Farm Advisory Services and Total Factor Productivity growth in the Irish dairy sector

Abstract

This paper investigates the impact of the engagement of individual farmers with Farm Advisory Services (FAS) on Total Factor Productivity (TFP) growth, as a relevant indicator of competitiveness under the vision of Sustainable Intensification (SI). Using farm-level data from the Irish dairy sector between 2008 and 2017, we estimate a random coefficients stochastic frontier model and construct a TFP growth index, extending Orea (2002) such that the contribution of FAS becomes an additional component of the index. The results indicate that the main driver of TFP growth was technical change and efficiency gains; a negative scale effect slowed down TFP growth, but this impact was counteracted by the positive contribution of FAS to productivity growth.

1 Introduction

Sustainable Intensification (SI) is an emerging production model in agriculture, which aims at increasing output volume with the same or smaller quantities of inputs, thus minimizing the environmental pressures resulting from production (e.g. Tilman et al. 2011; Garnett et al. 2013; Godfray and Garnett 2014; Campbell et al. 2014; Benton and Bailey 2019; Klerkx et al. 2019). Under this model, farmers are expected to adapt their production practices to various challenges, such as climatic change, and contribute to the Sustainable Development Goals for satisfying the increase in food demand from growth in world population (e.g. FAO 2009; Fedoroff et al. 2010; Campbell et al. 2014; United Nations 2014; Rossel and Bouma 2016; European Commision 2016).

For SI to be realised, farmers need to combine their own tacit knowledge, obtained through experience and learning by doing, with external information coming from various other actors, such as other farmers, Farm Advisory Services (FAS) and researchers (Rossel and Bouma 2016). The interactions between the various actors are combined to form Agricultural Innovation Systems (AIS), which play a central role in the process of innovation co-creation and knowledge transfer, using a "bottom-up" approach (e.g. Klerkx et al. 2012). Contrary to the linear "top-down" innovation model, where innovations are created and transferred from researchers to farmers, often neglecting local conditions and the particular objectives of the farmers, AIS are meant to support a shift from narrow production-based agricultural goals to wider sustainability in agricultural production and the rural communities. This is achieved by using knowledge instead of scarce (e.g. water) or harmful (e.g. chemicals) inputs, increasing productivity and maximizing farm incomes in a more sustainable manner (Bongiovanni and Lowenberg-DeBoer 2004; Mcbratney et al. 2005; Rossel and Bouma 2016; Lajoie-O'Malley et al. 2020). To foster these multi-actor innovation networks and knowledge co-production, FAS have switched from an expert role to broader facilitation and intermediation roles (Hall et al. 2003; Klerkx and Gildemacher 2012; Knierim et al. 2017; Turner et al. 2016; Nettle et al. 2018; Rijswijk et al. 2019). In this demand driven innovation system, farmers and advisors co-produce a solution, tailored to farmers' needs (Laurent et al. 2006; Labarthe and Laurent 2013b).

In the context of EU farming, CAP and the Farm to Fork strategy recognize the role of the European Innovation Partnership for Agricultural Productivity and Sustainability (EIP-AGRI) in ensuring that farmers' needs are communicated and linked better to other AIS actors and, in this way, promoting a "more competitive and sustainable agriculture that achieves more from less" (EIP-AGRI 2012). In particular, FAS is viewed as a key actor in fostering the uptake of relevant innovations at the farm level, which will assist farmers to become more competitive in a sustainable manner (European Commission 2018a; European Commission 2018b; EU SCAR 2019).

Despite the importance of FAS as a driver of productivity, previous empirical studies did not isolate the impact of FAS on Total Factor Productivity (TFP) growth. TFP is considered as one of the relevant indicators for monitoring the CAP objective of promoting viable food production, but also for evaluating the performance of the European Innovation Partnership (European Commision 2016). Consequently, TFP growth indices are widely used in empirical studies as reliable indicators of competitiveness, and to capture different dimensions of sustainability, such as social or environmental, at the farm level (e.g. Brümmer et al. 2002; Färe et al. 2005; Newman and Matthews 2006; Hadley 2006; Newman and Matthews 2006; Zhu and Oude Lansink 2010; Melfou et al. 2007; Emvalomatis 2012; Fuglie et al. 2016; O'Donnell 2012b; Murty et al. 2012; Sauer and Latacz-Lohmann 2015; Chambers and Serra 2018; Skevas et al. 2018; Coomes et al. 2018; Sidhoum et al. 2019; Skevas et al. 2021).

This paper addressed this gap by using Stochastic Frontier Analysis (SFA) to assess the impact of FAS on TFP growth in the Irish dairy sector between 2008 and 2017. The contribution to the literature is twofold. First, it provides a novel methodological approach for isolating the impact of FAS on TFP growth as an independent component, and further highlights the importance of accounting for unobserved heterogeneity. Second, it provides policy implications regarding TFP growth in the sector in the specific context of the CAP and Farm to Fork strategy.

The remainder of the paper is organized as follows: Section 2 provides the background to this study and presents a conceptual framework that links the Irish dairy sector to the TFP growth literature, while accounting for the contribution of FAS. The methodology is presented in Section 3, while Section 4 describes the data and the empirical specification. Section 5 reports the results and Section 6 concludes with policy implications for the Irish dairy sector and a generalization to the EU dairy sector.

2 Background and Conceptual Framework

The Irish dairy sector contributes significantly to the wider Irish economy (DAFM 2015). The main competitive advantage of this sector is the low cost natural grass based feed system (Thorne et al. 2017): due to favourable climatic conditions and suitable soils for grass growth (Hennessy and Roosen 2003), Irish dairy farming is mostly spring-calving, grass based feed system (Läpple et al. 2012). Nevertheless, the grass based feed system has also two main drawbacks. First, farmers are exposed to extreme weather events that can raise production costs, such as in the "fodder crisis" of 2012-13 (Hennessy et al. 2015; European Parliament 2018). Second, the opportunities for further expansion of milk production are bounded by the seasonality of grass production (Hennessy et al. 2015).

Land access and mobility (i.e. transferring of land between farmers and uses), presents a challenge for the growth of the Irish dairy sector (Hartvigsen 2014; O'Donoghue and Hennessy 2015). Unless smaller farmers exit the sector, there is little opportunity of acquisition of land and expansion of production in a post quota era without intensification, i.e. increase in the use of variable inputs, such as purchased feed and fertilizers. However, intensification may result in environmental pollution (e.g. FAO 2013; Coomes et al. 2018), while the exit of smaller farms may undermine the SI of the Irish dairy sector. This is because small (EU) farms play a number of socio-economic roles, producing a range of public goods (Davidova et al. 2013; Dillon et al. 2017). Specifically, small farms maintain rural welfare, keep rural areas populated, contribute to the rural non-farm economy, and provide environmental public goods, such as attractive landscapes (Dillon et al. 2017). Thus, the existence of smaller farms is important for SI. In contrast, the disappearance of small farms is often linked to increased poverty, losses to the rural non-farm economy and depopulation, especially in remote areas, and might result in environmental degradation (Dillon et al. 2017).

For these reasons, Ireland's strategic plan for the agri-food sector, FoodWise 2025, has set as an explicit objective to foster the competitiveness of the Irish dairy sector, under the vision of SI (DAFM 2015). Specifically, FoodWise 2025 suggests that Irish dairy farmers should increase output volume with the same or less inputs, by better exploiting through the use of innovations the sector's advantage in having access to the low cost grass based feed system (DAFM 2015). Ireland has one of the strongest and most integrated AIS in EU, in which, Teagasc FAS contributes to significant knowledge transfer to the Irish dairy sector (e.g. EIP-AGRI 2018; Läpple et al. 2016). In particular, Teagasc FAS, is delegated with supporting farmers in relation to their management and their technical demands and promotes innovations related to better grassland management, breeding techniques, cost management, i.e. the "core" technologies, that will allow farmers to make better use of their low cost feed system, without putting further pressures on the environment (e.g. O'Dwyer 2015; DAFM 2015; Läpple et al. 2019; Läpple and Sirr 2019).

Previous empirical studies examining the evolution of TFP of the Irish dairy sector found a negative efficiency change and a positive scale effect for the period 1996-2008 (Carroll et al. 2008; Kazukauskas et al. 2010; Gillespie et al. 2015). For productivity growth to be in line with the SI vision, this should be driven by technological and efficiency improvements (i.e. farmers learn to make better use of the newly acquired technology). Furthermore, it would be reasonable to expect negative scale effects in the period under investigation: although farms may become larger, the optimal scale of production due to technical progress may grow at a faster rate. In other words, given the importance of the grass based feed system and low land mobility, Irish dairy farmers cannot operate close to the optimal scale of production, thus slowing down TFP growth. Nevertheless, innovations such as those promoted by Teagasc FAS, can increase output from scarce resources (e.g. land) while minimising the use of harmful inputs (e.g. fertilizers and pesticides), enhancing productivity gains. This will ensure a more competitive and sustainable way of dairy farming, in line with SI.

A large empirical literature exists that links innovation and TFP growth in the agricultural sector. Studies usually proxy innovation at the farm level by investment expenditure or a transformation of this expenditure to deal with the possible zero or negative observed values, in case of disinvestment (e.g. Silva and Stefanou 2007; Serra et al. 2011; Emvalomatis et al. 2011; Sauer and Latacz-Lohmann 2015; Sauer 2017; Minviel and Sipiläinen 2018). Although innovations can increase productivity, this increase may come at the cost of another sustainability dimension, such as the environmental (e.g. FAO 2013). The impact of various AIS actors on productivity has been assessed using cross-sectional data, and as well as with panel data (e.g. Kalirajan 1981; Bravo-Ureta and Evenson 1994; Bravo-Ureta et al. 2012; Rao et al. 2012; Henningsen et al. 2015; Kumbhakar et al. 1991; O' Neill et al. 1999; Carroll et al. 2008; Martinez-Cillero et al. 2018; Martinez-Cillero et al. 2019). In a related stream of literature, accounting for the contribution of environmental goods and services (e.g. products or services that aim to prevent or minimise environmental pressures, restoration of environmental damage) or improvement of resource management through education and training, on TFP growth has been suggested as a way of building a better SI metric (Melfou et al. 2007; Fuglie et al. 2016; European Commision 2016; Coomes et al. 2018).

We extend the relevant literature by examining the impact of FAS, along with technical change, efficiency change and scale effect on TFP growth in the Irish dairy sector. Here, we further assume that farmers improve their technology by attaining better access to the "core" technologies, through contact with FAS (e.g. Dinar et al. 2007). FAS is viewed as a mediator in the process of adoption of innovations and practices that are consistent with the vision of SI. This implies that FAS contact can also result in better information flow, which can assist farmers regarding the input choices and access in technology embodied in inputs (Batte and Schnitkey 1989). The farmer's tacit knowledge is combined with the information shared from FAS, so applied information at the farmer level is better adjusted to the farmer's needs. From a theoretical perspective, tacit knowledge (passive learning, learning by doing), with farmer's initial ability and information flows could be seen as the three core determinants of management (Stefanou 2009; Shee and Stefanou 2016; Batte and Schnitkey 1989), where better management shifts the production frontier outwards (Triebs and Kumbhakar 2018). The decision of a farmer to initiate and maintain contact with FAS is determined by a number of farmer characteristics such as age, marital status (Läpple et al. 2015), and it is assumed to be exogeneous to the employed technology, in the sense that causality runs only in one direction, from contact with FAS to adoption of management practices.

3 Methodology

3.1 TFP growth decomposition

We use an output distance function to express mathematically the production technology while accounting for the multi-output nature of the production processes employed by Irish dairy farms (e.g. Newman and Matthews 2006). The output-oriented distance function can be stated as:

$$D_o(\mathbf{x}, \mathbf{y}, F, t) = \min \left\{ \theta : \frac{\mathbf{y}}{\theta} \; \in \text{output possibility set at time } t\text{, given } F \right\} \tag{1}$$

where the input and output vectors, $\mathbf{x} \in \mathbb{R}^N$ and $\mathbf{y} \in \mathbb{R}^M$, are implicitly defined as a functions of time, t, and F is a measure of the interaction between the farmer and FAS, which affects the production technology. In the empirical application F is defined as a time varying binary variable that indicates whether a farmer contacted FAS in a particular year. It is conventional in the relevant literature to capture technical progress exogenously by t. In this context, the passage of time reflects the shift in the production frontier due to improvements in the production technology.

The output oriented distance function returns the inverse of the maximum amount by which the output vector could be increased, but remain feasible, for a given level of inputs.[1] The range of the distance function is the unit interval and the combinations of $\mathbf{x}$, $\mathbf{y}$, F and t for which its value is equal to one define the boundary of the production possibilities set. Thus, the distance function itself can be used to define technical efficiency as a function of its arguments:

$$D_o\left(\mathbf{y}, \mathbf{x}, F, t\right) = TE \tag{2}$$

The TFP growth rate is defined as the weighted growth rate in outputs minus the weighted growth rate in inputs:

$$\frac{\mathrm{d}\log TFP}{\mathrm{d}t} = \sum_{m=1}^{M} \frac{\partial \log D_o}{\partial \log y_m} \hat{y}_m - \sum_{n=1}^{N} \frac{\varepsilon_n}{\varepsilon} \hat{x}_n \tag{3}$$

[1]The output-expanding view of efficiency taken here is in line with the vision of SI, in which farmers are encouraged to maximize the amount of output, given the available resources. Despite the quota scheme operating until 2014, we assume that the farmers' objectives are still consistent with output expansion, since quota was tradeable in Ireland (under some conditions) and between 2009-2014 dairy farmers were allowed to increase the amount of milk output, by up to 1% per year.

where $\varepsilon_n = \frac{\partial \log D_o}{\partial \log x_n}$, $\varepsilon = \sum_{n=1}^{N} \varepsilon_n$ and a "hat" over a variable denotes growth rate ($\hat{y}_m = \frac{\partial y_m}{\partial t}/y_m$ for example). The weights used for the aggregation of the growth rates of outputs in (3) are the respective distance elasticities, while for the inputs the weights are the shares of distance elasticities in scale elasticity (see Orea 2002; Lovell 2003). If the property of linear homogeneity in outputs of the distance function is imposed, then the weights for outputs sum to unity (e.g. O'Donnell and Coelli 2005), while the weights for inputs sum to unity by construction. Under a cost minimization assumption the distance elasticity with respect to each input is equal to the share of the respective input in total cost. Under revenue maximization, the distance elasticity with respect to each output is equal to the share of the respective output in total revenue. Thus, under profit maximization, the distance elasticities can replace the input shares in cost and output shares in revenues in a conventional Törnqvist index. When these strict assumptions fail, then the distance elasticities can be viewed as approximations to the weights required by the Törnqvist index.

Taking logs of both sides in (2), totally differentiating with respect to time, and rearranging gives:

$$\sum_{m=1}^{M} \frac{\partial \log D_o}{\partial \log y_m} \hat{y}_m + \sum_{n=1}^{N} \frac{\partial \log D_o}{\partial \log x_n} \hat{x}_n + \frac{\partial \log D_o}{\partial F} \dot{F} + \frac{\partial \log D_o}{\partial t} = \frac{\mathrm{d} \log TE}{\mathrm{d}t} \tag{4}$$

where $\dot{F} = \frac{\partial F}{\partial t} \approx F_{it} - F_{it-1}$. Finally, solving (4) for $\sum_{m=1}^{M} \frac{\partial \log D_o}{\partial \log y_m} \hat{y}_m$ and inserting this in (3) gives:

$$\frac{\mathrm{d} \log TFP}{\mathrm{d}t} = \frac{\mathrm{d} \log TE}{\mathrm{d}t} - \frac{\partial \log D_o}{\partial t} - (1+\varepsilon) \sum_{n=1}^{N} \frac{\varepsilon_n}{\varepsilon} \hat{x}_n - \frac{\partial \log D_o}{\partial F} \dot{F} \tag{5}$$

This is the usual decomposition of TFP growth into technical efficiency change (TE), technical change (TC) and scale effects (SE) as in Orea (2002), appending FAS, which is an additional component that explains TFP growth due to F. This component comes from isolating the impact of changes from one year to the next in farmer contact with FAS.

The employed TFP growth index, similarly to other popular indices, such as the Fisher, fails to satisfy key axioms from index theory, including monotonicity, identity, commensurability, proportionality, circularity and transitivity (Njuki et al. 2019). For example, the identity axiom states that two farms which use exactly the same amounts of inputs and produce exactly the same amounts of outputs should have exactly the same TFP index. The transitivity axiom states that the direct comparison of the TFP difference between two farms should result in the same TFP change as an indirect comparison through another farm (O'Donnell 2012b; O'Donnell 2012a). See O'Donnell (2018) for a formal presentation of the axioms, indices that satisfy these axioms, and summary of empirical applications that construct "proper" TFP indeces, i.e. indices which can be written as a proper output index divided by a proper input index (that satisfy the eight axioms).

3.2 Accounting for technological heterogeneity: Random Coefficients Model (RCM) specification

In a stochastic frontier framework, the description of the production technology will influence the measurement of TFP growth and its components (Kumbhakar et al. 2018) and although the assumption of homogeneous technology is convenient, it may not be realistic. Neglecting to account for technological heterogeneity may result in misleading characterizations of scale economies, elasticities of substitution and other measures of production structure (Kumbhakar et al. 2018). For example, Alvarez and del Corral (2010) compared the elasticity estimates produced by stochastic frontier models under homogenous and heterogeneous technologies, where heterogeneity was captured via a Latent Class Model (LCM). They used farm level data from the Spanish dairy sector between 1999 and

2006. The study showed that disregarding technological heterogeneity overestimates the marginal productivity of purchased feed per cow and the contribution of the scale effect to TFP growth for all farmers. We expect that not accounting for heterogeneity in our empirical investigation could result in biased estimates of the marginal productivity of inputs and the scale elasticity, as well as of the impact of FAS on productivity growth.

Following Alvarez and del Corral (2010) and Kumbhakar and Orea (2004), one could proceed by modelling technological heterogeneity through a LCM, in which class membership depends on farm size.[2] However, input marginal productivities as well as the effect of FAS on the production possibilities set could vary at the farm level, on many more factors, in addition to the scale of operations. For example, Teagasc FAS may also promote tacit technologies (Boyle 2012), in the sense that these are not fully embodied in purchased inputs, such as machinery or seeds (Evenson and Westphal 1995; Chatzimichael et al. 2014). In this regard, variability of marginal effects among farmers may be due to factors that are not observed by the researcher, such as diverse learning preferences, accessibility to inputs, risk perception, risk tolerance, etc. (Carroll et al. 2008; Kilpatrick and Johns 2003; Pasquini and Alexander 2005; Bowman and Zilberman 2013; Conradt et al. 2014; Saint-Cyr 2017; Trujillo-Barrera et al. 2016; Läpple et al. 2019). Instead, this paper uses an RCM, which can be viewed as a generalization of the LCM (Greene 2005). The main advantage of RCM is that it allows borrowing of strength from observations across farms and provides additional flexibility to the specification of the technology employed (Emvalomatis 2012). This is a particularly relevant issue when the sources of heterogeneity are not observed. In the RCM, the contribution of FAS, technical change and scale effects will depend on farm-specific unobserved heterogeneity, which is captured by farm-specific slope parameters (e.g. Kalirajan and Obwona 1994; Tsionas 2002; Emvalomatis 2012;

[2]Labarthe and Laurent (2013a) and Labarthe and Laurent (2013b) suggest that some EU farmers may proceed with different investments (e.g. in machinery, labour) in relation to the technology they employ. Thus, the effect of FAS may vary with the scale of operation. In general, technical progress, and especially embodied technical progress, tends to favour larger farms (e.g. Alvarez et al. 2012; Alvarez and del Corral 2010).

Njuki et al. 2019; Skevas 2019).

Even though RCMs account for unobserved heterogeneity and add flexibility to the data generating process, they do not correct entirely for endogeneity that stems from the appearance of the normalizing output in both the left- and right-hand sides of the equation to be estimated. This is a known issue with an extensive associated literature. As demonstrated by Roibás and Arias (2004), if random events affect the amounts of outputs non-proportionally, then the output ratios that appear as independent variables are measured with error and, thus, the observed ratios are correlated with the noise component of the error term, v_{it}. Given that farmers may attempt to reduce revenue variability by choosing output mixes such that random shocks counteract their effects on total revenue, it is rather unlikely that these shocks affect the two outputs proportionally. However, endogeneity is an especially difficult econometric problem in the SFA context, without an obvious solution (Mutter et al. 2013; Amsler et al. 2016). The Generalized Method of Momements (GMM) approach requires the selection of instrumental variables that are uncorrelated with the error term, which implies that is sensitive to the choice of instruments (O'Donnell 2010). A good choice for instruments are prices (e.g. Smith and Landry 2021), which are usually unavailable in most farm level production studies. To the best of our knowledge, studies provide a possible way to account for input endogeneity (not necessarily for an input distance function or a panel data model) (e.g. Gudbrand et al. 2018; Tsionas et al. 2015; Hung-pin and Kumbhakar 2021). The only possible methodological approach accounting for output endogeneity in distance function is O'Donnell (2014). This approach has appealing methodological features, but comes with a major limitation, given the aims of our paper: it does not accommodate unobserved heterogeneity. As argued above, neglecting to account for unobserved heterogeneity influences the estimates of TFP growth and its components, including the FAS effect, which is our main interest. Overall, accounting for endogeneity in an output distance function is still an open issue in the empirical literature.

Regarding the choice of the functional form of the distance function, translog appears to be preferred over Cobb Douglas in the literature, due to its flexibility with regards to the elasticity of substitution between inputs and outputs, as well as with respect to the curvature of the production possibilities set. Nevertheless, the RCM specification also allows for high flexibility (Emvalomatis 2012; Njuki et al. 2019), capturing the curvature of the underlying global distance function even when using restrictive local distance functions for each farm separately (Emvalomatis 2012). Using data from the German dairy sector, Emvalomatis (2012) found that formal test comparisons favour RCM specifications with fewer farm-specific parameters. This is crucial, since different functional forms may result in different elasticities and TFP growth results, and on the TFP index itself; and in turn, possibly different policy implications.

Therefore, three different specifications are considered, following Emvalomatis (2012): 1) a semitranslog RCM1, 2) a semitranslog RCM2 with fewer farm specific parameters that is used to assess whether the overparametarization of RCM1 affects estimates and 3) a semitranslog Common Frontier Model (CFM) to all farms, as a reference model to compare results under homogeneous technologies. RCM1 is specified as:

$$\begin{aligned}
-\log y_{it}^{M} = {} & \alpha_i + \sum_n \alpha_{in} \log x_{it}^{n} + \sum_m \beta_{im} \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \sum_\ell \sum_m \gamma_{i\ell m} \log\left(\frac{y_{it}^{\ell}}{y_{it}^{M}}\right) \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \eta_i F_{it} + \rho_i t F_{it} + \zeta_{1i} t + \zeta_{2i} t^2 + \sum_n \xi_i t \log x_{it}^{n} \\
& + \sum_m \varphi_i t \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) + v_{it} + u_{it}
\end{aligned} \tag{6}$$

where y_{it}^{M} is the normalizing output[3], α_i is a farm specific intercept, v_{it} captures statistical

[3]By definition, the distance function is homogeneous of degree one in outputs and imposing this property can be achieved by dividing all outputs and the value of the distance function by the amount of the normalizing output, y^M. Taking the natural logarithm of both sides of the resulting expression and

noise, assumed to be normally distributed with zero mean and precision (inverse variance) parameter τ, u_{it} is a one-sided non-negative error term that captures technical inefficiency, assumed to be exponentially distributed with rate parameter λ (van den Broeck et al. 1994). The technical efficiency of firm i in period t is defined as $TE_{it} = e^{-u_{it}}$ and is bounded between zero and unity. The dependent variable is negative and $\log(TE_{it})$ is subtracted from the right-hand side. This implies that the distance elasticities with respect to outputs should be positive and with respect to inputs negative.

Parameters associated with F and t are expected to be negative in the distance specification. The passage of time is expected to move the frontier of the production possibilities set outwards, reflecting a technological improvement over time. This will be reflected in equation (1) by a reduction in the value of the distance function, as a smaller value for θ is now necessary to project an observed combination of inputs and outputs onto the boundary of the extended production possibilities set. A similar interpretation exists for F: a farmer who is receiving advice in year t is expected to alter the employed technology and shift the frontier (or the farm specific frontier in the case of RCM) outwards.

A linear time trend captures neutral technical progress, and its interaction terms with inputs, F and normalised outputs are included to capture non-neutral technical progress. Specifically, the interaction term between t and F can be used to infer whether the impact of FAS on the frontier is increasing or decreasing over time. Second order terms for normalized outputs are also included so that the distance function is not restricted to be convex in the output dimension.

The specification of RCM2 is similar to the one for RCM1, but with the quadratic time-trend term, as well as the interaction terms between time and inputs and outputs

rearranging gives an expression where minus the logarithm of the normalizing output appears in the left-hand side and the logarithm of the ratio of other outputs to the normalizing output in the right hand side (e.g. Coelli and Perelman 1999).

dropped:

$$
\begin{aligned}
-\log y_{it}^{M} &= \alpha_i + \sum_n \alpha_{in} \log x_{it}^{n} + \sum_m \beta_{im} \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
&+ \sum_\ell \sum_m \gamma_{i\ell m} \log\left(\frac{y_{it}^{\ell}}{y_{it}^{M}}\right) \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
&+ \eta_i F_{it} + \rho_i t F_{it} + \zeta_{1i} t + v_{it} + u_{it}
\end{aligned} \tag{7}
$$

To reduce the RCM1 model to the CFM, all farm specific slope coefficients are replaced by parameters that are common to all farms, similar to Aigner et al. (1977).

The components of TFP growth are constructed after estimating RCM1, RCM2 and CFM. After estimating the parameters of the distance function, technical progress in the case of the RCM1 can be calculated as:

$$
\frac{\partial \log D_o}{\partial t} = \zeta_{1i} + 2\zeta_{2i} t + \sum_n \xi_{ni} \log x_{it}^{n} + \sum_m \varphi_i \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \tag{8}
$$

where ζ_{1i} and ζ_{2i} capture neutral technical progress and the ξ_ns non-neutral progress.

The contribution of FAS to TFP growth consists of two parts, which are obtained by totally differentiating with respect to time the terms of the distance function that involve F: $\eta_i F_{it} + \rho_i t F_{it}$. The first part comes from the partial derivative with respect to time and is equal to $\rho_i F_{it}$. The second part is due to the dependence of F on time and is obtained as $\frac{\partial \log D_o}{\partial F} \dot{F} \approx (\eta_i + \rho_i t) \cdot (F_{it} - F_{it-1})$. From a purely mathematical perspective, the first part could, alternatively, be included in the technical change component instead of the FAS effect. From a practical perspective, however, it is more appropriate to attribute this term to the FAS effect, as it comes to being only because farmers enter into contracts with FAS. Thus, we estimate and present a single aggregated FAS effect (in the results presented in Figure 1 below), that attributes both parts, $\rho_i F_{it}$ and $(\eta_i + \rho_i t) \cdot (F_{it} - F_{it-1})$ to the FAS effect.

Technical efficiency change is calculated as:

$$\frac{\mathrm{d}\log TE}{\mathrm{d}t} \approx \log TE_{it} - \log TE_{it-1} \tag{9}$$

capturing the change in the efficiency score of a farmer between two consequent periods. Finally, the scale effect is calculated as:

$$-(1+\varepsilon_{it})\sum_{n=1}^{N}\frac{\varepsilon_{nit}}{\varepsilon_{it}}\hat{x}_{nit} \tag{10}$$

with $\varepsilon_{nit} = \sum_n \alpha_{in} + \sum_n \xi_{n,i}t$, $\varepsilon_{it} = \sum_n \varepsilon_{nit}$ and $\hat{x}_{nit} = \log x_{nit} - \log x_{nit-1}$ approximates the rate of change in the quantity of input n used by farm i between two adjacent years. The components of TFP growth for RCM2 are calculated in a similar way, but after dropping the non-neutral technical progress components related to inputs and outputs.

Previous estimations of a RCM frontiers have been conducted in a classical/frequentist approach (Kalirajan and Obwona 1994) using simulated maximum likelihood (e.g. Greene 2005; Njuki et al. 2019), as well as using Bayesian inference (e.g. Tsionas 2002; Emvalomatis 2012; Skevas 2019). We proceed with Bayesian techniques, as these are better suited to compare the three candidate models via Bayes factors (Kass and Raftery 1995). Details about the priors are presented in Appendix A, along with some background on model comparison using Bayes factors.

4 Data

We use data from Teagasc's National Farm Survey (NFS) on Irish dairy specialist farms, observed between 2008 and 2017. The original dataset contains farms that are observed between 1 and all 10 years. To ensure that enough information per farm is available for precise estimation of the farm-specific parameters in the RCM models, we keep only farms that are observed for at least five years. This results in a sample of 2323 total observations on 277 farms that remain in the sample for and average of 8.7 years.

We define two categories of outputs: Milk output (y_1), measured as the total revenue from milk production, and other output (y_2), that consists of aggregate revenues from meat products, crops and other minor commodities (y_2). Regarding inputs, we consider four categories: Capital (K) includes the value of machinery and buildings and total livestock value, Labor (L) is measured in labour units working on the farm, including both paid and unpaid labour, Land (A) is the utilized agricultural area, measured in hectares, Materials (M) consists of expenditures in seeds and plants, fertilizers, crop protection, energy, contract work, purchased feed, upkeep of buildings, machinery hire and upkeep of land. A farmer's participation in FAS (F) is measured by a time varying binary variable, where unity denotes that a farmer had a contract with Teagasc FAS , without being obliged to do so by any scheme in any given year (e.g. Läpple et al. 2015). The qualification on the non-obligatory nature of the contract is imposed to ensure that F is related to the core technologies and not, for instance, with assistance to farms in schemes to fulfill bureaucratic requirements to receive subsidy payments (Cawley et al. 2018). For each aggregate that is measured in monetary terms (y_1, y_2, K, M), price indices from EUROSTAT with 2010 as a base year are used to construct a Törnqvist index. Then, each aggregate variable was deflated accordingly. Summary statistics of the relevant variables and price indices are presented in Table 1.

Table (1) Summary Statistics, Irish dairy farms 2008-2017

Variables	**Mean**	**Std. Dev**	**Min**	**Max**
Milk (1000 €)	132.93	89.38	1.13	623.69
Other Output (1000 €)	59.06	42.64	1.18	424.06
Labor (Units)	1.70	0.65	0.7	6.93
Capital (1000 €)	291.83	196.85	8.80	1066.93
Materials (1000 €)	79.41	54.48	4.67	383.43
Area (Ha)	62.21	32.18	3.7	222.61
F*	0.51	0.45	0	1
Milk price index	0.96	0.09	0.78	1.09
Other output price index	1.03	0.04	0.96	1.09
Capital price index	1.02	0.01	1.00	1.05
Materials price index	0.99	0.06	0.89	1.07

* F is a binary variable that is equal to one if a farmer had a contract with Teagasc FAS in each year. For example, the average number of years that farmers had a contract with Teagasc FAS over the total number of years that they remain in the sample is about 0.51, i.e. farmers on average had a contract for almost half of the years for which they are observed.

5 Results

The data for inputs and outputs are normalized by their geometric mean prior to estimation, leading to an interpretation of the parameters associated with the first-order terms as distance elasticities, evaluated at the geometric mean of the data. The results presented in this section are obtained using data augmentation techniques (Tanner and Wong 1987), which is a standard technique in Bayesian inference (Koop 2003) and a Markov chain with a burn-in phase of 40,000 iterations and a total of 80,000 retained draws from the posterior. Table 2 presents the posterior means of the parameters from the three specifications. The reported numbers for CFM are for the common slope parameters and the mean of the random intercepts. For the two RCMs the parameters for farm i are grouped in a vector θ_i and the numbers reported in Table 2 are the estimated means, for each model, of the distributions of the θ_is. In general, the chains converge relatively fast, without any issues. Details on the convergence performance of the samplers are given in Appendix B.

Table (2) Posterior means of the models' parameters

Variable	**CFM**	**RCM1**	**RCM2**
constant	-0.108*	-0.050*	-0.078*
$\log K$	-0.278*	-0.336*	-0.317*
$\log L$	-0.082*	-0.075*	-0.054*
$\log A$	-0.183*	-0.234*	-0.242*
$\log M$	-0.519*	-0.249*	-0.280*
$\log y_2$	0.312*	0.177*	0.189*
$\log^2 y_2$	0.109*	0.107*	0.088*
t	-0.014*	-0.024*	-0.023*
t^2	-0.002*	-0.004*	-
$t \cdot \log K$	0.000	-0.012	-
$t \cdot \log L$	-0.001	-0.009	-
$t \cdot \log A$	0.007	0.008	-
$t \cdot \log M$	0.000	0.010	-
$t \cdot \log y_2$	-0.004	-0.011	-
F	-0.050*	-0.028	-0.041*
$t \cdot F$	0.006*	0.012	0.004
Average TE	0.91	0.94	0.94
λ	10.56	17.72	17.478
σ^2	0.025	0.002	0.003
RTS	1.062	0.893	0.893
Marg. log-lik.	502.53	-50.60	857.829

*The corresponding 90% credible interval does not contain zero.

The elasticities from the RCMs can be viewed as estimates of average distance elasticities, depicting the shape of the "average" distance function, in which the farms' specific distance functions are located. For instance, the estimated distance elasticity with respect to y_2 in CFM shows that if the farmer produces 1% more of other output (holding inputs and milk output fixed) then the value of the distance function is increased by 0.31%, moving the farmer closer to the frontier. The distance elasticity with respect to y_2 in the two RCMs shows that if a farmer produces 1% more of other output, then, *ceteris paribus*, the value of the distance function is increased by 0.177% and 0.189% in RCM1 and RCM2, respectively.

The Bayes factors clearly favour RCM2 over CFM and RCM1, with the the marginal log-likelihood of RCM2 being much higher than the ones for CFM and RCM1. This suggests that RCM2 explains the data better compared to the other candidate models. It also suggests that increasing the number of farm-specific parameters to be estimated in a RCM may lead the Bayes factor to erroneously favour a homogeneous technology SF model.[4] We further evaluated the performance of the models in relation to the regularity conditions. For RCM2, which is favoured by the Bayes Factor, we found that motonicity conditions are satisfied for all inputs, but are violated for other outputs at 2%. Quasi-convexity in outputs is violated at 46% of the observations. Detailed results are presented on Appendix C. Hence, for the rest of the section we focus on discussing the results from the RCM2 model. In Appendix B, we discuss and compare the estimated elasticities between the three models.

The elasticity of the neutral component of technical progress is estimated at 0.23. The contribution of FAS to the outward shift of the frontier of the production possibilities set is estimated at 0.04. The model further shows that impact of FAS on the frontier is either increasing or decreasing over time (the credible intervals contain positive and negative values). We should, however, keep in mind that in the RCMs this is the average impact across farms and that farm-specific marginal effects could be significantly positive or negative.

[4] A full translog specification could have been used for the CFM. Here, the semi-translog form is kept to allow for an easier comparison of elasticities and TFP growth with RCM1. The semi-translog CFM is compared with and favoured by the data over a common frontier model with a full translog specification in inputs and outputs and including interactions with the time trend and FAS variables. The estimated marginal log-likelihood is 483.884 for the full translog specification.

One could possibly use investment levels as an alternative to the FAS variable, as these can be perceived as complementary innovation activities at the Irish dairy farm level (see Läpple et al. 2015). For example, FAS innovations are in the form of knowledge and information (e.g. Läpple and Hennessy 2015; Läpple et al. 2019) where a farmer may seek assistance from FAS to evaluate alternative plans of action before investing in the most appropriate one. Therefore, we further examine investment levels as a proxy of adjustment with respect to FAS technologies. Appendix C reports these results, showing that the above results are robust even when accounting for dynamic adjustments to production.

Regarding the remaining elasticities, there are some interesting differences compared to studies in other EU dairy sectors. Skevas et al. (2018) found the labour and land elasticities of the German dairy sector between 2001 and 2009 at 0.039 and 0.107 respectively. In the Dutch dairy sector for the 2009-2016 period, Skevas (2020) found the elasticity of labour at 0.092 and land at 0.146. The elasticity of land in RCM2 is much higher when compared to these studies, reflecting the importance of land in the Irish dairy sector. It is also interesting that the labour elasticitity for the case of Ireland is much lower than in the Dutch dairy sector, but higher than in the German dairy sector. Differences in the elasticities can also be attributed to the differences in the employed methodologies of these studies.

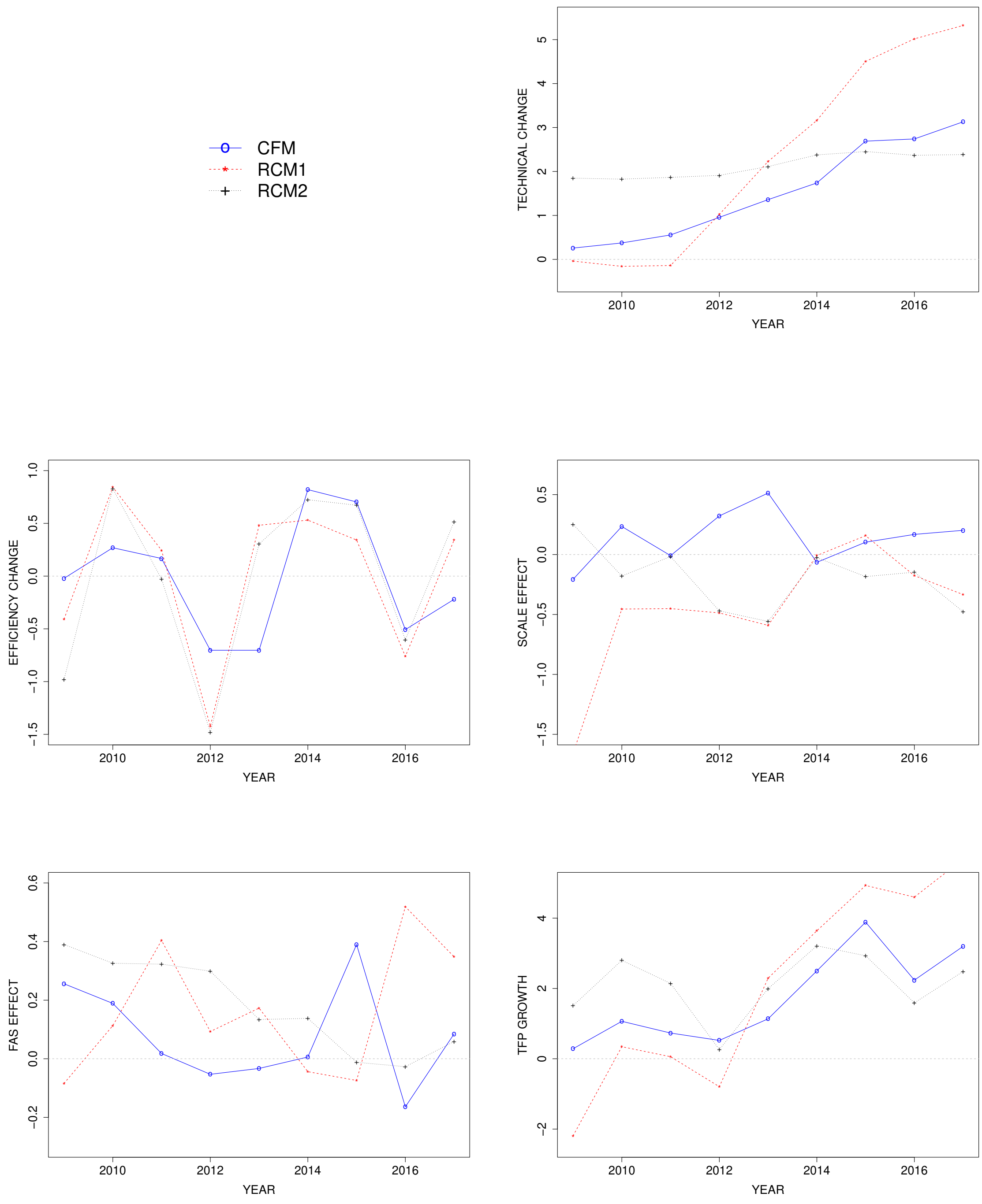
On average, the returns to scale (RTS) elasticity ($-\sum_n \varepsilon_n$) is 0.89. Please note that the RTS in the RCM models is a measure of local returns to scale, i.e. in the neighbourhood where the farms are currently operating (e.g. Emvalomatis 2012). Hence, decreasing the RTS indicate that the Irish dairy sector exhibits decreasing returns to scale, on average, at the points at which farms are observed. From an economic perspective, decreasing returns to scale may be attributed to limited accessibility to some inputs due to capital constraints, land fragmentation or limited access to raw materials or to regulated input markets (e.g. Karagiannis and Sarris 2005; Brümmer et al. 2006). Low land mobility is the most probable reason for the decreasing RTS in the results (O'Donoghue and Hennessy

2015).

The average efficiency score is 0.94, with similar minima and maxima, at 0.71 and 0.98, respectively, as in other relevant studies. Regarding efficiency scores in other EU dairy sectors, Skevas (2020) used a dynamic efficiency specification that accounts for spatial spillovers and estimated the average technical efficiency in the Dutch dairy sector at 0.843. For the 2001-2009 period in the German dairy sector, Skevas et al. (2018) estimated various stochastic frontier models, finding average technical efficiency scores to range between 86% and 95%. Using an RCM, Emvalomatis (2012) reported the average technical efficiency of German dairy farmers between 76% and 81% (1995-2004). An RCM was also used in Skevas (2019), who found the average efficiency of the German dairy sector at 0.921, that is close to the estimated average technical efficiency in RCM2.

In Figure 1 below, we present how the TFP growth evolution and its components differs between the three estimated models. In the rest of the section, we explain the TFP growth, using the obtained elasticities from RCM2, and its components Technical Change (TC), Technical Efficiency (TE) change, Scale Effect (SE), and FAS effect evolved during the period under consideration; while in Appendix B we extend this discussion to a comparison across the three models. The average annual TFP growth is 2.10%, which was mostly driven by TC (2.12%). In the first half of the period 2008-2012, the TC growth is on average 1.8% In this period, farmers proceeded with significant on-farm investments in infrastructure and livestock, preparing for the post quota era (O'Dwyer 2015; Läpple and Sirr 2019), while output was allowed to increase annually only by 1%. The TC in the second half of the period (between 2012 and 2017) increases considerably compared to the first half. Hence, Irish dairy farmers probably benefited from recent market reforms, i.e. "Soft Landing" and abolition of the quota scheme (European Parliament 2018), improving their technology at a faster rate. Similar arguments are presented by Gillespie et al. (2015).

Figure (1) Total Factor Productivity (TFP) growth decomposition



1 The efficiency change is almost 0.00%, on average. This is an important result that
2 shows that, although dairy farmers achieved a high rate of technical progress, there are

also important catch up effects that are possible, where farmers need to use better the adopted technical innovations.

Another interesting finding is that the scale effect varies considerably from year to year. The model shows a persistent negative sign for this component, which implies that farmers move further away from the optimal scale of the technology they utilize, i.e. they need further increases in scale to move closer to the constant-returns to scale part of the production technology. This could be attributed to low land mobility: as they start increasing milk output and herd size during this period, the lack of available land is possibly manifested as a negative scale effect. Finally, the model shows that FAS on average fostered TFP growth by 0.18%. This is close in magnitude to the average negative scale effect (-0.20%), suggesting that FAS "compensates" (almost) sufficiently for the low land mobility.

6 Conclusions

This paper extends previous empirical studies by examining the impact of Farm Advisory Services (FAS) on farm level Total Factor Productivity (TFP) growth in order to explain the contribution of FAS on competitiveness under the vision of Sustainabile Intensification (SI). Three TFP growth indices were constructed for the Irish dairy sector and for the period 2008-2017 period, using the estimates of a Common Frontier Model (CFM) and two Random Coefficient Models (RCMs). Model comparisons based on Bayes factors concluded that the data are explained better by RCM2, which accounts for unobserved heterogeneity in the slope coefficients of the distance function. This model also avoids overparametarizing the frontier specification, as is done by RCM1. RCM2 showed an average annual 2.10% TFP growth rate, driven overwhelmingly by 2.12% per annum growth in technical change over the period. TFP growth was inhibited by a negative scale effect (-0.20%), while the impact of efficiency change was 0.00%.

Furthermore, the impact of FAS on TFP growth was found to be positive on average (0.18%), but of diminishing magnitude over time. A possible explanation is that group discussion membership becomes counter-effective when groups continue operating with the same members for several years (discussion groups are widely used as a delivery method by Teagasc advisors (Läpple et al. 2019). This indicates the need for redesigning the discussion group members, for example, by mixing discussion groups with different members or group facilitators in order to enhance learning effects (Läpple et al. 2019). Another possible explanation for the declining effect of extension services on farm level performance may be due to the life-cycle effect of farmers growing older and beginning to disinvest (Läpple et al. 2019). Overall, RCM2 suggests that TFP growth was driven mainly through technology and efficiency gains, in line with the concept of SI.

The approach used in this paper has a few caveats. The first regards the arbitrary binary measure of FAS and its apparent limitations of not not describing whether the farmers used the acquired knowledge from FAS. For example, some farmers may be interested in obtaining knowledge and information with respect to breeding techniques, others for grassland management, or some other in all "core" technologies. In addition, not all farmers may choose the same delivery methods; for two farmers who are both interested in grassland management techniques, the impact of FAS may be higher for the farmer who will choose to participate in discussion group members as a delivery method, since he may learn faster to use this technology efficiently. It is noticed though in the data that most of the farmers who have a contract with FAS do so persistently, which implies that farmers indeed find advice and support helpful and perhaps adopt the FAS technologies. Furthermore, the binary measure does not provide more information regarding the quality of interaction between farmers and advisors. This is, however, partly dealt with the flexibility of the RCM specifications, which allow the impact of FAS to vary across farms.

Furthermore, there is an emerging literature that tries to explain the dynamics of weather (or climatic change etc.) on TFP growth (e.g. Njuki et al. 2020; Chambers and

Pieralli 2020). It would be interesting to include such determinants in our specification, and probably this topic would require a separate focus due to its individual importance. However, we did not have access to such variables: if such variables are available in the database of the Irish National Farm Survey collected by Teagasc, then these are not readily available for use. We are aware of a number of studies on farm level performance in Ireland that did not use such variables, possibly for the same reason: Newman and Matthews (2006), Newman and Matthews (2007), Carroll et al. (2008), Martinez-Cillero et al. (2018), Martinez-Cillero et al. (2019), Läpple et al. (2019), Balaine et al. (2020), and Bradfield et al. (2021). Furthermore, measures of weather conditions at the regional level would likely vary over time, but be relatively constant across farms for a given year. Thus, including them as independent variables could lead to a deterioration of the ability of the model to fit the data due to low variability in these variables and could possibly overparametetarize the model, as we discussed (e.g. resulting in most of farmers to being close to 100% efficient), leading to inconclusive results, as we explain above.

Moreover, given that the TFP index employed in this paper is not proper (as discussed previously), this could be dealt by explicitly accounting for statistical noise. O'Donnell (2018) conceptualizes statistical noise as a combination of four errors: functional form errors, measurement errors, omitted variable errors, and included variable errors. The TFP index could be written as the product of proper TFPI numbers and a statistical noise index (SNI), which accounts for functional form errors, measurement errors, and omitted and included variable errors (Njuki et al. 2019).

Methodologically, the paper also provides evidence that overaparametarizing an RCM in SFA estimation could lead to misleading results when model comparison is performed using Bayes factors, as shown previously in (Emvalomatis 2012). Furthermore, accounting for heterogeneity provides a different picture about the elasticities of purchased materials and the contribution of the scale effect to TFP growth. This is consistent with previous empirical findings (e.g. Alvarez and del Corral 2010; Emvalomatis 2012).

From a policy point of view, our findings provide support for the claim that FAS, indeed, fostered competitiveness under the vision of SI in a demand driven, "bottom up" process: Irish dairy farmers do not proceed with sufficient scale increases (possibly due to low land mobility), and hence, the negative scale effect inhibits overall TFP growth. However, the similar magnitudes (but opposite signs) of the scale and FAS effects indicates that the use of "core" technologies by Irish dairy farmers, counteracts the inhibiting impact of limited land availability: farmers are able to expand production volume without further environmental pressures that might arise from utilizing inputs such as purchased feeds, fertilizers, etc. This is achieved with use of the "core" technologies that allow Ireland's dairy farmers to exploit better the competitive advantage of the grass-based feed system and be less reliant on materials such as purchased feeds, and hence more resilient on input price volatility shocks.

From a EU policy perspective, TFP growth and its components indicate that the abolition of milk quotas increased competitiveness in the dairy sector, accelerating the technology update (as captured by the fast rate of technical progress). Furthermore, our results provide evidence that FAS fostered a more sustainable way of farming in line with the SI vision. FAS is part of the CAP 2021-2028 and Farm to Fork strategy for a more competitive and sustainable agriculture, highlighting that there is an ongoing need for improvement, for example, with the continuous promotion of technologies at the farm level through FAS and AIS. Otherwise, further increases in production volumes without improvements in resource use may result in higher GHG per unit product (GHG emissions intensity) (Lanigan et al. 2018). Future research could focus on examining the simultaneous impact of FAS on farm level productivity and environmental performance.

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Appendices

Appendix A

A multivariate normal distribution for θ is used in CFM, with mean zero and precision matrix P, whose diagonal elements are equal to 0.001. Gamma priors are used for the two precision parameters:

$$\tau \sim Gamma(a_\tau, b_\tau) \qquad \lambda \sim Gamma(a_\lambda, b_\lambda) \tag{11}$$

The shape, a_τ, and rate, b_τ, parameters are set equal to 0.001. An informative prior is required for λ, so a_λ is set to one and $b_\lambda = -\log(r_{it}^*)$, where r_{it}^* is the prior median efficiency and is equal 0.875 (van den Broeck et al. 1994).

A hierarchical structure is imposed on the vector of farm-specific parameters, θ_i, in the RCMs, where $\theta_i \sim \mathrm{N}\left(\tilde{\theta}, \Omega\right)$. A multivariate normal distribution is used as prior for $\tilde{\theta}$, with mean zero and precision matrix P, whose diagonal elements are equal to 0.001. A Wishart prior is used for Ω, with degrees-of-freedom n and scale matrix V. The density of the Wishart distribution integrates to unity only if $n \geq K$, where K is the number of independent variables. Therefore, n is set equal to K and V is diagonal (I_K) as in Tsionas (2002). The same priors for τ and λ are used in the RCMs as in CFM.

Model comparison is based on the Bayes factor, which summarizes "*the evidence provided by the data in favor of one scientific theory, represented by a statistical model, as opposed to another*" (Kass and Raftery 1995, p. 777). Assuming there are two competing models $\mathcal{M}_1$ and $\mathcal{M}_2$, the Bayes Factor (BF) can be expressed as their relative posterior probabilities:

$$\mathrm{BF} = \frac{p(\mathcal{M}_1|\mathcal{D})}{p(\mathcal{M}_2|\mathcal{D})} = \frac{p(\mathcal{D}|\mathcal{M}_1)}{p(\mathcal{D}|\mathcal{M}_2)} \frac{Prob(\mathcal{M}_1)}{Prob(\mathcal{M}_2)} \tag{12}$$

where $\mathcal{D}$ represents the observed data, $p(\mathcal{D}|\mathcal{M}_j)$ is the density of the data given $\mathcal{M}_j$, and $Prob(\mathcal{M}_j)$ is the prior probability of $\mathcal{M}_j$ being the true model. The marginal density $p(\mathcal{D}|\mathcal{M}_j)$ with respect to unobserved quantities and parameters is written as:

$$p(\mathcal{D}|\mathcal{M}_j) = \int p(\mathcal{D}|\theta_j, \mathcal{M}_j)\, \pi(\theta_j|\mathcal{M}_j)\, d\theta_j \tag{13}$$

where θ_j is the vector of parameters for model j and $\pi(\theta_j|\mathcal{M}_j)$ is the prior density of θ_j under model j. It is a common practice to set equal prior model probabilities for competing models, and hence, model comparison is conducted by simply calculating the ratio of marginal likelihoods between the competing models. We approximate the logarithm of the marginal likelihood for each model using the Laplace-Metropolis estimator (Lewis and Raftery 1997).

Appendix B

Table 3 presents values for Geweke's (1992) convergence diagnostic, as well as for the Simulation Inefficiency Factor (SIF) (Chib 2001) for each parameter and for each model. The convergence diagnostic tests whether the Markov chain used for sampling from the posterior distribution of the parameters has converged to its steady state. The convergence diagnostic Z is the difference between the means of the draws from the posterior for a parameter obtained, usually, from the first 10% of the draws and the last 50%, divided by the asymptotic standard error of their difference. If the chain has converged, the means of the values from the first and the second window should be similar and a Z-statistic smaller (in absolute value) than 1.96 leads to non-rejection of the hypothesis of convergence. According to Table 3, only the parameter associated with the $\log^2 y_2$ variable and only for the the CFM model presents issues of convergence.

Contrary to the convergence diagnostic, SIF is used to measure the performance of the sampler. By construction, a sampler that is based on a Markov chain produces autocorrelated draws from the posterior distribution. This autocorrelation, however, drops steadily as draws are further apart in the chain and SIF is the number of successive iterations needed to obtain approxiamtely independent draws (Kim et al. 1998). According to Feng et al. (2019) a sampler achieves reasonable mixing performance when SIF is smaller than 100. In our application all SIF values for all three models are much smaller than 100.

Table (3) Geweke diagnostics and SIF for each model

	CFM		**RCM1**		**RCM2**	
Variable	Z-score	SIF	Z-score	SIF	Z-score	SIF
$\log K$	1.46	1.83	-0.19	14.14	1.36	9.12
$\log L$	-0.08	2.28	1.45	14.98	-0.07	1.07
$\log A$	0.17	1.78	-0.04	16.51	-0.61	9.50
$\log M$	-1.96	1.69	-0.36	10.29	-0.29	5.81
$\log y_2$	1.12	1.73	0.65	6.26	0.27	5.09
$\log^2 y_2$	2.78	1.73	0.20	6.26	1.46	5.09
t	-0.24	1.70	0.18	1.16	-0.45	2.59
t^2	-0.10	1.66	-2.18	1.09	-	-
$t \cdot \log K$	-0.76	1.73	-0.69	1.14	-	-
$t \cdot \log L$	0.21	1.65	1.31	1.13	-	-
$t \cdot \log A$	0.84	1.61	-0.74	1.10	-	-
$t \cdot \log M$	-0.33	1.66	-0.37	1.17	-	-
$t \cdot \log y_2$	-0.06	1.79	-0.88	1.30	-	-
F	0.07	1.73	0.35	1.06	0.95	11.75
$t \cdot F$	0.30	1.71	-0.15	1.08	0.72	6.98

We further examine the differences between the three models in relation to their estimated elasticities, TFP growth and its components. There are differences in input elasticities between the models, which are reflected in the returns to scale (RTS) elasticity. The RTS elasticity in CFM is slightly higher than unity (1.06), indicating that, on average, farms operate under increasing RTS. According to the RCMs, the production technology of the Irish dairy sector exhibits decreasing returns to scale (0.89), on average, at the points at which farms are observed. The average efficiency score according to CFM is 0.91, with a minimum of 0.60 and a maximum of 0.94. Average efficiency in the RCMs is approximately 0.94, with similar minima and maxima at 0.71 and 0.98, respectively. Unsurprisingly, not accounting for unobserved heterogeneity deflates the estimated efficiency scores, as part of the heterogeneity is attributed to inefficiency (e.g. Kumbhakar et al. 2018; Njuki et al. 2019).

The three models also present striking differences in the estimated marginal effects of the technology components, t and F. The RCMs provide a much higher point estimate for the neutral component of technical progress compared to the CFM. Moreover, the contribution of F in the RCMs is much lower when compared to the CFM. Both RCMs show that the impact of FAS on the frontier is either increasing or decreasing over time (the credible intervals contain positive and negative values).

The three models exhibit some important differences about how TFP growth and its components Technical Change (TC) Technical Efficiency (TE) change, Scale Effect (SE), and FAS effect evolved during the period under consideration. The effect of technical progress is mostly positive in the first half of the period covered by the data, but it increases considerably between 2012 and 2017. Interestingly, the evolution of technical progress varies between the three models, with the over-parameterized RCM1 producing rather unrealistic results.[5] Similarly, CFM is too restrictive and the evolution of the technical progress component depends heavily on a common parameter (the one associated with t^2). A negative contribution of technical change on TFP growth is observed in the first three years of the period under consideration in RCM1. RCM2, which is favoured by the data, produces a much smoother evolution of the technical progress process during this adjustment period when compared to the very flexible RCM1 and the restrictive CFM.

Another interesting finding is that the scale effect varies notably across the three

[5]The growth rate of the technical progress component in the RCM1 model varies between -0.33% and 5.33%, which is much higher and more volatile compared to the results from the other two models. This is probably due to the high flexibility introduced by the many farm-specific parameters.

models. CFM reveals a positive scale effect, on average, over time (0.126%), with some negative estimates between 2010 and 2012. The positive scale effect shows that, on average, Irish dairy farmers experienced growth in TFP also because of increases in scale relative to the optimal scale implied by the model. The RCMs show a persistent negative sign for this component, which implies that farmers move further away from the optimal scale of the technology they utilize.

Appendix C

One could possibly use investment levels as an alternative to the FAS variable, as these can be perceived as complementary innovation activities at the Irish dairy farm level (see Läpple et al. 2015). For example, FAS innovations are in the form of knowledge and information (e.g. Läpple and Hennessy 2015; Läpple et al. 2019) where a farmer may seek assistance from FAS to evaluate alternative plans of action before investing in the most appropriate one. To capture this dynamic behaviour of farmers' production we use the enhanced hyperbolic distance function proposed in Minviel and Sipiläinen (2018), which is defined as:

$$D_{EH}(\mathbf{x}, \mathbf{y}, F, k, I, t) = inf\left\{\theta > 0 : (y\theta^{-1}, x\theta, I\theta^{-1}) \in T\right\} \tag{14}$$

where y is a vector of outputs, x a vector of variable inputs, k a vector of quasi-fixed inputs, and I a vector of gross investments. In addition, θ is a small positive scalar which allows a simultaneous expansion of outputs and investments and contraction of variable inputs, to reach the boundary of the technology set T. The empirical specification of (14) is:

$$
\begin{aligned}
-\log y_{it}^{M} = {} & \alpha_i + \sum_n \alpha_{in} \log(x_{it}^{n} y_{it}^{M}) + \sum_m \beta_{im} \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \sum_n \xi_{in} \log k_{it}^{n} + \sum_\ell \sum_m \gamma_{i\ell m} \log\left(\frac{y_{it}^{\ell}}{y_{it}^{M}}\right) \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \eta_i F_{it} + \rho_i t F_{it} + \zeta_{1i} t + \kappa_i \left(\frac{I}{y_{it}^{M}}\right) + v_{it} + u_{it}
\end{aligned}
\tag{15}
$$

where we use the hyperbolic sine transformation for the investment variable: $\log(I + \sqrt{I^2+1})$ and the quasi-fixed input k is the value of the farm capital (Minviel and Sipiläinen 2018). This specification (eq. 15) is labeled as Hyperbolic Distance Function 1 (HDF1). Furthermore, Minviel and Sipiläinen (2018) utilize a panel data that covers 20 years and assume that utilized agricultural area is variable input. Since the we utilize a shorter panel and given the low land mobility, we are going to estimate a model where capital and area are quasi-fixed; we label this model as Hyperbolic Distance Function 2 (HDF2). Each of the model is estimated in a Markov chain with a burn-in phase of 40,000 iterations and a total of 80,000 retained draws from the posterior, similar to the model in the main text. The results are presented in Table 4 below.

We observe that in both models, all the noise in this model is captured by the random coefficients: σ_{ν}^2 and the average Technical Efficiency (TE) are almost 0 (with the mass of efficiency distribution to be centered around the mean) (Table 4). According to the efficiency measurement literature these results suggest that the model fails to provide a good fit of the data in line with the economic theory (e.g. inefficiency is zero for the majority of farmers), and the results become unreliable and should not be trusted (e.g. Tsionas 2002; O'Donnell and Griffiths 2006; Galán et al. 2014). Furthermore, we further check for convergece using Geweke's (1992) convergence diagnostic for the model above. The Z scores below in Table (4) indicate problems with convergence, adding to the fact that the models does not provide reliable results.

Table (4) Posterior means and Geweke diagnostics for HDF1 and HDF2 at 95% credible intervals

	HDF1		**HDF2**	
Variable	Mean	Z-score	Mean	Z-score
$constant$	-0.00	0.35	0.00	1.98
$\log K$	-0.06*	0.34	0.00	-0.48
$\log(L \cdot y_2)$	0.19*	-1.65	0.32*	-0.07
$\log(A \cdot y_2)$	0.33*	1.44	-	-
$\log A$	-	-	0.01*	0.14
$\log(M \cdot y_2)$	0.14*	-1.11	0.25*	-0.39
$\log y_2$	-0.06*	0.36	-0.07*	1.19
$\log^2 y_2$	-0.03*	-1.84	-0.03*	0.14
t	0.00	0.42	0.00	-0.61
$\frac{I}{y_{it}^M}$	0.00	-0.46	0.00	-0.15
F	0.00	2.03	0.01	-0.54
$F \cdot t$	0.00	-1.13	0.00	0.50
σ_ν^2	0.00	0.93	0.00	0.33
Average TE	0.99		0.98	

*The corresponding 95% credible interval does not contain zero.

We estimate then again HDF1 and HDF2 using 10 Markov chains with a total of 120,000 interations in each of the chains. The first 40,000 iterations are dropped for the burn-in phase and 1 out of 10 from the rest 80,000 draws are retaind in order to remove the influence of the potential auto-correlation. The posterior means of the revised HDF1 and HDF2 (now labeled reHDF1 and reHDF2 respecitvely) are presented in Table 5 below. The results in Table 5 and in Table 4 are qualitatively similar. We report the

Geweke diagnostics for each of the model and each of estimated Markov chains in Table 6 and Table 7. The results in the Tables indicate that the models still fail to converge succesfully.

Table (5) Posterior means of HDF1 and HDF2 at 95%

Variable	**reHDF1**	**reHDF2**
cons	-0.00	-0.00
$\log K$	-0.06*	0.016
$\log(L \cdot y_2)$	0.19*	0.30
$\log(A \cdot y_2)$	0.33*	-
$\log A$	-	0.02
$\log(M \cdot y_2)$	0.14*	0.26
$\log y_2$	-0.06*	-0.07
$\log^2 y_2$	-0.03*	-0.03
t	0.00	0.00
$\frac{I}{y_{it}^M}$	0.00	0.00
F	0.00	0.01
$F \cdot t$	0.00	-0.00
σ_ν^2	0.00	0.00
Average TE	0.99	0.98

*The corresponding 95% credible interval does not contain zero.

Table (6) Geweke diagnostics for reHD1

Variable	Chain1	Chain2	Chain3	Chain4	Chain5	Chain6	Chain7	Chain8	Chain9	Chain10
cons	0.44	-1.60	-0.98	0.12	-1.83	-0.88	-1.34	-2.18	-0.91	-1.55
$\log K$	0.74	1.65	-0.19	3.53	-0.75	0.39	0.34	-0.24	-0.40	0.39
$\log(L \cdot y_2)$	1.28	0.05	-0.59	0.85	-0.27	1.08	1.07	1.75	-0.17	0.84
$\log(A \cdot y_2)$	1.05	0.44	1.65	-0.87	1.08	0.38	0.79	0.63	2.72	0.19
$\log(M \cdot y_2)$	-1.44	-1.28	-0.83	-0.09	0.98	-1.08	0.17	0.51	-2.11	-0.23
$\log y_2$	-0.00	-1.12	-0.22	1.38	-2.10	-0.61	1.4	0.53	0.34	0.18
$\log^2 y_2$	-0.56	-0.82	-0.21	0.67	-0.53	-1.18	0.08	2.95	-1.32	-0.73
t	0.01	-0.28	-1.70	-0.18	-0.12	0.64	-2.06	0.05	0.32	-0.97
$\frac{I}{y^M_{it}}$	-0.71	0.53	-1.38	-0.55	-0.04	-1.04	-0.92	-1.10	0.37	-1.68
F	0.12	-0.84	-2.04	-0.41	-0.50	0.04	-0.30	2.07	-0.41	-0.20
$t \cdot F$	-0.15	0.94	2.21	-0.95	0.57	0.01	1.37	0.12	-1.45	1.28
σ^2_ν	1.06	0.98	0.97	1.02	0.88	0.98	1.00	0.97	0.97	0.94

Table (7) Geweke diagnostics for reHD2

Variable	Chain1	Chain2	Chain3	Chain4	Chain5	Chain6	Chain7	Chain8	Chain9	Chain10
cons	-0.35	0.60	0.17	0.88	0.82	1.44	1.28	-0.63	-1.15	1.38
$\log K$	0.09	0.00	0.29	2.09	0.23	0.51	1.74	0.07	-1.60	-0.64
$\log(L \cdot y_2)$	-1.67	-1.39	-1.25	-0.53	-1.39	-1.18	0.75	-1.43	-0.81	-0.78
$\log A$	-0.36	-0.66	1.33	-0.98	1.26	0.40	0.41	0.45	-1.05	0.84
$\log(M \cdot y_2)$	1.05	1.78	1.10	-0.37	1.24	0.05	-1.68	1.25	1.21	0.23
$\log y_2$	-1.98	-0.44	-1.40	-0.11	-1.36	-0.25	0.49	0.24	0.29	-0.76
$\log^2 y_2$	-0.10	1.57	-1.35	0.26	-0.198	1.23	0.70	0.21	1.07	1.35
t	-0.38	0.40	0.96	-0.13	-2.41	0.62	1.05	1.22	1.23	1.12
$\frac{I}{y^M_{it}}$	0.40	-0.58	-0.08	2.02	-0.77	1.33	-1.71	-0.13	0.87	0.03
F	0.30	0.03	1.48	-1.30	-1.15	0.05	-1.09	-0.53	0.20	0.32
$t \cdot F$	0.22	-1.57	-0.75	0.24	1.10	-0.28	-0.90	0.15	-1.11	-0.47
σ^2_ν	1.53	0.99	0.50	0.79	0.81	0.78	0.41	0.99	1.22	0.92

We further explore the issue of investment and its impact on output in a dynamic setting using investments as an additional independent variable in an output distance function. We estimate Model 1 (M1), which is the RCM2 specification, but using the hyperbolic sine trasformation of gross investments levels (denoted by I) instead of the FAS variable; Model 2 (M2), which is the RCM2, including both I and FAS; Model 3 (M3), which is the RCM2 including I , normalized by other output, y_2 (i.e. $\frac{I}{y_{it}^M}$); Model 4 that is the RCM2, but replacing F with the logarithm of $\frac{I}{y_{it}^M}$. M3 is presented analytically as:

$$
\begin{aligned}
-\log y_{it}^{M} = {} & \alpha_i + \sum_n \alpha_{in} \log(x_{it}^{n}) + \sum_m \beta_{im} \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \sum_\ell \sum_m \gamma_{i\ell m} \log\left(\frac{y_{it}^{\ell}}{y_{it}^{M}}\right) \log\left(\frac{y_{it}^{m}}{y_{it}^{M}}\right) \\
& + \eta_i F_{it} + \rho_i t F_{it} + \zeta_{1i} t + \kappa_i \left(\frac{I}{y_{it}^{M}}\right) + v_{it} + u_{it}
\end{aligned}
\tag{16}
$$

The results are presented in Table (8). The models produce more resonable estimates in terms of efficiency, i.e. in line with economic theory. The elasticities with respect to I and $\frac{I}{y_{it}^M}$ are neglegible, i.e. very close to zero. This is not actually surprising in the case of the Irish dairy sector: information and knowledge are considered as more important than investments in the Irish AIS (see Läpple et al. 2015; Läpple et al. 2016; Läpple et al. 2019) for the SI of the Irish dairy sector (e.g. investments may result in productivity gains that might come at the cost of environmental pressures, investments may imply productivity gains mostly for larger farmers etc.). Investments may be more important for production in other EU dairy sectors, with different AIS structure and different production systems (e.g. more intensive). As an example, investments is an important innovation activity at the Dutch dairy sector, which is more intensive than the Irish dairy sector (see Reijs et al. 2013): e.g. less machinery in the Irish dairy sector is needed as cows are mostly grazing and thus, the demands for cow housing are also lower (see Reijs et al. 2013). The log marginal likelihood of these models are way less than the main model (RCM2) presented in the main text (857.829).

Table (8) Posterior means of M1, M2 and M3

Variable	**M1**	**M2**	**M3**	**M4**
cons	-0.065*	-0.074*	-0.071*	-0.073*
$\log K$	-0.347*	-0.348*	-0.351*	-0.345
$\log L$	-0.057*	-0.066*	-0.064*	-0.065
$\log A$	-0.238*	-0.227*	-0.230*	-0.216
$\log M$	-0.282*	-0.295*	-0.351*	-0.356
$\log y_2$	0.197*	0.198*	0.051*	0.051
$\log^2 y_2$	0.090*	0.090*	0.003	0.004
t	-0.022*	-0.020*	-0.021*	-0.020*
I	-0.005*	-0.006	-	-
$I \cdot I$	0.001	0.001	-	
F	-0.040*	-	-0.20	
$F \cdot t$	0.005*	-	-0.020	
$\frac{I}{y_{it}^M}$	-	-	0.004	-0.000
Average TE	0.91	0.94	0.93	0.93
λ	10.56	17.72	13.37	13.64
σ^2	0.025	0.002	0.005	0.005
Log Mag. Likelihood	502.53	-50.60	365.267	400.562

*The corresponding 90% credible interval does not contain zero.

Appendix D

The output distance function has to satisfy the theoretical regularity conditions of monotonicity and curvature. Monotonicity requires that the distance function is non-increasing in inputs (the the distance to the frontier cannot decrease by increasing only the amount of an input) and non-decreasing in outputs (the distance to the frontier cannot increase by increasing only the amount of an output). Mathematically:

$$\frac{\partial D_o}{\partial x_n} \leq 0 \Leftrightarrow \bar{k}_n \equiv \frac{\partial \log D_o}{\partial \log x_n} \leq 0 \qquad \text{and} \qquad \frac{\partial D_o}{\partial y_m} \geq 0 \Leftrightarrow \bar{r}_m \equiv \frac{\partial \log D_o}{\partial \log y_m} \geq 0 \tag{17}$$

Regarding curvature, $D_o(y, x, t)$ must be quasi-convex in inputs and convex in outputs (O'Donnell and Coelli 2005). For quasi-convexity in inputs we need to calculate the 5×5 bordered Hessian:

$$\mathbf{F} = \begin{bmatrix} 0 & \frac{s_{i1}D_{oi}}{x_{i1}} & \frac{s_{i2}D_{oi}}{x_{i2}} & \cdots & \frac{s_{iN}D_{oi}}{x_{iN}} \\ \frac{s_{i1}D_{oi}}{x_{i1}} & \frac{s_{i1}(s_{i1}-1)D_{oi}}{x_{i1}^2} & \frac{s_{i1}s_{i2}D_{oi}}{x_{i1}x_{i2}} & \cdots & \frac{s_{i1}s_{i2}D_{oi}}{x_{i1}x_{iN}} \\ \frac{s_{i2}D_{oi}}{x_{i2}} & \frac{s_{i2}s_{i1}D_{oi}}{x_{i2}x_{i1}} & \frac{s_{i2}(s_{i2}-1)D_{oi}}{x_{i2}^2} & \cdots & \frac{s_{i2}s_{iN}D_{oi}}{x_{i2}x_{iN}} \\ \vdots & & & & \\ \frac{s_{iN}D_{oi}}{x_{iN}} & \frac{s_{iN}s_{i1}D_{oi}}{x_{iN}x_{i1}} & \frac{s_{iN}s_{i2}D_{oi}}{x_{iN}x_{i2}} & \cdots & \frac{s_{iN}(s_{iN}-1)D_{oi}}{x_{iN}^2} \end{bmatrix}$$

where $s_{in} = \alpha_{in} + \xi_i t$ for RCM1 and $s_{in} = \alpha_{in}$ for RCM2 D_{oi} is an estimate of D_o for observtion i using the linear homogeneity restrictions. Because can be taken as a common factor in the matrix above and since $D_{oi} > 0$, there is no need to calculate this quantity, as it will not affect the sign of the principle minors. In short, we can ignore this term in all entries of $\mathbf{F}$.

For convexity in outputs we need to calculate the Hessian (for two outputs):

$$\mathbf{H} = \begin{bmatrix} \left(2\gamma_{i11} + r_{i1}\left(r_{i1}-1\right)\right)\frac{D_{io}}{y_{i1}^2} & \left(\gamma_{i12} + r_{i1}r_{i2}\right)\frac{D_{io}}{y_{i1}y_{i2}} \\ \left(\gamma_{i21} + r_{i2}r_{i1}\right)\frac{D_{io}}{y_{i2}y_{i1}} & \left(2\gamma_{i22} + r_{i2}\left(r_{i2}-1\right)\right)\frac{D_{io}}{y_{i2}^2} \end{bmatrix}$$

where:

- $r_{i1} = \beta_{i1} + 2\gamma_{i11}\log y_{i1} + \gamma_{i12}\log y_{i2} + \varphi_{i1}t$
- $r_{i2} = \beta_{i2} + 2\gamma_{i22}\log y_{i2} + \gamma_{i12}\log y_{i1} + \varphi_{i2}t$
- D_{oi} is an estimate of D_o for observtion i and, as in the previous case, can be ignored.

Thus D_o is convex in outputs if:

- $\left(2\gamma_{i11} + r_{i1}\left(r_{i1}-1\right)\right)\frac{D_{io}}{y_{i1}^2} \geq 0 \Leftrightarrow 2\gamma_{i11} + r_{i1}\left(r_{i1}-1\right) \geq 0$ and
- $\left(2\gamma_{i11} + r_{i1}\left(r_{i1}-1\right)\right)\frac{D_{io}}{y_{i1}^2}\left(2\gamma_{i22} + r_{i2}\left(r_{i2}-1\right)\right)\frac{D_{io}}{y_{i2}^2} - \left[\left(\gamma_{i12} + r_{i1}r_{i2}\right)\frac{D_{io}}{y_{i1}y_{i2}}\right]^2 \geq 0$ which is equivalent to $\left(2\gamma_{i11} + r_{i1}\left(r_{i1}-1\right)\right)\left(2\gamma_{i22} + r_{i2}\left(r_{i2}-1\right)\right) - \left[\left(\gamma_{i12} + r_{i1}r_{i2}\right)\right]^2 \geq 0$

Due to linear homogeneity $r_{i1} = 1 - r_{i2}$ and $\gamma_{i11} = \gamma_{i22} = -\gamma_{i12}$. So, the two conditions collapse to $2\gamma_{i22} \geq r_{i1}r_{i2}$. Notice here that we have $2 \cdot \gamma_{i22}$ on the left-hand side while O'Donnell and Coelli (2005) do not multiply the paramter by 2. This is because (O'Donnell and Coelli 2005) multiply the double sum of second-order terms in their translog specification by $\frac{1}{2}$.

Table (9) Regularity violations (at the posterior mean) (%)

Condition	**CFM**	**RCM1**	**RCM2**
Monotonicity			
$\bar{k}_K \leq 0$	0	2	0
$\bar{k}_L \leq 0$	0	20	0
$\bar{k}_A \leq 0$	0	7	0
$\bar{k}_M \leq 0$	0	8	0
$\bar{r}_{y_2} \geq 0$	0.4	3	2
$\bar{r}_{y_1} \geq 0$	0	0	0
Curvature			
Quasi-convex in inputs	-	-	-
Convex in outputs	44	21	46