters which can be directly observed, volatility has to be estimated. Also, as forces of supply and demand vie around the changing equilibrium, asset prices exhibit intrinsic variation.

In stock markets, if a share price series or a market index moves significantly and swings widely, it is said to be volatile. Instability of stock prices may provide opportunities for investors to earn capital gains, and also may cause their loss of fortune. Hence, share price volatility is one of most important measures of risk of holding the share. Economists generally assume that the standard deviation of returns is the best measure of the relative risk of a stock (Lintner, 1965). In fact, statistic standard deviation, and equally its square variance, is most often used indicator of stock market volatility.

Hotopp (1997) asserts that almost every interesting financial decision to make is interesting because of volatility. The expected volatility of stock markets is a key variable in many financial investment decisions. For instance, asset allocation decisions are usually reduced to a two-dimensional decision problem by focusing solely on the expected return and risk of an asset or portfolio, with risk being related to the volatility of the returns. In addition, the rapid development and prevalence of financial instruments based on asset price variation is another appropriate illustration. The volatility of returns plays a central role in the valuation of financial derivatives, such as options and futures, and can have a greater impact on the value of derivative securities than price movements in underlying assets.

Schwert (1989b) undertakes an extensive study and reports that volatility moves counter-cyclically, displaying spikes during recessions. Also, stock market volatility tends to increase dramatically during financial crises (such as the 1997 Asia crisis) and periods of uncertainty (such as the 1962 Cuban missile crisis). Moreover, once risen, volatility shows some inertia in the sense that it reverts slowly to its previous low level. This phenomenal is confirmed by the broadly reported evidence of volatility clustering in capital markets (see Mandelbrot, 1963 for example). Research across all asset markets generally finds that volatility shocks are highly persistent. In other words, volatility may depart from this mean for extended periods of time. However, volatility is typically stationary, in the sense that, over sufficiently long periods of time, it reverts back to a constant mean.

The generally accepted view is that asset price volatility is caused by the arrival of new information. According to the mixture of distribution hypothesis attributed to Clark (1973) and Epps and Epps (1976), the arrival of new information drives investors to adjust their portfolios; and consequently results in both the market price change and volume increase. The arrival of good or bad news leads to a price increase or fall; but both result in increased trading volume, as the market adjusts to a new equilibrium. In this way, Clark (1973) argues there is a positive correlation between volume and the absolute value of price changes due to arrival of new information. Epps and Epps (1976) provide a complementary explanation that the volume-volatility relationship is due to the extent how traders disagree on their reservation prices according to arrived information. More heterogeneous interpretations on the same news will cause more volatility, as dispersion of beliefs tend to create both more price variability and excess volume.

There is a multitude of ways to define volatility measures, among which the variance (or standard deviation) of returns is certainly the most usual measure of volatility. The simplest model of volatility is the historical estimate, which involves calculating the historical value of variance and using it as volatility forecast for future periods. However, from the later 1980's, researchers have reached a consensus on time variation of volatility. Since economic agents update their estimates each period using newly revealed information in last period, agents therefore have conditional expectations rather than homogeneous constant expectations of volatility of asset returns (Bollerslev *et al.*, 1988). The time-varying feature of volatility can be captured by the conditional variance of returns. Such conditional variance or time-varying volatility has crucial implications for asset pricing, asset allocation and risk management; and it therefore has become the central to the emerging field of financial econometrics (Diebold 2004, p. 382).

3.3.1.2 Autoregressive Conditional Heteroscedasticity (ARCH) Model

There is a common phenomenon to many series of financial asset returns that volatility occurs in bursts. Since Mandelbrot (1963) and Fama (1965), this time series behaviour has been reported by numerous studies, such as Chou (1988) and Schwert (1989b). Visually, there are clusters of extreme values in returns followed by periods without such extreme values. This broadly reported phenomenon is known as volatility clustering. Large positive and negative returns tend to gather in a short period of time, because volatility shocks are highly persistent. As a result, capital markets are sometimes tranquil and sometimes turbulent. Thus, volatility is not independent through time as large changes tend to be followed by large changes, of either sign, and small by small. Such dynamics of time-varying volatility is characterised by conditional heteroscedasticity, or conditional variance.

An important implication of stock market volatility clustering is that ability of forecasting stock prices or returns varies considerably from one time period to another. As Mandelbrot (1963) finds, for some time periods the forecast errors are relatively small, for some time periods they are relatively large, and then they are small again for another time period. This suggests that the variance of forecast errors is not constant, but varies from period to period with autocorrelation. Therefore, the forecast error is conditionally heteroscedastic with conditional variance. Conventionally, the problem of autocorrelation is a feature of time series data and heteroscedasticity is a feature of cross-sectional data. However, volatility clustering challenged the thought; and it was not fully exploited for modelling purposes until the introduction of autoregressive conditional heteroscedasticity (ARCH) model by Engle (1982), which provides a rigorous ways of empirically investigating issues involving the volatility of economic variables.

The ARCH model is a model that allows the conditional variance to be time dependent, while the unconditional variance is constant. In other words, it is a model with conditional heteroscedasticity, but unconditional homoscedasticity. Although Engle (1982) initially applies the ARCH model to time series on the rate of inflation, the model has since become predominantly popular in financial econometrics. As Franses and McAleer (2002) state, the Engle's (1982) ARCH paper had an enormous influence on both theoretical and applied econometrics; and was influential in the establishment of the discipline of Financial Econometrics. According to Harris and Sollis (2003), the prevalence of the ARCH model is due to that it not only allows for an estimate of the conditional variance of a time series to be obtained but also enables forecasts of future values of the conditional variance to be computed. For both market practitioners and financial econometricians, obtaining an estimate of the risk associated with a financial asset and being able to forecast the future risk is an extremely attractive feature of a time series model.

In general, the mean, variance and covariance of a time series are discussed on the basis of the long-run moments of the series, which is as $t \to \infty$. The distribution of a time series specified with its long-run moments is known as the unconditional distribution. While the mean, variance and covariance are calculated conditionally on precious values of the series, the corresponding distribution is the conditional distribution. The distinction between the conditional and the unconditional second order moments are the key insight offered by the ARCH model.

In order to simplify matters, white noise terms are usually assumed to follow independent identical normal distribution: $u_t \sim$ IID(0, σ^2). In this case, the disturbance u_t is strong white noise; and there is no distinction between its unconditional distribution and the distribution conditional upon its past. However, if u_t is dependent, its unconditional and conditional distributions differ. We denote the unconditional distribution by: $u_t \sim (0, \sigma^2)$; and the distribution conditional upon the information set Ω available at time $t-1$ by: $u_t \big| \Omega_{t-1} \sim (0, \sigma_t^2)$, where $\Omega_{t-1} = \{u_{t-1}, u_{t-2}, ...\}$.

To be more specific, the ARCH process assumes that the disturbance is conditionally distributed as: $u_t \sim N[0, (\alpha_0 + \alpha_1 u_{t-1}^2)]$. That is u_t follows the normal distribution with mean zero and time-varying variance of $(\alpha_0 + \alpha_1 u_{t-1}^2)$. Therefore, the simplest ARCH(1) model is

$$Var(u_t) = \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 \tag{3.30}$$

where α_0 and α_1 are constrained to be non-negative to ensure that the conditional variance will not be negative. According to the ARCH(1) model, the conditional variance σ_t^2 has two components: a constant and last period's information about volatility or the ARCH term. The ARCH term is modelled as last period's squared residual. Additionally, notice that equation (3.30) is only a partial model, without the conditional mean equation. The conditional mean equation can take a variety of forms as researchers wish.

More generally, the variance σ_t^2 can depend on any number of lagged volatilities. Consequently, an ARCH(q) equation can be written as

$$Var(u_t) = \sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \alpha_2 u_{t-2}^2 + \cdots + \alpha_q u_{t-q}^2 \tag{3.31}$$

Also, it is necessary to place restrictions on all parameter to be positive. By test the joint null hypothesis $H_0 : \alpha_1 = \alpha_2 = \cdots = \alpha_q = 0$, we can find whether ARCH effects are present or not. When the null is acceptable, the error variance is homoscedastic and simply equal to α_0; otherwise the ARCH effect exists[14]. Thus, in terms of specification, ARCH directly affects the error terms u_t; however, the dependent variable generated from a linear model with an ARCH error term is itself an ARCH process.

3.3.1.3 Generalised ARCH (GARCH) Model

The ARCH model has an influential contribution to financial econometrics, as it provides a framework for the analysis and development of non-linear models of volatility. However, ARCH models have been rarely used in practice, since the estimation of the ARCH model is not always straightforward. One of the most significant difficulties is that no clearly best approach is available to decide the number of lags of the squared residual (the value of q). Consequently, the basic ARCH specification has been extended in many ways in response to overcome observed problems. The Generalized ARCH (GARCH) model developed independently by Bollerslev (1986) and Taylor (1986) is the most prominent extension of an ARCH model. As Bollerslev (1986) demonstrates with an example, the virtue of this approach is that a GARCH model with a small number of terms appears to perform as well as or better than an ARCH model with many terms. In contrast with ARCH, GARCH models are widely employed in practice due to a more flexible lag structure.

Assume that the dependent variable y_t can be modelled as

$$y_t = \alpha + \sum_{i=1}^{n} \beta_i x_{it} + u_t \tag{3.32}$$

where x_{it} is the exogenous independent variable. In a GARCH(p, q) model, the disturbance u_t is defined as:

$$u_t = \varepsilon_t (\alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2)^{1/2} \tag{3.33}$$

where $\varepsilon_t \sim$ IID(0,1); $p \geq 0$, $q > 0$; $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1,...,q$ and $\beta_j \geq 0$, $j = 1,...,p$. Following from manipulation of (3.33), the conditional variance of the GARCH(p, q) is a function of lagged values of conditional variance as well as squared error terms:

[14] Notice that the test is a joint null hypothesis, using the test statistic nR^2 (the number of observation n multiplied by the coefficient of multiple correlation) compared to the critical value from the $\chi^2(q)$ distribution.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \qquad (3.34)$$

With the lag operator L, the GARCH(p, q) process can be rewritten as:

$$\sigma_t^2 = \alpha_0 + \alpha(L) u_t^2 + \beta(L) \sigma_t^2 \qquad (3.35)$$

where $\alpha(L) = \alpha_1 L + \alpha_2 L^2 + \cdots + \alpha_q L^q$ and $\beta(L) = \beta_1 L + \beta_2 L^2 + \cdots + \beta_p L^p$.

Defining $v_t = u_t^2 - \sigma_t^2$, the ARCH(q) model can be expressed in the form as:

$$u_t^2 = \alpha_0 + \alpha(L) u_{t-1}^2 + v_t \qquad (3.36)$$

Since the conditional mean $E(v_t|\Omega_{t-1})$ is equal to zero, the ARCH(q) model is exactly an AR(q) model for the squared residual terms. Consequently, the condition for covariance stationarity is that the sum of the positive autoregressive parameters is less than one, or $\sum_{i=1}^{q} \alpha_i < 1$. In this case, the white noise u_t has a zero unconditional mean; and its unconditional variance equals $\sigma_t^2 = \alpha_0 /(1 - \alpha_1 - \cdots - \alpha_q)$.

Similarly, substituted by the variable v_t, the GARCH(p, q) model in equation (3.35) can be rearranged as:

$$u_t^2 = \alpha_0 + [\alpha(L) + \beta(L)] u_{t-1}^2 - \beta(L) v_{t-1} + v_t \qquad (3.37)$$

Interestingly, this equation defines an ARMA[max(p, q),p] model for the squared error term. According to Bollerslev (1986), the model is weakly stationary if and only if all the roots of $\alpha(L) + \beta(L) = 1$ lie outside the unit circle. Therefore, for GARCH(p, q) model, parameters are required to satisfy $\sum \alpha_i + \sum \beta_i < 1$ for covariance stationarity. Thus, GARCH is to ARCH (for conditional variance dynamics) as ARMA is to AR (for conditional mean dynamics).

The simplest GARCH process is GARCH(1, 1), in which lagged values of both conditional variance and squared error term equal one.

The conditional variance of the GARCH(1,1) process can be modelled as

$$\sigma_t^2 = \alpha_0 + \alpha_1 u_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \qquad (3.38)$$

where all parameters are required to be non-negative, $\alpha_0 > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ to satisfy the non-negative constraint of the conditional variance. The GARCH(1,1) models interpret the current conditional variance as a function of a constant (α_0), the previous period's observed volatility (the ARCH term) and the previous period's forecasted variance (the GARCH term).

The autoregressive parameter of the ARCH term α_1 measures the ARCH effect, which describes the extent to which past news cause volatility today. If α_1 is significantly different from zero, it implies the existence of volatility clustering in the time series. Otherwise, the ARCH effect is absent in the data[15].

The GARCH term allows the time-varying variance to evolve over time in a way that is much more general than the simple specification of ARCH models. When β_1 is significantly greater than zero, the conditional variance is itself a serially correlated time series process. In this case, the GARCH effect exists; and this effect describes to what extent the forecasted variance of the previous period affect uncertainty of the current period. When $\beta_1 = 0$, the GARCH specification is essentially an ARCH process without the lagged variance or the GARCH effect. Bollerslev *et al.* (1992) report that GARCH effects are highly significant with daily and weekly financial data, while its effect tends to be much milder in less frequently sampled data, such as quarterly and yearly data.

The stationarity condition for the GARCH(1,1) model is $\alpha_1 + \beta_1 < 1$, which implies the process is weakly stationary with the unconditional mean $E(u_t) = 0$ and the unconditional variance $Var(u_t) = \alpha_0 /(1 - \alpha_1 - \beta_1)$. For $\alpha_1 + \beta_1 \geq 1$, the unconditional variance of the residual is not defined, which is termed as 'nonstationarity in variance'. Furthermore, the sum $\alpha_1 + \beta_1$ is an appropriate measure for persistence of a shock to volatility. In other words it determines the rate at which this effect dies over time. There are four categories of implications:

1. If $\alpha_1 + \beta_1 = 0$, there is no persistence of volatility.
2. If $0 < \alpha_1 + \beta_1 < 1$, the effect of the shock on volatility dies over time.

[15] Maddala (2001, p.468) summarises 'A large number of studies, particularly those of speculative price, have reported significant ARCH effects.'

3. If $\alpha_1 + \beta_1 = 1$, the model is known as integrated GARCH, or IGARCH (Engle and Bollerslev, 1986). In this extreme case, volatility shocks have a permanent persistent, in the sense that it remains important for future forecasts of all horizons.
4. If $\alpha_1 + \beta_1 > 1$, news has a magnifying effect on future volatility, which is alogical according to the market efficiency theory. Consequently, the conditional variance forecast will tend to infinite as the forecast horizon increases.

Therefore, $\alpha_1 + \beta_1 < 1$ is an important condition for predicting the conditional variance; and thus $\beta_1 < 1$ is an implicit constraint since α_1 is restrictedly non-negative. In the GARCH(1,1) case, $\beta_1 < 1$ is not only a requirement for the conditional variance forecast, but also explains why GARCH is more parsimonious than ARCH. As long as $\beta_1 < 1$, an infinite number of successive substitutions will change the form of the GARCH(1,1) equation (3.38) as

$$\sigma_t^2 = \frac{\alpha_0}{1-\beta_1} + \alpha_1 \sum_{i=1}^{\infty} \beta_1^{i-1} u_{t-i}^2 \tag{3.39}$$

This form says that the conditional variance today depends on all past volatilities, but with geometrically declining weights. In other words, the GARCH(1,1) model is equivalent to a restricted infinite order ARCH model. Therefore, the GARCH(1,1) model, containing only three components, is a very much parsimonious model, since it allows an infinite number of past squared errors to influence the current variance. As a result, this model has become the 'standard' model for describing changing variance for no obvious reason other than relative simplicity (Chatfield 2001, p. 64). Even among the GARCH family, as Brooks (2002, p. 455) states, a GARCH(1,1) model in general is sufficient to capture the volatility clustering in the data, and rarely is any higher order model estimated or even entertained in the academic finance literature. Furthermore, Bollerslev *et al.* (1992) review the empirical evidence on the ARCH modelling in finance and suggest the GARCH(1,1) is preferred in most cases.

3.3.2 Multivariate GARCH Models

The GARCH models discussed above are univariate GARCH models, in the sense that they are dealing with one error series. However, financial market volatility moves together over time across assets and markets. If researchers want to quantify the relationship between volatility of different time series, estimating a single equation ARCH or GARCH model would ignore the possible causality between the time-varying variances in both directions and would neglect the covariance between the series. In this case, a multivariate modelling framework may lead to obvious gains in efficiency, since the multivariate approach utilises information in the entire variance-covariance matrix of the errors and provides more precise estimates (Conrad *et al.*, 1991). A multivariate GARCH is applied to N different time series, in which the multivariate time series $y_t = (y_{1t}, y_{2t}, \ldots, y_{Nt})'$ and $N \geq 2$. Multivariate GARCH models giving estimates for the conditional covariance as well as the conditional variance, therefore, have a number of useful applications, such as forecasting hedge ratios and CAPM betas.

Kraft and Engle (1983), Engle *et al.* (1984) are the first to discuss multiple equation models with a multivariate ARCH structure. Baba *et al.* (1990) and Engle and Kroner (1995) introduce the theoretical framework for simultaneous equation models where the disturbances follow a GARCH behaviour. According to Engle and Kroner (1995), multivariate GARCH models are useful in multivariate finance and economic models, which require the modelling of both variance and covariance.

In particular, the bivariate GARCH model provides a more effective way to capture interactions between the volatility of two different time series, estimating a bivariate time series $y_t = (y_{1t}, y_{2t})'$. Here the label 'bivariate GARCH' refers to a model for a bivariate time series y_t in which the conditional variances of the individual series and the conditional covariance between the two series are estimated simultaneously.

3.3.2.1 Bivariate GARCH Model

To keep with the notation of the univariate GARCH model, the multivariate time series y_t is a vector of dimension $(N \times 1)$; and the conditional covariance of y_t is an $(N \times N)$ matrix H_t. The diagonal elements of H_t are the variance of each individual time series; and the off-diagonal elements of H_t are the covariance terms.

For the multivariate GARCH model, H_t is a symmetric matrix. The vech operator to a symmetric $(N \times N)$ matrix stacks the lower triangular elements into an $[N \times (N+1)/2]$ vector. Thus, with the vech transformation, the multivariate GARCH(*p*,*q*) model can be written as:

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(u_{t-i}u'_{t-i}) + \sum_{i=1}^{p} B_i vech(H_{t-i}) \quad (3.40)$$

where $u_t = (u_{1t}, \cdots, u_{Nt})'$ are the residual terms associated with the conditional mean equation for y_{1t} to y_{Nt}. C is a $(N \times N)$ positive matrix of parameters. A_i and B_i are $[N(N+1)/2 \times N(N+1)/2]$ matrices of parameters indicating ARCH effects and GARCH effects. In the case of $p = q = 1$, the bivariate GARCH model ($N = 2$) can be specified in full as:

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} & a_{12} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad (3.41)$$

where $h_{11,t}$ and $h_{22,t}$ are the conditional variance of the error term of time series y_{1t} and y_{2t}; $h_{12,t}$ is the conditional covariance between the residuals; also a_{ij}, b_{ij} and c_i ($i = 1,2,3; j = 1,2,3$) are elements of matrices A, B and C, standing for parameters of the ARCH term, the GARCH term and the long term average values. Notice that $h_{21,t}$ is omitted in the equation, since it also measures the conditional covariance. Such redundant term can be ignored without affecting the model.

Although bivariate GARCH(1,1) is the simplest multivariate GARCH model, it is still considerably complex with a fairly large number of parameters to be estimated. For instance, the VECH bivariate GARCH(1,1) has 21 parameters. If the number of variables increases to 3 and 4, the number of parameters will extremely boost to 78 and 210 respectively. Estimating a large number of parameters is not in theory a problem as long as the sample size is large enough. However, efficient estimation of the parameters in GARCH models is by maximum likelihood, which involves the numerical maximization of the likelihood function. As noted by Harris and Sollis (2003, p. 222), obtaining convergence of optimization algorithms can be very difficult when a large number of parameters are involved. Ding and Engle (1994) state that it is not computationally feasible for matrices of dimension N > 5, since there are too many parameters and they interact in a way that is too intricate for existing optimization algorithms to converge.

Additionally, as with univariate GARCH models, it is necessary to impose restrictions on the parameters to ensure the conditional variance of the individual variable is positive and definite, which can be difficult to do in practice. To resolve these difficulties, researchers have proposed various simplifying assumptions to reduce the number of unknown coefficients in the conditional variance matrix to a manageable level (see Bollerslev *et al.*, 1988 and Bollerslev, 1990 for example). As a typically parsimonious representation, the diagonal GARCH model suggested by Bollerslev *et al.* (1988) assumes that the A and B matrices are diagonal. Replacing all a_{ij} and b_{ij} ($i \neq j$) in A and B with zero, equation (3.41) evolves into a diagonal vech bivariate GARCH(1,1) model

$$\begin{bmatrix} h_{11,t} \\ h_{12,t} \\ h_{22,t} \end{bmatrix} = \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} + \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} u_{1,t-1}^2 \\ u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}^2 \end{bmatrix} + \begin{bmatrix} b_{11} & 0 & 0 \\ 0 & b_{22} & 0 \\ 0 & 0 & b_{33} \end{bmatrix} \begin{bmatrix} h_{11,t-1} \\ h_{12,t-1} \\ h_{22,t-1} \end{bmatrix} \quad (3.42)$$

The diagonal representation can further be expressed by three equations:

$$h_{11,t} = c_1 + a_{11}u_{1,t-1}^2 + b_{11}h_{11,t-1} \quad (3.42a)$$

$$h_{12,t} = c_2 + a_{22}(u_{1,t-1}u_{2,t-1}) + b_{22}h_{12,t-1} \quad (3.42b)$$

$$h_{22,t} = c_3 + a_{33}u_{2,t-1}^2 + b_{33}h_{22,t-1} \quad (3.42c)$$

Parameter a_{11} and a_{33} are the coefficients of the ARCH term of the two residuals series' conditional variance; while b_{11} and b_{33} are the coefficients of the GARCH process. The parameters a_{22} and b_{22} represent the covariance GARCH parameters, which measure interaction between two time series. It is still necessary to impose restrictions on parameters to ensure positive definiteness of H_t. The diagonal representation implies each conditional variance and covariance only depends on its lagged values and lagged squared residuals. Compared to VECH form of bivariate GARCH(1,1) model, the diagonal representation economises on parameters, reducing the number of parameters from 21 to 9. On the other hand, simplification has the cost of losing information on certain interrelationships, especially the interaction between conditional variance and conditional covariance.

3.3.2.2 Bivariate BEKK GARCH Model

The covariance matrix H_t is required to be positive definite, in order for any parameterisation to be sensible. However, this restriction can be difficult to check in the VECH and even in the diagonal representation. Accordingly, BEKK GARCH representation is proposed by Baba, Engle, Kroner and Kraft (1990) and name after the authors. Engle and Kroner (1995) further develop sufficient constraints to guarantee the positive definiteness of the covariance matrix. The model assumes that the following equation for H_t:

$$H_t = C'C + \sum_{i=1}^{q} A_i' u_{t-i} u_{t-i}' A_i + \sum_{i=1}^{p} B_i' H_{t-i} B_i \tag{3.43}$$

where A, B and C are $(N \times N)$ square matrices of parameters, and C is upper triangular. It is obvious from the equation above that the covariance matrix is guaranteed to be positive definite as long as the constant term $C'C$ is positive definite. For a bivariate BEKK GARCH(1,1) model, equation (3.43) can be specified as:

$$H_t = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} = \begin{bmatrix} c_{11} & 0 \\ c_{12} & c_{22} \end{bmatrix} \begin{bmatrix} c_{11} & c_{12} \\ 0 & c_{22} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}' \begin{bmatrix} u_{1,t-1}^2 & u_{1,t-1}u_{2,t-1} \\ u_{2,t-1}u_{1,t-1} & u_{2,t-1}^2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}' \begin{bmatrix} h_{11,t-1} & h_{12,t-1} \\ h_{21,t-1} & h_{22,t-1} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \tag{3.44}$$

The parameters in A reveal the extents to which the conditional variances of two variables are correlated with the past squared error. In particular, the off-diagonal elements measure how the past squared error of one variable affects the conditional variance of the other variable. The parameters in B depict the extents to which the current levels of conditional variances are correlated with past conditional variances. More specifically, the diagonal elements reflect the levels of persistence in the conditional variances. On the other hand, the off-diagonal elements in B indicate the extents to which the conditional variance of one variable is correlated with the lagged conditional variance of the other variable. According to Lee (1999), high values of off-diagonal elements imply a correlation between volatility of two variables. The BEKK representation requires 11 parameters to be estimated. Consistent estimates of the parameters can be obtained using the quasi maximum likelihood procedure suggested by Bollerslev and Wooldridge (1992).

The BEKK GARCH model improves on both the VECH and diagonal representations, since positivity of H_t is automatically guaranteed as long as $H_0 \geq 0$. Additionally, it avoids too many parameters to keep estimation feasible compared to the VECH GARCH model. Furthermore, it is more general than the diagonal VECH representation as it allows for interaction effects that the diagonal representation does not. In other words, the BEKK representation maintains enough flexibility in the dynamics of H_t.

The BEKK representation can also be extended to the diagonal BEKK models. Taking A and B as diagonal matrices, the expression of the model is simplified, since transpose does not change the diagonal matrix. Consequently, the diagonal version of model (3.44) can be written by a group of three equations

$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} \quad (3.44a)$$

$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}(u_{1,t-1}u_{2,t-1}) + b_{11}b_{22}h_{12,t-1} \quad (3.44b)$$

$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} \quad (3.44c)$$

The number of parameters to be estimated is significantly lower than equation (3.44). At the same time, it maintains the main advantage of this specification which is the positive definiteness of the conditional covariance matrix. However, some of the flexibility of the original BEKK model is lost.

3.3.2.3 Bivariate GARCH-GJR Model

Although parameterizations of GARCH models are successful and prevalent, these approaches cannot capture some features of financial time series, one of which is the asymmetric effect, also known as the leverage effect. First noted by Black (1976), the effect describes the phenomenon that negative shock to financial time series is likely to increase volatility more than a positive shock of the same magnitude. In other words, ‘bad news’ has a greater impact on volatility than ‘good news’.

In the case of stock returns, the asymmetric effect refers to the tendency for changes in stock prices to be negatively correlated with changes in stock volatility (Bollerslev *et al.*, 1994). Most evidence of volatility asymmetries is provided by studies on the US market (see French *et al.*, 1987; Schwert, 1989b for example). Although relative fewer studies have been concerned with other markets, volatility asymmetries have been broadly documented in other market indices. For example, the asymmetric effect is found in the UK market by Sentana (1993), in the Japanese stock market by Bekaert and Wu (2000), and in the Australian stock market by Brailsford and Faff (1993).

The usual claim of such asymmetries is the leverage effect hypothesis due to Black (1976) and Christie (1982). While a negative shock to a firm's stock price typically causes the value of the firm to fall and the debt to equity ratio to rise, the firm becomes more highly leveraged. Generally leverage is interpreted as an indicator of risk of a company. When the leverage ratio increases, the company is considered to be more risky. As a result, negative shocks raise equity returns volatility. Christie (1982) finds a strong correlation between the asymmetry and leverage. Additionally, his empirical work supports Black's (1976) argument that the leverage itself is not sufficient to explain the asymmetric effect.

An alternative explanation is the volatility feedback hypothesis due to Campbell and Hentschel (1992). The causality implied by this hypothesis runs from volatility to price that positive shocks to volatility increase future risk premium; and if the future dividends remain the same, then the stock price should fall. In addition, Campbell and Hentschel (1992) find supportive evidence for volatility feedback hypothesis. Furthermore, they find that the leverage effect also contributes to the asymmetric behaviour of stock market volatility. Therefore, as pointed out by Bekaert and Wu (2000), these two explanations are not in conflict; and each effect can account for partial reasons for the asymmetric effect.

GARCH models discussed above impose a symmetric response, since lagged error terms are squared in the equation for the conditional variance; hence their signs are irrelevant. Therefore, these models are not capable to allow for the asymmetric response. To overcome the restriction, Nelson (1991) specially designs an exponential GARCH (EGARCH) model to capture the asymmetry effect. In an EGARCH model, the natural logarithm of the conditional variance is assumed to depend on the lagged error terms rather than lagged squared error terms:

$$\ln(\sigma_t^2) = \omega + \beta \ln(\sigma_{t-1}^2) + \alpha f(z_{t-1}) \quad (3.45)$$

where

$$f(z_{t-1}) = \theta z_{t-1} + \gamma[|z_{t-1}| - E(|z_{t-1}|)] \quad (3.46)$$

α, β, ω, θ and γ are constant parameters; and z_{t-1} is the standardised residual at time $t-1$ defined as $z_{t-1} = u_{t-1} / \sigma_{t-1}$. Asymmetry is modelled by equation (3.46), where the sign of the errors is allowed to affect the conditional variance. Intuitively, asymmetry exists if θ is negative and statistically significant. Additionally, by making the natural logarithm, the conditional variance in an EGARCH model is always positive even if the parameters are negative. Thus, it avoids the problem with GARCH models that non-negativity constrains are artificially imposed on model parameters to ensure positivity of the conditional variance.

While Nelson (1991) introduces the univariate EGARCH model; its multivariate extensions have been extensively applied in the literature. Braun *et al.* (1992) proposes a bivariate version of the EGARCH model. The model is given by

$$\ln(\sigma_{i,t}^2) = c_i + \sum_{j=1}^{2} a_{i,j} f_j(z_{j,t-1}) + b_i \ln(\sigma_{i,t-1}^2) \quad (3.47)$$

$$f_j(z_{t-1}) = \theta_j z_{j,t-1} + \gamma_j |z_{j,t-1}| - E(|z_{j,t-1}|) \quad (3.48)$$

where $i, j = 1,2$; and $a_{i,j}, b_i, c_i$ are parameters of matrices A, B and C. The parameter $a_{i,j}$ (for $i \neq j$) captures the volatility interactions of the two series. The coefficient b_i measures persistence of volatility. The unconditional variance is finite if $b_i < 1$. If $b_i = 1$, the unconditional variance does not exist and the conditional variance follows an integrated process of order one. $z_{j,t-1}$ is defined as the same standardised innovation as above. Braun *et al.* (1992) find that the bivariate EGARCH provides a good description of the returns for a number of industry and size sorted portfolio.

In the EGARCH model, the dependent variable is the natural logarithm of the conditional variance which allows for the effect of the sign of the residual on the conditional variance. However, such specification is fundamentally different from the original GARCH framework and the question remains whether this asymmetric volatility model is practically superior to the original GARCH model. Empirical evidence is controversial, especially for the forecasting purpose (see Pagan and Schwert, 1990; Day and Lewis, 1992 for example). Moreover, the convergence difficulties are more general when estimating the EGARCH model. In some studies, convergence even cannot be reached with EGARCH models (see Jostova and Philipov, 2005 for example), which may be the most unfavourable disadvantage of the EGARCH model.

Alternatively, Glosten, Jagannathan and Runkle (1993) suggest that the asymmetry effect can also be captured simply by incorporating a dummy variable in the original GARCH. The model is named after the three authors as the GARCH-GJR representation

$$\sigma_t^2 = \alpha_0 + \alpha u_{t-1}^2 + \gamma u_{t-1}^2 I_{t-1} + \beta\sigma_{t-1}^2 \tag{3.49}$$

where $I_{t-1} = 1$ if $u_{t-1} < 0$; otherwise $I_{t-1} = 0$. Thus, the ARCH coefficient in a GJR model switches between $\alpha + \gamma$ and α, depending on whether the lagged error term is positive or negative. If the estimated value of γ is greater than zero, then the leverage effect is present in the data. Notice non-negative constrains should be imposed on parameters to ensure non-negativity of the conditional variance, which will be $\alpha_0 > 0$, $\alpha \geq 0$, $\beta \geq 0$ and $\alpha + \gamma \geq 0$.

Kim and Kon (1994) find that the GJR model is the most descriptive for individual stocks, while Nelson's (1991) EGARCH is the most appropriate for stock indexes. Engle and Ng (1993) argue that GARCH-GJR is the best parametric model because the conditional variance implied by the EGARCH model may be too high due to the exponential functional form. Additionally, according to Engle and Ng (1993), the GJR model is less sensitive to outliers than the EGARCH model. Moreover, the problem of convergence difficulty is less common to the GARCH-GJR representation than the EGARCH model, which is important for practical purposes. As a result, this thesis applies GJR rather than the EGARCH to model the time-varying beta.

The GJR model can also be applied to two variables to capture the conditional variance and covariance. Once again, a parsimonious representation can be obtained by imposing a diagonal restriction on parameter matrices. Consequently, the diagonal bivariate GJR GARCH(1,1) can be presented by the following equations

$$h_{11,t} = c_1 + a_1 u_{1,t-1}^2 + b_1 h_{11,t-1} + r_1 u_{1,t-1}^2 I_{t-1} \tag{3.50a}$$

$$h_{12,t} = c_2 + a_2 (u_{1,t-1} u_{2,t-1}) + b_2 h_{12,t-1} \tag{3.50b}$$

$$h_{22,t} = c_3 + a_3 u_{2,t-1}^2 + b_3 h_{22,t-1} + r_3 u_{2,t-1}^2 I_{t-1} \tag{3.50c}$$

where I is the same dummy variable as in the univariate model, a_i, b_i and c_i ($i = 1,2,3$) are parameters of the ARCH term, the GARCH term and the long term average value. In fact, only a few studies have involved the multivariate GJR model and there is no standardised bivariate GJR. Researchers usually apply the flexible bivariate GJR framework to look into cross-market interactions.

3.3.2.4 Bivariate GARCH-X Model

This extension of GARCH models links to the error-correction model of cointegrated series. To understand the concept of cointegration, assume two nonstationary time series y_t and x_t are both I(*d*). In general, the linear combination of the two series will also be I(*d*). In other words, the error term generated by regressing y_t on x_t ($u_t = y_t - \beta x_t$) is I(*d*). However, under some circumstances, the residuals u_t from the regression is of a lower order of integration, I(*d-b*). According to the definition of Engle and Granger (1987), y_t and x_t are cointegrated of order (*d-b*). In practice, many financial time series contain one unit root, such as stock prices; thus researchers typically focus on the case where $d = b = 1$. In this context, a set of variables is defined as cointegrated of order (1,1), if a linear combination of them is stationary, or $u_t \sim I(0)$. Such pairs of cointegrated variables are nonstationary, but in the long run they move together bound by some cointegrated relationship.

Several hypotheses may exist to explain the cointegrated relationship, among which researchers are particularly interested in a long term or equilibrium phenomenon. If two series are expected to hold an equilibrium relationship with one another, their association will be stable in the long run, although they may deviate from the relationship in the short run. In other words, the stochastic trend components of the two variables may exactly offset to compose a stationary linear combination. Thus, cointegration represents the existence of a long-run equilibrium to which an economic system converges over time.

As discussed in section 3.2.3, taking differences is a usual response to nonstationary time series. However, in the context of bivariate modelling, such as conditional covariance modelling, such a procedure is not advisable, even though the approach is statistically valid. The reason is taking differences can remove unit roots of variables, at the same time it may lose sight of the long run solution on the relationship between variables. Therefore, a class of models known as error correction mechanism were designed to overcome the problem by using combinations of first differenced and lagged levels of cointegrated variables[16]. The following equation is a simple example of the error correction model

$$\Delta y_t = \beta_1 \Delta x_t + \beta_2 (y_{t-1} - \theta x_{t-1}) + u_t \tag{3.51}$$

where β_1, β_2 and θ are parameters; u_t is the error term with usual properties; time series y_t and x_t are both I(1); and $(y_{t-1} - \theta x_{t-1})$ is known as the error correction term. Provided that y_t and x_t are cointegrated with cointegrating coefficient θ, the error correction term $(y_{t-1} - \theta x_{t-1})$ will be I(0); and thus OLS and standard procedures for statistical inference can be used on the error correction model. Furthermore, on the basis of financial theory, an intercept can be included in either the cointegrating term or in the model for Δy_t or for both.

The model for the error correction term can be generalised to a residual-based approach to test cointegration in regression, which is developed by Engle and Granger (1987). In the Engle-Granger framework, the cointegrated relationship between y_t and x_t is directly examined by testing whether the error term u_t generated by regressing y_t on x_t is I(1) against the alternative that u_t is I(0). Essentially, Engle and Granger (1987) advocate ADF tests on residuals. Notice, as a test on residuals, the critical values are different with those of a DF or an ADF test on a series of raw data. Thus, Engle and Granger (1987) tabulate a new set of critical values for this application. However, this residual-based ADF test for cointegration assumes that the

[16] The error correction model is sometimes termed the equilibrium correction model. It is first used by Sargan (1964) and later popularised by Engle and Granger (1987).

variables are I(1). The critical values should be changed when the variables are I(2) but still have the cointegrated relationship. According to Haldrup (1994), the critical values for the ADF test depend on the number of I(1) and I(2) regressors in the equation. Consequently, Haldrup (1994) provides the critical values for testing for cointegration when there is a mix of I(1) and I(2) variables. In this research, such critical values will be employed to test the cointegrated relationship between the market and individual firms.

For cointegrated series, the error correction term also can also be viewed as short term deviations from their long run equilibrium relationship. According to Engle and Yoo (1987), such error correction term has important predictive powers for the conditional mean of the series. Additionally, Lee (1994) suggests that if short term deviations influence the conditional mean, they may also have an effect on conditional variance and conditional covariance. A significant positive effect may imply that the further the series apart from each other in the short run, the harder they are to forecast. Therefore, Lee (1994) proposes the GARCH-X model to allow for the effect of short term deviations, with the lagged error correction term incorporated in equations. The GARCH-X(p,q) representation can be written as:

$$y_t = \alpha + \beta z_{t-1} + u_t - \theta u_{t-1} \tag{3.52a}$$

$$u_t | \Omega_{t-1} \sim N(0, H_t) \tag{3.52b}$$

$$vech(H_t) = vech(C) + \sum_{i=1}^{q} A_i vech(u_{t-i}u'_{t-i}) + \sum_{i=1}^{p} B_i vech(H_{t-i}) + \sum_{i=1}^{k} D_i vech(z_{t-1}) \tag{3.52c}$$

where z_t is the error correction term from the cointegration relationship between two series; and the other variables are as described in other GARCH representations. Notice that the error correction term is included in both the mean equation and the conditional heteroscedasticity equation. When the model is applied to time-varying betas, z_t in (3.52a) measures the effect of short term deviations on the firm and market returns; while the squared error correction term z_t^2 measures the influence of short term deviations on the conditional variance and covariance.

Lee (1994) advocates that the square of the error correction term lagged once should be applied in the GARCH-X(1,1) model. Similar to bivariate GARCH, the diagonal constraint is applicable to the GARCH-X(1,1) model, resulting in a diagonal vech bivariate GARCH-X(1,1) model:

$$h_{11,t} = c_1 + a_{11}(u_{1,t-1})^2 + b_{11}h_{11,t-1} + d_{11}(z_{t-1})^2 \tag{3.53a}$$

$$h_{12,t} = c_2 + a_{22}(u_{1,t-1}u_{2,t-1}) + b_{22}h_{12,t-1} + d_{22}(z_{t-1})^2 \tag{3.53b}$$

$$h_{22,t} = c_3 + a_{33}(u_{2,t-1})^2 + b_{33}h_{22,t-1} + d_{33}(z_{t-1})^2 \tag{3.53c}$$

where parameters d_{11}, and d_{33} respectively describe the influence of short term deviations on the conditional variance of the individual firm and the market index; the parameter d_{22} reveals the effect of short term deviations on the conditional covariance. Therefore, as long as d_{22} and d_{33} are significant, h_{12} and h_{22} will be different from those generated by the standard GARCH model. In this case, the time-varying beta estimated by GARCH-X will be different from the standard GARCH. In the same way, values of time-varying beta will be different when models are utilised to forecast.

3.3.3 Modelling Time-Varying Beta with GARCH

3.3.3.1 Estimation of Time-Varying Beta

The introduction of the CAPM promoted interests in the analysis of behaviour under uncertainty based on the second moments of return series, since the CAPM beta is defined as the ratio of the covariance between the market portfolio return and the equity return to the variance of the market portfolio return. Typically, time-varying betas are constructed using a set of historical data on conditional variance and covariance. However, the estimation of conditional betas in this fashion is backward-looking; while investors would be more interested in future beta values than historical values. An analytical framework to model second or possibly higher moments was absent until the emergence of GARCH models. To construct the conditional beta series from the bivariate GARCH model is a straightforward process, since the econometric specification provides time-varying estimates of the conditional covariance and the conditional variance.

Given all bivariate GARCH models discussed above, the beta can be calculated by

$$\beta_t = \frac{\hat{h}_{12,t}}{\hat{h}_{22,t}} \tag{3.54}$$

where $\hat{h}_{12,t}$ is the estimated conditional covariance between market returns and share returns; and $\hat{h}_{22,t}$ is the estimated conditional variance of market returns. Notice that conditional variances and covariances produced by various GARCH specifications are usually different; so are conditional betas.

3.3.3.2 Forecasting of Time-Varying Beta

There are vast practical applications of GARCH models in finance; and many of them attempt to produce out-of-sample variance and covariance forecasts. Although the forecasting functions of ARCH and GARCH models (for conditional variance) are less well documented than the forecasting function of the conventional ARIMA models (for conditional mean), the methodology used to obtain the optimal forecast of the conditional variance of a time series from a GARCH model is the same as that used to obtain the optimal forecast of the conditional mean (Harris and Sollis 2003, p. 246)[17]. The basic univariate GARCH(p, q) model (3.34) is utilised to illustrate the forecast function for the conditional variance of the GARCH process due to its simplicity.

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q} \alpha_i u_{t-i}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \tag{3.34}$$

Providing that all parameters are known and the sample size is T, taking conditional expectation the forecast function for the optimal h-step-ahead forecast of the conditional variance can be written:

$$E(\sigma_{T+h}^2 | \Omega_T) = \alpha_0 + \sum_{i=1}^{q} \alpha_i (u_{T+h-i}^2 | \Omega_T) + \sum_{j=1}^{p} \beta_j (\sigma_{T+h-i}^2 | \Omega_T) \tag{3.55}$$

where Ω_T is the relevant information set. For $i \le 0$, $E(u_{T+i}^2 | \Omega_T) = u_{T+i}^2$ and $E(\sigma_{T+i}^2 | \Omega_T) = \sigma_{T+i}^2$; for $i > 0$, $E(u_{T+i}^2 | \Omega_T) = E(\sigma_{T+i}^2 | \Omega_T)$; and for $i > 1$, $E(\sigma_{T+i}^2 | \Omega_T)$ is obtained recursively. Consequently, the one-step-ahead forecast of the conditional variance is given by:

$$E(\sigma_{T+1}^2 | \Omega_T) = \alpha_0 + \alpha_1 u_T^2 + \beta_1 \sigma_T^2 \tag{3.56}$$

[17] Harris and Sollis (2003, p. 247) discuss the methodology in details.

Although many GARCH specifications forecast the conditional variance in a similar way, the forecast function for some extensions of GARCH will be more difficult to derive. For instance, extra forecasts of the dummy variable *I* are necessary in the GARCH-GJR model. However, following the same framework, it is straightforward to generate forecasts of the conditional variance and covariance using bivariate GARCH models, and thus the conditional beta.

In this thesis, the forecasting performance of bivariate GARCH-type models are examined using daily data from UK and US firms. Their overall predictive ability will be compared in terms of a variety of forecast error measures. Candidates include the standard bivariate GARCH, the BEKK GARCH, GARCH-GJR, and GARCH-X models. There is no exact reason for the selection of the representative set of models in this research, since evidence from literature is conflicting and even absent for some representations.

The methodology of forecasting time-varying betas will include the following several steps[18].

1. Since beta values are not directly observable data from financial markets, constructing the time-varying beta series must be the precondition for further investigations. The actual beta series will be constructed by GARCH models and the non-GARCH approach for the whole sample (1989 to 2003). Although data series estimated by different method differ from each other, they will all be used as criteria to evaluate forecasting betas produced by means of the same model.
2. With some rational considerations about stationarity of time series data, the forecasting models will be used to forecast time-varying betas and be compared in terms of forecasting accuracy. The lack of *ex ante* beta values makes it impossible to evaluate the predictive ability of models according to the real future benchmarks. Consequently, *ex post* data must be used as remediation. For instance, sequences of beta will be 'predicted' for the year 2003 based on parameter values derived from 1989 to 2002. Forecasted betas then will be compared to estimated beta values in 2003.

[18] According to many previous studies (see Tse and Tung, 1992; Walsh and Tsou, 1998; Diodge and Wei, 1998 for example), a multi-period forecast may lead to serious GARCH convergence problem. Therefore, the static forecasting is the main scheme in the thesis.

3. The performance of various models will be evaluated on the basis of a variety of test statistics.

To avoid the sample effect and the overlapping issue, three forecast samples are considered, including two one-year samples (2001 and 2003) and one two-year horizon (2002 to 2003). Accordingly, time-varying betas need to be estimated in three periods, 1989 to 2000, 1989 to 2001 and 1989 to 2002, to generate model parameters. In this way, the forecasting procedure has the advantage of having three different sets of forecasting parameters. In addition, performance of alternative models can be compared in different forecast horizons, with either same or different length. Of course, another three forecast sample can be chosen without loss of these advantages, e.g. 2001 to 2002, 2002 and 2003. As argued by Brooks (2003, p. 279), "*where each of the in-sample and out-of-sample periods should start and finish is somewhat arbitrary and at the discretion of the researcher*". However, noticing the fact that the stock market suffered through the early 2000s due to a number of major events, such as the September 11 terrorist attach and scandals in US stock market, the extreme fluctuations throughout the years (2001 to 2003) may have an influence on the performance of alternative models.

Theoretically, this forecasting framework can be briefly evaluated in terms of Diebold's (2004) six basic considerations basic to successful forecasting.

(1) Decision Environment

Good predictions help to produce good decisions. Recognition and awareness of the decision-making environment is the key to effective design, use and evaluation of forecasting models. Many surveys show that the CAPM is the most often used model by financial managers for assessing the cost of equity and the risk of cash flows (see McLaney *et al.*, 1998; Lumby and Jones, 1999). Predictions of time-varying betas can guide to a number of decision-making processes, such as capital structure and investment appraisal decisions.

(2) Forecast Object

Clearly, the forecast object in this thesis is the value of beta. Quantitative data required for forecasting beta are available for a long sample. Additionally, such information is easily and publicly available.

(3) Forecast Statement

All researchers choose the means of point forecast, generating a single number for future beta value. A good point forecast provides a simple and easily digested guide to the future of a time series. On the other hand, it is more sensitive to random and unpredictable shocks, compared to the interval forecast or density forecast.

(4) Forecast Horizon

The out-of-sample prediction usually leads to one-step-ahead forecast of the beta (Hansen and Lunde, 2002). Therefore, the forecast horizon depends on the sampling frequency of data. Generally, high frequency data are preferred for producing the future beta due to the existence of volatility clustering. Furthermore, asset pricing tests are sensitive to the return interval. Three out-of-sample forecast horizons are considered in the study, to avoid the sample effect and overlapping issue

(5) The Information Set

The idea of an information set is fundamental to evaluate forecasts, since the forecast could be improved by either using more information or using given information more effectively. Forecasting time-varying betas is based on several series of price data from the financial market. Under the market efficiency hypothesis, share prices of UK and US firms have reflected a great deal of information[19]. Consequently, the forecast is directly based on price indices, and indirectly on a considerable set of information implied by the market efficiency.

(6) Method Complexity

In light of the obvious complexity of the real world phenomena, researchers and practitioners are seeking to answer the question what forecasting method is best suited to the need of forecast. Econometric models employed to forecast the time-varying beta are complex; and can only be implemented with specific mathematic software package. However question remains whether they are competent to forecast the conditional variance and covariance.

[19] Empirical tests broadly support the stock markets in the UK and the US as semi-strong form of efficiency, which implies share prices can reflect all public information.

The six considerations can help to design and evaluate the methodology of forecasting time-varying betas.

3.4 Kalman Filter Approach

In the engineering literature of the 1960s, an important notion called 'state space' was developed by control engineers to describe system that varies through time. The general form of a state space model defines an observation (or measurement) equation and a transition (or state) equation, which together express the structure and dynamics of a system. The convenient and powerful framework has been broadly applied to the statistics and econometrics research since the 1970s.

In a state space model, observation at time t is a linear combination of a set of variables, known as state variables, which compose the state vector at time t. Denote the number of state variables by m and the $(m \times 1)$ vector by θ_t, the observation equation can be written as

$$y_t = z_t' \theta_t + u_t \qquad (3.57)$$

where z_t is assumed to be a known the $(m \times 1)$ vector, and u_t is the observation error. The disturbance u_t is generally assumed to follow the normal distribution with zero mean, $u_t \sim N(0, \sigma_u^2)$. The set of state variables may be defined as the minimum set of information from present and past data such that the future value of time series is completely determined by the present values of the state variables. This important property of the state vector is called the Markov property, which implies that the latest value of variables is sufficient to make predictions.

In practice, it may be difficult to observe all elements of the state vector. However, it is reasonable to make assumptions about how the state vector θ_t evolves through time. A key assumption of the state space model is that the vector θ_t follows the time-varying process

$$\theta_t = G_t \theta_{t-1} + w_t \qquad (3.58)$$

where G_t is a known $(m \times m)$ matrix and w_t denotes an m-vector of error terms. The disturbance w_t is a multivariate normal with zero mean vector and known covariance matrix W_t. Equation (3.58) is called the transition equation. The error terms in the observation and transition equations are generally assumed to be uncorrelated with each other at all time periods, and also to be serially uncorrelated through time. Suppose future values of z_t and G_t are known, the h-step forecast formula of y_t becomes

$$\hat{y}_{t+h} = z_{t+h}' G_{t+h} \hat{\theta}_t \tag{3.59}$$

Since the exact value of θ_t will not be known in practice, it has to be estimated from information up to time t. Thus, θ_t is replaced by $\hat{\theta}_t$ in the forecast equation; and the computation of forecasts hinges on being able to obtain appropriate estimates of the present state vector θ_t.

In the unified framework of the state space form, the Kalman filter places a key role in providing optimal forecasts and a method of estimating the unknown model parameters. As its name suggested, the Kalman filter is primarily intended for filtering; while the approach is now used in a variety of statistical applications outside its original intention.

The Kalman filter is usually carried out in two stages. Suppose the estimate of the last period's state vector θ_{t-1} is known together with the estimate of its covariance matrix denoted by P_{t-1}. The first stage called the prediction stage is aiming to forecast θ_t using information up to time $(t-1)$, which can be modelled as

$$\hat{\theta}_{t|t-1} = G_t \hat{\theta}_{t-1} \tag{3.60}$$

where $\hat{\theta}_{t|t-1}$ is forecasted θ_t based on information up to time $(t-1)$. Using the notation of equation (3.58), the covariance matrix of $\hat{\theta}_{t|t-1}$ is given by

$$P_{t|t-1} = G_t P_{t-1} G_t' + W_t \tag{3.61}$$

where W_t is the covariance matrix of the error term of transition equation.

When the new observation at time t (y_t) becomes available, the second stage of the

Kalman filter is using the new observation y_t to update forecasts using the following equations

$$\hat{\theta}_t = \hat{\theta}_{t|t-1} + K_t e_t \tag{3.62}$$

and

$$P_t = P_{t|t-1} - K_t z_t' P_{t|t-1} \tag{3.63}$$

where $e_t = y_t - z_t' \hat{\theta}_{t|t-1}$ is the forecasting error at time t; and $K_t = P_{t|t-1} z_t / (z_t' P_{t|t-1} z_t + \sigma_u^2)$ is the Kalman gain matrix. The second stage is usually known as the updating stage. Using the two stages, the Kalman filter provides a powerful recursive algorithm for state space models (Hamilton, 1994). With the approach, the state space model has an important property that the latter quantity can readily be obtained as each new observation becomes available (Chatfield, 2001).

A state space model can be used to incorporate unobserved variables into, and estimate them along with, the observable model to impose a time-varying structure of the CAPM beta (Faff *et al.*, 2000). Additionally, the structure of the time-varying beta can be explicitly modelled within the Kalman filter framework to follow any stochastic process. The Kalman filter recursively forecasts conditional betas from an initial set of priors, generating a series of conditional intercept and beta coefficients for the CAPM. The technique has been used by some studies to forecast the time-varying beta (see Black *et al.*, 1992 and Well, 1994 for example).

The Kalman filter method estimates the conditional beta in the following way

$$R_{it} = \alpha_t + \beta_{it} R_{Mt} + \varepsilon_t \tag{3.64}$$

where R_{it} and R_{Mt} is the excess return on the individual share and the market portfolio at time t, and ε_t is the disturbance term. Equation (3.64) represents the observation equation of the state space model, which is similar to the CAPM model. However, the form of the transition equation depends on the form of stochastic process that betas are assumed to follow. In other words, the transition equation can be flexible, such as using AR(1), random coefficient, random walk and random walk with drift. These four potential dynamic processes of time-varying beta can be written as:

$$\beta_{it}^{AR} = \overline{\beta}_i + \phi\beta_{i,t-1}^{AR} + \eta_{it} \quad (3.65a)$$

$$\beta_{it}^{RC} = \overline{\beta}_i + \eta_{it} \quad (3.65b)$$

$$\beta_{it}^{RW} = \beta_{i,t-1}^{RW} + \eta_{it} \quad (3.65c)$$

$$\beta_{it}^{RWD} = \overline{\beta}_i + \beta_{i,t-1}^{RWD} + \eta_{it} \quad (3.65d)$$

Equations (3.64) and (3.65) form a state space model. In addition, prior conditionals are necessary for using the Kalman filter to forecast the future value, which can be expressed by

$$\beta_0 \sim N(\beta_0, P_0) \quad (3.66)$$

The first two observations can be used to establish the prior condition. Based on the prior condition, the Kalman filter can recursively estimate the entire series of conditional beta.

3.5 Measures of Forecast Accuracy

The evaluation of forecasts is an important part of any forecasting study. However, there is no simple answer to the question which is the best method of forecasting, since a variety of factors can be considered as criteria, such as forecast accuracy, cost and relevant contextual features. Thus, the answer directly depends on what is meant by 'best'. In this research, different econometric approaches follow the similar methodology; and thus it is reasonable to assume that 'best' means achieving the most accurate forecasts in this case. In other words, forecast accuracy is the appropriate criterion for the comparative assessment of various GARCH models and the Kalman filter approach.

3.5.1 Measures Derived from Forecast Errors

The time-varying beta forecasts of this thesis use *ex post* explanatory variables; and forecasted beta values are compared to their *ex post* values for accuracy evaluation. A group of measures derived from the forecast error are designed to evaluate *ex post* forecasts. The forecast error is defined as the difference between the actual value (y_t) and forecasted value ($\hat{y}_t$)

$$e_t = y_t - \hat{y}_t \tag{3.67a}$$

or the percent form of the forecast error

$$p_t = (y_t - \hat{y}_t) / \hat{y}_t \tag{3.67b}$$

Measures of forecast accuracy involved in this study include mean squared error (MSE), mean absolute error (MAE), mean absolute percent error (MAPE), mean error (ME), Theil U statistics.

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2 \tag{3.68}$$

$$MAE = \frac{1}{n}\sum_{t=1}^{n} |e_t| \tag{3.69}$$

$$MAPE = \frac{1}{n}\sum_{t=1}^{n} |p_t| \tag{3.70}$$

$$ME = \frac{1}{n}\sum_{t=1}^{n} e_t \tag{3.71}$$

$$U = \left(\sum_{t=1}^{y} e_t^2 \Big/ \sum_{t=1}^{y} y_t^2 \right)^{\frac{1}{2}} \tag{3.72}$$

Measures of MSE and MAE are importantly popular and easily computable. Very often, MAE preserves units, as it is in the same units as the measured variable and is better descriptive statistic than MSE. However, since the beta is a value without unit, MSE can be competent measures in this research. MAPE has the advantage of being dimensionless measure errors by taking the percentage form. Due to Theil (1961), Theil U statistics are also dimensionless and without scaling problem. Different measures indicate different loss function; and thus may generate different results. ME is not an appropriate measure for forecast accuracy, but it is helpful to evaluate whether the model tends to produce over or under prediction.

3.5.2 Test of Equal Forecast Accuracy

Certainly, except ME, the lower the forecast error measure, the better the forecasting performance. However, it does not necessarily mean that a lower MSE completely testifies superior forecasting ability, since the difference between the MSEs may be not significantly different from zero. Therefore, it is important to check whether any reductions in MSEs are statistically significant, rather than just compare the MSE of different forecasting models (Harris and Sollis 2003, p. 250).

Diebold and Mariano (1995) develop a test of equal forecast accuracy to test for whether two sets of forecast errors, say e_{1t} and e_{2t}, have equal mean value. Using MSE as the measure, the null hypothesis of equal forecast accuracy can be represented as $E[d_t] = 0$, where $d_t = e_{1t}^2 - e_{2t}^2$. Supposed *n*, *h*-step-ahead forecasts have been generated, Diebold and Mariano (1995) suggest the mean of the difference between MSEs $\bar{d} = \frac{1}{n}\sum_{t=1}^{n} d_t$ has an approximate asymptotic variance of

$$Var(\bar{d}) \approx \frac{1}{n}\left[\gamma_0 + 2\sum_{k=1}^{h-1}\gamma_k\right] \quad (3.73)$$

where γ_k is the *k*th autocovariance of d_t, which can be estimated as:

$$\hat{\gamma}_k = \frac{1}{n}\sum_{t=k+1}^{n}(d_t - \bar{d})(d_{t-k} - \bar{d}) \quad (3.74)$$

Therefore, the corresponding statistic for testing the equal forecast accuracy hypothesis is $S = \bar{d} / \sqrt{Var(\bar{d})}$, which has an asymptotic standard normal distribution. According to Diebold and Mariano (1995), results of Monte Carlo simulation experiments show that the performance of this statistic is good, even for small samples and when forecast errors are non-normally distributed. However, this test is found to be over-sized for small numbers of forecast observations and forecasts of two-steps ahead or greater.

Harvey *et al.* (1997) further develop the test for equal forecast accuracy by modifying Diebold and Mariano's (1995) approach. Since the estimator used by Diebold and Mariano (1995) is consistent but biased; Harvey *et al.* (1997) improve the finite sample performance of Diebold and Mariano (1995) test using an approximately unbiased estimator of the variance of $\bar{d}$. The modified test statistic is given by

$$S^* = \left[\frac{n + 1 - 2h + n^{-1}h(h-1)}{n}\right]^{1/2} S \quad (3.75)$$

Through Monte Carlo simulation experiments, this modified statistics is found to perform much better than the original Diebold and Mariano statistic at all forecast horizon and when the forecast errors are autocorrelated or have non-normal distribution.

3.6 Conclusion

The methodology chapter provides a comprehensive view of econometric methods involved in this research. Most econometric approaches are demonstrated by equations with necessary description. Additionally, their motivations, applications, advantages and weaknesses are critically discussed when appropriate. In this way, the chapter helps to understand the general principles behind the study and the appropriate procedures, rules and techniques to undertake time-varying beta forecasts.

Section 3.2 covers concepts and techniques consisting of the fundamental knowledge of time series models. We commence with the definition and notations of time series data. Features of standard models of stochastic process (including AR, MA, ARMA and ARIMA) are then discussed. This section continues to explain the concept of stationarity and specify the stationary condition for each standard time series model. Knowing many financial time series are practically nonstationary, it is important to test the existence of unit roots in time series. Section 3.2 finally describes the theory and application of classical DF, ADF tests for stationarity.

The section of GARCH models starts by reviewing some important features of stock market volatility, which are considerably relevant to forecasting the time-varying beta. Motivated by the broadly reported phenomenon of volatility clustering, Engle (1982) and Bollerslev (1986) propose the ARCH and GARCH model, which compose the foundation of the prevalent GARCH family. These models even play an influential role in the establishment of the discipline of financial econometrics. Formulations and implications of ARCH and GARCH models are discussed in details. several GARCH variations are also presented with their motivation to capture particular features of financial data. Among them, the BEKK representation overcomes the positive restriction of parameters. The GARCH-GJR model is able to capture the asymmetric effect in stock prices. The GARCH-X model links to the error-correction model, taking the cointegrated relationship between market and firm into account. Each extension of the GARCH has its predominance in particular aspect; thus in principle no particular GARCH specification is superior and preferred by all users. They are all employed to forecast conditional variance and covariance with the diagonal constraint. With a brief discussion on how the value of conditional beta is computed by conditional variance and covariance, the chapter extends application of GARCH model to estimate and forecast conditional beta in the conditional CAPM framework. At the end of this section, the steps to carry out the time-varying beta forecast are explained; and the forecasting framework is evaluated in terms of Diebold's (2004) six considerations basic to successful forecasting.

Section 3.4 is relatively compact, since it only contains the discussion of Kalman filter approach. Beginning with an introduction of state space model in nature, this section explains how the Kalman filter approach works in the state space framework. In addition, we particularly discuss the application of Kalman filter in estimating and forecasting the conditional beta. Based on previous empirical evidence, the random walk will be used for the transition equation to estimate and forecast the time-varying beta.

Section 3.5 focuses on techniques for the forecast evaluation. As different econometric models are compared in terms of conditional beta forecasts within the similar forecasting structure, forecast accuracy is the appropriate criterion of the evaluation. In order to measure the forecast accuracy, some statistics derived from the forecast error and tests for equal forecast accuracy are applied to examine the capability of each candidate model.

Chapter 4
Literature Review

4.1 Introduction

This chapter reviews existing literature relevant to forecasting time-varying betas, including both conceptual studies and methodological discussions. A rich body of literature exists on applying econometric techniques to forecast the financial time series, but considerably less studies on forecasting the time-varying beta. Therefore, the chapter begins with summarising studies on forecasting stock returns and forecasting stock market volatility. Such reviews help to put the thesis in an applicable context by considering the theoretical debate on stock price movements. In addition, the chapter discusses the development of forecasting beta values from both theoretical and empirical perspectives. Finally, studies on forecasting with GARCH models are presented to ensure that up-to-date techniques are suitably used in this research.

4.2 Forecasting Stock Returns

For many years, economists, statisticians and researchers of finance have been interested in developing and testing models of stock price behaviour, with a particular attention being paid to the construction and interpretation of return forecasts over last two decades due to the development of theoretical analysis and the increase of computer power. In retrospect, several theoretical hypotheses concerning stock market price movements had a great deal influence on the predictability issue of stock returns.

4.2.1 Randomness Debate and Predictability of Share Prices

In early stage, the random walk theory played a vital role on the stock price forecasts. Since the mathematics and statistics of Brownian motion developed by Bachelier (1900), the studies on stock price behaviour moved into the randomness debate and the development of the random walk hypothesis. Since the random walk hypothesis has significant implications on stock return forecasts, a primary focus of capital market research has been to determine whether or not the random walk is an accurate description of stock price movements (Bernstein, 1992). Over time, this line of studies evolved a more formal theory, the efficient markets hypothesis (Fama, 1970, 1991). The hypothesis argues that, in frictionless markets, and with random information flow, prices reflect all available information. Thus, the formulation of the efficient market hypothesis precludes predictable stock prices.

Among earlier empirical studies, the majority found supportive evidence for the random walk hypothesis (see Kendall, 1953; Osborne, 1959; Roberts 1959; Larson, 1960; Cowles, 1960 and Working, 1960). Kendall (1953) states that the weekly changes in a wide variety of financial prices could not be predicted from either past changes in the series themselves or from past changes in other price series. Osborne (1959) develops the proposition that it is not absolute price changes but logarithmic price changes which are independent of each other and that the changes themselves are normally distributed implying that prices follow a Brownian motion. These well known empirical studies had established the fact that markets were '*weak-form efficient*' in Roberts's (1967) terminology. Along with Fama (1965) concludes that "*there is no evidence of important dependence from either an investment or a statistical point of view*" after conducting an extensive empirical analysis of US stock returns from 1956 to 1962, the random walk theory has become an integral part of theories pertaining to stock price series. In this case, consequently most searchers agreed that past prices could not be used to forecast future prices changes. As Fama (1995) pointed out, if stock prices follow a random walk, the various technical or chartist procedures for predicting stock prices are completely without value.

It should be noticed that some early studies did find evidence against the random walk theory that stock returns are predictable from past returns. For example, Fama (1965) finds that the first-order autocorrelations of daily returns are positive for 23 of the 30 with Dow Jones Industrials. Fisher's (1966) results suggest that the autocorrelations of monthly returns on diversified portfolios are positive and larger than those for individual stocks. However, these findings were largely dismissed as statistical anomalies or not economically meaningful after accounting for transactions costs (see Cowles, 1960; Fama, 1970).

When the random walk property was taken for granted as gospel truth, it could be a remarkable negative motivation for forecasting stock returns studies. Such status had not been changed too much until 1980s. '*Stock prices do not follow random walks*' is the title of a heavily cited paper by Lo and MacKinlay (1988). Their paper presented that the weekly return on portfolios of NYSE stocks grouped according to size showed positive autocorrelation. Since this seminal work, a variety of studies have reconsidered testing the null hypothesis of a random walk for prices against a variety of alternative hypotheses. Although investigations still produce mixed evidence, there is mounting evidence that stock returns are predictable to some extent, as extensive work confirms the finding of Lo and MacKinlay (1988) (see Conrad and Kaul, 1988; Poterba and Summers, 1988; Lo and MacKinlay, 1990 and Chopra *et al.*, 1992 for example). Although most authors conducted research using data from well established stock markets, such as US, UK markets, some other researchers also find evidence to reject the random walk hypothesis in emerging markets. For instance, Urrutia (1995) tests Latin American emerging equity markets and rejects the hypothesis for Argentina, Brazil, Chile and Mexico, suggesting that there is predictability. In addition, Huang (1995) shows that the random walk hypothesis can be rejected for Korea, Malaysia, Hong Kong, Singapore and Thailand stock markets during the period from January 1988 to June 1992.

All evidence of the inefficiency of stock market returns led the researchers to investigate the sources of this inefficiency. In order to take advantage of the inefficiency, a growing body of research attempts to characterize stock return predictability, aided by the increased computer power. These studies can be classified into several categories in terms of the approach used, namely macroeconomic factor model, fundamental factor model, trading rules, nonlinear model and neural network.

4.2.2 Macroeconomic Factor Models

Financial researchers all agree that stock returns have a complex association with macroeconomic variables, since stock prices systematically react to changes in macroeconomic variables. A considerable amount of recent research seeks to predict stock market returns using models which contain macroeconomic variables (see Ferson and Harvey, 1993; Glosten *et al.*, 1993; Pesaran and Timmermann, 1995; Flannery and Protopapadakis, 2002 for example). Fama and French (1989) demonstrate that macroeconomic variables representing general business conditions can help predict the time series of stock returns. As reported by Campbell (1987) and Fama and French (1988), a variety of macroeconomic variables, including short-term interest rates, expected inflation, term spreads between long and short-term government bonds, default spreads between low grade and high grade bonds have some power to predict stock returns. In addition, Chen *et al.* (1986) find that changes in aggregate production, inflation, the short-term interest rates, the slope of term structure and risk premium are other macroeconomic factors have explanatory power to forecast stock returns. Since such macroeconomic variables have pervasive influence on all security prices, studies using the macroeconomic model generally focus on forecasting stock index rather than individual stock prices.

In particular, the interest rate is an important influencing macroeconomic variable. Kairys (1993) shows that changes in short-term interest rates (commercial paper) help explain excess stock returns in the US from the 1830s to the present. Moreover, Rapach *et al.* (2005) examine the predictability of stock returns using a variety of macroeconomic variables in terms of both in-sample and out-of-sample tests and find that interest rates are the most consistent and reliable predictors of stock returns across 12 industrialized countries. Additionally, Pesaran and Timmermann (1995) show that information about industrial production, inflation and monetary growth improves upon the predictability discovered by interest rate variables alone. Furthermore, Pesaran and Timmermann (1995) find that the predictive power of macroeconomic variables changed over time. Specifically, in the calm markets of the 1960s stock returns were less predictable than in the volatile markets of the 1970s.

Some research has been conducted outside the major stock markets. Bilson *et al.* (2001) extend the literature to an emerging markets context and find local variables, such as money supply, goods prices, real activity and exchange rates are significant in the association with equity returns. In particular, when a large set of variables is employed, the multifactor model is able to explain a large amount of return variation for most emerging markets. Gjerde and Sættem (1999) investigate the predictive power of macroeconomic variables, which are valid in major markets, in a small and open economy of Norway. They reported consistent finding with major markets that the real interest rate and oil price changes are accurate predictors of stock returns.

4.2.3 Fundamental Factor Models

While macroeconomic variables have power to predict stock index, microeconomic variables associated with firm-specific attributes are sources of predictability for individual stock returns. This type of studies reply on the empirical finding, for example, that company attributes such as dividend yield and book-to-market ratio explain a substantial proportion of common return (Connor, 1995). However, empirical studies have found conflicting evidence for predictability of fundamental variables. Some literature claims that the firm attributes can help investor to predict and gain excess returns (Lakonishok *et al.*, 1994). Contradictory opinion comes from researchers who believe that such measures are proxies of risk factors or that they might mostly reflect measurement problems and data mining (Fama and French, 1996; Campbell *et al.*, 1997)

Among the predetermined fundamental variables, dividend yield has received the most attention in the literature. Although Black and Scholes (1974) state that it is impossible to demonstrate the expected returns on high yield common stocks differ from the expected returns on low yield common stocks either before or after taxes, various studies have found that dividend yield has predictive power in both cross-section (Litzenberger and Ramaswamy, 1979; Kothari and Shanken, 1992) and time series (Fama and French, 1988; Fama and French, 1989). Fama and French (1988) find that dividend yield predicts monthly NYSE returns from 1941 to 1986. Kothari and Shanken (1992) conduct a cross-sectional experiment and show that nearly 90% of the portfolio return variation is explained by dividend and expected return variables. In the UK, Morgan and Thomas (1998) detect a significant positive relation between dividend yields and returns, controlling for firm size, seasonality and market risk. In Australia, Yao *et al.* (2005) find that the aggregate dividend-yield variable has influence on some of the industries.

The book-to-market and earning-to-price ratios share several common features with dividend yield, as they all measure stock prices relative to fundamentals. Lewellen (2004) suggests that they also share similar time-series properties; and their statistical properties have a big impact on tests of stock return predictability. Since the publication of Fama and French (1992, 1993), the book-to-market ratio has emerged as a strong contender as a determinant of expected returns. Chan *et al.* (1995) provide evidence that book-to-market significantly explains cross-sectional variation in average returns; although Kothari *et al.* (1995) show that the effect is weaker in large firms and argue that the magnitude and significance of the effect may be overstated due to data mining and selection biases in the data base. Kothari and Shanken (1997) find reliable evidence that both book-to-market and dividend yield track time-series variation in expected real stock returns over the period 1926 to 1991. Lamont (1998) finds no evidence that earning-to-price; by itself, predicts quarterly returns from 1947 to 1994. Together with book-to-market, earning-to-price can predict stock returns. Lewellen (2004) points out that book-to-market and earning-to-price forecast both equal- and value-weighted NYSE returns over the period 1963–1994. However, they predict only the equal-weighted index once data for 1995–2000 are included. Therefore, the evidence is less reliable compared to the predictive power of dividend yield.

Cremers (2002) summaries that attempts to characterize stock return predictability have not produced a consistent set of explanatory variables, giving rise to model uncertainty and data snooping fears. Accordingly, many studies have involved both macroeconomic variables and fundamental variables to improve accuracy of the stock return prediction. For example, the very influential study by Fama and French (1989) uses a model containing the P/E ratio, the slope of the term structure and the default spread to predict future excess returns on stock market. They found a clear pattern for expected returns on common stocks.

4.2.4 Technical Trading Rules

Another branch of the literature focuses on the predictability of stock returns using technical trading rules. Technical analysis attempts to detect a hidden trend in the movements of security prices by looking at the patterns of past prices. According to Gençay and Stengos (1998), technical trading analysis is based on two main premises. First, the behaviour pattern of stock markets does not change much over time, particularly the long-term trends. Thus, when the patterns incur in the future, they can be used for predictive purposes. Second, relevant investment information may be distributed fairly efficiently, but it is not distributed perfectly. Therefore, valuable information can be deduced by studying transaction activity.

Technical analysis has been popular among practitioners for several decades. However, among academics, most early empirical studies find that technical trade rules do not lead to profitable strategies (see Alexander, 1961, 1964; Fama and Blume, 1966 for example). In contrast, some recent studies provide evidence that some simple technical trading rules have considerable forecast power and are profitable. In a seminal paper, Brock *et al.* (1992) find that technical trading rules can predict future returns on the Dow Jones index over the 90-year period. Furthermore, their results from the bootstrap simulations indicate that none of the popular statistical models they examine are consistent with the trading rule profits.

Brock *et al.* (1992) propose the most popular moving average rule, which is the common component of many technical trading rules. A number of subsequent studies extend the work of Brock *et al.* (1992) to other markets, especially emerging markets. For instance, Bessembinder and Chan (1995) investigate the performance of trading rules using daily indices of six Asian equity markets, including Hong Kong, Japan, Korea, Malaysia, Thailand and Taiwan, over the period 1975 to 1991. The results indicated that technical trading rules were successful in Malaysia, Thailand and Taiwan stock markets; while these rules had less explanatory power in more developed markets, such as Hong Kong and Japan. Ratner and Leal (1999) analyse technical trading rules for 10 emerging equity markets from January 1982 through

April 1995 and find the predictive power can make substantial profits for Taiwan, Thailand and Mexico markets. Evidence for predictability of trading rules has been found in Chilean, South Asian stock markets by Parisi and Vasquez (2000) from 1987 to 1998. Ito (1999) applies the same trading rules as Brock *et al.* (1992) to the data on six Pacific-Basin countries and finds significant forecast power. In particular, Ito (1999) suggests that taking into account time-varying expected returns is important to evaluate the profitability of technical trading rules. In this way, profits on the trading rules can be explained by the risk-return relation implied by the asset pricing theory.

4.2.5 Non-Linear and Non-parameteric Models

Research on forecasting stock returns has indicated that stock returns are predictable using linear models (see Campbell and Shiller, 1988; Bollerslev and Hodrick, 1992). Furthermore, recent results, including Hiemstra (1996), Haefke and Helmenstein (1996), and Kanas and Yannopoulos (2001), suggest that nonlinear models tend to outperform linear models for stock returns forecasting. Once again, empirical evidence is mixed, which can be illustrated by the study of Bradley and Jansen (2004). Bradley and Jansen (2004) conduct an out-of-sample forecasting exercise and compare the performance of the nonlinear smooth transition autoregressive model with that of a linear model in terms of forecasting stock returns and industrial production. They find that the linear model generally does as well or better than any of our nonlinear models for stock return forecasts; but nonlinear models outperformed the linear competitor for industrial production forecasts.

A variety of non-linear extensions of the present value (PV) model have been proposed, following the failure of linear PV model to explain the behaviour of US stock prices. The fads model of Summers (1986) and the intrinsic bubbles model of Froot and Obstfeld (1991) are famous representatives that introduce nonlinearity in the relation between stock prices and fundamental variables, such as dividends and trading volume. According to Summers (1986), if there are fads in the stock market, one may observe long temporary price swings, which can be modelled as a slowly decaying stationary component in prices. The decay over time in the transitory component will entail mean reversion in stock prices. Van Norden and Schaller (1994) show how this fads model entails regime switching and thus, nonlinearity in the relation between stock price and dividend. The intrinsic bubbles model introduced by Froot and Obstfeld (1991) states an intrinsic bubble driven by a given level of dividends will remain constant over time. Stable and highly persistent dividends lead to stable and highly persistent departures from the linear PV model, thereby entailing nonlinearity in the stock price–dividend relation. To forecast stock returns, both the standard regime switching and Markov regime switching models can be used to assume specific dynamic processes for the underlying fundamental variables (Driffill and Sola, 1998; Kanas, 2003).

In particular, numerous studies have documented the successes and failures of artificial neural networks (ANNs) in forecasting time series and cross-sectional financial data. Neural networks are a set of non-parametric techniques useful for analysing non-linear data sets such as those that characterize stock price information. From 1990s, some first published studies generally reported that ANNs outperform traditional models, including ordinary least squares regression and logistic regression (see Kryzanowski *et al.*, 1993; Kuan and Lim, 1994; Haefke and Helmenstein, 1996). However, according to Episcopos and Davies (1995), Donaldson and Kamstra (1996) and many others, there is no guarantee that ANNs will dominate the linear model in terms of out-of-sample forecasts. Moreover, forecasters may want to build ANN models only if there is a strong a priori belief that additional complexity is warranted, since construction and implementation of the models is considerably more difficult and time consuming than using traditional techniques (Balkin and Ord, 2000; Darbellay and Slama, 2000). Nevertheless, several researchers dedicated to forecasting stock returns using the complex non-linear relationship of ANNs. Generally, their efforts have been rewarded as they found the superiority of the neural network models. Olson and Mossman (2003) indicate such translates superiority translate into greater profitability in Canadian stock market. **Similarly, Gençay** (1998) and Harvey *et al.* (2000) find strong evidence of predictability for stock market returns in different stock markets. Safer (2002) reveals the application of neural networks can help to maximise abnormal stock returns in several ways. Kanas (2001) find that the ANN forecasts are preferable to linear forecasts for the Dow Jones and FT data.

In summary, broad evidence of predictability for stock returns has been reported. Financial researchers have utilised various economic and financial information to forecast stock prices and returns. In addition to four categories listed above, many studies use other explanatory variables or econometric models for forecasting stock returns. For example, Jung and Boyd (1996) find the error correction model has better out-of-sample performance to forecast UK stock prices, compared to the vector autoregressive model and the Kalman filter mode. As Granger (1992) states, the literature on stock price forecasts is mainly concerned with the accuracy of such forecasts, with conflicting results predominantly stemming from data revisions and biases in aggregating data.

4.3 Forecasting Stock Market Volatility

Volatility forecasting permeates the world of economics and finance, since variations in market returns and other economy-wide risk factors are a main feature of asset and portfolio management and play a key role in derivatives pricing models. Since Engle (1982), financial econometrics has become a mature discipline over the last two decades, and one of its major research objects is the modelling and forecasting of volatility. Vast empirical and theoretical investigation has been conducted on stock and currency markets. We will focus on the literature modelling and forecasting stock market volatility. One significant feature of this line of prediction is that volatility, even measured by the standard deviation or variance, is unobservable. Therefore, as Engle (1993) indicated, volatility forecasting is a little like predicting whether it will rain; you can be correct in predicting the probability of rain, but still have no rain. Literature has observed that various models are appropriate to capture the stylised features of volatility.

4.3.1 History Volatility

According to Brooks (2002, p.441), the simplest model for volatility is the historical estimate, which simply uses the variance (or standard deviation) of returns over some historical period as the volatility forecast for future period. Although it is a relatively simple and naive model, Figlewski (1997) finds that forecast errors are generally lower if the historical variance is calculated over a much longer period. The study reached the conclusion that simple averages of historical volatility are preferable for predicting future volatility to more complex models. In fact, the historical average variance was traditionally used as the volatility input to options pricing models.

However, there is a growing body of research have found evidence against the historical volatility approach, especially on its out-of-sample forecasting performance (see Akgiray, 1989; Chu and Freund, 1996 for example). Walsh and Tsou (1998) argue that such an equally weighted model is inefficient, if recent observations are more important than long-distant observations. Their research found such '*naive approach*' is poor at forecasting volatility on Australian value-weighted indices. Furthermore, Alford and Boatsman (1995) studied historical volatility in predicting long-term stock return volatility. They found that lower frequency sampling should be used, when using historical volatility. Historical volatility method seems to break down with finer time partitioning, when dependency on lagged values of volatility appears to become greater (Walsh and Tsou, 1998).

Despite its theoretical and empirical inefficiency, historical volatility is still useful as a benchmark for comparing the forecasting ability of more complex models. Especially, when moving average approaches including simple or weighted moving averages are incorporated, the performance of historical average volatility measure can be improved (Dimson and Marsh, 1990). Thus, moving average models, which are essentially extensions of historical volatility model, have received some attention for forecasting stock market volatility (Jorion, 1995; Taylor, 1999).

4.3.2 Exponentially Weighted Moving Average

Among extensions of the historical volatility model, the exponentially weighted moving average (EWMA) has received the most attentions since its first use by Akgiray (1989) to forecast the volatility of stocks on the NYSE. By far, the most well known user of the EWMA is Riskmetrics, which utilizes it for its value-at-risk modeling (Riskmetrics, 1996).

The consensus of previous research is that the volatility model that has weighted recent observations more heavily than older observations, such as the EWMA, is more successful. This is clear from a number of studies, such as Tse (1991), Tse and Tung (1992), Corhay and Rad (1994). Some comparative studies supported the EWMA is the best model for out-of-sample volatility forecasting. Dimson and Marsh (1990) find that EWMA outperform historical volatility and GARCH models in UK markets. Tse (1991), Tse and Tung (1992) confirm this finding using data on Japan and Singapore stock markets. In Australia, Walsh and Tsou (1998) suggest that the EWMA appears to be the best volatility forecasting technique, closely followed by the appropriate GARCH specification. Furthermore, Taylor (2004) proposes an adaptive exponential smoothing method which allows smoothing parameters to change over time, and find the new model produces encouraging results when compared to fixed parameter exponential smoothing and a variety of GARCH models. Therefore, it can be argued that the moving average model, especially the EWMA is among the models for forecasting volatility, although its superiority is not agreed by all empirical studies.

4.3.3 Implied Volatility Models

In the framework of an option pricing model, such as the Black and Scholes (1973) model, the expected volatility of the asset over the life of the option is the volatility embedded in the price of the option. Therefore, given the price of a traded option, the option pricing formula can be inverted to compute the expected volatility over the life of the option. This implied volatility is the market's forecast of the volatility of underlying asset returns over the lifetime of the option, which is also known as market-based volatility.

If the option market is efficient and the valuation model is correctly specified, all relevant conditioning information is collapsed into the option price. The implied volatility, then, should represent a superior volatility forecast (se Jorion, 1995; Poon and Granger, 2003). A broad survey of recent papers by Poon and Granger (2003) indicates that, broadly speaking, forecasts based on implied volatility beat forecasts based on historical returns. However there exists only limited evidence of support, despite the strength of this implication.

Early studies including Latané and Rendleman (1976), Chiras and Manaster (1978), and Beckers (1981) found that the implied volatility indeed contained relevant information regarding future volatility. However, these studies were criticised for examining fairly small datasets and focusing on the cross-sectional relations within a select group of stocks (Fleming, 1998). More recent evidence, based on the analysis of overlapping time-series observations, is less supportive. Using S&P 100 index options, Day and Lewis (1992) find that the implied volatility contains useful information in forecasting volatility, but also that time-series models contain information incremental to the implied volatility. Lamoureux and Lastrapes (1993) find similar evidence using individual equity options. Canina and Figlewski (1993) conclude that the S&P 100 implied volatility is such a poor forecast that it is dominated by the historical volatility rate. Although the implied volatility model is unsuccessful to forecast stock market volatility, it performs well in currency markets. Jorion (1995) finds more favorable evidence in currency markets where the implied volatility outperforms both moving average and GARCH forecasts. Jorion attributed the poor performance in the stock market side to the measurement error which is due to bid-ask spread and non-continuous prices of stock index. In addition, he argued traditional regression analysis is biased and perhaps spurious in small samples. Based on the arguments of Jorion (1995), Fleming (1998) conducts a new examination of the forecast quality of the S&P 100 implied volatility and indicates that the implied volatility is an upward biased forecast, but also it contains relevant information regarding future volatility. In particular, Fleming (1998) suggests a linear model which corrects for the implied volatility's bias can provide a useful market-based estimator of conditional volatility. With many theoretical issues addressed, latest empirical studies have found more supportive evidence. For example, Ederington and Guan (2002) examine the relevance of implied volatility forecasts using S& P500 futures options data and conclude that '*implied volatility has strong predictive power and generally subsumes the information in historical volatility*'.

4.3.4 GARCH Models

The Autoregressive Conditional Heteroscedasticity (ARCH) models pioneered by Engle (1982) and generalized (GARCH) by Bollerslev (1986) have an enormous influence on both theoretical and applied econometrics, In respect of financial econometrics, Bollerslev (2001, p.41) states that the development of ARCH has been one of the two "*most important developments in the field over the past two decades*". As GARCH has become a key work in most research engines, it is the most popular statistical modelling approach to volatility forecasting.

Using US stock data, Akgiray (1989), Pagan and Schwert (1990) and Brooks (1998) find that GARCH models outperform most competitors. According to Ederington and Guan (2005), GARCH(1,1) generally yields better forecasts than the historical standard deviation and EWMA models, though there is no clear favorite between GARCH and EGARCH. Poon and Taylor (1992) finds similar evidence in UK stock markets. Brailsford and Faff (1996) find that the GARCH models are slightly superior to most simple models for forecasting Australian monthly stock index volatility. Some literature focuses on comparison between GARCH models and relatively sophisticated non-linear and non-parametric models, with the growth in popularity of these more complex approaches. For example, Pagan and Schwert (1990) compare GARCH, EGARCH, Markov switching regime and three non-parametric models for forecasting US stock return volatility. While all non-GARCH models produce very poor predictions; the EGARCH followed by the GARCH models perform moderately.

However, despite the empirical success of the GARCH model, some studies report that standard volatility models provide poor forecasts and explain little of the variability of *ex post* squared returns (see Cumby *et al.*, 1993; Figlewski, 1997; Jorion, 1995). A series of recent papers (see Andersen and Bollerslev, 1998; Andersen *et al.*, 1999) has revived the usefulness of GARCH models in providing accurate volatility forecasts has argued that the failure of the GARCH models to provide good forecasts is not a failure of the ARCH model per se, but a failure to specify correctly the 'true volatility' measure against which forecasting performance is measured. As discussed by Andersen and Bollerslev (1998), squared returns are noisy estimators of the actual variance dynamics and will thus limit the inference available regarding volatility forecast accuracy. McMillan and Speight (2004) propose an alternative measure for 'true volatility' and find the GARCH model outperforms smoothing and moving average techniques.

Despite the debate and inconsistence evidence, theoretical characteristics and attractions have led more and more researchers to employ GARCH models for forecasting stock market volatility, rather than the simple model. As Brooks (2002, p. 493) asserts, it appears that conditional heteroscedasticity models are among the best that are currently available. Meanwhile, a number of studies have compared different specifications of the GARCH model; these will covered in section 4.5.

4.4 Modelling Betas

4.4.1 Development of Theoretical and Empirical Analysis

Since the introduction of the CAPM by Sharpe (1964), the usefulness and validity of the beta coefficient has been a hotspot of academic research. In the early life of the CAPM, most empirical tests supported the beta and its results (see Black *et al.*, 1972 and Fama and MacBeth, 1973). However, the utility of beta as the only measure of systematic risk for a capital asset has been challenged by a number of succeeding studies. During the 1980s and 1990s, several deviations from the CAPM were discovered; and researchers began to look at other characteristics of stocks besides betas, such as firm size (Banz, 1981). The most challenging and heavily cited argument comes from Fama and French (1992), which indicates "*the relation between market 'beta' and average return is flat, even when 'beta' is the only explanatory variable*". Many explanations, both theoretical and empirical, have been proposed by researchers to answer these anomalies, which include data snooping (Lo and MacKinlay, 1990; White, 2000), sample effect (Black, 1993), limitations of the methodology (Clare *et al.*, 1998), inappropriate proxy of the market portfolio (Kandel, 1984; Kandel and Stambaugh, 1987). Although all studies still cannot produce complete conviction for the usefulness of betas and the academic debate continues; as Fama (1991) stated, "*market professionals (and academics) still think about risk in terms of market β*".

The traditional CAPM assumes the beta of a stock is constant over time. However, from the later 1980s, researchers have reached a consensus on time variation of betas. Empirical research has reported considerable evidence of beta instability. In US markets, Fabozzi and Francis (1978), Bos and Newbold (1984), Collins *et al.* (1987) and Kim (1993) provide evidence that betas are not only time-varying but can also be better described by some form of stochastic model. Similar evidence extends to international capital markets, as Bos and Fetherston (1992), Bos *et al.* (1995), Cheng (1997) and Faff *et al.* (1992) detect that beta values are dependent on time in Korean, Finnish, Hong Kong and Australian stock markets. Based on the broad evidence, Choudhry (2002) investigates the stochastic structure of time-varying betas in the UK market and find they are stationary and mean-reverting at a slow rate. The importance of studying time variability of betas is demonstrated by Berglund and Knif (1999), in which they propose an adjustment of the cross-sectional regressions to give larger weights to more reliable beta forecasts. Applying this approach to data, the study of Koutmos and Knif (2002) produces a significant positive relationship between returns and predictive beta, while the traditional Fama and MacBeth (1973) approach finds no relationship at all. In summary, empirical evidence suggests that the conditional beta has a satisfactorily explanatory power in the conditional version of CAPM; and therefore the prediction of the time-varying beta seems worthwhile.

4.4.2 Determinants of Time-Varying Betas

There are many studies providing theoretical explanations for the time-varying feature of beta. A fundamental statement was made by Bollerslev *et al.* (1988). Economic agents have conditional expectations rather than homogeneous constant expectations of the first and second moments of asset returns; because agents update their estimates of the mean and covariance of returns each period using newly revealed information in last period's asset returns. Conditional heteroscedasticity of both first and second moments will cause betas to be time dependent. According to Klemkosky and Martin (1975), betas will be time-varying if excess returns are conditionally heteroscedastic. Additionally, there is considerable evidence that returns on both individual stocks and market indices show time-varying second moments (Bollerslev *et al.*, 1992). Since beta is the ratio of covariance between market and stock returns to the variance of market returns, time variation in the second moments of returns can cause time variation in betas.

As mentioned before, for a well-diversified portfolio, sources of systematic risk are the factors which affect the entire market and cannot be avoided through diversification, such as interest rates, recession and wars. However, for individual stocks, the variation of systematic risk may arise through the influence of various macroeconomic and microeconomic factors (Bos and Newbold, 1984), as these factors may affect the relative risk of a firm's cash flow.

For individual shares, some macroeconomic factors are indicated to have influence on beta values, such as changes in risk-free returns and business cyclicality. Galai and Masulis (1976) interpret equity as a call option on the assets of the firm. They show that the beta of a stock is related to the beta of the firm's assets through a factor that depends on the level of risk-free interest rate. Cyclicality has been argued as an important determinant of the time-varying beta. The theoretical relationship between cyclicality and the beta has been well established by many authors, such as Conine (1983).

To identify beta determinants among a variety of microeconomic factors, many researchers have empirically investigated the relationship between beta and financial variables largely derived from accounting data. Many of those studies used multiple regressions in which beta was the dependent variable and financial variables were independent variables. As the first study in this category, Beaver *et al.* (1970) find sources of systematic risk in terms of beta may include financial leverage, dividend payout and earning yield instability measures. More empirical studies (Hamada, 1972; Rubinstein, 1973; Boness *et al.*, 1974) have been conducted on the relationship between financial leverage and beta. They all found that financial structure had an important influence on beta but disagreed over whether beta varied directly with the level of financial leverage. Chu (1986) proposes a theoretical explanation that as the financial leverage of a firm increases, its shareholders can be subjected to increased systematic risk. Moreover, Gahlon and Gentry (1982) analytically demonstrate that beta value is a function of the degree of operating and financial leverage, the coefficient of variation of the revenues, and the correlation coefficient between the firm's return and the aggregate market return. The joint impact of the degrees of operating and financial leverage is concluded to explain 38 to 48 percent of the cross-sectional variation in the beta by Mandelker and Rhee (1984). Chung (1989) supports the impact of the degrees of operating and financial leverage and asserts they are the major determinants of systematic risk of common stocks.

However, disappointing results are also reported by other researchers, who examined multivariate links between beta and a number of corporate risk factors (see Logue and Merville, 1972; Rosenberg and McKibben, 1973; Breen and Lerner, 1973 for example). Most studies find that many variables are not significant, and if they are, they are not consistently significant over time or differ greatly across industries. Therefore, as summarised by Thompson (1979), empirical evidence indicates that the systematic risk of a stock is related to corporate risk factors; however it is far from clear how these risk factors should be defined. Accordingly, more recent studies modelling betas has focused on capturing the dynamic process of time-varying betas by more complex time-series models, rather than analysing the beta determinants in a cross-sectional context. Forecasting involved in this study is also applied in a time series context.

4.4.3 Models to Estimate the Beta Value

According to Klemkosky and Martin (1975), betas will be time-varying if excess returns are conditionally heteroscedastic. The GARCH family provides powerful econometric techniques to model time-varying bets, since they are particularly able to capture time variation in conditional second moments that are conditional covariance between company and market returns and conditional variance of market returns (Engle and Kroner, 1995).

A variety of GARCH models have been employed by different researchers to estimate time-varying betas for different stock markets, such as Bollerslev *et al.* (1988), Engle and Rodriguez (1989), Ng (1991), Bodurtha and Mark (1991), Koutmos *et al.* (1994), Giannopoulos (1995), Braun *et al.* (1995), Gonzalez-Rivera (1996), Brooks *et al.* (2000) and Yu (2002). For example Bollerslev *et al.* (1988) use bivariate GARCH models to capture the dynamic beta. These models use time varying second moments of the market and index returns to obtain time varying betas. Braun *et al.* (1995) fit a bivariate E-GARCH model to monthly U.S. stock returns over the period and July 1926 to December 1990 to examine conditional covariances of stock returns, and find evidence of leverage effects. Gonzales-Rivera (1996) tests the conditional CAPM against the conditional residual risk model using US computer industry stock price data. Volatility in these models was captured using a bivariate GARCH-in-mean (GARCH-M) model, which was shown to provide superior performance over a univariate GARCH specification.

An alternative approach is the Kalman filter method, developed from the engineering literature of the 1960s. The Kalman filter recursively estimates the beta series from an initial set of priors, generating a series of conditional coefficients in the market model. The approach gives minimum mean square estimates of the state variable if the errors are normally distributed (Harvey, 1989b). The traditional Kalman filter assumes that the market model residual is Gaussian and homoscedastic. This is inconsistent with the considerable evidence which has accumulated about heteroscedasticity of financial returns (Bollerslev *et al.* 1988; Ng, 1991; Bollerslev *et al.* 1992). Harvey *et al.* (1992) derive the modified Kalman filter, which is quasi optimal when errors show conditional heteroscedasticity. Compared to other methods, the Kalman filter approach is less utilised by researchers. Black *et al.*, (1992) use a Kalman filter estimation of random walk betas for a sample UK unit trust. Similarly, Well (1994) employ the approach to obtain estimates of beta for a small sample of Swedish Stocks. Faff *et al.* (2000) consider three alternative potential dynamic processes of time dependence to calculate the time-varying beta and find the Kalman filter algorithm dominates complex GARCH and Schwert and Seguin forecasts.

While the Kalman filter approach may work well, Faff and Brooks (1998) argues that it lacks appeal due to the abstract nature of underlying models. Another approach, which associates time-varying beta directly with observable economic variables, is intuitively appealing. A few studies use the variable beta techniques (see Abell and Krueger, 1989; Shanken, 1990). Most important variables included by Abell and Krueger (1989) are interest rates, budget deficits, trade deficits, inflation rates and oil prices. Shanken (1990) has successfully model the beta by three state variables, including the monthly Treasure bill rate, a measure of Treasure bill volatility and a January dummy variable. However, according to Faff and Brooks (1998), the seeming lack of pervasiveness of any of these variables is disappointing.

The third approach is the time-varying beta market model suggested by Schwert and Seguin (1990). Schwert and Seguin (1990) augment the simple GARCH methodology by constructing an extended market model that incorporates a single factor model of security return heteroscedasticity. Such modified form provides estimates of time-varying market risk. However, this approach is not broadly adopted by researcher. Only a few studies utilise the Schwert and Seguin approach (see Koutmos *et al.*, 1994) and Episcopos, 1996). Koutmos *et al.* (1994) apply the Schwert and Seguin (1990) method to a world portfolio of several country indexes and find that markets with high volatility persistence exhibit higher systematic risk during periods of high world market volatility. Using both the Schwert and Seguin (1990) approach and E-GARCH model, Episcopos (1996) establishes that nonlinearities in the variance exist for most of the TSE300 sub-index portfolios, and finds that first order autocorrelations are negatively linked to conditional variance of returns and sub-index betas are time varying. Reyes (1999) extends the Schwert and Seguin approach by explicitly modelling conditional heteroscedasticity in the market model and find beta estimates of UK stocks are markedly differently from those when conditional heteroscedasticity is ignored.

Although various approaches have been used to model time-varying betas, they are mainly used for estimation rather than prediction. Thus, the performance of each model can only be examined by in-sample efficiency of the conditional beta. This research seeks to extend the prior research by investigating the out-of-sample beta forecasting ability of alternative models. More importantly, the performance of the candidate models are compared in terms of various error measurements.

4.5 Forecasting with GARCH Models

As GARCH becomes a popular model to forecast the conditional second moments of time series, its forecasting performance has been the focus of a number of studies. The existing literature contains conflicting empirical evidence regarding the predictive ability of GARCH and a variety of other models. Before we review and interpret the empirical evidence, a problem with respect to volatility forecast measures should be mentioned, since "*there is still a debate on how best to evaluate forecasts of the conditional variance*" (Harris and Sollis 2003, p. 245). When evaluating the accuracy of a model for forecasting the conditional mean, it is traditional to estimate the model using a subsample data and then compare the estimates with observed future data using a standard measure of forecast accuracy[20]. Whereas, when forecasting the conditional variance of a time series, such as stock returns, the observed values of the conditional variance is not available for comparison, even if the GARCH model is estimated using the subsample data. Traditionally, squared values of the data are used as a proxy for actual conditional variance values. Hence, the forecasts can be compared with the proxy and then be evaluated in the same way as the forecasts of the series itself. However, this approach is seriously doubted by Andersen and Bollerslev (1998) that the squared values of the series are sometimes a very poor proxy of the conditional variance. Additionally, they find that the misuse of proxy is the reason to cause several previous studies to conclude GARCH models with poor productive performance (see Jorion, 1995 and Figlewski, 1997 For example). Moreover, their analysis shows that the GARCH model is capable of producing very accurate forecasts of conditional variance of a time series. Similarly, Brailsford and Faff (1996, p. 419) claim "*volatility forecasting is a notoriously difficult task*", because the model selection is sensitive to the error statistic used to assess the accuracy of the forecasts. This idea is confirmed by the growing body of research using GARCH models to forecast conditional variance of a variety of financial data, such as exchange rates and stock returns.

The existing literature on out-of-sample forecasting ability of various models has reached inconsistent conclusions. Evidence can be found supporting the superiority of GARCH models; while there is also evidence supporting the superiority of more simple alternatives, such as the random walk model, the historical mean model, the moving average model and the exponentially weighted moving average (EWMA) model. Akgiray (1989) finds the GARCH(1,1) model specification exhibits superior forecasting ability to traditional ARCH, exponentially weighted moving average and historical mean models, using monthly US stock index returns. The apparent superiority of GARCH is also observed in forecasting exchange rate volatility by West and Cho (1995) for one week horizon, although for a longer horizon none of the models exhibits forecast efficiency. On the contrary, Dimson and Marsh (1990) in an examination of the UK equity market conclude that the simple models provide more accurate forecasts than GARCH models. However, theoretical characteristics and

20 Generally, measure of forecast accuracy attempts to find the model with smallest out of sample one-step-ahead mean squared error.

attractions have led more and more researchers to employ GARCH models in both univariate and multivariate cases, rather than the relative simple model.

More recently, empirical studies have been more emphasised on comparison between GARCH models with relatively sophisticated non-linear and non-parametric models, with the growth in popularity of these more complex approaches, such as non-parametric locally weighted regression and nearest neighbour forecasting. Pagan and Schwert (1990) compare GARCH, EGARCH, Markov switching regime and three non-parametric models for forecasting US stock return volatility. While all non-GARCH models produce very poor predictions; the EGARCH followed by the GARCH models perform moderately. As a representative applied to exchange rate data, Meade (2002) examines forecasting accuracy of linear AR-GARCH model versus four non-linear methods using five data frequencies and finds that the linear model is not outperformed by the non-linear models. Despite the debate and inconsistence evidence, as Brooks (2002, p. 493) says, "*it appears that conditional heteroscedasticity models are among the best that are currently available.*"

As mentioned above, a range of specifications of GARCH extend the original model in order to capture particular features of financial data. Thus, each extension has its predominance in particular aspect; whereas researchers cannot distinguish between various models and find which one provides the most accurate forecasts. Thus, in principle no particular GARCH specification will be preferred by all users. Investigations have been conducted to compare forecasting ability of the standard GARCH, GARCH-GJR and EGARCH models, while few compare that of the BEKK and GARCH-X extensions. Therefore, the following comparison will be focused on the GARCH and its two asymmetric specifications.

Franses and Van Dijk (1996) investigate the performance of the standard GARCH model and non-linear QGARCH and GARCH-GJR models for forecasting the weekly volatility of various European stock market indices[21]. The non-linear GARCH models are supposed to improve upon the standard GARCH model since they can cope with negative skewness. However, their results indicate that non-linear GARCH models

[21] QGARCH stands for the Quadratic GARCH model proposed by Engle and Ng (1993), which will not be used to forecast the time-varying beta in this research.

cannot beat the original model. In particular, the GJR model is not recommended for forecasting. In contrast to their result, Brailsford and Faff (1996) find the evidence favours the GARCH-GJR model for predicting monthly Australian stock volatility, compared with the standard GARCH model. Similar evidence is found for another asymmetric model EGARCH by Pagan and Schwert (1990) that EGARCH is slightly superior to GARCH models. However, Day and Lewis (1992) find limited evidence that, in certain instances, GARCH models provide better forecasts than EGARCH models by out of sample forecast comparison.

Engle and Ng (1993) introduce the news impact curve as a major analytical tool for measuring how new information is incorporated in volatility estimates by alternative GARCH models. They argue that GARCH-GJR is the best parametric model to incorporate the impact of new information in volatility estimates, because the conditional variance implied by the EGARCH model may be too high due to the exponential functional form. Empirical evidence is provided by Donaldson and Kamstra (1996) that GARCH-GJR seems more able to fit the asymmetric heteroscedasticity in data than either GARCH or EGARCH models. Friedmann and Sanddorf-Köhle (2002) re-examine the EGARCH and GARCH-GJR model using the concept of a redefined conditional news impact (CNI) curve. Unlike Engle and Ng, they argue that while the news impact for the EGARCH model does not depend on the volatility environment; both the standard and GARCH-GJR models display an acceleration of the news impact in periods of high volatility, thereby creating a potential for overshooting volatility predictions. Additionally, Friedmann and Sanddorf-Köhle (2002) compare the empirical performance of the EGARCH and the GARCH-GJR model fitted to daily Chinese stock returns. The empirical results confirm the theoretical comparison that EGARCH model is not inferior to the GJR model and the two approaches perform quite similarly.

One explanation to mixed empirical evidence is that out-of-sample performance of exchange rate volatility model depends on the criteria used to measure it (Lee, 1991). Yu (2002) evaluates the performance of nine models for predicting stock price volatility using New Zealand data and finds GARCH(3,2) is the best model among ARCH family. However, the conclusion is sensitive to the choice of evaluation measures. Additionally, Brailsford and Faff (1996) compare the predictive performance of several statistical methods with GARCH and TGARCH models. Using several loss functions, they are unable to identify a clearly superior model and suggest that the `best' forecasting model depends upon the subsequent application, as the ranking of models is sensitive to the choice of loss function.

4.6 Conclusion

There exists a rich body of literature applying econometric techniques to forecast the financial time series, as the ultimate aim of econometrics studies is using econometric models for prediction or policy purpose. However, when we narrow the subject to forecasting the time-varying beta, only a few studies have been found. As a consequence, the chapter broadly covers previous studies relevant to forecasting conditional betas, such as studies on forecasting stock returns and forecasting stock market volatility.

The first category of literature is regarding stock return forecasts, which have been the hotspot since the commencement of stock markets and received more attentions during the past two decades due to development of theoretical analysis and increase of computer power. A brief historical retrospect on theoretical hypotheses concerning stock market price movements helps to justify the motivation to forecast stock price behaviour. Most studies attempting to investigate the sources of market inefficiency and characterize stock return predictability are discussed in terms of the approach used, namely macroeconomic factor model, fundamental factor model, trading rules, nonlinear model and neural network. Evidence of stock return predictability has been found, but no certain approach dominates in terms of forecast accuracy.

Literature discussed in the following section is concerning forecasts of stock market volatility. As previously mentioned, one significant feature of this line of prediction is that volatility, even measured by the standard deviation or variance, is unobservable. Therefore, as stated by Engle (1993), volatility forecasting is a little like predicting whether it will rain; you can be correct in predicting the probability of rain, but still have no rain. However, various models are claimed appropriate to capture the stylised features of volatility, such as history volatility, EWMA, implied volatility and GARCH models. Literature has been summarised in terms of approach used, to highlight the strongpoint and the potential problem of each method. In particular, literature provides empirical evidence of the success of the GARCH model in volatility forecasts.

Section 4.4 focuses on the development of the conditional CAPM framework to justify the validity of time-varying beta. This issue has been discussed in chapter 2 in more details. Due to the shortage of existing literature, we review the studies dedicating to beta estimation rather than forecast. However, the application of these tools is fundamentally the same for either estimation or forecast purpose.

Section 4.5 sums up the literature on forecasting with GARCH models to examine the forecast performance of different GARCH representations. While few studies investigate the forecast ability of the BEKK and GARCH-X extensions, we focus on the standard GARCH and its two asymmetric specifications. Also, such literature is not restricted on the stock market because of the limited amount of existing studies. Overall, previous research has reached no consensus. There is yet no answer to 'who is the best among the GARCH family'.

Chapter 5
Data

5.1 Introduction

This chapter presents the data description and some related statistics. Two sets of daily data are applied in the empirical tests, from UK and US markets respectively. In fact, another two sets of UK and US weekly data are also involved in this research. Data with frequency lower than weekly are not considered, because GARCH effects at lower frequencies are not so apparent (Alexander, 2001). Additionally, results from intraday data are inappropriate to be used for practical reference. As a result, daily and weekly data are applied in this research. Empirical results indicate that both GARCH models and Kalman filter method behave better with daily data, as the models generally converge more easily and produce more robust coefficients than weekly data. Consequently the thesis only reports the empirical results of daily data due to the words limit.

In both UK and US markets, twenty companies are selected on a diversification basis for more reliable results. Diversification is considered in terms of three factors: types of service and product provided, the size of company and the location of origin. The data range from January 1989 to December 2003, a reasonable length of period backtracked from the initial stage of the research with enough observations. The FTSE All-Share and S&P 500 index are employed as the proxy for the market portfolio in UK and US data respectively. The returns on three-month Treasury bill represent of the returns on riskless assets in both UK and US markets. All data are obtained from DATASTREAM.

To provide data overview, the chapter mainly reports three categories of information in the following three sections. In section 5.2, a brief profile is presented for each company. Section 5.3 summaries statistical descriptions of the stock price and market index return series. Section 5.4 reports the cointegration test results between the log of firm price and the log of market index.

5.2 Company Profile

5.2.1 UK Company Profile

Table 5.1 summaries some details on the twenty UK companies. The firms come from a variety of industrial sectors, providing different products or services. The diversity of products and services provided includes airline services (British Airways), tobacco production (British American Tobacco), financial services (Legal and General, Barclays), telecommunications (BT Group, Cable and Wireless), oil production (Edinburgh Oil and Gas), alcohol production (Scottish and Newcastle) and many others. The firms are also selected in terms of different sizes based on market capitalisation. The size of the firms varies from 3.08 million pounds (Caldwell Investments) to 76153 million pounds (Glaxo Smith Kline). To provide more details on the firms, a brief profile is presented for each company as follows[22].

(1) British Airways

British Airways is the largest international scheduled airline in the United Kingdom, and the second largest airline in the world. Based at two airports in London (Heathrow and Gatwick), it manages airplanes flying to about one hundred countries. In addition, BA has holdings in other airlines, such as the Australian and the Spanish Iberia. Also, as one of the longest established airlines in the world, it has always been regarded as an industry leader.

(2) Tesco

Tesco is the leading supermarket in Britain. Since the company first used the trading name of Tesco in the mid 1920s, the group has expanded different markets and sectors. The principal activity of the group is food retailing, with over 2,000 stores worldwide including small grocery stores (Tesco Metro), big supermarkets outside cities (Tesco Extra) and twenty-four-hour stores. The long term strategy for growth of Tesco is based on four key parts: growth in the Core UK business, to expand by

[22] The profiles of the firms are mainly obtained from the website of Corporate Information.

growing internationally, to be as strong in non-food as in food and to follow customers into new retailing services.

(3) British American Tobacco

British American Tobacco was formed in 1902, as a joint venture between the UK's Imperial Tobacco Company and the American Tobacco Company. As the second largest harvester, producer and distributor of tobacco in the world, British American Tobacco runs 15% of the world tobacco market, with more than 300 brands sold in 180 markets, among which the company has leadership in more than 50 markets

(4)BT Group

BT is an integrated group of businesses that provide voice and data services in Europe, the Americas and Asia Pacific. As one of leading providers of communications solutions in the world, its principal activities include networked IT services, local, national and international telecommunications services, and higher-value broadband and internet products and services. In the UK, BT Group serves more than 20 million business and residential customers with more than 30 million exchange lines, as well as providing network services to other licensed operators.

(5) Legal and General

Established in 1836, Legal and General has made its mark on the insurance sector; as it becomes the most expert provider of insurance, investment and savings products in the UK in the UK. It offers life insurance and general insurance for health, property and other everyday areas, as well as other financial services, including fund management and individual savings accounts. The Group's primary focus is on the UK. However it also has operations in the USA, the Netherlands, France and Germany.

(6) Glaxo Smith Kline

Headquartered in the UK and with operations based in the US, GlaxoSmithKline ranks among the top five world-class companies in the pharmaceuticals sector. Essentially the group produces medicines that treat six major disease areas – asthma, virus control, infections, mental health, diabetes and digestive conditions. In addition, GlaxoSmithKline is a leader in the important area of vaccines and are developing new treatments for cancer.

(7) Edinburgh Oil and Gas

Edinburgh Oil and Gas is an independent oil and gas exploration and production company with significant operations and property interests in the United Kingdom. The company aims to increase its oil and gas reserves primarily through exploration and development drilling. Apart from a few oil fields and gas fields, Edinburgh Oil and Gas also has coal mine operation in Hem Heath.

(8) Boots Group

Boots is the leading health and beauty retailer in the UK. It operates in four segments: Boots The Chemists, Boots Opticians, Boots Healthcare International and Boots Retail International. Boots group has approximately 1,500 stores in the UK and Irish Republic serving around eight million customers every week.

(9) Barclays

Barclays began its operations as a bank in the 17th century in the financial centre of London. It has since then become a global financial services provider engaged in retail and commercial banking, credit cards, investment banking, wealth management and investment management services. As a financial services group domiciled in the UK, Barclays is also a strong entity in 60 international countries. It is now one of the major players in international financial services.

(10) Scottish and Newcastle

Scottish and Newcastle is an international brand-driven, beer-led drinks business with positions in 15 countries in Europe and Asia and exports to more than 60 countries around the world. In particular, Scottish and Newcastle has market leadership in three of the six largest beer markets in Europe: the UK, France and Russia. Its main brands include Foster's, Kronenbourg 1664, John Smith's, Strongbow and Baltika. The company also offers non-beer beverages, including soft drinks, water and other alcoholic drinks.

(11) Singet Group

Singet is one of the largest speciality retail jewellers in the world with operations in both the US and UK. Signet has an approximate 3.9% share of total jewellery market in the US, where the company trades as Kay Jewelers, Jared and under a number of regional names. In the UK, Signet is the largest speciality retailer of fine jewellery with an approximate 17% share of the total jewellery market, where the company trades as H. Samuel, Ernest Jones and Leslie Davis

(12) Goodwin

Goodwin is a company engaged in mechanical and refractory engineering through its manufacturing subsidiaries. Mechanical engineering is represented by Goodwin Steel Castings for castings, Goodwin International for machining, general engineering, valves and pumps and Easat Antennas Ltd for radar antennas. Refractory engineering is represented by Hoben International for powders, refractory cements and minerals and Hoben Minerals for mineral processing.

(13) British Vita

British Vita is a world class manufacturing producer of foam, plastics and non-woven products, including. Based in the UK, the company has over 113 manufacturing sites across 22 countries. British Vita is structured in five divisions based on the chemistry and versatility of polymers: Comfort Foam, Technical Foam, Compounding, Vitasheet Group and Nonwovens; each dedicated to a particular product group and its specific markets.

(14) Caldwell Investments

Caldwell Investments is a holding company based in the UK. The group's main activity is distributing underwear and Ninaclip products. As the creator of the NinaSun canopy, the subsidiary Ninaclip produces canopies, canopy furniture, parasols and minisols for baby buggies.

(15) Alvis

Alvis is a military vehicle manufacturer. The company designs, develops, manufactures and supplies tracked fighting vehicles, specialist wheeling vehicles, transmissions, simulators, explosive ordnance disposal equipment and other equipment and components for the defence and aerospace industries.

(16) Tottenham Hotspur

The principal activities of Tottenham Hotspur are operating a professional football club in England together with related commercial activities, such as merchandising a range of branded products and the sale of corporate and executive hospitality. In addition, the company undertakes various community projects to coach children and develop relationships.

(17)Care UK

Based in the UK, Care UK is engaged in the provision of person-centred care to a broad spectrum of service users throughout England and Scotland. Working in close partnership with local authorities, Care UK provides a range of health and social care solutions primarily to various public sector purchasers in four divisions: Residential Care, Specialist Care, Community Care and Clinical Care.

(18) Daily Mail and Gen Trust

Daily Mail and General Trust is one of the largest and most successful media companies in the UK, principally focusing on daily and weekly newspaper business. Over the last ten years, the company has expanded from its newspaper base into a variety of media forms, such as television, radio, exhibitions and information publishing, both in the UK and around the world.

(19) Cable and Wireless

Cable and Wireless is an international telecommunications company, serving customers in 80 countries through two standalone business units: international and UK. The company provides integrated conventional and internet protocol (IP) voice and data services to business and residential customers, and services to telecoms carriers, mobile operators and providers of content, applications and Internet services.

(20) BAE Systems

BAE Systems is one of the global leading providers of military equipment, having major operations across 5 continents and customers in over 130 countries. The company designs, manufactures and supports military aircraft, surface ships, submarines, combat vehicles, radar, avionics communications, electronics and guided weapon systems.

5.2.2 US Company Profile

Similarly, twenty US companies are picked up from different industrial sectors with various sizes. Their details are reported in Table 5.2. The industrial sectors include Utilities (American Electric Power, California Water Service), Financial (Bank of America), Aerospace (Boeing), Transportation (Alaska Air Group, Delta Air Lines), Automotive (Ford Motor), Application software (Microsoft), Publishing and newspapers (New York Times), Entertainment (Walt Disney), Restaurant (Wendy's International) and many others. The firms are also diversified in size, ranging from 12.23 million dollars (Florida Gaming) to 311755.30 million dollars (General Electric). To provide more details on the firms, a brief profile is presented for each company as follows[23].

(1) American Electric Power

Founded in 1906, American Electric Power is a public utility holding company engaged in the generation, transmission, and distribution of electric power. Although the company is based in Columbus and Ohio, it operates though a range of subsidiaries in the states of Arkansas, Indiana, Kentucky, Louisiana, Michigan, Ohio, Oklahoma, Tennessee, Texas, Virginia, and West Virginia. In December 31, 2006, it owned or leased approximately 35,000 megawatts of power generation capacity.

(2) Alaska Air Group

The principal activity of Alaska Air Group is to provide airline services through two subsidiaries namely Alaska Airlines and Horizon Air Industries. Alaska is a major airline, whose operating fleet consisted of 114 jet aircraft at the end of 2006. As a regional airline, Horizon had a operating fleet of 21 jets and 48 turboprop aircraft at the end of 2006. Alaska and Horizon integrate their flight schedules to provide connections between most points served by their systems.

(3) Bank of America

[23] The profiles of the firms are mainly obtained from the website of Corporate Information.

Bank of America Corporation was founded in 1874 and is headquartered in Charlotte, North Carolina. As a bank holding company, Bank of America provides banking and non-banking financial services and products both domestically and internationally. The services include deposit products, lending loans, investment banking, capital markets, and leasing and financial advisory services. The operations are carried out in the United States, Asia, Europe, Middle East, Africa, Mexico and Latin America.

(4) Boeing

Boeing is the world's leading aerospace company and the largest manufacturer of commercial jetliners and military aircraft combined, involved in the design, development, manufacturing, sale and support of commercial jetliners, military aircraft, satellites, missile defense, human space flight, and launch systems and services. Headquartered in Chicago, Boeing employs more than 150,000 people across the United States and in 70 countries. Additionally, Boeing has customers in more than 90 countries and is one of the largest U.S. exporters in terms of sales.

(5) California Water Service

Founded in 1926, California Water Service's principal activity is to provide water utility and related services in California, Washington, New Mexico, and Hawaii through its subsidiaries. The services include production, purchase, storage, treatment, testing, distribution and sale of water for domestic, industrial, public and irrigation uses, and for fire protection. The Group also provides non-regulated water-related services under agreements with municipalities and other private companies.

(6) Delta Air Lines

Delta Air Lines is an air carrier that provides scheduled air transportation for passengers and cargo worldwide. The Company offers customers service to over 300 destinations in 52 countries. Moreover, through its international alliance, and worldwide code share partners, Delta offers flights to 462 destinations worldwide in 98 countries. The Group's route network's hub airports are located in Atlanta, Cincinnati and Salt Lake City.

(7) Ford Motor

Based in Dearborn, Michigan, Ford Motor Company is a global automotive industry leader manufacturing and distributing automobiles in 200 markets across six continents. Basically, the company operates through both automotive and financial services sectors. The Automotive sector sells cars, trucks, and parts under Ford, Mercury, Lincoln, Volvo, Land Rover, Jaguar, and Aston Martin brand names; while finance services sector offers various automotive financing products to and through automotive dealers worldwide.

(8) General Electric

Founded in 1892, General Electric is a diversified industrial corporation. It is engaged in developing, manufacturing and marketing a variety of products for the generation, transmission, distribution, control and utilization of electricity. The products include major appliances, lighting products, industrial automation products, medical diagnostic imaging systems, bioscience assays and separation technology products, electrical distribution and control equipment. It also offers various financial products and services aviation and energy sectors.

(9) Honeywell International

Honeywell International was founded in 1920 and is headquartered in Morris Township, New Jersey. It operates as a diversified technology and manufacturing company in the United States, Europe, Canada, Asia, and Latin America. It operates in four segments: Aerospace, Automation and Control Solutions, Specialty Materials, and Transportation Systems.

(10) Microsoft

Microsoft was founded in 1975 by William H. Gates III and is headquartered in Redmond, Washington. Microsoft engages in the development, manufacture, licensing, and support of software products for various computing devices worldwide. Its software products include operating systems for servers, personal computers and intelligent devices, business solution applications and many other software development tools. It also provides consulting and product support services, and trains and certifies computer system integrators and developers.

(11) MGP Ingredients

MGP Ingredients develops and produces natural grain-based products in the United States. It has two reportable segments: ingredients and distillery products. Ingredients segment consist primarily of specialty wheat starches and proteins, commodity ingredients. Distillery products consist of food grade alcohol and fuel alcohol.

(12) New York Times

Founded in 1896, New York Times is a diversified media company. Its principal activity is to operate into two divisions namely News Media Group and Broadcasting Media Group. The products includes The New York Times, The Globe, the International Herald Tribune, the Worcester Telegram and Gazette, 14 regional newspapers, radio stations and more than 30 Web sites.

(13) Textron

Founded in 1923, Textron is a global multi-industry company. Headquartered in Providence, it has grown into a network of businesses with employees in 32 countries, serving a diverse and global customer base. The Company has operations in the aircraft, industrial and finance businesses through four segments: Bell, Cessna, Industrial and Finance.

(14) Utah Medical Products

Utah Medical Products was founded in 1978 and is based in Midvale, Utah. With a particular interest in healthcare for women and their babies, the company develops, manufactures, and markets a broad range of disposable and reusable specialty medical devices designed for better health outcomes for patients and their care-providers in the United States and internationally.

(15) Walt Disney

Since its founding in 1923, Walt Disney has committed to producing entertainment experiences based on its rich legacy of quality creative content and exceptional storytelling. The company is divided into four major business segments: Studio Entertainment, Parks and Resorts, Consumer Products, and Media Networks. Each segment consists of integrated, well-connected businesses that operate in concert to maximize exposure and growth worldwide.

(16) Wells Fargo & Company

Wells Fargo & Company is a financial holding company and a bank holding company. Its principal activities are to provide banking, insurance, investment, mortgage banking and consumer financing services. The Company provides retail, commercial and corporate banking services through banking stores located in 23 states; and it also provides other financial services through subsidiaries engaged in various businesses

(17) Wendy's International

Wendy's International was founded in 1969 and is based in Dublin, Ohio. The company is primarily engaged in the operation, development and franchising of quick-service restaurants. It has more than 6,300 Wendy's Old Fashioned Hamburgers restaurants in North America and more than 300 international Wendy's restaurants.

(18) Florida Gaming

Founded in 1976, the company was formerly known as Lexicon Corporation and was changed its name to Florida Gaming in 1994. Its principal activity is to own and operate three jai-alai fronton and inter-track pari-mutuel wagering facilities located in South and Central Florida. It is a relatively small on among the twenty US companies with 12.23 million dollars market capitalisation.

(19) Campbell Soup

Campbell Soup was founded in 1869 and is headquartered in Camden, New Jersey. Its principal activity is to manufacture and market soups, juice beverages, sauces, biscuits and confectionery products. It operates in four segments: U.S. Soup, Sauces, and Beverages; Baking and Snacking; International Soup and Sauces

(20) Bell Industries

Bell Industries provides technology lifecycle and outsourced services, distribute aftermarket products for recreational vehicles, motor cycles, snowmobiles and power boats and manufacture specialty electronic components. It operates in three reportable business segments: Technology Solutions, Recreational Products and Electronics Components.

5.3 Basic Statistics of Excess Stock Returns

The stock returns are defined as the first difference in the log of price indices, including both stocks and market indices. The excess stock returns are calculated as the nominal stock returns minus the returns on the risk-free assets.

5.3.1 Basic Statistics of UK Daily Excess Returns

Table 5.3 shows some basic statistics of the excess return on the twenty UK companies and the market portfolio. The mean of excess returns is significantly different from zero in nine cases. Daily Mail and Gen Trust has the highest mean return and lowest variance among the twenty companies. The market portfolio has the lowest variance overall. Care UK has the lowest and the only negative mean excess return among all return series, which also has the highest variance. Except British Airways, all return series are significantly skewed, including both positive and negative skewness. Thus, all returns with exception of British Airways are asymmetrically distributed. All excess returns have positive and significant kurtosis, implying fatter tails and higher peaks than a normal distribution. Consequently, all conditional betas are rejected for the null of normal distribution, which is confirmed by their Jarque-Bera statistics significant at 1% level.

5.3.2 Basic Statistics of US Daily Excess Returns

Some description statistics of US daily excess returns are presented in Table 5.4, including returns on the twenty US companies and the market portfolio. All mean values of the excess returns are positive and fifteen of them are statistically significant. Microsoft has the highest mean return and Bell Industries has the lowest mean return. According to variance, the return series of Florida Gaming is the most volatile; while the return series of market portfolio is the stablest. Except Alaska Air Group and Textron, all the other return series are significantly skewed. American Electric Power, Delta Air Lines and Bell Industries are the only firms with negative significant skewness. Statistics of excess kurtosis are positive and significant at 1% level for all return series, indicating higher peaks and fatter tails than the normal distribution in all cases. Therefore, all US daily excess returns are rejected for the null of normal distribution, as their Jarque-Bera statistics are all significant at 1% level.

5.4 Results of Cointegration Tests

As mentioned in section 3.3.2.4, cointegration tests between the log of stock price and the log of market index are required in order to implement the error-correction GARCH-X model. Before the cointegration tests are conducted, the stochastic structure of the individual price index has to be checked. According to results of DF and ADF tests presented in Table 5.5, 5.6, 5.7 and 5.8, logs of all UK and US price indices are nonstationary in levels but stationary after taking first difference. Consequently, the Engle and Granger (1987) two-step cointegration tests are applicable to all stocks and the market index[24]. Table 5.9 reports the cointegration test results between the price index of the twenty UK firms and the FTSE All-Share. Cointegrated relationship is found in five tests, which includes Legal and General, Glaxo Smith Kline, British Vita, Alvis and Care UK. Thus the GARCH-X model is only applicable to these five cases among the twenty UK firms. Similarly, Table 5.10 presents the cointegration test results of US daily data. Half of the twenty US firms are found to form a stationary cointegration relationship with S&P 500, including Alaska Air Group, Boeing, California Water Service, General Electric, Honeywell International, MGP Ingredients, Textron, Utah Medical Products, Walt Disney and Florida Gaming. Accordingly, GARCH-X is appropriate in these ten cases to capture the effect of short term deviations.

5.5 Conclusion

This chapter describes the data applied in the empirical tests, with a particular emphasis on the firms selected from UK and US stock markets. The firms are from different industrial sectors and range from small companies to giant industrial leaders. A profile is attached to each company to demonstrate the diversity of the firms under investigation. In addition, the basic statistics of excess returns on stocks and market

[24] Other forms of cointegration tests such as the multivariate Johansen and Juseliun (1990) generate the same results, which are not presented to save space.

indices are presented to summarise different time series properties of the return series. Finally, the chapter reports the results of cointegration tests. Five UK firms and ten US firms are found to have cointegration relationship with the market index, which indicates GARCH-X is appropriate to model the time-varying beta for these firms.

Table 5.1: UK Company Profile

Name	Industry	Products	Market Capitalisation[a]
British Airways	Transportation	Airline services	2517.50
TESCO	Retailer	Mass market distribution	18875.26
British American Tobacco	Tobacco	Cigars and Cigarettes	15991.70
BT Group	Utilities	Telecommunications	16269.67
Legal and General	Financial	Insurance	6520.12
Glaxo Smith Kline	Pharmaceutical	Medicines	76153.00
Edinburgh Oil and Gas	Energy Producer	Oil and gas	48.07
Boots Group	Retailer	Health and beauty products	5416.64
Barclays	Financial	Banking	32698.64
Scottish and Newcastle	Beverage	Beer	3380.12
Signet Group	Retailer	Jewellery and watches	1770.29
Goodwin	Metal Producer	Mental products	17.64
British Vita	Chemical	Polymers, foams and fibers	466.62
Caldwell Investments	Wholesaler	Ninaclip products	3.08
Alvis	Automotive	Military vehicles	189.68
Tottenham Hotspur	Recreation	Football club	28.57
Care UK	Service organization	Health and social care	146.84
Daily Mail and Gen Trust	Printing and Publishing	Media products	237.84
Cable and Wireless	Utilities	Telecommunications	3185.61
BAE Systems	Aerospace	Military equipments	5148.61

Notes:
[a] The unit of market capitalisation is million pounds.

Table 5.2: US Company Profile

Name	Products	Industry	Market Capitalisation
American Electric Power	Electric power	Utilities	79.64
Alaska Air Group	Airline services	Transportation	725.18
Bank of America	Financial services	Financial	119503.30
Boeing	Aircraft, satellites, missile	Aerospace	33721.10
California Water Service	Water related services	Utilities	463.94
Delta Air Lines	Airline services	Transportation	1458.07
Ford Motor	Cars and trucks	Automotive	28163.04
General Electric	Engines, turbines, generators	Conglomerates	311755.30
Honeywell International	Aerospace equipments	Aerospace	28818.35
Microsoft	Software	Application software	295937.20
MGP Ingredients	Ingredients and distillery	Consumer Goods	120.49
New York Times	Media products	Publishing and newspapers	7078.13
Textron	Aircraft, vehicles, finance	Conglomerates	780.03
Utah Medical Products	Medical devices	Healthcare	120.66
Walt Disney	Entertainment products	Entertainment	47718.27
Wells Fargo & Company	Financial services	Financial	99643.50
Wendy's International	Restaurant services	Restaurant	4470.80
Florida Gaming	Jai-Alai games	Gaming Activities	12.23
Campbell Soup	Convenience food	Consumer Goods	11016.59
Bell Industries	Electronics	Wholesaler	21.50

Notes:
[a] The unit of market capitalisation is million dollars.

Table 5.3: Basic Statistics of UK Daily Excess Returns

Company	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
British Airways	0.00041	0.000635	0.06350	5.58739[a]	5091.32249[a]
TESCO	0.00076[b]	0.000402	0.44199[a]	4.50321[a]	3432.82479[a]
British American Tobacco	0.00079[b]	0.000499	1.55206[a]	19.31672[a]	62391.73029[a]
BT Group	0.00031	0.000528	0.21075[a]	5.32768[a]	4655.58689[a]
Legal and General	0.00064[c]	0.000508	0.33306[a]	4.54442[a]	3438.55393[a]
Glaxo Smith Kline	0.00070[b]	0.000421	0.31173[a]	4.87958[a]	3944.43093[a]
Edinburgh Oil and Gas	0.00069	0.000739	2.73848[a]	33.89497[a]	192155.21263[a]
Boots Group	0.00058[b]	0.000344	0.55705[a]	6.06798[a]	6204.03768[a]
Barclays	0.00079[b]	0.000501	0.38973[a]	3.99694[a]	2703.04341[a]
Scottish and Newcastle	0.00031	0.000365	-0.15975[a]	9.95662[a]	16175.51929[a]
Signet Group	0.00017	0.001174	0.35038[a]	8.31725[a]	11355.8427[a]
Goodwin	0.00075[c]	0.000719	4.88334[a]	96.67526[a]	1538963.40[a]
British Vita	0.00046	0.000353	0.62036[a]	8.59559[a]	12294.02567[a]
Caldwell Investments	0.00022	0.000957	1.19043[a]	16.84906[a]	47198.17438[a]
Alvis	0.00032	0.000764	-0.72220[a]	45.33985[a]	335419.49115[a]
Tottenham Hotspur	0.00035	0.000405	1.04923[a]	14.45538[a]	34777.95104[a]
Care UK	-0.00018	0.001257	-0.27701[a]	86.38133[a]	1216312.61[a]
Daily Mail and Gen Trust	0.00091[a]	0.000214	1.56263[a]	22.64315[a]	85164.16686[a]
Cable and Wireless	0.00021	0.000825	-2.69853[a]	55.84310[a]	513055.53786[a]
BAE Systems	0.00042	0.000735	-2.77434[a]	53.64104[a]	474028.31845[a]
Market Portfolio	0.00052[b]	0.000186	0.97189[a]	14.13577[a]	33186.53811[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 5.4: Basic Statistics of US Daily Excess Returns

Company	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
American Electric Power	0.00069	0.001268	-0.46822[a]	7.79272[a]	10041.34242[a]
Alaska Air Group	0.00063	0.000739	0.02721	8.89084[a]	12885.15764[a]
Bank of America	0.00101[a]	0.000535	0.31752[a]	4.25838[a]	3021.54232[a]
Boeing	0.00084[b]	0.000504	0.09680[b]	6.98142[a]	7950.76595[a]
California Water Service	0.00075[a]	0.000546	0.37822[a]	3.14836[a]	1708.95208[a]
Delta Air Lines	0.00036	0.000846	-1.70982[a]	46.50704[a]	354459.57848[a]
Ford Motor	0.00069[c]	0.000559	0.60481[a]	5.46588[a]	5108.25922[a]
General Electric	0.00109[a]	0.000405	0.47708[a]	5.27694[a]	4687.30325[a]
Honeywell International	0.00091[b]	0.000580	0.20760[a]	10.18587[a]	16939.67630[a]
Microsoft	0.00165[a]	0.000684	0.14984[a]	3.72399[a]	2275.13849[a]
MGP Ingredients	0.00059	0.000878	0.08877[b]	5.02952[a]	4128.40368[a]
New York Times	0.00088[a]	0.000453	0.67194[a]	5.31739[a]	4903.14793[a]
Textron	0.00078[b]	0.000558	-0.05054	12.37030[a]	24944.64795[a]
Utah Medical Products	0.00124[a]	0.000942	0.37106[a]	9.91682[a]	16119.72854[a]
Walt Disney	0.00092[b]	0.000514	0.11602[a]	6.08791[a]	6049.98399[a]
Wells Fargo & Company	0.00124[a]	0.000455	0.42554[a]	3.69442[a]	2342.81744[a]
Wendy's International	0.00104[a]	0.000535	0.35524[a]	3.60991[a]	2206.40325[a]
Florida Gaming	0.00021	0.007149	0.18857[a]	8.39165[a]	11501.60929[a]
Campbell Soup	0.00088[a]	0.000455	0.50475[a]	5.63310[a]	5338.39899[a]
Bell Industries	0.00020	0.001033	-3.50143[a]	116.42285[a]	2217341.2737[a]
Market Portfolio	0.00091[a]	0.000235	1.01882[a]	12.39564[a]	25722.03597[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 5.5: Two Unit Root Tests for UK Daily Log Prices

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-57.73353[a]	-30.05021[a]	-23.57853[a]	-19.36345[a]
TESCO	-62.97755[a]	-33.75541[a]	-26.80778[a]	-21.66671[a]
British American Tobacco	-61.72705[a]	-34.16457[a]	-26.66141[a]	-21.26361[a]
BT Group	-59.18938[a]	-33.07308[a]	-26.80221[a]	-21.48803[a]
Legal and General	-64.29467[a]	-32.39989[a]	-26.16160[a]	-22.14146[a]
Glaxo Smith Kline	-60.89636[a]	-33.01063[a]	-26.34004[a]	-21.87390[a]
Edinburgh Oil and Gas	-60.62541[a]	-32.81689[a]	-24.19431[a]	-20.76050[a]
Boots Group	-61.12700[a]	-32.91673[a]	-25.58638[a]	-22.00654[a]
Barclays	-56.96656[a]	-32.46144[a]	-25.34044[a]	-20.25536[a]
Scottish and Newcastle	-62.59605[a]	-33.15883[a]	-25.45314[a]	-20.46293[a]
Signet Group	-55.53669[a]	-31.97811[a]	-24.49513[a]	-20.58974[a]
Goodwin	-58.19337[a]	-29.66328[a]	-24.07662[a]	-19.19809[a]
British Vita	-54.50790[a]	-29.13235[a]	-23.94705[a]	-20.60203[a]
Caldwell Investments	-56.09869[a]	-29.55080[a]	-23.36441[a]	-19.13389[a]
Alvis	-55.59940[a]	-28.92766[a]	-21.14511[a]	-18.99791[a]
Tottenham Hotspur	-56.57424[a]	-29.27951[a]	-22.26547[a]	-18.25358[a]
Care UK	-51.05556[a]	-29.74239[a]	-22.86514[a]	-17.67969[a]
Daily Mail and Gen Trust	-59.55904[a]	-29.10084[a]	-21.02573[a]	-16.48105[a]
Cable and Wireless	-57.03544[a]	-31.84139[a]	-25.36903[a]	-19.31076[a]
BAE Systems	-59.26780[a]	-32.35308[a]	-26.31819[a]	-21.19958[a]
Market Portfolio	-59.62619[a]	-30.81719[a]	-24.52010[a]	-19.38580[a]

Notes:
[a] Significant at the 1% level.

Table 5.6: One Unit Root Tests for UK Daily Log Prices

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-1.75701	-1.87816	-1.85131	-1.83784
TESCO	-1.54259	-1.50109	-1.44128	-1.37591
British American Tobacco	-1.68848	-1.63368	-1.54193	-1.48105
BT Group	-1.55528	-1.45029	-1.34248	-1.31903
Legal and General	-1.53776	-1.50644	-1.48377	-1.42821
Glaxo Smith Kline	-2.19662	-2.20922	-2.19417	-2.27341
Edinburgh Oil and Gas	-0.21153	-0.21483	-0.13367	-0.02019
Boots Group	-2.40284	-2.39916	-2.31154	-2.33761
Barclays	-1.21425	-1.20336	-1.12881	-1.12279
Scottish and Newcastle	-1.86143	-1.79289	-1.62709	-1.5982
Signet Group	-1.44786	-1.51083	-1.49458	-1.42897
Goodwin	-0.31026	-0.61252	-0.55055	-0.52584
British Vita	0.48878	0.36533	0.34182	0.37216
Caldwell Investments	-1.47853	-1.77479	-1.682	-1.72282
Alvis	-1.21959	-1.45207	-1.55909	-1.58048
Tottenham Hotspur	-0.95109	-1.03911	-1.08401	-1.08489
Care UK	-1.34504	-1.17921	-1.21804	-1.24930
Daily Mail and Gen Trust	-1.14064	-1.08596	-1.04337	-0.93193
Cable and Wireless	-1.06583	-1.12275	-1.00965	-1.07877
BAE Systems	-1.4147	-1.46427	-1.29923	-1.25933
Market Portfolio	-1.83797	-1.83323	-1.782	-1.72232

Table 5.7: Two Unit Root Tests for US Daily Log Price

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-25.85834[a]	-13.86438[a]	-9.92140[a]	-8.52629[a]
Alaska Air Group	-28.05279[a]	-13.85312[a]	-11.48460[a]	-8.93195[a]
Bank of America	-28.25552[a]	-14.49945[a]	-11.43180[a]	-8.76161[a]
Boeing	-30.39964[a]	-14.14299[a]	-11.35458[a]	-8.89114[a]
California Water Service	-32.35949[a]	-15.74607[a]	-12.55547[a]	-10.93653[a]
Delta Air Lines	-28.71825[a]	-13.90826[a]	-11.48326[a]	-9.65157[a]
Ford Motor	-29.41394[a]	-12.88143[a]	-9.61096[a]	-8.45568[a]
General Electric	-31.85612[a]	-14.16445[a]	-11.22224[a]	-8.98691[a]
Honeywell International	-28.70909[a]	-13.76842[a]	-10.79645[a]	-10.09779[a]
Microsoft	-28.41738[a]	-14.50367[a]	-11.92198[a]	-9.71009[a]
MGP Ingredients	-27.91517[a]	-13.75452[a]	-10.31757[a]	-8.60794[a]
New York Times	-29.04017[a]	-14.95148[a]	-10.37514[a]	-9.04154[a]
Textron	-27.60907[a]	-13.10071[a]	-10.75611[a]	-8.69069[a]
Utah Medical Products	-29.78357[a]	-14.43469[a]	-10.02388[a]	-8.06446[a]
Walt Disney	-28.84934[a]	-13.87908[a]	-9.94305[a]	-8.38354[a]
Wells Fargo & Company	-30.56193[a]	-15.60698[a]	-11.69691[a]	-9.25741[a]
Wendy's International	-29.83551[a]	-13.61441[a]	-10.20257[a]	-9.32242[a]
Florida Gaming	-31.57231[a]	-13.19528[a]	-10.02382[a]	-9.11631[a]
Campbell Soup	-30.36940[a]	-14.11490[a]	-10.45659[a]	-8.70591[a]
Bell Industries	-28.02045[a]	-15.58254[a]	-11.67521[a]	-9.44802[a]
Market Portfolio	-31.23278[a]	-14.60972[a]	-10.87070[a]	-8.98582[a]

Notes:
[a] Significant at the 1% level.

Table 5.8: One Unit Root Tests for US Daily Log Prices

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-1.82535	-1.88635	-1.92862	-1.94198
Alaska Air Group	-2.29640	-2.17954	-2.16883	-1.99830
Bank of America	-1.49621	-1.39401	-1.17380	-1.13237
Boeing	-2.23818	-2.15606	-2.16115	-2.03767
California Water Service	-1.87094	-1.50649	-1.48519	-1.32409
Delta Air Lines	-1.11178	-1.02973	-0.96909	-0.62956
Ford Motor	-1.20141	-1.20444	-1.34754	-1.30577
General Electric	-1.41586	-1.36606	-1.41570	-1.54078
Honeywell International	-1.52393	-1.48941	-1.46749	-1.47942
Microsoft	-2.21912	-2.17517	-2.37236	-2.63990
MGP Ingredients	-1.81608	-1.73939	-1.77983	-1.74521
New York Times	-0.55572	-0.43950	-0.45202	-0.40658
Textron	-1.43743	-1.42504	-1.29084	-1.26484
Utah Medical Products	-2.95542	-2.95560	-2.92076	-2.92640
Walt Disney	-2.36592	-2.19925	-2.20045	-2.20121
Wells Fargo & Company	-1.25949	-1.14919	-1.15645	-1.14836
Wendy's International	-1.32128	-1.32226	-1.25967	-1.23272
Florida Gaming	-1.81775	-1.51826	-1.50270	-1.47851
Campbell Soup	-1.92168	-1.80464	-1.75946	-1.77548
Bell Industries	-0.79670	-0.77256	-0.49340	-0.53188
Market Portfolio	-1.43226	-1.33068	-1.32766	-1.43001

Table 5.9: Cointegration Test Results of UK Daily Data

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-0.00110 (-1.42288)	-0.00119 (-1.53736)	-0.00116 (-1.49547)	-0.00114 (-1.47516)
TESCO	-0.00179 (-1.67159)	-0.00175 (-1.62736)	-0.00149 (-1.38492)	-0.00143 (-1.32404)
British American Tobacco	-0.00177 (-1.63759)	-0.00165 (-1.51947)	-0.00137 (-1.26573)	-0.00140 (-1.28815)
BT Group	-0.00118 (-1.23163)	-0.00107 (-1.11908)	-0.00089 (-0.92493)	-0.00076 (-0.79553)
Legal and General	-0.00553[b] (-3.34086)	-0.00507[b] (-3.05335)	-0.00480[b] (-2.88158)	-0.00460[c] (-2.75071)
Glaxo Smith Kline	-0.00352[c] (-2.71704)	-0.00362[c] (-2.79059)	-0.00355[c] (-2.72634)	-0.00373[c] (-2.85824)
Edinburgh Oil and Gas	-0.00025 (-0.37699)	-0.00025 (-0.38029)	-0.00018 (-0.27611)	-0.00012 (-0.17515)
Boots Group	-0.00284 (-2.48436)	-0.00275 (-2.40603)	-0.00259 (-2.26023)	-0.00245 (-2.14081)
Barclays	-0.00153 (-1.52975)	-0.00146 (-1.45276)	-0.00129 (-1.28852)	-0.00124 (-1.22909)
Scottish and Newcastle	-0.00228 (-1.93871)	-0.00223 (-1.88935)	-0.00199 (-1.68307)	-0.00191 (-1.61361)
Signet Group	-0.00082 (-1.30693)	-0.00087 (-1.39687)	-0.00087 (-1.38751)	-0.00082 (-1.31015)
Goodwin	-0.00093 (-0.85634)	-0.00123 (-1.13163)	-0.00113 (-1.03325)	-0.00109 (-0.99071)
British Vita	-0.00630[a] (-3.87701)	-0.00665[a] (-4.10422)	-0.00661[a] (-4.05566)	-0.00646[a] (-3.94214)
Caldwell Investments	-0.00114 (-1.48246)	-0.00136 (-1.78108)	-0.00129 (-1.68759)	-0.00133 (-1.73211)
Alvis	-0.00222[c] (-2.64141)	-0.00230[c] (-2.74146)	-0.00229[c] (-2.72554)	-0.00234[c] (-2.77994)
Tottenham Hotspur	-0.00133 (-1.64636)	-0.00140 (-1.73343)	-0.00141 (-1.73688)	-0.00152 (-1.87178)
Care UK	-0.00186[b] (-2.98095)	-0.00181[b] (-2.95138)	-0.00187[b] (-3.02352)	-0.00194[b] (-3.12304)
Daily Mail and Gen Trust	-0.00128 (-1.49754)	-0.00115 (-1.35023)	-0.00110 (-1.28640)	-0.00118 (-1.38306)
Cable and Wireless	-0.00034 (-0.48013)	-0.00039 (-0.55203)	-0.00026 (-0.36805)	-0.00033 (-0.46821)
BAE Systems	-0.00239 (-2.15694)	-0.00251 (-2.26511)	-0.00220 (-1.98874)	-0.00221 (-1.99407)

Notes: *t* statistics in parentheses.
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 5.10: Cointegration Test Results of US Daily Data

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
	-0.00197	-0.00182	-0.00222	-0.00206
American Electric Power	(-1.74052)	(-1.61338)	(-1.96714)	(-1.82395)
	-0.00417[b]	-0.00416[c]	-0.00419[b]	-0.00423[b]
Alaska Air Group	(-2.88102)	(-2.86292)	(-2.87902)	(-2.89463)
	-0.00247	-0.00251	-0.00252	-0.00260
Bank of America	(-2.14027)	(-2.18334)	(-2.18484)	(-2.25364)
	-0.00362[c]	-0.00354[c]	-0.00341[c]	-0.00353[c]
Boeing	(-2.74381)	(-2.68197)	(-2.57915)	(-2.66062)
	-0.02138[a]	-0.01510[a]	-0.01398[a]	-0.01405[a]
California Water Service	(-6.48637)	(-4.64256)	(-4.26891)	(-4.25816)
	-0.00101	-0.00085	-0.00079	-0.00076
Delta Air Lines	(-1.00498)	(-0.85323)	(-0.78770)	(-0.75602)
	-0.00289	-0.00274	-0.00269	-0.00272
Ford Motor	(-2.53878)	(-2.40702)	(-2.35696)	(-2.37821)
	-0.00726[a]	-0.00660[b]	-0.00644[b]	-0.00612[b]
General Electric	(-3.76729)	(-3.40910)	(-3.31130)	(-3.13948)
	-0.00351[c]	-0.00338	-0.00325	-0.00323
Honeywell International	(-2.58361)	(-2.48823)	(-2.38493)	(-2.36188)
	-0.00299	-0.00295	-0.00292	-0.00284
Microsoft	(-2.55917)	(-2.52389)	(-2.48769)	(-2.41927)
	-0.00515[b]	-0.00442[c]	-0.00441[c]	-0.00434[c]
MGP Ingredients	(-3.27378)	(-2.82795)	(-2.81181)	(-2.75909)
	-0.00260	-0.00234	-0.00228	-0.00225
New York Times	(-2.34913)	(-2.11727)	(-2.05674)	(-2.02807)
	-0.00315[c]	-0.00346[b]	-0.00353[b]	-0.00360[b]
Textron	(-2.62949)	(-2.89443)	(-2.93473)	(-2.98425)
	-0.00263[c]	-0.00266[c]	-0.00264[c]	-0.00266[c]
Utah Medical Products	(-2.71173)	(-2.73108)	(-2.70463)	(-2.71299)
	-0.00518[b]	-0.00485[b]	-0.00476[b]	-0.00471[b]
Walt Disney	(-3.38137)	(-3.15192)	(-3.08090)	(-3.03126)
	-0.00213	-0.00193	-0.00178	-0.00178
Wells Fargo & Company	(-1.94433)	(-1.75714)	(-1.61747)	(-1.61577)
	-0.00192	-0.00161	-0.00156	-0.00151
Wendy's International	(-1.87422)	(-1.56896)	(-1.51857)	(-1.46527)
	-0.00729[a]	-0.00524[c]	-0.00498[c]	-0.00482
Florida Gaming	(-3.79103)	(-2.78720)	(-2.64215)	(-2.55173)
	-0.00222	-0.00213	-0.00207	-0.00213
Campbell Soup	(-2.28627)	(-2.18953)	(-2.12275)	(-2.18744)
	-0.00103	-0.00098	-0.00097	-0.00094
Bell Industries	(-1.36531)	(-1.29492)	(-1.28581)	(-1.24443)

Notes: see Table 5.9.

Chapter 6
Empirical Results of UK Daily Data

6.1 Introduction

During the past two decades, the emphasis of financial economic research has shifted from the mean of stock market returns to the volatility of these returns. The CAPM beta is the systematic component of risk that is irreducible by diversification or hedging, and there is widespread evidence of the time variant feature of betas; forecasting time-dependent systematic risk is of considerable research interests. Accurate forecasts of conditional betas are clearly of particular value in decision making for both risk managers and investment analysts.

The success of univariate GARCH models in capturing short and medium term conditional second moments of financial and economic data has motivated many researchers to extend these models to various multivariate specifications. It is straightforward to calculate time-varying beta series with the conditional variance and covariance information generated by the bivariate GARCH models. However, among the vast literature, most studies utilise GARH models to formulate conditional betas for estimation but not for forecasting purpose. This thesis seeks to combine the time-varying beta estimation and forecasting outcomes from GARCH models and the Kalman filter method, with an emphasis on forecasting results. In particular, predictive performance of different models will be assessed in terms of accuracy of conditional beta forecasts. This chapter reports empirical results of forecasting time-varying betas using daily data of the twenty UK firms.

The rest of the chapter is as follows. Section 6.2 reports the estimation results of different models. In addition, the section presents the basic statistics and the stationary property of conditional betas constructed by alternative approach. Comparative analysis in section 6.3 provides an insight into the performance of different methods in estimating time-varying betas, by exploring beta estimates' similarities and differences. Section 6.4 describes the process of forecasting time-varying betas. Furthermore, this section evaluates the out-of-sample forecasting ability of alternative approaches, in terms of various forecast error statistics. Section 6.5 concludes the main findings from UK daily data.

6.2 Estimation of Time-Varying Betas

Construction of time-varying betas must be the first step for further investigations on forecasting accuracy, since betas are not directly observable in financial markets and can only be estimated in the context of a model. Consequently, the actual beta series are estimated by each model for the whole sample (1989 to 2003), using daily data of the market and the twenty UK firms[25]. The estimation results and distributional properties of UK daily beta series are reported model by model.

6.2.1 Bivariate GARCH(1,1) Model

6.2.1.1 Estimation Results

The diagonal bivariate GARCH model proposed by Bollerslev *et al.* (1988) is applied to estimate the time-varying beta series of UK firms. The diagonal version of GARCH model provides a parsimonious but efficient way to jointly capture volatility of asset returns. The details of the diagonal bivariate GARCH(1,1) representation are discussed in section 3.3.2.1. Table 6.1 reports the estimation results of bivariate GARCH(1,1) models. All time-varying betas are estimated by means of the BHHH algorithm[26]. For each firm, the estimation results include three types of information: the estimated parameters, the log-likelihood function value and the Ljung-Box statistics.

Since bivariate GARCH is the seminal model for other GARCH extensions, its estimation results are discussed in a bit more details. For a diagonal bivariate GARCH(1,1) model, there are nine parameters to be estimated. In Table 6.1, notations '*c*', '*a*' and '*b*' stand for the intercept term, the ARCH term and the GARCH term respectively. A significant coefficient of ARCH term implies the existence of volatility clustering or ARCH effect. A large GARCH term indicates that shocks to conditional variance take a long time to die out and volatility is persistent. Parameters of ARCH and GARCH effects are the key to assess and interpret the bivariate

[25] Actual beta and estimated beta are interchangeable in this chapter without particular indication.
[26] BHHH is an algorithm for optimisation due to Berndt *et al.* (1974).

GARCH model. Parameters with subscriptions '1' and '11' are those for the firm variance equation, '3' and '33' for the market variance equation, and '2' and '22' for the covariance parameters. In addition, Table 6.1 presents the log-likelihood function value, which is the optimum value from maximum likelihood estimation.

Part B of Table 6.1 reports the twelfth order of Ljung-Box Q statistics on the standardised residuals ($u_t / h_t^{1/2}$) and squared residuals (u_t^2 / h_t) of the firm and market equation respectively. A significant Ljung-Box statistics implies the presence of serial correlation and possible higher ARCH order. If Ljung-Box statistics are significant for most companies, further diagnostic tests can be carried out by calculating the Ljung-Box statistics on the products of standardised residuals of the firm and the market (Giannopoulos, 1995). The Ljung-Box statistics on the cross-product standardised residuals provide more comprehensive information to evaluate the general descriptive validity of a bivariate model, since combinations of residuals of the firm and the market are examined instead of individual fitted residuals.

According to Table 6.1, estimation results of bivariate standard GARCH(1,1) models are robust, since all estimated coefficient are positive and significant at 1% level. The positive and significant ARCH coefficients (a_{11} and a_{33}) provide strong evidence of volatility clustering in all twenty cases. Additionally, all the estimated ARCH coefficients are less than unity in size, implying that shocks of previous news to volatility are not explosive. Also GARCH coefficients (b_{11} and b_{33}) are all positive and significant at 1% level, which indicates the presence of GARCH effects. Except Daily Mail and Gen Trust, the sums of the ARCH and GARCH coefficients ($a_{11} + b_{11}$, $a_{33} + b_{33}$) are fairly close to unity, suggesting a high degree of volatility persistence for most companies. Nevertheless, the effect of the shock on volatility dies over time, since the sums are less than unity in size, which implies the returns process is stationary for every firm with a steady-state unconditional variance in the long term. For all firms, covariance coefficients (a_{22} and b_{22}) are positive and significant, which implies a positive and significant interaction between the firms and the market.

According to the twelfth order of Ljung-Box Q statistics, serial correlation is detected in eight cases (British Airways, TESCO, BT Group, Glaxo Smith Kline, Boots Group, Alvis, Care UK, Daily Mail and Gen Trust). Among them, two firms (TESCO, Daily Mail and Gen Trust) have significant Ljung-Box statistics for both standardised residuals ($u_t / h_t^{1/2}$) and squared standardised residuals (u_t^2 / h_t); while the others have significant Ljung-Box statistics for either standardised residuals or squared standardised residuals. However, according to further diagnostic tests on the products of standardised residuals of the firm and the market, significant Ljung-Box statistics of twelfth order is found in only three cases (BT Group, Edinburgh Oil and Gas Boots Group). As a result, according to Giannopoulos (1995), the general absence of serial correlation implies that there is no need to encompass a higher order ARCH process.

6.2.1.2 Basic Statistics of the Time-Varying Beta

Table 6.2 presents some basic statistics of the twenty time-varying beta series estimated by the bivariate GARCH(1,1) model. All the mean values are positive and significant at 1% level, ranging from 0.5772 (Tottenham Hotspur) to 1.22242 (Cable and Wireless). Six firms (British Airways, BT Group, Legal and General, Glaxo Smith Kline, Barclays, Cable and Wireless) fall into the category of aggressive shares, with their average beta values greater than unity. Most of the remaining firms are defensive shares, whose beta mean values are less than unity. Few are neutral stocks, as their beta values parallel that of the market portfolio. The conditional beta of Daily Mail and Gen Trust has the lowest variance; while the beta of Care UK has the highest variance. Most firms are found to have significantly skewed conditional betas, except Barclays and Caldwell Investments. Therefore, most conditional betas are asymmetrically distributed; among which ten are positively skewed and eight are negatively skewed. All time-varying beta series exhibit leptokurtic, with positive and significant excess kurtosis. Therefore all betas are rejected for the null of normal distribution. This is proved by the Jarque-Bera statistics which are significant at 1% level in all cases.

6.2.1.3 Unit Root Tests of the Time-Varying Beta

The classical Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests are applied to detect the presence of two and one unit roots in the conditional beta series. Both DF and ADF tests allow for a constant. ADF tests are conducted with 3, 6 and 9 lagged differences. Table 6.3 and 6.4 present the results of the DF tests and the ADF tests for two and one unit root respectively. All test statistics in both tables are significant at 1% level. Therefore, the null of both two and one unit root is rejected for all time-varying betas series. In consequence, all twenty conditional beta series constructed by the bivariate GARCH(1,1) model are stationary in levels.

6.2.2 Bivariate BEKK GARCH(1,1) Model

6.2.2.1 Estimation Results

Engle and Kroner (1995) propose the BEKK model and relate it to the VECH representation, which is regarded as an improvement to the first multivariate GARCH model introduced by Bollerslev *et al.* (1988), as the positive definiteness of the conditional variance matrix is guaranteed. Section 3.3.2.2 introduces the bivariate BEKK model in details. Despite its theoretical attractions, empirical applications of the unrestricted BEKK model encounter criticisms on misleading or biased parameters (see Tse, 2000 for example). By taking parameter metrics as diagonal, the diagonal BEKK model avoids some criticisms. The BEKK model is restricted to the diagonal specification in this thesis.

As asserted by Tse (2000), the unrestricted BEKK model has a main disadvantage that it is difficult to interpret the parameters. Additionally, the net effects of parameters on future variance and covariance are not readily seen; because there is no parameter in any equation that exclusively governs a particular conditional variance equation. Even for the diagonally restricted representation, the difficulty remains. The equation form of the diagonal BEKK model presented in chapter 3 clarifies this weakness. According to (3.44b), there is no parameter exclusively governing the conditional covariance equation; and the squared parameters measure the ARCH and GARCH effects in equation (3.44a) and (3.44c).

$$h_{11,t} = c_{11}^2 + a_{11}^2 u_{1,t-1}^2 + b_{11}^2 h_{11,t-1} \tag{3.44a}$$

$$h_{12,t} = c_{11}c_{12} + a_{11}a_{22}(u_{1,t-1}u_{2,t-1}) + b_{11}b_{22}h_{12,t-1} \tag{3.44b}$$

$$h_{22,t} = c_{12}^2 + c_{22}^2 + a_{22}^2 u_{2,t-1}^2 + b_{22}^2 h_{22,t-1} \tag{3.44c}$$

In addition, the statistical significance of estimated parameters is ambiguous due to the reason that squared parameters and combinations of different parameters act as new coefficients in BEKK models. As a result, the intuitional effect of parameters in a standard bivariate GARCH model is lost in BEEK.

Table 6.5 reports the estimation results of diagonal bivariate BEKK GARCH(1,1) models. Similar to those of bivariate GARCH(1,1), the estimation results contain three categories of outcome: estimated parameters, log-likelihood function values and Ljung-Box statistics. Unlike bivariate GARCH(1,1), the maximum likelihood method used to estimate BEKK GARCH models is BFGS algorithm [27]. BFGS is asymptotically equivalent to BHHH, but may produce different estimates for small samples (Brooks, 2002).

Although the coefficient estimates are not intuitional for interpretation, they generally indicate similar information to that of bivariate GARCH models. Volatility clustering is implied by significant coefficient a_{11} for most firms. Edinburgh Oil and Gas is the only firm having an insignificant coefficient a_{11}. For all firms, coefficient a_{22} is positive and significant. Since the values of coefficients a_{11} and a_{22} are all less than unity; it ensures that the squared coefficients a_{11}^2 and a_{22}^2 are also less than unity in size. This is important, because the squared coefficients a_{11}^2 and a_{22}^2 are the actual ARCH parameters of diagonal BEKK according to equations (3.44a) and (3.44c). Their values imply volatility clustering in most cases although the statistical significance of the real ARCH parameters is indefinite.

Coefficient b_{11} is positive and significant at 1% level in all cases. Since squared parameter b_{11}^2 is the actual GARCH coefficient for the firm variance equation, the GARCH effect is arguably significant for all firms but the exact statistical significance is indefinite. Similarly, coefficient b_{22} is positive and significant at 1% level for all firms, suggesting that BEKK captures the GARCH effect in the return series of the market. Additionally, GARCH coefficients (b_{11}^2 and b_{22}^2) are very high; and the sums of the ARCH and GARCH coefficients ($a_{11}^2 + b_{11}^2, a_{22}^2 + b_{22}^2$) are fairly close to unity. Therefore, there is generally a high degree of volatility persistence. Furthermore, the products of coefficients ($a_{11}*a_{22}$, $b_{11}*b_{22}$) act as the covariance parameters in equation

[27] BFGS algorithm developed by a series of studies (see Broyden 1965; Fletcher and Powell, 1963 for example) and named after Broyden, Fletcher, Goldfarb and Shannon.

(3.44b). Positive coefficients ensure that the products are also positive defined; thus indicating positive interactions between the firms and the market.

A battery of diagnostic tests based on the Ljung-Box statistics is employed to verify specification adequacy of BEKK. The same as bivariate GARCH, BEKK GARCH models are detected with significant Ljung-Box statistics in eight cases (British Airways, TESCO, BT Group, Glaxo Smith Kline, Boots Group, Care UK, Daily Mail and Gen Trust, BAE Systems). Among them, seven firms are found to have significant Ljung-Box statistics in bivariate GARCH results. Hence, the BEKK model exhibits similar descriptively ability to the standard GARCH model. Once again, further cross-product tests are used to assess the general descriptive validity of the BEKK model. Similarly, the cross-product tests indicate that BEKK GARCH(1,1) is generally sufficient to estimated the conditional beta, since the Ljung-Box statistics detect serial correlation in only four cases (TESCO, BT Group, Edinburgh Oil and Gas, Boots Group). Therefore, there is no need to encompass a higher ARCH process.

6.2.2.2 Basic Statistics of the Time-Varying Beta

Table 6.6 reports basic statistics of the time-varying beta series estimated by BEKK models. For all twenty firms, the mean value of the conditional beta is positive and significant at 1% level. Similar to estimation results of bivariate GARCH, Tottenham Hotspur has the smallest beta (0.58031); and Cable and Wireless has the largest beta (1.21609). Nevertheless, the variance column in Table 6.6 presents different results from bivariate GARCH, as the time-varying beta of Alvis is found to be most volatile; and the conditional beta of Barclays is most stable. Except Tottenham Hotspur, all time-varying betas are significantly skewed, thus implying asymmetrical distribution in most instances. For all beta series, excess kurtosis is positive and significant at 1% level. Consequently, all betas are distributed as leptokurtic. As a result, none of the conditional beta series can be accepted for the null of normal distribution, which is confirmed by the significant Jarque-Bera statistics in all cases.

6.2.2.3 Unit Root Tests of the Time-Varying Beta

Once again, DF and ADF tests are used to detect the presence of unit roots in the beta series. As done earlier, both DF and ADF tests allow for a constant. The nulls of two unit roots and one unit root are tested in order. The results are reported in Table 6.7 and 6.8 respectively. In Table 6.7, all test statistics from DF and ADF test for two unit roots are significant at 1% level, which indicates that all betas are free of two unit roots. Furthermore, the null of one unit root is rejected for all beta series, as all DF and ADF test statistics are significant at 1% level in Table 6.8. Therefore, DF and ADF tests provide the same evidence for conditional betas estimated by bivariate GARCH and BEKK, that time-varying betas are stationary in levels.

In terms of basic statistics and unit root tests, the daily conditional betas estimated by BEKK share several similarities with those generated by standard GARCH. For instance, both models find the highest and lowest mean value of conditional beta in the same companies; and unit root test statistics are all significant at 1% level in both cases. On the other hand, conditional beta series constructed by BEKK and bivariate GARCH exhibit different statistical features in terms of variance; as the two models find highest and lowest variance in different firms.

6.2.3 Bivariate GJR GARCH(1,1) Model

6.2.3.1 Estimation Results

Named after Glosten, Jagannathan and Runkle, the GARCH-GJR model is able to capture the leverage effect of financial time series. The leverage effect describes the broadly reported phenomenon that negative shocks to financial time series are likely to increase volatility more than positive shocks of the same magnitude. In other words, 'bad news' has a greater impact on volatility than 'good news'. Compared to the bivariate GARCH(1,1) model, the GJR extension has two additional parameters (r_1 and r_3) incorporated to allow for asymmetric responses. Section 3.3.2.3 introduces the bivariate form of GJR model in details.

Table 6.9 presents estimation results of bivariate GJR GARCH(1,1) models, which are estimated by means of BHHH algorithm. Estimation results include three categories of information: coefficient estimates, log-likelihood function values and Ljung-Box statistics. In addition to the nine standard GARCH parameters, GJR has two more parameter r_1 and r_3 incorporated to capture the leverage effect in the firm and the market respectively[28]. With the added parameters, the ARCH parameter in the GARCH-GJR model switches between a_1+r_1 and a_1 for the firm, and between a_3+r_3 and a_3 for the market, depending on whether the lagged error term is positive or negative. Therefore, the sign and significance of parameters r_1 and r_3 are of particular interest among the estimation results, as the other parameters are the same as those of standard GARCH models.

Positive and significant coefficients r_1 and r_3 imply the presence of leverage effects. However for six firms (TESCO, Legal and General, Boots Group, British Vita, Daily Mail and Gen Trust, BAE Systems), coefficient r_1 is negative, providing contrary evidence against the leverage effect. For these firms, 'good news' tends to have a greater impact on volatility than 'bad news'. Among them, the statistical significance of negative coefficient r_1 shows such abnormal features are only substantial in two cases (TESCO, Daily Mail and Gen Trust); as negative r_1 estimates are significant only for two firms. Moreover, the absolute values of negative r_1 coefficients are all less than the corresponding ARCH parameters a_1 that are all positive. Hence the sum of the two parameters (a_1+r_1) is always positive, which indicates positive ARCH terms and non-negative constrains in all cases. Among the fourteen positive r_1 estimates, twelve are significant; and two are insignificant. Therefore, the leverage effect is detected in more then half of the twenty UK firms.

Coefficient r_3 is also found to be negative in six cases (British Airways, Edinburgh Oil and Gas, Barclays, Goodwin, Cable and Wireless, BAE Systems). Except British Airways, the other negative coefficients r_3 are significant at 1% level. Therefore, adverse evidence against the leverage effect in the market is found in five cases. Additionally, all negative coefficients r_3 are less than corresponding ARCH parameters a_3, which ensures the sum of the two parameters (a_1+r_1) is positive; and thus the ARCH terms are positive. Among the fourteen positive r_3, seven coefficients are statistically significant; while the remaining seven are insignificant. Therefore, daily data provide conflicting evidence of the leverage effect in the market return.

[28]Likelihood ratio tests indicate that GARCH-GJR is superior to bivariate GARCH in estimating time-varying betas, with significant higher log-likelihood function values than bivariate GARCH. Results are not presented to save space.

The same as the results of bivariate GARCH models, the ARCH coefficients (a_1 and a_3) of GJR models are positive and significant at 1% level for all firms. As mentioned above, the size of coefficients a_1 and a_3 is substantially larger than the absolute values of r_1 and r_3. Therefore, GJR generally catches significant volatility clustering in all returns series. Similarly, all the GARCH coefficients (b_1 and b_3) are positively high and significant at the 1% level; indicate evident GARCH effects. Furthermore, the sums of the ARCH and GARCH coefficients ($a_1+r_1+b_1$ or a_1+b_1, $a_3+r_3+b_3$ or a_3+b_3) are all close to unity, suggesting a high degree of volatility persistence in all cases. For all firms, the covariance coefficients (a_2 and b_2) are positive and significant at 1% level, which implies a positive and significant interaction between the firm and the market.

The Ljung-Box statistics reported in Table 6.9 are similar to those of bivariate GARCH reported in Table 6.1. Serial correlation is detected in seven cases (British Airways, TESCO, BT Group, Glaxo Smith Kline, Boots Group, Care UK, Daily Mail and Gen Trust), one (Alvis) less than the results of standard GARCH models. Further joint tests for serial correlation in the product of the standardised residuals of the firm and the market are implemented for extra diagnosis. The Ljung-Box statistics from the cross-product tests reject the null of serial correlation in most cases; as the twelfth order statistics are significant in two cases (BT Group, Boots Group). The results of cross-product tests are the same as those of bivariate GARCH models. According to Giannopoulos (1995), lack of serial correlation implies absence a higher order ARCH process. Consequently, bivariate GJR GARCH(1,1) models are generally valid to estimated time-varying betas.

6.2.3.2 Basic Statistics of the Time-Varying Beta

Table 6.10 reports basic statistics of the time-varying beta series estimated by means of the GARCH-GJR model. The mean values of time-varying betas are all positive and significant at 1% level. The conditional beta of Tottenham Hotspur has the smallest mean (0.57606), while the beta of Cable and Wireless has the largest mean (1.21946). The conditional beta of Daily Mail and Gen Trust has the lowest variance; while the beta of Care UK has the highest variance. Six firms (British Airways, BT Group, Legal and General, Glaxo Smith Kline, Barclays, Cable and Wireless) are aggressive shares with their beta values greater than unity, while most of the rest are defensive stocks with the beta less than unity. All these are the same as the bivariate GARCH results. As bivariate GARCH and GARCH-GJR are nested models, it is reasonable to expect them to construct similar time-varying betas.

According Table 6.10, only the betas of Tottenham Hotspur and Goodwin are not rejected as symmetrical distribution with insignificant skewness statistics. All the other skewness statistics are significant, which imply that most conditional betas are asymmetrically distributed, either positively or negatively skewed. Statistics of excess kurtosis are positive and significant at 1% level for all beta series, indicating fatter tails than normal distribution in all cases. Therefore, none of the conditional beta can be accepted as normal distribution. Their nonnormality is confirmed by the Jarque-Bera statistics, which are significant at 1% level for all time-varying betas.

6.2.3.3 Unit Root Tests of the Time-Varying Beta

Once again, DF and ADF tests are used to detect the presence of unit root in the time-varying betas estimated by GJR. Table 6.11 and 6.12 respectively present the results tests for the nulls of two and one unit root. In Table 6.11, all test statistics are significant at 1% level, indicating the absence of two unit roots in all beta series. Moreover, the null of one unit root is also rejected as all statistics are significant at 1% level in Table 6.12. Therefore, time-varying betas estimated by the bivariate GARCH-GJR model are stationary in levels.

6.2.4 Bivariate GARCH-X(1,1) Model

6.2.4.1 Estimation Results

Proposed by Lee (1994), the bivariate GARCH-X model allows for the effect of short term deviations between two cointegrated series, with the lagged error correction term incorporated in conditional variance and conditional covariance equations. Details of cointegration and GARCH-X model are discussed in section 3.3.2.4 of this thesis.

As reported in chapter 5, the two-step Engle and Granger (1987) tests detect cointegrated relationship between the five firms and the market (Legal and General, Glaxo Smith Kline, British Vita, Alvis and Care UK). For these firms, short-run deviations from company and market indices may affect the conditional variance and conditional covariance; and thus they can also influence the time-varying beta. Hence, GARCH-X approaches are only applicable to these five companies. The estimation results are reported in Table 6.13, which contains three categories of information: estimated parameters, log-likelihood function values and Ljung-Box statistics.

The important part of interpreting the results of GARCH-X is the sign and statistical significance of the three parameters of error correction term (d_1, d_2 and d_3), which explain the influence of the short-run deviations between the share price and the market index on the conditional variance and covariance. According to Table 6.13, all the three parameters are positive and significant, which imply that short-run deviations impose a considerable effect on the conditional variance of returns of the firm and the market and also on the conditional covariance between the two returns. Such a considerable effect of short term deviations on volatility suggests that GARCH-X is successful in modelling the conditional variance and covariance with the extra parameters incorporated[29].

Besides the additional parameters, estimate results of elementary GARCH parameters are quite standard, as the nine parameters are positive and significant at 1% level for all firms. The ARCH coefficients (a_{11} and a_{33}) are significantly positive and less then unity in size. Thus the significance and size of ARCH coefficients implies volatility clustering returns of the firm and the market in all cases. The GARCH coefficients (b_{11} and b_{33}) are relative high and significant, presenting evidence of GARCH effects. In general, the sums of the ARCH and GARCH coefficients ($a_{11} + b_{11}$, $a_{33} + b_{33}$) are moderately close to unity, showing a high degree of volatility persistence. Since the covariance coefficients (a_{22} and b_{22}) are positive and significant at 1% level, all firms exhibit positive and significant interactions with the market.

Specification adequacy of the GARCH-X(1,1) model is verified through the serial correlation test of white noise. According to the Ljung-Box statistics of twelfth order,

[29]Likelihood ratio tests indicate that GARCH-X is superior to bivariate GARCH in estimating time-varying betas, with significant higher log-likelihood function values than bivariate GARCH. Results are not presented to save space.

serial correlation is detected in three cases with significant statistics on the standardised residual of the firm equation (Glaxo Smith Kline, Alvis, Care UK). The GARCH-X model seems to be descriptively inferior to other GARCH models, with significant Ljung-Box statistics in over half of the firms. Once again, further cross-product tests are employed to examine the serial correlation in the product of the standardised residuals of the firm and the market. The Ljung-Box statistics of cross-product tests are all insignificant, showing that model completely captures the ARCH pattern in series. Therefore, the diagnostic test results are satisfactory and the GARCH-X(1,1) model is valid in this case.

6.2.4.2 Basic Statistics of the Time-Varying Beta

Table 6.14 reports basic statistics of the five time-varying beta series estimated by means of GARCH-X models. The mean values of conditional betas are all positive and significant, ranging from 0.70334 (Care UK) to 1.12977 (Legal and General). According to variance, the time-varying beta of Glaxo Smith Kline is the most stable one; while the beta of Care UK is found to be most volatile. All conditional beta series are found to be significantly skewed with skewness statistics significant at 1% level; and thus they are all rejected as symmetries. In addition, statistics of excess kurtosis are positive and significant at 1% level for all time-varying betas, indicating fatter tails than normality. Therefore all conditional betas are rejected for the null of normal distribution. This is confirmed by their Jarque-Bera statistics that are all significant at 1% level.

6.2.4.3 Unit Root Tests of the Time-Varying Beta

Table 6.15 and 6.16 show the results from DF and ADF tests for two and one unit root of the conditional beta. ADF tests are conducted with 3, 6 and 9 lagged differences. All test statistics in both tables are significant at 1% level, showing the absence of unit root in conditional betas. Thus, the time-varying beta series estimated by the GARCH-X model are stationary in levels.

6.2.5 Kalman Filter Approach

6.2.5.1 Preliminary Analysis on Transition Equation

In this thesis, Kalman filter approach stands for non-GARCH models in competition with GARCH-type models for predicting the conditional beta. Based on the state space model, the Kalman filter method can be used to incorporate unobserved variables into, and estimate them along with, the observable model to impose a time-varying structure of the CAPM beta. Section 3.4 provides details for the Kalman Filter approach. Once again, BHHH algorithm is used as the optimisation method to estimate the twenty time-varying beta series.

As mentioned before, the time-varying characteristics of beta can me modelled by different dynamic approaches. For instance, Faff *et al.* (2000) use three types of transition equation to capture the dynamic process of beta, including AR(1), random coefficient and random walk. Beside the three dynamic processes, random walk with drift is also considered in this thesis.

Theoretically, there is no common view on the superiority of alternative dynamic approaches. Therefore, appropriate preliminary analysis on performance of different transition equations is helpful for further investigations. Two statistical criteria, Akaike information criterion (AIC) and Bayesian information criterion (BIC)[30], based on the log-likelihood function value are calculated to evaluate performance of alternative dynamic models. Both criteria follow the same rule for model selection. The preferred model is the one with the lowest AIC or BIC value.

Table 6.17 and 6.18 show the AIC and BIC derived from the estimation results of Kalman filter method based on four different state equations. Although the AR(1) model has a larger number of parameters to be estimated than other forms of state equations, both AIC and BIC are generally smaller than other dynamic processes. However, it encounters convergence difficulty in most cases and only has five optimal results. This is similar to Faff *et al.* (2000), where AR(1) seems to be worse than random walk and random coefficient parameterisations with the lowest convergence rate. Such an analysis of the convergence rate of each model can give a valuable insight into the underlying dynamic process of time-varying beta, as a low convergence rate is indicative of a misspecification of the transition equation. An

[30] Proposed by Akaike (1974) and Schwarz (1978), AIC and BIC are measures of the goodness of fit of an estimated statistical model. The models being compared need not be nested, unlike the case when models are being compared using a likelihood ratio test.

examination of Table 6.17 and 6.18 also shows that random walk with drift is not successful in describing the dynamic process of conditional beta. It has the largest AIC and BIC and a low convergence rate. Random coefficient and random walk seem to have the similar magnitude of AIC and BIC. However, random walk appears to give the best characterisation of the time-varying beta. It has no difficulty to converge in all twenty cases, which is important to be a reliable competitor with GARCH models. Therefore, random walk is the best transition equation; and random walk itself is sufficient to compete with GARCH models in forecasting the time-varying beta[31].

6.2.5.2 Basic Statistics of the Time-Varying Beta

Table 6.19 reports basic statistics of the time-varying beta series estimated by the Kalman filter approach. For all the firms, the mean of time-varying betas is positive significant. Similar to results of GARCH models, six firms (British Airways, BT Group, Legal and General, Glaxo Smith Kline, Barclays, Cable and Wireless) have betas greater than unity, among which Cable and Wireless has the largest mean value (1.24819). On the contrary, the beta of Daily Mail and Gen Trust has the smallest mean value (0.50925). Except two firms (Barclays, Caldwell Investments), the statistics of skewness are all significant, either positive or negative. Hence, only the conditional betas of Barclays and Caldwell Investments can be accepted as symmetric distribution. In addition, only the beta series of British Vita is found to have an insignificant excess kurtosis. Five conditional beta series exhibit peaked distribution with negative and significant excess kurtosis, which is not found in beta series estimated by GARCH models. The remaining fourteen conditional betas exhibit a flat distribution with positive and significant excess kurtosis. No beta series can be accepted as normal distributed, as the Jarque-Bera statistics are all significant at 1% level.

6.2.5.3 Unit Root Tests of the Time-Varying Beta

Table 6.20 and 6.21 report the results from DF and ADF tests for two and one unit root of the beta series estimated by Kalman filter models. In Table 6.20, all test

[31] Faff *et al.* (2000) find that dominance of Kalman filter method over GARCH models remains regardless of the form of transition equation, although random walk produces most accurate forecasts.

statistics are significant at 1% level. Thus, the null of two unit root is rejected for all the conditional betas. Tests results for one unit root presented in Table 6.21 provide diversified evidence on the stationarity of conditional betas. Not all test statistics are significant. Insignificant statistics are found in four time-varying betas (TESCO, British American Tobacco, Boots Group, Goodwin). However, only one of the ADF test results is insignificant for Boots Group, Goodwin; while the other three statistics are significant implying the absence of unit roots. For TESCO and British American Tobacco, there are at least two significant test statistics, rejecting the null of one unit root. Thus, all time-varying betas are stationary at first difference; and most of them are also rejected as having one unit root. However, results of DF and ADF with different lags are conflicting in few cases (TESCO, British American Tobacco), mixed of both significant and insignificant statistics. Therefore, conditional betas estimated by Kalman filter exhibit some different characteristics of dynamic structure from those estimated by GARCH class models.

In general, conditional betas estimated by GARCH models and Kalman filter method exhibit similarity in terms of distributional statistics, especially the mean value of time-varying betas. This implies the success of all these models. On the other hand, Kalman filter shows a few unique features with GARCH models in modelling time-varying betas, such as the presence of peak distribution and unit root in some cases. This is not surprising, given the fact that GARCH models and the Kalman filter method construct the conditional beta in distinguishing manners. In contrast to the GARCH models where the conditional beta series can only be calculated after the conditional variance and covariance have been obtained, the Kalman filter approach allows to estimate the time-varying structure of beta directly.

6.3 Comparison Analysis of Beta Estimates

Conditional beta series constructed by different modelling techniques can be easily compared in terms of the mean values and the visual graphs. Table 6.22 sums up the mean values of time-varying beta estimates calculated by various methods. The last column in Table 6.22 is the point estimates of beta by means of the market model, which provide a moderate reference for the precision of time-varying beta series. The first three columns show that all the conditional betas estimated by GARCH class models (bivariate GARCH, BEKK, GJR and GARCH-X) are fairly close to each other. Also, GARCH class models and the Kalman filter approach seem to generate conditional betas with similar mean values; although the similarity is less significant in some cases (see TESCO for example). Moreover the means of estimated time-varying betas are generally close to the point estimates of unconditional beta, showing that the modelling techniques not only capture the time variation feature of systematic risk but also measure systematic risk appropriately.

There may be considerable differences among time-varying beta estimates from the perspective of the whole range of series, although the mean values are reasonably close. Following Faff *et al.* (2000), a graphical analysis on the time series characteristics of the conditional betas is carried out for further perspective on the performance of the alterative models. For all the twenty firms, comparison based on visual observation leads to similar conclusion on the similarities and differences among the alternative models. The time-varying betas of two firms (Legal and General, Glaxo Smith Kline) are presented in Figure 6.1 and 6.2 to illustrate the result of graphic analysis. Both figures display graphs of beta series generated by different approaches on the same scale.

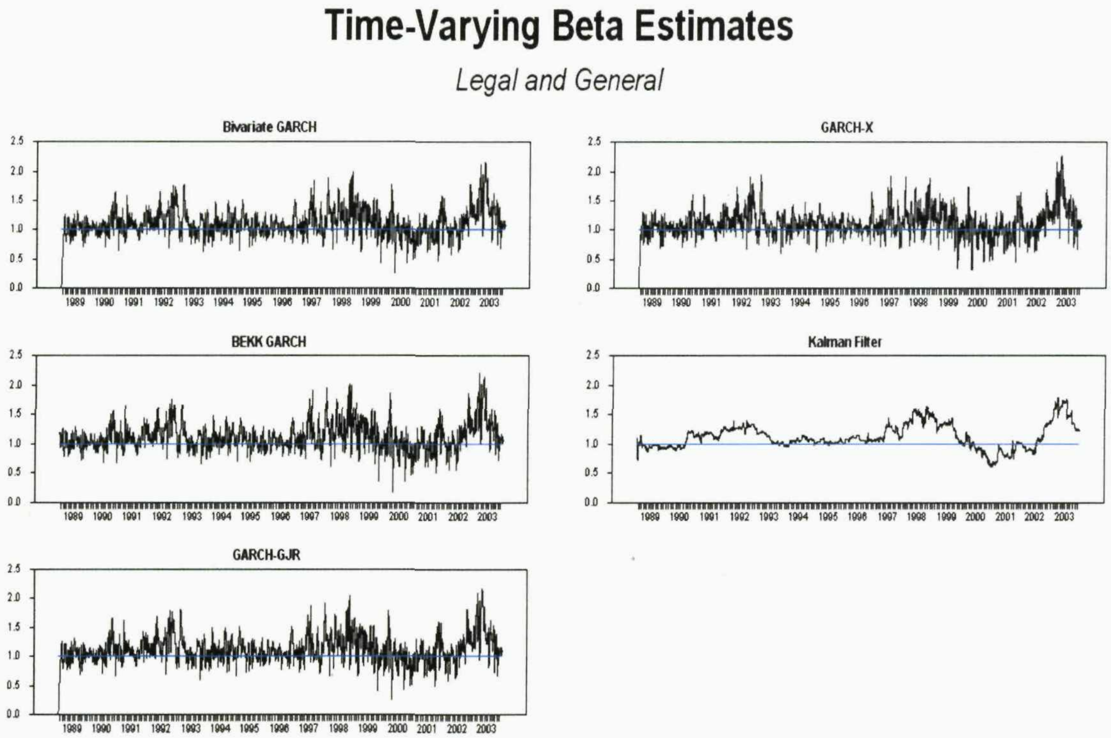

Figure 6.1: Time-Varying Beta Estimates (Legal and General)

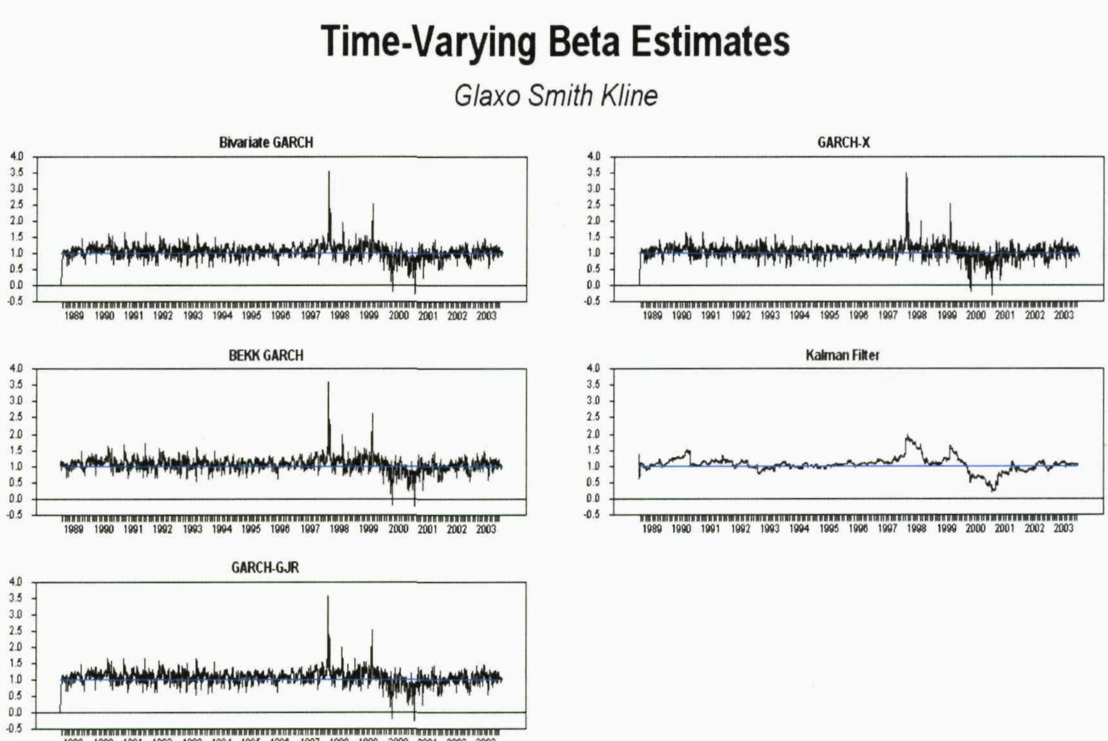

Figure 6.2: Time-Varying Beta Estimates (Glaxo Smith Kline)

The range of beta series shows that there are significant differences between the GARCH models and the Kalman filter approach. In Figure 6.1 and 6.2, the conditional betas estimated by the GARCH class models exhibit remarkable time variation features; while Kalman filter betas appear to be much more stable and smoother, showing much less sensitive to time variability. On the other hand, similarities are also found between GARCH models and Kalman filter methods; as conditional betas estimated by GARCH and Kalman filter models follow the comparable outline of movement over the estimation period. Visually, graphs of GARCH time-varying betas revert to the graph of Kalman filter betas, given that the extreme high and low values are equivalently removed. This once again indicates that both GARCH models and Kalman filter method provide a reasonable parameterisation of systematic risk.

Previous studies provide conflicting empirical evidence the relative degree of variation between conditional betas estimated by GARCH and Kalman filter. While Brooks *et al.* (1998) find that the GARCH beta exhibits a higher degree of variation than the Kalman filter beta; the opposite is true according to Faff *et al.* (2000). However both these studies used the constant correlation GARCH model by Bollerslev (1990), in which the correlation between the conditional variances is assumed to be constant to derive the conditional covariance equation. The assumption of constant correlation reduces the computational burden; however it also loses flexibility in modelling the conditional second moments. GARCH models in this study have no constraint correlation between the conditional variances; and therefore result in a more flexible description of volatility clustering and a higher degree of time variation in GARCH betas. Moreover, as the name suggested, the recursive algorithm of Kalman filter implicitly filters noisy observations and thus generates smoother results when used to construct time-varying beta series.

Finally, GARCH models exhibit different degrees of amplitude in some cases. Evidence is found that the standard GARCH model produce more moderate conditional betas relative to BEKK and GJR. In Figure 6.3, BEKK produces the most extreme values; while GJR generates the smallest amplitude.

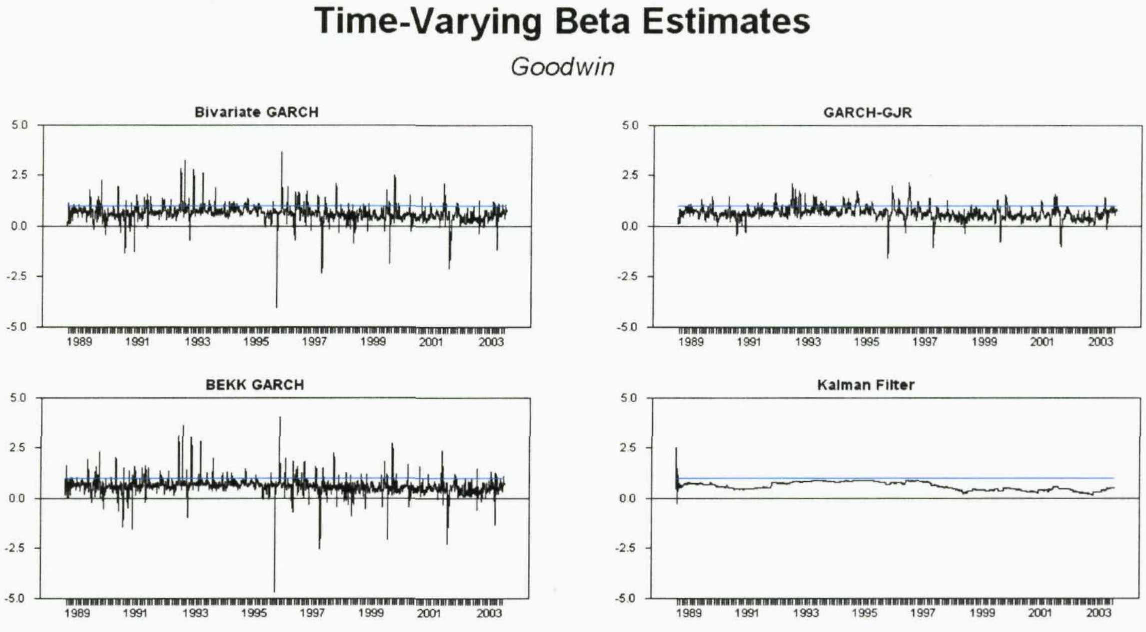

Figure 6.3: Time-Varying Beta Estimates (Goodwin)

In summary, the comparison analysis in terms of the mean values and graphs present an insight into the performance of alternative models in capturing the variation in beta series. Supportive evidence is found for the success of each method in providing parameterisations of systematic risk; as their mean values are fairly close to the point estimate of beta calculated by the market model. When observing the whole range of time-varying beta series, similarities of beta graphs suggest appropriate capability of alternative models to estimate conditional betas. On the other hand, a few dissimilarities are also noticeable. In general, the GARCH models are more sensitive to time variation than Kalman filter due to different model and algorithm features. In some cases, BEKK tends to produce more extreme values; while GJR tends to generate smoother results. However, we cannot complete the ranking of models simply based on comparison of their beta estimates. This is the task to be achieved in the next section, where comparison of forecast errors provides quantitative persuasion of the relative superiority of alternative forecasting methods.

6.4 Forecasting Time-Varying Betas

As mentioned before, three forecast periods are chosen in this thesis, including two one-year forecast periods (2001, 2003) and a two-year forecast horizon (2002 to 2003). As a result, each model is employed to estimate for three shorter periods (1989 to 2000, 1989 to 2001 and 1989 to 2002) and accordingly predict three forecast samples (2001, 2002 to 2003 and 2003)[32]. Forecasting with rolling and recursive windows are also considered to make beta out-of-sample forecasts. However, such multi-period schemes lead to serious GARCH convergence problem, which has also been reported in many previous studies (see Tse and Tung, 1992; Walsh and Tsou, 1998; Diodge and Wei, 1998 for example). Therefore, only the static forecasting is conducted.

6.4.1 Graphs of Beta Forecasts

To begin the comparison, examining the graphs of the forecasted beta and the real beta is helpful to approximately evaluate the performance of alternative model in an intuitive and straightforward way. In general, visualisation of beta forecasts provides similar evidence on the performance of different models. Figures 6.4, 6.5 and 6.6 illustrate the time-varying beta forecasts of three firms (Legal and General, Glaxo Smith Kline, British Vita) in the three forecast samples (2001, 2003, 2002 to 2003). All the figures indicate that GARCH-type models are extremely successful in predicting the conditional betas, with all the lines of forecasted betas and actual betas lapping over each other. Deviations between the real and the forecasted value are too small to be observed. On the other hand, the predictive ability of Kalman filter is intuitively inferior to GARCH models with significant gaps between the lines of forecasts and benchmarks. Especially in Figure 6.4 and 6.6, forecasting performance of Kalman filter is not as satisfactory as GARCH models. Graphs of the beta forecasts apparently diverge from those of the beta estimates, showing significant deviations between the forecasted and actual beta values.

[32] Estimation results of GARCH models for the shorter periods are similar to the results of the whole sample and are not presented to save space.

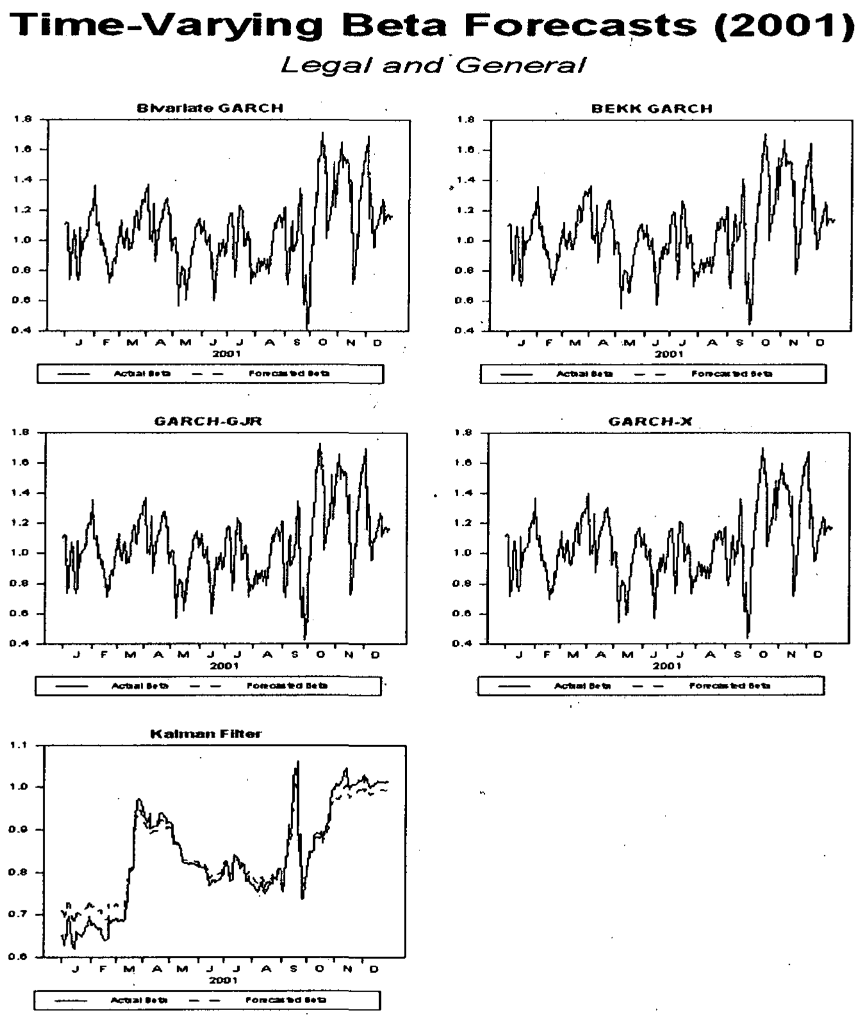

Figure 6.4: Time-Varying Beta Forecasts in 2001(Legal and General)

In summary, the graphical comparison of forecasting performance of alternative models suggests that GARCH type models outperform the Kalman filter approach. Among GARCH models, the visual inspection is not informative enough to rank the models, since all of them produce accurate and consistent conditional beta forecasts. As a result, the further comparative analysis based on quantitative forecast errors is necessary to rank the relative superiority of alternative models.

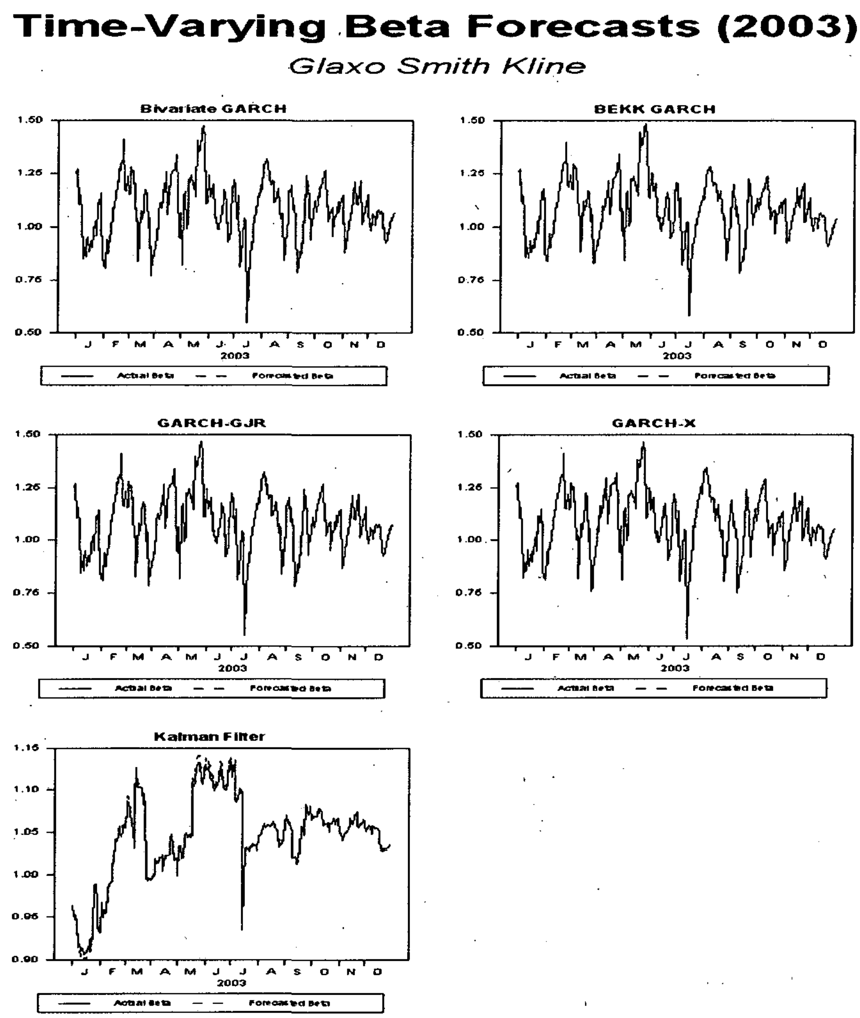

Figure 6.5: Time-Varying Beta Forecasts in 2003(Glaxo Smith Kline)

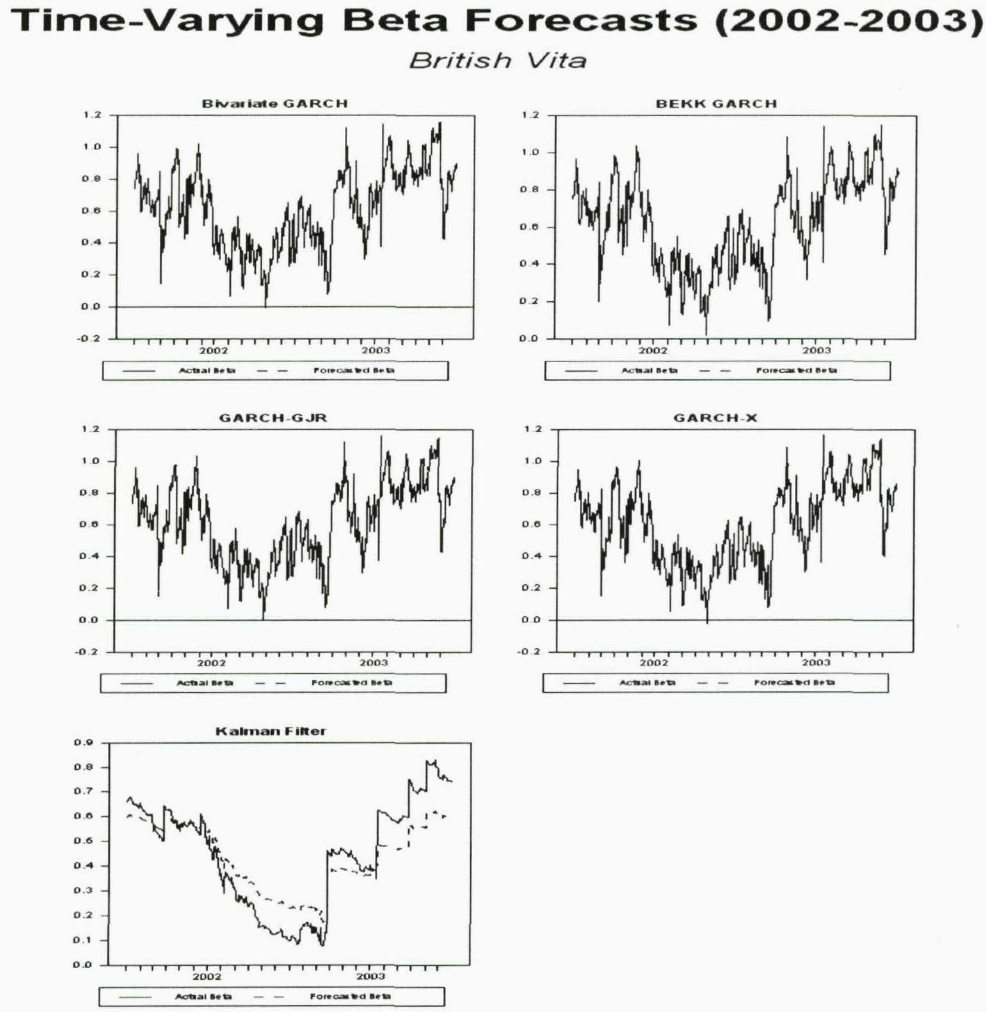

Figure 6.6: Time-Varying Beta Forecasts in 2002 to 2003 (British Vita)

6.4.2 Forecast Accuracy

6.4.2.1 Forecast Errors Based on Beta Forecasts

Forecast errors are employed to evaluate forecasting accuracy of alternative modelling techniques by investigating the level of deviations between conditional beta forecasts and estimates. As stated by Brailsford and Faff (1996), the ranking of forecasting models may be sensitive to the error statistic used. Therefore, a variety of measures are used to avoid bias, which include mean absolute error (MAE), mean square error (MSE), mean absolute percentage error (MAPE) and Theil U statistics. Different measures of forecast errors are discussed in section 3.5.1 and their results are reported in Table 6.23 to Table 6.34 over the three forecast samples.

(1) One year out-of-sample period: 2001

Tables 6.23, 6.24, 6.25 and 6.26 respectively report MAE, MSE, MAPE and Theil U of time-varying beta forecasts in the out-of-sample period 2001. According to the error statistics of MAE, BEKK seems to outperform other models with the smallest MAE in eight cases, followed by bivariate GARCH and GJR models with five and four smallest MAEs. Among the five applicable cases, GARCH-X has the smallest MAE in one forecast. In general, the MAE of GARCH forecasts are consistently low and fairly close to the smallest one for each company; also Kalman filter performs well in most cases and has three smallest MAE statistics. However, Kalman filter shows evidence of relative inferiority to GARCH models with a few extremely high errors. For instance, in the case of British Airways, the MAE of Kalman filter is about one hundred times greater than those of GARCH models.

The forecast error measure of MSE implicitly weights large forecast errors more heavily than small ones, since the quadratic loss function is considered. Thus, the ranking result may be different from MAE. MSE of beta forecasts reported in Table 6.24 confirms the superiority of BEKK and bivariate GARCH with seven and five smallest MSEs respectively. With respect to the ranking indicated by MAE, Kalman filter swaps its position with GJR with four smallest MSEs; while GJR models dominate others in three cases. Again, GARCH-X exhibits remarkable forecasting performance by having one the lowest value of MSE.

Statistics of MAPE measure errors in the percentage form and have the advantage of being dimensionless. MAPE reported in Table 6.25 draws to the same conclusion as MAE. Among the twenty beta forecasts, the BEKK GARCH model has seven lowest values of MAPE, followed by the standard GARCH model with five smallest MAPEs. GJR outperforms the other methods in four forecasts. Kalman is found to have an unusually large MAPE for TESCO; while it still dominates others in three cases. GARCH-X model has one smallest MAPE out of five applicable cases.

Similar to MAPE, Theil U statistics are dimensionless and have no scaling problem. Theil U statistics reported in Table 6.26 exhibit evidence of favouring BEKK and bivariate GARCH models. BEKK and bivariate GARCH dominate in nearly three fourths of the forecasts, with eight and six smallest statistics. GJR and Kalman filter have the best forecasting performance in three and two cases respectively. Again, GARCH-X only outperforms the other forecasting models in one case.

To summarise, the forecast error measures of out-of-sample 2001 suggest the BEKK GARCH model generates the most accurate beta forecasts in the forecasting horizon. The standard GARCH model has slightly inferior performance to BEKK. GARCH-GJR outperforms Kalman filter according to most error statistics. GARCH-X has a consistent predictive ability; it dominates in one forecast no matter which error criterion is used.

(2) One year out-of-sample period: 2003

Given the special market events of 11 September, the beta forecasts in 2001 may be insufficient to conclude the predictive ability of alternative models in one year forecast horizon. Another one year out-of-sample (2003) provides supplementary information on forecast accuracy of alternative methods. Measures of forecast errors are reported in Tables 6.27, 6.28, 6.29 and 6.30.

MAE reported in Table 6.27 indicates that no single model is favoured by the statistics. Bivariate GARCH, BEKK and Kalman filter exhibit equal superiority, as each model dominates for five firms. GJR also produce accurate forecasts with four smallest MAEs. GARCH-X outperforms others in one case according to MAE. Table 6.28 presents MSE of out-of-sample forecasts in 2003. With the quadratic loss function, bivariate GARCH and Kalman filter still exhibit relative superiority in forecasting conditional betas. Both bivariate GARCH and Kalman filter have six lowest values of MSE. BEKK has three smallest MSEs, losing three leading positions with respect to MAE. GJR dominates other forecasting models in four cases. GARCH-X is the best model in one forecast.

Results of MAPE are reported in Table 6.29. Unlike MAE and MSE, the percentage forecast error favours the GJR model; in the sense that GJR has six smallest MAPEs. Both bivariate GARCH and Kalman filter are superior to other models in five forecasts, followed by BEKK with four smallest MAPEs. GARCH-X fails to show its superiority in any forecasts. In Table 6.30, Theil U statistics find evidence for the superiority of bivariate GARCH, whose out-of-sample forecasts have six smallest Theil U statistics. GJR and Kalman filter each has five lowest Theil U statistics, showing a moderate forecasting performance. BEKK dominates other models in three forecasts; while GARCH-X is the most accurate model in one forecast.

In forecast horizon 2003, various measures of forecast errors present inconsistent rankings of the forecasting models. To sum up, bivariate GARCH is the most successful model in out-of-sample 2003; since it shows evidence of dominance in three out of four error statistics. However, the dominance is not significantly distinct, as bivariate GARCH shares the dominance with others in two cases. It is difficult to judge between GJR and Kalman filter; as their forecasting performances are so comparable. Both models dominate the others in two cases, and are ranked as the second best model in two forecasts. BEKK is approved to be successful in the out-of-sample 2001; however its performance deteriorates in 2003. GARCH-X performs consistently as it usually takes one leading position in the competitions.

(3) Two-year out-of-sample period: 2002 to 2003

Both forecast periods discussed above are one year horizon. Out-of-sample forecasts in the two-year period (2002 to 2003) help to assess the forecasting performance of models in a longer forecast sample. Table 6.31, 6.32, 6.33 and 6.34 report MAE, MSE, MAPE and Theil U statistics of conditional beta forecasts in 2002 to 2003. According to Table 6.31, Kalman filter is the best model with the smallest MAE for nine forecasted betas. BEKK outperforms the others in five cases in terms of MAE. Bivariate GARCH, GJR and GARCH-X each has two smallest MAEs. MSE reported in Table 6.32 confirms that Kalman filter is the most accurate model with best performance in ten forecasts. As the second best models, BEKK and GJR each has three smallest MSEs. Bivariate GARCH and GARCH-X have comparable performance, as they are most accurate models in two forecasts.

In terms of percentage error MAPE, Kalman filter is still the best model. It outperforms the other models in nine forecasts. BEKK is the second best model with five smallest MAPEs. In three cases, GJR show evidence of dominance with the smallest MAPE. Bivariate GARCH and GARCH-X have two and one lowest values of MAPE respectively. Theil U statistics presented in Table 6.34 indicate that the

Kalman filter method is superior in the longer forecast period, with lowest values of statistics in nine cases. Bivariate GARCH, BEKK and GJR exhibit similar predictive ability, as each has the smallest Theil U statistics in three cases. Again, GARCH-X has the smallest Theil U statistics in two forecasts.

To sum up the forecasting performance of alternative models in the two-year horizon, Kalman filter is the most accurate model, favoured by all error statistics. BEKK still has a remarkable performance when applied for longer sample forecasts. It is ranked as the second best model in terms of every statistics. Bivariate GARCH, GJR and GARCH-X are slightly inferior to the BEKK and Kalman filter. However, their predictive ability is acceptable, in the sense that each model outperforms others in a few forecasts.

Given empirical evidence on relative superiority of alternative models in different out-of-sample periods, it is difficult to conclude which model is superior to the others. Different models are found to dominate in different forecast samples. Also such empirical evidence from existing literature is absent, as no comparison in terms of beta forecasts and estimates has been done to our knowledge.

However it is reasonable to conclude that bivariate GARCH is the most accurate forecasting model in the one-year forecast sample. However, when the market is extremely volatile, BEKK becomes the most successful forecasting model, as it performs superiorly in out-of-sample 2001. For the two-year out-of-sample, Kalman filter is the most accurate model. Kalman filter fits to the market situation without significant volatility, but is less capable to predict betas with major market events (2001). This again indicates that the Kalman filter method is inferior to GARCH models in capturing time variation of beta series. The two elaborate GARCH extensions (GJR and GARCH-X) do not show improvements in one year out-of-sample forecasts. They both have similar performance to the standard GARCH in the longer forecast period by generating consistently accurate beta forecasts.

It is important to point out that the lack of benchmark is an inevitable weak point to compare time-varying beta forecasts; and it could be the reason for conflicting rankings in different samples. Since the beta value is not directly observable in the market, conditional beta estimates have to serves as the scale to evaluate conditional beta forecasts. However, forecast accuracy on the basis of comparing betas series estimated and forecasted by the same approach could be tricky and unreliable.

6.4.2.2 Forecast Errors Based on Return Forecasts

For better comparison analysis, Brooks *et al.* (1998) propose a logical extension to examine the accuracy of beta forecasts by comparing out-of-sample returns with actual returns. Using the following equation

$$\hat{r}_{i,t} = \hat{\beta}_{i,t} r_{m,t} \tag{6.1}$$

where $r_{m,t}$ is the risk premium of the market portfolio. Given out-of-sample forecasts of conditional betas ($\hat{\beta}_{i,t}$), out-of-sample forecasts of returns ($\hat{r}_{i,t}$) can be easily calculated using the equation above, in which the risk premium of the market portfolio is directly observable returns. The relative accuracy of conditional beta forecasts then can be assessed by comparing the market model return forecasts with the actual returns. The more the forecasted return series are close to actual values, the more accurate according forecasted betas are close to actual betas; and vice versa. In this way, investigation on forecast accuracy of forecasted betas is superseded by assessing precision of beta based returns; and the issue of missing benchmark for conditional betas can be resolved.

Similarly, the forecasting of returns is carried out in the three out-of-sample periods. To illustrate the forecasting results, Figure 6.7, 6.8 and 6.9 each displays the return forecasted by the different methods and the actual return for three firms (Legal and General, Glaxo Smith Kline, British Vita) in 2001, 2003 and 2002 to 2003. All estimates seem to move together with the actual return; but it is difficult to say which method shows the closest correlation between actual and forecasted returns. Generally, there is no model produces either perfect return forecast or extremely inaccurate forecast. In this case, relative superiority of forecasting methods can hardly be obtained from graphic analysis.

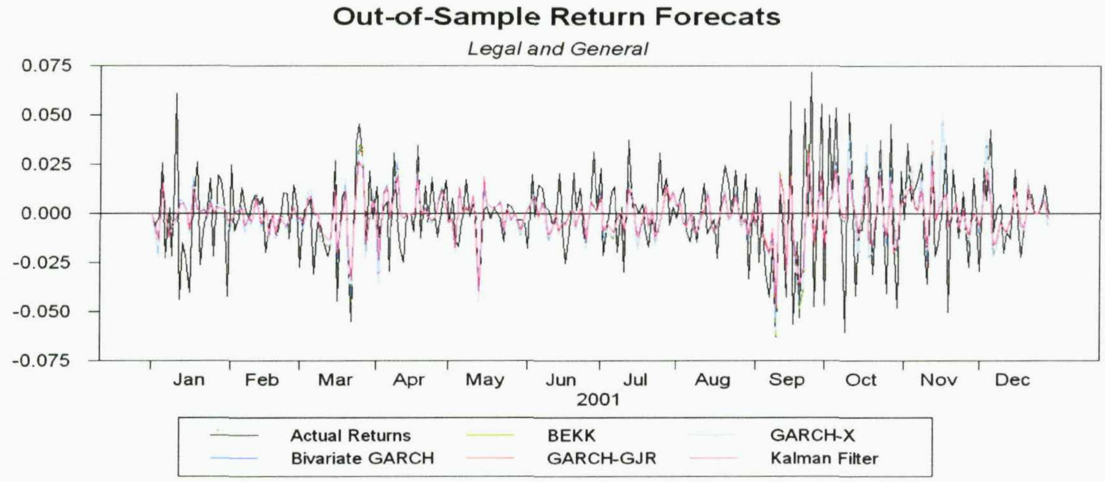

Figure 6.7: Time-Varying Return Forecasts in 2001 (Legal and General)

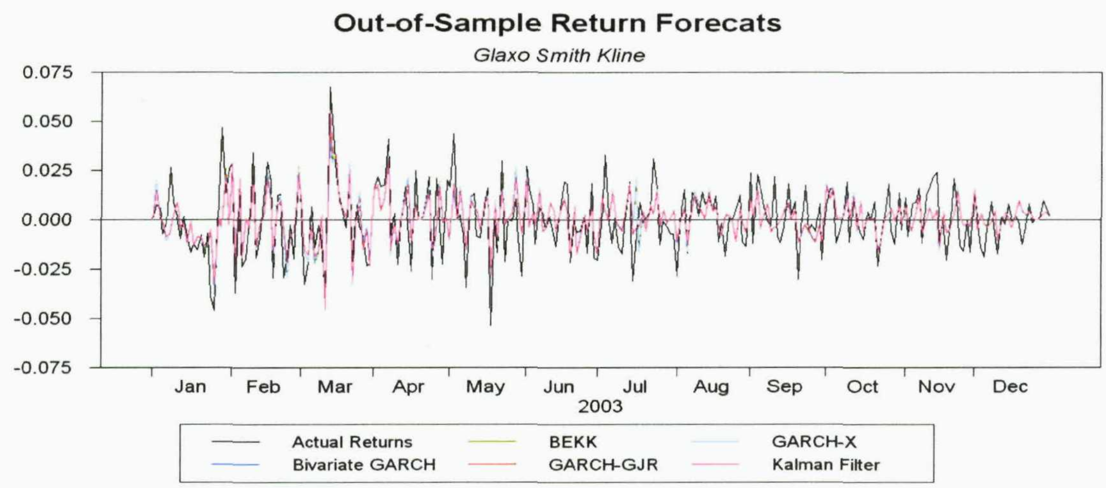

Figure 6.8: Time-Varying Return Forecasts in 2003 (Glaxo Smith Kline)

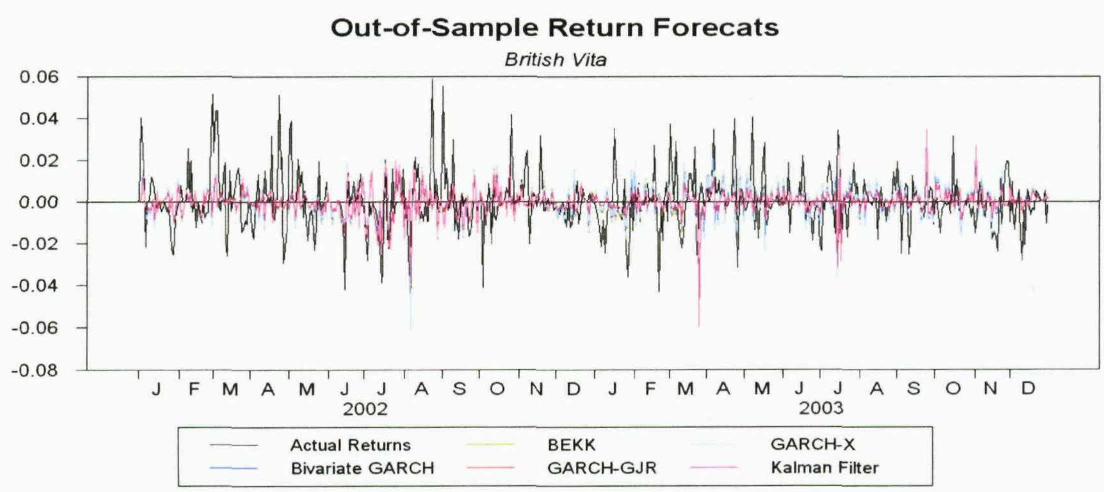

Figure 6.9: Time-Varying Return Forecasts in 2002-2003 (British Vita)

Another set of forecast error statistics are employed to assess the accuracy of return forecasts. Since the values of returns and return forecasts are relatively small in size and may take on opposite signs, measures of MAPE and Theil U statistics are not reliable in this case. Additionally, mean error (ME) is helpful to evaluate whether the model tends to over or under predict the return series, although ME is not a suitable measure for the forecasting accuracy.

(1) One year out-of-sample period: 2001

MAE, MSE and ME of return forecasts in out-of-sample 2001 are reported in Table 6.35, 6.36 and 6.37. According to MAE presented in Table 6.35, Kalman filter overwhelmingly dominates GARCH class models with eighteen lowest values of MAE. In the remaining two forecasts, GJR and bivariate GARCH share the glory by each having one smallest MAE. BEKK and GARCH-X has no smallest forecast error. However, it does not mean that they are inferior to other GARCH models. In fact, all MAEs are fairly close to each other. When forecast errors are compared only among GARCH models, BEKK and GARCH-X outperform the other GARCH models in five and two cases.

According to MSE reported in Table 6.36, Kalman filter approach is tremendously superior to GARCH models, with the lowest MSE values in nineteen forecasts. BEKK is found to be dominant in one case. Comparison among GARCH class models indicates that GJR is the most accurate model among GARCH models by outperforming the others in ten cases. Other GARCH models have similar performance, each dominating in three or four forecasts. ME statistics reported in Table 6.37 are not an appropriate measure for forecast errors, since the smallest ME does not guarantee the smallest forecast errors. However, the positive or negative sign of ME reveals the models over or under predict the return series. According to ME, all models tend to over predict the return values in 2001, since most ME are positive. The general over-prediction may be due to the reason that the financial market was significantly deteriorated by the tragic events of 11 September. In addition, all models tend to under forecast the returns in the same case with negative ME for the same company, exhibiting consistence in out-of-sample forecasts.

(2) One year out-of-sample period: 2003

Error statistics of out-of-sample forecasts in 2003 are reported in Table 6.38, 6.39 and 6.40. In Table 6.38, MAE statistics indicate that Kalman filter is overwhelmingly superior to GARCH models in terms of return forecasts, having the smallest MAEs in all twenty cases. BEKK is the most competent model among GARCH class models, with the ten lowest MAEs. MSE reported in Table 6.39 suggests that Kalman filter is still the most accurate models in forecasting returns when the quadratic loss function is used. In nineteen forecasts, Kalman filter has the smallest MSEs. GJR is the best one among GARCH type models in terms of MSE. According to ME reported in Table 6.40, no significant tendency of too high or too low forecasts is found. However, all models share the common tendency to under predict returns for the same firm, all the negative MEs appear at the same companies.

(3) Two-year out-of-sample period: 2002 to 2003

Measures of forecast error for the two-year out-of-sample return forecasts are reported in Table 6.41, 6.42 and 6.43. In Table 6.41, MAE statistics show that Kalman filter is dominant in all forecasts by having twenty smallest MAEs. BEKK is the best GARCH type models, outperforming other GARCH models in nine cases. Table 6.42 presents MSE of beta based return forecasts in the two-year forecast period. Once again, Kalman filter approach is favoured by MSE with the lowest value for all twenty forecasts. GJR becomes the best in GARCH models, when the quadratic loss function is used. In Table 6.43, positive and negative values of ME are mixed, implying that models do not tend to over or under forecast returns. However, GARCH, BEKK, GARCH-X and Kalman filter method together tend to under forecast returns for the same return series.

To summarise, it is a clear message that the Kalman filter approach is the most accurate forecasting technique, when forecasted returns are compared to actual returns. Kalman filter outperforms GARCH class models in most forecasts over different forecast samples. Therefore, forecast accuracy of return forecasts provides different evidence on relative superiority of alternative models from beta forecasts. This is understandable given the fact that Kalman filter and GARCH approaches model the conditional beta in contrasting ways. While volatility-based GARCH techniques construct conditional beta series indirectly via conditional variance and covariance, the state-space approach allows modeling the time-varying beta directly with the observation equation defined in form of the market model. In other words if the equation of conditional second moments cannot perfectly measure the value of beta, this deteriorates the forecasting performance of GARCH when compared in terms of return forecasts. In fact, the connection between beta and second movements implied by the CAPM is unfortunately imperfect in reality. Therefore, the distinguishing structure to model beta could be the main reason for the distinct performance between GARCH models and Kalman filter method in terms of return forecasts.

Among GARCH models, each produces comparably accurate return forecasts. More precisely, BEKK and GJR are slightly superior to other GARCH models. Interestingly, all models tend to over or under predict returns for the same firm.

6.4.2.3 Diebold and Mariano Tests

Evaluation of forecast accuracy of both beta forecasts and return forecasts above are based on a straightforward but reasonable principle that the lower the forecast error measure the better the forecasting performance. However when statistical significance is considered for the difference between the forecast errors, lowest values of forecast error cannot completely testifies superior forecasting performance, unless the forecast error is significantly smaller than the others. Diebold and Mariano (1995) develop a test of equal forecast accuracy to detect whether two sets of forecast errors have significantly different mean value. The modified test due to Harvey *et al.* (1997) improves the finite sample performance of Diebold-Mariano test. Both the original and the modified Diebold-Mariano tests are used to compare forecast errors of alternative models. However, both tests generate the same results at 10% significance level, since daily forecasts have a sufficient amount of observations. As a result, only the modified Diebold-Mariano test results are reported.

The modified Diebold-Mariano test is only valid for MSE and MAE derived from return forecasts. Each time, modified Diebold-Mariano tests are utilised to check superiority between two forecasting models through different forecast samples; thus there are ten groups of test for five models. For each group of test, there are six modified Diebold-Mariano tests for both MSE and MAE in three different forecast samples. Each modified Diebold-Mariano test generates two statistics, say S_1 and S_2, based on two hypotheses:

1. H_0^1 : there is no statistical difference between two sets of forecast errors.

   H_1^1 : the first set of forecasting errors is significantly smaller than the second.

2. H_0^2 : there is no statistical difference between two sets of forecast errors.

   H_1^2 : the second set of forecasting errors is significantly smaller than the first.

Once again, the significance of Diebold-Mariano statistics is defined as significant at least 10% level. Consequently, the statistics implies three possible answers to superiority between two forecasting models:

1. If S_1 is significant, then the former forecasting model is superior to the later one.
2. If S_2 is significant, then the later forecasting model is superior to the former one.
3. If neither S_1 nor S_2 is significant, then two models have equally accurate forecasts.

Table 6.44 to 6.53 present the proportion of firms giving the three answers, which provide reliable evidence for relative superiority of alternative forecasting models.

(1) Kalman filter and bivariate GARCH

Table 6.44 reports the percentage of firms accepting the three hypotheses about relative superiority of Kalman filter and bivariate GARCH. Over different forecast samples, Kalman filter approach is found to be significantly superior to bivariate GARCH model. In both one-year forecast horizons, 75% of the firms accept the first alternative hypothesis that the forecast errors of Kalman filter are significantly smaller than those of bivariate GARCH. In the two-year forecasts, the dominance of Kalman filter becomes more evident with the supportive evidence is found in more than 85% of the firms. No evidence is found that bivariate GARCH significantly outperforms Kalman filter method. The remaining firms all suggest that both forecasting models are found to produce equally accurate forecasts.

(2) Kalman filter and BEKK GARCH

Table 6.45 provides the evidence of dominance of Kalman filter over BEKK model. In the one year out-of-sample periods, at least 70% of the firms show that Kalman filter has significantly smaller MSE or MAE than BEKK. In remaining cases, BEKK shows evidence of equal accuracy with Kalman filter. In the two-year forecast sample, more than 80% of the firms significantly favour Kalman filter. None of the firms supports that BEKK is better forecasting model than Kalman filer.

(3) Kalman filter and GARCH-GJR

Results of modified Diebold-Mariano tests between Kalman filter and GARCH-GJR are reported in Table 6.46. Results are similar to those between Kalman filter and BEKK. For 70% of the firms, Kalman filter produces significant smaller forecast errors than GJR model. Evidence that GJR is better than Kalman filter is absent. Equal accuracy is also found to be in at least 10% cases in different forecast samples.

(4) Kalman filter and GARCH-X

Since GARCH-X can only be applied to five firms. Hence, a smaller group of forecast errors is available for Diebold-Mariano comparison tests between the two models. In Table 6.47, test results show that Kalman filter significantly dominates GARCH-X in forecast sample 2003 and 2002 to 2003, with more than 60% firms accepting the hypothesis of 'better'. In the volatile period 2001, most forecast errors are found to have no significant difference between each other, especially in terms of MSE that all firms present evidence of equal accuracy.

(5) Bivariate GARCH and BEKK GARCH

According to modified Diebold-Mariano test results in Table 6.48, the BEKK GARCH model has better forecasting performance than the bivariate GARCH model in 2003 and 2002 to 2003. However, bivariate GARCH is better than BEKK in 2001 with significant smaller forecast errors in one more cases. Over the three forecast samples, equal accuracy is supported by more than half of firms; thus the forecasting performance of these two models is rather close.

(6) Bivariate GARCH and GARCH-GJR

In Table 6.49, modified Diebold-Mariano tests provide evidence that both models may outperform the other in a few cases through different forecast periods. Moreover, GJR is slightly better than bivariate GARCH by having a higher percentage of dominance in most cases, expect MSE in 2001. On the other hand, most firms accept the hypothesis of 'equal accuracy'. Especially in 2001, 90% firms suggest that both models have similar levels of forecast errors, which implies that the additional parameters of GJR are not so functional in predicting severe price movements.

(7) Bivariate GARCH and GARCH-X

Table 6.50 present percentage of dominance of bivariate GARCH over GARCH-X. Comparison tests show there is no significant difference between errors in most cases. When MSE is used as the criterion, all differences between errors are insignificant in one year forecasts; and 95% differences are insignificant in 2002 to 2003. Therefore, both models have comparable forecasting ability.

(8) BEKK GARCH and GARCH-GJR

According to modified Diebold-Mariano test results in Table 6.51. In the forecast sample 2001 and 2002 to 2003, evidence is found that GJR outperforms BEKK with significantly smaller forecast errors in more firms. Nevertheless, BEKK is superior to GJR in forecast period 2003. Through different samples, at least 60% firms suggest the models generate equally accurate return forecasts.

(9) BEKK GARCH and GARCH-X

Results of modified Diebold-Mariano tests between BEKK GARCH and GARCH-X are reported in Table 6.52. In the forecast sample 2001 and 2002 to 2003, evidence is found that BEKK has significantly smaller forecast errors than GARCH-X. Nevertheless, their predictive accuracy becomes completely equal in 2003. Therefore, BEKK seems to be more capable than GARCH-X in the volatile and longer forecast period.

(10) GARCH-GJR and GARCH-X

Table 6.53 reports the results from the modified Diebold-Mariano tests between GJR and GARCH-X forecasting models. In the one year out-of-sample forecasts, Diebold-Mariano statistics provide evidence that the forecasting performance of GJR is slightly better than GARCH-X in terms of MAE, but both models are equally accurate in terms of MSE. In forecast period 2002 to 2003, GJR is favoured by MSE and not MAE. In general, most firms present evidence of equal accuracy for the models.

To sum up the Diebold-Mariano comparison tests, Kalman filter is the preeminent forecasting model, dominating all GARCH models with significantly smaller forecast errors. Among the GARCH models, the GJR specification is the best GARCH model, especially in the forecast period 2001 and 2002 to 2003. For a shorter forecast sample 2003 with less major market events, BEKK is found to be the most accurate one among GARCH class models. Bivariate GARCH and GARCH-X show somewhat a little inferior to GJR and BEKK. However, results suggest that the performance of GARCH models is comparable, as most firms indicate equal accuracy among their forecasts.

6.5 Conclusion

This chapter reports empirical results from forecasting time-varying betas of the twenty UK firms using daily data from 1989 to 2003. The whole chapter comprehensively discusses the performance of alternative modelling techniques and reports the process to determine the best forecasting model for time-varying betas.

Since the thesis seeks to combine the time-varying beta estimation results with forecasting outcomes in evaluating alternative models, estimation results of each model are described in details, in particular GARCH class models. For all GARCH models, the elemental GARCH coefficient estimates are all positive and significant, which implies the success of GARCH models in estimating time-varying betas on the daily basis. Additionally, the comparison analysis indicates that different GARCH models tend to construct similar beta series. In addition, the Kalman filter approach is less sensitive to time variation of systematic risk compared to GARCH models, which caused by the implicit filter feature of Kalman filter algorithm. However, each model is found to be successful in providing parameterisations of systematic risk, since the mean values of estimated betas are fairly close to the point estimates of CAPM beta.

A variety of comparison analyses are utilised to assess the modelling performance of alternative models. First, a visual inspection on graphs of conditional beta estimates and forecasts provides an intuitive perception of forecast accuracy of different models. The graphical comparison favours GARCH class model, as few deviations can be found between the graphs of estimated and forecasted betas.

Second, various measures of forecast errors based on beta forecasts are calculated to evaluate the relative superiority of alternative models. In generally, their performance varies in different samples. Bivariate GARCH is the most accurate forecasting model in the one year forecast sample 2003. However, the BEKK GARCH model generates more accurate beta forecasts than bivariate GARCH in 2001. When the out-of-sample forecast horizon becomes longer, the Kalman filter method outperforms its competitors. GJR and GARCH-X models do not show improvements on the standard GARCH model with additional parameters incorporated. The weak evidence of relative superiority is mainly due to the absence of the observable beta benchmark.

Third, forecast errors based on return forecasts are employed to evaluate out-of-sample predictive ability of both GARCH and Kalman filter models. Measures of forecast errors overwhelmingly support that the Kalman filter approach dominate other candidates. It is difficult to rank the performance of GARCH models. The last comparison technique used is modified Diebold-Mariano test. Taking statistical significance into account, modified Diebold-Mariano comparison tests find evidence in favour of the Kalman filter approach in terms of return forecasts. The dominance of Kalman filter can be due to distinguishing structure to model beta in which conditional betas are directly dependent on returns; while GARCH model construct conditional betas based on conditional variance and covariance. According to the modified Diebold-Mariano test, GJR is the best specification among GARCH models, especially in the out-of-sample period 2001 and 2002 to 2003. For a shorter forecast sample with less major market events 2003, BEKK is found to be the most accurate GARCH model. Bivariate GARCH and GARCH-X show similar forecasting performance and are slightly inferior to GJR and BEKK. However, as most firms indicate equal accuracy among GARCH models, the performance of GARCH models is fairly close in general.

As CAPM betas are widely used by market participators and researchers for various purposes, this thesis may be helpful for those who use the beta for practical decision-making or academic research conducting. Based on the UK daily empirical results, different models can be recommended for different purposes. Generally, GARCH class models are competent to estimate and forecast time-varying beta. Bivariate GARCH is ideal to model dynamic process of conditional betas in a relative normal market environment; while BEKK is appropriate for a more volatile situation.

However, they may produce extreme values of beta; and GJR is always an excellent reference as it produces fairly standard estimation results. To forecast the time-varying beta, different modelling techniques may be applied. If the purpose of forecasting time-varying beta is not directly associated with investment in stock markets, GARCH models, especially BEKK is a good choice, since BEKK effectively captures the time variation feature of the CAPM beta and produces moderately consistent and accurate forecasts of beta. Thus, BEKK is also suitable to establish measures for risk management purpose. If forecasted beta is used for decision making in stock markets, Kalman filter is an appropriate choice, since it is superior to GARCH models in terms of return forecasts.

Table 6.1 Part 1: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	British Airways	TESCO	British American Tobacco	BT Group	Legal and General
$c_1(10^{-5})$	1.6566[a]	2.6600[a]	1.2572[a]	7.3727[a]	1.4646[a]
	(11.73606)	(11.84477)	(18.67868)	(12.44745)	(14.41967)
a_{11}	0.0943[a]	0.1004[a]	0.1107[a]	0.0701[a]	0.0775[a]
	(24.91441)	(16.88117)	(20.22226)	(19.60477)	(17.68847)
b_{11}	0.8818[a]	0.8390[a]	0.8854[a]	0.9185[a]	0.8942[a]
	(180.93501)	(91.35937)	(198.63695)	(263.79824)	(184.31272)
$c_3(10^{-5})$	1.9197[a]	1.9150[a]	1.6013[a]	1.2466[a]	1.7815[a]
	(19.49672)	(20.09410)	(19.76441)	(22.55308)	(19.34802)
a_{33}	0.1477[a]	0.1542[a]	0.1582[a]	0.0990[a]	0.1117[a]
	(27.37178)	(28.23953)	(25.96353)	(30.07252)	(21.77310)
b_{33}	0.7617[a]	0.7589[a]	0.7818[a]	0.8433[a]	0.7963[a]
	(86.31893)	(89.53934)	(98.75096)	(175.42072)	(90.92689)
$c_2(10^{-5})$	1.5534[a]	1.5510[a]	1.1605[a]	0.8783[a]	1.2527[a]
	(14.01944)	(15.51582)	(21.19300)	(18.48230)	(19.14775)
a_{22}	0.1097[a]	0.1157[a]	0.1276[a]	0.0760[a]	0.0778[a]
	(22.72281)	(23.11335)	(28.40822)	(24.62380)	(19.93475)
b_{22}	0.8165[a]	0.8043[a]	0.8372[a]	0.8881[a]	0.8612[a]
	(97.34271)	(100.68495)	(167.60664)	(248.10980)	(152.78769)
L	28977.57	29638.81	29249.46	29833.51	29638.32

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	29.2294**	16.9872*	4.5018	24.4739**	7.5797
L-B(12)[e]	9.2729	9.6756	11.0250	13.4305	9.7878
$u_t / h_t^{1/2}$					
L-B(12)[d]	9.9196	34.0416**	6.7816	15.2652	9.9226
L-B(12)[e]	11.3795	13.6501	11.4934	14.7057	10.9318
L-B(12)[f]	8.5395	14.8163	11.1190	23.1013**	9.3622

Notes: t statistics in parentheses. L = log likelihood function value. L-B(12) = the Ljung-Box statistics of order 12. $u_t / h_t^{1/2}$ = standardised residuals. u_t^2 / h_t = standardised squared residuals.

[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.
[d] Ljung Box statistics for serial correlation of order 12 for the residuals of firm equations.
[e] Ljung Box statistics for serial correlation of order 12 for the residuals of market equations.
[f] Ljung Box statistics for serial correlation of order 12 for the cross products of standardised residuals of the firm and the market
* Significant at the 5% level.
** Significant at the 1% level.

Table 6.1 Part 2: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	Glaxo Smith Kline	Edinburgh Oil and Gas	Boots Group	Barclays	Scottish and Newcastle
$c_1(10^{-5})$	2.9374[a]	0.2039[a]	0.9438[a]	2.7557[a]	2.1170[a]
	(22.32817)	(12.97065)	(11.07525)	(13.43163)	(11.96624)
a_{11}	0.1074[a]	0.0211[a]	0.0681[a]	0.1159[a]	0.1586[a]
	(22.90642)	(46.60518)	(19.26014)	(19.50264)	(17.34974)
b_{11}	0.8326[a]	0.9792[a]	0.9102[a]	0.8276[a]	0.8080[a]
	(185.82379)	(2356.42254)	(216.97769)	(117.27087)	(83.87248)
$c_3(10^{-5})$	2.0197[a]	1.1966[a]	1.3537[a]	2.9226[a]	2.1229[a]
	(21.63542)	(19.96837)	(20.66824)	(23.90827)	(21.39358)
a_{33}	0.1459[a]	0.1127[a]	0.1228[a]	0.1696[a]	0.1957[a]
	(29.19433)	(29.58570)	(30.56312)	(28.39396)	(32.02490)
b_{33}	0.7576[a]	0.8353[a]	0.8182[a]	0.6830[a]	0.7239[a]
	(91.86291)	(161.67601)	(143.31186)	(67.94345)	(83.36624)
$c_2(10^{-5})$	1.7967[a]	0.4485[a]	0.9230[a]	2.3594[a]	1.6695[a]
	(19.97151)	(12.58315)	(18.18002)	(16.27429)	(18.69481)
a_{22}	0.1175[a]	0.0564[a]	0.0799[a]	0.1178[a]	0.1401[a]
	(26.89214)	(30.43543)	(24.52910)	(24.91950)	(20.48265)
b_{22}	0.7984[a]	0.9148[a]	0.8750[a]	0.7699[a]	0.7813[a]
	(129.99509)	(325.74221)	(207.98231)	(89.92873)	(91.69072)
L	29740.66	27822.98	30136.70	29900.26	29912.51

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	9.5615	7.3525	44.0837**	11.2028	5.7695
L-B(12)[e]	9.7079	12.3869	11.6864	6.4644	9.1146
$u_t / h_t^{1/2}$					
L-B(12)[d]	17.9163*	9.0535	10.4349	14.3032	6.6068
L-B(12)[e]	19.0957*	11.1182	10.6023	14.1752	11.4688
L-B(12)[f]	5.5300	33.8780**	17.6469*	9.2351	10.2060

Notes: see Table 6.1 part 1

Table 6.1 Part 3: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	Signet Group	Goodwin	British Vita	Caldwell Investments	Alvis
$c_1(10^{-5})$	1.0638[a]	14.7390[a]	2.6369[a]	16.0250[a]	9.9048[a]
	(17.55000)	(31.52721)	(18.70546)	(22.12451)	(24.42024)
a_{11}	0.0415[a]	0.1523[a]	0.0759[a]	0.0892[a]	0.1556[a]
	(24.06941)	(24.82217)	(18.74758)	(18.07203)	(25.07313)
b_{11}	0.9497[a]	0.6968[a]	0.8590[a]	0.7507[a]	0.7461[a]
	(644.13139)	(92.11259)	(143.78384)	(73.02719)	(97.59434)
$c_3(10^{-5})$	2.7554[a]	2.0019[a]	1.6286[a]	2.8073[a]	2.8814[a]
	(21.13430)	(19.44484)	(17.84337)	(20.20734)	(19.35035)
a_{33}	0.2027[a]	0.1863[a]	0.1470[a]	0.2091[a]	0.2008[a]
	(27.84572)	(30.96135)	(23.46811)	(27.38613)	(24.75473)
b_{33}	0.6712[a]	0.7356[a]	0.7847[a]	0.6635[a]	0.6603[a]
	(60.86209)	(82.06023)	(92.59923)	(56.38688)	(52.12588)
$c_2(10^{-5})$	1.2934[a]	1.2254[a]	1.2405[a]	2.1776[a]	3.5574[a]
	(8.42551)	(12.58056)	(22.32691)	(9.56292)	(11.43395)
a_{22}	0.0755[a]	0.1579[a]	0.1071[a]	0.1209[a]	0.0866[a]
	(10.50252)	(25.21380)	(27.09870)	(11.18138)	(9.85093)
b_{22}	0.8353[a]	0.7769[a]	0.8271[a]	0.6833[a]	0.5860[a]
	(57.29882)	(106.53156)	(152.02907)	(30.73925)	(16.06107)
L	27225.59	27700.74	29602.14	27080.58	27758.63
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	13.2890	0.5814	11.8636	5.9354	1.2835
L-B(12)[e]	6.7718	9.2517	10.7778	6.7736	6.3988
$u_t / h_t^{1/2}$					
L-B(12)[d]	4.9470	8.8513	13.0077	16.1106	19.4765*
L-B(12)[e]	11.6337	10.8211	11.8032	12.0188	11.8915
L-B(12)[f]	11.9134	8.5333	9.6200	7.5681	5.5783

Notes: see Table 6.1 part 1

Table 6.1 Part 4: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003					
Parameter	Tottenham Hotspur	Care UK	Daily Mail and Gen Trust	Cable and Wireless	BAE Systems
$c_1(10^{-5})$	7.9015[a]	0.1568[a]	6.7537[a]	1.5730[a]	3.5423[a]
	(26.26354)	(29.94013)	(36.98756)	(14.97568)	(15.81363)
a_{11}	0.1818[a]	0.0333[a]	0.2273[a]	0.0999[a]	0.1832[a]
	(21.75059)	(55.93504)	(21.22285)	(27.43151)	(28.06268)
b_{11}	0.6529[a]	0.9694[a]	0.5072[a]	0.8876[a]	0.7834[a]
	(55.09872)	(2618.32625)	(49.52770)	(297.37483)	(109.30792)
$c_3(10^{-5})$	2.6924[a]	3.0427[a]	2.3444[a]	2.0830[a]	2.9369[a]
	(20.86848)	(19.82837)	(19.75225)	(22.93679)	(23.57867)
a_{33}	0.1903[a]	0.1991[a]	0.1723[a]	0.1617[a]	0.2283[a]
	(28.50768)	(27.77346)	(28.49652)	(30.49332)	(29.46723)
b_{33}	0.6890[a]	0.6577[a]	0.7178[a]	0.7428[a]	0.6468[a]
	(64.78411)	(53.09453)	(73.46012)	(97.66595)	(60.43276)
$c_2(10^{-5})$	1.7544[a]	0.0473[a]	2.2753[a]	1.4439[a]	2.1817[a]
	(20.56579)	(6.62198)	(34.24181)	(15.47584)	(13.72810)
a_{22}	0.1819[a]	0.0072[a]	0.1617[a]	0.0907[a]	0.1165[a]
	(25.26014)	(10.67836)	(23.53297)	(19.97619)	(12.52931)
b_{22}	0.6931[a]	0.9869[a]	0.6590[a]	0.8439[a]	0.7610[a]
	(73.78202)	(845.36047)	(81.76062)	(153.48481)	(58.40927)
L	29136.83	28055.89	30755.95	28701.15	28601.49
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	2.6384	12.6200	19.0565*	4.1637	13.9568
L-B(12)[e]	6.9762	6.1575	7.6493	8.5713	7.0056
$u_t / h_t^{1/2}$					
L-B(12)[d]	11.0998	42.9017**	32.2447**	8.5488	14.3948
L-B(12)[e]	11.7825	12.1625	11.6681	10.9971	16.8462
L-B(12)[f]	9.7086	5.7711	4.3485	13.7366	13.0183

Notes: see Table 6.1 part 1

Table 6.2: Basic Statistics of the Time-Varying Beta (Bivariate GARCH)

Company	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
British Airways	1.08209[a]	0.084649	0.41671[a]	2.83087[a]	1419.10380[a]
TESCO	0.93109[a]	0.054702	-1.04892[a]	2.22015[a]	1520.40359[a]
British American Tobacco	0.98526[a]	0.122902	0.53342[a]	11.17150[a]	20523.06460[a]
BT Group	1.14696[a]	0.063448	0.51666[a]	2.43498[a]	1140.20000[a]
Legal and General	1.12486[a]	0.056125	0.48307[a]	2.06511[a]	847.07542[a]
Glaxo Smith Kline	1.06325[a]	0.049468	0.71735[a]	13.02190[a]	27968.24414[a]
Edinburgh Oil and Gas	0.65030[a]	0.111874	-1.40191[a]	8.19805[a]	12233.18800[a]
Boots Group	0.92020[a]	0.057333	-0.95280[a]	1.62986[a]	1024.64413[a]
Barclays	1.16440[a]	0.047046	-0.03023	2.77620[a]	1256.56343[a]
Scottish and Newcastle	0.86777[a]	0.075198	-1.28936[a]	3.45212[a]	3025.64663[a]
Signet Group	0.85000[a]	0.110922	0.20208[a]	2.76521[a]	1272.66005[a]
Goodwin	0.60953[a]	0.155806	-1.04418[a]	17.32578[a]	49627.96268[a]
British Vita	0.79515[a]	0.059406	-0.63474[a]	1.28365[a]	531.13881[a]
Caldwell Investments	0.61020[a]	0.088871	0.03380	7.55215[a]	9295.08040[a]
Alvis	0.68607[a]	0.054602	1.47833[a]	23.62906[a]	92409.44540[a]
Tottenham Hotspur	0.57720[a]	0.091552	0.06723[c]	5.79265[a]	5470.98730[a]
Care UK	0.68293[a]	0.158040	1.16073[a]	1.72251[a]	1361.70736[a]
Daily Mail and Gen Trust	0.60805[a]	0.039074	-0.96144[a]	6.41563[a]	7309.95264[a]
Cable and Wireless	1.22242[a]	0.136234	3.95500[a]	44.01636[a]	325917.94333[a]
BAE Systems	0.96733[a]	0.084653	-0.47160[a]	2.31246[a]	1016.38680[a]

Notes:
[a] Significant at the 1% level,
[c] Significant at the 10% level.

Table 6.3: Two Unit Root Tests for Time-Varying Betas (Bivariate GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-67.35228[a]	-35.51814[a]	-29.46404[a]	-26.06023[a]
TESCO	-66.09934[a]	-37.76172[a]	-30.23357[a]	-26.52619[a]
British American Tobacco	-67.23898[a]	-34.05770[a]	-28.19292[a]	-25.89655[a]
BT Group	-62.49721[a]	-34.23395[a]	-26.17402[a]	-21.72507[a]
Legal and General	-64.22113[a]	-33.15025[a]	-28.08439[a]	-25.67683[a]
Glaxo Smith Kline	-67.99646[a]	-37.52362[a]	-29.96849[a]	-27.21198[a]
Edinburgh Oil and Gas	-61.72748[a]	-31.95195[a]	-25.95603[a]	-22.32724[a]
Boots Group	-64.89195[a]	-34.25482[a]	-27.09057[a]	-23.55943[a]
Barclays	-68.76578[a]	-37.65031[a]	-32.91880[a]	-27.92658[a]
Scottish and Newcastle	-68.20934[a]	-37.34007[a]	-30.85116[a]	-27.04847[a]
Signet Group	-65.97942[a]	-36.54509[a]	-28.34836[a]	-26.40353[a]
Goodwin	-68.87081[a]	-38.71033[a]	-30.86041[a]	-26.93631[a]
British Vita	-68.80645[a]	-35.05982[a]	-27.08431[a]	-25.02334[a]
Caldwell Investments	-77.12043[a]	-41.20106[a]	-33.20985[a]	-27.72822[a]
Alvis	-74.04151[a]	-39.59313[a]	-34.57595[a]	-28.32509[a]
Tottenham Hotspur	-70.03382[a]	-40.25676[a]	-33.02560[a]	-28.98674[a]
Care UK	-67.46828[a]	-34.07130[a]	-28.69034[a]	-25.47597[a]
Daily Mail and Gen Trust	-71.71247[a]	-41.72072[a]	-33.28480[a]	-28.95910[a]
Cable and Wireless	-63.93085[a]	-34.69928[a]	-30.74379[a]	-26.63857[a]
BAE Systems	-72.68272[a]	-37.83822[a]	-31.94723[a]	-27.34888[a]

Notes:
[a] Significant at the 1% level.

Table 6.4: One Unit Root Tests for Time-Varying Betas (Bivariate GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-16.02020[a]	-14.62377[a]	-12.76383[a]	-11.43079[a]
TESCO	-16.84335[a]	-14.83125[a]	-13.20874[a]	-11.67911[a]
British American Tobacco	-14.18126[a]	-13.11113[a]	-12.40469[a]	-11.10767[a]
BT Group	-11.74049[a]	-11.99123[a]	-10.75659[a]	-10.91695[a]
Legal and General	-13.51543[a]	-13.31530[a]	-12.61439[a]	-11.46044[a]
Glaxo Smith Kline	-17.79441[a]	-16.15946[a]	-14.08083[a]	-12.45488[a]
Edinburgh Oil and Gas	-10.67476[a]	-11.17895[a]	-10.87632[a]	-10.20594[a]
Boots Group	-11.62603[a]	-10.92228[a]	-10.24692[a]	-9.68597[a]
Barclays	-20.54982[a]	-18.05055[a]	-16.23495[a]	-13.35810[a]
Scottish and Newcastle	-16.88267[a]	-14.04132[a]	-12.44605[a]	-10.86458[a]
Signet Group	-18.24021[a]	-17.04517[a]	-16.25472[a]	-14.54700[a]
Goodwin	-19.62486[a]	-16.71181[a]	-14.53361[a]	-13.46230[a]
British Vita	-14.34556[a]	-12.15950[a]	-11.77407[a]	-10.81489[a]
Caldwell Investments	-25.70797[a]	-19.03089[a]	-16.02855[a]	-14.26907[a]
Alvis	-23.85848[a]	-18.10796[a]	-14.67200[a]	-12.22311[a]
Tottenham Hotspur	-23.45934[a]	-20.08179[a]	-16.96587[a]	-14.59887[a]
Care UK	-11.01957[a]	-9.67129[a]	-8.66896[a]	-7.52293[a]
Daily Mail and Gen Trust	-20.88400[a]	-15.71470[a]	-12.49344[a]	-10.67191[a]
Cable and Wireless	-15.69123[a]	-15.42238[a]	-12.81726[a]	-11.97655[a]
BAE Systems	-21.77337[a]	-17.85786[a]	-16.41917[a]	-14.21501[a]

Notes:
[a] Significant at the 1% level.

Table 6.5 Part 1: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003					
Parameter	British Airways	TESCO	British American Tobacco	BT Group	Legal and General
c_{11}	0.003711[a]	0.004670[a]	0.003429[a]	0.002683[a]	0.003537[a]
	(4.73362)	(3.53363)	(5.96529)	(5.96613)	(13.94260)
a_{11}	0.292841[a]	0.296934[a]	0.322520[a]	0.261072[a]	0.263137[a]
	(6.23179)	(5.71178)	(12.03535)	(10.10488)	(15.23341)
b_{11}	0.946554[a]	0.929056[a]	0.944663[a]	0.959862[a]	0.952569[a]
	(56.77110)	(30.39288)	(102.20730)	(109.94434)	(168.99741)
c_{22}	0.002365[a]	0.002899[a]	0.002066[a]	0.001019[c]	0.002044[a]
	(4.38364)	(3.70512)	(4.71062)	(1.95707)	(5.49540)
a_{22}	0.384610[a]	0.368220[a]	0.387623[a]	0.312371[a]	0.332855[a]
	(5.73724)	(4.81962)	(16.79872)	(6.58603)	(17.23603)
b_{22}	0.871380[a]	0.889842[a]	0.890792[a]	0.922160[a]	0.896265[a]
	(15.66922)	(17.87506)	(113.86241)	(39.59495)	(215.02678)
c_{12}	0.003777[a]	0.002777[a]	0.003271[a]	0.003257[a]	0.003586[a]
	(2.87500)	(3.82853)	(17.90915)	(5.90871)	(9.17302)
L	21786.83	22458.75	22070.77	22648.87	22442.42
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	31.5668**	18.5579*	4.4462	24.8705**	7.9023
L-B(12)[e]	9.1364	11.0352	11.5046	13.8397	10.1823
$u_t / h_t^{1/2}$					
L-B(12)[d]	10.2028	33.7692**	6.6806	15.7772	9.6278
L-B(12)[e]	11.9352	13.9022	11.9731	14.3290	11.2769
L-B(12)[f]	9.1113	17.0806*	11.6999	23.6112**	9.8715

Notes: see Table 6.1 part 1.

Table 6.5 Part 2: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003					
Parameter	Glaxo Smith Kline	Edinburgh Oil and Gas	Boots Group	Barclays	Scottish and Newcastle
c_{11}	0.005179[a]	0.004080	0.002446[a]	0.004663[a]	0.003446[a]
	(5.31249)	(0.52050)	(2.87356)	(7.41082)	(3.08574)
a_{11}	0.314791[a]	0.247723	0.222789[a]	0.311135[a]	0.310732[a]
	(8.73625)	(0.88529)	(4.83064)	(11.74404)	(4.93958)
b_{11}	0.920265[a]	0.963015[a]	0.968224[a]	0.926814[a]	0.940749[a]
	(42.44752)	(9.65125)	(62.49807)	(67.77504)	(34.65664)
c_{22}	0.002885[a]	0.003375	0.001409[b]	0.002213[a]	0.002250[a]
	(4.25564)	(0.96628)	(2.36615)	(2.75781)	(2.71628)
a_{22}	0.363031[a]	0.385482[c]	0.291813[a]	0.392282[a]	0.351197[a]
	(4.59425)	(1.82247)	(3.39041)	(7.53665)	(3.58132)
b_{22}	0.887256[a]	0.888381[a]	0.936199[a]	0.825651[a]	0.915333[a]
	(15.67729)	(6.67854)	(24.94037)	(14.68865)	(18.05214)
c_{12}	0.002988[a]	0.001959	0.002584[a]	0.005186[a]	0.002448[a]
	(2.97832)	(1.40530)	(2.99679)	(3.96476)	(2.64187)
L	22558.00	20620.08	22949.80	22698.39	22691.30
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	9.3709	1.2926	57.3681**	14.3207	5.2519
L-B(12)[e]	11.1256	10.9523	14.8371	6.2982	13.1197
$u_t / h_t^{1/2}$					
L-B(12)[d]	18.2468*	8.8474	10.6029	14.8608	6.6533
L-B(12)[e]	19.2195*	10.8468	11.3127	14.7909	10.9720
L-B(12)[f]	6.5086	21.1218*	24.5470**	11.6549	16.8358

Notes: see Table 6.1 part 1.

Table 6.5 Part 3: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003					
Parameter	Signet Group	Goodwin	British Vita	Caldwell Investments	Alvis
c_{11}	0.003319[b]	0.012009[a]	0.004870[a]	0.012711[a]	0.013040[a]
	(2.13305)	(30.66335)	(4.05095)	(3.69151)	(2.63975)
a_{11}	0.205309[a]	0.402323[a]	0.270663[a]	0.296823[a]	0.427104[b]
	(3.47190)	(7.55716)	(6.33245)	(5.11942)	(2.51071)
b_{11}	0.974080[a]	0.834864[a]	0.931696[a]	0.866662[a]	0.800856[a]
	(59.31430)	(116.14228)	(34.37573)	(13.24947)	(5.34206)
c_{22}	0.002435	0.004745[a]	0.003001[a]	0.005033[a]	0.004626[a]
	(0.59195)	(7.26873)	(8.04048)	(7.90546)	(5.55060)
a_{22}	0.429078[a]	0.455763[a]	0.373142[a]	0.451521[a]	0.414457[a]
	(7.23707)	(8.58771)	(9.93241)	(8.81034)	(6.88675)
b_{22}	0.838712[a]	0.829614[a]	0.890597[a]	0.819809[a]	0.844941[a]
	(18.38145)	(20.75776)	(43.61324)	(21.19609)	(18.09301)
c_{12}	0.004298	0.001498[a]	0.002580[a]	0.001484[a]	0.001679[a]
	(1.42463)	(3.50714)	(7.80039)	(5.04886)	(5.52590)
L	20038.68	20492.90	22424.57	19896.79	20504.06
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	13.1468	0.5777	12.7100	5.9224	2.2030
L-B(12)[e]	7.5092	7.7044	11.0260	6.9236	7.7061
$u_t / h_t^{1/2}$					
L-B(12)[d]	5.1019	8.7114	11.5154	14.2264	13.4615
L-B(12)[e]	11.5571	11.4937	12.1035	12.3233	12.7478
L-B(12)[f]	12.4298	7.5231	10.1564	7.9224	6.4891

Notes: see Table 6.1 part 1.

Table 6.5 Part 4: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003

Parameter	Tottenham Hotspur	Care UK	Daily Mail and Gen Trust	Cable and Wireless	BAE Systems
c_{11}	0.008559[a]	0.001483[a]	0.009192[a]	0.003814[a]	0.004487[a]
	(7.34455)	(5.16586)	(6.74667)	(6.25496)	(21.62858)
a_{11}	0.416740[a]	0.192530[a]	0.502635[a]	0.298492[a]	0.383021[a]
	(10.28025)	(8.46781)	(6.41308)	(7.00889)	(116.30370)
b_{11}	0.819773[a]	0.982944[a]	0.633237[a]	0.948335[a]	0.919858[a]
	(21.47863)	(299.99100)	(5.22560)	(80.79380)	(478.37658)
c_{22}	0.004820[a]	-0.00000007	0.003672[a]	0.001772	0.003463[a]
	(6.82435)	(-0.00016)	(6.53519)	(0.96179)	(19.13672)
a_{22}	0.440700[a]	0.365505[a]	0.425012[a]	0.416975[a]	0.464209[a]
	(7.65556)	(5.61558)	(11.10764)	(4.81906)	(8.69812)
b_{22}	0.822530[a]	0.883688[a]	0.839093[a]	0.847074[a]	0.820343[a]
	(18.55634)	(22.10164)	(33.58359)	(11.71396)	(24.57830)
c_{12}	0.002243[a]	0.004179[a]	0.003456[a]	0.004558[b]	0.004052[a]
	(6.19329)	(5.68052)	(7.35465)	(2.33903)	(5.06074)
L	21953.67	20820.14	23535.38	21486.89	21346.84

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	2.8392	10.5532	18.3980*	3.6362	16.2668
L-B(12)[e]	6.6799	10.1965	7.3061	7.6928	7.5135
$u_t / h_t^{1/2}$					
L-B(12)[d]	10.3258	32.4465**	31.2114**	9.7820	15.1338
L-B(12)[e]	12.3567	13.1564	11.7559	11.5347	20.0662*
L-B(12)[f]	9.7671	7.5916	3.7779	13.4451	16.6889

Notes: see Table 6.1 part 1.

Table 6.6: Basic Statistics of the Time-Varying Beta (BEKK GARCH)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
British Airways	1.08630[a]	0.096390	0.58985[a]	2.66725[a]	1386.11081[a]
TESCO	0.93194[a]	0.054344	-1.04724[a]	2.18522[a]	1493.03623[a]
British American Tobacco	0.98876[a]	0.118873	0.51156[a]	11.21145[a]	20653.87970[a]
BT Group	1.15161[a]	0.061236	0.84286[a]	2.43257[a]	1427.36449[a]
Legal and General	1.12472[a]	0.055613	0.70984[a]	1.74377[a]	823.95173[a]
Glaxo Smith Kline	1.06361[a]	0.048745	1.03894[a]	14.81313[a]	36461.32054[a]
Edinburgh Oil and Gas	0.63689[a]	0.145364	-2.25770[a]	16.53491[a]	47875.89725[a]
Boots Group	0.92442[a]	0.051723	-1.03160[a]	1.36250[a]	996.20003[a]
Barclays	1.15795[a]	0.047517	0.46489[a]	2.58131[a]	1226.68958[a]
Scottish and Newcastle	0.85651[a]	0.079888	-1.41017[a]	3.61398[a]	3424.59366[a]
Signet Group	0.84889[a]	0.110990	-0.14612[a]	3.17946[a]	1661.25635[a]
Goodwin	0.58684[a]	0.142878	-1.18802[a]	26.07156[a]	111687.06271[a]
British Vita	0.80153[a]	0.054211	-0.68720[a]	1.31462[a]	589.45199[a]
Caldwell Investments	0.61539[a]	0.113568	0.07539[c]	7.56233[a]	9323.09138[a]
Alvis	0.69576[a]	0.167790	4.13609[a]	64.76350[a]	694649.10214[a]
Tottenham Hotspur	0.58031[a]	0.086913	0.02483	6.49079[a]	6865.88930[a]
Care UK	0.59538[a]	0.119956	2.42701[a]	15.80889[a]	44566.21926[a]
Daily Mail and Gen Trust	0.58766[a]	0.047867	-0.61202[a]	9.67197[a]	15488.43073[a]
Cable and Wireless	1.21609[a]	0.165759	5.61590[a]	72.55632[a]	878438.69571[a]
BAE Systems	0.96771[a]	0.143036	-0.25374[a]	3.54316[a]	2087.74049[a]

Notes:
[a] Significant at the 1% level,
[c] Significant at the 10% level.

Table 6.7: Two Unit Root Tests for Time-Varying Betas (BEKK GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-66.99047[a]	-35.18081[a]	-29.24107[a]	-25.88469[a]
TESCO	-65.48305[a]	-37.42160[a]	-29.73383[a]	-25.89214[a]
British American Tobacco	-67.32815[a]	-34.10289[a]	-28.21483[a]	-25.88146[a]
BT Group	-63.02024[a]	-34.69740[a]	-26.56440[a]	-22.06267[a]
Legal and General	-64.82560[a]	-34.15431[a]	-28.50698[a]	-26.14763[a]
Glaxo Smith Kline	-67.78534[a]	-37.23207[a]	-29.52398[a]	-26.86029[a]
Edinburgh Oil and Gas	-62.89201[a]	-34.05164[a]	-28.22021[a]	-24.45880[a]
Boots Group	-64.37816[a]	-33.85621[a]	-26.53617[a]	-22.85785[a]
Barclays	-69.13014[a]	-37.69926[a]	-32.90948[a]	-27.87171[a]
Scottish and Newcastle	-66.20594[a]	-35.41784[a]	-28.39738[a]	-24.40914[a]
Signet Group	-66.33895[a]	-37.55662[a]	-28.86577[a]	-26.85606[a]
Goodwin	-71.66916[a]	-41.33009[a]	-33.38111[a]	-28.66935[a]
British Vita	-68.90693[a]	-35.25170[a]	-27.25248[a]	-25.11408[a]
Caldwell Investments	-76.07894[a]	-40.34078[a]	-32.59730[a]	-27.38631[a]
Alvis	-71.32563[a]	-39.13066[a]	-34.71909[a]	-28.56893[a]
Tottenham Hotspur	-70.72130[a]	-40.85424[a]	-33.55026[a]	-29.41387[a]
Care UK	-68.36930[a]	-32.98104[a]	-27.49243[a]	-24.88979[a]
Daily Mail and Gen Trust	-75.17703[a]	-44.18543[a]	-35.37079[a]	-30.55376[a]
Cable and Wireless	-65.10581[a]	-36.08573[a]	-31.61340[a]	-27.81363[a]
BAE Systems	-74.26108[a]	-38.36636[a]	-31.70336[a]	-26.98277[a]

Notes:
[a] Significant at the 1% level.

Table 6.8: One Unit Root Tests for Time-Varying Betas (BEKK GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-15.02134[a]	-13.72236[a]	-11.90405[a]	-10.70199[a]
TESCO	-15.30305[a]	-13.46481[a]	-11.98016[a]	-10.74951[a]
British American Tobacco	-13.90522[a]	-12.76157[a]	-12.01953[a]	-10.74408[a]
BT Group	-11.69420[a]	-11.76681[a]	-10.23943[a]	-10.29900[a]
Legal and General	-13.42517[a]	-12.77793[a]	-11.81980[a]	-10.64838[a]
Glaxo Smith Kline	-16.74887[a]	-15.03334[a]	-13.15810[a]	-11.59449[a]
Edinburgh Oil and Gas	-14.25920[a]	-14.46193[a]	-13.51366[a]	-12.33444[a]
Boots Group	-8.91445[a]	-8.35470[a]	-7.82987[a]	-7.45424[a]
Barclays	-19.05795[a]	-16.20167[a]	-14.25255[a]	-11.71461[a]
Scottish and Newcastle	-12.01357[a]	-10.49828[a]	-9.61956[a]	-8.90953[a]
Signet Group	-18.94423[a]	-17.36590[a]	-16.29777[a]	-14.51345[a]
Goodwin	-24.33107[a]	-19.55503[a]	-16.14955[a]	-14.57370[a]
British Vita	-14.08739[a]	-11.80854[a]	-11.35879[a]	-10.43040[a]
Caldwell Investments	-24.58702[a]	-18.65349[a]	-15.96268[a]	-14.35263[a]
Alvis	-25.11205[a]	-20.78536[a]	-16.98587[a]	-14.68307[a]
Tottenham Hotspur	-24.43507[a]	-20.58297[a]	-17.19826[a]	-14.73516[a]
Care UK	-14.07694[a]	-12.07974[a]	-12.84443[a]	-10.95010[a]
Daily Mail and Gen Trust	-26.56681[a]	-19.00209[a]	-14.53887[a]	-12.05956[a]
Cable and Wireless	-17.16097[a]	-16.35725[a]	-13.08761[a]	-12.07022[a]
BAE Systems	-21.21146[a]	-16.91407[a]	-15.26368[a]	-13.57892[a]

Notes:
[a] Significant at the 1% level.

Table 6.9 Part 1: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	British Airways	TESCO	British American Tobacco	BT Group	Legal and General
$c_1(10^{-5})$	1.5817[a]	2.7046[a]	1.1443[a]	0.8086[a]	1.4768[a]
	(11.55850)	(11.76126)	(19.36058)	(12.25981)	(14.06651)
a_1	0.0807[a]	0.1102[a]	0.0853[a]	0.0630[a]	0.0818[a]
	(12.82513)	(14.84326)	(17.37205)	(16.26311)	(14.92976)
b_1	0.8868[a]	0.8370[a]	0.8927[a]	0.9151[a]	0.8933[a]
	(169.21383)	(88.61965)	(222.68725)	(250.22756)	(179.10510)
r_1	0.0183[a]	-0.0179[a]	0.0355[a]	0.0172[a]	-0.0070
	(3.46411)	(-2.67880)	(7.59694)	(5.26489)	(-1.43289)
$c_3(10^{-5})$	1.8884[a]	1.8852[a]	1.5561[a]	1.2600[a]	1.7877[a]
	(19.31577)	(20.01292)	(18.96008)	(21.82321)	(19.28410)
a_3	0.1435[a]	0.1375[a]	0.1429[a]	0.0960[a]	0.1074[a]
	(26.21423)	(26.73723)	(20.70383)	(29.34555)	(19.86584)
b_3	0.7663[a]	0.7595[a]	0.7895[a]	0.8413[a]	0.7952[a]
	(85.53463)	(90.60995)	(95.25072)	(164.46939)	(89.98531)
r_3	-0.0005	0.0366[a]	0.0075	0.0060	0.0105
	(-0.05161)	(4.43572)	(0.91749)	(1.41597)	(1.44661)
$c_2(10^{-5})$	1.5400[a]	1.5891[a]	1.2099[a]	0.9808[a]	1.2615[a]
	(14.16439)	(14.85288)	(23.39135)	(17.89414)	(18.74522)
a_2	0.1062[a]	0.1174[a]	0.1105[a]	0.0777[a]	0.0784[a]
	(20.50589)	(21.44779)	(26.69309)	(23.10401)	(19.66802)
b_2	0.8191[a]	0.8005[a]	0.8430[a]	0.8798[a]	0.8603[a]
	(98.85275)	(92.66378)	(178.26012)	(210.21408)	(149.50394)
L	28981.26	29643.26	29268.67	29847.83	29638.97

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	28.2871**	16.7438	4.6862	25.0866**	7.6253
L-B(12)[e]	9.3757	10.0489	11.0884	13.2570	9.8590
$u_t / h_t^{1/2}$					
L-B(12)[d]	9.5695	34.0373**	7.1610	14.9005	9.7619
L-B(12)[e]	11.4064	13.9514	11.4185	14.1514	11.0381
L-B(12)[f]	8.9767	14.7411	11.5184	22.5855**	9.2320

Notes: see Table 6.1 part 1.

Table 6.9 Part 2: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	Glaxo Smith Kline	Edinburgh Oil and Gas	Boots Group	Barclays	Scottish and Newcastle
$c_1(10^{-5})$	2.9345[a]	0.2551[a]	0.9297[a]	2.7238[a]	2.2505[a]
	(20.90935)	(12.36569)	(10.89259)	(13.36260)	(12.30218)
a_1	0.1059[a]	0.0163[a]	0.0698[a]	0.1082[a]	0.1394[a]
	(18.15567)	(29.24407)	(16.02315)	(16.18792)	(14.24445)
b_1	0.8326[a]	0.9754[a]	0.9112[a]	0.8281[a]	0.7968[a]
	(174.54623)	(1838.76402)	(213.61751)	(117.12678)	(80.12029)
r_1	0.0033	0.0211[a]	-0.0047	0.0150[b]	0.0551[a]
	(0.50367)	(19.13795)	(-1.24225)	(2.33984)	(6.56998)
$c_3(10^{-5})$	2.0450[a]	1.4226[a]	1.3185[a]	2.9016[a]	2.2169[a]
	(20.46470)	(20.19222)	(20.61479)	(23.67696)	(20.33457)
a_3	0.1451[a]	0.1297[a]	0.1125[a]	0.1884[a]	0.1879[a]
	(28.69811)	(29.68818)	(29.53829)	(26.26873)	(29.51654)
b_3	0.7549[a]	0.8168[a]	0.8216[a]	0.6845[a]	0.7157[a]
	(81.19493)	(137.50810)	(145.69089)	(66.53194)	(71.70242)
r_3	0.0050	-0.0258[a]	0.0173[a]	-0.0363[a]	0.0214[c]
	(0.58463)	(-4.47444)	(2.69221)	(-4.43955)	(1.88927)
$c_2(10^{-5})$	1.8241[a]	0.6163[a]	0.9110[a]	2.3194[a]	1.8287[a]
	(19.38416)	(12.84576)	(17.82510)	(16.20983)	(19.81103)
a_2	0.1187[a]	0.0712[a]	0.0786[a]	0.1183[a]	0.1466[a]
	(25.27578)	(44.25511)	(23.43204)	(24.14085)	(20.05696)
b_2	0.7959[a]	0.8885[a]	0.8769[a]	0.7715[a]	0.7640[a]
	(118.55955)	(287.91740)	(204.48349)	(89.58034)	(85.20614)
L	29741.00	27846.80	30138.32	29903.70	29927.15

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	9.5867	9.8940	44.0338**	11.5336	6.7648
L-B(12)[e]	9.6380	10.9596	12.0091	6.2859	8.9095
$u_t / h_t^{1/2}$					
L-B(12)[d]	17.9358*	9.2039	10.4396	14.2918	6.4271
L-B(12)[e]	19.0964*	10.3092	10.7299	13.6547	12.0753
L-B(12)[e]	5.4874	32.6096**	17.8580*	9.1853	10.4405

Notes: see Table 6.1 part 1.

Table 6.9 Part 3: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	Signet Group	Goodwin	British Vita	Caldwell Investments	Alvis
$c_1(10^{-5})$	0.8937[a]	1.3944[a]	2.6557[a]	15.931[a]	8.1359[a]
	(17.99066)	(40.78717)	(18.65980)	(21.98528)	(24.53985)
a_1	0.0236[a]	0.0185[a]	0.0774[a]	0.0883[a]	0.0976[a]
	(13.85577)	(21.14395)	(17.42262)	(16.40481)	(21.61855)
b_1	0.9572[a]	0.9509[a]	0.8581[a]	0.7520[a]	0.7689[a]
	(813.86901)	(1402.21196)	(142.72220)	(73.12122)	(108.22665)
r_1	0.0231[a]	0.0516[a]	-0.0024	0.0006	0.1478[a]
	(9.20788)	(28.59133)	(-0.47955)	(0.09715)	(14.51523)
$c_3(10^{-5})$	2.7034[a]	1.8649[a]	1.6148[a]	2.7826[a]	2.8781[a]
	(20.76689)	(23.38421)	(17.88941)	(19.44149)	(18.42124)
a_3	0.1972[a]	0.1681[a]	0.1426[a]	0.1939[a]	0.1819[a]
	(26.05659)	(27.31708)	(21.70004)	(26.42723)	(22.37706)
b_3	0.6751[a]	0.7649[a]	0.7857[a]	0.6629[a]	0.6610[a]
	(58.56240)	(111.95773)	(93.68438)	(51.86545)	(47.79493)
r_3	0.0075	-0.0278[a]	0.0084	0.0372[b]	0.0356[b]
	(0.53051)	(-2.86366)	(0.87381)	(2.27777)	(2.23272)
$c_2(10^{-5})$	1.2019[a]	0.6405[a]	1.2397[a]	2.1835[a]	4.2125[a]
	(7.95580)	(13.06958)	(22.05413)	(9.57457)	(10.19321)
a_2	0.0667[a]	0.0746[a]	0.1069[a]	0.1189[a]	0.1057[a]
	(10.52725)	(27.88091)	(27.10006)	(10.79012)	(11.20144)
b_2	0.8466[a]	0.8828[a]	0.8273[a]	0.6828[a]	0.5029[a]
	(59.23955)	(216.16009)	(151.10065)	(30.21227)	(10.50457)
L	27238.05	27671.45	29602.39	27081.41	27781.56

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	16.6069	1.2221	11.8576	5.9176	1.3377
L-B(12)[e]	6.9028	9.3550	10.9161	6.9927	6.5315
$u_t / h_t^{1/2}$					
L-B(12)[d]	4.8623	10.3648	12.9924	16.1170	14.8609
L-B(12)[e]	11.6026	11.2936	11.9235	12.1468	12.1655
L-B(12)[f]	11.8743	13.7340	9.6034	7.8380	6.2750

Notes: see Table 6.1 part 1.

Table 6.9 Part 4: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	Tottenham Hotspur	Care UK	Daily Mail and Gen Trust	Cable and Wireless	BAE Systems
$c_1(10^{-5})$	7.2989[a]	0.1872[a]	2.9979[a]	1.8454[a]	3.4970[a]
	(24.59120)	(29.34741)	(37.16414)	(14.66352)	(15.75161)
a_1	0.1562[a]	0.0221[a]	0.1558[a]	0.0745[a]	0.1849[a]
	(19.11516)	(28.11867)	(24.51130)	(14.43463)	(25.6108)
b_1	0.6736[a]	0.9689[a]	0.7608[a]	0.8805[a]	0.7851[a]
	(57.45802)	(2426.36989)	(139.31806)	(324.37668)	(110.63934
r_1	0.0332[a]	0.0215[a]	-0.0769[a]	0.0565[a]	-0.0056
	(3.89390)	(21.09563)	(-14.26300)	(9.08035)	(-0.64302)
$c_3(10^{-5})$	2.8333[a]	3.0676[a]	1.4003[a]	2.2081[a]	2.9199[a]
	(20.57474)	(18.89113)	(19.19742)	(23.32701)	(22.79214)
a_3	0.1955[a]	0.1880[a]	0.1128[a]	0.1879[a]	0.2485[a]
	(27.81598)	(26.59920)	(28.33032)	(28.68794)	(28.57359)
b_3	0.6747[a]	0.6533[a]	0.8125[a]	0.7334[a]	0.6503[a]
	(56.74326)	(46.85168)	(132.04735)	(92.26586)	(56.40514)
r_3	0.0048	0.0270[c]	0.0245[a]	-0.0471[a]	-0.0444[a]
	(0.40695)	(1.67083)	(3.17603)	(-6.19228)	(-3.61486)
$c_2(10^{-5})$	1.8082[a]	0.0047[a]	1.1452[a]	1.6510[a]	2.1513[a]
	(20.04363)	(6.79702)	(34.23753)	(16.08477)	(13.49259)
a_2	0.1781[a]	0.0066[a]	0.1008[a]	0.0965[a]	0.1174[a]
	(26.49176)	(10.69340)	(28.13745)	(21.57861)	(12.53823)
b_2	0.6879[a]	0.9872[a]	0.8174[a]	0.8281[a]	0.7633[a]
	(70.26366)	(857.47080)	(195.23097)	(148.33804)	(58.38271)
L	29139.46	28080.08	30786.01	28717.82	28603.47

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	2.7817	11.7665	10.1387	4.1267	14.1204
L-B(12)[e]	6.6877	6.1864	11.4087	7.7738	6.8534
$u_t / h_t^{1/2}$					
L-B(12)[d]	10.9578	39.5100**	33.0818**	8.9343	14.3566
L-B(12)[e]	11.9775	12.2305	10.6109	10.8628	16.0287
L-B(12)[f]	9.9232	5.7213	8.9089	13.1382	12.8573

Notes: see Table 6.1 part 1.

Table 6.10: Basic Statistics of the Time-Varying Beta (GARCH-GJR)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
British Airways	1.07933[a]	0.081367	0.41081[a]	2.87961[a]	1461.28581[a]
TESCO	0.93319[a]	0.056604	-0.98879[a]	2.08219[a]	1343.80787[a]
British American Tobacco	0.98789[a]	0.103739	0.44560[a]	10.34301[a]	17562.35513[a]
BT Group	1.14010[a]	0.058493	0.47151[a]	2.85070[a]	1469.19898[a]
Legal and General	1.12582[a]	0.057784	0.52772[a]	2.07531[a]	883.37800[a]
Glaxo Smith Kline	1.06307[a]	0.049754	0.73976[a]	13.30929[a]	29222.71306[a]
Edinburgh Oil and Gas	0.64222[a]	0.120078	-1.91031[a]	12.46722[a]	27707.60205[a]
Boots Group	0.92135[a]	0.058139	-0.91277[a]	1.52310[a]	921.11295[a]
Barclays	1.16111[a]	0.047373	-0.08203[b]	2.72742[a]	1216.60329[a]
Scottish and Newcastle	0.86134[a]	0.073474	-1.32496[a]	3.75785[a]	3445.49906[a]
Signet Group	0.83995[a]	0.102359	0.28130[a]	2.56154[a]	1120.82546[a]
Goodwin	0.63673[a]	0.106519	0.06055	4.51685[a]	3327.04795[a]
British Vita	0.79524[a]	0.059501	-0.62955[a]	1.27998[a]	525.32318[a]
Caldwell Investments	0.60978[a]	0.087873	0.11792[a]	7.75338[a]	9805.28821[a]
Alvis	0.67550[a]	0.061681	1.88278[a]	33.50633[a]	185259.75995[a]
Tottenham Hotspur	0.57606[a]	0.086075	0.03971	5.82300[a]	5526.50457[a]
Care UK	0.66500[a]	0.144947	1.12186[a]	1.62087[a]	1248.50584[a]
Daily Mail and Gen Trust	0.63067[a]	0.038880	-0.95484[a]	2.20425[a]	1386.05492[a]
Cable and Wireless	1.21946[a]	0.137193	4.67765[a]	58.68212[a]	575424.20088[a]
BAE Systems	0.96814[a]	0.085575	-0.46666[a]	2.38025[a]	1065.21066[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 6.11: Two Unit Root Tests for Time-Varying Betas (GARCH-GJR)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-67.31993[a]	-35.45055[a]	-29.37285[a]	-25.97461[a]
TESCO	-65.92624[a]	-37.67881[a]	-30.22925[a]	-26.54983[a]
British American Tobacco	-67.16353[a]	-33.90868[a]	-28.12071[a]	-25.76544[a]
BT Group	-63.10934[a]	-34.69655[a]	-26.63573[a]	-22.09680[a]
Legal and General	-64.23906[a]	-33.06642[a]	-28.08559[a]	-25.57338[a]
Glaxo Smith Kline	-68.05249[a]	-37.56075[a]	-30.07068[a]	-27.27335[a]
Edinburgh Oil and Gas	-62.14140[a]	-33.03155[a]	-27.01172[a]	-23.30809[a]
Boots Group	-64.66422[a]	-34.24910[a]	-26.99335[a]	-23.51352[a]
Barclays	-69.11565[a]	-37.69032[a]	-32.62806[a]	-28.12430[a]
Scottish and Newcastle	-68.74252[a]	-37.79893[a]	-31.35209[a]	-27.47317[a]
Signet Group	-65.97439[a]	-36.11638[a]	-28.13299[a]	-26.13631[a]
Goodwin	-64.69352[a]	-33.42667[a]	-26.63531[a]	-23.66299[a]
British Vita	-68.87504[a]	-35.05762[a]	-27.04313[a]	-24.99235[a]
Caldwell Investments	-77.11380[a]	-41.29990[a]	-33.34017[a]	-27.78676[a]
Alvis	-76.65791[a]	-41.03715[a]	-35.22156[a]	-29.01487[a]
Tottenham Hotspur	-70.12555[a]	-40.37236[a]	-33.08668[a]	-29.07188[a]
Care UK	-67.53026[a]	-34.23331[a]	-28.87280[a]	-25.59955[a]
Daily Mail and Gen Trust	-67.22111[a]	-37.50078[a]	-28.92858[a]	-25.38974[a]
Cable and Wireless	-64.41133[a]	-35.16729[a]	-31.16962[a]	-27.20962[a]
BAE Systems	-73.08437[a]	-37.79270[a]	-31.84497[a]	-27.19184[a]

Notes:
[a] Significant at the 1% level.

Table 6.12: One Unit Root Tests for Time-Varying Betas (GARCH-GJR)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-15.91344[a]	-14.54740[a]	-12.72276[a]	-11.41299[a]
TESCO	-16.92636[a]	-14.93612[a]	-13.31442[a]	-11.83075[a]
British American Tobacco	-13.99080[a]	-12.97089[a]	-12.29795[a]	-11.01659[a]
BT Group	-12.35424[a]	-12.45465[a]	-11.05719[a]	-11.12392[a]
Legal and General	-13.43007[a]	-13.24034[a]	-12.56009[a]	-11.39567[a]
Glaxo Smith Kline	-17.92005[a]	-16.26731[a]	-14.16011[a]	-12.49403[a]
Edinburgh Oil and Gas	-12.40245[a]	-12.80973[a]	-12.19000[a]	-11.31407[a]
Boots Group	-11.52347[a]	-10.82212[a]	-10.19426[a]	-9.66056[a]
Barclays	-20.66848[a]	-18.07791[a]	-16.28853[a]	-13.47762[a]
Scottish and Newcastle	-17.68644[a]	-14.54150[a]	-12.74169[a]	-11.04402[a]
Signet Group	-17.87283[a]	-16.74798[a]	-16.08238[a]	-14.43729[a]
Goodwin	-12.51419[a]	-11.88112[a]	-11.50397[a]	-11.00445[a]
British Vita	-14.33230[a]	-12.14215[a]	-11.78270[a]	-10.82005[a]
Caldwell Investments	-25.70360[a]	-18.99347[a]	-15.95894[a]	-14.13226[a]
Alvis	-26.65054[a]	-19.05571[a]	-15.44334[a]	-12.73030[a]
Tottenham Hotspur	-23.57914[a]	-20.12717[a]	-16.98524[a]	-14.59225[a]
Care UK	-11.26200[a]	-9.84047[a]	-8.79755[a]	-7.60547[a]
Daily Mail and Gen Trust	-13.24710[a]	-11.16478[a]	-9.79520[a]	-9.04281[a]
Cable and Wireless	-16.59498[a]	-16.12576[a]	-13.25335[a]	-12.29073[a]
BAE Systems	-21.77130[a]	-17.77129[a]	-16.41355[a]	-14.30453[a]

Notes:
[a] Significant at the 1% level.

Table 6.13: GARCH-X Estimation Results

A. GARCH-X results, sample period 1989-2003					
Parameter	Legal and General	Glaxo Smith Kline	British Vita	Alvis	Care UK
$c_1(10^{-5})$	1.4806[a]	2.8919[a]	2.5829[a]	9.2222[a]	0.1320[a]
	(13.97848)	(18.02210)	(16.81557)	(19.49517)	(16.75931)
a_{11}	0.0776[a]	0.1071[a]	0.0798[a]	0.1539[a]	0.0324[a]
	(17.12095)	(22.19240)	(18.29371)	(18.34144)	(50.03092)
b_{11}	0.8896[a]	0.8288[a]	0.8504[a]	0.6635[a]	0.9693[a]
	(170.73841)	(184.24258)	(121.98704)	(50.38526)	(2126.45820)
$d_1(10^{-5})$	7.8049[a]	5.9544[b]	9.9422[a]	27.2830[a]	0.1485[a]
	(3.09625)	(2.02987)	(4.26473)	(14.53342)	(6.47499)
$c_3(10^{-5})$	1.7823[a]	2.1178[a]	1.5064[a]	2.7274[a]	2.9802[a]
	(18.42652)	(19.25889)	(16.40697)	(16.29332)	(18.00128)
a_{33}	0.1162[a]	0.1493[a]	0.1463[a]	0.1876[a]	0.1987[a]
	(19.76647)	(27.95356)	(23.16333)	(18.90226)	(24.90062)
b_{33}	0.7662[a]	0.7310[a]	0.7855[a]	0.6709[a]	0.6578[a]
	(72.88859)	(70.71214)	(90.99770)	(46.04264)	(50.23194)
$d_3(10^{-5})$	18.3940[a]	8.7442[a]	5.0346[a]	0.5860[a]	0.0076[c]
	(10.59572)	(5.92612)	(3.33148)	(3.48770)	(1.66075)
$c_2(10^{-5})$	1.2069[a]	1.7875[a]	1.1148[a]	3.5772[a]	0.0045[a]
	(18.00160)	(15.62767)	(16.81126)	(5.21581)	(5.64439)
a_{22}	0.0779[a]	0.1178[a]	0.1090[a]	0.0617[a]	0.0069[a]
	(18.28450)	(25.58234)	(26.37499)	(5.70796)	(9.56625)
b_{22}	0.8465[a]	0.7828[a]	0.8241[a]	0.5589[a]	0.9866[a]
	(128.78452)	(111.55409)	(140.02598)	(7.04661)	(716.08722)
$d_2(10^{-5})$	13.5390[a]	8.2965[a]	6.2364[a]	2.5514[a]	0.0018[a]
	(6.76664)	(4.50321)	(4.04998)	(3.11076)	(3.43372)
L	29662.24	29751.68	29613.52	27918.19	28067.47
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	6.4662	9.3420	11.8913	1.8963	13.0953
L-B(12)[e]	9.3210	8.3033	11.0178	6.0642	5.1064
$u_t / h_t^{1/2}$					
L-B(12)[d]	10.4461	18.3160*	13.6212	24.5513**	37.1622**
L-B(12)[e]	10.8818	20.2929*	11.9756	11.5983	12.1435
L-B(12)[f]	8.2678	5.0839	10.2410	3.7766	5.5524

Notes: see Table 6.1 part 1.

Table 6.14: Basic Statistics of Time-Varying Betas (GARCH-X)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
Legal and General	1.12977[a]	0.055028	0.60673[a]	2.50730[a]	1264.40134[a]
Glaxo Smith Kline	1.06473[a]	0.047280	0.64660[a]	13.02330[a]	27911.28447[a]
British Vita	0.79066[a]	0.059445	-0.61208[a]	1.24951[a]	498.62742[a]
Alvis	0.71014[a]	0.059964	0.62784[a]	4.86054[a]	4106.80468[a]
Care UK	0.70334[a]	0.174471	1.09518[a]	1.23940[a]	1032.13614[a]

Notes:
[a] Significant at the 1% level,

Table 6.15: Two Unit Root Tests for Time-Varying Betas (GARCH-X)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Legal and General	-64.43451[a]	-33.49045[a]	-28.54154[a]	-25.87291[a]
Glaxo Smith Kline	-67.98938[a]	-37.74379[a]	-30.41867[a]	-27.59868[a]
British Vita	-69.14778[a]	-35.36320[a]	-27.23396[a]	-25.05314[a]
Alvis	-71.92869[a]	-37.55190[a]	-32.29266[a]	-26.49416[a]
Care UK	-67.33912[a]	-34.18026[a]	-28.95901[a]	-25.42002[a]

Notes:
[a] Significant at the 1% level.

Table 6.16: One Unit Root Tests for Time-Varying Betas (GARCH-X)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Legal and General	-14.20946[a]	-13.92497[a]	-13.04746[a]	-11.84252[a]
Glaxo Smith Kline	-18.51314[a]	-16.80439[a]	-14.58376[a]	-12.78011[a]
British Vita	-14.49358[a]	-12.14997[a]	-11.66684[a]	-10.74938[a]
Alvis	-18.24002[a]	-14.28115[a]	-11.94258[a]	-10.26299[a]
Care UK	-10.79273[a]	-9.46618[a]	-8.44099[a]	-7.29291[a]

Notes:
[a] Significant at the 1% level.

Table 6.17: Akaike information criterion for four transition equations

Company	AR(1)	Random Coefficient	Random Walk	Random Walk with Drift
British Airways	FTC	-5.026004	-5.001551	-5.001041
TESCO	FTC	-5.494316	-5.505300	-5.504799
British American Tobacco	FTC	-5.113172	-5.121012	-5.120808
BT Group	FTC	-5.318965	-5.319510	-5.319002
Legal and General	FTC	-5.446560	-5.437527	-5.437036
Glaxo Smith Kline	FTC	-5.562945	-5.565883	-5.565372
Edinburgh Oil and Gas	-4.489112	-4.486331	-4.464013	-4.463510
Boots Group	FTC	-5.662121	-5.677472	-5.676979
Barclays	FTC	-5.559008	-5.547856	-5.547348
Scottish and Newcastle	FTC	-5.519656	-5.524472	-5.523967
Signet Group	FTC	-4.045089	-4.039922	FTC
Goodwin	FTC	FTC	-4.490333	-4.489840
British Vita	FTC	-5.471271	-5.467581	-5.467077
Caldwell Investments	FTC	-4.192894	-4.188251	-4.187769
Alvis	-4.510596	-4.510887	-4.482909	-4.482418
Tottenham Hotspur	-5.209870	-5.208457	-5.180908	-5.180397
Care UK	-3.933540	-3.932203	-3.925021	FTC
Daily Mail and Gen Trust	-6.086694	-6.061594	-6.061610	-6.061101
Cable and Wireless	FTC	-4.736917	-4.674249	-4.673740
BAE Systems	FTC	-4.635179	-4.627606	-4.627162

Notes: FTC stands for 'failed to converge'

Table 6.18: Bayesian information criterion for four transition equations

Company	AR(1)	Random Coefficient	Random Walk	Random Walk with Drift
British Airways	FTC	-5.019592	-4.996741	-4.994628
TESCO	FTC	-5.487903	-5.500490	-5.498386
British American Tobacco	FTC	-5.106759	-5.116202	-5.114395
BT Group	FTC	-5.312552	-5.314700	-5.312589
Legal and General	FTC	-5.440147	-5.432717	-5.430623
Glaxo Smith Kline	FTC	-5.556532	-5.561074	-5.558959
Edinburgh Oil and Gas	-4.481096	-4.479918	-4.459203	-4.457098
Boots Group	FTC	-5.655708	-5.672662	-5.670567
Barclays	FTC	-5.552595	-5.543047	-5.540935
Scottish and Newcastle	FTC	-5.513244	-5.519662	-5.517554
Signet Group	FTC	-4.045089	-4.035112	FTC
Goodwin	FTC	FTC	-4.485523	-4.483427
British Vita	FTC	-5.464858	-5.462771	-5.460664
Caldwell Investments	FTC	-4.186481	-4.183441	-4.181357
Alvis	-4.502580	-4.504474	-4.478099	-4.476005
Tottenham Hotspur	-5.201854	-5.202044	-5.176098	-5.173984
Care UK	-3.925524	-3.925790	-3.920211	FTC
Daily Mail and Gen Trust	-6.078678	-6.055181	-6.056800	-6.054688
Cable and Wireless	FTC	-4.730504	-4.669439	-4.667327
BAE Systems	FTC	-4.628767	-4.622796	-4.620749

Notes: FTC stands for 'failed to converge'

Table 6.19: Basic Statistics of the Time-Varying Beta (Kalman Filter)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
British Airways	1.07209[a]	0.115166	0.09102[b]	2.53996[a]	1056.70729[a]
TESCO	0.90806[a]	0.071164	-1.37013[a]	1.96697[a]	1854.13711[a]
British American Tobacco	0.92564[a]	0.061546	-0.72419[a]	-0.59854[a]	400.23065[a]
BT Group	1.14976[a]	0.050016	1.44771[a]	2.76982[a]	2616.36263[a]
Legal and General	1.12516[a]	0.042993	0.48795[a]	0.27506[a]	167.52666[a]
Glaxo Smith Kline	1.04931[a]	0.052656	0.11382[a]	3.18857[a]	1665.23902[a]
Edinburgh Oil and Gas	0.58483[a]	0.055085	-0.17627[a]	-0.15508[b]	24.17115[a]
Boots Group	0.86368[a]	0.089443	-1.15266[a]	1.92089[a]	1467.33392[a]
Barclays	1.17733[a]	0.030063	0.03806	0.47980[a]	38.45907[a]
Scottish and Newcastle	0.80219[a]	0.110444	-1.82015[a]	2.89465[a]	3524.91774[a]
Signet Group	0.93146[a]	0.037155	1.54125[a]	13.16155[a]	29777.10806[a]
Goodwin	0.58962[a]	0.042937	0.35459[a]	2.08499[a]	790.37088[a]
British Vita	0.73707[a]	0.063464	-0.80230[a]	0.05265	420.02971[a]
Caldwell Investments	0.64591[a]	0.027790	-0.05460	0.72914[a]	88.57914[a]
Alvis	0.74390[a]	0.072038	0.43876[a]	2.73347[a]	1343.08784[a]
Tottenham Hotspur	0.61058[a]	0.066186	-0.15969[a]	-0.28572[a]	29.92615[a]
Care UK	0.75474[a]	0.116298	0.18388[a]	-1.02656[a]	193.76996[a]
Daily Mail and Gen Trust	0.50925[a]	0.115872	-0.57708[a]	-0.28880[a]	230.66789[a]
Cable and Wireless	1.24819[a]	0.144689	1.76700[a]	3.59514[a]	4141.45478[a]
BAE Systems	0.96369[a]	0.023567	0.72366[a]	9.48558[a]	15003.71549[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 6.20: Two Unit Root Tests for Time-Varying Betas (Kalman Filter)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-63.49464[a]	-30.45408[a]	-25.79095[a]	-20.98949[a]
TESCO	-58.59763[a]	-33.13054[a]	-26.89041[a]	-24.73387[a]
British American Tobacco	-49.83984[a]	-28.10411[a]	-20.32728[a]	-33.53931[a]
BT Group	-57.45071[a]	-34.11450[a]	-22.45443[a]	-18.87205[a]
Legal and General	-62.07455[a]	-34.69316[a]	-23.93416[a]	-23.04754[a]
Glaxo Smith Kline	-59.16422[a]	-38.43313[a]	-24.53414[a]	-20.55033[a]
Edinburgh Oil and Gas	-63.95792[a]	-33.40041[a]	-32.77253[a]	-24.71449[a]
Boots Group	-78.80929[a]	-34.05345[a]	-24.08977[a]	-24.60984[a]
Barclays	-65.88072[a]	-31.92170[a]	-25.08118[a]	-20.30024[a]
Scottish and Newcastle	-65.91796[a]	-31.94932[a]	-23.58330[a]	-19.76948[a]
Signet Group	-42.07776[a]	-51.18961[a]	-44.24169[a]	-22.23170[a]
Goodwin	-72.23046[a]	-38.40832[a]	-49.04800[a]	-41.64905[a]
British Vita	-61.96748[a]	-32.58456[a]	-22.69826[a]	-21.47364[a]
Caldwell Investments	-69.69420[a]	-40.78363[a]	-30.91158[a]	-23.56067[a]
Alvis	-81.11560[a]	-35.59331[a]	-24.20029[a]	-20.65389[a]
Tottenham Hotspur	-57.73718[a]	-35.19585[a]	-26.85063[a]	-21.21175[a]
Care UK	-58.71550[a]	-38.55352[a]	-33.95435[a]	-20.76338[a]
Daily Mail and Gen Trust	-64.75529[a]	-32.98975[a]	-26.18617[a]	-22.13277[a]
Cable and Wireless	-63.23847[a]	-31.12835[a]	-24.77916[a]	-20.80689[a]
BAE Systems	-56.70291[a]	-28.82734[a]	-25.25347[a]	-32.26326[a]

Notes:
[a] Significant at the 1% level,

Table 6.21: One Unit Root Tests for Time-Varying Betas (Kalman Filter)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
British Airways	-6.37393[a]	-6.11865[a]	-5.30381[a]	-5.14322[a]
TESCO	-2.45032	-2.74603[c]	-2.53473	-2.88236[b]
British American Tobacco	-2.88961[b]	-2.22084	-2.54119	-1.67399
BT Group	-3.57126[a]	-3.33147[b]	-3.60807[a]	-3.57554[a]
Legal and General	-3.14504[b]	-3.07534[b]	-2.80629[c]	-2.90648[b]
Glaxo Smith Kline	-3.72945[a]	-3.33462[b]	-3.21668[b]	-3.31898[b]
Edinburgh Oil and Gas	-3.99026[a]	-4.11332[a]	-3.39935[b]	-2.86987[b]
Boots Group	-7.29944[a]	-2.61374[c]	-3.17863[b]	-2.16402
Barclays	-3.99629[a]	-4.02428[a]	-3.67124[a]	-3.57726[a]
Scottish and Newcastle	-3.73017[a]	-3.32348[b]	-3.27850[b]	-3.25474[b]
Signet Group	-13.73460[a]	-3.03973[b]	-4.59839[a]	-7.61966[a]
Goodwin	-9.11296[a]	-5.32889[a]	-1.98572	-2.56840[c]
British Vita	-4.26349[a]	-3.01066[b]	-2.99008[b]	-3.08278[b]
Caldwell Investments	-6.42640[a]	-5.50879[a]	-4.45467[a]	-5.30243[a]
Alvis	-6.17811[a]	-4.34771[a]	-5.00069[a]	-3.71057[a]
Tottenham Hotspur	-3.97990[a]	-4.71054[a]	-4.01048[a]	-3.21042[b]
Care UK	-3.10896[b]	-2.94756[b]	-2.67894[c]	-2.86005[c]
Daily Mail and Gen Trust	-7.59942[a]	-6.73985[a]	-6.42242[a]	-6.32030[a]
Cable and Wireless	-4.21892[a]	-4.31487[a]	-3.91905[a]	-4.08168[a]
BAE Systems	-7.86545[a]	-8.43029[a]	-7.30041[a]	-3.68660[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 6.22: Mean Value of Beta Estimates

Company	GARCH	BEKK	GJR	GARCH-X	Kalman	Market Model
British Airways	1.08209	1.08630	1.07933		1.07209	1.12652
TESCO	0.93109	0.93194	0.93319		0.90806	0.92142
British American Tobacco	0.98526	0.98876	0.98789		0.92564	0.87438
BT Group	1.14696	1.15161	1.14010		1.14976	1.13519
Legal and General	1.12486	1.12472	1.12582	1.12977	1.12516	1.15979
Glaxo Smith Kline	1.06325	1.06361	1.06307	1.06473	1.04931	1.02731
Edinburgh Oil and Gas	0.65030	0.63689	0.64222		0.58483	0.58868
Boots Group	0.92020	0.92442	0.92135		0.86368	0.85192
Barclays	1.16440	1.15795	1.16111		1.17733	1.20711
Scottish and Newcastle	0.86777	0.85651	0.86134		0.80219	0.81358
Signet Group	0.85000	0.84889	0.83995		0.93146	0.89053
Goodwin	0.60953	0.58684	0.63673		0.58962	0.57454
British Vita	0.79515	0.80153	0.79524	0.79066	0.73707	0.72427
Caldwell Investments	0.61020	0.61539	0.60978		0.64591	0.62118
Alvis	0.68607	0.69576	0.67550	0.71014	0.74390	0.72646
Tottenham Hotspur	0.57720	0.58031	0.57606		0.61058	0.61376
Care UK	0.68293	0.59538	0.66500	0.703340	0.75474	0.70123
Daily Mail and Gen Trust	0.60805	0.58766	0.63067		0.50925	0.58916
Cable and Wireless	1.22242	1.21609	1.21946		1.24819	1.20858
BAE Systems	0.96733	0.96771	0.96814		0.96369	0.94268

Table 6.23: Mean Absolute Errors of Beta Forecasts (2001)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0084686	0.0168062	0.0074136		0.1282157
TESCO	0.0095412	0.0077338	0.0097130		0.0259571
British American Tobacco	0.0049125	0.0049499	0.0066632		0.0011952
BT Group	0.0026112	0.0031155	0.0019294		0.0449424
Legal and General	0.0100191	0.0149000	0.0110751	0.0078262	0.0199319
Glaxo Smith Kline	0.0134166	0.0161761	0.0108137	0.0158560	0.0055323
Edinburgh Oil and Gas	0.0067359	0.0606013	0.0070213		0.0539989
Boots Group	0.0044822	0.0102296	0.0195847		0.0326404
Barclays	0.0133022	0.0072333	0.0086839		0.0089580
Scottish and Newcastle	0.0063706	0.0164466	0.0049457		0.0724471
Signet Group	0.0071158	0.0054371	0.0121769		0.0088683
Goodwin	0.0034951	0.0034693	0.0023848		0.0056316
British Vita	0.0034018	0.0026321	0.0034122	0.0116327	0.0883260
Caldwell Investments	0.0141649	0.0188385	0.0172729		0.0206887
Alvis	0.0120857	0.0069104	0.0142171	0.0180816	0.0242078
Tottenham Hotspur	0.0032284	0.0029455	0.0035916		0.0546346
Care UK	0.0064999	0.0047806	0.0251933	0.0072353	0.0033839
Daily Mail and Gen Trust	0.0494063	0.0063821	0.0094027		0.1496226
Cable and Wireless	0.0103246	0.0184335	0.0110687		0.1243133
BAE Systems	0.0074434	0.0183744	0.0112785		0.0263841

Table 6.24: Mean Square Errors of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0001376	0.0006724	0.0000984		0.0245447
TESCO	0.0001334	0.0000804	0.0001303		0.0010002
British American Tobacco	0.0000293	0.0000296	0.0000728		0.0000022
BT Group	0.0000147	0.0000215	0.0000101		0.0024016
Legal and General	0.0001996	0.0004354	0.0002447	0.0001198	0.0006045
Glaxo Smith Kline	0.0003939	0.0004201	0.0002363	0.0005499	0.0000436
Edinburgh Oil and Gas	0.0001171	0.0110202	0.0001564		0.0044267
Boots Group	0.0000490	0.0001643	0.0007379		0.0014873
Barclays	0.0003015	0.0001028	0.0001292		0.0001284
Scottish and Newcastle	0.0000702	0.0004483	0.0000502		0.0100068
Signet Group	0.0000894	0.0000528	0.0003122		0.0001039
Goodwin	0.0000169	0.0000194	0.0000187		0.0000529
British Vita	0.0000279	0.0000165	0.0000253	0.0002969	0.0090600
Caldwell Investments	0.0008715	0.0016880	0.0013596		0.0005481
Alvis	0.0001981	0.0000581	0.0002923	0.0004871	0.0008504
Tottenham Hotspur	0.0000224	0.0000191	0.0000282		0.0039746
Care UK	0.0000787	0.0000312	0.0016764	0.0000952	0.0000208
Daily Mail and Gen Trust	0.0065895	0.0001940	0.0002274		0.0529513
Cable and Wireless	0.0003100	0.0009791	0.0003481		0.0215384
BAE Systems	0.0000968	0.0005543	0.0002477		0.0009257

Table 6.25: Mean Absolute Percentage Error of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0098324	0.0179261	0.0086332		0.1971840
TESCO	0.0259898	0.0159343	0.0209866		1.9537035
British American Tobacco	0.0141025	0.0178644	0.0458410		0.0028865
BT Group	0.0022037	0.0027288	0.0015381		0.0304860
Legal and General	0.0105802	0.0156244	0.0115684	0.0082000	0.0255635
Glaxo Smith Kline	0.0197898	0.0214860	0.0153031	0.0242599	0.0115882
Edinburgh Oil and Gas	0.0129400	0.3067038	0.0177402		0.1579803
Boots Group	0.0085581	0.0745465	0.0486433		0.4202224
Barclays	0.0113105	0.0058804	0.0074640		0.0074222
Scottish and Newcastle	0.0115005	0.0407082	0.0087261		0.3031698
Signet Group	0.0097631	0.0066747	0.0167899		0.0124647
Goodwin	0.0228771	0.0079339	0.0049237		0.0162980
British Vita	0.0107509	0.0050165	0.0108342	0.0447253	0.3589715
Caldwell Investments	0.0356812	0.0783948	0.0438557		0.0491595
Alvis	0.0233948	0.0141069	0.0270198	0.0375458	0.0608017
Tottenham Hotspur	0.0308284	0.0080950	0.0166161		0.1882548
Care UK	0.0156168	0.0123435	0.0903406	0.0166596	0.0113133
Daily Mail and Gen Trust	0.1651181	0.0321096	0.1367078		0.8363203
Cable and Wireless	0.0069528	0.0121303	0.0074356		0.0721755
BAE Systems	0.0131537	0.0492358	0.0194999		0.0394593

Table 6.26: Theil U Statistics of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.00900	0.01940	0.00766		0.12714
TESCO	0.01558	0.01231	0.01518		0.07400
British American Tobacco	0.00722	0.00730	0.01128		0.00320
BT Group	0.00281	0.00338	0.00235		0.03176
Legal and General	0.01304	0.01926	0.01440	0.01007	0.02908
Glaxo Smith Kline	0.02062	0.02140	0.01596	0.02383	0.00862
Edinburgh Oil and Gas	0.01259	0.11716	0.01474		0.11627
Boots Group	0.00953	0.01859	0.03732		0.09254
Barclays	0.01396	0.00808	0.00919		0.00959
Scottish and Newcastle	0.01108	0.02726	0.00935		0.18796
Signet Group	0.00998	0.00763	0.01889		0.01426
Goodwin	0.00605	0.00678	0.00638		0.01835
British Vita	0.00703	0.00542	0.00668	0.02231	0.20067
Caldwell Investments	0.04924	0.06739	0.06136		0.05344
Alvis	0.02125	0.01189	0.02617	0.03474	0.06251
Tottenham Hotspur	0.00862	0.00795	0.00967		0.14912
Care UK	0.01632	0.01052	0.07705	0.01830	0.01268
Daily Mail and Gen Trust	0.14556	0.02543	0.02664		0.53582
Cable and Wireless	0.01187	0.02084	0.01266		0.07793
BAE Systems	0.01086	0.02502	0.01752		0.04386

Table 6.27: Mean Absolute Errors of Beta Forecasts (2003)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0045983	0.0059847	0.0043704		0.0515983
TESCO	0.0068799	0.0076921	0.0074731		0.0027855
British American Tobacco	0.0047564	0.0048528	0.0040911		0.0013410
BT Group	0.0011102	0.0011546	0.0010064		0.0032679
Legal and General	0.0031213	0.0065911	0.0038696	0.0044011	0.0388119
Glaxo Smith Kline	0.0015708	0.0013815	0.0018276	0.0023252	0.0023246
Edinburgh Oil and Gas	0.0019670	0.0043035	0.0017061		0.0045907
Boots Group	0.0018547	0.0006080	0.0020537		0.0077447
Barclays	0.0056764	0.0087349	0.0045083		0.0004452
Scottish and Newcastle	0.0170112	0.0064285	0.0067307		0.0139968
Signet Group	0.0034560	0.0043387	0.0039627		0.0005482
Goodwin	0.0039920	0.0060818	0.0034736		0.0034858
British Vita	0.0028207	0.0029279	0.0028509	0.0027155	0.0495767
Caldwell Investments	0.0029663	0.0022079	0.0026260		0.0067177
Alvis	0.0061197	0.0022856	0.0062426	0.0203871	0.0349402
Tottenham Hotspur	0.0107096	0.0112547	0.0138342		0.0326198
Care UK	0.0024569	0.0041020	0.0259110	0.0071954	0.0111731
Daily Mail and Gen Trust	0.0011457	0.0026805	0.0020157		0.0795384
Cable and Wireless	0.0043440	0.0035129	0.0042065		0.0026282
BAE Systems	0.0039760	0.0055520	0.0050140		0.0187046

Table 6.28: Mean Square Errors of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0000447	0.0000701	0.0000490		0.0048096
TESCO	0.0000853	0.0001094	0.0001047		0.0000119
British American Tobacco	0.0000401	0.0000415	0.0000333		0.0000027
BT Group	0.0000027	0.0000027	0.0000022		0.0000156
Legal and General	0.0000220	0.0000854	0.0000357	0.0000423	0.0019072
Glaxo Smith Kline	0.0000034	0.0000028	0.0000045	0.0000068	0.0000106
Edinburgh Oil and Gas	0.0000083	0.0000353	0.0000066		0.0000308
Boots Group	0.0000054	0.0000006	0.0000069		0.0002836
Barclays	0.0000565	0.0001402	0.0000357		0.0000003
Scottish and Newcastle	0.0006660	0.0001142	0.0001087		0.0003261
Signet Group	0.0000202	0.0000411	0.0000238		0.0000004
Goodwin	0.0000465	0.0001306	0.0000331		0.0000166
British Vita	0.0000167	0.0000178	0.0000171	0.0000147	0.0029544
Caldwell Investments	0.0000125	0.0000129	0.0000097		0.0000556
Alvis	0.0000639	0.0000123	0.0000761	0.0006276	0.0014411
Tottenham Hotspur	0.0003615	0.0003669	0.0006573		0.0014620
Care UK	0.0000094	0.0000433	0.0015632	0.0000658	0.0001590
Daily Mail and Gen Trust	0.0000024	0.0000135	0.0000067		0.0084785
Cable and Wireless	0.0000442	0.0000283	0.0000492		0.0000123
BAE Systems	0.0000319	0.0000609	0.0000613		0.0004147

Table 6.29: Mean Absolute Percentage Error of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0032186	0.0042654	0.0031626		0.0304532
TESCO	0.0080631	0.0089799	0.0088327		0.0030066
British American Tobacco	0.0100603	0.0102013	0.0060223		0.0020923
BT Group	0.0009595	0.0010177	0.0008834		0.0030375
Legal and General	0.0022983	0.0049697	0.0029735	0.0031751	0.0259035
Glaxo Smith Kline	0.0014450	0.0012683	0.0017055	0.0021708	0.0022147
Edinburgh Oil and Gas	0.0036360	0.0124882	0.0033435		0.0111960
Boots Group	0.0020788	0.0006650	0.0023530		0.0102618
Barclays	0.0046474	0.0072334	0.0036679		0.0003484
Scottish and Newcastle	0.0241065	0.0397301	0.0093880		0.0172625
Signet Group	0.0066026	0.0078875	0.0062472		0.0007074
Goodwin	0.0302894	0.0272984	0.0097366		0.0103528
British Vita	0.0046640	0.0047855	0.0047247	0.0047346	0.1437389
Caldwell Investments	0.0112114	0.0055080	0.0060398		0.0183992
Alvis	0.0132807	0.0064161	0.0270597	0.0477765	0.1102146
Tottenham Hotspur	0.0527430	0.0730532	0.0411249		0.2654002
Care UK	0.0038220	0.0186503	0.0844401	0.0133712	0.0349534
Daily Mail and Gen Trust	0.0025751	0.0055633	0.0040571		0.6084532
Cable and Wireless	0.0048773	0.0069751	0.0049578		0.0021218
BAE Systems	0.0061243	0.0079628	0.0075013		0.0213760

Table 6.30: Theil U Statistics of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.00465	0.00568	0.00490		0.04255
TESCO	0.00952	0.01075	0.01057		0.00365
British American Tobacco	0.00750	0.00765	0.00677		0.00266
BT Group	0.00136	0.00138	0.00126		0.00333
Legal and General	0.00332	0.00651	0.00422	0.00457	0.02905
Glaxo Smith Kline	0.00168	0.00152	0.00195	0.00238	0.00312
Edinburgh Oil and Gas	0.00444	0.00944	0.00403		0.01194
Boots Group	0.00252	0.00085	0.00286		0.02127
Barclays	0.00605	0.00955	0.00482		0.00042
Scottish and Newcastle	0.02640	0.01090	0.01081		0.01976
Signet Group	0.00508	0.00734	0.00556		0.00076
Goodwin	0.01124	0.01935	0.00900		0.01120
British Vita	0.00552	0.00569	0.00559	0.00522	0.10200
Caldwell Investments	0.00619	0.00619	0.00545		0.01817
Alvis	0.01206	0.00539	0.01337	0.03428	0.07434
Tottenham Hotspur	0.03383	0.03405	0.04593		0.08478
Care UK	0.00473	0.01167	0.06251	0.01309	0.03630
Daily Mail and Gen Trust	0.00281	0.00683	0.00451		0.19841
Cable and Wireless	0.00542	0.00439	0.00574		0.00273
BAE Systems	0.00541	0.00708	0.00748		0.02371

Table 6.31: Mean Absolute Errors of Beta Forecasts (2002-2003)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0236427	0.0243810	0.0239083		0.0126694
TESCO	0.0044170	0.0031297	0.0055597		0.0031328
British American Tobacco	0.0052437	0.0049579	0.0050856		0.0000000
BT Group	0.0078565	0.0044705	0.0069262		0.0151559
Legal and General	0.0210524	0.0211435	0.0207527	0.0145962	0.0407126
Glaxo Smith Kline	0.0126473	0.0126276	0.0123341	0.0165558	0.0074735
Edinburgh Oil and Gas	0.0032825	0.0144478	0.0028304		0.0018991
Boots Group	0.0089044	0.0015787	0.0054030		0.0098713
Barclays	0.0102749	0.0024695	0.0112424		0.0015865
Scottish and Newcastle	0.0176912	0.0855151	0.0214859		0.0380506
Signet Group	0.0252728	0.0256021	0.0177374		0.0028286
Goodwin	0.0085991	0.0143034	0.0066745		0.0057926
British Vita	0.0047060	0.0046239	0.0047464	0.0044912	0.0835241
Caldwell Investments	0.0109865	0.0111852	0.0108563		0.0045828
Alvis	0.0197598	0.0144125	0.0223321	0.0259987	0.0372462
Tottenham Hotspur	0.0150951	0.0157761	0.0162875		0.0558911
Care UK	0.0064530	0.0062520	0.0066907	0.0065675	0.0076961
Daily Mail and Gen Trust	0.0100717	0.0101609	0.0031174		0.0486185
Cable and Wireless	0.0127927	0.0114645	0.0109396		0.0926153
BAE Systems	0.0098499	0.0050272	0.0099973		0.0017005

Table 6.32: Mean Square Errors of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0011394	0.0011836	0.0011224		0.0003165
TESCO	0.0000325	0.0000173	0.0000529		0.0000150
British American Tobacco	0.0000346	0.0000293	0.0000468		0.0000000
BT Group	0.0001225	0.0000399	0.0000983		0.0003469
Legal and General	0.0007872	0.0007773	0.0007781	0.0003902	0.0024890
Glaxo Smith Kline	0.0002362	0.0002437	0.0002234	0.0004333	0.0001018
Edinburgh Oil and Gas	0.0000327	0.0005960	0.0000210		0.0000047
Boots Group	0.0001245	0.0000075	0.0000592		0.0003968
Barclays	0.0001914	0.0000105	0.0002286		0.0000043
Scottish and Newcastle	0.0004554	0.0130250	0.0006946		0.0023305
Signet Group	0.0012229	0.0012482	0.0005409		0.0000126
Goodwin	0.0001171	0.0003420	0.0000701		0.0000491
British Vita	0.0000405	0.0000400	0.0000419	0.0000357	0.0095590
Caldwell Investments	0.0002069	0.0002068	0.0001973		0.0000289
Alvis	0.0004868	0.0003326	0.0006107	0.0008520	0.0019422
Tottenham Hotspur	0.0005204	0.0005620	0.0007284		0.0046124
Care UK	0.0000639	0.0000948	0.0000696	0.0000653	0.0000889
Daily Mail and Gen Trust	0.0001280	0.0001278	0.0000138		0.0044044
Cable and Wireless	0.0009688	0.0005179	0.0004169		0.0233508
BAE Systems	0.0001531	0.0000547	0.0001786		0.0000044

Table 6.33: Mean Absolute Percentage Error of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0203653	0.0211266	0.0205605		0.0078510
TESCO	0.0050824	0.0035774	0.0063627		0.0036893
British American Tobacco	0.0089764	0.0082775	0.0087430		0.0000000
BT Group	0.0069494	0.0039520	0.0061817		0.0135587
Legal and General	0.0179401	0.0179395	0.0176203	0.0123300	0.0301329
Glaxo Smith Kline	0.0125711	0.0124622	0.0122863	0.0163457	0.0078430
Edinburgh Oil and Gas	0.0080810	0.0569235	0.0073172		0.0038970
Boots Group	0.0118829	0.0019821	0.0075288		0.0139774
Barclays	0.0082891	0.0019091	0.0090234		0.0011665
Scottish and Newcastle	0.0347866	0.3727716	0.0722072		0.0877755
Signet Group	0.0557903	0.0482161	0.0359186		0.0034515
Goodwin	0.0382614	0.0675128	0.0215137		0.0178803
British Vita	0.0123615	0.0102507	0.0123594	0.0111530	0.3038819
Caldwell Investments	0.0639340	0.0559849	0.1034572		0.0116458
Alvis	0.0381092	0.0327124	0.0539383	0.0509191	0.1187485
Tottenham Hotspur	0.1190804	0.0807594	0.0724447		0.4647734
Care UK	0.0166124	0.0294248	0.0180452	0.0509191	0.0234495
Daily Mail and Gen Trust	0.0322697	0.0261830	0.0070284		0.7697512
Cable and Wireless	0.0107125	0.0126057	0.0100033		0.0614289
BAE Systems	0.0162566	0.0090538	0.0190893		0.0020270

Table 6.34: Theil U Statistics of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.02463	0.02450	0.02460		0.01146
TESCO	0.00609	0.00443	0.00782		0.00442
British American Tobacco	0.00765	0.00706	0.00879		0.00000
BT Group	0.00911	0.00519	0.00828		0.01596
Legal and General	0.02194	0.02169	0.02182	0.01546	0.03833
Glaxo Smith Kline	0.01461	0.01484	0.01423	0.01976	0.01008
Edinburgh Oil and Gas	0.00868	0.03739	0.00699		0.00375
Boots Group	0.01298	0.00319	0.00898		0.02677
Barclays	0.01098	0.00255	0.01197		0.00151
Scottish and Newcastle	0.02566	0.14027	0.03200		0.06669
Signet Group	0.03986	0.04037	0.02692		0.00441
Goodwin	0.01872	0.03258	0.01481		0.01858
British Vita	0.00967	0.00954	0.00986	0.00924	0.19610
Caldwell Investments	0.02442	0.02383	0.02400		0.01197
Alvis	0.03498	0.03015	0.03984	0.04409	0.09054
Tottenham Hotspur	0.04464	0.04612	0.05300		0.16553
Care UK	0.01319	0.01977	0.01413	0.01396	0.02527
Daily Mail and Gen Trust	0.02154	0.02156	0.00718		0.17914
Cable and Wireless	0.02126	0.01495	0.01385		0.10848
BAE Systems	0.01261	0.00705	0.01350		0.00262

Table 6.35: Mean Absolute Error of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0250654	0.0250783	0.0250556		0.0239367
TESCO	0.0127072	0.0126200	0.0127282		0.0123592
British American Tobacco	0.0158963	0.0158690	0.0158612		0.0149533
BT Group	0.0217160	0.0216911	0.0216962		0.0211849
Legal and General	0.0135475	0.0135365	0.0135500	0.0135202	0.0132796
Glaxo Smith Kline	0.0120847	0.0120971	0.0120946	0.0121054	0.0113994
Edinburgh Oil and Gas	0.0246765	0.0247832	0.0247145		0.0236882
Boots Group	0.0130286	0.0128844	0.0129654		0.0125544
Barclays	0.0116646	0.0117278	0.0115765		0.0116428
Scottish and Newcastle	0.0127709	0.0128278	0.0127671		0.0121875
Signet Group	0.0222870	0.0224033	0.0222272		0.0218283
Goodwin	0.0097306	0.0095619	0.0097586		0.0082519
British Vita	0.0128013	0.0128081	0.0128041	0.0128808	0.0117594
Caldwell Investments	0.0138171	0.0139182	0.0138177		0.0129173
Alvis	0.0101944	0.0101125	0.0101834	0.0100914	0.0092428
Tottenham Hotspur	0.0072708	0.0072775	0.0072741		0.0064840
Care UK	0.0080627	0.0081251	0.0079670	0.0080267	0.0071276
Daily Mail and Gen Trust	0.0086048	0.0085634	0.0085854		0.0065789
Cable and Wireless	0.0196925	0.0198110	0.0197132		0.0192077
BAE Systems	0.0172093	0.0175426	0.0172298		0.0172293

Table 6.36: Mean Square Error of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0013389	0.0013586	0.0013347		0.0011394
TESCO	0.0002835	0.0002808	0.0002834		0.0002540
British American Tobacco	0.0004635	0.0004619	0.0004606		0.0003998
BT Group	0.0008249	0.0008250	0.0008247		0.0008046
Legal and General	0.0003409	0.0003413	0.0003413	0.0003399	0.0003219
Glaxo Smith Kline	0.0002828	0.0002843	0.0002834	0.0002828	0.0002472
Edinburgh Oil and Gas	0.0016872	0.0017022	0.0016874		0.0016433
Boots Group	0.0003355	0.0003291	0.0003310		0.0003315
Barclays	0.0002305	0.0002317	0.0002275		0.0002252
Scottish and Newcastle	0.0002887	0.0002897	0.0002885		0.0002619
Signet Group	0.0009984	0.0010033	0.0009947		0.0009754
Goodwin	0.0003792	0.0003743	0.0003590		0.0003278
British Vita	0.0004229	0.0004223	0.0004230	0.0004220	0.0004003
Caldwell Investments	0.0011468	0.0011560	0.0011470		0.0011026
Alvis	0.0002843	0.0002920	0.0002847	0.0002832	0.0002583
Tottenham Hotspur	0.0001342	0.0001335	0.0001340		0.0001173
Care UK	0.0002361	0.0002391	0.0002338	0.0002359	0.0002121
Daily Mail and Gen Trust	0.0002740	0.0002791	0.0002675		0.0002275
Cable and Wireless	0.0008087	0.0008111	0.0008044		0.0007386
BAE Systems	0.0008113	0.0008279	0.0008134		0.0008000

Table 6.37: Mean Errors of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	-0.0028550	-0.0029574	-0.0028483		-0.0022965
TESCO	0.0008661	0.0008814	0.0008466		0.0006798
British American Tobacco	0.0016542	0.0016612	0.0016599		0.0015533
BT Group	-0.0006219	-0.0006024	-0.0005850		-0.0012019
Legal and General	0.0001009	0.0000999	0.0000937	0.0001057	0.0001798
Glaxo Smith Kline	0.0003708	0.0003619	0.0003713	0.0002976	0.0002685
Edinburgh Oil and Gas	0.0080494	0.0077197	0.0079651		0.0079126
Boots Group	0.0012936	0.0012733	0.0012795		0.0010650
Barclays	0.0007597	0.0006743	0.0007811		0.0005916
Scottish and Newcastle	0.0014537	0.0013173	0.0014463		0.0013265
Signet Group	0.0028814	0.0028569	0.0029184		0.0032929
Goodwin	0.0014684	0.0015833	0.0017190		0.0020193
British Vita	0.0007921	0.0008142	0.0007925	0.0007663	0.0010453
Caldwell Investments	0.0025542	0.0025114	0.0025411		0.0026286
Alvis	0.0019022	0.0016273	0.0019085	0.0019783	0.0016942
Tottenham Hotspur	0.0002197	0.0002232	0.0002170		0.0004564
Care UK	0.0001358	0.0001095	0.0001703	0.0001456	0.0000285
Daily Mail and Gen Trust	0.0012014	0.0013011	0.0011919		0.0009966
Cable and Wireless	-0.0034412	-0.0034538	-0.0034162		-0.0035626
BAE Systems	-0.0001008	-0.0003405	-0.0000692		0.0000606

Table 6.38: Mean Absolute Error of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0185217	0.0184925	0.0185243		0.0174962
TESCO	0.0115508	0.0115376	0.0115349		0.0111754
British American Tobacco	0.0094756	0.0094634	0.0094720		0.0092473
BT Group	0.0124291	0.0124298	0.0124075		0.0121858
Legal and General	0.0136021	0.0136231	0.0135874	0.0135725	0.0134095
Glaxo Smith Kline	0.0092898	0.0092855	0.0092826	0.0092693	0.0089191
Edinburgh Oil and Gas	0.0111239	0.0110298	0.0110831		0.0104659
Boots Group	0.0086942	0.0086368	0.0086988		0.0082768
Barclays	0.0096113	0.0096326	0.0096362		0.0093634
Scottish and Newcastle	0.0118202	0.0117944	0.0118191		0.0112943
Signet Group	0.0148877	0.0148152	0.0148815		0.0147425
Goodwin	0.0141553	0.0140250	0.0144543		0.0132958
British Vita	0.0102173	0.0102041	0.0102220	0.0102185	0.0093335
Caldwell Investments	0.0107620	0.0107959	0.0107899		0.0104671
Alvis	0.0103246	0.0103016	0.0102416	0.0105278	0.0094923
Tottenham Hotspur	0.0096136	0.0096069	0.0095960		0.0083703
Care UK	0.0105891	0.0103998	0.0104534	0.0105278	0.0099874
Daily Mail and Gen Trust	0.0046808	0.0045232	0.0048758		0.0034482
Cable and Wireless	0.0207460	0.0208721	0.0208465		0.0203655
BAE Systems	0.0176801	0.0177623	0.0177025		0.0174923

Table 6.39: Mean Square Error of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0006331	0.0006406	0.0006315		0.0005223
TESCO	0.0002471	0.0002469	0.0002469		0.0002290
British American Tobacco	0.0002122	0.0002118	0.0002110		0.0002022
BT Group	0.0002667	0.0002661	0.0002661		0.0002558
Legal and General	0.0003933	0.0003974	0.0003923	0.0003970	0.0003524
Glaxo Smith Kline	0.0001473	0.0001462	0.0001472	0.0001473	0.0001331
Edinburgh Oil and Gas	0.0003664	0.0003709	0.0003671		0.0003285
Boots Group	0.0001444	0.0001412	0.0001443		0.0001278
Barclays	0.0001966	0.0001975	0.0001962		0.0001888
Scottish and Newcastle	0.0002892	0.0002868	0.0002909		0.0002540
Signet Group	0.0005413	0.0005391	0.0005416		0.0005369
Goodwin	0.0007191	0.0007158	0.0007141		0.0006759
British Vita	0.0002072	0.0002061	0.0002075	0.0002076	0.0001821
Caldwell Investments	0.0005759	0.0005806	0.0005773		0.0005821
Alvis	0.0002730	0.0002781	0.0002704	0.0002772	0.0002548
Tottenham Hotspur	0.0003787	0.0003767	0.0003776		0.0003318
Care UK	0.0003384	0.0003436	0.0003340	0.0003384	0.0003255
Daily Mail and Gen Trust	0.0000646	0.0000627	0.0000700		0.0000316
Cable and Wireless	0.0009694	0.0009786	0.0009745		0.0009474
BAE Systems	0.0005922	0.0006060	0.0005939		0.0005685

Table 6.40: Mean Errors of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0008541	0.0007453	0.0008656		0.0010976
TESCO	0.0005148	0.0005189	0.0005417		0.0005750
British American Tobacco	0.0004762	0.0004827	0.0005118		0.0004493
BT Group	-0.0010086	-0.0010210	-0.0009668	-0.0011881	-0.0010095
Legal and General	-0.0011151	-0.0011598	-0.0011128	-0.0004524	-0.0008466
Glaxo Smith Kline	-0.0004206	-0.0004335	-0.0004209		-0.0003514
Edinburgh Oil and Gas	0.0003766	0.0004490	0.0003845		0.0001602
Boots Group	0.0001951	0.0002054	0.0002033		0.0001019
Barclays	-0.0001050	-0.0001792	-0.0001324		0.0002146
Scottish and Newcastle	-0.0015999	-0.0016963	-0.0015721		-0.0015868
Signet Group	0.0012082	0.0012771	0.0012154		0.0011735
Goodwin	0.0042030	0.0043311	0.0039376		0.0039010
British Vita	-0.0001062	-0.0000928	-0.0000940	-0.0000997	-0.0002253
Caldwell Investments	0.0014230	0.0013901	0.0014497		0.0011977
Alvis	0.0004216	0.0003823	0.0004670	0.0004373	0.0002654
Tottenham Hotspur	0.0003334	0.0003449	0.0003292		0.0004611
Care UK	0.0037610	0.0038407	0.0038370	0.0037613	0.0036281
Daily Mail and Gen Trust	-0.0001181	0.0000026	-0.0001972		-0.0003456
Cable and Wireless	0.0042403	0.0042297	0.0042192		0.0043160
BAE Systems	0.0003775	0.0000836	0.0003452		0.0006853

Table 6.41: Mean Absolute Error of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0194522	0.0194374	0.0194478		0.0183077
TESCO	0.0115321	0.0115203	0.0115558		0.0110978
British American Tobacco	0.0113774	0.0113632	0.0113505		0.0109130
BT Group	0.0152781	0.0152917	0.0152776		0.0148285
Legal and General	0.0133593	0.0133431	0.0133563	0.0133559	0.0131427
Glaxo Smith Kline	0.0106384	0.0106134	0.0106344	0.0106452	0.0101880
Edinburgh Oil and Gas	0.0162441	0.0162085	0.0162477		0.0157091
Boots Group	0.0098242	0.0097657	0.0098080		0.0093952
Barclays	0.0105626	0.0105632	0.0105720		0.0101433
Scottish and Newcastle	0.0113288	0.0114449	0.0113325		0.0107998
Signet Group	0.0167971	0.0168118	0.0167696		0.0165634
Goodwin	0.0122490	0.0121878	0.0124109		0.0117495
British Vita	0.0106529	0.0106525	0.0106504	0.0106209	0.0099815
Caldwell Investments	0.0121286	0.0121599	0.0121166		0.0119492
Alvis	0.0104867	0.0104779	0.0104293	0.0105788	0.0098177
Tottenham Hotspur	0.0087272	0.0087597	0.0087285		0.0077203
Care UK	0.0111680	0.0108088	0.0111211	0.0110808	0.0106860
Daily Mail and Gen Trust	0.0064148	0.0063268	0.0063524		0.0045348
Cable and Wireless	0.0222040	0.0223471	0.0222311		0.0216841
BAE Systems	0.0186652	0.0188155	0.0186760		0.0184089

Table 6.42: Mean Square Error of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0006930	0.0006974	0.0006919		0.0005966
TESCO	0.0002454	0.0002448	0.0002471		0.0002278
British American Tobacco	0.0002683	0.0002678	0.0002665		0.0002420
BT Group	0.0004146	0.0004149	0.0004137		0.0003871
Legal and General	0.0003600	0.0003627	0.0003598	0.0003615	0.0003332
Glaxo Smith Kline	0.0002060	0.0002049	0.0002059	0.0002068	0.0001884
Edinburgh Oil and Gas	0.0006673	0.0006776	0.0006711		0.0006235
Boots Group	0.0001831	0.0001809	0.0001825		0.0001663
Barclays	0.0002191	0.0002190	0.0002183		0.0002037
Scottish and Newcastle	0.0002523	0.0002574	0.0002525		0.0002254
Signet Group	0.0006068	0.0006087	0.0006050		0.0005853
Goodwin	0.0005617	0.0005604	0.0005591		0.0005394
British Vita	0.0002282	0.0002276	0.0002282	0.0002278	0.0002099
Caldwell Investments	0.0006868	0.0006915	0.0006869		0.0006676
Alvis	0.0002568	0.0002630	0.0002555	0.0002581	0.0002385
Tottenham Hotspur	0.0003182	0.0003175	0.0003175		0.0002894
Care UK	0.0003914	0.0003920	0.0003904	0.0003905	0.0003793
Daily Mail and Gen Trust	0.0001865	0.0001860	0.0001868		0.0001463
Cable and Wireless	0.0017665	0.0017860	0.0017706		0.0017174
BAE Systems	0.0008509	0.0008756	0.0008522		0.0008128

Table 6.43: Mean Errors of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
British Airways	0.0004641	0.0004110	0.0004587		0.0006915
TESCO	0.0002730	0.0002693	0.0002945		0.0001754
British American Tobacco	0.0006413	0.0006444	0.0006910		0.0005929
BT Group	-0.0003093	-0.0003134	-0.0002874		-0.0003337
Legal and General	-0.0006353	-0.0006531	-0.0006377	-0.0006514	-0.0006855
Glaxo Smith Kline	-0.0003075	-0.0003300	-0.0003052	-0.0003163	-0.0003852
Edinburgh Oil and Gas	0.0000690	0.0000487	0.0000468		0.0001021
Boots Group	0.0006446	0.0006651	0.0006721		0.0004782
Barclays	-0.0002239	-0.0002793	-0.0002581		0.0000552
Scottish and Newcastle	-0.0007615	-0.0008399	-0.0007439		-0.0008355
Signet Group	0.0004704	0.0005005	0.0004611		0.0004076
Goodwin	0.0023780	0.0024628	0.0022442		0.0021862
British Vita	0.0007593	0.0007719	0.0007640	0.0007464	0.0006015
Caldwell Investments	0.0013232	0.0013064	0.0013275		0.0010240
Alvis	0.0010089	0.0009209	0.0010175	0.0010170	0.0008068
Tottenham Hotspur	-0.0006784	-0.0006651	-0.0006786		-0.0007167
Care UK	0.0012710	0.0013698	0.0012644	0.0012552	0.0011024
Daily Mail and Gen Trust	-0.0006020	-0.0005306	-0.0006805		-0.0009812
Cable and Wireless	-0.0011612	-0.0012239	-0.0012058		-0.0008804
BAE Systems	-0.0009022	-0.0010300	-0.0009168		-0.0009998

Table 6.44: Percentage of Dominance of Kalman Filter over Bivariate GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	75	75	75	75	95	85
Worse	0	0	0	0	0	0
Equal Accuracy	25	25	25	25	5	15

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.45: Percentage of Dominance of Kalman Filter over BEKK GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	75	70	75	80	90	85
Worse	0	0	0	0	0	0
Equal Accuracy	25	30	25	20	10	15

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.46: Percentage of Dominance of Kalman Filter over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	75	70	70	80	90	80
Worse	0	0	0	0	0	0
Equal Accuracy	25	30	30	20	10	20

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.47: Percentage of Dominance of Kalman Filter over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	20	60	80	60	80
Worse	0	0	0	0	0	0
Equal Accuracy	100	80	40	20	40	20

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.48: Percentage of Dominance of Bivariate GARCH over BEKK GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	20	20	5	5	20	15
Worse	15	20	20	25	20	30
Equal Accuracy	65	60	75	70	60	55

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.49: Percentage of Dominance of Bivariate GARCH over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	5	0	10	15	10	5
Worse	5	10	25	20	20	15
Equal Accuracy	90	90	65	65	70	80

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.50: Percentage of Dominance of Bivariate GARCH over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	5	0	5	5	5
Worse	0	10	0	5	0	10
Equal Accuracy	100	85	100	90	95	85

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.51: Percentage of Dominance of BEKK GARCH over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	5	10	15	15	15	10
Worse	10	25	5	0	25	15
Equal Accuracy	85	65	80	85	60	75

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.52: Percentage of Dominance of BEKK GARCH over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	20	0	0	20	40
Worse	0	0	0	0	0	20
Equal Accuracy	100	80	100	100	80	40

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Table 6.53: Percentage of Dominance of GARCH-GJR over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	20	0	40	20	20
Worse	0	0	0	0	0	20
Equal Accuracy	100	80	100	60	80	60

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant different between forecast errors. The significance is defined as 'significant at least 10% level'.

Chapter 7
Empirical Results of US Daily Data

7.1 Introduction

The original Capital Asset Pricing Model (CAPM) assumes that the beta of a capital asset or portfolio is constant over time (Bos and Newbold, 1984). However since the 1970s, there has been considerable evidence indicating that the CAPM beta is time dependent rather than remains constant (see Fabozzi and Francis, 1978; Bos and Newbold, 1984 for example). Given that the beta is time-varying, empirical forecasting of the beta has become important for both market participators and corporate financial managers.

As stated by state Brailsford and Faff (1996), '*volatility forecasting is a notoriously difficult task*', the existing literature contains conflicting evidence regarding the relative superiority of volatility forecasts. Moreover, evidence of forecasting systematic risk is absent in US stock markets. Therefore, in order to provide a comprehensive investigation into the relative superiority of GARCH type models and Kalman filter approach, daily data from both UK and US stock markets are considered for comparison analysis. This chapter reports the empirical results of US daily data.

The structure of the chapter is as follows. In section 7.2, estimation results of each model are reported in details. Additionally, some distributional statistics of conditional beta series are reported to present time series characteristics of conditional betas generated by different models. Section 7.3 evaluates and compares the performance of different methods in estimating time-varying betas, mainly be graphic comparison. Section 7.4 presents the results of forecast accuracy evaluation, in terms of forecast error statistics and modified Diebold-Mariano tests. Section 7.5 summaries the main findings in the process of forecasting time-varying betas with US daily data.

7.2 Estimation of Time-Varying Betas

As stated earlier, the estimation of time-varying betas provides the foundation for further study on relative superiority of alternative models in forecasting time-varying betas. US daily beta series are estimated for the whole sample (1989 to 2003) by each candidate model. The estimation results of each GARCH model and time series characteristics of daily beta series constructed by different modelling techniques are discussed as follows.

7.2.1 Bivariate GARCH(1,1) Model

7.2.1.1 Estimation Results

Once again, the standard bivariate GARCH(1,1) model is the primary GARCH model to construct US daily time-varying betas. As done earlier, the bivariate GARCH model applied is the diagonal specification proposed by Bollerslev *et al.* (1988), which provides a parsimonious but efficient way to jointly capture the conditional second moments. Section 3.3.2.1 discusses the bivariate GARCH model in details.

Diagonal bivariate GARCH(1,1) models are all estimated by means of the BHHH algorithm, and the estimation results are reported in Table 7.1, which contains the coefficient estimates, the log-likelihood function value and the Ljung-Box statistics. Similar to UK results, the estimation of bivariate GARCH produces robust parameters. There is strong evidence of volatility clustering, as both ARCH coefficients (a_{11} and a_{33}) are positive and significant at the 1% level in all cases. The ARCH coefficients are all less than unity in size, showing that shocks of previous news to volatility are not explosive. Similarly, both GARCH coefficients (b_{11} and b_{33}) are positive and significant at 1% level in all estimation results, implying considerable GARCH effects in general. Additionally, all the sums of the ARCH and GARCH terms (a_{11} + b_{11}, a_{33} + b_{33}) are fairly high, signifying a high degree of volatility persistence in all return series. Moreover, the sums fall short of unity in most cases, satisfying the stability condition for GARCH models. However two atypical firms (Florida Gaming, Bell industries) has a sum of the ARCH and GARCH coefficients (a_{11} + b_{11}) greater than unity. Such extreme high value indicates that the model is not as robust as others that conform to the non-explosiveness conditional for GARCH models. Nevertheless, the results of the explosive ARCH process will not be excluded for further forecasting purposes. Finally covariance coefficients (a_{22} and b_{22}) are positive and significant for all firms, which implies a positive and significant interaction between the firm and the market index.

Unlike UK results, Ljung-Box test statistics provide evidence that serial correlation is generally present in the standardised residuals ($u_t / h_t^{1/2}$) of the market equation, suggesting the insufficiency of bivariate GARCH(1,1) models. Besides, thirteen firms have at least one more significant Ljung-Box statistics on other residuals. Consequently, following Giannopoulos (1995), further diagnostic tests are employed to investigate the higher ARCH process on the products of standardised residuals of the firm and the market. The higher ARCH process is found in only one case (Florida Gaming) on the cross-products of standardised residuals. As a result, GARCH(1,1) models are generally acceptable to capture the extra ARCH effects in the conditional variance process and valid in estimating US daily time-varying betas.

7.2.1.2 Basic Statistics of the Time-Varying Beta

Table 7.2 presents basic statistical characteristics of the twenty US time-varying daily beta series estimated by the standard bivariate GARCH model. All conditional betas have a positive mean value, which is statically significant at 1% level. The time-varying beta of Microsoft has the highest mean (1.12273). The conditional beta of Florida Gaming has the lowest mean value (0.49007) and the highest variance (0.193671). The conditional beta of General Electric is less volatile than all the others with the lowest variance (0.014668). The mean of conditional beta shows that five firms (Bank of America, General Electric, Honeywell International, Microsoft, Wells Fargo & Company) are aggressive shares. The remaining fifteen firms are defensive shares, whose beta means are less than unity.

All the twenty beta series are found to be asymmetrically distributed with significant skewness. Among them, two (Delta Air Lines and Utah Medical Products) are positively skewed; and the others are all negatively skewed. With exception of California Water Service whose excess kurtosis is insignificant, all beta series exhibit positive and significant excess kurtosis, indicating fatter tails than normal distribution for most beta series. Furthermore, all twenty daily conditional betas are rejected as normal distribution, as the Jarque-Bera statistics are significant at 1% level in all cases.

7.2.1.3 Unit Root Tests of the Time-Varying Beta

Once again, Dickey-Fuller (DF) and augmented Dickey-Fuller (ADF) tests with 3, 6 and 9 lagged differences are employed to examine the presence of unit root in the time-varying beta series constructed by bivariate GARCH models. Table 7.3 and 7.4 report the results from DF and the ADF tests for the null of two and one unit root respectively. In both Table 7.3 and Table 7.4, all test statistics are significant at 1% level, indicating the absence of either two unit roots or one unit root in the time-varying betas. Consequently, the US daily time-varying beta series estimated by the standard bivariate GARCH(1,1) model are stationary in levels.

7.2.2 Bivariate BEKK GARCH Model

7.2.2.1 Estimation Results

As discussed in section 6.2.2.1, the formulation of BEKK is an improvement on the basic GARCH model as it ensures the positivity of conditional variance matrix and significantly reduces the number of parameters to be estimated (Bollerslev *et al.*, 1994). Empirical evidence from UK stock market confirms its improvement in terms of forecasting performance. Once again, the BEKK model applied is restricted to the diagonal specification to eliminate misleading or biased parameters.

Estimation results of BEKK models reported in Table 7.5 embodies less information than those of bivariate GARCH models because the intuitional effect of parameters in a standard bivariate GARCH model is lost in BEEK. However, once again Table 7.5 contains three categories of estimation results: coefficient estimates, log-likelihood function value and the twelfth order Ljung-Box statistics. All results are estimated by means of BFGS algorithm.

For the BEKK model, the squared parameters (a_{11}^2 and a_{22}^2) are the measures of the ARCH effects. Thus, estimated parameters a_{11} and a_{22} cannot reflect the effect on the intertemporal dynamics of variances by themselves. However, the size and statistical significance of a_{11} and a_{22} can still provide some hints on volatility clustering. For all firms, coefficients a_{11} and a_{22} are significant at least 10% level. Except a_{22} of Bell Industries, all estimated coefficient a_{11} and a_{22} are positive. In addition, all squared coefficients (a_{11}^2 and a_{22}^2) are less than unity in size. Therefore, the size and significance of ARCH terms suggests volatility clustering in all return series, although the statistical significance of the real ARCH parameters is indefinite.

In the same way, the squared parameters (b_{11}^2 and b_{22}^2) are the actual GARCH terms of the BEKK model. Both b_{11} and b_{22} are in all positive and significant at 1% level. Thus, GARCH effect is arguably significant in all cases but the statistical significance is indefinite. Additionally, GARCH coefficients (b_{11}^2 and b_{22}^2) are fairly close to unity; and the sums of the ARCH and GARCH terms ($a_{11}^2 + b_{11}^2, a_{22}^2 + b_{22}^2$) are fairly close to unity except Florida Gaming. Hence, there is generally a considerable level of volatility persistence. Similar to bivariate GARCH results, Florida Gaming has the sum of ARCH and GARCH terms ($a_{11}^2 + b_{11}^2$) greater than unity, which indicates the BEKK is not robust and estimation may appear explosive[33]. Moreover, the products of coefficients (a_{11}*a_{22}, b_{11}*b_{22}) act as the covariance parameters of the BEKK model. Consequently, all firms have a positive interaction with the market index with exception of Bell Industries whose a_{22} is negative.

Once again, A battery of diagnostic tests based on the Ljung-Box statistics is employed to verify specification adequacy the bivariate BEKK GARCH(1,1) model. Similar to the results of the standard GARCH model, the twelfth order of Ljung-Box test statistics are significant on the standardised residuals ($u_t / h_t^{1/2}$) of all the market equations. In addition, serial correlation is detected in more other fitted residuals compared to the bivariate GARCH results (see Alaska Air Group for example), which suggests that the BEKK model is descriptively inferior to the standard GARCH model. Moreover, the likely incompetence of BEKK is also indicated by further tests on the cross-product of the standardised residuals, as the joint tests find serial correlation in six cases (California Water Service, Delta Air Lines, Textron, Walt Disney, Wendy's International, Florida Gaming). However, BEKK is descriptively sufficient to estimate

[33] Estimation results will be used for the forecasting accuracy comparison purpose.

the time-varying beta in the remaining fourteen cases. As a result, the estimation results of BEKK are arguably acceptable, as the majority outcomes imply that there is no need to encompass a higher ARCH process.

7.2.2.2 Basic Statistics of the Time-Varying Beta

Table 7.6 presents some basic statistics of the twenty time-varying betas series estimated by BEKK models. Generally, conditional betas estimated by the standard GARCH and BEKK GARCH have similar first moment characteristics. According to the mean, all the conditional betas have an expected value significantly larger than zero at 1% level. Same as bivariate GARCH results, Microsoft has the highest mean (1.13541); while Florida Gaming has the lowest mean (0.54251). In particular, the same group of five firms (Bank of America, General Electric, Honeywell International, Microsoft, Wells Fargo & Company) are classified as aggressive shares with beta values greater than unity. Moreover, the highest and lowest variance is found in the same company as the results of standard GARCH. Florida Gaming has the most unstable time-varying betas (0.489816) and General Electric has the most stable time-varying betas (0.008408).

Statistics of second moments show some differences between beta series constructed by bivariate GARCH and BEKK. Statistics of skewness is insignificant in three cases (American Electric Power, New York Times, Florida Gaming). For the rest, the time-varying beta series are significantly skewed, either positively or negatively. Thus, most conditional beta series are asymmetrically distributed. For all time-varying betas, excess kurtosis is significant at least 5% level. Among them, the betas of California Water Service and Wendy's International exhibit peaked distribution with negative and significant excess kurtosis; while the remaining conditional betas show flat distribution with positive and significant excess kurtosis. The Jarque-Bera statistics are significant for all time-varying betas. As a result, all the conditional beta series estimated by BEKK are rejected for the null of normal distribution.

7.2.2.3 Unit Root Tests of the Time-Varying Beta

Once again, classical DF and ADF tests are conducted to examine the presence of unit root in the estimated beta series. The nulls of two unit roots and one unit root are tested in order, and their test statistics are reported in Table 7.7 and 7.8 respectively. In both Table 7.7 and Table 7.8, all test statistics are significant at 1% level, which implies that all time-varying beta series are free of two or one unit root. As a result, DF and ADF tests draw to the same conclusion for conditional betas estimated by bivariate GARCH and BEKK that all time-varying betas are stationary in levels.

7.2.3 Bivariate GJR GARCH(1,1) Model

7.2.3.1 Estimation Results

Empirical evidence from the previous chapter shows that GARCH-GJR models are prominent with UK data. Empirical results of US daily data can be crucial to conclude the performance of GJR, as most evidence of leverage effect is provided by studies on the US market (see French *et al.*, 1987; Schwert, 1989 for example).

Once again, the GARCH-GJR model uses BHHH algorithm as the optimisation method to estimate the time-varying beta series; and the estimation results are reported in Table 7.9, which contains three categories of information: estimated parameters, log-likelihood function values and Ljung-Box statistics. Aside from the nine basic parameters of the standard bivariate GARCH, GJR has two additional parameters (r_1 and r_3) incorporated to capture the leverage effect in the firm and the market respectively[34]. Therefore, the sign and significance of r_1 and r_3 are of particular interest to interpret GJR results, as the ARCH term in the GJR model switches between a_1+r_1 and a_1 for the firm, and between a_3+r_3 and a_3 for the market, depending on whether the lagged error term is positive or negative.

In Table 7.9, estimated coefficient r_1 is significant in eleven cases, including eight positive and three negative r_1. Hence, evidence of leverage effects is found in the eight firms (see American Electric Power and Bank of America for example). On the contrary, three firms (California Water Service, Delta Air Lines, Utah Medical Products) provide opposite indication that 'good news' has a greater impact on volatility than 'bad news'. In the rest cases, insignificant r_1 are also mixed of positive

[34] Likelihood ratio tests indicate that GARCH-GJR is superior to bivariate GARCH in estimating time-varying betas, with significant higher log-likelihood function values. Results are not presented to save space.

and negative coefficient. Nevertheless, the absolute value of negative r_1 is all less than the corresponding ARCH coefficient a_1. In other words, the sum of the two parameters (a_1+r_1) is guaranteed to be positive; and thus the estimation results conform to the non-negative constrain.

Coefficient r_3 is found to be significant in ten cases, including eight positive and two negative r_3. .Nearly half of the results provide evidence of leverage effects in the market index. Similarly, the negative coefficient r_3 is always less than corresponding ARCH parameter a_3 in terms of the absolute value, which ensures that the sum of the two parameters (a_3+r_3) is positive.

In Table 7.9, the ARCH coefficients a_1 and a_3 are positive and significant at 1% level for all firms, which is similar to bivariate GARCH results. Additionally all the sums (a_1+r_1 and a_3+r_3) are positive. Therefore the size and significance of the ARCH coefficients imply volatility clustering in all returns series. Similarly, all the GARCH coefficients (b_1 and b_3) are positive and significant at the 1% level, indicating evident GARCH effects. For most firms, the sums of the ARCH and GARCH coefficients ($a_1+r_1+b_1$ or a_1+b_1, $a_3+r_3+b_3$ or a_3 $+b_3$) are all close to unity, suggesting a considerable degree of volatility persistence. Similar to bivariate GARCH, Florida Gaming, Bell industries still exhibit unusual spurious estimates with extremely high value sum $a_1+r_1+b_1$ larger than unity. In terms of covariance parameters (a_2 and b_2), all firms are found to have a positive and significant interaction with the market index, with positive and significant a_2 and b_2. In summary, the estimation results of the standard parameters of GJR models are generally similar to those of bivariate GARCH models.

In general, results of the Ljung-Box statistics reported in Table 7.9 are similar to those of bivariate GARCH models. Once again, serial correlation is generally detected in the standardised residuals of the market equations; as the twelfth order of Ljung-Box statistics are significant in all cases. However, when further cross-product tests are applied to assess the general specification sufficiency of the GJR model, serial correlation is found only in one case (Florida Gaming). Therefore, the joint diagnostic tests generally suggest the lack of serial correlation. In this case, the bivariate GJR GARCH(1,1) model is sufficient to estimate time-varying betas.

7.2.3.2 Basic Statistics of the Time-Varying Beta

Table 7.10 presents basic statistics of the US daily time-varying beta estimated by GARCH-GJR models. Descriptive information on mean and variance is similar to those reported in Table 7.2. Ranging from 0.49191 to 1.12059, the mean values of time-varying betas are all positive and significant. Same as the results of bivariate GARCH, Florida Gaming has the lowest and most volatile time-varying beta series; and General Electric has the most stable conditional beta. Additionally, the conditional betas of the same five firms (Bank of America, General Electric, Honeywell International, Microsoft, Wells Fargo & Company) are found to have a mean greater than unity in size. However, the beta of General Electric has the largest mean value suggesting some minor difference from bivariate GARCH results.

Statistics of skewness is significant at 1% level for all firms. Therefore, all estimated betas are rejected as symmetrical distribution. Among them, only two conditional betas (Delta Air Lines, Utah Medical Products) are positively skewed, which is the same as indicated by bivariate GARCH results. Statistics of excess kurtosis are significant at 1% level for all beta series. In particular, expect California Water Service, other excess kurtosis of conditional betas are positive, implying fatter tails than normal distribution in most cases. Thus, no time-varying beta series can be accepted as normal distribution. The significant Jarque-Bera statistics at the last column confirm the nonnormality of beta series.

7.2.3.3 Unit Root Tests of the Time-Varying Beta

Table 7.11 and 7.12 respectively report the test statistics of DF and ADF tests for the null of two and one unit root in the beta series. In both Table 7.11 and 7.12, test statistics are all significant at 1% level. Therefore, there is no unit root found in time-varying beta series. In other words, all time-varying betas estimated by the bivariate GARCH-GJR model are stationary in levels.

7.2.4 Bivariate GARCH-X Model

7.2.4.1 Estimation Results

Among US data, Engle and Granger (1987) tests find the ten firms having cointegrated relationship with the market index, including Alaska Air Group, Boeing, California Water Service, General Electric, Honeywell International, MGP Ingredients, Textron, Utah Medical Products, Walt Disney and Florida Gaming. As a result, the GARCH-X model is applicable to a larger sample of US data than UK data.

Once again, BHHH algorithm is used as the optimisation method to estimate the time-varying beta series by GARCH-X models. Table 7.13 present the estimation results in details, which contain the estimated parameters, the log-likelihood function value and the Ljung-Box statistics. Apart from the nine basic parameters, the GARCH-X model has three extra parameters of the error correction term (d_1, d_2 and d_3), which measure the impact of the short-run deviations between the share price and the market index on the conditional variance and covariance. Thus, the size and significance of the error correction terms count for much when interpreting the estimated parameters[35].

In Table 7.13, coefficient d_1 is significant in seven cases (Alaska Air Group, California Water Service, General Electric, Honeywell International, MGP Ingredients, Textron, Florida Gaming), implying that short term deviations generally have a significant effect on the conditional variance of firm returns. Such effects can be positive or negative, as there are three positive and four negative significant coefficient d_1. Similarly, evidence is found for the impact of the short-run deviations on the conditional variance of market returns, as coefficient d_3 is significant in eight cases (Alaska Air Group, Boeing, California Water Service, General Electric, Honeywell International, Textron, Walt Disney, Florida Gaming). Except General Electric, the significant d_3 is negative, showing a negative impact of the short-run deviations on the conditional variance of market in most cases. Additionally, estimated coefficient d_2 is significant in five cases, including one positive (General Electric) and four negative coefficients (Boeing, California Water Service, Honeywell

[35] Likelihood ratio tests indicate that GARCH-X is superior to bivariate GARCH in estimating time-varying betas, with significant higher log-likelihood function values. Results are not presented to save space.

International, Textron). Thus, short term deviations also have explanatory power on the conditional covariance.

According to Table 7.13, estimation results of the nine standard parameters are quite standard. First, both ARCH coefficients (a_{11} and a_{33}) are positive and significant at 1% level for all firms. In addition, they are less than unity in size, implying volatility clustering. Second, GARCH coefficients (b_{11} and b_{33}) are positive and statistically significant in all cases, exhibiting strong evidence of GARCH effects. Third, all covariance coefficients (a_{22} and b_{22}) are positive and significant at 1% level, showing that all these ten firms have a positive and significant interaction with the market.

Once again, specification adequacy of the GARCH-X model is verified through a battery of residual-based tests. According to the Ljung-Box test statistics reported in Table 7.13, results are similar to the standard bivariate GARCH models. The Ljung-Box statistics are significant on the standardised residuals of the market equation, apart from which there are significant Ljung-Box statistics on other fitted residuals. However, when further cross-product tests are employed to examine the serial correlation in the product of the standardised residuals of the firm and the market, only one firm (Florid Gaming) needs a higher order of ARCH process. As a result, the diagnostic test results generally imply the descriptive validity of bivariate GARCH-X(1,1) models.

7.2.4.2 Basic Statistics of the Time-Varying Beta

Table 7.14 reports some descriptive statistics of the ten time-varying betas estimated by bivariate GARCH-X models. In general, statistical characteristics of the first moment reflect that conditional betas estimated by GARCH-X are similar to those generated by GARCH-GJR model. The mean values are all positive and significant at 1% level. Among them, beta series of Florida Gaming has the lowest mean (0.51605) and highest variance (0.177556); while General Electric has the highest mean (1.08124) and the lowest variance (0.014365). In addition statistics of the second moments exhibit similarity to bivariate GARCH results. All conditional beta series are rejected as symmetries with significant skewness statistics. Except California Water Service, all time-varying betas exhibit positive and significant excess kurtosis, indicating fatness in most cases. Furthermore, all conditional betas are rejected for the null of normal distribution, with Jarque-Bera statistics significant at least 10% level.

7.2.4.3 Unit Root Tests of the Time-Varying Beta

Once again, DF and ADF tests are performed to detect the presence of unit roots in the time-varying beta series; and the test statistics for the null of two and one unit root are reported in Table 7.15 and 7.16 respectively. In both tables, test statistics are significant at 1% level. Thus, all conditional beta series are free of the unit root. As a result, the time-varying beta series estimated by the GARCH-X model are stationary in levels.

7.2.5 Kalman Filter Approach

Empirical evidence from UK data supports the outstanding performance of Kalman filter method in forecasting time-varying betas in terms of the CAPM-derived returns. On the other hand, empirical evidence shows that Kalman filter is not as capable as GARCH models to capture time variations of the conditional beta.

To apply Kalman filter to US daily data, preliminary considerations on the potential transition equation are also helpful for more efficient forecasting comparison. AIC and BIC derived from the estimation results of Kalman filter method based on four different state equations are reported in Table 7.17 and 7.18. Generally, all four dynamic processes exhibit evidence of misspecification. Although Kalman filter based on random walk has no difficulty to converge with UK data, it fails to converge in two cases (American Electric Power, Bell Industries) when applied to US data. Moreover the convergence difficulty is more serious for other forms of dynamic equation compared to UK results. For instance, state space model based on AR(1) has only three convergences in twenty cases, although it has the lowest AIC and BIC. The number of convergence failure based on random coefficient and random walk with drift has been doubled relative to UK results. In particular, no transition equation achieves convergence for Bell Industries. As a result, random walk and random coefficient have the same and highest convergence rate, both encountering convergence difficulty in the same companies. Also they have the comparable level of

AIC and BIC. Therefore, random walk is considered as the appropriate state equation to keep in line with UK data.

7.2.5.1 Basic Statistics of the Time-Varying Beta

Table 7.19 reports basic statistics of the eighteen time-varying betas estimated by the Kalman Filter approach. Generally, statistics of mean and variance reveal similar information to bivariate GARCH. Both Kalman filer and bivariate GARCH find the maximum and minimum values of mean and variance in the same firms. Ranging from 0.40695 (Florida Gaming) to 1.15981(Microsoft), the mean is all significant at 1% level. Once again, General Electric and Florida Gaming has the lowest and highest variance respectively. Besides all mean and variance values in Table 7.19 are fairly close to those in Table 7.2. Most time-varying betas are asymmetrically distributed with significant skewness, with exception of Microsoft and Wells Fargo & Company. According to statistics of excess kurtosis, three beta series (MGP Ingredients, Wells Fargo & Company, Wendy's International) exhibit peakedness with negative and significant excess kurtosis. The remaining conditional betas exhibit fatter tails with positive and significant excess kurtosis. Consequently, no time-varying beta can be accepted as normal distribution, which is confirmed by the significant Jarque-Bera statistics in all cases.

7.2.5.2 Unit Root Tests of the Time-Varying Beta

Table 7.20 and 7.21 report the results from DF and ADF tests for two and one unit root in the beta series estimated by Kalman filter models. In Table 7.20, the null of unit root is rejected for the first difference of all conditional betas, as all test statistics are significant at 1% level. Test statistics for one unit root in Table 7.21 provide similar evidence on the stationarity of conditional betas, as all DF and ADF test statistics are significant at least 10% level. Thus, the null of unit root is rejected for conditional betas and their first difference. As a result, all time-varying betas are stationary in level.

7.3 Comparison Analysis of Beta Estimates

As done earlier, the performance of different models in modelling time-varying betas can be revealed by comparing conditional betas in terms of both mean values and visual graphs over the whole sample (1989 to 2003). Table 7.22 presents the mean value of the twenty time-varying betas produced by each method. In addition, point estimates of beta calculated by means of the market model are reported in the last column as a magnitude reference to the time-varying betas.

In general, the conditional betas estimated by different GARCH type models (bivariate GARCH, BEKK, GJR and GARCH-X) have similar mean values, with few exceptions such as BEKK of Delta Air Lines. In addition, the Kalman filter approach also produces time-varying betas with similar mean value to GARCH models. However, the similarity is not as significant as among GARCH models (see MGP Ingredients and Florida Gaming for example). Furthermore, the mean values of time-varying betas are fairly close to the point estimate of unconditional beta in most cases, which indicates that both GARCH models and the Kalman filter method are able to measure systematic risk precisely in addition to capturing the time variation of systematic risk.

However, as found in UK daily results, there can be considerable differences among time-varying beta estimates from the perspective the whole sample, even although their mean values are reasonably similar. Once again, following Faff *et al.* (2000), graphical investigation on the time series characteristics of the conditional betas is conducted to achieve further insight on the differences and similarities among estimated outcomes of different modelling techniques.

In Figure 7.1 and 7.2, graphs of time-varying beta series estimated by different techniques for two firms (Alaska Air Group, Boeing) are displayed in the same scale. In both figures, beta series constructed by GARCH class models exhibit similar patterns, suggesting that different GARCH models describe the dynamic process of conditional second movements in a similar way. In particular, bivariate GARCH and its nested extensions (GJR and GARCH-X) show considerable similarity, while the time-varying beta estimated by BEKK GARCH tends to be spiky with extreme values in Figure 7.1. Similar to UK results, graphs of Kalman filter betas are generally smoother than their counterparties, implying that the Kalman filter approach is less sensitive to time variation (see Figure 7.1). Once again, graphs of conditional betas estimated by GARCH appear to revert to the Kalman filter beta over the estimation period, which confirms the noise filtering feature of Kalman filter algorithm.

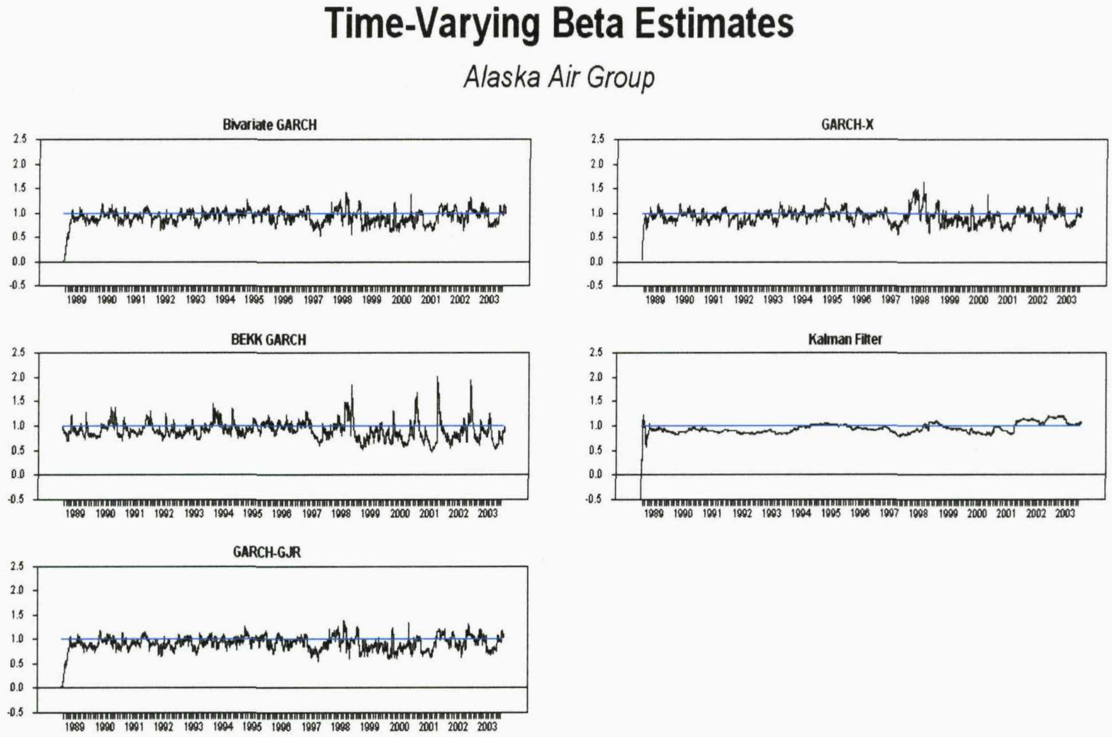

Figure 7.1: Time-Varying Beta Estimates (Alaska Air Group)

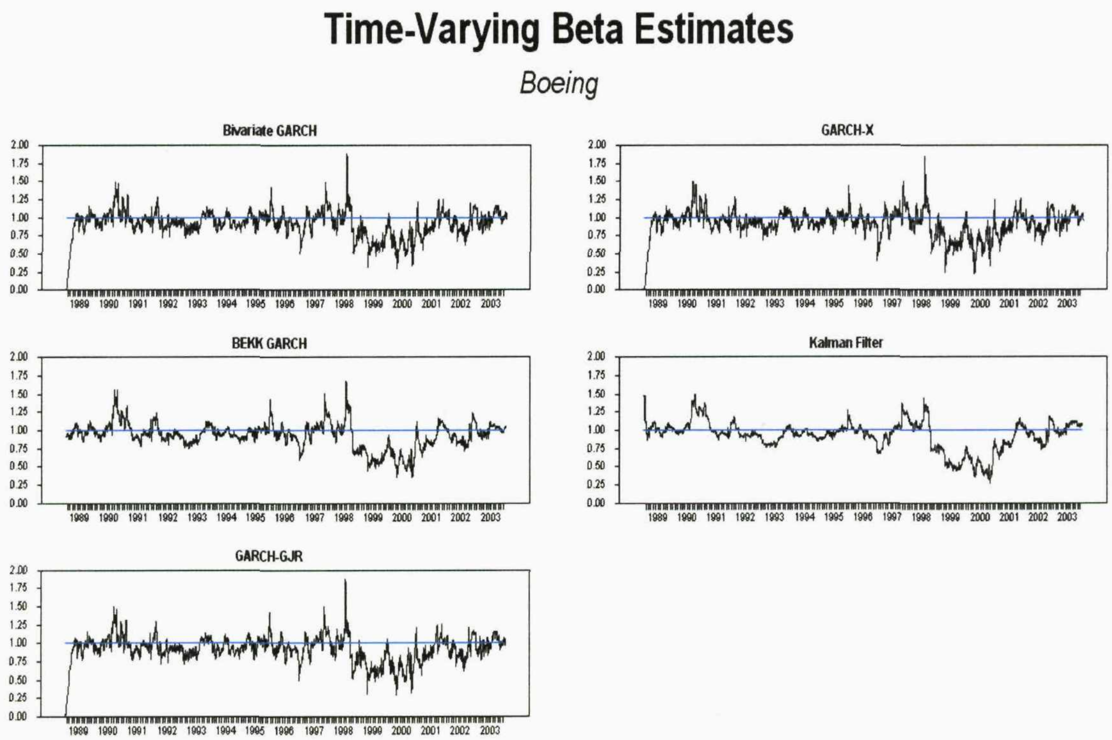

Figure 7.2: Time-Varying Beta Estimates (Boeing)

To summarise, the comparison analysis based on the mean values and graphs of conditional betas is informative on the performance of alternative models in modelling the time-varying beta. The similarity of mean values provide evidence that each method is capable in parameterisations of systematic risk, as their mean values are generally close to the point estimate of beta derived from the market model. According to graphs of time-varying betas, considerable similarities are found among GARCH models, especially among the three nested GARCH models. Additionally, BEKK betas show spiky features with the extreme values. Moreover, Kalman Filter seems to be less successful in capturing time variation of conditional beta due to the unique two-stage algorithm, as the graphs are generally smoother than those of GARCH betas.

7.4 Forecasting Time-Varying Betas

The same forecast periods are used to predict US time-varying betas (2001, 2003, and 2002 to 2003). Also results of rolling and recursive windows forecasting are not presented due to serious convergence problem; and only the static forecasting is reported in this thesis.

7.4.1 Graphs of Beta Forecasts

As done earlier, observing graphs of the forecasted beta and the actual beta helps to evaluate the forecast performance of alternative model in an intuitive and straightforward way. Therefore, graphs of forecasted and actual US daily betas generated by the same model are displayed to illustrate forecast accuracy.

Figures 7.3, 7.4 and 7.5 respectively illustrate the time-varying beta forecasts and estimates for three firms in three forecast horizons (2001, 2003 and 2002 to 2003). In general, GARCH-type model produce accurate time-varying beta forecasts. Especially in 2003 and 2002 to 2003, lines of forecasted betas and actual betas lap over each other with few visible deviations. Out-of-sample forecasts in 2001 are less successful, with several divergences between the forecasted and estimated betas. Intuitively, BEKK models produce less accurate conditional beta forecasts than other GARCH models in 2001 according to Figure 7.3.

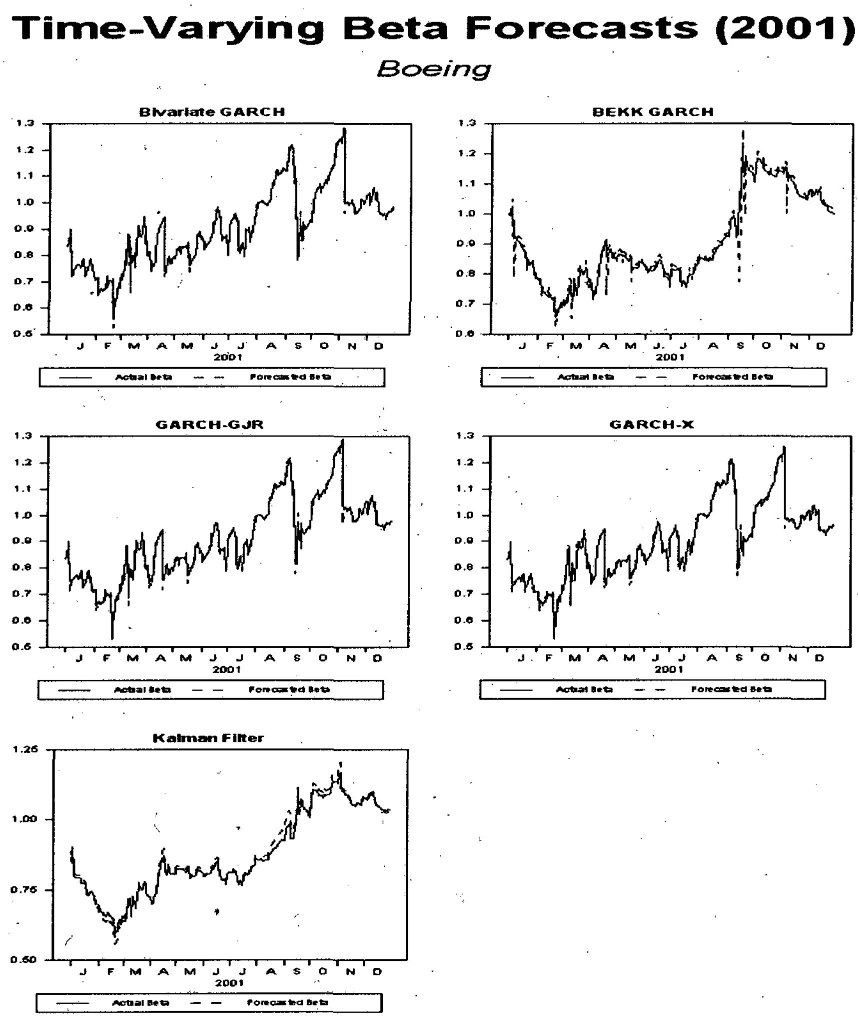

Figure 7.3: Time-Varying Beta Forecasts in 2001(Boeing)

Once again, the visual analysis suggests that the predictive ability of Kalman filter is inferior to GARCH class models with more visible deviations between beta forecasts and estimates. In all the three out-of-sample periods, Kalman filter produces relative poorer beta forecasts. In all figures, there are several significant divergences between conditional beta forecasts and estimates generated by the Kalman filter approach.

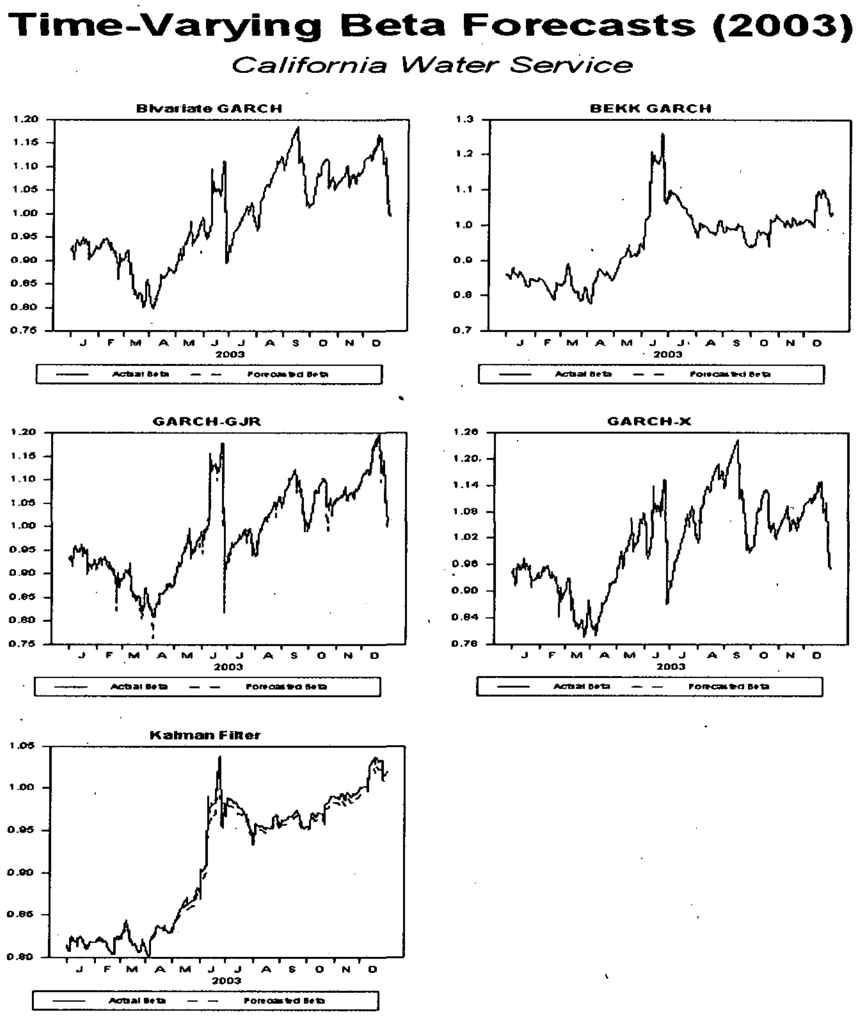

Figure 7.4: Time-Varying Beta Forecasts in 2003 (California Water Service)

Similar to UK results, the visual inspection on daily results is not informative enough to rank GARCH-type models, since they generally produce accurate and consistent conditional beta forecasts. However, there are some interesting findings from the graphical comparison. First, similar to UK results, the GARCH-type models exhibit dominance over the Kalman filter approach. Second, BEKK seems to be ineffective than other GARCH models in out-of-sample period 2001.

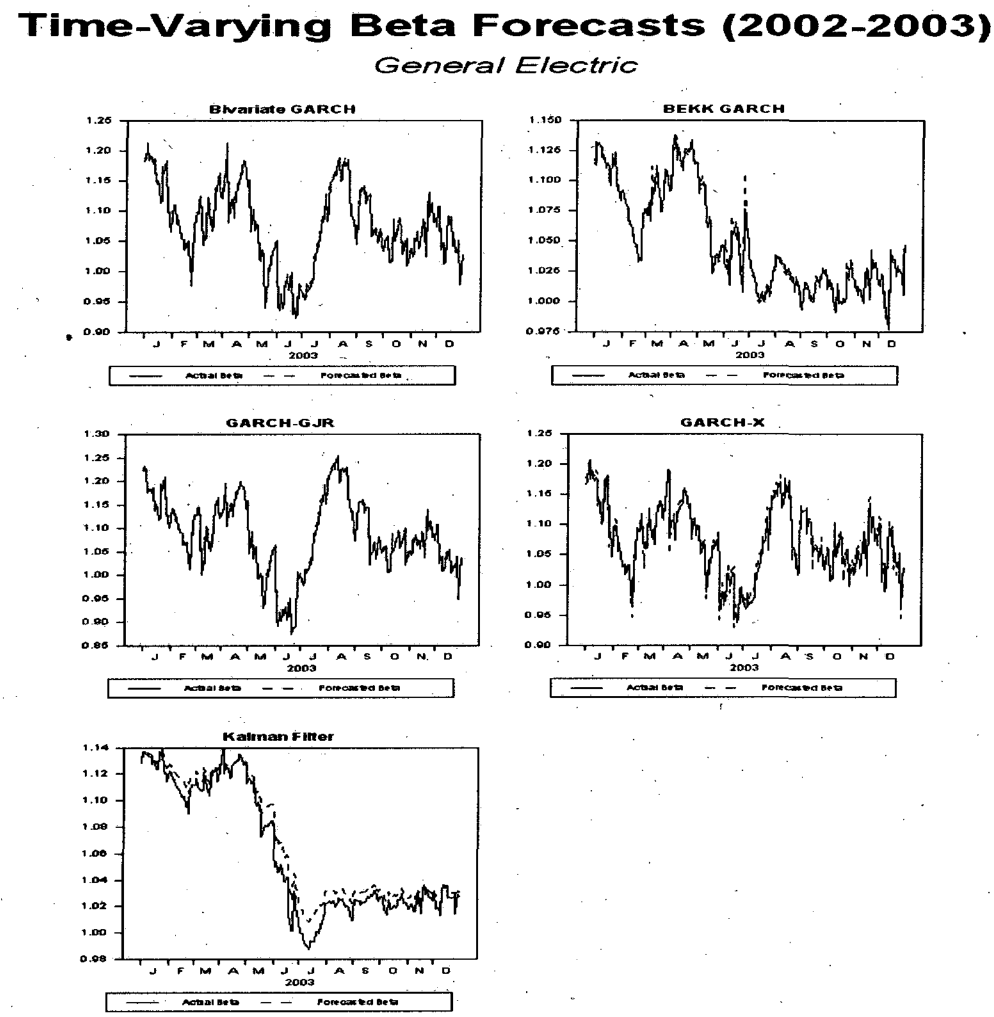

Figure 7.5: Time-Varying Beta Forecasts in 2002-2003 (General Electric)

7.4.2 Forecast Accuracy

7.4.2.1 Forecast Errors Based on Beta Forecasts

As done earlier, statistics of forecast errors are employed to evaluate the relative superiority of alternative models in forecasting time-varying betas, including mean absolute errors (MAE), mean square errors (MSE), mean absolute percentage errors (MAPE) and Theil U statistics. Results of forecast errors are reported in Table 7.23 to Table 7.34 for the three out-of-sample horizons.

(1) One year out-of-sample period 2001

Tables 7.23, 7.24, 7.25 and 7.26 respectively present MAE, MSE, MAPE and Theil U of the time-varying beta forecasted by alternative forecasting models in out-of-sample period 2001. In terms of MAE reported in Table 7.23, bivariate GARCH outperforms other forecasting models with seven smallest MAEs. BEKK, GARCH-X and Kalman filter show comparable forecast accuracy, each having the smallest MAE in four cases. GARCH-GJR model dominates other methods in only one case.

Using a quadratic loss function, dominance of standard bivariate GARCH is confirmed by MSE with six smallest MSEs. The Kalman filter approach also finds favour with MSE, as it outperform others in six cases. The superiority of GARCH-X remains in four conditional beta forecasts, followed by BEKK with three smallest MSEs. Same as MAE, GJR has the smallest MAE in only one case.

Compared to MAE, MAPE has the advantage of being dimensionless measure errors by taking the percentage form. According to MAPE reported in Table 7.25, the bivariate GARCH model is still the best forecasting model with seven lowest values of MAPE, followed by Kalman filter with five smallest MAPEs. GARCH-X seems to outperform BEKK and GJR in terms of MAPE, as GARCH-X has four smallest MAPEs. Both BEKK and GJR have two smallest MAPEs.

Like MAPE, Theil U statistics are dimensionless and without scaling problem. In Table 7.26, Theil U statistics suggest a similar ranking to MAPE. Bivariate GARCH and Kalman filter are superior to other models, with seven and five smallest statistics respectively. GARCH-X dominates the competition in four cases. BEKK is favoured by Theil U statistics relative to GJR; as BEKK has the smallest forecast errors in three cases, while GJR has only one lowest forecast error.

In terms of different forecast errors in out-of-sample period 2001, the standard bivariate GARCH is the most accurate forecasting model with the smallest forecast error in more cases than other models. Ranked as the second most successful model, Kalman filter also produce accuracy and consistent forecasts in 2001. GARCH-X is found to be competent, it has four most accuracy forecasts out of the ten applicable cases, no matter which statistics of forecast error is used. BEKK shows slightly inferior to GARCH-X; while GJR is the worst forecasting model in 2001.

(2) One year out-of-sample period 2003

Once again, out-of-sample forecasts in 2003 are valuable in providing supplementary information on forecast accuracy of alternative methods. Tables 7.27, 7.28, 7.29 and 7.30 report MAE, MSE, MAPE and Theil U of time-varying beta forecasts in 2003.

According to MAE in Table 7.27, BEKK is superior to other models with six smallest MAEs. Bivariate GARCH and Kalman filter both have five lowest forecast errors, showing comparable forecast ability. GARCH-GJR and GARCH-X are inferior to others forecasting models, having three and one leading positions respectively. According to MSE in Table 7.28, BEKK GARCH still outperforms its competitors with seven lowest values of MSE, followed by Kalman filter with six smallest MSEs. Bivariate GARCH also has a reasonable performance with the lowest MSE in five cases. GJR is favoured by MSE in two forecasts. There is no evidence that GARCH-X outperforms other forecasting models.

In terms of MAPE reported in Table 7.29, Kalman filter outperform GARCH type models in forecasting the time-varying beta, with six smallest percentage errors. Both bivariate GARCH and BEKK outperform other models in five cases. GJR dominates in three forecasts in terms of MAPE. GARCH-X has the lowest forecast error in one case. Theil U statistics presented in Table 7.30 suggest similar ranking, except that bivariate GARCH, BEKK and Kalman filter share the leading position. All the three models are dominant in six cases. GJR outperforms its competitors in two cases, while GARCH-X has no smallest Theil U statistics.

To sum up various forecast errors, the BEKK GARCH model is the most successful forecasting model in out-of-sample period 2003. Three out of the four forecast error measures indicate the superiority of BEKK models. Kalman filter is ranked as the second competent approach with consistently accurate forecasts. The performance of bivariate GARCH is slightly inferior to Kalman filter, but its out-of-sample forecasts are satisfactory. GJR and GARCH-X are the last two models in the ranking.

(3) Two-year out-of-sample period 2002 to 2003

The two-year out-of-sample forecasts (2002 to 2003) help to assess the performance of alternative models in a relative longer forecast horizon. Table 7.31, 7.32, 7.33 and

7.34 respectively report MAE, MSE, MAPE and Theil U statistics of beta forecasts in 2002 to 2003. According to MAE statistics in Table 7.31, bivariate GARCH is the preeminent forecasting model with ten smallest MAEs. BEKK models are still outstanding in the longer forecast horizon, with the smallest MAE in eight cases. Kalman filter outperforms others in two cases. Both GJR and GARCH-X have no smallest MAE. MSE reported in Table 7.32 confirms the remarkable dominance of bivariate GARCH with the lowest forecast error in twelve cases. BEKK and Kalman filter are also acceptable forecasting models with the smallest forecast errors in six and two cases. Once again, GJR and GARCH-X is outperformed by other models, without any lowest MSE.

In terms of percentage error MAPE, bivariate GARCH is still the best model, as it has the smallest MAPE in ten forecasts. In seven cases, BEKK shows evidence of dominance with the smallest MAPE. In the rest three forecasts, Kalman filter outperforms other models. No evidence is found to support GJR and GARCH-X. Theil U statistics reported in Table 7.34 provide the same ranking on relative superiority as MAPE. Bivariate GARCH is the superior forecasting model in twelve cases, followed by BEKK with six lowest values of statistics. In two forecasts, Kalman filter has the smallest Theil U statistics. Once again, both GJR and GARCH-X have no smallest error statistics.

In summary, two-year out-of-sample forecasts suggest that bivariate GARCH is the most accurate models, favoured by all error statistics. BEKK still has an outstanding performance in the longer forecast period, ranked as the second best model by all forecast errors. Kalman filter also outperforms other models in some instances. However, without any smallest forecast errors, both GARCH-GJR and GARCH-X models are found to be less effective in forecasting US daily betas, which is contrasting to their prominent performance with UK data.

Evidence from the three out-of-sample periods suggests similar conclusion to UK data. In general, the alternative superiority of alternative forecasting models varies in different samples. In addition, Bivariate GARCH, BEKK and Kalman filter seem to outperform GARCH-GJR and GARCH-X. Bivariate GARCH is the best forecasting model in 2001 and 2002 to 2003; while BEKK is found to be dominant in 2003. With consistently accurate forecasts, Kalman filter is the second best model in all forecast samples. Evidence of relative poor performance is found for the sophisticated GARCH GJR and GARCH-X extensions in different samples, except GARCH-X in 2001. Empirical evidence from both UK and US stock markets implies that additional parameters in GARCH models somewhat deteriorate the forecasting accuracy in terms of beta forecasts. However, such deterioration can be due to the issue of missing benchmark in the comparison analysis.

7.4.2.2 Forecast Errors Based on Return Forecasts

Once again, the comparison study is extended by analysing the forecast accuracy of returns instead of conditional betas. Thus, the evaluation of alternative forecasting models can be achieved by assessing the accuracy of return forecasts.

Figure 7.6, 7.7 and 7.8 each shows the return forecasted by the different methods and the actual return for three firms (Boeing, California Water Service, General Electric) in 2001, 2003 and 2002 to 2003. All estimates seem to move together with the actual return, but because of the high frequency of the data it is difficult to say which method shows the closest correlation.

As a result, visual comparison based on return forecasts are not as intuitive and informative as beta forecasts; as there is no perfect or return forecast wrong forecasts by particular forecasting approach. In this case, relative superiority of alternative models can hardly be obtained from graphic analysis. Therefore, a variety of forecast error statistics are employed to evaluate the forecast accuracy of the forecasting models, including mean absolute error (MAE), mean square error (MSE) and mean error (ME).

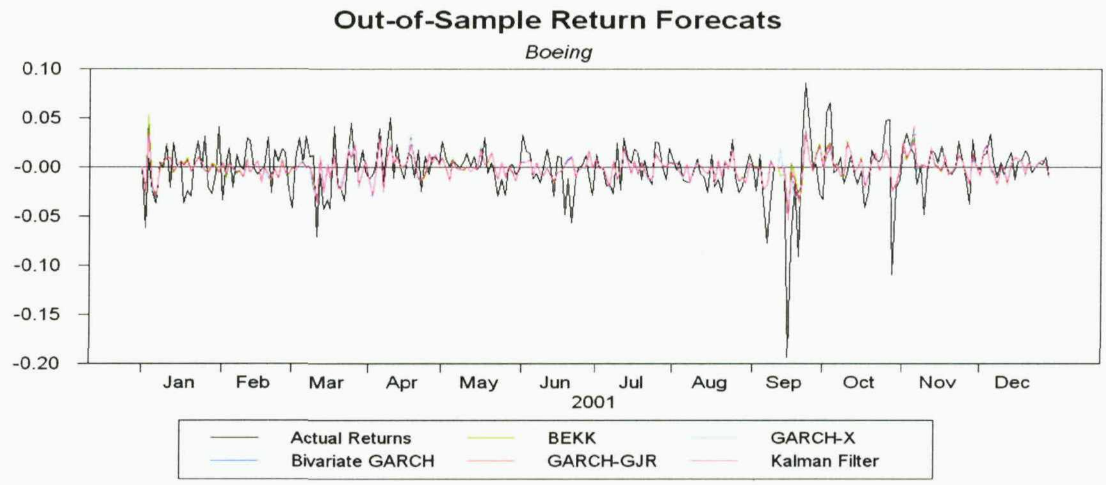

Figure 7.6: Time-Varying Return Forecasts in 2001 (Boeing)

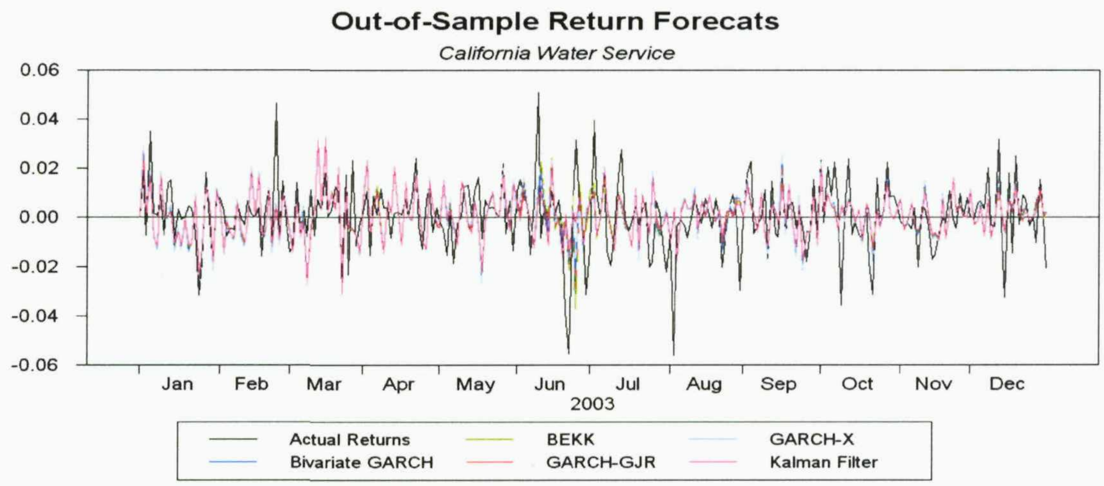

Figure 7.7: Time-Varying Return Forecasts in 2003 (California Water Service)

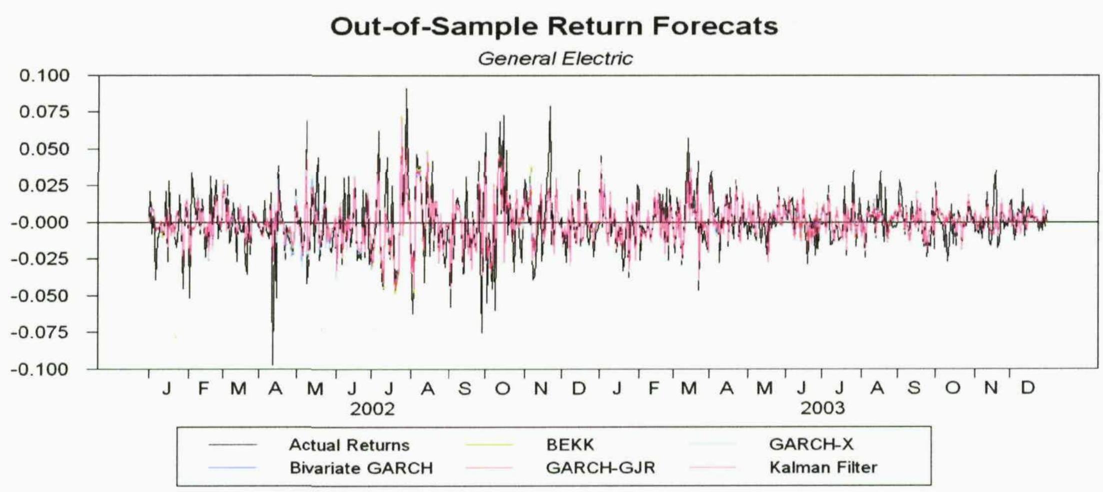

Figure 7.8: Time-Varying Return Forecasts in 2002-2003 (General Electric)

(1) One year out-of-sample period 2001

Table 7.35, 7.36 and 7.37 report the error statistics (MAE, MSE and ME) of return forecasts in out-of-sample period 2001. According to MAE in Table 7.35, Kalman filter is the dominant forecasting model in terms of return forecasts, with fourteen lowest values of MAE. Such a performance is remarkable, since Kalman filter is applicable in only sixteen out-of-sample forecasts. The GJR model finds favour with return forecasts, as it has five smallest MAEs. BEKK outperform others in one case. Comparison among GARCH class models suggests that BEKK and GJR are relative competent GARCH models, each having nine smallest MAEs, followed by GARCH-X with two smallest MAEs. Interestingly, bivariate GARCH is an inaccurate forecasting model in terms of return forecasts; although it produces the most accurate time-varying beta forecasts. According to MSE reported in Table 7.36, Kalman filter approach is overwhelmingly superior to GARCH type models, with the lowest MSE in all sixteen cases. BEKK and GJR each has two smallest MSEs, where Kalman filter fails to converge to a unique solution. The superiority of BEKK and GJR to other GARCH models is apparent, when forecast errors are compared only among GARCH models. GJR and BEKK outperform others in eleven and eight cases. ME statistics reported in Table 7.37 suggest that no tendency is found for particular model to over or under predict the return values.

In summary, Kalman filter is the remarkably superior model in out-of-sample period 2001. In addition, its superiority to GARCH models in terms of return forecasts is as overwhelming as indicated by UK data. Among GARCH models, GJR and BEKK models show evidence of dominance over bivariate GARCH and GARCH-X. No evidence is found on the tendency of over or under prediction of return values.

(2) One year out-of-sample period 2003

Table 7.38, 7.39 and 7.40 present MAE, MSE and ME of return forecasts in out-of-sample period 2003. In Table 7.38, MAE of return forecasts indicates that Kalman filter is the most successful forecasting approach, with fourteen smallest MAEs in eighteen applicable cases. BEKK is the second competent model with three smallest MAEs. Bivariate GARCH and GJR outperform other models in two and one cases. Comparisons among GARCH models suggest that all GARCH models produce comparably accurate forecasts in 2003. However, GJR and BEKK are slightly superior to other GARCH models with smaller MAE. In Table 7.39, the superiority of Kalman filter is more significant in terms of MSE. It has seventeen smallest forecast errors. Among GARCH models, MSE supports that GJR and BEKK are somewhat better than the standard GARCH and GARCH-X based on return forecasts. In Table 7.40, positive and negative values of ME are mixed; implying models do not tend to over or under forecast returns.

To sum up MAE and MSE, Kalman filter is still the best forecasting model in out-of-sample period 2003. Although all GARCH class models produce comparable forecast errors, GJR and BEKK are slightly superior to other GARCH models.

(3) Two-year out-of-sample period 2002 to 2003

Table 7.41, 7.42 and 7.43 report measures of forecast error (MAE, MSE and ME) for the two-year out-of-sample return forecasts. In Table 7.41, MAE indicates that Kalman filter is still the superior forecasting model in the two-year horizon, with sixteen smallest error statistics in seventeen forecasts. Comparison among GARCH models shows that GJR and BEKK outperform other GARCH models. According to MSE in Table 7.42, Kalman filter approach is favoured by MSE in all seventeen forecasts. Among GARCH type models, GJR is still the best when the quadratic loss function is considered, followed by BEKK, bivariate GARCH and GARCH-X. Unlike ME in 2001 and 2003, there are more positive MEs than negative MEs in Table 7.43, implying all models do not tend to over forecast returns in 2002 to 2003. In addition, bivariate GARCH, BEKK, GARCH-X and Kalman filter method together tend to under forecast returns for the same return series.

In summary, out-of-sample forecasts in different samples have drawn to the same conclusion that Kalman filter overwhelming dominates GARCH type models in terms of return forecasts. In addition, GJR and BEKK GARCH are slightly superior to bivariate GARCH and GARCH-X. Such predominance is consistent over different samples. As a result, evidence from US data coincides with UK results, which again indicates different model structure of Kalman filter and GARCH leading to different return forecasting performance.

7.4.2.3 Modified Diebold-Mariano Tests

Once again, the modified Diebold-Marian test is applied to MSE and MAE of return forecasts. Each time, the equal accuracy test checks the alternative superiority between two forecasting models in three forecast samples. Hence, there are ten groups of test for five forecasting models. In each group, there are six modified Diebold-Mariano tests for both MSE and MAE in three forecast horizons. Modified Diebold-Mariano test generates two statistics, say S_1 and S_2, based on two hypotheses:

1. H_0^1: there is no statistical difference between two sets of forecast errors.

   H_1^1: the first set of forecasting errors is significantly smaller than the second.

2. H_0^2: there is no statistical difference between two sets of forecast errors.

   H_1^2: the second set of forecasting errors is significantly smaller than the first.

It is clear that the sum of the P values of two statistics is equal to unity. Given that statistical significance is defined at least 10% level, each statistics provides three possible answers to superiority between two forecasting models:

1. If S_1 is significant, then the former forecasting model is superior to the later one.
2. If S_2 is significant, then the later forecasting model is superior to the former one.
3. If neither S_1 nor S_2 is significant, then two models have equally accurate forecasts.

The percentage of firms giving different answers are presented in Table 7.44 to 7.53

(1) Kalman filter and bivariate GARCH

Table 7.44 reports the percentage of firms accepting the three hypothesis regarding Kalman filter and bivariate GARCH, based on the modified Diebold-Marian test. Clearly, Kalman filter approach is superior to bivariate GARCH model. Over different forecast sample, most firms accept that Kalman filter produce significantly smaller forecast errors than bivariate GARCH. In forecast sample 2002 to 2003, although over 80% of the firms support the dominance of Kalman filter, there is contrary evidence in terms of MAE in few cases.

(2) Kalman filter and BEKK GARCH

Table 7.45 provides evidence of dominance of Kalman filter over BEKK model. In all forecast samples, the test statistics support that Kalman filter outperforms BEKK with significantly smaller forecast error in some cases. In particular, more than half forecasts in 2003 and 2002 to 2003 accept the hypothesis that Kalman filter is better than BEKK. Except MAE in 2002 to 2003, no evidence is found that BEKK outperforms Kalman filter. In 2001, most firms indicate equal accuracy between Kalman filter and BEKK.

(3) Kalman filter and GARCH-GJR

Table 7.46 reports the results of modified Diebold-Mariano tests between Kalman filter and GARCH-GJR, which are similar to those in Table 7.44. There are more than half of the firms favouring Kalman filter relative to GJR regardless of forecast sample and error criterion. Additionally, evidence that GJR is better than Kalman filter is only found in MAE of 2002 to 2003.

(4) Kalman filter and GARCH-X

There are less out-of-sample forecasts available for modified Diebold-Mariano comparison tests between GARCH-X and Kalman filter. According to Table 7.47, the majority of the firms indicate that Kalman filter is superior to GARCH-X in all forecast samples. Similar, evidence of GARCH-X outperforming Kalman filter is found in MAE of 2002 to 2003.

(5) Bivariate GARCH and BEKK GARCH

Table 7.48 reports the percentage of firms accepting the three hypotheses on the relative superiority of bivariate GARCH and BEKK GARCH. In one-year forecast sample 2001 and 2003, at least 80% firms indicate that both models produce equally accurate forecasts. In 2002 to 2003, about half of the firms suggest equal accuracy. Among those with significant forecast errors, bivariate GARCH dominates BEKK with a higher percentage of dominance across three forecast samples in terms of both MSE and MAE.

(6) Bivariate GARCH and GARCH-GJR

Table 7.49 reports the results from modified Diebold-Mariano tests between the standard bivariate GARCH and the GJR specification. Although both models produce forecast errors without insignificant difference in over 60% cases, GJR shows evidence of dominance over standard GARCH. Modified Diebold-Mariano tests provide evidence that bivariate GARCH outperforms GJR, as a higher percentage of firms accept the hypothesis of worse compared to those accept hypothesis of better.

(7) Bivariate GARCH and GARCH-X

According to modified Diebold-Mariano test results reported in Table 7.50, bivariate GARCH is slightly superior to GARCH-X, the proportion of firms indicating better is higher than firms indicating worse. However in most cases, there is no significant difference between MSE and MAE. Therefore, bivariate GARCH and GARCH-X exhibit comparable forecasting ability in most cases.

(8) BEKK GARCH and GARCH-GJR

Table 7.51 reports the results from Diebold-Mariano test between BEKK GARCH and GARCH-GJR. In 2001 and 2003, most firms suggest equal accuracy for the forecasts of both models. The rest firms show that GJR slightly outperforms BEKK. The dominance of BEKK becomes more common with a higher percentage of firms accepting the hypothesis of worse in longer forecast sample 2002 to 2003.

(9) BEKK GARCH and GARCH-X

According to modified Diebold-Mariano test results reported in Table 7.52, GARCH-X is somewhat superior to BEKK. However, most firms provide evidence of equal accuracy. In particular, all firms giving the answer to equal accuracy according to MSE in 2001. Additionally, MSE in another one-year forecast finds that same number of firms suggesting the dominance of either BEKK or GARCH-X.

(10) GARCH-GJR and GARCH-X

Table 7.53 reports the result from modified Diebold-Mariano comparison tests between GARCH-GJR and GARCH-X models. In general, GARCH-X is found to have dominance over GJR in different forecast samples. However, both models generate fairly close forecast errors, as over 70% accept the hypothesis of equal accuracy. In particular, MAE in 2003 indicates that both models have equal forecasting performance.

Modified Diebold-Mariano comparison tests find that Kalman filter is the most accurate forecasting model. Kalman filter dominate GARCH type models in terms of return forecasts, with statistical significance considered. According to modified Diebold-Mariano tests, forecast accuracy of the GARCH type models is not significantly distinguishing, since most firms provide evidence of equal accuracy among GARCH models. Within the firms indicating relative superiority of particular GARCH models, GJR is the most competent GARCH specification in terms forecast ability. The standard bivariate GARCH is the second best GARCH model, followed by GARCH-X. BEKK is inferior to other GARCH models. Therefore, both UK and US results support the dominance of Kalman filter over GARCH, which implies the advantage of Kalman filter being directly built upon the market model. Additionally, evidence is found in both UK and US results that GJR produce most accurate return forecasts among GARCH models, suggesting the important influence of leverage effect on systematic risk.

7.5 Conclusion

This chapter presents empirical results of forecasting time-varying betas with US daily data. The whole chapter discusses the performance of alternative models in both estimating and forecasting time-varying betas

The chapter begins with the discussion of estimation results for each model, as estimation builds the foundation for further forecast studies. In general, GARCH models produce robust estimated coefficients. However there are two extreme cases (Florida Gaming and Bell Industries), in which the sum of GARCH and ARCH terms is larger than unity. Such a great sum indicates that GARCH models appear explosive and may lead to spurious time-varying beta estimates. However, the estimation results will still be used for forecasting accuracy analysis. The residuals-based diagonal tests generally detect serial correlation in the standardised residuals of the market. Nevertheless, further cross-product tests provide supportive evidence for the descriptive validity of all GARCH models. Preliminary analysis on the potential transition equation finds random walk is an appropriate dynamic process to describe the US daily time-varying beta.

The mean values of conditional betas estimated by different models are highly correlated. Moreover, the mean values are fairly close to the point estimates of beta calculated by the market model, which suggests the satisfactory capability of the models in parameterisations of conditional systematic risk. However, the graphic comparison of beta estimates indicates there are both apparent similarities and differences among conditional betas estimated by different models. GARCH models generally construct comparable beta series; while Kalman filter approach seems to be less capable to capture the time variation of systematic risk.

Various comparison approaches are applied to evaluate relative superiority of alternative models in forecasting time-varying betas. First, as an intuitive and straightforward way, graphic inspection on forecasted and actual conditional betas is used to assess forecast accuracy of different models. Visually, all forecasting models produce accurate out-of-sample forecasts in most cases, while Kalman filter seems to produce less accurate forecasts than GARCH models.

Second, four forecast error measures, including MAE, MSE, MAPE and Theil U statistics, are calculated to assess the accuracy of different models in forecasting the time-varying beta. In 2001 and 2002 to 2003, bivariate GARCH is the best model with consistently accurate forecasts. In 2003, BEKK is found to be the best candidate model. Kalman filter seems to be less sensitive to different samples, as it is the second best forecasting techniques in all samples. The more elaborated GARCH extensions (GJR and GARCH-X) fail to exhibit improvement on bivariate GARCH.

Third, forecasting ability of alternative forecasting models are evaluated in terms of out-of-sample return forecasts. Statistics of forecast errors generally suggest that Kalman filter overwhelmingly outperforms GARCH type models. Among GARCH models, return forecasts produced by GJR are closest to the actual returns. However, the predominance of GJR over other GARCH models is not considerably evident.

Modified Diebold-Mariano comparison test is the last approach to evaluate the forecast ability of different models. Taken statistical significance into account, the error criteria (MSE and MAE) still support that the Kalman filter method is remarkably superior to GARCH models. Additionally, GJR is the best GARCH specification in forecasting US daily betas in terms of modified Diebold-Mariano test statistics.

CAPM betas are widely used by market participators and academic researchers for a variety of purposes. Therefore empirical evidence from this chapter is helpful for those who use the information of systematic risk for their decision making or research development in US stock markets. GARCH models are found to be more successful to estimate rather than to forecast the time-varying beta. Both bivariate GARCH and GARCH-GJR are appropriate models to capture the dynamic process of conditional betas.

Different benchmarks lead to different conclusions on forecast performance of alternative models, and thus different implications for different purposes regarding time-varying beta forecasts. If the purpose of beta forecast is not directly connected to decision making in the stock market, bivariate GARCH and BEKK are excellent choices, since they produces moderately accurate and consistent forecasts of systematic risk. If the forecasted beta is used for investment in stock markets, Kalman filter is a better choice than GARCH models, since it is considerably superior to GARCH models in terms of return forecasts.

Table 7.1 Part 1: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	American Electric Power	Alaska Air Group	Bank of America	Boeing	California Water Service
$c_1(10^{-5})$	9.4689[a]	3.5579[a]	1.9298[a]	1.0153[a]	0.2972[a]
	(16.15940)	(10.32033)	(15.34598)	(9.52937)	(6.83700)
a_{11}	0.1547[a]	0.0710[a]	0.0612[a]	0.0406[a]	0.0346[a]
	(32.80657)	(16.64412)	(16.28694)	(15.65354)	(14.68818)
b_{11}	0.7807[a]	0.8795[a]	0.9032[a]	0.9399[a]	0.9607[a]
	(105.28034)	(124.96508)	(171.03020)	(264.78200)	(365.30791)
$c_3(10^{-5})$	1.0401[a]	0.6135[a]	0.6387[a]	0.5023[a]	0.2835[a]
	(11.33686)	(10.49513)	(12.62614)	(10.58887)	(9.07912)
a_{33}	0.0655[a]	0.0369[a]	0.0497[a]	0.0434[a]	0.0325[a]
	(13.13850)	(13.90633)	(16.39569)	(16.58677)	(20.20845)
b_{33}	0.8927[a]	0.9363[a]	0.9263[a]	0.9372[a]	0.9572[a]
	(115.79367)	(202.16462)	(215.07607)	(235.00311)	(437.23939)
$c_2(10^{-5})$	0.8572[a]	0.6349[a]	0.7355[a]	0.4518[a]	0.1396[a]
	(5.48234)	(7.23932)	(13.07509)	(9.33498)	(7.36739)
a_{22}	0.0433[a]	0.0284[a]	0.0473[a]	0.0363[a]	0.0263[a]
	(6.94163)	(9.57333)	(16.36622)	(15.31861)	(19.62761)
b_{22}	0.9034[a]	0.9396[a]	0.9267[a]	0.9444[a]	0.9677[a]
	(66.70644)	(151.84674)	(223.65484)	(257.28218)	(603.96893)
L	26229.89	27601.10	29064.44	28672.92	28199.19

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	5.4134	54.6383**	8.0670	15.6245	17.0137*
L-B(12)[e]	1.4851	3.6570	2.2399	2.5665	4.0252
$u_t / h_t^{1/2}$					
L-B(12)[d]	27.3497**	16.0682	14.1717	31.1898**	5.1413
L-B(12)[e]	29.1669**	32.6788**	26.3977**	28.1509**	36.5582**
L-B(12)[f]	3.3531	3.8003	4.8378	4.8022	13.6586

Notes: t statistics in parentheses. L = log likelihood function value. L-B(12) = Ljung-Box statistics of order 12. $u_t / h_t^{1/2}$ = standardised residuals. u_t^2 / h_t = standardised squared residuals.

[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.
[d] Ljung Box statistics for serial correlation of order 12 for the residuals of firm equations.
[e] Ljung Box statistics for serial correlation of order 12 for the residuals of market equations.
[f] Ljung Box statistics for serial correlation of order 12 for the cross products of standardised residuals of the firm and the market
* Significant at the 5% level.
** Significant at the 1% level.

Table 7.1 Part 2: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003					
Parameter	Delta Air Lines	Ford Motor	General Electric	Honeywell International	Microsoft
$c_1(10^{-5})$	1.7807[a]	0.6874[a]	0.8953[a]	4.68778[a]	1.4447[a]
	(8.73334)	(6.33830)	(9.44178)	(14.17284)	(9.98744)
a_{11}	0.0866[a]	0.0447[a]	0.0503[a]	0.1383210742[a]	0.0385[a]
	(17.71813)	(14.77915)	(14.14104)	(29.41490)	(15.31603)
b_{11}	0.8919[a]	0.9443[a]	0.9282[a]	0.7911080892[a]	0.9396[a]
	(150.80809)	(259.04580)	(207.29741)	(89.22107)	(256.46681)
$c_3(10^{-5})$	0.7574[a]	0.7494[a]	0.9094[a]	1.58862[a]	0.4048[a]
	(11.27888)	(12.22713)	(13.28688)	(13.21462)	(10.45596)
a_{33}	0.0448[a]	0.0457[a]	0.0508[a]	0.0809173000[a]	0.0333[a]
	(13.57239)	(14.87972)	(15.36905)	(14.46239)	(20.25764)
b_{33}	0.9230[a]	0.9231[a]	0.9111[a]	0.8573454952[a]	0.9505[a]
	(169.33203)	(185.79590)	(166.78241)	(93.38109)	(340.55640)
$c_2(10^{-5})$	0.6931[a]	0.5189[a]	0.8170[a]	2.00485[a]	0.4094[a]
	(7.46505)	(8.33018)	(11.86956)	(13.19051)	(10.24301)
a_{22}	0.0328[a]	0.0356[a]	0.0463[a]	0.0879437514[a]	0.0339[a]
	(9.25342)	(13.39906)	(15.31335)	(19.01915)	(18.83812)
b_{22}	0.9337[a]	0.9420[a]	0.9221[a]	0.8373732695[a]	0.9512[a]
	(135.58205)	(212.58817)	(198.87976)	(91.57869)	(416.03588)
L	27888.97	28511.56	30429.65	28673.60	28253.51
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	385.1284**	10.6600	3.1964	6.1830	12.6282
L-B(12)[e]	2.7567	2.6657	2.4059	1.7581	4.0660
$u_t / h_t^{1/2}$					
L-B(12)[d]	23.1604**	15.7629	23.4701**	16.3145	20.3248*
L-B(12)[e]	33.3184**	29.3723**	28.3309**	37.8637**	27.4166**
L-B(12)[f]	6.8999	3.8392	1.6331	0.9381	14.5519

Notes: see Table 7.1 part 1

Table 7.1 Part 3: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	MGP Ingredients	New York Times	Textron	Utah Medical Products	Walt Disney
$c_1(10^{-5})$	4.8429[a]	2.8423[a]	2.0410[a]	14.5640[a]	1.7109[a]
	(13.32627)	(11.70026)	(13.16921)	(9.63457)	(11.38028)
a_{11}	0.1073[a]	0.0770[a]	0.0666[a]	0.0936[a]	0.0646[a]
	(14.22048)	(14.06893)	(20.07837)	(11.39572)	(16.15334)
b_{11}	0.8415[a]	0.8631[a]	0.8976[a]	0.7570[a]	0.9032[a]
	(89.17286)	(103.78978)	(198.83638)	(36.65301)	(179.15552)
$c_3(10^{-5})$	0.9557[a]	1.4838[a]	0.6445[a]	0.8143[a]	0.7279[a]
	(12.10653)	(13.12309)	(11.23996)	(10.54799)	(11.84090)
a_{33}	0.0618[a]	0.0689[a]	0.0441[a]	0.0505[a]	0.0476[a]
	(13.46217)	(13.72651)	(14.67825)	(13.05992)	(15.36856)
b_{33}	0.9006[a]	0.8697[a]	0.9293[a]	0.9151[a]	0.9226[a]
	(133.48188)	(102.74911)	(196.59396)	(142.35154)	(184.85518)
$c_2(10^{-5})$	0.9019[a]	1.1622[a]	0.4907[a]	0.8682[a]	0.7469[a]
	(8.65036)	(13.65022)	(10.58767)	(8.12787)	(10.65535)
a_{22}	0.0532[a]	0.0624[a]	0.0364[a]	0.0421[a]	0.0433[a]
	(7.98874)	(15.77643)	(14.68250)	(9.14441)	(14.04104)
b_{22}	0.8952[a]	0.8833[a]	0.9351[a]	0.9040[a]	0.9244[a]
	(82.96255)	(129.82396)	(224.10646)	(113.56101)	(176.18875)
L	26964.31	28781.40	28094.44	26624.22	28816.04
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	7.0400	6.6059	35.5158**	3.0335	18.8798*
L-B(12)[e]	1.9016	1.9640	2.6783	2.3091	2.4635
$u_t / h_t^{1/2}$					
L-B(12)[d]	11.9899	17.5112*	21.5455*	12.7270	14.0694
L-B(12)[e]	34.2473**	32.6221**	27.7084**	28.4566**	30.7703**
L-B(12)[f]	5.4978	2.6946	14.9441	13.0447	4.3246

Notes: see Table 7.1 part 1

Table 7.1 Part 4: Bivariate GARCH Estimation Results

A. Bivariate GARCH(1,1) results, sample period 1989-2003

Parameter	Wells Fargo & Company	Wendy's International	Florida Gaming	Campbell Soup	Bell Industries
$c_1(10^{-5})$	0.7175[a]	2.0900[a]	3.4572[a]	2.1927[a]	9.8390[a]
	(8.39670)	(9.26262)	(22.10984)	(15.23913)	(29.13498)
a_{11}	0.0450[a]	0.0492[a]	0.0955[a]	0.0673[a]	0.3550[a]
	(14.79397)	(13.58395)	(33.61148)	(15.85064)	(56.40126)
b_{11}	0.9421[a]	0.9134[a]	0.9132[a]	0.8879[a]	0.6723[a]
	(255.48046)	(153.19488)	(547.04092)	(157.50313)	(135.05762)
$c_3(10^{-5})$	0.3706[a]	1.0408[a]	0.1123[a]	1.0500[a]	0.8096[a]
	(9.58575)	(12.65515)	(8.33419)	(13.83479)	(11.20137)
a_{33}	0.0402[a]	0.0590[a]	0.0190[a]	0.0639[a]	0.0526[a]
	(17.53779)	(13.57152)	(24.54251)	(15.01733)	(13.47196)
b_{33}	0.9471[a]	0.8982[a]	0.9769[a]	0.8939[a]	0.9140[a]
	(294.30749)	(132.50124)	(1361.37046)	(141.43099)	(149.69936)
$c_2(10^{-5})$	0.3728[a]	0.8437[a]	0.0521	0.9878[a]	0.7578[a]
	(11.13986)	(10.70412)	(1.39158)	(14.15370)	(5.65681)
a_{22}	0.0395[a]	0.0436[a]	0.0144[a]	0.0546[a]	0.0248[a]
	(18.63311)	(13.41374)	(4.98329)	(16.79500)	(7.65981)
b_{22}	0.9491[a]	0.9159[a]	0.9794[a]	0.9025[a]	0.9227[a]
	(359.38735)	(149.82597)	(258.73927)	(170.47014)	(94.81420)
L	29363.89	28132.56	23102.27	28958.15	26655.26

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	10.2826	8.9296	14.6949	5.3291	0.5800
L-B(12)[e]	2.8005	2.0584	10.3722	1.8946	2.1874
$u_t / h_t^{1/2}$					
L-B(12)[d]	22.8212**	25.9040**	16.2466	15.1233	15.0573
L-B(12)[e]	31.7883**	29.4691**	29.9820**	26.6027**	28.7033**
L-B(12)[f]	5.9928	6.2205	27.1215**	5.3393	1.7127

Notes: see Table 7.1 part 1

Table 7.2: Basic Statistics of the Time-Varying Beta (Bivariate GARCH)

Company	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
American Electric Power	0.70815[a]	0.052191	-0.43791[a]	2.71535[a]	1326.51458[a]
Alaska Air Group	0.91923[a]	0.020592	-1.06657[a]	5.03357[a]	4870.36550[a]
Bank of America	1.08380[a]	0.024944	-1.11270[a]	7.32904[a]	9560.32861[a]
Boeing	0.91843[a]	0.032509	-0.67431[a]	3.21226[a]	1977.89391[a]
California Water Service	0.67902[a]	0.046895	-0.29062[a]	-0.06941	55.83945[a]
Delta Air Lines	0.95399[a]	0.035310	1.07914[a]	10.03899[a]	17182.24282[a]
Ford Motor	0.96423[a]	0.026140	-0.84486[a]	3.79894[a]	2817.08094[a]
General Electric	1.07990[a]	0.014668	-2.23341[a]	18.87528[a]	61309.61410[a]
Honeywell International	1.03078[a]	0.031637	-0.23495[a]	7.17102[a]	8415.88190[a]
Microsoft	1.12273[a]	0.032433	-1.25319[a]	6.52095[a]	7953.13923[a]
MGP Ingredients	0.68426[a]	0.055881	-0.18055[a]	2.02864[a]	691.88498[a]
New York Times	0.86927[a]	0.027353	-0.98585[a]	3.76556[a]	2944.16504[a]
Textron	0.73257[a]	0.029999	-0.69591[a]	2.65302[a]	1462.66043[a]
Utah Medical Products	0.73143[a]	0.050679	0.22297[a]	1.50400[a]	401.02004[a]
Walt Disney	0.96421[a]	0.023786	-1.61296[a]	6.70341[a]	9018.47584[a]
Wells Fargo & Company	1.04317[a]	0.031329	-1.14109[a]	6.04647[a]	6806.46145[a]
Wendy' s International	0.86391[a]	0.032520	-1.03838[a]	2.72353[a]	1911.58666[a]
Florida Gaming	0.49007[a]	0.193671	-0.63796[a]	2.92808[a]	1662.44011[a]
Campbell Soup	0.92211[a]	0.029903	-1.08965[a]	3.75792[a]	3075.24476[a]
Bell Industries	0.68871[a]	0.024860	-0.66497[a]	1.55579[a]	682.66798[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 7.3: Two Unit Root Tests for Time-Varying Betas (Bivariate GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-64.17813[a]	-35.88239[a]	-26.40108[a]	-23.75419[a]
Alaska Air Group	-64.71184[a]	-33.20676[a]	-25.42019[a]	-22.35964[a]
Bank of America	-64.01650[a]	-33.72063[a]	-25.72946[a]	-23.39145[a]
Boeing	-64.01321[a]	-31.94566[a]	-26.41914[a]	-22.67462[a]
California Water Service	-63.34452[a]	-31.66100[a]	-23.95280[a]	-20.73265[a]
Delta Air Lines	-61.74057[a]	-32.00551[a]	-25.54100[a]	-21.70267[a]
Ford Motor	-63.17185[a]	-32.27330[a]	-25.86844[a]	-21.64199[a]
General Electric	-64.10991[a]	-32.83121[a]	-25.39145[a]	-21.99248[a]
Honeywell International	-66.02343[a]	-37.43319[a]	-29.21742[a]	-25.69384[a]
Microsoft	-64.04504[a]	-33.51514[a]	-26.13952[a]	-22.54111[a]
MGP Ingredients	-63.59756[a]	-33.98661[a]	-27.46071[a]	-23.65870[a]
New York Times	-65.66422[a]	-34.74535[a]	-27.82687[a]	-23.23355[a]
Textron	-62.35842[a]	-32.02450[a]	-24.18283[a]	-21.73005[a]
Utah Medical Products	-62.55093[a]	-33.71930[a]	-27.31085[a]	-23.59063[a]
Walt Disney	-62.05769[a]	-33.12015[a]	-25.83233[a]	-22.20208[a]
Wells Fargo & Company	-62.17649[a]	-32.53379[a]	-26.28886[a]	-21.28636[a]
Wendy' s International	-64.60696[a]	-34.12064[a]	-26.11455[a]	-23.00224[a]
Florida Gaming	-64.38771[a]	-32.21891[a]	-23.53072[a]	-20.76333[a]
Campbell Soup	-64.02927[a]	-33.82249[a]	-26.74109[a]	-23.31931[a]
Bell Industries	-63.10121[a]	-33.35613[a]	-25.75082[a]	-23.62718[a]

Notes:
[a] Significant at the 1% level.

Table 7.4: One Unit Root Tests for Time-Varying Betas (Bivariate GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-12.87598[a]	-12.00353[a]	-11.47432[a]	-11.22572[a]
Alaska Air Group	-10.23586[a]	-9.92101[a]	-9.79349[a]	-9.67371[a]
Bank of America	-10.84711[a]	-10.44760[a]	-10.32735[a]	-10.06142[a]
Boeing	-8.09233[a]	-7.91974[a]	-7.54393[a]	-7.18275[a]
California Water Service	-5.92450[a]	-5.85841[a]	-5.99411[a]	-5.88043[a]
Delta Air Lines	-8.82533[a]	-8.89436[a]	-8.77085[a]	-8.87368[a]
Ford Motor	-9.43228[a]	-9.35761[a]	-8.95139[a]	-8.79092[a]
General Electric	-10.73474[a]	-10.87974[a]	-10.80689[a]	-10.68832[a]
Honeywell International	-17.44480[a]	-16.21649[a]	-15.15404[a]	-13.94310[a]
Microsoft	-8.75556[a]	-8.57188[a]	-8.25872[a]	-8.06270[a]
MGP Ingredients	-12.23643[a]	-11.73020[a]	-10.67263[a]	-10.05695[a]
New York Times	-12.67793[a]	-11.88427[a]	-11.02545[a]	-10.72759[a]
Textron	-9.07561[a]	-9.15108[a]	-9.22660[a]	-8.95284[a]
Utah Medical Products	-10.62572[a]	-10.14263[a]	-9.59576[a]	-8.95884[a]
Walt Disney	-10.14405[a]	-10.00838[a]	-9.68232[a]	-9.36108[a]
Wells Fargo & Company	-7.92872[a]	-7.85079[a]	-7.55224[a]	-7.41042[a]
Wendy’ s International	-10.06551[a]	-9.64994[a]	-9.31553[a]	-8.81948[a]
Florida Gaming	-5.87943[a]	-5.72594[a]	-5.63931[a]	-5.57170[a]
Campbell Soup	-10.86147[a]	-10.45413[a]	-9.69273[a]	-9.35600[a]
Bell Industries	-10.40375[a]	-9.92767[a]	-9.65530[a]	-8.80667[a]

Notes:
[a] Significant at the 1% level,

Table 7.5 Part 1: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003					
Parameter	American Electric Power	Alaska Air Group	Bank of America	Boeing	California Water Service
c_{11}	0.008919[a]	0.005142[a]	0.002612[b]	0.001964[b]	0.001410[a]
	(2.71260)	(3.51621)	(1.88409)	(2.15015)	(2.85416)
a_{11}	0.351035[a]	0.237362[a]	0.175073[a]	0.157563[a]	0.168219[a]
	(3.17220)	(3.87027)	(2.86854)	(3.88424)	(5.82101)
b_{11}	0.905558[a]	0.952332[a]	0.978132[a]	0.983984[a]	0.984286[a]
	(14.67949)	(37.92091)	(54.8169)	(100.02414)	(158.5002)
c_{22}	0.002088	-0.0000001	0.001075	0.000857	0.000941
	(1.47011)	(-0.00224)	(1.47210)	(1.14292)	(1.21381)
a_{22}	0.200358[b]	0.098002[a]	0.177171[a]	0.148520[b]	0.152622[b]
	(2.40477)	(6.57638)	(2.44793)	(2.19253)	(2.51268)
b_{22}	0.968672[a]	0.992042[a]	0.979078[a]	0.986467[a]	0.985692[a]
	(33.58363)	(437.4474)	(49.1937)	(69.09519)	(70.36955)
c_{12}	0.001052[a]	0.001167[a]	0.001274[a]	0.000791[c]	0.000785
	(2.62873)	(3.27411)	(1.69479)	(1.64671)	(1.63056)
L	19031.73	20369.62	21885.52	21512.64	21008.26
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	6.1976	77.7985**	14.9931	32.7880**	24.3537**
L-B(12)[e]	3.0293	31.9326**	4.1680	8.1085	7.5438
$u_t / h_t^{1/2}$					
L-B(12)[d]	27.1605**	16.9981*	16.5448	34.5066**	5.7392
L-B(12)[e]	31.1961**	39.6602**	28.0121**	32.2403**	41.7583**
L-B(12)[f]	5.1095	16.2116	10.6776	11.8469	22.4474**

Notes: see Table 7.1 part 1.

Table 7.5 Part 2: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003

Parameter	Delta Air Lines	Ford Motor	General Electric	Honeywell International	Microsoft
c_{11}	0.003522[a]	0.001447[a]	0.001428[b]	0.006586[a]	0.003210[a]
	(2.71811)	(2.92504)	(2.19837)	(4.43119)	(8.17759)
a_{11}	0.259646[a]	0.167830[a]	0.1620045[a]	0.364634[a]	0.188935[a]
	(4.05445)	(5.91553)	(3.34353)	(6.60145)	(12.38379)
b_{11}	0.957680[a]	0.984582[a]	0.984653[a]	0.896135[a]	0.974054[a]
	(44.03576)	(169.9826)	(97.3402)	(24.94670)	(275.8895)
c_{22}	0.000001	0.000644	0.000572	0.001502[c]	0.001429[a]
	(0.04762)	(1.17373)	(0.81503)	(1.71973)	(4.83939)
a_{22}	0.087204[a]	0.130856[a]	0.147619[c]	0.202988[a]	0.177200[a]
	(3.72338)	(3.15837)	(1.89048)	(3.15473)	(3.72959)
b_{22}	0.992520[a]	0.989528[a]	0.986557[a]	0.960777[a]	0.976506[a]
	(679.0149)	(128.7335)	(55.8259)	(54.86955)	(78.86374)
c_{12}	0.001222[a]	0.000774[a]	0.001026	0.002522[a]	0.001304[a]
	(2.91058)	(2.69019)	(1.08817)	(3.74740)	(7.04110)
L	20585.58	21357.97	23272.56	21443.37	21069.26

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	392.8918**	21.3368**	10.2919.	6.3638	13.9786
L-B(12)[e]	41.2787**	13.1649	8.3079	3.5034	4.3753
$u_t / h_t^{1/2}$					
L-B(12)[d]	26.0715**	15.7604	25.7089**	16.8610	20.1718*
L-B(12)[e]	38.8817**	34.3036**	30.3944**	35.3640**	28.1959**
L-B(12)[f]	36.0541**	11.7482	6.0981	1.1588	15.1225

Notes: see Table 7.1 part 1.

Table 7.5 Part 3: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003					
Parameter	MGP Ingredients	New York Times	Textron	Utah Medical Products	Walt Disney
c_{11}	0.006506[a]	0.002151	0.002664[a]	0.010965[a]	0.001160[b]
	(5.10396)	(1.35098)	(2.90118)	(3.64122)	(2.33208)
a_{11}	0.302879[a]	0.161289[c]	0.201997[a]	0.286486[a]	0.1264067[a]
	(8.66561)	(1.92134)	(3.44314)	(5.35165)	(4.20630)
b_{11}	0.929133[a]	0.981998[a]	0.973774[a]	0.891572[a]	0.990827[a]
	(48.52639)	(45.93742)	(64.18996)	(17.78766)	(202.3155)
c_{22}	0.002033[a]	0.001010	0.000855	0.002369[a]	0.000673
	(2.96531)	(0.96263)	(1.19454)	(3.05679)	(1.54465)
a_{22}	0.204744[a]	0.146062[c]	0.123282[a]	0.211600[a]	0.131748[a]
	(5.06062)	(1.87587)	(2.62086)	(5.34124)	(2.74070)
b_{22}	0.967994[a]	0.985555[a]	0.989666[a]	0.962009[a]	0.989771[a]
	(70.62202)	(50.75677)	(117.6406)	(60.08292)	(111.8361)
c_{12}	0.001179[a]	0.000895	0.000740[b]	0.001218[a]	0.000636
	(4.07437)	(1.25427)	(2.46067)	(3.32260)	(1.22295)
L	19774.75	21662.11	20889.65	19439.20	21632.14
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	7.9239	24.5490**	63.7892**	3.1923	95.2944**
L-B(12)[e]	2.8559	8.8347	16.1676	2.6736	12.7227
$u_t / h_t^{1/2}$					
L-B(12)[d]	10.8515	17.0184*	24.6654**	12.5838	18.2369*
L-B(12)[e]	35.1634**	35.4601**	32.0165**	29.9636**	35.1519**
L-B(12)[f]	6.9903	15.6215	40.7111**	14.5395	27.8443**

Notes: see Table 7.1 part 1.

Table 7.5 Part 4: BEKK GARCH Estimation Results

A. BEKK GARCH results, sample period 1989-2003

Parameter	Wells Fargo & Company	Wendy's International	Florida Gaming	Campbell Soup	Bell Industries
c_{11}	0.001405[c]	0.002231[a]	0.005644[a]	0.003931[a]	0.013826[b]
	(1.70174)	(3.69753)	(2.28897)	(3.59127)	(2.49263)
a_{11}	0.157920[a]	0.145137[a]	0.302098[a]	0.222408[a]	0.540429[b]
	(3.28237)	(5.01685)	(15.68004)	(5.25617)	(1.98552)
b_{11}	0.985912[a]	0.985125[a]	0.957412[a]	0.958582[a]	0.779960[a]
	(97.42705)	(147.4277)	(169.51345)	(52.23801)	(4.62090)
c_{22}	0.000638	0.000924[b]	0.000804	0.001814[b]	0.000002
	(1.39400)	(2.01982)	(0.70701)	(2.41203)	(0.00299)
a_{22}	0.156674[a]	0.140697[a]	0.119947[c]	0.214204[a]	-0.143191[a]
	(3.15697)	(3.05139)	(1.71346)	(3.42046)	(-4.05135)
b_{22}	0.985567[a]	0.987262[a]	0.991440[a]	0.963999[a]	0.964031[a]
	(89.19965)	(109.2405)	(82.47986)	(41.47392)	(52.52839)
c_{12}	0.000978[b]	0.000762[a]	0.000313	0.001718[a]	0.003410[a]
	(2.00980)	(2.88829)	(1.44479)	(2.64907)	(3.38732)
L	22212.92	21011.88	15895.01	21775.51	19339.65

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	24.2856**	24.2650**	15.3262	6.3018	0.2985
L-B(12)[e]	6.6930	10.1467	17.4616*	2.4720	17.2371*
$u_t / h_t^{1/2}$					
L-B(12)[d]	23.4812**	25.9414**	23.4376**	14.8443	13.9946
L-B(12)[e]	35.1951**	33.6761**	33.0779**	27.6600**	34.1087**
L-B(12)[f]	14.2090	26.2901**	30.9920**	7.3536	5.5529

Notes: see Table 7.1 part 1.

Table 7.6: Basic Statistics of the Time-Varying Beta (BEKK GARCH)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
American Electric Power	0.70971[a]	0.105467	0.00676	4.05305[a]	2676.97510[a]
Alaska Air Group	0.90801[a]	0.035491	1.09411[a]	3.41389[a]	2679.50675[a]
Bank of America	1.09189[a]	0.015582	0.29764[a]	0.50619[a]	99.50047[a]
Boeing	0.92919[a]	0.031805	-0.17899[a]	1.25222[a]	276.41271[a]
California Water Service	0.67933[a]	0.038875	-0.16894[a]	-0.15759[b]	22.65100[a]
Delta Air Lines	0.87714[a]	0.064120	2.88528[a]	14.07298[a]	37700.1212[a]2
Ford Motor	0.97276[a]	0.020537	-0.55086[a]	1.73817[a]	690.13024[a]
General Electric	1.08241[a]	0.008408	0.41936[a]	0.18257[b]	120.06234[a]
Honeywell International	1.02180[a]	0.039312	1.13799[a]	7.19854[a]	9288.45999[a]
Microsoft	1.13541[a]	0.023752	0.11803[a]	1.21926[a]	251.33220[a]
MGP Ingredients	0.68850[a]	0.080256	0.17400[a]	2.18057[a]	794.57997[a]
New York Times	0.87913[a]	0.018803	-0.00657	0.53876[a]	47.32945[a]
Textron	0.72997[a]	0.036129	0.55569[a]	3.01772[a]	1685.28288[a]
Utah Medical Products	0.73552[a]	0.067867	0.61284[a]	2.45398[a]	1226.14815[a]
Walt Disney	0.96018[a]	0.018830	-0.86825[a]	1.72119[a]	974.15717[a]
Wells Fargo & Company	1.05080[a]	0.020313	0.12057[a]	0.34615[a]	29.00228[a]
Wendy's International	0.88671[a]	0.025767	-0.36187[a]	-0.21956[a]	93.21527[a]
Florida Gaming	0.54251[a]	0.489816	-0.00718	6.23523[a]	6335.55107[a]
Campbell Soup	0.92404[a]	0.028246	-0.78115[a]	3.12924[a]	1993.45958[a]
Bell Industries	0.67290[a]	0.141214	-1.88606[a]	9.00121[a]	15521.90008[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level.

Table 7.7: Two Unit Root Tests for Time-Varying Betas (BEKK GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-65.46458[a]	-37.35607[a]	-27.35764[a]	-24.54567[a]
Alaska Air Group	-63.54589[a]	-32.05007[a]	-24.54981[a]	-20.77114[a]
Bank of America	-63.05540[a]	-32.78721[a]	-24.95508[a]	-22.53390[a]
Boeing	-62.00843[a]	-31.30769[a]	-25.39427[a]	-21.24854[a]
California Water Service	-63.30690[a]	-31.65058[a]	-23.93262[a]	-20.64816[a]
Delta Air Lines	-61.28032[a]	-29.31008[a]	-24.83176[a]	-20.31936[a]
Ford Motor	-62.38877[a]	-32.49154[a]	-25.45056[a]	-21.10309[a]
General Electric	-62.39251[a]	-31.83400[a]	-25.19408[a]	-21.15431[a]
Honeywell International	-64.71061[a]	-36.25903[a]	-28.22800[a]	-24.45083[a]
Microsoft	-64.54891[a]	-33.71398[a]	-26.34130[a]	-22.82542[a]
MGP Ingredients	-62.96032[a]	-33.89283[a]	-26.87240[a]	-23.21103[a]
New York Times	-63.23650[a]	-31.81911[a]	-24.95904[a]	-19.63705[a]
Textron	-61.84555[a]	-30.33747[a]	-22.07492[a]	-20.01403[a]
Utah Medical Products	-65.03410[a]	-34.95381[a]	-28.93369[a]	-25.39615[a]
Walt Disney	-61.37774[a]	-31.78060[a]	-24.09746[a]	-20.28113[a]
Wells Fargo & Company	-61.64100[a]	-31.84028[a]	-25.60079[a]	-20.79930[a]
Wendy's International	-62.42628[a]	-31.92290[a]	-24.37555[a]	-20.72235[a]
Florida Gaming	-65.23464[a]	-33.45013[a]	-24.75176[a]	-22.22450[a]
Campbell Soup	-63.37262[a]	-33.16169[a]	-26.19292[a]	-23.08058[a]
Bell Industries	-71.02580[a]	-38.31154[a]	-29.51647[a]	-26.32530[a]

Notes:
[a] Significant at the 1% level.

Table 7.8: One Unit Root Tests for Time-Varying Betas (BEKK GARCH)

Company	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-15.29282[a]	-13.62204[a]	-12.80532[a]	-12.29616[a]
Alaska Air Group	-8.44842[a]	-8.59361[a]	-8.32342[a]	-8.33818[a]
Bank of America	-8.17950[a]	-7.87666[a]	-7.76291[a]	-7.49368[a]
Boeing	-4.96804[a]	-4.96985[a]	-4.74009[a]	-4.55159[a]
California Water Service	-6.09543[a]	-5.97342[a]	-6.12366[a]	-6.04461[a]
Delta Air Lines	-7.19606[a]	-8.46143[a]	-8.21213[a]	-8.07904[a]
Ford Motor	-6.50914[a]	-6.44571[a]	-6.17080[a]	-5.84193[a]
General Electric	-5.84120[a]	-5.87724[a]	-5.60410[a]	-5.44831[a]
Honeywell International	-15.96497[a]	-15.33889[a]	-14.19566[a]	-13.51752[a]
Microsoft	-8.69369[a]	-8.22791[a]	-7.66199[a]	-7.29303[a]
MGP Ingredients	-12.02844[a]	-11.64969[a]	-10.71084[a]	-10.16493[a]
New York Times	-6.54581[a]	-6.47471[a]	-6.23364[a]	-6.43710[a]
Textron	-6.82400[a]	-7.03128[a]	-7.82654[a]	-7.70556[a]
Utah Medical Products	-13.91498[a]	-12.63177[a]	-11.45962[a]	-10.39850[a]
Walt Disney	-4.21555[a]	-4.19822[a]	-4.12860[a]	-4.00295[a]
Wells Fargo & Company	-5.42513[a]	-5.32706[a]	-5.03456[a]	-4.86395[a]
Wendy' s International	-5.43708[a]	-5.46060[a]	-5.36361[a]	-5.23630[a]
Florida Gaming	-9.52622[a]	-8.96591[a]	-8.58428[a]	-8.37978[a]
Campbell Soup	-9.30210[a]	-8.93797[a]	-8.28743[a]	-7.89912[a]
Bell Industries	-17.03168[a]	-13.27997[a]	-11.72164[a]	-10.54710[a]

Notes:
[a] Significant at the 1% level.

Table 7.9 Part 1: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	American Electric Power	Alaska Air Group	Bank of America	Boeing	California Water Service
$c_1(10^{-5})$	6.3380[a]	3.5358[a]	1.8160[a]	1.0079[a]	0.2799[a]
	(15.88426)	(10.12821)	(13.01552)	(9.39287)	(6.58641)
a_1	0.0699[a]	0.0673[a]	0.0516[a]	0.0398[a]	0.0374[a]
	(11.68089)	(14.39618)	(11.63973)	(13.17154)	(13.40760)
b_1	0.8374[a]	0.8790[a]	0.9054[a]	0.9401[a]	0.9628[a]
	(166.21119)	(121.09631)	(159.58979)	(265.87066)	(393.27062)
r_1	0.0994[a]	0.0095	0.0193[a]	0.0015	-0.0089[a]
	(12.31639)	(1.43296)	(5.14385)	(0.41188)	(-2.64636)
$c_3(10^{-5})$	0.8240[a]	0.5622[a]	0.6546[a]	0.5020[a]	0.2174[a]
	(10.71633)	(9.95835)	(12.12194)	(9.90409)	(8.72674)
a_3	0.0511[a]	0.0331[a]	0.0506[a]	0.0435[a]	0.0261[a]
	(12.05089)	(12.60643)	(15.88212)	(14.64345)	(18.90707)
b_3	0.9072[a]	0.9396[a]	0.9258[a]	0.9374[a]	0.9633[a]
	(143.22778)	(214.17301)	(207.40537)	(230.52028)	(552.39339)
r_3	0.0188[a]	0.0059	-0.0033	-0.0004	0.0065[b]
	(3.10743)	(1.59544)	(-1.08116)	(-0.11442)	(2.48532)
$c_2(10^{-5})$	0.6206[a]	0.6010[a]	0.74774[a]	0.499[a]	0.1131[a]
	(5.56694)	(7.18350)	(12.45031)	(8.97179)	(6.96935)
a_2	0.0349[a]	0.0270[a]	0.0462[a]	0.0361[a]	0.0244[a]
	(6.96770)	(8.90618)	(15.51681)	(14.93568)	(20.52080)
b_2	0.9265[a]	0.9425[a]	0.9266[a]	0.9446[a]	0.9712[a]
	(90.04321)	(156.97784)	(210.86493)	(245.53702)	(744.77063)
L	26253.38	27602.52	29071.57	28672.96	28215.66

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	6.5793	60.4010**	8.7617	15.6937	16.6032
L-B(12)[e]	2.2319	4.2255	2.2015	2.5658	5.5105
$u_t / h_t^{1/2}$					
L-B(12)[d]	25.7255**	16.3011	14.0979	31.1976**	5.5429
L-B(12)[e]	28.0256**	32.4483**	26.5913**	28.2076**	37.7435**
L-B(12)[f]	6.5217	4.2548	5.2549	4.8233	15.5504

Notes: see Table 7.1 part 1.

Table 7.9 Part 2: GARCH-GJR Estimation Results

Parameter	Delta Air Lines	Ford Motor	General Electric	Honeywell International	Microsoft
A. GJR GARCH(1,1) results, sample period 1989-2003					
$c_1(10^{-5})$	1.6154[a]	0.6811[a]	0.9395[a]	4.4310[a]	1.5977[a]
	(8.28160)	(6.29431)	(9.54097)	(14.34587)	(12.66215)
a_1	0.0987[a]	0.0438[a]	0.0453[a]	0.1157[a]	0.0370[a]
	(17.51301)	(13.26295)	(12.32042)	(17.23102)	(13.40905)
b_1	0.9013[a]	0.9445[a]	0.9265[a]	0.7995[a]	0.9319[a]
	(151.06742)	(258.53983)	(201.94138)	(96.37043)	(391.62439)
r_1	-0.0395[a]	0.0017	0.0114[a]	0.0378[a]	0.0133[a]
	(-6.84952)	(0.50894)	(3.68538)	(5.24001)	(4.14016)
$c_3(10^{-5})$	0.7239[a]	0.7670[a]	1.0397[a]	1.5851[a]	0.5526[a]
	(10.34207)	(11.89263)	(13.62095)	(12.91177)	(11.22940)
a_3	0.0432[a]	0.0470[a]	0.0570[a]	0.0839[a]	0.0393[a]
	(12.02839)	(13.77221)	(15.20375)	(13.50776)	(16.40551)
b_3	0.9247[a]	0.9220[a]	0.9041[a]	0.8604[a]	0.9396[a]
	(169.90025)	(182.76662)	(152.29486)	(93.11393)	(255.58767)
r_3	0.0030	-0.0018	-0.0106[a]	-0.0145[b]	-0.0034
	(0.69914)	(-0.43695)	(-3.24495)	(-2.54816)	(-1.20117)
$c_2(10^{-5})$	0.6818[a]	0.5233[a]	0.8914[a]	1.9122[a]	0.5258[a]
	(7.19054)	(8.35249)	(12.21464)	(13.27271)	(11.49243)
a_2	0.0344[a]	0.0357[a]	0.0467[a]	0.0824[a]	0.0368[a]
	(8.99069)	(13.11678)	(14.89578)	(16.56110)	(17.78067)
b_2	0.9332[a]	0.9417[a]	0.9185[a]	0.8454[a]	0.9435[a]
	(129.63754)	(211.52356)	(191.79891)	(94.21887)	(359.84767)
L	27899.92	28511.63	30434.43	28679.00	28251.84
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	359.6255	10.8049	3.4595	6.9441	13.2206
L-B(12)[e]	2.8385	2.5868	2.1781	1.6746	3.2491
$u_t / h_t^{1/2}$					
L-B(12)[d]	22.7246	15.7541	23.4373**	16.7070	19.3934*
L-B(12)[e]	32.6835	29.4438**	29.0487**	38.2271**	27.4630**
L-B(12)[f]	6.8828	3.8058	1.5949	0.9644	13.2542

Notes: see Table 7.1 part 1.

Table 7.9 Part 3: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003

Parameter	MGP Ingredients	New York Times	Textron	Utah Medical Products	Walt Disney
$c_1(10^{-5})$	4.6310[a]	1.1729[a]	2.1557[a]	14.9330[a]	1.7376[a]
	(12.86783)	(9.04648)	(12.47149)	(9.46683)	(11.24712)
a_1	0.0975[a]	0.0440[a]	0.0666[a]	0.1037[a]	0.0671[a]
	(12.05510)	(11.45896)	(16.01743)	(11.70205)	(14.60232)
b_1	0.8461[a]	0.9324[a]	0.8925[a]	0.7532[a]	0.9022[a]
	(91.30499)	(188.94363)	(171.71329)	(35.06373)	(174.88313)
r_1	0.0159[c]	-0.0025	0.0064	-0.0230[b]	-0.0043
	(1.66644)	(-0.72173)	(1.20449)	(-2.11715)	(-1.07752)
$c_3(10^{-5})$	0.8142[a]	0.4132[a]	0.6055[a]	0.7600[a]	0.7556[a]
	(11.24885)	(9.16403)	(9.85507)	(9.92769)	(11.52281)
a_3	0.0507[a]	0.0333[a]	0.0387[a]	0.0456[a]	0.0496[a]
	(12.23841)	(14.26416)	(11.63293)	(11.57457)	(14.33697)
b_3	0.9098[a]	0.9483[a]	0.9314[a]	0.9180[a]	0.9210[a]
	(151.66499)	(279.63364)	(190.64481)	(147.87107)	(179.70298)
r_3	0.0167[a]	0.0037	0.0104[a]	0.0095[c]	-0.0035
	(2.94203)	(1.18392)	(2.61818)	(1.70801)	(-0.96571)
$c_2(10^{-5})$	0.8250[a]	0.3608[a]	0.5128[a]	8.501[a]	0.7673[a]
	(8.23404)	(10.36559)	(10.59930)	(8.14504)	(10.62988)
a_2	0.0493[a]	0.0336[a]	0.0355[a]	0.0405[a]	0.0440[a]
	(8.03582)	(18.42142)	(13.35839)	(8.76907)	(13.39836)
b_2	0.9035[a]	0.9503[a]	0.9342[a]	0.9061[a]	0.9228[a]
	(87.50127)	(337.00475)	(219.69061)	(114.08942)	(169.96136)
L	26966.88	28835.77	28096.27	26625.47	28816.76

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	7.1709	11.5800	34.8788**	3.1344	17.8052*
L-B(12)[e]	2.2757	3.9504	3.1655	2.5775	2.3519
$u_t / h_t^{1/2}$					
L-B(12)[d]	12.0640	16.5952	21.6300**	12.8044	13.9473
L-B(12)[e]	33.4515**	31.6394**	27.3016**	27.9924**	30.8125**
L-B(12)[f]	6.4582	6.8866	16.6124	13.0018	4.1604

Notes: see Table 7.1 part 1.

Table 7.9 Part 4: GARCH-GJR Estimation Results

A. GJR GARCH(1,1) results, sample period 1989-2003					
Parameter	Wells Fargo & Company	Wendy' s International	Florida Gaming	Campbell Soup	Bell Industries
$c_1(10^{-5})$	0.7383[a]	2.1365[a]	3.2531[a]	2.1943[a]	10.5500[a]
	(8.39902)	(9.37398)	(19.76274)	(15.00066)	(30.38094)
a_1	0.0451[a]	0.0492[a]	0.0770[a]	0.0690[a]	0.3065[a]
	(13.36991)	(11.93784)	(24.34608)	(13.67114)	(42.69577)
b_1	0.9410[a]	0.9119[a]	0.9138[a]	0.8881[a]	0.6541[a]
	(248.71976)	(152.08725)	(494.64713)	(156.92633)	(117.42615)
r_1	0.0011	0.0013	0.0367[a]	-0.0039	0.1495[a]
	(0.35821)	(0.23554)	(6.08878)	(-0.89967)	(7.56041)
$c_3(10^{-5})$	0.3766[a]	0.9908[a]	0.4041[a]	1.0092[a]	0.5870[a]
	(9.50401)	(12.46801)	(8.41520)	(13.20397)	(9.60661)
a_3	0.0401[a]	0.0527[a]	0.0312[a]	0.0610[a]	0.0358[a]
	(16.65848)	(12.73327)	(13.06653)	(14.05477)	(11.56719)
b_3	0.9463[a]	0.9001[a]	0.9440[a]	0.8965[a]	0.9298[a]
	(289.60982)	(138.59736)	(241.73534)	(145.53097)	(186.07125)
r_3	0.0018	0.0144[a]	0.0187[a]	0.0042	0.0227[a]
	(0.71845)	(2.65476)	(3.86714)	(0.89808)	(4.14318)
$c_2(10^{-5})$	0.3817[a]	0.8424[a]	0.1133[c]	0.9737[a]	0.8011[a]
	(11.22977)	(10.57601)	(1.93617)	(13.91119)	(5.09722)
a_2	0.0397[a]	0.0433[a]	0.0157[a]	0.0541[a]	0.0254[a]
	(18.31039)	(13.26352)	(4.09976)	(16.83178)	(6.56112)
b_2	0.9485[a]	0.9161[a]	0.9712[a]	0.9036[a]	0.9194[a]
	(355.93284)	(147.71435)	(161.92689)	(171.84804)	(71.65846)
L	29364.69	28134.22	23097.71	28958.39	26662.62
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	10.2654	8.9445	16.4169	5.3033	0.6543
L-B(12)[e]	2.8080	2.3197	3.9504	1.9625	3.4206
$u_t / h_t^{1/2}$					
L-B(12)[d]	22.7727**	25.9242**	15.5772	15.1493	13.6659
L-B(12)[e]	31.6511**	29.1000**	25.7718**	26.4525**	27.4518**
L-B(12)[f]	5.9981	6.8195	22.6807**	5.3877	2.1010

Notes: see Table 7.1 part 1.

Table 7.10: Basic Statistics of the Time-Varying Beta (GARCH-GJR)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
American Electric Power	0.71043[a]	0.048622	-0.46469[a]	1.54154[a]	528.00118[a]
Alaska Air Group	0.91862[a]	0.021112	-1.08241[a]	4.92784[a]	4720.90772[a]
Bank of America	1.08105[a]	0.024129	-1.17639[a]	7.67038[a]	10489.66321[a]
Boeing	0.91852[a]	0.032448	-0.67672[a]	3.23280[a]	2001.57745[a]
California Water Service	0.67990[a]	0.045208	-0.15239[a]	-0.31219[a]	31.02046[a]
Delta Air Lines	0.95811[a]	0.038156	1.19922[a]	10.13080[a]	17662.33572[a]
Ford Motor	0.96430[a]	0.026251	-0.82408[a]	3.72392[a]	2702.50666[a]
General Electric	1.08004[a]	0.014975	-2.01071[a]	17.00102[a]	49735.93834[a]
Honeywell International	1.03161[a]	0.030695	-0.10530[a]	7.43027[a]	9003.99186[a]
Microsoft	1.12059[a]	0.030871	-1.11448[a]	6.08407[a]	6841.67004[a]
MGP Ingredients	0.68774[a]	0.056231	-0.16459[a]	1.68639[a]	481.09361[a]
New York Times	0.87121[a]	0.026630	-0.86932[a]	3.38697[a]	2361.98305[a]
Textron	0.73209[a]	0.028952	-0.72035[a]	2.62015[a]	1456.97280[a]
Utah Medical Products	0.73047[a]	0.051177	0.23120[a]	1.34549[a]	329.85304[a]
Walt Disney	0.96443[a]	0.024008	-1.53831[a]	6.52480[a]	8480.12974[a]
Wells Fargo & Company	1.04312[a]	0.031368	-1.11792[a]	5.96096[a]	6605.03113[a]
Wendy’ s International	0.86414[a]	0.034028	-0.97671[a]	2.45853[a]	1606.81347[a]
Florida Gaming	0.49191[a]	0.162605	-0.58942[a]	2.32729[a]	1109.08669[a]
Campbell Soup	0.92241[a]	0.030105	-1.09468[a]	3.72788[a]	3045.76036[a]
Bell Industries	0.69276[a]	0.029791	-0.40677[a]	0.81918[a]	217.20659[a]

Notes:
[a] Significant at the 1% level,
[c] Significant at the 10% level.

Table 7.11: Two Unit Root Tests for Time-Varying Betas (GARCH-GJR)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-63.56058[a]	-34.75557[a]	-25.61092[a]	-22.83869[a]
Alaska Air Group	-64.54749[a]	-32.78857[a]	-25.29078[a]	-22.20310[a]
Bank of America	-64.09510[a]	-33.77126[a]	-25.72142[a]	-23.36982[a]
Boeing	-64.02435[a]	-31.93508[a]	-26.40995[a]	-22.66565[a]
California Water Service	-63.28493[a]	-31.56225[a]	-23.74746[a]	-20.62348[a]
Delta Air Lines	-61.47074[a]	-31.88068[a]	-25.64387[a]	-21.69531[a]
Ford Motor	-63.19178[a]	-32.23275[a]	-25.81900[a]	-21.62806[a]
General Electric	-63.76474[a]	-32.62980[a]	-25.23162[a]	-22.00515[a]
Honeywell International	-65.70794[a]	-37.09860[a]	-29.22129[a]	-25.42023[a]
Microsoft	-64.33641[a]	-33.71038[a]	-26.27420[a]	-22.74195[a]
MGP Ingredients	-63.43217[a]	-33.70166[a]	-27.08658[a]	-23.34884[a]
New York Times	-63.02466[a]	-32.33290[a]	-25.41591[a]	-20.44092[a]
Textron	-62.51043[a]	-31.69506[a]	-24.13453[a]	-21.82581[a]
Utah Medical Products	-62.51020[a]	-33.62752[a]	-27.21521[a]	-23.59034[a]
Walt Disney	-62.17386[a]	-33.21206[a]	-25.83091[a]	-22.22548[a]
Wells Fargo & Company	-62.21163[a]	-32.52126[a]	-26.26148[a]	-21.30384[a]
Wendy' s International	-64.70207[a]	-34.04484[a]	-25.98850[a]	-22.87325[a]
Florida Gaming	-64.74417[a]	-32.66232[a]	-24.43391[a]	-21.06112[a]
Campbell Soup	-63.90624[a]	-33.82830[a]	-26.63109[a]	-23.23424[a]
Bell Industries	-63.09715[a]	-33.38613[a]	-25.78636[a]	-23.58380[a]

Notes:
[a] Significant at the 1% level.

Table 7.12: One Unit Root Tests for Time-Varying Betas (GARCH-GJR)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
American Electric Power	-11.09419[a]	-10.57837[a]	-10.24660[a]	-10.17204[a]
Alaska Air Group	-9.78880[a]	-9.57471[a]	-9.46153[a]	-9.33801[a]
Bank of America	-10.89905[a]	-10.47855[a]	-10.35960[a]	-10.09687[a]
Boeing	-8.08053[a]	-7.90836[a]	-7.53419[a]	-7.17713[a]
California Water Service	-5.59348[a]	-5.65858[a]	-5.87015[a]	-5.74770[a]
Delta Air Lines	-8.67748[a]	-8.81424[a]	-8.64587[a]	-8.74225[a]
Ford Motor	-9.44248[a]	-9.37761[a]	-8.97726[a]	-8.83184[a]
General Electric	-10.92903[a]	-11.05275[a]	-11.08252[a]	-10.87733[a]
Honeywell International	-16.94048[a]	-15.86029[a]	-14.87735[a]	-13.67598[a]
Microsoft	-9.52946[a]	-9.25586[a]	-8.90228[a]	-8.65378[a]
MGP Ingredients	-11.57196[a]	-11.13891[a]	-10.25258[a]	-9.70711[a]
New York Times	-8.17438[a]	-8.13851[a]	-7.86325[a]	-7.99724[a]
Textron	-9.13242[a]	-9.25115[a]	-9.38833[a]	-9.02922[a]
Utah Medical Products	-10.30284[a]	-9.87786[a]	-9.32770[a]	-8.70084[a]
Walt Disney	-10.27636[a]	-10.12321[a]	-9.78253[a]	-9.46971[a]
Wells Fargo & Company	-7.96782[a]	-7.88461[a]	-7.59551[a]	-7.44224[a]
Wendy’ s International	-9.89850[a]	-9.48383[a]	-9.17425[a]	-8.67419[a]
Florida Gaming	-6.77065[a]	-6.47705[a]	-6.23173[a]	-6.19519[a]
Campbell Soup	-10.74374[a]	-10.33538[a]	-9.62079[a]	-9.29372[a]
Bell Industries	-9.35873[a]	-8.88330[a]	-8.60575[a]	-7.74299[a]

Notes:
[a] Significant at the 1% level.

Table 7.13 Part 1: GARCH-X Estimation Results

A. GARCH-X results, sample period 1989-2003

Parameter	Alaska Air Group	Boeing	California Water Service	General Electric	Honeywell International
$c_1(10^{-5})$	3.5168[a]	1.1218[a]	0.3667[a]	0.7843[a]	4.9583[a]
	(9.74072)	(8.92097)	(7.17876)	(7.83251)	(12.86243)
a_{11}	0.0683[a]	0.0425[a]	0.0357[a]	0.0530[a]	0.1363[a]
	(16.88814)	(14.92937)	(14.26374)	(12.73218)	(29.43805)
b_{11}	0.8773[a]	0.9367[a]	0.9594[a]	0.9174[a]	0.7935[a]
	(117.33376)	(240.36527)	(349.66021)	(158.51426)	(88.88251)
$d_1(10^{-5})$	5.6721[a]	-0.7351	-6.8192[b]	49.3300[a]	-6.8692[a]
	(2.95375)	(-0.79585)	(-2.26225)	(6.01686)	(-3.47532)
$c_3(10^{-5})$	0.6158[a]	0.6324[a]	0.4583[a]	0.6876[a]	1.8188[a]
	(9.64818)	(11.71998)	(10.90594)	(11.56862)	(13.82112)
a_{33}	0.0338[a]	0.0466[a]	0.0375[a]	0.0512[a]	0.0785[a]
	(13.12198)	(15.51126)	(18.02949)	(12.49889)	(14.44210)
b_{33}	0.9420[a]	0.9323[a]	0.9487[a]	0.9047[a]	0.8611[a]
	(205.24358)	(214.49189)	(342.62860)	(145.52626)	(96.96600)
$d_3(10^{-5})$	-0.8849[a]	-1.8108[a]	-8.3578[a]	40.3330[a]	-5.9048[a]
	(-5.18619)	(-4.27526)	(-6.93485)	(12.92739)	(-7.71418)
$c_2(10^{-5})$	0.6490[a]	0.5611[a]	0.2085[a]	0.6419[a]	2.3276[a]
	(6.89580)	(9.24601)	(7.66458)	(10.01682)	(12.32256)
a_{22}	0.0257[a]	0.0384[a]	0.0279[a]	0.0477[a]	0.0857[a]
	(9.15023)	(14.81305)	(16.88089)	(13.42087)	(18.93731)
b_{22}	0.9433[a]	0.9405[a]	0.9642[a]	0.9134[a]	0.8399[a]
	(154.28695)	(233.55667)	(459.57151)	(165.87337)	(90.78219)
$d_2(10^{-5})$	-0.4790	-1.4832[a]	-3.4115[b]	40.8550[a]	-7.4645[a]
	(-1.31534)	(-2.80976)	(-2.55525)	(8.39019)	(-5.84508)
L	27636.81	28681.69	28221.75	30489.29	28694.92

B. Test for higher order ARCH effect

u_t^2 / h_t					
L-B(12)[d]	60.7053**	14.8606	17.2601*	3.2308	6.2786
L-B(12)[e]	4.3309	2.7974	3.6392	3.4122	2.0801
$u_t / h_t^{1/2}$					
L-B(12)[d]	16.4510	30.5316**	4.2065	22.1016**	16.7255
L-B(12)[e]	32.6789**	28.9653**	35.2823**	29.9397**	35.9886**
L-B(12)[f]	4.2633	4.5151	13.6923	2.1499	1.0400

Notes: see Table 7.1 part 1.

Table 7.13 Part 2: GARCH-X Estimation Results

Parameter	MGP Ingredients	Textron	Utah Medical Products	Walt Disney	Florida Gaming
A. GARCH-X results, sample period 1989-2003					
$c_1(10^{-5})$	5.4545[a]	6.0330[a]	14.8280[a]	1.7780[a]	3.0972[a]
	(12.46999)	(11.90610)	(9.43817)	(10.05122)	(15.48536)
a_{11}	0.1096[a]	0.1107[a]	0.0934[a]	0.0653[a]	0.0964[a]
	(14.07446)	(14.66697)	(10.98713)	(15.81218)	(31.41848)
b_{11}	0.8373[a]	0.7879[a]	0.7550[a]	0.9026[a]	0.9096[a]
	(85.74988)	(60.81874)	(35.38805)	(173.05782)	(429.10232)
$d_1(10^{-5})$	-5.5644[a]	-6.0626[a]	-0.4852	-2.4504	2.8647[a]
	(-3.45956)	(-3.66524)	(-0.55357)	(-0.95424)	(4.42896)
$c_3(10^{-5})$	0.9383[a]	2.2571[a]	0.7805[a]	0.7483[a]	0.8593[a]
	(11.72210)	(11.02002)	(10.37094)	(11.89227)	(10.56672)
a_{33}	0.0602[a]	0.0774[a]	0.0497[a]	0.0483[a]	0.0521[a]
	(13.29938)	(11.95521)	(13.06171)	(15.48283)	(13.36672)
b_{33}	0.9031[a]	0.8354[a]	0.9185[a]	0.9229[a]	0.9141[a]
	(135.86102)	(63.91839)	(148.35692)	(183.50448)	(148.88598)
$d_3(10^{-5})$	-0.1489	-2.7732[a]	-0.1052	-1.3430[c]	-0.0832[a]
	(-0.57786)	(-5.25789)	(-1.54306)	(-1.80828)	(-3.64189)
$c_2(10^{-5})$	0.8805[a]	1.7422[a]	0.8034[a]	0.7725[a]	0.1493
	(7.74515)	(9.59056)	(7.44783)	(9.52875)	(1.55609)
a_{22}	0.0510[a]	0.0634[a]	0.0405[a]	0.0444[a]	0.0189[a]
	(7.70143)	(11.13941)	(8.92488)	(13.99428)	(3.73423)
b_{22}	0.8961[a]	0.8414[a]	0.9084[a]	0.9228[a]	0.9606[a]
	(79.23247)	(64.65778)	(116.07166)	(166.13486)	(111.23190)
$d_2(10^{-5})$	0.1766	-1.9154[a]	0.0781	-0.4691	0.0963
	(0.33289)	(-2.84022)	(0.37849)	(-0.39538)	(0.94821)
L	26979.22	28067.90	26628.19	28824.11	23099.99
B. Test for higher order ARCH effect					
u_t^2 / h_t					
L-B(12)[d]	8.0504	18.0147*	3.0876	18.6114*	14.6587
L-B(12)[e]	1.9689	2.0985	2.3456	2.4363	2.0509
$u_t / h_t^{1/2}$					
L-B(12)[d]	11.6810	22.7031**	12.2568	13.9465	16.5905
L-B(12)[e]	33.8852**	29.4301**	28.3701**	30.2990**	26.6546**
L-B(12)[f]	5.5519	12.3612	13.3759	4.2921	19.8884*

Table 7.14: Basic Statistics of Time-Varying Betas (GARCH-X)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
Alaska Air Group	0.93261[a]	0.019852	0.23733[a]	2.57556[a]	1117.70179[a]
Boeing	0.91925[a]	0.033093	-0.71157[a]	3.06134[a]	1857.25925[a]
California Water Service	0.68835[a]	0.049719	-0.07846[b]	0.06941	4.79820[c]
General Electric	1.08124[a]	0.014365	-2.43278[a]	20.81073[a]	74432.85918[a]
Honeywell International	1.02680[a]	0.032752	-0.41960[a]	6.24903[a]	6478.34392[a]
MGP Ingredients	0.67658[a]	0.053115	-0.21692[a]	1.88329[a]	608.64848[a]
Textron	0.73352[a]	0.029409	-0.89284[a]	4.56238[a]	3911.65380[a]
Utah Medical Products	0.73106[a]	0.050123	0.20977[a]	1.40061[a]	348.35925[a]
Walt Disney	0.96292[a]	0.025057	-1.48938[a]	6.13024[a]	7569.88116[a]
Florida Gaming	0.51605[a]	0.177556	-0.30476[a]	2.48191[a]	1064.34248[a]

Notes:
[a] Significant at the 1% level.

Table 7.15: Two Unit Root Tests for Time-Varying Betas (GARCH-X)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Alaska Air Group	-64.17808[a]	-32.72297[a]	-25.26423[a]	-22.10686[a]
Boeing	-64.35037[a]	-32.12156[a]	-26.43326[a]	-22.65956[a]
California Water Service	-63.68161[a]	-31.78961[a]	-24.01216[a]	-20.59847[a]
General Electric	-64.09794[a]	-33.00714[a]	-25.60039[a]	-22.35088[a]
Honeywell International	-65.46641[a]	-37.60988[a]	-29.55524[a]	-25.30645[a]
MGP Ingredients	-63.63748[a]	-33.87343[a]	-27.34399[a]	-23.61026[a]
Textron	-65.80282[a]	-35.10227[a]	-27.13639[a]	-24.64355[a]
Utah Medical Products	-62.38908[a]	-33.57640[a]	-27.10495[a]	-23.34290[a]
Walt Disney	-62.06603[a]	-33.22134[a]	-25.90515[a]	-22.09642[a]
Florida Gaming	-65.53509[a]	-32.96529[a]	-24.69639[a]	-21.20012[a]

Notes:
[a] Significant at the 1% level.

Table 7.16: One Unit Root Tests for Time-Varying Betas (GARCH-X)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Alaska Air Group	-9.92779[a]	-9.43237[a]	-8.92717[a]	-8.63007[a]
Boeing	-8.37615[a]	-8.13672[a]	-7.75092[a]	-7.39167[a]
California Water Service	-6.31392[a]	-6.09446[a]	-6.32185[a]	-6.24367[a]
General Electric	-11.02355[a]	-11.20623[a]	-11.13400[a]	-11.04100[a]
Honeywell International	-17.18277[a]	-15.99415[a]	-14.82142[a]	-13.65521[a]
MGP Ingredients	-12.07258[a]	-11.57731[a]	-10.58768[a]	-9.94612[a]
Textron	-14.61104[a]	-13.43821[a]	-12.74696[a]	-12.14129[a]
Utah Medical Products	-10.30234[a]	-9.87342[a]	-9.38649[a]	-8.78990[a]
Walt Disney	-10.01816[a]	-9.84902[a]	-9.49720[a]	-9.21127[a]
Florida Gaming	-7.60573[a]	-7.18930[a]	-6.87530[a]	-6.87653[a]

Notes:
[a] Significant at the 1% level,

Table 7.17: Akaike information criterion for four transition equations

Company	AR(1)	Random Coefficient	Random Walk	Random Walk with Drift
American Electric Power	-3.935309	FTC	FTC	FTC
Alaska Air Group	FTC	-4.755960	-4.739618	-4.739165
Bank of America	FTC	-5.409009	-5.398860	-5.398358
Boeing	FTC	-5.277007	-5.270474	-5.269964
California Water Service	FTC	-4.918926	-4.913546	-4.913221
Delta Air Lines	-4.726321	-4.722431	-4.630551	-4.630013
Ford Motor	FTC	-5.190634	-5.164373	-5.163862
General Electric	FTC	-6.133390	-6.120464	-6.119968
Honeywell International	FTC	-5.184187	-5.169518	FTC
Microsoft	FTC	-5.065884	-5.059987	-5.059476
MGP Ingredients	FTC	-4.344973	-4.327555	-4.327053
New York Times	FTC	-5.393762	-5.378658	-5.378154
Textron	FTC	-4.998677	-4.969529	-4.969036
Utah Medical Products	FTC	-4.289368	-4.289876	-4.289375
Walt Disney	FTC	-5.324524	-5.308372	-5.307864
Wells Fargo & Company	FTC	-5.578983	-5.580489	-5.579991
Wendy' s International	FTC	-5.057184	-5.055225	-5.054717
Florida Gaming	-2.115389	-2.115431	-2.112909	FTC
Campbell Soup	FTC	-5.381097	-5.375624	-5.375120
Bell Industries	FTC	FTC	FTC	FTC

Notes: FTC stands for 'failed to converge'

Table 7.18: Bayesian information criterion for four transition equations

Company	AR(1)	Random Coefficient	Random Walk	Random Walk with Drift
American Electric Power	-3.927293	FTC	FTC	FTC
Alaska Air Group	FTC	-4.749547	-4.734808	-4.732753
Bank of America	FTC	-5.402596	-5.394050	-5.391945
Boeing	FTC	-5.270594	-5.265665	-5.263551
California Water Service	FTC	-4.912513	-4.908736	-4.906808
Delta Air Lines	-4.718305	-4.716018	-4.625741	-4.623600
Ford Motor	FTC	-5.184221	-5.159563	-5.157449
General Electric	FTC	-6.126977	-6.115654	-6.113555
Honeywell International	FTC	-5.177775	-5.164708	FTC
Microsoft	FTC	-5.059471	-5.055177	-5.053063
MGP Ingredients	FTC	-4.338561	-4.322745	-4.320640
New York Times	FTC	-5.387349	-5.373848	-5.371741
Textron	FTC	-4.992264	-4.964720	-4.962623
Utah Medical Products	FTC	-4.282955	-4.285066	-4.282962
Walt Disney	FTC	-5.318111	-5.303563	-5.301451
Wells Fargo & Company	FTC	-5.572570	-5.575680	-5.573578
Wendy's International	FTC	-5.050771	-5.050415	-5.048304
Florida Gaming	-2.107373	-2.109019	-2.108100	FTC
Campbell Soup	FTC	-5.374684	-5.370814	-5.368707
Bell Industries	FTC	FTC	FTC	FTC

Notes: FTC stands for 'failed to converge'

Table 7.19: Basic Statistics of the Time-Varying Beta (Kalman Filter)

	Mean	Variance	Skewness	Kurtosis	Jarque-Bera
Alaska Air Group	0.94283[a]	0.012350	-1.95803[a]	27.41011[a]	124931.91495[a]
Bank of America	1.08694[a]	0.008543	-1.99988[a]	8.48343[a]	14334.90031[a]
Boeing	0.92501[a]	0.039440	-0.62322[a]	0.80521[a]	358.83333[a]
California Water Service	0.62710[a]	0.036337	3.42778[a]	43.40357[a]	314651.17441[a]
Delta Air Lines	0.98389[a]	0.056387	1.34932[a]	4.83195[a]	4991.47587[a]
Ford Motor	0.97931[a]	0.011894	-0.85077[a]	1.76395[a]	978.85203[a]
General Electric	1.09057[a]	0.006066	0.27595[a]	0.84941[a]	167.20900[a]
Honeywell International	1.00255[a]	0.017392	-9.83869[a]	170.5938[a]	4805550.9037[a]
Microsoft	1.15981[a]	0.014452	0.03713	4.12818[a]	2778.01634[a]
MGP Ingredients	0.61493[a]	0.087893	-0.20854[a]	-0.43474[a]	59.14597[a]
New York Times	0.87231[a]	0.011082	0.04792	0.21777[a]	9.22530[a]
Textron	0.71676[a]	0.053395	-0.48135[a]	1.06778[a]	336.82316[a]
Utah Medical Products	0.76559[a]	0.077163	-0.26129[a]	0.44811[a]	77.22437[a]
Walt Disney	0.96023[a]	0.021419	-1.11776[a]	1.94739[a]	1432.38378[a]
Wells Fargo & Company	1.03454[a]	0.020676	-0.47847[a]	-0.17321[b]	154.11530[a]
Wendy's International	0.85500[a]	0.023534	-0.45915[a]	-0.44348[a]	169.46631[a]
Florida Gaming	0.40695[a]	0.211605	-1.52571[a]	4.10243[a]	4259.91405[a]
Campbell Soup	0.91957[a]	0.035701	-0.73833[a]	2.23510[a]	1169.41840[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level.

Table 7.20: Two Unit Root Tests for Time-Varying Betas (Kalman Filter)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Alaska Air Group	-65.06232[a]	-36.34547[a]	-21.29950[a]	-25.47190[a]
Bank of America	-64.83096[a]	-55.37047[a]	-27.66165[a]	-20.02363[a]
Boeing	-62.52893[a]	-29.78040[a]	-25.33580[a]	-20.59674[a]
California Water Service	-43.46723[a]	-37.08232[a]	-35.55344[a]	-31.01574[a]
Delta Air Lines	-63.83730[a]	-28.62393[a]	-24.12167[a]	-20.42871[a]
Ford Motor	-61.72125[a]	-33.83145[a]	-28.57473[a]	-34.11569[a]
General Electric	-66.23196[a]	-26.53121[a]	-27.42713[a]	-23.77894[a]
Honeywell International	-52.67807[a]	-30.20493[a]	-35.17955[a]	-23.90612[a]
Microsoft	-79.77131[a]	-37.71651[a]	-30.33901[a]	-25.86052[a]
MGP Ingredients	-63.85520[a]	-31.65582[a]	-24.34152[a]	-20.33577[a]
New York Times	-69.76541[a]	-30.88743[a]	-27.07615[a]	-21.39248[a]
Textron	-60.57045[a]	-29.90690[a]	-22.10495[a]	-19.84622[a]
Utah Medical Products	-70.91623[a]	-33.86803[a]	-23.70577[a]	-19.90316[a]
Walt Disney	-69.09070[a]	-32.03412[a]	-23.72742[a]	-18.70295[a]
Wells Fargo & Company	-61.09267[a]	-32.03243[a]	-26.92588[a]	-20.55062[a]
Wendy's International	-70.78773[a]	-48.41408[a]	-25.76506[a]	-22.66734[a]
Florida Gaming	-65.81180[a]	-33.35541[a]	-29.62800[a]	-19.02537[a]
Campbell Soup	-62.93638[a]	-30.75616[a]	-25.52855[a]	-23.35924[a]

Notes:
[a] Significant at the 1% level,

Table 7.21: One Unit Root Tests for Time-Varying Betas (Kalman Filter)

	DF Test	ADF (lag=3)	ADF (lag=6)	ADF (lag=9)
Alaska Air Group	-13.17311[a]	-13.75422[a]	-6.58457[a]	-7.22200[a]
Bank of America	-9.04659[a]	-4.35858[a]	-5.56391[a]	-7.35153[a]
Boeing	-3.85269[a]	-3.47087[a]	-3.49728[a]	-3.24193[b]
California Water Service	-14.69589[a]	-16.27783[a]	-2.96296[b]	-4.00892[a]
Delta Air Lines	-4.27615[a]	-4.42614[a]	-4.27464[a]	-4.10278[a]
Ford Motor	-7.14144[a]	-6.95713[a]	-4.77150[a]	-2.95936[b]
General Electric	-6.63081[a]	-6.74336[a]	-4.41632[a]	-3.78780[a]
Honeywell International	-22.41955[a]	-31.43929[a]	-11.44242[a]	-5.92107[a]
Microsoft	-7.48123[a]	-6.30715[a]	-5.02902[a]	-3.73242[a]
MGP Ingredients	-5.61200[a]	-5.44851[a]	-5.35969[a]	-5.28108[a]
New York Times	-3.84741[a]	-3.61866[a]	-3.31987[b]	-3.23791[b]
Textron	-4.98946[a]	-5.44047[a]	-5.99588[a]	-5.69792[a]
Utah Medical Products	-4.00864[a]	-3.61201[a]	-2.85435[c]	-3.70794[a]
Walt Disney	-4.70385[a]	-4.18639[a]	-4.23154[a]	-4.40645[a]
Wells Fargo & Company	-3.28782[b]	-2.64059[c]	-3.68286[a]	-2.94582[b]
Wendy's International	-5.76259[a]	-3.02922[b]	-3.83125[a]	-3.00944[b]
Florida Gaming	-3.91218[a]	-3.36373[b]	-4.20052[a]	-3.71669[a]
Campbell Soup	-5.21541[a]	-5.69935[a]	-3.71422[a]	-3.44493[a]

Notes:
[a] Significant at the 1% level,
[b] Significant at the 5% level,
[c] Significant at the 10% level.

Table 7.22: Mean Value of Beta Estimates

Company	GARCH	BEKK	GJR	GARCH-X	Kalman	Market Model
American Electric Power	0.70815	0.70971	0.71043			0.71810
Alaska Air Group	0.91923	0.90801	0.91862	0.93261	0.94283	0.98961
Bank of America	1.08380	1.09189	1.08105		1.08694	1.07569
Boeing	0.91843	0.92919	0.91852	0.91925	0.92501	0.92379
California Water Service	0.67902	0.67933	0.67990	0.68835	0.62710	0.69743
Delta Air Lines	0.95399	0.87714	0.95811		0.98389	1.06874
Ford Motor	0.96423	0.97276	0.96430		0.97931	0.98240
General Electric	1.07990	1.08241	1.08004	1.08124	1.09057	1.08680
Honeywell International	1.03078	1.02180	1.03161	1.02680	1.00255	1.02974
Microsoft	1.12273	1.13541	1.12059		1.15981	1.15784
MGP Ingredients	0.68426	0.68850	0.68774	0.67658	0.61493	0.66808
New York Times	0.86927	0.87913	0.87121		0.87231	0.88605
Textron	0.73257	0.72997	0.73209	0.73352	0.71676	0.79555
Utah Medical Products	0.73143	0.73552	0.73047	0.73106	0.76559	0.76731
Walt Disney	0.96421	0.96018	0.96443	0.96292	0.96023	0.98880
Wells Fargo & Company	1.04317	1.05080	1.04312		1.03454	0.99740
Wendy' s International	0.86391	0.88671	0.86414		0.85500	0.83014
Florida Gaming	0.49007	0.54251	0.49191	0.51605	0.40695	0.65896
Campbell Soup	0.92211	0.92404	0.92241		0.91957	0.87767
Bell Industries	0.68871	0.67290	0.69276			0.73551

Table 7.23: Mean Absolute Errors of Beta Forecasts (2001)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0114549	0.0061983	0.0132769		FTC
Alaska Air Group	0.0081732	0.0452556	0.0090512	0.0050966	FTC
Bank of America	0.0057226	0.0098266	0.0069064		FTC
Boeing	0.0063129	0.0163370	0.0069825	0.0057040	0.0125528
California Water Service	0.0159632	0.0148527	0.0178135	0.0125523	0.0818971
Delta Air Lines	0.0073822	0.2004987	0.0092151		0.1203343
Ford Motor	0.0096858	0.0118509	0.0107032		0.0110366
General Electric	0.0047359	0.0141161	0.0045192	0.0052524	0.0430843
Honeywell International	0.0183954	0.0246273	0.0213298	0.0159763	0.0721535
Microsoft	0.0207025	0.0181382	0.0210099		0.0073821
MGP Ingredients	0.0074330	0.0058878	0.0067159	0.0124305	0.0882067
New York Times	0.0108924	0.0113915	0.0130229		0.0072941
Textron	0.0090061	0.0673790	0.0119800	0.0096023	0.0205746
Utah Medical Products	0.0573399	0.0171382	0.0662782	0.0465971	0.0199398
Walt Disney	0.0158912	0.0315806	0.0166437	0.0114986	0.0099578
Wells Fargo & Company	0.0041308	0.0071960	0.0041528		0.0058456
Wendy’s International	0.0105686	0.0090718	0.0112882		0.0364550
Florida Gaming	0.0225070	0.0366495	0.0294528	0.0223624	0.0082489
Campbell Soup	0.0067707	0.0151425	0.0092795		0.0410966
Bell Industries	0.0069898	0.0565774	0.0101755		FTC

Table 7.24: Mean Square Errors of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0004571	0.0001452	0.0005608		FTC
Alaska Air Group	0.0001781	0.0126565	0.0002830	0.0000689	FTC
Bank of America	0.0001054	0.0002589	0.0001617		FTC
Boeing	0.0000896	0.0008891	0.0001194	0.0000711	0.0002597
California Water Service	0.0005127	0.0006639	0.0004855	0.0003056	0.0073508
Delta Air Lines	0.0001101	0.2373996	0.0001644		0.0244012
Ford Motor	0.0001937	0.0004377	0.0002694		0.0001657
General Electric	0.0000429	0.0005526	0.0000340	0.0000422	0.0020327
Honeywell International	0.0005561	0.0027604	0.0007819	0.0003592	0.0077049
Microsoft	0.0010779	0.0009032	0.0013644		0.0000816
MGP Ingredients	0.0001834	0.0000814	0.0001649	0.0006610	0.0126378
New York Times	0.0004317	0.0004212	0.0007436		0.0000607
Textron	0.0001579	0.0157650	0.0003373	0.0001811	0.0006028
Utah Medical Products	0.0050859	0.0006557	0.0070496	0.0042464	0.0005413
Walt Disney	0.0006137	0.0046165	0.0006982	0.0002921	0.0001411
Wells Fargo & Company	0.0000432	0.0001335	0.0000441		0.0000569
Wendy's International	0.0004066	0.0002028	0.0006586		0.0017583
Florida Gaming	0.0009551	0.0051493	0.0016516	0.0009311	0.0000736
Campbell Soup	0.0001141	0.0006070	0.0002411		0.0019734
Bell Industries	0.0001508	0.0147710	0.0006298		FTC

Table 7.25: Mean Absolute Percentage Error of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0178603	0.0135199	0.0191030		FTC
Alaska Air Group	0.0097144	0.0442205	0.0105203	0.0064188	FTC
Bank of America	0.0053319	0.0089024	0.0064506		FTC
Boeing	0.0072840	0.0177571	0.0079879	0.0066076	0.0146364
California Water Service	0.0187242	0.0173618	0.0198580	0.0144785	0.1049492
Delta Air Lines	0.0075335	0.1942008	0.0088619		0.1054968
Ford Motor	0.0098190	0.0117540	0.0108216		0.0119542
General Electric	0.0040850	0.0120475	0.0039711	0.0045319	0.0377256
Honeywell International	0.0175176	0.0220678	0.0200874	0.0156730	0.0650934
Microsoft	0.0164710	0.0148639	0.0169505		0.0055975
MGP Ingredients	0.4692523	0.0441356	0.0273384	0.0316736	1.4172697
New York Times	0.0128340	0.0135029	0.0145213		0.0085891
Textron	0.0126444	0.0994425	0.0160533	0.0134171	0.0272358
Utah Medical Products	0.0946979	0.0331116	0.1048190	0.0775261	0.0291767
Walt Disney	0.0162643	0.0290902	0.0169166	0.0116780	0.0096569
Wells Fargo & Company	0.0043399	0.0079648	0.0043981		0.0069864
Wendy' s International	0.0137286	0.0122827	0.0136529		0.0524938
Florida Gaming	0.0257121	0.0418470	0.0331783	0.0263442	0.0098783
Campbell Soup	0.0094734	0.0207422	0.0125859		0.0701881
Bell Industries	0.0130686	0.1260424	0.0176287		FTC

Table 7.26: Theil U Statistics of Beta Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.03092	0.01636	0.03304		FTC
Alaska Air Group	0.01515	0.12340	0.01875	0.00947	FTC
Bank of America	0.00935	0.01465	0.01163		FTC
Boeing	0.01032	0.03253	0.01186	0.00926	0.01819
California Water Service	0.02537	0.02987	0.02348	0.01915	0.11103
Delta Air Lines	0.01088	0.44113	0.01266		0.14762
Ford Motor	0.01372	0.02074	0.01618		0.01394
General Electric	0.00553	0.02014	0.00498	0.00553	0.03922
Honeywell International	0.02168	0.04921	0.02570	0.01776	0.08177
Microsoft	0.02591	0.02479	0.02963		0.00697
MGP Ingredients	0.01991	0.01329	0.01848	0.04068	0.17502
New York Times	0.02382	0.02368	0.03068		0.00930
Textron	0.01765	0.19408	0.02544	0.01874	0.02995
Utah Medical Products	0.11212	0.04244	0.12959	0.09896	0.03325
Walt Disney	0.02518	0.06648	0.02671	0.01726	0.01138
Wells Fargo & Company	0.00669	0.01250	0.00674		0.00873
Wendy’ s International	0.02606	0.01880	0.03209		0.06007
Florida Gaming	0.03066	0.06561	0.04084	0.03134	0.01042
Campbell Soup	0.01311	0.03107	0.01878		0.06642
Bell Industries	0.02024	0.16271	0.04062		FTC

Table 7.27: Mean Absolute Errors of Beta Forecasts (2003)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0135051	0.1621260	0.0102300		FTC
Alaska Air Group	0.0018346	0.0673429	0.0048383	0.0062931	0.0014987
Bank of America	0.0027170	0.0046178	0.0042158		0.0022385
Boeing	0.0022034	0.0017112	0.0038228	0.0024380	0.0035482
California Water Service	0.0012655	0.0019956	0.0053304	0.0016572	0.0053111
Delta Air Lines	0.0023307	0.0023257	0.0052010		0.0050403
Ford Motor	0.0023617	0.0011124	0.0052930		0.0044322
General Electric	0.0015448	0.0017440	0.0042539	0.0024454	0.0005652
Honeywell International	0.0067493	0.0086550	0.0062070	0.0089397	0.0105284
Microsoft	0.0037982	0.0025013	0.0017318		0.0045589
MGP Ingredients	0.0031012	0.0083197	0.0085645	0.0032200	0.0250479
New York Times	0.0094462	0.0015398	0.0057377		0.0059748
Textron	0.0041235	0.0713150	0.0077192	0.0161117	0.0076053
Utah Medical Products	0.0104410	0.0167001	0.0137579	0.0088238	0.00997453
Walt Disney	0.0019640	0.0113815	0.0025646	0.0027263	0.00140959
Wells Fargo & Company	0.0023035	0.0020757	0.0029098		0.00283223
Wendy' s International	0.0026297	0.0007866	0.0033551		0.0192158
Florida Gaming	0.0210035	0.0032021	0.0071772	0.0078159	0.0000000
Campbell Soup	0.0038494	0.0091425	0.0103489		0.0066225
Bell Industries	0.0045804	0.2080471	0.0085532		FTC

Table 7.28: Mean Square Errors of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0003260	0.0370339	0.0002462		FTC
Alaska Air Group	0.0000039	0.0100289	0.0000517	0.0000971	0.0000026
Bank of America	0.0000412	0.0000579	0.0000510		0.0000095
Boeing	0.0000140	0.0000078	0.0000531	0.0000163	0.0000187
California Water Service	0.0000042	0.0000116	0.0000917	0.0000107	0.0000370
Delta Air Lines	0.0000101	0.0000097	0.0000805		0.0000504
Ford Motor	0.0000158	0.0000042	0.0000727		0.0000271
General Electric	0.0000034	0.0000076	0.0000266	0.0000092	0.0000005
Honeywell International	0.0000701	0.0001501	0.0000642	0.0001158	0.0001530
Microsoft	0.0000301	0.0000133	0.0000157		0.0000273
MGP Ingredients	0.0000231	0.0002293	0.0002306	0.0000244	0.0010381
New York Times	0.0001761	0.0000037	0.0000963		0.0000562
Textron	0.0000254	0.0071981	0.0001620	0.0005692	0.0000719
Utah Medical Products	0.0004445	0.0004287	0.0008056	0.0002390	0.0001412
Walt Disney	0.0000070	0.0002932	0.0000188	0.0000210	0.0000029
Wells Fargo & Company	0.0000110	0.0000094	0.0000307		0.0000138
Wendy' s International	0.0000211	0.0000015	0.0000484		0.0004879
Florida Gaming	0.0009510	0.0000318	0.0001572	0.0003230	0.0000000
Campbell Soup	0.0000449	0.0002679	0.0003138		0.0001686
Bell Industries	0.0000285	0.1025197	0.0001519		FTC

Table 7.29: Mean Absolute Percentage Error of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0219807	0.3899037	0.0133290		FTC
Alaska Air Group	0.0019398	0.0709014	0.0052727	0.0068085	0.0013766
Bank of America	0.0026507	0.0046286	0.0040811		0.0023874
Boeing	0.0022739	0.0017124	0.0039360	0.0025362	0.0034900
California Water Service	0.0012756	0.0020542	0.0054767	0.0016537	0.0056260
Delta Air Lines	0.0020234	0.0020668	0.0044243		0.0035088
Ford Motor	0.0025305	0.0011402	0.0056367		0.0043383
General Electric	0.0014529	0.0016623	0.0040083	0.0023113	0.0005379
Honeywell International	0.0064144	0.0082331	0.0058915	0.00858067	0.0094823
Microsoft	0.0034768	0.0023395	0.0016239		0.0041725
MGP Ingredients	0.1458090	0.0371281	0.0459422	0.0143231	0.1201738
New York Times	0.0104788	0.0016656	0.0061287		0.0063185
Textron	0.0047239	0.0837245	0.0092774	0.02063184	0.0082311
Utah Medical Products	0.0157807	0.0252816	0.0215639	0.01270126	0.01405206
Walt Disney	0.0019527	0.0109396	0.0025360	0.00270237	0.00130452
Wells Fargo & Company	0.0022984	0.0020967	0.0029257		0.0030139
Wendy' s International	0.0029199	0.0008985	0.0038339		0.0215638
Florida Gaming	0.0265938	0.0236762	0.0100226	0.0123207	0.0000000
Campbell Soup	0.0045870	0.0108369	0.0120339		0.0074614
Bell Industries	0.0072456	1.0127240	0.0140043		FTC

Table 7.30: Theil U Statistics of Beta Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.02302	0.22434	0.02050		FTC
Alaska Air Group	0.00209	0.10294	0.00767	0.01053	0.00147
Bank of America	0.00639	0.00769	0.00706		0.00323
Boeing	0.00377	0.00278	0.00732	0.00408	0.00417
California Water Service	0.00205	0.00355	0.00963	0.00322	0.00662
Delta Air Lines	0.00267	0.00288	0.00742		0.00487
Ford Motor	0.00380	0.00205	0.00813		0.00512
General Electric	0.00172	0.00262	0.00475	0.00283	0.00068
Honeywell International	0.00799	0.01174	0.00759	0.01031	0.01112
Microsoft	0.00501	0.00342	0.00375		0.00474
MGP Ingredients	0.00708	0.02099	0.02282	0.00737	0.04479
New York Times	0.01437	0.00210	0.01051		0.00789
Textron	0.00586	0.10055	0.01528	0.02921	0.00889
Utah Medical Products	0.03005	0.02972	0.04123	0.02148	0.0151
Walt Disney	0.00261	0.01620	0.00427	0.00453	0.00158
Wells Fargo & Company	0.00331	0.00312	0.00555		0.00399
Wendy' s International	0.00507	0.00137	0.00780		0.02530
Florida Gaming	0.03852	0.00652	0.01601	0.02392	0.00081
Campbell Soup	0.00730	0.01784	0.01940		0.01413
Bell Industries	0.00814	0.46409	0.01978		FTC

Table 7.31: Mean Absolute Errors of Beta Forecasts (2002-2003)

Company	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0049221	0.0325351	0.0134542		FTC
Alaska Air Group	0.0032439	0.0080361	0.0067074	0.0084173	0.0156271
Bánk of America	0.0051297	0.0046351	0.0072017		FTC
Boeing	0.0038354	0.0017070	0.0038682	0.0033862	0.0003587
California Water Service	0.0037374	0.0029827	0.0090201	0.0047161	0.0095590
Delta Air Lines	0.0029976	0.0095839	0.0063450		0.0620185
Ford Motor	0.0020486	0.0023700	0.0028238		0.0099274
General Electric	0.0031631	0.0030721	0.0038361	0.0090688	0.0151495
Honeywell International	0.0049568	0.0097760	0.0095772	0.0070743	0.0312579
Microsoft	0.0020356	0.0044338	0.0158344		0.0051450
MGP Ingredients	0.0058977	0.0025121	0.0052506	0.0114387	0.0872756
New York Times	0.0086859	0.0034261	0.0090516		0.0165100
Textron	0.0064506	0.0964272	0.0101912	0.0189814	0.0267186
Utah Medical Products	0.0178173	0.0236102	0.0188719	0.0828557	0.0108321
Walt Disney	0.0028648	0.0176854	0.0041754	0.0088219	0.0086016
Wells Fargo & Company	0.0050003	0.0010325	0.0066582		0.0027160
Wendy’ s International	0.0134649	0.0015462	0.0133595		0.0117755
Florida Gaming	0.0332696	0.0020745	0.0158837	0.0215533	0.0084188
Campbell Soup	0.0018824	0.0065397	0.0094263		0.0031958
Bell Industries	0.0040066	0.4029485	0.0078573		FTC

Table 7.32: Mean Square Errors of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0000665	0.0035878	0.0004671		FTC
Alaska Air Group	0.0000223	0.0001232	0.0001233	0.0001810	0.0003081
Bank of America	0.0000788	0.0000716	0.0001744		FTC
Boeing	0.0000366	0.0000082	0.0000414	0.0000316	0.0000002
California Water Service	0.0000288	0.0000292	0.0001740	0.0000358	0.0001332
Delta Air Lines	0.0000244	0.0002175	0.0001346		0.0074887
Ford Motor	0.0000125	0.0000182	0.0000323		0.0001335
General Electric	0.0000131	0.0000296	0.0000199	0.0001259	0.0004551
Honeywell International	0.0000379	0.0002045	0.0002473	0.0000686	0.0012085
Microsoft	0.0000096	0.0000469	0.0006400		0.0000366
MGP Ingredients	0.0001143	0.0000099	0.0001262	0.0003540	0.0120480
New York Times	0.0001596	0.0000268	0.0002769		0.0004358
Textron	0.0000985	0.0127067	0.0003413	0.0008112	0.0009769
Utah Medical Products	0.0010323	0.0009860	0.0011799	0.0108648	0.0001780
Walt Disney	0.0000137	0.0007718	0.0000411	0.0001951	0.0001218
Wells Fargo & Company	0.0000353	0.0000038	0.0000959		0.0000107
Wendy's International	0.0004279	0.0000075	0.0004213		0.0001988
Florida Gaming	0.0026160	0.0000218	0.0007609	0.0015644	0.0000963
Campbell Soup	0.0000088	0.0001212	0.0002750		0.0000137
Bell Industries	0.0000384	0.4226255	0.0001474		FTC

Table 7.33: Mean Absolute Percentage Error of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0102453	0.0643221	0.0230358		FTC
Alaska Air Group	0.0032824	0.0091644	0.0070544	0.0090789	0.0141026
Bank of America	0.0053524	0.0048993	0.0074761		FTC
Boeing	0.0041017	0.0018289	0.0041372	0.0036641	0.0003733
California Water Service	0.0044114	0.0035740	0.0101356	0.0054239	0.0117556
Delta Air Lines	0.0023862	0.0076715	0.0054178		0.0450188
Ford Motor	0.0021582	0.0023563	0.0029496		0.0097750
General Electric	0.0028617	0.0027623	0.0034569	0.0082117	0.0131893
Honeywell International	0.0045667	0.0089249	0.0087017	0.00653825	0.0282648
Microsoft	0.0017738	0.0039577	0.0140687		0.0044522
MGP Ingredients	0.0993652	0.0101163	0.0132356	0.02904754	0.5036266
New York Times	0.0097111	0.0036872	0.0098173		0.0175581
Textron	0.0076103	0.1146349	0.0127654	0.02500922	0.0300232
Utah Medical Products	0.0289573	0.0364945	0.0315248	0.13118904	0.0166160
Walt Disney	0.0026938	0.0164563	0.0039715	0.00853358	0.0077322
Wells Fargo & Company	0.0054132	0.0011399	0.0072319		0.0030885
Wendy's International	0.0179524	0.0020581	0.0180014		0.0151199
Florida Gaming	0.0808058	0.0184826	0.0604873	0.1150553	0.0107393
Campbell Soup	0.0023836	0.0079257	0.0112254		0.0038610
Bell Industries	0.0085251	2.1996320	0.0149035		FTC

Table 7.34: Theil U Statistics of Beta Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.01189	0.08114	0.03184		FTC
Alaska Air Group	0.00486	0.01289	0.01154	0.01411	0.01578
Bank of America	0.00916	0.00885	0.01363		FTC
Boeing	0.00631	0.00297	0.00671	0.00588	0.00043
California Water Service	0.00587	0.00614	0.01441	0.00649	0.01339
Delta Air Lines	0.00416	0.01317	0.00963		0.06238
Ford Motor	0.00337	0.00421	0.00541		0.01138
General Electric	0.00319	0.00487	0.00390	0.00993	0.01926
Honeywell International	0.00572	0.01327	0.01450	0.00771	0.03161
Microsoft	0.00271	0.00612	0.02171		0.00524
MGP Ingredients	0.01545	0.00431	0.01642	0.02762	0.14392
New York Times	0.01369	0.00563	0.01792		0.02202
Textron	0.01184	0.13565	0.02265	0.03568	0.03375
Utah Medical Products	0.04909	0.04843	0.05339	0.1577	0.01774
Walt Disney	0.00354	0.02560	0.00611	0.01329	0.01010
Wells Fargo & Company	0.00634	0.00211	0.01048		0.00365
Wendy’ s International	0.02500	0.00337	0.02514		0.01753
Florida Gaming	0.07270	0.00596	0.03931	0.05882	0.01205
Campbell Soup	0.00343	0.01275	0.01921		0.00434
Bell Industries	0.01008	0.89848	0.02074		FTC

Table 7.35: Mean Absolute Error of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0179078	0.0181441	0.0178918		FTC
Alaska Air Group	0.0180536	0.0184080	0.0180394	0.0180960	FTC
Bank of America	0.0125451	0.0124521	0.0125267		FTC
Boeing	0.0157908	0.0159254	0.0157863	0.0157777	0.0154208
California Water Service	0.0139358	0.0138328	0.0141001	0.0140909	0.0136291
Delta Air Lines	0.0203384	0.0224459	0.0201979		0.0202046
Ford Motor	0.0163554	0.0162980	0.0163512		0.0158017
General Electric	0.0120464	0.0119510	0.0119852	0.0120402	0.0119416
Honeywell International	0.0156802	0.0161957	0.0156629	0.0156935	0.0156889
Microsoft	0.0144302	0.0141939	0.0144192		0.01410611
MGP Ingredients	0.0176497	0.0176907	0.0176380	0.0175855	0.01695922
New York Times	0.0131883	0.0130087	0.0130361		0.01288186
Textron	0.0156356	0.0163112	0.0154397	0.01563583	0.01481794
Utah Medical Products	0.0213807	0.0213219	0.0211859	0.0213497	0.0208958
Walt Disney	0.0155179	0.0155470	0.0155109	0.0155775	0.0151742
Wells Fargo & Company	0.0106426	0.0105511	0.0106163		0.01027426
Wendy's International	0.0121062	0.0120742	0.0121442		0.0119400
Florida Gaming	0.0394641	0.0397400	0.0392890	0.0393699	0.0381095
Campbell Soup	0.0114857	0.0113491	0.0114323		0.0112891
Bell Industries	0.0235061	0.0258070	0.0232171		FTC

Table 7.36: Mean Square Error of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0007120	0.0007342	0.0007109		FTC
Alaska Air Group	0.0009063	0.0008800	0.0008876	0.0009164	FTC
Bank of America	0.0002769	0.0002748	0.0002758		FTC
Boeing	0.0005199	0.0005178	0.0005186	0.0005196	0.0004906
California Water Service	0.0003639	0.0003627	0.0003708	0.0003752	0.0003522
Delta Air Lines	0.0017399	0.0033129	0.0017138		0.0016952
Ford Motor	0.0005411	0.0005302	0.0005405		0.0005104
General Electric	0.0002830	0.0002822	0.0002813	0.0002832	0.0002799
Honeywell International	0.0005737	0.0006150	0.0005723	0.0005732	0.0005608
Microsoft	0.0003930	0.0003790	0.0003915		0.0003715
MGP Ingredients	0.0006840	0.0007174	0.0006917	0.0006903	0.0006299
New York Times	0.0003292	0.0003263	0.0003256		0.0003167
Textron	0.0006768	0.0006778	0.0006544	0.0006778	0.0005886
Utah Medical Products	0.0008531	0.0008429	0.0008377	0.0008454	0.0008137
Walt Disney	0.0005126	0.0005047	0.0005116	0.0005182	0.0004866
Wells Fargo & Company	0.0002041	0.0002035	0.0002026		0.0001937
Wendy' s International	0.0002652	0.0002683	0.0002628		0.0002593
Florida Gaming	0.0056703	0.0056365	0.0056603	0.0056787	0.0055663
Campbell Soup	0.0002619	0.0002613	0.0002581		0.0002507
Bell Industries	0.0011102	0.0014835	0.0010893		FTC

Table 7.37: Mean Errors of Return Forecasts (2001)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	-0.0004051	-0.0006754	-0.0005233		FTC
Alaska Air Group	0.0008805	0.0005688	0.0007850	0.0009238	FTC
Bank of America	0.0015359	0.0014015	0.0015500		FTC
Boeing	-0.0014245	-0.0014771	-0.0014426	-0.0013938	-0.0013575
California Water Service	0.0009751	0.0008709	0.0007765	0.0009013	0.0015124
Delta Air Lines	-0.0016381	-0.0039676	-0.0019065		-0.0014685
Ford Motor	-0.0010625	-0.0012000	-0.0010600		-0.0008558
General Electric	-0.0008089	-0.0007711	-0.0007688	-0.0008127	-0.0005665
Honeywell International	-0.0012271	-0.0012032	-0.0012463	-0.0011097	-0.0007478
Microsoft	0.0010074	0.0011058	0.0011094		0.00109302
MGP Ingredients	0.0030911	0.0030986	0.0030055	0.0033036	0.00306857
New York Times	0.0014367	0.0013081	0.0013659		0.00132731
Textron	0.0025595	0.0033199	0.0024923	0.00251181	0.00179286
Utah Medical Products	0.0044282	0.0046767	0.0043546	0.0046193	0.0037923
Walt Disney	-0.0006390	-0.0009923	-0.0006623	-0.0006477	-0.0009065
Wells Fargo & Company	-0.0002181	-0.0000905	-0.0002438		0.00001282
Wendy' s International	0.0018679	0.0018156	0.0017460		0.0020165
Florida Gaming	0.0026014	0.0019145	0.0025588	0.0026184	0.0022313
Campbell Soup	0.0008708	0.0008210	0.0008056		0.0011904
Bell Industries	0.0016143	0.0032969	0.0015175		FTC

Table 7.38: Mean Absolute Error of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0264975	0.0264269	0.0262572		FTC
Alaska Air Group	0.0161561	0.0162016	0.0161669	0.0161757	0.0159207
Bank of America	0.0059137	0.0058689	0.0059270		0.0057046
Boeing	0.0107117	0.0106185	0.0106979	0.0107258	0.0103341
California Water Service	0.0097717	0.0098446	0.0097584	0.0097743	0.0096984
Delta Air Lines	0.0247718	0.0254688	0.0247600		0.0240932
Ford Motor	0.0145989	0.0146121	0.0145898		0.0144595
General Electric	0.0073391	0.0073443	0.0074342	0.0073312	0.0071982
Honeywell International	0.0094757	0.0100312	0.0094994	0.00946761	0.00916139
Microsoft	0.0084255	0.0083898	0.0083686		0.008315
MGP Ingredients	0.0205940	0.0207288	0.0206291	0.0206071	0.01981913
New York Times	0.0072519	0.0072473	0.0071685		0.007094
Textron	0.0122314	0.0123825	0.0122217	0.01235249	0.01163594
Utah Medical Products	0.0134204	0.0135571	0.0134436	0.0133786	0.0130733
Walt Disney	0.0099433	0.0099296	0.0099454	0.0099607	0.00976433
Wells Fargo & Company	0.0054020	0.0053780	0.0054054		0.0053916
Wendy' s International	0.0101921	0.0101467	0.0102226		0.01017381
Florida Gaming	0.0310624	0.0317222	0.0313403	0.0313881	0.0310751
Campbell Soup	0.0080703	0.0079760	0.0080253		0.0080899
Bell Industries	0.0246520	0.0260652	0.0246527		FTC

Table 7.39: Mean Square Error of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0015801	0.0016052	0.0015624		FTC
Alaska Air Group	0.0004979	0.0004953	0.0004973	0.0004983	0.0004847
Bank of America	0.0001032	0.0001017	0.0001057		0.0000961
Boeing	0.0002008	0.0001964	0.0002002	0.0002017	0.0001870
California Water Service	0.0001779	0.0001831	0.0001796	0.0001796	0.0001696
Delta Air Lines	0.0014226	0.0014924	0.0014194		0.0013317
Ford Motor	0.0004411	0.0004345	0.0004411		0.0004215
General Electric	0.0000902	0.0000898	0.0000926	0.0000904	0.0000870
Honeywell International	0.0001539	0.0001691	0.0001539	0.0001542	0.0001447
Microsoft	0.0001503	0.0001507	0.0001502		0.0001464
MGP Ingredients	0.0010301	0.0010327	0.0010299	0.0010320	0.0009483
New York Times	0.0001034	0.0001033	0.0001022		0.0001002
Textron	0.0002969	0.0003010	0.0002966	0.0003019	0.0002724
Utah Medical Products	0.0003400	0.0003528	0.0003405	0.0003394	0.0003183
Walt Disney	0.0001945	0.0001909	0.0001948	0.0001948	0.0001837
Wells Fargo & Company	0.0000511	0.0000512	0.0000512		0.0000517
Wendy' s International	0.0002039	0.0002016	0.0002040		0.0002003
Florida Gaming	0.0035006	0.0035964	0.0035205	0.0035437	0.0034241
Campbell Soup	0.0001353	0.0001351	0.0001350		0.0001343
Bell Industries	0.0010833	0.0012198	0.0010897		FTC

Table 7.40: Mean Errors of Return Forecasts (2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	-0.0016182	-0.0022392	-0.0017238		FTC
Alaska Air Group	0.0000600	-0.0000066	0.0000800	0.0000653	-0.0002974
Bank of America	-0.0005712	-0.0005188	-0.0006164		-0.0002778
Boeing	0.0001610	0.0001018	0.0001711	0.0001764	0.0000091
California Water Service	-0.0002950	-0.0002686	-0.0002630	-0.0003240	-0.0001682
Delta Air Lines	-0.0016209	-0.0015401	-0.0016721		-0.0022664
Ford Motor	0.0014737	0.0013814	0.0014809		0.0012242
General Electric	-0.0000860	-0.0001216	-0.0000830	-0.0001009	-0.0001194
Honeywell International	0.0002479	-0.0000136	0.0002574	0.0002491	0.00015418
Microsoft	-0.0008091	-0.0007961	-0.0007688		-0.0008516
MGP Ingredients	0.0027728	0.0026589	0.0027491	0.0027437	0.0026065
New York Times	-0.0005452	-0.0005493	-0.0005413		-0.0005862
Textron	0.0012712	0.0012519	0.0013217	0.00128488	0.00104026
Utah Medical Products	0.0006709	0.0003813	0.0007226	0.0006474	0.0008565
Walt Disney	0.0006855	0.0003781	0.0006852	0.0006764	0.00039444
Wells Fargo & Company	-0.0001416	-0.0000934	-0.0001275		0.0000835
Wendy' s International	0.0007255	0.0007363	0.0007653		0.00085703
Florida Gaming	-0.0023277	-0.0024922	-0.0022761	-0.0020803	-0.0023206
Campbell Soup	-0.0002570	-0.0003394	-0.0002376		-0.0002125
Bell Industries	0.0016217	0.0019097	0.0016144		FTC

Table 7.41: Mean Absolute Error of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0249337	0.0252098	0.0248111		FTC
Alaska Air Group	0.0174149	0.0177442	0.0174146	0.0174200	0.0171147
Bank of America	0.0073808	0.0073699	0.0073762		FTC
Boeing	0.0127333	0.0126572	0.0127245	0.01274254	0.0123097
California Water Service	0.0110301	0.0110912	0.0110243	0.0110141	0.0109778
Delta Air Lines	0.0257587	0.0261014	0.0257318		0.0253199
Ford Motor	0.0165526	0.0165451	0.0165534		0.01633917
General Electric	0.0093267	0.0093416	0.0093467	0.0093193	0.0091947
Honeywell International	0.0128400	0.0131180	0.0128525	0.0128306	0.0123422
Microsoft	0.0103888	0.0103753	0.0104794		0.0102782
MGP Ingredients	0.0228170	0.0229773	0.0227840	0.0228070	0.02221745
New York Times	0.0084310	0.0083873	0.0083570		0.00820353
Textron	0.0134798	0.0136421	0.0134724	0.01362836	0.01285009
Utah Medical Products	0.0139857	0.0141556	0.0139920	0.0142242	0.0139379
Walt Disney	0.0123389	0.0123146	0.0123362	0.0123319	0.0120862
Wells Fargo & Company	0.0066166	0.0065747	0.0066200		0.00651427
Wendy' s International	0.0125063	0.0172271	0.0124875		0.0122906
Florida Gaming	0.0319132	0.0321153	0.0319751	0.0319677	0.0326706
Campbell Soup	0.0099294	0.0098934	0.0098927		0.0096850
Bell Industries	0.0239441	0.0248895	0.0239027		FTC

Table 7.42: Mean Square Error of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	0.0017906	0.0018591	0.0017730		FTC
Alaska Air Group	0.0006252	0.0006503	0.0006248	0.0006260	0.0006080
Bank of America	0.0001286	0.0001278	0.0001283		FTC
Boeing	0.0002967	0.0002949	0.0002963	0.0002975	0.0002766
California Water Service	0.0002229	0.0002254	0.0002239	0.0002231	0.0002174
Delta Air Lines	0.0014449	0.0014957	0.0014404		0.0013969
Ford Motor	0.0005825	0.0005774	0.0005829		0.0005599
General Electric	0.0001702	0.0001703	0.0001704	0.0001698	0.0001664
Honeywell International	0.0003840	0.0003945	0.0003835	0.0003838	0.0003637
Microsoft	0.0002131	0.0002134	0.0002142		0.0002081
MGP Ingredients	0.0011908	0.0012081	0.0011867	0.0011885	0.0011219
New York Times	0.0001329	0.0001309	0.0001312		0.0001261
Textron	0.0003524	0.0003581	0.0003523	0.0003570	0.0003270
Utah Medical Products	0.0003907	0.0004007	0.0003913	0.0004126	0.0003810
Walt Disney	0.0002937	0.0002891	0.0002939	0.0002933	0.0002775
Wells Fargo & Company	0.0000849	0.0000841	0.0000853		0.0000818
Wendy's International	0.0003018	0.0002968	0.0003003		0.0002915
Florida Gaming	0.0034129	0.0034084	0.0034003	0.0034069	0.0033753
Campbell Soup	0.0001954	0.0001963	0.0001948		0.0001857
Bell Industries	0.0010855	0.0011910	0.0010871		FTC

Table 7.43: Mean Errors of Return Forecasts (2002-2003)

	GARCH	BEKK	GJR	GARCH-X	Kalman
American Electric Power	-0.0014532	-0.0018411	-0.0014062		FTC
Alaska Air Group	-0.0001324	-0.0001615	-0.0001164	-0.0001271	-0.0002548
Bank of America	0.0003082	0.0003416	0.0003290		FTC
Boeing	0.0003146	0.0002579	0.0003184	0.00033215	0.0001564
California Water Service	0.0002316	0.0002082	0.0002577	0.0002099	0.0003351
Delta Air Lines	-0.0022338	-0.0024020	-0.0022785		-0.0025946
Ford Motor	0.0004181	0.0002576	0.0004221		0.00018816
General Electric	-0.0004592	-0.0005355	-0.0004507	-0.0004598	-0.0005471
Honeywell International	-0.0001207	-0.0003269	-0.0000994	-0.0001047	-0.0001345
Microsoft	-0.0003538	-0.0003524	-0.0003185		-0.0003799
MGP Ingredients	0.0010869	0.0009664	0.0010822	0.0010729	0.0012651
New York Times	0.0003579	0.0003065	0.0003489		0.00030484
Textron	0.0002899	0.0002443	0.0003248	0.00028919	0.00010941
Utah Medical Products	0.0016234	0.0014444	0.0016471	0.0016536	0.00173455
Walt Disney	0.0002938	0.0001737	0.0002950	0.0002992	0.0002364
Wells Fargo & Company	0.0005253	0.0005911	0.0005398		0.00072471
Wendy’ s International	0.0009232	0.0123518	0.0009279		0.0008941
Florida Gaming	0.0018966	0.0015555	0.0018900	0.0019134	0.0018348
Campbell Soup	0.0001096	0.0000091	0.0001169		0.0000170
Bell Industries	0.0009602	0.0009910	0.0009463		FTC

Table 7.44: Percentage of Dominance of Kalman Filter over Bivariate GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	62.50	75	72.22	55.56	88.24	82.35
Worse	0	0	0	0	0	5.88
Equal Accuracy	37.50	25	27.78	44.44	11.76	11.76

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.45: Percentage of Dominance of Kalman Filter over BEKK GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	37.50	37.50	72.22	66.67	88.24	76.47
Worse	0	0	0	0	0	5.88
Equal Accuracy	62.50	62.50	27.78	33.33	11.76	17.65

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.46: Percentage of Dominance of Kalman Filter over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	62.50	56.25	72.22	55.56	88.24	82.35
Worse	0	0	0	0	0	5.88
Equal Accuracy	37.50	43.75	27.78	44.44	11.76	11.76

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.47: Percentage of Dominance of Kalman Filter over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	77.78	77.78	80	70	80	80
Worse	0	0	0	0	0	10
Equal Accuracy	22.22	22.22	20	30	20	10

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.48: Percentage of Dominance of Bivariate GARCH over BEKK GARCH

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	10	15	15	20	35	35
Worse	0	5	5	0	20	15
Equal Accuracy	90	80	80	80	45	50

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.49: Percentage of Dominance of Bivariate GARCH over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	5	5	10	5	5
Worse	20	35	10	20	20	20
Equal Accuracy	80	60	85	70	75	75

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.50: Percentage of Dominance of Bivariate GARCH over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	15	10	15	10	15	10
Worse	0	5	0	5	0	0
Equal Accuracy	85	85	85	85	85	90

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.51: Percentage of Dominance of BEKK GARCH over GARCH-GJR

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	5	10	0	10	15
Worse	15	20	15	20	45	40
Equal Accuracy	85	75	75	80	45	45

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.52: Percentage of Dominance of BEKK GARCH over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	0	0	10	0	10	10
Worse	0	20	10	10	20	20
Equal Accuracy	100	80	80	90	70	70

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Table 7.53: Percentage of Dominance of GARCH-GJR over GARCH-X

Hypothesis	2001		2003		2002-2003	
	MSE	MAE	MSE	MAE	MSE	MAE
Better	30	20	30	20	30	30
Worse	0	0	10	20	0	0
Equal Accuracy	70	80	60	60	70	70

Note:
This table presents the proportion of firms that accept the three hypotheses. The statistic is the modified Diebold-Mariano test statistic, using MSE and MAE as the error criterion. Better means the former model dominate the later; while worse means the later model significantly outperform the former. Equal accuracy indicates no significant difference between forecast errors. The significance is defined as at least 10% significance level of *t* distribution.

Chapter 8
Conclusion

8.1 Major Findings

Beta stands for the systematic component of risk and is one of the most stylised measures of volatility. Forecasting the time-varying beta is an interesting and attractive task for both academic researchers and market practitioners. This study empirically tests the modelling ability of a collection of econometric techniques with an emphasis on their forecasting performance. These modelling techniques include standard bivariate GARCH, bivariate BEKK GARCH, bivariate GARCH-GJR, the bivariate GARCH-X and the Kalman filter approach. The study employs the four GARCH-type models and the Kalman filter method to model the time-varying beta. The data applied in the empirical tests include UK and US daily data ranging from January 1989 to December 2003.

The selection of candidate modelling techniques put an emphasis on GARCH models, since the superiority of Kalman filter method has been found in existing literature. Several refined GARCH specifications are considered to compete with Kalman filter. The BEKK model is an improvement to the standard GARCH, as the positive definiteness of the conditional variance matrix is guaranteed. GARCH-GJR allows for the asymmetry effect with two additional parameters incorporated in the model. The GARCH-X model allows for the effect of short term deviations between two cointegrated series, with the lagged error correction term incorporated in conditional variance and conditional covariance equations. The Kalman filter method can be used to capture the beta with a time-varying structure with a flexible transition equation. Comparison based on convergence rate and model selection criteria (AIC and BIC) suggest random walk is an appropriate characterisation of the time-varying beta.

8.1.1 Estimating Ability of Alternative Models

In generally, both UK and US results indicate similar evidence on the estimation performance of the alternative models. In particular, estimation results imply the success of GARCH models in capturing conditional variance and covariance; as the elemental GARCH coefficient estimates are all positive and significant in both UK and US results. Additionally, comparison among the beta estimates indicates that the mean values of conditional betas estimated by different models are highly correlated. Moreover, the mean values are fairly close to the point estimates of beta calculated by the market model, which indicates the reasonable capability of all models in the parameterisation of conditional systematic risk. However, the graphic comparison of beta estimates indicates there are both apparent similarities and differences among conditional betas estimated by different models. GARCH class models generally construct comparable beta series; while Kalman filter approach is less sensitive to time variation of systematic risk. The different estimation results indicate the distinct structure and algorithm of underlying models. GARCH models attempt to capture volatility clustering with a flexible framework; and result in a more flexible description of volatility clustering and a higher degree of time variation in GARCH betas. Moreover, as the name suggested, the recursive algorithm of Kalman filter implicitly filters noisy observations and thus generates smoother results when used to construct time-varying beta series.

The similarity of time-varying betas constructed by different models is also confirmed by some basic statistics of the beta series. In addition, the time-varying beta series generated by GARCH type models are all found to be stationary in levels. Four conditional betas generated by Kalman filter is found to be nonstationary in UK results, implying that conditional betas estimated by Kalman filter exhibit some different characteristics of dynamic structure from betas estimated by GARCH class models.

8.1.2 Forecasting Ability of Alternative Models

To avoid the sample effect and the overlapping issue, three out-of-sample forecast horizons are considered, including two one-year forecast horizons (2001 and 2003) and a two-year forecast horizon (2002 to 2003). To conduct the out-of-sample forecasting, each model is employed to estimate three shorter periods (1989 to 2000, 1989 to 2001 and 1989 to 2002) and accordingly predict the time-varying beta in three forecast samples (2001, 2003 and 2002 to 2003) with coefficient estimates.

Various methods are considered to evaluate forecasting performance of alternative techniques. First, visual inspection on the graphs of the forecasted and the actual beta provides an intuitive perception of forecast accuracy of alternative models. In both UK and US results, the graphical comparison favours GARCH class model, as fewer deviations can be found between the graphs of actual betas and forecasted betas compared to the Kalman filter approach.

Second, a variety of measures including MSE, MAE, MAPE and Theil U statistics are utilised to assess the level of forecast errors of alternative models. The bivariate GARCH and BEKK are found to be superior to other forecasting models in terms of beta forecasts. In UK empirical results, BEKK outperforms other models in 2001; and bivariate GARCH is the most successful model in 2003. In US results, bivariate GARCH is the best model with consistently accurate forecasts in 2001 and 2002 to 2003; while BEKK is the best forecasting technique in 2003. Kalman filter exhibit prominent performance in 2002 to 2003 among UK results.

Third, following Brooks *et al.* (1998) accuracy of beta forecasts can be investigated by comparing the actual returns with the out-of-sample returns which is directly calculated by the conditional CAPM using out-of-sample forecasts of conditional betas. Measures of forecast errors including both MSE and MAE overwhelmingly support the Kalman filter approach in all out-of-sample periods with both data sets.

The last comparison technique used is modified Diebold-Mariano test, which is a test of equal forecast accuracy designed to detect whether two sets of forecast errors have significantly different mean value. Evidence from both UK and US data shows that Kalman filter is the most accurate forecasting model in terms of return forecasts, which implies the advantage of Kalman filter being directly built upon the market model. Ranking of the GARCH type models is different with UK and US data. However in both UK and US results, evidence of equal accuracy among GARCH models is found to be common with most firms indicating equal accuracy. However GARCH-GJR is slightly superior to the other GARCH specifications in both stock markets, suggesting the important explanatory power of leverage effect on beta.

8.2 Implication and Suggestion

As CAPM betas are widely used by market participators and academic researchers for a variety of purposes, this thesis may be helpful for those who use the beta for their decision making or research development. Based on the empirical results, different models can be applied for different purposes, as their modelling ability varies according to different criteria.

GARCH models are found to be more successful in estimating the time-varying beta than the Kalman filter method. Although both GARCH and Kalman filter methods are capable parameterisations of systematic risk, Kalman Filter seems to be less competent in capturing the time variation of systematic risk. The recursive algorithm of Kalman filter has a smoothing and filtering property and result in a less prompt response to changes of beta. Bivariate GARCH is an ideal model to construct the dynamic process of conditional betas in a normal market circumstances. Taken the leverage effect into account, GJR is always an excellent model to estimate the conditional beta as it consistently produces fairly standard estimation results in different market circumstances.

Different modelling techniques may be recommended to forecast the time-varying beta for different reasons[36]. If the purpose of forecasting time-varying beta is not directly associated with the calculation of expected returns, GARCH models, especially bivariate GARCH and BEKK are more appropriate choices, since they produces moderately accurate and consistent out-of-sample forecasts of systematic risk. If the forecasted beta is used to provide information of expected returns, Kalman filter is a better choice than GARCH models, since it is considerably superior to GARCH models in terms of return forecasts. However, the Kalman filter may encounter the difficulty of converge in some cases, where GARCH-GJR is an appropriate replacement as the GJR model provides somewhat more accurate forecasts than the other GARCH specifications.

The success of GARCH type models in forecasting the time-varying beta also implies

[36] The recommendation is also considered upon the empirical results of UK and US weekly data, which is similar to daily results but not reported in the thesis to save space.

their competence in forecasting conditional second movement, which is crucial in a wide range of decision-making processes involving information of variance and covariance, such as derivative pricing and risk management.

Results presented in this thesis advocate further research in this field, applying different markets, time periods, data frequency and modelling techniques. Although research has been conducted in Australian market (see Brooks *et al.*, 1998 for example), evidence from other Pacific Basin markets or other non-UK/US markets may add to the accumulated evidence to date. There are potential insights to be gained from examining markets with different institutional features. Similarly, other time periods such as those before the international stock market crash of 1987 and the Asian financial crisis of 1997 may also provide an opportunity to complement empirical evidence, since the stock market suffered through the early 2000s due to a number of major events.

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