

Worksheet Correlators in Black Hole Microstates

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(Dated: March 25, 2022)

Light probes interacting with heavy bound states such as black holes give rise to interesting observables containing valuable dynamical information. Recently, a family of black hole microstates was shown to admit an exact string worldsheet description. We construct the physical vertex operators of these models, and derive their form in the low-energy AdS₃ limit. We then compute an extensive set of novel heavy-light correlators in these backgrounds. A subset of these provide the first match between worldsheet correlators in black hole microstates and the holographically dual conformal field theory, in a highly non-trivial way. We further conjecture a closed formula for heavy-light correlators with an arbitrary number of light insertions. As an application, we compute the analogue of the Hawking radiation rate for these microstates.

Introduction.—In recent years we have seen ground-breaking observations of black holes, as well as a sharpening of problems that arise in their quantum description, in particular the information paradox [1, 2]. There are strong reasons to expect that black hole evaporation is unitary. However, a microscopic description of the quantum physics of general black holes, including singularity resolution and evaporation, is presently out of reach.

In String Theory, black holes are bound states of strings and branes with an exponential number of internal microstates. Large classes of such microstates are well-described by smooth horizonless supergravity solutions [3–6], whose intricate horizon-scale structure appears black-hole-like for most purposes, while nevertheless allowing for potentially observable signatures [7, 8]. Holography provides strong evidence for the microscopic interpretation of these solutions [9–12].

Studying how light probes interact with a heavy background such as a black hole microstate yields valuable dynamical information, for instance about the unitary microscopic process underlying black hole evaporation. In holographic models, such processes can be understood in terms of correlators in the dual field theory involving two heavy operators and a number of light insertions, often referred to as heavy-light (HL) correlators [13, 14].

While supergravity solutions are very useful descriptions, there is a wealth of string-theoretic physics that they do not capture. There are growing expectations that such stringy physics will be necessary to obtain a complete description of black hole microstructure. Recently, a novel set of worldsheet models describing black hole microstates has been constructed and studied [15–

19]. A subset of these models are exactly solvable gauged Wess-Zumino-Witten (WZW) models. They provide a rare and valuable window into stringy physics in highly-excited backgrounds such as black hole microstates. In this Letter we construct the physical vertex operators of these models, and initiate the systematic study of their correlation functions. These describe the dynamics of perturbative strings in the given heavy backgrounds, and are exact in α' . The main building blocks are given by correlators based on $SL(2, \mathbb{R})$ and $SU(2)$. The former have a complicated structure due to spectral flow [20–22], on which progress has been made recently [23, 24].

There are two instances of holography relevant to the models we study. On the gravity side, in the UV we have linear-dilaton NS5-brane asymptotics, and the dual theory is Little String Theory (LST) [25, 26]. Thus correlators of the vertex operators we construct holographically define LST amplitudes. In the IR, we have an AdS₃/CFT₂ duality. We will refer to the holographic CFT₂ as the HCFT.

Due to the gauging, the identification of the coordinates on the boundary of AdS₃ is subtle. By carefully making this identification, we derive all HLLH correlators for light operators in chiral multiplets, in the AdS₃ limit. We find exact and highly non-trivial agreement with the small subset of known examples [13, 27, 28]. We further identify the precise relation with the HCFT at the symmetric orbifold locus. Moreover, we conjecture a closed-form expression for all higher-point HL correlators on these backgrounds, Eq. (15) below.

As an application, we compute the emission rate for the unitary analogue of Hawking radiation from these

backgrounds. Further applications of our results include studying the Penrose process [29] and scrambling of in-falling perturbations [30]. Our results also lay foundations for analyzing the stringy phenomenology of black hole microstates, which will be an important endeavour in the coming years.

Worldsheet models for black hole microstates.—

We work in type IIB string theory on $\mathbb{R}^{4,1} \times \mathbb{S}^1 \times \mathbb{T}^4$ with a microscopic \mathbb{T}^4 , and a macroscopic \mathbb{S}^1 with asymptotic radius R_y . We consider n_5 NS5-branes wrapped on $\mathbb{T}^4 \times \mathbb{S}^1$, n_1 F1-strings wound on \mathbb{S}^1 , and n_p units of momentum charge along \mathbb{S}^1 . We work in the NS5-brane decoupling limit, in which the string coupling g_s and the radial coordinate r scale to zero with g_s/r fixed; we choose units in which $\alpha' = 1$.

We consider a set of horizonless supergravity solutions collectively known as spectral flowed supertubes. The non-BPS solutions in this family are known as the JMaRT solutions [31], and the family also contains BPS [32–34] and/or two-charge solutions. These solutions can be recast in a simple form in terms of three integers m, n, k , associated to the angular momenta and the orbifold structure near $r = 0$, plus R_y [19].

Flowing further to the IR, one can take the AdS₃ decoupling limit, after which the geometries are related by a large gauge transformation to global $(\text{AdS}_3 \times \mathbb{S}^3)/\mathbb{Z}_k \times \mathbb{T}^4$. The holographically dual CFT states are heavy in the sense that their conformal dimensions scale linearly with the central charge. At the locus in moduli space at which the HCFT is realized as the symmetric product orbifold $\text{Sym}^N(\mathbb{T}^4)$ with $N = n_1 n_5$, the states dual to the JMaRT solutions are related to the k -twisted vacuum state by fractional spectral flow with parameters $(m \pm n)/k$ [35].

An exact worldsheet description of these configurations was constructed in [16]. The target space is the coset

$$\frac{\text{SL}(2, \mathbb{R}) \times \text{SU}(2) \times \mathbb{R}_t \times \text{U}(1)_y}{\mathbb{R} \times \text{U}(1)} \times \mathbb{T}^4, \quad (1)$$

where the gauging involves two null chiral currents,

$$\begin{aligned} J &= J^3 + s_+ K^3 + i\mu \partial_t + ik_+ \partial_y, \\ \bar{J} &= \bar{J}^3 + s_- \bar{K}^3 + i\mu \partial_t + ik_- \bar{\partial}_y, \\ s_{\pm} &= n \pm m, \quad k_{\pm} = \mp k R_y + n_5 m n / (k R_y), \end{aligned} \quad (2)$$

where J^3 and K^3 are the Cartan generators of $\text{SL}(2, \mathbb{R})$ and $\text{SU}(2)$, respectively, while $n_5(1 - s_{\pm}^2) + \mu^2 - k_{\pm}^2 = 0$ and $s_{\pm} \in 2\mathbb{Z} + 1$. Supersymmetry is preserved in the left/right sector when $|s_{\pm}| = |k_{\pm}/\mu| = 1$. The gauging effectively generates a $J\bar{J}$ marginal deformation on the worldsheet, similar to that of [36, 37].

Vertex operators.—The string spectrum contains both short and long strings, built upon the discrete and continuous series of $\text{SL}(2, \mathbb{R})$. In this Letter, we shall primarily focus on supergravity operators, dual to light op-

erators in chiral multiplets of the HCFT. The corresponding discrete series vertex operators with zero (worldsheet) spectral flows and y -winding are excitations of the center-of-mass wave-function $\Phi_0 = V_{j m \bar{m}} V'_{j' m' \bar{m}'} e^{i(-E t + P_y y)}$, where $V_{j m \bar{m}}$ is a bosonic $\text{SL}(2, \mathbb{R})$ primary field of spin j and J^3 eigenvalue m , and similarly for the $\text{SU}(2)$ vertex $V'_{j' m' \bar{m}'}$, while $P_y = n_y/R_y$ with $n_y \in \mathbb{Z}$. First-excited states must satisfy the Virasoro and gauge constraints

$$\begin{aligned} 0 &= [-j(j-1) + j'(j'+1)]/n_5 - (E^2 - P_y^2)/4, \quad (3) \\ 0 &= m + s_+ m' + (\mu E + k_+ P_y)/2, \\ 0 &= \bar{m} + s_- \bar{m}' + (\mu E + k_- P_y)/2, \quad (4) \end{aligned}$$

which couple probe and background angular momenta. Sub-families of solutions to these conditions were analyzed in [15, 16, 18].

We now construct the states polarized in the directions involved in the gauging. Importantly, the $\text{AdS}_3 \times \mathbb{S}^3$ isometries are broken in the full model, and only emerge in the IR. Thus, vertex operators need not have definite $\text{SL}(2, \mathbb{R})$ and $\text{SU}(2)$ spins or chiralities, which complicates the analysis. From now on we mostly write only holomorphic expressions, and assume normal ordering. The chiral BRST charge contains the usual terms plus additional contributions $\tilde{c}J$ and $\tilde{\gamma}\lambda$ associated to the gauging, where $\lambda = \psi^3 + s_+ \chi^3 + \mu \lambda^t + k_+ \lambda^y$.

In the NS sector, before gauging (“upstairs”) we start from a generic linear combination of excitations in the $\text{AdS}_3 \times \mathbb{S}^3 \times \mathbb{R}_t \times \mathbb{S}_y^1$. Of these eight polarizations, two are removed by the γG and $\tilde{\gamma}\lambda$ constraints. Another two are BRST-exact combinations of $G\Phi_0$ and $\lambda\Phi_0$. This gives the correct four transverse polarizations for a massless field in the six physical directions orthogonal to the gauging and the \mathbb{T}^4 . We obtain the physical operators

$$\begin{aligned} \mathcal{W} &= e^{-\varphi} (\psi_{\perp} V_j)_{j m} V'_{j' m'} e^{i(-E t + P_y y)}, \\ \mathcal{X} &= e^{-\varphi} V_{j m} (\chi_{\perp} V'_{j'})_{j' m'} e^{i(-E t + P_y y)}, \end{aligned} \quad (5)$$

where $(\psi_{\perp} V_j)$ and $(\chi_{\perp} V'_{j'})$ are generalizations of Eq. (A.6) in [38] with the replacements $\psi^3 \rightarrow \psi^3 + c^t \lambda^t + c^y \lambda^y$ and $\chi^3 \rightarrow \chi^3 + d^t \lambda^t + d^y \lambda^y$, where

$$c^t = \frac{d^t}{s_+} = \frac{n_5 P_y}{k_+ E + \mu P_y}, \quad c^y = \frac{d^y}{s_+} = -\frac{n_5 E}{k_+ E + \mu P_y}.$$

To take the AdS₃ limit, we define the rescaled energy $\mathcal{E} = E R_y$, and hold \mathcal{E} , n_y , and s_{\pm} fixed as $R_y \rightarrow \infty$. The gauging is then concentrated mostly in $\mathbb{R}_t \times \text{U}(1)_y$, and parametrically less in $\text{SL}(2, \mathbb{R}) \times \text{SU}(2)$. As a result, $c^{t,y}$, $d^{t,y}$ become $\mathcal{O}(R_y^{-1})$, and we recover the AdS₃ expressions.

In the R sector, before gauging, the polarizations involve a spin field that is a 64-component spinor of $O(10, 2)$, $S_{\varepsilon_1 \dots \varepsilon_6}$ with $\varepsilon_i = \pm 1$. Implementing GSO, we fix an overall spinor parity. The BRST procedure reduces the d.o.f. by a further factor of four, leaving the

correct eight physical polarizations. Again, the \mathbb{T}^4 does not participate in the gauging, so the corresponding signs $\varepsilon_{4,5}$ are constrained only by overall parity. Therefore, we consider an ansatz whose form we write schematically as

$$e^{-\frac{\varphi-\tilde{\varphi}}{2}} \sum_{\varepsilon_1, \varepsilon_2, \varepsilon} F_{\varepsilon_1 \varepsilon_2}^\varepsilon S_{\varepsilon_1 \dots \varepsilon_6} V_{j, m - \frac{\varepsilon_1}{2}} V'_{j', m' - \frac{\varepsilon_2}{2}} e^{i(-Et + P_y y)},$$

where we parametrize $\varepsilon_3 = \varepsilon \varepsilon_1 \varepsilon_2$ and $\varepsilon_6 = \varepsilon \varepsilon_4 \varepsilon_5$. The field $\tilde{\varphi}$ arises from the bosonization of the $\tilde{\beta}\tilde{\gamma}$ ghosts for null gauging. In the $(-\frac{1}{2}, \frac{1}{2})$ picture for $(\varphi, \tilde{\varphi})$, there are no exact states. The BRST procedure mixes the different ε -chirality sectors and leaves only two independent components in $F_{\varepsilon_1 \varepsilon_2}^\varepsilon$, say $F_{\pm\pm}^+$. Acting with $\tilde{\gamma}\lambda$ gives

$$\frac{F_{\varepsilon_1 \varepsilon_2}^-}{F_{\varepsilon_1 \varepsilon_2}^+} = i\sqrt{n_5} \frac{1 - \varepsilon_1 \varepsilon_2 s_+}{\mu + \varepsilon_4 \varepsilon_5 k_+} = \frac{i}{\sqrt{n_5}} \frac{\mu - \varepsilon_4 \varepsilon_5 k_+}{1 + \varepsilon_1 \varepsilon_2 s_+}, \quad (6)$$

which allows us to decouple the remaining constraints. The explicit relations will be given in a forthcoming companion work [39]. Finally, we can choose $F_{\pm\pm}^\pm$ by asking that the R states reduce to the usual AdS₃ ones in the IR. Interestingly, in this regime Eq. (6) implies that $F^\pm \sim 1$ and $F^\mp \sim \mathcal{O}(R_y^{-1})$ for $\varepsilon_4 \varepsilon_5 = \pm 1$. We thus recover definite ε -chirality states.

We derive the (worldsheet) spectral flowed versions of the above short string states analogously. For this, we use the same left/right spectral flow charge ω in SL(2, \mathbb{R}) and SU(2). The Virasoro condition is modified accordingly, while the gauge conditions should be understood in terms of $m_w = \pm(J_w + n)$, where J_w is the (flowed) spin and $n \in \mathbb{N}$. A residual discrete gauge symmetry allows us to recast ω units of spectral flow in terms of $k\omega$ units of y -winding [19].

Heavy-light correlators and spacetime basis.—

In the IR, correlators of the above vertex operators correspond to heavy-light n -point functions of the HCFT. In the remainder of this Letter we work in the AdS₃ limit. The characteristics of the heavy states are encoded in the construction of the coset theory (1) itself. Hence, we can compute $\langle O_H(0) O_L(1) \bar{O}_L(x) \bar{O}_H(\infty) \rangle$ HCFT four-point functions by computing worldsheet two-point functions.

So far we have constructed vertex operators in the representation in which the Cartan currents are diagonalized, which provides the natural framework for our gauging procedure. Their correlators are the gauge-invariant subset of the so-called m -basis correlators of the upstairs theory, where the quantum numbers are related through Eq. (4) [40, 41]. In particular the two-point functions factorize, the only non-trivial contributions being the SL(2, \mathbb{R}) propagators [21].

Holographic applications of AdS₃ string models involve the conjugate x -basis, in which the SL(2, \mathbb{R}) currents act as differential operators associated to the spacetime Virasoro generators. The complex label x is identified with the coordinate of the HCFT. More explicitly, in global AdS₃, given an operator $\mathcal{V}_{hm\bar{m}}$ of spacetime weight h ,

with $m = h + n$, $\bar{m} = h + \bar{n}$, $n, \bar{n} \in \mathbb{N}$, one defines $\mathcal{V}_h(x) = \sum_{m, \bar{m}} x^{m-h} \bar{x}^{\bar{m}-h} \mathcal{V}_{hm\bar{m}}$ [20, 21]. Here $\mathcal{V}_{hm\bar{m}}$ contains any excitations, such as fermions or spin fields. However, in the gauged models (1), the identification of the x variable is obscured by the gauging, since J^\pm do not commute with the BRST charge.

We now show that a careful reconstruction of the local basis in these cosets leads to a considerable set of interesting new results, a subset of which match very non-trivially to two families of known HLLH correlators.

We choose a gauge in which the upstairs SL(2, \mathbb{R}) time and angular direction are fixed. Then, importantly, t/R_y and y/R_y parametrize the asymptotic boundary of the downstairs AdS₃ at a fixed point on the \mathbb{S}^3 . We define $m_y = (\mathcal{E} + n_y)/2$ and $\bar{m}_y = (\mathcal{E} - n_y)/2$, and interpret these as the asymptotic AdS₃ mode labels of the cosets.

At large R_y , the modified Virasoro condition (3) is solved by the usual relation $j = j' + 1 + \mathcal{O}(R_y^{-2})$. Although the weights of the t, y -exponentials scale as $\mathcal{O}(R_y^{-2})$, the associated quantum numbers \mathcal{E} and n_y enter non-trivially in the leading-order gauge conditions (4) because $\mu \sim k_+ \sim -k_- \sim -kR_y$. Hence, the vertex operators reduce to the familiar AdS₃ ones, however their quantum numbers must satisfy the gauge conditions

$$0 = m + s_+ m' - k m_y = \bar{m} + s_- \bar{m}' - k \bar{m}_y. \quad (7)$$

Therefore, m_y and \bar{m}_y take values in \mathbb{Z}/k . Crucially, momentum quantization along the y -circle imposes that $m_y - \bar{m}_y \in \mathbb{Z}$. This implies that local operators are built only out of the subset of SL(2, \mathbb{R}) modes that satisfy

$$m - \bar{m} \equiv s_+ m' - s_- \bar{m}' \pmod{k}. \quad (8)$$

Since m_y and \bar{m}_y label the spacetime modes, we define the x -basis operators in the coset as

$$O_h(x) = \frac{1}{k^{2h}} \sum_{m_y, \bar{m}_y} x^{m_y - h} \bar{x}^{\bar{m}_y - h} \mathcal{V}_{hm\bar{m}} \mathcal{V}'_{h'm'\bar{m}'}, \quad (9)$$

where h' is related to h , and the left- and right-handed quantum numbers are constrained by Eqs. (7)–(8). Coset HLLH correlators are accordingly expanded as power series in x, \bar{x} , where the coefficients are the m -basis two-point functions.

We now have everything in place to compute a large set of correlators. We first present a simple example involving the untwisted $h = 1/2$ chiral primary of the orbifold CFT, denoted by $O_L = O^{++}$. This is associated to a RR worldsheet state for which the SL(2, \mathbb{R}) mode propagators are trivial. We write $s_+ = 2s + 1$, $s_- = 2\bar{s} + 1$, which makes explicit that s_\pm are odd, and connects to the notation of previous works. We decompose $s = pk + \hat{s}$ and $\bar{s} = \bar{p}k + \tilde{s}$ with $0 \leq \{\hat{s}, \tilde{s}\} < k$, and define $\delta = \tilde{s} - \hat{s}$. Then we obtain

$$\langle O^{++}(1) O^{--}(x) \rangle_H = \frac{1}{k^2} \frac{(1-x)|x|^{\delta/k} + (1-\bar{x})|x|^{-\delta/k} - |1-x|^2 |x|^{-|\delta|/k}}{x^{(2s+\delta)/2k} \bar{x}^{(2\bar{s}-\delta)/2k} |x| |1-x|^2 (1-|x|^{2/k})}, \quad (10)$$

where $\langle \dots \rangle_H$ stands for the two-point function between heavy states $O_H(0)|0\rangle = |s_\pm, k\rangle$. Eq. (10) matches the results of [13, 27, 42] for the appropriate values of s_\pm and k . It also significantly extends these results to the full set of non-BPS JMaRT backgrounds.

HL correlators and the $\text{Sym}^N(\mathbb{T}^4)$ CFT.— By enforcing the constraint (8) using a Kronecker comb, $\sum_{q \in \mathbb{Z}} \delta_{m-\bar{m}, kq} = k^{-1} \sum_{r=0}^{k-1} e^{2\pi i r \frac{m-\bar{m}}{k}}$, the sum in (9) over *restricted* powers of the physical cross-ratio x becomes a sum over *unrestricted* powers of its k^{th} roots, i.e. a sum over powers of all u such that $u^k = x$. Then the RHS of Eq. (10) takes the simple and elegant form

$$\langle O^{++}(1)O^{--}(x) \rangle_H = \frac{1}{k^3} \sum_{u^k=x} \frac{u^{-\frac{s_+}{2}} \bar{u}^{-\frac{s_-}{2}} |u|^{1-k}}{|1-u|^2}. \quad (11)$$

This formula is extremely instructive. It makes the relation with symmetric orbifold physics more transparent, and is readily generalized to light insertions with higher weights h and generic R-charges. Indeed, the appearance of the k^{th} roots of x invites a comparison with dual HCFT computations at the symmetric orbifold point using the Lunin-Mathur covering space method [43, 44].

We now consider a worldsheet operator with definite charge m' , and express the coset spacetime modes as $m_y = x\partial_x + h$. Proceeding similarly for the *upstairs* $\text{SL}(2, \mathbb{R})$ modes $m = u\partial_u + h - \beta$, for the auxiliary variable u and shift β , we see that in order to obtain $u^k = x$, we should fix β and the analogous right-moving $\bar{\beta}$ to be

$$\beta = h(1-k) + s_+ m', \quad \bar{\beta} = h(1-k) + s_- \bar{m}'. \quad (12)$$

Then the worldsheet operator (9) is re-expressed as

$$O_h(x) = \frac{1}{k^{2h+1}} \sum_{u^k=x} u^\beta \bar{u}^{\bar{\beta}} \mathcal{V}_h(u) \mathcal{V}'_{h'm'\bar{m}'}. \quad (13)$$

The role of β is two-fold: it gives the Jacobian factor for the coordinate change from x to u , and also the appropriate rescaling under spectral flow [27]. Our worldsheet computation then shows that HLLH correlators with generic light insertions in chiral multiplets take the surprisingly simple form

$$\langle O_L(x_1) \bar{O}_L(x_2) \rangle_H = \frac{1}{k^{4h+2}} \sum_{u_i^k=x_i} \frac{u_1^{\beta_1} \bar{u}_1^{\bar{\beta}_1} u_2^{\beta_2} \bar{u}_2^{\bar{\beta}_2}}{|u_1 - u_2|^{4h}}. \quad (14)$$

For untwisted light operators, it is natural to identify u with the coordinate on the k -fold covering space that trivializes the twist operators involved in the definition of the heavy states. This coincides precisely with the symmetric orbifold covering space method for such correlators. In the presence of twist- k operators, modes take values in \mathbb{Z}/k [44]. Moreover, when mapping S_k -invariant untwisted operators to the k -fold covering space, one obtains an expression analogous to Eq. (13).

By contrast, for twisted operators, the interpretation of our result (14) is more involved: the covering map for such correlators is *not* $x = u^k$. To exhibit the precise relation, we focus on the singlet marginal deformation operator of twist two, and take $s_\pm = 0$. More general light operators and values of s_\pm can be treated analogously. The relevant four-point function was studied recently in [28, 45, 46] in the $\text{Sym}^N(\mathbb{T}^4)$ CFT. At large N the correlator is dominated by a contribution from a covering space with genus zero. The relation between the physical-space cross-ratio and its covering-space images is $x = (u+1)^{2k}(u-1)^{-2k}$. The relevant correlator takes the form $G(u) = u^{-2}(u+1)^{2+2k}(u-1)^{2-2k}$. The final result is obtained by summing over the $2k$ pre-images of x , and including the appropriate combinatorial factor. However, due to the $u \rightarrow 1/u$ symmetry of the map, there are actually only k distinct contributions. Upon inserting the explicit solutions $u = u(x)$, the final expression remarkably coincides precisely with (14).

From the point of view of the worldsheet theory, there is no strong distinction between vertex operators associated to different twist sectors of the HCFT at the orbifold point, and our general result depends only on their weight and charges. In this sense, the only *covering space* that appears is that associated to the heavy operators $x = u^k$.

Moreover, given the structure of our worldsheet amplitudes, we conjecture that HL correlators with n light chiral-multiplet insertions of weights h_i and charges m'_i, \bar{m}'_i are given by

$$\langle O_1(x_1) \dots O_n(x_n) \rangle_H = \frac{1}{k^{2H+n}} \sum_{u_i^k=x_i} \left(\prod_{i=1}^n u_i^{\beta_i} \bar{u}_i^{\bar{\beta}_i} \right) \langle O_1(u_1) \dots O_n(u_n) \rangle, \quad (15)$$

with $\beta_i, \bar{\beta}_i$ as in Eq. (12), and $H = h_1 + \dots + h_n$. In this way, these HL correlators are directly determined from the *vacuum* n -point function evaluated at the k^{th} roots of the physical insertion points. Interestingly, this suggests that the large N fusion rules for these correlators within our heavy states coincide with the vacuum ones, including those involving spectrally flowed states.

Hawking radiation from the worldsheet.— The asymptotically flat JMaRT solutions emit ergoregion radiation [47]. This has been interpreted as an enhanced analogue of Hawking radiation, since both are described by the same microscopic process in the HCFT [27, 48]. As an application, we now rederive the amplitude controlling the rate of emission of supergravity quanta from the JMaRT solutions. This is obtained by taking $x_2 \rightarrow 0$ and $x_1 = x$ in Eq. (14), obtaining

$$\mathcal{A}(x) = \frac{1}{k^{2h}} \frac{\sum_{\ell \in \mathbb{Z}} \delta_{s_+ m' - s_- \bar{m}', k\ell}}{x^{h(1+\frac{1}{k}) - m' \frac{s_+}{k}} \bar{x}^{h(1+\frac{1}{k}) - \bar{m}' \frac{s_-}{k}}}. \quad (16)$$

The numerator ensures the correct spectrum of emission. This amplitude gives a direct worldsheet derivation of the emission rates computed in [27, 35, 42, 48].

Discussion.— We have obtained closed-form expressions for a comprehensive family of HL correlators of the HCFT by using the exact worldsheet description of the JMaRT black hole microstates. We did so by carefully identifying the correct spacetime modes and the local coordinate on the asymptotic AdS_3 boundary emerging in the IR. Our findings are a significant improvement over previous work, providing a powerful general method that is much less computationally intensive than the Lunin-Mathur covering space method in the HCFT, as well as a set of results that substantially expand upon the small number of earlier case-by-case studies.

The match we found to the HCFT for particular sub-cases of our results (10) and (14) is highly non-trivial. Perhaps most strikingly, even for the non-supersymmetric amplitudes (16), we find a precise agreement with the HCFT at the orbifold point. There is no known non-renormalization theorem protecting the latter amplitudes. However the matching we find is consistent with previous results, and is almost certainly due to the relation between these heavy states and the NS vacuum involving orbifolding and fractional spectral flow [35].

Our results open up many interesting directions for future work. Eqs. (14) and (15) can be used to obtain a deeper understanding of factorization channels, conformal blocks and anomalous dimensions of the HCFT [28, 46]. Furthermore, the vertex operators we have constructed will enable the explicit study of correlators in the full coset models. In the UV, the holographically dual LST is non-local, and we see that our construction of the x coordinate breaks down, as it must. Computing these correlation functions might very well take us one step closer to flat space holography [49].

Finally, our methods can be used to study a wealth of black hole microstate physics including the Penrose process [29], tidal forces [30] and multipole ratios [7], all of which can be probed with our higher-point functions. Our work lays the foundations for computing more general heavy-light correlators from the string worldsheet, which will contribute to the emerging programme of exploring the stringy phenomenology of black hole microstates in this exciting era of direct detections of gravitational waves [50].

ACKNOWLEDGMENTS

For discussions, we thank I. Bena, S. Chakraborty, N. Čeplak, A. Dei, S. Giusto, M. Graña, M. Guica, Y. Li, E. Martinec, S. Massai, S. Rawash, R. Russo and M. Santagata. D.B. is supported by Royal Society Research Grant RGF\R1\181019. N.K. is supported by ERC Consolidator Grant 772408-Stringlandscape. D.T. is supported by Royal Society Tata University Research Fellowship UF160203. For hospitality, D.B thanks CEA Saclay and N.K. thanks the University of Southampton.

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