



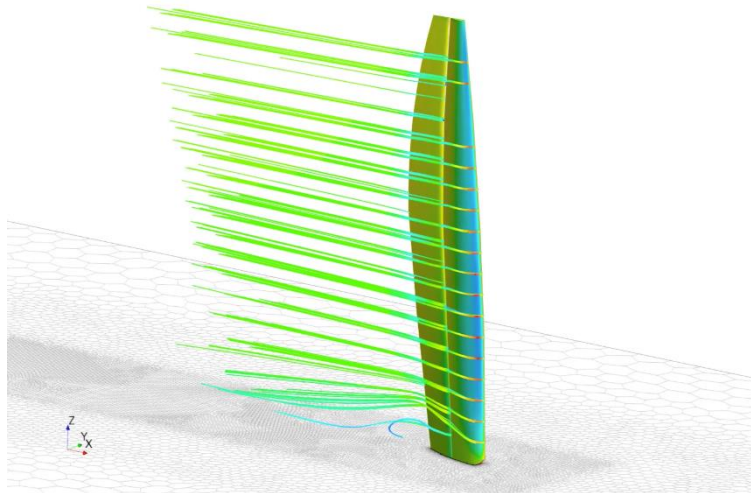
**THE 24<sup>TH</sup> CHESAPEAKE SAILING YACHT SYMPOSIUM**  
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## **Wingsail Profile Optimisation Using Computationally Efficient Methods**

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### **ABSTRACT**

On any race yacht, having the ability to maximise boat speed is key to obtain race winning performances. To achieve this the sail or wing must be set at its optimum profile. To find the best wingsail profile the trend recently has been towards more computationally expensive approaches, but can we use less intensive methods to contribute to the design and optimisation process when time and resource may be limited? With an extensive number of different flying shapes, a computationally efficient approach at accurately finding optimum wingsail profiles for any given wind speed and direction is required. Using a two-dimensional section of the wingsail, lift and drag characteristics were found using Reynolds Averaged Navier-Stokes (RANS) simulations within Star-CCM+. A modified lifting line (LL) model was programmed in Python which used the two-dimensional characteristics to give fast and accurate predictions of drive force and heeling moment for a twisted inflow. The LL code was verified using experimental data, and showed that with analytical corrections, accurate predictions of lift and induced drag could be obtained. 3D RANS simulations confirmed that the LL model with correct tuning of the root vortices could predict driving forces and heeling moments within 1% and 5% respectively for a typical range of angle of attacks (AoA) and wing shapes. LL predictions took ~8 seconds on a laptop compared to ~6 hours for 3D RANS simulations running on a High-Performance Computing cluster. A machine learning algorithm using Kernel ridge multivariate regression was trained to produce a surrogate model of the wingsail giving accurate predictions within 1% of the LL results. Using the surrogate model, performance predictions could be obtained in ~0.001 seconds showcasing the large computational savings. This method permitted an exhaustive search of different wingsail profiles, giving information on parameter trends such as AoA, camber, and twist. This provides a tool that could be adopted in a velocity prediction program (VPP) and used by sailors or designers to aid in the setup and trimming of wingsails for maximum performance.

### **NOTATION**

|         |  |
|---------|--|
| AWA     | Apparent wind angle ( $^{\circ}$ )                       |
| AWS     | Apparent wind speed ( $\text{m s}^{-1}$ )                |
| AoA     | Angle of Attack ( $^{\circ}$ )                           |
| $\beta$ | Local flap angle w.r.t main element chord ( $^{\circ}$ ) |

|                   |   |
|-------------------|---|
| CFD               | Computational Fluid Dynamics                          |
| $C_i$             | Local chord (m)                                       |
| $C_D$             | Coefficient of drag                                   |
| $C_L$             | Coefficient of lift                                   |
| $C_M$             | Coefficient of moment                                 |
| CoE               | Centre of effort                                      |
| D                 | Drag force (N)  |
| $F_D$             | Drive force (N)                                       |
| $F_S$             | Side force (N)  |
| h                 | Height of CoE above origin (m)                        |
| HPC               | High Performance Computer                             |
| $H_M$             | Heeling moment (N m)                                  |
| L                 | Lift fore (N)   |
| LL                | Lifting Line  |
| L/D               | Lift to drag ratio                                    |
| RANS              | Reynolds Averaged Navier-Stokes                       |
| Re                | Reynolds Number                                       |
| TWA               | True wind angle ( $^{\circ}$ )                        |
| TWS               | True wind speed ( $\text{m s}^{-1}$ )                 |
| $\Gamma$          | Vortex strength ( $\text{m}^3 \text{s}^{-1}$ )        |
| u                 | Incident flow velocity ( $\text{m s}^{-1}$ )          |
| VPP               | Velocity Prediction Programme                         |
| $V_{\text{indC}}$ | Induced velocity at point $P_C$ ( $\text{m s}^{-1}$ ) |
| $Z_C$             | Vertical collocation point position (m)               |

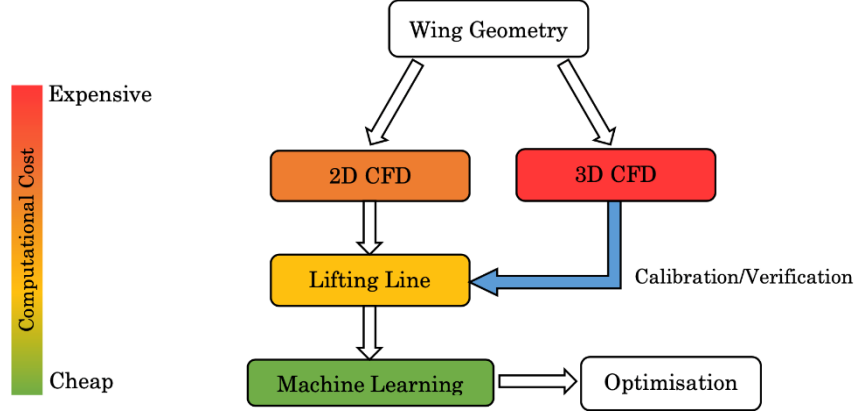
## INTRODUCTION

Ever since the successful 27<sup>th</sup> America's Cup (AC) win by Stars and Stripes in 1988 wingsails have been showcased to the sailing world on scale. Such devices featured in the 33<sup>rd</sup>, 34<sup>th</sup> and 35<sup>th</sup> AC and more recently SailGP on the F50 catamaran. Wingsails have several advantages over conventional soft sails by having higher aerodynamic efficiency, improved control, and the added ease of profile measurement (Whidden & Levitt, 2016). The wingsail on the F50 is controlled by a wing sheet that changes the angle of attack (AoA) of the wing, and buttons that effect camber, twist, and twist profile. Therefore, knowing optimum target wing profiles for a given true wind speed (TWS) and angle (TWA) allow teams to maximize boat speed and race performance. To accelerate the development of new teams, tools for helping them optimize and understand the boats performance are vital.

Recent advances in computational fluid dynamics (CFD) provide accurate tools for evaluating the forces and moments a wingsail produces, however, this accuracy comes at a computational cost by demanding more resource and time. An example of this is highlighted by Team New Zealand who used a 567 core Dell high performance computing (HPC) cluster to run RANS simulations to create an aerodynamic database during the 34<sup>th</sup> AC (Collie, et al., 2015). Therefore, unless you have the resource of a Cup team, 3D CFD is not often feasible for full design space exploration.

To reduce computational cost a lifting line (LL) model can be used to approximate the lift distribution and hence forces on a full 3D wingsail. A modified LL model using an iterative approach to make use of non-linear lift coefficients obtained from 2D RANS has been shown to give close agreements of drive force ( $F_D$ ) and heeling moment ( $H_M$ ) when compared to 3D RANS simulations at low AoA (Graf, et al., 2014). Graf found slight overpredictions of lift close to maximum lift coefficient ( $C_L$ ) because LL theory struggles to capture the complex flow separation that occurs near stall. Graf used the model to optimise wingsail trim settings such as sheeting angle and camber using the 'Generalised Reduced Gradient' solver within Microsoft Excel, however, the optimisation was conducted for a wing with fixed AWA and twist.

To truly find the best wingsail profile, all configurations must be considered, hence the design space is greatly increased, therefore, an approach focused on computational speed is needed. Once 2D lift and drag coefficients have been obtained from CFD, the LL model can be tuned using a free vortex weighting at the root and necessary corrections to best correlate it to 3D CFD results. Predictions can be made in a few seconds compared to hours needed for 3D CFD giving huge savings. However, to compare thousands of different profiles an even faster tool is needed. This is where the use of a trained machine learning model will be introduced. A machine learning algorithm using Kernel ridge regression was trained to interpolate the relationship between AoA, camber, twist,  $F_D$ , side force ( $F_S$ ), and  $H_M$ . The resulting surrogate model permitted an exhaustive search of different wingsail profiles, providing information on parameter trends such as AoA, camber, and twist. Wingsail profiles that exceeded a given heeling moment limit were excluded and the profile that maximized drive force was chosen to be optimal for that given TWS and TWA, see Figure 1.

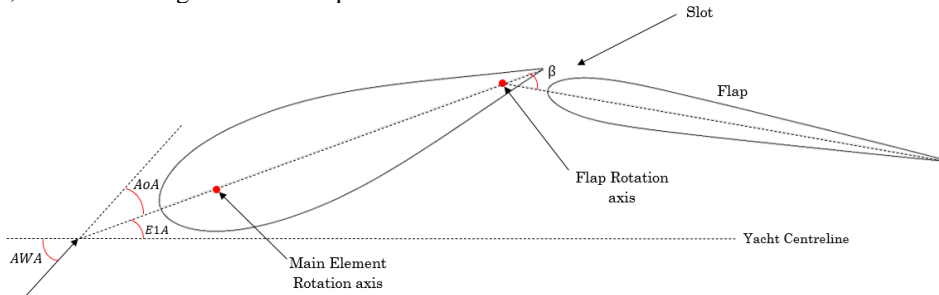


**Figure 1 - Hierarchical approach to evaluating wingsail profiles**

The method can be applied to other sailing yachts with similar sail plans. The combined use of a LL model and machine learning to produce surrogate models is shown to be valuable for the use in velocity prediction programmes (VPPs) (Peart, et al., 2021). The approach is not seen as a substitute for accuracy and resolution afforded using 3D RANS, but as a tool that could be ‘practically’ applied as part of a yacht’s performance development process in terms of costs and timescales. The method is shown to have the ‘potential’ to bring good/reasonably accurate predictions to teams without so much dependency in terms of time, licensing cost, and computation resource for extensive 3D RANS.

## WINGSAILS

Wingsails consist of two symmetrical airfoils: a leading-edge element and trailing edge flap. A cross-section of the wingsail and its terminology is shown in Figure 2. The 24 m wingsail on the F50 catamaran from SailGP is chosen for analysis. The wing has 5 hydraulic control arms that control 4 trailing edge flaps. The chord length varies along the span, with an equal chord distribution between the leading and trailing elements. The trailing edge flap rotates about a point which is 90% along the leading elements chord. Camber refers to the flap angle ( $\beta$ ) at the root of the wing, and twist is the difference between  $\beta$  at the root and tip. The twist profile distribution is controlled by a Bezier curve meaning non-linear profiles are possible, however, for this investigation a linear profile was used.



**Figure 2 – Wingsail section terminology**

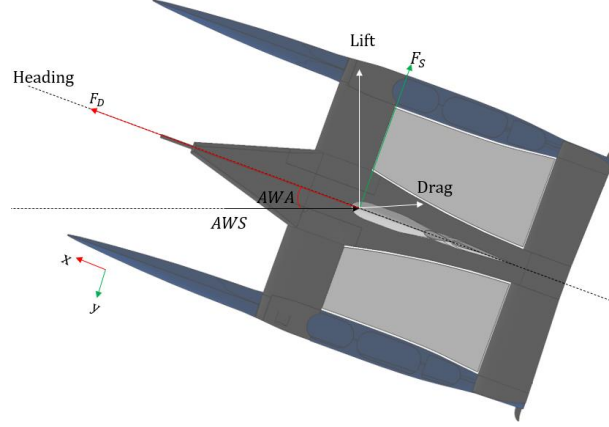
## THEORETICAL METHOD

### Physics of Sailing

The forces created by the wingsail are resolved in the  $x$  and  $y$  directions of the boat coordinate system, as shown in Figure 3. Lift is defined as the component of the aerodynamic force ( $F_A$ ) perpendicular to the onset flow direction (Houghton, 2013). Drag is the component of  $F_A$  in the direction of the onset flow. Yachts encounter a twisted incident wind due to earths boundary layer meaning that both apparent wind speed (AWS) and apparent wind angle (AWA) change with height. Therefore, lift and drag are resolved into  $F_D$  and  $F_S$ :

$$F_D = L \sin(AWA) - D \cos(AWA)$$

$$F_S = L \cos(AWA) + D \sin(AWA)$$



**Figure 3 - Plan view of aerodynamic forces acting on the wing**

$F_A$  is simplified to act at a single point on the sail plan called the centre of effort (*CoE*). The position of this is dependent on many factors adjusted by sail trim. The vertical height of the *CoE* above the mast ball is denoted by  $h$ . This force induces a moment around the mast ball causing the yacht to heel. This is labelled  $H_M$ :

$$H_M = F_S \times h$$

### Upwind Requirements

This paper will focus on upwind sailing, i.e.  $TWA < 90^\circ$ , to limit the number of optimisation cases, but the methods are also applicable to downwind cases. It is reasonable to assume that when foiling upwind  $F_D$  is to be maximised without exceeding a maximum  $H_M$  constraint. This approximation neglects the induced hydrodynamic drag created by  $F_S$ . The side force is balanced by the appendages in the water, therefore high values of  $F_S$  create large amounts of hydrodynamic drag. Ultimately  $F_S$  causes the boat to sail with leeway. In stronger winds, to obtain large drive forces without exceeding the  $H_M$  constraint, wingsail inversion is used to give a negative camber at the top of the sail providing desirable righting moment  $R_M$  (Whidden & Levitt, 2016). The control arms make it possible for the wing trimmer to easily add negative twist at the top of the wing. Wingsail inversion has been proven to obtain the best  $F_D/H_M$  ratio for strong wind conditions (Wood & Tan, 1978).

### Computational Fluid Dynamics

The commercial software package Star-CCM+ 2020.3 has been chosen as the fluid dynamics solver in this study. Star-CCM+ is widely used throughout the marine industry and provides an integrated environment for computer aided design (CAD) import, geometry repair, automated meshing, post-processing, and design exploration.

### 2D Analysis

A cross-section of the F50 wing with maximum chord length of 4.9 m was used for the 2D section analysis. An *AWS* of 40 knts was chosen for an upwind sailing case in approximately 16 knts *TWS*. The calculated *Re* number is  $6.85 \times 10^6$ , using combined chord as the length scale. The air properties used for the 2D simulations are standard air properties taken at sea level, see Table 1. A design sweep of *AoA* and  $\beta$  were conducted using the Design Manager tool within Star-CCM+. A domain study was carried out resulting in a chosen domain size of 4 chords upstream/either side and 8 chords downstream of the mast ball coordinate. At this size the change in drag coefficient ( $C_D$ ) and  $C_L$  with respect to increased domain size had converged to  $< 2\%$ . The boundary conditions were setup with a velocity inlet, pressure outlet and symmetry boundary conditions on the sides.

**Table 1 – Air properties used for 2D simulation at sea level and  $15^\circ\text{C}$**

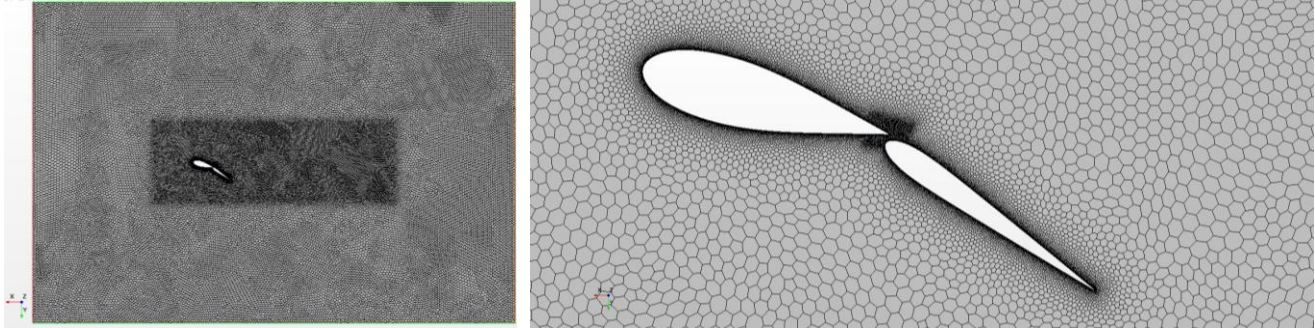
| Property          |   |
|-------------------|---|
| Pressure          | $101.3 \times 10^3 \text{ N m}^{-2}$      |
| Density           | $1.225 \text{ kg m}^{-3}$                 |
| Dynamic Viscosity | $1.802 \times 10^{-5} \text{ N s m}^{-2}$ |
| Flow Speed        | $20.578 \text{ m s}^{-1}$                 |

Table 2 presents the assumptions and appropriate parameters used for the simulation setup. Unsteady RANS simulations have been conducted with 1<sup>st</sup> order time integration, and a time step of 0.015 seconds to better solve the N-S equations and achieve accurate results. A maximum physical simulation time of 8.5 seconds has been chosen which allows a flow particle in the free stream to travel approximately 3 times the domain length. The SST  $k-\omega$  model (Menter, 1994) was chosen after reviewing previous literature regarding aerodynamic wingsail analyses (Collie, et al., 2015). Comparing different models was beyond the scope of this project therefore the SST  $k-\omega$  model was chosen for both 2D and 3D simulations.

**Table 2 - Simulation assumptions used for 2D CFD**

| Category             | Model                     |
|----------------------|---------------------------|
| Time                 | Implicit Unsteady         |
| Time step            | 0.015 s                   |
| Max physical time    | 8.5 s                     |
| Flow                 | Segregated flow           |
| Equation of state    | Constant density          |
| Viscous Regime       | Turbulent                 |
| Turbulence model     | SST (Menter) $k - \omega$ |
| Wall treatment       | All $y^+$                 |
| Turbulence intensity | 1%                        |

Following the domain study, a mesh sensitivity analysis was undertaken to obtain grid-invariant simulation results. A polygonal mesher within Star-CCM+ was chosen because polygonal mashers are numerically more stable and give higher accuracy compared to an equivalent tetrahedral mesh (Siemens Digital Industries Software, 2020). A base size of 0.3 m was chosen, equating to approximately 155000 cells, as this provided a good balance between accuracy and solver time.



**Figure 4 - Domain and mesh used for 2D analysis**

An inner region sized 1 chord upstream, 1 chord either side and 5 chords downstream were used for mesh refinement with the target size being set to 50% of base size, see Figure 4. A second area of refinement was added around the wing profile and in the slot, seen in Figure 4. The boundary layer has been resolved to a  $y^+ \approx 1$  using 20 structured prism layers to yield more accurate  $C_L$  and  $C_D$  values. The prism layers were defined by a starting layer thickness of  $1.5 \times 10^{-5}$  m and growth rate of 1.2. The first cell height has been chosen to lie within the desired  $y^+$  range to reduce errors related to the wall functions used. Final mesh properties are shown in Table 3.

**Table 3: Mesh properties used in 2D analysis**

| Property           |              |
|--------------------|--------------|
| Base size          | 0.3 m        |
| Cell no.           | 156893       |
| Prism layer no.    | 20           |
| Volume Refinement  | 50% of base  |
| Slot refinement    | 2.5% of base |
| Airfoil refinement | 0.8%         |

Validation was achieved by comparing results from the chosen CFD setup with experimental data from a wind tunnel test of a NACA0012 airfoil at a Reynolds number ( $Re$ ) of  $1.76 \times 10^6$  (Sheldehl & Klimas, 1981). The NACA0012 profile was simulated using the same  $Re$  number, domain, mesh, and setup used in the 2D analysis. In conclusion, the setup was found to provide accurate lift and drag predictions within the linear region, however, the values close to and around stall must be viewed with caution as the turbulence model fails to capture the complex wake and shed vortices present. Results can be seen in Figure 5.

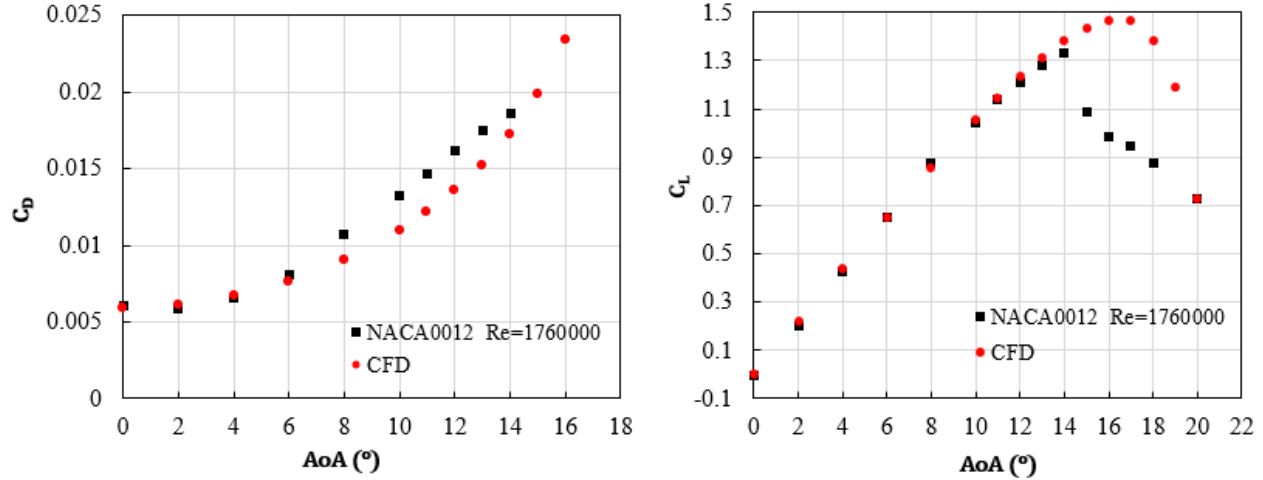


Figure 5 – Validation of 2D setup against NACA0012 airfoil

### 3D Analysis

Due to time constraints, full mesh and domain sensitivity analyses were not able to be conducted, but the simulation assumptions and models used were the same as that in the 2D analysis. Differences were a coarser mesh and smaller time step due to computational limits. Meshing was conducted using the University of Southampton's Iridis 5 HPC cluster using three nodes with  $2.0\text{ GHz}$  Intel Xeon processors totaling 120 cores and 486 GB Ram. A typical simulation took ~6 hours. The wing shape was manipulated in the CAD software package Rhino via a Grasshopper script. Throughout this study the aeroelastic effects of the wing have been neglected for simplicity. The domain size was increased from the 2D section simulations to 5 chords upstream, 5 chords either side, and 10 chords downstream to ensure there was no reversed flow on the boundaries. The domain height was 28.5 m which equated to 1 chord length above the wing tip, as shown in Figure 6.

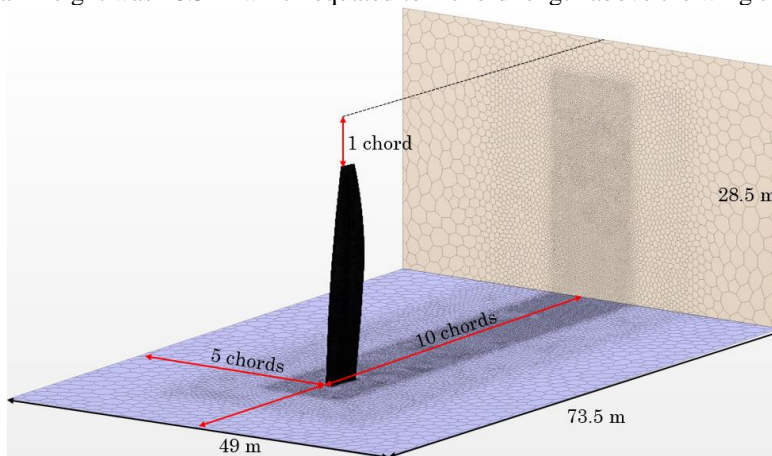


Figure 6 – 3D domain and mesh in Star-CCM+

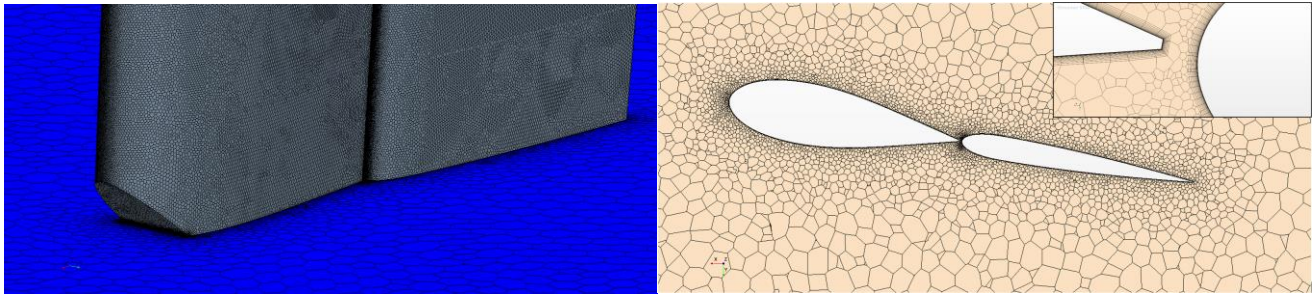
Boundary conditions were as follows: a velocity inlet, pressure outlet, outlet on sides and top. For simplicity, the wing platform was not considered in the analysis, therefore as an acceptable assumption, the floor was modelled as a symmetry plane. For initial comparisons with the LL, the velocity gradient of the wind was ignored to simplify the setup. Table 4 shows the models used in the simulations.



**Table 4 – Simulation assumptions used for 3D CFD**

| Category             | Model                                       |
|----------------------|---|
| Time                 | Implicit Unsteady                           |
| Time step            | 0.1 s                                       |
| Max physical time    | 8.5 s                                       |
| Flow                 | Segregated flow                             |
| Equation of state    | Constant density                            |
| Viscous Regime       | Turbulent                                   |
| Turbulence model     | <i>SST (Menter) <math>k - \omega</math></i> |
| Wall treatment       | All $y^+$                                   |
| Turbulence intensity | 1%  |

Final 3D simulations were run with a twisted incident flow to replicate the true apparent wind the wing experiences when sailing. The twisted velocity gradient was achieved using field functions within Star-CCM+ and changes to the inflow boundaries were made to ensure simulations converged. A base size of 2 m was used with refinements around the wing, and on the wing surface itself to accurately capture the curved surface of the leading edge. Figure 8 shows the mesh refinement on the wing surface. A cross section of the wing at 12 m was taken to inspect the mesh around the wingsail seen in Figure 8. A total of 15 prism layers were used to resolve the boundary layer and no mesh refinement was used in the region of the slot to save time and computational cost. Even with a coarse base size of 2 m, the number of cells exceeded 20 million. Table 5 shows a summary of the mesh properties.

**Figure 8 – Mesh at wing root and cross-section of mesh in 3D domain****Table 5 – Mesh properties used in 3D analysis**

| Property            |              |
|---------------------|--------------|
| Base size           | 2.0 m        |
| Cell no.            | 20.9 million |
| Prism layer no.     | 15           |
| Volume Refinement 1 | 30% of base  |
| Volume Refinement 2 | 10% of base  |
| Airfoil refinement  | 1%           |

### Lifting Line Theory

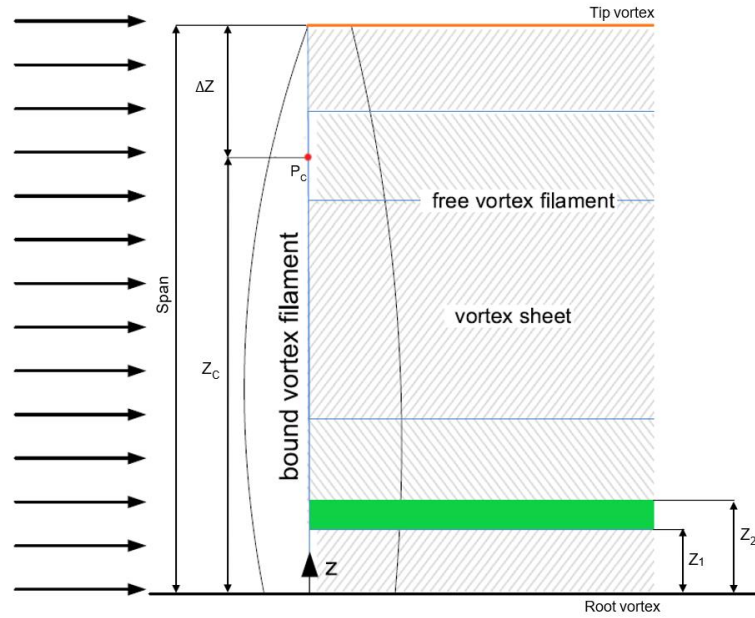
LL theory is a simple method of quickly establishing the performance of an airfoil. The method has the advantage that it provides good estimates of spanwise loading and induced drag, whilst remaining simple to implement and computationally inexpensive (Molland & Turnock, 2021). Flow around an airfoil can be represented by a combination of free stream and vortex flow circulations around the airfoil. This vortex can be represented by a line vortex of strength ( $\Gamma$ ), called a lifting line, lying perpendicular to the chord (Claughton, 1998). The Kutta-Joukowski theorem states that the force ( $F$ ) on this vortex is given by:

$$F = \rho U \times \Gamma$$

This only calculates the lift force perpendicular to the direction of the incoming flow, and neglects drag. By expanding the lifting vortex from the Kutta-Joukowski theorem, a bound vortex filament along the span of the wing is created. This vortex filament is extended to infinity along the incident flow to form a horseshoe vortex at the wing tips to comply with Thompson's rule. A change in lift along the span ( $y$ ) can be represented by changing the strength of the bound vortex and any associated shedding of differential velocity through a free vortex filament ( $\Gamma_{vff}$ ).

$$\Gamma_{vff} = \frac{d\Gamma}{dy}$$

The Biot-Savart law states that shed vorticity induces a velocity normal to the incident flow and that reduces the effective  $AoA$ . The net effect is a downwash which is added to the freestream velocity and causes the inflow to rotate. This induced velocity causes induced drag and is used to model the three-dimensional flow effects around an airfoil. Graf showed that a wingsail can be accurately modelled using a modified LL method described below. The wing is discretized into  $N$  number of panels depicted in Figure 9. Induced velocity is calculated in the centre of each panel from  $j = 1, 2 \dots N$  by summing up the induced wind generated by any free vortex wake sheet and the discrete vortex filaments at root and tip (Graf, et al., 2014).



**Figure 9 – Wing discretization for the LL method**

The root vortex is dampened by a weighting factor ( $\omega_{FVF}$ ) which considers to what degree the root  $\Gamma_{vff}$  is suppressed by a wall. For no gap at the root,  $\omega_{FVF} = 0$ , and if a large gap is present  $\omega_{FVF} = 1$ . Induced velocity is calculated by:

$$V_{indc} = \frac{-1}{4\pi} \sum_{i=1}^N \frac{\Gamma_i - \Gamma_{i-1}}{Z_i - Z_{i-1}} \ln\left(\frac{Z_i - Z_{cj}}{Z_{i-1} - Z_{cj}}\right) + \omega_{FVF} \frac{\Gamma_0}{4\pi} \frac{1}{z_0 - Z_{cj}} + \frac{-\Gamma_N}{4\pi} \frac{1}{Z_N - Z_{cj}}$$

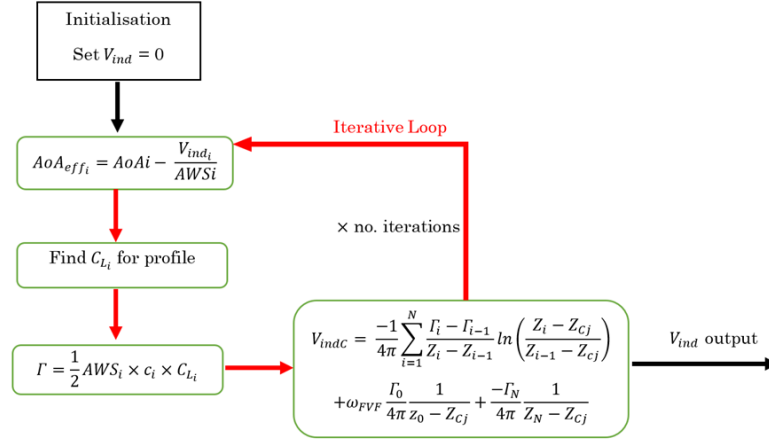
Vortex strength is calculated from the equation below. 2D section lift coefficients must be known in advance and are a function of effective angle of attack and  $Re$  number. Since  $V_{indc}$  and  $\Gamma_i$  are coupled, the solution is found iteratively using under relaxation to achieve convergence (Graf, et al., 2014). Once the effective  $AoA$  is found, the drag can be calculated from tabulated 2D profile  $C_D$  values.

$$\Gamma_i = \frac{1}{2} v_{ind_i} c_i C_{L_i}(AoA_i - \frac{v_{ind_i}}{AWS_i}, Re_i)$$

This method was implemented in the form of a Python script with user inputs for different wing profiles, plotting options, wind shear assumptions and vortex weighting factors. The Pandas Python module was used as it provided fast and powerful data storage and manipulation tools. Wingsail profile data containing  $x$  and  $z$  coordinates discretized the wing into 90 panels of approximately 0.26 m in height. The  $C_L$  and  $C_D$  values from 2D RANS were non-dimensionalised by  $Re$  number to account for changes in chord length and  $AWS$  along the span.



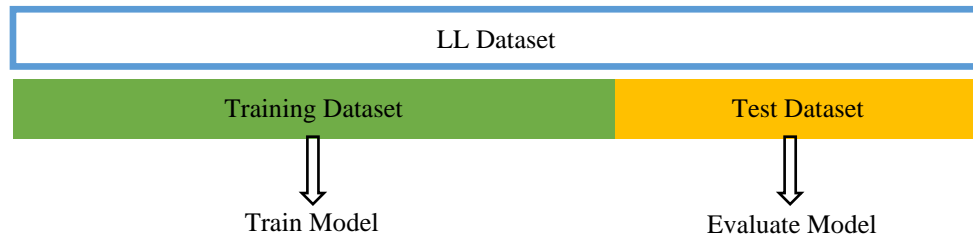
A function was written to read the 2D RANS values and create a cubic surface of  $AoA$  against  $\beta$  via least squares regression. To aid with the surface fitting, coefficients are only plotted up to stall. Beyond stall the code assumes large decreases in  $C_L$  and large increases in  $C_D$  in the form of an equation. However, such large  $AoA$ 's are not expected to give optimal profile shapes. The induced wind loop, see Figure 10, uses this surface to find corresponding coefficients for each profile from its effective  $AoA$  and  $\beta$  before multiplying this by the local  $Re$  number. After induced wind is calculated, the lift and drag contributions of each profile are calculated from their associated coefficients.  $F_D$  and  $F_S$  are calculated from the boats  $AWA$  and integrated along the span.  $H_M$  and the height of  $CoE$  are subsequently found. Predictions took  $\sim 8$  seconds on a standard laptop.



**Figure 10 – Iterative loop for calculating induced wind at each profile height**

### Machine Learning Surrogate Model

Results from the LL code were used to train a machine learning algorithm producing a surrogate model of the wingsail that could efficiently predict performance using several input variables. Scikit-learn, an open source, widely used machine learning library for Python offered the necessary tools for implementing this. The Kernel Ridge regression (KRR) model was chosen as it provided support for multi-variate regression and had capability for polynomial mapping (Scikit-Learn, n.d.). KRR combines standard ridge regression and the kernel method for pattern analysis. Firstly, the LL data is split into two sets, a training dataset, and a test dataset. This is essential for unbiased evaluation of prediction performance. The training to test split is 3:1, see Figure 11.



**Figure 11 – Training to testing data split**

The KRR model learns the relationships between the  $AoA$ , camber, twist,  $F_D$ ,  $F_S$ , and  $H_M$  for a given  $TWA$  and  $TWS$ . The accuracy of the model can be evaluated by looking at the coefficient of determination,  $R^2$  value. The maximum value of  $R^2$  is 1, with a higher score resulting in a better fit. The model is compared against the known test data and scores are compared. For an accurate surrogate model, sufficient training data is needed and hence this is where the previous approach of a LL code is useful. Training the model from 3D RANS data would be computationally expensive given the vast number of different wingsail configurations that would need to be evaluated. The best training set size can be determined by looking at the learning curve of the model. A learning curve shows the validation and training score of the model for varying numbers of training samples. It is a useful tool for finding out how much benefit there is from adding more training data and whether the estimator suffers more from a variance error or a bias error (Scikit-Learn, n.d.). Once the model has been trained to sufficient accuracy, predictions of  $F_D$ ,  $F_S$  and  $H_M$  can be made from three inputs, see Figure 12. Once trained, typical model predictions took  $< 0.001$  seconds which is over 8000 times faster than the LL code.

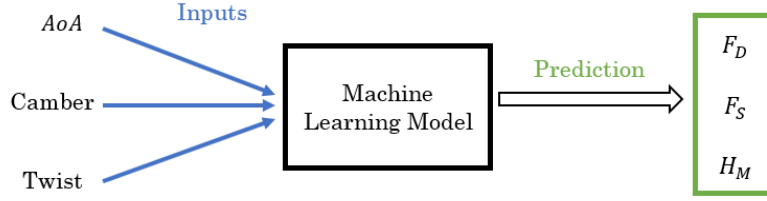


Figure 12 – Model inputs and predicted outputs using KRR model

## RESULTS AND DISCUSSION

### 2D CFD Coefficients

Initially, time averaged RANS simulations were employed to obtain 2D section results. On inspecting the data it was evident that stall was predicted very low at  $8^\circ$   $AoA$  and the residuals showed poor closure of the Navier-Stoke equations. This is partly due to the complex flow that occurs through the slot which influences flow attachment on the trailing flap (Graf, et al., 2014). Therefore, unsteady RANS were chosen to give more accurate results and give better predictions of stall. This meant simulation times were longer, at approximately 16 minutes, but the results could be better trusted. Figure 13 shows  $C_L$  and  $C_D$  for a range of flap angles and  $AoAs$ .  $\beta$  ranges from  $0^\circ$  to  $30^\circ$  and  $AoA$  is plotted up to stall. Unsurprisingly as  $\beta$  increases,  $C_L$  for a given  $AoA$  increases and stall occurred earlier, agreeing with the findings of (Chail, 1949).

All flap angles show a linear region between  $-8^\circ$  and  $12^\circ$  but it's worth noting that for  $\beta > 10^\circ$ ,  $C_L$  stays positive even at an  $AoA$  of  $-8^\circ$  due to the asymmetry of the profile. Stall occurs between  $13.5^\circ$  and  $15^\circ$  after which the simulations have stopped due to poor RANS convergence. In total over 200 2D RANS simulations were conducted equating to approximately 54 CPU hours. There is an expected quadratic increase in drag against increasing  $AoA$  between  $-8^\circ$  and  $13^\circ$ , after which the drag increases at a rapid rate close to stall. Increasing  $\beta$  results in a larger  $C_D$  for the same  $AoA$  except for at  $\beta = 0^\circ$  which gives larger values of  $C_D$  compared to larger flap angles at a negative  $AoA$  caused by the symmetry of the profile resulting in the  $C_D$  curve being mirrored in the y axis about  $0^\circ$   $AoA$ .

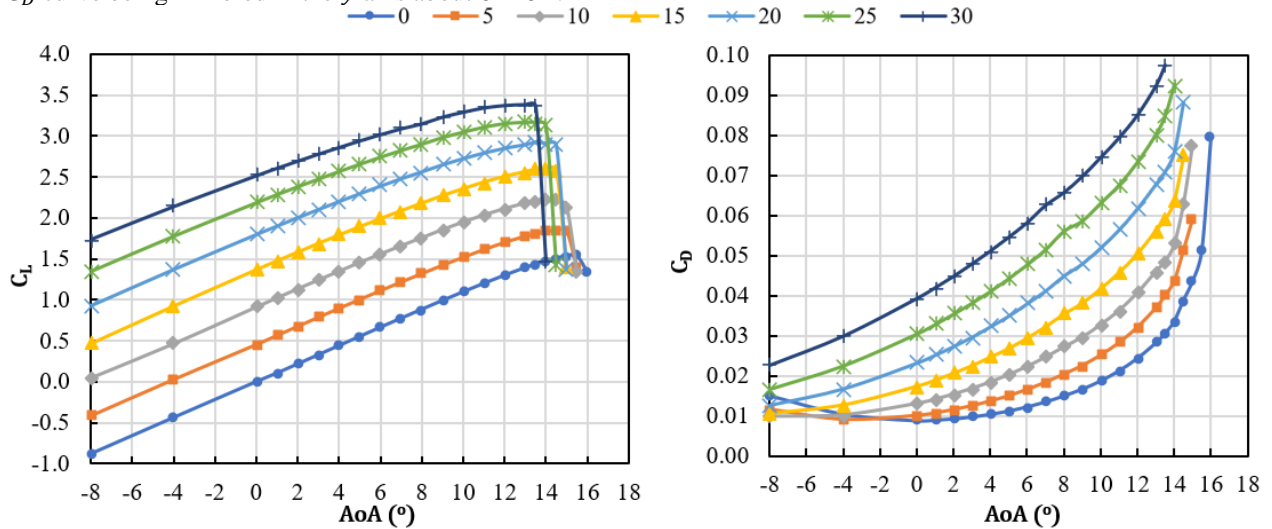
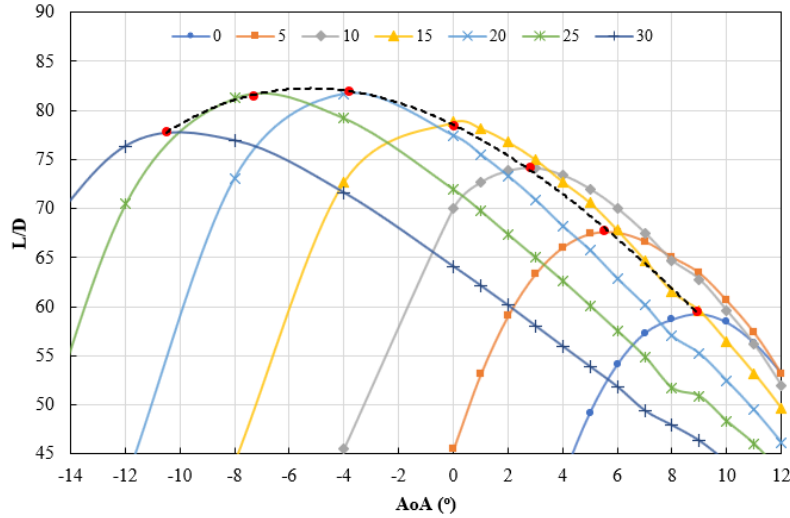


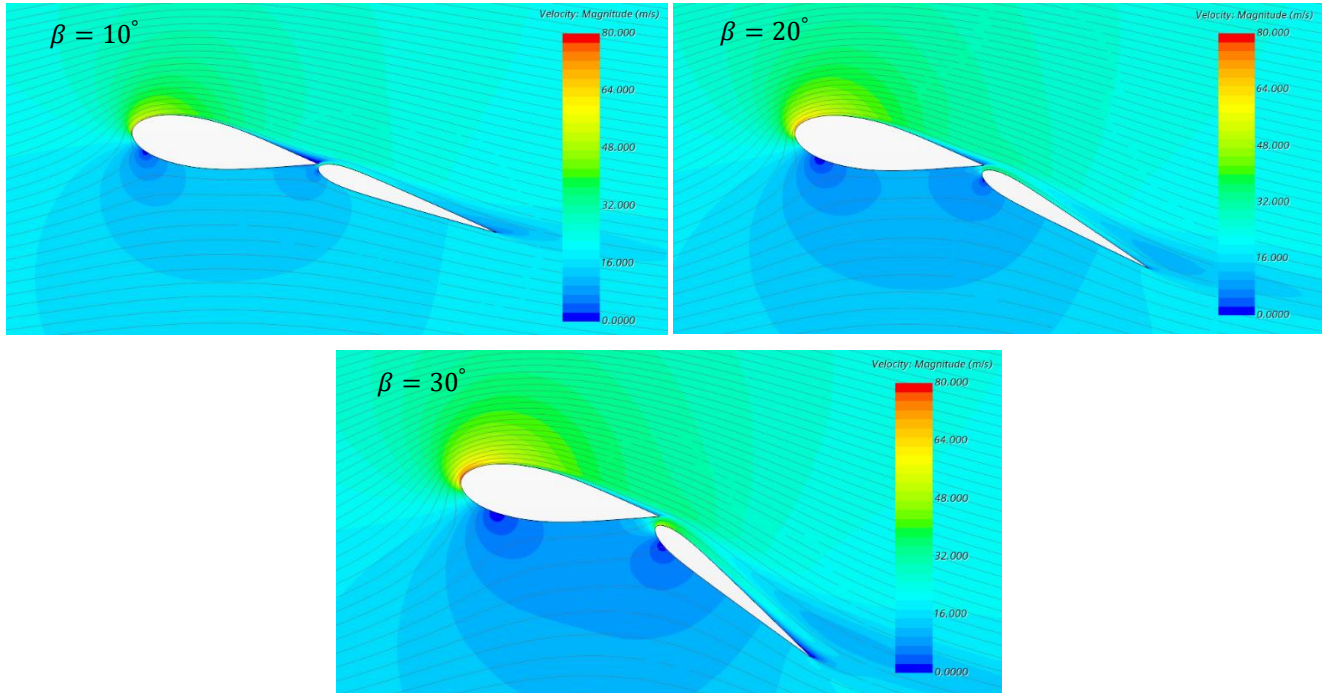
Figure 13 –  $C_L$  and  $C_D$  against  $AoA$  for different  $\beta$ 's up to stall

The lift to drag ratio (L/D) for each  $\beta$  is plotted against  $AoA$  in Figure 14, and the maximum L/D ratio at each flap angle is overlaid. Knowing the relationship between L/D ratio and  $\beta$  is important in understanding the trade-offs when searching for extra lift. Figure 14 shows that by increasing  $\beta$ , the  $AoA$  at which maximum L/D occurs, decreases until a maximum achievable L/D for the section is hit. This occurs at  $\approx -5^\circ$   $AoA$  and  $\beta \approx 22.5^\circ$ . The change in  $AoA$  up the wing caused by wind shear suggests small amounts of wing twist are preferable to maintain optimal section performance. It is worth noting, however, that for finite span lifting surfaces the L/D ratio is likely to be dominated by the induced drag. Therefore, while the 2D L/D ratios are higher than what is expected for 3D, the trends are still likely applicable.



**Figure 14 – L/D ratio for different flap angles and AoA.**

Velocity contours and streamlines at  $10^\circ$  AoA and varying flap angles are shown in Figure 15 to illustrate the flow around the wing. By increasing  $\beta$ , the velocity on the leeward leading edge of the main element increases and the stagnation point for both elements shifts aft. At smaller flap angles, airflow is restricted through the slot matching the findings in (Grassi, et al., 2013) (Turnock, et al., 2014) (Smith, 1975). At larger flap angles of  $20^\circ$  and  $30^\circ$ , the relative AoA of the flap is  $30^\circ$  and  $40^\circ$  respectively. At such large AoA, separation would be expected, however, the slot channels flow from the high-pressure side to the low-pressure side as seen by the streamlines, this delays separation on the trailing flap in accordance with the literature. This flow phenomenon allows the large L/D ratios to be achieved.



**Figure 15 – Velocity plot and streamlines at  $10^\circ$  AoA for  $\beta = 10^\circ$ ,  $20^\circ$  and  $30^\circ$**

Assuming an AWA of  $12^\circ$  for an upwind case,  $F_D$  is plotted against  $F_S$  in Figure 16 from  $-8^\circ$  AoA up until stall for a range of flap angles to view the relationship. Figure 16 shows that for each  $\beta$  there is a linear relationship between  $F_D$  and  $F_S$ . Unsurprisingly, as  $\beta$  increases, the forces produced also increase. The gradient of this linear trend is 1.9 meaning that for every increase in  $F_D$ ,  $F_S$  approximately doubles.

Therefore, the limit of  $H_M$  is quickly reached and ways of depowering the wing are needed. What is noticeable is that the same values of  $F_D$  are possible to achieve with different  $\beta$  operating at different  $AoAs$ . For optimal wingsail performance, large driving forces generated by large flap angles are desirable low in the wing whereby the penalty paid in  $H_M$  caused by increased side force is reduced due to the smaller lever arm on the yacht.

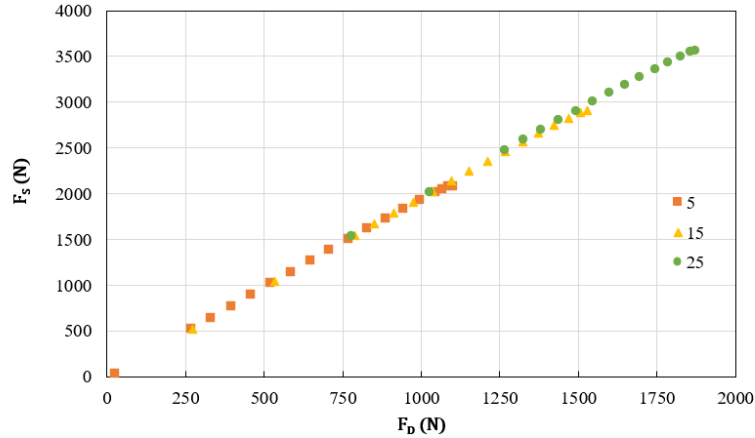


Figure 16 –  $F_D$  vs  $F_S$  for  $\beta = 5^\circ, 15^\circ$  and  $25^\circ$

### Lifting Line Model

Figure 17 shows how varying twist changes  $F_D$ ,  $F_S$  and  $CoE$  height. The results presented are for an upwind case in 12 knts  $TWS$  at  $60^\circ TWA$  with 31.6 knts of boat speed, as per typical F50 upwind performance.  $F_D$  is represented by the lines on the left of the graph and  $F_S$  on the right. Increasing the twist reduced the  $CoE$  height as expected. As twist is increased, not only is  $F_{Smax}$  reduced, but this maximum occurs at a lower location. This confirms that twist is a powerful tool for reducing  $H_M$  in accordance with (Collie, et al., 2015) (Whidden & Levitt, 2016) (Graf, et al., 2014). Furthermore, the increase in twist, reduces the maximum value of  $F_D$  and shifts the distribution to lower down the wing subsequently reducing the pitching moment and improving the boats longitudinal stability. With  $35^\circ$  of twist,  $F_S$  changes sign at 87% span indicating that the top 13% of the wing is producing positive righting moment.

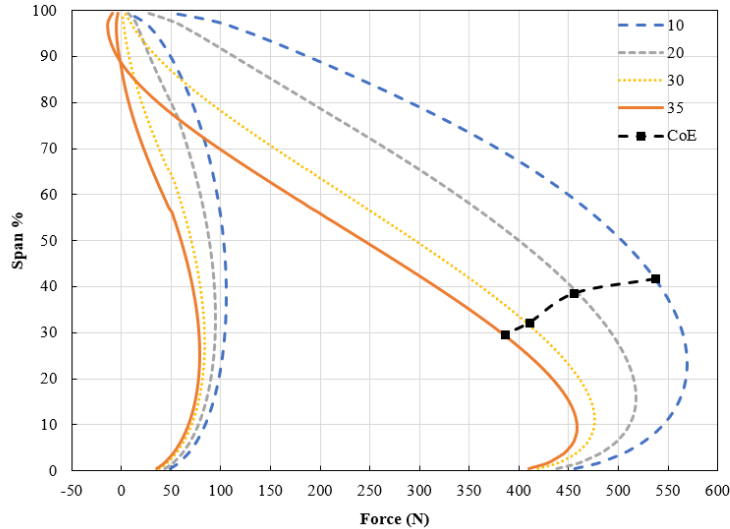
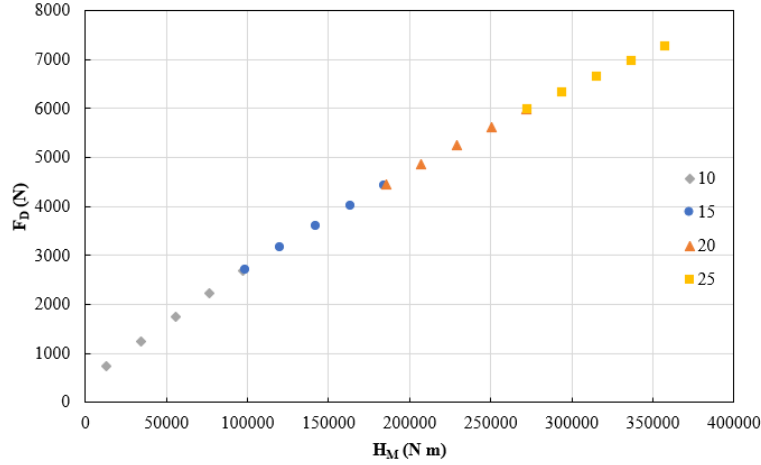


Figure 17 –  $F_D$  (left) and  $F_S$  (right) distribution for wing at  $4^\circ AoA$ ,  $20^\circ$  camber and varying twists

Figure 18 shows the relationship between  $F_D$  and  $H_M$  in 12 knts  $TWS$  at  $60^\circ TWA$  for different camber configurations. The twist is kept constant at  $20^\circ$  and each point represents an  $AoA$  which ranges from  $-1^\circ$  to  $3^\circ$ . The relationship between  $F_D$  and  $H_M$  is approximately linear for each camber configuration. As camber increases, the values of  $F_D$  and  $H_M$  increase and follow the trend from the previous configuration. There are points where different cambers will give similar values of  $F_D$  and  $H_M$  just at different  $AoAs$ . An example of this is at  $-1^\circ AoA$  for  $15^\circ$  camber and,  $-3^\circ AoA$  for  $10^\circ$  camber. The same relationship was also shown above for  $F_D$  and  $F_S$  in Figure 16.



**Figure 18 –  $F_D$  vs  $H_M$  for different  $AoA$ 's and camber configurations**

Interestingly, at higher camber configurations, the range of  $F_D$  decreases for the same range of  $AoA$ s, see Table 6. This suggests that at smaller cambers, the change in  $F_D$  is more sensitive to changes in wing  $AoA$ . Therefore, lower camber configurations are less forgiving to changes in  $AoA$  caused by wing sheet movement. However, this is only considering the case where twist is kept constant for all cambers.

**Table 6 – Change in  $F_D$  for different cambers between  $-1^\circ \leq AoA \leq 3^\circ$**

| Camber Setting ( $^\circ$ ) | 10   | 15   | 20   | 25   |
|-----------------------------|------|------|------|------|
| $\Delta F_D$ (N)            | 1935 | 1730 | 1521 | 1282 |

### 3D CFD Comparisons

For initial comparisons wind shear was excluded from the CFD results to simplify the model setup. An assumption of the root being end plated to the deck has been made, allowing little to no pressure loss from the leeward to windward side. Increasing  $\omega_{FVF}$  results in larger losses of lift at the root and increases induced drag. Initial comparisons for a wing with  $0^\circ$  camber and  $0^\circ$  twist showed that a  $\omega_{FVF} = 0.1$  gave the closest trends against  $C_L$ ,  $C_D$ ,  $CoE$ , and moment coefficient ( $C_M$ ) when compared to other  $\omega_{FVF}$  factors, see Figure 19. Higher  $\omega_{FVF}$ 's gave better predictions of  $C_D$  but compromised  $C_L$ ,  $C_M$  and  $CoE$  predictions.

On average the LL gave a difference of -5%  $C_L$  and -20%  $C_D$  from  $2^\circ \leq AoA \leq 14^\circ$  demonstrating how powerful the LL is when computational speed is a priority, see Figure 19. A single LL prediction took  $\leq 8$  seconds on a laptop, compared to a 3D simulation that took  $\sim 6$  hours. At  $AoA$ 's  $\geq 15^\circ$  the LL began to deviate from the CFD results by overestimating lift and underestimating drag. This is likely due to flow separation that occurs close to stall being poorly captured by the LL, agreeing with the findings of (Graf, et al., 2014).

Comparisons with  $C_M$  show an average difference of -10%. With increasing  $AoA$  the difference becomes larger with the reasons stemming from the smaller lift and drag predictions above. Figure 19 shows  $CoE$  height is a more sensitive result than  $C_M$  because it is affected by the shape of the lift distribution. As  $AoA$  increases,  $CoE$  is expected to rise and this can be seen in the CFD. Strangely this does not occur with the LL, whereby the  $CoE$  reduces in height. On average the difference in predicted  $CoE$  is -8% and indicates a difference in lift distribution.



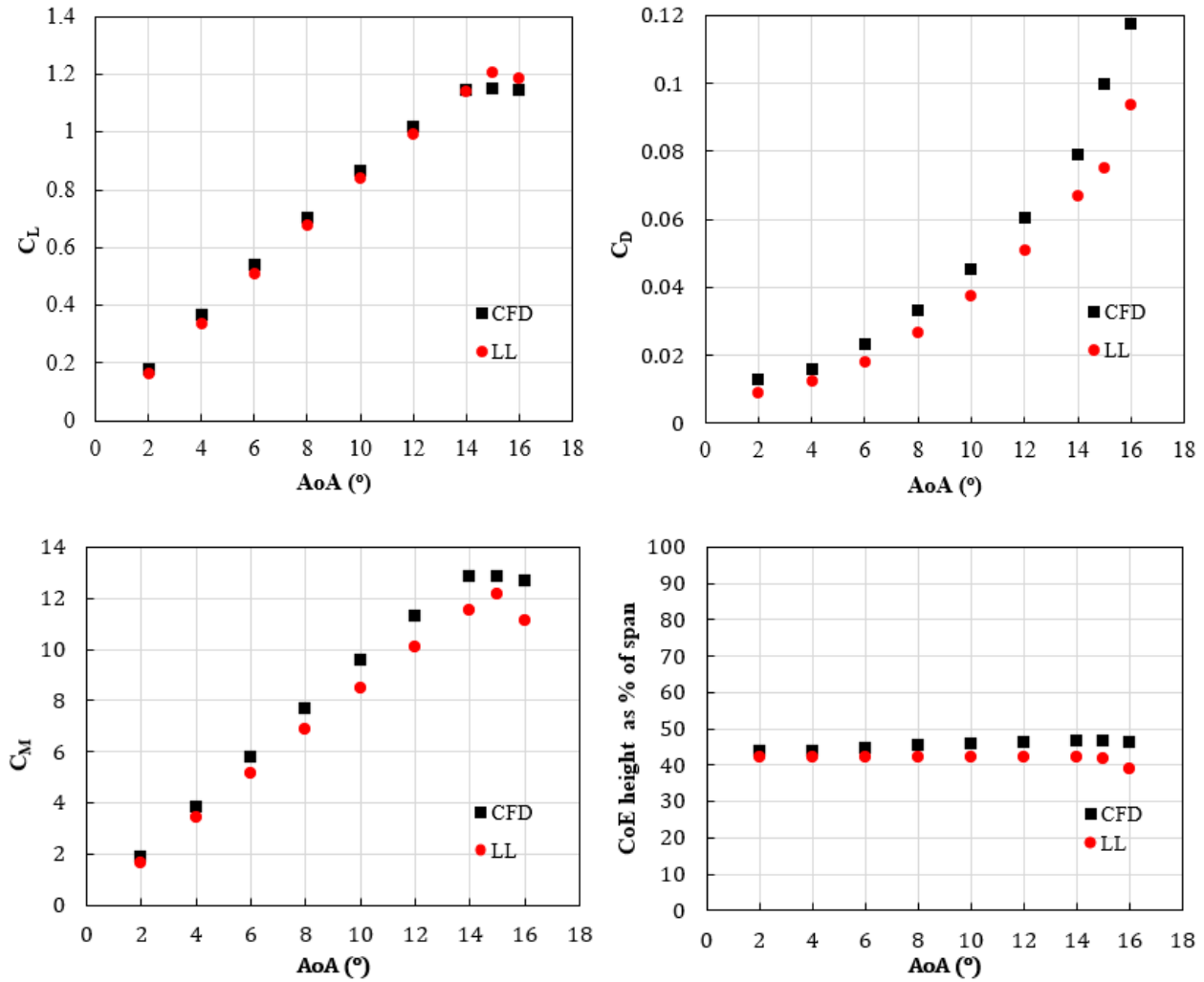


Figure 19 –  $C_L$ ,  $C_D$ ,  $C_M$  and  $CoE$  against  $AoA$  for both LL and CFD ( $0^\circ$  camber,  $0^\circ$  twist)

The LL is overestimating the tip washout causing a lower percentage of lift predicted at the wing tip, causing the  $CoE$  to be lower, see Figure 20. The LL predicts higher lift than the CFD up to 60% span. Beyond 60% span, the LL shows a rapid decrease in spanwise load towards the tip caused by the underprediction of downwash. Empirical corrections by (Molland & Turnock, 2021), and increases in  $\omega_{FVF}$  gave improved spanwise loading but did not translate into better overall prediction of  $C_L$  and  $C_D$ . The presence of tip and root vortices is shown in Figure 21.

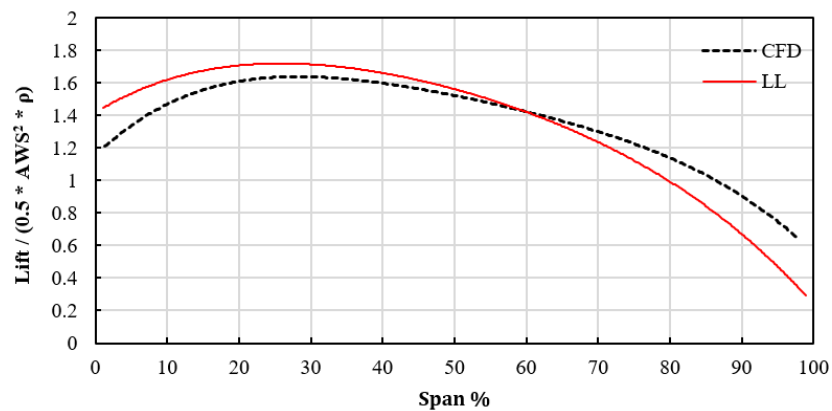
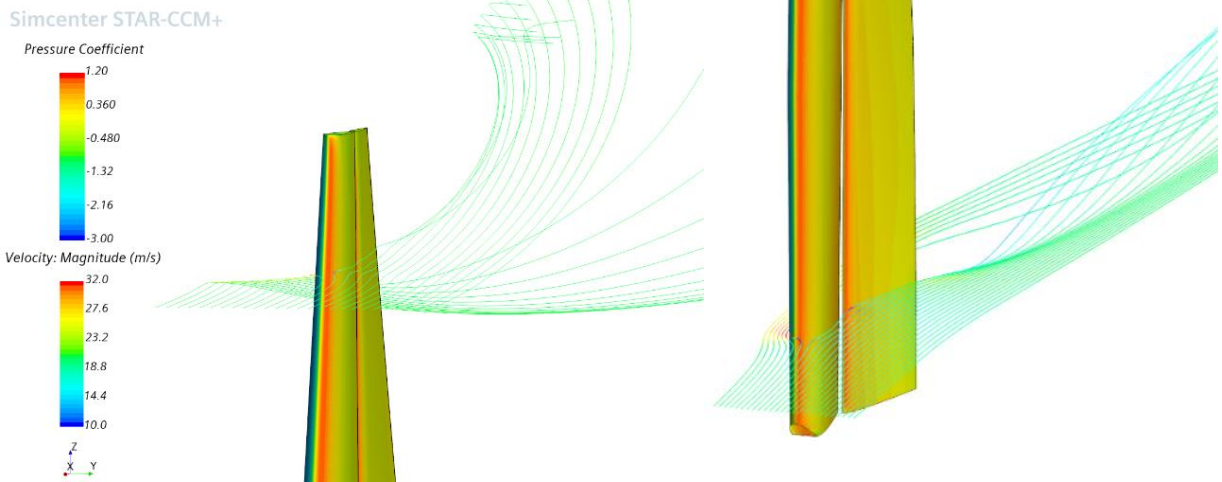
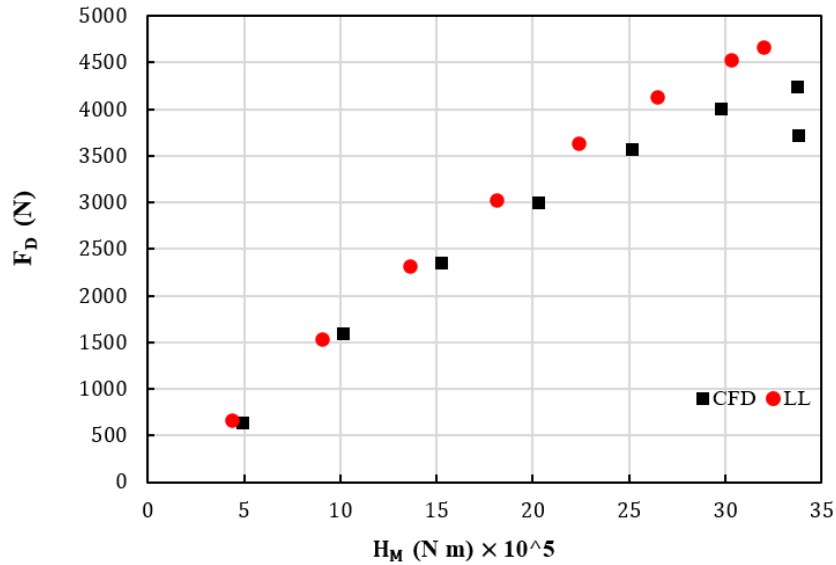


Figure 20 – Spanwise  $C_L$  distribution for both CFD and LL



**Figure 21 – Pressure coefficient and streamline plot showing the tip and root vortex**

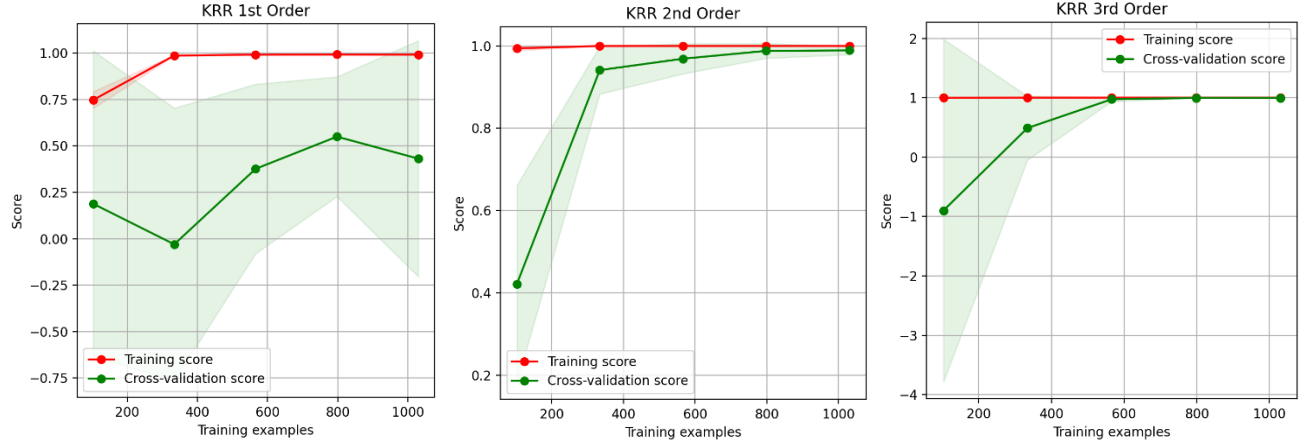
The driving force and heeling moment are plotted for  $2^\circ \leq AoA \leq 15^\circ$  in Figure 22. Up to  $14^\circ$   $AoA$ , the LL underpredicts  $H_M$  on average by 5% and overpredicts  $F_D$  by 1% showing impressive accuracy. Past  $14^\circ$  the wing starts to stall and so there is a drop in  $F_D$  meaning that sheeting the wing in further is counterproductive as  $H_M$  remains high. This is not predicted by the LL and once again shows its limitations at high  $AoA$ 's. Overall the trends of  $C_L$ ,  $C_D$  and  $C_M$  match up for both CFD and LL predictions, therefore there can be confidence in the LL code allowing wingsail optimisations to be carried out. With more investigation the accuracy can be improved by factoring in additional corrections to the LL model.



**Figure 22 –  $F_D$  against  $H_M$  for both LL and CFD at  $2^\circ \leq AoA \leq 15^\circ$**

### Surrogate Model

Using the LL model, a sweep of  $AoA$ , camber and twist configurations were conducted to produce a dataset of results. The size of this dataset was varied to investigate the influence of training set size on performance for different order KRR models. The learning curves for each order model are seen below in Figure 23. Both the mean and standard deviation of scores are shown. Different training set sizes ranging from 100 to 1000 were tested. The LL allows such large data sets to be created for a relatively cheap computational cost. 1000 LL evaluations took approximately 2.2hrs which is vastly quicker than if the data were to come from 3D RANS simulations.



**Figure 23 – Learning curves for 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup> order KRR models**

For the first order model, cross validation scores are low with a high variance for a range of training sizes showing that the first order approximation is not a good choice. For small numbers of training data, the 2<sup>nd</sup> order model scores much higher and has a smaller standard deviation suggesting it is a much better fit than the 1<sup>st</sup> order model. Adding more training samples will most likely increase generalisation whereby the score converges just below 1 with a sample size of 1000. The 3<sup>rd</sup> order model shows the worst performance with small training sizes but quickly converges to a score of 1 with just 566 training points, after which the model does not benefit from more training data. The 3<sup>rd</sup> order model has the longest fit times of ~0.05s when compared to the 1<sup>st</sup> and 2<sup>nd</sup> order models, but this time is negligible when compared to LL evaluations. Once built, predictions from the model take even less time at ~0.001s showcasing its attractiveness for implementation into VPPs. Other regression models were not investigated due to time limitations, but full comparisons of other techniques would be interesting to investigate, especially when using data with non-linear wing twist profiles whereby the relationships between input and output might not be so straightforward. In this study the 3<sup>rd</sup> order model was chosen with 900 data points for training giving an  $R^2$  value of 0.999.

## OPTIMISATION

### Compute Resource

Table 7 shows the computational resource and time taken for each method discussed. The 3D CFD takes the most time at ~6 hours per simulation and requires the most compute resource by a large margin using the University of Southampton's HPC cluster. 2D CFD took slightly less time at ~16 minutes per simulation but can be run at much higher fidelity. Ultimately the 2D simulations are necessary as the LL code is dependent on them, however, once the sectional information is obtained the number of different wing configurations that can be predicted is endless. Arguably the CFD runs can be run in parallel which would shorten the time taken for multiple evaluations, but the computational resource needed remains comparable.

**Table 7 – Computational cost for each method**

| Method                              | Machine | Available Resource                                 | CPU Time<br>(1 sim) | CPU Time<br>(500 sims) |
|-------------------------------------|---------|--|---------------------|------------------------|
| 2D CFD                              | HPC     | × 3 2.0 $GH_z$ Intel Xeon<br>120 cores, 486 GB Ram | ~16 mins            | ~5.6 days              |
| 3D CFD                              | HPC     | × 3 2.0 $GH_z$ Intel Xeon<br>120 cores, 486 GB Ram | ~360 mins           | ~125 days              |
| LL code                             | Laptop  | 2.2 $GH_z$ Quad-core Intel i7<br>16 GB Ram         | ~8 s                | ~1 hr                  |
| Machine Learning<br>Surrogate Model | Laptop  | 2.2 $GH_z$ Quad-core Intel i7<br>16 GB Ram         | ~0.001 s            | ~0.5 s                 |

A single simulation using the LL code took  $\sim 8$ s which is over 2700 times faster than a 3D CFD simulation. This time saving is impressive and when scaled up to 500 simulations, the saving speaks for itself. The physical cost of such computation, which can be broken down into the cost of components, operation/maintenance, and price of electricity, would be substantially higher than that of a desktop PC. Once the LL model has been leveraged to produce a large sparse data set of wingsail profiles, the machine learning algorithm can be trained producing a surrogate model of the wing. An exhaustive search in a fraction of the time is now possible and implementation of this model into a VPP would be the next logical step.

### Upwind Profile Optimisation

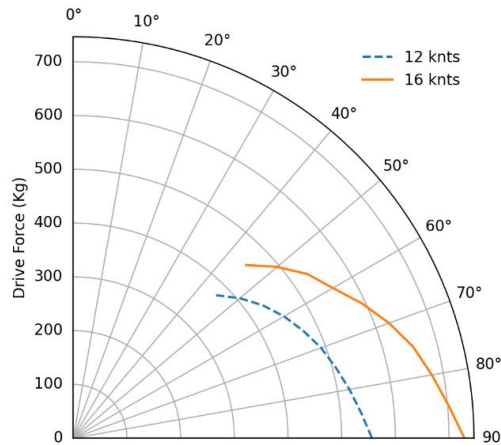
The LL code is set to produce a sparse set of data points of different wingsail profiles. The goal is to have enough data points so that the machine learning algorithm can accurately interpolate the data. Using the surrogate model, an exhaustive sweep is possible due to its computational speed. Results that exceed  $H_M$  constraints are filtered out and then profiles giving the largest drive force are presented. An optimum wing shape is to be found for an upwind case at  $60^\circ$  TWA in 12 and 16 knts TWS. Wind shear is considered, and a linear twist profile is assumed for the optimisation. The data input to the LL code is shown in Table 8. After speaking with designers from SailGP, a  $H_M$  limit of  $18 T m$  has been set.

**Table 8 – Data input to LL model**

| Input       | Case 1       | Case 2       |
|-------------|--------------|--------------|
| TWS         | 12 knts      | 16 knts      |
| TWA         | $60^\circ$   | $60^\circ$   |
| $V_s$       | 31.6 knts    | 39.6 knts    |
| Leeway      | $3.15^\circ$ | $3.26^\circ$ |
| $H_M$ limit | 18 T m       | 18 T m       |

### Results

The LL produced 1200 data points for each case taking a time of  $\sim 2$  hours. The machine learning algorithm then interpolated the data with an  $R^2$  value of 0.999, showing an extremely accurate model fit. The algorithm performed an exhaustive sweep with  $0^\circ \leq \text{camber} \leq 30^\circ$ ,  $0^\circ \leq \text{twist} \leq 50^\circ$ , and  $-5^\circ \leq \text{AoA} \leq 7^\circ$ . 14500 different wing profiles were evaluated in an impressive 2.85 seconds. A polar plot of the maximum  $F_D$  for the given  $H_M$  constraint is plotted from  $40^\circ$  to  $90^\circ$  TWA for both wind speeds in Figure 24.

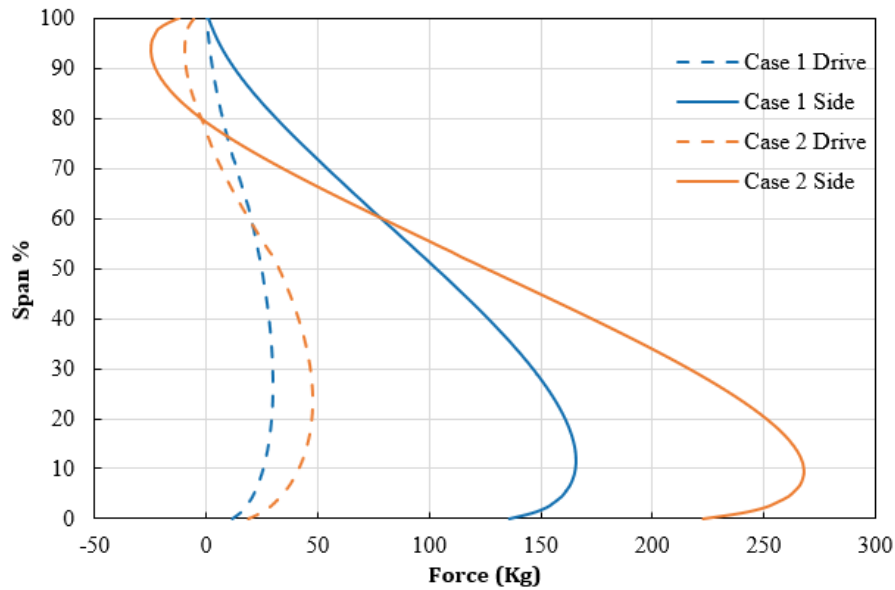


**Figure 24 – Polar plot of maximum  $F_D$  against TWA for 12 and 16 knts TWS**

Table 9 shows results of the optimum profiles found. The profiles were entered into the LL to check interpolation accuracy of the algorithm. The algorithm's output was within 0.5% of the LL predictions proving accurate interpolation had been achieved. For Case 2, the profile found had less camber and more twist compared with Case 1. This is expected as the  $AWS$  is higher in Case 2 and hence the wing needs depowering more to not exceed the  $H_M$  constraint. The increased twist reduced the  $CoE$  by  $1.86 m$  and meant the top 20% of the wing was providing positive  $R_M$ . This allowed for a higher overall drive force to be achieved, see Figure 25.

**Table 9 – Optimum profiles found via the surrogate model**

|               | Case 1   | Case 2   |
|---------------|----------|----------|
| <i>AoA</i>    | 2°       | 2°       |
| <i>Camber</i> | 20°      | 18°      |
| <i>Twist</i>  | 28°      | 34°      |
| $F_D$         | 446 kg   | 552 kg   |
| $F_S$         | 2263 kg  | 2932 kg  |
| $H_M$         | 17.9 T m | 17.8 T m |
| <i>CoE</i>    | 7.91 m   | 6.05 m   |

**Figure 25 – Distribution of  $F_D$  (dashed) and  $F_S$  (solid) against % span****Final CFD Comparison**

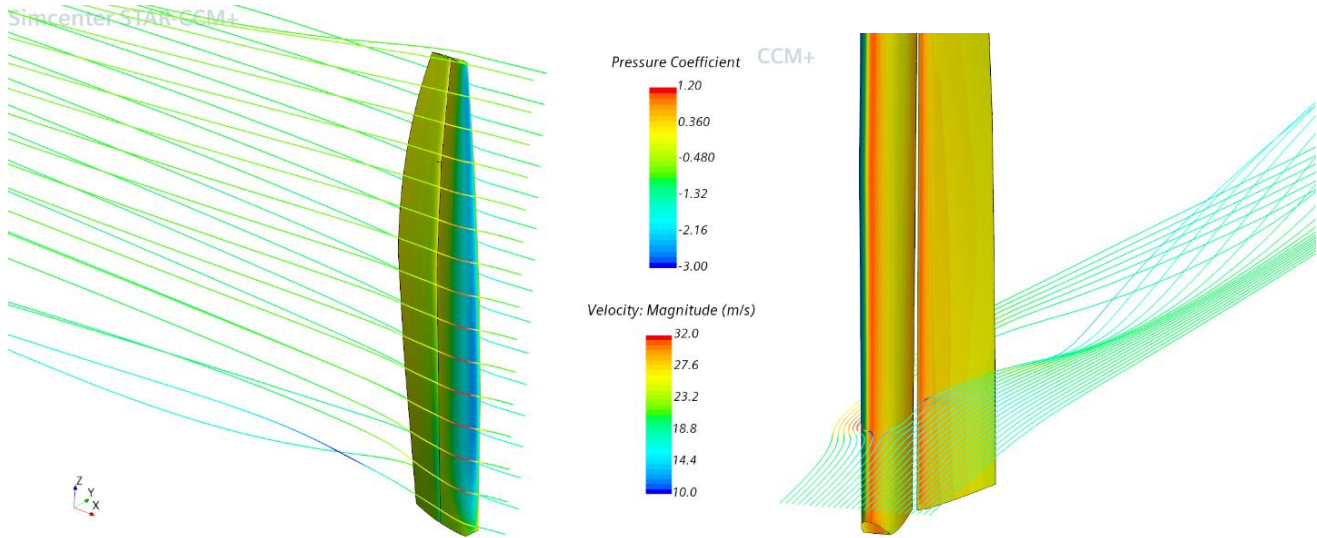
A final 3D simulation implementing a twisted inflow velocity was conducted to compare results for Case 1. Table 10 shows the output from the CFD along with the LL model prediction.

**Table 10 – CFD and LL comparison of optimised profile at 60° TWA in 12 knts TWS**

|            | CFD                    | LL Model               | % Difference |
|------------|------------------------|------------------------|--------------|
| $F_D$      | 486 kg                 | 446 kg                 | -8.2         |
| $F_S$      | 2272 kg                | 2263 kg                | -0.4         |
| $H$        | 17.7 T m <sup>-1</sup> | 17.9 T m <sup>-1</sup> | 1.1          |
| <i>CoE</i> | 7.82 m                 | 7.91 m                 | 1.2          |

The prediction of  $H_M$  and *CoE* height was close with the LL model overpredicting by 1%. The  $F_S$  prediction was even closer at -0.4%. There was a larger difference in  $F_D$  with the model underpredicting by 8%. Figure 26 show the pressure coefficient on the surface of the wingsail along with streamlines depicting the flow at the root and tip. There is a large negative pressure coefficient close to the leading-edge showing an increase in air speed over the leeward side. The pressure coefficient decreases along the span also indicating that less lift is being produced near the tip, because of the wing twist.





**Figure 26 – Pressure coefficient and streamline plot of the wing for Case 1 showing large negative pressure coefficients on leading edge and influence of root vortex**

## CONCLUSION

In summary, a computationally efficient approach for finding optimum wingsail profiles has been developed. A range of different methods have been used to show that computationally fast code run on a laptop can adequately predict wingsail performance when compared to expensive 3D CFD executed on a HPC cluster. This method isn't a substitute for existing approaches but offers a solution to select design areas that could then be investigated more thoroughly with 3D RANS or similar. The speed of this method would also allow this to be integrated within a VPP where full yacht optimisation could be carried out. The project has shown that:

- i.) Unsteady RANS simulations give improved flow predictions when finding 2D section data of wingsails. Steady RANS simulations give poor CFD residuals convergence, predict stall earlier, and give lower values of  $C_{L\ max}$ .
- ii.) Resolving lift and drag forces into  $F_D$  and  $F_S$  show a linear relationship between the two up to stall.
- iii.) Using 2D profile data of the wingsail, a LL model taking  $\sim 8$  seconds per simulation on a laptop, can predict  $F_D$  and  $H_M$  within 1% and 5% respectively when compared to 3D RANS simulations taking 6 hours on a HPC cluster. This gives confidence in using the model for wingsail optimisation.
- iv.) For fixed wing profiles, increasing the  $AoA$  results in a linear increase in  $F_D$  and  $H_M$  up to stall. For a fixed value of twist, lower camber configurations result in  $F_D$  being more sensitive to changes in  $AoA$  caused by wing sheet movement.
- v.) A simple machine learning algorithm can produce an accurate surrogate model using LL data, giving predictions within 0.5%. This accuracy allows exhaustive wingsail profile sweeps 8000 times faster than the LL code demonstrating the power of leveraged surrogate models.
- vi.) The combined LL and machine learning algorithm approach provides an efficient and powerful tool for evaluating a large number of different wingsail configurations with acceptable levels of accuracy.
- vii.) Using the approach, optimum wingsail profiles polars can be found for a range of  $TWA$  and  $TWS$ , accelerating the learning of new teams, designers and sailors ultimately improving yacht performance.

## ACKNOWLEDGMENTS

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