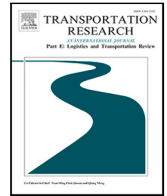


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The economic ship speed under time charter contract—A cash flow approach

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ABSTRACT

Various deterministic models for economic ship speed optimisation exist in the literature, but none considered the time charter contract, and in particular the influence of the redelivery time. This paper studies the economic optimal speed of a ship on a (time) charter contract through the development of an Operational Research (OR) optimisation model. The ship charterer's objective is to maximise the Net Present Value (NPV) of a cash-flow function of the ship's activities over a relevant future horizon H , where H can be interpreted as any possible day within the redelivery time window as specified in the time charter clause. We develop a general time charter contract model $P_M(H)$, and three special cases P_1 , P_∞ and $P_M(H \rightarrow \infty)$, each model mapping onto different contractual contexts, and present algorithms to each of these models for finding optimal ship speeds for any journey structure. While ships on time charter contracts may travel to any series of ports during the charter contract, examining the models' behaviour when the ship repeatedly executes a roundtrip journey allows us to reach some important general insights about the impact of contract type for any journey structure. In particular, economic speeds in $P_M(H)$ follow a very different pattern than those in the classic models from the literature, as well as in the recent class of NPV models $\mathcal{P}(n, m, G_o)$ from Ge et al., (2021). We prove that P_1 and P_∞ map quite generally to the classes $\mathcal{P}(1, n, 0)$ and $\mathcal{P}(\infty, n, -)$, respectively, while $\mathcal{P}(n, m, 0)$ shows behaviour in approximation equal to the special case $P_M(H \rightarrow \infty)$. We prove that two main strands of speed optimisation models from the literature, which did not consider the contract type nor used the NPV approach, show equivalence under mild conditions to P_1 and P_∞ , respectively. These results facilitate matching models to contract types. None of these models, however, matches the general time charter contract model $P_M(H)$ introduced in this paper. In general, the paper demonstrates how optimal economic speed is dependent on the (time) charter contract type, and that this should thus be reflected in the speed optimisation model developed.

1. Introduction

The optimisation of seaborne transportation has received much attention in the literature. We refer to the systematic reviews in Ronen (1983, 1993), Christiansen et al. (2004, 2013), Wang et al. (2013) and Meng et al. (2014). Most of the studies focus on ship routing and scheduling, where ship speed is either a constant or determined approximately from a cost structure that is linear in ship speed. This assumption removes the flexibility of decision making deemed important in many practical applications, according

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to Psaraftis and Kontovas (2013, 2014), who also provide comprehensive reviews on speed optimisation models in maritime OR literature.

Close to 80% of world trade is carried by ships. Bulk carriers and tankers make up 32% of the total merchant fleet of 53,000 vessels, and 70% of the global fleet capacity of 1190 million deadweight tonnage (maribus, 2010). Two main actors in this industry are ship owners (operators) and charterers. The charter contract specifies how the owner hires out the ship (and its crew) to the charterer. Several types of contract exist. In this paper we focus on the speed optimisation problem for the ship charterer in this industry when the ship is subject to a time charter contract. In a time charter contract, the charterer rents the ship for a certain time and compensates the owner with a daily hire (in USD/day). The charterer determines the vessel's route(s) but pays for certain operational costs, including fuel costs. Ships under these contracts are commonly used for repetitive tasks, see also Wu et al. (2021).

Optimal ship speed in maritime economics and operations research literature, as well as in this paper, is to be seen as *an average speed value over a particular leg*, and as a proxy for the time that a ship should ideally take when travelling from one particular port to another particular port, so as to help maximise the ship's commercial success. These strategic models differ in scope from operational models that aim to account for weather conditions, as also explained in e.g. Yan et al. (2020), or to design voyage charter arrangements as in Sun et al. (2021).

It is known that ship speed affects overall profitability. Lowering ship speed reduces daily fuel consumption, but less profit generating trips can be undertaken by the ship over time. If speed is a variable, it should thus be set in the context of this economic trade-off. Economic theory to date (Ronen, 1982; Stopford, 2009; Devanney, 2010; Psaraftis, 2017) finds that the optimal speed is largely determined from a non-linear function of the ratio between the freight rate and the fuel price (Adland and Jia, 2018; Psaraftis, 2019a).

According to Psaraftis (2019b), slow steaming is typically observed in periods of depressed market conditions or high fuel prices, and this is reported in every market. In the context of ongoing climate change concerns, ship speed has become an important consideration since slow steaming saves fuel and reduces emissions (Psaraftis and Kontovas, 2014). Psaraftis (2019b) investigates whether reducing speed by imposing a limit is better than doing the same by imposing a bunker levy, and concludes that having ship speed flexibility remains important (see also Section 3.3.3). It thus seems that speed flexibility and speed optimisation may continue to serve an important role in maritime optimisation (Psaraftis, 2017).

The aim of this paper is to present a modelling approach and analyse its properties so as to investigate how this economic trade-off plays out for ships under (time) charter contracts. Speed optimisation models which consider contract type explicitly are scarce, see also Table 1. Actually, Devanney (2010), in an industrial report, claims that contract types or details would not affect the optimal economic (speed) choices. This may be questionable, as Adland and Prochazka (2021) have examined and shown the value of optionality in a time charter contract, which unfortunately is not based on speed optimisation.

To this end, we follow the OR approach and develop a framework of speed optimisation models with the consideration of time charter contracts and its relevant time horizon. We adopt the principle that optimal economic speeds maximise the Net Present Value (NPV) of this activity for the decision maker, and apply this to a continuous time formulation based on cash-flow functions, an approach that Grubbström (1967) presented as highly suitable to economic problems in which decisions and impacts arise over longer time horizons. While the techniques of NPV evaluation and cash-flow accounting are not new to the economics of shipping, see e.g. Stopford (2009) (Chapter 6), the approach has, to our knowledge, not yet been widely applied to the shipping industry as a methodology to find optimal ship speed decisions. To the best of our knowledge, ship operators do currently not make explicit use of the NPV methodology for speed decisions. Our discussions with an international ship charterer, operating a large number of chartered vessels, give us a good level of confidence that the NPV models developed, and the insights derived from them, exhibit the characteristics and trade-offs experienced decisions makers in this industry take into account. Recently, this NPV approach has also been adopted in Ge et al. (2021), where an unconstrained ship speed optimisation problem without the consideration of charter contract is studied. An NPV perspective is also adopted in Zhang et al. (2020) in the context of cold chain shipping for perishable goods.

In this paper we develop the general model $P_M(H)$ where the charterer has to decide the optimal usage of the ship over the duration of the charter contract H . Prior to signing the contract, deciding on the duration of the contract forms part of the decision whether to hire this ship. Once signed, the charterer has to return the ship to the owner within a time-frame as specified in the redelivery clause. The model $P_M(H)$ also applies to ships already sailing under a time charter contract as to make adjustments to the remaining journey plan, and to decide on the actual date of redelivery. H is then the time remaining on the contract until the intended actual date of redelivery.

Three additional models, P_1 , P_∞ , and $P_M(H \rightarrow \infty)$, are each a special case of $P_M(H)$. They map to specific types of charter contracts and situations that are also commonly found in the industry. Optimal speeds can be very dependent on which model is used, supporting the view that more attention may be needed in the speed optimisation literature to contract type.

That said, some of the model types in the literature can be used in certain charter contract situations. We are able to demonstrate this by proving, under mild conditions, that P_1 and P_∞ map onto two main types of 'classic' (not based on NPV principles, and not explicitly considering the charter contract) ship speed optimisation models from the literature. We henceforth call these models of type 'PK' and 'R', respectively. See also Table 1.

In particular, P_1 shows equivalence to speed models like Psaraftis and Kontovas (2014), where the objective function is to maximise the *total* profit or to minimise the total cost per trip, per set of routes, or per nautical mile. We henceforth denote with 'Model PK' the type of models from the literature using this kind of objective function. Model PK is found in e.g. Corbett et al. (2009), Norstad et al. (2011), Fagerholt et al. (2015), Wen et al. (2017), Theocharis et al. (2019), Adland and Prochazka (2021). The P_∞ class, on the other hand, shows equivalence to another class of speed models, where the objective function is to maximise

Table 1
Related economic speed optimisation modelling literature.

Authors	Objective or criterion	Time value of money	Charter contract	Fuel consumption	Harbour Cost/Time	Incentive of revenue ⁽¹⁾
Psaraftis and Kontovas (2014)	Model PK ⁽²⁾	X	X	Payload Dependent	X	X
Adland and Prochazka (2021)	Model PK	X	✓ ⁽³⁾	Constant	X	X
Theocharis et al. (2019)	Model PK	X	X	Payload Dependent	X	X
Wen et al. (2017)	Model PK	X	X	Payload Dependent	X	X
Fagerholt et al. (2015)	Model PK	X	X	Piece-wise Linear Approximation of Data	X	X
Norstad et al. (2011)	Model PK	X	X	Quadratic Approximation of Data	X	X
Corbett et al. (2009)	Model PK	X	X	Cubic Law	X	X
Ronen (1982)	Model R ⁽⁴⁾	X	X	Cubic Law	✓	✓
Devanney (2010)	Model R	X	✓ ⁽⁵⁾	Data	X	✓
Ronen (2011)	Model R	X	X	Cubic Law	✓	✓
Magirou et al. (2015)	Model R	✓ ⁽⁶⁾	X	Cubic Law	✓	✓
Lee and Song (2017)	Model R	X	X	Cubic Law	✓	✓
Adland et al. (2020)	Model R	X	X	Speed dependent Elasticity	X	X
Ge et al. (2021)	NPV Profits	✓	X	Payload dependent + Ballast	✓	✓
This Paper	NPV Profits	✓	✓	Payload dependent + Ballast	✓	✓

(1) The model captures the impact of revenue on speed choices, c.f. Psaraftis (2017); (2) Model PK refers to the type of (non-NPV based) models that optimises the total profits/costs per route; (3) Their model is about route selection rather than speed optimisation; (4) Model R refers to the type of (non-NPV based) models that optimises the total profits/costs per unit of time; (5) Devanney (2010) however claimed that charter contract would not affect the optimisation problem; (6) Magirou et al. (2015) proposed a discounted model as an extension to the standard model.

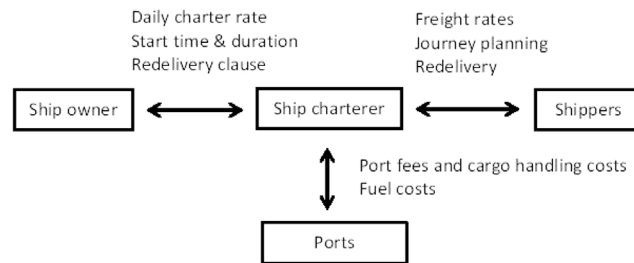


Fig. 1. Time-charter players and interactions.

the daily profit, or minimise the daily cost, as in Ronen (1982), and also Devanney (2010), Ronen (2011), Magirou et al. (2015), Lee and Song (2017), Adland et al. (2020). Models in this class divide the profit (or cost) earned on a route by total travel time of the route to arrive at their objective function. ‘Model R’ in this paper denotes models that follow Ronen’s approach to constructing the objective function. Because we show the equivalence of Model PK to P_1 and of Model R to P_∞ (under mild conditions), we can from the study of the properties of the NPV models learn about the properties and underlying assumptions of the classic models.¹ We also show how the general class of models $\mathcal{P}(n, m, G_0)$ introduced in Ge et al. (2021), and which place no constraint on the available contract duration, behave similar to the special case $P_M(H \rightarrow \infty)$, and thus differently from the general model $P_M(H)$.

The differences between speed optimisation model types have not been explicitly discussed until Psaraftis (2017) and Ge et al. (2021). In this paper, we will enrich this discussion by showing the links of these models to charter contracts.

2. Problem description

This paper considers the time charter agreement in the bulk cargo/tanker industry. Fig. 1 illustrates the players and their interactions. The three major players are ship owners, ship charterers, and shippers. The charterers rent ships (including crew) from ship owners and receive revenues from shippers for the transport of cargo between ports.

We are concerned with determining the profitability of a particular time charter offer available to the ship charterer. The time charter party is the contract between ship owner and charterer, and typically allows charterers to decide on which journeys are undertaken, and at which speeds the ship will travel.

Our primary purpose is to develop an understanding of the degree by which the time charter contract affects how fast a vessel should ideally travel on the different journeys. We assume that the ship is not limited in its choice of speeds by time windows imposed by external players (e.g. ports, shippers). At the moment of considering chartering a vessel in this industry, time windows are not an exogenous input. Only when (voyage) contracts with shippers are signed may these become (soft) constraints, see also Psaraftis

¹ The study of equivalence between models derived from classic principles and models based on NPV principles finds its roots in Grubbström (1980) (in the context of production–inventory systems), see also Beullens and Janssens (2014).

(2017) and Prochazka and Adland (2021). At this point the ship is already planned to travel according to a chosen optimal speed range. This still allows for the charterer, if so desired, to alter speeds on later voyages of the ship within the time charter contract.²

Most of the examples in this paper consider a repeated ballast-laden journey between two ports. Journeys of ships under time charters in the tanker and bulk shipping industry are often of such a relatively simple structure, as are those found in a Contract of Affreightment (COA) agreed between a charterer and a shipper. A COA may run from a few months to several years, and has become the most commonly used contract in tramp shipping, see also Wu et al. (2021). The models and algorithms presented, however, can find optimal ship speed for each of the legs on a journey of arbitrary structure, with or without repetitions. More importantly, the repeated laden-ballast situation as considered in this paper facilitates the analysis and extraction of general insights that equally apply to journeys of arbitrary structure under time charter contracts.

It is perhaps worthwhile to stress that deterministic models have formed the basis of economic understanding of ship speed optimisation and routing in general, and this paper follows in this tradition. We indeed consider deterministic freight rates as in most ship routing optimisation literature. This assumption fits reality often quite well. It is true that both the time charterer's costs of renting the ship, as well as the revenues received from shipping goods, follow freight rate market indices, and future values of these should be regarded as random variables. Typically, however, charterers fix the renting cost in an agreement with the particular ship owner for the duration of the contract. Furthermore, freight rates received for each delivery by the ship may have been agreed upon with the shippers, as would be typical in a COA. Alternatively, if operating on the spot market, the freight rates can be hedged through the adoption of a freight futures contract with a broker. In both cases the adoption of deterministic rates would then be correct.³

2.1. Charter party characteristics

When a ship charterer hires a ship from an owner through a typical time charter contract, the ship owner carries the costs of crew, repair, maintenance, lubricants, supplies, and capital costs. The charterer is concerned with revenues and costs associated with cargo handling, including costs of loading and unloading at ports, and main and auxiliary fuel costs. The agreement is made at decision time 0 for a certain duration during which the charterer pays a daily rate to the owner. The main variables involved are:

- H : initial approximate duration of charter party (in days);
- t_{FS} : start time of charter relative to signing contract (in days);
- f^{TCH} : daily charter rate (USD/day);
- H^* : actual duration of charter party (in days).

The rate f^{TCH} is known as the Time Charter Hire (TCH), and t_{FS} as the Forward Start. TCH is a function of charter time duration and market conditions. In late November 2017, for example, TCH estimates for a Suezmax oil tanker published by the Hellenic Shipping News Worldwide were increasing with contract duration: 18,750 (USD/day) for 1 year; 20,000 (USD/day) for 2 years; and 23,500 (USD/day) for 3 to 5 years, respectively. At other times, these estimates may be constant or decreasing. TCH can be a function of t_{FS} . Angolucci et al. (2014) found, for example, that rates in the Panamax market for the period 2008–2012 reduced on average by 60 USD per extra day Forward Start. The agreed TCH will also depend on the ship's condition and negotiation.

The process of negotiating the time charter contract may require some iterations, and typically occurs through mediation by intermediate brokers. We refer to Ge (2021) (Chapter 3, Section 3.1). In this article, we assume that the f^{TCH} is agreed based on the contract length H and t_{FS} , and will be accepted to hold for the actual value H^* that represents the actual redelivery day.

We remark that time charter contracts include a redelivery clause with details about the time window in which the charterer must return the vessel. We show in Section 3.3.1 that the model $P_M(H)$ can indeed account for this redelivery flexibility. In addition, the model can also be used for ships already travelling on a time charter contract, see Section 3.3.2.

2.2. Journey characteristics

During the time of the charter, the ship transports cargo between ports, see also Fig. 2. The charterer can adjust the speed of the ship on its voyages within an allowed range of speeds. This freedom, as discussed in Section 1, is important in order to help maximise the charterer's profits.

To characterise the logistics tasks to be undertaken, we adopt a modelling logic based on Psaraftis and Kontovas (2014), but which extends this in various respects.

We define a *journey* of the ship as a scenario in which the ship visits a set of $n + 1$ ports numbered $0, 1, 2, \dots$ to n in this given order, see also Fig. 3. Based on demand from an origin port i to a destination port j , the ship will, at each port i , unload cargo destined for this port and picked up from previous ports $j < i$ and pickup loads to be delivered to ports $j > i$. These amounts are such that at any point in the journey the cargo load remains feasible with respect to the ship's deadweight carrying capacity.

² In liner services, speed optimisation is also important, but additional constraints, see e.g. Christiansen et al. (2019), may make it difficult to implement optimal speeds identified by the models presented in this paper.

³ The assumption of fixed freight rates when on the spot market will only hold for a limited time period. Ship owners operating in the spot market may benefit from models using stochastic freight rates, see also (Magirou et al., 2015).

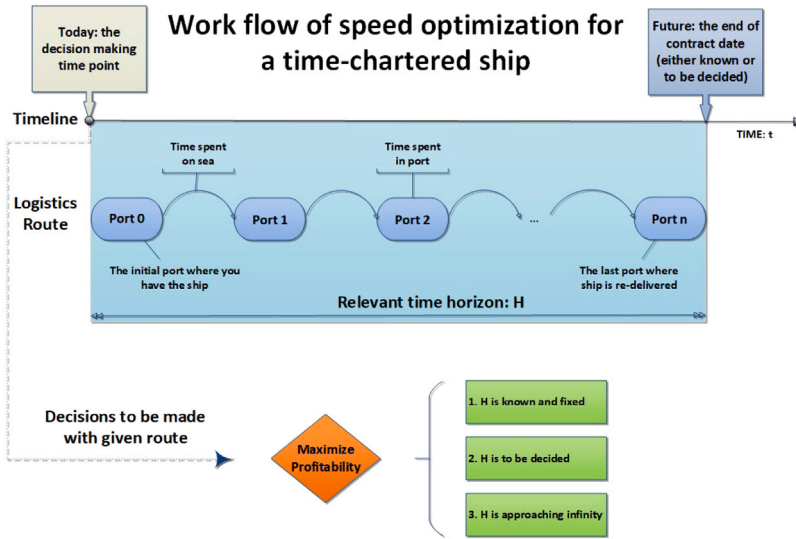


Fig. 2. The timeline and workflow of the speed optimisation problem for a time-chartered ship.

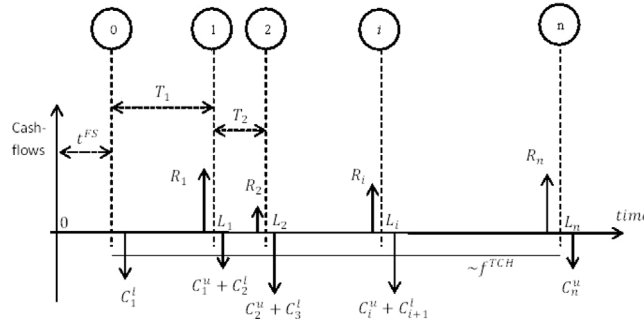


Fig. 3. Cash-flows for a Journey of n -legs between $n + 1$ ports.

We define a leg i ($i = 1, \dots, n$) as the process of the ship moving from port $i - 1$ to port i . The time T_i (in years) corresponding to leg i consists of: loading time T_i^l at port $i - 1$, sea voyage time to port i denoted by T_i^s , waiting time T_i^w at port i , and unloading time T_i^u at port i :

$$T_i(v_i) = T_i^l + T_i^s(v_i) + T_i^w + T_i^u, \tag{1}$$

where v_i the sailing speed (in kn) on leg i . The (un)loading time at a port is linearly dependent on the tonnage to be (un)loaded. For oil, for example, those times depend on the pump capacity of the harbour and the ship, respectively. During unloading and loading at port i , the ship will also replenish supplies and remove waste, take the main fuel and auxiliary fuel, and adjust the amount of ballast water needed to cover leg $i + 1$. The travel time (in days) is given by:

$$T_i^s(v_i) = \frac{S_i}{(24)v_i}, \tag{2}$$

where S_i is the distance (in nm) from port $i - 1$ to i .

Each visit to a port incurs costs. Without loss of generality, we will split the total costs incurred at a port i into two components. The first component C_i^u represents costs associated with entering the port for unloading, the second component C_{i+1}^l represents costs at port i associated with loading so that the ship is ready to undertake the sea voyage of leg $i + 1$. We take C_i^u at port i to include the costs of waiting to enter port and dock at berth, fixed port charges, any mooring dues, and unloading charges. Cost C_{i+1}^l incurred at port i include cargo loading costs, cost of readjusting ballast, and the costs of main and auxiliary fuel intake.

The amount of main fuel to cover leg i depends on the sailing speed v_i , and the deadweight tonnage (dwt) w_i carried, which includes cargo, ballast, supplies, and main and auxiliary fuels carried. We adopt a fuel consumption function (tonne/day) proposed in Psaraftis and Kontovas (2014):

$$F(v_i, w_i) = k(p + v_i^g)(w_i + A)^h, \tag{3}$$

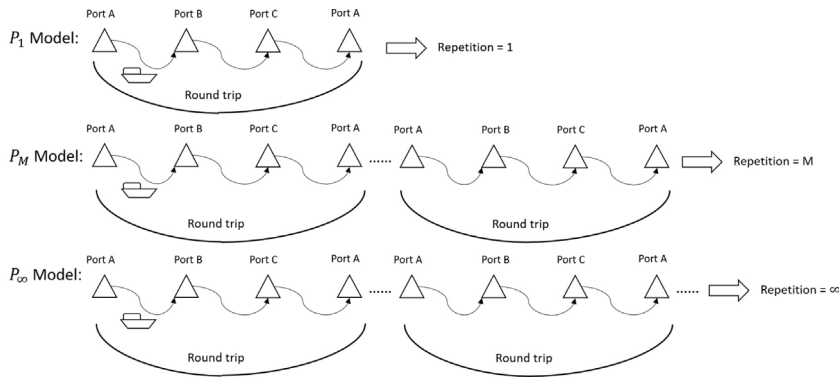


Fig. 4. Example of a round-trip journey from Port A to B to C, and back to A.

where w_i is the dwt carried on leg i , A the lightweight of the ship, and typically $g \geq 3$ and $h \approx 2/3$. We impose a stability constraint that dwt carried should be at least a given percentage of the design dwt of the vessel, see also David (2015); any shortcoming arising from zero or low amounts of cargo carried is to be made up from ballast water. The loading cost can be stated as:

$$C_i^l(v_i) = C_i^h + c_i^f F(v_i, w_i) \frac{S_i}{24v_i}, \tag{4}$$

where c_i^f is the fuel cost (USD/tonne) at port $i - 1$, and C_i^h represent the sum of other costs associated with the loading operations.

The sum of C_i^u and C_{i+1}^l is due within a number of days d_i^c after leaving the port i . The time lapse of this payment (in days), relative to the completion time of the leg i , is given by:

$$\epsilon_i = T_i^l + d_i^c. \tag{5}$$

Revenues are based on freight rates, i.e. (market) unit prices of the cargo type and the distance between origin and destination ports, and negotiations with shippers. For the journey described with known tonnages transported, we can thus calculate the total revenues R_i from unloading cargo at each port i . This value is not dependent on the actual sailing distances or leg speeds. Most ship charterers will demand payment a number of days d_i^r prior to the start of unloading the cargo at the destination port. Let us thus define the time of payment, relative to the completion time of leg i , as :

$$-\delta_i = -(T_i^u + d_i^r). \tag{6}$$

The negative signs indicate that payments are made earlier. The time charter hire payments can be modelled as an annuity stream at rate f_{TCH} over the period of the contract,⁴ see Fig. 3.

Note that where we differ from Psaraftis and Kontovas (2014) in the modelling of the logistics process is primarily in the following aspects: (a) we include the costs and times of harbour activities, (b) we include the revenues earned from cargo transport, (c) we specify the actual timing of when costs and revenues are occurring, (d) we do not explicitly incorporate the carrying charges of payload and goods yet to be picked up. For further discussion on these differences, see Section 6.1.

2.3. Model formulations: $P_M(H)$, P_1 , $P_M(H \rightarrow \infty)$ and P_∞

We consider the situation in which the charterer desires to use the ship for the repeated execution of the journey between ports, as illustrated in Fig. 4. As the journey represents a round-trip, port n equals port 0 and, without loss of generality, we can choose $R_0 = C_0^u = C_{n+1}^l = 0$.

We develop one general model $P_M(H)$, and derive three special cases. Define $V_i^- = \{v_1, \dots, v_i\}$ and $V_i^+ = \{v_i, \dots, v_n\}$ ($i = 1, 2, \dots, n$). Let us define variable L_j ($j = 1, \dots, n$) as the time (in days) from the start of the charter party t_{FS} to the moment that leg j is completed:

$$L_j(V_j^-) = \sum_{i=1}^j T_i(v_i). \tag{7}$$

⁴ The actual charter hire payment times could be e.g. twice every month on day 1 and 15. These details are not relevant since the payments scheme is not linked to any intermediate events in the model. Any actual payments plan can be converted to an equivalent annuity stream value in the model with simple modifications.

We refer to these as *leg completion times*, and L_n represents the duration of the journey. The Net Present Value (NPV) of the first journey relative to decision time 0 is therefore:

$$NPV_1(V_n^-) = \left[\sum_{j=1}^n \left(R_j e^{-\alpha(L_j - \delta_j)} - C_j^u e^{-\alpha(L_j + \epsilon_j)} - C_j^l e^{-\alpha(L_{j-1} + \epsilon_{j-1})} \right) - \frac{f^{TCH}(1 - e^{-\alpha L_n})}{\alpha} \right] e^{-\alpha t_{FS}}, \quad (8)$$

where α is the (continuous) opportunity cost of capital against which the charterer wishes to evaluate this activity. Note that α is not an interest rate but the profit rate of the next best alternative foregone (Grubbström, 1967). Typical values are larger than the interest rate on a risk-free investment. The most appropriate time unit in this study is per day: we use a daily rate $\alpha = 0.000219$ (per day), equivalent to a cost of capital at 8% (per annum), a value consistent with marine industry values, see also Yagerman (2015). See also Section 5.1, where we examine the model's sensitivity to α .

Let integer M represent the number of times the ship will repeat this journey. The NPV of this is given by (using geometric series):

$$NPV_M = NPV_1 \left[\sum_{i=0}^{\infty} e^{-i\alpha L_n} - \sum_{i=0}^{\infty} e^{-(i+M)\alpha L_n} \right] = NPV_1 \frac{1 - e^{-\alpha M L_n}}{1 - e^{-\alpha L_n}}. \quad (9)$$

Note the dependencies of loading costs and completion times on speed variables, i.e. $C_i^l(v_i)$ and $L_i(V_i^-)$, whereas R_i and C_i^u are constants ($i = 1, 3, \dots, n$). Let v^- and v^+ indicate the minimum and maximum sailing speeds allowed for the ship.

We can now formulate Model $P_M(H)$:

$$\text{Max } NPV_M(V_n^-, M), \quad (10)$$

and subject to

$$v^- \leq v_i \leq v^+, \forall v_i \in V_n^-, \quad (11)$$

$$ML_n(V_n^-) \leq H, \quad (12)$$

$$V_n^-, M \geq 0. \quad (13)$$

The decisions variables are leg speeds v_i and number of journey repetitions M where $ML_n(V_n^-) = H^*$ (recall Section 2.1). After completion of the work at time H^* , the charterer will return the vessel to the owner and stops paying the time charter hire. We assume that other (future) projects for the charterer do not depend on the completion time of this contract.⁵

Taking $M = 1$ and dropping constraint (12) (as if taking $H \rightarrow \infty$), we get as a special case the decision problem P_1 . This model optimises the NPV of a single journey. Decisions about ship speed will affect the duration L_n of the journey. If the ship charterer would like to undertake further journeys with this ship, it would have to account for this in the formulation. The fact that P_1 does not do so means that after completion of the journey, the ship is returned to the owner and that $L_n = H^*$, i.e. L_n is the duration of the charter contract. P_1 is thus a model for what is known as a *trip time charter*, which is a time charter contract of comparatively short duration agreed for a specified route, where route in this context refers to the n -leg journey. For further information about charter contract types, see Section 6.2. We can formulate it as Model P_1 :

$$\text{Max } NPV_1(V_n^-), \quad (14)$$

and subject to

$$v^- \leq v_i \leq v^+, \forall v_i \in V_n^-, \quad (15)$$

$$V_n^- \geq 0, \quad (16)$$

where the time constraint (12) can be added if, for example, a time window is considered. We do not explicitly consider this scenario in this paper, but such time constrained P_1 model can be solved in the similar manner as our $P_M(H)$ model; see Algorithm 2 in Section 2.4.

A second special case is when $H \rightarrow \infty$ in $P_M(H)$, but is M predetermined and fixed. This can be seen as the model $P_M(H)$ where constraint (12) is removed, and M is a parameter, not a decision variable. We refer to this model as $P_M(H \rightarrow \infty)$. With $H \rightarrow \infty$ in $P_M(H)$, but M yet unspecified, we arrive at a third special case. For a charter contract to be considered, it must be that repeating the journey once must be profitable, i.e. $NPV_1 \geq 0$, for at least some speed values. Then (9) shows it is optimal that the journey is infinitely repeated ($M \rightarrow \infty, H^* \rightarrow \infty$). We call this model P_∞ . For NPV problems with infinite horizons, it is more convenient to maximise the Annuity Stream (AS) value, where $AS = \alpha NPV$, representing the constant stream of cash (USD/day) with the same NPV value. We can formulate it as Model P_∞ :

$$\text{Max } AS(V_n^-) = \alpha NPV_\infty = NPV_1 \frac{\alpha}{1 - e^{-\alpha L_n}}, \quad (17)$$

⁵ This is valid if the charterer has access to capital so it can hire other ships at any other time if other profitable business is on the horizon. This is different to the viewpoint of a ship owner, who may need this ship to complete before undertaking some other activities. For further information about how to handle the comparison of investments of different duration, when the completion time affects future activities, see e.g. Brealey and Myers (2003), p.132.

and subject to

$$v^- \leq v_i \leq v^+, \forall v_i \in V_n^- \quad (18)$$

$$V_n^- \geq 0, \quad (19)$$

Model P_∞ represents an idealised case of a time charter of very long duration.

2.4. Algorithms

For solving P_1 or P_∞ models, use Algorithm 1 where $M = 1$ and $H \rightarrow \infty$, or $M \rightarrow \infty$ and $H \rightarrow \infty$, respectively, and where minimum and maximum speeds on each of the legs correspond to the minimum and maximum speeds allowed for the ship, v^- and v^+ , respectively. For $P_M(H \rightarrow \infty)$, use Algorithm 1 where M is the required number of repetitions, and $H \rightarrow \infty$.

Algorithm 1: Grid search algorithm to solve P_1 , $P_M(H \rightarrow \infty)$ (with given M), or P_∞

Result: $\{v_1^*, \dots, v_n^*\}$, F^* , H^*

(F^* is the optimal NPV and H^* is the number of days used by the ship)

Input: M , H , $\{(v_i^{\min}, v_i^{\max}), i = 1, \dots, n\}$;

Step 1: For $i = 1, \dots, n$, set $\epsilon > 0$ and $\mathbb{V}_i =: \{v_i^{\min}, v_i^{\min} + \epsilon, \dots, v_i^{\max}\}$;

Step 2: Let F be given by equation (9) (If $M \rightarrow \infty$, let F be given by equation (17)), and L_n is given by equation (7);

Step 3: $\{v_1^*, \dots, v_n^*\} = \operatorname{argmax}_{v_i \in \mathbb{V}_i, \forall i} F(v_1^*, \dots, v_n^*)$ s.t. $ML_n \leq H$;

$$F^* = F(v_1^*, \dots, v_n^*);$$

$$H^* = \lceil (ML_n(v_1^*, \dots, v_n^*)) \rceil$$

Finding the maximum of $F(v_1^*, \dots, v_n^*)$, for small values of n , can be done by grid search. For a 7 knot range and a laden-ballast journey, as considered in this paper, this requires $(7/\eta)^2$ function evaluations, where the factor η (knots) is speed accuracy required. Note that M and H values are given, and even large values for these will not complicate the algorithm. For cases where $H \rightarrow \infty$, the constraint does not need to be checked.

If the ship cannot run at certain intermediate values of speed on one or more of the legs (e.g. to avoid ship vibrations), the algorithm can be adjusted in Step 1, by eliminating these invalid speed ranges in \mathbb{V}_i . Likewise, if the ship has different upper- and lowerbounds on speed for different legs of the journey, it is straightforward to make the necessary adjustments. For large values of n , one may need to resort to other approaches, for example a dynamic programming formulation as in Ge et al. (2021).

For solving the general time charter contract problem $P_M(H)$, one can use Algorithm 2. It will generate speed menu profiles (introduced in Section 3.2) as in e.g. Fig. 5, starting from the minimum possible horizon H_0 , corresponding to the time needed by the ship to execute the journey when travelling at maximum speed v^+ . H^{\max} is the latest possible redelivery date considered. The algorithm iteratively calls Algorithm 1, each time feeding it specific values for M , H , and minimum and maximum speeds on each of the legs. Algorithm 2 produces as output a record, for every possible redelivery horizon H within H_0 to H^{\max} , the actual optimal redelivery day (which can be less than H), optimal leg speeds, optimal number of journey repetitions, and corresponding NPV value.

Algorithm 2: Grid search algorithm to solve time charter $P_M(H)$ (with unknown M)

Result: Recorded for each $H_0, H_0 + 1, \dots, H^{\max}$: $\{v_1^*, \dots, v_n^*\}$, M'^* , F'^* , H'^*

Input: $H_0^{(*)}$, H^{\max}

Initialisation: Set $\theta > 0$, $H = H_0 + 1$. Call Algorithm 1 ($1, H_0, \{(v_i^-, v_i^+), \forall i\}$) and record $H'^* = H^*$, $M'^* = 1$ and

$$\{v_1^*, \dots, v_n^*\} = \{v_1^*, \dots, v_n^*\};$$

while $H \leq H^{\max}$ **do**

 Call Algorithm 1 ($M'^*, H, \{(v_i^-, \min(v_i^+, v_i^* + \theta)), \forall i\}$);

 record $F'^* = F^*$, $H'^* = H^*$, $\{v_1^*, \dots, v_n^*\} = \{v_1^*, \dots, v_n^*\}$;

 Call Algorithm 1 ($M'^* + 1, H, \{(max(v_i^-, v_i^* - \theta), v_i^+), \forall i\}$);

if $F^* > F'^*$ **then**

 | update $F'^* = F^*$, $H'^* = H^*$, $\{v_1^*, \dots, v_n^*\} = \{v_1^*, \dots, v_n^*\}$, $M'^* = M'^* + 1$;

$H = H + 1$;

end

(*) H_0 is the minimum days required to complete the journey when travelling at maximum speed.

In Algorithm 2, we are considering that the journey takes, at maximum sailing speeds, considerably more time than 1 day. Therefore, when extending the allowable horizon with 1 additional day in the next iteration of the while loop, the optimal number of journey repetitions either stays the same, or increases at most with one.⁶

⁶ This is supported e.g. by Fig. 5 as otherwise we would observe there a jump from some value M to $M + 1$ at some day H , being followed to a jump to $M + 2$ on day $H + 1$. The fact that the solutions in Fig. 5 show each time a steady plateau of H values where one M value is optimal proves the optimality in Algorithm 2 of only considering one additional repetition for each increment in H .

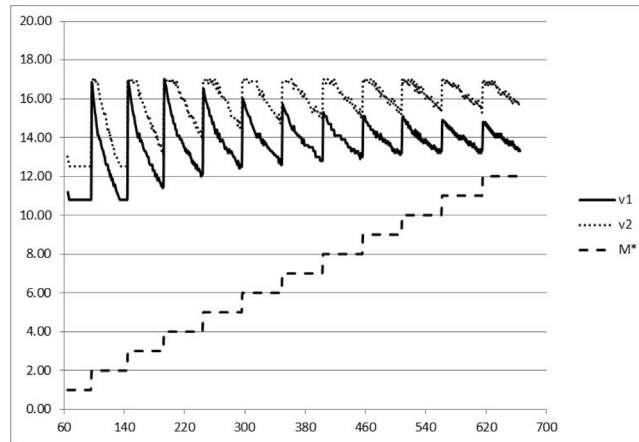


Fig. 5. Speed menu curve: Optimal leg speeds (kn) and repetitions as functions of H (days), in the base case example.

When extending the allowable horizon with one day, and if we keep the number of repetitions the same, then intuitively there is no point for the ship to speed up a lot. Likewise, when extending the allowable horizon with one day but the number of journey repetitions increases, then intuitively it does not make sense that the ship would slow down a lot. For this reason we can adjust the speed boundaries in the two calls to Algorithm 1 and using θ in the range of 1 knot or lower.

The output from Algorithm 2 is of course also applicable for a time charter contract with a redelivery time window from (H_1, H_2) , where $H_0 \leq H_1 < H_2 \leq H^{max}$.

3. General model P_M

In this section, we examine the general model $P_M(H)$, given by (10)–(13), and show that the charter party horizon H strongly influences speed decisions of NPV profit-maximising ship charterers under deterministic conditions. We introduce the novel concept of a *speed menu curve* to show the possible solutions to the general problem $P_M(H)$ as a function of available days H . We discuss differences with the models in Ge et al. (2021), and the differences with other deterministic models from the literature are discussed in Section 4, and also Section 6. The impact of redelivery flexibility and uncertainty are discussed in Section 3.3.

Throughout the paper, we consider the following *base case example*: a Suezmax ship of 157,880 dwt is to transport 1,200,000 barrels of light crude oil over a distance of 8,293 nm, and return to the first port in ballast. Minimum and maximum speeds are 10 and 17 kn. Further details are given in Appendix A.

3.1. When subject to a given number of journey repetitions M

Before we examine the general model, it is useful to show the special case $P_M(H \rightarrow \infty)$ and show its similarity with the unconstrained model in Ge et al. (2021). In $P_M(H \rightarrow \infty)$ there is no constraint on maximum available days H but we adopt a given number of journey repetitions M , and return the vessel at $H^* = ML_n^*$. Table 2 shows optimal return times, speeds, and NPV values for increasing M in the base case example. We observe that the economic principle that ships react to improved market conditions (increased freight rate) by speeding up is recognised in $P_M(H \rightarrow \infty)$. Also it shows that an increase in the daily hire cost (TCH) will tend to speed up the ship so as to conclude the work in less time. These findings are no different to what is concluded in Ge et al. (2021) and also the classic models ‘PK’.

Indeed, this special case $P_M(H \rightarrow \infty)$ is similar to the $\mathcal{P}(n, m, G_0)$ model⁷ developed in Ge et al. (2021), where $n = M$, $m = 2$ and $G_0 = 0$ here. In particular, the laden and ballast speeds obtained from $P_M(H \rightarrow \infty)$ can be viewed as the approximations of the averaged speeds on laden and ballast legs, respectively, in $\mathcal{P}(M, 2, 0)$.

More interestingly, and similar to the *chain effect* observed in Ge et al. (2021), Table 2 indicates that ships would ideally travel faster (on average) on both laden and ballast leg with the increase of repetitions M . Indeed, the ship’s optimal speeds of a series of repetitive journeys would converge to a stationary state if we extend the repetitions $M \rightarrow \infty$. Intuitively, we may see that this converges to the solution of P_∞ (as $P_{M \rightarrow \infty}(H \rightarrow \infty) \equiv P_\infty$). We note that the chain effect was not observed in the classic models ‘PK’ and ‘R’.

⁷ The difference is that in $P_M(H)$ model, the speed on a particular leg is kept identical in each repetition to reserve the same frequency, while in Ge et al. (2021), speeds at each repetition would vary.

Table 2
Performance of $P_M(H \rightarrow \infty)$ for a given number of journey repetitions M .

Variable/M-values	$M = 5$	$M = 10$	$M = 20$	$M = 30$
Base case				
H^* (days)	331.85	656.59	1,289.79	1,996.83
v_1 (kn) (laden)	11.0	11.1	11.3	11.5
v_2 (kn) (ballast)	12.6	12.8	13.1	13.3
NPV (USD)	7,829,768	15,113,504	28,219,741	37,580,022
Freight rates $\times 1.5$				
H^* (days)	322.12	637.58	1,218.99	1,770.21
v_1 (kn) (laden)	11.2	11.5	12.1	12.5
v_2 (kn) (ballast)	12.8	13.2	13.9	14.5
NPV (USD)	19,826,413	38,300,872	71,709,624	101,078,409
TCH $\times 1.5$				
H^* (days)	299.80	597.90	1,187.91	1,770.21
v_1 (kn) (laden)	12.3	12.3	12.4	12.5
v_2 (kn) (ballast)	14.2	14.3	14.4	14.60
NPV (USD)	4,789,051	9,273,999	17,413,688	24,569,661

Table 3
Optimal speeds for a laden–ballast journey (base case example).

Variable/Model	P_1	P_∞	Model PK	Model R
v_1 (kn) (laden)	10.9	14.0	12.9	17.0
v_2 (kn) (ballast)	12.5	16.3	12.5	12.5
L_n (days)	66.9	53.4	62.0	55.5
NPV (USD)	1,616,189	10,069,976 ^a	1,576,804	8,828,658 ^a
f^{TCE} (USD/day)	44,344	47,589	45,621	44,188

^aAS (USD/year).

3.2. When subject to a finite horizon H : speed menu curves

Fixing the number of journeys a priori, however, is not necessarily optimal in the case that H is finite, i.e., the optimisation problem $P_M(H)$ is subject to (12), that is $H^* = M^* L_n^* \leq H$. We will show why the value of H has a significant impact on optimal speeds. We also introduce a novel representation of optimal solutions to model $P_M(H)$ as a function of H that we refer to as ‘speed menu curves’.

Though it appears that only one additional constraint H (12) is added to the model, the complexity of the optimisation problem $P_M(H)$ is much increased as we need to also decide the optimal value of repetition M simultaneously. We can visualise the optimal results obtained from model $P_M(H)$ to illustrate how, for each possible choice of H , the corresponding optimal speed would vary for each optimal value of M and how it would converge to the optimal solution of P_∞ . For the base case example, using Algorithm 2 and iteratively updating the H^{max} , we depict these results in Fig. 5. It displays the optimal laden speeds (v_1) and ballast speeds (v_2) and corresponding optimal number of journey repetitions (M^*) as functions of integer values for H , where $H \in [65, 665]$. Fig. 6 plots the corresponding NPV profits; this is a non-decreasing function of H . Optimal speeds, broadly speaking, follow a saw-tooth pattern where each cycle corresponds to a constant optimal M^* value. It is clear why: when M^* remains constant (and $H^* \approx H$), an increase in available days H allows the ship to travel slower and slower, up to the point where an increase in M^* would achieve a higher NPV value rather than further reducing the ship speed. In that case, however, the ship will have to speed up on all planned journeys so as to accommodate the extra journey. It is for this reason that one cannot derive an optimal strategy for a single journey through decomposition, as used in the classic models ‘PK’ and ‘R’. Also, the model in Ge et al. (2021), while not using decomposition, still does not identify these solutions: for any given M in Fig. 5 it would fail to find any of the optimal speeds as from $P_M(H)$. Instead, it would typically execute at a much more conservative low speed. For $M = 5$, for example, the Ge et al. (2021) model would produce average speeds of 11.0 and 12.6 kn on laden and ballast legs, respectively, as reported in Table 2. From Fig. 5, the slowest optimal average speeds for $M^* = 5$ are 12.5 and 14.4 kn, respectively.

We also observe that the amplitude of the saw-tooth slowly reduces for larger M^* values, i.e. that the range of optimal speeds within a cycle reduces with H . As H values increase, although not shown in the figure, optimal speeds converge slowly to unique values for ballast and laden leg speeds, respectively. These limits correspond to the solution for P_∞ (displayed in Table 3), i.e., when $H \rightarrow \infty$, $M \rightarrow \infty$, we have $P_M(H) \equiv P_M(H \rightarrow \infty) \equiv P_\infty$. This pattern of convergence is essentially very different than our $P_M(H \rightarrow \infty)$ model as well as the chain effect reported in Ge et al. (2021), due to the impact of duration constraint H (12).

We provide three examples to help clarify how the model $P_M(H)$ and its solutions over a range of possible H values, presented as speed-menu curves, can be utilised.

Example 1. The charterer considers hiring a ship from an owner, who needs the ship back in 200 days. From Fig. 5 and Fig. 6, we can see that the highest NPV that could be achieved is 4 repetitions of the journey, at high speeds of 15.7 kn and 16.9 kn for all laden and all ballast legs, respectively. If adopted, the ship will only complete on day 200, and it would not have the ability

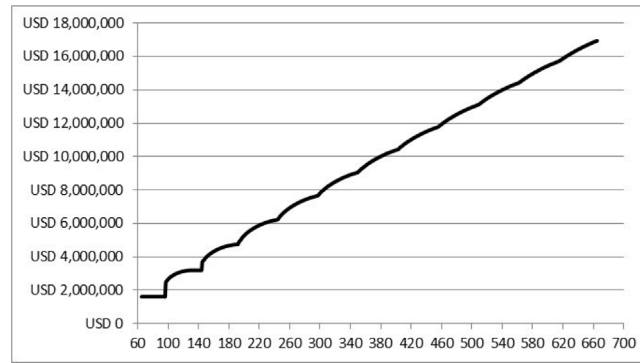


Fig. 6. Optimal NPV profit (USD) as a function of H (days) in the base case example.

to make up any time lost during the four journeys. The more sensible solution is for the ship charterer to aim for 3 repetitions at the lowest optimal speeds of 11.5 kn and 13.1 kn, needing 192 days in total (including harbour time), or 64.0 days per roundtrip (without harbour time). \square

Example 2. As in Example 1 but now the ship can be returned in 300 days maximum. From Figs. 5 and 6, we deduce that, with this ship, it would be optimally executed at 12.6 kn laden speed and 14.5 kn ballast speed, requiring 294 days in total (including harbour time), or 58.8 days per roundtrip (without harbour time), thus a 9% decrease in the optimal roundtrip time! \square

Example 3. From Table 2, we see that the $P_M(H \rightarrow \infty)$ model suggests it is optimal to complete 5 journeys in 331.85 days. However, for $H = 331$ days, see Fig. 5, $P_M(H)$ arrives at speeds of 13.5 kn and 15.7 kn in order to complete 6 journeys. The NPV of this is USD 8,751,115, which is much higher (i.e., an increase of 11.8%) than the results of the $P_M(H \rightarrow \infty)$ model. It is clear that given a number of available days H , the charterer will greatly benefit from seeking an optimal value M^* using model $P_M(H)$. \square

The above examples consider an identical ship, the same daily hiring cost, the same journey structure, and under identical economic conditions. Yet, the range of optimal speeds for the ship clearly depend on the time charter contract horizon.

We note that the solutions of $P_M(H)$, and represented in the speed menu curves, as illustrated in the above examples, do not require the ship to flexibly adjust its speeds over a 7 knots range from one voyage to the next. Rather, the ship would operate within quite a narrow range: in each of the laden legs over the time charter contract duration, it would travel at constant speed. The differences in optimal speeds on a laden and ballast leg is within a range of 2 kn, or less, and it would be optimal for the ship to travel faster on the ballast legs: this will happen quite naturally when the ship uses a similar power setting as used on the laden leg.

3.3. Further usage of $P_M(H)$

The speed menu curves as in Fig. 5 plot solutions to $P_M(H)$, i.e. the optimal speeds on each leg, and number of journey repetitions as a function of the available horizon H . In this section we illustrate further possible scenarios in which the model and the speed menu curves can be of usage.

3.3.1. Redelivery clause

The solutions from $P_M(H)$ as depicted in Fig. 5 are also applicable for a ship which would already be on a time charter contract by reinterpreting H as the time remaining on the contract. It also allows for the consideration of flexible redelivery time windows, often present in contracts, showing how the choice of actual redelivery date and optimal journey choices and leg speeds are linked decision problems.

As introduced in Section 2.1, the redelivery clause of a charter party stipulates when the charterer's right to use the vessel terminates. Such clauses exist in many variations, but most include some flexibility. The example considered here is the approach whereby the ship is to be returned up to 45 days earlier or later, at the charterer's discretion, relative to an agreed termination date.⁸

To examine the impact of such a redelivery clause, we can re-use the results of Section 3.2. We focus on $H \in [320, 410]$ in Fig. 5 and it is re-interpreted as displaying the range of optimal strategies available to the charterer when the agreed termination date is one year ahead (365 days), plus or minus 45 days. Depending on the actual completion date, possible speed choices for ballast legs range from 17 kn down to 15 kn, and laden leg speeds from 15.8 kn to 12.6 kn. Which speeds from these 'speed menus' are optimal? Recall that the NPV within this range of available horizons is strictly increasing (see Fig. 6). Does this mean that charterers, when having agreed to this contract, will select the solution as to arrive at day 410?

⁸ This is a typical arrangement in for example the ExxonMobil2000 charter contracts.

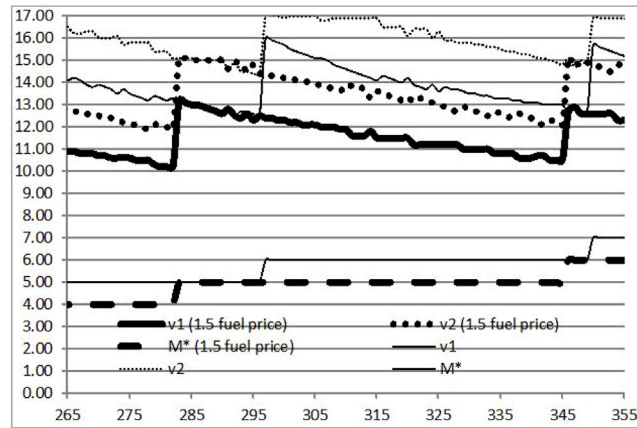


Fig. 7. Optimal leg speeds (kn) and number of repetitions (fuel prices \times 1.5) for $H \in [265, 355]$.

Example 4. When aiming to complete 8 journeys and arrive at day 410, the ship would travel at 17 kn in ballast and 15 kn laden. However, any delays means that the ship would not be able to complete this. Very few charterers would take this risk, and the hassle and penalties associated with overrunning the contract duration. Travelling at these high speeds but completing only 7 journeys is by far out-competed by targeting initially 7 journeys only (see next example).

Example 5. When targeting 7 journeys, several choices can be made. For example, a solution ($v_1 = 14.0$ kn, $v_2 = 16.3$ kn)⁹ would lead to a completion by day 375. But this still gives 35 days of buffer relative to day 410. Some charterers may think this buffer can be smaller, since as the NPV curve indicates, travelling slower can increase profits. Therefore, a charterer may perhaps conclude that a solution ($v_1 = 13.0$ kn, $v_2 = 15.1$ kn), which will complete the 7 journeys by day 399, with a buffer of 11 days, would be a decent compromise between profit maximisation and risk. \square

3.3.2. Decision dynamics

The model $P_M(H)$ is a deterministic model, like many other classic speed optimisation models in the literature. While there are situations in which prices can be agreed over a longer-term contract, recall also the discussion in Section 2, uncertainty during voyages from e.g. bad weather, or port delays, may introduce unanticipated disruptions. These situations may call for a re-optimisation. Considering the present state of the ship, and the redelivery time window remaining on the contract, as above, one can repopulate the model with the new parameter values and re-assess the remaining strategy for the ship on this contract.

From the discussion in Section 3.3.1 follows that, under identical conditions, charterers may make different initial choices about vessel speeds. This also sets them each on a different course towards addressing future dynamic changes.

Consider two charterers operating under identical conditions, aiming to complete 7 journeys in a one-year contract: charterer CI adopts ($v_1 = 14.0$ kn, $v_2 = 16.3$ kn, $L_n = 53.4$ days), but charterer CII adopts ($v_1 = 13.0$ kn, $v_2 = 15.0$ kn, $L_n = 57$ days).

Example 6. Consider that near the time of completing the first journey, fuel prices rise by a factor 1.5. CI then has $410 - 53.4 = 356$ days left on the contract, and CII $410 - 57 = 353$ days. Fig. 7 displays the new optimal solutions to $P_M(H)$ under the fuel price rise. Having completed the first journey as planned, CI adjusts its estimate of needed buffer down to 25 days, while CII to 8 days. They thus target completion by days 332 and 345, respectively. Fig. 7 shows that they each can now only complete another 5 journeys, and CI selects ($v_1 = 11.0$ kn, $v_2 = 12.6$ kn, $L_n = 66.4$ days), while CII chooses ($v_1 = 10.5$ kn, $v_2 = 12.1$ kn, $L_n = 69.0$ days). The fuel price change has had a dramatic impact on the chosen speeds of both charterers. \square

Example 7. Consider that the fuel price rise occurs only when they have completed 4 journeys according to plan. They then have, respectively, 196 and 182 days left on the contract. It is reasonable to assume that with only 3 more journeys to go, without disruptions for previous journeys, CI and CII may become embolden to reduce the buffer more drastically to, say, 6 days and 2 days, respectively. This means CI now targets completion by day $196 - 6 = 190$, and CII by day $182 - 2 = 180$. We see on Fig. 8 that CI selects ($v_1 = 11.6$ kn, $v_2 = 13.3$ kn, $L_n = 63.3$ days), while CII chooses ($v_1 = 12.3$ kn, $v_2 = 14.2$ kn, $L_n = 60$ days). Note that CI now steers the ship at a lower speed than CII, and from Fig. 8 it is also clear that a fuel price rise would now have no impact on their speed choices. \square

Even if we assume identical ship, journey, and charter contract characteristics, we can thus still expect different charterers to arrive at different strategies to (adjusting) ship speeds when subject to a change in external data.

⁹ This is the solution obtained from P_{∞} , see Section 4.3.

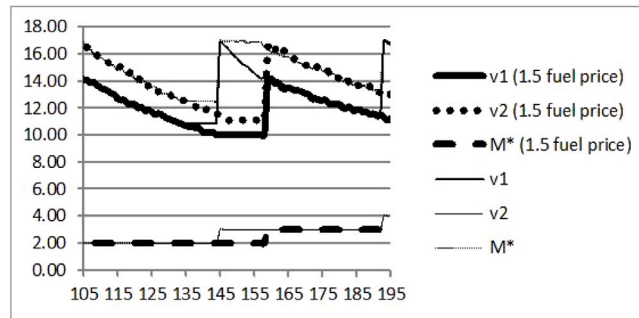


Fig. 8. Optimal leg speeds (kn) and number of repetitions (fuel prices \times 1.5) for $H \in [105, 195]$.

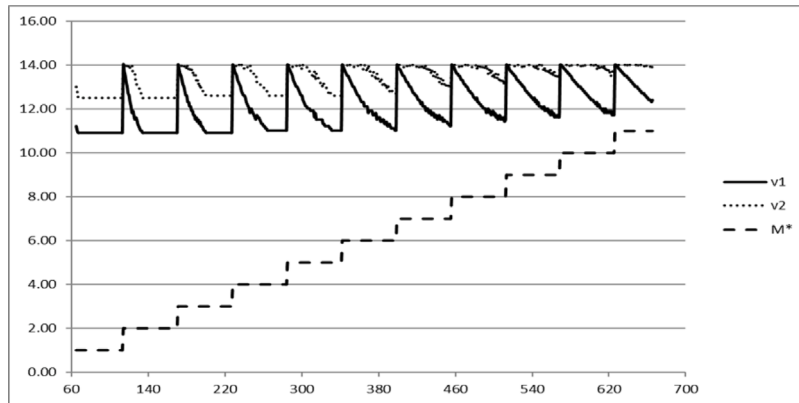


Fig. 9. Optimal leg speeds (kn) and repetitions as functions of H (days), maximum ship speed 14 kn.

3.3.3. Environmental constraints: speed limits

The modelling approach developed in this study may help in the assessment of developments in emission related regulatory and technological initiatives when these affect the conditions under which ship speed decisions are made. Note that the ship's speed range limitations corresponding to Fig. 5 is 10 to 17 kn.

Example 8. An often discussed maritime regulation to decarbonisation is to impose strict restrictions on the maximum speed of a ship. Fig. 9 plots the speed menu curve in the case that maximum speed v^+ is reduced from 17 kn to 14 kn. Comparing with Fig. 5, the ship with this speed limitation can do, generally speaking, one less repetition when given the same number of days (H). The charterer's NPV is much reduced (when rates remain the same), and the charter market would in total need more ships to perform the same amount of cargo movements per day. Notice in Fig. 9 that on ballast legs in longer time charter contracts the ship already travels close to the maximum allowed speed. The ship may thus have much greater difficulty making up for any lost time. When considering the potential profitability of a time charter contract under such speed limits, the ship charterer thus may feel it necessary to build in safety against such risks by considering only solutions where v_1 and v_2 are set even lower than the 14 kn. \square

An alternative is for legislation to define the speed limit as an average over different legs, but this seems difficult to clearly define, check and enforce. For a thorough discussion of speed limits, see Psaraftis (2019a).

4. Special cases P_1 and P_∞

Recall from Section 2.3 that P_1 optimises the NPV of a single journey and assumes the journey time can be freely decided like in a trip time charter, while P_∞ optimises the stationary speeds per unit of journey time, assuming the journey would be repeated infinitely under static economic conditions. These are two special cases of our proposed $P_M(H)$, namely when $H \rightarrow \infty$, and with fixed $M = 1$ and $M \rightarrow \infty$, respectively.

We analyse these two special cases by comparing them to models from the literature. Ge et al. (2021) showed that Model PK and Model R, without the consideration of charter contract, can be also mapped to two special cases in their model. We therefore consider to map our P_1 and P_∞ first to the $\mathcal{P}(n, m, G_\theta)$ framework in Ge et al. (2021), and then compare to Model PK and Model R, respectively.

By demonstrating this equivalence, an understanding about the differences between Model PK and Model R can be obtained by studying the differences between P_1 and P_∞ , and it also shows how PK and R models map onto charter contract types.

4.1. Equivalence to $\mathcal{P}(1, n, 0)$ and $\mathcal{P}(\infty, n, -)$

Consider model P_1 maximising (8) for a journey of arbitrary number of legs n and payment times δ and ϵ . Let us denote the NPV for a single leg j of the journey, discounted to the start time of that leg, by $h_j(T_j, \delta_j, \epsilon_j, \epsilon_{j-1})$. We can then re-write (8) as follows:

$$h_j(T_j, \delta_j, \epsilon_j, \epsilon_{j-1}) = R_j e^{-\alpha(T_j - \delta_j)} - C_j^u e^{-\alpha(T_j + \epsilon_j)} - C_j^l e^{-\alpha \epsilon_{j-1}} - \int_0^{T_j} f^{TCH} e^{-at} dt, \tag{20}$$

$$NPV_1(V_j^-) = NPV_1(V_{j-1}^-) + h_j(T_j, \delta_j, \epsilon_j, \epsilon_{j-1}) e^{-\alpha L_{j-1}}. \tag{21}$$

Comparing to Ge et al. (2021), if $\delta = \epsilon = 0$, it is clear that (20) is identical to (6) in their paper, and (21) is a recursive function that is equivalent to their dynamic programming formulation of model $\mathcal{P}(1, n, 0)$. It implies that a trip time charter model P_1 , in the formalism of Ge et al. (2021), considers a zero value for the future profit potential. This result is formalised in Lemma 1.

Lemma 1 (Equivalence of $\mathcal{P}(1, n, 0)$.) (a) The objective function of model $\mathcal{P}(1, n, 0)$ is identical to P_1 when $\delta = \epsilon = G_0 = 0$, which can be solved by Algorithm 1; (b) Solving $\mathcal{P}(1, n, 0)$ is equivalent to solving a problem in P_1 , i.e. maximising the Goodwill of the current journey subjected to a scenario like in a trip time charter with zero future profit potential.

We now consider model P_∞ that maximises (17) for a journey of arbitrary number of legs n and payment times δ and ϵ . Given (20) and (21), if $\delta = \epsilon = 0$, (17) can be re-written as:

$$AS(V_n^-) = \alpha NPV_\infty = \frac{\alpha NPV_1}{1 - e^{-\alpha L_n}} = AS(\Gamma_1), \tag{22}$$

where Γ_1 is used in Ge et al. (2021) that is equivalent to our V_n^- . We thus see that P_∞ maps onto $\mathcal{P}(\infty, n, -)$ and there is no need to consider a future profit potential.

Lemma 2 (Equivalence of $\mathcal{P}(\infty, n, -)$.) (a) The objective function of model $\mathcal{P}(\infty, n, -)$ is identical to P_∞ when $\delta = \epsilon = 0$, which can be solved by Algorithm 1; (b) Solving $\mathcal{P}(\infty, n, -)$ is equivalent to solving a problem in P_∞ , i.e. maximising the Goodwill of the repeating current journey infinitely under static economic conditions subjected to a contract of very long duration.

The above results prove that, unlike model $P_M(H)$, assuming identical speeds for each repetition does not make P_1 and P_∞ any different to $\mathcal{P}(1, n, 0)$ and $\mathcal{P}(\infty, n, -)$, respectively.

4.2. Equivalence to model PK and model R

Model PK refers to the class of models as presented by Psaraftis and Kontovas (2014). Using our notation, their formulation (after (1) on p. 59) can be stated as:

$$\text{Min } \frac{1}{v_j} \left[c_j^f F(v_j, w_j) + f^{TCH} + \alpha_s u + \beta_s w \right]. \tag{23}$$

In this model, u refers to volume of cargo yet to be picked up in ports $j + 1$ to $n - 1$ that would be waiting in these ports, and w to the volume of the cargo currently on the ship. The α_s and β_s represent the carrying charges for the shipper, and from the example on p.64 in that paper, we observe that e.g. $\beta_s = \alpha G$, where G is taken (in the examples on p. 64 of that paper) to be the CIF value of the cargo. The objective function (USD/nm) expresses costs per unit of distance travelled on the leg. Since distance is a constant, this is equivalent to minimising (USD/leg). The Model PK is adopted in e.g. Wen et al. (2017). When not considering holding costs ($\alpha_s = \beta_s = 0$) and replacing $F(V, W)$ by the simpler propeller law $F(V) \sim V^g$, the model is also representative of those used to optimise speeds in earlier literature, as in e.g. Corbett et al. (2009). More generally, Model PK represents the models from the literature that use the criterion (USD/nm), (USD/journey), or (USD/leg).

Model R refers to another class of models following the approach developed in the income generating leg model in Ronen (1982). When we replace the propeller law by (3), and use our notation, Ronen’s problem statement is that optimal speed for leg j is found from:

$$\text{Max } \frac{1}{T_j(v_j)} \left[R - T_j(v_j)C - c_j^f F(v_j, w_j) T_j^g(v_j) \right], \tag{24}$$

where R is the income from the leg, and C the (constant) daily cost of the vessel (which can be dropped from the objective function). This formulation thus seeks to maximise the profits per day on the leg (USD/day). Model R, using the propeller law, can be recognised in e.g. the models of Devanney (2010) and Fagerholt and Psaraftis (2015).

Equivalence analysis has also been used in Ge et al. (2021), where the authors find equivalence of these two classic models, namely Model PK and Model R, to their $\mathcal{P}(1, n, 0)$ and $\mathcal{P}(\infty, n, -)$ models, respectively. From Lemmas 1 and 2, it is easy to verify also the equivalence of our P_1 and P_∞ to these classic models. Therefore, applying the conclusion of Ge et al. (2021)[Section 5.1], classic models that maximise daily profitability as in Model R should only consider journeys of a roundtrip structure, as does P_∞ , see Fig. 4. As indicated by the chain effect in Ge et al. (2021)[Section 4.3], the decomposition principle explicitly proposed by Ronen (1982) and Psaraftis and Kontovas (2014) in their model setting does not produce optimal results in the NPV framework. One thus needs to solve the NPV models simultaneously like in this paper.

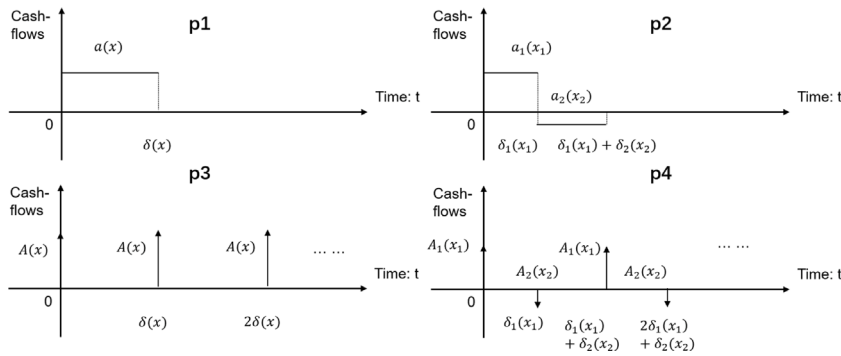


Fig. 10. Cash-flows functions of the abstract representations of 4 different ship journeys p_1 , p_2 , p_3 and p_4 .

To better appreciate such equivalence, we use a high level setting that is rather general to illustrate the rationale of why (the linear approximations of) our P_1 and P_∞ can be mapped to Model PK and Model R types of classic models.

Consider 4 logistics scenarios, see also Fig. 10, that may represent the cash flows arising during trip time or long term time charters, respectively.

Projects p_1 and p_2 are simplified representation of a one-leg journey and two-leg journey model, respectively. Most models from the literature do not specify when revenues and costs are incurred. The more it is reasonable to assume that profits earned from a leg i arrive at a uniform rate a_i over the leg’s duration, for which only its magnitude would depend on $x_i \in X$, the more p_1 and p_2 are fair characterisations of such models. Projects p_3 and p_4 represent infinite repetitions of the one-leg journey project p_1 and the two-leg journey project p_2 , respectively.

Let $a(x)$ denote a cash-flow function associated with a leg when the ship travels at speed x with a corresponding duration $\delta(x)$, then consider four possible journeys:

1. p_1 : a laden leg where it generates daily profit $a(x)$ for $t = [0, \delta(x)]$, and 0 ($t \geq \delta(x)$);
2. p_2 : a laden leg where it generates daily profit $a_1(x_1)$ for $t = [0, \delta_1(x_1)]$, followed by a ballast leg where it incurs daily cost $a_2(x_2)$ for $t = [\delta_1(x_1), \delta_1(x_1) + \delta_2(x_2)]$, and 0 ($t \geq \delta_1(x_1) + \delta_2(x_2)$).
3. p_3 : an infinite repetition of a laden leg where it generates profit at times 0, $\delta(x)$, $2\delta(x)$, \dots , of magnitude $A(x)$, where $A(x)$ equals the NPV of p_1 ;
4. p_4 : an infinite repetition of a laden leg followed by a ballast leg, where it generates profit $A_1(x_1)$ at times 0, $(\delta_1(x_1) + \delta_2(x_2))$, $2(\delta_1(x_1) + \delta_2(x_2))$, \dots and it incurs cost $A_2(x_2)$ at times $\delta_1(x_1)$, $\delta_1(x_1) + (\delta_1(x_1) + \delta_2(x_2))$, $\delta_1(x_1) + 2(\delta_1(x_1) + \delta_2(x_2))$, \dots , where $A_1(x_1)$ is the NPV of $a_1(x_1)$ and $A_2(x_2)$ is the NPV of $a_2(x_2)$ of p_2 .

Let $\overline{Y(\alpha)}$ represent the linear approximation of a function in α ; and ‘Max x ’ shorthand for maximising a function x .

Theorem 1. (proof in Appendix B):

- (I) (Model PK) Max $a\delta$ of $p_1 \equiv (\text{Model } P_1) \text{ Max } \overline{NPV(\alpha)}$ of p_1 ;
- (II) (Model PK with decomposition) Max $a_1\delta_1 + a_2\delta_2$ of $p_2 \equiv (\text{Model } P_1) \lim_{\alpha \rightarrow 0} \text{Max } \overline{NPV(\alpha)}$ of p_2 ;
- (III) (Model R) Max $a(x)$ of $p_1 \equiv (\text{Model } P_\infty) \text{ Max } \overline{AS(\alpha)}$ of p_3 ;
- (IV) (Model R without decomposition) Max $(a_1\delta_1 + a_2\delta_2)/(\delta_1 + \delta_2)$ of $p_2 \equiv (\text{Model } P_\infty) \lim_{\alpha \rightarrow 0} \text{Max } \overline{AS(\alpha)}$ of p_4 ;
- (V) (a) (Model R with decomposition) Max $\frac{a_1\delta_1}{\delta_1} + \frac{a_2\delta_2}{\delta_2}$ of $p_2 \not\equiv (\text{Model } P_\infty) \text{ Max } \overline{AS(\alpha)}$ of p_4 , even if $\alpha = 0$, and (b) Max $a_1 + a_2$ will skew the optimisation by placing too much emphasis on the cash-flow a_i of leg i with the smallest value for $\delta_i/(\delta_i + \delta_j)$ ($i, j \in \{1, 2\}$).

In the theorem, the left-hand sides of \equiv or $\not\equiv$ represent (possible) methods we can apply when the objective function is set up following the classic models. The right-hand sides follow the NPV approach — but possibly applied to the linear approximation of the NPV function. Because this approximation is often of good accuracy, a result of equivalence (\equiv) implies a good correspondence between the classic method and the NPV approach. We can then also expect to be able to learn about the classic method through the study of the equivalent NPV model. See also (Beullens and Janssens, 2014).

In particular we derive the following insights. (I) and (II) indicate that an approach as in Model PK that maximises profits per nautical mile per leg (by decomposition), will come close to maximising the NPV for the ship charterer of single journey only. This implies that Model PK implicitly assumes that the time of completing the journey does not affect future investment opportunities for the ship charterer (as this is indeed what optimising the NPV of p_1 or p_2 establishes).

(III) to (V) show properties of models such as Model R that maximise daily profits over legs of a single finite journey. (III) indicates that Model R is equivalent to maximising the AS (USD/day) of a model where this journey, that Model R considers, is in fact infinitely repeated. It is thus important to realise that such models implicitly adopt this assumption. (IV) and (V) show how to best construct Model R when legs are not uniform in their profit characteristics and/or duration. (V) indicates that decomposition will not

Table 4
Optimal speeds for a ballast-laden journey.

Variable/Model	P_1	P_∞	Model PK	Model R
v_1 (kn) (ballast)	12.6	16.4	15.1	12.5
v_2 (kn) (laden)	10.9	14.1	12.5	17.0
L_n (days)	65.7	52.1	57.2	55.5
NPV (USD)	1,608,351	10,246,502 ^a	1,517,854	8,674,053 ^a
f^{TCE} (USD/day)	44,672	48,073	46,701	43,765

^aAS (USD/year).

work well. For example, if leg 1 requires 1 month and leg 2 half of that, (III) indicates that the decomposed model will account for 12 repetitions of the first and 24 repetitions of the second leg in one year. This is wrong, since both legs can only be repeated an equal number of times (8 in a year). Using decomposition will thus skew the results by putting too much weight on the importance of leg 2. (IV) shows that an approach which maximises the weighted profits over the total journey duration will achieve much better results.

4.3. Numerical comparison

The purpose of this section is to compare solutions of P_1 , P_∞ , Model PK and Model R on the base case example. Table 3 compares the models when the journey starts with the laden leg. In Model PK, inventory costs are at CIF value of 63 USD/barrel and an opportunity cost of 0.08 per annum. In Model R, we use the revenue generating leg model for both laden and ballast legs. To determine journey times L_n , we evaluate the solution of Model PK and Model R into our journey model (which also accounts for harbour times). We evaluate the speed values obtained from Model PK and Model R using the NPV and AS functions of models P_1 and P_∞ , respectively.

Observe that the models arrive at different speed recommendations. Both P_1 and P_∞ find ballast optimal speeds to be higher than laden leg speeds, corresponding to observed industry practice. P_∞ arrives at much higher speeds, completing the journey (including port times) in about 13 days less, and also the gap between the optimal laden and ballast speed is widened.

Comparing journey times and speeds, we observe that Model PK is closer to P_1 and Model R is closer to P_∞ . As P_1 maximises (USD/journey) while P_∞ maximises (USD/year), and given the equivalence results in Section 4.2, this is not surprising. While Model PK arrives at a higher TCE¹⁰ than P_1 , it is the result of shorter journey time, and its NPV value is lower. Model R arrives at a journey time close to that of P_∞ , but the very different laden and ballast speeds will achieve one million USD less in AS value. Both Model R and Model PK find ballast speeds to be lower than laden speeds.

Table 4 shows results when the journey starts with the ballast leg. P_∞ and Model R give each the same speed results they obtained under the laden-ballast case, while P_1 slightly raises the speed on the ballast leg. The largest difference is observed in Model PK, where ballast speed shows a large increase and laden speed is slightly lowered. NPV and AS values are lower because costs arise earlier and revenues later compared to the laden-ballast situation. TCE values are therefore also lower, except in Model PK due to the much smaller journey time.

The example illustrates that optimal speeds depend on the model used. With respect to the differences between P_1 and P_∞ , we see that charter contract type matters. Optimal speeds on a time charter contract of very long duration (P_∞) are significantly higher (in this example) than those on a trip time charter (P_1), and the gap between laden and ballast speed is significantly larger. Whether or not the decision maker has an interest in what happens with the ship after completion of a journey, and thus the type of the charter contract, influences speed decisions.

Further comparison with results for $P_M(H)$ and $P_M(H \rightarrow \infty)$ on the same base case example, as shown in Fig. 6 and Table 2, respectively, shows that optimal speeds are highly model dependent, despite the fact that in each case the logistics task, and the basic economic parameters and ship's characteristics, always remain the same. In each model, however, the contract type differs.

5. Sensitivity analysis

This section reports on the sensitivity of the models P_1 , $P_M(H)$ and P_∞ to various model parameters.

5.1. Sensitivity to the opportunity cost α

It is useful to give an indication of the importance of using appropriate estimates for the value of α . Table 5 lists optimal speeds for α ranging from 0 to 0.3 (per year) for P_1 , P_∞ , and P_{10} ($H \rightarrow \infty$) (as in Section 3.1). The optimal speed v_1 on the laden leg is lower than that on the ballast leg v_2 .

Profitability (NPV or AS) is determined by the value of α . If the next best available alternative is more profitable, α is higher, and the relative attractiveness of this project reduces, and its NPV reduces. This matters most in P_{10} , and much less in P_1 and P_∞ .

¹⁰ Time Charter Equivalent (TCE) earnings is an industry standard expressing the difference between voyage revenues and voyage costs (excluding charter hire costs), divided by the total number of voyage days. We calculate TCE with time discounted cash-flows, for details, see Ge (2021), Chapter 3.

Table 5
Sensitivity analysis of α for $M = 1$, $M = 10$ and $M \rightarrow \infty$.

Variable/Model	P_1	P_{10} ($H \rightarrow \infty$)	P_∞ ($M \rightarrow \infty$)
$\alpha = 0$ (/year)			
v_1 (kn)	10.8	10.8	14.0
v_2 (kn)	12.5	12.5	16.3
NPV (USD)	1,630,374	16,303,740	10,084,697 ^a
$\alpha = 0.05$ (/year)			
v_1 (kn)	10.8	11.0	14.0
v_2 (kn)	12.5	12.7	16.3
NPV (USD)	1,621,467	15,568,458	10,075,487 ^a
$\alpha = 0.1$ (/year)			
v_1 (kn)	10.9	11.2	14.0
v_2 (kn)	12.5	12.9	16.3
NPV (USD)	1,612,675	14,885,105	10,066,268 ^a
$\alpha = 0.2$ (/year)			
v_1 (kn)	11.0	11.5	14.0
v_2 (kn)	12.5	13.1	16.3
NPV (USD)	1,595,239	13,654,616	10,047,317 ^a
$\alpha = 0.3$ (/year)			
v_1 (kn)	11.1	11.8	14.0
v_2 (kn)	12.4	13.4	16.2
NPV (USD)	1,578,092	12,577,983	10,027,717 ^a

^aAS (USD/year).

In P_1 the timeframe is short (less than 70 days) and reduced sensitivity is thus as expected. In P_∞ , AS values change very little, in part due to optimal speeds in that model being quite insensitive to α .

Optimal speeds on both laden and ballast legs are typically increasing with α in P_1 and P_{10} . Interestingly, the difference in optimal speeds of these models in comparison to P_∞ is the largest when α is the smallest, while the difference between P_1 and P_{10} is the largest for the largest values of α .

From the table, we can infer that for an estimate of α (per year) within an accuracy of ± 0.05 (or $\pm 5\%$), we find optimal speeds (in knots) within a $\pm 5\%$ accuracy. Given that optimal speeds thus remain fairly insensitive to the value of α , we have conducted all experiments in this study using the marine industry average value of α at 8% (per year) reported in Yagerman (2015).

5.2. Sensitivity to other economic parameters in model $P_M(H)$

The sensitivity is greatly dependent on whether the constraint (12) is binding in the optimal solution. Two examples illustrate:

Example 9. Fig. 7 illustrates the impact of a rise in fuel prices by a factor 1.5 for $H \in [265, 355]$. For $H \in [297, 345]$, optimal speeds drop by 2 to 2.5 kn since M^* reduces with 1. In certain ranges, such as $H \in [283, 297]$ the optimal speeds are not significantly affected by the fuel price change. This is because M^* remains constant and also $M^* L_n^* = H^* \approx H$, so the journey time L_n^* is unaffected. For fuel price rises much smaller than by a factor 1.5, the ranges of H for which we would see no significant impact from fuel price changes will become more prominent. \square

Example 10. Sometimes $H^* \ll H$, i.e. the constraint is non-binding, see Fig. 8. In the base case scenario, such non-binding solutions arise for $H \in [134, 144]$. The increase in fuel prices moves the region of non-binding solutions to $H \in [145, 159]$. For $H \in [134, 144]$, the fuel price increase would thus result in the ship slowing down slightly and the solution moves from non-binding to binding, but for $H \in [145, 159]$, the ship's speed reduces drastically and the solution moves from binding to non-binding. \square

Further sensitivity analysis in Ge (2021) indicates that we can draw similar conclusions with respect to the sensitivity to changes in other cost or revenue parameters. If as a consequence of the change in parameter value, the solution remains binding, and M^* keeps its original value, optimal speeds remain largely unaffected. If harbour times increase, however, then under these conditions the ship must speed up to realise the same total journey time. These insights are summarised in the second column of Table 6.

For some data inputs, a change in value of a parameter produces a new optimal value for M^* . An increase in freight rates, ceteris paribus, may thus include one extra journey within H , with usually a large increase in ballast and laden speeds. Conversely, a decrease in M^* usually results in much lower speeds, and may be the result from increases in fuel prices, TCH values, fixed (un)loading costs, or harbour times. This is summarised the last two columns in Table 6.

The above situations are most commonly arising. Less often occurring, but possible, are situations where an optimal non-binding solution remains non-binding at the same M^* value. Then, speeds tend to decrease with increases in fuel prices, and increase with increased TCH values. This is summarised in the third column of Table 6.

To summarise, the impacts of model parameters are largely affected by the time constraint as well. They may play completely opposite roles if we are looking at different time periods; recall Figs. 7 and 8.

Table 6
Main drivers of ship speeds in model P_M .

Parameter/Model	P_M speeds ^a	P_M speeds ^b	M^* ^c	P_M speeds ^c
Fuel prices	≈ 0	-	-	--
Freight rates	≈ 0	≈ 0	+	++
Time Charter Hire	≈ 0	+	-	--
Fixed (un)loading costs	≈ 0	≈ 0	-	-
Harbour times	+	≈ 0	-	-

How an increase in parameter value (e.g. Freight rates) affects model outcome (e.g. optimal speeds): -- and - indicate, respectively, a large and small reduction in outcome value; ++ and + indicate a large and small increase in outcome value; 0 indicates no impact.

^aFor constant M^* such that $H^* \approx H$.

^bFor constant M^* such that $H^* \ll H$.

^cFor changes leading to new optimal M^* value.

Table 7
Main drivers of ship speeds and profitability in P_1 and P_∞ .

Parameter	P_1 speeds	P_1 NPV	P_∞ speeds	P_∞ AS
Fuel prices	--	--	--	--
Dwt carried	--	n/a	--	n/a
Freight rates	+	++	++	++
Time Charter Hire	++	--	0	--
Leg distance (with linearly increasing revenues)	$+e$	+	+	+
Leg distance (at constant revenues)	0	--	--	--
Fixed (un)loading costs	$-e$	--	--	--
Harbour times	0	-	-	-

How an increase in parameter value (e.g. Freight rates) affects model outcome (e.g. optimal speeds): $-e$ indicate a minimal reduction in outcome value; $+e$ indicate a minimal increase in outcome value.

5.3. Sensitivity to economic parameters in P_1 and P_∞

A similar sensitivity analysis was conducted for model P_1 and P_∞ , see Ge (2021). The main drivers of optimal speed values and profitability are reported in Table 7. In both models, fuel price increases will greatly lower optimal speeds.

From this we derive that speed decisions under trip charter contracts may differ from those under time charter contracts of very long duration. The first main difference is the impact of the time charter hire: higher values drive up optimal speeds in P_1 , as this helps to reduce the total number of contract days. In P_∞ , this has no effect because it is simply a constant term in the objective function. A second difference is in how port related events affect speeds: higher (un)loading costs will greatly reduce the optimal speeds in P_∞ , but have only a minimal effect in P_1 , while an increase of the time spend in harbours reduces speeds in P_∞ , but has no impact in P_1 . Port events affect speed decisions, but not in the way as hypothesised in some of the literature, see also Example 11. A third main difference is freight rate sensitivity: in P_∞ optimal speeds are highly sensitive to increases in freight rates, but this sensitivity remains small in P_1 . Given this, it is intuitive why an increase in leg distance (with linearly increasing revenues) increases optimal speeds in P_∞ , but has only a minimal impact in P_1 .

Example 11. It has been postulated in literature, see Johnson and Styhre (2015), that reducing harbour time may encourage slow-steaming. What would the models predict? From Table 7 we see that reducing harbour times in P_1 has no effect on optimal speed. Reducing harbour times does not seem to be an effective incentive towards slow steaming for ships on trip time charter contracts. In P_∞ , it has the opposite effect. Thus, slow steaming is encouraged by increasing port times, or increasing port costs, at the expense of reducing the profitability of the ship charterers. In P_M , the hypothesised effect will only be seen when the number of journeys as well as the optimal contract duration remains unaltered (Case (*) in Table 6). This is logical, as then the ship has to spend more time at sea. In the other cases (**) and (***), the impact would be either of no significance (as in P_1), or opposite (as in P_∞). □

We note that Johnson and Styhre (2015) observed in their study that shippers made no adjustment of ship speeds. The conditions of that study (shorter distances, possibly fixed liner routes) also differ from the situations we consider.

In order to develop some intuition for this difference in freight rate sensitivity, we can use an analytical process of approximation, see Ge (2021)[Chapter 3], and write the linear approximation in α of the objective function (8) as $NPV_1 = \sum_{j=1}^n \overline{NPV}_{1(j)}$, that is, as a sum of terms with information that is mostly related to each of the legs, and which allows us to associate also an interpretation to the terms in this objective function that are influenced by data from other legs. This process of simplification arrives at the following minimisation problem for each leg j (Ge, 2021):

$$\text{Min } \frac{1}{v_j} \left[c_j^f F(v_j, w_j) + f^{TCH} + \alpha \sum_{i=j}^n (R_i - C_i^u - C_{i+1}^l(v_{i+1})) \right]. \tag{25}$$

Table 8
Performance of repeated laden–ballast journeys having a maximum of H available days.

Variable/Model	P_1	P_∞	$P_M(H)$
$H = 335$ (days)			
H^* (days)	334.4	320.5	334.8
M^* (–)	5	6	6
v_1 (kn) (laden)	10.9	14.0	13.4
v_2 (kn) (ballast)	12.5	16.3	15.5
NPV (USD)	7,828,927	8,514,068	8,829,191
$H = 268$ (days)			
H^* (days)	267.5	267.1	268.0
M^* (–)	4	5	5
v_1 (kn) (laden)	10.9	14.0	13.9
v_2 (kn) (ballast)	12.5	16.3	16.3
NPV (USD)	6,308,705	7,136,261	7,158,921
$H = 365$ (days)			
H^* (days)	334.4	320.5	364.6
M^* (–)	5	6	7
v_1 (kn) (laden)	10.9	14.0	14.4
v_2 (kn) (ballast)	12.5	16.3	16.8
NPV (USD)	7,828,927	8,514,068	9,618,736

This function is dependent on v_j , and only on V_{j+1}^+ through the C_{i+1}^l terms. The first two terms of the function (25) show that important drivers of speed decisions in P_1 are fuel price, deadweight carried, and time charter hire. The third term shows a dependency on the opportunity cost of deferred profits from all *subsequent legs* only. It can be inferred that the greatest impact of future legs on the speed of the ship will be witnessed on the first legs of the journey, and in particular when n grows to larger values. This *chain effect* is further formally analysed in Ge et al. (2021). It also intuitively explains the difference in freight rate sensitivity between P_1 and P_∞ , because the latter model can be viewed as the model P_1 with an increasing number of legs n growing to infinity. Note that, as a consequence of the approximations, the impact of harbour times and leg distances are no longer present in (25), although have an impact on speed in the unapproximated NPV models (see Table 7).

In conclusion, the impact of the main drivers on optimal speeds works quite differently in each of the models P_1 , P_∞ , and P_M . This contributes to the understanding of *general economic principle* in maritime economics that ship speed is determined by a nonlinear function of the ratio of freight rates to fuel prices. In trip time charters (P_1), the sensitivity to freight rates remains very small, assuming that the ship charterer aims to maximise the NPV of the ship to execute the planned for (multiple-leg) single journey, and can sign the contract for the time required to do so. The underlying assumption here is that the charterer’s other business opportunities are not dependent on this contract’s termination date. What would happen in P_1 if the single journey would start to take longer and longer time? Then the importance of the freight rate would increase. This can be deduced from the results presented in Section 3.1 about $P_M (H \rightarrow \infty)$, since repeating M times a short journey can be reinterpreted as performing a long multiple-leg journey once. This is also in line with the finding that P_∞ is highly sensitive to freight rates. In the general model $P_M(H)$ the economic principle is more nuanced, see Table 6.

5.4. What if we use the wrong model?

In this section we illustrate that, when having a maximum of H available days, deriving solutions from $P_M(H)$ is superior to finding solutions based on speeds found from models P_1 (or Model PK) or P_∞ (or Model R).

We illustrate the impact of a finite time horizon in base case example. We test for: $H = 335$ days, or 5 times the journey duration obtained from model P_1 ; $H = 268$ days, or 5 times the journey duration obtained from P_∞ ; and finally, an arbitrary horizon set at $H = 365$ days.

In Table 8, we compare the performance of $P_M(H)$ with that obtained from using a feasible number of repetitions of optimal journey speeds obtained from P_1 and P_∞ , respectively. For each of the three possible horizons, the table lists the optimal values of the agreed duration H^* , the corresponding number of feasible repetitions M^* , the leg speeds, and the NPV. For P_1 and P_∞ , the optimal leg speeds are as reported in Table 3. In P_M , optimal leg speeds are determined simultaneously with the optimal values for M^* and H^* .

It is a striking observation how much better a solution from $P_M(H)$ is compared to those obtained from P_1 or P_∞ .

Note that in each solution, the charterer only pays the charter rate f^{TCH} during $H^*(\leq H)$. In P_∞ for $H = 335$, for example, the charter rate only applies to 320.5 days. $P_M(H)$ completes an equal number of journeys at lower speeds such that $H^* \approx H$. For $H = 365$ days, however, optimal leg speeds increase relative to the P_∞ solution as to include one additional journey within the time horizon, increasing NPV profits by over 1 million USD, or an increase of 15%.

Depending on the horizon, P_1 leads to solutions which are between 0.8 and 1.8 million USD worse in NPV, or about 11% to 23% worse. The reason for this bad performance of P_1 can by now be understood since trip time charter contracts assume the ship is returned to the owner after a single journey is completed, and does not account for future repetitions. Model P_∞ is on average

Table 9
Comparison between P_1 , P_∞ and $P_M(H)$ models.

Contract Length	Objective	Model
Short or Undefined (a few months, e.g. Spot Contract)	maximise the profitability over given leg(s) (USD/Nautical Miles)	P_1
Very long (> 5 years, e.g. the decision maker owns the ship)	maximise the daily profitability over a round-trip (USD/Day)	P_∞
In between, with specified duration H (e.g. Time Charter)	maximise the profitability over given horizon H (USD/Journey)	$P_M(H)$

better equipped as it considers journey repetition, although depending on the horizon constraint, can still be quite inferior relative to $P_M(H)$.

In brief, the example illustrates that the horizon of the charter contract is a factor of significant impact on speed decisions. The impact of the horizon is not explicitly considered in the special cases: in P_1 it is given as a result of speed choices on the journey legs, i.e. $H \equiv H^* = L_n^*$, and in P_∞ is it taken as $H \equiv H^* \rightarrow \infty$. Model $P_M(H)$, by explicitly accounting for the impact of H to determine H^* in conjunction with optimal leg speeds, can significantly outperform the more naive approaches of multiplying the optimal journey speeds obtained from either of the special cases.

6. Comments on literature

6.1. Models PK and R revisited

Because of the equivalence, most of the insights developed in Section 5 transfer to Model PK and Model R (for roundtrips). Yet there are a few distinctions we here further address.

The approximate optimisation function (25) for P_1 and the objective (23) of Model PK show great similarity. The interpretation we need to associate with the terms in α are, however, very different. In Model PK, these terms account for holding costs incurred by shippers. Although usually not charged to the charterer, they might be relevant to the charterer’s speed decision, as argued in Psaraftis and Kontovas (2014), in a context where competition for demand makes transport time a competitive service component. In P_1 , the opportunity cost rate is that of the charterer, and the opportunity costs are based on freight rate, not CIF values. In addition, the model considers the opportunity rewards from deferred fixed (un)loading costs and fuel costs for future legs, a component that is missing from Model PK, but should always be relevant.

As discussed, the opportunity cost components of P_1 and the holding costs of Model PK consider different aspects of a problem context. If service requires the consideration of holding costs for shippers, these should be added to the model *in addition to* the opportunity costs of deferred profits identified in P_1 . According to our understanding, the bulk cargo markets are not (yet) that sophisticated, and rather use the mechanism of changes to freight rates to implicitly reflect imbalances between ship availability and demand pressures. Model P_1 will then better reflect how speed decisions are made on trip charter journeys.

The optimisation function for P_∞ differs in structure from the function (24) of the R model. Theorem (IV) establishes that contributions of individual legs should be valued per unit of time over the whole roundtrip journey, and such a modification can be easily implemented in Model R. Further analysis in Ge (2021) shows that it is best to take $R = R_k - C_k^u - C_k^h$ in the Model R. When testing with these modifications only, we find that the so adapted R model will give speeds only marginally above those found from P_∞ , with AS values that remain typically within 1%.

It is worthwhile to restate that the equivalence analysis shows that Model R, by maximising the USD/day on a single (round-trip) journey, implicitly adopts the assumption that the *same journey* is infinitely repeated under *identical economic conditions*. This strong assumption makes model R (and P_∞) a more rough approximation of reality when compared to Model PK (and P_1). This difference between the models, however, has no bearing on the relative value of trip time charters versus long time charters to a ship charterer.

6.2. Selecting models based on contract type

We have demonstrated the importance of choosing the correct model in previous sections. In this section, we would like to summarise common types of contracts that can be modelled by our P_1 , $P_M(H)$, $P_M(H \rightarrow \infty)$ and P_∞ (and also Model PK and Model R according to the equivalence).

Table 9 lists the main relationship between objective type and contract length. When optimising speeds over a journey choosing between the criterion (USD/nm) or (USD/day) is crucial as they imply adopting very different underlying assumptions about the future use of the vessel. In the former, the decision maker has no interest at all in its future use, while in the latter, it is assumed to be repeatedly used for an infinite repetition of identical journeys under identical circumstances. This difference in assumptions is only implicitly present in Model PK and Model R, but made explicit in P_1 and P_∞ .

Table 10 lists a number of key model characteristics. In $P_M(H)$, the decision variables are the leg speeds V_n^- , and the number of journey repetitions M . The final contract horizon length then follows from: $H^* = M^* L_n(V_n^{*-}) \leq H$. In the special cases P_1 and P_∞ , leg speeds are the only decision variables, and there is no constraint on the horizon. The table also lists how the three models may be matched to other contract types found in Stopford (2009), and the corresponding decision maker. For example: Model P_∞ can also represent the situation in which the ship owner would not engage in a time charter party, but use the ship to attract business from shippers and plan its own (infinite series of) *voyage charters*. Then f^{TCH} represents the owner’s daily costs of crew, lubricants, supplies, etc. If, however, the owner has signed a contract for a time charter to *start* at a time H into the future, then model $P_M(H)$ is a better representation of the owner’s decision problem of how to use the ship up to that time H .

Table 10
Types of Contracts for Models.

Model	Decision variables	Contract horizon H^*	Possible contract types ⁽¹⁾	Decision maker
P_1	V_a^-	Not a decision variable but follows from solving P_1 , since $H^* = L_a(V_a^-)$, where L_a is a function of V_a^-	Trip Time Charter Spot Market or various Voyage Charters (before retiring the ship) ⁽²⁾	Charterer Owner
$P_M(H \rightarrow \infty)$	V_a^-	Not a decision variable but follows from solving $P_M(H \rightarrow \infty)$. M^* is given, and $H^* = M^* L_a(V_a^-)$	Any contract in which one has the power to freely decide its length	Any
$P_M(H)$	V_a^-, M	It is now part of the decision: $H^* = M^* L_a(V_a^-) \leq H$, where M^* is solved from $P_M(H)$	Time Charter Consecutive Voyage Charter Contract of Affreightment ⁽³⁾ Spot Market or various Voyage Charters (limited availability to H) ⁽⁴⁾	Charterer Charterer Charterer Owner
P_∞	V_a^-	Not a decision variable and we have infinite $H^* \rightarrow \infty$	Time Charter (long duration) Consecutive Voyage Charter (long duration) Bare Boat Charter Spot Market (voyage charters)	Charterer Charterer Charterer ⁽⁵⁾ Owner ⁽⁵⁾

(1) For definitions, see Stopford 2009 p. 176; (2) The time charter hire f^{TCH} in the models is replaced by the operations costs, see Stopford, 2009, p. 182; (3) When using only a single ship. (4) H : e.g. the time at which the owner is required by law to lay ship up for major overhaul/maintenance operations, or the time at which the owner is bound to deliver the ship to a charterer under an agreed time charter contract. (5) The last leg includes the cost (or revenue) to the owner of positioning the ship.

6.3. Impact on related literature

We have spent considerable effort in analysing the differences between the decision contexts of P_1 , P_∞ , and $P_M(H)$, and on the equivalence with Model PK and R. We believe that this helps us better understand the results found in previous literature, and determine avenues for further research. We give three examples to illustrate this point.

Fagerholt and Psaraftis (2015) study speed choices for a ship using a model that maximises daily profit over the (variable) duration of a journey. The underlying assumptions are therefore that of a model like P_∞ . They argue that the time in port can be left out as it is constant, and that their one-leg journey model can be adapted to account for roundtrips if the ship’s payload on all legs would be the same. From Table 7, we conclude that harbour times would indeed have no effect in models akin P_1 , but do affect speed choices in P_∞ . Further, Theorem 1, (IV)-(V), in Section 4.2 shows that erroneous speed decisions will be found in models that maximise daily profits of an individual leg.

As a second example, consider Corbett et al. (2009). The paper examines how speeds are to be found for a (variable) fleet of ships that must meet a certain annual demand rate for freight between origin and destination pairs. The approach consists of modelling, for each ship, the objective function as maximising the profits per trip from a particular origin to a particular destination. The number of trips possible within a year is then determined after the optimisation of this function. The optimisation in essence thus considers a model such as P_1 . We have seen in Section 4.2 the importance of accounting for the future use of a ship when it is to be used repeatedly, and thus a much better choice of model would be based on P_∞ , maximising the annual profitability (or USD/day) from each ship instead.

As a final example, consider Devanney (2010), who claims that all ship operators (either being the owner or as a charterer) will basically react in the same manner to changes in freight rates, independent of the particular charter contract type and its duration. The proof consists of developing two objective functions, one from the viewpoint of a ship owner operating in the spot market, the second from the viewpoint of a charterer. However, both models are based on maximising (USD/day) of a roundtrip journey, which implicitly assumes an infinite repetition of this activity. As indicated in Section 6.2, model P_∞ can indeed also be interpreted as the situation of a ship owner, operating the ship on voyage charters in the spot market. However, our analysis of model $P_M(H)$ in Section 5.4 leads us to a very different conclusion when the time remaining on the charter contract would be finite, as then the charterer would face a different optimisation problem than that of the owner, with different optimal strategies to react to an increase in freight rates.

7. Conclusions

Current deterministic models for economic ship speed optimisation in the literature have not considered the time charter contract, and in particular the influence of the redelivery time. The charterer’s objective is to maximise the Net Present Value (NPV) of a cash-flow function of the ship’s activities over a relevant future horizon H , where H can be interpreted as any possible day within the redelivery time window as specified in the time charter clause. We develop a general time charter contract model $P_M(H)$, and three special cases P_1 , P_∞ and $P_M(H \rightarrow \infty)$, each model mapping onto different contractual contexts (Table 10), and present algorithms for finding optimal ship speeds for any journey structure.

In this paper we have also established equivalence results between our models and those from the literature, and this is summarised in Table 11. We show in Sections 3.1 and 4.1 that the three special cases P_1 , $P_M(H \rightarrow \infty)$ and P_∞ are equivalent to particular NPV models in Ge et al. (2021) that do not explicitly consider the contract type. Theorem 1 in Section 4.2 maps the models PK and R of classic speed optimisation literature to P_1 and P_∞ , respectively. Through the analysis of this equivalence, in Section 6.1, the insights developed in Section 5 can transfer to Model PK and Model R (for roundtrips). It therefore improves our understanding of the applicability of the classic models by also mapping these models to specific contract types in Section 6.2. In Section 6.3, we further show that how these findings can help us better understand the results found in previous literature. As observed from Table 11, model $P_M(H)$ has no equivalent model. Optimal leg speeds of a ship working for a profit maximising charterer are different from situations in which the relevant time horizon considered is not pre-specified.

Table 11
Equivalence table of models.

This paper	Ge et al. (2021)	Classic models
P_1	$\mathcal{P}(1, n, 0)$	Model PK
$P_M(H \rightarrow \infty)$	$\mathcal{P}(M, n, 0)^a$	–
P_∞	$\mathcal{P}(\infty, n, 0)$	Model R ^b
$P_M(H)$	–	–
–	$\mathcal{P}(M, n, G_o)$	–

^a $P_M(H \rightarrow \infty)$ provides approximations of the averaged speeds on laden and ballast legs, respectively, in $\mathcal{P}(M, n, 0)$.

^bOnly valid for Model R of roundtrips without decomposition.

When a ship is hired on a time charter, the charterer's wish is to let it perform as much profit generating work as possible during the time the ship is used, up to the point of actual redelivery. We can examine these characteristics, which should also play their role for journeys of any structure, with or without repetitions, through the consideration of a repeated laden–ballast journey structure in which, next to legs speeds, also the number of repetitions and the actual redelivery date (as available within the contract) are optimised. All these features are present in the general model $P_M(H)$, but some of these features are fixed or absent in the three special cases. This explains why the structure of optimal solutions, and their sensitivity to economic and ship-specific parameters, in each of these models differ so much, as discussed in Section 5.

Using the general model $P_M(H)$, we visualise the optimal solutions via the *speed menu curve*, a saw-tooth pattern which plots optimal solutions to $P_M(H)$ as a function of available time left on the charter contract H . The examples in Section 3.2 illustrate how the charterer will want to choose the best possible speeds from these speed menus so as to realise the preferred number of journeys given the trade-off between profitability and risk within an allowable horizon. The *speed menu curve* demonstrates that the optimal speed range depends on horizon length: when contracts of longer duration are considered, these saw-tooth patterns slowly converge to particular values for laden leg and ballast leg speeds, respectively, in a way that is not identified in any classic deterministic speed optimisation models that have not considered the time charter contract. The *economic principle* from maritime economics theory that ship speed is determined by a nonlinear function of the ratio of freight rate to fuel price is therefore more nuanced when ships travel under a time charter contract: it seems that the shorter the remaining available time on the contract, the less the impact of freight rate (see Section 5.3).

The use of the $P_M(H)$ model was further discussed in Section 3.3. We considered three cases: (1) the valid range of H represents the available time remaining according to the redelivery clause; (2) reaction to exogenous changes; (3) impact of speed limitations. From these examples we see that ship speed is not only determined by the ship's characteristics, the journey characteristics, and the economic parameters, but also by the time charter contract. For example, we showed how maximising profits may imply that some ships will not speed up when freight rates increase. It is also concluded that identical ships may want to travel at different economic speeds. This is not in contradiction to the mixed signals found in the empirical data study of Adland and Jia (2018) and the empirical findings of Prakash et al. (2016).

Restricting ourselves to static conditions in this paper allows us to make direct comparisons with classic deterministic models from the literature. The NPV models developed may form the basis for further research into various areas, including: the consideration of cargo load decisions (Adland et al., 2018); redelivery port selection or contract extensions (Adland et al., 2020); impact of underperformance quotation by ship owner on the actual value of time charter contracts (Veenstra and van Dalen, 2011); alternative or refined treatment in the models of how a ship's power setting translates into fuel consumption and travel time (Meng et al., 2016); and the interactions with various financial instruments and strategies in stochastic settings. The use of such models may also help in the assessment of how changes in the economic and regulatory frameworks may impact ship design and contract selection.

CRediT authorship contribution statement

Patrick Beullens: Conceptualisation, Methodology, Software, Formal analysis, Investigation, Writing – original draft, Writing – review & editing, Supervision, Funding acquisition. **Fangsheng Ge:** Conceptualisation, Methodology, Software, Formal analysis, Investigation, Writing – original draft, Visualisation, Writing – review & editing. **Dominic Hudson:** Conceptualisation, Writing – review & editing, Supervision, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Data of the base case example

Ship characteristics: 157,800 tonne dwt capacity (scantling) and 145,900 tonne dwt (design); $A = 49,000$ tonne lightweight; $k = 3.910^{-6}$, $p = 381$, $g = 3.1$ and $a = 2/3$, corresponding to 60.5 tonne bunker fuel at nominal speed 15.2 kn. Auxiliary fuel (in ports) at 5 tonne/day. Max. speed $v^+ = 17$ kn and min. speed $v^- = 10$ kn. Cargo discharge at 3000 m³/h. Ballast stability condition: minimum fill-rate of vessel at 30% of design dwt (ballast tank capacity 54,500 m³). **Journey characteristics:** Distance between the two ports is 8,293 nm (symmetric). Waiting time (prior to unloading) 24 h in first port, 48 h in second port; loading rates 3000 m³ per hour in both ports. **Demand characteristics:** parcel of 1,200,000 barrels of light crude oil at 1.07 m³/tonne, and 1 barrel equals 0.136 m³. **Revenue structure:** Freight rate at 0,50 USD per barrel and 1,000 nm. Revenue received 12 h before start of unloading at destination port. **Cost structure:** Total fixed port costs of a journey towards the first port equals 300,000 USD, while being 500,000 USD for a journey towards the second port. Unloading and loading charges are at a rate of 4000 USD/hour in both ports. Main bunker fuel at 63 USD/barrel and auxiliary fuel at 590 USD/tonne in both ports. Port and fuel costs due within 72 h after leaving port. **Charter party data:** Zero forward start, and TCH at 20,000 USD/day. **Opportunity cost of capital:** $\alpha = 0.08$ per year.

Appendix B. Proof of Theorem 1

Proof (I). The NPV of p1 is:

$$NPV(x, \alpha) = \frac{a(x)}{\alpha}(1 - e^{-\alpha\delta(x)}) = \frac{a(x)}{\alpha}\left(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta(x))^k}{k!}\right). \tag{B.1}$$

With \overline{NPV} representing the linear approximation of NPV, we get:

$$\overline{NPV}(x, \alpha) = \frac{a(x)}{\alpha}(1 - (1 - \alpha\delta(x))) = a(x)\delta(x). \tag{B.2}$$

(II) The NPV of p2 is:

$$NPV(x, \alpha) = \frac{a_1}{\alpha}(1 - e^{-\alpha\delta_1}) + \frac{a_2}{\alpha}(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1} \tag{B.3}$$

$$= \frac{a_1}{\alpha}\left(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta_1)^k}{k!}\right) + \frac{a_2}{\alpha}\left(1 - \sum_{k=0}^{\infty} \frac{(-\alpha\delta_2)^k}{k!}\right) \sum_{k=0}^{\infty} \frac{(-\alpha\delta_1)^k}{k!}. \tag{B.4}$$

Algebraic manipulation shows that:

$$\overline{NPV}(x, \alpha) = a_1(x_1)\delta_1(x_1) + a_2(x_2)\delta_2(x_2) + \alpha a_2(x_2)\delta_2(x_2)\left(\delta_1(x_1) - \frac{\delta_2(x_2)}{2}\right) \tag{B.5}$$

$$\neq a_1(x_1)\delta_1(x_1) + a_2(x_2)\delta_2(x_2), \text{ unless } \alpha = 0 \text{ or } \delta_2(x_2) = 2\delta_1(x_1). \tag{B.6}$$

(III) The NPV of p1 is: $NPV(a, \alpha) = \frac{a}{\alpha}(1 - e^{-a\delta})$. Therefore, if $A = NPV(a, \alpha)$:

$$a = NPV(a, \alpha) \frac{\alpha}{1 - e^{-a\delta}} = NPV(a, \alpha) \sum_{i=0}^{\infty} \alpha e^{-ia\delta} = \alpha NPV(A, \alpha) = AS(A, \alpha). \tag{B.7}$$

Because both projects have the same $\delta(x)$ ($x \in X$), maximising a of p1 is thus equal to maximising the AS of p3.

(IV) The AS of p4, given A_1 and A_2 being the NPV of a_1 and a_2 , is:

$$AS = \left[\frac{a_1}{\alpha}(1 - e^{-\alpha\delta_1}) + \frac{a_2}{\alpha}(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1} \right] \frac{\alpha}{1 - e^{-\alpha(\delta_1+\delta_2)}} = \frac{a_1(1 - e^{-\alpha\delta_1})}{1 - e^{-\alpha(\delta_1+\delta_2)}} + \frac{a_2(1 - e^{-\alpha\delta_2})e^{-\alpha\delta_1}}{1 - e^{-\alpha(\delta_1+\delta_2)}}. \tag{B.8}$$

Note that:

$$\frac{\alpha}{1 - e^{-\alpha X}} = \frac{1}{X} + \frac{\alpha}{2} + \frac{\alpha^2 X}{12} + O(\alpha^3).$$

Therefore, algebraic manipulation shows that:

$$\overline{AS} = \frac{1}{(\delta_1 + \delta_2)} \left[a_1\delta_1 + a_2\delta_2 + \frac{\alpha}{2}(a_1(\delta_1)^2 + a_2(\delta_2)^2) - \alpha a_2\delta_1\delta_2 \right] + \frac{\alpha}{2}(a_1\delta_1 + a_2\delta_2) \tag{B.9}$$

$$\neq \frac{a_1\delta_1 + a_2\delta_2}{\delta_1 + \delta_2}, \text{ unless } \alpha = 0. \tag{B.10}$$

(V) (a) follows immediately from the proof of part (IV). For (b) observe from proof (IV) that:

$$\lim_{\alpha \rightarrow 0} \overline{AS} = \frac{a_1\delta_1 + a_2\delta_2}{\delta_1 + \delta_2} = \frac{a_1\delta_1}{\delta_1 + \delta_2} + \frac{a_2\delta_2}{\delta_1 + \delta_2} \neq \frac{1}{2}(a_1 + a_2), \text{ unless } \delta_1 = \delta_2. \tag{B.11}$$

The contributions of a_1 and a_2 need to be weighted by their relative duration to the total journey duration. An objective function $a_1 + a_2$ places too much emphasis on the process with the shortest relative duration. This completes the proof. \square

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