The influence of the modes of surrounding buildings on the ground vibration from railways

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ABSTRACT

To improve the accuracy of railway induced ground vibration predictions, the influence of surrounding buildings should be considered. For this reason, a fast semi-analytical model is developed to investigate the vibration of the ground and the influence of a nearby building. The model contains two sub models: a finite element model which can simulate the building, and a semi-analytical model which can simulate the infinite free-field ground. The dynamic stiffness matrix method is used to simulate the ground as a homogeneous or layered half-space. The ground model works in the frequency-wavenumber domain and is based on the Green's functions developed for a half-space. Focusing on a building with a piled foundation, the coupling forces due to soil-structure interaction at coupled pile nodes are calculated assuming that the soil and pile displacements are identical. The results from the proposed model show that the vertical modes of the building, especially those that involve the vertical movement of the floors, can significantly influence the vibration of the surrounding ground. The findings suggest that when predicting the ground surface response induced by the railway, the surrounding buildings and their vertical modes should be included in the calculations.

1. INTRODUCTION

Railway-induced ground vibration in an urban environment is affected by the existence of the surrounding buildings between the load position and target building. Current prediction models for railway-induced vibration generally divide the problem into the vibration source, transmission path, and the receiver [1]. While the soil-structure interaction [2] and coupling loss [3] of the receiver structure are often considered, the transmission path is usually seen as the free field. In other words, the current prediction models neglect the effects of neighbouring structures. However, due to the soil-structure interaction, the vibration of the free field is affected by the vibration of the buildings. The effect of adjacent buildings should therefore be taken into account to increase the accuracy of railway-induced ground vibration predictions.

In this study, a rapid semi-analytical model is built to analyse the effect of a neighbouring structure on train-induced ground vibration. In this model, the dynamic stiffness matrix (DSM) method in the wavenumber-frequency domain [4] is used to simulate the ground response. The building structure is viewed as a column-plate system that can be modelled using a finite element (FE) model; for this the Stabil FE MATLAB toolbox is used [5] which can calculate the building modal frequencies and mode

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shapes conveniently. The DSM and FE models are coupled in the frequency domain at the node points of the FE model that coincide with the ground. The whole model approach is described first, followed by details on each sub-model, and then model validation and results from a case study.

2. METHODOLOGY

The problem studied is illustrated in Figure 1. A unit harmonic load is applied to the ground surface and the vibration response at a receiver point behind a building is determined. This can be seen as the superposition of two parts: the response in the free field to the unit load, and the response due to the reaction forces caused by the building at the coupled nodes (coupling the soil and the foundation). The effects of these reaction forces are equivalent to the influence of the building on the ground.

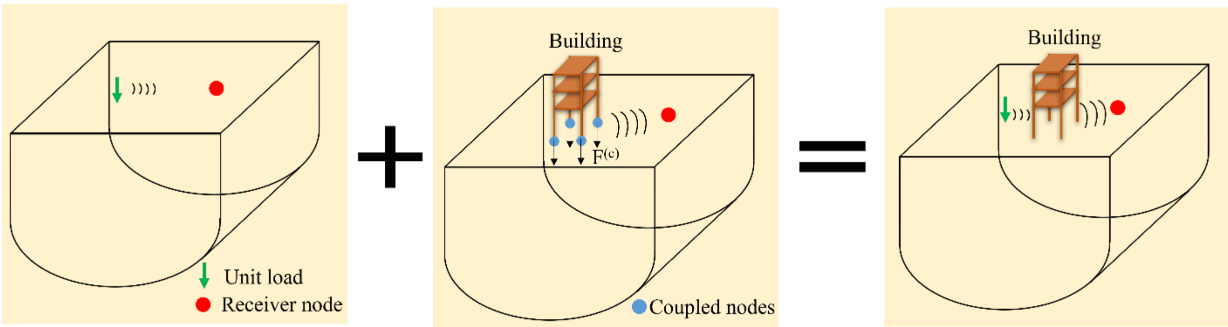

Figure 1: Schematic diagram of the modelling approach

The sub-models are introduced in the following, including the building structure FE model and the ground frequency-wavenumber domain semi-analytical model based on DSM.

2.1. Sub-models

The building is simulated by an FE model and consists of columns represented by beam elements and floor plates represented by shell elements. More details of these elements are provided in references [5], [6] and [7]. The beam elements include bending in two directions and axial extension. The building model is shown schematically in Figure 2. The beam elements below the ground floor are considered to be piles buried in the ground. Those above the ground are used to simulate the columns in the building.

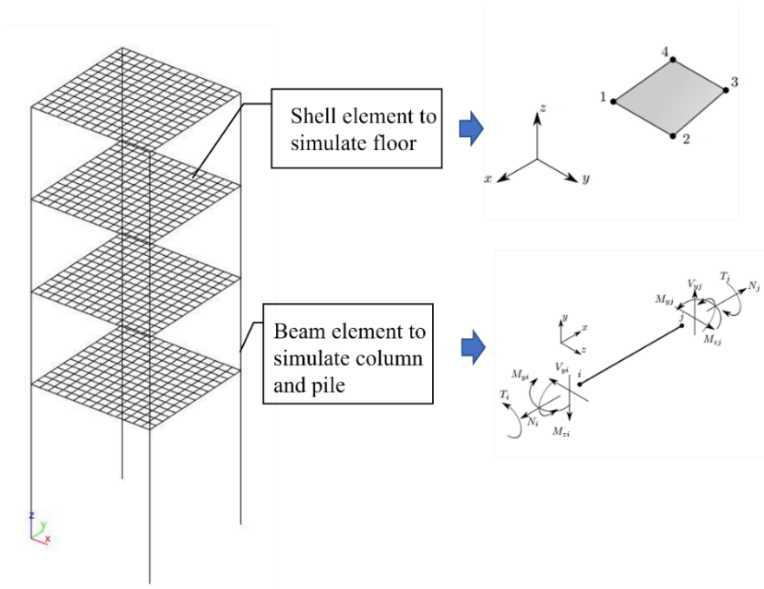

Figure 2: Building model sketch

The ground is considered here as a layered half-space, for which the response is calculated based on Green's functions which are used to construct a dynamic stiffness matrix of the soil [4]. In the example in Section 3 all the layers have the same properties to represent a homogeneous half-space. The response is expressed in terms of wavenumbers k_x and k_y in the two horizontal directions, x and y, and is transformed to the spatial domain using a double Fourier transform. The global stiffness matrix of the layered soil can be assembled from the element stiffness matrix of each layer so that the dynamic equation of the soil model can be expressed as

$$\begin{bmatrix} \tilde{\mathbf{K}}_{11}^1 & \tilde{\mathbf{K}}_{12}^1 & & & \\ \tilde{\mathbf{K}}_{21}^1 & \tilde{\mathbf{K}}_{22}^1+\tilde{\mathbf{K}}_{11}^2 & \tilde{\mathbf{K}}_{12}^2 & & \\ & \tilde{\mathbf{K}}_{21}^2 & \tilde{\mathbf{K}}_{22}^2+\tilde{\mathbf{K}}_{11}^3 & \tilde{\mathbf{K}}_{12}^3 & \\ & & \tilde{\mathbf{K}}_{21}^3 & \tilde{\mathbf{K}}_{22}^3+\tilde{\mathbf{K}}_{11}^4 & \cdots \\ & & & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \tilde{\mathbf{u}}^1 \\ \tilde{\mathbf{u}}^2 \\ \tilde{\mathbf{u}}^3 \\ \tilde{\mathbf{u}}^4 \\ \tilde{\mathbf{u}}^5 \end{bmatrix} = \begin{bmatrix} \tilde{\mathbf{p}}^1 \\ \tilde{\mathbf{p}}^2 \\ \tilde{\mathbf{p}}^3 \\ \tilde{\mathbf{p}}^4 \\ \tilde{\mathbf{p}}^5 \end{bmatrix} \tag{1}$$

where $\mathbf{K}_{ij}^e$ refers to the element stiffness matrix of the e^{th} layered element that relates the displacement at interface j due to the load at interface i. $\tilde{\mathbf{u}}^i$ and $\tilde{\mathbf{p}}^i$ denote the displacement and force at interface i respectively. The tilde over each symbol means that value is in the wavenumber-frequency domain. This approach yields the stiffness matrix of the soil, as well as the displacement response of any point in the soil when boundary conditions are applied.

2.2. Transfer matrix of the model

The total displacement at a response point (x_0, y_0, z_0) induced by a unit load at the position (0, 0, 0) can be divided into two components, the free field ground response induced by the load, and the response caused by the equivalent forces at coupled nodes. It is denoted by

$$\begin{aligned} \hat{u}(x_0,y_0,z_0,\omega) = & \sum_{k=1}^{n} \hat{H}^G(x_{c,k}-x_0, y_{c,k}-y_0, z_{c,k}-z_0,\omega)\hat{F}_{\text{eq}}(x_{c,k},y_{c,k},z_{c,k},\omega) \\ & +\hat{u}_{\text{s0}}(x_0,y_0,z_0,\omega) \end{aligned} \tag{2}$$

where $\hat{H}^G$ means the transfer receptance (Green's function) of the soil. $\hat{F}_{\text{eq}}(x_{c,k},y_{c,k},z_{c,k},\omega)$ means the equivalent reaction force in the frequency domain at coupled node k, with Cartesian coordinates ($x_{c,k}$, $y_{c,k}$, $z_{c,k}$). It can simulate the effects on the ground from the building structure. $\hat{u}_{\text{s0}}(x_0,y_0,z_0,\omega)$ means the ground vibration at (x_0, y_0, z_0) induced by a unit load transmitted through the free field.

Referring to the sub-modelling technique from Hussein et al. [2], the displacement matrix $\hat{\mathbf{U}}_{\text{g}}(\omega)$ at the coupled nodes is given by

$$\begin{aligned} \hat{\mathbf{U}}_{\text{g}}(\omega) &= \left(\mathbf{I}+\hat{\mathbf{H}}^{\text{G}}(\omega)\hat{\mathbf{K}}'_{\text{b}}(\omega)\right)^{-1}\hat{\mathbf{U}}_{\text{s0}}(\omega) \\ &= \left(\mathbf{I}+\hat{\mathbf{H}}^{\text{G}}(\omega)\left(\hat{\mathbf{K}}_{\text{gg}}(\omega)-\hat{\mathbf{K}}_{\text{gi}}(\omega)\hat{\mathbf{K}}_{\text{ii}}(\omega)^{-1}\hat{\mathbf{K}}_{\text{ig}}(\omega)\right)\right)^{-1}\hat{\mathbf{U}}_{\text{s0}}(\omega) \end{aligned} \tag{3}$$

where $\hat{\mathbf{K}}'_{\text{b}}(\omega)$ is the global stiffness matrix of the FE building model containing only DOFs of the coupled nodes. It can be calculated from the full stiffness matrix including the internal element nodes (subscript i) and ground-coupled nodes (subscript g) as described below. $\hat{\mathbf{U}}_{\text{s0}}(\omega)$ is the ground displacement at the position of the coupled nodes calculated by DSM in the free field. $\hat{\mathbf{H}}^{\text{G}}(\omega)$ means the soil transfer receptance matrix for the coupling nodes, which can be denoted as

$$\hat{\mathbf{H}}^{\mathrm{G}}(\omega)=\begin{bmatrix}\hat{\mathbf{H}}_{11}(\omega) & \hat{\mathbf{H}}_{12}(\omega) & \cdots & \hat{\mathbf{H}}_{1k}(\omega)\\ \hat{\mathbf{H}}_{21}(\omega) & \hat{\mathbf{H}}_{22}(\omega) & \cdots & \hat{\mathbf{H}}_{2k}(\omega)\\ \vdots & \vdots & \ddots & \vdots\\ \hat{\mathbf{H}}_{l1}(\omega) & \hat{\mathbf{H}}_{l2}(\omega) & \cdots & \hat{\mathbf{H}}_{lk}(\omega)\end{bmatrix} \tag{4}$$

where

$$\hat{\mathbf{H}}_{lk}(\omega)=\hat{H}^{G}(x_{c,l}-x_{c,k},y_{c,l}-y_{c,k},z_{c,l}-z_{c,k},\omega) \tag{5}$$

The relationship between the equivalent forces $\hat{\mathbf{F}}_{\mathrm{g}}(\omega)$ and displacements at the coupled nodes $\hat{\mathbf{U}}_{\mathrm{g}}(\omega)$ of the pile foundation is given by

$$\hat{\mathbf{F}}_{\mathrm{g}}(\omega)=-\hat{\mathbf{K}}'_{\mathrm{b}}(\omega)\hat{\mathbf{U}}_{\mathrm{g}}(\omega)=-\left(\hat{\mathbf{K}}_{\mathrm{gg}}(\omega)-\hat{\mathbf{K}}_{\mathrm{gi}}(\omega)\hat{\mathbf{K}}_{\mathrm{ii}}(\omega)^{-1}\hat{\mathbf{K}}_{\mathrm{ig}}(\omega)\right)\hat{\mathbf{U}}_{\mathrm{g}}(\omega) \tag{6}$$

Thus, the equivalent force at Equation 2 can be calculated from Equation 6:

$$\hat{\mathbf{F}}_{\mathrm{g}}(\omega)=\begin{bmatrix}\hat{F}(x_{c,1},y_{c,1},z_{c,1},\omega)\\ \hat{F}(x_{c,2},y_{c,2},z_{c,2},\omega)\\ \vdots\\ \hat{F}(x_{c,k},y_{c,k},z_{c,k},\omega)\end{bmatrix} \tag{7}$$

In summary, when determining the impact of a building on the ground, the ground response may be estimated using the DSM approach to simulate the soil, the FE model to simulate the building and the coupling procedure to allow for the soil-structure interaction.

3. RESULTS

3.1. Model

To illustrate the model and explore the influence of building modes on the ground response, results are calculated for a simple example. The building model consists of a three-storey structure with a single span in each direction. Figure 3 depicts the size of the building model, the position of the excitation point and the receiver point. The columns and piles have a rectangular cross-section area of 0.6 × 0.6 = 0.36 m^2 and a second moment of area of 0.0108 m^4. Each beam element is 0.5 m long for the piles and 0.3 m long for the columns. Thus, there are 10 elements for each 5 m long pile embedded in the soil and for each 3 m long column in one storey. The floor plates have a thickness of 0.5 m. Each shell element is 0.3 × 0.3 = 0.09 m^2 in size. There are a total of 784 shell elements and 160 beam elements and 1065 nodes altogether. This model is used to focus on the frequency range 1-80 Hz, which is relevant to human feelable vibration and environmental vibration.

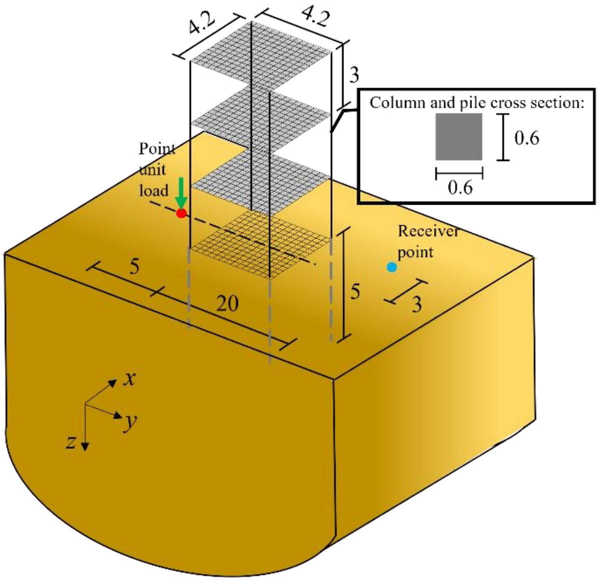

Figure 3: The main dimensions of the model

The building is assumed to be constructed of concrete with a Young's modulus of 3×10^{10} Pa, density of 2500 kg/m^3, and Poisson's ratio of 0.3. Timoshenko beam elements are used to simulate the columns and piles, with a shear factor of 0.83. The soil is assumed to be homogeneous with a compressional wave velocity of 500 m/s, shear wave velocity of 250 m/s and density of 2100 kg/m^3. The shear and dilatational damping loss factor is 0.1.

3.2. Modes of the building

Modal analysis is used to study the modes of the building and their natural frequencies. These are calculated with all nodes on the piles fully constrained. The natural frequencies of the first 20 modes and the corresponding mode shapes are shown in Figure 4. Because the structure is symmetrical in both the *x* and *y* directions, some modes occur in pairs, only one of which is displayed. These modes cover the frequency range up to 80 Hz and are important for the dynamic response of the building in this frequency range.

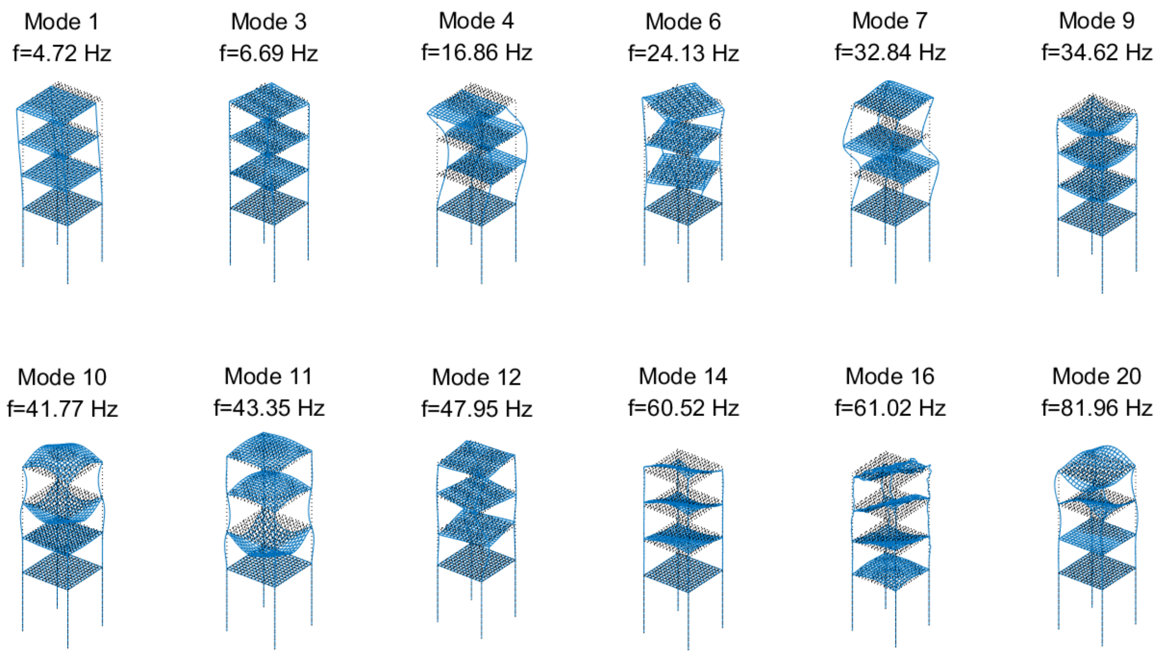

Figure 4: Mode shapes of the building with constrained foundation

From the modes shape shown in Figure 4, it can be seen that the first (and second) mode for this structure involves horizontal bending of the columns, as does the fourth (and fifth) mode with more complex movement. These modes occur in pairs with identical natural frequencies. The third and sixth modes involve torsional motion. The vertical motion of the floors is included in modes 9-11. More complicated floor motions are contained in higher modes, but considering the whole building, the vertical movement of one part is offset by others and there is no net vertical reaction on the ground. The vertical reaction on the ground is therefore expected to be largest in modes 9-11, between 34 and 44 Hz.

3.3. Ground response

Ground vibration is calculated for a unit force applied at a point 5 m in front of the building and a response point located 15.8 m behind the building and 3 m from the centreline (see Figure 3). Figure 5 shows the vertical displacement level (in dB re 10^{-12} m/N) at the receiver point behind the building in comparison with the corresponding result for the free field case. The left figure shows the displacement spectrum with the building present and the contributions to this from the free field result and the reaction forces at the building, i.e. the two terms in Equation 2. The vibration level in the free field is approximately constant for frequencies up to about 30 Hz, before dropping at higher frequencies due to the soil damping, whereas the vibration induced by the reaction forces is much lower in the low frequency region. There is a peak at about 4 Hz, which corresponds to the first two natural frequencies of the building, and another at about 40 Hz which reduces the vibration relative to the free field. In the right figure, the ground response is shown with and without the building present at frequencies corresponding to the modes of the building. The difference between these two cases (i.e. the insertion loss due to the building) is also shown. The insertion loss at the 10^{th} and 11^{th} modal frequencies is larger than others, corresponding to the modes with vertical floor vibration.

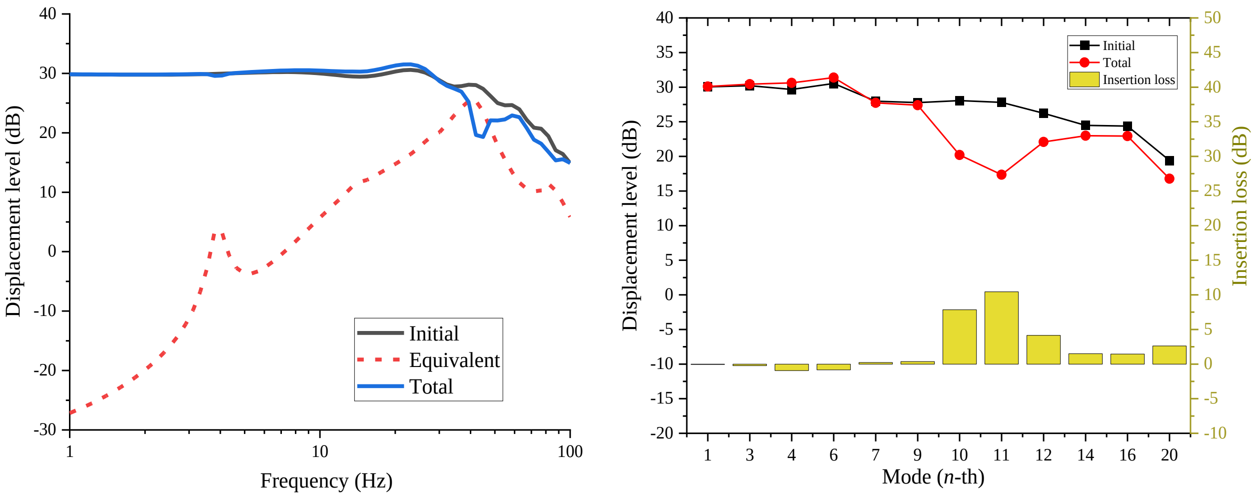

Figure 5: Displacement level of ground in vertical direction (dB ref=10^{-12} m/N)

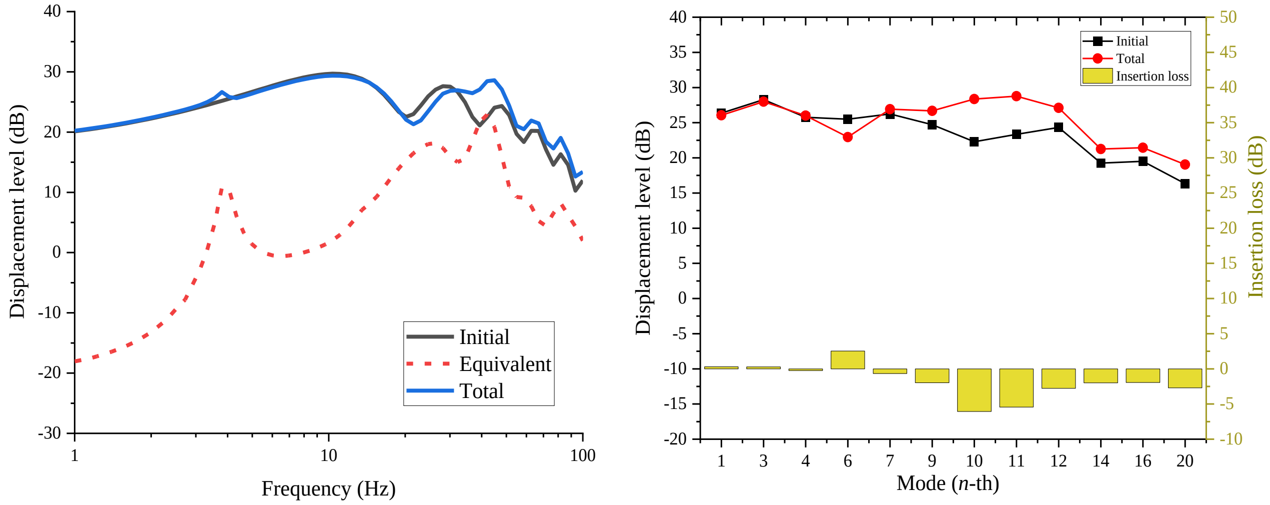

Figure 6: Displacement level of ground in horizontal direction (dB ref=10^{-12} m/N)

The corresponding results for the horizontal direction (*y* direction) due to a vertical unit force are shown in Figure 6. Again, the left figure shows the displacement spectrum with the building and the contributions to this from the free field result and the reaction forces at the building. There is again a peak at around 4 Hz. This is stronger than in the vertical direction, but the response is still much lower than free-field response and the effect on the total vibration is negligible. Hence, in the frequency region below 10 Hz, the building makes no significant difference. Around 30-45 Hz, which is close to the natural frequencies of the floor vertical bending modes, the building causes an amplification in the horizontal direction. The right figure shows the response with and without the building present at the modal frequencies. Especially for the 9^{th}-12^{th} modes, the vibration is amplified by the presence of the building.

4. CONCLUSION

A rapid semi-analytical model of the ground has been coupled with a finite element model of a building and used to determine the ground vibration behind the building caused by a unit harmonic load. A modal analysis of the building constrained at the foundation is used to illustrate the building modes. The coupled model is used to show the effect on the ground response of the presence of the building.

From the calculation results, it is shown that the vibration of the ground in the vicinity of the building can be considerably influenced by the modes of the building, particularly those that entail vertical movement of the floors coupled to extension of the columns. This can affect the vibration in the particular frequency region associated with these modes. This implies that the adjacent structures and their vertical modes should be considered when calculating the ground vibration generated by the railway.

5. REFERENCES

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