

Brillouin assisted slow-light enhancement via Fabry-Perot cavity effects

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Abstract:

We show that the presence of a cavity modifies the behaviour of Brillouin-assisted slow light and can be used to significantly enhance the achievable pulse delays. Moreover, the cavity introduces an additional wavelength dependence into the delay versus gain relationship which can be used to provide an extra degree of control within a slow light system. The degree of delay enhancement depends critically both on the cavity finesse and the Brillouin pump power. Our experiments show that delay enhancements greater than 100% can be obtained accompanied by only relatively modest increases in pulse distortion.

1. Introduction

The topic of slow light has become of increasing interest since the first experimental demonstrations in the early 1980s [1]. The term slow light refers to the possibility of reducing the group velocity of an optical pulse [2] by placing its wavelength near an optical resonance (either natural or induced). In these spectral regions the group index of refraction is altered leading to a modification in the group velocity of the pulse. Slow light generation has been reported in a variety of systems, most of them based on exotic media, such as ultracold atoms [3,4] or planar photonic crystal microcavity arrays [5]. However, recently a major advance was reported when it was shown that significant speed reduction factors could be achieved through Brillouin-scattering in optical fibers [6,7] using readily available off-the-shelf components. These experiments opened up the possibility of developing slow-light based devices, such as optically controlled delay lines, that are inherently compatible with existing optical fiber sensor and communications systems.

Brillouin scattering is a very well known nonlinear effect in fibers in which an intense light beam (pump) interacts with an acoustic wave giving rise to a backscattered lightwave [8] down shifted in frequency by a quantity known as the Brillouin frequency shift. The fact that this is an induced resonance offers several benefits for slow light generation. For example it can be positioned at any desired optical frequency through appropriate choice of the pump wavelength and can be dynamically controlled by suitable modulation of the pump intensity. Moreover, since the resonance is a gain peak, amplification as well as delay of the signal, can be obtained. However, there are a number of significant practical limitations associated with this new slow light technique that need to be addressed. Firstly, the inherently narrow (~30 MHz) gain bandwidth of the silica Brillouin resonance makes it difficult to accommodate a broadband communications signal. This problem has recently been solved by modulating the pump with a pseudorandom signal [9,10] in order to broaden the effective Brillouin linewidth. The other issue is the need to reduce the pump power requirements. In order to keep the power requirements to practical levels long fibers have been used, however this tends to give rise to other unwanted effects that degrade the performance and stability of the system. Recently we have addressed this problem by using high nonlinearity fibers based on Bismuth oxide glass [11] which allow a two orders of magnitude reduction in pump power compared to standard single mode silica fibers. Further reductions in pump power should be achievable using fibers made of even higher nonlinearity materials [12].

Whilst our early experiments with Bismuth fiber clearly showed the expected benefits of using high nonlinearity fibers we observed significant deviations from the delay versus gain behavior predicted by the standard theory [13]. Specifically we observed increasingly larger delays than expected for high values of Brillouin gain. In this paper we present the results of a systematic study of the pulse delay characteristics when using our 2m long Bismuth oxide fiber sample, along with the associated characterization and numerical modeling results, which have allowed us to determine the physical origin of the observed delay enhancement. Our work shows that our previous observations can be explained by the presence of partially reflective splices at the end of the fiber which serve to create a low-finesse cavity about the Brillouin gain medium. The presence of the cavity modifies both the gain dynamics [14, 15] and the effective profile of the Brillouin gain spectrum experienced by the signal beam which can result in substantial increases in the achievable delays. With this system we have observed induced delays that are more than twice those obtained in the absence of the cavity. Use of cavity effects is thus seen to present a convenient way of further enhancing Brillouin assisted slow light delays without recourse to higher pump powers, or new glass types. Whilst we present the effect in the context of Brillouin-assisted slow-light in optical fibers, this approach is not restricted to this slow light approach. Cavity enhancement should work with any means of generating slow-light that involves gain, and is thus potentially applicable to many other slow-light systems. In particular, we envisage the possible use of this effect in semiconductor

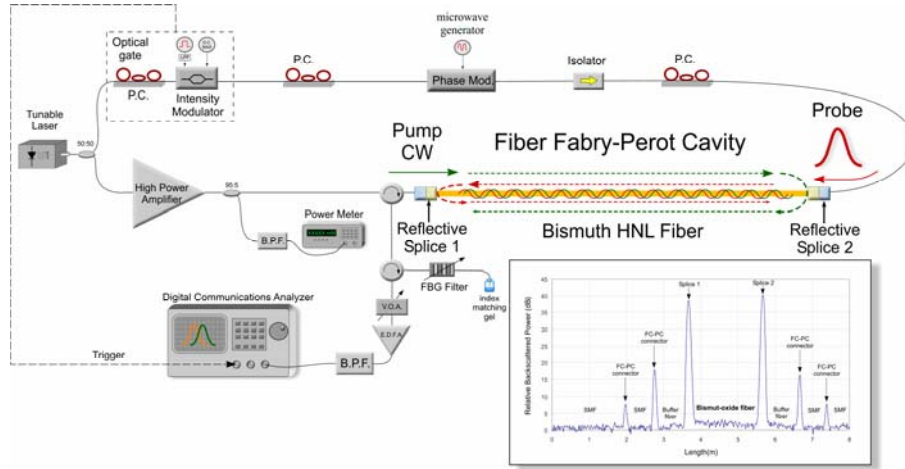


Figure 1. The experimental setup for the cavity induced delay enhancement. A 2m long highly nonlinear Bismuth-oxide fiber is used as the active medium to generate Brillouin assisted slow-light using a continuous wave pump and a counter-propagating pulsed probe. Partially reflecting splices between the Bismuth-oxide fiber and conventional single-mode fibers are used to define a low finesse Fabry-Perot cavity. The inset shows a trace from a high resolution OTDR used to locate and characterise the cavity. In the figure: P.C.- polarization controller; B.P.F.- bandpass filter; V.O.A.- variable optical attenuator; FBG-fiber Bragg grating; E.D.F.A.- Erbium-doped fiber amplifier.

based devices to obtain enhanced delays at bandwidths appropriate for telecommunications applications.

2. Experimental Setup

Our experimental setup is depicted in figure 1. The output of a single-frequency laser source operating at 1549nm is first divided into two branches. In the lower branch the optical power is boosted using an erbium doped fiber amplifier (EDFA) to generate an intense pump beam. In the upper branch the light is chopped into Gaussian-like pulses using an electro-optic intensity modulator. These pulses are then phase modulated using an electro-optic phase modulator to generate probe pulses that can be tuned to any desired optical frequency about the Brillouin gain peak. Both the pump and probe beams are launched in a counter-propagating sense into the Bismuth-oxide fiber. The delayed probe pulses are then directed via a circulator to the detection system. This system comprises a tunable fiber Bragg grating (FBG) filter that eliminates all unwanted spectral components leaving only the amplified and delayed probe pulses. The combination of an EDFA and an optical attenuator are then used to ensure that the power levels reaching the digital sampling oscilloscope are more or less constant regardless of the probe signal gain.

The 2 metre long fiber was spliced at both ends to conventional single-mode fibers. These splices present a small reflectivity which consequently creates a Fabry-Perot cavity. The inset of figure 1 shows a trace from a high resolution Optical Time Domain Reflectometer (OTDR) which confirms the presence of the cavity formed by the partially-reflective splices at the extremes of the Bismuth-oxide fiber. From these measurements we can infer a rough reflectivity estimate of $2.5 \pm 0.5\%$ for each facet. Although the reflectivity of these splices is small, the presence of this cavity has a substantial effect on the Brillouin interaction once the Brillouin gain is sufficiently high to ensure that the small percentage of power reflected by the splice at the pump's entry point, back-propagated through the fiber and reflected by the probe input splice is of comparable magnitude to that of the incoming probe. If the pulse duration is more than twice (3 to 4 times in practice) the length of time taken for the light to make a

single pass through the fiber then significant intrapulse interference effects can take place. The most obvious consequence of such intrapulse interference is that the effective gain of the system can be increased (or decreased) relative to the single pass Brillouin gain, depending on whether this interference is constructive (or destructive). Thus two mechanisms combine in this system to create the final effective gain: Brillouin gain and interference. Likewise it should also be expected that the effective group velocity of the pulses is affected by the presence of the cavity, as we demonstrate herein.

For our experiments we used 200ns pulses and the propagation time through the fiber was ~ 14 ns. Note that the fiber was placed in a thermally insulated box for the duration of our experiments but that no attempt was made to either actively stabilize the temperature of the fiber, or the position of the cavity modes with respect to the frequency of the pump/probe beams.

3. Experimental Results

Using this setup, we set the probe frequency to the centre of the Brillouin gain bandwidth, applied different pump powers to the bismuth-oxide fiber cavity and measured the output probe waveforms as a function of the measured effective gain (the total pulse energy gain resulting from the interplay between the Brillouin scattering and, in this case, constructive interference processes). Fig 2a shows the delayed waveforms. As can be seen, large delays are attainable with only modest induced distortions. The results in terms of both delay and pulse duration as a function of the effective gain are shown in figure 2b. Most significantly, we see that the measured delays for a given overall gain are considerably larger than those predicted by the standard theory [6,7] which considers only a single pass through the bismuth oxide fiber. The measured delay enhancement is more than 100%. The maximum delay observed in our measurements is around ~ 66 ns, meaning that the effective group velocity is just ~ 24500 km/s, i.e. less than $c/12$. This, as far as we know, is the lowest value of group velocity yet reported in a solid core optical fiber. Note also that the maximum experimentally determined effective gain is some 6 dB higher than the expected single-pass Brillouin gain for the maximum level of pump power used (490mW).

In Fig. 2 we see that these large delay enhancements are accompanied by significant increases in pulse distortion, and again that single pass theory cannot explain these observations. These results highlight the fact that a compromise needs to be struck in terms of delay enhancement versus tolerable pulse distortion. In our experiments a pulse broadening of just $\sim 16\%$ was obtained at the maximum delay of 66ns.

In order to accurately predict and better understand the measurement results shown in Fig. 2 we developed a new model based on solving the set of differential equations that govern the Brillouin scattering process subject to appropriate boundary conditions that describe the presence of the cavity. This system of equations was solved using the method of characteristics [16]. The results of our simulations (which assume a splice reflectivity of 2.3% in good accord with our rough OTDR based estimates of $\sim 2.5\%$) fit the measured delay and pulse width values extremely well (red curves in Fig. 2b). We attribute the seeming discrepancy between theory and experiment at low gain values to be due simply to the relatively poor signal to noise ratios of the output pulses at low gain, which compromised our ability to make accurate pulse width measurements.

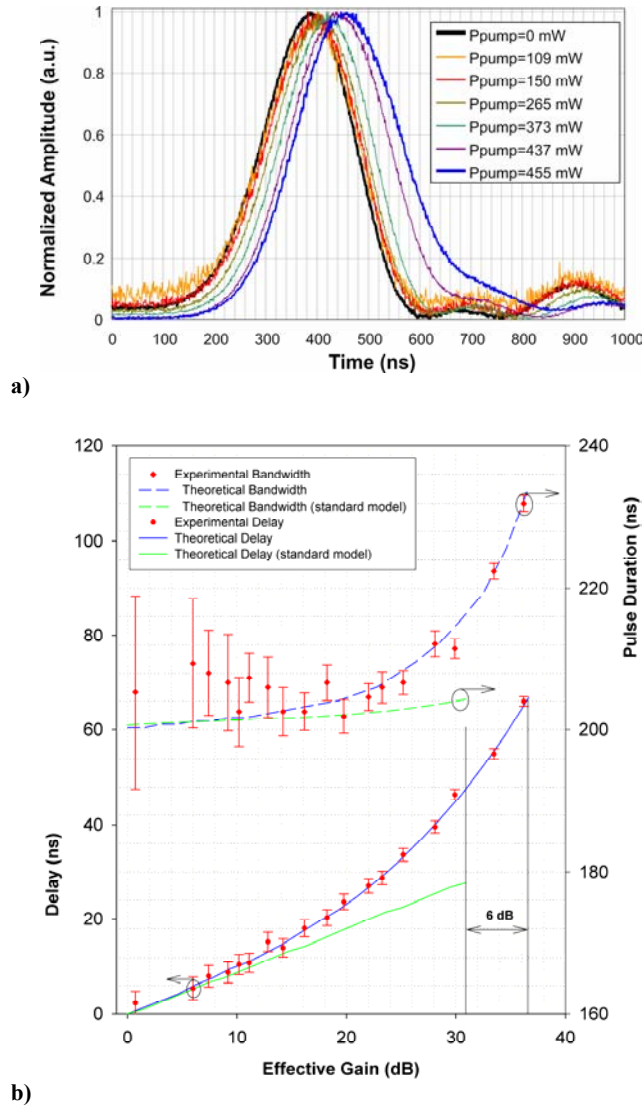


Figure 2. Experimental results. a) the waveforms are progressively delayed as the pump power is increased. The modest distortion introduced in the waveforms is clearly visible. b) A comparison of the experimental results with the theoretical predictions (that assume splice reflectivities of 2.3%). The lower plot shows the measured delay versus Brillouin gain (red dotted line) together with the theoretical prediction from the cavity model (blue solid line) and that from a simple analytic model that describes a single pass through the fiber (green solid line). The nonlinear delay enhancement due to the cavity is evident. The upper plot shows the pulse duration as a function of the Brillouin gain as measured in our experiments (red line with diamonds) and as predicted by our model (blue dashed line) compared to that of the single pass theory (green dashed line). It can be seen that an increasing amount of pulse broadening accompanies the delay enhancement.

4. Theoretical model

The model we developed to study the effect of the cavity on the generation of slow-light is based on solving the set of differential equations that govern the interaction of the multiple waves propagating in the medium. The starting point is the system of equations describing the Brillouin scattering for the field amplitudes [8]:

$$\begin{aligned}
\frac{\partial A_p^+}{\partial z} + \frac{1}{v_g} \frac{\partial A_p^+}{\partial t} &= -\frac{\alpha}{2} A_p^+ + j\gamma \left(|A_p^+|^2 + 2|A_s^-|^2 \right) A_p^+ - \frac{g_B}{2} A_s^- Q \\
-\frac{\partial A_s^-}{\partial z} + \frac{1}{v_g} \frac{\partial A_s^-}{\partial t} &= -\frac{\alpha}{2} A_s^- + j\gamma \left(|A_s^-|^2 + 2|A_p^+|^2 \right) A_s^- + \frac{g_B}{2} A_p^+ Q^* \\
T_B \frac{\partial Q}{\partial t} + (1 + j\delta) Q &= A_p^+ A_s^-^*
\end{aligned} \tag{1}$$

These equations assume plane waves and propagation in a medium of constant refractive index.. The effect of fiber dispersion has been neglected because we are generally dealing with relatively short fibers and long pulses. Moreover, the Brillouin-induced dispersion that gives rise to slow-light is much higher than the fiber dispersion. On the other hand, equation (1) is valid for quasi-monochromatic pulses (that is, pulses longer than 1ps).

In equation (1) g_B is the Brillouin gain coefficient (6.43e-11 m/W for the Bismuth fiber), A_p is the amplitude of the pump electric field, A_s is the amplitude of the probe electric field (where the superscripts + and - represent propagation in the positive or negative z -direction). For simplicity we have dropped the explicit dependence of $A_p(t,z)$ and $A_s(t,z)$ on time (t) and fiber length (z) from the notation. Q is related to the amplitude of the density oscillations in the optical fiber, v_g is the group velocity of the light in the fiber, α is the attenuation coefficient of the fiber (0.9 dB/m in this case) and γ its nonlinear coefficient (1100 W⁻¹km⁻¹). δ is the detuning parameter defined as:

$$\delta = (\omega_p - \omega_s - \Omega_B) T_B \tag{2}$$

Where $\omega_p = 2\pi c / \lambda_p$ is the angular frequency of the pump (of wavelength λ_p , which is 1549nm in our simulations), $\omega_s = 2\pi c / \lambda_s$ is the angular frequency of the probe (of wavelength λ_s), $\Omega_B = 2\pi\nu_B$ where ν_B is the Brillouin frequency shift (8.824GHz in the Bismuth fiber). Finally $T_B = 2 / \Gamma_B$ is the phonon life time in the fiber, and Γ_B is the full width at half maximum of the Brillouin gain peak (40 MHz in the Bismuth fiber).

For pump pulses longer than 10ns, $\frac{\partial Q}{\partial t}$ can be neglected [8] simplifying the solution of equation 1. This approximation is valid in our case because we are working with a continuous wave pump.

However, the description of our problem is not complete with equation 1. Since we have a fiber cavity we need to consider two more waves interacting in the system: the portions of pump and probe reflected by the end facets of the cavity (A_p^- and A_s^+ respectively). Since the reflectivities of the end facets of the cavity are considered to be small, the reflected pump and probe intensities will also be small and, therefore, no noticeable Brillouin interaction will take place between them. Thus, the equation governing the propagation of these reflected waves is:

$$\begin{aligned}
\frac{\partial A_s^+}{\partial z} + \frac{1}{v_g} \frac{\partial A_s^+}{\partial t} &= -\frac{\alpha}{2} A_s^+ + j\gamma \left(|A_s^-|^2 + 2|A_p^+|^2 \right) A_s^+ \\
-\frac{\partial A_p^-}{\partial z} + \frac{1}{v_g} \frac{\partial A_p^-}{\partial t} &= -\frac{\alpha}{2} A_p^- + j\gamma \left(|A_s^-|^2 + 2|A_p^+|^2 \right) A_p^-
\end{aligned} \tag{3}$$

Thus equations (1) and (3) form the set of equations that govern the interaction of the five waves present in the system. These equations have to be solved subject to appropriate boundary conditions that describe the presence of the cavity. These are:

$$\begin{aligned}
A_s^+(t, 0) &= r_1 A_s^-(t, 0) e^{j\beta L} \\
A_s^-(t, L) &= r_2 A_s^+(t, L) e^{j\beta L} + A_{si}(t) \\
A_p^+(t, 0) &= r_1 A_p^-(t, 0) e^{j\beta L} + A_{pi}(t) \\
A_p^-(t, L) &= r_2 A_p^+(t, L) e^{j\beta L}
\end{aligned} \tag{4}$$

Where r_1 and r_2 are the reflection coefficients of each of the cavity end facets (considered identical in our simulations and of value 2.3%), β is the propagation constant, L is the length of the fiber (2 metres), and $A_{pi}(t)$ and $A_{si}(t)$ are the input waveforms for pump and probe signals respectively. As can be deduced from equation (3), we have taken $z=L$ as the input end for the probe and $z=0$ as the insertion end for the counter-propagating pump.

The system of equations given by (1), (3) and (4) completely describes the interactions taking place in the cavity. We have solved this system of equations using the method of characteristics [16] in order to derive the simulation results presented in this paper.

5. Physical insight

The reason behind the delay enhancement can be understood physically by considering the simultaneous processes that occur within the fiber cavity as schematically illustrated in the inset of Fig.3. Firstly, the intense pump power generates acoustic waves within the fiber giving rise to Brillouin amplification. This interaction creates a narrow gain peak (~40 MHz wide) frequency downshifted from the pump wavelength by the Brillouin frequency shift (~8.824 GHz for the bismuth-oxide fiber). In addition to this there is the transmission spectrum of the cavity whose relative frequency position with respect to both the pump and the probe plays a critical role in determining the behaviour of the slow-light system.

An optical cavity *selectively passes* some wavelengths whilst rejecting others - the *preferred* wavelengths are those that fit an integer number of times in the optical path length of the cavity. These wavelengths undergo constructive interference when circulating around the cavity. Although the (unpumped) cavity used within our experiments has a low finesse, it can be substantially affected by the presence of Brillouin gain within the fiber. In particular, by centering the Brillouin gain peak on one of the cavity transmission maxima, progressively large increases in finesse can be achieved as the Brillouin gain gets closer and closer to compensating the cavity round trip loss. As the finesse is increased this then reduces the bandwidth of the transmission maxima thereby progressively narrowing the effective-gain bandwidth experienced by probe light passing through the cavity. Since the maximum achievable delay is inversely proportional to the bandwidth, the gain bandwidth narrowing results in a nonlinear delay enhancement and a higher peak gain relative to the cavity-less case (see Fig. 2b). However, if the gain peak does not coincide with the cavity transmission maxima then there is a complex interplay between the effects of the gain-induced finesse

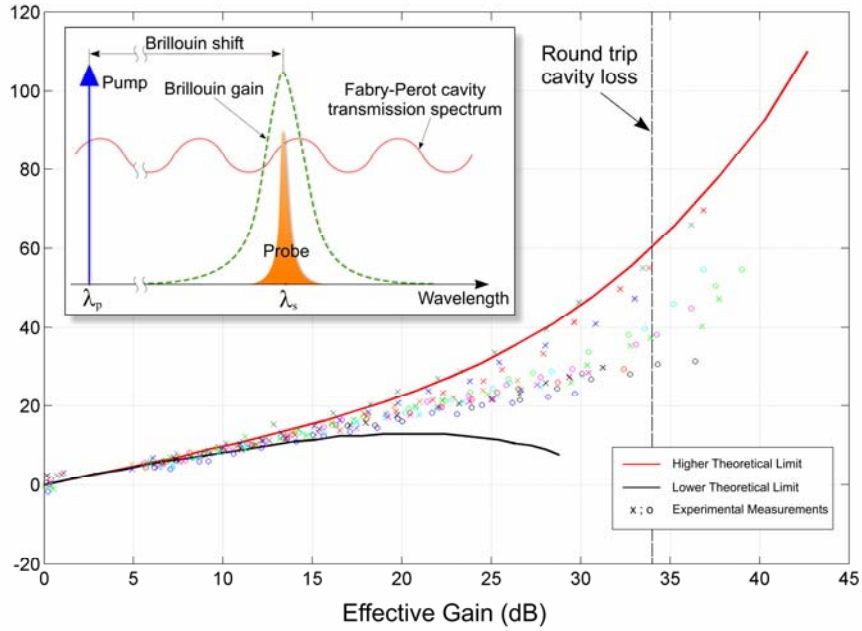


Figure 3. (inset) A schematic illustrating all the interactions simultaneously taking place in the Bismuth-oxide fiber cavity. The Brillouin interaction results in an energy transfer between the pump and the probe that leads to probe amplification. However, this process is affected by the relative positions of both the pump and probe with respect to the Fabry-Perot transmission spectrum. (main plot) Dependence of the delay versus gain relationship on the relative position of the pump wavelength with respect to the cavity spectrum. The graph shows the degree of variation attainable in the delay-gain profile when changing the relative position of the pump with respect to the spectrum of the cavity. The solid lines give the theoretical predictions for maximum (red line) and minimum (black line) delays obtained at positions of transmission maxima and minima for the cavity respectively. The crosses and circles represent experimental measurements for different relative positions of the pump with respect to the spectrum of the cavity. It can be seen that the experimental measurements are contained between the limits given by the theoretical predictions.

enhancement and the higher rejection factor of the cavity, which can result in significant reductions in the overall effective gain and reduced delays relative to the cavity-less case.

Thus the relative frequency of the pump beam relative to the cavity modes is seen to be of primary importance. This is borne out by the data shown in figure 3 where we plot the change in the delay with gain for several arbitrarily selected positions of the pump wavelength relative to the fiber cavity transmission spectrum, along with theoretical upper and lower bound predictions (gain centred on cavity maxima or minima respectively). These calculations assume a maximum pump power of 200mW, and facet reflectivities of 2.3%. Note that during these experiments the probe was always kept at the wavelength of maximum single pass Brillouin gain. As can be seen extremely different delay versus effective gain properties can be obtained depending on the detuning of the pump wavelength relative to the cavity (delays that range from more than double to less than half those of the cavity-less case). Therefore, this wavelength sensitivity provides another means of delay control. Additionally, the effective gain also exhibits this wavelength dependence. Our experimental data is seen to be well contained within the limits given by the theoretical predictions. The inset of Fig. 3 illustrates the modest levels of distortion that the pulses suffer even near the theoretical upper bound prediction.

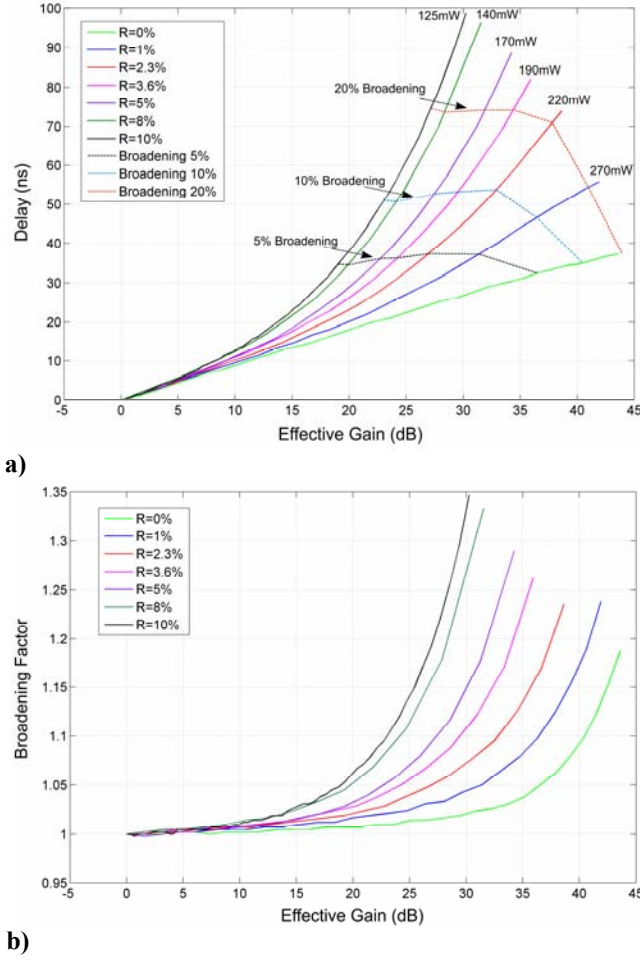


Figure 4. Dependence of pulse delay (a) and broadening factor (b) on the reflectivity of the end facets of the cavity. The maximum pump power inside the cavity is given by the labels in each case and roughly corresponds to the lasing threshold.

6. Limitations and potential extensions of the technique

The delay enhancement obtained using a fiber cavity is an attractive approach since it allows access to much higher delays without recourse to higher pump powers – albeit with slightly reduced bandwidths. However, there are limits to the maximum delay that can be obtained for a given cavity. A first limit is imposed by the onset of stimulated Brillouin scattering for which the Brillouin threshold is itself reduced in the presence of a cavity [14]. However, a more restrictive limit is imposed by the onset of lasing within the cavity which restricts the maximum single-pass Brillouin gain to values below the cavity round trip loss. This can be set to reasonably high values using a low finesse cavity as in our experiments. It is important to note that the effective gain of the system can exceed this value due to the contribution arising from intrapulse interference (which results from multiple passes through the cavity) as seen in Fig. 3.

As stated previously, there is a limit on the maximum effective gain that can be obtained for a given cavity. In order to obtain higher delay enhancements for a given single pass Brillouin

gain/pump power it is necessary to increase the reflectivity of the cavity end facets. We illustrate this in figure 4a where we plot the predicted pulse delay as a function of effective gain for different values of facet reflectivity. However, as shown in figure 4b, this enhancement is accompanied by a rapid increase in pulse broadening. Therefore a compromise between maximum delay and acceptable pulse distortion must be reached. This fact will ultimately limit the maximum practical delay. For example, for our 2 meter long Bismuth fiber, if we want to keep the broadening factor below 20% this limit will be around 75ns for a reasonably wide range of reflectivities (as illustrated by the contour plot). Furthermore, the use of higher reflectivities reduces the maximum usable single pass Brillouin gain due to lasing which ultimately limits the maximum achievable delay. Despite these limitations, we reiterate the fact that the use of a cavity allows delay enhancements greater than 100% with little pulse distortion (as observed in Fig. 2a) and can be used to provide both an extra degree of control on the delay and reduced pump power requirements.

As we have shown, implementation of cavity based time delay enhancement simultaneously requires the use of high gains and pulses that are much longer than the cavity transit time. This obviously imposes significant bandwidth restrictions when using Brillouin gain in fibers since device lengths of order 1m are still required even when using ultrahigh nonlinearity fiber variants. It is hard to envisage significant fiber length reductions beyond this. Thus such systems will always be constrained to operate in the ns pulse duration regime. However, we consider there to be considerable scope for applying similar concepts to other slow-light media capable of far higher gains per unit length than fiber (e.g. within semiconductor devices).

Acknowledgements

The authors acknowledge F. Koizumi and N. Sugimoto of Asahi Glass Company, Japan, for providing the Bi-HNLF, also José Miguel López-Higuera for useful discussions, and Hirotaka Ono for his help in the characterization of Brillouin gain. This work was partially supported by the FP6 Network of Excellence E-Photon/ONe IST-027497. The first author also acknowledges the *Fundación Ramón Areces* for financial support.

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