A TWO-SECTOR GROWTH MODEL FOR A DEVELOPING ECONOMY:

THE CASE OF GREECE

by

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Submitted for the degree of
Doctor of Philosophy

University of Southampton

September 1976

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UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF SOCIAL SCIENCES

ECONOMICS AND ECONOMETRICS

Doctor of Philosophy

A TWO-SECTOR GROWTH MODEL FOR A DEVELOPING ECONOMY: THE CASE OF GREECE

By Anastasios Vasiliou Katos

This thesis has three purposes. These are, firstly, to bridge the gap between growth models for the developed and the less developed economies; secondly, to bridge the very large gap between abstract growth theory and empirical analysis; and thirdly, to provide a numerical analysis for optimal control policies.

The first objective is tackled by constructing a two-sector growth model for a 'developing' economy, the first sector being the agricultural and the second sector the non-agricultural, by building on the assumptions of the dual development. The principal characteristics of the model are the use of general production functions, the introduction of capital in the first sector, the bias in technological change between sectors and factors of production, the introduction of three income classes with three different propensities to save and the existence of a wage differential between the two sectors which produces labour migration and which is decreasing over time. This model together with the presented demographic dualism, the mechanics of labour reallocation and the rate of urbanisation includes as special cases models for developed and less-developed economies.

The empirical validity of the model was the second objective. The experience of the Greek economy for the period 1950–72 was studied and the structural parameters of the model using Cobb-Douglas production functions estimated. This application permitted the comparison of the behaviour of the ratios of economics of growth and the estimated parameters for the Greek economy between theory and reality.

Finally, the last objective was to investigate numerically the sensitiveness of the optimal paths for the key variables of the economy under consideration by extending theoretically the descriptive growth model to a normative one and by applying in it the values of the parameters already estimated.

ACKNOWLEDGEMENTS

I am very grateful to my supervisors Mr. P.J. Simmons and Mr. A.Ingham for their guidance during the period of research.

My thanks go to Mr. G.J. Anderson and Mr. T.J.W. Harrison for their assistance during various stages of this study.

I also wish to express my thanks to my colleagues Mr. B. Hilliard and Mr. V. Borooah for their helpful comments.

Finally, I thank Mrs. W. Rogers for typing this thesis.

INTRODUCTION

The theory of economic growth has been one of the fastest growing branches of economics over the past twenty years. Growth, after the Second World War, was everybody's concern. The war-damaged economies were trying hard to reconstruct quickly, the underdeveloped countries were attempting to initiate economic development, the advanced capitalistic countries being relatively free from periodic slumps were trying to concentrate on raising the long-run rate of growth, and the socialist countries were determined to overtake the richer capitalist economies by fast economic expansion, Sen (1971).

The phenomena that are presented by a developing (changing) economy are immensely complex; any theory about them is bound to simplify, and at least in some way to over-simplify. In order to deal with those aspects with which it is possible to deal, it must neglect others; there is no known approach which is not based upon omissions, omissions that can easily prove to be of critical importance, Hicks (1972).

Among the various attempts to avoid critical omissions were the attempts to disaggregate the economy. In this thesis we assume that the economy has been divided into two sectors. The introduction of two sectors immediately raises a number of special problems such as, the shift of the allocation ratios of factors over the growth path, the change in the relative product prices as the economy grows, the different capital intensities into different sectors and so on.

The economist who is trying to provide a systematic analysis of the growth path of these two-sector economies faces two different groups of growth theories or models (for the relation of growth theories to growth models see Jones (1975)). The first group is that in which the two sectors of the economy are the consumption-good and capital-good sectors, Meade (1961), both goods produced with labour and capital and is applied mostly

to the developed economies of the advanced Western countries. The second group is that in which the two sectors are a backward and an advanced sector, the output of the first sector is used for consumption and is produced with labour whilst the output of the second sector is used for consumption and investment and is produced with labour and capital, and is applied to the underdeveloped economies or oriental economies in Boeke's (1953) terms.

In the real world we cannot be so sure in dividing the economies into the developed and underdeveloped ones because there exist some economies which are clearly between these two groups. Often, international organisations, such as the United Nations or the OECD, using different criteria, face difficulties in sectoring the economies into these two groups and it is not rare to see the same economy to be characterized as a developed one by the criteria of one organisation and at the same time as a less developed one by the criteria of another organisation, Maddison (1971).

The aim of the present thesis is to bridge the gap between these two extreme groups, the developed and the underdeveloped economies, and the corresponding growth theories (or models), introducing a two-sector growth model for a "developing"economy.

In Chapter one, we discuss the concept of development and related topics as for example the existence of disguised unemployment, trying to survey the existing theories and models in this area. Our survey is mainly based on the works of Lewis (1954), Fei and Ranis (1964), Jorgenson (1961) and Dixit (1973). In this chapter we are discussing solely descriptive models even though planning models are probably far more important to development economics, and this is due to our belief that we must know the structure of an economy before we try to use any planning policy.

In Chapter two, we present the mechanics of labour migration between the two

sectors of the economy and we are discussing the hypothesis of the existence of a gap between the wages of the two sectors. Of course, in the same chapter we investigate the migration decision due to wages differential.

In Chapter three, we present our "general" two-sector growth model for a developing economy. With the word "general" we mean that our model is not bound to any specific production functions, as the majority of the models in the survey of Chapter one are, and which use for example, production functions of the Cobb-Douglas type. In this model the first sector is the agricultural sector whilst the second sector is the non-agricultural one. The inputs used for production in both sectors are the inputs of labour and capital as in Kelley, Williamson and Cheetham (1972) model. It is assumed that factor-augmenting technical change applies to both capital and labour and it can be of different bias for the two sectors whilst it is of the same bias for the model of the latter writers. By assuming different wage rates between the two sectors and different propensities to save between the three income classes, ie. workers of the two sectors and capitalists, we try to investigate the structure of the developing economy and to find the behavioural and technical sufficient conditions for the properties of existence and uniqueness of the momentary solution and stability of the balanced growth of our model. Migration of labour force between the two sectors enters into our model by trying to investigate the growth process through time.

The aim of Chapter four is twofold. Firstly, to provide us with a simple version of the general two-sector growth model of Chapter three, by using production functions of the Cobb-Douglas type, suitable for comparisons with the models presented in the survey of Chapter one, and secondly, to be applied to a specific developing economy, ie. precisely to the economy of Greece.

Before we applied the model to the Greek economy we thought it would be better first to examine whether the Greek economy behaves in the same way as our model. Therefore, in Chapter five we present and examine all the relevant available data of the economy under consideration summarizing in the last section the main conclusions of this investigation.

In Chapter six we estimate the model. An important omission in the literature is the actual estimation of the various parameters involved in a two-sector growth model and the aim of this Chapter is to fill this gap. Firstly, we start with the estimation (or interpolation) of the data which are missing and are necessary for our application. The relation between the specification of the actual forms of the functions and parameters used in the model with that of reality is investigated by the examination of the great ratios of economics of growth, Klein & Kosobud (1961), as a second step. Finally, we are doing the actual estimation explaining firstly the econometrics used and secondly, the criteria under which we chose the "best" estimates.

In the first six chapters of the present thesis we discuss (and estimate) positive aspects of the economics of development. For the exercise to be complete we think that it was necessary to extend from positive economics into normative economics. This is the aim of the last chapter, Chapter seven. In this chapter we not only present and investigate the optimal solution of the general or specific two-sector growth models of the previous chapters but also provide the actual numerical values of the variables of the optimal path of the model for the Greek Economy. The sensitivity of this optimal path to changes in the various parameters is finally investigated.

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CHAPTER 1

GROWTH MODELS FOR DEVELOPING ECONOMIES: A SURVEY

1.1 Introduction
1.2 The Dual Economy
1.3 The Backward Sector
1.4 The Advanced Sector
1.5 The Complete Model of the Dual Economy
1.6 Trade of Goods between the Two Sectors
1.7 Growth Models with more than Two Consumption Goods
1.8 Conclusions
1.9 References

1.1 Introduction

There are several criteria according to which an economy can be characterized either as a developed economy or as a less developed one. Examples of these are the level of per capita income, the level of per capita consumption, the allocation of output between the different sectors of the economy and the corresponding allocation of the labour force and so on.

From empirical data it has been shown that the high income per head is associated with a relatively large proportion of the total population working in the industry, while low income per head is associated predominantly with employment in the agricultural sector.

Comparing the theories of economic growth and development we can see that ever since the fifties there have been a large number of theories and models forwarded for the advanced economies and only a small number for the less developed economies. We could divide the part of economics which deals with 'economic growth and development' in such a way that we use 'economic growth' to refer to the advanced economies and 'economic development' to refer to the less developed economies.

In the models of a developed economy which are highly aggregative the analysis is concentrated on the allocation of national product between consumption and investment, while for the less developed economies it is concentrated on the relationship between the growth of income and the growth of population, Jorgenson (1961).

In the theory of economic growth the balance between investment and savings is all important and the growth of population is treated as constant or put aside as a qualification to the main argument. In the theory of development, emphasis is put on the balance between capital accumulation and the growth of population, each adjusting to the other,

Jorgenson (1967).

As we said above, the models of economic growth for the advanced economies are highly aggregative with only one putput and a single production function. The post-keynesian models of Harrod (1948) and Domar (1957) and the neo-classical models of Solow (1956) and Swan (1956), fall into this category and have little relevance to the first stages of growth in the less developed economies. In 1961 Uzawa published a paper of a model of economic growth introducing two sectors, one producing a capital good and the other a consumption good. Uzawa's two-sector growth model and later Inada's (1963), Drandakis's (1963) and others which originated in Meade's (1961) book use one consumption good only. Only a few models use more than one consumption good, for example, Jorgenson (1961) in his model of a dual economy and Dhrymes (1962) in his multi-sectoral growth model, and they are very special cases using production functions of the Cobb-Douglas-type. Recently, Kelley–Williamson–Cheetham (1972) and Marino(1975) have generalized Jorgenson's model by introducing general production functions.

Most of the two-sector growth models in which one sector is relatively less developed than the other are referred to as models of 'dual economies'. Of course, there are a number of possible definitions and interpretations of the term 'dualism', but in the main it refers to economic and social divisions in an economy, such as differences in the level of technology between sectors or regions, differences in the degree of geographic development, and differences in social customs and attitudes between an indigenous and imported social system, Thirlwall (1972). The models of economic development of Lewis (1954, 1958), Fei and Ranis (1961), Jorgenson (1961), Dixit (1973) and others fall into this category.

Dual economy models provide a significantly better description and understanding of the problems of development than any aggregative model can, not because two sectors

are better than one, but because the sectoral division chosen reflects several vital social and economic distinctions in the type of economy being analysed, Dixit (1973).

The division of the economy into sectors poses a difficult problem regarding the classification of activities into sectors. Following Dixit (1973) we classify the backward (or alternatively 'traditional', 'rural' or 'peasant-owned') sector to include agricultural production and the advanced (or alternatively 'modern', 'urban' or 'capitalistic') sector to include industrial production. According to Lewis (1954) we should classify in the advanced sector those services which are available on the market and leave domestic services in the backward sector. Stiglitz (1973) writes that the agricultural-industrial division is not helpful, more appropriate would be a 'traditional' (including services) and 'advanced' classification which, eg. in Israel, Kenya and New Zealand would include agriculture. It is the institutional arrangements and techniques used that determine the division. Spaventa (1973) agrees that the service sector should be included in the traditional sector. Reynolds (1969) uses four sectors, of these, two are traditional; the agricultural sector and the urban trade-service sector. The other two sectors, industry and government, are the modern sectors.

In what follows we will use Dixit's sectoral division.

1.2 The Dual Economy

In this section we will try to present the basic notions met in the theories of dual growth, leaving for the next sections the detailed examination of these notions.

As we said in the previous section, one of the two sectors in a dual economy is the backward or agricultural sector. This sector produces food, which, apart from some requirement for seed, can only be used for consumption. The advanced or industrial sector produces a different kind of good which can be used as a consumption good or as an

investment good in the two sectors.

Productive activity in the two sectors may be characterised by the production functions which relate the two different outputs to the factors of production. From the existing literature one should note the assymetry of the two production functions. In the agricultural sector there are two factors of production, labour and land, while in the industrial sector the corresponding factors of production are labour and capital. So, in the first sector we do not have capital accumulation, except where investment takes the form of land reclamation and in the second sector the role of land is negligible. Of course, there are cases in which capital accumulation played an important role in increasing the productivity in agriculture as for example in Greece, Japan and Taiwan. Recently growth models have been developed including the capital factor in the agricultural sector, Mundlak–Mosenson (1970-72), Kelley–Williamson–Cheetham (1972), Niho (1974) and Mass-Collell–Razin (1974). Another point is that the supply of land in the first sector is fixed, so that it can be subtracted from the production function in the agricultural sector.

Agricultural activity is characterised by the use of traditional methods of production with the consequence of high ratios of labour to land (or labour to capital for the models in which capital is an explicit factor of production, in agriculture), very low labour productivity and generally diminishing returns to scale. In contrast, industrial activity is characterized by a higher productivity of labour and constant returns to scale.

Most of the theories of the dual economy use the notion of neutral technical progress which means that each production function will shift over time so that a given bundle of inputs will produce a higher level of output at a given point of time than at an earlier one.

The growth of population is taken as exogenous or depending upon the per capita

supply of food and the force of mortality. The latter is assumed to be given – it may be altered only by an improvement in medical techniques. The birth rate depends upon the per capita supply of food. However, it may attain a physiological maximum, ie. a maximum given by a particular social situation and by the state of medical knowledge, provided that the supply of food is sufficient. McIntosh (1975) proposes a non-Malthusian population theory which is based on the hypothesis that changes in fertility rates are intimately connected with migration patterns and that the rate of population growth is determined by the distribution of the population between the two sectors. If the food supply is more than sufficient there exists an agricultural surplus and labour may be freed from the land for employment in industry. Labour is divided between the two sectors in a straight forward manner: if there is no agricultural surplus, all labour remains on the land; if an agricultural surplus can be generated, the labour force available for employment in the industry grows at a rate which is equal to the rate of growth of the agricultural surplus, Jorgenson (1961).

A much-debated concept is the existence of "disguised unemployment". There are various definitions of disguised unemployment: Rosenstein–Rodan (1957), Raj (1957), Leinbenstein (1957), Cho (1963) and others, but the simplest definition of it implies that some labour could be withdrawn from subsistence farming without reducing the volume of farm output. According to Nurkse (1953), in technical terms the marginal productivity of labour is believed to be zero. But is the marginal productivity of labour in agriculture really zero? Lewis (1954) and Fei and Ranis (1964) in their models use the concept that the marginal product of agricultural labour is near zero, and therefore withdrawal of labour from this sector will not reduce the level of output. In contrast, Viner (1957) finds it difficult to believe that the marginal product of labour in agriculture is zero. Also, Jorgenson

(1961) in his neo-classical model argues that the marginal product of labour in agriculture is in fact substantially above zero so that withdrawal of labour from this sector would decrease output and raise the marginal productivity of the remaining workers. But this is not the case if we use man-hours of work instead of workers, Sen (1960), Myint (1964) and Robinson (1971). In fact, Sen (1966) showed that the marginal productivity of a worker could be zero even though the marginal productivity of a man-hour is substantially above zero. To establish this Sen argued that if the marginal utility to income is constant, and the marginal disutility of work is also constant, then the withdrawal of a worker from the agricultural sector leads others in this sector to work more hours to make up for the loss. Thus, Sen's assumption that the indifference curve between leisure and income is a straight line (ie. assuming perfect substitutability) implies that the existence of surplus labour is consistent with the zero marginal productivity of the worker and the non-zero marginal productivity of the man-hour in agriculture. Wellisz (1968), Berry and Soligo (1968), Hymer and Resnick (1969), Uppal (1969), Robinson (1969, 1971), Zarembka (1972) Lal (1973) and others also examined the underlyings of the leisure-income choice.

At the same time there has also been a considerable debate on the empirical evidence in favour or against the concept of disguised unemployment. Mellor and Stevens (1956), Sarkar (1957), Mehra (1966), Sanghvi (1969) and others seem to have found evidence of its existence. On the contrary, Oshima (1958), Schultz (1964), Hansen (1966), Jorgenson (1966) and others have presented opposing evidence.

Finally, Sen (1975) argues that the debate above is based on the production approach whilst we should examine the income approach (ei. is this man's income a reward for his work, and will he cease to get income if he stops work?) and the recognition approach (ie. does he think of himself as being employed? Do others?).

In calculating the level of industrial wages we assume that the workers receive a wage equal to the marginal product. There is some difficulty over the earnings of the agricultural workers. In Fei and Ranis model (1961), the land is owned by landlords who hire labour and impose a constant real wage, because there is surplus labour (for this period the wages in industry are constant also). In Jorgenson's (1961) model, the land is owned by family farmers and they receive their average product. According to Zarembka (1972), if the typical worker who migrates is landless or sells or rents land that he owns, his wage is his marginal product. If the migrant owns land but loses it when he moves (eg. it is typical to pass land on to relatives), or if he worked in a family farm, then his wage is the return labour input and the implicit rent of land. In other words, the farm net revenue per worker is the opportunity cost of his moving to an urban area, so that farm workers wages are simply the average agricultural product.

There is also a considerable debate about the existence of equality or not in the earnings of the two labour classes, ie. the workers in either sector, but we will discuss this concept in depth in the next chapter. Regardless of whether or not a differential in the wages of the two sectors exists it should be clear that the dual economy's total saving fund is composed of two kinds of surpluses; industrial profits and agricultural surplus, Fei and Ranis (1966). Surplus food is sold in the intersectoral commodity market and the proceeds invested in either sector.

The existing theories and models for the development of the dual economies can be divided into two categories according to the way by which they approach the conditions governing the supply of labour to the industrial sector. Thus, according to Jorgenson's (1967) classification the two approaches are classical and neoclassical. In the classical approach to the theory, the real-wage rate is assumed to be fixed in terms of the agricultural

goods; from the point of view of industry, labour is available in unlimited amounts at a fixed real wage. In the neo-classical approach, labour is never available to the industrial sector without sacrificing agricultural output. In the industrial sector, the real wage-rate rises steadily over time depending upon the rates of technological progress in both sectors and the rate of capital accumulation. The theories of Lewis, and Fei and Ranis belong to the classical approach whereas, the theory of Jorgenson belongs to the neoclassical one.

In the survey presented below, the similarities and differences between the two approaches will be stressed.

1.3 The Backward Sector

In spite of the differences between the various approaches in dividing the economy into two sectors, most of them classify agricultural activity as being in the backward sector. So, as we pointed out in section 1.1, we will use the word 'agricultural' synonymously with 'backward'.

In most of the early theories, classical and neo-classical, the production function of the agricultural sector is the same and is taken to be (in the majority of the existing literature), of the Cobb-Douglass type form:

$$Y_1(t) = A_1 e^{\lambda_1 t} E(t)^{\alpha_1} L_1(t)^{1-\alpha_1} \qquad (1.3.1)$$

where $Y_1(t)$ is the agricultural output, $E(t)$ is the input of land, $L_1(t)$ is the input of labour, all measured in their own physical units, λ_1 is the constant proportional rate of technological progress, α_1 is the constant elasticity of output with respect to land, and A_1 is a constant positive parameter.

Because we assume that land is fixed in supply the production function in (1.3.1) can be reduced to the following:

$$Y_1(t) = A_1 e^{\lambda_1 t} L_1(t)^{1-\alpha_1} \qquad (1.3.2)$$

Capital accumulation and technological progress are the main sources of increasing agricultural productivity. Since the above production function does not explicitly include the input of capital we assume that capital is implicitly included in the exogenous exponential effect of technological change.

The differences between the classical and the neo-classical approaches start with the assumption about population growth; or labour force growth, taking the labour force as a constant fraction of total population. Suppose that the economy is in the very early stage of development inwhich everyone is working in agriculture.

In the classical approach, the growth of population is exogenous, the population being divided into two parts. The first consists of that part of the labour force employed at a positive marginal productivity and the other being that part of the labour force which has zero marginal productivity and is called the 'redundant' labour force. So, if the total population (or labour force) is L(t) then we have the result that:

$$L(t) = L_1(t) + R(t) \qquad (1.3.3.)$$

where R(t) is the redundant labour force. If now we write the maximum labour force which may be employed at a positive marginal productivity by $L_1(t)^+$, then the agricultural labour force is the minimum of the total labour force of the economy and the maximum labour force which is employed at a positive marginal productivity. Thus,

$$L_1(t) = \min \begin{cases} L(t) \\ L_1^+(t) \end{cases} \qquad (1.3.4)$$

Of course, if the agricultural labour force is equal to the total labour force, disguised unemployment is zero. If the agricultural labour force is equal to the maximum level, $L_1(t)^+$, redundant labour is equal to the difference between total population and this maximum level, Jorgenson (1967).

The term 'disguised unemployment', as we have already seen, is commonly used to designate a situation in which the removal of some units of labour from a working combination of factors nothing else of consequence being changed, will leave the aggregate product of the working combination undiminished, and may even increase it. To say that there is 'disguised unemployment' is therefore equivalent to that part of the labour force employed in working combinations whose marginal productivity of labour is almost zero, zero, or even negative, Viner (1957). Lewis (1951) writes that disguised unemployment occurs only for peasant or self-employed labour and not for the plantation labour, and also, that the most disguised unemployment is amongst self-employed labour although with some amongst hired agricultural labour. Nurkse (1953), has suggested that where disguised unemployment prevails in agriculture, it would be desirable to transfer the surplus labour from the farms to industry, while keeping the consumption of food, by the population as a whole, constant through taxation or direct controls.

The classical thoery for the agricultural sector, under the assumption of disguised unemployment existing, can be described, with the help of figure 1.1, which is taken from Fei and Ranis (1961, 1964),as follows.

Figure 1.1a shows total physical productivities (TPP) while figure 1.1b shows marginal physical productivities (MPP). The curve OP R' X' in 1.1a represents the TPP of labour. The concave portion OP' R' shown a gradually diminishing marginal productivity, (corresponding to ABR in 1.1b), and a horizontal portion R' X', where marginal physical product vanishes, (corresponding to RX in 1.1b).

The quantity of labour force in excess of OR may be considered redundant in that its withdrawal from agriculture would not affect agriculture output. If all the population, OX, is employed in the agricultural sector and all the output is consumed by this population,

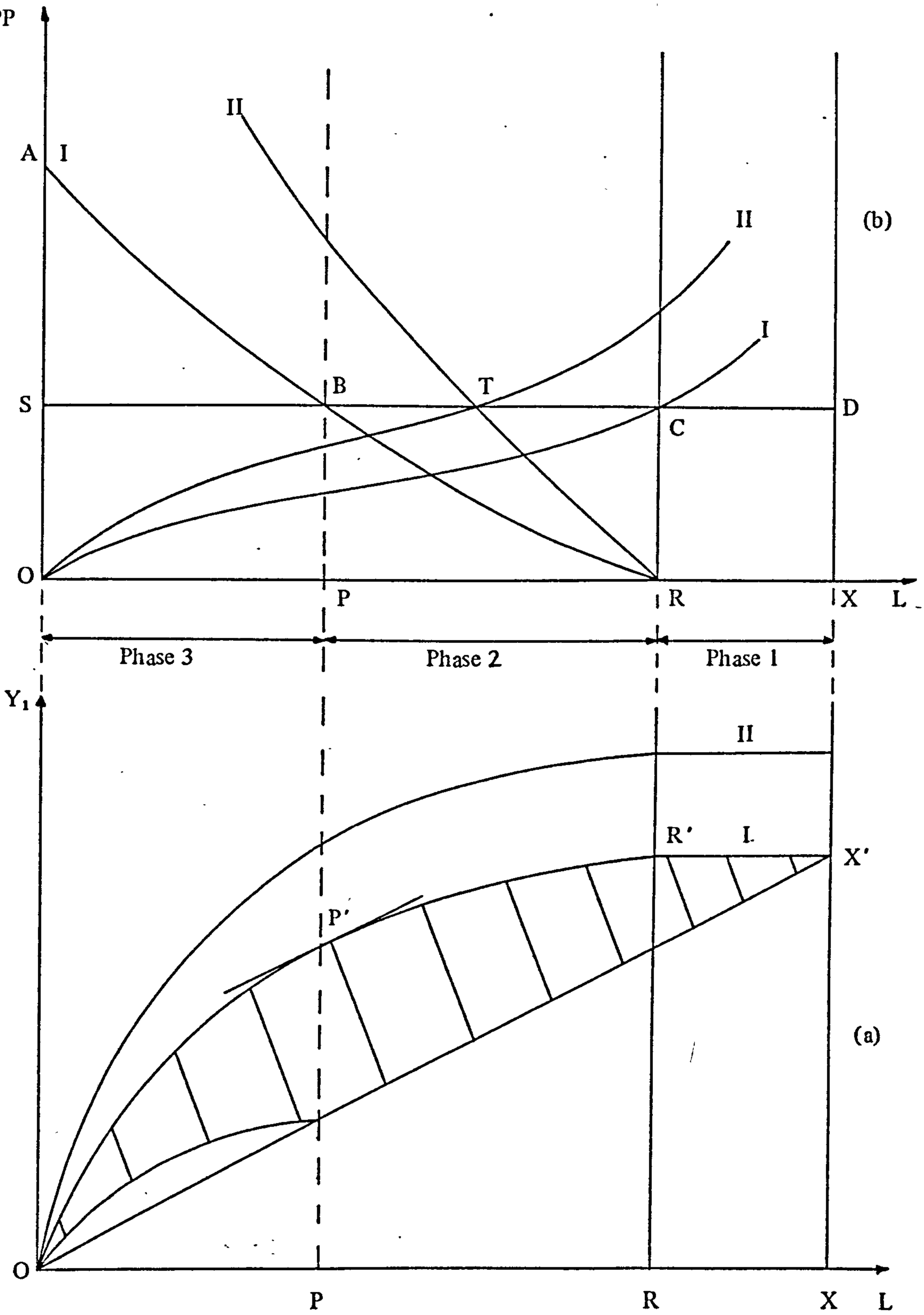
MPP
A
I
II
B
T
S
D
C
O
P
R
X
L
Phase 3
Phase 2
Phase 1
(b)
Y_1
II
I
R'
I.
X'
P'
(a)
O
P
R
X
L

Figure 1.1

then the real wage is equal to the average productivity, which is equal to the slope of OX' in 1.1a. The existence of this non-zero wage level is due to institutional or non-market forces since under competitive assumptions the real wage would fall to zero, at equality with MPP. This wage is called by Fei and Ranis, (1961), the 'institutional wage'.

If point P' on the total output curve is the point at which the MPP equals the institutional wage, Fei and Ranis define as PX the disguised unemployed agricultural labour force since beyond P, MPP is less than the institutional wage. In figure 1.1b we can see the problem in terms of MPP. The curve ABRX is the marginal physical productivity curve of labour in agriculture. Suppose that OS is equal to the institutional wage. We can distinguish three phases. Phase one is the range for which MPP = 0, which means that the TPP curve in figure 1.1a is horizontal. This phase marks off the redundant labour force RX. Phase two is the range for which MPP is positive but less than the institutional wage. Phase one and phase two together mark the size of the disguised unemployed labour force PX. Phase three is the range for which MPP is greater than the institutional wage rate.

Because of the withdrawal of agricultural workers a surplus of agricultural goods begins to appear. Once all the redundant labour has been withdrawn from the agricultural sector, the economy has reached the first turning point R, which is called the 'shortage point'. Once all the disguised labour has been withdrawan the economy has reached the second turning point P, which Fei and Ranis call the 'commericalization point'. This point marks the end of the take-off process with the economy going into phase three where labour has been a scarce resource and the economy is fully competitive in both sectors.

We define total agricultural surplus (TAS) as the agricultural output in excess of the consumption requirements of the agricultural labour force at the institutional wage, thus:

TAS = TPP – (inst. wage) x (total labour force – withdrawn labour force)

The TAS in figure 1.1a is the shaded area which is the area between the straight line OX' and curve OPRX' in all phases except phase three where the MPP of labour is larger than the institutional wage. We define the average agricultural surplus (AAS) as TAS per unit of withdrawn labour and in figure 1.1b this is presented by the curve OC D.

Increases in the productivity of labour causes the TPP, the MPP and the AAS curves to move upwards, as shown in figure 1.1, by the labels I and II. A consequence of this is that phase three starts earlier (point T compared with point B), keeping constant the institutional wage, Fei and Ranis (1961, 1964), and the shortage points move to the left (point T compared with point C). So, increases in productivity cause the commericalisation points and the shortage points to move in opposite directions until they coincide at one turning point (point T).

From the above description we conclude that in the classical approach during the supply-labour phases (phases 1 and 2) an absolute decline in agricultural employment takes place. Jorgenson (1966) shows that this result is inconsistent with the empirical evidence, and subsequently, Dixit (1970) showed that it would be possible not to have an absolute decline in agricultural employment during these two phases by taking account of the shift upwards over time of point R due to technological progress.

The growth of population or labour force is taken by some authors, not as exogenous, but as depending on some other variables. For example, Niehans (1963) studied in an aggregate model the growth of population as a function of capital accumulation. Recently McIntosh (1975), states that the rate of population growth is determined by the distribution of the population between the two sectors, with the population distribution being determined by relative prices.

According to Jorgenson (1961) a major characteristic feature of the agricultural system is the function which governs the growth of population. It is assumed that if there is literally no agricultural production the reproduction rate falls to zero and the force of mortality is constant at the level O. Secondly, it is assumed that the rate of gross reproduction is our increasing function of agricultural output per head up to some physiological maximum, say $\epsilon + \Theta$. It is simplest to assume that the rate of increase in the gross reproduction rate is constant as per capita income increases, so that population theory may be written in the form:

$$\frac{\dot{L}_1(t)}{L_1(t)} = \min\begin{cases}\gamma y_1(t) - \Theta \\ \\ \epsilon\end{cases} \qquad (1.3.5)$$

where γ is the rate of increase of the gross reproduction rate with respect to an increase in the output of food per man $y_1(t)$. The net reproduction rate $\dot{L}_1(t) / L_1(t)$ is the gross reproduction rate less the force of mortality, where the gross reproduction rate is the minimum of the two rates determined by the physiologically maximum rate of reproduction and the rate determined by the output of food per head.

From (1.3.2) we have:

$$y_1(t) = \frac{Y_1(t)}{L_1(t)} = A_1 e^{\lambda_1 t} L_1(t)^{-\alpha_1}$$

or

$$\frac{y_1(t)}{y_1(t)} = \lambda_1 - \alpha_1 \frac{\dot{L}_1(t)}{L_1(t)} \qquad (1.3.6)$$

Substituting (1.3.5) into (1.3.6), for the first phase of development we derive the fundamental differential equation for the theory of development of the agricultural sector in in the neo-classical approach.

$$\dot{y}_1(t) = (\lambda_1 + \alpha_1 \Theta)\, y_1(t) - \alpha_1 \gamma y_1(t)^2 \qquad (1.3.7)$$

Substituting the static solution $y_{12} = (\lambda_1 + \alpha_1 \Theta) / \alpha_1 \gamma$ (the other solution $y_{11} = 0$ is not important) from (1.3.7) into (1.3.5) we obtain:

$$\frac{\dot{L}_1(t)}{L_1(t)} = \gamma \left[(\lambda_1 + \alpha_1 \Theta) / \alpha_1 \gamma \right] - \delta = \frac{\lambda_1}{\alpha_1} > 0 \qquad (1.3.8)$$

which in combination with (1.3.6) gives the result that the food per capita does not increase. So we see that output per head remains stagnant and population is increasing.

This fact is a special case of Nelson's (1956) or more generally of Leibenstein's (1957) low-level equilibrium trap. Leinbenstein's approach is illustrated in figure 1.2.

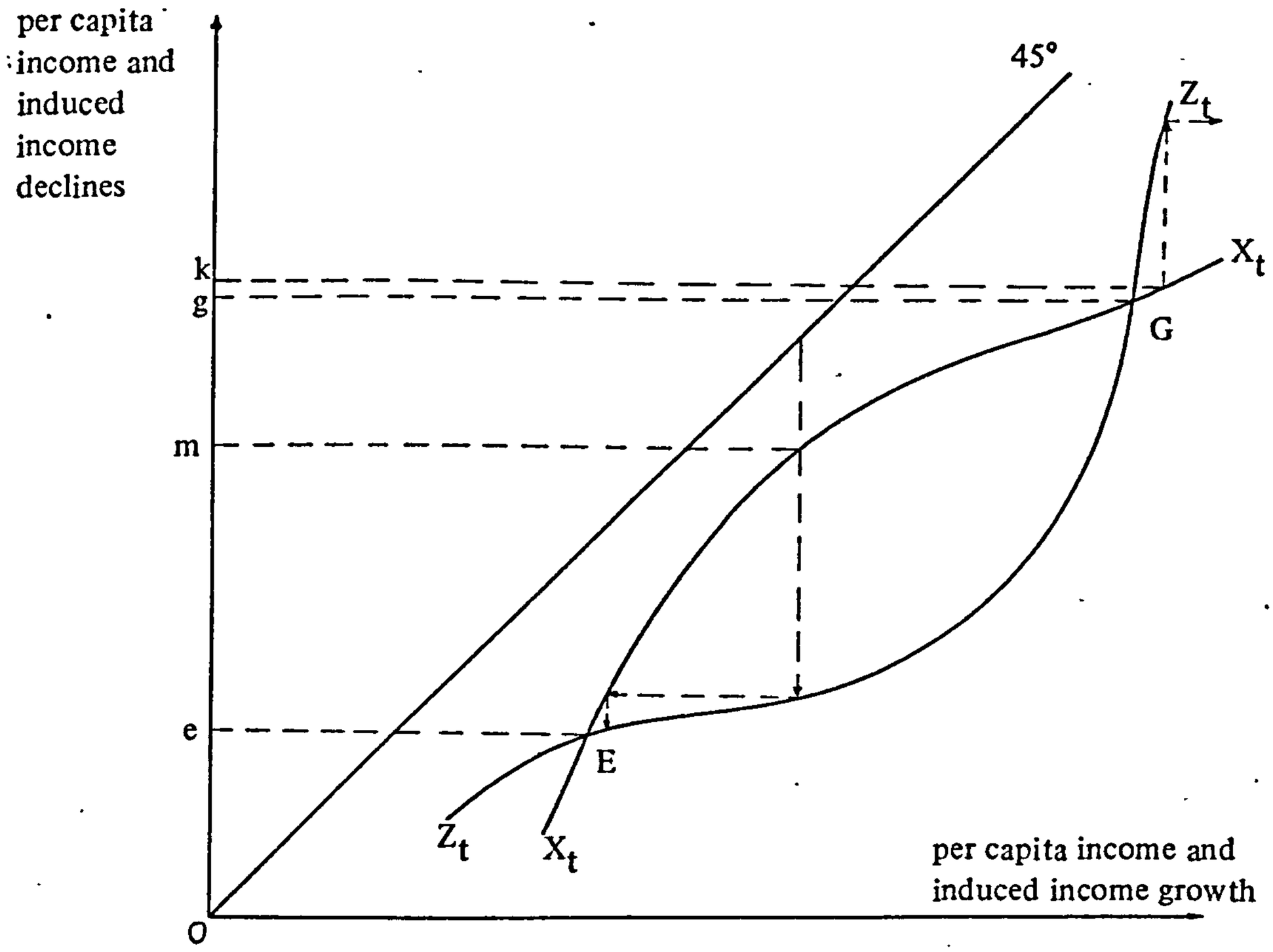

Figure 1.2

Curve Z_t represents the effects of all the per capita income-depressing forces measured horizontally from the 45° line and curve X_t represents the effects of all the per capita

income-raising forces measured vertically from the 45° line. If the initial per capita income was at the level 0m, then this would generate the path towards point E (the only point of stable equilibrium), because between income levels Oe and Og, income-depressing forces are greater than income-raising forces. Similarly, if the initial per capita income was greater than Og, say Ok, then this will generate an explosive growth path, because beyond level Og income-raising forces are greater than income-depressing forces. Hence, level Og is the critical per capita income necessary to escape the low-level equilibrium trap.

Furthermore, Jorgenson denotes as $y_1^+(t)$ the minimum level of income at which $\dot{L}_1(t) / L_1(t)$ attains its physiological maximum. So, from (1.3.5) we can define:

$$\frac{\dot{L}_1(t)}{L_1(t)} = \gamma\, y_1^+(t) - \Theta = \epsilon$$

or

$$y_1^+(t) = \frac{\epsilon + \Theta}{\gamma} \qquad (1.3.9)$$

If now, the critical level of income, $y_1^+(t)$, is below y_{12}, the stationary level, assuming that population growth has not attained its maximum, y_{12} will not be attained, since it would require a rate of growth of population higher than the physiological maximum. In the event that y_{12} exceeds $y_1^+(t)$ the growth path will be given instead by the expression:

$$\frac{\dot{y}_1(t)}{y_1(t)} = \lambda_1 - \alpha_1 \epsilon$$

which has the general solution

$$y_1(t) = e^{(\lambda_1 - \alpha_1 \epsilon)t}\, y_1(0) \qquad (1.3.10)$$

where the rate of growth of per capita income, $\lambda_1 - \alpha_1 \epsilon > 0$, will be attained from any positive initial level of output.

Dixit (1973) illustrates the above discussion as follows with the help of figure 1.3.

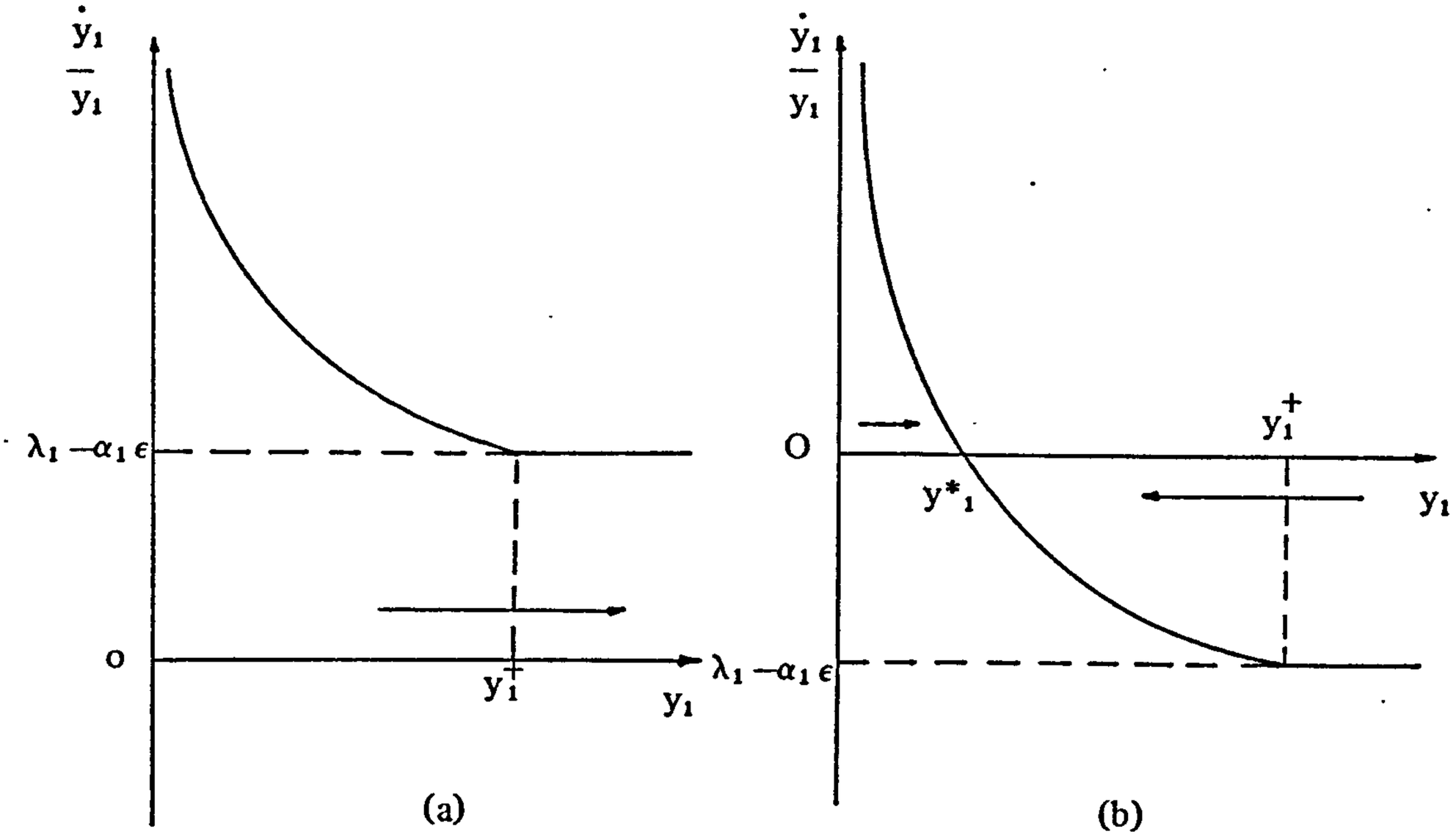

Figure 1.3

If $\lambda_1 - \alpha_1 \epsilon < 0$, we have a y^* such that $y_1(t)$ decreases when above y^* and increases when below it, thus forming a stable "trap" for the economy. With technological progress, we do not have Malthusian trap in the strict sense: population is growing although at a rate below the maximum. This is shown in Figure 1.3b. In this case, output per capita is stationary, and clearly agriculture will not be able to release any labour to industry without making matters even worse. Thus the agrarian economy must remain in that state for ever, and if an industrial sector exists initially, it must decay. So, we assume that the condition which ensures sustained growth in $y_1(t)$, is:

$$\lambda_1 - \alpha_1 \epsilon > 0 \qquad (1.3.11)$$

It must be noted that the condition (1.3.11) will ensure growth rather than decay in the standard of living even in the case where labour is not growing endogenously.

Now assuming that the condition (1.3.11) holds it can be seen from (1.3.10) that the per capita output will surpass the level $y_1^+(t)$. Jorgenson assumes that no one wants

more food than this, so that beyond this point in time a portion of population suffices to meet the food requirements of the whole, thus releasing some labour for industrial work.

1.4 The Advanced Sector

Industry is the main activity in the advanced sector. Suppose that the production function is of the Cobb-Douglas form again, thus:

$$Y_2(t) = A_2\, e^{\lambda_2 t} K_2(t)^{\alpha_2} L_2(t)^{1-\alpha_2} \qquad (1.4.1)$$

where $Y_2(t)$ is the output of the industrial sector, $K_2(t)$ is the capital stock engaged in industry, $L_2(t)$ is the input of labour, all measured in their own units, α_2 is the elasticity of output with respect to capital, λ_2 is the constant proportional rate of technological progress and A_2 is a constant positive parameter. As can be seen from (1.4.1) it is assumed that the returns to scale are constant.

The role of industry is the expansion of employment opportunities to the level required for the absorption of the surplus labour force released by the agricultural sector and implies a gradual expansion of industrial productive capacity and output, Fei & Ranis (1964). So, in this section we are going to study the growth of the industrial sector, with the only role of the agricultural sector being the supply of labour to the industrial sector.

Suppose that either the wage-rate $w_2(t)$ in the industry is exogenously fixed and equal to the institutional wage-rate in agriculture, or that it is fixed at a slightly higher level from that of the institutional wage in order to compensate labour for the cost of transferring and to induce labour to leave the traditional life of the backward sector (see next chapter). Thus, we have a perfectly elastic supply of labour for the industrial sector at an existing fixed wage in the industry, and the demand curve is simply the

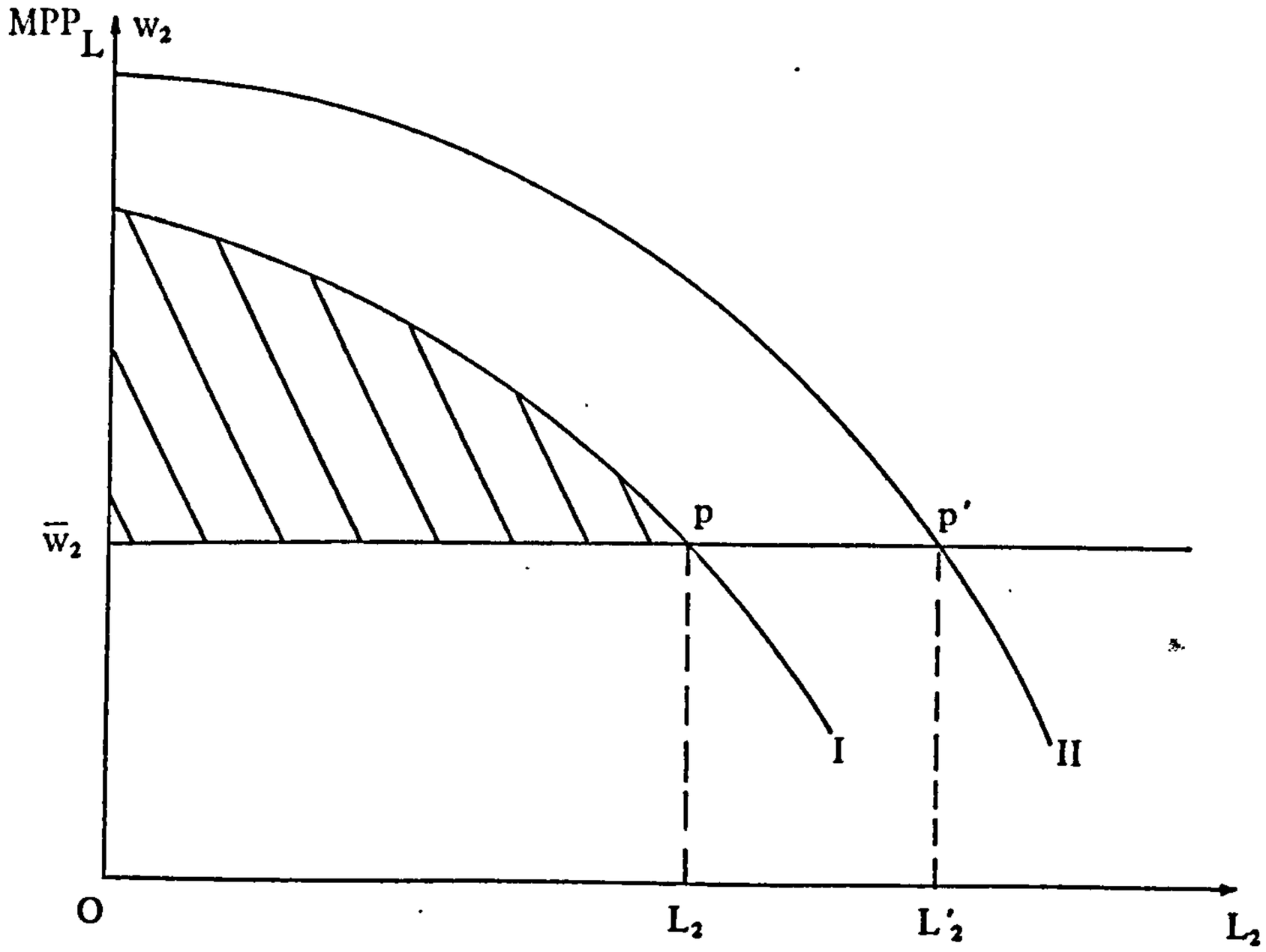

Figure 1.4

marginal product curve.

This situation can be illustrated in figure 1.4 which was firs t introduced by Lewis (1954). The straight line $\bar{w}_2$ = constant represents the constant wage-rate in the industry while the curve I represents the marginal productivity of labour for a fixed amount of capital. If we assume profit maximization then at the point of intersection P of the two curves we have the marginal productivity of labour equal to the current wage-rate and so we obtain the equilibrium quantity of labour L_2. Then, the industrial profits are shown in figure 1.4 as the area between the MPP_L curve I and the fixed real wage curve w_2 (shaded area).

If the engaged capital in industry was not at the previous quantity, but at a higher level, then the corresponding MPP_L curve will not be the same but curve II, which lies higher and to the right of I. So, the new equilibrium position for labour in the industry will be L'_2, and not L_2, which means that the larger is the investment in industry the larger

will be the quantity of labour that can be absorbed. Of course, the situation is very different when there is no redundant labour in agriculture and thus labour becomes a scarce resource.

From (1.4.1) the marginal productivity of labour is:

$$\frac{\partial Y_2(t)}{\partial L_2(t)} = (1-\alpha_1) \quad \frac{Y_2(t)}{L_2(t)} = w_2 \qquad (1.4.2)$$

(we assume for the moment, as does Lewis, that the terms of trade are fixed). From (1.4.1) and (1.4.2) we take:

$$Y_2(t) = A_2^{\frac{1}{\alpha_2}} \left(\frac{1-\alpha_2}{w_2}\right)^{\frac{1-\alpha_2}{\alpha_2}} e^{\frac{\lambda_2}{\alpha_2} t} K_2(t) \qquad (1.4.3)$$

If we now ignore depreciation and assume that savings are equal to the share of profits in the industrial sector, then:

$$\dot{K}_2(t) = \alpha_2 Y_2(t)$$

so that the rate of growth of capital, after substitution from (1.4.3) is equal to:

$$\frac{\dot{K}_2(t)}{K_2(t)} = \alpha_2 A_2^{\frac{1}{\alpha_2}} \left(\frac{1-\alpha_2}{w_2}\right)^{\frac{1-\alpha_2}{\alpha_2}} e^{\frac{\lambda_2}{\alpha_2} t} \qquad (1.4.4)$$

Using the production function (1.4.1) and the fact that the output per man remains constant, the rate of growth of industrial output may be written:

$$\frac{\dot{Y}_2(t)}{Y_2(t)} = \frac{\lambda_2}{\alpha_2} + \alpha_2 A_2^{\frac{1}{\alpha_2}} \left(\frac{1-\alpha_2}{w_2}\right)^{\frac{1-\alpha_2}{\alpha_2}} e^{\frac{\lambda_2}{\alpha_2} t} \qquad (1.4.5)$$

The rate of growth of industrial employment is, of course, equal to the rate of growth of industrial output.

Dixit (1973) develops a simple model assuming that wage income is consumed and that a constant fraction s of profits is saved and invested. He takes a general production

function with constant return to scale and purely labour-augmenting technical progress at a constant rate b. Thus the basic equations are:

$$Y_2(t) = F(K_2(t),\ e^{bt} L_2(t)) \tag{1.4.6}$$

$$w_2(t) = e^{bt} F_{M_2}(K_2(t), M_2(t)) \tag{1.4.7}$$

where $M_2(t) = e^{bt} L_2(t)$ is the labour force in efficiency units, and

$$\dot{K}_2(t) = I_2(t) = s(Y_2(t) - w_2(t)\, L_2(t)) =$$

$$= s\,[\,F(K_2(t), M_2(t)) - M_2(t)\, F_{M_2}(K_2(t), M_2(t))\,] =$$

$$= s\, K_2(t)\, F_{K_2}(K_2(t), M_2(t)).$$

If π is the elasticity of F with respect to K_2 and σ is the elasticity of substitution for F, then, the growth equations are:

$$\dot{K}_2(t) / K_2(t) = s\, F_{K_2}(K_2(t), M_2(t)) \tag{1.4.8}$$

$$\dot{Y}_2(t) / Y_2(t) = s\, F_{K_2}\ b\, \sigma (1 - \pi) / \pi \tag{1.4.9}$$

and

$$\dot{L}_2(t) / L_2(t) = s\, F_{K_2} + b\,(\sigma / \pi - 1) \tag{1.4.10}$$

From the expressions above after some manipulations we find:

$$\left(\frac{\dot{Y}_2(t)}{L_2(t)}\right) \Big/ \left(\frac{Y_2(t)}{L_2(t)}\right) = b\,(1 - \sigma) \tag{1.4.11}$$

$$\left(\frac{\dot{K}_2(t)}{Y_2(t)}\right) \Big/ \left(\frac{K_2(t)}{Y_2(t)}\right) = -b\,\sigma\,(1 - \pi) / \pi \tag{1.4.12}$$

and

$$\left(\frac{\dot{K}_2(t)}{M_2(t)}\right) \Big/ \left(\frac{K_2(t)}{M_2(t)}\right) = -b\,\sigma / \pi \tag{1.4.13}$$

In interpreting the above growth rates we take the following results for the classical model. The capital/output ratio falls over time and so does the ratio of capital to labour in efficiency units. Hence, the marginal product of capital rises and, from (1.4.8), so does

the rate of growth of capital stock. Industrial capital, in other words, experiences accelerated growth. Industrial labour productivity behaves ambiguously, growing at a positive rate if $\sigma < 1$. Jorgenson in his models assumes $\sigma = 1$ and concludes that industrial productivity remains constant in the classical approach. Of course, Jorgenson's conclution depends crucially on the use of a Cobb-Douglas production function. If the production function was of the constant-elasticity of substitution variety, Arrow, Chenery, Minhas, Solow (1956), the classical assumption of a constant wage is perfectly consistent with the existence of different rates of growth of output and employment, Marglin (1966).

Of course, everything can be explained by allowing wages to grow over time as a result of increases in agricultural productivity or shifts in the terms of trade. If we assume an exogenous rate of growth $\dot{w}_2(t) / w_2(t) = a$, then we would find:

$$\left(\frac{Y_2(t)}{L_2(t)}\right)^{\cdot} \Big/ \left(\frac{Y_2(t)}{L_2(t)}\right) = b(1-\sigma) + a\sigma \qquad (1.4.14)$$

In the beginning of this section we said that the natural criterion of success for industry would be its ability to absorb labour, and for this to happen industrial employment should grow faster than total labour force, a fact which is easily obtained from (1.4.10).

Fei and Ranis (1964) introduce the "critical minimum effort" criterion, which allows a different bias in technological progress, to investigate the various factors influencing the rate of labour absorption into the industrial sector. They derive the demand curve for labour by the industrial sector as

$$\eta_{L_2} = \eta_{K_2} + \frac{B_{L_2} + J_2}{\epsilon_{L_2 L_2}} \qquad (\eta_{w_2} = 0) \qquad (1.4.15)$$

where η indicates rate of growth, J_2 innovation intensity, B_{L_2} the degree of labour-using bias and $\epsilon_{L_2 L_2}$ the elasticity with respect to labour of the MPP of labour in the industry. Equation (1.4.15) may thus be viewed as the industrial sector's labour absorption equation.

If a particular labour surplus economy is to meet the above natural criterion of success, $L_2(t) / L(t)$ must be increasing through time, where L(t) is the total labour force; that is $\eta_{L_2/L} > 0$ which can be written as:

$$\eta_{L_2} > \eta_L \tag{1.4.16}$$

Combining now this criterion for success, (1.4.16), and the labour absorption equation (1.4.15), we obtain the "critical minimum effort criterion" (CMEC) for successful development:

$$\eta_L < \eta_{L_2} = \eta_{K_2} + \frac{B_{L_2} + J_2}{\epsilon_{L_2 L_2}} \qquad \text{for } \eta_{w_2} = 0 \tag{1.4.17}$$

Equation (1.4.17) states that the national development effort reflected in the magnitude of the labour causal factors on the right-hand side must, together, exceed population growth. In other words, the rate of industrial capital accumulation η_{K_2} must be large enough, the intensity of innovation J_2 high enough, the labour-using bias of innovation B_{L_2} strong enough, and the low of diminishing returns to labour $\epsilon_{L_2 L_2}$ weak enough so that their combined effect on the demand for labour exceeds the population growth rate.

Clearly, an underdeveloped economy may choose not to use this criterion from time to time. But no development effort can be termed successful if the CMEC is consistently violated over time. Such violation would imply that the pool of agricultural surplus population is being augmented faster by demographic pressures than it is being diminished by the withdrawal of workers reallocated to the industrial sector. The centre of gravity of the economy can be gradually shifted in the direction of the industrial sector if and

only if (1.4.17) is consistently satisfied over the long run. Fei and Ranis assert that (1.4.17) is a powerful equation summarising a whole constellation of forces. Dixit, on the contrary writes that we really want higher standards of living to be the result of successful development and not a mere transfer of labour.

Furthermore, Dixit (1973) uses his model above to tackle what Lewis (1954) calls the "central problem in the theory of economic development", namely "to understand the process by which a community which was previously saving and investing 4 or 5 per cent of its national income or less, converts itself into an economy where voluntary saving is running at about 12 to 15 per cent of national income or more." The saving ratio out of national income can be written as a product of two factors: the saving ratio out of industrial income, $I_2(t) / Y_2(t)$, and the share of industry in national income. He shows that:

$$\left(\frac{\dot{I}_2(t)}{Y_2(t)}\right) \Big/ \left(\frac{I_2(t)}{Y_2(t)}\right) = (b - a)(1 - \sigma)(1 - \pi)/\pi \qquad (1.4.18)$$

which means that if wages rise more slowly than the rate of labour-augmenting technological progress and if $\sigma < 1$, then there is not enough substitution of labour for capital and the share of profit goes up. This seems to have been what was understood to happen in the earlier phases of industrialisation, according to the classical theory.

The other component, the share of industrial income in the national income, goes up in the course of development, according to all the theories. An extreme classical theory with zero marginal product of labour would keep food output constant and the terms of trade constant, and then industrial growth would lead to an increase in industry's share. Of course, a rise in both factors is sufficient but not necessary for a rise in their product. Jorgenson, for instance, uses a Cobb-Douglas production function in the industrial sector

thus keeping its profits share constant and relying entirely on the rise in the relative weight of the industrial sector to explain the rise in the economy-wide saving ratio. Moreover, in the course of development, agricultural savings would become more important as financial intermediation becomes better, and this would contribute further to a rise in the savings ratio.

1.5 The Complete Model of the Dual Economy

In section 1.3 we concentrated on the backward sector and in section 1.4 on the advanced sector. In this section we shall consider both the backward and advanced sectors together noting the interactions between these two sectors.

So, for the classical approach, the total population is the sum of the agricultural labour force, the industrial labour force and the redundant labour, ie.

$$L(t) = L_1(t) + L_2(t) + R(t) \qquad (1.5.1)$$

The agricultural labour force is the minimum of total labour force less the industrial labour force and the maximum labour force which may be employed at a positive marginal productivity in agriculture.

$$L_1(t) = \min \begin{cases} L(t) - L_2(t) \\ + \\ L_1(t) \end{cases} \qquad (1.5.2)$$

So long as there is redundant labour in the agricultural sector, industrial output and industrial employment grow at a rate which is positive and increasing. Capital in the industry also grows at a rate which is positive and increasing, but always less than the rate of growth of output. This implies that the capital-output ratio is always falling, Jorgenson (1967), result which has also been obtained by Fei and Ranis (1963).

Fei and Ranis (1961), explain the interactions between the two sectors using the following figure 1.5. Figure 1.5b is figure 1.1a rotated through 180 degrees. Suppose

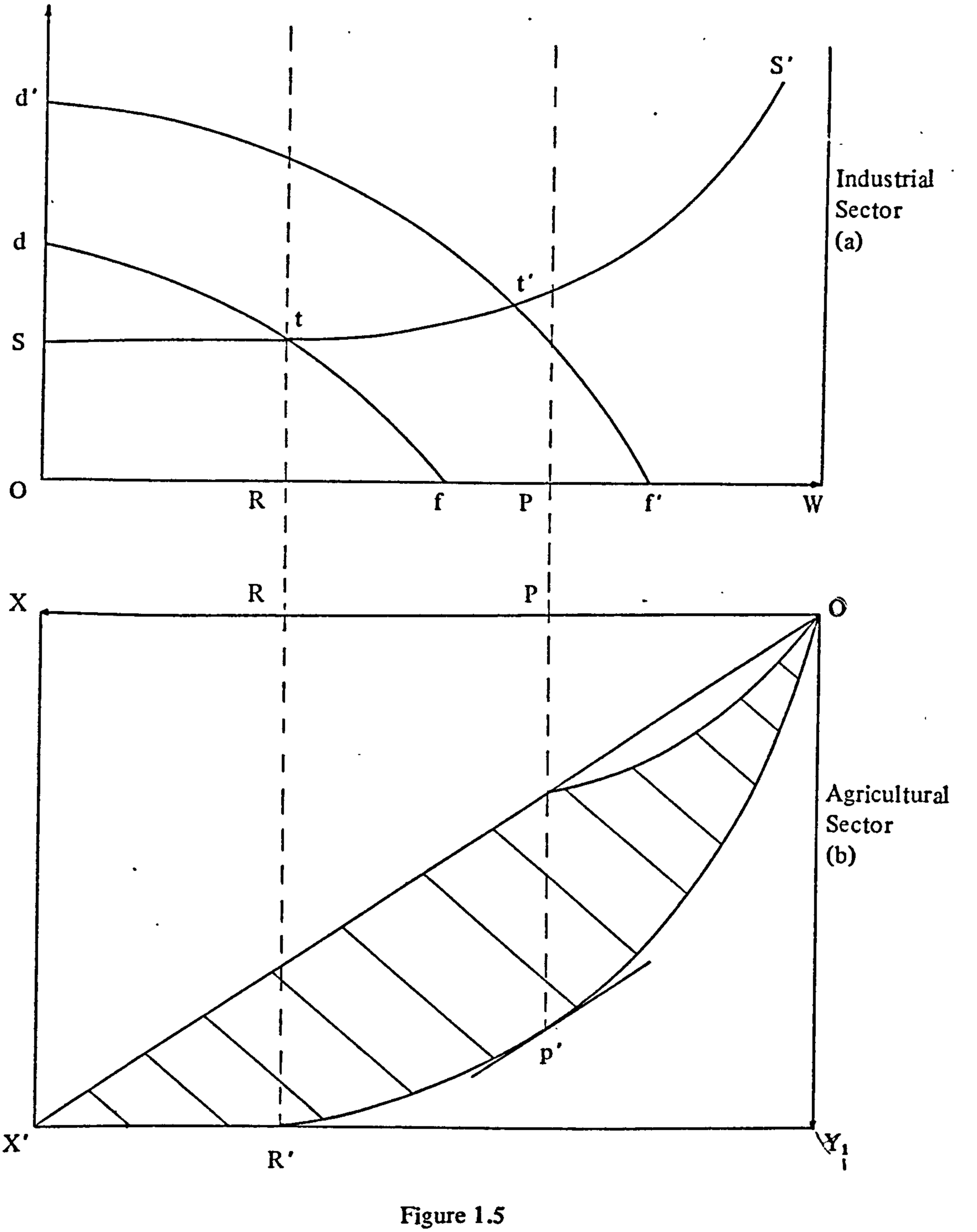

Figure 1.5

that the total labour force is constant and represented in figure 1.5 by the segment OX = OW. The industrial labour force is measured from O to W and the agricultural labour force from O to X. The explanation of figure 1.5b is similar to that of figure 1.1a. The point R

is the first turning point while point P is the second turning point. The segment XR = OR represents the redundant labour for which the marginal physical productivity of labour in agriculture is zero.

Every labourer in agriculture receives the constant institutional wage which is equal to the slope of the straight line OX',(or the slope of the tangent of the TPP curve at point P'). The departure of labour from the agricultural sector as long as the labour force in agriculture exceeds OR will not decrease the total output in agriculture. If the total labour force in agriculture is between OP and OR then a departure of a labourer will decrease agricultural output by an amount less than the institutional wage, so the landlord will not mind losing the services of the labourer. If the agricultural labour force continues to decline it will eventually fall to the level OP. At the point P the agricultural labour market starts to be fully commercialized and enter into competition with the industrial labour market because any departure of a labourer from the agricultural sector to the industrial sector will have the result that agricultural output will decrease by an amount greater than the wage and so the landlord will be willing to bid up the agricultural wage to keep the man on the farm.

The continuous movement of labour from agriculture to industry augments the supply of labour to industry. If we assume that, at some point in time, all the labour was in agriculture and none in industry then the start of this movement would create an agricultural surplus of output. If, for instance, L'(t) labourers went to industry then the total agricultural surplus would be $[TPP - \bar{y}\ (L(t) - L'(t)\ )\ ]$, where $\bar{y}$ is the institutional wage. If figure 1.56 the total agricultural surplus is represented by the shaded area (as explained in figure 1.1a).

The landlord who is receiving the agricultural surplus as a consequence of the

decrease in the total wage bill sells it on the urban market. So, we can say that the amount of food available per industrial worker is still $\bar{y}$. According to these assumptions it is obvious why the supply curve of labour in the industry has the horizontal segment St at the level of OS, equal to the institutional wage, $\bar{y}$, (ie. every labourer who migrates to industry takes with him his own food budget).

After the point R the withdrawal of more labour from agriculture leads to some decrease in food output with the consequence that the average surplus of food falls. This means that the relative price of food rises at the point where the supply curve of labour in the industry begins to rise.

After the point P the agricultural surplus falls more rapidly causing the wage to increase more quickly. Also after this point, the supply curve is much steeper than between points R and P.

The amount of labour that can be engaged in industry is defined by the intersection of the supply curve of labour SS' and its demand curve df (the marginal productivity curve for a fixed amount of capital). But as the agricultural surplus per head is decreasing and the wages are increasing the profits in the industry decrease with the consequence that in the next period investment is lower and so development is slower. In contrast, the increase of agricultural productivity due to technological progress (and also implicitly capital accumulation) and the increase of industrial productivity also due to technological progress and capital accumulation (explicitly), pushes the demand curve of labour in the industry higher and to the right. The landlord can sell the excess surplus (due to the increase in productivity) in the urban market and afterwards he transfers his savings to industry to strengthen its development.

Fei and Ranis (1964) discuss the quantity and the quality of the enterpreneurs and their

part in the process of development. Also, they advocate balanced growth with a constant industrial wage until the economy is commercialised. The horizontal portion of the industrial sector supply curve of labour St may be thus interpreted as an ex post supply curve, defined as the locus of all the points belonging to an infinite number of short-run equilibrium positions generating the balanced growth path under continuous increases of agricultural productivity. The economy's actual growth path will, of course, only very rarely follow the balanced growth path exactly; in any real world situation continuous deviations from the strictly horizontal pattern are bound to occur.

Jorgenson (1967) asserts, for the classical approach (an analytical description of the Fei and Ranis model using the Cobb-Douglas production function form of the previous section) that, since agricultural output is increasing at a constant rate, equal to the rate of technological progress in agriculture, population is increasing at this same rate. Whatever the initialvalue for the rate of growth of industrial output, this rate of growth eventually exceeds any fixed rate of growth. The sum of redundant labour and industrial employment grows at a rate which exceeds the rate of growth of population; but this rate must fall to the rate of growth of population. Hence, the rate of growth of industrial employment eventually becomes so large as to force the rate of growth of redundant labour to become negative and decreasing. Under this condition redundant labour eventually disappears altogether. This concludes the description of the first phase of development of the Fei and Ranis theory.

After the first turning-point is reached (Lewis turning point) and under the assumption that the real wage-rate remains fixed when measured in agricultural goods, the rate of growth of population is equal to the rate of technological change in the agricultural sector less the elasticity of agricultural output with respect to labour multiplied by the rate of

decline of the agricultural labour force. Where ℓ_1 is the proportion of the agricultural labour force in the total population and ℓ_2 is the proportion of the industrial labour force, this condition on the rate of the population growth implies:

$$\lambda_1 + (1 - \alpha_1) \frac{\dot{L}_1(t)}{L_1(t)} = \ell_1 \frac{\dot{L}_1(t)}{L_1(t)} + \ell_2 \frac{\dot{L}_2(t)}{L_2(t)}$$

or simply:

$$\lambda_1 + (\ell_2 - \alpha_1) \frac{\dot{L}_1(t)}{L_1(t)} = \ell_2 \frac{\dot{L}_2(t)}{L_2(t)} \qquad (1.5.3)$$

For this condition to be satisfied, the industrial proportion ℓ_2, must be such that the rate of growth of the agricultural labour force is negative at the Lewis turning-point. Furthermore, the rate of growth of the agricultural labour force must remain negative until the labour force itself reaches the level at which the marginal product of labour in agriculture, measured in agricultural goods, is equal to the real wage. For this to happen it suffices to assume that the share of the industrial sector in the total labour force exceeds the elasticity of agricultural output with respect to land. Under this condition the agricultural labour force declines at an increasing rate until the marginal product of labour is equal to the real wage. At this point the third phase of development of the dual economy is reached, where the wage-rate of labour is the same in the two sectors. This phase is going to be discussed below as a part of the neoclassical rather than the classical approach because in this phase all the factors of production are scarce, in the sense that their supply is inelastic.

As we saw in the previous section for the neoclassical theory of economic development the system depends upon the conditions of production and the net reproduction rate. It is characterised either by a low-level equilibrium, in which output per head is

constant and population is growing at less than its physiologically maximum rate, or by a steady growth equilibrium, in which output per head is rising and population is growing at its physiologically maximum rate. Where output per head is constantly rising, an agricultural surplus is generated, Jorgenson (1961, 1967). So, if s denotes the agricultural surplus, then:

$$y_1(t) - y_1^{+}(t) = s(t) \qquad (1.5.4)$$

If agricultural output per head exceeds the critical level, $y_1^{+}(t)$, part of the labour force may be released from the land to produce industrial goods with no reduction in the rate of growth of total population.

The demographic theory for development of a dual economy, which we have seen in section 1.3, when all the population was in the agricultural sector, can be described according to Jorgenson, as follows.

The net rate of reproduction is the minimum of the rate corresponding to the minimum force of mortality ϵ and a rate which corresponds to output of food per head; the basic demographic relationship may be written:

$$\frac{\dot{L}(t)}{L(t)} = \min \begin{cases} \gamma y_1(t) \dfrac{L_1(t)}{L(t)} - \theta \\ \epsilon \end{cases} \qquad (1.5.5)$$

where $y_1(t)\, L_1(t) / L(t)$ is output of food per head for the whole economy. For an economy with an agricultural surplus, total food consumption is the critical level, $y_1^{+}(t)$, multiplied by total population; the proportion of the total labour force employed in agriculture is the ratio of this critical level of agricultural production per head to the actual level of output per head

$$\frac{y_1^{+}(t)}{y_1(t)} = \frac{L_1(t)}{L(t)} \quad (\text{when } y_1(t) > y_1^{+}(t)\ ) \qquad (1.5.6)$$

Under these assumptions, the relationship governing the distribution of labour between the backward sector and the advanced sector may be represented by:

$$\frac{L_1(t)}{L(t)} = \min \left\{ \begin{matrix} 1 \\ \\ y_1^{+}(t) / y_1(t) \end{matrix} \right. \tag{1.5.7}$$

It can be proved that a necessary and sufficient condition for the viability of the advanced sector is $\lambda_1 - \alpha_1 \epsilon > 0$ which is also both the sufficient and the necessary condition for the emergence of the agricultural surplus.

From the fact that population is growing at a constant rate and that consumption of food per head is stationary, we obtain the following expression for the growth of the agricultural labour force:

$$L_1(t) = L(0)\, e^{\left[\frac{\epsilon - \lambda_1}{1 - \alpha_1}\right] t} = L_1(0)\, e^{\left[\frac{\epsilon - \lambda_1}{1 - \alpha_1}\right] t} \tag{1.5.8}$$

Agricultural population may grow, decline, or remain constant, depending on the magnitude of the parameters ϵ, the rate of growth of total population and λ_1 the rate of technological progress in agriculture. Then, the industrial population can easily be found as:

$$L_2(t) = L(0) \left[e^{\epsilon t} - e^{\left[\frac{\epsilon - \lambda_1}{1 - \alpha_1}\right] t} \right] \tag{1.5.9}$$

Bearing in mind the assumption $\lambda_1 - \alpha_1 \epsilon > 0$ we obtain that

$$\epsilon > \frac{\epsilon - \lambda_1}{1 - \alpha_1} \tag{1.5.10}$$

From this it can be seen that the rate of growth of population is greater than that of the agricultural population alone; hence the industrial labour force is growing at a rate which exceeds that of total population and which is always declining and approaches, as a limit, the rate of growth of total population, ϵ.

Once the share of labour in industrial output is distributed to workers in the form of food and consumption goods, and agricultural workers have received the proportion of industrial output which must be traded for food, the remainder of industrial output is available for capital accumulation, or more properly, for investment. Capital accumulation is defined as investment less depreciation, where depreciation is a constant fraction of capital stock:

$$\dot{K}_2(t) = I_2(t) - \delta K_2(t) \qquad (1.5.11)$$

where δ is the rate of depreciation, $I_2(t)$ is investment and $\dot{K}_2(t)$ is the net capital accumulation. Then, by definition, total industrial output is equal to consumption, plus investment:

$$Y_2(t) = (1-\alpha_2)\ Y_2(t) + I_2(t) \qquad (1.5.12)$$

Substituting in the last expression from (1.5.11) and (1.4.1) we obtain the fundamental differential equation for the development of a dual economy, which is:

$$\dot{K}_2(t) = -\delta K_2(t) + \alpha_2 A_2 K_2(t) L_2(0)^{1-\alpha_2} e^{\lambda_2 t} e^{\epsilon t} \left[e^{\left(\frac{\epsilon-\lambda_1}{1-\alpha_1}\right) t (1-\alpha_2)} - e^{\epsilon t (1-\alpha_2)} \right] \qquad (1.5.13)$$

From (1.5.13) it may be deduced immediately that there is no stationary situation for any economy in which the advanced sector is economically viable; that is, provided that there is a positive and growing agricultural surplus, the advanced sector must continue to grow. It can be shown, Jorgenson (1961), that only the initial size of population has any effect on the long-run pattern of growth of the economy while the influence of the initial capital stock required for sustained economic growth eventually dies out. Furthermore, it msut be noted that given any positive initial capital stock, no matter how small, the existence of a positive and growing agricultural surplus generates sustained economic growth.

For the neoclassical theory of development of a dual economy capital and output grow at the same rate in the long-run, namely, $\lambda_2 / (1 - \alpha_2) + \epsilon$. Population grows at the rate ϵ ; since the share of labour in industrial output is constant, the wage rate of the industrial labour force eventually grows at the rate $\lambda_2 / (1 - \alpha_2)$. In the short-run the beginning of the growth of the advanced sector is always characterized by a "big push" that is, an extraordinary high rate of growth of industrial output. From the view point of the neoclassical theory of development of a dual economy, such a high initial rate of growth may be interpreted as a statistical artifact. The rate of growth of industrial output is:

$$\frac{\dot{Y}_2(t)}{Y_2(t)} = \lambda_2 + \alpha_2 \frac{\dot{K}_2(t)}{K_2(t)} + (1 - \alpha_2) \frac{\dot{L}_2(t)}{L_2(t)} \qquad (1.5.14)$$

This rate of growth declines gradually, approaching a long-run equilibrium value equal to the rate of growth of total population.

The industrial wage-rate is determined by the usual marginal productivity condition.

$$\frac{\partial Y_2(t)}{\partial L_2(t)} = (1 - \alpha_2) \frac{Y_2(t)}{L_2(t)} = w_2(t) \qquad (1.5.15)$$

which is a necessary condition for the maximization of profit. Real wages in the industry must therefore rise at a rate $\lambda_2 / (1 - \alpha_2)$ in the long-run steady growth situation; this can be derived as follows:

$$\frac{\dot{w}_2(t)}{w_2(t)} = \frac{\dot{y}_2(t)}{y_2(t)} = \left[\frac{\dot{Y}_2(t)}{Y_2(t)} - \frac{\dot{L}_2(t)}{L_2(t)} \right] = \left[\frac{\lambda_2}{1-\alpha_2} + \epsilon \right] - \epsilon = \frac{\lambda_2}{1-\alpha_2} \qquad (1.5.16)$$

From (1.5.16) we see that the real wage will rise more rapidly, the more rapid is the rate of technological progress and less rapidly the higher is the share of labour in the product of any given period.

The real rate of interest attains a constant level whether or not there is technical

progress and it is:

$$r(t) = \frac{\partial Y_2(t)}{\partial K_2(t)} = \alpha_2 \frac{Y_2(t)}{K_2(t)} \qquad (1.5.17)$$

where the capital-output ratio, $K_2(t) / Y_2(t)$, in long-run sustained growth is given by:

$$\frac{K_2(t)}{Y_2(t)} = \frac{\alpha_2 (1 - \alpha_2)}{\lambda_2 + (1 - \alpha_2)(\epsilon+\delta)} \qquad (1.5.18)$$

which increases with the saving ratio and diminishes with the rate of depreciation on capital goods, the rate of growth of population and the rate of technical progress.

For the model of Dixit, introduced in section 1.4, we obtain the similar asymptotic results, of constancy of the capital-output ratio, with wages and productivity raising at the rate of labour-augmenting technical progress, ie.:

$$\frac{\dot{L}_2(t)}{L_2(t)} = \epsilon \qquad (1.5.19)$$

$$\frac{\dot{K}_2(t)}{K_2(t)} = \frac{\dot{Y}_2(t)}{Y_2(t)} = b + \epsilon \qquad (1.5.20)$$

$$\frac{\dot{w}_2(t)}{w_2(t)} = \left(\frac{Y_2(t)}{L_2(t)}\right)^{\cdot} \Big/ \left(\frac{Y_2(t)}{L_2(t)}\right) = b \qquad (1.5.21)$$

Having now the above results and that of (1.4.11) – (1.4.13), Jorgenson claims superiority for the neoclassical model, for its results are closer to the nineteenth-century Japanese data. In Dixit's (1970) opinion the main difficulty with such a comparison is that the asymptotic results of the neoclassical model are being compared with the short-term results of the classical model and he studies the neoclassical model in finite time as follows. Since,

$$r(t) = \frac{\dot{K}_2(t)}{K_2(t)} = \alpha_2 \frac{Y_2(t)}{K_2(t)}$$ we take

$$\frac{\dot{r}(t)}{r(t)} = \frac{\dot{Y}_2(t)}{Y_2(t)} - \frac{\dot{K}_2(t)}{K_2(t)} = (1-\alpha_2)\left(\frac{\dot{L}_2(t)}{L_2(t)} + b_2 - \frac{\dot{K}_2(t)}{K_2(t)}\right)$$

or

$$\dot{r}(t) = (1-\alpha_2)\, r(t)\left(\frac{\dot{L}_2(t)}{L_2(t)} + b_2 - r(t)\right) \qquad (1.5.22)$$

The solution of (1.5.22) can be shown to be of the form depicted in figure 1.6.

For small t, r(t) is small and positive. It rises steadily until, at time t^*, it crosses the declining curve of $\dot{L}_2(t) / L_2(t) + b_2$. From (1.5.22) we see that $\dot{r}(t)$ must be zero at this point. Then r(t) begins to decline, staying above $\dot{L}_2(t) / L_2(t) + b_2$, and the two

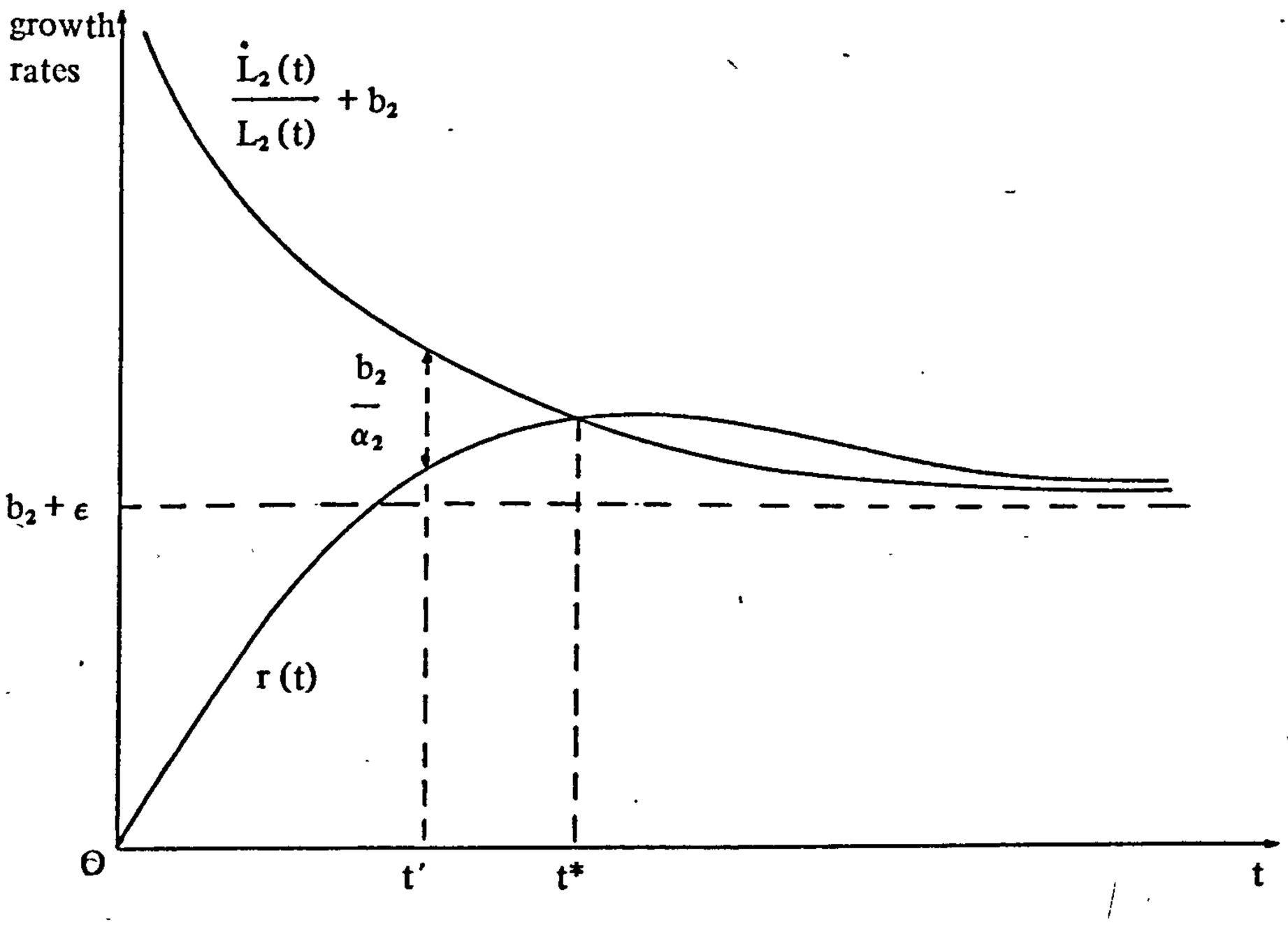

Figure 1.6

have the same asymptotic value $b_2 + \epsilon$. Up to t^*, r(t) is increasing and below $\dot{L}_2(t) / L_2(t) + b_2$. Thus, industrial capital is undergoing accelerated growth in this phase. The ratios of capital to output and capital to labour in efficeincy units are both falling, and hence the marginal product of capital is rising.

All these results are identical with those of the surplus-labour phase of the classical theory derived earlier. The Japanese data, cited by Jorgenson, if they are to be relied upon to reject the classical approach, must now reject the neoclassical approach as well. But the data, especially those pertaining to capital stock, are surely too crude to be used as a basis for passing such judgements on these simple and basically qualitative models.

We can derive the expression for the rate of change of labour productivity in the neoclassical model to be:

$$\left(\frac{\dot{Y}_2(t)}{L_2(t)} \right) / \left(\frac{Y_2(t)}{L_2(t)} \right) = b_2 - \alpha_2 \left(\frac{\dot{L}_2(t)}{L_2(t)} + b_2 - r(t) \right) \qquad (1.5.23)$$

This is ambiguous; in the early phases of development when $\dot{L}_2(t) / L_2(t) + b_2$ is large and $r(t)$ is small, it is certainly negative. It becomes positive and remains so from the instant t' when the gap between these two is reduced to b_2 / α_2. We know that t' is less than t^*, and if we carry out simulations with reasonable parameter values in the model, we find that values of t^* are too large to be neglected while values of t' are quite small and can be ascribed to be "statistical artifacts" prevailing in the very early phases of the model. On the whole, the neoclassical model does provide a somewhat better explanation of increases in industrial productivity.

Jorgenson's neoclassical model has been extended by Ramanathan (1967). He examined the properties of the model under the assumption that industrial workers may also save but save a different proportion than property owners. Secondly, he relaxed the assumption of constant per capita food consumption by assuming that per capita food consumption grows at a constant rate. So, if s_w is the proportion of income saved by industrial workers and s_r the proportion of income saved by capital owners, then, the level of investment is given in equilibrium by:

$$I_2(t) = s_w(1-\alpha_2)\ Y_2(t) + s_r\alpha_2\ Y_2(t) = s\,Y_2(t) \tag{1.5.24}$$

and per capita food consumption is given by:

$$\dot{y}(t) \ / \ y(t) = p \tag{1.5.25}$$

Having these new equations together with the basic functions of Jorgenson's model, we obtain:

$$L_1(t) = L(0)\ e^{\left(\frac{\epsilon-\lambda_1+p}{1-\alpha_1}\right)t} \tag{1.5.26}$$

from which we have the viability condition

$$\frac{\epsilon-\lambda_1+p}{1-\alpha_1} < \epsilon \text{ or } \lambda_1 - p - \alpha_1\epsilon > 0 \tag{1.5.27}$$

The manufacturing labour force then is given by

$$L_2(t) = L(0)\ [\ e^{\epsilon t} - e^{\left(\frac{\epsilon-\lambda_1+p}{1-\alpha_1}\right)t}\ ] \tag{1.5.28}$$

Following Jorgenson's method it can be proved that:

$$K_2(t) \to \left[\frac{s\,L(0)^{1-\alpha_2}(1-\alpha_2)}{\lambda_2 + \epsilon(1-\alpha_2)}\right]^{\frac{1}{1-\alpha_2}} e^{\left(\epsilon+\frac{\lambda_2}{1-\alpha_2}\right)t} \tag{1.5.29}$$

and

$$\frac{\dot{Y}_2(t)}{Y_2(t)} \to \epsilon + \frac{\lambda_2}{1-\alpha_2} \tag{1.5.30}$$

Thus, in the long-run, capital and industrial output both grow at the rate $\epsilon+\lambda_2/(1-\alpha_2)$, which is evidently the same as obtained by Jorgenson, even under the assumptions of different saving proportions and a growing per capita food consumption. The rate of growth is independent of the savings ratios.

The capital-output ratio is given by:

$$\frac{K_2(t)}{Y_2(t)} \to \frac{(1-\alpha_2)\,[\,s_w(1-\alpha_2)+s_r\alpha_2\,]}{\lambda_2 + \epsilon(1-\alpha_2)} \tag{1.5.31}$$

Thus, the long-run capital-output ratio decreases if the population and technological growth rates increase, and increases if the savings ratios increases.

The rate of growth of real wages is also the same as obtained by Jorgenson, and it is:

$$\frac{\dot{w}_2(t)}{w_2(t)} = \frac{\dot{Y}_2(t)}{Y_2(t)} - \frac{\dot{L}_2(t)}{L_2(t)} = \frac{\lambda_2}{1-\alpha_2} \qquad (1.5.32)$$

The real rent is also constant,

$$r(t) = \frac{\alpha_2 \left[\epsilon + \frac{\lambda_2}{1-\alpha_2}\right]}{(1-\alpha_2)\, s_w + s_r\, \alpha_2} \qquad (1.5.33)$$

Thus, the assumptions of different saving propensities for industrial workers and property owners has not altered the long-run growth rates of capital, output and real wages. Since the long-run rates of growth of capital, output, the capital-output ratio, real wages and rent are all independent of p, these variables are totally unaffected by the assumption that per capita food consumption is growing at a constant rate. This result is somewhat surprising because intuitively we would expect that a growing per capita food consumption will release less workers to the industrial sector, and therefore industrial output is likely to grow at a smaller rate. However, the reconciliation is straightforward. In the short-run, capital and output do grow at a smaller rate because of the above fact, but as t increases without limit, the effects of p die out, giving rise to the same long-run growth as was obtained before.

1.6 Trade of Goods between the two Sectors

In the previous sections we did not mention the terms of trade between agriculture and industry, leaving them for this section in which we will also discuss the marketable surplus of food.

Jorgenson, (1961, 1967), assumes that the wage rates in the backward sector of a

dual economy are proportional to those of the advanced sector. Using this relationship and the saving function it is possible to determine the terms of trade between agriculture and industry.

The balance of trade between agriculture and industry requires that the value of labour income in both sectors is equal to the value of industrial output not used for additions to capital together with the value of total agricultural output. This balance relation may be written:

$$w_2(t)L_2(t) + \mu w_2(t)L_1(t) = (1 - \alpha_2)\, Y_2(t) + q(t)\, Y_1(t) \qquad (1.6.1)$$

where q(t) is the terms of trade between agriculture and industry and μ is the constant of proportionality between wage rates in the agricultural sector and wage rates in the industrial sector expressed in industrial goods.

There are many interpretations of μ. For instance, in a neo-classical theory of growth μ is equal to unity because wage-rates in the two sectors are equal. Alternatively, if the process of development of a dual economy is characterized by a steady flow of labour from agriculture to industry, a differential between agricultural and industrial wages may be required to sustain this flow, Jorgenson (1967). In another way, if land is owned by the cultivators but the full value of land cannot be realized by outright sale, the industrial wage rate must be sufficiently high to cover both labour and property income for a member of the agricultural labour force, Lewis (1954). For an analytical interpretation of μ see next chapter.

Taking now that μ is a fixed constant, from (1.6.1) we obtain:

$$\mu w_2(t)\, L_1(t) = q(t)\, Y_1(t) \qquad (1.6.2)$$

so that

$$\frac{\dot{w}_2(t)}{w_2(t)} + \frac{\dot{L}_1(t)}{L_1(t)} = \frac{\dot{q}(t)}{q(t)} + \frac{\dot{Y}_1(t)}{Y_1(t)}$$

and finally

$$\frac{\dot{q}(t)}{q(t)} = \left(\frac{\epsilon - \lambda_1}{1 - \alpha_1} - \epsilon \right) + \frac{\dot{w}_2(t)}{w_2(t)} \qquad (1.6.3)$$

In the long-run the rate of growth of the wage-rate in the industry is equal to $\lambda_2 / (1 - \alpha_2)$, so that the rate of growth of the terms of trade is the sum of a negative and a positive quantity; hence the terms of trade may turn in favour of agriculture or industry, depending on the relative magnitude of the two quantities.

In the extension of Jorgenson's model by Ramanathan (1967), the growth rate of the terms of trade is given by:

$$\frac{\dot{q}(t)}{q(t)} = \frac{\alpha_1 (p + \epsilon) - \lambda_1}{1 - \alpha_1} + \frac{\dot{w}_2(t)}{w_2(t)} \qquad (1.6.4)$$

From (1.6.4) we conclude that if per capita food consumption is steady and industrial technical progress is fast, then the supply of industrial goods increases at a rapid rate, thereby lowering their price. This accounts for the fact that when λ_2 is large q(t) has a positive rate of growth. If per capita food consumption grows at a rate p this tends to raise the price of agricultural goods, thereby leading to a faster growth of q, ie., the terms of trade move in favour of agriculture. From the above we see that the assumptions of different saving propensities for industrial workers and property owners do not affect the terms of trade.

Generally, if μ is not a constant we would have

$$\frac{\dot{q}(t)}{q(t)} = \frac{\dot{w}_2(t)}{w_2(t)} + \frac{\dot{\mu}(t)}{\mu(t)} + \frac{\alpha_1 (p + \epsilon) - \lambda_1}{1 - \alpha_1} \qquad (1.6.5)$$

Of course, from (1.6.5) we can no longer determine the growth rate of the terms of trade unless we have some additional information about $\mu(t)$.

In what follows we will discuss the marketable surplus of food.

Jorgenson (1961) assumes that agriculture is peasant-owned and that each peasant receives his average product, so that the distribution among peasants is completely equal and beyond the level $y_1^+(t)$ the income elasticity of demand for food is zero.

Fei and Ranis (1961) assume that farm workers are paid an institutional wage and that all the surplus is appropriated and sold by landlords.

It must be mentioned that the greater the inequality of income within the agricultural sector, the greater will be the supply of marketable surplus out of a given level of output. Of course, the truth lies between the extreme models of Jorgenson and Fei & Ranis and the income elasticity of demand for food must be between zero and one in accordance with Engel's law.

Dixit (1973) presents a model with income effects. Suppose that, once food output per capita has passed $y_1^+(t)$ and population is growing exponentially at a rate ϵ, the per capita demand for food is a function of income in terms of food, with a constant elasticity v between 0 and 1. We shall confine ourselves to the case where the income in terms of food is equal for workers in the two sectors, each being equal to the average product in agriculture. Then, the total demand for food in the economy is simply:

$$c\,L(t)\left(\frac{Y_1(t)}{L_1(t)}\right)^{v}$$

where c is a constant. This must equal food output on the equilibrium growth path, hence

$$Y_1(t) = c\,L(t)\left(\frac{Y_1(t)}{L_1(t)}\right)^{v} \qquad (1.6.6)$$

The production function for food is

$$Y_1(t) = A_1\, e^{b_1 t}\, L_1(t)^{\beta_1} \qquad (1.6.7)$$

From the above two functions we find that

$$\frac{\dot{L}_1(t)}{L_1(t)} = \frac{\epsilon - b_1 + b_1 v}{\beta_1 + v - v\beta_1} \qquad (1.6.8)$$

In the case with the population initially totally agrarian, the productivity condition which enables the release of labour to industry is $\dot{L}_1(t) / L_1(t) < \epsilon$, which is easily seen to reduce to

$$(1 - v)(b_1 - \epsilon(1 - \beta_1)) > 0 \qquad (1.6.9)$$

Since $0 < v < 1$, this is equivalent to (1.3.11). Thus Engel's law is consistent with industrialisation and imposes no further restriction to allow the possibility of sustained industrial growth. This does not mean that the marketable surplus problem is irrelevant. It is easy to see that

$$\partial \left(\frac{\dot{L}_1(t)}{L_1(t)} \right) / \partial v = (b_1 - \epsilon(1 - \beta_1)) / (\beta_1 + v - v\beta_1)^2 \qquad (1.6.10)$$

If the productivity condition is satisfied, this is positive so that higher values of v entail higher values of $\dot{L}_1(t) / L_1(t)$, and consequently a smaller $\dot{L}_2(t) / L_2(t)$. In other words, when the dual economy has to reckon with the problem of a positive propensity to retain farm output for farm consumption, it must be content with a slower rate of release of labour to industry. Although industrialisation is ultimately possible, the lower level of command over the marketable surplus slows the rate at which it takes place. This is the crux of the marketable surplus problem.

Zarembka (1970, 1972) explains the trade of surplus agricultural products in the rural areas for the industrial products of the urban areas, allowing a gap between the incomes of the workers in the two sectors. The two production functions that he uses are of the Cobb-Douglas form which for notational convenience we write again as

$$Y_1(t) = e^{b_1 t} L_1(t)^{\beta_1}, \quad 0 < \beta_1 < 1 \qquad (1.6.11)$$

$$Y_2(t) = e^{b_2 t} K_2(t)^{1-\beta_2} L_2(t)^{\beta_2}, \quad 0 < \beta_2 < 1 \qquad (1.6.12)$$

First, the consumption of food in rural areas depends upon the consumption levels of two separate groups: those who are landless, $L'_1(t)$, and those who live on the farm as part of a farm family, $L''_1(t)$, (ie. $L'_1(t) + L''_1(t) = L_1(t)$. The average income level of the former group is their wage-rate, equal to the marginal product of farm labour $\beta_1 Y_1(t)/L_1(t)$; the average income of the latter group is the average product per farm, net of wage payments to landless labour, or $(Y_1(t) - L'_1(t)\,\beta Y_1(t)/L_1(t)\,)\,/L''_1(t)$. Therefore, if food consumption depends upon real income (defined in terms of food pruchasing power) and upon the relative food price q(t), then food consumption is

$$L'_1(t)\, f_1\left(\frac{\beta_1 Y_1(t)}{L_1(t)}, q(t)\right) + L''_1(t)\, f_1\left(\frac{Y_1(t) - L'_1(t)\beta_1 Y_1(t)/L_1(t)}{L''_1(t)}, q(t)\right) \tag{1.6.13}$$

where f_1 is the rural consumption function for food.

In the secondary, urban sector food consumption is also formulated as depending upon real income (defined, again, in terms of food purchasing power) and upon the relative price of food. Thus, if only capital income is invested, the urban food demand is:

$$L_2(t)\, f_2\left(\frac{\beta_2 Y_2(t)}{L_2(t)\, q(t)}, q(t)\right) \tag{1.6.14}$$

where f_2 is the food consumption function in urban areas.

The marketable surplus of the primary sector is its sectoral output minus its own consumption. In equilibrium between the primary and secondary sectors, the supply of food to the urban areas must be equated to the food demand in the cities. So, from (1.6.13) and (1.6.14) taking into account that

$$\frac{q(t)\,\beta_1 Y_1(t)}{L_1(t)} = \frac{\mu\,\beta_2 Y_2(t)}{L_2(t)}, \quad 0 < \mu \leq 1 \tag{1.6.15}$$

we obtain

$$Y_1(t) - L'_1(t)\, f_1\left(\frac{\beta_1 Y_1(t)}{L_1(t)},\ q(t)\right) - L_1''(t)\, f_1\left(\frac{(1+(1-\beta_1)L'_1(t)/L_1''(t))\, Y_1(t)}{L_1(t)},\ q(t)\right) =$$

$$= L_2(t)\, f_2\left(\frac{\mu^{-1}\beta_1 Y_1(t)}{L_1(t)},\ q(t)\right) \qquad (1.6.16)$$

In the special case where the consumption functions for the sectors owe the same and the implicit demand functions for industrial goods are such that the consumption function for food has a constant elasticity in both income α_1 and relative price $-\eta_1$, from (1.6.16) after substitution we obtain

$$\frac{Y_1(t)}{L(t)} = c_1 \left(\frac{Y_1(t)}{L_1(t)}\right)^{\alpha_1} q(t)^{-\eta_1} \qquad (1.6.17)$$

where c_1 is taken as a constant. From (1.6.17) and after substitution from (1.6.11) and assuming that the capital-output ratio in the industry is constant, which amounts to studying the asymptotic characersitics of growth, we can solve for the other variables of the model. In particular the growth of the relative price is given by:

$$\frac{\dot{q}(t)}{q(t)} = \frac{b_2}{\beta_2} - \frac{(1-\beta_1)\,\eta_1\,(b_2/\beta_2) + (b_1 - (1-\beta_1)\,\epsilon)}{1-(1-\beta_1)(1-\alpha_1-\eta_1)} \qquad (1.6.18)$$

where ϵ is the exogenous growth rate of the total labour force, as previously.

As we said above, the constraint on the marketable surplus slows the rate of industrialisation. If a government has total control over the economy and is able to depress current consumption as low as is compatible with biological subsistence then there is no problem about the growth of the surplus. Things are quite different in the "no-control" economies and a policy is required to control the surplus. Thus, Bose (1968) shows that direct taxation of agriculture will always be worth while so long as there are no singificant disincentive effects on output. Hornby (1968) shows that policies of investment and indirect taxation for agriculture are complementary and are the most effective for fast

growth. Finally, Dixit (1969) obtains the optimum policies for rapid industrialisation.

1.7 Growth Models with more than Two Consumption Goods

We mentioned in section 1.1, that the growth models are divided, with respect to the consumption goods which they include, into three categories, Wan (1971):

1. The models with no consumable goods. The Von-Neumann model (1937) of balanced growth belongs to this category. Consumption is regarded as a necessary input to produce labour and little discretion is left to the consumer. We are not going to consider these models.

2. The models with one consumption good. This is the Meade case, with outputs divided into several capital goods and one consumption good. Most two-sector growth models belong to this type. We will not consider those models either.

3. More than one consumption good. So far, there have been few models of this kind. Amongst the exceptions are the Jorgenson (1961) model of a dual economy and the Dhrymes (1962) model of N-sector balanced growth. Both rely on the very special assumptions of a Cobb-Douglas production function.

The models that we discussed in the previous sections all refer to two consumption goods. In this section we will discuss two models using more than two consumption goods. The first has been presented by Zarembka (1972) and is a model for a developing economy with three sectors. The first two sectors of this model are exactly the same as those of his model presented in section 1.6. The third sector is a labour-intensive service sector.

In this model Zarembka assumes that the price elasticities of food demand are zero.

So, equation (1.6.17) is changed to:

$$\frac{Y_1(t)}{L(t)} = c_1 \left(\frac{Y_1(t)}{L_1(t)}\right)^{\alpha_1} \qquad 0 \leqslant \alpha_1 \leqslant 1 \qquad (1.7.1)$$

The equation which allocates total population becomes

$$L_2(t) = L(t) - L_1(t) - L_3(t) \tag{1.7.2}$$

and the production function is assumed to depend only upon labour as a variable factor of production, and to have constant returns to scale,

$$Y_3(t) = e^{b_3 t} L_3(t) \tag{1.7.3}$$

where b_3 is the rate of exogenous technological change.

Consumption of tertiary sector goods is equal to tertiary production and is specified to depend upon real consumer income (defined, as always, in terms of food), and upon its own relative price $q'(t)$, (the tertiary sector price relative to the secondary sector price which is taken as the numeraire). Therefore,

$$\frac{Y_3(t)}{L(t)} = c_3 \left(\frac{Y_1(t)}{L_1(t)} \right)^{\alpha_3} q'(t)^{-\eta_3} \tag{1.7.4}$$

where α_3 is the income elasticity of demand, $-\eta_3$ is the price elasticity, and c_3 is a constant.

The real wage-rate in the tertiary sector is simply equal to the average product $q'(t)Y_3(t) / L_3(t)$. This wage-rate is observed to be less than the secondary sector wage-rate in developing countries, perhaps owing to low skill levels. We have here the same expression for the wage-rates:

$$\frac{q'(t)\, Y_3(t)}{L_3(t)} = \frac{\mu' \beta_2 Y_2(t)}{L_2(t)}, \qquad 0 < \mu' \leq 1 \tag{1.7.5}$$

The solution to this model is not as complex as the number of equation suggests, primarily because we have made the model block-recursive by assuming the price elasticities of food demand to be zero. Here we only present the results for the rate of growth of prices:

$$\frac{\dot{q}(t)}{q(t)} = \frac{b_2}{\beta_2} - \frac{b_1 - (1-\beta_1)\epsilon}{1 - (1-\beta_1)(1-\alpha_1)} \tag{1.7.6}$$

and

$$\frac{\dot{q}'(t)}{q'(t)} = \frac{b_2}{\beta_2} - b_3 \qquad (1.7.7)$$

Drymes (1962) presented a multisectoral growth model which is a generalization of Solow's (1956) model. His sectoral production functions are given by:

$$Q_i(t) = A_i e^{\lambda_i t} K_i(t)^{\alpha_i} L_i(t)^{\beta_i} \qquad i = 1, \ldots\ldots, n \qquad (1.7.8)$$

where Q, K and L denote, respectively, output, capital and labour, all stated in their own physical units. Since we suppose one homogenous capital good and one homogenous type of labour, no ambiguity arises. We also impose the usual restrictions that $\alpha_i + \beta_i = 1$, for all i ; the A_i , λ_i are non-negative constants and λ_i signifying the productivity factor in the ith sector.

The demand functions employed are:

$$Q_j^{\,d}(t) = m_j I(t) \qquad j = 2, \ldots\ldots, n \qquad (1.7.9)$$

where we define aggregate income by

$$I(t) = r(t) K(t) + w(t) L(t) \qquad (1.7.10)$$

ie. as the sum of the income of capitalists, r(t) K(t), and the income of workers, w(t) L(t).

We require that $m_j > 0$, $j = 2, \ldots\ldots, n$ which, of course, implies that

$$\sum_{j=2}^{n} m_j < 1 \qquad (1.7.11)$$

The condition of equilibrium, assuming perfect competition, are

$$r(t) = p_i(t) \frac{\partial Q_i(t)}{\partial K_i(t)} \qquad i = 1, \ldots\ldots, n \qquad (1.7.12)$$

$$w(t) = p_i(t) \frac{\partial Q_i(t)}{\partial K_i(t)} \qquad i = 1, \ldots\ldots, n \qquad (1.7.13)$$

In (1.7.12) it is to be understood that $p_1(t) \equiv 1$, identically in t, ie. the first good is the

numeriare.

The clearance of the market equations takes the form

$$Q_j(t) = p_i(t)^{-1} m_j I(t) \qquad j = 2, \ldots\ldots, n \qquad (1.7.14)$$

The full employment resources assumptions are

$$K(t) = \sum_{i=1}^{n} K_i(t) \qquad (1.7.15)$$

$$L(t) = \sum_{i=1}^{n} L_i(t) \qquad (1.7.16)$$

To complete the structure of the system we must add the following equations

$$\frac{dK(t)}{dt} = Q_1(t) \qquad (1.7.17)$$

and

$$L(t) = L(0)\, e^{st} \qquad (1.7.18)$$

where s is the exogenously determined rate of growth of labour.

The analytical solution to this model is given by

$$\begin{aligned}
K_i(t) &= a_i'\, e^{(\frac{\lambda_1}{\beta_1} + s)\, t} \\
L_i(t) &= b_i'\, e^{st} \\
p_i(t) &= D_j'\, e^{[(\lambda_1 - \lambda_j) + \frac{\lambda_1}{\beta_1}(\alpha_1 - \alpha_j)]\, t} \\
r(t) &= c_1' = \frac{\sum_{i=1}^{n} m_i \alpha_i}{m_1} \left(\frac{\lambda_1}{\beta_1} + s\right) \\
w(t) &= c_2'\, e^{\frac{\lambda_1}{\beta_1} t}
\end{aligned} \qquad (1.7.19)$$

where the a_i', b_i', D_j' and c_k' are appropriate constants which depend only on A_i, m_i and α_i.

The above system experiences "balanced growth" in the sense that the ratio of resources diverted into the ith and jth sectors is independent of time, ie. constant; precisely:

$$\frac{L_i(t)}{L_j(t)} = \frac{\beta_i m_i}{\beta_j m_j} \quad , \quad \frac{K_i(t)}{K_j(t)} = \frac{\alpha_i m_i}{\alpha_j m_j} \quad , \qquad i,j = 1, \ldots\ldots, n \qquad (1.7.20)$$

If income is defined as in (1.7.10) and if we denote the share of labour by

$$\mu(t) = \frac{w(t)\, L(t)}{I(t)} \qquad (1.7.21)$$

then $\mu(t) = c_1 / (c_1 + c_2)$.

It is obvious, that the share of capital is $1 - \mu(t)$. The interesting feature of (1.7.21) is that when the relation is expressed in terms of the parameters of the system, it yields

$$\mu(t) = \sum_{i=1}^{n} m_i \beta_i \quad , \qquad 1 - \mu(t) = \sum_{i=1}^{n} m_i \alpha_i \qquad (1.7.22)$$

Equations (1.7.19) give a complete characterisation of the equilibrium properties of the system. We easily obtain from them the result that, asymptotically the rent-wage ratio vanishes, and that the rate of growth of output and the capital stock are equal, results which were also obtained by Solow (1956).

It is of interest to examine briefly the behaviour of (physical) output-capital and output-labour ratios. Thus, from (1.7.19) we find

$$Q_i(t) = E_i\, e^{(\lambda_i + \frac{\lambda_1 \alpha_i}{\beta_1} + s)\, t} \qquad (1.7.23)$$

$$\frac{Q_i(t)}{K_i(t)} = F_i\, e^{(\lambda_i - \frac{\beta_i}{\beta_1} \lambda_1)\, t} \qquad (1.7.24)$$

$$\frac{Q_i(t)}{L_i(t)} = G_i\, e^{(\lambda_i + \frac{\lambda_1 \alpha_i}{\beta_1})\, t} \qquad (1.7.25)$$

where E, F and G are appropriate constants. Now, from (1.7.23) it may be easily deduced that the relative rate of change of the equilibrium output of the jth good may be decomposed into two parts; one

$$(\lambda_j \beta_1 - \lambda_1 \beta_j) / \beta_1$$

which is of opposite sign but equal in magnitude to the rate of change of its own (relative) price and the other

$$(\lambda_1 / \beta_1) + s$$

which is identical with the rate of growth of income implied by the choice of the first good as the numeraire.

In both (1.7.24) and (1.7.25) we find that output per limit of capital and output per unit of labour will grow more rapidly the more capital-intensive the sector is. In view of the "balanced growth" feature of the equilibrium configuration, this result is a reflection of the fact that capital is a "more important intrinsic" contributor to the growth of sectoral outputs than is labour, the former being produced under conditions of technological progress, whilst the latter is not.

Finally, we may remark that the capital-labour ratios tend to increase in all sectors, and this again in another aspect of the increasing (relative) scarcity of labour.

1.8 Conclusions

In section 1.2 we have presented the main notions and topics met in the theories of dual growth. Therefore, there is little left to be said in this section.

Here, we will summarise the major points which Jorgenson (1967) pointed out and which are presented in Dixit (1970).

1. In the classical theory of the development of a dual economy it is assumed that labour is growing exogenously, that there is some level at which the marginal

productivity of labour in agriculture becomes zero and after that level labour is available to industry without loss of agricultural output.

2. In contrast, in the neoclassical theory, labour is a function of the per capita food supply until a critical level is reached, whereupon it becomes exogenous. The marginal productivity of labour in agriculture is always positive and labour is not available to industry without losing agricultural output.

3. The classical (or labour-surplus) approach implies a fall in the industrial capital-output ratio as the economy develops. The ratio is asymptotically constant under the neoclassical model.

4. The rate of growth of capital increases over time in the classical model, but is asymptotically constant in the neoclassical model.

5. In the classical model, output and employment in industry grow at the same rate so that productivity is constant while it rises in the neoclassical model.

6. There must be an absolute decline in the agricultural labour force before the surplus labour phase ends in the classical approach; this is not a necessary consequence of the neoclassical approach.

In spite of the various differences mentioned above, the dualistic models have a surprising amount in common. For example, in both, Jorgenson's and Fei & Ranis model, the production functions being of the restrictive Cobb-Douglas type, hypothesise in the industrial sector two inputs, those of capital and labour, whilst in the agricultural sector they assume only one input, that of labour (the input of land, being constant in supply, has been absorbed in the units of measurement). In both approaches the returns to scale are constant for the industrial sector and diminishing for the agricultural one, with the technological change being of the neutral disembodied type.

Finally, Singh (1975) proposes that it is analytically better to conceive of the classical and the neoclassical approaches as being just different stages in the same conceptual model of development and therefore, in this framework, the classical versus neoclassical debate really disappears.

1.9 References

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CHAPTER 2

MIGRATION AND THE WAGES DIFFERENTIAL

2.1 Introduction
2.2 Determinants of Migration
2.3 The Costs of Migration
2.4 The Migration Decision
2.5 The Labour Allocation Curve
2.6 The Supply of Labour
2.7 Conclusions
2.8 References

2.1 Introduction

In this chapter we do not try to develop a complete theory of labour migration from the agricultural to the non-agricultural sector of an economy; rather we try to express some ideas and to investigate some ad hoc hypotheses which have been taken by some theorists.

Therefore, in the first two sections of this chapter we summarize, in some way, the existing theoretical and empirical work in this area of research, and in the following five sections, we try to extend it by investigating in a more analytical way the determinants and the mechanics of labour migration between the two sectors.

We believe that we can not study migration alone without introducing a complete two-sector growth model. This relation of migration to the two-sector growth model will be discussed in the next chapter.

In this chapter we investigate the wages differential between the two sectors, a much debated notion in the literature, and its relation to labour migration. The wages differential plays a crucial role in the model of the next chapter, and for that reason we want to define it explicitly before including it in the dynamics of the two-sector growth model of the next chapter.

2.2 Determinants of Migration

Economic history is replete with evidence of persistent sectoral differentials in labour incomes (or wages), for example in the studies of Borts-Stein (1964), Watanabe (1965), Williamson (1965), Reynolds (1965), Taira (1966), and Ohkowa-Rovosky (1972).

The question which the economist usually faces is why the earnings of the workers in the various sectors of the economy (ie. agricultural and non-agricultural for our case) are not equal, in spite of the mass of labour migration between the two sectors.

In trying to find a plausible answer to this question, the economists who deal with migration research work along two lines. The first line, which has received more attention, concerns the direction and magnitude of the response of migrants to labour earnings differentials over space. The second line pertains to the connection between migration and those earnings, that is, the effectiveness of migration in equalizing inter-regional earnings of comparable labour.

For these lines of work, research has been focussed to two different approaches. The first approach takes into account expected earnings, introducing an urban unemployment variable. The theoretical works of Todaro (1968, 1969, 1970) and Zarembka (1970) fall into this category. The second approach uses the hypothesis that current wage differentials cause labour transfer between sectors. The theoretical works of Lewis (1954, 1958), Fei-Ranis (1961, 1964) and Jorgenson (1961, 1967) fall into this category.

Most studies concerned with the first line of research, ie. the direction and magnitude of the response of migrants to labour earnings differentials over space, have found a relationship between income and migration, and usually in the expected direction, that is, high earnings are associated with in-migration and low earnings with out-migration. Of course, the significance of wage differentials (current or expected) is not the only determinant of migration, and not all determinants are necessarily economic.

We will mention here some empirical works concerning the significance of the various determinants of migration, Zarembka (1972). Beals, Levy and Moses (1967) studied internal migration in Ghana, and concluded that migration is responsive to income differentials and that geographic distance -- as a surrogate for differences in culture, social organization and language as well as for transport cost -- is a strong deterrent to migration. Also, regions of large population are attractive to migrants, but the educational level does

not seem to directly affect migration.

The major findings of Sahota (1968) in a study of Brazil were again that internal migration is highly responsive to earning differentials and that distance is a strong deterrent to migration. However, he found that pecuniary costs of moving are more important than cultural and social differences. Further, Sahota found evidence that education is important to migration, even abstracting from its influence on other variables, and that urbanization and industrialization, as well as population density turn out to be related to migration. In sum, he asserts that 'economic costs and returns appear, on the whole, to dominate the behaviour of migrants, though some evidence of the non-economic 'push' and 'pull' factors is not denied'.

Greenwood (1963) analyzed migration in Egypt and similarly concluded that 'distance acts as an important impediment to migration' and that 'migration is away from low-wage and toward high-wage regions. His conclusions concerning urbanization, population density, and education were much the same as those of Sahota.

In studying Colombia, Schultz (1971), also found that migration responds to the market forces of wage differentials and employment opportunities and that distance to the city and schooling influence migration in the expected direction. He found, additionally, that migration in Colombia is increased with the frequency of rural violence.

Sjaastad (1961), in his study of interstage migration, shows that the observed relationship between net migration and wages differential is quite small and weak. He shows that over the decade 1940–1950, an increase in per capita labour earnings of $ 1000 (1947–49 dollars) induced net in-migration or retarded net out-migration by only 4 or, at most, 5 per cent of the population aged between fifteen and twenty-four years at the end of the decade. The percentage was lower for other ages and hence lower for the total

population.

Hanna (1959) concludes that the low income states are dominated by occupations with relatively low earnings at the national level, and the earnings within particular occupations in low-income states tend to be lower than the national average. Opposite relationships characterize the high-income states. Hanna's study, together with the observed relation between income and net migration, supports the hypothesis that migration does constitute a response to spatial earnings differentials; moreover, this evidence is consistent with the hypothesis that migration is a search for opportunities in higher-paying occupations, Sjaastad (1962).

Are these wages differentials real or not? There has been considerable debate in the literature about the question above. Recent studies confirm the hypothesis of real income differentials (Reynolds, (1965), Johnston-Nielsen (1966), Hellemer (1968), Warren (1966)).

The second line of research, ie. how effective is migration in equalizing inter-regional earnings of comparable labour, has received much less attention than the first line. The models of dual development of Fei-Ranis (1964) and Jorgenson (1961) assume that the migrant's response to an emerging wage gap is sufficiently rapid, compared to the forces causing the divergence in wages, to permit a convergence to short-run equilibrium in every period. Furthermore, Jorgenson argues that the differential which is necessary to cause a movement of the agricultural labour force into the non-agricultural sector is roughly proportional to the industrial wage, and he assumes that:

$$\frac{\bar{w}_1(t)}{w_2(t)} = \mu \qquad \text{where } 0 < \mu < 1$$

$w_1(t)$ = agricultural wage
$w_2(t)$ = industrial wage

or
$$\bar{w}_2(t) - w_1(t) = (1+\mu)\, w_2(t) \qquad (2.2.1)$$

According to Todaro (1969), in his low-income economy, or Harris and Todaro (1970)

in their two-sector model, migration follows a two-stage process. In the first stage the unskilled rural worker migrating to an urban area initially spends a period of time in the "urban traditional" sector (or stays unemployed). In the second stage, he obtains a more permanent job in the modern sector. The above authors assume that the decision to migrate from rural to urban areas is a function of the real wage differential and the probability of obtaining a job in the urban sector; thus the determinants of migration are expected earnings differentials rather than real current earnings differentials. These authors argue that the real wage in the urban sector is constrained to be greater than or equal to a politically fixed minimum wage, which is higher than the earnings in the agricultural sector, and that migration is still to be viewed as an equilibrium process, since labour migration continues in each period until the expected income differential is zero..

Sjaastad (1962) and DeVoretz (1969) utilize a capital-theoretic framework which explicitly introduces calculations of present values and migration costs. According to Sjaastad migration is treated as an investment increasing the productivity of human resources, an investment which has costs and which also renders returns. Treating migration as an investment, there exists a ready-made criterion to test the effectiveness of migration in reducing earnings differentials over space. That criterion is, of course, the rate of return on resources allocated to migration.

Wellisz (1974), follows Kenen (1963) and Hagen (1958) by using their short-run migration function which is:

$$M(t) = f(w_2(t)/w_1(t)) \text{ with } f(1) = 0 \text{ and } dM/d(\frac{w_2}{w_1}) > 0 \qquad (2.2.2)$$

Presented in diagrammatic terms the argument is that if the growth in demand for urban labour relative to the net increase in the number of urban-born workers is faster than the

growth in demand for rural labour relative to the net increase in the number of rural-born workers, an equilibriating migration stream is generated.

Suppose, in Fig. 2.1, that the initial agricultural wage $O_1 w_1^0$ equals the initial urban wage $O_2 w_2^0$ if full equilibrium is assumed. Two events now take place: (1) capital accumulation (or technological change) drives up the marginal products of the initial stocks of labour $O_2 w_2^1$ and $O_1 w_1^1$ respectively, and (2), the stocks of the two kinds of labour increase from natural causes by $O_2 O$ and $O_1 O$ respectively. No workers are hired during this period.

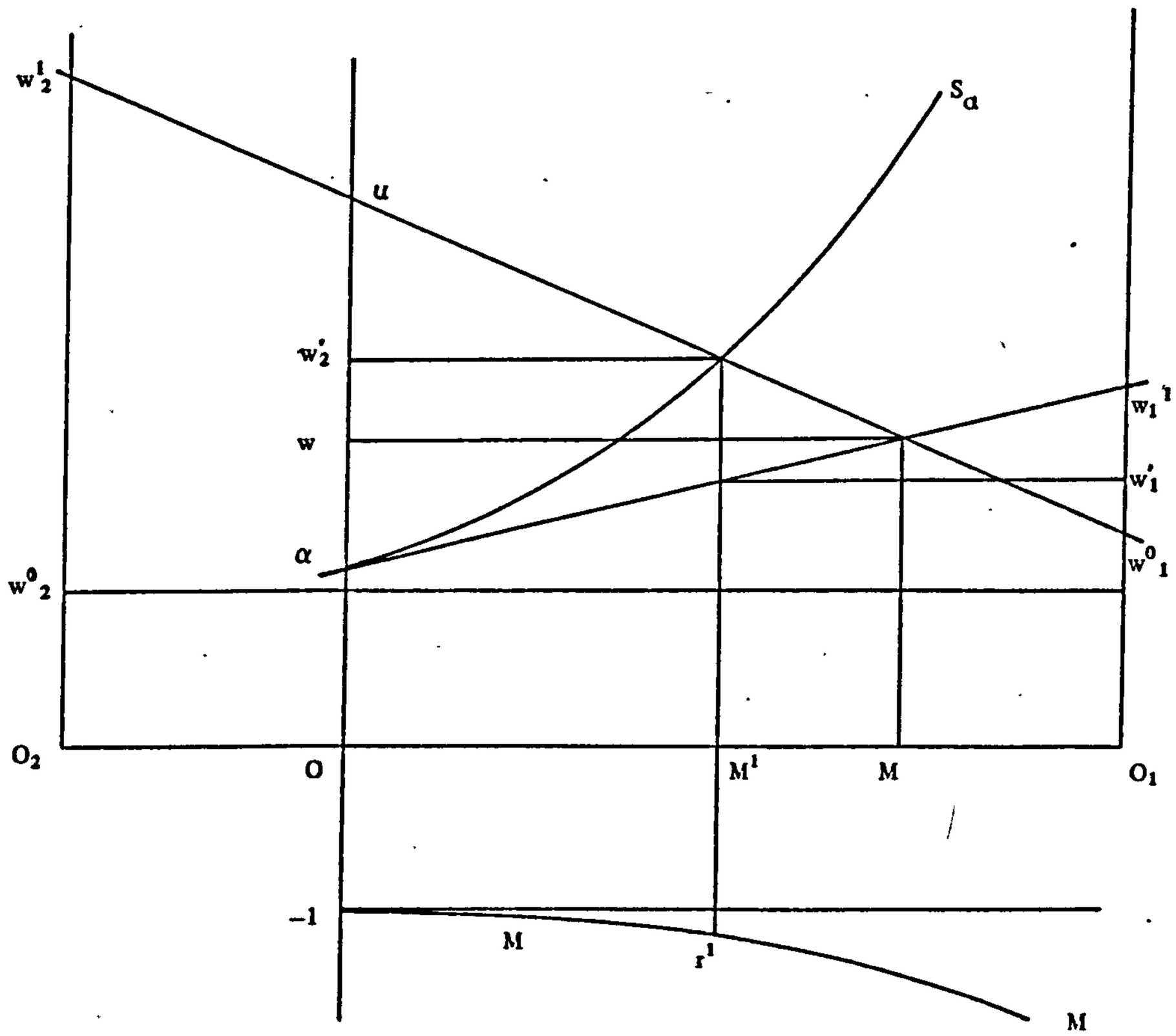


Figure 2.1

During the subsequent period workers are being hired, but there is no capital accumulation and no natural growth of labour. For the time being, it will be assumed

that the labour markets are competitive and that the wages announced at the beginning of the period clear the markets by the end of the period.

If migration from rural to urban areas was not permitted, or if it were prohibitively expensive, the urban wage would settle at Ou and the rural wage at Oa. If migration was economically and physically costless, a migration flow $O\hat{M}$ would restore the static equilibrium and establish a new common wage $O\hat{w}$. With positive but finite migration costs, an intermediate situation will be reached within a finite and arbitrarily short time span: the migration volume will be less than $O\hat{M}$, the urban wage higher than $O\hat{w}$ but lower than Ou, and the agricultural wage lower than $O\hat{w}$ but higher than Oa.

Let MM relate the migration volume to the wage differential w_2/w_1 indicated on the negative y axis. If $w_1/w_2 = 1$ (shown by $y = -1$), then $M = 0$; if w_2/w_1 equals, say $M^1 r^1$ then OM^1 workers will migrate. If such a volume of out-migration from agriculture occurs, the marginal product of the remaining stock of agricultural workers will be $O_1 w_1'$. The urban wage required to effect the transfer of OM^1 workers is equal to $O_1 w_1'$ multiplied by the differential $M^1 r^1$, ie. it is equal to Ow_2'. It follows that wages $O_1 w_1'$ and Ow_2' satisfy the short-term equilibrium conditions for the two markets. If the urban wage were set at a higher level, the migration volume would be higher, and there would be urban unemployment at the end of the period; if it were set at a lower level, the migration volume would be lower and there would be unfilled vacancies.

The 'supply of in-migrants curve', αS_a, is obtained by relating successive values of w_2, and by obtaining the corresponding volumes of M, given the migration function, the initial stock of rural labour and the demand for agricultural labour. The shape of the supply of in-migrants curve thus depends crucially on the shape of the function relating the volume of migration to the wage differential.

2.3 The Costs of Migration

According to Sjaastad (1962) the costs of the people who migrate can be broken down into money and non-money costs. The former include the out-of-pocket expenses of movement, while the latter include foregone earnings and the "psychic" costs of changing one's environment.

Increased expenditures for food, lodging, transportation etc. constitute the money costs. There is no doubt that there is no data on these expenses incurred by migrants in the course of moving. Of course, an estimate can be found by questionning the migrants themselves or by taking into account specific distances. Maddox (1960), for example, estimates that many farm people can travel as far as five hundred miles from their home, take ten days to find non-farm jobs, and wait a week for their first pay-cheque after they start work, with a nest egg of no more than $100 per person. Nevertheless, the order of magnitude of the money cost is sufficiently small so that it cannot account for the large earnings differentials observed between the agricultural and non-agricultural sectors.

In his study of internal migration in the United States in 1949-50, Sjaastad (1961, 1962) suggests that the marginal costs associated with additional distance are considerably higher than could be attributed to the costs considered in Maddox's estimate. The migration variable was defined as the number of (net) migrants going from state i to state j as a function of all (net) migrants from state i. Regression coefficients obtained indicate that the attractiveness of a given destination was unaffected by a 10 per cent gain in annual per capita labour earnings and a simultaneous 16 per cent increase in distance. At the mean of the income and distance variables these percentages imply that the typical migrant would be indifferent between two destinations, one of which was 146 miles more distant than the other, if the average annual labour earnings were $106 (1947-49

dollars) higher in the more distant one. Nelson (1957) and others, tried to appeal to market imperfections such as lack of information to explain the apparently high distance cost of migration, but it seems that there is no simple way of testing such hypothesis.

The non-money costs may be divided into the opportunity costs and "psychic" costs. The opportunity costs consist from the loss of income during the migration process itself, searching for, and learning a new job. This size of the foregone earnings can be estimated if we take into account the time taken to move from the rural to the urban area and to search for or learn a new job. Of course, the time spent in the migration process itself is a function of the distance and the time spent in searching for an urban job is function of the level of urban unemployment.

The "phychic" costs, are those costs which are relevant with the psychic world of the migrant, ie. the psychological pressure on the migrant when he leaves his family, friends, familiar surroundings etc. These costs are difficult to quantify. Psychic costs together with money costs and opportunity costs, could explain the existence of earnings differentials.

Given the earnings levels at all other places, there is some minimum earning level at the place under consideration which will cause a given individual to be indifferent between migrating and remaining at home. In other words, the wages differential reflects the belief that farm workers require some premium over their agricultural wage to induce them to incur the costs of moving to a perhaps less well-known and more risky urban area, Zarembka (1970a). The difference in life-style between the city and the remote hinterland is much greater than that between the city and the surrounding area, and the costs and time of travel preclude frequent visits of the out-migrant back to his distant native home. For an inhabitant of a remote region, migration means a sharper and more permanent break than that for a migrant from a suburban area. Therefore, psychic costs apparently

increase very sharply with distance, Schwartz (1973), Wellisz (1974).

Since direct migration costs are relatively small in many developing regions, the major determinant of costs is the loss of income during the migration process itself. This suggests that c(t), the current cost of migration, might be a function of the income in the sector from which the migrant departed. In fact, we shall assume that the cost of migrating from rural to urban areas is a linear function of income per labourer in agriculture, but that there is no cost associated with movement in the opposite direction, Kelley-Williamson-Cheetham (1972). Thus we have:

$$c(t) = \beta(t) \cdot w_1(t) - \theta \qquad (2.3.1)$$

where $\beta(t) \geqslant 0$ is a decreasing function of time showing that as the economy becomes more developed, the migration process becomes quicker, and $\theta \geqslant 0$ is a fixed parameter over time. When $\theta = 0$, the costs of migration are proportional to the rural wage; when $\beta(t) = 0$, the migration costs are constant over time.

2.4 The Migration Decision

We could say that this section is a continuation of section 2.2 because it also deals with determinants of migration. But we prefer to separate it from section 2.2 because we would like to present here some new points on the decision to migrate and the wage differentials.

Suppose, for the moment, that there exists a wage differential between the agricultural and the non-agricultural sector, the two wages being $w_1(t)$ and $w_2(t)$ respectively. Suppose also that each individual's utility function is given by:

$$u(x) = \frac{1}{1-v} x^{1-v}, \quad 0 \leqslant v < 1 \qquad (2.4.1)$$

We assume that the direct cost of moving between the two sectors is very small, so that it may be omitted from the cost function (2.3.1). Therefore, (2.3.1) becomes (for $\theta = 0$)

$$c(t) = \beta(t) \cdot w_1(t) \tag{2.4.2}$$

or in utility terms

$$u(c(t)) = \frac{1}{1-v} (\beta(t)\, w_1(t))^{1-v} = \frac{1}{1-v} \beta(t)^{1-v} w_1(t)^{1-v} \tag{2.4.3}$$

(2.4.3) shows the utility lost by the individual who migrates.

We define P(t) as the probability of an individual finding a job in the non-agricultural sector. If we assume that employment and unemployment in the non-agricultural sector are two mutually exclusive events, then the individual who migrates to the urban area has a probability of 1–P(t) of not finding a job or, equivalently, of not earning any wage.

Therefore, the individual in agriculture migrates to the non-agricultural sector if and only if he assumes that he will gain greater utility than by staying in the agricultural sector. Hence, the decision to migrate is given by the expression:

$$(1-P(t))\, u(0) + p(t) \cdot u(w_2(t)) - u(c(t)) > u(w_1(t)) \tag{2.4.4}$$

From (2.4.1) and (2.4.3) it is known that

$$\begin{aligned} u(0) &= 0 \\ u(w_i(t)) &= \frac{1}{1-v} w_i(t)^{1-v} \qquad i = 1, 2 \\ \text{and} \qquad u(c(t)) &= \frac{1}{1-v} \beta(t)^{1-v}\, w_1(t)^{1-v} \end{aligned} \tag{2.4.5}$$

We substitute (2.4.5) into(2.4.4) and obtain:

$$P(t)\frac{1}{1-v} w_2(t)^{1-v} - \frac{1}{1-v}\beta(t)^{1-v} w_1(t)^{1-v} > \frac{1}{1-v} w_1(t)^{1-v}$$

or

$$P(t)\, w_2(t)^{1-v} > (1 + \beta(t)^{1-v})\, w_1(t)^{1-v}$$

or

$$\frac{w_1(t)}{w_2(t)} < \left(\frac{P(t)}{1 + \beta(t)^{1-v}}\right)^{\frac{1}{1-v}} \tag{2.4.6}$$

From (2.4.6) we see that, at time t, given the probability of somebody finding a job in the non-agricultural sector and the costs of migration, an individual in the agricultural sector migrates to the non-agricultural sector if and only if:

$$\frac{w_1(t)}{w_2(t)} < \mu(t) \tag{2.4.7}$$

where $$\mu(t) = \left(\frac{P(t)}{1+\beta(t)^{1-v}}\right)^{\frac{1}{1-v}} \tag{2.4.8}$$

In the case where $w_1(t)/w_2(t) \geq \mu(t)$, nobody is willing to migrate. We call this case the "relative equilibrium". Thus, the lowest wages ratio for the relative equilibrium is:

$$\frac{w_1(t)}{w_2(t)} = \left(\frac{P(t)}{1+\beta(t)^{1-v}}\right)^{\frac{1}{1-v}} = \mu(t) \tag{2.4.9}$$

Let's see now some properties of (2.4.9) or (2.4.8). $\mu(t)$ always lie between zero and one. We obtain this by giving to P(t) and $\beta(t)$ extreme values. Thus, for $P(t) = 0$ and $0 \leq \beta(t) \leq \infty$ we have that $\mu(t) = 0$; for $P(t) = 1$ and $\beta(t) = 0$ we have that $\mu(t) = 1$; for $P(t) > 0$ and $\beta(t) > 0$ it is $0 < \mu(t) < 1$.

We further assume that as the economy becomes more developed, P(t) tends to one, which means that, as time passes, it becomes easier for the individual of the agricultural sector to find a job in the non-agricultural sector. At the same time, $\beta(t)$ tends to zero, that is, the costs of migration are a decreasing function of time. Therefore, as $t \to \infty$, $P(t) \to 1$ and $\beta(t) \to 0$, or in other words, $\mu(t) \to 1$.

In the Harris-Todaro (1970) model and in Harberger (1971), P(t) is given by the ratio of the employed to the total urban labour force. A more plausible determinant of the chances of a single migrant is given by the number of vacancies, V(t), occurring per period t divided by the number of candidates for these vacancies, that is the urban

unemployed, U(t), Todaro (1969), Lal (1973). However, for either formulation of P(t), its equilibrium value $P_e(t)$ will be determined by the equilibrium migration condition, which the above authors found to be:

$$P_e(t) = \frac{w_1(t)}{w_2(t)} \qquad (2.4.10)$$

Of course, the probability P(t) is a theoretical concept, and differs from that which the prospective rural migrants into urban areas take into account. Speare (1971), in a questionnaire survey which he conducted among the immigrants into the city of Taichung in Taiwan, reports that the immigrants from rural areas had only very hazy notions about urban jobs. Also, Speare (1971) and Wilkinson (1970) point out that an *a priori* probability presumably reflects the actual experience of earlier migrants.

Comparing (2.4.10) with (2.4.9) or with the equivalent:

$$P_e(t) = (1+\beta(t)^{1-v})\left(\frac{w_1(t)}{w_2(t)}\right)^{1-v} \qquad (2.4.11)$$

we see that our expression for the equilibrium probability is more general than (2.4.10) because it does not depend solely on the rural-urban wage differential, as does the expression (2.4.10), but also on the costs of migration $\beta(t)$ and the utility parameter v, which we could say reflects the tastes of the individuals.

In the case where we have full employment, which means that the vacancies at time t are always equal to the amount of unemployment, and hence P(t) = 1, and where v = 0 then:

$$\mu(t) = \frac{w_1(t)}{w_2(t)} = \frac{1}{1+\beta(t)} \qquad (2.4.12)$$

From (2.4.12) we see that, even in this simple special case of full employment, the wages in the two sectors cannot be the same if there exists a positive cost of migration, which can be expressed by $\beta(t) > 0$. This result has also been obtained by Wellisz (1974), as we have already seen in section 2.2 with the help of figure 2.1.

2.5 The Labour-Allocation Curve

For any period of time t we define

$L_i(t)$ is the stock of labour in the ith sector, (i = 1, 2)

$L(t)$ is the total stock of labour in the economy

$L_{ii}(t)$ is the natural increase of labour in the ith sector, (i = 1, 2)

$M_i(t)$ is the number of labourers who migrate from the ith sector, (i = 1, 2)

The truth of the following expressions is obvious

$$L(t) = L_1(t) + L_2(t) \tag{2.5.1}$$

$$L_1(t) = L_1(t-1) + L_{11}(t) - M_1(t) + M_2(t) \tag{2.5.2}$$

$$L_2(t) = L_2(t-1) + L_{22}(t) + M_1(t) - M_2(t) \tag{2.5.3}$$

We now define

$\ell_i(t) = \frac{L_i(t)}{L(t)}$ the proportion of labour engaged in the ith sector, (i = 1, 2)

$\eta_i(t) = \frac{L_{ii}(t)}{L_i(t)}$ the natural or physiological rate of increase of labour in the ith sector, (i = 1, 2).

$m_i(t) = \frac{M_i(t)}{L_i(t)}$ the rate of migration from the ith sector, (i = 1, 2).

Under the above notation, the expressions (2.5.1), (2.5.2) and (2.5.3) are written:

$$\frac{\dot{L}(t)}{L(t)} = \eta_1(t)\,\ell_1(t) + \eta_2(t)\,\ell_2(t) \tag{2.5.4}$$

$$\frac{\dot{L}_1(t)}{L_1(t)} = \eta_1(t) + \frac{1}{\ell_1(t)}[m_2(t)\,\ell_2(t) - m_1(t)\,\ell_1(t)] \tag{2.5.6}$$

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta_2(t) + \frac{1}{\ell_2(t)}[m_1(t)\,\ell_1(t) - m_2(t)\,\ell_2(t)] \tag{2.5.7}$$

where of course

$$\ell_1(t) + \ell_2(t) = 1 \tag{2.5.8}$$

In our case, where we assume that migration takes place from the argicultural to the non-agricultural sector only, $M_2(t) = 0$. Therefore (2.5.6) and (2.5.7) may be simplified

as the following:

$$\frac{\dot{L}_1(t)}{L_1(t)} = \eta_1(t) - m_1(t) \qquad (2.5.9)$$

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta_2(t) + m_{12}(t) \qquad (2.5.10)$$

where $m_{12}(t) = m_1(t) \frac{\ell_1(t)}{\ell_2(t)} = \frac{M_1(t)}{L_2(t)}$ (2.5.11)

Suppose now that labour force grows at a constant (or variable) rate η, ie.

$$\dot{L}(t) / L(t) = \eta \qquad (2.5.12)$$

determined by:

$$\eta_1(t)\,\ell_1(t) + \eta_2(t)\ell_2(t) = \eta \qquad (2.5.13)$$

Solving the system (2.5.8) and (2.5.13) for $\ell_1(t)$ and $\ell_2(t)$, we find:

$$\ell_1(t) = \frac{\eta - \eta_2(t)}{\eta_1(t) - \eta_2(t)} \qquad (2.5.14)$$

and

$$\ell_2(t) = \frac{\eta_1(t) - \eta(t)}{\eta_1(t) - \eta_2(t)} \qquad (2.5.15)$$

(2.5.14) and (2.5.15) have meaning only for the case where $\eta_1(t) \neq \eta_2(t)$. Of course, this is a mathematical restriction necessary for the existence of the solution of the system above, but it can be justified in practice by the higher fertility of the people in the agricultural sector, that is:

$$\eta_1(t) > \eta > \eta_2(t) \qquad (2.5.16)$$

We assume that the rates of natural change of the labour in the two sectors, $\eta_i(t)$, $(i = 1, 2)$, are variable, and $\eta_1(t)$ is decreasing whilst $\eta_2(t)$ is increasing over time. We have to introduce two types of exogenous rates of fertility:

Type I:

$$\eta_1(t) = (\eta + \theta_1) + \eta_1\underline{(0)}\, e^{-\eta_1 t} \qquad (2.5.17)$$

$$\eta_2(t) = (\eta - \theta_2) - \eta_2(0)\, e^{-\eta_2 t} \tag{2.5.18}$$

and Type II:

$$\eta_1(t) = \eta + \eta_i(0)\, e^{-\eta_1 t} \tag{2.5.19}$$

$$\eta_2(t) = \eta - \eta_2(0)\, e^{-\eta_2 t} \tag{2.5.20}$$

where η_i and θ_i for i = 1, 2 are parameters that are constant over time. It can be seen that Type II is a special case of Type I where $\theta_1 = \theta_2 = 0$.

In type one the fertility of people in the argricultural sector is decreasing over time, but it never reaches the average fertility rate of the whole economy, η. In the non-agricultural sector the fertility of people is increasing over time, but it never reaches the common rate η. Therefore, for type one:

$$\lim_{t \to \infty} \eta_1(t) = \eta + \theta_1 \text{ and } \lim_{t \to \infty} \eta_2(t) = \eta - \theta_1 \tag{2.5.21}$$

It is obvious that, for type two where $\theta_1 = \theta_2 = 0$, we have:

$$\lim_{t \to \infty} \eta_1(t) = \eta \text{ decreasingly}$$

$$\lim_{t \to \infty} \eta_2(t) = \eta \text{ increasingly} \tag{2.5.22}$$

Let us now see how (2.5.14) and (2.5.15) behave as $t \to \infty$. Thus, using (2.5.21) we obtain:

$$\lim_{t \to \infty} \ell_1(t) = \lim_{t \to \infty} \frac{\theta_2 + \eta_2(0)\, e^{-\eta_2 t}}{(\theta_1 + \theta_2) + \eta_1(0)\, e^{-\eta_1 t} + \eta_2(0)\, e^{-\eta_2 t}}$$

$$= \frac{\theta_2}{\theta_1 + \theta_2} = p_1 \tag{2.5.23}$$

and

$$\lim_{t \to \infty} \ell_2(t) = \lim \frac{\theta_1 + \eta_1(0)\, e^{-\eta_1 t}}{(\theta_1 + \theta_2) + \eta_1(0)\, e^{-\eta_1 t} + \eta_2(0)\, e^{-\eta_2 t}}$$

$$= \frac{\theta_1}{\theta_1 + \theta_2} = p_2 \tag{2.5.24}$$

From (2.5.23) and (2.5.24) we see that, as time passes, or, as the economy becomes more developed, for the first type of exogenous change in fertility of people in the two sectors, the proportions of labour engaged in the agricultural and the non-agricultural sectors, tend to the constants $0 < p_1 < 1$ and $0 < p_2 < 1$ respectively, where $p_1 + p_2 = 1$. From (2.5.23) and (2.5.24) one may derive:

$$\lim_{t\to\infty} \frac{\ell_1(t)}{\ell_2(t)} = \lim_{t\to\infty} \frac{L_1(t)}{L_2(t)} = \frac{\theta_2}{\theta_1} = \frac{p_1}{p_2} = p \gtreqless 1 \qquad (2.5.25)$$

Using (2.5.22) we obtain:

$$\lim_{t\to\infty} \ell_1(t) = \lim_{t\to\infty} \frac{\eta_2(0)\, e^{-\eta_2 t}}{\eta_1(0) e^{-\eta_1 t} + \eta_2(0)\, e^{-\eta_2 t}} = \lim_{t\to\infty} \frac{1}{1 + \frac{\eta_1(0)}{\eta_2(0)} e^{(\eta_2 - \eta_1)t}} \qquad (2.5.26)$$

and

$$\lim_{t\to\infty} \ell_2(t) = \lim_{t\to\infty} \frac{\eta_1(0)\, e^{-\eta_1 t}}{\eta_1(0)\, e^{-\eta_1 t} + \eta_2(0) e^{-\bar{\eta}_2 t}} = \lim_{t\to\infty} \frac{1}{1 + \frac{\eta_2(0)}{\eta_1(0)} e^{(\eta_1 - \eta_2)t}} \qquad (2.5.27)$$

In (2.5.26) and (2.5.27), it is more reasonable to assume that $\eta_2 > \eta_1$, which means that the decrease in the rate of fertility of the people in the agricultural sector is less than the increase of the same rate in the non-agricultural sector. Therefore, for $\eta_2 - \eta_1 > 0$ the above limits are equal to:

$$\lim_{t\to\infty} \ell_1(t) = 0 \quad , \quad \lim_{t\to\infty} \ell_2(t) = 1 \qquad (2.5.28)$$

and so:

$$\lim_{t\to\infty} \frac{\ell_1(t)}{\ell_2(t)} = 0 \qquad (2.5.29)$$

From (2.5.28), we see that as time passes, the agricultural sector decreases in size relative to the non-agricultural sector. At $t = \infty$, the agricultural sector has disappeared,

or preferably, we could say that it has become fully developed, and so it has been absorbed in the non-agricultural sector, so that the economy has only one sector.

In the case where $\eta_1 = \eta_2$, it is always true that

$$\ell_1(t) = \frac{\eta_2(0)}{\eta_1(0) + \eta_2(0)}, \quad \ell_2(t) = \frac{\eta_1(0)}{\eta_1(0) + \eta_2(0)} \tag{2.5.30}$$

and

$$\frac{\ell_1(t)}{\ell_2(t)} = \frac{\eta_2(0)}{\eta_1(0)} \tag{2.5.31}$$

which means that, in this case where the changes in the rates of fertility in the two sectors are equal, the proportions of the labour force engaged in the two sectors remain constant over time.

From the above discussion we can derive geometrically the labour allocation curves between the two sectors as in figure 2.2, for the two types of fertility rates. In figure 2.2

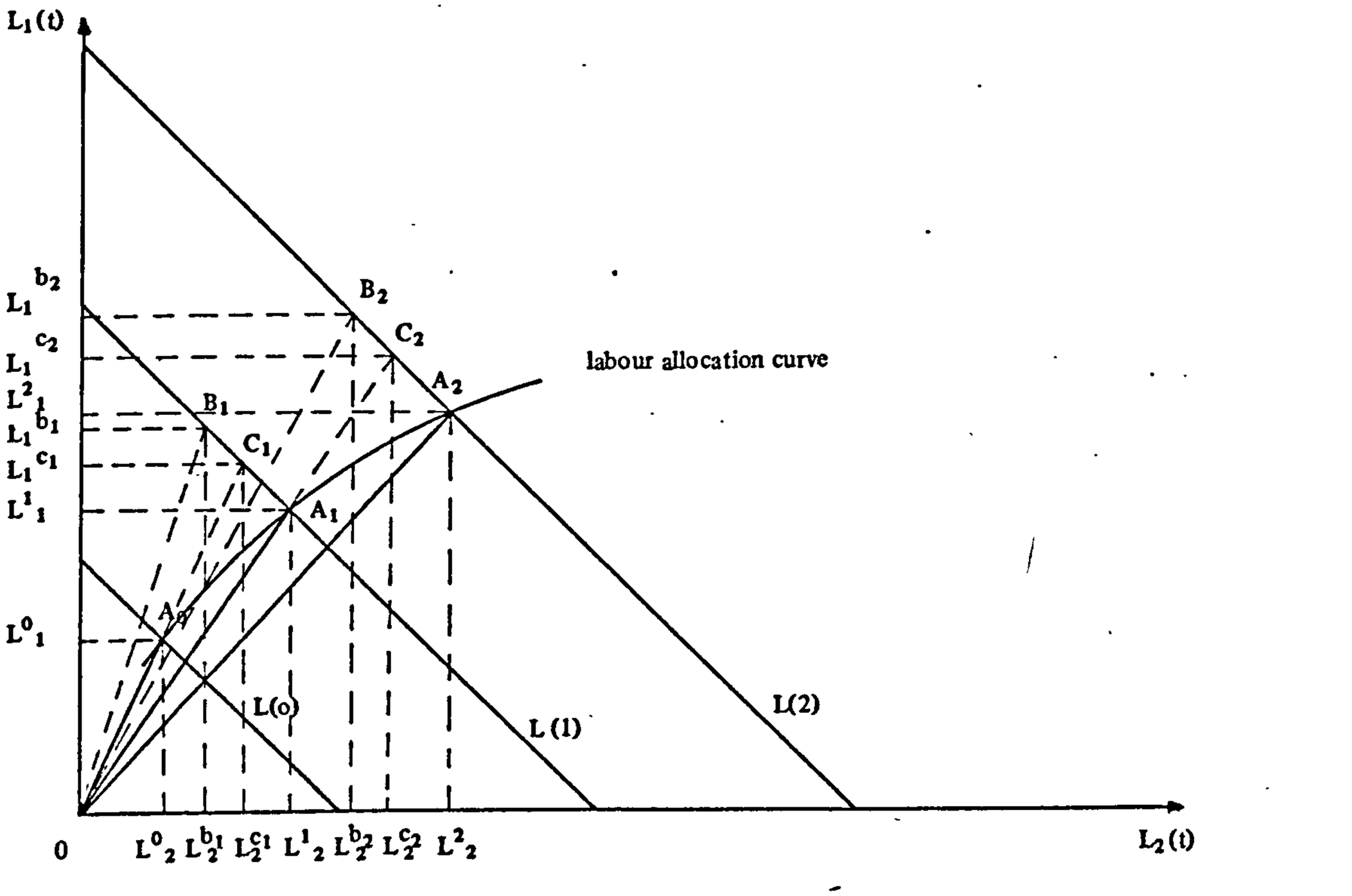


Figure 2.2

the exogenous increase in labour force L(t) is depicted by the 45° lines. The lines show the allocations of labour between the two sectors possible at a particular time. For example, the point A_0 on the line L(0) shows that the labour allocation of the L(0) workers, is $L_1(0)$ in the agricultural sector (on vertical axis) and $L_2(0)$ in the non-agricultural sector (on horizontal axis). The ratio $L_1(0)/L_2(0)$ or $\ell_1(0)/\ell_2(0)$ is shown by the slope of the straight line from the origin to point A_0.

Suppose now that, at time t = 0, the actual labour force engaged in the two sectors is L_1^0 and L_2^0 for the agricultural and non-agricultural sectors respectively. If we suppose now that the natural rate of increase of labour in the two sectors was the same, then at time t = 1, where the total labour force in the economy is L(1), the point which shows the natural labour allocation between the two sectors is C_1, on the L(1) line, with $L_1^{c_1}$ and $L_2^{c_1}$ the corresponding levels of labour force for the two sectors. In this case it is:

$$\frac{L_1^0}{L_2^0} = \frac{L_1^{c_1}}{L_2^{c_1}}$$

Of course, this is not the case in reality, where the natural rate of increase of labour in the agricultural sector is greater than that in the non-agricultural sector, and so (2.5.16) applies. This means that the point, in figure 2.2, which shows the natural labour allocation between the two sectors at time t = 1, is not the point C_1 but another, say B_1, which lies on the left of point C_1 and on the line L(1), and which gives $L_1^{b_1}$ and $L_2^{b_1}$ workers for the agricultural and non-agricultural sectors respectively. It is obvious that:

$$\frac{L_1^{b_1}}{L_2^{b_1}} > \frac{L_1^{c_1}}{L_2^{c_1}} \qquad (2.5.32)$$

Until now, the discussion has been based on natural rates of change of labour force, and not on actual changes. By the word 'actual' we mean that the change in the labour

force in the two sectors is the combined result of the natural (exogenous) change of labour plus the (endogenous) net migration between the two sectors. Therefore, suppose that $L_1^{b_1} L_1^1$ ($= L_2^{b_1} L_2^1$) workers migrated in time t = 1, from the agricultural to the non-agricultural sector. This change in the volume of the labour force in the two sectors passes point B_1 down and to the right along the line L(1), say to point A_1. Thus, point A_1 shows the actual allocation of labour force between the two sectors, with L_1^1 and L_2^1 workers for the agricultural and non-agricultural sectors respectively for the time period t = 1. It is clear that:

$$\frac{L_1^{b_1}}{L_2^{b_1}} > \frac{L_1^1}{L_2^1}$$

In figure 2.2 point A_1 lies on the right of point C_1. Of course it is possible for point A_1 to lie on the left of point C_1 or to coincide with it. The position of A_1 depends on the volume of migration. The greater is the migration, the further is point A_1 down from point B_1, along the line L(1).

The same analysis applies to the points A_2, A_3, etc. in figure 2.2. The only point that we must note here is that the angles $C_1 \hat{O} B_1$, $C_2 \hat{O} B_2$, etc. are decreasing towards zero or almost zero according to the adopted type of natural rate of fertility in the two sectors.

In figure 2.3 is shown the complete labour-allocation curve as $t \to \infty$ for the two types. If we assume that, at time t = 0, all the labour force was engaged in the agricultural sector ($L_1(0) = L(0)$, $L_2(0) = 0$), then for t = 0 the labour allocation ratio is given by:

$$\frac{L_1(0)}{L_2(0)} = \tan (L_2 \hat{O} L_1) = \tan 90° = \infty \qquad (2.5.33)$$

If we now assume that at time t = 0, there were some people working in the non-agricultural sector, then the labour allocation ratio, for t = 0, is given by:

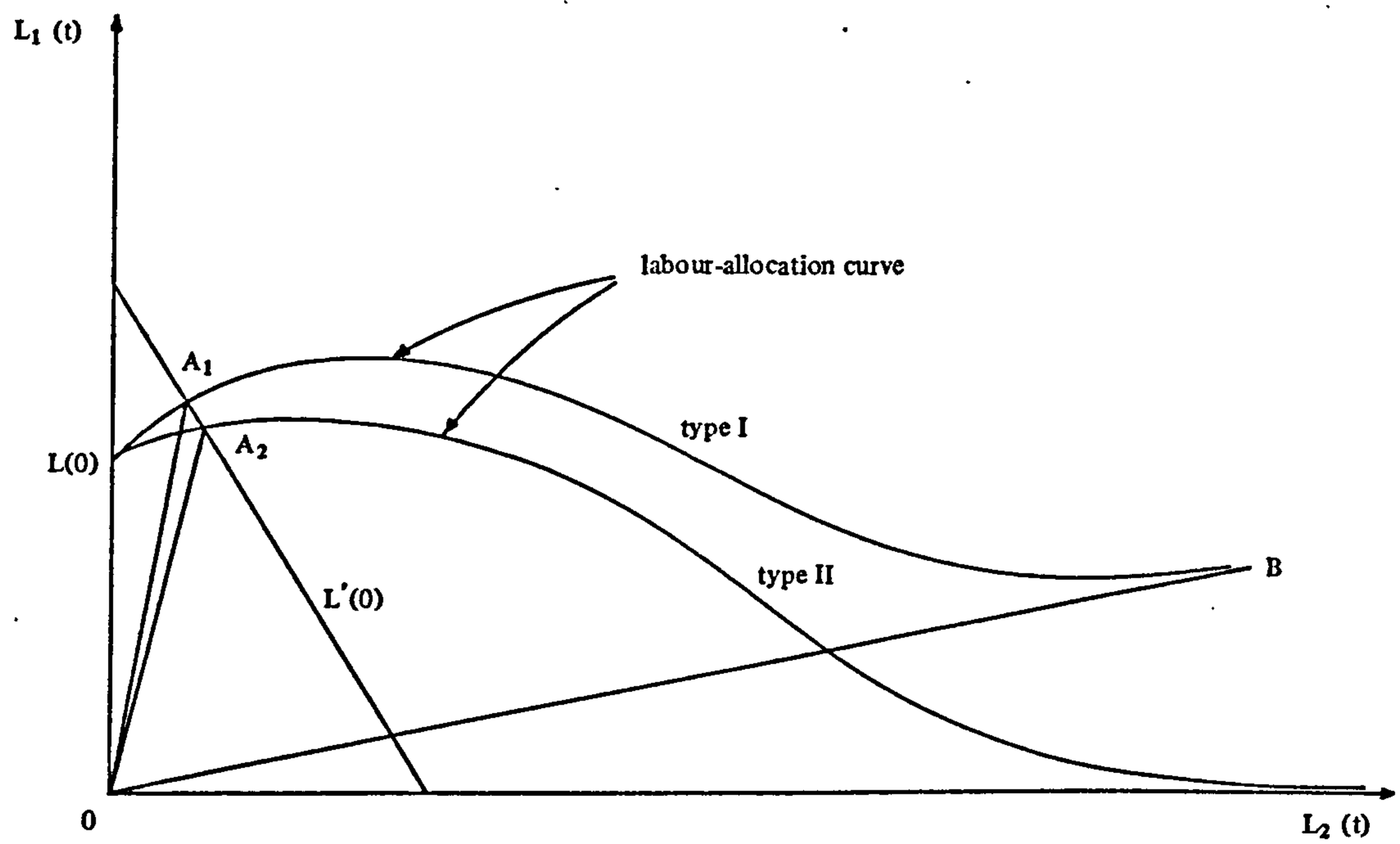


Figure 2.3

$$\text{type I} : \frac{L_1(0)}{L_2(0)} = \tan(L_2\hat{O}A_1) = \frac{\theta_2 + \eta_2(0)}{\theta_1 + \eta_1(0)} \tag{2.5.34}$$

$$\text{type II} : \frac{L_1(0)}{L_2(0)} = \tan(L_2\hat{O}A_2) = \frac{\eta_2(0)}{\eta_1(0)} \tag{2.5.35}$$

As $t \to \infty$ the labour-allocation ratio tends to:

$$\text{type I} : \frac{L_1(\infty)}{L_2(\infty)} = \tan(L_2\hat{O}B) = \frac{\theta_2}{\theta_1} = p \tag{2.5.36}$$

$$\text{type II} : \frac{L_1(\infty)}{L_2(\infty)} = \tan 0^\circ = 0 \tag{2.5.37}$$

In figure 2.3, the relative positions of the two labour-allocation curves and of the points A_1 and A_2 depend, of course, on the values of the θ's and the η's.

2.6 The Supply of Labour

In section 2.5 we analyzed the labour-allocation curve. In this section we are going a step further by trying to find the supply of labour functions for each of the two sectors.

Form (2.5.10) and (2.5.12) we obtain:

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} = \frac{\dot{L}_2(t)}{L_2(t)} - \frac{\dot{L}(t)}{L(t)} = \eta_2(t) + m_{12}(t) - \eta \qquad (2.6.1)$$

Substituting $\eta_2(t)$ into (2.6.1), using the expressions (2.5.18) and (2.5.20) for types I and II respectively we obtain:

$$\text{type I} : \frac{\dot{\ell}_2(t)}{\ell_2(t)} = m_{12}(t) - \theta_2 - \eta_2(0)\, e^{-\eta_2 t} \qquad (2.6.2)$$

and

$$\text{type II} : \frac{\dot{\ell}_2(t)}{\ell_2(t)} = m_{12}(t) - \eta_2(0)\, e^{-\eta_2 t} \qquad (2.6.3)$$

The limiting values which (2.6.2) takes are:

$$\left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=0} = m_{12}(0) - \theta_2 - \eta_2(0), \left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=\infty} m_{12}(\infty) - \theta_2 \qquad (2.6.4)$$

and the limiting values of (2.6.3) are:

$$\left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=0} = m_{12}(0) - \eta_2(0), \left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=\infty} = m_{12}(\infty) \qquad (2.6.5)$$

In both case I and case II we have, from (2.5.25) and (2.5.29) respectively, that as $t \to \infty$, $\ell_1(t) / \ell_2(t)$ tends to a constant , p and 0 respectively. This means that in both (2.6.4) and (2.6.5):

$$\left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=\infty} = 0 \qquad (2.6.6)$$

$$\text{or for type I} : m_{12}(\infty) = \theta_2 \qquad (2.6.7)$$

$$\text{and for type II} : m_{12}(\infty) = 0 \qquad (2.6.8)$$

From (2.6.7) we see that for the first type of assumption about the natural fertility of the people, at time $t = \infty$, there still exists a constant rate of migration of workers from the agricultural to the non-agricultural sector, equal to the gap between the natural rates of increase of population in the total economy and that in the non-agricultural sector. So,

there is migration to fill this gap.

For the second type, where at $t = \infty$ there is no such gap, we see from (2.6.8) that migration at time $t = \infty$ is equal to zero.

We will try now to introduce wages differentials into our formulation, using a similar urbanization function as those of Todaro (1969) and Zarembka (1970). The urbanisation function is of the general form:

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} = F\left(\frac{w_2(t)}{w_1(t)}\right) \tag{2.6.9}$$

which must obey the restriction

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} \to \infty \quad \text{as} \quad \frac{w_2(t)}{w_1(t)} \to \infty \tag{2.6.10}$$

One possible such function is that used by Mass-Colell and Razin (1974). It is of the simple form:

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} = \delta\left[\frac{w_2(t) - w_1(t)}{w_1(t)}\right] \tag{2.6.11}$$

where δ is a positive constant.

(2.6.11) can be written as

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} = \delta\left(\frac{w_2(t)}{w_1(t)} - 1\right) = \delta\left(\frac{1}{\mu(t)} - 1\right) \tag{2.6.12}$$

or using, (2.4.12), which is the case for full employment and $v = 0$, we obtain:

$$\frac{\dot{\ell}_2(t)}{\ell_2(t)} = \delta\,\beta(t) = \delta\,\beta(0)\,e^{-\beta t} \tag{2.6.13}$$

where $\beta(t) = \beta(o)e^{-\beta t}$, $\beta(0), \beta \geq 0$.

Comparing (2.6.13) with (2.6.1) we derive a function for the rate of migration, ie.:

$$m_{12}(t) = \eta - \eta_2(t) + \delta\,\beta(0)\,e^{-\beta t} \tag{2.6.14}$$

From (2.6.14) we obtain for the two types of $\eta_2(t)$ the results that:

type I : $m_{12}(t) = \theta_2 + \eta_2(0)\, e^{-\eta_2 t} + \delta\, \beta(0)\, e^{-\beta t}$ (2.6.15)

$m_{12}(0) = \theta_2 + \eta_2(0) + \delta\, \beta(0)$ (2.6.16)

$m_{12}(\infty) = \theta_2$. (2.6.17)

and type II : $m_{12}(t) = \eta_2(0)\, e^{-\eta_2 t} + \delta\, \beta(0)\, e^{-\beta t}$ (2.6.18)

$m_{12}(0) = \eta_2(0) + \delta\, \beta(0)$ (2.6.19)

$m_{12}(\infty) = 0$ (2.6.20)

From the above we see that (2.6.17) does not contradict (2.6.7) and (2.6.20) does not contradict (2.6.8). Also from (2.6.16) and (2.6.19), after substitution into (2.6.4) and (2.6.5) respectively, we obtain that at time t = 0, for both types of $\eta_2(t)$:

$$\left.\frac{\dot{\ell}_2(t)}{\ell_2(t)}\right|_{t=0} = \delta\, \beta(0) \qquad (2.6.21)$$

Before we try to find the supply function of labour in the two sectors, we must note some points which can be derived from the discussion above. From section 2.4 we know that as $t \to \infty$, then $\beta(t) \to 0$ or $w_1(t)/w_2(t) \to 1$. But from (2.6.11) or (2.6.13), we also know that as $t \to \infty$, or $w_1(t)/w_2(t) \to 1$, $\dot{\ell}_2(t)/\ell_2(t) \to 0$. Therefore, we conclude from (2.6.17) and (2.6.20), that for type I of $\eta_2(t)$, there still exists migration of workers between the two sectors, even for the case where the two wages become equal when $t \to \infty$. This illustrates the case, observed in reality, that in some countries where there is no wage gap between the two sectors, there is still migration. Of course, for the type II of $\eta_2(t)$, we do not have migration when $w_1(t) = w_2(t)$.

We are now able to derive the supply functions of labour for the two sectors. It is known from (2.5.10) that:

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta_2(t) + m_{12}(t) \qquad (2.6.22)$$

We substitute $m_{12}(t)$ from (2.6.14) into (2.6.22) and obtain:

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta + \delta\,\beta(0)\,e^{-\beta t} \qquad (2.6.23)$$

Taking the integrals of both sides of (2.6.23) from $t_1=0$ to $t_1=t$, we have:

$$\int_0^t \frac{dL_2(t_1)}{L_2(t_1)} = \int_0^t (\eta + \delta\,\beta(0)\,e^{-\beta t_1})\,dt_1$$

or

$$\ln\ L_2(t_1)\Big|_0^t = \eta t_1 - \frac{\delta\,\beta(0)}{\beta}\,e^{-\beta t_1}\Big|_0^t$$

or

$$\ln\ L_2(t) - \ln\ L_2(0) = \eta t - \frac{\delta\,\beta(0)}{\beta}\,(e^{-\beta t} - 1)$$

or

$$L_2(t) = \exp\,[\ln\ L_2(0) + \eta t - \frac{\delta\,\beta(0)}{\beta}\,(e^{-\beta t} - 1)]$$

or

$$L_2(t) = L_2(0)\,e^{\eta t} - \frac{\delta\,\beta(0)}{\beta}\,(e^{-\beta t} - 1) \qquad (2.6.24)$$

which is the supply function of labour in the non-agricultural sector over time. The supply function of labour for the agricultural sector can be found from:

$$L_1(t) = L(t) - L_2(t)$$

substituting $L(t)$ and $L_2(t)$ from (2.5.12) and (2.6.24) respectively, ie. it is:

$$L_1(t) = [\,L(0) - L_2(0)\,.\,e^{-\frac{\delta\,\beta(0)}{\beta}(e^{-\beta t}-1)}\,]\,e^{\eta t}. \qquad (2.6.25)$$

We can see the mechanics of labour migration and supply curves of labour with the help of figure 2.3.

In the case of full employment ($P(t) \equiv 1$) and $v = 0$, the expression (2.4.4.), which shows the condition under which workers migrate from the agricultural to the non-agricultural sector, can be written in the simple form:

$$w_2(t) - w_1(t) > c(t) \qquad (2.6.26)$$

(2.6.26) means that if the gap between the current wages in the two sectors is less or equal to the current cost of migration, then nobody is willing to migrate.

Suppose now that at time $t = t_1$ the marginal productivity of labour curves for the two sectors are given, in figure 2.3, by the curves $MPL_1(t_1)$ and $MPL_2(t_1)$.

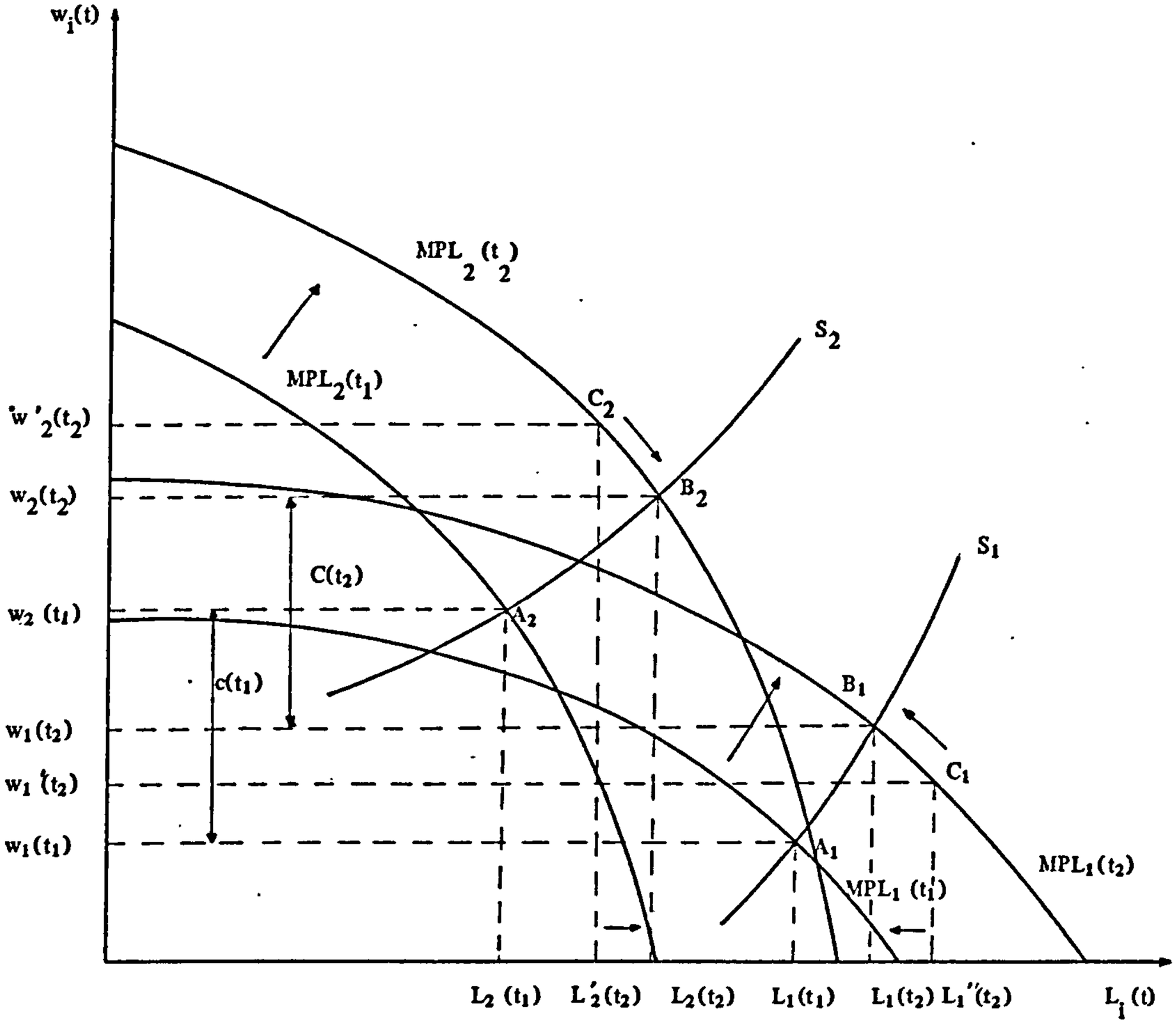


Figure 2.3

The workers engaged in the two sectors are $L_1(t_1)$ and $L_2(t_2)$, with corresponding wages $\hat{w}_1(t_1)$ and $w_2(t_1)$. We say that the system is in "relative equilibrium" if the gap in the wages is less than or equal to c(t), (ie. $w_2(t) - w_1(t) \leqslant c(t)$), at which level nobody is willing to migrate from the agricultural to the non-agricultural sector; it is in "relative

disequilibrium" if the gap in the two wages is greater than c(t), ie. if migration is taking place.

Suppose now that at time $t = t_1$, $\Delta w(t_1) \leqslant c(t_1)$, which means that, at time t_1, the system is in relative equilibrium. Due to capital accumulation and technological change in both sectors, suppose that at time $t = t_2$ the marginal productivity curves of labour shift to $MPL_1(t_2)$ and $MPL_2(t_2)$. In the same period the labour force in the two sectors has increased by the amounts $L_1(t_1)\, L'_1(t_2)$ and $L_2(t_1)\, L'_2(t_2)$, which means that the wages of the two sectors are now $w'_1(t_2)$ and $w'_2(t_2)$. If the difference between the new wages is greater than $c(t_2)$, the critical value, then the system is in relative disequilibrium. Migration starts from sector one to sector two. This means that as workers leave sector one, the wage rate in that sector increases and that in sector two decreases. Therefore, in relative disequilibrium situations, we see that the wages move in opposite directions and, after some time, the difference between them reaches the critical value c(t), at which migration stops and the system is again in relative equilibrium.

In figure 2.3, the points $A_1(L_1(t_1), w_1(t_1))$ and $A_2(L_2(t_1), w_2(t_2))$ show the relative equilibrium position of the system at time $t = t_1$. At time $t = t_2$ the points $B_1(L_1(t_2), w_2(t_1))$ and $B_2(L_2(t_2), w_2(t_2))$ similarly show the relative equilibrium position of the system. Points between (C_1, B_1) and (C_2, B_2) show the relative disequilibrium positions of the system during time period t_2, in which the number of workers that migrate from the first to the second sector is equal to $L'_1(t_2)\, L_1(t_2) = L_2(t_2)\, L'_2(t_2)$. If we now join the relative equilibrium points of each sector we derive the supply functions of labour for the two sectors, denoted by S_1 and S_2. Each of these supply curves looks like a conventional upward-sloping labour-supply curve, but is actually the locus of demand-supply intersections at successive time periods, as shown in figure 2.4. This seems

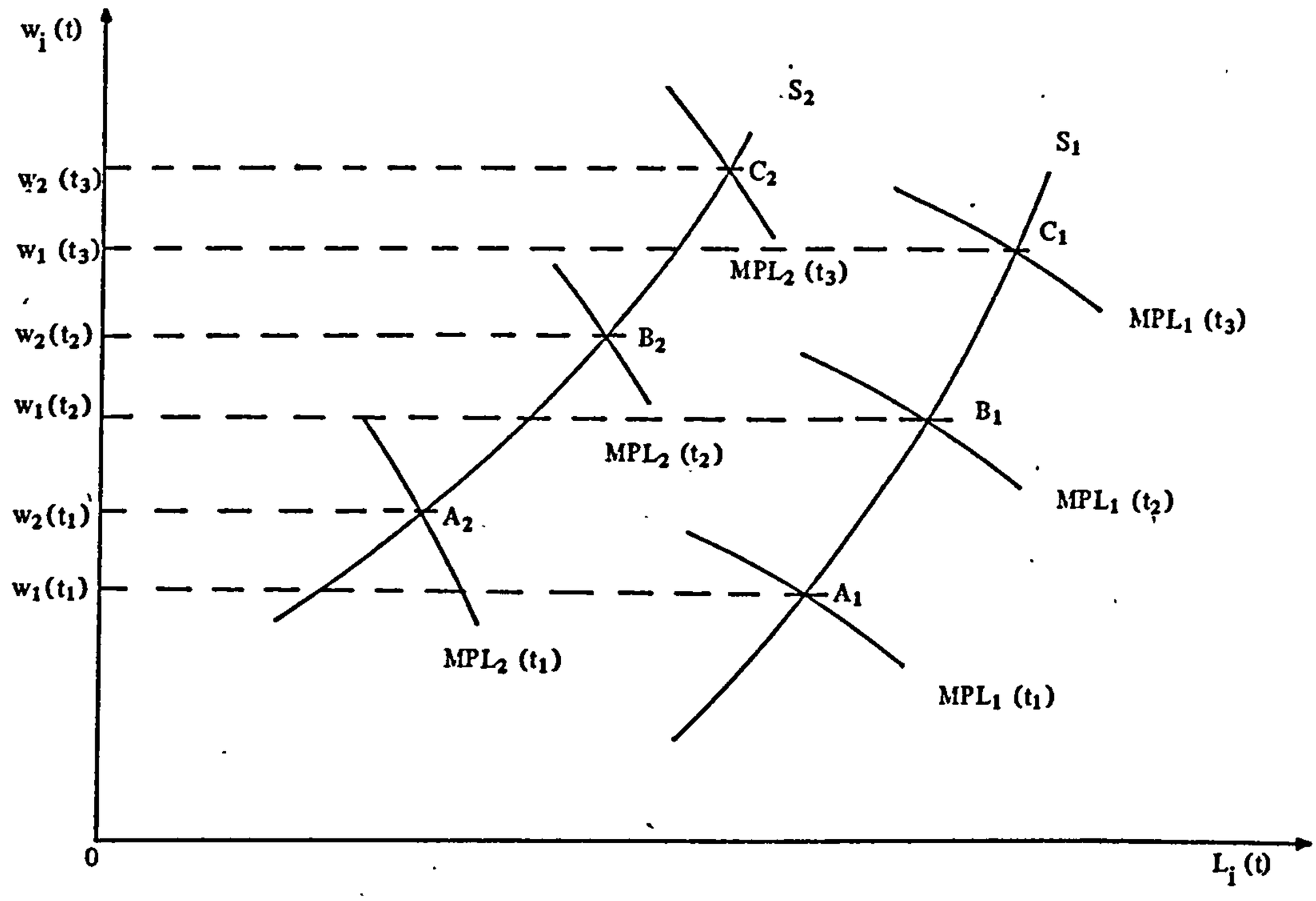


Figure 2.4

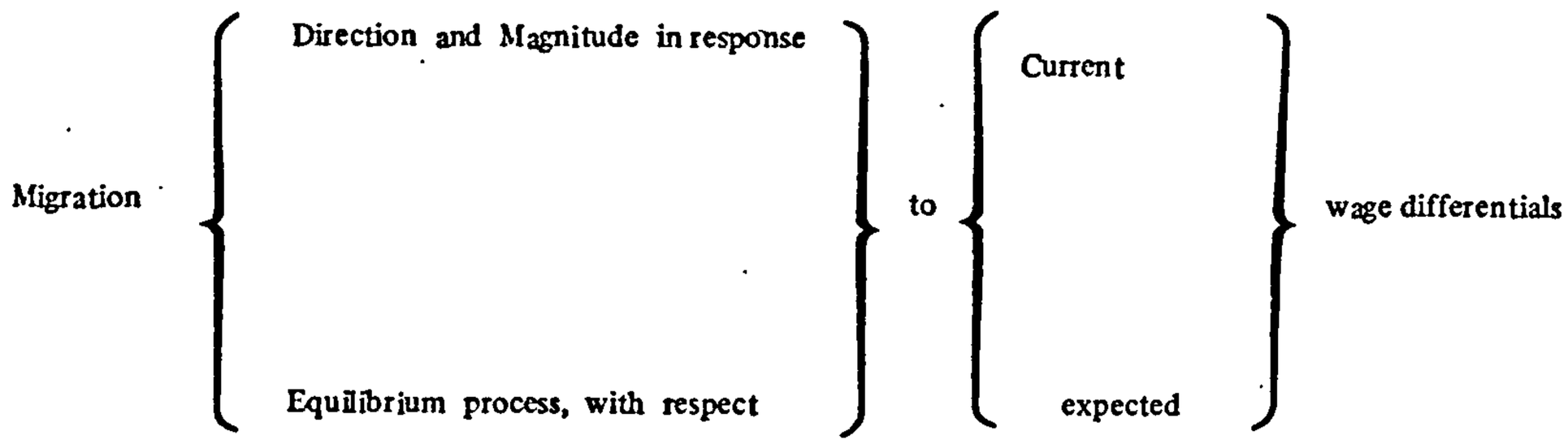
to be in agreement with Mazumbar (1959), who writes that the wage-rate cannot be determined simply by the intersection of supply and demand curves for labour, because the supply curve itself varies with the wage level.

In the above exposition, it must be noted that the shape and the position of the marginal productivity of labour curves depend upon the level of capital accumulation and on the kind of technological progress. Also, the wage rate depends upon the rate of increase in labour force. If, for example, in period $t = t_2$ the wages differential was less than the critical value $c(t_2)$, then no migration occurs from one sector to the other. Another extreme case is that in which in period $t = t_2$ the wages differential is negative, which means that there is migration of workers, but from the second sector to the first. This seems to be quite unrealistic for a developing economy and contradicts (2.4.8).

2.7 Conclusions

In this chapter we have tried to classify migration research according to the adopted

methods and instruments. The following chart shows this classification:



The main conclusion of this chapter is that there always exists a wage gap between the agricultural and non-agricultural sectors in the cases where there exist costs of migration whether monetary or psychic or both.

We generalized Todaro's equilibrium value of probability of finding a job into the non-agricultural sector, (2.4.10), including not only the wages of the two sectors, but also the costs of migration and a utility parameter of the individuals as in (2.4.11). From there, we see that even in the case of full employment there still exists a wage differential between the sectors, if, of course, there is a positive cost of migration.

In our formulation of wages differentials we find that as $t \to \infty$, the probability of finding a job in the non-agricultural sector tends to one and the costs of migration tend to zero, so the wages tend to be equal.

Investigating the mechanics of labour migration between the two sectors, we conclude that the assumptions about the natural or physiological rates of change of the labour force in the two sectors play a basic role in determining the results obtained. In particular, if we assume that as $t \to \infty$ the natural rates of change of the labour forces in the two sectors tend to two distinct values around the average rate of change of the labour force in the total economy, then the ratio (of allocation) of the labour force engaged in the agricultural

sector by that engaged in the non-agricultural sector tends to a positive value. If we assume that the two natural rates of change of labour tend to become equal as $t \to \infty$, then we conclude that as $t \to \infty$, the agricultural sector disappears, which means that it has been fully developed and so the economy behaves like a one-sector economy or, if we still want to take two sectors into account, the economy must be divided into the consumption goods sector and into the capital goods sector, as has been done by Uza wa (1961, 1963) and others.

Finally, from our migration function, we conclude that migration from the agricultural to the non-agricultural sector stops when the two wage rates are equal and the two natural rates of increase of labour are equal. In the other case, of different natural rates of increase of labour, we find that, even with equal wage rates in the two sectors, migration is still positive and its constant rate is equal to the constant deviation of the natural rate of change of the labour force in the non-agricultural sector from that of the labour force as a whole.

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CHAPTER 3

A GENERAL TWO-SECTOR GROWTH MODEL FOR A DEVELOPING ECONOMY

3.1 Introduction
3.2 The Model
3.3 The Static Economy (Existence & Uniqueness)
3.4 Properties of the Static Model (Comparative Statics)
3.5 The Dynamic Economy (Stability)
3.6 Structural Change and the Growth Process
3.7 Conclusions
3.8 References

3.1 Introduction

The usual procedure in constructing a theory of economic development of a two-sector economy is to view growth as a movement by the economy through a sequence of short-run equilibrium positions in which sectoral wages and rental rates are equated. In other words, these theories are based on a fundamental proposition of the theory of general equilibrium and the theory of the competitive market according to which the discrepancies in factor payments lead to appropriate re-allocations of productive factors until equilibrium is restored. In these models we assume that capital and labour adjust instantaneously to any differentials between sectors and that there are no costs associated with the transfer.

Undoubtedly, from the empirical data, we can see that in the real world there exist sectoral differentials in wages and in rental rates, Borts-Stein (1964), Watanabe (1965), Williamson (1965), Reynolds (1965), Taira (1966) and Ohkawa-Rovosky (1972). The persistence of such differentials has led Higgins (1959), Bruton (1965) and others to argue that economic development is not an equilibrium process, but rather a process of change that is subject to continuous disequilibrating forces. A variety of hypotheses have been advanced to explain why discrepancies in sectoral incomes and factor prices may exist over extended periods. Most of these are based on the notion of sluggish adjustments in factor allocation in response to changes in the industrial output mix and labour demand, Nourse (1968). The slow adjustment of the capital and labour markets is attributed to a variety of factors that include the existence of high transactions cost, imperfect information, risk and uncertainty, and sectoral capital specificity, Kelley-Williamson-Cheetham (1972).

The aim of this chapter is to take explicit account of the phenomena of continuing

disequilibria in the labour market constructing a two-sector growth model for a developing economy. By relaxing the assumptions of instantaneous and costless adjustments to differentials in the labour market and by incorporating the assumptions of chapter 2 on the movement of labour between the two production sectors we provide an analytical framework that is particularly useful for analysing structural change under conditions of disequilibrium growth.

We use the notion of dualistic growth, Lewis (1954), Fei-Ranis (1964), Jorgenson (1961) and Dixit (1973) as a suitable basis for our model but at the same time we avoid the assymetry in the two production functions assuming that the two sectors use the same inputs, capital and labour, in the production of their own output but combined in different proportions, Kelley-Williamson-Cheetham (1972), Niho (1974). This is due to the historical evidence that capital in the agricultural sector was an important element in the successful development of many economies and because in introducing capital into the agricultural sector we add another policy dimension to these models.

Under the above assumptions we try to present the structure of a developing economy to find behavioural and technical sufficient conditions for the properties of existence and uniqueness of the momentary solution and stability of balanced growth of our model and finally to analyse the structural change and the growth process through time.

3.2 The Model

A. Basic Assumptions

A1. There is a single malleable capital good used as an input in both sectors.

A2. The total labour force, L(t), grows exponentially.

$$L(t) = L(0)\, e^{\eta t} \qquad (3.2.1)$$

where L(0) is the stock of labour at time $t = 0$; η is the exogenously given constant rate

of growth of labour.

A3. Both agricultural and non-agricultural goods are assumed to be composed of homogeneous quantities and to be produced by two homogeneous factors of production, labour and capital. The quantities of the two outputs, $Y_i(t)$, $(i = 1,2)$, produced at time t are related to the quantities of labour, $L_i(t)$, and capital, $K_i(t)$, by the corresponding production functions, $F_i(\ )$, ie:

$$Y_i(t) = F_i\,(A_i(t)\,K_i(t)\,,\,B_i(t)\,L_i(t)\,) \qquad i = 1, 2 \tag{3.2.2}$$

where $A_i(t) > 0$ and $B_i(t) > 0$ are variables showing the technological progress of capital and labour respectively.

It is:

$$\dot{A}_i(t) / A_i(t) = \lambda_{1i} \qquad i = 1, 2 \tag{3.2.3}$$

$$\dot{B}_i(t) / B_i(t) = \lambda_{2i} \qquad i = 1, 2 \tag{3.2.4}$$

where λ_{1i} and λ_{2i} , $(i = 1, 2)$, denote the constant rates of the technological change of the two factors of production, in the two sectors respectively. We denote:

$$\bar{K}_i(t) = A_i(t)\,K_i(t) \qquad i = 1, 2 \tag{3.2.5}$$

$$\bar{L}_i(t) = B_i(t)\,L_i(t) \qquad i = 1, 2 \tag{3.2.6}$$

and we call $\bar{K}_i(t)$ the capital in efficiency units or "efficiency capital" and $\bar{L}_i(t)$ the labour in efficiency units or "efficiency labour". Therefore, (3.2.2) is written:

$$Y_i(t) = F_i\,(\bar{K}_i(t), \bar{L}_i(t)\ ) \qquad i = 1, 2 \tag{3.2.7}$$

In each sector, production is subject to constant returns to scale, ie. for any positive scale factor ζ , it is:

$$F_i\,(\zeta\,\bar{K}_i(t)\,,\,\zeta\,\bar{L}_i(t)\ ) = \zeta\,F_i\,(\bar{K}_i(t)\,,\,\bar{L}_i(t)\ ) = \zeta\,Y_i(t) \quad i = 1, 2 \tag{3.2.8}$$

and diminishing marginal rates of substitution. Joint products are excluded and external economies (diseconomies) do not exist.

A4. The saving/income ratios for the profits, s_3, and for the wages in the two sectors, s_1 and s_2 respectively, are such that:

$$0 \leqslant s_1 \leqslant s_2 \leqslant s_3 \leqslant 1 \qquad (3.2.9)$$

Extreme cases:

(a) The keynesian special case: $s_1 = s_2 = s_3 = s$. This was adopted by Harrod, Domar, Solow, and Swan.

(b) The Kaleckian special case: $s_1 = s_2 = 0 < s_r = 1$. This was adopted by Mrs. Robinson and Kaldor in some of their models.

B. Sectoral assumptions

B1. The output of the second sector is used as capital input in both sectors and as consumption good in both sectors. It is:

$$Y_2(t) = C_2(t) + I(t) \qquad (3.2.10)$$

where $C_2(t)$ is consumption of the second sector good in both sectors, and $I(t)$ is investment in both sectors.

B2. Capital depreciates exponentially over time

$$\dot{K}(t) = I(t) - \delta K(t) \qquad (3.2.11)$$

where $K(t)$ is the total capital in the economy and δ is the constant over time depreciation rate.

B3. The output of the first sector is all consumed

$$Y_1(t) = C_1(t) \qquad (3.2.12)$$

where $C_1(t)$ is consumption of the first sector good in both sectors.

B4. Gross investment, in terms of the second good, equals ex ante savings:

$$I(t) = s_3 r(t) K(t) + s_2 w_2(t) L_2(t) + s_1 w_1(t) L_1(t) \qquad (3.2.13)$$

where $r(t)$ is the return on capital and $w_i(t)$ is the wage-rate ($i = 1, 2$) in sector i,

all in terms of the second sector goods.

C. Special conditions

C1. Inada "derivative conditions".

The production functions are assumed invariant over time and twice differentiable, where, for all positive factor inputs:

$$\frac{\partial F_i}{\partial K_i} > 0 \quad , \quad \frac{\partial^2 F_i}{\partial K_i^2} < 0 \tag{3.2.14}$$

$$\frac{\partial F_i}{\partial L_i} > 0 \quad , \quad \frac{\partial^2 F_i}{\partial L_i^2} < 0 \tag{3,2.15}$$

and, taking limits:

$$\lim_{K_i \to 0} \frac{\partial F_i}{\partial K_i} = \infty \quad , \quad \lim_{K_i \to \infty} \frac{\partial F_i}{\partial K_i} = 0 \tag{3.2.16}$$

$$\lim_{L_i \to 0} \frac{\partial F_i}{\partial L_i} = \infty \quad , \quad \lim_{L_i \to \infty} \frac{\partial F_i}{\partial L_i} = 0 \tag{3.2.17}$$

so that both marginal products start at infinity and diminish at zero.

D. Market conditions

D1. The allocation of capital existing at any moment in time is assumed perfectly competitive, so that in each sector the rentals, $r'_i(t)$, of capital goods are equal to the marginal product of capital, ie:

$$r'_i(t) = \frac{\partial Y_i(t)}{\partial K_i(t)} \qquad i = 1, 2 \tag{3.2.18}$$

In terms of the second sector goods, it is

$$r_1(t) = p(t) \cdot r'_1(t) \tag{3.2.19}$$

and

$$r_2(t) = r'_2(t) \qquad (3.2.20)$$

where p(t) is the relative price (ratio of the price of the first sector over the second sector goods). We assume that

$$r_1(t) = r_2(t) = r(t) \qquad (3.2.21)$$

where r(t) is the common rate of return on capital in both sectors and in terms of the second sector goods (see B.4.)

D2. In a way similar to D1 we define the rate of return on capital in efficiency units, $\bar{r}'_i(t)$, ie:

$$\bar{r}'_i(t) = \frac{\partial Y_i(t)}{\partial \bar{K}_i(t)} \qquad i = 1, 2 \qquad (3.2.22)$$

$$\bar{r}_1(t) = p(t)\,\bar{r}'_1(t) \qquad (3.2.23)$$

$$\bar{r}_2(t) = \bar{r}'_2(t) \qquad (3.2.24)$$

and

$$A_1(t)\,\bar{r}_1(t) = A_2(t)\,\bar{r}_2(t) = \bar{r}(t) \qquad (3.2.25)$$

Because from (3.2.18), (3.2.22), (3.2.5) and (3.2.3) we can derive that

$$r'_i(t) = \frac{\partial Y_i(t)}{\partial \bar{K}_i(t)} \quad \frac{\partial \bar{K}_i(t)}{\partial K_i(t)} = \bar{r}'_i(t) \cdot A_i(t)$$

or

$$r'_i(t) = A_i(0)\, e^{\lambda_1 i\, t}\, \bar{r}'_i(t) \qquad i = 1, 2 \qquad (3.2.26)$$

D3. For the allocation of labour existing at any moment in time we assume that there is not perfect mobility of labour and there are transfer costs and ohter similar imperfections according to which the allocation of labour takes place, as we have seen analytically in chapter two. Since the proposed model refers to those developing economies in which the wage rate is above the subsistence level, we assume that the prevailing wage rate, $w'_i(t)$, is equal to the marginal product

of labour, in each sector respectively, ie:

$$w_i'(t) = \frac{\partial Y_i(t)}{\partial L_i(t)} \qquad i = 1, 2 \tag{3.2.27}$$

In terms of the second sector goods, it is

$$w_1(t) = p(t)\ w_1'(t) \tag{3.2.28}$$

and

$$w_1(t) = w_2^*(t) \tag{3.2.29}$$

We have seen in chapter two that

$$w_1(t) = \mu(t)\, w_2(t) \tag{3.2.30}$$

where $\mu(t)$ is given by

$$\mu(t) = \left(\frac{P(t)}{1 + \beta(t)^{1-v}}\right)^{\frac{1}{1-v}} \qquad 0 \leqslant v < 1 \tag{3.2.31}$$

D4. In similar to D3 way we define the wage-rates in the two sectors in efficiency units, $\bar{w}_i'(t)$, ie:

$$\bar{w}_i'(t) = \frac{\partial\, Y_i(t)}{\partial\, \bar{L}_i(t)} \qquad i = 1, 2 \tag{3.2.32}$$

$$\bar{w}_1(t) = p(t)\, \bar{w}_1'(t) \tag{3.2.33}$$

$$\bar{w}_2(t) = \bar{w}_2'(t) \tag{3.2.34}$$

and

$$B_1(t)\, \bar{w}_1(t) = \mu(t) \cdot B_2(t)\, \bar{w}_2(t) \tag{3.2.35}$$

because from (3.2.27), (3.2.32), (3.2.6) and (3.2.4) we can derive that:

$$w_i'(t) = \frac{\partial\, Y_i(t)}{\partial\, \bar{L}_i(t)} \frac{\partial\, \bar{L}_i(t)}{\partial\, L_i(t)} = \bar{w}_i'(t) \cdot B_i(t)$$

or

$$w_i'(t) = B_i(0) \cdot e^{\lambda_{2i} t}\, \bar{w}_i'(t) \qquad i = 1, 2 \tag{3.2.36}$$

From the conditions in D we conclude that in our economy there exist three

classes of income levels; the capitalists in both sectors, the workers in the agricultural sector and the workers in the non-agricultural sector. Therefore, it is quite reasonable to assume that the propensities to invest (consume) out of income may be different amongst these three classes as in A4 and B4. Of course, under (3.2.26) and (3.2.36) B4 is written:

$$I(t) = s_3 \bar{r}(t) K(t) + s_2 \bar{w}_2(t) \bar{L}_2(t) + s_1 \bar{w}_1(t) \bar{L}_1(t) \qquad (3.2.37)$$

In (3.2.37) we must note that it is:

$$\bar{w}_1(t) \bar{L}_1(t) = w_1(t) L_1(t)$$

$$\bar{w}_2(t) \bar{L}_2(t) = w_2(t) L_2(t)$$

$$\bar{r}_1(t) \bar{K}_1(t) = r_1(t) K_1(t)$$

$$\bar{r}_2(t) \bar{K}_2(t) = r_2(t) K_2(t)$$

$$\bar{r}(t) \bar{K}(t) \neq r(t) K(t) = \bar{r}(t) K(t)$$

In the case where $A_1(t) = A_2(t) = A(t)$ then

$$\bar{r}_1(t) = \bar{r}_2(t) = \bar{r}_0(t)$$

and

$A(t) \bar{r}_0(t) = \bar{r}(t)$, therefore

$$r(t) K(t) = \bar{r}_0(t) \bar{K}(t)$$

E. Comodity demand conditions

E1. Suppose that all individuals belonging to the same income class have the same utility function $U_j(t)$, $j = 1, 2, 3$, given for the hth individual by:

$$U_{jh}(t) = \Gamma_{jh} C_{1jh}(t)^{\gamma_{jh}} C_{2jh}^{\epsilon_{jh}}(t) \qquad \begin{matrix} j = 1, 2, 3 \\ h = 1, \ldots\ldots \end{matrix} \qquad (3.2.38)$$

where $C_{ijh}(t)$ (for $i = 1, 2; j = 1, 2, 3; h = 1, 2, \ldots\ldots$) is the amount of good i, which is consumed by the hth individual of the jth income class and $\Gamma_{jh}, \gamma_{jh}, \epsilon_{jh}$ are parameters such that $\Gamma_{jh} > 0$, $0 < \gamma_{jh} < 1$ and $0 < \epsilon_{jh} < 1$.

From A4 we have that each individual spends the fraction $(1-s_j)$, $(j = 1, 2, 3)$ of his income on consumption. Below we will work for the workers in the agricultural sector $(j = 1)$; the same analysis applies for $j = 1, 2$. Therefore, the budget constraint of the hth agricultural worker is given by:

$$p(t)\, C_{11\,h}(t) + C_{21\,h}(t) = (1-s_1)\, w_1(t) \tag{3.2.39}$$

We maximize the hth individual's utility function subject to his budget constraint to find his demand functions for the two goods. So, using the normal Lagrangean procedure, we maximize instead the function $V_{1\,h}(t)$ which is:

$$V_{1\,h}(t) = \Gamma_{1\,h}\, C_{11\,h}^{\gamma_{1\,h}}(t)\, C_{21\,h}^{\epsilon_{1\,h}}(t) - \\ - \lambda_h (p(t)\, C_{11\,h}(t) + C_{21\,h}(t) - (1 - s_1)\, w_1(t)) \tag{3.2.40}$$

where λ_h is the Langrangean coefficient. Taking the partial derivatives of the function (3.2.40) with respect to $C_{11\,h}$, $C_{21\,h}$ and λ_h respectively we obtain:

$$\frac{\partial V_{1\,h}(t)}{\partial C_{11\,h}(t)} = \gamma_{1\,h} \frac{\Gamma_{1\,h} C_{11\,h}^{\gamma_{1\,h}}(t)\, C_{21\,h}^{\epsilon_{1\,h}}(t)}{C_{11\,h}(t)} - \lambda_h p(t) \tag{3.2.41}$$

$$\frac{\partial V_{1\,h}(t)}{\partial C_{21\,h}(t)} = \epsilon_{1\,h} \frac{\Gamma_{1\,h} C_{11\,h}^{\gamma_{1\,h}}(t)\, C_{21\,h}^{\epsilon_{1\,h}}(t)}{C_{21\,h}(t)} - \lambda_h \tag{3.2.42}$$

and

$$\frac{\partial V_{1\,h}(t)}{\partial \lambda_h} = -(p(t)\, C_{11\,h}(t) + C_{21\,h}(t) - (1-s_1)\, w_1(t)) \tag{3.2.43}$$

Putting the above derivatives equal to zero and solving the system of equations for the unknown $C_{11\,h}(t)$, $C_{21\,h}(t)$ and λ_h we obtain the following individual demand functions for the workers in the agricultural sector.

$$C_{11\,h}(t) = \frac{\gamma_{1\,h}}{\gamma_{1h} + \epsilon_{1h}} (1 - s_1) \frac{w_1(t)}{p(t)} \tag{3.2.44}$$

and

$$C_{21\,h}(t) = \frac{\epsilon_{1\,h}}{\gamma_{1\,h} + \epsilon_{1\,h}} (1-s_1)\, w_1(t) \qquad (3.2.45)$$

Assuming that $\gamma_{1\,h}$ and $\epsilon_{1\,h}$ are equal to γ_1 and ϵ_1 respectively, for all h, and summing up the individual demand functions we obtain total demand functions for the first income class and for the two goods.

$$C_{11}(t) = \sum_{\forall h} C_{11\,h}(t) = \frac{\gamma_1}{\gamma_1 + \epsilon_1} (1 - s_1) \frac{w_1(t)}{p(t)} L_1(t) \qquad (3.2.46)$$

and

$$C_{21}(t) = \sum_{\forall h} C_{21\,h}(t) = \frac{\epsilon_1}{\gamma_1 + \epsilon_1} (1 - s_1)\, w_1(t) L_1(t) \qquad (3.2.47)$$

Working in the same way, as above, we find the general results.

E2. Total demand functions for each of the two outputs and for the two worker classes:

$$C_{ij}(t) = m_{ij} (1 - s_j) \frac{w_j(t)}{p(t)} L_j(t) \qquad j = 1, 2 \qquad (3.2.48)$$

$$C_{2j}(t) = m_{2j} (1 - s_j)\, w_j(t)\, L_j(t) \qquad j = 1, 2 \qquad (3.2.49)$$

where $m_{ij} = \gamma_j/(\gamma_j + \epsilon_j)$ and $m_{2j} = \epsilon_j/(\gamma_j + \epsilon_j)$ $\quad j = 1, 2$.

E3. Total demand functions for each of the two outputs of the capitalists in both sectors:

$$C_{13}(t) = m_{13}(1 - s_3) \frac{r(t)}{p(t)} K(t) \qquad (3.2.50)$$

$$C_{23}(t) = m_{23}\, (1 - s_3)\, r(t)\, K(t) \qquad (3.2.51)$$

where $m_{13} = \gamma_3/(\gamma_3 + \epsilon_3)$ and $m_{23} = \epsilon_3/(\gamma_3 + \epsilon_3)$.

(3.2.48) to (3.2.51) can of course be expressed in terms of efficiency units.

F. Accounting identities

F1. Assume full employment of capital

$$K_1(t) + K_2(t) = K(t) \qquad (3.2.52)$$

F2. Assume full employment of labour

$$L_1(t) + L_2(t) = L(t) \qquad (3.2.53)$$

This assumption simplifies (3.2.31) in which P(t) becomes equal to one, and if we also put v = 0, (transform to current wages), then (3.2.31) is written:

$$\mu(t) = \frac{1}{1+\beta(t)} \qquad (3.2.54)$$

As we have seen in chapter two, $\beta(t)$ measures the costs of migration and that for $t \to \infty$, $\beta(t) \to 0$. As far as we know, there is no explicit method of deriving $\beta(t)$. Therefore, we take $\beta(t)$ to vary exogenously in the form given in chapter two.

This means that for our model $\mu(t)$ is an exogeneous function which tends to one as $t \to \infty$.

F3. The gross national product in terms of the second sector goods, Y(t), is given by:

$$Y(t) = p(t)\,Y_1(t) + Y_2(t) \qquad (3.2.55)$$

Having now completed the structure of the growth model we find that the static model is a system of 18 equations in 17 endogeneous variables. The behaviour of the static economy is completely described by the 17 equations, ignoring one of the commodity market clearance equations, (3.2.10) or (3.2.12), Kelley-Williamson-Cheetham (1972), so the model becomes a system of 17 equations in 17 endogeneous variables. The endogeneous variables are $Y_i(t)$, $K_i(t)$, $L_i(t)$, $w_i(t)$, r(t), p(t), $C_{ij}(t)$ and I(t) for i = 1, 2 and j = 1, 2, 3; the exogenous variables are K(t) and L(t).

3.3 The Static Economy: Existence and Uniqueness

In this section we shall discuss under which conditions the momentary solution of quantities and prices, given the capital stock, K(t), and the available labour force, L(t), exists and is uniquely determined.

We define for i = 1, 2 and j = 1, 2, 3:

$y_i(t) = \frac{Y_i(t)}{L_i(t)}$ the sectoral output-labour ratios

$\bar{y}_i(t) = \frac{Y_i(t)}{\bar{L}_i(t)}$ the sectoral output-efficiency labour ratios

$k(t) = \frac{K(t)}{L(t)}$ the overall capital-labour ratio

$\bar{k}(t) = \frac{\bar{K}(t)}{\bar{L}(t)}$ the overall efficiency capital-efficiency labour ratio

$k_i(t) = \frac{K_i(t)}{L_i(t)}$ the sectoral capital-labour ratios

$\bar{k}_i(t) = \frac{\bar{K}_i(t)}{\bar{L}_i(t)}$ the sectoral efficiency capital-efficiency labour ratios

$\ell_i(t) = \frac{L_i(t)}{L(t)}$ the proportion of labour engaged in each sector

$\bar{\ell}_i(t) = \frac{\bar{L}_i(t)}{\bar{L}(t)}$ the proportion of efficiency labour engaged in each sector

$q_i(t) = \frac{Y_i(t)}{L(t)}$ the overall per capita output of the ith sector

$\bar{q}_i(t) = \frac{Y_i(t)}{\bar{L}(t)}$ the overall per efficiency labour unit output of the ith sector

$\omega_i(t) = \frac{w_i(t)}{r_i(t)}$ the sectoral wage-rentals ratio

$\bar{\omega}_i(t) = \frac{\bar{w}_i(t)}{\bar{r}_i(t)}$ the sectoral efficiency wage-rentals ratio

$c_i(t) = \frac{C_i(t)}{L(t)}$ the overall per capita consumption of the ith good

$\bar{c}_i(t) = \frac{C_i(t)}{\bar{L}(t)}$ the overall per efficiency labour unit consumption of the ith good

$c_{ij}(t) = \frac{C_{ij}(t)}{L_j(t)}$ the per capita consumption of the ith good by individuals of the jth income class

$\bar{c}_{ij}(t) = \frac{C_{ij}(t)}{\bar{L}_j(t)}$ the per efficiency unit of labour consumption of the ith good by labour of the jth income class

$$i(t) = \frac{I(t)}{L(t)}$$ the overall investment-labour ratio

$$\bar{i}(t) = \frac{I(t)}{\bar{L}(t)}$$ the overall investment-efficiency labour ratio

$$\tilde{k}(t) = \frac{K(t)}{\bar{L}(t)}$$ the overall capital-efficiency labour ratio

$$\tilde{\ell}_i(t) = \frac{L_i(t)}{\bar{L}(t)}$$ the sectoral labour-total efficiency labour ratios

Below, all variables are for brevity denoted without explicitly referring to time t.

From the constancy of the returns to scale, in (3.2.8), if we choose $\zeta = 1/\bar{L}_i(t)$

we have that:

$$\frac{Y_i(t)}{\bar{L}_i(t)} = F_i \left(\frac{\bar{K}_i(t)}{\bar{L}_i(t)}, 1 \right) \qquad i = 1, 2$$

or

$$\bar{y}_i(t) = f_i(\bar{k}_i(t)) \qquad i = 1, 2 \qquad (3.3.1)$$

where

$$F_i \left(\frac{\bar{K}_i(t)}{\bar{L}_i(t)}, 1 \right) = f_i(\bar{k}_i(t)) \qquad i = 1, 2$$

We have from (3.2.5), (3.2.6), (3.2.3) and (3.2.4) that:

$$\bar{k}_i(t) = \frac{A_i(t)\, K_i(t)}{B_i(t)\, L_i(t)} = \frac{A_i(0)}{B_i(0)} e^{(\lambda_{1i} - \lambda_{2i})t} k_i(t) \qquad i = 1, 2 \qquad (3.3.2)$$

therefore (3.3.1) is written:

$$y_i(t) = B_i(0)\, e^{\lambda_{2i}} f_i \left(\frac{A_i(0)}{B_i(0)} e^{(\lambda_{1i} - \lambda_{2i})t} k_i(t) \right) \qquad i = 1, 2 \qquad (3.3.3)$$

C1 applies for (3.3.3) or (3.3.1) also. Therefore, the "Inada derivative conditions", for $f_i(\bar{k}_i)$ which assume to be continuously twice differentiable for all $\bar{k}_i > 0$, can be written:

(a) $$f_i(\bar{k}_i) > 0,\ f_i'(\bar{k}_i) > 0,\ f_i''(\bar{k}_i) < 0, \forall \bar{k}_i > 0 \qquad i = 1, 2 \qquad (3.3.4)$$

(b) $f_i'(0) = \infty \ , \ f_i'(\infty) = 0 \qquad i = 1, 2 \qquad (3.3.5)$

(c) $f_i(0) = 0 \ , \ f_i(\infty) = \infty \qquad i = 1, 2 \qquad (3.3.6)$

In economic terms there highly restrictive assumptions imply, in efficiency units, in turn that:

(a) The marginal product of capital is always positive, yet diminishes with higher and higher capital/labour ratios.

(b) As the capital/labour ratio decreases towards zero, the marginal product of capital approaches infinity. As capital per worker increases without limit, capital saturation leads to zero marginal product of capital.

(c) Without any capital good, output per worker is zero. When capital per worker increases without limit, any level of output per worker can be reached (Wan, 1971).

From (3.2.53) we have:

$$\frac{L_1(t)}{L(t)} + \frac{L_2(t)}{L(t)} = 1$$

or $$\ell_1(t) + \ell_2(t) = 1 \qquad (3.3.7)$$

Also, from (3.2.52) we have:

$$k(t) = \frac{K_1(t)}{L_1(t)} \quad \frac{L_1(t)}{L(t)} + \frac{K_2(t)}{L_2(t)} \quad \frac{L_2(t)}{L(t)}$$

or $$k(t) = k_1(t)\,\ell_1(t) + k_2(t)\,\ell_2(t) \qquad (3.3.8)$$

This shows that the overall capital-labour ratio is the weighted average of sectoral capital-labour ratios with labour allocation ratios $\ell_1(t)$ and $\ell_2(t)$, in either sector as weights.

In the same way we can derive:

$$\bar{\ell}_1(t) + \bar{\ell}_2(t) = 1 \qquad (3.3.9)$$

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and $\quad \bar{k}_1(t)\,\bar{\ell}_1(t) + \bar{k}_2(t)\,\bar{\ell}_2(t) = \bar{k}(t)$. (3.3.10)

The relations between $\ell_i(t)$ and $\bar{\ell}_i(t)$, and $k_i(t)$ and $\bar{k}_i(t)$ can be found as follows:

We substitute (3.2.6) into $\bar{L}_1 + \bar{L}_2 = \bar{L}$ and obtain:

$$B_1 L_1 + B_2 L_2 = \bar{L} \qquad (3.3.11)$$

It is

$$\bar{\ell}_1 = \frac{\bar{L}_1}{\bar{L}} = \frac{B_1 L_1}{B_1 L_1 + B_2 L_2} = \frac{\dfrac{B_1 L_1}{L}}{\dfrac{B_1 L_1 + B_2 (L - L_1)}{L}}$$

and finally

$$\bar{\ell}_1 = \frac{B_1 \ell_1}{B_2 + (B_1 - B_2)\,\ell_1} \qquad (3.3.12)$$

similarly

$$\bar{\ell}_2 = \frac{B_2 \ell_2}{B_1 + (B_2 - B_1)\,\ell_2} \qquad (3.3.13)$$

Dividing (3.2.5) by (3.2.6) we have:

$$\bar{k}_i = \frac{A_i}{B_i} k_i \qquad i = 1, 2 \qquad (3.3.14)$$

In the case where the technological change for labour is in both sectors the same,

ie. $B_1(t) = B_2(t)$, then from (3.3.12) and (3.3.13) it is obvious that:

$$\bar{\ell}_1 = \ell_1 \quad \text{and} \quad \bar{\ell}_2 = \ell_2 \qquad (3.3.15)$$

Solving the system of the equations (3.3.7) and (3.3.8) or (3.3.9) and (3.3.10) we

derive respectively:

$$\ell_1 = \frac{k - k_2}{k_1 - k_2} \quad \text{and} \quad \ell_2 = \frac{k_1 - k}{k_1 - k_2} \qquad (3.3.16)$$

or

$$\bar{\ell}_1 = \frac{\bar{k} - \bar{k}_2}{\bar{k}_1 - \bar{k}_2} \quad \text{and} \quad \bar{\ell}_2 = \frac{\bar{k}_1 - \bar{k}}{\bar{k}_1 - \bar{k}_2} \qquad (3.3.17)$$

For the existence of the solutions (3.3.16) and (3.3.17) must be $k_1 \neq k_2$ and

$\bar{k}_1 \neq \bar{k}_2$ respectively.

In the cases where $A_1 = A_2 = A$ and $B_1 = B_2 = B$ over time then:

$$\bar{k} = \frac{A}{B} k \qquad \text{and} \qquad \bar{k}_i = \frac{A}{B} k_i \tag{3.3.18}$$

and so

$$\ell_1 = \bar{\ell}_1 = \frac{k - k_2}{k_1 - k_2} \qquad \text{and } \ell_2 = \bar{\ell}_2 = \frac{k_1 - k}{k_1 - k_2} \tag{3.3.19}$$

Having now defined the two production functions per efficiency unit of labour output, in (3.3.1), we derive that:

$$\bar{r}_1 = p \cdot \bar{r}_1 = p \cdot \frac{\partial Y_1}{\partial \bar{K}_1} = p \cdot \frac{\partial \bar{y}_1}{\partial \bar{k}_1} = p \cdot f'_1(\bar{k}_1) \tag{3.3.20}$$

and similarly

$$\bar{r}_2 = f'_2(\bar{k}_2) \tag{3.3.21}$$

From (3.3.20), (3.3.21) and (3.2.26) it is:

$$r_1 = A_1 \; p \; f'_1(\bar{k}_1) \tag{3.3.22}$$

$$\text{and} \qquad r_2 = A_2 \; f'_2(\bar{k}_2) \tag{3.3.23}$$

In the same way we have that:

$$\bar{w}_1 = p \cdot \bar{w}'_1 = p \cdot \frac{\partial Y_1}{\partial \bar{L}_1} = p \cdot [f_1(\bar{k}_1) - \bar{k}_1 f'_1(\bar{k}_1)] \tag{3.3.24}$$

$$\text{and} \qquad \bar{w}_2 = f_2(\bar{k}_2) - \bar{k}_2 \cdot f'_2(\bar{k}_2) \tag{3.3.25}$$

From (3.3.24), (3.3.25) and (3.2.36) we have that:

$$w_1 = B_1 \; p \cdot [f_1(\bar{k}_1) - \bar{k}_1 f'_1(\bar{k}_1)] = B_1 \bar{w}_1 \tag{3.3.26}$$

$$\text{and} \qquad w_2 = B_2 [f_2(\bar{k}_2) - \bar{k}_2 f'_2(\bar{k}_2)] = B_2 \bar{w}_2 \tag{3.3.27}$$

Dividing (3.3.24) by (3.3.20) and (3.3.25) by (3.3.21) we derive the sectoral efficiency wage-rentals ratios, $\bar{\omega}_i$, ie.:

$$\bar{\omega}_i = \frac{\bar{w}_i}{\bar{r}_i} = \frac{f_i(\bar{k}_i) - \bar{k}_i f'(\bar{k}_i)}{f'_i(\bar{k}_i)}$$

or
$$\bar{\omega}_i = \frac{f_i(\bar{k}_i)}{f'_i(\bar{k}_i)} - \bar{k}_i \qquad i = 1, 2 \tag{3.3.28}$$

In the same way, dividing (3.3.26) by (3.3.22) and (3.3.27) by (3.3.23) we find the sectoral wage-rentals ratios, ω_i, ie:

$$\bar{\omega}_i = \frac{w_i}{r_i} = \frac{B_i}{A_i} \quad \frac{f_i(\bar{k}_i) - \bar{k}_i f'(\bar{k}_i)}{f'_i(\bar{k}_i)}$$

or
$$\omega_i = \frac{B_i}{A_i} \left(\frac{f_i(\bar{k}_i)}{f'_i(\bar{k}_i)} - \bar{k}_i \right) \tag{3.3.29}$$

Comparing (3.3.28) and (3.3.29) we see that
$$\omega_i = \frac{B_i}{A_i} \bar{\omega}_i \tag{3.3.30}$$

Dividing (3.2.30) by (3.2.21) we obtain that
$$\omega_1 = \mu \omega_2 \tag{3.3.31}$$

or after substitution from (3.3.30)
$$\bar{\omega}_1 = \mu \cdot \frac{A_1 B_2}{A_2 B_1} \cdot \bar{\omega}_2 \tag{3.3.32}$$

From (3.2.25) and (3.2.26) we find the relative price p, ie:
$$p = \frac{A_2 \bar{r}'_2}{A_1 \bar{r}'_1} = \frac{r'_2}{r'_1} \tag{3.3.33}$$

or substituting from (3.3.20) and (3.3.21) we have that
$$p = \frac{A_2}{A_1} \quad \frac{f'_2(\bar{k}_2)}{f'_1(\bar{k}_1)} \tag{3.3.34}$$

We will find now the per capita consumption of the two goods and for the three income classes. Therefore, from (3.2.48) we have that
$$c_{ij} = \frac{C_{ij}}{L_j} = m_{ij}(1 - s_j) \quad \frac{w_j}{p} \qquad j = 1, 2 \tag{3.3.35}$$

Substituting (3.3.26) into (3.3.35) for j = 1, we have

$$c_{11} = m_{11}(1 - s_1)\frac{B_1\, p\,[\, f_1(\bar{k}_1) - \bar{k}_1\, f_1'(\bar{k}_1)\,]}{p} =$$

$$= m_{11}(1 - s_1) B_1\, f_1'(\bar{k}_1)\,[\,\frac{f_1(\bar{k}_1)}{f_1'(\bar{k}_1)} - \bar{k}_1\,]$$

or after substitution of (3.3.28) into the last

$$c_{11} = m_{11}(1-s_1) B_1\, f_1'(\bar{k}_1)\,\bar{\omega}_1 \qquad (3.3.36)$$

which is equal to

$$c_{11} = m_{11}(1 - s_1) f_1'(\bar{k}_1)\, A_1\, \omega_1 \qquad (3.3.37)$$

if we take into account (3.3.30).

For j = 2 we have from (3.3.35), under the substitution of (3.3.27) in it, that

$$c_{12} = m_{12}(1 - s_2)\frac{1}{p} B_2\,[\, f_2(\bar{k}_2) - \bar{k}_2\, f_2'(\bar{k}_2)\,]$$

Substituting (3.3.34) into the last expression we have

$$c_{12} = m_{12}(1 - s_2)\, B_2 \frac{A_1}{A_2}\, f_1'(\bar{k}_1)\,[\,\frac{f_2(\bar{k}_2)}{f_2'(k_2)} - \bar{k}_2\,]$$

$$= m_{12}(1 - s_2)\, B_2 \frac{A_1}{A_2}\, f_1'(\bar{k}_1)\,\bar{\omega}_2 \qquad (3.3.38)$$

$$= m_{12}(1 - s_2) f_1'(\bar{k}_1)\, A_1\, \omega_2 \qquad (3.3.39)$$

From (3.2.49) we have

$$c_{2j} = \frac{C_{2j}}{L_j} = m_{2j}(1 - s_j) w_j \qquad j = 1, 2 \qquad (3.3.40)$$

Substituting (3.3.26) into (3.3.40), for j = 1, we have

$$c_{21} = m_{21}(1 - s_1)\, B_1\, p\,[\, f_1(\bar{k}_1) - \bar{k}_1\, f_1'(\bar{k}_1)\,]$$

$$= m_{21}(1 - s_1)\, B_1 \frac{A_2}{A_1}\, f_2'(\bar{k}_2)\,[\,\frac{f_1(\bar{k}_1)}{f_1'(\bar{k}_1)} - \bar{k}_1\,]$$

$$= m_{21}(1 - s_1)\, B_1 \frac{A_2}{A_1}\, f_2'(\bar{k}_2)\,\bar{\omega}_1 \qquad (3.3.41)$$

$$= m_{21}(1 - s_1)\, f_2'(\bar{k}_2) A_2\, \dot{\omega}_1 \qquad (3.3.42)$$

For j = 2 we have from (3.3.40), under the substitution of (3.3.27) in it, that

$$c_{22} = m_{22}(1-s_2)B_2[f_2(\bar{k}_2) - \bar{k}_2 f'_2(\bar{k}_2)]$$

$$= m_{22}(1-s_2)B_2 f'_2(\bar{k}_2)\bar{\omega}_2 \qquad (3.3.43)$$

$$= m_{22}(1-s_2)f'_2(\bar{k}_2)A_2\omega_2 \qquad (3.3.44)$$

From (3.2.50) we have that

$$c_{13} = \frac{C_{13}}{L} = m_{13}(1-s_3)\frac{r}{p}k$$

$$= m_{13}(1-s_3)A_2 f'_2(\bar{k}_2)\frac{A_1}{A_2}\frac{f'_1(\bar{k}_1)}{f'_2(\bar{k}_2)}k$$

$$= m_{13}(1-s_3)A_1 f'_1(\bar{k}_1)k \qquad (3.3.45)$$

and from (3.2.51) we have that

$$c_{23} = \frac{C_{23}}{L} = m_{23}(1-s_3)rk$$

$$= m_{23}(1-s_3)A_2 f'_2(\bar{k}_2)k \qquad (3.3.46)$$

Working in the same way as above we can find the per efficiency unit of labour consumption for the two goods and the three income classes. The results obtained are:

$$\bar{c}_{11} = m_{11}(1-s_1)f'_1(\bar{k}_1)\bar{\omega}_1 \qquad (3.3.47)$$

$$= m_{11}(1-s_1)f'_1(\bar{k}_1)\frac{A_1}{B_1}\omega_1 \qquad (3.3.48)$$

$$\bar{c}_{12} = m_{12}(1-s_2)f'_1(\bar{k}_1)\frac{A_1}{A_2}\bar{\omega}_2 \qquad (3.3.49)$$

$$= m_{12}(1-s_2)f'_1(\bar{k}_1)\frac{A_1}{B_2}\omega_2 \qquad (3.3.50)$$

$$\bar{c}_{21} = m_{21}(1-s_1)\frac{A_2}{A_1}f'_2(\bar{k}_2)\bar{\omega}_1 \qquad (3.3.51)$$

$$= m_{21}(1-s_1)f'_2(\bar{k}_2)\frac{A_2}{B_1}\omega_1 \qquad (3.3.52)$$

$$\bar{c}_{22} = m_{22}(1-s_2)f'_2(\bar{k}_2)\bar{\omega}_2 \qquad (3.3.53)$$

$$= m_{22}(1-s_2)f'_2(\bar{k}_2)\frac{A_2}{B_2}\omega_2 \qquad (3.3.54)$$

$$\bar{c}_{13} = m_{13}(1 - s_3)\ A_1\ f'_1\ (\bar{k}_1)\ \tilde{k} \qquad (3.3.55)$$

$$\bar{c}_{23} = m_{23}(1 - s_3)\ A_2\ f'_2\ (\bar{k}_2)\ \tilde{k} \qquad (3.3.56)$$

We can find $\tilde{k} = K / \bar{L}$ after substitution of $\bar{L}$ from (3.3.11), ie:

$$\tilde{k} = \frac{K_1 + K_2}{\bar{L}} = \frac{K_1 + K_2}{B_1 L_1 + B_2 L_2} = \frac{k_1 + k_2 \frac{\ell_2}{\ell_1}}{B_1 + B_2 \frac{\ell_2}{\ell_1}}$$

substituting ℓ_1 and ℓ_2 into the last expression from (3.3.16), we have

$$\tilde{k} = \frac{k\,(k_1 - k_2)}{B_1\,(k - k_2) + B_2\,(k_1 - k)} \qquad (3.3.57)$$

(3.3.55) and (3.3.56) can be written in the following way:

$$\bar{c}^{13} = \frac{C^{13}}{\bar{L}} = m_{13}(1 - s_3)\frac{r}{p}\frac{K}{\bar{L}} = m_{13}(1 - s_3)\frac{1}{p}\ \frac{r_1 K_1 + r_2 K_2}{\bar{L}} =$$

$$= m_{13}(1 - s_3)\frac{1}{p}\ \frac{\bar{r}_1 \bar{K}_1 + \bar{r}_2 \bar{K}_2}{\bar{L}} =$$

$$= m_{13}(1 - s_3)\frac{1}{p}\ (\bar{r}_1\ \bar{k}_1\ \bar{\ell}_1 + \bar{r}_2\ \bar{k}_2\ \bar{\ell}_2) =$$

$$= m_{13}(1 - s_3)\ (\frac{p \cdot f'_1\ (\bar{k}_1)}{p}\ \bar{k}_1\ \bar{\ell}_1 + \frac{A_1}{A_2}\ \frac{f'_1\ (\bar{k}_1)}{f'_2\ (\bar{k}_2)}\ f'_2(\bar{k}_2)\,\bar{k}_2\ \bar{\ell}_2)$$

$$= m_{13}(1 - s_3)\,f'_1(\bar{k}_1)\ [\ \bar{k}_1\ \bar{\ell}_1 + \frac{A_1}{A_2}\ \bar{k}_2\ \bar{\ell}_2\ ] \qquad (3.3.55')$$

and in the same way

$$\bar{c}_{23} = m_{23}(1 - s_3)\,f'_2\ (\bar{k}_2)\ (\frac{A_2}{A_1}\ \bar{k}_1\ \bar{\ell}_1 + \bar{k}_2\ \bar{\ell}_2) \qquad (3.3.56')$$

We will find now the overall investment-labour ratio. From (3.2.13) we have that

$$i = \frac{I}{L} = s_3\ r\,k + s_2\,w_2\,\ell_2 + s_1\,w_1\,\ell_1$$

$$= r\,(s_3\ k + s_2\ \omega_2\ \ell_2 + s_1\ \omega_1\ \ell_1)$$

Substituting (3.3.23) into the last expression we have

$$i = A_2\ f'_2\ (\bar{k}_2)\,(s_3 k + s_2\,\omega_2\,\ell_2 + s_1\,\omega_1\,\ell_1 \qquad (3.3.58)$$

$$= A_2\ f'_2\,(\bar{k}_2)\,(s_3 k + s_2\,\frac{B_2}{A_2}\ \bar{\omega}_2\ \ell_2 + s_1\,\frac{B_1}{A_1}\ \bar{\omega}_1\ \ell_1) \qquad (3.3.59)$$

and

$$\bar{i} = A_2 \; f'_2 \; (\bar{k}_2)(s_3\tilde{k} + s_2\omega_2\tilde{\ell}_2 + s_1\omega_1\tilde{\ell}_1) \qquad (3.3.60)$$

(3.3.60) can be written in the following way

$$\bar{i} = \frac{I}{\bar{L}} = s_3 \frac{\bar{r}_1\bar{K}_1 + \bar{r}_2\bar{K}_2}{\bar{L}} + s_2 \frac{\bar{w}_2\bar{L}_2}{\bar{L}} + s_1 \frac{\bar{w}_1\bar{L}_1}{\bar{L}}$$

$$= s_3 (\bar{r}_1\bar{k}_1\bar{\ell}_1 + \bar{r}_2\bar{k}_2\bar{\ell}_2) + s_2\bar{w}_2\bar{\ell}_2 + s_1\bar{w}_1\bar{\ell}_1$$

$$= \bar{r}_1\bar{\ell}_1(s_3\bar{k}_1 + s_1\bar{\omega}_1) + \bar{r}_2\bar{\ell}_2(s_3\bar{k}_2 + s_2\bar{\omega}_2)$$

$$= p \cdot f'_1(\bar{k}_1)\bar{\ell}_1(s_3\bar{k}_1 + s_1\bar{\omega}_1) + f'_2(\bar{k}_2)\bar{\ell}_2(s_3\bar{k}_2 + s_2\bar{\omega}_2)$$

$$= \frac{A_2 \cdot f'_2(\bar{k}_2)}{A_1 \; f'_1(\bar{k}_1)} \cdot f'_1(\bar{k}_1)\bar{\ell}_1(s_3\bar{k}_1 + s_1\bar{\omega}_1) + f'_2(\bar{k}_2)\bar{\ell}_2(s_3\bar{k}_2 + s_2\bar{\omega}_2)$$

$$= f'_2(\bar{k}_2)\left[\frac{A_2}{A_1}\bar{\ell}_1(s_3\bar{k}_1 + s_1\bar{\omega}_1) + \bar{\ell}_2(s_3\bar{k}_2 + s_2\bar{\omega}_2)\right] \qquad (3.3.60)$$

where

$$\tilde{\ell}_1 = \frac{L_1}{\bar{L}} = \frac{L_1}{B_1L_1 + B_2L_2} = \frac{\ell_1}{B_1\ell_1 + B_2\ell_2} \qquad (3.3.61)$$

and

$$\tilde{\ell}_2 = \frac{\ell_2}{B_1\ell_1 + B_2\ell_2} \qquad (3.3.62)$$

We will find now the overall per capita consumption of the ith good. It is:

$$c_1 = \frac{C_1}{L} = \frac{Y_1}{L} = \frac{Y_1}{L_1}\frac{L_1}{L} = y_1\ell_1$$

or $$c_1 = B_1 f_1(\bar{k}_1)\ell_1 \qquad (3.3.63)$$

Also, $$\bar{c}_1 = \frac{C_1}{\bar{L}} = \frac{Y_1}{\bar{L}_1}\frac{\bar{L}_1}{\bar{L}} = \bar{y}_1\bar{\ell}_1$$

or $$\bar{c}_1 = f_1(\bar{k}_1)\bar{\ell}_1 \qquad (3.3.64)$$

From (3.2.10) we have:

$$c_2 = \frac{C_2}{L} = \frac{Y_2}{L} - \frac{I}{L} = \frac{Y_2}{L_2}\frac{L_2}{L} - \frac{I}{L} = y_2\ell_2 - i$$

or $$c_2 = B_2 f_2(\bar{k}_2)\ell_2 - i \qquad (3.3.65)$$

Also,

$$\bar{c}_2 = \frac{C_2}{\bar{L}} = \frac{Y_2}{\bar{L}_2} \frac{\bar{L}_2}{\bar{L}} - \frac{I}{\bar{L}} = \bar{y}_2 \bar{\ell}_2 - \bar{i}$$

or $$\bar{c}_2 = f_2 (\bar{k}_2) \bar{\ell}_2 - \bar{i} \qquad (3.3.66)$$

We will summarise now the results obtained above. Thus, the static model can be stated as follows:

$$y_i = B_i f_i \left(\frac{A_i}{B_i} k_i\right) \qquad i = 1, 2 \qquad (3.3.67)$$

$$\ell_1 + \ell_2 = 1 \qquad (3.3.68)$$

$$k = k_1 \ell_1 + k_2 \ell_2 \qquad (3.3.69)$$

$$\omega_i = \frac{B_i}{A_i} \left(\frac{f_i \left(\frac{A_i}{B_i} k_i\right)}{f_i' \left(\frac{A_i}{B_i} k_i\right)} - \frac{A_i}{B_i} k_i \right) \quad i = 1, 2 \qquad (3.3.70)$$

$$\omega_1 = \mu \omega_2 \qquad (3.3.72)$$

$$p = \frac{A_2}{A_1} \frac{f_2' \left(\frac{A_2}{B_2} k_2\right)}{f_i' \left(\frac{A_1}{B_1} k_1\right)} \qquad (3.3.72)$$

$$c_{ij} = m_{ij} (1 - s_j) f_i' \left(\frac{A_i}{B_i} k_i\right) A_i \omega_j \qquad \begin{matrix} i = 1, 2 \\ j = 1, 2 \end{matrix} \qquad (3.3.73)$$

$$c_{i3} = m_{i3} (1 - s_3) f_i' \left(\frac{A_i}{B_i} k_i\right) A_i k \qquad i = 1, 2 \qquad (3.3.74)$$

$$i = A_2 f_2' \left(\frac{A_2}{B_2} k_2\right) (s_3 k + s_2 \omega_2 \ell_2 + s_1 \omega_1 \ell_1) \qquad (3.3.75)$$

$$c_1 = B_1 \ell_1 f_1 \left(\frac{A_1}{B_1} k_1\right) \qquad (3.3.76)$$

$$c_2 = B_2 \ell_2 f_2 \left(\frac{A_2}{B_2} k_2\right) - i \qquad (3.3.77)$$

The static model can also be stated in efficiency units as follows:

$$\bar{y}_i = f_i (\bar{k}_i) \qquad i = 1, 2 \qquad (3.3.78)$$

$$\bar{\ell}_1 + \bar{\ell}_2 = 1 \tag{3.3.79}$$

$$\bar{k} = \bar{k}_1 \bar{\ell}_1 + \bar{k}_2 \bar{\ell}_2 \tag{3.3.80}$$

$$\bar{\omega}_i = \frac{f_i(\bar{k}_i)}{f'_i(\bar{k}_i)} - \bar{k}_i \qquad i = 1, 2 \tag{3.3.81}$$

$$\bar{\omega}_1 = \mu \frac{A_1 B_2}{A_2 B_1} \bar{\omega}_2 \tag{3.3.82}$$

$$p = \frac{A_2}{A_1} \frac{f'_2(\bar{k}_2)}{f'_1(\bar{k}_1)} \tag{3.3.83}$$

$$\bar{c}_{ij} = \begin{cases} m_{ij}(1 - s_j) f'_i(\bar{k}_i) \bar{\omega}_j & i = j \\ m_{ij}(1 - s_j) f'_i(\bar{k}_i) \frac{A_i}{A_j} \bar{\omega}_j & i \neq j \end{cases} \qquad \begin{matrix} i = 1, 2 \\ j = 1, 2 \end{matrix} \tag{3.3.84}$$

$$\bar{c}_{i3} = m_{i3}(1 - s_3) A_i f'_i(\bar{k}_i) \left(\frac{\bar{k}_1 \bar{\ell}_1}{A_1} + \frac{\bar{k}_2 \bar{\ell}_2}{A_2}\right), \qquad i = 1, 2 \tag{3.3.85}$$

$$\bar{I} = A_2 f'_2(\bar{k}_2) \left(\frac{\bar{\ell}_1}{A_1}(s_3 \bar{k}_1 + s_1 \bar{\omega}_1) + \frac{\bar{\ell}_2}{A_2}(s_3 \bar{k}_2 + s_2 \bar{\omega}_2)\right) \tag{3.3.86}$$

$$\bar{c}_1 = \bar{\ell}_1 f_1(\bar{k}_1) \tag{3.3.87}$$

$$\bar{c}_2 = \bar{\ell}_2 f_2(\bar{k}_2) - \bar{I} \tag{3.3.88}$$

The static model expressed in "actual" or in "efficiency" units is a system of 17 equations in 16 endogenous variables and can be reduced into a system of 16 equations in 16 endogenous variables, ignoring, as in section 3.2, one of the commodity market clearance equations. The endogenous variables for the "actual" model are: y_i, k_i, ℓ_i, ω_i, p, c_{ij}, i, for i = 1,2 and j = 1,2,3; while k is the exogenous variable. The endogenous variables for the "efficiency" model are: $\bar{y}_i$, $\bar{k}_i$, $\bar{\ell}_i$, $\bar{\omega}_i$, p, $\bar{c}_{ij}$, $\bar{I}$, for i = 1,2 and j = 1,2,3; while $\bar{k}$ is the exogenous variable.

We will try now to find under which conditions the solution to the static model above exists and is uniquely determined. For this reason we add up (3.2.48) and (3.2.50); we have:

$$C_1 = C_{11} + C_{12} + C_{13} = m_{11}(1 - s_1) \frac{w_1}{p} L_1 +$$

$$+ m_{12}(1 - s_2)\frac{w_2}{p} L_2 + m_{13}(1 - s_3)\frac{r}{p} K$$

We divide both sides of the above expression by rL:

$$m_{11}(1 - s_1)\frac{w_1}{r}\frac{L_1}{L} + m_{12}(1 - s_2)\frac{w_2}{r}\frac{L_2}{L} + m_{13}(1 - s_3)\frac{r}{r}\frac{K}{L} =$$

$$= \frac{p}{r}\frac{C_1}{L}$$

or $m_{11}(1 - s_1)\omega_1 \ell_1 + m_{12}(1 - s_2)\omega_2 \ell_2 + m_{13}(1 - s_3) k =$

$$= \frac{p}{r} c_1$$

We substitute p by (3.3.33) and c_1 by (3.3.76) into the last expression:

$$m_{11}(1 - s_1)\omega_1 \ell_1 + m_{12}(1 - s_2)\omega_2 \ell_2 + m_{13}(1 - s_3) k =$$

$$= \frac{B_1}{A_1} \ell_1 \frac{f_1(\bar{k}_1)}{f'_1(\bar{k}_1)}$$

Finally, if we substitute (3.3.70) into the last we have:

$$m_{11}(1 - s_1)\omega_1 \ell_1 + m_{12}(1 - s_2)\omega_2 \ell_2 + m_{13}(1 - s_3) k = \ell_1(\omega_1 + k_1) \quad (3.3.89)$$

We solve now for k after substitution of (3.3.16) into (3.3.89), ie:

$$m_{11}(1 - s_1)\omega_1 \frac{k - k_2}{k_1 - k_2} + m_{12}(1 - s_2)\omega_2 \frac{k_1 - k}{k_1 - k_2} +$$

$$+ m_{13}(1 - s_3) k = \frac{k - k_2}{k_1 - k_2} (\omega_1 + k_1)$$

or

$$m_{11}(1 - s_1)\omega_1 (k - k_2) + m_{12}(1 - s_2)\omega_2 (k_1 - k) +$$

$$+ m_{13}(1 - s_3)(k_1 - k_2) k = (k - k_2)(\omega_1 + k_1)$$

or

$$k = \frac{m_{11}(1 - s_1)\omega_1 k_2 - m_{12}(1 - s_2)\omega_2 k_1 - (\omega_1 + k_1) k_2}{m_{11}(1 - s_1)\omega_1 - m_{12}(1 - s_2)\omega_2 + m_{13}(1 - s_3)(k_1 - k_2) - (\omega_1 + k_1)} \quad (3.3.90)$$

(3.3.90) can be written in the form:

$$k = \frac{(m_{11}(1-s_1)-1)\,\omega_1 k_2 - (m_{12}(1-s_2)\omega_2 + k_2)\,k_1}{(m_{11}(1-s_1)-1)\,\omega_1 - m_{12}(1-s_2)\,\omega_2 + m_{13}(1-s_3)(k_1-k_2) - k_1} \quad (3.3.91)$$

In (3.3.91) the numerator, N, is always negative. For k to be positive the denominator D, must be negative. A sufficient condition for the denominator to be negative, in other words for the existence of k, is $k_1 - k_2 < 0$, or the second sector (non-agricultural) must be more capital intensive than the first sector (agricultural).

In the case where $s_3 = 1$ (all the profits invested) and $s_1 = s_2 = 0$ (all the wages consumed) – classical saving function – (3.3.91) is written:

$$k = \frac{(m_{11} - 1)\,\omega_1 k_2 - (m_{12}\,\omega_2 + k_2)\,k_1}{(m_{11} - 1)\,\omega_1 - m_{12}\,\omega_2 - k_1} \quad (3.3.92)$$

In (3.3.92), k is always positive, without imposing any restriction on the capital intensities of the two sectors.

Equation (3.3.91) uniquely determines the wage-rental ratio ω_2 (or $\omega_1 = \mu\omega_2$). To see this we differentiate (3.3.91) with respect to ω_2 . Thus,

$$\frac{dk}{d\omega_2} = \frac{N'D - N\,D'}{D^2} \quad (3.3.93)$$

We define σ_i the sectoral elasticity of factor substitution, which is equal to:

$$\sigma_i = \frac{dk_i}{d\omega_i}\ \frac{\omega_i}{k_i} \qquad i = 1, 2 \quad (3.3.94)$$

we have

$$N' = (m_{11}(1-s_1)-1)\,(\mu k_2 + \mu\omega_2\ \frac{dk_2}{d\omega_2}) -$$

$$-(m_{12}(1-s_2) + \frac{dk_2}{d\omega_2})\,k_1 - (m_{12}\,(1-s_2)\,\omega_2 + k_2)\,\mu\,\frac{dk_1}{d\omega_1}$$

In the last expression we substitute (3.3.94):

$$N' = (m_{11}(1-s_1)-1)\,\mu\,(k_2 + k_2\,\sigma_2) - (m_{12}(1-s_2) + \frac{k_2}{\omega_2}\ \sigma_2)\,k_1 -$$

$$-(m_{12}(1-s_2)\,\omega_2 + k_2)\,\mu\,\frac{k_1}{\omega_1}\ \sigma_1$$

or finally:

$$N' = (m_{11}(1-s_1)-1)\mu(1+\delta_2)k_2 - m_{12}(1-s_2)(1+\delta_1)k_1 - \frac{k_1 k_2}{\omega_2}(\delta_1+\delta_2) \tag{3.3.95}$$

Also, $$D' = (m_{11}(1-s_1)-1)\mu - m_{12}(1-s_2) + m_{13}(1-s_3)\left(\mu\frac{dk_1}{d\omega_1} - \frac{dk_2}{d\omega_2}\right) - \mu\frac{dk_1}{d\omega_1}$$

or after substitution by (3.3.94) we have:

$$D' = (m_{11}(1-s_1)-1)\mu - m_{12}(1-s_2) + (m_{13}(1-s_3)-1)\mu\,\delta_1\frac{k_1}{\omega_1} - m_{13}(1-s_3)\,\delta_2\frac{k_2}{\omega_2} \tag{3.3.96}$$

Therefore,

$$N'D - N\,D' = (m_{11}(1-s_1)-1)^2\,\mu\,(1+\delta_2)k_2\,\omega_1$$

$$-(m_{11}(1-s_1)-1)\mu\,(1+\delta_2)k_2\,m_{12}(1-s_2)\,\omega_2$$

$$+(m_{11}(1-s_1)-1)\mu\,(1+\delta_2)k_2\,(m_{13}(1-s_3)-1)\,k_1$$

$$-(m_{11}(1-s_1)-1)\mu\,(1+\delta_2)k^2{}_2\,m_{13}(1-s_3)$$

$$-m_{12}(1-s_2)(1+\delta_1)k_1\,(m_{11}(1-s_1)-1)\,\omega_1$$

$$+m_{12}(1-s_2)(1+\delta_1)k_1\,m_{12}(1-s_2)\,\omega_2$$

$$-m_{12}(1-s_2)(1+\delta_1)k_1{}^2\,(m_{13}(1-s_3)-1)$$

$$+m_{12}\,(1-s_2)(1+\delta_1)k_1\,m_{13}(1-s_3)k_2$$

$$-\frac{k_1 k_2}{\omega_2}(\delta_1+\delta_2)(m_{11}(1-s_1)-1)\omega_1 + \frac{k_1 k_2}{\omega_2}(\sigma_1+\sigma_2)\,m_{12}(1-s_2)\,\omega_2$$

$$-\frac{k_1{}^2 k_2}{\omega_2}(\sigma_1+\sigma_2)(m_{13}(1-s_3)-1) + \frac{k_1 k^2{}_2}{\omega_2}(\sigma_1+\sigma_2)\,m_{13}(1-s_3)$$

$$-(m_{11}\,(1-s_1)-1)^2\,\mu\,\omega_1 k_2 + (m_{11}(1-s_1)-1)\mu\,m_{12}(1-s_2)\,\omega_2 k_1$$

$$+(m_{11}\,(1-s_1)-1)\mu\,k_1 k_2 + m_{12}(1-s_2)(m_{11}(1-s_1)-1)\,\omega_1 k_2$$

$$- m_{12}^2 (1 - s_2)^2 \omega_2 k_1 - m_{12}(1 - s_2) k_1 k_2 -$$

$$- (m_{13}(1 - s_3) - 1) \mu \sigma_1 \frac{k_1}{\omega_1} (m_{11}(1 - s_1) - 1) \omega_1 k_2$$

$$+ (m_{13}(1 - s_3) - 1) \mu \sigma_1 \frac{k_1}{\omega_1} m_{12}(1 - s_2) \omega_2 k_1$$

$$+ (m_{13}(1 - s_3) - 1) \mu \sigma_1 \frac{k_1}{\omega_1} k_1 k_2$$

$$+ m_{13}(1 - s_3) \sigma_2 \frac{k_2}{\omega_2} (m_{11}(1 - s_1) - 1) \omega_1 k_2 -$$

$$- m_{13}(1 - s_3) \sigma_2 \frac{k_2}{\omega_2} m_{12}(1 - s_2) \omega_2 k_1 - m_{13}(1 - s_3) \sigma_2 \frac{k_2}{\omega_2} k_1 k_2$$

or $$N' D - N D' = (m_{11}(1 - s_1) - 1)^2 \sigma_2 k_2 \omega_2 +$$

$$+ (m_{11}(1 - s_1) - 1) \mu (m_{13}(1 - s_3) - 1) (1 + \sigma_2 - \sigma_1) k_1 k_2 -$$

$$- m_{13}(1 - s_3) (m_{11}(1 - s_1) - 1) \mu k_2^2 - (m_{13}(1 - s_3) - 1) m_{12}(1 - s_2) k_1^2 -$$

$$- (m_{12}(1 - s_2))^2 \sigma_1 \omega_2 k_1 - (m_{11}(1 - s_1) - 1) m_{12}(1 - s_2) \sigma_2 k_2 \omega_1 -$$

$$- m_{12}(1 - s_2) (m_{11}(1 - s_1) - 1) \mu \sigma_1 k_1 \omega_2 - (m_{13}(1 - s_3) - 1) \sigma_2 \frac{k_1^2 k_2}{\omega_2} +$$

$$+ m_{13}(1 - s_3) \sigma_1 \frac{k_1 k_2^2}{\omega_2} + m_{12}(1 - s_2) (1 + \sigma_1 - \sigma_2) m_{13}(1 - s_3) k_1 k_2 +$$

$$+ ((m_{11}(1 - s_1) - 1) \mu - m_{12}(1 - s_2)) (1 - \sigma_1 - \sigma_2) k_1 k_2$$

Therefore (3.3.93) is written:

$$\frac{dk}{d\omega_2} = D^{-2} \cdot \Big\{ (m_{11}(1 - s_1) - 1)^2 \sigma_2 k_2 \omega_2 - m_{13}(1 - s_3) (m_{11}(1 - s_1) - 1) \mu k_2^2 -$$

$$- (m_{13}(1 - s_3) - 1) m_{12}(1 - s_2) k_1^2 - (m_{11}(1 - s_1) - 1) m_{12}(1 - s_2) \sigma_2 k_2 \omega_1 -$$

$$- (m_{13}(1 - s_3) - 1) \sigma_2 \frac{k_1^2 k_2}{\omega_2} + m_{13}(1 - s_3) \sigma_1 \frac{k_1 k_2^2}{\omega_2} +$$

$$+ (m_{11}(1 - s_1) - 1) \mu (1 - \sigma_1 - \sigma_2) k_1 k_2 + m_{12}(1 - s_2) (\sigma_1 + \sigma_2 - 1) k_1 k_2 -$$

$$- m_{12}(1 - s_2) \sigma_1 k_1 \omega_2 (m_{12}(1 - s_2) + \mu (m_{11}(1 - s_1) - 1)) +$$

$$+ (m_{12}(1 - s_2) m_{13}(1 - s_3) + (m_{11}(1 - s_1) - 1) (m_{13}(1 - s_3) - 1) \mu) k_1 k_2$$

$$+ (m_{12}(1 - s_2) m_{13}(1 - s_3) - (m_{11}(1 - s_1) - 1) (m_{13}(1 - s_3) - 1) \mu) (\sigma_1 - \sigma_2) k_1 k_2 \Big\}$$

(3.3.97)

Sufficient conditions for (3.3.97) to be positive, or in other words for the monotonicity of k (ω_2) are the following:

(i) $\quad \sigma_1 + \sigma_2 \geq 1$

(ii) $\quad \mu > \dfrac{m_{12}(1 - s_2)}{1 - m_{11}(1 - s_1)}$

(3.3.98)

(iii) a) $\sigma_1 - \sigma_2 > 0$ with $\mu \begin{cases} < \\ > \end{cases}$ than $\dfrac{m_{12}(1 - s_2)}{1 - m_{11}(1 - s_1)} \cdot \dfrac{m_{13}(1 - s_3)}{1 - m_{13}(1 - s_3)}$

b) $\sigma_1 - \sigma_2 < 0$

c) $\sigma_1 - \sigma_2 = 0$ with $\forall \mu$

Therefore sufficient conditions for the unique determination of k(ω_2) are (3.3.98 i & ii) in combination with any of (3.3.98 iii).

A closer examination of (3.3.98) shows that in (3.3.98 iiic) μ must obey (3.3.98 ii). Also, the coefficients of comsumption of agricultural goods for the two worker classes, m_{11} and m_{12}, together with the propensities to consume out of income for the two worker classes, put a lower limit in the wages ratio $\mu(t)$ over time as shown in (3.3.98 ii).

From the above exposition we conclude that in the case of non-classical saving function ($0 \leq s_1 \leq s_2 \leq s_3 \leq 1$) for any positive capital-labour ratio, k, the wage-rental ratio $\omega_2 = \omega_2(k)$ is uniquely determined by solving the equation (3.3.90), under the purely behavioural and technical sufficient conditions of capital intensities, $k_1 - k_2 < 0$, and that of (3.3.98), respectively.

Finally, we must note that we obtain the same conditions, as was expected, even when we work with the static model in efficiency units.

3.4 Properties of the Static Model (Comparative Statics)

In the static model, equations (3.3.67) – (3.3.77) or equations (3.3.78) – (3.3.88), all the endogenous variables can be expressed in terms of the wage-rental ratios, which as

we have already shown in section 3.3.can be uniquely determined under the sufficient conditions of (3.3.98). Below we will analyse in more detail some of their properties.

(i) From (3.3.30) we have:

$$\frac{d\omega_i}{dk_i} = \frac{B_i}{A_i} \frac{d\bar{\omega}_i}{d\bar{k}_i} \frac{d\bar{k}_i}{dk_i} \tag{3.4.1}$$

but $$\frac{d\bar{\omega}_i}{d\bar{k}_i} = \frac{f_i'(\bar{k}_i) f_i'(\bar{k}_i) - f_i''(\bar{k}_i) f_i(\bar{k}_i)}{[f_i'(\bar{k}_i)]^2} - 1$$

$$= - \frac{f_i(\bar{k}_i) f_i''(\bar{k}_i)}{[f_i'(\bar{k}_i)]^2} \quad \text{(from (3.3.81))} \tag{3.4.2}$$

and $$\frac{d\bar{k}_i}{dk_i} = \frac{A_i}{B_i} \quad \text{(from (3.3.14))} \tag{3.4.3}$$

· Therefore, (3.4.1) is written:

$$\frac{d\omega_i}{dk_i} = - \frac{f_i(\bar{k}_i) f_i''(\bar{k}_i)}{[f_i'(\bar{k}_i)]^2} = \frac{d\bar{\omega}_i}{d\bar{k}_i} \qquad i = 1, 2 \tag{3.4.4}$$

which under the Inada derivative conditions is positive. This proves the monotomicity of $\omega_i(k_i)$. Thus, we conclude that, for any wage-rental ratios ω_i, the optimal capital-labour ratios k_i in each sector are uniquely determined by the relations (3.3.70) or (3.3.81) and therefore we can invert these relations to obtain the capital per worker in either sector as a function of the wage-rental ratios, respectively:

$$k_i = k_i(\omega_i) \ , \ k_i'(\omega_i) > 0 \ , \qquad i = 1, 2 \tag{3.4.5}$$

where the assumptions on the production functions imply that:

$$k_i(0) = 0 \ , \quad k_i(\infty) = \infty \ , \qquad i = 1, 2 \tag{3.4.6}$$

The unique determination of $k_i(\omega_i)$ can be represented in diagrammatic terms with the help of figure 3.1.

In figure 3.1 the values of the wage-rental ratios are measured on the vertical axis

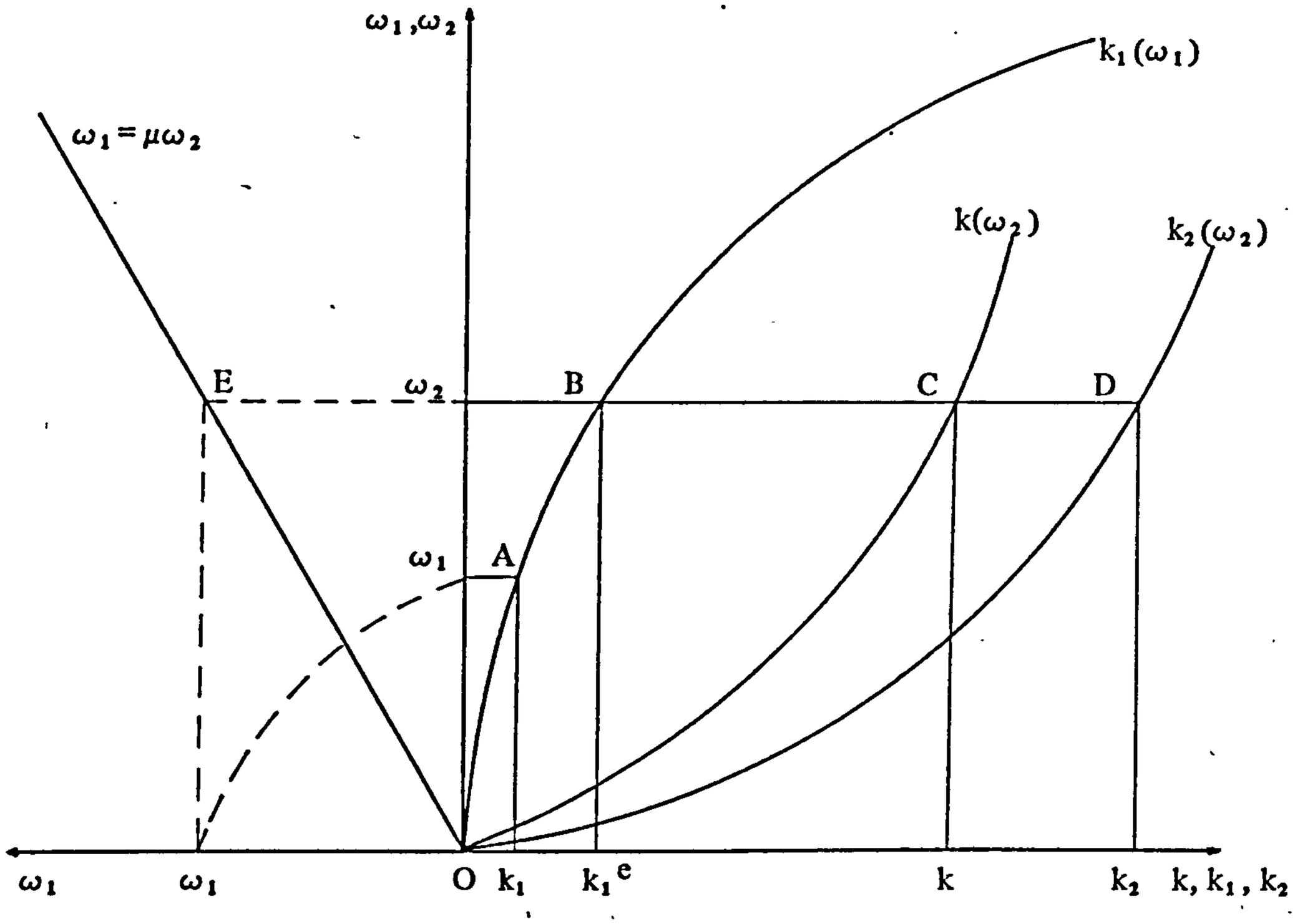

Figure 3.1

and also leftward from the origin along the horizontal axis. The capital-labour ratios are measured rightward on the horizontal axis. In the first quadrant the curves of $k_1(\omega_1)$, $k_2(\omega_2)$ and $k(\omega_2)$ are plotted which corresponds to functions (3.4.5, i = 1, 2) and (3.3.90). In the second quadrant the relation (3.3.71) is plotted.

Suppose now that the overall capital-labour ratio is k. Then the corresponding wage-rental ratio through the curve $k(\omega_2)$ is ω_2. For that ω_2 there is a unique k_2 which can be found with the help of curve $k_2(\omega_2)$. If the wage-rental ratios were the same in both sectors then k_1^e should be the corresponding capital-labour ratio in the first sector. Because this is not our case, then we can find ω_1 with the help of $\omega_1 = \mu\omega_2$ line in the second quadrant of figure 3.1. We then rotate $O\omega_1$ around O to find ω_1 on the vertical axis. The corresponding capital-labour ratio for the first sector to ω_1 can be found with

the help of the curve $k_1(\omega_1)$ and it is $k_1 < k_1^e$.

In figure 3.1 we plotted $k_i(\omega_i)$ according to (3.4.6) where the Inada derivative conditions apply. Of course, this is not always the case. For example, if

$$f_i(\bar{k}_i) = \log(\bar{k}_i + 2) \text{ , Burmeister-Dobell (1972)}$$

then $$\bar{\omega}_i = \frac{f_i(\bar{k}_i)}{f'_i(\bar{k}_i)} - \bar{k}_i = (\bar{k}_i + 2)\log(\bar{k}_i + 2) - \bar{k}_i$$

or $$\omega_i = (k_i + 2\frac{B_i}{A_i})\log(\frac{A_i}{B_i}k_i + 2) - \frac{A_i}{B_i}k_i$$

then $$\lim_{k_i \to 0} \omega_i = 2\frac{B_i}{A_i}\log 2 > 0$$

which means that although k_i can assume any value between zero and infinity, the range of ω_i may not include all positive numbers.

The difficulty is avoided by considering only values of ω contained in the open interval $(\omega_{min}, \omega_{max})$ where:

$$\omega_{min} = \max\left\{\lim_{k_2 \to 0}\omega_2, \lim_{k_1 \to 0}\omega_1\right\}$$

and

$$\omega_{max} = \min\left\{\lim_{k_2 \to \infty}\omega_2, \lim_{k_1 \to \infty}\omega_1\right\}$$

(ii) From (3.3.83) we find the behaviour of the realtive price. Therefore, taking the logs (3.3.83) we obtain:

$$\log p = \log\frac{A_2}{A_1} + \log f'_2(\bar{k}_2) - \log f'_1(\bar{k}_1)$$

Differentiating the last wigh respect to ω_2 we obtain:

$$\frac{1}{p}\frac{dp}{d\omega_2} = \frac{1}{f'_2(\bar{k}_2)} f''_2(\bar{k}_2)\frac{d\bar{k}_2}{dk_2}\frac{dk_2}{d\omega_2} -$$

$$- \frac{1}{f'_1(\bar{k}_1)} f''_1(\bar{k}_1)\frac{d\bar{k}_1}{dk_1}\frac{dk_1}{d\omega_1}\frac{d\omega_1}{d\omega_2} \qquad (3.4.7)$$

but $\frac{d\bar{k}_i}{dk_i} = \frac{A_i}{B_i}$, $\frac{d\omega_1}{d\omega_2} = \mu$ and $\frac{dk_i}{d\omega_i} = -\frac{[f'_i(\bar{k}_i)]^2}{f_i(\bar{k}_i) f''_i(\bar{k}_i)}$

therefore (3.4.7) is written

$$\frac{1}{p}\frac{dp}{d\omega_2} = \frac{1}{f'_2(\bar{k}_2)} f''_2(\bar{k}_2)\frac{A_2}{B_2}\left[-\frac{[f'_2(\bar{k}_2)]^2}{f_2(\bar{k}_2) f''_2(\bar{k}_2)}\right] -$$

$$-\frac{1}{f'_1(\bar{k}_1)} f''_1(\bar{k}_1)\frac{A_1}{B_1}\left[-\frac{[f'_1(\bar{k}_1)]^2}{f_1(\bar{k}_1) f''_1(\bar{k}_1)}\right]\mu$$

or

$$\frac{1}{p}\frac{dp}{d\omega_2} = -\frac{A_2}{B_2}\frac{f'_2(\bar{k}_2)}{f_2(\bar{k}_2)} + \frac{A_1}{B_1}\frac{f'_1(\bar{k}_1)}{f_1(\bar{k}_1)}\mu$$

or substituting (3.3.70) into the last expression we get

$$\frac{1}{p}\frac{dp}{d\omega_2} = -\frac{1}{\omega_2 + k_2} + \frac{\mu}{\omega_1 + k_1}$$

which is written

$$\frac{1}{p}\frac{dp}{d\omega_2} = \frac{\mu k_2(\omega_2) - k_1(\omega_1)}{(k_2(\omega_2) + \omega_2)(k_1(\omega_1) + \omega_1)} \qquad (3.4.8)$$

(3.4.8) is positive, zero or negative according to whether $\mu k_2 - k_1$ is greater, equal or less than zero.

In the case where $\mu k_2 - k_1 > 0$ or $k_1 / k_2 < \mu$ we see from (3.4.8) that as ω_2 increases, or as labour becomes relatively more expensive than capital, then the agricultural goods become relatively more expensive than the non-agricultural goods. Because μ is always less than one for $0 \leqslant t < \infty$ we see that $k_1 / k_2 < 1$ which means that the second sector is more capital intensive, which does not contradict the sufficient conditions of existence and uniqueness of section 3.3.

In the case where $\mu k_2 - k_1 < 0$ or $k_1 / k_2 > \mu$ we see that as ω_2 increases the agricultural goods become relatively less expensive than the non-agricultural goods. In

this case it is possible for k_1/k_2 to be greater than one but that contradicts the capital intensity hypothesis; so we only consider the range $1 > k_1/k_2 > \mu$.

For the case where $\mu k_2 - k_1 = 0$ or $k_1/k_2 = \mu$ we see that the relative positions of the prices of the two goods remain the same with a change in ω_2.

From the above discussion we conclude that the value of μ is of critical importance because it determines the behaviour of the terms of trade.

(iii) For the percentages of urbanization, ℓ_i (ω_i), we have from (3.3.16) that

$$\frac{d\ell_1}{d\omega_1} = \frac{1}{(k_1 - k_2)^2} \left[- \frac{dk_2}{d\omega_1} (k_1 - k_2) - (k - k_2) \left(\frac{dk_1}{d\omega_1} - \frac{dk_2}{d\omega_1} \right) \right]$$

or

$$\frac{d\ell_1}{d\omega_1} = \frac{-1}{k_1 - k_2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} \frac{k_1 - k}{k_1 - k_2} + \frac{dk_1}{d\omega_1} \frac{k - k_2}{k_1 - k_2} \right]$$

or finally, substituting back (3.3.16) into the above expression, we find

$$\frac{d\ell_1}{d\omega_1} = \frac{-1}{k_1 - k_2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1 \right] \qquad (3.4.9)$$

Working in the same way as above we find

$$\frac{d\ell_2}{d\omega_2} = \frac{\mu}{k_1 - k_2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1 \right] \qquad (3.4.10)$$

From (3.4.9) and (3.4.10) we have that

$$\frac{d\ell_1}{d\omega_1} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad k_1 - k_2 \begin{matrix} < \\ > \end{matrix} 0 \qquad (3.4.11)$$

and

$$\frac{d\ell_2}{d\omega_2} \begin{matrix} > \\ < \end{matrix} 0 \quad \text{if} \quad k_1 - k_2 \begin{matrix} > \\ < \end{matrix} 0 \qquad (3.4.12)$$

Thus, in the case where the second sector is more capital intensive, $(k_1 - k_2 < 0)$, we find that as labour becomes relatively more expensive than capital, urbanization decreases (ℓ_2 decreases, ℓ_1 increases), which means that substitution of labour for capital

takes place.

(iv) For the per capita demand of workers, in both sectors, and for both goods we have from (3.3.73) that

$$\frac{dc_{ij}}{d\omega_j} = m_{ij}(1-s_j)\, A_i \left[f_i''(\bar{k}_i) \frac{d\bar{k}_i}{d\omega_j}\, \omega_j + f_i'(\bar{k}_i) \right] \qquad (3.4.13)$$

In (3.4.13) for $i = j\ (= 1, 2)$ we have

$$\frac{dc_{ii}}{d\omega_i} = m_{ii}(1-s_i)\, A_i \left[f_i''(\bar{k}_i) \frac{d\bar{k}_i}{d\bar{\omega}_i} \frac{d\bar{\omega}_i}{d\omega_i}\, \omega_i + f_i'(\bar{k}_i) \right]$$

We substitute (3.4.4.) and (3.3.30) into the last expression:

$$\frac{dc_{ii}}{d\omega_i} = m_{ii}(1-s_i)\, A_i \left[f_i''(\bar{k}_i) \left(- \frac{[f_i'(\bar{k}_i)]^2}{f_i(\bar{k}_i) f_i''(\bar{k}_i)} \right) \frac{A_i}{B_i}\, \omega_i + f_i'(\bar{k}_i) \right]$$

or after substitution by (3.3.81)

$$\frac{dc_{ii}}{d\omega_i} = m_{ii}(1-s_i)\, A_i f_i'(\bar{k}_i) \frac{\bar{k}_i}{\bar{\omega}_i + \bar{k}_i}$$

or after substitution by (3.3.14) and (3.3.30)

$$\frac{dc_{ii}}{d\omega_i} = m_{ii}(1-s_i)\, A_i f_i' \left(\frac{A_i}{B_i} k_i \right) \frac{k_i}{\omega_i + k_i} \qquad (3.4.14)$$

In (3.4.13) for $i = 2 \neq j = 1$ we have

$$\frac{dc_{21}}{d\omega_1} = m_{21}(1-s_1)\, A_2 \left[f_2''(\bar{k}_2) \frac{d\bar{k}_2}{d\bar{\omega}_2} \frac{d\bar{\omega}_2}{d\omega_2} \frac{d\omega_2}{d\omega_1}\, \omega_1 + f_2'(\bar{k}_2) \right]$$

Substituting (3.4.4), (3.3.30) and (3.3.31) into the last expression we have

$$\frac{dc_{21}}{d\omega_1} = m_{21}(1-s_1)\, A_2 f_2'(\bar{k}_2) \frac{k_2}{\omega_2 + k_2} \qquad (3.4.15)$$

Working in the same way we find that

$$\frac{dc_{12}}{d\omega_2} = m_{12}(1-s_2)\,A_1\,f'_1(\bar{k}_1)\,\frac{k_i}{\omega_1 + k_1} \tag{3.4.16}$$

We can summarize (3.4.14), (3.4.15) and (3.4.16) as follows:

$$\frac{dc_{ij}}{d\omega_j} = m_{ij}(1-s_j)\,A_i\,f'_i\left(\frac{A_i}{B_i}\,k_i\right)\,\frac{k_i}{\omega_i + k_i} \qquad \begin{matrix} i=1,2 \\ j=1,2 \end{matrix} \tag{3.4.17}$$

(3.4.17) is always positive and means that an increase in the wage-rental ratio implies an increase in each wage-earner's income and so increase in the demand for all consumption goods.

In the opposite case, for the per capita demand of the capitalists for both goods, we have from (3.3.74) that

$$\frac{dc_{i3}}{d\omega_i} = m_{i3}(1-s_3)\,A_i\,k\,f''_i(\bar{k}_i)\,\frac{d\bar{k}_i}{d\bar{\omega}_i}\,\frac{d\bar{\omega}_i}{d\omega_i}$$

or after substitution of (3.4.4) and (3.3.30) into the last expression:

$$\frac{dc_{i3}}{d\omega_i} = -m_{i3}(1-s_3)\,A_i\,f'_i\left(\frac{A_i}{B_i}\,k_i\right)\,\frac{k}{\omega_i + k_i}, \quad 1=1,2 \tag{3.4.18}$$

which is always negative and means that an increase in the wage-rental ratio implies a decrease in the capitalist's income and so a decrease in the demand for all consumption goods.

(v) We will try now to investigate the per capita investment of the economy. From (3.3.75) we have

$$\frac{di}{d\omega_2} = A_2\,f''_2(\bar{k}_2)\,\frac{d\bar{k}_2}{d\bar{\omega}_2}\,\frac{d\bar{\omega}_2}{d\omega_2}(s_3 k + s_2\omega_2\ell_2 + s_1\omega_1\ell_1) +$$

$$+ A_2\,f'_2(\bar{k}_2)\left[s_2\left(\ell_2 + \omega_2\frac{d\ell_2}{d\omega_2}\right) + s_1\left(\mu\,\ell_1 + \omega_1\frac{d\ell_1}{d\omega_1}\,\mu\right)\right]$$

Substituting (3.4.4) and (3.3.30) into the above expression we have:

$$\frac{di}{d\omega_2} = -A_2 f''_2(\bar{k}_2) \frac{[f'_2(\bar{k}_2)]^2}{f_2(\bar{k}_2) f''_2(\bar{k}_2)} \frac{A_2}{B_2} (s_3 k + s_2 \omega_2 \ell_2 + s_1 \omega_1 \ell_1) +$$

$$+ A_2 f'_2(\bar{k}_2) [(s_1 \mu \ell_1 + s_2 \ell_2) + \omega_2 (s_1 \frac{d\ell_1}{d\omega_1} + s_2 \frac{d\ell_2}{d\omega_2})]$$

We substitute $\frac{d\ell_i}{d\omega_i}$ from (3.4.9) and (3.4.10), therefore:

$$\frac{di}{d\omega_2} = -A_2 f'_2(\bar{k}_2) \frac{1}{\omega_2 + k_2} [s_3 k + \omega_2 (s_1 \mu \ell_1 + s_2 \ell_2)] +$$

$$+ A_2 f'_2(\bar{k}_2) [(s_1 \mu \ell_1 + s_2 \ell_2) + \frac{\omega_2}{k_1 - k_2} (-s_1 + \mu s_2) (\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1)]$$

or

$$\frac{di}{d\omega_2} = \frac{A_2 f'_2(\bar{k}_2)}{\omega_2 + k_2} [-s_3 k - \omega_2 (s_1 \mu \ell_1 + s_2 \ell_2) +$$

$$+ (\omega_2 + k_2) (s_1 \mu \ell_1 + s_2 \ell_2)] +$$

$$+ A_2 f'_2(\bar{k}_2) \frac{\omega_2}{k_1 - k_2} (\mu s_2 - s_1) (\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1)$$

or finally

$$\frac{di}{d\omega_2} = \frac{A_2}{\omega_2 + k_2} f'_2(\bar{k}_2) [k_2 (s_1 \mu \ell_1 + s_2 \ell_2) - s_3 k] +$$

$$+ \frac{A_2 \omega_2}{k_1 - k_2} f'_2(\bar{k}_2) (\mu s_2 - s_1) (\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1) \qquad (3.4.19)$$

In our model we assume that all income classes invest part of their incomes. Therefore, we cannot be so sure that an increase in the wage-rental ratios will decrease the per capita investment. We need further behavioural or technical restrictions. Thus, sufficient conditions for (3.4.19) to be negative, under the general capital intensity condition of $k_1 - k_2 < 0$, are:

$$\text{(i)} \quad \mu s_2 - s_1 \geq 0$$

$$\text{(ii)} \quad \frac{k}{k_2} \geq \frac{s_1 \mu \ell_1 + s_2 \ell_2}{s_3} \qquad (3.4.20)$$

(3.4.20i) does not only show that $s_1 / s_2 \leqslant 1$, which we have already assumed in A4.

but that the relative level of s_1 compared with that of s_2 must be less than $\mu < 1$. (3.4.20ii) shows that under the capital intensity hypothesis ($k_1 < k < k_2$) it is $s_1 \mu \ell_1 + s_2 \ell_2 \leqslant s_3$. Of course, in the usual case of the Uzawa type models where $\mu = 1$, $s_1 = s_2 = s_W < s_3 = s_I$, (3.4.20) relations reduce to the very simple one

$$1 > \frac{k}{k_2} \geqslant \frac{s_\omega}{s_I} \tag{3.4.21}$$

We will try now to investigate the per capita investment of the economy but starting from the (3.3.77) and not the (3.3.75) function. We have:

$$i = B_2 \ell_2 f_2(\bar{k}_2) - c_2 \tag{3.4.22}$$

Adding up (3.2.49) for $j = 1, 2$ and (3.2.51) and dividing by L we have:

$$c_2 = \frac{C_{21} + C_{22} + C_{23}}{L} = m_{21}(1 - s_1) w_1 \ell_1 + m_{22}(1 - s_2) w_2 \ell_2 + m_{23}(1 - s_3) r k$$

$$\text{or} \quad c_2 = A_2 f'_2(\bar{k}_2)(m_{21}(1 - s_1)\omega_1 \ell_1 + m_{22}(1 - s_2)\omega_2 \ell_2 + m_{23}(1 - s_3) k) \tag{3.4.23}$$

Substituting (3.4.23) into (3.4.22) we have

$$i = A_2 f'_2(\bar{k}_2) \left[\frac{B_2}{A_2} \ell_2 \frac{f_2(\bar{k}_2)}{f'_2(\bar{k}_2)} - (m_{21}(1 - s_1)\omega_1 \ell_1 + m_{22}(1 - s_2)\omega_2 \ell_2 + m_{23}(1 - s_3) k \right]$$

Substituting now (3.3.81), (3.3.30) and (3.3.14) into the last expression we obtain

$$i = A_2 f'_2(\bar{k}_2) [\ell_2(\omega_2 + k_2) - (m_{21}(1 - s_1)\omega_1 \ell_1 + m_{22}(1 - s_2)\omega_2 \ell_2 + m_{23}(1 - s_3) k]$$

or

$$i = A_2 f'_2(\bar{k}_2) [\ell_2 k_2 - m_{21}(1 - s_1)\omega_1 \ell_1 + (1 - m_{22}(1 - s_2))\omega_2 \ell_2 - m_{23}(1 - s_3) k] \tag{3.4.24}$$

Differentiating (3.4.24) with respect to ω_2 we have

$$\frac{di}{d\omega_2} = A_2 f''_2(\bar{k}_2) \frac{d\bar{k}_2}{d\bar{\omega}_2} \frac{d\bar{\omega}_2}{d\omega_2} [\ell_2 k_2 - m_{21}(1-s_1)\omega_1 \ell_1 + (1 - m_{22}(1-s_2))\omega_2 \ell_2$$

$$-m_{23}(1-s_3)k] + A_2 f'_2(\bar{k}_2) [\frac{d\ell_2}{d\omega_2} k_2 + \ell_2 \frac{dk_2}{d\omega_2} -$$

$$- m_{21}(1-s_1)\mu(\ell_1 + \omega_1 \frac{d\ell_1}{d\omega_1}) + (1 - m_{22}(1-s_2))(\ell_2 + \omega_2 \frac{d\ell_2}{d\omega_2})]$$

Substituting (3.4.24), (3.4.4), (3.3.30) and (3.3.40) into the last expression we have

$$\frac{di}{d\omega_2} = A_2 f''_2(\bar{k}_2) \left(-\frac{[f'_2(\bar{k}_2)]^2}{f_2(\bar{k}_2) f''_2(\bar{k}_2)}\right) \frac{A_2}{B_2} \frac{i}{A_2 f'_2(\bar{k}_2)} +$$

$$+ A_2 f'_2(\bar{k}_2) [(1 - m_{22}(1-s_2))\omega_2 \frac{d\ell_2}{d\omega_2} - m_{21}(1 - s_1)\omega_2 \frac{d\ell_1}{d\omega_1} +$$

$$+ ((1-m_{22}(1-s_2))\ell_2 - m_{21}(1-s_1)\mu \ell_1)' + \ell_2 \frac{dk_2}{d\omega_2} +$$

$$+ k_2 \frac{\mu}{k_1 - k_2} \left(\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1\right)']$$

or

$$\frac{di}{d\omega_2} = -\frac{i}{\omega_2 + k_2} + A_2 f'_2 \left(\frac{A_2}{B_2} k_2\right) [(1-m_{22}(1-s_2))\omega_2 \frac{d\ell_2}{d\omega_2} -$$

$$- m_{21}(1-s_1)\omega_2 \frac{d\ell_1}{d\omega_1} + ((1-m_{22}(1-s_2))\ell_2 - m_{21}(1-s_1)\mu\ell_1)' +$$

$$+ \frac{1}{k_1 - k_2} [\mu k_2 \ell_1 \frac{dk_1}{d\omega_1} + k_1 \ell_2 \frac{dk_2}{d\omega_2}]] \qquad (3.4.25)$$

Under the capital intensity condition $k_1 - k_2 < 0$, a sufficient condition for (3.4.25) to be negative is

$$\frac{\ell_1}{\ell_2} \geq \frac{1 - m_{22}(1 - s_2)}{m_{21}(1 - s_1)\mu} \qquad (3.4.26)$$

(vi) We will try now to analyze the effect of a change in the wage-rental ratio upon the gross output per head, sectoral and total. We have

$$q_i = \frac{Y_i}{L} = \frac{Y_i}{L_i}\,\frac{L_i}{L} = y_i\,\ell_i \qquad i = 1, 2 \tag{3.4.27}$$

For i = 1 it is

$$q_1 = B_1 f_1(\bar{k}_1)\,\frac{k - k_2}{k_1 - k_2} \tag{3.4.28}$$

and for i = 2 it is

$$q_2 = B_2 f_2(\bar{k}_2)\,\frac{k_1 - k}{k_1 - k_2} \tag{3.4.29}$$

Taking logs of (3.4.28) and (3.4.29) and after differentiating with respect to ω_1 and ω_2 respectively we derive that:

$$\frac{1}{q_1}\,\frac{dq_1}{d\omega_1} = \frac{1}{f_1(\bar{k})}\,f_1'(\bar{k}_1)\,\frac{d\bar{k}_1}{d\bar{\omega}_1}\,\frac{d\bar{\omega}_1}{d\omega_1} + \frac{1}{k - k_2}\left(-\frac{dk_2}{d\omega_2}\,\frac{d\omega_2}{d\omega_1}\right) -$$

$$-\frac{1}{k_1 - k_2}\left(\frac{dk_1}{d\omega_1} - \frac{dk_2}{d\omega_2}\,\frac{d\omega_2}{d\omega_1}\right) \tag{3.4.30}$$

and

$$\frac{1}{q_2}\,\frac{dq_2}{d\omega_2} = \frac{1}{f_2(\bar{k}_2)}\,f_2'(\bar{k}_2)\,\frac{d\bar{k}_2}{d\bar{\omega}_2}\,\frac{d\bar{\omega}_2}{d\omega_2} + \frac{1}{k_1 - k}\left(\frac{dk_1}{d\omega_1}\,\frac{d\omega_1}{d\omega_2}\right) -$$

$$-\frac{1}{k_1 - k_2}\left(\frac{dk_1}{d\omega_1}\,\frac{d\omega_1}{d\omega_2} - \frac{dk_2}{d\omega_2}\right) \tag{3.4.31}$$

Substituting (3.3.81), (3.4.4), (3.3.30) and (3 3.31) into (3 4.30) and (3.4.31) we obtain:

$$\frac{1}{q_1}\,\frac{dq_1}{d\omega_1} = \frac{-(\omega_1 + k_2)}{(\omega_1 + k_1)(k_1 - k_2)}\,\frac{dk_1}{d\omega_1} + \frac{1}{\mu}\,\frac{k - k_1}{(k - k_2)(k_1 - k_2)}\,\frac{dk_2}{d\omega_2}$$

(3.4.32)

and

$$\frac{1}{q_2}\,\frac{dq_2}{d\omega_2} = \mu\,\frac{k - k_2}{(k_1 - k)(k_1 - k_2)}\,\frac{dk_1}{d\omega_1} + \frac{\omega_2 + k_1}{(\omega_2 + k_2)(k_1 - k_2)}\,\frac{dk_2}{d\omega_2} \tag{3.4.33}$$

From (3.4.32) and (3.4.33) we see that under the capital intensive hypothesis, $k_1 < k < k_2$, we have that

$$\frac{1}{q_1}\frac{dq_1}{d\omega_1} > 0 \qquad \text{and} \qquad \frac{1}{q_2}\frac{dq_2}{d\omega_2} < 0 \tag{3.4.34}$$

Things are not so easy when we want to find the effect of a change in ω_2 upon the total gross output per head. From (3.2.55) we have that

$$q = \frac{Y}{L} = p\frac{Y_1}{L} + \frac{Y_2}{L} = pq_1 + q_2 \tag{3.4.35}$$

Differentiating (3.4.35) we have

$$\frac{dq}{d\omega_2} = \frac{dp}{d\omega_2} q_1 + p\frac{dq_1}{d\omega_1}\frac{d\omega_1}{d\omega_2} + \frac{dq_2}{d\omega_2} \tag{3.4.36}$$

Substituting (3.4.8), (3.4.32), (3.4.33) and (3.4.35) into (3.4.36) we obtain

$$\frac{dq}{d\omega_2} = (q-q_2)\frac{\mu k_2 - k_1}{(k_2+\omega_2)(k_1+\omega_1)} +$$

$$+\mu(q-q_2)\left[\frac{-(\omega_1+k_2)}{(\omega_1+k_1)(k_1-k_2)}\frac{dk_1}{d\omega_1} + \frac{1}{\mu}\frac{k-k_1}{(k-k_2)(k_1-k_2)}\frac{dk_2}{d\omega_2}\right]$$

$$+q_2\left[\mu\frac{k-k_2}{(k_1-k)(k_1-k_2)}\frac{dk_1}{d\omega_1} + \frac{\omega_2+k_1}{(\omega_2+k_2)(k_1-k_2)}\frac{dk_2}{d\omega_2}\right]$$

or

$$\frac{dq}{d\omega_2} = (q-q_2)\frac{\mu k_2 - k_1}{(k_2+\omega_2)(k_1+\omega_1)} +$$

$$+\frac{dk_1}{d\omega_1}\mu\left[(q-q_2)\frac{-(\omega_1+k_2)}{(\omega_1+k_1)(k_1-k_2)} + q_2\frac{k-k_2}{(k_1-k)(k_1-k_2)}\right] +$$

$$+\frac{dk_2}{d\omega_2}\left[(q-q_2)\frac{k-k_1}{(k-k_2)(k_1-k_2)} + q_2\frac{\omega_2+k_1}{(\omega_2+k_2)(k_1-k_2)}\right]$$

or

$$\frac{dq}{d\omega_2} = (q-q_2)\frac{\mu k_2 - k_1}{(k_2+\omega_2)(k_1+\omega_1)} +$$

$$+\frac{dk_1}{d\omega_1}\ \frac{\mu}{(\omega_1+k_1)}\ [-q\ \frac{\omega_1+k_2}{k_1-k_2}+q_2\ \frac{\omega_1+k}{k_1-k}]\ +$$

$$+\frac{dk_2}{d\omega_2}\ \frac{1}{k-k_2}\ [\ q\ \frac{k-k_1}{k_1-k_2}+q_2\ \frac{\omega_2+k}{\omega_2+k_2}\ ] \qquad (3.4.37)$$

Under the capital intensity conditions, $\mu k_2 - k_1 \geqslant 0$ and $k_1 < k < k_2$, a sufficient condition for (3.4.37) to be positive is:

$$\frac{q_2}{q} \leqslant \frac{k_1-k}{k_1-k_2}\ \frac{\omega_2+k_2}{\omega_2+k} \qquad (3.4.38)$$

Therefore, the gross national product per capita is an increasing function of the wage-rental ratio if the sufficient conditions of capital intensities and that of (3.4.38) are satisfied.

We will try now to find sufficient conditions for the gross national product per capita to be an increasing function of the wage-rental ratio but working with different variables. Therefore, we have that

$$q = pq_1 + q_2 = \frac{A_2}{A_1}\ \frac{f_2'(\bar{k}_2)}{f_1'(\bar{k}_1)}\ B_1 f_1(\bar{k}_1)\ell_1 + B_2 f_2(\bar{k}_2)\ell_2$$

or

$$q = A_2 f_2'(\bar{k}_2)\ [\ \frac{B_1}{A_1}\ \frac{f_1(\bar{k}_1)}{f_1'(\bar{k}_1)}\ \ell_1 + \frac{B_2}{A_2}\ \frac{f_2(\bar{k}_2)}{f_2'(\bar{k}_2)}\ell_2\ ]$$

Substituting (3.3.81), (3.3.14), (3.3.30) and (3.3.16) into the last expression we have

$$q = \frac{A_2 f_2'(\bar{k}_2)}{k_1-k_2}\ [\ (\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)\ ] \qquad (3.4.39)$$

We take the logs of (3.4.39) and after differentiate with respect to ω_2, ie.:

$$\frac{1}{q}\ \frac{dq}{d\omega_2} = \frac{1}{f_2'(\bar{k}_2)} \cdot f_2''(\bar{k}_2)\ \frac{d\bar{k}_2}{d\bar{\omega}_2}\ \frac{d\bar{\omega}_2}{d\omega_2} - \frac{1}{k_1-k_2}\ (\frac{dk_1}{d\omega_1}\ \frac{d\omega_1}{d\omega_2} - \frac{dk_2}{d\omega_2})$$

$$+\frac{1}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\ [\ (\ \frac{d\omega_1}{d\omega_2}+\frac{dk_1}{d\omega_1}\ \frac{d\omega_1}{d\omega_2}\ )(k-k_2)$$

$$+(\omega_1+k_1)\left(-\frac{dk_2}{d\omega_2}\right)+\left(1+\frac{dk_2}{d\omega_2}\right)(k_1-k)+(\omega_2+k_2)\frac{dk_1}{d\omega_1}\frac{d\omega_1}{d\omega_2}\Big]$$

or

$$\frac{1}{q}\frac{dq}{d\omega_2}=-\frac{1}{\omega_2+k_2}-\frac{1}{k_1-k_2}\left(\frac{dk_1}{d\omega_1}\mu-\frac{dk_2}{d\omega_2}\right)$$

$$+\frac{1}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\Big[\left(1+\frac{dk_1}{d\omega_1}\right)\mu(k-k_2)-$$

$$-(\omega_1+k_1)\frac{dk_2}{d\omega_2}+\left(1+\frac{dk_2}{d\omega_2}\right)(k_1-k)+(\omega_2+k_2)\frac{dk_1}{d\omega_1}\mu\Big]$$

or

$$\frac{1}{q}\frac{dq}{d\omega_2}=\Big[-\frac{1}{\omega_2+k_2}+\frac{\mu(k-k_2)+(k_1-k)}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\Big]+$$

$$+\frac{dk_1}{d\omega_1}\Big[-\frac{\mu}{k_1-k_2}+\frac{\mu(k-k_2)+\mu(\omega_2+k_2)}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\Big]$$

$$+\frac{dk_2}{d\omega_2}\Big[\frac{1}{k_1-k_2}+\frac{-(\omega_1+k_1)+(k_1-k)}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\Big]$$

or

$$\frac{1}{q}\frac{dq}{d\omega_2}=\frac{(k-k_2)(\mu k_2-k_1)}{(\omega_2+k_2)\left[(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)\right]}$$

$$+\frac{(\omega_2-\omega_1)}{(k_1-k_2)\left[(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)\right]}\Big[\mu(k-k_2)\frac{dk_1}{d\omega_1}$$

$$+(k_1-k)\frac{dk_2}{d\omega_2}\Big]$$

or

$$\frac{1}{q}\frac{dq}{d\omega_2}=\frac{1}{(\omega_1+k_1)(k-k_2)+(\omega_2+k_2)(k_1-k)}\Big[\frac{(k-k_2)(\mu k_2-k_1)}{\omega_2+k_2}+$$

$$+\frac{\dot{\omega}_2(1-\mu)}{k_1-k_2}\Big[\mu(k-k_2)\frac{dk_1}{d\omega_1}+(k_1-k)\frac{dk_2}{d\omega_2}\Big] \qquad (3.4.40)$$

A sufficient condition for (3.4.40) to be positive, under the capital intensity hypothesis, is:

$$\frac{(k-k_2)(\mu k_2-k_1)}{\omega_2+k_2} < \frac{\omega_2(\mu-1)}{k_1-k_2}\left(\mu(k-k_2)\frac{dk_1}{d\omega_1}+(k_1-k)\frac{dk_2}{d\omega_2}\right)$$

or after substitution by (3.3.94) and (3.3.16) into the last expression

$$\ell_1 k_1 \sigma_1 + \ell_2 k_2 \sigma_2 < \frac{(k_2-k)(\mu k_2-k_1)}{(\omega_2+k_2)(1-\mu)} \qquad (3.4.41)$$

In the case of wages' equilibrium $\omega_1 = \omega_2 = \omega$, ie. $\mu = 1$, (3.4.40) is equal to

$$\frac{1}{q}\frac{dq}{d\omega_2} = \frac{k_2-k}{(\omega+k_2)(\omega+k)} \qquad (3.4.42)$$

which understhe capital intensity condition $k_1 - k_2 < 0$ is always positive. This case is equivalent to that of the Uzawa type models.

(vii) Finally, we shall interpret the overall gross saving ratio, s, due to a change in ω_2.

It is:

$$s = \frac{s_3 rK + s_1 w_1 L_1 + s_2 w_2 L_2}{rK + w_1 L_1 + w_2 L_2}$$

or

$$s = \frac{s_3 k + s_1 \omega_1 \ell_1 + s_2 \omega_2 \ell_2}{k + \omega_1 \ell_1 + \omega_2 \ell_2} \qquad (3.4.43)$$

Differentiating (3.4.43) with respect to ω_2 we obtain

$$\frac{ds}{d\omega_2} = (k+\omega_1\ell_1+\omega_2\ell_2)^{-2} \cdot \Big[\Big[s_1\Big(\mu\ell_1+\omega_1\mu\frac{d\ell_1}{d\omega_1}\Big)+ s_2\Big(\ell_2+\omega_2\frac{d\ell_2}{d\omega_2}\Big)\Big](k+\omega_1\ell_1+\omega_2\ell_2) -$$

$$-(s_3k+s_1\omega_1\ell_1+s_2\omega_2\ell_2)\Big[\Big(\mu\ell_1+\omega_1\mu\frac{d\ell_1}{d\omega_1}\Big)+\Big(\ell_2+\omega_2\frac{d\ell_2}{d\omega_2}\Big)$$

or

$$\frac{ds}{d\omega_2} = (k+\omega_1 \ell_1 + \omega_2 \ell_2)^{-2} \left[k \left[\mu \ell_1 (s_1 - s_3) + \ell_2 (s_2 - s_3) \right] + \right.$$

$$+ \mu \omega_1 \frac{d\ell_1}{d\omega_1} \left[k (s_1 - s_3) + \omega_2 \ell_2 (s_1 - s_2) \right] +$$

$$\left. + \omega_2 \frac{d\ell_2}{d\omega_2} \left[k (s_2 - s_3) + \omega_1 \ell_1 (s_2 - s_1) \right] \right] \qquad (3.4.44)$$

A sufficient condition for (3.4.44) to be negative, under the capital intensity hypothesis, is:

$$\frac{k}{\omega_1 \ell_1} \leqslant \frac{s_2 - s_1}{s_3 - s_2} \qquad (3.4.45)$$

ie. under $k_1 - k_2 < 0$ and (3.4.45) an increase in the wage-rental ratio ω_2 causes a decrease in the gross saving ratio, s.

From (3.4.43) we have

$$\lim_{\omega_2 \to 0} s = s_3 \qquad (3.4.46)$$

and

$$\lim_{\omega_2 \to \infty} s = \lim_{\omega \to \infty} \frac{s_3 \frac{k}{\omega_2} + s_1 \mu \ell_1 + s_2 \ell_2}{\frac{k}{\omega_2} + \mu \ell_1 + \ell_2} = \frac{s_1 \mu \ell_1 + s_2 \ell_2}{\mu \ell_1 + \ell_2} \qquad (3.4.47)$$

Comparing (3.4.46) and (3.4.47) we cannot predict with certainty which is greater s_3 or $(s_1 \mu \ell_1 + s_2 \ell_2) / (\mu \ell_1 + \ell_2)$. So we do not know if $s(\omega_2)$ is a decreasing or an increasing function of ω_2.

In the case where $\omega_1 = \omega_2$, ie. $\mu = 1$ and $s_1 = s_2 = s_\omega$, then

$$\lim_{\omega_2 \to \infty} s = s_\omega \qquad (3.4.48)$$

In this case $s(\omega_2)$ is a decreasing function of ω_2 because we have already assumed that $s_\omega \leqslant s_3$.

From the discussion above we conclude that, given the overall capital-labour ratio, k, and the constant propensities to invest $0 < s_1 < s_2 < s_3 < 1$, the overall gross saving ratio, s, depends on factor prices, which means that we have multiple gross saving ratio for the same k and s_i, but various ω_2.

3.5 The Dynamic Economy (Stability)

We have seen in section 3.2 that the dynamic equations of our model are the (3.2.1) and (3.2.11) which we write below as:

$$\dot{L}(t) / L(t) = \eta \tag{3.5.1}$$

and

$$\dot{K}(t) = I(t) - \delta K(t) \tag{3.5.2}$$

Taking the derivative of $k = K/L$ with respect to time we have

$$\dot{k} = \frac{\dot{K}L - K\dot{L}}{L^2} = \frac{\dot{K}}{L} - \frac{K}{L}\frac{\dot{L}}{L} = \frac{\dot{K}}{L} - k\eta \tag{3.5.3}$$

From (3.5.2) by division by L we take

$$\frac{\dot{K}(t)}{L(t)} = \frac{I(t)}{L(t)} - \delta \frac{K(t)}{L(t)} = i(t) - \delta k(t)$$

and substituting the above into (3.5.3) we derive the fundamental dynamic equation

$$\dot{k} = i - (\delta + \eta) k = i - \lambda k \tag{3.5.4}$$

where $\lambda = \delta + \eta$, or the equivalent

$$\frac{\dot{k}}{k} = \frac{i}{k} - \lambda \tag{3.5.5}$$

Let k^* be a trajectory of the capital-labour ratio such that

$$\frac{i(\omega_2^*)}{k^*} = \lambda \tag{3.5.6}$$

where ω_2^* is the appropriate trajectory of the wage-rental ratio given by (3.3.90) for $k = k^*$. Such a trajectory k^* may be referred

to as the trajectory of the "balanced capital-labour ratio".

We will now see under which conditions this trajectory of the balanced capital-labour ratio is uniquely determined. We define

$$\varphi(\omega_2) = \frac{i(\omega_2)}{k(\omega_2)} \tag{3.5.7}$$

where $i(\omega_2)$ is given in (3.3.75) or in (3.4.24) and $k(\omega_2)$ in (3.3.90).

Differentiating (3.5.7), after transforming it into logs, we have

$$\ln \varphi(\omega_2) = \ln i(\omega_2) - \ln k(\omega_2)$$

and

$$\frac{1}{\varphi(\omega_2)} \frac{d\varphi(\omega_2)}{d\omega_2} = \frac{1}{i(\omega_2)} \frac{di(\omega_2)}{d\omega_2} - \frac{1}{k(\omega_2)} \frac{dk(\omega_2)}{d\omega_2} \tag{3.5.8}$$

In (3.5.8) $-dk/d\omega_2$ is negative under the sufficient conditions of (3.3.98) and $di/d\omega_2$ is negative under the sufficient conditions of (3.4.20) or (3.4.26). Therefore, under these sufficient conditions, we have that

$$\frac{d\varphi(\omega_2)}{d\omega_2} < 0 \ , \ \forall \omega_2 \tag{3.5.9}$$

Hence, under the sufficient conditions (3.3.98) and (3.4.20) or (3.4.26), for any λ, there exists one and only one trajectory, ω_2^* for which $\varphi(\omega_2^*) = \lambda$ and the corresponding trajectory $k^* = k(\omega_2^*)$ is uniquely determined by λ. Also, under the same sufficient conditions the relations (3.5.9) and (3.3.97) yield the following stability theorem, which is that along an arbitrary path of growth of K(t) and L(t), the capital-labour ratio k(t) asymptotically approaches the uniquely determined trajectory of the balanced capital-labour ratio k^*. The approach of k(t) to k^* is monotonic; ie. if k(0) is greater than k^*, k(t) decreasingly approaches k^*, and if k(0) is less than k^*, k(t) increasingly approaches k^*. Of course, this stability theorem applies to all the endogenous variables

of the system.

In section 3.4 we have seen that the overall gross saving ratio, s, can take multiple values with the same overall capital-labour ratio. This multiplicity of s causes multiplicity of i, which in turn is possible to produce causal indeterminacy in the model (multiple $\dot{k}$ for the same k). Therefore, the Inada derivative conditions which guarantee the existence of at least one nontrivial balanced growth path cannot assure the uniqueness of it without additional assumption. For our model these additional assumptions are the (3.3.98) and (3.4.20) or (3.4.26). Of course, indeterminacy rules out, if we adopt the keynesian saving assumption, workers and capitalists save the same proportion of income, since under that assumption the gross saving ratio is unique and given.

3.6 Structural Change and the Growth Process

In the previous sections of this chapter we discussed the static solution of our two-sector growth model, ie. under which sufficient conditions a solution of the static model exists and is unique, and also under which sufficient conditions the balanced growth path of our model is stable. We did not say anything about the viability of our model.

The definition of viability of an economy can be studied under several, but not independent, ways: Zarembka (1972). For example, first, an economy can be defined as viable if per-capita agricultural output tends to rise; second, if the labour force grows faster in urban than in rural areas; third, if per-capita disposable income increases and fourth, if industrial output grows faster than population.

Before we try to analyse the question of viability we find it necessary to define some commodities.

In (3.3.94) we have already defined the elasticity of substitution between the factors, σ_i, i = 1, 2, for the two sectors, which has been introduced by Hicks (1932), and is:

$$\sigma_i = \frac{dk_i}{d\omega_i} \quad \frac{\omega_i}{k_i} \qquad i = 1, 2 \qquad (3.6.1)$$

and which, according to (3.4.4) is also equal to

$$\sigma_i = \frac{d\bar{k}_i}{d\bar{\omega}_i} \quad \frac{\bar{\omega}_i}{\bar{k}_i} \qquad i = 1, 2 \qquad (3.6.2)$$

In the case of constant returns to scale, as in our model, the above elasticity of substitution between the factors is written, Allen (1973),

$$\sigma_i = -\frac{f'_i (f_i - \bar{k}_i f'_i)}{\bar{k}_i f_i f''_i} > 0 , \qquad i = 1, 2 \qquad (3.6.3)$$

The current elasticity of output with respect to capital input in the ith sector is given by:

$$\epsilon_{1i} = \frac{\partial Y_i}{\partial K_i} \quad \frac{K_i}{Y_i} \qquad i = 1, 2 \qquad (3.6.4)$$

and the current elasticity of output with respect to labour input in the ith sector is given by:

$$\epsilon_{2i} = \frac{\partial Y_i}{\partial L_i} \quad \frac{L_i}{Y_i} \qquad i = 1, 2 \qquad (3.6.5)$$

Substituting (3.3.22) and (3.3.23) into (3.6.4) we have that

$$\epsilon_{1i} = r'_i \frac{K_i}{Y_i} = A_i f'_i(\bar{k}_i) \frac{k_i}{y_i} \qquad i = 1, 2$$

or after substitution of (3.3.67) and (3.3.81) into the last expression, we have

$$\epsilon_{1i} = A_i k_i \frac{f'_i(\bar{k}_i)}{B_i f_i(\bar{k}_i)} = \frac{A_i}{B_i} k_i \frac{1}{\bar{\omega}_i + \bar{k}_i} = \frac{k_i}{\omega_i + k_i} \quad i = 1, 2 \qquad (3.6.6)$$

Substituting (3.3.26) and (3.3.27) into the (3.6.5) we have

$$\epsilon_{2i} = w'_i \frac{L_i}{Y_i} = B_i \left[f_i(\bar{k}_i) - \bar{k}_i f'_i(\bar{k}_i) \right] \frac{1}{y_i}$$

Substituting (3.3.67) and (3.3.81) into the last expression, we obtain

$$\epsilon_{2i} = 1 - \bar{k}_i \frac{f'_i(\bar{k}_i)}{f_i(\bar{k}_i)} = 1 - \bar{k}_i \frac{1}{\bar{\omega}_i + \bar{k}_i} = \frac{\omega_i}{\omega_i + k_i} \qquad i = 1, 2 \qquad (3.6.7)$$

From (3.6.6) and (3.6.7) we have that

$$\epsilon_{1i} + \epsilon_{2i} = 1 \qquad \text{for} \qquad i = 1, 2 \qquad (3.6.8)$$

which is Euler's theorem.

In our model technological progress is growing exogenously, as in (3.2.3) and (3.2.4). There have been some attempts to endogenise technological progress Conlisk (1968), but it is still in primitive stages of development. In our analysis we distinguish the two characteristics of technological progress, ie. the intensity of innovation and the factor-saving bias, Fei-Ranis (1964).

With the first characteristic of technological progress, the intensity of innovation, π_i, we define the increase in output due only to the passage of time and not to any changes in the quantity of the two inputs, capital and labour. Therefore, the intensity of innovation is written as

$$\pi_i(t) = \frac{1}{Y_i(t)} \frac{dY_i(t)}{dt} \qquad i = 1, 2 \qquad (3.6.9)$$

or

$$\pi_i = \frac{1}{Y_i} \left[\frac{\partial Y_i}{\partial \bar{K}_i} \frac{\partial \bar{K}_i}{\partial t} + \frac{\partial Y_i}{\partial \bar{L}_i} \frac{\partial \bar{L}_i}{\partial t} \right] \qquad i = 1, 2 \qquad (3.6.10)$$

Taking the derivatives of (3.2.5) and (3.2.6) with respect to time and substituting the results together with (3.2.22) and (3.2.32) into (3.6.10) we obtain:

$$\pi_i = \frac{1}{Y_i} \left[\bar{r}'_i (\dot{A}_i K_i + A_i \dot{K}_i) + \bar{w}'_i (\dot{B}_i L_i + B_i \dot{L}_i) \right]$$

Taking into account that $\dot{K}_i = \dot{L}_i = 0$ and substituting (3.3.20), (3.3.21), (3.3.24) and (3.3.25) into the last expression we obtain:

$$\pi_i = \frac{L_i}{Y_i} \left[f'_i(\bar{k}_i) \dot{A}_i \frac{K_i}{L_i} + \left[f_i(\bar{k}_i) - \bar{k}_i f'_i(\bar{k}_i) \right] \dot{B}_i \right]$$

or

$$\pi_i = \frac{B_i}{y_i} \left[f'_i \frac{A_i}{B_i} \lambda_{1i} k_i + (f_i - \bar{k}_i f'_i) \lambda_{2i} \right]$$

or via (3.3.67),

$$\pi_i = \left[\lambda_{1i} \bar{k}_i \frac{f'_i}{f_i} + \lambda_{2i} (1 - \bar{k}_i \frac{f'_i}{f_i}) \right]$$

Substituting (3.3.81) into the last expression we have

$$\pi_i = \lambda_{1i} \frac{\bar{k}_i}{\bar{\omega}_i + \bar{k}_i} + \lambda_{2i} (1 - \frac{\bar{k}_i}{\bar{\omega}_i + \bar{k}_i})$$

or, via (3.3.30) and (3.3.14) we have

$$\pi_i = \lambda_{1i} \frac{k_i}{\omega_i + k_i} + \lambda_{2i} \frac{\omega_i}{\omega_i + k_i}$$

which, under (3.6.6) and (3.6.7) is written

$$\pi_i(t) = \lambda_{1i} \epsilon_{1i}(t) + \lambda_{2i} \epsilon_{2i}(t) , \qquad i = 1, 2 \qquad (3.6.11)$$

From (3.6.11) it follows that the intensity of innovation in each sector is a weighted average of the rates of factor augmentation, the weights being the output elasticities of the two inputs. This result can also be found in Amano (1964), Fei-Ranis (1964).

With the second characteristic of technological progress, the factor-saving bias, λ_i, we define the proportionate rate of change in the marginal rate of factor substitution in the ith sector, Hicks (1932). Therefore it is:

$$\lambda_i(t) = \frac{\partial (\frac{\partial F_i}{\partial K_i})}{\partial t} \frac{1}{\frac{\partial F_i}{\partial K_i}} - \frac{\partial (\frac{\partial F_i}{\partial L_i})}{\partial t} \frac{1}{\frac{\partial F_i}{\partial L_i}} \qquad i = 1, 2 \qquad (3.6.12)$$

or $$\lambda_i(t) = \lambda_i^K(t) - \lambda_i^L(t) \qquad i = 1, 2 \qquad (3.6.13)$$

where the substitution is obvious.

From (3.6.13) we see that for any given capital-labour ratio at time t in the ith sector, technological progress is Hicks-neutral if $\lambda_i(t) = 0$, or $\lambda_i^K(t) = \lambda_i^L(t)$, is Hicks-labour saving if $\lambda_i(t) > 0$, or $\lambda_i^K(t) > \lambda_i^L(t)$ and is Hicks-capital saving if $\lambda_i(t) < 0$, or $\lambda_i^K(t) < \lambda_i^L(t)$.

From (3.6.13) we have that

$$\lambda_i^K = \frac{\partial r'_i}{\partial t}\ \frac{1}{r'_i} = \frac{\partial(A_i f'_i)}{\partial t}\ \frac{1}{A_i f_i'} = \frac{\dot{A}_i f'_i + A_i \dot{f}'_i}{A_i f'_i} \tag{3.6.14}$$

and

$$\lambda_i^L = \frac{\partial w'_i}{\partial t}\ \frac{1}{w'_i} = \frac{\partial (B_i (f_i - \bar{k}_i f'_i)}{\partial t}\ \frac{1}{B_i (f_i - \bar{k}_i f'_i)}$$

$$= \frac{\dot{B}_i (f_i - \bar{k}_i f'_i) + B_i (\dot{f}_i - \dot{\bar{k}}_i f'_i - \bar{k}_i \dot{f}'_i)}{B_i (f_i - \bar{k}_i f'_i)} \tag{3.6.15}$$

In the above expression we have that

$$\dot{\bar{k}}_i = \frac{\partial \bar{k}_i}{\partial t} = (\frac{\dot{A}_i}{B_i}) k_i + (\frac{A_i}{B_i}) \dot{k}_i = (\lambda_{1i} - \lambda_{2i}) \frac{A_i}{B_i} k_i = (\lambda_{1i} - \lambda_{2i}) \bar{k}_i \tag{3.6.16}$$

because $\dot{k}_i = 0$,

$$\dot{f}_i = \frac{\partial f_i}{\partial t} = \frac{\partial f_i}{\partial \bar{k}_i}\ \frac{\partial \bar{k}_i}{\partial t} = f'_i (\lambda_{1i} - \lambda_{2i}) \bar{k}_i \tag{3.6.17}$$

via (3.6.16), and

$$\dot{f}'_i = \frac{\partial f'_i}{\partial t} = \frac{\partial f'_i}{\partial \bar{k}_i}\ \frac{\partial \bar{k}_i}{\partial t} = f''_i (\lambda_{1i} - \lambda_{2i}) \bar{k}_i \tag{3.6.18}$$

via (3.6.16).

Substituting (3.6.16), (3.6.17) and (3.6.18) into (3.6.14) and (3.6.15) we obtain:

$$\lambda_i^K = \lambda_{1i} + \frac{\dot{f}'_i}{f'_i} = \lambda_{1i} + \frac{(\lambda_{1i} - \lambda_{2i}) \bar{k}_i f''_i}{f'_i} \tag{3.6.19}$$

and

$$\lambda_i^{L} = \lambda_{2i} + \frac{(\lambda_{1i}-\lambda_{2i})\,\bar{k}_i f'_i - (\lambda_{1i}-\lambda_{2i})\,\bar{k}_i f'_i - (\lambda_{1i}-\lambda_{2i})\, f''_i\, \bar{k}_i^{\,2}}{f_i - \bar{k}_i f'_i}$$

or

$$\lambda_i^{L} = \lambda_{2i} - (\lambda_{1i}-\lambda_{2i})\,\frac{f''\bar{k}_i^{\,2}}{f_i - \bar{k}_i f'_i} \qquad (3.6.20)$$

Substituting (3.6.19) and (3.6.20) back into (3.6.13) we have

$$\lambda_i = \lambda_{1i}-\lambda_{2i} + (\lambda_{1i}-\lambda_{2i})\,\bar{k}_i f''_i \left(\frac{1}{f'_i} + \frac{k_i}{f_i - \bar{k}_i f'_i}\right)$$

or

$$\lambda_i = (\lambda_{1i}-\lambda_{2i}) + (\lambda_{1i}-\lambda_{2i})\,\frac{\bar{k}_i f_i f''_i}{f'_i (f_i - \bar{k}_i f'_i)}$$

Substituting (3.6.3) into the last expression we have

$$\lambda_i = (\lambda_{1i} - \lambda_{2i}) - (\lambda_{1i}-\lambda_{2i})\,\frac{1}{\sigma_i}$$

and finally

$$\lambda_i(t) = (\lambda_{1i}-\lambda_{2i})\left(\frac{\sigma_i(t)-1}{\sigma_i(t)}\right), \qquad i = 1, 2 \qquad (3.6.21)$$

In (3.6.21) we see that given $\lambda_{1i}-\lambda_{2i}$ the sign of (3.6.21) and so the nature of the factor-bias in the ith sector depends upon the current elasticity of factor substitution in that sector. The result of (3.6.21) can be found (among others), in Fei-Ranis (1965), Kotowitz (1968), Williamson (1971) and Hamberg (1971).

We will try now to interpret the question of viability of the economy considering (i) the per capita agricultural output, (ii) the composition of output among the two sectors and finally (iii) the labour redistribution.

(i) The per capita agricultural output is given in (3.3.76) by

$$q_1(t) = B_1(t)\,.\,f_1(\bar{k}_1(t))\,.\,\ell_1(t) \qquad (3.6.22)$$

Differentiating (3.6.22) with respect to time t, after taking its logs, we have:

$$\frac{1}{q_1}\frac{dq_1}{dt} = \frac{1}{B_1}\frac{dB_1}{dt} + \frac{1}{f_1}\frac{df_1}{dt} + \frac{1}{\ell_1}\frac{d\ell_1}{dt}$$

or

$$\frac{1}{q_1}\frac{dq_1}{dt} = \frac{1}{B_1}\frac{dB_1}{dt} + \frac{1}{f_1}\frac{df_1}{d\bar{k}_1}\frac{d\bar{k}_1}{dt} + \frac{1}{\ell_1}\frac{d\ell_1}{d\omega_1}\frac{d\omega_1}{dt} \qquad (3.6.23)$$

Substituting (3.2.4) , (3.6.16) without $\dot{k}_i = 0$, and (3.3.81) into (3.6.23) we have

$$\frac{1}{q_1}\frac{dq_1}{dt} = \lambda_{21} + \frac{1}{\omega_1 + k_1}\left[(\lambda_{11} - \lambda_{21})k_1 + \dot{k}_1\right] + \frac{1}{\ell_1}\frac{d\ell_1}{d\omega_1}\frac{d\omega_1}{dt}$$

Substituting (3.6.6) into the last expression we have

$$\frac{1}{q_1}\frac{dq_1}{dt} = \lambda_{21} + \epsilon_{11}\left[(\lambda_{11} - \lambda_{21}) + \frac{\dot{k}_1}{k_1}\right] + \frac{1}{\ell_1}\frac{d\ell_1}{d\omega_1}\frac{d\omega_1}{dt}$$

or

$$\frac{1}{q_1}\frac{dq_1}{dt} = \epsilon_{11}\lambda_{11} + (1 - \epsilon_{11})\lambda_{21} + \epsilon_{11}\frac{\dot{k}_1}{k_1} + \frac{1}{\ell_1}\frac{d\ell_1}{d\omega_1}\dot{\omega}_1 \qquad (3.6.24)$$

In the last expression we have that $\epsilon_{21} = 1 - \epsilon_{11}$ and

$$\sigma_i = \frac{dk_i}{d\omega_i}\frac{\omega_i}{k_i} = \left(\frac{dk_i}{dt} \Big/ \frac{d\omega_i}{dt}\right)\frac{\omega_i}{k_i} = \frac{\dot{k}_i}{\dot{\omega}_i}\frac{\omega_i}{k_i} \quad i = 1, 2$$

or

$$\frac{\dot{\omega}_i}{\omega_i} = \frac{1}{\sigma_i}\frac{\dot{k}_i}{k_i} \qquad i = 1, 2 \qquad (3.6.25)$$

Also, from (3.4.9), after substitution by (3.6.1) we have that

$$\frac{d\ell_1}{d\omega_1} = \frac{1}{k_2 - k_1}\left[\frac{1}{\mu}\frac{dk_2}{d\omega_2}\ell_2 + \frac{dk_1}{d\omega_1}\ell_1\right] =$$

$$= \frac{1}{k_2 - k_1}(\sigma_2 k_2 \ell_2 + \sigma_1 k_1 \ell_1)\frac{1}{\omega_1} \qquad (3.6.26)$$

Substituting (3.6.25), (3.6.26) and (3.6.11) into (3.6.24) we have

$$\frac{\dot{q}_1(t)}{q_1(t)} = \pi_1(t) + [\epsilon_{11}(t)\sigma_1(t) + \frac{1}{k_2(t) - k(t)} (\sigma_2(t)k_2(t)\ell_2(t) + \sigma_1(t)k_1(t)\ell_1(t))] \frac{\dot{\omega}_1(t)}{\omega_1(t)} \quad (3.6.27)$$

(3.6.27), under the general condition of capital intensities in the two sectors, $k_1(t) < k(t) < k_2(t)$, and the condition of increase in the wage-rental ratios, is positive; it is assumed that $\lambda_{11} \geqslant 0$ and $\lambda_{21} \geqslant 0$. Unfortunately, from the interpretation of (3.6.27) we cannot draw conclusions about the type of increase of the per capita agricultural output, ie. if the increase of $q_1(t)$ is at an increasing, constant or decreasing rate.

(ii) The composition of output among the two sectors can be defined by the share of the non-agricultural output in the total output, v(t), which is:

$$v(t) = \frac{Y_2(t)}{p(t)Y_1(t) + Y_2(t)} = \frac{Y_2(t)}{Y(t)} \quad (3.6.28)$$

From (3.6.28) we have that

$$\frac{\dot{v}}{v} = \frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}}{Y} = \frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1 p + \dot{p}Y_1 + \dot{Y}_2}{Y}$$

Substituting (3.6.28) into the last expression we have

$$\frac{\dot{v}}{v} = \frac{\dot{Y}_2}{Y_2} - v \frac{\dot{Y}_1 p + \dot{p}Y_1 + \dot{Y}_2}{Y_2}$$

or

$$\frac{\dot{v}}{v} = (1 - v) \frac{\dot{Y}_2}{Y_2} - v \frac{\dot{Y}_1 p + \dot{p}Y_1}{Y_2}$$

but from (3.6.28) it is $Y_2 = \frac{v}{1-v} pY_1$, therefore the last expression is written

$$\frac{\dot{v}}{v} = (1 - v) \frac{\dot{Y}_2}{Y_2} - (1 - v) \frac{\dot{Y}_1 p + \dot{p}Y_1}{pY_1}$$

or

$$\frac{\dot{v}}{v} = (1-v) \left[\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} - \frac{\dot{p}}{p} \right] \qquad (3.6.29)$$

In (3.6.29) we will analyse the price and quantity effects; ie. for the price effects we have that

$$p(t) = \frac{A_2(t)}{A_1(t)} \frac{f'_2(\bar{k}_2(t))}{f'_1(\bar{k}_1(t))} \qquad (3.6.30)$$

and therefore, taking logs of (3.6.30) and after its derivatives we have

$$\frac{\dot{p}}{p} = \frac{\dot{A}_2}{A_2} \div \frac{\dot{A}_1}{A_1} + \frac{\dot{f}'_2}{f'_2} - \frac{\dot{f}'_1}{f'_1} \qquad (3.6.31)$$

In (3.6.31) we have that

$$\frac{\dot{f}'_i}{f'_i} = \frac{1}{f'_i} f''_i \frac{d\bar{k}_i}{d\bar{\omega}_i} \frac{d\bar{\omega}_i}{d\omega_i} \dot{\omega}_i =$$

$$= \frac{1}{f'_i} \left(- \frac{(f'_i)^2}{f_i} \right) \frac{A_i}{B_i} \dot{\omega}_i = - \frac{\omega_i}{\omega_i + k_i} \frac{\dot{\omega}_i}{\omega_i}$$

or finally

$$\frac{\dot{f}'_i}{f'_i} = -\epsilon_{2i} \frac{\dot{\omega}_i}{\omega_i} = - \frac{\epsilon_{2i}}{\sigma_i} \frac{\dot{k}_i}{k_i} \qquad i = 1, 2 \qquad (3.6.32)$$

via (3.6.7) and (3.6.25). Substituting now (3.6.32) and (3.2.3) into (3.6.31) we have

$$\frac{\dot{p}}{p} = \lambda_{12} - \lambda_{11} - \frac{\epsilon_{22}}{\sigma_2} \frac{\dot{k}_2}{k_2} + \frac{\epsilon_{21}}{\sigma_1} \frac{\dot{k}_1}{k_1}$$

or

$$\frac{\dot{p}}{p} = (\lambda_{12} - \lambda_{11}) - \epsilon_{22} \frac{\dot{\omega}_2}{\omega_2} + \epsilon_{21} \frac{\dot{\omega}_1}{\omega_1}$$

Because

$$\frac{\dot{\omega}_1}{\omega_1} = \frac{\dot{\mu}}{\mu} + \frac{\dot{\omega}_2}{\omega_2}$$

the last expression is written

$$\frac{\dot{p}}{p} = (\lambda_{12} - \lambda_{11}) + (\epsilon_{21} - \epsilon_{22}) \frac{\dot{\omega}_2}{\omega_2} + \epsilon_{21} \frac{\dot{\mu}}{\mu} \qquad (3.6.33)$$

The sign of p/p is ambigous because the signs of the first two quantities in brackets of the R.H.S. of (3.6.33) are ambiguous.

In an equilibrium model, where $\lambda_{12} = \lambda_{11}$ and $\omega_1 = \omega_2$, Kelley-Williamson-Cheetham (1972), (3.6.33) is equal to

$$\frac{\dot{p}}{p} = (\epsilon_{12} - \epsilon_{11}) \frac{\omega_2}{\omega_2} \qquad (3.6.34)$$

Since the non-agricultural sector is more capital intensive than the agricultural sector, $\epsilon_{11} < \epsilon_{12}$ and thus, with rising wage-rental ratios the relative price of agricultural to non-agricultural products is always rising.

In our more general case of (3.6.33) we have the same result like (3.6.34) if we assume that $\lambda_{12} \geq \lambda_{11}$, ie. if we assume that the rate of growth of technological progress of capital is greater in the non-agricultural sector than that of the agricultural sector.

For the quantity effects of (3.6.29) we have from (3.6.11) that

$$\frac{\dot{Y}_i}{Y_i} = \pi_i + \frac{1}{Y_i} [\, \bar{r}_i' A_i \dot{K}_i + \bar{w}_i' B_i \dot{L}_i \,]$$

or

$$\frac{\dot{Y}_i}{Y_i} = \pi_i + \frac{L_i}{Y_i} [\, A_i f_i' \frac{\dot{K}_i}{L_i} + B_i (f_i - \bar{k}_i f_i') \frac{\dot{L}_i}{L_i} \,]$$

or

$$\frac{\dot{Y}_i}{Y_i} = \pi_i + \frac{A_i}{B_i} \frac{f_i'}{f_i} \frac{\dot{K}_i}{K_i} \frac{K_i}{L_i} + (1 - \bar{k}_i \frac{f_i'}{f_i}) \frac{\dot{L}_i}{L_i}$$

or

$$\frac{\dot{Y}_i}{Y_i} = \pi_i + \frac{A_i}{B_i} \frac{k_i}{\bar{\omega}_i + \bar{k}_i} \frac{\dot{K}_i}{K_i} + (1 - \frac{\bar{k}_i}{\bar{\omega}_i + \bar{k}_i}) \frac{\dot{L}_i}{L_i}$$

$$= \pi_i + \frac{k_i}{\omega_i + k_i} \left(\frac{\dot{K}_i}{K_i} - \frac{\dot{L}_i}{L_i} \right) + \frac{\dot{L}_i}{L_i}$$

and finally

$$\frac{\dot{Y}_i}{Y_i} = \pi_i + \epsilon_{1i} \frac{\dot{k}_i}{k_i} + \frac{\dot{L}_i}{L_i} = \pi_i + \epsilon_{1i} \sigma_i \frac{\dot{\omega}_i}{\omega_i} + \frac{\dot{L}_i}{L_i} \qquad i = 1, 2 \qquad (3.6.35)$$

via (3.6.25), (3.6.6), (3.3.81) and (3.3.67).

From (3.6.35) we obtain the quantity effect to be

$$\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} = (\pi_2 - \pi_1) + \left(\epsilon_{12}\sigma_2 \frac{\dot{\omega}_2}{\omega_2} - \epsilon_{11}\sigma_1 \frac{\dot{\omega}_1}{\omega_1}\right) + \left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right)$$

or

$$\frac{\dot{Y}_2}{Y_2} - \frac{\dot{Y}_1}{Y_1} = (\pi_2 - \pi_1) + (\epsilon_{12}\sigma_2 - \epsilon_{11}\sigma_1)\frac{\dot{\omega}_2}{\omega_2} - \epsilon_{11}\sigma_1 \frac{\dot{\mu}}{\mu} + \left(\frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}\right) \qquad (3.6.36)$$

Unfortunately, we cannot draw general conclusions about the nature of the quantity effect because the sign of (3.6.36) is ambiguous.

Substituting (3.6.33) and (3.6.36) into (3.6.29) we derive the growth rate of the share of the non-agricultural output with respect to the total output. ie.

$$\frac{\dot{v}}{v} = (1 - v) \left[\lambda_{12}\epsilon_{12} + \lambda_{22}\epsilon_{22} - \lambda_{11}\epsilon_{11} - \lambda_{21}\epsilon_{21} - \lambda_{12} + \lambda_{11} + (\epsilon_{12}\sigma_2 - \epsilon_{11}\sigma_1 - \epsilon_{21} + \epsilon_{22})\frac{\dot{\omega}_2}{\omega_2} - (\epsilon_{11}\sigma_1 + \epsilon_{21})\frac{\dot{\mu}}{\mu} + \frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1} \right]$$

or

$$\frac{\dot{v}}{v} = (1 - v) \left[(\lambda_{22} - \lambda_{12})\epsilon_{22} + (\lambda_{11} - \lambda_{21})\epsilon_{21} + [\epsilon_{12}(\sigma_2 - 1) - \epsilon_{11}(\sigma_1 - 1)]\frac{\dot{\omega}_2}{\omega_2} - [\epsilon_{11}(\sigma_1 - 1) + 1]\frac{\dot{\mu}}{\mu} + \right.$$

$$+ \frac{\dot{L}_2}{L_2} - \frac{\dot{L}_1}{L_1}] \qquad (3.6.37)$$

(3.6.37) is too complicated to derive general conclusions about the share of non-agricultural output in the total output of the economy. Even for the simpler case of equilibrium in the labour market the corresponding expression to (3.6.37) is still ambiguous for deriving general results (see Kelley-Williamson-Cheetham (1972)). The only thing that we should add here is the behaviour of the elasticities of output with respect to input in accordance to the elasticity of factor substitution. Therefore, dividing (3.6.5) by (3.6.4) we have

$$\frac{\epsilon_{2i}}{\epsilon_{1i}} = \omega_i \quad \frac{1}{k_i} \qquad i = 1, 2$$

or

$$\omega_i = \frac{\epsilon_{2i}}{\epsilon_{1i}} \; k_i \qquad i = 1, 2 \qquad (3.6.38)$$

Taking the logs of (3.6.38) and after its derivative with respect to time we have that

$$\frac{\dot{\omega}_i}{\omega_i} = \frac{\dot{k}_i}{k_i} + \frac{\dot{\epsilon}_{2i}}{\epsilon_{2i}} - \frac{\dot{\epsilon}_{1i}}{\epsilon_{1i}} \qquad i = 1, 2$$

which according to (3.6.25) is written

$$\frac{\dot{k}_i}{k_i} + \frac{\dot{\epsilon}_{2i}}{\epsilon_{2i}} - \frac{\dot{\epsilon}_{1i}}{\epsilon_{1i}} = \frac{1}{\sigma_i} \quad \frac{\dot{k}_i}{k_i} \qquad i = 1, 2$$

or

$$(1 - \frac{1}{\sigma_i}) \frac{\dot{k}_i}{k_i} = \frac{\dot{\epsilon}_{1i}}{\epsilon_{1i}} + \frac{\dot{\epsilon}_{1i}}{1 - \epsilon_{1i}}$$

and finally

$$\frac{\dot{\epsilon}_{1i}}{\epsilon_{1i}} = (1 - \epsilon_{1i}) \quad \frac{\sigma_i - 1}{\sigma_i} \quad \frac{\dot{k}_i}{k_i} \qquad i = 1, 2 \qquad (3.6.39)$$

From (3.6.39) we see that $\epsilon_{1i}(t)$ is an increasing function of time, with increasing capital-labour ratio, in the case where $\sigma_i > 1$.

(iii) Finally, we will try to analyse the redistribution of labour between the two sectors, ie. to find the behaviour of the migration process from the agricultural to non-agricultural sector.

It is known that $\omega_1 = \mu\omega_2$, from which we have that

$$\frac{\dot{\omega}_1}{\omega_1} = \frac{\dot{\mu}}{\mu} + \frac{\dot{\omega}_2}{\omega_2} \qquad (3.6.40)$$

We substitute (3.6.25) into (3.6.40)

$$\frac{1}{\sigma_1}\ \frac{\dot{k}_1}{k_1} = \frac{\dot{\mu}}{\mu} + \frac{1}{\sigma_2}\ \frac{\dot{k}_2}{k_2}$$

or

$$\frac{1}{\sigma_1}\ \left(\frac{\dot{K}_1}{K_1} - \frac{\dot{L}_1}{L_1}\right) = \frac{\dot{\mu}}{\mu} + \frac{1}{\sigma_2}\ \left(\frac{\dot{K}_2}{K_2} - \frac{\dot{L}_2}{L_2}\right) \qquad (3.6.41)$$

Recall from chapter 2 that

$$\frac{\dot{L}_1(t)}{L_1(t)} = \eta_1(t) - m_1(t) \qquad (3.6.42)$$

and

$$\frac{\dot{L}_2(t)}{L_2(t)} = \eta_2(t) + m_1(t)\frac{\ell_1(t)}{\ell_2(t)} \qquad (3.6.43)$$

Substituting (3.6.42) and (3.6.43) into (3.6.41) we have

$$\frac{1}{\sigma_1(t)}\ \left(\frac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t) + m_1(t)\right) = \frac{\dot{\mu}(t)}{\mu(t)} + \frac{1}{\sigma_2(t)}\left(\frac{\dot{K}_2(t)}{K_2(t)} - \right.$$

$$\left. - \eta_2(t) - m_1(t)\frac{\ell_1(t)}{\ell_2(t)}\right) \qquad (3.6.44)$$

or

$$m_1(t) = \ell_2(t)\ \frac{\dfrac{\dot{\mu}(t)}{\mu(t)} + \dfrac{1}{\sigma_2(t)}\left(\dfrac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t)\right) - \dfrac{1}{\sigma_1(t)}\left(\dfrac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t)\right)}{\dfrac{\ell_2(t)}{\sigma_1(t)} + \dfrac{\ell_1(t)}{\sigma_2(t)}} \qquad (3.6.45)$$

Substituting $\dot{\mu}(t) / \mu(t)$ from (3.6.33) into (3.6.44) we have

$$\frac{1}{\sigma_1(t)} \left(\frac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t) + m_1(t) \right) = \frac{1}{\epsilon_{21}(t)} \frac{\dot{p}(t)}{p(t)} - \frac{\lambda_{12} - \lambda_{11}}{\epsilon_{21}(t)} +$$

$$+ \frac{\epsilon_{22}(t)}{\epsilon_{21}(t)} \frac{1}{\sigma_2(t)} \left(\frac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t) - m_1(t) \frac{\ell_1(t)}{\ell_2(t)} \right)$$

or

$$\left(\frac{1}{\sigma_1(t)} + \frac{\epsilon_{22}(t)}{\epsilon_{21}(t)} \frac{1}{\sigma_2(t)} \frac{\ell_1(t)}{\ell_2(t)} \right) m_1(t) = \frac{1}{\epsilon_{21}(t)} \cdot \frac{p(t)}{p(t)} - \frac{\lambda_{12} - \lambda_{11}}{\epsilon_{21}(t)} +$$

$$+ \frac{\epsilon_{22}(t)}{\epsilon_{21}(t)} \frac{1}{\sigma_2(t)} \left(\frac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t) \right) - \frac{1}{\sigma_1(t)} \left(\frac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t) \right)$$

and finally

$$m_1(t) = \ell_2(t) \frac{\frac{\dot{p}(t)}{p(t)} + \lambda_{11} - \lambda_{12} + \frac{\epsilon_{22}(t)}{\sigma_2(t)} \left(\frac{\dot{K}_2(t)}{K_2(t)} - \eta_2(t) \right) - \frac{\epsilon_{21}(t)}{\sigma_1(t)} \left(\frac{\dot{K}_1(t)}{K_1(t)} - \eta_1(t) \right)}{\frac{\epsilon_{21}(t)}{\sigma_1(t)} \ell_2(t) + \frac{\epsilon_{22}(t)}{\sigma_2(t)} \ell_1(t)} \qquad (3.6.46)$$

(3.6.45) and (3.6.46) are two alternative expressions of the rate of migration from the agricultural sector.

In (3.6.46) we see that the rate of migration depends upon the existing labour allocation, the elasticities of factor substitution, the elasticities of outputs with respect to inputs, the rates of technological progress, the rates of exogenous physiological labour change and the rates of change of the relative price and capital in the two sectors. We see that an increase in the capital of the second sector produces a positive effect on migration whilst on the contrary an increase in the capital of the first sector produces a negative effect on migration. Also, increases in the physiological rates of fertility of labour produce positive and negative effects for the first and the second sector respectively. In the same way the rate of technological progress of capital in the first sector has a positive effect on migration

whilst that of the second sector has a negative effect, as was expected. The behaviour of terms of trade is ambiguous, as we have seen in (3.6.33), therefore we cannot conclude anything about its effect on migration. We should add here that even in the case where we knew the sign of $\dot{p}(t) / p(t)$ we shouldn't derive any conclusions for migration using (3.6.46) only. For example, suppose that $\dot{p}(t) / p(t)$ is positive, a usual assumption for developing economies. This positive sign means that the terms of trade are in favour of the agricultural goods. But, from (3.6.29) we see that this positive sign in $\dot{p}(t) / p(t)$ has a negative effect on the share of output of non-agricultural goods in the output of the total economy. Therefore, this relative reduction of v(t) can produce a different allocation of capital between the two sectors, which as we have seen above causes opposite effects on migration.

Finally, using (3.6.45) we see that as $t \to \infty$, or as the economy becomes more developed, $\dot{\mu}(t) / \mu(t) \to 0$, which means that the effect of wages differential on migration is decreasing over time.

3.7 Conclusions

The basic characteristics of our model is the introduction of capital into the agricultural sector, the notion of wage differential between the two sectors which causes labour reallocation, the different savings behaviour between the three income classes, namely the two worker classes and the capitalists, and the introduction of the technological progress of different rates into general production functions.

Kelley-Williamson-Cheetham (1972), argue that the industrial sector must be more capital intensive than the agricultural sector. Our model with its sufficient condition, $k_1 - k_2 < 0$, for existence of the momentary solution agrees with the above authors.

Eckaus (1955) has argued that in underdeveloped economies the production process

in the secondary industry is more capital intensive than in agriculture, which, together with the differences in elasticities of factor substitution, gives rise to the phenomenon of technological dualism. The above authors assume that $0 < \sigma_2 < 1$ and $1 \leqslant \sigma_1 < \infty$ so that $\sigma_1 > \sigma_2$. In our model the sufficient conditions for uniqueness (3.3.98a) agree with the above writers. Moreover, in our model it is possible to be $\sigma_1 \leqslant \sigma_2$ but at the same time we must add some restrictions which can be seen in (3.3.98).

At the same sufficient conditions, (3.3.98), we have that $\sigma_1 + \sigma_2 \geqslant 1$. This is a result obtained by Hahn (1965) in his two sector growth model of one consumption-good and one capital-good sectors with classical saving function.

In similarity with the latter model, Uzawa (1961), Shinkai (1960) and Inada (1963) found that the consumption-good sector must be more capital intensive than the capital-good sector and that Takayama (1963), Drandakis (1963) and Amano (1965) that as long as the elasticity of factor substitution in at least one sector is equal to or greater than unity, the two-sector growth model is globally stable. Of course, these results are different from ours but we must take into account the different structure of our model concerning the commodity market.

The introduction of the wage differential or of the wages ratio $\mu(t)$ into our model appears relevant, as was expected, to all the sufficient conditions with labour allocation, (3.4.26), savings behaviour (3.3.98ii) and (3.4.20i) and consequently capital allocation, (3.4.20ii), and trade of goods between the two sectors, (3.4.8). Thus, from (3.4.26) we have that the ratio $L_1(t) / L_2(t)$ must have a lower limit, one of the conclusions of Chapter 2; from (3.3.98ii) and (3.4.20i) we have that $s_2 > s_1$ which is true if we remember that the workers of the second sector are in higher income level than the workers in the first sector; from (3.4.20ii) we see that in the allocation of capital enters both the savings

behaviour and the wages differential which constitute the determinants of investment; and finally from (3.4.8) we see the critical importance of $\mu(t)$ in determining the behaviour of the terms of trade.

In analysing the structural change and the growth process through time of our model we concluded that our economy is viable in the case where the non-agricultural sector is more capital intensive than the agricultural sector and the intensity of innovation $\pi_1(t)$ is positive, with increasing the wage-rental ratios through time. Also, we found that the migration rate $m_1(t)$ depends on most of the variables and parameters of our model, with the expected sign, and that $m_1(t)$ is a decreasing function of the wage-differential as $t \to \infty$.

If in our model we put that $\mu(t)$ is constant and that there is no capital in the agricultural sector, then our model, after the appropriate changes in the savings function, behaves as the "conventional" theories of dual development, Niko (1974). If now, we put that $\mu(t) = 1$ and assume that the output of the second sector is completely invested in both sectors, then our model behaves as the two-sector growth models for the "advanced" economies, Sato (1969). Therefore, in spite of the more complicated results of our model, compared with those of other writers, we believe that the present model is more general and tries to bridge the gap between the models for the developed and the less-developed economies.

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CHAPTER 4

A SPECIFIC TWO-SECTOR GROWTH MODEL FOR A DEVELOPING ECONOMY

4.1 Introduction
4.2 The Cobb-Douglas Case
4.3 The Static Solution of the Model for the Cobb-Douglas Case
4.4 The Time Path Solution for the Cobb-Douglas Case
4.5 The Long-run Solution for the Cobb-Douglas Case
4.6 Conclusions
4.7 References

4.1 Introduction

It seems to us that in the early models of economic development (in the fifties and early sixties) the most popular functions in the various models, classical or neoclassical were those of the Cobb-Douglas type. Throughout the development of this part of economic theory these "simple", in mathematical terms, models have recently been generalized in various ways. Therefore, we believe that a comparison between the models of the major contributors to development theory and more recent ones is going to be a little "unfair" to the former, if we do not reduce the latter models to common units of comparison, say to Cobb-Douglas production functions.

This is one of the reasons why we found the explicit solution of the general two-sector growth model of chapter three using Cobb-Douglas production functions. A second reason is that we will try to find actual estimates of the various parameters of this model, something which is really missing in the theory of development, using data for the Greek economy.

In section 4.2 we introduce the model with the specific Cobb-Douglas production functions. Following the steps of the general model, we find the static solution of the present model in section 4.3. In section 4.4 we present the time path solution of the model for the case where $\mu(t)$ is constant over time. This is because the fundamental differential equation of our model (ie. the capital accumulation equation) cannot be solved in closed form. A similar difficulty can also be found in Jorgenson's (1961) model, due to the presence of some other variables, and not of $\mu(t)$ which is taken to be constant over time. After an investigation of the time path solution in this section, as regards the behaviour of the variables and the notion of balanced growth, we present the long-run solution of the model in section 4.5. By the term long-run we mean that $\mu(t)$ is not constant but follows

an exogenous path through time. Fianlly, in the same section we investigate the similarities between the time-path and the long-run solution of the model.

4.2 The Cobb-Douglas Case

In this section we will try to present the two-sector growth model of Chapter 3 using Cobb-Douglas type production functions. Therefore, adopting the same notation as in chapter 3, the two production functions of the economy are:

$$Y_i(t) = \bar{K}_i(t)^{\alpha_i}\ \bar{L}_i(t)^{\beta_i} \qquad i = 1, 2 \tag{4.2.1}$$

where α_i, β_i are constants.

Substituting (3.2.5) and (3.2.6) into (4.2.1) we get:

$$Y_i(t) = [A_i(t)\,K_i(t)\,]^{\alpha_i}\,[B_i(t)\,L_i(t)]^{\beta_i}$$

$$= [A_i(0)^{\alpha_i} B_i(0)^{\beta_i}]\, e^{(\alpha_i\lambda_{1i} + \beta_i\lambda_{2i})\,t}\ K_i(t)^{\alpha_i}\,L_i(t)^{\beta_i}$$

Recalling that for Cobb-Douglas production functions $\epsilon_{1i} = \alpha_i$ and $\epsilon_{2i} = \beta_i$ for $i = 1, 2$, we substitute (3.6.11) into the expression above, ie:

$$Y_i(t) = \Gamma_i\, e^{\pi_i t}\ K_i(t)^{\alpha_i}\ L_i(t)^{\beta_i},\ i = 1, 2 \tag{4.2.2}$$

where $\Gamma_i = A_i(0)^{\alpha_i} B_i(0)^{\beta_i}$, $i = 1, 2$

(4.2.2) is written in per capita terms, $(\alpha_i + \beta_i = 1)$, ie:

$$y_i(t) = \Gamma_i\, e^{\pi_i t}\ k_i(t)^{\alpha_i},\ i = 1, 2 \tag{4.2.3}$$

From (3.3.67) and (4.2.1) we obtain the equivalent forms for the production function as:

$$f_i(\bar{k}_i) = \bar{k}_i^{\ \alpha_i} \qquad i = 1, 2 \tag{4.2.4}$$

Using (4.2.4) we will present the two-sector growth model in the form of the per capita equations of (3.3.67) to (3.3.77). Therefore, substituting (4.2.4) into (3.3.70) gives us the wage-rental ratio:

$$\omega_i(t) = \frac{B_i}{A_i}\left(\frac{\bar{k}_i^{\alpha_i}}{\alpha_i \bar{k}_i^{\alpha_i - 1}} - \frac{A_i}{B_i} k_i\right) =$$

$$= \frac{B_i}{A_i} \bar{k}_i \left(\frac{1}{\alpha_i} - 1\right) = \frac{\beta_i}{\alpha_i} k_i \qquad i = 1, 2 \tag{4.2.5}$$

From (3.3.72) we have the terms of trade:

$$p(t) = \frac{A_2}{A_1} \frac{\alpha_2 \bar{k}_2^{\alpha_2 - 1}}{\alpha_1 \bar{k}_1^{\alpha_1 - 1}} = \frac{A_2}{A_1} \frac{\alpha_2}{\alpha_1} \frac{A_2^{\alpha_2 - 1} B_1^{\alpha_1 - 1}}{B_2^{\alpha_2 - 1} A_1^{\alpha_1 - 1}} \frac{k_2^{\alpha_2 - 1}}{k_1^{\alpha_1 - 1}}$$

$$= \frac{\alpha_2}{\alpha_1} \frac{A_2(0)^{\alpha_2} B_2(0)^{\beta_2}}{A_1(0)^{\alpha_1} B_1(0)^{\beta_1}} e^{[(\alpha_2 \lambda_{12} + \beta_2 \lambda_{22}) - (\alpha_1 \lambda_{11} + \beta_1 \lambda_{21})]t} \frac{k_1^{\beta_1}}{k_2^{\beta_2}}$$

or

$$p(t) = \frac{\alpha_2}{\alpha_1} \frac{\Gamma_2}{\Gamma_1} e^{(\pi_2 - \pi_1)t} \frac{k_1(t)^{\beta_1}}{k_2(t)^{\beta_2}} \tag{4.2.6}$$

From (3.3.73) we have the per capita consumption of the workers:

$$c_{ij} = m_{ij}(1 - s_j)\alpha_i \bar{k}_i^{\alpha_i - 1} A_i \omega_j \qquad \begin{matrix} i = 1, 2 \\ j = 1, 2 \end{matrix}$$

Substituting (4.2.5) into the last expression yields

$$c_{ij} = m_{ij}(1 - s_j)\alpha_i \frac{\beta_j}{\alpha_j} A_i^{\alpha_i} B_i^{\beta_i} k_j k_i^{\alpha_i - 1}$$

or

$$c_{ij}(t) = m_{ij}(1 - s_j) \frac{\alpha_i \beta_j}{\alpha_j} \Gamma_i e^{\pi_i t} k_j k_i^{\alpha_i - 1} \qquad \begin{matrix} i = 1, 2 \\ j = 1, 2 \end{matrix} \tag{4.2.7}$$

while (3.3.74) gives us the per capita consumption of the capitalists, that is:

$$c_{i3} = m_{i3}(1 - s_3)\alpha_i \bar{k}_i^{\alpha_i - 1} A_i k$$

$$= m_{i3}(1 - s_3)\alpha_i A_i^{\alpha_i} B_i^{\beta_i} k_i^{\alpha_i - 1} k$$

or

$$c_{i3} = m_{i3}(1 - s_3)\alpha_i \Gamma_i e^{\pi_i t} k_i^{\alpha_i - 1} k, \quad i = 1, 2 \tag{4.2.8}$$

(3.3.75) yields the per capita investment for the total economy:

$$i = A_2 \alpha_2 \bar{k}_2^{\alpha_2 - 1} (s_3 k + \omega_2 (s_2 \ell_2 + \mu s_1 \ell_1))$$

or substituting (4.2.5) into the expression above:

$$i = \alpha_2 \Gamma_2 e^{\pi_2 t} k_2^{\alpha_2 - 1} (s_3 k + \frac{\beta_2}{\alpha_2} k_2 (s_2 \ell_2 + \mu s_1 \ell_1)) \quad (4.2.9)$$

For the total per capita consumption, of the agricultural and non-agricultural goods, we have from (3.3.76) and (3.3.77) that:

$$c_1 = B_1 \ell_1 \bar{k}_1^{\alpha_1} = \Gamma_1 \ell_1 e^{\pi_1 t} k_1^{\alpha_1} \quad (4.2.10)$$

and

$$c_2 = B_2 \ell_2 \bar{k}_2^{\alpha_2} - i \quad \text{or}$$

$$c_2 = \Gamma_2 e^{\pi_2 t} k_2^{\alpha_2} [\ell_2 - \alpha_2 k_2^{-1} (s_3 k + \frac{\beta_2}{\alpha_2} k_2 (s_2 \ell_2 + \mu s_1 \ell_1))] \quad (4.2.11)$$

Equations (4.2.3), (4.2.5), (4.2.6), (4.2.7), (4.2.8), (4.2.9), (4.2.10) and (4.2.11) together with the equations (3.3.68), (3.3.69) and (3.3.71) constitute the two-sector growth model in per capita terms with the Cobb-Douglas type production functions.

4.3 The Static Solution of the Model for the Cobb-Douglas Case

In section 4.2 all the endogenous variables of the system can be written as functions of the wage-rental ratio $\omega_2(t)$. From the general model of chapter 3, and especially from equation (3.3.91), we see that the overall capital-labour ratio, k(t), can also be written as a function of $\omega_2(t)$. Therefore, if the function of the Cobb-Douglas type model the corresponding to function (3.3.91) is inversable, then the wage-rental ratio $\omega_2(t)$ can be written as a function of the exogenous overall capital-labour ratio k(t). Thus, all the endogenous variables of the model can be expressed as functions of the exogenous variable k(t).

We will find now the function $\omega_2 = \omega_2(k)$ for the Cobb-Douglas case. The numerator, N, in (3.3.91), after substitution from (4.2.5) can be written:

$$N = (m_{11}(1-s_1)-1)\mu\omega_2\frac{\alpha_2}{\beta_2}\omega_2 - m_{12}(1-s_2)\omega_2\frac{\alpha_1}{\beta_1}\mu\omega_2 - \frac{\alpha_2}{\beta_2}\omega_2\frac{\alpha_1}{\beta_1}\mu\omega_2 =$$

$$= [(m_{11}(1-s_1)-1)\frac{\alpha_2}{\beta_2} - m_{12}(1-s_2)\frac{\alpha_1}{\beta_1} - \frac{\alpha_1\alpha_2}{\beta_1\beta_2}]\mu(t)\omega_2{}^2(t) \qquad (4.3.1)$$

and the denominator, D, in (3.3.91), after substitution from (4.2.5) as:

$$D = (m_{11}(1-s_1)-1)\mu\omega_2 - m_{12}(1-s_2)\omega_2 + (m_{13}(1-s_3)-1)\frac{\alpha_1}{\beta_1}\mu\omega_2 -$$

$$- m_{13}(1-s_3)\frac{\alpha_2}{\beta_2}\omega_2$$

or

$$D = [[(m_{11}(1-s_1)-1) + (m_{13}(1-s_3)-1)\frac{\alpha_1}{\beta_1}]\mu(t) -$$

$$- [m_{12}(1-s_2) + m_{13}(1-s_3)\frac{\alpha_2}{\beta_2}]]\omega_2(t) \qquad (4.3.2)$$

Dividing (4.3.1) by (4.3.2) we obtain:

$$k(t) = \frac{\Delta_1\mu(t)}{\Delta_2\mu(t)-\Delta_3}\omega_2(t) \qquad (4.3.3)$$

where $\Delta_1 = (m_{11}(1-s_1)-\frac{1}{\beta_1})\frac{\alpha_2}{\beta_2} - m_{12}(1-s_2)\frac{\alpha_1}{\beta_1}$ (4.3.4)

$$\Delta_2 = (m_{11}(1-s_1)-\frac{1}{\beta_1}) + m_{13}(1-s_3)\frac{\alpha_1}{\beta_1} \qquad (4.3.5)$$

and $\Delta_3 = m_{12}(1-s_2) + m_{13}(1-s_3)\frac{\alpha_2}{\beta_2}$ (4.3.6)

From (4.3.3) we find that:

$$\omega_2(t) = \frac{\Delta_2\mu(t)-\Delta_3}{\Delta_1\mu(t)}k(t) \qquad (4.3.7)$$

Substituting (4.3.7) into (4.2.5) we obtain for the capital/labour ratios for each sector:

$$k_1(t) = \frac{\alpha_1}{\beta_1}\frac{\Delta_2\mu(t)-\Delta_3}{\Delta_1}k(t) \qquad (4.3.8)$$

$$k_2(t) = \frac{\alpha_2}{\beta_2}\frac{\Delta_2\mu(t)-\Delta_3}{\Delta_1\mu(t)}k(t) \qquad (4.3.9)$$

The proportions of labour engaged in each sector can be found by substitution of (4.3.3) and (4.2.5) into (3.3.16). Thus:

$$\ell_1(t) = \frac{k(t) - k_2(t)}{k_1(t) - k_2(t)} = \frac{\dfrac{\Delta_1 \mu(t)}{\Delta_2 \mu(t) - \Delta_3} - \dfrac{\alpha_2}{\beta_2}}{\dfrac{\alpha_1}{\beta_1}\mu(t) - \dfrac{\alpha_2}{\beta_2}} =$$

$$= \frac{(\Delta_1 \beta_2 - \Delta_2 \alpha_2)\mu(t) + \Delta_3 \alpha_2}{\beta_2 (\Delta_2 \mu(t) - \Delta_3)(\frac{\alpha_1}{\beta_1}\mu(t) - \frac{\alpha_2}{\beta_2})} \qquad (4.3.10)$$

But from (4.3.4), (4.3.5) and (4.3.6) we have:

$$\Delta_1 \beta_2 - \Delta_2 \alpha_2 = -m_{12}(1 - s_2)\frac{\alpha_1}{\beta_1}\beta_2 - m_{13}(1 - s_3)\frac{\alpha_1}{\beta_1}\alpha_2 =$$

$$= -(m_{12}(1 - s_2) + m_{13}(1 - s_3)\frac{\alpha_2}{\beta_2})\frac{\alpha_1}{\beta_1}\beta_2$$

$$= -\Delta_3 \frac{\alpha_1}{\beta_1}\beta_2$$

Therefore, (4 3 10) is written

$$\ell_1(t) = \frac{\Delta_3 \dfrac{\alpha_1}{\beta_1}\beta_2 \mu(t) - \Delta_3 \alpha_2}{\beta_2(\Delta_3 - \Delta_2 \mu(t))(\dfrac{\alpha_1}{\beta_1}\mu(t) - \dfrac{\alpha_2}{\beta_2})} =$$

$$= \frac{\Delta_3 \beta_2 (\dfrac{\alpha_1}{\beta_1}\mu(t) - \dfrac{\alpha_2}{\beta_2})}{\beta_2 (\Delta_3 - \Delta_2 \mu(t)(\dfrac{\alpha_1}{\beta_1}\mu(t) - \dfrac{\alpha_2}{\beta_2})}$$

and finally:

$$\ell_1(t) = \frac{\Delta_3}{\Delta_3 - \Delta_2 \mu(t)} \qquad (4\,3\,11)$$

For the proportion of labour engaged in the second sector we have:

$$\ell_2(t) = 1 - \ell_1(t) = 1 - \frac{\Delta_3}{\Delta_3 - \Delta_2 \mu(t)} = \frac{\Delta_3 - \Delta_2 \mu(t) - \Delta_3}{\Delta_3 - \Delta_2 \mu(t)}$$

or

$$\ell_2(t) = \frac{-\Delta_2 \mu(t)}{\Delta_3 - \Delta_2 \mu(t)} \qquad (4.3.12)$$

From (4.3.11) and (4.3.12) we see that the labour allocation ratio $L_1(t) / L_2(t)$ between the two sectors depends upon the wages ratio $\mu(t)$; ie.

$$\frac{L_1(t)}{L_2(t)} = \frac{\ell_1(t)}{\ell_2(t)} = -\frac{\Delta_3}{\Delta_2}\,\frac{1}{\mu(t)} \qquad (4.3.13)$$

But we have seen previously that $\mu(t)$ is an increasing function of time with an upper limit equal to one and so $\mu(t)^{-1}$ is a decreasing function of time with lower limit equal to one. Thus, from (4.3.13) we can easily deduce that the ratio $L_1(t) / L_2(t)$ is a decreasing function of time towards $-\Delta_1 / \Delta_2$, implying that the labour force is growing more rapidly in the non-agricultural than in the agricultural sector.

The sectoral output/labour ratio can be found, in terms of k(t), by substitution of (4.3.8) and (4.3.9) into (4.2.3); ie.,

$$y_1(t) = \Gamma_1 e^{\pi_1 t}\,\left[\frac{\alpha_1}{p_1}\,\frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1}\,k(t)\right]^{\alpha_1}$$

or

$$y_1(t) = \left[\frac{\Gamma_1}{\Delta_1^{\alpha_1}}\left(\frac{\alpha_1}{\beta_1}\right)^{\alpha_1}\right](\Delta_2\mu(t) - \Delta_3)^{\alpha_1} e^{\pi_1 t} k(t)^{\alpha_1} \qquad (4.3.14)$$

and

$$y_2(t) = \left[\frac{\Gamma_2}{\Delta_1^{\alpha_2}}\left(\frac{\alpha_2}{\beta_2}\right)^{\alpha_2}\right](\Delta_2\mu(t) - \Delta_3)^{\alpha_2}\,\mu(t)^{-\alpha_2} e^{\pi_2 t} k(t)^{\alpha_2} \qquad (4.3.10)$$

The relative price p(t) can be found by substitution of (4.3.8) and (4.3.9) into (4.2.6); ie.,

$$p(t) = \frac{\alpha_2}{\alpha_1}\,\frac{\Gamma_2}{\Gamma_1}\,e^{(\pi_2 - \pi_1)t}\,\frac{\left[\frac{\alpha_1}{\beta_1}\,\frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1}\,k(t)\right]^{\beta_1}}{\left[\frac{\alpha_2}{\beta_2}\,\frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1\mu(t)}\,k(t)\right]^{\beta_2}}$$

or

$$p(t) = \left[\frac{\Gamma_2}{\Gamma_1}\,\Delta_1^{\beta_2-\beta_1}\,\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}\right](\Delta_2\mu(t) - \Delta_3)^{\beta_1-\beta_2}\,\mu(t)^{\beta_2} e^{(\pi_2-\pi_1)t} k(t)^{\beta_1-\beta_2} \qquad (4.3.11)$$

In the same way, substituting k_j and k_i for i, j = 1, 2 from (4.3.8) and (4.3.9) into (4.2.7) and (4.2.8) we find the per capita consumption of the two goods for the three income classes. Substituting (4.3.9), (4.3.11) and (4.3.12) into (4.2.9) we find the overall investment/labour ratio as a function of the overall capital/labour ratio; ie.:

$$i = \alpha_2 \Gamma_2 e^{\pi_2 t} \left[\frac{\alpha_2}{\beta_2} \frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1\mu(t)} k(t)\right]^{\alpha_2 - 1} \left\{ s_3 k(t) + \right.$$

$$\left. + \frac{\beta_2}{\alpha_2} \frac{\alpha_2}{\beta_2} \frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1\mu(t)} k(t) \left[s_2 \frac{-\Delta_2\mu(t)}{\Delta_3 - \Delta_2\mu(t)} + s_1\mu(t) \frac{\Delta_3}{\Delta_3 - \Delta_2\mu(t)} \right] \right\}$$

or

$$i = \alpha_2 \Gamma_2 e^{\pi_2 t} \left(\frac{\alpha_2}{\beta_2} \frac{\Delta_2\mu(t) - \Delta_3}{\Delta_1\mu(t)}\right)^{\alpha_2 - 1} k(t)^{\alpha_2 - 1} k(t) \left(s_3 + \frac{s_2\Delta_2 - s_1\Delta_3}{\Delta_1}\right)$$

and finally:

$$i = \left[\frac{\Gamma_2}{\Delta_1^{\alpha_2}} \alpha_2^{\alpha_2} \beta_2^{\beta_2} (s_3\Delta_1 + s_2\Delta_2 - s_1\Delta_3)\right] (\Delta_2\mu(t) - \Delta_3)^{-\beta_2} \mu(t)^{\beta_2} e^{\pi_2 t} k(t)^{\alpha_2} \qquad (4.3.12)$$

or

$$i = G\,[\Delta_2\mu(t) - \Delta_3]^{-\beta_2}\, \mu(t)^{\beta_2} e^{\pi_2 t} k(t)^{\alpha_2} \qquad (4.3.13)$$

where

$$G = \frac{\Gamma_2}{\Delta_1^{\alpha_2}} \alpha_2^{\alpha_2} \beta_2^{\beta_2} (s_3\Delta_1 + s_2\Delta_2 - s_1\Delta_3)$$

The expression $\Delta_3 - \Delta_2\,\mu(t)$ enters into some of the variables found as functions of k(t) and t. We have to note that if $\Delta_3 - \Delta_2\,\mu(t) = 0$ then these variables are not defined. For example, if $\Delta_3 - \Delta_2\,\mu(t) = 0$ then $\ell_1(t)$, from (4.3.11), is equal to infinity. Therefore, a necessary condition for the existence of the static solution is $\Delta_3 - \Delta_2\,\mu(t) \neq 0$. This condition is of course another way of writing that $k_1 - k_2 \neq 0$.

4.4 The Time Path Solution for the Cobb-Douglas Case

As we have seen in the last section the variables in the momentary solution of the

model depend on the capital/labour ratio k(t). The actual path of k(t) over time can be found by solving the fundamental differential equation of the system which is (3.5.5) or after substitution from (4.3.13):

$$\dot{k}(t) = G[\Delta_2 \mu(t) - \Delta_3]^{-\beta_2} \mu(t)^{\beta_2} e^{\pi_2 t} k(t)^{\alpha_2} - \lambda k(t) \qquad (4.4.1)$$

or

$$\dot{k}(t) = -\lambda k(t) + G\Delta_2 \left[1 - \frac{\Delta_3}{\Delta_2} \frac{1}{\mu(t)}\right]^{-\beta_2} e^{\pi_2 t} k(t)^{\alpha_2} \qquad (4.4.2)$$

To solve the differential equation (4.4.2) we use the transformation:

$$k(t) = \Phi(t)^{\frac{1}{\beta_2}} \Rightarrow \dot{k}(t) = \frac{1}{\beta_2} \Phi(t)^{\frac{\alpha_2}{\beta_2}} \dot{\Phi}(t)$$

So, (4.4.2) becomes:

$$\frac{1}{\beta_2} \Phi(t)^{\frac{\alpha_2}{\beta_2}} \dot{\Phi}(t) = -\lambda \Phi(t)^{\frac{1}{\beta_2}} + G\Delta_2 \left[1 - \frac{\Delta_3}{\Delta_2} \frac{1}{\mu(t)}\right]^{-\beta_2} e^{\pi_2 t} \Phi(t)^{\frac{\alpha_2}{\beta_2}}$$

or

$$\dot{\Phi}(t) + \lambda \beta_2 \Phi(t) = G\Delta_2\beta_2 \left[1 - \frac{\Delta_3}{\Delta_2} \frac{1}{\mu(t)}\right]^{-\beta_2} e^{\pi_2 t} \qquad (4.4.3)$$

which is linear in $\Phi(t)$ and can be integrated as follows:

$$\Phi(t) = \Phi(0) e^{-\beta_2 \lambda t} + e^{-\beta_2 \lambda t} G \Delta_2 \beta_2 \int_o^t \left(1 - \frac{\Delta_3}{\Delta_2} \frac{1}{\mu(t_1)}\right)^{-\beta_2} e^{(\pi_2 + \beta_2 \lambda) t_1} dt_1 \qquad (4.4.4)$$

Unfortunately, it is impossible to evaluate the integral in (4.4.4) in closed form (except for special cases). It is necessary to expand $\left(1 - \frac{\Delta_3}{\Delta_2} \mu(t_1)^{-1}\right)^{-\beta_2}$ in a Maclaurin series taking the remainder of the series as tending to zero as $t \to \infty$. But, this means that we are computing the long-run values. To solve the integral in closed form it is necessary to assume that $\mu(t)$ is independent of time, taking the constant value of μ. We leave, for the next section, the solution of the differential equation where $\mu(t)$ is a function of time.

For $\mu(t) = \mu$ = constant, (4.4.4) is transformed to:

$$\Phi(t) = \Phi(0) e^{-\beta_2 \lambda t} + e^{-\beta_2 \lambda t} G\Delta_2\beta_2 \left(1 - \frac{\Delta_3}{\Delta_2 \mu}\right)^{-\beta_2} \int_o^t e^{(\pi_2 + \beta_2 \lambda) t_1} dt_1 \qquad (4.4.5)$$

and so

$$\Phi(t) = \Phi(0)\, e^{-\beta_2 \lambda t} + e^{-\beta_2 \lambda t}\, \frac{G\Delta_2\beta_2(1-\frac{\Delta_3}{\Delta_2\mu})^{-\beta_2}}{\pi_2 + \beta_2\lambda}\, (e^{(\pi_2\beta_2\lambda)t} - 1)$$

or substituting back from the transformation in the above we find:

$$k(t)^{\beta_2} = k(0)^{\beta_2}\, e^{-\beta_2\lambda t} + e^{-\beta_2\lambda t}\, T\, (e^{(\pi_2+\beta_2\lambda)t} - 1)$$

where

$$T = \frac{G\Delta_2\beta_2(1-\frac{\Delta_3}{\Delta_2\mu})^{-\beta_2}}{\pi_2+\beta_2\lambda}$$

and finally

$$k(t) = e^{-\lambda t}\, [\, k(0)^{\beta_2} + T(e^{(\pi_2+\beta_2\lambda)t} - 1)\,]^{\frac{1}{\beta_2}} \qquad (4.4.6)$$

Given the momentary solution of the last section, and the time path of k(t) under the assumption of the constancy of $\mu(t)$ over time, we can find by substitution the time path solution of the system. The time path solution is given by:

$$y_1(t) = T_1\, D(t)^{\frac{\alpha_1}{\beta_2}}\, e^{(\pi_1-\lambda\alpha_1)t} \qquad (4.4.7)$$

$$y_2(t) = T_2\, D(t)^{\frac{\alpha_2}{\beta_2}}\, e^{(\pi_2-\lambda\alpha_2)t} \qquad (4.4.8)$$

$$k_1(t) = T_3\, D(t)^{\frac{1}{\beta_2}}\, e^{-\lambda t} \qquad (4.4.9)$$

$$k_2(t) = T_4\, D(t)^{\frac{1}{\beta_2}}\, e^{-\lambda t} \qquad (4.4.10)$$

$$\ell_1(t) = T_5 \qquad (4.4.11)$$

$$\ell_2(t) = T_6 \qquad (4.4.12)$$

$$\omega_1(t) = T_7\, D(t)^{\frac{1}{\beta_2}}\, e^{-\lambda t} \qquad (4.4.13)$$

$$\omega_2(t) = T_8\, D(t)^{\frac{1}{\beta_2}}\, e^{-\lambda t} \qquad (4.4.14)$$

$$p(t) = T_9\, D(t)^{\frac{\beta_1-\beta_2}{\beta_2}}\, e^{(\pi_2-\pi_1-\lambda(\beta_1-\beta_2))t} \qquad (4.4.15)$$

$$i(t) = T_{10}\, D(t)^{\frac{\alpha_2}{\beta_2}}\, e^{(\pi_2-\lambda\alpha_2)t} \qquad (4.4.16)$$

The $c_{ij}(t)$ can be found in the same way. In the solution above it is:

$$D(t) = k(0)^{\beta_2} + T\,(e^{(\pi_2+\beta_2\lambda)t} - 1)$$

$$T_1 = \frac{\Gamma_1}{\Delta_1^{\alpha_1}} \left(\frac{\alpha_1}{\beta_1}\right)^{\alpha_1} (\Delta_2\mu - \Delta_3)^{\alpha_1}$$

$$T_2 = \frac{\Gamma_2}{\Delta_1^{\alpha_2}} \left(\frac{\alpha_2}{\beta_2}\right)^{\alpha_2} (\Delta_2\mu - \Delta_3)^{\alpha_2} \mu^{-\alpha_2}$$

$$T_3 = \alpha_1(\Delta_2\mu - \Delta_3)/\beta_1\Delta_1$$

$$T_4 = \alpha_2(\Delta_2\mu - \Delta_3)/\beta_2\Delta_1\mu$$

$$T_5 = \Delta_3/(\Delta_3 - \Delta_2\mu)$$

$$T_6 = -\Delta_2\mu/(\Delta_3 - \Delta_2\mu)$$

$$T_7 = (\Delta_2\mu - \Delta_3)/\Delta_1$$

$$T_8 = (\Delta_2\mu - \Delta_3)/\Delta_1\mu$$

$$T_9 = \frac{\Gamma_2}{\Gamma_1}\Delta_1^{\beta_2-\beta_1}\frac{\alpha_2^{\alpha_2}\beta_2^{\beta_2}}{\alpha_1^{\alpha_1}\beta_1^{\beta_1}}(\Delta_2\mu - \Delta_3)^{\beta_1-\beta_2}\mu^{\beta_2}$$

$$T_{10} = G(\Delta_2\mu - \Delta_3)^{-\beta_2}\mu^{\beta_2}$$

In the above time path solution of the system for the case where $\mu(t)$ is constant over time we see that the sectoral capital/labour ratios grow at the same rate over time and the proportion of labour engaged in each sector is constant over time.

The solution above is very complicated because of the constants of integration and it is not easy to interpret the behaviour of the variables over time. Thus, we will try to simplify this solution by integrating (4.4.5) on $(-\infty, t)$ instead of $(0, t)$ as suggested by Dhrymes (1962). In this way we get rid of the constant of integration by putting $k(-\infty)=0$. This is an approximation for a negligible amount of capital in time $-\infty$; we do not mean that $k(-\infty)$ is actually equal to zero, as this would cause problems about the existence of the output at time $-\infty$.

So, from (4.4.5), assuming always that $\mu(t)$ is constant over time, we have:

$$\Phi(t) = \Phi(-\infty)e^{-\beta_2 \lambda t} + e^{-\beta_2 \lambda t} T \int_{-\infty}^{t} e^{(\pi_2 + \beta_2 \lambda)t_1} d(\pi_2 + \beta_2 \lambda)t_1$$

and so

$$\Phi(t) = \Phi(-\infty)\, e^{-\beta_2 \lambda t} + e^{-\beta_2 \lambda t} T \; e^{(\pi_2 + \beta_2 \lambda)t}$$

and after using the transformation:

$$k(t)^{\beta_2} = k(-\infty)^{\beta_2} e^{-\beta_2 \lambda t} + T e^{\pi_2 t}$$

or

$$k(t) = T^{\frac{1}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.17}$$

If we now substitute (4.4.17) into the solution of the last section, we arrive at much more simple solutions (see below):

$$y_1(t) = T_1 T^{\frac{\alpha_1}{\beta_2}} e^{(\pi_1 + \frac{\alpha_1}{\beta_2}\pi_2)t} \tag{4.4.18}$$

$$y_2(t) = T_2 T^{\frac{\alpha_2}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.19}$$

$$k_1(t) = T_3 T^{\frac{1}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.20}$$

$$k_2(t) = T_4 T^{\frac{1}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.21}$$

$$\ell_1(t) = T_5 \tag{4.4.22}$$

$$\ell_2(t) = T_6 \tag{4.4.23}$$

$$\omega_1(t) = T_7 T^{\frac{1}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.24}$$

$$\omega_2(t) = T_8 T^{\frac{1}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.25}$$

$$p(t) = T_9 T^{\frac{\beta_1 - \beta_2}{\beta_2}} e^{(\frac{\beta_1}{\beta_2}\pi_2 - \pi_1)t} \tag{4.4.26}$$

$$i(t) = T_{10} T^{\frac{\alpha_2}{\beta_2}} e^{\frac{\pi_2}{\beta_2} t} \tag{4.4.27}$$

From (4.4.18) – (4.4.21) we see that a balanced growth path, in the sense of the per capita output and the per capita capital in the two sectors being constant over time, does not exist. $y_2(t)$, $k_1(t)$ and $k_2(t)$ are obviously growing at the same positive constant

rate π_2/β_2 whilst $y_1(t)$ is growing at the constant positive rate $(\pi_1 + \frac{\alpha_1}{\beta_2}\pi_2)$. Of course, from (4.4.18) – (4.4.21) we see that if there is no technological change in the two sectors ($\pi_1 = \pi_2 = 0$) then we have balanced growth in the above sense.

We could say that the system experiences balanced growth in the sense that the ratios of the capital/labour ratio and of the proportions of labour engaged in each sector are constant over time; ie.:

$$\frac{k_1(t)}{k_2(t)} = \frac{T_3}{T_4} \quad \text{and} \quad \frac{\ell_1(t)}{\ell_2(t)} = \frac{T_5}{T_6} \tag{4.4.28}$$

We must mention here that the constancy of $\ell_1(t)/\ell_2(t)$ and $k_1(t)/k_2(t)$ in (4.4.28) is a consequence of the assumption that $\mu(t)$ is constant over time. So, in the Cobb-Douglas case, the constancy of $\mu(t)$ entails constancy of $k_1(t)/k_2(t)$ and $\ell_1(t)/\ell_2(t)$ and thus balanced growth in the latter sense.

From (4.4.24) and (4.4.25) we see that the wage/rental ratios in the two sectors are always growing at the same constant positive rate π_2/β_2. This shows, as can be seen from (4.4.20) and (4.4.21), the relatively increasing scarcity of labour.

From (4.4.26) we see that the terms of trade depend upon the factor $\beta_1\pi_2/\beta_2 - \pi_1$, ie. we have:

$$p(t) \text{ is } \begin{cases} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{cases} \quad \text{wherever} \quad \frac{\beta_1}{\beta_2} \quad \text{is} \quad \begin{matrix} > \\ = \\ < \end{matrix} \quad \text{than} \frac{\pi_1}{\pi_2} \tag{4.4.29}$$

Finally, it is of interest to examine the sectoral output/capital ratios. From (4.4.18), (4.4.19), (4.4.20) and (4.4.21) we obtain that:

$$\frac{Y_1(t)}{K_1(t)} = \frac{y_1(t)}{k_1(t)} = \frac{T_1}{T_3} \, T^{-\frac{\beta_1}{\beta_2}} \, e^{(\pi_1 - \frac{\beta_1}{\beta_2}\pi_2)t} \tag{4.4.30}$$

and

$$\frac{Y_2(t)}{K_2(t)} = \frac{y_2(t)}{k_2(t)} = \frac{T_2}{T_4}\ T^{-1} \qquad (4.4.31)$$

From the above two expressions we see that the output/capital ratio in the non-agricultural sector is constant over time whilst that in the agricultural sector depends upon the factor $(\pi_1 - \beta_1 \pi_2 / \beta_2)$, ie. we have that:

$$\frac{y_1(t)}{k_1(t)} \quad \text{is} \quad \begin{cases} \text{increasing} \\ \text{constant} \\ \text{decreasing} \end{cases} \quad \text{wherever} \ \frac{\beta_1}{\beta_2} \ \text{is} \ \begin{matrix} < \\ = \\ > \end{matrix} \ \text{than} \ \frac{\pi_1}{\pi_2} \qquad (4.4.32)$$

From (4.4.29) and (4.4.32) we see that the terms of trade and the output/capital ratio in the agricultural sector move in opposite directions.

Of course, in the case where there is no technological progress ($\pi_1 = \pi_2 = 0$) the output/capital ratio is constant in the agricultural sector also.

4.5 The Long-run Solution for the Cobb-Douglas Case

In section 4.4 we have seen that we cannot evaluate the integral in the solution of the fundamental differential equation (4.4.4) in closed form. We have to expand the expression $(1 - \frac{\Delta_3}{\Delta_2}\ \frac{1}{\mu(t_1)})^{-\beta_2}$ in a Maclaurin series. Therefore, assuming that:

$$\mu(t_1) = \frac{1}{1+\beta(t_1)} = \frac{1}{1+\beta_0 e^{-gt_1}} \qquad \beta_0 > 0,\ g > 0,\ 0 < \mu(0) \leq \frac{1}{1+\beta_0} < 1 \qquad (4.5.1)$$

$$\mu(+\infty) = 1$$

we have that

$$[\,1 - \frac{\Delta_3}{\Delta_2}\ \frac{1}{\mu(t_1)}\,]^{-\beta_2} = [\,1 - \frac{\Delta_3}{\Delta_2}\,(1+\beta_0\, e^{-gt_1})\,]^{-\beta_2} =$$

$$= [\,(\frac{\Delta_2 - \Delta_3}{\Delta_2}) - \frac{\Delta_3}{\Delta_2}\,\beta_0\, e^{-gt_1}\,]^{-\beta_2} =$$

$$= \frac{\Delta_2 - \Delta_3}{\Delta_2}\ [\,1 + \gamma e^{-gt_1}\,]^{-\beta_2}$$

$$\text{where} \quad \gamma = \frac{\Delta_3\,\beta_0}{\Delta_3 - \Delta_2}$$

Analysing the expression in the brackets above, in the Maclaurin series we have:

$$(1+\gamma e^{-gt_1})^{-\beta_2} = 1 - \beta_2\,\gamma e^{-gt_1} + \frac{\beta_2(\beta_2+1)}{2!}\,\gamma^2\,e^{-2gt_1} + \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ (-1)^m\ \frac{\beta_2(\beta_2+1)\ldots\ldots(\beta_2+m+1)}{m!}\,\gamma^m\,e^{-mgt_1} + \ldots\ldots\ldots\ldots$$

Integrating term by term we obtain:

$$\int_0^t (1+\gamma e^{-gt_1})^{-\beta_2}\ e^{(\pi_2+\beta_2\lambda)t_1}\,dt_1 =$$

$$= \int_0^t [\,e^{(\pi_2+\beta_2\lambda)t_1} - \beta_2\gamma e^{(\pi_2+\beta_2\lambda-g)t_1} + \frac{\beta_2(\beta_2+1)}{2!}\,\gamma^2 e^{(\pi_2+\beta_2\lambda-2g)t_1} -$$

$$\ldots\ldots\ldots\ (-1)^m\ \frac{\beta_2(\beta_2+1)\ldots\ldots(\beta_2+m-1)}{m!}\ \gamma^m\,e^{(\pi_2+\beta_2\lambda-mg)t_1} + \ldots\ldots\ldots]\,dt_1 =$$

$$= \frac{1}{\pi_2+\beta_2\lambda}\,[\,e^{(\pi_2+\beta_2\lambda)t} - 1] - \frac{\beta_2\,\gamma}{\pi_2+\beta_2\lambda-g}\,[\,e^{(\pi_2+\beta_2\lambda-g)t} - 1] +$$

$$+ \frac{\beta_2(\beta_2+1)\,\gamma^2}{2!\,(\pi_2+\beta_2\lambda-2g)}\,[e^{(\pi_2+\beta_2\lambda-2g)t} - 1] - \ldots\ldots\ldots\ldots$$

$$\ldots\ldots\ldots\ (-1)^m\ \frac{\beta_2(\beta_2+1)\ldots\ldots(\beta_2+m-1)}{m!\,(\pi_2+\beta_2\lambda-mg)}\,[e^{(\pi_2+\beta_2\lambda-mg)t} - 1] + \ldots\ldots\ldots$$

Hence,

$$\Phi(t) = \Phi(0)\,e^{-\beta_2\lambda t} + \frac{G(\Delta_2-\Delta_3)\beta_2}{\pi_2+\beta_2\lambda}\ e^{\pi_2 t} + R(t) \qquad (4.5.2)$$

where R(t) is a remainder, defined as the sum of the appropriate infinite series. Each term of R(t) grows at a rate which is strictly less than the rate π_2, which is the "dominant" term in the expression for $\Phi(t)$. If $\pi_2 + \beta_2\lambda > g$, $R(t) \to 0$ as t increases without limit; otherwise R(t) grows over time, diminishing relative to the dominant term. We must notice here from $\Phi(0) = k(0)^{\beta_2}$ that the effects of the initial endowment of capital/labour ratio die out, absolutely, and also in relation to the dominant term, (Jorgenson, 1961).

To compute the long-run value of the rate of growth for the capital/labour ratio we note first that:

$$\dot{\Phi}(t) / \Phi(t) \to \pi_2 \quad \text{as} \quad t \to \infty$$

and from the transformation we have:

$$\frac{\dot{k}(t)}{k(t)} = \frac{1}{\beta_2} \quad \frac{\dot{\Phi}(t)}{\Phi(t)} \to \frac{\pi_2}{\beta_2} \quad \text{as} \quad t \to \infty \tag{4.5.3}$$

Given the long-run value of the rate of growth of the capital/labour ratio (4.5.3), we can compute the long-run values of the rates of growth for all the variables of the system after taking their differentials of the solution of the model in section (4.3).

Thus, for $t \to \infty$ we have that:

$$\dot{y}_1(t) / y_1(t) \to \pi_1 + \frac{\alpha_1}{\beta_2} \pi_2 \tag{4.5.4}$$

$$\dot{y}_2(t) / y_2(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.5}$$

$$\dot{k}_1(t) / k_1(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.6}$$

$$\dot{k}_2(t) / k_2(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.7}$$

$$\dot{\ell}_1(t) / \ell_1(t) \to 0 \tag{4.5.8}$$

$$\dot{\ell}_2(t) / \ell_2(t) \to 0 \tag{4.5.9}$$

$$\dot{\omega}_1(t) / \omega_1(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.10}$$

$$\dot{\omega}_2(t) / \omega_2(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.11}$$

$$\dot{p}(t) / p(t) \to \frac{\beta_1}{\beta_2} \pi_2 - \pi_1 \tag{4.5.12}$$

$$\dot{i}(t) / i(t) \to \frac{\pi_2}{\beta_2} \tag{4.5.13}$$

Comparing the rates of growth of the variables in the long-run solution, (4.5.4) – (4.5.13), with those of the solutions in which $\mu(t)$ is constant over time, (4.4.7) – (4.4.16)

or (4.4.18) – (4.4.27), we see that they are exactly the same. This result is to be expected because in the long-run $\mu(t) \to 1$, ie. to a constant, whilst in the other solutions $\mu(t)$ is constant already.

4.6 Conclusions

The main models of dualistic development which we discussed in chapter one were those of Lewis (1954) and Fei-Ranis (1964) classical approach and Jorgenson's (1961) neoclassical approach. As we said in the introduction to this chapter these models have the common feature that they use Cobb-Douglas production functions; we must note that Lewis does not do so explicitly in his article but they follow from his assumptions about the constant partial elasticity of the output function.

In the last section of the same chapter we presented Dhryme's (1962) model of multisectoral growth which also used Cobb-Douglas production functions for the different sectors.

In the present chapter we restricted ourselves to the case of the Cobb-Douglas function using the analytical solution of the general model presented in chapter three. The reason for this was twofold. First, because we want to see the similarities and the differences between our model and those of the first chapter and second because we want to apply this model to the Greek economy in the next chapters.

We believe that our model is more general than that of Jorgenson and Drymes in the sense that by introducing extra assumptions we can restrict it to these two cases. For example, if we put in the assumptions that the wages are proportional in the two sectors, that investment takes place only in the industrial sector and that only the capitalists invest, our model behaves in the same way as the Jorgenson's neoclassical model. Alternatively, by assuming that the wages are equal in the two sectors and that the output of the industrial

sector is completely invested in both sectors, then our model behaves as the Drymes ($\eta = 2$) model.

There is a major difference in the necessary conditions for existence of the static solution between the two-sector growth models for developing and advanced economies. For the developing economies we must have $k_1 - k_2 < 0$ as in Keley and Williamson and Cheetham (1972) and in our model (chapter 3), whilst for the advanced economies, $k_1 - k_2 > 0$ is required, as in the Uzawa (1961) type models (we must bear in mind the differences in the specification of the two sectors for the two kinds of model).

In our Cobb-Douglas two-sector growth model this condition is $k_1 - k_2 \neq 0$, as found in section 4.3, which can also be written as:

$$\frac{\alpha_1}{\beta_1}\ \mu(t) - \frac{\alpha_2}{\beta_2} \neq 0 \qquad (4.6.1)$$

We know that $\mu(t)$ is increasing exogenously towards one, which means that the economy is becoming more developed with the passage of time.

A closer investigation of (4.6.1) shows that if the value of $\frac{\alpha_1}{\beta_1}\ \mu(t)$ is less than $\frac{\alpha_2}{\beta_2}$ then the necessary condition is $k_1 - k_2 < 0$ (as for the developing economies) whilst if $\frac{\alpha_1}{\beta_1}\ \mu(t) > \frac{\alpha_2}{\beta_2}$ then it is $k_1 - k_2 > 0$ (as for the advanced economies). Therefore, if the economy is at a level in which $k_1 - k_2 < 0$ then, because of the increase in $\mu(t)$, it is possible to switch to the case in which $k_1 - k_2 > 0$, if, and only if, $\frac{\alpha_2}{\beta_2} / \frac{\alpha_1}{\beta_1} < 1$. The time period t^* in which the economy will switch from the first case into the second is given by, $\mu(t^*) = \frac{\alpha_2}{\beta_2} / \frac{\alpha_1}{\beta_1} < 1$.

From the above investigation of (4.6.1) we obtained a critical time t^* according to which we can implicitly decide whether an economy is advanced or developing. Of course, this time period t^* depends not only on the ratio $\frac{\alpha_2}{\beta_2} / \frac{\alpha_1}{\beta_1}$ but on the actual

expression of $\mu(t)$.

Finally, we must not take the definition of t* with respect to the level of development of an economy as being absolute. This definition alone is too arbitrary to classify economies into developed or developing ones.

4.7 References

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CHAPTER 5

THE ECONOMY OF GREECE: 1950–1972

5.1 Introduction
5.2 Population and Employment
5.3 Gross Domestic Product and Income
5.4 Consumption
5.5 Investment
5.6 The Behaviour of Prices
5.7 The Foreign Sector
5.8 Conclusions
5.9 References

5.1 Introduction

The period after 1950, which we will try to examine, immediately followed one of the most disastrous periods in the long history of Greece. The world depression had hit Greece very severely, war with Italy had been costly, invasion and occupation by Germany had been worse than humiliating, and the fractricidal civil war, which lasted until late in 1949, was bloody and divisive.

The problems that faced Greece in 1950 were immense. About 700,000 people – or as 10% of the population – had to be resettled in their damaged villages; this work ended in 1951. As a result of the civil war about one-third of the population was on relief. Much land was in poor condition and productive capacity had to be reconstructed. Inflation was rampant. Extreme balance of payments difficulties were kept in check only by a system of rigid import and foreign exchange controls, and public administration was in chaos.

Amazingly enough, by 1954 most of these problems had been brought under control so that attention could then be shifted to the even greater problem of economic development. The devaluation of the drachma in April 1953 (and its subsequent successful stabilization), and American aid which in the postwar period has amounted to over three million dollars (roughly half of which was military aid, while the other half was used mostly for resettlement and reconstruction), were undoubtedly the two most important factors in returning Greece to order, stability and to the normal prewar level of production – or even higher level by 1954. So, it would be logical to take as a starting point, from which to evaluate progress in the economic development of Greece, the year 1954, (Papandreou, 1962; Nugent, 1966).

In the following sections of this chapter we will present the data on the most

important among the variables indicative of economic growth in Greece which are relevant to our theoretical two-sector growth model of the previous chapters.

5.2 Population and Employment

The population of Greece (mid-year estimates) and its density is given in table 5.1.

TABLE 5.1

Mid-Year estimates of Population and Density

Mid-year	Population	Changes		Inhabitants per sq. km.
		Absolute numbers	%	
1950	7566028	83280	1.11	57.3
1951	7646402	80374	1.06	58.0
1952	7733250	86848	1.14	58.6
1953	7817095	83845	1.08	59.2
1954	7893412	76317	0.98	59.8
1955	7965538	72126	0.91	60.4
1956	8031013	65475	0.82	60.9
1957	8096218	65205	0.81	61.4
1958	8173129	76911	0.95	61.9
1959	8278162	85033	1.04	62.6
1960	8327405	69243	0.84	63.1
1961	8398050	70645	0.85	63.6
1962	8448233	50183	0.60	64.0
1963	8479625	31392	0.37	64.3
1964	8510429	30804	0.36	64.5
1965	8550333	39904	0.47	64.8
1966	8613651	63318	0.74	65.3
1967	8716441	102790	1.19	66.1
1968	8740765	24324	0.28	66.2
1969	8772764	31999	0.36	66.5
1970	8792806	20042	0.23	66.6
1971	8851913	59107	0.67	67.1
1972	8950000	98087	1.11	67.8

Source: ESYE, Statistical Yearbook of Greece, Athens, 1972.

From table 5.1 we see that the annual average compound growth rate of population was 0.99% in the fifties while it was 0.65% in the sixties. This reduction in the rate of

growth of population is a result of the relatively high rate of emigration and of the deliberate reduction of the number of children per family, (Table 5.3).

Population density is relatively high, more than 66 inhabitants per square km. This number is in fact misleading as an index of population density if the characteristics of the land are taken into account. Most of the land area is mountainous, and only 30% is cultivated. Moreover, a large part of the cultivated land is not very fertile, while the possibilities of expanding the area under cultivation appear very limited. To this one should also add the limitations imposed by the exceptionally dry weather, making the raising of summer crops impossible without irrigation. An indication of the population density is provided by the available data for the average farm size. There are approximately 1,000,000 farms of which 57% are between 2.5 and 12.5 acres and 27.5% are between 0.25 and 2.5 acres (Pavlopulos, 1966).

The distribution of the population between urban, semi-urban and rural, according to the censuses (1951, 1961, 1971), is given in table 5.2.

TABLE 5.2

Urban, Semi-Urban or Rural Population

Census Year	Population				%			
	Total	Urban	Semi-urban	Rural	Total	Urban	Semi-urban	Rural
1951	7632801	2879994	1130188	3622619	100.0	37.7	14.8	47.5
1961	8388553	3628105	1085856	3674592	100.0	43.3	12.9	43.8
1971	8768641	4667489	1028769	3072383	100.0	53.2	11.7	35.1

Source: ESYE, Statistical Yearbook of Greece, Athens, 1972.

From table 5.2 we see that there has been a shift of population from both semi-urban and rural towards urban areas.

In table 5.3 below we present for comparison figures on the active population, births, deaths, emigration and immigration.

TABLE 5.3

Active Population, Births, Deaths, Emigration and Immigration

Year	Active Population (thousands)	Births	Deaths	Emigration		Immigration
				Permanent	Temporary	
1950	4877	151134	53755			
1951	4952	155422	57508			
1952	5059	149637	53377			
1953	5153	143765	56680			
1954	5217	151892	55625			
1955	5282	154263	54781	29787	14465	
1956	5322	158203	59460	35349	21849	
1957	5345	155940	61664	30428	22540	
1958	5380	155359	58160	24521	16287	
1959	5429	160199	60852	23684	19999	
1960	5477	157239	60563	47768	27454	
1961	5502	150716	63955	58837	26426	
1962	5549	152158	66554	84054	26668	
1963	5569	148249	66813	100072	35437	
1964	5576	153109	69429	105569	47616	
1965	5591	151448	67269	117167	59241	
1966	5634	154613	67912	86896	61518	
1967	5695	162839	71975	42730	59732	
1968	5701	160338	73309	50866	64138	18882
1969	5702	154077	71825	91552	67123	18132
1970	5656	144928	74009	92681	70570	22665
1971	5611	141126	73819	61745	75229	24709
1972	5566	148910	76859	43397	72741	27522

Sources: 1) ESYE, Statistical Yearbook of Greece, Athens, 1972
2) OECD.

Unfortunately, the data for emigration and immigration in table 5.3 are not complete. Data on immigration has been collected only since 1968, when the Passport Control services started collecting data at the traveller's entrance in the country. The net migration can be derived as follows:

NET MIGRATION = BIRTHS–DEATHS–INCREASE IN TOTAL POPULATION

In this way the OECD constructed a series of net migration statistics for Greece from 1949. Again, unfortunately if we use the data for the most recent years, to construct such a series, we do not get the same results as OECD. This discrepancy arises because the data on "Increase in total population" from Table 5.1 are mid-year estimates whilst the figures in Table 5.3 constitute actual observations.

In table 5.3 we observe that the active population in Greece is increasing at a decreasing rate and, after 1968 decreasing absolutely. This is due to high emigration, and because the emigrants seem to have been the youngest, most enterprising and adaptable elements of the labour force. These characteristics can also be seen by examining the proportions of deaths and births per 1000 inhabitants. In 1951, 1961 and 1971 the proportions of deaths were 7.5‰, 7.6‰ and 8.3‰ and the proportions of births were 20.3‰, 17.9‰ and 15.9‰ respectively. These figures show that in spite of the drastic improvement in medical standards in Greece the death rate is slightly increasing. This is due to the emigration of the younger people whilst older people have remained at home. Another factor accounting for the decrease in the active population is the decrease in the birth rate as can be seen from the figures given above.

Unemployment in Greece according to the census data as 5.5% out of the economically active population in 1951, 5.91% in 1961 and 3.34% in 1971. Except for the seasonal unemployment in agriculture, Pepelasis and Yotopoulos (1962) estimated the disguised unemployment in agriculture for the years 1953–1960 to be about 3%. Also, Germidis and Negreponti-Delivanis (1975), adopting a method of Taylor (1972) which is based on the assumption that the difference between output per person employed and its potential level (with full employment) can be used to measure under-employment,

estimated under-employment for the years 1960–1970 to be about 7.6% of active population and for the economy as a whole. But one must say that the above figures for unemployment reflect the true situation inadequately as they have been estimated with a zero net emigration rate. If we take into account the high emigration rates then the level of unemployment is much higher than that presented above.

Of course, emigration has been a short-run solution to the unemployment problem keeping domestic unemployment from reaching alarming proportions. It has also contributed to the reduction of population pressure which in turn facilitates accumulation of savings, training and acquisition of skills abroad, but these benefits are by far out-weighed by adverse effects which may be felt strongly in the coming years, Zolotas (1965).

It is not only that a great social cost is involved in spending to develop a labour force which is not going to participate in home production, but also that the emigrants come usually from the most dynamic part of the population.

In the second place, emigration reduces significantly the importance of the abundance of labour as an incentive for the attraction of foreign enterprenuerial capital, and rapidly reducing wage differentials between Greece and the European countries.adversely affects the competitiveness of her products.

Thirdly, this results in an unfavourable age composition of the population and in serious social imbalances which, apart from the unhappiness which it gives rise to in the short-run, will affect seriously the physical increase of the population. This will lead to a labour shortage causing a serious bottleneck to the economic growth of the country in the long-run.

Fourth, emigration seems to have reached a high level, as was shown in table 5.3, while its structure is growing worse. Wage differentials are large enough to encourage

emigration. It is more than certain that such developments will be felt soon, and may prove themselves an unexpected bottleneck for the attainment of the development plan targets. There are indications that in certain areas and occupations labour shortages have already appeared (Pavlopoulos, 1966).

Table 5.4 shows the distribution of the labour force between the agricultural and non-agricultural sectors for the three census years. We observe that the distribution of the labour force between the two sectors, agricultural and non-agricultural, has moved in favour of the non-agricultural sector between the years 1951 and 1971. The ratio of the labour force in the agricultural to that of the non-agricultural sector is 1.318 for 1951, 1.168 for 1961 and 0.681 for 1971, ie. this ratio is decreasing over time, which is a fact compatible with the assumption we made about labour behaviour in the two-sector growth models of the previous chapters.

TABLE 5.4

Distribution of total labour force; 1951, 1961, 1971.

Activity	1951		1961		1971	
	Absolute numbers	%	Absolute numbers	%	Absolute numbers	%
Agriculture	1864000	56.9	1960446	53.9	1330320	40.4
Non-agriculture	1414000	43.1	1678155	46.1	1953560	59.6
Total	3278000	100.0	3638601	100.0	3283880	100.0

Sources: ESYE, Statistical Yearbooks of Greece, Athens, 1952, 1962, 1972.

Unfortunately, the figures in table 5.4 are the only existing data on labour allocation between the two sectors, agricultural and non-agricultural. In table 5.5 we present data on the industrial labour force which is a part of the non-agricultural sector. To construct the index of the labour force given in table 5.5, we used three sources with different

base years. The labour index of table 5.5 is the result of transforming all the available data according to a common base year which is taken to be 1963.

In table 5.5 we see that for the decade 1950–1960 there was little or no change in the labour-force in the industrial sector in contrast with the period after 1960 where we observe a steady increase, except for the years 1967, 1968 (years of political instability in Greece).

TABLE 5.5

Labour-force Index in the Industrial Sector

Year	Index	Year	Index
1950	91.5 [1]	1962	96.1 [3]
1951	91.5	1963	100.0
1952	92.4	1964	102.7
1953	88.8	1965	106.4
1954	92.6 [2]	1966	109.6
1955	93.5	1967	108.9
1956	91.6	1968	108.2
1957	91.6	1969	112.2
1958	93.5	1970	117.5
1959	88.8	1971	124.7
1960	90.7	1972	129.8
1961	94.5		

Sources: 1) OECD, General Statistics, Part II, Paris, 1961.
2) U.N., Statistical Yearbook, N.Y., 1962.
3) Yearbook of Labour Statistics, Geneva, 1973.

If we assume now that there is some relationship between the industrial sector and the whole non-agricultural sector as far as concerns the trend in labour force then the small increase in the percentage of the labour force in industry during the years 1950–1960 is shown by the small decrease of the labour allocation ratio among the two sectors (agricultural and non-agricultural) and the steady increase of the industrial labour force after 1960 is shown by the heavy decrease of the labour allocation ratio, as found in

table 5.4.

5.3 Gross Domestic Product and Income

The distribution of the labour force between the different activities does not of course reflect absolutely the degree of economic development without taking into account the composition of the economy's GDP.

In table 5.6 we present the distribution of the GDP between the agricultural and the non-agricultural sectors. From this table we can see that the contribution of the agricultural sector in the total GDP of the economy is diminishing in favour of the non-agricultural sector. In 1951 the share of the agricultural sector in GDP was 30.3% (in constant 1958 prices) while in 1961 and 1971 it was 27.4% and 17.4% respectively, which shows that the decrease was much faster between 1961–1971 than it was during the previous decade (1951–1961).

Comparing now the distribution of labour and the distribution of product from tables 5.4 and 5.6 we see that in 1951 the agricultural sector absorbed about 57% of the total labour force whilst its share in the gross domestic product was about 30% and for 1971 the corresponding figures were 40% and 17%. These results show that the composition of the two sectors has changed significantly but there is still room for improvement as the proportion of the total labour force engaged in agriculture is still very high.

The average annual rate of increase of the GDP for the decade 1951–1961 was 5.7% in constant prices while for the decade 1961–1971 it was 7.0%. The agricultural sector contributed to this increase with an increase of its own product by 4.6% per annum for the decade 1951–61, whilst non-agricultural product increased by 6.2%. For the decade 1961–1971 the increases were 2.4% and 8.4% respectively. These rates of

TABLE 5.6

Gross Domestic Product (in million drs.)

Year	(at current prices)					(at constant 1958 prices)				
	Agriculture		Non-agriculture		Total	Agriculture		Non-agriculture		Total
	Absolute numbers	%	Absolute numbers	%		Absolute numbers	%	Absolute numbers	%	
1950	9169	31.0	20420	69.0	29589	15250	28.8	37695	71.2	52945
1951	11002	30.7	24791	69.3	35793	17319	30.3	39818	69.7	57137
1952	10935	23.4	26310	70.6	37245	16301	28.6	40765	71.4	57066
1953	16491	33.8	32343	66.2	48834	20596	31.3	43874	68.1	64470
1954	17377	31.0	38760	69.0	56137	19974	30.1	46367	69.9	66341
1955	19747	30.6	44765	69.4	64512	21351	30.1	49486	69.9	70837
1956	22511	30.1	52195	69.9	74706	21944	29.2	53183	70.8	75127
1957	24279	30.5	55319	69.5	79598	24808	30.7	56075	69.3	80883
1958	22946	27.5	60572	72.5	83518	22946	27.5	60572	72.5	83518[
1959	22876	26.3	64049	73.7	86925	24095	27.6	63077	72.4	87172
1960	22853	24.5	70387	75.5	93240	22281	24.7	67767	75.3	90048
1961	28665	27.3	76513	72.7	105178	27246	27.4	72310	72.6	99556
1962	28246	25.4	83085	74.6	111331	25278	24.8	76587	75.2	101865
1963	31472	25.7	91168	74.3	122640	27226	24.9	82057	75.1	109283
1964	34762	25.4	102187	74.6	136949	28956	24.4	89797	75.6	118753
1965	38448	25.0	115608	75.0	154056	29726	23.1	99031	76.9	128757
1966	39942	23.6	129075	76.4	169017	29755	21.8	106585	78.2	136340
1967	42450	23.2	140156	76.8	182606	31507	21.9	112076	78.1	143583
1968	39835	20.2	156946	79.8	196781	29084	19.1	123481	80.9	152565
1969	42608	19.4	177591	80.6	220199	30536	18.3	136046	81.7	166582
1970	46700	18.9	200271	81.1	246971	33173	18.3	148303	81.7	181476
1971	51215	18.6	223686	81.4	274901	34035	17.4	161808	82.6	195843
1972	59213	18.6	259399	81.4	318612	35196	16.2	181435	83.8	216631

Sources: NAS, National Accounts of Greece, Nos. 21, 22, Athens, 1972, 1973.

increase in production in the two sectors show very clearly why the distribution of their product in the total GDP changed much more quickly in the decade 1961–71 than in the decade 1951–61.

If one considers the annual rate of increase of the GDP, Greece was one of the nine countries which were in the high growth league, their output having grown at a rate of more than 6 per cent a year since 1950. The other countries were Israel, Mexico, South Korea, Spain, Taiwan, Thailand, Venezuela and Yugoslavia. Six of these experienced accelerated growth in the 1960's and the growth rate of only one, Venezuela, fell back below 6 per cent. One other country, Malaya, joined the group in the 1960's (Maddison, 1971).

Another criterion for interpreting the development of a country is of course the development of the per capita product (or income) of the economy. Table 5.7 shows the

TABLE 5.7

Per capita GDP at factor cost

Year	at current prices		at constant 1958 prices		NOTE:
	In drachmas	In USA $	In drachmas	In USA $	30 drs=1$
1960	11197	373	10813	360	
1961	12524	417	11855	395	
1962	13178	439	12058	402	
1963	14463	482	12888	430	
1964	16092	536	13954	465	
1965	18018	601	15059	502	
1966	19622	654	15828	528	
1967	20950	698	16473	549	
1968	22512	750	17454	582	
1969	25100	837	18989	633	
1970	28080	936	20639	688	
1971	31157	1038	22197	740	
1972	35936	1198	24434	814	

Source: NAS, National Accounts of Greece, N°=22, Athens, 1973.

per capita gross domestic product at factor cost. Table 5.8 shows the per capita gross national income at factor cost and table 5.9 shows the per capita gross national product at market prices. In all these tables the figures are given in drachmas and in US dollars.

TABLE 5.8

Per capita GNI at factor cost

Year	at current prices		at constant 1958 prices	
	In drahmas	In US $	In drahmas	In US $
1960	11439	381	11061	369
1961	12808	427	12151	405
1962	13486	450	12384	413
1963	14810	494	13264	442
1964	16486	550	14367	479
1965	18442	615	15494	516
1966	20074	669	16288	543
1967	21443	715	16977	566
1968	23090	770	18036	601
1969	25667	856	19562	652
1970	28689	956	21219	707
1971	31987	1066	22991	766
1972	37024	1234	25394	846

Source: NAS, National Accounts of Greece, N°= 22, Athens, 1973.

From tables 5.7, 5.8 and 5.9 it can clearly be seen that the per capita product (or income) has increased dramatically in comparison with other developing countries. The year 1970/71 was the critical time period in which the per capita product (or income) surpassed the level of 1000 USA dollars in current prices and in 1972 the per capita GNP at market prices was $ 980; even at constant prices, not far from the $ 1000 level.

In table 5.10 we present the net national income as the sum of the agricultural income (including both wages and profits in the agricultural sector), the wages and salaries of the non-agricultural sector, and the net profits of the non-agricultural sector (obtained as a residual).

TABLE 5.9

Per capita GNP at market prices

Year	at Current prices		at constant 1958 prices	
	In drahmas	In US $	In drahmas	In US $
1960	12781	426	12352	412
1961	14364	479	13622	454
1962	15225	508	14032	468
1963	16789	560	15087	503
1964	18714	624	16402	547
1965	20973	699	17740	591
1966	23034	768	18790	626
1967	24732	824	19711	657
1968	26891	896	21185	706
1969	29928	998	23006	767
1970	33326	1111	24858	829
1971	36869	1229	26712	890
1972	42502	1417	29398	980

Source: NAS, National Accounts of Greece, N^0 = 22, Athens, 1973

From table 5.10 we see that the share of net national income received by agriculture is decreasing over time. The share of wages plus salaries and that of net profits in the non-agricultural sector, on the other hand, is increasing over time. From the same table we can also see that the share of wages plus salaries out of the net non-agricultural income is slightly increasing with that increase being a little more rapid in the second decade.

Unfortunately, there are no figures on wages in the agricultural sector. Therefore, we cannot test directly the validity of our assumptions about the wage-rates in the two sectors, ie. that there exists a gap between the two wage-rates and that the ratio of the wage-rates is increasing towards one. Instead we will try to give some indirect evidence about the truth of our assumptions.

From Maddison (1971) we have the information that for the year 1950 the per

TABLE 5.10

Net National Income (in million drs. and in current prices).

Year	Agriculture Net Income	%	Non-agriculture Wages & Salaries	%	Net Profits	%	Net National Income
1954	17120	31.8	17625	32.8	19032	35.4	53777
1955	19485	31.4	20573	33.2	21899	35.4	61957
1956	22232	30.7	24883	34.4	25256	34.9	72380
1957	23957	30.9	26025	33.6	27525	35.5	77507
1958	22548	28.0	27086	33.5	31024	38.5	80658
1959	22419	26.8	28982	34.7	32239	38.5	83640
1960	22333	24.8	31606	35.1	36013	40.1	89952
1961	28079	27.6	34405	33.8	39367	38.7	101851
1962	27589	25.6	37358	34.7	42632	39.7	107579
1963	30766	26.0	40682	34.4	46890	39.6	118338
1964	33976	25.7	46139	34.9	52107	39.4	132222
1965	37567	25.3	53588	36.0	57555	38.7	148710
1966	38918	23.9	60903	37.4	62897	38.7	162718
1967	41283	23.5	67890	38.6	66484	37.9	175657
1968	38565	20.4	75496	39.9	75186	39.7	189247
1969	41200	19.5	84279	39.9	85695	40.6	211174
1970	45201	19.1	93859	39.7	97261	41.2	236321
1971	49518	18.8	105193	39.9	108971	41.3	263682
1972	57338	18.7	121276	39.6	127581	41.7	306195

Source: NAS, National Accounts of Greece, N° = 21, 22, Athens, 1972, 1973.

capita level of output in agriculture and in non-agriculture was 223 and 1470 US 1965 $ respectively with their ratio being 0.152. These figures for the year 1965 were 493 and 2699 US 1965 $ respectively with their ratio being 0.188. Of course, these figures are far from being synonymous with wages in the two sectors, but at least they show the economic dualism between the agricultural and non-agricultural sectors and that this dualism vanishes over time.

Perhaps a better approximation to use in order to compare the incomes of the workers in the two sectors can be obtained from the table 5.11 taken from Geronymakis

(1963) and Germidis-Delivanis (1975).

TABLE 5.11

Average Annual Income by Socio-occupational Groups
(in drachmae at constant 1958 prices)

Group	1951	Index	1961	Index	1971*	Index	Percentage change 1951/61	1961/71	1951/71
Farmers	5870	100.0	14508	100.0	33371	100.0	+147	+130	+468
Self-employed persons in non-agriculture	23704	403.8	54624	376.5	123083	368.8	+130	+125	+419

Sources: Geronymakis S. (1963), *Germidis, D – Negreponti–Delivanis, M. (1975).

Comparing the average annual income of the farmers and of the self-employed persons in non-agriculture we clearly see that the income of the self-employed persons is about four times greater than that of the farmers and also, that this gap in the two incomes is decreasing over time, ie. the ratio of the farmers income to that of the self-employed persons in non-agriculture is increasing over the whole period, from 0.247 for 1951 to 0.271 for 1971.

We believe that the comparison of the incomes of the two groups above is valid for the following reasons. An individual who migrates from the agricultural to the non-agricultural sector if he does not find a job in the non-agricultural sector he remains there unemployed, or returns back to the agricultural sector, or migrates abroad, or becomes self-employed. Therefore, with the comparison above we compare in some sense incomes of the individuals before and after the migration process, ie. incomes of the farmers and the self-employed persons in the non-agricultural sector.

5.4 Consumption

Total consumption expenditure, which is presented in tables 5.12 and 5.13, forms

TABLE 5.12

Consumption and Gross Expenditure of the Economy
(In million drs. and at current prices)

Year	Private	% in total C	Government	% in total C	Total	% in total C	Gross expenditure of the economy
1950	27183	88.0	3715	12.0	30898	83.5	37012
1951	31558	86.2	5032	13.8	36590	83.7	43713
1952	33606	86.3	5334	13.7	38940	87.9	44295
1953	42292	88.3	5605	11.7	47897	84.5	56697
1954	51760	87.9	7135	12.1	58895	89.1	66080
1955	56499	87.9	7812	12.1	64311	85.7	75030
1956	64588	87.0	9649	13.0	74237	84.1	88296
1957	69303	87.7	9733	12.3	79036	83.3	94859
1958	72132	87.1	10678	12.9	82810	82.5	100387
1959	73241	86.5	11438	13.5	84679	82.3	102935
1960	79975	86.6	12366	13.4	92341	82.5	111894
1961	87586	86.8	13304	13.2	100890	79.7	126650
1962	94606	86.7	14536	13.3	109142	80.3	135890
1963	104138	86.8	15810	13.2	119948	79.8	150240
1964	113219	85.8	18668	14.2	131887	77.2	170937
1965	129378	85.8	21361	14.2	150739	76.9	195932
1966	141596	85.4	24203	14.6	165799	79.1	209582
1967	152380	84.1	28827	15.9	181207	79.8	227169
1968	164845	84.4	30378	15.6	195223	77.9	250551
1969	182064	84.1	34540	15.9	216604	77.0	281413
1970	201084	83.8	38764	16.2	239848	76.7	312705
1971	216459	83.5	42647	16.5	259106	75.1	345130
1972	244768	84.0	46734	16.0	291502	73.1	398899

Sources: NAS, National Accounts of Greece, N^0=21, 22 Athens, 1972, 1973.

the major component of gross expenditure of the economy. From these tables we see that in 1951 total consumption accounted for 83.0% of total expenditure (in constant prices) while in 1961 and 1971 it was 79.6% and 75.9% respectively. Thus, the part of gross expenditure of the economy absorbed by consumption has been continuously falling since 1951 and consequently the remaining part which constitutes the gross asset formation of the economy has been continuously increasing. This behaviour shows that the standard

TABLE 5.13

Consumption and Gross Expenditure of the Economy
(In million drs. and in constant 1958 prices)

Year	Private	% in total C	Government	%in total C	Total	% in total C	Gross expenditure of the economy
1950	47412	84.4	8735	15.6	56147	82.1	68402
1951	50330	85.3	8702	14.7	59032	83.0	71089
1952	51039	85.2	8896	14.8	59935	87.1	68830
1953	52987	86.0	8600	14.0	61587	83.1	74134
1954	59192	86.4	9306	13.6	68498	89.1	76902
1955	60325	86.4	9466	13.6	69791	86.0	81129
1956	65088	86.9	9815	13.1	74903	84.0	89198
1957	69273	87.1	10291	12.9	79564	83.2	95629
1958	72132	87.1	10678	12.9	82810	82.5	100387
1959	73622	86.8	11155	13.2	84777	82.6	102684
1960	77755	87.1	11554	12.9	89309	82.5	108275
1961	83922	87.5	12006	12.5	95928	79.6	120573
1962	89376	87.6	12609	12.4	101985	80.8	126210
1963	97244	88.2	13001	11.8	110245	80.7	136680
1964	103806	87.9	14253	12.1	118059	77.8	151726
1965	114639	88.3	15182	11.7	129821	77.0	168634
1966	119957	88.2	16039	11.8	135996	78.8	172517
1967	128562	87.9	17635	12.1	146197	79.6	183766
1968	138761	88.9	17349	11.1	156110	77.8	200759
1969	150537	88.7	19169	11.3	169706	76.8	220887
1970	161180	88.7	20444	11.3	181624	76.5	237421
1971	171789	88.9	21471	11.1	193260	75.9	254675
1972	187054	89.2	22547	10.8	209601	74.7	280733

Sources: NAS, National Accounts of Greece, N^0 = 21, 22, Athens, 1972, 1973.

of living was improving and so permitting an increase in gross asset formation during this period.

Also, from these tables we see that the part of consumption absorbed by the private sector out of total consumption is increasing over time and consequently that of the Government sector is decreasing over time, both in constant prices. The proportions above move in opposite directions in terms of current prices; ie. the part of consumption absorbed

TABLE 5.14

Composition of Private Consumption Expenditure.
(In million drs. and in current prices)

Year	.Food & Drink	%	. Tobacco	. Clothing etc.	. Housing Heating	. Durables	. Services	. Private Domestic Consumption Expenditure
1950	14027	48.5	1106	2993	5360	700	4721	28907
1951	15867	47.9	1290	3242	6419	780	5493	33091
1952	17092	47.6	1361	3344	7225	840	6026	35888
1953	21535	49.0	1427	4338	8314	1119	7178	43911
1954	25260	48.8	1576	5071	9788	1600	8451	51746
1955	27559	47.0	1904	6139	11050	2074	9949	58675
1956	31485	47.5	2176	7060	12024	2312	11207	66264
1957	33099	46.6	2368	7719	12994	2651	12173	71004
1958	34279	44.9	2611	8528	13993	2906	13956	76273
1959	35823	45.3	2741	8446	14600	2779	14774	79163
1960	36711	43.7	2872	9564	15708	3141	16035	84031
1961	40273	44.2	3117	10408	16585	3476	17233	91032
1962	41291	42.7	3289	11403	17938	3859	19007	96787
1963	45804	43.1	3628	12162	19296	4306	21121	106317
1964	49008	42.5	4014	13637	20721	4896	23004	115280
1965	54592	42.6	4708	15559	22433	5624	25270	128186
1966	59695	42.2	5309	16787	24428	6175	28930	141324
1967	62928	41.2	6068	19184	26806	6964	30893	152843
1968	65511	39.9	6463	20450	29319	7697	34744	164184
1969	69821	39.0	6814	22413	32044	8844	38870	178806
1970	74084	37.8	7303	25551	34411	10259	44375	195983
1971	81558	37.3	7762	29384	37671	12310	50231	218916
1972	90139	36.5	8419	33792	41508	14156	59093	247107

Sources: NAS, National Accounts of Greece, N° = 21, 22, Athens, 1972, 1973.

by the private sector out of total consumption is decreasing and consequently that of the Government sector is increasing over time. This behaviour of consumption in terms of current and constant prices for the private and the Government sectors shows that there is price discrimination for the consumption goods purchased by the two sectors and reflects the efforts of the various governments to keep prices down for the consumers.

The improvement in the standard of living during the period under consideration is also reflected in the composition of private consumption. Tables 5.14 and 5.15

TABLE 5.15

Composition of Private Consumption Expenditure.
(In million drs. and in constant 1958 prices).

Year	Food & Drink	%	Tobacco	Clothing etc.	Housing, Heating	Durables	Services	Private Domestic Consumption Expenditure
1950	25511	49.5	1702	5511	10094	962	7809	51589
1951	26264	49.5	1901	5417	10373	948	8178	53081
1952	26924	49.1	1929	5467	10961	970	8543	54794
1953	28313	48.6	2001	5862	11400	1203	9472	58251
1954	29873	48.9	1972	6210	11792	1607	9678	61132
1955	31311	47.9	2119	7065	12092	2053	10784	65444
1956	32585	47.4	2176	7333	12774	2238	11639	68745
1957	33947	46.7	2368	7990	13382	2619	12360	72666
1958	34279	44.9	2611	8528	13993	2906	13956	76273
1959	34898	45.2	2678	8248	14258	2714	14466	77262
1960	35369	43.8	2783	9313	15035	3067	19238	80805
1961	38095	44.2	2824	10224	15642	3839	16105	86230
1962	38559	42.4	2888	11301	16699	3768	17634	90849
1963	40918	41.9	3068	11959	17625	4193	19934	97697
1964	43135	41.1	3235	13265	18694	4823	21731	104883
1965	44903	39.8	3468	14946	20116	5492	24016	112941
1966	47118	39.4	3794	15486	21557	5793	25977	119725
1967	49118	38.6	4002	17037	23411	6478	27093	127139
1968	51371	37.7	4262	18145	25033	7200	30171	136182
1969	52149	36.1	4493	19764	27071	8327	32827	144631
1970	55035	35.2	4816	22102	28710	9368	36130	156161
1971	56909	33.8	5058	24860	31256	10510	39937	168530
1972	58867	32.5	5550	27918	33835	11180	43768	181118

Sources: NAS, National Accounts of Greece, No. 21, 22, Athens, 1972, 1973.

represent private consumption in current and in constant prices. From these tables we see that the expenditure on food and beverages absorbed 49.5% of total domestic private consumption in 1951 and 44.2% and 33.8% in 1961 and 1971 respectively, in constant 1958 prices. Therefore we see that the expenditure on food and beverages is decreasing over time, with respect to total private consumption. The same conclusions about the share of expenditure on food and beverages out of total private consumption follow from the figures in current prices.

5.5 Investment

In this section we will present total gross domestic asset formation in three ways, ie. according to the way into which it is divided between various sectors and activities. Firstly, total gross domestic asset formation is divided according to whether it is financed by the private and public sectors. Secondly, it is divided into the agricultural and non-agricultural sectors, and thridly, according to whether it is financed by domestic and foreign resources.

Tables 5.16 and 5.17 represent the first way of describing investment.

TABLE 5.16

I. Gross Asset Formation
(In million drs. and in current prices).

Year	Private	% in GAF	Public	% in GAF	Total	% in GAF	Depreciation	% in GAF
1950	3753	61.4	2361	38.6	6114	16.5	1486	24.3
1951	5271	74.0	1852	26.0	7123	16.3	1844	25.9
1952	3873	72.3	1482	27.7	5355	12.1	2044	38.2
1953	6884	78.2	1916	21.8	8800	15.5	2417	27.5
1954	5659	78.8	1526	21.2	7185	10.9	3233	45.0
1955	8195	76.5	2524	23.5	10719	14.3	3493	32.6
1956	10366	73.7	3693	26.3	14059	15.9	3847	27.4
1957	11595	73.3	4228	26.7	15823	16.7	4180	26.4
1958	12553	71.4	5024	28.6	17577	17.5	4319	24.6
1959	13169	72.1	5087	27.9	18256	17.7	4821	26.4
1960	13072	66.9	6481	33.1	19553	17.5	5302	27.1
1961	17685	68.7	8075	31.3	25760	20.3	5711	22.2
1962	17513	65.5	9235	34.5	26748	19.7	6353	23.8
1963	22751	75.1	7541	24.9	30292	20.2	7244	23.9
1964	26505	67.9	12545	32.1	39050	22.8	8077	20.7
1965	32864	72.7	12329	27.3	45193	23.1	8976	19.9
1966	31559	72.1	12224	27.9	43783	20.9	10197	23.3
1967	32595	70.9	13367	29.1	45962	20.2	11249	24.5
1968	41934	75.8	13394	24.2	55328	22.1	12576	22.7
1969	46200	71.3	18609	28.7	64809	23.0	13999	21.6
1970	51793	71.1	21064	28.9	72857	23.3	15938	21.9
1971	59211	68.8	26813	31.2	86024	24.9	18541	21.6
1972	74137	69.0	33260	31.0	107397	26.9	22058	20.5

Sources: National Accounts of Greece, Nos. 21 and 22;
Ministry of coordination, Athens, 1972, 1973.

TABLE 5.17

Gross Asset Formation
(in million drs. and in constant 1958 prices).

Year	Private	% in GAF	Public	% in GAF	Total	% in GAF	Depreciation	% in GAF
1950	7605	62.1	4650	37.9	12255	17.9	2950	24.1
1951	8969	74.4	3088	25.6	12057	17.0	3153	26.2
1952	6337	71.2	2558	28.8	8895	12.9	3302	37.1
1953	9657	77.0	2890	23.0	12547	16.9	3388	27.0
1954	6562	78.1	1842	21.9	8404	10.9	3523	41.9
1955	8619	76.0	2719	24.0	11338	14.0	3674	32.4
1956	10472	73.3	3823	26.7	14295	16.0	3898	27.3
1957	11717	72.9	4348	27.1	16065	16.8	4133	25.7
1958	12553	71.4	5024	28.6	17577	17.5	4319	24.6
1959	12968	72.4	4939	27.6	17907	17.4	4588	25.6
1960	12683	66.9	6283	33.1	18966	17.5	4915	25.9
1961	16698	67.8	7947	32.2	24645	20.4	5242	21.3
1962	15675	64.7	8550	35.3	24225	19.2	5623	23.2
1963	19623	74.2	6812	25.8	26435	19.3	6019	22.8
1964	22763	67.6	10904	32.4	33667	22.2	6561	19.5
1965	27530	70.9	11283	29.1	38813	23.0	7171	18.5
1966	26358	72.2	10163	27.8	36521	21.2	8097	22.2
1967	26207	69.8	11362	30.2	37569	20.4	8683	23.1
1968	33521	75.1	11128	24.9	44649	22.2	9351	20.9
1969	36511	71.3	14670	28.7	51181	23.2	10334	20.2
1970	38919	69.8	16878	30.2	55797	23.5	11330	20.3
1971	41815	68.1	19600	31.9	61415	24.1	12319	20.1
1972	48398	68.0	22734	32.0	71132	25.3	13432	18.9

Sources: National Accounts of Greece, Nos. 21 & 22, Ministry of Coordination, Athens, 1972, 1973.

From tables 5.16 and 5.17 (current or constant 1958 prices) we see that the overall average propensity to save (our of gross expenditure of the economy) has increased from a level of 15% to 30%! Many developing countries are very happy to achieve propensities to save of 15%, (Nugent, 1966).

The contribution of the private and public sectors to total gross asset formation was 74.4% and 25.6% respectively for 1951. In 1961 these proportions were 67.8%

TABLE 5.18

II. Gross Asset Formation
(in million drs. and in current prices)

Year	Agriculture			Non-Agriculture		
		% in GAF	% in GDP_1		% in GAF	% in GDP_2
1950	337	5.5	3.7	5777	94.5	28.3
1951	1886	26.5	17.1	5237	73.5	21.1
1952	270	5.0	2.5	5085	95.0	19.3
1953	2140	24.3	13.0	6660	75.7	20.6
1954	311	4.3	1.2	6874	95.7	17.7
1955	1971	18.4	10.0	8748	81.6	19.5
1956	1643	11.7	7.3	12416	88.3	23.8
1957	3410	21.6	14.0	12413	78.4	22.4
1958	2276	12.9	9.9	15301	87.1	25.3
1959	2798	15.3	12.2	15458	84.7	24.1
1960	2490	12.8	12.2	17055	87.2	24.2
1961	6747	26.2	23.5	19013	73.8	24.8
1962	2301	8.6	8.1	24447	91.4	29.4
1963	7197	23.8	22.9	23095	76.2	25.3
1964	2813	7.2	8.1	36237	92.8	35.5
1965	7498	16.6	19.5	37695	83.4	32.6
1966	3271	7.5	8.2	40512	92.5	31.4
1967	7211	15.7	17.0	38751	84.3	27.6
1968	5494	9.9	13.8	49834	90.1	31.7
1969	6913	10.7	16.2	57896	89.3	32.6
1970	9585	13.2	20.5	63272	86.8	31.6
1971	11682	13.6	22.8	74342	86.4	33.2
1972	11584	10.8	19.6	95813	89.2	36.9

Sources: National Accounts of Greece, Ibid.

and 32.2%, and in 1971 68.1% and 31.9% respectively for the two sectors.

From these percentages we see that the public sectors' contribution to gross asset formation increased during the period 1951–1961 and was roughly constant during 1961–1971. In accordance with this the contribution of the private sector to gross asset formation has decreased for the period 1951–1961 and was roughly the same for the period 1961–1971.

TABLE 5.19

II. Gross Asset Formation
(in million drs. and in constant 1958 prices).

Year	Agriculture			Non-Agriculture		
		% in GAF	% in GDP_1		% in GAF	% in GDP_2
1950	1338	10.9	8.8	10917	89.1	29.0
1951	3062	25.4	17.7	8995	74.6	22.6
1952	263	3.0	1.6	8632	97.0	21.2
1953	3025	24.1	14.7	9522	75.9	21.7
1954	353	4.2	1.8	8051	95.8	17.4
1955	1785	15.7	8.4	9553	84.3	19.3
1956	1755	12.3	8.0	12540	87.7	23.6
1957	3383	21.1	13.6	12682	78.9	22.6
1958	2276	12.9	9.9	15301	87.1	25.3
1959	2758	15.4	11.4	15149	84.6	24.0
1960	2582	13.6	11.6	16384	86.4	24.2
1961	6369	25.8	23.4	18276	74.2	25.3
1962	2175	9.0	8.6	22050	91.0	28.8
1963	5958	22.5	21.9	20477	77.5	25.0
1964	2592	7.7	9.0	31075	92.3	34.6
1965	6341	16.3	21.3	32472	83.7	32.8
1966	3426	9.4	11.5	33095	90.6	31.1
1967	5962	15.9	18.9	31607	84.1	28.2
1968	5124	11.5	17.6	39525	88.5	32.0
1969	5949	11.6	19.5	45232	88.4	33.2
1970	7367	13.2	22.2	48430	86.8	32.3
1971	8496	13.8	25.0	52919	86.2	32.7
1972	8203	11.5	23.3	62929	88.5	34.7

Sources: National Accounts of Greece, ibid.

Finally, from these tables we see that the proportion of depreciation out of total gross asset formation was about 29% for the period 1950–1960 and that for the period 1961–1972 was much lower, of the order of about 20%.

Tables 5.18 and 5.19 show the second way of expressing investment.

Unfortunately, from these tables we cannot see explicitly the contribution of each sector's gross asset formation to the total. On average we could say that for the period

1950–1972 the percentage of investment out of total investment for agriculture and non-agriculture was about 14.0% and 86.0% respectively.

In contrast to this, we see from table 5.19 (in constant 1958 prices) that gross asset formation in non-agriculture is steadily increasing as a percentage of its gross domestic product. For the agricultural sector for the period 1950–1960 the average percentage of gross asset formation out of this sector's gross domestic product was 9.8%; this percentage increased to 18.5% for the period 1961–1972. Therefore, we conclude that on average the percentage of investment out of GDP in agriculture is also increasing over time. A large part of the investment in agriculture was devoted to irrigation which was one of the important reasons for fast growth in the Greek agricultural sector. Irrigation increases yields, permits new seed to be used and adds greatly to the reliability of harvests. The major source of finance for irrigation was the public sector, whilst for the mechanisation of agriculture private investment was used, Maddison (1971).

Finally, table 5.20 represents the third way of expressing investment. According to this table the percentage of gross asset formation which was financed by the rest of the world out of total investment was more than 50%, in the early fifties (in current prices). This percentage fell to about 5% in the early seventies. Also, gross asset formation financed by the rest of the world as a percentage of Gross Domestic Product was over 10% in the early fifties, falling to about 2% in the early seventies. On the contrary, the percentage of gross asset formation financed by domestic sources out of total investment or GDP was drastically increased during the time period under consideration. The large amounts of foreign investment in Greece in comparison with domestic investment for the early fifties was of course, due to the weakness of the Greek economy to raise the standard of living after the war.

TABLE 5.20

III. The Finance of Gross Domestic Asset Formation
(in million drs. and in current prices)

Year	By Domestic Sources	% in GAF	% in GDP	By the rest of the World	% in GAF	% in GDP
1950	1662	27.0	5.6	4462	73.0	15.1
1951	2998	42.1	8.4	4125	57.9	11.5
1952	2512	46.9	6.7	2843	53.1	7.6
1953	7466	84.8	15.3	1334	15.2	2.7
1954	5235	72.9	9.3	1950	27.1	3.5
1955	9883	92.2	15.3	836	7.8	1.3
1956	11994	85.3	16.1	2065	14.7	2.8
1957	13911	87.9	17.5	1912	12.1	2.4
1958	14564	82.9	17.4	3013	17.1	3.6
1959	16478	90.3	19.0	1778	9.7	2.0
1960	17002	87.0	18.2	2551	13.0	2.7
1961	23159	89.9	22.0	2601	10.1	2.5
1962	23923	89.4	21.5	2825	10.6	2.5
1963	27582	91.1	22.5	2710	8.9	2.2
1964	32724	83.8	23.9	6326	16.2	4.6
1965	34798	77.0	22.6	10395	23.0	6.7
1966	39579	90.4	23.4	4204	9.6	2.5
1967	41213	89.7	22.6	4749	10.3	2.6
1968	46840	84.7	23.8	8488	15.3	4.3
1969	54063	83.4	24.6	10746	16.4	4.9
1970	63356	87.0	25.7	9501	13.0	3.8
1971	80121	93.1	29.1	5903	6.7	2.1
1972	102278	95.2	32.1	5119	4.8	1.6

Sources: National Accounts of Greece, Ibid.

5.6 The Behaviour of Prices

We can study the behaviour of prices indirectly by deriving and interpreting implicit price indexes for the two sectors. Therefore, dividing GDP expressed in current prices by GDP figures expressed in constant prices, from table 5.6, we derive the price indexes for the agricultural and non-agricultural sectors, table 5.21. In table 5.21 we present the relative price index by taking the ratio of the price indexes of the agricultural to the non-

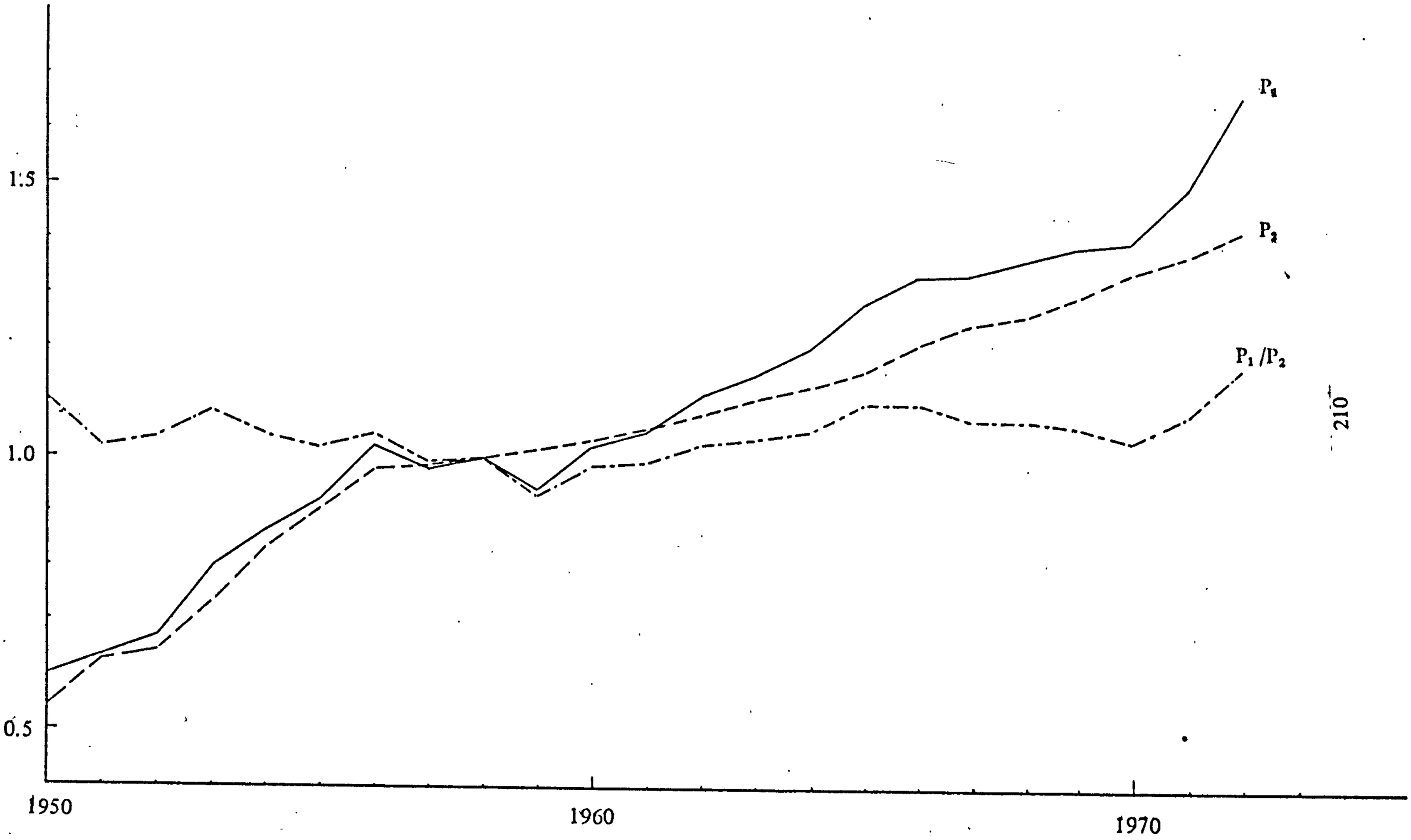
TABLE 5.21

Prices Indexes

Year	Agriculture	-Non-agriculture	Relative
1950	0.601	0.542	1.110
1951	0.635	0.623	1.020
1952	0.671	0.645	1.039
1953	0.801	0.737	1.086
1954	0.870	0.836	1.041
1955	0.925	0.905	1.022
1956	1.026	0.981	1.045
1957	0.979	0.987	0.992
1958	1.000	1.000	1.000
1959	0.949	1.015	0.935
1960	1.026	1.039	0.987
1961	1.052	1.058	0.994
1962	1.117	1.085	1.030
1963	1.156	1.111	1.040
1964	1.201	1.138	1.055
1965	1.293	1.167	1.108
1966	1.342	1.211	1.108
1967	1.347	1.251	1.077
1968	1.370	1.271	1.078
1969	1.395	1.305	1.069
1970	1.408	1.350	1.042
1971	1.505	1.382	1.089
1972	1.682	1.430	1.177

agricultural sector.

In figure 5.1 we present the price indexes of the two sectors P_1 and P_2 and also the relative price index $P = P/P_2$. In this graph we see that both the price indexes are increasing over time. One could possibly say that the rate of increase of the price index of the non-agricultural sector is more steady than that of the agricultural sector. The trend of the relative price index is not very clear. We see that for the period from 1950 to 1959 the relative price index is decreasing whilst after the year 1959 it is increasing over time, with



Eigure 5.1

the only exception for the period being 1967–1970, a period of political instability in Greece. Therefore, the terms of trade for the period 1950–1959 move in favour of the non-agricultural sector whilst after the year 1959 they move in favour of the agricultural sector.

5.7 The Foreign Sector

Table 5.22 represents the exports and imports of goods and services of the Greek economy.

TABLE 5.22

Exports and Imports of Goods and Services
(in million drs. and in current prices)

Year	Exports				Imports			
	Goods (f.o.b.)	Services	Total	% in GDP	Goods (f.o.b.)	Services	Total	% in GDP
1950	1355	346	1701	5.749	5713	1039	6752	22.819
1951	1524	321	1845	5.155	5647	1057	6704	18.730
1952	1799	393	2192	5.885	5004	954	5958	15.997
1953	3417	1069	4486	9.186	5595	1203	7798	15.967
1954	4803	1424	6227	11.093	9526	1915	11441	20.380
1955	6033	2187	8220	11.003	10535	2128	12663	19.628
1956	6163	2346	8509	11.390	12544	2872	15416	20.636
1957	6874	2987	9861	12.389	14742	2844	17586	22.094
1958	7197	2764	9961	11.927	14434	2658	17092	20.465
1959	6567	2869	9436	10.855	12913	2682	15595	17.941
1960	6283	3357	9640	10.339	14036	3082	17118	18.359
1961	6954	3834	10788	10.257	15764	3426	19190	18.245
1962	7675	4278	11953	10.736	17923	3894	21817	19.517
1963	8740	5371	14111	11.506	20471	4456	24992	20.325
1964	9352	5121	14473	10.568	24250	5242	29492	21.535
1965	9960	6162	16122	10.465	30190	6169	36359	23.601
1966	14999	7380	22379	13.241	30836	6614	37450	22.176
1967	15500	7525	23025	12.609	31602	7321	38923	21.315
1968	14272	8207	22479	11.423	35181	7840	43021	21.862
1969	17058	8841	25899	11.762	40307	9427	49734	22.586
1970	19378	10610	29988	12.142	44828	10127	54955	22.252
1971	19976	13906	33882	12.325	49407	11635	61042	22.205
1972	26191	17890	44081	13.853	61024	14776	75800	23.791

Sources: National Accounts of Greece, Nos. 21, 22.

From this table we see that Greek exports of goods and services for the period after the year 1953 (the year in which the drachma was devalued) represent about 11.5% of the gross domestic product at factor cost whilst the imports of goods and services for the same period represent about the 21.0% of GDP. From these figures we see that the Greek economy is extremely open to the outside world and to a much greater extent for imports than for exports.

Greek exports consist predominantly of agricultural goods, although this proportion is decreasing over time. The proportion of agricultural goods imported out of total imports is also decreasing over time as can be seen from table 5.23.

TABLE 5.23

Percentage of agricultural goods out of Exports and Imports of goods

Year	Exports	Imports	Year	Exports	Imports
1950	82.6	41.4	1962	86.3	20.9
1951	83.0	44.3	1963	88.9	23.7
1952	81.0	40.0	1964	88.3	24.2
1953	84.0	40.0	1965	84.9	24.2
1954	84.0	36.0	1966	79.7	24.4
1955	86.3	32.1	1967	73.6	24.4
1956	85.1	33.7	1968	66.6	21.7
1957	87.0	31.5	1969	60.8	21.3
1958	86.4	26.1	1970	57.3	18.2
1959	85.7	22.1	1971	60.1	19.6
1960	81.3	19.3	1972	55.0	18.9
1961	89.1	21.5			

Sources: FAO, Trade Yearbooks

The resulting large trade deficit (see table 5.22) becomes a little less marked due to the net transfers of income from abroad shown in table 5.24.

TABLE 5.24

Income payments from and to the rest of the world
(in million drs. and in current prices)

Year	Payments from abroad			Payments to abroad			Net payments	
	Profits	Wages & Salaries	Total	Profits	Wages & Salaries	Total	Current prices	Constant 1958 prices
1950	12	255	267	64	–	64	203	379
1951	17	351	368	44	–	44	324	555
1952	25	428	453	40	–	40	413	703
1953	33	582	615	86	–	86	529	597
1954	63	912	975	102	–	102	873	845
1955	76	1027	1103	165	–	165	938	910
1956	174	1425	1599	78	–	78	1521	1422
1957	206	1960	2166	77	–	77	2089	1909
1958	749	852	1601	89	53	142	1459	1459
1959	852	874	1726	105	85	190	1536	1531
1960	1201	1077	2278	174	90	264	2014	2063
1961	1613	1193	2806	297	125	422	2384	2487
1962	1801	1244	3045	342	102	444	2601	2755
1963	2204	1263	3467	351	174	525	2942	3195
1964	2908	1326	4234	395	489	884	3350	3514
1965	3067	1321	4388	537	221	758	3630	3722
1966	3625	1319	4944	696	350	1046	3898	3958
1967	3876	1737	5613	936	377	1313	4300	4396
1968	4368	2116	6484	1069	373	1442	5042	5088
1969	4623	2076	6699	1311	414	1725	4974	5028
1970	5315	2247	7562	1812	462	2274	5288	5094
1971	6857	3233	10090	2172	596	2768	7322	7010
1972	7919	5045	12964	2480	843	3323	9641	8508

Sources: National Accounts of Greece, Nos. 21, 22.

5.8 Conclusions

The variation in the definition of developing countries poses quite a difficult problem. For some purposes, UN agencies leave out Southern Europe, which is included in OECD statistics. According to Maddison (1971), the OECD definition of developing countries is currently the most useful. Southern Europe (ie. Greece, Spain, Turkey and Yugoslavia) consists of countries relatively well-off by Asian and African standards, but if we were

to exclude them on these grounds it would also be necessary to exclude some Latin American countries, Israel and the Middle East oil producers which are as prosperous as Southern Europe.

In spite of these various definitions of what constitutes a developing country, from the examination of the data presented in the previous sections of this chapter the following points must be admitted:

1. The distribution of the labour force between the agricultural and non-agricultural sector, has moved in favour of the latter, a fact which is compatible with our assumptions about labour allocation imposed on our theoretical growth model.
2. The contribution of the agricultural sector in the total GDP of the economy is diminishing in favour of the non-agricultural sector.
3. The composition of the two sectors as far as each sector's product and labour force are concerned show that on average the agricultural labour force is 48.5% of the labour employed in the economy, whilst its product is 23.5% of total GDP, which shows the low productivity of labour in the agricultural sector.
4. The per capita income is increasing over time .
5. There exists an income differential between the people of the two sectors. This differential is slowly decreasing over time, which is compatible with our model.
6. The increase in the standard of living has been reflected by the falling of that part of gross expenditure in the economy absorbed by consumption and with the composition of that private consumption.
7. The overall average propensity to save has increased from a level of 15% to 30%, indicating the continuing economic development effort.
8. The percentage of investment out of GDP in the two sectors is increasing over time.

9. The terms of trade move on average in favour of the agricultural sector.

10. The contribution of agricultural goods to exports or imports in the economy is decreasing over time.

From the above ten points it is, we believe, clear that the theoretical dual economies in the sense of our two-sector growth model are relevant to the economy of Greece. Therefore, we agree with Higgins (1956) who opposes Boeke (1953) that dualism is not only an Eastern phenomenon but also applies to Western societies.

5.9 References

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CHAPTER 6

ESTIMATION OF THE MODEL

6.1 Introduction
6.2 The Model Once More
6.3 Interpolation of Labour
6.4 The Consumption Allocation Parameter of the Two Goods
6.5 Shares of Labour and Capital in GDP for the Two Sectors
6.6 Net Capital Stock, Rental Rates and the Propensity to Invest Out of Profits for the Two Sectors
6.7 The Econometrics Used
6.8 Estimation of the Model
6.9 Which Estimate?
6.10 Conclusions
6.11 References

6.1 Introduction

In the first four chapters of this thesis we presented and discussed various models of economic development including our own model. But theorizing is really an unending process. Are these theories relevant to the real world? Do the key variables and parameters in the develcped countries actually behave in the manner that they are supposed to behave in the various models of growth? What is the actual structure of these economies? These and other related questions cannot be answered by pure theorizing.

The aim of this chapter, together with the findings in chapter 5, is to ensure that our model is empirically relevant to the Greek economy. We will try in this chapter to estimate the structural parameters of the model. An important lacuna in the theory of economic development is the empirical analysis of observed experience. We hope in this chapter to make a modest attempt of bridging the gap between the theory and the empirical evidence of the economics of growth.

In the first sections of this chapter we try to estimate with various methods the missing data which are relevant to our empirical tests. Unfortunately, in developing economies published data is relatively scanty and of uncertain quality. Therefore, the results, which will be presented in the latter sections of this chapter are heavily dependent on the data that was approximated. So, any conclusion must only be a crude index of the actual activities of the economy.

Before doing the actual estimation of the various parameters of the model we thought that it would be better to present and to try to investigate the great ratios of economics of growth, Klein-Kosobud (1961). This work is really yielding richer insights into the key variables of the growth model and prevent us of using mispecified equations in our econometric study of the model.

Finally, in the last section of this chapter we compare our results with results for similar studies existing in the literature.

6.2 The Model Once More

In the previous chapter we presented the available data for the Greek economy relevant to our two sector growth model of Chapters 3 and 4. However owing to the insufficiency of data we were unable to estimate our model using this data. Consequently simplifying assumptions had to be introduced into the model in order to make estimation possible. Therefore, we must simplify our model enough to be able to produce an approximate estimation of it.

The simplifications that we impose on the model are as follows:

(i) The workers of both the sectors do not invest any part of their income but they consume all of it. This implies that $s_1 = s_2 = 0$.

(ii) For every income level the proportion of income spent on agricultural goods is the same. This implies that $m_{11} = m_{12} = m_{13} = m_1$ and $m_2 = 1 - m_1 = m_{21} = m_{22} = m_{23}$.

With these simplifications the structural form for our model (for the Cobb-Douglas case) is written:

$$y_1(t) = \Gamma_1 e^{\pi_1 t} k_1(t)^{\alpha_1} \tag{6.2.1}$$

$$y_2(t) = \Gamma_2 e^{\pi_2 t} k_2(t)^{\alpha_2} \tag{6.2.2}$$

$$\omega_1(t) = \frac{\beta_1}{\alpha_1} k_1(t) \tag{6.2.3}$$

$$\omega_2(t) = \frac{\beta_2}{\alpha_2} k_2(t) \tag{6.2.4}$$

$$\ell_1(t) + \ell_2(t) = 1 \tag{6.2.5}$$

$$k_1(t)\,\ell_1(t) + k_2(t)\,\ell_2(t) = k(t) \tag{6.2.6}$$

$$\omega_1(t) = \mu(t)\,\omega_2(t) \tag{6.2.7}$$

$$p(t) = \frac{\alpha_2 \Gamma_2}{\alpha_1 \Gamma_1} e^{(\pi_2 - \pi_1)t} \frac{k_1(t)^{\beta_1}}{k_2(t)^{\beta_2}} \tag{6.2.8}$$

$$c_1(t) = \ell_1(t)\, y_1(t) \tag{6.2.9}$$

$$i(t) = s_3\, (\alpha_2 \Gamma_2\, e^{\pi_2 t} k_2(t)^{\alpha_2 - 1}\, k(t)| \tag{6.2.10}$$

$$c_1(t) = m_1\, p(t)^{-1} [\, (1-s_3)\, \alpha_2 \Gamma_2\, e^{\pi_2 t} k_2(t)^{\alpha_2 - 1} k(t) +$$

$$+ \omega_1(t)\, \ell_1(t) + \omega_2(t)\, \ell_2(t)\,] \tag{6.2.11}$$

$$c_2(t) = m_2\, [\, (1 - s_3)\, \alpha_2 \Gamma_2\, e^{\pi_2 t} k_2(t)^{\alpha_2 - 1} k(t) +$$

$$+ \omega_1(t)\, \ell_1(t) + \omega_2(t)\, \ell_2(t)\,] \tag{6.2.12}$$

In (6.2.11) and (6.2.12) the expression in the brackets is the part of total income of the economy which is consumed on goods of both the sectors expressed in units of the non-agricultural goods.

In (6.2.10) the product in parenthesis on RHS is the total profits in the economy, ie. profits of capitalists in both sectors.

Equation (6.2.9) shows that the goods of the agricultural sector are all consumed.

All the other equations are self explanatory.

The reduced form of the above system of equations is written:

$$y_1(t) = N_1 \Delta(t)^{\alpha_1}\, e^{\pi_1 t} k(t)^{\alpha_1} \tag{6.2.13}$$

$$y_2(t) = N_2 \Delta(t)^{\alpha_2}\, \mu(t)^{-\alpha_2}\, e^{\pi_2 t} k(t)^{\alpha_2} \tag{6.2.14}$$

$$k_1(t) = N_3 \Delta(t)\, k(t) \tag{6.2.15}$$

$$k_2(t) = N_4 \Delta(t)\, \mu(t)^{-1}\, k(t) \tag{6.2.16}$$

$$\ell_1(t) = N_5\, \Delta(t)^{-1} \tag{6.2.17}$$

$$\ell_2(t) = N_6\, \Delta(t)^{-1}\, \mu(t) \tag{6.2.18}$$

$$\omega_1(t) = N_7\, \Delta(t)\, k(t) \tag{6.2.19}$$

$$\omega_2(t) = N_8\, \Delta(t)\, \mu(t)^{-1}\, k(t) \tag{6.2.20}$$

$$p(t) = N_9\, \Delta(t)^{\beta_1 - \beta_2}\, \mu(t)^{\beta_2}\, e^{(\pi_2 - \pi_1)t}\, k(t)^{\beta_1 - \beta_2} \tag{6.2.21}$$

$$i(t) = N_{10}\, \Delta(t)^{-\beta_2} \mu(t)^{\beta_2}\, e^{\pi_2 t} k(t)^{\alpha_2} \tag{6.2.22}$$

$$c_1(t) = N_{11}\,\Delta(t)^{-\beta_1}\,e^{\pi_1 t}\,k(t)^{\alpha_1} \tag{6.2.23}$$

$$c_2(t) = N_{12}\,\Delta(t)^{-\beta_2}\,\mu(t)^{\beta_2}\,e^{\pi_2 t}\,k(t)^{\alpha_2} \tag{6.2.24}$$

In the above equations the N's are expressions involving the parameters of the model and therefore independent of time. Also, $\Delta(t)$ is given by:

$$\Delta(t) = [m_1\beta_1(1+(1-s_3)\frac{\alpha_2}{\beta_2}) + (1-m_1\beta_1(1+(1-s_3)\frac{\alpha_1}{\beta_1})\,\mu(t)\ ] \tag{6.2.25}$$

The above system of equations is a system of 12 endogenous variables; $y_i(t)$, $k_i(t)$, $\ell_i(t)$, $\omega_i(t)$, $p(t)$, $i(t)$ and $c_i(t)$ for $i = 1, 2$, in 12 equations. The exogenous variables of the system are the overall capital-labour ratio $k(t)$, the wages ratio $\mu(t)$ and the time t.

In the following sections of this chapter we will try to estimate our model in the forms above using data for the Greek economy.

6.3 Interpolation of Labour

In Chapter 5 and in section 5.2 we have seen that the only available data on the labour force employed in the two sectors are those given in table 5.4. In that table we see that we have only three observations (corresponding to the censuses of 1951, 1961 and 1971) for the period 1950–1972. More precisely, the figures in table 5.4 do not refer to the labour force actually engaged in the various activities but to the active population. The model to be estamated is one of full employment of the resources and therefore we take active population to represent the actual labour force. We must remember that the unemployment rate for the period under consideration was about 4.5% of the active population.

Various researchers (Papandreou,(1962), Prodromidis (1973)), faced the problem of missing observations in labour or population by interpolating between the census figures using average annual growth rates. Here, we will try to estimate the labour force in the two sectors using Friedman's (1962) method. Friedman's method is interpolation

of time series by related series. Let X be the series to be interpolated, Y, the related series to be used in interpolation. Although this method does not explicitly use any information on the degree of correlation between Y and X, a particular series Y is of course chosen for use in interpolation because its interyearly movements are believed to be highly correlated with the intrayearly movements of X. This belief may be based on nonquantitative considerations (eg. that employment in different firms producing the same product or vault cash in different classes of banks in the same geographic area will be affected by common forces and hence are likely to move together). Alternatively, it may be based on an observed high correlation between movements in Y and in X for the time units to be interpolated but for a different period than that to be interpolated; or between movements in Y and in X for different time units for the same period (eg. a high correlation between annual observation may be taken as evidence that there is also a high correlation between the unknown intrayearly movement); or between movements in two series other than Y and X but analogous to them for the same time units and for the same period (eg. a high correlation between vault cash in national and state weekly reporting member banks may be taken as evidence that there is also a high correlation between vault cash in weekly-reporting and non-weekly-reporting member banks).

A straight-line trend is computed bewteen the knwon values of X, also between the corresponding values of Y. The proportional d.fference between each value of Y and the corresponding trend value of Y is then added to the trend value of X.

Let xo and x_n be two successive known values of X; x_1, ..., x_i, ..., x_{n-1}, the unknown values to be estimated by interpolation; and y_0, y_1, ..., y_i, ..., y_{n-1}, y_n, the values of Y for the corresponding period. Let x_i^* be an estimate of x_i for $i = 1, ..., n-1$. So, any x_i^* is given by:

$$x_i^* = \left[\frac{(n-i)\,x_o + i\,x_n}{n} \right] \frac{y_i}{\left[\frac{(n-i)\,y_o + i\,y_n}{n} \right]} = \frac{Tx_i}{Ty_i}\, y_i \qquad (6.3.1)$$

where Tx_i and Ty_i are the trend values of X and Y respectively. It is easy to see that (6.3.1) is equivalent to:

$$x_i^* = Tx_i + \frac{Tx_i}{Ty_i}(y_i - Ty_i) = \frac{Tx_i}{Ty_i}\, y_i \qquad (6.3.2)$$

The factor Tx_i/Ty_i is to be interpreted as a scale factor used to convert Y values to X values.

Returning now to our model we will try to estimate the sectoral labour force using the above method and the available data from tables 5.1 for the total population, 5.3 for the active population, 5.4 for the distribution of labour among sectors according to the three censuses and 5.5 for the labour-force index in the industrial sector.

We will either use the formula (6.3.2) as it stands or in logarithmic form. We assume that the related series for the interpolation of the non-agricultural labour force is the index of the labour-force in the industrial sector. For the total labour-force of the economy we take as our related series the total population or move precisely the total active population of the economy. The labour force in the agricultural sector is taken to be the difference between the estimated total labour force of the economy and the estimated labour force of the non-agricultural sector. The results of the two methods ie. absolute numbers and logarithms, are shown in tables 6.1 and 6.2.

We now face the problem of choosing the best time series for the sectoral labour force. In each table, comparing the data for the agricultural labour force, columns (2) and (3), we believe that the results in columns (3) are more reliable than the results in columns (2) because they are derived from the use of the total active population as a

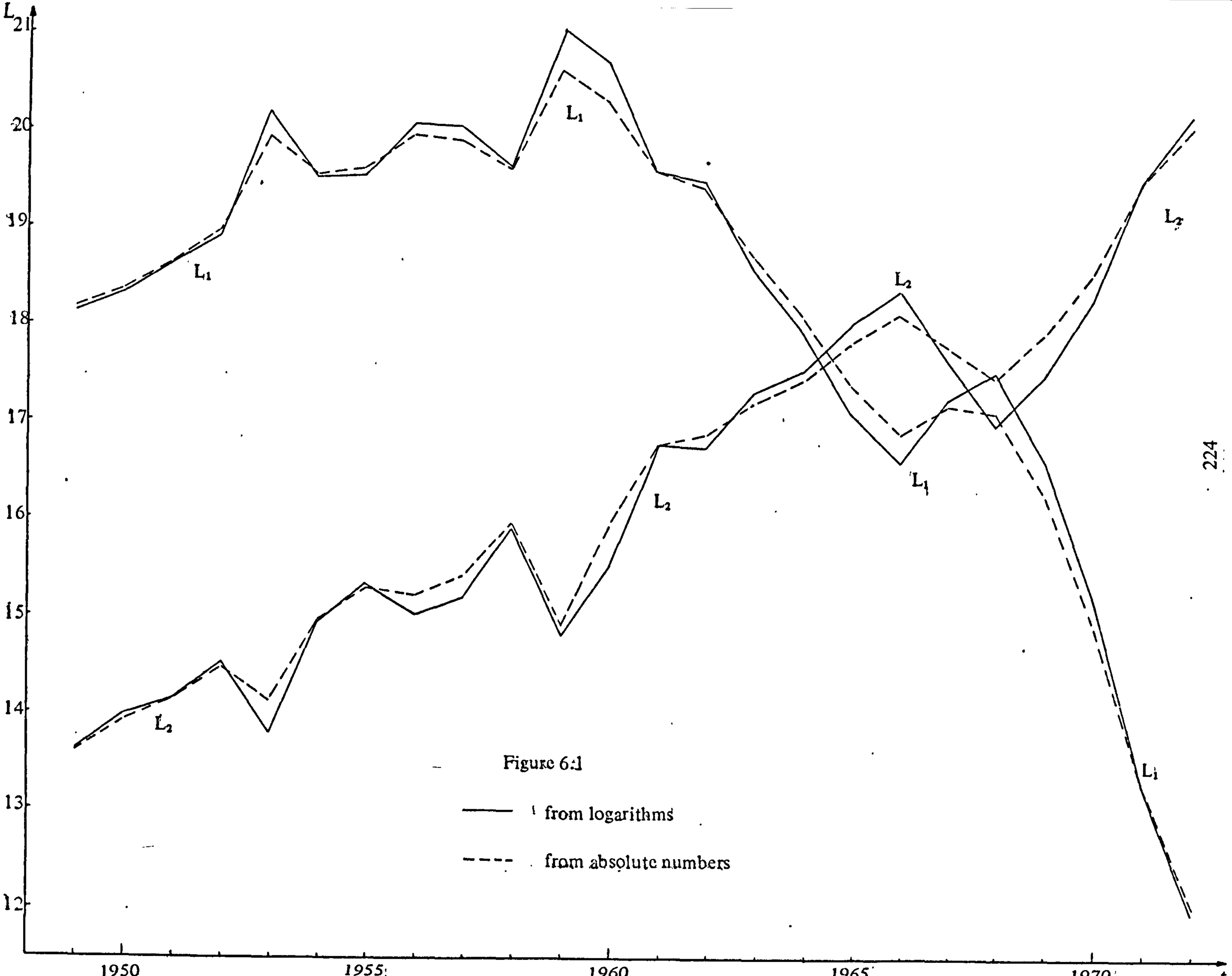

Figure 6:1

——— from logarithms

– – – – from absolute numbers

TABLE 6.1

Interpolation of Labour-Force Using Absolute Numbers

Year	Non-Agriculture	Agriculture		Total	
		(4) – (1)	(5) – (1)	Using total population	Using total population
	(1)	(2)	(3)	(4)	(5)
1950	1392149	1847560	1836550	3239709	3228699
1951	1414000	1864000	1864000	3278000	3278000
1952	1449830	1869244	1898648	3319074	3348478
1953	1414274	1944596	1996072	3358870	3410346
1954	1496479	1898957	1955877	3395435	3452355
1955	1532777	1897420	1962249	3430197	3495026
1956	1522803	1939285	1998351	3462088	3521155
1957	1543841	1950014	1992198	3493854	3536039
1958	1597200	1933468	1961665	3530668	3558865
1959	1537048	2033948	2053906	3570996	3590954
1960	1590369	2014128	2032013	3604497	3622382
1961	1678155	1960446	1960446	3638601	3638601
1962	1680859	1924317	1945865	3605176	3626723
1963	1723927	1839864	1872923	3563791	3596850
1964	1746154	1776188	1812388	3522343	3558542
1965	1785297	1699490	1740042	3484786	3525339
1966	1815857	1640836	1693659	3456694	3509517
1967	1782508	1661465	1721776	3443972	3504284
1968	1750567	1649458	1714302	3400025	3464869
1969	1795123	1564152	1627407	3359275	3422531
1970	1859856	1454324	1492630	3314180	3352486
1971	1953560	1330320	1330320	3283880	3283880
1972	2013364	1254285	1202744	3267649	3216108

series related to the total labour force of the economy. We believe that total labour force is better related to the total active population than to the total population. Comparing now the data in the two tables we see that there are no major differences between the figures. We have chosen the data in table 6.2 because it shows a slightly higher movement of labour than the data of table 6.1. These trends of labour movements are shown in figure 6.1. Hence, the time series for the labour force are chosen from table 6.2; column (3) for the agricultural sector; column (1) for the non-agricultural sector; and

TABLE 6.2

Interpolation of Labour-Force Using Logarithms

Year	Non-Agricultural	Agricultural		Total	
		(4) – (1)	(5) – (1)	Using total population	Using total population
	(1)	(2)	(3)	(4)	(5)
1950	1397198	1843276	1832205	3240473	3229403
1951	1414000	1864000	1864000	3278000	3278000
1952	1453693	1864454	1893058	3318147	3346751
1953	1379901	1977335	2027412	3357236	3407313
1954	1494021	1899481	1954982	3393502	3449004
1955	1535762	1892474	1955572	3428236	3491334
1956	1503365	1957104	2014583	3460470	3517948
1957	1521294	1971395	2012583	3492689	3533877
1958	1591622	1937961	1965745	3529583	3557367
1959	1480979	2088785	2108699	3569765	3589679
1960	1551050	2052809	2070323	3603859	3621373
1961	1678155	1960446	1960446	3638601	3638601
1962	1673807	1929928	1950136	3603735	3623944
1963	1733047	1828898	1859456	3561946	3592503
1964	1756482	1763913	1796942	3520396	3553424
1965	1805467	1677180	1714070	3482647	3519538
1966	1838609	1615076	1663785	3453685	3502394
1967	1767776	1670977	1727736	3438752	3495512
1968	1700272	1695814	1756498	3396086	3456770
1969	1750494	1606084	1665151	3356578	3415644
1970	1830211	1483276	1518680	3313487	3348891
1971	1953560	1330320	1330320	3283880	3283880
1972	2022173	1244876	1197854	3267049	3220027

column (5) for the total labour force of the economy.

6.4 The Consumption Allocation Parameter of the Two Goods

In this section we will try to interpret the behaviour of the consumption allocation parameter, ie. the parameter m_1 in equation (6.2.11) which measures the proportion of the agricultural goods consumed out of total consumption. Of course, an easy way of finding out m_1 is to divide the consumption of agricultural goods by the total consumption, both expressed in the same units. Unfortunately, the available data for consump-

tion in tables 5.12 to 5.15 do not contain as a category "consumption of agricultural goods." The figures shown in these tables contain some general consumption categories as for example "food and beverages" from which it is impossible to separate consumption of agricultural products without additional information.

Another general difficulty is that although our model refers to closed economies the available data refers to the open economy of Greece. Therefore, we must determine our data in such a way as to be compatible with our model.

Having these points in mind we will try to develop time series for the two consumption goods using some accounting identities used by the "National Accounts of Greece".

We define:

GNE	=	Gross National Expenditure
GNP	=	Gross National Product
GNI	=	Gross National Income
NNI	=	Net National Income
GDP	=	Gross Domestic Product
NIA	=	Net Income from Abroad
C	=	Total Consumption
I	=	Total Investment
D	=	Depreciation
XG	=	Export of Goods
XS	=	Export of Services
XI	=	Payments to the rest of the world
MG	=	Imports of goods
MS	=	Import of Services
MI	=	Payments from the rest of the world.

Accounting identities used are the following:

$$GNE = G+I + (XG + XS + MI) - (MG + MS + XI) \qquad (6.4.1)$$

$$GNE = GNP = NNI + D = GNI = GDP + NIA \qquad (6.4.2)$$

$$NIA = MI - XI \qquad (6.4.3)$$

From the above identities we have that:

$$GDP + NIA = C + I + (XG + XS) - (MG + MS) + (MI - XI)$$

or

$$GDP = C + I + (XG + XS) - (MG + MS) \tag{6.4.4}$$

Denoting by the subscript 1 the agricultural quantities and by the subscript 2 the non-agricultural quantities (6.4.4) can be written:

$$GDP_1 + GDP_2 = (C_1 + C_2) + I + (XG_1 + XG_2) - (MG_1 + MG_2) + \\ + (XS - MS) \tag{6.4.5}$$

Assuming now that all the transactions of services with the rest of the world is a non-agricultural activity, (6.4.5) can be divided into two identities:

$$C_1 = GDP_1 - XG_1 + MG_1 \tag{6.4.6}$$

$$C_2 = GDP_2 - I - XG_2 + MG_2 - (XS - MS) \tag{6.4.7}$$

Of course, in accordance with the theoretical model of the previous chapters we have assumed in the above two identities that total investment of the economy consists only of goods from the non-agricultural sector.

The accounting identities (6.4.6) and (6.4.7) express the values of the agricultural and non-agricultural goods respectively domestically consumed.

Table 5.6 represents GDP_i, (i = 1, 2), in constant 1958 prices. Table 5.17 shows I in constant 1958 prices, also. Table 5.22 represents XG, XS, MG and MS but in current prices. We transformed these current price figures into constant 1958 price figures using price indexes for exports and imports shown in table 6.3. The data transformed in such a way (table 5.22) are shown in table 6.4.

With the help of table 5.23, ie. the proportion of agricultural goods in total exports and imports of goods we divided the exports and imports of goods of table 6.4 into the agricultural and non-agricultural goods. Therefore, we see that we have all the data that we need to compute consumption of agricultural and non-agricultural goods using the

TABLE 6.3

Price Indexes for Exports and Imports

Year	Exports	Imports	Year	Exports	Imports
1950	0.548	0.469	1962	0.930	0.938
1951	0.545	0.534	1963	1.000	0.957
1952	0.524	0.544	1964	1.014	0.986
1953	0.763	0.835	1965	1.000	0.988
1954	0.925	1.059	1966	1.023	1.026
1955	0.988	1.049	1967	1.009	0.994
1956	1.063	1.107	1968	1.000	0.997
1957	1.053	1.154	1969	1.000	0.995
1958	1.000	1.000	1970	1.031	1.036
1959	0.931	0.973	1971	1.050	1.048
1960	0.914	0.950	1972	1.075	1.089
1961	0.915	0.938			

Source: U.N. National Accounts Statistics

expressions (6.4.6) and (6.4.7); the results are shown in table 6.5.

The last column in table 6.5, ie. the proportion of agricultural goods consumed out of total consumption, m_1, calculated as the division of the first column by the third column.

Unfortunately, a slight problem arises in the above calculations with respect to the value of exports in table 6.4. These values are expressed in f.o.b. units. According t o the definition of exports in f.o.b. units, given in the Statistical Yearbooks of Greece, these values include some indirect taxes less subsidies. It was impossible to obtain from the available data the value of these indirect taxes and subsidies. However, these values are small compared to the volume of exports and therefore, we tried to approximate these values in the following way.

We assumed that total indirect taxes less subsidies are proportional to the value of total consumption (in market prices) and exports of goods (in f.o.b. units). Therefore,

TABLE 6.4

Exports and Imports of Goods and Services
(in million drs. and in constant 1958 prices)

Year	Exports			Imports		
	Goods (f.o.b.)	Services	Total	Goods (f.o.b.)	Services	Total
1950	2472.6	631.4	3104.0	12174.8	2214.2	14389.0
1951	2796.3	589.0	3385.3	10570.7	1978.6	12549.3
1952	3433.2	750.0	4183.2	9195.2	1753.0	10948.2
1953	4478.4	1401.0	5879.4	7902.9	1441.6	9344.4
1954	5192.4	1539.5	6731.9	8992.2	1807.7	10799.9
1955	6106.3	2213.6	8319.9	10040.7	2028.2	12068.9
1956	5797.7	2207.0	8004.7	11329.7	2594.0	13923.7
1957	6528.0	2836.7	9364.7	12778.5	2465.2	15243.7
1958	7197.0	2764.0	9961.0	14434.0	2658.0	17092.0
1959	7053.7	3081.6	10135.3	13271.0	2756.4	16027.4
1960	6874.2	3672.9	10547.1	14782.2	3245.8	18028.0
1961	7600.0	4190.2	11790.2	16802.5	3651.7	20454.2
1962	8252.7	4600.0	12852.7	19115.6	4153.1	23268.7
1963	8740.0	5371.0	14111.0	21395.7	4657.3	26053.0
1964	9222.9	5050.3	14273.2	24604.5	5318.6	29923.1
1965	9960.0	6162.0	16122.0	30555.3	6243.7	36799.0
1966	14661.8	7214.1	21875.9	30056.1	6446.7	36502.8
1967	15361.7	7457.9	22819.6	31806.3	7368.3	39174.6
1968	14272.0	8207.0	22499.0	35281.6	7862.4	43144.0
1969	17058.0	8841.0	25899.0	40512.0	9475.0	49987.0
1970	18795.3	10291.0	29086.3	43259.6	9772.7	53032.3
1971	19024.8	13243.8	32268.6	47166.3	11107.3	58273.6
1972	24363.7	16641.9	41005.6	56036.3	13568.3	69604.6

the value of exports of goods at factor cost units is given by:

$$XG = XG_{fob} - \frac{XG_{fob}}{G_{mp}} \times (ITLS) \qquad (6.4.8)$$

where XG_{fob} = exports of goods in fob units

G_{mp} = total consumption in market prices

ITLS = indirect taxes less subsidies.

Using formula (6.4.8) we estimated total exports of goods at factor cost as presented

TABLE 6.5

I. Consumption of Agricultural and Non-agricultural Goods
(in million drs. and in constant 1958 prices)

Year	Agricultural	Non-Agricultural	Total	m_1
1950	18248.0	33727.0	51975	0.351
1951	19680.9	34563.1	54244	0.363
1952	17198.2	37737.8	54936	0.313
1953	19995.3	35392.7	55388	0.361
1954	18849.6	43155.4	62005	0.304
1955	19304.3	43943.7	63248	0.305
1956	20828.2	45922.8	66751	0.312
1957	23153.8	47543.2	70697	0.328
1958	20495.1	52576.9	73072	0.280
1959	20982.9	54174.1	75157	0.279
1960	19545.3	59017.7	78563	0.249
1961	24086.9	59488.1	83575	0.288
1962	22151.1	65904.9	88056	0.252
1963	24526.9	70265.1	94790	0.259
1964	26766.5	73969.5	100736	0.266
1965	28664.4	81956.6	110621	0.259
1966	25403.3	89042.7	114446	0.222
1967	27961.5	94407.5	122369	0.229
1968	27235.0	101346.0	128581	0.212
1969	28793.8	110695.2	139489	0.206
1970	30276.5	119348.5	149625	0.202
1971	31845.7	128587.3	160433	0.198
1972	32386.8	141711.2	174098	0.186

in table 6.6. In the same table we present also the total import duties because in formula (6.4.8) we did not use the variable "Indirect taxes less subsidies" but instead, for a better approximation, the variable "Indirect taxes less subsidies less import duties". Import duties were originally in current prices and we converted them into constant 1958 prices using as an implicit deflator the ratio of current to constant 1958 prices of indirect taxes less subsidies.

Applying now the new figures for exports of goods, of table 6.6, into the formula (6.4.6) and (6.4.7) we found new series for the consumption of agricul-

TABLE 6.6

Exports of Goods, Indirect taxes less Subsidies and Import Duties
(in million drs. and in constant 1958 prices)

Year	Exports of goods at factor cost	Indirect taxes less subsidies	Import duties Current	Import duties Constant 1958 prices	Tax Deflator
1950	2358.3	4172	935	1576.7	0.593
1951	2647.2	4788	1094	1637.0	0.667
1952	3258.5	4999	1368	1948.9	0.702
1953	4171.2	6199	1511	1975.3	0.765
1954	4854.6	6493	1792	2036.3	0.880
1955	5734.2	6543	2358	2290.4	1.029
1956	5399.8	8153	2905	3010.2	0.965
1957	6071.3	8867	3257	3300.2	0.987
1958	6655.6	9738	3508	3508.0	1.000
1959	6553.9	9620	3700	3613.2	1.024
1960	6344.3	10746	4016	3861.5	1.040
1961	6961.0	12353	4536	4287.2	1.058
1962	7514.3	13929	5068	4803.8	1.055
1963	7917.3	15455	5514	5077.4	1.086
1964	8327.6	17323	6420	5862.9	1.095
1965	9014.7	19200	7752	6878.2	1.127
1966	13167.7	21550	9099	7691.4	1.183
1967	13706.5	23828	9714	8074.8	1.203
1968	12538.9	27529	10346	8571.5	1.207
1969	14968.9	30217	11669	9433.2	1.237
1970	16746.1	31999	15539	12196.9	1.274
1971	17036.5	32827	16570	12629.6	1.312
1972	21879.5	35503	19332	14131.6	1.368

Sources: for columns 2 and 3, National Accounts of Greece, Nos. 21, 22.

tural and non-agricultural goods with the corresponding consumption allocation parameter m_1, all presented in table 6.7.

We believe that the figures in table 6.7 are higher than those of the real world, because of the assumption that we have adopted about the proportional allocation of indirect taxes between total consumption and export of goods. However, we think that the allocation of taxes is less than proportional for the exports of goods, (a policy which

TABLE 6.7

II. Consumption of Agricultural and Non-Agricultural Goods.
(in million drs. and in constant 1958 prices)

Year	Agricultural Goods	Non-Agricultural Goods	Total	m_1
1950	18342.4	33746.9	52089.3	0.352
1951	19804.6	34588.5	54393.1	0.364
1952	17339.2	37771.0	55110.7	0.315
1953	20253.3	35441.8	55695.1	0.364
1954	19133.3	43209.5	62342.8	0.307
1955	19625.4	43994.6	63620.0	0.308
1956	21166.9	45982.1	67149.0	0.315
1957	23551.2	47602.5	71153.7	0.331
1958	20962.9	52650.6	73613.5	0.285
1959	21411.2	54245.6	75656.8	0.283
1960	19976.1	59116.8	79092.9	0.253
1961	24656.3	59557.7	84214.0	0.293
1962	22788.3	66006.1	88794.4	0.257
1963	25258.3	70354.4	95612.7	0.264
1964	27557.0	74074.3	101631.3	0.271
1965	29466.9	82099.4	111566.3	0.264
1966	26594.1	89346.0	115940.1	0.229
1967	29179.8	94844.5	124024.3	0.235
1968	28389.2	101924.9	130314.1	0.218
1969	30064.0	111514.1	141578.1	0.212
1970	31450.7	120223.5	151674.2	0.207
1971	33040.6	129380.6	162421.2	0.203
1972	33753.1	142829.1	176582.2	0.191

makes exports cheaper). Therefore, we decided to accept as the final approximation of consumption of agricultural and non-agricultural goods the figures shown in table 6.8, which have been estimated as the middle-point between the corresponding figures of the variables of tables 6.5 and 6.6.

Comparing the consumption allocation parameters of the tables 6.5, 6.6 and 6.8 we see that the differences amongst them are very small (of the order of the third decimal point) and therefore we believe that the notion of indirect taxes less subsidies

TABLE 6.8

III. Consumption of Agricultural and Non-agricultural Goods
(in million drs. and in constant 1958 prices)

Year	Agricultural Goods	Non-agricultural Goods	Total	m_1
1950	18295.2	33737.0	52032.2	0.352
1951	19742.8	34575.8	54318.6	0.363
1952	17268.9	37754.4	55023.3	0.314
1953	20124.3	35417.3	55541.6	0.362
1954	18991.4	43182.5	62173.9	0.305
1955	19464.9	43969.1	63434.0	0.307
1956	20997.6	45952.4	66950.0	0.314
1957	23352.5	47572.8	70925.3	0.329
1958	20729.0	52613.8	73342.8	0.283
1959	21197.0	54209.9	75406.9	0.281
1960	19760.7	59067.3	78828.0	0.251
1961	24371.6	59522.9	83894.5	0.291
1962	22469.7	65955.5	88425.2	0.254
1963	24892.6	70308.7	95201.3	0.261
1964	27161.8	74021.9	101183.7	0.268
1965	29065.6	82028.0	111093.6	0.262
1966	25998.7	89194.4	115193.1	0.226
1967	28570.6	94626.0	123196.6	0.232
1968	27812.1	101635.5	129447.6	0.215
1969	29428.9	111104.7	140533.6	0.209
1970	30863.6	119786.0	150649.6	0.205
1971	32443.2	128984.0	161427.2	0.201
1972	33070.0	142270.1	175340.1	0.189

in the export of goods variable did not introduce considerable errors in the estimation of the consumption series for the two goods.

We would like now to test the results of the consumption allocation parameter with the input-output table 6.9 for the year 1958 (the only available input-output table for the Greek economy).

Table 6.9 shows the proportion of the agricultural and non-agricultural production for each of the various sectors of the final demand; ie. column one,

TABLE 6.9

Allocation of Final Demand Among the Productive Activities (1958)

Production	Final Demand					
	Food & Beverages	Tobacco	Clothing etc.	Housing Heating	Durables	Services
Agriculture	0.643	0.347	0.012	0.152	0.077	0.025
Non-agriculture	0.357	0.653	0.988	0.848	0.923	0.975

Source: Suits, D., (1965), An Econometric Model of the Greek Economy, Center of Planning and Economic Research, Athens.

of food and beverages, shows that the demand of food and beverages of value of 1 million drs. presupposes 0.643 million of agricultural production and 0.357 million of non-agricultural production. Therefore, if we want to find the final demand for agricultural goods, we must multiply each figure of the first line of table 6.9 by the corresponding figures of table 5.15, (for the composition of consumption expenditure, for the year 1958) and sum the results of the multiplication; ie:

$$C_1 = 0.643 \text{ x (Food \& Beverages)} + 0.347 \text{ x (Tobacco)} + 0.012 \text{ x (Clothing)} +$$

$$+ 0.152 \text{ x (Housing, etc.)} + 0.077 \text{ x (Durables)} + 0.025 \text{ x (Services)} \qquad (6.4.9)$$

Dividing now the result above by the total consumption for the year 1958 we find that $m_1 = 0.301$. Our estimate of m_1 for the same year in table 6.8 is equal to 0.283. The discrepancy between these two values we believe arose because in expression (6.4.9) the data of table 6.9 was estimated from factor cost units whilst those of table 5.15 are expressed in market prices. In fact, this discrepancy is not too large if one also takes into account that the general discrepancy in the data with reference to the whole system is about 5%. Therefore, we believe that the difference between the two consumption allocation parameters is not significant (at least for the year 1958) and so the figures in

table 6.8 we accept to be reliable.

In the following pages we will try to interpret the parameter m_1. In table 6.8 the series for m_1 is clearly decreasing over time. Its time trend is given by the expression:

$$m_1(t) = m_1(0)(1+j)^t \qquad (6.4.10)$$

where j is the average annual rate of change, which turns out to be:

$$\ln m_1(t) = \underset{(0.0178)}{-1.096080} - \underset{(0.0017)}{0.027635} \times t, \quad R^2 = 0.9186$$

or

$$m_1(t) = 0.334178 \times (0.972743)^t \qquad (6.4.11)$$

From (6.4.11) we deduce that $m_1(t)$ is decreasing over time at an average annual rate of $j = 0.027257$.

The economic interpretation of this decrease is that as the economy develops people can consume more things that do not belong to the category of "indispensable" goods, ie. the agricultural goods.

In our model, m_1 has been taken to be constant, (equations 6.2.11 and 6.2.12). In reality, at least for the Greek economy, this parameter seems to be a decreasing function of time. Therefore, keeping m_1 constant and trying to estimate the various parameters of our model using the reduced form (equations 6.2.13–6.2.24) we are introducing mispecification errors into our estimates, because in the reduced form the constants N_i are not really constants but vary over time. This mispecification can be altered by allowing m_1 to vary over time; but this assumption makes the parameters N_i in the reduced form so complicated as to make estimation very difficult. For this reason and for some other reasons which we will discuss in the following sections we decided not to use the reduced form of the model in our estimations but instead to use the structural form of the model

where the possible mispecification in one equation does not carry over to others.

6.5 Shares of Labour and Capital, in GDP for the Two Sectors

Table 5.10 of the previous chapter represents the net national income for the two sectors; more precisely it represents the wages and salaries and the net profits for the non-agricultural sector. Of course, the figures in table 5.10 are net values as far as depreciation of capital and transations with the rest of the world are concerned.

From the National Accounts of Greece we have the following accounting identities:

$$GDP = NNI + D - NIA \tag{6.5.1}$$

$$NIA = MI - XI = (MP + MW) - (XP + XW) \tag{6.5.2}$$

where MP = imports of profits from the rest of the world

MW = imports of wages from the rest of the world

XP = exports of profits to the rest of the world

XW = exprots of wages to the rest of the world

the other variables have been explained in 6.4.

Assuming now that all the transactions of profits and wages with the rest of the world belong to the non-agricultural sector, the above identities can be split into the following accounting identities:

$$GDP_1 = NNI_1 + D_1 = NW_1 + (NP_1 + D_1) = W_1 + GP_1 \tag{6.5.3}$$

$$GDP_2 = NNI_2 + D_2 - NIA =$$

$$= (NW_2 - MW + XW) + (NP_2 - MP + XP + D_2)$$

$$= W_2 + GP_2 \tag{6.5.4}$$

where W = wages and salaries

NP = net profits

GP = gross profits

NW = net wages and salaries

Using the accounting identity (6.5.4) and the available data we can construct time series for wages and salaries and gross profits for the non-agricultural sector. Unfortunately, for the agricultural sector, we do not have any data which can be applied directly to (6.5.3) to produce the desired variables.

In table 5.10 the wages and salaries in the non-agricultural sector are given in current prices. Using the implicit price index of table 5.21, for the non-agricultural sector, we transformed this data into constant 1958 prices. Also, in table 5.24 the various income payments are in current prices. Using an implicit price deflator for net payments (dividing current by constant 1958 prices net payments) we transformed this data into constant 1958 prices. The results are given in table 6.10.

Using now the data of tables 5.24 and 6.10 and that of GDP from table 5.6 and with the help of the accounting identity (6.5.4) we calculated wages and salaries and gross profits for the non-agricultural secotr which we have presented in table 6.11. In calculating wages and salaries for the agricultural sector we used the expression:

$$w_1(t) = \mu(t)\, w_2(t) \qquad (6.5.5)$$

For the exogenous variable $\mu(t)$, which shows the ratio of the two wages, we used the data from table 5.11. As we see in table 5.11 the existing figures are for three years only, 1951, 1961 and 1971, and so the corresponding $\mu(t)$ takes the values $\mu(51) = 0.2476$, $\mu(61) = 0.2656$ and $\mu(71) = 0.2711$ which we found by dividing the income of the farmers and the income of the self-employed persons in the non-agricultural sector. For the values of $\mu(t)$ for the intermediate years we used straight-line trends between 1951 and 1961 and between 1961 and 1971. The time series for $\mu(t)$ so constructed is shown in table 6.12. In the same table it is shown the wages and salaries per worker, found by

TABLE 6.10

Income payments from and to the rest of the world
(in million drs. and in constant 1958 prices)

Year	Deflator	Payments from abroad		Payments to abroad	
		Profits	Wages & Salaries	Profits	Wages & Salaries
1950	0.536	22.4	476.1	119.5	–
1951	0.584	29.1	601.3	75.4	–
1952	0.587	42.6	728.5	68.1	–
1953	0.886	37.2	656.8	97.1	–
1954	1.033	61.0	882.7	98.7	–
1955	1.031	73.7	996.3	160.1	–
1956	1.070	162.7	1332.2	72.9	–
1957	1.094	188.3	1791.1	70.4	–
1958	1.000	749.0	852.0	89.0	53.0
1959	1.003	849.2	871.2	104.7	84.7
1960	0.976	1230.2	1103.2	178.2	92.2
1961	0.959	1682.7	1244.5	309.8	130.4
1962	0.944	1907.6	1317.7	362.2	108.0
1963	0.921	2393.5	1371.6	381.2	189.0
1964	0.953	3050.4	1390.9	414.3	512.9
1965	0.975	3144.7	1354.5	550.6	226.6
1966	0.985	3680.8	1339.3	706.7	355.4
1967	0.978	3962.5	1775.8	956.9	385.4
1968	0.991	4407.9	2135.3	1078.8	376.4
1969	0.989	4673.2	2098.5	1325.2	418.5
1970	1.038	5120.0	2164.6	1745.5	445.1
1971	1.045	6564.8	3095.2	2079.4	570.6
1972	1.133	6988.4	4452.1	2188.6	743.9

6.5.5 for the agricultural sector and using the data of tables 6.11 and 6.2 for the non-agricultural sector. We must note here that the estimates of wages and salaries in the agricultural sector depend heavily on the estimate of $\mu(t)$. Unfortunately, there was no other reasonable way in estimating these figures. From tables 6.12 and 5.6 we can derive the wages and salaries and gross profits for the agricultural sector in constant 1958 prices, shown in table 6.13.

Dividing now the wages and salaries and the gross profits by the GDP in both sectors,

TABLE 6.11

Wages–Salaries and Gross Profits for the Non-agricultural Sector
(in million drs.)

Year	Current Prices		Constant 1958 prices	
	Wages & Salaries	Gross Profits	Wages & Salaries	Gross Profits
1954	16713	22047	20199.8	26167.2
1955	19546	25219	21736.3	27749.7
1956	23458	28737	24032.7	29150.3
1957	24065	31254	24576.7	31498.3
1958	26287	34285	26287.0	34285.0
1959	28193	35856	27767.3	35309.7
1960	30619	39768	29408.6	38358.4
1961	33337	43176	31404.8	40905.2
1962	36216	46869	33221.7	43365.3
1963	39593	51575	35434.8	46622.2
1964	45302	56885	39666.0	50131.0
1965	52488	63120	44791.6	54239.4
1966	59934	69141	49307.6	57277.4
1967	66530	73626	52878.2	59197.8
1968	73753	83193	57640.0	65841.0
1969	82617	94974	62901.6	73144.4
1970	92074	108197	67805.7	80497.3
1971	102556	121130	73591.9	88216.1
1972	117074	142325	81100.2	100334.8

we can find the shares of wages and salaries and gross profits out of GDP respectively for each sector. In deriving the gross profits we used the accounting identity:

$$GP_i = GDP_i - W_i \qquad (i = 1, 2) \tag{6.5.6}$$

ie. the gross profits is the residual of GDP less wages and salaries. Therefore, the shares of wages and salaries and gross profits add up to one, which means, a priori, constant returns to scale. Hence, we cannot test our production functions for the returns to scale because we impose a priori constancy of the returns to scale. The shares are shown in table 6.14.

An investigation of the series of figures in table 6.14 show that in general the shares

TABLE 6.12

Wages & Salaries per Worker for the Two Sectors
(in constant 1958 prices)

Year	Wages & Salaries per worker		Ratio of wages
	Agricultural	Non-agricultural	$\mu(t)$
1954	3423	13520	0.2530255
1955	3603	14153	0.2548215
1956	4107	15986	0.2566175
1957	4166	16155	0.2584135
1958	4296	16516	0.2602095
1959	4913	18749	0.2620055
1960	5004	18960	0.2638015
1961	4962	18714	0.2655975
1962	5289	19848	0.2661503
1963	5461	20447	0.2667031
1964	6043	22583	0.2672559
1965	6642	24809	0.2678087
1966	7207	26818	0.2683615
1967	8042	29912	0.2689143
1968	9140	33900	0.2694671
1969	9701	35934	0.2700199
1970	10026	37048	0.2705727
1971	10234	37671	0.2711259
1972	10901	40105	0.2716783

of labour in the agricultural sector are smaller than those of the non-agricultural sector, (the opposite for the shares of capital); a quite reasonable result if we bear in mind that the agricultural sector is less developed than the non-agricultural one. Furthermore, using the general form (6.4.10) for time trends, we find for the share of capital in the agricultural sector:

$$\ln \alpha_1(t) = -0.420930 - 0.010372 \times t, \quad R^2 = 0.2787$$
$$(0.0436) \qquad (0.0038)$$

or

$$\alpha_1(t) = 0.656436 \times (0.989682)^t \tag{6.5.7}$$

and for that in the non-agricultural sector:

TABLE 6.13

Wages & Salaries and Gross Profits in the Agricultural Sector
(in million drs. and in constant 1958 prices)

Year	Wages & Salaries	Gross Profits	Year	Wages & Salaries	Gross Profits
1954	6691.3	13282.7	1964	10858.5	18097.5
1955	7045.8	14305.2	1965	11385.1	18340.9
1956	8272.9	13671.1	1966	11991.3	17763.7
1957	8385.1	16422.9	1967	13894.6	17612.4
1958	8444.1	14501.9	1968	16054.4	13029.6
1959	10360.9	13734.1	1969	16153.5	14382.5
1960	10360.7	11920.3	1970	15226.4	17946.6
1961	9726.8	17519.2	1971	13614.0	20421.0
1962	10313.4	14964.6	1972	13057.7	22138.3
1963	10155.3	17070.7			

TABLE 6.14

Shares of Labour and Capital out of GDP for the two Sectors

Year	Agricultural Sector		Non-agricultural sector	
	Labour	Capital	Labour	Capital
1954	0.335	0.665	0.436	0.564
1955	0.330	0.670	0.439	0.561
1956	0.377	0.623	0.452	0.548
1957	0.338	0.662	0.438	0.562
1958	0.368	0.632	0.434	0.566
1959	0.430	0.570	0.440	0.560
1960	0.465	0.535	0.434	0.566
1961	0.357	0.643	0.434	0.566
1962	0.408	0.592	0.434	0.566
1963	0.373	0.627	0.432	0.568
1964	0.375	0.625	0.442	0.558
1965	0.383	0.617	0.452	0.548
1966	0.403	0.597	0.463	0.537
1967	0.441	0.559	0.472	0.528
1968	0.552	0.448	0.467	0.533
1969	0.529	0.471	0.462	0.538
1970	0.459	0.541	0.457	0.543
1971	0.400	0.600	0.455	0.545
1972	0.371	0.629	0.447	0.553

$$\ln \alpha_2(t) = \underset{(0.0084)}{-0.565927} \underset{(0.007)}{- 0.002644} \times t, \quad R^2 = 0.4068$$

or

$$\alpha_2(t) = 0.567833 \times (0.997359)^t \qquad (6.5.8)$$

From (6.5.7) and (6.5.8) we see that both shares of capital show a decreasing time trend with that of the agricultural sector, which is $j_1 = 0.010318$, being higher than that of the non-agricultural sector, which is $j_2 = 0.002641$.

In fact, the decreasing rate of the share of capital in the non-agricultural sector is very small so we could take this share as constant for the period under study. For the agricultural sector the capital share is decreasing till the year 1968. After that year it is increasing (possibly due to the political instability). This result is depicted into the small R^2.

The measurement of the wage share of national income has received much attention. One of the problems faced concerns magnitudes that the notion of wages must include and the aggregate quantities in which the share must be measured. Dunlop (1950) has charted labour's share for a great variety of alternative concepts but his graphs show no statistical grounds for choosing between alternative formulations. Johnson (1954) imputing a wage to self-employed persons found from the first decade of this century to the post World War II period, for the American Economy, a growth in labour's share of about 5 per centage points. Kravis (1959) is a study of Johnson's and later figures, adduces reasons for an increasing trend in labour's share. Weintraub (1959) and Klein and Kosobud (1961) use the notion of the constancy of labour's share. Our results for the share of labour in the two sectors for the Greek economy show an increasing trend for the agricultural sector whilst that of the non-agricultural sector is practically constant.

6.6 Net Capital Stock, Rental Rates and the Propensity to Invest out of Profits for the Two Sectors

Probably, one of the most difficult data problems is the problem of existing data for the net capital stock in Greece. There were some attempts of estimating capital stock for the manufacturing sector, Krengel-Mertens (1967) and for the non-agricultural sector, Germidis-Negreponti-Delivanis (1975). Others, took an arbitrary capital stock/GDP ratio for a basic year and derived the subsequent figures by cumulating net capital formation for the total economy. For example, Adelman and Chenery (1966) assumed that the capital stock /GDP ratio for the Greek economy for the year 1950 was equal to three.

We will start our analysis by trying to estimate net capital formation for the two sectors. First, we have split out into the two sectors total depreciation which has been presented into tables 5.16 and 5.17. For this reason we employ the accounting identities:

$$D_1 = GDP_1 - NNI_1 \qquad (6.6.1)$$

$$D_2 = GDP_2 - NNI_2 + NIA \qquad (6.6.2)$$

where the symbols have been explained in the previous sections.

Using the figures for NNI_i (i = 1, 2) in table 5.10 (or transforming NNI_1 into constant 1958 prices by the price deflator for the agricultural sector of table 5.21), those of NIA from table 5.24 and those of GDP_i (i = 1, 2) from table 5.6 we found depreciation in the two sectors which has been presented in the table 6.15. In fact having found depreciation in the agricultural sector, we subtracted those figures from total depreciation to find depreciation in the non-agricultural sector.

Subtracting now depreciation from gross investment we find net investment in constant 1958 prices as presented in table 6.16.

To calculate net capital stock in the non-agricultural sector we need an initial value,

TABLE 6.15

Depreciation in the Two Sectors (in million drs.)

Year	Current Prices		Constant 1958 prices	
	Agricultural	Non-agricultural	Agricultural	Non-agricultural
1950	94	1392	156.3	2793.7
1951	119	1725	187.3	2965.7
1952	136	1908	202.7	3099.3
1953	183	2234	228.6	3159.4
1954	257	2976	295.4	3227.6
1955	262	3231	283.3	3390.7
1956	279	3568	272.0	3626.0
1957	322	3858	329.0	3804.0
1958	398	3921	398.0	3921.0
1959	457	4364	481.4	4106.6
1960	520	4782	507.0	4408.0
1961	586	5125	557.0	4685.0
1962	657	5696	588.0	5035.0
1963	706	6538	610.8	5408.2
1964	786	7291	654.7	5906.3
1965	881	8095	681.1	6489.9
1966	1024	9173	762.8	7334.2
1967	1167	10082	866.2	7816.2
1968	1270	11306	927.2	8423.8
1969	1408	12591	1009.1	9324.9
1970	1499	14439	1064.8	10265.2
1971	1697	16844	1127.7	11191.3
1972	1875	20183	1114.5	12317.5

ie. for a base year. We are taking 1950 as our base year for which the capital stock in the non-agricultural sector is 97223.0 million drs. in constant 1958 prices as has been estimated by Germidis and Negreponti-Delivanis (1975). The subsequent figures derived by cumulating net investment, table 6.16.

Unfortunately, for the agricultural sector there is no such estimate for a base year. Therefore, we estimated net capital stock in the agricultural sector in the following way. We assumed that for the base year the rate of return on capital is the same in both sectors (as in our model). This rate of return can be found by dividing gross profits by net capital

TABLE 6.16

Net Investment and Net Capital Stock in the Two Sectors
(in million drs. and in constant 1958 prices).

Year	Agricultural Sector		Non-agricultural sector	
	Net investment	Net capital stock	Net investment	Net capital stock
1950	1181.7	55140.9	8123.3	97223.0
1951	2874.7	58015.6	6029.3	103252.3
1952	60.3	58075.9	5532.7	108785.0
1953	2796.4	60872.3	6362.6	115147.6
1954	57.6	60929.9	4823.4	119971.0
1955	1501.7	62431.6	6162.3	126133.2
1956	1483.0	63914.6	8914.0	135047.2
1957	3054.0	66968.6	8878.0	143925.2
1958	1878.0	68846.6	11380.0	155305.2
1959	2276.6	71123.2	11042.4	166347.6
1960	2075.0	73198.2	11976.0	178323.6
1961	5812.0	79010.2	13591.0	191914.6
1962	1587.0	80597.2	17015.0	208929.6
1963	5347.2	85944.4	15068.8	223998.4
1964	1937.3	87881.7	25168.7	249167.1
1965	5659.9	93541.6	25982.1	275149.2
1966	2663.2	96204.8	25760.8	300910.0
1967	5095.8	101300.6	23790.2	324880.2
1968	4196.8	105497.4	31101.2	355981.4
1969	4939.9	110437.3	35907.1	391888.5
1970	6302.2	116739.5	38164.8	430053.3
1971	7368.3	124107.8	41727.7	471781.0
1972	7088.5	131196.3	50611.5	522392.5

stock in the non-agricultural sector. For the year 1954, (this is the year for which the time series in gross profits of the non-agricultural sector starts) the rate of return on capital was found to be equal to $r_2(1954) = 0.218$. Dividing now gross profits of the agricultural sector for the year 1954 by $r_2(1954) = 0.218$ we found a starting value of the net capital stock in the agricultural sector, and therefore we estimated the whole time series as in table 6.16.

The time series for the rates of return on capital, using the formula:

$$r_i(t) = GP_i/NKS_i \qquad (i = 1, 2) \tag{6.6.3}$$

where NKS = net capital stock, are shown in table 6.17. From this table we have to do some comments. Firstly, the two rates of return are not equal, as it has been assumed for our theoretical two sector growth model. It is possible this difference to be real or to be fictional, because a lot of the data have been estimated with one or other way. Therefore,

TABLE 6.17

Rates of Return on Capital

Year	Agricultural $r_1(t)$	Non-agricultural $r_2(t)$	Rental ratio $r_1(t)/r_2(t)$	Wage/rental ratio, $\omega_1(t)/\omega_2(t)$
1954	0.218	0.218	1.000	0.253
1955	0.229	0.220	1.041	0.244
1956	0.214	0.216	0.991	0.259
1957	0.245	0.219	1.119	0.230
1958	0.211	0.221	0.955	0.273
1959	0.193	0.212	0.910	0.288
1960	0.163	0.215	0.758	0.349
1961	0.222	0.213	1.042	0.255
1962	0.186	0.208	0.894	0.298
1963	0.199	0.208	0.957	0.280
1964	0.206	0.201	1.025	0.261
1965	0.196	0.197	0.995	0.269
1966	0.185	0.190	0.974	0.277
1967	0.174	0.182	0.956	0.282
1968	0.124	0.185	0.670	0.404
1969	0.130	0.187	0.695	0.387
1970	0.154	0.187	0.824	0.330
1971	0.165	0.187	0.882	0.309
1972	0.169	0.192	0.880	0.309

if this approximation of the missing data is not quite correct then it will produce further problems in the estimation of the parameters. If we assume that the missing data have been estimated in the best possible way then we have to accept that the difference in the two rental rates is real and not fictional. Of course, here we will not explain the rationale for such difference in the rental rates but the interested reader can look in Kelley-Williamson

& Cheetham (1972) for more details.

Secondly, both rental rates are decreasing over time with the rental rate in the agricultural sector decreasing faster. From the data on the private investment we see that for the decade 1954–1963 the percentage of investment in agriculture out of total investment was an average 11.25% and that for the decade 1963–1972 it was 9.96%, (estimated by constant 1958 prices values). This means that the faster decrease of rental rate in agriculture was the reason for the switch of the individuals preferences in investing more in the non-agricultural sector. This behaviour of the individual supports the case that there is a gap in the two rental rates.

Thirdly, the decrease of the rental rates is an expected result because the input capital is increasing more than the input labour.

Employing again the form for time trends (6.4.10) we have that:

$$\ln r_1(t) = -1.440331 - 0.024313 \times t, \quad R^2 = 0.5578$$
$$(0.0566) \quad (0.0050)$$

or

$$r_1(t) = 0.236849 \times (0.975980)^t \tag{6.6.4}$$

and

$$\ln r_2(t) = -1.483202 - 0.011335 \times t, \quad R^2 = 0.8493$$
$$(0.0125) \quad (0.0011)$$

or

$$r_2(t) = 0.226910 \times (0.988729)^t \tag{6.6.5}$$

from which we can see that the average annual decrease of the rental rate in agriculture is $j_1 = 0.02402$ which is greater than that of the non-agricultural sector, $j_1 = 0.011271$.

The difference in the two rental rates causes the ratio of the wage/rental ratios of the two sectors to be different from the ratio of the wages $\mu(t)$. This new ratio, $\mu^*(t)$

is shown in table 6.17 and its time trend is given by:

$$\ln \mu^*(t) = \underset{(0.0541)}{-1.409490} + \underset{(0.0047)}{0.016895} \times t, \quad R^2 = 0.4006$$

or

$$\mu^*(t) = 0.244268 \times (1.017039)^t \tag{6.6.6}$$

From (6.6.6) we see that this ratio is increasing with rate j = 0.017039.

Table 5.20 represents as we have seen the finance of gross domestic asset formation in current prices. Using an implicit deflator for total investment (ie. dividing total investment expressed in current prices by the corresponding expressed in constant 1958 prices), we transformed the data of table 5.20 in those of table 6.18 of constant 1958 prices. Dividing now gross domestic asset formation financed domestically by the total profits we find the propensity to invest out of profits, as shown in table 6.18.

We see, that the investment/profits ratio in the economy as a whole shows a steady upward-trend. This trend has been estimated by:

$$\ln s_3(t) = \underset{(0.0526)}{-1.503427} + \underset{(0.0046)}{0.050819} \times t \quad , \qquad R^2 = 0.8646$$

or

$$s_3(t) = 0.222367 \times (1.052132)^t \tag{6.6.7}$$

The growth of this ratio which the average annual increase is j = 0.052132 is one of the most promising ratios for the development of Greece. This type of savings, out of capital gains, has been discussed by Shell-Sidrauski–Stiglitz (1969).

Returning now to our model we see from equation (6.2.10) that the propensity to invest s_3 is constant over time. Therefore, this equation is also mispecified as far as the Greek data is concerned. For this reason and for the other two, ie. the variable consumption allocation parameter and the difference in the return of capital for the two sectors,

TABLE 6.18

Propensity to Invest Out of Profits and Other Relevant Data
(data in million drs. and in constant 1958 prices)

Year	Investment finance from Domestic	Abroad	Total Profits	Deflator of Investment	Propensity to Invest
1950	3311.3	8943.7	–	0.499	–
1951	5074.7	6982.3	–	0.591	–
1952	4172.6	4722.4	–	0.602	–
1953	10645.0	1902.0	–	0.701	–
1954	6123.2	2280.8	39449.9	0.855	0.155
1955	10453.7	884.3	42054.9	0.945	0.249
1956	12195.3	2099.7	42821.4	0.983	0.285
1957	14123.8	1941.2	47921.2	0.985	0.295
1958	14564.0	3013.0	48786.9	1.000	0.299
1959	16163.0	1744.0	49043.8	1.019	0.330
1960	16491.6	2474.4	50278.7	1.031	0.328
1961	22156.6	2488.4	58424.4	1.045	0.379
1962	21666.5	2558.5	58329.9	1.104	0.371
1963	24070.1	2364.9	63692.9	1.146	0.378
1964	28213.0	5454.0	68228.5	1.160	0.414
1965	29885.5	8927.5	72580.3	1.164	0.412
1966	33014.3	3506.7	75041.1	1.199	0.440
1967	33687.2	3881.8	76810.2	1.223	0.439
1968	37799.3	6849.7	78870.6	1.239	0.479
1969	42694.7	8486.3	87526.9	1.266	0.488
1970	48520.7	7276.3	98443.9	1.306	0.493
1971	57200.7	4214.3	108637.1	1.401	0.527
1972	67741.5	3390.5	122473.1	1.510	0.553

we cannot use the reduced form of the model in estimating the other parameters, and therefore we will try to estimate the parameters from the structural form.

Finally, we will investigate other four great ratios of economics of growth; the capital/labour ratio and the capital/output ratio, presented in table 6.19 and in figures 6.2 and 6.3.

The time series for the capital/labour ratios in the two sectors show a steady upward growth. Our trend formulas are respectively:

TABLE 6.19

Capital/labour and Capital/output Ratios

Year	Capital/labour ratios in constant 1958 prices		Capital/output ratios	
	Agriculture	Non-agriculture	Agriculture	Non-agriculture
1950	30095	69584	3.616	2.579
1951	31124	73021	3.350	2.593
1952	30678	74834	3.563	2.669
1953	30025	83446	2.956	2.625
1954	31166	80301	3.050	2.587
1955	31925	82131	2.924	2.549
1956	31726	89830	2.913	2.539
1957	33275	94607	2.699	2.567
1958	35023	97577	3.000	2.564
1959	33728	112323	2.952	2.637
1960	35356	114970	3.285	2.631
1961	40302	114360	2.900	2.654
1962	41329	124823	3.188	2.728
1963	46220	129251	3.157	2.730
1964	48906	141856	3.035	2.775
1965	54573	152398	3.147	2.778
1966	57823	163662	3.233	2.823
1967	58632	183779	3.215	2.899
1968	60061	209367	3.627	2.883
1969	66323	223873	3.617	2.881
1970	76869	234975	3.519	2.900
1971	93292	241498	3.646	2.916
1972	109526	258332	3.728	2.879

$$\ln k_1(t) = \underset{(0.0402)}{3.362569} + \underset{(0.0039)}{0.054157} \times t, \quad R^2 = 0.8948$$

or

$$k_1(t) = 28.863245 \times (1.055650)^{t} \qquad (6.6.8)$$

and

$$\ln k_2(t) = \underset{(0.0199)}{4.337976} + \underset{(0.0019)}{0.061764} \times t, \quad R^2 = 0.9784$$

or

$$k_2(t) = 76.552440 \times (1.063711)^{t} \qquad (6.6.9)$$

From (6.6.8) and (6.6.9) we see that the capital/labour ratio for the non-agricultural sector is growing with the rate of $j_2 = 0.063711$ which is greater than that of the agricultural sector, $j_1 = 0.05565$.

Also, from table 6.19 or from figure 6.2 it is obvious that the capital/labour ratio, or the capital intensity, in the non-agricultural sector is greater than that of the agricultural sector for every year. This result is consistent with a basic assumption of our two-sector growth model of the previous chapter, ie. with the assumption that the non-agricultural sector is the more capital intensive.

The series for the capital/output ratios in the two sectors are also increasing over time.

According to table 6.19 or to figure 6.3 the capital/output ratio in the non-agricultural sector showed an upward trend. We could say that this increase may be attributed to the continuation of capital formation at an accelerated rate and to the rather unsatisfactory distribution of total investment among subsectors of economic activity of the non-agricultural sector, which reduces the return on capital for the sector as a whole. Our trend formula is:

$$\ln \frac{K_2(t)}{Y_2(t)} = \underset{(0.0067)}{0.944571} + \underset{(0.0006)}{0.006515} \times t, \quad R^2 = 0.8184$$

or

$$\frac{K_2(t)}{Y_2(t)} = 2.571710 \times (1.006536)^t \qquad (6.6.10)$$

In contrast to the steady increase of the capital/output ratio in the non-agricultural sector this ratio for the agricultural sector is on average increasing after the year 1957 but not at a steady rate, as we can see from figure 6.3. The behaviour of this ratio depends heavily on a lot of factors. Firstly, in the first decade for the period under consideration,

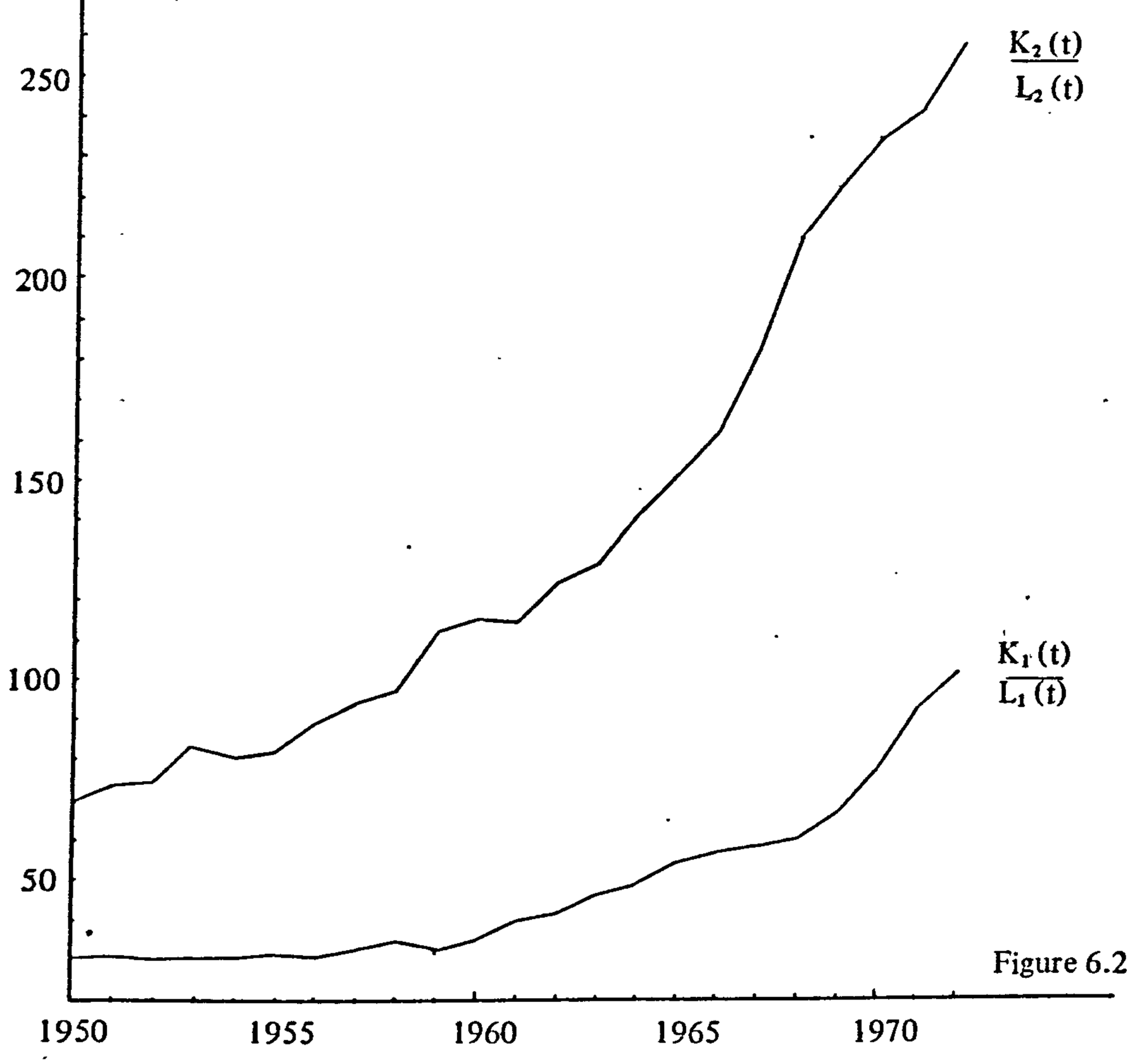

Figure 6.2

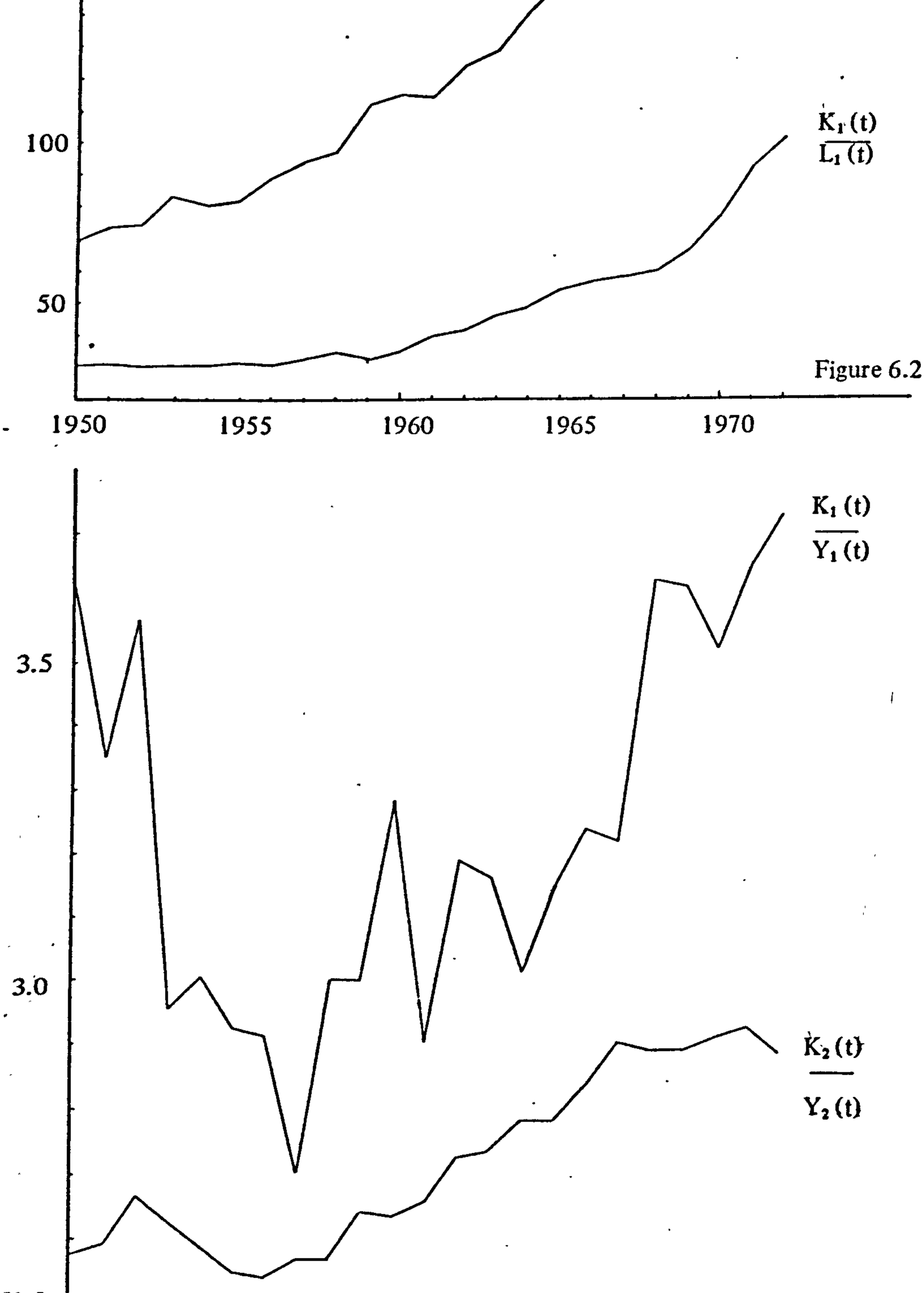

Figure 6.3

the net investment in this sector was rather small and unstable (table 6.16) with the consequence that the capital/output ratio may be decreasing until the year 1957. Secondly, the behaviour of the capital/output ratio depends upon the type of investment; after the sixties investment went more into land improvement. Thirdly, on the weather and on the cycle of oil production which is one of the most important goods in agricultural production. Of course, there are also other unstable factors to be added to the three factors mentioned above. The trend of this ratio, without excluding the years before 1957 for which this ratio is decreasing, is given by:

$$\ln \frac{K_1(t)}{Y_1(t)} = \underset{(0.0268)}{1.123415} + \underset{(0.0026)}{0.005676} \times t, \quad R^2 = 0.1744$$

or

$$\frac{K_1(t)}{Y_1(t)} = 3.075339 \times (1.005692)^t \qquad (6.6.11)$$

The very small R^2 is (6.6.11) shows also the unstable behaviour of this ratio. Of course, we cannot compare (6.6.11) with (6.6.10) because in (6.6.11) the trend coefficient is significant.

From these four ratios of economic growth we must finally note that these increases are the natural consequence of the continuing economic development effort.

6.7 The Econometrics Used

A particular feature of the system that we wish to estimate is that the same parameters appear in many equations. We need to incorporate this restriction into the estimation technique adopted. To do this we amalgamate all the equations into a single block.

For example, say we had a two equation system with the same parameter α_2 appearing in both equations, ie.:

$$\left.\begin{aligned} y_1 &= \alpha_0 + \alpha_1 x_1 + \alpha_2 x_2 + \epsilon_1 \\ y_2 &= \beta_0 + \quad\; + \alpha_2 x_2 + \epsilon_2 \end{aligned}\right\} \qquad (6.7.1)$$

We can then incorporate this restriction by writing the system in the following way:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} i & o & x_1 & x_2 \\ o & i & o & x_3 \end{bmatrix} x \begin{bmatrix} \alpha_0 \\ \beta_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \end{bmatrix} \qquad (6.7.2)$$

So, we will estimate the system in the form:

$$y = X\,\beta + \epsilon \qquad (6.7.3)$$

where $y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix}$ is an nmx1 vector; n = number of observations in each equation and m = number of equations,

$\epsilon = \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon m \end{bmatrix}$ is an nmx1 vector; the vector of the disturbance terms,

X is an nmxk matrix of the restricted form described above and β is an kx1 vector; vector of the parameters of the system, where k is the number of the parameters in the system.

In the case of a single equation:

$$y_i = X_i\,\beta_i + \epsilon_i \qquad i = 1, \ldots\ldots, m \qquad (6.7.4)$$

with $E(\epsilon_i) = 0$ and $E(\epsilon_i\,\epsilon'_i) = V_i$

where the disturbance follows a first-order autoregresive scheme:

$$\epsilon_{it} = p_i\,\epsilon_{i,t-1} + u_{it}\,, \qquad \begin{matrix} i = 1, \ldots\ldots, m \\ t = 1, \ldots\ldots, n \end{matrix} \qquad (6.7.5)$$

where $|p| < 1$ and u_t satisfies the assumptions

$$E(u_{it}) = 0$$

$$E(u_{it}\ u_{i,t+s}) = \sigma^2_{ui} \quad \text{for } s = 0 \quad \text{for all } t \qquad (6.7.6)$$
$$= 0 \quad \text{for } s \neq 0$$

It is known that

$$V_i = \sigma^2_{\epsilon_i} \begin{bmatrix} 1 & p_i & p_i^2 & \cdots & p_i^{n-1} \\ p_i & 1 & p_i & \cdots & p_i^{n-2} \\ \vdots & \vdots & \vdots & & \vdots \\ p_i^{n-1} & p_i^{n-2} & p_i^{n-3} & \cdots & 1 \end{bmatrix} = \sigma^2_{\epsilon_i}\ \Omega_i \qquad (6.7.7)$$

Since the dispersion matrix for ϵ_i is not scalar, ordinary leas-squares (OLS) is inefficient, and best linear unbiased estimates of β_i are provided by generalized least-squares (GLS), Aitkens (1934). If, for example, it is known that the disturbance follows a first-order scheme and if the value of the parameter p_i is known then GLS may be applied directly and the estimator is:

$$b_i = (X_i'\ \Omega_i^{-1}\ X_i)^{-1}\ X_i'\Omega_i^{-1}\ y_i \qquad (6.7.8)$$

The estimated dispersion matrix for b_i is

$$\text{var}(b_i) = s_i^2\ (X_i'\Omega_i^{-1}\ X_i)^{-1} \qquad (6.7.9)$$

where

$$s_i^2 = e_i'\ \Omega_i^{-1}\ e_i / (n-k_i) \text{ and } e_i = y_i - X_i\ b_i \qquad (6.7.10)$$

and

$$\Omega_i^{-1} = \frac{1}{1-p_i^2} \begin{bmatrix} 1 & -p_i & o & \cdots & o & o & o \\ -p_i & 1+p_i^2 & -p_i & \cdots & o & o & o \\ o & -p_i & 1+p_i^2 & \cdots & o & o & o \\ \vdots & \vdots & \vdots & & \vdots & \vdots & \vdots \\ o & o & o & \cdots & -p_i & 1+p_i^2 & -p_i \\ o & o & o & \cdots & o & -p_i & 1 \end{bmatrix} \qquad (6.7.11)$$

The above estimates can be obtained by a simple two-stage procedure, as follows: First, we transform the original observations by appropriate use of p_i which is assumed to be known and second, we apply OLS to the transformed data.

The transformation matrix T_i such that the relation:

$$T_i y_i = T_i X_i \beta_i + T_i \epsilon_i \qquad (6.7.12)$$

will have a scalar dispersion matrix, ie.:

$$E(T_i \epsilon_i \epsilon_i' T_i') = \sigma_{ui}^2 I \qquad (6.7.13)$$

is found (Kadiyala, 1968) to be:

$$T_i = \begin{bmatrix} \sqrt{1-p^2}_i & o & o & \ldots\ldots & o & o & o \\ -p_i & 1 & o & \ldots\ldots & o & o & o \\ o & -p_i & 1 & \ldots\ldots & o & o & o \\ & & & & & & \\ o & o & o & \ldots\ldots & -p_i & 1 & o \\ o & o & o & \ldots\ldots & o & -p_i & 1 \end{bmatrix} \qquad (6.7.14)$$

so that the OLS may be applied to the transformed data $T_i y_i$ and $T_i X_i$ of (6.7.12).

Instead of using the matrix T_i a more simple case is to use an other one, T_i^*, which is merely T_i with the first row deleted. Therefore, using T_i^*, we are losing one observation and each transformed observation, say Z_{it}^* of Z_{it}, is given by:

$$Z_{it}^* = Z_{it} - p_i Z_{i,t-1} \ , \ t = 2, \ldots\ldots, n \qquad (6.7.15)$$

It can be shown that applying OLS in (6.7.12) we will obtain the estimator (6.7.8) and applying OLS in (6.7.12) using the matrix T_i^* we will obtain a close approximation of (6.7.8).

In the above methods we assumed that we know the autocorrelation structure and the value(s) of the parameter(s) p_i. In general, neither the order of the autocorrelation structure is known nor the values of the p_is. Of course, various methods of approximating the structure and the values of parameters of the autocorrelation scheme have been proposed.

The Cochrane-Orcutt (1949) method is a two-step procedure according to which in the first step we compute $\hat{p}_i$ from the OLS (applied to 6.7.4) residuals using the formula:

$$\hat{p}_i = \sum_{i=1}^{n} \hat{\epsilon}_i(t)\ \hat{\epsilon}_i(t-1) \Big/ \sum_{i=1}^{n} [\hat{\epsilon}_i(t-1)]^2 \qquad (6.7.16),$$

and the second step is of estimating β_i by applying OLS to the transformed data using (6.7.15). This method corresponds to GLS estimation using the T_i^* matrix. Also, this method can be extended to more than one iterations computing successive p_i's from the corresponding residuals; ie. we obtain a new set of residuals by $\hat{\epsilon}_i^{(1)} = y_i - X_i \hat{\beta}_i^{(1)}$ and calculate a new value of $\hat{p}_i$, $\hat{p}_i^{(1)}$, and so on.

The Durbin (1960) method is also a two-step procedure. The first step is to compute an estimate of p_i by applying OLS to:

$$y_{it} = p_i y_{i,t-1} + \beta_{i1} X_{i1}t - p_i\beta_{i_1} X_{i_1,t-1} + \ldots\ldots + \beta_{ik_i} X_{ik_i}t - p_i \beta_{ik_i} X_{ik_i,t-1} + u_{it} \qquad \begin{matrix} i = 1, \ldots\ldots, m \\ t = 2, \ldots\ldots, n \end{matrix} \qquad (6.7.17)$$

where k_i is the number of parameters to be estimated. The second step is to estimate β_i by applying OLS to:

$$y_{it} - \hat{p}_i y_{i,t-1} = \beta_{i_1} (X_{i_1}t - \hat{p}_i X_{i_1,t-1}) + \ldots\ldots + \beta_{ik_i} (X_{ik_i}t - \hat{p}_i X_{ik_i,t-1}) + u_{it} \qquad (6.7.18)$$

This method can be extended quite simply to higher order autoregressive schemes.

Finally, Phillips (1956) method which is also a two-step procedure. The first step is that of Durbin's method, ie. estimating p_i and β_i, say $\hat{p}_i^0$ and $\hat{\beta}_i^0$, using (6.7.17), Using a formal Taylor expansion we have:

$$p_i \beta_i = \hat{p}_i^0 \hat{\beta}_i^0 + (\beta_i - \hat{\beta}_i^0)\hat{p}_i^0 + (p_i - \hat{p}_i^0)\hat{\beta}_i^0 + (p_i - \hat{p}_i^0)(\beta_i - \hat{\beta}_i^0) \qquad (6.7.19)$$

and neglecting the last term of (6.7.19), (6.7.17) is transformed into the following:

$$[y_{it} - \hat{p}_i^0(\hat{\beta}_{i_1}^0 X_{it,t-1} + \ldots + \hat{\beta}_{ik_i}^0 X_{ik_i,t-1})] = \beta_{i_1}(X_{i_1}t - \hat{p}_i^0 X_{i_1,t-1}) + \ldots\ldots +$$

$$+ \beta_{ik_i}(X_{ik_i}t - \hat{p}_i^0 X_{ik_i,t-1}) + p_i(y_{i,t-1} - \hat{\beta}_{i_1}^0 X_{i_1,t-1} - \ldots\ldots - \hat{\beta}_{ik_i}^0 X_{ik_i,t-1}) + u_{it} \qquad (6.7.20)$$

from which applying OLS we estimate the parameters. This method can be extended to

more than one iterations computing successive p_i's and β_i's from the equations (6.7.20) and so repeating the whole procedure.

Sargan (1964) and Oberhofer and Kmenta (1974) show that these methods represent a convenient way of calculating the maximum likelihood estimates for the autoregressive model. Also, Griliches and Rao (1969) showed that the above methods give a gain in efficiency, compared with OLS even in small samples and especially when $|p_i| > 0.3$.

For further discussion on the above methods see Theil (1971) and Johnston (1972).

In our case, where we do not have a single equation but a system of equations, as in equation (6.7.2), we apply the above methods to each equation individually and then amalgamate all the transformed equations into a single block. Therefore, instead of having the single transformation equation (6.7.12) we have the transformed system:

$$Ty = TX\beta + T\epsilon \tag{6.7.21}$$

where

$$T = \begin{bmatrix} T_1 & o & \cdots\cdots & o \\ o & T_2 & \cdots\cdots & o \\ \vdots & \vdots & & \vdots \\ o & o & \cdots\cdots & T_m \end{bmatrix} \tag{6.7.22}$$

In the case where we estimate a common p for all the equations in the system (6.7.21), treating system (6.7.21) as a single equation, then in (6.7.22) we have that:

$$T_1 = T_2 = \cdots\cdots = T_m = \bar{T} \tag{6.7.23}$$

If we apply (6.7.21) into our simple example of (6.7.2) we obtain:

$$\begin{bmatrix} T_1 y_1 \\ \\ T_2 y_2 \end{bmatrix} = \begin{bmatrix} T_1 i & O & T_1 x_1 & T_1 x_2 \\ \\ O & T_2 i & O & T_2 x_3 \end{bmatrix} x \begin{bmatrix} \alpha_0 \\ \beta_0 \\ \alpha_1 \\ \alpha_2 \end{bmatrix} + \begin{bmatrix} T_1 \epsilon_1 \\ \\ T_2 \epsilon_2 \end{bmatrix} \tag{6.7.23}$$

We must find the necessary and sufficient conditions for (6.7.21) to have

$$E(T \epsilon \epsilon' T') = \sigma_u^2 I_{mn} \tag{6.7.24}$$

we have that:

$$E(T\,\epsilon\,\epsilon'\,T) = \begin{bmatrix} E(T_1\,\epsilon_1\,\epsilon'_1\,T'_1) & E(T_1\,\epsilon_1\,\epsilon'_2\,T'_2) & \cdots\cdots\cdots & E(T_1\,\epsilon_1\,\epsilon'_m\,T'_m) \\ E(T_2\,\epsilon_2\,\epsilon'_1\,T'_1) & E(T_2\,\epsilon_2\,\epsilon'_2\,T'_2) & \cdots\cdots\cdots & E(T_2\,\epsilon_2\,\epsilon'_m\,T'_m) \\ \vdots & \vdots & & \vdots \\ E(T_m\,\epsilon_m\,\epsilon'_1\,T'_1) & E(T_m\,\epsilon_m\,\epsilon'_2\,T'_2) & \cdots\cdots\cdots & E(T_m\,\epsilon_m\,\epsilon'_m\,T'_m) \end{bmatrix} \quad (6.7.25)$$

We assume that in (6.7.25) the off-diagonal submatrices, eg. $E(\epsilon_i\,\epsilon'_j)$ are all zero matrices. These matrices are of the form $E(T_i\,\epsilon_i\,\epsilon'_j\,T'_j)$ where $i \neq j$ and deal with the disturbances of different equations. Therefore, for (6.7.25) to be equal to $\sigma_u^2\,I_{mn}$ we must have that:

$$E(T_i\,\epsilon_i\,\epsilon'_i\,T'_i) = \sigma_{ui}^2\;\; I_n\,,\; i = 1, \ldots\ldots, m \qquad (6.7.26)$$

which is obvious due to (6.7.13), and to further assumption that

$$\sigma_{u_1}^2 = \sigma_{u_2}^2 = \ldots\ldots = \sigma_{u_m}^2 = \sigma_u^2\,.$$

In applying the various methods of estimating the autoregressive coefficients into the system (6.7.3) of the stacked equations we have to note the following:

In the case where we assume that the autoregressive coefficient is the same for all the equations, then in using the formula (6.7.16) of the Cochrane-Orcutt method we introduce an incorrect product term in the estimate between the elements of the residual vector, of the form $\epsilon_1(n)\,\epsilon_2(1), \epsilon_2(n)\,\epsilon_3(1), \ldots\ldots, \epsilon_{m-1}(n)\,\epsilon_m(1)$. We can avoid this inconsistency by not taking into account these elements and so losing m observations.

In the case where we are using the other two methods, ie. Durbin and Phillips, we have a similar inconsistency by taking lags of the various variables.

Although we are aware of these inconsistencies, we retain the methods in the interest of simplicity.

In the case now where there is a separate first order autoregressive process for each equation we obtain the estimates of the p_i's without introducting the above inconsistencies. We treat each equation individually using the various methods for single equations. Of course, in estimating the p_i's we are losing one observation for each equation.

Having found the autoregressive coefficients with the one or the other method (ie. same or different p_i's) we continue our estimation, using the transformation matrix (6.7.22), as presented previously.

Together with the various methods mentioned above we will also perform two-stages least-squares (2SLS) estimation, Theil (1953), Basmann(1957).

6.8 Estimation of the Model

In this section we will present the results of our estimation for the model presented in section 6.2 without commenting on these, but leaving the comments for the next section.

In the model presented in section 6.2 we distinguished two subsystems of simultaneous equations. These two subsystems are the production functions with the corresponding marginal productivity conditions, for each section:

$$y_i(t) = \Gamma_i \, e^{\pi_i t} \, k_i(t)^{\alpha_i} \qquad i = 1, 2 \tag{6.8.1}$$

$$\omega_i(t) = \frac{1-\alpha_i}{\alpha_i} \, k_i(t) \tag{6.8.2}$$

From equation (6.8.1) taking its logs we have:

$$\ln y_i(t) = \ln \Gamma_i + \pi_i t + \alpha_i \ln k_i(t) \tag{6.8.3}$$

and from equation (6.8.2) we obtain:

$$\frac{k_i(t)}{\omega_i(t) + k_i(t)} = \alpha_i \tag{6.8.4}$$

Stacking equations (6.8.3) and (6.8.4) into one block, according to (6.7.3), we have:

$$\begin{bmatrix} \ln y_i(t) \\ \\ \dfrac{k_i(t)}{\omega_i(t) + k_i(t)} \end{bmatrix} = \begin{bmatrix} i & t & \ln k_i(t) \\ \\ o & o & i \end{bmatrix} x \begin{bmatrix} \ln \Gamma_i \\ \\ \pi_i \\ \alpha_i \end{bmatrix} + \begin{bmatrix} \epsilon_{1\,i}(t) \\ \\ \epsilon_{2\,i}(t) \end{bmatrix} \qquad i = 1, 2 \tag{6.8.5}$$

(i is the unit vector). In the above equations it is obvious that for the estimation we added an error term.

In the system (6.8.5) we will apply ordinary least squares (OLS) using the data of the previous sections for the agricultural (i = 1) and non-agricultural (i = 2) sectors. Also, we will apply generalized least squares using the common autocorrelation coefficient estimated by the residuals of OLS estimation (GLS:p), and generalized least squares using different autocorrelation coefficients estimated by the residuals of the OLS application to the individual equations of the system (GLS:p_i). The GLS:p method is that of applying the transformation matrix (6.7.22) with same T_i's whilst the GLS:p_i is that with different T_i's. The results of the methods above are given in table 6.20, where R^2 is calculated as the simple correlation between y and $\hat{y}$ (= $X\hat{\beta}$), D.W. is the Durbin-Watson statistic of autocorrelation, F is the F ratio, p is the simple first order auto-correlation coefficient of the estimated residuals, $\hat{v}$, $\bar{\hat{v}}$ is the mean of the $\hat{v}$, $\hat{v}_{S.D.}$ is the standard deviation of $\hat{v}$ and $\hat{v}_{R.M.S.E.}$ is the root mean squared error of $\hat{v}$. Numbers in parentheses show the standard deviation of the corresponding estimated parameter.

In the same system (6.8.5) we will perform two-stages least-squares (2SLS) for its equations. The exogenous variables of the system are taken to be the overall capital/labour ratio, k(t), the ratio of the wage/rental ratios of the two sectors, $\mu^*(t)$, and the time trend, t. Substituting the predicted values of $k_i(t)$ (the only endogenous variable lying on the R.H.S. of equations (6.8.1) and (6.8.2)) which we found by regressing $k_i(t)$ onto the exogenous variables of the system, into (6.8.5), we applied OLS, GLS:p which we denote (2SGLS:p) and GLS:p_i which we call (2SGLS:p_i), into system (6.8.5). The results of these methods are given in table 6.21.

Till now we presented the results of the estimated parameters of the model by using the two subsystems of the form (6.8.5). But in this way there is an equation, namely the relative price equation which is left outside of any estimation. Therefore, from (6.2.8)

TABLE 6.20

Agricultural Sector

Method	$\ln \Gamma_i$	π_i	α_i	R^2	D.W.	F	p	$\hat{\bar{v}}$	$\hat{v}_{S,D}$	$\hat{v}_{R.M.S.E.}$
OLS	0.276307	0.012330	0.597675	0.9973	1.1720	6399.0	0.4080	−0.0013	0.0565	0.0523
	(0.0496)	(0.0025)	(0.0129)							
GLS:p	0.272753	0.011545	0.601270	0.9946	1.9655	3148.3	0.0101	−0.0016	0.0521	0.0734
	(0.0798)	(0.0040)	(0.0194)							
GLS:p_i	0.216111	0.011030	0.616555	0.9986	1.6532	12674.8	0.1666	−0.0043	0.0497	0.0375
	(0.0857)	(0.0026)	(0.0257)							

Non-Agricultural Sector

OLS	0.984091	0.021537	0.553968	0.9999	0.5029	158469.6	0.7485	−0.0004	0.0179	0.0105
	(0.0195)	(0.0008)	(0.0041)							
GLS:p	0.958890	0.023098	0.555790	0.9997	1.5464	61636.8	0.2192	−0.0007	0.0118	0.0167
	(0.0526)	(0.0018)	(0.0092)							
GLS:p_i	0.936272	0.021016	0.564900	0.9999	1.0224	164962.2	0.4862	−0.0012	0.0131	0.0104
	(0.0559)	(0.0012)	(0.0124)							

TABLE 6.21

Agricultural Sector

Method	$\ln \Gamma_i$	π_i	α_i	R^2	D.W.	F	p	$\hat{\bar{v}}$	$\hat{v}_{S.D.}$	$\hat{v}_{R.M.S.E.}$
2 SLS	0.275941 (0.0496)	0.012331 (0.0025)	0.597759 (0.0129)	0.9973	1.0559	6407.7	0.4688	–0.0014	0.0565	0.0522
2 SGLS:p	0.265218 (0.0861)	0.012442 (0.0043)	0.601445 (0.0207)	0.9942	1.8889	2918.4	0.0482	–0.0022	0.0503	0.0763
2 SGLS:p_i	0.227767 (0.0839)	0.012474 (0.0033)	0.610332 (0.0238)	0.9962	1.8036	4584.9	0.0954	–0.0034	0.0497	0.0621

Non-Agricultural Sector

2 SLS	0.983245 (0.0232)	0.021805 (0.0010)	0.553586 (0.0049)	0.9998	1.0154	112321.0	0.4730	–0.0002	0.0213	0.0125
2 SGLS:p	0.994706 (0.0386)	0.020469 (0.0016)	0.554646 (0.0076)	0.9997	2.030	51630.8	–0.0225	–0.0002	0.0184	0.0181
2 SGLS:p_i	0.971970 (0.0507)	0.021379 (0.0013)	0.556792 (0.0116)	0.9998	1.1778	107523.3	0.4083	–0.0008	0.0213	0.0128

we obtain by taking its logs and rearranging:

$$[\ln p(t) - \ln k_1(t) + \ln k_2(t)] = \ln \frac{\alpha_2 \Gamma_2}{\alpha_1 \Gamma_1} - \pi_1 t + \pi_2 t - \alpha_1 \ln k_1(t) + \alpha_2 \ln k_2(t) \quad (6.8.6)$$

From (6.8.6) we see that this equation includes parameters which are parameters of the two subsystems. Hence, we have to take into account restrictions for the whole system and not independently for its subsystems. We therefore, amalgamate equations (6.8.3), (6.8.4) and (6.8.6) into a single block as below:

$$\begin{bmatrix} \ln y_1(t) \\ \frac{k_1(t)}{\omega_1(t)+k_1(t)} \\ \ln y_2(t) \\ \frac{k_2(t)}{\omega_2(t)+k_2(t)} \\ [\ln p(t) - \ln k_1(t) + \ln k_2(t)] \end{bmatrix} = \begin{bmatrix} i & t & \ln k_1(t) & o & o & o & o \\ o & o & i & o & o & o & o \\ o & o & o & i & t & \ln k_2(t) & o \\ o & o & o & o & o & i & o \\ o & -t & -\ln k_1(t) & o & t & \ln k_2(t) & i \end{bmatrix} x \begin{bmatrix} \ln \Gamma_1 \\ \pi_1 \\ \alpha_1 \\ \ln \Gamma_2 \\ \pi_2 \\ \alpha_2 \\ \alpha_2 \Gamma_2 \\ \ln \frac{\alpha_2 \Gamma_2}{\alpha_1 \Gamma_1} \end{bmatrix} + \begin{bmatrix} \epsilon_1(t) \\ \epsilon_2(t) \\ \epsilon_3(t) \\ \epsilon_4(t) \\ \epsilon_5(t) \end{bmatrix} \quad (6.8.7)$$

We will apply now all the methods mentioned above into system (6.8.7). The results are given in table 6.22.

Table 6.23 represents the results of the application of the Cochrane-Orcutt method into system (6.8.7) by using the original data (CO) and after transformation for two-stages least-squares (2SCO).

Also, in the same table we presented the results of the application of the Phillips method into system (6.8.7) by using the original data (P) and after they have been transformed for two stages least-squares (2SP). For the Phillips method we present two auto-correlation coefficients; one derived from the residuals of the estimation (as in all the other

TABLE 6.22

System (6.8.7)

Method	$\ln \Gamma_1$	π_1	α_1	$\ln \Gamma_2$	π_2	α_2	$\ln \Gamma_3$	R^2	DW	F	p	$\hat{\bar{v}}$	$\hat{v}_{S.D}$	$\hat{v}_{R.M.S.E.}$
OLS	0.275284	0.011966	0.598879	0.979224	0.021790	0.554440	0.592888	0.9991	1.1334	16293.8	0.4297	–0.0010	0.0405	0.0300
	(0.0340)	(0.0015)	(0.0092)	(0.0429)	(0.0015)	(0.0093)	(0.0521)							
GLS:p	0.258043	0.011747	0.604472	0.979045	0.021942	0.554074	0.612674	0.9982	1.9772	8022.6	0.0088	–0.0012	0.0366	0.0425
	(0.0521)	(0.0023)	(0.0139)	(0.0640)	(0.0021)	(0.0139)	(0.0775)							
GLS:p_i	0.204269	0.010741	0.620355	0.899197	0.020535	0.573374	0.582163	0.9991	1.6755	17229.0	0.1596	–0.0030	0.0352	0.0294
	(0.0597)	(0.0017)	(0.0179)	(0.1485)	(0.0028)	(0.0344)	(0.1594)							
2SLS	0.276156	0.011991	0.598582	0.980593	0.021809	0.554126	0.593384	0.9990	1.1938	15340.2	0.4010	–0.0009	0.0417	0.0309
	(0.0350)	(0.0016)	(0.0095)	(0.0442)	(0.0016)	(0.0096)	(0.0537)							
2SGLS:p	0.265228	0.011877	0.602814	0.973446	0.021880	0.555267	0.603285	0.9982	1.9863	7858.8	0.0005	–0.0014	0.0382	0.0430
	(0.0523)	(0.0023)	(0.0139)	(0.0643)	(0.0021)	(0.0140)	(0.0779)							
2SGLS:p_i	0.225275	0.011498	0.613486	0.955342	0.021666	0.559579	0.621145	0.9991	1.8586	16043.0	0.0664	–0.0022	0.0377	0.0303
	(0.0613)	(0.0020)	(0.0178)	(0.0892)	(0.0020)	(0.0205)	(0.1062)							

TABLE 6.23

System (6.8.7)

Method	$\ln\Gamma_1$	π_1	α_1	$\ln\Gamma_2$	π_2	α_2	$\ln\Gamma_3$	R^2	DW	F	p	$\hat{\bar{v}}$	$\hat{v}$.S.D	$\hat{v}$.R.M.S.E.
C.O.	0.257186	0.011750	0.604690	0.978964	0.021952	0.554068	0.613490	0.9982	1.9923	7877.2	0.0011	−0.0013	0.0366	0.0429
	(0.0527)	(0.0024)	(0.0140)	(0.0647)	(0.0021)	(0.0140)	(0.0783)							
2 SCO	0.264692	0.01184	0.602957	0.973142	0.021885	0.555314	0.603656	0.9981	1.9989	7741.3	−0.0059	−0.0014	0.0382	0.0433
	(0.0528)	(0.0024)	(0.0140)	(0.0649)	(0.0021)	(0.0141)	(0.0786)							
P	0.240024	0.011227	0.610094	0.979168	0.021698	0.554762	0.629823	0.9997	2.0061	41805.1	-0.0060	0.0001	0.0363	0.0171
	(0.0206)	(0.0009)	(0.0056)	(0.0255)	(0.0008)	(0.0055)	(0.0312)				0.177802			
											(0.0381)			
2 SP	0.257840	0.011677	0.606185	0.959323	0.021658	0.559365	0.600987	0.9999	2.0033	163579.6	-0.0081	0.0002	0.0380	0.0087
	(0.0101)	(0.0004)	(0.0027)	(0.0126)	(0.0004)	(0.0027)	(0.0154)				0.098338			
											(0.0226)			

TABLE 6.24

System (6.8.7)

Method	$\ln\Gamma_1$	π_1	α_1	$\ln\Gamma_2$	π_2	α_2	$\ln\Gamma_3$	R^2	DW	F	p	$\bar{v}$	v.s.D	v.r.m.s.e.
D : 1	0.254521	0.011759	0.605368	0.978710	0.021983	0.554055	0.616017	0.9981	2.0353	7463.8	–0.0206	–0.0013	0.0366	0.0441
	(0.0546)	(0.0024)	(0.0145)	(0.0669)	(0.0022)	(0.0145)	(0.0809)							
D : 2	0.225933	0.012275	0.609971	0.971421	0.022109	0.555426	0.630206	0.9977	1.8842	6123.3	0.0573	–0.0016	0.0369	0.0484
	(0.0644)	(0.0029)	(0.0169)	(0.0776)	(0.0024)	(0.0169)	(0.0928)							
2SD : 1	0.260549	0.011937	0.604066	0.970799	0.021921	0.555693	0.606500	0.9979	2.0821	6964.7	–0.0483	–0.0015	0.0383	0.0457
	(0.0564)	(0.0025)	(0.0150)	(0.0693)	(0.0022)	(0.0150)	(0.0837)							
2SD : 2	0.230565	0.012780	0.608515	0.955533	0.022128	0.558407	0.617037	0.9973	2.0282	5303.6	–0.0210	–0.0020	0.0386	0.0521
	(0.0701)	(0.0031)	(0.0183)	(0.0845)	(0.0025)	(0.0184)	(0.1006)							

methods) and another one, with its standard deviation, derived directly as the estimate of the coefficient of the lagged dependent variable expression in equation (6.7.20).

Table 6.24 represents the results of the application of the Durbin method into system (6.8.7) by using the original data (D:i) and after they have been transformed for two-stages least-squares (2SD:i), where i = 1, 2 show the degree of the autoregressive scheme.

We applied the last three methods (CO, P, D) by using common autoregressive coefficients for all the equations of the system. We took this decision because in this way we avoid extra computation which as it can be seen from the other previous methods does not significantly change the results.

Finally, the depreciation rate of capital has been estimated by:

$$\dot{K}(t) = I(t) - \delta K(t)$$

or

$$D(t) = \delta K(t)$$

which gave the results

$$D(t) = 0.020180\ K(t) \qquad R^2 = 0.9744$$
$$(0.0005) \qquad D.W. = 1.5572$$

6.9 Which Estimate?

In the previous section we presented the results of the various econometric methods using in estimating our two-sector growth model using data from the Greek economy. Now, is time to explain why we used these methods and finally to choose one of the sets of estimates, presented previously, as the best.

First of all, we have to investigate if the Cobb-Douglas production function used in our growth model is valid for the Greek economy. From the facts that the sectoral ratios

of capital/output and capital/labour have not been constant over time we have to accept some substitution between labour and capital over time. Therefore, we must reject any kind of fixed proportions production function. Thus, having established that the production function is of the continuous type, we chose the Cobb-Douglas for simplicity, and due to the relative constancy of the shares of the two factors of production (Klein-Kosobud (1961)).

The production functions used are of constant returns to scale. Also, our model implies imputation of wages according to marginal productivity doctrine. Therefore, from these two conditions we cannot appeal to profit maximization.

The form of technical progress is of the neutral type and so is not apportioned to any one factor. Of course, if we employ the shares of labour and capital out of the sectoral outputs of table 6.14 we see that the ratios of labour share/capital share are increasing over time with that of the agricultural sector increasing faster. This is the only way to obtain non-neutral technical change with the Cobb-Douglas function, Wallis (1973), and in our case, because the ratios β_i/α_i are increasing over time, the technical change is labour-using, a quite good policy for a country with high unemployment.

The first method used in our estimation was that of ordinary least squares (OLS) in the two sectors separately. The statistics of these results (table 6.20), ie. the DW statistic and the autocorrelation coefficient, showed a high positive autocorrelation in the residuals. To get rid of the autocorrelation we used the generalized least-squares method (GLS : p and GLS : p_i).

To capture the problem of simultaneity in the equations we used two-stages least-squares method (2 SLS). The statistics of these results (table 6.21), also showed positive autocorrelation. Therefore, we used the GLS:p and GLS:p_i methods. This method is

similar to that of Wallis (1967) in which he proposes estimation using instrumental variables as a first step, and consequently, since the estimates of the coefficients are consistent, the residuals can be used to estimate the correlation of the errors which is going to be used to GLS.

The introduction of the relative price equation into the system caused problems of identification. This is because the parameter Γ_3 is a function of the parameters α_i and Γ_i which can be estimated directly from all the other equations; it is:

$$\Gamma_3 = \frac{\alpha_2 \Gamma_2}{\alpha_1 \Gamma_1} \qquad (6.9.1)$$

Unfortunately, the restrictions imposed were not enough to get rid of the overidentification in the relative price equation. In spite of that, we performed all the estimating methods. Having found the estimates of all the parameters of the system we calculated a $\tilde{\Gamma}_3$ parameter using (6.9.1), for each method. These calculated values of Γ_3 together with the directly estimated values of Γ_3 are presented in table 6.25. In this table we see that the differences between the calculated and the estimated $\ln \Gamma_3$ are all (except one) very small compared to the standard deviations of the estimated $\ln \Gamma_3$. We believe that this is a sign that the overidentification problem did not produce any large misestimation in the various parameters.

We performed the same methods, ie. OLS, GLS:p, GLS:p_i and 2 SLS, 2 SGLS:p, 2 SGLS:p_i, to the stacked system of all the equations as we have previously done for the two subsystems.

Furthermore, in spite of the theoretical equivalence of the Cochrane-Orcutt (CO and 2 SCO), Phillips (P and 2 SP) and Durbin (D : i and 2 SD : i) methods, we performed all these methods to see if there is any significant difference between their actual estimates.

The 2 SLS methods possess the merit of giving consistent estimates, compared to

TABLE 6.25

Identification in $\ln \Gamma_3$

Method	$\ln \hat{\Gamma}_3$	$\ln \tilde{\Gamma}_3$	$\ln \tilde{\Gamma}_3 - \ln \hat{\Gamma}_3$
OLS	0.592888 (0.0521)	0.626839	0.03395
GLS : p	0.612674 (0.0775)	0.633945	0.021271
GLS : p_i	0.582163 (0.1594)	0.583174	0.001011
2 SLS	0.593384 (0.0537)	0.627266	0.033882
2 SGLS : p	0.603285 (0.0779)	0.626058	0.022773
2 SGLS : p_i	0.621145 (0.1062)	0.638087	0.016942
CO	0.613490 (0.0783)	0.634349	0.020859
2 SCO	0.603656 (0.0786)	0.626138	0.022482
P	0.629823 (0.0312)	0.644070	0.014247
2 SP	0.600987 (0.0154)	0.621100	0.020113*
D : 1	0.616017 (0.0809)	0.635619	0.019600
D : 2	0.630206 (0.0928)	0.651812	0.021606
2 SD1	0.606500 (0.0837)	0.626783	0.020283
2 SD2	0.617037 (0.1006)	0.639035	0.021998

* difference greater than the standard deviation of the estimate.

the biased and inconsistent estimates of the OLS methods (Haavelmo, 1947). But consistency is an asymptotic property and given the smallness of the sample size with which we work it is of limited importance as a criterion for chosing between the various

methods. Choice of a particular estimator should therefore be based upon its small sample properties of bias and efficiency.

The OLS method has been shown to give biased estimates, but at the same time, these estimates possess the merit of having smaller variance around their mean than other particular unbiased estimates. Comparing the results of the OLS method with all the other methods (except the P and 2SP where explicitly there is more than one degree of freedom) we see that the standard deviations of the estimates obtained by OLS are smaller than in the others. Moreover, the estimates of the other methods will also have small sample bias, although smaller than the OLS (as the results of the available Monte Carlo studies show). To decide therefore upon which estimation method to choose, the overall performance of each estimator should be considered, combining bias and efficiency, and this in association with the purposes for which the econometric model was constructed (ie. the interest may be centred in estimating structural coefficients, or in the behaviour of the model as a whole), Pavlopulos, (1966).

One real advantage of the OLS method is in the use of the model for prediction. If we are not particularly interested in the values of the structural parameters but we are really concerned about predicting as accurately as we can an endogenous variable then the OLS is unbiased and best. This is the basic theorem of least-squares prediction. Of course, the least-squares estimate of the coefficients will be biased but this does not matter if we are only interested in prediction. Paradoxically this bias in the coefficients makes the prediction efficient, Walters (1970).

Table 6.26 represents the tracking errors, R;

$$R = \sum_{i=1}^{m} \ln \sum_{t=1}^{n} \hat{\epsilon}_{it}^2 \qquad (6.9.2)$$

where $\hat{\epsilon}_{it}$ are the residuals corresponding to the estimates of the various methods, i runs

over the equations and t runs over the data period used. In table 6.26 we see the truth of the discussion above; ie. comparing the tracking errors of OLS and 2 SLS, GLS : p and 2 SGLS : p, CO and 2 SCO, P and 2 SP, and D : i and 2 SD : i we see that in all these cases the tracking errors of the methods used the original data are smaller than those used the two-stages least-squares data. Paradoxically this is not the case for GLS : p_i and 2 SGLS : p_i. This "paradox" together with the smaller tracking error of GLS : p compared to that of GLS : p_i is explained by the fact that the program used for calculation of the tracking errors does not use the actual observations but it uses calculated observations by simultaneous solution of the equations in the model. Obviously, the best set of estimates as far as concern the overall prediction of the model is that of D : 1 for which the tracking error is the smallest of all the others.

TABLE 6.26

Tracking Errors

Method	Errors	Method	Errors
OLS	17.11204877	2 SCO	17.14736362
GLS:p	17.06962167	P	17.18140449
GLS:p_i	18.51829963	2 SP	17.48443809
2 SLS	17.11333098	D : 1	17.06269822
2 SGLS:p	17.14518586	D : 2	17.15513136
2 SGLS:p_i	17.05091742	2 SD : 1	17.16650263
CO	17.06784413	2 SD : 2	17.34579810

To chose a set of estimates as far as concern the structural parameters we have also to take into account that our system is overidentified. In practice the two-stages least-squares method often constitutes the best estimation method for overidentified models Malinvaud (1970). In table 6.22 the results on 2 SLS show positive autocorrelation and so inefficient estimates. Therefore, we have to choose between the two-stages iterative

methods 2 SCO and 2 SP and also the two-stages Durbin method 2 SD : i. Theoretically, these methods possess the same properties. But in practice, given that our sample is very small, we choose as the best set of estimates that with the smallest tracking error, ie. the two-stages Cochrane-Orcutt method (2 SCO) in table 6.23.

Finally, we have to test if the autocorrelation scheme imposed on our model is valid For this reason we performed the Durbin method using first and second order autoregressive schemes on the errors. From the results obtained in table 6.24 we see that D:1 is superior to D:2 and 2 SD:1 is superior to 2 SD:2, because in both cases the F ratio for the first order autoregressive scheme is higher to that of the second order. We derive the same conclusion by comparing the tracking errors of the above methods where the tracking errors of D:1 and 2 SD:1 are smaller than those of D:2 and 2 SD:2. But this investigation shows that the first order autoregressive scheme is superior to that of the second order without giving any information about the validity of this autoregressive restriction.

To test the validity of the autoregressive restriction imposed on our model we will perform the Hendry(1974) test. According to him we denote by S_1, S_2 and S_3 the sums of squares of the residuals of the three sets of estimates, ie. first is the OLS in the structural form, second is the estimation using one year lags in all the variables except those which are redundant (constant terms, time trends) and third is the estimation under the auto-regressive transformation. Then we can perform the two tests:

1. Test for the significance of the autoregressive coefficient p (which can be also seen by the D.W. statistic)

$$T \log (S_1/S_3) \underset{A}{\sim} \chi^2_{(1)} \text{ on } H_0 : p = 0 \qquad (6.9.3)$$

2. Test for the validity of the autoregressive restriction

$$T \log (S_3/S_2) \underset{A}{\sim} \chi^2_{(N)} \qquad (6.9.4)$$

where T is the number of observations and N is the number of restrictions imposed on the second estimation to obtain the third one. The results of the two tests are given in table 6.27 with level of significance 0.05.

TABLE 6.27

Hendry tests

Test	D : 1	2 SD : 1
for significance of p	1.359065	1.142469
for validity of autor. restriction	0.850712	0.810804

In table 6.27 the figures presented show the ratio of the χ^2 computed by (6.9.3) and (6.9.4) by the corresponding χ^2 (for 0.05) from the tables and for the cases of D:1 and 2 SD:1 mentioned above (for all the other methods of estimation the above procedure gives similar results). Therefore, testing the significance of p we found that the χ^2 ratio is greater than one in both cases of D : 1 and 2 SD : 1 and so rejecting the hypothesis that there is no autocorrelation in the residuals (fact which can be seen from the low D.W. statistic in the OLS and 2 SLS estimation in table 6.22). Testing now the validity of the autoregressive restriction we found that the χ^2 ratio is less than one in both cases and so accepting the hypothesis that the transformation used is valid.

6.10 Conclusions

In this chapter we tried to estimate our two-sector growth model using annual data from the Greek economy. For the following reasons we do not suggest that a high (or low) degree of fit of our model to the data implies that it is completely realistic. First, our model is related to a closed economy, which is a major limitation and greatly restricts

the applicability of our analysis. Undoubtedly, the Greek economy is an open economy, which implies that we had to transform the existing data before we try any application to our model. Second, there is a lot of important data which is missing which implies that our results are heavily based on the methods that we used for estimation of the missing data. Third, our results depend on the specification of the model, ie. Cobb-Douglas production functions, constant returns to scale and so on. Therefore, in this application we tried to find an approximate index, showing the structure of the Greek economy, of course under the assumptions of our two-sector growth model.

From the results that we obtained by estimating the model using the structural form we note the difference between the two rates of technological change and the shares of capital in output in the two sectors. Thus, in the agricultural sector the rate of technological change is lower than that in the non-agricultural sector and the share of capital in output in the agricultural sector is higher than that in the non-agricultural sector. These two differences show that the agricultural sector is less developed compared with the non-agricultural sector.

Unfortunately, as far as we know, the existing literature on these kind of results for the Greek economy is very limited, making comparisons difficult.

Kintis (1970) estimated time trends for C.E.S. production functions for the manufacturing sector. His results for the different industries, are between –0.007 (for the beverages industry) and 0.063 (for the cigarettes industry) with an average of all industries being equal to 0.026706. Our time trend for the non-agricultural sector is 0.021952 which is not significantly different from the figure above.

Adelman and Chenery's (1966) estimate for the depreciation rate, taking one year's lag for capital stock and using a constant, is 0.0213 which is not far from ours, 0.02018,

if we bear in mind the different specification of the capital accumulation functions.

Koutsoyianni-Kokkova (1964) estimated Cobb-Douglas production functions for Greek industry. She took a sample of large-scale industries and her results for the capital share are around 0.30. Our estimate for the non-agricultural sector is 0.554068. We believe that the difference in the two results is based mainly on the fact that our non-agricultural sector includes both large-scale and small-scale industries and any other activity which is not included in the agricultural sector. These activities have a high percentage of unpaid family workers. The result of 0.30 for the capital share is typical for the advanced economies or for the modern large-scale industries of less developed countries like Greece. If we apply to our estimate the method developed by Professor J. Lecaillon who is taking explicit account of the percentage of wage earners out of wages earners plus entrepreneurs then according to his formulation the share of capital in output is given by:

$$\alpha_2 = 1 - \frac{\text{share of labour in output}}{(\text{wage earners/wage earners + entrepreneurs})}$$

The denominator in this formula has the average value, for the period 1951 to 1971, of 0.683 (Germidis-Negreponti-Delivanis, 1975).Therefore, the new estimate of the capital's share in output is 0.346668 which is not too far from that of Koutsoyianni-Kokkova.

Unfortunately, for the agricultural sector there are no estimates in the literature with which to make comparison.

Also, we have to note that the general property of existence of the solution of our model, ie. that the non-agricultural sector is the most capital intensive, is satisfied with our data, as we have already seen in section 6.6. The sufficient conditions for uniqueness of the monentary solution for our model are the (3.3.98) (i), (ii) and (iii; c). From these (i) and (iii, c) are satisfied because $\sigma_1 = \sigma_2 = 1$; (ii) is satisfied for the years after 1967

as we can see comparing the data in tables 6.8 and 6.17. The sufficient conditions for the uniqueness of the balanced growth, (3.4.20) are also satisfied because in our application we took $s_1 = s_2 = 0$.

Finally, in section 4.4.7 we found that for the case where the wages ratio is constant over time,

$$\frac{\beta_1}{\beta_2} > \frac{\pi_1}{\pi_2} \quad \Rightarrow \quad p(t) \text{ increasing and } \frac{Y_1(t)}{K_1(t)} \text{ decreasing.}$$

If now in our data assume that $\mu(t)$ is constant (in fact it is increasing very slowly), then using our estimates we have that $\beta_1/\beta_2 = 0.886480$ which is greater than $\pi_1/\pi_2 = 0.535259$. From tables 5.21 and 6.19 we see that the relative price is increasing whilst the output/capital ratio in the agricultural sector is decreasing. Therefore, the move of p(t) and $Y_1(t)/K_1(t)$ into opposite directions is compatible with our estimates.

6.11 References

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CHAPTER 7

OPTIMAL CONTROL THEORY FOR THE TWO-SECTOR GROWTH MODEL

7.1 Introduction
7.2 Optimal Wage/Rental Ratio and Consumption Maximisation Over Time
7.3 The Specific Two-sector Growth Model and Optimization Over Time
7.4 Optimal Growth and the Case of Greece
7.5 Conclusions
7.6 References

7.1 Introduction

The reduction of unemployment and the raising of the income level are the major targets in a development policy for underdeveloped countries. In order to achieve these targets one must define a policy allocating investment each year between the capital goods and consumer goods industries. Such a choice is also equivalent to a decision about the marginal saving ratio, Frankel (1961). The problem of optimal growth can then be expressed as follows:

"In what proportion should net capital formation be divided between capital goods industries and consumer good industries?", Goodwin (1961). The usual answer to this problem is to relate welfare to consumption (aggregate or per capita) so that optimal development is defined as maximizing some increasing function of consumption; in so doing, one can determine the optimal policy. This is known in the literature as the "optimal savings" problem, and since a pioneering article of Ramsey (1928), the issue has been discussed many times, in particular by Tinbergen (1956, 1960), Goodwin (1961), Dobb (1959), Sen (1961), Chakravarty (1962) and Srinivasan (1962, 1962a).

Stoleru (1965) discussed the problem of reaching full employment in minimum time.

Sen (1968) has shown that decisions for the problem of optimal investment and employment in the advanced sector of underdeveloped countries must be taken togehter and Marglin (1966) and Dixit (1968) have developed optimal growth models to indicate how these decisions might be made. The last three writers have adopted the Lewis (1954) assumption of a perfectly elastic supply of labour from the traditional sector at an institutionally fixed wage.

Stern (1972) developed an optimum growth model in a dual economy taking into

account the output of the traditional sector and the intra-generational distribution consequences of employment policies whilst the above three writers did not.

Srinivasan (1964), Uzawa (1964) and Shell (1967) have investigated the problem of optimal economic growth in the two-sector model, giving conditions for the times of switching from specialized to non-specialized production and verifying the existence of a Modified Golden Rule equilibrium having the same characteristics as the one-sector model.

Shell (1967), Mirrlees (1967), Sheshinski (1967), Stiglitz (1968) and Kamien & Schwartz (1968) discussed the problem of optimal growth under the assumption of technological change of various kinds.

There is a considerable debate (in which we will not get involved) concerning the length of the planning horizon; ie. finite or infinite. For both lengths of the planning horizon there are merits and de-merits, Malinvaud (1953, 1961), Fisher (1965), Koopmans (1957, 1965, 1967), Samuelson (1958, 1967), Radner (1967), Wan (1971), and others.

We must note that in both cases the conditions for maximization are the same and because we believe that the issues connected with different specifications of the planning horizon are not unambiguously in favour of one approach or the other we decided to adopt the infinite time approach in the planning horizon for our analysis below, so avoiding the determination of the capital stock at the end of the planning horizon for the finite case.

In section 7.2 we investigate the problem, for the general two-sector model, assuming constant marginal utility and using as state and control variables the overall capital-labour ratio and the wage-rental ratio respectively. In section 7.3 we discuss the problem for the specific Cobb-Douglas model assuming decreasing marginal utility and using as state and control variables the overall capital-labour ratio, again, and per capita consumption

respectively. Finally, in section 7.4 we produce some results of the application of the optimal model of section 7.3 to the Greek economy.

7.2 Optimal Wage/Rental Ratio and Consumption Maximization Over Time

The problem of economic growth for the two-sector growth model, in the case of an infinite time horizon and constant marginal utility, is the problem of choosing paths $[\ell_1(t)]$, $[\ell_2(t)]$, $[k_1(t)]$ and $[k_2(t)]$, so as to

$$\max W = \int_0^{\infty} e^{-pt} c \, dt \qquad (7.2.1)$$

st $\dot{k} = i - \lambda k$

$k(0) = k_0$

$q_i = B_i f_i(\bar{k}_i) \ell_i$, $\qquad (i = 1, 2)$

$\ell_1 + \ell_2 = 1$

$k = k_1 \ell_1 + k_2 \ell_2$

$\ell_1, \ell_2, k_1, k_2 \geq 0$

$c, \ell_1, \ell_2, k_1, k_2$ piecewise continous

where c, the overall percapita total consumption, is the objective functional; k, the overall capital/labour ratio is the state variable; ℓ_1, ℓ_2, k_1 and k_2 are the control variables; $f_1(\bar{k}_1)$ and $f_2(\bar{k}_2)$ are given strictly concave functions; p and k_0 are given parameters.

In (7.2.1) c can be obtained by

$$c = pc_1 + c_2 \qquad (7.2.2)$$

Substituting (3.3.72), (3.3.76) and (3.3.77) into (7.2.2) we obtain

$$c = \frac{A_2}{A_1} \frac{f_2'}{f_1'} B_1 \ell_1 f_1 + B_2 \ell_2 f_2 - i$$

and after substitution into the above of (3.3.70) and (3.3.75) we obtain:

$$c = A_2 f'_2 \left[\frac{B_1}{A_1} \frac{f_1}{f'_1} \ell_1 + \frac{B_2}{A_2} \frac{f_2}{f'_2} \ell_2 \right] - i =$$

$$= A_2 f'_2 \left[(\omega_1 + k_1) \ell_1 + (\omega_2 + k_2) \ell_2 \right] -$$

$$- A_2 f'_2 \left[s_3 k + s_2 \omega_2 \ell_2 + s_1 \omega_1 \ell_1 \right]$$

and finally

$$c = A_2 f'_2 \left[(1 - s_3)k + (1 - s_2) \omega_2 \ell_2 + (1 - s_1) \omega_1 \ell_1 \right] \qquad (7.2.3)$$

which is a function of k, ω_2 and t (see chapter 3).

Therefore, the solution to the problem (7.2.1) can be obtained after the transformation of the variables in terms of k, ω_2 and t. Thus, the problem of optimal economic growth can be rewritten as:

$$\max_{[\omega_2(t)]} W(k, \omega_2, t) = \int_0^{\infty} e^{-pt} \, c \, dt \qquad (7.2.4)$$

s.t. $\dot{k} = i - \lambda k$

$k(0) = k_0$

$k_i = k_i(\omega_2)$

$\ell_i = \ell_i(\omega_2)$

$\omega_{min}(k) < \omega_2 < \omega_{max}(k)$

$\omega_2(t)$ piecewise continuous

where c is the objective functional, k is the state variable as before, but now the control variable is the wage/rental ratio ω_2.

In order for ω_2 to be a control variable we need to relax the equilibrium model, as we have already shown that only one value of ω_2 is compatible with equilibrium of savings and investment. Let us allow nonequality of investment and private savings decisions and suppose that the discrepancy between them to be made up by the govern-

ment. The government saves, or dissaves so as to maintain equilibrium between total savings and investment, which in this case will be the optimal investment. This government saving is financed by a tax on wages and rentals in a way such that the ratio is unchanged. This is done by having a proportional tax on both. If dissaving were to be necessary the tax would become a subsidy. We now have a degree of freedom so as to control the economy and the control variable we take is, instead of total savings, the wage rental ratio ω_2. There is a diffeomorphic relationship between them.

The optimization will be carried out using the maximum principle of Pontryagin and others (1962). The Hamiltonian for this problem is:

$$H(k, \omega_2, \tau, t) \equiv e^{-pt}[c + \tau(i - \lambda k)] \qquad (7.2.5)$$

where $\tau(t)$ is the costate variable. (7.2.5) is written explicitly as:

$$H = e^{-pt} [A_2 f'_2 [(1-s_3)k + (1-s_2)\omega_2 \ell_2 + (1-s_1)\omega_1 \ell_1] +$$

$$+\tau [A_2 f'_2 [s_3 k + s_2 \omega_2 \ell_2 + s_1 \omega_1 \ell_1] - \lambda k]]$$

or

$$H = e^{-pt} A_2 f'_2 [[1+s_3(\tau-1)] k + [[1 + s_2(\tau-1)] \ell_2 +$$

$$+ [1 + s_1(\tau-1)] \mu \ell_1] \omega_2] - e^{-pt} \tau \lambda k \qquad (7.2.6)$$

The canonical equations are

$$\dot{k} = A_2 f'_2 [s_3 k + [s_2 \ell_2 + s_1 \ell_1 \mu] \omega_2] - \lambda k \qquad (7.2.7)$$

and

$$\dot{\tau} = (\lambda + p) r - A_2 f'_2 [[1 + s_3(\tau-1)] +$$

$$+ \frac{\omega_2}{k_1 - k_2} [[1+s_1 (\tau-1)] \mu - [1 + s_2(\tau - 1)]]] \qquad (7.2.8)$$

where r(t) has the interpretation of the shadow price of capital accumulation.

The optimal wage/rental ratio ω_2 must maximize the Hamiltonian or equivalently, maximize $H e^{pt}$. Thus,

$$\frac{\partial}{\partial \omega_2}(H e^{pt}) = A_2 f''_2 \frac{d\bar{k}_2}{d\omega_2} [[1+s_3(\tau-1)] k +$$

$$+ [[1 + s_2(\tau-1)]\ell_2 + [1 + s_1(\tau-1)] \mu \ell_1] \omega_2] +$$

$$+ A_2 f'_2 [(1+s_2(\tau-1)] \ell_2 + [1+s_1(\tau-1)] \mu \ell_1 +$$

$$+ [[1+s_2(\tau-1)] \frac{d\ell_2}{d\omega_2} + [1+s_1(\tau-1)] \mu \frac{d\ell_1}{d\omega_2}] \omega_2]$$

Substituting now (3.4.4), (3.4.9), (3.4.10) and (3.3.70) into the last expression we obtain:

$$\frac{\partial(\cdot)}{\partial \omega_2} = \frac{A^2_2}{B_2} f''_2 (- \frac{(f'_2)^2}{f_2 f''_2}) [\quad \cdots \quad] +$$

$$+ A_2 f'_2 [[1+s_2(\tau-1)] \ell_2 + [1+s_1(\tau-1)] \mu \ell_1 +$$

$$+ \left[\left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} \ell_2 + \frac{dk_1}{d\omega_1} \ell_1 \right] \frac{1}{k_1 - k_2} \left[\left[1+s_2(\tau-1) \right] \mu - \left[1+s_1(\tau-1) \right] \right] \right] \omega_2 \Big]$$

For an extremum the last expression must be equal to zero, ie:

$$\frac{\partial(\cdot)}{\partial \omega_2} = 0$$

Therefore, we have

$$- \frac{1}{\omega_2 + k_2} \left[\left[1+s_3(\tau-1) \right] k + \frac{\omega_2}{k_1 - k_2} \left[\left[1+s_1 (\tau-1) \right] \mu(k - k_2) + \right. \right.$$

$$+ \left[1+s_2 (\tau-1) \right] (k_1 - k) \Big] + \frac{1}{k_1 - k_2} \left[\left[1+s_1 (\tau-1) \right] \mu (k - k_2) + \right.$$

$$+ \left[1+s_2 (\tau-1) \right] (k_1 - k) \Big] + \frac{\omega_2}{(k_1 - k_2)^2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} (k_1 - k) + \right.$$

$$+ \left. \frac{dk_1}{d\omega_1} (k - k_2) \right] \left[\left[1+s_2(\tau-1) \right] \mu - \left[1+s_1(\tau-1) \right] \right] = 0$$

or

$$- \frac{1}{\omega_2 + k_2} \left[k + \frac{\omega_2}{k_1 - k_2} \left[\mu (k - k_2) + (k_1 - k) \right] + \right.$$

$$+ \left[s_3 k + \frac{\omega_2}{k_1 - k_2} \left[s_1 \mu (k - k_2) + s_2 (k_1 - k) \right] \right] (\tau-1) \Big] +$$

$$+ \frac{1}{k_1 - k_2} \left[\mu(k - k_2) + (k_1 - k) + \left[s_1 \mu(k - k_2) + s_2 (k_1 - k) \right] (\tau-1) \right] +$$

$$+ \frac{\omega_2 (\mu-1)}{(k_1 - k_2)^2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} (k_1 - k) + \frac{dk_1}{d\omega_1} (k - k_2) \right] +$$

$$+ \frac{\omega_2 (s_2 \mu - s_1)}{(k_1 - k_2)^2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} (k_1 - k) + \frac{dk_1}{d\omega_1} (k - k_2) \right] (\tau-1) = 0$$

Solving the last equation for τ we obtain

$$\left[\frac{1}{\omega_2 + k_2} \left[\frac{k_2}{k_1 - k_2} \left[s_1 \mu (k - k_2) + s_2 (k_1 - k) \right] - s_3 k \right] + \right.$$

$$+ \left. \frac{\omega_2 (s_2 \mu - s_1)}{(k_1 - k_2)^2} \left[\frac{1}{\mu} \frac{dk_2}{d\omega_2} (k_1 - k) + \frac{dk_1}{d\omega_1} (k - k_2) \right] \right] (\tau-1) =$$

$$= \left[\frac{1}{\omega_2+k_2}\left[\,k-\frac{k_2}{k_1-k_2}\left[\,\mu(k-k_2)+(k_1-k)\,\right]\,\right]\,-\right.$$

$$\left.-\frac{\omega_2(\mu-1)}{(k_1-k_2)^2}\left[\frac{1}{\mu}\,\frac{dk_2}{d\omega_2}\,(k_1-k)+\frac{dk_1}{d\omega_1}\,(k-k_2)\,\right]\,\right]$$

or

$$A(\tau-1) = B$$

and

$$\tau = 1+\frac{B}{A} \tag{7.2.9}$$

where the substitution is obvious for A and B.

Equations (7.2.7), (7.2.8) and (7.2.9) describe the differential equations for the state and the control variables. Especially for the control variable ω_2 we have to take the derivative of (7.2.9) with respect to time, to substitute into (7.2.8) and after to solve analytically for $\dot{\omega}_2$, which is rather tedious but not difficult.

Here, we will try to interpret a simpler case in which $0 \leqslant s_3 \leqslant 1$, $s_1 = s_2 = 0$ and $\omega_1 = \omega_2 = \omega$, which implies that $\mu = 1$. The difference between this case and the usual two-sector growth models is that in this case the output of the second sector is not all invested but only a part of it. Therefore, the canonical equations become

$$\dot{k} = A_2 f'_2\, s_3 k - \lambda k \tag{7.2.10}$$

and

$$\dot{r} = (\lambda+p)\tau - A_2 f'_2\,[1+s_3(\tau-1)\,] \tag{7.2.11}$$

and for the control variable we have that

$$A = -\frac{s_3\, k}{\omega + k_2} \tag{7.2.12}$$

and

$$B = \frac{1}{\omega+k_2}\left[\,k-\frac{k_2}{k_1-k_2}\,(k-k_2+k_1-k)\,\right] =$$

$$= \frac{k-k_2}{\omega+k_2} \tag{7.2.13}$$

hence,

$$r = 1 - \frac{k-k_2}{s_3\ k} \tag{7.2.14}$$

Taking the derivative of (7.2.14) with respect to time t we obtain

$$\dot{r} = - \frac{(\dot{k} - \frac{dk_2}{d\omega}\ \dot{\omega})\ s_3 k - (k-k_2)\ s_3\ \dot{k}}{(s_3\ k)^2}$$

or

$$r(s_3 k)^2 = - s_3 k \dot{k} + s_3 \frac{dk_2}{d\omega}\ \dot{\omega}\ k + s_3 k \dot{k} - s_3 k_2 \dot{k}$$

$$= s_3 \frac{dk_2}{d\omega}\ \dot{\omega}\ k - s_3\ k_2\ \dot{k}$$

or

$$\dot{\omega} = \frac{\dot{r}\ s_3 k^2 + k_2\ \dot{k}}{\frac{dk_2}{d\omega}\ k} = \frac{d\omega}{dk_2}\ (s_3\ \dot{r}\ k + k_2 \frac{\dot{k}}{k}) \tag{7.2.15}$$

We substitute (7.2.10), (7.2.11) and (7.2.14) into (7.2.15)

$$\dot{\omega} = \frac{d\omega}{dk_2}\ [\ s_3\ [\ (\lambda+p)(1 - \frac{k-k_2}{s_3\ k}) - A_2 f'_2\ (1 - \frac{k-k_2}{k})\ ]\ k +$$

$$+ k_2\ (A_2 f'_2 s_3 - \lambda)\ ]$$

and finally

$$\dot{\omega} = \frac{f_2\ f''_2}{f'^2_2}\ [\ (\lambda+p)(s_3-1)\ k + pk_2\ ] \tag{7.2.16}$$

Thus, we derived the differential equations for the state, (7.2.10) and the control, (7.2.16), variables of our model in its simpler case where $0 \leqslant s_3 \leqslant 1$, $s_1 = s_2 = 0$ and $\omega_1 = \omega_2 = \omega$.

By putting $\dot{k} = \dot{\omega} = 0$ in (7.2.10) and (7.2.16) we derive the balanced growth solution, along which k and ω are stationary. This solution exists and is unique for $\omega = \omega^*$ and $k = k^*$, where:

$$f_2' (\bar{k}_2(\omega^*)) = \frac{\lambda}{s_3 A_2} \tag{7.2.17}$$

and

$$k^* = \frac{p k_2(\omega^*)}{(\lambda + p)(1 - s_3)} \tag{7.2.18}$$

The introduction of technical change into our two-sector growth model causes the Hamiltonian to be non-autonomous (ie. depending on time) with the consequence that the equations (7.2.10) – (7.2.18) become non-autonomous. Therefore, the equilibrium values (k^*, ω^*) are the unique levels of the overall capital per worker and the wage-rental ratio for a specific given time period only.

The analysis could be presented in a (k, ω, t) – phase space. But since such presentation is graphically difficult in three dimensions, we will present instead the case in which there is no technical change, ie. B_i (t) and A_i (t) are constants over time and the system is autonomous, leaving the analysis of the system with technological change for the end of this section.

We define, from (7.2.17) and (7.2.18), that

$$\omega^* = \frac{s_3 B_2 f_2}{\lambda} - k_2(\omega^*) \tag{7.2.19}$$

and

$$\hat{k}(\omega) = \frac{p\, k_2(\omega)}{(\lambda + p)(1 - s_3)} \tag{7.2.20}$$

From (7.2.16) we have that for

$$\dot{\omega} = 0 \Rightarrow (\lambda + p)(s_3 - 1)k = -pk_2 \Rightarrow k = \frac{pk_2}{(\lambda + p)(1 - s_3)}$$

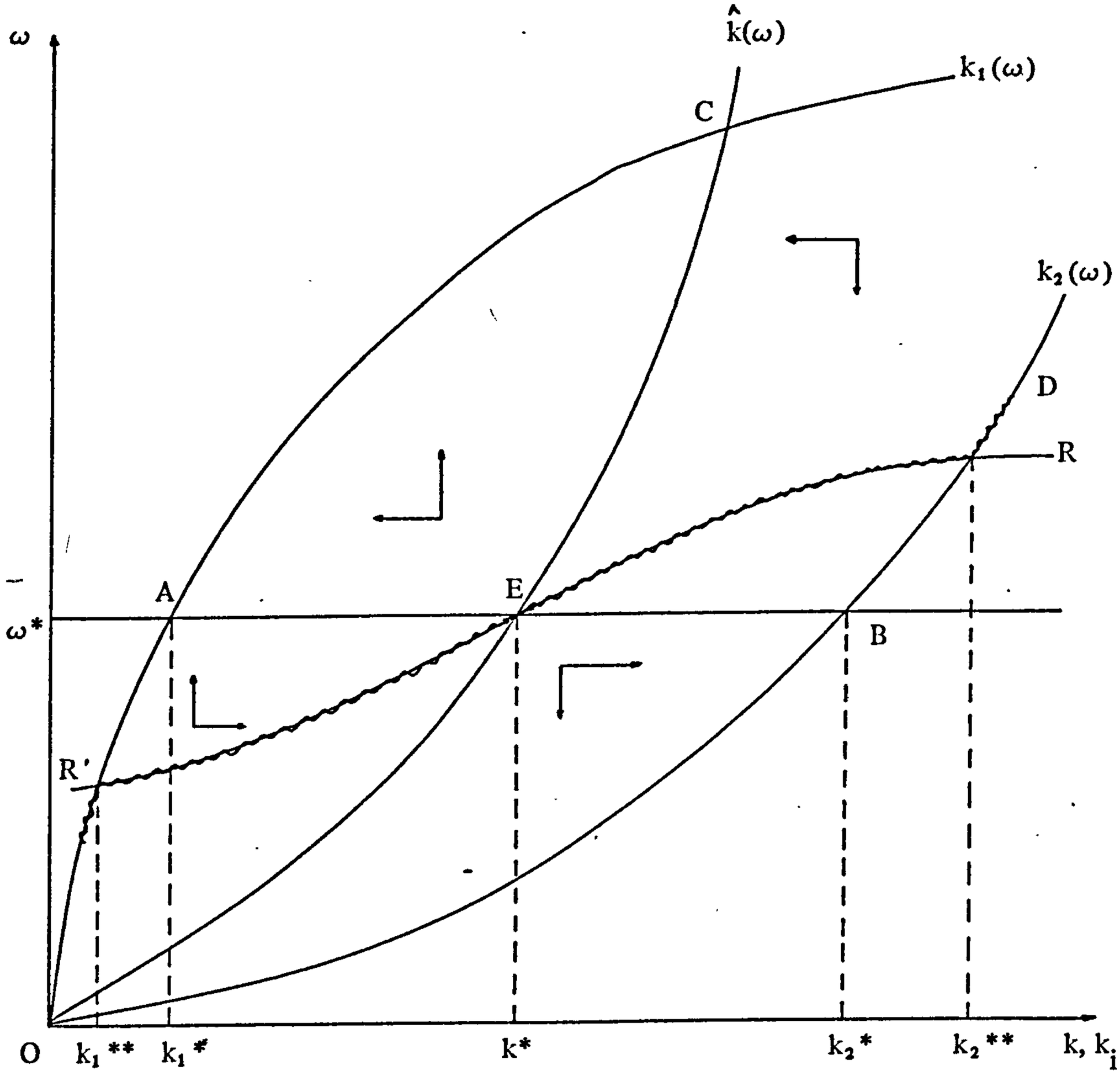

Figure 7.1

for

$$\dot{\omega} > 0 => (\lambda+p)(s_3-1)k > -pk_2 \;\; => k < \frac{pk_2}{(\lambda+p)(1-s_3)}$$

and for

$$\dot{\omega} < 0 => (\lambda+p)(s_3-1)k < -pk_2 \;\; => k > \frac{pk_2}{(\lambda+p)(1-s_3)}$$

Also, from (7.2.10) we have that for

$$\dot{k} = 0 => f'_2 = \frac{\lambda}{s_3 A_2} \;\; => \frac{f_2}{f'_2} = \frac{s_3 A_2 f_2}{\lambda} \;\; => \omega = \frac{s_3 B_2 f_2}{\lambda} - k_2$$

for

$$\dot{k} > 0 \Rightarrow f_2' > \frac{\lambda}{s_3 A_2} \Rightarrow \frac{f_2}{f_2'} < \frac{s_3 A_2 f_2}{\lambda} \Rightarrow \omega < \frac{s_3 B_2 f_2}{\lambda} - k_2$$

and for

$$\dot{k} < 0 \Rightarrow f_2' < \frac{\lambda}{s_3 A_2} \Rightarrow \frac{f_2}{f_2'} > \frac{s_3 A_2 f_2}{\lambda} \Rightarrow \omega > \frac{s_3 B_2 f_2}{\lambda} - k_2$$

To summarise, inspection of (7.2.16) and (7.2.10) together with (7.2.19) and (7.2.20) shows that

$$\dot{\omega} \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0 \qquad \text{iff} \qquad k \begin{bmatrix} < \\ = \\ > \end{bmatrix} \hat{k} \tag{7.2.21}$$

and

$$\dot{k} \begin{bmatrix} > \\ = \\ < \end{bmatrix} 0 \qquad \text{iff} \qquad \omega \begin{bmatrix} < \\ = \\ > \end{bmatrix} \omega^* \tag{7.2.22}$$

Figure 7.1, in the (k, ω) plane, represents our case in which $k_1(\omega) < k_2(\omega)$. Along the horizontal line $\omega = \omega^*$ there is no change in ω, and along the curve $k = \hat{k}(\omega)$ there is no change in k. The intersection of $k = \hat{k}(\omega)$ and $\omega = \omega^*$ determines the balanced growth equilibrium $k^* = \hat{k}(\omega^*)$, point E in figure 7.1.

The feasible area in figure 7.1, ie. the area between the curves $k_1(\omega)$ and $k_2(\omega)$ is divided into four subareas, namely OAE, EAC, DBEC and BOE, by the $\omega = \omega^*$ and $k = \hat{k}(\omega)$ curves. (7.7.21) and (7.7.22) are shown by the arrow signs and indicate how a solution curve of (7.2.10) and (7.2.16) will move in the (k, ω) plane, given the initial point for k and ω, say (k_0, ω_0). Hence, if the initial point (k_0, ω_0) falls into the region EAC (or BOE) then the system ends up on the arc AC (or OB). On the contrary, if the initial point (k_0, ω_0) falls into the area OAE (or DBEC) then the system ends up on the balanced growth equilibrium point E.

Precisely, point E is a saddle point. To see this first we linearize the two differential

equations (7.2.10) and (7.2.16), taking also into account that

$$\text{sign}\,(\dot{\omega}) = \text{sign}\,(\ (\lambda + p)(s_3 - 1)\,k + pk_2\ ),$$

at the point $(k^*\ ,\ \omega^*)$ and examine the characteristic roots of the resulting coefficient matrix:

$$\begin{bmatrix} \dfrac{\partial \dot{k}}{\partial k} & \dfrac{\partial \dot{k}}{\partial \omega} \\[2ex] \dfrac{\partial \dot{\omega}}{\partial k} & \dfrac{\partial \dot{\omega}}{\partial \omega} \end{bmatrix}$$

which is explicitly written as

$$\begin{bmatrix} 0 & \dfrac{A_2{}^2}{B_2}\ s_3 f_2'' \left.\dfrac{dk_2}{d\omega}\right|_{(k^*\ ,\ \omega^*)} \\[2ex] (\lambda+p)(s_3-1) & p \left.\dfrac{dk_2}{d\omega}\right|_{(k^*\ ,\ \omega^*)} \end{bmatrix}$$

The characteristic equation of the matrix above is

$$x^2 - p\,\frac{dk_2}{d\omega}\ x + (\lambda+p)(1-s_3)\,\frac{A_2{}^2}{B_2}\ s_3 f_2''\,\frac{dk_2}{d\omega} = 0 \qquad (7.2.23)$$

The roots of the equation (7.2.23), ie. the characteristic roots are given by

$$x_{1\ ,2} = \tfrac{1}{2}\left[-p\,\frac{dk_2}{d\omega} \pm \sqrt{(p\,\frac{dk_2}{d\omega})^2 - 4(\lambda+p)(1-s_3)\,\frac{A_2{}^2}{B_2}\ s_3 f_2''\,\frac{dk_2}{d\omega}}\ \right] \qquad (7.2.24)$$

Taking into account that $f_2'' < 0$, the expression under the square root in (7.2.24) is positive and therefore the roots are real; moreover, since

$$\sqrt{(p\,\frac{dk_2}{d\omega}\ )^2 - 4(\lambda+p)(1-s_3)\,\frac{A_2{}^2}{B_2}\ s_3 f_2''\,\frac{dk_2}{d\omega}} > p\,\frac{dk_2}{d\omega}$$

the roots are of opposite sign. Hence, by definition, $(k^*,\ \omega^*)$ is a saddle point in the set

of pairs of functions (k(t), ω(t)) which satisfy the necessary conditions and we may be sure that there exist two trajectories which tend asymptotically to the point (k* , ω*). These trajectories which are called the stable branches of the saddle are labelled in figure 7.1 by RE and R'E, Pontryagin (1962), Hadley and Kemp (1971).

Therefore, the unique optimal growth path in the vicinity of the balanced growth equilibrium lies along the stable branch of the saddle. However, if the initial capital stock per capita is extremely small ($k_0 < k_1$ **) or extremely large ($k_0 > k_2$ **) there can be an initial phase of specialisation. In the case where $k_0 < k_1$ **, then the initial phase will be one of specialization in the production of agricultural goods, followed by nonspecialized movement along the stable branch past k_1**, while if $k_0 > k_2$ **, initial specialization in the production on non-agricultural goods will be followed by nonspecialized movement down the stable branch past k_2 **. Thus, in figure 7.1, the heavy shaded curve indicates paths of optimal growth.

We return now to the case of nonzero technical change. From the equation (7.2.19) which is nonautonomous (ie. $B_2(t)$ varying over time), it is seen that ω* is simply a function of time, ie. $\omega^* = \omega^*(t)$. Therefore, from (7.2.18) it is $k^* = k^*(t)$. Below, we will try to analyse the behaviour of $k^*(t)$ and $\omega^*(t)$ over time.

From equation (7.2.18) we have taking its derivative with respect to time that

$$\dot{k}^*(t) = \frac{p}{(\lambda+p)(1-s_3)} \dot{k}_2^*(t) \qquad (7.2.25)$$

therefore,

$$\text{sgn } \dot{k}^*(t) = \text{sgn } \dot{k}_2^*(t) \qquad (7.2.26)$$

Thus, we have to investigate the sign of $\dot{k}_2^*(t)$.

It is known from (7.2.17) that

$$A_2(t) f'_2 (\bar{k}_2^*(t)) = \frac{\lambda}{s_3} \qquad (7.2.27)$$

Differentiating (7.2.27) with respect to time we have

$$\dot{A}_2(t)\, f'_2(\bar{k}_2^*(t)) + A_2(t)\, \dot{f}'_2(\bar{k}_2^*(t)) = 0 \qquad (7.2.28)$$

But $$\dot{A}_2(t) = A_2(t)\,\lambda_{12} \qquad (7.2.29)$$

and $$\dot{f}'_2(\bar{k}_2^*(t)) = f''_2(\bar{k}_2^*(t))\,\dot{\bar{k}}_2^*(t) =$$

$$= f''_2(\bar{k}_2^*(t))\left[\left(\frac{\dot{A}_2(t)}{B_2(t)}\right) k_2^*(t) + \frac{A_2(t)}{B_2(t)}\,\dot{k}_2^*(t)\right]$$

$$= f''_2(\bar{k}_2^*(t))\left[\frac{A_2(t)}{B_2(t)}(\lambda_{12}-\lambda_{22})\,k_2^*(t) + \frac{A_2(t)}{B_2(t)}\,\dot{k}_2^*(t)\right] \qquad (7.2.30)$$

Substituting (7.2.29) and (7.2.30) into (7.2.28) we obtain

$$A_2(t)\lambda_{12} f'_2(\bar{k}_2^*(t)) + A_2(t)\frac{A_2(t)}{B_2(t)}\, f''_2(\bar{k}_2^*(t))\left[(\lambda_{12}-\lambda_{22})\,k_2^*(t) + \right.$$

$$\left. + \dot{k}_2^*(t)\right] = 0$$

or dropping the index of time and rearranging

$$B_2\lambda_{12} f'_2 + A_2 f''_2(\lambda_{12}-\lambda_{22})\,k_2^* + A_2 f''_2\,\dot{k}_2^* = 0$$

which finally yields

$$\dot{k}_2^* = \frac{B_2\lambda_{12} f'_2 + A_2(\lambda_{22}-\lambda_{12})(-f''_2)\,k_2^*}{A_2(-f''_2)} \qquad (7.2.31)$$

From (7.2.31) we see that

$$\operatorname{sgn}\dot{k}_2^* = \operatorname{sgn}(\text{numerator of 7.2.31}) \qquad (7.2.32)$$

Investigation of (7.2.32) yields

1.1) If $\lambda_{22} = \lambda_{12} \neq 0$, ie. technical change is neutral in the second sector, then $\dot{k}_2^*$ is positive.

1.2) If $\lambda_{22} > \lambda_{12}$, ie. technical change is labour-saving in the second sector, then $\dot{k}_2^*$ is positive.

1.3) If $\lambda_{22} = \lambda_{12} = 0$, ie. there is no technical change in the second sector (as in figure 7.1), then $\dot{k}_2^*$ is zero.

1.4) If $\lambda_{22} < \lambda_{12}$, ie. technical change is capital-saving in the second sector, then the sign of $\dot{k}_2^*$ is ambiguous.

To specify the sign of $\dot{k}_2^*$ in the last case we need more assumptions about the various parameters and variables of (7.2.31). $\dot{k}_2^*$ is positive, zero or negative according to whether:

$$B_2 \lambda_{12} f'_2 \gtreqless A_2 (\lambda_{12} - \lambda_{22})(-f''_2)\, k_2^*$$

or

$$\frac{f'_2}{-f''_2} \frac{1}{k_2^*} \gtreqless \frac{A_2}{B_2} \frac{\lambda_{12} - \lambda_{22}}{\lambda_{12}} \tag{7.2.33}$$

But from (3.6.1), (3.4.4), (3.3.70) and (3.6.7) it is known that

$$\sigma_2 = \frac{dk_2}{d\omega} \frac{\omega}{k_2} = \frac{-f'^2_2}{f_2 f''_2} \frac{\omega}{k_2} = \frac{f'_2}{-f''_2} \frac{f'_2}{f_2} \frac{\omega}{k_2}$$

or

$$\frac{f'_2}{-f''_2} = \frac{f_2}{f'_2} \frac{k_2}{\omega} \qquad \sigma_2 = \frac{A_2}{B_2} (\omega + k_2) \frac{k_2}{\omega} \sigma_2$$

$$= \frac{A_2}{B_2} \frac{\sigma_2}{\epsilon_{22}} k_2 \tag{7.2.34}$$

Substituting (7.2.34) into (7.2.23) we obtain

$$\frac{A_2}{B_2} \frac{\sigma_2^*}{\epsilon_{22}^*} k_2^* \frac{1}{k_2^*} \gtreqless \frac{A_2}{B_2} \frac{\lambda_{12} - \lambda_{22}}{\lambda_{12}}$$

or

$$\frac{\sigma_2^*}{\epsilon_{22}^*} \gtreqless 1 - \frac{\lambda_{22}}{\lambda_{12}} \tag{7.2.35}$$

therefore (1.4) is written as

1.4) $$\dot{k}_2^* \left[\begin{matrix}>\\=\\<\end{matrix}\right] 0 \quad \text{if} \quad \frac{\sigma_2^*}{\epsilon_{22}^*} \left[\begin{matrix}>\\=\\<\end{matrix}\right] 1 - \frac{\lambda_{22}}{\lambda_{12}} \tag{7.2.36}$$

(remembering that σ_2 is the elasticity of substitution between the two factors of production and ϵ_{22} is the current elasticity of output with respect to labour input, both in the second sector).

Turning now to (7.2.26) we see that the sign of $\dot{k}^*(t)$, ie. the behaviour of $k^*(t)$ over time, depends on the assumptions 1 to 4', above.

For the investigation of $\omega^* = \omega^*(t)$ we differentiate (7.2.19) with respect to time, ie.

$$\dot{\omega}^*(t) = \frac{s_3}{\lambda} \left(\dot{B}_2(t)\, f_2(\bar{k}_2^*(t)) + B_2(t)\, \dot{f}_2(\bar{k}_2^*(t)) \right) - \dot{k}_2^*(t) \qquad (7.2.37)$$

It is known that $\dot{B}_2(t) = B_2(t)\,\lambda_{22}$

and $\dot{f}_2(\bar{k}_2^*(t)) = f'_2(\bar{k}_2^*(t))\,\dot{\bar{k}}_2^*(t) =$

$$= \frac{A_2(t)}{B_2(t)}\, f'_2(\bar{k}_2^*(t))\cdot[(\lambda_{12}-\lambda_{22})\,k_2^*(t) + \dot{k}_2^*(t)]$$

Therefore (7.2.37) is written

$$\dot{\omega}^* = \frac{s_3}{\lambda} B_2 \lambda_{22} f_2 + \frac{s_3}{\lambda} A_2 f'_2 (\lambda_{12}-\lambda_{22})\, k_2^* +$$

$$+ \frac{s_3}{\lambda} A_2 f'_2 \dot{k}_2^* - \dot{k}_2^*$$

or

$$\dot{\omega}^* = \frac{s_3}{\lambda} A_2 f'_2 k_2^* \left[\frac{B_2}{A_2} \frac{f_2}{f'_2} \frac{\lambda_{22}}{k_2^*} + (\lambda_{12}-\lambda_{22}) \right] +$$

$$+ \left(\frac{s_3}{\lambda} A_2 f'_2 - 1\right) \dot{k}_2^*$$

which according to (3.3.70), (3.3.6) and (7.2.17) is written

$$\dot{\omega}^* = \frac{s_3}{\lambda} A_2 f'_2 k_2^* \left[\frac{\omega^* + k_2^*}{k_2^*} \lambda_{22} + (\lambda_{12}-\lambda_{22}) \right]$$

$$= \frac{s_3}{\lambda} A_2 f'_2 k_2^* \left[\frac{\lambda_{22}}{\epsilon_{12}^*} + (\lambda_{12}-\lambda_{22}) \right]$$

and finally with the help of (3.6.8) and (3.6.11)

$$\dot{\omega}^* = \frac{s_3}{\lambda} A_2 f'_2 k_2^* (\lambda_{12} \epsilon_{12}^* + \lambda_{22} \epsilon_{22}^*) \tag{7.2.38}$$

or

$$\dot{\omega}^* = \frac{s_3}{\lambda} A_2 f'_2 k_2^* \pi_2^* \tag{7.2.39}$$

(remembering that π_2 is the intensity of innovation for the second sector defined in (3.6.9)).

Investigation of (7.2.38) or (7.2.39) yields:

2.1) $\dot{\omega}^*$ is always positive if there is technological progress in at least one input in the second sector, ie. $\lambda_{12} > 0$ or $\lambda_{22} > 0$ or both > 0.

2.2) $\dot{\omega}^*$ is always zero if both λ_{12} and λ_{22} are zero, ie. there is no technological change in the second sector (as in figure 7.1).

The analysis is summarised in the table 7.. .

TABLE 7.1

Cases	1.1	1.2	1.3	1.4'
2.1	ω^* increasing k^* increasing	ω^* increasing k^* increasing	– –	ω^* increasing k^* ? (7.2.36)
2.2	– –	– –	ω^* constant k^* constant	– –

In conclusion we would like to note that at any given instant of time $t = t^*$, equations (7.2.17) and (7.2.18) uniquely determine the wage/rental ratio $\omega(t^*) = \omega^*(t)$ and the capital labour ratio $k(t^*) = k^*(t)$. In the case of zero technical change the equilibrium values $(k^*(t), \omega^*(t))$ if once obtained would optimally be maintained forever, whilst in the case of nonzero technical progress the equilibrium values $(k^*(t), \omega^*(t))$ follow paths according to table 7.1, ie. $\omega^*(t)$ is always increasing whilst for the deter-

mination of the path of k*(t) we need further assumptions.

7.3 The Specific Two-sector Growth Model and Optimization Over Time.

In this section we will present the optimal control problem for the specific two-sector growth model of the Cobb-Douglas type, presented in chapter four.

The problem is similar to that presented in the previous section with the only difference being the objective functional. There, we used the notion of constant marginal utility and therefore the objective functional was per capita consumption, whilst here we use a particular utility index of current consumption per capita u(c) with the characteristic of decreasing marginal utility, Tinbergen (1960).

This particular concave utility index u(c) which we will employ is given by

$$u(c) = \frac{1}{1-v} \; c^{\,1-v} \tag{7.3.1}$$

where v is a constant positive parameter, a form which has been used a great deal by Ramsey, Frich, Tinbergen and others.

The question now is whether it is necessary to be involved with the complications of the previous section, in trying to solve the optimal control problem, or to try to simplify it by taking into account the specific functions of the Cobb-Douglas model.

On practical grounds we follow the second approach.

In chapter four we found that the per capita output of the two sectors and the relative price are given respectively by the following expressions:

$$q_1(t) = \frac{Y_1(t)}{L(t)} = \zeta_1 \, [\, a + b\,\mu(t)\,]^{-\beta_1} \, e^{\pi_1 t} \, k(t)^{\alpha_1} \tag{7.3.2}$$

$$q_2(t) = \frac{Y_2(t)}{L(t)} = \zeta_2 \, [\, a + b\,\mu(t)\,]^{-\beta_2} \, \mu(t)^{\beta_2} \, e^{\pi_2 t} \, k(t)^{\alpha_2} \tag{7.3.3}$$

and

$$p(t) = \zeta_3 \, [\, a + b\,\mu(t)\,]^{\beta_1 - \beta_2} \, \mu(t)^{\beta_2} \, e^{(\pi_2 - \pi_1)t} \, k(t)^{\beta_1 - \beta_2} \qquad (7.3.4)$$

Therefore, the overall per capita output of the economy (in terms of the second sector goods) is given by

$$\begin{aligned} q(t) &= p(t)\; q_1(t) + q_2(t) \\ &= \zeta_1 \zeta_3 \, [\, a + b\,\mu(t)\,]^{-\beta_2} \, \mu(t)^{\beta_2} \, e^{\pi_2 t} \, k(t)^{\alpha_2} \\ &\quad + \zeta_2 \, [\, a + b\,\mu(t)\,]^{-\beta_2} \, \mu(t)^{\beta_2} \, e^{\pi_2 t} \, k(t)^{\alpha_2} \end{aligned}$$

or finally

$$q(t) = [\, \zeta_1 \zeta_3 + \zeta_2 \,] \, [\, a + b\,\mu(t)\,]^{-\beta_2} \, \mu(t)^{\beta_2} \, e^{\pi_2 t} \, k(t)^{\alpha_2} \qquad (7.3.5)$$

where the parameters $\zeta_1, \zeta_2, \zeta_3$; a and b are given by

$$a = m_1 \beta_1 \, \left[\, 1 + (1 - s_3)\, \frac{\alpha_2}{\beta_2} \,\right] \qquad (7.3.6)$$

$$b = 1 - m_1 \beta_1 \, \left[\, 1 + (1 - s_3)\, \frac{\alpha_1}{\beta_1} \,\right] \qquad (7.3.7)$$

$$\zeta_1 = \frac{\Gamma_1 \, (\alpha_1 \beta_2)^{\alpha_1} \, a}{(\alpha_1 \beta_2 \, a + \alpha_2 \beta_1 \, b)^{\alpha_1}} \qquad (7.3.8)$$

$$\zeta_2 = \frac{\Gamma_2 \, (\alpha_2 \beta_1)^{\alpha_2} \, b}{(\alpha_1 \beta_2 \, a + \alpha_2 \beta_1 \, b)^{\alpha_1}} \qquad (7.3.9)$$

and

$$\zeta_3 = \frac{\Gamma_2 \, \alpha_2^{\,\alpha_2} \, \beta_2^{\beta_1} \, [m_1 \, (\alpha_1 \beta_2 - \beta_1 \alpha_2) + \beta_2]^{\beta_2}}{\Gamma_1 \, \alpha_1^{\,\alpha_1} \, [\alpha_1 \beta_2 \, a + \beta_1 \alpha_2 \, b]^{\beta_1}} \qquad (7.3.10)$$

(it is also assumed that $s_1 = s_2 = 0$, $0 \leqslant s_3 \leqslant 1$)

Comparing (7.3.5) and (7.3.3) we see that these two production functions are identical, except of course for their constant parameter. This result is similar to that of Mundlak and Mosenson (1970-72) who write that "when the production functions in both sectors are Cobb-Douglas and when the income and price elasticities are unitary, the aggregate production function, including the technical change component, will be identical, up to a

scalar, to that of sector two. As such, of course it is also a Cobb-Douglas function. The asymmetry with respect ot the two sectors is a result of the choice of numeraire". Hence, by using the aggregate production function (7.3.5), instead of the two production functions (7.3.2) and (7.3.3), we can reduce our optimal control problem to that of the one sector models.

Doing so, our optimal control problem becomes:

$$\max_{[c(t)]} \int_0^{\infty} e^{-pt} u(c(t))\, dt \qquad (7.3.11)$$

s.t.

$$\dot{k}(t) = f(k,t) - c(t) - \lambda k(t) \qquad (7.3.12)$$

$$k(0) = k_0$$

$$0 \leqslant c(t) \leqslant f(k, t)$$

$$c(t) = \text{piecewise continuous}$$

where f (k, t) is the aggregate production function (7.3.5), k(t) is the state variable and c(t) is the control variable.

The Hamiltonian for this problem (dropping the time index from the variables) is given by

$$H(k, c, t) = e^{-pt} [\, u(c) + \tau [\, f(k) - \lambda k - c] \,] \qquad (7.3.13)$$

where r(t) is the costate variable as before.

The canonical equations are

$$\dot{k} = f(k) - \lambda k - c \qquad (7.3.14)$$

and

$$\dot{\tau} = -[f'(k) - (\lambda + p)\,]\, \tau \qquad (7.3.15)$$

The optimal per capita consumption c(t) must maximise the Hamiltonian or, equivalently, maximize H e^{pt}. Hence,

$$\frac{\partial}{\partial c}(H e^{pt}) = u'(c) - r = 0$$

ie. $$r = u'(c) \tag{7.3.16}$$

From (7.3.16) we have by taking its derivative with respect to time

$$\frac{\dot{r}}{r} = \frac{u''(c)}{u'(c)} \dot{c} = -\sigma(c) \frac{\dot{c}}{c} \tag{7.3.17}$$

where

$$\sigma(c) = -c \frac{u''(c)}{u'(c)} \tag{7.3.18}$$

is the elasticity of marginal utility.

Substituting (7.3.15) into (7.3.17) we have

$$\dot{c} = \frac{c}{\sigma(c)} [\, f'(k) - (\lambda + p) \,] \tag{7.3.19}$$

By substituting (7.3.1) into (7.3.18) we have

$$\sigma(c) = -c \frac{(-v) c^{-v-1}}{c^{-v}} = v$$

and therefore (7.3.19) becomes

$$\dot{c} = \frac{c}{v} [\, f'(k) - (\lambda + p) \,] \tag{7.3.20}$$

Equations (7.3.14) and (7.3.20) describe the differential equations of the state, k(t), and the control, c(t), variables respectively.

By putting $\dot{k} = \dot{c} = 0$ in (7.3.14) and (7.3.20) we derive the balanced growth solution of the optimal control problem. We have three cases to distinguish.

The first is that in which there is no technological progress and the $\mu(t)$ is constant

over time. Putting these restrictions into the model, it behaves as the neoclassical optimal economic growth models, and investigation of it, Intriligator (1971), shows that

$$\dot{c} \left\{ \begin{matrix} = \\ > \\ < \end{matrix} \right\} 0 \qquad \text{if} \qquad k \left\{ \begin{matrix} = \\ < \\ > \end{matrix} \right\} k^* \tag{7.3.21}$$

and

$$\dot{k} \left\{ \begin{matrix} = \\ > \\ < \end{matrix} \right\} 0 \qquad \text{if} \qquad c \left\{ \begin{matrix} = \\ < \\ > \end{matrix} \right\} f(k) - \lambda k \tag{7.3.22}$$

where k^* is the balanced-growth capital-labour ratio.

In the second case only $\mu(t)$ is constant over time. This is the case studied by Inagaki (1966) according to which there is exogenous exponential technological pregress of the product-augmenting type. He found that for the existence of the optimal path the following conditions must hold, Koopmans(1967).

$$v = -\lim_{c \to \infty} \frac{cu''(c)}{u'(c)} \quad \text{and} \quad p > \pi_2 \frac{1-v}{1-\alpha_2} \tag{7.3.23}$$

where

$$\alpha_2 = \lim_{k \to \infty} \frac{kf'(k)}{f(k)} \tag{7.3.24}$$

and π_2 is the rate of technological progress.

In the third case we deal with the balanced-growth problem more generally assuming that our system is nonautonomous due to technological progress and to the variation of $\mu(t)$ over time. Therefore, from (7.3.20) we have that

$$\zeta \alpha_2 (a + b\mu)^{-\beta_2} \mu^{\beta_2} e^{\pi_2 t} k^{*-\beta_2} - (\lambda + p) = 0$$

or finally, solving for k^*

$$k^* = \left(\frac{\lambda + p}{\zeta \alpha_2}\right)^{\frac{1}{\beta_2}} (a + b\mu)^{-1} \mu \, e^{\frac{\pi_2}{\beta_2} t} \tag{7.3.25}$$

where $\zeta = \zeta_1 \zeta_3 + \zeta_2$.

From (7.3.14) we obtain that

$$\zeta (a + b\mu)^{-\beta_2} \mu^{\beta_2} e^{\pi_2 t} k^{*\alpha_2} - c^* - \lambda k^* = 0$$

or after substitution from (7.3.25), the last expression can be written as

$$c^* = \frac{p + \beta_2 \lambda}{\alpha_2} k^* \qquad (7.3.26)$$

where c^* is the balanced-growth per capita consumption.

From (7.3.25) and (7.3.26), it is obvious that the balanced-growth capital-labour ratio, k^*, and the per capita consumption, c^*, are not stationary, due to the nonautonomous effects of the technological progress and of $\mu(t)$.

Having found the optimal paths for c and k from the equations (7.3.20) and (7.3.14) the corresponding paths of the other variables of the model are given by

$$\ell_1(t) = a(a + b\mu(t))^{-1} \qquad (7.3.27)$$

$$\ell_2(t) = b(a + b\mu(t))^{-1} \quad \mu(t) = 1 - \ell_1(t) \qquad (7.3.28)$$

$$k_1(t) = \frac{\alpha_1 \beta_2}{\alpha_1 \beta_2 a + \alpha_2 \beta_1 b} (a + b\mu(t)) \quad k(t) \qquad (7.3.29)$$

$$k_2(t) = \frac{\alpha_2 \beta_1}{(\alpha_1 \beta_2 a + \alpha_2 \beta_1 b)} \frac{(a + b\mu(t))}{\mu(t)} \quad k(t) \qquad (7.2.30)$$

$$\omega_2(t) = \frac{\beta_2 (a + b\mu(t))}{(m_1 (\alpha_1 \beta_2 - \beta_1 \alpha_2) + \beta_2)\mu(t)} k(t) \qquad (7.3.31)$$

From the expressions above we see that the allocation of labour does not depend explicitly on the optimal path of the overall capital-labour ratio, a result which was expected, because of the specification of the Cobb-Douglas model (see chapter four)

Therefore, the optimal control problem becomes:

Given people's behaviour (or with some small changes in it through change in $\mu(t)$

what are the optimal paths of all the other variables of the economy, that maximise the national welfare through time.

7.4 Optimal Growth and the Case of Greece

In this section we will apply the analysis of section 7.3 to the case of the Greek economy. Therefore, the parameters of the optimal growth problem are those which correspond to the estimated ones for the Greek economy in chapter six. These parameters, which are taken to be constant through the various experiments below, are:-

$$\begin{aligned} \Gamma_1 &= 1.303030 & \Gamma_2 &= 2.646246 \\ \alpha_1 &= 0.602957 & \alpha_2 &= 0.555314 \\ \beta_1 &= 0.397043 & \beta_2 &= 0.444686 \\ \pi_1 &= 0.011884 & \pi_2 &= 0.021885 \\ & \delta = 0.020180 & & \\ & \beta_0 = 2.333333 & & \end{aligned} \tag{7.4.1}$$

The initial conditions for the state, k(t), and the control, c(t), variables are taken to be equal to the corresponding values of the year 1972 for the Greek economy. Year 1972 is the last year of the sample period used in the estimation of the two-sector growth model in chapter six. These values, which are also kept unchanged, in the exercise below, are

$$k(0) = 203.0$$

both in thousand drs. and in constant 1958 prices.

$$c(0) = 30.9 \tag{7.4.2}$$

The remaining parameters are not estimated within the model. We try a variety of values of these parameters to examine the sensitivity of the system to changes in these values. For example, in our exercise these parameters take the values

$$\begin{aligned} g &= 0.040 \ , \ 0.060 \\ s_3 &= 0.553 \ , \ 0.600 \ , \ 0.650 \\ m_1 &= 0.189 \ , \ 0.170 \ , \ 0.150 \\ \eta &= 0.000 \ , \ 0.010 \ , \ 0.020 \end{aligned} \tag{7.4.3}$$

In the set of values (7.4.3), parameter g shows the speed of development, first used in (4.5.1), and together with β_0 give that $\mu(0) = 0.300$ which is equal to the value of $\mu(t)$ for the year 1972. Also, the values of $s_3 = 0.553$ and $m_1 = 0.189$ are those of the corresponding parameters for the year 1972. The growth rate of the labour force, η, in Greece is really very small, and with the return of the Greek workers who migrated abroad it is not believed that the rate will exceed the value of 0.01 or 0.02.

For the solution of the differential equations (7.3.14) and (7.3.20) numerically we used Merson's method and Newton's iteration method, Haselgrove (1961).

As we said in section 7.3 we will solve the optimal problem assuming an infinite time planning horizon. With this assumption we overcome the problem of putting terminal conditions on the variables but it is possible to have problems about the existence of the optimal path solution. In any case convergence is assured if the initial stock of capital per worker is less than the maximum sustainable level, Intriligator (1971).

For the developing economies, most of the various parameters, which are taken to be constant over time, are not in fact constant in the long-run. Therefore, the results that we present below must be accepted with a little care after the 20th period.

In tables 7.2–7.5 and 7.6–7.9 we present the optimal path solution to our model found directly by solving the differential equations (7.3.14) and (7.3.20) for various values of v and p. Values for v both, less than and greater than one are used. There is no reason to believe that in this normative exercise we have to use values of v less than one, Chakravarty (1969). The other parameters have the values

$$g = 0.04 \quad , s_3 = 0.553 \ , m_1 = 0.189 \quad , \eta = 0.01 \tag{7.4.4}$$

and are always the same for the tables presented, except if otherwise mentioned.

Across each of the tables we can see the sensitivity of the optimal solution with respect

TABLE 7.2

(v = 0.5): Sensitivity of optimal capital/labour ratios to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	203.0	203.0	203.0	203.0	203.0	203.0
1	208.6	209.3	210.0	210.7	211.3	212.0
2	208.6	211.9	215.1	218.0	220.9	223.5
3	200.6	209.4	217.5	224.9	231.7	238.0
4	180.9	199.6	216.1	230.8	244.0	255.7
5	143.5	179.4	209.7	235.6	257.8	277.0
6	76.1	143.5	196.0	238.4	273.4	302.5
7		81.2	171.8	238.9	291.1	332.8
8			131.4	235.7	311.0	368.4
9			61.9	227.6	333.6	409.9
10				212.3	359.2	458.1
15					550.5	824.8
20					914.3	1471.1
25					1575.4	2497.4
30					2688.3	4018.7
35					4426.9	6178.2
40					6990.3	9160.2
45					10619.6	13203.1
50					15619.3	18614.4

to changes in p whilst between tables we can see the sensitivity of the optimal solution with respect to changes in v. For example, in table 7.2, where v = 0.5, we see that for p being less than or equal to 0.06 the optimal path of k(t) shows decumulation of capital stock per worker; this decumulation starts earlier the less is the value of p. The fact of the decumulation of capital stock per worker is shown by the comparison of the tables 7.6 and 7.10 where we can see that with the passage of time consumption per worker absorbs all the output per worker and so nothing is left for investment. This fact can also be seen from table 7.14 where the propensity to save is decreasing over time for p less than or equal to 0.06.

For values of p greater than 0.06 the optimal paths of the variables change completely.

TABLE 7.3

(v = 1.5): Sensitivity of optimal capital/labour ratios to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	203.0	203.0	203.0	203.0	203.0	203.0
1	210.7	211.0	211.2	211.4	211.6	211.8
2	218.4	219.3	220.2	221.2	222.1	222.9
3	225.7	228.0	230.3	232.4	234.5	236.6
4	232.7	237.1	241.3	245.4	249.4	253.2
5	239.1	246.5	253.6	260.3	266.8	273.0
6	244.6	256.2	267.1	277.4	287.2	296.4
7	249.1	266.2	282.1	297.0	310.9	324.0
8	252.0	276.4	298.7	319.4	338.5	356.2
9	253.0	286.7	317.2	344.9	370.3	393.5
10	251.5	297.1	337.7	374.0	406.8	436.5
15	165.8	344.8	480.7	590.5	681.9	759.3
20		357.2	729.0	983.4	1175.8	1327.8
25		213.2	1159.4	1655.0	1994.4	2245.8
30			1887.0	2737.0	3267.0	3637.8
35			3070.0	4392.4	5150.7	5656.2
40			4915.1	6822.5	7840.5	8491.5
45			7682.5	10278.3	11581.1	12385.8
50			11699.4	15075.2	15536.3	17647.7

For example, in the same table 7.2, where v = 0.5, and for p = 0.08 we have that the capital stock per worker is increasing over time, a fact which can also be seen by the comparison of tables 7.6 and 7.10 or by the increasing propensity to save in table 7.14.

By now keeping p constant we make a comparison of the optimal time paths between tables. This comparison shows that the lower is v the higher is the per capita consumption and consequently the lower is the capital stock per worker over time.

From the analysis above we see that the two parameters, v and p, play a crucial part in the process of planning economic growth. A complete picture of this part is shown in tables 7.14–7.17 where we can see the sensitivity of the propensities to save to changes in p and v. In these tables we see that the propensity to save moves towards either zero

TABLE 7.4

(v = 2.0): Sensitivity of optimal capital/labour ratios to changes in p

t \ p	0.00	0.02	0.04	0.06	0.08	0.10
0	203.0	203.0	203.0	203.0	203.0	203.0
1	211.0	211.2	211.3	211.5	211.6	211.8
2	219.5	220.2	220.8	221.5	222.2	222.9
3	228.4	230.1	231.7	233.3	234.9	236.4
4	237.8	241.0	244.1	247.1	250.0	252.8
5	247.8	253.1	258.2	263.1	267.9	272.5
6	258.2	266.4	274.1	281.6	288.8	295.7
7	269.2	281.0	292.3	303.0	313.2	322.9
8	280.7	297.3	312.9	327.6	341.5	354.7
9	292.7	315.2	336.2	355.9	374.2	391.5
10	305.3	335.2	362.7	388.2	411.9	433.8
15	375.0	473.2	556.8	629.7	694.0	751.1
20	455.8	709.8	905.5	1064.2	1194.5	1308.5
25	529.1	1115.1	1503.1	1793.8	2023.1	2209.2
30	545.0	1794.6	2474.7	2946.7	3300.9	3577.4
35	321.8	2896.0	3980.4	4682.4	5186.0	5566.4
40		4614.7	6220.6	7198.1	7872.8	8367.7
45		7202.5	9447.1	10742.1	11606.2	12224.4
50		10979.4	13977.3	15628.0	16697.5	17445.9

or one over time. The direction in which s(t) is moving over time depends crucially on the values of the parameters v and p. For example, in table 7.15, where v = 1.5, we have that, for p = 0.0, s (t) moves towards zero and reaches it after about 15 years; for p = 0.02, s(t) reaches zero after about 25 years; for p = 0.04 and over, it moves towards one. The greater is p, above the level of 0.04, the higher is the rate of increase in s(t).

By comparing the tables 7.14–7.15, we see that for the cases where s(t) moves towards zero the greater is v the lower is the rate of change of s(t). In the other cases where s(t) moves towards one we see that, for example, in tables 7.15 and 7.17, for intermediate values of p, ie. $0.02 \leqslant p < 0.08$, the greater is v the higher is the rate of growth of s(t), and for higher values of p, ie. $p \geqslant 0.08$, the greater is v the lower is the rate of growth of s(t). These results come as a consequence of the behaviour of the

TABLE 7.5

(v = 2.5): Sensitivity of optimal capital/labour ratios to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	203.0	203.0	203.0	203.0	203.0	203.0
1	211.1	211.3	211.4	211.5	211.7	211.8
2	220.1	220.7	221.2	221.7	222.3	222.8
3	230.0	231.3	232.6	233.8	235.1	236.3
4	240.8	243.3	245.7	248.1	250.4	252.7
5	252.7	256.8	260.8	264.7	268.5	272.2
6	266.0	272.1	278.2	284.1	289.7	295.2
7	280.4	289.4	298.1	306.5	314.5	322.3
8	296.4	309.0	320.9	332.5	343.3	353.8
9	314.1	331.0	347.0	362.2	376.6	390.3
10	333.7	355.8	376.7	396.3	414.8	432.3
15	468.9	536.2	596.7	651.3	700.9	746.1
20	698.9	857.5	991.6	1107.2	1208.1	1296.9
25	1089.9	1407.2	1658.7	1865.4	2038.8	2186.6
30	1741.7	2304.8	2723.4	3052.3	3319.0	3539.6
35	2794.6	3705.5	4343.5	4825.2	5203.8	5509.1
40	4436.1	5806.4	6716.5	7380.0	7887.3	8287.3
45	6909.7	8857.0	10091.4	10963.5	11614.5	12117.6
50	10528.0	13173.2	14782.6	15888.8	16696.7	17310.1

differential equations (7.3.14) and (7.3.20). In (7.3.20) for example, the increase in v or p produces a decrease in $\dot{c}$ with the result that $\dot{k}$ increases more quickly than before. But the faster increase in $\dot{k}$ produces a faster decrease in f'(k), the marginal productivity of capital, and so on. These "feedback" effects are shown in the tables presented thus far and in particular in the tables 7.14–7.17. Therefore, before we choose any value for v or p we must look at the path of s(t) that they generate.

The sensitivity of the relative price, p(t), depends absolutely on the behaviour of the optimal path of the capital stock per worker, as can be seen from (7.3.4). From tables 7.18 and 7.19, for example, we see that the terms of trade are increasing in favour of the agricultural sector. The higher is p the slower is the increase of p(t) for a given v. On the other hand, for values of p less than or equal to 0.06 the higher is v the lower is

TABLE 7.6

(v = 0.5): Sensitivity of optimal per capita consumption to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	30.9	30.9	30.9	30.9	30.9	30.9
1	37.2	35.8	34.4	33.1	31.8	30.5
2	45.1	41.6	38.4	35.4	32.6	30.1
3	55.1	48.7	43.0	38.0	33.6	29.7
4	68.3	57.2	48.5	41.0	34.6	29.3
5	87.1	69.1	55.2	44.3	35.7	28.8
6	118.7	85.3	63.6	48.1	36.8	28.3
7		112.3	74.7	52.6	37.9	27.6
8			90.9	57.9	39.0	26.9
9			120.6	64.4	40.1	26.0
10				72.6	41.1	25.1
15					44.8	19.0
20					44.2	12.6
25					38.7	7.5
30					30.3	4.2
35					21.6	2.2
40					14.4	1.1
45					9.1	0.5
50					5.5	0.2

the rate of increase in p(t), whilst for values of p greater than 0.06 the higher is v the higher is the rate of increase in p(t).

As we said in section 7.3 the balanced growth path of the capital stock per worker, $\hat{k}(t)$, and the per capita consumption, $c^*(t)$, depend on time because our system is nonautonomous, eqs. (7.3.25) and (7.3.26). Also, from these equations we see that both $k^*(t)$ and $c^*(t)$ do not depend on v. They do depend on the other parameters of the model and p. The sensitivity of $k^*(t)$ with respect to changes in p is shown in table 7.20. In this table we see that $k^*(t)$ is increasing over time and is increasing more slowly the higher is p. In table 7.20 we presented $c^*(t)$. From equation (7.3.26) we see that the higher is $c^*(t)$ the higher is p. But at the same time we find that the higher is p the lower is $k^*(t)$.

TABLE 7.7

(v = 1.5): Sensitivity of optimal per capita consumption to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	30.9	30.9	30.9	30.9	30.9	30.9
1	32.9	32.5	32.1	31.6	31.2	30.8
2	35.1	34.1	33.2	32.3	31.5	30.7
3	37.4	35.9	34.5	33.1	31.8	30.5
4	39.9	37.7	35.8	33.9	32.1	30.4
5	42.6	39.7	37.1	34.7	32.4	30.2
6	45.5	41.9	38.5	35.5	32.7	30.1
7	48.7	44.1	40.0	36.3	32.9	29.9
8	52.2	46.6	41.5	37.1	33.1	29.6
9	56.1	49.2	43.1	37.9	33.3	29.3
10	60.5	51.9	44.8	38.7	33.5	29.3
15	93.8	69.3	53.6	42.1	33.5	26.8
20		95.9	62.8	44.3	32.1	23.6
25		147.0	71.5	44.9	29.7	20.1
30			78.8	44.1	26.7	16.7
35			84.1	42.1	23.4	13.6
40			87.3	39.3	20.1	10.8
45			88.7	36.1	17.1	8.6
50			88.1	32.8	14.4	6.7

Therefore, the behaviour of $c^*(t)$ depends really on the combined effects of p on $c^*(t)$ directly and through $k^*(t)$. This combined effect is shown in table 7.21 where we see that the higher is p the lower is the rate of increase in $c^*(t)$.

In the tables presented thus far we assumed that the rate of increase in labour force was $\eta = 0.01$. In table 7.22 we present some results for the leading variables of the optimal problem for the case where $\eta = 0.00$ and $\eta = 0.02$. Comparing now the results in table 7.22 with the results of the previous tables we see that the higher is η the lower are k(t), c(t) and s(t), as was expected.

In table 7.23 we present the optimal k(t), c(t) and s(t) for the cases where $s_3 = 0.060$ and $s_3 = 0.65$. Comparing these results with those of the previous tables where $s_3 = 0.553$,

TABLE 7.8

(v = 2.0): Sensitivity of optimal per capita consumption to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	30.9	30.9	30.9	30.9	30.9	30.9
1	32.4	32.1	31.8	31.5	31.1	30.8
2	34.0	33.3	32.6	32.0	31.4	30.7
3	35.6	34.6	33.5	32.5	31.6	30.6
4	37.4	35.9	34.5	33.1	31.8	30.5
5	39.2	37.3	35.4	33.7	32.0	30.4
.6	41.2	38.7	36.4	34.2	32.2	30.3
7	43.3	40.2	37.4	34.8	32.4	30.1
8	45.5.	41.8	38.5	35.4	32.5	30.0
9	47.8	43.4	39.5	35.9	32.7	29.7
10	50.3	45.1	40.5	36.4	32.9	29.5
15	65.1	54.4	45.7	38.6	32.7	27.9
20	85.3	64.6	50.3	39.8	31.7	25.4
25	113.3	75.2	54.0	39.8	29.8	22.5
30	154.6	85.4	56.4	39.0	27.5	19.6
35	229.0	94.8	57.8	37.4	24.9	16.8
40		101.5	58.2	35.4	22.2	14.2
45		110.1	57.8	33.1	19.7	11.9
50		115.9	56.8	30.7	17.3	9.9

we see that the higher is s_3, the higher is k(t) and the lower is c(t) as was expected. For s(t) we cannot make a direct comparison because by changing s_3 the initial value of s(t) is also changed. In any case comparing the rates of growth of s(t) we see that the lower is s_3 the higher is the rate of growth of s(t), reflecting the greater efforts for development.

In table 7.24 we present the sensitivity of the optimal solution with respect to changes in the consumption allocation parameter, m_1. We see that the lower is m_1 the higher is k(t) and the lower is c(t), as was expected. On the other hand the higher is m_1 the higher is the rate of growth of s(t), because higher m_1 is associated with less developed economies.

TABLE 7.9

(v = 2.5): Sensitivity of optimal per capita consumption to changes in p

t \ p	0.00	0.02	0.04	0.06	0.08	0.10
0	30.9	30.9	30.9	30.9	30.9	30.9
1	32.1	31.9	31.6	31.3	31.1	30.9
2	33.3	32.8	32.3	31.8	31.3	30.8
3	34.6	33.8	33.0	32.2	31.4	30.8
4	36.0	34.8	33.7	32.7	31.6	30.6
5	37.4	36.0	34.5	33.1	31.8	30.5
6	38.9	37.0	35.2	33.5	31.9	30.4
7	40.4	38.1	36.0	34.0	32.1	30.3
8	42.0	39.2	36.8	34.4	32.2	30.2
9	43.6	40.5	37.5	34.8	32.3	30.0
10	45.4	41.7	38.3	35.2	32.4	30.0
15	54.8	47.9	41.9	36.8	32.3	28.4
20	65.6	54.1	44.9	37.5	31.4	26.4
25	77.2	59.8	47.1	37.5	30.0	24.1
30	89.2	64.9	48.5	36.7	28.0	21.5
35	101.2	69.2	49.1	35.4	26.0	19.0
40	113.0	72.8	49.1	33.9	23.7	16.7
45	124.3	75.6	48.6	32.1	21.4	14.5
50	135.1	77.9	47.8	30.5	19.3	12.5

Finally, in tables 7.25 and 7.26 we present the sensitivity of the optimal solution to changes in the speed of development, g. As we have seen in (7.3.27) and (7.3.28) the allocation of labour depends on $\mu(t)$, ie. on g. The higher is the g the faster is the decrease in ℓ_1 and consequently the faster is the increase in ℓ_2.

The change in g has its effects on the optimal capital/labour ratio, ie. the higher is g the higher is k(t), as can be seen in table 7.26. In the same table are shown the optimal sectoral capital/labour ratios, $k_i(t)$, as a combined effect of the change in ℓ_i's and the increase in k(t). Both $k_i(t)$ are increasing over time with $k_1(t)$ increasing faster due to the decrease in the labour-force engaged in the agricultural sector. In particular, for g = 0.06, $k_1(t)$ overtakes $k_2(t)$ at about the year 40. This is a result which is associated

TABLE 7.10

(v = 0.5): Sensitivity of the optimal per capita output to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	44.9	44.9	44.9	44.9	44.9	44.9
1	46.7	46.8	46.9	47.0	47.1	47.1
2	47.9	48.4	48.8	49.1	49.5	49.8
3	48.1	49.3	50.3	51.3	52.1	52.9
4	46.6	49.2	51.4	53.3	55.0	56.5
5	42.0	47.6	51.9	55.3	58.2	60.5
6	30.3	43.1	51.2	57.1	61.6	65.2
7		32.2	48.4	58.6	65.4	70.5
8			43.1	59.7	69.6	76.4
9			29.1	60.0	74.2	83.2
10				59.1	79.2	90.7
15					113.4	142.0
20					169.6	220.8
25					258.2	333.5
30					390.5	488.1
35					578.2	695.8
40					835.7	971.0
45					1181.0	1332.8
50					1638.2	1805.8

with the time period after which an economy can be characterized as entering the state of being a developed economy (see chapter 4). Of course this critical time depends explicitly on β_0 (starting level), on g (speed of development) and on the methods of production (α_i , β_i), taking into account the Cobb-Douglas specification of the model.

TABLE 7.11

(v = 1.5): Sensitivity of the optimal per capita output to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	44.9	44.9	44.9	44.9	44.9	44.9
1	47.0	47.0	47.0	47.1	47.1	47.1
2	49.2	49.3	49.4	49.5	49.6	49.7
3	51.4	51.7	51.9	52.2	52.5	52.7
4	53.6	54.1	54.7	55.2	55.7	56.2
5	55.8	56.7	57.6	58.5	59.3	60.0
6	57.9	59.4	60.8	62.1	63.3	64.5
7	60.0	62.3	64.3	66.2	67.9	69.4
8	61.9	65.2	68.0	70.6	72.9	75.0
9	63.6	68.2	72.1	75.6	78.6	81.3
10	65.0	71.3	76.5	81.0	84.9	88.3
15	58.3	87.5	105.2	118.0	127.8	135.6
20		100.6	149.5	176.5	195.0	208.6
25		85.0	217.6	265.4	294.3	314.4
30			320.8	394.4	435.1	461.9
35			471.9	575.7	629.0	662.5
40			687.9	824.5	890.7	931.0
45			986.7	1159.8	1239.8	1286.4
50			1395.3	1606.2	1699.2	1753.1

TABLE 7.12

(v = 2.0): Sensitivity of the optimal per capita output to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	44.9	44.9	44.9	44.9	44.9	44.9
1	47.0	47.0	47.1	47.1	47.1	47.1
2	49.3	49.4	49.5	49.6	49.6	49.7
3	51.7	51.9	52.1	52.3	52.5	52.7
4	54.2	54.6	55.0	55.4	55.8	56.1
5	56.9	57.6	58.2	58.8	59.4	60.0
6	59.7	60.7	61.7	62.6	63.5	64.4
7	62.6	64.2	65.6	66.9	68.1	69.3
8	65.7	67.9	69.8	71.6	73.3	74.9
9	69.0	71.9	74.5	76.9	79.1	81.1
10	72.4	76.2	79.6	82.7	85.5	88.0
15	91.8	104.3	114.2	122.2	129.0	134.8
20	115.2	147.2	168.6	184.5	196.9	206.9
25	140.9	213.1	251.5	277.5	296.7	311.5
30	161.0	312.0	372.9	410.9	437.6	457.6
35	134.8	456.8	545.1	596.5	631.4	656.7
40		663.6	783.3	849.4	892.7	923.0
45		952.0	1106.7	1188.6	1240.8	1277.0
50		1346.9	1540.2	1638.7	1700.0	1741.9

TABLE 7.13

(v = 2.5): Sensitivity of optimal per capita output to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	44.9	44.9	44.9	44.9	44.9	44.9
1	47.0	47.1	47.1	47.1	47.1	47.1
2	49.4	49.5	49.5	49.6	49.7	49.7
3	51.9	52.1	52.2	52.4	52.5	52.6
4	54.6	54.9	55.2	55.5	55.8	56.1
5	57.5	58.0	58.5	59.0	59.5	59.9
6	60.7	61.5	62.2	62.9	63.6	64.3
7	64.1	65.2	66.3	67.3	68.3	69.2
8	67.8	69.3	70.8	72.2	73.5	74.7
9	71.7	73.8	75.8	77.6	79.3	80.9
10	76.0	78.8	81.3	83.6	85.8	87.8
15	103.8	111.8	118.6	124.5	129.7	134.3
20	146.0	163.6	177.4	188.6	197.6	205.9
25	210.4	242.5	265.7	283.6	297.9	309.7
30	306.8	358.5	393.3	419.0	438.9	454.9
35	447.9	523.8	572.2	606.6	632.6	652.9
40	649.2	753.8	817.3	861.2	893.6	918.5
45	930.9	1067.8	1148.0	1202.1	1241.3	1270.8
50	1315.9	1490.3	1588.8	1653.8	1700.0	1734.4

TABLE 7.14

(v = 0.5): Sensitivity of Savings ratios to changes in p

t	0	5	10	15	20	25	30	35	40	45	50
p											
0.00	0.310	−1.073									
0.02	0.310	−0.452									
0.04	0.310	−0.064									
0.06	0.310	0.199	−0.228								
0.08	0.310	0.386	0.481	0.605	0.739	0.850	0.922	0.963	0.983	0.992	0.997
0.10	0.310	0.524	0.724	0.866	0.943	0.977	0.991	0.997	0.999	1.000	1.000

TABLE 7.15

(v = 1.5): Sensitivity of Savings ratios to changes in p

t	0	5	10	15	20	25	30	35	40	45	50
p											
0.00	0.310	0.237	0.070	−0.609							
0.02	0.310	0.300	0.271	0.208	0.047	−0.728					
0.04	0.310	0.356	0.415	0.491	0.580	0.672	0.754	0.822	0.873	0.910	0.937
0.06	0.310	0.407	0.523	0.643	0.749	0.831	0.888	0.927	0.952	0.969	0.980
0.08	0.310	0.454	0.606	0.738	0.835	0.899	0.939	0.963	0.977	0.986	0.992
0.10	0.310	0.436	0.671	0.803	0.887	0.936	0.964	0.980	0.988	0.993	0.996

TABLE 7.16

(v = 2.0): Sensitivity of Savings ratios to changes in p

t p	0	5	10	15	20	25	30	35	40	45	50
0.00	0.310	0.311	0.305	0.290	0.260	0.196	0.039	–0.698			
0.02	0.310	0.352	0.408	0.479	0.562	0.647	0.726	0.792	0.845	0.884	0.914
0.04	0.310	0.391	0.491	0.600	0.702	0.785	0.849	0.894	0.926	0.948	0.963
0.06	0.310	0.428	0.559	0.684	0.784	0.856	0.905	0.937	0.958	0.972	0.981
0.08	0.310	0.461	0.616	0.746	0.839	0.899	0.937	0.961	0.975	0.984	0.990
0.10	0.310	0.493	0.665	0.794	0.877	0.928	0.957	0.974	0.985	0.991	0.994

TABLE 7.17

(v = 2.5): Sensitivity of Savings ratios to changes in p

t p	0	5	10	15	20	25	30	35	40	45	50
0.00	0.310	0.350	0.403	0.472	0.551	0.633	0.709	0.774	0.826	0.866	0.897
0.02	0.310	0.382	0.471	0.572	0.670	0.753	0.819	0.868	0.903	0.929	0.948
0.04	0.310	0.411	0.529	0.647	0.747	0.823	0.877	0.914	0.940	0.958	0.970
0.06	0.310	0.439	0.579	0.705	0.801	0.868	0.912	0.942	0.961	0.973	0.982
0.08	0.310	0.466	0.623	0.751	0.841	0.899	0.936	0.959	0.974	0.983	0.989
0.10	0.310	0.491	0.661	0.788	0.872	0.922	0.953	0.971	0.982	0.989	0.993

TABLE 7.18

(v = 2;0): Sensitivity of optimal relative price to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	0.874	0.874	0.874	0.874	0.874	0.874
1	0.891	0.891	0.891	0.891	0.891	0.891
2	0.909	0.909	0.908	0.908	0.908	0.891
3	0.926	0.926	0.926	0.925	0.925	0.925
4	0.944	0.944	0.943	0.943	0.942	0.942
5	0.962	0.961	0.960	0.960	9.959	0.958
6	0.980	0.979	0.978	0.976	0.975	0.974
7	0.999	0.997	0.995	0.993	0.992	0.990
8	1.017	1.015	1.012	1.010	1.008	1.006
9	1.036	1.032	1.029	1.026	1.024	1.022
10	1.055	1.050	1.046	1.043	1.040	1.037
15	1.152	1.139	1.131	1.124	1.119	1.115
20	1.254	1.228	1.213	1.204	1.197	1.192
25	1.362	1.314	1.295	1.285	1.277	1.972
30	1.481	1.399	1.378	1.367	1.359	1.354
35	1.648	1.484	1.462	1.450	1.443	1.438
40		1.569	1.547	1.536	1.529	1.525
45		1.654	1.633	1.623	1.617	1.613
50		1.740	1.720	1.711	1.706	1.702

TABLE 7.19

($v = 2.5$): Sensitivity of optimal relative price to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	0.874	0.874	0.874	0.874	0.874	0.874
1	0.891	0.891	0.891	0.891	0.891	0.891
2	0.909	0.909	0.908	0.908	0.908	0.908
3	0.926	0.926	0.926	0.925	0.925	0.925
4	0.944	0.943	0.943	0.942	0.942	0.942
5	0.961	0.961	0.960	0.959	0.959	0.958
6	0.979	0.978	0.977	0.976	0.975	0.974
7	0.997	0.995	0.994	0.993	0.991	0.990
8	1.015	1.013	1.011	1.009	1.008	1.006
9	1.033	1.030	1.028	1.026	1.024	1.022
10	1.051	1.047	1.044	1.042	1.040	1.038
15	1.140	1.133	1.127	1.122	1.118	1.115
20	1.228	1.217	1.208	1.202	1.197	1.193
25	1.315	1.300	1.289	1.282	1.277	1.273
30	1.401	1.383	1.372	1.364	1.359	1.355
35	1.486	1.467	1.456	1.448	1.443	1.439
40	1.572	1.552	1.541	1.534	1.529	1.526
45	1.657	1.638	1.628	1.621	1.617	1.614
50	1.744	1.725	1.716	1.710	1.706	1.703

TABLE 7.20

Sensitivity of the optimal balanced capital/labour ratio to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	4756.9	1516.3	713.1	405.8	258.6	177.7
1	5039.6	1606.3	755.5	429.9	274.0	188.3
2	5337.6	1701.3	800.2	455.3	290.3	199.4
3	5651.7	1801.5	847.3	482.1	307.3	211.2
4	5982.9	1907.0	896.9	510.3	325.3	232.5
5	6361.8	2018.2	949.2	540.1	344.2	236.2
6	6699.6	2135.5	1004.3	571.5	364.2	250.3
7	7087.0	2258.9	1062.4	604.5	385.3	264.8
8	7495.1	2389.0	1123.6	639.3	407.5	280.0
9	7925.0	2526.1	1188.0	676.0	430.8	296.1
10	8378.0	2670.4	1255.9	714.6	455.5	313.0
15	11026.4	3514.6	1653.0	940.6	599.5	412.0
20	14446.4	4604.7	2165.7	1232.3	785.4	539.7
25	18853.0	6009.3	2826.3	1608.2	1025.0	704.4
30	24521.6	7816.1	3676.1	2091.7	1333.1	916.2
35	31804.3	10137.5	4767.8	2712.9	1729.1	1188.2
40	41151.7	13116.9	6169.1	3510.3	2237.2	1537.5
45	53139.8	16938.1	7966.3	4532.9	2889.0	1985.4
50	68505.9	21835.0	10269.9	5843.6	3724.4	2559.5

TABLE 7.20

Sensitivity of the optimal balanced per capita consumption to changes in p

t / p	0.00	0.02	0.04	0.06	0.08	0.10
0	115.0	91.3	68.6	53.6	43.5	36.3
1	121.8	96.7	72.7	56.8	46.1	38.5
2	129.0	102.4	77.0	60.2	48.8	40.7
3	136.6	108.4	81.5	63.7	51.7	43.1
4	144.6	114.8	86.3	67.5	54.7	45.7
5	153.0	121.5	91.3	71.4	57.9	48.3
6	161.9	128.5	96.6	75.6	61.3	51.1
7	171.3	136.0	102.2	79.9	64.8	54.1
8	181.1	143.8	108.1	84.5	68.6	57.2
9	191.5	152.0	114.3	89.4	72.5	60.5
10	202.5	160.7	120.8	94.5	76.6	63.9
15	266.5	211.5	159.0	124.4	100.8	84.1
20	349.1	277.1	208.3	162.9	132.1	110.2
25	455.6	361.7	271.4	212.6	172.4	143.9
30	592.6	470.4	353.6	276.6	224.3	187.1
35	768.6	610.1	458.7	358.7	290.9	242.7
40	994.5	789.4	593.5	464.1	376.4	314.0
45	1284.3	1019.4	766.3	599.3	486.0	405.5
50	1655.6	1314.2	987.9	772.6	626.6	522.8

TABLE 7.22

($v = 2.5, p = 0.02$): Sensitivity of optimal solution to changes in η

	$\eta = 0.00$			$\eta = 0.02$		
t	k	c	s	k	c	s
0	203.0	30.9	0.310	203.0	30.9	0.310
1	213.4	31.0	0.324	209.2	31.7	0.322
2	225.1	33.1	0.339	216.3	32.6	0.334
3	238.2	34.2	0.354	224.5	33.4	0.347
4	253.0	35.3	0.371	233.9	34.3	0.361
5	269.7	36.5	0.388	244.6	35.3	0.375
6	288.5	37.7	0.405	256.7	36.3	0.391
7	309.7	39.0	0.424	270.5	37.3	0.407
8	333.5	40.3	0.443	286.1	38.3	0.423
9	360.4	41.6	0.463	303.8	39.4	0.441
10	390.8	42.9	0.483	323.8	40.5	0.459
15	610.6	49.7	0.586	470.2	46.2	0.556
20	1001.2	56.5	0.683	733.0	51.9	0.654
25	1668.0	62.9	0.764	1185.7	57.1	0.741
30	2756.1	68.7	0.827	1928.4	61.6	0.810
35	4456.4	73.8	0.873	3089.2	65.3	0.862
40	7015.8	78.2	0.907	4828.6	68.2	0.900
45	10751.8	81.9	0.931	7346.3	70.3	0.927
50	16071.3	85.2	0.949	10891.9	71.7	0.947

TABLE 7.23

($v = 2.5, p = 0.02$): Sensitivity of optimal solution to changes in s_3

	$s_3 = 0.60$			$s_3 = 0.65$		
t	k	c	s	k	c	s
0	203.0	30.9	0.313	203.0	30.9	0.316
1	211.5	31.9	0.326	211.7	31.9	0.329
2	221.1	32.8	0.340	221.5	32.8	0.343
3	231.9	33.8	0.354	232.6	33.8	0.357
4	244.2	34.9	0.369	245.1	34.9	0.373
5	258.0	35.9	0.385	259.3	36.0	0.389
6	273.6	37.0	0.402	275.3	37.1	0.405
7	291.2	38.2	0.419	293.3	38.2	0.423
8	311.6	39.3	0.437	313.6	39.4	0.441
9	333.6	40.5	0.456	336.5	40.5	0.460
10	359.0	41.7	0.475	362.4	41.7	0.479
15	542.7	47.9	0.576	549.6	47.9	0.580
20	869.2	54.1	0.673	881.8	54.0	0.676
25	1426.7	59.8	0.756	1447.6	59.7	0.759
30	2335.0	64.8	0.821	2367.3	64.7	0.823
35	3749.6	69.0	0.869	3796.7	68.9	0.871
40	5867.8	72.6	0.904	5933.3	72.3	0.905
45	8939.4	75.4	0.930	9027.1	75.1	0.931
50	13280.4	77.6	0.948	13394.5	77.3	0.949

TABLE 7.24

($\nu = 2.5, p = 0.02$): Sensitivity of optimal solution to changes in m_1

	$m_1 = 0.15$			$m_1 = 0.17$		
t	k	c	s	k	c	s
0	203.0	30.9	0.326	203.0	30.9	0.318
1	212.4	31.9	0.339	211.8	31.9	0.331
2	222.9	32.9	0.353	221.7	32.8	0.345
3	234.8	33.9	0.368	233.0	33.8	0.359
4	248.3	35.0	0.383	245.7	34.9	0.374
5	263.4	36.0	0.400	260.0	36.0	0.390
6	280.4	37.2	0.417	276.1	37.1	0.407
7	299.7	38.3	0.435	294.3	38.2	0.425
8	321.3	39.5	0.453	314.9	39.4	0.443
9	345.7	40.6	0.472	338.0	40.6	0.462
10	373.3	41.9	0.492	364.2	41.8	0.481
15	571.9	48.0	0.592	553.3	47.9	0.582
20	922.0	54.0	0.687	888.5	54.0	0.678
25	1514.2	59.5	0.767	1458.7	59.7	0.760
30	2469.8	64.3	0.829	2384.4	64.6	0.824
35	3945.4	68.3	0.875	3821.5	68.8	0.871
40	6139.4	71.6	0.909	5967.7	72.2	0.906
45	9302.5	74.3	0.933	9073.2	75.0	0.931
50	13752.1	76.4	0.950	13454.4	77.1	0.949

TABLE 7.25

The speed of development and the allocation of labour force

	g = 0.04			g = 0.06		
t	μ	ℓ_1	ℓ_2	μ	ℓ_1	ℓ_2
0	0.300	0.308	0.692	0.300	0.308	0.692
1	0.308	0.302	0.698	0.313	0.300	0.700
2	0.317	0.297	0.703	0.326	0.291	0.709
3	0.326	0.291	0.709	0.339	0.283	0.717
4	0.335	0.286	0.714	0.353	0.275	0.725
5	0.344	0.280	0.720	0.367	0.267	0.733
6	0.353	0.275	0.725	0.381	0.260	0.740
7	0.362	0.270	0.730	0.395	0.253	0.747
8	0.371	0.265	0.735	0.409	0.246	0.754
9	0.381	0.260	0.740	0.424	0.240	0.760
10	0.390	0.255	0.745	0.439	0.234	0.766
15	0.439	0.234	0.766	0.513	0.207	0.793
20	0.488	0.215	0.785	0.587	0.186	0.814
25	0.538	0.199	0.801	0.658	0.169	0.831
30	0.587	0.186	0.814	0.722	0.156	0.844
35	0.635	0.174	0.826	0.778	0.147	0.853
40	0.680	0.164	0.836	0.825	0.139	0.861
45	0.722	0.156	0.844	0.864	0.134	0.866
50	0.760	0.150	0.850	0.896	0.130	0.870

TABLE 7.26

($v = 2.5, p = 0.02$): Sensitivity of the optimal sectoral capital stock per worker to changes in g.

	g = 0.04			g = 0.06		
t	k	k_1	k_2	k	k_1	k_2
0	203.0	92.1	252.4	203.0	92.1	252.4
1	211.3	97.7	260.5	211.3	98.7	259.5
2	220.7	104.1	269.9	220.8	106.2	267.9
3	231.3	111.2	280.6	231.7	114.6	277.9
4	243.3	119.2	292.9	244.1	124.2	289.5
5	256.8	128.3	306.9	258.1	135.1	303.0
6	272.1	138.5	322.9	274.0	147.4	318.5
7	289.4	150.1	341.0	292.1	161.5	336.3
8	309.0	163.2	361.5	312.6	177.5	356.7
9	331.0	178.1	384.7	335.7	195.8	379.9
10	355.8	195.0	411.0	362.0	216.7	406.3
15	536.2	321.0	601.9	553.5	374.5	600.1
20	857.5	557.8	939.6	895.7	675.5	945.8
25	1407.2	988.9	1511.2	1480.6	1225.6	1532.5
30	2304.8	1738.3	2433.9	2431.7	2175.8	2479.1
35	3705.5	2978.7	3858.6	3907.7	3725.8	3939.0
40	5806.4	4940.9	5976.7	6109.2	6128.5	6106.1
45	8857.0	7924.9	9029.8	9288.3	9697.8	9224.9
50	13173.2	12315.1	13324.3	13763.3	14823.1	13605.0

7.5 Conclusions

Before trying to enact any kind of optimal policy we must know the actual parameter values in the functions that explain the behaviour of the economy. The specification of these functions and the estimation of the parameters was the aim of the previous chapters.

In this chapter, where we have investigated the normative properties of the models presented in the earlier chapters, we followed the same two paths, ie. the analysis of the theoretical model and the application of the theory to the economy of Greece.

Unfortunately, the analysis of the general model could not be continued in a general form after the derivation of the differential equations of the variables governing the optimal policies, due to their highly complicated formulation. To continue the analysis we had to make some drastic simplifications, ie. $\mu = 1$ and $s_1 = s_2 = 0$. Even in this simpler case our optimal model differs from the usual two-sector growth models because it takes into account that only part of the output of the second sector is invested and that it includes technological change that is different between sectors and factors of production. The results of this analysis have been presented in the table 7.1, where we see that in the cases where there is technological change (nonautonomous system) the balanced-growth capital/labour and wage/rental ratios are increasing over time (except for the case of 1.4' and 2.1 which is ambiguous for $k^*(t)$) and in the case where there is no technological change both k^* and ω^* are stationary.

In section 7.3 the simplification of the model by the introduction of Cobb-Douglas production functions allows us to keep the variability in μ (t). Having found the differential equations for the control and the state variables we applied optimal control analysis, using the estimated parameters from chapter six, for the Greek economy.

From the analysis of the results of this application, in section 7.4, we see that all the variables are moving very smoothly and this is a consequence of the use of an infinite planning horizon. The difficulty for the planning authority is in fact not so much the derivation of the optimal paths for the different variables but the selection of the appropriate values for the two parameters v and p which cannot be estimated directly. The great sensitivity of the optimal solution to changes in v and p makes things more difficult.

A possible guide to the selection of the "best" values for v and p is undoubtedly the savings ratio s(t). There are two extreme possibilities: either all income is consumed which means that s(t) moves towards zero or it is all invested which means that s(t) moves towards one. Between these two extreme cases there are an infinite number of possibilities and for each of these the speed with which s(t) moves towards zero or one will be different.

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CONCLUDING REMARKS

It is easy to be pessimistic about economic development. This is for two reasons. Firstly, by observing that three quarters of the world's population lives in a degree of poverty which rich countries have not known for generations, and the income gap has widened in spite of $ 80 billion of foreign aid, Maddison (1971). Secondly, because it is true that in spite of the intensive intellectual study of the past thirty years economists did not achieve the "general" model which explains reality in the best way, Sen (1970).

Our own view of the development process and of future prospects is not so pessimistic and we answer the above points as follows:

Firstly, even a brief survey makes it clear that the developing world has moved decisively into a new era; political independence has permitted experiment with a wide variety of growth policies; colonial types of exploitation have virtually disappeared; aid has been outstandingly successful in countries where it was given in large quantities; if post-war performance is measured in the light of economic history, the acceleration of growth has been remarkable almost everywhere; per capita income has risen about three times as fast as in the past, and progress has been significant enough for this generation to have experienced very tangible evidence of change for the better, Maddison (1971).

Secondly, reality is so complex that it would be naive to imagine that it could be completely captured by modern growth theory, Hahn (1971). Instead, a whole series of new models and policy ideas answered many questions about economic development which twenty years ago seemed unanswerable.

In order to justify the optimism of the previous paragraph we constructed a descriptive model, building on the contributions of Fei and Ranis (1964), Jorgenson (1967), Kelley,

Williamson and Cheetham (1972) and Dixit (1973) and applied it to the economy of Greece for the period between the years 1950 and 1972.

One of the major characteristics of our model is that there exists a wage differential between the two sectors of the economy, ie. the agricultural and non-agricultural sector, and that this wage gap is decreasing over time. This characteristic has been established both analytically, by extending the Harris and Todaro (1970) equilibrium probability of finding a job in the non-agricultural sector, the Jorgenson's (1961) constancy of the wage rates ratio of the two sectors, and the Fei and Ranis (1964) assumption of equal wage rates, and empirically, by using the available data for the Greek economy.

Our model falls into the category of growth models which describe development using the concept of production dualism. Production dualism appears into our model in the form of different sectoral parameters, ie. bias in technological change between sectors and factors of production, different elasticities of output and substitution, and in the form of differences in the ratio variables, ie. different capital intensities and different wage/rental ratios between the two sectors. Undoubtedly, by observing the Greek reality we see that both kinds of duality exist for the period under consideration. In fact, from the original data we see that the sectoral capital/labour and wage/rental ratios are different (precisely the capital/labour ratio in the non-agricultural sector is greater than that of the agricultural sector which is consistent with the necessary condition for existence of the analytical solution of the model) and from the estimation of the model we find that the elasticities of output with respect to inputs of production and the rate of change of technological progress is different between the two sectors. Empirical evidence for the duality in the elasticities of substitution cannot be supported because the production functions used for our test were those of the Cobb-Douglas type.

Between the models of economic growth and development there exists a difference in the degree of generalisation adopted concerning the theoretical analysis and the empirical applications of them. In the models of economic growth, mathematical difficulties restrict the theoretical analysis to relatively simple cases, compared to the existing data, whilst in the models of economic development the existing empirical information is limited making applications difficult. In our theoretical model we have introduced capital and labour in the two sectors which has been extended to our empirical test thus, avoiding duality in the factors of production and so extending the models of dual development which use only labour (and land) in the agricultural sector. At the same time we have generalised our theory by introducing three income classes and consequently three (constant over time) propensities to save. This assumption cannot be extended in the application due to the lack of data. However, for the Greek economy we have seen that the propensity to save out of profits is not constant but in fact strictly increasing over time.

Another problem which the models of economic development face is the problem of viability of the economy or in other words whether the variables of the model appear in the analysis with the expected sign. The usual procedure of solving this problem is by investigating some variables as for example the per capita agricultural output, the composition of output among the two sectors and the labour redistribution. Testing analytically and empirically we have seen that the per capita agricultural output is increasing, the composition of output moves in favour of the non-agricultural sector, and the allocation of labour moves also in favour of the non-agricultural sector.

It is true that the specific production functions restrict our application and thus we cannot test some of the findings of the more general model. However, the introduction of

the Cobb-Douglas production functions can be supported by the observation that the capital/output and capital/labour ratios have found to be variable over time and by the relative constancy of the shares of capital and labour out of sectoral output. By using the Cobb-Douglas production functions we have seen that a necessary condition for the terms of trade to be increasing in favour of the agricultural sector and the sectoral output/ capital ratios to be decreasing, is that the ratio of the wages share in the agricultural to the non-agricultural sector must be greater than that of the corresponding ratio of the rates of change of the technological progress of the two sectors. This condition has been empirically supported by the estimates of the parameters of the model. Another aspect of the use of the Cobb-Douglas production functions is the possible switch of capital intensities from the non-agricultural sector to the agricultural one, depending of course on the elasticities of the production functions and on the speed of development.

One must be sceptical of applying the specific models of this thesis to actual planning decisions. This is because we have seen, by applying the theoretical optimal control problem to the Greek economy, that the results obtained about the optimal paths of the variables of the economy are very sensitive in changes of the various parameters. Therefore, misestimation of a parameter due to inadequate data of the developing economy will produce misleading results. However, in our case we tried various optimal control experiments to derive an appropriate answer to the optimal control problem for the Greek economy.

Finally, we believe that our work supports the contention of Higgins (1956) that dualism is not only an Eastern phenomena but also exists in Western countries. In our work the Greek economy is shown to be a dual economy which behaves in a manner similar to the theoretical model presented.

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