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FACULTY OF LAW, ARTS & SOCIAL SCIENCES

School of Social Sciences

Three Essays in New Open Economy Macroeconomics with Multiple Countries, Non-tradable Goods, Capital Accumulation and Distribution Sector

by

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ABSTRACT

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THREE ESSAYS IN NEW OPEN ECONOMY MACROECONOMICS WITH MULTIPLE COUNTRIES, NON-TRADABLE GOODS, CAPITAL ACCUMULATION AND DISTRIBUTION SECTOR

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This thesis aims at developing and applying the New Open Economy Macroeconomics (NOEM) models to the open economy issues.

Chapter 1 is a non-technical introduction of my research.

In Chapter 2, Do non-tradable sector and capital stock matter in New Open Economy Macroeconomics models? I evaluate the effects of adding non-tradable sector and capital stock on improving NOEM models' ability to capture the features of the data. I first construct and estimate a benchmark model with tradable, non-tradable sectors and capital stock with Bayesian techniques. The benchmark model is then compared with other two models: the model without non-tradable sector and the model without capital. The model comparison shows that the two models with capital outperform the model without it. Meanwhile, the performance of the benchmark model is only slightly better than the model without non-tradable sector. The results state that, the capital stock has important effects in improving the NOEM models performance, but the effects of having the non-tradable sector are small.

In Chapter 3, What determines the pound-euro real exchange rate fluctuations? I study the nature of the shocks that drive the pound-euro real exchange rate movements. Theoretically, if the real shocks dominate monetary shocks in variance decomposition of real exchange rates, then it could inform policy makers that a flexible exchange rate regime is preferable to a fixed exchange rate regime. Therefore, a decomposition of the pound-euro real exchange rate fluctuations is an important criterion on the issue whether the U.K. should join the Euro area. The result shows that real shocks are the predominant deliver of both nominal and real exchange rate volatilities for the U.K.-Euro area. This supports the argument that the U.K. should not join the Euro area.

In Chapter 4, The welfare effects of various exchange rate regimes in a three-country New Open Economy Macroeconomics model, I compare the welfare effects of the monetary and technology shocks under floating exchange rate regime, fixed exchange rate regime, basket peg regime and currency union in a three-country NOEM model. The results show that the welfare depends not only upon the choice of exchange rate regime but also upon the nature of shocks. The three-country framework is superior to the traditional two-country framework as the welfare effects of the shocks in the third country play an important role in the comparison of various exchange rate regimes. With only two-country framework, the effects of the third country cannot be exhibited, and the results might be incomplete.

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Chapter 1

Introduction

In this thesis, I aim to develop and apply the New Open Economy Macroeconomics (NOEM) framework to the following issues: 1) do non-tradable goods and capital accumulation have an important effects in improving the NOEM models' ability to capture the features of the data; 2) the relative importance of real and nominal shocks on determining the pound-euro real exchange rate movements; and 3) the welfare effects of the monetary and technology shocks under various exchange rate regimes in a three-country NOEM model.

The NOEM refers to a growing body of literatures which addresses open economy issues in a microfounded dynamic general equilibrium version with nominal rigidities and market imperfections. It has a number of advantages over the classic Mundell-Fleming-Dornbusch model. The nominal rigidities and market imperfections provide an explicit justification for the Keynesian assumption of short-run demand-determined output. The presentation of utility maximising agents allows the researchers to conduct to normative analysis, thereby evaluating the welfare implications of policy decisions in the context of the model.

The first paper, "Do non-tradable sector and capital stock matter in New Open Economy Macroeconomics models?" explores whether including non-tradable sector or capital stock can help improving NOEM models' ability to fit the features of macroeconomic data, thereby guiding the future NOEM modelling strategies. This is motivated by the fact that many NOEM models in the literature choose to subtract the non-tradable goods or capital from the model for simplicity, although both factors are actually important components of the economy.[1]

The reason that most NOEM models do not include both non-tradable sector and capital simultaneously is related with the classic Blanchard-Kahn condition in solving DSGE models. This condition states that the number of eigenvalues of the linearised system that are larger than 1 in modulus must equal to the number of forward-looking variables in order to have a unique stable equilibrium in the neighbourhood of the steady state. In the experiments I did, I found that this condition does not hold with the presence of capital in the production functions of both tradable and non-tradable sectors. However, by assuming the capital stock is only used in the tradable sector, the Blanchard-Kahn condition does hold. Given that the non-tradable sector is more labour intensive than the tradable sector, this assumption is plausible.[2] Of course, this non-existence or indeterminacy problem in DSGE can also be solved with different modelling approach.

In this paper, I estimate a small-scale two country NOEM model with tradable, non-tradable sectors and capital (the TNT model). The benchmark model is then compared with another two models: the model with only tradable sector (the T model), and the model with both tradable and non-tradable sectors, but without capital (the NK model).

The results show that the TNT and T model with capital outperform the NK model which does not include capital. The posterior odds ratio between the TNT model and the NK model (in favour of the TNT model) is 8.01; and the ratio between the T model and NK model (in favour of the T model) is 6.17. This means that a prior that favours the NK model over the TNT or T model by a factor of 8.01 or 6.17 is needed in order to accept it after observing the data. This shows that including capital accumulation with adjustment costs do improve the model's ability to fit the data.

The model comparison between the TNT and T model is not that obvious: the TNT model does a better job than the T model, but only slightly. This result is consistent with Engel (1999) and Chari, Kehoe and McGrattan (2002), which state that

[1]For example, see Chari, Kehoe and McGrattan (2002), Lubik and Schorfheide (2005), and Bergin (2006).

[2]For example, Rebelo and Végh (1995) also assume that the production of non-tradable goods does not require capital in order to 'improve the model's implications for the behaviour of the real exchange rate'.

adding non-tradable sector has small effects in improving the model's ability to explain the data. However, this might be because sectoral price index data and sectoral productivity shock are not used in the model construction and estimation. Nevertheless, different data set will be needed in estimation process if these two changes are implemented, which will make the model comparison infeasible.

The second paper, "What determines the pound-euro real exchange rate fluctuations?" studies the contributions of the real and nominal shocks that driving the dynamics of the pound-euro real exchange rates. The choice of the optimal exchange rate regime is a long-standing question in international economics. In a series paper, Mundell (1960, 1961a, 1961b, 1963) argue in favour of the floating exchange rate regime, as the pass-through of exchange rates to import prices generates an expenditure-switching effect between home and foreign goods and lends a stabilisation role to exchange rates in the face of country-specific real shocks. However, Corsetti and Pesenti (2001) and Devereux and Engel (2003) argue that fixed exchange rate regime is preferable in absence of the expenditure-switching effects, as fixed exchange rate help reducing real exchange rate volatilities when prices are preset and exporters price on local currencies. Taking both arguments into consideration, Devereux and Engel (2006) conclude that the choice of exchange rate policy is a trade-off between the desire to smooth fluctuations in real exchange rate in order to achieve smaller cross-country deviations in consumer prices, and the need to allow flexibility in the nominal exchange rate so as to facilitate terms of trade adjustment.

Theoretically, the fixed exchange rate can help reducing the real exchange rate volatilities, while the floating exchange rate regime provides the expenditure-switching effects. Therefore, if the real shocks dominate monetary shocks in driving the movements of real exchange rates, the fixed exchange rate regime would has only limited effects on real exchange rate stabilisation. In this case, the floating exchange rate regime would be more preferable as it provides the expenditure-switching effects. Consequently, if the real shocks dominate monetary shocks, then it could inform policy makers that a flexible exchange rate system is preferable to fixed rate regime, or at least would highlight an important potential drawback associated with maintaining a system of fixed exchange rates. Thus, what determines the pound-euro real exchange rate fluctuations has become an important question since the U.K. government is currently considering about whether the U.K. should join the Euro area.

On the other hand, while the dollar-euro real exchange rates have been studied intensively in NOEM literatures, there are few papers that address on the fluctuations of pound-euro real exchange rates. Therefore, in this paper, I construct and estimate a NOEM model using data for the U.K. and Euro area. An important feature of this model is the presence of distribution sector. The distribution cost accounts for the differences between producer prices and consumer prices, which leads to the deviation from law of one price[3]. Moreover, I assume that the non-tradable goods are the only input in the distribution sector, which delivers tradable goods for consumption and investment. This model is different from traditional NOEM models by assuming that the non-tradable goods are only used in the distribution sector, but not in the final consumption and investment. This setting simplifies the model by assuming that the non-tradable goods do not enter the consumption index, without sacrificing the model's ability to capture real exchange rate volatilities. This is because the prices of non-tradable goods can still affect the consumer price index through distribution cost. Therefore, the movements in the relative prices of non-tradable to tradable goods can still account for the fluctuations in the real exchange rates.

The results show that the real shocks are much more important than nominal shock in capturing the fluctuations in real exchange rates. The variance decomposition of real exchange rates shows that the productivity shocks in aggregate explain around 50.2% of the real exchange rate variations, while the monetary policy shock can only explain 0.9%. The historical decomposition of real exchange rates also shows that the contributions of the real shocks are more important than that of monetary policy shock. Therefore, a flexible exchange rate system is preferable to fixed rates or currency union for the U.K. and Euro area economy. This result does not lend the support to the argument that the U.K. should join the Euro area.

In the third paper, "The welfare effects of various exchange rate regimes in a three-country New Open Economy Macroeconomics model", I extend the two-country framework commonly used in the NOEM models to a three-country framework. There are some advantages to the three-country framework. First, although a fixed exchange rate

[3] Firms' local currency pricing behaviour also leads to deviation from law of one price in this model.

regime country can peg its currency to the anchor country[4], its exchange rate still floats against other floating exchange rate regime countries. Therefore, the fixed exchange rate regime is actually a partial fix (to the anchor country) and a partial flexible (to other floating countries) regime. Then, the advantages and disadvantages of the fixed exchange rate regime compared to the floating exchange rate regime can be very different when the third country is taken into consideration. In other words, shocks in these floating countries can affect the economic status and welfare of the fixed country. And if these effects are big enough, it can have very important welfare effects on the fixed country, which may even change the benefits and costs of adopting the fixed exchange rate regime. Thus, to fully understand the effects of the fixed exchange rate regime in this context and compare the welfare effects between fixed and floating regimes, a three-country framework would be preferable. Second, the three-country framework is a useful model setting to study the welfare effects of the basket-peg exchange rate regime. As the basket peg regime country chooses to fix its currency to a basket of foreign currencies, a two-country framework is inadequate to model this situation. Third, the three-country framework also lends a lever to study the welfare effects of currency union, where the monetary policy and the welfare in the union are not only affected by the shocks in the monetary union, but also the shocks in other countries.

The three-country model is an extension of the classic Obstfeld and Rogoff (1995a) exchange rate dynamics redux model. In order to study the welfare effects of different exchange rate regimes, I assume that domestic monetary policy is endogenously determined by different monetary policy rules. The monetary policy rules, on the other hand, are implied by the choice of exchange rate regimes. This is different from Obstfeld and Rogoff (1995a), who simply treat the monetary policy as exogenous.

The results show that household welfare depends not only upon the choice of the exchange rate regime, but also upon the nature of the technology shocks and the foreign monetary shocks. A fixed exchange rate regime country is most exposed to the monetary shocks of its anchor country, and the least to that of the third country. This result implies that the fixed exchange regime provides some insulation from the monetary shocks in the third country. Therefore, the fixed exchange rate regime is preferable

[4]In this paper, country A is called the anchor of country B, if country B chooses to fix its currency to the currency of country A.

than the floating regime when the monetary shocks are less volatile in the anchor country and more volatile in the third country. On the other hand, the fixed exchange rate regime helps to reduce the fixed country's exposure to its anchor country's technology shock, although at the cost of increasing the exposure to the technology shocks of the third country. With only two-country framework, the welfare effects of the shocks in the third country cannot be shown, thus the policy implications under this framework might be inappropriate.

It is also worth noting that the basket peg regime country always exposes more to the foreign monetary shocks than the floating regime country, while exposes less to the foreign technology shocks. In the currency union case, a country benefits the most from its union partner country's technology improvement compare to other regimes, and of course, will suffer the most if the technology shocks are negative. However, although exposing the most to the technology shocks in its partner country, the currency union country exposes the least to the technology shocks of other floating countries. Again, the three-country framework is preferable to fully study the welfare effects of currency union regime.

This paper shows that the three-country framework is preferable than the traditional two country framework as the effects of the shocks in the third country cannot be shown in the two-country model. Given that the shocks in the third country have important effects on the welfare of the country of interest, the result implied by the two-country framework can be very incomplete.

Chapter 2

Do Non-Tradable Sector and Capital Stock Matter in New Open Economy Macroeconomics Models?

Abstract

How to improve the model's ability to capture the features of the data is still a problem for New Open Economy Macroeconomics. This paper evaluates the effects of having non-tradable sector and capital stock by how much they can improve the NOEM model's performance. I first construct and estimate a benchmark model with tradable, non-tradable sectors and capital stock using Bayesian techniques. The model is different from traditional NOEM models by including non-tradable goods and capital stock simultaneously. This benchmark model is then compared with other two models: the model without non-tradable sector and the model without capital. The model comparison shows that the models with capital outperform the model without it. Meanwhile, the performance of the model with non-tradable sector is only slightly better than the model without it. The results state that, the capital stock has important effects in improving the NOEM models performance, while the effects of having the non-tradable sector are small.

Key Words: New Open Economy Macroeconomics, Dynamic Stochastic General Equilibrium, Bayesian estimation, Real exchange rate, Non-tradable sector, Capital accumulation, Model comparison

JEL Classification: C11, F31, F41

2.1 Introduction

A central goal of international macroeconomics is the explanation of real exchange rate[1] volatility and persistence. Recent theoretical development allows us to consider the real exchange rate within the rich framework of the New Open Economy Macroeconomics (NOEM). Unlike the traditional open economy approach, the NOEM model attempts to address open economy issues in a micro-founded dynamic general equilibrium version, which allows us to write more exacting models and analyse the welfare implications of policy decisions within the context of the model.

However, beyond its advantages, the NOEM model is still far from a complete success[2]. Recent studies in NOEM focus on how to improve the model's ability to fit the features of the data, especially its ability to capture the volatility and persistence of the real exchange rates in the data. There is a wide debate about the different modelling approach, for example, the choice between local currency pricing (LCP) or producer currency pricing (PCP), complete asset market or incomplete asset market, deviations from the uncovered interest rate parity or not. In this paper, I study the effects of including the non-tradable goods and capital stock in the NOEM model. This is motivated by the fact that many NOEM models choose to sacrifice the non-tradable goods or capital accumulation from the model for simplicity, although both factors are actually important components of the economy.[3]

There is a wide debate about the role of the non-tradable sector in determining the fluctuations of the real exchange rates. Traditional open economy theory classifies all goods as either tradable or non-tradable. Tradable goods are normally assumed to be internationally exchangeable at zero or negligible cost, while non-tradable goods cannot be exchanged in this manner. Theoretically, movements in the real exchange rates can arise from two sources: 1) deviations from the law of one price for tradable

[1] The real exchange rate is defined as the relative cost of the common reference basket of goods across countries.

[2] For example, Bergin (2006) estimates a NOEM model with maximum likelihood algorithm. His model beats a random walk model in in-sample predictions for some variables. But its performance is not as good as a standard unidentified vector autoregression (VAR) model. Lubik and Schorfheide (2005) estimate a small-scale two country NOEM model with Bayesian methods using data for the U.S. and Euro area. They find that, for a tight prior distribution, the Bayesian VAR model clearly dominates all specifications of DSGE models.

[3] For example, see Chari, Kehoe and McGrattan (2002), Lubik and Schorfheide (2005), and Bergin (2006).

goods across countries[4]; 2) movements in the relative prices of non-tradable to tradable goods across countries.

Under the classic Harrod-Balassa-Samuelson real exchange rate theory, tradable goods are assumed to obey the law of one price. Therefore, all volatilities in real exchange rates are attributable to the second factor. However, as the law of one price does not hold empirically[5], the effects of the first factor cannot be ignored. Obstfeld (2001) states that 'international divergences in the relative consumer price of tradables are so huge that the theoretical distinction between supposedly arbitraged tradables prices and completely sheltered non-tradables prices offers little or no help in understanding U.S. real exchange rate movements, even at long horizons.'

Therefore, previous empirical researches normally emphasised the first factor, and treated the second factor as unimportant. For example, Rogers and Jenkins (1995) empirically assess the relative importance of the two sources in real exchange rate movements using disaggregated price data from 11 OECD nations. They find the presence of non-tradable goods play a role, but the effect is 'limited in scope, at least in our span of data.' Engel (1999) measures the proportion of the U.S. real exchange rate movements that can be accounted for by movements in the relative prices of non-tradable goods with data horizons from one month to up to 30 years. He shows that relative prices of non-tradable goods appear to account for almost none of the movements of the U.S. real exchange rates. Chari, Kehoe and McGrattan (2002) also show that fluctuations in the relative prices of non-tradable goods account for only about 2% of the fluctuations of real exchange rates in variance decomposition of the U.S. bilateral real exchange rates with a number of OECD countries. Therefore they choose to subtract non-tradable goods from the model and focus on fluctuations in the real exchange rates which arise solely from deviations from the law of one price for tradable goods.

However, recent empirical developments show that the relative importance of the two factors in the real exchange rate volatilities depends crucially on the choice of price series used to measure relative prices and on the choice of trade partner. These findings have re-emphasised an important role for the non-tradable sector in generating the real

[4] The first source also includes the movements in the real exchange rates caused by the different preference on tradable goods across countries. Therefore, it can be understand as the deviation to the law of one price for tradable good consumption composite across countries.

[5] For example, see Crucini, Telmer and Zachariadis (2001).

exchange rate fluctuations.

Betts and Kehoe (2006) decompose the variance of real exchange rates between the U.S. and five of its trading partners: Canada, Germany, Japan, Korea and Mexico. They find large differences in the relationships between the U.S. bilateral real exchange rates and the bilateral relative prices of non-tradable goods across alternative price measures and across alternative trade partners. For some bilateral trade relationships and some measures of the relative prices of non-tradable goods, fluctuations in the relative prices constitute a large fraction of the fluctuations in the real exchange rates. Burstein, Eichenbaum and Rebelo (2005) construct the relative prices of tradable goods across countries using a weighted average of import and export price indices. They analyse the source of real exchange rate fluctuations at cyclical frequencies and find that more than half of these fluctuations are accounted for by movements in the prices of non-tradable goods relative to the prices of tradable goods. Their findings suggest that equal attention should be paid to modelling movements in the relative price of non-tradable to tradable goods.

The debates bring a natural question to the NOEM modelling strategy: whether we should having non-tradable goods in the NOEM model to improving the models performance or ignoring it from the model for simplicity and tractability reasons as the role of non-tradable goods in real exchange rate determination is unimportant. However, this problem has yet not been assessed in a NOEM framework. Therefore, in this paper, I try to contribute to this question with a small-scale two country model following the NOEM paradigm. In particular, I compare the performance of two models: a model including both tradable and non-tradable sectors (the TNT model), and the model with only tradable sector (the T model).

Whether capital should be added into the NOEM model is another issue. It is commonly believed in the open economy macroeconomics literatures that the transitional dynamics of the model could be enriched by modelling capital accumulation subject to adjustment costs, which might also improve the model's ability to match the moments of the data. Surprisingly, almost all existing NOEM models which accommodate both tradable and non-tradable sectors choose to subtract capital stock.

The reason that most NOEM models do not include non-tradable sector and capital simultaneously is related with the classic Blanchard-Kahn condition in solving DSGE models. This condition states that the number of eigenvalues of the linearised system that are larger than 1 in modulus must equal to the number of forward-looking variables in order to have a unique stable equilibrium in the neighbourhood of the steady state. In the experiments that I did, I found that this condition does not hold with the presence of capital in the production functions of both tradable and non-tradable sectors. But by assuming the capital stock is only used in the tradable sector, the Blanchard-Kahn condition does hold. Given that the non-tradable sector is more labour intensive than the tradable sector, this assumption is plausible.[6] Of course, this non-existence or indeterminacy problem in DSGE models can also be avoided with different modelling approach.

Therefore, in this paper, I also include the capital in tradable goods production. Having non-tradable sector and capital stock simultaneously makes this model different from others NOEM models. Moreover, In order to evaluate the effects of including capital accumulation, I also estimate a model without capital (the NK model), and compare it with the benchmark model.

The purpose of this paper is to assess whether including non-tradable sector or capital stock can improve the ability of the NOEM models to fit the macroeconomic data, especially the real exchange rate data. The benchmark model shares many common features of the NOEM models, such as monopolistically competitive firms, sluggish local currency price setting, capital accumulation subject to adjustment costs and monetary policies in the form of interest rate setting rules. Four shocks are introduced to the DSGE system: an economy wide technology shock, a consumption preference shock, an uncovered interest rate parity (UIP) shock and a monetary policy shock.

The model is then estimated using Bayesian techniques, which have several advantages over traditional GMM-based estimation. Lubik and Schorfheide (2005) state:

'First, unlike GMM estimation of monetary policy rules and first-order conditions, the Bayesian analysis is system-based and fits the solved DSGE model to a vector of aggregate time series. Second, the estimation is based on the likelihood function generated by the DSGE model rather than, for instance, the discrepancy between DSGE

[6] For example, Rebelo and Végh (1995) also assume that the production of non-tradable goods does not require capital in order to 'improve the model's implications for the behaviour of the real exchange rate'.

model impulse response functions and identified VAR impulse responses as in Rotemberg and Woodford (1997) and Christiano, Eichenbaum, and Evans (2005). Third, prior distributions can be used to incorporate additional information into the parameter estimation.'

Bayesian techniques also have advantages over the maximum likelihood methods which are used in some NOEM papers[7]. With maximum likelihood methods, if the likelihood is completely flat in some direction, the associated parameter is not identified. However, the prior information used in the Bayesian techniques can help identification by assuaging the problem of nearly flat likelihoods.

The data used for estimation are quarterly data from 1971:1 - 2006:4, with the U.S. as Home country and the E.U. as Foreign country. Because of the data availability, the aggregate of France, Germany, Italy and the U.K. are used as an approximation of the E.U.. In order to avoid the typical singularity problem of DSGE models, the number of observed variables is the same as that of the shocks. Therefore, four time series data for Home and Foreign countries are used in the empirical analysis, which are real GDP per capita, consumer price index, nominal short-run interest rates and real exchange rates.

The three models (the TNT, T and NK models) are then compared with various model comparison criterions. The results show that the TNT and T models with capital outperform the NK model which does not include capital. The posterior odds ratio between the TNT model and the NK model (in favour of the TNT model) is 8.01; and the ratio between the T model and NK model (in favour of the T model) is 6.17. This means that a prior that favours the NK model over the TNT or T model by a factor of 8.01 or 6.17 is needed in order to accept it after observing the data. This shows that including capital accumulation with adjustment costs do improve the model's ability to fit the data, which implies that subtracting of capital from the model for simplicity is inappropriate.

The model comparison between the TNT and T model is not that obvious: the TNT model does a better job than the T model, but only slightly. This result is consistent with Engel (1999) and Chari et al. (2002), which state that non-tradable sector is

[7]For example, see Bergin (2006).

not important in explaining the real exchange rate volatilities. However, this is likely because sectoral price index data and sectoral productivity shocks are not used in the TNT model. However, if these two changes are implemented, different data are needed for model estimation, which would make the model comparison infeasible.

The paper is organised as follows. Section 2 illustrates the structure of the benchmark model. In Section 3, I first describe the data and Bayesian techniques used for estimation, and then report the results from the estimation, with interpretations. In section 4, the three models are compared with various criterions. Section 5 is the conclusion.

2.2 Structure of the TNT Model

The world economy consists of two countries of equal size, Home and Foreign. Each country is populated by a large number of identical infinitely-lived households. The Home economy produces a continuum of differentiated, intermediate tradable goods and a continuum of differentiated intermediate non-tradable goods, indexed by i, $j \in [0,1]$, respectively. The Foreign country also produces two continuums of intermediate goods, indexed by i^*, $j^* \in [0,1]$, respectively. All trades between countries are intermediate tradable goods that are produced by monopolists who can charge different prices in the two countries. In addition, in each country, there are perfectly competitively final goods firms, which combine tradable intermediate goods and domestic non-tradable goods to form country-specific final goods. The final good is used for domestic consumption and investment and cannot be traded internationally. Goods produced in the Home country are subscripted with an H, while those produced in the Foreign country are subscripted with an F. Allocations and prices in the Foreign country are denoted with an asterisk.

2.2.1 The Final Good Producers

The Home final good producers use Home tradable and non-tradable intermediate goods together with Foreign tradable intermediate goods to produce Home final goods. The aggregate technology for producing the final goods is:

$$Y_t = \left(a_T^{\frac{1}{\phi}} \left(Y_{T,t}\right)^{\frac{\phi-1}{\phi}} + \left(1-a_T\right)^{\frac{1}{\phi}} \left(Y_{N,t}\right)^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}}, \phi > 0, \tag{2.1}$$

where, Y is the final good, Y_T is an aggregate of tradable intermediate goods and Y_N is an aggregate of Home non-tradable intermediate goods. a_T measures the relative weight of tradable goods in final goods production, ϕ is the intratemporal elasticity of substitution between Y_T and Y_N.

The aggregation technology for Y_T can be written as:

$$Y_{T,t} = \left(a_H^{\frac{1}{\rho}} \left(Y_{H,t}\right)^{\frac{\rho-1}{\rho}} + \left(1-a_H\right)^{\frac{1}{\rho}} \left(Y_{F,t}\right)^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \rho > 0, \tag{2.2}$$

where Y_H and Y_F are again the aggregate tradable intermediates goods produced in the Home and Foreign countries, respectively. a_H is the home bias coefficient, which measures the relative weight of Home tradable goods in total tradable goods used at Home, ρ denotes the intratemporal elasticity of substitution between Y_H and Y_F.

Moreover, Y_H, Y_F and Y_N can be further divided as:

$$Y_{H,t} = \left(\int_0^1 y_{H,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \theta > 1, \tag{2.3}$$

$$Y_{F,t} = \left(\int_0^1 y_{F,t}(i^*)^{\frac{\theta-1}{\theta}} di^* \right)^{\frac{\theta}{\theta-1}}, \tag{2.4}$$

$$Y_{N,t} = \left(\int_0^1 y_{N,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \tag{2.5}$$

where $y_H(i)$, $y_F(i^*)$ and $y_N(i)$ represent individual Home tradable, Foreign tradable, and Home non-tradable goods consumed in Home country; θ is the constant intratemporal elasticity of substitution across varieties.

Final good producers behave competitively, maximising profits each period:

$$\max P_t Y_t - \int_0^1 p_{H,t}(i) y_{H,t}(i) di - \int_0^1 p_{F,t}(i^*) y_{F,t}(i^*) di - \int_0^1 p_{N,t}(j) y_{N,t}(j) dj,$$

subject to the aggregate technology (2.1) – (2.5), where P is the overall price index of the final good, $p_H(i)$, $p_F(i^*)$ and $p_N(j)$ are the Home currency price of individual Home tradable goods, Foreign tradable goods and Home non-tradable goods, respectively.

Solving the maximising problem above gives the demand functions for each intermediate good as:

$$y_{H,t}(i) = a_H a_T \left(\frac{P_{H,t}}{p_{H,t}(i)}\right)^{\theta} \left(\frac{P_{T,t}}{P_{H,t}}\right)^{\rho} \left(\frac{P_t}{P_{T,t}}\right)^{\phi} Y_t, \tag{2.6}$$

$$y_{F,t}(i^*) = (1 - a_H) a_T \left(\frac{P_{F,t}}{p_{F,t}(i^*)}\right)^{\theta} \left(\frac{P_{T,t}}{P_{F,t}}\right)^{\rho} \left(\frac{P_t}{P_{T,t}}\right)^{\phi} Y_t, \tag{2.7}$$

$$y_{N,t}(j) = (1 - a_T) \left(\frac{P_{N,t}}{p_{N,t}(j)}\right)^{\theta} \left(\frac{P_t}{P_{N,t}}\right)^{\phi} Y_t. \tag{2.8}$$

Then, using the zero-profit condition, the price index can be derived as:

$$P_{H,t} = \left(\int_0^1 p_{H,t}(i)^{1-\theta} \, di\right)^{\frac{1}{1-\theta}}, \tag{2.9}$$

$$P_{F,t} = \left(\int_0^1 p_{F,t}(i^*)^{1-\theta} \, di\right)^{\frac{1}{1-\theta}}, \tag{2.10}$$

$$P_{N,t} = \left(\int_0^1 p_{N,t}(j)^{1-\theta} \, dj\right)^{\frac{1}{1-\theta}}, \tag{2.11}$$

and

$$P_{T,t} = \left(a_H P_{H,t}^{1-\rho} + (1 - a_H) P_{F,t}^{1-\rho}\right)^{\frac{1}{1-\rho}}, \tag{2.12}$$

$$P_t = \left(a_T P_{T,t}^{1-\phi} + (1 - a_T) P_{N,t}^{1-\phi}\right)^{\frac{1}{1-\phi}}. \tag{2.13}$$

Analogous conditions hold for Foreign country.

2.2.2 The Tradable Intermediate Good Producers

Each tradable intermediate good firm is a monopolistic producer of a single differentiated good, who uses labour and capital as inputs. The technology of the firm that produces domestic tradable intermediate good "i" is:

$$y_{H,t}(i) + y_{H,t}^*(i) = \Lambda_t \Lambda_H k_{H,t-1}(i)^{\alpha} \left(l_{H,t}(i)\right)^{1-\alpha}, \tag{2.14}$$

where Λ_t is a stationary stochastic aggregate productivity shock to the Home economy, Λ_H represents the technology common to all Home tradable intermediate good production firms, $k_H(i)$ and $l_H(i)$ are the inputs of capital and labour used in the tradable goods production.

Price stickiness is introduced with quadratic menu costs, which take the form:[8]

$$AC_{H,t}(i) = \frac{\chi_p}{2} \frac{[p_{H,t}(i) - p_{H,t-1}(i)]^2}{P_t p_{H,t-1}(i)} y_{H,t}(i), \tag{2.15}$$

$$AC^*_{H,t}(i) = \frac{\chi_p}{2} \frac{s_t \left[p^*_{H,t}(i) - p^*_{H,t-1}(i)\right]^2}{P_t p^*_{H,t-1}(i)} y^*_{H,t}(i), \tag{2.16}$$

where χ_p is the price adjustment parameter and s_t is the nominal exchange rate.

The problem solved by Home tradable intermediate good producer i is to choose prices $p_H(i)$, $p^*_H(i)$, capital stocks $k_H(i)$, and labour inputs $l_H(i)$ to maximise its lifetime profit:

$$E_0 \sum_{t=0}^{\infty} \rho_t [p_{H,t}(i) y_{H,t}(i) + s_t p^*_{H,t}(i) y^*_{H,t}(i) - P_t \omega_t l_{H,t}(i)$$
$$-P_t r_t k_{H,t-1}(i) - P_t AC_{H,t}(i) - P_t AC^*_{H,t}(i)],$$

subject to the demand function (2.6) and its Foreign counterpart, production technology (2.14) and menu costs equations (2.15) and (2.16). ρ_t is the price kernel used to value random date t payoffs. Since firms are assumed to be owned by the representative household, it is assumed that firms value future payoffs according to the households' intertemporal marginal rate of substitution in consumption, and thus $\rho_t = \beta^t U'_{c,t}/U'_{c,0}$. Note that the local currency pricing is adopted by the tradable firms. And thus, they can price differently across the Home and Foreign countries.

Solving this maximising problem, we obtain the optimal price setting rule for the

[8] The choice between quadratic adjustment costs and the Calvo adjustment rules is made for analytical convenience. Benigo and Thoenissen (2002) state that 'From quantitative point of view the two adjustment mechanisms are identical'.

domestic market as:

$$(1-\theta)-\chi_p\frac{p_{H,t}(i)-p_{H,t-1}(i)}{p_{H,t-1}(i)}+\frac{\theta\chi_p}{2}\frac{[p_{H,t}(i)-p_{H,t-1}(i)]^2}{p_{H,t}(i)\,p_{H,t-1}(i)}$$
$$=-\frac{\theta P_t mc_{H,t}}{p_{H,t}(i)}-\frac{\chi_p}{2}E_t\frac{\rho_{t+1}}{\rho_t}\frac{y_{H,t+1}(i)}{y_{H,t}(i)}\frac{p_{H,t+1}(i)^2-p_{H,t}(i)^2}{p_{H,t}(i)^2}, \quad (2.17)$$

where

$$mc_{H,t}=\frac{\omega_t}{(1-\alpha)\,\Lambda_t\Lambda_H\,(k_{H,t-1}(i)/l_{H,t}(i))^{\alpha}}.$$

The optimal price setting rule for local currency pricing exports is:

$$(1-\theta)\,s_t-\chi_p s_t\frac{p^*_{H,t}(i)-p^*_{H,t-1}(i)}{p^*_{H,t-1}(i)}+\frac{\theta\chi_p}{2}s_t\frac{\left[p^*_{H,t}(i)-p^*_{H,t-1}(i)\right]^2}{p^*_{H,t}(i)\,p^*_{H,t-1}(i)}$$
$$=-\frac{\theta P_t mc_{H,t}}{p^*_{H,t}(i)}-\frac{\chi_p}{2}E_t\frac{\rho_{t+1}}{\rho_t}s_{t+1}\frac{y^*_{H,t+1}(i)}{y^*_{H,t}(i)}\frac{p^*_{H,t+1}(i)^2-p^*_{H,t}(i)^2}{p^*_{H,t}(i)^2}. \quad (2.18)$$

The optimal trade-off between capital and labour inputs depends upon their relative cost, according to the following condition:

$$\frac{r_t}{\omega_t}=\frac{\alpha}{1-\alpha}\left(\frac{k_{H,t-1}(i)}{l_{H,t}(i)}\right)^{-1}. \quad (2.19)$$

Analogous conditions hold for Foreign country.

2.2.3 The Non-Tradable Intermediate Good Producers

In their famous paper, Blanchard and Kahn (1980) state the so called Blanchard-Kahn conditions: the conditions for the existence and uniqueness of the solution for a linear difference model under rational expectations. The linear difference model with rational expectation can be arranged in the Blanchard-Kahn form as:

$$\begin{bmatrix} E_t V_{t+1} \\ X_t \end{bmatrix} = A \begin{bmatrix} V_t \\ X_{t-1} \end{bmatrix} + \digamma_t,$$

where X_t is an n-vector of variables predetermined at data t, V_t is an m-vector of variables non predetermined at t and $\digamma_t$ is a $n+m$ vector of exogenous variables, E_t is the mathematical expectation conditional on time t, A is a $n+m$ square matrix.

The Blanchard-Kahn conditions state if the number of eigenvalues of A outside the unit circle is equal to the number of non-predetermined variables, then there exists a unique solution; if the number of eigenvalues larger than one more or less than the forward looking variables, there is no solution (non-existence) or infinite solutions (indeterminacy).

In the experiments I did, I found that if capital is included in both the tradable and non-tradable sectors, the Blanchard-Kahn conditions do not hold. In particular, the number of eigenvalues larger than one in modulus is more than the number of forward-looking variables, implying that there is no solution. This might be the reason that most NOEM models only include one of the two elements: non-tradable sector and capital stock.

This problem can be avoided by assuming the capital is only used in the tradable sector, but not in the non-tradable sector[9]. Rebelo and Végh (1995)[10] also assume that the production of non-tradables does not require capital in order to 'improve the model's implications for the behaviour of the real exchange rate'. They also state that, given that the non-tradable sector is more labour intensive than the tradable sector, this assumption is plausible. And thus, the technology of the firm that produces domestic non-tradable intermediate good j is:

$$y_{N,t}(j) = \Lambda_t \Lambda_N l_{N,t}(j), \tag{2.20}$$

where Λ_N is the technology common to all Home non-tradable goods firms.

The quadratic menu cost for non-tradables is:

$$AC_{N,t}(j) = \frac{\chi_p}{2} \frac{\left[p_{N,t}(j) - p_{N,t-1}(j)\right]^2}{P_t p_{N,t-1}(j)} y_{N,t}(j). \tag{2.21}$$

The problem solved by Home non-tradable intermediate good producer i is to choose

[9]There are other modelling strategies to solve this non-existence or indeterminacy problem. For example, Laxton and Pesenti (2003) develop a large-scale two country Global Economic Model (GEM) with several inputs including capital. However, size creates computational challenges and up to now, it can only been calibrated to conduct simulation experiments, and not yet formally estimated. Selaive and Tuesta (2006) calibrate a small-scale model with both non-tradable sector and capital, but with the cost of assuming no nominal rigidities.

[10]The model constructed by Rebelo and Végh (1995) is not based on NOEM framework, and thus do not share the common features of NOEM models, such as nominal rigidities.

price $p_{N,t}(j)$ and labour input $l_N(j)$ to maximise:

$$E_0 \sum_{t=0}^{\infty} \rho_t [p_{N,t}(j) y_{N,t}(j) - P_t \omega_t l_{N,t}(j) - P_t AC_{N,t}(j)],$$

subject to the demand function (2.8), the production function (2.20) and menu cost (2.21). Solving this problem, the price setting rule for non-tradable firm i are derived as:

$$\begin{aligned} & (1-\theta) - \chi_p \frac{p_{N,t}(j) - p_{N,t-1}(j)}{p_{N,t-1}(j)} + \frac{\theta \chi_p}{2} \frac{[p_{N,t}(j) - p_{N,t-1}(j)]^2}{p_{N,t}(j)\, p_{N,t-1}(j)} \\ = & -\frac{\theta P_t \omega_t}{p_{N,t}(j)\, \Lambda_t \Lambda_N} - \frac{\chi_p}{2} E_t \frac{\rho_{t+1}}{\rho_t} \frac{y_{N,t+1}(j)}{y_{N,t}(j)} \frac{p_{N,t+1}(j)^2 - p_{N,t}(j)^2}{p_{N,t}(j)^2}. \end{aligned} \tag{2.22}$$

Analogous conditions hold for Foreign non-tradable firms.

2.2.4 The Households

The household utility function is positively related to consumption and leisure. Households generate income from supplying labour, renting capital to firms, and receiving profits from intermediate goods firms. Also, households can hold two types of risk-free bonds, one denominated in Home currency B_t with gross nominal interest rate R_t, another one is denominated in Foreign currency $-B_t^*$ (the negative sign means the Home borrowing from the Foreign) with interest rate R_t^*. The representative Home household chooses consumption C, labour supply L, investment I and bond holding B and $-B^*$ to maximise its lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\tau_t C_t^{1-\sigma_1}}{1-\sigma_1} - \frac{\sigma_2}{1+\sigma_2} L_t^{\frac{1+\sigma_2}{\sigma_2}} \right),$$

subject to the sequence of budget constraint:

$$\begin{aligned} & P_t C_t + B_t - s_t B_t^* + P_t I_t - R_{t-1} B_{t-1} + s_t R_{t-1}^* B_{t-1}^* \\ = & P_t \omega_t L_t + P_t r_t K_{t-1} + \Pi_{H,t} + \Pi_{N,t}, \end{aligned}$$

and the law of motion for capital:

$$K_t = (1-\delta)K_{t-1} + I_t - AC_{k,t}.$$

τ is the preference shock, Π_H and Π_N are total profits from tradable and non-tradable sectors respectively, δ is the depreciation rate, and AC_k is the quadratic capital adjustment cost of the form:

$$AC_{k,t} = \frac{\chi_k}{2}\frac{(K_t - K_{t-1})^2}{K_{t-1}},$$

where, χ_k is the capital adjustment cost parameter.

Solving the household maximising problem, the following optimal conditions are obtained. The consumption Euler equation is:

$$\frac{\tau_t C_t^{-\sigma_1}}{P_t} = R_t \beta E_t \frac{\tau_{t+1} C_{t+1}^{-\sigma_1}}{P_{t+1}}. \tag{2.23}$$

The consumption leisure trade-off is:

$$L_t^{\frac{1}{\sigma_2}} = \omega_t \tau_t C_t^{-\sigma_1}. \tag{2.24}$$

Capital accumulation is set to equate the costs and expected benefits:

$$1 + \chi_k \frac{K_t - K_{t-1}}{K_{t-1}} = \beta E_t \left\{ \frac{\tau_{t+1}}{\tau_t} \frac{C_{t+1}^{-\sigma_1}}{C_t^{-\sigma_1}} \left(1 - \delta + r_{t+1} + \frac{\chi_k}{2} \frac{K_{t+1}^2 - K_t^2}{K_t^2}\right)\right\}. \tag{2.25}$$

Finally, optimal portfolio choices imply the uncovered interest rate parity (UIP) condition:

$$R_t E_t \frac{U'_{c,t+1}}{U'_{c,t}} \frac{P_t}{P_{t+1}} = R_t^* E_t \frac{U'_{c,t+1}}{U'_{c,t}} \frac{P_t}{P_{t+1}} \frac{s_{t+1}}{s_t}.$$

Log-linearise the UIP equation above, we obtain:

$$\left(\widehat{R}_t - \widehat{R}_t^*\right) = E\widehat{s}_{t+1} - \widehat{s}_t,$$

where, $\widehat{x}_t = (x_t - \overline{x})/\overline{x}$ is the percentage changes from the initial steady state, and $\overline{x}$ is the steady-state value. Given the empirical evidence of the departure from UIP[11],

[11] For example, see Lewis (1995).

following Bergin (2006), the UIP condition is modified by adding a risk premium term to the right hand side of the equation above:

$$\widehat{R}_t - \widehat{R}_t^* = (E\widehat{s}_{t+1} - \widehat{s}_t) - \psi_{RP}\left(\frac{B_t - s_t B_t^*}{P_t Y_t}\right) + \xi_t. \tag{2.26}$$

The first component of this term is a function of the debt of a country, which is a device to remove an element of non-stationarity in the model. Bergin (2006) states that under incomplete asset markets, shocks can lead to permanent wealth reallocations that would induce non-stationary in the model. Introducing this risk premium term as a function of debt forces wealth allocations return to their initial distribution in the long run. The second component is a stochastic shock to the risk premium aiming at capturing any time-varying deviations form UIP. The equation above implies that Home households are charged a premium over the Foreign interest rate if the Home economy as a whole is a net borrower, and receive a lower interest on the savings if the Home economy is a net lender.

2.2.5 Monetary Policy

I assume the central banks in both countries adopt Taylor-type instrument rules[12]. An alternative way of closing the model is to assume that monetary policy aims at optimising a specific objective function: the targeting rules. This does not imply that instrument rules are more accurate description of monetary policy in either the U.S. or E.U., but they do offer a convenient way to capture an active monetary policy. And also from empirical viewpoint and welfare perspective, there is no obvious evidence that instrument rules perform substantially worse than targeting rules.[13]

Following Lubik and Schorfheide (2005), the monetary authorities are assumed to adjust the short-term interest rate in response to inflation, output growth and exchange

[12] Instrument rules refer to some formulas prescribing settings for the monetary policymaker's instrument as a function of currently observed variables.

[13] It is still controversial which rules are better in modelling monetary policy. For example, Svensson (2003) argues strongly that targeting rules are normatively superior to instrument rules for the conduct of monetary policy. However, McCallum and Nelson (2005) argue that 'Svensson (2003) does not develop any compelling reasons for preferring targeting rules over instrument rules, from a normative perspective. We also suggest, regarding the positive perspective, that no actual central bank has expressed explicitly the magnitude of objective function parameters that are essential for the utilization of a targeting rule.' Onatski and Williams (2004) also find that instrument rules perform relatively well compared to targeting rules, and they are more robust to different parameter estimates.

rate depreciation:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + a_1(1-\rho_R)\left(\widehat{P}_t - \widehat{P}_{t-1}\right) + a_2(1-\rho_R)\left(\widehat{Z}_t - \widehat{Z}_{t-1}\right) + a_3(1-\rho_R)\left(\widehat{s}_t - \widehat{s}_{t-1}\right) + \varepsilon_{\mu,t}, \tag{2.27}$$

where Z is the total output (GDP), ε_μ is the monetary policy shock.

For Foreign country, analogous condition holds.

2.2.6 Additional Equilibrium Conditions

Equilibrium requires several market clearing conditions. The resource constraint is:

$$Y_t = C_t + I_t + \int_0^1 AC_{H,t}(i)di + \int_0^1 AC^*_{H,t}(i)di + \int_0^1 AC_{N,t}(j)dj, \tag{2.28}$$

where Y is the final good which are used for consumption expenditure, investment expenditure and menu costs.

The total output GDP equals to the home produced tradable and non-tradable goods:

$$\overline{P}Z_t = \int_0^1 \overline{p}_H(i)\, y_{H,t}(i)di + \int_0^1 \overline{p}^*_H(i)\, y^*_{H,t}(i)di + \int_0^1 \overline{p}_N(j)\, y_{N,t}(j)dj, \tag{2.29}$$

where Z_t is the GDP of Home country, which is measured using steady-state relative prices. The Home labour market clearing condition is given by:

$$L_t = \int_0^1 l_{H,t}(i)di + \int_0^1 l_{N,t}(j)\, dj. \tag{2.30}$$

And the Home capital market clearing condition is given by:

$$K_t = \int_0^1 k_{H,t}(i)di. \tag{2.31}$$

Combining the budget constraint of the household and intermediate goods production firms, we can write the balance of payment condition:

$$TB_{H,t}+R_{t-1}\left(B_{t-1}-s_tB^*_{t-1}\right)=\left(B_t-s_tB^*_t\right), \tag{2.32}$$

where TB_H is the Home country trade balance, which is the total value of exports minus the value of imports:

$$TB_{H,t}=s_tP^*_{H,t}Y^*_{H,t}-P_{F,t}Y_{F,t}. \tag{2.33}$$

Finally, the real exchange rate is defined as:

$$q_t=\frac{s_tP^*_t}{P_t}. \tag{2.34}$$

2.2.7 The Exogenous Law of Motions

Technology shock follows the following process:

$$\log\Lambda_t=\rho_\Lambda\log\Lambda_{t-1}+\varepsilon_{\Lambda,t},\ \varepsilon_{\Lambda,t}\sim N\left(0,\sigma_\Lambda\right).$$

Shock to preference follows the process:

$$\log\tau_t=\rho_\tau\log\tau_{t-1}+\varepsilon_{\tau,t},\ \varepsilon_{\tau,t}\sim N\left(0,\sigma_\tau\right).$$

Shock to the modified UIP condition follows the process:

$$\xi_t=\rho_\xi\xi_{t-1}+\varepsilon_{\xi_t,t},\varepsilon_{\xi,t}\sim N\left(0,\sigma_\xi\right).$$

The shock to the monetary policy rule is $\varepsilon_{\mu,t}\sim N\left(0,\sigma_\mu\right)$. All shocks are stationary, therefore autoregressive coefficients on shock processes ρ_Λ, ρ_τ, ρ_ξ are restricted to be greater than zero and less than unity.

The shocks for Foreign country are analogous.

2.2.8 The Equilibrium

An equilibrium for this economy is a collection of allocations for Home households C_t, K_t, L_t, TB_t, B_t and $-B_t^*$; allocations for Foreign households C_t^*, K_t^*, L_t^*, TB_t^*, $-B_t$ and B_t^*; allocations and prices for Home tradable intermediate good producers $y_{H,t}(i)$, $y_{H,t}^*(i)$, $k_{H,t-1}(i)$, $l_{H,t}(i)$, $p_{H,t}(i)$ and $p_{H,t}^*(i)$ for $i \in [0,1]$; allocations and prices for Foreign tradable intermediate good producers $y_{F,t}^*(i^*)$, $y_{F,t}(i^*)$, $k_{F,t-1}^*(i^*)$, $l_{F,t}^*(i^*)$, $p_{F,t}^*(i^*)$ and $p_{F,t}(i^*)$ for $i^* \in [0,1]$; allocations and prices for Home non-tradable intermediate good producers $y_{N,t}(j)$, $l_{N,t}(j)$ and $p_{N,t}(j)$ for $j \in [0,1]$; allocations and prices for Foreign non-tradable intermediate good producers $y_{N,t}^*(j^*)$, $l_{N,t}^*(j^*)$ and $p_{N,t}^*(j^*)$ for $j^* \in [0,1]$; and allocations for Home and Foreign final good producers Y_t and Y_t^*, final good prices P_t and P_t^*, real wages ω_t and ω_t^*, nominal interest rates R_t and R_t^* that satisfy the following conditions:

1. the household allocations solve the household's problem;
2. the prices of tradable intermediate good producers solve their maximisation problem;
3. the prices of non-tradable intermediate good producers solve their maximisation problem;
4. the final good producers' allocations solve their problem; and
5. the market clearing conditions hold.

2.3 Estimation

Because of its richness, this model does not yield simple closed-form solutions. However, the model's dynamics can be obtained by taking a linear approximation around the steady state. In the specific steady state used in this paper, the firms in both tradable and non-tradable sectors charge the same prices in the domestic and Foreign markets. I also assume the inflation rate is zero and nominal and real exchange rates are one in the steady state. And thus, in the steady state, all prices in the economy are equal. Moreover, in this steady state, all variables in Home and Foreign countries are symmetric and the net Foreign asset position is equal to zero.

The model is then linearised around this deterministic steady state. The deviation of the variable from its steady state is denoted with $\widehat{x}_t = (x_t - \overline{x})/\overline{x}$. Furthermore, in order to reduce the dimensions of the data set and parameter space, I follow Bergin (2006) to write the variables as country difference, that is, Home minus the Foreign counterparts. The appendix 5.1.1 lists the linearised equations.[14]

2.3.1 Data

Data for the U.S. and the E.U. are used to estimate the model. The U.S. is treated as Home country, while the E.U. is the Foreign country. In this paper, the aggregate of France, Germany, Italy and the U.K. are used as the approximation of the E.U.. The series are constructed with data for individual countries collected from the International Monetary Fund (IMF), International Financial Statistics (IFS) and the Organization for Economic Co-operation and Development (OECD), Main Economic Indicators (MEI). All data series are seasonally adjusted quarterly series at quarterly rate[15] for the period 1971:1 - 2006:4. Seven time series data are used in the estimation, which are real output (GDP) per capita, price index and nominal short-term interest rate for both Home and Foreign countries, and real exchange rate data. Foreign aggregate variables are computed as a geometric weighted average, where time-varying weights are based on each country's share of total real GDP. The details of construction of the data set are provided in Appendix 5.1.2.

The model has implication for the log deviation from steady state of all these variables, and thus the data is pre-processed before the estimation stage. In particular, data are logged and then detrended using Hodrick-Prescott filter[16]. The filtered data are then transformed into country differences, that is: Home minus the Foreign counterpart. The last transformation of course does not apply to the real exchange rate

[14]The standard tools developed in DYNARE are used to estimate and study the properties of linear DSGE models. Dynare is a pre-processor and a collection of MATLAB or SCILAB routines which solve non–linear models with forward looking variables. It has been built in order to study the transitory dynamics of non-linear models with consistent expectations. More information about Dynare can be found at the official website: http://www.cepremap.cnrs.fr/dynare/.

[15]U.S. publishes the data at annual rate, while E.U. countries report data at quarterly rate. According to OECD Quarterly National Accounts Statistics, data which are not in quarterly rate are transformed to quarterly rate by dividing by four.

[16]The multiplier λ is set to 1600, which is reasonable for quarterly data as advised by Hodrick and Prescott.

data.

Finally, there are four data series, which are used to estimate the model parameters. As the number of the shocks is equal to that of the observable variables, the typical singularity problem of DSGE models is avoided.

2.3.2 Prior Distributions

A few parameters will not be estimated here, but are pinned down ahead of time, as the data used in estimation contains only limited information about them. This can be seen as a very strict prior. The discount factor β is calibrated at 0.99; the elasticity of substitution between varieties θ is set equal to 6, implying a steady state markup of 1.2; the labour supply elasticity $1/\sigma_2$ is calibrated at unity; the capital share in intermediate goods production α is set at 0.33; the depreciation rate δ is set at 0.025.

In principle, priors can be gleaned from personal introspection to reflect held beliefs about the validity of economic theories. Lubik and Schorfheide (2005) state that prior distributions play an important role in the system-based estimation of DSGE models. In this paper, the prior distributions are chosen from related recent empirical researches, such as Laxton and Pesenti (2003), Lubik and Schorfheide (2005) and Cristadoro, Gerali, Neri and Pisani (2006). Prior distributions for the parameters are summarised in Table 2.1. Parameters are also assumed to be *a priori* independent.

2.3.3 Parameter Estimates

The Bayesian techniques are used to estimate the posterior distribution of the parameters. The computation of the posterior distribution of the parameters proceeds in the following steps. First, the log posterior kernel are maximised with respect to the parameters needed to be estimated to find the posterior mode. Next, a Random Walk Metropolis-Hastings algorithm is used to explore the parameter space in a neighbourhood of that point. The algorithm generates a sequence of dependent draws from the posterior distribution that can be used to approximate the posterior distribution of any quantity of interest. Table 2.2 reports the parameter estimates which are delivered by a 500,000 run of Metropolis-Hastings algorithm.

The parameter estimates generally are reasonable. The data seem to be particularly informative about home bias coefficient a_H, with the posterior mean approximately 0.97, a value that is much higher than the prior mean. Cristadoro et al. (2006) find the similar home bias coefficient and they explain it as the attempt of the model trying to capture the exchange rate volatilities of the data.[17]

The posterior mean of a_T is about 0.62, which implies that tradable goods weight around 62% in the final goods.[18] The estimated value of a_T in this model suggests that the weight of non-tradable goods in the total consumption and investment is around 40%, which is non-negligible.

The estimates of the elasticity of substitution between Home and Foreign tradable goods, ρ, and the elasticity of substitution between tradable and non-tradable goods, ϕ, are close to their prior means, indicating that the sample is uninformative about these two parameters. The posterior mean of intertemporal elasticity of substitution, $1/\sigma_1$ is about 0.56, with the 90% probability interval ranging from about 0.46 to 0.73. This range is close to the values used in most Real Business Cycle literature, which assume the elasticity between 0.5 and 1.

The prior distribution for the investment adjustment cost parameters χ_k and price adjustment cost χ_P are updated. The posterior mean of χ_k is around 25.95, which is close to the value of 21.52 in Bergin (2006). This value implies that if investment increases 1% above the steady state, about 0.32% of this investment will be used to pay the adjustment cost. The price adjustment cost parameter, χ_P, reduces from its prior mean 50 to 16.03, indicating that lower nominal rigidities is sufficient for the model to generate the right volatilities of the data.

The sensitivity of a country's interest rate premium to the changes in net Foreign assets, ψ_{RP}, is around 0.0064, which implies that for the U.S. and the E.U., if the proportion of net Foreign asset relative to GDP decreases by 10%, its domestic interest rate would increase by 6.4 basis points. The estimate of ψ_{RP} is reasonable and consistent with relevant literatures.[19]

[17]For details about the relationship between home bias and exchange rate volatilities see Dornbusch (1976) and Warnock (2003).

[18]Stockman and Tesar (1995) suggest roughly half of nation's output consists of non-tradable goods for the seven largest OECD countries. In an empirical study of the U.S. industry from 1961-2001, Belo, Cerda and Santos (2004) find the share of tradable goods on total consumption expenditures is below 50% and decreases over time.

[19]Lane and Milesi-Ferretti (2001) estimate a value of 0.0107 from cross-sectional regressions and a

Table 2.2 also provides estimates for parameters in the monetary policy reaction function. The persistence coefficient ρ_R is around 0.70, a value quite similar to Lubik and Schorfheide (2005), who find a value of 0.76 for the U.S. - Euro Area economy. The estimated values of the weight parameters in the monetary policy rule are also consistent with the findings of Lubik and Schorfheide (2005). These estimates imply that the monetary policy makers react strongly to inflation and output growth movements, but only weakly to nominal exchange rate changes.

2.3.4 Impulse Response Analysis

Impulse response offers a greater understanding of the inherent dynamics and the relative importance of different shocks. The responses of endogenous variables of interest to one-standard deviations (the posterior mean of shock variance shown in Table 2.2) of structural shocks are reported in Figure 2.1 to 2.4. As the model is estimated in terms of country difference, the response in the graphs is in terms of the differentials between Home and Foreign variables: $\widehat{x}_t - \widehat{x}_t^*$.

Figure 2.1 shows the impulse response of the five variables to a one standard deviation of Home monetary policy shock. The response of the economy to the monetary shock is as expected. Home interest rate increase immediately against the Foreign after this shock according to the monetary policy rule. Price and output differentials decrease in response to this contraction monetary policy. The increase in relative interest rate sharply appreciates the nominal exchange rate. The nominal exchange rate appreciation counteracts the effect of the decrease in the relative price level, and the real exchange appreciates.

Figure 2.2 plots the impulse response to a one standard deviation of preference shock to the Home households. The taste shock implies an immediate increase in Home relative consumption, and consequently, raises Home relative price level and output. Given the higher relative price level and output, the Home monetary authority increases the nominal interest rate. Consequently, the interest rate differential between Home and Foreign appreciates Home currency, nominal exchange rate decreases. Moreover,

value of 0.0254 from panels using data from 66 countries between 1970-1998. Nason and Rogers (2006) use a value of 0.0035 for Canadian data; Bergin (2006) estimates a value of 0.0038 with data from the E.U. and the U.S..

the lower nominal exchange rate and higher relative price level cause the appreciation in real exchange rate.

Figure 2.3 illustrates the response of the economy to a positive risk-premium shock in the context of this model. The Home relative interest rate increases after this shock as implied by the modified UIP equation (2.27). The interest rate differential implies that the Home output and consumption decreases with respect to Foreign. The interaction between supply and demand decides the price level. In Figure 2.3, Home relative price level decreases at the beginning and starts to rise after a few periods. The response of the nominal exchange rate is similar to that of Bergin (2006), which is because of the nature of the risk-premium shock. Bergin (2006) states 'such a shock permits the Home interest rate to rise relative to the Foreign rate, even though the value of the domestic currency is appreciating over time, as is often observed empirically'.

Figure 2.4 plots the impulse response to the positive productivity shock. This shock raises Home relative output, lowers Home relative price level and interest rate. The nominal and real exchange rates depreciate, which is expected after a Home productivity growth.

2.4 Model Comparison

2.4.1 The T and NK model

In this section, I construct and estimate a model without non-tradable sector (the T model). This T model is then compared with the benchmark model, which has both tradable and non-tradable sectors (the TNT model). The T model can be easily derived from the TNT model by setting $a_T = 1$, and shutting down non-tradable intermediate good sectors for both Home and Foreign countries.

The benchmark model is different from other NOEM models by including non-tradable goods sector and capital accumulation subject to adjustment cost. It is commonly believed in the NOEM literatures that the transitional dynamics of the model could be enriched by modelling capital accumulation subject to adjustment costs, which might also improve the model's ability to match the moments of the data.[20] However,

[20]For example, see Benigno and Thoenissen (2003).

almost all existing NOEM models with both tradable and non-tradable sectors choose to subtract capital from the model. In order to evaluate the effects of capital in NOEM model, I also construct a model with both tradable and non-tradable sectors, but without capital (the NK model).

The T model and NK model are then estimated using the same Bayesian techniques and prior distributions. The results are reported in Table 2.3 and 2.4.

The posterior parameter estimates for T model and NK model in general are similar to these of the TNT model. The T model reports a lower estimate of investment adjustment cost parameter. Its posterior mean decreases from 24.95 in the TNT model to 17.45 in the T model. The parameter values the monetary policy response to exchange rate depreciation a_3 is around half in the T model than in the TNT model, which indicates an even lower effect of nominal exchange rate changes on central bank's interest rate setting decision. It is worth noting that in the T model the shocks become more volatile than in the TNT model. This implies that greater shocks are required for the T model to fit the volatilities of the data. This might be because excluding the non-tradable sector shuts down the second sources of the movements of real exchange rates, and thus, greater shocks are needed to compensate the lost.

For the NK model, the estimated mean of a_T, which is the weight of tradable goods in the final consumption basket, decreases from 0.62 in the TNT model to about 0.41. This shows that, without capital accumulation subject to adjustment cost in the model, the non-tradable sector becomes even more important for the model fit the data, thus a large weight of non-tradable goods is required.

2.4.2 Posterior Odds Ratio

One advantages of Bayesian estimation is the posterior distribution offers a particularly natural method of comparing models. The so called posterior odds ratio is used to compare the two models. The formula used to calculate the posterior odds ratio between model H_0 and H_1(in favour of H_0) is:

$$\frac{p(H_0 \mid y)}{p(H_1 \mid y)} = \frac{p(H_0)}{p(H_1)} \frac{p(y \mid H_0)}{p(y \mid H_1)},$$

where y is the observations of the data, $p(I)$ $(I = H_0,\ H_1)$ is the prior density, and $p(I \mid y)$ are the posterior distribution over models. $\frac{p(y|H_0)}{p(y|H_1)}$ is called the Bayes factor, where $p(y \mid I)$ is the marginal density of the data conditional on the model. If one does not have strong preference on which model is the best one, the prior density should be equal, i.e. $p(H_0) = p(H_1)$. And thus, the posterior odds ratio is equal to the Bayes factor. So the only complication to calculate the posterior odds ratio falls on finding the marginal data density $p(y \mid I)$.

In this paper, the Harmonic Mean Estimator[21] is used as the estimator of the marginal data density. Table 2.2, 2.3 and 2.4 report the log marginal data density for the three models, which are numerically computed from the posterior draws using the modified harmonic mean estimator.

The result shows that the TNT model yields the highest log marginal data density among the three models. And the two models with capital yield higher marginal likelihood than the one without it. It is important to note that the marginal data density penalises the likelihood fit by a measure of model complexity, and thus guard against overfitting. For the TNT model, the improvement in model fit dominates the penalty for increased model complexity and the marginal data density improves.

However, the marginal data density for the TNT model is only slightly higher than the T model, which is around 0.26 in log term. In terms of posterior odds ratio, the value between the TNT model and T model (in favour of the TNT model) is 1.30. This means that a prior that favours the T model over the TNT model by a factor of 1.3 is needed in order to accept it after observing the data. The value greater than one does suggest accepting the TNT model upon the T model. But, since this is not a big number, I can only conclude that the TNT model outperforms the T model slightly.

The posterior odds ratio between the TNT model and the NK model (in favour of the TNT model) is 8.01; and the ratio between the T model and NK model (in favour of the T model) is 6.17. Again, this means that a prior that favours the NK model over the TNT or T models by a factor of 8.01 or 6.17 is needed in order to accept it. Since these are not small numbers, the models with capital do outperform the one without it.

[21] For details, see Geweke (1999).

2.4.3 Kalman Filtered In-Sample Forecast

In order to have an clear idea of how well the models fit the data, Figure 2.5 plots the actual data and Kalman filtered estimates of the observables for the TNT model, computed at the mean of the posterior distribution of the estimated parameters. The graphs for the other two models are quite similar, and thus are not presented here. Kalman filtered variables are built based on knowing past information, and thus can be used to measure one period ahead prediction errors.[22] The figure shows that the overall in-sample fit for the 4 variables is satisfactory.

In order to have a more intuitive way to compare the in-sample fit of the three models, I also report root mean squared error (RMSE) for one-step ahead in-sample forecasts of all models in table 2.5. The RMSE of one period forecast of observed variables for each model is

$$RMSE_m = \sqrt{\sum_{t=T+1}^{t+H} \frac{\{z_t - E\left(z_t \mid \Theta_{t-1}, m\right)\}^2}{H}},$$

where, m represents different the models, z_t is the vector of observed variables, and Θ_{t-1} denotes the information set up to period $t-1$. Therefore, $E\left(z_t \mid \Theta_{t-1}, m\right)$ denotes the one-period ahead forecast.

The result shows that the RMSE for one-period ahead in-sample forecasts is satisfactory for all models. The NK model performs the worst when trying to forecast the real exchange rate, while the TNT model and the T model do a better job. In particular, the T model produces smallest RMSE for real exchange rate, but it is only slightly lower than the TNT model. The RMSE for other variables are relatively small compared with that of real exchange rate. The T model performs the best in forecasting relative price level and nominal interest rate, and the NK model predicts the smallest RMSE for relative output.

[22]The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are used to compute the estimate for the current state.

2.4.4 Unconditional Second Moments

The models can also be compared by how well they capture the unconditional moments of the data. Table 2.6 reports the unconditional second moments from the data and the models. All second moments are obtained by simulating the model at the posterior mean of estimated parameters and applying the HP filter.

The results show that all three models do a good job in fitting the data's moments. All models generate the right standard deviation of price and interest rate differentials as in the data. The TNT model performs the best in matching the volatilities of the relative consumption (0.0138 compared to 0.0133 in the data), while the T model obtains a higher value (0.0153) and the NK model predicts a lower value (0.0117). All three models predict right but somewhat smaller nominal and real exchange rate volatility and persistence. The NK model does the best in matching the standard deviation of real exchange rate, but it is only slightly better than others.

For the autocorrelations, all three models behave quite similar: they generate similar autocorrelations for price and interest rate differentials as in the data, but somewhat smaller values for other variables of interest.

The bottom panel of Table 2.6 shows cross-correlations among the variables of interest. For the cross-correlation between real and nominal exchange rate, relative output and consumption, all three models perform quite well. There is also no evidence of the *consumption real exchange rate anomaly*[23]: all three models correctly predict negative correlation between real exchange rate and relative consumption. The TNT model performs the best here. It generates a correlation of -0.1902, which is close to the data (-0.1973). The T model predicts a higher negative correlation (-0.2383), while the NK model reports a lower value (-0.0798).

In general, the results show that different model capture a particular moment of the data better than the others. It seems that the benchmark model is preferred since it fits most features of the data best, although the difference between models is not large. This confirms the results obtained from the posterior odds ratio comparison. The TNT

[23]Chari et al. (2002) construct and calibrate a NOEM with complete financial market. Their model predicts a high and positive correlation between real exchange rate and relative consumption; while in the data, this correlation is negative. They refer to this large discrepancy as the *consumption real exchange rate anomaly.*

model ranks the first using the posterior odds ratio, but it outperforms the T model only slightly.

2.4.5 Discussion

Since Obstfeld and Rogoff (1995), capital stock is normally subtracted from the modelling approach for the model simplicity and tractability. The advantage of including capital accumulation subject to adjustment costs is obvious: it can enrich the transitional dynamics of the model. And also, given the large role of capital in production, it is unrealistic to exclude it from the models. The evidence of model comparison in this paper does show that models with capital outperform the model without it, which emphasises the important effects of including capital stock in NOEM models.

In the particular experiment I did in this paper, the performance of the TNT model is only slightly better than the T model, which might suggest that the non-tradable sector plays an unimportant role in NOEM models. However, the performance of the TNT model might be improved by using the price index data at the sector level. That is: the price indexes for the tradable and non-tradable goods, respectively. However, so far, there is no commonly accepted method to calculate the tradable or non-tradable price indexes. Although the definition of tradable and non-tradable is theoretically clear, these two goods interlock with each other in reality.[24] Another important reason that I do not use sectoral prices is because the using of tradable price index makes the model comparison between TNT and T models infeasible as different data sets are then needed to be used in the estimation process.

The performance of the TNT model can also be enhanced by assuming that there are sector-specific productivity shocks. Different sectoral productivity shocks lead to higher variability of the relative prices between tradable and non-tradable goods, and therefore can improve the model's ability to generate higher real exchange rate volatilities as in

[24] Chari et al. (2002) present two ways of constructing the price index data for tradable goods. In the first way, they follow the OECD MEI to disaggregate the CPI for all items into indices for food, all goods less food, rent, and services less rent. Then, the tradable price index is the weighted average of the price indices for food and for all goods less food. The second way is to divide the consumption expenditure for four categories: durable goods, semi-durable goods, non-durable goods and services, which are then used to construct tradable and non-tradable goods price indices.

However, they also admit that their measures of the price of tradable goods are clearly imperfect in another way. '...the price paid by the final user of the goods and, hence, incorporate the value of such nontraded services, such as distribution and retailing...'

the data. However, the same model comparison problem still exists in this case.

2.5 Conclusion

In this paper, I constructed a NOEM model with both tradable and non-tradable sectors, together with capital accumulation subject to adjustment costs. The model shares many features of the standard NOEM model, including monopolistically competitive firms, sluggish price, local currency pricing and monetary policy in the form of interest rate setting rules. Bayesian techniques are used to estimate the model with the data from 1971:1 to 2006:4 for the U.S. and the E.U..

This model differs from other NOEM models by including both non-tradable sector and capital stock simultaneously. The effects of non-tradable sector and capital accumulation are considered to be important in terms of explaining real exchange rate volatilities and persistence in some empirical research in open economy macroeconomics, but the importance has not yet been evaluated in the NOEM framework. And thus, in this paper, I try to assess this question in an NOEM model, which hopefully can have some implications for future NOEM modelling strategy.

The model comparison in this paper shows clearly that the TNT and T model both outperform the NK model, which suggests that including capital accumulation subject to adjustment cost can improve the NOEM model's ability to fit the data. On the other side, the comparison between the TNT model and T model is not that clear, the TNT model performs only slightly better than the T model according to the Bayesian posterior odds ratio criterion. Nevertheless, the performance of the TNT model could potentially be improved by using sectoral price index data or introducing sector specific productivity shocks.

This model is of course based on a number of simplifying assumptions which facilitate the analysis considerably. The results of estimates are contingent on the model settings and to some extent on the chosen prior distributions. Moreover, additional or alternative shocks which are not included in the model may also affect the movements of real exchange rates. The supply side could be enriched by introducing wholesale and retail sectors and including distribution costs. Together, such modifications could help improving the model performance, albeit at the cost of increased complexity.

Table 2.1 Prior Distribution*

Parameters	Domain	Density	Mean	SD
Structural Parameters				
σ_1	$\mathbb{R}^+$	Gamma	2.0	0.25
a_T	$[0,1)$	Beta	0.5	0.2
a_H	$[0,1)$	Beta	0.75	0.2
ρ	$\mathbb{R}^+$	Gamma	1.1	0.2
ϕ	$\mathbb{R}^+$	Gamma	1.2	0.2
χ_P	$\mathbb{R}^+$	Gamma	50	25
χ_x	$\mathbb{R}^+$	Gamma	50	25
ψ_{RP}	$[0,1)$	Beta	0.005	0.005
Monetary Policy Rule				
ρ_R	$[0,1)$	Beta	0.75	0.10
a_1	$\mathbb{R}^+$	Gamma	1.50	0.25
a_2	$\mathbb{R}^+$	Gamma	0.50	0.25
a_3	$\mathbb{R}^+$	Gamma	0.10	0.05
Shock Autocorrelations				
ρ_Λ	$[0,1)$	Beta	0.9	0.05
ρ_τ	$[0,1)$	Beta	0.9	0.05
ρ_ξ	$[0,1)$	Beta	0.9	0.05
Shock Standard Deviations				
σ_Λ	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_τ	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_ξ	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_μ	$\mathbb{R}^+$	InvGamma	0.01	*inf*

*Shock variances are assumed to have inverted gamma-1 distribution, $y \sim IG1(v,s)$, where the density is

$$f(y) = \varsigma(\frac{v}{2},\frac{2}{s})^{-1} * y^{-\frac{v}{2}-1} e^{-\frac{s}{2y}},$$

where $\varsigma(\alpha,\beta) = \Gamma(\alpha)\beta^{\alpha}$ is the constant of integration, and $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$ is the gamma function. The mean (u) and standard deviation (σ) are thus defined as

$$u = \left(\frac{s}{2}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)}, \quad \sigma = \left(\frac{s}{v-2} - u^2\right)^{\frac{1}{2}}.$$

Note, *inf* means infinite standard deviation, in this case, there is a closed form solution of v and s, which are: $v = 2$ and $s = \frac{2}{\pi}u^2$.

Table 2.2 Posterior Estimates for TNT Model

Log data density: 1885.17

Parameters	Mean	90% Probability intervals
Structural parameters		
σ_1	1.7890	(1.3669, 2.1816)
a_T	0.6198	(0.2840, 0.9379)
a_H	0.9677	(0.9335, 0.9996)
ρ	1.1009	(0.7825, 1.3944)
ϕ	1.1993	(0.8730, 1.5071)
χ_P	16.0360	(5.6085, 29.6315)
χ_x	25.9490	(4.8124, 57.2655)
ψ_{RP}	0.0064	(0.0000, 0.0153)
Monetary policy rule		
ρ_R	0.7033	(0.6182, 0.7949)
a_1	1.8779	(1.5548, 2.1885)
a_2	0.9784	(0.6813, 1.2754)
a_3	0.0462	(0.0179, 0.0727)
Shock autocorrelations		
ρ_Λ	0.7270	(0.6116, 0.8329)
ρ_τ	0.8510	(0.7705, 0.9330)
ρ_ξ	0.8770	(0.8236, 0.9317)
Shock variances		
σ_Λ	0.0123	(0.0081, 0.0174)
σ_τ	0.0242	(0.0158, 0.0322)
σ_ξ	0.0068	(0.0043, 0.0093)
σ_μ	0.0037	(0.0029, 0.0045)

Table 2.3 Posterior Estimates for T Model

Log data density: 1884.91

Parameters	Mean	90% Probability intervals
Structural parameters		
σ_1	1.8452	(1.4094, 2.2409)
a_H	0.9773	(0.9597, 0.9983)
ρ	1.0939	(0.7878, 1.4082)
χ_P	16.4144	(5.5594, 29.3646)
χ_x	17.4468	(5.9789, 33.2286)
ψ_{RP}	0.0068	(0.0000, 0.0135)
Monetary policy rule		
ρ_R	0.6704	(0.5821, 0.7709)
a_1	1.9127	(1.6169, 2.2453)
a_2	0.9788	(0.6477, 1.2609)
a_3	0.0257	(0.0107, 0.0399)
Shock autocorrelations		
ρ_Λ	0.7274	(0.61840.8343)
ρ_τ	0.8430	(0.79540.9228)
ρ_ξ	0.8741	(0.82380.9270)
Shock variances		
σ_Λ	0.0177	(0.0114, 0.0237)
σ_τ	0.0286	(0.0202, 0.0378)
σ_ξ	0.0070	(0.0045, 0.0093)
σ_μ	0.0041	(0.0031, 0.0049)

Table 2.4 Posterior Estimates for NK Model

Log data density: 1883.09

Parameters	Mean	90% Probability intervals
Structural parameters		
σ_1	1.7386	(1.36552.1107)
a_T	0.4135	(0.05330.7195)
a_H	0.9543	(0.8737, 1.0000)
ρ	1.1080	(0.7948, 1.4095)
ϕ	1.1929	(0.8671, 1.5109)
χ_P	15.0984	(4.9076, 27.2461)
ψ_{RP}	0.0053	(0.0000, 0.0109)
Monetary policy rule		
ρ_R	0.7403	(0.6840, 0.8006)
a_1	1.8365	(1.5195, 2.1466)
a_2	0.9442	(0.6679, 1.2451)
a_3	0.0453	(0.0177, 0.0729)
Shock autocorrelations		
ρ_Λ	0.7092	(0.6018, 0.8194)
ρ_τ	0.8578	(0.7902, 0.9274)
ρ_ξ	0.8792	(0.8279, 0.9350)
Shock variances		
σ_Λ	0.0131	(0.0084, 0.0190)
σ_τ	0.0194	(0.0142, 0.0248)
σ_ξ	0.0066	(0.0040, 0.0090)
σ_μ	0.0033	(0.0028, 0.0039)

Table 2.5 Root Mean Squared Errors of One Period Ahead Forecasts

(In percent)

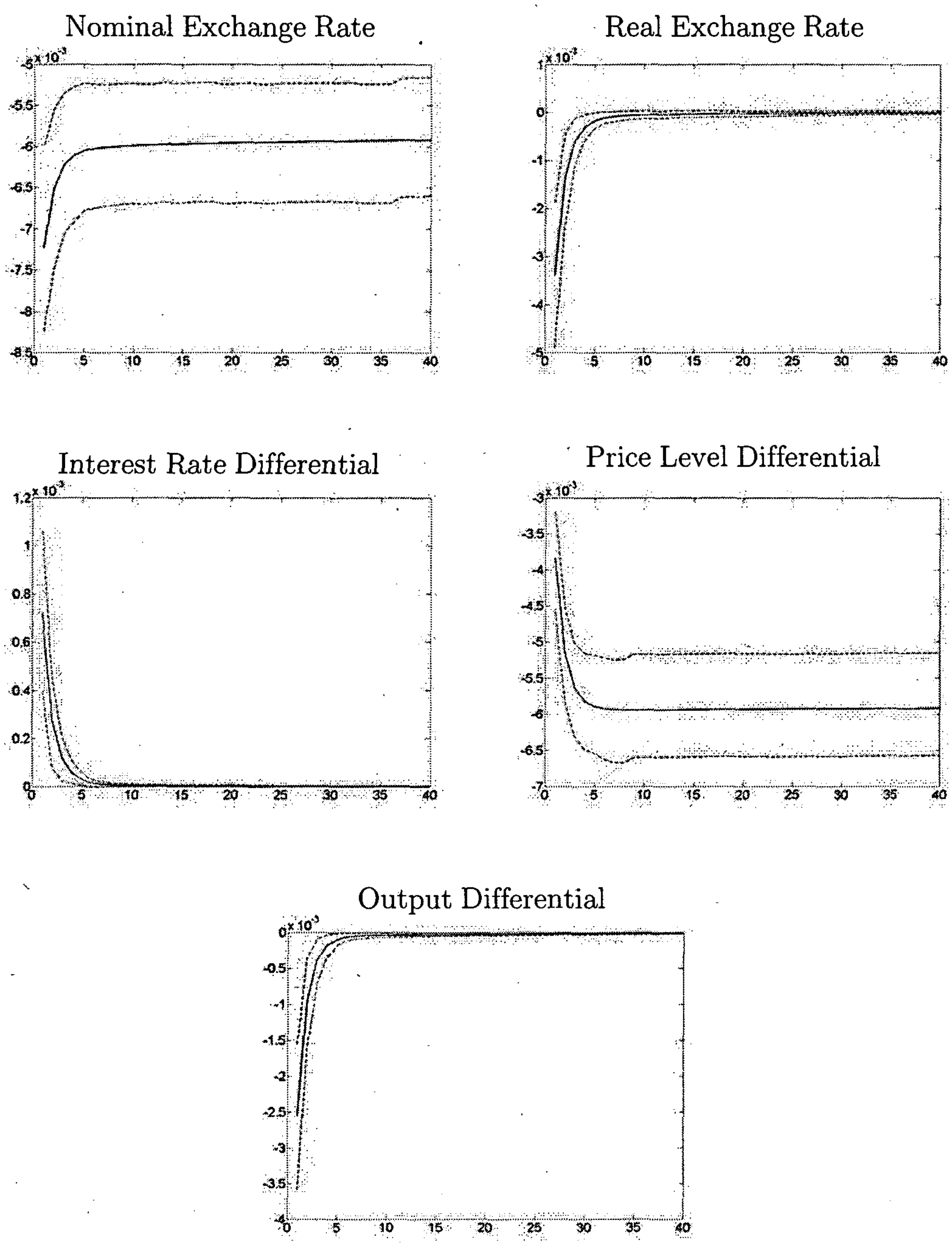
	TNT Model	T Model	NK Model
Output differential	0.917	0.918	0.914
Price differential	0.604	0.598	0.613
Interest rate differential	0.234	0.231	0.236
Real exchange rate	4.683	4.662	4.703

Table 2.6 Business Cycle Statistics*

Statistic	Data	TNT Model	T model	NK Model
Standard Deviations				
Output differential	0.0142	0.0116	0.0115	0.0115
Con. differential	0.0133	0.0138	0.0153	0.0117
Price differential	0.0010	0.0106	0.0105	0.0107
Interest rate differential	0.0033	0.0031	0.0031	0.0032
Nominal exchange rate (NER)	0.0784	0.0601	0.0597	0.0612
Real exchange rate (RER)	0.0757	0.0611	0.0607	0.0621
Autocorrelations				
Output differential	0.7806	0.6858	0.6899	0.6874
Con. differential	0.7853	0.6843	0.6987	0.6869
Price differential	0.8178	0.8273	0.8271	0.8299
Interest rate differential	0.7141	0.6765	0.6860	0.6729
Nominal exchange rate	0.8074	0.6787	0.6771	0.6860
Real exchange rate	0.7928	0.6799	0.6775	0.6868
Cross-Correlations Between				
RER and Con. differential	−0.1973	−0.1902	−0.2383	−0.0798
RER and NER	0.9922	0.9849	0.9850	0.9849
Output and Con. differential	0.7946	0.9024	0.9162	0.9878

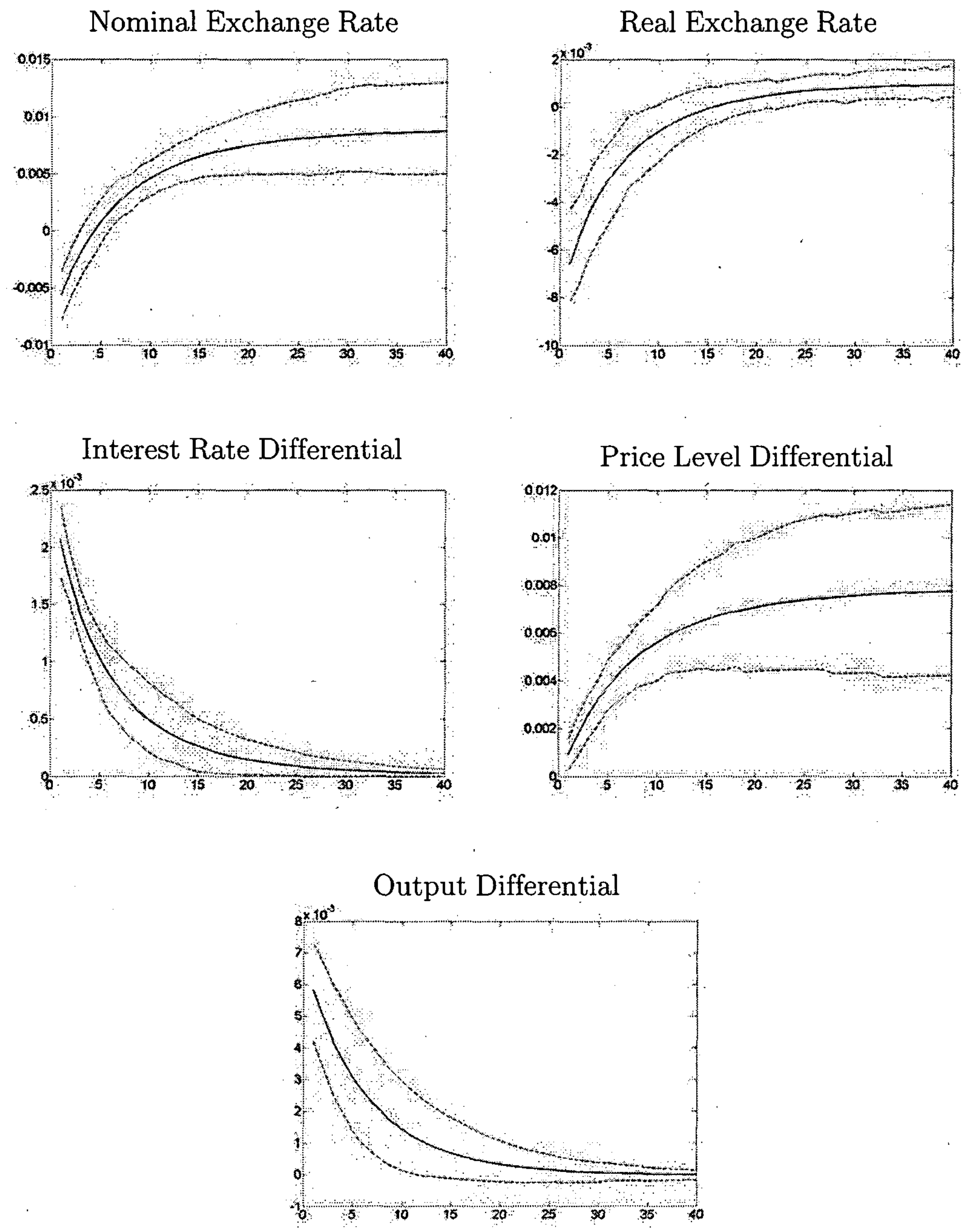
*Con. is the short for consumption.

Figure 2-1 Impulse Responses to a Monetary Policy Shock*



* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 2-2 Impulse Responses to a Preference Shock*



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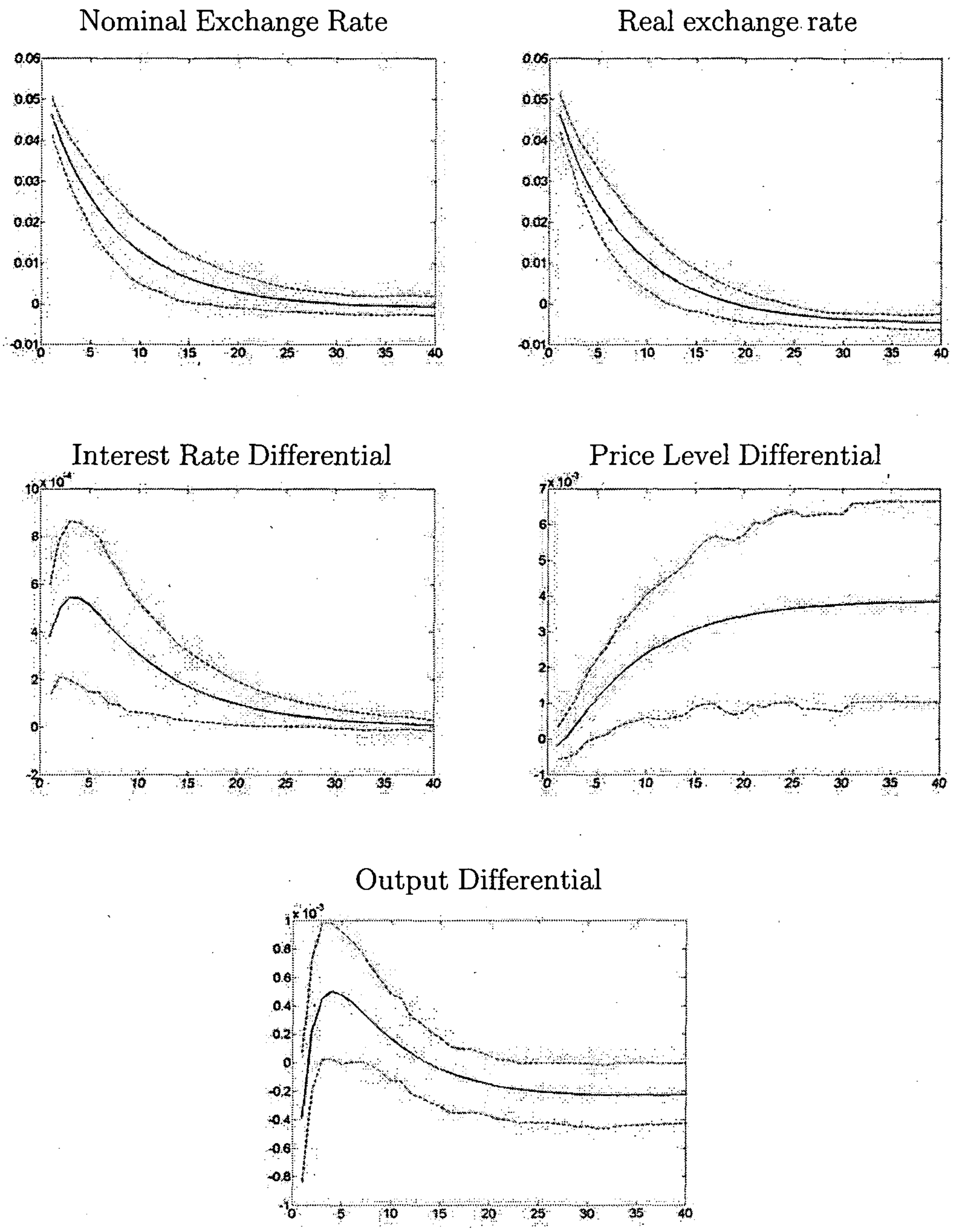
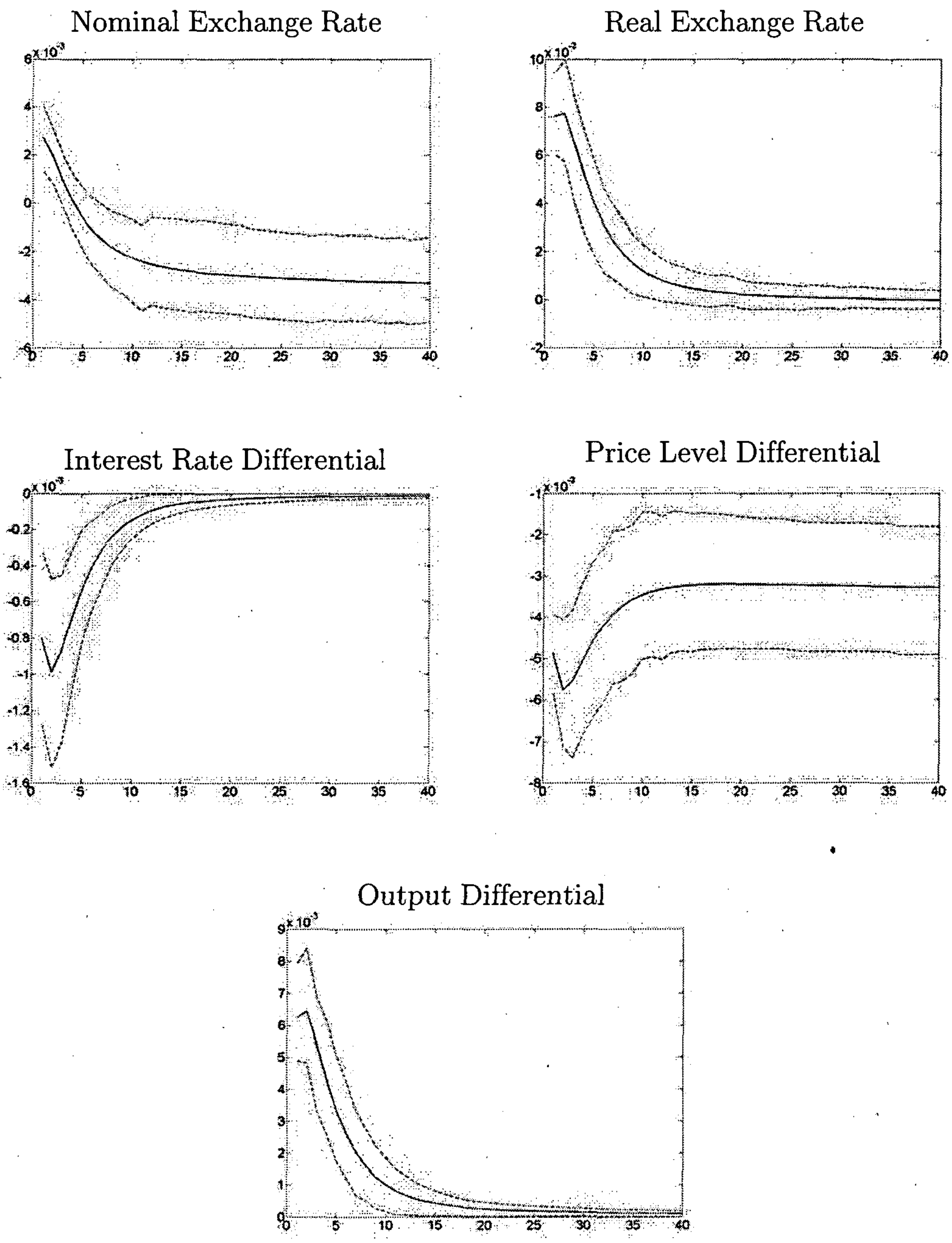
* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 2-3 Impulse Responses to an UIP Shock*



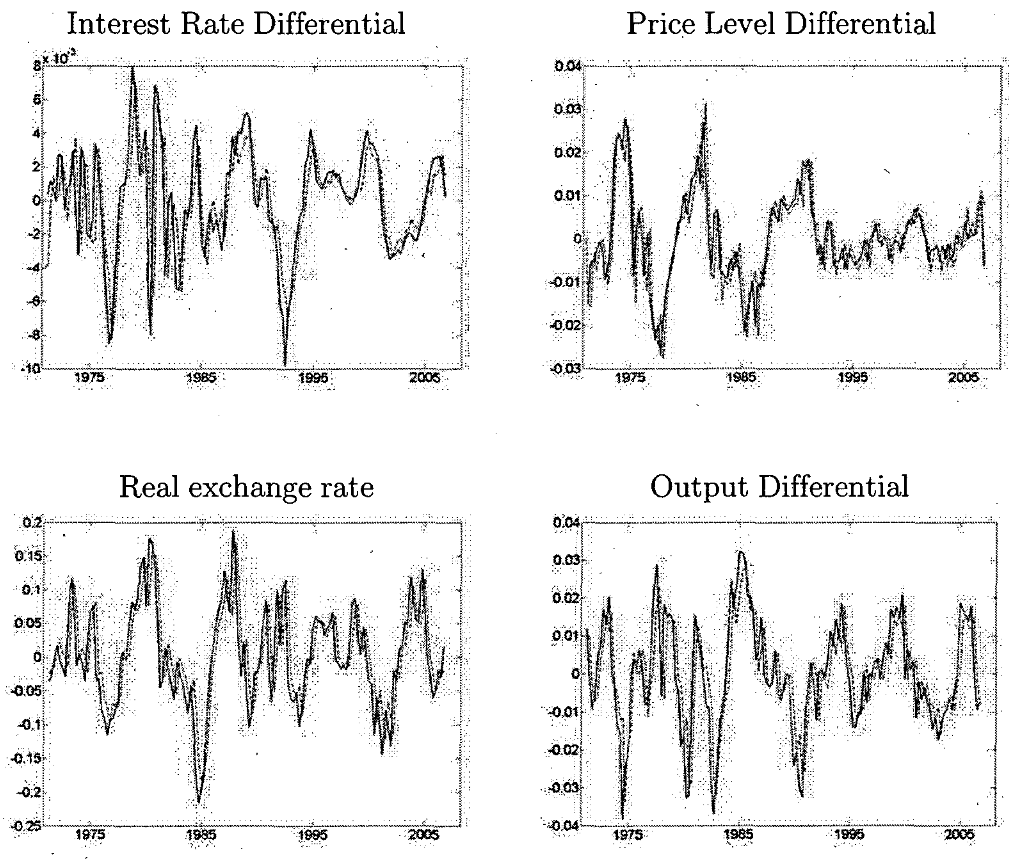
* Figure depicts posterior means (solid lines) and pointwise 90% pôsterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 2-4 Impulse Responses to a Tradable Technology Shock*



* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 2-5 In-Sample Fit of the TNT Model*



* The solid lines represent the data, and dash lines are the Kalman filtered estimates of the variables.

Chapter 3

What Determines the Pound-Euro Real Exchange Rate Fluctuations?

Abstract

What is the nature of the shocks that drives the real exchange rate movements is an important question for policy makers. Theoretically, if the real shocks dominate monetary shocks, then it could inform policy makers that a flexible exchange rate system is preferable to a fixed rate regime. Therefore, a decomposition of the pound-euro real exchange rate fluctuations is an important criterion on the issue whether the U.K. should join the Euro area. However, while the dollar-euro exchange rates have been intensively studied in New Open Economy Macroeconomics, there are few papers focusing on the behaviour of the pound-euro exchange rates. Thus, in this paper, I construct and estimate a two-country New Open Economy Macroeconomics model for the U.K.-Euro area economy. I find that real shocks are the predominant deliver of both nominal and real exchange rate volatilities for the U.K.-Euro area. This result supports the argument that the U.K. should not join the Euro area.

Keywords: New Open Economy Macroeconomics; Real exchange rate; Variance decomposition; Distribution sector

JEL classification: C11; F31; F41;

3.1 Introduction

The choice of the optimal exchange rate regime is a long-standing question in international economics. In a series paper, Mundell (1960, 1961a, 1961b, 1963) argue in favour of the floating exchange rate regime, as the pass-through of exchange rates to import prices generates an expenditure-switching effect between home and foreign goods and lends a stabilisation role to exchange rates in the face of country-specific real shocks. However, Corsetti and Pesenti (2001) and Devereux and Engel (2003) argue that fixed exchange rate regime is preferable in absence of the expenditure-switching effects, as fixed exchange rate help reducing real exchange rate volatilities when prices are preset and exporters price on local currencies. Taking both arguments into consideration, Devereux and Engel (2006) conclude that the choice of exchange rate policy is a trade-off between the desire to smooth fluctuations in real exchange rate in order to achieve smaller cross-country deviations in consumer prices, and the need to allow flexibility in the nominal exchange rate so as to facilitate terms of trade adjustment.

Theoretically, the fixed exchange rate can help reducing the real exchange rate volatilities, while the floating exchange rate regime supplies the expenditure-switching effects. Therefore, if the real shocks dominate monetary shocks in driving the movements of real exchange rates, the fixed exchange rate regime would has only limited effects on real exchange rate stabilisation. In this case, the floating exchange rate regime would be more preferable as it provides the expenditure-switching effects. Consequently, if the real shocks dominate monetary shocks, then it could inform policy makers that a flexible exchange rate system is preferable to fixed rate regime, or at least would highlight an important potential drawback associated with maintaining a system of fixed exchange rates. Thus, what determines the pound-euro real exchange rate fluctuations has become an important question since the U.K. government is currently considering about whether the U.K. should join the Euro area.

Since Dornbusch (1976), many economists have long suspected that monetary shocks together with sluggish price adjustment might play an important role in accounting for real exchange rate volatilities. However, a number of empirical studies with different identifying assumptions show that the real shocks are more important than or at least as important as nominal shocks.

Lastrapes (1992) use the Blanchard-Quah approach to estimate structural VARs using monthly nominal exchange rate and price level series from March 1973 to December 1989 for Canada, Germany, Italy, Japan, the U.K. and the U.S.. The results show that real shocks dominate nominal shocks for real exchange rates over short and long frequencies. Clarida and Gali (1994) also use the Blanchard-Quah identification strategy to identify the sources of real exchange fluctuations for the same countries except Italy since the collapse of Bretton Wood. Theyn also find that monetary shocks are less important. Bergman, Cheung and Lai (2000) use VEC model to study an alternative perspective on the individual roles of monetary shocks and productivity shocks in real exchange rates fluctuation using the data from April 1973 to December 1998 for Germany, Japan and the U.K.. They find that real shocks play a significant role in real exchange rate fluctuations.

The development of the study in international business cycles leads this debate to the New Open Economy Macroeconomics (NOEM) framework, but this question still remains controversial. For example, Chari, Kehoe and McGrattan (2002) calibrate a DSGE model with complete markets and a high degree of risk aversion. They find that monetary shocks, along with price stickiness are enough to account for real exchange rate volatilities. However, Cristadoro, Gerali, Neri and Pisani (2006) estimate a DSGE model using data for the U.S. and the Euro area with Bayesian approach. They find that the contribution of monetary shocks to real exchange rate volatilities is very low. Using the same Bayesian approach, Rabanal and Tuesta (2006) find that monetary shocks have played a minor role in explaining the behaviour of the dollar-euro real exchange rates, while real shocks have been important.

While the dollar-euro real exchange rates have been studied intensively in NOEM models, there are few papers that address on the fluctuations of pound-euro real exchange rates. So, in this paper, I try to contribute to this debate with a NOEM model using data for the U.K. and Euro area. The model shares lots of common characteristics of the NOEM literatures, such as sluggish local currency price setting (LCP),[1] monopolistically competitive firms, capital accumulating subject to adjustment cost,

[1]Engel (2000) shows that there is local currency pricing for consumer goods for nine European countries: Belgium, Denmark, Germany, Spain, France, Italy, the Netherlands, Portugal and the U.K., and consumer goods prices do not respond to exchange rate changes.

deviation to the uncovered interest rate parity, and Taylor-style monetary policy rule. An important feature of this model is the presence of distribution sector. The distribution cost accounts for the differences between producer prices and consumer prices, which leads to the deviation from law of one price[2].

Following Burstein, Neves and Rebelo (2003), and Laxton and Pesenti (2003), I assume that non-tradable goods are the only input in the distribution sector, which delivers tradable goods for consumption and investment. This model is different from traditional NOEM models by assuming that the non-tradable goods are only used in the distribution sector, but not in the final consumption and investment.[3] This setting simplifies the model by subtracting non-tradable goods from the consumption basket, but without sacrificing the model's ability to capture real exchange rate volatilities. This is because the prices of non-tradable goods affect the consumer price index through distribution cost. Therefore the movements in the relative prices of non-tradable to tradable goods still account for the fluctuations in the real exchange rates.[4]

Five shocks are introduced to the DSGE system: the technology shocks from tradable and non-tradable sectors, the households' preference shock, the uncovered interest rate parity shock and the monetary policy shock. The model is then estimated using data for the U.K. and the Euro area during 1971:1-2006:4 with Bayesian techniques. Because of data availability, the Euro area data are approximated by the geometric weighted average of the France, Germany and Italy.

The variance decomposition tells the relative importance of the shocks to the real exchange rate fluctuations. The results show that the real shocks are much more important than nominal shocks in capturing the fluctuations in real exchange rates. The productivity shocks in aggregate explain around 50.2% of the real exchange rate variations, while the monetary policy shock can only explain 0.9%. The historical decomposition of real exchange rates also shows that the contributions of the real shocks are more important than that of monetary policy shock. Therefore, a flexible exchange rate system is preferable to fixed rates for the U.K. and Euro area economy. This result does not lend the support to the argument that the U.K. should join the

[2]Firms' local currency pricing behaviour also leads to deviation from law of one price in this model.

[3]For example, Laxton and Pesenti (2003) and Selaive and Tuesta (2006) assume the non-tradable goods are used in both consumption and distribution.

[4]For details, see section 3.2.3 and Appendix 5.2.1.

Euro area.

However, this results may seem contradict with Corsetti and Pesenti (2001) and Devereux and Engel (2003), who find that the fixed exchange rate is always preferable when the prices are preset and the exporters choose to price in local currency. Their argument is that, under the LCP case, as prices are preset in local currency, the nominal exchange rate adjustment has no effects on the prices, and thus it plays no role in altering patterns of expenditure between home and foreign goods. The optimal monetary policy no longer tries to move the exchange rates to achieve an optimal terms of trade adjustment, since the terms of trade do not influence the real side of the economy. The optimal exchange rate choice will solely focus on the real exchange rate stabilisation. As the consumer price indexes are preset, the optimal monetary policy is to keep nominal exchange rates constant, which also fixes real exchange rates.

Nevertheless, in our model, the prices are sticky, but they are not completely fixed in the short-run. Therefore, the export prices can still change if there are movements in nominal exchange rates, although with adjustment costs. In this case, the consumer price index does respond to movements in the exchange rates. As a result, the expenditure-switching effects of the flexible exchange rate still exist.

Moreover, Duarte and Obstfeld (2007) show that the presence of the different preferences across countries upsets the fixed exchange rate prescription even in absence of expenditure-switching effects. This is because with different preferences, the consumption indexes across countries would respond disproportionately strongly to country-specific productivity shocks. In this case, it is optimal for monetary authorities to affect domestic aggregate demand differently in response to country-specific real shocks, which implies a flexible exchange rate regime.

The paper is organised as follows. Section 2 introduces the theoretical model. Section 3 estimates the model with Bayesian approach using data from the U.K. and the Euro area. In section 4, I evaluate the model fit. Section 5 is dynamic properties of the model. Section 6 is the discussion. Section 7 is the conclusion.

3.2 Structure of the Model

The world economy consists of two countries, Home and Foreign, of equal size[5]. Each country is populated by a large number of identical infinitely lived households. The Home economy produces a continuum of differentiated intermediate tradable and non-tradable goods, indexed by $i \in [0,1]$ and $j \in [0,1]$, respectively. The Foreign country also produces two continuum of intermediate goods indexed by $i^* \in [0,1]$ and $j^* \in [0,1]$. All trade between countries is in intermediate tradable goods that are produced by monopolists who can charge different prices in the two countries. In addition, there is a distribution sector in each country, which uses domestic non-tradable goods to distribute tradable goods to the final good producers. The final good producers then combine Home and Foreign tradable goods to produce country-specific final goods which are used for consumption and investment. Goods produced in the Home country are subscripted with an H, while those produced in the Foreign country are subscripted with an F. Allocations and prices in the Foreign country are denoted with an asterisk.

3.2.1 The Households

In each country, there is a continuum of symmetric households. The households derive the utility from consumption, and leisure[6]. Households generate their income from supplying labour, renting capital and receiving profit from intermediate goods firms[7]. There is an incomplete asset market, where Home households can hold two types of risk free bonds, one denominated in Home currency B_t paying gross interest rate R_t, and another denominated in Foreign currency B_t^* paying gross interest rate R_t^*. It is also assumed that only the Foreign currency bond is traded internationally, the Home currency bond is only traded within Home country. In particular, B_H^* is the Foreign bonds held by Home households and B_F^* is held by Foreign households. The representative Home household chooses consumption C, labour supply L, investment

[5]For simplicity, the population in each country is normalized at unity. It is straightforward to allow for different population in each country as in Clarida, Gali and Gertler (2002), and Benigno and Benigno (2003).

[6]Therefore, the utility is negative related with labour effort.

[7]The distribution and final goods firms face perfect competition and thus have zero profit.

I, capital K, bonds B and B_H^* to maximise its lifetime utility:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{\tau_t}{1-\sigma_1} C_t^{1-\sigma_1} - \frac{\sigma_2}{1+\sigma_2} L_t^{\frac{1+\sigma_2}{\sigma_2}} \right),$$

The budget constraint is:

$$\begin{aligned} & B_t + s_t B_{H,t}^* - R_{t-1} B_{t-1} - s_t R_{t-1}^* B_{H,t-1}^* \\ & = P_t \omega_t L_t + P_t r_t K_{t-1} - P_t C_t - P_t I_t + \Pi_{H,t} + \Pi_{N,t}, \end{aligned} \tag{3.1}$$

where P is the price level, τ is the preference shocks, Π_H and Π_N are the profit from tradable and non-tradable sectors, respectively, δ is the depreciation rate.

The law of motion for capital is

$$K_t = (1-\delta) K_{t-1} + I_t - AC_{k,t}, \tag{3.2}$$

where AC_k is the capital adjustment cost with the form:

$$AC_{k,t} = \frac{\chi_k}{2} \frac{(K_t - K_{t-1})^2}{K_{t-1}}. \tag{3.3}$$

χ_k is the capital adjustment parameter.

Solving the household maximising problem gives the following optimality conditions. The consumption Euler equation is:

$$\frac{\tau_t C_t^{-\sigma_1}}{P_t} = R_t \beta E_t \frac{\tau_{t+1} C_{t+1}^{-\sigma_1}}{P_{t+1}}. \tag{3.4}$$

The consumption leisure trade-off:

$$L_t^{\frac{1}{\sigma_2}} = \omega_t \tau_t C_t^{-\sigma_1}. \tag{3.5}$$

Capital accumulation is set to equate the costs and expected benefits:

$$\tau_t C_t^{-\sigma_1} \left(1 + \chi_k \frac{K_t - K_{t-1}}{K_{t-1}}\right) = \beta E_t \tau_{t+1} C_{t+1}^{-\sigma_1} \left(1 - \delta + r_{t+1} + \frac{\chi_k}{2} \frac{K_{t+1}^2 - K_t^2}{K_t^2}\right). \tag{3.6}$$

Finally, the optimal portfolio choice implies an interest rate condition:

$$R_t E_t \frac{\tau_{t+1} C_{t+1}^{-\sigma_1}}{P_{t+1}} = R_t^* E_t \frac{s_{t+1}}{s_t} \frac{\tau_{t+1} C_{t+1}^{-\sigma_1}}{P_{t+1}}.$$

Log-linearising the equation above gives the Uncovered Interest Parity (UIP) condition:

$$\widehat{R}_t - \widehat{R}_t^* = E_t \widehat{s}_{t+1} - \widehat{s}_t,$$

where, $\widehat{x}_t = (x_t - \overline{x}) / \overline{x}$ is the percentage changes from the initial steady state, and $\overline{x}$ is the steady-state value. Since the well-documented strong departures from UIP[8], following Bergin (2006), I add a risk premium term to the right hand side of the above equation,

$$RP_t = -\psi_{RP} \left(\frac{B_t + s_t B_{H,t}^*}{P_t Y_t} \right) + \xi_t.$$

The first component of this term is a function of the debt of a country, which helps to remove the element of non-stationarity in the model. The second component is a mean-zero disturbances aiming at capturing time-varying deviations from UIP. And thus, the modified UIP condition is:

$$\left(\widehat{R}_t - \widehat{R}_t^* \right) = \left(E_t \widehat{s}_{t+1} - \widehat{s}_t \right) - \psi_{RP} \left(\frac{B_t + s_t B_{H,t}^*}{P_t Y_t} \right) + \xi_t. \tag{3.7}$$

The utility maximisation problem and optimal conditions for Foreign households is similar except they only trade the bond denominated in local currency. And thus, there is no modified UIP condition for Foreign households.

3.2.2 The Final Good Sector

The final good producers face a perfect competitive market. They use Home and Foreign tradable goods to produce the final good Y, which is then used for domestic consumption and investment. The technology adopted by the final good producer is:

$$Y_t = \left(a_H^{\frac{1}{\rho}} Y_{H,t}^{\frac{\rho-1}{\rho}} + (1 - a_H)^{\frac{1}{\rho}} Y_{F,t}^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}, \rho > 0,$$

[8] For example, see Lewis (1995).

where Y_H and Y_F are the aggregate of Home and Foreign tradable goods used in Home country, ρ denotes the intratemporal elasticity of substitution between Home and Foreign aggregate tradable goods, a_H is the Home bias, which measures the weight of Home tradable goods in the consumption basket.

The aggregate technology for Home and Foreign tradable goods are

$$Y_{H,t} = \left(\int_0^1 y_{H,t}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}}, \theta > 1,$$

$$Y_{F,t} = \left(\int_0^1 y_{F,t}(i^*)^{\frac{\theta-1}{\theta}} di^* \right)^{\frac{\theta}{\theta-1}},$$

where $y_H(i)$, and $y_F(i^*)$ represent individual Home and Foreign tradable good consumed in Home country, respectively; θ is the constant elasticity of substitution.

Final good producers behave competitively, maximising their profits each period:

$$\max P_t Y_t - \int_0^1 p_{H,t}(i) y_{H,t}(i) di - \int_0^1 p_{F,t}(i^*) y_{F,t}(i^*) di^*,$$

subject to the aggregate technology. P is the overall price index of the final good, $p_H(i)$, and $p_F(i^*)$ are the Home price of individual Home and Foreign tradable goods, respectively. The prices above are all denominated in Home currency.

The demand functions for individual intermediate goods are derived by solving the final good producer's maximising problem.

$$y_{H,t}(i) = a_H \left(\frac{P_{H,t}}{p_{H,t}(i)} \right)^{\theta} \left(\frac{P_t}{P_{H,t}} \right)^{\rho} Y_t, \tag{3.8}$$

$$y_{F,t}(i^*) = (1 - a_H) \left(\frac{P_{F,t}}{p_{F,t}(i^*)} \right)^{\theta} \left(\frac{P_t}{P_{F,t}} \right)^{\rho} Y_t. \tag{3.9}$$

The price index can be derived by imposing the zero-profit condition as the final goods market is perfect competitive:

$$P_{H,t} = \left(\int_0^1 p_{H,t}(i)^{1-\theta} di \right)^{\frac{1}{1-\theta}}, \tag{3.10}$$

$$P_{F,t} = \left(\int_0^1 p_{F,t}(i^*)^{1-\theta} di^* \right)^{\frac{1}{1-\theta}}, \tag{3.11}$$

and

$$P_t = \left(a_H P_{H,t}^{1-\rho} + (1-a_H) P_{F,t}^{1-\rho}\right)^{\frac{1}{1-\rho}}. \tag{3.12}$$

The analogous equations hold for Foreign country.

3.2.3 The Distribution sector

Following Burstein, Neves and Rebelo (2003), this paper assumes a perfect competitive distribution sector in the economy. The distribution cost implies a wedge between producer and consumer prices. In this case, the final good firms cannot purchase the intermediate goods directly from the intermediate good producers. Instead, firms in the distribution sector purchase the intermediate goods and distribute them to the final good producing firms using country-specific non-tradable goods. The aggregate technology and price index of the non-tradable good are analogous as the tradable good firm:

$$Y_{N,t} = \left(\int_0^1 y_{N,t}(j)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}},$$

$$P_{N,t} = \left(\int_0^1 p_{N,t}(j)^{1-\theta} dj\right)^{\frac{1}{1-\theta}},$$

where Y_N and P_N are the aggregate of Home non-tradable goods and Home non-tradable price index, respectively.

With competitive firms in the distribution sector, the consumer prices of the intermediate tradable goods are:

$$p_{H,t}(i) = \widetilde{p}_{H,t}(i) + \kappa P_{N,t}, \tag{3.13}$$

$$p_{F,t}(i^*) = \widetilde{p}_{F,t}(i^*) + \kappa P_{N,t}, \tag{3.14}$$

where κ are the units of a basket of differentiated non-tradable goods necessary to bring one unit of tradable goods to the consumers, $\widetilde{p}_{H,t}(i)$ and $\widetilde{p}_{F,t}(i^*)$ are the prices of Home and Foreign tradable goods at the producer level, $p_{H,t}(i)$ and $p_{F,t}(i^*)$ are the prices at the consumer level.

Thus, the demand of Home non-tradable goods can be derived as

$$y_{N,t}(j) = \left(\frac{P_{N,t}}{p_{N,t}(j)}\right)^{\theta} \kappa \left(a_H \left(\frac{P_t}{P_{H,t}}\right)^{\rho} + (1-a_H)\left(\frac{P_t}{P_{F,t}}\right)^{\rho}\right) Y_t. \tag{3.15}$$

The analogous demand function holds for Foreign non-tradable goods.

It is worth noting that this model is different from traditional NOEM models by assuming that the non-tradable goods are only used in the distribution sector, but not in the final consumption and investment. This setting simplifies the model by assuming the non-tradable goods do not enter the consumption index, without sacrificing the model's ability to capture real exchange rate volatilities. This is because the prices of non-tradable goods can still affect the consumer price index through distribution cost. Therefore the movements in the relative prices of non-tradable to tradable goods can still account for the fluctuations in the real exchange rates.

This argument can be seen clearly with the decomposition of the real exchange rate fluctuations. First, under the case where non-tradable goods are used in both final goods production and distribution sector, the real exchange rate fluctuations can be expressed as:

$$\begin{aligned}\widehat{q} &= \left[a_H\left(\widehat{s} + \widehat{\widehat{P}}_F^* - \widehat{\widehat{P}}_H\right) + (1-a_H)\left(\widehat{s} + \widehat{\widehat{P}}_H^* - \widehat{\widehat{P}}_F\right)\right] \\ &\quad + (1 - a_T + \kappa a_T)\left[\left(\widehat{\widehat{P}}_T - \widehat{P}_N\right) - \left(\widehat{\widehat{P}}_T^* - \widehat{P}_N^*\right)\right],\end{aligned}$$

where, $\widehat{\widehat{P}}_T$ and $\widehat{\widehat{P}}_T^*$ are the price indices at the producer level of the aggregate tradable goods used in Home and Foreign countries respectively. The solution is derived in the Appendix 5.2.1.

The first source of the real exchange rate movements is the deviations to the law of one price to tradable goods across countries. Without international price discrimination, this source is equal to zero. The second source is the movements in the relative price of tradable to non-tradable goods across countries. In absence of any of these deviations, the real exchange rate should be constant.

In this paper, I assume the non-tradable goods are only used to distribute tradable goods for consumption and investment, but not used directly in the final goods production. In this case, the real exchange rate can be expressed as:

$$\begin{aligned}\widehat{q} &= \left[a_H\left(\widehat{s}+\widehat{\widetilde{P}}_F^* - \widehat{\widetilde{P}}_H\right) + (1-a_H)\left(\widehat{s}+\widehat{\widetilde{P}}_H^* - \widehat{\widetilde{P}}_F\right)\right] \\ &+\kappa\left[\left(\widehat{\widetilde{P}}_T - \widehat{P}_N\right) - \left(\widehat{\widetilde{P}}_T^* - \widehat{P}_N^*\right)\right].\end{aligned}$$

It is clear that the movements of the relative price of tradable to non-tradable goods still affect the real exchange rate movements. The two equations above show that assuming that the non-tradable goods are not used directly in consumption does not affect the second source of the real exchange rate fluctuations. As mentioned before, this is because the movements in the relative price of tradable to non-tradable goods can still affect the real exchange rate fluctuations through the distribution sector.

3.2.4 The Tradable Intermediate Good Sector

In each country, there is a monopolistic competitive tradable good sector. Each tradable intermediate good firm is a monopolistic producer of a single differentiated good, who uses labour and capital as inputs. The technology of the firm that produces domestic tradable intermediate good "i" is a standard Cobb-Douglas production function:

$$y_{H,t}(i) + y_{H,t}^*(i) = \Lambda_{H,t} k_{H,t-1}(i)^{\alpha} l_{H,t}(i)^{1-\alpha}, \qquad (3.16)$$

where Λ_H is the technology common to all Home intermediate tradable good production firms, α is the capital share in the production, $k_H(i)$ and $l_H(i)$ are capital and labour used in the tradable goods production, respectively.

The price stickiness is introduced by assuming the quadratic menu costs. Following Ireland (1997), Dedola and Leduc (2001) and Bergin (2006), the adjustment cost is set as:

$$AC_{H,t}(i) = \frac{\chi_H}{2}\frac{\left[\widetilde{p}_{H,t}(i) - \left(1+\overline{\overline{\pi}}_H(i)\right)\widetilde{p}_{H,t-1}(i)\right]^2}{P_t\widetilde{p}_{H,t-1}(i)} y_{H,t}(i), \qquad (3.17)$$

$$AC_{H,t}^*(i) = \frac{\chi_F}{2}\frac{s_t\left[\widetilde{p}_{H,t}^*(i) - \left(1+\overline{\overline{\pi}}_H^*(i)\right)\widetilde{p}_{H,t-1}^*(i)\right]^2}{P_t\widetilde{p}_{H,t-1}^*(i)} y_{H,t}^*(i), \qquad (3.18)$$

where χ_H and χ_F measure the size of Home tradable goods price stickiness in the Home and Foreign countries, respectively. $\overline{\overline{\pi}}_H(i)$ and $\overline{\overline{\pi}}_H^*(i)$ are the steady state values of Home tradable good inflation rate in Home and Foreign countries, respectively. Thus, there is no adjustment cost when the steady state inflation rate prevails.

Engel (2000) show that there is local currency pricing for consumer goods for nine European countries: Belgium, Denmark, Germany, Spain, France, Italy, the Netherlands, Portugal and the U.K.. Taken that into consideration, I assume LCP behaviours for tradable good firms. Therefore, the law of one price does not need to hold even without distribution cost. The problem solved by Home tradable intermediate good producer i is to choose prices $\widetilde{p}_H(i)$, $\widetilde{p}^*_H(i)$, capital stocks $k_H(i)$, and labour inputs $l_H(i)$ to maximise its lifetime profit:

$$E_0 \sum_{t=0}^{\infty} \rho_t [\widetilde{p}_{H,t}(i) y_{H,t}(i) + s_t \widetilde{p}^*_{H,t}(i) y^*_{H,t}(i)$$
$$-P_t \omega_t l_{H,t}(i) - P_t r_t k_{H,t-1}(i) - P_t AC_{H,t}(i) - P_t AC^*_{H,t}(i)],$$

subject to the demand function (3.8) and its Foreign counterpart, production technology (3.16) and menu costs (3.17) and (3.18). Follow Bergin (2006), the price kernel ρ_t is used to value random date t payoffs. Since firms are assumed to be owned by the households, it is assumed that firms value future payoffs according to the households' intertemporal marginal rate of substitution in consumption, which is $\rho_t = \beta^t U'_{c,t}/U'_{c,0}$.

Solving this maximising problem, the optimal price setting rule for domestic sale of Home good "i" can be derived as:

$$\begin{aligned} & 1 - \theta \frac{\widetilde{p}_{H,t}(i)}{p_{H,t}(i)} + \frac{\theta \chi_H}{2} \frac{\left[\widetilde{p}_{H,t}(i) - \left(1 + \overline{\overline{\pi}}_H(i)\right) \widetilde{p}_{H,t-1}(i)\right]^2}{p_{H,t}(i) \widetilde{p}_{H,t-1}(i)} \\ & -\chi_H \frac{\widetilde{p}_{H,t}(i) - \left(1 + \overline{\overline{\pi}}_H(i)\right) \widetilde{p}_{H,t-1}(i)}{\widetilde{p}_{H,t-1}(i)} + \frac{\theta P_t}{p_{H,t}(i)} mc_{H,t} \\ = & -\frac{\chi_H}{2} E_t \frac{\rho_{t+1}}{\rho_t} \frac{y_{H,t+1}(i)}{y_{H,t}(i)} \frac{\widetilde{p}_{H,t+1}(i)^2 - \left[\left(1 + \overline{\overline{\pi}}_H(i)\right) \widetilde{p}_{H,t}(i)\right]^2}{\widetilde{p}_{H,t}(i)^2}, \end{aligned} \tag{3.19}$$

where the marginal cost of labour is:

$$mc_{H,t} = \frac{\omega_t}{(1-\alpha) \Lambda_{H,t} \left(k_{H,t-1}(i)/l_{H,t}(i)\right)^{\alpha}}. \tag{3.20}$$

The price setting rule for export is:

$$\left(1-\theta\frac{\widetilde{p}^{*}_{H,t}(i)}{p^{*}_{H,t}(i)}\right)s_t+\frac{\theta\chi_F}{2}s_t\frac{\left[\widetilde{p}^{*}_{H,t}(i)-\left(1+\overline{\overline{\pi}}^{*}_{H}(i)\right)\widetilde{p}^{*}_{H,t-1}(i)\right]^2}{p^{*}_{H,t}(i)\widetilde{p}^{*}_{H,t-1}(i)}$$
$$-\chi_F s_t\frac{\widetilde{p}^{*}_{H,t}(i)-\left(1+\overline{\overline{\pi}}^{*}_{H}(i)\right)\widetilde{p}^{*}_{H,t-1}(i)}{\widetilde{p}^{*}_{H,t-1}(i)}+\frac{\theta P_t}{p^{*}_{H,t}(i)}mc_{H,t}$$
$$=-\frac{\chi_F}{2}E_t\frac{\rho_{t+1}}{\rho_t}s_{t+1}\frac{y^{*}_{H,t+1}(i)}{y^{*}_{H,t}(i)}\frac{\widetilde{p}^{*}_{H,t+1}(i)^2-\left[\left(1+\overline{\overline{\pi}}^{*}_{H}(i)\right)\widetilde{p}^{*}_{H,t}(i)\right]^2}{\widetilde{p}^{*}_{H,t}(i)^2}. \tag{3.21}$$

And the optimal trade-off between capital and labour inputs is:

$$\frac{r_t}{\omega_t}=\frac{\alpha}{1-\alpha}\left(\frac{k_{H,t-1}(i)}{l_{H,t}(i)}\right)^{-1}. \tag{3.22}$$

The analogue conditions hold for Foreign intermediate tradable goods firms.

3.2.5 The Non-Tradable Intermediate Good Sector

In this paper, I assume that the non-tradable goods are only used to distribute the intermediate tradable goods and do not enter the final consumption and investment directly. As the non-tradable goods are labour intensive[9], I also assumed labour is the only input for non-tradable goods production. The technology of the firm that produces domestic non-tradable intermediate good "j" is:

$$y_{N,t}(j)=\Lambda_{N,t}l_{N,t}(j), \tag{3.23}$$

where Λ_N is the technology common to all Home non-tradable good firms, l_N is the labour input.

The quadratic menu cost is:

$$AC_{N,t}(j)=\frac{\chi_N}{2}\frac{\left[p_{N,t}(j)-(1+\overline{\pi}_N(j))\,p_{N,t-1}(j)\right]^2}{P_t p_{N,t-1}(j)}y_{N,t}(j). \tag{3.24}$$

Again, χ_N is the price stickiness parameter for Home non-tradable sector and $\overline{\pi}_N(j)$ is the steady state inflation rate. The problem solved by Home tradable intermediate

[9]For example, see Rebelo and Végh (1995).

goods producer j is to choose prices $p_N(j)$ and labour inputs $l_N(j)$ to maximise:

$$E_0 \sum_{t=0}^{\infty} \rho_t [p_{N,t}(j) y_{N,t}(j) - P_t \omega_t l_{N,t}(j) - P_t AC_{N,t}(j)],$$

subject to the demand function (3.15), the production function (3.23)and menu cost equation (3.24). Solving this problem, the optimal price setting rule for Home non-tradable goods firm is given as:

$$\begin{aligned} & 1 - \theta + \frac{\theta \chi_N}{2} \frac{[p_{N,t}(j) - (1 + \overline{\pi}_N(j))\, p_{N,t-1}(j)]^2}{p_{N,t}(j)\, p_{N,t-1}(j)} \\ & -\chi_N \frac{p_{N,t}(j) - (1 + \overline{\pi}_N(j))\, p_{N,t-1}(j)}{p_{N,t-1}(j)} + \frac{\theta P_t \omega_t}{p_{N,t}(j)\, \Lambda_{N,t}} \\ = & -\frac{\chi_N}{2} E_t \frac{\rho_{t+1}}{\rho_t} \frac{y_{N,t+1}(j)}{y_{N,t}(j)} \frac{p_{N,t+1}(j)^2 - [(1 + \overline{\pi}_N(j))\, p_{N,t}(j)]^2}{p_{N,t}(j)}. \end{aligned} \tag{3.25}$$

The optimal price setting rule for Foreign non-tradable firms is analogous.

3.2.6 The Monetary Policy

In this paper, I assume the monetary authorities in both countries adopt a Taylor-style interest rate feedback rule. In particular, the central banks in both countries adjust the nominal interest rate in response to deviation of inflation, a measure of output gap and nominal exchange rate depreciation:

$$\widehat{R}_t = \rho_R \widehat{R}_{t-1} + (1 - \rho_R) \left[a_1 \pi_t + a_2 \left(\widehat{Z}_t - \widehat{Z}_{t-1} \right) + a_3 \Delta \widehat{s}_t \right] + \varepsilon_{u,t}, \tag{3.26}$$

where Z is the output level measured as country's GDP, $\pi_t \equiv \ln(P_t/P_{t-1})$ is the aggregate inflation rate and $\Delta \widehat{s}_t \equiv \ln s_t - \ln s_{t-1}$ measures the nominal exchange rate depreciation rate. ε_u is the stochastic monetary policy shocks.

The monetary policy rule for Foreign country is analogue.

3.2.7 Market Clearing

The model is closed by imposing the following resource constraints and market clearing conditions. The Home resource constraint for final goods is:

$$Y_t = C_t + I_t + \int_0^1 AC_{H,t}(i)di + \int_0^1 AC^*_{H,t}(i)di + \int_0^1 AC_{N,t}(j)dj. \tag{3.27}$$

The total output is:

$$\overline{P}Z_t = \int_0^1 \overline{p}_H\,(i)\,y_{H,t}(i)di + \int_0^1 \overline{s}\overline{p}^*\,(i)\,y^*_{H,t}(i)di + \int_0^1 \overline{p}_N\,(j)\,y_{N,t}(j)dj, \tag{3.28}$$

where the GDP Z_t is evaluated using steady state relative prices. Note that Y is the total goods consumed in the Home country, while Z is the total goods produced in the Home country. The Home labour market clearing condition is given by:

$$L_t = \int_0^1 l_{H,t}(i)di + \int_0^1 l_{N,t}\,(j)\,dj. \tag{3.29}$$

And the Home capital market clearing condition is given by:

$$K_t = \int_0^1 k_{H,t}(i)di. \tag{3.30}$$

The bonds market clearing condition is

$$B^*_{H,t} + B^*_{F,t} = 0. \tag{3.31}$$

The trade balance TB is the total value of exports minus the value of imports:

$$TB_{H,t} = \int_0^1 s_t p^*_{H,t}\,(i)\,y^*_{H,t}(i)\,di - \int_0^1 p_{F,t}\,(i^*)\,y_{F,t}\,(i^*)\,di^*. \tag{3.32}$$

Combining the budget constraint of the households and firms together with the equation above, the nominal current account can be derived as:

$$s_t B^*_{H,t} - s_t R^*_{t-1} B^*_{H,t-1} = TB_{H,t}, \tag{3.33}$$

Finally, the real exchange rate q is defined as:

$$q_t = \frac{s_t P_t^*}{P_t}. \tag{3.34}$$

3.2.8 The Exogenous Law of Motion

The productivity shocks in the tradable and non-tradable are specified as:

$$\log \Lambda_{H,t} = \rho_H \log \Lambda_{H,t-1} + \varepsilon_{H,t}, \ \varepsilon_{H,t} \sim N(0, \sigma_H); \tag{3.35}$$

$$\log \Lambda_{N,t} = \rho_N \log \Lambda_{N,t-1} + \varepsilon_{N,t}, \ \varepsilon_{N,t} \sim N(0, \sigma_N). \tag{3.36}$$

The shock to the households' preference is:

$$\log \tau_t = \rho_\tau \log \tau_{t-1} + \varepsilon_{\tau,t}, \ \varepsilon_{\tau,t} \sim N(0, \sigma_\tau). \tag{3.37}$$

The modified UIP shock follows the process:

$$\xi_t = \rho_\xi \xi_{t-1} + \varepsilon_{\xi,t}, \ \varepsilon_{\xi,t} \sim N(0, \sigma_\xi). \tag{3.38}$$

Moreover, there is shock to the monetary policy rule: $\varepsilon_{\mu,t} \sim N(0, \sigma_\mu)$. The autoregressive coefficients on shock processes ρ_H, ρ_N,ρ_τ, ρ_ξ are restricted to be greater than zero and less than unity.

The analogous conditions hold for Foreign country.

3.3 Estimation

In this section, the model will be first log-linearised around its steady-state. And then, the Bayesian techniques of the DSGE model will be used to estimate the model.

In particular, the model will be linearised around a global symmetric steady state. In this steady state, all corresponding variables are equal across countries and the net Foreign asset position is zero. For simplicity, it is also assumed that the prices in the consumer level are equal, and thus the relative price between producer level and consumer level is $1-\kappa$, where κ is the distribution cost parameter. In the steady state, the inflation rate is set to zero; and the nominal interest rate is bound to the households' subjective discount rate, β.

The model is then log-linearised. The deviation of the logarithm of a variable from its steady state is denoted with $\widehat{x}_t = \ln x_t - \ln \overline{x}$. The model is further written as country difference in order to reduce the dimensions of the data set and parameter space. This operation is necessary to make the sizeable empirical exercise tractable. The details of the log-linearised model can be found in the appendix.

3.3.1 Data

Data for the U.K. will be used for the Home country, and the data for Euro area will be used for the Foreign country. Because of data availability, the aggregate of France, Germany and Italy are used as the approximation of the Euro area. All data are seasonally adjusted quarterly series for the period 1971:1-2006:4, obtained from IMF International Financial Statistics and OECD Main Economic Indicators. Nine time series data are used in the estimation, which are real GDP per capita, real consumption per capita, inflation rate and short-run nominal interest rate for both the U.K. and Euro area, and the pound-euro real exchange rates. The Euro area aggregate data are computed as a geometric weighted average, where the time-varying weights are based on each country's share of total real GDP. The real exchange rate is calculated using

$$q_t = s_t \frac{CPI_{EA}}{CPI_{UK}},$$

where CPI_{EA} and CPI_{UK} are the consumer price index of Euro area and the U.K., respectively. Staring in 1999:1, the official pound-euro exchange rates are used. Prior to that, a synthetic bilateral exchange rate series are constructed with the form:

$$s_t = \Pi_{i=1}^{n} \left(f_i^* s_{i,t}\right)^{w_{i,t}},$$

where $w_{i,t}$ is the time-varying weight corresponding to the real GDP weights, f_i^* is the fixed conversion rates between the national currency units and the euro, $s_{i,t}$ is exchange rate between each country's currency to pound.

The model has implication that all variables are in log-deviation from the steady state, and thus the data need to be pre-processed before the estimation stage. In particular, real GDP per capita, real consumption per capita, inflation rate and nominal interest rate are logged and detrended. For inflation rate, the data are derived from the consumer price index

$$\pi_t \equiv \ln CPI_t - \ln CPI_{t-1},$$

which are then detrended as well. Hodrick-Prescott filter with multiplier equals to 1600 is used to detrend data.[10] The last step is to transform the data into country differentials, that is, Home minus the Foreign. This transformation of course does not apply to real exchange rate data.

3.3.2 Prior Distribution

A few parameters will not be estimated here, but instead are pinned down ahead of time. This can be viewed as a very strict prior, which would not be updated at all. The discount factor β is calibrated at 0.99, implying an annual real interest rate around 4%. The elasticity of substitution between varieties, θ is set to match existing estimate of steady-state price markup $\theta/(\theta-1)$. Martins, Scarpetta and Pilat (1996) estimate the average markup for manufacturing sectors at around 1.2 in OECD countries over the period 1980-92, which suggest setting θ equal to 6. The capital share parameter $\alpha = 0.33$ and the depreciation rate $\delta = 0.025$ are from Laxton and Pesenti (2003). The latter implies an annual depreciation rate for capital of about 10%. The labour supply elasticity $(1/\sigma_2)$ is calibrated at unity as suggested by Bergin (2006).

Prior distributions for the parameters are listed in Table 3.1. Following Cristadoro et al. (2006), the prior mean of Home bias a_H is set to 0.75, with standard deviation 0.2; the intratemporal elasticity of substitution between Home and Foreign tradable goods ρ is set equal to 1.1, with standard deviation 0.2; the mean of price rigidities and capital adjustment cost parameters are set equal to 50, with standard deviation 25.

Most Real Business Cycle literatures assume the intertemporal elasticity of substitution $1/\sigma_1$ between 0.5 and 1. And thus, the mean of σ_1 is set equal to 2, with standard deviation 0.25. Empirical analyses suggest values from 0.0035 to 0.0254 for ψ_{RP}, which measures the interest rate premium paid by Home agents for their net Foreign asset position. In this paper, the prior mean of ψ_{RP} is set to 0.01, with standard

[10]Hodrick and Prescott advise that, for quarterly data, a value of $\lambda = 1600$ is reasonable.

deviation 0.005.

According to the evidence for the U.S. economy in Burstein et al. (2003), the distribution costs are large and account for about $40-60$ percent of the retail price. Following their calibration, the mean of the distribution cost parameter κ is set equal to 0.5 with the stand deviation 0.2.

The priors for the coefficients in the monetary policy rule are from Lubik and Schorfheide (2005), which are loosely centred around values typically associated with the Taylor rule. In particular, the prior mean of the nominal interest rate persistent coefficient ρ_R is set to 0.5, with standard deviation 0.2. The responses of the interest rate to inflation rate a_1 and output growth a_2 are set to be 1.5 and 0.5, respectively, with the same standard deviation 0.25. The response of the interest rate to nominal exchange rate depreciation rate a_3 is set to 0.10, with the standard deviation 0.05.

Finally, the autoregressive parameters of the shocks are assumed to follow a beta distribution with mean 0.9 and standard deviation 0.05. All the shock variances coefficient are assumed to have inverse gamma-1 distribution, with mean 0.01. Parameters are also assumed to be *a priori* independent.

3.3.3 Parameter Estimates

Markov Chain Monte Carlo Methods are used to generate draws from the posterior distribution of the model parameters. The posterior means and 90% probability intervals reported in Table 3.2 are delivered by 500,000 run of Metropolis-Hastings algorithm.

The data seems to be particularly informative about the elasticity of substitution between Home and Foreign tradable goods ρ and the relative risk aversion coefficient σ_1. The posterior mean of ρ is around 1.78, which is essential updated from its prior mean of 1.1. The higher value of ρ corresponds to a higher level of substitution between Home and Foreign tradable goods in the utility function. The estimated mean of σ_1 is about 2.48, with 90% probability interval ranges from about 2.08 to 2.86.

The posterior mean of Home bias coefficient a_H is around 0.65, which is updated from its prior mean of 0.75. Some empirical literatures in NOEM find a very high value of Home bias, for example Lubik and Schorfheide (2005) estimate a value of 0.87 for the U.S.-Euro area economy. Cristadoro et al. (2006) report an even high value of 0.98, and they explain it as the attempt of the model trying to capture the exchange rate volatility of the data. Theoretically, higher Home bias leads to greater difference in consumption basket across countries, which is an important source of real exchange rate volatilities. In this model, the introduction of the distribution sector amplifies the deviations of law of one price for tradable goods[11]. Therefore, the model can match the volatilities of the real exchange rates by requiring small but more reasonable Home bias coefficient.[12]

The distribution cost parameter κ is also updated. The posterior mean is around 0.62, with the 90% probability interval ranges from about 0.55 to 0.70, which is similar to the finding of Burstein et al. (2003)[13].

The data are very informative about the price adjustment parameters. The posterior mean of local produced tradable price adjustment cost parameter χ_H is about 59.49, indicating a high price rigidity for local produced tradable goods. However, the posterior distribution for the exported or imported tradable price adjustment cost parameter χ_F and non-tradable price adjust cost parameter χ_N is very low. The results show that the price of local produced tradable goods is less volatile than the international traded tradable goods and non-tradable goods as the later have much lower price adjustment cost.

The estimates of capital adjustment cost parameter, χ_k is around 47.62, which implies that if the investment increases 1% above the steady state, about 0.50% of this investment is used to pay the adjustment cost.

The estimate mean of the risk premium paid by Home households for taking a position in the international financial markets, ψ_{RP}, is 0.0075, with 90% probability intervals range from 0.0013 to 0.0140. This is consistent with relevant empirical literatures[14]. This value implies that when the U.K. runs a net Foreign debt that is 10% of GDP, its domestic interest rate would rise by 7.5 basis points.

[11]Another source of deviations from law of one price for tradable goods is the tradable firms' local currency pricing behaviours.

[12]Another possible reason is that the home bias between U.K. and Euro Area are smaller than that between the E.U. and the U.S..

[13]Burstein et al. (2003) finds that the distribution costs accounts for about 40% – 60% of the retail price for the U.S. economy.

[14]Lane and Milesi-Ferretti (2001) estimate a value of 0.0107 from cross-sectional regressions and a value of 0.0254 from panels using data from 66 countries between 1970-1998. Nason and Rogers (2006) use a value of 0.0035 for Canadian data; Bergin (2006) estimates a value of 0.0038 with data from the E.U. and the U.S..

The posterior mean of persistence coefficient, ρ_R, is around 0.28, which indicates low persistence of short-run nominal interest rate. The posterior distribution of the interest rate response parameters is consistent with existing literatures. The estimates imply that the monetary policy makers react strongly to inflation and output movement, but only weakly to nominal exchange rate depreciation.

3.4 Model Fit

The fit of the model is assessed along several dimensions. I first compare the fitted series with the observed ones. Then, the model is compared with standard VAR models and Random walk models in terms of one-period ahead predictions. At last, I take a look at how well the model does in replicating the theoretical moments shown in the data, which is a common practice in the Real Business Cycle literatures.

3.4.1 Kalman Filter of the Observed Variables

In figure 3.1, I report the data and benchmark model's Kalman filtered one period ahead forecast of the observed variables, computed at the mean of posterior distribution. The in-sample fit of the model appears to be satisfactory, with the exception of the inflation rate. The model appears to predict low inflation rate in the periods around 1975, 1980 and 1990, which is not surprising given the high relative inflation rate during these periods observed from the data. In figure 3.2, I plot the historical inflation rate for the U.K. and the Euro area (the weighted average of France, Germany and Italy). Figure 3.2 shows that the inflation rate in the Euro area is relatively flat during the sample periods, while the inflation in the U.K. is very volatile. In particular, the U.K. suffered from chronic inflation during the periods from 1974-1977 (average annual interest rate around 16.37%, and 22.54% in 1975), 1979-1981 (annual average around 13.80%), and a high inflation in 1990 and 1991 (annual average around 7.37%).

3.4.2 Root Mean Squared Errors

In this section, I compare the one-period ahead forecast errors of the benchmark model with VAR models of various lags and random walk models. The root mean squared errors (RMSE) are reported in table 3.3. The result shows that all versions of the VAR models perform better than the structural model, which is not surprise given the same results are normally observed in NOEM papers.[15] However, it is worth noting that, the VAR with just one lags performs only slightly better than the benchmark models, and it is only after including four lags in the VAR that the RMSE for the observed variables drop significantly.

Bergin (2006) suggests that random walk model is 'a more fair comparison, given the VAR's extra parameters'. In table 3.3, the benchmark model does beat the random walk model and the random walk model with drift in forecasting one-period ahead real exchange rate: the benchmark model generates a RMSE of 3.696%, while the random walk model obtains a RMSE of 4.074%, and the random walk model with drift predicts a RMSE of 4.067%.

3.4.3 Unconditional Moments

The unconditional moments are the typical measure of fit used in Real Business Cycle exercises. The unconditional moments of the variables obtained from the data and model are reported in table 3.4. In general, the model does a good job in fitting the unconditional moments of the data. In particular, the model performs well in explaining real exchange rate volatility and persistence. The standard deviations of the relative consumption and inflation generated from the model are close to the data, although the model predicts higher volatility for the relative output and lower volatility for the relative interest rate. The autocorrelation of the relative inflation rate from the data is low (0.0046). Although the value generated from the model is also low (0.0902), it is still much larger than the data in relative terms. This again shows that the model does not manage to fit the relative inflation data successfully because of the high inflation rate for the U.K. in the periods around 1975, 1980 and 1990.

The model performs quite well in matching the cross-correlation between real exchange rate with relative inflation rate and interest rate. It is worth noting that the opposite of the classic *consumption real exchange rate anomaly*[16] happens: the cor-

[15] For example, see Bergin (2006), and Rabanal and Tuesta (2006).

[16] Chari et al. (2002) construct and calibrate a NOEM with complete financial market. Their model predicts a high and positive correlation between real exchange rate and relative consumption; while in the data, this correlation is negative. They refer to this large discrepancy as the *consumption real exchange rate anomaly*.

relation between the relative consumption and real exchange rate is positive, while the model generates a small negative relationship. The reason is the assumption of incomplete markets, which obstacles risk-sharing across countries.[17]

3.5 Dynamic Properties

3.5.1 Impulse Response Analysis

Impulse response functions are the expected future path of the endogenous variables conditional on a shock of one standard deviation in period 1. The impulse response functions show the inherent dynamics and the relative importance of different shocks. And thus, they contain important information about the linkage between the shocks and the fluctuations of the variables of interest. The impulse responses of endogenous variables of interest to the structural shocks are reported in Figure 3.3 to 3.7. As the model is log-linearised and transformed into country difference, all variables and shocks except nominal depreciation rate and real exchange rate are in terms of country differentials: $\widehat{x}_t$-$\widehat{x}_t^*$.

Figure 3.3 plots the impulse responses to a one standard deviation shock to the monetary policy. The relative Home monetary shock increases the Home relative interest rate through the monetary policy feedback rule. The monetary contraction in Home country also decreases relative Home inflation rate. As a result, the relative Home output and consumption decrease. The increase in relative interest rate sharply appreciates the Home currency, but the effects diminish quickly after a few periods. The real exchange rate also decreases; indicating pound appreciates against euro in real terms.

Figure 3.4 shows the impulse responses of the economy to a one standard deviation preference shock. This shock increases Home relative consumption immediately. Consequently, Home relative inflation rate and output increase. The Home monetary authority reacts positively to the inflation and output change, and thus, the Home relative nominal interest rate rises. The nominal and real exchange rate both decrease after this shock, indicating pound appreciates against euro in both nominal and real

[17] For details, see Chari et al. (2002) and Rabanal and Tuesta (2006).

terms. Note that, there is a small decrease in the relative inflation at the initial period before it starts to increase. This is because the initial Home currency appreciation improves the Home terms of trade. Therefore, the prices of Home imports decrease. If this effect denominates the price increase in Home tradable goods, the Home consumer price index should decrease. As the appreciation in nominal exchange rate diminish quickly, this effect also fades away quickly after the initial period.

Figure 3.5 illustrates an UIP shock to the model economy. This shock can be understand as a type of portfolio shift away from Home assets, such that an excess return is required to make households willing to hold home bonds in equilibrium. Therefore, the Home relative interest rate increases as implied by the modified UIP equation. The higher interest rate differential implies that consumption in the Home country decreases with respect to the Foreign country. The sharp depreciation of the Home currency in the initial period deteriorates the Home terms of trade: Home imports are more expensive and Home exports become cheaper. As a result, Home relative output increases because of the higher export; and Home inflation increase as the price increase of the imported goods. Note that, the change to the inflation and output differentials are small as these effects are partly counteracted by the increase of the interest rate differential.

Figure 3.6 shows the impulse responses of the six variables to a one standard deviation of Home relative tradable sector productivity shock. This shock increases the Home relative output and consumption and decrease the inflation. In the monetary policy rule, the increase in output growth overwhelms the decrease in the inflation, the nominal interest rate raises. The tradable sector productivity shock also depreciates the nominal and real exchange rate as expected.

Figure 3.7 plots the impulse responses to the productivity shock in the non-tradable sector. This shock leads to similar responses of the economy as the tradable productivity shock. The only difference is the interest rate. After the non-tradable productivity shock, the interest rate decreases as the effect of the negative inflation rate dominates in the monetary policy rule.

The impulse response functions show that the monetary policy shock has very limited effects on the real exchange rate. The real shocks, on the other hand, exhibit much greater effects on the real exchange rate: the tradable productivity shock generates a strong and very persistent adjustment in the real exchange rate; the effect of the non-tradable productivity shock on real exchange rate movements is also strong: it is about five times as much as that of the monetary policy shock.

3.5.2 Variance Decomposition

The Bayesian approach allows decomposing of exchange rate volatility into individual components explained by the five shocks in the model. Table 3.5 reports the decomposition of the forecast error variance of the nominal and real exchange rates in an infinitely long simulation.

The results show that the UIP shock explains most of the variance in real exchange rates (43.1%) and nominal depreciation rates (70.3%). Lubik and Schorfheide (2005) add error terms to either the UIP or the PPP equations to assess the degree of model misspecification in explaining real exchange rate dynamics. As in their model, the UIP shock in this paper is not contained in the model's primitives, and thus is not strictly structural. In this case, the UIP shock can capture all deviations from the modified UIP equation that has not been explained by the structure shocks of the model. This shock essentially captures model misspecification. If the estimated variance of this shock is small, the model can explain most of the observed real exchange rate fluctuations.

In Lubik and Schorfheide (2005), the non-structural shock explains more than 90% of the variation of the depreciation rates, which means their model can only explains less than 10% of the movements in the depreciation rates. A more similar model to our model is Cristadoro et al. (2006)[18], which find the non-structural UIP shock contributes to around 83.1% of real exchange rate volatilities. In this paper, the non-structural shock explains 43.1% of the variations in real exchange rates and 70.3% of the fluctuations in nominal depreciation rates. The proportion is still high, but it shows that this model does make great progress from Lubik and Schorfheide (2005) and Cristadoro et al. (2006).

The second largest components of real exchange rate variations come from the real shocks. The non-tradable productivity shock contributes to around 21.8%, and the tradable productivity shock explains about 28.4%. The posterior distribution of

[18]The differences of Cristadoro et al. (2006) from this model are that they do not include distribution cost and capital.

variance decomposition also shows that the preference and monetary policy shocks only account for only a small proportion of the real exchange rate variations (5.7% and 0.9%, respectively).

For the nominal depreciation rates, the non-tradable productivity shock contributes to the second largest proportion of the variations, around 16.1%. The monetary policy shock ranks third, which explains about 8.2% of the variations in nominal depreciation rates. The contributions of tradable productivity and preference shocks are small, with only 1.5% and 3.9%, respectively.

Overall, the result does show that the pound-euro real exchange rate dynamics are largely driven by real shocks, the effects of nominal shock are small. This finding is consistent with Cristadoro et al. (2006) and Rabanal and Tuesta (2006). Within real shocks, it is also worth noting that the contribution of non-tradable productivity shock to the real exchange rate volatility is as important as tradable productivity shock.

3.5.3 Historical Decomposition

Figures 3.8 to 3.12 report the historical contribution of the estimated shocks to the dynamics of the observable variables. The historical decomposition is based on the best estimates of the shocks and the posterior mean of the estimated parameters.

The result of historical decomposition of the real exchange rates is not different from the variance decomposition in the infinite horizon. The interest rate parity shock, tradable and non-tradable productivity shocks contributes the most to the dynamics of the real exchange rate. The effects of the preference and monetary policy shocks are small. This confirms that the pound-euro real exchange rate dynamics are largely driven by the productivity shocks, rather than monetary policy shocks.

For other observable variables, the results are also as expected. The consumption is mainly explained by the preference shock, followed by the non-tradable productivity shock. The inflation rate is mostly accounted for by the monetary policy shock. The interest rate is mostly explained by the interest rate parity shock, non-tradable productivity shock and preference shock. Tradable and non-tradable productivity shocks, together with preference shock, contribute the most to the dynamics of the output.

3.6 Discussion

Whether the U.K. should abandon the pound and adopt euro is a complicated question, which is related with various benefits and costs. One of the important benefits to adopt common currency is the insulation from monetary disturbances that might lead to temporary unnecessary fluctuations in real exchange rates. If the effect of monetary shock turns to be small in generating real exchange fluctuations, then this benefit is less important. The variance decomposition shows that the real shocks overwhelm the monetary policy shock in explaining the volatilities in both real and nominal exchange rates. Therefore, a flexible exchange rate system is preferable to fixed rates or currency union, or at least would highlight an important potential drawback associated with maintaining a system of fixed exchange rates. The results in this model do not lend support to the argument that the U.K. should join the Euro area.

In a closed solution NOEM model, Devereux and Engel (2003) study the optimal level of exchange rate flexibilities under two price settings rules: producer currency pricing (PCP) or local currency pricing (LCP). Under the assumption of preset prices, they find that the floating exchange rate regime is preferable under PCP, while the fixed exchange rate is always desirable under LCP. In our paper, however, the result shows that the floating exchange rate regime is preferable, even with the presence of LCP.

What derives the different results in this model with Devereux and Engel (2003)? To answer this question, we first need to learn the target of the exchange rate policy. Devereux and Engel (2006) treat the exchange rate policy as a trade-off between the desire to smooth fluctuations in real exchange rates in order to achieve smaller cross-country deviations in consumer prices on the one hand, and the need to allow flexibility in the nominal exchange rates so as to facilitate terms of trade adjustment on the other hand.

Under the PCP case, because prices are preset and preferences are identical across countries, the law of one price always holds. Therefore, the real exchange rates remain constant as shown in equation (3.34). In this case, the exchange rate policy only focuses on the expenditure-switching effects, thus the flexible exchange rate regime is preferable.

However, under the LCP case, as prices are preset in local currency, the nominal exchange rate adjustments have no effects on the prices, and thus play no roles in altering patterns of expenditure between home and foreign goods. The optimal monetary policy no longer tries to move the exchange rate to achieve an optimal terms of trade adjustment, since the terms of trade do not influence the real side of the economy. The optimal exchange rate choice will solely focus on the real exchange rate stabilisation. As the consumer's preferences are identical across countries, the real shocks lead to the same movements in the consumption indexes and the consumer price indexes. Since the consumer price indexes between Home and Foreign change proportionally, the optimal monetary policy is to keep nominal exchange rates constant, which fixes real exchange rates.

Nevertheless, in our model, the prices are sticky, but they are not completely fixed in the short-run. Equation (3.20) shows that the export prices can still change if there is a movement in nominal exchange rates, although with adjustment costs. In this case, the consumer price indexes in both counties will response to the movements in the exchange rates. As a result, the expenditure-switching effects of the flexible exchange rates still exist.

Moreover, even in the absence of expenditure-switching effects of exchange rate changes, the exchange rate flexibility is still desirable in response to country-specific real shocks. Duarte and Obstfeld (2007) show that the presence of the different preferences across countries, such as the presence of non-tradable goods or domestic preference for domestic tradable goods, upsets the fixed exchange rate prescription even prices are preset. This is because with different preferences, the consumption indexes across countries would response disproportionately strongly to country-specific productivity shocks. In this case, it is optimal for monetary authorities to affect domestic aggregate demand differently in response to country-specific real shocks, which implies a flexible exchange rate regime.

Therefore, the contradictions are original from the different model settings. Devereux and Engel (2003) assume that all prices are set prior to the realisation of shocks and can fully adjust to all shocks after one period; *i.e.* there is no persistence to the price adjustment process. This assumption, however, is made so that monetary rules can be derived analytically. For a more realistic design of the model, it would be necessary to introduce more persistent price stickiness.[19] Moreover, given the empirical evidence of home bias and non-tradable goods in consumption[20], the identical households' preference assumption made by Devereux and Engel (2003) is also less realistic. Taken these into consideration, the results in this model should have more implications for policy makings.

3.7 Conclusion

This paper studies what determines the fluctuations of the pound-euro real exchange rates. This question has important empirical meaning for the issue whether the U.K. should join the Euro area. A two-country NOEM model is employed to assess this question. A key feature of the model is to include distribution sector, which generates a wedge between producer prices and consumer prices. The model is different from traditional NOEM models by assuming that the non-tradable goods are only used in the distribution sector, but not in the final consumption basket. This setting simplifies the consumption index without affecting the model's ability to capture real exchange rate volatilities. The theoretical model is then estimated using data from the U.K. and the Euro area with Bayesian techniques. Some results emerge from the empirical analysis.

The variance decomposition and historical decomposition exhibit the contributions of the shocks to the real exchange rate fluctuations. The results show that the real shocks are much more important than monetary policy shock in capturing the fluctuations in real exchange rates. The productivity shocks in aggregate explain around 50.2% of the real exchange rate variations, while the monetary shock can only explain 0.9%. This finding does not lend support to the argument that the U.K. should join the Euro area as the real exchange rate stabilisation benefit of common currency is small.

However, the results might be heavy relied upon the model assumptions, such as LCP, incomplete asset market, identical country size. As suggested by Devereux and Engel (2003), and Duarte and Obstfeld (2007), we should be caution in evaluating the policy effects in open economies with different model settings. And also, the data

[19] For details, see Devereux and Engel (2003.)

[20] For example, see McCallum (1995), Wolf (2000) and Hillberry and Hummels (2002), Duarte and Obstfeld (2007).

used in this paper is from 1971 to 2006, which is a fairly long horizon. During this period, there are some structural changes occur to the E.U. economy, especially to the monetary system. For example, the European Exchange Rate Mechanism (ERM) was introduced by the European Community in March 1979, and was taken place by the single currency on 1 January 1999. The U.K. entered the ERM in 1990, but was forced to exit the programme in 1992 after the pound sterling came under major pressure from currency speculators. The empirical study on these sub-sample data with various model settings will help building more confidence on the policy choice.

Table 3.1 Prior Distribution*

Parameters	Domain	Density	Mean	SD
Structural Parameters				
σ_1	$\mathbb{R}^+$	Gamma	2.0	0.25
κ	$[0,1)$	Beta	0.5	0.2
a_H	$[0,1)$	Beta	0.75	0.2
ρ	$\mathbb{R}^+$	Gamma	1.1	0.2
χ_H	$\mathbb{R}^+$	Gamma	50	25
χ_F	$\mathbb{R}^+$	Gamma	50	25
χ_N	$\mathbb{R}^+$	Gamma	50	25
χ_k	$\mathbb{R}^+$	Gamma	50	25
ψ_{RP}	$[0,1)$	Beta	0.01	0.005
Monetary Policy Rule				
ρ_R	$[0,1)$	Beta	0.50	0.20
a_1	$\mathbb{R}^+$	Gamma	1.50	0.25
a_2	$\mathbb{R}^+$	Gamma	0.50	0.25
a_3	$\mathbb{R}^+$	Gamma	0.10	0.05
Shock Autocorrelations				
ρ_H	$[0,1)$	Beta	0.9	0.05
ρ_N	$[0,1)$	Beta	0.9	0.05
ρ_T	$[0,1)$	Beta	0.9	0.05
ρ_ξ	$[0,1)$	Beta	0.9	0.05
Shock Standard Deviations				
σ_H	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_N	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_T	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_ξ	$\mathbb{R}^+$	InvGamma	0.01	*inf*
σ_μ	$\mathbb{R}^+$	InvGamma	0.01	*inf*

*Shock variances are assumed to have inverted gamma-1 distribution, $y \sim IG1(v,s)$, where the density is

$$f(y) = \varsigma(\frac{v}{2}, \frac{2}{s})^{-1} * y^{-\frac{v}{2}-1} e^{-\frac{s}{2y}},$$

where $\varsigma(\alpha,\beta) = \Gamma(\alpha)\beta^{\alpha}$ is the constant of integration, and $\Gamma(n) = \int_0^\infty x^{n-1}e^{-x}dx$ is the gamma function. The mean (u) and standard deviation (σ) are thus defined as

$$u = \left(\frac{s}{2}\right)^{\frac{1}{2}} \frac{\Gamma\left(\frac{v-1}{2}\right)}{\Gamma\left(\frac{v}{2}\right)}, \quad \sigma = \left(\frac{s}{v-2} - u^2\right)^{\frac{1}{2}}.$$

Note, *inf* means infinite standard deviation, in this case, there is a closed form solution of v and s, which are: $v = 2$ and $s = \frac{2}{\pi}u^2$.

Table 3.2 Posterior Estimates

		Log data density: 2003.82
Parameters	**Mean**	90% **Probability intervals**
Structural parameters		
σ_1	2.4753	(2.0845, 2.8609)
κ	0.6245	(0.5531, 0.6985)
a_H	0.6545	(0.5540, 0.7567)
ρ	1.7819	(1.4563, 2.1131)
χ_H	59.4928	(18.2786, 99.7037)
χ_F	1.0708	(0.3947, 1.7728)
χ_N	3.8794	(1.5999, 6.7033)
χ_k	47.6204	(10.1066, 82.9547)
ψ_{RP}	0.0075	(0.0013, 0.0140)
Monetary policy rule		
ρ_R	0.2835	(0.13070.4334)
a_1	2.1816	(1.82662.5079)
a_2	0.9782	(0.72431.2290)
a_3	0.0762	(0.03960.1137)
Shock autocorrelations		
ρ_H	0.5465	(0.4404, 0.6474)
ρ_N	0.7456	(0.6709, 0.8191)
ρ_τ	0.7747	(0.7048, 0.8444)
ρ_ξ	0.7508	(0.6878, 0.8126)
Shock variances		
σ_H	0.1425	(0.0878, 0.1948)
σ_N	0.0239	(0.0167, 0.0312)
σ_τ	0.0377	(0.0311, 0.0442)
σ_ξ	0.0116	(0.0091, 0.0140)
σ_μ	0.0162	(0.0129, 0.0195)

Table 3.3 Root Mean Squared Errors of One Period Ahead Forecasts*

(In percent)

	RER	OUT	CON	INF	INT
Benchmark model	3.696	1.393	1.334	0.955	0.908
VAR (1)	3.577	1.272	1.277	0.876	0.827
VAR (2)	3.516	1.248	1.221	0.839	0.783
VAR (4)	3.380	1.082	1.091	0.716	0.752
VAR (6)	3.048	1.009	1.018	0.659	0.686
Random walk model	4.074				
Random walk with drift	4.067				

*For random walk model and random walk model with drift model, logarithm real exchange rate data is used instead of HP filtered data. The five variables of interest are: real exchange rate (RER), output differential (OUT), consumption differential (CON), inflation rate differential (INF) and interest rate differential (INT).

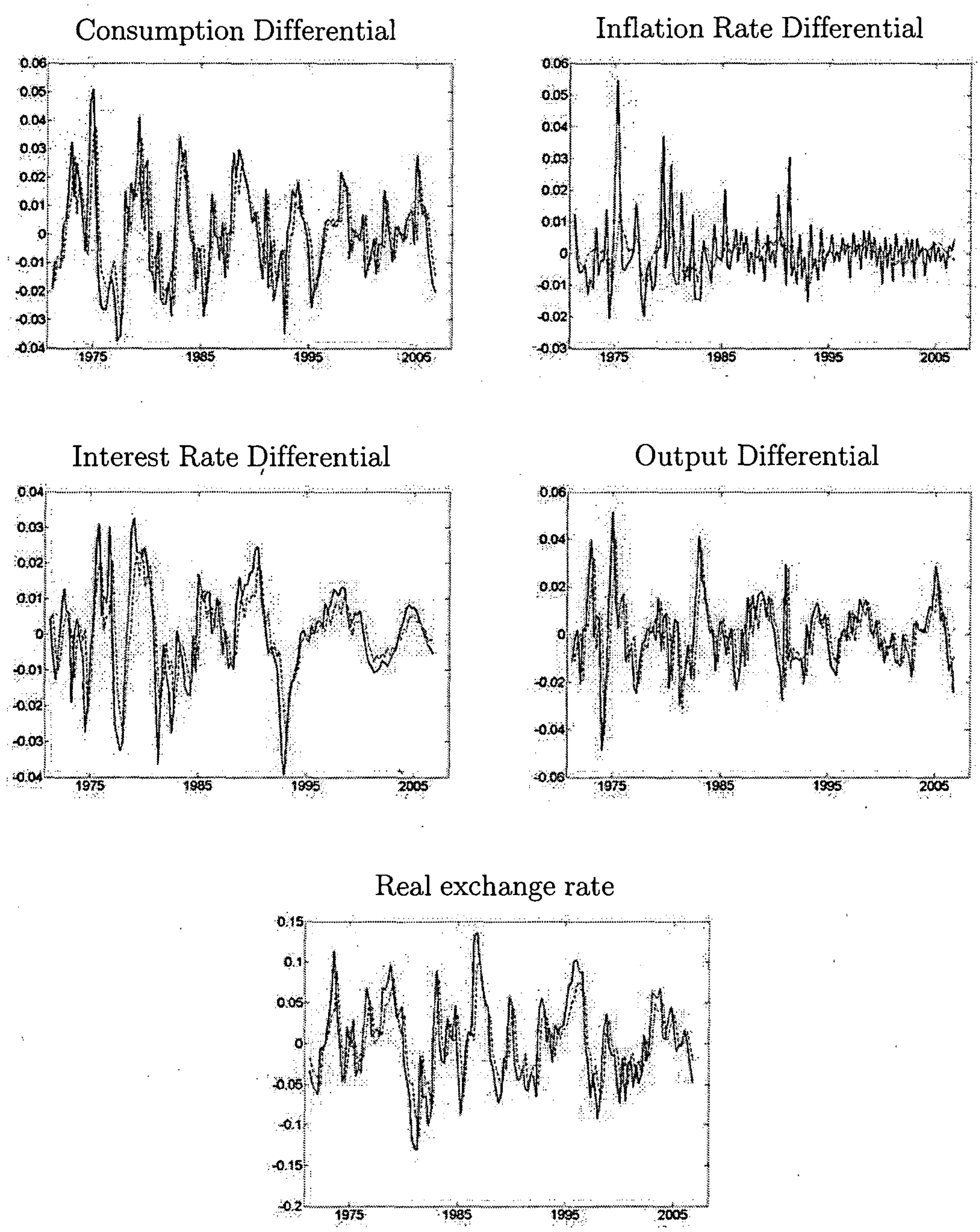
Table 3.4 Business Cycle Statistics

Statistic	Data	Model
Standard Deviations		
Output differential	0.0145	0.0177
Consumption differential	0.0171	0.0176
Interest rate differential	0.0140	0.0099
Inflation rate differential	0.0100	0.0101
Real exchange rate	0.0532	0.0485
Autocorrelations		
Output differential	0.4630	0.6284
Consumption differential	0.6429	0.6179
Interest rate differential	0.7719	0.5670
Inflation rate differential	0.0046	0.0902
Real exchange rate	0.7253	0.6218
Cross-Correlations Between Real exchange rate and		
Output differential	0.0796	0.3577
Consumption differential	0.1611	−0.0014
Interest rate differential	0.0557	0.0498
Inflation rate differential	−0.0694	−0.0760

Table 3.5 Variance Decompositions

	Real Exchange Rate		Nominal Depreciation Rate	
Shocks	Mean	90% Interval	Mean	90% interval
Tradable	0.284	(0.052, 0.497)	0.015	(0.004, 0.029)
Non-tradable	0.218	(0.061, 0.359)	0.161	(0.108, 0.230)
Monetary Policy	0.009	(0.003, 0.014)	0.082	(0.048, 0.109)
UIP	0.431	(0.269, 0.593)	0.703	(0.633, 0.783)
Preference	0.057	(0.019, 0.094)	0.039	(0.025, 0.057)

Figure 3-1 In-Sample Fit of the Benchmark Model*



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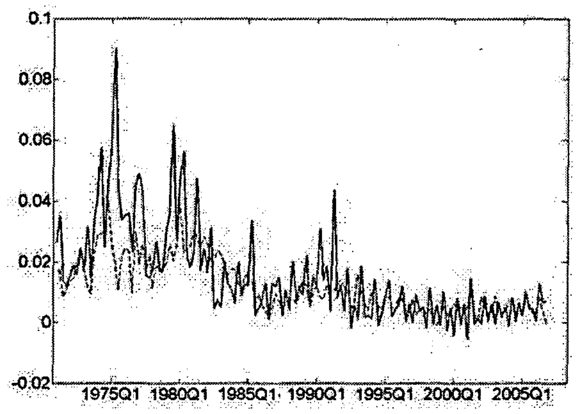
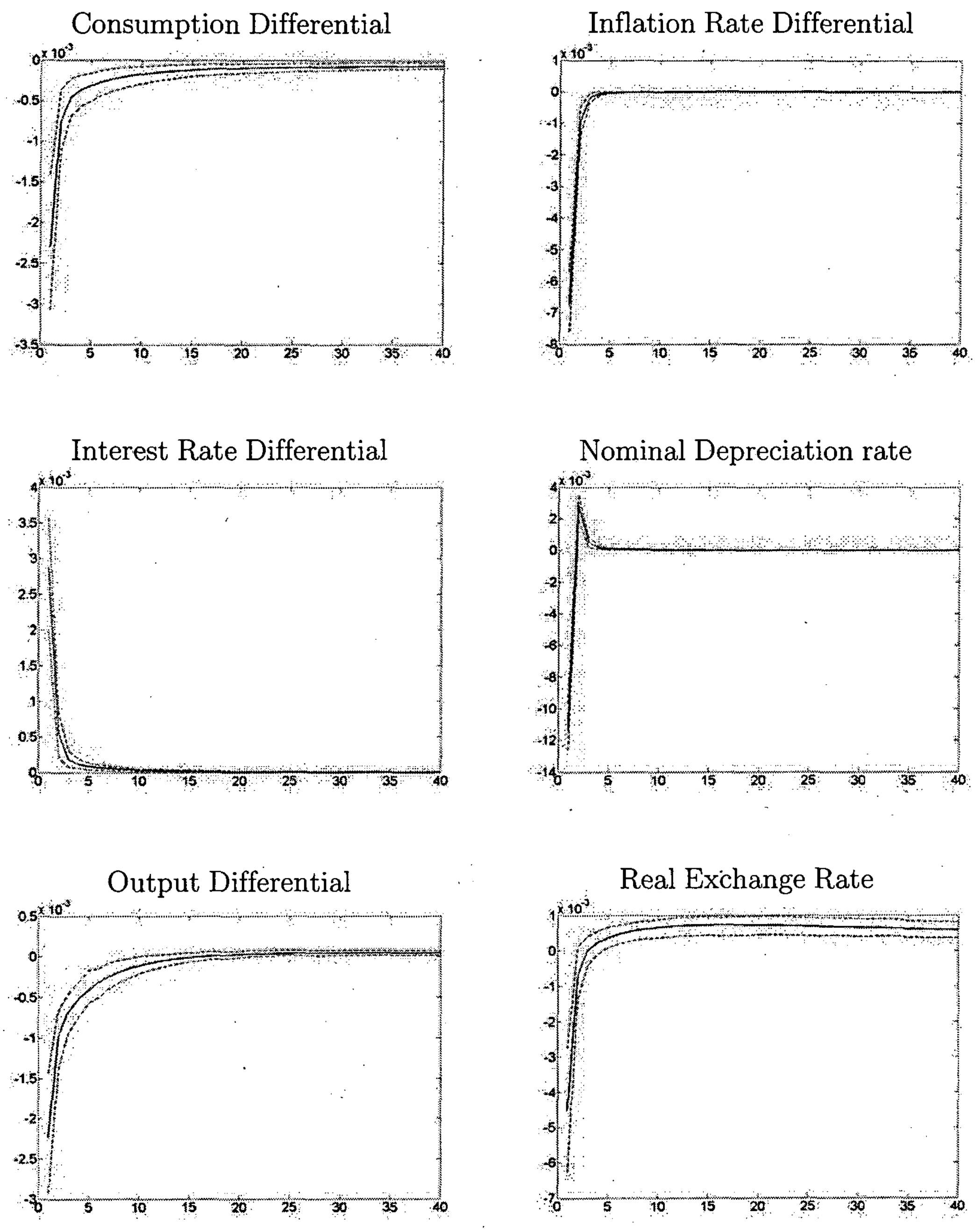
* The solid lines represent the data, and dash lines are the Kalman filtered estimates of the variables.

Figure 3-2 The U.K. and Euro Area Quarterly Inflation Rate*



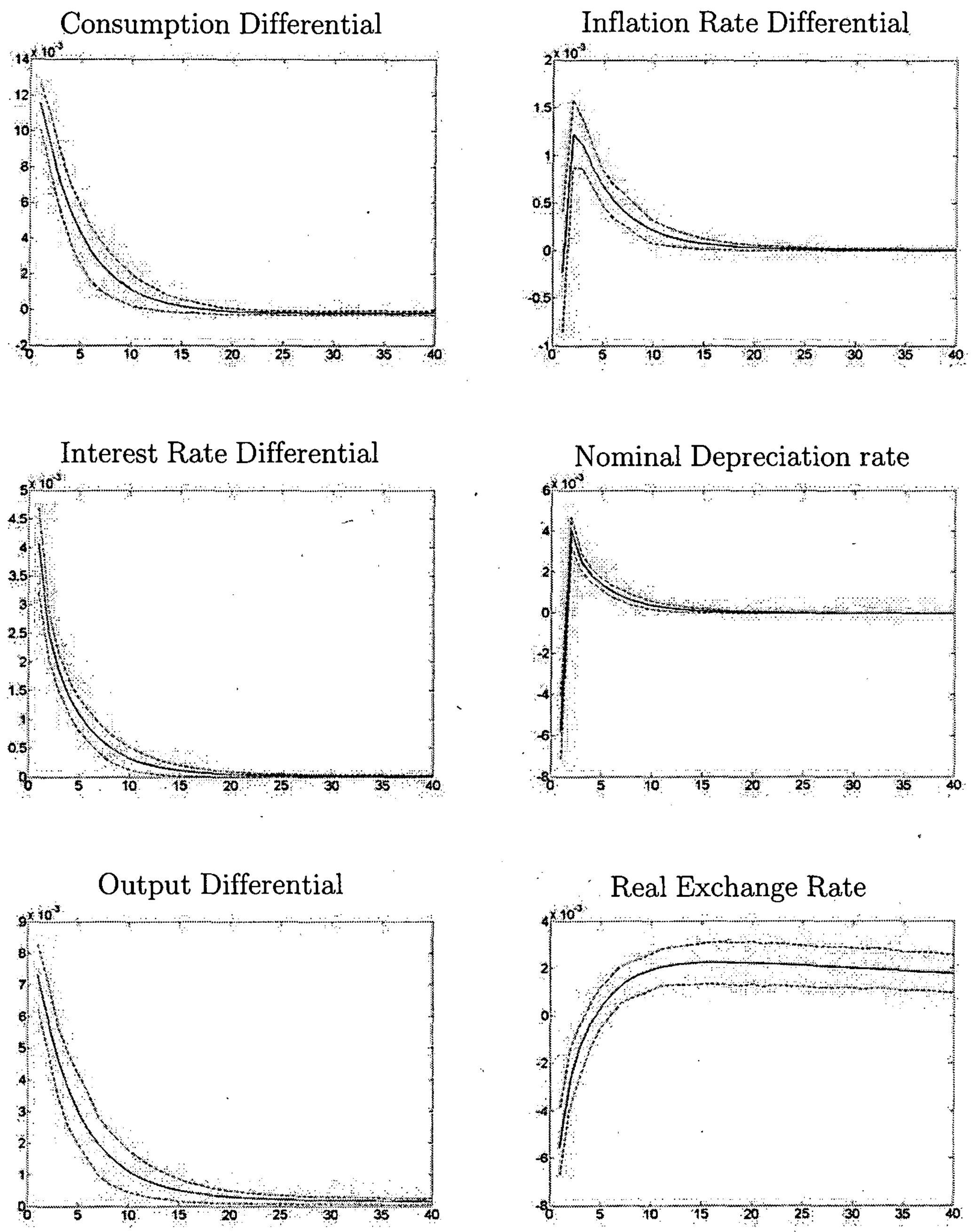
* The solid lines represent the U.K. inflation rate, and dash line is the Euro Area inflation. The Euro Area inflation rate is approximately by the weighted average of France, Germany and Italy. Data are from OECD MEI (series code: CPALTT01.IXOB).

Figure 3-3 Impulse Responses to a Monetary Policy Shock*



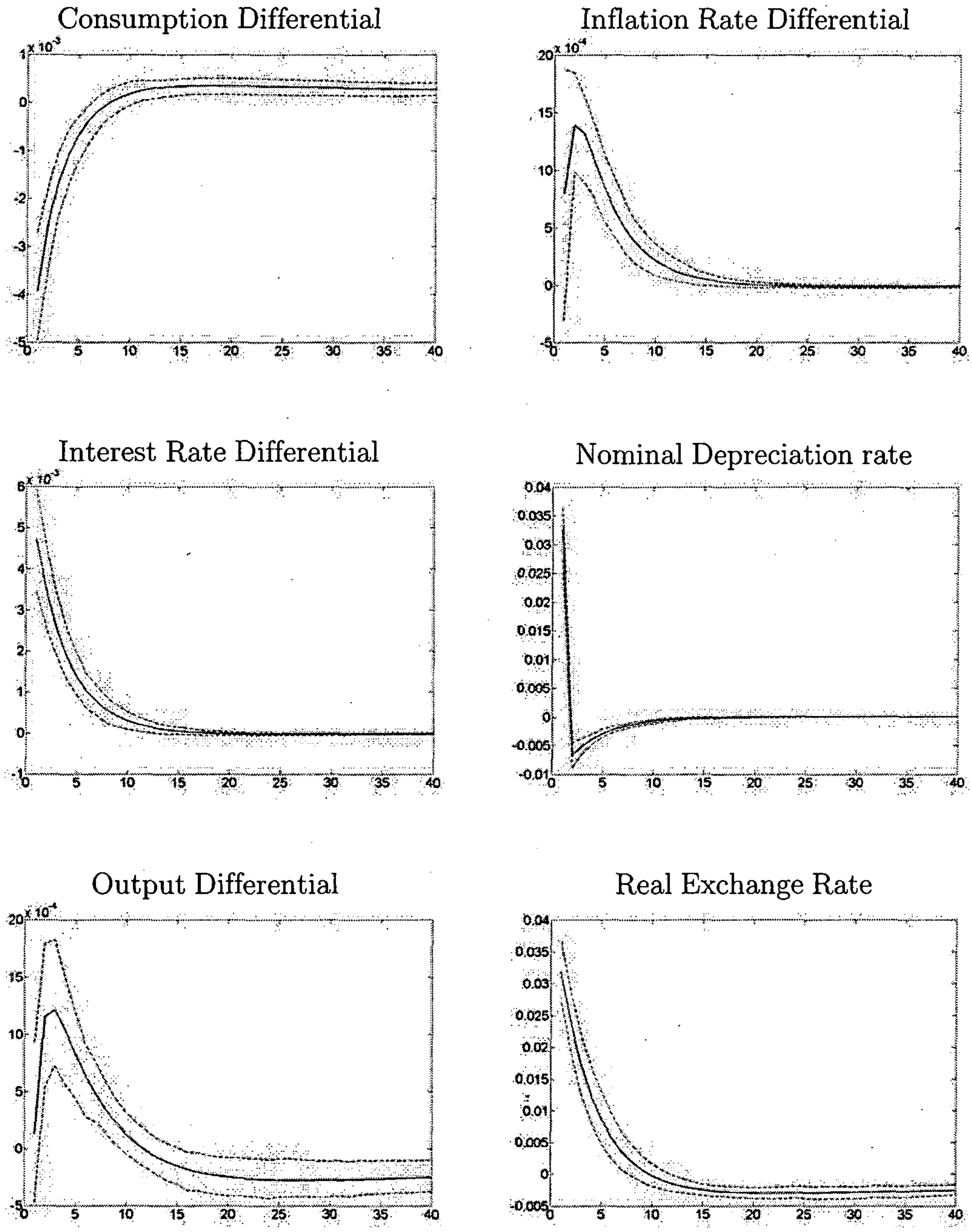
* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 3-4 Impulse Responses to a Preference Shock*



* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 3-5 Impulse Responses to an UIP Shock*



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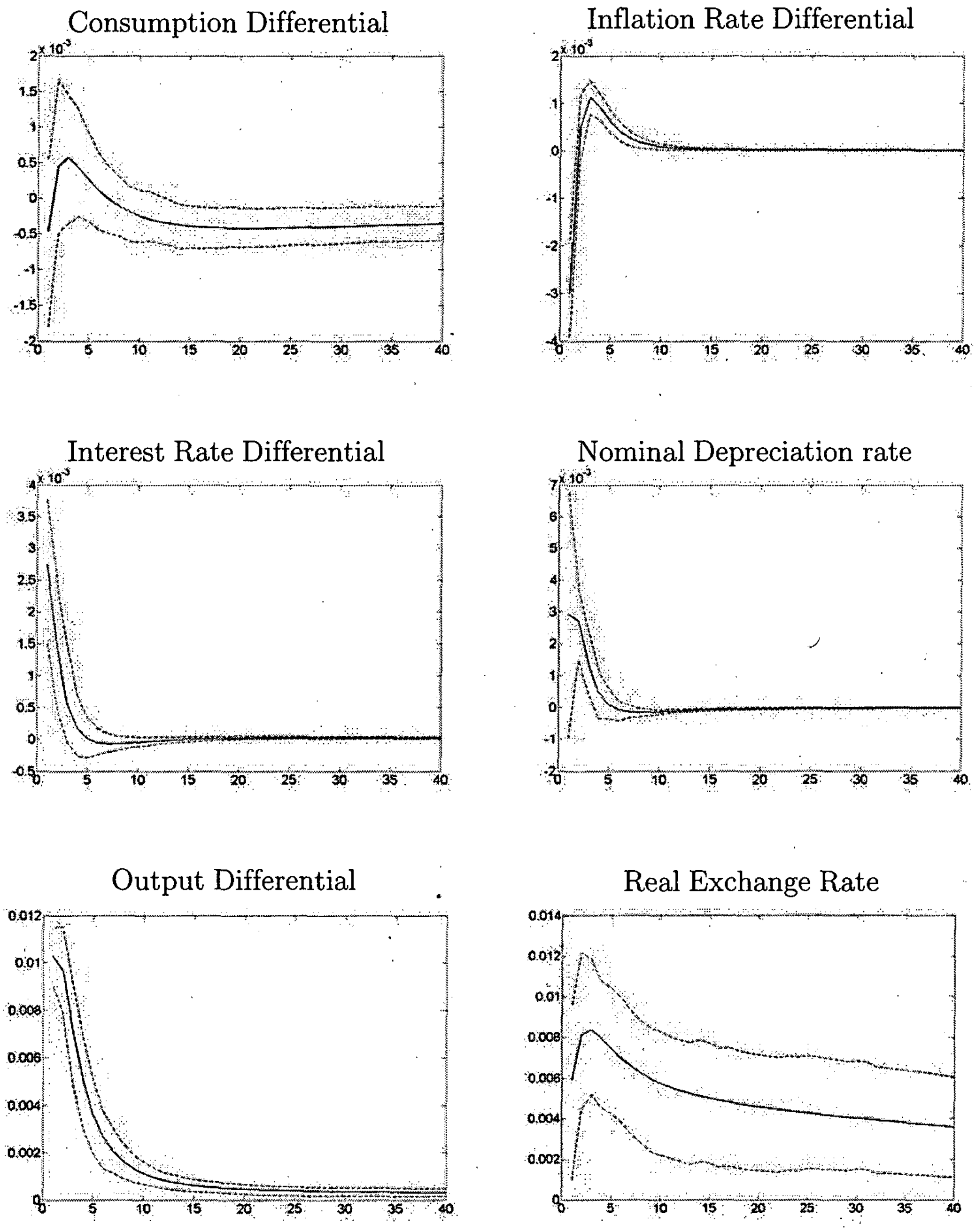
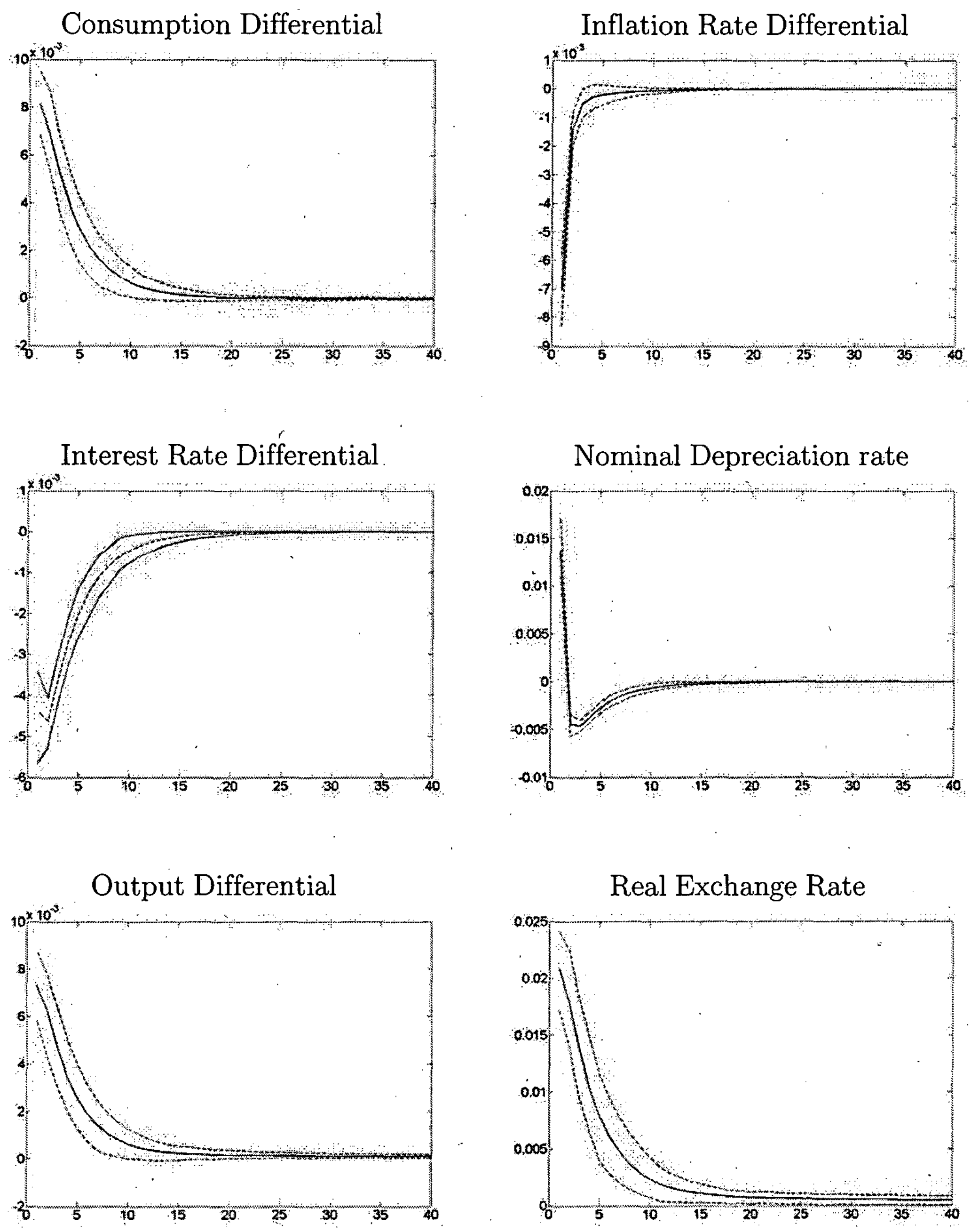
* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 3-6 Impulse Responses to a Tradable Technology Shock*



* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 3-7 Impulse Responses to a Non-Tradable Technology Shock*



* Figure depicts posterior means (solid lines) and pointwise 90% posterior probability intervals (dashed lines) for impulse responses of endogenous variables to one-standard deviation structural shocks.

Figure 3-8 Historical Decomposition of Real Exchange Rate

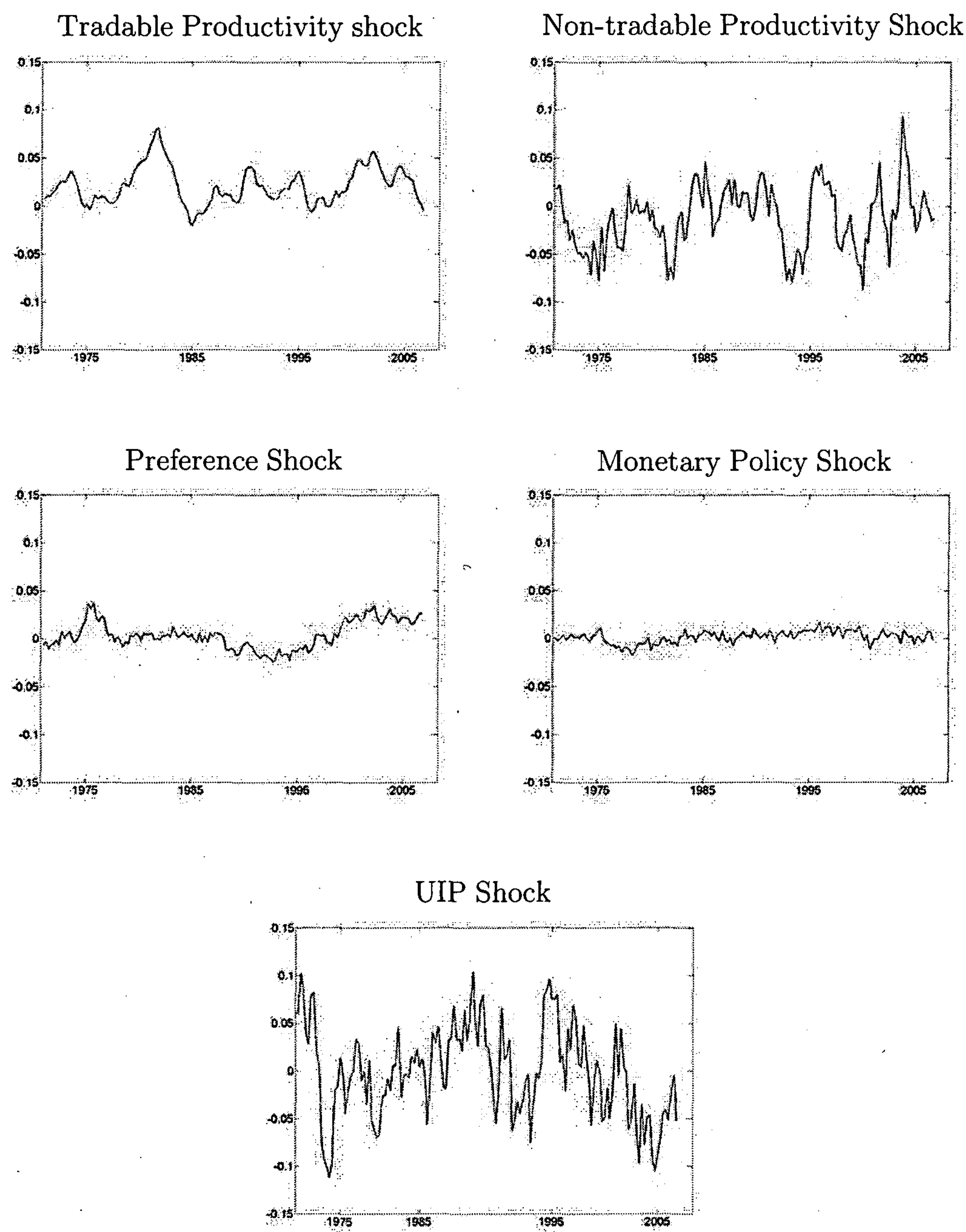


Figure 3-9 Historical Decomposition of Consumption Differential

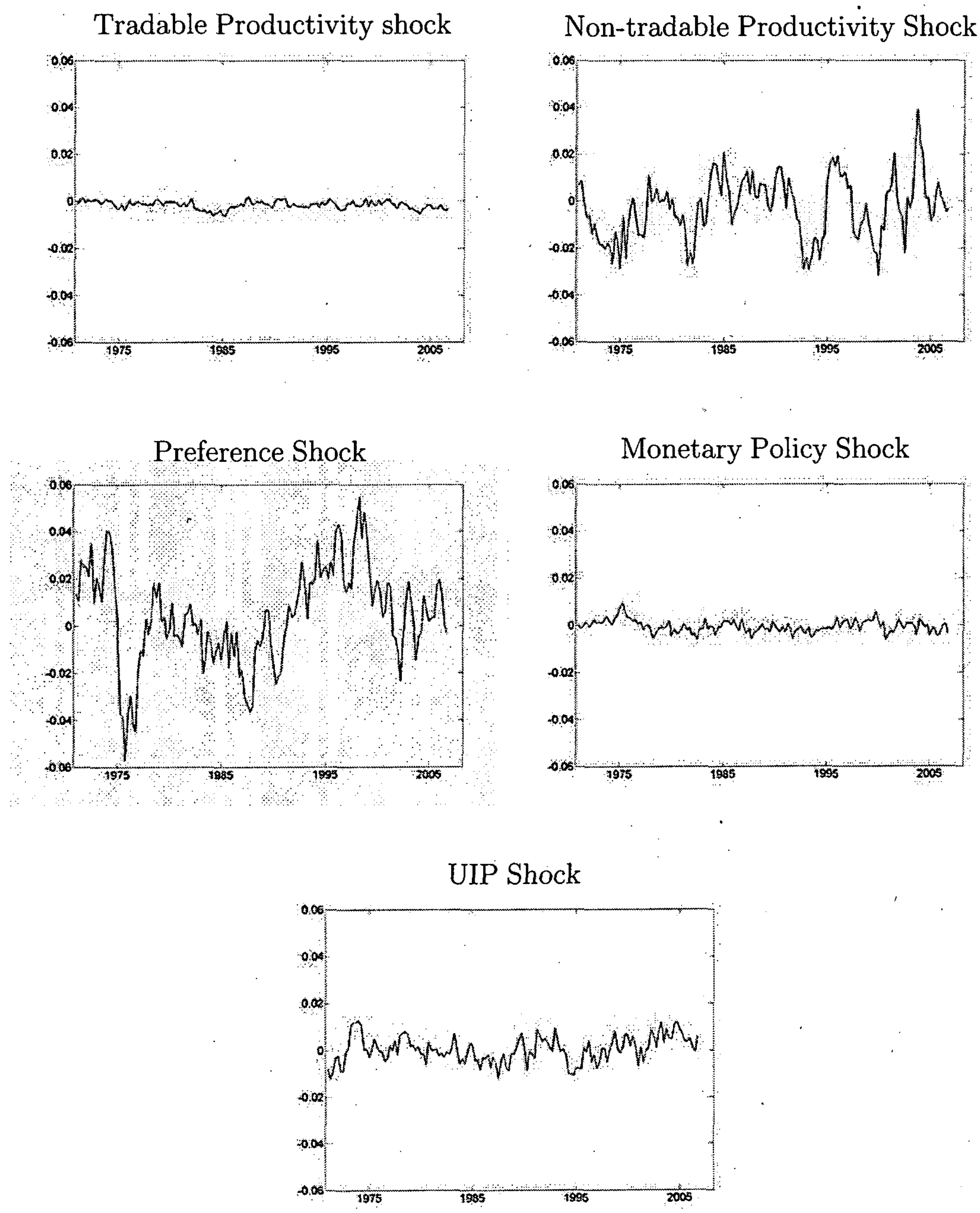


Figure 3-10 Historical Decomposition of Inflation Rate Differential

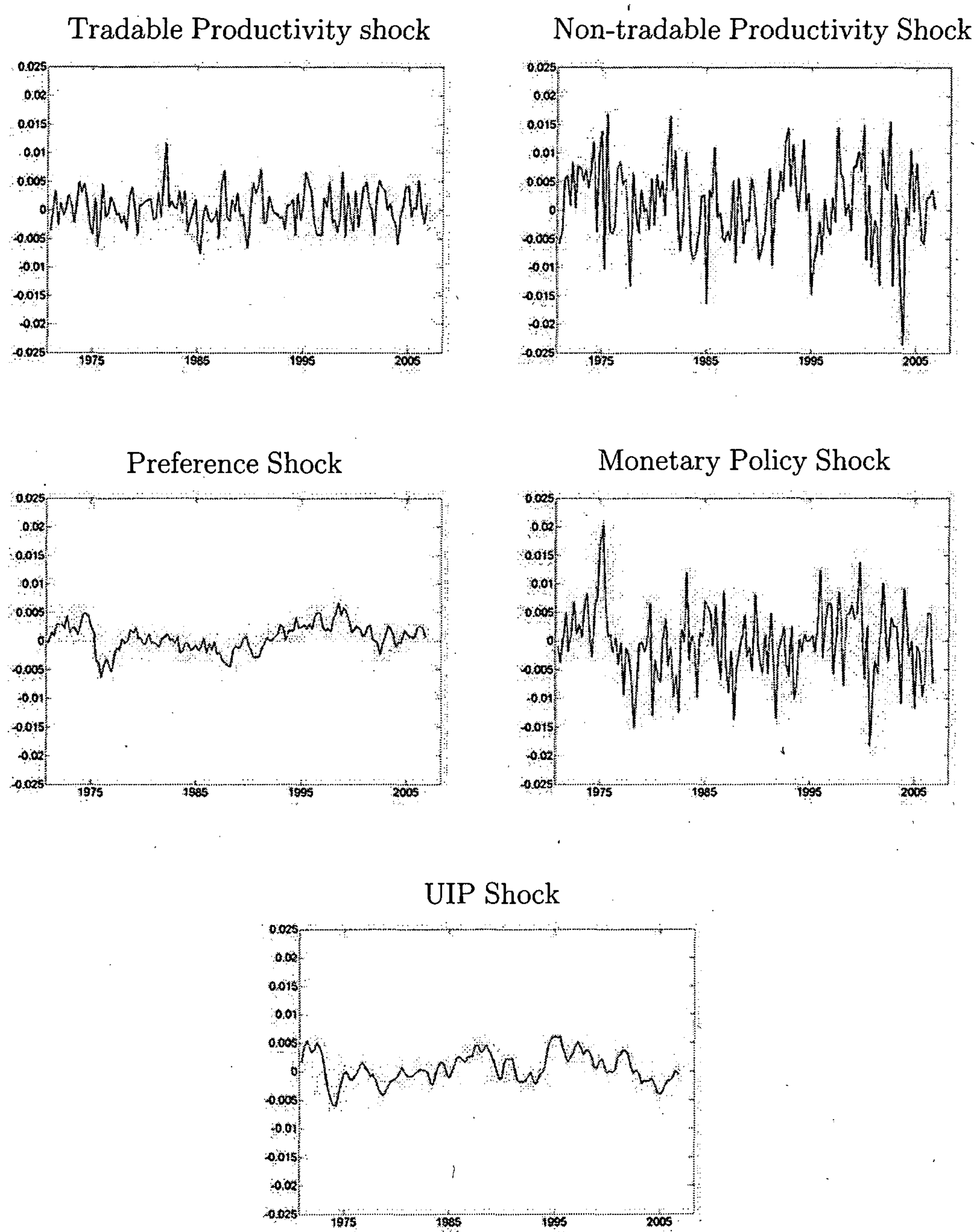


Figure 3-11 Historical Decomposition of Interest Rate Differential

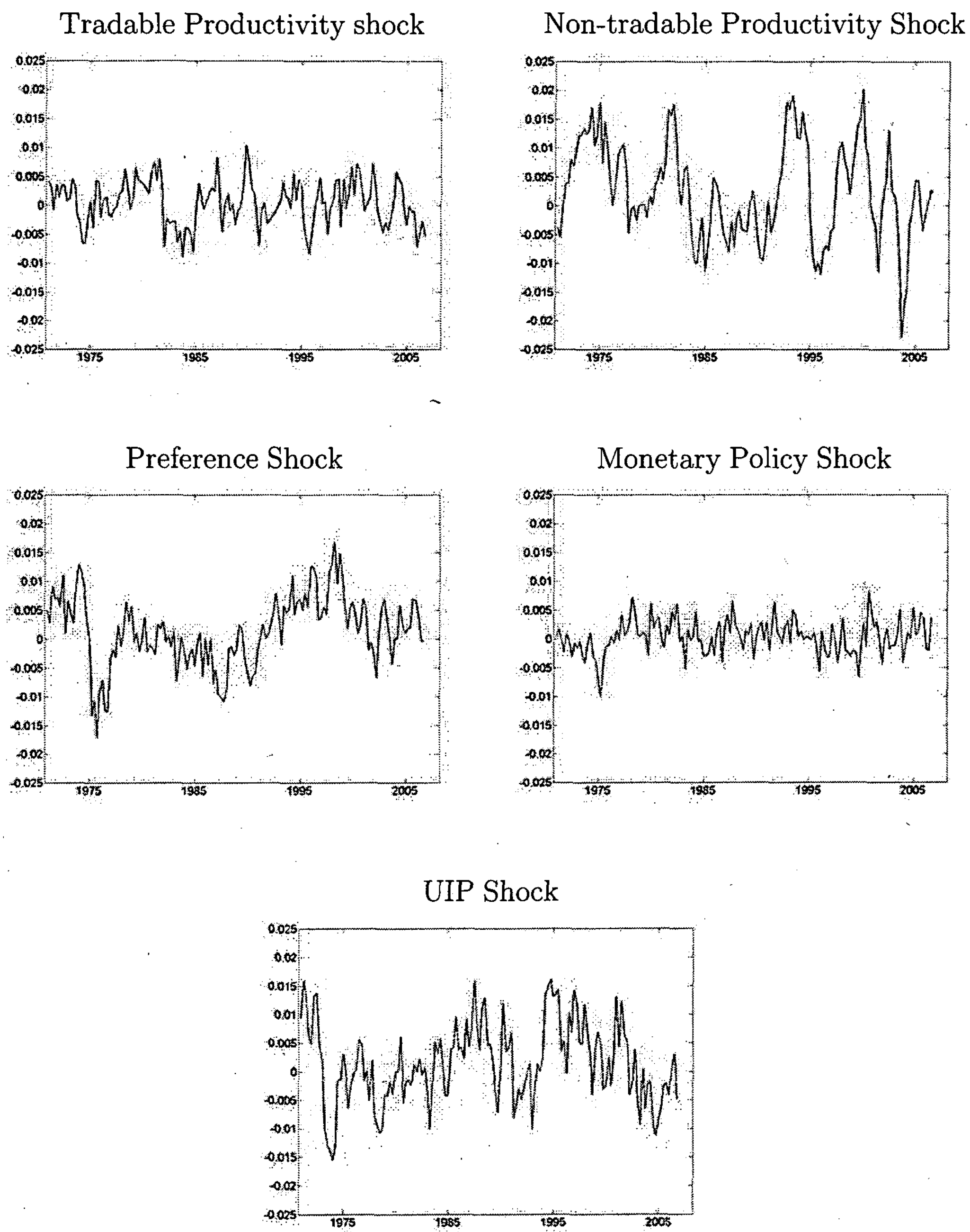
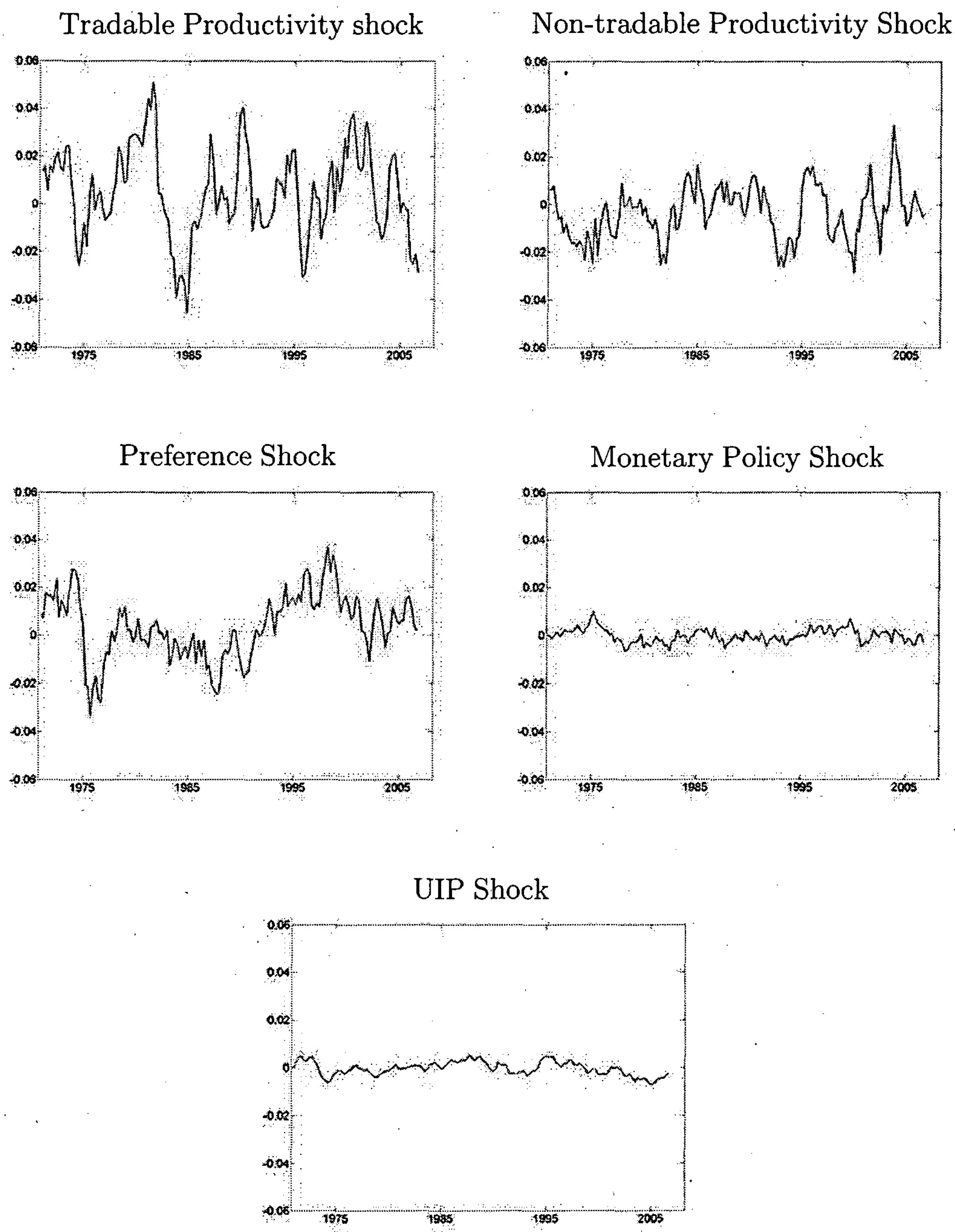


Figure 3-12 Historical Decomposition of Output Differential



Chapter 4

The Welfare Effects of Various Exchange Rate Regimes in a Three-Country New Open Economy Macroeconomics Model

Abstract

The optimal choice of various exchange rate systems is always an interesting topic in open-economy macroeconomics. In this paper, I construct a three-country NOEM model, and use it to study the welfare effects of monetary and technology shocks under various exchange rate regimes. In particular, I compare the welfare effects of the floating exchange rate regime, fixed exchange rate regime, basket peg regime and currency union. The results show that the welfare depends not only on the choice of exchange rate regime but also on the nature of shocks that hit the economy. The three-country framework is superior to the traditional two-country framework as the welfare effects of the shocks in the third country play an important role in the comparison of various exchange rate regimes. With only two-country framework, the effects of the third country cannot be exhibited, and the results might be inappropriate.

Keywords: Exchange rate regimes, New Open Economy Macroeconomics, Monetary policy, Welfare effects

JEL classification: F31 F33 F41

4.1 Introduction

In this paper, I construct a three-country NOEM model, and use it to study the welfare effects of various exchange rate regimes. In particular, I compare the welfare effects of the monetary and technology shocks under floating exchange rate regime, fixed exchange rate regime, basket peg regime and currency union.

The optimal choice of exchange rate system is always an interesting topic in open-economy macroeconomics. The modern study of the exchange rate regimes is from Friedman (1953). Since then, almost all papers in this area adopt a two-country framework. However, in the real world, the fixed and floating exchange rate regimes coexist. Although a fixed regime country can peg its currency to an anchor country[1], its exchange rates still float against other floating exchange rate regime countries. Therefore, the fixed exchange rate regime is actually a partial fix (to the anchor country) and a partial flexible (to other floating countries) regime. Then, the advantages and disadvantages of the fixed exchange rate regime compared to the floating exchange rate regime can be very different when the third country is taken into consideration. In other words, shocks in these floating countries can affect the economic status and welfare of the fixed country. And if these effects are big enough, it can have very important welfare effects on the fixed country, which may even change the benefits and costs of adopting the fixed exchange rate regime. And thus, to fully understand the effects of the fixed exchange rate regime and compare the welfare effects between fixed and floating regimes, a three-country framework is preferable.

Another advantage of the three-country framework is that it can be used directly to investigate the behaviour of the basket peg exchange rate regime. The basket peg country chooses fixing its exchange rate to a weighted average of several currencies rather than a single currency. And thus, this regime cannot be modelled in a two-country framework. Moreover, the three-country framework can also be used to compare the welfare effects of a currency union arrangement with other regimes. The same advantage of three-country framework exists here: the two currency union countries could also be affected by the shocks in the third country, which can be viewed as the rest of

[1] In this paper, country A is called the anchor of country B, if country B chooses to fix its currency to country A's currency.

the world.

The three-country model here is an extension of the classic Obstfeld and Rogoff (1995a) two-country New Open Economy Macroeconomics (NOEM) model. In order to study the welfare effects of different exchange rate regimes, I assume the domestic monetary policy is endogenously determined by different monetary policy rules. The monetary policy rules, on the other hand, are implied by the choice of exchange rate regimes. This is different from Obstfeld and Rogoff (1995a), who simply treat the monetary policy as exogenous.

Following Obstfeld and Rogoff (1995a), I assume households all over the world have identical preferences, which depend upon consumption, leisure and real balance. A monopolistically competitive supply sector is introduced by assuming each producer produces a single differentiated product. Moreover, there is an integrated world capital market in which a real bond is the only asset that can be traded across countries. There are three countries in this model, which are country C, U and E. For the benchmark model, I assume all three countries choose floating exchange rate regime. Other exchange rate regimes can be derived directly from the benchmark model.

I first solve for the long-run equilibrium when prices are fully flexible, and then the short-run equilibrium assuming the existence of nominal prices rigidities. In particular, short-run nominal rigidities are introduced in the way such that nominal producer prices are set a period in advance but can be fully adjusted after one period. Therefore, in the short-run, the producer prices do not change at all. With the solution of the long-run and short-run equilibrium, I can write the welfare change in terms of monetary and technology shocks.

In order to compare the welfare effects of shocks under different exchange rate regimes, the benchmark model is then modified to reflect three other exchange rate arrangements. In the fixed exchange rate regime, it is assumed that country C pegs its currency with country U, while both country U and E still choose floating exchange rate regimes. In the basket peg regime, country C fixes to a basket of country U and E currencies, and country U and E are still floating. For the currency union case, country C and U form a monetary union and country E still adopts floating regime. Furthermore, assuming country C takes technology shocks and foreign monetary policy shocks as given, the monetary supply change in country C can be solved in terms of these shocks.

After solving for the monetary supply change of country C under various exchange rate regimes, the welfare effects of different exchange rate regimes can be derived in terms of technology shocks in each country and foreign monetary supply shocks. The results show that no exchange rate regime is always better or worse than others. The welfare change are not only depends on the choice of the exchange rate regime, but also on the nature and relative importance of the technology and monetary shocks. For example, the best exchange rate regime under the positive shocks can be the worst one if these shocks are negative.

A fixed exchange rate regime country is most exposed to the monetary shocks of its anchor country, while the least to that of the third country. This result implies that the fixed exchange regime provides some insulation to monetary shocks in the third country. Therefore, the fixed exchange rate regime is preferable than the floating regime when the monetary shocks are less volatile in the anchor country and more volatile in the third country. On the other hand, fixed exchange rate regime reduces the exposure of the fixed country to its anchor country's technology shock, but with the cost of increasing the exposure to other floating countries. With only two-country framework, the country C welfare change in correspondence to the country E monetary and technology shocks cannot be shown. In this case, one may simply conclude that country C exposes more to foreign monetary shocks under the fixed regime than floating regime, and thus the floating exchange rate regime is always better than the fixed exchange rate regime. The two-country framework cannot show the welfare effects of the shocks in the third country, so the policy implications under this framework might be inappropriate.

It is also worth noting that the basket peg regime country always exposes more to the monetary shocks of its anchor countries (country U and E in the model) than the floating regime country, while exposes less to the technology shocks of the anchor countries. In the currency union case, country C benefits the most from country U technology improvements compare to other regimes, and, of course, will suffer the most if there are negative country U technology shocks. However, under currency union, although exposing the most to the technology shocks of its partner country (country U), country C exposes the least to that of the third countries. Again, the three-country framework is preferable to fully study the welfare effects of currency union regime.

The paper is organised as follows. In section 2, I develop the three-country benchmark model with floating exchange rate regimes. Section 3 solves for the long-run equilibrium when prices are fully flexible. In section 4, I solve for the short-run equilibrium with preset prices. Section 5 is the welfare comparison across various exchange rate regimes. Section 6 is the conclusion.

4.2 A Three-Country Model with Monopolistic Competition and Flexible Prices

4.2.1 Preference and Technology

There are three countries in this world, which are country C, U and E. The world is inhabited by a continuum of individual agents, indexed by $z \in [0, 1]$, each of whom produces a single differentiated perishable product. Country C agents are indexed by numbers in the interval $[0, m]$, while country U agents are indexed in $(m, n]$, and the remaining $(n, 1]$ agents reside in the country E.

Suppose agents everywhere in the world have the same preferences, which depends positively on consumption index and real money balances, but negatively on work effort. Preference of the representative country C agent is given by

$$U_t = \sum_{t=0}^{\infty} \beta^t \left[\log C_t + \varkappa \log \frac{M_t}{P_t} - \frac{\kappa_t}{2} y_t(z)^2 \right], \tag{4.1}$$

where $0 < \beta < 1$.[2]

C is the country C per capita consumption index, on which utility depends:

$$C = \left[\int_0^1 c(z)^{(\theta-1)/\theta} dz \right]^{\theta/(\theta-1)}, \theta > 1, \tag{4.2}$$

where $c(z)$ is individual's consumption of product z in country C and the parameter θ is the price elasticity of demand faced by each monopolist, i.e. the elasticity of

[2]This version of utility function assumes that the elasticity of intertemporal substitution is 1 and the elasticity of disutility from output is 2. The separability of the utility function may be problematic for analysing some aspects of exchange rate and current account dynamics, although it is helpful in making the model more tractable analytically. Bacchetta and Wincoop (1998) have emphasised (in a two-period framework) the importance of non-separabilities of leisure and consumption for the impact of the choice of exchange rate regime on the volume of trade.

substitution. Utility depends positively on individual domestic real money balance, M/P, because of the role of money in reducing transaction costs. Residents of a country derive utility from their country's currency only, and not from foreign currency. Country U and E agents have identical preferences except they hold their own currency M^* and M^{**}, and products y^* and y^{**}[3].

Utility depends negatively on individual output, y, because production requires irksome labour effort. The production technology in this paper is:

$$y(z) = \Lambda l(z),$$

where Λ is the technology common in country C which is naturally negative related with κ.

4.2.2 Prices, Demand, and Budget Constraints

First, I assume that all three countries in the economy adopt floating exchange rate regime, other exchange rate regimes can be developed directly from this benchmark model. The exchange rate between country C and U is s_1, and the exchange rate between country U and E is s_2. Thus, the exchange rate between country C and E is $s_1 * s_2$. Assuming there are no impediments or costs to trade between countries, the law of one price holds for every good:

$$p(z) = s_1 p^*(z) = s_1 s_2 p^{**}(z), \tag{4.3}$$

where $p(z)$, $p^*(z)$ and $p^{**}(z)$ are the local currency prices of good z in country C, U and E, respectively.

[3]Throughout the paper, one asterisk denotes country U variables and two asterisks represent country E variables.

The consumption-based price index in each country is[4]

$$\begin{aligned} P &= \left[\int_0^1 p(z)^{1-\theta} dz\right]^{1/(1-\theta)} \\ &= \left\{\int_0^m p(z)^{1-\theta} dz + \int_m^n [s_1 p^*(z)]^{1-\theta} dz + \int_n^1 [s_1 s_2 p^{**}(z)]^{1-\theta} dz\right\}^{1/(1-\theta)}, \end{aligned} \quad (4.4)$$

$$\begin{aligned} P^* &= \left[\int_0^1 p^*(z)^{1-\theta} dz\right]^{1/(1-\theta)} \\ &= \left[\int_0^m \left[\frac{p(z)}{s_1}\right]^{1-\theta} dz + \int_m^n p^*(z)^{1-\theta} dz + \int_n^1 [s_2 p^{**}(z)]^{1-\theta} dz\right]^{1/(1-\theta)}, \end{aligned} \quad (4.5)$$

$$\begin{aligned} P^{**} &= \left[\int_0^1 p^{**}(z)^{1-\theta} dz\right]^{1/(1-\theta)} \\ &= \left\{\int_0^m \left[\frac{p(z)}{s_1 s_2}\right]^{1-\theta} dz + \int_m^n \left(\frac{p^*(z)}{s_2}\right)^{1-\theta} dz + \int_n^1 p^{**}(z)^{1-\theta} dz\right\}^{1/(1-\theta)}. \end{aligned} \quad (4.6)$$

Since residents all over the world have the same preferences, equations (4.3) – (4.6) imply the purchasing power parity condition:

$$P = s_1 P^* = s_1 s_2 P^{**}. \quad (4.7)$$

Given the sub-utility function (4.2), a country C representative household's demand

[4]The price index is defined as the minimal expenditure of domestic money needed to purchase one unit of C. Formally, the price index P solves the problem
$\min_{c(z)} \int_0^1 p(z)c(z)dz$,
subject to
$\left[\int_0^1 c(z)^{(\theta-1)/\theta} dz\right]^{\theta/(\theta-1)} = 1$.

for product z in period t is:[5]

$$c_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t,$$

therefore, θ is the intratemporal elasticity of substitution.

Assume that there is an integrated world capital market, in which residents in the economy can borrow and lend. Real bond is the only asset traded in the world market, which is denominated in the composite consumption good. Let r_t denotes the real world interest rate earned on bonds between dates t and $t+1$, and b_t denotes the stocks of bonds held by residents in country C entering date $t+1$, b_t^* and b_t^{**} are the bonds held by country U and E, respectively.

The country C budget constraint of the representative household is

$$P_t b_t + M_t = P_t(1+r_{t-1})b_{t-1} + M_{t-1} + p_t(z)y_t(z) - P_t C_t + P_t T_t,$$

where M_t is the money stock held by country C residents at the end of period t and T denotes the government transfer.

Since Ricardian equivalence holds in this model, I can simply assume that government runs a balanced budget each period.[6] That is, all seigniorage revenues are rebated to the public in the form of transfer:

$$T_t = \frac{M_t - M_{t-1}}{P_t}. \tag{4.8}$$

As there is no government spending, the private consumptions are the total world

[5]The demand function for product z is derived by maximising

$$C = \left[\int_0^1 c(z)^{\frac{\theta-1}{\theta}}\, dz\right]^{\frac{\theta}{\theta-1}},$$

subject to the nominal budget constraint

$$\int_0^1 p(z)c(z)dz = Z,$$

where Z is any fixed total nominal expenditure on goods.

[6]As Woodford (1996) points out , Ricardian equivalence holds in monetary models with nominal rigidities only under somewhat more stringent assumptions than real models.

demand. Therefore, the good z producer faces the period t world demand curve:

$$y_t(z) = \left[\frac{p_t(z)}{P_t}\right]^{-\theta} C_t^w, \tag{4.9}$$

where

$$C_t^w \equiv mC_t + (n-m)C_t^* + (1-n)C_t^{**}, \tag{4.10}$$

is world consumption demand, which producers take as given. The budget constraint for country U and E are analogous.

4.2.3 Households Maximisation

First, the gross nominal interest rate R_t for country C between date t and $t+1$ is defined as

$$R_t = \frac{P_{t+1}}{P_t}(1+r_t). \tag{4.11}$$

Note that, because purchasing power parity holds[7], real interest rate equality implies uncovered interest parity across the world:

$$R_t = \frac{s_{1,t+1}}{s_{1,t}} R_t^* = \frac{s_{1,t+1}}{s_{1,t}} \frac{s_{2,t+1}}{s_{2,t}} R_t^{**}. \tag{4.12}$$

Solving the country C household utility maximisation problem, we can obtain the following first order conditions:

$$C_{t+1} = \beta(1+r_t)C_t, \tag{4.13}$$

$$\frac{M_t}{P_t} = \chi C_t \left(\frac{R_t}{1-R_t}\right), \tag{4.14}$$

$$y_t(z)^{(\theta+1)/\theta} = \left(\frac{\theta-1}{\theta}\right) \kappa_t^{-1} C_t^{-1} (C_t^w)^{1/\theta}. \tag{4.15}$$

Equation (4.13) is the standard first-order consumption Euler equations for the case where the intertemporal elasticity of substitution is 1. Equation (4.14) are money

[7]Purchasing power parity holds for consumer price indexes (equation (4.7)), but only because both countries consume identical commodity baskets. Purchasing power parity does not hold for national output deflators, and thus the terms of trade can change.

market equilibrium conditions in the form of money in the utility function model. It arises from the equilibrium condition that the marginal rate of substitution of composite consumption for the services of real money balances should equate the consumption opportunity cost of holding real balance. Equation (4.15) is the labour-leisure trade-off condition, which states that the marginal cost of producing an extra unit of output equals the marginal utility of consuming the added revenue that an extra unit of output brings. The first order conditions for country U and E representative households are analogously.

To fully characterise the equilibrium, the following transversality condition is also needed:

$$\lim_{T\to\infty}\left[\prod_{t=0}^{T}\left(\frac{1}{1+r_t}\right)\right]\left(b_T+\frac{M_T}{P_T}\right)=0.$$

The analogous transversality conditions are held for country U and E.

4.2.4 Market Clearing Conditions

In aggregate, the domestic nominal money supply must equal domestic nominal money demand in each country. And the asset-market-clearing condition indicates that global net foreign assets must be zero:

$$mb_t+(n-m)b_t^*+(1-n)b_t^{**}=0.$$

Then, the goods market clearing conditions can be derived by using the asset market clearing condition together with country C budget constraint and the corresponding foreign budget constraints:

$$\begin{aligned} C_t^w &\equiv mC_t+(n-m)C_t^*+(1-n)C_t^{**} \\ &= m\frac{p_t(c)}{P_t}y_t+(n-m)\frac{p_t^*(u)}{P_t^*}y_t^*+(1-n)\frac{p_t^{**}(e)}{P_t^{**}}y_t^{**}, \end{aligned} \tag{4.16}$$

where $p(c)$, $p^{**}(u)$ and $p^{**}(e)$ are the local currency prices of a typical local produced good in country C, U and E respectively, and y, y^* and y^{**} are the corresponding output levels. The market prices are used to value all goods in terms of composite real consumption, thus the equation above states that world real consumption equals world real income.

4.2.5 A Global Symmetric Steady State

Because of monopoly pricing and endogenous output, the model here does not yield simple closed-form solutions for general paths of the exogenous variables.[8] Although, the effects of exogenous shocks can be analysed through numerical simulations, the intuition of the model can be more easily studied with linearised version.

Following Obstfeld and Rogoff (1995a), the economy is assumed to start from a global symmetric steady state, where all shocks are zero. Since consumption and output are constant in the steady state, the real interest rate $\overline{r}$ is tied down by the consumption Euler conditions (4.13): $\overline{r} = \frac{1-\beta}{\beta}$.[9] In the special case of global symmetric steady state, all corresponding variables are equal across countries and the net foreign assets position is zero. There is a closed-form solution for the steady state, which is denoted by zero subscripts:

$$\frac{\overline{p}_0(c)}{\overline{P}_0} = \frac{\overline{p}_0^*(u)}{\overline{P}_0^*} = \frac{\overline{p}_0^{**}(e)}{\overline{P}_0^{**}} = 1,$$

$$\overline{C}_0 = \overline{C}_0^* = \overline{C}_0^{**} = \overline{y}_0 = \overline{y}_0^* = \overline{y}_0^{**} = \overline{C}_0^w = (\frac{\theta - 1}{\theta \overline{\kappa}_0})^{1/2}.$$

And by assuming zero inflation in the steady state, the steady-state real balances are

$$\frac{\overline{M}_0}{\overline{P}_0} = \frac{\overline{M}_0^*}{\overline{P}_0^*} = \frac{\overline{M}_0^{**}}{\overline{P}_0^{**}} = \frac{\chi}{1-\beta}(\frac{\theta - 1}{\theta \overline{\kappa}_0})^{1/2}.$$

4.2.6 The Linearised Model

In order to allow for asymmetries in policies and current account, the model are then log-linearised around the initial global symmetric steady state. This linearisation is implemented by expressing the model in terms of deviations from the baseline steady-state path. Denoting percentage changes from the baseline by hats: $\widehat{x}_t \equiv dx_t/\overline{x}_0$, where $\overline{x}_0$ is the initial steady-state value.

Starting with price index equations (4.4), (4.5) and (4.6), small percentage devia-

[8] Corsetti and Pesenti (1997) present a model in which current account imbalances are always zero in equilibrium under the assumption that the elasticity of net output demand with respect to relative prices and elasticity of intratemporal substitution among goods are equal to one, and thus a closed-form solution are achievable. However, in this model, these elasticities are larger than one: world demand for goods is more sensitive to prices than in the setup of Corsetti and Pesenti (1997).

[9] Throughout the paper, steady-state values are marked by overbars.

tions of consumer price levels from their initial paths are given by

$$\widehat{P}_t = m\widehat{p}_t(c) + (n-m)\widehat{p}^*(u) + (1-n)\widehat{p}^{**}(e) + (1-m)\,\widehat{s}_{1,t} + (1-n)\widehat{s}_{2,t}, \tag{4.17}$$

$$\widehat{P}^*_t = m\widehat{p}_t(c) + (n-m)\widehat{p}^*(u) + (1-n)\widehat{p}^{**}(e) - m\widehat{s}_{1,t} + (1-n)\widehat{s}_{2,t}, \tag{4.18}$$

$$\widehat{P}^{**}_t = m\widehat{p}_t(c) + (n-m)\widehat{p}^*(u) + (1-n)\widehat{p}^{**}(e) - m\widehat{s}_{1,t} - n\widehat{s}_{2,t}. \tag{4.19}$$

The purchase power parity equation can be derived using the equations above:

$$\widehat{s}_{1,t} = \widehat{P}_t - \widehat{P}^*_t, \text{ and } \widehat{s}_{2,t} = \widehat{P}^*_t - \widehat{P}^{**}_t. \tag{4.20}$$

Equation (4.20) implies that the percentage changes of the exchange rate equals to the relative price change between countries. This equation can be used to show one of the advantages of pegged exchange rate regime: anti-inflation. That is, if the exchange rate is fixed ($\widehat{s}_{1,t} = 0$), the price change in two countries should be the same. Thus, a high-inflation country can reduce its inflation by fixing it's currency to a low-inflation country.

Then, the log-linearised global goods market equilibrium condition (4.16) is,

$$\begin{aligned} \widehat{C}^w_t &= m\left[\widehat{p}_t(c) + \widehat{y}_t - \widehat{P}_t\right] + (n-m)\left[\widehat{p}^*(u) + \widehat{y}^*_t - \widehat{P}^*_t\right] \\ &\quad + (1-n)\left[\widehat{p}^{**}_t(e) + \widehat{y}^{**}_t - \widehat{P}^{**}_t\right]. \end{aligned} \tag{4.21}$$

The log-linearised version of equation (4.9), interpreted as world demand schedules for typical local domestic products, are

$$\widehat{y}_t = \theta\left[\widehat{P}_t - \widehat{p}_t(c)\right] + \widehat{C}^w_t, \tag{4.22}$$

$$\widehat{y}^*_t = \theta\left[\widehat{P}^*_t - \widehat{p}^*_t(u)\right] + \widehat{C}^w_t, \tag{4.23}$$

$$\widehat{y}^{**}_t = \theta\left[\widehat{P}^{**}_t - \widehat{p}^{**}_t(e)\right] + \widehat{C}^w_t. \tag{4.24}$$

Equation (4.15) and its foreign counterparts, which describe the optimal flexible-price output levels, are approximated by

$$(\theta+1)\,\widehat{y}_t = -\theta\widehat{C}_t - \theta\widehat{\kappa}_t + \widehat{C}^w_t, \tag{4.25}$$

$$(\theta+1)\,\widehat{y}^*_t = -\theta\widehat{C}^*_t - \theta\widehat{\kappa}^*_t + \widehat{C}^w_t, \tag{4.26}$$

$$(\theta+1)\,\widehat{y}^{**}_t = -\theta\widehat{C}^{**}_t - \theta\widehat{\kappa}^{**}_t + \widehat{C}^w_t. \tag{4.27}$$

The consumption Euler equation (4.13) and its foreign counterparts take the log-linear form near the initial steady-state path:

$$\widehat{C}_{t+1} = \widehat{C}_t + (1-\beta)\widehat{r}_t, \tag{4.28}$$

$$\widehat{C}^*_{t+1} = \widehat{C}^*_t + (1-\beta)\widehat{r}_t, \tag{4.29}$$

$$\widehat{C}^{**}_{t+1} = \widehat{C}^{**}_t + (1-\beta)\widehat{r}_t. \tag{4.30}$$

Finally, the money demand equation (4.14) together with nominal interest rate equation (4.11) show the linearised version of money demand equation:

$$\widehat{M}_t - \widehat{P}_t = \widehat{C}_t - \beta(\widehat{r}_t + \frac{\widehat{P}_{t+1} - \widehat{P}_t}{1-\beta}), \tag{4.31}$$

$$\widehat{M}^*_t - \widehat{P}^*_t = \widehat{C}^*_t - \beta(\widehat{r}_t + \frac{\widehat{P}^*_{t+1} - \widehat{P}^*_t}{1-\beta}), \tag{4.32}$$

$$\widehat{M}^{**}_t - \widehat{P}^{**}_t = \widehat{C}^{**}_t - \beta(\widehat{r}_t + \frac{\widehat{P}^{**}_{t+1} - \widehat{P}^{**}_t}{1-\beta}). \tag{4.33}$$

With the preceding log-linearised equations in hand, we are now ready to solve the model, first for the steady state that is reached when prices are full flexible, and then for short-run dynamics due to temporary price rigidities.

4.3 Solving for the Long-Run Equilibrium

Let $\widehat{\overline{x}} \equiv d\overline{x}/\overline{x}_0$ denotes the percentage change of the new steady-state value from the initial steady-state value. First, the intertemporal budget constraints can be obtained from log-linearising the country C budget constraint and its foreign counterparts:

$$\widehat{C} = \frac{1-\beta}{\beta}\frac{d\overline{b}}{\overline{C}_0^w} + \widehat{p}(c) + \widehat{y} - \widehat{P}, \tag{4.34}$$

$$\widehat{C}^* = \frac{1-\beta}{\beta}\frac{d\overline{b}^*}{\overline{C}_0^w} + \widehat{p}^*(u) + \widehat{y}^* - \widehat{P}^*, \tag{4.35}$$

$$\widehat{C}^{**} = \frac{1-\beta}{\beta}\frac{d\overline{b}^{**}}{\overline{C}_0^w} + \widehat{p}^{**}(e) + \widehat{y}^{**} - \widehat{P}^{**}. \tag{4.36}$$

The next step in solving for the steady state is to observe that equations (4.21) - (4.27) hold across steady states, so that they remain valid after time-subscripted changes are replaced by steady state changes. Together with (4.34), (4.35) and (4.36), there are ten equations and ten unknown variables, $\widehat{C}$, $\widehat{C}^*$, $\widehat{C}^{**}$, $\widehat{y}$, $\widehat{y}^*$, $\widehat{y}^{**}$, $\widehat{p}(c) - \widehat{P}$, $\widehat{p}^*(u) - \widehat{P}^*$, $\widehat{p}^{**}(e) - \widehat{P}^{**}$, and $\widehat{C}^w$. Therefore, the long-run equilibrium can be solved in terms of current account and exogenous shocks.

The solutions for consumption are

$$\widehat{C}^w = -\frac{1}{2}\left(m\widehat{\kappa} + (n-m)\widehat{\kappa}^* + (1-n)\widehat{\kappa}^{**}\right), \tag{4.37}$$

$$\widehat{C} = \frac{1+\theta}{2\theta}\frac{\overline{r}d\overline{b}}{\overline{C}_0^W} + \frac{1-\theta-m}{2\theta}\widehat{\kappa} - \frac{n-m}{2\theta}\widehat{\kappa}^* - \frac{1-n}{2\theta}\widehat{\kappa}^{**}, \tag{4.38}$$

$$\widehat{C}^* = \frac{1+\theta}{2\theta}\frac{\overline{r}d\overline{b}^*}{\overline{C}_0^W} - \frac{m}{2\theta}\widehat{\kappa} + \frac{1-\theta-n+m}{2\theta}\widehat{\kappa}^* - \frac{1-n}{2\theta}\widehat{\kappa}^{**}, \tag{4.39}$$

$$\widehat{C}^{**} = \frac{1+\theta}{2\theta}\frac{\overline{r}d\overline{b}^{**}}{\overline{C}_0^W} - \frac{m}{2\theta}\widehat{\kappa} - \frac{n-m}{2\theta}\widehat{\kappa}^* + \frac{n-\theta}{2\theta}\widehat{\kappa}^{**}. \tag{4.40}$$

Equations (4.37) implies that positive technology shocks increase the world total consumption, as technology shock is negatively related with κ.

To see the effects of net foreign assets on outputs and the terms of trade, observe that equations (4.17) - (4.27), together with (4.38), (4.39), and (4.40) imply

$$\widehat{y} = -\frac{\theta}{1+\theta}\widehat{C} - \frac{2\theta+m}{2(\theta+1)}\widehat{\kappa} - \frac{n-m}{2(\theta+1)}\widehat{\kappa}^* - \frac{1-n}{2(\theta+1)}\widehat{\kappa}^{**}, \tag{4.41}$$

$$\widehat{y}^* = -\frac{\theta}{1+\theta}\widehat{C}^* - \frac{m}{2(\theta+1)}\widehat{\kappa} - \frac{2\theta+n-m}{2(\theta+1)}\widehat{\kappa}^* - \frac{1-n}{2(\theta+1)}\widehat{\kappa}^{**}, \tag{4.42}$$

$$\widehat{y}^{**} = -\frac{\theta}{1+\theta}\widehat{C}^{**} - \frac{m}{2(\theta+1)}\widehat{\kappa} - \frac{n-m}{2(\theta+1)}\widehat{\kappa}^{*} - \frac{2\theta+1-n}{2(\theta+1)}\widehat{\kappa}^{**}; \tag{4.43}$$

and

$$\widehat{p}(c) - \widehat{p}^{*}(u) - \widehat{s}_1 = \frac{1}{\theta}(\widehat{y}^{*} - \widehat{y}) = \frac{1}{1+\theta}\left(\widehat{C} - \widehat{C}^{*}\right) + \frac{1}{1+\theta}(\widehat{\kappa} - \widehat{\kappa}^{*}), \tag{4.44}$$

$$\widehat{p}(c) - \widehat{p}^{**}(e) - \widehat{s}_1 - \widehat{s}_2 = \frac{1}{\theta}(\widehat{y}^{**} - \widehat{y}) = \frac{1}{1+\theta}\left(\widehat{C} - \widehat{C}^{**}\right) + \frac{1}{1+\theta}(\widehat{\kappa} - \widehat{\kappa}^{**}), \tag{4.45}$$

$$\widehat{p}^{*}(u) - \widehat{p}^{**}(e) - \widehat{s}_2 = \frac{1}{\theta}(\widehat{y}^{**} - \widehat{y}^{*}) = \frac{1}{1+\theta}\left(\widehat{C}^{*} - \widehat{C}^{**}\right) + \frac{1}{1+\theta}(\widehat{\kappa}^{*} - \widehat{\kappa}^{**}). \tag{4.46}$$

Equations (4.44), (4.45), and (4.46) show that the increase in the domestic terms of trade (the rise in the relative price of home products) is proportional to both the increase in relative foreign output and the increase in relative home consumption. Note that because the infinitely lived households in all countries have equal constant discount rates, an international transfer of assets leads to a permanent change in the consumption (As shown in Equations (4.38), (4.39), and (4.40)) and thus in the terms of trade.

Note that, the long-run real equilibrium of the economy can be determined without reference to the money-demand equations because changes in the level of the money supply have no effect on real variables here when prices are flexible. Across steady state, the interest rate and inflation do not change, so the solutions for steady-state equilibrium price levels follow directly from the linearised money-demand equations (4.31), (4.32), and (4.33):

$$\widehat{\overline{P}} = \widehat{\overline{M}} - \widehat{\overline{C}}, \tag{4.47}$$

$$\widehat{\overline{P}}^{*} = \widehat{\overline{M}}^{*} - \widehat{\overline{C}}^{*}, \tag{4.48}$$

$$\widehat{\overline{P}}^{**} = \widehat{\overline{M}}^{**} - \widehat{\overline{C}}^{**}. \tag{4.49}$$

Equation (4.47), (4.48) and (4.49) together with the purchasing power parity equation (4.20) gives the long-run exchange rate as:

$$\widehat{\overline{s}}_2 = \left(\widehat{\overline{M}}^{*} - \widehat{\overline{M}}^{**}\right) - \left(\widehat{\overline{C}}^{*} - \widehat{\overline{C}}^{**}\right), \tag{4.50}$$

$$\widehat{\overline{s}}_1 = \left(\widehat{\overline{M}}^{*} - \widehat{\overline{M}}^{**}\right) - \left(\widehat{\overline{C}}^{*} - \widehat{\overline{C}}^{**}\right). \tag{4.51}$$

4.4 The Short-Run Equilibrium with Prices Rigidities

4.4.1 Short-Run Equilibrium Conditions

In this paper, it is assumed when there is either one-period (temporary) or permanent changes from the baseline policies, the world economy reaches its new steady state after a single period. Thus, all $t+1$ subscripted variables in the linearised system can be replaced with steady-state changes. All t-subscripted variables are now interpreted as short-run values.

In the short run, nominal producer prices $p(c)$, $p^*(u)$ and $p^{**}(e)$ are predetermined. That is, they are set a period in advance but can be adjusted fully after one period. With price rigidities, output becomes demand determined for small enough shocks. Because a monopolist always prices above marginal cost, it is profitable to meet unexpected demand at the preset price. In short run, therefore, the equations equating marginal revenue and marginal cost in the flexible-price case, (4.25), (4.26), and (4.27), need not hold. Instead, output is determined by the demand equations, (4.22), (4.23), and (4.24).

Although prices are preset in terms of the producers' own currencies, the foreign currency prices of a producer's output must change if the exchange rate moves. With rigid output prices, equations (4.17), (4.18) and (4.19) imply

$$\widehat{P} = (1-m)\,\widehat{s}_1 + (1-n)\widehat{s}_2, \tag{4.52}$$

$$\widehat{P}^* = -m\widehat{s}_1 + (1-n)\widehat{s}_2, \tag{4.53}$$

$$\widehat{P}^{**} = -m\widehat{s}_1 - n\widehat{s}_2. \tag{4.54}$$

In (4.52), (4.53) and (4.54) and henceforth, the hated variables without time subscripts or overbars are used to denote short-run deviations from the symmetric steady state path. Combining these price changes with (4.22), (4.23), and (4.24) show that short-run aggregate demands can be expressed as

$$\widehat{y} = \theta\left[(1-m)\,\widehat{s}_1 + (1-n)\widehat{s}_2\right] + \widehat{C}^w, \tag{4.55}$$

$$\widehat{y}^* = \theta\left[-m\widehat{s}_1 + (1-n)\widehat{s}_2\right] + \widehat{C}^w, \tag{4.56}$$

$$\widehat{y}^{**} = \theta\left[-m\widehat{s}_1 - n\widehat{s}_2\right] + \widehat{C}^w. \tag{4.57}$$

The remaining equations of short-run equilibrium include (4.28) - (4.33), which always hold.

In the last section, we solved for the model as a function of the changes in money supplies, technology shocks and net foreign assets (the current account). The change in net foreign assets, however, is endogenous and can be determined only in conjunction with a full solution of the model's intertemporal equilibrium.

In the long-run, current accounts are balanced. However, in the short-run, income need not equal expenditure, and thus countries may run the current account imbalance. Since the initial steady-state assets position is zero, the linearised short-run current account equations are

$$\frac{d\bar{b}}{\overline{C}_0^w} = \widehat{y} - \widehat{C} - (1-m)\,\widehat{s}_1 - (1-n)\widehat{s}_2, \tag{4.58}$$

$$\frac{d\bar{b}^*}{\overline{C}_0^w} = \widehat{y}^* - \widehat{C}^* + m\widehat{s}_1 - (1-n)\widehat{s}_2, \tag{4.59}$$

$$\frac{d\bar{b}^{**}}{\overline{C}_0^w} = \widehat{y}^{**} - \widehat{C}^{**} + m\widehat{s}_1 + n\widehat{s}_2, \tag{4.60}$$

where we have made use of (4.52), (4.53) and (4.54). Note that $d\bar{b}$, $d\bar{b}^*$and $d\bar{b}^{**}$ appear above because the asset stocks at the end of period t are steady-state levels.

4.4.2 Solution of the Model

In order to solve for the short-run equilibrium, the remaining 16 short-run variables $\widehat{C}$, $\widehat{C}^*$, $\widehat{C}^{**}$, $\widehat{y}$, $\widehat{y}^*$, $\widehat{y}^{**}$, $\widehat{P}$, $\widehat{P}^*$, $\widehat{P}^{**}$, $\widehat{s}_1$, $\widehat{s}_2$, $\widehat{C}^w$, $\widehat{r}$, $d\bar{b}$, $d\bar{b}^*$ and $d\bar{b}^{**}$ need to be solved in terms of monetary and productivity shocks. Although a direct solution is available, the implication of the model can be better explored by an intuitive approach as suggested by Obstfeld and Rogoff (1995a).

For simplicity, I assume there are only two kinds of shocks hitting the economy, which are permanent monetary supply shocks and technology shocks.[10] Following Ob-

[10]The case of temporary (one-period) monetary shock can be solved analogously. For the technology shock, as outputs are demand determined in the short-run, a temporary technology shock has no effects on the economy: the firms produce the same quantity of output, and households simply supply less

stfeld and Rogoff (1996), I assume the productivity shock is: $\widehat{\Lambda}_t = -\frac{\kappa_t - \overline{\kappa}_0}{\overline{\kappa}_0} = -\widehat{\kappa}_t$.

The exchange rate between two countries is determined solely by these two countries. In this section, I will show how to derive s_2, which is the exchange rate between country U and E. The exchange rate between country C and U can be derived analogously. The exchange rate is jointly determined by the MM and GG schedules as shown in Obstfeld and Rogoff (1995a).

The MM schedule shows how relative consumption changes affect the exchange rates by changing relative money demand. Subtracting equation (4.30) from (4.29) shows the consumption difference between country U and E:

$$\widehat{C}^* - \widehat{C}^{**} = \widehat{\overline{C}}^* - \widehat{\overline{C}}^{**}, \tag{4.61}$$

where the left-hand side is the short-run difference and the right hand side is the long-run difference. Equations (4.61) shows that changes in relative consumption levels are permanent, even though short-run real interest rate changes can tilt individual-country consumption profiles. The reason is that with integrated bond markets and identical consumption baskets, individuals in each country face the same real interest rate. Therefore, interest rate changes tilt their consumption profiles proportionately.

A similar operation on the money demand equations (4.32) and (4.33) leads to

$$\left(\widehat{M}^* - \widehat{M}^{**}\right) - \widehat{s}_2 = \left(\widehat{C}^* - \widehat{C}^{**}\right) - \frac{\beta}{1-\beta}(\widehat{\overline{s}}_2 - \widehat{s}_2). \tag{4.62}$$

Then leading (4.62) by one period to obtain

$$\widehat{\overline{s}}_2 = \left(\widehat{\overline{M}}^* - \widehat{\overline{M}}^{**}\right) - \left(\widehat{\overline{C}}^* - \widehat{\overline{C}}^{**}\right).$$

As $\widehat{C}^* - \widehat{C}^{**} = \widehat{\overline{C}}^* - \widehat{\overline{C}}^{**}$ by (4.61) and $\widehat{M}^* - \widehat{M}^{**} = \widehat{\overline{M}}^* - \widehat{\overline{M}}^{**}$ (since the monetary supply shock is permanent), the MM schedule can be derived by using the above expression to substitute for $\widehat{\overline{s}}_2$in (4.62):

$$\widehat{s}_2 = \left(\widehat{M}^* - \widehat{M}^{**}\right) - \left(\widehat{C}^* - \widehat{C}^{**}\right). \tag{4.63}$$

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labour.

Note that $\widehat{s}_2 = \widehat{\overline{s}}_2$, the exchange rate jumps immediately to its long-run levels despite the inability of prices to adjust in the short run.

The GG schedule shows the domestic currency depreciation needed to raise relative home output enough to justify a given permanent rise in relative home consumption. Using the current account equations (4.59) and (4.60) together with the long-run consumption equations (4.39) and (4.40) to write the long-run consumption difference as

$$\widehat{\overline{C}}^* - \widehat{\overline{C}}^{**} = \frac{\overline{r}(1+\theta)}{2\theta}\left[(\widehat{y}^* - \widehat{y}^{**}) - \left(\widehat{C}^* - \widehat{C}^{**}\right) - \widehat{s}_2\right] - \frac{1-\theta}{2\theta}\left(\widehat{\Lambda}^* - \widehat{\Lambda}^{**}\right).$$

Equations (4.56) and (4.57) show that country U output rises relative to country E as the country U currency depreciates and makes its products cheaper in the short run: $\widehat{y}^* - \widehat{y}^{**} = \theta\widehat{s}_2$, combining this equation with the one preceding it and with the relative Euler equation (4.61), we arrive at the GG schedule:

$$\widehat{s}_2 = \frac{\overline{r}(1+\theta)+2\theta}{\overline{r}(\theta^2-1)}\left(\widehat{C}^* - \widehat{C}^{**}\right) - \frac{1}{\overline{r}(\theta+1)}\left(\widehat{\Lambda}^* - \widehat{\Lambda}^{**}\right). \tag{4.64}$$

Together, MM schedule (4.63) and GG schedule (4.64) imply the exchange rate change is

$$\widehat{s}_2 = \frac{\overline{r}(1+\theta)+2\theta}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{M}^* - \widehat{M}^{**}\right) - \frac{(\theta-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{\Lambda}^* - \widehat{\Lambda}^{**}\right). \tag{4.65}$$

The exchange rate between country C and U can be derived analogously:

$$\widehat{s}_1 = \frac{\overline{r}(1+\theta)+2\theta}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{M} - \widehat{M}^*\right) - \frac{(\theta-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{\Lambda} - \widehat{\Lambda}^*\right). \tag{4.66}$$

The relative consumption change can also be derived as:

$$\widehat{C} - \widehat{C}^* = \frac{\overline{r}(\theta^2-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{M} - \widehat{M}^*\right) + \frac{(\theta-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{\Lambda} - \widehat{\Lambda}^*\right). \tag{4.67}$$

$$\widehat{C}^* - \widehat{C}^{**} = \frac{\overline{r}(\theta^2-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{M}^* - \widehat{M}^{**}\right) + \frac{(\theta-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{\Lambda}^* - \widehat{\Lambda}^{**}\right). \tag{4.68}$$

Note that

$$m\frac{d\overline{b}}{\overline{C}_0^w} + (n-m)\frac{d\overline{b}^*}{\overline{C}_0^w} + (1-n)\frac{d\overline{b}^{**}}{\overline{C}_0^w} = 0.$$

The equilibrium current account can be derived by combining the equation above with equation (4.38), (4.39), (4.40), (4.61), (4.67) and (4.68):

$$\frac{d\bar{b}}{\overline{C}_0^w} = \frac{2(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{M}-\widehat{M}^w\right) - \frac{(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{\Lambda}-\widehat{\Lambda}^w\right), \tag{4.69}$$

$$\frac{d\bar{b}^*}{\overline{C}_0^w} = \frac{2(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{M}^*-\widehat{M}^w\right) - \frac{(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{\Lambda}^*-\widehat{\Lambda}^w\right), \tag{4.70}$$

$$\frac{d\bar{b}^{**}}{\overline{C}_0^w} = \frac{2(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{M}^{**}-\widehat{M}^w\right) - \frac{(\theta-1)}{\bar{r}(\theta+1)+2}\left(\widehat{\Lambda}^{**}-\widehat{\Lambda}^w\right). \tag{4.71}$$

With the solution of current account, all the steady-state values of other variables can be derived. For example, the long-run terms of trade are found by combining (4.70) and (4.71) with (4.38), (4.39), (4.40) and (4.52), (4.53), (4.54):

$$\widehat{\overline{p}}(c) - \widehat{\overline{p}}^*(u) - \widehat{\overline{s}}_1 = \frac{\bar{r}(\theta-1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{M}-\widehat{M}^*\right) - \frac{(\bar{r}\theta+1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{\Lambda}-\widehat{\Lambda}^*\right),$$

$$\widehat{\overline{p}}(c) - \widehat{\overline{p}}^{**}(e) - \widehat{\overline{s}}_1 - \widehat{\overline{s}}_2 = \frac{\bar{r}(\theta-1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{M}-\widehat{M}^{**}\right) - \frac{(\bar{r}\theta+1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{\Lambda}-\widehat{\Lambda}^{**}\right),$$

$$\widehat{\overline{p}}^*(u) - \widehat{\overline{p}}^{**}(e) - \widehat{\overline{s}}_2 = \frac{\bar{r}(\theta-1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{M}^*-\widehat{M}^{**}\right) - \frac{(\bar{r}\theta+1)}{\theta[\bar{r}(\theta+1)+2]}\left(\widehat{\Lambda}^*-\widehat{\Lambda}^{**}\right).$$

The equations above show that the terms of trade are positive related with domestic monetary expansion and foreign productivity improvement; while negative related with domestic productivity improvement and foreign monetary expansion.

The last step is to solve the effect of an unanticipated permanent monetary shock on the world real interest rate. Using the short-run price equations (4.52), (4.53), (4.54) and the long-run equations (4.47), (4.48), (4.49) to express the money market equilibrium conditions (4.31), (4.32), (4.33) as

$$\beta\widehat{r} = \widehat{C} + \frac{\beta}{1-\beta}\widehat{\overline{C}} - \frac{1}{1-\beta}\left(\widehat{M} - (1-m)\widehat{s}_1 - (1-n)\widehat{s}_2\right), \tag{4.72}$$

$$\beta\widehat{r} = \widehat{C}^* + \frac{\beta}{1-\beta}\widehat{\overline{C}}^* - \frac{1}{1-\beta}\left(\widehat{M}^* + m\widehat{s}_1 - (1-n)\widehat{s}_2\right), \tag{4.73}$$

$$\beta\widehat{r} = \widehat{C}^{**} + \frac{\beta}{1-\beta}\widehat{\overline{C}}^{**} - \frac{1}{1-\beta}\left(\widehat{M}^{**} + m\widehat{s}_1 + n\widehat{s}_2\right). \tag{4.74}$$

Then, taking a population-weighted sum of the equations above, we can derive:

$$\widehat{r} = -\frac{1}{1-\beta}\widehat{M}^{w} + \frac{1}{2(1-\beta)}\widehat{\Lambda}^{w}, \tag{4.75}$$

$$\widehat{C}^{w} = \widehat{M}^{w}. \tag{4.76}$$

A monetary expansion in any country will reduce the world real interest rate in proportion to the increasing in the world monetary supply $\widehat{M}^{w}$ and thus, increases global consumption demand. Therefore, global money shocks are not a zero-sum game, although the effects may be unequal across countries. The welfare analysis now is available with the solution of the short-run equilibrium.

4.5 Welfare Analysis

In this section, I will first derive the welfare change in terms of monetary and technology shocks. Rather than treat monetary shocks exogenous, I then back out monetary shocks using different monetary policy rules, which is implied by the different exchange rate regimes. The last step is to derive and compare the welfare across various exchange rate regimes.

The represent households' total utility can be written as $U = \overline{U}_0 + dU$, where $\overline{U}_0$ is the individual's utility at the initial global symmetric steady-state. As $\overline{U}_0$ is equal for all households across the world, the welfare analysis only need to focus on comparing dU, which is the welfare change after shocks. dU can be divided into two parts by writing the intertemporal utility function (4.1) as $dU = dU^{R} + dU^{M}$, where dU^{R} consists of the terms depending on consumption and output and dU^{M} depends on real money balance.

Following Devereux and Engel (2003), I assume that the real balances part of the utility function is ignored[11]. Therefore, the welfare comparison only concentrated on U^{R}. From equation (4.1), U^{R} can be written as,

[11] Obstfeld and Rogoff (1995a) and Corsetti and Pesenti (2001) make the same assumption. Obstfeld and Rogoff (1996) explain the reason: 'one can show that as long as the derived utility from real balance is not too large as a share of total utility- a very reasonable assumption that will hold as long as χ is not too large-changes in U^{R} dominate total changes in utility.'

$$U_t^R = \sum_{t=o}^{\infty} \beta^t \left[\log C_t - \frac{\kappa_t}{2} y_t(z)^2 \right].$$

Then, the utility change is the log-linearised version of the above equation:

$$dU^R = \widehat{C} - \frac{\theta - 1}{\theta}\widehat{y} + \frac{\beta}{1-\beta}\left(\widehat{\widehat{C}} - \frac{\theta - 1}{\theta}\widehat{\widehat{y}}\right) + \frac{1}{1-\beta}\widehat{\Lambda}. \tag{4.77}$$

which use the fact that the economy reaches a steady state after one period. $\widehat{C}$'s value follows from (4.67), (4.68) and (4.76).

$$\widehat{C} = \frac{\overline{r}\left(\theta^2 - 1\right)}{\overline{r}\theta\left(\theta + 1\right) + 2\theta}\widehat{M} + \frac{\overline{r}\left(\theta + 1\right) + 2\theta}{\overline{r}\theta\left(\theta + 1\right) + 2\theta}\widehat{M}^w - \frac{\left(\theta - 1\right)}{\overline{r}\theta\left(\theta + 1\right) + 2\theta}\left(\widehat{\Lambda}^w - \widehat{\Lambda}\right). \tag{4.78}$$

Equation (4.55) shows the value of $\widehat{y}$.

$$\widehat{y} = \frac{\left[\overline{r}\left(\theta + 1\right) + 2\theta\right]}{\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{M} - \frac{2\left(\theta - 1\right)}{\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{M}^w - \frac{\left(\theta - 1\right)}{\left[\overline{r}\left(\theta + 1\right) + 2\right]}\left(\widehat{\Lambda} - \widehat{\Lambda}^w\right). \tag{4.79}$$

The long-run country C consumption change $\widehat{\widehat{C}}$ can be derived from (4.38) and (4.69):

$$\widehat{\widehat{C}} = \frac{\overline{r}\left(\theta^2 - 1\right)}{\theta\left[\overline{r}\left(\theta + 1\right) + 2\right]}\left(\widehat{M} - \widehat{M}^w\right) + \frac{\left(\theta - 1\right)}{\theta\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{\Lambda} + \frac{\overline{r}\theta\left(\theta + 1\right) + 2}{2\theta\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{\Lambda}^w. \tag{4.80}$$

Equation (4.41) shows that the country C long-run output change is:

$$\widehat{\widehat{y}} = -\frac{\overline{r}\left(\theta - 1\right)}{\left[\overline{r}\left(\theta + 1\right) + 2\right]}\left(\widehat{M} - \widehat{M}^w\right) + \frac{\left(\theta\overline{r} + 1\right)}{\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{\Lambda} - \frac{\overline{r}\left(\theta - 1\right)}{2\left[\overline{r}\left(\theta + 1\right) + 2\right]}\widehat{\Lambda}^w. \tag{4.81}$$

Returning to (4.77), the utility change can be derived in terms of monetary and technology shocks.

$$dU^R = \frac{1}{\theta}\widehat{M}^w + \frac{\beta}{2\theta\left(1 - \beta\right)}\widehat{\Lambda}^w + \frac{1}{1-\beta}\widehat{\Lambda}. \tag{4.82}$$

The country U and E welfare changes can be derived analogously:

$$dU^{*R} = \frac{1}{\theta}\widehat{M}^w + \frac{\beta}{2\theta\left(1 - \beta\right)}\widehat{\Lambda}^w + \frac{1}{1-\beta}\widehat{\Lambda}^*,$$

$$dU^{**R} = \frac{1}{\theta}\widehat{M}^w + \frac{\beta}{2\theta\left(1 - \beta\right)}\widehat{\Lambda}^w + \frac{1}{1-\beta}\widehat{\Lambda}^{**}.$$

The results show that different countries share the same benefits from world monetary expansion and technology improvement. The only difference is that country benefits more from the technology improvement in their own country than from other countries.

It is worth noting that the world monetary expansion can raise welfare, and all countries benefit from it equally. As the monopolistic producer always price above marginal cost, it is profitable for them to meet unexpected demand at preset prices. Because price exceeds marginal cost in a monopolistic equilibrium, the monetary supply expansion that coordinates higher work effort move the economy closer to efficient production. As households have no special preference on domestic produced goods, all countries benefit from world the monetary expansion equally.[12]

4.5.1 Monetary Policies under Various Exchange Rate Regimes

In this section, I will study the welfare effects of various exchange rate regimes for country C, while other countries still choose floating exchange rate regime[13]. In particular, I assume country C chooses among four different kinds of exchange rate regimes, which are floating exchange rate regime, fixed exchange rate regime, basket peg regime, and currency union.

Moreover, it is also assumed that the technology shocks and country U and E monetary shocks are exogenous in this model. Then, the monetary supply change in country C can be back out from different monetary rules which are implicitly bound with the choice of exchange rate regimes. Therefore, the country C monetary supply change is endogenous, and can be solved in terms of exogenous shocks.

[12]Corsetti and Pesenti (2001) show that despite the presence of monopolistic distortions and sticky prices, a monetary shock boosting short-run demand for domestic output can be welfare-reducing in an open economy with monopoly power on its terms of trade. This is because with different preference on home and foreign produced goods, the monetary expansion can reduce domestic households' purchasing power (real exchange rate depreciation) in the global markets. If this effect dominates the positive welfare effect of the domestic monetary expansion, the monetary expansion can reduce welfare. However, in this model, as households across the world have the same preferences, the purchasing power parity equation (4.7) always holds. Therefore, the negative effects on domestic monetary expansion do not exist.

[13]The only exception is the currency union, in which case country C and U have same monetary policy rule.

Floating Exchange Rate Regime

In this case, all countries adopt floating exchange rate regime. For country C, I assume it adopts the traditional Taylor-style monetary feedback rule:

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\left[\lambda_1\left(\widehat{P}_t - \widehat{P}_{t-1}\right) + \lambda_2\left(\widehat{y}_t - \widehat{y}_{t-1}\right)\right],$$

where R_t is the gross nominal interest rate. The monetary policy rule shows that country C monetary authorities are assumed to adjust the short-run nominal interest rate in response to inflation and output change. As the economy is in the steady state before period t, the equation above can be simplified as:

$$\widehat{R} = a\widehat{P} + b\widehat{y},$$

where $a = (1-\rho)\lambda_1$ and $b = (1-\rho)\lambda_2$. Therefore, the short-term nominal interest rate change can be solved in terms of monetary and technology shocks:

$$\begin{aligned}\widehat{R} &= -\frac{a\left[\overline{r}(1+\theta)+2\theta\right]+2b\theta(\theta-1)}{\theta\left[\overline{r}(\theta+1)+2\right]}\widehat{M}^w \\ &+\frac{\left[\overline{r}(\theta+1)+2\theta\right](a+b\theta)}{\theta\left[\overline{r}(\theta+1)+2\right]}\widehat{M} - \frac{(\theta-1)(a+b\theta)}{\theta\left[\overline{r}(\theta+1)+2\right]}\left(\widehat{\Lambda}-\widehat{\Lambda}^w\right). \qquad (4.83)\end{aligned}$$

Also, the log-linearised version of equation (4.11) gives:

$$\widehat{R} = \widehat{\overline{P}} - \widehat{P} + (1-\beta)\widehat{r},$$

where $\widehat{\overline{P}}$ is from (4.47), $\widehat{P}$ is from (4.52) and $\widehat{r}$ is from (4.75). Therefore, the short-run gross nominal interest rate change is

$$\widehat{R} = 0. \qquad (4.84)$$

Together, (4.83) and (4.84) solve the monetary change in country C as a function of technology shocks and foreign monetary shocks:

$$\widehat{M} = \frac{\varphi(n-m)}{1-\varphi m}\widehat{M}^* + \frac{\varphi(1-n)}{1-\varphi m}\widehat{M}^{**} + \frac{(\theta-1)}{(1-\varphi m)\left[\overline{r}(\theta+1)+2\theta\right]}\left(\widehat{\Lambda}-\widehat{\Lambda}^w\right). \qquad (4.85)$$

where $\varphi = \frac{[a[\overline{r}(\theta+1)+2\theta]+2b\theta(\theta-1)]}{[\overline{r}(\theta+1)+2\theta](a+b\theta)} < 1$. The equation above shows that country C monetary supply change responses positively to country U and E monetary expansion, while responses negatively to country U and E technology improvements.

Fixed Exchange Rate Regime

Under this regime, it is assumed that country C fixes its exchange rate with country U, while country U and E still adopt floating regime. As the exchange rate between countries C and U is fixed: $\widehat{s}_1 = 0$, the MM and GG schedules between these two countries imply:

$$\widehat{M} = \widehat{M}^* + \frac{(\theta - 1)}{\overline{r}\,(1+\theta) + 2\theta}\left(\widehat{\Lambda} - \widehat{\Lambda}^*\right). \tag{4.86}$$

This results show that the monetary supply change in the fixed country depends on the monetary shock of the anchor country and the technology shocks differentials across countries. Given that the technology shocks are exogenous, country C monetary policy are actually decided by country U monetary supply shock. This verifies the relevance of the impossible trinity: a country can only choose two of the three desirable outcomes simultaneously: open capital markets, monetary independence, and exchange rates stability. In this three-country open economy model, country C which fixes the exchange rate with country U, loses the independence of its monetary policy.

The reason is because if the exchange rates cannot change and capital is mobile, the domestic nominal interest rates must equal the foreign nominal interest rates. But this obviously implies that domestic interest rates are determined abroad, not by domestic monetary policy. Any attempt to expand the money supply would leave people holding more money than they desire at the prevailing interest rate. Rather than bidding the interest rate down, agents simply sell their excess money holdings to the home central bank for foreign currency at the fixed exchange rates. They then invest the proceeds abroad and thereby restore their initial portfolio balance between domestic money and bonds. The key lesson is that monetary policy in the fixed exchange rate country is bound to its anchor country through the interest rate parity condition.

Basket Peg

Under this regime, the monetary authorities in country C choose to peg to a basket of currencies. In other words, they try to keep the weighted average of the exchange rate between country C and the basket currency countries (country U and E in this model) constant. In particular, I assume:

$$\frac{n-m}{1-m}\widehat{s}_1 + \frac{1-n}{1-m}\left(\widehat{s}_2 + \widehat{s}_1\right) = 0.$$

So, the relative weight is equal to the relative country size.

In this case, the monetary policy in country C can be derived as a function of foreign monetary shocks and productivity shocks:

$$\widehat{M} = \frac{n-m}{1-m}\widehat{M}^{*} + \frac{1-n}{1-m}\widehat{M}^{**} + \frac{(\theta-1)}{(1-m)\left[\overline{r}\,(1+\theta)+2\theta\right]}\left(\widehat{\Lambda} - \widehat{\Lambda}^{w}\right). \qquad (4.87)$$

The above equation shows that the monetary policy in the basket peg country is similar as the fixed regime country. The only difference is as the basket peg country tries to fix to a basket of currencies. It responses to the monetary shocks of basket countries proportionally, where the proportion equals to the weight of the currency in the currency basket.

Currency Union

Under this regime, it is assume that country C and U form a currency union. Therefore, there is a unique central bank for country C and U, which adopt the same interest rate feedback rules as the floating exchange rate regime.

$$\widehat{R}_t^u = \rho \widehat{R}_{t-1}^u + (1-\rho)\left[\lambda_1\left(\widehat{P}_t^u - \widehat{P}_{t-1}^u\right) + \lambda_2\left(\widehat{y}_t^u - \widehat{y}_{t-1}^u\right)\right],$$

where $\widehat{R}^u$ is the gross nominal interest rate in the currency union, $\widehat{P}^u$ is the prices index, $\widehat{y}^u$ is the per capita output of the currency union. Following the same strategy of the floating exchange rate regime, the monetary change in the currency union can be derived as:

$$\widehat{M}^u = \frac{\varphi(1-n)}{1-\varphi n}\widehat{M}^{**} + \frac{(\theta-1)}{(1-\varphi n)\left[\overline{r}(\theta+1)+2\theta\right]}\left(\widehat{\Lambda}^u - \widehat{\Lambda}^w\right), \tag{4.88}$$

where $\widehat{M}^u$ is the monetary supply shock in the currency union, which country C and U follows:

$$\widehat{M}^u = \widehat{M} = \widehat{M}^*.$$

$\widehat{\Lambda}^u$ is the average productivity shock in the currency union:

$$\widehat{\Lambda}^u = \frac{m}{n}\widehat{\Lambda} + \frac{n-m}{n}\widehat{\Lambda}^*.$$

4.5.2 Welfare Effects of Various Exchange Rate Regimes

In this section, I will compare the welfare effects of different exchange rate regimes under monetary and technology shocks. For simplicity and comparability, I choose to calibrate some of the parameters before the calculation of country C welfare. However, the main results are irrelevant to the parameter values chosen. In particular, the discount factor β is calibrated at 0.99 (and thus $\overline{r} = 0.0101$); the elasticity of substitution between varieties θ is set equal to 6, implying a steady state markup of 1.2. The parameter values of the monetary feedback equation follows Lubik and Schorfheide (2005), who set $\rho = 0.5$, $\lambda_1 = 1.5$, $\lambda_2 = 0.5$, (thus $a = 0.75$, $b = 0.25$). At last, I assume all three countries have equal size, therefore, $m = 1/3$, $n = 2/3$.

Using the utility equation (4.82) and monetary policies under various exchange rate regime, the country C utility change can be derived in terms of the technology shocks and foreign monetary shocks:

$$dU^R_{\text{floating}} = 0.0788\widehat{M}^* + 0.0788\widehat{M}^{**} + 102.7718\widehat{\Lambda} + 2.7391\widehat{\Lambda}^* + 2.7391\widehat{\Lambda}^{**}, \tag{4.89}$$

$$dU^R_{\text{fix}} = 0.1111\widehat{M}^* + 0.0556\widehat{M}^{**} + 102.7730\widehat{\Lambda} + 2.7270\widehat{\Lambda}^* + 2.75\widehat{\Lambda}^{**}, \tag{4.90}$$

$$dU^R_{\text{b\_peg}} = 0.0833\widehat{M}^* + 0.0833\widehat{M}^{**} + 102.7730\widehat{\Lambda} + 2.7385\widehat{\Lambda}^* + 2.7385\widehat{\Lambda}^{**}, \tag{4.91}$$

$$dU^R_{\text{c\_union}} = 0.1356\widehat{M}^{**} + 102.7687\widehat{\Lambda} + 2.7687\widehat{\Lambda}^* + 2.7125\widehat{\Lambda}^{**}. \tag{4.92}$$

The results show that no exchange rate regime is always better or worse than others. Utility changes depend not only upon the choice of the exchange rate regime, but also on the nature of the shocks hitting the economy. For example, the best exchange rate regime under the positive shocks can be the worst one if the shocks are negative.

As shown in equation (4.90), the welfare effects of country U monetary shock is the largest under the fixed exchange rate regime. This implies that a fixed regime country exposes the most to its anchor country's monetary shocks, and thus can benefit (suffer) the most from its anchor's monetary expansion (contraction). There are two factors of the foreign monetary shocks which affect the domestic welfare. Equation (4.82) shows the first factor: the aggregate world monetary shocks can affect country C welfare, and thus the monetary shocks in country U and E can affect the country C welfare directly. The second factor comes from the country C domestic monetary supply change. Under the fixed exchange rate regime, given the technology shocks are exogenous, equation (4.86) shows that there is a one to one response of country C monetary supply change to country U monetary shocks. Of course, this response is smaller in the case of floating regime and basket peg regime as the monetary policy of country C response not only to the monetary shocks in country U, but also to that in country E under these regimes.[14]

So far, I have checked country C welfare effect of country U monetary shocks. The results are reasonable and standard as in a two-country framework: country C exposes the most to country U monetary shocks; the exposure is smaller for basket peg regime and the floating regime. However, the results are incomplete without including the effects of the third country. The equations above show that the welfare effects of country E monetary shocks under the fixed regime case are the smallest. This is because country C monetary policy does not response to country E monetary shocks at all in this case. And thus, the country C welfare is only affected by country E monetary shocks through the aggregate world monetary shocks. This result implies that the fixed exchange regime provides country C insulation to monetary shocks in the third country. Therefore, the fixed exchange rate regime is preferable than the floating regime when the monetary shocks are less volatile in the anchor country and more volatile in other

[14]As $\varphi < 1$, the following inequality hold: $\frac{\varphi(n-m)}{1-\varphi m} < \frac{n-m}{1-m} < 1$. Therefore, equations (4.85), (4.86) and (4.87) show that the response of the domestic monetary policy to country U monetary shocks is the greatest under the fixed regime, and the least under the floating regime. Note that, this result is irrelevant to the chosen values of parameters.

countries (for instance, the rest of the world).

The equations above also show the welfare effects of technology shocks. Country C exposes the most to its own technology shocks under fixed regime and basket peg regime. Under these two regimes, the monetary policy depends on its anchor countries' monetary shocks, and thus cannot adjust in response to domestic technology shocks.[15] Note that, fixed exchange rate regime reduces the exposure of the fixed country to its anchor country's technology shocks, but with the cost of increasing the exposure to these of other floating countries.

With only two-country framework, the country C welfare change in correspondence to the country E monetary and technology shocks cannot be shown. Therefore, country C welfare change is only affected by its own technology shocks and country U shocks. In this case, one may simply conclude that country C exposes more to foreign monetary shocks (or less to foreign technology shocks) under the fixed regime than floating regime. The two-country framework cannot show the welfare effects of the other floating countries, and thus, the results might be inappropriate.

It is also worth noting that the effects of foreign monetary expansion is always greater in the basket peg regime than in the floating regime as shown in equation (4.89) and (4.91). Equation (4.85) and (4.87) show the reason: the country C monetary policy responses to country U and E monetary shocks under the basket peg regime more than under the floating regime.[16] In this case, the basket peg regime country always exposes more to its anchor countries' (country U and E) monetary shocks than the floating regime country. On the other hand, the basket peg country always expose less to foreign technology shocks compare to the floating regime as the basket peg regime supplies an insulation to foreign technology shocks. However, as in the fixed regime case, the basket regime country would expose less (more) to the monetary shocks (technology shocks) of other floating countries, which, of course, can only be shown algebraically in a richer framework.

In the currency union case, country C benefits the most from the country U tech-

[15]The monetary policy equations under different exchange rate regimes show the effects. Note that, for the floating and currency union regime, country C does not need to follow the foreign monetary shocks as the monetary policy is jointly determined by the foreign shocks and the values of parameters in the interest rate feedback rule.

[16]This is because $\frac{\varphi(n-m)}{1-\varphi m} < \frac{n-m}{1-m}$ as mentioned before.

nology improvement compare to other regimes, and, of course, will suffer the most if there is a negative country U technology shock. As in the case of currency union, the positive technology shock in country U leads to a monetary expansion in the union as shown in equation (4.88). As discussed before, the monetary expansion could increase the aggregate demand and output and move the monopolistic equilibrium closer to the efficient level of production. However, under currency union, although exposing the most to the technology shock of its partner country (country U), country C exposes the least to the technology shocks of other floating countries (country E). Therefore, currency union regime is preferable in the case of less volatile technology shocks in the currency union and more volatile technology shock in the rest of the world. As we see, the three-country framework is still desirable to study the welfare effects of this regime.

4.6 Conclusion

Base on the technology developed by Obstfeld and Rogoff (1995a), I construct a three-country NOEM model, which can be used to study the welfare effects of monetary and technology shocks under various exchange rate regimes. The results show that no exchange rate regime is always better or worse than others. They utility changes depend not only on the choice of the exchange rate regime, but also on the types of the technology shocks and foreign monetary shocks. In particular, I find that different exchange rate regimes provide insulation to certain monetary and technology shocks, but with the cost of exposing more to other monetary and technology shocks.

The results show that the three-country framework is preferable than the traditional two country framework as the effects of the shocks in the third country cannot be shown in the two-country model. Given that the shocks in the third country have important effects on the welfare of the country of interest (country C), the result implied by the two-country framework can be very different from the three-country framework. For example, the two-country framework would always suggest adopting the floating exchange rate regime as it reduces the exposure to the foreign monetary shocks. However, in the three-country framework, it is clear that although the fixed exchange rate regime country exposes the most to the monetary shocks of its anchor country, it helps reducing the exposure to the monetary shocks of the third country. Therefore, the fixed exchange rate regime might be preferable than the floating regime when the monetary shocks are less volatile in the anchor country and more volatile in the third country.

Chapter 5

Appendix

5.1 Appendix to Chapter 2

5.1.1 The Log-linearised Equilibrium Conditions

The non-linear model equations are then log-linearised around the specific steady state, and further expressed as differences between the Home country variables and Foreign counterparts. The system can be written in the following 22 variables: $\widehat{P}_t - \widehat{P}_t^*$, $\widehat{p}_{H,t} - \widehat{p}_{F,t}^*$, $\widehat{p}_{F,t} - \widehat{p}_{H,t}^*$, $\widehat{p}_{N,t} - \widehat{p}_{N,t}^*$, $\widehat{Y}_t - \widehat{Y}_t^*$, $\widehat{y}_{H,t} - \widehat{y}_{F,t}^*$, $\widehat{y}_{F,t} - \widehat{y}_{H,t}^*$, $\widehat{y}_{N,t} - \widehat{y}_{N,t}^*$, $\widehat{K}_t - \widehat{K}_t^*$, $\widehat{k}_{H,t} - \widehat{k}_{F,t}^*$, $\widehat{mc}_{H,t} - \widehat{mc}_{F,t}^*$, $\widehat{L}_t - \widehat{L}_t^*$, $\widehat{l}_{H,t} - \widehat{l}_{F,t}^*$, $\widehat{l}_{N,t} - \widehat{l}_{N,t}^*$, $\widehat{\omega}_t - \widehat{\omega}_t^*$, $\widehat{R}_t - \widehat{R}_t^*$, $\widehat{C}_t - \widehat{C}_t^*$, $\widehat{Z}_t - \widehat{Z}_t^*$, $b_t - b_t^*$, $\widehat{s}_t$, $\widehat{q}_t$, $Tb_{H,t}$.

First the linearised form of the Home final goods price index can be written as:

$$\widehat{P}_t = a_T a_H \widehat{p}_{H,t} + a_T (1 - a_H) \widehat{p}_{F,t} + (1 - a_T) \widehat{p}_{N,t},$$

where, the linearised form of the Home tradable goods price index is used:

$$\widehat{P}_{T,t} = a_H \widehat{p}_{H,t} + (1 - a_H) \widehat{p}_{F,t}.$$

Together with the Foreign counterpart, the log-linearised price index in country difference can be obtained as:

$$\widehat{P}_t - \widehat{P}_t^* = a_T \left[a_H \left(\widehat{p}_{H,t} - \widehat{p}_{F,t}^*\right) + (1 - a_H) \left(\widehat{p}_{F,t} - \widehat{p}_{H,t}^*\right)\right] + (1 - a_T) \left(\widehat{p}_{N,t} - \widehat{p}_{N,t}^*\right).$$

The Home demand functions can be linearised as:

$$\widehat{y}_{H,t} = [(\rho - \phi)\, a_H - \rho]\, \widehat{p}_{H,t} + (\rho - \phi)\,(1 - a_H)\, \widehat{p}_{F,t} + \phi \widehat{P}_t + \widehat{Y}_t,$$

$$\widehat{y}_{F,t} = (\rho - \phi)\, a_H \widehat{p}_{H,t} + [(\rho - \phi)\,(1 - a_H) - \rho]\, \widehat{p}_{F,t} + \phi \widehat{P}_t + \widehat{Y}_t,$$

$$\widehat{y}_{N,t} = -\phi \widehat{p}_{N,t} + \phi \widehat{P}_t + \widehat{Y}_t,$$

which can be combined with Foreign counterparts to form:

$$\begin{aligned}&\left(\widehat{y}_{H,t} - \widehat{y}^*_{F,t}\right) - (\rho - \phi)\,(1 - a_H)\left(\widehat{p}_{F,t} - \widehat{p}^*_{H,t}\right)\\ &= [(\rho - \phi)\, a_H - \rho]\left(\widehat{p}_{H,t} - \widehat{p}^*_{F,t}\right) + \phi\left(\widehat{P}_t - \widehat{P}^*_t\right) + \left(\widehat{Y}_t - \widehat{Y}^*_t\right),\end{aligned}$$

$$\begin{aligned}&\left(\widehat{y}_{F,t} - \widehat{y}^*_{H,t}\right) - [(\rho - \phi)\,(1 - a_H) - \rho]\left(\widehat{p}_{F,t} - \widehat{p}^*_{H,t}\right)\\ &= (\rho - \phi)\, a_H\left(\widehat{p}_{H,t} - \widehat{p}^*_{F,t}\right) + \phi\left(\widehat{P}_t - \widehat{P}^*_t\right) + \left(\widehat{Y}_t - \widehat{Y}^*_t\right),\end{aligned}$$

$$\left(\widehat{y}_{N,t} - \widehat{y}^*_{N,t}\right) = -\phi\left(\widehat{p}_{N,t} - \widehat{p}^*_{N,t}\right) + \phi\left(\widehat{P}_t - \widehat{P}^*_t\right) + \left(\widehat{Y}_t - \widehat{Y}^*_t\right).$$

The production functions for both tradable and non-tradable goods firms are:

$$\begin{aligned}&\frac{\overline{y}_H}{\overline{y}_H + \overline{y}^*_H}\left(\widehat{y}_{H,t} - \widehat{y}^*_{F,t}\right) - \frac{\overline{y}^*_H}{\overline{y}_H + \overline{y}^*_H}\left(\widehat{y}_{F,t} - \widehat{y}^*_{H,t}\right)\\ &= \left(\widehat{\Lambda}_t - \widehat{\Lambda}^*_t\right) + \alpha\left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right) + (1 - \alpha)\left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right),\end{aligned}$$

$$\widehat{y}_{N,t} - \widehat{y}^*_{N,t} = \left(\widehat{\Lambda}_t - \widehat{\Lambda}^*_t\right) + \left(\widehat{l}_{N,t} - \widehat{l}^*_{N,t}\right).$$

The tradable goods firm's optimal price setting conditions can be linearised as:

$$\begin{aligned}&\beta \chi_p E_t\left(\widehat{p}_{H,t+1} - \widehat{p}^*_{F,t+1}\right) + \left[1 - \theta - (1 + \beta)\,\chi_p\right]\left(\widehat{p}_{H,t} - \widehat{p}^*_{F,t}\right)\\ &= -\chi_p\left(\widehat{p}_{H,t-1} - \widehat{p}^*_{F,t-1}\right) - (\theta - 1)\left(\widehat{P}_t - \widehat{P}^*_t\right) - (\theta - 1)\left(\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t}\right),\end{aligned}$$

$$\begin{aligned}&-\beta \chi_p E_t\left(\widehat{p}_{F,t+1} - \widehat{p}^*_{H,t+1}\right) - \left[1 - \theta - (1 + \beta)\,\chi_p\right]\left(\widehat{p}_{F,t} - \widehat{p}^*_{H,t}\right) + 2\,(1 - \theta)\,\widehat{s}_t\\ &= \chi_p\left(\widehat{p}_{F,t-1} - \widehat{p}^*_{H,t-1}\right) - (\theta - 1)\left(\widehat{P}_t - \widehat{P}^*_t\right) - (\theta - 1)\left(\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t}\right),\end{aligned}$$

where

$$\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t} = (\widehat{\omega}_t - \widehat{\omega}^*_t) - \alpha\left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right) + \alpha\left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right) - \left(\widehat{\Lambda}_t - \widehat{\Lambda}^*_t\right).$$

And the labour-capital trade-off:

$$(\widehat{r}_t - \widehat{r}^*_t) - (\widehat{\omega}_t - \widehat{\omega}^*_t) = \left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right) - \left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right).$$

The optimal conditions for non-tradable goods firm are:

$$\begin{aligned}&\beta\chi_p E_t\left(\widehat{p}_{N,t+1} - \widehat{p}^*_{N,t+1}\right) + \left[1-\theta-(1+\beta)\chi_p\right]\left(\widehat{p}_{N,t} - \widehat{p}^*_{N,t}\right) + \chi_p\left(\widehat{p}_{N,t-1} - \widehat{p}^*_{N,t-1}\right)\\ &= -(\theta-1)\left(\widehat{P}_t - \widehat{P}^*_t\right) - (\theta-1)(\widehat{\omega}_t - \widehat{\omega}^*_t) + (\theta-1)\left(\widehat{\Lambda}_t - \widehat{\Lambda}^*_t\right),\end{aligned}$$

The household's optimal conditions are:

$$\sigma_1\left(\widehat{C}_t - \widehat{C}^*_t\right) + \frac{1}{\sigma_2}\left(\widehat{L}_t - \widehat{L}^*_t\right) - (\widehat{\omega}_t - \widehat{\omega}^*_t) - (\tau_t - \tau^*_t) = 0,$$

$$\begin{aligned}&\beta\chi_k E_t\left(\widehat{K}_{t+1} - \widehat{K}^*_{t+1}\right) - \sigma_1 E_t\left(\widehat{C}_{t+1} - \widehat{C}^*_{t+1}\right) + E_t\left(\tau_{t+1} - \tau^*_{t+1}\right)\\ &+ \left[1-\beta(1-\delta)\right]E_t\left(\widehat{r}_{t+1} - \widehat{r}^*_{t+1}\right) - (1+\beta)\chi_k\left(\widehat{K}_t - \widehat{K}^*_t\right)\\ &+ \sigma_1\left(\widehat{C}_t - \widehat{C}^*_t\right) + \chi_k\left(\widehat{K}_{t-1} - \widehat{K}^*_{t-1}\right) - (\tau_t - \tau^*_t) = 0,\end{aligned}$$

$$\begin{aligned}&\sigma_1 E_t\left(\widehat{C}_{t+1} - \widehat{C}^*_{t+1}\right) + E_t\left(\widehat{P}_{t+1} - \widehat{P}^*_{t+1}\right) - E_t\left(\tau_{t+1} - \tau^*_{t+1}\right)\\ &+ (\tau_t - \tau^*_t) - \sigma_1\left(\widehat{C}_t - \widehat{C}^*_t\right) - \left(\widehat{P}_t - \widehat{P}^*_t\right) - \left(\widehat{R}_t - \widehat{R}^*_t\right) = 0.\end{aligned}$$

Define $b_t = \frac{B_t}{P_tY_t}$, and $b^*_t = \frac{s_tB^*_t}{P_tY_t}$, the interest rate parity equation can be written as:

$$\widehat{R}_t - \widehat{R}^*_t = (E_t\widehat{s}_{t+1} - \widehat{s}_t) - \psi_{RP}(b_t - b^*_t) + \varepsilon_{\xi,t}.$$

The resource constraint is:

$$\widehat{Y}_t - \widehat{Y}^*_t = \frac{\overline{C}}{\overline{Y}}\left(\widehat{C}_t - \widehat{C}^*_t\right) + \frac{\overline{K}}{\overline{Y}}\left(\widehat{K}_t - \widehat{K}^*_t\right) - (1-\delta)\frac{\overline{K}}{\overline{Y}}\left(\widehat{K}_{t-1} - \widehat{K}^*_{t-1}\right).$$

The goods market condition is:

$$\widehat{Z}_t - \widehat{Z}_t^* = a_H a_T \left(\widehat{y}_{H,t} - \widehat{y}_{F,t}^*\right) - (1-a_H)\, a_T \left(\widehat{y}_{F,t} - \widehat{y}_{H,t}^*\right) + (1-a_T) \left(\widehat{y}_{N,t} - \widehat{y}_{N,t}^*\right).$$

The labour and capital market clearing conditions are given by

$$\widehat{L}_t - \widehat{L}_t^* = \frac{\bar{l}_H}{\bar{L}} \left(\widehat{l}_{H,t} - \widehat{l}_{F,t}^*\right) + \frac{\bar{l}_N}{\bar{L}} \left(\widehat{l}_{N,t} - \widehat{l}_{N,t}^*\right),$$

$$\widehat{K}_t - \widehat{K}_t^* = \left(\widehat{k}_{H,t} - \widehat{k}_{F,t}^*\right).$$

Define, $Tb_{H,t} = \frac{TB_{H,t}}{P_t Y_t}$, the trade balance can be write as:

$$\frac{1}{(1-a_H)\, a_T} Tb_{H,t} = s_t - \left(\widehat{P}_{F,t} - \widehat{P}_{H,t}^*\right) - \left(\widehat{Y}_{F,t} - \widehat{Y}_{H,t}^*\right).$$

The real exchange rate is:

$$\widehat{q}_t = \widehat{s}_t - \left(\widehat{P}_t - \widehat{P}_t^*\right).$$

The budget constraint is:

$$Tb_{H,t} - (b_t - b_t^*) + \frac{1}{\beta} \left(b_{t-1} - b_{t-1}^*\right) = 0.$$

Finally, the monetary policy rule is:

$$\begin{aligned}\left(\widehat{R}_t - \widehat{R}_t^*\right) &= \rho_R \left(\widehat{R}_{t-1} - \widehat{R}_{t-1}^*\right) + (1-\rho_R)\{a_1 \left[\left(\widehat{P}_t - \widehat{P}_t^*\right) - \left(\widehat{P}_{t-1} - \widehat{P}_{t-1}^*\right)\right] \\ &+ a_2 \left[\left(\widehat{Z}_t - \widehat{Z}_t^*\right) - \left(\widehat{Z}_{t-1} - \widehat{Z}_{t-1}^*\right)\right] + 2a_3 \left(\widehat{s}_t - \widehat{s}_{t-1}\right)\} + \left(\mu_t - \mu_t^*\right).\end{aligned}$$

5.1.2 Data Set

- The GDP data is from IMF IFS (series code: 99B. CZF). For E.U. countries, the nominal GDP data series are transformed to real data by deflating the CPI index (2000 as the base year) of each country. The data are then transformed into 2000 year international dollar terms using PPP exchange rate 2000. The E.U. real GDP is calculated as the aggregate of France, Germany, Italy and U.K..

  The labour force data is used to construct the per capita real GDP. The U.S. labour force data is from IMF IFS (series code: 67D.. ZF). U.K. data is from UK national statistic labour Force Survey 2007. France labour force data is from MEI (series code: FRA. PLLFTLTT. ST). For France, quarterly data is not available, so the annual data are used. The annual data is converted to quarterly frequency using a quadratic interpolation (quadratic match average). Germany data is from OECD MEI (series code: DEU. PLLFTLTT. STSA) during 1971:1-2004:4 and IMF IFS (series code: 67D.. ZF) afterwards. Italy data is from MEI (series code: ITA. PLLFTLTT. STSA). The E.U. labour force data is calculated as the aggregate of France, Germany, Italy and U.K..

Per capita real output data is then constructed as the real GDP divided by labour force.

- The CPI data is from OECD MEI (series code: CPALTT01. IXOB). The E.U. CPI is constructed as the geometric average of the four countries:

$$CPI_t = \Pi_{i=1}^{n} \left(CPI_{i,t}\right)^{w_{i,t}},$$

where the time-varying weight $w_{i,t}$ correspond to the real GDP weight

$$w_{i,t} = \frac{GDP_{i,t}}{\Sigma_{i=1}^{n} GDP_{i,t}}.$$

- The U.S. interest rate is the three-month CDs rate from OECD MEI (series code: USA. IR3TCD01. ST). The interest rate data for UK is the three-month treasury bills rate from OECD MEI (series code: GBR. IR3TTS01. ST). The France data is the three-month treasury bills rate from IMF IFS (series code: 60C.. ZF). The IFS data ends at 2004:3, thus the data (series code: QS. M. IFRTRF3M) from bank of France is used after this period. The Germany interest rate is the call money rate from IMF IFS (series code: 60B.. ZF). The data for Italy is the money market rate from IMF IFS (series code: 60B.. ZF). The E.U. nominal short-term interest rate are again the geometric average of France, Germany, Italy and U.K.. The short-term interest rate is then divided by four to obtain its quarterly equivalent.

- The nominal exchange rate data are from OECD MEI (series code: CCUSSP01.

ST), data are the bilateral exchange rate between the U.S. and other countries. The bilateral exchange rate between the U.S. and E.U. is then constructed as the form

$$s_t = \Pi_{i=1}^{n} (\frac{s_{i,t}}{s_{i,0}})^{w_{i,t}},$$

where $s_{i,t}$ is the exchange rate for country i in period t, $e_{i,0}$ is the exchange rate for country i in the first quarter of 1971. The real exchange rate is then constructed as:

$$q_t = s_t \frac{CPI_t^*}{CPI_t}.$$

5.2 Appendix to Chapter 3

5.2.1 Decomposition of the Real Exchange Rate Fluctuations

Under the case where non-tradable goods are used in both final goods production and distribution, the price indices in the Home country will be:

$$P_T = \left(a_H P_H^{1-\rho} + (1 - a_H) P_F^{1-\rho}\right)^{\frac{1}{1-\rho}},$$

$$P = \left(a_T P_T^{1-\phi} + (1 - a_T) P_N^{1-\phi}\right)^{\frac{1}{1-\phi}}.$$

where the P_H and P_F are the consumer price of the representative intermediate tradable goods from Home and Foreign respectively. The log-linearised form is:

$$\widehat{P}_T = a_H \widehat{P}_H + (1 - a_H) \widehat{P}_F, \tag{5.1}$$

$$\widehat{P} = a_T \widehat{P}_T + (1 - a_T) \widehat{P}_N. \tag{5.2}$$

The intermediate goods are delivered to the final good producer with the distribution cost:

$$P_H = \widetilde{P}_H + \kappa P_N,$$

$$P_F = \widetilde{P}_F + \kappa P_N,$$

where $\widetilde{P}_H$ and $\widetilde{P}_F$ are the prices at the producer level, κ is the distribution cost parameter. The log-linearised form is

$$\widehat{P}_H = (1-\kappa)\,\widehat{\widehat{P}}_H + \kappa\widehat{P}_N, \tag{5.3}$$

$$\widehat{P}_F = (1-\kappa)\,\widehat{\widehat{P}}_F + \kappa\widehat{P}_N. \tag{5.4}$$

The real exchange rate would be

$$q = s\frac{P^*}{P}.$$

And the log-linearised form is

$$\widehat{q} = \widehat{s} + \widehat{P}^* - \widehat{P}. \tag{5.5}$$

Using equations (5.1) – (5.4) and their Foreign counterparts, the real exchange rate can be expressed as

$$\begin{aligned}\widehat{q} \;=\; & \left[a_H\left(\widehat{s}+\widehat{\widehat{P}}{}^*_F-\widehat{\widehat{P}}_H\right)+(1-a_H)\left(\widehat{s}+\widehat{\widehat{P}}{}^*_H-\widehat{\widehat{P}}_F\right)\right] \\ & +(1-a_T+\kappa a_T)\left[\left(\widehat{\widehat{P}}_T-\widehat{P}_N\right)-\left(\widehat{\widehat{P}}{}^*_T-\widehat{P}^*_N\right)\right],\end{aligned} \tag{5.6}$$

where $\widehat{\widehat{P}}_T$ and $\widehat{\widehat{P}}{}^*_T$ are the price indices at the producer level of the aggregate tradable goods used in Home and Foreign respectively:

$$\widehat{\widehat{P}}_T = (1-\kappa)\,\widehat{\widehat{P}}_T + \kappa\widehat{P}_N,$$

$$\widehat{\widehat{P}}{}^*_T = (1-\kappa)\,\widehat{\widehat{P}}{}^*_T + \kappa\widehat{P}^*_N.$$

5.2.2 Log-linearised System

Since there is positive inflation in the model, the prices need to be normalised in order to make the system of equations stationary. Following Cristadoro et al. (2006), the prices are transformed into following relative price:

$$T_t \equiv \frac{P_{H,t}}{P_{F,t}},\; T_t^* \equiv \frac{P^*_{F,t}}{P^*_{H,t}},$$

$$N_t \equiv \frac{P_{N,t}}{P_{H,t}}, \; N_t^* \equiv \frac{P_{N,t}^*}{P_{F,t}^*},$$

T and T^* represent the price of domestic tradable composite good in terms of the imported tradable composite good in the Home and Foreign country, respectively. N and N^* are the price of non-tradable composite good in terms of domestic tradable composite good in the Home and Foreign country, respectively.

Then, the relative price equations above can be log-linearised as:

$$\widehat{T}_t = \widehat{T}_{t-1} + \pi_{H,t} - \pi_{F,t},$$

$$\widehat{T}_t^* = \widehat{T}_{t-1}^* + \pi_{F,t}^* - \pi_{H,t}^*,$$

$$\widehat{N}_t = \widehat{N}_{t-1} + \pi_{N,t} - \pi_{H,t},$$

$$\widehat{N}_t^* = \widehat{N}_{t-1}^* + \pi_{N,t}^* - \pi_{F,t}^*.$$

Then re-write the above equations in country difference to obtain:

$$\widehat{T}_t - \widehat{T}_t^* = \left(\widehat{T}_{t-1} - \widehat{T}_{t-1}^*\right) + \left(\pi_{H,t} - \pi_{F,t}^*\right) - \left(\pi_{F,t} - \pi_{H,t}^*\right),$$

$$\widehat{N}_t - \widehat{N}_t^* = \left(\widehat{N}_{t-1} - \widehat{N}_{t-1}^*\right) + \left(\pi_{N,t} - \pi_{N,t}^*\right) - \left(\pi_{H,t} - \pi_{F,t}^*\right).$$

The price index equations (3.12) and its Foreign counterpart can be linearised as:

$$\pi_t - \pi_t^* = a_H \left(\pi_{H,t} - \pi_{F,t}^*\right) + (1 - a_H)\left(\pi_{F,t} - \pi_{H,t}^*\right).$$

The producer level inflation rate of intermediate goods can be obtained by linearising equations (3.13), (3.14) and the Foreign counterparts,

$$\widetilde{\pi}_{H,t} - \widetilde{\pi}_{F,t}^* = \frac{1}{1-\kappa}\left(\pi_{H,t} - \pi_{F,t}^*\right) - \frac{\kappa}{1-\kappa}\left(\pi_{N,t} - \pi_{N,t}^*\right),$$

$$\widetilde{\pi}_{F,t} - \widetilde{\pi}_{H,t}^* = \frac{1}{1-\kappa}\left(\pi_{F,t} - \pi_{H,t}^*\right) - \frac{\kappa}{1-\kappa}\left(\pi_{N,t} - \pi_{N,t}^*\right).$$

Also, the linearised version of real exchange rate (3.34) is:

$$\widehat{q}_t = \widehat{q}_{t-1} + \Delta\widehat{s}_t + \pi_t^* - \pi_t.$$

For the demand block of the economy, using equations (3.8), (3.9), (3.15) and their Foreign counterparts, the demand functions for the intermediate goods are:

$$\widehat{y}_{H,t} - \widehat{y}^*_{F,t} = \left(\widehat{Y}_t - \widehat{Y}^*_t\right) - \rho\left(1 - a_H\right)\left(\widehat{T}_t - \widehat{T}^*_t\right),$$

$$\widehat{y}_{F,t} - \widehat{y}^*_{H,t} = \left(\widehat{Y}_t - \widehat{Y}^*_t\right) + \rho a_H\left(\widehat{T}_t - \widehat{T}^*_t\right),$$

$$\widehat{y}_{N,t} - \widehat{y}^*_{N,t} = \left(\widehat{Y}_t - \widehat{Y}^*_t\right).$$

For the supply side of the economy, the equilibrium conditions of the tradable goods firms (3.16), (3.20), (3.21), (3.22) and their Foreign counterparts can be linearised as:

$$\begin{aligned} & a_H\left(\widehat{y}_{H,t} - \widehat{y}^*_{F,t}\right) - \left(1 - a_H\right)\left(\widehat{y}_{F,t} - \widehat{y}^*_{H,t}\right) \\ & = \alpha\left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right) + (1-\alpha)\left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right) + \left(\widehat{\Lambda}_{H,t} - \widehat{\Lambda}^*_{F,t}\right), \end{aligned}$$

$$\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t} = \left(\widehat{\omega}_t - \widehat{\omega}^*_t\right) - \alpha\left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right) + \alpha\left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right) - \left(\widehat{\Lambda}_{H,t} - \widehat{\Lambda}^*_{F,t}\right),$$

$$\begin{aligned} & \frac{\theta\kappa}{\theta(1-\kappa)-1}\left(\widehat{N}_t - \widehat{N}^*_t\right) - \left(1 - a_H\right)\left(\widehat{T}_t - \widehat{T}^*_t\right) + \left(\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t}\right) \\ & = \frac{1}{\theta(1-\kappa)-1}\left[\chi_H\left(\widetilde{\pi}_{H,t} - \widetilde{\pi}^*_{F,t}\right) - \beta\chi_H E_t\left(\widetilde{\pi}_{H,t+1} - \widetilde{\pi}^*_{F,t+1}\right)\right], \end{aligned}$$

$$\begin{aligned} & \left(\frac{\theta\kappa}{\theta(1-\kappa)-1} + a_H\right)\left(\widehat{T}_t - \widehat{T}^*_t\right) + 2\widehat{q}_t + \frac{\theta\kappa}{\theta(1-\kappa)-1}\left(\widehat{N}_t - \widehat{N}^*_t\right) \\ & = \left(\widehat{mc}_{H,t} - \widehat{mc}^*_{F,t}\right) + \frac{1}{\theta(1-\kappa)-1}\left[\chi_F\left(\widetilde{\pi}_{F,t} - \widetilde{\pi}^*_{H,t}\right) - \beta\chi_F E_t\left(\widetilde{\pi}_{F,t+1} - \widetilde{\pi}^*_{H,t+1}\right)\right], \end{aligned}$$

$$\left(\widehat{r}_t - \widehat{r}^*_t\right) - \left(\widehat{\omega}_t - \widehat{\omega}^*_t\right) = \left(\widehat{l}_{H,t} - \widehat{l}^*_{F,t}\right) - \left(\widehat{k}_{H,t-1} - \widehat{k}^*_{F,t-1}\right).$$

For the non-tradable goods firms, the production function (3.23), price setting equations (3.25), (3.27) and their Foreign counterparts are written as:

$$\left(\widehat{y}_{N,t} - \widehat{y}^*_{N,t}\right) = \left(\widehat{l}_{N,t} - l^*_{N,t}\right) + \left(\widehat{\Lambda}_{N,t} - \widehat{\Lambda}^*_{N,t}\right),$$

$$(1-a_H)\left(\widehat{T}_t-\widehat{T}_t^*\right)+\left(\widehat{N}_t-\widehat{N}_t^*\right)-\left(\widehat{\omega}_t-\widehat{\omega}_t^*\right)+\left(\widehat{\Lambda}_{N,t}-\widehat{\Lambda}_{N,t}^*\right)$$
$$=-\frac{1}{\theta-1}\left[\chi_N\left(\pi_{N,t}-\pi_{N,t}^*\right)-\beta\chi_N E_t\left(\pi_{N,t+1}-\pi_{N,t+1}^*\right)\right].$$

The optimal conditions for households are:

$$\widehat{K}_t-\widehat{K}_t^*=(1-\delta)\left(\widehat{K}_{t-1}-\widehat{K}_{t-1}^*\right)+\frac{\overline{I}}{\overline{K}}\left(\widehat{R}_t-\widehat{R}_t^*\right),$$

$$E_t\left(\pi_{t+1}-\pi_{t+1}^*\right)+\sigma_1 E_t\left(\widehat{C}_{t+1}-\widehat{C}_{t+1}^*\right)-E_t\left(\widehat{\tau}_{t+1}-\widehat{\tau}_{t+1}^*\right)$$
$$=\sigma_1\left(\widehat{C}_t-\widehat{C}_t^*\right)+\left(\widehat{R}_t-\widehat{R}_t^*\right)-\left(\widehat{\tau}_t-\widehat{\tau}_t^*\right),$$

$$\sigma_1\left(\widehat{C}_t-\widehat{C}_t^*\right)+\frac{1}{\sigma_2}\left(L_t-L_t^*\right)-\left(\widehat{\omega}_t-\widehat{\omega}_t^*\right)-\left(\widehat{\tau}_t-\widehat{\tau}_t^*\right)=0,$$

$$\beta\chi_k E_t\left(\widehat{K}_{t+1}-\widehat{K}_{t+1}^*\right)-(1+\beta)\chi_k\left(\widehat{K}_t-\widehat{K}_t^*\right)+\chi_k\left(\widehat{K}_{t-1}-\widehat{K}_{t-1}^*\right)$$
$$-\sigma_1 E_t\left(\widehat{C}_{t+1}-\widehat{C}_{t+1}^*\right)+\left[1-\beta(1-\delta)\right]E_t\left(\widehat{r}_{t+1}-\widehat{r}_{t+1}^*\right)$$
$$=-E_t\left(\widehat{\tau}_{t+1}-\widehat{\tau}_{t+1}^*\right)+\left(\widehat{\tau}_t-\widehat{\tau}_t^*\right)-\sigma_1\left(\widehat{C}_t-\widehat{C}_t^*\right).$$

The modified UIP parity, as discussed in equation (3.7) is

$$\widehat{R}_t-\widehat{R}_t^*=E_t\Delta\widehat{s}_{t+1}-\psi_{RP}b_{H,t}^*+\xi_t,$$

where, $b_{H,t}^*=\frac{s_tB_{H,t}^*}{P_tY_t}$ and $b_{F,t}^*=\frac{s_tB_{F,t}^*}{P_tY_t}$.

The monetary policy (3.26) and its Foreign counterpart can be written as:

$$\widehat{R}_t-\widehat{R}_t^*=\rho_R\left(\widehat{R}_{t-1}-\widehat{R}_{t-1}^*\right)+(1-\rho_R)\{a_1\left(\pi_t-\pi_t^*\right)$$
$$+a_2\left[\left(\widehat{Z}_t-\widehat{Z}_t^*\right)-\left(\widehat{Z}_{t-1}-\widehat{Z}_{t-1}^*\right)\right]+2a_3\Delta\widehat{s}_t\}+\left(\mu_t-\mu_t^*\right).$$

The last part is the market clearing conditions. Using (3.37) and its Foreign counterpart, the resource constraint for final goods can be linearised as:

$$\widehat{Y}_t-\widehat{Y}_t^*=\frac{\overline{C}}{\overline{Y}}\left(\widehat{C}_t-\widehat{C}_t^*\right)+\frac{\overline{I}}{\overline{Y}}\left(\widehat{R}_t-\widehat{R}_t^*\right).$$

The log-linearised GDP equation (3.28) is:

$$\widehat{Z}_t - \widehat{Z}_t^* = (1-\kappa)\, a_H \left(\widehat{y}_{H,t} - \widehat{y}_{F,t}^*\right) - (1-\kappa)\,(1-a_H)\left(\widehat{y}_{F,t} - \widehat{y}_{H,t}^*\right) + \kappa\left(\widehat{y}_{N,t} - \widehat{y}_{N,t}^*\right).$$

The labour and capital market clearing conditions can be re-written as:

$$\widehat{L}_t - \widehat{L}_t^* = \frac{\bar{l}_H}{\overline{L}}\left(\widehat{l}_{H,t} - \widehat{l}_{F,t}^*\right) + \frac{\bar{l}_N}{\overline{L}}\left(\widehat{l}_{N,t} - \widehat{l}_{N,t}^*\right),$$

$$\widehat{K}_t - \widehat{K}_t^* = \left(\widehat{k}_{H,t} - \widehat{k}_{F,t}^*\right).$$

The Foreign bond market clearing condition is:

$$b_{H,t}^* + b_{F,t}^* = 0.$$

The trade balance equation (3.32) is log-linearised as:

$$\frac{1}{(1-\kappa)(1-a_H)} Tb_{H,t} = \widehat{q}_t - \left(\widehat{y}_{F,t} - \widehat{y}_{H,t}^*\right)$$
$$+\frac{1}{1-\kappa}\left[(a_H + \kappa - \kappa a_H)\left(\widehat{T}_t - \widehat{T}_t^*\right) + \kappa\left(\widehat{N}_t - \widehat{N}_t^*\right)\right],$$

where $Tb_{H,t} = \frac{TB_{H,t}}{P_t Y_t}$. At last, the log-linearised version of current account equation (3.33) is:

$$b_{H,t}^* - \frac{1}{\beta} b_{H,t-1}^* = Tb_{H,t}.$$

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