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Uncertainty in Migration Scenarios

Deliverable 9.2



QuantMig has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 870299.

History of changes

Version	Date	Changes
1.0	8 September 2021	Issued for Consortium Review
1.1	7 October 2021	First version submitted as official deliverable to the EC

Suggested citation

Barker E. R. and Bijak J (2021) Uncertainty in Migration Scenarios. QuantMig Project Deliverable D9.2. Southampton: University of Southampton.

Dissemination level

PU Public

Key words

Complexity, DSGE models, Macroeconomics, Migration modelling, Prediction, Shocks, Uncertainty, Forecasting, Panel VAR, Bayesian Estimation, Automation

Acknowledgments

This work has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No. 870299 QuantMig: Quantifying Migration Scenarios for Better Policy. We are very grateful to Mathias Czaika and Valentina Di Iasio, for their comments that helped improve an earlier draft. The authors acknowledge the University of Southampton's Iridis high performance computing resources (<https://www.southampton.ac.uk/isolutions/staff/iridis.page>) made available for conducting the research reported in this paper. All the remaining errors and inaccuracies are ours. This document reflects the authors' view and the Research Executive Agency of the European Commission are not responsible for any use that may be made of the information it contains.

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Technical Report:

D9.2: Uncertainty in migration scenarios

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October 7, 2021

Abstract

In this report, we propose ways of looking at the uncertainty of migration forecasts and scenarios across a range of time horizons, through the lens of macroeconomic modelling. As an illustration, for short-term horizons, we present the results of empirical models aiming to assess different aspects of the uncertainty in migration and economic dynamics following exogenous shocks. To that end, we estimate Bayesian panel vector autoregressive (VAR) models to generate forecasts, which can be also used in scenario-setting. We also examine the effects of an exogenous increase to migration on the macroeconomy. By looking at the forecast errors for different migration indicators, and for a range of models and groups of European countries, we assess the usefulness of VAR models for generating short- and long-range migration forecasts and scenarios and for estimating their uncertainty. For longer horizons, we also look into dynamic stochastic general equilibrium (DSGE) models, which are used here to generate theoretically-informed migration scenarios. In particular, we look at a scenario of job automation, examining inequalities in migration processes, either modelled as exogenous or, in a two-country model, with fully endogenous migration decisions, depending on the labour market conditions and costs. The results of modelling offer coherent migration scenarios and provide a tool for assessing the uncertainty of both migration and its impacts. We also identify and discuss several important remaining research gaps and methodological challenges of using modern macroeconomic approaches for forward-looking migration studies, and propose some practical solutions.

Keywords: International migration, Migration Forecasting, Open Economies, Scenarios

JEL Classification: C52, C53, E32, F22, F42, J11

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1 Introduction

The different types of uncertainty about the size, timing, type, duration and impact of migration flows, and their interplay within the economy and the society, are paramount contemporary challenges, which become increasingly important for policy and planning. The drivers of migration flows are variable, and are easily subject to change, sometimes at a very short notice and with hardly any warning signs ([Bijak and Czaika, 2020](#)). In that context, the aim of this report is to explore a range of methods for assessing the uncertainty of migration forecasts and scenarios for short-, mid- and longer-term horizons.

Some of the complexity of migration can be described, at least approximately, by applying formal relationships in models, ideally equipped with microfoundations corresponding to the individual decisions of people and other units, such as firms. That possibility, which is explored in the current report, is offered by approaches used in contemporary macroeconomics, especially by dynamic stochastic general equilibrium (DSGE) models (see, for example [Rao et al., 2010](#)), and time series models, which examine the dynamic relationships between migration and its different drivers. The models dealing with very short-term horizons, such as early warnings and nowcasts (e.g. [Napierała et al., 2021](#)), are out of the scope of this study, and will be dealt with in a separate report.

Adopting the macroeconomic perspective for scenario setting is appealing, as some canonical pull factors of migration include improved employment opportunities, salary levels, and other job-related factors, and the same holds for push factors, including economic downturns, low wages, or high unemployment ([Massey et al., 1993](#)). Such factors can be modelled with macroeconomic data. Of course, there are also circumstances in which migration changes unexpectedly, and which cannot be adequately explained or predicted by changes in macroeconomic structures and processes alone. Hence, crises, such as the so-called European asylum crisis of 2015–16, policy shifts, or political uncertainty, such as the one around Brexit in the United Kingdom, are also out of scope of this study.

One key advantage of modern macroeconomic modelling in the context of migration forecasting is its dynamic nature. Here, many possibilities are available, from gravity (for example [Beine et al., 2016](#)) to dynamic stochastic general equilibrium models. A desirable feature of the latter modelling techniques is a coherent inclusion of the behaviour of

individuals and firms, and their expectations of the future based on the present and past information¹. Of course, some circumstances, which do end up showing in macroeconomic data, such as the COVID-19 pandemic, cannot be largely foreseen in advance.

Hence, as there are reasons beyond macroeconomics that make people migrate, the different aspects of uncertainty inherent in migration forecasting are even more challenging to capture. Besides unpredictable events, these also include migration policy changes (tightening or relaxation of relevant legislation, see [Czaika et al., 2021](#)), and people’s reactions to them at the micro level. That notwithstanding, macroeconomic approaches can offer coherent description of at least *some* of the relevant drivers, and help set plausible migration scenarios, and analyse their uncertainty and sensitivity to shocks. One example, which we further explore in this report, is how labour migration could be changed by job automation, if a significant number of migrants in destination countries are employed in low-skilled occupations, and what effects automation has on labour markets in general.

To investigate possible future migration and its uncertainty we employ two established techniques from the macroeconomic toolkit: Bayesian panel vector autoregressive (VAR) models and dynamic stochastic general equilibrium (DSGE) models in the context of the job automation scenarios. In the panel VAR, we employ a large data series across 26 European countries² to assess the impact of increases to net migration and produce forecasts with in-sample and out-of-sample assessment of their errors. The DSGE models employed here also enable exploring a range of policy-relevant topics related to the automation scenario, such as the income inequalities in a small closed economy, and labour market effects in a two-country model, both being a result of migration shocks.

In this report, Section 2 presents the econometric analysis of migration and short-term forecasts based on VAR models, and Section 3 presents two DSGE models as tools of migration scenario setting, which enable examining migration both as an exogenous and endogenous process. In both these sections, the focus on migration forecasts or scenarios is presented first, before turning to the analysis of uncertain impacts of migration on the rest of the economy. Section 4 includes reflections and some empirical analysis for longer-range forecasts, while Section 5 provides further discussion and concludes.

¹For empirical representation of expectations, there is (limited) data availability of forecasts in the [OECD Economic Outlook](#), as well as in forecasts of statistical organisations, central banks, and so on.

²The countries included are from the EU, EFTA, plus the United Kingdom, with a few exceptions.

2 Short-range Analysis: Time-Series VAR Approaches

In this section, we introduce Bayesian panel vector autoregressive (VAR) models and present forecasts for 26 European countries from the European Union (EU), the European Free Trade Agreement (EFTA), and the United Kingdom.³ The first part (Section 2.1) is devoted to the use of VAR models for forecasting net migration *per se*, and the second one (Section 2.2) to examining the impacts of migration and migration shocks on other areas of the economy. In this way, we examine two aspects of the future migration uncertainty: the magnitude and calibration of forecast errors, as well as the way in which the uncertain migration shocks are propagated through the broader economic system.

2.1 Forecasting Net Migration with VAR Models

As mentioned before, forecasting migration is a challenge due to a sheer number of drivers and push and pull factors for different types of migration, as well as the irreducible (*aleatory*) uncertainty of the future (Bijak and Czaika, 2020). In this section, we take a look at migration forecasting through the prism of macroeconomic drivers, using Bayesian panel VAR models, extending the previous work of Gorbey et al. (1999) and Bijak (2010). We apply these models to quarterly data, and evaluate the forecasts by predicting migration indicators for eight quarters, 2018Q1:2019Q4, and comparing them *ex post* with the values observed in the data for that period. By so doing, we are also trying to assess to what extent including theoretical macroeconomic relationships and regularities encoded in the models can shed some light on the *epistemic* migration uncertainty, related to imperfect knowledge.

The focus on macroeconomic variables among the many theoretical considerations about migration drivers is of course to some extent arbitrary (see e.g. Massey et al., 1993; Czaika and Reinprecht, 2020). Our motivation here is threefold. First, these drivers form a very important part of the whole driver environments (or *complexes*, Czaika and Reinprecht 2020), especially given the prominent role of labour migration amongst all population flows. Second, the impacts of migration on different aspects of the economy and the labour markets are also intensely studied (e.g. d’Albis et al., 2019; Furlanetto and

³Croatia, Cyprus, Iceland, Liechtenstein, and Malta are excluded from the analysis due to data limitations, and the Netherlands are excluded due to issues with data uncertainty.

Robstad, 2019), and their uncertain features are sometimes contested (e.g. Borjas, 2006; Card, 2005). Third, even though the examples presented here focus on the macroeconomy, the tools used in the analysis can be transferable to other drivers and their environments, as long as appropriate data are available.

In our examples, the main series of macroeconomic data cover the period 2002Q1:2019Q4.⁴ In addition, to enable conditional forecasts, we use GDP (plus private consumption and investment) and population forecasts which, country dependent, stretch up to 2022Q4 and 2024Q4 respectively.⁵ Our dataset features both annual and quarterly data, so that the models need to be analysed by using mixed-frequency tools proposed by Canova and Ferroni (2020) and Dieppe et al. (2016). From the national accounts, we use GDP, private consumption, private investment, exports, and imports, all sourced from the OECD data. Fiscal data, also collected from the OECD, include: government final consumption, fixed capital formation, social security benefits paid and received, taxes on production and imports, other current receipts, total direct taxes, property income paid and received, and other current outlays. The data selection follows the suggestions by d’Albis et al. (2019). For national accounts and government accounts, the variables are expressed in real terms, per working-age population, and are log-transformed. For population and labour market, the unemployment rate, total employment and population data are also sourced from the OECD, with the percentage of the population aged 15-64 originating from the World Bank’s World Development Indicators. The latter indicator approximates the share of the working-age population according to a consistent definition across countries. Finally, migration estimates are sourced from Eurostat, the IMEM project (Raymer et al., 2013), or national statistics authorities.

We conduct the in-sample forecasting exercise over the period 2018Q1:2019Q4. The evaluation time frame was limited to these two years, to avoid potential problems with the effects of the COVID-19 pandemic on the migration measures used. The indicators being forecast are relative measures of migration intensity, expressed per 1,000 working-age inhabitants of a given country. For emigration, these measures approximate proper

⁴The annual data for 2020 were still unpublished at time of writing. Further work will assess the implications of the COVID-19 pandemic on both migration flows and the way they were measured. It has yet to be seen whether some of the migration decisions have been postponed or cancelled.

⁵The OECD Economic Outlook only includes GDP forecasts for some countries. Where there are no forecasts available, data up to 2021Q2 are used (where available).

demographic rates, while for immigration and net migration, these are analogous measures, which we label as ‘rates’, to reflect that they do not correspond to the correct populations at risk. In addition, we conduct an out-of-sample forecasting exercise, for the period 2018Q1:2022Q4. Table 1 lists the different variables used in the forecasting models, their brief description, source, and any transformation that was made to the data before their inclusion.

Table 1: Data Variables and Descriptions

Variable	Description	Source	Transformation
<i>Migration</i>			
NM	Net Migration	Eurostat, IMEM, Nat. Stat.	Per 1000 WA Residents
Emig	Emigration	Eurostat, IMEM, Nat. Stat.	Per 1000 WA Residents
Immig	Immigration	Eurostat, IMEM, Nat. Stat.	Per 1000 WA Residents
<i>National Accounts - Expenditure</i>			
GDP	Gross Domestic Product	OECD - CARSA, DNBSA	Real, Per WA, Logged
Cons	Private Consumption	OECD - CARSA, DNBSA	Real, Per WA, Logged
X	Private Investment	OECD - CARSA, DNBSA	Real, Per WA, Logged
<i>National Accounts - (General) Government</i>			
GovCons	Gov Final Consumption	OECD - CARSA, DNBSA	Real, Per WA, Logged
GovInv	Gov Fixed Capital Formation	OECD - CARSA	Real, Per WA, Logged
SocSecP	Social security benefits paid GG	OECD - CARSA	Real, Per WA, Logged
SocSecR	Social security benefits received GG	OECD - CARSA	Real, Per WA, Logged
TaxProdImp	Taxes on production and imports	OECD - CARSA	Real, Per WA, Logged
TaxOther	Other current receipts	OECD - CARSA	Real, Per WA, Logged
TaxDir	Total direct taxes	OECD - CARSA	Real, Per WA, Logged
PropIncP	Property income paid	OECD - CARSA	Real, Per WA, Logged
PropIncR	Property income received	OECD - CARSA	Real, Per WA, Logged
OutOther	Other current outlays	OECD - CARSA	Real, Per WA, Logged
GovPur	Government Purchases	GovCons+GovInv	
TaxRev	Tax Revenues	TaxProdImp + TaxDir + SocSecR + PropIncR + TaxOther	
Transf	Transfers	SocSecP + PropIncP + OutOther	
PubSpn	Public Spending	GovPur + Transf	
<i>Labour Market</i>			
Unemp	Unemployment 15-64 %	Eurostat	
Emp	Employment 15-64 %	Eurostat	
WageSal	Wages and Salaries	Eurostat	Real, Per WA, Logged
WagePre	Wage Premium to EU 15	Eurostat	WageSal to WageSal-EU15

Variables, description, sources, and transformation for data included in the estimation. Data apart from those in percentages are logged (log-transformed) during the estimation. **Abbreviations used:** WA = working-age population. Nat. Stat. = National statistic offices. CARSA = National currency, current prices, annual levels, seasonally adjusted. DNBSA = Deflator, national base year, seasonally adjusted. Variables are deflated by using the GDP deflator unless a corresponding one is available. *Wage premium to EU-15* is a ratio of wage and salary data taken from the national accounts via Eurostat, transformed into real terms, per working-age population, relative to that of the EU-15 countries.

2.1.1 Formulation and Evaluation of Models

In the forecasting exercise, we have forecast immigration, emigration, as well as net migration series separately. Different models were initially considered, as discussed further in Section 2.2, but in this section we report the illustrative results for the model which

performed well enough across all country samples, defined in 2.2 and 2.3 below.

To formulate and estimate the forecasting models, we use toolboxes by [Canova and Ferroni \(2020\)](#) and [Dieppe et al. \(2016\)](#) for mixed-frequency data transformation and Bayesian panel VAR respectively (the latter via the BEAR toolbox: Bayesian Estimation, Analysis and Regression). We employ the approach of [Canova and Ferroni \(2020\)](#) relying on mixed-frequency VAR (MF-VAR), based on both annual and quarterly data from the national accounts, government accounts, unemployment and migration statistics. The Gibbs sampler is used in the reduced-form VAR, to estimate the quarterly observations of the variables ([Canova and Ferroni, 2020](#), p 54). The [Dieppe et al. \(2016\)](#) toolbox is used for the panel VAR modelling. The analysis contains N countries, n variables, p lags, and covers T quarters. The element of the panel VAR model related to country i ($i \in 1, 2 \dots N$) is specified as:

$$y_{i,t} = \sum_{j=1}^N \sum_{k=1}^p \Psi_{ij,t}^k y_{j,t-k} + \epsilon_{i,t} \quad (2.1)$$

where $y_{i,t}$ is a $n \times 1$ vector of n endogenous variables for country i at time t . The matrix of coefficients is given by $\Psi_{ij,t}$, of size $n \times n$, and $\epsilon_{i,t}$ is a vector of $n \times 1$ vector white noise error terms with $\epsilon_{i,t} \sim N(0, \Sigma_t)$ as specified by [Dieppe et al. \(2016\)](#). The model is estimated with four lags, $p = 4$, corresponding to one year. The contemporaneous mutual impacts of different variables for each country are introduced through the covariance matrix Σ_t , allowing for reverse (reinforcing or dampening) feedback effects to occur simultaneously in the same period. To take advantage of the panel structure, the borrowing of strength between different countries occurs not only through Σ_t , but also via the matrices of vector autoregressive parameters $\Psi_{ij,t}$.

As we are using Bayesian methods, we need to make a number of prior assumptions. The approach relies on a pooled estimator, with data for all countries pooled together to estimate a single, homogenous VAR model, with four lags and no constant in each model, with estimation based on 5000 iteration runs of the Gibbs sampler (following a burn-in of 500 iterations). The parameters and hyperparameters follow standard values from macroeconomic literature encoded in the BEAR package with the conjugate multivariate normal-inverse Wishart model structure: the normal priors for the autoregressive

parameters are centred around 0.8, indicating a belief *a priori* in relatively large autocorrelations, whereas the marginal priors for the residual and factor variances are assumed to follow (tightening) inverse Gamma distributions with the shape parameter 1000 and scale parameter 1, to prevent the forecasts from exploding too fast (Dieppe et al., 2021).

In our panel VAR, we model four groups of countries: high-income and high-net immigration Western European countries (Group 1); lower-high-income low-net immigration Western European Countries (Group 2); Central and Eastern European (CEE) countries with positive net migration (Group 3), and CEE countries with negative net migration (Group 4). The inclusion of a group of countries with *negative net migration* in a macroeconomic context is novel, as such migration has not been investigated before in detail across a number of countries.

We group the countries in the analysis to overcome problems arising from the short time period covered by the data. Using a panel VAR with eleven, five, four, and six countries in Groups 1–4 respectively, increases the number of observations (the sample size) used to estimate the parameters, while allowing to take the advantage from the similarities in macroeconomic and migration patterns between the countries in each group. The list of countries and a data snapshot is provided in Table 2. As previously mentioned, Croatia, Cyprus, Iceland, Liechtenstein, and Malta are excluded from the analysis due to data limitations and the Netherlands is excluded due to problems with data uncertainty.

To calculate the forecasts, we employ a vector of endogenous variables y_t describing the macroeconomy and labour market. Predictions for immigration and emigration are estimated separately, as in (2.2), and net migration is predicted independently, as a separate process, as in (2.3). The non-migration variables include some of the push and pull factors of migration identifiable at a macroeconomic level in data (see e.g. Massey et al., 1993), with i.e. higher levels of private consumption, wages, and employment being pull factors, and lower levels – push factors. GDP represents the overall state of the economy, with an economy above the trend being an attractive destination (pull factor). There are also other variables in the dataset in Table 1) which could individually describe different migration drivers, but for consistency we use the same models to estimate emigration and immigration.⁶

⁶The variables which are excluded via block exogeneity (i.e. exogenous variables) are not listed here.

Table 2: Case Study: Selected Summary Statistics for Selected 26 European Countries

Country	Net Migration Rate	Real GDP PC (€)	Unemployment Rate	Wage Premium to EU15	Fiscal Balance to GDP %
<i>Group 1: Western Europe Countries - high net immigration and wage premiums</i>					
Austria	7.35	58,273	5.05	1.18	-3.36
Belgium	6.38	55,158	7.67	1.07	-4.34
Denmark	3.55	69,385	5.77	1.83	-2.10
Finland	3.72	53,961	8.12	1.23	-4.02
Germany	5.27	53,464	6.81	1.17	-2.12
Ireland	7.39	72,600	8.62	1.14	-4.24
Luxembourg	22.39	116,251	5.07	2.89	-0.27
Norway	9.37	109,214	3.63	2.02	6.76
Sweden	9.23	68,302	7.07	1.32	-5.12
Switzerland	9.20	92,371	4.61	2.34	-1.78
United Kingdom	5.09	55,583	5.73	1.19	-5.05
<i>Group 2: Western Europe Countries - low net immigration and wage premiums</i>					
France	1.84	50,468	9.09	1.02	-6.49
Greece	0.11	27,849	16.19	0.41	-8.34
Italy	6.79	44,044	9.37	0.66	-4.49
Portugal	0.62	25,676	9.76	0.48	-6.08
Spain	8.00	34,320	16.32	0.66	-5.71
<i>Group 3: CEE Countries - net receiver of migrants</i>					
Czechia	2.75	20,567	5.84	0.37	-6.39
Hungary	2.87	12,348	7.33	0.32	-6.01
Slovakia	0.98	19,519	12.65	0.28	-7.32
Slovenia	3.87	25,077	6.91	0.59	-5.21
<i>Group 4: CEE Countries - net sender of migrants</i>					
Bulgaria	-3.08	8,688	9.61	0.14	-3.93
Estonia	-0.21	22,036	8.49	0.40	-3.10
Latvia	-9.78	16,430	11.10	0.32	-7.78
Lithuania	-11.90	16,675	10.05	0.29	-6.27
Poland	-1.11	14,073	10.54	0.25	-6.69
Romania	-5.15	9,871	6.53	0.18	-4.93

Average values for 2002:2019. Source: [Barker \(2021a\)](#) Authors' calculations using data from [Eurostat](#), [IMEM database](#), [OECD](#), and national statistics institute.

$$y_t = [\text{Emig}_t(\text{or Immig}_t), \text{Const}_t, \text{WageSal}_t, \text{Emp}_t, \text{GDP}_t]' \quad (2.2)$$

$$y_t = [\text{NM}_t, \text{Const}_t, \text{WageSal}_t, \text{Emp}_t, \text{GDP}_t]' \quad (2.3)$$

2.1.2 Short-Range Migration Forecasts: Summary of the Results

In this section we present a selection of forecasts from models (2.2) and (2.3), which have performed reasonably well for all groups of countries. Their forecasts include country-specific variables with block exogeneity controlled either by the total size of the labour market of Europe, the size of the common labour market, the size of the common labour market per working-age population, or with no control variable. We present the forecasts of the immigration 'rate', emigration rate, and net migration 'rate' below.⁷ Plotting

⁷The forecasts for immigration and emigration were estimated at the (natural) logarithmic scale, but are presented in the level form for the ease of interpretation.

the forecasts for immigration and emigration separately helps identify, in which case the process dynamics was the most uncertain and volatile. Figure 1 presents the forecasts of immigration, Figure 2 of emigration, and 3 the forecasts of net migration ‘rates’ produced directly by model (2.3). On the whole, the presented forecasts performed reasonably well, although the model was generally able to predict values which were within the 67% predictive intervals for most countries, for most of the time. All the in-sample forecasts presented in this section are unconditional. In addition, the 90% predictive intervals are presented in Appendix A: even though the 67% predictive intervals seem calibrated relatively well, the 90% ones are too broad to enable meaningful conclusions.

The predictive intervals for the three migration indicators are relatively wide, although visibly wider for some countries than for other ones. Unsurprisingly, this correlates with the historical migration trends and the presence of visible shocks and high volatility in the data series – for countries with more stable migration patterns, the intervals are narrower. This suggests that the role of uncertainty driving the migration processes, including their unpredictable (aleatory) aspects, plays a greater role in the forecasting uncertainty than, for example, parametric uncertainty of panel VAR models used for prediction.

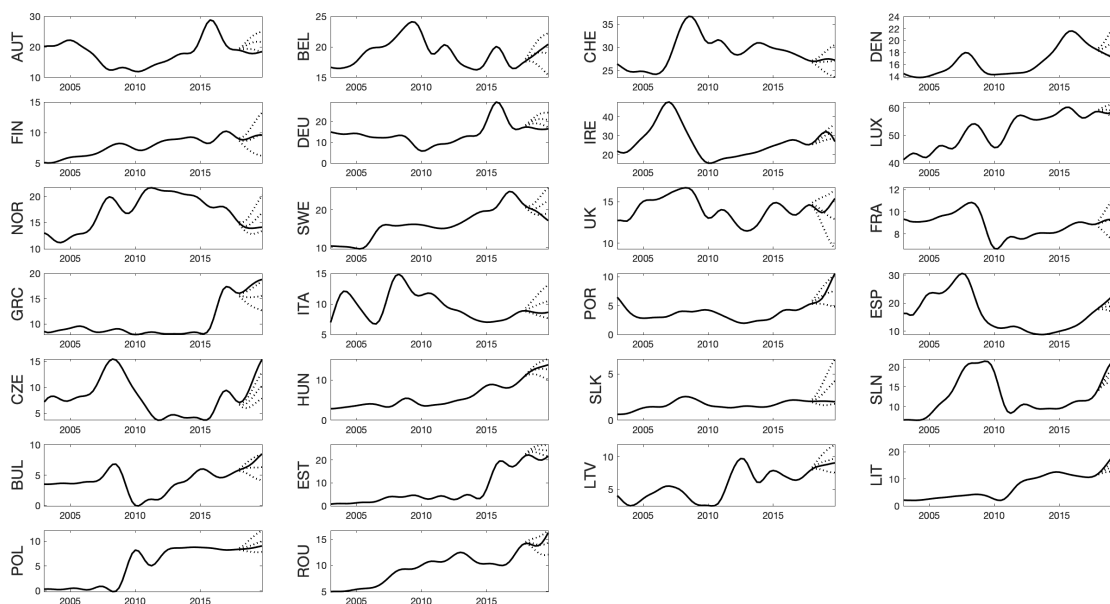


Figure 1: In-sample Forecasting Exercise for Immigration ‘Rates’, 2018–19
The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

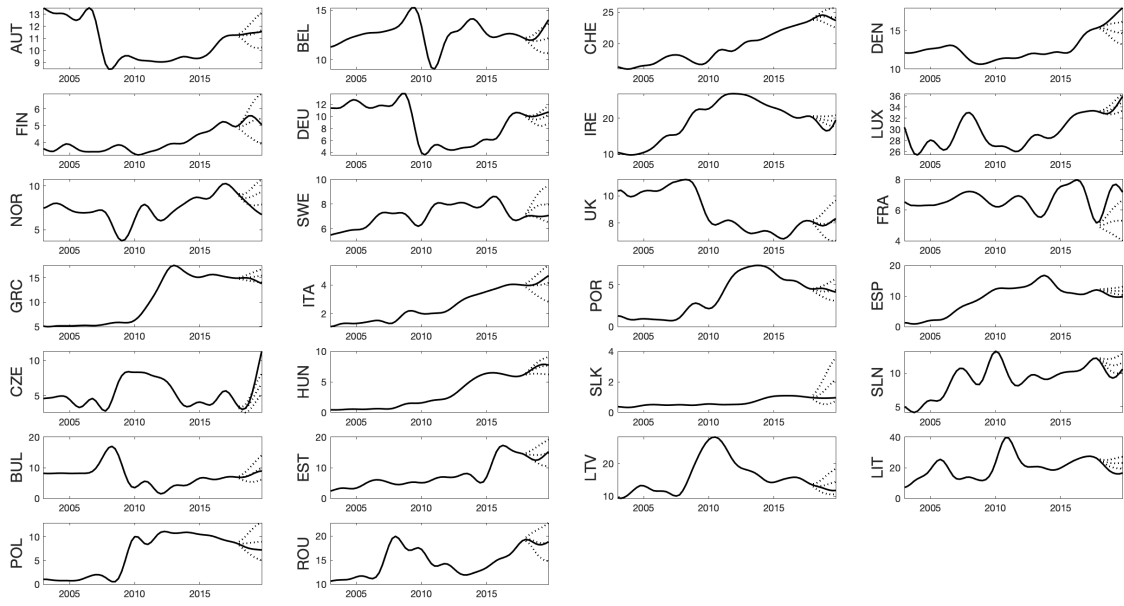


Figure 2: In-sample Forecasting Exercise for Emigration Rates, 2018–19
 The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

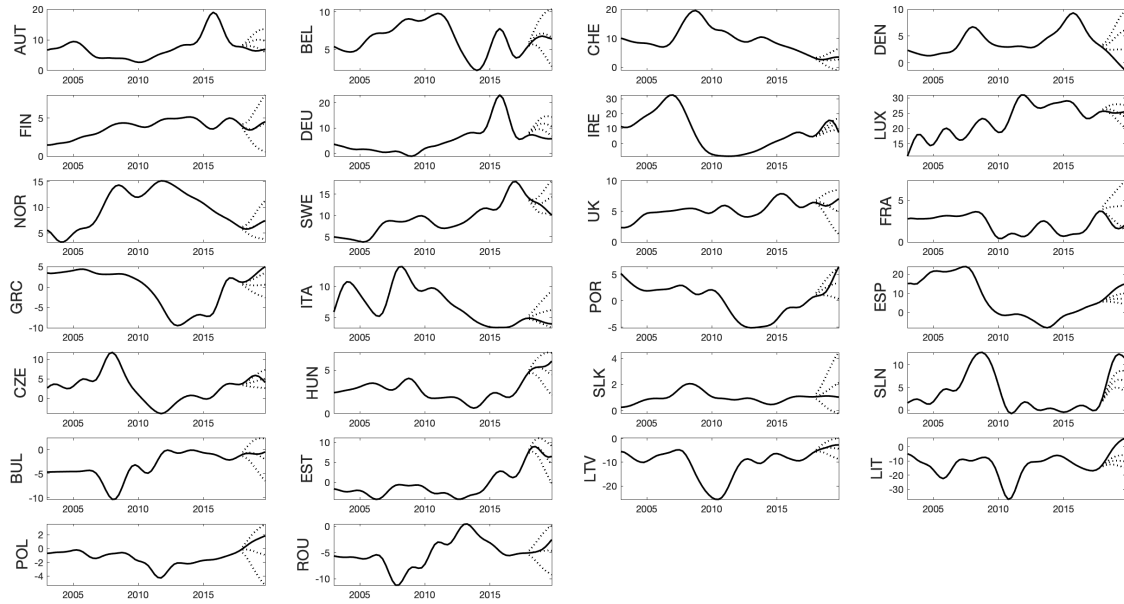


Figure 3: In-sample Forecasting Exercise for Net Migration 'Rates', 2018–19
 These forecasts are produced directly by model (2.3). The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

To show how the error of forecasts compare, we present a selection of the error measures for each forecasts in Table 3, and the calibration of the predictive intervals in Table

4. As the values are expressed per working-age population, there is not a need to additionally weigh any of the estimates. For each group, an arithmetic average is taken for each of the scores across the eight periods analysed and all the countries in that group.

Table 3: Analysis of Forecast Errors, 2018–19: Selected Summary Measures

	Group 1			Group 2		
	Immigration	Emigration	Net Mig.	Immigration	Emigration	Net Mig.
RMSE	0.9826	0.4665	1.1292	0.9140	0.5794	1.2937
MAE	0.8424	0.3950	0.9830	0.7792	0.5049	1.1512
MAPE	4.40%	3.33%	42.69%	5.91%	6.62%	24.61%
Theil's U	0.0245	0.0192	0.0897	0.0358	0.0401	0.1275

	Group 3			Group 4		
	Immigration	Emigration	Net Mig.	Immigration	Emigration	Net Mig.
RMSE	0.5451	1.0456	0.6810	0.9378	1.0209	1.3103
MAE	0.4408	0.8878	0.4798	0.7886	0.8886	1.1037
MAPE	10.93%	11.34%	7.98%	5.22%	6.35%	72.36%
Theil's U	0.0656	0.0684	0.0732	0.0330	0.0329	0.2384

RMSE: Root mean square error; MAE: Mean Absolute Error; MAPE: Mean Absolute Percentage Error.

Table 4: Forecast Calibration: Observation Shares within 67% Predictive Intervals

	Group 1			Group 2		
	Immigration	Emigration	Net Mig.	Immigration	Emigration	Net Mig.
Coverage	0.68	0.84	0.39	0.72	0.72	0.00
Abs. diff.	0.01	0.17	0.28	0.05	0.05	0.67

	Group 3			Group 4		
	Immigration	Emigration	Net Mig.	Immigration	Emigration	Net Mig.
Coverage	0.50	0.72	0.47	0.65	0.81	0.33
Abs. diff.	0.17	0.05	0.20	0.02	0.14	0.34

Coverage: the average proportion of quarterly forecasts where the observation was within the bounds of 67% predictive intervals; Abs. diff.: absolute difference from the nominal coverage probability, 0.67.

The errors in Table 3 show that, from that point of view alone, with an exception of Group 3, smaller errors are obtained for the forecasts of immigration and emigration than for net migration, which supports the notion of forecasting each flow separately

rather than net migration (Rogers, 1990; Bijak, 2010), and using flow-specific data for that purpose. The results presented in Table 4 show a similar picture with respect to the calibration of error distributions, based on the 67% predictive intervals: for immigration and emigration, the empirical coverage of these intervals (that is, the fraction of the observed data points that fell within the intervals) was on average closer to the nominal value of 0.67 than for net migration, except in Group 2.

Overall, in terms of the *ex-post* analysis of errors, short-range forecasts utilising macroeconomic drivers have yielded results with acceptable errors and reasonably well-calibrated 67% predictive intervals between 2018Q1-2019Q4, with immigration forecasts, especially for the traditionally receiving countries, generally outperforming emigration forecasts. This is expected due to the models including additional variables for the home economies, rather than the origin or destination countries of the migrants. To investigate model performance for different countries further, we would need to examine them in more detail to see if there were microeconomic, political, policy, or other reasons that could not be modelled by using these variables, which goes beyond the scope of this study.

2.1.3 Out-of-Sample Analysis of Migration Forecasts

A complete set of quarterly data for all migration flows under study is available up to 2019Q4, with some variables available up to 2021Q2, and forecasts available through the OECD Economic Outlook. In this section, we present out-of-sample forecasts, with migration predicted jointly for 2018–22, so with a five-year horizon, *conditional* on the predicted trajectories of the covariates, wherever available. We proceed with the same vector of endogenous variables as in Section 2.1.1. Figure 4 presents the predictions for immigration, Figure 5 for emigration, and Figure 6 for net migration. The migration data for 2018–2019 are not used for the conditional forecasts, only the covariates, through which we can capture the effects of the COVID-19 pandemic and subsequent recovery.

To deal with the COVID-19 pandemic and the fallout from recent political events, such as Brexit, we thus use conditional forecasts for GDP and country specific variables for the size of the common labour market per working-age population. Hence, for 2020Q1:2020Q3, the common market per working-age capita is set to zero as borders were mostly closed. As far as Brexit is concerned, the United Kingdom is removed from the

EU common market (with an exception of Ireland, thanks to the bilateral arrangements) and, conversely, the common market for the UK from 2021 onwards only includes Ireland.

The modelling results show that there is some effect from the labour market conditions imposed by the pandemic, but until more data for 2020 and 2021 become available, we cannot make more precise statements. It was expected that migration would drop during the pandemic, as borders were closed and a lot of the normal economic activity was suspended, but the trajectory of the post-pandemic recovery remains uncertain. In addition, modelling and forecasting would need to be confirmed based on harmonised migration data: this would require an update to the earlier estimates, such as those of [Raymer et al. \(2013\)](#), which at the moment cover only the period 2002–2008.

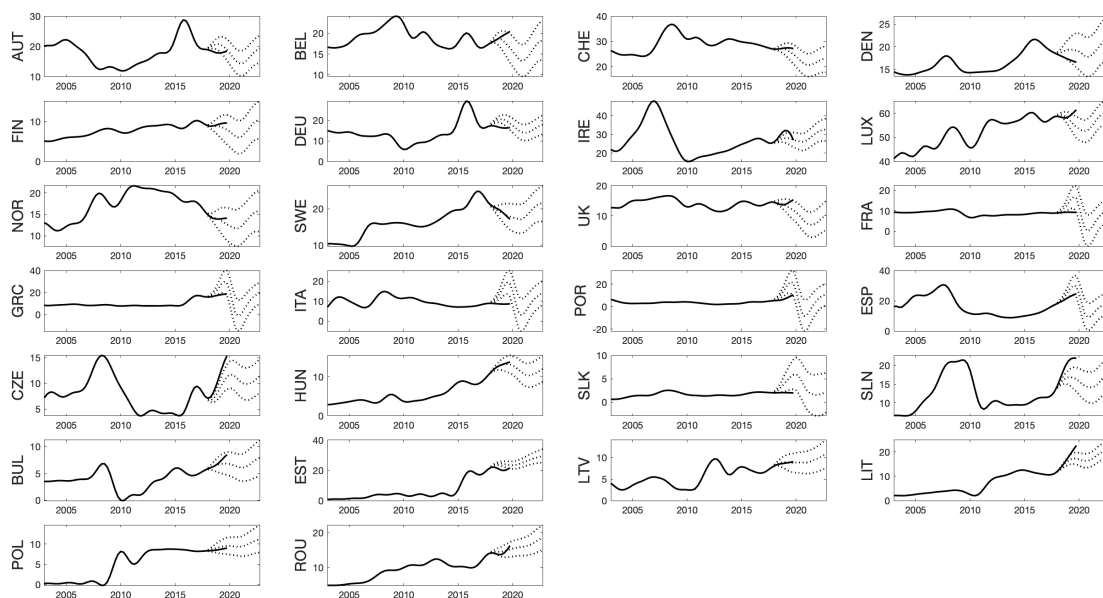


Figure 4: Out-of-Sample Forecasts of Immigration ‘Rates’, 2018–22

The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

As before, the forecasts presented in Figures 4, 5 and 6 have tighter predictive intervals for some countries than others. Here again, this has to do predominantly with the presence of shocks in the past data series. Luxembourg is an understandable anomaly: the volatility of migration rates is driven by the small size of the country being at the same time at the ‘core’ of the EU migration system. Some other instances can be down to significant economic and migration developments in these countries over a relatively short time period, whether it be joining the European common labour market as in the CEE

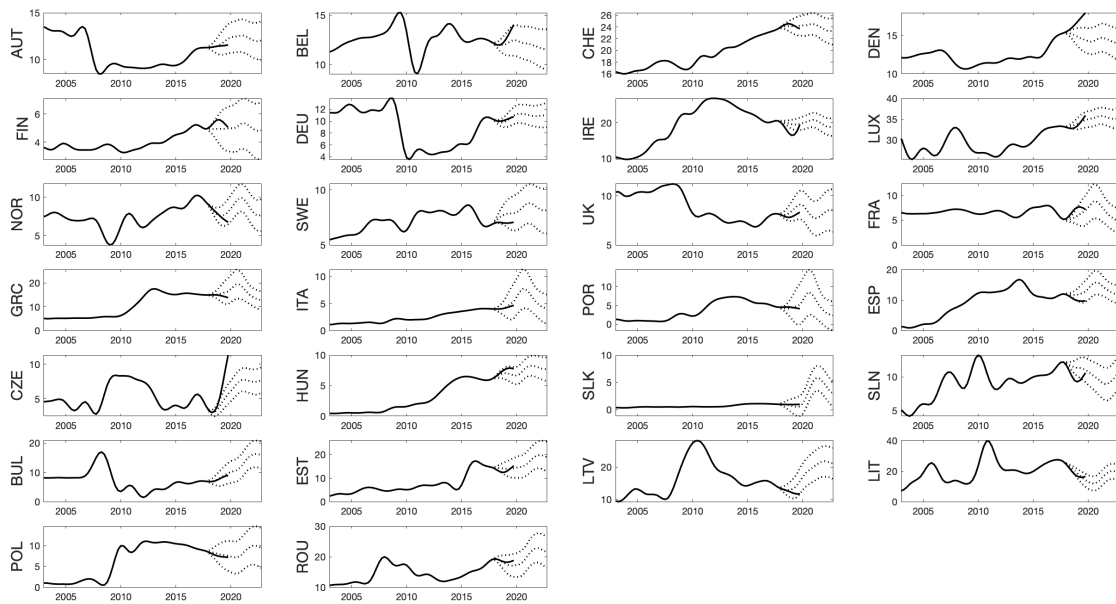


Figure 5: Out-of-Sample Forecasts of Emigration Rates, 2018–22

The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

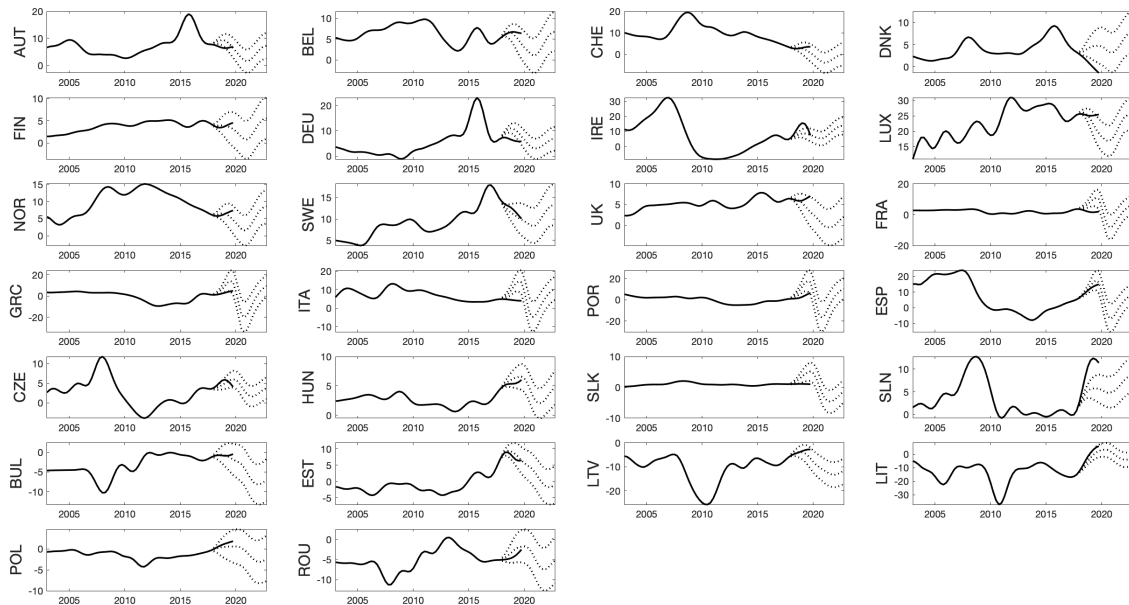


Figure 6: Out-of-Sample Forecasts of Net Migration 'Rates', 2018–22

The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 67% predictive intervals.

countries, or prolonged effects from the financial crises in Southern European countries.⁸

⁸Greece, Italy, Portugal, and Spain, suffered longer effects to the macroeconomy as a result of the Great Financial Crisis 2007–2008 than other countries did, and experienced these effects on the labour

An examination of the raw data helps shed light on this. Greece has two factors that explain the effects on the immigration rate – firstly, during the migration crisis they were a large receiver of migrants due to their location. Greece was the first country where many of the asylum seekers arrived, and where they were meant to claim asylum as per the Dublin Regulation.⁹ The second recent development is the return of working-age Greek citizens who left during the financial crisis in search of better opportunities. There have been fiscal programmes to support their return and as the economy improves from the effects of the financial crisis and COVID-19 pandemic, this can be expected to continue.

As seen in the data for 2002–2019, immigration tends to be quite volatile. However, emigration needs not be necessarily similarly well explained by the data, as it also depends on other countries offering better opportunities. There are outside factors, such as Brexit, which has been one potential reason for the increase in EU emigration from the United Kingdom. The United Kingdom is also strongly affected by the COVID-19 pandemic and the reduction in access to the common labour market, as shown in the immigration forecasts.

Another major question is continuing emigration from the CEE countries from Group 4. These countries have experienced large emigration since joining the European labour market, however, in some cases the rate of departures (for example from Estonia and Poland) is diminishing, so that in the future they may be better described as belonging to Group 3. Their economies have closed the gap to Southern European countries, which would make them attractive destinations when compared to other CEE countries. Other members of Group 4 still have high net emigration, which makes them distinct from the rest of Europe – only potential new members of the EU would have similar characteristics.

Besides providing ready uncertainty statements (predictive intervals), short-range forecasts presented in this section can also help us assess what would happen after exogenous shocks, such as a further expansion of the EU. One of the exogenous variables used in analysis is the size of the common labour market, where significant changes only arise when new countries are added – or removed, as in the case of Brexit. For example, for Austria, this variable increased in 2011 when eight CEE accession countries gained access

markets for a number of years after the majority of countries had recovered to pre-crisis levels.

⁹For further details on the Dublin Regulation, see the [European Commission website](#).

to the labour market, in 2014 when restrictions were lifted for the nationals of Bulgaria and Romania, and in 2020 when the remaining restrictions for Croatian citizens were removed. For the related analysis of wage premium changes and other drivers of migration for CEE migrants, conditional forecasts provide a tool of choice, as demonstrated above.

2.2 The Impacts of Uncertain Net Migration Shocks

To analyse the effects of a possible increase (shock) to net migration at a macroeconomic level, we can also use a panel VAR in the spirit of [d’Albis et al. \(2019\)](#)¹⁰ to consider the effects of net immigration into and net emigration out of European countries. We consider four models, where the central focus for each is the macroeconomy, fiscal budget, labour market, and drivers of migration. To estimate the four models introduced in Section 2.1, and the related impulse response functions (IRFs), we also use the tools provided by [Canova and Ferroni \(2020\)](#) and [Dieppe et al. \(2016\)](#) for mixed-frequency data – here, annual as well as quarterly.

The IRFs presented in this section are shown together with their 67-per cent confidence (*credible*) intervals, demonstrating the uncertainty of the responses of individual variables to migration shocks in a particular scenario. The shocks are of the magnitude of one standard deviation estimated for the observed series, but of course for a policy analysis, this parameter can be arbitrarily changed, depending on the user needs. As is standard in the macroeconomic literature, we look at one-time shocks in the first period under study. As before, default priors are used for estimating the Bayesian panel VAR models.

Model 1: The Macroeconomy

In the first model, besides migration, we look at the variables from the national accounts (expenditure approach), such as the GDP, investment and consumption, with the vector of endogenous variables in Model 1 defined as follows (all variables are listed in Table 1):

$$y_t = [NM_t, C_t, X_t, GDP_t]'$$

¹⁰[d’Albis et al. \(2019\)](#) examines OECD countries. In comparison, we drop the OECD Pacific (Australia, Japan, New Zealand, and South Korea), Canada and the United States. We continue with 13 of the 15 OECD European countries and add 13 other European countries.

The IRFs for Model 1 are shown in Figure 7. The axes are normalised so that we can see the relative scale of the effects on each of the four groups of countries. The responses to the net emigration shock in Group 4 are inverted to aid comparison of responses between different groups of countries. One period is equal to one quarter. At $t = 0$ there is a shock, or rather increase, to net immigration (emigration for Group 4). The IRFs show the responses to each of the variables listed in the vector of endogenous variables.

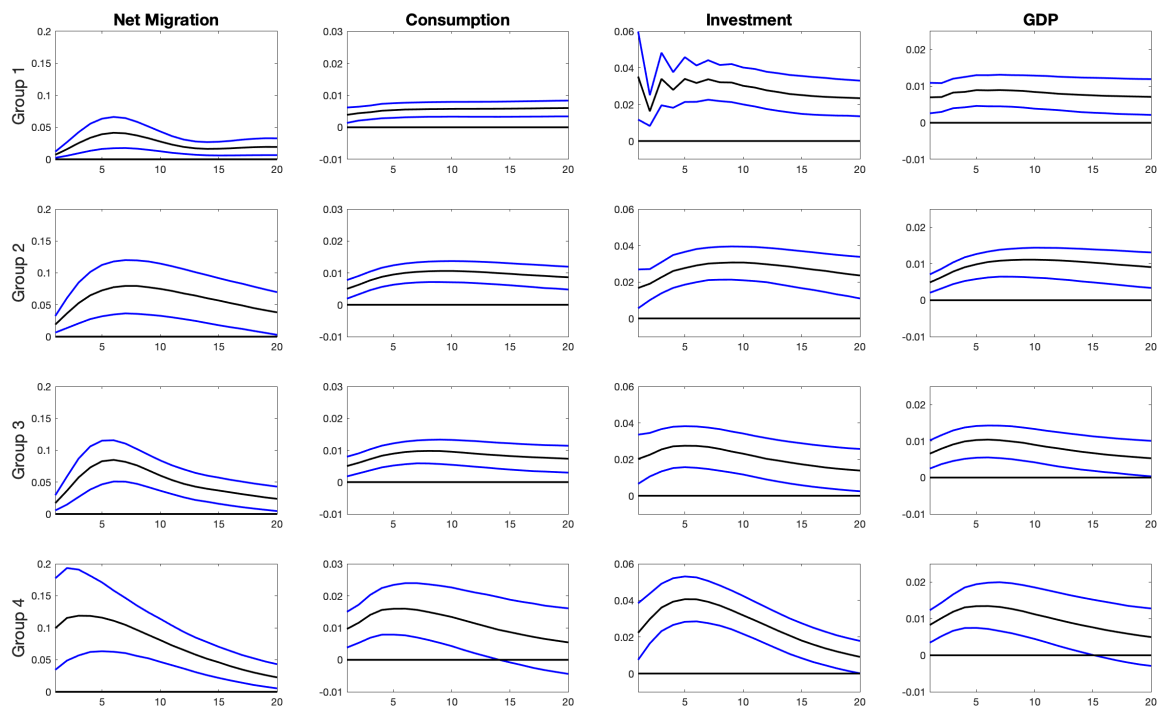


Figure 7: Impulse Responses for a Net Migration Shock in Model 1

The responses to a one standard deviation net immigration shock for Groups 1-3 and a net emigration shock for Group 4. The vertical axis identifies the responses in percentage deviations from trend. The horizontal axis identifies the quarter after the shock, up to five years (20 quarters). The column identifies the response of a variable which is in the column heading, and the row corresponds to the country group in the row heading. The responses to the variables of Group 4 are inverted to aid comparison.

An important thing to note is the size of net migration shock in each group. The shock size in Group 1 is relatively smaller than any of the others, while in Group 4 it is the largest. Overall, the impulse responses indicate that, in macroeconomic terms, net immigration proves to be expansionary, and net emigration contractionary. The effects on investment are particularly differing in magnitude between the different groups of countries. The increase in Group 1 and large decrease in Group 4 shows that investment is relatively sensitive to changes in migration flows. The large fall in private investment

for Group 4 countries suggests that the economy is particularly vulnerable to financially active individuals leaving these countries.

Model 2: The Fiscal Budget

The second model considers the effects of migration on the fiscal budget, and vice versa. The variables included are similar to the ones in [d’Albis et al. \(2019\)](#). The vector of endogenous variables therefore in Model 2 is set as follows:

$$y_t = [NM_t, GovPur_t, Transf_t, NetTaxRev_t, FisBal_t, GDP_t]'$$

The fiscal balance is calculated thus, with hats denoting impulse responses of variables ([d’Albis et al., 2019](#)): $100 * (NT/GDP(\widehat{NetTax}_t - \widehat{GDP}_t) - GP/GDP(\widehat{GovPur}_t - \widehat{GDP}_t))$. The impulse response functions corresponding to migration shocks are shown in Fig. 8 :

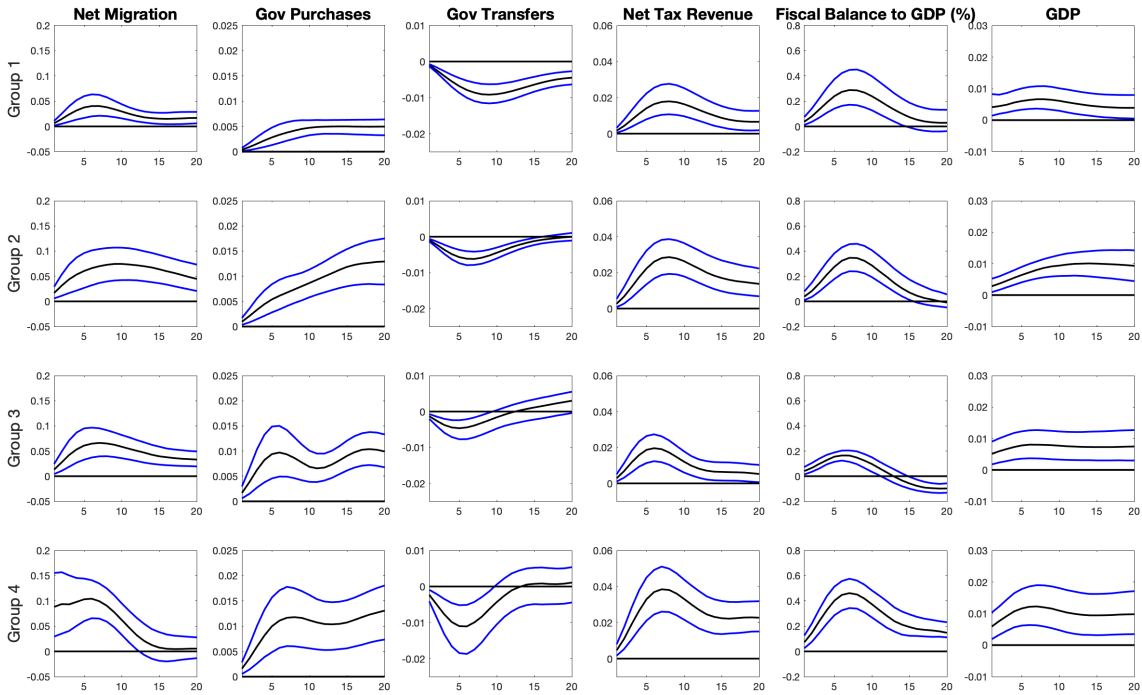


Figure 8: Impulse Responses for a Net Migration Shock in Model 2

The responses to a one standard deviation net immigration shock for Groups 1-3 and a net emigration shock for Group 4. The vertical axis identifies the responses expressed as percentage deviations from trend. For $FisBal_t$, the changes are expressed in percentage points. The horizontal axis identifies the quarter after the shock, up to five years (20 quarters). The column identifies the response of a variable which is in the column heading, and the row corresponds to the country group in the row heading. The responses to the variables of Group 4 are inverted to aid comparison.

The results show that net immigration is expansionary to the fiscal budget, by increas-

ing (net) tax revenues and reducing government transfers.¹¹ The effects on government transfers are prolonged for Group 1, whilst the effects for Groups 2–4 only decrease for three years. For the Western European nations, in Groups 1 and 2, the improvement in the fiscal balance is larger and longer-lasting. For Group 3, the effects are only short-lasting, and in addition, for Group 4, they can be very large. The effects for Group 4 would be concerning from a policy perspective, as they could indicate brain drain, or at least that the higher earners and thus larger contributors to government finances are emigrating. Group 1 gains the most in terms of net tax revenue (tax revenue minus transfers), as well as a relatively small increase in government purchases. These results are consistent with existing literature (d’Albis et al., 2019; Furlanetto and Robstad, 2019). The increase in tax revenues occurs both through direct and indirect means – directly from the receipts of consumption and labour taxes due to new migrants being more likely employed, and indirectly through the stimulation and expansionary effects on the economy.¹²

Model 3: The Labour Market

Expansionary or improving labour markets are some of the largest migration pull factors within Europe. As such, in the third model, we incorporate the unemployment and employment rate as the key labour market-related drivers of migration.¹³ The vector of endogenous variables in Model 3 is therefore as follows:

$$y_t = [NM_t, Unemp_t, Emp_t, WageSal_t, GDP_t]'$$

The impulse response functions are shown in Figure 9. This analysis is potentially one of the most important from a policy (and political) perspective, since one key argument of anti-immigration parties is that immigrants take the jobs of natives and decrease or suppress wages, at least in some segments of the labour market.

The results for this model show that increases in net immigration (respectively, net emigration) reduce (increase) unemployment and increase (decrease) employment. The

¹¹Further studies with *Tax Revenue* rather than *Net Tax Revenue* show increases as well.

¹²As shown in Figure 7, there is an increase in private consumption – as tax rates remain largely unchanged (or even increase during expansionary periods) – higher levels of consumption generate more income for governments. There is an equivalent argument for labour market revenues (see Figure 9).

¹³The unemployment rate is calculated the percentage of unemployed persons of all economically active persons. Employment rate is the employment as a percentage of all working-age people.

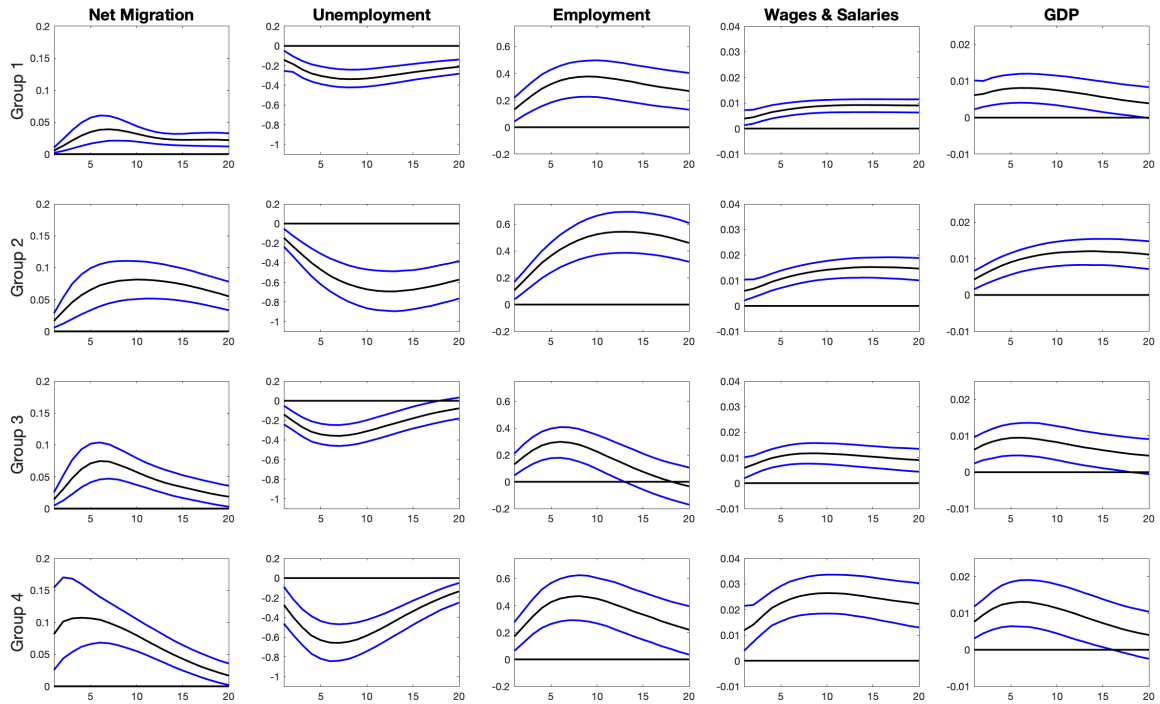


Figure 9: Impulse Responses for a Net Migration Shock in Model 3

The responses to a one standard deviation net immigration shock for groups 1-3 and a net emigration shock for group 4. The vertical axis identifies the responses in percentage deviations from trend. For $Unemp_t$ and Emp_t , the changes are expressed in percentage points. The horizontal axis identifies the quarter after the shock, up to five years (20 quarters). The column identifies the response of a variable which is in the column heading, and the row corresponds to the country group in the row heading. The responses to the variables of Group 4 are inverted to aid comparison.

changes for unemployment significantly differ from zero for all countries, with between 0.2% decrease on impact, rising to up to 0.8% after three years (Group 2). The increase in employment helps demonstrate that the effects of migration do not cause an exit of people from the labour market, but rather lead to an increase of overall labour supply. For Group 3, the impacts are much quicker to return to average (un)employment levels, whilst the wages and salaries show more long lasting effects. On the whole, the improvements in the labour market for net immigration countries, are mirrored by the labour market deterioration for the countries with net emigration. The effects visibly differ from zero, even allowing for errors, and are persistent.

The reduction in unemployment is consistent with the findings of [Furlanetto and Robstad \(2019\)](#), who looked at the example of Norway. One reason is that immigrants are likely to enter the labour market as employed – would-be immigrants will likely

search for a job in their current location then move to the receiving country once they have found employment. In addition, the peaks in the responses to immigration shocks are after generally after two years (8 quarters), which allows the business cycle effects, or feedback from expansion to GDP, to play a role in expanding the economy. The results are in line with the theoretical and empirical models presented in [Barker \(2020a\)](#).

At the same time, the effects on unemployment and wages, in particular, are also of interest due to the [Borjas \(2006\)](#) and [Card \(2005\)](#) debate about the complementary (or, conversely, substitute) nature of migrant and native labour. Whilst this debate comes from a microeconomic standpoint, rather than a macroeconomic one, it is nonetheless relevant, despite the focus on the United States, where the role of migrants can be different to that in Europe, especially in two key aspects. Firstly, the migration policies differ: the European common labour market makes achieving legal migration status much easier than visa-based routes. Secondly, the skill level of migrants provides another key distinction factor, even more notable on a macroeconomic scale than on a microeconomic one. If a new migrant earns an above-average wage, this increases the average wage slightly, and to a similar extent increases the level of complementarity between natives and migrants.

As this is such a country-dependent topic, highly- or low-skilled migrants may be either substitute or complementary, depending on the exact circumstances. The challenge of incorporating this issue into a formal analysis remains beyond the scope of this report, although arguably issue this constitutes yet another source of uncertainty, this time in the theoretical description of migration patterns, their drivers and impacts. We explore some of these aspects in [Section 3](#) in the context of job automation scenarios.

Model 4: Drivers of Net Migration

There is a large literature dedicated to examining the macro-level pull and push factors and drivers of migration, including [Massey et al. \(1993\)](#) or [Grogger and Hanson \(2011\)](#). Still, most of the pull factors of immigration operate at a microeconomic level, even those with the labour market focus. Workers are more likely to migrate to countries that have better employment opportunities, and higher labour income, which enables them to experience higher consumption levels. We investigate this in the fourth model, with focus on the wage premium variable. The wage premium is particularly effective for Group 1

and Group 4 countries, as it explains the large wage gaps observed for these countries. The vector of endogenous variables in Model 4 is set as follows:

$$y_t = [\text{NM}_t, \text{Cons}_t, \text{WagePre}_t, \text{Emp}_t, \text{GDP}_t]'$$

For the final model, including the drivers of net migration, the impulse response functions are presented in Figure 10. Here, the effect of a migration shock on wage premium is only significant for Group 1. The underlying model has been adjusted from the respective model used to forecast migration in Section 2.1, with the wage premium substituting the wages and salaries used therein.¹⁴ No other country group seems to be noticeably closing the wage premium gap, and there may be a slight increase of the gap for Group 4. From a broader perspective, continued migration into Group 1, increasing the wage premium, may be counterproductive for economic convergence between different country groups.

In evaluating the role of migration on the macroeconomy, important data limitations need to be mentioned. A significant portion of migration can be attributed to economic (labour) migration. Since a migrant needs to apply for a job to gain employment, this must be in response to a posted vacancy. There are data available for vacancies, however, the required detail does not cover the required sample period or enough countries. The number of vacancies would be an interesting indicator for the effect of the financial crisis and pandemic in particular: while fiscal policies employed by governments limited the impacts of the crisis on unemployment levels, vacancies were significantly impacted.

These limitations notwithstanding, the methods presented above, based on the analysis of Bayesian VAR models, with a set of co-varying indicators informed by theoretical considerations about migration drivers, provide a useful way of assessing the uncertainty of short-term future migration flows and their impacts. Conditional forecasts allow for making informed ‘what-if’ statements, and the analysis of impulse response functions enables stress-testing different elements of the migration and macroeconomic systems. The short-term predictive errors presented in this section proved to be reasonable in terms of their magnitude and calibration, while at the same time identifying idiosyncratic features of migration trends and patterns in individual countries. Further work might addition-

¹⁴In Section 2.1, wages and salaries were used to estimate immigration and emigration as they are conditional on the macroeconomy of the country rather than that of the EU-15 in addition.

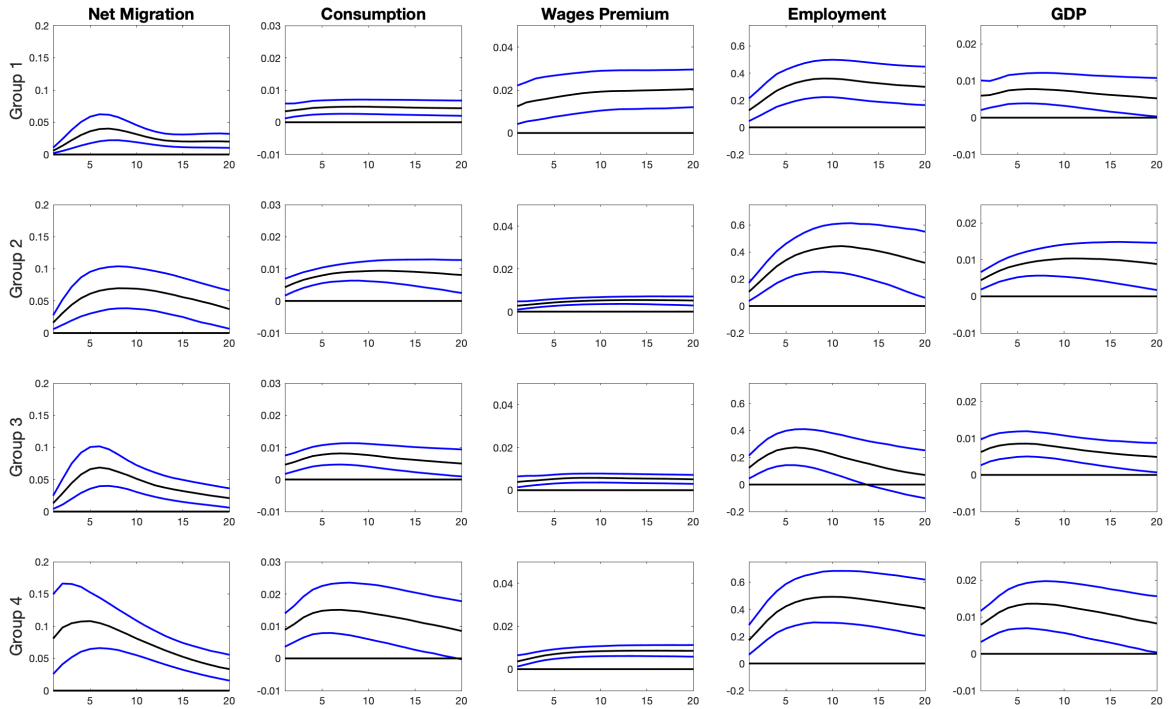


Figure 10: Impulse Responses for a Net Migration Shock in Model 4

The responses to a one standard deviation net immigration shock for Groups 1-3 and a net emigration shock for Group 4. The vertical axis identifies the responses in percentage deviations from trend. The horizontal axis identifies the quarter after the shock, up to five years (20 quarters). The column identifies the response of a variable which is in the column heading, and the row corresponds to the country group in the row heading. The responses to the variables of Group 4 are inverted to aid comparison.

ally involve assessing some aspects of the model uncertainty, including the sensitivity to the choice of priors, variables, and estimation methods. Having a harmonised migration dataset would additionally strengthen the potential of these methods by ensuring comparability of different indicators.

On the whole, Bayesian VAR methods presented here enabled assessing, even if partially, both the aleatory, as well as epistemic features of the migration uncertainty, both in the context of short-term predictions and responses to shocks. The former is approximated by the stochastic component of the VAR process, while some aspects of the latter by the parameters of the model (and their uncertainty), which can be identified from the data and – where available – prior knowledge, for example elicited from experts. In the next section, we try to shed more light on some of the other aspects of epistemic uncertainty, related to the *mechanisms* of migration dynamics, by analysing theoretically-informed models of migration processes and their potential in scenario-setting.

3 Short- to Mid-range Scenarios: Macroeconomic DSGE Models

Dynamic stochastic general equilibrium (DSGE) models are structural theoretical models which are formulated at the macro level, but have microeconomic foundations. These models are not widely known in population and migration studies beyond macroeconomics, and yet they offer an appealing way of describing a theoretical mechanisms by using formal models across multiple levels of aggregation. In this way, DSGE models provide one possible answer to the main challenge identified by [Burch \(2018\)](#) in contemporary demography, which is the paucity of formal theoretical reflection.

The variables in DSGE models can be perturbed by a series of (predominantly macroeconomic) shocks, the effects of which can give some indication as to the extent of uncertainty and sensitivity of the underlying processes. A natural extension of this analysis, which remains beyond the scope of the current, illustrative study, would be an explicit modelling of *uncertainty shocks* ([Bloom, 2009](#)) – the changes to volatility (second- and higher-order moments of the processes under study) or to the uncertainty of decisions inherent in the modelled microfoundations. A brief introduction into their use in the context of migration modelling is in [Barker and Bijak \(2020, Sect. 4 and pp. 88–90\)](#).

In this section, we present two models, one of which ([Section 3.1](#)) examines migration as an endogenous process, in the context of migration scenario-setting, and one ([Section 3.2](#)) as an exogenous process, with focus on the impacts of migration shocks on the wider economy. In the former case we examine the migration decisions in a two-country model calibrated to the economies of Germany and Poland, and in the latter we look at the effects of migration in a small open economy model. In both instances, the scenario narrative – the context in which the modelling exercise is embedded – is related to the automation of low-skilled jobs and the impact it may have on different migration streams. In [Appendix B](#), we describe more detailed features of the models by presenting the main microeconomic foundations and their linkages to macroeconomics.

3.1 Job Automation Scenario: Endogenous Migration and Robots

In this section, we look at a range of ‘what-if’ short- to mid-term scenarios focused on the effects of job automation on migration flows. Automation capital is used in an equivalent form to traditional physical capital. Here, we analyse the effect of automation on migration flows in a two-country model similar to [Barker \(2020b\)](#)¹⁵. The primary country of interest, modelled on the German economy, is a net receiver of migrants from another country, modelled on Poland as the largest representative of CEE countries, which is also – or has been until recently – a net sender of migrants. In this section, we will therefore refer to these countries as ‘Germany’ and ‘the East’, for brevity. Rather than a migration shock changing the stock of migrants, in this model workers are able to endogenously choose their location. The decision is based on labour market drivers, existing employment conditions in their home country and those at the destination.

3.1.1 Model and Scenario Outline

The two countries differ in terms of their economic profiles. [Figure 11](#) illustrates the decisions made by different agents in each country. Household members are all economically active so can either be employed or unemployed. In Germany, each household feature highly-skilled and low-skilled members who cannot transfer between the forms of employment. In the East, the skill type is assumed to be homogeneous, and the workers are able to be employed in the East or they can choose to migrate to find work in Germany.

There are four types of workers across the two countries. In the East, there is one type of labour $j = E$. In Germany, there is highly-skilled labour $j = G$, low-skilled native labour, $j = S$, and migrant labour $j = M$. We assume that there is no migration from Germany to the East of native Germans, only return migration. Highly-skilled German workers can only work in section $j = G$, and equivalently low-skilled German workers can only work in $j = S$, which is the secondary type of employment for the natives. For worker types $j \in [E, S, M]$, the firms in the respective countries can employ a worker or a robot to do the job. They are perfect substitutes. The modelling of automation is based on the model presented by [Leduc and Liu \(2019\)](#).

Firms employ workers. In Germany, the highly-skilled workers do not have jobs that

¹⁵The full version of this research is available in [Barker \(2021b\)](#).

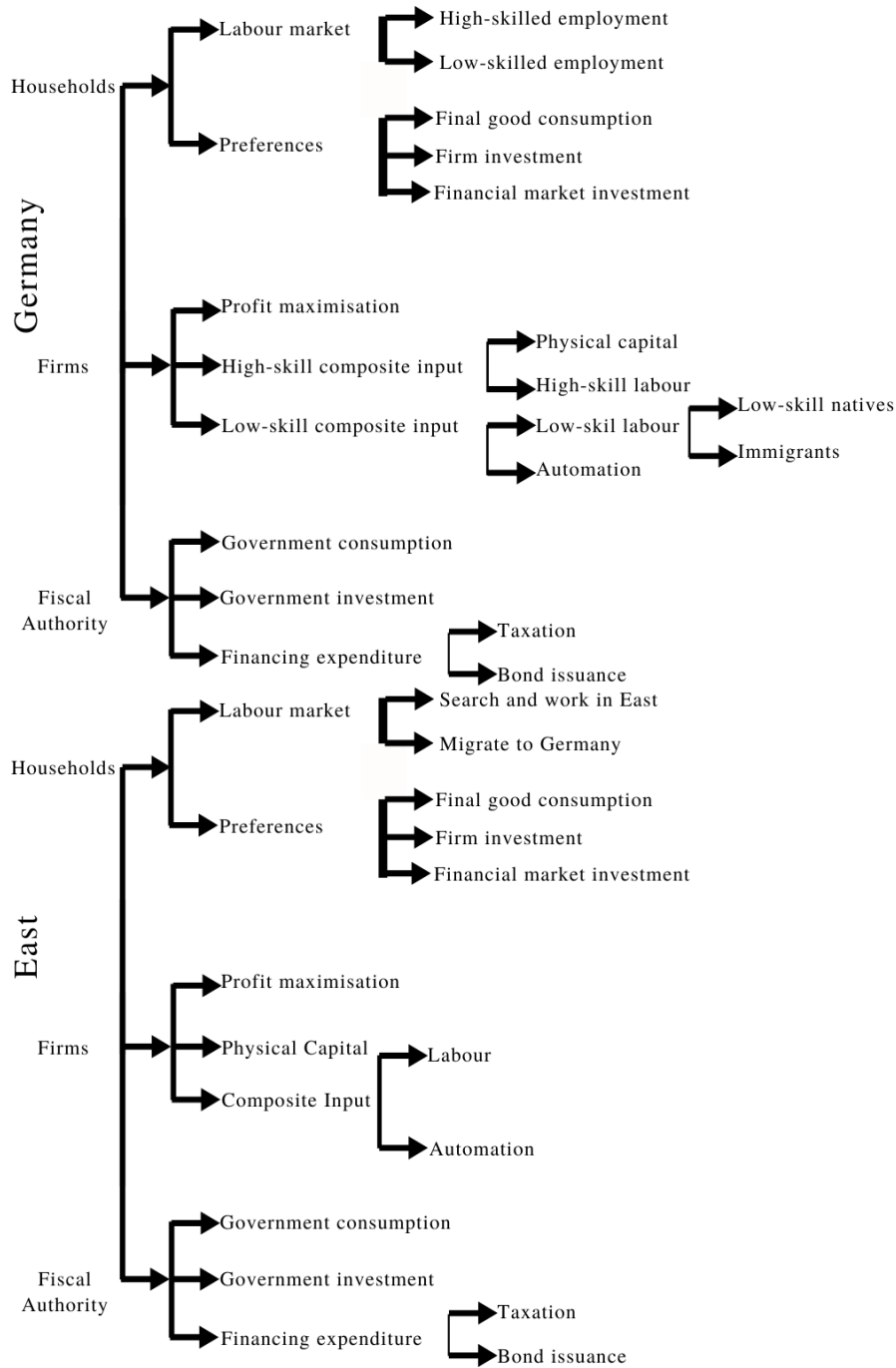


Figure 11: DSGE with Endogenous Migration: Overview of the Model Setup

can be replaced by robots, such that they are complementary to the physical capital. The low-skilled composite input, in turn, consists of low-skilled workers and robots, which in this case are assumed to be perfect substitutes. We include the distinction of highly-skilled workers being complementary to capital and low-skilled workers being perfect substitutes to robots, so that there assumption of capital-skill complementarity holds and is close

to real-life scenario. In fact, some low-skilled jobs have been fully automated, but this extent has not reached the highly-skilled inputs to production. Berg et al. (2018) present a model with a similar set-up, though robots are *imperfect* substitutes to low-skilled labour. In the East, the physical capital can be substituted with the composite input of robots and labour. The firm in Germany has a production function of the form:

$$y_t^G = \psi_t^{a^G} \left(\aleph (I_t^G)^{\alpha^G} + (1 - \aleph) (L_t^{MS})^{\alpha^G} \right)^{\frac{1}{\alpha^G}},$$

with the low-skilled inputs, L_t^{MS} , of low-skilled native labour $j = S$, low-skilled migrant labour $j = M$ and their corresponding robot substitutes A_t^S and A_t^M . Again, robots and low-skilled labour are assumed to be perfect substitutes. The intensity of highly-skilled input, I_t^G , is given by \aleph , with the elasticity of substitutions between the highly- and low-skilled inputs given by $\varrho_{IG, LMS}$, where $\varrho_{IG, LMS} = \frac{1}{1 - \alpha^G}$.

$$L_t^{MS} = \left[n_t^M h_t^M \psi_t^{l^M} + A_t^M \psi_t^{A^G} + n_t^S h_t^S \psi_t^{l^S} + A_t^S \psi_t^{A^G} \right]$$

The highly-skilled services provided to the firms come from high;y-skilled natives $j = G$ and the physical capital, k_t^G :

$$I_t^G = \left[\nu^G (n_t^G h_t^G)^{\Phi^G} + (1 - \nu^G) (k_{t-1}^G)^{\Phi^G} \right]^{\frac{1}{\Phi^G}}$$

Where the intensity of highly-skilled labour is governed by ν and the elasticity of substitution between the highly-skilled labour and traditional physical capital is given by $\varrho_{n^G h^G, k^G} = \frac{1}{1 - \Phi^G}$.

The fiscal authority (the government) consumes, invests and sources their expenditures through taxation or issuing bonds to the financial market. We assume that prices are perfectly flexible, and the interest rate is endogenously determined, so there is no role for monetary policy.

3.1.2 The Migration Decision

Agents residing in the East have the option to migrate to Germany in search of work. They make an inter-temporal decision using their knowledge of current and discounted future labour market conditions, as demonstrated in Figure 12. In each market, they take the probability of finding employment ζ_t^j in labour market $j \in [E, M]$ for the East

and migration markets respectively, which will give them income w_t^j . Then, they take into account the expectations for the next period of remaining employed or becoming unemployed once more. Alternatively, being unemployed with a probability of $1 - \zeta_t^j$, and receiving unemployment benefits ub^j , then account for the next period of finding employment or remaining unemployed.

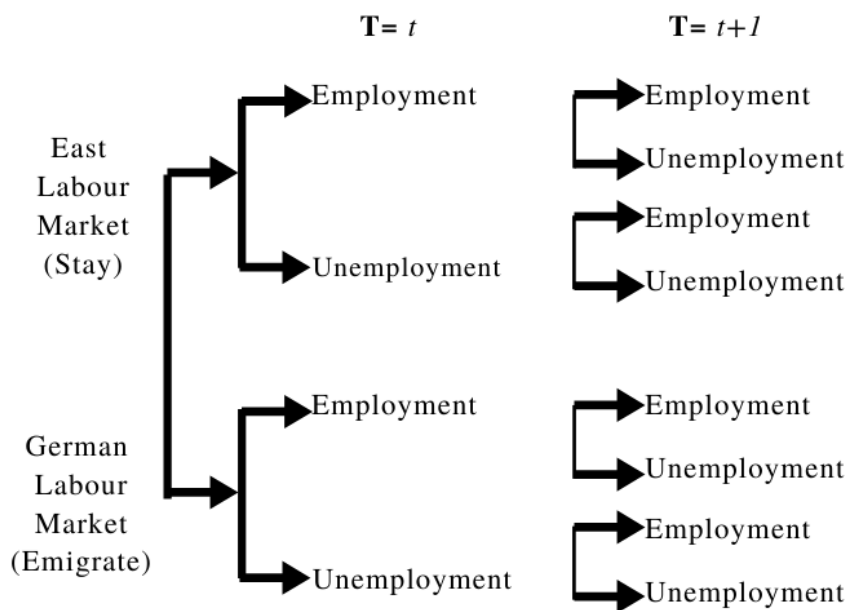


Figure 12: Framework for Modelling Endogenous Migration Decisions

The migration decision is changeable and reversible in any period, and depends on the search intensity. An increase to search intensity results in an increase of unemployed agents searching for employment in the East, whilst a negative response to search intensity is an increase in the number of agents searching for employment in Germany.

3.1.3 Migration Scenarios: Selected Results

The scenarios that are produced by DSGE models can be expressed in terms of ‘what-if’ responses to a range of shocks to the other parts of the macroeconomic system. In Figure 13 we present selected results of applying the DSGE model described above for that purpose, which has been calibrated and estimated based on the German and Polish data. There are six shocks presented: the total factor productivity (TFP) shock in Germany and in the East, an increase to the productivity of robots in Germany and in the East,

and an increase to low-skilled labour productivity for $j = [S, M]$. As these scenarios result from theoretical modelling, they are standalone, rather than being equipped with additional uncertainty bounds, although these could in principle be obtained either by estimating a corresponding VAR model in parallel, or, as suggested above, by explicitly modelling of shocks to the higher-order uncertainty (volatility) of the observed processes (Bloom, 2009).

The impulse responses show the percentage change (except from the variables already in percentages, e.g. unemployment) from steady state, or the state of the economy that is not subject to any shocks. At $t = 0$, a shock occurs to one part of the economy, with propagation and amplification resulting in effects throughout the economy. For simplicity, only one shock can occur at a time, such that their cross-correlations are equal to zero.

As all agents are economically active, there is no equivalent to enjoying leisure (as in Section 3.2) or being out of the labour market. There are small effects of TFP shocks on the low-skilled German employment, with larger ones for the highly-skilled natives. Both of these groups are relatively unaffected by migration shocks, with TFP in Germany, and labour productivity having the greatest impacts. The TFP shock in Germany has a slight negative effect on the low-skilled employment of natives whilst for highly-skilled, the effect is positive. If we look at the employment of robots after a TFP shock, this partially explains why: following a TFP shock, there is a much larger increase of the number of – robots and similarly of migrants. However, migrants have lower wages and bargaining power so in this model would be willing to accept lower wages. As for citizens of the East, their employment opportunities are located in two economies so the pull effects of one is detriment to the other. When there is a TFP shock in the East, some workers return there, as there are greater job search opportunities. Notably, the increase in productivity of robots in Germany has an insignificant effect on employment, whereas it does positively effect output. However, in the East, an increase in productivity of robots has a negative effect on employment.

The labour productivity shock of low-skilled workers in Germany (natives and migrants) has a similar effect on the employment of highly- and low-skilled natives but a significantly larger one for migrants. The impact on both highly- and low-skilled sectors follows through to the wages, though it is only short lived. The migration shock has

slightly different effects than before, as it only have a negative effect on the wage of migrants. The setup of the model can provide an explanation for it, as workers of the same skill level in the previous model were assumed to be imperfect substitutes, whereas here they are perfect substitutes (see also the discussion in Section 2.2).

The larger effects for migrants from the East on employment and unemployment are due to the fact they have two markets to search in, whereas the labour markets of highly- and low-skilled German natives are constrained by the overall population size. Employment matches between vacancies and unemployed citizens are made using the matching technology which respects the search and matching frictions in force. It is not necessarily the fact that migrants and workers in the East are fired more, as they all have an exogenously determined job destruction rate. What is affected is how quickly job matches are made. Highly-skilled workers are assumed to have access to better matching technologies than low-skilled workers, and natives to better ones than migrants.

As we examine the effect on robots, the responses for the low-skilled German workers and migrant jobs are relatively similar. Though the shocks which affect migrants more display different paths. For example, when there is a TFP shock in the East, there is a larger increase in the number of robots for the migrant sector than the low-skilled natives. In the East, however, where there is a much lower stock of robots per worker, the responses in each of the shocks are much larger, and all positive. As shown in the search intensity, only the TFP shock in East causes a large shift to employment and searchers in East, all the others have workers migrating which requires robots to replace the workers. A TFP shock would increase the number of robots as it is expansionary across the firm.

The last variable subject to shocks in this scenario is the automation ratio, A_t^G/A_t^E . The lower stock of robots in each and direction of workers increase the need for robots more in East than in Germany, and as such the gap closes. The problem is two-fold for countries in East. There is such a wage gap between countries such as Poland relative to Germany that even with the presence of robots in the short-run, there is still the incentive to migrate, as a lower wage due to job automation in Germany is still far greater than what the migrants would receive at home, as shown by the wage premium in Table 2.

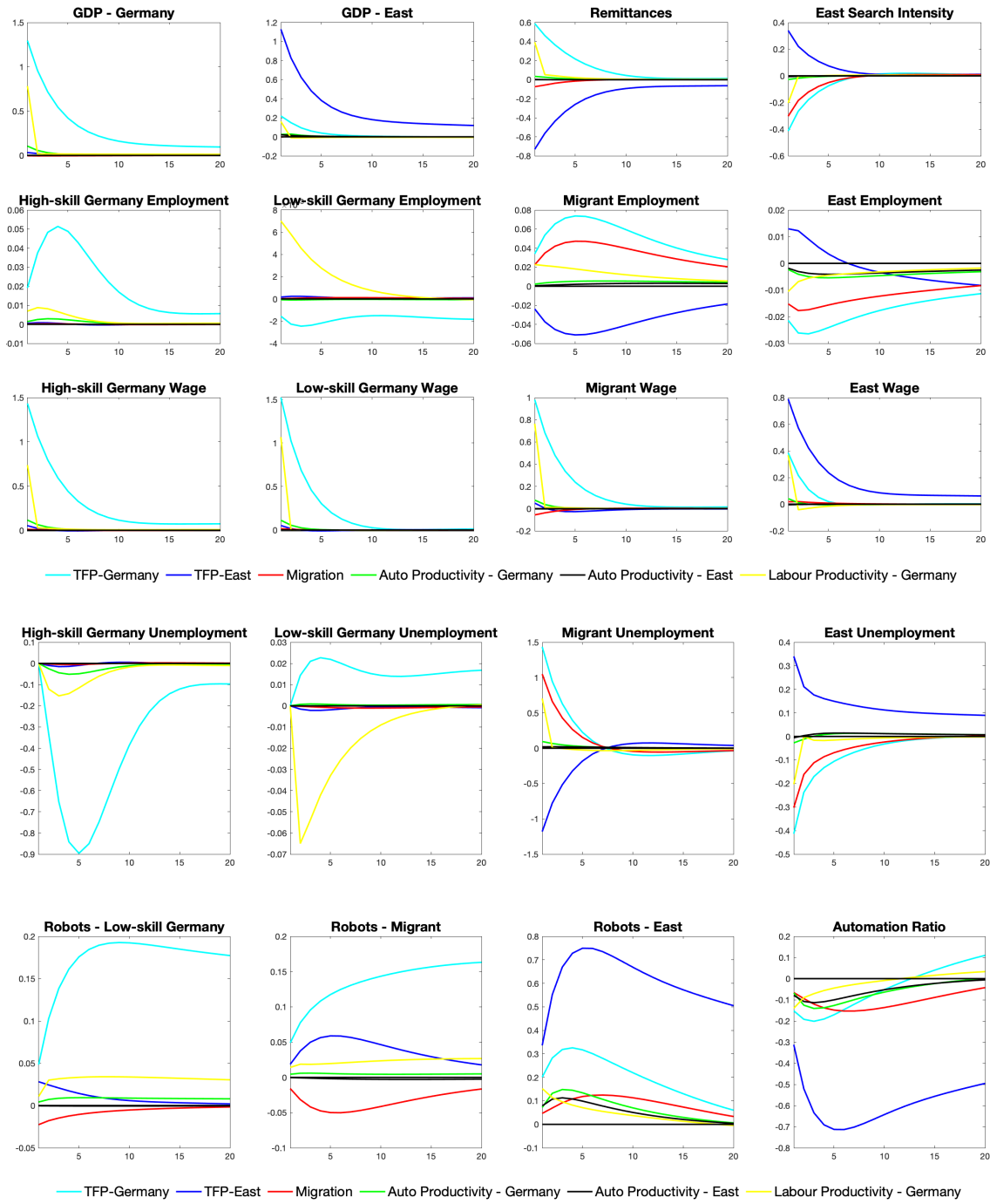


Figure 13: Endogenous Migration Model: Selected Impulse Response Functions

The horizontal axis identifies the quarter. The vertical axis shows the deviation from steady state. The economy aggregates (output, skilled and low-skilled inputs, and consumption) are in per capita terms, whilst employment and unemployment are in aggregate terms rather than per household, i.e. N_t^j . **Key:** Cyan: TFP Germany; dark-blue: TFP East; red: Migration; green: Robot productivity Germany; black: Robot productivity East; yellow: low-skilled labour productivity in Germany

Scenario Extension: Increasing Employment of Robots

In an extension to the model, we introduce a variable growth rate of the number of robots. The increase of use of robots is present anyway, to which there is an increase to the growth rate of automation (i.e the steady state growth rate of automation is not equal to one). Here we provide two shocks, one to each country, that increases the growth rate of automation. What this model has shown is that as the gap closes between wages, migration will be lower and when there are increases to the automation stock in Germany, fewer workers will be attracted to the market as there are fewer opportunities.

In substantive terms, the results of the scenarios presented above, summarised in Figure 14, show that a shift towards automation of low-skilled jobs, whether with robots as perfect or imperfect substitutes of workers, has negative consequences for low-skilled workers and the likely migration flows. With robots as perfect substitutes, there is another option than posting a vacancy. Anything that makes robots relatively more productive and/or attractive, or equivalently workers less productive/attractive will see a shift towards further automation of jobs. The reduction in need for workers to do the lower-skill jobs that are repetitive results in lower migration flows.

In methodological terms, the scenarios above enabled the analysis and stress-testing of various ‘what-if’ policy questions in response to uncertain shocks, the magnitude of which, as in Section 2, can be varied by users, depending on their needs. One crucial advantage of models such as DSGEs is that, as they include microfoundations, modelled at the level of individuals, households and firms, and their decisions, the resulting scenario dynamics is coherently defined across different levels of aggregation.

3.2 Job Automation Scenario: Exogenous Migration and Automation Capital

As was the case with VAR models presented in Section 2.2, the DSGE framework can be also used for examining the impact of migration shocks on the economy and different groups – migrants and non-migrants alike. An example, still remaining in the framework of the broader job automation scenario, is presented in this section. The methods proposed here can again serve as stress-testing tools to examine possible responses of other variables of interest to migration shocks of a pre-defined magnitude. In reality, this

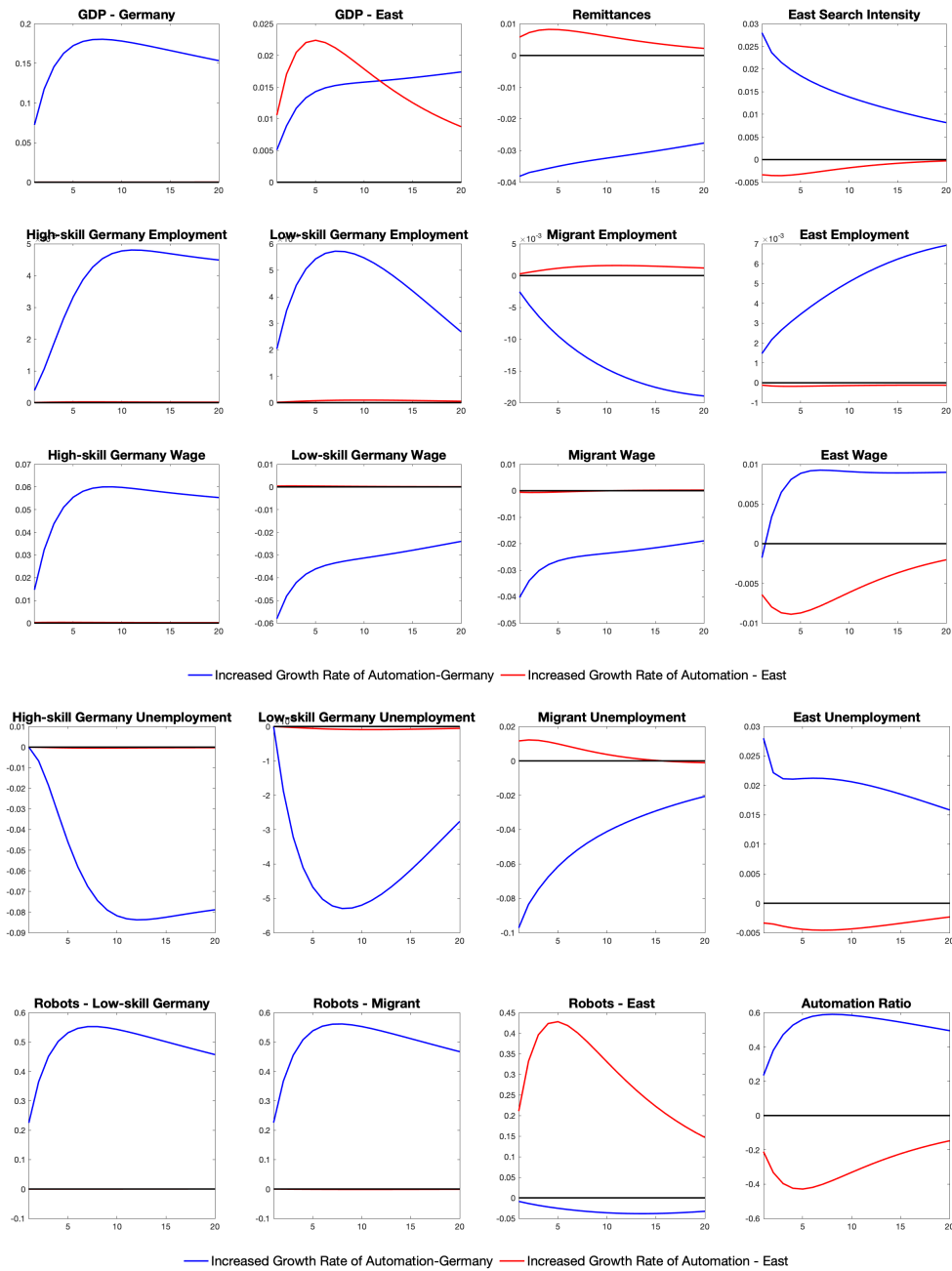


Figure 14: Endogenous Migration Model: Impulse Response Functions, Automation Growth Rate

The horizontal axis identifies the quarter. The vertical axis shows the deviation from steady state. The economy aggregates (output, skilled and low-skilled inputs, and consumption) are in *per capita* terms, whilst employment and unemployment are in aggregate terms rather than per household, i.e. N_t^j
Key: dark-blue: growth rate of automation (Germany); red: growth rate of automation (East)

magnitude is of course uncertain, but by examining a range of possible scenario-based ‘worlds’ (or by modelling the higher-order uncertainty explicitly) the analysts and users can gain insights as to the seriousness of the shocks and their impacts.

3.2.1 Model Outline

In our example, in the context of the broad scenario narrative related to job automation, there are two aspects of automation that are of importance to addressing this research question: skill level and the magnitude of international flows. The first issue is how each type of skill level is affected by the automation process. The second is, how the increasing levels of automation affect international migration flows?

To answer these questions, we model a small, open economy, calibrated to the German data. We assess the impact of migration on the four types of households: highly-skilled natives, low-skilled natives, highly-skilled immigrants, and low-skilled immigrants. The highly- and low-skilled households provide different forms of labour to the firms, where natives and immigrants at the same skill level are imperfect substitutes. Figure 15 provides an outline of the model, and the summaries of its different elements are presented below. The full model is described in detail in [Barker et al. \(2021\)](#).

Labour market. There are four households in the model, with the population size normalised to one, with the relative sizes denoted by φ^j , such that $\sum \varphi^j = 1$. Each household has three options for their labour market status: employed, n_t^j ; unemployed but searching for employment, u_t^j ; and out of the labour market enjoying leisure l_t^j .

$$1 = n_t^j + u_t^j + l_t^j$$

The aggregate values for each type of employment and unemployment are given by $N_t^j = \varphi_t^j n_t^j$ and $U_t^j = \varphi_t^j u_t^j$. The employment status is determined by frictions of the labour market, for which the law of motion evolves as follows:

$$N_t^j = (1 - \rho_n^j) N_{t-1}^j + m_t^j.$$

In this equation, the parameter ρ_n^j is the exogenous job destruction rate and m_t^j is the number of employment matches each period.

Firm Dynamics The output of a firm, y_t is a function of highly-skilled inputs of labour and physical capital, X_t and low-skilled inputs of labour and automation capital, V_t which is based on the model put forth by [Berg et al. \(2018\)](#). The function is a nested constant

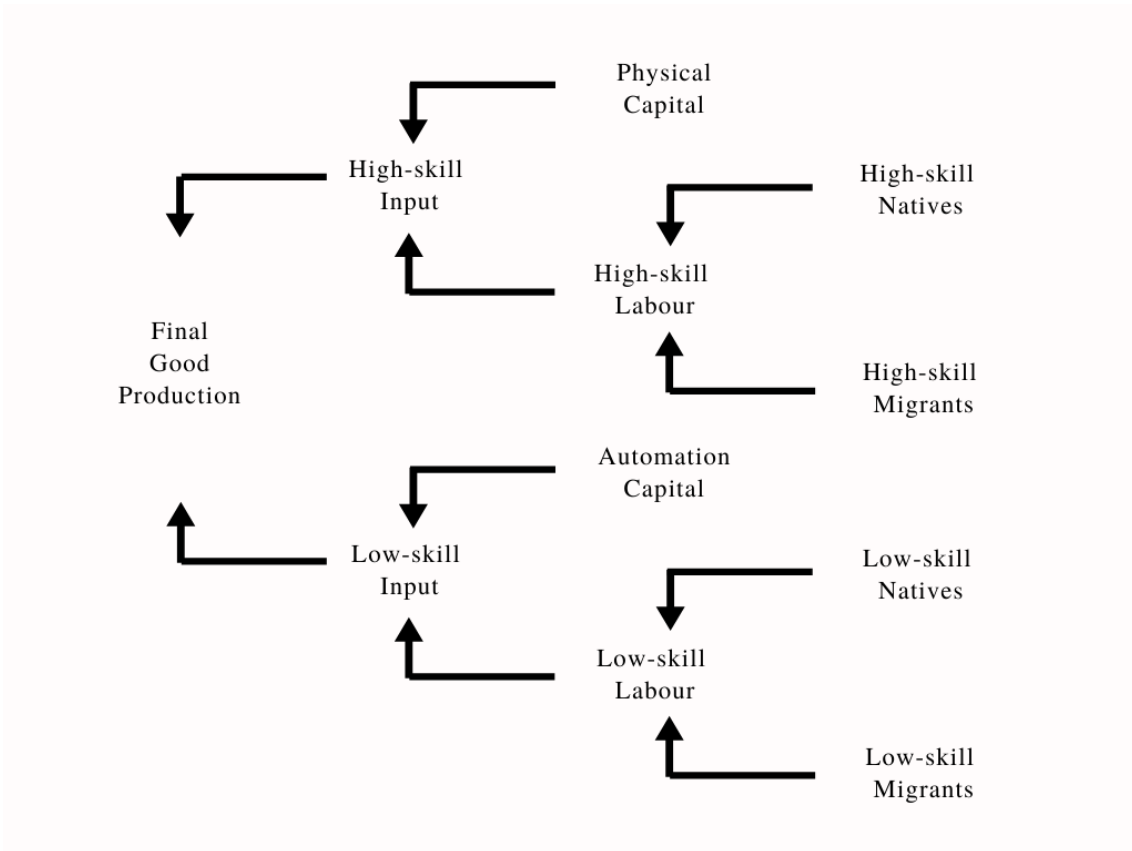


Figure 15: Exogenous Migration Model: Overview of the Production Setup

A diagram to show how the inputs to production of the final goods in the economy follow through at each level of the model.

elasticity of substitution (CES) function¹⁶ where a, ζ, e are the intensities of low-skilled input of total production, physical capital services of highly-skilled input, and low-skilled labour of low-skilled input, respectively. The corresponding elasticities of substitution are given by ε, ρ , and σ . Thus:

$$y_t = A_t \left[a^{\frac{1}{\varepsilon}} (V_t)^{\frac{\varepsilon-1}{\varepsilon}} + (1-a) (X_t)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}},$$

where

$$X_t = \left[\zeta^{\frac{1}{\rho}} k_{t-1}^{\frac{\rho-1}{\rho}} + (1-\zeta) (l_t^H)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}$$

and

$$V_t = \left[e^{\frac{1}{\sigma}} (L_t^L)^{\frac{\sigma-1}{\sigma}} + (1-e)^{\frac{1}{\sigma}} (b_t Z_t)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}}$$

Automation capital, or robots, is given by Z_t , while b_t determines the relative productivity of robots, which here can be subject to a shock. Robots compete with low-skilled workers L_t^L and increase productivity of highly-skilled inputs. The equations assume capital-skill complementarity, for which $\rho < \varepsilon$ is imposed [Krusell et al. \(2000\)](#). By following this, we see the distinction between automation capital and physical capital with the substitution of labour, and relative to the macroeconomy.¹⁷

3.2.2 Results

We focus the results on the impacts of migration shocks on highly-skilled and low-skilled migrants respectively. In addition, the impact of automation capital productivity, automation capital investment, physical capital investment and a standard technology shock (total factor productivity, TFP), are also included. The model is first estimated by using Bayesian techniques and then calibrated to German data. In particular, the model parameters, standard deviation of the shocks and their persistence parameters of the shocks are estimated from the data, with other values either calibrated to the data or taken from the literature.

Figure 16 shows the responses of key variables to the six shocks described above. As

¹⁶A CES function is an equation that incorporates the factors of production providing constant elasticities of substitution *e.g.* $\varrho_{X,V}$ for the elasticity of substitution between high and low-skill inputs, and factor shares *e.g.* a which total 1, such that can be written as $a, (1-a)$. Other production functions such as Cobb–Douglas and linear are special cases of the CES function.

¹⁷For a short explanation, see [Dolado et al. \(2021\)](#).

discussed in Section 3.1, only the central tendencies of the impulse response functions are shown here, as assessing their uncertainty in turn would require some additional modelling. Among the series of shocks, output increases on impact for all of them, to varying degrees with the response to the TFP shock being the greatest. Notably, both types of migration shocks increase significantly, and the response to automation capital shocks is greater than traditional physical capital. This can be explained by the way it feeds through into the high and low-skilled inputs. Even though the shocks are of the same size, there is a much greater impact of automation capital.¹⁸ Aggregate consumption decreases following an automation shock which is because the highly-skilled native household switches their disposable income otherwise spent on consumption to automation investment due to the higher returns.

At a household level, employment increases for all households with migration shocks leading to the greatest increase. This is due to migrants entering the labour markets with the same participation rate and unemployment rates as their household. However, some of the migrants do not join the migrant households at the corresponding skill level, a small number join the native households, while other are returning emigrants or other immigrants that have the ability to be immediately recognised as natives. Other reasons for the increase in employment is the high increase in output by the firms.

Wages are largely dependent on marginal product of labour, and due to the increase in employment and number of labour hours provided to the firm, this parameter is lower. On aggregate, factoring in the wage level, number of employed in the household, and hours worked by each employed person, the labour income generally increases, as shown in Figure 17. There are exceptions to this, in particular for the migrant households for whom, following exogenous increase to the size of their household and thus employment, the decrease in marginal product of labour is dominant.

The automation effect comes into to play when we look at the effect on wages. There are distinct differences between the highly- and low-skilled workers. When there is a productivity increase to automation capital, the wages increase of the highly-skilled workers but decrease for low-skilled workers. This is due to the capital-skill complementar-

¹⁸The investment shock for each type of capital is the same for simplicity. The available data do not allow obtaining comparable estimates for automation investment.

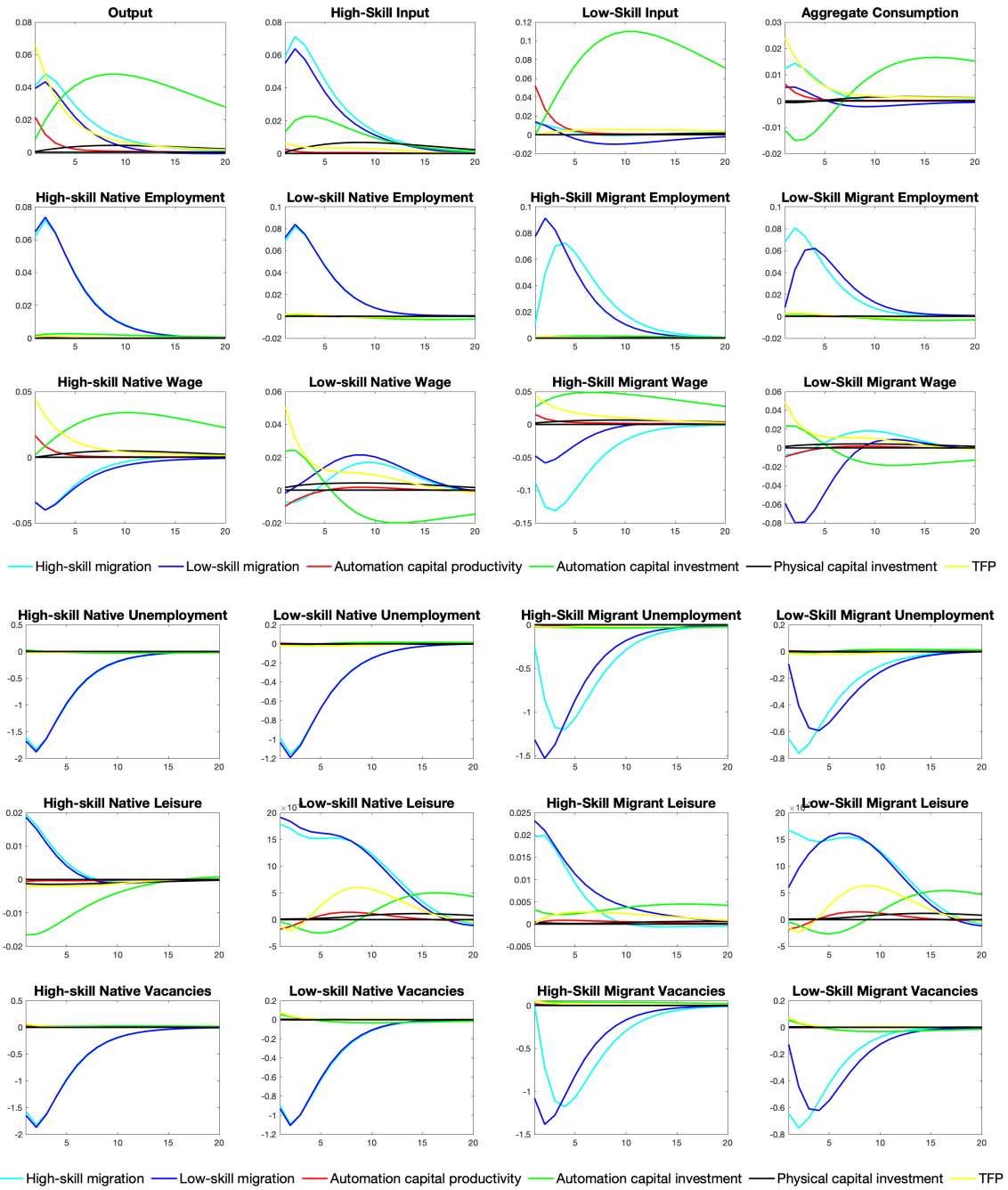


Figure 16: Exogenous Migration Model: Selected Impulse Response Functions
 The horizontal axis identifies the quarter. The vertical axis shows the deviation from steady state. The economy aggregates (output, highly-skilled and low-skilled inputs, and consumption) are in per capita terms, whilst employment and unemployment are in aggregate terms rather than per household, i.e. N_t^j
Key: Cyan: highly-skilled migration; dark-blue: low-skilled migration; red: automation capital productivity; green: automation capital investment; black: physical capital investment; yellow: TFP

ity. Highly-skilled workers are complementary to automation capital whereas low-skilled workers are substitutes. If the productivity of the substitute increases, the firm will

switch more resources to the automation capital than the workers. However, when there is more automation capital investment, the wages increase. Just as with more workers, the marginal productivity decreases, and the same is true for capital.

One large effect of migration is the decrease in unemployment. This is not fully explained by the increase in employment, as people have the option not to be active in the labour market, equivalently enjoying leisure. The effects on unemployment and employment are also reflected in the results from the panel VAR shown in Section 2.2. Workers exit the labour market when they prefer enjoying leisure over searching for employment and receiving unemployment benefits. The effects are only small, however, even though the low-skilled workers are more likely to switch to leisure, as they get greater utility from it, the dynamics of automation can reduce this effect.

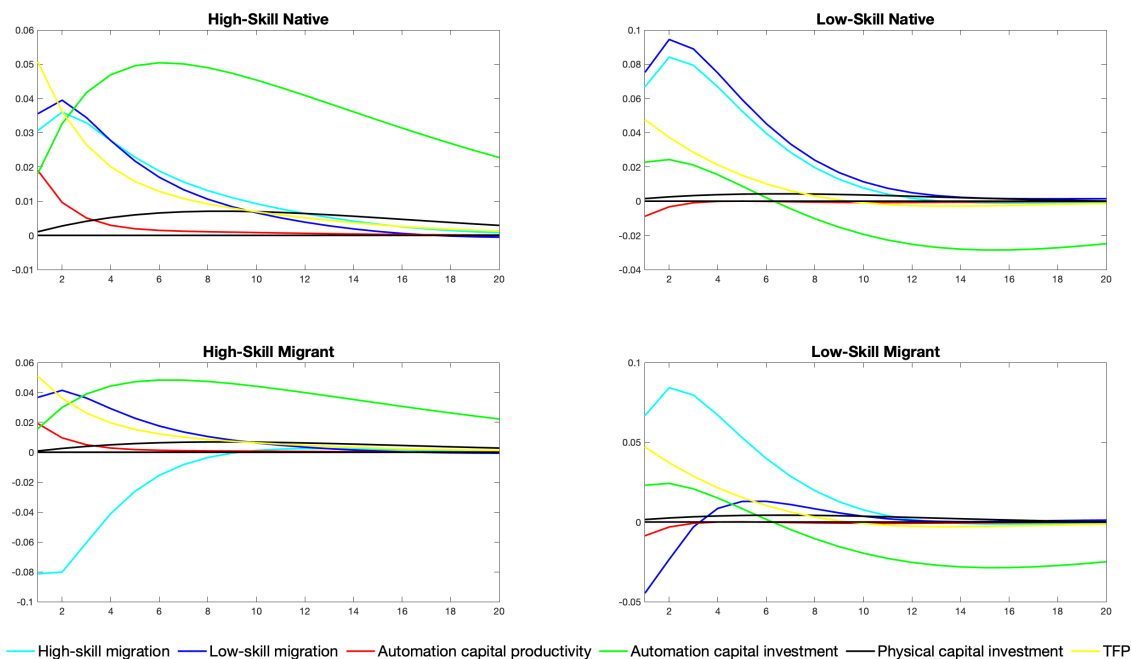


Figure 17: Exogenous Migration Model: Impulse Response Functions, Labour Income. The horizontal axis identifies the quarter. The vertical axis shows the deviation from steady state. Labour income is given by $w_t^j n_t^j h_t^j$.

Key: Cyan: highly-skilled migration; dark-blue: low-skilled migration; red: automation capital productivity; green: automation capital investment; black: physical capital investment; yellow: TFP

The results in Figure 18 show the changes in wages following each of the shocks, for example, the first sub-plot presents the responses to $WP^N = \frac{w^H}{w^L}$. The impact of wages is two-fold, as explained above, which explains the link between the IRF for wages and

salaries increasing as shown in Figure 17. Assessing the effect on a wage premium provides a further aspect to examining the impacts of migration. If the wage level decreases overall, but remains on par for natives and migrants, then the distinction does not matter a lot. However, in our example (as in real economies), this is not the case. Natives have higher bargaining power over their wages, and so are less likely to feel the effect of any decrease and more likely to gain from the increase. The same goes for the advantage of highly-skilled over low-skilled workers.

The first row of Figure 18 depicts the impulse response functions rates for the highly-skilled vs low-skilled premium for the natives and the migrants, whilst the second row depicts the premium effects of being a native at each skill level. These results show the varying size of responses for different groups of workers. We can combine these results with those from Figure 17 to help explain these findings. Thus, the highly-skilled workers are more likely to switch to employment over leisure or increase hours of work, which can in turn help explain the effects of migration shocks. With the migrant skill premium, the highly-skilled migrants gain in wage terms for all but the highly-skilled migration shock.

For the native premium, the effect from automation capital investment is the change in highly-skilled native employment relative to that of migrants. Otherwise, the natives tend to gain in wage premium terms, although the size of the changes is small. The greatest effects are following the migration shock, where the natives experience a wage increase relative to their skill counterparts. For the low-skilled native premium, the only significant change is with the low-skilled migration shock. These two differing effects show how the different shocks affect the wages relative to one another.

Hence, there is a distinct earnings inequality between highly- and low-skilled natives and migrants. Part of this effect is due job to automation, which itself is likely to increase in the future. The other is migration itself. As discussed before (Section 3.1), there are clear links between automation and migration, so how will the dynamics of automation (which negatively affects low-skilled workers) impact migration in terms of profile of migrants and total migration flows? Migration and automation, are both expansionary to the economy with the workers who are substitutes to automation capital, affected negatively by advances whilst those who are complements benefiting.

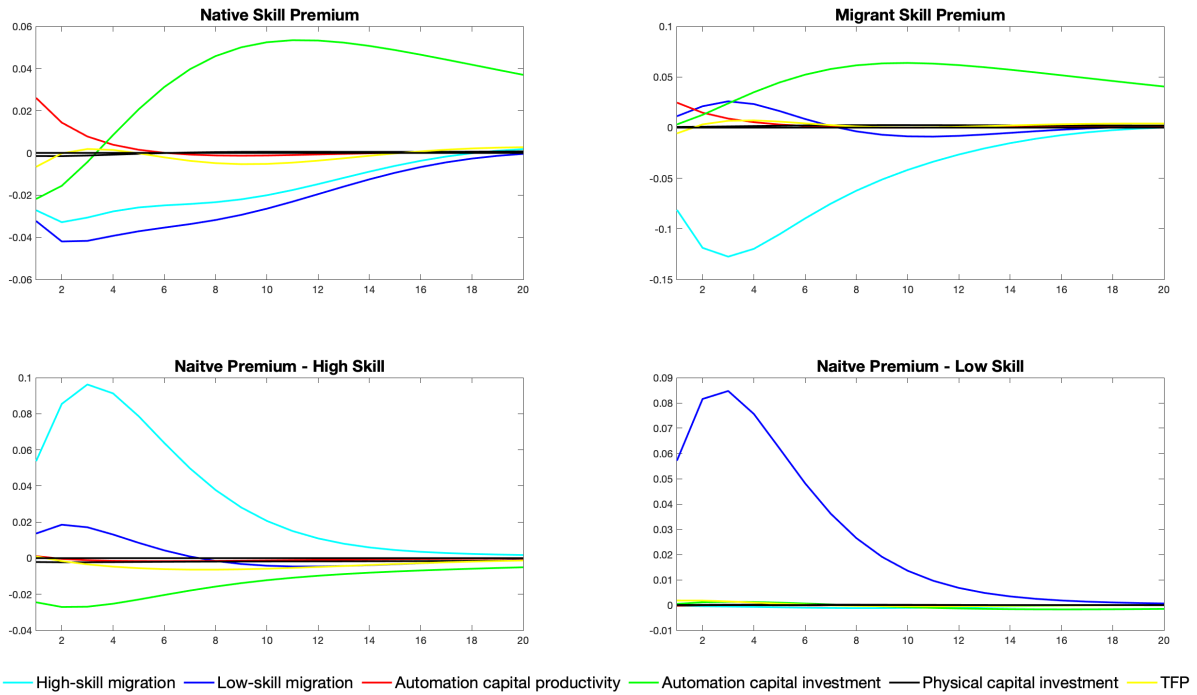


Figure 18: Exogenous Migration Model: Impulse Response Functions, Wage Premiums
The horizontal axis identifies the quarter. The vertical axis shows the deviation from steady state. Wage premium is given by $WP = w_t^{j^1} / w_t^{j^2}$.
Key: Cyan: highly-skilled migration; dark-blue: low-skilled migration; red: automation capital productivity; green: automation capital investment; black: physical capital investment; yellow: TFP

On the whole, with the DSGE model presented in this section, we were able to assess the effects of migration as an exogenous shock to a small open economy with high-levels of automation. The discussion and interpretations above demonstrated, how the models can present coherent scenario narratives of changes in drivers, migration flows, and their impacts. In this particular example, with respect to the main sources of *uncertainty* in responses to migration shocks, the key factor proved to be the skill level of the newly-arriving migrants, and the composition of migration flows. The analysis of various shocks enabled to explore this scenario uncertainty through the means of different possible variant trajectories of economic and migration developments. The differences between these scenarios describe not only the range of uncertainty, at least in a qualitative sense, but also enable to check the sensitivity (or robustness) of various economic variables to a range of individual shocks.

4 Long-Range Scenarios: From VAR to Approximating Uncertainty

4.1 Towards Long-Term VAR Model-Based Migration Scenarios

By using the Bayesian panel VAR models introduced in Section 2, we also produced long-range forecasts for immigration, emigration and net immigration up to 2050Q4, so with the horizon of 30 years – around one generation ahead. In this exercise, instead of estimating immigration and emigration simultaneously, we estimate them individually. The extensions of forecasts for the 26 countries included in the analysis in Section 2 are presented in Figure 19 for immigration, Figure 20 for emigration, and Figure 21 for net migration. As in Section 2, we predict relative measures of migration intensity per 1,000 inhabitants of a given country: proper demographic rates for emigration, and ‘rates’ for immigration and net migration, given the incorrect populations at risk. In all figures, central tendencies are shown alongside the 67-per cent predictive intervals.

As expected (Bijak, 2010; Bijak et al., 2019), the predictability of migration over such long horizons appears to be generally poor. This is especially visible for emigration, and even more so for net migration. This seems to be a problem continuing from the short-range forecasts presented in Section 2, only exacerbated by a longer prediction horizon. The uncertainty is visibly higher for countries that are net senders of migrants, such as many of the CEE nations. Besides, in more general terms, without controlling for population and, equivalently, the total labour market for the entire forecast period, the predicted emigration (and thus net migration) processes exhibit great variability.

In general, how the population picture will look in ten, 20, or 30 years’ time, is hard to predict. Besides, it has to be stressed that the forecasts presented in this section are conditional, based on external predictions of drivers where available. Here, for example for the GDP, some forecasts were available from the OECD Economic Outlook, but only up to 2024, and are replaced by model-generated forecasts thereafter, compounding the predictive uncertainty. In the longer term, the role of additional factors is becoming increasingly more likely – one example of such a factor is job automation, as shown in the DSGE model in Section 3, the advancements of which are expected to negatively affect migration flows from Central and Eastern European countries to Western Europe.

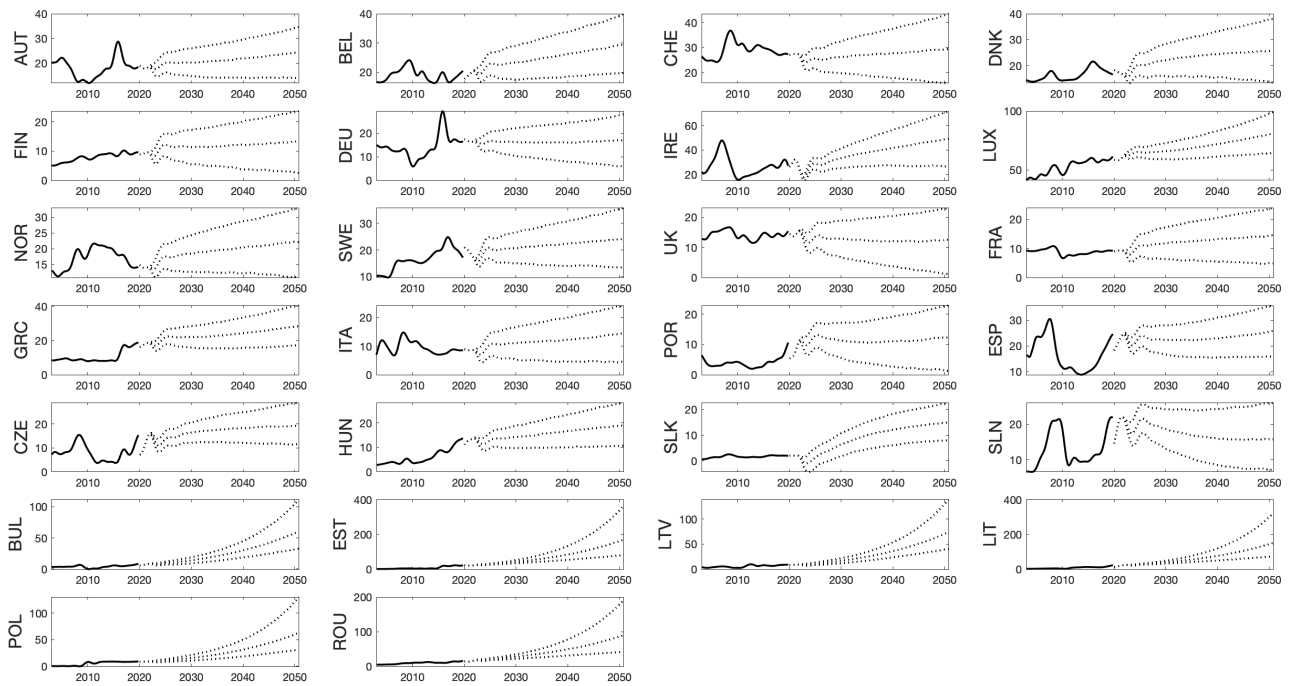


Figure 19: Long-Range VAR Forecasts of Immigration 'Rates', 2020–2050

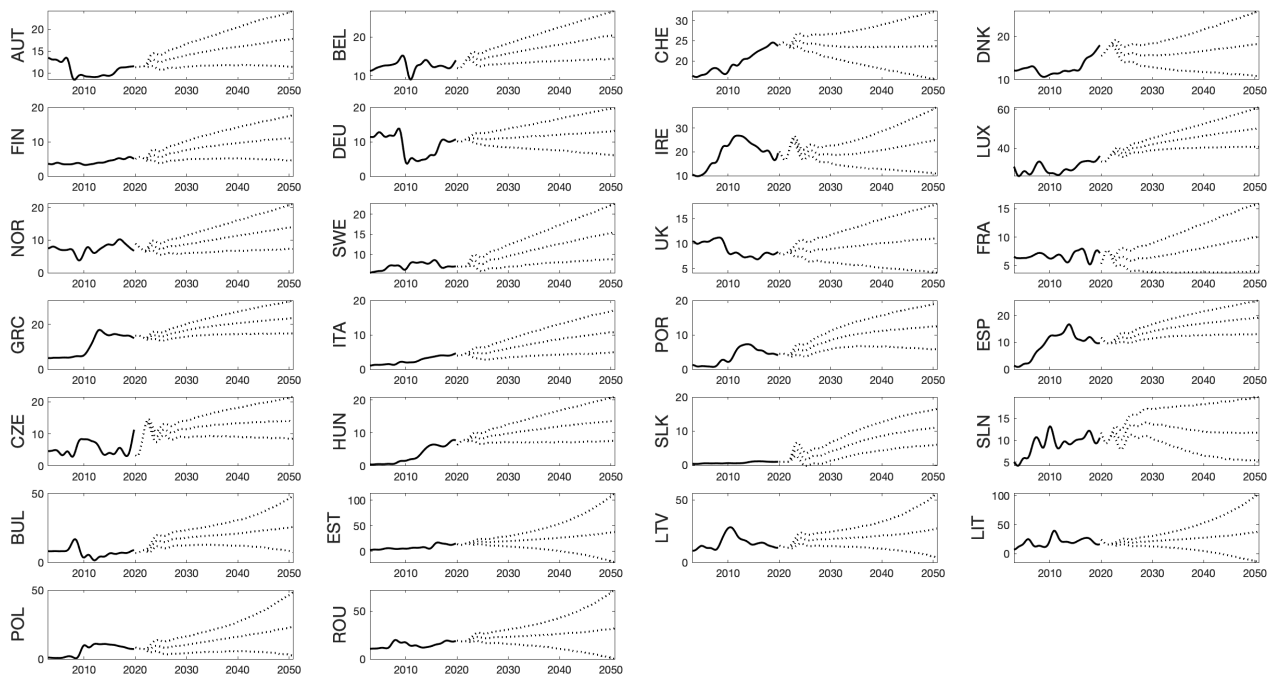


Figure 20: Long-Range VAR Forecasts of Emigration 'Rates', 2020–2050

In the examples presented above, some countries have significantly large error bands towards the end of the sample. As a general conclusion, these are mainly countries which

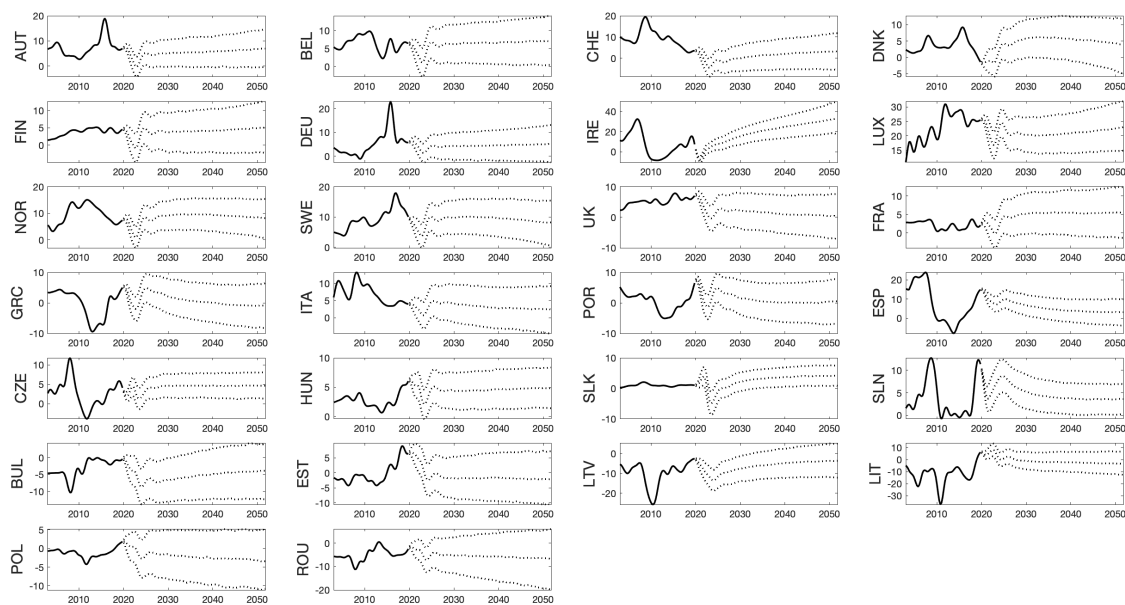


Figure 21: Long-Range VAR Forecasts of Net Migration ‘Rates’, 2020–2050

have had volatile periods of migration in the sample period, not fully captured by the macroeconomic or labour market data data. The forecast begins from 2020Q1, conditional on GDP, employment, consumption, wages and salaries, and labour market. The migration data thus end in 2019Q4, with forecasts conditional on data and expectations impacted by the COVID-19 pandemic, which explains some of the large changes at the beginning of the forecast horizons. Interestingly, the forecasts of net migration exhibit relatively most stable long-term tendencies in terms of their uncertainty bounds, similar to those obtained by [Azose and Raftery \(2015\)](#) based on univariate hierarchical models.

In general, the models presented in this report seem to have been able to identify different patterns of predictive uncertainty for various groups of countries. In particular, there are clear differences between countries, where the presence of shocks in otherwise relatively stable trends is increasing the predictive errors comparatively slowly (Groups 1–3), as opposed to countries with visible changes in trends implying clearly non-stationary processes and wide, as well as rapidly increasing predictive intervals (mainly Group 4). This analysis allows to identify, even if in qualitative sense, in which countries the predictive uncertainty is the greatest, as they may be (or soon become) at the verge of a migration transition, for example from being net senders to net receivers.

4.2 Alternative Methods: Expert Input and Weighting Scenarios

As discussed in the previous section, somewhat predictably, the predictive performance of models equipped with macroeconomic drivers was mixed, and in many cases, such as for Central and Eastern European countries, undergoing a transition from net senders to net receivers, outright unsatisfactory. Whenever good data are available, and the underlying processes are relatively stable, as in the pioneering study of [Gorbey et al. \(1999\)](#) of migration between Australia and New Zealand, Bayesian VAR models offer an appealing way of providing coherent scenario trajectories describing the future uncertainty, including theoretical insights into structural mechanisms shaping population flows. In a general case, however, difficulties with predicting migration beyond a five- to ten-year horizon are well known (e.g. [Bijak and Wiśniowski, 2010](#)), which leaves the question of how to predict migration in the longer term to a large extent open.

There exist several solutions to the challenge of long-term forecasting and uncertainty description. These include approximations by time series processes, modelled at a low temporal resolution (e.g. with five-yearly data), and possibly embedded in a Bayesian hierarchical framework, allowing for the borrowing of strength across countries ([Azose and Raftery, 2015](#)). Another option is to rely on expert opinion, which can be expressed as a prior distribution – either on the future migration processes themselves ([Wiśniowski et al., 2013](#)), their statistical features ([Bijak and Wiśniowski, 2010](#)), or various possible migration scenarios and their relative likelihood of occurrence ([Wiśniowski et al., 2014](#)). The elicitation of expert opinion can be also formally applied to the scenario approach, as presented for example in a recent study by [Acostamadiedo et al. \(2020\)](#).

Even with expert-based input about future migration flows available, a technical question remains how to link the short- or mid-term uncertainty assessment, for example based on time series or panel VAR models, such as those presented in Section 2, with the expert input elicited for the longer term. One appealing option is to use dynamic weighting to bridge these two approaches, as suggested by [Wiśniowski et al. \(2014\)](#) for migration, or recently by [Dodd et al. \(2021\)](#) in the context of mortality forecasting.

In practical terms, we can describe the short-term uncertainty by applying established models or techniques, such as a time series or a VAR model, until a certain time period

$T + t$, with T denoting the last year of observed data, and t being in the range of 5–10 years. Let the resulting predictive distribution be denoted by $f(T + t)$. Let us further assume that the elicited expert input relates to a future horizon $T + H$, where $H > t$, and has a form of a probabilistic distribution, $f_{ex}(T + H)$. The distribution for any interim period $T + t'$, $f(T + t')$, such that $t < t' < H$ can then be obtained by weighting:

$$f(T + t') = \alpha(t')f(T + t) + [1 - \alpha(t')]f_{ex}(T + H) \quad (4.1)$$

In the above formula, the weights $\alpha(t')$ have to fulfil the boundary conditions, $\alpha(t) = 1$ and $\alpha(H) = 0$, and can have additional constraints imposed on them, such as monotonicity. The weights α can be also assigned probability distributions, in which case the resulting bridging densities would be mixture distributions. The trajectories of the weights can additionally reflect expert knowledge about the possible paths of transition from the short-term to long-term dynamics, or even about their combinations weighted by their subjective probabilities of occurrence, as suggested by [Wiśniowski et al. \(2014\)](#).

The approach presented above offers a way of providing at least approximate probabilistic description of the uncertainty of migration across a range of time horizons, with the level of detail and predictability decreasing with the time horizon. In this way, not surprisingly, the longer were the horizon of future scenarios, the more approximate would be their uncertainty assessment. For a social process of such complex nature as migration, this may well be the epistemological limit of the statements that can be made today about the migration flows in the more distant future ([Bijak and Czaika, 2020](#)).

The short- and long-run forecasts presented here have extended the previous attempts of using Bayesian VAR methods to forecast migration ([Gorbey et al., 1999](#); [Bijak, 2010](#)). The novel contribution has been the use a much broader set of macroeconomic and labour market data. These models have proved more successful for some countries than others – unsurprisingly, especially for those who were relatively unaffected by the shocks of economic and migration nature. Still, the large residual uncertainty indicates that the macro-level picture of future migration is far from complete, and that approximations, such as those suggested above, can offer the only realistic alternative solution.

5 Summary and Conclusions

The research presented in this report has put forth theoretical and empirical models to examine the effects of macroeconomic drivers on migration, and conversely, of migration on the economy. In addition, we have presented in-sample and out-of-sample analysis of the related migration forecasts, which have indicated relatively good, yet variable performance, depending on the indicator used – with immigration bearing on average the lowest errors, and net migration the highest – as well as on the countries, with uncertainty larger in the countries undergoing migration transition than in the more established economies. In the light of previous research (e.g. [Bijak et al., 2019](#)), these findings are not surprising, as they confirm earlier intuitions as to the key role of the stability of the migration processes being predicted.

Still, with the short- and mid-term scenarios using DSGE models, we were able to produce a coherent picture of changes in migration under the conditions of continuing job automation, and assess the extent of uncertainty in the responses of migration to a range of shocks. We identified some key effects of automation on migrants and migration flows, with a number of results that can be helpful towards shaping migration and macroeconomic policies. They include not only qualitative statements on the impact of automation on low-skilled jobs, and on the skill mix of migration on other parts of the economy, but also the analysis of a range of possible outcomes under different assumptions, which can serve as a basis for scenario planning. In that respect, this research adds to the existing literature by carrying out a full macroeconomic analysis of automation and migration, enhanced by a large empirical evidence from a broad range of European countries, including net senders.

In general, the forecasts and scenarios presented in this report, both for the long- and short-range horizons, offer new prospects to forward-looking migration studies. Unlike for other components of population change, births and deaths, there are important macroeconomic drivers which represent the push and pull factors of migration. This allows us to obtain at least some indications of the future direction of change of migration flows, albeit often with very wide prediction intervals, by applying macroeconomic tools and techniques. There are, of course, outstanding issues related to systemic shocks (policy

changes, lowering or raising of migration and trade barriers, or crisis events, such as the recent ones in Syria or Afghanistan), which are foreseeable only to some extent. Still, even for such shocks, the analysis of impulse response functions, such as those presented in Section 3, offers an appealing option for research and policy alike. One caveat here is that, as DSGE models can be prone to misspecification, especially with respect to their microfoundations, it is recommended to model the processes in question with empirical VAR models in parallel, to ensure that the results are robust (Drautzburg, 2020).

In substantive terms, the results from the panel VAR models in Section 2 confirm that net immigration is expansionary for the macroeconomy and labour market and improves fiscal finances, whilst net emigration leads to opposite effects. In this case, the evaluation of in-sample forecasts indicated good performance for the majority of countries, while the out-of-sample forecast analysis highlighted promising avenues for further research. In particular, the results of the DSGE modelling exercise presented in Section 3 showed that job automation increases the income inequality through selective migration, and leads to a predicted reduction in migration when the jobs which migrants perform are (imperfect) substitutes for robots. Both these approaches offer a way of describing theoretically-informed structural relationships between migration and its key drivers in scenarios, as postulated for example by van Wissen (2012). In particular the DGSE models can fill the well-recognised formal theoretical void in demography, and broader population and migration studies (see Burch, 2018). As argued above, these models and the uncertainty analysis they offer can be further extended by an explicit modelling of *uncertainty shocks* and responses to them (Bloom, 2009). In turn, the next step in fine-tuning the empirical analysis and making its finding more robust would be to use harmonised data on migration flows once they become available, for example extending the work by Raymer et al. (2013), and conducting a comprehensive sensitivity analysis of model results.

In this way, both the empirically-grounded predictive models, such as panel VARs, as well as theoretical models, such as DSGEs, can shed light on different aspects of migration uncertainty. The former can at least approximate the intrinsic randomness (the aleatory uncertainty) through probabilistic description, and the latter can contribute to reducing the knowledge-related epistemic uncertainty by providing insights into the mechanisms of migration processes, their drivers and impacts. Both approaches can be useful for

formally describing uncertainty in future migration – either probabilistically or in the form of scenario-based trajectories – in the short-to-mid-term horizons of a few years ahead. Some analytical tools, such as impulse-response functions, can also help stress-test different aspects of migration scenarios or policy decisions.

As discussed in Section 4, the key challenges in forecasting and scenario setting for migration remain the methods that would work over longer horizons. For those, the approaches based on conditional forecasting can be of limited use, given the necessary reliance on a range of other variables and drivers, which need to be predicted themselves and conditioned upon. At the moment, modelling options based on simple time series methods, possibly coupled with expert-based assessment of long-range scenario uncertainty, offer a plausible, pragmatic solution. Future work can additionally explore hybrid solutions, fusing different approaches. Such methods could start from driver-based forecasts and scenarios in the short- to mid-term horizons, derived from univariate or multivariate time series, and then use some dynamic weighting to morph the associated probability distributions into purely expert-based ones for the long run. In addition, the relative advantages and limitations of using net migration rather than individual flows in the long run (as in [Azose and Raftery 2015](#) or Section 4 above) warrant further exploration.

In the otherwise very uncertain world of migration and its forecasts and scenarios, one thing seems relatively certain: the assessment of migration uncertainty over longer time horizons remains a heroic exercise, and at the current state of knowledge, the best solutions we can hope for are only approximate. Still, the analysis presented in this report allows for making pragmatic choices: shorter-term forecasts and analysis of migration impacts, especially for countries with more stable migration processes and history, allow for using more complex models describing the economic and other aspects in finer detail, while longer-term scenarios, where the past information is less relevant, call for expert input at a very high level of abstraction. In either case, the crucial epistemological limits of any statements about future migration flows, and the associated caveats for the forecasters and forecast users alike (e.g. [Bijak and Czaika, 2020](#)), remain fully in force.

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Appendices

A Short-term Migration Forecasts: 90% Predictive Intervals

In addition to the forecasts presented in Section 2.1, in this Appendix we present the estimations with 90% predictive intervals. As previously mentioned, the predictive intervals prove to be too wide, and increase too rapidly to aid useful analysis or decision advice. This confirms earlier intuitions on the limited usefulness of predictive intervals with too high nominal probabilities attached to them, especially if the predictive distributions have ‘heavy tails’, as is often the case in realistic migration forecasting (e.g Bijak, 2010). The predictions for relative measures of immigration, emigration and net migration are shown in Figures A.1, A.2, and A.3, respectively.

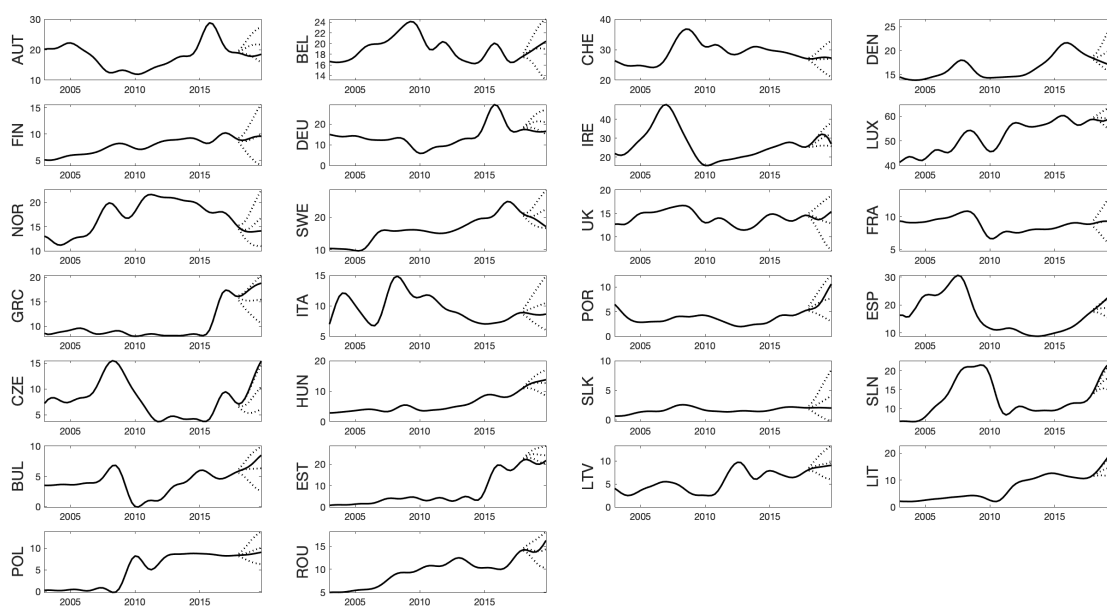


Figure A.1: In-sample Forecasts of Immigration ‘Rates’, 2018–19: 90% Predictive Intervals

The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 90% predictive intervals.

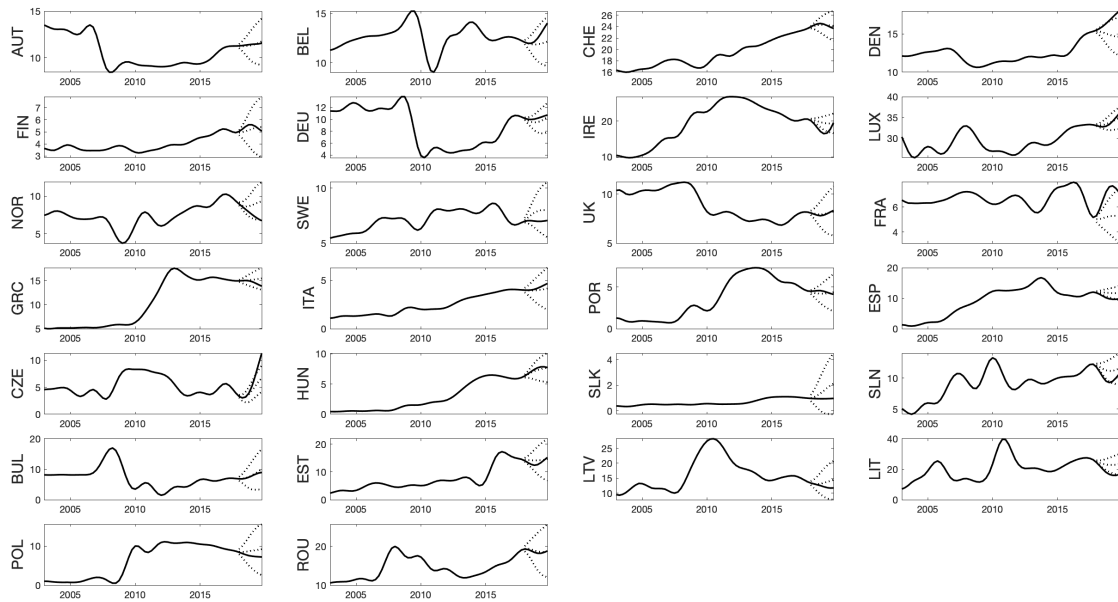


Figure A.2: In-sample Forecasts of Emigration Rates, 2018–19: 90% Predictive Intervals
 The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 90% predictive intervals.

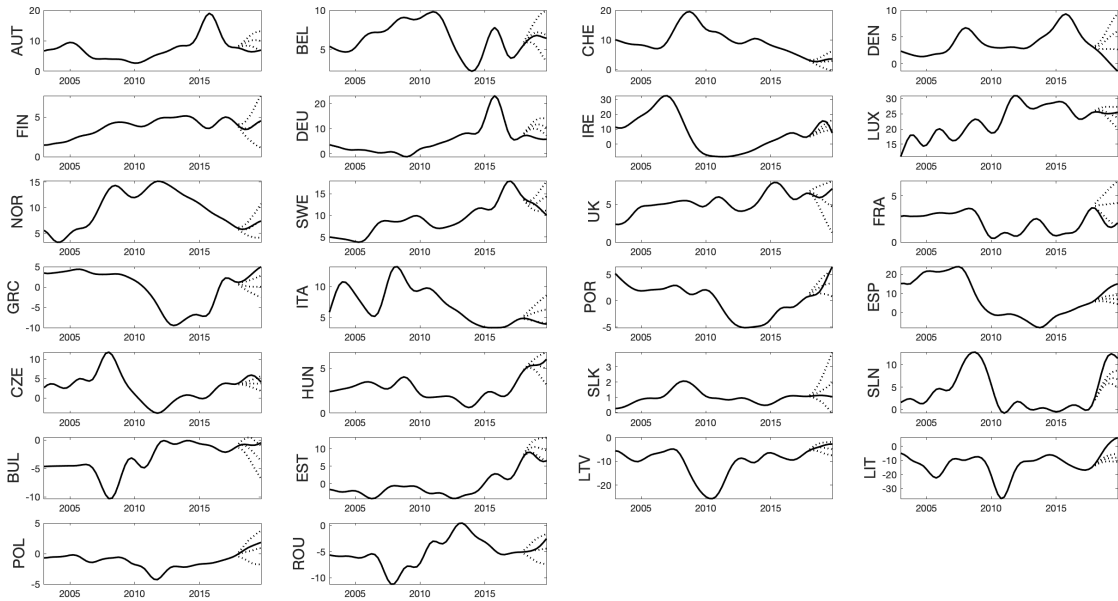


Figure A.3: In-sample Forecasts of Net Migration ‘Rates’, 2018–19: 90% Predictive Intervals
 The solid black line represents the data used for estimation. The dotted lines depict the mean forecast and the 90% predictive intervals.

B DSGE Models: Further Details

In Section 3, we presented two models, with endogenous and exogenous migration. As a further explanation to the models, and how they fit into the framework for assessing uncertainty of migration, we present further equations and their relationships. In Table B.1, we list a small number of equations to explain the main foundations and mechanisms of the models employed in Section 3. In the left column of Table B.1, we list the main equations related to the two-country model with endogenous migration, presented in Section 3.1, whereas in the right column, we show the model of a small open economy with exogenous migration, discussed in Section 3.2. The narrative of the main mechanism of how the model works workings is also presented below.

Let's start with the microfoundations of the DSGE models. There are two main types of agents in the economy that make decisions: households and firms. They each make decisions on a per household or per firm basis – the micro-scale decisions. As such, all of the relevant variables are expressed on a per capita, or working-age basis. First, let's consider households, which aim to maximise utility or happiness/enjoyment. People get utility from consumption of goods, c_t , and leisure, l_t , and in general, *disutility* (negative utility) from supplying labour to firms, h_t .

In order to buy goods for the purpose of consumption, people need to have an income. Since the DSGE models study working-age people (younger and older dependents are not considered in this type of DSGE model), the sources of income are wages from supplying labour, w_t or unemployment insurance, ub dependent on the employment n_t and unemployment u_t levels respectively. The households make choices on the levels of consumption, labour market decisions, and investment/borrowing in financial markets, b_t , by maximising their utility function subject to a number of constraints. They also pay taxes, either lump sum Tax_t , or at a proportional rate to consumption levels, τ^c and income τ^w , as well as receiving household transfers z_t .

Table B.1: DSGE Equations

Two Country Model - Model 1	Small Open Economy - Model 2
<i>Household Utility</i>	
$U_t = \beta^t E_t \sum_{t=0}^{\infty} \left(\frac{(c_t)^{1-\sigma_c}}{1-\sigma_c} - \frac{\phi_0 (h_t)^{1+\phi}}{1+\phi} \right)$	$U_t = \beta^t E_t \sum_{t=0}^{\infty} \left(\frac{(c_t)^{1-\theta}}{1-\theta} - \frac{\phi_0 (h_t)^{1+\eta}}{1+\eta} + \Phi \frac{(l_t)^{1-\zeta}}{1-\zeta} \right)$
<i>Budget Constraint of Intertemporally Optimising Agents</i>	
$c_t + b_t - b_{t-1}(1+r_{t-1}) + Tax_t =$ $w_t h_t n_t + u_t u b + d_t$	$(1+\tau^c)c_t + b_t - (1+r_{t-1})b_{t-1} =$ $w_t h_t n_t (1-\tau^w) + u_t u b + z_t + d_t$
<i>Law of Motion of Employment (common to both)</i>	
$n_t = (1-\rho_n^j)n_{t-1} + m_t$	$n_t = (1-\rho_n^j)n_{t-1} + \zeta_t u_t$
<i>Firm Profit</i>	
$\Pi_t = \beta^t E_t \sum_{t=0}^{\infty} (y_t - w_t h_t n_t - \kappa_v v_t - \kappa_a A_t - x_t)$	$\Pi_t = \beta^t E_t \sum_{t=0}^{\infty} (y_t - w_t h_t n_t (1+\tau^f) - \kappa_v v_t - x_t - x_t^z)$
<i>Law of Motion of Capital</i>	
$k_t = (1-\delta)k_{t-1} + \iota(x_t, x_{t-1})$ $Z_t = (1-\delta^Z)Z_{t-1} + \iota(x_t^Z, x_{t-1}^Z)$	$k_t = (1-\delta)k_{t-1} + \iota(x_t, x_{t-1})$
<i>Fiscal Policy</i>	
$g_{c_t} = \left(\frac{GDP_t^i}{GDP^i} \right)^\Theta \bar{g}^c$	$g_{c_t} = \vartheta^{g^c} g_{c_{t-1}} +$ $(1-\vartheta^{g^c})\bar{g}^c - \zeta^{g^c, y}(y_{t-1} - \bar{y}) + \zeta^{g^c, b^g}(b_{t-1}^g - \bar{b}^g)$
	$z_t = \vartheta^z z_{t-1} +$ $(1-\vartheta^z)\bar{z} - \zeta^{z, y}(y_{t-1} - \bar{y}) + \zeta^{z, b^g}(b_{t-1}^g - \bar{b}^g)$
<i>Fiscal Budget Constraint</i>	
$G_t + (1+r_{t-1})b_{t-1}^G = Tax_t + b_t^G$	$g_{c_t} + z_t + u_t u b + b_{t-1}^g (1+r_{t-1}) = tax_t^w + tax_t^c + tax_t^f + b_t^g$

Variables for the household, and parameters can be household-specific. For instance, highly-skilled household place less weight on leisure than low-skilled households do.

The primary constraint is the budget constraint, with other constraints being the *law of motion* of employment in the labour market, and capital constraints for those investing in firms. Households can have access to credit and financial markets enabling consumption smoothing, as such they are intertemporal optimisers, or Ricardian households. They own the firms and received dividends in the form of profits, d_t . Some households cannot access credit or financial markets as such, all income is used during the period to maximise utility. The law of motion followed by the labour markets is such that the matches are formed between unemployed workers, u_t , and vacancies posted, v_t . The probability of finding employment is $\zeta_t = m_t/u_t$.

The second type of agent is the firm. We model firms that are perfectly competitive (other models can for example look at monopolistically competitive firms). Firms are profit-maximisers. The output is determined by the quantity of inputs in the production function: labour, capital, k_t , automotive capital, Z_t , and robots A_t , and is scaled by the total factor productivity (TFP). However, each of these factors of production comes at a cost: labour needs wages to be paid to the workers, and posting vacancies to hire workers also involves a cost, κ . Capital has maintenance and adjustment costs, as well as further investments x_t where required. Robots have similar features to capital and labour, with either a fixed cost, κ_a , or investment in automotive capital, x_t^z . The optimisation problem takes the profit equation, subject to costs of the input factors of production, a capital accumulation constraint, laws of motion of employment, and adjustment costs.

The government is another agent within the model, tasked with maintaining an equilibrium. Aside from shocks, there are no major roles to be played by the government, or fiscal policy, in these models. They collect taxes, consume gc_t , provide household transfers z_t , and invest in financial markets.