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The power of investors' optimism and pessimism in oil market forecasting

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ABSTRACT

By modelling dynamism in the global oil market by three essential market-centric observables (viz., *Market Expansion, Market Regime, and Market Liquidity*), we study forecasting potential of the future oil markets within a memory-driven interdependence setting. We combine spot prices with our derived market proxies to produce representative global market proxies. The latter are used to quantify the extent of static and dynamic persistence within the system. Our extensive empirical investigation exploits the rich features of fractionally cointegrated vector autoregression, where the rate of disequilibrium error correction within the system is modelled to be slow, approximating real life system dynamics. An advantage is that it explains why we often experience a slow response of a policy intervention. We present robust evidence of both system-wide long-memory and a long-memory in the market-centric observables. We introduce a *memory of memory* estimation to discern the magnitude of the relative rate of acceleration/deceleration of shocks within each observable, which reflects on the overall stability of the system. Our results show significant degree of non-linear error dissipation and high degree of informational inefficiency. Rigorous out-of-sample forecasting exercise produces robust predictions and demonstrate superiority of our approach.

1. Introduction

1.1. Motivation

Investors' relative degree of optimism and pessimism can significantly govern the growth trajectories of the oil future markets. Considered from political-economic and environmental perspectives, crude oil is one of the world's most important strategic resources. A volatility in its price is tightly interlinked with the price variations of other commodities, major world's indices, currencies and the macroeconomics factors across many countries (Nandha and Faff, 2008; Silvennoinen and Thorp, 2013; Ratti and Vespignani, 2016; Boldanov et al., 2016; Morana, 2013; Wang et al., 2019; Ahmad et al., 2020; Boako et al., 2020). Therefore, a reliable prediction of the price of oil can lend answer to many important questions and offers a valuable guide to hedging and trading strategies among investors. Furthermore, because major world economic and financial indices are strongly correlated with oil price movement, producing a robust approach to forecasting the price of oil remains high on the agenda of policymakers, economists and researchers worldwide.

Various techniques have been developed over the years to forecast crude prices but the search for an informative forecasting technique still remains a focus of the research community. The reviews of the oil price forecasting techniques by Bashiri Behmiri and Manso (2013) as well as Gabralla and Abraham (2013) suggest that researchers consider two main approaches in forecasting price of crude, viz., quantitative and qualitative methods with latter being the least favourable. The authors conclude that although different data sets and tools produced accurate predictions, more research should be dedicated to the development of efficient methods in feature selection and prediction of the oil price.

In a quest for characterising and forecasting the dynamics between various factors that influence the price of oil, the research community began to look for new techniques that could account for both linear and nonlinear dynamics in the data. Among others, for example, "Qual", "VAR", "wavelet analysis" and "Threshold models" have been applied to model nonlinear characteristic of West Texas Intermediate (WTI) and forecast its price (Gupta and Wohar, 2017; Uddin et al., 2019; Bekiros et al., 2020; Rubaszek et al., 2020). The authors show that the new approach outperforms both the Random Walk and the standard VAR models. Other researchers employ machine learning and artificial intelligence techniques to account for the dynamics that run through various dimensions of data and its interdependence (Chiroma et al., 2015, 2016). In summary, extant studies report obvious advantages and improved forecast accuracy compared with the traditional forecasting models.

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Despite an increasing access to competing models in oil price forecasting, the major challenge the research community is still facing is the selection of predictors. The problem arises as the source of crude oil price changes can be attributed to a large number of variables that affect the price of oil. Accounting for each one of these variables poses a "dimensionality" problem and as such is impractical from the overfitting point of view (Gonzalo and Pitarakis, 1999). To circumvent this conundrum, large dimensional system of variables needs to be strategically decomposed into a finite number of variables that jointly capture the overall dynamics in the oil market. To achieve this aim, a market system needs to be considered where all factors related to the oil are fixed besides the price and the time horizon for physical delivery of the commodity. One such system is the oil futures market.

It has been long recognised that the futures markets are effective vehicles for aggregating market information which is fully justifiable in view of efficient market hypothesis. Concerted attempts have been made to extract such information. One of such work in this regard is by Coppola (2008). By examining a cointegration relationship between the spot price and the prices of 1, 2, 3 and 4 – *month* maturities WTI futures contracts, Coppola used a vector error correction model (VECM) to confirm the value of the information embedded in the long-run spot-futures relationship. Other researches have also considered incorporating information contained in the futures market in the analysis of the price of oil, (Roache and Reichsfeld, 2011). It has been noted that the forecast from the futures market is hard to beat and show that the futures-based model consistently outperformed the benchmark. Furthermore, the forecasting performance of futures was shown not to depend on the slope of the futures curve.

In the latest attempts to exploit predictive power embedded in the price curve of oil futures, researchers began to take holistic perspective on the dynamic observed in the futures term structure as it responds to exogenous shocks. For example (Baruník and Malinska, 2016) and Bredin et al. (2021) proposed to model the oil futures term structure based on three Nelson–Siegel (NS) factors (Nelson and Siegel, 1987) which parametrise price curve shapes into level, slope and curvature factors and assess their predictive power through generalised regression framework based on neural networks and LASSO modelling approach, respectively. Both papers conclude that forecasting strategies based on NS factors produce the lowest errors across all considered forecast horizons compared to benchmark models. Bredin et al. (2021) also conclude that decomposition of futures term structure into NS factors and exploiting them in forecasting result in trading strategies with higher Sharpe ratios and better skewness properties than buy and hold strategies and historical mean strategies. This suggests that significant amount of information is embedded within oil futures prices which can lend itself well for forming reliable forecasts of future oil price trajectory.

1.2. Contribution

This study contributes to the literature in two specific ways. *Firstly*, we propose a methodology for *information extraction* from the market and expressing it in terms of proxies for $Market\ Expansion$, $Market\ Regime$ and $Market\ Liquidity$. What economic rationale justifies extraction of aggregated information contained in the futures markets associated with major crude oil benchmarks[1]?

[1] Crude oil price benchmark is the standard which serves a reference point for determining the price of the crude related raw materials and products. Crude oil benchmark is a blend of several crude oils extracted in various locations and combined in proportions to achieve industry standard chemical compositions to which various refineries are collaborated to. Crude oil benchmarks are traded freely in futures and money markets in accordance with the terms and conditions of specific laws. The major oil benchmarks are West Texas Intermediate in North America (USA) and Brent (North Sea).

It is a common knowledge that commodity futures markets offer effective mechanisms for absorbing large volume of information associated with the supply demand fundamentals as well as geopolitical factors impacting the price of a specific commodity. As information arrives into the commodity market, the prices of futures contracts with different terms to maturity continuously adjust to compensate for the effect this information has on their current and future values. We propose this mechanism ensures that the futures prices remain in constant equilibrium with a certain economic force we refer to as $global\ economic\ mean$, that is driven by geopolitical and economic conditions in the world. Although this $global\ economic\ mean$ is exogenously determined and therefore, cannot be directly observed, it can be inferred from the relationship it has with a specific commodity market, namely through the impact its dynamic exerts on the price structure of futures.

We suggest that the effect of this force on individual futures market is eclipsed by the noise originating from inter-market dynamics that exist in every market which has a detrimental affects on the purity of its inference. However, the inference can be improved by bringing together analysis from a range of commodity futures markets that share similar characteristics (traded commodity, contractual volumes, trading currency and comparable term structure lengths). This is because investors use futures markets to fix commodity prices at a certain time horizon (which they do so at the best possible price) and this process ensures that the information on the perceived value and trajectory of the $global\ economic\ mean$ is injected into the term structure across various futures markets that involved in trading the same commodity. Being able to quantify the effect of $global\ economic\ mean$ on global futures market a crucial aspect from forecasting perspective. Therefore, detailed understanding of how the information is absorbed into the term structure of futures is essential.

The process of information absorption that drives structural changes in the futures market can be examined through trading dynamics. Generally, we identify two stages in the process. In the first stage, all information arriving into the futures market is aggregated and converted into $market\ sentiment$. This happens when market participants (comprising of both physical and algorithmic speculators and hedgers) while competing for the best price, continuously assimilate developments in the information space and form their $believes$ on the effect these developments will have on the current and the future commodity price. In the second stage of the process, $market\ sentiment$ is converted into commodity futures prices and $open\ interest$ across the term structure of futures. This happens when market participants express their $believes$ by either buying or selling futures contracts across the term structure at a level they deem $optimal$ based on the information available at the time.

As the prices of futures contracts with different terms to maturity adjust to compensate for the effect of new developments in the information space, following changes happen in the structure of futures market: (i) market*expands* or *contracts* (up or down parallel shift in the price curve), (ii) market regime fluctuates between the states of *Backwardation* or *Contango* (increase or decrease of the price curve slope) (iii) market liquidity levels *increase* or *decrease* (up or down parallel shift in the open interest curve). We construct market indicators by quantifying these structural changes in the price structure of future contracts.

From crude oil market perspective, proposed proxies extracted from the spot and futures markets associated with major crude oil benchmarks explain the relationship between structural changes in the market and corresponding crude oil spot prices or prices of crude oil derivatives such that:

$$X_{t+1} = E_t + L_t + R_t + \epsilon_t \tag{1}$$

where X_t — Price of crude oil or their derivatives, E_t — Global Oil Market Expansion (GE), L_t — Global Oil Market Liquidity (GL), R_t — Global Oil Market Regime (GR) and ϵ_t — is the error term. In this paper these variables will also be referred to as *market vectors*, *market proxies* or *market variables* which will be used interchangeably throughout the text.

Further, there is an economic significance of the proposed proxies for $market\ expansion$, $market\ liquidity$ and $market\ regime$ extracted from specific commodity market. This approach enables us to capture and summarise collective sentiment prevailing among investors in the commodity market the proxies are extracted from. We propose that these proxies are the representations of principal vectors that unify commodity market constituents via equilibrium relationships in a cointegrated system where the change in one of the variables triggers adjustment in the whole system.

To examine this theory, consider following example from the oil market. When demand for the oil increases, holding longer maturity futures becomes less valuable compared to the investment into $spot$ which causes the slope of the futures curve (the measure for $market\ regime$ as discussed later) to drop. This triggers price shift along the entire future price curve causing the oil market to $expand$ and $open\ interest$ to raise. As this happens, the oil producers begin to shift their production to capitalise on raising price which causes the oil supply to increase, in effect, acting as a balancing mechanism for the oil price in the market. Reverse process takes place when oil demand oil drops.

Because $spot's$ correlation with the prices of futures varies depending on how far the futures contracts are on the price curve relative to $spot$, the level of change in the measure of $market\ regime$ relative to $market\ expansion$ and $market\ liquidity$ purely depends on the outlook investors have for the delivery period the futures contracts are associated with. Therefore, the relationships among the proxies and market constituents by nature have to be non linear. Logic suggests to replicate such market dynamic the system must be cointegrated with at least one long run relationship amongst the variables. We examine this aspect in detail in the following sections of this study.

The *second* contribution of our work concerns proposing a long-memory framework to model dynamic interdependence among the price series of one month Brent future contract (BF1), the WTI spot (WSP) as traded on Intercontinental Exchange (ICE), and the New York Mercantile Exchange (NYMEX), respectively. The presence of short or long 'memory' within a time series can be captured by estimating its fractional integration order, d. Since the late 1970s, thanks to the path breaking research of Dickey and Fuller (1979), testing for the degree of integration order of economic and financial time series is an established phenomenon. While an $I(1)/I(0)$ framework attributed to Dickey and Fuller with d assuming either 0 (stationary) or 1 (non-stationary), captures some aspects of the persistence profiles of shocks in the series, the knife-edge assumption of d being either 0 or 1, overlooks much of the real life situations where a series can display a range of persistence features (such as, short-memory: mean-convergent shocks, stationary long-memory: long-term persistence with convergence of shocks, and non-stationary long-memory: the unit root persistence features). All these realistic dynamics, which have tremendous implications for predicting the trajectory of a time series or a system of time series, such as the case of ours. Depending on the value of d, shocks display hyperbolic or geometric decay — whether considered in a single time series or a vector of time series. These have implications for long-run relationship. For instance, when the integration parameter $0 < d < 0.5$, shocks would dissipate faster (but not instantaneously) so that within a shorter time horizon a stable relationship can be predicted. In case, $0.5 < d < 1$, the system may take long-time to stabilise but given other things constant, the system will converge to a long-run stable state. In other words, when $0 < d < 1$, the system depicts varied degrees of long-memory persistence of shocks with defined convergence speed of shocks for $0 < d < 0.5$ and $0.5 < d < 1$ having distinct temporal specifications for long-run stability within the interacting system.

Motivated by these dynamic implications of the fractional integration parameter, we perform a Fractional Cointegrated Vector Autoregressive model (FCVAR) estimation for a system that uses our proposed market proxies. We show that the FCVAR offers superior predictive ability of the oil market compared to that of conventional Cointegrated Vector Autoregressive (CVAR) model. An imposing feature of the FCVAR is that the process of adjustment of disequilibrium shocks in a dynamic interdependent system is slow, in contrast to being either instantaneous (when the integration parameter $d = 0$) or permanent (when $d = 1$). The latter two types are further from reality. Thanks to the complex interactions of agents over time, shocks have an innate tendency to dissipate very slowly than being hypothesised in a conventional cointegrated VAR (CVAR) framework (with $d = 0$). It should be noted that the presence of cointegration among growing variables indicate a theory of endogenous nature of growth, the trajectory of which can be manipulated by policy interventions (in technical sense, policies that can accommodate long-memory shocks within the decision making process and design strategies to neutralise them in the long-run.

The rest of the paper is planned as follows. Section 2 discusses our strategy of oil futures market decomposition into the proposed proxies. This section also discusses the importance and the approach in capturing inter-market dynamics and covers rationale for data selection and process of derivation of global proxies based on extracted components from individual markets. Section 3 introduces methodological architecture behind FCVAR model estimation, out-of-sample forecasting based on it and forecast evaluation. Section 4 presents empirical analysis results. Part 4.1 of this section presents concept of the long memory. Here memory parameters of individual variables are examined from static perspective (Part A) and from dynamic persistence standpoint using rolling window estimation (Part B). Analysis of speed of adjustment in individual memory parameters presented in Part C. Section 4.2 of Section 4 presents FCVAR model construction process (Part i), model estimations (Part ii & Part iii), out-of-sample forecasting results and relative forecast performance evaluation between FCVAR and CVAR models (Part iv). In Section 5 we present results for robustness evaluation of FCVAR approach and demonstrate absence of spurious memory in our sample. Finally, Section 6 concludes the paper.

2. Construction of market proxies and data selection

2.1. Market proxies

This section outlines a strategy to decompose oil futures market into our proposed proxies. For our subsequent empirical analyses, these proxies will play central roles. Fig. 1 summarises our strategy of constructing Global Market Proxies through schematic presentation. The first step is to collect daily data on closing prices and open interest rate series. In the second step, we derive benchmark proxies (the details of which are presented shortly). The following steps involve spot price data collection and dimensionality reduction through a principal component analysis, the technique that allows to extracts prevailing vectors in the data set. Finally, newly constructed proxies are used in our empirical exercise for a memory-embedded FCVAR estimation leading to a robust forecasting exercise.

(a) Market regime

$Market\ Regime$ is the first vector proxy proposed in this study. We define it by gradient of the futures price curve. The slope of the price curve is widely used by the economists, traders and researchers as an indicator of the $Market\ Regime$, *Backwardation* or *Contango*. The gradient describes the relationship across the price structure and able to capture the sentiment component in the market. When the price of commodity future contract with the long time to maturity is lower than the price of future contract with short time to maturity, the commodity market is referred to be in *Backwardation*, $i.e.$ down sloping price curve. In essence, this means that the cash asset is more valuable than holding the futures contract. When the price curve is upward sloping, the market is believed to be in *Contango*. Depending on the type of the market, different price curve regimes are considered as indicators of

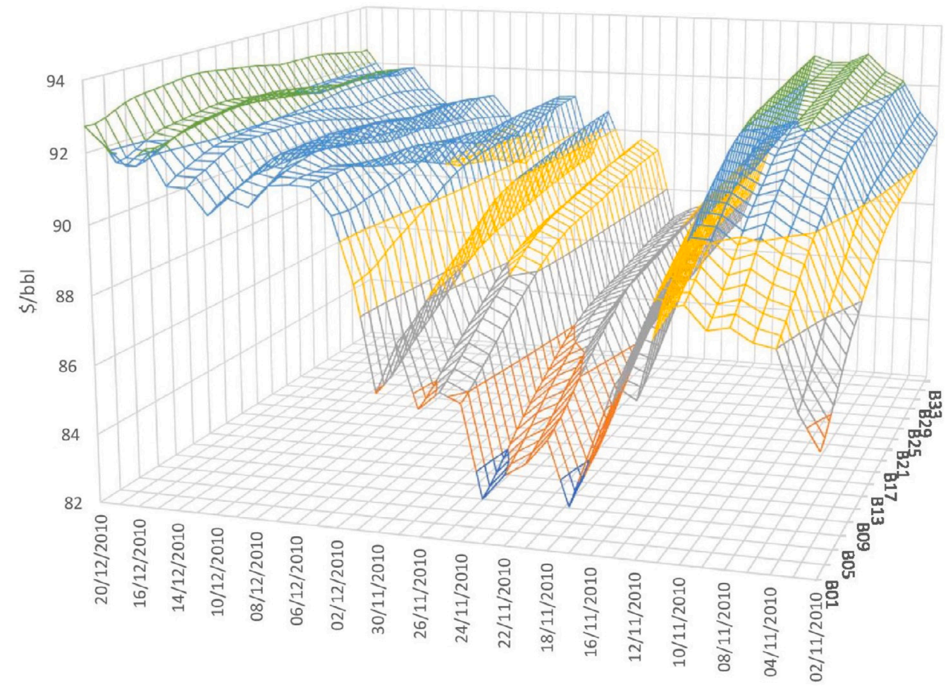

Fig. 1. Daily change in Brent futures price term structure.
Note: $3D$ representation of daily fluctuations in the price structure of Brent futures with respect to time. Graph based on 36 individual time series represented by Brent futures contracts prices up to 36 months to maturity on z-axis ($B01$, $B02$ … $B36$). Sample of daily frequency for the period between 02/11/2010 and 20/12/2010.

stable conditions. In the oil market downward sloping curve serves as such indication. The example presented in Fig. 1 shows transition of the of Brent term structure from *Contango* regime into *Backwardation* which was driven by improving economic outlook in the last quarter of 2010.

(b) Market expansion

The second proxy proposed in this study is $Market\ Expansion$ vector. We define it by area under the futures price curve. Although this proxy is a more precise measure of price curve dynamic, it is unable to capture whether the market expanding or contacting while the market is in *Contango* or in *Backwardation*, which is an important aspect and should not be ignored.

(c) Market liquidity

The final proxy proposed in this study is $Market\ Liquidity$. We define it by area under the curve of open interest term structure. As seen in Fig. 2 open interest term structure is more complex compared to the price curve of oil futures. Open interest reports the total number of unsettled contracts in the futures market and can be viewed as a measure of liquidity. The change in the term structure represents the difference in the number of buyers and sellers with the open positions. For example, when a trader takes a position in the market, the open interest increases by the number of open contracts and when the trader unwinds open position the open interest decreases. Physical delivery resulting from the trader exercising the conditions outlined in the futures contracts also decreases the open interest. However, as these instances are rare in the oil futures market, this aspect does not have a major impact on the measure of liquidity and therefore can be ignored. A sharp increase in open interest indicates arrival of the new information into the market but it does not provide on its own the direction the market is likely to move in and needs to be considered in conjunction with the other proxies defined in previous sections.

2.2. Inter-market dynamics

As mentioned previously, each commodity market follows certain $global\ economic\ mean$ which cannot be directly observed in the market but can be inferred from the relationship it has with a particular commodity, namely through the impact its dynamic exerts on the price structure of futures. While derived proxies reasonably reflect the dynamics prevailing in a market associated with Brent crude benchmark they still have a limited forecasting power. This is because the affect that $global\ economic\ mean$ has on the price structure of futures can be eclipsed by the noise originating from inter-market dynamics. To negate this effect, analysis from a range of commodity futures markets that share similar characteristics (such as traded commodity, contractual volumes, trading currency and comparable term structure lengths) would need to be factored into its inference. We establish Brent crude counterpart and extract the equilibrium relationship both benchmarks follow.

One such counterpart is WTI crude. This benchmark plays a major role in global oil price formation. Although Brent crude benchmark is used in pricing of 2/3 of the world's oil and therefore its price captures the regional information associated with these crudes, WTI is driven by the dynamics prevailing in the North America, home of the two major economies which are also major producers and consumers of the crude oil. The correlation between the spot price of Brent and WTI is above 0.95. Similar trend is observed in the corresponding futures contracts where correlation between the futures with identical term to maturity in both markets is consistently above 0.9. This suggests presence of equilibrium both benchmarks follow which provides a valuable perspective on the price formation mechanism in both markets. In order to examine this adjustment mechanism, WTI market proxies were extracted using the same methodology as in Brent market case.

Another important aspect which needed to be considered in inter-market analysis is the dynamic associated with the spot markets of the corresponding benchmarks. This is because commodity spot prices are often found to be more long-run dominant in the price discovery process than the futures prices, as demonstrated by Dolatabadi et al.

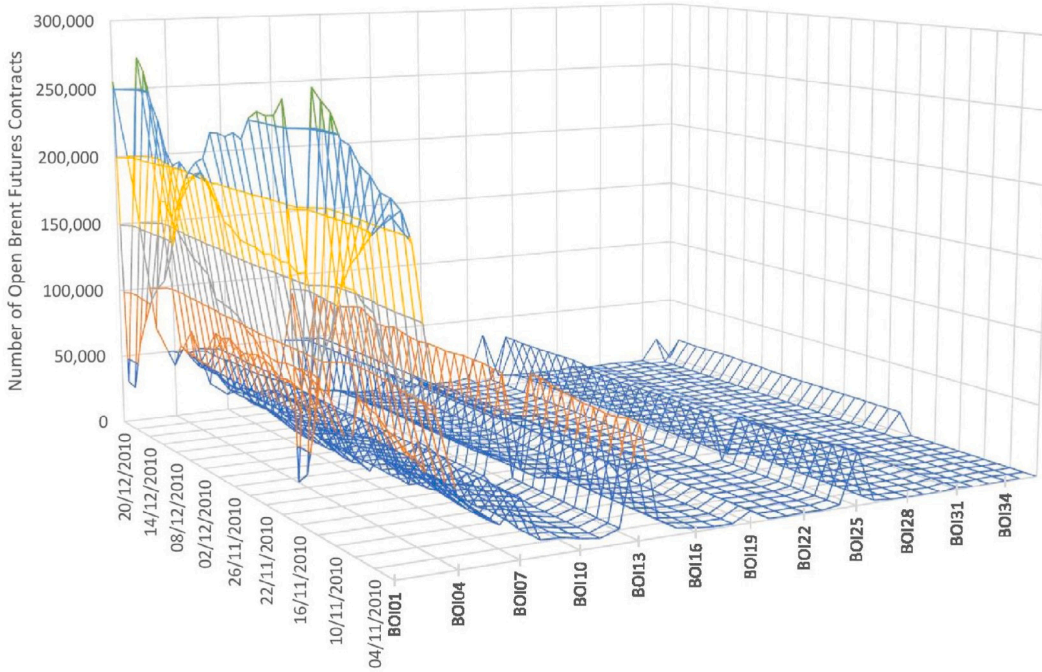

Fig. 2. Daily change in the number of open Brent futures contracts.
Note: 3D representation of the open interest term structure daily fluctuations with respect to time. Graph based on 36 individual time series represented by the Brent Open Futures contracts with 36 months to maturity on *z*-axis (BOI01, BOI02 ... BOI36). Sample period between 02/11/2010 and 20/12/2010.

(2015). Therefore, it is logical to conclude that there also exists an equilibrium relationship between the futures and the spot markets. To examine this aspect, Brent and WTI spot series were also considered in the analysis. Due to the absence of the term structure in the spot markets, raw spot series were used for this purpose.

2.3. Data and construction of proxies

To derive proposed proxies, individual price series of the futures contracts with up to 36 months to maturity were obtained for Brent and WTI crude benchmarks which are traded on the Intercontinental Exchange (ICE) and New York Mercantile Exchange (NYMEX), respectively. The considered sample span is between 02/01/2003 and 30/04/2021. Note, although the entire Brent futures market is defined by 87 months and WTI futures market is defined by 99 months to maturity futures, the span of 36 months futures represents the most liquid part of the futures curve and for this reason, it was considered as a good representation of the overall market dynamic. Brent Spot Price FOB[2] and NYMEX WTI spot price series were obtained from US Energy Information Administration (EIA). This sums to a total of 74 individual time series.

Transformation process applied to the obtained data reduces the overall number of series to just eight, two series for Brent and WTI spot prices and the remaining six representing the pairs of proposed market proxies from each of the corresponding futures markets. It is worth noting that to avoid introduction of error associated with excessive data manipulation, no log transformation was applied to any of the series prior to data transformation. Summary statistics for the set of derived variables presented in Table 1. (See Figs. 3 and B.1).

[2] Free on Board — a logistic term that indicates which party assumes liability for the damage or loos of goods. FOB origin — indicates that the purchaser bares the shipping cost from storage location and assumes ownership of the goods as soon as it is dispatched. FOB destination — indicates the seller retains the risk of loss until the goods reach the buyer.

Construction of global proxies

Decomposition of Brent and WTI spot and futures markets resulted in the total of eight variables which jointly capture the overall oil market dynamics. However, from modelling perspective using all eight as explanatory variables would present a problem of multicollinearity. This is due to a high correlation that exists amongst the corresponding pairs of spots and proxies. For instance, results presented in Table 2 show the correlation between BE and WE stands at 0.99, BL and WL at 0.86 and correlation of BR with WR at 0.90. The results also show that while there is a high correlation between Brent and WTI spot price series, both variables are also highly correlated with the proxies for $Market\ Expansion$ from both benchmarks. This indicates that all four variables follow the same equilibrium relationship.

Results suggest the existence of interdependencies amongst the market variable and implies that each pair follows a certain equilibrium characteristic. To extract these equilibrium relationships, we apply Principal Component Analysis (PCA). Due to a difference in scales of underlying data, Principal Components (PC) were calculated based on correlation (end-to-end process for derevation of global proxies presented in Fig. 3).

Dimension reduction results presented in Table 3 indicate that PC1, PC2 and PC3 jointly account for 98% of variation in the data; 56% attributed to PC1; 25% and 17% of variation accounted by PC2 and PC3 respectively. The residual 2% of variation in the data is described by the remaining five principal components which were discarded from the analysis due to low significance. Further examination established that the three significant principal components correspond to the proposed global proxies in the following order:

- $(PC1)$ - E_t — Oil Market Expansion
- $(PC2)$ - L_t — Oil Market Liquidity
- $(PC3)$ - R_t — Oil Market Regime

As previously mentioned, focus of the forecasting exercise in this study is aimed at demonstrating that the prices of spot and of futures contracts for both Brent and WTI benchmarks can be successfully forecasted using proposed proxies as explanatory variables. Therefore,

Table 1
Summary statistic: Benchmark Proxies and Brent & WTI spot prices.

	Brent market proxies			WTI market proxies			Spot prices	
	BE	BL	BR	WE	WL	WR	BSP	WSP
Mean	2531.14	1325956	−0.03	2399.28	1469121	−0.03	69.73	66.24
Median	2373.92	1044164	−0.05	2273.17	1443196	−0.06	64.44	61.67
Maximum	5288.50	2736704	0.88	5202.61	2681700	0.73	143.95	145.31
Minimum	843.29	47662	−0.71	871.69	437816	−0.70	9.12	−36.98
Std. Dev.	890.55	746014.1	0.26	814.63	536237.4	0.24	27.33	23.90
Skewness	0.25	0.37	0.26	0.25	0.08	0.19	0.42	0.42
Kurtosis	2.38	1.57	2.72	2.62	2.18	2.93	2.18	2.54
Observations	4565	4565	4565	4565	4565	4565	4565	4565

Note: Summary statistic of the derived variables with daily frequency. Date range between 02/01/2003 and 30/04/2021. B — Brent crude oil, W — WTI crude oil, R — market regime, L — market liquidity, E — market expansion. BSP — Price series of Europe Brent Spot Price FOB, WSP — price series of WTI Spot Price as reported on NYMEX.

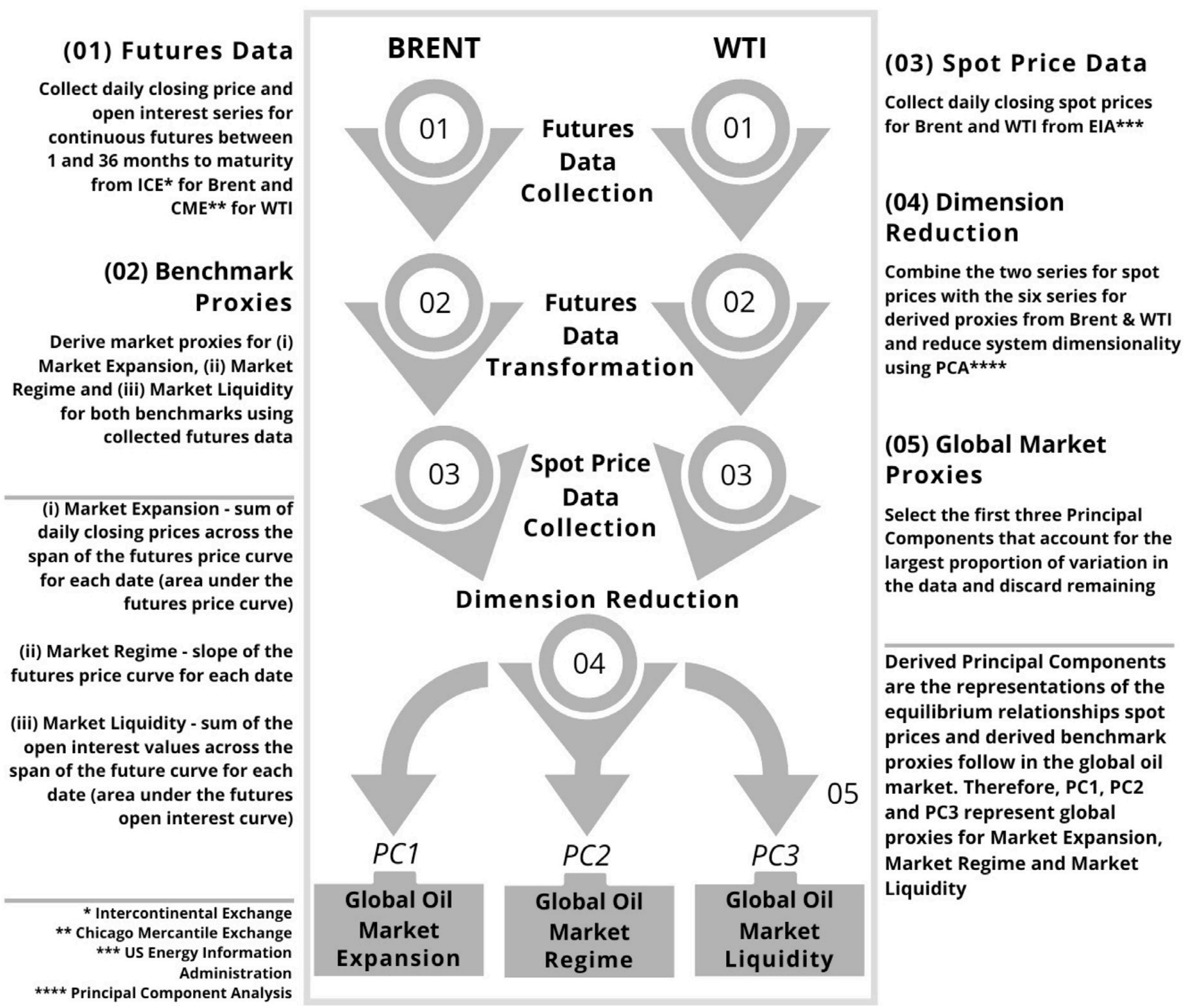

Fig. 3. The architecture of generating Global Proxies for world's oil market.

Table 2
Correlation matrix: Benchmark Proxies vs Brent & WTI spots.

Pc Numbers	BE	BL	BR	WE	WL	WR	BSP	WSP
BE	1	−0.16	−0.39	0.99	0.23	−0.21	0.97	0.96
BL	−0.16	1	−0.06	−0.26	0.86	−0.11	−0.12	−0.22
BR	−0.39	−0.06	1	−0.33	−0.18	0.90	−0.59	−0.52
WE	0.99	−0.26	−0.33	1	0.15	−0.16	0.94	0.96
WL	0.23	0.86	−0.18	0.15	1	−0.13	0.24	0.16
WR	−0.21	−0.11	0.90	−0.16	−0.13	1	−0.40	−0.40
BSPOT	0.97	−0.12	−0.59	0.94	0.24	−0.40	1	0.97
WSPOT	0.96	−0.22	−0.52	0.96	0.16	−0.40	0.97	1

Note: B — Brent crude oil, W — WTI crude oil, R — market regime, L — market liquidity, E — market expansion.

Table 3
Dimension reduction of Benchmark Proxies combined with Brent & WTI spots using PCA.

Pc numbers	Value	Difference	Proportion	Cumulative value	Cumulative proportion
PC 1	4.477	2.473	55.96%	4.48	55.96%
PC 2	2.004	0.657	25.05%	6.48	81.01%
PC 3	1.347	1.253	16.84%	7.83	97.85%
PC 4	0.094	0.031	1.18%	7.92	99.03%
PC 5	0.063	0.054	0.78%	7.98	99.81%
PC 6	0.009	0.004	0.11%	7.99	99.92%
PC 7	0.005	0.003	0.06%	8.00	99.98%
PC 8	0.002	–	0.02%	8.00	100.00%

Note: PCA analysis of Brent & WTI spots and Benchmark Proxies. Table presents dimensionally reduced representation of the global market where: PC 1 — market expansion, PC2 — market liquidity and PC 3 — market regime. Date range between 02/01/2003 and 30/04/2021. Calculation type: correlation.

Table 4
Summary statistics: Global oil market proxies, One month brent future contract and WTI spot.

	BF1	WSP	GE	GL	GR
Mean	70.266	66.243	0.000	0.000	0.000
Median	64.830	61.670	−0.348	−0.276	0.030
Maximum	146.080	145.310	5.924	3.465	2.795
Minimum	19.330	−36.980	−5.484	−3.307	−2.668
Std. Dev.	27.123	23.896	2.116	1.416	1.161
Skewness	0.413	0.419	0.415	0.357	−0.436
Kurtosis	2.192	2.544	2.186	2.090	2.571
Observations	4565	4565	4565	4565	4565

Note: Summary statistic of the proposed global market proxies of daily frequency. Date range between 02/01/2003 and 30/04/2021. BF1 — ICE One Month Brent Future price; WSPOT — NYMEX WTI SPOT price; GE — oil market expansion; GL — oil market liquidity; GR — oil market regime.

we include summary statistics for the two price series selected for the forecasting exercise to demonstrate this aspect, i.e. ICE one month Brent future contract as well as NYMEX WTI Spot and present them together with global proxies for Market Expansion, Market Liquidity and Market Regime for ease of contextualisation, as presented in Table 4.

3. Model set-up and estimation

(A) Long memory

Since we aim to analyse forecasting potential of our model through the prism of a memory-embedded system, we need to introduce briefly the key concepts related to the memory properties of a time series. This is important as we seek to understand the dynamic nature of persistence of the market-centric observable in our data. One of the time series property of interest from forecasting perspective is the presence of long memory process which is observed either in the mean or the variance. This process governs how quickly the shocks on financial series are dissipated through the time. The presence of long memory process implies that the order of integration of time series lies somewhere between $I(0)$ and $I(1)$. In this case the time series are referred to as fractionally integrated and can be modelled using fractional operator d. Consider the following process:

$$\Delta^d y_t = (1-L)^d y_t + \varepsilon_t \tag{2}$$

Where ε_t is an independent identically distributed (iid) error term with $E(\varepsilon_t) = 0$ and $var = (\varepsilon_t) = \sigma^2$. When $d = 0$, then y_t is the white noise and is stationary. Under this scenario y_t is said to be integrated to the order of zero and referred to as $I(0)$ process. When $d = 1$, then y_t is non-stationary random walk process. In this case y_t is integrated of order 1 and is referred to as $I(1)$ process. However, when $0 \leq d \leq 1$, then this process referred to as fractionally integrated (Appendix A (section 1) presents a detailed discussion of persistence profiles of various types of memory related to the fractional integration order, d).

(B) Fractional cointegrated VAR

We now move from an individual series to a system. In our case, we have a system of three market-centric variables, which we assume to be dynamically interdependent and possess slow-convergent memory. An important approach to model such a mechanism is the fractionally cointegrated Vector Autoregressive (FCVAR) system. In particular, we study interdependences between one month to maturity Brent future contract as well as WTI spot and market proxies E, L and R using FCVAR system — an approach which was first proposed by Johansen (2008) and further developed by Johansen and Nielsen (2012) as a generalisation of Johansen's cointegrated vector autoregressive model (CVAR) (Johansen, 1995). It allows for fractional process d to be integrated to the order of $d-b$ and represented by the following:

$$\Delta^d (X_t - \mu) = \alpha\beta' L_d (X_t - \mu) + \sum_{i=0}^{k} \Gamma_i \Delta^d L_d^i (X_t - \mu) + \varepsilon_t, \qquad t = 1, 2, \ldots, T \tag{3}$$

Where ε_t is an independent identically distributed (iid) $p-dimensional$ error term with $E(\varepsilon) = 0$ and $var(\varepsilon_t) = \sigma^2$. Parameters α and β are $(p \times r)$ matrices with $0 \leq r \leq p$. Coefficients in α represent the rate of adjustment of each element in the system to the equilibria β is a column vector of co-integrating relationships in the system. Stationary combinations or the long run equilibria is represented by the elements of $\beta' X_t$ and short run behaviour is governed by parameter Γ_i. Parameter μ shifts each series by a constant which allows for truncating the fractional differences by terminating the summation in (2) at $k = t-1$. Johansen and Nielsen (2016). Detailed derivation of the model is presented in Appendix A (section 2).

(C) Forecasting

Due to the autoregressive nature of FCVAR model, its forecasting methodology is relatively straight forward. Similar to VAR methodology, it allows to generate estimates for every variable in the system with every forecast iteration, such that n-step ahead forecast can be formed recursively using following relationship:

$$\hat{X}_{t+j|t} = \hat{\mu} + L_{\hat{d}}(X_{t+j|t} - \hat{\mu}) + \hat{\alpha}\hat{\beta}' L_{\hat{b}} \Delta^{\hat{d}-\hat{b}} (X_{t+j|t} - \hat{\mu}) + \sum_{i=o}^{k} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{b}}^i (X_{t+j|t} - \hat{\mu}) \tag{4}$$

(D) Forecast performance

To measure forecasting accuracy of the FCVAR model with respect to CVAR model we use several techniques, namely assessment of: (i) relative forecast errors, (ii) equal predictive ability, (iii) test of symmetry in the loss, and (iv) test of forecast rationality under asymmetric loss.

We first use the approach proposed by Nielsen and Shibaev (2018). This method allows to target's model forecasting performance by calculating root mean squared forecasting errors ($RMSFE$) which is formulated as:

$$RMSFE = \left\{ \frac{1}{K} \sum_{i=1}^{K} (\hat{Y}_{i,t+h|t} - Y_{i,t+h|t})^2 \right\}^{1/2} \tag{5}$$

Where $\hat{Y}_{i,t+h|t}$ represents forecasting values of Y_i over the period between t+1 to t+h conditional on the information set Y_i being available at the time t. Subscript i denotes specific included variables in the target model where $i = 1, 2, \ldots, K$. Subscript h denotes the specified forecast horizon. The above formula calculates the average value of RMSFE for each of the individual series incorporated into the multivariate model and can be used for calculating errors produced by the system.

We then perform test of equal predictive ability between the models introduced by Giacomini and White (2006). We use unconditional variation of the test which restricts attention on establishing superiority of comparable forecast models with null hypothesis assessing if forecasts generated by different models are equally accurate. Sign convention of the test statistic allows to establish dominant model where negative value indicates that the model one dominates over the model two and vice versa.

Next, we test for the symmetry in the loss using loss function introduced by Elliott et al. [2005]. We do that through estimation of loss coefficient α and testing the null that loss is symmetrical. The loss function takes the following form:

$$L(e) = [\alpha + (1 - 2\alpha)I(e < 0)]|e|^p, \qquad \alpha \in [0, 1] \tag{6}$$

where I is an indicator function which equals to one if the forecast error $e < 0$, and zero otherwise. Asymmetry in the loss is given when $\alpha \neq 0.5$. Here the values of α that exceed 0.5 indicate greater aversion to positive forecast errors and vice versa. Parameter p is a positive integer and setting $p = 1$ results in a lin–lin loss function which allows for different slopes under asymmetric loss. The quadratic asymmetric loss function is given for the special setting $p = 2$ and $\alpha \neq 0.5$. In our test we use $p = 1$ condition due to its robustness.

To finalise the forecast evaluation exercise, we perform rationality test introduced by Elliott et al. [2005]. The objective of this test is to examine whether the forecasts are rational under estimated alpha.

4. Empirical analysis

4.1. Estimates of 'memory'

In order to establish if FCVAR methodology is the most appropriate approach in describing the dynamics in the multivariate system, we further examine univariate time series for the presence of long memory, the backbone of FCVAR theory. We use fractional integration analysis based on the methodology proposed by Shimotsu (2010) who calculated fractional integration of univariate time series with unknown mean and time trend across range of bandwidths using different modifications of Local Whittle estimator.

(A) Static long memory estimates

We first calculate the average long memory of the variables by estimating Local Whittle d parameters using fixed data sample between 02/01/2003 and 30/04/2021. In this analysis we consider bandwidth ranges between 0.40 and 0.80. Results presented in Table 5 indicate that the long memory of the variables appear to be following different trends. In the case of one month to maturity $Brent\ future\ contract$, $WTI\ Spot$ and $Market\ Expansion$, d estimates are slightly above one and have similar values across bandwidth values and Local Whittle variations, which is due to a high correlation amongst the three variables. In the case of $Market\ Liquidity$ and $Market\ Regime$s, d estimates appear to be averaging below one which suggests that the shocks in the market do not affect their memory parameters in the same way as they do memory parameters of one month $Brent\ future\ contract$, $WTI\ Spot$ and $Market\ Expansion$. This aspect indicates that $Market\ Liquidity$ and $Market\ Regime$ could play an important role in the adjustment mechanism that exists amongst the endogenous variables in the system, the result echoed in the analysis of cointegration relationships using FCVAR methodology as discussed later.

Table 5
(Static) memory estimates of ICE one month Brent future, NYMEX WTI Spot & derived Global Oil Market Proxies.

Variable	BF1				WSP			
Bandwidth	LW	ELW	FELW	TSELW	LW	ELW	FELW	TSELW
0.40	0.864	0.913	0.894	0.894	0.799	0.851	0.824	0.824
0.45	0.971	0.973	0.977	0.977	0.961	0.960	0.966	0.966
0.50	1.103	1.138	1.136	1.136	1.090	1.116	1.116	1.116
0.55	1.167	1.176	1.191	1.191	1.131	1.135	1.147	1.147
0.60	1.075	1.091	1.092	1.092	1.064	1.073	1.076	1.076
0.65	1.071	1.078	1.083	1.083	1.053	1.059	1.062	1.062
0.70	1.059	1.075	1.076	1.076	1.017	1.029	1.027	1.027
0.75	1.039	1.063	1.062	1.062	0.999	1.020	1.016	1.016
0.80	1.020	1.050	1.050	1.050	0.971	0.996	0.993	0.993
Variable	GE				GL			
Bandwidth	LW	ELW	FELW	TSELW	LW	ELW	FELW	TSELW
0.40	0.826	0.852	0.854	0.854	0.879	0.919	0.929	0.929
0.45	0.938	0.957	0.946	0.946	0.897	0.875	0.862	0.862
0.50	1.095	1.098	1.119	1.119	0.995	0.968	0.957	0.957
0.55	1.179	1.175	1.193	1.193	1.045	1.015	1.001	1.001
0.60	1.073	1.075	1.085	1.085	1.018	1.009	1.004	1.004
0.65	1.079	1.081	1.089	1.089	0.937	0.911	0.907	0.907
0.70	1.060	1.064	1.072	1.072	0.964	0.949	0.947	0.947
0.75	1.042	1.052	1.063	1.063	0.951	0.935	0.931	0.931
0.80	1.027	1.049	1.057	1.057	0.927	0.918	0.914	0.914

Variable	GR			
Bandwidth	LW	ELW	FELW	TSELW
0.40	1.014	1.001	0.956	0.956
0.45	0.844	0.828	0.793	0.793
0.50	0.943	0.933	0.917	0.917
0.55	0.980	0.974	0.966	0.966
0.60	0.948	0.946	0.931	0.931
0.65	0.897	0.897	0.874	0.874
0.70	0.919	0.921	0.910	0.910
0.75	0.928	0.938	0.922	0.922
0.80	0.925	0.948	0.934	0.934

Note: Fixed univariate d estimates for the bandwidth range between 0.40 and 0.80 and Local Whittle estimators: LW — Local Whittle estimator, ELW — Exact Local Whittle estimator; FELW — Feasible Exact Local Whittle estimator; TSELW — 2-step ELW estimator. Sample period: 02/01/2003–30/04/2021.

(B) Dynamic persistence: Rolling window estimates of long-memory

The condition when $0 < d < 0.5$ implies the existence of long memory in the variable, but one that disappears from the trajectory after a period of time (a covariance stationary process). If $d > 0.5$, then it is still a long memory and the impact of shocks in this case is felt longer and the system might take very long time before it settles to a long-run mean or a steady state. Good thing: such a process still can converge to a steady state. Bad thing: if there are further shocks in between then the expected convergence rate may be zero and the shocks can start diverging again and this is when the d values may appear slightly greater than one.

Due to the fact that various endogenous and exogenous shocks that can affect the overall trajectory of the whole process in the span of the sample, the system may encounter different perturbations in d estimators. In order to demonstrate the sensitivity of d to the roll of the sample a range of rolling window sizes needed to be considered — $small$ or big. While in all cases converging or diverging state of individual memory indicators is captured, the trend and magnitude of the dynamic would purely depend on the number and size of the shocks falling into the chosen rolling window span. For example, selecting a big rolling window would imply that more fluctuations are expected to fall within the rolling window which would have a compounding effect on the memory parameters in the system forcing them to reflect market dynamics that corresponding proxy's memory is sensitive to. Under this scenario, if with the roll of the window number of shocks entering the system become seldom and low in magnitude, d parameters will begin to converge to a long run mean and vice versa. Selecting a $small$ rolling window size, on the other hand, would imply greater possibility that d estimates are sensitive to fluctuations. This means that while reflecting the market dynamics associated with the smaller rolling window, d estimates would display different behaviour and levels when compared to the behaviour of d estimates based on big rolling window size. Both measures, however, are useful for interpretation of short and long market dynamics.

In order to demonstrate this aspect, we choose two rolling window sizes, $small$ — 490 observations and big — 1470 observations, which

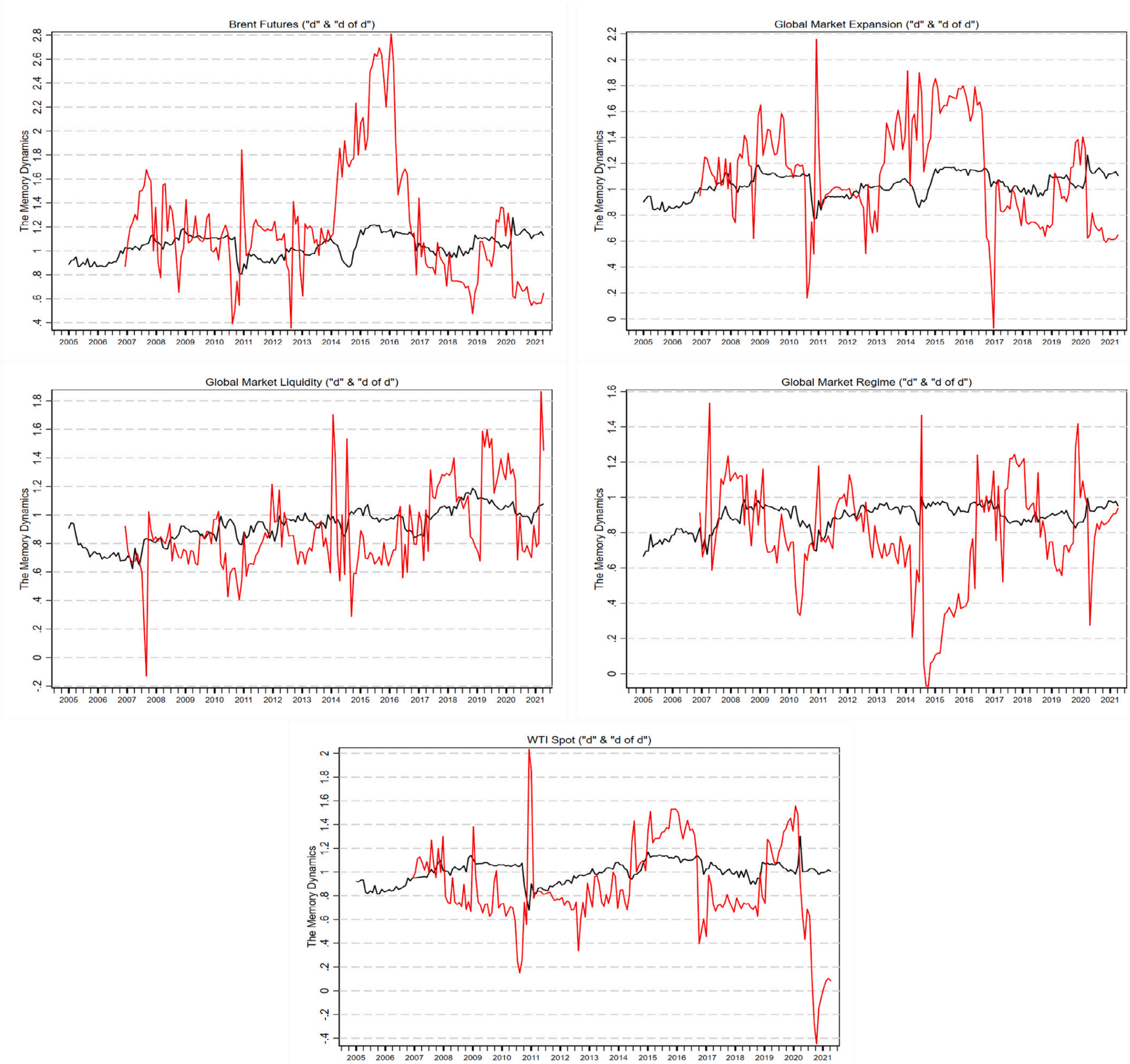

Fig. 4. Dynamic Estimates of Memory (Rolling windows of estimated d of Global Proxies, One Month Brent Future and WTI Spot (490 Window Size)
Note: Estimation of the long memory parameter d (black line) and d of d (red line) using rolling window on Feasible Exact Local Whittle with the bandwidth of 0.60. Window size is 490 days and estimation increments are 20 days. The top left corner graph is the estimated (d) of Oil Spot Price, top right plot is the estimated (d) of Market expansion, left bottom corner is the estimated (d) of Market Liquidity and right bottom figure is the estimated (d) of Market Regime. The x-axis represents the date and ticks are quarterly.

equate to 2-year and 6-year business cycles respectively. We record calculated d in the sequence of 20 observations which equates to monthly frequency. Fig. 4 demonstrates the results of estimated d for the set of considered variables based on Feasible Exact Local Whittle calculation. In our analysis we detect the sensitivity of each d through $m = T^{0.6}$ bandwidth and check different bandwidths (e.g., 0.5, 0.7 and 0.8) to insure the accuracy of our tests.

Results based on *small* rolling window size (2-year business cycle), indicate the stability of long memory parameters over time. We can see that the estimated d of $BF1$ on average fluctuating above 1.00 with three major shocks, one starting in September 2010, second starting in December 2013 and the third shock starting in December 2019. While d values dropped significantly below *one* during the first two shocks, the third shock associated with the onset of $Covid$ 19 pushed it into *explosive memory* territory when d reached 1.3 level. This dynamic was a result of substantial level of overreaction in the market. Similar dynamics are observed in the evolution of d parameters of WSP and GE. An interesting observation that December 2013 shock in WSP is far less pronounced compared to that of $BF1$ and GE for the same period. Since this shock was driven by the events around Ukraine and Crimea, the impact of these developments on North America's oil market with which WSP is associated with, was perceived by investors as less significant compared to that of $BF1$ which is associated with Europe oil market. Results also show that the estimated d of GE co-moves with the long memory of the $BF1$ and WSP. This aspect is not surprising as these variables correlated with each other and play important role in absorbing information arriving into the market and

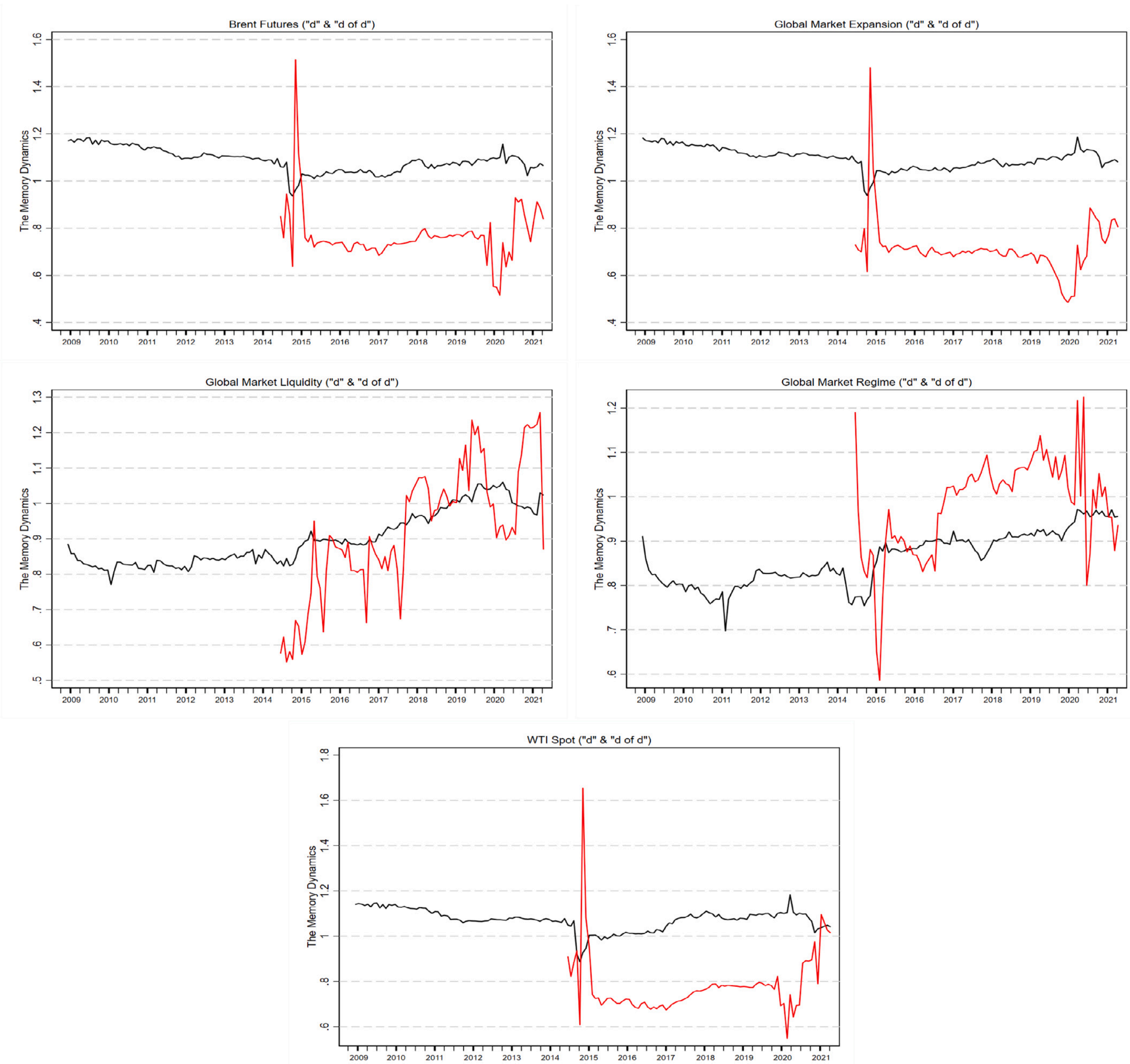

Fig. 5. Dynamic Estimates of Memory (Rolling windows of estimated d of Global Proxies, One Month Brent Future and WTI Spot Price (1470 Window Size).
Note: Estimation of the long memory parameter d (black line) and d of d (red line) using rolling window on Feasible Exact Local Whittle with the bandwidth of 0.60. Window size is 1470 days and estimation increments are 20 days. The top left corner graph is the estimated (d) of Oil Spot Price, top right plot is the estimated (d) of Market expansion, left bottom corner is the estimated (d) of Market Liquidity and right bottom figure is the estimated (d) of Market Regime. The x-axis represents the date and ticks are quarterly.

transmitting its effect onto the reaming variables, as highlighted in FCVAR analysis section below.

Comparing the results of long memory estimation based on *small* rolling window suggests that d parameters of $BF1$, WSP and GE are negatively correlated with the oil prices post financial crisis of 2008. This implies that in the lower oil price environment, below \$80 per barrel, d of the variable averages above *one* and for the oil prices above \$80 per barrel, d averages below *one*. Bearing in mind that forecast power of the series deteriorates as d approaches *one* and the process becomes explosive when $d > 1.0$, this implies that in the low oil price environment oil market becomes less forecastable as there is a higher effort amongst market participants to establish the *best* price. This appear to make processes in the oil market more efficient.

$Market\ Liquidity$ d estimates for the same time period, on the other hand, demonstrate opposite dynamic. In the lower oil price environment, below \$80 per barrel, its d values average at 0.8 and when prices are above \$80 per barrel, they average at 0.9. As we calculate GL based on *open interest* term structure, this may indicate that in the lower price environment market participants act less efficiently when entering or closing positions in the market and vice versa. The suggests that inefficiencies in long memory processes present in $BF1$, WSP and GE are being balanced by the processes in d parameter of GL which stabilises the system. In the context of this study, this case provides an illustrative example of the adjustment mechanism amongst the long memory parameters in the presence of shocks in the system.

With regards to GR, its estimated parameter d has remained consistently below 1 throughout the considered period. Results show that the long memory was trending toward the unity in the pre-financial crisis period of 2008, and had an opposite momentum when economy began to recover reaching 0.7 level in December 2010. It then began opposite dynamic trending toward the unity which lasted till the second half of 2016 when it reversed and began to slide. This effect could have been caused by the stance US adopted towards the oil price after presidential elections held at the end of 2016, thereby throwing the system off balance. Similar effect is observed during the same period in d parameters of $BF1$, WSP, GL and GE where they saw a sudden drop.

Results based on *big* rolling window size (6-year business cycle) portray different behaviour. Due to significantly larger rolling window, d estimates in this case reflect long run market trends. Behaviour of d estimates associated with $BF1$, WSP and GE show that post financial crisis of 2008 they were in the state of convergence. This behaviour could have been achieved if the number and the size of the shocks entering the rolling window was in continuous decline signalling stabilising conditions in the oil market. However, the trend in d estimates for these variables reverses in 2014 following the shock. The values of d in the estimators begin to increase which would have been caused by the increasing number of shocks entering rolling window.

Memory parameters of GL and GR, on the other hand, apart from the shock in 2014, do not share similar dynamics. Results show that post financial crisis of 2008 both d parameters, while being significantly below 1.00, have been continuously increasing with a brief interruption in 2014 when memory estimates dropped. As mentioned previously, this behaviour indicates that GL and GR play an important role in adjustment mechanism that exists amongst endogenous parameters in the system.

(C) Convergence of memory: Speed of acceleration (deceleration)

To finalise univariate analysis, we examine the long memory of the memory parameter of each variable in the system. The goal of this test is to demonstrate memory's persistence. The question here is, does the memory really disappear over time? If so, at what rate? Is there a rate of acceleration or deceleration? The test on *memory* of *memory* or d of d implies that the memory in the system may accelerate, decelerate or depict a mixed tendency over time. In case, d of d exhibit decelerating tendency over time, that then shows that the system is inherently a stable one as at a certain rate, keeping other things constant, the system tends to stabilise. If d of d values rise or fluctuate, the system displays unstable or unpredictable pattern.

We apply rolling window calculation onto series of d estimates generated based on *small* and *big* rolling windows covered in previous section. For consistency, we keep the window size to 2 and 6 years and record values in monthly increments. The resultant series superimposed on the series of d parameters presented in Figs. 4 and 5 demonstrate the dynamics of individual memory estimates. In our analysis we mainly detect the sensitivity of each d using FELW estimator through $m = T^{0.6}$ bandwidth and check different bandwidths (e.g., 0.5, 0.7 and 0.8) to insure the accuracy of our tests.

Results of the test, based on two year business cycle, show that the estimated values of d for all variables range between 0.0 and 3.0 as in the case of $BF1$ variable. This implies that the long memory characteristics of d in all cases evince inherently instability of the system. These are dominated by the shocks which push the system into disequilibrium in the chosen window span. However, d of d associated with GL and GR variables display relatively high negative correlations with d of d of the remaining variables (GE, $BF1$ and WSP). This suggests that the system of proposed variables follows self-adjusting dynamic where the effects of shocks are being dissipated by the co-movement amongst the variables which brings the system into equilibrium.

Results based on six year business cycle paint a different picture which leads to a similar conclusion, however. They show that d parameters of the long memory in $BF1$, WSP and GE apart from the shocks in 2014 and 2020 when their values saw a sharp increase, in general average around 0.7. This suggests that the number and magnitude of the shocks entering the rolling window during periods is roughly constant which indicates stable conditions in the oil market. Long memory of d parameters associated with GL and GR on the other hand, fluctuate between the values of 0.5 and 1.3 for GL and between 0.8 and 1.2 for GR. This dynamic is being balanced by the long memory processes in d parameters of the remaining variables which ensures the system stabilises to an equilibrium. The next section focuses on demonstrating this aspect using FCVAR methodology.

4.2. FCVAR results: Understanding dynamics within memory-embedded system

(i) Model identification

Based on the analysis presented in previous sections, we considered FCVAR methodology as the most appropriate approach for modelling time-varying processes amongst Global Market Proxies and oil market members, such as the price of one month to maturity Brent future contract and WTI spot price which we use to demonstrate this aspect. The following section presents empirical results of the analysis on long-run equilibrium using FCVAR framework as defined by (8) which we use to construct models for BF1 and for WSP. Both models and their parameters are estimated using Matlab package developed by Nelsen and Popiel (Popiel et al., 2018).

From the model specification perspective, the biggest challenge after variable selection is to establish lag augmentation term, k. We use Akaike information criterion to establish statistically significant highest order lag of the model. Special attention was paid to the residuals to ensure FCVAR did not suffer from serial correlation. Multivariate Ljung–Box Q-test, $Q_\varepsilon(h)$, with 12 lags was used for this purpose. Based on Akaike information criterion the lag order for BF1 model was chosen at $k = 5$ with P value of 0.00 and $Q_\varepsilon(h)$ P value of 0.99. The lag order for WSP model using the same information criterion was chosen at $k = 6$ with P value of 0.00 and $Q_\varepsilon(h)$ P value of 0.98, see results in Table 6.

The next step in model estimation is to identify the rank r of the system which indicates the number of cointegration relationships that exist amongst the variables and related to matrix $\Pi = \alpha\beta'$ in (8). If the system contains p number of $I(1)$ variables, then Π is a p x p matrix where $0 < r < p$. Under this scenario there are r number of cointegrating relationships amongst the p variables which have to be $I(0)$ for the relationship to be balanced. When $r = 0$, Π is a *null* matrix and the optimal model to describe the relationship amongst the variables would be VAR in first difference without level term.

The approach outlined in Johansen (1995) was used to establish the rank of the system which is based on sequential LR tests of the rank $= r$ against the alternative of rank $= p$, with $p \geq r$. In this approach first non-rejected value of r in the sequence of tests is the estimated rank of the system. Test results indicate that there are two cointegration relationships present in each system considered, with P value of 0.11 for BF1 and 0.63 for WSP model. Results presented in Table 7.

(ii) Model 1: BF1, Global Market Expansion, Global Market Liquidity and Global Market Regime

Unrestricted model describing relationship amongst Global Market Proxies & One Month Brent Future price is derived based on $k = 5$ and $r = 2$. The results of FCVAR model are presented in (6) where both $X_t - \mu$ on the left side and α on the right side of the equation are expanded in the matrix form. The column vector v_t represents $\beta' L_d(X_t - \mu)$ in (3) and the highest estimated lag of the short-run dynamic is 5. The estimated long memory parameter d of the model is found to be 0.567 with standard error of 0.02, pointing towards the conclusion of fractional cointegration order. This value agrees with the

Table 6
Lag-Order Selection: (1) FCVAR of Global Proxies & Brent One Month Future; (2) FCVAR of Global Proxies & WTI spot.

Model	p	r	$\hat{d}$	$Log L$	LR	$P-value$	AIC	BIC	$PmvQ$
BF1	5	4	0.562	9983.24	58.10	0.00	−19764.48	−19117.74	0.99
WSP	6	4	0.650	8284.25	54.45	0.00	−16334.51	−15585.31	0.98

Note: (i) p — number of lags; (ii) order of autocorrelation test (white noise test) = 12. (iii) r — number of ranks. (iv) $\hat{d}$ — the memory system.

Table 7
Rank Tests: (1) FCVAR of Global Proxies & Brent One Month Future; (2) FCVAR of Global Proxies & WTI Spot.

Model	*BF1*				*WSP*			
Rank	$\hat{d}$	*Log-likelihood*	*LR statistic*	*P-value*	$\hat{d}$	*Log-likelihood*	*LR statistic*	*P-value*
0	0.878	9939.268	87.94	0.00	0.732	8250.245	68.02	0.00
1	0.864	9956.404	53.67	0.00	0.784	8266.821	34.86	0.00
2	0.568	9979.460	7.56	0.11	0.654	8282.926	2.65	0.62
3	0.562	9982.822	0.83	0.36	0.650	8284.120	0.27	0.61
4	0.562	9983.238	–	–	0.650	8284.253	–	–

Note: (i) r — number of ranks; (ii) $\hat{d}$ — the memory system.

Table 8
Hypothesis test on model's long memory parameter, d of Global Proxies & WTI Spot.

$Parameter$	$\mathcal{H}_d$
df	1
LR statistic	85.205
P value	0

Note: (i) df: degree of freedom; (ii) LR: Likelihood Ratio test. Null Hypothesis: $d = 1$ strongly rejected.

obtained d value in Table 7 when $r = 2$ and $p = 5$, which equals to 0.568. The P value of the Ljung–Box Q-test with 12 lags is 0.988 shown in the parenthesis below the test statistic, strongly indicating that the residuals in (6) are stable with no autocorrelation.

Estimated Unrestricted FCVAR model:

$$\Delta^{\hat{d}}\left(\begin{bmatrix} BF1 \\ E \\ L \\ R \end{bmatrix} - \begin{bmatrix} 29.211 \\ -2.776 \\ -0.712 \\ -2.461 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} 0.014 & -0.180 \\ 0.002 & -0.029 \\ 0.001 & -0.007 \\ -0.004 & 0.054 \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} + \sum_{i=1}^{5} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^{i} (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{7}$$

$\hat{d} = 0.567$ ($s.e. = 0.02$), $Q_\epsilon = 150.576$ ($p = 0.988$), $LogL = 9945.007$

Long-run Equilibrium Relationships:

$$BF1_t = -443.871 + 0.23L_t + 192.17R_t + v_{1t} \tag{8}$$

$$E_t = -38.719 + 0.024L_t + 14.598R_t + v_{2t} \tag{9}$$

A single inference was tested on derived model to determine whether the time varying processes prevailing in the proposed variables are best modelled using FCVAR or CVAR methodology where the long memory parameter d is assumed to be 1. The null hypothesis of the test $d = 1$ strongly rejected with a P value of 0.000, results presented in Table 8.

The long-run equilibrium relationships represented by (7) and (8) are built with β normalised by BF1 and coefficient for Global Expansion. They demonstrate how L_t and R_t drive the equilibrium in BF1 and E_t. For example, the first relationship denoted by (7) demonstrates how L_t and R_t drive the equilibrium in BF1 when $v_{1t} = 0$; the second relationship represented by (8) describes how the same factors determine the equilibrium in E_t when $v_{2t} = 0$. The long-run equilibrium relationship in (7) indicates that the increase in L_t positively affects BF1. Specifically, a 1% unit change of the growth of L_t induces a 0.23 unit increase in BF1. On the other hand, the increase in R_t also has a positive impact on BF1 where a 1% unit change of the growth of R_t induces a 192.17 unit increase. The second equilibrium relationship denoted by (8) suggests that the increase in L_t also has a positive impact on E_t, where a 1% unit change of the growth of L_t induces a 0.024 unit increase in E_t; the increase change in R_t has a positive impact on E_t. Specifically, a 1% unit change of the growth of R_t leads to a 14.598 unit increase in its value. In comparison with (7), however, the impacts of $regime$ factor on $Market\ Expansion$ is much smaller in this equilibrium relationship.

The results of stationary equilibrium relationship in (7) and (8) appear to be consistent with the theoretical expectation. Specifically, in oversupplied oil market or in the scenario when the demand for crude is on decline due to economic slowdown, spot becomes less valuable and R_t begins to experience upward pressure as investors shift their interest into contracts with longer maturity (increase in the slope of the futures price curve). Under these conditions holding longer maturity futures becomes more valuable compared to the investment into spot. Therefore, because the spot price is highly correlated with the short part of the futures curve, any downward pressure on the spot price triggers price adjustment along the entire future price curve causing oil market to contract. As oil market oversupply eases or the economic outlook improves, the reverse process begins to happen exerting downward pressure on R_t and upward pressure on spot causing oil market to expand. In other words investment into cash asset becomes more valuable which triggers prices across the price curve to increase and oil market to expand.

(iii) Model 2: WSP, Global Market Expansion, Global Market Liquidity and Global Market Regime

Following parameters estimation, the unrestricted model that describes relationship amongst Global Market Proxies & WTI Spot Price was derived based on $k = 6$ and $r = 2$. The results of general FCVAR are presented in (18) where both $X_t - \mu$ on the left side and α on the right side of the equation are expanded in the matrix form. The column vector v_t represents $\beta' L_d(X_t - \mu)$ in (3) and the highest estimated lag of the short-run dynamic is 6. The estimated long memory parameter d of the model is found to be 0.650 with standard error of 0.024, pointing towards the conclusion of fractional cointegration order. It also confirms with the obtained d value in Table 7 when $r = 2$ and $p = 6$, which is equals to 0.654. The P value of the Ljung–Box Q-test with 12 lags is 0.987 shown in the parenthesis below the test statistic, strongly denoting that the residuals in (18) are stable with no autocorrelation.

Table 9
Hypothesis test on model's long memory parameter, d of Global Proxies & WTI Spot.

Parameter	$\mathcal{H}_d$
df	1
LR statistic	55.924
P value	0

Note: (i) df: degree of freedom; (ii) LR: Likelihood Ratio test. Null Hypothesis: $d = 1$ strongly rejected.

Estimated Unrestricted FCVAR model:

$$\Delta^{\hat{d}}\left(\begin{bmatrix} WSP \\ E \\ L \\ R \end{bmatrix} - \begin{bmatrix} 30.571 \\ -2.819 \\ -0.718 \\ -2.434 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.027 & 0.280 \\ -0.002 & 0.024 \\ -0.002 & 0.017 \\ 0.002 & -0.024 \end{bmatrix} \begin{bmatrix} v_{1t} \\ v_{2t} \end{bmatrix} + \sum_{i=1}^{6} \hat{\Gamma}_i \Delta^{\hat{d}} L^i_{\hat{d}} (X_t - \hat{\mu}) + \hat{\epsilon}_t \tag{10}$$

$\hat{d} = 0.650$ ($s.e. = 0.024$), $Q_\epsilon = 151.152$ ($p = 0.987$), $Log L = 8246.031$
Long-run Equilibrium Relationships:

$$WSP_t = -145.35 + 7.049 L_t + 70.199 R_t + v_{1t} \tag{11}$$

$$E_t = -20.69 + 0.58 L_t + 7.173 R_t + v_{2t} \tag{12}$$

A single inference was tested to determine whether time varying processes prevailing in the proposed variables are best modelled by the FCVAR or CVAR methodology where the long memory parameter d is assumed to be 1. The null hypothesis of the test $d = 1$ strongly rejected with a P value of 0.000, results presented in Table 9.

Similar to BF1 model, the long-run equilibrium relationships represented by (10) and (11) are built with β normalised by WTI Spot and coefficient for Global Expansion, respectively. Here the relationships denote the levels of WTI Spot price and $Market\ Expansion$ in the equilibrium condition achieved through $Market\ Liquidity$ and $Market\ Regime$ when both v_{1t} and v_{2t} are equal to zero. Results show that the increase in the oil market liquidity and the slope of the oil futures price curve positively affects WTI spot price and the oil market expansion. Specifically, a 1% unit change of the growth of L_t induces a 7.049 and 0.58 unit increase in WSP and E_t, respectively. Similarly, a 1% unit change in R_t induces a 70.199 and 7.173 growth in the value of WSP and E_t.

The stationary equilibrium relationships described by (10) and (11) also demonstrate consistency with theoretical expectations and follow a very similar pattern as in the model derived to explain dynamism between proposed market proxies and Brent One Month Futures price. This aspect reinforces the notion that the proposed proxies are able to accurately depict dynamism in the oil market and therefore help to forecast their future values and trends which we demonstrate in the next section.

(iv) Out-of-Sample Forecast Evaluation

This section presents results for out-of-sample forecast[3] evaluation exercise. We demonstrate forecasting power of proposed market proxies and assess reliability of estimated FCVAR models and the effects long-run equilibrium has on the forecast performance. Comparative predictive performance of FCVAR model with respect to conventional CVAR counterpart is also discussed here.

To assess forecast performance of FCVAR and CVAR models, the data is partitioned into two parts, $estimation\ period$ and $test\ period$. $Estimation\ period$ is based on the sample span between 02.01.2003 and 30.11.2020 and $test\ period$ for out-of-sample forecast performance evaluation is set to period between 01.12.2020 and 30.04.2021 totalling to 104 observations. Once FCVAR and CVAR models were estimated, a set of 104 consecutive forecasts with 100 day horizon were generated on each day in the $test\ period$. Forecasting errors were calculated for each forecast horizon as defined in (5). Relative forecasting performance (RFE) of each BF1 and WTI FCVAR and CVAR models were assessed by calculating the percentage change between their RMSFE values as defined below:

$$RFE = 100 \times \left\{ \frac{FCVAR_{RMSFE}}{CVAR_{RMSFE}} - 1 \right\} \tag{13}$$

Here a negative RFE indicates relative forecasting performance superiority of FCVAR model with respect to CVAR model; and a positive value indicates superiority of the CVAR model over FCVAR counterpart. Arbitrary set of ten horizons were considered for forecast evaluation of BF1 and WSP models, namely: 1, 5, 10, 20, 30, 40, 50, 60, 80 and 100-step ahead forecasts which correspond to: 1-day, 1-week, 2-weeks, 1-month, 1.5-month, 2-months, 2.5-months, 3-months, 4-months and 5-months, respectively.

The magnitudes of RMSFE values of BF1 and WSP models reported in section (a) and (b) of the Table 10 which reports goodness of fit between forecasted and actual values, suggest that FCVAR methodology provides superior forecast estimation compared to CVAR approach for all forecast horizons considered in this study. Results also demonstrate that for forecast horizons between T+5 and 40-steps, WTI Spot benefits from FCVAR modelling more compared to that of the price of Brent Future.

Examination of section (a) shows that the accuracy of WTI Spot FCVAR does not deteriorate at the same rate as for One Month Brent Future FCVAR. For example, while RMSFE values for 1-step ahead forecast of BF1 and WSP models are 1.310 and 1.321 respectively, the gap widens for 40-steps ahead forecast where they are 5.281 and 4.671. The increasing spread between the outputs of the two models indicate that using proposed market variables in FCVAR framework are able to replicate WTI Spot dynamics better at shorter horizons than for price of Brent One Month Future. However, for longer forecast horizons between 50-steps and 100-steps ahead FCVAR methodology delivers more consistent forecasts for Brent Future prices than for WTI spot.

Relative forecasting performance presented in section (c) of Table 10 demonstrates the level of improvement FCVAR methodology delivers over its CVAR counterpart. Negative percentage increase with increase in forecast horizon in both models suggests that FCVAR methodology is better approach for replicating long-run equilibrium relationship that exists amongst the variables in the system. In both models, FCVAR clearly yield more accurate forecasts compared to CVAR counterpart with improvement of 59% for BF1 and 49% for WSP models at 100-steps ahead forecast horizon. Results also show that at forecast horizons between 5 and 60-steps ahead, WTI forecasting appears to benefit from FCVAR methodology the most with relative forecast improvement higher thank in BF1 forecasting. This indicates that CVAR methodology performs significantly worse in forecasting WTI spot using proposed proxies than in forecasting Brent One Month Future Price.

Section (d) of Table 10 presents results of the test for equal predictive ability between forecast models under consideration introduced by Giacomini and White (2006). Sign convention of the test statistic allows to establish which model dominates in producing more accurate forecasts. Results show that at shorter forecast horizons for up to $T + 50$ for Brent and $T + 20$ for WTI, we cannot reject the null of equal accuracy between outputs of FCVAR and CVAR models. This result is not surprising as methodology of both models closely related and only differ in the order of integration both methodologies allow. In FCVAR methodology fractional difference allows to better mimic the effect of memory embedded in the variables compared to that of CVAR approach. Therefore, observable difference between outputs of both models should manifest at longer horizons, precisely what is observed

[3] Out-of-sample forecasting is a dynamic form of forecast methodology where posteriors are used are as priors in generating next-step ahead forecast.

Table 10
RMSFE values and relative performance of FCVAR and CVAR models for BF1 and WSP.

Model		Forecast horizon									
		T+1	T+5	T+10	T+20	T+30	T+40	T+50	T+60	T+80	T+100
(a) FCVAR RMSFE magnitudes											
	BF1	1.310	2.376	3.215	4.418	5.246	5.281	5.642	5.372	4.529	5.120
	WSP	1.321	2.363	3.198	4.055	4.599	4.671	5.785	5.776	5.826	8.098
(b) CVAR RMSFE magnitudes											
	BF1	1.330	2.510	3.523	5.124	6.692	8.059	9.578	10.021	10.552	12.612
	WSP	1.336	2.561	3.696	5.571	7.592	9.431	11.430	12.320	13.555	15.935
(c) RFE — FCVAR vs CVAR											
	BF1	−1.50%	−5.33%	−8.73%	−13.78%	−21.61%	−34.47%	−41.09%	−46.39%	−57.08%	−59.41%
WSP		−1.13%	−7.74%	−13.50%	−27.21%	−39.42%	−50.48%	−49.39%	−53.12%	−57.02%	−49.18%
(d) FCVAR vs CVAR predictive equality											
BF1	Test stat	−0.74	−0.58	−0.31	−0.24	−0.45	−0.99	−1.88	−3.39	−7.30	−5.68
	P Value	0.390	0.447	0.580	0.627	0.502	0.320	0.170	0.066	0.007	0.017
WSP	Test stat	−0.44	−1.34	−0.96	−1.54	−3.23	−5.82	−9.37	−18.15	−34.67	−19.08
	P Value	0.509	0.247	0.328	0.215	0.072	0.016	0.002	0.000	0.000	0.000

Note: (i) Model's forecasting performance is measured by the RMSFE values. (ii) Section (a) reports the RMSFE values for the multivariate FCVAR models of BF1 and WSP. (iii) Section (b) reports the RMSFE values for the multivariate CVAR models of BF1 and WSP. (iv) Section (c) reports the relative performance of the FCVAR with respect to CVAR model in terms of RMSFE values of BF1 and WSP; negative values signify FCVAR model superiority and vice versa. (v) Section (d) presents results of unconditional test of equal predictive ability between FCVAR and CVAR models proposed by Giacomini and White (2006). H0: forecasts are equally accurate. Negative values of test statistic indicate that FCVAR dominates CVAR model output.

in the results of the tests for equal predictive ability between models. For example, in the case of One Month Brent Future forecast, the differences between outputs of FCVAR and CVAR models begin to skew towards superiority of FCVAR model from $T+40$ and null gets strongly rejected at $T+80$ horizon. Similar dynamic is observed in the forecasts of WTI spot with only difference that the variation between outputs of FCVAR and CVAR models begin to skew towards superiority of FCVAR model significantly earlier, from $T+10$ and null is strongly rejected from $T+50$ horizon.

We also evaluate forecasting power of estimated models, through symmetry analysis of forecast errors (loss) and the test for forecast rationality. Both objectives are achieved using the test introduced by Elliott et al. (2005). Results in section (a) of Table 11 report symmetry analysis of the forecast errors. Here symmetry parameter $\hat{\alpha}$ as defined in (6), is estimated for each forecast horizon and tested against the null hypothesis: $\hat{\alpha} = 0.5$. Results show that the null of symmetric loss cannot be rejected in the forecasts of FCVAR model for horizon of up-to 10-steps ahead in case of Brent One Month Future price forecast with p-value of 0.626, and up-to 5-steps ahead for WTI spot forecast with p-value of 0.961. The null is rejected for every subsequent forecast horizon where aversion to positive errors occurs in the case of Brent forecasts and negative forecast errors in the case of WTI forecasts using FCVAR methodology.

The CVAR forecasts show that in both oil benchmarks the symmetry in the loss gets skewed towards negative errors. This suggests that CVAR methodology is inadequate in interpreting dynamics that exists between the oil prices under consideration and the market proxies proposed in this study. Logic suggests that this is due to the setup of CVAR model which approximates process in the system of variables as unit root which clearly is not true as seen in the output of the FCVAR model. Also, in the case of WTI spot CVAR model, the test statistics could not be calculated for horizons of 40-steps ahead and above due to the matrix inversion failure. As the test statistic could not be defined for these horizons, this points to instability in forecast outputs for WTI spot using proposed proxies in CVAR methodology framework which implies CVAR methodology should be avoided for forecasting oil price dynamics using explanatory variables suggested in this study.

Section (b) of Table 11 reports results of forecast rationality test. Results show that forecast rationality is achieved for both FCVAR and CVAR models but only for 1-step ahead horizon and only for Brent price where CVAR forecast tested higher for rationality with p-value of 0.185 than for FCVAR with p-value of 0.080. Rationality for WTI spot forecast is rejected for all horizons. The lack of rationality in higher forecast horizons of Brent forecasts and absence of such in all of WTI forecasts could be explained by the forecast methodology adopted in this study. The objective of this research is to evaluate the relationship between the prices of Brent and WTI benchmarks & proposed market proxies and explicitly demonstrate the robustness of the approach. For that reason all forecasts were calculated based on single estimation of FCVAR and CVAR models. With this in mind, the strength of test results of forecast rationality for 1-step ahead horizon in Brent example perfectly demonstrates this power for at least 104 forecast iterations conducted based on a single estimation of both models. Therefore, it is also logical to conclude that the test results of forecasts rationality (as well as other forecast evaluation metrics covered up-to now) would be significantly improved should the models be estimated at each iteration similar to the approach followed in the finance industry.[4]

Graphical representations of relative forecasting performance provided in Figs. 6 and 7 echo the findings presented in Tables 10 and 11 and show that compared to CVAR model, FCVAR methodology is a better approach for replicating price dynamics in both oil benchmarks based on proposed proxies.

Overall findings show that FCVAR methodology is more superior in capturing the long-term price trends. This is due to the model's ability to better characterise the long run equilibrium that exists amongst the variables compared to CVAR technique. We conclude that using proposed proxies in FCVAR framework allows to model the dynamism in the oil market with respect to individual market members. This leads to improvement in forecast accuracy as demonstrated using prices of WTI Spot and One Month Brent Future contract. We also conclude that forecasting performance based on proposed global proxies varies depending on which oil market member is selected for the exercise.

5. Robustness

In this section, we study sensitivity of our baseline results to, for instance, the presence of a possible break point in the data? Two potential issues may bias our inference of FCVAR estimates and its predictions; the estimates may be spurious. Indeed, the presence of such break points may confuse the behaviour of the time series as a long-memory process. Further, we also report density forecasts of the original series and compare the same with FCVAR forecasts.[5]

[4] Due to computation complexity and significant duration it takes to estimate each FCVAR model based on considered span in the data, this approach has not been considered in this study.

[5] Many thanks to an anonymous referee to suggest this additional exercise.

Table 11
Loss symmetry & forecast rationality evaluation of FCVAR & CVAR outputs.

Test criteria		*Forecast horizon*									
		T+1	*T+5*	*T+10*	*T+20*	*T+30*	*T+40*	*T+50*	*T+60*	*T+80*	*T+100*
(a) Test of symmetric loss											
(i) BF1 — FCVAR	$\hat{\alpha}$	0.49	0.477	0.524	0.881	0.912	0.94	0.936	0.933	0.954	0.964
	Test stat	−0.211	−0.47	0.487	11.9	14.7	18.8	18	17.5	21.9	25.1
	P Value	0.833	0.638	0.626	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(ii) BF1 — CVAR	$\hat{\alpha}$	0.409	0.269	0.120	0.037	0.020	0.004	0.002	0.000	0.001	0.000
	Test stat	−1.88	−5.25	−11.8	−24.7	−34.6	−76.2	−130	−426	−153	−4e+10
	P Value	0.060	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(iii) WSP — FCVAR	$\hat{\alpha}$	0.522	0.547	0.392	0.686	0.175	0.166	0.095	0.050	0.007	0.001
	Test stat	0.45	0.961	−2.24	4.05	−8.64	−9.04	−13.9	−20.9	−61.9	−146
	P Value	0.652	0.337	0.025	0.000	0.000	0.000	0.000	0.000	0.000	0.000
(iv) WSP — CVAR	$\hat{\alpha}$	0.441	0.265	0.101	0.0147	0.005	Not defined	Not defined	Not defined	Not defined	Not defined
	Test stat	−1.2	−5.39	−13.4	−40.8	−70.9					
	P Value	0.232	0.000	0.000	0.000	0.000					
(b) Forecast rationality under $\hat{\alpha}$											
(i) BF1 — FCVAR	J-stat	3.07	29.5	30.2	24.7	29.1	31.7	36.1	26.7	4.76	20.8
	P Value	0.080	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.029	0.000
(ii) BF1 — CVAR	J-stat	1.76	18.5	14.1	15.4	14.5	8.63	6.87	1.99	2.9	0.627
	P Value	0.185	0.000	0.000	0.000	0.000	0.003	0.009	0.159	0.089	0.428
(iii) WSP -FCVAR	J-stat	6.11	29.9	32.5	42.9	40.2	37.5	27.5	17.7	6.42	3.89
	P Value	0.014	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.011	0.049
(iv) WSP -CVAR	J-stat	8.49	10.6	12.2	11.9	10.6	Not defined	Not defined	Not defined	Not defined	Not defined
	P Value	0.004	0.001	0.000	0.001	0.001					

Note: (i) Symmetry in the forecast loss and the test of forecast rationality are examined using methodology proposed by Elliott et al. (2005). (ii) Section (a) reports estimated loss coefficient $\hat{\alpha}$ and results of the test for loss symmetry in each forecast horizon. H0: $\hat{\alpha} = 0.5$ [Loss is symmetrical]. (iii) $\hat{\alpha} > 0.5$ indicates aversion to positive forecast error and vice versa. (iv) Section (b) reports test results for forecast rationality under estimated $\hat{\alpha}$ for each model and oil benchmark under consideration. H0: forecast is rational.

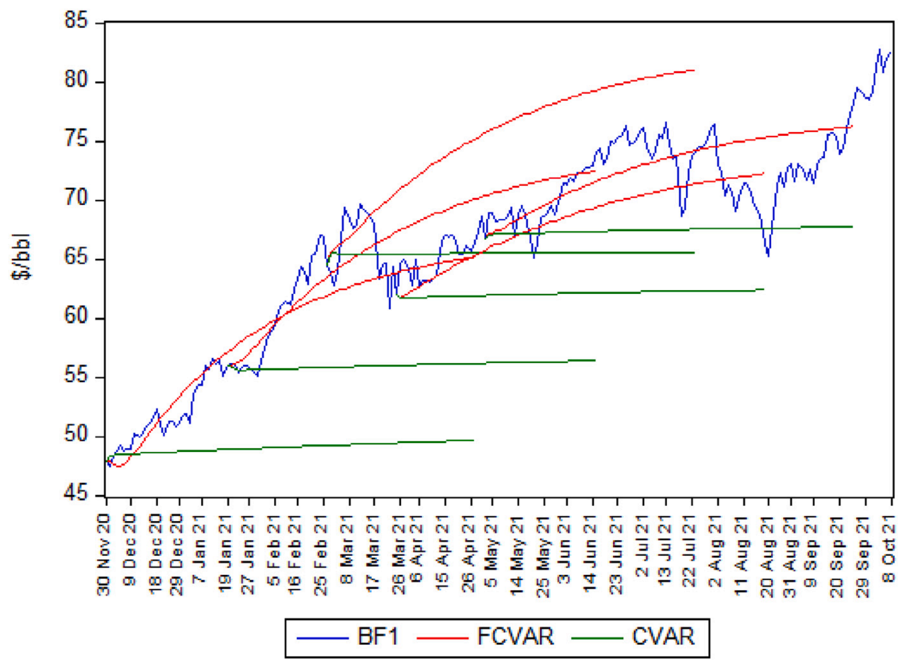

Fig. 6. FCVAR vs. CVAR out-of-sample forecasts: One Month Brent Future price (T+100 horizon).
Note: Graphical representation of the out-of-sample forecast results for 100-step ahead horizon versus actual data for Brent One Month Future Contract price (blue line). Forecast results produced by FCVAR (red line) and CVAR (green line) models estimated based on data sample between 02.01.2003–30.11.2020. Forecast evaluation period between 01.12.2020–30.04.2021.

To test whether our long memory estimates are spurious, we carry out a Break point test (using innovation outlier) for Brent One Month Future and WTI Spot series. We find two structural break points: 02.07.2008 & 20-06-2014 for BF1 and 14.07.2008 & 28.07.2014 for WSP (breakpoint analysis setup is presented in Table B.2). A Chisquare value of 28.35 shows that our null hypothesis of no break point in the data is rejected at 1% significance level. Could the presence of these break points invalidate our estimates of long-memory? To test, therefore, possible spuriousness in estimated values of long-memory, we employ the procedure of Shimotsu and Phillips (2006). This approach exploits certain time domain properties of true I(d) processes and distinguish the same with a spurious I(d) process. Results based on Phillips–Perron (PP), Kwiatkowski–Phillips–Schmidt–Shin (KPSS) and Modified Wald Test for BF1 and WSP series (in Appendix B, Table B.1) clearly indicate that there is no spuriousness in the long-memory estimates in our data.

The second issue concerns whether the FCVAR results would produce different inference on system memory before and after the break

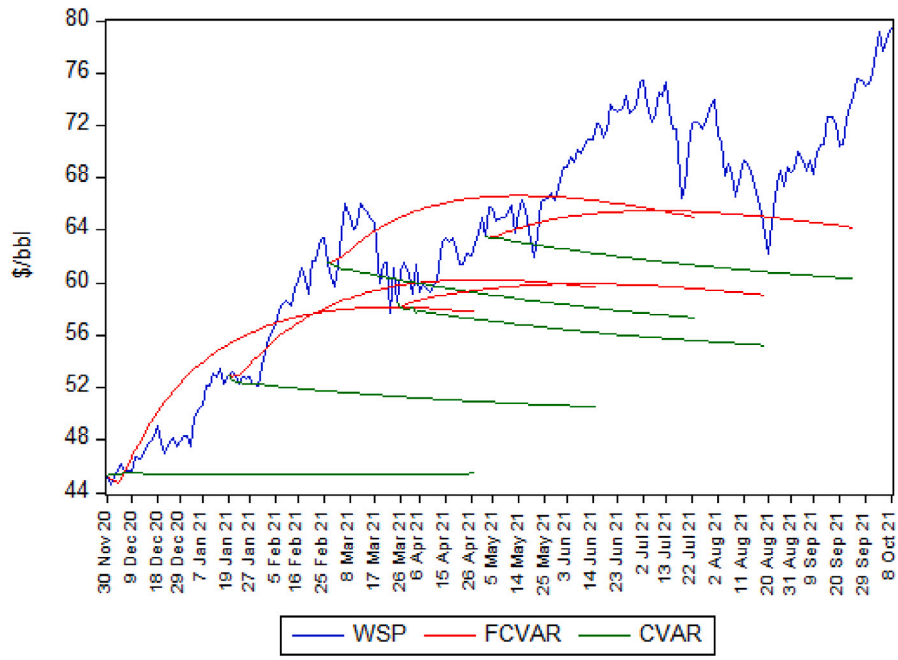

Fig. 7. FCVAR vs. CVAR out-of-sample forecasts: WTI Spot price (T+100 horizon).
Note: Graphical representation of the out-of-sample forecast results for 100-step ahead horizon versus actual data for WTI Spot price (blue line). Forecast results produced by FCVAR (red line) and CVAR (green line) models estimated based on data sample between 02.01.2003–30.11.2020. Forecast evaluation period between 01.12.2020–30.04.2021.

Table 12
BF1 & WSP FCVAR models based on breakpoint intervals.

Model	*Interval selection*		Model parameters						Long-Run equilibrium relationship				
	Span	*LogL* (*P − val*)	*Obs*	*Lag*	*Rank*	*d* (*S.E.*)	*Qϵ* (*P − val*)	*LogL*	*ρ*	X_t	E_t	L_t	R_t
BF1	02.01.03–03.07.08	−7995.97 (0.00)	1361	7	2	0.784 (0.026)	118.09 (1.00)	3,850.22	76.53 0.57	1 0	0 1	1.98 0.42	−20.64 −1.53
	11.12.08–20.06.14	−6050.52 (0.00)	1382	2	2	0.884 (0.020)	138.53 (1.00)	2,948.07	452.70 19.61	1 0	0 1	2968.49 147.08	4046.54 201.43
	13.01.15–06.01.20	–	1242	3	2	0.942 (0.032)	192.16 (0.48)	4,587.19	370.16 21.27	1 0	0 1	142.68 10.14	200.69 14.60
WSP	02.01.03–14.07.08	−9174.99 (0.00)	1368	4	1	0.41 (0.049)	155.96 (0.97)	3,617.01	66.88	1	−11.29	0.18	−1.84
	23.12.08–28.07.14	−5408.57 (0.00)	1399	3	1	0.631 (0.039)	112.27 (1.00)	2,936.08	63.78	1	−9.78	−7.07	−8.89
	28.01.15–06.01.20	–	1232	4	1	0.427 (0.029)	146.57 (0.99)	4,374.42	67.36	1	−11.09	1.89	−1.31

Note: Intervals in Brent and WTI price series identified using Breakpoint Unit Root Test. Series are in first difference. Test parameters: basic trend specification — trend and intercept; breaking — intercept; lag length identification — *t*-statistic with 30 lags and *p*-value of 0.1; break type — innovation outlier; break point selection — Intercept break $min - t$. While first two intervals identified using above methodology, the third interval taken from the last break point identified in the test and up to the last date considered in the analysis. Therefore, on Log Likelihood details reported for this interval. Sample period between 02.01.2003 and 06.01.2020 of daily frequency. Equilibrium relationship parameters: ρ — restricted constant ($\beta'\mu$), X_t — forecastable variable (One Month Brent Future price in BF1 FCVAR model & WTI spot in WSP FCVAR model). Variables E_t, L_t and R_t are the proxies for global oil market expansion, liquidity and regime, respectively.

points in the data. To test this, we have re-estimated FCVAR for three periods in each BF1 and WSP model. The results for individual periods are presented in Table 12. The system-estimated $\hat{d}$ in each period are 0.784, 0.884 and 0.942 for BF1 and 0.410, 0.631 and 0.427 in case of WSP FCVAR models, respectively. These estimates of memory together with the long-run equilibrium relationship in individual periods also confirm that our estimation for the whole sample are robust.

Differences in magnitudes and sign convention of coefficients in the equations describing the long-run equilibrium in FCVAR model based on individual breakpoint intervals compared to the model based on the entire data span, is due to economic conditions prevailing in each interval between the breakpoints. These economic conditions force market expansion, market regime and market liquidity in the oil market to deviate from general convention which applies stress on all agents linked to the oil market forcing the whole system to adjust and bring it back to equilibrium.

Results show that while each breakpoint FCVAR is still able to capture the price trend (Fig. B.1), suggesting degree of explanatory power, it is significantly weaker than what exists in the model based on the whole data span. This aspect is demonstrated using forecast evaluation of Brent and WTI FCVAR models derived based on intervals between breakpoints and models based on the whole sample span, results presented in Appendix B.

We now check for the consistency of our results using density forecasts. Fig. B.2 and Table B.4 (in Appendix B) summarise our results. Essentially, we plot the Kernel density for each forecast (e.g. FCVAR and CVAR) across all time horizons (Fig. B.2). To numerically assess the relative spread of the distributions plotted in Fig. B.2, we have presented mean, standard deviation, skewness and kurtosis (four moments) of each forecast (corresponding to various forecast horizons). Our idea is to study, whether there is a substantial variations in the density plots of the predicted values for each time horizon, which we compare against the density of the original series. As we can clearly see, the FCVAR kernel density plots for each horizon, especially for long horizons, tend to mimick the original spot price kernel density. Comparing the original spot prices kernel density with the CVAR forecasted kernel densities, we can conclude that for both Brent and WTI CVAR forecasts densities shift to the right, which are different from the FCVAR and spot prices densities. The results hold for majority of the time horizons (e.g. 20 to 100 days ahead).

6. Conclusion

This paper examines global oil market dynamism as encoded by proxies for market expansion, liquidity and regime and assesses their contribution to reliable forecast of future oil price. We argue that while it is impossible to account for every factor that influences the price of oil, an alternative mechanism exists that can robustly quantify the arrival of information in the market at any given point of time. We propose that the process of information aggregation in the commodity market causes market to either expand or contract, it affects levels of liquidity in the market and causes market regime to fluctuate between the states of *contango* and *backwardation*. We hold that the approach of quantifying these effects present a comprehensive and robust set of variables to enable out-of-sample forecast of the spot price of oil as well as of other financial instruments whose value is derived or closely linked with the price of oil. We first demonstrate the procedure for deriving the proxies for $Market\ Expansion$, $Market\ Regime$ and $Market\ Liquidity$ from the spot and futures markets of WTI and Brent benchmarks and then examine the memory characteristics of proposed variables.

Using derived proxies to describe oil future markets, we design a memory-driven cointegrated VAR system to capture real world possibility of slow-convergence of shocks to the long-run equilibrium. This possibility of non-linear dissipation of shocks is important to produce realistic predictions. A series of tests for univariate static and dynamic persistence behaviour show that the estimated memory averages slightly above *one* for One Month Brent Future, WTI Spot and proxy for $Market\ Expansion$. The long memory estimates for $Market\ Liquidity$ and $Market\ Regime$s appear to average below *one*, which suggests that the shocks in the market do not affect their memory parameters in the same way as they do d parameters of One Month Brent Future, WTI Spot and $Market\ Expansion$. We conclude that this effect is due to adjustment mechanism that exists amongst the variables.

The rolling window estimation of long memory of each variable (capturing dynamic persistence) gives us interesting results. We find that each window captures different aspects of market dynamics driven by the number and size of the shocks entering the rolling window. Results show that 2 year rolling window captures adjustment mechanism amongst the memory parameters in the system which is driven by the short-term goals of market participants. Results based on 6 year rolling window, on the other hand, appear to suggest the memory estimates of One Month Brent Future, WTI Spot and proxy for $Market\ Expansion$ in particular, are function of geopolitical situation in the world.

To finalise univariate series analysis, we touch upon the concept of *memory* of *memory* or d of d, which reflects memory persistence in the series. We approach this aspect by examining the sensitivity of the long memory of d estimates of the corresponding series to the roll of the sample using two and six year rolling window estimation. Our results show that *memory* of *memory* estimates in both cases depict pattern of non-mean convergence. In other words, the system reveals a tendency to remain unstable, following the intervention of a stochastic shock. The conventional theorisation of long memory model for a growing system holds that the system should be inherently stable, despite stochastic shocks exhibiting a persistence feature for a period of time. However, if one studies further the relative degrees of acceleration or deceleration of shocks (the d of d estimates), this should establish an internal capability of the system to correct itself. In other words, the d of d should depict, on average a tendency to taper-off in the long-run.

Our estimation d of d for various window sizes show varied patterns of the convergence of shocks; while for a specific window size and a defined time frame, the shocks appear to converge, there is a significant evidence for divergence for other periods. An implication is that oil market experiences far greater degree of volatility than other asset markets, due to sensitivity of oil prices to macroeconomic surprises, war, political regime changes and others, so much so that the fluctuations in oil prices happen almost on a daily basis. From theory, we know that for a system to stabilise upon the intervention of a specific shock, requires a condition that either the emergence of unexpected shocks during the period of study disappear very fast or do not appear at all. Shocks in the oil market, which occur frequently, come from sources such as war, differences in political opinion across countries or regions, among others. These shocks, by nature, do not leave the system immediately, rather they have a tendency to persist for some time. In this typical situation, the estimated long-memory of oil prices depicting mean-convergence may show a tendency to diverge with the introduction of a new shock (small or big). These results lend strong conclusion of the nature of inefficiency of the energy market, which we exploit to build a cointegrated system to produce out-of-sample forecasts.

By using proposed proxies in forecasting exercise on One Month Brent Future and WTI Spot prices we show that in both cases the system dynamics are best modelled using FCVAR approach and reject model based on a unit-root cointegration framework. In both models we identified two effect-transmitting cointegration relationships through the variables of *BF1* & *Market Expansion* in Brent model and *WSP* & *Market Expansion* in WTI Model. We quantify their impacts on the dynamic of variables in the systems simultaneously. Our models are able to account for the impact of shocks on fractionally cointegrated variables and bring relationships back into equilibrium, which replicates the adjustment mechanism variables follow in the real life. This aspect allows to make reliable inferences on the future dynamics in the market (represented by market proxies) and therefore, produce reliable out-of-sample forecasts.

We confirm superiority of forecasting power of FCVAR model compared to CVAR framework. This is because of FCVAR's model ability to better capture long-run equilibrium amongst the variables compared to that of CVAR model. Results also show that WTI Spot price forecasting using FCVAR methodology appears to be more accurate particular at longer horizons where spread increases in favour of WTI Spot price accuracy. This suggests that forecasting performance based on proposed global proxies in FCVAR framework varies depending on which oil market member is selected for the exercise. Robustness exercise lead us to conclude that our estimates of memory are non-spurious and we find significant system-wide memory.

CRediT authorship contribution statement

Dmitri Mustanen: Conceptualization, Validation, Formal analysis. **Ahmad Maaitah:** Conceptualization, Methodology, Software, Formal analysis. **Tapas Mishra:** Supervision, Writing – review & editing. **Mamata Parhi:** Methodology, Estimation, Analysis.

Appendix A

(i) Measuring Dynamic Persistence

Dynamic persistence can be gauged by studying how estimated memory of a time series evolves over time (with various windows). When a time series exhibits slow decay in its auto correlation function

(ACF), this indicates the presence of long memory process and indicates persistence behaviour. This also remains true even in the case of weakly stationary time series, *i.e.* as d approaches 1. The impact of the variation of d on the ACF decay can be demonstrated using Newton's Binomial Theorem which allows for $(1-L)^d$ to be expressed as infinite series:

$$\Delta^d = (1-L)^d = \sum_{k=0}^{\infty} \left[\frac{d}{k}\right](-L)^k \tag{B.1}$$

Where,

$$\left[\frac{d}{k}\right] = \frac{d(d-1)(d-2)\cdots(d-k+1)}{k!} \tag{B.2}$$

Hence,

$$(1-L)^d = 1 - dL + L^2 \cdot \frac{d(d-1)}{2!} - L^3 \cdot \frac{d(d-1)(d-2)}{3!} + \cdots \tag{B.3}$$

The above implies that when the value of d is an integer, the infinite series is truncated at the value of d. In ACF terms this can be reflected through rapid exponential decay. When $0 \le d \le 1$, however, the decay is slow and defined by hyperbolic decay in ACF. It turns out that the condition for stationarity is $0 \le d \le 0.5$. In this scenario the system is described as covariance stationary, meaning that the forecasting power of the time series is strong in this case. When the process is described by $0.5 \le d \le 1$ it is referred to as mean revering process where forecasting power of the time series deteriorate and becomes negligible as the value of d approaches 1. In the scenario when $d > 1$ this process is described as explosive, implying that the effect of the shocks in the system is permanent and any forecasting based on this relationship would make little sense.

(ii) Fractional cointegrated vector auto regression model

Model Derivation

In order to derive FCVAR the easiest approach is to consider CVAR model first:

$$\Delta Y_t = \alpha\beta' Y_{t-1} + \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-i} + \varepsilon = \alpha\beta' L^i Y_t + \varepsilon_t. \tag{C.1}$$

Then, to derive the FCVAR model from the above equation we can replace the difference and lag operators Δ and $L = 1 - \Delta$ by their fractional counterparts, Δ_b and $L_b = 1 - \Delta^b$

$$\Delta^b Y_t = \alpha\beta' L_b Y_t + \sum_{i=0}^{k} \Gamma_i \Delta^b L_b^i Y_t + \varepsilon_t, \tag{C.2}$$

Applying it to $Y_t = \Delta^{d-b} X_t$ it becomes:

$$\Delta^d X_t = \alpha\beta' L_b \Delta^{d-b} X_t + \sum_{i=o}^{k} \Gamma_i \Delta^d L_b^i X_t + \varepsilon_t \tag{C.3}$$

Where ε_t is an independent identically distributed (iid) $p - dimensional$ error term with $E(\varepsilon) = 0$ and $var(\varepsilon_t) = \sigma^2$.

Similar to CVAR model, the parameters have the following interpretations, α and β are $(p \times r)$ matrices with $0 \le r \le p$. Coefficients in α represent the rate of adjustment of each element in the system to the equilibria β is a column vector of co-integrating relationships in the system. Stationary combinations or the long run equilibria is represented by the elements of $\beta' X_t$ and short run behaviour is governed by parameter Γ_i.

To account for the bias introduced into the model by the assumption that fractional differences are defined by the infinite series expansion, whereas the actual data sample consists of a finite number of observations, a further modification is required. Johansen and Nielsen proposed to include a level parameter μ that shifts each series by a constant (Johansen and Nielsen, 2016):

$$\Delta^d(X_t - \mu) = \alpha\beta' L_d (X_t - \mu) + \sum_{i=0}^{k} \Gamma_i \Delta^d L_d^i (X_t - \mu) + \varepsilon_t, \qquad t = 1, 2, \dots, T \tag{C.4}$$

Effectively, this addition allows for truncating the fractional differences by terminating the summation in (2) at $k = t - 1$.

The hypothesis testing approach on the model parameters is adopted from the $CVAR$ model (Johansen, 1995) which remains unchanged when carried over to the $FCVAR$ model. Using this approach degrees of freedom is equal to the number of restrictions under null hypothesis. The general hypothesis testing framework on α and β is as follows:

$$\beta = H\varphi \tag{C.5}$$

Where H is a $p \times s$ matrix of restriction coefficients and φ is a $s \times r$ matrix which defines freely varying parameters. Under this scenario the same restriction is imposed on every co-integrating relationship and degrees of freedom is defined as $df = (p - s)r$. When $r > 1$ different restrictions can be imposed on different columns of β such that the degrees of freedom in this case defined as $df = \sum_{i=1}^{r}(p - s - s_i + 1)$.

$$\alpha = A\varphi \tag{C.6}$$

Where A is a $p \times m$ matrix of coefficients and φ is a $m \times r$ matrix of freely varying parameters with $m \ge r$. Degrees of freedom of this test is defined as $df = (p - r)r$.

Forecasting Using FCVAR

Consider the following model:

$$\Delta^d(X_{t+1} - \mu) = X_{t+1} - \mu - (X_{t+1} - \mu) + \Delta^d(X_{t+1} - \mu) = X_{t+1} - \mu - L_d(X_{t+1} - \mu) \tag{C.7}$$

Rearranging for X_{t+1} gives the following relationship:

$$X_{t+1} = \mu + L_d(X_{t+1} - \mu) + \alpha\beta' L_b \Delta^{d-b}(X_{t+1} - \mu) + \sum_{i=o}^{k} \Gamma_i \Delta^d L_b^i (X_{t+1} - \mu) + \varepsilon_{t+1} \tag{C.8}$$

Since $L_b = 1 - \Delta^b$ is a lag operator, so that $L_b^i X_{t+1}$ is known at the time t for $i \ge 1$, the Eq. (10) is the foundation of FCVAR forecasting. Expressing the above relationship based on coefficient estimates using conditional expectation notation for all given information at time t, a one step ahead forecast of $\hat{X}_{t+1|t}$ is defined as:

$$\hat{X}_{t+1|t} = \hat{\mu} + L_{\hat{d}}(X_{t+1} - \hat{\mu}) + \hat{\alpha}\hat{\beta}' L_{\hat{b}} \Delta^{\hat{d}-\hat{b}}(X_{t+1} - \hat{\mu}) + \sum_{i=o}^{k} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{b}}^i (X_{t+1} - \hat{\mu}) \tag{C.9}$$

Since VAR methodology with every forecast iteration allows to generate estimates for every variable in the system, n-step ahead forecast can be formed recursively based on generalised (C.8):

$$\hat{X}_{t+j|t} = \hat{\mu} + L_{\hat{d}}(X_{t+j|t} - \hat{\mu}) + \hat{\alpha}\hat{\beta}' L_{\hat{b}} \Delta^{\hat{d}-\hat{b}}(X_{t+j|t} - \hat{\mu}) + \sum_{i=o}^{k} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{b}}^i (X_{t+j|t} - \hat{\mu}) \tag{C.10}$$

Where $\hat{X}_{s|t} = X_s$ for $s \le t$. In this case the forecasts are calculated recursively from (C.9) for $j = 1, 2, \dots, h$ to generate h-step ahead forecast, $\hat{X}_{t+j|t}$.

Appendix B

B.1. Spurious memory analysis of timeseries for one month brent future and WTI spot prices

See Table B.1.

Table B.1
Spurious memory analysis of one month brent future and WTI spot price series.

Series	ω	m	$\hat{d}$							Average $\hat{d}$			PP crit			KPSS crit			Test Stat		Wald stat	
	Interval		1	2		4				1	2	4	1	2	4	1	2	4	PP	KPSS	2	4
BF1	0.50	67	1.13	1.31	1.08	0.85	1.62	1.26	1.20	1.13	1.20	1.23	−2.56	−2.85	−3.43	0.35	0.46	0.73	−3.39	0.04	2.60	11.20
	0.55	102	1.18	1.22	1.16	0.86	1.39	1.21	1.15	1.18	1.19	1.15	−2.56	−2.85	−3.43	0.35	0.46	0.73	−4.72	0.04	0.27	9.60
	0.60	156	1.09	1.08	1.13	0.92	1.10	1.23	1.06	1.09	1.10	1.08	−2.56	−2.85	−3.42	0.35	0.46	0.74	−3.01	0.06	0.25	5.50
	0.65	239	1.08	1.11	1.08	1.01	1.15	1.08	1.07	1.08	1.09	1.08	−2.56	−2.85	−3.42	0.35	0.46	0.74	−2.87	0.08	0.24	1.82
WSP	0.50	67	1.11	1.29	1.07	0.86	1.58	1.31	1.17	1.11	1.18	1.23	−2.56	−2.85	−3.43	0.35	0.46	0.73	−4.39	0.04	2.39	10.13
	0.55	102	1.14	1.18	1.12	0.85	1.35	1.25	1.06	1.14	1.15	1.13	−2.56	−2.85	−3.43	0.35	0.46	0.73	−5.92	0.04	0.26	9.62
	0.60	156	1.07	1.05	1.12	0.91	1.08	1.22	1.07	1.07	1.09	1.07	−2.56	−2.85	−3.42	0.35	0.46	0.74	−3.98	0.05	0.54	5.59
	0.65	239	1.06	1.07	1.08	0.94	1.12	1.13	1.05	1.06	1.07	1.06	−2.56	−2.85	−3.42	0.35	0.46	0.74	−3.75	0.07	0.00	4.13

Note: Spurious memory test performed by splitting series into sections of two and four and comparing d estimates across different bandwidths (ω) for the considered intervals against the d estimate based on the entire length of the series. Parameter (m) is the number of the periodogram ordinates used in the objective function. Results based on PP, KPSS and Modified Wald Test show that series does not suffer from spurious long memory. PP — Phillips–Perron test — Null: Time series follow I(d) process. $KPSS$ — Kwiatkowski–Phillips–Schmidt–Shin test — Null: Series trend stationary. W_c — Modified Wald Test — Null: d is consistent (critical value: 7.815). Test is based on the sample span between 02.01.2003 and 30.04.2021 of daily frequency.

Table B.2
BF1 & WSP forecast evaluation setup.

FCVAR model	Breakpoint period	Breakpoint interval [Dates]		Breakpoint interval [Obs]	Estimation interval [Dates]		Estimation interval [Obs]	Forecast interval [Dates]		Forecasting interval [Obs]
BF1	Period 1	02.01.03	02.07.08	1–1361	02.01.03	22.04.08	1–1311	23.04.08	12.09.08	1312-1411
	Period 2	11.12.08	20.06.14	1–1382	11.12.08	09.04.14	1–1332	10.04.14	02.09.14	1333-1432
	Period 3	13.01.15	06.01.20	1–1242	13.01.15	22.10.19	1–1192	23.10.19	18.03.20	1193-1292
WSP	Period 1	02.01.03	14.07.08	1–1368	02.01.03	01.05.08	1–1318	02.05.08	23.09.08	1319-1418
	Period 2	23.12.08	28.07.14	1–1399	23.12.08	15.05.14	1–1349	16.05.14	07.10.14	1350-1449
	Period 3	28.01.15	06.01.20	1–1232	28.01.15	22.10.19	1–1182	23.10.19	18.03.20	1183-1282

Note: Intervals in Brent and WTI price series identified using Breakpoint Unit Root Test. Series are in first difference. Test parameters: basic trend specification — trend and intercept; breaking — intercept; lag length identification — t-statistic with 30 lags and p-value of 0.1; break type — innovation outlier; break point selection — Intercept break $min-t$. First two intervals identified using above methodology, the third interval taken from the last break point identified in the test and up to the last date considered in the analysis. Therefore, on Log Likelihood details reported for this interval. Sample period between 02.01.2003 and 06.01.2020 of daily frequency.

Table B.3
Relative forecast performance of BF1 & WTI FCVAR models estimated based on full sample span vs. FCVAR models based on intervals between breakpoints.

(A) Brent FCVAR comparison	*Forecast horizon*									
	T+1	T+5	T+10	T+20	T+30	T+40	T+50	T+60	T+80	T+100
(a) RMSFE magnitudes										
Full Span FCVAR	1.310	2.376	3.215	4.418	5.246	5.281	5.642	5.372	4.529	5.120
Breakpoint Period 1 FCVAR	3.034	6.347	8.986	14.060	21.962	30.035	38.326	47.047	63.223	75.759
Breakpoint Period 2 FCVAR	2.596	3.546	4.527	6.095	7.767	9.810	11.061	12.537	19.930	29.222
Breakpoint Period 3 FCVAR	1.861	5.029	8.396	13.046	17.053	20.759	23.189	24.758	26.551	25.681
(b) DRMSFE — Full Span FCVAR vs:										
Breakpoint 1 FCVAR	−56.8%	−62.6%	−64.2%	−68.6%	−76.1%	−82.4%	−85.3%	−88.6%	−92.8%	−93.2%
Breakpoint 2 FCVAR	−49.5%	−33.0%	−29.0%	−27.5%	−32.5%	−46.2%	−49.0%	−57.1%	−77.3%	−82.5%
Breakpoint 3 FCVAR	−29.6%	−52.7%	−61.7%	−66.1%	−69.2%	−74.6%	−75.7%	−78.3%	−82.9%	−80.1%

(B) WTI FCVAR comparison	*Forecast Horizon*									
	T+1	T+5	T+10	T+20	T+30	T+40	T+50	T+60	T+80	T+100
(a) RMSFE Magnitudes										
Full Span FCVAR	1.321	2.363	3.198	4.055	4.599	4.671	5.785	5.776	5.826	8.098
Breakpoint Period 1 FCVAR	3.887	7.326	9.974	15.872	22.797	29.101	34.817	41.909	55.738	65.682
Breakpoint Period 2 FCVAR	3.829	4.661	5.962	7.385	8.236	9.522	11.947	15.269	22.397	26.456
Breakpoint Period 3 FCVAR	1.771	4.642	8.476	14.084	20.751	24.411	26.854	28.748	31.054	30.858
(b) DRMSFE — Full Span FCVAR vs:										
Breakpoint 1 FCVAR	−66.0%	−67.7%	−67.9%	−74.5%	−79.8%	−84.0%	−83.4%	−86.2%	−89.5%	−87.7%
Breakpoint 2 FCVAR	−65.5%	−49.3%	−46.4%	−45.1%	−44.2%	−51.0%	−51.6%	−62.2%	−74.0%	−69.4%
Breakpoint 3 FCVAR	−25.4%	−49.1%	−62.3%	−71.2%	−77.8%	−80.9%	−78.5%	−79.9%	−81.2%	−73.8%

Note: (i) Forecasting performance of derived models measured by the RMSFE values. (ii) Subsections (a) reports the RMSFE values for multivariate BF1 FCVAR models derived based on individual breakpoint intervals and one derived based on the entire span of the data. (iii) Subsections (b) reports relative forecast performance in terms of DRMSFE values of FCVAR model derived based on individual breakpoint intervals with respect to the model based on the entire span of the data; negative values signify full span FCVAR model's superiority and vice versa.

Table B.4
Characteristics of momentum: WSP and BSP under both CVAR & FCVAR models.

Original, FCVAR, CVAR												
BSP	Mean			St. Dev.			Skewness			Kurtosis		
BSP,BF1&BC1	59.84	59.90	59.70	6.57	6.65	6.71	−0.32	−0.31	−0.31	1.71	1.72	1.72
BSP,BF5&BC5	60.62	60.80	59.75	6.36	6.62	6.72	−0.39	−0.33	−0.30	1.77	1.82	1.72
BSP,BF10&BC10	61.53	62.39	59.80	6.03	6.31	6.71	−0.49	−0.29	−0.30	1.88	1.88	1.72
BSP,BF20&BC20	63.17	65.31	59.90	5.22	5.79	6.69	−0.72	−0.19	−0.30	2.30	1.99	1.72
BSP,BF30&BC30	64.93	67.64	59.99	4.72	5.46	6.66	−0.60	−0.09	−0.30	2.58	2.09	1.72
BSP,BF40&BC40	66.73	69.52	60.07	4.49	5.24	6.64	−0.16	0.00	−0.30	2.98	2.18	1.71
BSP,BF50&BC50	68.34	70.67	60.17	4.16	5.11	6.57	0.35	0.11	−0.28	2.09	2.18	1.68
BSP,BF60&BC60	69.17	71.87	60.24	4.14	5.00	6.54	0.08	0.17	−0.28	1.89	2.24	1.68
BSP,BF80&BC80	70.11	73.62	60.36	3.84	4.85	6.49	−0.12	0.24	−0.28	1.95	2.32	1.68
BSP,BF100&BC100	71.77	74.71	60.46	2.98	4.76	6.43	−0.27	0.28	−0.28	2.06	2.37	1.68
WSP	Mean			St. Dev.			Skewness			Kurtosis		
WSP,WF1&WC1	56.54	56.55	56.34	6.47	6.49	6.53	−0.34	−0.29	−0.29	1.71	1.71	1.69
WSP,WF5&WC5	57.30	57.33	56.22	6.28	6.16	6.46	−0.42	−0.24	−0.27	1.80	1.74	1.69
WSP,WF10&WC10	58.22	58.66	56.05	5.95	5.60	6.35	−0.52	−0.12	−0.26	1.93	1.77	1.69
WSP,WF20&WC20	59.89	60.62	55.72	5.21	4.85	6.14	−0.72	0.10	−0.26	2.42	1.88	1.68
WSP,WF30&WC30	61.78	61.87	55.43	4.80	4.39	5.96	−0.38	0.27	−0.25	2.61	2.03	1.68
WSP,WF40&WC40	63.70	62.63	55.16	4.81	4.06	5.80	0.17	0.40	−0.24	2.86	2.15	1.68
WSP,WF50&WC50	65.46	62.66	54.92	4.86	3.92	5.65	0.53	0.45	−0.24	2.14	2.20	1.68
WSP,WF60&WC60	66.43	62.82	54.70	4.86	3.71	5.51	0.23	0.50	−0.23	1.83	2.27	1.67
WSP,WF80&WC80	67.53	62.61	54.32	4.57	3.37	5.28	−0.07	0.54	−0.22	1.86	2.36	1.67
WSP,WF100&WC100	69.24	61.97	54.01	3.40	3.10	5.08	−0.18	0.53	−0.21	2.04	2.37	1.60

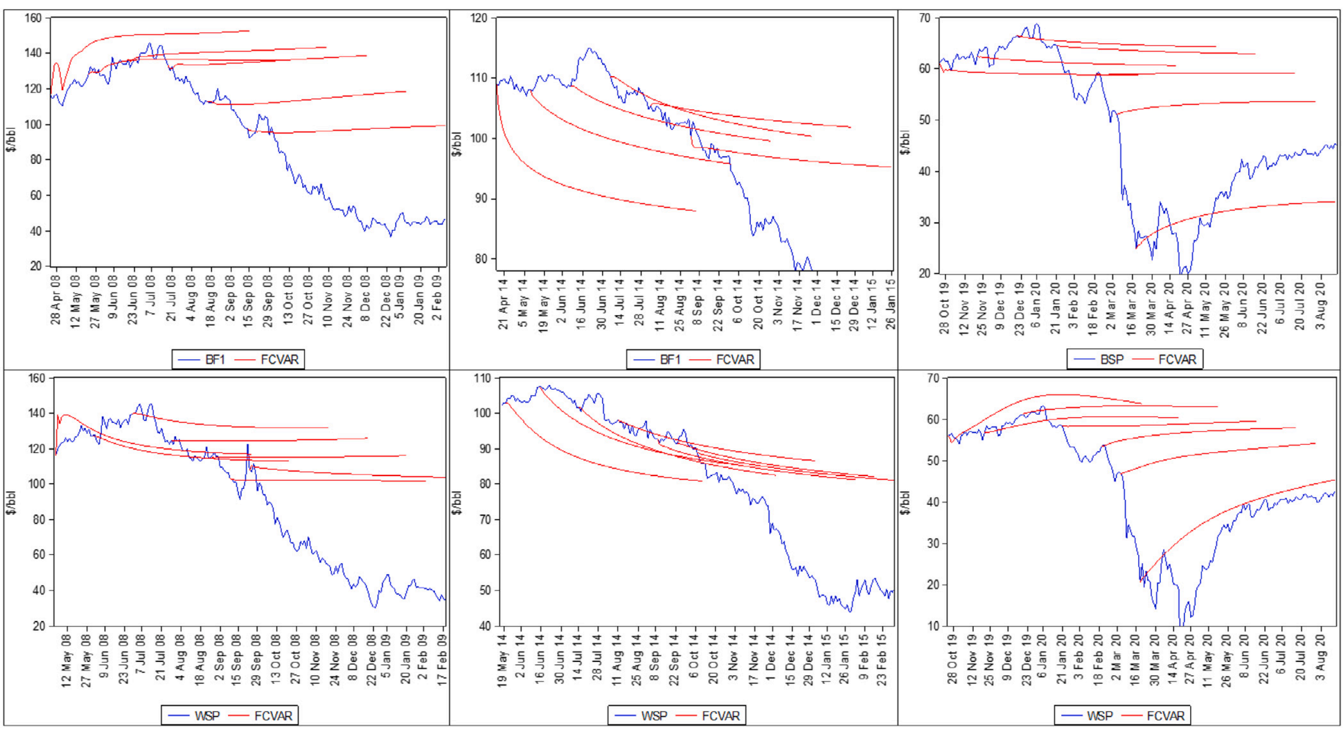

Fig. B.1. Out-of-Sample FCVAR forecast in break-point intervals with T+100 horizon.
Note: Graphical representation of the out-of-sample forecast results for 100-step ahead horizon versus actual data for Brent One Month Future price (top three graphs) and WTI Spot (bottom three graphs). FCVAR forecast (red lines) show performance of the models estimated based on break-point intervals as defined in Table D/A.

B.2. Forecasting evaluation of FCVAR models estimated based on break-point intervals vs. model based on the entire data span

Test Setup

The data for forecast evaluation robustness was partitioned and arranged in the format outlined in Table D/A below. All models were re-estimated based on individual breakpoint interval minus the last 50 observations which were included into forecast validation set along with 50 observations that followed directly after each breakpoint. This was done to emphasise FCVAR methodology robustness in context of proposed market proxies. Relative forecast evaluation was then performed with respect to FCVAR model derived based on the whole data sample using DRMSFE calculations as outlined in (13) (See Table B.2).

Results

Forecast performance of Brent and WTI FCVAR models estimated based on breakpoint intervals are presented in Fig. B.1 and summarised in Table B.3. The table reports RMSFE values of each FCVAR model and presents analysis of relative forecast evaluation performed against FCVAR models that use the entire span of the data.

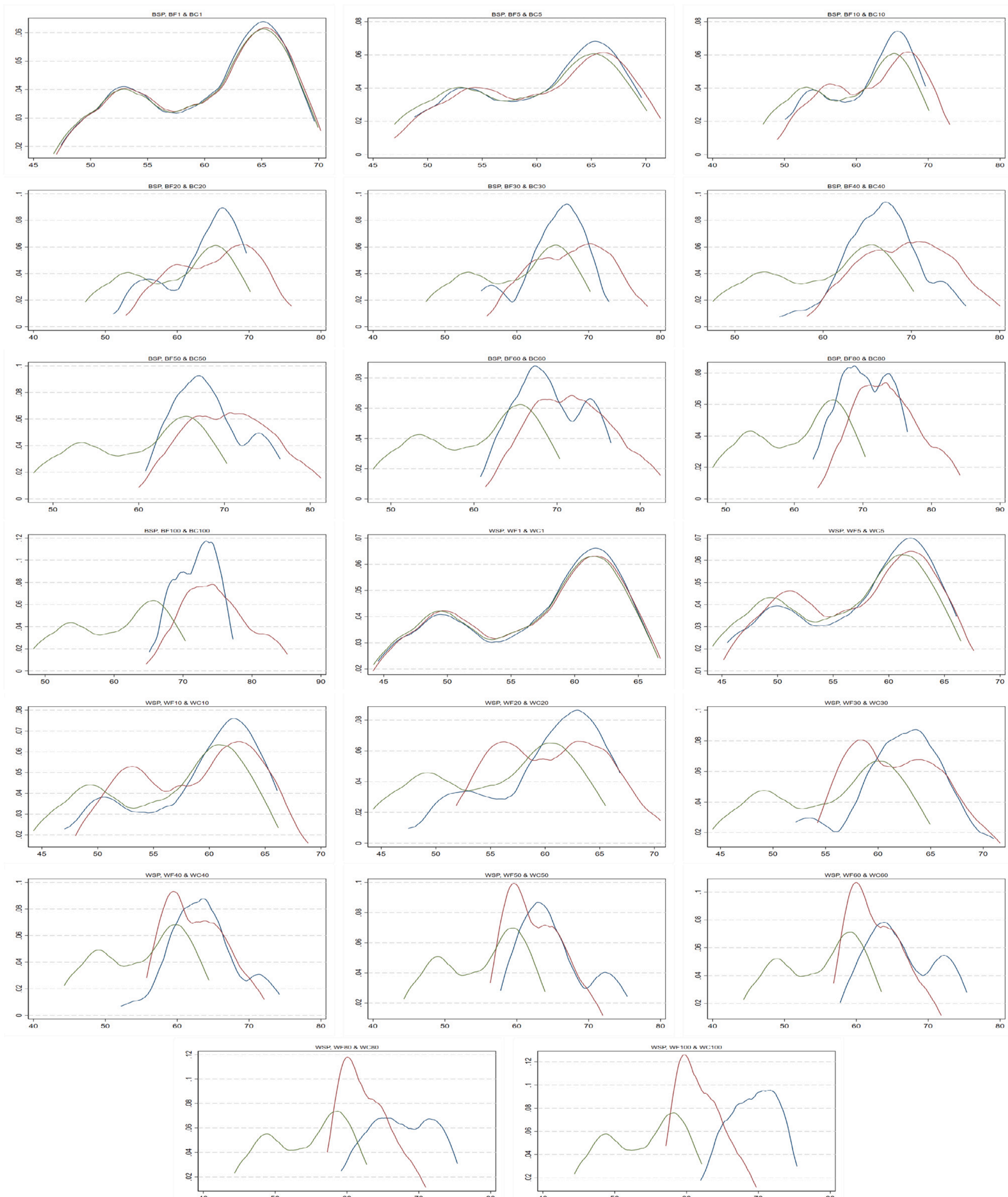

Fig. B.2. Kernel Density for Original and Forecast-ed BSP & WSP in Different Horizons. Note: BSP blue, BF1 red, BC1 green. Y is the kernel density.

Appendix C. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.eneco.2022.106273.

References

Ahmad, W., Prakash, R., Uddin, G.S., Chahal, R.J.K., Rahman, M.L., Dutta, A., 2020. On the intraday dynamics of oil price and exchange rate: What can we learn from China and India?. Energy Econ. 91, 104871.

Baruník, J., Malinska, B., 2016. Forecasting the term structure of crude oil futures prices with neural networks. Appl. Energy 164, 366–379.

Bashiri Behmiri, N., Manso, J., 2013. Crude oil price forecasting techniques: A comprehensive review of literature. SSRN Electron. J. http://dx.doi.org/10.2139/ssrn.2275428.

Bekiros, S., Arreola Hernandez, J., Salah Uddin, G., Muzaffar, A.T., 2020. On the predictability of crude oil market: A hybrid multiscale wavelet approach. J. Forecast. 39 (4), 599–614.

Boako, G., Alagidede, I.P., Sjo, B., Uddin, G.S., 2020. Commodities price cycles and their interdependence with equity markets. Energy Econ. 91, 104884.

Boldanov, R., Degiannakis, S., Filis, G., 2016. Time-varying correlation between oil and stock market volatilities: Evidence from oil-importing and oil-exporting countries. Int. Rev. Financ. Anal. 48, 209–220.

Bredin, D., O'Sullivan, C., Spencer, S., 2021. Forecasting WTI crude oil futures returns: Does the term structure help? Energy Econ. 105350.

Chiroma, H., Abdul-kareem, S., Shukri Mohd Noor, A., Abubakar, A.I., Sohrabi Safa, N., Shuib, L., Fatihu Hamza, M., Ya'u Gital, A., Herawan, T., 2016. A review on artificial intelligence methodologies for the forecasting of crude oil price. Intell. Autom. Soft Comput. 22 (3), 449–462.

Chiroma, H., Abdulkareem, S., Herawan, T., 2015. Evolutionary neural network model for west texas intermediate crude oil price prediction. Appl. Energy 142, 266–273.

Coppola, A., 2008. Forecasting oil price movements: Exploiting the information in the futures market. J. Futures Mark. 28 (1), 34–56.

Dickey, D., Fuller, W., 1979. Distribution of the estimators for autoregressive time series with a unit root. J. Amer. Statist. Assoc. 74 (366), 427–431.

Dolatabadi, S., Nielsen, M.Ø., Xu, K., 2015. A fractionally cointegrated VAR analysis of price discovery in commodity futures markets. J. Futures Mark. 35 (4), 339–356.

Elliott, G., Timmermann, A., Komunjer, I., 2005. Estimation and testing of forecast rationality under flexible loss. Rev. Econom. Stud. 72 (4), 1107–1125.

Gabralla, L.A., Abraham, A., 2013. Computational modeling of crude oil price forecasting: A review of two decades of research. Int. J. Comput. Inf. Syst. Ind. Manage. Appl. 5, 729–740.

Giacomini, R., White, H., 2006. Tests of conditional predictive ability. Econometrica 74 (6), 1545–1578.

Gonzalo, J., Pitarakis, J.-Y., 1999. Dimensionality effect in cointegration analysis. In: Cointegration, Causality, and Forecasting. A Festschrift in Honour of Clive WJ Granger. Oxford University Press, Oxford, pp. 212–229.

Gupta, R., Wohar, M., 2017. Forecasting oil and stock returns with a qual VAR using over 150 years off data. Energy Econ. 62, 181–186.

Johansen, S., 1995. Likelihood-Based Inference in Cointegrated Vector Autoregressive Models. Oxford University Press.

Johansen, S., 2008. A representation theory for a class of vector autoregressive models for fractional processes. Econom. Theory 24 (3), 651–676.

Johansen, S., Nielsen, M.Ø., 2012. Likelihood inference for a fractionally cointegrated vector autoregressive model. Econometrica 80 (6), 2667–2732.

Johansen, S., Nielsen, M.Ø., 2016. The role of initial values in conditional sum-of-squares estimation of nonstationary fractional time series models. Econom. Theory 32 (5), 1095.

Morana, C., 2013. Oil price dynamics, macro-finance interactions and the role of financial speculation. J. Bank. Financ. 37 (1), 206–226.

Nandha, M., Faff, R., 2008. Does oil move equity prices? A global view. Energy Econ. 30 (3), 986–997.

Nelson, C.R., Siegel, A.F., 1987. Parsimonious modeling of yield curves. J. Bus. 473–489.

Nielsen, M., Shibaev, S., 2018. Forecasting daily political opinion polls using the fractionally cointegrated vector auto-regressive model. J. R. Stat. Soc. Ser. A.

Popiel, M.K., et al., 2018. A Matlab Program And User's Guide For The Fractionally Cointegrated VAR Model. Tech. Rep..

Ratti, R.A., Vespignani, J.L., 2016. Oil prices and global factor macroeconomic variables. Energy Econ. 59, 198–212.

Roache, S.K., Reichsfeld, D.A., 2011. Do Commodity Futures Help Forecast Spot Prices?. IMF Working Papers, pp. 1–25.

Rubaszek, M., Karolak, Z., Kwas, M., Uddin, G.S., 2020. The role of the threshold effect for the dynamics of futures and spot prices of energy commodities. Stud. Nonlinear Dyn. Econom. 24 (5).

Shimotsu, K., 2010. Exact local Whittle estimation of fractional integration with unknown mean and time trend. Econom. Theory 26 (2), 501–540.

Shimotsu, K., Phillips, P.C., 2006. Local Whittle estimation of fractional integration and some of its variants. J. Econometrics 130 (2), 209–233.

Silvennoinen, A., Thorp, S., 2013. Financialization, crisis and commodity correlation dynamics. J. Int. Financial Mark. Instit. Money 24, 42–65.

Uddin, G.S., Gençay, R., Bekiros, S., Sahamkhadam, M., 2019. Enhancing the predictability of crude oil markets with hybrid wavelet approaches. Econom. Lett. 182, 50–54.

Wang, Y., Pan, Z., Liu, L., Wu, C., 2019. Oil price increases and the predictability of equity premium. J. Bank. Financ. 102, 43–58.