UNIVERSITY OF SOUTHAMPTON

ABSTRACT

FACULTY OF ENGINEERING, SCIENCE & MATHEMATICS

SCHOOL OF ENGINEERING SCIENCES

Doctor of Philosophy

AN INVESTIGATION OF LAGRANGIAN RIEMANN METHODS INCORPORATING MATERIAL STRENGTH

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The application of Riemann Methods formulated in the Lagrangian reference frame to the numerical simulation of non-linear events in solid materials is investigated. Here, solids are characterised by their ability to withstand shear distortion since they possess material strength. In particular, numerical techniques are discussed for simulating the transient response of solids subjected to extreme loading. In such circumstances, the response of solids will often be highly non-linear, displaying elastic and plastic behaviour, and even moderate compressions will produce strong shock waves.

This work reviews the numerical schemes or 'hydrocodes' which have been adopted in the past in order to simulate such systems, identifying the advantages and limitations of such techniques. One of the most prominent limitations of conventional Lagrangian methods is that the computational mesh or grid has fixed-connectivity i.e. mesh nodes are connected to the same nodes for all time. This has significant disadvantages since the computational mesh can easily become tangled as the simulated material distorts. The majority of conventional hydrocodes are also constructed using outdated artificial viscosity schemes which are known to diffuse shock waves and other steep features which may be present in the solution.

In the work presented here, a novel two-dimensional Lagrangian solver has been developed *Vucalm-EP* which overcomes many of the limitations of conventional techniques. By employing the Free-Lagrange Method, whereby the connectivity of the computational mesh is allowed to evolve as the material distorts, problems of arbitarily large deformation can be simulated. With the implementation of a spatially second-order accurate, finite-volume, Godunov-type solver, non-linear waves such as shocks are represented with higher resolution than previously possible with contemporary schemes. The *Vucalm-EP* solver simulates the transient elastic-perfectly plastic response of solids and displays increased accuracy over alternative Lagrangian techniques developed to simulate large material distortion such as Smoothed particle Hydrodynamics (SPH). Via a variety of challenging numerical simulations the *Vucalm-EP* solver is compared with contemporary Euler, fixed-connectivity Lagrangian, and meshless SPH solvers. These simulations include the solution of one- and two-dimensional shock tube problems in aluminium, simulating the collapse of cylindrical shells and modelling high-velocity projectile impacts. Validation against previously published results, solutions obtained using alternative numerical techniques and analytical models illustrates the versatility and accuracy of the technique. Thus, the *Vucalm-EP* solver provides a numerical scheme for the Lagrangian simulation of extensive material distortion in materials with strength, which has never previously been possible with mesh-based techniques.