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UNIVERSITY OF Southampton

Multi-scale Damage Analysis of Composite Structures

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ABSTRACT

The number of applications of composite materials in different industries including aerospace, marine, automotive and civil are increasing due to the several advantages they provide. However, there are still some applications that composite materials are not used because of the lack of deep knowledge and fast methods in composite modelling. Therefore deeper understanding of damage behaviour and faster methodologies are required in order to increase the usage of composites.

The aim of this research is to take the advantage of the multi scale nature of the material in order to define structural behaviour of composites. The structure can be considered in three scales: macro-level, meso-level and micro-level. Composite materials gain most of the advantages becoming heterogeneous material, however this feature have significant effects on the behaviour of the macroscopic level. An accurate and fast approach at macro level is required to estimate the structural response and effect of micro level mechanisms should be investigated to define this macro level response.

In this thesis, the studies are conducted in two level: Macro and Micro. A composite stiffened plate is modelled with finite element approach at macro level. The model is generated with geometrically non-linear and materially linear approximations. The results show good agreement with experiments in literature. But, the accuracy of the modelling is improved by including material non-linearity. Therefore, a progressive damage model is developed and applied to a composite plate at meso-level. The results presented the requirement of damage modelling. In order to have an accurate way of predicting the damage behaviour of structures, micro-level mechanisms are modelled. An RVE model is generated in order to model micro level responses of composite materials. The model provided good approximations for material properties compared to experimental estimations with numerical modelling approach and seem promising to model damage behaviour of composites. Despite the increased usage composites, manufacturing variability is still not well understood. To increase structural safety it will be important to understand the effect of inherent variations on the failure, how do variations from different manufacturing processes effect the structural reliability? Increasingly material variations are being investigated, with an assumption that these variations are more important than topological defects. In this study, the impact of material and topological variations on structural integrity are compared via a reliability assessment. By using direct monte-carlo simulations, the reliability of top-hat stiffened plates is explored. Halpin-Tsai and Representative Volume Elements are utilised to characterise the material and an empirically adapted Navier grillage method is employed for structural analysis. The reliability of structures with only material variations (up to 20%) exhibited significant differences when equivalent variations are applied for both levels. Material variations have an order of magnitude higher impact on structural reliability of stiffened plates than topological variations (up to 20%), meaning that the focus on materials is justified but that topological defects should be further investigated.

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Acknowledgments

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Nomenclature

E_c: Composite Young's modulus

E_f: Fibre modulus of elasticity

E_m: Matrix modulus of elasticity

WWFE: World Wide Failure Exercise

v_c: Volume of a composite specimen

V_f: Fibre volume fraction

V_m: Matrix volume fraction

1. Introduction

1.1. Background

The number of applications of composite materials in engineering structures are increasing. These materials are being used in different industries including aerospace, marine, energy automotive, civil, sport and leisure. The reason for this trend is the advantages composite materials provide. Initially, composite materials have high specific strength and stiffness which enables excellent weight reduction at the structures. They have good corrosion resistance which is an important advantage for application in the marine environment. Moreover, different composite materials have the advantages of acoustic transparency, thermal insulation and the lack of magnetic signatures. Beside these advantages, the most important property of composite materials is the flexibility in tailoring the structure to its application by selection of materials, lay-up or curing. The different properties: stiffness, strength, toughness, impact resistance and damping can be customised within the structure by changing the architecture or by using different type of materials such as woven, bi-axial or unidirectional. This gives the designers a large design space to optimize their product. Finally, composite materials with different manufacturing techniques enables one to optimize the design and produce complex components without losing any benefit of the material.

In order to increase the usage of composites, two main areas require improvement: firstly, a deeper understanding of the composite behaviour is needed and secondly, faster methodologies are required during the design process.

The main goal of all design approaches is to ensure the safety. There are three basic design principles that are applied in structural systems according to Lloyd Register [1]: safe life design, damage tolerance design and fail safety design. Safe life design assures the life of a component in one of two ways: either the structural component will never fail during the operational conditions while the design limits are not exceeded or the component will have a certain replacement date before ultimate failure occurs. An example to this is polymer timing belt used in many cars. This component will eventually fail and should be replaced before damage occurs. Damage-tolerant design of a structure is intended to ensure that should some form of degradation occur during the operational life, the structure should be able to withstand the design load, at least until the next inspection. This damage may occur during the operation time or during manufacture. The initiation and growth of damage in engineering structures are inevitable during their service life. The important thing is to be aware of the probability of damage and be able to detect damage during the inspection schedule. In order to be able detect the damage during the inspections one needs to have prior information about where the damage will occur. An example of this approach is aircraft designs as metal fatigue will initiate some cracks in components. The crucial part is to make sure growing crack is detected and repaired if necessary. The final design principle, fail-safe design, prescribes that a structural system contains some redundancy; therefore failure of a component will not cause a catastrophic failure. Nuclear power designs or space craft designs are based on this design approach.

The performance of composite structures depends on their mechanical and geometrical properties. Each of the design principles has one main concern: damage. During the design process damage properties of the structure should be investigated to have a more reliable, safe and economic product. In order to design “reliable” structures, damage characteristics should be implemented to design process. The structures can be “safe” and able to operate during their service life without failure by investigating the damage tolerance of the structure. Without any knowledge about damage behaviour of the structures, designs become more conservative which eliminates one of the most important advantages of composite materials. Having knowledge about damage performances of the composite materials will enable us to design more “economic” structures. Therefore implementing damage investigation into the design process is crucial for composite materials.

There has been a lot of research relating to composite materials including damage. However, there is not a whole, trustable and fast way for investigating the damage behaviour of composite structures. It is mostly because of the complex nature of composites. The general and damage behaviour of composites depend on different parameters such as geometry, material type, lay-up, loading conditions, loading history and failure modes [2]. Besides the complex nature of composite materials, the structures designed and the marine environment have complexities and variations. Due to the all these complexities, they includes some variations related to manufacturing, material properties and life time effects. Therefore, structural behaviour of composites needs to be studied in more detail by including these variabilities [3]. Besides these variations, the anisotropy of the material, the non-linear response, and the difficulty of failure mechanisms increase the complexity of the damage modelling of composites, including the initiation and the evaluation of the damage.

Composite materials gain most advantages by their heterogeneity. However this feature has significant effects on the behaviour of the structure which makes the design and production of these structures very challenging tasks. Because of the heterogeneities, composite materials have random characteristics at micro scale which affects the overall behaviour of the structure [4]. These probabilistic behaviours of the composite structure cause complex responses especially in terms of failure and damage. The heterogeneity at the micro level of composites leads to several damage mechanisms such as matrix crack, fibre breakage, fibre-matrix debonding and delamination [5]. Any of these failure types causes a reduction in the performance of the structure. Additionally, different types of damage mechanisms may interact with each other, which leads to a more complex response of the structure. Therefore, there is a need to investigate the micro mechanisms of composite materials to have a better prediction of the responses.

Like many other materials, composites have a structural hierarchy including different scales starting from the micro-level by considering the continuum level which assumes no gaps between lower scale elements [6] . At the microscale, composites composed of three different “environments”: fibre, matrix and the interface between those two. Failure initiation and progression may have different routes between those three environments. Usually it depends on the loading, fibre orientation and stacking sequence as well as the micro-mechanisms. Initially, the cracks/damage starts as a simple mode, usually in the matrix or at the interface [7]. Then the crack propagates to another environment by combining these two failures. Afterwards, the fibre failure occurs when the matrix is not able to carry the loads between fibres any longer. This scenario shows us there are three basic steps in the development of damages in composite materials. Crack initiation, crack growth and localization of cracks leading to ultimate failure. It can be seen that the first failure doesn't always mean ultimate failure. From first failure to the ultimate failure the structure can continue to carry a load and operate even if it loses performance. The time and behaviour between first failure and final failure represents the damage tolerance of the structure. Talreja [8] defines a “criticality” term where each composite part has its own “criticality” which doesn't need to be a failure stage as conventionally described. The existing cracks/failures cause material property degradation which result in stiffness degradation at the macro scale of the structure. When the stiffness decreases to a certain level, the structure is considered to have failed. On the other hand, the degradation of load carrying capability may be considered as failure. In any case, the initiation of failure at micro-level and its consequences at the macro level should be analysed in order to have comprehensive information about damage tolerance of composites.

Cox and Yang [4] divide failure prediction methods into two approaches: Bottom-Up and Top-down. Bottom-up models predict the failure by modelling the structure at molecular or micro scale while the top-down models focuses on macro scale, structural needs. During the structural analysis at the macro level the failure models identify the ply failures not the damage at the fibre matrix level. In order to be able to define the damage mechanisms at the fibre/matrix level, the micro-level should be included in the modelling. Ideally, the whole microstructure will be modelled as including the heterogeneities in the material but it is an impossible task when considering the current computational resources [9].

Another way to determine the mechanical and damage behaviour of the composites is by experiment. The experiments can be applicable for the determination of the material properties of composites and damage behaviour. However, performing parametric studies of composites structures experimentally is time consuming and expensive. Experiments on a number of different material samples for orientation angles, volume fractions, and stacking sequences will induce a huge amount of time and cost [9]. In terms of safety requirements in airplane certifications, a typical airframe requires around ~10,000 tests of material specimens, along with tests of material components and structures up to entire tails, wing boxes, and fuselages [4]. Many testing needs to be carried out either on full scale prototypes or small scale samples and this also generates size and scale effects which are reviewed in [10]. The number of experiments required should be reduced by the help of reliable numerical methods across various industries to have more efficient design and production processes.

Another parameter which affects the damage behaviour and tolerance of composite structures is the geometrical design [2]. In order to carry in-plane loadings, thin composite plates suffice. However, if the loading is compressive or out-of-plane, composite plates will not be sufficient. In order to improve the buckling and other characteristics of the thin plate structure stiffeners are used which provides more effective results rather than increasing the plate thickness [11]. Stiffeners provide stiffer structures and lower impact on the weight. Mainly top-hat stiffeners are used in aerospace, marine and civil industries as they are able to provide extra resistance to buckling and increased torsional rigidity [12]. There are several studies [12-15] which focus on specific stiffened composite structures, but there is still lack of knowledge on the effect of micro mechanisms on the stiffened composite structure to define the overall damage tolerance.

1.2. Problem statement

Based on this background, the problems requiring further study are summarized below:

Although the usage of composites have increased excessively there are still a lack of deep knowledge about damage behaviour. Despite the quantity of research looking at damage behaviour, it is not successfully understood [8].

In composite materials, the onset and growth mechanisms of damage are different to isotropic materials. Due to the complex nature of composites, the response varies for different micro structures. Therefore, the micro mechanisms have to be considered in structural analysis in order to increase the reliability of the structure.

Most studies focusing on damage behaviour of structures are either computationally expensive [13] or don't provide enough accuracy [14] because of not enabling multi scale effects. There is a need for solution methods which are computationally efficient without sacrificing the necessary accuracy. Therefore, multiscale methods, which provide high fidelity results at a reduced computational cost should be investigated.

Top-hat stiffened composite plates are one of the main structures used in engineering applications. However, there is a lack of knowledge about the damage behaviour of them which leads conservative designs of components. Implementation of micro level mechanisms into analysis of top-hat stiffened plate is essential to improve the accuracy on failure prediction responses.

There is a lack of understanding about the sensitivity of different variations on response of composite structures. In the available literature the variation of the material properties is investigated by itself or the impact of topological variations are only considered for simplified geometries. This means that the material and topological variations are not explored together or compared. Therefore the impact of material characterisation and topological design variations on composite structural components should be investigated. Reliability assessment methods are used to incorporate stochastic variations from geometric and material variations. A comparison is made between different methods for predicting the properties at two scales: fibre and ply.

1.3. Research aim and objectives

The aim of this study is to investigate the structural response of composite materials including the effect of micro-level mechanisms to the macro-level structure in order to increase the reliability of the structure in a computationally efficient way.

In order to achieve this main aim, the study will progress through a number of objectives and sub-objectives:

- A literature review including modelling types, damage mechanisms, and the methods available to model damage behaviour of composites will be conducted.
- Application of a macro-scale analysis to composite marine grillages (stiffened plates):
  - A finite element model of a typical composite grillage will be constructed and verified against experiments available in the literature.
  - A surrogate model will be constructed of the grillage to rapidly investigate the macroscopic behaviour.
- Perform micro-scale modelling in order to obtain accurate material properties and microscopic mechanisms of the structure.
- Multiscale reliability analysis of composite stiffened plates.

Having better knowledge about the damage behaviour of a composite structure will enable:

- optimized design process by decreasing the time and sources spent for composite vessels and structures design.
- increase performance of the structures by implementing the damage tolerance phenomena
- decreased costs of inspections of composite vessels with prior information about damage times and locations.

1.4. Scope of work

This project will primarily focus on investigating damage behaviour of composite structures. Investigation of failure and the effect to the structure is crucial for the performance of the composites [15]. In order to have better understanding of damage behaviour of composite structures, lower scales will be considered. Micro-level damage mechanisms will be implemented to define the failure response of the macroscopic structure. Another important requirement of design process of composite structures is faster methodologies. This study will be searching for faster methodologies with required fidelity by taking advantage of the multi-scale nature of the composites.

1.5. Research Novelty

The novelty of this study comes from the consideration of micro-level effects on the macro-level structures in terms of failure response in a computationally efficient way. Despite the many studies, the effect of micro-mechanisms couldn't be understood on damage behaviour of macro structures. Especially, the studies about the complex composite structures such as top-hat stiffened structures are not sufficient. The damage tolerance of top-hat stiffened structures will be investigated in this study. To achieve this, multiscale methods will be used. The study will focus on investigating methods which provide high fidelity whilst require low computational effort.

1.6. Outline of the Study

After the introduction in this chapter, a literature review will be given in Chapter-2 about damage modelling of composites. Chapter-3 will represent the methodology of the study. Chapter-4 includes the macro level analysis of composite stiffened panel using finite element method and implementation of damage modelling. Chapter-5 investigates the micro level modelling of unidirectional composite material. The last section, Chapter-6, will provide an application of micro and macro level modelling on reliability assessment of composite structures.

2. Literature Review

A review of literature is conducted under this section to understand the damage phenomena and modelling techniques. The failure types and criteria are summarised in the first part, followed by the review of strength based damage modelling approaches: continuum damage and progressive damage modelling techniques. Afterwards, the multiscale methods are reviewed and finally a general summary of the review is given.

2.1. Damage Mechanisms:

Failure, by definition is the fact that the structure is not working or stopped working as well as it should be. Composite materials have a variety of failure mechanisms due to their complex nature. These failures may occur during the manufacturing process or during service life. The failure occurrence doesn't mean final failure of the structure as the structure continues to work even if there occurs different type of failures. Although computational power is an important shortcoming while dealing with composite damage problems, it is not the only one. The more challenging area is to define damage mechanisms and characterize them into a realistic model [16]. The main failure mechanisms for long continuous fibre composites are: fibre failure, matrix cracking, interfacial debonding and delamination. These failure modes can be varied from simple loss of structural rigidity due to ply-level failure through the loss of load carrying capacity because of damage growth, or complete failure because of the interactions of several damage types. Having knowledge of these failure mechanisms will enable us to choose suitable approach to determine the damage behaviour of the structure. Therefore investigation of the damage mechanisms is essential for the rest of the study.

2.1.1. Matrix Crack

Matrix cracks are the damages occurs inside the layer and describes the failures between fibres. They are crucial because of being initial damage form in composite laminates [17] which usually develops along fibres in that ply. Different fibre orientations may cause different form of matrix cracking but the most common one is in the case of 0 degree loading to the 90 plies. They usually form initially in the plies which are oriented off-axis to the loading. Different terms such as matrix micro cracks, matrix cracks, transverse cracks, intra-laminar cracks, and ply cracks may be used to define this phenomena. The initiation and growth of the matrix crack accumulate at the micro-level of the structure. However, matrix crack doesn't cause a macro-level failure immediately. The formation of micro-cracks result material property degradation. If the design of the structure can't tolerate the degradation, the failure may occur at the macro-level. The another important effect of matrix micro cracks is that they lead different types of damages like delamination, fibre breaks, causing pathways for the entry of the fluids which is critical for marine structures [18]. The World Wide Failure Exercises studies emphasizes that the matrix micro cracks should be taken into account for the final failure of macro-structure [19].

2.1.2. Fibre failure

The main load carrying constituent of the composite material is fibres which has crucial impact on the response of the composite structures. Fibre breakage is the main failure which is very critical when there is not enough fibre to carry design loads which usually occurs at the tension loads. The breaking fibre results a stress concentration around vicinity of the failure. The stress is transferred by the matrix material to other fibres but stress may be high to break more fibres which lead damage localization and finally catastrophic failure. Because of the consequences of fibre breakage, it is essential to define the first fibre failure at the micro-level. Another fibre failure types are micro buckling and buckling of the fibres under compression. The compressive strength of the fibres are much less than tensile strengths. For example, the compressive strengths of unidirectional carbon fibre laminates less than 60% of the tensile strength [20]. Evaluation of micro buckling, kink band, micro crushing cause the structure not to perform as its design requirements [21].

2.1.3. Interfacial Debonding

Failure between matrix and fibre interface is another failure mechanism. This kind of failure initially attributed by the inappropriate choose of matrix/fibre combinations. Environmental effects may also cause fibre matrix interface failure. An individual matrix crack may result in matrix/fibre interface debonding or itself may cause matrix crack. Therefore, interfacial debonding can be considered as a separate failure mechanism or maybe considered with matrix crack [22].

2.1.4. Delamination

Failure of the interface between two layers in a laminate called as delamination/inter-laminar cracking. Inter-laminar tension and shear forces are the main reasons of the delamination [23]. Free edges, the discontinuities within the structure, local impacts during manufacture or service, the change at the moisture and temperature, the other failure mechanisms individually or combined may cause delamination [Fig-1].

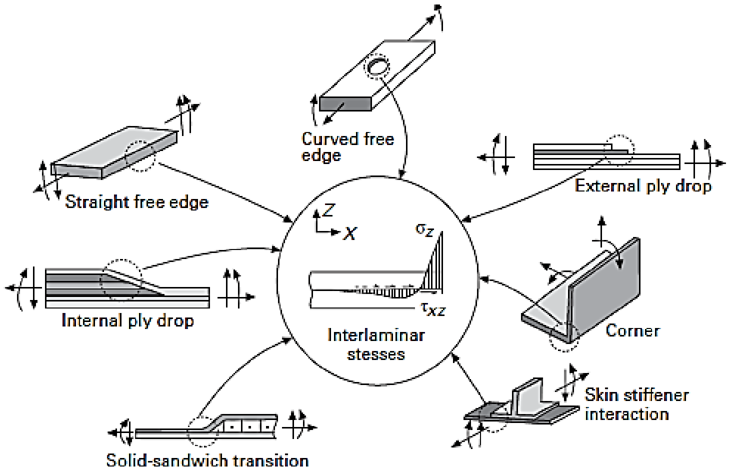

Figure 1 Delamination sources [23]

There are many inspection methods such as ultrasonic scans to detect delamination however it cannot always be inspected easily during manufacturing or in service as they occur between layers and some core material prevents easy detection. Once the delamination occurs it can grow very fast with the effect of loading and reduce the stiffness properties. During the design process of composite structures, the causes of the delamination must be considered to provide real performance of the composite by controlling the initiation and growth of delamination.

2.1.5. Failure Criteria and Damage Modelling

Matrix crack, fibre failure, interfacial debonding and delamination are the main failure types as reviewed in the previous sections and needs to be studied in order to determine the damage initiation and growth in the composite materials. Currently several studies have been done to investigate the failure theories and damage characterisation. There is also continuous effort keep improving the failure theories with World Wide Failure Exercises which has started as an expert meeting to investigate the available failure theories [24]. The main aims of the study can be summarized as establishing the current status of failure criteria, make a connection between academia and industry by providing design engineers more robust and accurate failure prediction methods. The organizers graded the each failure criteria according to their accuracy compared to experimental results. The overall exercise showed that there is no single failure criterion able to predict for different failure modes under various loading conditions. The organizers concluded that the composite structural design requires more understanding of damage onset, propagation, ultimate failure and the interaction of stress-strain behaviour and failure modes. The first world wide failure exercise investigated the 2-D failure criteria. The second exercise extended the assessment of failure criterion to 3-D with the same methodology. The results of second exercise were similar to the first one. The third

WWFE is underway in order to investigate the current status of cracking and damage models which might be able to model the progressive failure of structure.

In most studies maximum stress or maximum strain criterion are applied using limit values extracted from experiments. Hashin [25], Hoffman [26], Tsai-Hill [27] , Tsai Wu [28], Puck [29] have developed new formulations for different failure mechanism under different loading conditions. A survey conducted by The American Institute of Aeronautics and Astronautics (AIAA) has shown that [Fig-2] 80% of the participants had been using one of Maximum Stress, Maximum Strain, Tsai-Hill and Tsai-Wu. More importantly, around 70% of the composite designers are not applying Tsai-derived criteria which can identify failure types [30]. The continuing studies and the big number of the approaches to define the failure prove that it is still an important research area.

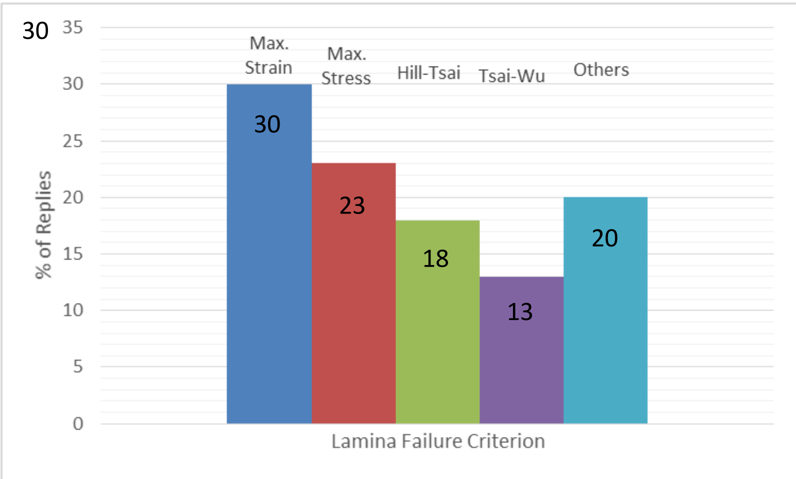

Figure 2 The Usage of Failure Criteria [30]

In addition to the failure criteria, the approaches to damage characterisation varies. It is possible to characterise all the different methodologies in two general approaches: 1) the methodologies based on strength theories and 2) fracture mechanics based methodologies [31]. The material strength is one of the basic approach to determine the damage initiation and progress. With application of load, one or more failure criteria are satisfied which means the material is exposed to an irreversible failure. Strength based characterisation is mostly suitable for to define the damage initiation. The damage initiation and ultimate failure of structure can be predicted related to material, geometry, loading conditions with the help of a specified failure criterion [32]. While strength based approaches are usually used for damage onset and growth, using fracture mechanics approach allows researchers to study on the existing damage. Although this approach is not applicable for the most of damage types [2], it can be applied for debonding and delamination problems successfully [12]. The progression of damage depends on the rate of strain energy released. Fracture mechanics has three mechanisms for damage progression: Mode-I opening, Mode-II Shearing, Mode III Tearing as seen in [Fig-3].

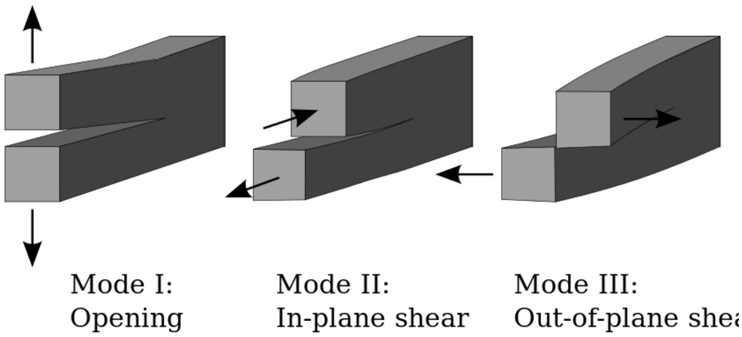

Figure 3 Damage mechanisms in fracture mechanics

These mechanisms have developed for application to isotropic materials but it is also very suitable to apply composite structure delamination without need of any exception or addition. The important point is the definition of the strain energy rate which exhibited by mode. There are different approaches to define strain energy release rates such as J-integral offered by Rice [33], Virtual Crack Closure Technique initially proposed by Rybicki and Kanninen [34], [35] and using interface elements like Cohesive Element proposed by Dugdale [36] and Barenblatt [37]. It is important to emphasize that generally the fracture mechanics approaches don't identify damage initiation and progress. They assume an existing damage within the structure and analyse the behaviour of this damage.

In summary, in this section different damage modes and the failure criteria to define the damage occurrence is reviewed. The different failure types may result under different loading conditions and it is essential to consider the various damage types and failure criteria in order to have better predictions of damage behaviour.

2.2. Strength Based Damage Modelling

The estimation of residual strength of composite materials depends on progression of damage within the structure. Therefore, damage initiation and growth should be investigated with appropriate damage evaluation and stiffness degradation laws. Strength based damage modelling is a practical way of predicting the load bearing capability. This modelling technique can generally classified into two approaches depends on the selection of damage evaluation and material degradation methods. These are continuum damage mechanics based approach and progressive damage modelling approach.

2.2.1. Continuum Damage Mechanics Approach

Kachanov [38] first applied Continuum Damage Mechanics (CDM) on the investigation of creep rupture of metals. CDM applies different mechanical properties to the structures when a damage occurs. The change of the mechanical properties is defined as a function of the damage type. Stiffness degradation and damage evaluation by introducing a damage tensor depends on the damage type. Different damage mechanisms have different effects on mechanical properties. This approach has been applied to isotropic materials. Chaboche [39], Ladeveze [40] have been extensively used for creep and fatigue damage modelling after the introduction by Kachanov. On the other hand, the use of continuum damage modelling in the analysis of orthotropic or transversely isotropic composite materials includes further complications. The direction of the cracks are not only depend on the loading, geometry or boundary condition at composite materials. The nature of the composites causes different preferred crack directions [41].

Damage modelling in composite materials includes difficulties because of the number of parameters to identify the anisotropic behaviour. Before the initiation of the damage the homogenized ply can be simplified to transversally isotropic which reduces the number of independent engineering constants to five [42]. However when the damage starts material will not behave as having one plane of symmetry anymore which increase the number of constants to at least 9 (representing an orthotropic material). Therefore at least nine independent constants should be investigated depending on the amount of the damage. Characterization of the damage degradation with experiments is not practical because of the limitations on the loading and material scenarios.

The usage of CDM basically depends on the determination of the damage variable D related to failure mechanisms. This damage tensor is used to define the stiffness degradation from the intact state. Cauchy stress tensor σ is replaced by an effective stress σ^{eff}.

$$\sigma^{eff} = \frac{\sigma}{1-D} \qquad (1)$$

Damage variable D varies between 0 and 1. D=0 represents the undamaged material and D=1 means completely damaged material. According to Lemaitre and Chaboche [39] the elastic strain will be same for the stress acting on damaged material and effective stress on the undamaged material.

$$\varepsilon = \frac{\sigma}{E^d} = \frac{\sigma^{eff}}{E^{init}} \qquad (2)$$

E^d and E^{init} are the Young's modulus of the damaged and undamaged material, respectively. Therefore, the relationship between undamaged and damaged material`s Young's modulus is as follows:

$$E^d = (1-D)E^{init} \qquad (3)$$

Generalizing the Continuum Damage Mechanics approach to the tri-axial stress states requires to use tensor representations [43]:

$$\boldsymbol{\sigma}^{eff} = \boldsymbol{M}(\boldsymbol{D})\boldsymbol{\sigma} \qquad (4)$$

The constitutive law for damaged material as defined as:

$$\boldsymbol{\sigma} = \boldsymbol{E}^d \varepsilon \qquad (5)$$

Undamaged and damaged material elasticity tensors are related with $\boldsymbol{M}(\boldsymbol{D})$ which is a tensorial function of damage tensor $\boldsymbol{D}$.

$$\boldsymbol{E}^d = \frac{\boldsymbol{E}^{init}}{\boldsymbol{M}(\boldsymbol{D})} \qquad (6)$$

The damage variables are generally represented by tensors. If these tensors are zero-order, they are scalar values or the tensors might be second-order such as stress/strain tensors or fourth-order like elasticity (compliance and stiffness) tensors. Generally, damage tensor $\boldsymbol{D}$ is a fourth-order tensor which contains crack direction [44]. The damage tensor consists of scalar damage variables where the number of variables depends upon the damage model and full form of damage tensor can be found in [43]. In continuum damage mechanics, the evaluation of damage variables and $\boldsymbol{M}(\boldsymbol{D})$ function should be determined to define the material law. There are two ways to define the influence of damage on material behaviour: phenomenological methods which are reviewed under progressive damage modelling section and thermo-dynamical methods.

Thermodynamically consistent models is the approach in order to determine the evaluation of the damage variables based on energy considerations. Schapery [45], Murakami and Kamiya [46] have introduced damage evaluation laws by introducing internal state variables. These variables are formulated in the framework of the thermodynamics of irreversible processes. Each damage mode (e.g. fibre breakage, matrix cracking, interfacial debonding) includes specified internal variables and evaluation of damage growth requires dissipation energy on the basis of thermodynamical analysis of the system. Therefore different damage modes for different materials have different internal state variables to track the damage. As an example, Hild et al. [47] demonstrated internal state variables for matrix cracking, fibre breakage and matrix/fibre interface debonding to investigate the fibre and matrix breakage in ceramic matrix fibre reinforced composites. Matzenmiller et al [48] introduced a damage model in order to relate the effective elastic properties with damage. They define two damage terms: active damage ("compatible with the sign of the corresponding response") and passive damage ("related to the opposite sign of the current stresses") to define the internal state variables for tension and compression.

Maimi et al [32] introduced a thermodynamic model by CDM approach in order to predict the initiation and growth of intra-laminar damages and final collapse of structures. Their damage model has its foundation in irreversible thermodynamics and applies LaRC04 failure criterion as damage determination. The model also accounts for the effects of the ply thickness and configuration on the matrix crack initiation. In their second part of the work [41] they developed a computational model for their constitutive model.

Another example to the thermodynamically consistent approach is the studies of Ladeveze and co-workers [49, 50]. They modelled a single layer to investigate the intralaminar damage mechanisms and an interface which provides stress and displacement transfers between layers. Damage energy release rates are defined with the help of experiments to determine the damage evaluation. Similar approach is applied in order to model the delamination. The main disadvantage of the model is the high number of additional tests required to define the model parameter and determination of interface stiffness properties [43].

Talreja [51], has also investigated anisotropic composite materials with several cracks by classical framework of thermodynamics with internal variables. Damage mode tensors are used as internal variables. Damage is derived from two constituents: damage entity vector ***n*** and damage influence vector ***a*** [Fig-4].

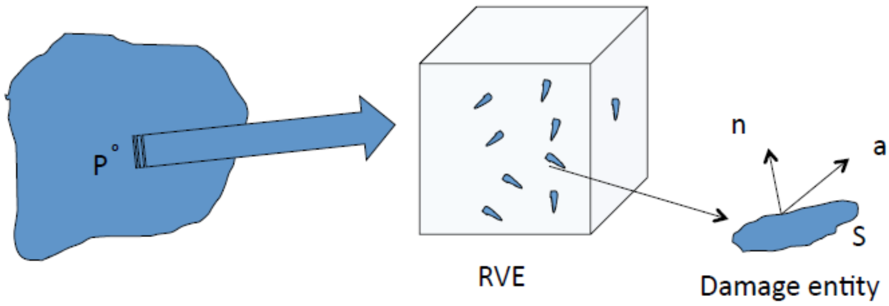

Figure 4 Damage entitiy vector [8]

In this modelling, ***n*** represents the surface normal of a flaw and ***a*** denotes the effect of this flaw on material response which is a crack opening displacement. Second-order damage tensor is calculated by using ***a*** and ***n*** on a crack surface. The number of the cracks inside the volume element is used to evaluate the damage laws. But, the evaluation of the damage laws is very laborious because of the determination of the actual crack opening displacements and experimental evaluation of the constraint parameter for each mode is not practical.

This section has attempted to provide a brief summary of the literature relating to continuum damage modelling. Although the studies provide good predictions for the damage behaviour of the composites, application of thermodynamically consistent methods is tedious and mostly derivation of the damage evaluation rules requires experiments. However, the main reason of the material property degradation at the macro scale is the initiation and growth of the crack at the micro-level which shows that applying only CDM is not satisfactory to model the damage performance in an accurate way [52].

2.2.2. Progressive Damage Modelling

The damage behaviour of composite structures depends on different circumstances such as loading configuration, material orientation, stacking sequence, volume fractions of each layer. These properties of composite materials results different in failure modes which is not the case for metallic structures. The damage initiation occurs within a ply and propagate to the rest of the lamina and laminate under different failure modes. In order to fully exploit the capabilities of composite materials, many efforts have been devoted to understand failure progression. Especially in the aerospace industry the term "black aluminium" is used for the composite materials which are used to replace aluminium by simplifying the design rules and not considering all the failure modes. Progressive damage modelling (PDM) is an efficient way to investigate the residual load bearing capacity of composite structure from initiation to the ultimate failure. Phenomenological PDM models which apply direct degradation of material constants generally depends on the first ply failure theories and experiments in composite materials, working on only meso-level ply scale. These approaches usually use first ply failure criteria like Tsai-Wu, Tsai-Hill, Maximum Stress/ Strain or Hashin criterion [53]. After a failure criterion has been satisfied, material constants reduced to a certain value. The determination of degraded properties of damaged material may be achieved in three ways. First is the total discount approach [54] which sets the stiffness and strength of the damaged ply directly to zero. This method underestimates the strength of the ply. Second approach is the limited discount method which the failure mode specifies the reduction of the stiffness. Third way is the residual property method which selected stiffnesses are reduced to non-zero values [55].

Chang and Chang [56] is one of the early studies of progressive damage modelling by using a finite element approach to deal with progressive damage of a plate with a cut-out. They presented a failure criterion and property degradation models that depend on failure mode. The proposed method reduced the stiffness with the failure criteria which considers the possible matrix crack, fibre-matrix shearing and fibre failure. The material nonlinearity is included with a parameter that was derived from experiments whereas geometric nonlinearity is ignored. The all material properties are reduced to zero except longitudinal Young's modulus for matrix cracking. The same degradation method is applied for the fibre failure and fibre-matrix shear failure. Longitudinal Young's modulus and longitudinal shear modulus are reduced to a value generated from a Weibull distribution and the other two moduli are reduced to zero. The final strength of plates with a cut-out are compared with experiments under tensile loading. Chang and Lessard [57] applied a similar approach for plates with holes under compressive loading. A progressive damage modelling is used which employs stress analyses and failure analyses. Shahid and Chang [58] developed a progressive damage model with a constitutive approach and damage accumulation procedure for plates under tensile and shear loadings. Although, they proposed a complete approach for the progressive damage, the model has several material properties such as crack density function, crack density saturation, material degradation parameter which make the modelling complicated. Lee and Chen [59] is one of the early studies investigating the progressive damage behaviour of composite plates under in plane loading. They used the maximum stress and maximum strain criterion with sudden degradation of material properties upon failure.

Padhi et al [53] investigated the first ply failure load and damage growth of laminated composite plates under non-linear deformation. Limited stiffness reduction method has been applied depending upon the failure mode by using different failure criteria such as, Hashin, Tsai Wu and Tsai Hill implemented in different analyses. The material degradation model was based on Chang and Lessard's approach and experimental observations. Transverse modulus E_y and Poisson ratio v_{yx} are reduce to zero for matrix cracking while longitudinal modulus and shear modulus remain unchanged. When fibre/matrix shearing occurs transverse modulus and Poisson ratio reduced to zero while longitudinal modulus and transverse modulus remain unchanged. If fibre failure occurs the stiffness is reduced to zero at that integration point. The analysis results and experiments are in good agreement for the application of laminated composite plates with all edges fixed under transverse pressure loading.

Kam et al. [60] studied the deformation and first ply failure of graphite/epoxy plates under centrally applied load. They used maximum stress criterion to apply their unique material degradation model. They assumed that, once a matrix crack occurs within the ply, a crack line is created along the plate. Hence, all the relevant material properties of integration points along this line is reduced. It is reported that the actual failure process of the damaged plates could not be simulated by this approach. Liu and Mahadevan [61] applied a probabilistic progressive failure analysis to predict the ultimate strength failure probability of composite structures. First order shear deformation theory is used for the structural analysis with the implementation of Lee`s simple failure criteria [62]. A simply supported laminated plate under uniform pressure is analysed. It is concluded that the method is capable of modelling realistic structural behaviour and assessing the ultimate strength failure of composite structures.

Davila et al. [63] investigated the damage initiation and progress in stiffened graphite/epoxy composite panels under axial compression. The proposed method is validated with experiments. Hashin criterion is used to define the failure modes separately and elastic stiffness degradation model developed by Chang and Lessard is employed to model the damage growth. In order to increase the computational efficiency superposed shell elements are used. All of the plies with same orientation are added together and combined into one shell element. The results are found to be good correlation with experimental results. However, it is observed that once the damage reaches to the stiffener, the model becomes less accurate due to damage progression differs when it reaches a damage stopper like a stiffener. Ambur et al. [64] also used the same methodology with same failure criterion and degradation model to study progressive failure of stiffened composite panels with and without a notch under in-plane shear loading. The progressive failure results are in good agreement with the experimental results in most part of the loading. However, the panels' final failure occurs suddenly while the numerical results present a gradual decrease.

Wang et al. [65] applied progressive damage analysis to the un-notched and notched carbon/epoxy laminate coupons under in plane tension and compression loading. The degradation model they employed is using the 1% of the original value of material constants.

In their study Kim et al. proposed a progressive failure method for composite beams by using Beam finite element with layer-wise constant shear in bending. Maximum stress and Tsai-Wu failure criteria are used to predict the failure. Once the failure occurs within the layer, the stiffeness of the damaged layer over a discrete length is reduced by a homogeneous degraded layer.

Wolford and Hyer [66] investigated the influences of the noncircular geometry and orthotropy on the damage onset and distribution at the internally-pressurized laminated composite cylinders. Three different failure criteria is used namely Maximum Stress, Tsai-Wu and Hashin to identify the damage initiation. However they reduced the material properties by applying a factor, β, with different degradation schemes. Kweon et al. [67] also studied the composite cylinders behaviour to define the initial buckling load and ultimate collapse of the composite cylinders under compression. Hinton et al. and Kaddour et al. [68] used five different failure criteria in order to define the initial failure but only maximum stress criterion is used in progressive failure analysis of filament wound composite tubes in conjunction with laminate theory to estimate the damage onset and final failure under biaxial loading. They addressed the significant advance of progressive damage analysis on initial failure theories.

Another important part of composite structures are the joints such as T-joint, Π-joints, L-joint. A number of researchers have investigated the progressive failure of composite joints. Blake et al. [69] investigated the structural behaviour of composite tee joints by conducting three point bending tests. A progressive damage methodology is presented by using Tsai-Hill failure criterion. The study showed the different failure responses of joints when various type of inserts used. Bai et al. [70] investigated the structural behaviour and failure mechanism of hybrid RTM-made Π-joints through four point flexure tests. They proposed a progressive damage model based on mixed failure criteria to model the onset and progress of failure. Experimental results correlated well with numerical results by using different failure modes.

This review showed that the investigation of damage progression of composite materials stated with geometrically linear approximations. With the development of computational power and implementation of finite element, recent applications include geometric nonlinearities. Finite element approach provides good approximation for the damage governors such as geometry, material properties, boundary conditions. The current capabilities of available finite element codes provide efficient and accurate way of modelling damage behaviour of composites considering the nonlinearities and contacts if needed. Two dimensional and three dimensional finite element approaches are available for structural models of composite materials. Although, 3-D approaches provide out-of-plane stress components, it is computationally expensive. However, 2-D models provide efficient solutions for the composite plates and shells [47, 62]. The structure type and geometry with the loading type has a huge effect on damage evaluation. Different studies have investigated the progressive damage behaviour with phenemological methods under various loading conditions. Table-1 exhibits some of the studies. To the author's knowledge, the studies investigating the progressive damage behaviour of composite stiffened plates, which are commonly used in different engineering structures, are limited. The failure criteria employed is another stage of progressive damage modelling and the studies differentiate from each other depends on the failure criterion applied which can be interactive or non-interactive failure criteria. The material degradation model is an important step to model the effect of damage. Therefore different researcher applied different material degradation models to predict the damage behaviour of composites in a best way. Garnich and Akula [71] gave an extensive review of the material degradation models. Moreover, the damage behaviour of composite structures also depends on the structure type and loading condition which make differences at the response of the material. The size of the structure which the proposed methods applied gives an idea about the applicability of the approach to the real size structures.

Table 1 Progressive Damage Studies

	Laminate	Stiffened Plate	Tube	Joints
Post-Buckling	[72], [73], [74]	[75], [76], [77]	[67]	n/a
In-Plane Loading	[56], [57], [58], [59], [65], [78]	[63], [64], [79], [75]	[68]	n/a
Transverse Loading	[53], [60], [61], [80]	[81]	[66], [82]	[69], [70], [83]

Strength based methods: continuum based damage modelling and progressive damage modelling methods are reviewed in this section. The review showed that CDM provides good approximations but includes some modelling complications because of introducing of damage tensor. Several studies are conducted to avoid these complications by employing phenomological degradation approach. Both of these techniques have advantages and disadvantages in terms of efficiency and fidelity. While CDM based methods provide high fidelity results, phenomological methods are more efficient in terms of computation. However, neither of them considers the effects of different length scales, specifically micro mechanisms of damage. Therefore, the multiscale methods are reviewed in next section.

2.3. Multiscale Methods

Laminated composite structures can be considered at different levels starting from the atomic scale to the whole structure including sub-lamina, lamina, laminate and structural parts. The response of the structure to any type of loading is the combination of the all responses of the lower scales. The choice of the scales to determine the response is crucial to have a realistic and efficient analysis. Only continuum level models are reviewed here since there is not lower material scale in engineering problems which structural behaviour plays a significant and measurable role in the response of the structure. There are mainly three length scales in continuum scale: Macro scale (structure level), Meso scale (Ply level) and Micro Scale (Constituent level). The material properties and damage behaviour are interactive across different levels of structure as well as with each other. The scale that has the major effect on the structure response should be decided carefully since most of the structural responses are averaged at the lower scales and the effect at the structure level may not be significant. The choice of the dominant scale enables to simplify the problem. However there isn't a rule of thumb to define the dominant scale. The interaction between different levels and different damage mechanisms should be considered carefully to have a better understanding of the behaviour of the structure without causing any information loss.

Multiscale methods provide bridge between different length scales by passing material information between scales. There are two main ways in order to achieve information passing between length scales: Sequential and Concurrent [84]. Sequential multiscale approaches apply micro to macro homogenization separately from structural analysis [85]. The information of effective properties or material behaviour is carried from lower scales to upper scales. Different homogenization methods and unit cell methods in which representative volume elements are used to induce the macroscopic model can be classified under the sequential multiscale methods. Concurrent approaches are the integrated multiscale methods which considers the local microstructure during the analysis of the macrostructure by applying direct analysis to each scale simultaneously. The information transfer is achieved both ways from micro to macro and vice versa during the analysis. According the definition of scales, it is not necessary to define a meso scale. Computational multiscale techniques which apply direct micro-macro methods are classified under concurrent approaches.

2.3.1. Sequential Approaches

Under sequential approaches the homogenization methods and direct micro structure evaluation by using representative volume element are reviewed. Homogenization methods uses an appropriate averaging schemes to predict the bulk material responses [84]. They vary in the way they transfer information between scales. These methods are reviewed under Analytical, Asymptotic, Mean Field and Higher-Order homogenization methods. Another way to bridge the micro level with upper levels is to use representative volume elements (RVE) to model the microstructure. RVE approach is reviewed after homogenization methods.

2.3.1.1. Analytical Homogenization Techniques

The first attempts to define the material behaviour at macro scale were done by Voigt [86] and Reuss [87] who calculated effective material behaviour of composites with prescribed volume fractions. The composite structure is assumed as blocks according to volume fractions of matrix and fibre as shown in Figure 5.

The Voigt approach is based on the assumption of uniform strain throughout the micro structure under uniaxial tensile. When a load in a direction parallel to the fibres is applied the strains will be equal for matrix and fibre. Further, these strains are equal to the total strain on the composite. This assumption enables to calculate the Young's modulus of the composite as shown in Eq. 7, which is also known as Rule of Mixtures [88].

$$E_c = v_f E_f + v_m E_m \qquad (7)$$

Reuss proposed a uniform stress assumption under transverse loading of microstructure. It is assumed that the average stress of matrix and fibre is equal to the applied stress to the composite. This assumption enable to calculate the Young's modulus as shown in Eq.8 which is also known as the Inverse Rule of Mixtures [88].

$$E_c = (\frac{v_f}{E_f} + \frac{v_m}{E_m})^{-1} \qquad (8)$$

The Voigt uniform strain model works well in terms of axial loading since the mechanical properties of the fibre are much higher than the resin properties. However, when the load is applied to composite material perpendicular to the fibres, to estimate the transverse modulus, the estimation gives poor results: The stress distribution is not uniform and the Reuss equal stress approach between constituents is not correct since the stresses on the fibre and resin are not uniform at the same time. The main reason for this non-uniformity is the effect of fibres on the surrounding matrix which causes lower stress values than in other remote parts of resin.

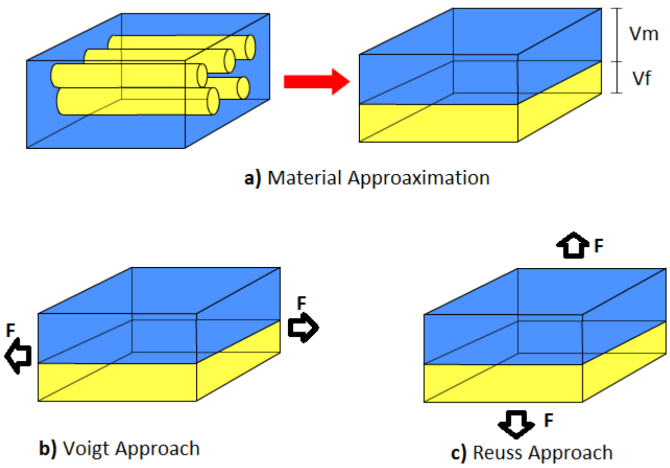

Figure 5 Voigt and Reuss approximations

Voigt and Reuss tried to provide exact predictions on the material properties of composite materials with different assumptions. However, Hill [89] showed that the predictions give upper (Voigt) and lower (Reuss) bounds for elastic moduli of composite which includes the real values inside.

Despite the attractiveness of bounds providing quick approximation, the wideness of the bounds grows with the increasing volume fractions [Fig-6]. The information transfer from material level to the macro level is not accurate enough with this modelling to model damage behaviour at macro level [88]. These approaches give rough estimations for the effective elastic material properties. Thus, these models may result in highly inaccurate predictions for the macro level damage behaviour [84].

Hashin and Shtrikman [90] extended Voigt and Reuss methods with variational approach to define narrower bounds for the effective properties of a composite material. Their approach gives tighter bounds than Voigt and Reuss bounds. Fig-6 gives a comparison of Voigt Reuss and Hashin-Shtrikman bounds [91]. However, this approach depends on the sample size and assumes microstructure is isotropic [85].

Another improvement to these bounds is given by the Self Consistent Method developed by Hill [92] and Budiansky [93]. The method considers a single particle and infinite matrix to overcome the interaction problem amongst particles but the method doesn't work well for the fibre volume fraction above 40% [94, 95]. Although in general the self-consistent method gives reasonable results for the poly crystal microstructure, it is less accurate for two-phase composites [96]. In order to overcome the drawbacks of self-consistent method [97] developed Generalized Self Consistent method by encasing the particle in a shell of matrix material surrounded by the effective medium. But in terms of elasto-plastic behaviour of the material this method becomes very tedious [98].

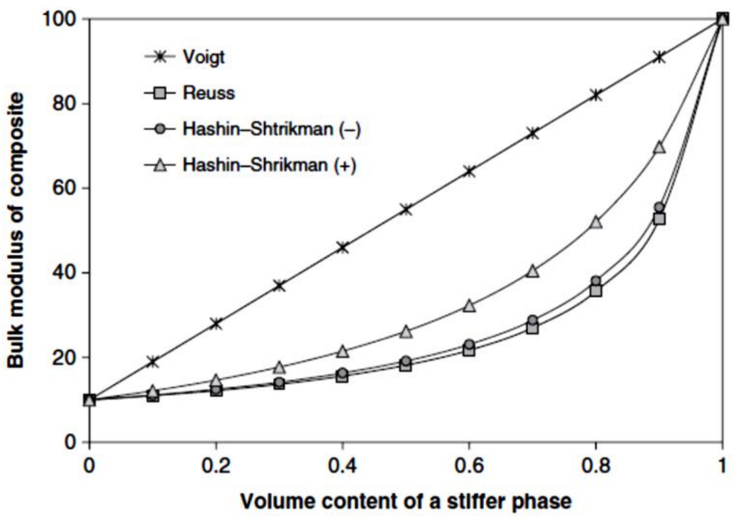

Figure 6 The comparison of different bounds [91]

2.3.1.2. Asymptotic Homogenization

If the structure has a regular or almost regular microstructure it is possible to obtain the effective properties by assuming the structure is made up of several identical representative volumes. Asymptotic homogenization method or mathematical homogenization theory is based on the “asymptotic expansions of displacement and strain fields about macroscale values” [91].When the structure is modelled by number of Representative Volume Elements which are under far field loading, the overall material properties can be derived by analysing these representative volume elements. In this method two different levels should be identified: micro- and macro-levels [Fig-7]. The macro level modelling should be made up with repetitive elements and there should be significant difference between length scales. This difference between microstructure and considered macro level enables one to carry out asymptotic series expansion of the variables [91]. The ratio of the period of structure to a typical length in the region should tend to zero for more accurate results (b/a) [85, 99]. Since the ratio is assumed to be small, the considered parameters converge towards a homogeneous macro-level solution. The details of the mathematical homogenization process can be found in [99]. Pinho de Cruz et al. [100] has also explained the mathematical homogenization process for linear elasticity problems in a detailed form, implementing the finite element equations into the mathematical homogenization process. However there are problems with controlling some parameters such as reinforcement volume fraction, geometry and distribution within the matrix which are overcome in [101]. The study shows that the method is an accurate and efficient tool to derive effective material properties and multiscale analysis. In order to derive more accurate results with asymptotic homogenization simple microscopic geometries and material models are considered mostly under small strain conditions [102].

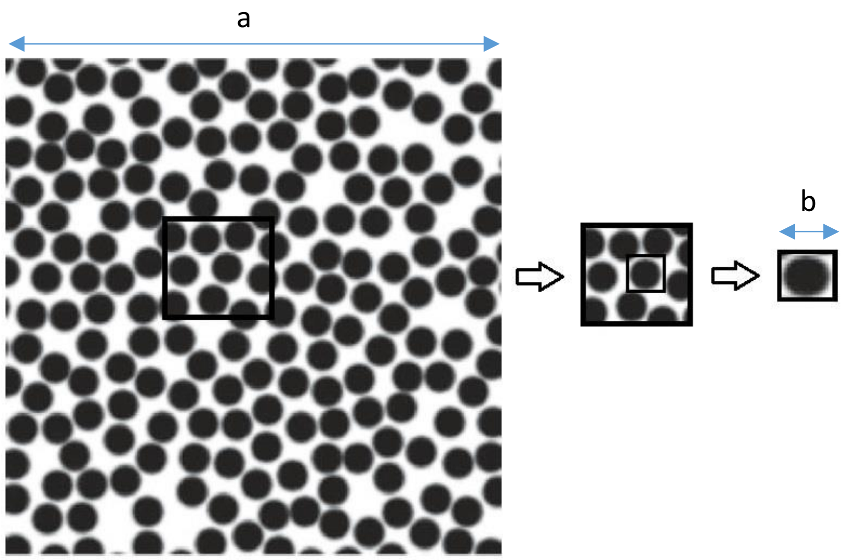

Figure 7 Micro-structure to unit cell [103]

2.3.1.3. Mean Field Approaches

There are also several studies to develop homogenization methods which include the nonlinear effective behaviour of heterogeneous materials. Mean Field Approaches make an assumption on the interaction laws between different constituents to define the effective non-linear behaviour. These approaches consider the effect of the local average fields in the different phases on the micro and macro behaviour by assuming the heterogeneous materials as an initially homogeneous material or matrix with different material inclusions [104]. Eshelby [105] gave the solution of these heterogeneous inclusion problem in terms of Eigen strains. This approach is a more powerful and faster tool compared to the analytical methods mentioned above. The main idea of the approach is to define a transformation strain to equalize the stress within the inclusion and within the matrix which is transformed by a polarization tensor [106]. Eshelby`s solution is applicable for the case of single inclusion and infinite matrix assumption. In order to use Eshelby`s solution different homogenization schemes have been proposed because of the drawbacks of Eshelby`s solution. Firstly, Eshelby assumes that the inclusions don't affect each other. Secondly, although this approach works well for low volume fractions (diluted concentrations), it doesn't provide the same accuracy for the high volume fractions.

Mori and Tanaka [107] have used Eshelby's tensor as the basis for their approach. This method considers the interactions between inclusions by defining an additional transformation matrix to Eshelby`s matrix. By this addition the Mori-Tanaka approach is able to consider the interactions between inclusions and may be applicable at higher volume fractions [108]. Bohm [109] showed that although the overall Young's and shear moduli predictions made by Mori-Tanaka approach are lower than experimental results, they are still very close. There are also some other studies which show how the Mori-Tanaka is a powerful approach to predict the material effective properties of different type of composites; Nallim et al. [110] have used this method to predict the mechanical properties of each lamina of long fibre reinforced laminated plates as a function of elastic properties of constituents and volume fractions. Skrzat et al. [111] have used the method to derive the effective material properties of the composite with regular patterns of parallel fibres. A thin-walled tank under internal pressure is analysed by using this micromechanics approach. Although the starting point of the Eshelby and Mori-Tanaka approaches is in metal matrix composites, it can also be applied to different types of composite materials like textile composites. Gommers et al. [112] used the Mori Tanaka method to derive the 2-D elastic properties of different textile composites such as braided, woven and knitted.

2.3.1.4. Semi Analytical Methods

Another way of taking into account multiscale effects is with Semi-Analytical methods which describe the fields with uniform sub-cells. Thus this class of homogenization approach can consider the spatial variations in the different fields which results in more accurate estimations [84]. These methods connect the micro-level to the macro by using constitutive equations directly at the micro-level. The analytical relations are based on mean field approaches [85].

The Method of Cells (MOC) [113, 114]] is one of the most popular semi analytical methods. According to this method the representative volume element of composite material consists of four square sub-cells. One of the sub-cell represents the fibre, the other three represent the matrix material [Fig-8]. Aboudi derived the expansion coefficients and predicted the macro and micro stress fields by approximating the displacement fields in the sub-cells and applying continuity condition between sub-cells.

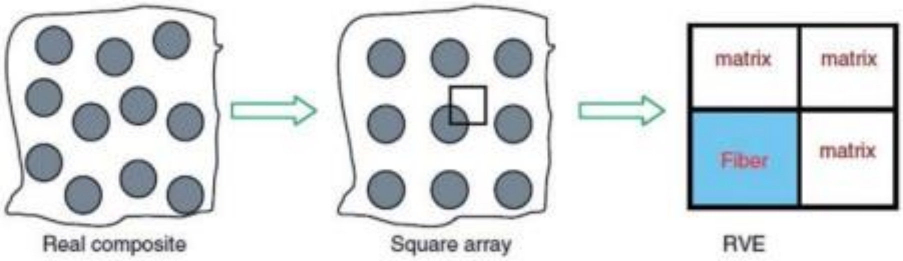

Figure 8 The method of cells [115]

Paley and Aboudi [116] generalised the Method of Cells (GMOC) by using an arbitrary number of sub-cells as NxN rectangular sub-cells along coordinate axis x and y direction. This generalization increases the capability of the method to model the variable fibre shapes, different fibre arrays and to include the porosities and damage to the model [117].

Later, Aboudi et al [118] suggested high fidelity generalized method of cells (HFGMC) which doesn't assume uniform displacement fields within sub-cells. They consider linear stress and strain fields by approximating the displacement field with quadratic functions of local coordinates [119]. According to Mishnaevsky [91] the HFGMC provide very good results not only for the determination of effective properties of composites but also in the determination of localization of strain and stresses within the representative volume elements.

2.3.1.5. Representative Volume Element (RVE)

Another sequential approach is to use representative volume elements in order to transfer the material behaviour to upper scales. Representative volume element first introduced by Hill [120] as a statistical sample of the material. It is not possible to model a whole composite structure at micro level with current computational resources, modelling a representative part of the whole structure is possible. They take advantage of the periodic nature of the fibrous composites to investigate the thermal and mechanical behaviour of the structure. The main aim of using RVEs is to take into consideration the micro level structure in a macro scale analysis by using the smallest representative of the whole structure. There are two important steps using RVEs for multiscale analysis: definition and boundary conditions.

Several researchers emphasize that the definition of the RVE has an important effect on the properties of the material. Hashin [90] indicates that RVE "should be large enough to contain sufficient information about the microstructure however it should be much smaller than the macroscopic structure". Drugan and Willis [121] define the RVE as "the smallest material volume element for which the overall modulus macroscopic representation is sufficiently accurate to represent the mean constitutive response". All of the definitions include that RVE should contain enough information/heterogeneity to represent the whole structure and should be sufficiently smaller than macro-level dimensions. Several examples exist defining

RVE shapes at the micro-level analysis of fibrous composite materials but in general it has been found that unidirectional composites are easier to define due to their repetitive microstructure. Different RVE arrangements in 2-D can be seen from Fig-9. From this figure left upper configuration is also called unit cell which is a specific RVE. This configuration includes a non-divided fibre with surrounding matrix whereas other configurations might have different fibre configurations.

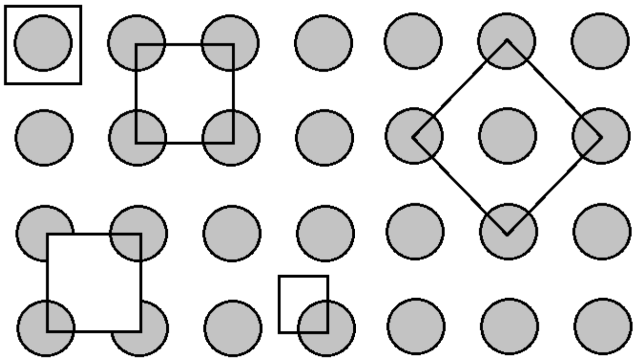

Figure 9 Possible RVE arrangements

Another important issue for the RVE-based multiscale analysis is the applied boundary conditions which have a direct effect on the material properties and mechanical behaviour prediction [122]. One of the boundary conditions is the displacement boundary condition which is also called the Kinematic Uniform Boundary Condition (KUBC); which applies uniform displacements to the boundaries (Dirichlet BC). Another boundary condition is the traction boundary condition which is also called the Static Uniform Boundary Condition, where a uniform stress field is applied to the boundaries (Neumann BC). However for periodic microstructures, periodic boundary conditions (PBC) give better approximations [123] in comparison to KUBC and SUBC. PBCs assume that strains and stresses are periodic at the level of RVE which uses mixed Neumann and Dirichlet boundary conditions together.

Unit-cell methods are one of the most used sequential approach which perform finite element calculations on detailed model of RVEs to induce macro level behaviour. One of the first applications of the unit cell approach was done by Christman et al [124]. They derived deformation characteristics of ceramic whisker and particulate reinforced metal matrix composites which are in good agreement with experimental results. Li [125] applied micromechanical analyses to unidirectional fibre-reinforced composites by considering different unit cell layouts and symmetry conditions. He proposed to use trapezoidal unit cells to take advantage of the existing symmetries which enable the derivation of macroscopic stresses and strains directly from the results of unit cell analyses.

Kari et al [126] used the RVE approach to evaluate the effective material properties of short fibre composite structures and to investigate the influence of different parameters such as aspect ratio, volume fraction, fibre orientation angles. According to them effective material properties of randomly distributed short fibre composites mainly depend on the volume fraction. They have compared their results with Hashin-Shritkman bounds and Self-Consistent method which have close predictions

RVE approaches or so called Unit cell methods can also be used to transfer the damage behaviour to upper scales in order to analyse the damage initiation and evaluation. Damaged elements can be used to model the RVE or unit cell in order to analyse the effect of micro-level damages on macro-level behaviour [91]. Xia et al [128] created 3-D multilayer unit-cells at meso-micro level of a glass-fibre/epoxy laminate. The numerical results for overall stress/strain response and damage initiation showed good agreement with experimental results. However the damage modelling using unit cells were based on the assumption of uniformly distributed micro damage mechanisms within the structure and captured the damage progression.

2.3.2. Concurrent Approaches

Another multiscale approach is the concurrent method which can be called the computational multiscale method. Among multiscale methods, computational methods are some of the most accurate with regards to micro-level properties of the structures [129]. Generally, computational methods apply direct micro-macro calculations. It assumes each macroscopic point represent an RVE and applies separate calculations for each of the scales and each of the RVEs. These methods provide accurate macro level response of heterogeneous material with arbitrary micro-level properties with the help of RVEs located at each integration point. However, these methods are computationally expensive to apply to engineering structures with ordinary computational resources [85].

Different researchers proposed different methods to apply direct micro-macro calculations. They mainly differ from each other in two ways: the analysis of the RVE and the method of bridging between macro and micro level. Finite element method, Voronoi Cell method and Fast Fourier transfer are the main approaches to analyse the RVE. In order to transfer the information between scales asymptotic homogenization and volume average methods are being used.

2.3.2.1. FE Based Computational Methods

The first application of direct finite element discretization of the microstructure linked to macrostructure was done by Renard et al [130]. After first attempts Guedes and Kikuchi [131] implemented a multiscale method via finite element technique to model the linear elastic composite material by using a mathematical asymptotic homogenization theory of Bensoussan [99].

Feyel [13] developed a generalized multiscale finite element method called FE^2 to consider the non-homogeneous behaviour of the fibre/matrix level of structures more accurately. In this method a representative volume element is assigned to each Gauss point of the macro level structure`s mesh. A finite element computation is applied at macro and micro level concurrently. Three main steps are included in the method: RVE modelling, localization and homogenization. The main drawback of this approach is the computational time depending on the macroscopic structure`s complexity. This method requires high computational costs as a complete boundary value problem has to be solved for every macroscopic Gauss point [13].

Smit et al [132] also developed a similar multi-level finite element approach for large deformation cases with complex micro-level structures. The local micro-structure is defined by RVEs at each integration point of the discretised homogenized macro-structure. The local deformation and stress tensors which are derived from macro-level homogenized finite element analysis are assumed to be equal to the RVE averaged deformations and stress tensors. The finite element analysis provides both macro-level global responses through averaged stress deformation field and RVE responses through local stress deformation fields.

Miehe et al [133] presented a theoretical and computational approach for the analysis of a homogenized macro-level structure with locally attached micro-level structures at large strains under non-isothermal conditions. The macro scale is defined as a homogenized continuous medium whereas the micro scale is associated with the RVE. The deformation of the RVE is coupled with the local deformation of the material. There are three coupling constraints that can be applied which are: “zero fluctuations in the domain”, “zero fluctuations on the boundary” and “periodic fluctuations on the boundary”. The macro-level thermal and mechanical variables are defined as volume averages of their micro level counterparts. Miehe has proposed here evaluating these approaches in a straight forward manner by setting and solving two locally coupled boundary value problems for the finite deformations of the micro continuum and associated micro-structure, respectively. Obviously, this method requires a large scale computation which depends on the mesh size of the micro-structure.

Kouznetsova et al [102] also applied a direct micro-macro approach by assigning an RVE to each integration point of the macro level structure`s mesh. The boundary conditions are determined from the macroscopic deformation tensors and applied on the RVE boundaries. By applying volume averaging over the RVE, the macroscopic stress tensor is determined. In this study, the constitutive behaviour at the macroscopic integration point is obtained from averaged behaviour of the RVE associated with that point like as in [132]. Kouznetsova has evaluated this approach for time dependent material behaviour for an elasto-visco-plastic model with the application of pure bending of porous aluminium.

Mosby and Matous [134], in a recent study, has demonstrated the direct numerical method by employing computational homogenization at extreme scales in terms of both physical scales and computing resources. They employed fully coupled multi scale simulations to predict the damage properties from micro level constituents. In three different numerical examples around 54 billion finite elements which contain 28.1 billion non-linear equations are employed. The calculations are achieved by using up to 262,144 computing cores. The study showed the promising way of computational homogenization for the future use however it is not practical with current computational power.

2.3.2.2. Voronoi Cell FE Based Computational Methods

Ghosh and co-workers [135] have developed another computational method for arbitrary heterogeneous materials called Voronoi Cell Finite Element method (VCFEM). The micro-level representative volume element involves different number of multisided convex "Voronoi" polygons [Fig-10]. Ghosh and Mukhopadhyay [136] have developed a mesh generator to create these polygons with different shape-size and location to represent the heterogeneities more accurately by assigning different properties to each one. Ghosh et al [137] have presented a two-dimensional coupled multiscale method by applying asymptotic homogenization theory between different length scales for linear elastic structures.

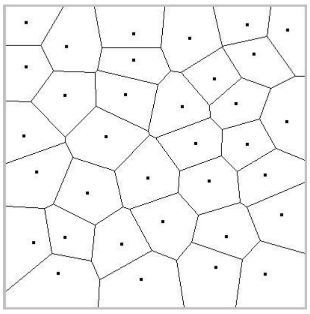

Figure 10 Voronoi cell polygons

Thus far, the available multiscale methods in continuum level are reviewed. A summarized representation of the methods is shown in Table-2. Another important parts of the multiscale method is the way of transferring information between scales and the analysis of the microstructure. Table-3 groups the studies regarding the analysis of the RVE and the bridging between micro and macro scales.

As mentioned before, under extreme conditions or even under normal operating conditions different failure mechanisms can occur within the matrix, fibre or fibre-matrix interface at the micro scale. This failure may cause loss of stiffness and strength of the material and cause final catastrophic failure. Investigation of failure mechanisms: the onset of failure, the growth of damage and the determination of the maximum loads before the first failure or before the final failure are crucial to be able to design reliable and safe structures. In order to have deep understanding of the micro-mechanisms on damage behaviour of composites, several studies have been applied by using multi-scale methods mentioned earlier. The importance of failure and damage analysis on the constituent level using micro-mechanisms in order to understand damage onset and growth is indicated by [138].

Table 2 Classification of Multi-Scale Methods

Sequential Methods	Analytical Homogenization Techniques	Rule of Mixtures Hashin and Shtrikman Self Consisten Method
	Asymptotic Homogenization	Bensoussan
	Mean Field Approaches	Eshelby Mori-Tanaka
	Semi Analytical Methods	Method of Cells(MOC) Generalized MOC High Fidelty GMOC
	Representative Volume Element	Unit Cell Methods
Concurrent Methods	Finite Element Based Comp. Methods	FE2 Multi-Level FE
	Voronoi Cell FE Based Comp. Methods	VCFEM

Table 3 Classification of studies: Analyse of the RVE and Bridging

The Analysis of RVE	FEM	Terada-Kikuchi (1995) Smit (1998) Smit et al. (1998) Miehe et al. (1999) Feyel and Chaboche (2000) Kouznetsova et al (2001)
	Voronoi Cell	Ghosh et al. (1995) Ghosh et al (1996)
	Fourier Transform	Moulinec Suquet (1998)
The Bridge between Micro and Macro Level	Asymptotic Homogenization	Fish et al (1999) Ghosh et al. (1995) Terada Kikuchi (1995)
	Volume Averaging	Smit et al. (1998) Kouznetsova et al (2001)

The main aim of using multiscale methods for damage analysis is realizing a transition of damage entities between macro-scale and fibre-matrix level. By analysis at the micro scale, failure criteria and effective macro material properties are predicted. After obtaining homogenized material properties and damage implementation to the micro-level model, the macro level analysis is applied. Beside a number of basic studies including the studies mentioned earlier, within this framework there are different researchers using different methods, different computational approaches for different types of materials and structures.

Ivancevic and Smojver [139] have studied the damage and failure prediction of laminated composite structure by applying high fidelity generalized method of cells which is an upgraded version of generalized method of cells by Abouidi [119]. Equivalent material properties are calculated at the beginning of the analysis and are used for the macroscopic constitutive equations. The micro-level failure analysis and macro level failure analysis results are compared. The results showed that the micro-level failure modelling give more accurate results than macroscopic approach.

Another study by using Generalized Method of Cells is done by Ye et al [140]. The initial and final failure strength analysis of composite materials has been investigated. The effects of thermal residual stresses on initial and final failure strength are examined. The results obtained from multiscale analysis show good agreement with the experimental data.

Kwon and Park [141] have developed a general purpose micro mechanics model to be able apply to different composite materials such as particle reinforced composites, long fibre and short fibre composites. The model calculates the effective properties of these different materials. They developed a 3-D brick shape unit cell to calculate the material properties. Those effective properties are used for the multiscale analysis.

In a recent study, Murari and Upadhyay [142], presented a ply level continuum damage model by employing a stiffness degradation with microlevel damage. The effect of volume fraction and micro level damage size is accounted for in the continuum material model. The effect of volume fraction and damage parametres on the elastic material properties are calculated with a micromechanical model of asymptotic homogenization approach. Zhang et al. [143] proposed a micromechanics-based degradation model to evaluate the damage behaviour of bolted joint composites. The proposed method is based on homogenization and rule of mixture with the simplified fibre and matrix microstructure. Fibre tension/compression failures, matrix tension/compression failures and fibre matrix shear-out failure modes are considered by using the modified version of Hashin failure criterion. The study introduced material degradation models for each failure modes either calculated from rule of mixture approach or sudden degradation approximations. The predicted stiffness agrees with the experiments for the initial stages of the loading however there are significant differences on the predictions of final failures.

Tay et al, proposed a finite element based element failure method to model progressive damage of composite materials. They modified the nodal forces of a damaged element to represent the damaged behaviour instead of degradation of material properties. The stiffness matrix remains unaltered and no reformulation of stiffness matrix is required which avoid computational problems. They employed the strain invariant failure theory (SIFT) which strain invariant quantities are used to determine the failure through micromechanical analysis of RVE.

Using Eshelby-Mori Tanaka homogenization method, Nguyen and Simons [144] used fibre and matrix constituent properties to investigate the macroscopic response to analyse filament wound composite pressure vessels. They have compared their multiscale model which has been implemented failure prediction with experiments available in the literature. The results showed good agreement for different parameters of composite pressure vessel with a thick aluminium liner.

Another study on composite pressure vessels is performed by Liu et al [145] . A multiscale damage model is applied to investigate the failure initiation and final failure of the vessel. In this study RVE approach is used for the analysis of the composite pressure vessel. RVEs associated to each Gauss point are employed to define the stiffness properties. At the macro scale finite element analysis is applied under given material properties, boundary conditions and loading. Then a progressive failure analysis of each RVE is applied for the stress values at each Gauss point of macroscopic mesh. After determining failed number of microscopic matrix elements they performed macroscopic stiffness degradation for each Gauss point until the vessel fails.

On the other hand, textile composites have wide area of application with its advantages like higher delamination and impact strength. Material inhomogeneties in lower scales have higher impact on mechanical properties of textile composites than that final composite structure. Ernst et al [146] have used a multiscale algorithm to model the effect of the lower scale inhomogeneties on macro scale behaviour by using a unit cell approach. Their approach is information passing multiscale modelling which provides more efficient computational time according to fully-coupled multi-scale methods. For this approach they generated micro-mechanical unit cells at micro-level consisting of fibre and matrix and meso mechanical unit cells for the fibre bundles at meso-scale. This model requires the determination of material properties.

Another example to the application of multi scale modelling to the different engineering structures is flexible risers. Chi et al [147] have developed a nested multiscale procedure in which macro level structure and RVEs of flexible pipes run in parallel. They proposed to apply local analysis at specified regions in parallel with the macro level analysis in order to save computational time compared to fully coupled multi scale modelling.

One of the challenging points of the multi scale analysis is the application of the computational schemes irrespective of multiscale approach is being applied: hierarchical, semi-concurrent or concurrent. The majority of authors have developed their in house codes. Feyel [13] has developed Zebulon which is an object oriented code making it more portable with different libraries for their multilevel finite element modelling FE^2. Nguyen and Simmons [144] also developed their in house code called EMTA-NLA for their multiscale modelling approach. This code is using Eshelby and Mori Tanaka homogenization scheme. Talebi et al [148] have presented an open source multiscale framework called PERMIX for modelling and simulation of material failure. Their code is able to use different multiscale approaches like semi-concurrent and concurrent. Nezamabadi et al [149] developed their in-house Matlab code by applying multilevel nonlinear finite element analysis. These types of codes have two main drawbacks. Firstly, the accessibility to these codes is limited, which is unattractive to researchers and industry. Secondly, these codes mostly need access to commercial finite element codes but these commercial products don't give full access to their structure. Yuan and Fish [150] have made first attempts to overcome these difficulties by integrating computational homogenization techniques into conventional finite element codes. Tchalla et al [151] have also propped a comprehensive procedure to implement such computational approach in Abaqus for linear and non-linear problems.

2.4. Summary

The literature review of composite damage modelling showed the requirement of multiscale approach for more accurate predictions. There are many methods and studies that are applied for the damage modelling implementing the effects of different length scales. The micro structures of composites has a crucial influence on macro level response and needs to be considered to identify the damage behaviour of macro level structures.

Determination of ultimate failure of composites is essential since the structure is able to carry load beyond initial failure. Progressive damage models offer an important capability on predicting failure process. However, most of the models cannot be re-established for different cases, such as loading, material configurations, constraint conditions [19]. The efforts on modelling damage evaluation of composite structures aims to improve one of the main three parts of progressive damage modelling: The stress analysis to determine the stress and strain fields within the structures, failure criterion in order to decide damage occurrence and the degradation of material properties. The studies are trying to make improvements in one of the parts of progressive damage modelling in terms of accuracy and efficiency. Finite element analysis of the structures give good predictions for the definition of stress fields with high accuracy. However, there is a need to speed up analysis in order to apply for heavy computations, such as large structures, high non-linearities, reliability analysis. WWFE and other independent researchers trying to improve the efficiency of failure detection. In this area, micro level effects and constituents properties in lieu of homogenized lamina properties are trying to be implemented to the failure criteria. The constituent properties and micro level mechanisms are essential aspects in the area of composite damage modelling. Finally, the review showed the need of improvement of degradation methods to estimate more appropriate residual strengths.

3. Outline

The background section and the literature review highlighted that estimation of residual strength of composite materials are crucial in order to determine the realistic strengths of structures. There is a need for greater understanding of load bearing capabilities of stiffened composite structures. Progressive failure analysis is a practical way of predicting the load bearing capability of the composite structures and composed of three main steps, namely: *i)* stress analysis, *ii)* damage evaluation and *iii)* material property degradation. The literature review has shown that despite the number of studies on damage modelling, current state of art requires improvements in terms of fidelity and accuracy. In this study, the progressive damage behaviour of composite structures is investigated under these steps in order to obtain more accurate estimations of composite behaviour in an efficient way.

Composite materials gain its advantages from heterogeneous nature. Using different materials with different properties together gives flexibility to designer. However, the same complex nature cause complex responses of the macro-structure. Generally composite marine structures can be divided three different length scales in continuum scale as shown in Fig-11. Top hat composite stiffened plates are considered as macro-level structure. Micro level structure is the fibre and matrix level including two different constituents. There is also one transition level between macro and micro, called as meso-level. This level considered as ply level. The interaction between different length scales are not understood well due to the lack of maturity of research in this area.

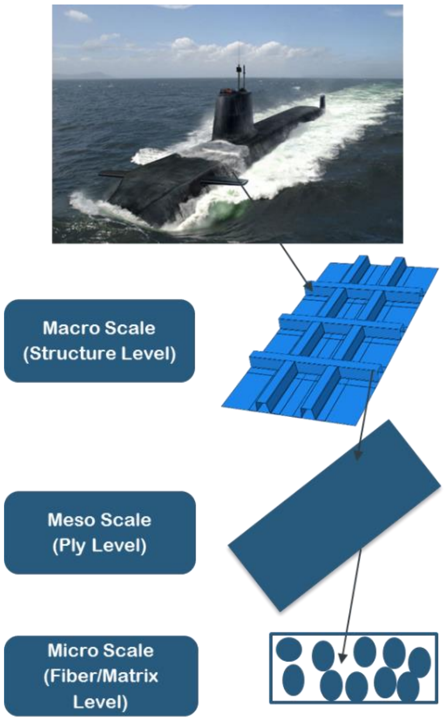

Figure 11 Multi-scale methodology

This study investigates the effect of length scales on macro level response of composite stiffened structures. The macro level behaviour of the composite structure directly depends on the micro level mechanisms. In order to have better predictions at the macro scale, micro level mechanisms should be investigated. The transfer information from micro level to the macro level will enable one to have more accurate predictions for macro level linear or non-linear response. The first thing is the evaluation of material properties to predict the composite behaviour and basic analytical methods are being used to determine it in most cases. Then, damage accumulation at micro level should be implemented to improve the predictions of non-linear response. Multi-scale methodology will be used in order to investigate the macro level behaviour including more detailed micro-structure mechanisms. In order to have deeper understanding of composite behaviour, the information transfer will be applied between different length scales.

Fig-12 shows the methodology for multiscale damage analysis of composite structures including macro- and micro-level analysis.

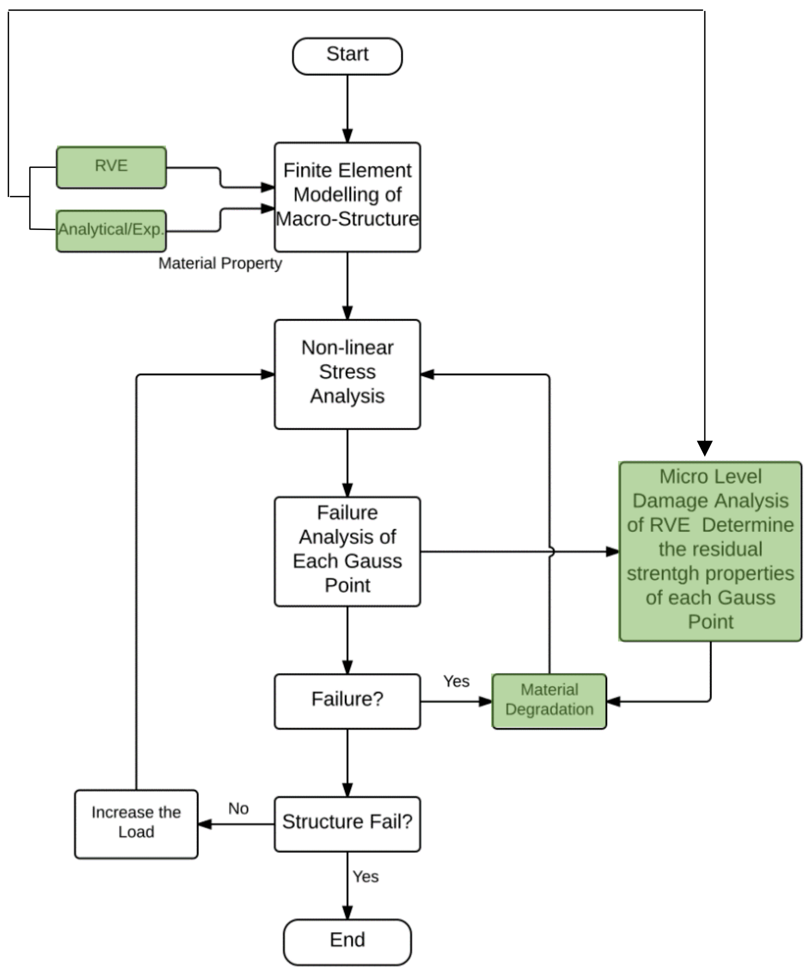

Figure 12 Multi-scale damage analysis

3.1. Macro level analysis

This thesis starts with analyses of composite stiffened plate for marine applications. Top-hat stiffened structures have complex geometry and behaviours. Finite element modelling techniques is the main way to investigate the overall behaviour of these structures. Firstly, finite element analysis will be applied for cross stiffened composite plates and the results will be compared with experiments from literature [152]. Finite element modelling of the structure is replicate as close as possible to the experiments in order to have a realistic modelling techniques. Abaqus CAE [153] , is used for finite element analysis with an automated Python [154] code which generates the model in parametric way and run the analyses. This section will show the response of top hat stiffened composite plates under pressure loading with geometrically non-linear but materially linear modelling.

In order to implement material non-linearity of composite structures under pressure loading, a progressive damage modelling will be applied to a composite plate. Macro level modelling of damage progression is achieved by using Abaqus and user subroutines written in Fortran languages. Material property degradation is applied to the each gauss points when failure criterion is satisfied. Tsai-Hill failure criterion and Chang-Lessard failure criterion are applied. The results are compared with numerical [53] and experimental results [155].

3.2. Micro level analysis

In order to have better predictions of composite behaviour, the micro level mechanisms will be investigated. Material effective properties and overall material stiffness tensor will be investigated by using numerical methods. Micro scale analysis will be applied by using representative volume elements which will be modelled by finite element technique to derive detailed information of micro structure. The purpose of the RVE analysis is to evaluate weighted or effective composite material property or behaviour which can be used to obtain macro-level constitutive relations.

The micro level modelling of composite microstructure is composed of three steps: *i)* RVE modelling, *ii)* application of boundary conditions and *iii)* homogenization. The shape of the representative volume element will be decided according to accuracy provided and computational time requirements. Achieving a representative structure with the possible smallest volume is crucial. Next step is selection of boundaries of the RVE which describes the response of the microstructure. Periodic boundary conditions will be applied to the RVE by assuming the stresses and strains are periodic in a periodic architecture. Following having the stress strain fields within RVE, homogenization will be executed by volume averaging approach.

The effective material properties will compared with analytical and experimental results from literature. The main aim is to define the effective material properties with high fidelity and low computational effort which can be used instead of experiments. The material properties derived from micro level analysis will be used directly on macro level analysis and compared with the results.

3.3. Multi Scale Damage Analysis

The multi scale nature of the composite will be implemented to the damage analysis of the structures by using RVE approach. The stress and strain values for the matrix and fibre constituents will be calculated at the micro structure. The failure initiation and progression of the damage will be investigated by stiffness degradation. Material degradation will be implement in a progressive damage process in order to determine the damage tolerance of the structures. Progressive stiffness degradation will be applied according to damage mode in order to have better predictions of damage behaviour. The damage modelling process is shown in Fig [12].

Although finite element modelling is one of the accurate way to investigate the thermal or mechanical behaviour of the structures, it doesn't provide enough efficiency to implement high number of simulations. Surrogate models can be used instead of this expensive and complex finite element simulations to implement in multi scale analysis which will provide an efficient way to transfer the information between scales. A surrogate model will be created for micro level RVEs and macro level stiffened structures by using response surface methodology since surrogate modelling techniques gives higher accurate results when compared to analytical methods [156].

Creating a surrogate model using response surface methodology involves following steps:

- Sampling Plan: The first step and the most important step of surrogate modelling is definition of sample points to solve. The quality of the surrogate model depends on the sampling plan [157]. The number of the sample points and the locations affect the accuracy of the model as well as the computational effort. To obtain the best approximations Latin Hypercube [158, 159] methodology is used.
- Constructing Surrogate: The results from first step are used to construct the surrogate model. Kriging [160] is used to construct the surrogate model which provides a good generalisation of the solution space [161].
- Exploring Surrogate: The last step of surrogate modelling is exploring the accuracy of the model, by using test points which are different from the ones used for constructing the surrogate. Initially, average prediction errors and root mean square errors are calculated in order to measure the accuracy of the models.

In summary, the methods have been chosen due to their high efficiency and high accuracy modelling capabilities.

4. Macro Level Modelling

4.1. Introduction

Stiffened plates are one of the most used structures in different engineering structures due to the advantages they provide such as stiffer structure with less weight increase. Top-hat stiffened plates are used as structural elements in different aerospace, civil and marine applications since they provide extra strength to the buckling loads [12]. Top-hat stiffened structures are geometrically and mechanically are complex which cause inaccurate results when analytical methods are used. Therefore, finite element modelling has crucial importance to model such complex structures among different numerical methods [162]. In this chapter therefore the macro level behaviour of top hat stiffened composite plates are developed with the help of finite element modelling. The aim of this chapter is to determine the macroscopic behaviour of composite structures incorporating conventional modelling way such as using analytically or experimentally derived material properties. Firstly, top hat stiffened composite stiffened plates are modelled using a general purpose finite element software Abaqus CAE. The structure is modelled as geometrically non-linear and materially linear. This modelling is initially applied in order to determine the general response of the structures by designers and researchers. Later, in order to have detailed response of macro structure, the modelling technique is improved by incorporating material non-linearity. A progressive damage modelling technique is employed to understand the damage behaviour of composite plates.

4.2. Finite Element Analysis of Stiffened Composite Plate

A top-hat cross-stiffened composite plate is analysed with finite element by using Abaqus CAE. Finite element modelling usually provides to user an accurate estimations of structural behaviour. However, the results doesn’t necessarily mean legitimate all the time. Modelling approach and the approximations done have significant effects on outputs and should be selected carefully. In order to verify the modelling approach, the results are compared with experiments of Eksik et al [163]. The other weak point of finite element analysis is the time spent for modelling and solving the problem. In order to reduce the time spending on modelling and to ensure an opportunity to change the design parameters without difficulty, an automated Python code is developed. A one by one top-hat stiffened composite plate is investigated.

4.2.1. Geometry and Modelling

The composite panel analysed has 800 mm breadth and 850 mm length. In the experiments, panel dimensions are originally 1 m. by 1 m., however because of the test equipment and application of boundary conditions, the load was only applied to the area of 800x850 mm of the single cross stiffened glass/polyester panel [Fig-13]. X-direction represents the longitudinal direction and the stiffeness here is continuous, the Y-direction is the transverse direction and the stiffener is joint to the other one. In order to determine the stress and strain values during the loading, 14 strain gages are implemented to both sides of the plate: loaded surface and stiffened surface. The positions of all strain gauges' are displayed in Fig-13. The strain gauges number 3 and 11 to 14 are biaxial rosettes and the rests are uniaxial strain gauges. The plate has two identical top-hat stiffeners which Fig-14 shows the cross section. The stiffened plate is categorized into four areas. The top of the stiffener is crown, the side faces are flange, and the attached part of stiffener to the plate is flange whereas plate represent the whole lamination under the stiffeners. The panel was made up of glass-fibre reinforced polyester resin and finite element modelling is performed with Abaqus CAE using an automated script for modelling.

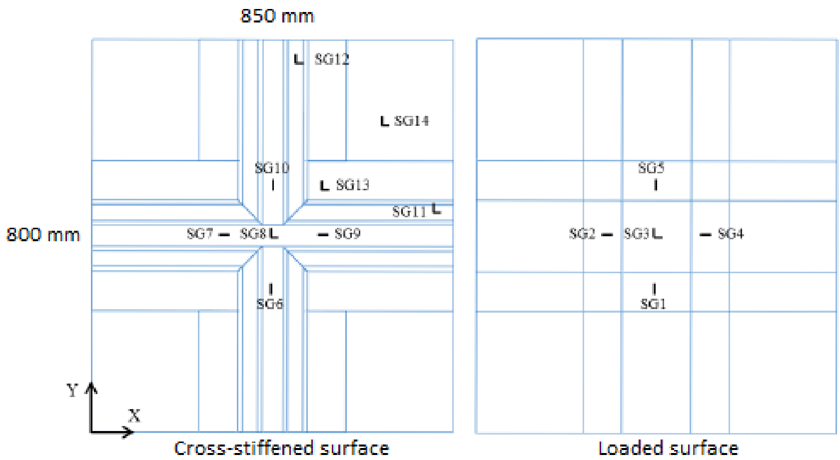

Figure 13 Cross stiffened plate and SG locations

4-node general-purpose quadrilateral shell elements with reduced integration points are used for modelling. Abaqus CAE shell element library offers elements the modelling of curved, intersecting shells that can exhibit nonlinear material response and allows large translations and rotations as well good modelling predictions on the bending behaviour of composite materials. The general purpose shell elements can be applied for both thin and thick shell problems and provide robust and accurate predictions under different loading conditions. It provides nonlinear solution with computational efficiency under finite membrane strains [153]. During the analysis geometric nonlinearity is considered while the material properties are assumed linear elastic. The connection between the plate and stiffeners and between the stiffeners themselves are critical decisions for numerical modelling. The longitudinal stiffener is continuous while the transverse one is discontinuous. The connection between those is made by using resin and extra fibre during production to make it as strong as possible. So it is prevented to have any joint failure in this connection. Therefore, in numerical modelling the cross stiffeners are assumed to have perfect tie constraints. As physically observed by Eksik et al, the dominant behaviour will be longitudinal continuous stiffener. Therefore, it is ensured that the elements of transverse stiffener connection area is behaving related to longitudinal ones. The contact between the plate and stiffeners are applied as tie constraints similarly. Two different approaches are tried out in order to model the stiffened plate realistically: plate dominant behaviour and stiffener dominant behaviour. It is shown that the top-hat stiffened plate behaves stiffener dominated way. The stiffeners have trapezoidal polyurethane non-structural foam inside in order to laminate the stiffener easily. As this foam is not structural, it is not included in modelling.

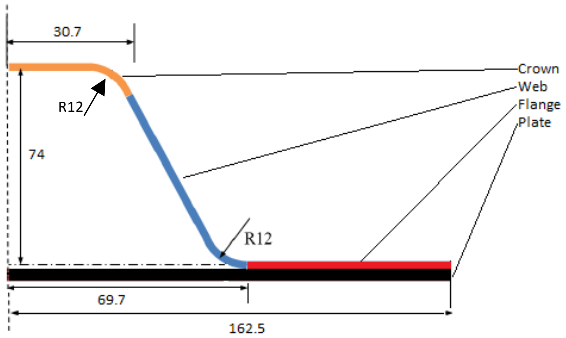

Figure 14 Stiffener cross section

The stiffened plate includes 14 layers totally both in the plate and stiffeners. The thickness of the plate is 11.35 mm while the thickness of the crown of stiffeners is 6.5 mm. It is modelled with shell layers for each ply. The layup properties of the stiffened plate shown in Table-4. The material properties of the five different material used in the structure is showed in Table-5.

Table 4 Structure layup properties (CSM: Chopped Strand Mat, WR: Woven Roving, UD:Unidirectional)

	Layer No	Material	Thickness(mm)	Angle
Plate	1	300 g/m^2CSM	0.75	-
	2	600 g/m^2CSM	1.5	-
	3	600 g/m^2WR	1	0
	4	600 g/m^2CSM	1.5	-
	5	600 g/m^2CSM	1.5	-
	6	600 g/m^2WR	1	90
	7	450 g/m^2CSM	0.8	-
	8	600 g/m^2CSM	1.5	-
	9	600 g/m^2WR	1	90
	10	450 g/m^2CSM	0.8	-
Flange	1	600 g/m^2CSM	1.5	-
	2	600 g/m^2CSM	1.5	-
	3	600 g/m^2CSM	1.5	-
Web	1	600 g/m^2CSM	1.5	-
	2	600 g/m^2CSM	1.5	-
	3	600 g/m^2CSM	1.5	-
Crown	1	600 g/m^2CSM	1.5	-
	2	600 g/m^2CSM	1.5	-
	3	1600 g/m^2 UD	2	0
	4	600 g/m^2CSM	1.5	-

4.2.2. Loading and Boundary Conditions

The top-hat stiffened panels are generally subjected to hydrostatic pressure during operation in marine structures. Therefore, the experiments are performed under uniform pressure which is applied from unstiffened side of the plates. The test panels were sandwiched between steel frames which were then bolted. The intention was to simulate fixed boundary conditions but as the pressure increased the friction between plate and test rig has overcome and the panel was pulled inwards [163]. Therefore, in order to replicate the experimental conditions, the corner points of the plate are fully clamped while the plate edges are pinned boundary conditions.

The structure is meshed by using standard large strain S4R quadrilateral four-node shell elements. Under this loading and boundary conditions a mesh convergence is applied to the finite element model which shows around 80000 elements are enough to achieve convergence. It takes around 150mins with 16GB memory PC.

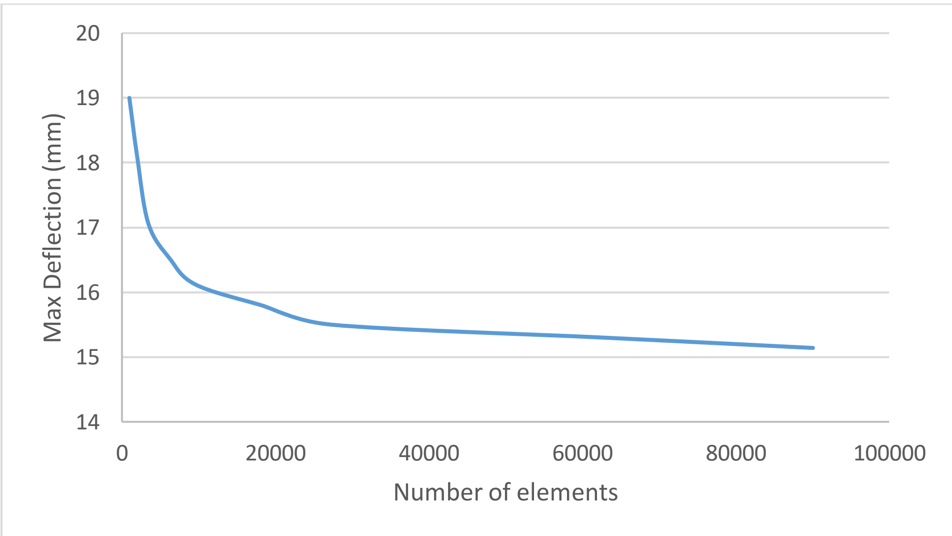

Figure 15 Mesh Convergence

4.2.3. Results and Discussion

The strain states from 14 different strain-gages are measured during the experiments. These strain values are compared with non-linear finite element modelling results of this study and finite element linear solutions of Eksik et al [163]. The strain comparisons from strain-gages number 1 and 5, strain-gages number 2 and 4 are presented in Fig 16 and Fig 17, respectively. The strain comparisons from all available positions are presented in APPENDIX B. The figures include the strain change until maximum loading of 0.3 MPa. Numerical modelling predictions presented with only one curve because of the symmetry of the strain-gages. However, the experimental results presented for each location and some differences are observed despite the symmetry. These may be due to the small differences at the strain-gage locations or differences occurred while loading. The predictions of linear finite element modelling done by Eksik et al. [163] demonstrated significant differences as expected. Because this model didn't include any non-linearity. The current geometrically non-linear model represented with green-triangle curves showed good predictions for the strain development. There are some significant differences are observed at some of the strain-gage locations such as number 8. These differences may be due to the application of boundary conditions or manufacturing defects. The stiffened plates are produced by hand lay-up system. Therefore, it is likely to have some flaws around these locations which will have influence on strain distribution.

Table 5 Material Properties of cross stiffened plate [152]

Material	Property	Value
300 g/m^2CSM	E_x G_{xy} Poisson Ratio	8 GPa 3.1 GPa 0.3
450 g/m^2CSM	E G_{xy} Poisson Ratio	7.3 GPa 2.8 GPa 0.3
600 g/m^2CSM	E G_{xy} Poisson Ratio	6.8 GPa 2.6 GPa 0.3
600 g/m^2WR	E_x E_y G_{xy} Poisson Ratio	14.8 GPa 14.8 GPa 2.4 GPa 0.092
1600 g/m^2 UD	E_x E_y G_{xy} Poisson Ratio	24.6 GPa 24.6 GPa 2.3 GPa

In addition to these reasons, the damage development within the structure generate differences between numerical estimations and experimental results. Once the loading progress, damage initiate first in the matrix and propagate. Accumulation of damage doesn't necessarily mean a catastrophic failure but has considerable effect on the response of the structure. The main reason of fluctuations at the experimental results might be the consequence of the damage accumulation. Overall, these results indicate that the non-linear finite element modelling provides good predictions in order to determine the macro level behaviour especially for the initial research and design process. However, material non-linearity and damage modelling is required to be included into numerical modelling for more comprehensive predictions.

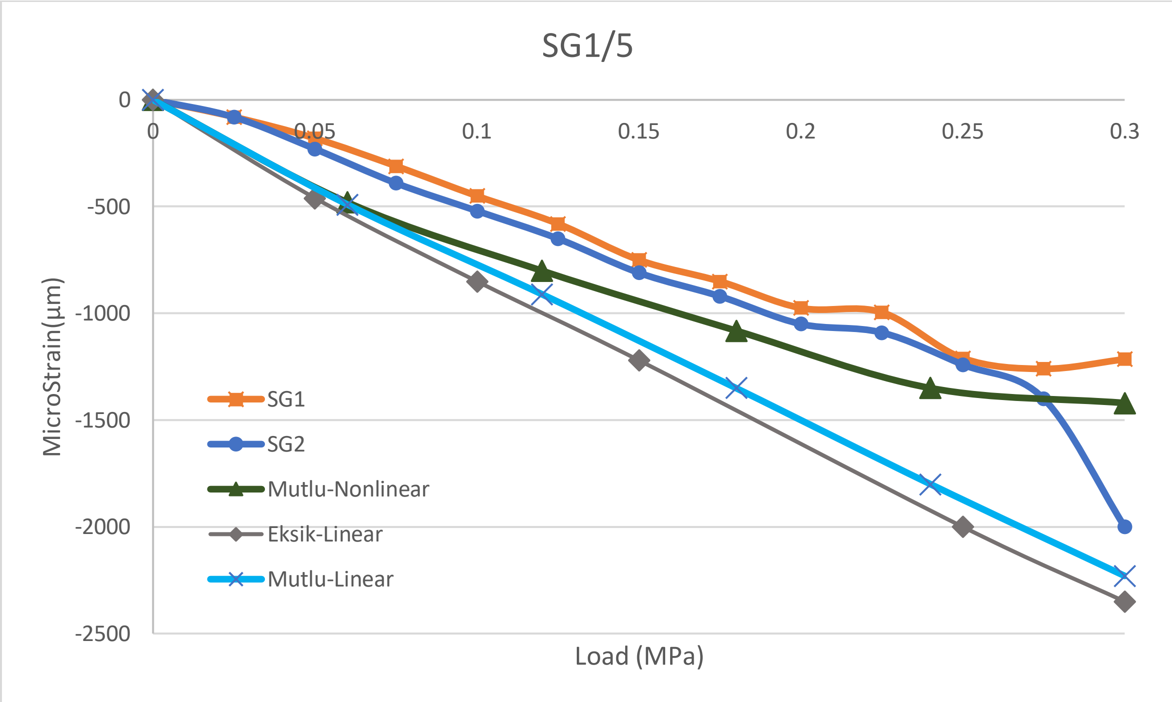

Figure 16 Strain-Load SG1/5

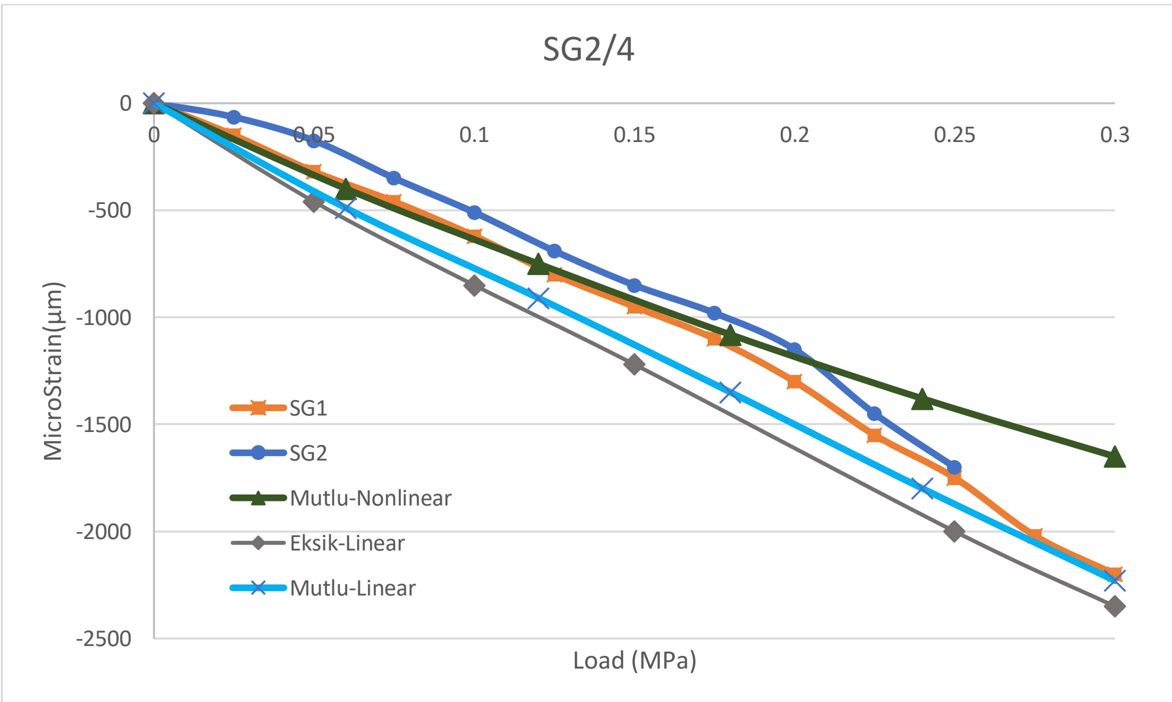

Figure 17 Strain-Load SG2/4

4.2.4. Progressive Damage Modelling

In order to have more accurate prediction of structural behaviour of composite structures, a non-linear material characterisation is required. Material non-linearity can be included with fully non-linear finite element modelling. In earlier part of this study, stiffened plate is investigated with geometrically non-linear but materially linear models. Although this modelling approach gives good results for initial design and research process, it is not comprehensive to determine the genuine response of the structure. Therefore, a materially non-linear model is presented in this section. Materially linear modelling assumed no failure occurs during the loading within structure. However, in reality, several failure mechanisms form that affect the characteristics of the macro level response.

Progressive damage modelling is an efficient way to involve the failure mechanisms into the structural modelling as reviewed in Chapter-2. Progressive damage modelling composed of two main steps: determination of damage initiation and the evaluation of damage. In this section a progressive damage model is developed and applied to a composite plate as re-production of works of Moy et al [155] and Padhi et al [53].

4.2.5. Damage Modelling Methodology

Progressive failure analysis depends on the degradation of material properties at the locations where the failure occurs. The general progressive damage algorithm is shown in Fig 18. Initially, a non-linear finite element model is generated of the macro level structure. Under suitable boundary conditions and loading stress analysis is performed. A non-linear analysis is completed when the analysis is converged at the load step. Then, the stresses or strains at each Gauss point of the each element is determined. The stress values of each Gauss point is evaluated according to failure criterion. If a failure is detected, the material properties of that Gauss point are degraded accordingly. A non-linear stress analysis is again performed with the new material properties to establish the equilibrium. If no more failure is detected, the load increased and material degradation is applied again if needed. The process continues until the ultimate failure of the structure.

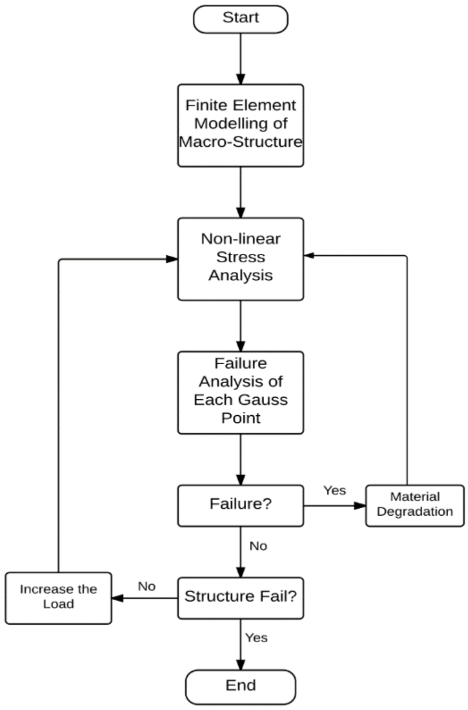

Figure 18 Progressive Damage Algorithm

4.2.6. Failure Criteria and Material Degradation

The progressive damage algorithm includes two critical steps. First one is the definition of the failure and secondly is the material degradation according to failure detected. Generally, the failure criteria can be classified into two categories: independent and interactive failure criteria. Independent failure criteria defines the mode of failure but it doesn't include the stress interactions in the failure mechanism. Interactive failure criteria includes stress interactions however it doesn't give the mode of failure. Most failure criteria can be expressed with a single tensor polynomial failure criterion in two-dimensional form such as:

$$F_1\sigma_1 + F_2\sigma_2 + 2F_{12}\sigma_1\sigma_2 + F_{11}\sigma_1^2 + F_{22}\sigma_2^2 + F_{66}\sigma_6^2 \geq 1 \qquad (9)$$

where σ_1, σ_2 and $\sigma_6(\sigma_{12})$ are the in-plane stresses and F_{ij} terms are the failure indices. Tsai-Hill interactive failure criterion is used in order to determine the failure. The failure indices derived from eq. 9 for Tsai-=Hill failure criterion is:

$$F_1 = 0, \qquad F_2 = 0, \qquad F_{12} = -\frac{1}{2X^2}$$

$$F_{11} = \frac{1}{X^2}, \qquad F_{22} = \frac{1}{Y^2}, \qquad F_{66} = \frac{1}{S_c^2} \qquad (10)$$

where: if $\sigma_1 > 0, \quad X = X_T$, otherwise $X = X_C$. If $\sigma_2 > 0, \quad Y = Y_T$, otherwise $Y = Y_C$. Since the interactive failure criterion doesn't give the mode of failure, the expressions in Eq.11 are used to determine the failure mode:

$$\begin{aligned} &H_1 = F_1\sigma_1 + F_{11}\sigma_1^2 \qquad H_2 = F_2\sigma_2 + F_{22}\sigma_2^2 \\ &H_6 = F_{66}\sigma_6^2 \end{aligned} \tag{11}$$

The largest H_i terms identifies the dominant failure mode and corresponding modulus reduced to zero. Here, H_1 refers to fibre failure, H_2 refers to matrix failure and H_6 corresponds to fibre-matrix shear failure. Another failure criterion applied in this study is proposed by Chang and Lessard [57] which defines different failure modes with different expressions. Matrix tensile failure is a result of combined transverse tensile stress and shear stress. When the failure index e_m exceeds 1, failure is assumed to occur. The failure index is like this without non-linear material behaviour:

$$e_m^2 = (\frac{\sigma_y}{Y_T})^2 + (\frac{\sigma_{xy}}{S_C})^2 \tag{12}$$

With non-linear shear behaviour taken into consideration, the failure index takes the form:

$$e_m^2 = (\frac{\sigma_y}{Y_T})^2 + \frac{2\sigma_{xy}{}^2 / G_{xy} + 3\sigma_{xy}{}^4}{2S_C^2 / G_{xy} + 3S_C^4} \tag{13}$$

The same failure index is used since the previous two failure mechanisms cannot occur simultaneously at the same point. Fibre-matrix shearing failure results from a combination of fibre compression and matrix shearing. The failure criterion has essentially the same form as the other two criteria:

$$e_{fs}^2 = (\frac{\sigma_x}{X_C})^2 + \frac{2\sigma_{xy}{}^2 / G_{xy} + 3\sigma_{xy}{}^4}{2S_C^2 / G_{xy} + 3S_C^4} \tag{14}$$

The fibre failure occurs when compressive stress in the fibre direction exceeds fibre buckling strength or when tensile stress exceeds the fibre breakage strength.

$$e = \frac{\sigma_x}{X_C} \tag{15}$$

In order to degrade material properties upon failure stiffness reduction method is applied which is based on work of Chang and Chang [56]. Padhi et al [53] proposed an effective material degradation based on experimental observations and the work of Chang and Chang. For matrix cracking at a Gauss integration point, transverse modulus and Poisson ratio are reduced to zero. However, the longitudinal modulus and the shear modulus remain unchanged. When fibre-matrix shear failure occurs, the transverse modulus and Poisson ratio are reduced to zero while the longitudinal modulus and transverse modulus remain unchanged. When fibre failure is detected, the material properties are reduced to zero

4.2.7. Verification of Progressive Damage Modelling

Progressive damage methodology is generated as stated in Padhi et al [53]. The experimental procedure and results can be found in Moy et al [155]. A number of composite plates are tested under pressure loading in order to determine the effects of material properties, manufacturing method aspect ratio of the panel. The selected panel`s material properties and laminated plate specification and material properties are presented in Table 6 and 7, respectively.

Table 6 Laminated plate specifications

Plate **[155]**	Lay-up	No of plies	Length (mm)	Width (mm)	Thickness (mm)
12-A	[0/45/90/-45/0]	5	600	600	3.43

Table 7 Mechanical properties of glass/polyester unidirectional lamina [52]

Moduli Parameters	
Longitudinal modulus $E_x (GPa)$	23.6
Transverse modulus $E_y (GPa)$	10.0
Shear modulus $G_{xy} (GPa)$	1.0
Poisson's ratio ν_{xy}	0.23
Strength Parameters	
Longitudinal tension X_T (MPa)	735
Longitudinal compression X_C (MPa)	600
Transverse tension Y_T (MPa)	45
Transverse compression Y_C (MPa)	100
In-plane shear S_C (MPa)	45

The composite plate is produced by E-glass/polyester material by using the hand lay-up method. The fibre weight fraction is determined as 0.415. Boundary conditions are performed by a heavy steel frame which is clamped together with the plate between them. The frame gives in-plane and rotational restraint to the edges of the panel according to Moy et al [155]. Therefore fixed boundary conditions are applied to all edges of the panel.

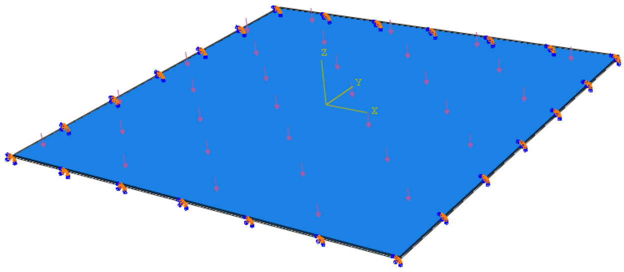

Figure 19 Finite element model of plate

The finite element model of the plate is showed in Fig. 19. A 20x20 (400 elements) are used to achieve the mesh convergence. A user subroutine USDFLD written in Fortran in order to apply progressive damage modelling. This algorithm allows the user to define material properties as functions of the field variables at an integration point. Three field variable are introduced to cover the three failure mechanisms. The first field variable is the contribution of fibre failure (H_1), the second and third field variables are the contributions of matrix (H_2) and fibre-matrix shear failure (H_6), respectively. The material degradation and field variables are shown in Table 8.

Table 8 Field variables with material degradation

Material Properties				*Field Variables*		
E_x	E_y	ν_{xy}	G_{xy}	FV1	FV2	FV3
-	-	-	-	0	0	0
0	0	0	0	1	0	0
-	0	0	-	0	1	0
-	-	0	0	0	0	1
0	0	0	0	1	1	0
0	0	0	0	1	0	1
-	0	0	0	0	1	1
0	0	0	0	1	1	1

4.2.8. Results and Discussion

A laminated composite plate under lateral uniform pressure loading is investigated. The central deflection values of the plate compared for different modelling techniques. Firstly, the plate is modelled without damage but geometrically non-linear. The deflection reached 34.16 mm at maximum loading. This deflection value shows good agreement with [53]. Thye experimental results are compared with progressive damage modelling with Tsai-Hill and Chang-Lessard failure criteria. Maximum deflection at the centre location of the plate at the end of the loading is same for the experimental and numerical results. The first failure is detected as matrix failure at 0.0219 MPa.

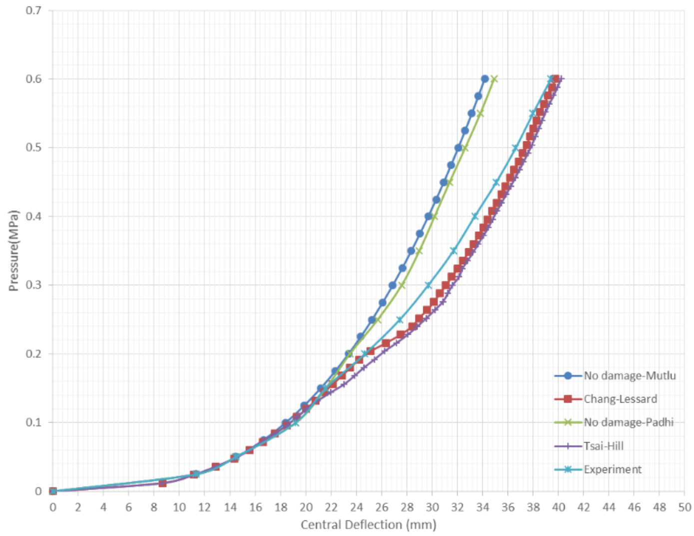

Figure 20 Load deflection comparison

The results, as shown in Fig 20, indicate that progressive damage modelling technique with specified failure criteria provide accurate results for damage modelling. It is demonstrated that implementing of damage modelling gives more accurate results compared to materially linear models. However, it is noticeable that the selection of failure criteria is critical. There is some differences between the predictions of the failure criteria. More detailed study is required on the selection of failure criteria employed.

4.3. Summary

In this chapter, firstly, a top hat stiffened composite stiffened plate modelled and analysed under uniform pressure loading. The structure is modelled as a representative of a marine structure. A geometrically non-linear model is applied. The boundary conditions and the connection between different parts of structure is modelled as close as possible to the experimental conditions. The results have good agreement with experiments. Some differences between finite element and experiments are observed which are interpreted as the result of the boundary conditions and manufacturing. The finite element modelling showed that; the modelling of connection of stiffeners with each other and with plate plays an important role on the behaviour of the structure. The replication of boundary conditions are as close as possible to the experiments or real conditions. This macro level analysis is performed using material properties which are derived from large number of coupon tests under tension and bending. It is the one of the main drawbacks of direct macro level analysis. In order to define material properties for any change at the material, experiments should be applied again which is time consuming and expensive. Micro level modelling is an effective way to determine the fibre matrix mechanisms and material properties. It will allow better predictions of macro level behaviour without need of high number of experiments. Another suggestion coming out from these results is the necessity of damage modelling. Therefore progressive damage modelling is presented. A laminated composite plate is examined under pressure loading including damage. The progressive failure is carried out by degradation of material properties of failed integration points of structure. The damage modelling agreed well experimental and numerical results in the literature.

This study also showed the requirement of faster analysis methodologies. Although finite element modelling gives accurate results for macro level behaviour, the computational times for large structures are high. Therefore, application of direct finite element codes to the reliability or optimisation analyses of composite structures is not practical since these analyses requires several calls to the models. Therefore, surrogate modelling of macro level structures are investigated and present in Appendix C for the application of composite stiffened plates.

5. Micro Level Modelling

5.1. Introduction

Composite structures include several level of materials which each of them has different amount of effect on structural response. Micro level mechanisms has one of the greatest contribution to the macro level responses. Therefore it is essential to model the micro level of composite materials to explain the macroscopic behaviour. Representative volume element (RVE) approach is an effective way to model the micro level mechanisms to model the macro level behaviour and effective properties. In this chapter, RVE based micro level modelling of composite material is carried out in three steps. First, RVE is identified in composite materials. Finite element modelling of RVE is performed by using Abaqus CAE. Secondly, periodic boundary conditions are investigated in detail since the boundary conditions applied to the finite element model is crucial to be able to model the microstructure as a representative of macro. Then homogenization process is conducted by using micro level finite element analysis results. Finally, the RVE model is verified with literature.

5.2. Modelling of RVE

Heterogeneous composite materials represent periodicity at the meso-level and can be modelled using RVEs. Determination of material properties of unidirectional composites by using RVEs reduce the computational time to investigate the macro level behaviour. A square packed RVE model is used to model the microstructure of the composite which provides easy application of boundary conditions while establishing adequate microstructural modelling [164].

The physical and geometrical properties of the microstructure are described by the RVE model which has two distinct materials: fibre and matrix. The matrix material is modelled as isotropic homogeneous material where the fibre is can be modelled as orthotropic or isotropic material depends on the type of the fibre. The RVE is one of the representatives of the macro structure where it has perfect boundaries with other RVEs. The RVE should be large enough to represent the whole structure and small enough to avoid computational expense [9]. Therefore, the dimensions at each edge of the RVE are chosen as 1mm which provides enough information about the microstructure [25] and is sufficiently smaller than the macro level structure [121]. The diameter of the fibre is varied to represent the volume fraction of the composite.

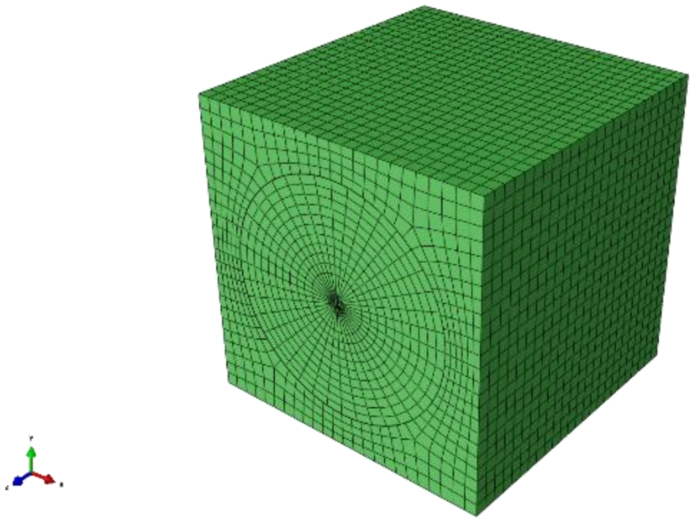

Figure 21 Finite Element RVE Model

20 nodded brick elements (C3D20R) and 8 nodded brick elements (C3D8R) are used to mesh the model. The element distribution on opposite surfaces of the RVE are a mirror image to be able to apply the periodic boundary conditions. A mesh convergence study is applied to the RVE showing that mesh sizes with elements of 0.04 mm and smaller converged. The RVE is modelled geometrically non-linear whereas material non-linearity is not included. Therefore, the interface between the matrix and fibre is modelled as perfect bonding by using tied constraints.

5.3. Boundary conditions

One of the important steps of RVE modelling id the application of boundary conditions. The boundary conditions applied have direct effect on the macro level material response. The RVE should be treated as a dependent element which is within the macro level structure, not as an independent element. This develops same response for RVE to the macro level structure under same loading condition. The selection of proper boundary conditions produce the realistic behaviour of the RVE which is in the middle of the macro structure. Fig-22 shows an RVE without any constraining boundary condition under axial tension loading. Because of the higher elasticity modulus of the fibre, the bottom and top surface of the RVE is expanded. This behaviour is not an expected response of an RVE in a periodic structure. Hence, application of boundary conditions are required on the RVE to obtain appropriate response. There are different applications of boundary conditions to apply RVE such as kinematic uniform boundary conditions, static uniform boundary conditions, periodic boundary conditions and homogeneous boundary conditions. There are different studies applied different boundary condition on RVEs such as displacement boundary condition [165] which is applied solely to model the response of RVE. This approach applies displacement field to the surface in the form of Eq. 16.

$$U_i\big|_s = \bar{\varepsilon}_{ij} x_j \qquad (16)$$

where s denotes the surface and x_j is the length of the RVE in one direction. It is assumed that the effective strain $\bar{\varepsilon}_{ij}$ is equal to the applied macroscopic strain ε_{ij}^0 by considering perfect bonding between fibre and matrix. This boundary condition causes flat surface to remain flat after loading. However, the surface of the RVE cannot stay flat after deformation because of the heterogeneous structure. This assumption results mostly higher stress values within the RVE since the surfaces are forced to remain flat. In order to achieve periodicity conditions, not only displacement boundary condition but also traction boundary condition should be employed. Application of these two boundary conditions ensure the periodicity of the structure and give better approximations in comparison to other boundary conditions on periodic microstructures [123, 166].

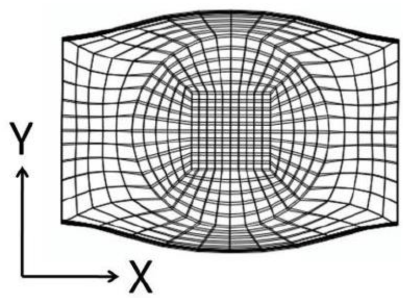

Figure 22 RVE without any constrain

The application of PBCs on the RVE is given by Suquet [167]. The displacement field for the periodic structure, consisting of a periodic array of representative unit cells, can be defined in eq.1 as:

$$u_i(x_1, x_2, x_3) = \varepsilon_{ij}^0 x_j + u_i^*(x_1, x_2, x_3) \qquad (17)$$

Here ε_{ij}^0 is the macroscopic strain tensor of the periodic structure, $\varepsilon_{ij}^0 x_j$ represents a linear displacement field and $u_i^*(x_1, x_2, x_3)$ represents the periodic function between RVEs [168]. Two continuities must be satisfied at the boundaries of the RVE which are the displacement continuity, to ensure the RVE surfaces cannot be separated or penetrated, and the traction equity at the opposite faces of the RVE, which ensures the periodicity. These two conditions mean, the RVE is representative of a continuous physical body. Although eq.1 satisfies the displacement continuity, it cannot be directly applied to the surfaces of the RVE as the periodic function is unknown. Therefore, the displacements for any nodes on the combined parallel surfaces are written as Eqs. 18 and 19:

$$u_i^{k+} = \varepsilon_{ij}^0 x_j^{k+} + u_i^*, \qquad (18)$$

$$u_i^{k-} = \varepsilon_{ij}^0 x_j^{k-} + u_i^*. \qquad (19)$$

The indices $k+$ and $k-$ represents the k^{th} pair of two nodes on opposite parallel surfaces of the RVE; these surfaces are paired between the top and bottom, left and right, and front and rear. The periodic function $u_i^*(x_1, x_2, x_3)$ is equal at the two opposite surfaces of the RVE, therefore the difference between the Eq. 18 and Eq. 19 becomes:

$$u_i^{k+} - u_i^{k-} = \varepsilon_{ij}^0 (x_j^{k+} - x_j^{k-}) = \varepsilon_{ij}^0 \Delta x_j^{k}. \qquad (20)$$

A schematic representation of periodicity of undeformed (a) and deformed (b) RVEs under periodic boundary conditions in 2-D representation with periodic function, u^*, from one RVE to another.

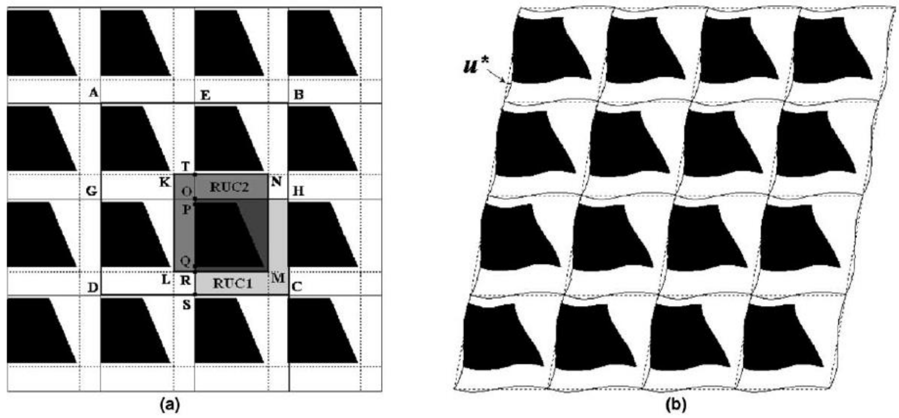

Figure 23 Periodicity in 2-D

The pair of parallel surfaces of the RVE has constant $\Delta x_j^{\ k}$ with a prescribed ε_{ij}^0. In this manner, the boundary condition can be applied to the finite element analysis as a nodal displacement constraint equation. Eq. 4 is a special type of boundary condition which establishes displacement-differences between two combined surfaces instead of applying prescribed boundary displacements. This equation satisfies the displacement continuity at the surfaces of the boundary conditions. The other condition that needs to be satisfied is the traction continuity which can be written as Eqs. 21 and 22:

$$\sigma_n^{k+} - \sigma_n^{k-} = 0\,, \qquad (21)$$

$$\tau_{nt}^{k+} - \tau_{nt}^{k-} = 0\,. \qquad (22)$$

However, Xia et al. [168] proved that once the displacement continuity, eq. 20, is satisfied, the traction boundary condition is automatically satisfied.

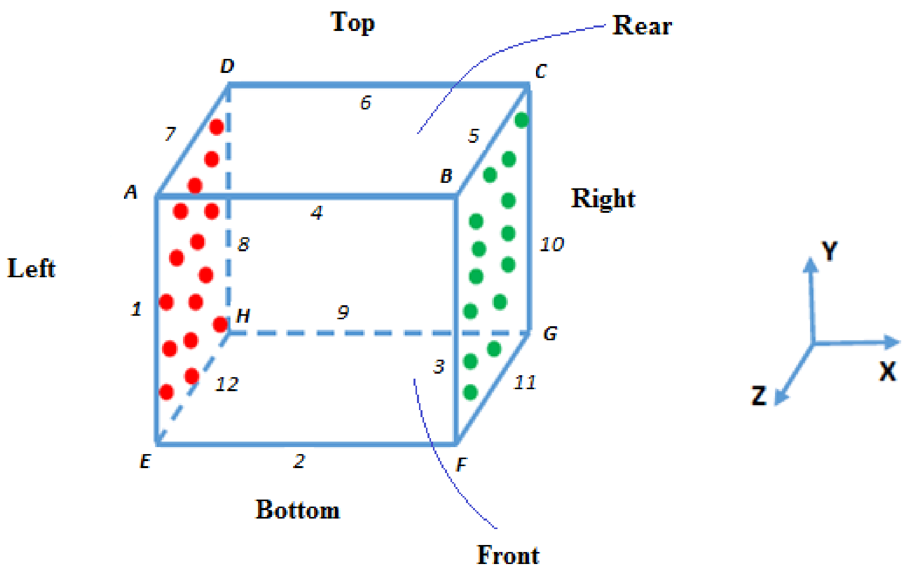

Figure 24 PBC Application

An automated Python [154] code is written for the construction of the RVE and the periodic boundary conditions are applied through linear constraints in Abaqus. A linear multi point constraint is used between combined pairs of surfaces of the RVE to set the system of equations. A linear combination of nodal variables, such as displacements, is equal to zero, which are shown in eq. 23 [169]:

$$A_1 u_i^P + A_2 u_i^Q + \ldots + A_N u_k^R = 0\,, \qquad (23)$$

where R is the node, k is the degree of freedom, and A_N is a constant coefficient to define the relative motion of nodes. Eq. 23 is an interpretation of Eq. 20 to apply the periodic boundary condition in Abaqus. An abstract 'dummy node' concept is introduced to be able to apply the periodic boundary condition in Abaqus which is showed in eq. 24:

$$u_i^{k+} - u_i^{k-} + u_i^{dummy} = 0\,. \qquad (24)$$

By using the dummy node concept, periodic boundary condition is applied to the opposite faces, edges and corner points of the RVE. Firstly, the nodes on each faces of the RVE should be combined with the node at the opposite side. The degrees of freedom of each node will be combined with the corresponding node at another face. The definition of the faces of RVEs showed in Fig. 24. Top and Bottom surfaces, Right and Left surfaces, and Rear and Front surfaces are paired together. The nodes on the surfaces excluding the ones on edges and corners are considered. The formulation for the Left and Right faces is represented in Eq. 25:

$$\begin{aligned} & u_1^{Left} - u_1^{Right} - u_1^{dummy} = 0 \\ & u_2^{Left} - u_2^{Right} = 0 \\ & u_3^{Left} - u_3^{Right} = 0 \end{aligned} \tag{25}$$

Where u_i represents the degree of freedom for the each nodes on the surfaces. The applied far-field strains are employed by the help of dummy nodes, u_i^{dummy}. The formulation for the other faces represented in the Eqs. 26 and 27.

$$u_2^{Top} - u_2^{Bottom} - u_2^{dummy} = 0 \tag{26}$$

$$u_3^{Front} - u_3^{\mathrm{Re}ar} - u_3^{dummy} = 0 \tag{27}$$

Since the nodes on the edges of the RVE are overlapped, the formulation should be established separately in order to avoid over constrain of the nodes. The edges of the RVE are numbered as in Fig. 24 from 1 to 12. The nodes along one edge combines with the ones on parallel edge. The formulation of edges number 10 and number 3 is shown as:

$$\begin{aligned} & u_1^{edge10} - u_1^{edge3} = 0 \\ & u_2^{edge10} - u_2^{edge3} = 0 \\ & u_3^{edge10} - u_3^{edge3} + u_3^{dummy} = 0 \end{aligned} \tag{28}$$

The nodes on the rest of the edges are combined as shown below.

$$\begin{aligned} & u_1^{edge3} - u_1^{edge1} + u_1^{dummy} = 0 \\ & u_3^{edge1} - u_3^{edge8} + u_3^{dummy} = 0 \\ & u_3^{edge6} - u_3^{edge4} + u_3^{dummy} = 0 \\ & u_2^{edge4} - u_2^{edge2} + u_2^{dummy} = 0 \\ & u_3^{edge2} - u_3^{edge9} + u_3^{dummy} = 0 \\ & u_1^{edge7} - u_1^{edge5} + u_1^{dummy} = 0 \\ & u_2^{edge5} - u_2^{edge11} + u_2^{dummy} = 0 \\ & u_1^{edge11} - u_1^{edge12} + u_1^{dummy} = 0 \end{aligned} \tag{29}$$

As mentioned earlier, not to over constrain the each degree of freedom of the nodes on the corners, the formulation should be as follow according to notation shown in Fig.24:

$$
\begin{aligned}
&u_3^{cornerB} - u_3^{cornerC} - u_3^{dummy} = 0 \\
&u_2^{cornerC} - u_2^{cornerG} + u_2^{dummy} = 0 \\
&u_3^{cornerG} - u_3^{cornerF} + u_3^{dummy} = 0 \\
&u_3^{cornerA} - u_3^{cornerD} - u_3^{dummy} = 0 \\
&u_2^{cornerD} - u_2^{cornerH} + u_2^{dummy} = 0 \\
&u_3^{cornerH} - u_3^{cornerE} + u_3^{dummy} = 0 \\
&u_1^{cornerE} - u_1^{cornerF} + u_1^{dummy} = 0
\end{aligned} \qquad (30)
$$

5.4. Homogenization

Not only composite materials but also all other materials used in engineering applications are heterogeneous when the material scale is small enough. For instance, metallic materials are heterogeneous at molecular level while composites are heterogeneous at micro level. All the material properties are homogenized at one of the levels to investigate the behaviour and each homogenization step means changing the level of materials from lower to higher.

Mean field theory is employed in this study which represents the larger scale with volume average of corresponding lower scales. Some of the analytical methods mentioned earlier also applies average field theory with different homogenization approaches. However the averaging with analytical methods is not flexible since these methods are only valid under specific conditions. Therefore, implementing parametric studies, modelling complex geometric effects and nonlinear behaviours are challenging whereas numerical homogenization methods provides better solutions with the growing computational power.

The homogenization process based on the following general assumption: material properties of heterogeneous composite material can be represented by the averaged properties of an equivalent homogeneous one. Following the finite element analysis of the RVE, homogenization process is applied by using the Hill approach [120] which employs volume integration over the RVE. Averaged-macro stress and strain are derived by averaging across each node to determine the homogenous medium by using eqs. 31 and 32.

$$\overline{\sigma_{ij}} = \frac{1}{V} \int_V \sigma_{ij}(x, y, z) dV \qquad (31)$$

$$\overline{\varepsilon_{ij}} = \frac{1}{V} \int_V \varepsilon_{ij}(x, y, z) dV \qquad (32)$$

Where $\overline{\sigma_{ij}}$ and $\overline{\varepsilon_{ij}}$ denote average stress and V indicates the RVE volume. Since RVE composed of several finite elements, Gaussian quadrature integration is applied for each element to calculate the averaged values:

$$\overline{\sigma_{ij}} = \frac{1}{V}\sum_{i=1}^{NE}\int_V \sigma_{ij}(x,y,z)dV = \frac{1}{V}\sum_{e=1}^{ele}\sum_{k=1}^{int}\left(\sigma_{ij}\right)_k V_e \qquad (33)$$

$$\overline{\varepsilon_{ij}} = \frac{1}{V}\sum_{i=1}^{NE}\int_V \varepsilon_{ij}(x,y,z)dV = \frac{1}{V}\sum_{e=1}^{ele}\sum_{k=1}^{int}\left(\varepsilon_{ij}\right)_k V_e \qquad (34)$$

The stress and strain values at 8 integration points of each finite element in RVE is calculated by using Abaqus and Python. Later the averaged stress and strain values are determined using eq. VV and VV. The periodic unidirectional composite structure is assumed transversely isotropic as the fibre direction is considered axis of symmetry. In this case, any plane containing the fibre direction, is the plane of symmetry. A transversely isotrpic material has five constants and when the axis of symmetry is the fibre direction (z-direction) the Hooke's law reduces to:

$$\begin{Bmatrix} \overline{\sigma_{11}} \\ \overline{\sigma_{22}} \\ \overline{\sigma_{33}} \\ \overline{\sigma_{12}} \\ \overline{\sigma_{13}} \\ \overline{\sigma_{23}} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{12} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{23} & 0 & 0 & 0 \\ C_{12} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11}-C_{12}) \end{bmatrix} \begin{Bmatrix} \overline{\varepsilon_{11}} \\ \overline{\varepsilon_{22}} \\ \overline{\varepsilon_{33}} \\ \overline{\gamma_{12}} \\ \overline{\gamma_{13}} \\ \overline{\gamma_{23}} \end{Bmatrix} \qquad (35)$$

Once the components of the stiffness tensor, C_{ij}, are known, elastic properties of the homogenized composite materials can be calculated. In order to obtain the values of stiffness matrix, the RVE is subjected to proper strains. Six separate strains are applied in order to evaluate the stiffness tensor where each analyses determines one column of the tensor. For the homogenized composite material, the relationship between average stress and strain is:

$$\overline{\sigma_{\alpha}} = C_{\alpha\beta}\overline{\varepsilon_{\beta}} \qquad (36)$$

Therefore, the components of the stiffness tensor are evaluated by solving six elastic models of the RVE under periodic boundary conditions. Only one strain component is different from zero for each of the six models. By applying a unit value of applied strain under periodic boundary conditions, it is a trivial matter to calculate the corresponding component of stiffness matrix. For instance, the first column of the stiffness matrix is computed by the applied strain in the fibre direction while the rest of the strains remain zero:

$$\overline{\varepsilon_{11}} = 1 \qquad \overline{\varepsilon_{22}} = \overline{\varepsilon_{33}} = \overline{\gamma_{12}} = \overline{\gamma_{13}} = \overline{\gamma_{23}} = 0 \tag{37}$$

The strains are applied in a sequence as shown above while remain of the strains are kept zero. Thus, the stiffness matrix and effective material properties Longitudinal and transverse Young's moduli, longitudinal and transverse shear moduli, Poisson's ratios are computed.

5.5. RVE Verification

Verification of the RVE model is performed replicating Sun and Vaidya [170] for two different materials, Boron/Aluminium and Graphite/Epoxy where the material properties are shown in Table 9 and Table 10 respectively. The RVE model's estimations are compared with analytical solutions and available experimental results. Sun and Vaidya [170] applied finite element analysis to the RVE in order to obtain material properties of unidirectional composites. Sun and Chen [171] and Chamis [172] estimations depends on mechanics approach with displacement continuity and force equilibrium conditions. Whitney and Riley [173] employed the classical elasticity theory with the use of energy balance. Hashin and Rosen [174] approach is based on energy variational principles and provided lower and upper bounds for the material properties. The experimental results are taken from Kenaga et al. [175], Daniel and Lee [176] and Sun and Zhou [177]. All the results including the estimations of current study are shown in Table 11 and 12 for Boron Aluminium and AS4/3501-6 graphite epoxy, respectively. The RVE is created with the diameter of the fibre is calculated to ensure a volume fraction of 0.47 Boron and 0.6 AS4 for two cases.

Table 9 Boron/Aluminium Material Properties

	E(GPa)	ν
Boron	379.3	0.1
Aluminium	68.3	0.3

Table 10 Graphite/Epoxy Material Properties

	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	ν_{12}	ν_{23}
AS4	235	14	28	0.2	0.25
3501-6	4.8	4.8	1.8	0.34	0.34

The RVE stress states for each of the loading conditions are presented in Fig. 25 and 26, for boron/aluminium composite and graphite/epoxy composite, respectively.

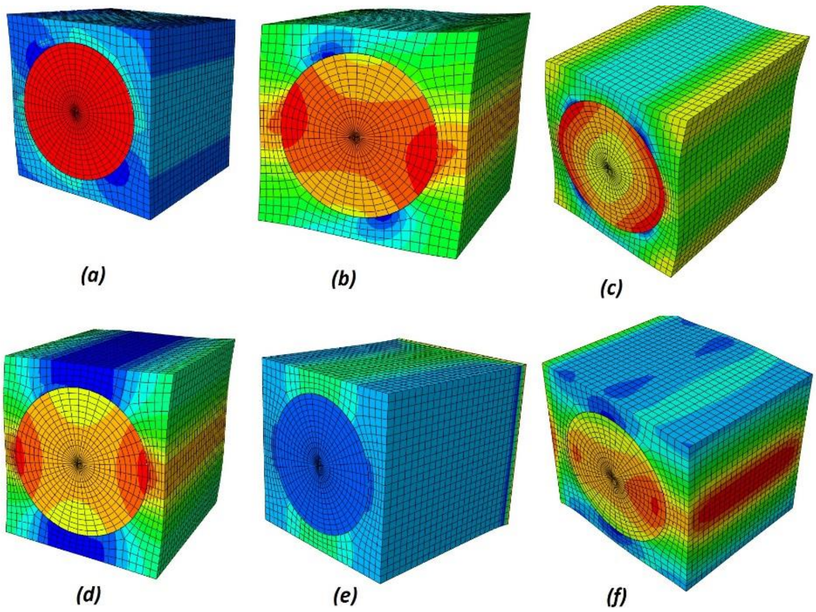

Figure 25 RVEs for Boron/Aluminium composite (a) Long. Young's Mod. (b) Trans. Young's Mod. (c) Trans. Shear Mod. (d)Long. Shear Mod. (e) Poisson Rat. 12 (f) Poisson Rat. 23

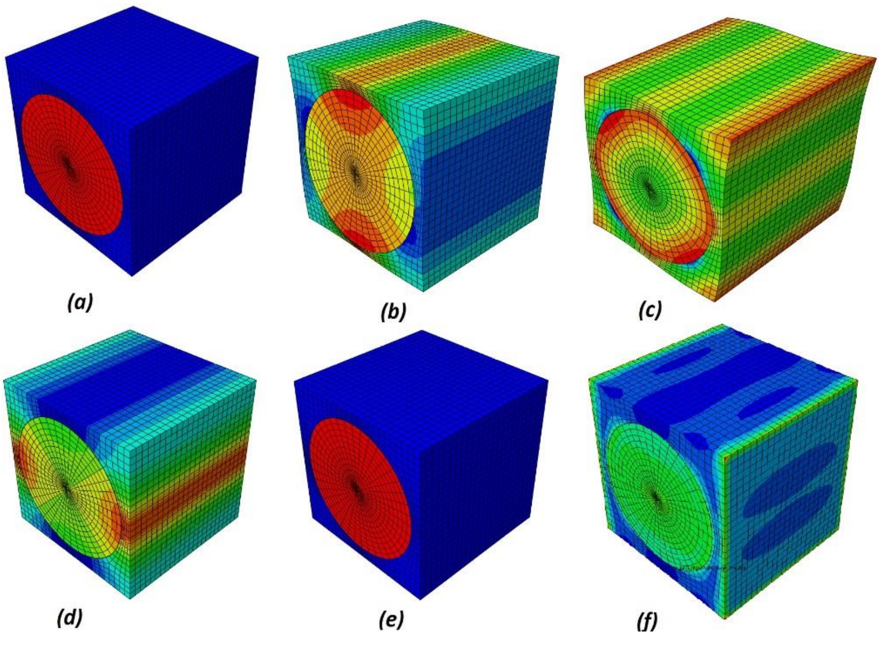

Figure 26 RVEs for Graphite/Epoxy composite (a) Long. Young's Mod. (b) Trans. Young's Mod. (c) Trans. Shear Mod. (d)Long. Shear Mod. (e) Poisson Rat. 12 (f) Poisson Rat. 23

The material properties are compared with numerical, experimental and analytical results from literature. In general, the RVE model provides good agreement with experimental and other numerical methods. RVE model gives consistent data when compared to analytical results.

Table 11 Material Properties of boron/aluminium composite

Elastic Constants (GPa)	RVE Model	FE Model [170]	Mechanics Approach [171]	Strain Energy Approach [172]	Energy Balance Approach [173]	Energy Variation Principle [174]	Experiment [175]
E_1	214.2	215	214	214	215	215	216
E_2	144	144	135	156	123	139.1-131.4	140
G_{12}	55	57.2	51.1	62.6	53.9	53.9	52
G_{23}	46.2	45.9	-	43.6	-	54.6-50.0	-
ν_{12}	0.20	0.19	0.19	0.20	0.19	0.195	0.29
ν_{23}	0.28	0.29	-	0.31	-	0.31-0.28	-

Table 12 Material Properties of Graphite/Epoxy composite

Elastic Constants (GPa)	RVE Model	FE Model [170]	Mechanics Approach [171]	Strain Energy Approach [172]	Energy Balance Approach [173]	Energy Variation Principle [174]	Experiment [176]	Experiment [177]
E_1	143.2	142.6	142.9	142.9	142.9	142.9	142	139
E_2	10	9.6	9.2	9.79	9.78	9.40-9.10	10.3	9.85
G_{12}	5.84	6.0	5.5	6.53	5.8	5.8	7.6	5.25
G_{23}	2.93	3.1	-	3.01	-	3.42-3.26	3.8	-
ν_{12}	0.24	0.25	0.26	0.26	0.25	0.25	-	0.3
ν_{23}	0.31	0.35	-	0.42	-	0.39-0.37	-	-

5.6. Summary

In this chapter, micro level modelling of composite structures is studied with finite element RVE modelling. Abaqus CAE general purpose finite element programme is used to model the RVEs and periodic boundary conditions are applied in order to perform periodicity of microstructures. The homogenized material properties are compared with the available data in literature and observed good agreement. The positive results on determination of material properties is encoring to continue to damage characterisation of micro structures. RVE model will be improved in order to determine the damage mechanisms at micro scale and provide material degradation models.

6. Multiscale Reliability Analysis: Sensitivity of structural integrity to material and topological variations

6.1. Introduction

The behaviour of composite structures depend on a number of parameters such as the geometry, material type, lay-up and failure modes which vary based on the initial manufacture. Besides the complex nature of the material, operating in the marine environment provides additional complexities and uncertainties. Therefore structural behaviour of composites needs to be studied in more detail to incorporate these variabilities to increase the realibility. Sriramula and Chryssanthopoulos [178] quantified the variability in composite materials and reviewed different stochastic methods suggested by researchers. They classified the uncertainties starting at constituent (micro) level to ply (meso) level and component (macro) level. They concluded that multi scale modelling approaches are promising for stochastic analysis of composite structures. This is because the uncertainties at the micro level propagate to higher scales and cause significant effects on macro scale stiffness and strength properties. Thus, accounting for these micro level properties should increase the understanding of macro level behaviours of composites structures.

Despite the number of applications, the production of composite materials and structures still involves many uncertainties regardless of the industry in which they are employed. This can lead to more conservative designs, higher costs and a negative environmental impact as the advantages of the composites can't be fully exploited. Each manufacturing methods in different industries presents sources of uncertainty and initial defects such as resin rich and dry spots, voids, fibre distortions, dimensional inaccuracies, delamination which can be categorised under two different types of variabilities: material properties and topological features [179].For example, hand lay-up process can cause resin rich zones in angled composite parts which cause lower volume fractions and variations at material properties. In RTM process, because of the uneven flow, similar sharp corners contain voids or broken fibres. Liu et al. [180] reported 3.2% void content reduces flexural and tensile strength around 20%. Despite the required high investment, automated methods also include uncertainties. Croft et al. [181] showed automated fibre placement (AFP) method can generate gaps, overlaps, missing or twisted tows within the whole structure that cannot be avoided and these defects cause a reduction of ultimate strength up to 13% at laminate level.

Defects in the material can be difficult to trace as there are a number of different issues that can arise and they are often too small to be easily spotted [178]. Therefore, a robust design process which can quantify the effect of variations on structural integrity is required to prevent unwanted events at composite applications.

Composite materials demonstrate high heterogeneity compared to homogenous counterparts not only because of manufacturing processes but also due to the nature of the material. Behaviour of the composite structures depend on a number of parameters such as constituent properties, lay-up characteristics and failure modes. In addition, the manufacturing processes also cause topological variations in the final products. Potter et al. in [182] and [183] investigates the variabilities in the mass and fibre alignments to understand these uncertainties emphasizing the importance of the design process, which is a key to improve structural performance both in terms of material description and topology. Fernlund [184] investigates the uncertainties in geometric properties of larger and integrated composite structures designed for the aerospace industry and underlines the importance of dimensional accuracy on structural behaviour. Larger structures lead to greater uncertainties and despite the high cost investments to the manufacturing process in the aerospace industry there are still difficulties to produce components within their specifications.

As a consequence of these material and topological variabilities, design principles for different engineering structures define acceptability of components by assessing the reliability of them. Reliability assessment of structures is a good indicator of structural behaviour of composites. There has already been much progress in reliability assessments of composite structures. The effect of material variability on structural response of composites is investigated in different reliability and optimisation studies [9-17] and is continuing to grow. A comprehensive review of available reliability analysis methods is given for different levels of structures such as ply and laminate level by Chiachio et. al. [185]. The final behaviour of the structural components is the accumulation of the effects from various sources taking effect at multiple length scales. These sources of variability have a crucial effect on structural behaviour, and vary between different structures, and should be quantified in the design process to fully exploit the capabilities of composites. Despite the existence of these different uncertainties and mechanisms most of the studies do not account for the multiscale nature of the composite and are usually conducted at one single level, mostly ply or laminate level. Thus, accounting for the constituent properties and the tolerances they have should increase the understanding of behaviour of composite structures.

In order to evaluate the material properties and capture the effects of different length scales a number of virtual experimental approaches have been developed [4] [120] [9] [167]. Kanoute et al. [85] and in a more recent study Matous et al. [186] give a comprehensive review of these approaches. The scope of virtual experiments is to develop combined computational methods that are capable of providing high accuracy analysis within reasonable timeframes by providing a bridge between different length scales to transfer material information. There are some researchers employing virtual experiments to analyse the effect of material variation on the reliability of different composite components. Although these studies provide a useful insight, they are mostly limited by simple geometries such as composite rods [187], single plies and laminates [188] and composite plates [189]. However, these are not representative of the structural components often seen in engineered artefacts, these components tend to be more complex. When the topological complexity increases, the variations and the impact of these variations becomes more significant.

There is a lack of understanding about the sensitivity of different variations on response of composite structures. In the available literature the variation of the material properties is investigated by itself or the impact of topological variations are only considered for simplified geometries. This means that the material and topological variations are not explored together or compared. This paper therefore explores the impact of material characterisation and topological design variations on composite structural components, top-hat stiffened plates are used as an example. Reliability assessment methods are used to incorporate stochastic variations from geometric and material variations. A comparison is made between different methods for predicting the properties at two scales: fibre and ply.

6.2. Reliability Assessment of Structural Components

The structural integrity of the composite stiffened plate is quantified by using a Monte Carlo simulation based reliability analysis. In order to evaluate the impact of material and topological variations simulations are run where only the material properties are varied, only the topological dimensions are varied and a combination of the two. First, the material properties and effects of the constituent properties are calculated by the micro level RVE modelling. Second, the material properties are calculated using the analytical approximation of Halpin-Tsai from constituent properties.

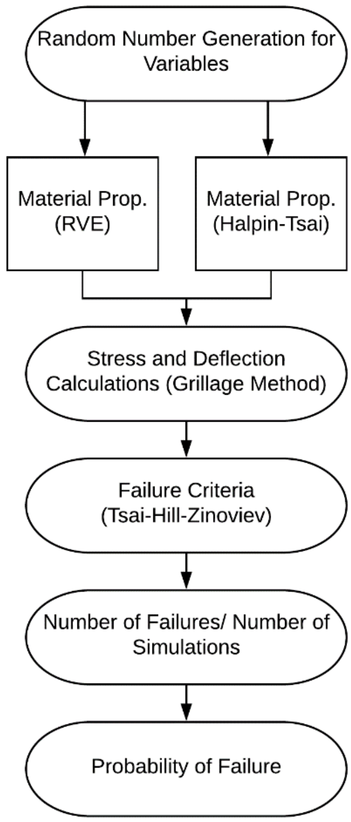

Figure 27. Reliability analysis approach to determine structural integrity

After determination of material properties, a closed-form grillage analysis method based on Navier's energy approach is employed at structural level to calculate stress and deflection values to investigate the effect of topological variations.

6.2.1 Determination of material properties

There are several methods available for the evaluation of material properties. Using analytical techniques is the common way in composite industry. In this study Rule of Mixture (RoM) and Halpin-Tsai (HT) methods are used for the estimation of effective material properties and compared with a Representative Volume Element (RVE) based virtual experimental approach for the material characterisation by modelling the interaction between constituents are employed. The material properties from experiments by Kaddour et al. [190] for the test cases used in the third world-wide failure exercise are used, with comparisons to the laminate properties derived in these experiments.

6.2.2 RVE based laminate properties

A square packed RVE model is utilised to model the microstructure of the composite. A sample RVE created using Abaqus CAE [153] is showed in Fig.28. The RVE represents one fibre which is assumed to have perfect boundaries with other fibres. The fibre is modelled using the dimensions documented from the experimentation and the surrounding matrix is determined in accordance with the volume fraction of the composite. This modelling approach provides enough information about the microstructure [25] and the FEA model is sufficiently smaller than the macro level structure [121] meaning the RVE is the minimum size that is representative of the structure.

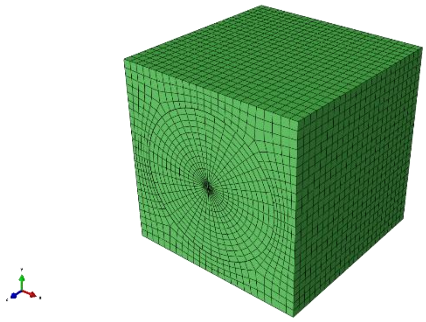

Figure 28. RVE mesh element distribution

20 nodded brick elements (C3D20R) and 8 nodded brick elements (C3D8R) are used to mesh the model. Periodic boundary conditions (PBC) are used as they give better approximations in comparison to kinematic uniform boundary conditions and static uniform boundary conditions on periodic microstructures [123]. The application of PBCs to an RVE is given by Suquet [167] Where the element distribution on opposite surfaces of the RVE are a mirror image of each other. The RVE is modelled geometrically non-linear whereas the material non-linearity is not included. The interface between the matrix and fibre is modelled as perfect bonding by using tied constraints. A mesh convergence study is applied to the RVE showing that it converges when 24 or more elements are used along one edge. An automated Python [154] code is written for the construction of the RVE and the periodic boundary conditions are applied through a linear multi point constraint between combined pairs of surfaces of the RVE to set the system of equations. After the analysis of the RVE, a homogenization process is applied by using the Hill approach [120] which applies volume integration over the RVE. Averaged-macro stress and strain are derived by averaging across each node to determine the homogenous medium.

Material properties comparing an analytical approach and RVE model are presented along with experimental predictions. As expected Halpin-Tsai and Rule of Mixtures has exhibited good predictions for longitudinal Young's modulus.

Table 13 Comparison of evaluated material properties

Material	Method	E_1(GPa)	E_2(GPa)	G_{12}(GPa)	G_{23}(GPa)
Glass-Epoxy(LY556)	Experiment	45.6	16.2	5.83	5.7
	RVE Model	47.4	16.5	4.56	3.24
	RoM	45.74	7.84	2.92	n/a
	HT	45.74	14.47	2.92	n/a
Carbon(G40-800)-Epoxy(3501-6)	Experiment	173	10	6.94	3.35
	RVE Model	175.9	10.9	4.59	2.65
	RoM	175.68	10.27	3.60	n/a
	HT	175.68	21.22	3.60	n/a

Surrogate modelling is performed for the RVE to reduce the computational time and provide a rapid enough evaluation of the material properties for use in the Monte Carlo Simulation. A surrogate model is constructed using Kriging with 60 sample points provided by a Latin Hypercube to construct the surrogate and another 15 test points are used to check the accuracy of the model. Root mean square errors (RMSE) and average prediction errors are used to check the accuracy of the model. Fig 29 shows the accuracy of the predictions made by surrogate model for the calculation of the longitudinal Young modulus which is in good agreement. The accuracy of the surrogate model, the average prediction errors and root mean square errors (RMSEs) are presented in Table 14. The verification studies of the RVE model show that it can be used to implement the material properties into the reliability analysis.

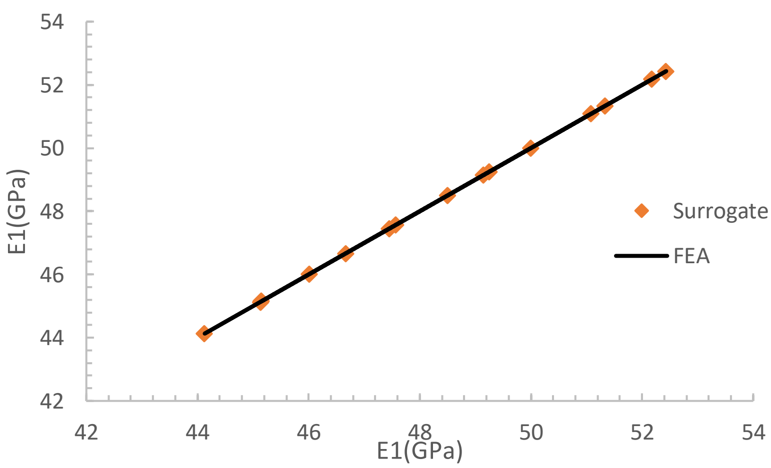

Figure 29 FEA and Surrogate prediction for E1

Table 14 Accuracy of surrogate model

	Avg. Pred. Error (%)	RMSE
E_1	8.78E-06	2.35E-05
E_2	-9.4E-07	2.7E-05
G_{12}	0.000122	2.97E-05

6.3. Structural Analysis

A two by two top-hat stiffened e-glass plate with 0.60 volume fraction is analysed as an example, the dimensions are selected to be those which represent those typically used in marine applications, and is shown in Fig 30 with the dimensions and material properties given in Table 15. A grillage is analysed based on Navier's Energy Method to determine the macro level stresses. Sobey et al. [191] applied this method for the reliability analysis of composite stiffened plates showing it to be suitable for modelling composite grillages. The main steps for calculating the stress are given. Deflection of the stiffened plate is calculated by:

$$w(x,y)=\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}a_{mn}\sin\frac{m\pi x}{L}\sin\frac{n\pi y}{B}\quad, \qquad (5)$$

where m and n wave numbers, L and B, length and breadth of the plate and a_{mn} are coefficients which can be calculated by minimising the potential energy and equating it to the work done, using;

$$a_{mn}=\frac{16PLB}{\pi^6 mn\{m^4(g+1)\frac{D_g}{L^3}+n^4(b+1)\frac{D_b}{B^3}\}}\quad, \qquad (6)$$

where P is the pressure, b and g number of beams and girders, D is the structural rigidity of the stiffeners. The moments can be found from eq. 7:

$$M=-D\frac{\partial^2 w}{\partial x^2}\quad. \qquad (7)$$

Finally the maximum stresses on the stiffeners can be calculated using eq. 8:

$$\sigma_{\max}=\frac{E_{(i)}MZ}{D}\quad, \qquad (8)$$

where $E_{(i)}$ is the Young modulus of the i^{th} element of the stiffener, M is the moment and Z is the vertical distance of the centroid of element to the neutral axis..

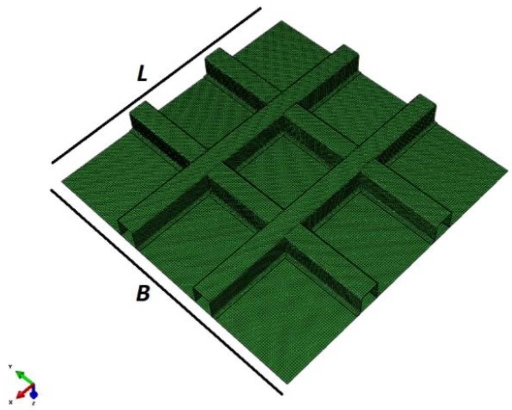

Figure 30 Composite stiffened plate

Table 15 Topological Variables

Variable	Definition	Value (mm)	CoV (%)
L	Length	5000	0-10
B	Breadth	5000	0-10
P	Pressure (kPa)	137	-
a	Crown width	211	0-10
b	Crown thickness	12	0-10
c	Web-Flange thick.	12	0-10
d	Web height	258	0-10
e	Flange width	211	0-10
	Ply thickness	1.2	-
	Number of plies	10	-

Table 16 Material variables

Variable	Definition	Value(mm)	CoV(%)
Ef	Longitudinal modulus of fibre	74	0-20
Gf	In-plane shear modulus of fibre	30.8	0-20
υ_f	Major Poisson's ratio	0.2	-
Em	Elastic modulus of matrix	3.35	0-20
Gm	Elastic shear modulus of matrix	12	0-20
υ_m	Elastic Poisson's ratio	12	0-20

6.4 Analysis Methodology

10^7 simulations are conducted for each study. The required number of simulations are evaluated according to [192] with a confidence level 99.75%. Variations from 0% to 20% are simulated according to literature which mainly depends on manufacturing processes. Mustafa et al. [193] used 20% variation for a wind turbine blade structure similar to top hat stiffened plates and Liu et al. [194] employed 3% material variations for a stiffened plate.

Target Annual Failure Probabilities (DNV GL Rules)

	Failure Consequence		
Material	Low Safety Class	Normal Safety Class	High Safety Class
Composite Material (Brittle Failure Type)	$P_F = 10^{-4}$	$P_F = 10^{-5}$	$P_F = 10^{-6}$

It is also important to understand the context of probability of failures and society's general reaction to hazards. 10^{-3} is unacceptable to everyone. When probability approaches this level, immediate action should be taken to reduce the hazard. When probability of failure reach 10^{-4}, people are willing to spend public money to control hazards at this level. 10^{-5} PoF is rare however people still recognize these hazards and some might accept inconvenience to avoid to avoid similar hazards such as avoiding air travel. 10^{-6} PoF is not of great concern to average person. People are aware of these hazards, but feel "it can never happen to me".

6.5 Sensitivity of Structural Failure to Manufacturing Defects

The impact of different manufacturing defects on structural integrity of composite stiffened plates is investigated. Three subjects; modelling techniques, material variations and topological variations which has an impact on evaluation of structural reliability of composite stiffened plates is explored.

6.5.1 Effect of modelling techniques

RVE models are being utilised to a greater extent for material property evaluation and damage behaviour prediction as reviewed by [186]. However, the effect of using a materially linear RVE on structural reliability evaluation is not well explored. The performance of RVE based reliability prediction of stiffened plates is not compared with other analytical methods. Therefore, in this section, a comparison is presented between surrogate RVE approaches and Halpin-Tsai analytical approximations in which both methods are used to derive material properties. For both approaches up to 10^{6} simulations conducted which is a sufficient number to compare the methods and to avoid high computational times.

The methods are compared using different material variations. The first failure is observed with Halpin-Tsai at 10% material variations where the probability of failure is $2x10^{-6}$. The surrogate RVE approach does not show any failure up to 12% variation. The non-failure region is presented with dashed lines in Figure 31 which shows the probability of failure in this area is lower than 10^{-6}. When the material variation increases to 12%, Halpin-Tsai exhibits $1.97x10^{-4}$ probability of failure and surrogate model shows a probability of $8.2x10^{-5}$. The probability of failure calculated by the Halpin-Tsai is around 2 times higher than the one calculated by surrogate RVE models. This results do not present a significant difference between two methods in terms of reliability predictions. Similar trend is observed for all material variations, for example when material variation becomes 20%, surrogate RVE estimates $4.58x10^{-2}$ probability of failure when Halpin-Tsai prediction is $7.49x10^{-2}$ which is 1.6 times higher. The results indicated that when a materially linear RVE approach is used to evaluate composite properties, the probability of failures don't vary with estimations when analytical Halpin-Tsai method is used. Although RVE model provides good estimations for material properties, using Halpin-Tsai approach is less complex in terms of pre-process and analysis times to determine reliability of composite structures.

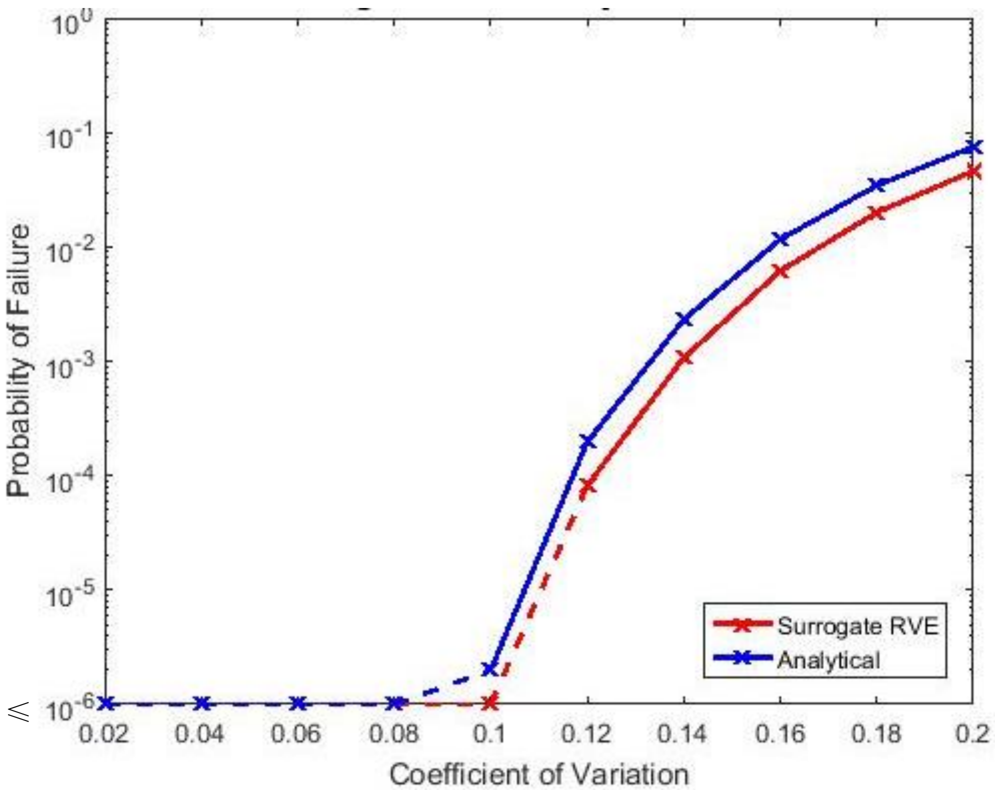

Figure 31 Effect of modelling techniques on structural reliability

6.5.2 Effect of Material Variation

The impact of material variation on the structural integrity is investigated at material and ply level to explore the changes when uncertainties originated from constituent properties and ply properties. The constituent properties and experimentally derived ply properties by [190] are varied from 2% to 20%. When same amount of variation are applied to the constituent properties, the calculated CoV is different than ply level CoV. Monte Carlo simulations, up to 10^7, are conducted to calculate the variations. Two different cases are investigated, employing variations to constituent properties with a constant volume fraction and with a varied volume fraction. It is observed that when a varied volume fraction is employed, the coefficient of variations at ply level increases significantly (Figure 33). For instance, having 20% variation at constituent properties and volume fraction together means 27% CoV at ply level. However, under same conditions, if volume fraction is kept fixed with no variation, the ply level material variation is calculated around 19%. This indicates the importance of volume fraction on CoV calculations. Therefore, it is important to have accurate information how the manufacturing method performs in terms of volume fraction variation. Hand layup is a typical example since it is difficult to control the process and mainly human dependant. This can cause higher variations at volume fractions which leads to unexpected reliability predictions.

For the ply level approach the lowest coefficient of variation at which failure occurs is 8%; for the micro level model this is at 10% variation giving a probability of failure of $2.9x10^{-6}$. At 10% variation the ply level approach shows a significantly higher probability of failure of $1.94\text{x}10^{-3}$, almost 700 times higher. No failures occur in the number of simulations at the lower probabilities of failure, indicating that the probability of failure is likely to be 10^{-7} or lower. The increase in failures with higher variations is not a linear relationship. The gap between the estimations of two approaches closes with higher variations. When the coefficient of variation reaches 20%, the probability of failures are $7.49\text{x}10^{-2}$ for the micro model and $2.59\text{x}10^{-1}$ for the ply model respectively. At the medium manufacturing capabilities it is observed that small improvements to reduce the variations on the material properties can make significant changes to the reliability of stiffened plates. An improvement from 10% variation to 8% gives a significant drop in the probability of failure; from $1.94\text{x}10^{-3}$ to $5.95\text{x}10^{-5}$. There are similar gain for the structural reliability with small improvements for the medium manufacturing capabilities. However, when higher coefficient of variables are investigated an improvement, for example from 16% to 14%, doesn't make an important change at the prediction of probability of failures. The probability of failure drops from $1.01\text{x}10^{-1}$ to $4.57\text{x}10^{-2}$. Similar behaviour is observed for the micro level approach.

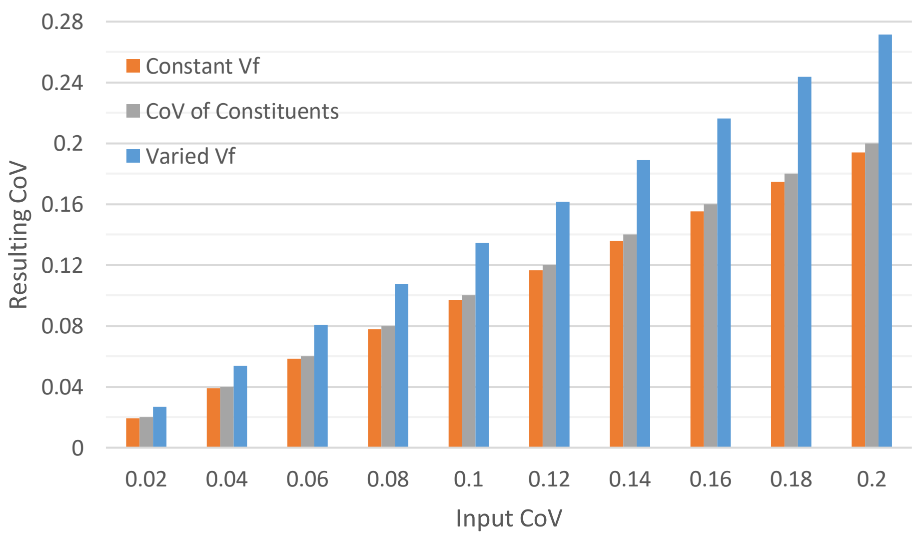

Figure 32 The change at coefficient of variations

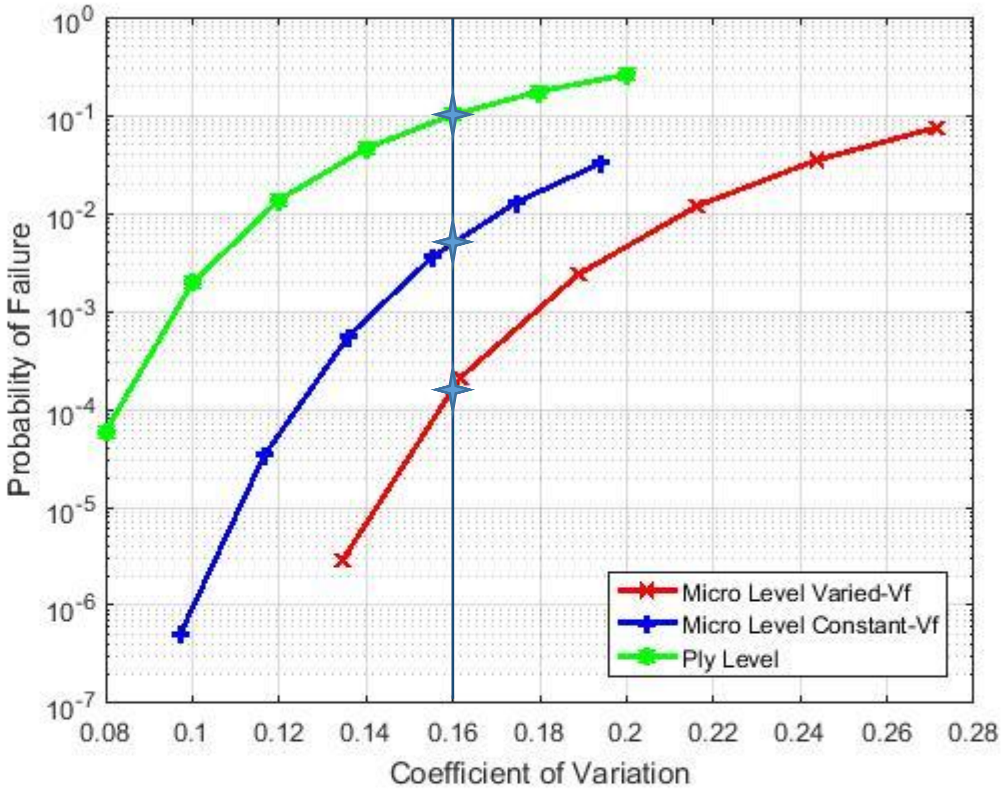

Figure 33 Effect of material variation

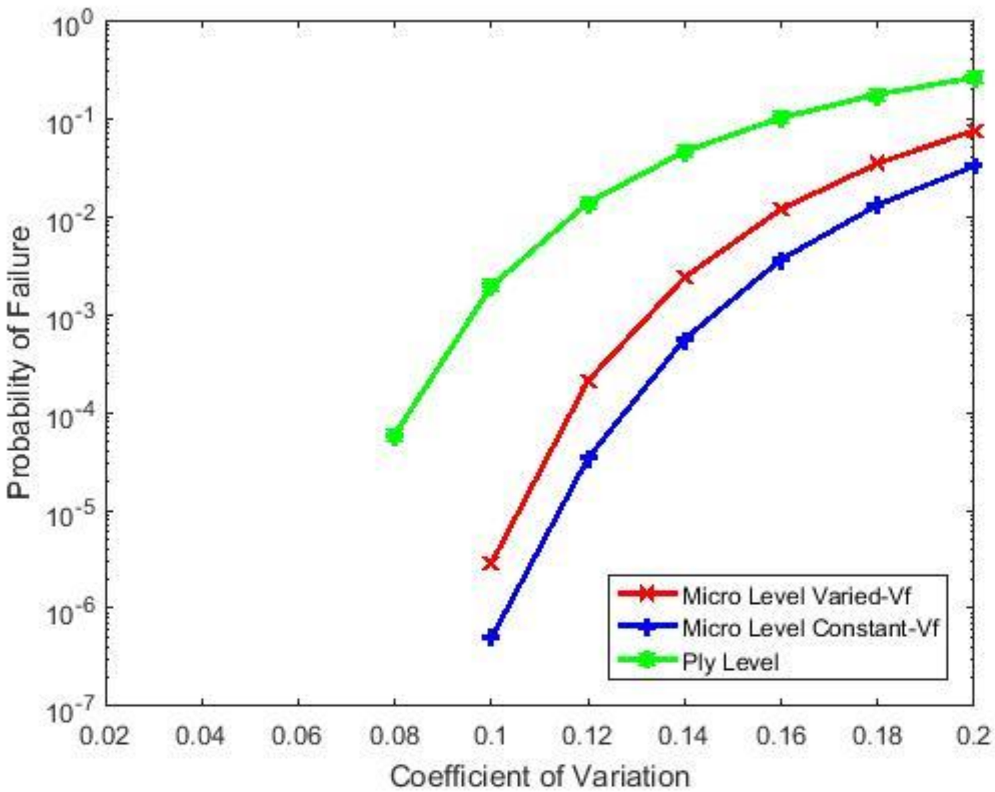

Figure 34 Equivalent CoV

6.5.3 Effect of Topological Variation

Only variations from 1% to 10% are considered for topological properties using 10^7 simulations. Akula [195] applied a 10% variation for topological variations of composite stiffened plate and it can be as low as 0.7% [196] depends on the part size, geometry and manufacturing process. No failures observed are observed at variations below 6% for either the micro level and ply level approach. The first probabilities of failure observed are 8.0×10^{-7} for the micro-level model and 3.7×10^{-6} for the ply level approach. The difference between the two modelling methods is smaller compared to the impact of material variation. When the coefficient of variation becomes 10% the probability of failure reaches to 2.28×10^{-3} for the micro-level model and 3.96×10^{-3} for the ply level approach. When only topological variations are considered the improvements in low capability and medium capability manufacturing techniques does not make a considerable difference, as opposed to the material variations.

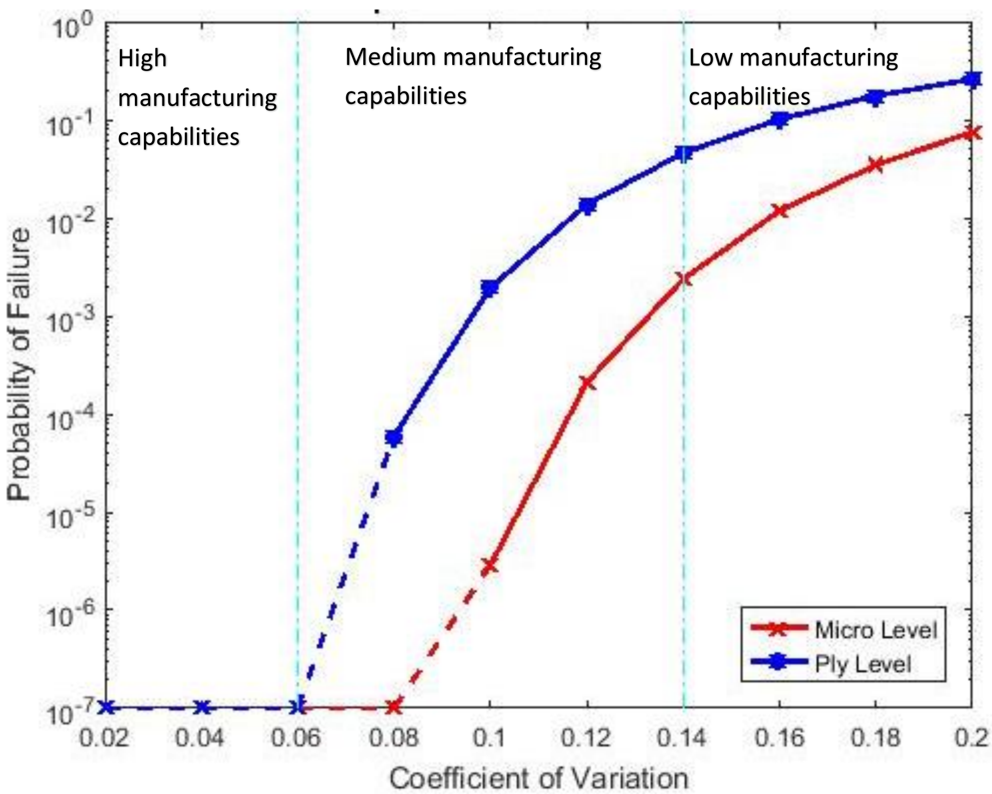

Figure 35 Effects of material variation on structural reliability

The coefficient of variations applied for the constituents' properties are identical to the ply properties. The reason is to understand the effect of different level approach with same amount of variations. If the same variations is achieved at material properties, it exhibits less probability of failure. When a coefficient of variation applied at micro level, it transfers to ply level as a different level of variation as seen in Table 17. The coefficient of variation of Longitunal Young's modulus increases usually around 34% for each CoV from 2% to 20%. However, for transverse Young's modulus and in-plane shear modulus, the change increases to around 80% more than initial value.

Table 17 Micro to ply variation change

COV	0.02	0.04	0.06	0.08	0.1	0.12	0.14	0.16	0.18	0.20
E1	0.0269	0.0537	0.0806	0.1076	0.1346	0.1617	0.1889	0.2162	0.2436	0.2714
E2	0.0363	0.0728	0.1098	0.1475	0.1862	0.2265	0.2691	0.3156	0.4829	3.1492
G12	0.0331	0.0666	0.1007	0.1358	0.1725	0.212	0.2577	2.3978	4.4639	35.689

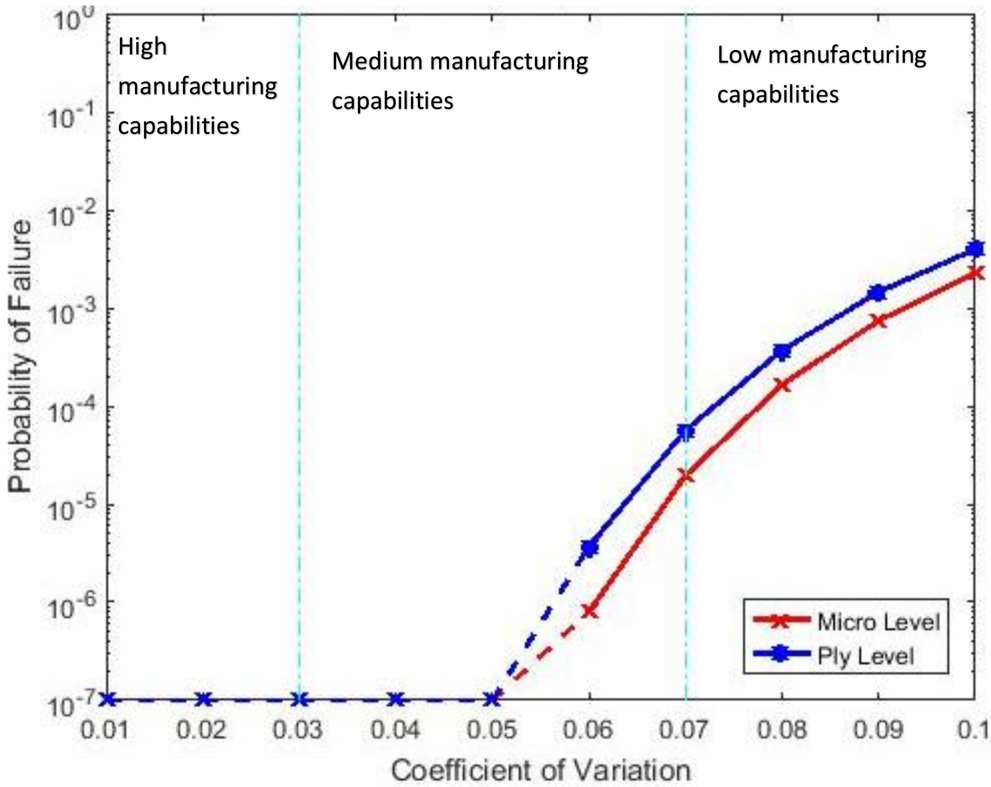

Figure 36 Efect of topological variations on structural reliability

6.5.4 Investigation of material and topological variations together

After exploring the impact of material and topological variations separately this section investigated the joint effect of the variations using both the micro and ply level approaches. Firstly, the impact of material variation with constant topological variations: 0%, 4% and 8%, is investigated. Figure 37 shows that when there is no toplogical variation the first failure is observed at 10% material variation. When topological variations of 4% are added to the structures the first failure is detected at 8% material variation giving a probability of failure of $7.1x10^{-6}$. When a fixed 8% topological variation is added the first failure is observed at 2% material variation with a high probability of $2.23x10^{-4}$. When no or only a small amount of topological variations are considered the impact of the material variations is most significant and remains largely unaffected. However, when topological variations are increased to higher values, impact of material variation becomes less significant.

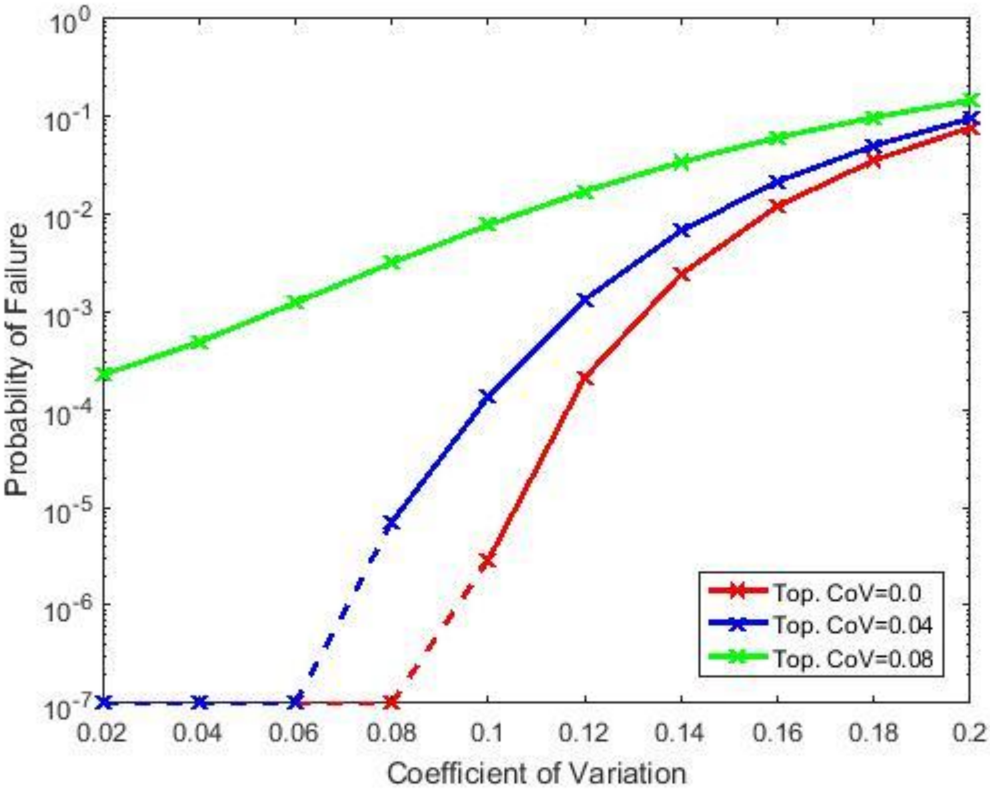

Figure 37 Micro level material and topological variations together

The impact of joint variations on reliability is also investigated with constant material variations, of 0%, 6% and 12%. Increases in material variations results in higher differences in probabilitiy of failure than for the previous constant topological variation cases. When the topological variations are larges, 10%, the probablity of failures are $2.28x10^{-3}$ at 0% variation, $7.60x10^{-3}$ at 6% and $3.78x10^{-2}$ at 12%. This represents larger differences compared constant material variations.

6.6 Summary

Despite the number of studies investigating the reliability of composite structures, composite stiffened plates and the impact of different types of variations is not widely explored. In this study, the impact of topological and material variations on structural integrity of top-hat stiffened composite plates is investigated by using Monte Carlo reliability analysis. As expected the material variations have a significant effect on structural reliability, particularly if the variations are between 0.06 and 0.14 where small improvements to production processes makes a significant change in the reliability.

- Topology should be incorporated more carefully.
- There is a significant a transition region. Similar region is observed with different geometries.
- Small mistakes or improvements in this region have significant consequences.

7 Conclusions

The literature review of the study showed that damage behaviour of composite materials not understood well yet and faster methodologies are required. In order to understand composite structures behaviour more accurately damage behaviour should be investigated with including micro-level damage mechanisms. Moreover, computational efficient approaches are needed in the design process. Therefore, multi scale nature of the composite material should be implemented to the design process.

In this study, composite structures are divided into three length scales: macro-, meso and micro. Initially, macro level behaviour of composite stiffened structures are investigated by using finite element modelling. Finite element model for cross stiffened structures has good agreement with experiments but also showed the requirement for the damage modelling for better approximations. Therefore, a progressive damage model is generated and applied for a composite plate to be validated from literature. A laminated composite plate is examined under pressure loading including damage. The progressive failure is carried out by degradation of material properties of failed integration points of structure. The damage modelling agreed well experimental and numerical results in the literature. However, this study also showed the requirement of faster analysis methodologies. Although finite element modelling gives accurate results for macro level behaviour, the computational times for large structures are high. Therefore, application of direct finite element codes to the reliability or optimisation analyses of composite structures is not practical since these analyses requires several calls to the models.

Micro level is modelled by generating a finite element RVE model. This model exhibited good predictions for material properties which showed it can be used instead of experiments to evaluate the material properties. The homogenized material properties are compared with the available data in literature and observed good agreement.

By employing the micro and macro level modelling techniques, reliability of composite marine structures are investigated to understand the impact of topological and material variations on structural integrity of top-hat stiffened composite plates is investigated by using Monte Carlo reliability analysis. As expected the material variations have a significant effect on structural reliability, particularly if the variations are between 0.06 and 0.14 where small improvements to production processes makes a significant change in the reliability.

In conclusion, an efficient and accurate multiscale modelling approach is presented. This approach can optimise design process by decreasing the time and resources used for the development of structural components. The performance of the structures can be increased by using multiscale reliability analysis during design and development stages.

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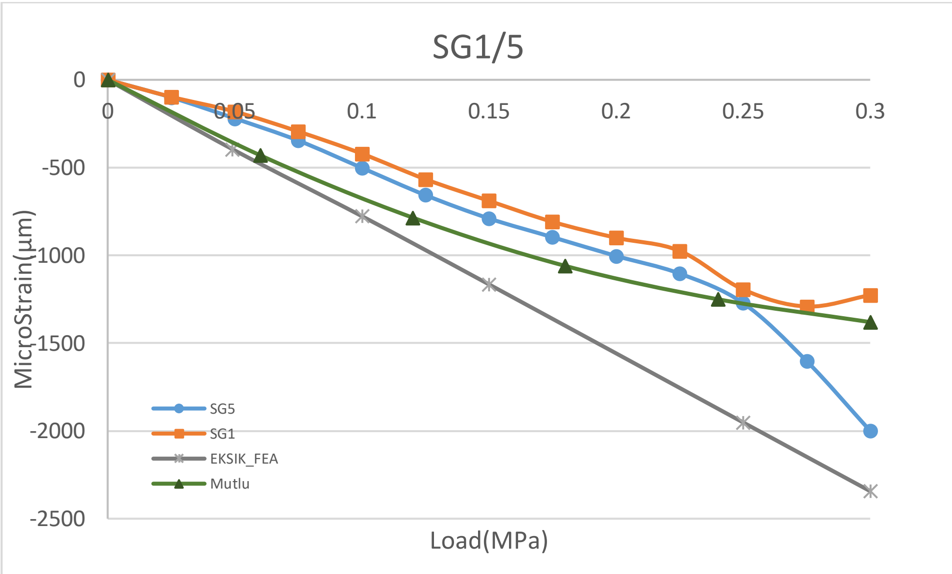
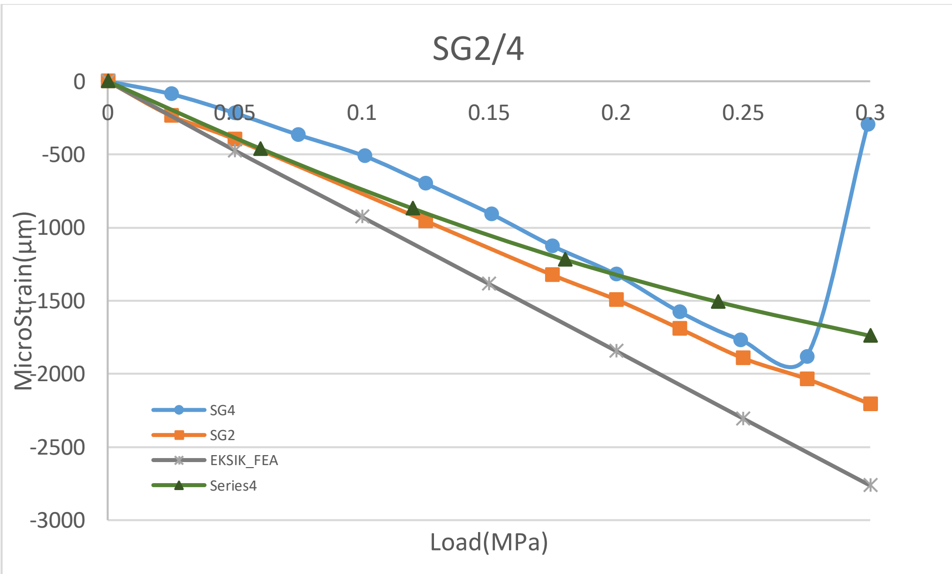
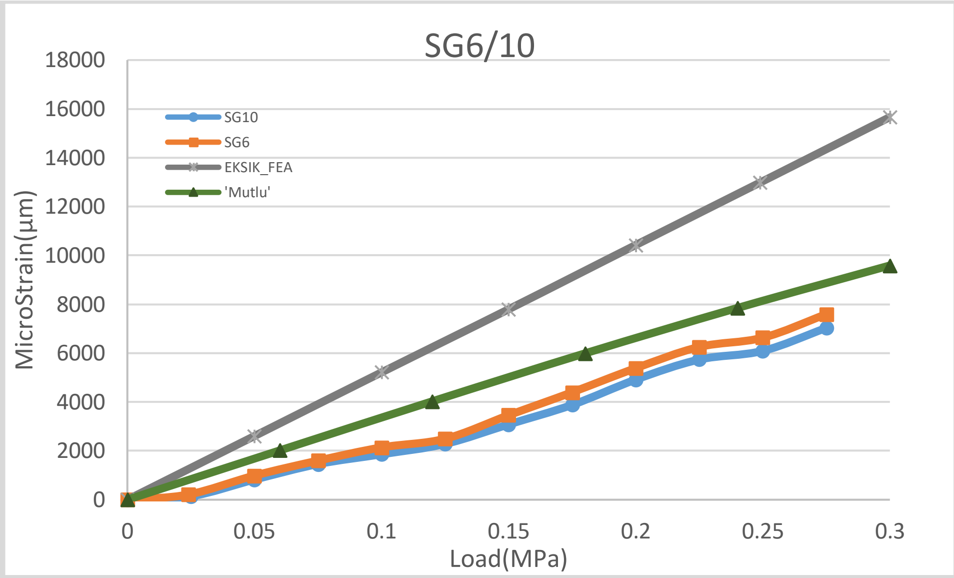
Appendix A

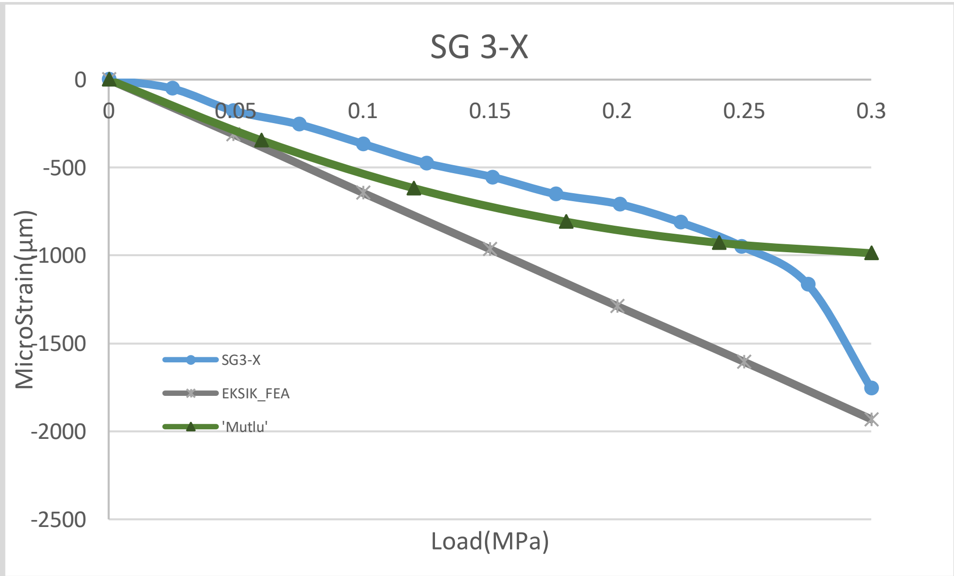
Macro Level Load/Strain Graphs

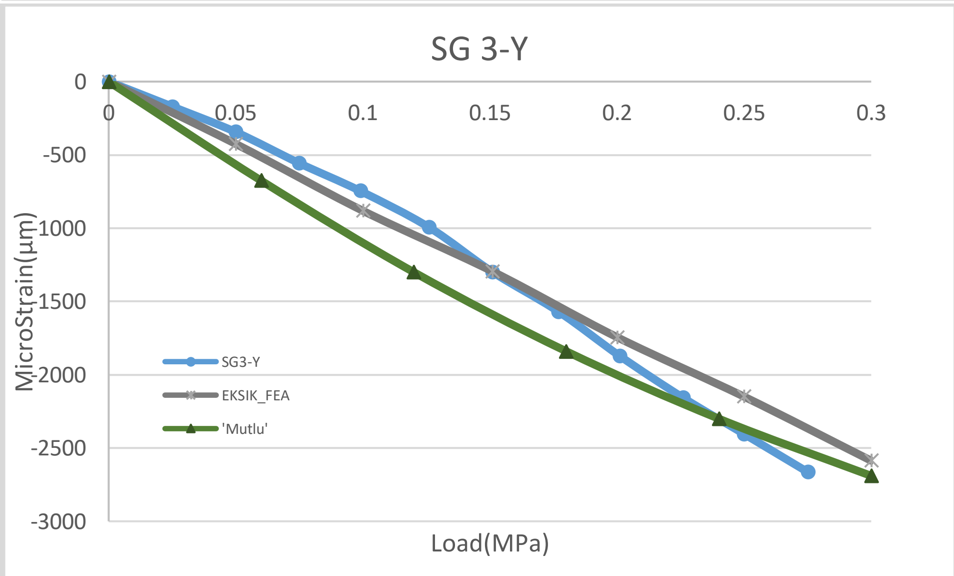
The comparison of experimental and numerical results of Eksik et al. [163] with geometrically non-linear, materially linear model results of this study are given below for 14 strain gages locations:

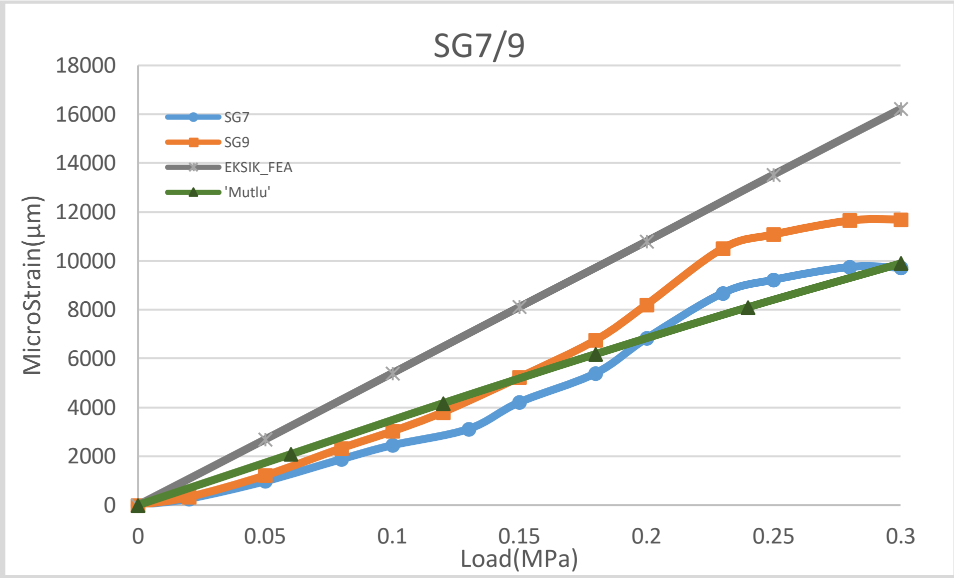


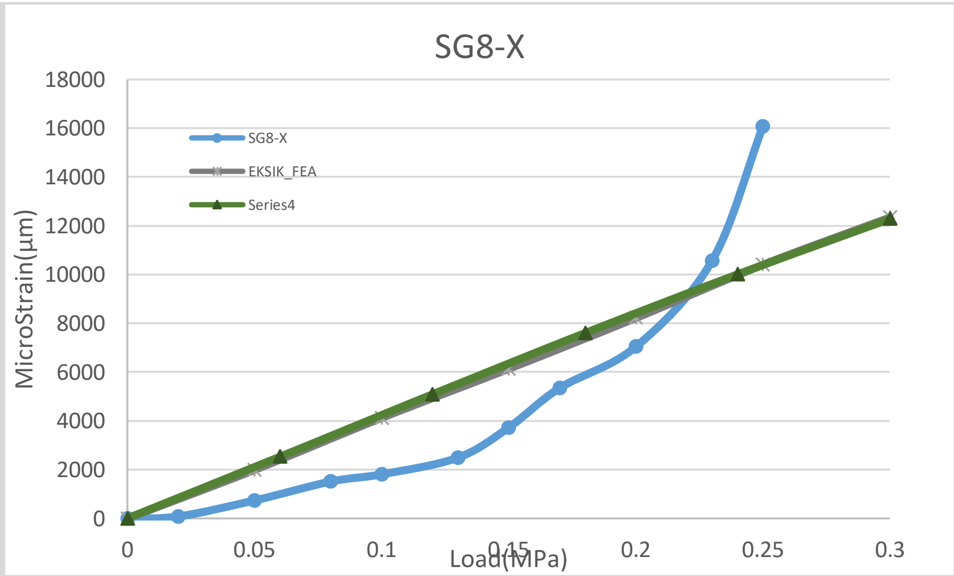


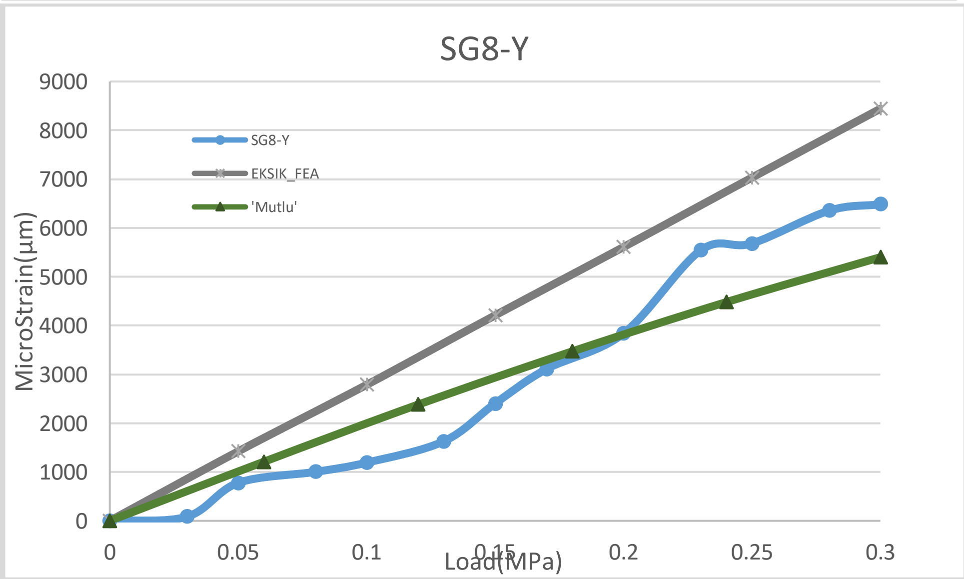

SG6/10
MicroStrain(µm)
Load(MPa)
SG10
SG6
EKSIK_FEA
'Mutlu'
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
0.05
0.1
0.15
0.2
0.25
0.3

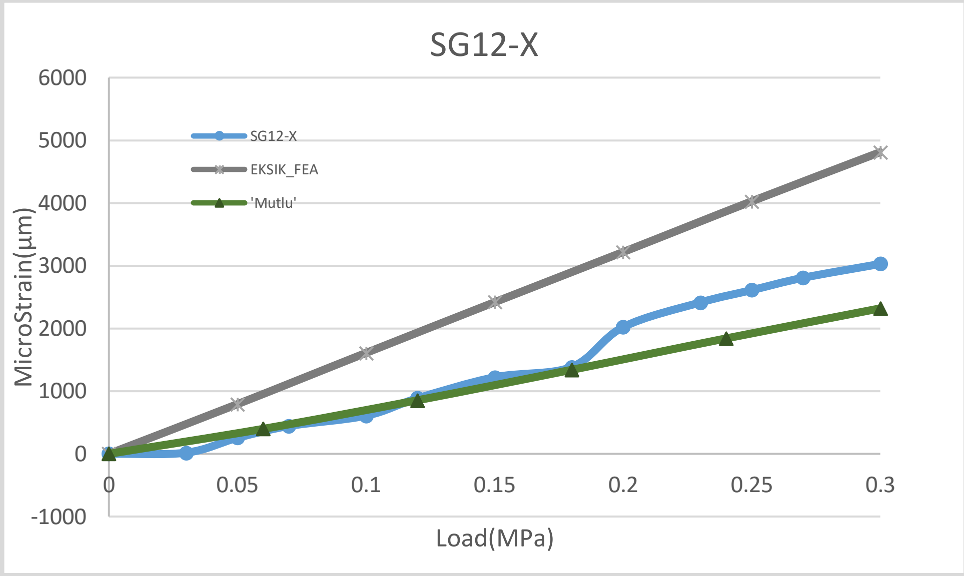

SG 3-X
MicroStrain(µm)
Load(MPa)
SG3-X
EKSIK_FEA
'Mutlu'
0
-500
-1000
-1500
-2000
-2500
0
0.05
0.1
0.15
0.2
0.25
0.3

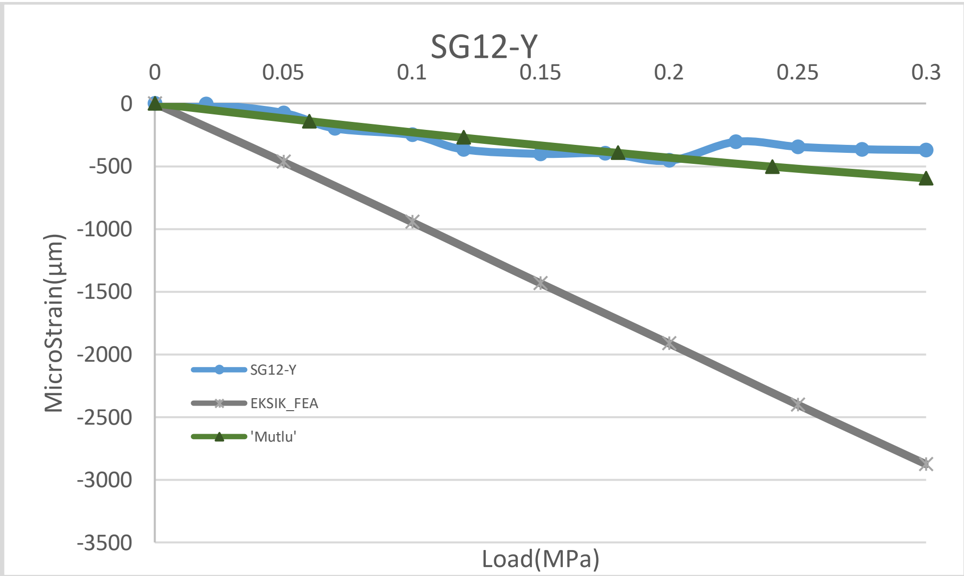

SG 3-Y
MicroStrain(µm)
Load(MPa)
SG3-Y
EKSIK_FEA
'Mutlu'
0
-500
-1000
-1500
-2000
-2500
-3000
0
0.05
0.1
0.15
0.2
0.25
0.3


SG7/9
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
0.05
0.1
0.15
0.2
0.25
0.3
MicroStrain(µm)
Load(MPa)
SG7
SG9
EKSIK_FEA
'Mutlu'


SG8-X
18000
16000
14000
12000
10000
8000
6000
4000
2000
0
0
0.05
0.1
0.15
0.2
0.25
0.3
MicroStrain(µm)
Load(MPa)
SG8-X
EKSIK_FEA
Series4


SG8-Y
9000
8000
7000
6000
5000
4000
3000
2000
1000
0
0
0.05
0.1
0.15
0.2
0.25
0.3
MicroStrain(µm)
Load(MPa)
SG8-Y
EKSIK_FEA
'Mutlu'


SG12-X
6000
5000
4000
3000
2000
1000
0
-1000
0
0.05
0.1
0.15
0.2
0.25
0.3
MicroStrain(µm)
Load(MPa)
SG12-X
EKSIK_FEA
'Mutlu'


SG12-Y
0
0.05
0.1
0.15
0.2
0.25
0.3
0
-500
-1000
-1500
-2000
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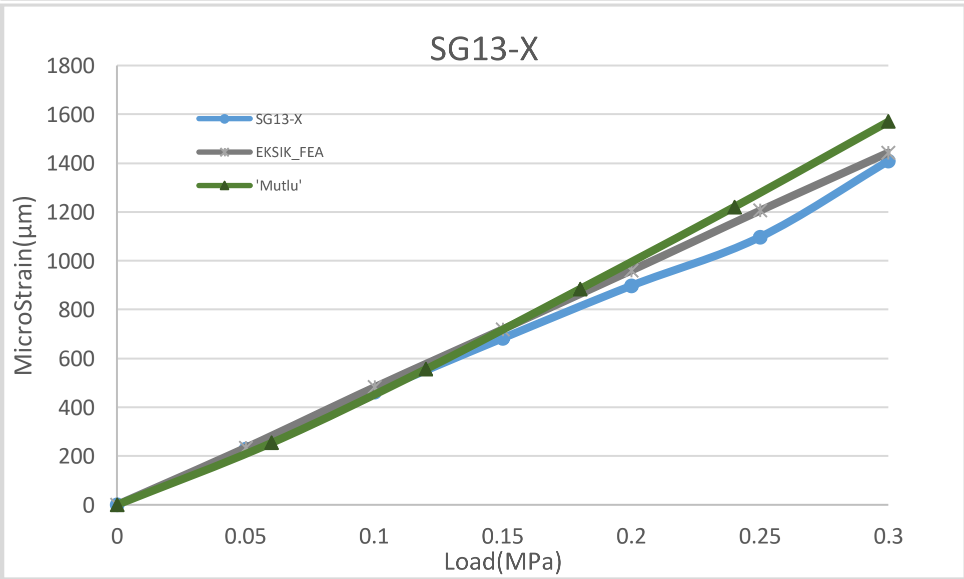
-3000
-3500
MicroStrain(µm)
Load(MPa)
SG12-Y
EKSIK_FEA
'Mutlu'


SG13-X
1800
1600
1400
1200
1000
800
600
400
200
0
0
0.05
0.1
0.15
0.2
0.25
0.3
MicroStrain(µm)
Load(MPa)
SG13-X
EKSIK_FEA
'Mutlu'

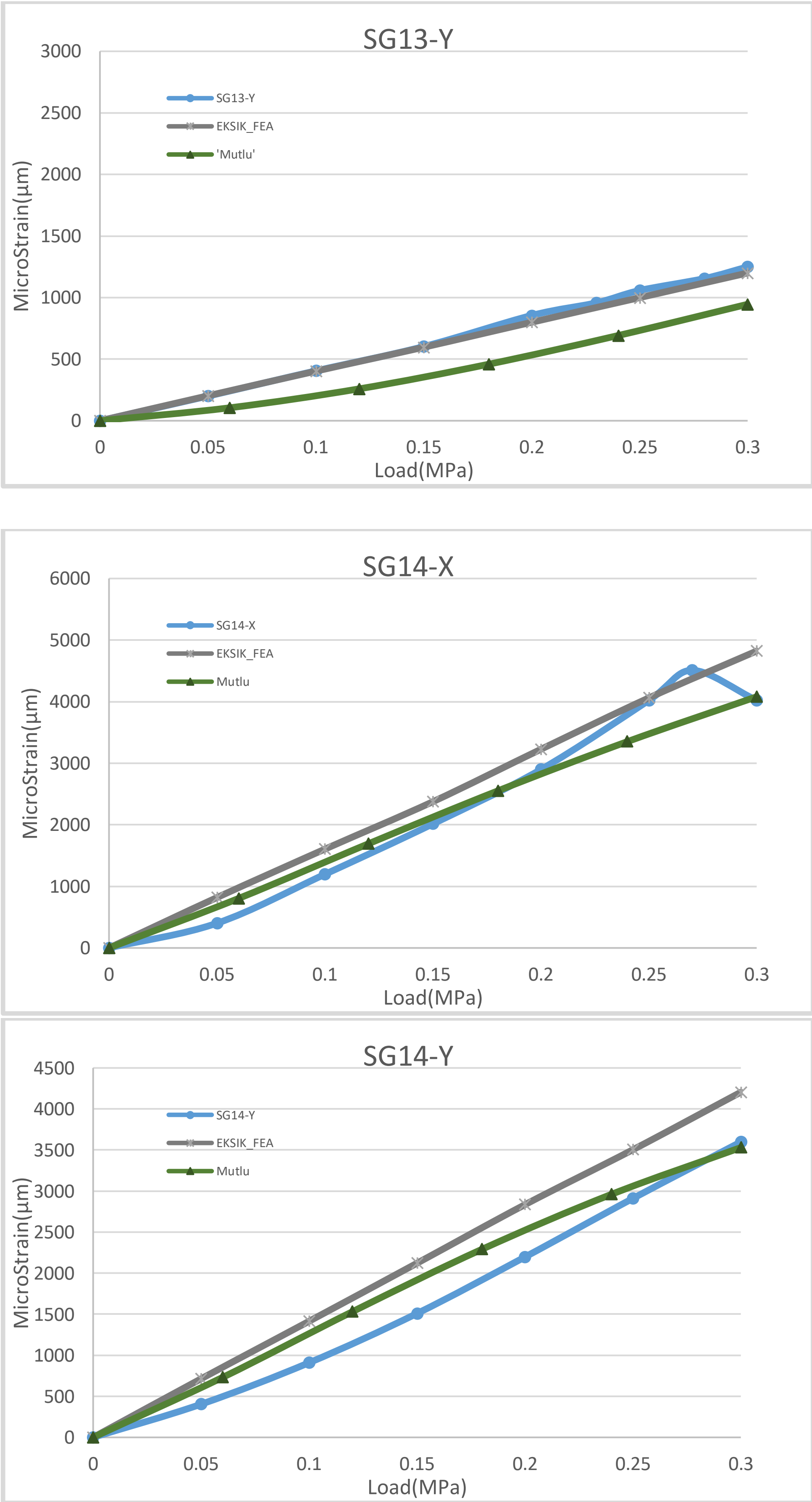


Figure 38 Strain comparisons from different SG locations

APPENDIX B

Surrogate Modelling of Macro Level

Surrogate modelling can be used at any stage of the design process which requires faster methodologies to solve the engineering problem. Initial design or detailed design stages, reliability and optimization studies are the main application areas of surrogate modelling techniques. Surrogate modelling using Krigging approach also allows to establish the order of importance of variables which is important for the designers.

The first application of surrogate modelling performed to the cross stiffened composite structure which is validated in Chapter-4. The parameters chosen as material properties and the lower and upper limits for the material properties is shown in Table-18.

The structure consist of 5 different materials and each material has 6 different variables which totally the system has 30 different variables. In order to construct the surrogate model ten sample points for each variable is used as recommended by [197]. Totally 300 sample points which are defined by using Latin Hypercube is used. The surrogate model is constructed in order to investigate the maximum stress values of the structure under uniform pressure 0.3 kPa as applied during the experiments. Kriging method is applied for the construction of the response surface.

Table 18 Material properties and upper and lower bounds for cross stiffened panel

Variable	Value (MPa)	Lower Limit (MPa)	Upper Limit (MPa)
300 CSM E1 = E2 = E3	8000	7500	8500
300 CSM G12 =G13 =G23	3100	2850	3350
450 CSM E1 = E2 = E3	7300	6800	7800
450 CSM G12 =G13 =G23	2800	2550	3050
600 CSM E1 = E2 = E3	6800	6300	7300
600 CSM G12 =G13 =G23	2600	2350	2850
600 WR E1 = E2	14800	14300	15300
600 WR E3	3000	2500	3500
600 WR G12	3400	3150	3650
600 WR G13 =G23	3600	3350	3850
1600 UD E1	24600	24100	25100
1600 UD E2 =E3	7300	6800	7800
1600 UD G12	4200	3950	4450
1600 UD G13 =G23	3500	3250	3750

The response surface is tested for 41 different points. The comparison between real results and surrogate model predictions are showed in Fig-19.

The root mean square error and average prediction errors also showed the model has enough accuracy in order to replace the original finite element modelling within defined boundaries to investigate the maximum stress values. Root mean square error is 1.229 MPa which is very low value when compared to max stress values which are around 900 MPa. Average prediction error also shows that

$$RMSE = \sqrt{\frac{\sum_{i=0}^{n_t}\left(y^{(i)} - \hat{y}^{(i)}\right)^2}{n_t}} = \mathbf{1.229}$$

Average Prediction Error= **0.11 %**

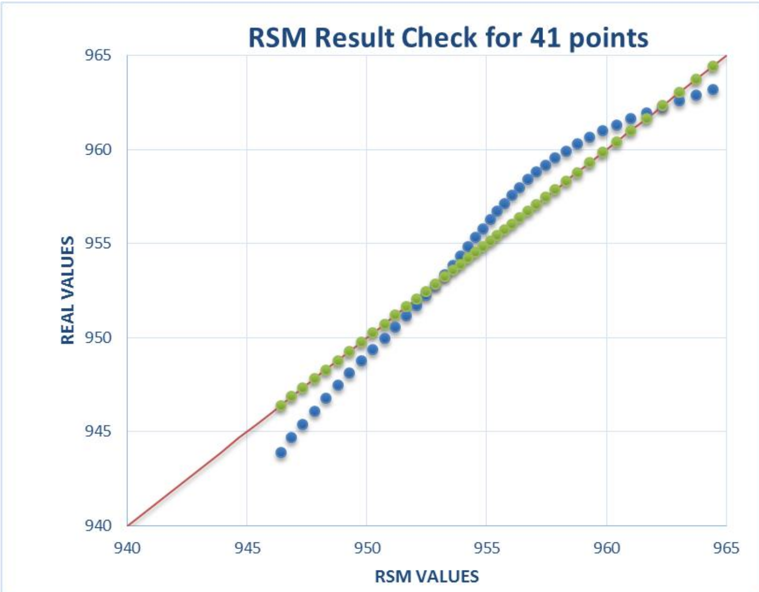


Figure 39 Surrogate model accuracy check for cross stiffened plate

Surrogate Modelling of 2by3 Stiffened Plate

The second application of surrogate modelling is performed for more realistic marine structure which is a two by three top hat stiffened plate [Fig-20]. The parameters are chosen as material properties, geometric properties and loading which has big differences with the first application. The variables and the boundaries are shown in Table-4.

This composite stiffened plate is constructed by single material and as variable Young modulus, shear modulus is chosen as variables. As geometric properties, the length of the plate, the beam, the height and width of the stiffener is chosen as geometric variables. Besides, the pressure loading is applied as parameter which is between 60kPa and 160 kPa in order to capture the uncertainties of the loading.

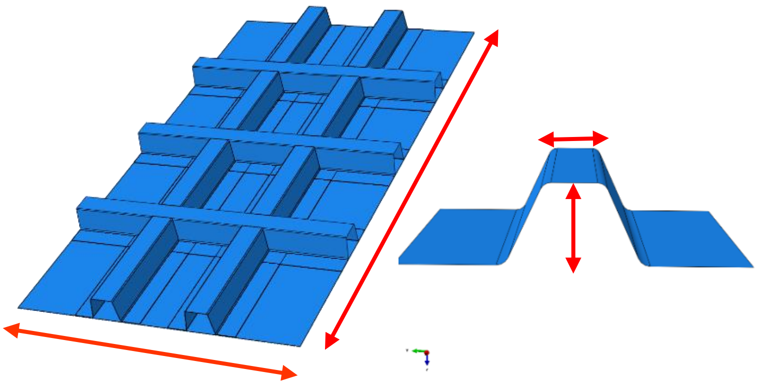


Figure 40 2 by 3 stiffened plate and geometric variables

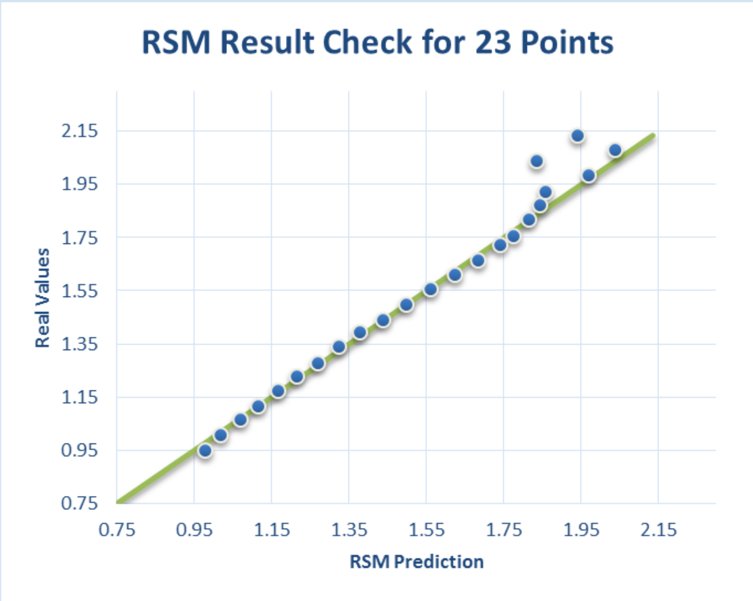
In order to construct the surrogate model 80 sample points are used which are derived by using Latin Hypercube. Two different surrogate model is constructed for maximum stress and maximum deflection within the stiffened plates. The surrogate models are tested from 23 test points which shows good agreement with real results.

Table 19 Material properties, upper and lower limits

Variable	Mean Value (μ)	Standard Deviation(σ)	Lower Limit	Upper Limit
Material Properties				
$E_1 = E_2$ (GPa)	17.814	1.79	**12**	**24**
E_3 (GPa)	10.176	1.02	**7**	**14**
$G_{12} = G_{13} = G_{23}$ (GPa)	2.396	0.09	**2.12**	**2.66**
Poisson Ratio	0.33	-	-	-
Geometric properties				
L (mm)	2825	84.75	**2570**	**3080**
B (mm)	6021	180.63	**5480**	**6565**
Crown Width (mm)	211	3.38	**201**	**221**
Crown Thickness (mm)	6.4	-	-	-
Web Height (mm)	258	4.90	**243**	**273**
Web Thickness (mm)	6.4	-	-	-
Plate Thickness (mm)	6.4	-	-	-
Loading (kPa)	110	16.5	**60**	**160**

Max. Stress Results of Surrogate Model

The first surrogate model is constructed to investigate the maximum stress of structure within the design space. Afterwards, the surrogate model is tested by using different 23 test points. These points are selected as evenly distributed like full factorial design, and added extra points very close the boundaries. The results have showed good agreement with real finite element results as shown in Fig. 21. The root mean square error is 0.063 GPa where the maximum stresses are around 2 GPa. The average prediction error also showed the surrogate model have high accuracy.



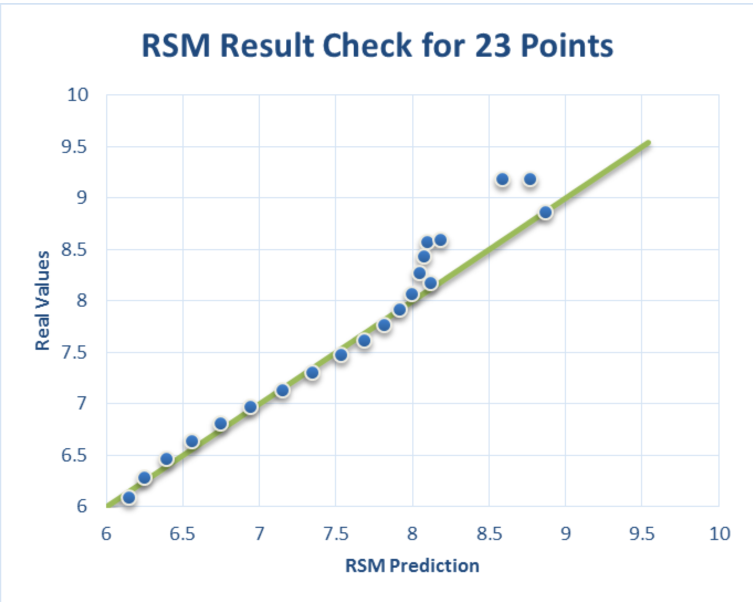
$$RMSE = \sqrt{\frac{\sum_{i=0}^{n_t}\left(y^{(i)}-\hat{y}^{(i)}\right)^2}{n_t}} = 0.063$$

Average Prediction Error= 1.8 %

Figure 41 Max. stress accuracy test for 2by3 stiffened plate

Max. Deflection Results of Surrogate Model

Another surrogate model is constructed for the same structure but for the maximum deflection results with same parameters within the same design space [Fig.22]. The root mean square error and average prediction error showed that same surrogate model can be used also to investigate the maximum deflection of the structure in fast way.



$$RMSE = \sqrt{\frac{\sum_{i=0}^{n_t}\left(y^{(i)}-\hat{y}^{(i)}\right)^2}{n_t}} = 0.24$$

Average Prediction Error= 2.12 %

Figure 42 Max. deflection accuracy test for 2by3 stiffened plate

Summary

The results for both applications provide high accurate predictions. For the first application, 30 material properties for five different materials are used as variables. Although the number of variables is too high to gain this much accuracy, the surrogate model which is constructed for 30 variables gave high accurate results. It is interpreted that there two main reasons for this performance with high number of variables, firstly shear modulus doesn't effective as high as young modulus and secondly the defined boundaries for the variables relatively narrow. The surrogate model for the 2 by 3 stiffened panels also presented accurate predictions. Fig-19 and Fig-20 also shows some higher errors for some of the test points. These points are chosen at the boundaries on purpose in order to check the performance of the surrogate model close the boundaries. The accuracy can be increased by adding extra sample points locally if the higher accuracy required.