Cite as: Little, C. & Jones, K. (2007), Contexts for Pure Mathematics: an analysis of A-level mathematics papers, Proceedings of the British Society for Research into Learning Mathematics, 27(1), 48-53.

CONTEXTS FOR PURE MATHEMATICS: AN ANALYSIS OF A-LEVEL MATHEMATICS PAPERS

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While there has been some research into the use of context in mathematics assessments pre-16, little, if any, work exists on the role of context in post-16 mathematics. For A- and AS-level mathematics courses in the UK, assessment schemes are required to include questions that test candidates' abilities to apply mathematical models to real-life contexts, and to translate real-life contexts into mathematics. This paper explores the ways in which context occurs in 'pure' mathematics questions and, through this, suggests a framework for analysis that encompasses issues such as authenticity, accessibility and fitness for purpose.

INTRODUCTION

The use of context in mathematics test items is now accepted practice in many forms of national assessment in the UK, with, on occasion, as many as 50% of questions in a particular set of papers involving some mention of a context external to mathematics. Yet that the use of context is not entirely straightforward is borne out by research. For example, Silver, Shapiro & Deutsch (1993) researched the now famous 'bus' item, reporting how children, when asked to work out how many 36-seater buses would be required to transport 1128 soldiers, included fractions of a bus in their answers. Similarly, Verschaffel, De Corte & Lasure (1994) found that children can fail to apply realistic considerations to their solutions of word problems.

In the UK, Cooper and Dunne (2000) studied National Curriculum test items for mathematics at Key Stage 2 (when pupils are 11) and, while they found a similar range of 'misinterpretations' as Silver *et al* and Verschaffel *et al*, they, interestingly, carried the analysis a stage further by looking at responses in relation to family social class. What they concluded was that the way children applied mathematical procedures was subject to class bias, implying, for Cooper and Dunne, that National Curriculum test items are unreliable. An alternative explanation might be that the test items analysed were flawed in the sense that the degree of realism brought to each item by those taking the tests invited a range of responses that were not taken sufficiently into account by the assessment mark schemes. This raises the issue of the nature and degree of 'realism' presented in assessment items and what influence this might have on the range of responses obtained. While this existing research seems relevant to the situation in A-level mathematics, in surveying the literature, no equivalent research appears to have been carried out in relation to the use of context in post-16 examinations.

CONTEXT IN ADVANCED LEVEL MATHEMATICS

In England, AS and A-level courses in mathematics contain a balance of pure and applied topics, with the current specification delineating two-thirds pure mathematics (see, QCA, 2003). In terms of this pure mathematics, the extent to which A-level mathematics examinations embed such content in real-life contexts could well vary across topics, and perhaps across Examination Boards. While it is likely that some 'pure' mathematics topics might seem intrinsically 'pure' in nature (perhaps topics such as the language of function, binomial series, and techniques of differentiation and integration), other topics (such as arithmetic and geometric series, calculus as rates of change, three-dimensional vector geometry, trigonometry, and exponential growth and decay) might be capable of being treated either as pure mathematics, or as embedded in real-life contexts.

The use of such contexts raises a number of research issues which existing research does appear not to address. This paper focuses on how it might be possible to analyse the use of context in advanced post-16 mathematics examination questions.

THEORETICAL FRAMEWORK AND METHODOLOGY

As Vappula and Clausen-May (2006, p100) say "defining what constitutes a context in a maths test question is more difficult than may at first appear" in that "contexts may serve at least two different functions". One function, according to Vappula and Clausen-May, relates to the match that the selected context might have with the 'reality' of those tackling the examination question, while the second, and quite different function, relates to what Clausen-May (2005, p.39) calls a "model to think with". In this latter function, the context within which the examination question is said to act as mental scaffolding for the student.

Given that these two functions were identified through an analysis of relatively elementary mathematics, the extent to which such functions apply to more advanced post-16 mathematics remains, given the lack of existing research, an open question.

In what follows, the use of context in the specimen pure mathematics papers of two current UK specifications is analysed, with the two specifications coming from two different UK Examination Boards, Edexcel and OCR. In the analysis, while the functions indicated by Vappula and Clausen-May are utilised, the possibility of other functions is kept open via the use of a grounded theory approach (Glaser & Strauss, 1967) through which there are possibilities of building theoretical formulations from the data. The approach is to analyse, and re-analyse, the selected set of examination papers and identify relevant categories and their interrelationships.

ANALYSIS OF A/AS PURE SPECIMEN PAPERS

The C1-C4 core Edexcel papers from 2004 (Edexcel, 2004) contain six questions that include some use of context. For example, Figure 1 shows a question in which an arithmetic series is applied to savings (C1, Q7). In other questions, a badge is used to describe an area formed using the arc of a circle (C2, Q5), geometric series are used to model depreciation and a loan with compound interest (C2, Q6), an exponential decay function is used to model cooling of a substance (C3, Q6), and a differential equation and the chain rule are used to model the growth of a stain (C4, Q5).

7. Ahmed plans to save £250 in the year 2001, £300 in 2002, £350 in 2003, and so on until the year 2020. His planned savings form an arithmetic sequence with common difference £50.

(a) Find the amount he plans to save in the year 2011.

(2)

(b) Calculate his total planned savings over the 20 year period from 2001 to 2020.

(3)

Ben also plans to save money over the same 20 year period. He saves £A in the year 2001 and his planned yearly savings form an arithmetic sequence with common difference £60.

Given that Ben's total planned savings over the 20 year period are equal to Ahmed's total planned savings over the same period,

(c) calculate the value of A.

(4)

Fig 1: Edexcel C1 specimen paper, Arithmetic series model applied to savings

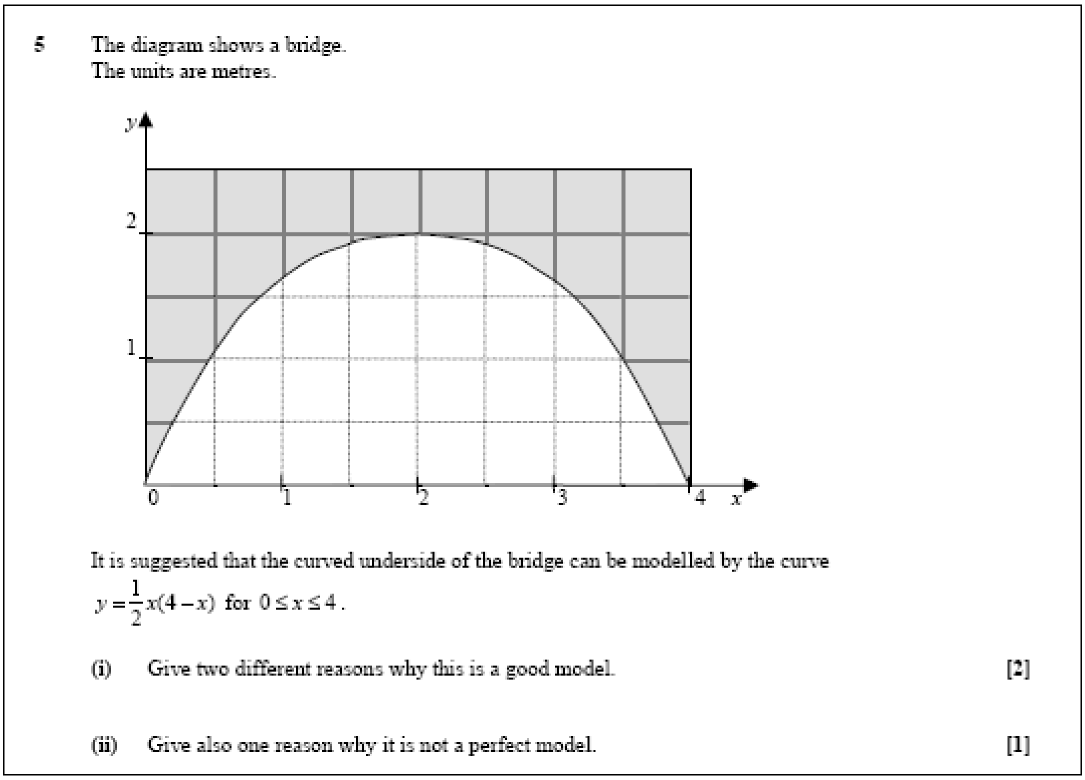
From analysis (and re-analysis) the contexts in these questions appear to be *accessible* to students. For example, in the case of the badge and the geometric series questions, the context would appear to enhance the comprehensibility of the questions to candidates by providing a mental image or scaffolding for students to picture the mathematics.

The contexts could be said to be *realistic*. In C1 Q7 (see Figure 1), the solver is asked to project an arithmetic increase in savings each year, which is not unreasonable. Similarly, in C2 Q6, geometric progressions are natural models for depreciation and compound interest, the 'badge' in C2 Q5 is shaped like a real badge, and in C3 Q6, exponential decay is predicted by Newton's Law of Cooling and is therefore realistic.

The *authenticity* of the arithmetic and geometric progression questions could be said to satisfactory in that the calculations required are germane to the context. For example, the exponential decay model (C3 Q6) predicts results about temperature and cooling rate at different times, which seems worthwhile.

The OCR core pure papers for the MEI specification for 2004 (OCR, 2004) include six contextualised questions. For example, in Figure 2, a quadratic function is used to model the underside of a bridge. In other questions, triangle trigonometry is used to estimate the angle of a leaning tree (C2 Q8), an arithmetic series models a relay race in which skittles are picked up in turn (C2 Q9), reduction to linear form models the spread of a virus (C2 Q10), differential equations providing alternative models of population growth in a city (C4 Q7), and 3D vector geometry is used to model the path of a helicopter.

In this corpus of questions, none of the contexts would seem to offer problems of *accessibility*, although the 'skittles' context might be unfamiliar to some candidates, and the helicopter context requires knowledge of bearings, and confers some advantage to candidates used to using vectors in kinematics.


5 The diagram shows a bridge.
The units are metres.

It is suggested that the curved underside of the bridge can be modelled by the curve
$y = \frac{1}{2}x(4 - x)$ for $0 \le x \le 4$.

(i) Give two different reasons why this is a good model. [2]

(ii) Give also one reason why it is not a perfect model. [1]

Fig 2: OCR/MEI C1 specimen paper, Quadratic to model the underside of a bridge

It is worth considering in what way these contexts are *realistic*. In Figure 2, for example, the underside of the bridge might look similar to a parabola - however, the questions asks candidates to compare the parabola to the curve drawn in the diagram, which itself is a model of the real bridge. There is a potential confusion here between the efficacy of the quadratic as a model of the curve shown in the diagram, or the curve shown in the diagram as a model of a real bridge.

The questions could be said to have varying degrees of *authenticity*. Most bridge arches (see Figure 2) are likely to be circular, not parabolic, and the purpose of modelling this with a quadratic function is not immediately clear. However, in general terms, relating curves found in real-life contexts to mathematical functions would seem to be an appropriate modelling exercise, and at A- and AS level, the range of functions which can be used is clearly limited.

TOWARDS A FRAMEWORK FOR EVALUATING CONTEXT

In the absence of existing research on the use of context in A-level mathematics, and utilising a grounded theory approach informed by the work of Vappula and Clausen-May (2006), we propose an initial framework for evaluating contexts in advanced level questions.

First, there is the issue of the *accessibility* of the context to the learner. Some contexts might be considered as cultural constructs, assumed to be familiar to all students in UK society. For example, financial concepts such as monetary value, simple and compound interest, profit and loss, etc. might be assumed to be understood by all students. In contrast, some questions may make assumptions about students' knowledge of sport, for example in modelling, mathematically, the conversion of a try in rugby in order to maximise the angle between the posts. In other A-level mathematics questions, there may be assumptions about scientific knowledge, for example calculus questions in the context of dynamics, or exponential growth and decay in economics, or the physics of radioactive decay. Such questions might try to overcome a lack of universal familiarity by attempting to explain the context from first principles; however, this explanatory text has to be weighed against the increase in the demands of comprehension it places on the solver in that the more wordy the question, the less accessible it might become, especially to those whose first language is not English.

Secondly, there is the issue of the *realism* of the context. A context might be considered as *realistic* if it models real-life in a way which appears to accord with experience. The demands of *realism* can conflict with *accessibility*. For example, on the one hand, if, in order to tailor the question to the mathematics demands being tested, the context filters out, or over-simplifies, reality, the outcome is likely to be artificiality. On the other hand, if the context adds too much extra-mathematical detail in order to strengthen its claim to be realistic, this can lead to problems of accessibility and wordiness.

Thirdly, there is the dimension of *authenticity*. This idea has been used before in models of assessment, see, for example, Pandey (1990). In the context of timed written paper questions, *authenticity* might be taken as a measure of whether the questions posed by the item are worthwhile and interesting within the real-life context, as well as testing the mathematics. In a sense, we are asking if there is some closure of the modelling cycle, so that results obtained by applying pure mathematics can be re-applied to the context in a meaningful and interesting way. Thus, a question may appear to provide an accessible and realistic context for doing some mathematics, but can be constructed in such a way that the answers are essentially irrelevant to the context, and such questions would score low on a scale of authenticity.

To sum up, our three theoretical measures of context are as follows:

- *Accessibility*: the familiarity and comprehensibility of the context
- *Realism*: the fit of the mathematical model to students' perceptions of real life
- *Authenticity*: how relevant and useful the solution of the question is to the context

CONCLUSIONS

Although the proposed model requires considerable refinement, some tentative conclusions can be drawn from the above analysis. The use of context in the questions analysed is heavily constrained by design considerations: they must be fit for the purpose of testing pure mathematical syllabus content; they must be accessible and comprehensible to candidates; they must strive for realism, albeit sanitised from the complexity of real life; and they must strive for authenticity, where this is possible. The questions analysed in this paper clearly do not give our students the opportunity to engage in genuine mathematical modelling, and it might be unrealistic to expect them to do so. However, longer questions can provide candidates with a 'flavour' of how mathematics can be used to model reality. Given the strong backwash of formal assessment on the teaching and learning of students engaged in advanced courses, it would seem to be important to construct examination questions which remind candidates, albeit artificially, of the notion that mathematics, even pure mathematics, serves a purpose beyond its own boundaries.

Notes

1. MEI stands for *Mathematics in Education and Industry*, a project which, as its title suggests, promotes the teaching and learning of mathematics within an industrial context.

Acknowledgements

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