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A Fuzzy Approach to Inventory Replenishment Systems

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ABSTRACT

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A FUZZY INVENTORY REPLENISHMENT SYSTEM

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Current business trends affecting inventory replenishment, such as Just-In-Time and Efficient Consumer Response, often aim to reduce inventories to a minimum. On the other hand, the commercial emphasis on good customer service may make it important to keep inventories as high as possible. The tension between these two conflicting objectives makes the decision of how much inventory to hold ambiguous. In practice, inventory managers remove this ambiguity by stating clear assumptions upon which inventory replenishment systems base their calculations. For example, managers may input the desired level of customer service, measured as a simple percentage of out-of-stocks. Therefore, the conventional, precise mathematics of current inventory systems does not reflect the ambiguous nature of inventory replenishment, but rather manages around it. This thesis develops a new type of inventory replenishment system that better reflects this reality.

The thesis begins by identifying, through an historical analysis, the major conventional approaches to inventory replenishment. These approaches are then evaluated and their strengths and weaknesses identified. This evaluation concludes that a heuristic approach to inventory replenishment that can cope with the competing objectives of cost and customer service may prove beneficial. Such an approach may be based on fuzzy logic. Hence, a simple fuzzy-based inventory replenishment system is developed and its performance analysed through simulation against a simple, conventional system. The fuzzy-based inventory replenishment system developed achieves higher customer service for the same cost, as well as providing other practical benefits, such as more stable stock levels and order sizes. This opens up a potentially rich area for inventory research and practice.

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Figures

Tables

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Chapter 1

AN INTRODUCTION
Taking stock of inventory

Inventory management

The word 'inventory' stems from the medieval Latin word *'inventorium'*[1] meaning *'a list of what is found'*[1]. The word 'management' stems from the Latin word *'manus'*[1] meaning *'hand'*[1]. Thus, the average medieval Roman's approach to inventory replenishment in any business may well have been to maintain *by hand, a list of what is found.* This approach may have been acceptable in Italian businesses of the Middle Ages but is unlikely to be successful today. Just as the words have evolved so has inventory management.

Today, 'inventory' may be defined as:

'the quantity of stock or goods etc. which are or may be made the subject of an inventory [a detailed list of items ...with a statement giving the nature and value of each item].'[1]

'Management' may be defined as:

'the application of skill or care in the manipulation, use, treatment, or control of things or persons.'[1]

Thus, inventory management may be defined as:

the application of skill or care in the manipulation, use, treatment, or control of a quantity of stock or goods etc..

Inventory supply chains

All products are supplied through a chain of processes and organisations. For example, a product may originate as raw materials. These may be sold by various suppliers to a manufacturer who transforms them into finished goods. These finished products may then be distributed via a number of retailers to consumers. Such supply chains are often complex and involve a network of organisations. Within a complete supply chain there are various points where product (or inventory) may be stored for many reasons (see Table 1). The types of inventory stored at these various points may be classified as follows[2]:

- **Raw materials** - materials supplied to an organisation prior to
- **Work-in-progress** - goods transformed in some way from those raw materials supplied yet not ready to be released as finished products to a customer.
- **Finished goods** - product available for the next customer in the supply chain.

Not all organisations hold inventory in all of these formats. For example, retailers generally hold inventory only in the form of finished goods.

The need for inventory

Inventory is required at many points within supply chains due to differences between supply and demand. These differences may relate primarily to supply or demand, or a combination of both, and may be split between factors that are internal or external to the organisation. Table 1 shows some examples of reasons to hold inventory.

	Supply related issues	***Demand related issues***
Internal factors	• Large batch sizes for economies of scale. • Unreliable machinery. • Poor management/labour. • Even batch sizes for ease of production. • Factory/distribution/retail locations.	• Demand stimulation. • Agreed contract for specific customer. • Merchandising effect. • Promotional stocks. • Poor demand forecasting systems. • Impulse purchases.
External factors	• Variable supplier delivery. • Price discount on bulk order. • Expected price increase. • Variable product availability.	• Variable customer demand. • Available for customer to try out. • Emergency inventory for immediate use. • Seasonal demand. • Change in economic conditions.

Table 1: Examples of reasons to hold inventory

Inventory trends

It is clear that inventory levels have generally reduced over the last 30 years or more. For example, in the UK inventory levels have fallen from around 40% of Gross Domestic Product (GDP) in the early 1960's to around 20% of GDP in the early 1990's[3]. At the same time customer service expectations of supply have generally increased[4]. Perhaps a common misconception of inventory is that it no longer exists in successful businesses. Although inventories have reduced, they still exist and are substantial, being estimated at £119 billion in the UK alone in 1991[5].

Inventory replenishment strategies adopted over the last 30 years may have helped reduce inventories within supply chains but they have not eliminated them completely. Much of this reduction in inventory is often attributed to the introduction of *Just-In-Time*[6] or *JIT* principles, which focuses on the elimination of inventory throughout the supply chain. This approach can provide both benefits and costs to organisations. For example, a Just-In-Time approach may reduce warehousing and storage requirements, while increasing transport and distribution requirements.

One of the major benefits often argued for employing Just-In-Time principles is that:

'In the past, we have covered up our problems by means of a sea of inventory.'[7]

Thus, the theory of Just-In-Time suggests that by eliminating inventory the previously hidden problems of the business will come to the surface, such as late deliveries, work-in-progress queues, unstable demand and so on (see Table 1).

However, it can be argued that while Just-In-Time practices have helped to improve many areas of inventory replenishment, Just-In-Time is not a panacea for all the potential ills of inventory replenishment. Indeed there may be a danger that the Just-In-Time gospel of the elimination of inventory, having spread through supply chains and across industries, becomes a dogmatic chant resulting in ever increasing and unsustainable distribution requirements. Furthermore, inventory replenishment problems may be hidden elsewhere. For example, delivery frequency may be increased to such an extent that if an error occurs in one delivery it appears not to matter because the next delivery will arrive shortly afterwards. Reducing 'the sea of inventory' may have helped see new areas for business improvement, but as Archimedes found out, the water is only displaced elsewhere.

Scope of this thesis

The broad aim of this thesis is to consider the future development of inventory replenishment in general, and to evaluate a new approach to inventory replenishment in particular. The thesis begins by discussing inventory replenishment in the broadest business context: both in terms of the supply chain, from raw material supplier to consumer, and in terms of product range, from commodities to luxury goods. The thesis then focuses on developing a new inventory replenishment system that may be benchmarked against an example of current theory and practice.

In summary, this thesis aims to answer the following research questions:

1. What are the major approaches to inventory replenishment?

2. What are the strengths and weaknesses of current approaches?

3. How might a new replenishment system build on the strengths and negate some of the weaknesses of current approaches?

4. How does such an original replenishment system compare with a typical, current system?

5. Is there a future for such an inventory replenishment system?

Therefore, this research has the following aims and objectives.

Aim

This research aims:

1. To understand the major approaches to inventory replenishment

2. To propose, develop and evaluate a new inventory replenishment system

Objectives

This research has the following major objectives:

1. To identify, classify and critically evaluate the major approaches to inventory replenishment (Chapters 2 and 3).

2. To develop an alternative approach to inventory replenishment systems (Chapter 4).

3. To compare and contrast the alternative approach developed with a typical, current inventory replenishment system (Chapters 5 and 6).

Overall Methodology

The methodology is primarily deductive: formulating a hypothesis and then testing it based in the empirical world. This is appropriate and relevant to inventory management by definition, though other approaches are employed as appropriate. The overall methodology consists of six main stages:

1. *Historical review of inventory replenishment literature* to identify the major philosophical approaches to inventory management.

2. *Survey of current practitioners combined with a review of current trade and journal articles* to evaluate the theory and practice of the major approaches to inventory replenishment, and to identify the approach most likely to yield successful development in the future.

3. *Literature review of relevant theory and practice that may be newly applied to inventory replenishment systems* to provide the foundation knowledge on which to develop an original inventory replenishment system within the philosophical approach identified in 2.

4. *Development of an original inventory replenishment system.*

5. *Development of a model* to test the performance of an example of current inventory replenishment systems against the original inventory replenishment system developed in 4. The methodology employed here is discussed in more detail in Chapter 5.

6. *Reflections on the results and research process* to highlight the new understanding gained and to identify future research potential.

Chapter 2

CLASSIFYING APPROACHES TO INVENTORY REPLENISHMENT
Time to say farewell to EOQs and Just-In-Time

Inventory replenishment decisions may be considered to be as varied as the people, products and businesses that utilise them. By identifying the underlying paradigms of inventory replenishment over time and placing them into a common framework, it is possible to understand the historical development of theory and practice, and to point the way towards future profitable research and implementation.

Fundamental paradigms

Although current classifications of inventory replenishment systems exist, they do not provide an integrating framework for further evaluation. Typical of such classifications are those found in Inventory Management or Operations Management texts and journals, such as Silver & Peterson[8], Wild[9], Buffa & Sarin[10] or Chikán[11]. These classifications are based on the replenishment systems employed, rather than on conceptual or philosophical grounds. Although precise definitions may vary, three categories of replenishment system are usually discussed:

- **Independent demand based replenishment systems.** For example, optimising costs by calculating Economic Order Quantities (EOQs).
- **Dependent demand based replenishment systems.** For example, satisfying demand objectives by employing Materials Requirements Planning (MRP) techniques.
- **Time based replenishment systems.** For example, reacting to external supply and demand by employing a Just-In-Time (JIT) approach.

Any classification that can contribute to a deeper understanding of inventory replenishment should be seen to be durable (philosophically based) rather than transient (current process or technique based) - as well as exhaustive.

A common framework

From a basic viewpoint, any management decision can be considered to consist of two key questions.

1. What is the business objective?

2. How can it be achieved?

For a retailing example, see Myers, Daugherty & Autry[12]. For a production example, see Li & O'Brien[13]. A durable conceptual framework for identifying the fundamental paradigms of inventory replenishment systems must therefore take account of both the objective and the method employed for achieving the objective. The key business objectives of inventory replenishment can be distilled to minimising cost and maximising customer service. The methods for achieving these objectives may be divided between direct methods and indirect methods. Direct methods focus solely on achieving the primary objective. Indirect methods focus on a key driver related to the primary objective. For example, costs may be minimised by focusing on costs directly or by focusing, indirectly, on cost drivers, such as the amount of physical inventory. By combining these viewpoints, a simple framework for identifying the fundamental paradigms of inventory replenishment can be proposed, as follows.

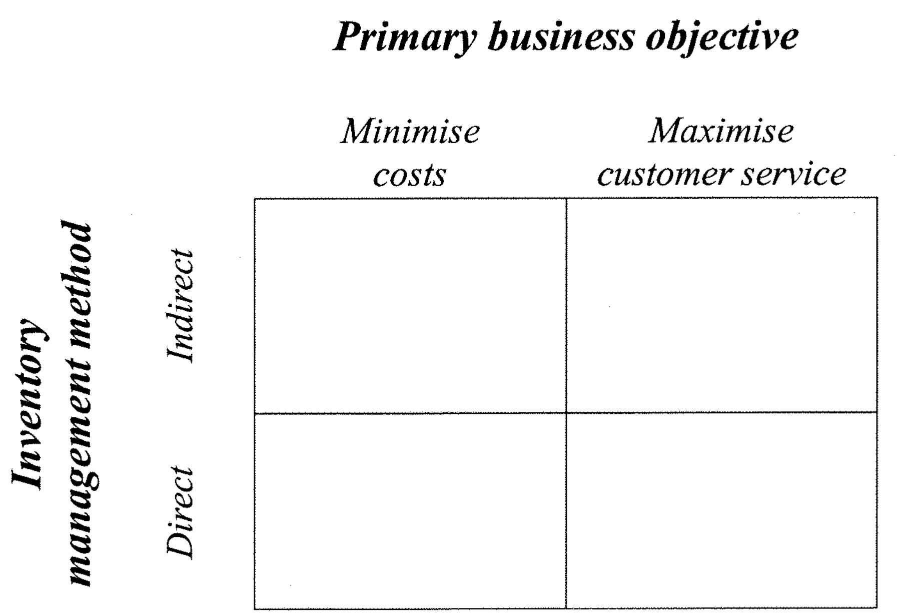


Figure 1: Inventory replenishment paradigms

It is important to note that all inventory replenishment systems will take account of both cost and customer service issues. For example, a replenishment system may exist that aims to minimise cost while maximising customer service. However, there is usually a primary or dominant focus. For example, an inventory replenishment system may exist that aims to minimise costs that include the cost of stockouts or poor customer service. Though such a system does take account of some customer service issues, the dominant objective is to minimise costs and hence the primary business objective is to minimise costs.

Fundamental approaches

From this simple framework, the fundamental approaches to inventory replenishment can be identified.

A direct method for minimising inventory costs can be described as an *economic* approach, where the aim is to apply models that minimise the total costs of inventory management subject to defined constraints. Similarly, a direct method for maximising probable customer service levels can be described as a *stochastic* approach, where the aim is to apply statistical models that maximise customer service criteria subject to defined constraints.

Indirectly, lower inventories imply lower costs and higher inventories imply higher customer service levels. Hence, an indirect method for minimising inventory costs can be described as a *minimal* approach, where the primary aim is to minimise physical inventories. Conversely, an indirect method for maximising inventory customer service levels can be described as a *maximal* approach, where the primary aim is to maximise physical inventories.

One final paradigm should be added, where tacit knowledge guides the decision making. This is where there is no obvious primary business objective or method, as may be common in many small enterprises. In this situation the inventory replenishment system employed may be based on experience, trial and error or 'rules of thumb', where the primary business objective or method is not easily categorised. Such an approach to inventory replenishment can be considered to be a *heuristic* one.

These approaches can be incorporated into the previously developed framework, with the heuristic approach potentially represented in all paradigms.

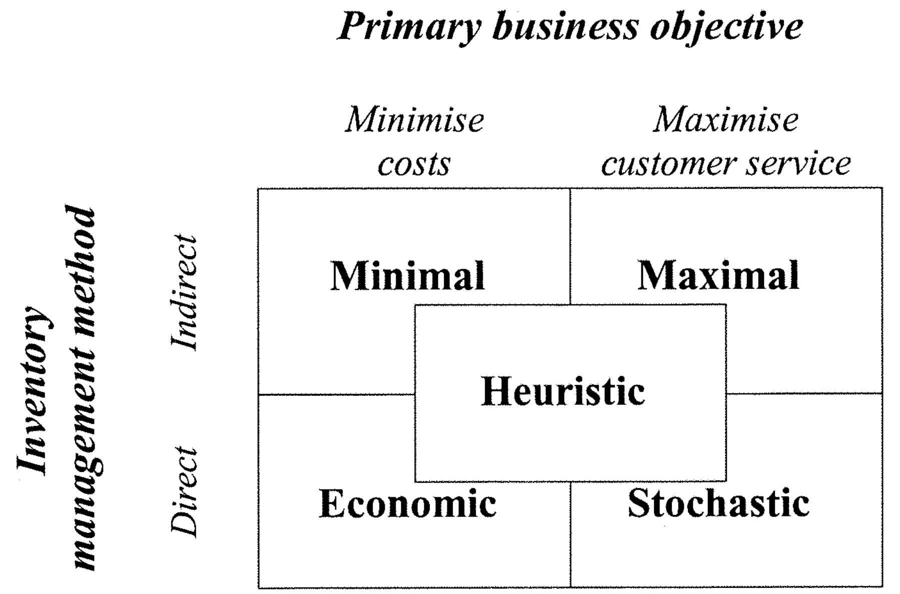


Figure 2: Major approaches to inventory replenishment

These proposed approaches are exhaustive but may not be exclusive. An inventory replenishment system may be based on a single approach or a combination of approaches. For example, a minimal approach to cost control may be used in conjunction with a stochastic approach to customer service.

To validate these paradigms and their associated approaches to inventory management an historical analysis of the relevant literature is carried out. This analysis also leads to suggest the dominant paradigms over time and so indicates future areas for profitable inventory research and implementation.

Historical analysis

It should be noted that the following historical analysis of the development of inventory replenishment systems is particularly influenced by documentation originating from the USA and the UK. This is especially true of the history portrayed prior to World War II, when there was arguably much more focus on national rather than international developments in business. Many developments in inventory replenishment systems may have occurred in other parts of the world quite independently either before or after the dates specified here.

Pre-1800: Maximise your assets

People throughout the world have always managed inventory. Even pre-historic people managed inventory. They would stock food at times of plentiful supply, such as berries and nuts, to be available for demand when supply was difficult, if not impossible. This constituted a maximal approach to inventory replenishment systems. For all of pre-history and most of history large inventories were generally seen as advantageous.

Until the Industrial Revolution, inventory or stock was also generally considered to be a measure of wealth and was therefore to be maximised as far as possible. This is illustrated by Papillion[14] writing in 1677:

> *'... usually the measure of stock and riches is accounted by money; but that is rather in imagination than reality ... The stock or riches of the kingdom doth not only consist in our money, but also in our commodities and ships for trade, and in our ships for war, and magazines furnished with all the necessary materials.'*

This view also implies a maximal approach to inventory replenishment systems (i.e. keep stocks as high as possible). Further examples of this view of inventory are provided by Viner[15]. Higher inventory reduces the risks associated with inconsistent supply or demand, as well as being associated with higher wealth.

The 19th Century: Sales forecasting

During the 1800's, the supply and distribution of goods became more organised. Merchants typically went to purchase product once or twice a year. These merchants aimed to buy just enough product to satisfy their customers until the next trip.[16] This represented a move away from a maximal approach towards a more heuristic approach to inventory replenishment systems, as evidenced by Hunt's Merchant's Magazine[17] in 1843.

> *'The dealer, aware that the only means in his power to meet the obligations is by making cash sales to a corresponding amount, becomes careful not to buy more than he thinks he can sell.'*

This can be considered to be a heuristic approach to inventory replenishment systems since experience through trial and error may result in 'rules of thumb' for good practice.

The early 1900's: Scientific management

During the early 20th Century, 'rules of thumb' began to be replaced by a more scientific approach to management in general. (For a seminal example see Taylor's 'Principles of Scientific Management'[18]). The earliest evidence of a more scientific approach to inventory replenishment systems dates from 1904.[19] However, the first well documented scientific approach appeared in 1912 when George Babcock,[20] of the H.H. Franklin Manufacturing Company in Syracuse, New York, attempted to balance the cost of changing a production machine against the inventory costs incurred (which depended upon the size of the production run). This could be considered to be the first documented economic approach to inventory replenishment systems. This example concerns the management of inventory in a production scenario, which is often referred to as the Economic Lot Size. In more general terms, this became known as the Economic Order Quantity or EOQ.

The original Economic Order Quantity is generally credited to Ford Harris,[21] who in 1915 defined the EOQ as follows:

$$Q = \sqrt{\frac{P \cdot S}{c} \cdot k}$$

where Q = the order quantity or lot size
P = the total cost of preparation for manufacture or order of a lot
S = the sales over the period
c = the cost of producing one unit of product
k = a constant factor including interest costs (assumed to be 10% originally)

Morse[22] provides an early example of this formula in practice, at the General Electric Company in 1917.

According to Raymond[23], by 1918 it had become more common for the interest rate cost (usually assumed to be around 6%) to be separated out from the constant factor. Thus, the equation became as follows:

$$Q = \sqrt{\frac{P \cdot S}{c \cdot i} \cdot k}$$

where i = interest rate

k = a constant factor excluding interest costs

However, like many of the early developments of the basic EOQ model, each development was not necessarily published and many practitioners were developing their own EOQ models independently and may have been unaware of other simultaneous developments.

It should be noted that financial considerations per se do not necessarily imply an economic approach to inventory replenishment systems. All firms wish to consider the financial implications of inventory management. An economic approach focuses on the total cost of inventory replenishment, whereas a minimal (or maximal) approach focuses on the cost of the physical stock.

While the theoretical foundations and some early practical applications of the EOQ were being developed in manufacturing, the First World War was creating difficulties in product supply.

After the war many supply difficulties remained, though the demand for products was high. This combination of low supply and high demand meant that whenever product was available it was bought in large quantities. Thus during and after the First World War, a maximal approach to inventory replenishment systems returned in many industries. This created pressures on suppliers to expand their own manufacturing or buying processes, which eventually led to a glut of inventory within the supply chain at the beginning of the 1920s.

The 1920's: Inventory depression

The overstock within supply chains eventually resulted in the 'inventory depression' of 1920-1921, particularly in the USA. At this time, all elements of the supply chain, from retailers to raw materials suppliers, were generally heavy with inventory. Eventually, this oversupply triggered a rapid reduction in prices with many inventories suddenly becoming worth less than originally paid for, and many businesses went bust as a result. Leverett S. Lyon describes the mood at the time:

> *'Inventories which had made men rich in the preceding period bankrupted them now. Forced sales, cut prices, liquidations were rife.'*[24]

The 'inventory depression' provided a violent swing away from the maximal approach towards a more minimal approach to inventory replenishment systems, while simultaneously encouraging research into more scientific (often economic based) inventory replenishment.

In the early 1920's, the EOQ was adjusted to include other variable costs such as insurance, rent and taxes. In 1926, Benjamin Cooper[25] added obsolescence and deterioration to this list of costs and also considered different rates of production.

In the middle to late 1920's, the economic approach to inventory replenishment systems was being developed both in theory and practice within manufacturing. Meanwhile the minimal approach was often being adopted further down the supply chain, by retailers, wholesalers and distributors. This approach was often referred to as 'hand-to-mouth buying', which was described by McGill[3] as 'steady but more frequent buying'. As time progressed, a system of low inventories and rapid turnover was increasingly affecting the whole supply chain.[26] It became common practice to aim for high stock-turn.

During the late 1920's, there were growing arguments about the move towards a minimal approach in inventory replenishment systems. Arguments both for and against this approach were often put in economic terms. For example, in favour of the minimal approach McGill[3] argued in the following way:

> *'If you have $100,000 invested in merchandise and do a business of only $100,000 yearly, you do not make a profit that is legitimately yours because the interest on the investment is $6,000 and that is an expense just the same as your rent or clerk higher [sic]. What you should do is to carry only $20,000 worth of merchandise but still sell $100,000 worth of merchandise annually. You can accomplish this by ordering in small quantities and often. True, your freight bills will be a little more, but you have a tremendous saving in the interest on your investment.'*

On the other hand, Stanley Goodman[27] argued that aiming purely for high stock-turn might result in inefficient ordering, transport and manufacture due to diseconomies of scale. Lyon[24] also noted this fact when he wrote:

> *'The conclusion is inescapable that the trend toward the transaction of trade in smaller orders and small shipments has brought added costs.'*

Thus a need to balance the costs of inventory replenishment systems was still thought useful, and so the development of an economic approach to inventory replenishment continued.

A major development occurred in the latter half of the 1920's, when Thornton Fry[28] researched into inventory systems where demand is unknown, and therefore uncertain. Fry introduced probability into his inventory models to take account of this uncertainty and so began the stochastic approach to inventory replenishment systems. This approach enabled an inventory manager to take a scientific approach to customer service requirements when demand was unknown, thus aiming to provide adequate safety stock. This stochastic approach was not exclusive and could be combined with the other approaches.

Many other developments to the basic economic and stochastic approaches have been created since this time, though the fundamental concepts remain.

The 1930's: The seminal textbook

In 1931, Fairfield Raymond[29], while working at Massachusetts Institute of Technology, wrote what may be considered to be the seminal textbook on an economic approach to inventory replenishment systems.

Howard Lewis[30] argued against the scientific approach:

> *'Dissertations on purchasing [and inventory] have appeared, particularly by those who are perhaps unduly engineering minded or mathematically inclined, which devote page after page to the statement, elaboration, and proof of some formula, the value of which is ruined by a very necessary statement, at the end, to the effect that varying conditions alter the character of the formula and that under no condition is it to be taken as a substitute for human judgement.'*

Indeed, throughout history, it can be assumed that 'human judgement' has often provided for a heuristic approach to inventory replenishment systems that has not been fully documented.

The theoreticians may have largely ignored Lewis' arguments but the literature suggests that practitioners were still generally following a minimal based approach to inventory replenishment systems in the early half of the 1930's. This was largely due to the often-documented failure of businesses due to surplus stocks. In America in particular, inventory became associated with dead or dying businesses, as recorded in a paper by Trundle[31] entitled 'Your inventory a graveyard?' In the depression years of the late 1930's, inventories generally declined even further as business finances became increasingly constrained.

The 1940's: War and commerce

The Second World War brought a more immediate challenge to the management of inventory, namely scarcity of supplies, and so the maximal approach to inventory replenishment systems reappeared. In Europe, in particular, governments rationed many products in order to tightly control stocks of those products and materials that were increasingly difficult to procure. This rationing process meant that many organisations in Europe continued to follow a maximal approach to inventory replenishment systems both during and after the war years.

The importance of good inventory replenishment systems was emphasised during the war years of the early 1940's, and not only in terms of the supply difficulties. For example, within the American Navy alone, the number of stocked items multiplied twelve-fold from a quarter of a million to 3 million.[32]

In a commercial context, developments in scientific inventory replenishment systems focused on consolidation rather than new approaches, as reflected in publications of that time, such as Wilson's[33] combined economic and stochastic approach to inventory replenishment systems. Other examples include Alford and Banks'[34] *Production Handbook* and Hannon's[35] case study of substantial inventory savings brought about by the introduction of EOQ-based methods.

Towards the end of the 1940's the bargaining power of different players within the supply chain was beginning to be recognised. In the automotive industry, large manufacturers began to apply pressure to suppliers to provide quick deliveries so that the manufacturer did not have to hold large inventories. This allowed the manufacturer to follow a minimal approach to inventory replenishment systems, while the supplier might take a heuristic approach or a combined economic and stochastic approach. In other industries, such as food, retailers were often making inventory savings at the expense of the manufacturers,[36] through adopting a minimal approach.

The late 1940's brought about the first major marketing of inventory replenishment systems (initially paper-based and later computer-based). One early example in 1949 is the 'Wilson Inventory Management Plan'[37] which was based on an economic and stochastic approach to inventory replenishment systems. The Wilson Plan was implemented by major companies, such as General Foods Corporation and Westinghouse Electric Corporation. It became so well known that the EOQ formula used became known as the 'Wilson formula'.

The 1950's: Inventory management systems

The 1950's could be considered to be the start of the modern era of inventory replenishment systems. It is suggested by Whitin[38] that until the 1950s new theoretical developments in inventory replenishment systems had had little impact on business behaviour in general. Ford Dickie's seminal paper on ABC inventory analysis[39] played a major role in focusing inventory managers on their key product lines rather than all product lines. The 1950's also saw the proliferation of special cases of the EOQ to the point where it sometimes becomes almost unrecognisable from Harris'[21] original EOQ formula of 1915.

The economic and stochastic approaches to inventory replenishment systems were often developed in combination. The economists Kenneth Arrow, Theodore Harris and Jacob Marschak[40] wrote the influential paper entitled 'Optimal Inventory Policy' in 1951, which provided a rigorous mathematical analysis of a combined economic and stochastic approach. Later, the mathematicians Dvoretzky, Kiefer and Wolfowitz[41,42] also wrote important, and often quoted papers, on the combined economic and stochastic approach. Other techniques that were developed for managing inventory include linear programming.[43] This was generally based on an economic approach, and provided a useful technique when planning production under certain constraints. In 1956, Lewis, Neeland and Gourary[44] assembled a bibliographic summary of combined economic and stochastic approaches.

Thomson Whitin,[45] who wrote a major text on inventory replenishment systems published in 1953, argued that there were three major reasons for the increasing transition to scientific (economic and stochastic) inventory control:

> *'First, the increasing size of business establishments....*

Second, during the past century there has been an enormous increase in the amount of business training. ...

A third factor ... is the increased emphasis that has been placed on the importance of engineers in business.'

It was suggested in the mid to late 1950's that:

'More operations research has been directed towards inventory control than toward any other problem area in business and industry'[46]

Whitin[45] also noted that:

'The formulas are, in general, applicable to merchandise of a staple, durable nature, and not to style goods or 'soft' lines.'

This was an important point since the number of style goods had greatly increased during the boom years following the Second World War. The high demand for style goods and the impact of industrial relations problems interrupting supply caused some businesses to hold higher safety stocks, and so move towards a more maximal approach to their inventory replenishment systems. Indeed, wherever demand or supply can vary considerably then safety stocks could represent the major part of inventory investment.

To counter high safety stocks and other related issues, Taiichi Ohno and Eiji Toyoda of the Toyota Motor Corporation in Japan were developing the Just-In-Time approach to manufacturing. As Ohno later explained[47],

'The basis of the Toyota production system is the elimination of waste. ... Costs really decrease when goods are produced singly or in small lots.'

'... provide only what is needed in the needed amount at the needed time.'

This is clearly a minimal approach to inventory replenishment systems, allowing for greater product differentiation and production flexibility.

Meanwhile in the USA, the status of the economic approach to inventory replenishment systems was recognised in 1957 when the Government actually recommended its adoption in an official publication[48]. It would be over two decades before the US automobile industry fully adopted Just-In-Time techniques, by which time Japanese companies, such as Toyota, would have substantially less inventory compared to their competitors[49].

The 1960's: Costs and computers

The 1960's brought fresh impetus to the implementation of inventory replenishment systems due to the combined effect of increased business opportunities, economic pressures and the increasing use of computers, as described by Dennett[50] at the beginning of this decade:

> *'Today the whole pattern of trading and manufacturing has changed and quickened; locally, nationally and internationally, for in each of those spheres competition has become more and more fierce.*
>
> *In addition to facing this ever increasing pressure from without, both manufacturer and retailer must grapple with overheads that are soaring at an alarming pace, and to aggravate the position yet further, interest on capital is higher than it has been for many years.'*
>
> *'In fully automatic methods...progress has been breathtaking. Electronic computers in particular have come down from the mathematical clouds in which they were born to the solid earth of commercial use by even medium sized firms.'*

In 1961, Eliezer Naddor[51] reviewed the current status of research into the methods and techniques of inventory control. The review concentrated on developments of Harris'[21] basic EOQ formula and made some important observations.

Naddor[51] observed at this time that:

> *'In most industrial applications costs are used as measures of effectiveness.'*

This appears to suggest that primarily an economic approach to inventory replenishment systems was taken during this period. This is further emphasised by Starr and Miller[52] who state:

> *'The analysis of inventory problems is fundamentally based on a very simple common-sensical observation. This is that in any genuine inventory problem whatsoever there* ***must*** *be opposing costs.'*

Hadley and Whitin[53] support the economic and stochastic approach, since their 1963 book entitled *'Analysis of Inventory Management Systems'* only really considers *'an operating doctrine which will minimize some cost expression for a specified stochastic output.'*

In his survey, Naddor[51] also recognised the difficulties in taking an economic approach in that determining the cost parameters, particularly shortage costs

'turns out to be a difficult research problem.'

Naddor[51] also lists a number of areas where there were problems with inventory replenishment systems of the time. These include conceptual difficulties, such as defining optimal rules and parameters; practical and operational difficulties, such as obtaining data or dealing with shortages in delivery; as well as touching on some of the more strategic difficulties, such as the relationship between supply and distribution. Naddor[51] also noted that:

'When an inventory control system is put on a digital computor [sic], ...opportunities for human judgement decisions [sic] are reduced considerably...'

Hadley and Whitin[53] support the practical difficulties of some of the inventory replenishment systems of this era when they state:

'Originally, the development of inventory models had practical application as an immediate objective. To a large extent this is still true, but as the subject becomes older, better developed, and more thoroughly explored, an increasing number of individuals are working with inventory models because they present interesting theoretical problems in mathematics.'

For example, in 1966 Feeney and Sherbrooke[54] developed a multi-echelon stochastic inventory model using compound Poisson distributions. This model was developed despite the fact that buyers and suppliers within the existing supply chains could not realistically share the required data (and would not be able to until the advent of Electronic Data Interchange in the 1980's and beyond). Thus, in practice, the heuristic approach was still widespread. Of course, the individual (rather than generic) nature of such an approach lends itself to a lack of formal documentation.

The 1960's also saw the further development of Just-In-Time systems, primarily still within Japan. In the USA and Europe, the focus was on developing Materials

Requirements Planning[55] (MRP). This is a dependent demand based inventory replenishment system, which allows for parts or component orders by manufacturers to be scheduled so that they arrive when required. In essence this was usually a minimal approach but initially very different to Just-In-Time (though the two systems have converged to some extent since that time). MRP was aimed at batch production methods, was often computerised and accepted certain constraints, such as inventory build-up due to order lead times. Just-In-Time was aimed at continuous production, tended to use low levels of technology and did not readily accept constraints such as order lead times.

The 1970's: Increasing information technology

The 1970's was a very important decade in terms of inventory replenishment systems, mainly due to the increasing influence of information technology allowing previous theoretical concepts or prototypes for inventory replenishment systems to be implemented.

Major developments occurred in the 1970's in terms of the computerisation of stock control systems. Lines and Beart[56] provide an example of lectures from an early 1970's management development programme explaining the new computer hardware for stock control of the time, including:

> *'a central processor, capable of between 5,000 and 50,000 multiplications per second,...magnetic tape units...or...magnetic disc units,... a card reader...or... a paper tape reader... a teletypewriter...or... a visual display unit'*

This hardware was used to implement the '*scientific approach*' to stock control: basically Fry's stochastic enhancement[28] of Harris' original EOQ[21]. The use of computers allowed for more frequent and potentially a more accurate calculation of the EOQ for many different product lines. This increase in the number and complexity of mathematical calculations enabled the widespread practical application of many previously developed theories. Prior to this decade it was

> *'often said of Inventory Control that the available theory leads the current practice by up to a decade.'*[57]

New heuristic enhancements to Harris' original EOQ[21] were also being developed, such as Naddor's Heuristic.[58] Sani and Kingsman[59] provide a recent discussion of such techniques.

The 1970's also saw somewhat divergent development in different parts of the world. In the USA, the 1970's saw the maturity of Materials Requirements Planning (MRP), while in Japan the Just-In-Time approach to inventory replenishment systems in manufacturing environments was continuing its development.

Although MRP had been around for some time,[55] it was not until the MRP concepts were combined with significant computing power that MRP really 'came of age'. This is reflected in what is often considered to be the seminal book on MRP by Orlicky[55] (an IBM employee) appearing in 1975 - over a decade after Orlicky first discussed the technique.

Fundamentally MRP is a deterministic technique to inventory replenishment systems but it is easily combined with a stochastic approach to deal with uncertainty.[60] Oliver Wight[61] stated in 1970 when comparing dependent demand inventory systems, such as MRP, with independent demand inventory systems, such as those based on EOQ:

> *'It is a sign of the adolescence of our field that the literature available is in inverse proportion to the applicability of the techniques.'*

Just-In-Time (or the Toyota Production System as it was sometimes called) also became well-known in the 1970's. Like MRP, Just-In-Time had been around for some time both in concept and practice. In what might be considered to be the seminal book on Just-In-Time, entitled *'Toyota Production System'*[62], Ohno describes one of the major goals as the reduction of inventory to as near zero as possible, emphasising minimal approach.

The mid-1970's saw the further development of MRP applied to independent demand items for maintenance and service inventories, and later for finished goods inventories,[63] usually known as Distribution Requirements Planning (DRP). Xerox is generally attributed[64] as developing the first large scale DRP system. DRP uses the MRP deterministic scheduling logic and 'inverts it' to break down inventory requirements from different distribution outlets rather than build up the inventory requirements of components to manufacture a product.

The 1980's: Lean supply

The 1980's saw a continuation of the developments from the 1970's, including the US and Europe adopting Just-In-Time based methods.[65] The Massachusetts Institute of Technology in the USA was an important supplier of information about Just-In-Time through two major research programs: the Future of the Automobile (1980-85) and the International

Motor Vehicle Program (1986-1990). The International Motor Vehicle Program provided around 160 research papers as well as many spin- off projects and culminated in the influential text *'The Machine That Changed The World'* by Womack, Jones and Roos.[66] The generic name used in this text to describe the overall best practice was 'lean production' emphasising the minimal nature of the production and supply methods involved.

The 1980's also saw a general trend (associated with Just-In-Time and MRP) towards the increased integration of supply chains, including further development of complex, multi-echelon inventory models. For example, see Clark, Trempe and Trichlin.[67]

The MRP/DRP approach was also developed further (usually under the name of Manufacturing Resource Planning, MRP II, or Distribution Resource Planning[68]), both in terms of extensions to functionality and in terms of increasing computerisation[69]. An example of this type of approach is that of the Lotus Development Corporation that claimed to have reduced overall inventories by 10%[70] through the adoption of a DRP system in the mid-1980's.

MRP was still essentially a minimal approach, though often with stochastic or heuristic enhancements designed to take account of variations in supply or demand. For example, see Brown[71] (1986) or the later paper of Coleman & McKnew[72] (1991).

The late 1980's brought the further application of Just-In-Time down the supply chain, via 'quick response' (QR) logistics.[73] This is a minimal approach to inventory replenishment systems (though often with stochastic enhancements) that first began in the clothing industry. Early examples of the QR approach include Levi Strauss and JC Penney in the USA.[74] Quick response is heavily dependent on the quick exchange of sales information between the retailer and supplier. This quick exchange of information became possible during the 1980's through the use of Electronic Data Interchange (EDI).

Some problems with a minimal approach to retail inventory replenishment systems were also identified at this time. Retail inventory does not exist solely for demand servicing (satisfying sales) but also for demand stimulating (creating sales), thus a minimal approach must take account of this requirement for 'psychic stock'. [75]

The 1990's: Minimise stock; maximise service

In the 1990's most of the effort into inventory replenishment systems continued to focus on a minimal approach, though often combined with a stochastic approach to provide a counterbalance in terms of customer service requirements.

As Donald Waters[76] (1994) states:

> *'The main objective of inventory managers now is to minimise stock levels, while maintaining an acceptable level of service.'*

Waters goes on to show that since the 1960's stocks have declined - presumably as a result of the dominant minimal approach. This is further reflected in the fact that techniques used in inventory replenishment systems have largely converged. For example, the 'time buckets' used in MRP or DRP calculations are now so small that they resemble (and are often combined with) a Just-In-Time or QR method for inventory replenishment systems. These reductions in lead times are mainly the result of the implementation of EDI and bar-coding systems allowing information to be shared easily, accurately and quickly within a supply chain. The latest 'label' for a range of inventory management techniques requiring such technology is Efficient Consumer Response (ECR).

ECR was developed in the early 1990's by Kurt Salmon Associates[77] in the USA for the grocery industry, though many businesses, such as Sainsbury's in the UK, claimed to have already employed many of the techniques involved[78]. ECR is a combination of many processes that usually include: EDI, Continuous Replenishment, Computer-Assisted Ordering, Flow-Through Distribution (cross-docking), Activity Based Costing and Category Management.[79] ECR focuses on a minimal approach to inventory replenishment systems, though with some stochastic elements and has laid claim to substantial reductions in inventory.[80]

ECR also recognises the role of demand management[81] in minimising inventory, particularly in areas such as product promotion. Demand management is not a new concept, with documented elements in both marketing (for example, Kotler[82] in 1973) and distribution (for example, Higgins[64] in 1980). However, this has become increasingly possible to manage as information systems have developed. For example, see Landvater[83]. This is likely to grow further as more customer-specific communication links and information becomes available via emerging channels, such as the Internet and digital television.

Dominant paradigms and associated approaches

The historical analysis appears to validate the proposed framework and classification of the major approaches employed in inventory management. Furthermore, it is clear that fundamentally new approaches to inventory replenishment have not been developed since the 1920s. However, the importance of any particular approach has varied over time.

Historical synthesis

Before mass distribution and mass production, the dominant inventory issue was the relative scarcity of products, and so inventory replenishment focused on procurement. Hence, during this time, the maximal approach to inventory replenishment was dominant. As distribution channels began to evolve, the dominance of customer service issues reduced. With scant information available, inventory replenishment often followed a heuristic approach.

As a result of the development of mass production and the associated 'scientific management' in the first half of the 20th Century, direct measurements of inventory objectives were developed, and so the economic and stochastic approaches to replenishment came to dominate inventory management thinking. However, in practice, much of this thinking could only be implemented on a relatively small-scale, either due to the lack of relevant information, such as consumer demand or accurate costings, or due to the lack of processing power that was required for some of the more sophisticated developments. Meanwhile, the development of mass production and distribution channels had resulted in inventory surpluses, triggering a devaluation of inventory and a focus on costs. Hence, the minimal approach to replenishment came to dominate, eventually leading to the 'Depression Years'.

World War II, and to a lesser extent the First World War, interrupted the commercial evolution of inventory management, by cutting existing supply lines. Thus, a temporary return to a maximal approach dominated at these times.

Following the Second World War, developments in inventory management diverged. The reasons for this divergence are not fully explored here, though they are probably due to cultural and technological differences. At this time, Western cultures and their associated IT development may have been more aligned to reductionist inventory replenishment systems, and hence the economic, stochastic and minimal (MRP-based) approaches were important. On the other hand, Eastern cultures and their lack of funds may have been more aligned with holistic inventory replenishment, and hence the dominance of the minimal (Just-In-Time) approach. In any case, these views eventually converged. Western companies came to recognise the importance of a holistic view in improving product flow within the supply chain, and Eastern firms began to develop leading-edge IT and thus adopt successful reductionist practices. Thus a combined minimal and stochastic approach appears to have become dominant in large organisations that actively manage product flow through the supply chain and possess powerful information processing capability. This combines an indirect method of minimising costs through improving product flow, with a direct method of maximising customer service through employing information processing power. This allows the often competing objectives of minimising costs and maximising customer service to be pursued at the same time. However, the minimal approach may be seen as more dominant than the stochastic approach because the indirect, minimal approach may cover all aspects of inventory management, including structural aspects of the supply chain, while the direct, stochastic approach is only focused on customer service measurements.

This historic analysis is summarised in the following diagram:

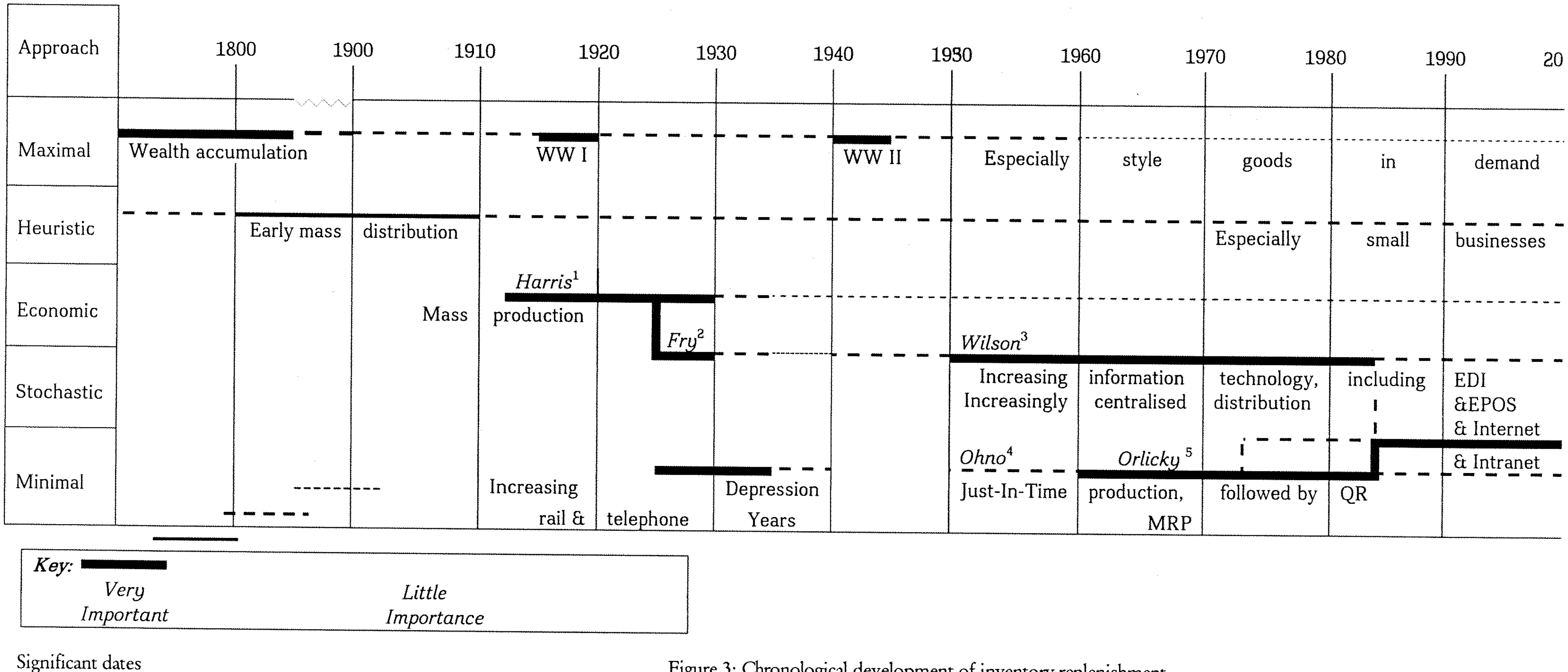


Figure 3: Chronological development of inventory replenishment

[1] Economic Order Quantity (1915)

[2] Stochastic EOQ (1928)

[3] Wilson Plan (1949)

[4] Toyota Production System (published 1978)

[5] Materials Requirements Planning (published 1975)

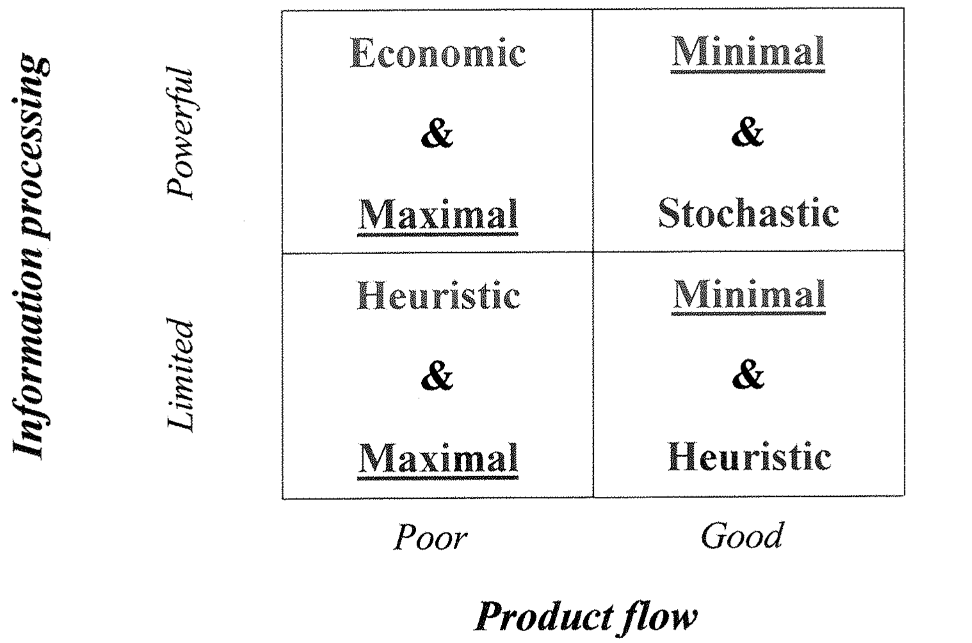
Dominant approaches

This historical analysis of commercial inventory management suggests that the dominance of any particular paradigm and associated approach appears to depend primarily upon product flow within the supply chain and information processing capability. Indirect methods relate to product flow, while direct methods relate to information processing. In particular:

- A *minimal* approach dominates inventory management where product flow is good.
- A *maximal* approach dominates inventory management where product flow is poor.
- An *economic* approach may dominate inventory management where powerful information processing is available. However, where product flow is good, then a minimal approach will take precedence.
- A *stochastic* approach may dominate inventory management where powerful information processing is available. However, where product flow is poor, then a maximal approach will take precedence.
- A *heuristic* approach may be so individual that no universal statement can be made. However, where information processing is poor, then a heuristic approach may be employed.

Product flow is more important than information processing because it may cover all aspects of inventory management. However, the ease of product flow may partially depend upon information processing power. Moreover, product flow and information processing capability are dependent upon many external factors, such as economic development, wars and so on.

In general, issues of both cost and customer service must be considered. Hence, the relationship between the dominant approaches to inventory replenishment and product flow and information processing can be generalised as follows.



Key
Approaches focused on minimising costs are in red.
Approaches focused on maximising customer service are in blue.
The likely dominant approach of any pair of approaches is underlined.

Figure 4: Dominant approaches to inventory replenishment

Inventory replenishment systems in the 21st Century

When contrasting inventory replenishment in the early and the late 20th Century, arguably the most important developments have occurred in information technology. These developments have enabled information to be shared throughout the supply chain, thus improving product flow, and the realisation of many sophisticated techniques to be applied to very large numbers of products, through increased information processing capability. In the 21st Century, as technologies improve further allowing for learning and adaptation rather than just calculation, this opens up the real possibility of generic systems finding their own solutions to inventory replenishment problems. This may enable a heuristic approach to be developed that is able to deal with the competing (though complementary) business objectives of minimising costs and maximising customer service. Furthermore, there is relatively little documented research on pure heuristic approaches to inventory replenishment. Therefore, heuristic approaches to inventory replenishment represent both an important and potentially rich area for future research. Before considering such an approach, it is important to critically evaluate the strengths and weaknesses of all the approaches identified.

Chapter 3

EVALUATING THE MAJOR APPROACHES
The mismatch between theorists and practitioners

Five major approaches to inventory replenishment have been identified: maximal, minimal, economic, stochastic and heuristic. By considering the relevant literature, this chapter evaluates these major approaches in theory and practice. A survey of current inventory practice helps to understand the reality of implementing the approaches. A survey of current research papers then considers whether researchers are addressing this reality, and hence are likely to produce some relevant, new inventory solutions in the near future.

Documented strengths and weaknesses

Based on published evidence, each of the defined approaches to inventory replenishment is now evaluated with respect to both theoretical and practical considerations.

Economic approach

The economic approach aims to minimise the total costs of inventory replenishment. In theory, this minimisation of overall inventory costs appears reasonable. Any business wants to minimise total costs where possible in order to make best use of available resources. The basic EOQ proposed by Harris[84] has been developed to cover a multitude of situations, as any good text or journal on inventory replenishment will show, for example Silver & Peterson[8] or the Journal of the Operational Research Society.

However, this approach is based on many impractical assumptions. Some of these assumptions may be valid in theory but are difficult to apply in practice. Fundamentally this approach assumes that all business issues can be accurately measured in financial terms, including areas of cost shared with other business operations as well as 'softer' issues such as customer goodwill or business strategy.

The major criticism from practitioners of any economic approach is that cost estimations are often debatable, difficult to calculate and imprecise as a result. It should be noted that inventory managers always use a financial approach of some kind, though this is not usually the rigorous application of economic theory.

St. John[85] referring to economic lot sizing studies calculating order quantities or order periods makes this strongly worded plea:

> *'...authors, researchers, software companies and practitioners alike ... [should] direct their attention to subjects that really need attention. There are some significant payoffs to be achieved in other areas that are going unexplored while some of the most brilliant minds in our profession continue to pollute the literature with more and more lot sizings studies.'*

It is worth noting that researchers have recognised these problems from the early development of the economic approach. In 1931, Raymond[86] noted many difficulties, including the following:

> *'A controversy rose among accountants...as to just what cost figures should be employed for the evaluation of inventories.'*
>
> *'There arose a desire to eliminate, if possible, the necessity of employing the rate of consumption or sales demand in the formula in order to avoid errors in estimating future sales requirements.'*
>
> *'Objections were raised over the fact that economic lots often indicated as economical an increase in inventory values when experience and financial facts showed conclusively that an increase in capital turnover was paramount.'*

In summary, the following major criticisms can be made of the economic approach in practice:

- The cost of ordering inventory is usually based on a relatively crude calculation, such as the total cost of order processing staff and estimated relevant overheads divided by the total number of orders placed. This calculation not only suffers from providing average cost figures but also requires judgements as to which costs should be included and which excluded.

- The relationship between the cost of ordering inventory and the numbers of orders placed is usually assumed to be proportional and therefore continuously linear, though in reality such costs are more likely to be incremental and discontinuous. For example, staff costs tend to change incrementally as one staff member is recruited or made redundant.

- The cost of holding inventory is generally based on a crude calculation such as interest rates plus an opportunity cost and perhaps an obsolescence charge. All of this calculation relies on the judgement of the manager involved, and so completely different figures could be equally defensible.

- The relationship between the cost of holding inventory and the value of inventory is usually assumed to be proportional and therefore continuously linear, though in reality such costs are more likely to be incremental and discontinuous. For example, warehouse costs tend to change incrementally as extra warehouse space is employed.

- The cost of being out of stock is often based on an easily accountable short term cost such as lost profit on the sale, perhaps with some extra cost representing the loss of 'goodwill'. However, in reality the cost of being out of stock may range from the complete loss of that customer and potential customers to no effect whatsoever.

It is clear that there are many practical difficulties in applying the economic theory of minimising total inventory costs.

Stochastic approach

The stochastic approach to inventory replenishment aims for customer service criteria to be met by modelling uncertain demand using probability distributions. In theory, these probability distributions allow for the random process of demand to be described using statistics of the long-run averages. However, this assumes that demand is random.

In practice, the uncertainty of demand may not be random due to marketing intervention, which is designed to stimulate demand. For example, sales targets and marketing promotions aim to influence demand[87] and therefore alter demand in a non-random way. Similarly merchandising factors, such as the amount or location of inventory displayed, may alter demand[88] in a non-random way. In competitive environments, similar initiatives by competing retailers may also influence demand. Even in parts supply, where there might be expected to be random events, such as machine failures, there is still great scope to influence demand. For example, the rate of machine failure may be influenced by some planned maintenance, the introduction of a new replacement part or a change in the machine operator.

It can be argued therefore that since organisations generally want to try to influence demand (usually in order to increase sales) they will often intervene and hence often alter demand in a non-random way, thus making a stochastic approach to inventory replenishment fundamentally flawed in theory. Evidence of this flaw might be recognised by the large number of extensions to stochastic (and often economic) replenishment systems (for example, see Feng & Xiao[89]). This suggests that the fundamental model is weak and therefore requires frequent modification to cope with each individual situation.

Furthermore, stochastic models rely on being able to agree measurement criteria for customer service. However, customer service can be measured in a number of ways, not all of them quantitative. Stochastic models therefore tend to focus on a few (and sometimes a single) quantitative aspects of customer service, such as number of stockouts or percentage of time stock is available.

On the other hand, the increased use of information technology makes it relatively easy to employ stochastic inventory replenishment. Frequent calculations applying complex probability distributions are made easy through the use of up-to-date software and hardware. Though this does not override the theoretical flaw identified above, in practice it may be that a stochastic approach sometimes provides a reasonable way of managing inventory-based customer service.

Minimal approach

The minimal approach aims to minimise inventory levels. In theory, the minimal approach encourages changes in supply to more closely (if not exactly) match changes in demand thus reducing the need for inventory. In effect, this theory reduces the 'gap' between supply and demand and so appears fundamentally sound.

However, this theoretical approach ignores the fact that holding stock may influence demand[87]. It might be argued that holding stock might not influence demand if supply lead times were zero but this ignores the potential need to 'try out' a product before acquiring it. For example, most clothes are bought in shops where they can be tried on for size, texture, colour and so on before purchase. This reflects some of the different reasons for holding stock.

The minimal approach may also have a potentially detrimental impact on cost. In theory, it may be argued that minimising inventory reduces rather than increases cost but this is not necessarily the case. For example, transport frequency and associated costs may be increased disproportionately to meet a reduction in inventory.

In practice the minimal approach is often an important element of a wider management strategy such as Just-In-Time[90] manufacturing or Quick Response[90] retailing. There is evidence that this approach has reduced costs and improved customer service simultaneously[91]. However there is some disagreement with this claim,[92] especially over how minimum stocks and costs should be measured. Furthermore, a minimal approach is not useful in circumstances where the cost of holding inventory is seen as secondary in importance to the prevailing conditions of supply or demand, such as:

- *Emergency demand,* for example, holding drugs in stock to meet unscheduled demand may be the difference between life and death.
- *Convenient demand,* for example, holding a bottle of lemonade in stock to meet unscheduled demand may be the difference between a sale or no sale.
- *Impulsive demand,* for example, holding sweets in stock may be the difference between attracting a customer to purchase and not.
- *Economic supply,* for example, holding stock as a result of efficient batch production at a factory may be the difference between profit and loss.

It is clear that there are many reasons for not applying the minimal approach to inventory replenishment.

Maximal approach

A maximal approach aims to maximise inventory levels. The maximal approach is justified in theory where inventory costs are outweighed by the customer benefit. This is often the case where a product may appreciate in value by stocking it, for example, historic works of art. Of course, it is a poor theory where inventory costs outweigh the customer benefit, for example, where products are likely to depreciate in value by holding them in stock, such as Information Technology or dairy products.

The main difficulty in employing a maximal approach lies in deciding upon which goods will appreciate and by how much; in other words, risk management. In practice the maximal approach may therefore be used in conjunction with a stochastic or heuristic approach in order to take account of the risk involved.

Heuristic approach

It is difficult to generalise about the myriad of heuristic approaches developed. Each individual heuristic approach may be difficult to justify in theory and practice, except in terms of results, assuming it works successfully. Moreover, any heuristic approach might only be applicable to the specific business situation for which it has been developed.

Summary

By summarising the main documented criticisms of the major approaches to inventory replenishment, it can be argued that no one approach is fundamentally sound both in theory and practice for all business situations.

In theory, the economic approach appears sound, while the maximal and minimal approaches appear sound only in particular business circumstances. The stochastic approach appears to be fundamentally weak, while the heuristic approach tends to lack theoretical underpinning, by definition, and may not be generally applicable in any case.

In practice, the economic approach can be difficult if not impossible to employ effectively. However, the stochastic approach is relatively easy to apply, and may be seen as successful despite its theoretical flaw. The minimal, maximal and heuristic approaches all usually require an element of compromise in practice, which may be difficult to justify objectively.

In summary, none of the current approaches appear strong both in theory and practice. The economic and stochastic approaches appear to have fundamental flaws, while the minimal and maximal approaches are theoretically sound only under certain conditions. The heuristic approach may offer the potential to cope with the competing objectives of minimising cost and maximising customer service, while being easy to implement, but it may be considered difficult to justify or apply generically.

Current inventory replenishment systems in practice

To consider how inventory replenishment is currently practised in business, both the practitioners of inventory replenishment and the providers of inventory replenishment software were surveyed. The survey of practitioners aims to provide a general understanding of inventory management issues and to show which approaches to inventory replenishment are most likely to be applied. The survey of inventory replenishment software providers aims to show which methods are commercially available and so may suggest current potential practice.

Method of analysis

The survey of practitioners was carried out in a three-stage process:

1. *Initial discussions* were held with 6 inventory managers in order to develop a postal survey.

2. *A postal survey* was then sent out to 186 inventory managers with whom the University had current links and therefore who might be more responsive and frank in their replies. This survey aimed to explore the major practical issues involved in inventory management. Completed replies were obtained from 34 respondents (see Appendix A for details). These respondents represent different levels of the supply chain: manufacturers and suppliers (44%), wholesalers and distributors (12%), and retailers (50%). A range of market sectors are also represented, as shown in Figure 5.

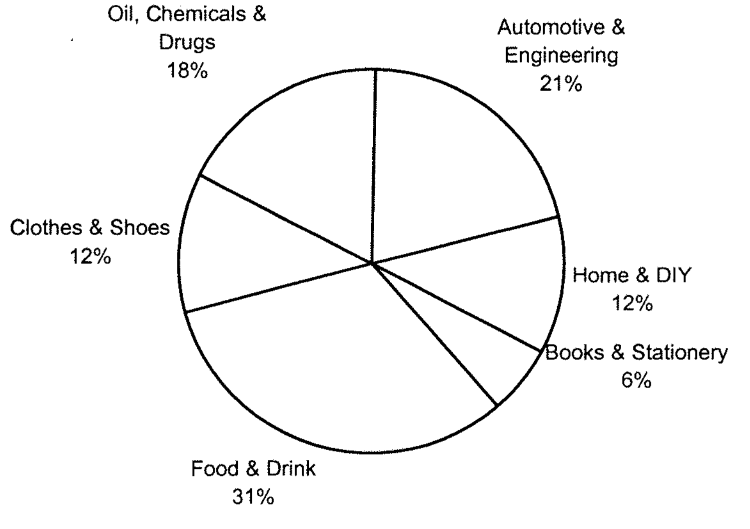


Figure 5: Major market sectors sampled

All of the respondents represented significant businesses, having a minimum of £1 million annual turnover and many having an annual turnover of several £100 million. Thus the survey considered businesses with major inventory replenishment requirements and resources.

3. *Follow-up telephone discussions* were used to confirm information and (following the classifications developed in the historical analysis) to classify the primary approach(es) to inventory replenishment employed by the practitioner.

The survey of inventory replenishment software providers was also carried out in a three-stage process:

1. *Initial discussions* took place with 12 inventory software providers, chosen at random at the Logistics and Supply Chain Systems trade show, to understand the mechanics of their particular software. The vendors chosen offered packages or major elements of integrated packages focusing on inventory replenishment (see Appendix B for details).

2. *A review of trade literature* provided on the 14 packages covered was then undertaken.

3. *Telephone discussions* were used to confirm information and (following the classifications developed in the historical analysis) to classify the primary approach(es) to inventory replenishment potentially available using the specified software.

The size of the samples are limited primarily by practical constraints, though this is not fundamental as the results are intended to be indicative rather than statistically significant. The samples are also limited in range, in that only major businesses are represented in order to deliberately bias the research towards businesses with major inventory replenishment decisions or software and the resources to invest in them.

Current practice

The results of the survey of major practitioners showed that inventory replenishment practice focused on the following areas:

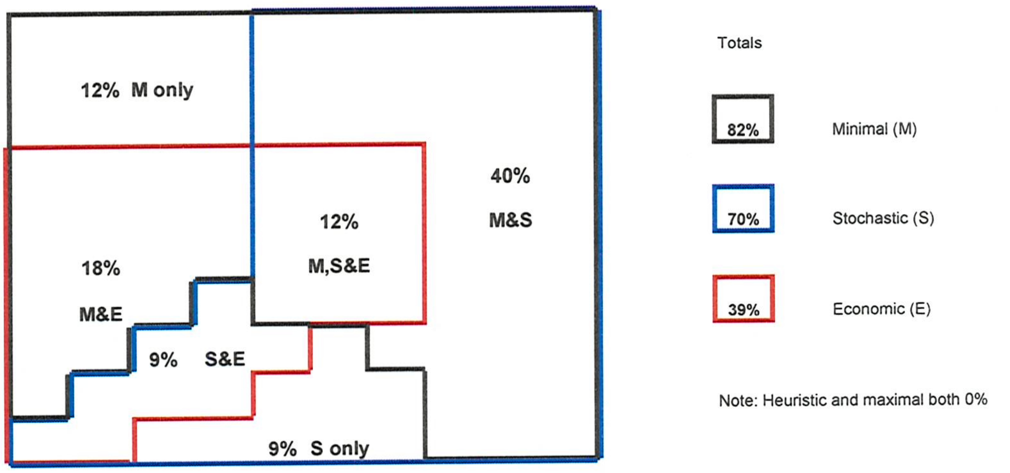


Figure 6: Primary approach used

The most common approach to inventory replenishment in practice appears to be to reduce physical inventories as much as possible, often by increasing supply frequency or smoothing supply schedules. This is essentially a minimal approach. This approach is often used in conjunction with software that employs a stochastic approach, usually by defining a desired level of customer service. For example by defining a 95% customer service level or 5% stockout level. The lack of heuristic or maximal approaches may reflect the nature of the companies surveyed, i.e. large and dealing with large numbers of goods that tend to depreciate in value over time. However, there are both heuristic and economic elements involved in the decision making process, particularly in deciding upon the frequency of supply or production.

Current software

The survey of providers of inventory replenishment software shows that all software potentially (depending on operational use) falls into just two categories as follows:

43% focus on a stochastic and minimal approach.

57% focus on a minimal, economic and stochastic approach.

Comparing current practice and the current software available suggests that software providers are well matched to practitioners' likely requirements. This survey also emphasises the flexibility of software provision to cater for all types of practitioners, regardless of approach to inventory replenishment. It also reflects the potential of EDI and Extranets to enable a minimal approach.

It should be noted that some overlap between the software survey and the practitioners survey may exist. However, practitioners may not use all of the software facilities available to them. Hence this survey of software providers indicates potential rather than actual practice.

Comparing the survey of practitioners with the evaluation, it appears that organisations implement what is currently practical rather than what is theoretically rigorous.

Current theories on inventory replenishment

In order to consider current research issues, recent journal publications were analysed to identify the current focus of inventory replenishment research.

Method of analysis

This analysis was carried out in three stages:

1. *Keyword analysis* – Keywords used in the ANBAR online indexes of abstracts of recent journal articles (see Appendix C for details) were analysed to identify an appropriate sample of recent research papers. The keyword analysis on the ANBAR system resulted in 764 journal articles found which mentioned inventory. Of these articles, 91 had 'inventory' as their main subject area of which 5 were not related to product/material inventory.

2. *Text analysis* - The text for each of these remaining 86 papers was read in order to identify the major approach(es) under investigation. This classification is based only on the approach under investigation and not the context of the paper. For example, a paper investigating an EOQ model that assumes a stochastic context would be identified as research into the economic approach only. On the other hand, a paper investigating stochastic customer service measures albeit in the context of an EOQ-based system would be identified as research into the stochastic approach only. A paper would have to investigate aspects of both a stochastic and an economic nature for it to be considered as research into both of these approaches.

This method of analysis is limited by the journals considered and the sample size. However, the journals were chosen to cover a wide range of subject areas from the specialist and technical to the more general management journals. Overall it is considered that neither of these limitations are fundamental since no statistical significance is derived from the results. The results are an indication of the dominant inventory replenishment theories being researched today, and therefore may be an indication of inventory replenishment practice in the future.

Current research

These articles provided the following results:

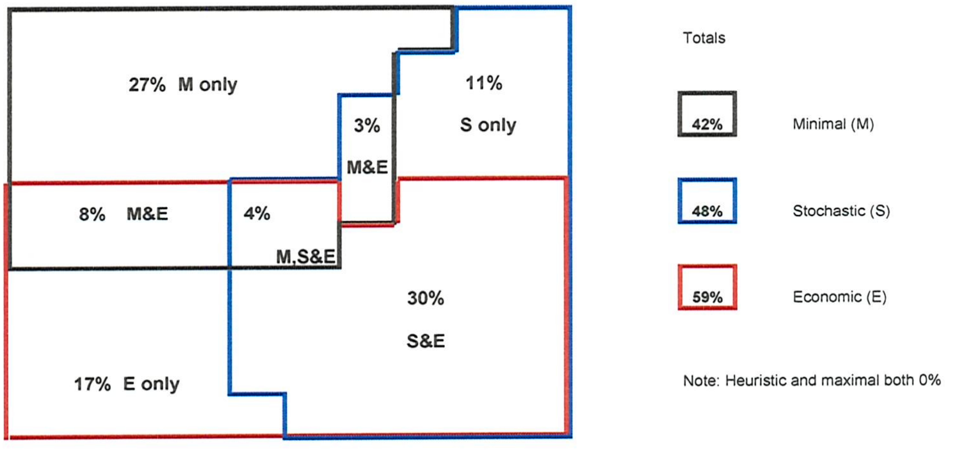


Figure 7: Focus of articles surveyed

The most popular research topics are those based on a joint economic and stochastic approach (usually extensions to existing EOQ-based models) and those based on a minimal approach (often based on Just-In-Time or a related concept).

There are no articles identified where the heuristic or maximal approaches are dominant, though there may be sub-elements of these approaches in some papers. Elements of the heuristic approach are often included where there is incomplete information. For example, when deciding on replenishment frequency in a Periodic-Review system. Elements of the maximal approach are included by implication, where there are customer service issues that imply holding as much stock as possible within existing constraints.

This analysis suggests that research is concentrated on areas where there is already strong mathematical underpinning, allowing for rigorously testing new hypothesises under 'laboratory' conditions.

Current research versus practice

Comparing current theory and practice suggests that there is a potential mismatch, with the practitioners focusing on the joint minimal and stochastic approach to inventory while the researchers focus on a joint economic and stochastic approach to inventory replenishment.

The relationship between theory and practice can be illustrated by plotting the surveyed incidence of the major approaches in theory versus practice, as follows:

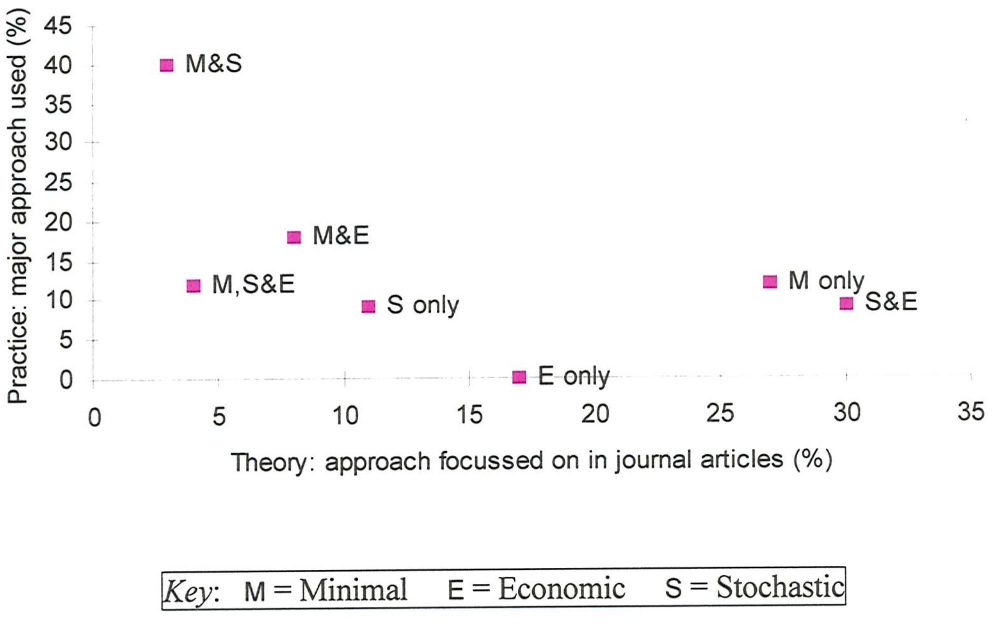


Figure 8: Theory versus practice

Figure 8 suggests an inversely proportional relationship between theory and practice. This suggests that researchers are publishing most papers (and perhaps spending most time and effort) on approaches that practitioners apply the least.

It should be noted that this mismatch may be due to the small samples, bias or poor information. It might also be expected that practitioners over-report the more visible approaches, such as physical inventory reduction in the minimal approach, and under-report the more invisible approaches, such as the stochastic approach applied within inventory replenishment software. It may also be expected that researchers write more frequently on the more universal concepts of the economic and stochastic approaches rather than the more individually applied joint minimal and stochastic approaches to inventory replenishment. Furthermore, the more clearly structured economically-based approaches could be considered to be more straightforward to research when compared to the potentially more open questions of minimally-based approaches.

Assuming that current theory is an indication of future practice, suggests a move towards a joint economic and stochastic approach or a single-minded minimal approach. However, from the previous evaluation, it is suggested that the economic approach is fundamentally impractical, the stochastic approach fundamentally lacking in theory, and the minimal approach requires compromises to be made in practice. Thus, rather than focusing on developing new theories that may be put into practice in the future, researchers appear to be focusing on developing old concepts that are fundamentally flawed, either in theory or practice.

On the other hand, a dominant heuristic approach appears to have been largely ignored by researchers. While there are issues of universal applicability, a heuristic approach may offer the most potential to cope with the competing objectives of inventory replenishment systems (minimise costs, maximise customer service) and practicality. This chapter therefore concludes that future research into inventory replenishment systems should move away from the current focus towards a more heuristic approach, which may better reflect the reality of inventory management.

Chapter 4

A HYPOTHESIS

Fuzzy notions

Successful inventory management requires a replenishment system that will cope with the often-competing objectives of minimum cost, maximum customer service and practicality. This requires the ability to integrate competing concepts. For example, an individual product may be considered to be both overstocked and understocked simultaneously. A product may be overstocked in terms of operations criteria, where the objective may be to minimise costly inefficiencies.[93] At the same time the product may be understocked in terms of marketing criteria, where the objective may be to provide high levels of product availability and hence good customer service.[94]

This apparent conflict is a reflection of the complexity of business and can be illustrated in the following diagram, which provides a simple example of how a firm may have a common objective at the strategic level but conflicting objectives at an operational level.

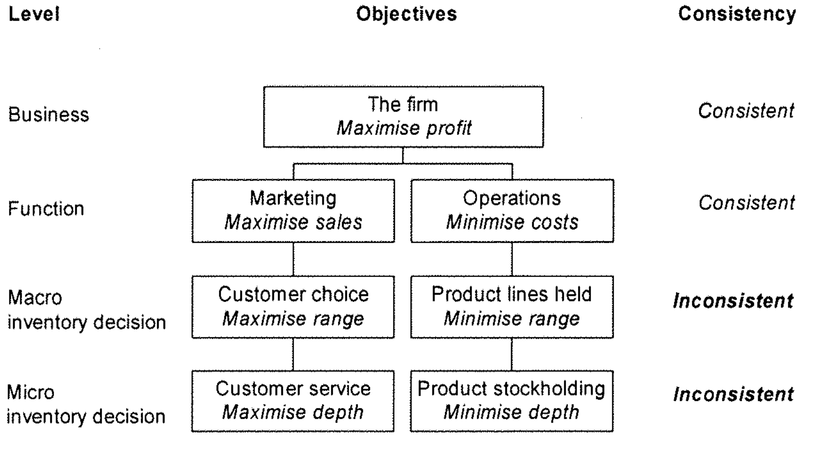


Figure 9: Business objectives and contrasting inventory requirements

Firms recognise these conflicting requirements, and often manage them by a series of trade-offs, such as lower customer service for lower cost. These trade-offs may be conscious choices or the result of focussing on one or more criteria at the expense of other factors. For example, by concentrating on reducing lead times, transport costs may rise as a result. While such trade-offs may be modelled and new scenarios simulated, the chosen level of trade-offs is based on a combination of market conditions, the firm's business strategy and ultimately management judgement and experience. This decision making process is founded on imprecise reasoning rather than a wholly precise science, which enables the decision making process to simultaneously cope with the competing objectives of minimum cost and maximum customer service while remaining practical.

This thesis aims to explore whether inventory replenishment systems may be developed that better reflect this fuzzy reality rather than current conventional inventory replenishment systems which are based on bivalent logic.

The bivalent weaknesses of current approaches

Bivalent logic assumes that every statement is true or false. In the early part of this century, Alfred North Whitehead and Betrand Russell[95] showed that most mathematics reduces to this form of logic. For example, the simple statement $5 + 6 = 11$ is considered true whereas $9 + 3 = 2$ is considered false. The major approaches to inventory replenishment systems used today are based on this bivalent logic. However, this bivalent logic can be seen to be a consistent source of weakness within the major approaches to inventory management.

The economic approach to inventory replenishment assumes that costs can be measured precisely. In reality, this fails when two different but equally valid costing systems suggest different stock holding levels. This is a result of imposing a precise, bivalent 'straitjacket' on what is a much fuzzier problem. Similarly, the stochastic approach assumes that customer service can be measured precisely. Furthermore, this approach assumes that demand is randomly distributed over the long-term, when in reality the demand pattern is much fuzzier because it is subject to a great deal of human intervention. The minimal approach to inventory replenishment focuses on treating inventory as a cost to avoid without necessarily recognising conflicting objectives, such as high levels of customer service. At its purest, the minimal approach assumes a precise, bivalent environment rather than imprecise, multivalent and fuzzy reality. Similarly, the maximal approach to inventory replenishment focuses on treating inventory as an asset without necessarily recognising conflicting objectives, such as other opportunities for investment. So, in its purest form, the maximal approach also assumes a precise, bivalent environment rather than imprecise, multivalent and fuzzy reality. In contrast to the other approaches, the heuristic approach to inventory replenishment includes such a wide variety of techniques that some may be based on bivalent mathematics while others may be more fuzzy and less well-defined. It is therefore impossible to make an all-inclusive statement regarding this approach.

In summary, conventional inventory replenishment systems that tend to rely upon bivalent mathematics appear weak. The use of a heuristic approach based on fuzzy rather than binary logic may prove to be more beneficial.

Bivalent and fuzzy models

In classical set theory, based on bivalent mathematics, an individual object is either a member of a particular set or not. It cannot be both a member and a non-member of a particular set at the same time. For example, the number 2 is a member of the set of all integers. In bivalent logic it cannot be a member and a non-member of this set. Neither can it be a partial member of a set. It is wholly a member or not at all.

On the other hand, a member of a fuzzy set may be a partial member rather than restricted to being a whole member or a non-member. (In fact, bivalent sets are special cases of fuzzy sets where membership is either 100%, a whole member, or 0%, a non-member.) This multivalent mathematics allows for varying degrees of membership of the set on the real continuous interval [0,1]. A fuzzy set is defined by its membership function. (See Ross[96] for a general discussion of fuzzy sets in theory and application). Zadeh[97] developed the notation that is commonly used to describe fuzzy membership functions as follows:

$$\mu_{\underset{\sim}{A}}(x) \in [0,1]$$

where $\underset{\sim}{A}$ (the conventional set symbol A with a tilde underneath) represents the fuzzy set and the symbol $\mu_{\underset{\sim}{A}}(x)$ represents the degree of membership of element x in fuzzy set $\underset{\sim}{A}$.

Bivalent overstock and understock

In conventional inventory replenishment systems, which use bivalent mathematics, any quantity of product can be classed as either overstocked, understocked or neither, but never a combination, even though this may be a better reflection of different viewpoints and circumstances of the business. The level at which product is considered overstocked or understocked may be defined by the acceptable maximum and minimum stock levels. These levels are the result of the type of replenishment system employed, the level of safety stock held, the target customer service level and the assumed distribution of demand (for some simple examples see Waters[98]). For example, an order up-to level may define the acceptable maximum stock level while the acceptable minimum level may be defined by the lead time and safety stock required. This view of overstock and understock derived from conventional inventory replenishment systems can be defined in mathematical terms.

Let V be the set of all overstocked product and U be the set of all understocked product within the universe of inventories held I where i is any element. The bivalent approach employed in conventional inventory replenishment systems may view the membership of inventory held i in the sets of overstock (V) and understock (U) as:

$$\chi_V(i) = \begin{cases} 1, i \in V \\ 0, i \notin V \end{cases} \qquad \chi_U(i) = \begin{cases} 1, i \in U \\ 0, i \notin U \end{cases}$$

For example, bivalent overstock may be defined as:

$$\chi_V(i) = \begin{cases} 1, i > v \\ 0, i \leq v \end{cases}$$ where v = maximum acceptable stock level

and bivalent understock may be defined as:

$$\chi_U(i) = \begin{cases} 1, i < u \\ 0, i \geq u \end{cases}$$ where u = minimum acceptable stock level

This example can be depicted as follows:

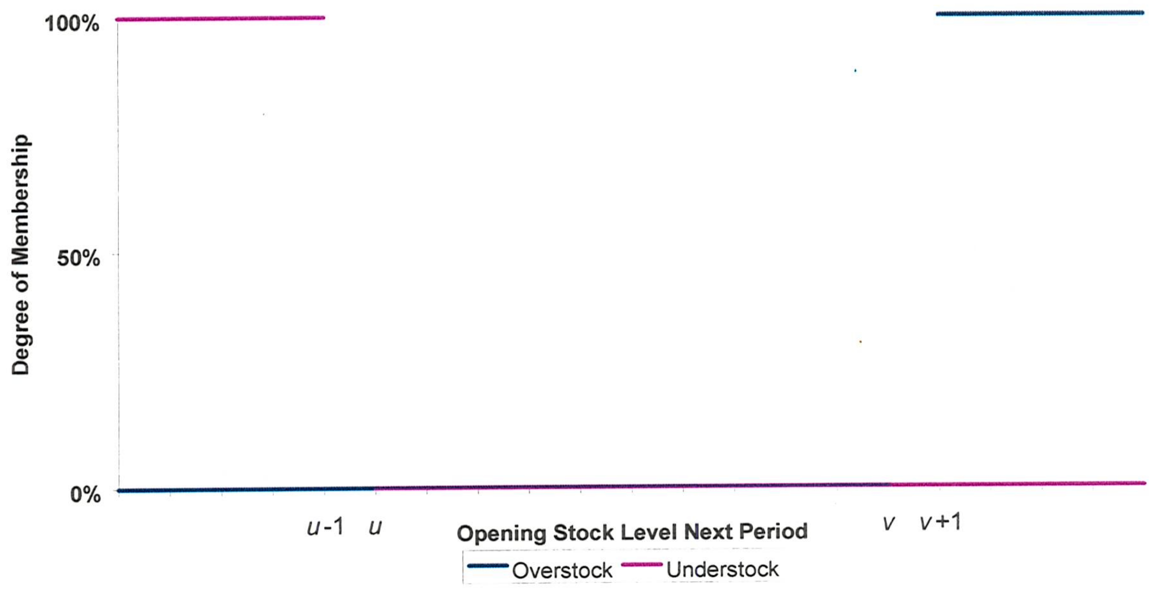


Figure 10: Bivalent overstock and understock

Note that the function is discontinuous, reflecting the sudden switch between acceptable and non-acceptable (overstock and understock) stock levels.

Fuzzy overstock and understock

On the other hand, a similar but fuzzy definition of overstock may be as follows.

$$\mu_{\underset{\sim}{V}}(i)=\begin{cases} 1, i \geq v+1 \\ \dfrac{i-u}{v+1-u}, u < i < v+1 \\ 0, i \leq u \end{cases}$$

while a similar but fuzzy definition of understock may be:

$$\mu_{\underset{\sim}{U}}(i)=\begin{cases} 1, i \leq u-1 \\ 1-\dfrac{i-(u-1)}{v-(u-1)}, u-1 < i < v \\ 0, i \geq v \end{cases}$$

This provides simple (linear) example definitions of overstock and understock, which can be depicted as follows:

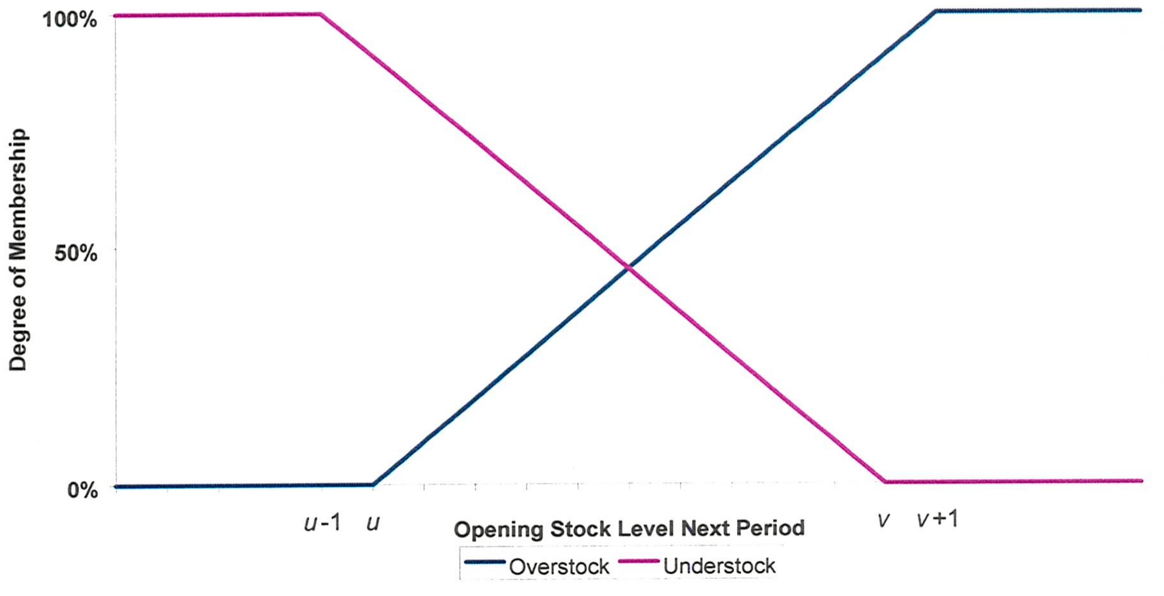


Figure 11: Fuzzy overstock and understock

The partial membership of a fuzzy set allows for gradual changes in the view of overstock or understock and for opposing concepts to overlap, as they often do in reality. This is in direct contrast to the view of inventory using bivalent logic where an increase of 1 item may cause a change in status from not being overstocked to being overstocked.

It is sometimes argued[99] that in inventory theory

> *'... when vagueness enters it is normally of the stochastic kind which can be properly modelled using probability theory.'*

However, the distinction between overstock and understock is not a matter of chance or probability but one of ambiguity and therefore fuzziness. (For more detail, Kosko[100] provides a proof of the distinction between fuzziness and probability.)

Therefore, it is argued that fuzzy sets better reflect the reality of inventory replenishment practice compared to the conventional use of bivalent mathematics in current inventory replenishment systems.

Fuzzification

The process described above is known as fuzzification. Fuzzification takes precise measurable inputs and converts them to fuzzy approximations. So, in the example introduced above, it may be that a precise measurement of inventory such as 15 units is considered (in fuzzy terms) mostly to be overstocked and partly to be understocked. Fuzzification requires the relevant fuzzy membership functions, such as overstock, to be defined. The definition of fuzzy membership functions lies in the academic domain of knowledge acquisition and knowledge engineering.

There are increasingly many methods available to define membership functions as well as many different ways to classify fuzzy measures. Over the years some authors have tried to summarise these methods including Saaty[101] (1974), Sugeno[102] (1977), Nowakowska[103] (1979), Banon[104] (1981), Smithson[105] (1987), Ezhkova[106] (1989), Sim & Wang[107] (1990), Stojakovic[108] (1992), Wang & Klir[109] (1992), Klir & Yuan[110] (1995) and Ross[111] (1995). While these authors provide various classifications of the fuzzification methods available, broadly speaking the methods divide into two groups: those defined either indirectly or directly by expert(s), and those defined by analysis of the available data. Expert-derived methods include intuition[112], inference[113] and rank ordering[114]. Data-derived methods are increasing in number and include neural networks[115], genetic algorithms[116] and inductive reasoning[117]. The advantages and disadvantages of each method depend upon the individual application.

Fuzzy systems

In order to apply this fuzzy logic to inventory replenishment it is necessary to understand how a fuzzy system operates. A fuzzy system is[118]:

> *'A set of fuzzy rules that converts inputs into outputs. In the simplest case an expert states the rules in words or symbols. In the more complex case a neural system learns the rules from data or from watching the behaviour of human experts.'*

Fuzzy rules are conditional statements that may be considered vague or multi-valued. This means that everything is a matter of degree.

As demonstrated earlier (see page 47), an example of a fuzzy statement is 'the product is overstocked'. In this case 'overstocked' is a vague term which may be interpreted in a number of ways both in terms of extent (greatly overstocked, only just overstocked, and so on) as well as definition (overstocked in operational terms, or in marketing terms, and so on).

Fuzzy inference

Fuzzy inference rules are used to convert fuzzy inputs (such as levels of overstock) into fuzzy outputs (such as large or small orders). These rules are determined using the same methods to define fuzzy membership functions: human 'experts' or available data. In particular the use of neural networks is common when using a data driven approach.[119]

These rules may be relatively simple, such as:

> *If overstocked then place a small order.'*

or complex, such as the following example:

> *If overstocked, interest rates are high, unit cost of product is high and importance of product to overall range is low then place a very small order.*

Note that all of the qualifying terms may be considered fuzzy. Hence 'high' interest rates, 'high' unit costs, 'low' importance of the product to the overall range and 'small' order size may all be defined by fuzzy membership functions similar to the examples of overstock and understock. Formal definitions of fuzzy logic operations can be found in the appendices.

Defuzzification

When these fuzzy rules are calculated the resulting fuzzy output set represents the decision or action to be implemented. So, in the example above it would represent the size of the next replenishment order. For example, the resulting output set may be 'a small replenishment order'. This output set is fuzzy, yet (as with most decisions) a binary solution is required. For example, a precise, binary replenishment figure is required, such as 15 units. Therefore, before a decision can be implemented the fuzzy output must be converted to a precise, binary output. This process is known as defuzzification.

> Mathematically, defuzzification can be thought of as a 'rounding' process - similar to rounding 4.836 up to 5. However unlike 'rounding' in bivalent mathematics there is no universally accepted method for defuzzification.

The most common methods are documented by Hellendoorn and Thomas.[120] These are briefly summarised below.

Centre of gravity - (also known as centroid or centre of area). This is probably the most popular method of defuzzification.

Max-membership - (also known as the height method). The output is simply the value with the highest membership.

Weighted average - weight each membership function in the output by its respective maximum membership value. This method is restricted to symmetrical functions.

Mean-max membership - (also known as middle of the maxima). This is similar to the max-membership method except that there can be more than one value with the highest membership.

Centre of sums - similar to the weighted average method except that the weight used is the area of the respective membership function rather than the height.

Centre of the largest area - similar to the centre of gravity method except that the largest convex sub-region is used rather than the whole area.

First (or last) of maxima - similar to the max-membership method except that there may be more than one value with the highest membership and the first (or last) of these maxima is chosen.

Of these methods the first two are the most important since all other methods can be considered to be variations of them.

The most appropriate method depends on the individual circumstances concerned. Hence, the relative advantages and disadvantages of each method relate to the individual situation. While no defuzzification method is universal, Hellendoorn and Thomas[120] suggest five criteria to help compare defuzzification methods and so identify an appropriate method for a particular situation. These criteria are as follows:

Continuity - a small change in the input of a fuzzy process should not produce a large change in the output.

Disambiguity - defuzzification should produce only one output.

Plausibility - an output should lie 'near the middle' of the fuzzy set and have a high degree of membership.

Computational simplicity - the more time-consuming the calculation, the less the value of the defuzzification process.

The weighting method - the difference between the method chosen and most other methods should be small.

For the purposes of this thesis, the relevant fuzzy functions are simple, linear and symmetrical (see pages 53 to 61). Therefore, a simple weighted average is employed for defuzzification. This satisfies the key criteria proposed above.

Developing fuzzy systems

There is no universally accepted approach to developing fuzzy systems. At each stage of the process there are many options to consider. Any evaluation of options can only take place when developing a system for a particular situation (see Cox[121], 1992 for further general discussion).

Previous fuzzy inventory research

Previous research into fuzzy inventory management is limited. An extensive literature review provides only 32 papers (see Appendix E for details), many by the same authors. Many of these papers employ fuzzy logic to deal with uncertainty within a conventional inventory replenishment system. For example, a fuzzy rather than stochastic approach may treat demand as high or low rather than distributed Normally with a particular mean (such as Petrovic & Sweeney[122]). Costs may be treated similarly as "high" or "low" within a conventional economic approach, rather than as precisely defined figures (for example, see Vujosevic et al[123]). Such adaptations are similar to the heuristic approaches discussed earlier (see Chapter 2), where the (fuzzy) heuristic is employed within a primarily economic and/or stochastic approach.

Of the other papers identified, the relevance to inventory replenishment systems is either indirect (for example, Chang & Yih[124] who discuss a fuzzy approach to the control of a kanban system) or relatively insignificant (for example, Yenradee et al[125] who discuss the situational factors regarding production and inventory systems).

Thus, there is the opportunity to develop a new inventory replenishment system that is primarily (rather than secondarily) based on a fuzzy, heuristic approach.

Developing a FIRS

Developing a fuzzy inventory replenishment system (abbreviated as FIRS) is an ill-defined process. A fuzzy approach has been chosen as potentially beneficial because of the huge flexibility provided by the infinite choice of descriptive functions. There are many ways to define a fuzzy system, from the completely qualitative to the fully quantitative. The same is true for a FIRS. The relevant fuzzy membership functions and rules could be developed via a qualitative-based process such as Soft Systems Methodology[126] or a heavily quantitative process such as via neural networks[127].

The FIRS chosen for evaluation within this thesis was developed through an evolutionary process with the aim of keeping it both simple and intuitive. By keeping the FIRS simple enables the concept to be evaluated universally, without the need for developing special cases or further sophistication (though this may obviously be desired for particular applications). By keeping the FIRS intuitive aims to provide some logical foundations on which to base any results and subsequent development. A fuller description of the FIRS development process for this thesis can be found in the appendices. Once an acceptable FIRS is developed, then a range of different business scenarios can be evaluated and compared to existing inventory replenishment systems (see Chapter 5).

The proposed FIRS for evaluation

The FIRS developed for this thesis employs just two fuzzy input variables of overstock and understock, relevant to the inventory held (*i*), and two fuzzy output variables of small order and large order, relevant to the order placed (*p*).

$$\textit{If overstocked, } \mu_{\underset{\sim}{V}}(i)\textit{, then place a small order, } \mu_{\underset{\sim}{S}}(p)$$

and

$$\textit{If understocked, } \mu_{\underset{\sim}{U}}(i)\textit{, then place a large order, } \mu_{\underset{\sim}{L}}(p)$$

More fuzzy sets and associated rules could be used. However, this thesis is focused on the question of whether FIRS may be beneficial rather than the more particular question of what is the best FIRS for a specific situation. In order to ensure that all of the fuzzy membership functions are generic and practical, as well as simple and intuitive, they are defined in relation to sales history. To keep the definitions simple and fuzzy, they are all defined as linear and sloping ('triangular') functions. Thus the fuzzy membership functions for overstock, understock, small and large orders must be defined.

Overstock and understock fuzzy membership functions

The overstock and understock fuzzy membership functions chosen are similar to those previously described in Figure 11, but based on historic sales.

Let S_t = historic sales between orders in time period t

where t = -1, -2, -3, ...

then the minimum acceptable stock level must be equal to the minimum historic sales.

So,

minimum acceptable stock level = $min(S_t)$

The maximum acceptable stock level must be at least the maximum historic sales and is possibly greater if some sales may have been lost due to stockouts.

Hence,

if there are no historical stockouts

then the maximum acceptable stock level = $max(S_t)$

else (if stockouts)

maximum acceptable stock level $\geq max(S_t)$

Where stockouts do occur the actual value of the maximum acceptable stock level is difficult (if not impossible) to predict. In order to take some account of the possibility of extra sales while minimising the risk of over exaggerating the value, the maximum acceptable stock level is set at $max(S_t) + 1$.

Therefore in general terms,

maximum acceptable stock $= max(s_t) + x$ where $x = 0$ if no historical stockouts

$x = 1$ if stockouts exist

So, using the assumptions described previously (see page 46),

$$\mu_{\underset{\sim}{V}}(i) = \begin{cases} 1, i \geq max(s_t) + 1 + x \\ \dfrac{i - min(s_t)}{max(s_t) + 1 + x - min(s_t)}, min(s_t) < i < max(s_t) + 1 + x \\ 0, i \leq min(s_t) \end{cases}$$

$$\mu_{\underset{\sim}{U}}(i) = \begin{cases} 1, i \leq min(s_t) - 1 \\ 1 - \dfrac{i - (min(s_t) - 1)}{max(s_t) + x - (min(s_t) - 1)}, min(s_t) - 1 < i < max(s_t) + x \\ 0, i \geq max(s_t) + x \end{cases}$$

This is depicted as follows:

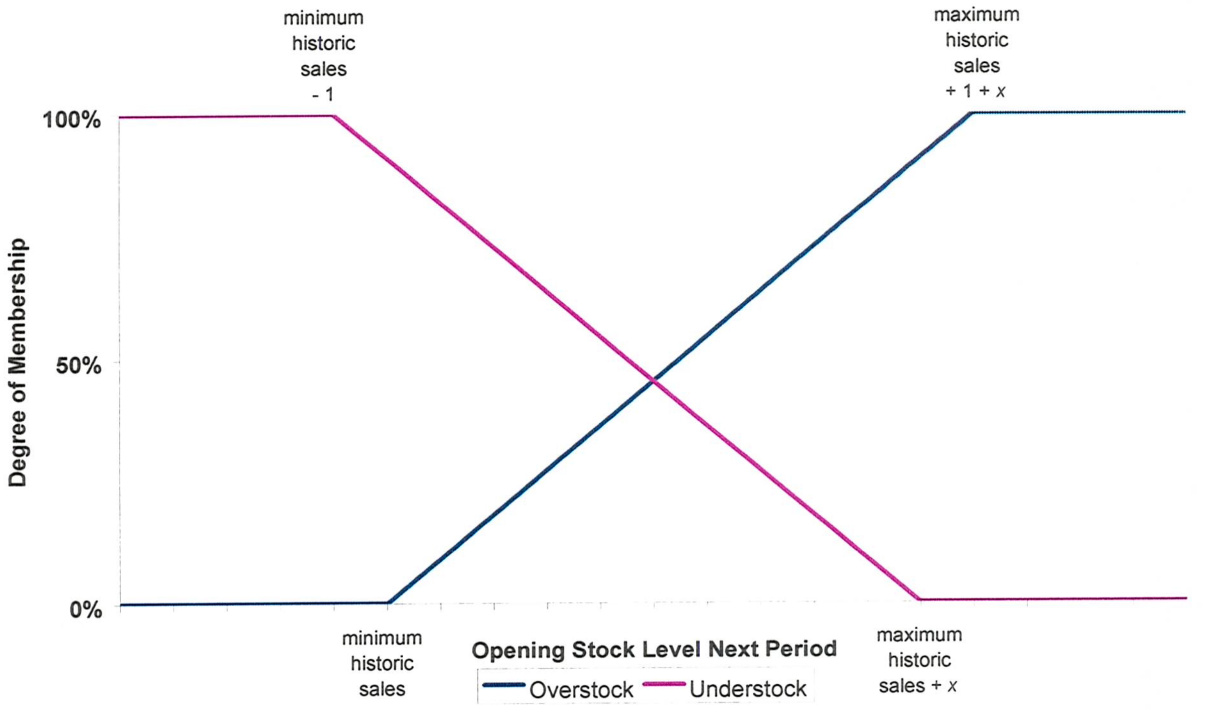


Figure 12: Overstock & understock fuzzy membership functions

These equations can be simplified to

$$\mu_{\underline{V}}(i) = \begin{cases} 1, i > max(s_t) + x \\ \dfrac{i - min(s_t)}{max(s_t) + 1 + x - min(s_t)}, min(s_t) < i \leq max(s_t) + x \\ 0, i \leq min(s_t) \end{cases}$$

$$\mu_{\underline{U}}(i) = \begin{cases} 1, i < min(s_t) \\ 1 - \dfrac{i - min(s_t) + 1}{max(s_t) + x - min(s_t) + 1}, min(s_t) \leq i < max(s_t) + x \\ 0, i \geq max(s_t) + x \end{cases}$$

Large and small order fuzzy membership functions

The choice of fuzzy membership functions to represent small and large orders is unlimited. For the purposes of this thesis some simple but logical definitions are required.

The rationale for the chosen definitions of the fuzzy membership functions for large and small orders begins with defining the small order because logically this is finite, while the large order theoretically could be infinite (which does not lend itself to simple definition).

Small orders

As a starting point, the fuzzy membership function for a small order could start at zero because any order close to zero must be considered small. However, zero itself represents a non-order and so the fuzzy membership of a small order at zero must be 0%. Given that a small order cannot, by definition, be infinite, then the small order fuzzy membership function must rise and then fall back to zero. In order to achieve the aim of simplicity, it is assumed that this rise and fall may be defined by two further points, at 100% membership and back to 0% membership. Without any other intuitive reference point and with the aim of simplicity, the return to 0% and the original 0% starting point are assumed to be equidistant from the peak at 100%, thus constructing a simple, symmetrical triangle as depicted in Figure 13.

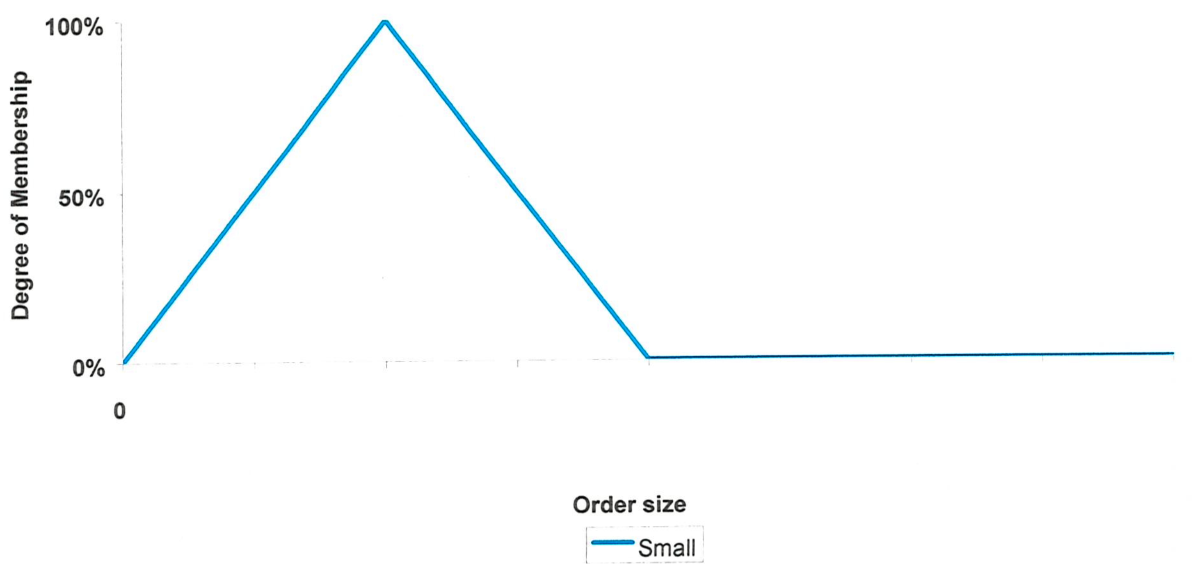


Figure 13: Simple small order fuzzy membership function

Hence the concept of a fuzzy small order is defined.

Large orders

In order to achieve the aim of simple and intuitive fuzzy membership functions and to limit the large order, it is assumed that the membership functions of both the small and large orders are homogenous (and that the large order is bigger than the small order), as depicted in Figure 14.

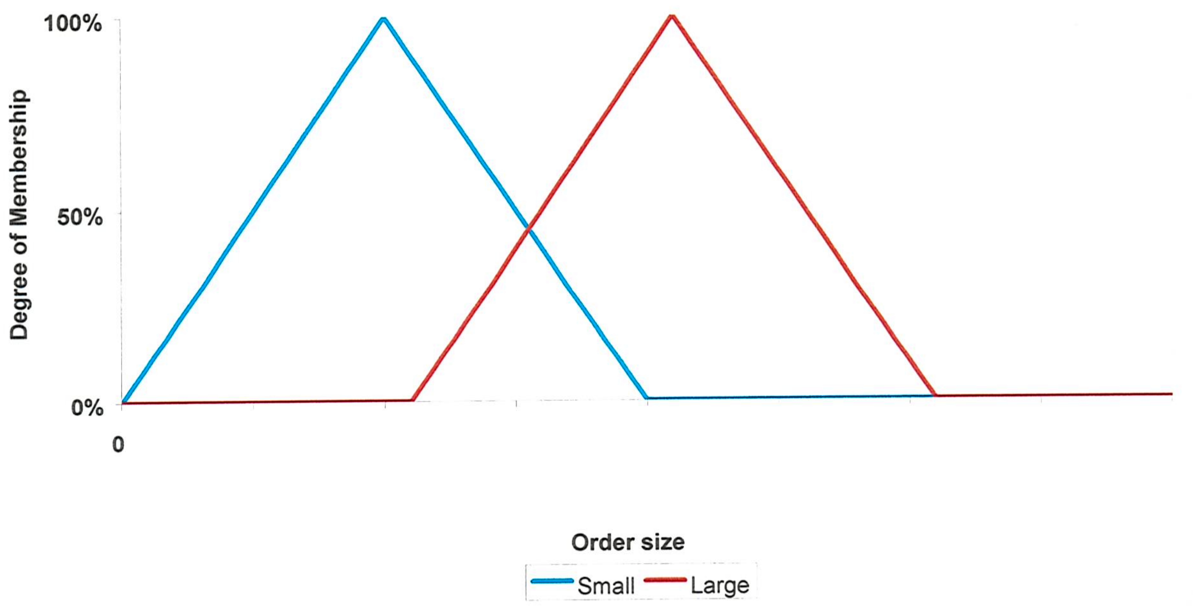


Figure 14: Simple small and large order membership functions

Hence the concept of a fuzzy large order is defined.

The point at which the orders reach 100% membership could be at any point on the scale. However, if the points chosen are too great then the system will continually over-order and thus continually increase stock levels, resulting in ever-increasing costs. On the other hand, if the points chosen are too low then the system may continually under-order and thus reduce stock levels towards zero, resulting in lost sales and poor customer service. Each of these scenarios is considered in turn.

Extreme overstock

Assuming the inventory system is totally overstocked then by definition the system is 100% overstocked and 0% understocked.

So,

$$\mu_{\underset{\sim}{V}}(i) = 1 \text{ and } \mu_{\underset{\sim}{U}}(i) = 0$$

and employing the proposed fuzzy rules defined earlier (see page 52), then

$$\mu_{\underset{\sim}{V}}(i) \bullet \mu_{\underset{\sim}{S}}(p) + \mu_{\underset{\sim}{U}}(i) \bullet \mu_{\underset{\sim}{L}}(p) = \mu_{\underset{\sim}{S}}(p)$$

Hence, if the system is totally overstocked then place a 100% small order.

Given the choice of a symmetric fuzzy membership function for small orders then a 100% small order is represented by its midpoint. This midpoint must be lower than the mean historical sales, otherwise the system would continue to add to the overstock if sales statistically remain the same or decrease.

Thus,

$$midpoint\left(\mu_{\underset{\sim}{S}}(p)\right) < mean(S_t)$$

Similarly,

$$midpoint\left(\mu_{\underset{\sim}{S}}(p)\right) \geq min(S_t)$$

otherwise the inventory system will order less than the minimum historical sales. Without this limitation there is a danger that the system will reduce inventory levels below the minimum historical sales, and so result in lost sales.

Hence,

$$min(S_t) \leq midpoint\left(\mu_{\underset{\sim}{S}}(p)\right) < mean(S_t)$$

Assuming only whole units (integer values) of product may be ordered, this may be written as

$$min(s_t) \le midpoint\left(\mu_{\underset{\sim}{S}}(p)\right) \le Roundup(\,mean(s_t) - 1,0\,)$$

or

$$min(s_t) \le midpoint\left(\mu_{\underset{\sim}{S}}(p)\right) \le Roundup(\,mean(s_t),0\,) - 1$$

This may be depicted as follows:

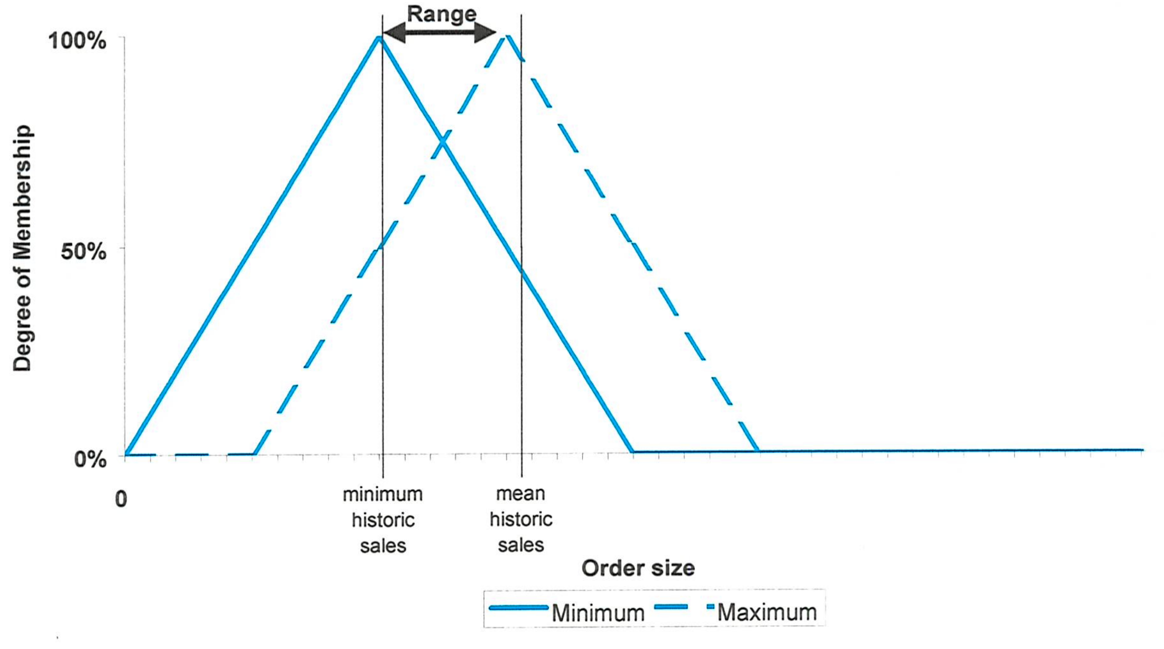


Figure 15: Range of small order membership functions

Extreme understock

On the other hand, assuming the inventory system is totally understocked then by definition

$$\mu_{\underset{\sim}{U}}(i) = 1 \text{ and } \mu_{\underset{\sim}{V}}(i) = 0$$

so

$$\mu_{\underset{\sim}{V}}(i) \bullet \mu_{\underset{\sim}{S}}(p) + \mu_{\underset{\sim}{U}}(i) \bullet \mu_{\underset{\sim}{L}}(p) = \mu_{\underset{\sim}{L}}(p)$$

Hence, if the system is totally overstocked then place a 100% large order. Again, given the choice of a symmetric fuzzy membership function for large orders, then a 100% large order is represented by its midpoint. This midpoint must be able to exceed the maximum historic sales otherwise future sales are limited to historical sales and there is no opportunity for growing sales.

Thus,

$$midpoint\left(\mu_{\underset{\sim}{L}}(p)\right) > max(s_t)$$

Again, assuming only whole units of product may be ordered, this may be written as

$$max(s_t) + 1 \leq midpoint\left(\mu_{\underset{\sim}{L}}(p)\right)$$

For simplicity it is assumed that the large orders are homogenous with the small orders and thus

$$max(s_t) + 1 \leq midpoint\left(\mu_{\underset{\sim}{L}}(p)\right) \leq max(s_t) + 1 + (Roundup(mean(s_t), 0) - 1 - min(s_t))$$

which simplifies to

$$max(s_t) + 1 \leq midpoint\left(\mu_{\underset{\sim}{L}}(p)\right) \leq max(s_t) + Roundup(mean(s_t), 0) - min(s_t)$$

This may be depicted as follows:

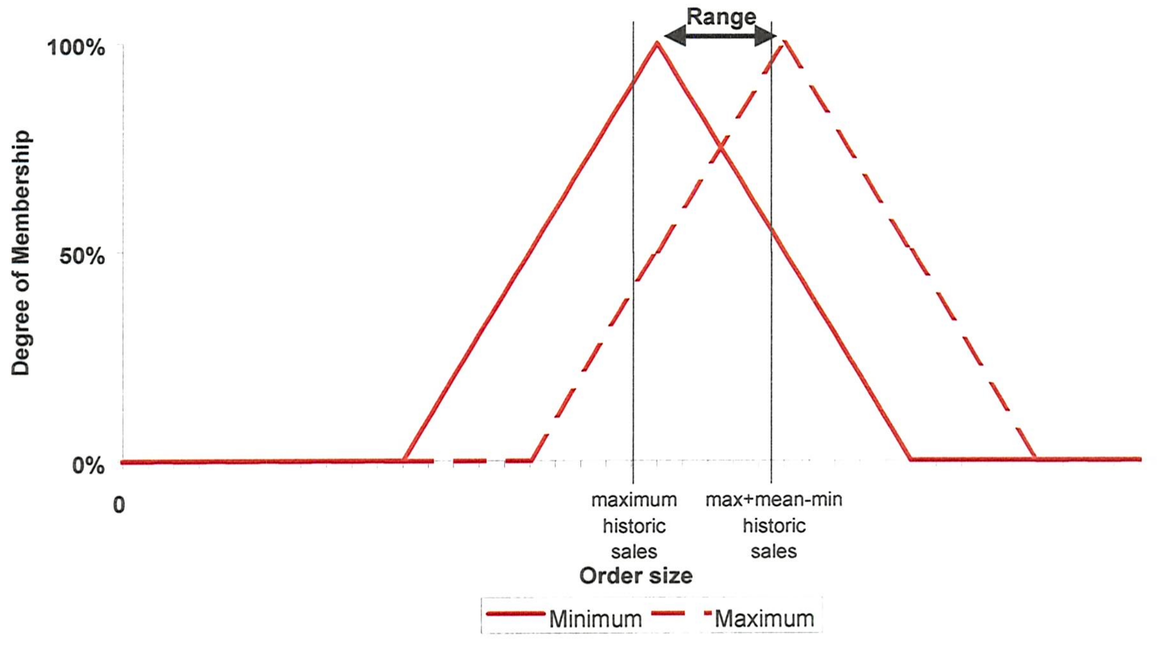


Figure 16: Range of large order membership functions

Combining membership functions

The arguments and assumptions above define small and large order fuzzy membership functions. The most relevant definition depends upon the business scenario under consideration. For a particular business scenario the fuzzy membership functions may be uniquely defined. For the purpose of this thesis, however, a simple generic model is required that can be evaluated across a number of business scenarios, it is therefore proposed to pair up the functions to form a range of large and small order sizes to be evaluated. This process may be considered similar to the conventional idea of customer service or stock availability targets employed in typical inventory replenishment systems used today. For example, the lower bound of the membership functions defined can be considered to represent a minimum customer service level, while the upper bound membership functions represent a maximum level of customer service. Thus a customer service strategy may be chosen within these bounds. This may be measured as the distance from the lower bound (0%) to the upper bound (100%).

Thus, a general set of equations may be written to define small and large orders based on a specified customer service strategy, as follows:

$$\mu_{\tilde{S}_y}(p)=\begin{cases} 0, p \le y \bullet \left(mean(S_t)-min(S_t)\right) \\ \dfrac{S_t - y \bullet mean(S_t)}{min(S_t)}, y \bullet \left(mean(S_t)-min(S_t)\right) < p \le y \bullet mean(S_t)+(1-y)\bullet min(S_t) \\ i-\dfrac{S_t - y \bullet mean(S_t)}{min(S_t)}, y \bullet mean(S_t)+(1-y)\bullet min(S_t) < p \le y \bullet mean(S_t)+(2-y)\bullet min(S_t) \\ 0, p > y \bullet mean(S_t)+(2-y)min(S_t) \end{cases}$$

$$\mu_{\tilde{L}_y}(p)=\begin{cases} 0, p \le max(S_t)+y\bullet mean(x)-(1+y)min(x) \\ 1+y+\dfrac{p-max(S_t)-y\bullet mean(S_t)}{min(S_t)}, max(S_t)+y\bullet mean(S_t)-(1+y)\bullet min(S_t) < p \le max(S_t)+y\bullet mean(S_t)-y\bullet min(S_t) \\ 1+y-\dfrac{p-max(S_t)-i\bullet mean(S_t)}{min(S_t)}, max(S_t)+y\bullet mean(S_t)-y\bullet min(S_t) < p \le max(S_t)+y\bullet mean(S_t)+(1-y)\bullet min(S_t) \\ 0, p > max(S_t)+y\bullet mean(S_t)+(1-y)\bullet min(S_t) \end{cases}$$

where y = customer service strategy chosen and $0\% \le y \le 100\%$

These functions can be depicted as follows:

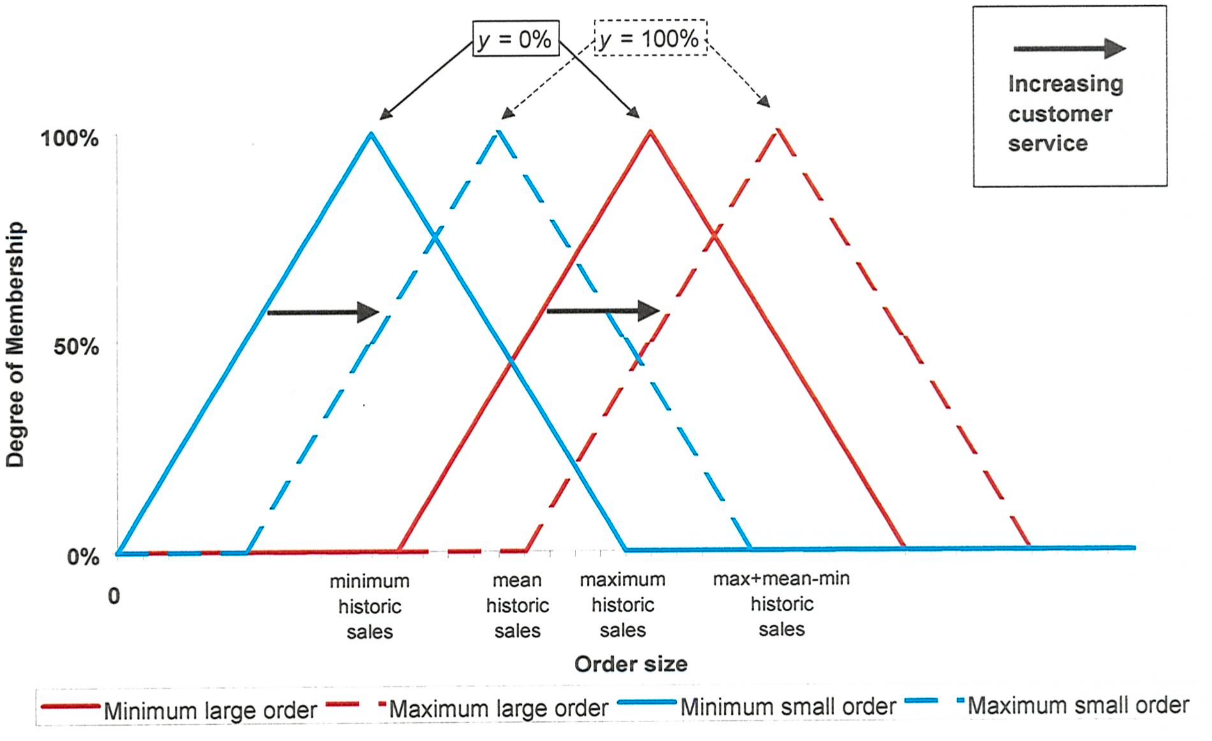


Figure 17: Order sizes dependent upon customer service strategy

Thus, a proposed conceptual FIRS has been defined, which can now be evaluated.

Chapter 5

THE RESEARCH MODEL
Quantifying fuzzy effects

Given the quantitative nature of replenishment systems, it seems appropriate to employ deductive research methods to investigate the concept of the fuzzy inventory replenishment system developed in the previous chapter. This chapter explains the empirical testing of this concept. It is recognised that inductive research methods may be more useful in researching the implementation and practicalities of such a system. These issues are considered only briefly in this thesis (see Chapter 7).

According to Gill and Johnson[128], the design of such empirical research entails four basic elements:

1. **Delineate the problem** the research is attempting to tackle, by identifying the 'theoretically dependent variables'.

2. Identify the 'theoretically independent variables' whose variations cause changes in the dependent variables, according to **the hypothesis being tested**.

3. **Design a method** for making changes to the independent variables and observing changes to the dependent variables.

4. **Control (or exclude) any 'extraneous variables'**, which may otherwise affect the outcome of the experiment.

Each one of these steps is now considered with respect to researching fuzzy inventory replenishment systems.

Delineating the problem

This research considers the following question:

> *'Does a fuzzy inventory replenishment system perform better than a conventional inventory replenishment system?'*

The number of different combinations of inventory systems and business scenarios is limitless. In order to make this investigation manageable, the variables need to be constrained allowing for results that simultaneously provide theoretical insight as to the credibility of fuzzy inventory replenishment systems while avoiding such narrowness that the results are of no practical use.

The research focus is on the replenishment system (fuzzy or conventional) and not on supply and/or demand issues. Therefore, in individual experiments, supply and demand are treated as extraneous variables. However, it is important to understand the impact of variations in such variables, particularly in terms of demand, and so a range of experiments with varying demand conditions are carried out. The research focus is summarised in the following diagram.

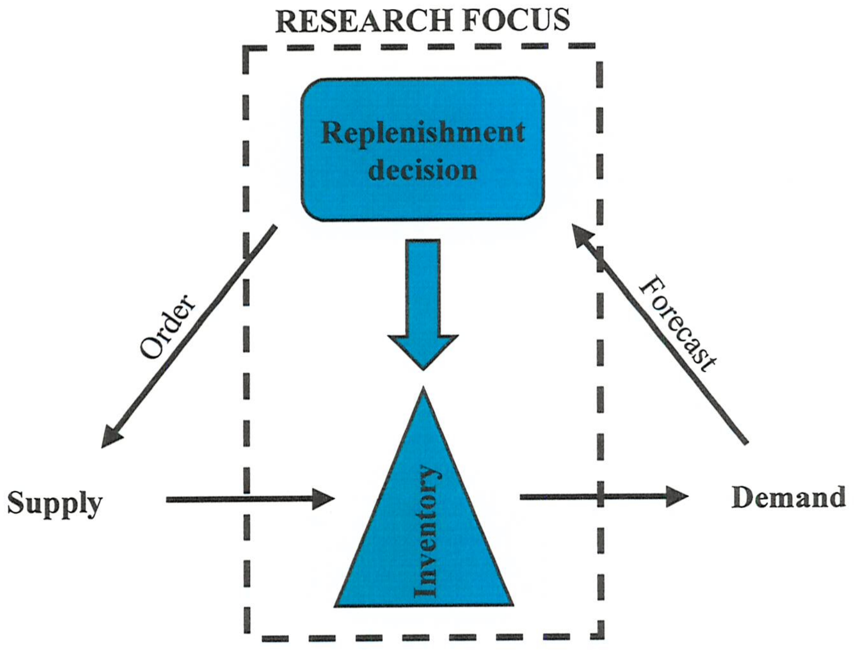


Figure 18: Area of research focus

In summary, for any individual experiment, forecasting demand and the supplier order cycle are considered extraneous to the central hypothesis. This is discussed in more detail later (see pages 81 to 88).

The research hypothesis considers the performance of the inventory management that is dependent upon the replenishment decisions made. Therefore the dependent variables need to reflect inventory management performance. Although there are many different approaches to inventory management, there are consistent performance measures that are generally employed, by both practitioners and academics. Inventory management performance measures relate broadly to either cost or customer service.

Cost

Cost may be measured in a number of direct and indirect ways. However, whether taking a Just-In-Time approach (for example, see Harrison[129]) or a purely economic approach (for example, see Silver & Peterson[8]), it is generally agreed that the amount of inventory held is a major determinant of cost. This is particularly true when focusing on a comparison of two conceptual replenishment systems (fuzzy and conventional), where extraneous factors are considered the same. In this case it may be assumed that, with the exception of the number of replenishments, other cost drivers are basically the same in both systems. These other cost drivers include cost of the product, delivery lead-time, delivery costs, order costs, damage, waste and so on. In these circumstances it is assumed that there are no significant cost differences between the two systems other than those costs related to the quantity of inventory held and the number of replenishments (excluding customer service costs, which are considered below).

Common measures of inventory held include the absolute quantity (both in physical and financial terms) and a ratio of stock-turn or stock cover. From the survey of practitioners (see Appendix A), 100% used stock-turn or stock cover as a key inventory management performance measure, and 82% used the total value of stock. For easy comparison, stock cover is a robust measure when there are variations in demand, because it is a ratio related to the demand per period. In reality, true demand is often unknown and so stock cover is typically measured as a percentage of actual sales rather than demand. The number of replenishments may also be converted to a ratio by dividing by the number of time periods represented.

Customer service

Customer service may also be measured in many different ways, though the major criteria usually relate to products out of stock, either in terms of frequency (periods out of stock, commonly referred to as stockouts) or volume (lost sales). Lost sales can only be measured when the absolute demand is known (regardless of whether a sale is made as a result of the demand). In practice, the absolute level of demand is often unknown and in these circumstances lost sales are difficult, if not impossible, to measure accurately. Stockouts, however, can always be measured where inventory records are correct. From the survey of practitioners (see Appendix A), 71% measured stockouts.

The dependent variables

In summary, therefore, in this research two primary variables are measured: stock cover as a measure of cost, and lost sales as an accurate (theoretically possible, though generally impractical) measure of customer service. Two secondary variables are also considered: number of replenishments as a further measure of cost, and number of stockouts as a further measure of customer service. All of these variables are dependent upon the replenishment system employed, whether it is fuzzy or conventional.

The performance measures are therefore calculated as follows:

Parameter	***Measure***	***Units***
Stock cover (based on demand)	$\frac{inventoryheld}{demand}$	%
Stock cover (based on sales)	$\frac{inventoryheld}{sales}$	%
Replenishments	$\frac{replenishments}{timeperiods}$	%
Customer service (based on lost sales)	$1-\frac{lostsales}{demand}$	%
Customer service (based on stockouts)	$1-\frac{stockouts}{periods}$	%

Table 2: The dependent variables

Although simple, these are well-accepted performance measurements in typical business scenarios. Furthermore, they can be seen to satisfy Caplice and Sheffi's eight metric evaluation criteria,[130] which are summarised in Table 3.

Criterion	*Description*
Validity	The metric accurately captures the events and activities being measured and controls any exogenous factors.
Robustness	The metric is interpreted similarly by the users, is comparable across time, location and organisations and is repeatable.
Usefulness	The metric is readily understandable by the decision-maker and provides a guide for action to be taken.
Integration	The metric includes all the relevant aspects of the process and promotes co-ordination across functions and divisions.
Economy	The benefits of using the metric outweigh the costs of data collection, analysis and reporting.
Compatibility	The metric is compatible with the existing information, material, and cash flows and systems in the organisation.
Level of detail	The metric provides a sufficient degree of granularity or aggregation for the user.
Behavioural soundness	The metric minimises incentives for counter-productive acts or game playing and is presented in a useful form.

Table 3: Caplice & Sheffi's eight logistics metric evaluation criteria

The hypothesis being tested

The central hypothesis of this thesis is that a fuzzy inventory replenishment system may perform better than a conventional one. Any inventory replenishment system (fuzzy or conventional) is defined by the relevant variables employed. This research is focused on the variables that are combined to calculate the size of the replenishment order. Thus these are the independent variables under investigation. The fuzzy inventory replenishment system variables were discussed in detail in Chapter 4. The conventional variables are now discussed.

The conventional replenishment system

A conventional replenishment system includes many of the variables defining the overall approach employed (such as stochastic, economic and so on) as described in any general inventory text, such as Silver & Peterson[8], or in case studies, such as Cuthbertson and Moore[131]. To compare the performance of a fuzzy replenishment system against an economic, minimal or maximal approach is self-defeating because these approaches are based on certain assumptions with singular objectives (minimise cost, minimise time, maximise investment respectively). If the assumptions are valid then by definition these approaches must perform better than any other approach, including a fuzzy-based one. On the other hand, if the assumptions are debatable (see page 29 onwards), then 'better performance' is also debatable. It is possible to compare a fuzzy-based approach against another heuristic approach. However, these approaches are varied, often selective and poorly documented. Therefore, it is proposed to compare the fuzzy replenishment system against a stochastic replenishment system. The conventional, stochastic, periodic review, order up-to system is the basis for many popular inventory replenishment systems in use today (see Silver & Peterson[8] for more details). This approach also has the added advantage of being dependent on only one major parameter (customer service), which is typically calculated based on the Normal distribution.

'The formula for calculating the safety stock level is:

$$\textit{Safety stock} = z\,\sigma_d \sqrt{L}$$

where z represents the desired service level [expressed as the number of standard deviations form the mean demand], σ_d *is the standard deviation of demand, and L is the order lead time.' (Sandvig, 1998)* [132]

For example, a customer service target of 98% equates to a z-score[133] of 1.64 standard deviations from the mean, assuming a Normal distribution. Thus, given an order lead time of one period and a standard deviation of daily demand of 10 units, then

safety stock $= 1.64 x 10 x \sqrt{1}$ = 16.4 units. In the same situation, 90% and 99% customer service targets would equate to 12.8 and 23.1 units respectively.

Note that the customer service target is not necessarily the same as the customer service achieved, due to the time lag between placing and receiving an order. This is clearly demonstrated in Chapter 7.

In summary, a conventional, stochastic, inventory replenishment system only requires consideration of a range of customer service targets, say 50% to 99%, to reflect typical business scenarios.

This type of system should provide a rigorous benchmark against which to compare a simple fuzzy-based replenishment system. However, the relative performance of conventional inventory management systems relies to a large extent on the accuracy of the forecasting technique employed, and so this must also be considered here.

Forecasting demand

Common forecasting techniques employed in inventory software include Holt[134] and Winters[135] multiplicative forecasting models and Box-Jenkins[136] autoregressive integrated moving average (ARIMA) forecasting models. Different techniques may suit different circumstances (for some detailed analysis, see Makridakis et al[137]). However, in this research, arguments about the accuracy of the forecasting techniques employed by the conventional inventory replenishment system are avoided by providing the best possible forecast of the demand under consideration. Where theoretical demand is employed, then the conventional system is provided with the underpinning theoretical model of demand (except for any random variation). Where actual demand is employed, the conventional system is provided with a forecast model of demand based on the actual future demand data (rather than historical data).

Note that the fuzzy inventory replenishment system does not require a forecast of demand, but instead relies upon the extremities of demand (see Appendix G for details).

The fuzzy replenishment system

For the fuzzy replenishment system, the theoretically independent variables include the definition of the fuzzy membership functions and the fuzzy rules employed. For the purposes of this evaluation, these are those defined in Chapter 4.

Zeigler[138] differentiates between the theoretically independent variables and the theoretically dependent variables by calling them variables and parameters respectively. In this primary research, there are many variables, defining the inventory system employed, and few parameters, defining customer service and cost. The parameters used within the model are clearly defined. However, there are many possible variable settings creating the potential for infinite variations. Furthermore, there is an infinite range of extraneous settings. In order to design a practical research experiment it is therefore necessary to limit the number and scope of all of the variables under investigation.

The independent variables

In considering the conventional replenishment system employed, it is first necessary to limit the overall approach taken and then limit the variables within the limited approach. For the purposes of these analyses the following overall system is adopted.

Variable	***Potential range of values***	***Range considered***
Conventional system employed	Infinite variations	Stochastic, order up-to system employed

Table 4: Overall conventional replenishment system

The following variables are considered within this conventional system.

Variable	***Potential range of values***	***Range considered***
Customer service target	0% → 100%	99%, 90%, 80%, 70%, 60%, 50% All Normally distributed

Table 5: Conventional replenishment system variables

Six customer service levels are chosen to provide a resulting curve, rather than a point, against which to compare the fuzzy approach. These levels have been chosen to reflect a wide range of business scenarios.

Evaluating the fuzzy replenishment system proposed is an ill-defined process. This thesis proposes a new concept and therefore this has not been defined previously. Moreover, a fuzzy approach has been chosen as potentially beneficial because of the huge flexibility provided by the infinite choice of descriptive functions. The following table summarises the overall system chosen for this research.

Variable	*Potential range of values*	*Range considered*
Fuzzy approach employed	Infinite variations	Fuzzy inventory replenishment system defined in Chapter 4

Table 6: Overall fuzzy replenishment system

The following variables are considered within this fuzzy system.

Variable	*Potential range of values*	*Range considered*
Customer service strategy, as defined earlier (see page 61)	0 → 1	1.0, 0.8, 0.6, 0.4, 0.2, 0.0

Table 7: Fuzzy replenishment system variables

Six customer service strategies are again chosen to provide a resulting curve, rather than a point, against which to compare the fuzzy approach. These levels have been chosen to provide comparable results with the conventional customer service models (see Chapter 6).

Designing a method for investigation

Having identified the major variables to be investigated, the method for investigation must be designed.

Metamodelling

In order to consider a range of scenarios, this investigation takes a metamodelling approach. A metamodel is a higher order model of the results derived from another model[139], which, in turn, is trying to create 'a representation of reality'[140], so that it can be analysed. This metamodelling approach allows for incremental development in terms of the range of business scenarios (extraneous variables) considered, under which the conventional and fuzzy approach to inventory replenishment systems may be compared.

Modelling

Inventory replenishment management can be modelled in many ways from soft, qualitative interpretative approaches to hard, quantitative, deterministic approaches, depending upon the focus of decision-making. For example, the process of setting customer service targets for inventory may be modelled via a soft approach such as Soft Systems Methodology[126], whereas the replenishment of stock may be most usefully modelled via a hard approach, such a simulation. While many approaches to modelling are available, this research employs simulation to model inventory replenishment because this concurs with the features that tend to characterise the systems best suited to simulation, as identified by Pidd[141]: namely that they are dynamic, interactive and complicated. Replenishment systems are dynamic due to the many variables that vary over time, especially customer demand. They are interactive because current stock levels affect future replenishment decisions, which, in turn, affect future stock levels and so on. This feedback loop combined with the dynamic nature of the system creates high complexity. Furthermore, the quantitative nature of inventory leads towards employing a hard approach, while actual experiments within a business are intolerable without prior investigation. Therefore, simulation is chosen as the most appropriate tool to understand and compare the results of a conventional and fuzzy approach to inventory replenishment.

Simulation

The simulation model employed is built using a Microsoft Excel spreadsheet combined with Palisade's @RISK risk analysis software. This combination of software adequately provides for the uncertainty in demand to be defined and modelled, different scenarios to be simulated and the results compared. The overall results are compared as a single value, such as average stockholding, and as a distribution, for example the distribution of stockholding levels over the simulation period. The model uses a Monte Carlo-based [142] simulation approach, whereby the distribution of possible outcomes is generated by automatically recalculating the worksheet over and over again (iterations), each time sampling from the demand distribution.

Model overview

The simulation model was developed through an evolutionary process. This development process involved four main stages. Firstly, a model was created to enable the comparison of results. This initially compared the conventional replenishment system with a fixed order system. The fixed order system was then replaced with a fuzzy order system, based on known demand. The model was then further developed to adapt to varying demand. Finally, the model was 'tidied up' to make it simple to use and explain.

Once an acceptable final model was developed, then a wide range of different business scenarios was considered by varying the extraneous variables (see pages 81 to 88). Printouts of the spreadsheet model (values and equations), along with a glossary of the variables used, are found in the appendix.

The simulation model is summarised in the following diagram.

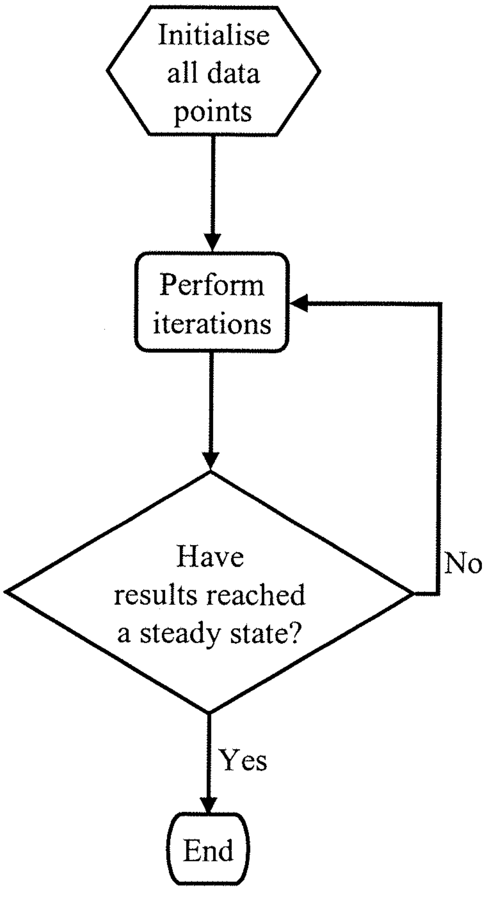


Figure 19: Overview of simulation model

There are three main stages: initialisation, iterations and steady state testing.

Initialisation

Initialisation sets all variables to their starting values. Figure 20 summarises the key inventory related variables (i.e. those not associated with the monitoring and running the simulation process). A complete list of key variables can be found in the appendix.

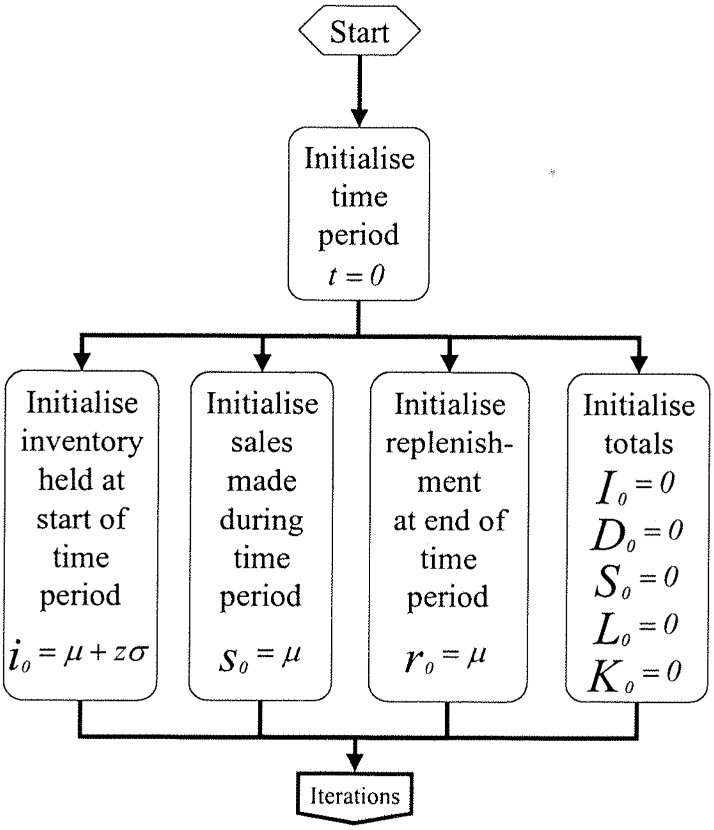


Figure 20: Initialisation

The initialisation process sets the same values in both the fuzzy and conventional systems in order to ensure that they both start from the same point. The initialisation process primarily consists of the following:

- Set the time period (t) to 0.
- Set the initial inventory (i) in both the conventional and fuzzy systems to the mean demand plus safety stock, appropriate to the conventional customer service level target (see page 69).

- Set the initial sales figure (***s***) in both the conventional and fuzzy systems to the mean demand. This figure is used as the sales figure for any historic demand prior to the simulation building its own sales history. This assumes the ideal situation that any forecast of demand prior to the start of the simulation is the best possible, except for any random variation.

- Set the initial replenishment (***r***) in both the conventional and fuzzy systems to the mean demand.

- Set the initial totals for inventory (***I***), demand (***D***), sales (***S***), lost sales (***L***) and stockouts (***K***) to 0.

Once the initialisation process is complete, then iterations are performed.

Iterations

The iteration process carries out complete cycles of demand, sales, ordering and replenishment. There are three groups of variables which can be differentiated within the model relating to the differences and similarities between the conventional and fuzzy inventory replenishment systems.

- **Same calculation, same values**: In order to ensure an exact comparison, the simulated time period and the sampled demand are always the same for both the conventional and the fuzzy inventory replenishment system. Likewise, initial variables are also set at exactly the same values (see page 75).

- **Different calculation, different values**: The purpose of the simulation model is to compare conventional and fuzzy replenishment orders, thus the replenishment orders are calculated quite differently (see pages 69 and 70) and subsequently produce different results (see Chapter 6).

- **Same calculation, different values**: To ensure consistent analysis, all other calculations are the same. However, due to the different replenishment order sizes produced by the conventional and fuzzy systems, the same calculations produce different results.

All relevant calculations are rounded to the nearest integer to reflect a discrete inventory replenishment system. This reflects the majority of product or component based inventory management systems.

Every 100 iterations the @RISK software automatically compares results (stock cover and customer service) to see if a steady state, when results are not changing significantly, appears to have been reached (see page 79). If the results have not reached a steady state, then further iterations are required. The iteration process is essentially the same for both the conventional and fuzzy inventory replenishment systems. An overview of the iteration process follows.

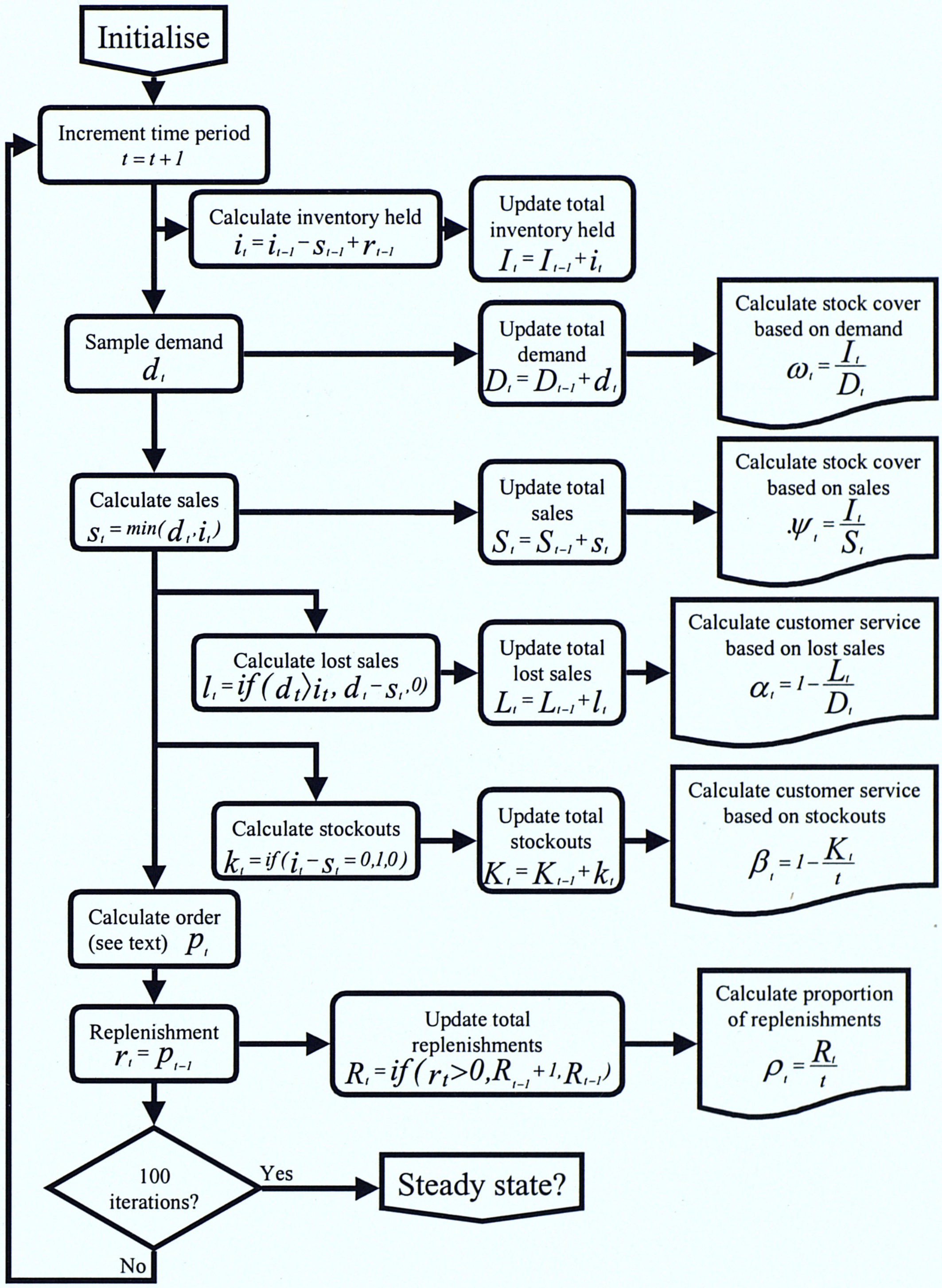


Figure 21: Iterations

- The iteration process begins with increasing the time period (***t***) by one increment.

- The inventory held (***i***) at the start of the period is then calculated by taking the inventory held at the start of the previous period, subtracting sales during the previous period and adding the replenishment delivered at the end of the previous period. The total inventory held (***I***) over the whole simulation is then updated.

- The demand (***d***) is then sampled from the relevant demand distribution (see pages 81 to 87). The total demand (***D***) over the whole simulation is then updated. From this figure and the inventory total, the number of periods stock cover based on demand (ω) is calculated.

- Actual sales (***s***) are then calculated, which are the same as the demand only if there is inventory available to satisfy the demand. The total sales (***S***) over the whole simulation is then updated. From this figure and the inventory total, the number of periods stock cover based on sales (ψ) is calculated. No back orders are accepted and thus if demand exceeds inventory held, then these are lost sales (***l***). The total lost sales (***L***) over the whole simulation is then updated. From this figure and the demand total, the customer service level based on lost sales (α) is calculated. Similarly, where demand exceeds or equals inventory available, then this is a stockout (***k***) situation. The total number of stockouts (***K***) over the whole simulation is then updated. From this figure and the number of time periods, the customer service level based on stockouts (β) is calculated.

- The replenishment order to be placed (***p***) is then calculated. This is the crucial difference between the simulated inventory replenishment systems. It is assumed that the inventory replenishment system is for components or finished goods and therefore the replenishment order placed is rounded up to the nearest integer.

- The previous order is then received (***r***), ready for the next iteration. The total number of replenishments (***R***) over the whole simulation is then updated. From this figure the percentage of replenishments (ρ) is calculated.

- Finally, the software includes a counter to determine when another 100 iterations have been carried out. The results are then compared with those from 100 iterations previously. This is a test of whether a steady state has been reached or not.

Steady state testing

The @RISK software includes a convergence monitoring facility to observe the stability of the output distributions created during a simulation. The key outputs considered are the major performance measures: stock cover, the number of replenishments, lost sales and stockouts. As more iterations are run, output distributions of the performance measures may become more 'stable' as the statistics describing each distribution change less and less with additional iterations. At this point, additional iterations will not markedly change the shape or statistics of the performance measures. The software automatically monitors 3 convergence statistics on each output distribution during a simulation. During monitoring, the software calculates a set of statistics for each output after every 100 iterations throughout the simulation. These statistics are then compared with the same statistics calculated at the prior interval. The change in the statistics due to the additional iterations is then calculated. As more iterations are run, the degree of change in the statistics becomes less and less as it converges towards zero. When the statistics have changed less than the chosen threshold percentage, the simulation ends. The statistics monitored on each output distribution are:

- the average percent change in percentile values (0% to 100% in 5% steps),
- the mean
- the standard deviation.

The length of the simulation runs are varied and are stopped when there is less than a 2.5% change in the standard deviation of the outputs under consideration (or in the case of decreasing demand, when demand has reduced to zero if this occurs earlier). This percentage change threshold is chosen because it is relatively small, implying a stable result. Beyond the 2.5% figure chosen there is no discernible difference when graphing the results. Furthermore, only the standard deviations are still subject to any degree of variation. At this point in the simulations, the means and percentile values have generally stabilised such that any changes in these statistics are well below 1%. In practice, this is generally equivalent to around 2,200 simulated periods (see simulation results in Appendix I for more details), which in turn could be interpreted as over 6, 42 or 183 years of data depending upon whether the period length represents daily, weekly or monthly replenishment respectively. In any case, this aims to provide an evaluation based on long term rather than short term results.

Accurate results for the output distributions depend on a complete sampling of input distributions. Where theoretical demand distributions are used, the sampling techniques employed aim to ensure the effect of random demand data. With the relatively large samples (around 2,200) generally employed, true Monte Carlo sampling is generally used. However, where demand is decreasing the number of simulated time periods is low and the experiments are repeated using Latin Hypercube[143] sampling. This is based on Monte Carlo sampling but without replacement and thus ensures that samples more quickly reflect the underlying distribution. However, in these experiments, there are no differences in the overall statistical results whether using Latin Hypercube or Monte Carlo sampling.

When the output distributions reflect a steady state, the simulation ends.

End

At the end of a complete simulation, the results for one business scenario are produced. these may then be compared with other business scenarios to establish whether the comparable performance of the inventory replenishment systems is robust. These other business scenarios are defined by the extraneous variables.

Controlling extraneous variables

For any individual experiment, issues relating to either supply or demand for the product are considered as extraneous variables since these are not the focus of the hypothesis under investigation (see page 64). However, it is recognised that some extraneous factors, such as demand, may have a critical impact on the overall performance of a replenishment system and so require further investigation. Therefore, individual experiments are run with different extraneous variables, representing different demand scenarios.

Demand variables

A controlled examination of the impact of demand on the performance of the replenishment systems may be carried out by defining demand theoretically in terms of the typical descriptors found in any forecasting text[144], such as mean, variance, distribution, trend, seasonality and cycle, or by using actual business demand data. This research considers three groups of demand: theoretical, actual business data and modified theoretical demand.

The initial simulations are based on theoretical random demand. However, as discussed previously (see page 31), there may be many interventions in the demand process that mean that demand is not random in practice and so actual demand data provided by Tesco is used to consider the results based on non-random 'real-life' data. This then leads to some further analysis based on random theoretical demand but with modifications based on the statistical distribution of the Tesco data.

Theoretical demand

The initial simulations employ the Poisson distribution. This assumes the following:

- *'The number of occurrences [of units demanded] in one interval of time is unaffected by (statistically independent of) the number of occurrences in any other non-overlapping time interval. ...*

- *The expected (or average) number of occurrences[of units demanded] over any time period is proportional to the size of the time interval. ...*

- *Events [demand] cannot occur exactly at the same time. ...'* [145]

These are all reasonable theoretical assumptions for modelling random demand over time.

The theoretical demand variables considered for the analyses are mean, variance and trend. The values chosen on the grounds of practicality and likely relevance to reality are listed in the following table along with their potential range of values.

Demand variable	***Potential range of values***	***Range considered***
Distribution	Infinite	Poisson
Mean	Infinite	1, 10, 100 per period (the order lead time)
Variance	Infinite	Same as mean above
Trend	Infinite	-0.1%, 0%, 0.1% of the initial mean, per period

Table 8: Demand variables

The Poisson distribution represents random demand and so is theoretically unbiased. If anything, this distribution should favour the conventional stochastic inventory replenishment systems that are based on such assumptions. The mean values used represent low (1), medium (10) and high (100) demand. The demand variance is the same as the mean, due to choice of Poisson distribution. The trends used include stable demand (0%), as well as growth (+ve) and decline (-ve). These trends represent different annual rates of change, depending on the length of the order lead time. For example, assuming weekly lead times, these factors represent annual rates of change in demand of 5.2% (= 52 x 0.1%). If other lead times are assumed then annual rates of change alter accordingly. So, these figures represent annual rates of change in demand of 1.2% (= 12 x 0.1%) assuming monthly lead times and 36.5% (= 365 x 0.1%) assuming daily lead times. Many scenarios are catered for by this choice of variables. Some broad indications of outcomes might be implied from the simulations with constant trend. Seasonality and cyclical variations are among the many extensions to these simulations that maybe considered in the future.

Actual business demand

Actual business data from Tesco is considered in separate scenarios. This demand data may include many 'hidden' factors, such as in-store promotions. The data is therefore much more variable. Tesco provide data on 6 products at 3 stages within the supply chain.

Tesco are chosen for these analyses on three criteria:

1. A range of business scenarios is required. Tesco stock a very wide range of products (>30,000) with a wide variety of demand patterns.

2. The retail end of the supply chain is likely to provide the most challenging inventory replenishment scenarios, because it is based on a large number of individual (consumer) purchases, rather than a few transactions. This means that unit sizes are at their smallest; so there is less aggregate demand and more variation. Furthermore, the most complex interventions in the demand process are likely to occur at the retail level. This may take the form of promotions and associated marketing activity, as well as stock loss due to theft etc..

3. Tesco agreed to release the relevant data.

The 6 products chosen by Tesco to reflect a range of demand levels and stability are:

- Heinz baked beans in tomato sauce (415g)
- Persil biological washing powder (E3)
- Hellmann's real mayonnaise (400g)
- Tesco Value low sugar lemonade (2l)
- Tesco 60W pearl light bulbs (twin pack)
- Red Stripe strong lager (4x400ml)

The three stages within the supply chain are:

- Consumer purchases at a store
- Store orders on the retailer warehouse
- Warehouse orders on the supplier

While these links within the supply chain are obviously related, they have very different demand distributions. This is mainly a reflection of different batch sizes, the effects of aggregation and supply lead times.

These 18 (= 6 products x 3 stages) business scenarios are very different to the theoretical analysis based on random demand. For example, the weekly demand for Heinz baked beans at one Tesco store over a year is depicted in Figure 22.

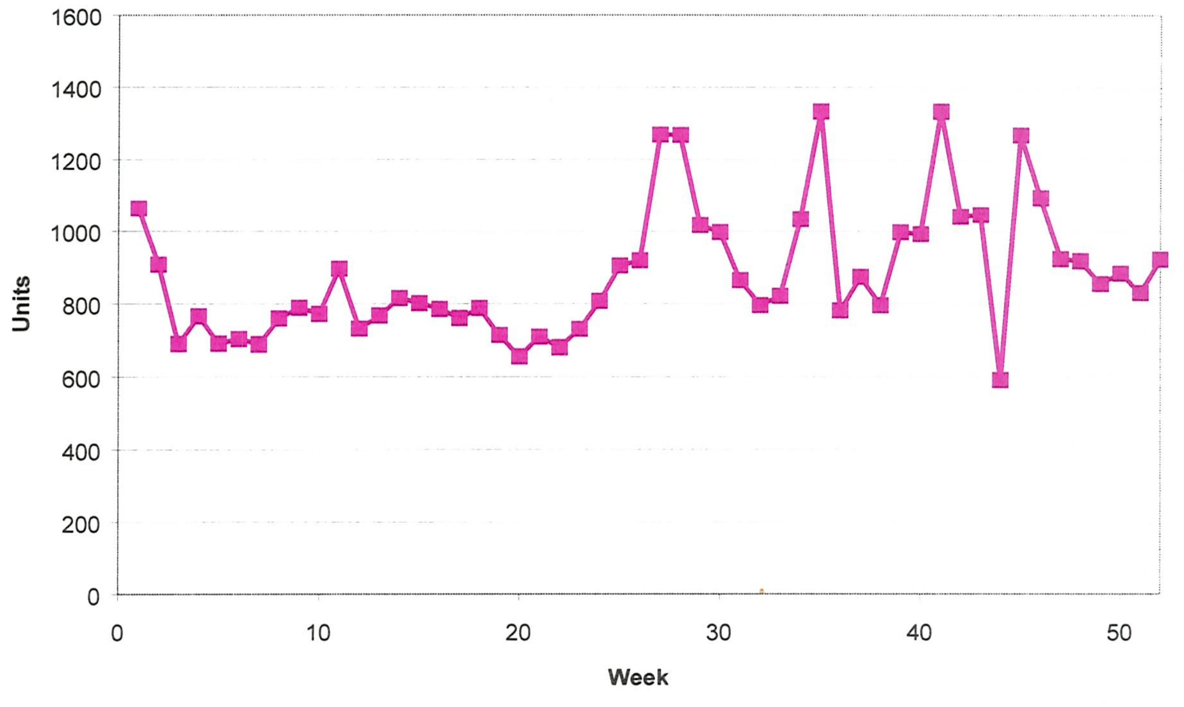


Figure 22: Weekly demand for Heinz baked beans at a Tesco store

The major characteristics of this 'real-life' demand distribution are very different from a random scenario with the same mean - although it is recognised that the 'real-life' distribution is based on a relatively small sample of 52 weeks of data. These differences are emphasised in Table 9.

Measure	***Tesco data: baked beans (practice)***	***Random data: with same mean (theory)***	***Ratio of practice against theory (%)***
Maximum	590	792	**74%**
Minimum	1334	972	**137%**
Mean	882	882	**100%**
Standard deviation	180	30	**600%**
Variance	32304	882	**3600%**
Skew	1.02	0.00	-
Kurtosis	0.58	3.00	**19%**

Table 9: Example characteristics of demand: theory versus practice

As can be seen in this example, the Tesco data provides some very different scenarios to the analysis undertaken using the theoretical random scenario. The major differences are in the spread and skew of the data. The wider spread of the Tesco data is reflected in the much higher variance and much lower kurtosis values. This may be the result of consumer influences. For example, sales of baked beans may be heavily influenced by external factors that may change throughout the year or even from week to week, such as the weather or when a consumer receives their pay packet or salary. These influences may produce much wider variations that might be expected with truly random data. The positive skew of the data is likely to be the result of marketing initiatives (by the retailer and/or manufacturer). These marketing initiatives may boost sales for a short period above their 'natural' level, thus providing positive skew. This may be seen occurring in weeks 27, 28, 35, 41 and 45 in Figure 22, above. In reality, where these are planned promotions, it may be possible to separate out the effects of such interventions and thus calculate inventory replenishments based on 'normal' and 'promotional' demand (see Cuthbertson and Moore[131] for further details based on a Sainsbury's case study).

The analysis of actual Tesco data is discussed in the next chapter.

Highly variable demand
The data sets kindly provided by Tesco are small and therefore to underpin any findings further theoretical analysis is undertaken. This analysis is based on the variances of the Tesco data and defined using the @RISK functions that define the relevant probability distributions. These are discussed in the next chapter and the detailed results may be found in the appendices.

Forecasting demand

In the analyses undertaken employing a theoretical demand (unmodified and modified) the conventional system is provided with the exact parameters underlying the theoretical model of demand. Thus, the conventional inventory replenishment system employs both the mean and variance of the theoretical demand, allowing for random variations.

In the practical analyses of 'real' data from Tesco, the complete data set is analysed prior to simulation such that the forecast error is minimised through the use of regression. Thus the conventional inventory replenishment system is again provided with the best possible forecast, allowing for random variation.

Therefore, in both theory and practice, the simulated conventional replenishment system is provided with the best possible forecast, allowing for random variation.

Note that the proposed fuzzy system does not require a forecast as such but relies on the extremities of historical demand (minimum and maximum). The length of the historical demand considered may have an impact upon the results, depending on the demand data. For the purposes of this research, 20 periods of historical data are chosen as a reasonable sample of past demand (see Appendix G for more details).

Thus, the simulated fuzzy replenishment system is left to adapt as necessary. Therefore, the simulation results should, if anything, be biased in favour of the conventional rather than the proposed fuzzy system. This is designed to demonstrate that any results in favour of the fuzzy system would appear to be significant.

Supply variables
A controlled examination of the impact of supply on the performance of the replenishment system is another extension to the range of simulations possible and may be carried out by defining the supply process in a similar way to the demand process. However, the major

supply issue concerns the replenishment lead-time, statistically described by its mean and variance. The time period chosen could represent any scenario, such as a daily, weekly or monthly delivery schedule. In practice there may be some variation, though in business situations the variation is limited as much as possible (sometimes successfully and sometimes not). For practical reasons, it is assumed to be managed successfully, and is therefore fixed for the purposes of this research.

The supply variable used in these simulations is summarised in Table 10.

Variable	*Potential range of values*	*Range considered*
Replenishment lead time	Infinite	1 time period with no variation

Table 10: Supply variable

Other extraneous variables

This research method is designed to exclude other extraneous variables, such as quality of product, so that the impacts of any replenishment decisions made are as clear as possible. However, it is recognised that in reality other extraneous variables may play an important role in the relative performance of different inventory replenishment systems.

Summary of the research model

The research model may be summarised in terms of the research process and the key variables considered.

Summary of the primary research process

The overall primary research process is summarised in the following stages:

1. ***Performance measures*** – identify the outputs to be compared.
2. ***Replenishment model*** – define the replenishment systems which produce the outputs.
3. ***Simulation model*** – run the simulation model to compare the impact of the replenishment decisions chosen on the performance measures.

4. ***Metamodel*** – re-run the simulation for various combinations of extraneous variables to provide the overall metamodel results.

Summary of the key variables

The key variables considered within this primary research are summarised in the following table.

Type	*Variables*
Dependent	*Cost measures* – Stock cover (%) – Replenishments (%) *Customer service measures* – Lost sales (%) – Stockouts (%)
Independent	*Conventional inventory decision* – Approach employed (stochastic) – Type of system (order up-to) – Customer service target (50% to 99%) *Fuzzy inventory decision* – Variables or 'nouns' employed (overstock, understock, small order, large order) – Fuzzy sets, i.e. fuzzy definition of the 'nouns' employed – Fuzzy rules, i.e. the relationship between the fuzzy sets
Extraneous	*Demand related variables* – Pattern of demand (distribution, mean, variance, trend or actual data) *Supply related variables* – Replenishment lead time (fixed)

Table 11 : Key variables used in the primary research

The results of all of the various business scenarios considered are discussed in Chapter 6.

Chapter 6

RESULTS
Comparing fuzzy and conventional replenishment systems

The results of the simulations described in the Chapter 5 may be analysed in a number of ways. However, given the definitions of the performance criteria required of the inventory replenishment systems, the primary analysis is clear: i.e. which system provides better customer service (primarily measured based on lost sales) at lower cost (primarily measured as stock cover). More importantly, the analysis aims to explain why one system should perform better than another system, and how this may lead to further improvements in inventory replenishment. Spreadsheet data, analysis and results can be found on the CD attached. The relevant filenames for all graphs and tables found in this chapter are referenced in Appendix I.

Random demand around a static mean

A picture of the relative performance of the fuzzy and conventional inventory replenishment systems when simulating random demand around a static mean (using spreadsheet model *FIRS model v8*) can be built up by plotting the simulated stock cover against the relevant customer service levels achieved.

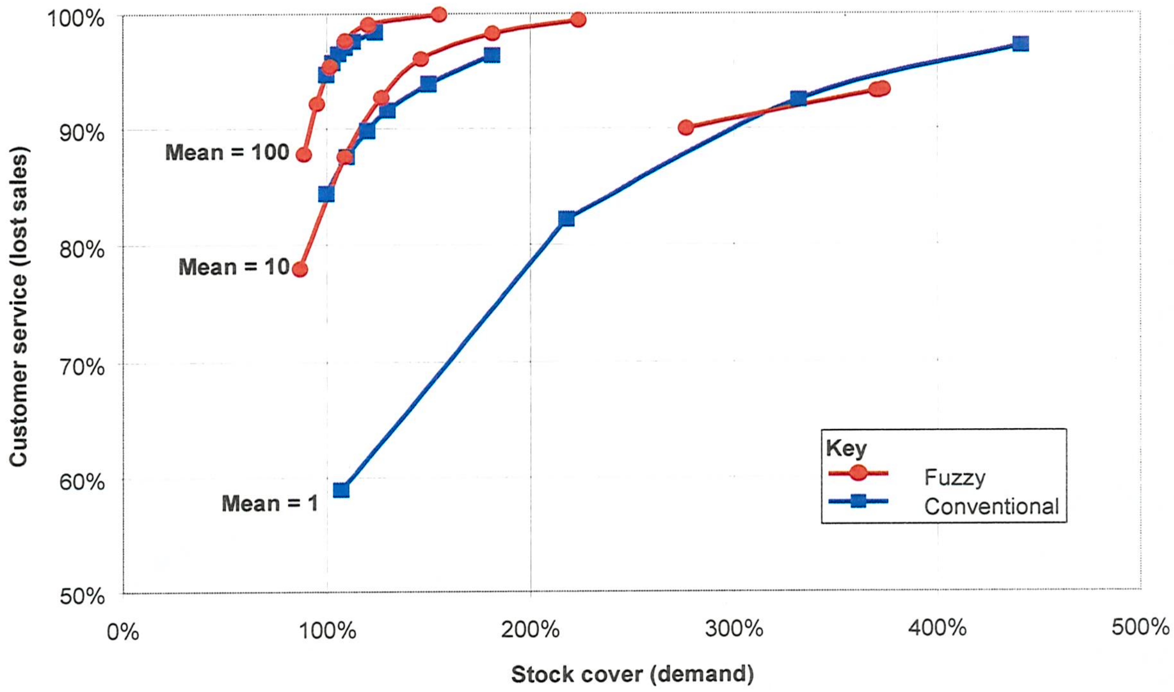


Figure 23: Conventional versus fuzzy - static demand

The results show a family of curves as the mean demand varies, from 1 to 10 to 100. The conventional system appears to produce a consistent set of curves. The fuzzy system produces a set of curves that appear consistent for higher demand (mean = 100 and mean = 10), but not for the low demand (mean = 1).

Note that the customer service target is different to the actual customer service level achieved. The customer service target in the conventional system is based on the number of times the system may run out of stock during the order lead time, while the actual customer service level achieved is based on the number of lost sales at any time. However, the stock cover does reflect the customer service target in the conventional system. For example, a 50% customer service target implies no safety stock and hence roughly equates to 100% stock cover in all cases. As the customer service target increases, this has a proportionally greater effect on stock cover where demand is low, due to the assumed underlying Normal distribution.

All of these results will now be considered in turn and analysed in order to explain the differences in performance.

High, static demand

For the relatively high demand items (mean demand = 100 and mean demand = 10), the results show that the fuzzy inventory replenishment system performs better than the conventional system, except at low levels of conventional customer service where performance is similar. These results are illustrated in Figure 24 and Figure 25.

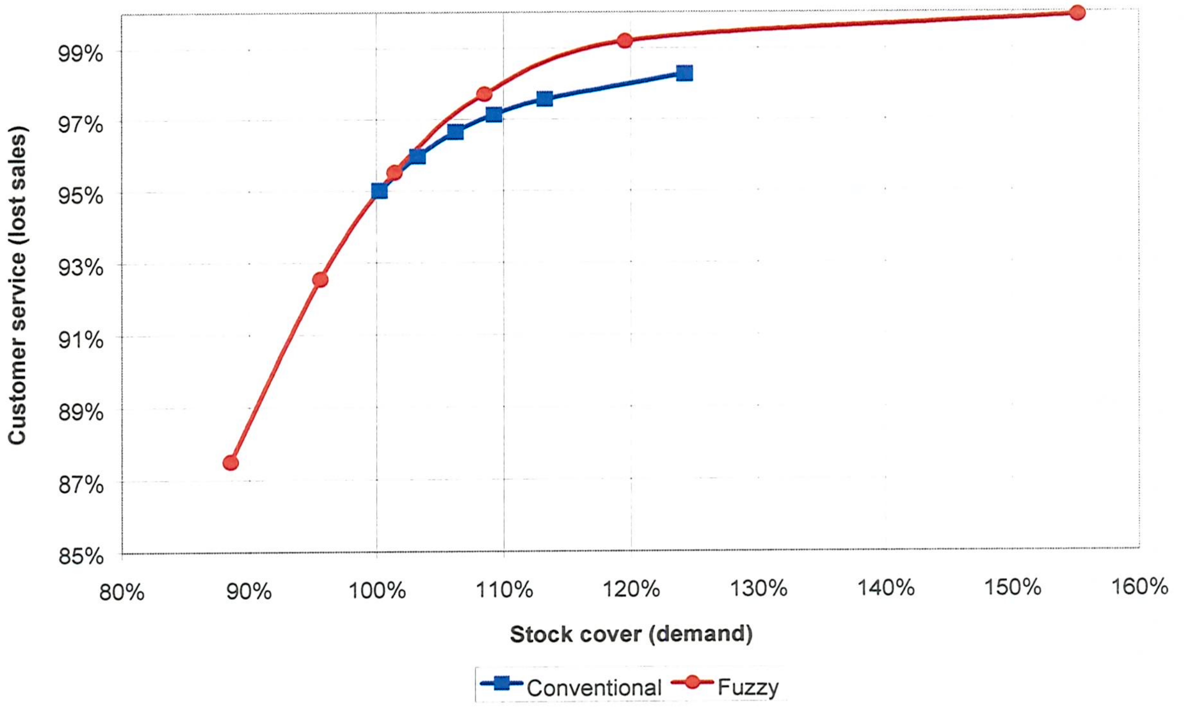


Figure 24: Conventional versus fuzzy - static demand, mean =100

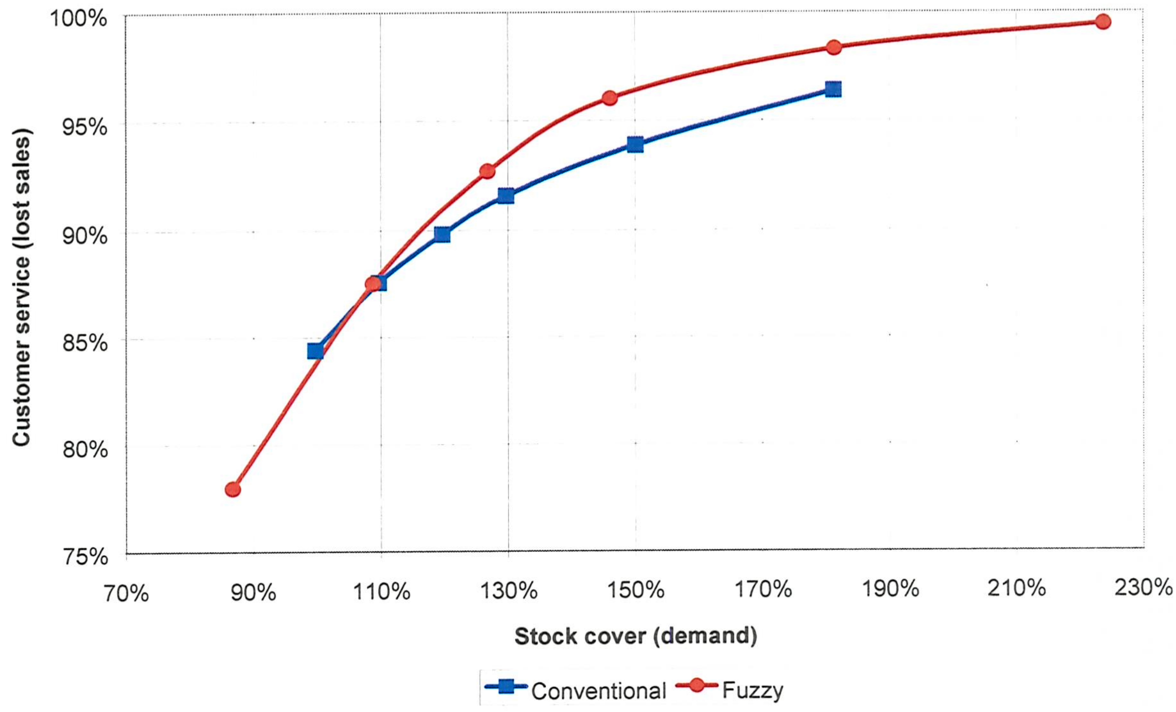


Figure 25: Conventional versus fuzzy - static demand, mean = 10

In order to focus in on the area of overlap between the two systems for high demand products, further simulations are carried out for the highest demand (mean =100). These simulations retain the conventional customer service targets (50% to 99%) but the range of fuzzy customer service strategies (see page 61 for definition) are narrowed to between 0.35 and 0.85, in steps of 0.1. This produces the following results.

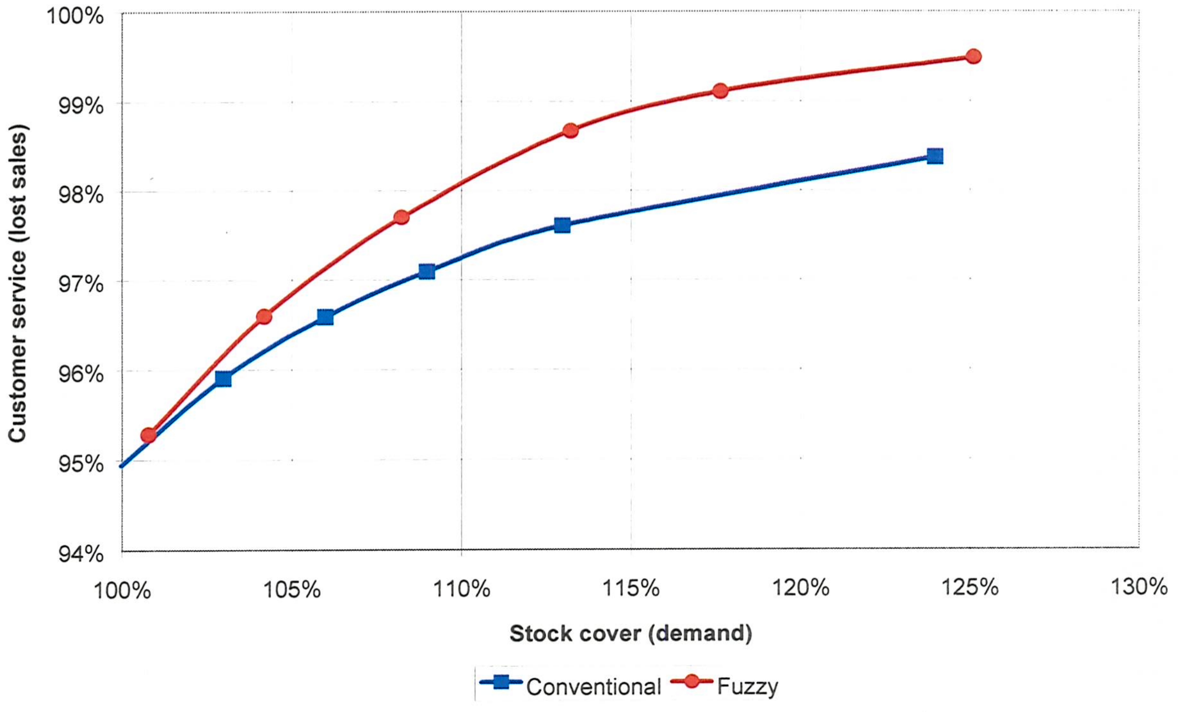


Figure 26: Conventional versus fuzzy - static demand, mean = 100, overlap

It is clear from this graph that the fuzzy system is superior at all levels of customer service, with the possible exception of the lowest overlapping point, where the results appear to be very similar.

Further simulations with a narrower range of fuzzy customer service strategies (0.1 to 0.8 in steps of 0.14) for demand =10 result in a similar graph.

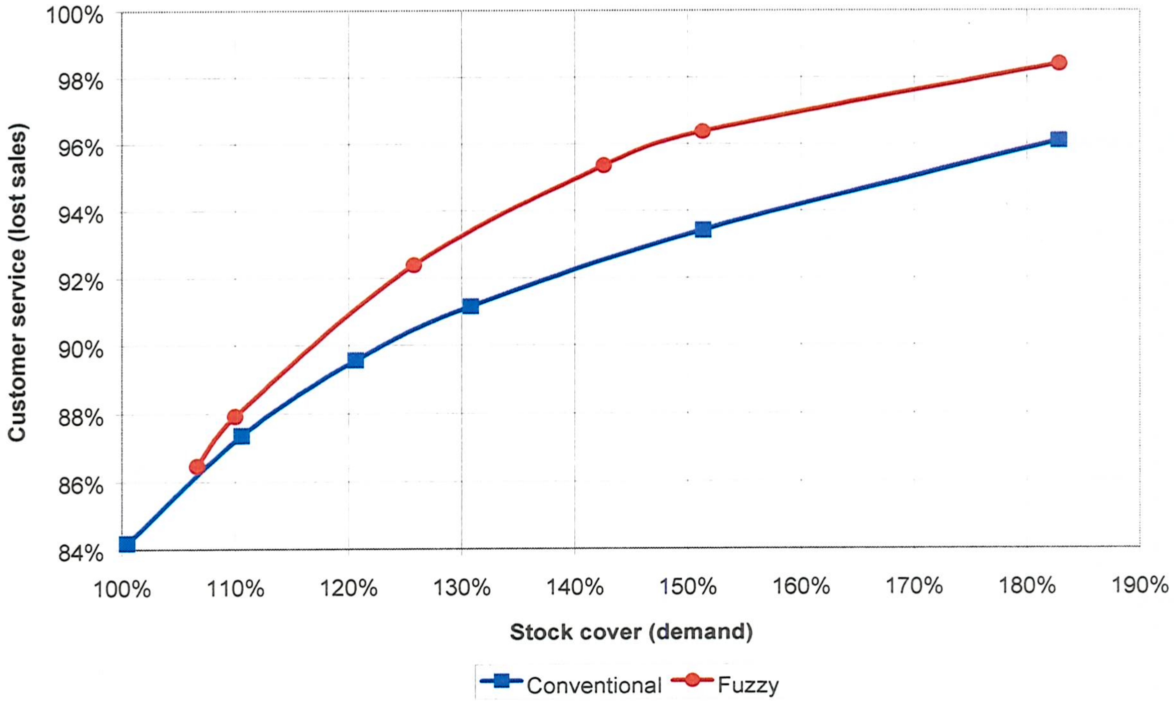


Figure 27: Conventional versus fuzzy – static demand, mean = 10, detail

Although there are no directly comparable pairs of data, the obvious differences are statistically significant due to the small variances (usually well below 1%) resulting from the long simulation runs.

The closest example of two comparable data points is found when comparing the final two data points in Figure 27, above (demand = 10). Here, both the fuzzy and conventional stock cover values round to 183%. Despite this lack of difference in stock cover, the difference between the customer service levels provided, 96.3% using conventional replenishment and 98.4% using fuzzy replenishment, provides a Z-statistic of 222.92, when 3.00 would be considered significant at the 99.9% confidence level. Thus this is clearly a significant difference between the two systems! This even more impressive given that the conventional system has been provided with the best possible demand forecast (see page 70).

Low, static demand

For the low demand items (mean demand = 1), the results are mixed. These results are illustrated in Figure 28.

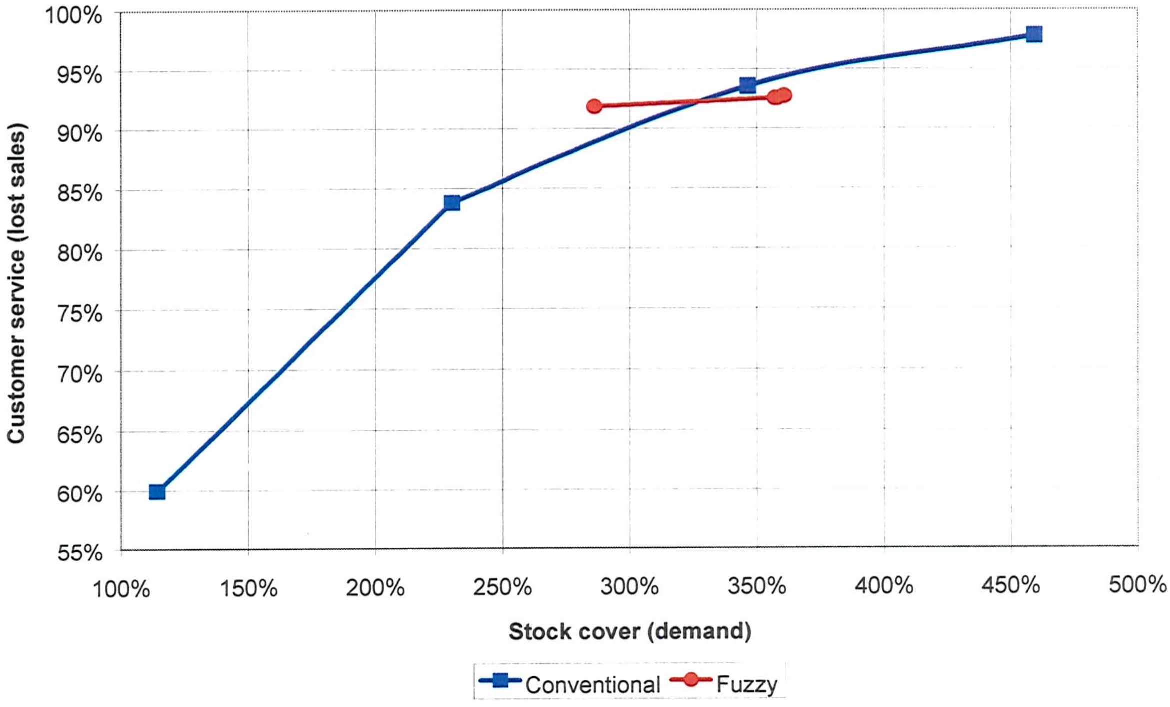


Figure 28: Conventional versus fuzzy - static demand, mean =1

In order to focus in on the area of overlap between the two systems for low demand products, further simulations are carried out. These simulations retain the fuzzy customer service targets (0 to 1) but the range of conventional customer service strategies considered are narrowed to between 80% and 99%, in steps of 4%. This produces the following results.

Conventional customer service target	*Fuzzy customer service strategy*	*Conventional stock cover (demand)*	*Fuzzy stock cover (demand)*	*Conventional customer service (lost sales)*	*Fuzzy customer service (lost sales)*
99.0%	1.0	469.8%	381.2%	98.1%	94.3%
96.0%	0.8	356.1%	380.5%	94.4%	94.2%
92.0%	0.6	356.1%	380.0%	94.4%	94.2%
88.0%	0.4	356.1%	379.9%	94.4%	94.2%
84.0%	0.2	235.2%	379.0%	83.9%	94.2%
80.0%	0.0	235.2%	297.2%	83.9%	92.2%

Table 12: Conventional versus fuzzy - static demand, mean = 1, overlap

It is clear from this table that both systems struggle to cover a wide range of stock levels due to the lack of choice, effectively 2 to 5 units. Results from the conventional system shows 3 differing levels of stock cover (235%, 356% and 470%), while the fuzzy system shows even less variability with just two major points (297% and around 380%). This can be depicted as follows.

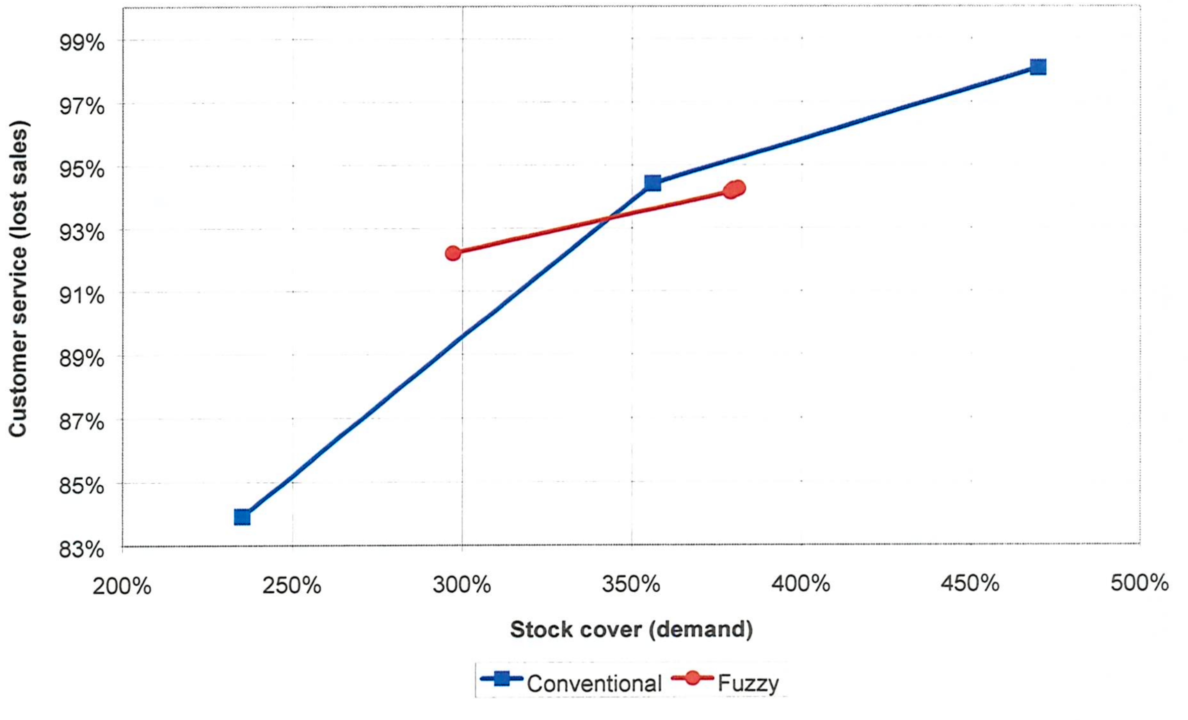


Figure 29: Conventional versus fuzzy - static demand, mean = 1, overlap

These results are mixed, with neither system showing overall superiority. Given the long simulation runs these results are statistically significant. However, there is a lack of directly comparable pairs of data. The closest pair of points lie between 350% and 400% stock cover. Here, the fuzzy system requires more stock and provides a lower customer service than the conventional system. Hence, within this region, it seems reasonable to conclude that the conventional system may provide better performance. Any other conclusions about the relative performance of the two systems are debatable.

Summary, static demand

The relative performance of the inventory replenishment systems, depending upon the precise definitions of the demand level and customer service requirements, may be broadly summarised in the following table.

Customer service requirements				
	High	**Unknown**	**Fuzzy**	**Fuzzy**
	Medium	**Conventional**	**Fuzzy**	**Fuzzy**
	Low	**Unknown**	**Neither**	**Neither**
		Low	*Medium*	*High*
			Demand	

Table 13: Better performing inventory replenishment system

Further analysis is required to understand the nature of these differences.

Analysis of differences in performance

The only difference between the two systems lies in the orders generated and the resulting stock levels. Therefore, an analysis of the orders generated and their resulting stock levels should give some insight into the reasons for significant differences in performance.

Ideal orders and stock levels

Orders and stock levels have two dimensions: size and timing. By definition, the ideal order pattern would result in stock levels that exactly satisfy the requested demand both in terms of size and timing, taking account of any required lead time. However, the exact timing of orders and the resulting stock levels may only be known if the actual demand is known in advance of the lead time. If this is the case, then there is no need to hold any inventory - a true Just-In-Time system can be employed. In most business situations (as in these simulations) the actual demand is unknown and some inventory must be held. Therefore, the distribution of orders and resulting stock levels, rather than the timing of orders and stock levels, must be considered here. It is worth noting that to some extent the resulting stock levels are a reflection of the timing of orders.

The major contrasts identified in inventory performance relate to the extremes of demand or customer service requirements. Thus, in the first instance this analysis will focus on the following:

high demand, high customer service requirements – This is where there is the most extreme difference between the two systems. The fuzzy system very significantly outperforms the conventional system under this scenario.

low demand, medium customer service requirements – This is the only comparable example found of the conventional system outperforming the fuzzy system. Hence, this should provide a contrasting viewpoint.

high demand, low customer service requirements – This provides a third and final example to balance the two views. Under these conditions there appears to be little difference in performance between the two systems.

Each scenario is considered in turn. By overlaying the orders and resulting stock distributions on top of the demand distribution, the major characteristics of the inventory replenishment systems may become clear and their differing performance levels understood. A final synthesis is then provided.

High demand, high customer service

First, the demand that the orders are trying to satisfy is considered. At high levels (mean = 100), random demand is Normally distributed, as illustrated in Figure 30.

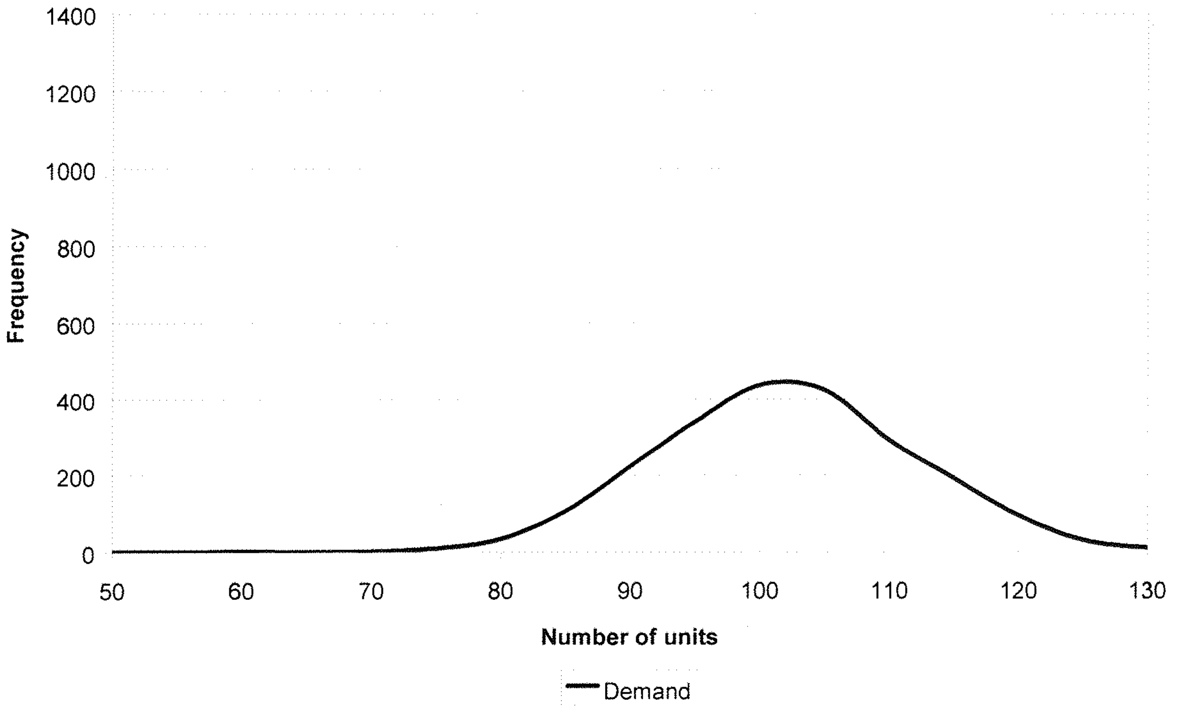


Figure 30: Demand - static demand, mean = 100

The order and resulting stock level distributions for the conventional system with a high customer service target (99%) appears as follows.

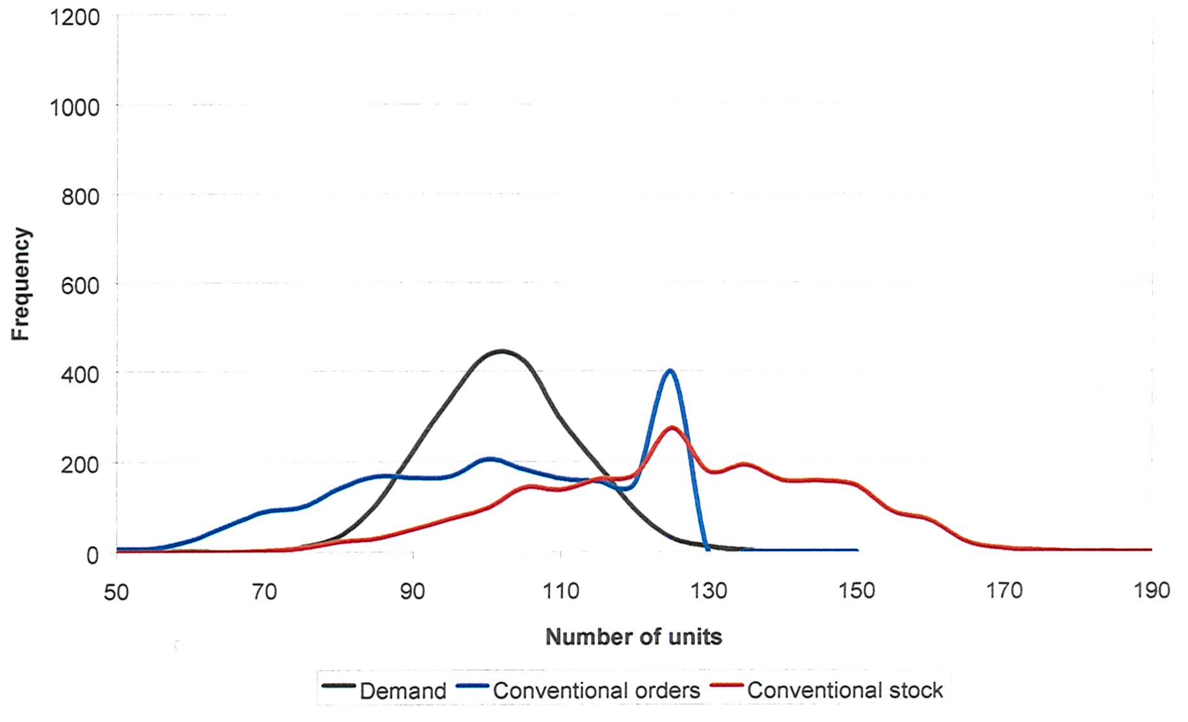


Figure 31: Conventional orders & resulting stock – high demand, high customer service

(static demand, μ = 100, customer service target = 99%)

This clearly illustrates the major characteristic of an order up-to inventory replenishment system, namely that orders do not exceed the 'up-to' level (in this case, usually around 125 units). Assuming that the order up-to point is not excessively high in relation to demand, this results in the most frequently placed orders being made around the order up-to point. This order distribution clearly shows the strategy underpinning such a system when trying to achieve high customer service. The strategy aims to shift the peak of orders to the right of the demand curve. The resulting stock is well spread out (variance = 394) with a peak coinciding with the peak of the order distribution.

By contrast the comparable fuzzy order distribution is a much smoother distribution in general – see Figure 32. This illustrates the expected characteristic of this fuzzy inventory replenishment system, namely that there is a gradual (fuzzy) transition between overstock and understock and so orders are 'smoothed' and tend to a central point (partly overstocked and partly understocked). This is very different to orders generated by the conventional order up-to system. Furthermore, the orders peak around the mean demand (99.51 compared to the demand mean of 100) and are heavily positively skewed (skew[146] = +1.39, which is significant at a 92% confidence level).

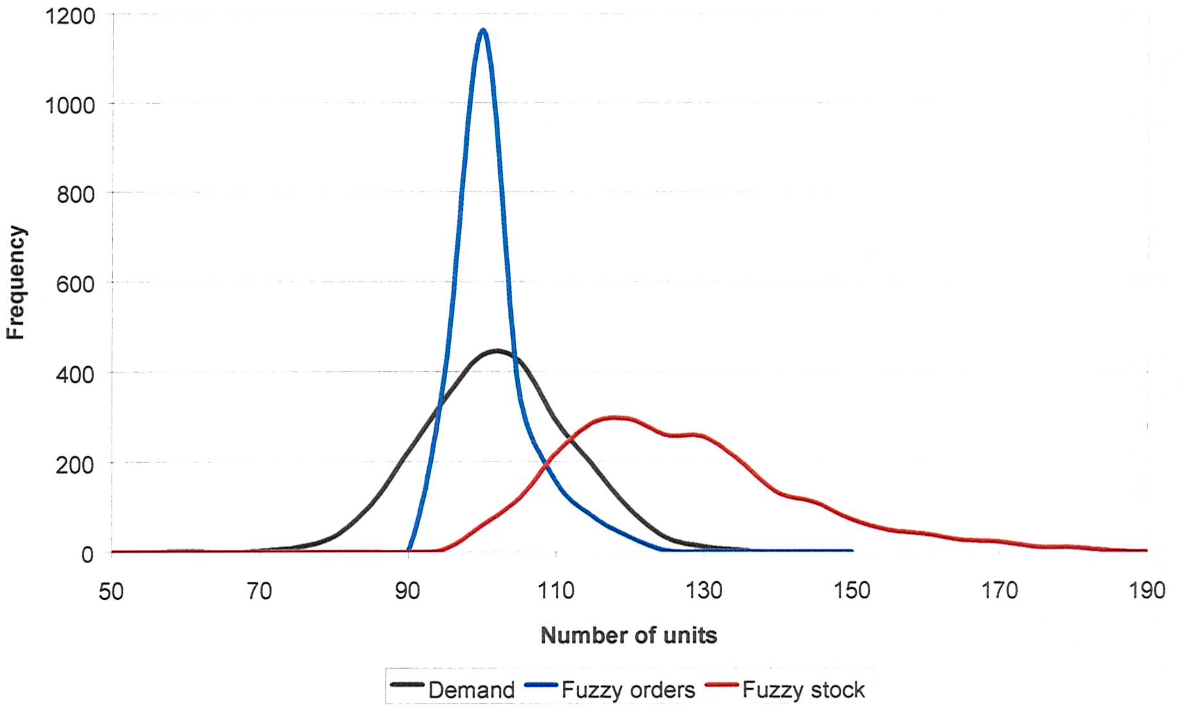


Figure 32: Fuzzy orders & resulting stock – high demand, high customer service

(static demand, μ = 100, customer service strategy = 0.85)

The resulting distribution of stock, for the fuzzy system, does bear some similarity to the conventional one, though again it is much smoother. However, a more detailed inspection of the two distributions of stock (see Figure 33) reveals some marked differences, especially bearing in mind that the two systems mean stock holding is very similar (124 units in the conventional system and 125 in the fuzzy system).

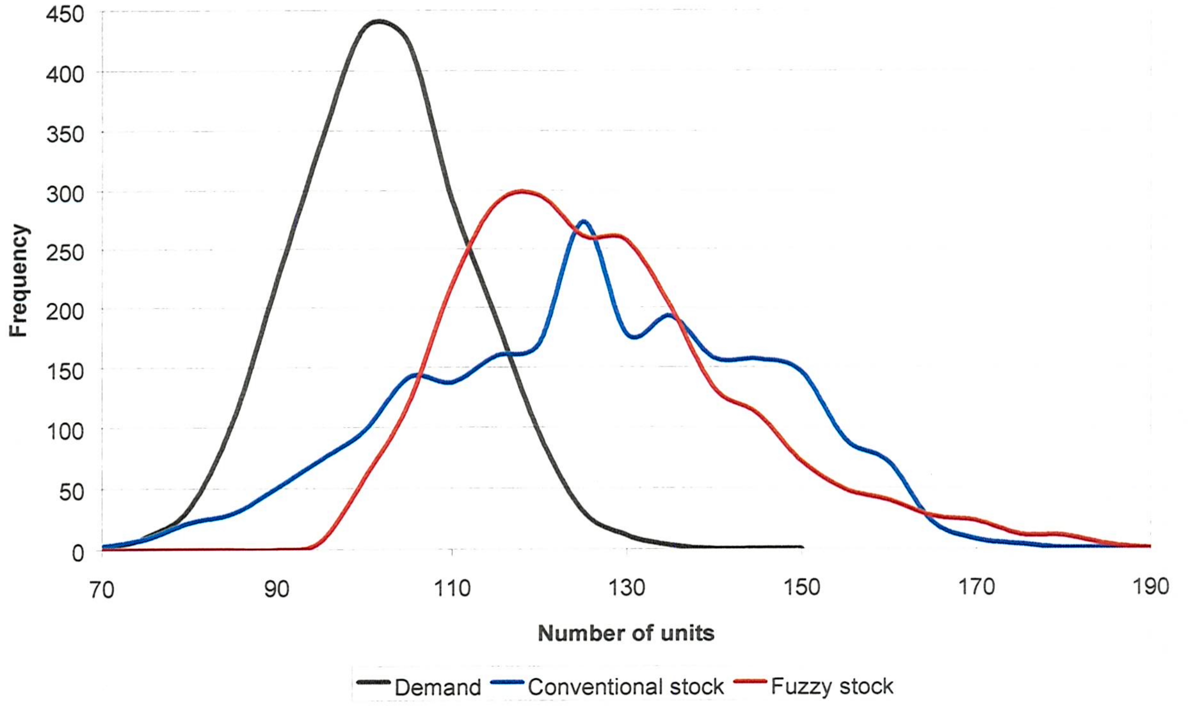


Figure 33: Conventional & fuzzy stock – high demand, high customer service

(static demand, μ = 100, cst =99%, css = 0.85)

The following table summarises the key parameters of the stock distributions.

Measure	***Conventional system stock distribution cst=99%***	***Fuzzy system stock distribution css=0.85***
Maximum	181	219
Minimum	68	94
Mean	124	125
Standard deviation	19.9	16.8
Variance	394	282
Skew	-0.16	1.03
Kurtosis	2.55	4.71

Table 14: Conventional v. fuzzy stock – high demand, high customer service

(static demand, μ = 100, cst =99%, css = 0.85)

The major difference between the two stock distributions is the positive skew of the fuzzy one (fuzzy skew = +1.03, which is significant at an 85% confidence level) compared to the slightly negative skew of the conventional one (conventional skew = -0.16). So, although the fuzzy stock distribution has a similar mean to the conventional one, it is skewed to the higher stock values. Thus, the fuzzy stock distribution for a similar level of stock cover (125% versus 124%), possesses a higher maximum (219 versus 181) and a higher minimum (94 versus 68) number of units, and so provides a significantly higher level of customer service (99.5% versus 98.4%, based on lost sales).

This explanation compares and contrasts with the analysis of the low demand distribution.

Low demand, medium customer service

First, the demand that the orders are trying to satisfy is considered. At low levels (mean = 1), random demand forms a Poisson distribution, as illustrated in Figure 34.

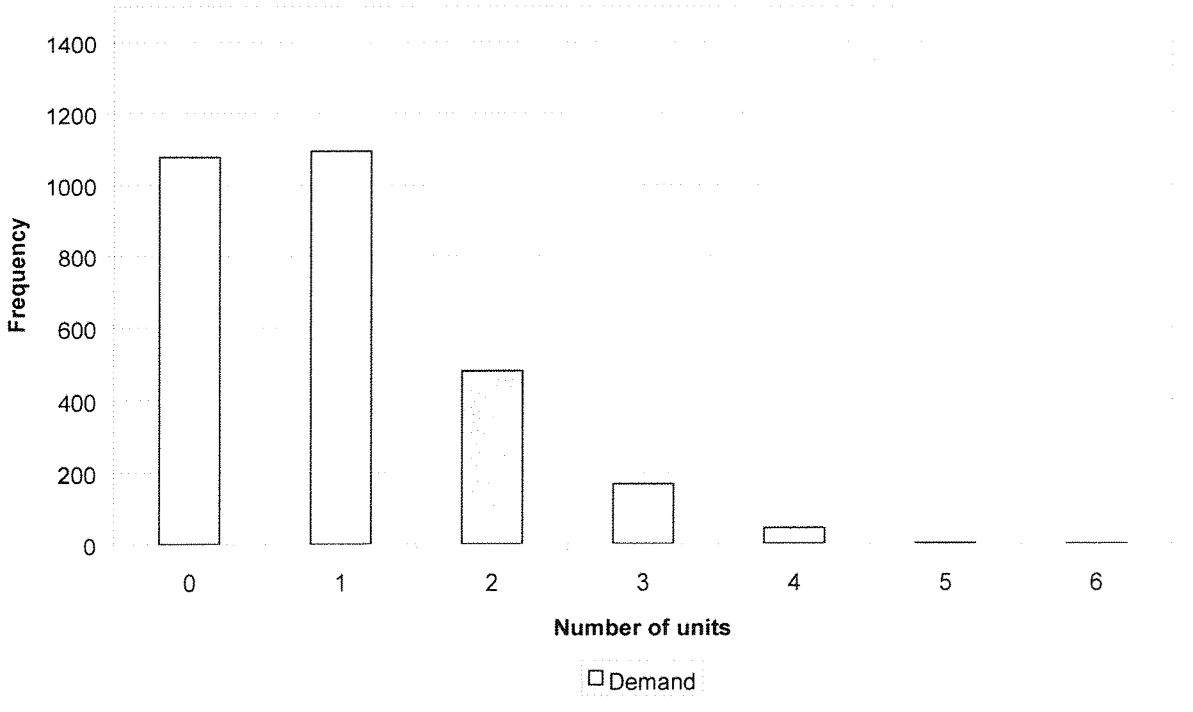


Figure 34: Demand - static demand, mean = 1

An overlay of the comparable order and stock distributions at a medium customer service level (customer service target 92%) of the conventional system is shown in Figure 35.

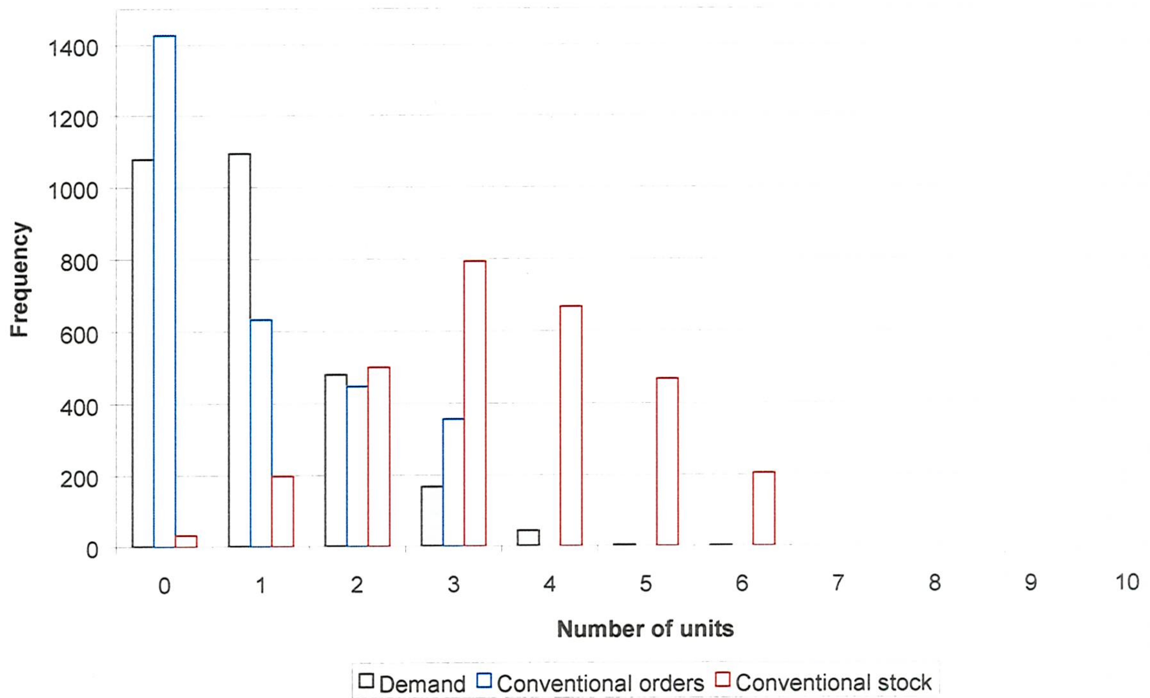


Figure 35: Conventional orders & stock – low demand, medium customer service

(static demand, μ = 1, customer service target = 92%)

The major characteristics of the conventional order up-to inventory replenishment system are not as clear as in the previous high demand example. The order up-to point appears to be around 3 units and certainly no orders exceed this. Note that the most frequently placed orders are no longer near the order up-to point. In fact, most orders are non-orders (i.e. order zero quantity).

The stock distribution appears symmetrical with a mean of around 3.4 units. Given the distribution of demand (mean of 0.96), this concurs with a relatively high customer service level (94.4%) provided by very high stock cover (356%).

This compares and contrasts with the results from the fuzzy inventory management system.

As with the conventional system, the characteristics of the order distribution generated by the fuzzy system are not as clear as in the high demand example. However, the order distribution is still 'smoother' than the comparable conventional one, as it tails away rather than suddenly stops. Also, as with the previous high demand example, the fuzzy orders peak around the mean demand (0.91 compared to the demand mean of 0.96) and are positively skewed (skew = +0.98, which is significant at an 84% confidence level).

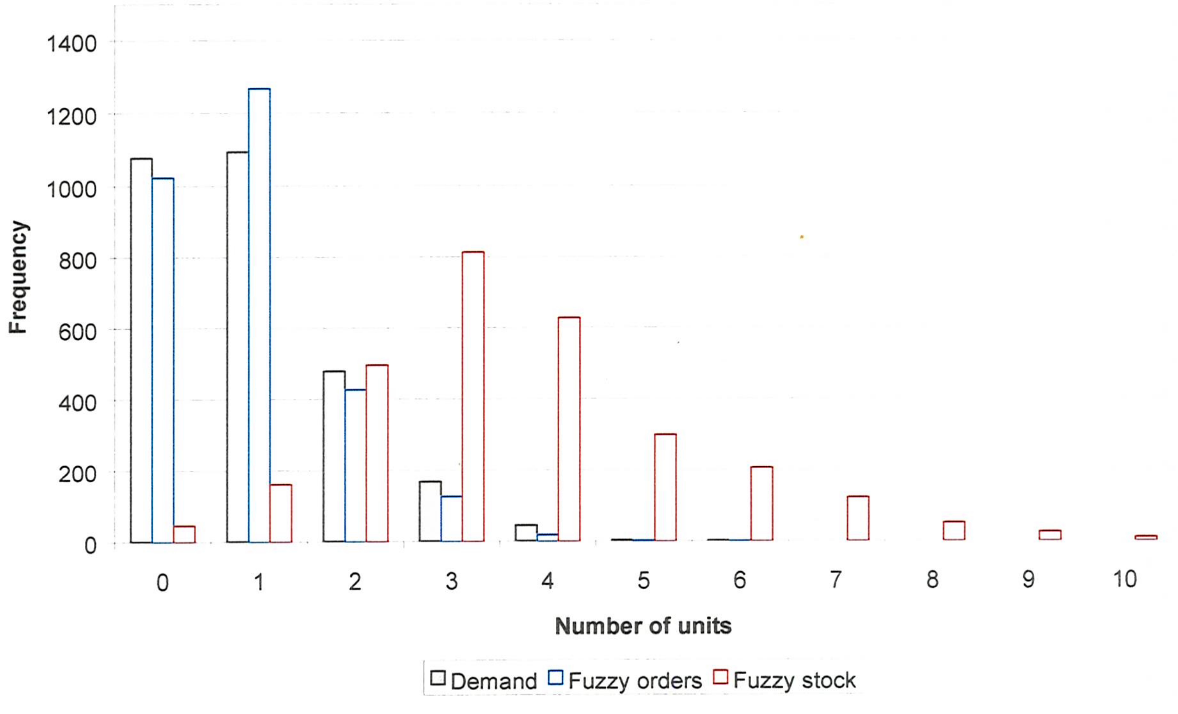


Figure 36: Fuzzy orders & stock – low demand, medium customer service

(static demand, μ = 1, customer service strategy = 0.85)

The resulting distribution of stock, for the fuzzy system, again bears some similarity to the conventional one, though it is much more variable than the conventional distribution of stock (a variance of 3.08 compared to 1.91 for the conventional system).

A closer analysis (see Figure 37) reveals further marked differences, especially bearing in mind that, again, the systems mean stock holding is similar (3.43 units in the conventional system and 3.65 in the fuzzy system).

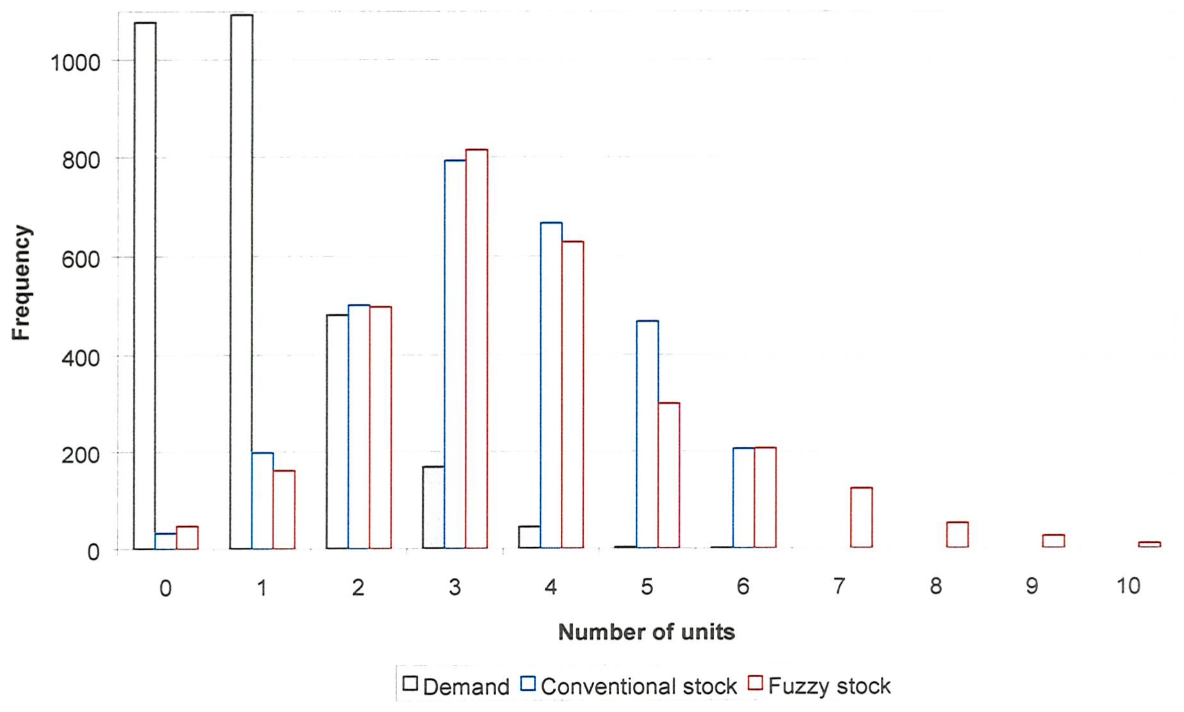


Figure 37: Conventional & fuzzy stock – low demand, medium customer service

(static demand, μ = 1, cst =92%, css = 0.60)

Again, the major difference between the two stock distributions is the positive skew of the fuzzy one (fuzzy skew = +0.75, which is significant at a 77% confidence level) compared to the slightly negative skew of the conventional one (conventional skew = -0.03). So, although the fuzzy stock distribution has a similar mean to the conventional one, it is again skewed to the higher stock values. Thus, the fuzzy stock distribution for a similar level of stock cover (125% versus 124%), again possesses a much higher maximum (11 versus 6). In contrast to the high demand example discussed previously (see Table 14), the minimum is the same in both cases (zero). This is due to the constraint caused by being so close to zero. Given the previous high demand example, this appears to particularly constrain the conventional system.

Unlike in the high demand example, the higher maximum of the fuzzy system is not an advantage in this low demand scenario. In the previously discussed high demand scenario (pages 99 to 105), the positively skewed distribution and associated higher maximum helps provide a significantly higher level of customer service. In a very low demand scenario, rather than adding to customer service, this extra stock is surplus to requirements. There is no imminent demand for it. In the high demand example, the maximum fuzzy stock is

119% higher than the mean demand, implying that it could quickly be sold to avoid overstocking. In the low demand example discussed here, the maximum fuzzy stock is 1,046% higher than the mean demand, implying an unlikely quick sale, and subsequent overstocking cannot be avoided.

It is worth noting that if the maximum demand figures were used rather than the mean figures, then the figures become almost comparable. In the high demand scenario, the maximum fuzzy stock is 65% higher than the maximum demand, whereas in the low demand scenario, the comparable figure is 83%. This reflects the relative importance of the maximum sales figure rather than the mean sales figure in the fuzzy system calculations. This may be an important consideration in situations where the maximum demand is effectively an outlier and not representative of the distribution as a whole, as in this scenario of low demand.

This analysis appears to explain the major reasons for the different performance of the two inventory replenishment systems. However, before synthesising this discussion, it is worth providing validation from the third and final viewpoint – where the relative performance of the different systems is similar.

High demand, low customer service

Figure 38 shows the orders placed and resulting stock levels of the conventional inventory replenishment system where the customer service target is low, (50%).

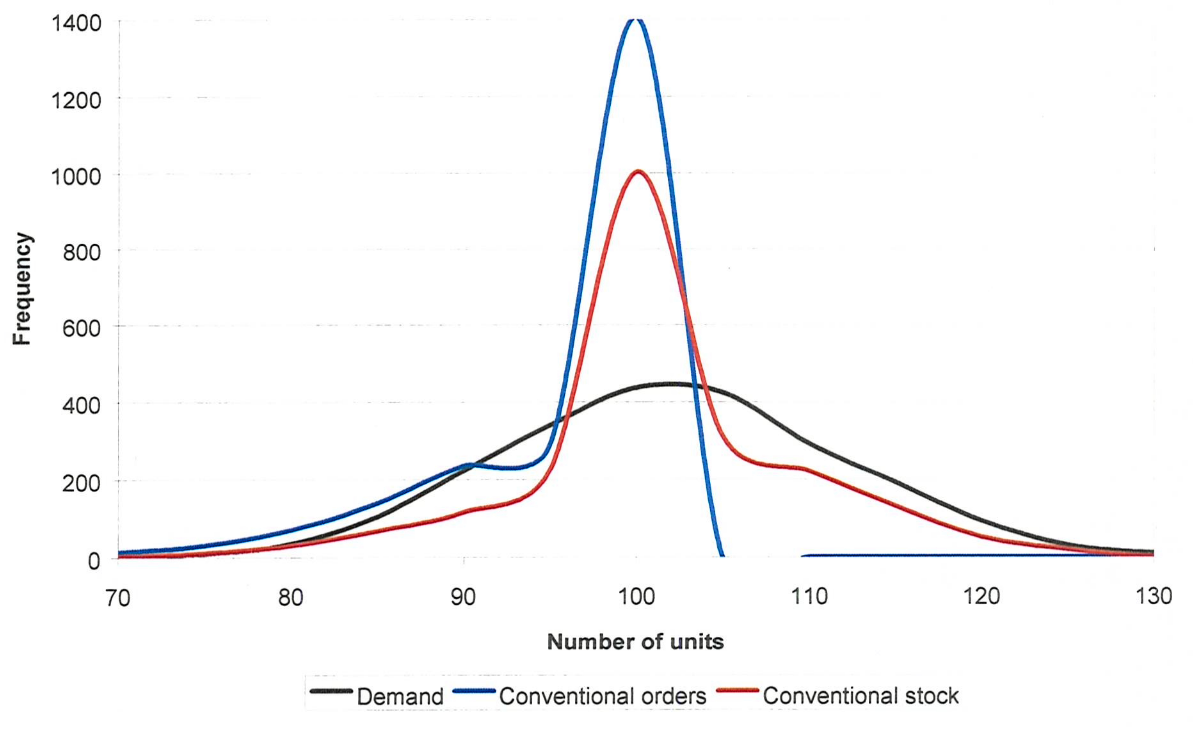


Figure 38: Conventional orders & resulting stock – high demand, low customer service

(static demand, μ = 100, customer service target = 50%)

This again illustrates the major characteristic of this order up-to inventory replenishment system, namely that orders do not exceed the 'up-to' level (in this case, around 100). Again, this results in the most frequently placed orders being around the order up-to point and this coincides with the peak of the resulting stock distribution. The resulting distribution of stock is similar to the distribution of demand, with slightly wider spread in terms of maximums (143 versus 133 for demand) and minimums (46 versus 57 for demand), but much narrower in terms of variance (68 versus 100 for demand). This stock distribution provides a 94.9% customer service level based on lost sales.

In contrast, the fuzzy inventory replenishment system, again, provides a quite different distribution of orders, though there are some comparisons to be made in terms of the stock distribution.

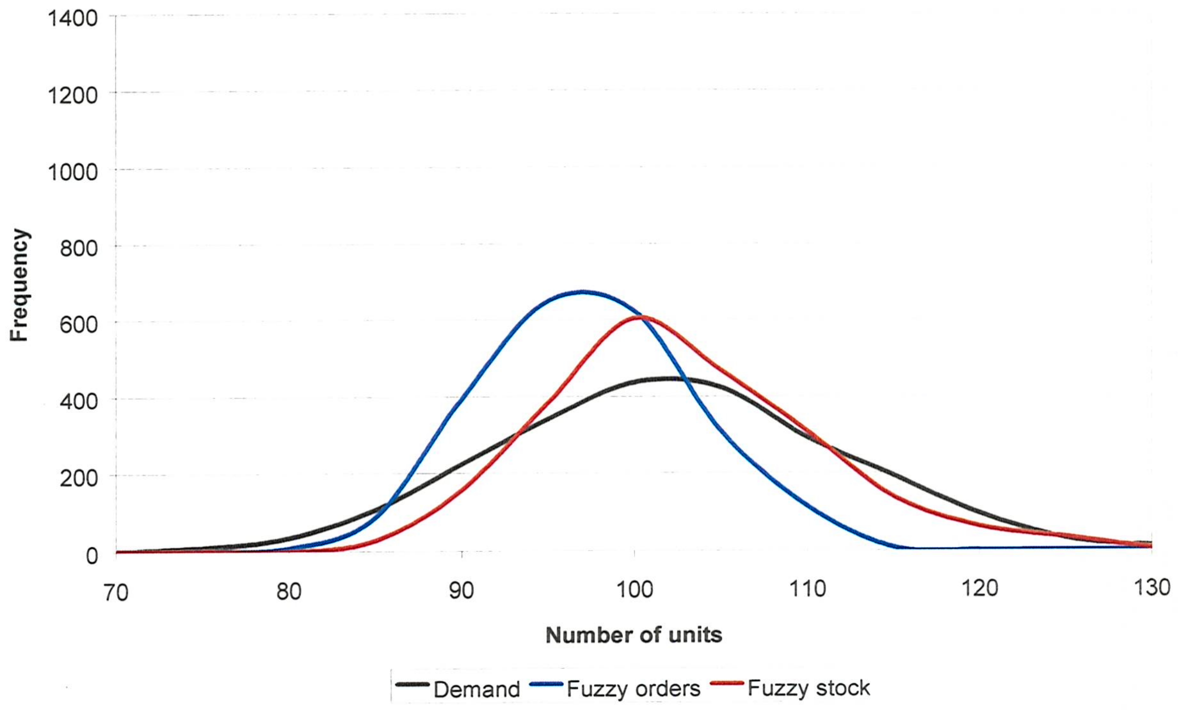


Figure 39: Fuzzy orders & resulting stock – high demand, low customer service

(static demand, μ = 100, customer service strategy = 0.35)

Again, the order distribution is much 'smoother' than the comparable conventional one, as it tails away rather than suddenly stops. As with the previous examples, the fuzzy orders peak around the mean demand (95 compared to the demand mean of 100). However, the fuzzy orders are much less positively skewed than previously (skew = +0.12, which is only significant at a 55% confidence level). This is due to the low customer service strategy adopted (0.35), which results in order sizes that are usually close to mean sales volumes, rather than being 'pushed' up towards the larger volumes. This, in turn, impacts the resulting stock, which although still positively skewed (skew = +0.69, which is significant at a 79% confidence level), does not 'get away' from the demand curve. Orders are usually sold immediately and thus stocks are not able to build up sufficiently. While this helps avoid overstocking, it does not increase customer service.

Further analysis of this issue helps explain the similar performances of the fuzzy and conventional systems.

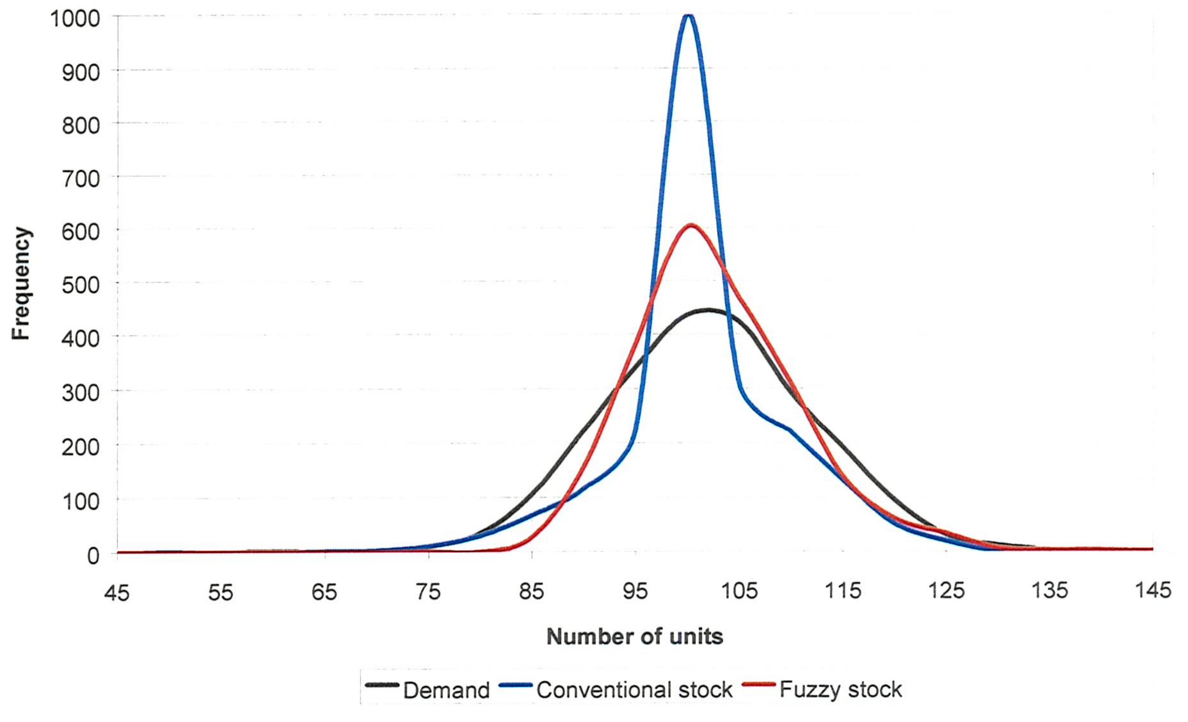


Figure 40: Conventional & fuzzy stock – high demand, low customer service

(static demand, μ = 100, cst = 50%, css = 0.35)

The following table summarises the key parameters of the stock distributions.

Measure	*Conventional system stock distribution cst=50%*	*Fuzzy system stock distribution css=0.35*
Maximum	143	144
Minimum	46	73
Mean	100	101
Standard deviation	8.22	8.20
Variance	68	67
Skew	-0.27	0.69
Kurtosis	5.79	4.11

Table 15: Conventional versus fuzzy stock distributions – high demand, low customer service

(static demand, μ = 100, cst =50%, css = 0.35)

From the graph, it is clear that the distributions of fuzzy and conventional stock are very similar. The means (101 versus 100), variances (67 versus 68) and maximums (144 versus 143) are all very close. The major differences between the two distributions of stock are the frequency peaks and the minimum stock levels. The peaks are highly significant in both cases (kurtosis[146] values of 4.11 and 5.79), but much higher in the conventional distribution. The extreme peak of the conventional stock distribution greatly reduces the impact of very different minimum stock levels (46 in the conventional case and 73 in the fuzzy one).

A synthesis of the three examples discussed is now possible.

Fuzzy versus conventional performance

The differences in performance between the fuzzy and conventional inventory replenishment systems are explained by analysing the distributions of orders and resulting stocks. This analysis is split into two main areas: unconstrained results and constrained results (discussed on page 117). The unconstrained results concern those simulations where neither inventory replenishment system is hampered by any factor, such as limited demand or customer service requirements.

Unconstrained order distributions

The previous analysis of high demand and high customer service requirements illustrates unconstrained order distributions for both the conventional and fuzzy inventory replenishment systems. In particular, they have the following contrasting measurements:

Measurement	***Conventional***	***Fuzzy***
Peak order frequency	Close to but not exceeding the typical order up-to level	Close to but not exceeding the mean sales volume
Skew of order distribution	Slightly negative	Heavily positive

Table 16 : Unconstrained order distributions – conventional versus fuzzy

These order distributions give rise to unconstrained stock distributions.

Unconstrained stock distributions

The previous analysis of high demand and high customer service requirements also illustrates the resulting unconstrained stock distributions for both the conventional and fuzzy inventory replenishment systems. In particular, they have the following contrasting measurements:

Measurement	***Conventional***	***Fuzzy***
Skew of stock distribution	Close to zero	Positive
Variance of stock distribution	High	Medium

Table 17 : Unconstrained stock distributions – conventional versus fuzzy

These stock distributions give rise to the unconstrained performance characteristics.

Unconstrained performance

The fuzzy inventory replenishment system outperforms the conventional system in terms of the customer service level provided for any given stockholding, as long as there are no constraints. This is due to the positive skew of the fuzzy stockholdings compared to the zero skew of the conventional stockholding. This difference arises from the different order strategies.

To increase customer service in the conventional inventory replenishment system, the order strategy adopted is to move the peak of orders (the order up-to level) to a larger order size.

This accomplishes the aim of increasing the mean stock held. However, this also has the effect of reducing the stability of the order size because the mean order size must remain below the mean demand - otherwise stock levels will continually increase. Hence, the variance of the order sizes must increase to accommodate this change and so the order sizes become less stable. This effect can be illustrated using the previous examples of high demand.

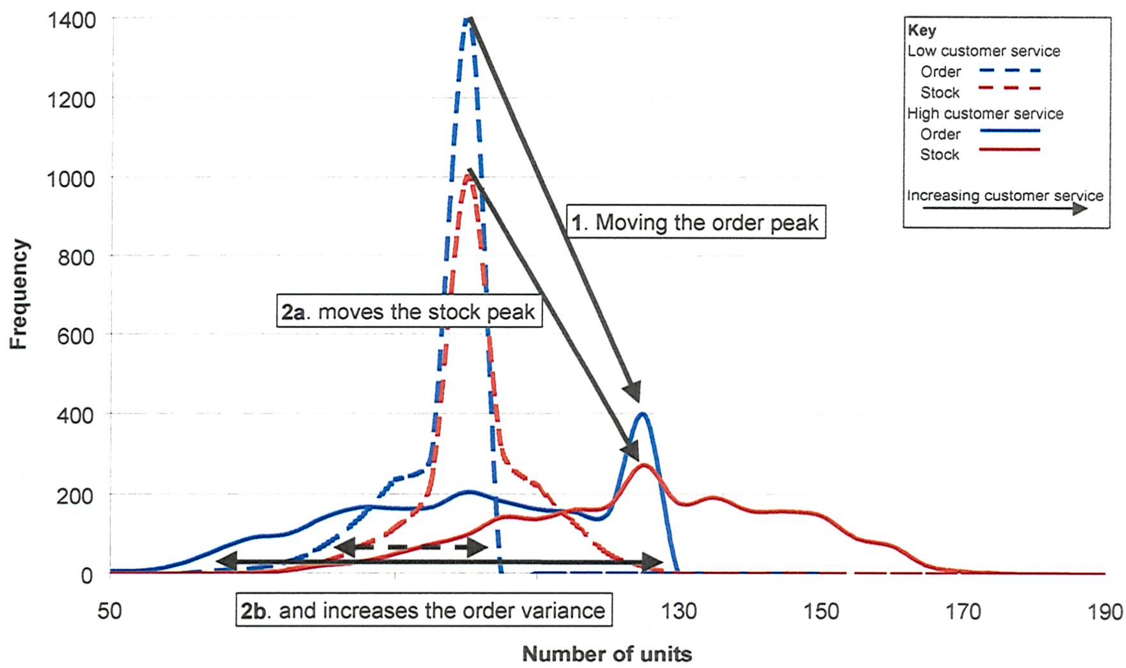


Figure 41: Increasing customer service - conventional system

This is in stark contrast to the fuzzy system.

To increase customer service in the fuzzy inventory replenishment system, the order strategy adopted is to shift the skew of the order distribution towards the larger order sizes. This accomplishes the aim of increasing the mean stock held by positively skewing the stock distribution. The peak of the order distribution remains below the mean demand and so the variance of orders is reduced and ordering becomes more, not less, stable.

This can be illustrated as follows:

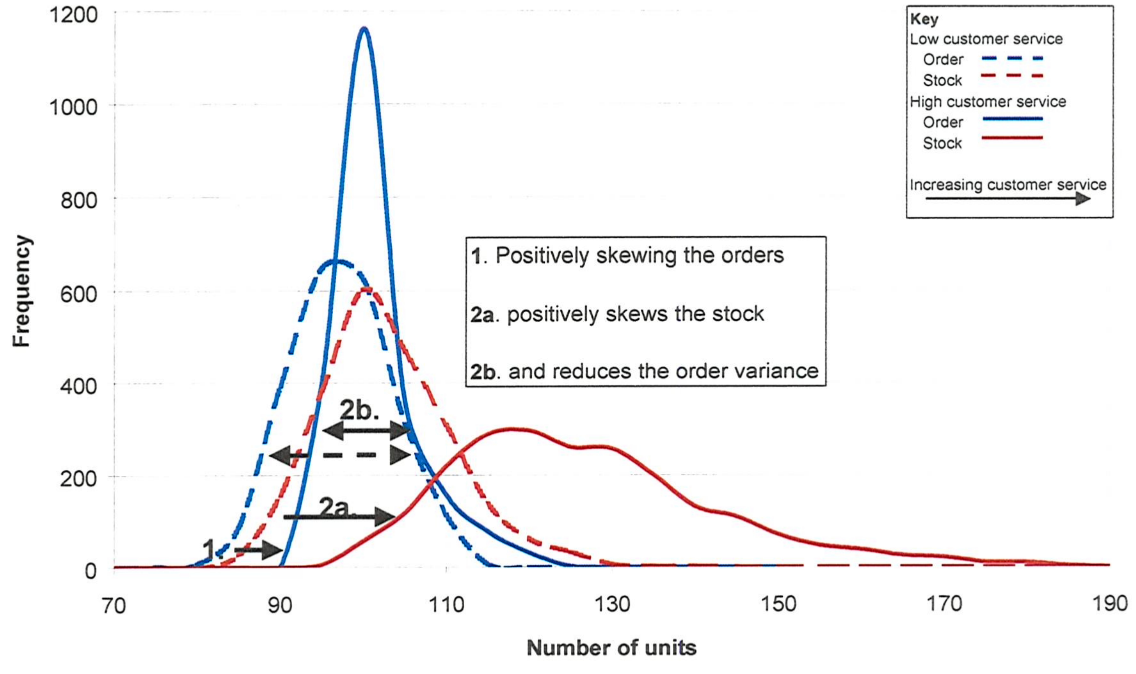


Figure 42: Increasing customer service - fuzzy system

This increased stability of ordering provides a further advantage of the fuzzy inventory replenishment system over the conventional system because more stable ordering usually leads to easier and cheaper supply, for example in terms of production or delivery schedules.

Summary, unconstrained performance

Free from constraints, the performance of the fuzzy inventory replenishment system is superior to the conventional system. This is due to the different order strategies applied. To increase customer service, the conventional system appears to pull its most frequent order size towards the larger order sizes. In contrast, the fuzzy system attempts to push all of the order sizes towards the larger order sizes. This focus on skewing the whole, rather than moving the peak makes the fuzzy system more efficient.

However, there are situations where constraints exist which alter the relative performances of these systems.

Constrained performance

There are two situations in the examples analysed where constraints have affected the relative performance of the inventory replenishment systems. In one case the constraint is obvious; in the other, the nature of the constraint is more subtle.

Demand constraints

In a low demand scenario, the positively skewed stock distribution and associated high maximum of the fuzzy inventory replenishment system generates stock that is surplus to requirements. There is no imminent demand for it. For example, in the simulated low demand scenarios (mean = 0.96), 38% of periods experienced zero demand. All stock held during these periods is surplus to requirements. This undermines the advantage of a positively skewed distribution of stock, which characterises the fuzzy system. In higher demand scenarios, at least some of the stock would be sold and so any excess stock as a result of the positively skewed stock distribution would quickly be reduced.

Customer service constraints

Where there is a low customer service requirement, fuzzy inventory replenishment systems do not generate enough stock to 'get away' from the demand curve. Orders are relatively low and are therefore usually sold immediately. In these circumstances, the fuzzy system cannot build up enough stockholding to generate the characteristic heavily skewed distribution of stock. Therefore, the performance of the fuzzy inventory replenishment system is not significantly different from the performance of the conventional system.

Implications for further analysis

Given the findings described above, the relative performance of the fuzzy and conventional inventory replenishment systems appears reasonably predictable.

The analysis suggests that:

> the fuzzy inventory replenishment systems will outperform the conventional system, except where there are circumstances that make the characteristic positively skewed distribution of orders either:
>
> a disadvantage, as in the case of very low demand
>
> or
>
> impossible to generate, as in the case of very low customer service requirements.

Further analysis follows to test whether the fuzzy system copes differently in a dynamic situation or reality, where demand more varied.

Random demand with constant trend

Similar theoretical analyses of random scenarios, but this time with a constant trend show results consistent with the previous findings.

Increasing demand

As expected, starting with random demand at a high level (100 units per period), where the fuzzy system already performs better, and then increasing this mean demand at a constant rate (0.1% per period) continues to show the fuzzy system outperforming the conventional one, as shown in Figure 43.

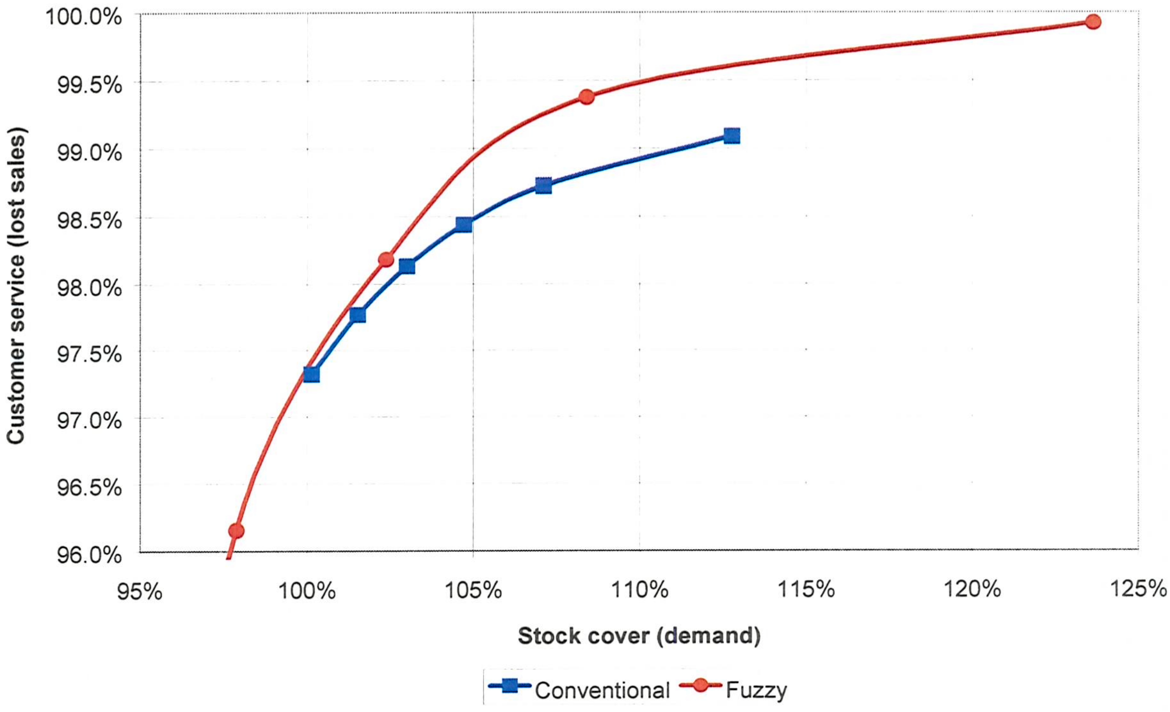


Figure 43: Conventional versus fuzzy – increasing demand, starting with high demand (random demand, +0.1% per period, starting at 100 units per period)

Predictably, starting at a low demand (1 unit per period), where the fuzzy system underperforms the conventional system, and then increasing this demand at a constant rate (0.1% per period) shows the fuzzy system improving and eventually outperforming the conventional one at higher customer service levels, as shown in Figure 44.

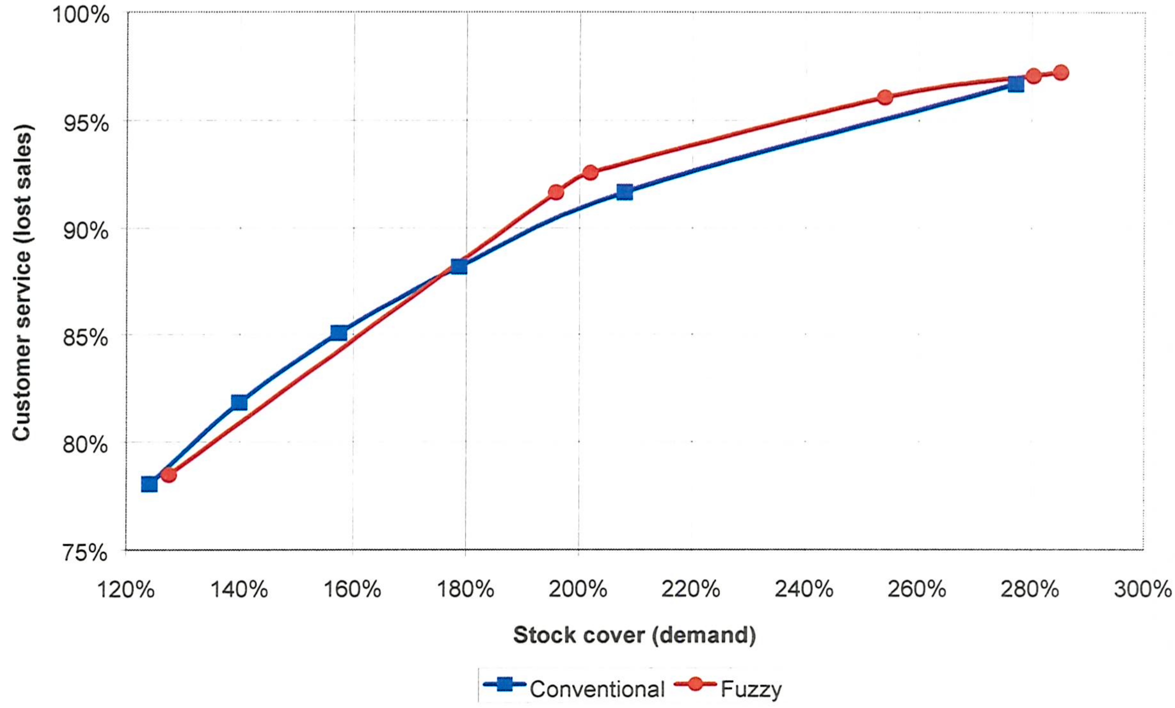


Figure 44: Conventional versus fuzzy – increasing demand, starting with low demand (random demand, +0.1% per period, starting at 1 unit per period)

This is predictable as these results sit between the relevant static results, as shown in Figure 45.

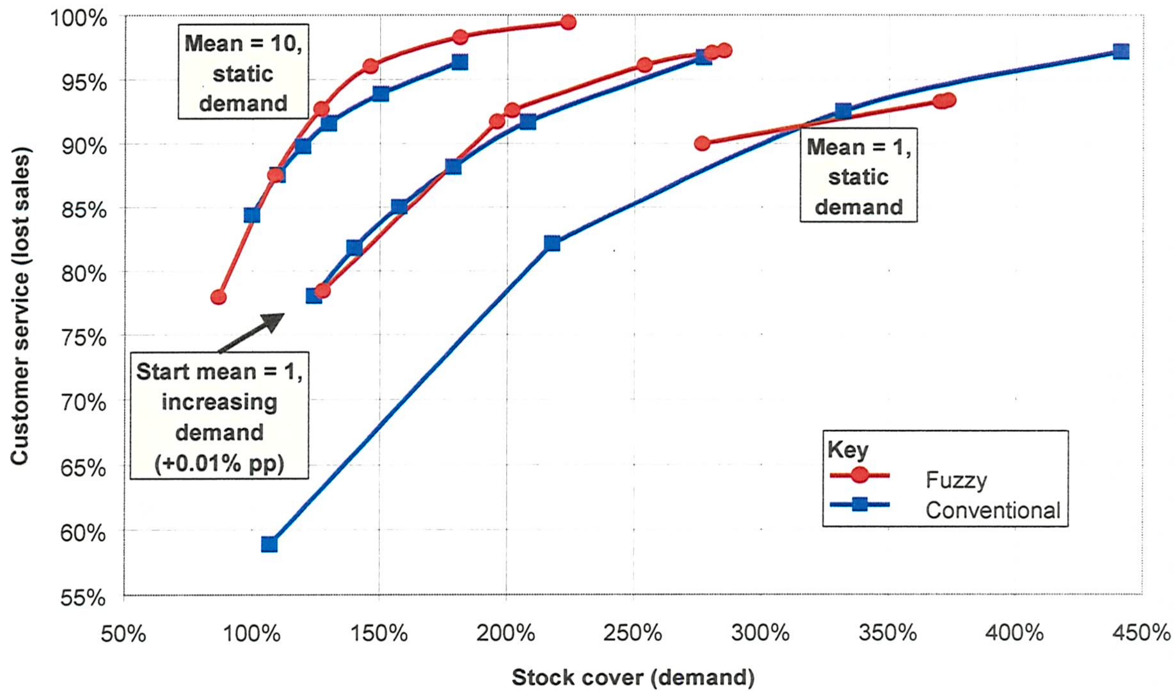


Figure 45: Conventional v fuzzy – comparison of increasing low demand with static results

Thus the pattern of results for increasing demand at a constant rate is entirely consistent with the previous analysis.

Furthermore, the fuzzy inventory replenishment system is able to cope with increasing demand. This is done without any direct forecasting, unlike the conventional system which is provided the best forecast possible (see page 70).

Decreasing demand

The simulations of random but decreasing demand (at a constant rate 0.1% per period) produce unsurprising results, with the exception of one data point. As shown in Figure 46, the results are again a family of curves that fit with the curves generated in the analyses of both static and increasing demand.

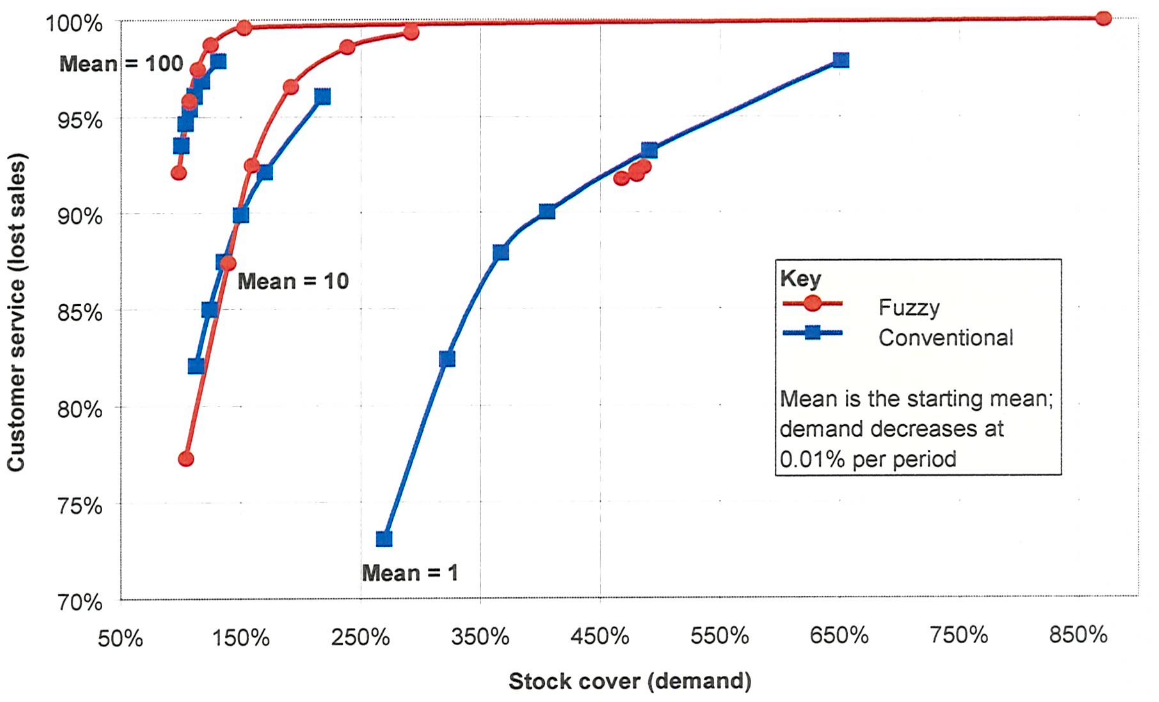


Figure 46: Conventional versus fuzzy – decreasing demand, starting with high demand (random demand, -0.1% per period)

The one exceptional point concerns the fuzzy system when aiming for a very high level of customer service (customer service strategy = 1.0), in a high demand scenario. This is illustrated in Figure 47.

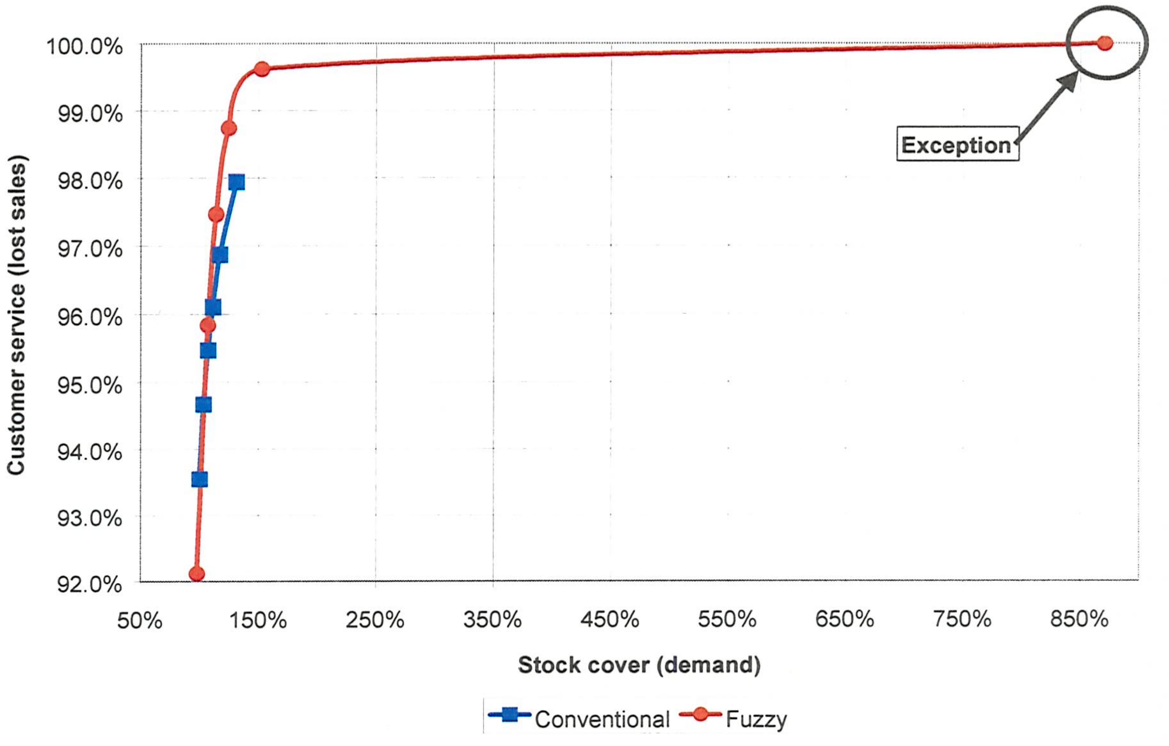


Figure 47: Conventional versus fuzzy – decreasing demand

(random demand, -0.1% per period, starting at 100 units per period)

This data point is the result of constant overstocking. Before discussing this, it is worth showing the graph without this data point, to illustrate that it fits the usual pattern of the fuzzy system generally outperforming the conventional system.

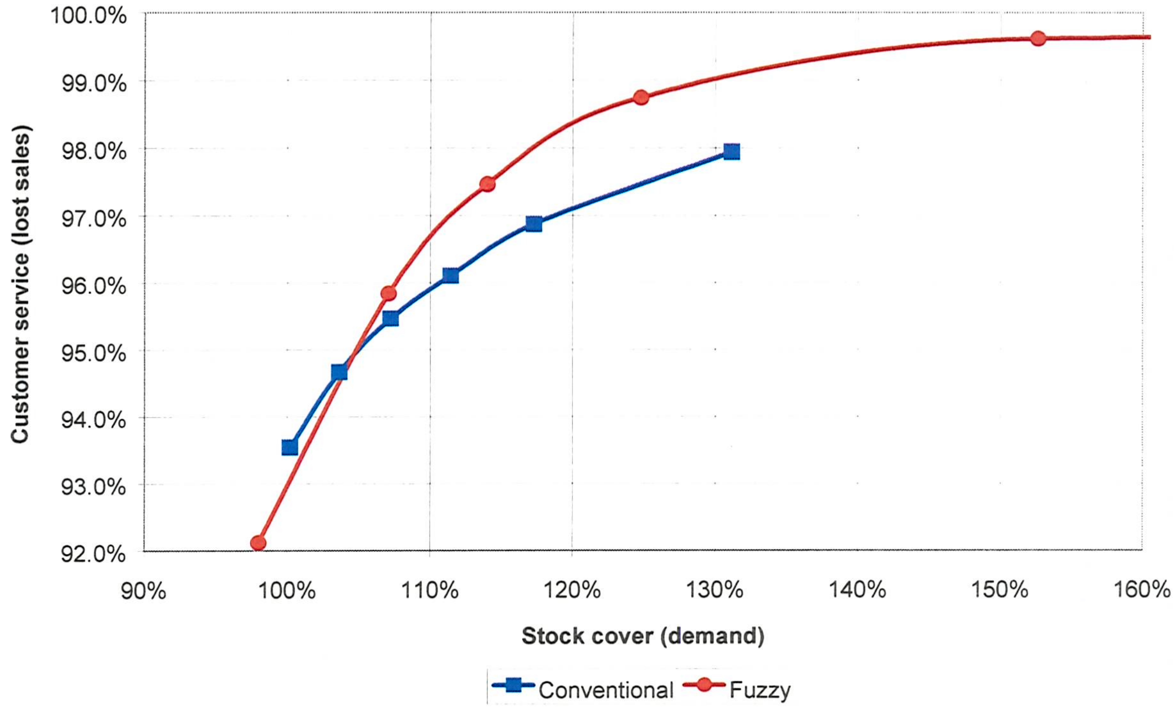


Figure 48: Conventional versus fuzzy – decreasing demand (excludes fuzzy overstocking)

(random demand, -0.1% per period, starting at 100 units per period)

The reason for the overstocking is that when the fuzzy inventory replenishment system has a customer service strategy equal to 1.0, then the small order size is set to one unit below (100% of) the mean sales in the last 20 periods. If demand is falling so quickly that it does not reach this value, then excess stock is added. When overstocked, the fuzzy system places a small order and so continues to add to the existing overstock unless the fall in demand slows down (or demand rises).

A solution to this problem that is consistent with the fuzzy approach is to add another fuzzy input variable, output variable and associated rule. The input variable might be called 'massively overstocked', the output variable 'a very small order' and the rule would simply be:

If massively overstocked, then place a very small order.

This *very small order* would be below the minimum order size in an attempt to reduce stocks. A further extension could include the rule:

If ridiculously overstocked, then order nothing.

This would ensure that stocks cannot continue to grow under any circumstances.

All of these rules would be in keeping with the fuzzy approach. As a crude, bivalent example of this rule in action, the simulation can be re-run using the previously proposed rule where 'ridiculously overstocked' is defined as stock in excess of twice the maximum sales. The definition is reasonable on the basis that if stock is at twice the maximum sales, then there should be enough stock held to cover maximum sales in the period about to start and the following period, when the zero order is delivered. The results of this simulation (using spreadsheet model *FIRS model v8cutoff*) are shown in Figure 49.

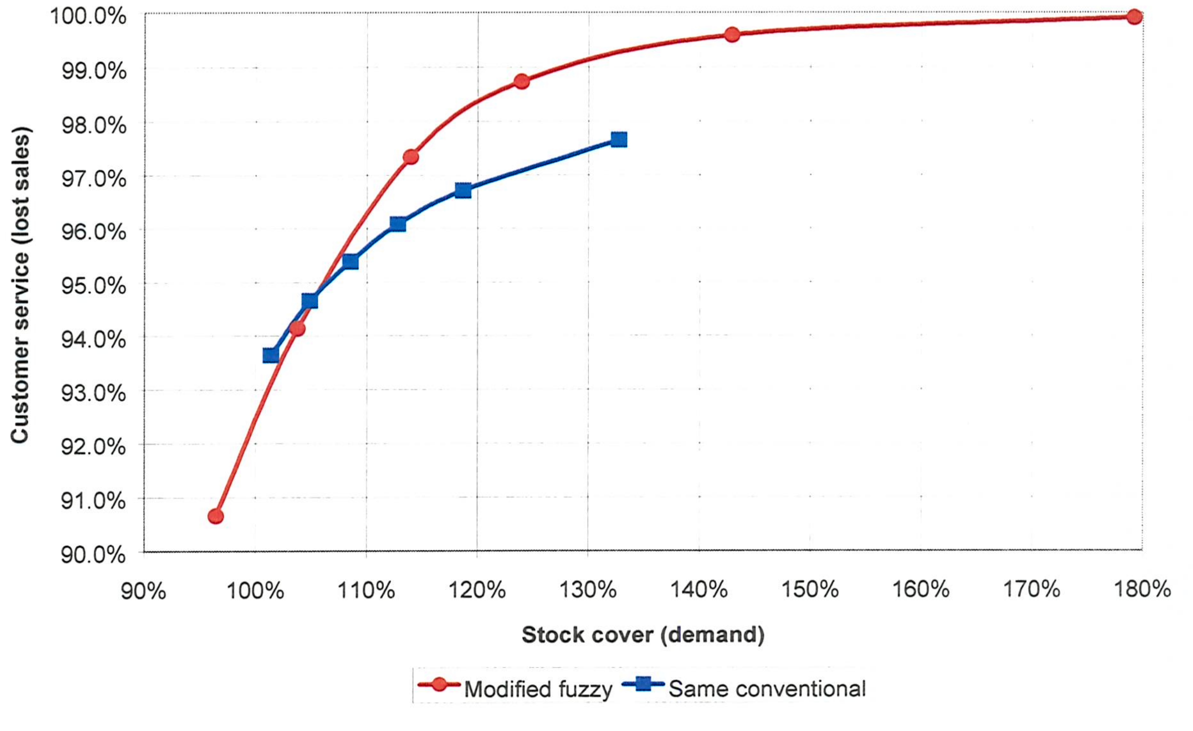


Figure 49: Conventional versus fuzzy – decreasing demand (modified fuzzy system)

(random demand, -0.1% per period, starting at 100 units per period)

Hence, the pattern of results remains consistent throughout all of the simulated theoretical scenarios.

Implications for further analysis

Given the findings above, the relative performance of the fuzzy and conventional inventory replenishment systems appears increasingly predictable. The modification of the fuzzy inventory replenishment system in the way suggested also reduces the overstock experienced under low demand conditions – see Figure 50.

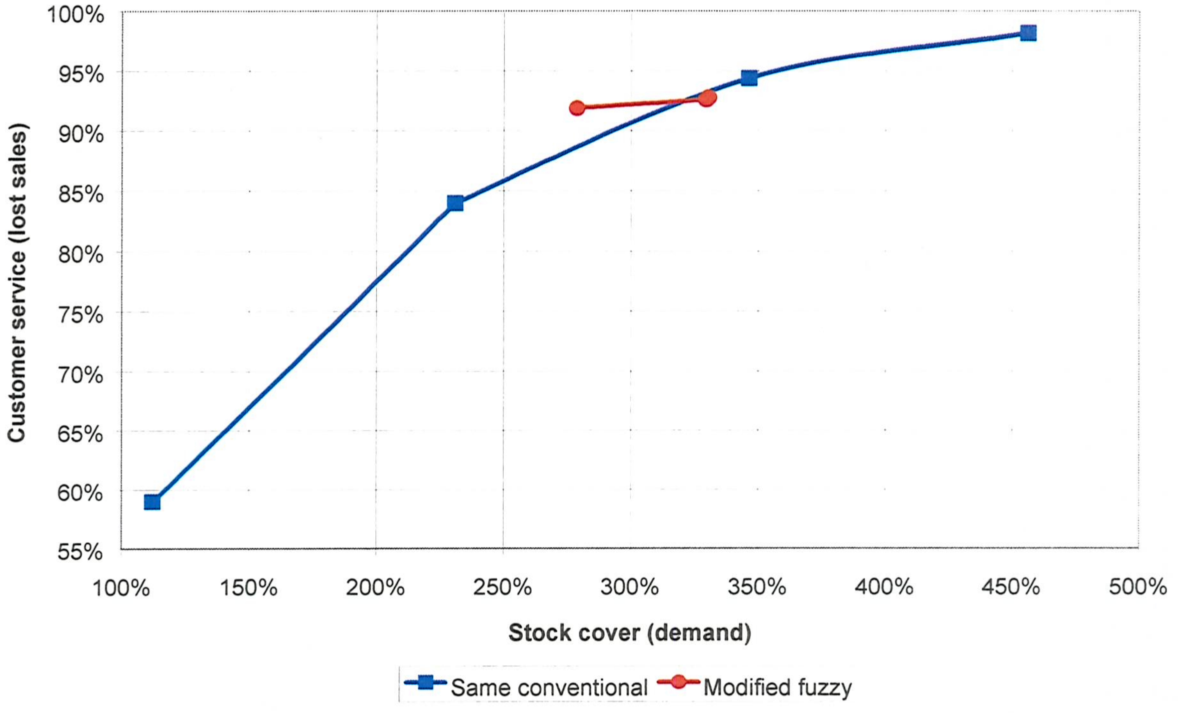


Figure 50: Conventional versus fuzzy – low demand (modified fuzzy system)

(random static demand, $\mu = 1$)

Thus this analysis suggests that:

the fuzzy inventory replenishment system, including an order cut off point of twice the maximum sales, will outperform, or perform at least as well as, the conventional system under all the theoretical scenarios investigated.

An analysis of some practical scenarios now follows.

Tesco data analysis

The Tesco demand data is far from being random around a static or even dynamic mean. While much of the data contains variation around a mean, the variation is generally of a higher magnitude that would be considered statistically random (see spreadsheet *Tesco data analysed* for more details). Also, while the demand data is generally Normally distributed, there are many exceptional points. The discussion here focuses on two key products: lemonade, with a particularly high peak, and light bulbs, with a particularly wide variation in demand. The other products are more subdued variations on these themes.

In these simulations (using spreadsheet model *FIRS Tesco v8cutoff*), it is again assumed that the conventional inventory system uses the known mean and variance of the demand. With only 52 weeks of data, it is also necessary to assume that the fuzzy inventory system uses the known mean, maximum and minimum of the demand. In such a short period, there is not enough time for the fuzzy system to build up historic sales data. In both systems, such parameters would usually be based on historic sales and/or marketing judgement. However, it is assumed that all parameters are estimates and not actual measures of the data in question. Neither system uses the actual demand data, except as part of the simulations themselves.

On the one hand, these assumptions weaken any conclusions reached. Using the same data to set parameters and test the arguments is weak. On the other hand, the general principle of the fuzzy inventory replenishment system and its comparability with a conventional system has been established. This section is more focused on the practicality of such a system compared to a conventional one. Under these conditions the assumptions made are reasonable. Furthermore, without such assumptions then the arguments become less about the inventory system employed and more about the forecasting system employed – a topic which is extensively discussed elsewhere.

Demand with a high peak

The pattern of demand for the lemonade at store level is as follows.

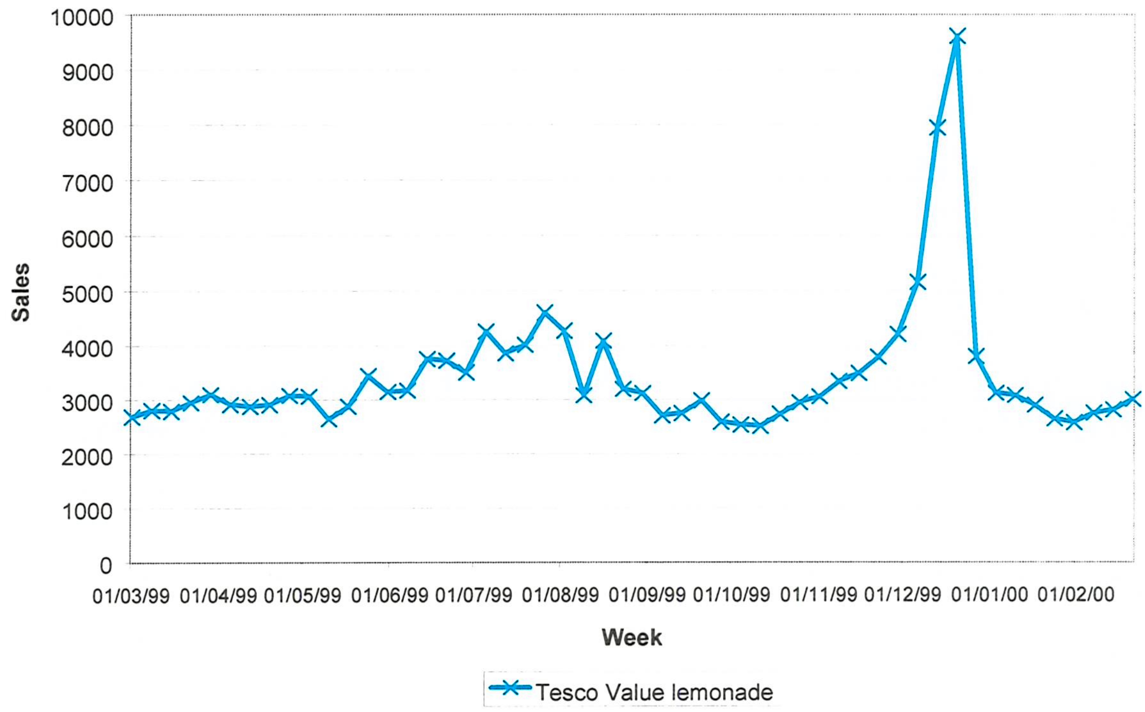


Figure 51: Demand for lemonade at store level

The demand data has a particularly high variance (1,528,995) compared to it's mean (3,444) and experiences a huge peak of demand just before Christmas.

A comparison of how the fuzzy and conventional systems deal with this peak, without intervention is as follows.

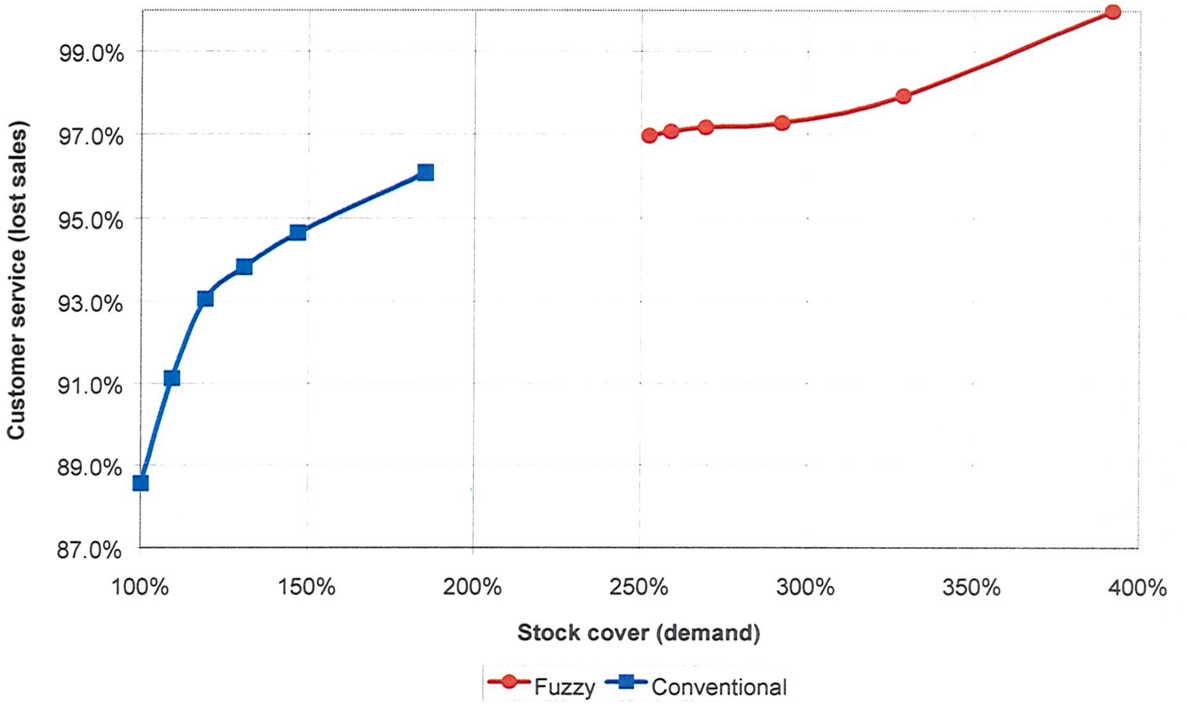


Figure 52: Conventional versus fuzzy - high peak demand

The two systems are impossible to compare due to the lack of overlap in the results. The reason for this lack of overlap is the very different approaches of the two systems. The fuzzy system stocks up waiting for the Christmas peak, while the conventional system attempts to mirror the demand albeit at an exaggerated level to try and ensure customer levels. These different approaches are graphically illustrated as follows.

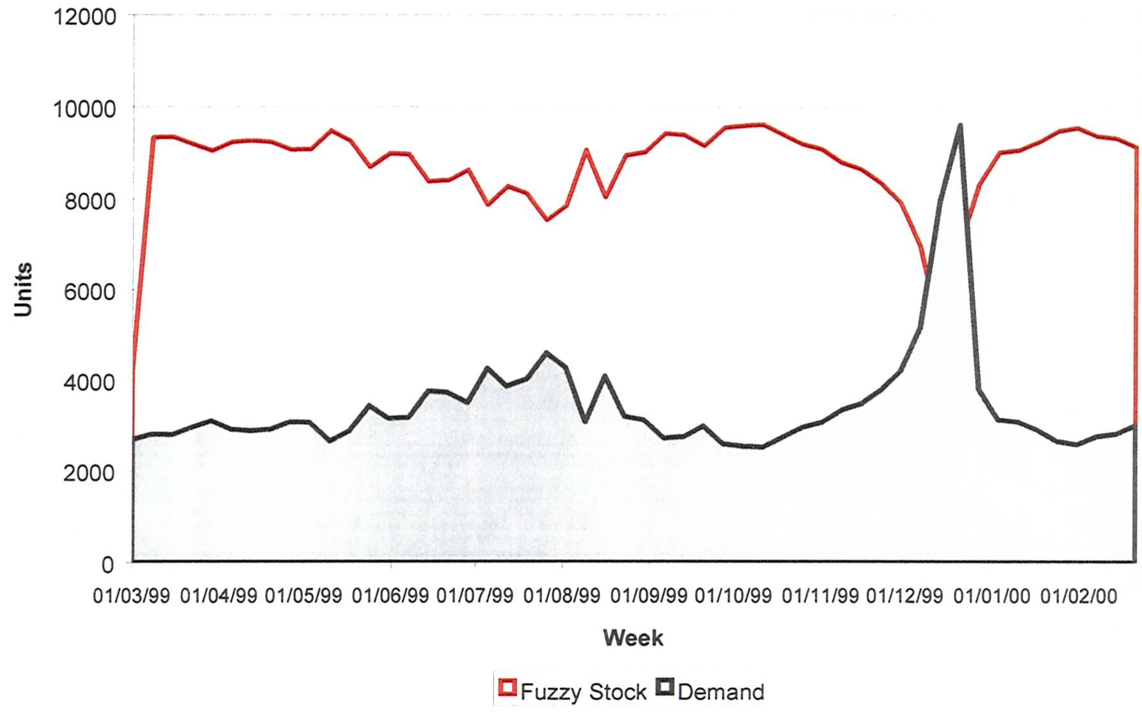


Figure 53: Fuzzy stock - high peak demand

(Lemonade, customer service strategy = 0.0)

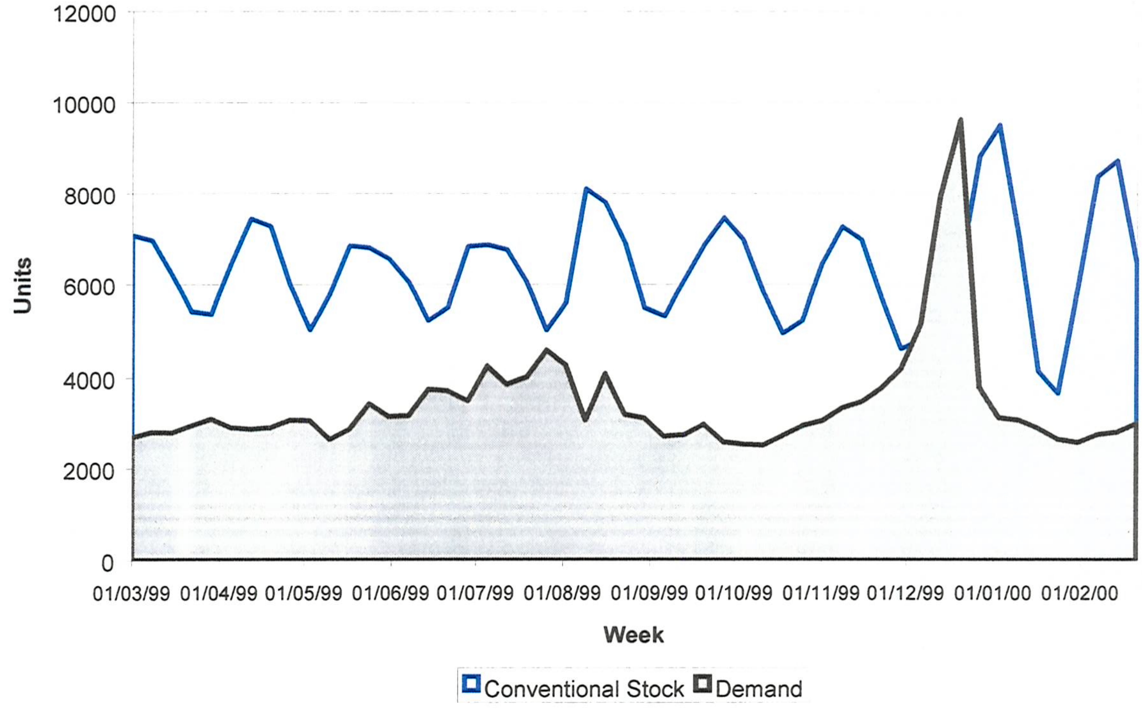


Figure 54: Conventional stock - high peak demand

(Lemonade, customer service target =99%)

Although the fuzzy system obviously has to carry more stock due to its underlying strategy, the conventional system loses sales by not anticipating the main Christmas sales.

In practice, an estimate is made of the Christmas sales uplift and this is taken into account when deciding upon appropriate stock levels. This can be accounted for in the simulations by treating sales higher than 1.64 times the standard deviation from the mean, a 95% confidence interval assuming Normal distribution, as different from other demand. This picks out the two weeks before Christmas as different. Thus, the data can be divided into two groups: the pre-Christmas fortnight and the rest of year. Each of these groups has their own mean, variance, maximum and minimum and can be simulated accordingly. Moreover, this pre-Christmas peak is a known phenomenon for a product such as lemonade. Hence the change in demand from one group (the rest) to another (the pre-Christmas fortnight) can be anticipated in supply terms, i.e. orders can be made a period in advance to arrive on-time for the peak demand, unlike the situation for the conventional system shown above.

This analysis (using spreadsheet model *FIRS Tesco v8cutoff seasonal*) results in the following performances from the two systems.

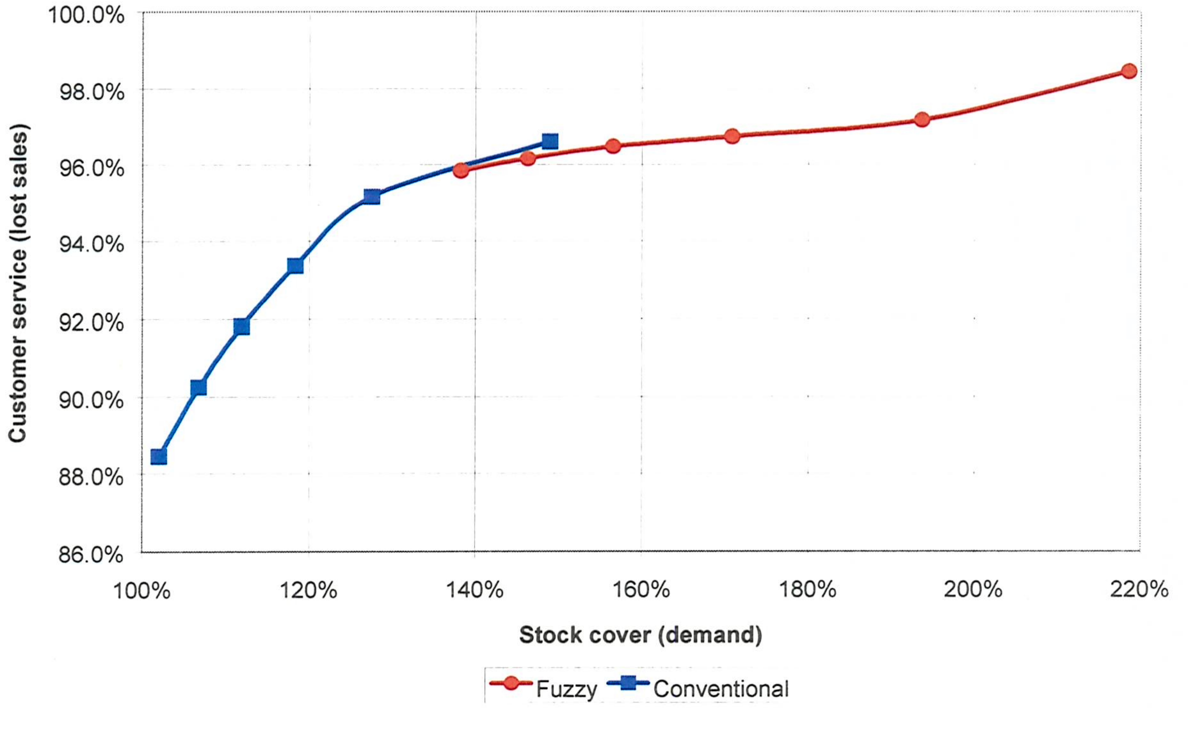


Figure 55: Conventional versus fuzzy - high peak modified

As can be seen, the performances still differ, with the focus more on customer service in the fuzzy system and more on stock in the conventional system. Where the results do overlap, these are not significantly different given the relatively small sample size (52 weeks).

So, without further modification it appears that the two systems are comparable. However, it is worth noting that although the two systems are comparable in terms of performance there is still a major contrast in terms of how the performance is achieved. As seen previously in the theoretical experiments, the fuzzy system has a more stable order size and stock level. These issues may have important customer service or cost implications. For example, where vehicle delivery size is an issue then the fuzzy system may be seen as superior with more stable order sizes.

Further sophistication of the two systems may yet provide a discernible difference in performance. This will be considered after the next scenario is discussed: demand with a high variance.

Demand with a high variance

The pattern of demand for the light bulbs at store level is as follows.

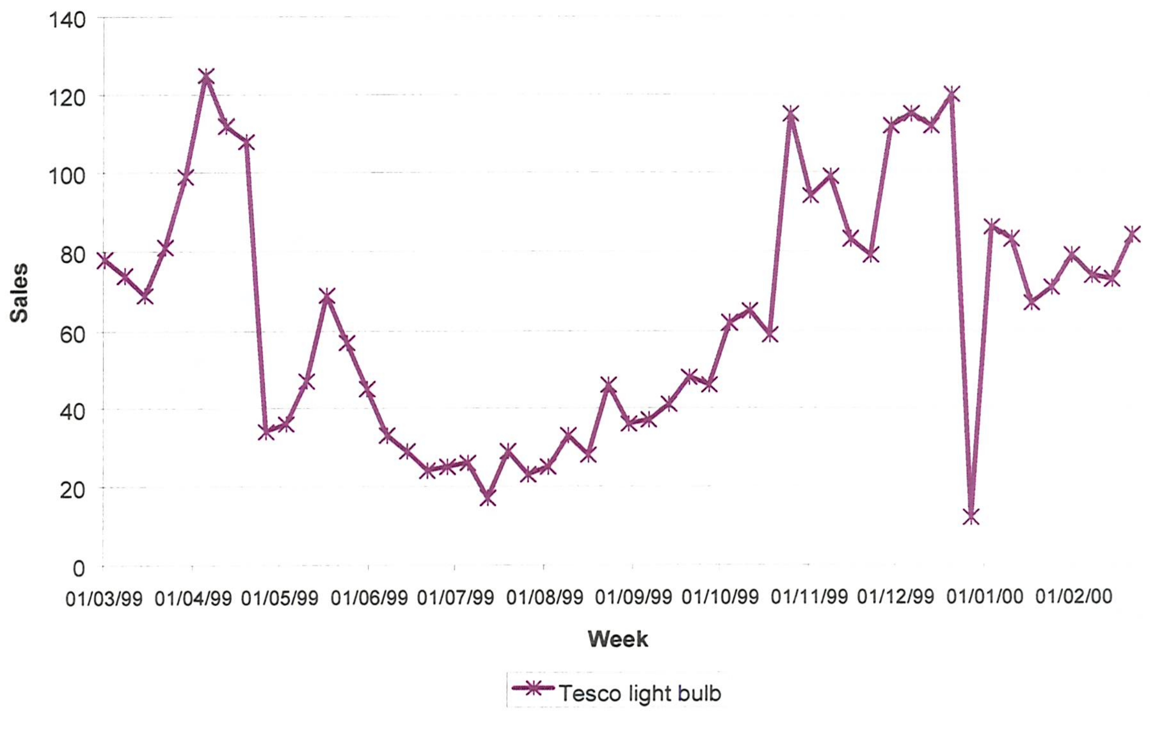


Figure 56: Demand for light bulbs at store level

This data shows particularly wide variations in demand. Although mean demand is around 64, there are few occasions when demand is close to this mean.

A comparison of how the fuzzy and conventional systems deal with this variation, without intervention is as follows.

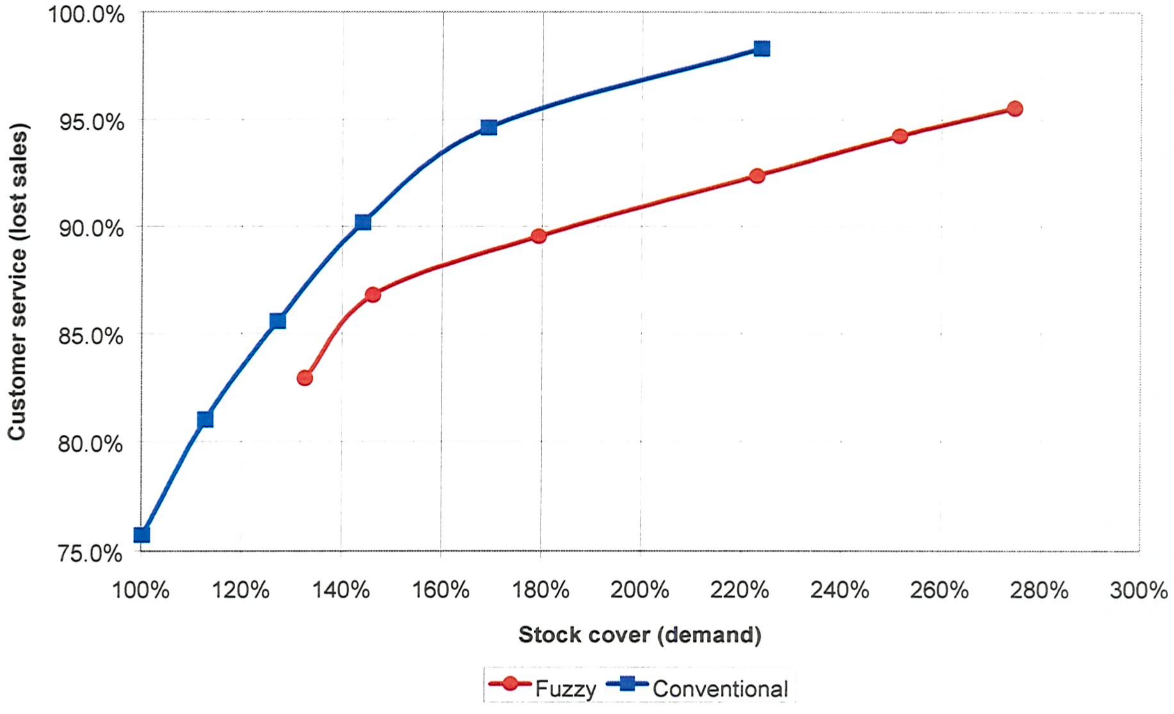


Figure 57: Conventional versus fuzzy - high variation in demand

On this evidence, the fuzzy inventory replenishment system is clearly outperformed by the conventional one.

Again, the different approaches underlying the different inventory strategies of the two systems are clearly illustrated in terms of how stock is held. The following two graphs show similar average inventory, but with very different patterns of stockholding.

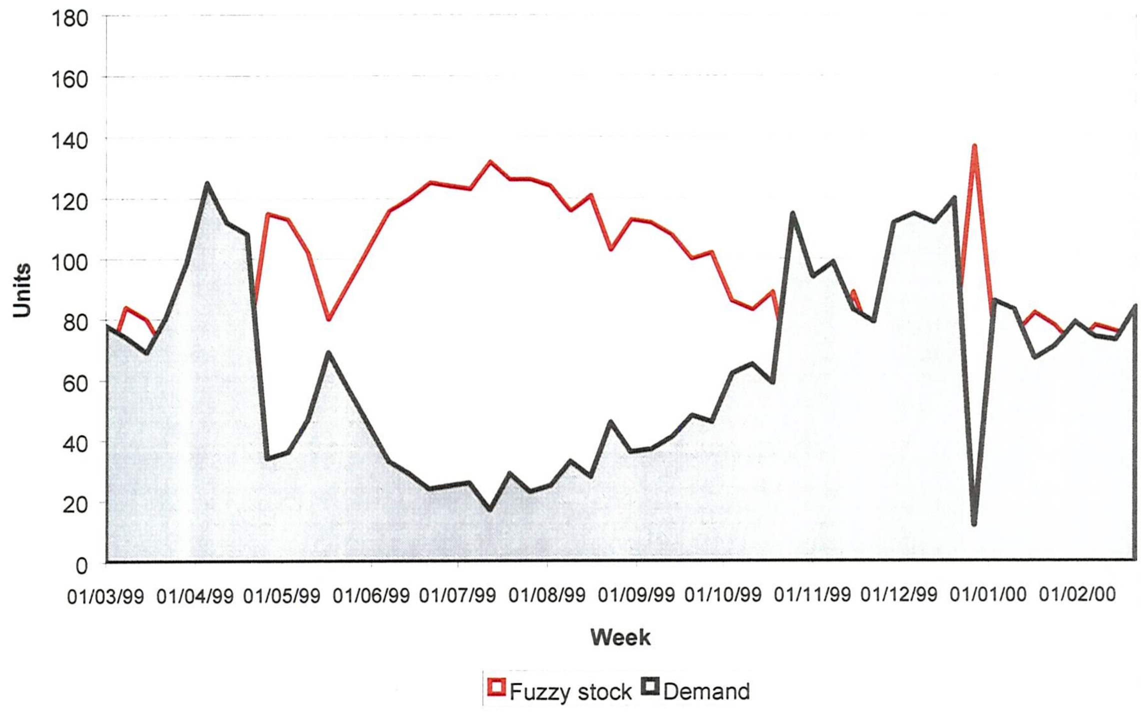


Figure 58: Fuzzy stock - high variation in demand

(Light bulbs, customer service strategy = 0.20)

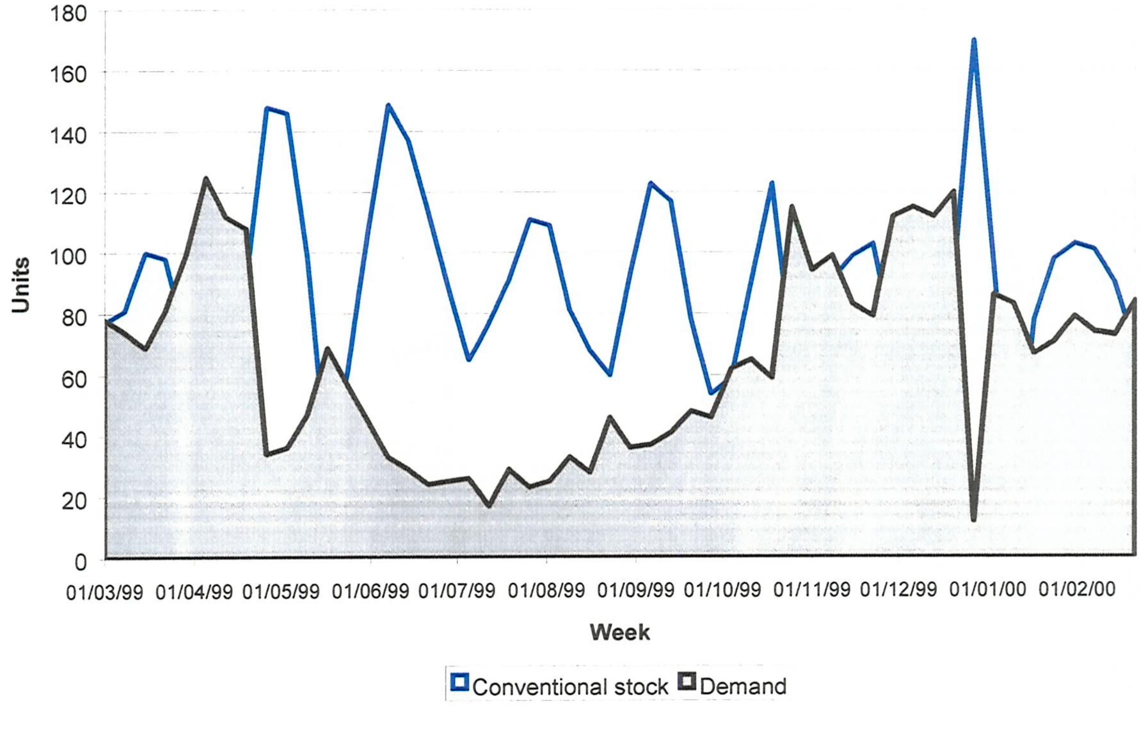


Figure 59: Conventional stock - high variation in demand

(Light bulbs, customer service target = 80%)

Again, the conventional system tries to track demand, while the fuzzy system aims to cover the peaks. However, with this wide variation in demand the fuzzy system cannot compete at the same stock level as the conventional system. This is shown by inverting the previous graphs, so that the lost sales are illustrated by the demand that can still be seen above the inventory.

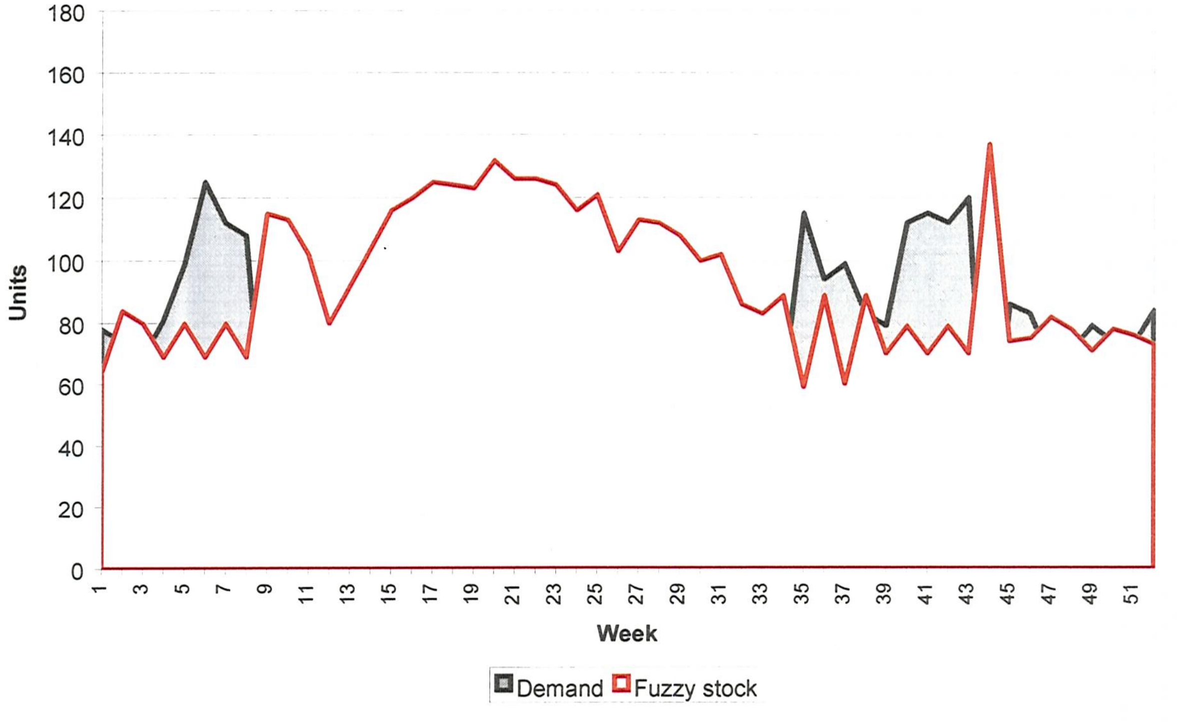


Figure 60: Fuzzy lost sales - high variation in demand

(Light bulbs, customer service strategy = 0.20)

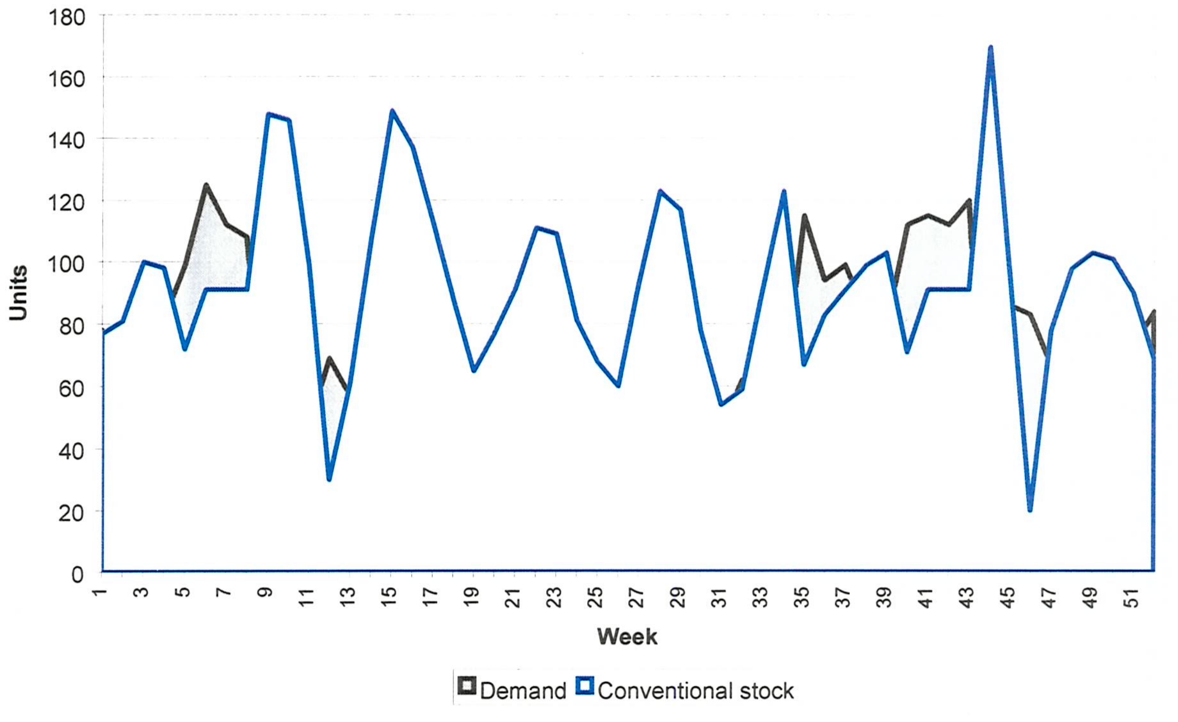


Figure 61: Conventional lost sales - high variation in demand

(Light bulbs, customer service target = 80%)

The fuzzy system completely misses the peaks in demand, whereas the conventional system at least tries to follow these peaks.

Again, in practice, there is more analysis of the underlying demand data and an understanding of product characteristics so that better estimates of demand are employed. In particular, it is clear from both the demand data and known characteristics of this product that this is a seasonal item. People turn more lights on in winter and hence use more light bulbs!

With this information and recognising that the data effectively splits above and below the average (64), the data can be divided into two groups: summer (from 26th April to 24th October) and winter (the rest of the year). Each of these groups has their own mean, variance, maximum and minimum and can be simulated accordingly. Again, this changeover is a known phenomenon and hence it is assumed that the change in demand from winter to summer to winter can be anticipated in supply terms.

This analysis (using spreadsheet model *FIRS Tesco v8cutoff sea LB*) results in the following performances from the two systems.

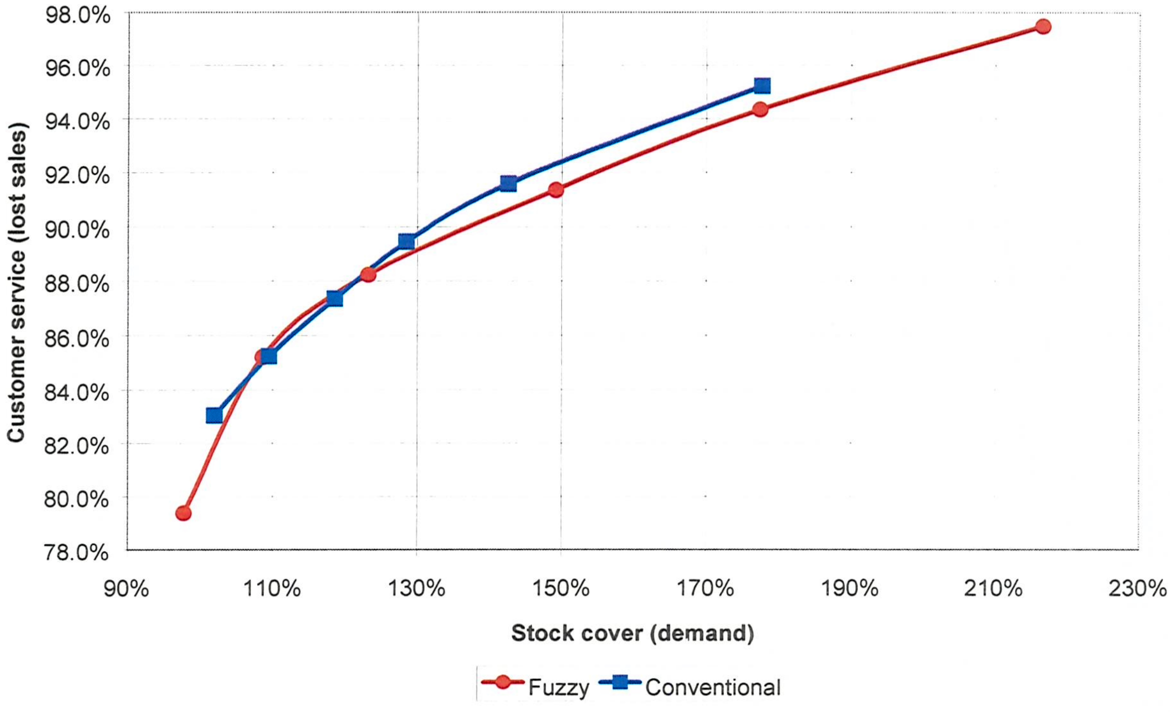


Figure 62: Conventional versus fuzzy - high variation modified for seasonality

Having taken account of seasonality, the results are similar, especially given the relatively small sample size (52 weeks).

So, without further modification it appears that the two systems are again comparable, though the fuzzy system still provides more stable patterns of ordering and stocking, as illustrated in the examples of ordering in Figure 63 and Figure 64. These examples provide similar customer service for similar stock cover.

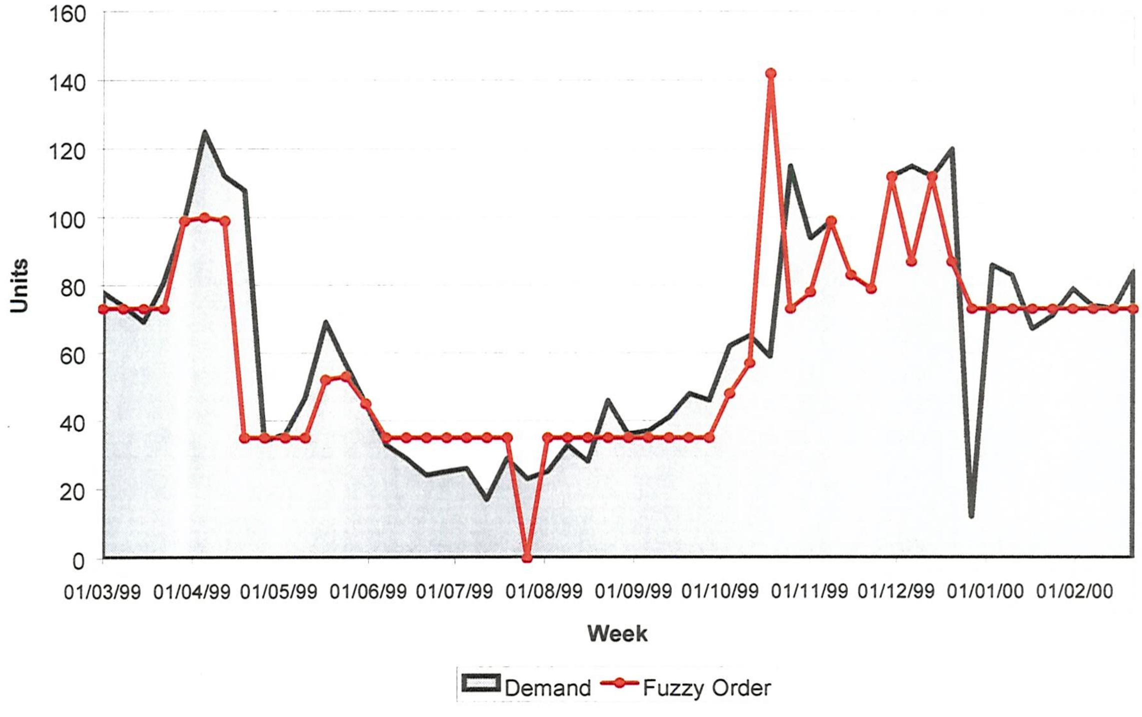


Figure 63: Fuzzy orders - high variation modified for seasonality

(Light bulbs, customer service strategy = 0.80)

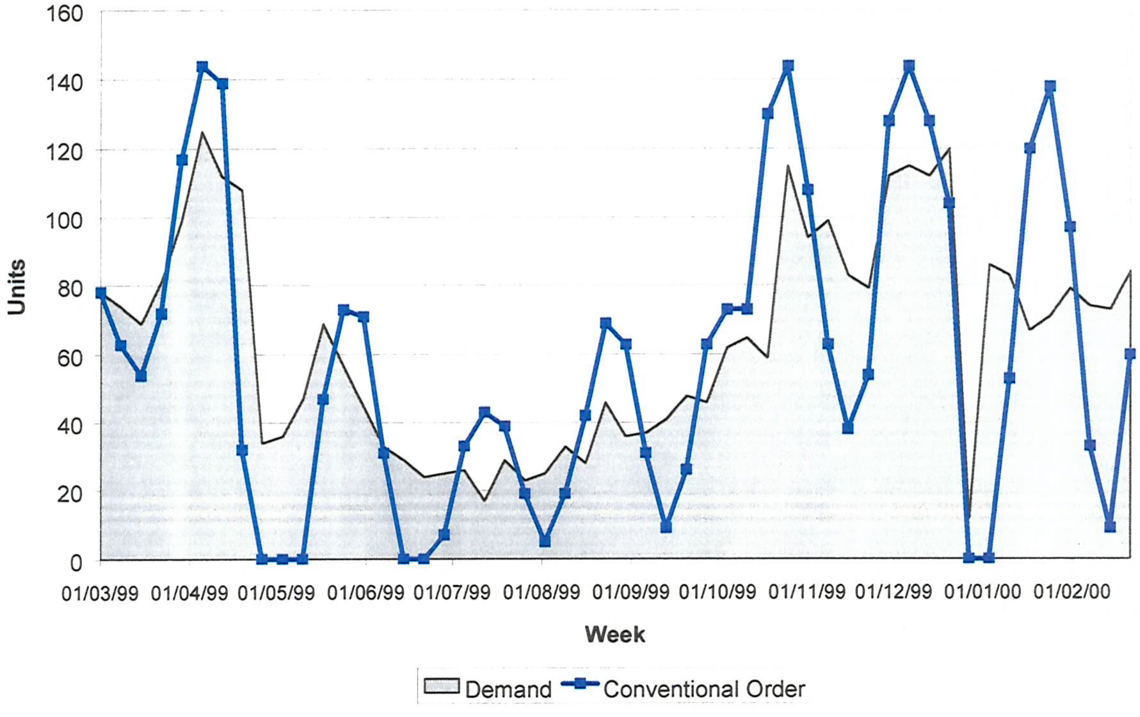


Figure 64: Conventional orders - high variation modified for seasonality

(Light bulbs, customer service target = 99%)

Further sophistication of the two systems may yet provide a discernible difference in performance, as well as the obvious existing difference in order and stock variability. This can now be considered by combining seasonality and Christmas peak (or trough).

Further modifications

Looking more closely at the demand data for the lemonade it is clear that this data also has seasonality. Conversely, the light bulb demand contains a post-Christmas trough - rather than a pre-Christmas peak. These are both clear and known characteristics of the particular products sales. By dividing the demand data into three groups: summer, winter and Christmas, the simulations (using spreadsheet model *FIRS Tesco v8cutoff trigroup*) are reworked, which provides the following results.

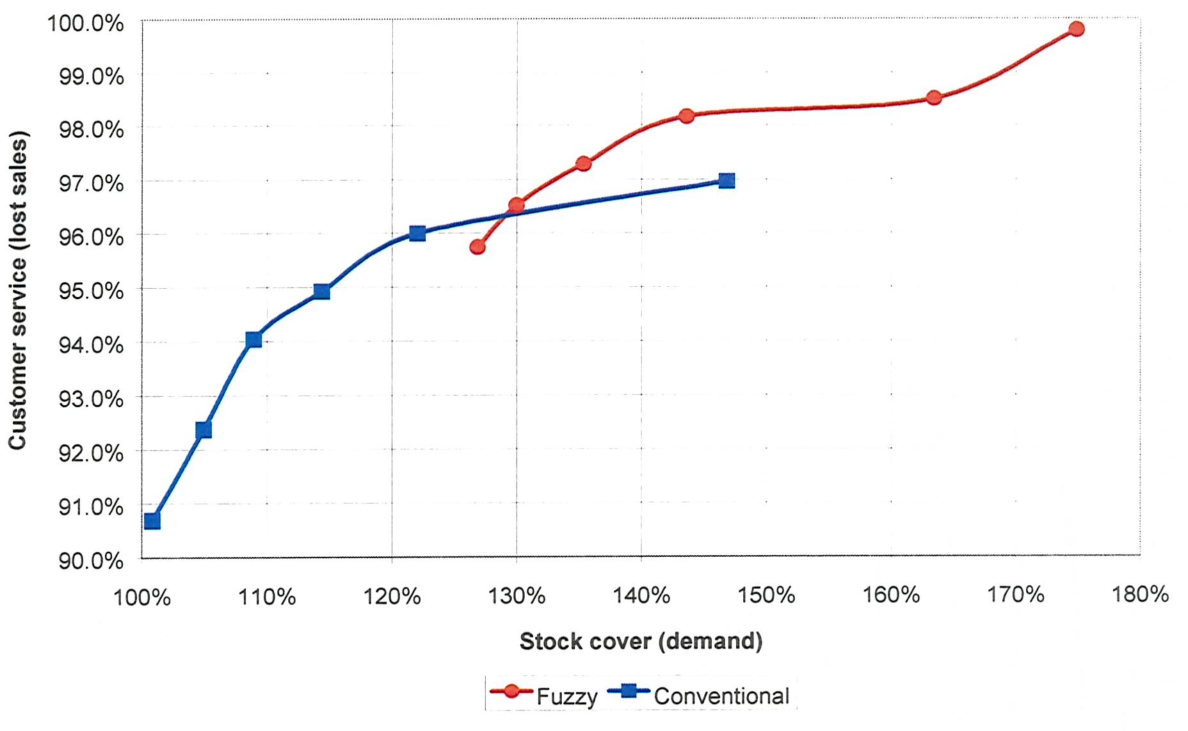


Figure 65: Conventional versus fuzzy – lemonade

(Taking crude account of seasonality and Christmas)

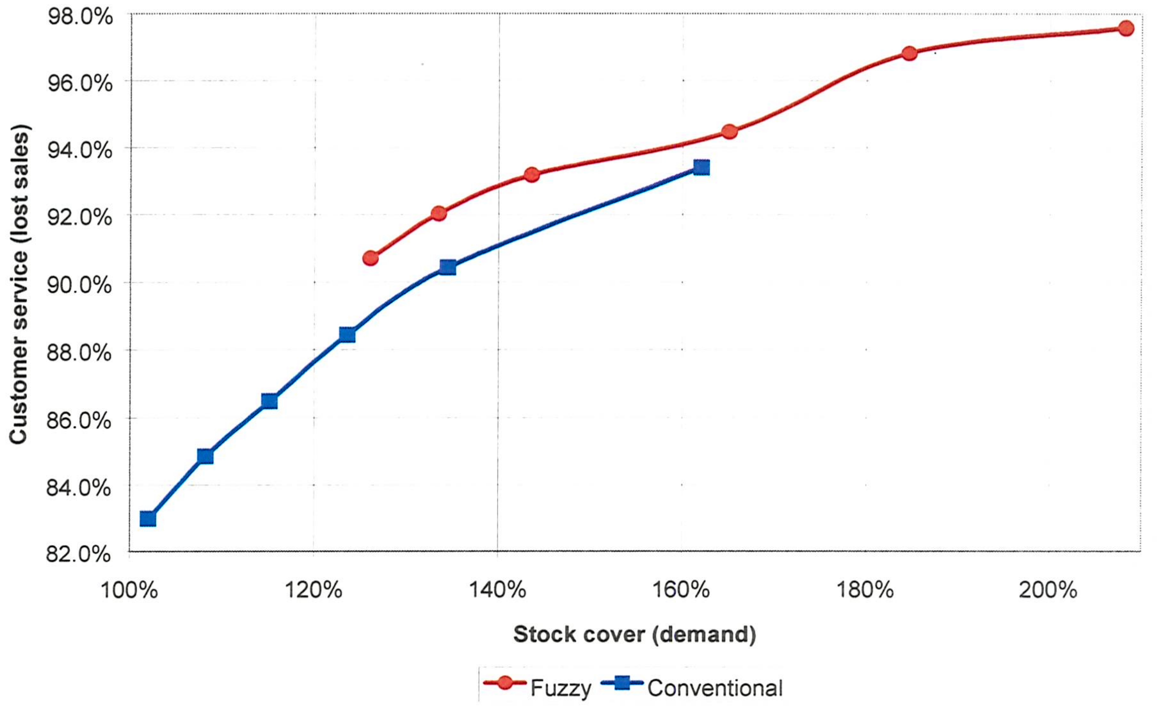


Figure 66: Conventional versus fuzzy – light bulb

(Taking crude account of seasonality and Christmas)

From these graphs, it appears that with a few crude changes the fuzzy inventory replenishment system becomes superior to the conventional system. However, this has only been possible by segmenting the data into similar groups of demand. This helps the conventional system by reducing the variance, but appears to help the fuzzy system more by making the maximum and minimum parameters more relevant to the data.

Practical implications

In all cases, the fuzzy system focuses more on customer service and provides for more stable ordering and stock levels. Therefore, if any of these issues are of prime importance then the fuzzy system has built-in advantages over the conventional system.

Where demand is more structured the fuzzy inventory replenishment system also appears to perform better than the conventional system. The degree of tolerable variance is investigated in the next section.

Highly variable demand

By running simulations with Normally distributed demand but with high variance, the tolerance of the fuzzy and conventional systems to different degrees of variance may be considered. The Normal distribution is chosen to reflect the distribution used within the conventional inventory management system and thus provide it with the best theoretical opportunity to perform against the fuzzy system. The range of variances have been chosen based on the demand data supplied by Tesco, where the variance ranges from 6 to 450 times the mean. A number of simulations are carried out (using spreadsheet model *FIRS model v8cutoff Normal*), which provide consistent results, as illustrated in Figure 67.

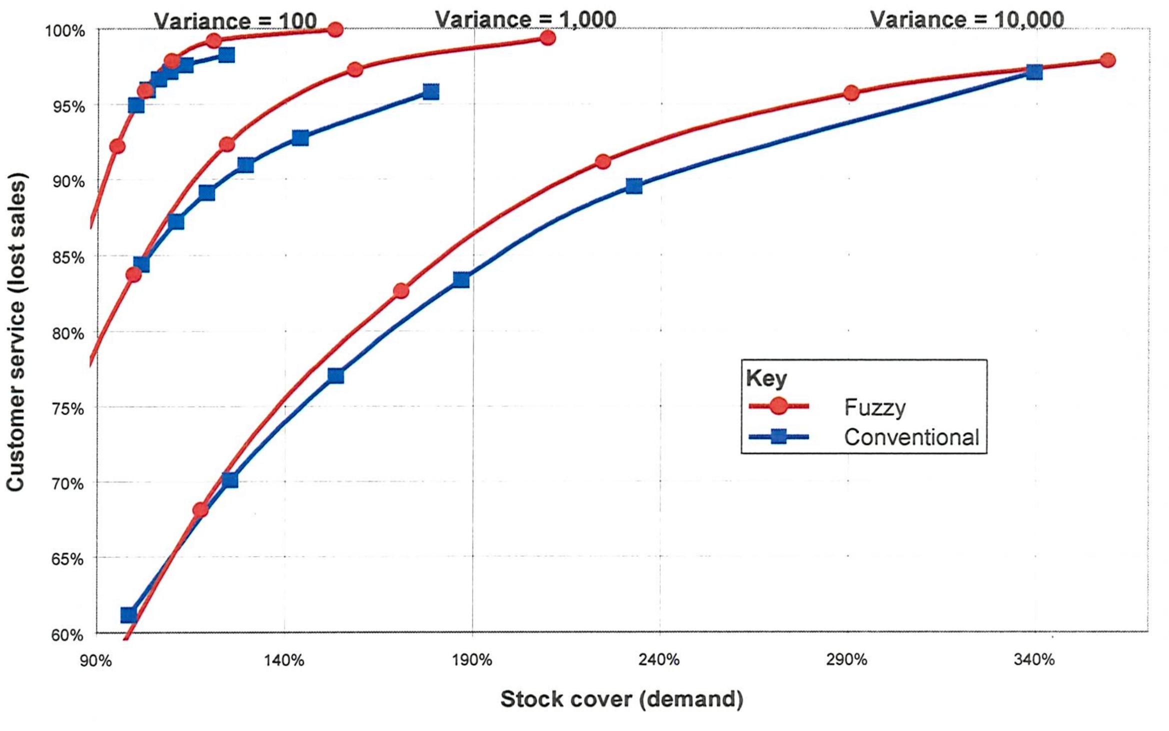


Figure 67: Conventional versus fuzzy – different variances

(Normally distributed, $\mu = 100$)

The fuzzy system outperforms the conventional system in all cases unless the demand becomes very low when the results are more mixed. This is entirely consistent with all of the previous analyses. It is clear that the fuzzy system does not find high variances any more difficult to deal with than the conventional system.

Conclusion of results

This analysis shows that a basic fuzzy inventory replenishment system will outperform a basic conventional system in many different scenarios. Apart from very low demand where results are mixed, the major area where the basic fuzzy system developed does not perform well occurs when there is a step change in demand. While this is a common occurrence in many situations, it is usually expected - even if actual forecasts may be difficult. Moreover, actual forecasts are not required in the fuzzy system; instead the bounds of demand are the key drivers of the fuzzy system. Thus, a demand range is the key forecast required rather than the actual measure. This fuzzy approach, therefore has many benefits not only in terms of performance but also in terms of more stable order sizes, more stable stock levels and easier forecast requirements.

However, this analysis only considers very simple conventional and fuzzy inventory replenishment systems. Further sophistication of either system may change the results. These results, therefore, provide a platform on which to build future research.

Chapter 7

REFLECTIONS AND FUTURE DIRECTIONS
Past, present and future fuzzy inventory research

This final chapter aims to reflect on the results as well as the research process, and suggests some possible directions for future research into inventory replenishment systems, which builds on the results of this thesis.

The research questions

This thesis has answered the research questions identified in Chapter 1

1. *What are the major approaches to inventory replenishment?*

   A set of approaches to inventory replenishment have been identified and validated through the historical analysis (see Chapter 2). They are: minimal, maximal, economic, stochastic and heuristic.

2. *What are the strengths and weaknesses of current approaches?*

   All approaches have strengths and weaknesses either in theory or practice (see Chapter 3). Moreover, with the potential exception of a heuristic approach, it is necessary to combine or compromise approaches in order to abate some of the weaknesses identified. This appears to be primarily due to their bivalent nature and hence a focus on either cost or customer service but not both.

3. *How might a new replenishment system build on the strengths and negate some of the weaknesses of current approaches?*

   A new replenishment system taking a heuristic approach and based on fuzzy logic may be able to negate some of the weaknesses by allowing multi-valent handling of the competing inventory objectives of minimising cost and maximising customer service (see Chapter 4). This could be built upon the strengths of one or more of the existing approaches. For example, it can be argued that the fuzzy inventory replenishment system developed in this thesis is based on the underlying structure of a typical, combined minimal and stochastic approach to inventory replenishment.

4. *How does such an original replenishment system compare with a typical, current system?*

   Compared to a simple stochastic system of today (see Chapter 5), the fuzzy-based system developed achieves higher customer service for the same cost, as well as providing many practical benefits in terms of more consistent order sizes and resultant inventory levels (see Chapter 6).

5. *Is there a future for such an inventory replenishment system?*

   Given the results generated (see Chapter 6), there appears to be a future for fuzzy-based replenishment systems. Moreover, the huge flexibility associated with a fuzzy system implies that it could be applied to a wide range of inventory scenarios.

Therefore, this thesis has provided some valuable answers to the research questions posed. Reflecting on the results and methodological process may suggest some directions for future research in this area.

Reflecting on the results

Through an historical analysis, this thesis has argued that future research into inventory management should take a more heuristic approach, focusing not just on cost or customer service, nor adopting a singular method (see page 28). Instead the approach proposed is to see inventory as open to multiple viewpoints, and never having any singular characteristic for all time. After all, inventory management is very time dependent, as demonstrated in the historical analysis (see page 24). This thesis rejects the Just-In-Time approach as a 'cure all' for inventory managers and instead aims to place inventory in a multi-dimensional perspective, allowing for theoretical contradictions and practical realities.

An example of such a fuzzy approach to inventory replenishment has been developed (see Chapter 4). The approach appears to have potential, not only in terms of results (see Chapter 6) but also in terms of coping with contradictions and reality (see page 42).

However, the approach may also be criticised. It clearly relies on some understanding of the demand for a product, as it copes badly with step changes in demand (see page 132). It does not react to these changes quickly as part of its 'natural' process. However, it can deal with these changes if they are flagged up beforehand and there is some understanding of the general change to be experienced (see page 139). There is still some debate as to whether fuzzy logic is pseudo-probability, though it has been argued that this is not the case (see page 46). Certainly, this thesis has generated very different results taking a fuzzy rather than a stochastic approach (see Chapter 6). More subtlety, it could be argued that the whole approach is actually some form of (exponential) smoothing, just a forecasting method in disguise rather than an inventory replenishment system. While, on the surface this argument may seem to have some weight, particularly given the smooth nature of the results (see page 140), this cannot be the case. After all, exponential smoothing is often employed to forecast demand in an order up-to system, which would still fluctuate in terms of orders and stock level despite a smooth forecast. Indeed in many of the experiments carried out, the forecast has been completely smooth, static even, being based on a theoretical distribution around a static mean, and yet the order up-to system has still shown a tendency to fluctuate greatly (see page 99 onwards).

In the final analysis, the fuzzy inventory replenishment system developed in this thesis appears to perform better than the conventional system to which it is compared (see Chapter 6). Furthermore, it is highly adaptable and could be enhanced in many different ways. Before considering some of these potential developments (see page 146), it is worth considering the research methodology employed here.

Reflecting on the methodology

The research methodology employed here is perhaps an obvious one, from idea to literature review, to survey and evaluation, to conceptual model and experimentation and finally conclusion. An alternative research approach may have provided other benefits.

An apparent weakness of the research methodology chosen is the lack of focus on individual issues, markets or organisations. The approach taken has been to develop thoughts and models that are universal. This has been informed, or biased, by experience, discussion with business managers and academics and by the product examples supplied by Tesco, as well as time. The elapsed time from idea to thesis documentation has been long, which while bringing problems of its own, has allowed for much reflection and informal validation of views and ideas.

On the other hand, a major strength of the research methodology has been the rigour required, especially when making proposals regarding universal phenomenon. This has included the need to focus on the detailed issue under investigation rather than get trapped into explaining by-products of the research. For example, it would have been quite possible to end up discussing the relative merits of different forecasting systems, rather than the relative merits of inventory replenishment systems.

Finally, it must be recognised that this thesis is limited to only comparing the fuzzy model developed with a simple stochastic model. However, this research methodology has opened up possibilities for future research rather than closed them down.

Future research into inventory replenishment systems

Future research in this area is potentially very rich, in terms of both process and content. This is just the beginning of defining, developing and enhancing a fuzzy approach to inventory replenishment systems. In particular, there is great potential for research into developing fuzzy membership functions and rules for inventory management. This may include the application of a wide-range of different tools and techniques, some focused on content while others focus on process. For example, the application of neural networks to solving fuzzy problems may result in some valuable new inventory replenishment techniques. In contrast, Soft Systems Methodology (SSM) may be employed to review inventory decision making within a firm from multiple viewpoints, enabling the development of consensual fuzzy rules or even clarifying how power within the organisation or supply chain affects the view of overstock or understock..

The development possibilities of the fuzzy approach to inventory management are literally endless. Also, unlike early developments of the EOQ, there is enough computing power to put successful ideas into immediate widespread practice. Thus, theory and practice can and need to work together in developing this potentially valuable area of research.

Richard Cuthbertson, May 2001.

APPENDICES

A Practitioner survey

Companies surveyed

The practitioner companies surveyed who replied are as follows:

Alldays

Allied Carpets

Booker Belmont Wholesale

Brother International

Budgens Stores

Clarks International

Courtaulds Chemicals

Dillons

Do It All

Dunnes Stores

Eastman Kodak

Fiat

Fosters Trading Company

GlaxoWelcome

Great Mills

Guinness

Habitat

Halfords

Hodgkinsons Stationers

Iceland Frozen Foods

ITT Automotive

Nike

Oddbins

Rank Xerox

Sainsburys

Scottish Courage

Somerfield

The Body Shop International

Total Oil Marine

Unipart Group of Companies

United Distillers

Vauxhall

YHA Adventure Shops

Zeneca

Survey letter

Figure 68 shows a survey letter to a practitioner:

23 May 1997

Dear Stock Controller / Inventory Manager,

I am a Senior Lecturer at Bournemouth University researching inventory management for a MPhil/PhD at Southampton University. I would like a few minutes of your time in filling out the short questionnairre attached. In return, I will provide you with a summary of responses from this survey, within which it will not be possible to identify an individual company's response. Indeed, any information you provide will be treated as confidential and will not be identifiable to any other party, though I may list your company as helping with this research.

Please complete as much of the questionnaire as possible and feel free to add comments at any point. The information you provide is likely to be collated in mid-June so an early response is appreciated, even if incomplete.

I look forward to your reply.

Yours Sincerely,

Richard Cuthbertson.

Senior Lecturer.

Figure 68: Practitioner survey letter

Example questionnaire

Figure 69, below and overleaf, shows an example of a returned questionnaire.

Inventory Management Survey

Who Are You?

Company	ICELAND FROZEN FOODS
Name of respondent	ROD JONES
Job title	INVENTORY M.I.S. MANAGER
Telephone number	01244 842869

What major market sectors is your company involved in?

Fashion	
Food	✓
Electrical goods	
Stationary	
Other (please state)	

What is your annual company turnover?

		(please specify if known)
Under £0.5 million		
£0.5 to £5 million		
Over £5 million	✓	£1.5 bn

ALL THE REMAINING QUESTIONS RELATE TO THE RETAIL STORE INVENTORY MANAGEMENT SYSTEM

Name of major inventory management system(s) used	E3 SLIM
Supplier of major inventory management system(s) used	E3 UK LTD.

What are the main factors used for classifying stock?

Demand	✓	
Perishability	✓	
Size		
Value	✓	
Other (please state)		

What are the major forecasting techniques used?

Exponential smoothing		
Manual		
Moving average	✓	
Other (please state)	✓	SEASONAL

What are the main order frequencies? (by type of stock if different)

More than once a day		
Daily	✓	
Weekly		
Monthly		
Less than once a month		

Page 1

What are order quantity calculations based on? (by type of stock if different)

Economic order sizes	[ ]	
Maximum order sizes	[ ]	
Minimum order sizes	[ ]	
Order frequency	[✓]	
Other (please state)	[ ]	

What are safety stock calculations based on? **(by type of stock if different)**

Cost of supplying "emergency" orders	[ ]	
Customer service level target (please state)	[✓]	99.5% ON SHELF.
Other (please state)	[✓]	LEAD TIME + DEMAND VARIABILITY + PRESENTATION STOCK

Inventory Management Performance

Why might too much stock be held? **(please state particular reasons where currently occurring)**

Large discount in buying stock	[ ]	
Large safety stock required	[ ]	
Low sales	[ ]	
Poor sales forecast	[✓]	PROMOTIONAL SALES
Promotional/merchandising stock	[ ]	
Other (please state)	[ ]	

Why might stockouts occur? **(please state particular reasons where currently occurring)**

Safety stock too small	[ ]	
High sales	[✓]	PROMOTIONAL SALES.
Poor sales forecast	[✓]	
Delivery/supply problems	[ ]	
Other (please state)	[ ]	

How is inventory management performance measured?
(Please state actual figures if possible)

(by type of stock if different)

Stock turn	[✓]	
Total value of stock	[✓]	
Total sales	[✓]	
Stockout frequency	[✓]	
Other (please state)	[ ]	

Thank you for your time and co-operation
Please return to R Cuthbertson. Department of Retail Management, Bournemouth University, Fern Barrow, Poole, BH12 5BB.

Page 2

Figure 69: Example of returned practitioner questionnaire

Practitioners telephone survey

The structure of the telephone survey of practitioners is shown below.

1. Introduce myself.

2. Thanks for participating in survey.

3. Explain require confirmation of understanding and identification of main approach to inventory management.

4. From survey, confirm use/non-use of economic order quantity.

5. From survey, confirm use/non-use of customer service level targets based on probability.

6. Ask "what is your major approach to inventory replenishment?" Discuss this to confirm major approach taken.

7. Thanks and goodbye.

B Software survey

Companies surveyed

The software companies surveyed are as follows:

Apex Systems

Control Group

Dataday Computer Systems

Eclipse Software

Execulink Inventory replenishment

Interactive

International Business Systems

Kalamazoo

Kerridge Computer Company

LogicLine

Lucas Bear and Prescient Systems

Minerva Industrial Systems

Panacea

Siemens Nixdorf Information Systems

Software telephone survey

The structure of the telephone survey of inventory management software providers is shown below.

1. Introduce myself.

2. Thanks for participating in survey.

3. Explain require confirmation of understanding and identification of main approach(es) to inventory management supported by their software.

4. Ask "what are the major approaches to inventory replenishment that can be supported by your software?" Discuss this to confirm the potential major approach(es) available using this software.

5. Thanks and goodbye.

C Survey of literature

The following papers were included in the survey of literature:

ANBAR Brief Citations

| Anbar Home | Database Menu | Browse Index | Form Search | Expert Search | Shopping Basket | Help |

Results for: *su=inventory*
91 records matched your search criteria. Returning: 91

A stock answer (efficient customer response)
Field C
Computer Weekly; 20 Feb 97;

Buffering from material recovery uncertainty in a recoverable manufacturing environment
Guide V D R, Srivastava R
Journal of the Operational Research Society; May 97 (48/5);

Deterministic inventory models for variable production
Bhunia A K, Maiti M
Journal of the Operational Research Society; Feb 97 (48/2);

Don't automate inventory tracking, eliminate it
Stahl R A
APICS - The Performance Advantage; Apr 97 (7/4);

Estimating the demand pattern for C category items
Bradford J W, Sugrue P K
Journal of the Operational Research Society; May 97 (48/5);

Getting lean: a new strategy for remanufacturers
Cooper M H, Wellens P L
APICS - The Performance Advantage; Mar 97 (7/3);

Jackpot: how to win the migration game
Freeman E
Datamation; Mar 97 (43/3);

Measuring forecast accuracy: some practical suggestions
Sanders N R
Production and Inventory Management Journal; Vol 38 No 1 97;

Optimal provisioning strategies for slow moving spare parts with small lead times
Klein Haneveld W K, Teunter R H
Journal of the Operational Research Society; Feb 97 (48/2);

Optimized inventory management
Harris T
Production and Inventory Management Journal; Vol 38 No 1 97;

Retailing logistics: all change at the checkout
Peck F
Logistics Focus; Apr 97 (5/3);

Safeguarding your assets (risk management)
Sparrow A
Professional Manager; May 97 (6/3);

Watch your assets (asset management software)
Hobby J
Computer Weekly; 9 Jan 97;

A backlog inventory model during restricted sale periods

Anll-Hyde R I.
Journal of the Operational Research Society; Sep 96 (47/9);

A graphical aid for the initial purchase of 'insurance type' spares
Walker J
Journal of the Operational Research Society; Oct 96 (47/10);

A heuristic algorithm for the capacitated multiple supplier inventory grouping problem
Syam S S, Shetty B
Decision Sciences; Autumn 96 (27/4);

A model for a mixed continuous-periodic review one-warehouse, N-retailer inventory system
Ahire S L, Schmidt C P
European Journal of Operational Research; 5 Jul 96 (92/1);

A new optimal algorithm for the joint replenishment problem
Viswanathan S
Journal of the Operational Research Society; Jul 96 (47/7);

A note on 'an approximate solution to deterministic kanban systems'
Vemuganti G, Batta R, Zhu Y
Decision Sciences; Autumn 96 (27/4);

A note on 'an approximate solution to deterministic kanban systems': a commentary and further insights
Moeeni F, Chang Y-L
Decision Sciences; Autumn 96 (27/4);

A note on the newsboy problem with an emergency supply option
Khouja M
Journal of the Operational Research Society; Dec 96 (47/12);

A stocking policy for spare part provisioning under age-based preventive replacement
Kabir A B M Z, Al-Olayan A S
European Journal of Operational Research; 5 Apr 96 (90/1);

Alternative inventory and distribution policies of a food manufacturer
Mercer A, Tao X
Journal of the Operational Research Society; Jun 96 (47/6);

An EOQ model for deteriorating items with time varying demand and costs
Giri B C, Goswami A, Chaudhuri K S
Journal of the Operational Research Society; Nov 96 (47/11);

An interactive dynamic inventory-production control system
Polomsinnos J, Orman A J
Journal of the Operational Research Society; Aug 96 (47/8);

An inventory model with two-component demand rate and shortages
Paul K, Datta T K, Chaudhuri K S, Pal A K
Journal of the Operational Research Society; Aug 96 (47/8);

Capacity planning under different inspection strategies
Gurnani H, Drezner Z, Akella R
European Journal of Operational Research; 8 Mar 96 (89/2);

Cash flow optimization in delivery scheduling
Dror M, Trudeau P
European Journal of Operational Research; 8 Feb 96 (88/3);

Delivering more than the customer demands (software)
Schroeder B, Sems P
APICS - The Performance Advantage; Jun 96 (6/6);

Distributed planning for a divergent depotless two-echelon network under service constraints

Vernijdt J H C M, de Kok A G
European Journal of Operational Research; 8 Mar 96 (89/2);

Fibre maker uses RF to improve inventory tracking
Schultz G
Managing Automation; May 96 (11/5);

Forecasting for items with intermittent demand
Johnston F R, Boylan J E
Journal of the Operational Research Society; Jan 96 (47/1);

Integrated multi-item production-inventory systems
Arreola-Risa A
European Journal of Operational Research; 8 Mar 96 (89/2);

Internal, vertical and horizontal logistics integration in Italian grocery distribution
Caputo M, Minlano V
International Journal of Physical Distribution & Logistics Management; Vol 26 No 9 96;

Methods of inventory monitoring and measurement
Ballard R L
Logistics Information Management; Vol 9 No 3 96;

Mixture inventory model with backorders and lost sales for variable lead time
Ouyang L-Y, Yeh N-C, Wu K-S
Journal of the Operational Research Society; Jun 96 (47/6);

Networked inventory management information systems: materializing supply chain management
Verwijmeren M, van der Vlist P, van Donselaar K
International Journal of Physical Distribution & Logistics Management; Vol 26 No 6 96;

Object-oriented model construction in production scheduling decisions
Pillutla S N, Nag B N
Decision Support Systems; Nov 96 (18/3&4);

On an inventory model for deteriorating items with increasing time-varying demand and shortages
Benkherouf L, Mahmoud M G
Journal of the Operational Research Society; Jan 96 (47/1);

On approximating lead time demand distributions using the generalized l-type distribution
Kumaran M, Achary K K
Journal of the Operational Research Society; Mar 96 (47/3);

Optimal batch size and raw material ordering policy for a production system with a fixed-interval, lumpy demand delivery system
Sarker B R, Parija G R
European Journal of Operational Research; 22 Mar 96 (89/3);

Optimal EOQ models for deteriorating items with time-varying demand
Hariga M
Journal of the Operational Research Society; Oct 96 (47/10);

Optimal time varying lot-sizing models under inflationary conditions
Hariga M A, Ben-Daya M
European Journal of Operational Research; 8 Mar 96 (89/2);

Performance indicators in distribution
van Amstel R P, D'hert G
The International Journal of Logistics Management; Vol 7 No 1 96;

Reducing inventory costs by order splitting in the sole sourcing environment
Chiang C, Chiang W-C
Journal of the Operational Research Society; Mar 96 (47/3);

Simple models and insights for warehouse sizing
Cormier G, Gunn E A
Journal of the Operational Research Society; May 96 (47/5);

Tailoring your MRP system to meet your needs
Estep J A
IIE Solutions; Sep 96 (28/9);

Take a year 2000 inventory
de Jager P
Datamation; Aug 96 (42/14);

~~The Herrmann brain dominance inventory and development goals~~
~~Estienne M~~
~~*Organisations & People: Successful Development*; Nov 96 (3/4);~~

The impact of demands' correlation on the effectiveness of component commonality
Eynan A
International Journal of Production Research; Jun 96 (34/6);

The impact of restricting the flow of inventory in serial production systems
Atwater J B, Chakravorty S S
International Journal of Production Research; Sep 96 (34/9),

The newsboy problem with multiple discounts offered by suppliers and retailers
Khouja M
Decision Sciences; Summer 96 (27/3);

The newsstand problem: a capacitated multiple-product single-period inventory problem
Lau H-S, Lau A H-L
European Journal of Operational Research; 11 Oct 96 (94/1);

The trended inventory lot sizing problem with shortages under a new replenishment policy
Goyal S K, Hariga M A, Alyan A
Journal of the Operational Research Society; Oct 96 (47/10);

Utilizing inventory flow models with suppliers
Farris M T
Journal of Business Logistics; Vol 17 No 1 96;

Warehouse location with uncertain stock availability
Meshkat H, Ballou R H
Journal of Business Logistics; Vol 17 No 2 96;

You make the choice (inventory management)
Harris T
APICS - The Performance Advantage; Jun 96 (6/6);

A re-examination of the effect of just-in -time on inbound logistics
Tracey M, Tan C L, Vonderembse M, Bardi E J
The International Journal of Logistics Management; Vol 6 No 2 95;

Inventory - asset or liability?
Crandall R E
APICS - The Performance Advantage; Apr 95 (5/4)c;

The new training: operations methods for high performance
Bassett G
Human Resource Development Quarterly, Autumn 95 (6/3);

An integrated DSS for global logistics
Min H, Eom S B
International Journal of Physical Distribution & Logistics Management, Vol 24 No 1 94;;

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Norris D M, Swanson R D, Chu Y-L
Production and Inventory Management, Vol 35 No 2 94:

Measuring inventory management performance
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Senln S, McGrath R
Information & Management, Aug 93 (25/2)::

Buyer-supplier relations in the UK automotive industry
Turnbull P
Strategic Management Journal: Feb 92 (13/2):;

When JIT is not JIT
Luthmaus R R, Duclos-Wilson L
Production and Inventory Management, Vol 33 No 2 92*:;

A case of Japanization? (accounting and manufacturing)
Munday M
Management Accounting; Mar 91 (69/3)::

Make room, make room (JIT)
Fashbaf M R
Manufacturing Engineer, Sep 91 (70/7)::

Managing uncertainty in multi-level manufacturing sytems
Vargas G A, Dear R G
Integrated Manufacturing Systems; Vol 2 No 4 91:;

Pillow talk for productivity
Klein L, Jacques R M
Management Accounting; Feb 91 (72/8)::

Inventory accuracy - is it worth it?
Meyer H
Production and Inventory Management, Vol 31 No 2 90::

JIT implementation within a service industry
Mehra S, Inman R A
International Journal of Service Industry Management, Vol 1 No 3 90::

Managing European logistics assets
Braithwaite A
Logistics Today; Jul/Aug 90 (9/4): p. 22 +;

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Larson P D, DeMarais R A
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Ramsay M L
Industrial Engineering; Feb 90 (22/2)::

Recasting the traditional inventory model to implement JIT
Ramasesh R V
Production and Inventory Management; Vol 31 No 1 90::

Schedule stability and the implementation of just-in-time
Chapman S N
Production and Inventory Management; Vol 31 No 3 90::

Successful supplychain management
Stevens G C
Management Decision; Vol 28 No 8 90::

The component chart
Livne Z A, Ronen B
Production and Inventory Management; Vol 31 No 1 90::

Time to take stock
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BPICS Control; Feb/Mar 90 (16/2)::

Better service with less inventory
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P&IM Review; Oct 89 (9/10)::

Evaluation of the effects of holding excess inventories
Cheatham L R
Managerial Finance; Vol 15 No 6 89::

Inventory management for hospitals
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Information Strategy: the Executive's Journal; Autumn 89 (6/1)::

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Rosenfield D B
Harvard Business Review; Jul/Aug 89 (67/4)::

Trade-offs to consider in industrial distribution
Powers T L
Industrial Marketing Management; Aug 89 (18/3)::

Is today's office receiving full value from its computers?
Hammer C
Information & Management; Aug 88 (15/1)::

JIT for small manufacturers
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Journal of Small Business Management; Oct 88 (26/4)::

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Industrial Marketing Digest; Vol 13 No 3 88::

Short-range production-based forecasting
Chase C W
The Journal of Business Forecasting; Winter 88/89 (7/4)::

Throughput accounting
Waldron D, Galloway D
Management Accounting; Nov 88 (66/10): p. 34 + Dec 88 (66/11)::

ANBAR

D Fuzzy logic operations

This appendix explains the basic fuzzy logic operations. In order to explain fuzzy logic operations it is first necessary to define fuzzy sets and then define the operations on those sets.

Defining fuzzy sets

Fuzzy sets may be defined by their membership functions. This can be formalised for a discrete and finite universe X as follows:

$$\underset{\sim}{A} = \left\{ \frac{\mu_{\underset{\sim}{A}}(x_1)}{x_1} + \frac{\mu_{\underset{\sim}{A}}(x_2)}{x_2} + \ldots.. \right\} = \left\{ \sum_i \frac{\mu_{\underset{\sim}{A}}(x_i)}{x_i} \right\}$$

where the horizontal bar is not a quotient but a delimiter showing the membership value (the numerator) for the relevant element (the denominator). Also the '+' sign represents an aggregation of the terms (the function-theoretic union) rather than the algebraic 'addition'. For example the classical sets represented by current inventory replenishment systems maybe written as a special case of fuzzy set as follows.

For a continuous and infinite universe X the notation is as follows:

$$\underset{\sim}{A} = \left\{ \int \frac{\mu_{\underset{\sim}{A}}(x)}{x} \right\}$$

where again the horizontal bar represents a delimiter and the integral sign ($\int$) represents a (continuous) function-theoretic union.

Fuzzy set operations

The basic set operations of union, intersection and complement are defined in the same way regardless of whether the set is fuzzy or classical[147]. Let $\underset{\sim}{A}$ and $\underset{\sim}{B}$ be fuzzy sets within the universe of X, then

the **union** of two fuzzy sets is described by their membership functions as

$$\mu_{\underset{\sim}{A} \cup \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \vee \mu_{\underset{\sim}{B}}(x)$$

the **intersection** of two fuzzy sets is described by their membership functions as

$$\mu_{\underset{\sim}{A} \cap \underset{\sim}{B}}(x) = \mu_{\underset{\sim}{A}}(x) \wedge \mu_{\underset{\sim}{B}}(x)$$

and the **complement** of two fuzzy sets is described by their membership functions as

$$\mu_{\overline{\underset{\sim}{A}}}(x) = 1 - \mu_{\underset{\sim}{A}}(x)$$

Subsets, the universe set and the null set are also defined in the same way regardless of whether using fuzzy or classical set theory. So,

the **null set** is described by the membership function as

$$\forall x \in X, \mu_{\varnothing}(x) = 0$$

the **universe set** is described by the membership function as

$$\forall x \in X, \mu_X(x) = 1$$

the **subset** is described by the membership function as

$$\underset{\sim}{A} \subseteq X \Rightarrow \mu_{\underset{\sim}{A}}(x) \leq \mu_X(x)$$

DeMorgan's laws also hold for fuzzy sets[147] in the same way they do for classical sets, so

$$\overline{\underset{\sim}{A} \cap \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cup \overline{\underset{\sim}{B}}$$

and

$$\overline{\underset{\sim}{A} \cup \underset{\sim}{B}} = \overline{\underset{\sim}{A}} \cap \overline{\underset{\sim}{B}}$$

However the **excluded middle laws** do not hold for fuzzy sets[147] (except in the special case of classical sets). So in general for fuzzy sets,

$$\underset{\sim}{A} \cup \overline{\underset{\sim}{A}} \neq X$$

and

$$\underset{\sim}{A} \cap \overline{\underset{\sim}{A}} \neq \varnothing$$

Properties of fuzzy sets

Let $\underset{\sim}{A}$, $\underset{\sim}{B}$ and $\underset{\sim}{C}$ be fuzzy sets within the universe of X, then commonly used properties of fuzzy sets are as follows.[148]

Associativity

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cup \underset{\sim}{C}$$

$$\underset{\sim}{A} \cap \left(\underset{\sim}{B} \cap \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cap \underset{\sim}{B}\right) \cap \underset{\sim}{C}$$

Distributivity

$$\underset{\sim}{A} \cup \left(\underset{\sim}{B} \cap \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cup \underset{\sim}{B}\right) \cap \left(\underset{\sim}{A} \cup \underset{\sim}{C}\right)$$

$$\underset{\sim}{A} \cap \left(\underset{\sim}{B} \cup \underset{\sim}{C}\right) = \left(\underset{\sim}{A} \cap \underset{\sim}{B}\right) \cup \left(\underset{\sim}{A} \cap \underset{\sim}{C}\right)$$

Idempotency

$\underset{\sim}{A} \cup \underset{\sim}{A} = \underset{\sim}{A}$ and $\underset{\sim}{A} \cap \underset{\sim}{A} = \underset{\sim}{A}$

Identity

$\underset{\sim}{A} \cup \varnothing = \underset{\sim}{A}$ and $\underset{\sim}{A} \cap X = \underset{\sim}{A}$

$\underset{\sim}{A} \cap \varnothing = \varnothing$ and $\underset{\sim}{A} \cup X = X$

Transitivity

If $\underset{\sim}{A} \subseteq \underset{\sim}{B} \subseteq \underset{\sim}{C}$, then $\underset{\sim}{A} \subseteq \underset{\sim}{C}$

Involution

$$\overline{\overline{\underset{\sim}{A}}} = \underset{\sim}{A}$$

E Fuzzy inventory papers

The following papers were identified including both "fuzzy" and "inventory" as key words:

Chang S-C, Yao J-S & Lee H-M (1998). Economic reorder point for fuzzy backorder quantity. *European Journal of Operational Research,* **109**:1 183-202.

Chang T-M & Yih Y (1998). A fuzzy rule-based approach for dynamic control of kanbans in a generic kanban system. *International Journal of Production Research,* **36**:8 102-107.

Chen S-H & Wang C-C (1996). Backorder fuzzy inventory model under function principle. *Intelligent Systems,* **95** 71-79.

Chu C-H & Hayya JC (1991). A fuzzy clustering approach to manufacturing cell formation. *International Journal of Production Research,* **29**:7 1475-1487.

Fang XD & Biglari FR (1995). Real-time fuzzy logic control for maximising the tool life of small-diameter tools. *Fuzzy Sets and Systems,* **72**:1 91-101.

Ishii H, & Konno T (1998). A stochastic inventory problem with fuzzy shortage cost. *European Journal of Operational Research,* **106**:1 90-94.

Kim DB, Hwang H & Yoon WC (1995). Developing a dispatching rule for an Automated Guided Vehicle System using fuzzy logic. *Engineering Optimisation,* **24**:1 39-57.

Lam SM & Wong DS (1996). A fuzzy mathematical model for the joint economic lot size problem with multiple price breaks. *European Journal of Operational Research,* **95**:3 611-622.

Lee H-M & Yao J-S (1998). Economic production quantity for fuzzy demand quantity and fuzzy production quantity. *European Journal of Operational Research,* **109**:1 203-211.

Lee YY, Kramer BA & Hwang CL (1991). A comparative study of three lot sizing methods for the case of fuzzy demand. *International Journal of Operations and Production Management,* **11**:7 72-80.

Liu B & Esogbue AO (1996). Fuzzy criterion set and fuzzy criterion dynamic programming. *Journal of Mathematical Analysis and Application,* **199** 293-311.

Liu B (1999). Fuzzy criterion models for inventory systems with partial backorders. *Annals of Operational Research,* **87**:1 117-126.

Liu B & Odanaka T (1999). Dynamic fuzzy criterion model for reservoir operations and case study. *Computers and Mathematics with Applications,* **37**:11/12 65-75.

Mandal M, Roy TK & Maiti M (1998). A fuzzy model of deteriorating items with stock dependent demand. *Opsearch,* **35**:4 323-337.

Nagata M, Yamaguchi T & Kono Y (1995). An interactive method for multi-period multiobjective production-transportation programming problems with fuzzy coefficients. *Japanese Journal of Fuzzy Theory and Systems,* **7**:1 81-95.

Pappis CP & Karacapilidis NI (1995). Lot size scheduling using fuzzy numbers. *ITOR,* **2**:2 205-212.

Park JW (1988). Development of a company-tailored part classification and coding system using fuzzy logic. *Journal of the Korean OR-MS Society,* **13**:1 31-38.

Pandharkar PC (1997). A fuzzy linear programming model for production planning in coal mines. *Computers and Operational Research,* **24**:12 1141-1149.

Petrovic D, Petrovic R & Vujosevic M (1996). Fuzzy models for the newsboy problem. *International Journal of Production Economics,* **45** 435-441.

Petrovic D, Roy R & Petrovic R (1998). Modelling and simulation of a supply chain in an uncertain environment. *European Journal of Operational Research,* **109**:2 299-309.

Petrovic D, Roy R & Petrovic R (1998). Supply chain modelling using fuzzy sets. *International Journal of Production Economics,* **59**:1/3 443-453.

Petrovic D & Sweeney E (1994). Fuzzy knowledge-based approach to treating uncertainty in inventory control. *Computer Integrated Manufacturing Systems,* **7**:3 147-152.

Roy TK & Maiti M (1995). A fuzzy inventory model with constraint. *Opsearch,* **32**:4 287-298.

Roy TK & Maiti M (1997). A fuzzy EOQ model with demand-dependent unit cost under limited storage capacity. *European Journal of Operational Research,* **99**:2 425-432.

Roy TK & Maiti M (1998). Multi-objective inventory models of deteriorating items with some constraints. *Computers and Operational Research,* **25**:12 1085-1095.

Sengupta JK (1992). A fuzzy-systems approach in data envelopment analysis. *Computers and Mathematics with Applications,* **24**:8/9 259-266.

Singh N & Mohanty BK (1991). A fuzzy approach to multi-objective routing problem with applications to process planning in manufacturing systems. *International Journal of Production Research,* **29**:6 1161-1170.

Tuma A, Haaisis H-D & Rentz O (1996). Development of emission orientated production control strategies using fuzzy logic. *Fuzzy Sets and Systems,* **77**:3 255-264.

Vujosevic M, Petrovic D & Petrovic P (1996). EOQ formula when inventory cost is fuzzy. *International Journal of Production Economics,* **45** 499-504.

Wang M-JJ & Chang T-C (1995). Tool steel materials selection under fuzzy environment. *Fuzzy Sets and Systems,* **72**:3 263-270.

Yao J-S & Lee H-M (1996). Fuzzy inventory with backorder for fuzzy order quantity. *Information Sciences,* **93** 283-319.

Yao J-S & Lee H-M (1999). Fuzzy inventory with or without backorder for fuzzy order quantity with trapezoid fuzzy number. *Fuzzy Sets and Systems,* **105**:3 311-337.

Yenradee P, Van Oudheusden DL & Tabucanon MT (1995). Sequence for managing the situational factors to improve the performance of production and inventory control system. *International Journal of Production Research,* **33**:12 3349-3366.

F FIRS development process

The three major stages of the FIRS development were as follows:

1. **Basic model**: to build an evaluation model comparing a basic fuzzy inventory replenishment system with a conventional one.

2. **Simulation model**: to create a model that is able to compare the fuzzy and conventional systems over a range of business scenarios defined by the extraneous variables.

3. **Final model**: to develop the fuzzy inventory replenishment system so that it automatically adapts the fuzzy functions.

Basic model

The major objective for this model is to build a basic fuzzy inventory system to illustrate that the basic fuzzy concept is worth investigating.

The initial fuzzy model consisted of the following fuzzy rules:

If sales from stock are high then increase the order size

If sales from stock are about right then slightly reduce the order size

If sales from stock are low then significantly reduce the order size

The fuzzy membership functions for sales from stock were defined as simple linear functions, within the following ranges: high 90% to 100%, about right 67% to 95% and low 0% to 80%.

Simulation model

The fuzzy model defined was then modelled on a spreadsheet. A simple, conventional stochastic inventory replenishment system was then added to the spreadsheet. Using simulated demand data, through trial and error the fuzzy ranges for high, about right and low stock were adjusted to test whether the fuzzy inventory replenishment system could provide better results than a traditional bivalent system.

With the bivalent outputs of the fuzzy rules set at 18%, -5% and –50% to increase, slightly reduce and significantly reduce the order size respectively, then this basic fuzzy model could outperform a basic conventional inventory replenishment system under certain circumstances.

Thus the concept of a fuzzy inventory replenishment system appeared to have potential.

Final model

The final fuzzy model consisted of further simplifying the fuzzy rules to:

If understocked then order a lot

and

If overstocked then order a little

Only two rules are used in order to make it simpler to develop them as self-adapting. In order to make them self-adapting they relate to the demand history. Overstocked is defined as 100% if there is more stock than the maximum historical demand and 0% if there is only enough stock to cover the minimum historical demand. Understocked is defined as 100% if there is less stock than the minimum historical demand and 0% if there is enough stock to cover the maximum historical demand. In order to make the resulting changes in order size self-adapting it was decided to set them according to a customer service strategy.

Thus, the basic concept of a new inventory replenishment system was developed.

G Spreadsheet model and macros

Each spreadsheet model (for example, see *FIRS model v8* on the CD provided) consists of 6 sheets. The first five sheets (labelled 99%, 90%, 80%, 70%, 60% and 50%) represent the main calculation sheets for each of the customer service levels specified. These are discussed on pages 173 to 187. (Note that the name of the sheets always refers to the original customer service target for the conventional model.) The final sheet contains the summary parameters and results - as it is labelled.

Summary parameters and results

The main parameters set for each simulation consist of:

1. The mean potential sales per period

2. Any increase in the mean potential sales per period

3. A set of 5 customer service targets for the conventional model

4. A set of 5 customer service strategies for the fuzzy model

From 1 and 2 above, the mean potential sales in the next period are calculated, as shown in the extract from a spreadsheet model in Figure 70.

Mean Sales per period	100	Next period 100	Increase per period	0

Figure 70: Key sales parameters – example extract from a spreadsheet model

Simulations may then be carried out, based on the customer service levels set (3 and 4 above), with the following example summary of results.

Conventional customer service level	Fuzzy customer service strategy	Conventional Stock cover (demand)	Fuzzy Stock cover (demand)	Conventional Stock cover (sales)	Fuzzy Stock cover (sales)	Conventional Cust serv (lost sales)	Fuzzy Cust serv (lost sales)	Conventional Cust serv (stockouts)	Fuzzy Cust serv (stockouts)	Conventional Replenishment rate	Fuzzy Replenishment rate
99%	100%	124.5%	148.1%	127.0%	148.3%	98.0%	99.9%	81.6%	96.8%	100.0%	100.0%
90%	80%	113.4%	119.7%	116.5%	120.8%	97.3%	99.1%	71.2%	84.8%	100.0%	100.0%
80%	60%	109.3%	109.8%	112.9%	112.1%	96.8%	98.0%	66.4%	73.6%	100.0%	100.0%
70%	40%	106.3%	104.4%	110.3%	108.2%	96.4%	96.5%	60.8%	53.6%	100.0%	100.0%
60%	20%	103.2%	100.3%	107.9%	105.8%	95.7%	94.8%	52.0%	46.4%	100.0%	100.0%
50%	0%	100.2%	94.5%	105.8%	103.1%	94.8%	91.7%	44.0%	35.2%	100.0%	100.0%
		Number of time periods		125							

Figure 71: Customer service levels and summary results – example extract from a spreadsheet model

These results also show the number of time periods simulated.

The results are derived from the calculation sheets.

Calculation sheets

The calculation sheets consist of 3 major components:

1. Input parameters

2. Calculations, for both the conventional and the fuzzy inventory replenishment models

3. Results

Input parameters

The input parameters are shown in Figure 72. The customer service target for the conventional model and the customer service strategies for the fuzzy model are taken from the 'Parameters and results' sheet. From the conventional customer service target, the customer service criteria z-statistic is calculated, using the Normal distribution. From the fuzzy customer service strategy, the mid-points of the symmetrical small order and large order membership functions are calculated, as defined earlier (see page 61).

Input Parameters		
Conventional Model		
(Poisson Distributed) Customer service target	99%	Note: must be 50% or above for this model
Customer Service Criteria	2.33	s.d. from mean
Fuzzy Model		
Customer service strategy	100%	
Small order mid-point	101.00	
Large order mid-point	138.00	

Figure 72: Calculation input parameters – example extract from a spreadsheet model

Calculations

The calculations for the conventional model (see Figure 73) begin with the starting stock (the end stock from the previous period). Demand is then sampled from the relevant distribution, and actual sales calculated.

Actual sales = minimum (start stock, demand)

The remaining intermediate stock is then calculated.

Intermediate stock = start stock – actual sales

The new forecast for demand is derived from the user-defined mean potential sales on the 'Parameters and results' sheet. The order up-to point is then calculated based on the relevant user-defined customer service target on the 'Parameters and results' sheet. The order placed is then calculated.

Order placed = order up to point – intermediate stock

The delivery of stock is the order placed in the previous period. The end stock is then calculated.

End stock = intermediate stock + delivery

The number of incremental replenishments, lost sales and stockouts are then calculated.

Replenishments = if(delivery>0,1,0)

Lost sales = demand – actual sales

Stockouts = if(intermediate stock =0,1,0)

Calculations

Conventional Model

Period	Start Stock	Demand (Sales Requested)	Actual Sales	Intermediate Stock	New Forecast Mean	S.d.	Order Upto Point	Order Placed	Delivery	End Stock	Replenishment	Lost Sales
125	145	100	100	45	100	10	124	79	103	148	1	0
Previous period	124	103	103	21	100	10	124	103	124	145	1	0

Figure 73: Conventional calculation – example extract from a spreadsheet model

The calculations for the fuzzy model (see Figure 74) also begin with the starting stock (the end stock from the previous period). The demand is the same as for the conventional model. Actual sales are then calculated.

$$Actual\ sales = minimum\ (start\ stock,\ demand)$$

The remaining intermediate stock is then calculated.

$$Intermediate\ stock = start\ stock - actual\ sales$$

The degree of overstock and understock is then calculated (see page 53), based on the mean, maximum and minimum sales statistics (see Figure 75) of the last 20 periods sales history (see Figure 76). The order placed is then calculated, as defined previously (see page 52).

Fuzzy Model Period	Start Stock	Demand (Sales Requested)	Actual Sales	Intermediate Stock	Over Stocked	Under Stocked	Order Placed	Delivery	End Stock	Replenishment	Lost Sales
125	164	100	100	64	100%	0%	101	100	164	1	0
Previous period	167	103	103	64	100%	0%	100	100	164	1	0

Figure 74: Fuzzy calculations – example extract from a spreadsheet model

Mean sales in last 20 periods	101.4
Minimum of sales in last 20 periods	83
Maximum of sales in last 20 periods	119

Figure 75: Fuzzy sales statistics – example extract from a spreadsheet model

History															
Sales	103	101	110	92	92	110	83	105	104	107	97	86	100	119	108
Period	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15
Stockouts	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Figure 76: Fuzzy sales history – example extract from a spreadsheet model

Results

The results section captures the key performance measures. Results from the previous period are added to the relevant results for the current simulation period to give the overall results.

Results		
	Traditional	Fuzzy
Number of time periods	125	125
Total inventory (end stock) held	15521	18466
Total demand	12470	12470
Total sales	12226	12452
Total lost sales	244	18
Total stockouts	23	4
Total replenishments	125	125
Stock cover (demand)	124%	148%
Stock cover (sales)	127%	148%
Customer service (lost sales)	98%	100%
Customer service (stockouts)	82%	97%
Replenishment rate	100%	100%
Results - Previous period		
	Traditional	Fuzzy
Number of time periods	124	124
Total inventory (end stock) held	15373	18302
Total demand	12370	12370
Total sales	12126	12352
Total lost sales	244	18
Total stockouts	23	4
Total replenishments	124	124
Stock cover (demand)	124%	148%
Stock cover (sales)	127%	148%
Customer service (lost sales)	98%	100%
Customer service (stockouts)	81%	97%
Replenishment rate	100%	100%

Figure 77: Results - example extract from a spreadsheet model

Initialisation

Before any simulations can be carried out, all relevant parameters need to be set to their starting values (see page 75). This is done by running a macro named "Setup".

```
Sub Setup()
'
' Setup Macro
' Macro recorded 26/11/98 by Richard Cuthbertson Sets up initial conditions
'
' Keyboard Shortcut: Ctrl+s
'
' Select all worksheets
    Sheets(Array("99%", "90%", "80%", "70%", "60%", "50%")).Select
' Reset Periods
    Range("B29,B36").Select
    Selection.FormulaR1C1 = "=1"
' Reset Start Stocks
    Sheets("99%").Select
    Range("C29").Select
    ActiveCell.FormulaR1C1 = _
        "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'99%'!R[-17]C,0)"
    Range("C36").Select
    ActiveCell.FormulaR1C1 = "=R[-7]C"
    Sheets("90%").Select
    Range("C29").Select
    ActiveCell.FormulaR1C1 = _
        "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'90%'!R[-17]C,0)"
    Range("C36").Select
    ActiveCell.FormulaR1C1 = "=R[-7]C"
    Sheets("80%").Select
    Range("C29").Select
    ActiveCell.FormulaR1C1 = _
```

```
    "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'80%'!R[-17]C,0)"
  Range("C36").Select
  ActiveCell.FormulaR1C1 = "=R[-7]C"
  Sheets("70%").Select
  Range("C29").Select
  ActiveCell.FormulaR1C1 = _
    "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'70%'!R[-17]C,0)"
  Range("C36").Select
  ActiveCell.FormulaR1C1 = "=R[-7]C"
  Sheets("60%").Select
  Range("C29").Select
  ActiveCell.FormulaR1C1 = _
    "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'60%'!R[-17]C,0)"
  Range("C36").Select
  ActiveCell.FormulaR1C1 = "=R[-7]C"
  Sheets("50%").Select
  Range("C29").Select
  ActiveCell.FormulaR1C1 = _
    "=ROUNDUP('Parameters and Results'!R[-26]C[-1]+SQRT('Parameters and
Results'!R[-26]C[-1])*'50%'!R[-17]C,0)"
  Range("C36").Select
  ActiveCell.FormulaR1C1 = "=R[-7]C"
'Reset Order Deliveries
  Sheets("Parameters and Results").Select
  Range("B3").Select
  Selection.Copy
  Sheets(Array("99%", "90%", "80%", "70%", "60%", "50%")).Select
  Sheets("99%").Activate
  Range("K29").Select
  ActiveSheet.Paste
  Range("K36").Select
```

```
  ActiveSheet.Paste
' Reset Previous Fuzzy Sales and Stockouts
  Range("E39:X39").Select
  ActiveSheet.Paste
  Range("E41:X41").Select
  Selection.FormulaR1C1 = "0"
' Reset Previous Results Table
  Range("C61:D72").Select
  Selection.FormulaR1C1 = "0"
' Go back to parameter sheet
  Sheets("Parameters and Results").Select
End Sub
```

Figure 78: Setup macro – example extract from a model spreadsheet

Iterations

Between iterations, certain information must be updated (see page 76). This is done by running the macro "Nextpinc".

```
Sub Nextpinc()
'
' Nextpinc Macro
' Macro recorded 14/10/99 by Richard Cuthbertson
'  Updates period and start stock and increases mean sales value
'
' Capture previous period values
  Previousperiod
' Select all worksheets
  Sheets(Array("99%", "90%", "80%", "70%", "60%", "50%")).Select
' Copy Previous Results
  Range("C46:D57").Select
  Selection.Copy
  Range("C61:D72").Select
```

```
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
' Update Periods
    Range("B29").Select
    ActiveCell.FormulaR1C1 = "=R[32]C[1]+1"
    Range("B36").Select
    ActiveCell.FormulaR1C1 = "=R[-7]C"
'Update Start Stocks
    Range("L30").Select
    Selection.Copy
    Range("C29").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
    Range("L37").Select
    Selection.Copy
    Range("C36").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
' Update Deliveries
    Range("J30").Select
    Selection.Copy
    Range("K29").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
    Range("J37").Select
    Selection.Copy
    Range("K36").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
'Update Previous Fuzzy Sales and Stockouts
    Sheets("99%").Select
    Range("E39:W39").Select
    Selection.Copy
    Range("F39:X39").Select
```

```
ActiveSheet.Paste
Range("E41:W41").Select
Selection.Copy
Range("F41:X41").Select
ActiveSheet.Paste
Sheets("90%").Select
Range("E39:W39").Select
Selection.Copy
Range("F39:X39").Select
ActiveSheet.Paste
Range("E41:W41").Select
Selection.Copy
Range("F41:X41").Select
ActiveSheet.Paste
Sheets("80%").Select
Range("E39:W39").Select
Selection.Copy
Range("F39:X39").Select
ActiveSheet.Paste
Range("E41:W41").Select
Selection.Copy
Range("F41:X41").Select
ActiveSheet.Paste
Sheets("70%").Select
Range("E39:W39").Select
Selection.Copy
Range("F39:X39").Select
ActiveSheet.Paste
Range("E41:W41").Select
Selection.Copy
Range("F41:X41").Select
ActiveSheet.Paste
Sheets("60%").Select
Range("E39:W39").Select
```

```
  Selection.Copy
  Range("F39:X39").Select
  ActiveSheet.Paste
  Range("E41:W41").Select
  Selection.Copy
  Range("F41:X41").Select
  ActiveSheet.Paste
  Sheets("50%").Select
  Range("E39:W39").Select
  Selection.Copy
  Range("F39:X39").Select
  ActiveSheet.Paste
  Range("E41:W41").Select
  Selection.Copy
  Range("F41:X41").Select
  ActiveSheet.Paste
  Sheets(Array("99%", "90%", "80%", "70%", "60%", "50%")).Select
  Range("E37").Select
  Selection.Copy
  Range("E39").Select
  Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
     False, Transpose:=False
  Range("O37").Select
  Selection.Copy
  Range("E41").Select
  Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
     False, Transpose:=False
'Show Results Table
  Sheets("Parameters and Results").Select
' Increase mean sales value'
  Range("C3").Select
  Selection.Copy
  Range("B3").Select
  Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
```

```
    False, Transpose:=False
' Go back to parameter sheet
    Sheets("Parameters and Results").Select
End Sub
```

Figure 79: Nextpinc macro – example extract from a spreadsheet model

The "Nextpinc" macro calls on another macro, named "Previousperiod" (see Figure 80), to copy the outcomes from the current period to the previous period in readiness for the next iteration.

```
Sub Previousperiod()
' Previousperiod Macro
' Macro recorded 11/02/01 by Richard Cuthbertson
'
' Keyboard Shortcut: Ctrl+c
'
    Sheets("99%").Select
    Range("C29:O29").Select
    Application.CutCopyMode = False
    Selection.Copy
    ActiveWindow.SmallScroll ToRight:=-7
    Range("C30").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
    Range("C36:O36").Select
    Application.CutCopyMode = False
    Selection.Copy
    ActiveWindow.SmallScroll ToRight:=-2
    Range("C37").Select
    Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
        False, Transpose:=False
    Sheets("90%").Select
```

```
Range("C29:O29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C30").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Range("C36:O36").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C37").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Sheets("80%").Select
Range("C29:O29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C30").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Range("C36:O36").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C37").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Sheets("70%").Select
Range("C29:O29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
```

```
Range("C30").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Range("C36:O36").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C37").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Sheets("60%").Select
Range("C29:O29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C30").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Range("C36:O36").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C37").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
Sheets("50%").Select
ActiveWindow.LargeScroll ToRight:=-1
Range("C29:O29").Select
Application.CutCopyMode = False
Selection.Copy
ActiveWindow.LargeScroll ToRight:=-1
Range("C30").Select
Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
```

```
  Range("C36:O36").Select
  Application.CutCopyMode = False
  Selection.Copy
  ActiveWindow.LargeScroll ToRight:=-1
  Range("C37").Select
  Selection.PasteSpecial Paste:=xlValues, Operation:=xlNone, SkipBlanks:= _
    False, Transpose:=False
  Sheets("Parameters and Results").Select
End Sub
```

Figure 80: Previousperiod macro – example extract from a spreadsheet model

H Definitions of key variables

α_t = customer service based on lost sales (α) in period (t)

β_t = customer service based on stockouts (β) in period (t)

μ = mean demand

ω_t = stock cover based on demand (ω) in period (t)

ρ_t = proportion of replenishments (ρ) in period (t)

σ = standard deviation of demand

$_t$ = stock cover based on sales (ψ) in period (t)

d_t = demand (d) in period (t)

D_t = total demand (D) in period (t)

i_t = inventory (i) at start of period (t)

I_t = total inventory held (I) in all periods up to and including period (t)

k_t = stockouts (k) in period (t)

K_t = total stockouts (K) in all periods up to and including period (t)

l_t = lost sales (l) in period (t)

L_t = total lost sales (L) in all periods up to and including period (t)

$\underset{\sim}{L}$ = fuzzy set of large orders

p_t = order placed (p) in period (t)

$p(c)_t$ = conventional (c) order placed (p) in period (t)

$p(f)_t$ = fuzzy (f) order placed (p) in period (t)

s_t = sales (s) in period (t)

$\underset{\sim}{S}$ = fuzzy set of small orders

t = period (t)

$\underset{\sim}{U}$ = fuzzy set of understock

$\underset{\sim}{V}$ = fuzzy set of overstock

y = customer service strategy in fuzzy inventory replenishment system

z = z-statistic from Normal distribution tables

I Key spreadsheet filenames

The following tables identify the relevant spreadsheet files for the figures (graphs) and tables discussed in Chapter 6. These files can be found on the attached CD.

Theoretical demand

The following files are found in the folder labelled "*1. Theoretical demand*".

Figures	***Filename***
23, 24, 25 & 28	Initial Theoretical Results v8
26, 27 & 29	Initial Theoretical v8 focus
30, 31, 32, 33 & 38, 39, 40, 41, 42	Initial theo focus 100 data v8
34, 35, 36, 37	Initial theo focus 1 data v8
43, 44, 45	Increasing Theoretical results v8
46, 47, 48, 49, 50	Decreasing Theoretical results v8

Table 18: Filenames for theoretical demand figures (graphs) in Chapter 6

Tables	***Filename***
12	Initial Theoretical Results v8
14, 15	Initial theo focus v8 statistics

Table 19: Filenames for theoretical demand tables in Chapter 6

Actual business demand

The following files are found in the folder labelled "*2. Actual business demand*".

Figures	***Filename***
51 & 56	Tesco data analysed
52, 53, 54, 55 & 65	Tesco TVL results v8 cutoff
57, 58, 59, 60, 61, 62, 63, 64 & 66	Tesco LB results v8 cutoff
67	Normal results v8

Table 20: Filenames for figures (graphs) in Chapter 6

Highly variable demand

The following file is found in the folder labelled "*3. Highly variable demand*".

Figures	***Filename***
67	Normal results v8

Table 21: Filenames for figures (graphs) in Chapter 6

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