Measuring market integration during crisis periods

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Abstract

Pukthuanthong and Roll (2009) measure the degree of market integration by the percentage of a market's returns explained by global risk factors. However, during periods of crisis characterised by high volatility, their measure may be biased. This paper investigates the determinants of the explanatory power in a multi-factor model during global crises. We show that the explanatory power is influenced by factor heteroscedasticity, changes in factor loadings and residual heteroscedasticity. Using a counterfactual analysis, we establish an empirical framework to examine the effects of each element on integration for 53 financial markets during six recent crisis periods. We find the unconditional market integration is much lower for most markets during a period of crisis than implied. Both factor heteroscedasticity and the existence of contagion during crises account for this difference.

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1 Introduction

Over the past decades, financial liberalisation and increased openness have led to greater integration across global capital markets (Lewis, 2011; Akbari et al., 2020). Concomitant to the development of financial and economic openness, researchers, practitioners and policy makers have questioned and continue to question how to measure market integration (Billio et al., 2017; Akbari and Ng, 2020). The degree of market integration is central in establishing a reasonable international asset pricing model.[2] From the perspective of international investors, the measure of market integration across markets has portfolio diversification implications and hence accurate measurement leads to make better asset allocation decisions (Evans and Hnatkovska, 2014; Li and Liu, 2018; Batten, et al., 2019; Bessler, et al., 2021). For policy makers, the effects of increased market integration on the real economy may determine the policy markers' decisions on market openness. A high level of market integration could lower cost of capital and increase risk-sharing opportunities and welfare benefits (see, Bekaert and Harvey, 2003; Carrieri et al., 2007; Colacito and Croce, 2010; Jappelli and Pistaferri, 2011; Suzuki, 2014; Yu, 2015). However, high integration also can reduce diversification benefits and make markets more vulnerable to crises (see, *inter alia*, Goetzmann et al., 2005; Mendoza and Quadrini, 2010; Christoffersen et al., 2012; Berger and Pukthuanthong, 2012, 2016; Fratzscher, 2012; Donadelli and Paradiso, 2014; Gkillas et al. 2019).

Since the early 1980s, researchers have investigated market integration and its dynamics across markets with recent focus on estimating the fraction of total risk due to global factors (see, *inter alia*, Bekaert and Harvey, 1997; Schotman and Zalewska, 2006; Pukthuanthong and Roll, 2009; Yu et al., 2010; Berger et al., 2011; Volosovych 2011, 2013; Donadelli and Paradiso, 2014; Eiling and Gerard, 2015).[3] This paper concentrates on the approach advocated by Pukthuanthong and Roll

[2] Traditionally, asset pricing models have been based on either fully segmented markets (such as Sharpe, 1964; Lintner, 1965; Black, 1972) or fully integrated markets (such as Harvey, 1991; Dumas and Solnik, 1995; Harvey et al., 2002). Yet, most recent literature considers markets are partially integrated with expected returns determined by both global and local factors (Bekaert and Harvey, 1995; Carrieri et al., 2007; Bali and Cakici, 2010; Bekaert and Mehl, 2019; Bekaert and De Santis, 2021).

[3] Alternatives include conditional asset pricing models typically employing different advanced econometric techniques. However, there is no consensus on a single approach, (e.g. Errunza and Losq, 1985; Errunza et al., 1992; Bekaert and Harvey, 1995; DeJong and DeRoon, 2005; Hardouvelis et al., 2006; Carrieri et al., 2007; Chambet and Gibson, 2008; Carrieri et al., 2013). The measurement of correlation coefficients between markets is also widely employed to evaluate market integration (e.g. Meric and Meric, 1989; Mauro et al., 2002; Goetzmann et al., 2005; Quinn and Voth, 2008). However, many papers criticise the simple correlation as a proxy for market integration (e.g. Forbes and Rigobon, 2002; Dumas et al., 2003; Carrieri et al., 2007; Pukthuanthong and Roll, 2009; Volosovych, 2011; Eiling and Gerard, 2015).

(2009): a measure of market integration based on the explanatory power of a multi-factor model. This method is attractive since it is easily interpretable without parameter estimation. In integrated markets, total risk comes from global factors and thus market integration is close to one, vice versa. However, contributing to the recent debate on this approach (e.g. Akbari et al., 2020), this paper demonstrates that while Pukthuanthong and Roll (2009)'s method may be a reliable measure of integration during normal times, there is considerable bias in the measure during financial crises.

Pukthuanthong and Roll (2009) propose an intuitive idea to measure the degree of market integration as the estimated percentage of a market's returns explained by global risk factors, i.e, R-squared (R^2) in a regression based on a multi-factor model. Subsequently, this approach has been widely adopted in the literature, see *inter alia*, Berger, et al. (2011), Christiansen (2014), Lehkonen (2015) and Cordella and Ospino Rojas (2017). There exist several key concerns within the literature on integration and interdependence/comovement. First, Forbes and Rigobon (2002) highlight the need to correct the bias in their integration measure (i.e., correlation coefficients) due to heteroscedasticity between two market returns. Pukthuanthong and Roll (2009) discuss this well-known issue in their measure (e.g., higher factor volatilities could induce a higher degree of market integration) but argue that, comparing integration across markets, time-varying volatilities in the global factors may not be a serious issue in their sample periods. This may be entirely reasonable during periods of normality when there is insignificant change in global factor volatilities. However, during crisis periods such as the 2007-2009 Global Financial crisis (GFC), global factor volatilities are observed to dramatically increase and become much higher than normal. These significant changes in factor volatility could be important when we consider the determinants of R^2. Cordella and Ospino Rojas (2017) extend Pukthuanthong and Roll (2009)'s integration measure adopting the adjustment approach in Forbes and Rigobon (2002) to correct for factor heteroscedasticity documenting that the increasing trend of market integration is slower after adjustment for global volatility. Moreover, Hou et al. (2013) relax Forbes and Rigobon (2002)'s limitation and argue that apart from common factor exposure, common and local-specific factor volatility also can affect the market integration in a multi-factor model. Recently, Akbari et al. (2020) critique Pukthuanthong and Roll (2009)'s approach for failing to account for the spurious link between volatility and correlation and propose an alternative which accounts for time varying volatility. Second, many researchers suggest that time-variation in integration may be due to contagion effects that exist during crises. For example, Bekaert et al. (2014) demonstrate how different forms of contagion influence the level of interdependence and comovement between markets. Thus, during crises, the existence of contagion is also an important consideration when estimating market integration via R^2.

Estimation of Pukthuanthong and Roll (2009)'s method shows that R squared becomes high during the period around the global financial crisis (2007-2012).[4] However, this result is largely inconsistent with recent empirical findings on market integration using other measures. Bekaert et al. (2014) illustrate risk aversion or a risk premium is a main determinant of valuation convergence, so valuation differentials should diverge, and market integration should decrease during highly-volatile periods. Bekaert et al. (2011) find that average segmentation increases at the end of 2008 but then falls back in 2009 and Lehkonen (2015) suggests that developed markets became less integrated while emerging markets witnessed a slight increase in market integration during the global financial crisis. Further, Akbari et al. (2020) compare the performance of their financial integration R^2 with the P&R measure arguing that in both developed and emerging markets, integration is much lower during crisis periods than implied by P&R. The differences in the results appear to indicate that the P&R's R^2 measure of market integration may overstate the true level of integration during crises, and it is necessary to eliminate the potential effects of factor volatility and contagion before measuring the degree of market integration.

Our theoretical derivation and adjustments to P&R's R^2 are motivated by these concerns. Our focus is different to Akbari et al., (2020) as we seek to document how volatility and contagion affect the P&R measure, and we extend beyond Cordella and Ospino Rojas (2017) to account for contagion (via factor loadings and residual heteroscedasticity) in addition to factor heteroscedasticity.[5] We investigate the potential bias in the explanatory power of an international multi-factor model due to factor heteroscedasticity and contagion during global crises. Following Bekaert et al. (2005) and Bekaert et al. (2014), contagion is defined as unexplained changes in factor loadings and residuals correlation, that is, excess correlation (not implied by economic fundamentals) between financial markets. Conversely, integration is the correlation between markets determined by economic fundamentals. In integrated markets, assets with same risks should have the same expected returns, irrespective of their location. Bekaert et al. (2014) define four types of contagion determined by changes in factor exposures and residuals. Inspired by Bekaert et al. (2014), we categorise contagion during crises into two types: one reflecting factor

[4] Results are available upon request.

[5] Notwithstanding, methodological differences (e.g. Cordella and Ospino Rojas (2017) only use the first principal component (one global factor) as the global risk factor) their integration measure is equivalent to our R^2_{FH} which corrects for factor heteroscedasticity without considering the effects of contagion on market integration.

loadings and one reflecting residual heteroscedasticity, defining these as 'exposure contagion' and 'residual contagion' respectively. We adopt a counterfactual analysis using an empirical framework to measure the bias in the R^2 methodology proposed by Pukthuanthong and Roll (2009) during six crises: the 1987 US crisis, the 1994-1995 Mexican crisis, the 1997 Asian crisis, the 1998 Russian/LTCM (Long Term Capital Management) crisis, the 2007-2009 Global Financial crisis (GFC) and the 2009-2014 European Sovereign Debt crisis (ESDC), documenting the differences in market integration across 53 markets by market development, region and time over the period 1973-2017. We find that during most crises, the explanatory power in a multi-factor model becomes lower after adjusting for all bias caused by factor heteroscedasticity and contagion. More specifically, four crises witness dramatic increases in factor volatilities and result in the large bias in R squared. The 1987 US crisis and 1997 Asian crisis drive changes in factor loadings and consequently cause upward changes in R squared, which shows the existence of 'exposure contagion'. Moreover, residual heteroscedasticity largely affects R squared and 'residual contagion' occurs during the 1998 Russian/LTCM crisis and 2007-2009 GFC. We also examine time-varying market integration across markets and find that most markets increase and after adjusting for all bias during crises, the increasing trend becomes smoother and more gradual. Consistent with Bekaert and Mehl (2019), this suggests integration did not peak during the GFC. Finally we compare our adjusted R squared measure and the original P&R measure with the approach of Akbari et al. (2020), highlighting that adjusted measure implies lower integration during the GFC and ESDC.

Our study makes three main contributions to the literature. First, we theoretically and empirically demonstrate that during financial crises when investors desire to diversify their portfolios, the R-squared method is biased when measuring market integration. We propose a new method to correct the bias during crises and after adjusting all bias caused by factor heteroscedasticity and contagion, the observed level of market integration becomes much lower. Second, we investigate the transmission mechanism of a shock from equity markets to one another. We consider two types of contagion: exposure contagion, which causes unexpected changes in global factor exposures, and residual contagion, which causes residual heteroscedasticity. Through estimating the bias caused by changes in factor loadings and residual heteroscedasticity, we can clearly identify and understand how contagion affects financial markets. Third, this paper provides a comprehensive analysis to test the extent and dynamics of market integration across 53 markets, including 21 developed markets, 25 emerging markets and 7 frontier markets. The panel results provide a whole picture of market integration across markets for international investors and may help improve diversification performance.

The reminder of this paper is as follows. Section two discusses how the Pukthuanthong and Roll (2009) R-squared methodology is biased in measuring the level of market integration during financial crises. Section three proposes an empirical framework to test the bias caused by each factor and estimate adjusted market integration. The data is also presented in section three and the empirical analysis is in section four. Section five concludes.

2 The Pukthuanthong and Roll (2009) R^2 measure of market integration and bias

This section presents the R^2 measure of market integration of Pukthuanthong and Roll (2009) and illustrates how it suffers bias caused by factor heteroscedasticity and contagion during periods of crisis characterised by high volatility.

Assume a multi-factor model

$$y_t = \alpha + \sum_{i=1}^{10} \beta_i x_{it} + \varepsilon_t \tag{1}$$

where, x_{it} $(i = 1,2, \ldots.,10)$ are global risk factor, y_t is the stock index return of an individual market.

Pukthuanthong and Roll (2009) form the covariance matrix of 17 developed market returns and estimate the first 10 principal components as global risk factors (expressed x in equation (1)). The R^2 of the multi-factor model is the level of market integration (P&R R squared). Pukthuanthong and Roll (2009) note that decreases in residual volatility can increase values of their R squared and that the multi-factor R^2 might not rise linearly with the level of market integration due to factor heteroscedasticity. These issues have been highlighted previously in the literature. Forbes and Rigobon (2002) argue that higher factor volatility may result in higher correlation coefficients concluding that correlation is biased by heteroscedasticity. Bekaert et al. (2005) argue residual heteroscedasticity may cause high correlation which is not determined by fundamentals with the excessive correlation in residuals measured as contagion. Corsetti et al. (2005) state that higher return volatility during a crisis may be due to country-specific noise in addition to higher common factor variance and show the variance of the country-specific factor severely biases the correlation coefficients. Similarly, Eiling and Gerard (2015) conclude that the risk exposures to global factors and factor volatility increase cross-country correlations while idiosyncratic risk decreases them. Hou et al. (2013) demonstrate R^2 in a multi-factor model depends not only on common factor exposure but also on common factor volatility, firm-specific factor volatility and investor sentiment. Meanwhile, Bekaert et al. (2014) argue that no matter which factor model is used, correlation coefficients are affected by unexpected factor exposure and returns unrelated to factors. While Pukthuanthong and Roll (2009) acknowledge many of these issues, they elect not to adjust their measure since they state its reliability as an indicator of integration is unaffected. Here, we shed light on the role factor volatility and contagion play in the measurement of R squared during crisis periods.

The informal derivation presented below shows how factor heteroscedasticity, factor loadings and residual heteroscedasticity determine R^2 in the multi-factor model. Appendix provides the formal proof.[6]

Consider the general N-factor model. Assume x_{it} $(i = 1,2, \ldots., N)$ are global risk factors, y_t is the stock market return, and the N-factor model is:

$$y_t = \alpha + \sum_{i=1}^{N} \beta_i x_{it} + \varepsilon_t \qquad (2)$$

where, $E[\varepsilon_t] = 0$, $E[x_{it}\varepsilon_t] = 0$, $E[x_{it}] = 0$ and $E\left[x_{it}x_{jt}\right] = 0$ ($j = 1,2, \ldots, N$ for $i \neq j$).[7]

Assume the full sample is divided into two according to the variance of y_t. The first group (l) captures the low-variance period and the second group (h) captures the high-variance period, such that, $\sigma_{yy}^h > \sigma_{yy}^l$. Then we assume

$$\sigma_{x_i x_i}^h = \left(1 + \delta_{x_i}\right)\sigma_{x_i x_i}^l \qquad (3)$$

$$\sigma_{\varepsilon\varepsilon}^h = (1 + \delta_\varepsilon)\sigma_{\varepsilon\varepsilon}^l \qquad (4)$$

[6] The proof builds on Forbes and Rigobon (2002) and Hou et al. (2013). Forbes and Rigobon (2002) illustrate the correlation coefficient between two markets is conditional on market volatility. Hou et al. (2013) relax Forbes and Rigobon (2002)'s limitation. These two papers investigate the existence of contagion while our focus is the effects of contagion on market integration.

[7] In this paper, we use PCA to estimate the N global risk factors. Since the data is mean-centred, the risk factors have zero means.

$$\beta_i^h = (1+\delta_{\beta_i})\beta_i^l \tag{5}$$

where $\sigma_{x_i x_i}$ is the variance of x_i, $\sigma_{\varepsilon\varepsilon}$ is the variance of residuals ε, δ_{x_i}, δ_ε and δ_{β_i} are separately the relative changes in the variance of x, in the variance ε and in β. For identification, $\delta_{x_i}, \delta_\varepsilon > -1$ and at least one of δ_{β_i} is not equal to -1 $(i = 1,2,\dots,N)$.[8] Moreover, we assume $\sigma_{\varepsilon\varepsilon}^h = c^h < \infty$ and $\sigma_{\varepsilon\varepsilon}^l = c^l < \infty$, where c^h and c^l are constant.

Since the N global risk factors are orthogonal to each other, R squared in (2) is the sum of correlations between each factor and market return, which can be written as

$$R^2 = \sum_{i=1}^{N} \left[\rho_{x_i y}\right]^2 \tag{6}$$

where $\rho_{x_i y}$ refers to the correlation between x_i $(i = 1,2,\dots,N)$ and y.

The correlation $\rho_{x_i y}$ $(i = 1,2,\dots,N)$ is given by:

- The covariance between x_i and y in the high-variance period: $\sigma_{x_i,y}^h$ $(i = 1,2,\dots,N)$

$$\sigma_{x_i y}^h = (1+\delta_{\beta_i})(1+\delta_{x_i})\sigma_{x_i y}^l \tag{7}$$

- The variance of y in the high-variance period: σ_{yy}^h

$$\sigma_{yy}^h = \sigma_{yy}^l \left[1 + \sum_{i=1}^{N} \left[(1+\delta_{\beta_i})^2(1+\delta_{x_i}) - 1\right] (\rho_{x_i y}^l)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right] \tag{8}$$

- From equation (3), (7) and (8), the correlation is

$$\rho_{x_i y}^h = \frac{\sigma_{x_i y}^h}{\sigma_{x_i}^h \sigma_y^h} = \rho_{x_i y}^l \frac{(1+\delta_{\beta_i})\left[(1+\delta_{x_i})\right]^{1/2}}{\left\{1+\sum_{i=1}^{N}\left[(1+\delta_{\beta_i})^2(1+\delta_{x_i})-1\right](\rho_{x_i y}^l)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right\}^{1/2}} \tag{9}$$

Equation (9) shows that the estimated correlation coefficients between each global risk factor (x_i) and market return (y) increase when either the factor volatility increase ($\delta_{x_i} > 0$) or the

[8] Here, we do not consider the extreme case when $\delta_{\beta_1} = \delta_{\beta_2} = \cdots = \delta_{\beta_N} = -1$, that is, the market is fully segmented.

residual volatility decreases ($\delta_\varepsilon < 0$) even if the true correlation does not change. In other words, the correlation in equation (9) is conditional on factor heteroscedasticity (δ_{x_i}), changes of factor loadings (δ_{β_i}) and residual heteroscedasticity (δ_ε). We consider estimated values based on equation (9) as 'conditional' below.

Based on equation (6), the Appendix derives the conditional R squared:

$$R^{2^*} = R^2 \frac{\sum_{i=1}^{N}\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)\rho_{x_i y}^2}{\sum_{i=1}^{N}\rho_{x_i y}^2 \sum_{i=1}^{N}\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)\rho_{x_i y}^2+\sum_{i=1}^{N}\rho_{x_i y}^2\left(1-\sum_{i=1}^{N}\rho_{x_i y}^2+\frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right)} \tag{10}$$

where, R^{2^*} is the conditional R squared and R^2 is the unconditional R squared.

Holding other variables constant, R^{2^*} is increasing in δ_{x_i} and decreasing in δ_ε. It is increasing in δ_{β_i} when $\delta_{\beta_i} > -1$ and decreasing in δ_{β_i} when $\delta_{\beta_i} < -1$ $(i = 1, 2, \dots, N)$. Therefore, the estimated R^{2^*} can be higher, lower or equal to R^2. In other words, even if the R^2 remains constant during both the low-variance period and the high-variance period, R^{2^*}may not equal to R^2 during high-variance period.

As a result, the estimated R^2 is determined by changes in factor loadings, changes in factor volatility and changes in residual volatility.[9] Therefore, estimating market integration based on the explanatory power of a multi-common factor model is potentially misleading. The estimated values during a high-variance period are biased and conditional on loading changes, factor volatility changes and residual volatility changes compared to the low-variance period.

The equation (10) provides a direct implication for the measurement of market integration based on R^2 in a multi-factor model. Normally, financial crisis periods experience higher return volatility than stable periods and hence factor volatility, residual volatility and factor loadings may change due to crises, generating the bias in measuring the underlying market integration using R squared. In other words, even if the unconditional R squared is the same as during the stable periods, the conditional R squared will tend to change during crises. More specifically, the

[9] Changes in regional or country-specific factors' volatility can cause residual heteroscedasticity. The existence of contagion during crisis also can cause residual heteroscedasticity.

conditional R squared becomes higher than the unconditional R squared with higher factor volatility or lower residual volatility during crises.

Bekaert et al. (2014) establish an international factor model with three factors: a U.S. factor, a global financial factor and a domestic market factor. They separately define excessive correlation caused by unexpected increases of these three factor exposures as 'U.S. contagion', 'global contagion' and 'domestic contagion', and any unexplained or residual excessive correlation is classed as 'residual contagion'. Hence, during crises, contagion can both affect factor loadings and residual volatility. In our model, we consider global risk factors as explanatory variables, so changes in residual volatility may be influenced by regional or country-specific factors or returns unrelated to factors. Also, the existence of contagion not only affects changes in global risk factor loadings, but also affects residual heteroscedasticity. We define significant changes in global risk factor loadings as 'exposure contagion' and define significant changes in residual volatility as 'residual contagion'. During a crisis, factor loadings are determined by fundamentals, that is, market integration, and non-fundamentals, that is, contagion. We measure the level of market integration on an annual basis and assume it is constant during each year. Therefore, within a crisis year, it is reasonable to assume that changes in factor loadings are caused by the crisis and nothing else.[10] In other words, during a crisis, contagion causes any observed changes in factor loadings.

3 The empirical framework

The previous section mathematically illustrates the bias in R squared due to factor heteroscedasticity and contagion during high-variance periods. In this section, we develop an implementable testing framework to test and correct the bias.

We divide the full sample into two periods: a low-variance or stable period and a high-variance or crisis period. Then, we separately estimate the following regression in each period:

$$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_{10} x_{10,t} + \varepsilon_t \quad (11)$$

where, x_{it} $(i = 1,2, \ldots ,10)$ are the global risk factors; y_t is the stock index return of an individual market; α is a constant term and ε_t is a residual vector.

[10] We assume any changes in factor volatility, factor loadings and residual volatility during a crisis period are due to the crisis.

Recall equation (9) for the correlation in the crisis period $\rho^h_{x_i y}$. Rewriting in terms of the conditional correlation gives:

$$\rho^*_{x_i y} = \rho_{x_i y} \frac{\left(1+\delta_{\beta_i}\right)\left[\left(1+\delta_{x_i}\right)\right]^{1/2}}{\left\{1+\sum_{i=1}^{N}\left[\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)-1\right]\rho^2_{x_i y}+\frac{\delta_\varepsilon \sigma^l_{\varepsilon\varepsilon}}{\sigma^l_{yy}}\right\}^{1/2}} \qquad (i = 1,2,..,10) \qquad (9')$$

For the stable period l, the betas β^l_i, the residuals, ε^l, and the variances $\sigma^l_{\varepsilon\varepsilon}$, σ^l_{yy} and $\sigma^l_{x_i x_i}$. can be estimated by regressing the ten global risk factors on the stock index return in (11). The regression is then re-estimated for the crisis period h, to give β^h_i, ε^h, and the variances $\sigma^h_{\varepsilon\varepsilon}$ and $\sigma^h_{x_i x_i}$.

Rearranging equations (3) to (5):

$$\delta_{x_i} = \frac{\sigma^h_{x_i x_i}}{\sigma^l_{x_i x_i}} - 1 \qquad (3')$$

$$\delta_\varepsilon = \frac{\sigma^h_{\varepsilon\varepsilon}}{\sigma^l_{\varepsilon\varepsilon}} - 1 \qquad (4')$$

$$\delta_{\beta_i} = \frac{\beta^h_i}{\beta^l_i} - 1 \qquad (5')$$

From equations (3') to (5'), given the known values $\sigma^h_{x_i x_i}$, $\sigma^l_{x_i x_i}$, $\sigma^h_{\varepsilon\varepsilon}$, $\sigma^l_{\varepsilon\varepsilon}$, β^h_i, and β^l_i, we can obtain the delta values (δ_{x_i}, δ_ε and δ_{β_i}). Now, all values in equation (9') are known except for the unconditional correlation $\rho_{x_i y}$ $(i = 1,2,\ldots,10)$. However, due to the orthogonality of 10 global risk factors, the unconditional R squared (expressed as R^2_{ALL}) is the sum of unconditional correlations $\rho_{x_i y}$ $(i = 1,2,\ldots,10)$, given by equation (6).

In order to examine the potential effects of factor heteroscedasticity and contagion on R squared in a multi-factor model, a series of adjusted R squared are estimated.

I. Factor Heteroscedasticity

To test the effects of factor heteroscedasticity on R squared, i.e. factor heteroscedasticity exists, but contagion does not, let $\delta_{\beta_1} = \delta_{\beta_2} = \cdots = \delta_{\beta_{10}} = \delta_{\varepsilon} = 0$ but at least one of $\delta_{x_i} \neq 0\ (i = 1,2,\ldots,10)$ such that equation (9') becomes:

$$\rho^*_{x_i y} = \rho_{x_i y} \frac{\left[\left(1+\delta_{x_i}\right)\right]^{1/2}}{\left\{1+\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)-1\right]\rho^2_{x_i y}\right\}^{1/2}} \tag{12}$$

According to equation (3'), $\delta_{x_i}\ (i = 1,2,\ldots,10)$ can be calculated by estimating the variances of global risk factors during stable period ($\sigma^l_{x_i x_i}$) and crisis period ($\sigma^h_{x_i x_i}$). $\rho^*_{x_i y}$ is the correlation of x_i and y during crisis period. We can solve the unconditional correlation $\rho_{x_i y}$ from equation (12). The sum of unconditional correlations is unconditional R squared, denoted as R^2_{FH}.

II. Exposure (Beta) Contagion (Factor Loadings)

To measure the effects of the changes of factor loadings on R squared , i.e. exposure contagion exists but factor heteroscedasticity and residual contagion do not, let $\delta_{x_1} = \delta_{x_2} = \cdots = \delta_{x_{10}} = \delta_{\varepsilon} = 0$ but at least one of $\delta_{\beta_i} \neq 0\ (i = 1,2,\ldots,10)$ such that equation (9') can be written as:

$$\rho^*_{x_i y} = \rho_{x_i y} \frac{1+\delta_{\beta_i}}{\left\{1+\sum_{i=1}^{N}\left[\left(1+\delta_{\beta_i}\right)^2-1\right]\rho^2_{x_i y}\right\}^{\frac{1}{2}}} \tag{13}$$

As before, the factor loadings $\beta^h_i, \beta^l_i\ (i = 1,2,\ldots,10)$ for the high- and low-variance periodsare estimated from equation (11). Then, the delta value are calculated from equation (5') and the unconditional correlation $\rho_{x_i y}$s can be obtained from equation (13). The R squared is the summation of the unconditional correlations and denoted as R^2_{BC}.

III. Factor Heteroscedasticity and Exposure Contagion

To measure the combined effects of factor heteroscedasticity and exposure contagion on R squared, i.e. factor heteroscedasticity and exposure contagion exist but residual contagion does not, let $\delta_{\varepsilon} = 0$ but for at least one , $\delta_{x_i} \neq 0$ and $\delta_{\beta_i} \neq 0\ (i = 1,2,\ldots,10)$ such that equation (9) becomes:

$$\rho^*_{x_i y} = \rho_{x_i y} \frac{\left(1+\delta_{\beta_i}\right)\left[\left(1+\delta_{x_i}\right)\right]^{\frac{1}{2}}}{\left\{1+\sum_{i=1}^{N}\left[\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)-1\right]\rho^2_{x_i y}\right\}^{\frac{1}{2}}} \tag{14}$$

Following to the first two scenarios, we can estimate the delta values of δ_{β_i}, δ_{x_i} from equations (3') and (5') and the unconditional correlation $\rho_{x_i y}$ from equation (14). The unconditional R squared is the sum of the ten unconditional correlations, and we denote this as R_{FB}^2.

Therefore, R_{ALL}^2 is the R squared after adjusting for all bias and captures the unconditional market integration during the high-variance or crisis period. R_{FH}^2 is the R squared after adjusting for the bias only caused by factor heteroscedasticity and measures the pure bias of factor heteroscedasticity in market integration. Within our framework, this is equivalent to the Cordella and Ospino Rojas (2017) measure.[11] R_{BC}^2 is the R squared after adjusting for the bias only caused by beta changes and estimates the pure bias of beta changes in market integration. R_{FB}^2 is the R squared after adjusting for the bias both caused by factor heteroscedasticity and beta changes and measures the total bias caused by both influential factors.

4 Data and Empirical Analysis

4.1 Data

We obtain data on 53 financial markets from DataStream. All indexes are denominated in the U.S. dollars. The data frequency is daily, and the sample period is from 1 January 1973 to 31 December 2017. To estimate accurate levels of market integration, we adjust the sample filtering out non-trading periods and holidays. From the cross-section of 53 markets, 17 markets have data which spans the full sample period: 1973-2017. They are Canada, the US, Australia, Austria, Belgium, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherland, Singapore, South Africa, Switzerland, the UK.

This paper focuses on six main financial crises widely investigated in the existing literature: the 1987 US crisis, the 1994-1995 Mexican crisis, the 1997 Asian crisis, the 1998 Russian/LTCM crisis, the 2007-2009 Global Financial crisis (GFC) and the 2009-2014 European Sovereign Debt crisis

[11] Similar to Cordella and Ospino Rojas (2017)'s measure that adjusts for the bias caused by factor volatility only, our R_{FH}^2 measure is the R^2 after adjusting for the bias caused by factor heteroscedasticity. However, while Cordell and Ospino Rojas (2017) only use a single global factor in their model, we follow Pukthuanthong and Roll (2009)'s procedure and include 10 global factors in our multi-factor model, since Pukthuanthong and Roll (2009) and Hou et al. (2011) argue that using one global factor cannot fully capture the extent of market integration.

(ESDC).[12] Table 1 reports the crisis period and the stable period used as the baseline for the six financial crises. Three crises span more than one calendar year: the 1994-1995 Mexican crisis, the 2007-2009 GFC and the 2009-2014 ESDC. Since the Russian crisis and the LTCM crisis occur at roughly the same time, this renders the separation of their effects impossible and therefore we treat them as a single crisis.

<Insert Table 1 here>

4.2 Estimating global factors with principal components

In this paper, we employ PCA to estimate the global risk factors. First, we establish the covariance matrix using the 17 markets available for the full sample period.[13] These 17 markets are a reasonable representation of global markets, accounting for over 70% of global market capitalization across the entire sample period as shown in Figure 1. At the beginning of the sample, the world market consists entirely of the 'main' markets. With the development of other equity markets, the percentage of the world market capitalization captured by the 17 markets decreases slowly from the mid-1980s until 2009 after which is rises again slightly. To account for potential non-synchronicity across markets, we add the one-day lagged returns of the North American markets (Canada and the US) to the covariance matrix. Finally, when any of the 17 market returns is the dependent variable, principal components are estimated excluding its returns from the calculation.

<Insert Figure 1 here>

The eigenvectors and eigenvalues are computed using the full sample data and sorted from the highest to lowest value, to calculate the principal components for each calendar year. To compare R squared between our approach and Pukthuanthong and Roll (2009), we follow Pukthuanthong and Roll (2009) and adopt the first 10 principal components as proxies for the global risk factors. Those 10 principal components explain almost 90% of total variance, capturing most global shocks.

[12] The selection of stable periods and crisis periods for each crisis follows the prior literature: Forbes and Rigobon (2002), Rigobon (2003), Bekaert et al. (2014) and Filoso et al. (2017).

[13] Either the covariance matrix or the correlation matrix can be used to estimate principal components. We follow Pukthuanthong and Roll (2009) and use the covariance matrix.

4.3 Return regressions and unconditional R^2

The 10 estimated global risk factors form the explanatory variables in regression equation (11). For each market, the stock market return is the dependent variable in each calendar year. The regression is estimated separately during the stable period and the crisis period to obtain the various R squared measures based on equations (9), (12), (13) and (14).

Rather than reporting the results of 53 markets individually, we assign markets according to their economic level or market development and by geographic region. At the economic level, we divide the 53 markets into three subsamples: emerging markets, developed markets and frontier markets. Based on geographic location, we categorise markets into 6 regions: North America, Latin America, Asia-Pacific and Europe which is further separated into Emerging Europe and Developed Europe. The categories are taken from Standard&Poor's and Dow Jones.[14] Table 2 presents the countries included in each category.

<Insert Table 2 here>

4.3.1 The 1987 US crisis

The first crisis analysed in our sample is the 1987 US crisis. We follow Forbes and Rigobon (2002) in defining the crisis period from 17 October 1987 to 4 December 1987 and the stable period from 1 January 1986 to 17 October 1987. Due to data availability, we consider 20 markets in total.

Table 3 presents the various R squared measures highlighting the bias caused by influential factors. The first six columns report the R squared measures. They are, sequentially, the P&R R squared in the crisis year, (R^2_{PRy}), the P&R R squared during the crisis period, (R^2_{PR}), the R squared after adjusting for the bias caused by factor heteroscedasticity during the crisis period, (R^2_{FH}), the R squared after adjusting for the bias purely caused by beta changes during the crisis period, (R^2_{BC}), the R squared after adjusting for the bias purely caused by factor heteroscedasticity and beta changes, (R^2_{FB}), and the unconditional R squared after adjusting for all bias caused by crises, (R^2_{ALL}). The final four columns present the size of the bias caused by factor heteroscedasticity, beta

[14] The Middle East & Africa is also a region identified by Standard & Poor's and Dow Jones. However, among 53 markets, there are only two markets located in the region: South Africa and Israel. Therefore, we do not consider these two markets to reflect the diversity of the region here.

changes, both factor heteroscedasticity and beta changes and all influential factors respectively. The value in parentheses reports the number of markets included in each category.

Comparing the first two R squared measures, it can be seen that the P&R R squared during the crisis period is higher than the P&R R squared over the whole crisis year, which means the crisis increases the conditional R squared of the whole year. The R squared after adjusting for the bias caused by factor heteroscedasticity is lower than the conditional R squared, meaning changes in factor volatility during this crisis cause a measurement bias which increases the conditional R squared. Also, the R squared after adjusting for the bias caused by changes in factor loadings is lower than the conditional R squared and even lower than the R squared adjusting for the bias caused by factor heteroscedasticity, especially for emerging markets and markets in the Asia-Pacific region. This interesting finding provides the evidence that during the 1987 US crisis, contagion exists in the global financial market via increasing global factor exposures unexpectedly in the crisis ('exposure contagion'), which implies excess comovements of country portfolios with global factors, especially in Asia. The existence of exposure contagion increases R squared in the multi-factor model. Further, the combination of the bias caused by both factor heteroscedasticity and changes in factor loading are the largest. After adjusting for the combined bias, the R squared dramatically drops to a lower level. The final R squared is market integration after adjusting for all bias caused by the crisis. Relative to the penultimate R squared (R_{FB}^2), the unconditional R squared also adjusts for the bias caused by residual heteroscedasticity. The adjusted R squared value becomes a little higher than R_{FB}^2. As discussed previously, residual heteroscedasticity may increase due to changes in regional or country-specific factors. Meanwhile, higher residual heteroscedasticity results in a lower conditional R squared. Thus, after adjusting for the bias caused by residual heteroscedasticity, R squared become higher than prior to correction. Yet the effects of residual heteroscedasticity (residual contagion) are less prominant than exposure contagion. Compared to the conditional R squared (R_{PR}^2), the unconditional R squared is lower. Hence the P&R R squared overestimates the level of market integration during the 1987 US crisis since it does not consider the bias caused by changes in factor volatility, in factor loading and in residual volatility caused by the crisis. From the last four columns in the table, we observe how much bias is caused by each factor. The combined bias caused by factor heteroscedasticity and changes in factor loadings are the largest. The total bias at a global level is 0.4714, which means the unconditional R squared is 0.4714 lower than the conditional R squared on average.

At the economic level, the conditional R squared in developed markets (0.7998) is higher than the conditional R squared in emerging markets (0.6661). However, the unconditional R squared becomes similar between these two types of markets, which is below 0.32. In other words, all markets are less integrated during the 1987 US crisis. More specifically, although there are similar effects of factor heteroscedasticity on R squared in emerging and developed markets, exposure contagion has greater influence in emerging markets. Comparing the last two columns, we can see that residual contagion exists in emerging markets but not in developed markets. Across regions, we can see that except North America, all regions have similar market integration during the 1987 US crisis, which is around 0.8 before adjustments and about 0.3 after. In North America (Canada and the US) the total bias is 0.2843 which is much less than other regions. Hence the level of market integration remains high.

<Insert Table 3 here>

4.3.2 The 1994-1995 Mexican crisis

The Mexican crisis occurred in December 1994 and ended in March 1995 lasting three and half months. We define the stable period as from 1 June 1994 to 16 December 1994.[15] We investigate 48 financial markets in total during the crisis.

Table 4 summarises the results of the different R squared measures and the bias during the crisis. The first two R squared measures show that on the whole the conditional R squared during the crisis period (0.3046) is little lower than the value during the whole crisis year (0.3873), which shows on average the 1994-1995 Mexican crisis does not hugely affect market correlations. The total bias showed in the last column also confirms this point (0.0569). At the economic level, emerging and frontier markets witness huge effects of this crisis but developed markets do not. Developed markets maintain a high level of market integration due to minimal total bias. Compared to the conditional R squared, the adjusted value in emerging markets becomes lower mainly after considering the effects of factor heteroscedasticity and changes of factor loadings and the total bias is 0.1289. Although the difference between unadjusted and adjusted values is small in frontier markets, the effects of exposure contagion and residual contagion on R squared are clear (0.1206 and 0.1264 separately). We observe the bias caused by beta changes in Latin America is 0.1230, which also reflects the existence of exposure contagion. After adjusting for all bias,

[15] The definition of stable period and crisis period for the 1994-1995 Mexican crisis follows Rigobon (2003).

emerging markets and frontier markets have the similar unconditional R squared. For emerging markets and markets in Latin America, the total bias, 0.1289 and 0.1220 respectively, caused by the Mexican crisis are significant. We posit that the reason for this is that at the start of the crisis, Mexico first experienced a devaluation of the peso and the crisis quickly extended to other emerging markets and especially other Latin American markets. The crisis drives those markets are highly correlated to the world financial market during this period by the existence of exposure contagion. Besides, an interesting finding is that the markets influenced by this crisis are mainly located in Latin America where the crisis starts and partly in Emerging Europe. In other words, the effects of the 1994-1995 Mexican crisis were localised.

<Insert Table 4 here>

4.3.3 The 1997 Asian crisis

Many papers focus on the 1997 Asian crisis but adopt slightly different crisis periods. Here, we follow Forbes and Rigobon (2002) and define the crisis period from 17 October 1997 to 16 November 1997. The stable period starts from 1 January 1996 and ends on 16 October 1997. Although the Thai baht sharply collapsed in June 1997, its effects were not widespread. However, after the Hong Kong market crashed in October 1997, the crisis quickly spread to other markets, especially in Southeast Asia. Since, before October developed markets were largely unaffected, we consider this period as the stable period. Due to data availability, we exclude Bulgaria, Russia, Romania and Slovenia and investigate 49 financial markets during the Asian crisis.

Table 5 documents the potential impact of bias on the measurement of market integration during the Asian crisis. First, the initial two R squared measures show that the P&R R squared during the crisis period is much larger than the whole level in 1997 across all categories. Compared to the conditional R squared, the unconditional R squared after adjusting for all bias caused by the crisis become lower in all cases. The values at the global level fall from 0.8016 to 0.2542 with a total bias of 0.5475 caused by this crisis. The factor heteroscedasticity and exposure contagion make the main contribution on the total bias and the residual heteroscedasticity is not obvious during Asian crisis. The P&R method shows the level of market integration during the crisis is quite high which is 0.8937 but after adjustments the value drop dramatically to 0.3842. The big difference is mainly because of the bias caused by factor volatility and beta changes. The emerging and frontier markets also get a high P&R R squared values during this crisis, but those two types of markets get greater decrease after adjusting the bias by three potential factors than developed markets because exposure contagion exists and affects more in those markets. From the results of adjusted R squared, we can find that developed markets still exhibit higher market integration than emerging markets while frontier markets have the lowest level of market integration (0.0959). Moreover, the integration has bigger differences among economic levels than the P&R measure showed. Markets in North America retain a high level of market integration (0.7532) and are only impacted minimally by the crisis (0.2239). Consistent with the prior literature, the most affected regions by the crisis are Latin America and Asia-Pacific because the crisis first happened in Southeast Asia and quickly extended to emerging markets especially in Latin America. Moreover, the factor loadings increase substantially in most markets during the crisis which implies an unexpected excess comovement of domestic market portfolio and global factors. The exposure contagion drives up P&R R squared values dramatically and after considering its bias, the R squared drops to a lower level.

<Insert Table 5 here>

4.3.4 The 1998 Russian/LTCM crisis

Shortly after the 1997 Asian crisis, the 1998 Russian/LTCM crisis occurred. The crisis is short, lasting for three months from 2 March 1998. Due to the preceding Asian crisis, the stable period is chosen to be from 3 August 1998 to 15 October 1998.[16] Due to data availability, we exclude Bulgaria, Russia and Slovenia from the sample.

Table 6 presents the conditional and unconditional R squared measures and the bias caused by each influential factor. The first two R squared show that the R squared during the crisis period is largely higher than during the whole crisis year when applying Pukthuanthong and Roll (2009)'s method, which partly demonstrates the crisis causes a bigger value of P&R R squared. The total bias is 0.1424 at global level and further confirms the argument. After adjusting for all bias, the R squared falls. The market integration is estimated to be 0.4646 at the global level compared to a level of market integration as measured by Pukthuanthong and Roll (2009) of 0.6070 during the crisis period. As expected, developed markets display a high level of market integration, followed by emerging markets with frontier markets having the lowest values. The differences of market integration across economic levels are still obvious. Similarly, North America (including the US and Canada) maintains a high level of market integration (0.7699) while markets in Latin America

[16] Rigobon (2003) adopts the same crisis period and stable period.

are least integrated (0.3133) on average. The Emerging Europe and Asia-Pacific regions are also at a low level of market integration.

From Table 6, we can see that two influential factors cause the bias of P&R R squared during the 1998 Russian/LTCM crisis: factor heteroscedasticity and residual heteroscedasticity. The bias caused by changes in factor volatility is positive and significant across categories suggesting that the 1998 Russian/LTCM crisis increases factor volatility globally. Examining the last two columns in the table shows that residual heteroscedasticity caused by the crisis affects most markets which reflects the existence of residual contagion globally. In other words, the Russian/LTCM crisis not only increases the volatility of global risk factors, but also increases volatility of regional or country-specific risk factors. As discussed in the theoretical section, an increase of global volatility cause a high P&R R squared value and high residual volatility drives the value down. Thus the bias caused by factor heteroscedasticity (0.2443) is partly traded off by the bias from high residual volatility (0.1392). Besides, the bias caused by beta changes implies that 'exposure contagion' largely only affects Latin America and Emerging Europe. The results of the total bias illustrate that Emerging Europe suffers the biggest bias in total. This is perhaps unsurprising, given that the majority of emerging markets are in Eastern Europe and Russia is a dominant neighbour. Hence the Russian/LTCM crisis has significant effects on surrounding financial markets.

<Insert Table 6 here>

4.3.5 The 2007-2009 GFC

The 2007-2009 GFC first originated from the subprime mortgage market in the US and quickly spread to the world. Following Bekaert et al. (2014), we choose the crisis period from 7 August 2007 to 15 March 2009 and the stable period from 1 January 2003 to 31 December 2006.

Table 7 shows the different R squared measures and the bias during the 2007-2009 GFC. The first two R squared values show no big difference whether measured during the crisis years or across the entire crisis period since much of the period remained highly volatile. Compared to the conditional R squared, values for market integration after adjusting for all bias caused by the crisis become lower. The P&R R squared is on average biased by 0.2230 in total. Developed markets and markets in North America still exhibit a high level of market integration on average while frontier markets continue to exhibit low levels of market integration. There still exists great gaps in market integration among economic levels. Meanwhile, developed Europe region is much more integrated than Emerging Europe region.

The 2007-2009 GFC drives high factor volatilities worldwide and thus results in the high conditional R squared. Consequently, after adjusting for the bias caused by factor heteroscedasticity, the R squared dramatically drops. The crisis does not appear to change factor loadings much and hence the measured bias is small. In most cases the bias caused by factor loadings is less than 0.1, reflecting little evidence of 'exposure contagion'. The total bias suffered is much greater for emerging and frontier markets than developed markets, which suggests that the differences in market integration between developed markets and emerging or frontier markets are larger than that implied by the Pukthuanthong and Roll (2009) measure. This suggests that during the GFC, Pukthuanthong and Roll (2009) largely overestimate the level of market integration for emerging markets and frontier markets. Similarly, across regions, North America and developed markets, integration faces less adjustment due to bias but for Latin America, Emerging Europe and Asia-Pacific the adjustment is more substantial. After considering the effects of residual heteroscedasticity, the R squared increases for all categories suggesting the presence of residual contagion during the crisis. At the global level, we observe value increases from 0.2513 (adjusting for factor heteroscedasticity and beta changes) to 0.4116 (adjusting for all bias including residual heteroscedasticity) indicating that the rise of residual volatility during the crisis decreases the conditional R squared in a multi-factor model. We can conclude that this crisis causes the excess comovement of domestic market portfolios with local factors but not with global factors. Moreover, developed markets are the most influenced by residual contagion compared to emerging and frontier markets.

<Insert Table 7 here>

4.3.6 The 2009-2014 ESDC

The 2009-2014 ESDC happened shortly after the GFC and is the final crisis period examined here. There is no obvious single event causing or terminating the crisis and consequently often prior papers define slightly different crisis periods. We follow Filoso et al. (2017) and define the crisis period from 1 June 2009 to 23 June 2014. The stable period is from 1 January 2015 to 31 December 2017.

Table 8 summarizes the results of the various R squared measures for the 2009-2014 ESDC. Again, after adjusting for the bias, the R squared becomes lower than the P&R R squared. However, compared with the results during the 2007-2009 GFC, adjustments to R squared are relatively small during this crisis with little evidence of contagion. More specifically, the bias caused by factor heteroscedasticity is minimal in most categories except for Emerging Europe region (0.1103). From the bias of beta changes, we know that there is no obvious unexpected comovments of domestic market portfolios with global risk factors during this crisis. Meanwhile, the effects of residual heteroscedasticity are also negligible across markets. Therefore, two types of contagion are not found in this crisis. The P&R R squared values in European markets get large adjustments and the total bias is 0.1048. The emerging European markets have higher drop on R squared when applying our adjusted measure (0.1836). The findings are not surprising given the epicentre of the ESDC. Across market groupings the usual patterns in integration hold. Developed markets have higher market integration than emerging markets or frontier markets. Markets in North America and in Developed Europe remain highly integrated even after adjusting for all bias.

<Insert Table 8 here>

4.4 Trends in market integration

In order to examine how market integration has evolved over time, we estimate the level of market integration each year and plot average values for each category across time. To compare with Pukthuanthong and Roll (2009), we classify the 53 markets into four cohorts based on sample length and data availability: the pre-1973 cohort, the 1974-1983 cohort, the 1984-1993 cohort and the post 1993 cohort. The constituents of each cohort are reported in Table 9.

<Insert Table 9 here>

Figure 2 plots average market integration across time for each of the four cohorts. We observe that average market integration becomes less volatile after adjusting for all bias during six main financial crises due the differences between the P&R R squared and R squared adjusted for all bias during crisis periods, particularly the GFC and ESDC. While adjusted market integration increases across time in each cohort, the trend is not as pronounced as suggested by Pukthuanthong and Roll (2009).

<Insert Figure 2 here>

Figure 3 plots average market integration across time for the economic and geographic groupings. The first four graphs illustrate changes in market integration for all markets and then split by market development. The remaining figures plot the dynamics of market integration in each region. Consistently, after adjusting for the bias caused by crises, market integration increases more smoothly over time than implied by the P&R measure in each category. The largest adjustments occur during the GFC and ESDC which are the longer and deeper crises in our sample period. In line with expectations, developed markets exhibit the highest average market integration and frontier markets display low levels of integration. Similarly, the markets of North America and Developed Europe display relatively higher levels of market integration with lower levels of market integration observed in Latin America, Asia-Pacific and Emerging Europe.

<Insert Figure 3 here>

In order to establish whether integration has increased over time we regress the unconditional R squared for each category on a time trend. Table 10 supports the view of increasing market integration over the period 1973 to 2017, consistent with Bekaert, et al. (2007), Carrieri, et al. (2007), Batten, et al. (2015), Bekaert and Mehl (2019) and others. Both measures of R squared show that market integration increases with time but only gradually. However, after adjusting for all bias, the observed increase in market integration is less. At the global level, market integration increases by only 0.0035 each year on average. Individually, 45 markets experience an increase in market integration and just 8 markets have no significant increase or exhibit a decrease in market integration.[17]

<Insert Table 10 here>

Akbari et al. (2020) investigate the dynamics of market integration across markets decomposing total returns into cash-flow expectation and risk-pricing adjustments and employing a smooth transition dynamic conditional correlation (STDCC) model to separately measure economic and financial integration controlling for factor volatilities. In the terms of econometric methods, by comparison, our approach is easily implementable and computationally simple.. More importantly, as discussed above, just considering factor volatilities is unlikely to be sufficient when measuring market integration via R squared. Contagion is a non-negligible factor.

[17] Results available on request.

To compare our method with theirs, we employ the STDCC approach in Akbari et al. (2020) on our data.[18] More specifically, we use the STDCC model to measure the correlations of each of 53 market returns with global market returns from February 1973 to December 2017. Figure 4 compares the average R squared values of the 53 markets using the Pukthuathong and Roll (2009) method, our approach and Akbari et al. (2020)'s STDCC model. We find that for the majority of the sample period, STDCC market integration is lower than our adjusted value, especially in the period between 1980 and 2000.. We postulate that this is due to employing only a single global factor in the STDCC estimation which is unable to capture all global information. In fact Akbari et al (2020) confirm the use of more global factors leads to an increase in level estimated level of market integration. Comparing the results of Table 2 and 3 in Akbari et al. (2020) shows that market integration estimated using five global factors rather than a single global factor yields higher mean integration. Consistent with Akbari et al. (2020), the STDCC measurement suggests a lower level of market integration than the P&R (2009) method for most of the sample and particularly in periods of crisis. However during the period of the GFC and ESDC, our adjusted measure suggests the actual level of integration may be even lower due to additionally controlling for residual volatilities and beta changes.

<Insert Figure 4 here>

5 Conclusions

Pukthuanthong and Roll (2009) employ the explanatory power of a multi-factor model to measure the level of market integration. We investigate the extent to which the R squared in a multi-factor model is affected by factor heteroscedasticity, changes in factor loadings and residual heteroscedasticity. High factor volatility or increases in factor loadings can cause high R squared values while high residual volatility drives down R squared in a multi-factor model. In this paper, we estimate and adjust for the bias caused by these three potential factors on the R squared and employ an unconditional R squared after adjusting for all bias as the measure of market integration during six financial crises: the 1998 US crisis, the 1994-1995 Mexican crisis, the 1997 Asian crisis, the 1998 Russian/LTCM crisis, the 2007-2009 GFC and the 2009-2014 ESDC.

Our intuition is inspired by Forbes and Rigobon (2002) and Bekaert et al. (2014). Forbes and Rigobon (2002) derive the bias in correlation coefficients between two markets caused by factor

[18] We thank Amir Akbari for graciously providing data and code to facilitate our comparison.

heteroscedasticity and propose a methodology to correct for the bias before examining contagion during crises. Bekaert et al. (2014) demonstrate that correlation coefficients during crises are affected by unexpected factor exposures and returns unrelated to factors. This paper similarly considers two types of contagion: 'exposure contagion', which causes unexpected changes in factor exposures, and 'residual contagion', which causes changes in residual volatility. We theoretically and empirically demonstrate the bias in R squared caused by factor heteroscedasticity and contagion and estimate market integration after adjusting for all the bias caused by crises. By identifying the sources of the bias we extend our understanding of how and why the Pukthuanthong and Roll (2009) measure potentially overstates the level of market integration during periods of crisis.

We demonstrate that for most crises, R squared are characterised by adjustments after filtering out the bias caused by factor heteroscedasticity, changes in factor loadings and residual heteroscedasticity. The unconditional R squared, that is, market integration becomes lower than that implied by the Pukthuanthong and Roll (2009) measure and trends in market integration are much more gradual although most markets still witness increasing market integration. More specifically, aside from the 1994-1995 Mexican crisis and the 2009-2014 ESDC, other crises face dramatic increases in factor volatilities, which result in the upward bias in R squared. During the 1987 US crisis and the 1997 Asian crisis, changes in factor loadings drive the increase in the conditional R squared and there is clear positive bias caused by beta changes during these crises. In other words, there is evidence of global 'exposure contagion' during these two crises. The 1998 Russian/LTCM crisis and the 2007-2009 GFC witness a significant rise in residual volatility and lower R squared in the multi-factor models. Residual heteroscedasticity during these two crises causes negative bias in R squared and the unconditional R squared becomes higher after adjusting for the bias caused by residual heteroscedasticity, suggesting the presence of 'residual contagion'. Moreover, after adjustments we do not find evidence of reversion in market integration after the 2007-2009 GFC, consistent with Bekaert and Mehl (2019).

Our evidence highlights that during crisis periods one should be cautious when measuring market integration. Neglecting to account for the presence of factor heteroscedasticity and contagion may lead to overstating the actual level of integration. We provide alternative measures of R squared which adjust for the presence of measurement bias and better measure the level of integration during times of high volatility and crisis. We also demonstrate how our rather simple approach compares to recent evidence from Akbari et al. (2020) who also suggest that the P&R measure over states the level of market integration. The focus of our analysis has been to highlight how crisis periods impact a widely adopted measure of market integration, and that ignoring the impact of crises and contagion through factor heteroscedasticity, changes in factor loadings and residual heteroscedasticity can lead to incorrect inference. This remains an important issue. For instance, in March 2020, the covid-19 outbreak dramatically affected the global financial markets. The pandemic led to panic-selling triggering circuit breakers. For investors accurate measurement of market integration is essential as they seek to diversify their asset portfolios. Future work could consider investigating the effects of the Covid-19 pandemic on market integration, to provide an out-of-sample examination that the bias adjusted measure of market integration implies lower levels of integration that the traditional P&R measure.

References

Akbari, A. and Ng, L., 2021. International market integration: A survey. *Asia-Pacific Journal of Financial Studies* 49(2), 161-195

Akbari, A., Ng, L. and Solnik, B., 2020. Emerging markets are catching up: economic or financial integration?. *Journal of Financial and Quantitative Analysis* 55(7), 2270-2303.

Bali, T.G. and Cakici, N., 2010. World market risk, country-specific risk and expected returns in international stock markets. *Journal of Banking and Finance* 34(6), 1152-1165.

Batten, J.A., Kinateder, H., Szilagyi, P.G. and Wagner, N.F., 2019. Time-varying energy and stock market integration in Asia. *Energy Economics* 80, 777-792.

Batten, J.A., Morgan, P. and Szilagyi, P.G., 2015. Time varying Asian stock market integration. *The Singapore Economic Review* 60(01), 1550006.

Bekaert, G. and De Santis, R.A., 2021. Risk and return in international corporate bond markets. *Journal of International Financial Markets, Institutions and Money* 72, 101338.

Bekaert, G., Ehrmann, M., Fratzscher, M. and Mehl, A., 2014. The global crisis and equity market contagion. *Journal of Finance* 69(6), 2597-2649.

Bekaert, G. and Harvey, C.R., 1995. Time-varying world market integration. *Journal of Finance* 50, 403-444.

Bekaert, G. and Harvey, C.R., 1997. Emerging market volatility. *Journal of Financial Economics* 43, 29-77.

Bekaert, G. and Harvey, C.R., 2003. Emerging markets finance. *Journal of Empirical Finance* 10(1), 3-55.

Bekaert, G., Harvey, C.R. and Ng, A., 2005. Market integration and contagion. *Journal of Business* 78(1), 39-69.

Bekaert, G., Harvey, C.R., Lundblad, C.T. and Siegel, S., 2011. What segments equity markets? *Review of Financial Studies* 24, 3841-3890.

Bekaert, G. and Mehl, A., 2019. On the global financial market integration "swoosh" and the trilemma. *Journal of International Money and Finance* 94, 227-245.

Berger, D. and Pukthuanthong, K., 2012. Market fragility and international market crashes. *Journal of Financial Economics* 105(3), 565-580.

Berger, D. and Pukthuanthong, K., 2016. Fragility, stress, and market returns. *Journal of Banking & Finance* 62, 152-163.

Berger, D., Pukthuanthong, K. and Yang, J.J., 2011. International diversification with frontier markets. *Journal of Financial Economics* 101(1), 227-242.

Bessler, W., Taushanov, G. and Wolff, D., 2021. Optimal asset allocation strategies for international equity portfolios: A comparison of country versus industry optimization. *Journal of International Financial Markets, Institutions and Money* 72, 101343.

Black, F., 1972. Capital market equilibrium with restricted borrowing. *Journal of Business* 45(3), 444-455.

Billio, M., Donadelli, M., Paradiso, A. and Riedel, M., 2017. Which market integration measure?. *Journal of Banking and Finance* 76, 150-174.

Carrieri, F., Errunza, V. and Hogan, K., 2007. Characterizing world market integration through time. *Journal of Financial and Quantitative Analysis* 42, 915-940.

Carrieri, F., Chaieb, I. and Errunza, V., 2013. Do implicit barriers matter for globalization? *Review of Financial Studies* 26, 1694-1739.

Chambet, A. and Gibson, R., 2008. Financial integration, economic instability and trade structure in emerging markets. *Journal of International Money and Finance* 27, 654-675.

Christiansen, C., 2014. Integration of European bond markets. *Journal of Banking and Finance* 42, 191-198.

Christoffersen, P., Errunza, V., Jacobs, K. and Langlois, H., 2012. Is the potential for international diversification disappearing? A dynamic copula approach. *Review of Financial Studies* 25(12), 3711-3751.

Colacito, R. and Croce, M.M., 2010. The short and long run benefits of financial integration. *American Economic Review* 100(2), 527-31.

Cordella, T. and Ospino Rojas, A., 2017. Financial globalization and market volatility: an empirical appraisal. Policy research working paper. World Bank Group, Washington, D.C.

Corsetti, G., Pericoli, M. and Sbracia, M., 2005. 'Some contagion, some interdependence': More pitfalls in tests of financial contagion. *Journal of International Money and Finance* 24(8), 1177-1199.

DeJong, F. and DeRoon, F.A., 2005. Time-varying market integration and expected returns in emerging markets. *Journal of Financial Economics* 78, 583-613.

Donadelli, M. and Paradiso, A., 2014. Is there heterogeneity in financial integration dynamics? Evidence from country and industry emerging market equity indexes. *Journal of International Financial Markets, Institutions and Money* 32, 184-218.

Dumas, B., Harvey, C.R. and Ruiz, P., 2003. Are correlations of stock returns justified by subsequent changes in national outputs?. *Journal of international Money and Finance* 22(6), 777-811.

Dumas, B. and Solnik, B., 1995. The world price of foreign exchange rate risk. *Journal of Finance* 50, 445-479.

Eiling, E. and Gerard, B., 2015. Emerging equity market comovements: trends and macroeconomic fundamentals. *Review of Finance* 19(4), 1543-1585.

Errunza, V. and Losq, E., 1985. International asset pricing under mild segmentation: Theory and test. *Journal of Finance* 40, 105-124.

Errunza, V., Losq, E. and Padmanabhan, P., 1992. Tests of integration, mild segmentation and segmentation hypotheses. *Journal of Banking and Finance* 16(5), 949-972.

Evans, M.D. and Hnatkovska, V.V., 2014. International capital flows, returns and world financial integration. *Journal of International Economics* 92(1), 14-33.

Filoso, V., Panico, C., Papagni, E., Purificato, F. and Suarez, M.V., 2017. *Causes and timing of the European debt crisis: An econometric evaluation* (No. EERI RP 2017/03). Economics and Econometrics Research Institute (EERI), Brussels.

Forbes, K.J. and Rigobon, R., 2002. No contagion, only interdependence: measuring stock market comovements. *Journal of Finance* 57(5), 2223-2261.

Fratzscher, M., 2012. Capital flows, push versus pull factors and the global financial crisis. *Journal of International Economics* 88(2), 341-356.

Gkillas, K., Tsagkanos, A. and Vortelinos, D.I., 2019. Integration and risk contagion in financial crises: Evidence from international stock markets. *Journal of Business Research* 104, 350-365.

Goetzmann, W.N., Li, L. and Rouwenhorst, K.G., 2005. Long-term global market correlations. *Journal of Business* 78(1), 1-38.

Hardouvelis, G.A., Malliaropulos, D. and Priestley, R., 2006. EMU and European stock market integration. *Journal of Business* 79, 365-392.

Harvey, C.R., 1991. The world price of covariance risk. *Journal of Finance* 46, 111-157.

Harvey, C.R., Solnik, B. and Zhou, G., 2002. What determines expected international asset returns?. *Annals of Economics and Finance* 3(2), 249-298.

Hou, K., Peng, L. and Xiong, W., 2013. Is R2 a measure of market inefficiency? Working Paper. Ohio State University, City University of New York, Princeton University, and National Bureau of Economic Research.

Jappelli, T. and Pistaferri, L., 2011. Financial integration and consumption smoothing. *The Economic Journal* 121(553), 678-706.

Jian, Z. and Li, X., 2021. Skewness-based market integration: A systemic risk measure across international equity markets. *International Review of Financial Analysis* 74, 101664.

Lehkonen, H., 2015. Stock market integration and the global financial crisis. *Review of Finance* 19, 2039-2094.

Lewis, K.K., 2011. Global asset pricing. *Annual Review Financial Economics* 3(1), 435-466.

Li, Z. and Liu, L., 2018. Financial globalization, domestic financial freedom and risk sharing across countries. *Journal of International Financial Markets, Institutions and Money* 55, 151-169.

Lintner, J., 1965. The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets. *Review of Economics and Statistics* 47(1), 13-37.

Mauro, P., Sussman, N. and Yafeh, Y., 2002. Emerging market spreads: then versus now. *The Quarterly Journal of Economics* 117(2), 695-733.

Mendoza, E.G. and Quadrini, V., 2010. Financial globalization, financial crises and contagion. *Journal of Monetary Economics* 57(1), 24-39.

Meric, I. and Meric, G., 1989. Potential gains from international portfolio diversification and inter-temporal stability and seasonality in international stock market relationships. *Journal of Banking and Finance* 13(4-5), 627-640.

Pukthuanthong, K. and Roll, R., 2009. Global market integration: An alternative measure and its application. *Journal of Financial Economics* 94(2), 214-232.

Quinn, D. P. and Voth, H. J., 2008. A century of global equity market correlations. *American Economic Review* 98(2), 535-40.

Rigobon, R., 2003. On the measurement of the international propagation of shocks: is the transmission stable? *Journal of International Economics* 61(2), 261-283.

Schotman, P.C. and Zalewska, A., 2006. Non-synchronous trading and testing for market integration in Central European emerging markets. *Journal of Empirical Finance* 13, 462-494.

Sharpe, W.F., 1964. Capital asset prices: A theory of market equilibrium under conditions of risk. *Journal of Finance* 19(3), 425-442.

Suzuki, Y., 2014. Financial integration and consumption risk sharing and smoothing. *International Review of Economics and Finance* 29, 585-598.

Volosovych, V., 2011. Measuring financial market integration over the long run: is there a U-shape? *Journal of International Money and Finance* 30, 1535–1561.

Volosovych, V., 2013. Learning about financial market integration from principal components analysis. *CESifo Economic Studies* 59 (2), 360–391.

Wheatley, S., 1988. Some tests of international equity integration. *Journal of Financial Economics* 21, 177-212.

Yu, C., 2015. Evaluating international financial integration in a center-periphery economy. *Journal of International Economics* 95(1), 129-144.

Yu, I.W., Fung, K.P. and Tam, C.S., 2010. Assessing financial market integration in Asia–equity markets. *Journal of Banking and Finance* 34(12), 2874-2885.

Tables and figures

Table 1: Crisis and Stable Periods

The table lists the six main financial crises reporting their crisis dates and corresponding stable periods during 1973-2017. LTCM is Long-Term Capital Management, GFC refers to the Global Financial crisis and ESDC refers to the European Sovereign Debt crisis;.

Crisis	Crisis Period	Stable Period
1987 US crisis	17/10/1987-04/12/1987	01/01/1986-17/10/1987
1994-1995 Mexican crisis	19/12/1994-31/03/1995	01/06/1994-16/12/1994
1997 Asian crisis	17/10/1997-16/11/1997	01/01/1996-16/10/1997
1998 Russian/LCTM crisis	03/08/1998-15/10/1998	02/03/1998-01/06/1998
2007-2009 GFC	07/08/2007-15/03/2009	01/01/2003-31/12/2006
2009-2014 ESDC	01/06/2009-23/06/2014	01/01/2015-31/12/2017

Table 2: Country Classification

The table presents the constituent markets in each category. 'All World' includes all 53 markets investigated in this paper. The 53 markets are then divided into groupings based on market development and geographic location. The markets are separated into developed markets, emerging markets, and frontier markets. Based on geographic location, markets are grouped as North America, Latin America, Europe which is further split into Developed Europe and Emerging Europe, and Asian-Pacific. The market classification is based on Standard & Poor's and Dow Jones.

Categories	Markets	No.
All World	Argentina, Australia, Austria, Belgium, Brazil, Bulgaria, Canada, Chile, China A shares, China H+B shares, Colombia, Cyprus, Czech, Denmark, Finland, France, Germany, Greece, Hungary, Hong Kong, India, Indonesia, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Malaysia, Mexico, Netherlands, New Zealand, Norway, Pakistan, Philippines, Peru, Poland, Portugal, Romania, Russia, Singapore, Slovenia, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Taiwan, Thailand, Turkey, the UK, the US, Venezuela	53
Developed	Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Hong Kong, Ireland, Israel, Italy, Japan, Korea, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Singapore, Spain, Sweden, Switzerland, the UK, the US	25
Emerging	Brazil, Chile, China A shares, China H+B shares, Colombia, Czech, Greece, Hungary, India, Indonesia, Malaysia, Mexico, Peru, Philippines, Poland, Russia, South Africa, Taiwan, Thailand, Turkey, Venezuela	21
Frontier	Argentina, Bulgaria, Cyprus, Pakistan, Romania, Slovenia, Sri Lanka	7

(Table 2 continued)

Categories	Markets	No.
North America	Canada, the US	2
Latin America	Argentina, Brazil, Chile, Colombia, Mexico, Peru, Venezuela	7
Europe	Austria, Belgium, Bulgaria, Cyprus, Czech, Denmark, Finland, France, Germany, Greece, Hungary, Ireland, Italy, Luxembourg, Netherlands, Norway, Poland, Portugal, Romania, Russia, Slovenia, Spain, Sweden, Switzerland, Turkey, the UK	26
Developed Europe	Austria, Belgium, Denmark, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, the UK	6
Emerging Europe	Czech, Greece, Hungary, Poland, Russia, Turkey	16
Asia-Pacific	China A shares, China H+B shares, Hong Kong, India, Indonesia, Japan, South Korea, Malaysia, Pakistan, Philippines, Singapore, Sri Lanka, Taiwan, Thailand, Australia, New Zealand	16

Table 3: Market Integration and bias during the 1987 US Crisis

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the 1987 US crisis. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (20)	0.5221	0.7864	0.6009	0.4813	0.2896	0.3150	0.1855	0.3051	0.4968	0.4714
Developed (18)	0.5287	0.7998	0.6144	0.5018	0.3040	0.3149	0.1853	0.2979	0.4957	0.4849
Emerging (2)	0.4627	0.6661	0.4789	0.2961	0.1595	0.3167	0.1871	0.3700	0.5066	0.3494
Frontier (0)	---	---	---	---	---	---	---	---	---	---
North America (2)	0.8388	0.9272	0.7968	0.7880	0.6236	0.6429	0.1304	0.1391	0.3036	0.2843
Latin America (0)	---	---	---	---	---	---	---	---	---	---
Europe (12)	0.4908	0.7774	0.5707	0.5075	0.2599	0.2868	0.2067	0.2699	0.5174	0.4906
Developed Europe (12)	0.4908	0.7774	0.5707	0.5075	0.2599	0.2868	0.2067	0.2699	0.5174	0.4906
Emerging Europe (0)	---	---	---	---	---	---	---	---	---	---
Asia-Pacific (5)	0.5194	0.7963	0.6398	0.3666	0.2744	0.2879	0.1564	0.4297	0.5219	0.5084

Table 4: Market Integration and bias during the 1994-1995 Mexican crisis

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the 1994-1995 Mexican crisis. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (48)	0.3046	0.3873	0.3182	0.4044	0.3291	0.3303	0.0690	-0.0171	0.0582	0.0569
Developed (25)	0.4524	0.4785	0.4031	0.5650	0.4685	0.4700	0.0754	-0.0865	0.0100	0.0085
Emerging (19)	0.1559	0.3049	0.2393	0.2598	0.2015	0.1760	0.0656	0.0452	0.1035	0.1289
Frontier (4)	0.1238	0.2081	0.1628	0.0875	0.0640	0.1904	0.0453	0.1206	0.1441	0.0177
North America (2)	0.6194	0.5789	0.5029	0.7477	0.6634	0.7097	0.0760	-0.1688	-0.0845	-0.1308
Latin America (6)	0.1135	0.2776	0.2197	0.1546	0.1194	0.1556	0.0579	0.1230	0.1582	0.1220
Europe (22)	0.3928	0.4328	0.3656	0.5071	0.4215	0.4041	0.0671	-0.0743	0.0113	0.0286
Developed Europe (16)	0.4883	0.5095	0.4352	0.6080	0.5077	0.4871	0.0744	-0.0985	0.0018	0.0225
Emerging Europe (5)	0.1338	0.2031	0.1602	0.2626	0.2128	0.1564	0.0429	-0.0595	-0.0097	0.0467
Asia-Pacific (16)	0.2530	0.3633	0.2871	0.3382	0.2612	0.2709	0.0762	0.0251	0.1021	0.0924

Table 5: Market Integration and bias during the 1997 Asian crisis

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the 1997 Asian crisis. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (49)	0.4103	0.8016	0.6902	0.5312	0.2802	0.2542	0.1114	0.2704	0.5214	0.5475
Developed (25)	0.5449	0.8937	0.7833	0.7063	0.4240	0.3842	0.1104	0.1874	0.4698	0.5095
Emerging (20)	0.2777	0.6987	0.5754	0.3278	0.1244	0.1232	0.1233	0.3709	0.5743	0.5754
Frontier (4)	0.2315	0.7410	0.6823	0.4541	0.1607	0.0959	0.0587	0.2869	0.5803	0.6451
North America (2)	0.8336	0.9771	0.9049	0.9628	0.7884	0.7532	0.0723	0.0143	0.1888	0.2239
Latin America (7)	0.3742	0.8602	0.6875	0.5391	0.1663	0.1412	0.1726	0.3211	0.6939	0.7190
Europe (22)	0.4616	0.8108	0.7036	0.6248	0.3842	0.3223	0.1072	0.1861	0.4267	0.4885
Developed Europe (16)	0.5612	0.9037	0.8061	0.8003	0.5050	0.4162	0.0976	0.1034	0.3987	0.4875
Emerging Europe (4)	0.2127	0.5914	0.4112	0.1197	0.0408	0.0737	0.1802	0.4717	0.5506	0.5177
Asia-Pacific (16)	0.2919	0.7309	0.6359	0.3727	0.1491	0.1640	0.0951	0.3583	0.5818	0.5669

Table 6: Market Integration and bias during the 1998 Russian/LTCM crisis

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the 1998 Russian/LTCM crisis. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (50)	0.4537	0.6070	0.3627	0.5736	0.3254	0.4646	0.2443	0.0334	0.2816	0.1424
Developed (25)	0.5933	0.7062	0.4536	0.7077	0.4367	0.5896	0.2527	-0.0015	0.2696	0.1166
Emerging (20)	0.3564	0.5549	0.3010	0.4713	0.2356	0.3613	0.2539	0.0836	0.3193	0.1936
Frontier (5)	0.1450	0.3194	0.1557	0.3125	0.1287	0.2526	0.1637	0.0068	0.1907	0.0668
North America (2)	0.8106	0.8839	0.7081	0.7730	0.5868	0.7699	0.1757	0.1109	0.2971	0.1140
Latin America (7)	0.3329	0.4988	0.2164	0.3028	0.1147	0.3133	0.2824	0.1959	0.3841	0.1854
Europe (23)	0.5438	0.6907	0.4325	0.6781	0.4190	0.5484	0.2582	0.0126	0.2717	0.1423
Developed Europe (16)	0.6415	0.7643	0.4911	0.7827	0.5077	0.6532	0.2732	-0.0184	0.2566	0.1111
Emerging Europe (5)	0.4204	0.6588	0.3737	0.5329	0.2755	0.3794	0.2851	0.1260	0.3833	0.2795
Asia-Pacific (16)	0.3372	0.4970	0.2889	0.5381	0.2724	0.3833	0.2081	-0.0411	0.2246	0.1137

Table 7: Market Integration and bias during the 2007-2009 GFC

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the GFC. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (53)	0.6255	0.6346	0.2979	0.5786	0.2513	0.4116	0.3368	0.0560	0.3833	0.2230
Developed (25)	0.7673	0.7706	0.4197	0.7243	0.3659	0.6066	0.3509	0.0463	0.4047	0.1640
Emerging (21)	0.5486	0.5650	0.2116	0.5176	0.1771	0.2780	0.3534	0.0475	0.3879	0.2870
Frontier (7)	0.3501	0.3578	0.1215	0.2418	0.0648	0.1157	0.2363	0.1160	0.2931	0.2421
North America (2)	0.8665	0.8666	0.5896	0.7775	0.4836	0.7168	0.2770	0.0891	0.3830	0.1498
Latin America (7)	0.4724	0.4687	0.1454	0.5172	0.1570	0.2277	0.3233	-0.0485	0.3117	0.2410
Europe (26)	0.7106	0.7245	0.3605	0.6447	0.3102	0.5052	0.3640	0.0797	0.4143	0.2193
Developed Europe (16)	0.8054	0.8074	0.4492	0.7750	0.4159	0.6616	0.3582	0.0324	0.3916	0.1458
Emerging Europe (6)	0.6200	0.6484	0.2411	0.5572	0.1890	0.3193	0.4073	0.0912	0.4593	0.3290
Asia-Pacific (16)	0.5287	0.5409	0.2385	0.4749	0.1757	0.3175	0.3024	0.0660	0.3652	0.2234

Table 8: Market Integration and bias during the 2009-2014 ESDC

The table reports the different R squared measures and the estimated bias caused by influential factors across categories during the 2009-2014 ESDC. R^2_{PRy} is (average) R^2 during the crisis year(s) using Pukthuanthong and Roll (2009); R^2_{PR} is the P&R R^2 during the crisis period; R^2_{FH} is R^2 after adjusting for the bias caused by factor heteroscedasticity during the crisis period, which is calculated by the basic equation (28); R^2_{BC} is R^2 after adjusting for the bias caused by beta changes during the crisis period, which is calculated by the basic equation (29); R^2_{FB} is R^2 after adjusting for the bias caused by factor heteroscedasticity and beta changes during the crisis period, which is based on the equation (30); R^2_{ALL} is the unconditional R^2 after adjusting for all bias caused by the crisis, including factor heteroscedasticity, beta changes and residual heteroscedasticity, which is calculated based on the general equation (24). The last four columns calculate the bias caused by influential factors. $\Delta R^2_{FH} = R^2_{PR} - R^2_{FH}$, measures the pure bias caused by factor heteroscedasticity during the crisis; $\Delta R^2_{BC} = R^2_{PR} - R^2_{BC}$, measures the pure bias caused by beta changes during the crisis; $\Delta R^2_{FB} = R^2_{PR} - R^2_{FB}$, measures the bias caused by factor heteroscedasticity and beta changes; $\Delta R^2_{ALL} = R^2_{PR} - R^2_{ALL}$, measures the total bias caused by the crisis. The values are averaged across markets in each category and the value in the parentheses reports the number of markets in each category.

Regions	R^2_{PRy}	R^2_{PR}	R^2_{FH}	R^2_{BC}	R^2_{FB}	R^2_{ALL}	ΔR^2_{FH}	ΔR^2_{BC}	ΔR^2_{FB}	ΔR^2_{ALL}
All World (53)	0.6047	0.5802	0.5001	0.5600	0.4763	0.4923	0.0801	0.0202	0.1039	0.0879
Developed (25)	0.7596	0.7539	0.6776	0.7271	0.6408	0.6648	0.0763	0.0268	0.1131	0.0891
Emerging (21)	0.5250	0.4901	0.3977	0.4951	0.3994	0.3912	0.0924	-0.0050	0.0907	0.0989
Frontier (7)	0.2907	0.2302	0.1735	0.1581	0.1198	0.1796	0.0567	0.0721	0.1104	0.0506
North America (2)	0.8767	0.8740	0.8072	0.8707	0.8028	0.8053	0.0668	0.0033	0.0712	0.0687
Latin America (7)	0.4385	0.3879	0.2913	0.4828	0.3871	0.3205	0.0966	-0.0949	0.0008	0.0673
Europe (26)	0.7002	0.6846	0.6011	0.6199	0.5403	0.5798	0.0834	0.0647	0.1443	0.1048
Developed Europe (16)	0.8218	0.8206	0.7458	0.7945	0.7122	0.7435	0.0747	0.0261	0.1084	0.0771
Emerging Europe (6)	0.5773	0.5543	0.4439	0.4619	0.3559	0.3707	0.1103	0.0924	0.1984	0.1836
Asia-Pacific (16)	0.4980	0.4661	0.4031	0.4560	0.3728	0.3942	0.0630	0.0102	0.0934	0.0720

Table 9: Sample period cohorts

The table documents the constituents of the sample period cohorts.

Category	Market included	No.
Pre-1973 Cohort	Canada, the US, Australia, Austria, Belgium, Denmark, France, Germany, Hong Kong, Ireland, Italy, Japan, Netherlands, Singapore, South Africa, Switzerland, the UK	17
1974-83 Cohort	Norway, Sweden	2
1984-93 Cohort	Argentina, Chile, China A, Colombia, Cyprus, Finland, Greece, Hungary, India, Indonesia, Israel, Korea, Luxembourg, Malaysia, Mexico, New Zealand, Pakistan, Philippines, Portugal, Spain, Sri Lanka, Taiwan, Thailand, Turkey, Venezuela	25
Post-1993 Cohort	Brazil, Bulgaria, China H+B, Czech, Peru, Poland, Romania, Russia, Slovenia	9

Table 10: Trends in Market Integration

The table presents the time trend in conditional and unconditional R squared for each category. For each category it reports the sample length in years and the coefficient on a linear time trend and its corresponding t-statistic for both R^2_{PRy}, the conditional R squared measured by Pukthuanthong and Roll (2009), and R^2_{ALLy}, the unconditional R squared which adjusts for all bias. *** and ** indicate significance at the 1% and 5% level.

Panel A categories based on sample availability					
Categories	Years	R^2_{PRy}		R^2_{ALLy}	
		Coefficient	t-Statistics	Coefficient	t-Statistics
Pre-1973 Cohort	45	0.0109***	11.2495	0.0098***	11.3679
1974-83 cohort	38	0.0142***	8.5880	0.0123***	9.5616
1984-93 cohort	32	0.0055**	2.2815	0.0036**	1.8519
Post-1993 cohort	24	0.0144***	3.7151	0.0105***	5.2249
Panel B categories based on market development and geographic location					
Categories	Years	R^2_{PRy}		R^2_{ALLy}	
		Coefficient	t-Statistics	Coefficient	t-Statistics
All World	45	0.0050***	4.6811	0.0035***	4.3003
Developed	45	0.0098***	10.1766	0.0086***	10.5850
Emerging	45	0.0066***	4.5918	0.0046***	3.9611
Frontier	30	0.0075***	4.8742	0.0049***	5.9365
North America	45	0.0035***	3.5370	0.0025***	3.1730
Latin America	30	0.0119***	5.5982	0.0094***	7.3133
Europe	45	0.0082***	7.4343	0.0066***	7.2819
Developed Europe	45	0.0125***	12.3755	0.0116***	12.4252
Emerging Europe	30	0.0147***	4.9767	0.0103***	5.6042
Asia-Pacific	45	0.0046***	4.0049	0.0031***	3.5530

Figure 1: Global Market Capitalisation

This figure shows percentage of total market capitalisation accounted by the main 17 markets across time. The market values are from DataStream and expressed with U.S. dollars.

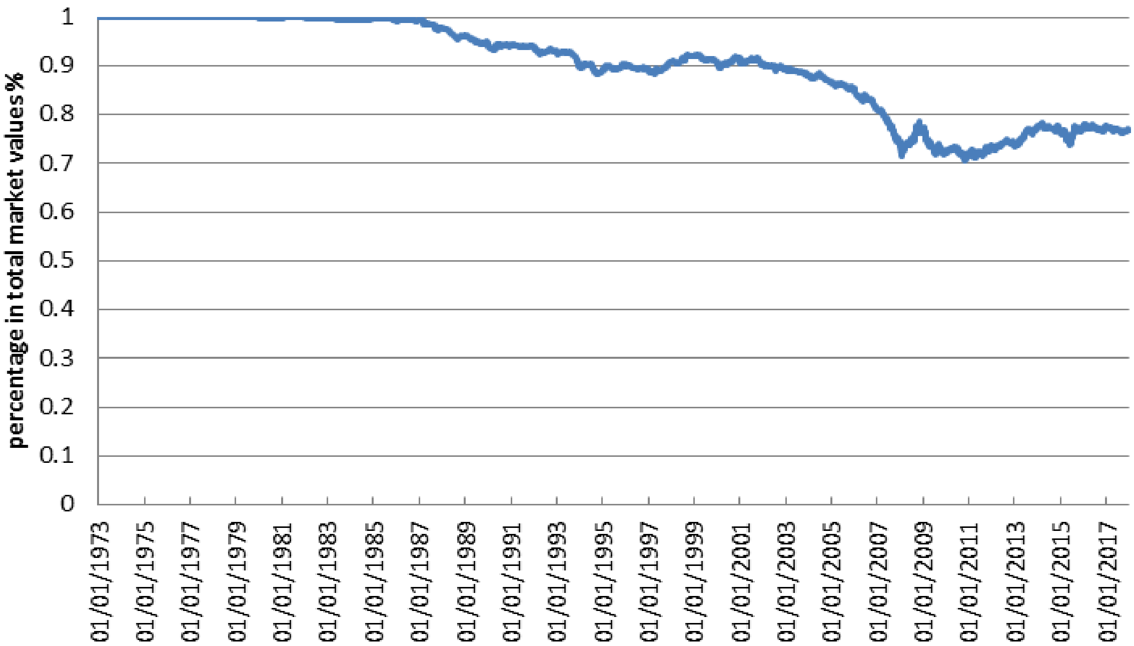


Figure 2: Market Integration across Time Cohorts

This figure shows the trend of average market integration for four categories: pre-1973 cohort, 1974-83 cohort, 1984-93 cohort, and post-1993 cohort. The categories are based on the time of data availability.

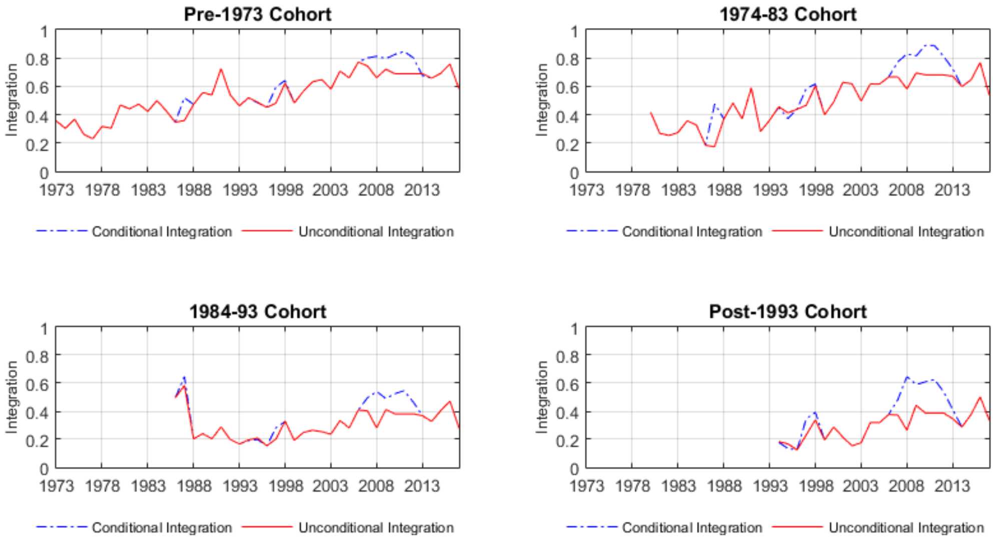
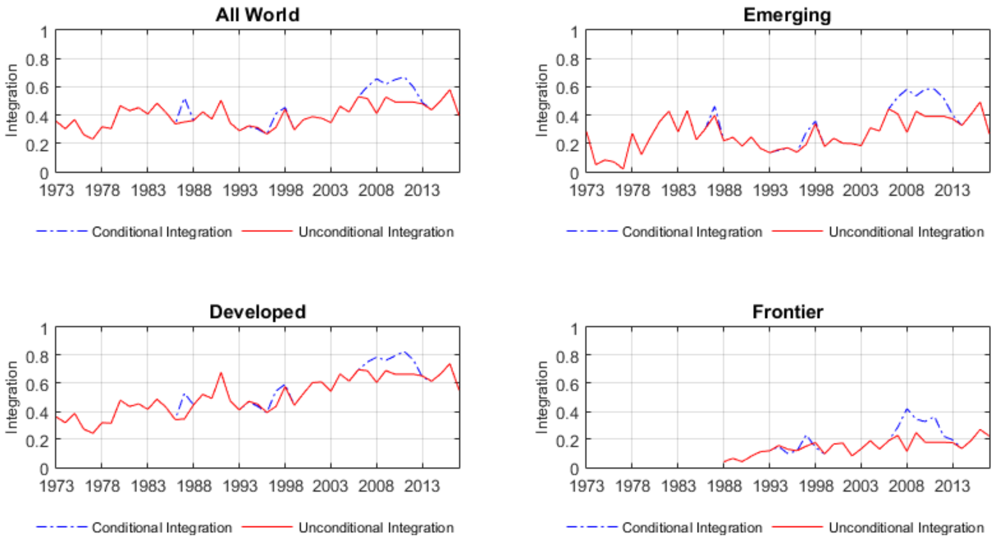


Figure 3: Market Integration across Economic Levels and Regions

This figure shows the trend of market integration for two groupings: economic level and geographical location. The economic categories include All World markets, Emerging markets, Developed markets and Frontier markets. The geographical categories include North America markets, Latin America markets, Europe markets, Asia-Pacific markets, Emerging Europe markets and Developed Europe markets.



(Figure 3 continued)

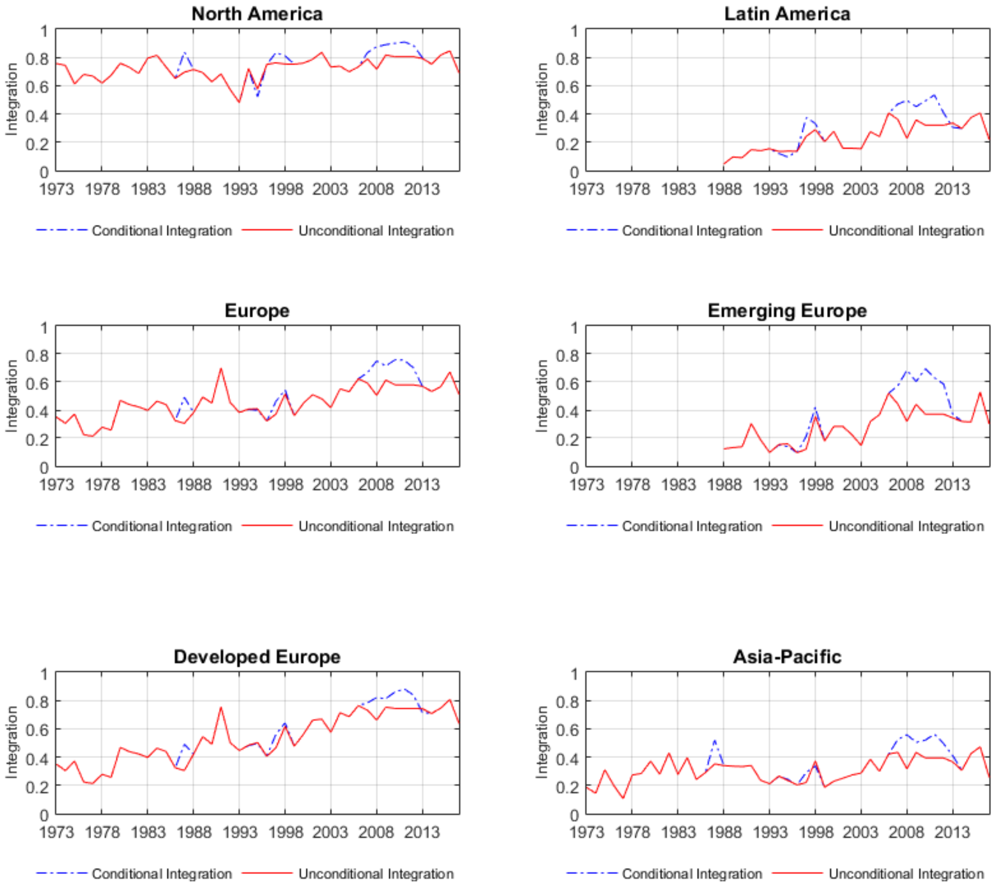
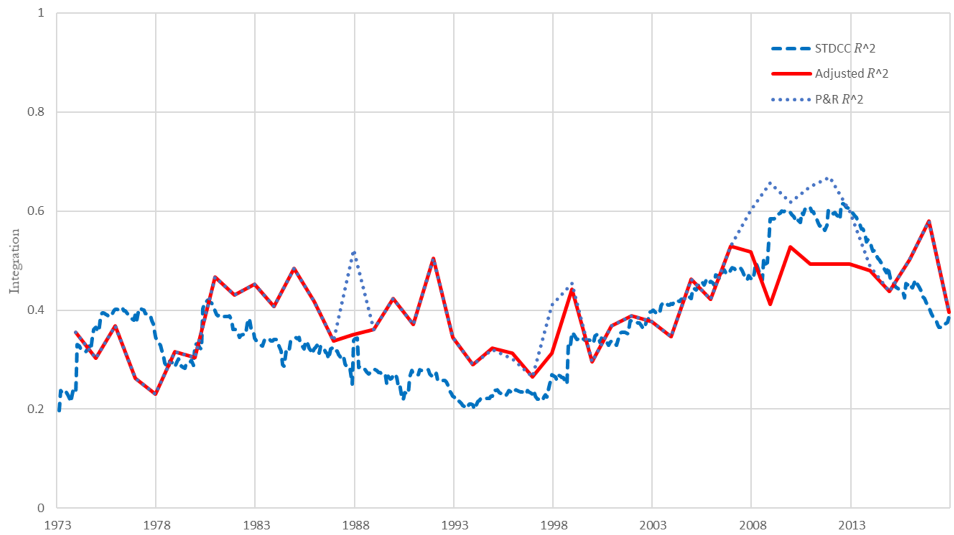


Figure 4: Comparison of Financial Integration Measure (P&R R^2 vs. Adjusted R^2_{ALL} vs. STDCC R^2)

This figure shows the dynamics of average market integration across 53 markets separately employing the Pukthuathong and Roll (2009) measure (P&R R^2), our adjusted measure (Adjusted R^2_{ALL}), and the measure of Akbari et al. (2020) (STDCC R^2).



Appendix

Given a N-factor model

$$y_t = \alpha + \beta_1 x_{1t} + \beta_2 x_{2t} + \cdots + \beta_N x_{Nt} + \varepsilon_t \tag{A.1}$$

where, $x_{1t}, x_{2t}, \dots, x_{Nt}$ are N global risk factors, y_t is the stock market return, $E[\varepsilon_t] = 0$, $E[x_{it}\varepsilon_t] = 0$, $E[x_{it}x_{jt}] = 0$ and $E[x_{it}] = 0$ where $i, j = 1,2, \dots, N$ but $i \neq j$. Data is mean-centred data, so all PCA factors have zero means.

Divide the full sample into two sets so that the variance of y_t is lower in the first group (l, called the low-variance period below) and higher in the second group (h, called the high-variance period below), such that $\sigma_{yy}^h > \sigma_{yy}^l$.

The relation between factor variance, residual variance and the factor loadings in the high and low variance periods is assumed to be: $\sigma_{x_i x_i}^h = (1 + \delta_{x_i})\sigma_{x_i x_i}^l$, $\sigma_{\varepsilon\varepsilon}^h = (1 + \delta_\varepsilon)\sigma_{\varepsilon\varepsilon}^l$, $\beta_i^h = (1 + \delta_{\beta_i})\beta_i^l$, where $\sigma_{x_i x_i}$ is the variance of x_i, $\sigma_{\varepsilon\varepsilon}$ is the variance of residuals ε, δ_{x_i}, $\delta_\varepsilon > -1$ and at least one of δ_{β_i} is not equal to -1 $(i = 1,2, \dots, N)$[19]. Moreover, we assume $\sigma_{\varepsilon\varepsilon}^h = c^h < \infty$ and $\sigma_{\varepsilon\varepsilon}^l = c^l < \infty$, where c^h and c^l are constant.

To derive the bias in R^2 during crisis period under an N-factor model:

The R squared can be written as:

$$R^2 = (\rho_{x_1 y}, \rho_{x_2 y}, \dots, \rho_{x_N y}) \begin{pmatrix} 1 & \rho_{x_1 x_2} & \cdots & \rho_{x_1 x_N} \\ \rho_{x_2 x_1} & 1 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{x_N x_1} & \cdots & \cdots & 1 \end{pmatrix}^{-1} \begin{pmatrix} \rho_{x_1 y} \\ \rho_{x_2 y} \\ \vdots \\ \rho_{x_N y} \end{pmatrix} \tag{A.2}$$

where, $\rho_{x_i y}$ refers to the correlation between x_i and y; $\rho_{x_i x_j}$ are the correlation between x_i and x_j, $i, j = 1,2, \dots, N$.

[19] Here, we do not consider the extreme case when $\delta_{\beta_1} = \delta_{\beta_2} = \cdots = \delta_{\beta_N} = -1$, that is, the market is fully segmented.

Since the N global risk factors are orthogonal by construction, $\rho_{x_i x_j} = 0$ ($i, j = 1,2, \dots N$ and $i \neq j$). Therefore, the R squared can be rewritten as:

$$R^2 = \left(\rho_{x_1 y}\right)^2 + \left(\rho_{x_2 y}\right)^2 + \cdots + \left(\rho_{x_N y}\right)^2 = \sum_{i=1}^{N}\left(\rho_{x_i y}\right)^2 \tag{A.3}$$

Here, we assume the market is neither fully integrated nor fully segmented, so $0 < R^2 < 1$, that is, $0 < \sum_{i=1}^{N}\left(\rho_{x_i y}\right)^2 < 1$. Also, based on the assumption, we know $\left(\rho_{x_i y}\right)^2 < 0$ ($i = 1,2, \dots N$).

Step 1: Deriving the covariance between $\boldsymbol{x_i}$ and $\boldsymbol{y}$ in the high-variance period: $\boldsymbol{\sigma^h_{x_i,y}}$ ($\boldsymbol{i = 1, 2, \dots, N}$)

Since $x_1, x_2, \dots, x_N$ are uncorrelated, the same coefficients β_i ($i = 1,2, \dots, N$) are obtained from the above multiple regression (A.1) and the following bivariate regressions.

A simple bivariate regression of y_t on x_{it} $(i = 1,2, .., N)$

$y_t = \alpha + \beta_i x_{it} + v_{it}$ where $v_{it} = \sum_{m=1, m\neq i}^{N} \beta_m x_{mt} + \varepsilon_t$

We know $E[v_{it}] = 0$, $E[x_{it} v_{it}] = 0$ and $E[{v_{it}}^2] < \infty$.

$$\rho_{x_i y} = \frac{\sigma_{x_i y}}{\sigma_{x_i}\sigma_y} = \frac{\sigma_{x_i y}}{\sigma_{x_i x_i}}\frac{\sigma_{x_i}}{\sigma_y} = \beta_i \frac{\sigma_{x_i}}{\sigma_y} \tag{A.4}$$

Moreover, since $\beta_i^h = \left(1 + \delta_{\beta_i}\right)\beta_i^l$ and $\beta_i = \frac{\sigma_{x_i y}}{\sigma_{x_i x_i}}$, this can be rewritten as $\frac{\sigma^h_{x_i y}}{\sigma^h_{x_i x_i}} = \left(1 + \delta_{\beta_i}\right)\frac{\sigma^l_{x_i y}}{\sigma^l_{x_i x_i}}$. Rearranging gives $\frac{\sigma^h_{x_i y}}{\sigma^l_{x_i y}} = \left(1 + \delta_{\beta_i}\right)\frac{\sigma^h_{x_i x_i}}{\sigma^l_{x_i x_i}} = \left(1 + \delta_{\beta_i}\right)\left(1 + \delta_{x_i}\right)$. Therefore, $\sigma^h_{x_i y}$ can be expressed as:

$$\sigma^h_{x_i y} = \left(1 + \delta_{\beta_i}\right)\left(1 + \delta_{x_i}\right)\sigma^l_{x_i y} \tag{A.5}$$

Step 2: Deriving the variance of $\boldsymbol{y}$ in the high-variance period: $\boldsymbol{\sigma^h_{yy}}$

$$\sigma_{yy}^h = \sum_{i=1}^{N} \left(\beta_i^h\right)^2 \sigma_{x_i x_i}^h + \sigma_{\varepsilon\varepsilon}^h = \sum_{i=1}^{N} \left(1+\delta_{\beta_i}\right)^2 \left(\beta_i^l\right)^2 \left(1+\delta_{x_i}\right) \sigma_{x_i x_i}^l + (1+\delta_\varepsilon)\sigma_{\varepsilon\varepsilon}^l$$

$$= \sum_{i=1}^{N} \left(\beta_i^l\right)^2 \sigma_{x_i x_i}^l + \sigma_{\varepsilon\varepsilon}^l + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\beta_i^l\right)^2 \sigma_{x_i x_i}^l + \delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l$$

$$= \sigma_{yy}^l \left[1 + \frac{\sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\beta_i^l\right)^2 \sigma_{x_i x_i}^l + \delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right]$$

$$= \sigma_{yy}^l \left[1 + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right] \quad \text{(A.6)}$$

Moreover, since $\sigma_{yy}^h > \sigma_{yy}^l$, $\sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l} > 0$ (A.7).

Step 3: Deriving the R squared in the high-variance period: $\boldsymbol{R^{2,h}}$

Given the correlation between factors and the return in the high-variance period:

$$\rho_{x_i y}^h = \frac{\sigma_{x_i y}^h}{\sigma_{x_i}^h \sigma_y^h} = \frac{\left(1+\delta_{\beta_i}\right)\left(1+\delta_{x_i}\right)\sigma_{x_i y}^l}{\left[\left(1+\delta_{x_i}\right)\right]^{\frac{1}{2}} \sigma_{x_i}^l \sigma_y^l \left\{1 + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right\}^{\frac{1}{2}}}$$

$$= \rho_{x_i y}^l \frac{\left(1+\delta_{\beta_i}\right)\left[\left(1+\delta_{x_i}\right)\right]^{1/2}}{\left\{1 + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right\}^{1/2}} \qquad (i = 1,2,..,N) \quad \text{(A.8)}$$

The R squared can be written as:

$$R^{2,h} = \sum_{i=1}^{N} \left(\rho_{x_i y}^h\right)^2 = \frac{\sum_{i=1}^{N} \left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) \left(\rho_{x_i y}^l\right)^2}{1 + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}}$$

$$= R^{2,l} \frac{\sum_{i=1}^{N} \left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) \left(\rho_{x_i y}^l\right)^2}{\sum_{i=1}^{N} \left(\rho_{x_i y}^l\right)^2 \left\{1 + \sum_{i=1}^{N} \left[\left(1+\delta_{\beta_i}\right)^2 \left(1+\delta_{x_i}\right) - 1\right] \left(\rho_{x_i y}^l\right)^2 + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right\}}$$

$$= R^{2,l} \frac{\sum_{i=1}^{N}\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)\left(\rho_{x_iy}^l\right)^2}{\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2 \sum_{i=1}^{N}\left(1+\delta_{\beta_i}\right)^2\left(1+\delta_{x_i}\right)\left(\rho_{x_iy}^l\right)^2+\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2\left(1-\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2+\frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}\right)} \quad (A.9)$$

To better understand the expression, several examples are presented below:

Example 1:

If there is no difference in factor variances, residual variance and factor loadings between the high-variance and low-variance period, then:

$\delta_{\beta_1} = \delta_{\beta_2} = \cdots = \delta_{\beta_N} = \delta_{x_1} = \delta_{x_2} = \cdots = \delta_{x_N} = \delta_\varepsilon = 0$, and $R^{2,h} = R^{2,l}$.

Example 2

If there is no difference in residual variance and factor loadings between the high-variance and low-variance period, but one of the factor variances differs then:

$\delta_{\beta_1} = \cdots = \delta_{\beta_N} = \delta_{x_1} = \cdots = \delta_{x_{j-1}} = \delta_{x_{j+1}} = \cdots = \delta_{x_N} = \delta_\varepsilon = 0$ with $\delta_{x_j} \neq 0$,

1) Equation (A.7) becomes $\delta_{x_j}\left(\rho_{x_jy}^l\right)^2 > 0$, so $\delta_{x_j} > 0$;
2) $R^{2,h}$ can be rewritten as:

$$R^{2,h} = R^{2,l} \frac{\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2+\delta_{x_j}\left(\rho_{x_jy}^l\right)^2}{\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2+\delta_{x_j}\left(\rho_{x_jy}^l\right)^2 \sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2} \quad (A.10)$$

Assuming $a = \sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2$, $b = \left(\rho_{x_jy}^l\right)^2$ $(0 < a, b < 1)$, then

$$R^{2,h} = R^{2,l} \frac{a+\delta_{x_j}b}{a+\delta_{x_j}ba} \quad (A.11)$$

Since $0 < a, b < 1$ and $\delta_{x_j} > 0$, $\frac{a+\delta_{x_j}b}{a+\delta_{x_j}ba} > 1$. Thus, $R^{2,h} > R^{2,l}$ $(j = 1, \ldots, N)$;

3) The partial derivative of $R^{2,h}$ with respect to δ_{x_j} is

$$\frac{\partial R^{2,h}}{\partial \delta_{x_j}} = R^{2,l}\frac{b\left(a+\delta_{x_j}ba\right)-(a+\delta_{x_j}b)ba}{\left(a+\delta_{x_j}ba\right)^2} = R^{2,l}\frac{ab(1-a)}{\left(a+\delta_{x_j}ba\right)^2} > 0 \qquad \text{(A.12)}$$

So, $R^{2,h}$ is an increasing function of δ_{x_j} $(j = 1, \dots, N)$.

Example 3:

If there is no difference in factor variances and residual variance between the high-variance and low-variance period, but one of the factor loadings differs then:

$$\delta_{x_1} = \cdots = \delta_{x_N} = \delta_{\beta_1} = \cdots = \delta_{\beta_{j-1}} = \delta_{\beta_{j+1}} = \cdots = \delta_{\beta_N} = \delta_{\varepsilon} = 0 \text{ with } \delta_{\beta_j} \neq 0$$

1) Equation (A.7) becomes $\left(2\delta_{\beta_j} + \delta_{\beta_j}^2\right)\left(\rho_{x_jy}^l\right)^2 > 0$, so $\delta_{\beta_j} > 0$ or $\delta_{\beta_j} < -2$;
2) $R^{2,h}$ can be rewritten as:

$$R^{2,h} = R^{2,l}\frac{\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2 + \left(2\delta_{\beta_j} + \delta_{\beta_j}^2\right)\left(\rho_{x_jy}^l\right)^2}{\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2 + \left(2\delta_{\beta_j} + \delta_{\beta_j}^2\right)\left(\rho_{x_jy}^l\right)^2\sum_{i=1}^{N}\left(\rho_{x_iy}^l\right)^2}$$

$$= R^{2,l}\frac{\left[\left(\rho_{x_1y}^l\right)^2+\left(\rho_{x_2y}^l\right)^2\right]+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)\left(\rho_{x_jy}^l\right)^2}{\left[\left(\rho_{x_1y}^l\right)^2+\left(\rho_{x_2y}^l\right)^2\right]+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)\left(\rho_{x_jy}^l\right)^2\left[\left(\rho_{x_1y}^l\right)^2+\left(\rho_{x_2y}^l\right)^2\right]} \qquad \text{(A.13)}$$

Assuming $a = \left(\rho_{x_1y}^l\right)^2 + \left(\rho_{x_2y}^l\right)^2$, $b = \left(\rho_{x_jy}^l\right)^2$ $(0 < a, b < 1)$, then

$$R^{2,h} = R^{2,l}\frac{a+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)b}{a+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)ba} = R^{2,l}\left(\frac{1}{a} - \frac{1-a}{a+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)ba}\right) \qquad \text{(A.14)}$$

Since $0 < a, b < 1$ and $2\delta_{\beta_j} + \delta_{\beta_j}^2 > 0$, $R^{2,h} > R^{2,l}$ $(j = 1, \dots, N)$;

3) The partial derivative of $R^{2,h}$ with respect to δ_{β_j} is

$$\frac{\partial R^{2,h}}{\partial \delta_{\beta_j}} = R^{2,l}\frac{(1-a)ba(2+2\delta_{\beta_j})}{\left(a+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)ba\right)^2} = R^{2,l}\frac{2(1-a)ba}{\left(a+\left(2\delta_{\beta_j}+\delta_{\beta_j}^2\right)ba\right)^2}(1+\delta_{\beta_j}) \qquad \text{(A.15)}$$

If $\delta_{\beta_j} > 0, \frac{\partial R^{2,h}}{\partial \delta_{\beta_j}} > 0$, which means $R^{2,h}$ is an increasing function of δ_{β_j} ($j = 1, \dots, N$);

If $\delta_{\beta_j} < -2, \frac{\partial R^{2,h}}{\partial \delta_{\beta_j}} < 0$, which means $R^{2,h}$ is a decreasing function of δ_{β_j} ($j = 1, \dots, N$).

Example 4:

If there is no difference in factor variances and factor loadings between the high-variance and low-variance period, but the residual variance differs then:

When $\delta_{x_1} = \cdots = \delta_{x_N} = \delta_{\beta_1} = \cdots = \delta_{\beta_N} = 0$ with $\delta_\varepsilon \neq 0$,

1) Equation (A.7) becomes $\frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l} > 0$, so $\delta_\varepsilon > 0$;
2) $R^{2,h}$ can be rewritten as:

$$R^{2,h} = R^{2,l} \frac{1}{1+\frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}} \tag{A.16}$$

Since $\delta_\varepsilon > 0$, $R^{2,h} < R^{2,l}$;

3) The partial derivation of $R^{2,h}$ with respect to δ_ε is

$$\frac{\partial R^{2,h}}{\partial \delta_\varepsilon} = R^{2,l} \frac{-\sigma_{yy}^l \sigma_{\varepsilon\varepsilon}^l}{\left(\sigma_{yy}^l + \delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l\right)^2} < 0 \tag{A.17}$$

So, $R^{2,h}$ is a decreasing function of δ_ε;

In **general**, assuming $a = \sum_{i=1}^N \left(\rho_{x_i y}^l\right)^2$, $f(\delta_\varepsilon) = \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l}$ and $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right) = \sum_{i=1}^N \left(1 + \delta_{\beta_i}\right)^2 \left(1 + \delta_{x_i}\right) \left(\rho_{x_i y}^l\right)^2$, the $R^{2,h}$ can be written as (Note: $0 < a < 1$):

$$R^{2,h} = R^{2,l} \frac{f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)}{af\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right) + a\left(1 - a + f(\delta_\varepsilon)\right)} \tag{A.18}$$

1) The partial derivation of $R^{2,h}$ with respect to $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)$ is

$$\frac{\partial R^{2,h}}{f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)}$$

$$= \frac{f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right) + 1 - a + f(\delta_\varepsilon) - f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)}{\left(f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right) + 1 - a + f(\delta_\varepsilon)\right)^2}$$

$$= \frac{1-a+f(\delta_\varepsilon)}{\left(f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)+1-a+f(\delta_\varepsilon)\right)^2} \tag{A.19}$$

Now, show $1 - a + f(\delta_\varepsilon) > 0$.

$$\rho_i = \beta_i \frac{\sigma_{x_i}}{\sigma_y} \to \sum_{i=1}^{N} \left(\rho_{x_i y}^h\right)^2 = \sum_{i=1}^{N} \beta_i^2 \frac{\sigma_{x_i x_i}}{\sigma_{yy}} = \frac{\sum_{i=1}^{N} \beta_i^2 \sigma_{x_i x_i}}{\sigma_{yy}} = \frac{\sigma_{yy} - \sigma_{\varepsilon\varepsilon}}{\sigma_{yy}}$$

$$\to \sum_{i=1}^{N} \left(\rho_{x_i y}^h\right)^2 \sigma_{yy} = \sigma_{yy} - \sigma_{\varepsilon\varepsilon}$$

$$\to \frac{\left(\sum_{i=1}^{N}\left(\rho_{x_i y}^h\right)^2 - 1\right)\sigma_{yy}}{\sigma_{\varepsilon\varepsilon}} = -1 \tag{A.20}$$

Since $\delta_\varepsilon > -1$,

$$\delta_\varepsilon > \frac{\left(\sum_{i=1}^{N}\left(\rho_{x_i y}^h\right)^2 - 1\right)\sigma_{yy}^l}{\sigma_{\varepsilon\varepsilon}^l} \to (1-a)\sigma_{yy}^l + \delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l > 0 \to 1 - a + \frac{\delta_\varepsilon \sigma_{\varepsilon\varepsilon}^l}{\sigma_{yy}^l} > 0$$

$$\to 1 - a + f(\delta_\varepsilon) > 0 \tag{A.21}$$

Since $1 - a + f(\delta_\varepsilon) > 0$, $R^{2,h}$ is an increasing function of $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)$.

- Since $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)$ is an increasing function of δ_{x_i}, $R^{2,h}$ is an increasing function of δ_{x_i}, where $\delta_{x_i} > -1$ and $i = 1, 2, \dots, N$;
- When $\delta_{\beta_i} > -1$ $(i = 1,2,\dots,N)$, then $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)$ is an increasing function of δ_{β_i}, which means $R^{2,h}$ is an increasing function of δ_{β_i} $(i = 1,2,\dots,N)$;

  When $\delta_{\beta_i} < -1$ $(i = 1,2,\dots,N)$, then $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right)$ is a decreasing function of δ_{β_i}, which means $R^{2,h}$ is a decreasing function of δ_{β_i} $(i = 1,2,\dots,N)$.

2) The partial derivative of $R^{2,h}$ with respect to $f(\delta_\varepsilon)$ is

$$\frac{\partial R^{2,h}}{\partial f(\delta_\varepsilon)} = \frac{-1}{\left(f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)+1-a+f(\delta_\varepsilon)\right)^2} < 0 \tag{A.22}$$

Since $f(\delta_\varepsilon)$ is an increasing function of δ_ε, $R^{2,h}$ is a decreasing function of δ_ε ($\delta_\varepsilon > -1$);

3) Based on Equation (A.7), we know

$$\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)\left(1+\delta_{\beta_i}\right)^2 - 1\right]\left(\rho_{x_i y}^{l}\right)^2 > -f(\delta_\varepsilon) \tag{A.23}$$

Then $R^{2,h}$ can be rewritten as:

$$R^{2,h} = R^{2,l}\frac{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)}{af\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)+a\left(1-a+f(\delta_\varepsilon)\right)}$$

$$= R^{2,l}\frac{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)}{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-\left\{\left[f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-a\right](1-a)-af(\delta_\varepsilon)\right\}}$$

$$= R^{2,l}\frac{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)}{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-\psi} \tag{A.24}$$

where, $\psi = \left[f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-a\right](1-a)-af(\delta_\varepsilon)$.

- When $-1 < \delta_\varepsilon \le 0$, according to Equation (A.23) and $0 < a < 1$, we can get

$$\psi = \left[f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-a\right](1-a)-af(\delta_\varepsilon) = \left[\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)\left(1+\delta_{\beta_i}\right)^2 - 1\right]\left(\rho_{x_i y}^{l}\right)^2\right](1-a)-af(\delta_\varepsilon) > -f(\delta_\varepsilon)(1-a)-af(\delta_\varepsilon) = -f(\delta_\varepsilon) > 0 \tag{A.25}$$

Meanwhile, according to (A.7), $f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-a+f(\delta_\varepsilon) > 0$. So, $f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-\psi = af\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)+a\left(1-a+f(\delta_\varepsilon)\right) = a\left(1+f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-a+f(\delta_\varepsilon)\right) > a$. Besides, $f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right) > \psi + a > 0$, so $\frac{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)}{f\left(\delta_{\beta_1},\dots,\delta_{\beta_N},\delta_{x_1},\dots,\delta_{x_N}\right)-\psi} > 1$. Therefore, $R^{2,h} > R^{2,l}$.

- When $\delta_\varepsilon > 0$, if $\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)\left(1+\delta_{\beta_i}\right)^2 - 1\right]\left(\rho_{x_i y}^{l}\right)^2 \le 0$, we can get ($0 < a < 1$)

$$\psi = \left[\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)\left(1+\delta_{\beta_i}\right)^2 - 1\right]\left(\rho_{x_i y}^{l}\right)^2\right](1-a) - af(\delta_\varepsilon) \leq -af(\delta_\varepsilon) < 0$$

(A.26)

Meanwhile, $f\left(\delta_{\beta_1}, \dots, \delta_{\beta_N}, \delta_{x_1}, \dots, \delta_{x_N}\right) > 0$, thus $R^{2,h} < R^{2,l}$; if $\sum_{i=1}^{N}\left[\left(1+\delta_{x_i}\right)\left(1+\delta_{\beta_i}\right)^2 - 1\right]\left(\rho_{x_i y}^{l}\right)^2 > 0$, t