Orthogonal Time Frequency Space (OTFS) for Reconfigurable Intelligent Surface (RIS)
(Including Fading Characteristics and Waveforms)

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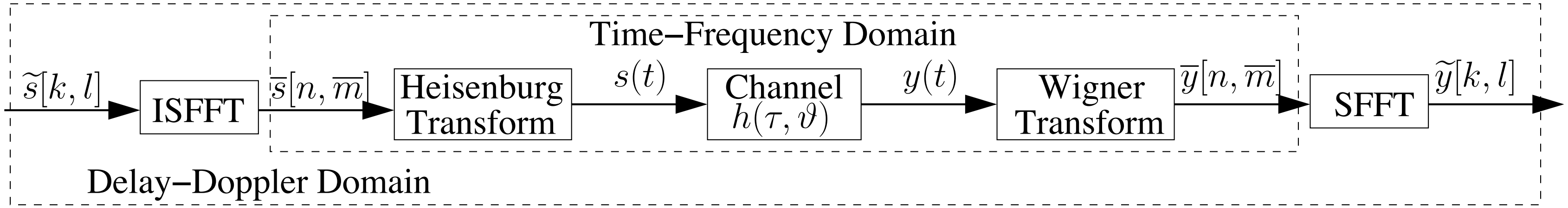


- From the generic received signal model:

$$y(t) = \int\int \widetilde{h}(\tau,\vartheta)s(t-\tau)e^{j2\pi\vartheta(t-\tau)}d\tau d\vartheta + v(t)|_{t=\frac{nT}{M}=\frac{n}{M\Delta f}}$$

where $\quad \widetilde{h}(\tau,\vartheta) = \sum_{p=0}^{P-1} \widetilde{h}_p \delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p)|_{\tau_p=\frac{l_p}{M\Delta f},\vartheta_p=\frac{k_p}{NT}}$

- Orthogonal Time Frequency Space (OTFS) modulation is specifically designed for doubly selective fading, where P propagation paths are resolvable and time-invariant in the delay-Doppler (DD) domain:

$$\widetilde{y}[k,l] = \sum_{p=0}^{P-1} \widetilde{h}_p e^{\frac{-j2\pi l_p k_p}{MN}} \widetilde{s}[< k-k_p >_N, < l-l_p >_M]$$

- Issues to consider first: (1) Relationship between frequency/time selectivity and waveforms; (2) Relationship between determinstic fading model and stochastic Ricean/Rayleigh fading models; (3) Channel estimation; (4) Differential encoding and non-coherent detection; (5) Reconfigurable intelligent surface (RIS) applications;

- From the generic received signal model:

$$y(t) = \int\int \widetilde{h}(\tau, \vartheta) s(t - \tau) e^{j2\pi\vartheta(t-\tau)} d\tau d\vartheta + v(t)|_{t=nT=\frac{n}{\Delta f}}$$

where $\widetilde{h}(\tau, \vartheta) = \sum_{p=0}^{P-1} \widetilde{h}_p \delta(\tau - \tau_p)\delta(\vartheta - \vartheta_p)|_{\tau_p=0}$

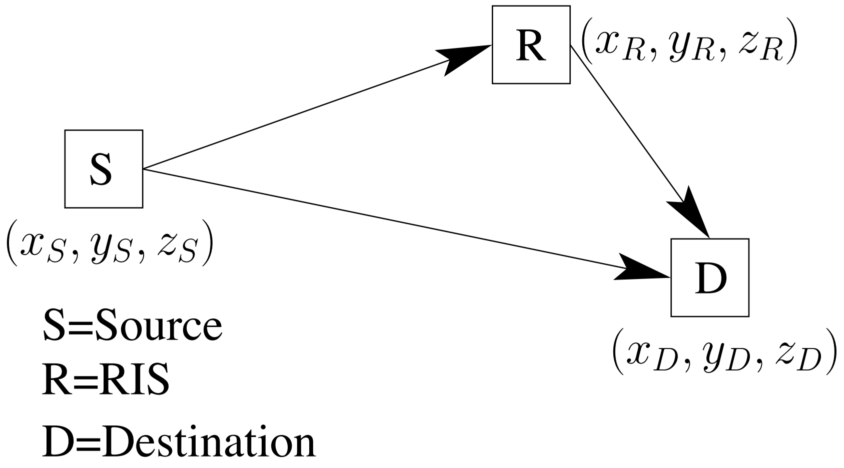
  - P propagation paths with delays $\{\tau_p\}_{p=0}^{P-1}$ and Doppler shifts $\{\vartheta_p\}_{p=0}^{P-1}$.
  - T and Δf are symbol period and signal bandwidth, respectively.

- Frequency non-selective (flat): $\{\tau_p = 0\}_{p=0}^{P-1}$
  - All paths arrive within one symbol period T.
  - This requires signal bandwidth to be smaller than coherent bandwidth $\Delta f < B_c$.

- Time invariant (slow): $\{\vartheta_p << \Delta f\}_{p=0}^{P-1}$
  - This implies that $\widetilde{h}_p e^{j2\pi\vartheta_p nT} = \widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{\Delta f}}$ remains near-constant over a symbol period of T.
  - This requires signal period to be smaller than coherent time $T < T_c$.

- Block fading: $\{N_f\vartheta_p << \Delta f\}_{p=0}^{P-1}$ so that $\{\widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{\Delta f}}\}_{n=0}^{N_f-1}$ remains near-constant over a frame duration $N_f T$.

- The discrete-time received signal model: $y_n = \left(\sum_{p=0}^{P-1} \widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{\Delta f}}\right) s_n + v_n = h_n s_n + v_n$
- SAGIN Ricean fading: $h_n = h_n^{\text{LoS}} + h_n^{\text{NLoS}}$
  - LoS associated with $(p=0)$: $h_n^{\text{LoS}} = \sqrt{\frac{K}{K+1}} e^{\frac{j2\pi\vartheta^{\text{LoS}} n}{\Delta f}}$, where $\widetilde{h}_0 = \sqrt{\frac{K}{K+1}}$ and $\vartheta_0 = \vartheta^{\text{LoS}} = f_D \cos(\theta_0)$. The maximum Doppler frequency is $f_D = \frac{v f_c}{c}$.
  - NLoS associated with $(p \neq 0)$: $h_n^{\text{NLoS}} = \left(\sum_{p=1}^{P-1} \widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{\Delta f}}\right) \sim \mathcal{CN}(0, \frac{1}{K+1})$ for large P. The correlation based on Jakes model is $E\left[h_n^{\text{NLoS}}(h_{n+\Delta n}^{\text{NLoS}})^*\right] = \frac{1}{K+1} J_0\left(\frac{2\pi f_D \Delta n}{\Delta f}\right)$.
- Channel estimation: pilots and MMSE interpolation based on the known LoS h_n^{LoS} and NLoS correlation.
- Differential encoding and non-coherent detection:
  - Differential encoding: $s_n = s_{n-1} x_{n-1}$
  - Received signal: $y_n = h_n s_{n-1} x_{n-1} + v_n \approx (y_{n-1} - v_{n-1}) x_{n-1} + v_n$ when $h_n \approx h_{n-1}$.
  - More robust non-coherent detectors operate based on the known LoS and NLoS correlation.



- The source-destination (SD) link: h_n^{SD}.
- The source-RIS (SR) link: $h_n^{\text{SR}_r}$, $r = 1, \cdots, R$.
- The RIS-destination (RD) link: $h_n^{\text{RD}_r}$, $r = 1, \cdots, R$.
- The RIS phase rotations: α_n^r, $r = 1, \cdots, R$.

- The received signal at the destination node:

$$y_n = \left(h_n^{\text{SD}} + \sum_{r=1}^{R} \alpha_n^r h_n^{\text{SR}_r} h_n^{\text{RD}_r} \right) s_n + v_n$$

- The receive SNR is maximized when $\angle \alpha_n^r = \angle h_n^{\text{SD}} - \angle h_n^{\text{SRD}_r}$, where $h_n^{\text{SRD}_r} = h_n^{\text{SR}_r} h_n^{\text{RD}_r}$. This is equivalent to $\alpha_n^r = \frac{h_n^{\text{SD}} (h_n^{\text{SRD}_r})^*}{|h_n^{\text{SD}} h_n^{\text{SRD}_r}|}$. In this way, the received signal becomes:

$$y_n = h_n^{\text{SD}} \left(1 + \frac{\sum_{r=1}^{R} |h_n^{\text{SRD}_r}|}{|h_n^{\text{SD}}|} \right) s_n + v_n$$

- Summary on suitabilities: (1) 5G uplink; (2) Low range so that $\tau_{\text{max}} < T$; (3) Low rate so that $\Delta f < B_c$; (4) Suitable for high-mobility with robust non-coherent detection; (5) Suitable for energy-efficient techniques with low PAPR; (6) Suitable for single-/reduced-RF MIMO including SM and STSK.

- OFDM parameters: (1) Subcarrier spacing (SCS): Δf; (2) Total bandwidth: $N\Delta f$; (3) OFDM duration: $T = \frac{1}{\Delta f}$; (4) Sampling period: $\frac{1}{N\Delta f} = \frac{T}{N}$.
- From the generic received signal model:

$$y(t) = \int\int \widetilde{h}(\tau,\vartheta)s(t-\tau)e^{j2\pi\vartheta(t-\tau)}d\tau d\vartheta + v(t)|_{t=\frac{nT}{N}=\frac{n}{N\Delta f}}$$

where $\widetilde{h}(\tau,\vartheta) = \sum_{p=0}^{P-1}\widetilde{h}_p\delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p)|_{\tau_p=\frac{lT}{N}=\frac{l}{N\Delta f}}$

- Frequency selective: $\frac{T}{N} < \tau_{\text{max}} < T$
  - The P paths fall into L resolvable TDL $\tau_p = \frac{lT}{N} = \frac{l}{N\Delta f}$, where $l = 0, \cdots, L-1$.
  - The total bandwidth may exceed coherent bandwidth $N\Delta f > B_c$, but each sub-channel is flat $\Delta f < B_c$.
  - Frequency selectivity imposes inter-symbol interference (ISI) in the time-domain (TD), hence frequency-domain (FD) signal processing.
- Time invariant (slow): $\{\vartheta_p << \Delta f\}_{p=0}^{P-1}$
  - This implies that $\{\widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{N\Delta f}} \approx \widetilde{h}_p e^{\frac{j2\pi\vartheta_p}{\Delta f}}\}_{n=0}^{N-1}$ remains near-constant over a OFDM period of T.
  - This requires OFDM period to be smaller than coherent time $T < T_c$.

- TD received signal of the i-th OFDM symbol: $y_n^i = \sum_{l=0}^{L-1} h_l^i s_{n-l}^i + v_n^i$
- SAGIN Ricean fading:
  - LoS: $h_0^{i,\text{LoS}} = \sqrt{\frac{K}{K+1}} e^{\frac{j2\pi\vartheta^{\text{LoS}} i}{\Delta f}}$ is in $l = 0$.
  - NLoS: $h_l^i = \left(\sum_{p=P_{l-1}}^{P_l - 1} \widetilde{h}_p e^{\frac{j2\pi\vartheta_p i}{\Delta f}} \right) \sim \mathcal{CN}(0, \frac{1}{(K+1)L})$ for large P_l over $0 \le l \le L-1$, where $P = \sum_{l=0}^{L-1} P_l$. The NLoS correlation (Jakes model): $E\left[h_l^i (h_l^{i+\Delta i})^* \right] = \frac{1}{(K+1)L} J_0 \left(\frac{2\pi f_D \Delta i}{\Delta f} \right)$.
- OFDM operations
  - Frequency-domain (FD) modulation $\overline{\mathbf{s}} \in \mathcal{C}^{N\times 1}$.
  - IFFT at transmitter $\mathbf{s} = \mathbf{W}_N^H \overline{\mathbf{s}}$, where $\mathbf{W}_N \in \mathcal{C}^{N\times N}$ denotes DFT matrix.
  - TD received signal $\mathbf{y} = \mathbf{H}_c \mathbf{s} + \mathbf{v}$, where $\mathbf{H}_c$ is a circulant matrix, i.e. row $n+1$ is right-shift of row n.
  - FFT at receiver: $\overline{\mathbf{y}} = \mathbf{W}_N \mathbf{y} = \overline{\mathbf{D}}_H \overline{\mathbf{s}} + \mathbf{W}_N \mathbf{v}$, where $\overline{\mathbf{D}}_H = \mathbf{W}_N \mathbf{H}_c \mathbf{W}_N^H$ is a diagonal matrix.
  - FD input-output: $\overline{y}_k = \overline{h}_k \overline{s}_k + \overline{v}_k$, where $\overline{h}_k = \sum_{l=0}^{L-1} h_l w_N^{-kl}$ is the k-th diagonal element in $\overline{\mathbf{D}}_H$.
- Channel estimation: FD/TD pilots and MMSE interpolation based on known LoS and NLoS correlation.
- Differential encoding: $\overline{s}_k^i = \overline{s}_k^{i-1} \overline{x}_k^{i-1}$ or $\overline{s}_k^i = \overline{s}_{k-1}^i \overline{x}_{k-1}^i$.

- RIS application
  - SD link: $\sum_{l_0=0}^{L^{\text{SD}}-1} h_{l_0}^{\text{SD}} s_{n-l_0}$
  - SR link: $y_n^{\text{SR}_r} = \sum_{l_1=0}^{L^{\text{SR}}-1} \alpha_n^r h_{l_1}^{\text{SR}_r} s_{n-l_1}$
  - SRD link: $\sum_{l_2=0}^{L^{\text{RD}}-1} h_{l_2}^{\text{RD}_r} y_{n-l_2}^{\text{SR}_r} = \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha_{n-l_2}^r h_{l_1}^{\text{SR}_r} h_{l_2}^{\text{RD}_r} s_{n-l_1-l_2}$
  - Overall received signal: $y_n = \sum_{l_0=0}^{L^{\text{SD}}-1} h_{l_0}^{\text{SD}} s_{n-l_0} + \sum_{r=1}^{R} \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha_{n-l_2}^r h_{l_1}^{\text{SR}_r} h_{l_2}^{\text{RD}_r} s_{n-l_1-l_2} + v_n = \sum_{l=0}^{L-1} h_l s_{n-l} + v_n$, where $L = \max(L^{\text{SD}}, L^{\text{SR}} + L^{\text{RD}} - 1)$.
  - Configure RIS based on LoS: $\angle\alpha^r = \angle h_0^{\text{SD,LoS}} - \angle h_0^{\text{SR}_r\text{,LoS}} h_0^{\text{RD}_r\text{,LoS}}$ for large Ricean K.
- Other multi-carrier waveforms:
  - OFDM-IM can improve OFDM throughput and improve noise resilience – at the cost of IM complexity.
  - DFT-S-OFDM can improve OFDM's PAPR and achieve frequency-diversity order of L (improve Doppler resilience) – at the cost of out-of-band emission.
  - CE-OFDM can improve OFDM's PAPR – at the cost of non-linear noise.
  - ZT/UW can eliminate OFDM's CP overhead – at the cost of throughput loss for guard interval.
  - UFMC can improve OFDM's out-of-band emission – at the cost of compromising SC orthogonality.

- From the generic received signal model:

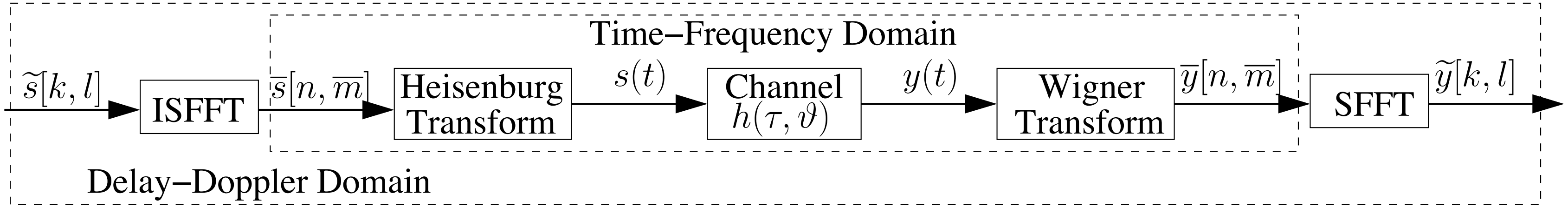
$$y(t) = \int\int \widetilde{h}(\tau,\vartheta)s(t-\tau)e^{j2\pi\vartheta(t-\tau)}d\tau d\vartheta + v(t)|_{t=\frac{nT}{M}=\frac{n}{M\Delta f}}$$

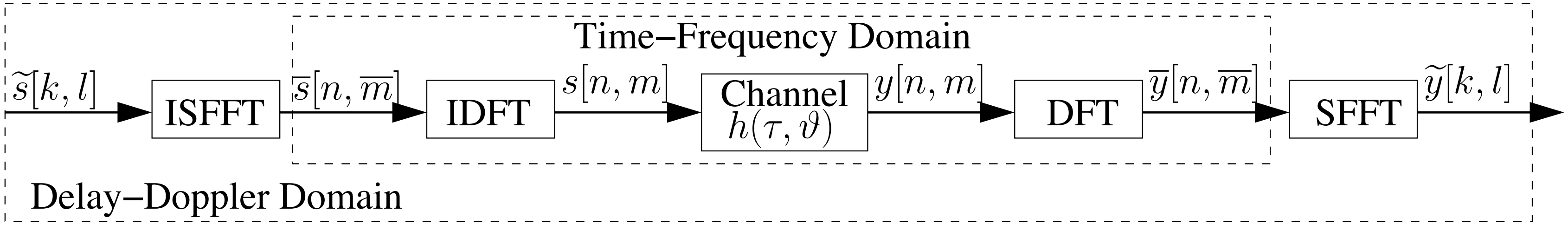
$$\text{where} \quad \widetilde{h}(\tau,\vartheta) = \sum_{p=0}^{P-1}\widetilde{h}_p\delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p)|_{\tau_p=\frac{lT}{M}=\frac{l}{M\Delta f}}$$

- Frequency selective: $\tau_{\text{max}} > \frac{T}{M}$
  - The P paths fall into L resolvable TDL $\tau_p = \frac{lT}{M} = \frac{l}{M\Delta f}$, where $l = 0, \cdots, L-1$.
  - Frequency selectivity imposes ISI in the TD.

- Time varying: f_D becomes compariable to Δf
  - Assuming $\{\vartheta_p << M\Delta f\}_{p=0}^{P-1}$ – $\widetilde{h}_p e^{\frac{j2\pi\vartheta_p n}{M\Delta f}}$ still remains near-constant over a sampling period $\frac{T}{M}$ but varies for $n = 0, \cdots, M-1$ within an OFDM period T, i.e. $\frac{T}{M} < T_c$.
  - This implies that the circulant $\mathbf{H}_c$ becomes time-varying from row to row, and the FD CFR matrix $\overline{\mathbf{D}}_H = \mathbf{W}_N\mathbf{H}_c\mathbf{W}_N^H$ is no longer diagonal, which imposes inter-carrier interference (ICI) in the FD.

- TD received signal: $y_n = \sum_{l=0}^{L-1} h_{n,l} s_{n-l} + v_n$
- SAGIN Ricean fading:
  - LoS: $h_{n,0}^{\text{LoS}} = \sqrt{\frac{K}{K+1}} e^{\frac{j2\pi\vartheta^{\text{LoS}} n}{M\Delta f}}$ in $l = 0$.
  - NLoS: $h_{n,l} = \left(\sum_{p=P_{l-1}}^{P_l - 1} \widetilde{h}_p e^{\frac{j2\pi\vartheta_p (n-l)}{M\Delta f}} \right) \sim \mathcal{CN}(0, \frac{1}{(K+1)L})$ for large P_l over $0 \leq l \leq L-1$, $P = \sum_{l=0}^{L-1} P_l$. The NLoS correlation (Jakes model): $E(h_{n,l} h_{n+\Delta n,l}^*) = \frac{1}{(K+1)L} J_0 \left(\frac{2\pi f_D \Delta n}{M\Delta f} \right)$.
- Channel estimation and signal detection in FD/TD suffer from ICI/ISI.
- Differential encoding and non-coherent detection in FD/TD suffer from ICI/ISI.
- RIS application
  - SD link: $\sum_{l_0=0}^{L^{\text{SD}}-1} h_{n,l_0}^{\text{SD}} s_{n-l_0}$
  - SR link: $y_n^{\text{SR}_r} = \sum_{l_1=0}^{L^{\text{SR}}-1} \alpha_n^r h_{n,l_1}^{\text{SR}_r} s_{n-l_1}$
  - SRD link: $\sum_{l_2=0}^{L^{\text{RD}}-1} h_{n,l_2}^{\text{RD}_r} y_{n-l_2}^{\text{SR}_r} = \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha_{n-l_2}^r h_{n-l_2,l_1}^{\text{SR}_r} h_{n,l_2}^{\text{RD}_r} s_{n-l_1-l_2}$
  - Overall received signal: $y_n = \sum_{l_0=0}^{L^{\text{SD}}-1} h_{n,l_0}^{\text{SD}} s_{n-l_0} + \sum_{r=1}^{R} \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha_{n-l_2}^r h_{n-l_2,l_1}^{\text{SR}_r} h_{n,l_2}^{\text{RD}_r} s_{n-l_1-l_2} + v_n = \sum_{l=0}^{L-1} h_{n,l} s_{n-l} + v_n$, where $L = \max(L^{\text{SD}}, L^{\text{SR}} + L^{\text{RD}} - 1)$.
  - Configure RIS based on LoS: $\angle\alpha_n^r = \angle h_{n,0}^{\text{SD,LoS}} - \angle h_{n,0}^{\text{SR}_r\text{,LoS}} h_{n,0}^{\text{RD}_r\text{,LoS}}$.





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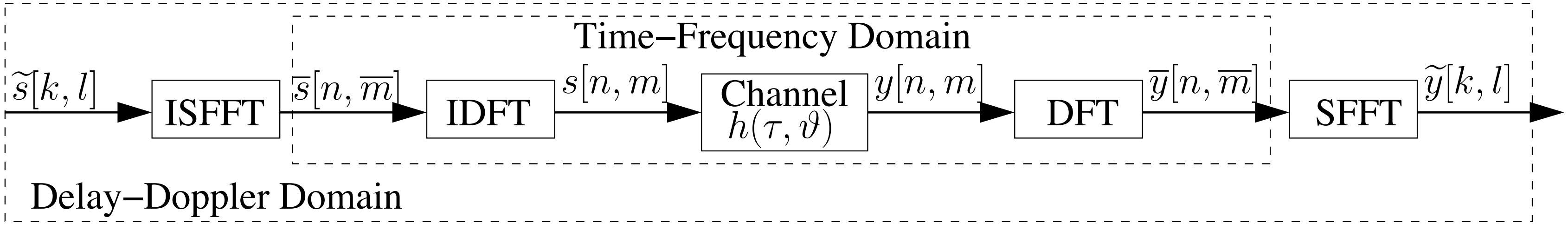
where $\widetilde{h}(\tau,\vartheta) = \sum_{p=0}^{P-1}\widetilde{h}_p\delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p)|_{\tau_p=\frac{l_p}{M\Delta f},\vartheta_p=\frac{k_p}{NT}}$

- Orthogonal Time Frequency Space (OTFS) modulation is specifically designed for doubly selective fading, where P propagation paths are resolvable and time-invariant in the delay-Doppler (DD) domain:

$$\widetilde{y}[k,l] = \sum_{p=0}^{P-1}\widetilde{h}_p e^{\frac{-j2\pi l_p k_p}{MN}}\widetilde{s}[<k-k_p>_N, <l-l_p>_M]$$



- ISFFT at the transmitter:

$$\overline{s}[n,\overline{m}] = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \widetilde{s}[k,l] w_N^{nk} w_M^{-\overline{m}l} \qquad \overline{\mathbf{S}} = \mathbf{F}_N^H \widetilde{\mathbf{S}} \mathbf{F}_M$$

- IDFT at the transmitter:

$$s[n,m] = \frac{1}{\sqrt{M}} \sum_{\overline{m}=0}^{M-1} \overline{s}[n,\overline{m}] w_M^{m\overline{m}} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \widetilde{s}[k,m] w_N^{nk} \qquad \mathbf{S} = \overline{\mathbf{S}} \mathbf{F}_M^H = \mathbf{F}_N^H \widetilde{\mathbf{S}}$$

- Fading channel: $\widetilde{h}(\tau,\vartheta) = \sum_{p=0}^{P-1} \widetilde{h}_p \delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p)\big|_{\tau_p=\frac{l_p}{M\Delta f},\vartheta_p=\frac{k_p}{NT}}$

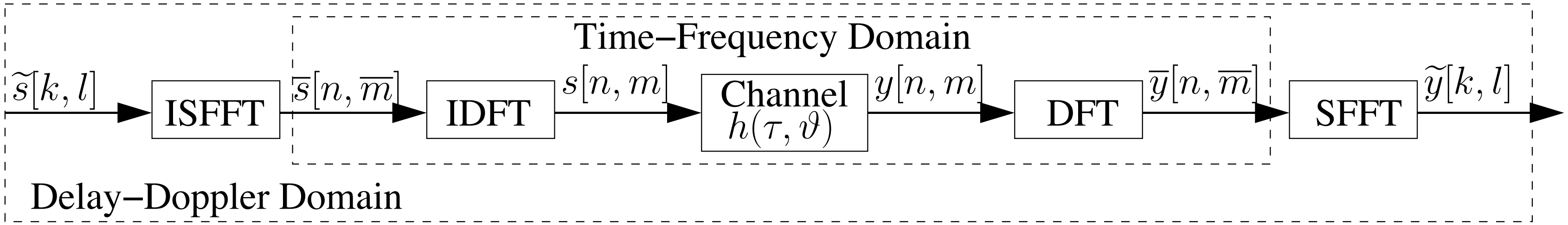
$$y[n,m] = \sum_{l=0}^{L-1} h_{n,m,l} s[n,<m-l>_M] + v[n,m]$$

where $\quad h_{n,m,l} = \sum_{p=P_{l-1}}^{P_l-1} \widetilde{h}_p e^{\frac{j2\pi\vartheta_p[n(M+M_{cp})+m-l_p]}{M\Delta f}} = \sum_{p=P_{l-1}}^{P_l-1} \widetilde{h}_p w_{MN}^{k_p[n(M+M_{cp})+m-l_p]}|_{l=l_p}$

$\mathbf{Y}[n,:] = \mathbf{S}[n,:]\mathbf{H}_{\text{CIR},n}^T + \mathbf{V}[n,:]$, where $\quad \mathbf{H}_{\text{CIR},n}(r,c) = h_{n,r,<r-c>_M}$

P paths fall into L resolvable TDL $\tau_p = \frac{l_p}{M\Delta f}$, where each TDL has P_l paths.

OTFS based on OFDM



- DFT at the receiver:

$$\begin{aligned}\overline{y}[n,\overline{m}] &= \frac{1}{\sqrt{M}}\sum_{m=0}^{M-1} y[n,m]w_M^{-m\overline{m}} \\ &= \frac{1}{\sqrt{M}}\sum_{m=0}^{M-1}\sum_{p=0}^{P-1}\widetilde{h}_p s[n,<m-l_p>_M]w_{MN}^{k_p[n(M+M_{cp})+m-l_p]}w_M^{-m\overline{m}}\end{aligned}$$

$$\overline{\mathbf{Y}} = \mathbf{Y}\mathbf{F}_M$$

- SFFT at the receiver:

$$\begin{aligned}\widetilde{y}[k,l] &= \frac{1}{\sqrt{MN}}\sum_{n=0}^{N-1}\sum_{\overline{m}=0}^{M-1}\overline{y}[n,\overline{m}]w_N^{-nk}w_M^{\overline{m}l} \\ &= \frac{1}{M\sqrt{N}}\sum_{n=0}^{N-1}\sum_{\overline{m}=0}^{M-1}\sum_{m=0}^{M-1} y[n,m]w_N^{-nk}w_M^{\overline{m}(l-m)} = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1} y[n,l]w_N^{-nk} \\ &= \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{k_p\left[n(M+M_{cp})+l-l_p\right]} s[n,<l-l_p>_M]w_N^{-nk} + \widetilde{v}[k,l]|_{M_{cp}\approx 0} \\ &\approx \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_N^{n(k_p-k)}w_{MN}^{k_p(l-l_p)} s[n,<l-l_p>_M] + \widetilde{v}[k,l] \\ &= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k'=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_N^{n(k_p-k)}w_{MN}^{k_p(l-l_p)}\widetilde{s}[k',<l-l_p>_M]w_N^{nk'} + \widetilde{v}[k,l]|_{k'=<k-k_p>_N} \\ &= \sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{k_p(l-l_p)}\widetilde{s}[<k-k_p>_N,<l-l_p>_M] + \widetilde{v}[k,l]\end{aligned}$$

$$\widetilde{\mathbf{Y}} = \mathbf{F}_N\overline{\mathbf{Y}}\mathbf{F}_M^H = \mathbf{F}_N\mathbf{Y}$$

- If one CP is added to the entire OTFS frame, the TD circular convolution becomes MN-periodic:

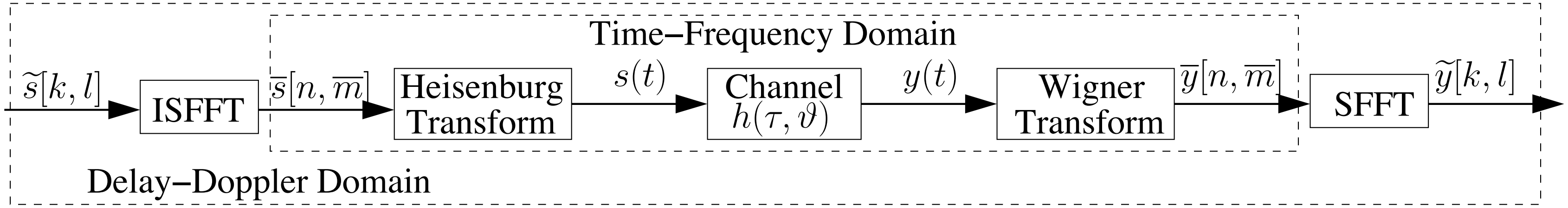
$$y[n,m] = \sum_{l=0}^{L-1} h_{n,m,l}\, s[< nM + m - l >_{MN}] + v[n,m]$$

$$s[< nM + m - l >_{MN}] = \begin{cases} s[n, < m - l >_M], & m \geq l \\ s[n-1, < m - l >_M], & m < l \end{cases}$$

- As a result, the input-output relationship becomes:

$$\widetilde{y}[k,l] = \sum_{p=0}^{P-1} \widetilde{h}_p \widetilde{T}(k,l,k_p,l_p)\widetilde{s}[< k - k_p >_N, < l - l_p >_M] + \widetilde{v}[k,l]$$

$$\widetilde{T}(k,l,k_p,l_p) = \begin{cases} w_{MN}^{k_p(< l - l_p >_M)}, & l \geq l_p \\ w_N^{-(k-k_p)} w_{MN}^{k_p(l-l_p)} = w_N^{-k} w_{MN}^{k_p(< l - l_p >_M)}, & l < l_p \end{cases}$$



- ISFFT at the transmitter:

$$\bar{s}[n,\bar{m}] = \frac{1}{\sqrt{NM}} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \tilde{s}[k,l] w_N^{nk} w_M^{-\bar{m}l} \qquad \bar{\mathbf{S}} = \mathbf{F}_N^H \tilde{\mathbf{S}} \mathbf{F}_M$$

- Heisenburg Transform at the transmitter:

$$s(t) = \frac{1}{\sqrt{M}} \sum_{n=0}^{N-1} \sum_{\bar{m}=0}^{M-1} \bar{s}[n,\bar{m}] g_{tx}(t-nT) e^{j2\pi\bar{m}\Delta f(t-nT)} \Big|_{t=\frac{n(M+M_{cp})+m}{M}T \approx nT+\frac{m}{M}T}$$

$$s[n,m] = \frac{1}{\sqrt{M}} \sum_{\bar{m}=0}^{M-1} \bar{s}[n,\bar{m}] g_{tx}(\tfrac{m}{M}T) w_M^{\bar{m}m} = s'[n,m] g_{tx}(\tfrac{m}{M}T)$$

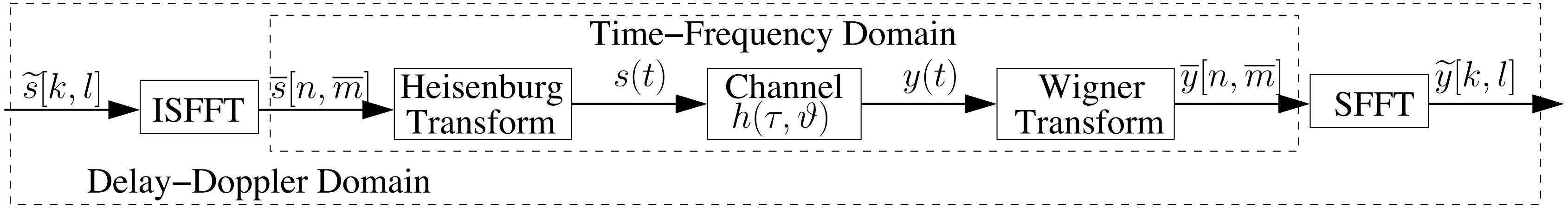
where $s'[n,m] = \frac{1}{\sqrt{M}} \sum_{\bar{m}=0}^{M-1} \bar{s}[n,\bar{m}] w_M^{\bar{m}m} = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \tilde{s}[k,m] w_N^{nk}$, $\mathbf{S}' = \bar{\mathbf{S}}\mathbf{F}_M^H = \mathbf{F}_N^H \tilde{\mathbf{S}}$.

- Received signal:

$$y(t) = \int\int \sum_{p=0}^{P-1} \tilde{h}_p \delta(\tau-\tau_p)\delta(\vartheta-\vartheta_p) s(t-\tau) e^{j2\pi\vartheta(t-\tau)} d\tau d\vartheta + v(t) \Big|_{\substack{t = \frac{n(M+M_{cp})+m}{M}T, \\ \tau = \frac{l}{M\Delta f}, \vartheta = \frac{k}{NT}}}$$

$y[n,m] = \sum_{l=0}^{L-1} h_{n,m,l} s[n, <m-l>_M] + v[n,m]$ where $h_{n,m,l} = \sum_{p=P_{l-1}}^{P_l-1} \tilde{h}_p w_{MN}^{k_p[n(M+M_{cp})+m-l_p]}$

$\mathbf{Y}[n,:] = \mathbf{S}[n,:]\mathbf{H}_{\text{CIR},n}^T + \mathbf{V}[n,:]$, where $\mathbf{H}_{\text{CIR},n}(r,c) = h_{n,r,<r-c>_M}$



- Cross-ambiguity function of ideal waveform with bi-orthogonal property:

$$\begin{aligned} A_{rx,tx} &= \int g_{rx}^*(t'-t)g_{tx}(t')e^{-j2\pi f(t'-t)}dt'\Big|_{t=nT+\frac{m}{M}T,t'=n'T+\frac{m'}{M}T,f=\overline{m}\Delta f+\frac{k}{NT}} \\ &= \delta[n]\delta[m] \quad \text{for } k\in[\min_p k_p,\max_p k_p] \text{ and } l\in[\min_p l_p,\max_p l_p] \end{aligned}$$

- Wigner Transform at the receiver:

$$\overline{y}(t,f) = \int g_{rx}^*(t'-t)y(t')e^{-j2\pi f(t'-t)}dt'|_{t=nT,f=\overline{m}\Delta f,M_{cp}\approx 0}$$

$$\overline{y}[n,\overline{m}] = \frac{1}{\sqrt{M}}\sum_{n'=0}^{N-1}\sum_{m'=0}^{M-1} g_{rx}^*\left((n-n')T+\frac{m'}{M}T\right)y[n',m']w_M^{-\overline{m}[(n'-n)M+m']}$$

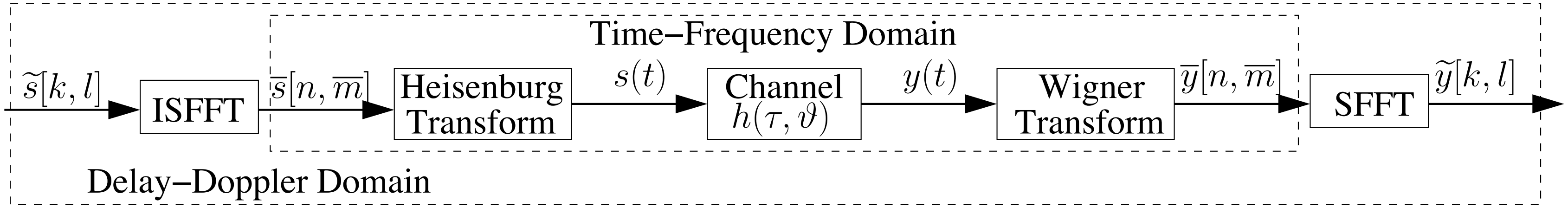
$$=\frac{1}{\sqrt{M}}\sum_{n'=0}^{N-1}\sum_{m'=0}^{M-1}\sum_{p=0}^{P-1} g_{rx}^*\left((n'-n)T+\frac{m'}{M}T\right)\widetilde{h}_p s[n',<m'-l_p>_M]w_{MN}^{k_p[nM+m'-l_p]}w_M^{-\overline{m}[(n'-n)M+m']}+\overline{v}[n,\overline{m}]$$

$$=\frac{1}{\sqrt{M}}\sum_{n'=0}^{N-1}\sum_{m'=0}^{M-1}\sum_{p=0}^{P-1} g_{rx}^*\left((n'-n)T+\frac{m'}{M}T\right)g_{tx}\left(\frac{<m'-l_p>_M}{M}T\right)w_{MN}^{-[(\overline{m}'-\overline{m})N-k_p][(n'-n)M+m']}$$

$$\times\widetilde{h}_p s'[n',<m'-l_p>_M]w_{MN}^{[(\overline{m}'-\overline{m})N-k_p][(n'-n)M+m']}w_{MN}^{k_p[nM+m'-l_p]}w_M^{-\overline{m}[(n'-n)M+m']}+\overline{v}[n,\overline{m}]|_{n=n',\overline{m}=\overline{m}'}$$

$$=\frac{1}{\sqrt{M}}\sum_{m'=0}^{M-1}\sum_{p=0}^{P-1}\widetilde{h}_p s'[n,<m'-l_p>_M]w_{MN}^{k_p(nM-l_p)}w_M^{-\overline{m}m'}+\overline{v}[n,\overline{m}] = \frac{1}{M}\sum_{m=0}^{M-1}y'[n,m]w_M^{-\overline{m}m}$$

where $y'[n,m]=\sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{k_p(nM-l_p)}s'[n,<m'-l_p>_M]+v[n,m]$, $\overline{\mathbf{Y}}=\mathbf{Y}'\mathbf{F}_M$, $\mathbf{Y}'[n,:]=\mathbf{S}'[n,:](\mathbf{H}'_{\text{CIR},n})^T+\mathbf{V}[n,:]$ and $\mathbf{H}'_{\text{CIR},n}(r,c)=h_{n,0,<r-c>_M}$, which is no longer time varying within the n-th OFDM symbol.



- SFFT at the receiver:

$$\begin{aligned}
\widetilde{y}[k,l] &= \frac{1}{\sqrt{MN}}\sum_{n=0}^{N-1}\sum_{\overline{m}=0}^{M-1}\overline{y}[n,\overline{m}]w_N^{-nk}w_M^{\overline{m}l}\\
&= \frac{1}{M\sqrt{N}}\sum_{n=0}^{N-1}\sum_{\overline{m}=0}^{M-1}\sum_{m=0}^{M-1}y'[n,m]w_N^{-nk}w_M^{\overline{m}(l-m)}\\
&= \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}y'[n,l]w_N^{-nk}\\
&= \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{k_p(nM-l_p)}s'[n,<l-l_p>_M]w_N^{-nk}+\widetilde{v}[k,l]\\
&= \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_N^{n(k_p-k)}w_{MN}^{-k_pl_p}s'[n,<l-l_p>_M]+\widetilde{v}[k,l]\\
&= \frac{1}{N}\sum_{n=0}^{N-1}\sum_{k'=0}^{N-1}\sum_{p=0}^{P-1}\widetilde{h}_p w_N^{n[k'-(k-k_p)]}w_{MN}^{-k_pl_p}\widetilde{s}[k',<l-l_p>_M]+\widetilde{v}[k,l]\\
&= \sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{-k_pl_p}\widetilde{s}[<k-k_p>_N,<l-l_p>_M]+\widetilde{v}[k,l]
\end{aligned}$$

$$\widetilde{\mathbf{Y}}=\mathbf{F}_N\widetilde{\mathbf{Y}}\mathbf{F}_M^H=\mathbf{F}_N\mathbf{Y}'$$

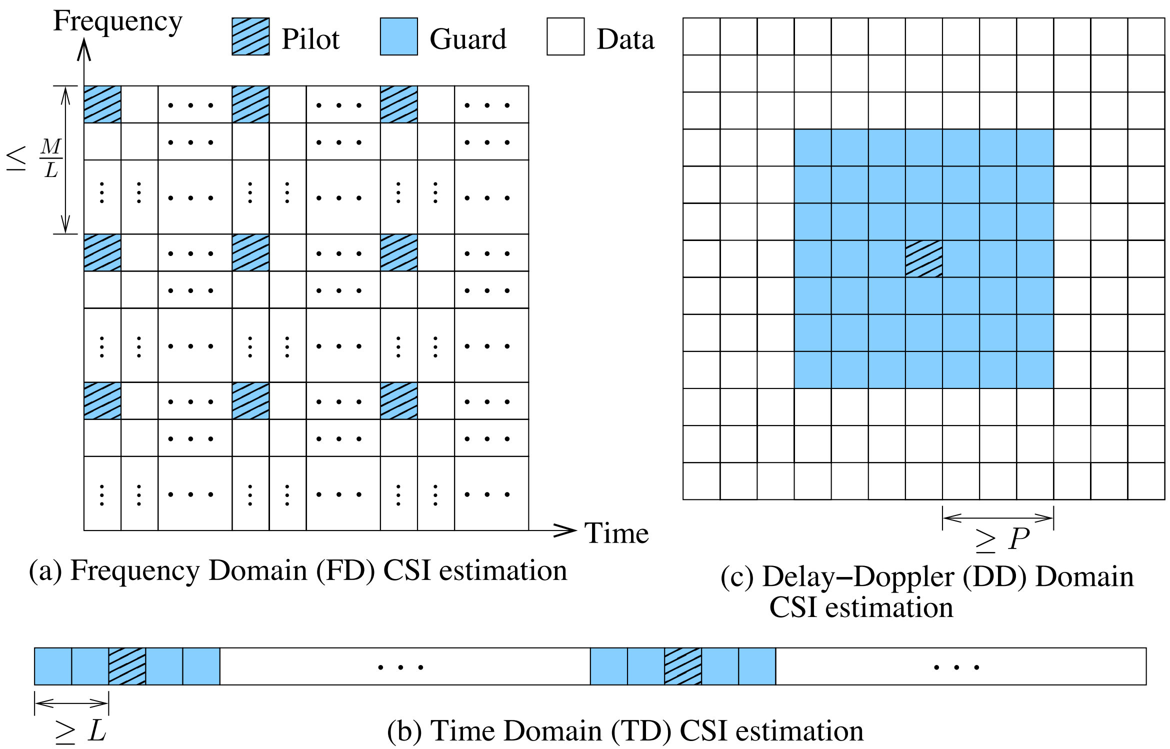
- OTFS input-output relationship in matrix form: $\widetilde{\mathbf{y}} = \widetilde{\mathbf{H}}\widetilde{\mathbf{x}} + \widetilde{\mathbf{v}}$, $\widetilde{\mathbf{y}} \in \mathcal{C}^{MN\times 1}$, $\widetilde{\mathbf{y}}_\kappa = \widetilde{y}[k,l]$, $\widetilde{\mathbf{x}} \in \mathcal{C}^{MN\times 1}$, $\widetilde{\mathbf{x}}_\kappa = \widetilde{x}[k,l]$, $\widetilde{\mathbf{v}} \in \mathcal{C}^{MN\times 1}$, $\widetilde{\mathbf{v}}_\kappa = \widetilde{v}[k,l]$, $k = \lfloor \frac{\kappa}{M} \rfloor$, $l = \kappa - kM$, $\widetilde{\mathbf{H}} \in \mathcal{C}^{MN\times MN}$
  - OTFS based on PS-OFDM: $\widetilde{\mathbf{H}}_{\kappa,\iota} = \widetilde{h}_p w_{MN}^{-k_p l_p}$, $\iota = M\times < k - k_p >_N + < l - l_p >_M$.
  - OTFS based on OFDM (Symbol CP): $\widetilde{\mathbf{H}}_{\kappa,\iota} = \widetilde{h}_p w_{MN}^{k_p(l-l_p)}$.
  - OTFS based on OFDM (Frame CP): $\widetilde{\mathbf{H}}_{\kappa,\iota} = \widetilde{h}_p \widetilde{T}(k,l,k_p,l_p)$.
  - MMSE detector: $\widetilde{\mathbf{z}} = (\widetilde{\mathbf{H}}^H\widetilde{\mathbf{H}} + N_0\mathbf{I}_{MN})^{-1}\widetilde{\mathbf{H}}^H\widetilde{\mathbf{y}}$.
  - Message Passing (MP) detector exploits the sparsity of $\widetilde{\mathbf{H}}$.
- OFDM and DFT-S-OFDM:
  - FDE: $\overline{z}_m = \overline{y}_m/\overline{h}_m$, $0 \le m \le M-1$.
  - FD-MMSE: $\overline{\mathbf{z}} = (\overline{\mathbf{H}}^H\overline{\mathbf{H}} + N_0\mathbf{I}_M)^{-1}\overline{\mathbf{H}}^H\overline{\mathbf{y}}$, where $\overline{\mathbf{H}} = \mathbf{W}_N\mathbf{H}_{\text{CIR}}\mathbf{W}_N^H$.
  - TD-MMSE: $\mathbf{z} = (\mathbf{H}_{\text{CIR}}^H\mathbf{H}_{\text{CIR}} + N_0\mathbf{I}_M)^{-1}\mathbf{H}_{\text{CIR}}^H\mathbf{y}$.
  - Message Passing (MP) detector can also exploit the sparsity of $\mathbf{H}_{\text{CIR}}$.

- Example: $N=3$, $M=2$, $P=3$, $[k_p,l_p]=\{[2,0],[-1,1],[-2,1]\}$.
- OTFS based on OFDM (Symbol CP):
  $\widetilde{y}[k,l]=\sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{k_p(l-l_p)}\widetilde{s}[<k-k_p>_N,<l-l_p>_M]+\widetilde{v}[k,l]$

$$\begin{bmatrix}\widetilde{y}[0,0]\\\widetilde{y}[0,1]\\\widetilde{y}[1,0]\\\widetilde{y}[1,1]\\\widetilde{y}[2,0]\\\widetilde{y}[2,1]\end{bmatrix}=\begin{bmatrix}0 & 0 & \widetilde{h}_0 & \widetilde{h}_1w_6^1 & 0 & \widetilde{h}_2w_6^2\\ 0 & 0 & \widetilde{h}_1 & \widetilde{h}_0w_6^2 & \widetilde{h}_2 & 0\\ 0 & \widetilde{h}_2w_6^2 & 0 & 0 & \widetilde{h}_0 & \widetilde{h}_1w_6^1\\ \widetilde{h}_2 & 0 & 0 & 0 & \widetilde{h}_1 & \widetilde{h}_0w_6^2\\ \widetilde{h}_0 & \widetilde{h}_1w_6^1 & 0 & \widetilde{h}_2w_6^2 & 0 & 0\\ \widetilde{h}_1 & \widetilde{h}_0w_6^2 & \widetilde{h}_2 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\widetilde{s}[0,0]\\\widetilde{s}[0,1]\\\widetilde{s}[1,0]\\\widetilde{s}[1,1]\\\widetilde{s}[2,0]\\\widetilde{s}[2,1]\end{bmatrix}$$

- OTFS based on Pulse-Shaped OFDM:
  $\widetilde{y}[k,l]=\sum_{p=0}^{P-1}\widetilde{h}_p w_{MN}^{-k_pl_p}\widetilde{s}[<k-k_p>_N,<l-l_p>_M]+\widetilde{v}[k,l]$

$$\begin{bmatrix}\widetilde{y}[0,0]\\\widetilde{y}[0,1]\\\widetilde{y}[1,0]\\\widetilde{y}[1,1]\\\widetilde{y}[2,0]\\\widetilde{y}[2,1]\end{bmatrix}=\begin{bmatrix}0 & 0 & \widetilde{h}_0 & \widetilde{h}_1w_6^1 & 0 & \widetilde{h}_2w_6^2\\ 0 & 0 & \widetilde{h}_1w_6^1 & \widetilde{h}_0 & \widetilde{h}_2w_6^2 & 0\\ 0 & \widetilde{h}_2w_6^2 & 0 & 0 & \widetilde{h}_0 & \widetilde{h}_1w_6^1\\ \widetilde{h}_2w_6^2 & 0 & 0 & 0 & \widetilde{h}_1w_6^1 & \widetilde{h}_0\\ \widetilde{h}_0 & \widetilde{h}_1w_6^1 & 0 & \widetilde{h}_2w_6^2 & 0 & 0\\ \widetilde{h}_1w_6^1 & \widetilde{h}_0 & \widetilde{h}_2w_6^2 & 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\widetilde{s}[0,0]\\\widetilde{s}[0,1]\\\widetilde{s}[1,0]\\\widetilde{s}[1,1]\\\widetilde{s}[2,0]\\\widetilde{s}[2,1]\end{bmatrix}$$



(a) Frequency Domain (FD) CSI estimation

(c) Delay–Doppler (DD) Domain CSI estimation

(b) Time Domain (TD) CSI estimation

- Advantages of OTFS:
  - OTFS achieves a diversity order of P, which improves Doppler resilience.
  - Channel estimation in DD domain is lower complexity than channel estimation in TD.
- Disadvantages of OTFS:
  - Pilot percentage and detection complexity increase with P.
  - Equalization is required for signal detection.
- Differential encoding and non-coherent detection of OTFS will have to ignore NLoS taps.
- RIS application and configuration can be done in TD based on LoS in the same way as OFDM.

- Assume one antenna at source node, one antenna at destination node and R RIS elements.
- SD link: $\sum_{l_0=0}^{L^{\text{SD}}-1} h^{\text{SD}}_{n,m,l_0} s_{n,m-l_0} = \sum_{p_0=0}^{P^{\text{SD}}-1} \widetilde{h}^{\text{SD}}_{p_0} w_{MN}^{k^{\text{SD}}_{p_0}(nM+m-l_{p_0})} s_{n,m-l_{p_0}}|_{\mathfrak{b}(k^{\text{SD}}_{p_0},l_{p_0})=1,l_{p_0}=l_0}$
- SR link:
$y^{\text{SR}_r}_{n,m} = \sum_{l_1=0}^{L^{\text{SR}}-1} h^{\text{SR}_r}_{n,m,l_1} s_{n,m-l_1} = \sum_{p_1=0}^{P^{\text{SR}}-1} \widetilde{h}^{\text{SR}_r}_{p_1} w_{MN}^{k^{\text{SR}_r}_{p_1}(nM+m-l_{p_1})} s_{n,m-l_{p_1}}|_{\mathfrak{b}(k^{\text{SR}_r}_{p_1},l_{p_1})=1,l_{p_1}=l_1}$
- SR-RD link:

$$\sum_{l_2=0}^{L^{\text{RD}}-1} h^{\text{RD}_r}_{n,m,l_2} \alpha^r_{n,m-l_2} y^{\text{SR}_r}_{n,m-l_2} = \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha^r_{n,m-l_2} h^{\text{SR}_r}_{n,m-l_2,l_1} h^{\text{RD}_r}_{n,m,l_2} s_{n,m-l_1-l_2}$$
$$= \sum_{p_1=0}^{P^{\text{SR}}-1} \sum_{p_2=0}^{P^{\text{RD}}-1} \alpha^r_{n,m-l_{p_2}} \widetilde{h}^{\text{SR}_r}_{p_1} w_{MN}^{k^{\text{SR}_r}_{p_1}(nM+m-l_{p_1}-l_{p_2})} \widetilde{h}^{\text{RD}_r}_{p_2} w_{MN}^{k^{\text{RD}_r}_{p_2}(nM+m-l_{p_2})} s_{n,m-l_{p_1}-l_{p_2}}$$

where $\mathfrak{b}(k^{\text{SR}_r}_{p_1}, l_{p_1}) = 1$, $l_{p_1} = l_1$, $\mathfrak{b}(k^{\text{RD}_r}_{p_2}, l_{p_2}) = 1$, $l_{p_2} = l_2$.

- Received signal in TD:

$$y_{n,m} = \sum_{l_0=0}^{L^{\text{SD}}-1} h^{\text{SD}}_{n,m,l_0} s_{n,m-l_0} + \sum_{r=0}^{R} \sum_{l_1=0}^{L^{\text{SR}}-1} \sum_{l_2=0}^{L^{\text{RD}}-1} \alpha^r_{n,m-l_2} h^{\text{SR}_r}_{n,m-l_2,l_1} h^{\text{RD}_r}_{n,m,l_2} s_{n,m-l_1-l_2}$$
$$+ v_{n,m} = \sum_{l=0}^{L-1} h_{n,m,l} s_{n,m-l} + v_{n,m}$$

where the total number of overall TDL taps is $L = \max(L^{\text{SD}}, L^{\text{SR}} + L^{\text{RD}} - 1)$.

- How to format the time-varying RIS phase rotation $\alpha^r_{n,m}$ in DD domain?
  - General rule for $h_{n,m,l}$ is $\widetilde{h}_p w_{MN}^{k(nM+m-l)}$: time-invariant $\widetilde{h}_p$, Doppler index k and delay index l.
  - Similarly, $\alpha^r_{n,m}$ can be represented by $\widetilde{\alpha}^r w_{MN}^{k^{\text{RIS}}(nM+m)}$ in DD domain with a time-invariant tap $\widetilde{\alpha}^r$ and a virtual Doppler index k^{RIS_r}.
  - RIS is frequency non-selective, i.e. it cannot be tuned for different TDL taps, hence no delay index.
  - RIS can now tune the Doppler difference between the SD link and the RIS-reflected links.
  - RIS configuration is simplified to setting the time-invariant $\widetilde{\alpha}^r$ and k^{RIS_r}.
- SR-RD link with $\mathfrak{b}(k_{p_1}^{\text{SR}_r}, l_{p_1}) = 1$ and $\mathfrak{b}(k_{p_2}^{\text{RD}_r}, l_{p_2}) = 1$:

$$\sum_{p_1=0}^{P^{\text{SR}}-1} \sum_{p_2=0}^{P^{\text{RD}}-1} \widetilde{\alpha}^r \widetilde{h}_{p_1}^{\text{SR}_r} \widetilde{h}_{p_2}^{\text{RD}_r} w_{MN}^{(k_{p_2}^{\text{RD}_r}+k^{\text{RIS}_r})l_{p_1}} w_{MN}^{(k_{p_1}^{\text{SR}_r}+k_{p_2}^{\text{RD}_r}+k^{\text{RIS}_r})(nM+m-l_{p_1}-l_{p_2})} s_{n,m-l_{p_1}-l_{p_2}}$$

- Received signal with DD representation: $y_{n,m} = \sum_{p=0}^{P-1} \widetilde{h}_p w_{MN}^{k_p(nM+m-l_p)} s_{n,m-l_p} + v_{n,m}$

- The time-invariant tap in DD domain:

$$\widetilde{h}_p = \widetilde{h}_p^{\mathsf{SD}} \mathfrak{b}(k_p^{\mathsf{SD}}, l_p) + \sum_{r=0}^{R} \sum_{\forall l_{p_1} + l_{p_2} = l_p} \widetilde{\alpha}^r \widetilde{h}_{p_1}^{\mathsf{SR}_r} \widetilde{h}_{p_2}^{\mathsf{RD}_r} w_{MN}^{(k_{p_2}^{\mathsf{RD}_r} + k^{\mathsf{RIS}_r}) l_{p_1}} \mathfrak{b}(k_{p_1}^{\mathsf{SR}_r}, l_{p_1}) \mathfrak{b}(k_{p_2}^{\mathsf{RD}_r}, l_{p_2})$$

- The Doppler index and delay index:

$$k_p = \begin{cases} k_p^{\mathsf{SD}}, & \mathfrak{b}(k_p^{\mathsf{SD}}, l_p) = 1, \\ k_{p_1}^{\mathsf{SR}_r} + k_{p_2}^{\mathsf{RD}_r} + k^{\mathsf{RIS}_r}, & \mathfrak{b}(k_{p_1}^{\mathsf{SR}_r}, l_{p_1}) \mathfrak{b}(k_{p_2}^{\mathsf{RD}_r}, l_{p_2}) = 1. \end{cases} \qquad l_p = \begin{cases} l_p^{\mathsf{SD}}, & \mathfrak{b}(k_p^{\mathsf{SD}}, l_p) = 1, \\ l_{p_1} + l_{p_2}, & \mathfrak{b}(k_{p_1}^{\mathsf{SR}_r}, l_{p_1}) \mathfrak{b}(k_{p_2}^{\mathsf{RD}_r}, l_{p_2}) = 1. \end{cases}$$

- The total number of resolvable paths in DD domain $(k_p^{\mathsf{SD}}, l_p)_{\forall p} \cup (k_{p_1}^{\mathsf{SR}_r} + k_{p_2}^{\mathsf{RD}_r} + k^{\mathsf{RIS}_r}, l_{p_1} + l_{p_2})_{\forall p_1 \forall p_2}$.

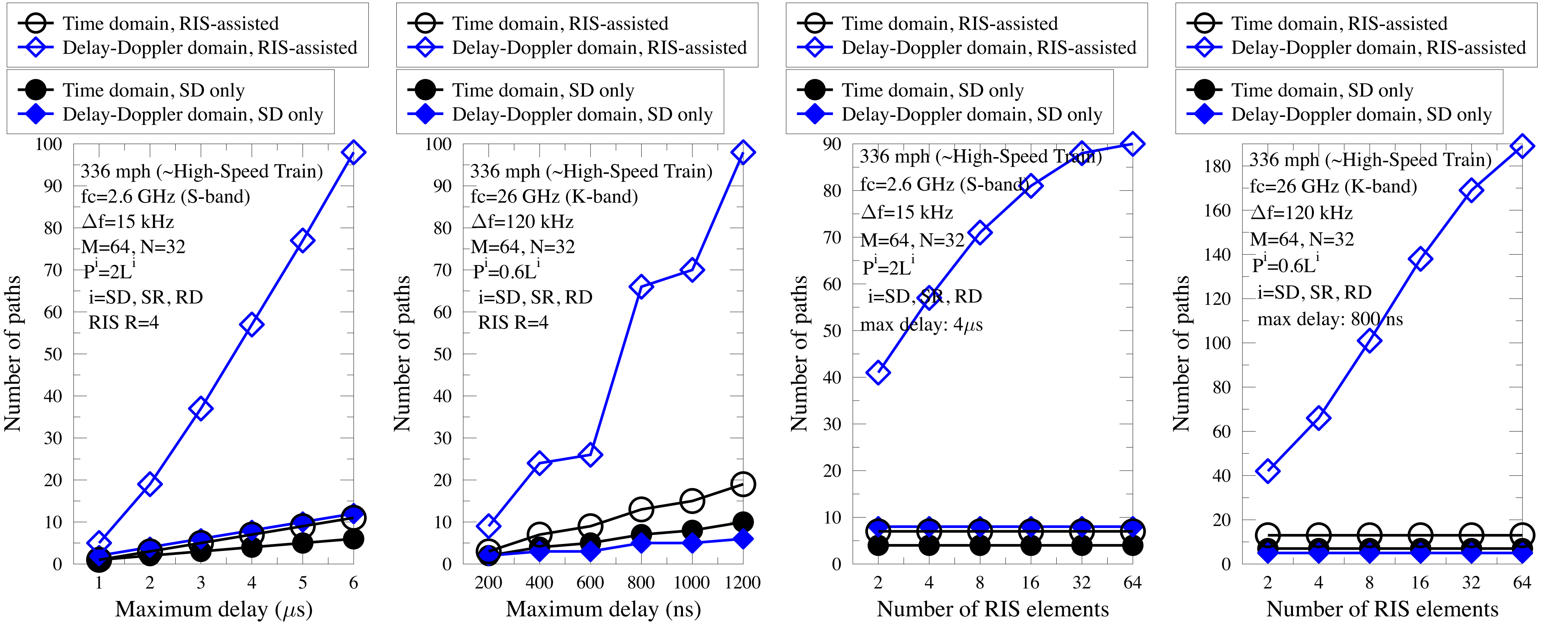
- Configure RIS based on LoS:

$$k^{\mathsf{RIS}_r} = k_0^{\mathsf{SD}} - k_0^{\mathsf{SR}_r} - k_0^{\mathsf{RD}_r}, \quad \angle \widetilde{\alpha}^r = \angle \widetilde{h}_0^{\mathsf{SD}} - \angle \widetilde{h}_0^{\mathsf{SR}_r} \widetilde{h}_0^{\mathsf{RD}_r}$$

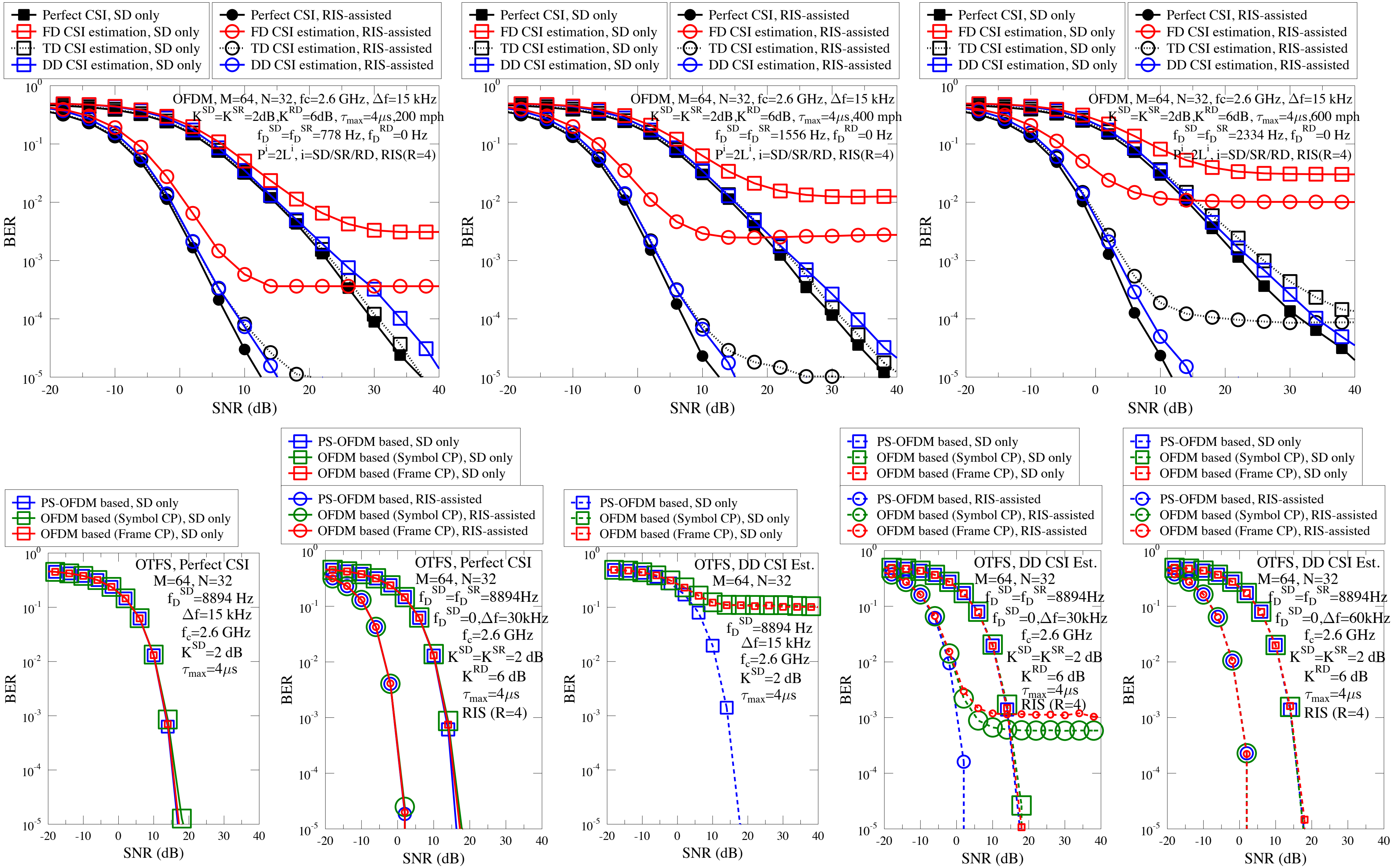
  - LoS tap of SD link: $h_{n,m,0}^{\mathsf{SD,LoS}} = \sqrt{\frac{K^{\mathsf{SD}}}{K^{\mathsf{SD}}+1}} e^{\frac{j2\pi\vartheta^{\mathsf{SD,LoS}}(nM+m)}{M\Delta f}} = \widetilde{h}_0^{\mathsf{SD}} w_{MN}^{k_0^{\mathsf{SD}}(nM+m)}$.
  - LoS tap of SR link: $h_{n,m,0}^{\mathsf{SR}_r,\mathsf{LoS}} = \sqrt{\frac{K^{\mathsf{SR}}}{K^{\mathsf{SR}}+1}} [\mathbf{a}_{\mathsf{RIS\text{-}AoA}}]_r w_{MN}^{k_0^{\mathsf{SR}_r}(nM+m)} = \widetilde{h}_0^{\mathsf{SR}_r} w_{MN}^{k_0^{\mathsf{SR}_r}(nM+m)}$.
  - LoS tap of RD link: $h_{n,m,0}^{\mathsf{RD}_r,\mathsf{LoS}} = \sqrt{\frac{K^{\mathsf{RD}}}{K^{\mathsf{RD}}+1}} [\mathbf{a}_{\mathsf{RIS\text{-}AoD}}]_r w_{MN}^{k_0^{\mathsf{RD}_r}(nM+m)} = \widetilde{h}_0^{\mathsf{RD}_r} w_{MN}^{k_0^{\mathsf{RD}_r}(nM+m)}$.

OTFS for RIS: Increased number of paths

- The maximum delay indices of SD, SR and RD links: L^{SD}, L^{SR}, L^{RD}. The maximum delay of the RIS-assisted system: $L = \max(L^{\text{SD}}, L^{\text{SR}} + L^{\text{RD}} - 1)$.
- The maximum Doppler indices of SD, SR and RD links: $k^{\text{SD}}_{\text{max}}$, $k^{\text{SR}}_{\text{max}}$, $k^{\text{RD}}_{\text{max}}$. The maximum Doppler index of RIS configuration: $k^{\text{RIS}}_{\text{max}} = k^{\text{SD}}_{\text{max}} + k^{\text{SR}}_{\text{max}} + k^{\text{RD}}_{\text{max}}$. The maximum Doppler index of the RIS-assisted system: $k_{\text{max}} = k^{\text{SR}}_{\text{max}} + k^{\text{RD}}_{\text{max}} + k^{\text{RIS}}_{\text{max}}$.
- The total number of resolvable paths in delay-Doppler domain: $P^{\text{SD}} \leq L^{\text{SD}}(2k^{\text{SD}}_{\text{max}} + 1)$ without RIS and $P^{\text{SD}} \leq L(2k_{\text{max}} + 1)$ with RIS.



OTFS for RIS: Channel Estimation



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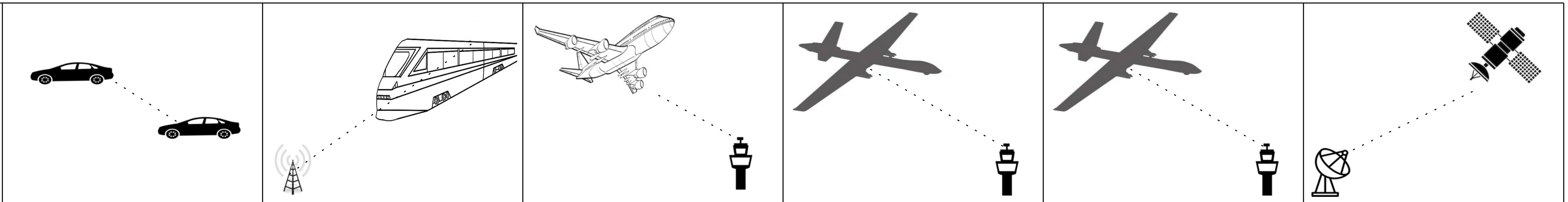
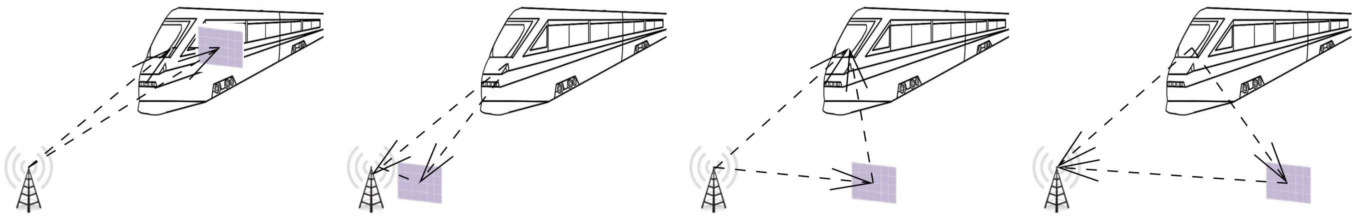


	Vehicle–to–Vehicle	Train	Civial Aviation	Supersonic UAV	Hypersonic UAV	Low Earth Orbit
Distance	300 m	500 m	10 km	20 km	20 km	2000 km
Speed	100 mph	336 mph	671 mph	Mach 3	Mach 12	Mach 25
Ricean K	$K^{\mathrm{SD}} = -3$ dB $K^{\mathrm{SR}} = -3$ dB $K^{\mathrm{RD}} = 3$dB	$K^{\mathrm{SD}} = -3$ dB $K^{\mathrm{SR}} = -3$ dB $K^{\mathrm{RD}} = 3$ dB	$K^{\mathrm{SD}} = 3$ dB $K^{\mathrm{SR}} = 3$ dB $K^{\mathrm{RD}} = 6$ dB	$K^{\mathrm{SD}} = -3$ dB $K^{\mathrm{SR}} = -3$ dB $K^{\mathrm{RD}} = 6$ dB	$K^{\mathrm{SD}} = -3$ dB $K^{\mathrm{SR}} = -3$ dB $K^{\mathrm{RD}} = 6$ dB	$K^{\mathrm{SD}} = 2$ dB $K^{\mathrm{SR}} = 2$ dB $K^{\mathrm{RD}} = 6$ dB
$\tau_{\max}$	S–band: 4000 ns K–band: 800 ns	S–band: 4000 ns K–band: 800 ns	S–band: 600 ns K–band: 250 ns	S–band: 600 ns K–band: 250 ns	S–band: 400 ns K–band: 120 ns	S–band: 100 ns K–band: 40 ns
P^i i=SD/SR /RD	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.6\tau_{\max}}{\Delta f} \rceil$	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.6\tau_{\max}}{\Delta f} \rceil$	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.6\tau_{\max}}{\Delta f} \rceil$	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.4\tau_{\max}}{\Delta f} \rceil$	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.4\tau_{\max}}{\Delta f} \rceil$	S–band: $P^i = \lceil \frac{2\tau_{\max}}{\Delta f} \rceil$ K–band: $P^i = \lceil \frac{0.4\tau_{\max}}{\Delta f} \rceil$
Δf	S–band: 15 kHz K–band: 60 kHz	S–band: 15 kHz K–band: 120 kHz	S–band: 30 kHz K–band: 240 kHz	S–band: 60 kHz K–band: 960 kHz	S–band: 240 kHz K–band: 3840 kHz	S–band: 480 kHz K–band: 7680 kHz



(a) Near–field, downlink
$f_D^{\mathrm{SD}} = f_D^{\mathrm{SR}}, f_D^{\mathrm{RD}} = 0$
$K^{\mathrm{SD}} = K^{\mathrm{SR}} < K^{\mathrm{RD}}$

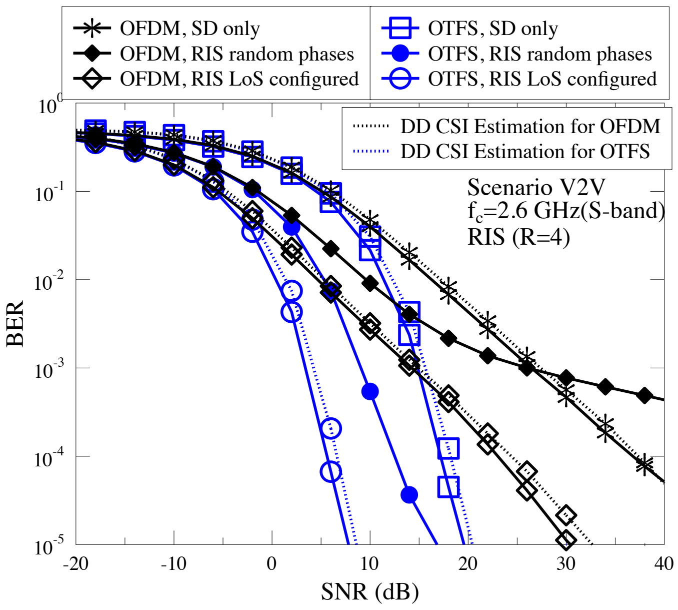
(b) Near–field, uplink
$f_D^{\mathrm{SD}} = f_D^{\mathrm{SR}}, f_D^{\mathrm{RD}} = 0$
$K^{\mathrm{SD}} = K^{\mathrm{SR}} < K^{\mathrm{RD}}$

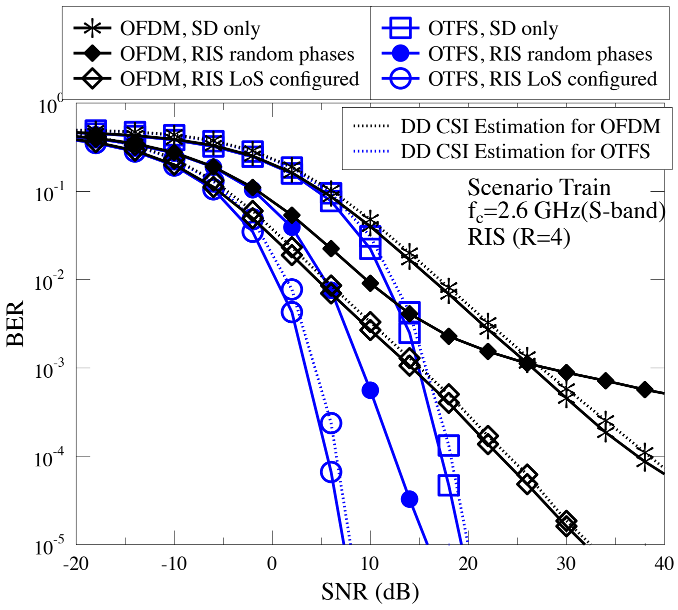
(c) Far–field, downlink
$f_D^{\mathrm{SD}} = f_D^{\mathrm{RD}}, f_D^{\mathrm{SR}} = 0$
$K^{\mathrm{SD}} < K^{\mathrm{SR}} = K^{\mathrm{RD}}$

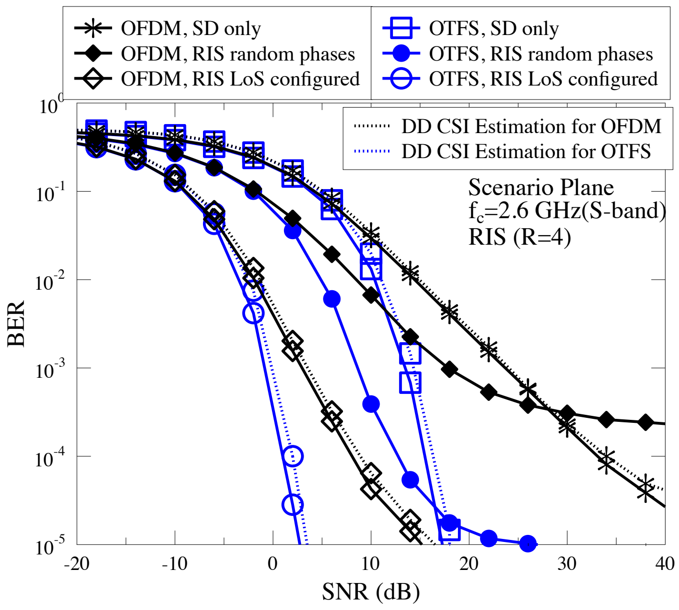
(d) Far–field, uplink
$f_D^{\mathrm{SD}} = f_D^{\mathrm{SR}}, f_D^{\mathrm{RD}} = 0$
$K^{\mathrm{SD}} < K^{\mathrm{SR}} = K^{\mathrm{RD}}$

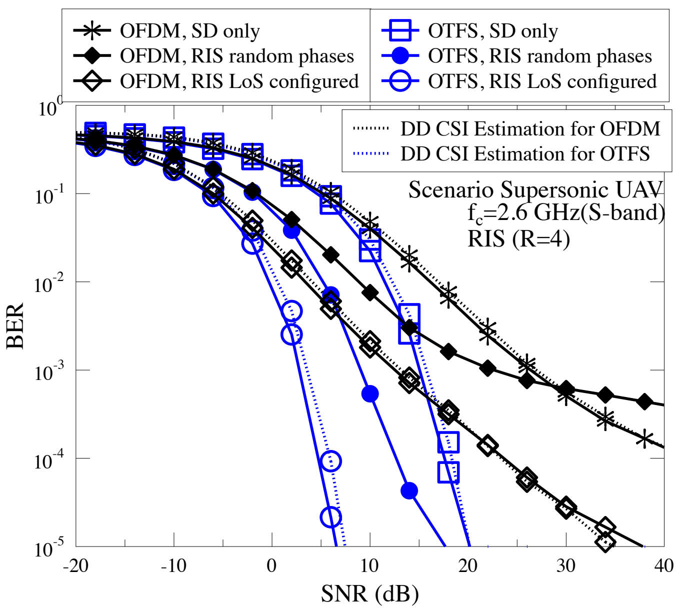
- Common assumptions
  - $L^i = \frac{\tau_{\text{max}}}{\Delta f}$, $i =$SD/SR/RD.
  - 5G FR1: 0.8 GHz (UHF-band), 1.5 GHz (L-band), 2.6 GHz (S-band) and 4.7 GHz (C-band) share the same parameters for τ_{max}, L^i and P^i.
  - 5G FR2: 26 GHz (K-band) and 28.5 GHz (Ka-band) share the same parameters for τ_{max}, L^i and P^i.
  - SCS Δf is adjusted for different carriers f_c to facilitate DD channel estimation, i.e. $\Delta f > 2f_D^{\text{max}}$.
- Assumptions for near-field
  - RIS is considered part of the transmitter or the receiver.
  - The direct link and the reflected links have approximately the same path loss for $d^{SD} \approx d^{SR} + d^{RD}$.
- Assumptions for far-field
  - Coordinates: $(x_S, y_S, z_S) = (0, 0, 0)$, $(x_R, y_R, z_R) = (500, 4, 0)$, $(x_D, y_D, z_D) = (500, -2, 0)$.
  - Path loss of SD/SR/RD link: PL=$-10 \log_{10} \gamma \log_{10} d - 20 \left(\frac{4\pi}{\lambda}\right) + G_e^{Tx} + G_e^{Rx}$, where antenna gain is available at the BS and user $G_e = \frac{4\pi A_e}{\lambda^2}$ with aperture $A_e^{BS} = 80$cm^2 and $A_e^{user} = 40$cm^2.
  - $K^{\text{SD}} = -6$ dB, $K^{\text{SR}} = K^{\text{RD}} = 3$ dB.
  - Path loss factors $\gamma^{SD} = 3.8$, $\gamma^{SR} = \gamma^{RD} = \gamma^{SRD} = 2.0$.
  - Receiver sensitivity: -174 dBm/Hz.

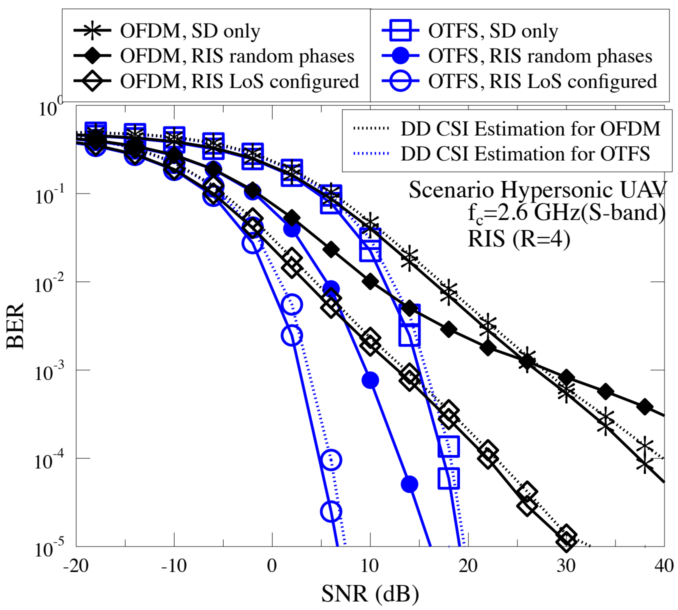
OTFS for RIS (Near-Field): SAGIN Scenarios (S-band)

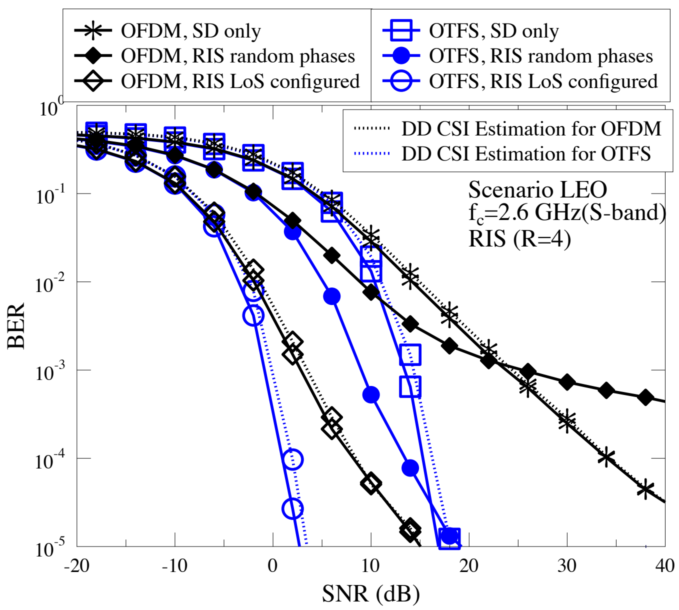




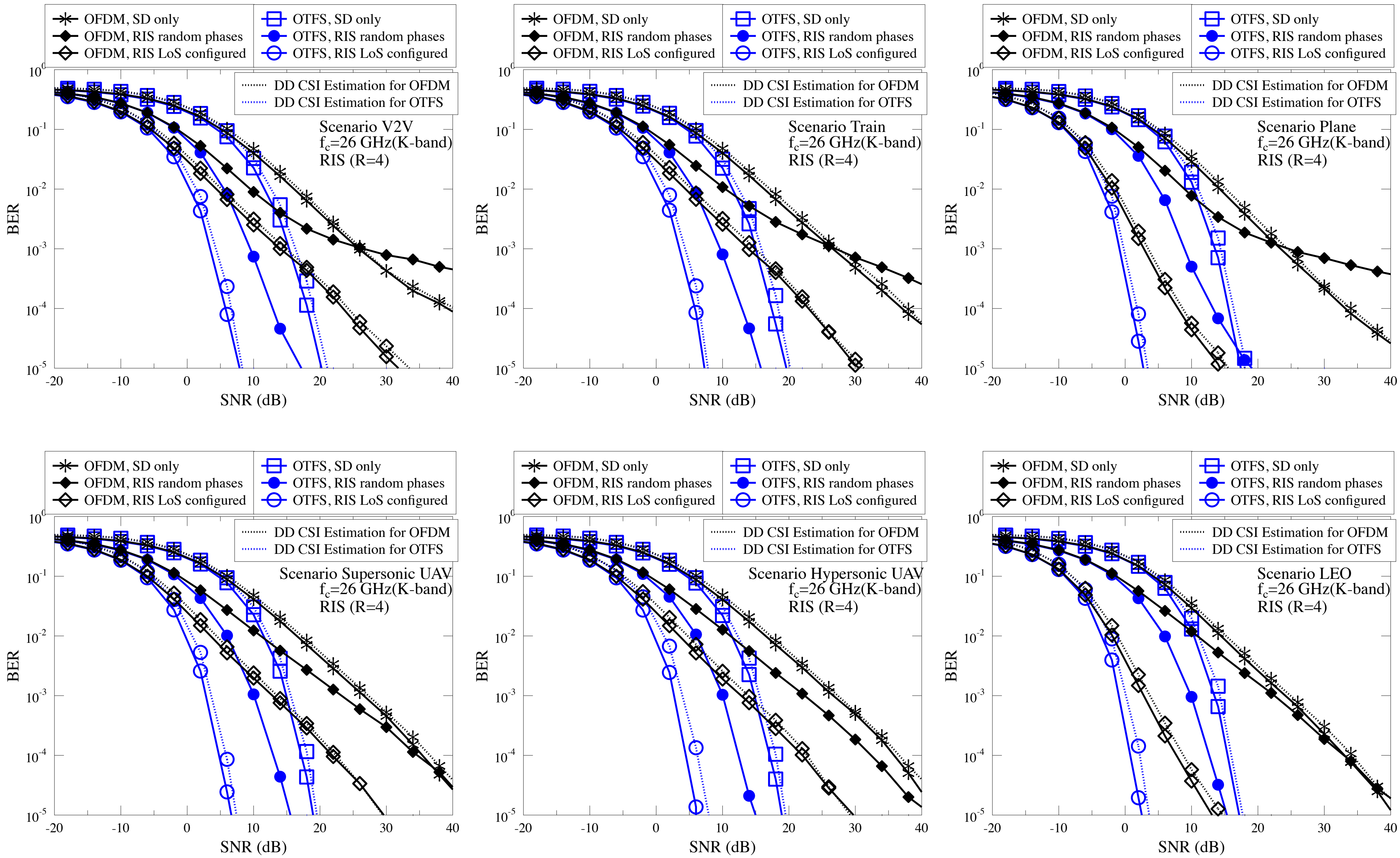








OTFS for RIS (Near-Field): SAGIN Scenarios (K-band)



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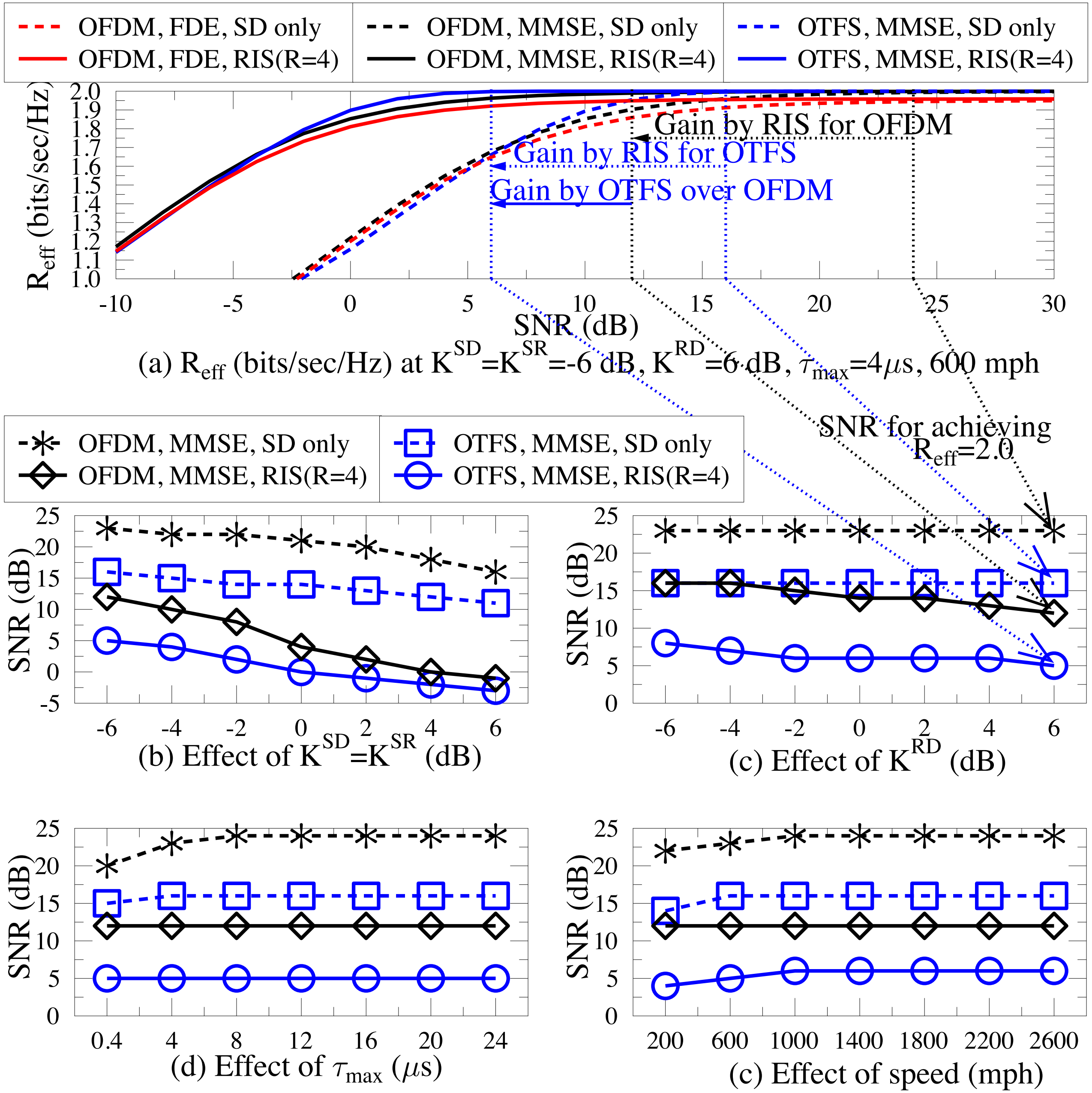
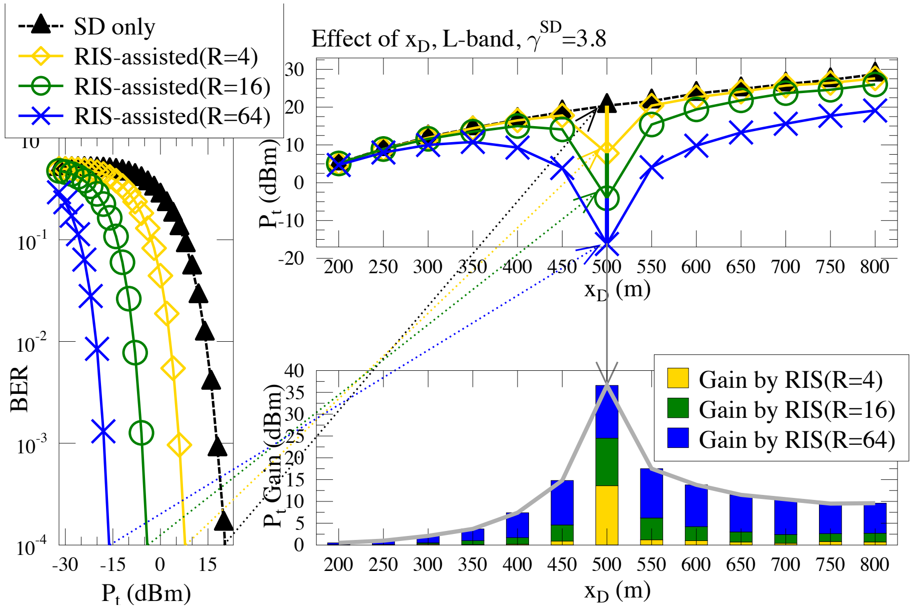
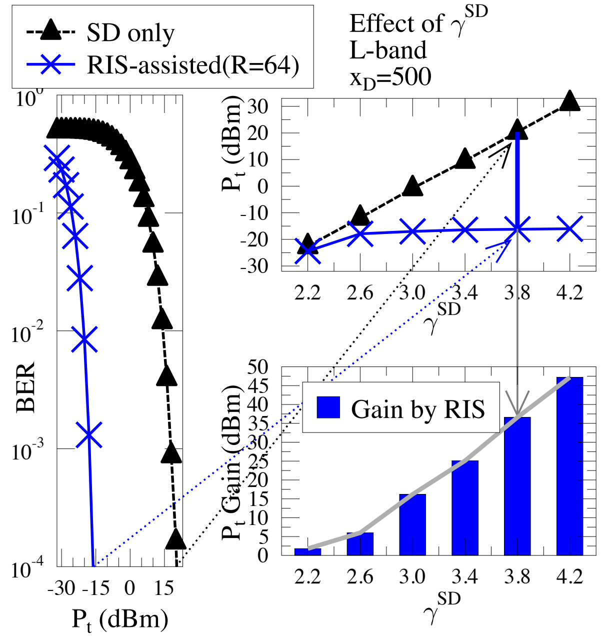
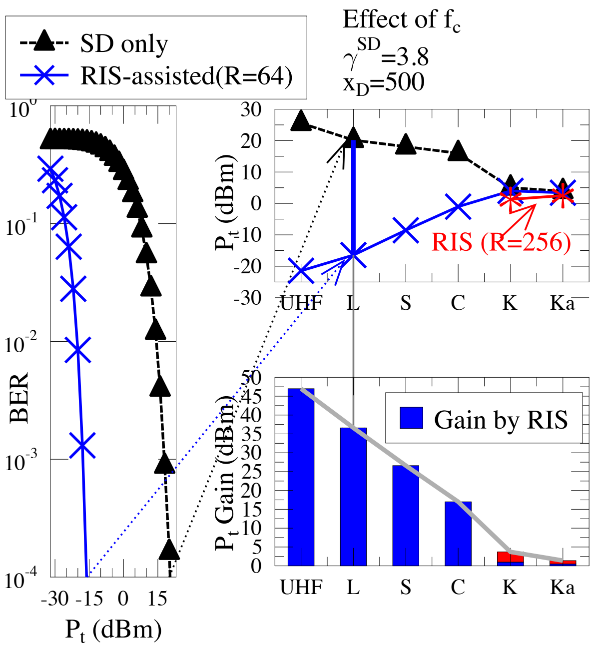
OTFS for RIS (Near-Field): Effective Throughput

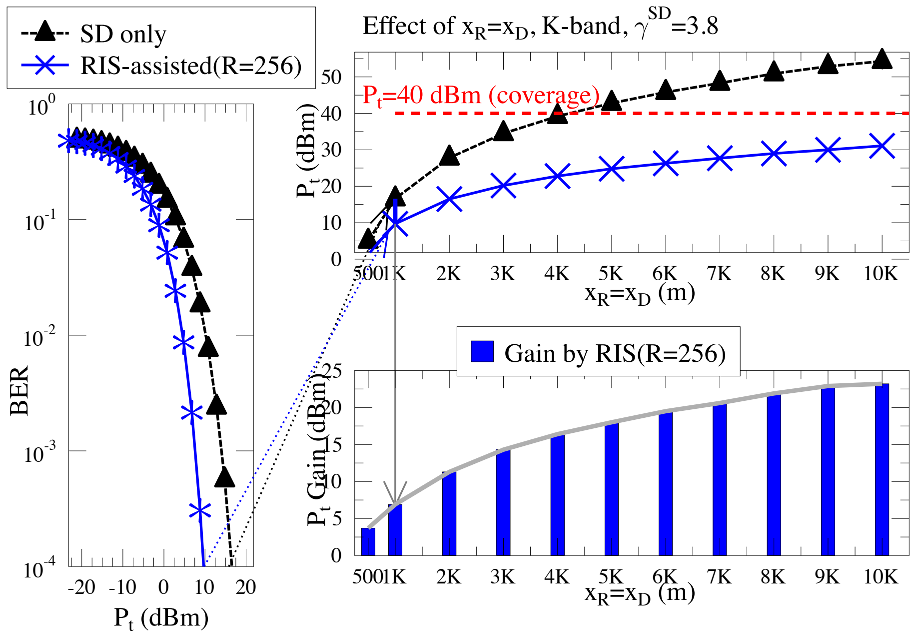


OTFS for RIS (Far-Field)









Thank You !