Information generation in vertically differentiated markets*

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Abstract

In a model of vertical competition, two firms draw costly public signals that are informative about the quality of their products and then competitively set prices. When each firm generates information independently from the other, there will be overinvestment (underinvestment) in information generation if the market share of the quality follower in the subsequent market equilibrium is high (low). Moreover, information generation by one firm has a positive externality on the other firm. Hence, coordination (e.g. via industry associations) increases information generation. When product qualities are endogenous, information generation may prevent quality degradation and thus have an additional social benefit.

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1 Introduction

Do competing sellers generate the efficient level of information about their goods? We contribute to answering this fundamental question by examining the case of a competitive market with vertically differentiated products. We do so by considering a canonical model of price competition with vertically differentiated products (as in Shaked and Sutton, 1982), in which consumers and firms have identical beliefs regarding the products qualities. Our innovation is to allow each firm, before setting its price, to generate a costly, unbiased signal, informative about the quality of the products available on the market, observed publicly by all consumers and all firms.

For example, a product's technical specifications may be perfectly known to all consumers and all firms. However, the consumption utility generated by the product may depend on harder-to-measure attributes such as its aesthetic appeal, its ergonomics and ease of use, the presence of unexpected bugs or defects, both in absolute terms and relative to other products. These attributes can be (partially) learned via informative public signals: industry competitions, trade shows, industry classifications, reviews by experts in the media (examples are *Consumer Reports* in the US or *Which?* in the UK), quality tests and certification by professional agencies (e.g., rating agencies for financial products, TÜV for industrial goods). These signals are often commissioned by firms, either individually or via industry bodies tasked with organizing industry competitions or maintaining classification systems.[1]

As a preliminary step we examine the pricing equilibrium in Shaked and Sutton (1982)'s model. We show that under mild assumptions (separable consumer preferences in taste and quality, log concavity of the distribution of the taste parameter and sufficiently high minimum taste for quality ensuring the market is covered) the equilibrium market shares are independent of the distribution of qualities, and are determined exclusively by the taste

[1] A case in point are wine classification systems, usually maintained by associations of vintners. See, for example, the Bordeaux wine classification of 1855, and its more recent and regularly updated offshoot Cru Bourgeois. Similar systems in Burgundy, Champagne, Douro, and other regions. While observable variables such as soil quality of the vineyard and the weather of the vintage determine expected qualities, the true quality of a fine wine often only realizes after years of storage.

distribution. This is a convenient result, as it implies that the arrival of new information affecting the quality distribution does not change the equilibrium market shares.

We then turn to the benefit of generating information, in the form of generating unbiased signals about the firms' product qualities. Both from the private and the social viewpoint new information is valuable if and only if it is possible to reverse the quality ranking and thus alter equilibrium consumption choices. However, its private value will, in general, differ from its social value. The social value of new information depends on the difference between the aggregate valuation of the quality leader's customers and the aggregate valuation of the quality follower's customers. This difference measures the change in social welfare when the allocation of goods to consumers changes.

By contrast, the private benefit from new information depends on the market share of the quality follower. The reason is that, after drawing a signal, either firm may become the quality follower or the quality leader. This implies that each firms' private benefit of information generation increases with *both* firms' profits. Moreover, under the market segmentation implied by log-concavity, in equilibrium the market share of the quality follower is always smaller than that of the quality leader, but the closer these shares are the higher are aggregate profits. Hence, all else equal, firms' market power and aggregate profits increase in the market share of the quality follower, and with it the private incentive to generate information.

Because the market share of the quality follower is endogenous, comparing social and private value of information generation is no trivial task. Nonetheless, we can show that if the mass of consumers close to the taste distribution's lower bound is sufficiently large, the quality follower's market share will be close to zero. In this case the social benefit of information generation exceeds the private benefit and firms underinvest in information relative to the social optimum.[2] This underinvestment is most severe when

[2]Interestingly, in the underinvestment case, consumers may benefit from generating additional information. This paper focuses on firm behavior, see Terstiege and Wasser (2019) for optimal information generation from the consumers' perspective.

the quality leader captures the entire market and effectively becomes a monopolist.[3]

We provide comparative statics results linking the taste distribution to information generation. We show that a change of the taste distribution that preserves it locally around the threshold consumer (i.e. the consumer who is indifferent between purchasing from either firm) only affects the social benefits of information generation, but leaves the private benefit and the equilibrium unchanged. For instance, a right tail spread of the taste distribution, affecting only tastes above the median, increases the social value of information, because the difference in aggregate valuations of consumers of the two firms increases. Such a spread thus makes under-provision of information more likely. Using a similar method, but focusing on the left tail of the taste distribution, we can link inefficiencies in the pricing equilibrium to inefficiencies in information generation. We identify a family of taste distributions for which under-investment (over-investment) in information generation is more likely when the taste distribution generates less (more) inefficiencies in the pricing equilibrium.

Interestingly, public information generates a positive externality among firms, because drawing *any* signal increases a firm's expected profits, including signals about the competing product. Hence, even if prices are set competitively, firms can soften competition by cooperating in generating unbiased, publicly available information about product quality, for example, by introducing a classification system or an industry-wide competition. Such coordination may decrease consumer rent and aggregate surplus in our setting, providing a novel perspective on the regulation of industry cooperation.

Finally we allow firms to achieve vertical differentiation also via costless quality degradation. It is well known that, absent information generation, in equilibrium firms will increase the expected quality distance in the product market by way of quality degradation. We show that the possibility of generating public information mitigates strategic quality degradation, because new information provides an alternative means to generate quality dispersion.

[3]We use the term *monopoly* as in Baumol (1982)'s *contestable monopoly*: there are two competing firms and, in equilibrium, a single one serves the entire market.

The remainder of the paper proceeds as follows. The next subsection discusses the relevant literature. In Section 2 we present the model. In Section 3 we derive the equilibrium in the pricing game for given expected qualities. In Section 4 we solve the full game, in which firms can invest to generate information before setting prices. Section 5 presents an extension with quality degradation. All mathematical derivations missing from the text are in the appendix.

Related literature.

We contribute to the literature studying information generation in monopoly settings and in competitive settings. We also relate our results to the literature on information disclosure.

Public information generation in single-firm settings. Information generation by a single seller has been extensively studied. As we consider heterogeneous buyers, our paper is closely related to the auctions literature. In their seminal paper Milgrom and Weber (1982) study the auctioneer's incentives to disclose signals affiliated with bidders' valuations, which intuitively are those generating a symmetric reaction in the bidders' valuations. They show that the auctioneers should always commit to fully and publicly disclose those signals. Ganuza and Penalva (2010) consider the auctioneer's incentive to generate private idiosyncratic signals that affect different bidders asymmetrically. They show that the amount of information generated by the auctioneer will fall short of the social optimum, because information increases the dispersion in buyers' valuations and information rents.[4]

[4]Also related is Ottaviani and Prat (2001), who consider an environment with a monopolist and a *single* buyer. They also find that a monopolist always benefits from generating and disclosing signals that are affiliated to the buyer's valuation. Roesler and Szentes (2017) consider a monopolist and a single buyer, and identify the optimal information environment from the buyer's point of the view. There is also a literature studying the incentive to acquire *private* information. For example, Bergemann and Välimäki (2002) consider a mechanism design problem in which players can covertly obtain private information at a cost. They show that if the mechanism is ex-post efficient (for example, an auction), then information acquisition is also efficient.

In our setup buyers are heterogeneous and signals are affiliated with the consumers' valuations, too, but while monopoly may be an equilibrium outcome, the *identity* of the monopolist may depend on the realizations of the information generated. Thus we provide an additional reason why a monopolist may invest in information generation, which complements those already proposed in the literature. We also show that the equilibrium level of information generation crucially depends on whether sellers generate information independently from each other or coordinate (in particular, it will be higher when firms can coordinate). As a consequence, despite the fact that signals are affiliated with consumers' valuations, in our model information is under-provided if there is a monopolist and firms act independently.

Information generation in competitive settings. The existing literature studying information generation in competitive settings has mostly focused on horizontal competition (see, in particular, Anderson and Renault, 2000, 2009, Levin, Peck, and Ye, 2009). As in our setup, under horizontal competition increasing the distance between quality levels via information generation increases firms' market power. However, the welfare implications differ markedly between horizontal and vertical competition.[5]

Both Moscarini and Ottaviani (2001) and Armstrong and Zhou (2019) study the problem of information generation by firms when the signal generated is *privately* observed by consumers. Armstrong and Zhou (2019) consider a model of horizontal competition, derive both firms' and consumers' optimal information structures, and compare them with the efficient information structure. They find that the firms' optimal information structure is socially efficient. By contrast, with public signals we find that firms may generate more or less information than efficient, depending on the distribution of the taste parameter and whether they act independently or jointly.[6]

[5]There is, of course, the classical paradox that by construction fully revealing market equilibrium prices will not provide incentives for costly information generation (Grossman and Stiglitz, 1980), which has been resolved by allowing agents to take into account the effect of their actions on prices and other agents' beliefs (Milgrom, 1981; Verrecchia, 1982).

[6]Armstrong and Zhou (2019) consider a single consumer and firms' joint decision on information structure. By contrast in our setup the average consumer may have different valuation for quality than the marginal consumer, which will matter because the social

The small literature on public information generation with vertical differentiation tends to rely on very specific informational environments. E.g. Bouton and Kirchsteiger (2015) examine the role of reliable rankings of sellers and show that their presence can reduce consumers' welfare. Bergemann and Välimäki (2000) consider a dynamic setting, in which information is generated through repeated purchases, and find that information generation increases firms' market power and may reduce social welfare. By contrast we consider a generic form of information generation (i.e., any unbiased signal correlated with the quality distribution) and examine incentives for over- or underinvestment in information generation and the role of coordination.

Information disclosure. A related problem is that of a privately informed firm deciding how much information to disclose to consumers. Jovanovic (1982) studies costly disclosure of information by a monopolist, and shows that a monopolist will disclose too much information to consumers relative to the social optimum. Also, Matthews and Postlewaite (1985) consider the problem of costly information generation and subsequent (costless) disclosure by a monopolist. They show that mandatory disclosure rules may decrease the amount of information generated by the monopolist. Lewis and Sappington (1994) and Johnson and Myatt (2006) study the disclosure of information by a monopolist who know the true quality of its good, and can choose the precision of a signal observed by consumers (but not its realization). In this case, information disclosure leads to a demand rotation. Typically, the monopolist wants to disclose either everything or nothing.

With respect to information disclosure in competitive settings, to the best of our knowledge, Meurer and Stahl (1994) are the first to point out that information disclosure by one firm generates a positive externality on competing firms. However, they only consider horizontal competition and a very specific information structure (i.e., informative advertising).[7] Ivanov (2013)

value of information will depend on the average consumer valuation, but its private value on the marginal consumer's valuation.

[7]See also Vives (1999), chapter 8, discussing incentives of firms to share (but not generate) private information in different models of oligopolistic competition. Related to our findings, both Meurer and Stahl (1994) and Vives (1999) argue that firms may benefit

and Boleslavsky et al. (2019), also study competitive information disclosure about horizontally differentiated goods. Finally, Board (2009) studies information disclosure by firms under vertical competition. In his model, there is always an equilibrium with full disclosure. Next to this equilibrium there could also be an equilibrium with partial disclosure.[8]

2 Model

Our starting point is the canonical model of a duopoly with vertically differentiated products (see Gabszewicz and Thisse, 1979, Shaked and Sutton, 1982, and Chapter 7 of Tirole, 1988's textbook). The market consists of 2 firms and a mass 1 of buyers. Each firm produces a good of quality $s_i \in [\underline{s}, \overline{s}]$ for $i \in \{1, 2\}$. A buyer's utility is given by

$$U = \begin{cases} \theta s_i - p_i & \text{if good } i \text{ is purchased} \\ 0 & \text{in case of no purchase,} \end{cases}$$

where p_i is the price of the good produced and $\theta \in \mathbb{R}_+$ is an i.i.d. taste parameter with cumulative distribution function $F(x) = \text{pr}(\theta \leq x)$ that is continuous, differentiable, and has a continuous first derivative. We assume that the support of θ has a minimum $\underline{\theta}$ (so that $F(x) = 0$ for all $x \leq \underline{\theta}$), but may or may not have a maximum. If a maximum exists, we call it $\overline{\theta} > \underline{\theta}$, otherwise we write $\overline{\theta} = \infty$.

Each firm has zero marginal cost of production, so that profit is given by price times quantity sold.

Information and Learning

We depart from the canonical model by assuming that the quality levels s_i are unknown to both buyers and firms, who have common ex-ante beliefs

from organizing a trade association to gather information from their members and then share it.

[8]It turns out that, unlike in Board (2009), in our model a standard unraveling argument holds: if firms first generate information and then decide whether to disclose, in equilibrium they always disclose fully (results available upon request).

about s_i. Call $q_i = E[s_i]$ the initial expected quality of firm i's product, and assume, without loss of generality, that $q_1 \geq q_2$.Firm i can generate information by paying a cost k and drawing a signal σ_i, which is informative with respect to s_i and may be informative with respect to s_{-i} as well.[9]

We allow the two firms to differ in their information-generation technology, so that σ_1 and σ_2 may have different distributions. We also allow for any possible correlation between σ_1 and σ_2, except for perfect correlation (negative or positive) to avoid trivial cases. Information generated is public: all market participants receive the signal and update their belief about quality. We adopt the convention that $\sigma_i = \emptyset$, if firm i does not generate information. We therefore write $\sigma = (\emptyset, \emptyset)$ if no firm generates information, $\sigma = (\emptyset, \sigma_i)$ when firm $i \in \{1, 2\}$ generates information but not firm $-i$, and $\sigma = (\sigma_1, \sigma_2)$ when both firms generate information.

Starting from a common prior over each firm's quality s_i and a specific distribution of signals, each realization of σ corresponds to an *ex-post expected quality* which we denote by $\hat{q}_i$. By iterating expectations $E\left[\hat{q}_i|\sigma\right] = q_i$ for any signal configuration σ (where the expectation is taken over the possible realizations of σ). Hence, ex ante, before any signal is drawn, the expected ex-post quality is equal to the initial expected quality. Finally, since consumption choices are based on expected qualities after signals are realized, we can simply assume that each σ gives rise to an exogenous distribution of $\hat{q}_i$ with $E\left[\hat{q}_i|\sigma\right] = q_i$ (i.e., we do not need to derive explicitly the distribution of $\hat{q}_i$ from the prior and the distribution of signals).

Timing

To summarize, the timing of the game is as follows.

[9] We abstract away from the choice of precision of the signal (as in the Bayesian persuasion literature, see in particular Gentzkow and Kamenica, 2016) as well as from the possibility of signal jamming. We will show below that both firms' expected profits increase in the precision of both signals. Hence, firms have no incentive to jam each other's signal, and, for given cost of drawing a signal, firms prefer the most precise signal available. Hence, our results carry over to such a case. However, if signals of different precision differ in their cost, firms will face a trade off. This trade off depends crucially on the details of the cost function, and we prefer to leave this extension for future research.

1. Given the initial beliefs about qualities, firms simultaneously decide whether to acquire information at cost k, yielding a vector of signals σ.

2. Realizations of signals are publicly revealed, leading to a revision of the beliefs about the products' qualities and to $\hat{q}_1, \hat{q}_2$.

3. Firms announce prices simultaneously. Consumers decide if and from whom to buy and consume. Payoffs are realized.

Solution Concept

We solve the model by backward induction. For a given signal realization, firms simultaneously and independently choose a price. Anticipating this in the previous stage firms simultaneously and independently choose whether to draw signals. We thus derive the subgame perfect Nash equilibrium of signal choices σ_1 and σ_2 and price choices p_1 and p_2 depending on the signals.

Assumptions

We conclude the model description by introducing some restrictions on the distribution of the taste parameter θ, which will guarantee the existence and uniqueness of a pure strategy Nash equilibrium in the pricing game (stage 3 in the timeline above).

Assumption 1 (Log-concavity)**.** *The density $f(\theta)$ is log-concave.*

This assumption comes with only a very modest loss of generality, as log-concavity is satisfied by a host of widely used distributions. Nonetheless, it puts some useful structure on $F(\theta)$ and $f(\theta)$. For example, log concavity implies that $f(\theta)$ is unimodal (see e.g. Dharmadhikari and Joag-Dev, 1988). Hence, $f(\theta)$ is strictly positive for $\theta \in (\underline{\theta}, \overline{\theta})$ (remember that if there is no upper bound, we write $\overline{\theta} = \infty$). Furthermore, log concavity of $f(\theta)$ ensures that both $F(\theta)$ and $1 - F(\theta)$ are log-concave (see Prékopa, 1973 and Bagnoli and Bergstrom, 2005). This in turn implies that $F(\theta)/f(\theta)$ increases, $(1-F(\theta))/f(\theta)$ decreases, and $(1-2F(\theta))/f(\theta)$ also decreases, all facts that we will use extensively in our derivations.

Finally, we assume that there is enough potential revenue in the left tail of the taste distribution, in the sense that $\underline{\theta}$ is sufficiently high and the taste distribution has enough mass at or near $\underline{\theta}$.

Assumption 2 (Covered Market)**.** *Either* $\underline{\theta} \cdot f(\underline{\theta}) > 1$*, or*

$$\underline{\theta} \cdot m \geq \frac{\overline{s} - \underline{s}}{\overline{s}}, \tag{A2}$$

where $m \equiv \min_{\theta \in [\underline{\theta}, \theta^*]} f(\theta)$ *and* θ^* *is implicitly defined as* $\theta^* = \frac{1-F(\theta^*)}{f(\theta^*)}$.

Note that, because of log-concavity, $\frac{1-F(\theta)}{f(\theta)}$ is strictly decreasing and hence θ^* exists and is unique as long as $f(\underline{\theta})\underline{\theta} \leq 1$.

Condition (A2) is a generalization of the standard *covered market* condition.[10] As we will show, it guarantees that in equilibrium all consumers prefer purchasing from one of the firms to not purchasing.[11] Indeed, any distribution that is bounded below with $f(\underline{\theta}) > 0$ satisfies (A2), if appropriately scaled up. This is because increasing $\underline{\theta}$ decreases $\theta^* - \underline{\theta}$ and, therefore, (weakly) increases m. If the new $\underline{\theta}$ is sufficiently large relative to the maximum possible dispersion in quality $\overline{s} - \underline{s}$, then Condition (A2) will hold. Furthermore, because a truncation of a log-concave distribution is also log-concave (Bagnoli and Bergstrom, 2005, Theorem 7), any log-concave distribution that is unbounded below or bounded below but with mass equal to zero at the lower bound satisfies Assumptions 1 and 2, if appropriately truncated.

[10] For example, Chapter 7 of Tirole (1988)'s textbook uses a uniform distribution of the taste parameter with $\overline{\theta} - \underline{\theta} = 1$, solving the model assuming the covered market condition $\frac{|\hat{q}_1 - \hat{q}_2|}{\max\{\hat{q}_1, \hat{q}_2\}} \leq \underline{\theta}$. Condition (A2) is a generalization of this condition, because it applies to all possible distributions of the taste parameter, and to all possible quality levels (in Tirole, 1988 the quality levels are given exogenously).

[11] To the best of our knowledge, (1)-(2) are the weakest conditions existing in the literature guaranteeing existence, uniqueness and full analytical characterization of the pricing equilibrium with covered market. Studies examining the non covered-market case (Moorthy, 1988, Choi and Shin, 1992) or not imposing ex-ante whether the market will be covered (Wauthy, 1996) restrict their attention to uniform taste distributions.

3 The Pricing Game

Consider a given realization of the signal vector σ. Since information generation may reverse the initial quality ranking of firms, we will refer to the quality leader by L and the follower by F, so that $\hat{q}_L \equiv \max\{\hat{q}_1, \hat{q}_2\} > \hat{q}_F \equiv \min\{\hat{q}_1, \hat{q}_2\}$.

Denote a firm i's posted price by p_i. We introduce two thresholds. The first threshold X is the consumer type that is indifferent between purchasing from either firm, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer L (F):

$$X \equiv \begin{cases} \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \geq \overline{\theta}. \end{cases}$$

The second threshold Y is the consumer type that is indifferent between the lower quality firm F and not consuming, if there is such a consumer, and by $\underline{\theta}$ ($\overline{\theta}$) if all consumers weakly prefer F (not to consume):

$$Y \equiv \begin{cases} \frac{p_F}{\hat{q}_F} & \text{if } \frac{p_F}{\hat{q}_F} \in (\underline{\theta}, \overline{\theta}) \\ \underline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \leq \underline{\theta} \\ \overline{\theta} & \text{if } \frac{p_F}{\hat{q}_F} \geq \overline{\theta}. \end{cases}$$

Using these thresholds we can derive the two best responses in the pricing game:

Lemma 1. *The quality leader's best response is implicitly defined as:*

$$p_L(p_F) = \max\left\{\frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\},$$

and is a continuous function. The quality follower's best response is implicitly *defined as:*

$$p_F(p_L) = \begin{cases} [0, +\infty) & \textit{if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) \\ \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} & \textit{otherwise.} \end{cases}$$

and is an upper-hemicontinuous, compact valued, convex correspondence.

The proof of the Lemma uses Assumption 1 to establish existence and uniqueness of the best responses. In addition, Assumption 2 implies that the market is covered: for every p^L the follower's optimal price is such that $Y = \underline{\theta}$.

Thus the demand faced by the quality leader is $1 - F(X)$ and the demand faced by the quality follower is $F(X)$. Note that, if both firms' profit maximization problems have interior solutions, optimal prices are given by:

$$p_L(p_F) = \frac{1 - F(X)}{f(X)}(\hat{q}_L - \hat{q}_F) \text{ and } p_F(p_L) = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F).$$

These expressions have a straightforward intuition: firms' market power under vertical differentiation allows them to set prices to ensure that their own price elasticity of demand equals -1.

A pure strategy Nash equilibrium is a pair p^* such that $p_i(p_{-i}(p_i^*)) = p_i^*$ for $i = L, F$.

Proposition 1 (Market Equilibrium Outcome)**.**

(i) If $1 \leq \underline{\theta} \cdot f(\underline{\theta})$*, then in the unique pure strategy Nash equilibrium* $X^* = Y^* = \underline{\theta}$*, i.e. the quality leader supplies the entire market, and prices are* $p_F^* = 0$ *and* $p_L^* = \underline{\theta}(\hat{q}_L - \hat{q}_F)$.

(ii) If instead $1 > \underline{\theta} \cdot f(\underline{\theta})$*, then in the unique pure strategy Nash equilibrium* $Y^* = \underline{\theta}$ *and* $X^* > \underline{\theta}$ *where*

$$X^* = \frac{1 - 2F(X^*)}{f(X^*)} \tag{1}$$

Equilibrium prices are $p_L^* = \frac{1-F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$ *and* $p_F^* = \frac{F(X^*)}{f(X^*)}(\hat{q}_L - \hat{q}_F)$.

For intuition, note that $\underline{\theta} \cdot f(\underline{\theta})$ is a measure of the aggregate willingness to pay at the bottom of the taste distribution. If it is sufficiently high the quality leader will find it profitable to serve the entire market, leading to a monopoly. If instead it is low (i.e., $\underline{\theta} \cdot f(\underline{\theta}) < 1$) then the quality leader will optimally serve only the high-valuation consumers and the remaining consumers purchase from the quality follower, leading to a duopoly.

In case of a duopoly, by (1) the threashold cutoff X^*, separating consumers buying from L from those buying from F, cannot be greater than the median. This is a consequence of log concavity of the taste distribution, which limits the degree to which the distribution can grow. Also, the equilibrium demand faced by leader and follower does *not* depend on expected qualities $\hat{q}_L$ and $\hat{q}_F$. This is because the firms' optimal prices ensure their own price demand elasticity equals -1. Since these elasticities are linear in $\hat{q}_L - \hat{q}_F$, the threshold X^* at which they both equal -1 does not depend on $\hat{q}_L - \hat{q}_F$.[12] This fact will be very convenient, implying that the signal realizations and thus also the signal configurations, only affect the identity of quality leader and follower and market prices, but not demand.

For illustration of Proposition 1 suppose the taste parameter θ follows a uniform distribution. In this case, the quality leader captures the market if $\overline{\theta} \leq 2\underline{\theta}$, otherwise there is a duopoly with $X^* = \frac{\underline{\theta}+\overline{\theta}}{3}$. Which case will occur depends mainly on two intuitive effects. First, fixing either $\overline{\theta}$ or $\underline{\theta}$, the duopoly becomes more likely as $\overline{\theta}-\underline{\theta}$ increases (and with it the variance of the distribution). This is because lowering the price to attract the least quality sensitive consumers will become more costly for the quality leader, leaving demand for the follower. By contrast, for a given range of the support $\overline{\theta}-\underline{\theta}$ an increase in the mean makes a duopoly less likely, as the least quality-sensitive consumer becomes more quality sensitive and accepts a higher relative price.

[12] Note that this result relies on separability of taste and quality in consumers' utility and a covered market, but does not require log concavity, which instead guarantees existence and uniqueness of X^*.

4 Information generation

Equipped with the properties of the pricing equilibrium we turn now to the firms' choices of information generation. Depending on the type distribution, either the quality leader will corner the market (monopoly case) or both firms will supply some consumers. Below we consider each case separately.

4.1 Case 1: Monopoly ($1 \leq \underline{\theta} f(\underline{\theta})$)

We start by considering the case $1 \leq \underline{\theta} f(\underline{\theta})$, in which the quality leader covers the entire market. Since all consumers consume the good with higher expected quality, the pricing equilibrium is efficient.

Social value of information generation. Given the expected qualities $\hat{q}_1$ and $\hat{q}_2$ the expected social welfare is given by:[13]

$$S(\hat{q}_1, \hat{q}_2) = \max\{\hat{q}_1, \hat{q}_2\} E[\theta] \, .$$

By the law of iterated expectation, the two expected qualities are independent from σ. This implies that if new information cannot reverse the quality ranking (i.e. if firm 1 remains the leader for any signal realization), then the expected social welfare is the same with or without information generation. If instead new information can reverse the quality ranking, then a straightforward application of Jensen's inequality implies social welfare is higher with new information. The next lemma derives the exact expression for the social value of information.

Lemma 2. *For any two signal configurations σ' and σ'', the social benefit generated from moving from σ' to σ'' is given by*

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = E[\theta] \left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2) \right),$$

[13] We assume throughout the paper that the investment in information generation k by itself is not socially valuable.

where

$$\Delta(\sigma, q_1, q_2) \equiv pr\{\hat{q}_2 \geq \hat{q}_1|\sigma\}E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma], \tag{2}$$

is the expected quality gain.

That is, the expected social gain from changing the signal configuration is just the expected welfare gain from replacing one product by the other, *conditional* on a change in the product ranking. Hence, information is only socially valuable, if it can lead to a change of optimal choice.

For intuition consider the case $\sigma' = \emptyset$ but $\sigma'' \neq \emptyset$, i.e. a move from no information generation to some information generation. This move is socially beneficial if and only if the quality ranking reverses for some signal realizations. If positive, the social benefit increases in the expected quality gain $\Delta(\sigma'', q_1, q_2)$, which can be rewritten as

$$\text{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma''\} \left(E[\theta]E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma''] - E[\theta]E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma'']\right).$$

That is, the value of information is the value of changing the allocation following a reversal in the quality ranking. It is given by the difference between social welfare when reallocation is possible ($E[\theta]E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma'']$) and social welfare when reallocation is not possible ($E[\theta]E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma'']$).

Note that $\Delta(\sigma, q_1, q_2)$ depends both on the signal σ and on the quality distribution. For given $\sigma \neq (\emptyset, \emptyset)$, as the priors q_1 and q_2 get closer the expected quality gain increases. Similarly, the next lemma shows that for given q_1 and q_2, drawing more signals leads to higher expected quality gain.

Lemma 3. *The expected quality gain increases in the number of signals:*

$$\Delta((\emptyset, \emptyset), q_1, q_2) \leq \Delta((\emptyset, \sigma_i), q_1, q_2) \leq \Delta((\sigma_1, \sigma_2), q_1, q_2). \textit{ for } i \in \{1, 2\} \tag{3}$$

Hence, more information (in form of two signals rather than one) increases the dispersion in the posterior distribution.[14] By doing so, it increases the

[14]This is because, as the proof exploits, the expected quality distribution for two signals drawn is a mean preserving spread of the expected quality distribution for only one signal drawn (independently from the signals' correlation structure, as long as they are not perfectly correlated). Drawing a second signal after the realization of the first one adds "noise", but the expected quality remains the same.

probability of a change in the quality ranking and the distance between qualities in case of such quality ranking.

The socially optimal investment in information generation then solves

$$\max_{\sigma} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - \begin{cases} 2k \text{ if } \sigma = (\sigma_1, \sigma_2), \\ k \text{ if } \sigma = (\emptyset, \sigma_i) \text{ for } i \in \{1, 2\}, \\ 0 \text{ if } \ \sigma = (\emptyset, \emptyset). \end{cases}$$

Whether it is optimal to learn about only the quality leader, only the quality follower, or both will depend on the two expected qualities, on the two signals, and on the cost parameter k.

Private value of information generation. Given the outcome of the pricing game in Proposition 1, firm $i \in \{1, 2\}$ profits are

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \begin{cases} \underline{\theta}|\hat{q}_i - \hat{q}_{-i}| & \text{if } \hat{q}_i > \hat{q}_{-i} \\ 0 & \text{otherwise,} \end{cases}$$

which increase in the distance between quality levels, strictly so for the quality leader.

Note that profits depend on the marginal consumer $\underline{\theta}$, who in equilibrium is indifferent between buying from either firm. Thus each firm's benefit from generating information depends on $\underline{\theta}$, while the social value of information depends on $E[\theta]$. The next proposition shows that, as a consequence, the private value of information generation is below its social value.

Proposition 2. *Suppose there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations σ' and σ'' a firm i's gain in payoffs from moving from σ' to σ'' is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \underline{\theta}\left(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)\right), \quad (4)$$

and is proportional to the gain in social welfare with a factor $\underline{\theta}/E[\theta] < 1$.

Proof. Rewrite firm i's profits as

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = \underline{\theta}(\max\{\hat{q}_i, \hat{q}_{-i}\} - \hat{q}_{-i}) = \frac{\underline{\theta}}{E[\theta]} S(\hat{q}_1, \hat{q}_2) - \underline{\theta}\hat{q}_{-i},$$

and use Lemma 2. □

Again, information is beneficial if and only if it may alter the consumption choice. For intuition, suppose that $\sigma' = \emptyset$ but $\sigma'' \neq \emptyset$, that is moving from zero to some information generation. In this case the private benefit of information generation for firm 1 can be rewritten as

$$E[\pi_1(\hat{q}_1, \hat{q}_2)|\sigma''] - \pi_1(q_1, q_2) = \underline{\theta}\text{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma''\} \left(-E[|\hat{q}_1 - \hat{q}_2||\hat{q}_2 \geq \hat{q}_1, \sigma'']\right),$$

which is the difference between its equilibrium profits in case of a quality ranking reversal (zero) and a counterfactual in which firm 1 remains serving the whole market, despite having a worse quality than firm 2. To retain demand firm 1 needs to set a negative price and hence has a negative profit, see Figure 1 for an illustration. Similarly, the private benefit of information generation for firm 2 can be rewritten as

$$E[\pi_2(\hat{q}_2, \hat{q}_1)|\sigma''] = \underline{\theta}\text{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma''\} \left(E[|\hat{q}_2 - \hat{q}_1||\hat{q}_2 \geq \hat{q}_1, \sigma'']\right),$$

which is the difference between its profit in case of a quality ranking reversal and its counterfactual profit from remaining the follower. This reveals a symmetry between the firms, in that the value of information is identical for both firms, since firm 2's gains from becoming a monopolist equals the loss firm 1 avoids by not having to retain its demand at inferior quality.

Finally, note that by (3) the benefit will be non-negative, if σ'' contains strictly more signals than σ'. The proposition thus confirms that the private benefit of information generation is lower than the social benefit, for any increase in the number of signals and given any signal configuration. Since social welfare is the sum of profits and consumer surplus, information generation must increase consumer surplus.

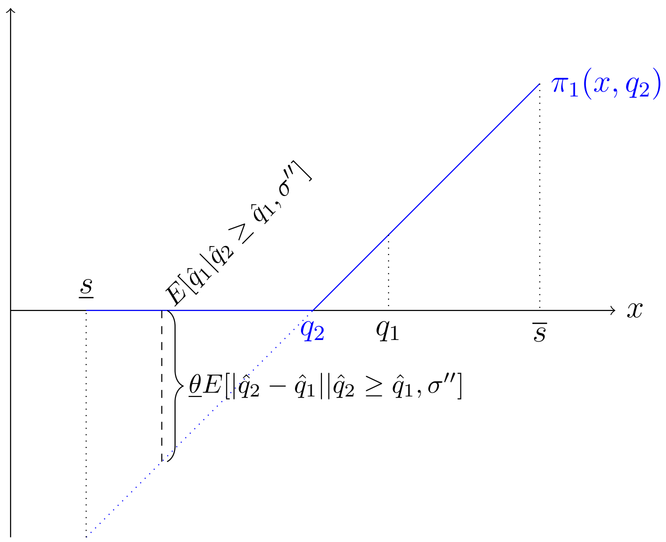

Figure 1: Value of information for firm 1. An illustration for the case in which the signal is informative only about product 1 (monopoly case).

Subgame Perfect Nash Equilibrium A firm's optimal choice of signal acquisition, and thus the outcome of the two stage game, depends on whether the expected increase in profits computed above outweighs the signal cost k. That is, the subgame perfect Nash equilibrium of the information generation cum pricing game depends on the quality distribution q_1 and q_2, the signal technology, and the cost k. We list the pure strategy equilibria below:

- If $k > \underline{\theta}\left(\Delta(\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$, there is an equilibrium, in which only firm $i \in \{1, 2\}$ generates information. If the inequalities hold for both $i = 1$ and $i = 2$, there are two equilibria (each corresponding to a different firm generating information).

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$ and $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, there is a unique equilibrium in which both firms generate information.

- if $k \leq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right)$, but $\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, there are two equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise, there is no information generation in equilibrium.

Therefore multiple equilibria are possible in two cases. First, if acquiring a signal is profitable for each firm individually, but not jointly, there may be two equilibria, one each for each firm to acquire a signal but not the other one. Second, if the expected quality gain $\Delta(\sigma, q_1, q_2)$ increases in the number of signals drawn, then the firms' signal choices will be strategic complements. Suppose, for instance, that acquiring signal is not profitable for one firm only, but it is if the other firm does as well, say if one signal cannot perturb the posterior quality distribution sufficiently to reverse the quality ranking, but two signals can. If the cost k is sufficiently small, this case would produce a familiar coordination failure: an equilibrium without information generation, Pareto dominated by another one, in which both firms acquire information. In what follows, if there are multiple equilibria that can be Pareto ranked, we focus on the Pareto-preferred one.

Which of the different cases will emerge depends both on the signal structure and on the distance in expected qualities $|q_1 - q_2|$. If, for given signals, this distance is sufficiently small, information generation by at least one firm is more likely in equilibrium. For intermediate $|q_1 - q_2|$, there may be multiple equilibria, in which either both firms generate information or neither does; the former equilibrium Pareto dominates the latter. If the distance is sufficiently large, neither firm will acquire any signal.

The characterization of the Nash Equilibrium and Proposition 2 imply the following proposition, derived in the appendix, stating that the equilibrium level of information generation is inefficiently low.

Proposition 3. *Suppose* $1 \leq \underline{\theta} f(\underline{\theta})$*, i.e. there is a monopoly.*

(i) There are values of k *for which the number of signals drawn in equilibrium is strictly lower than socially optimal. For all other values of* k *the efficient number of signals is drawn.*[15]

[15] Note that for some k there could be coordination failure: there are multiple Nash equilibria, each with one firm drawing a signal, but not the other. The inefficient equilibrium is the one in which the firm with the less informative signal generates information.

(ii) *Consider any two taste parameter distributions $F(\theta)$ and $F'(\theta)$ that either have equal mean but different lower bounds $\underline{\theta} > \underline{\theta}'$, or have equal lower bounds but $F(\theta)$ has lower mean than $F'(\theta)$. The set of k for which there is an inefficient equilibrium under $F'(\theta)$ contains the set of k for which there is an inefficient equilibrium under $F(\theta)$.*

Hence, firms are more likely to draw fewer signals in equilibrium than efficient if the difference between private benefit (as measured by $\underline{\theta}$) and social benefit (as measured by $E(\theta)$) of information generation is large.

Coordination in information generation. An implication of Proposition 2 is that drawing a signal benefits *both firms in the same way* and there is a positive externality in information generation across firms. Therefore firms may wish to coordinate their choice of information generation, e.g. via an industry body.[16]

When firms can coordinate in information generation, they will choose a signal configuration that maximizes joint profits. By the previous derivations, the joint benefit of information generation by firm i is:

$$2\underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2),$$

and the joint benefit of information generation by both firms is

$$2\underline{\theta}\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

Because information generation by one firm imposes a positive externality on the other one, the firms' joint benefit from information generation is larger than each firm's individual benefit. Hence, there are cost parameters k, for which no firm generates information in any equilibrium described above, but information generation by one or both firms will occur when firms jointly decide on information generation and share its cost. Similarly, for some level of k only one firm generates information in equilibrium, and both firms generate information when they can coordinate.

[16] As already noted, industry bodies are often responsible for creating and maintaining public information generation mechanisms such as classifications and competitions.

The increase in information generation is socially desirable, if the social benefit of information generation remains higher than the joint private benefits, that is, if $2\underline{\theta} \leq E[\theta]$. In this case there is underinvestment in information generation, both with and without coordination, but the underinvestment is less severe when firms coordinate.[17] These observations yield the following corollary to our results above.

Corollary 1. *Firms that coordinate their choice of information generation generate more information than do individual choices in the Nash equilibrium. If $2\underline{\theta} \leq E[\theta]$, coordination increases social welfare and consumer surplus.*

4.2 Case 2: Duopoly $(1 > \underline{\theta} f(\underline{\theta}))$

Turn now to the case of a duopoly, i.e., $1 > \underline{\theta} f(\underline{\theta})$ and thus both firms sell to some consumers, jointly covering the market.

Social benefit of information generation. In contrast to the case above, now the pricing equilibrium is inefficient and expected social welfare is:

$$\begin{aligned} S(\hat{q}_1, \hat{q}_2) &= \hat{q}_L \int_{X^*}^{\overline{\theta}} \theta dF(\theta) + \hat{q}_F \int_{\underline{\theta}}^{X^*} \theta dF(\theta) \\ &= \max\{\hat{q}_1, \hat{q}_2\}(1-F(X^*))E[\theta|\theta>X^*]+\min\{\hat{q}_1, \hat{q}_2\}F(X^*)E[\theta|\theta<X^*] \\ &= \max\{\hat{q}_1, \hat{q}_2\}E[\theta] - |\hat{q}_1 - \hat{q}_2|F(X^*)E[\theta|\theta<X^*]. \end{aligned} \tag{5}$$

The first part of this expression is the first-best social welfare, (i.e. when all consumers consume the higher quality good). The second part is the deadweight loss generated by positive demand for the lower quality good.

Information generation therefore has two competing effects on social welfare. Similar to the monopoly case, drawing a signal increases the expected highest quality in the market, which increases social welfare. By contrast to the monopoly case, information generation also increases the expected

[17]When $2\underline{\theta} > E[\theta]$, coordination by firms may lead to overinvestment in information generation and may reduce social welfare, see the next subsection.

quality distance, which in turn increases the deadweight loss. The strength of the second effect depends on the quality follower's market share (i.e., on $F(X^*)$) and on the average taste for quality of the lower quality good consumers (i.e., $E[\theta|\theta < X^*]$). Both quantities strictly increase in X^*, which is therefore a sufficient statistics for the social cost of information generation. The following lemma states the social benefit of information generation.

Lemma 4. *For any two signal configurations σ' and σ'', the social benefit generated from moving from σ' to σ'' is given by*

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = ((1 - F(X^*))E[\theta|\theta > X^*] - F(X^*)E[\theta|\theta < X^*]) \times (\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)) .$$

Recall that, by definition of X^*, the majority of consumers purchase the high quality good ($F(X^*) < 1/2$), so that the above expression is positive. It follows that increasing the number of signals increases social welfare, strictly so when increasing the number of signals strictly increases the expected quality gain. The social value of information increases linearly in the difference between the aggregate valuation of those consuming the higher quality good ($(1 - F(X^*))E[\theta|\theta > X^*]$) and those consuming the lower quality good ($F(X^*)E[\theta|\theta < X^*]$). This is because the social benefit of information is only driven by a reversal of the quality ranking, and the difference in welfare resulting from the switch of consumption goods between lower and higher valuation consumers.

Private benefit of information generation. To compare private and social returns, recall the firms' profits:

$$\pi_i(\hat{q}_i, \hat{q}_{-i}) = |\hat{q}_i - \hat{q}_{-i}| \begin{cases} \frac{(1-F(X^*))^2}{f(X^*)} & \text{if } \hat{q}_i \geq \hat{q}_{-i} \\ \frac{F(X^*)^2}{f(X^*)} & \text{if } \hat{q}_i \leq \hat{q}_{-i}. \end{cases} \tag{6}$$

Both firms' profits increase linearly in the quality distance, but because $F(X^*) < \frac{1}{2}$ this increase is steeper for the quality leader.

The next proposition shows that, as a consequence of positive and increasing profits of the quality follower, the private benefit of information generation is larger than in the monopoly case.

Proposition 4. *Suppose there is a duopoly (i.e., $1 > \underline{\theta} f(\underline{\theta})$). Then for any two signal configurations σ' and σ'' a firm i's gain in payoffs from moving from σ' to σ'' is given by*

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)) \,. \tag{7}$$

As above the benefit will be non-negative, if σ'' contains strictly more signals than σ'. For intuition, suppose again $\sigma' = \emptyset$ but $\sigma'' \neq \emptyset$. In this case each firm's private value of information generation can be written as

$$\text{pr}\left\{\hat{q}_2 > \hat{q}_1|\sigma''\right\} E\left[\pi_1(\hat{q}_1, \hat{q}_2) + \pi_2(\hat{q}_1, \hat{q}_2)|\hat{q}_2 > \hat{q}_1, \sigma''\right].$$

The intuition is similar to the monopoly case. Each firm's benefit from generating information stems from a possible quality ranking reversal. Conditional on a quality ranking reversal, the private value of information is the difference between profits if consumers switch suppliers and profits in a counterfactual in which consumers stay put, which may require a negative price (and profit) to ensure that demand remains constant. In this example profits for the quality leader would be $-\frac{(1-F(X^*))^2}{f(X^*)} E[|\hat{q}_1 - \hat{q}_2|\hat{q}_2 > \hat{q}_1, \sigma'']$, see Figure 2.

An interesting observation is that, for any σ' and σ'', if $\Delta(\sigma'', q_1, q_2) > \Delta(\sigma', q_1, q_2)$, then the value of information is strictly increasing in

$$X^* + 2\frac{F(X^*)^2}{f(X^*)} = \frac{\pi_1(\hat{q}_1, \hat{q}_2) + \pi_2(\hat{q}_1, \hat{q}_2)}{|\hat{q}_1 - \hat{q}_2|}$$

Hence, the sensitivity of each firm's profits to new information is equal to the sensitivity of aggregate profits to the quality distance. This is an intuitive measure of firms' market power and strictly increases in X^* due to log concavity. Recall that X^* must be below the median of the taste distribution. Hence, log-concavity implies a form of market segmentation: the closer each firm is to serve half of the market, the higher are aggregate profits, given the

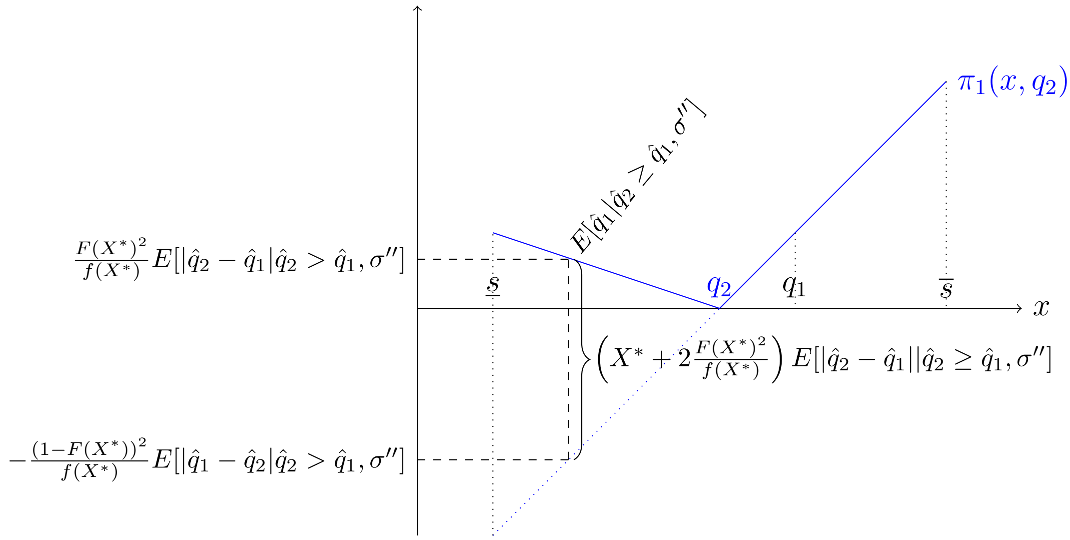

Figure 2: Value of information for firm 1. An illustration for the case in which the signal is informative only about product 1 (duopoly case).

quality distribution. Indeed firms' *aggregate* profits determine each firm's *individual* benefit of information. This is because information is valuable if and only if either firm may become leader or follower, implying that profits both of leader and follower determine the benefit of drawing a signal.

The following corollary compares private and social benefits of information generation.

Corollary 2. *Each firm's private benefit from generating information is strictly higher than its social benefit if, and only if:*

$$2\frac{F(X^*)^2}{f(X^*)} + X^* > (1 - F(X^*))E[\theta|\theta > X^*] - F(X^*)E[\theta|\theta < X^*]. \quad (8)$$

Note that condition (8) reduces to $\underline{\theta} > E[\theta]$, when there is a monopoly (i.e., $1 \leq \underline{\theta} f(\underline{\theta})$ and thus $X^* = \underline{\theta}$), so that no qualifier is needed. The condition becomes more likely to hold as X^*, $F(X^*)$ and $E[\theta|\theta < X^*]$ increase, reflecting an increase in the follower's market share. An important takeaway is that the social benefit of information generation depends on the *global* properties of the taste distribution (i.e. the two conditional expectations), but the private benefit depends on the threshold X^* and on the *local* properties of the taste distribution around X^*.

The next proposition links Condition (8) to equilibrium information generation relative to the social optimum. Its proof contains the derivation of the subgame perfect equilibria of the information generation and pricing game (which is analogous to the monopoly case).

Proposition 5. *Suppose $1 > \underline{\theta} f(\underline{\theta})$, i.e. there is a duopoly. If Condition (8) holds, there are values of k for which the number of signal drawn in equilibrium is strictly greater the socially optimal one. For all other values of k the efficient number of signals is drawn. If condition (8) does not hold, then Proposition 3(i) applies.*[18]

Hence, whether a change in the taste distribution makes over- or investment in information generation more likely depends on whether the resulting local change around X^* dominates the resulting global change in the two conditional expectations in Condition (8).

Comparative statics. To illustrate the mechanism at work, consider again a uniform distribution. This allows for straightforward computation of the effects of distributional changes on Condition (8). We consider first an increase in the mean, keeping the variance constant, increasing X^* and decreasing $F(X^*)$, leading to an decrease in the market share of the quality follower. While both private (LHS of (8)) and social (its RHS) benefits of information generation increase, the social benefit increases by more, i.e. the aggregate effect (increasing all consumers' valuations) dominates the local effect (on prices through the marginal consumer's valuation), making underinvestment more likely.

By contrast a mean-preserving spread of a uniform distribution keeps X^* constant, decreases $f(X^*)$ and increases $F(X^*)$, so that the market share of the quality follower increases. A mean-preserving spread also affects the

[18]Also here, for some k there could be coordination failure: there are multiple Nash equilibria, one with each firm drawing a signal, but not the other. One of these equilibria is inefficient, because the firm with the less informative signal generates information.

social value of information. The overall effect is to increase the private more (or decrease less) than the social benefit and the local effect, on prices and profits through the change in own price elasticity of demand, dominates, and overinvestment becomes more likely. We summarize these observations in the following lemma, see appendix for the computations.

Lemma 5. *Suppose the taste parameter is uniformly distributed. Increasing its mean while keeping constant its variance decreases the follower's market share and makes underinvestment in information generation more likely. Conversely, a mean preserving spread increases the follower's market share and makes overinvestment in information generation more likely.*

Consider now any log-concave distribution $F(\theta)$ for which (A2) holds strictly. A distribution $F'(\theta)$ is an X^* *preserving spread (XPS)* of $F(\theta)$ if $F'(\theta) = F(\theta)$ for $\theta \in [X^* - \epsilon, X^* + \epsilon]$ for some $\epsilon > 0$, while either $E_{F'}[\theta|\theta < X^*] < E_F[\theta|\theta < X^*]$ or $E_{F'}[\theta|\theta > X^*] > E_F[\theta|\theta > X^*]$ or both. Since $F'(X^*) = F(X^*)$ and $f(X^*) = f(X^*)$ the LHS of (8) and market shares remain constant when moving from $F(.)$ to $F'(.)$. This implies that the private incentive to generate information (and hence the Nash equilibrium) is unaffected by an XPS. However, the RHS of (8) increases, because the social value of information increases. Hence, an XPS may cause (8) to no longer hold, as stated in the following lemma.

Lemma 6. *Let two distributions $F(\theta)$ and $F'(\theta)$ satisfy Assumptions 1 and 2. If $F'(\theta)$ is a X^*-preserving spread of $F(\theta)$ then:*

- *If there is overinvestment in information generation under $F'(\theta)$, then there is overinvestment in information generation under $F(\theta)$ as well.*
- *If there is underinvestment in information generation under $F(\theta)$, there is underinvestment in information generation under $F'(\theta)$ as well.*
- *There could be overinvestment in information generation under $F(\theta)$ and underinvestment in information generation under $F'(\theta)$, but not the other way around.*

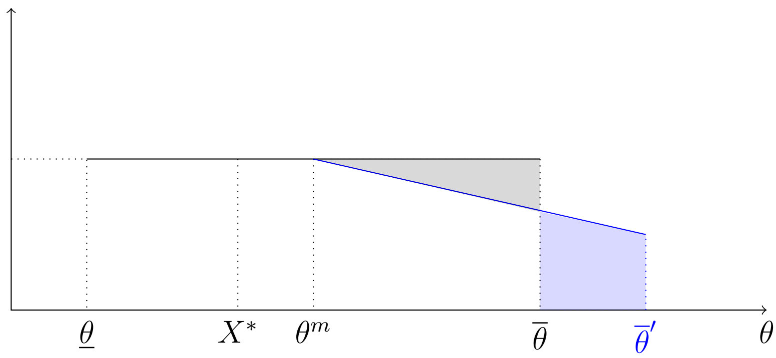

Figure 3: Right-tail shift from a uniform distribution. Note that the two shaded areas are equal.

For illustration consider an XPS that affects only the upper half of the distribution, that is, only consumers with valuation above the median. Formally, let $F'(\theta)$ be a *right-tail spread (RTS)* of $F(\theta)$ if they are identical below the median taste θ_m, (that is, $F'(\theta) = F(\theta)$ for $\theta \leq \theta_m$, which means X^*, $f(X^*)$ and $F(X^*)$ all remain constant), but $E_{F'}[\theta|\theta > \theta^m] > E_F[\theta|\theta > \theta^m]$. Intuitively, an RTS increases the quality sensitivity of the more quality-sensitive consumers (see Figure 3). Since an RTS is a version of XPS, by Lemma 6, it will make underinvestment in information more likely.

As a second illustration consider the opposite, a left tail XPS (LXPS): an XPS in which $E[\theta|\theta > X^* + \epsilon]$ is constant and only $E[\theta|\theta < X^* - \epsilon]$ changes (see Figure 4). By contrast to an RTS, an LXPS affects not only incentives for information generation (Lemma 6), but also the efficiency of the pricing equilibrium, because an LXPS decreases the deadweight loss in the pricing equilibrium for given $\hat{q}_1, \hat{q}_2$. At the same time, an LXPS increases the social value of information generation, while keeping its private value (and hence its equilibrium level) constant. An LXPS thus provides a connection between the inefficiencies in the pricing equilibrium and the direction of the inefficiency in information generation: the higher (lower) the inefficiencies in the pricing equilibrium, the more likely becomes overinvestment (underinvestment) in information generation.

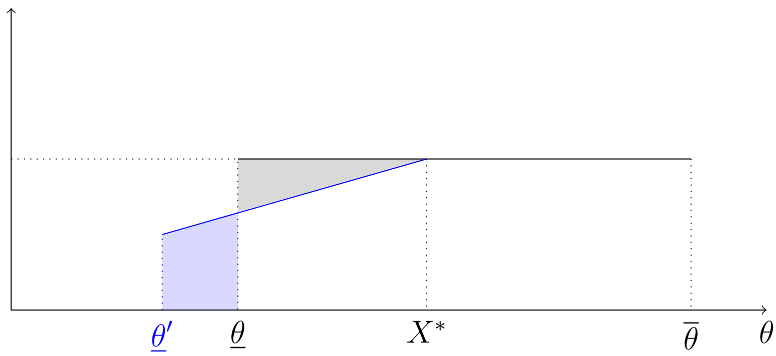

Figure 4: Left XPS from a uniform distribution. Note that the two shaded areas are equal.

Coordination in information generation. The same logic as in the monopoly case applies and information generation by one firm imposes a positive externality on the other firm, because of (7). Hence, the firms' joint benefit of information generation exceeds each firm's private benefit. Allowing firms to coordinate in information generation can thus only increase the number of signals drawn. If condition (8) holds, the number of signals drawn in a Nash equilibrium is already higher than socially optimal, and thus coordination decreases social surplus, implying the following corollary.

Corollary 3. *If (8) holds, then coordination in information generation decreases aggregate welfare and consumer surplus.*

If instead (8) does not hold, then similarly to the monopoly case examined above coordination in information generation may increase welfare.

5 Extension: Endogenous quality

In our model information generation is the only way for firms to differentiate vertically. A relevant question is therefore whether and how the possibility of vertical differentiation by other means changes our results.

In this extension firms can degrade the quality of their product at zero cost before generating information. A standard result in the literature (see, for example, Tirole, 1988) is that, absent information generation, the quality follower will degrade its quality as much as possible as to achieve maximum distance from the quality leader. While quality degradation has been observed,[19] it is far from ubiquitous. We show that information generation may act as a strategic substitute for quality degradation, and thus explain why quality degradation is rarely observed. This also means that information generation may have an added social benefit by preventing harmful quality degradation.

Denote by $q_i^0 \in [\underline{s}, \overline{s}]$ firm i's initial quality with the convention that $q_1^0 > q_2^0$. Before the market opens both firms simultaneously can decrease their expected quality at zero cost to any $q_i \in [\underline{s}, q_i^0]$.[20] Quality degradation is publicly observable. Recall that firms' profits increase in the distance between their expected quality levels. Hence, absent information generation, in the pure strategy Nash equilibrium of a quality degradation game the quality leader will maintain the initial quality $q_1 = q_1^0$, but the follower will degrade as much as possible to $q_2 = \underline{s}$.[21]

Turn now to a quality degradation and information generation game: after deciding whether to degrade its quality each firm can acquire information at a cost k. Introducing information generation may affect the choice quality degradation, because information generation provides an alternative means to increase the quality distance between firms. However, in contrast to degradation, information generation allows for upward as well as downward revisions of the expected quality, increasing the expected highest quality and thus aggregate surplus.

[19]For example, several producers of electronic devices are known to intentionally reduce the performance and functionality of their products; e.g. the case of IBM printers.

[20]For example, as discussed above, the "consumption utility" generated by consuming a product s_i is unknown, but the product's technical specifications are publicly known and determine the expectation of s_i. With this interpretation in mind, quality degradation can be achieved by designing a product with worse technical specifications.

[21]The pure-strategy Nash equilibrium in which the quality follower degrades always exists. A second pure-strategy Nash equilibrium, in which the quality leader fully degrades its quality, but the quality follower does not, exists for some q_1^0, q_2^0. In case both equilibria exist, they can be ranked in terms of efficiency, because the welfare loss is smaller when the quality follower degrades than when the quality leader degrades. For ease of exposition, we only discuss the Nash equilibrium in which the quality follower degrades.

If at least one firm generates information, the quality follower's profit is:

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] = \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2) + \pi_2(q_1, q_2). \qquad (9)$$

Note that $\pi_2(q_1, q_2)$ decreases in q_2, but $\Delta(\sigma, q_1, q_2)$ increases in q_2, and strictly so if $\Delta(\sigma, q_1, q_2) > 0$. Hence, if $\Delta(\sigma, q_1, q_2) = 0$ and information generation has no value, the above result carries over and the quality follower is better off by degrading as much as possible to maximize the distance to the quality leader. When $\Delta(\sigma, q_1, q_2) > 0$, however, it is possible that $E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma]$ increases in q_2, and hence that there is no incentive to degrade quality. That is, quality degradation and information generation can be alternative ways to achieve vertical differentiation.

To provide a sufficient condition for this case to occur note that

$$\pi_2(q_1, q_2) = \frac{F(X^*)^2}{f(X^*)}(q_1^0 - q_2),$$

is arbitrarily close to zero, if X^* is close to $\underline{\theta}$, because then the demand for the quality follower's good is arbitrarily small. It is also arbitrarily close to zero if q_1^0 is close to $\underline{s}$, because the maximum distance that can be achieved between quality leader and follower is also arbitrarily small. In either of these cases, we have that

$$E[\pi_2(\hat{q}_1, \hat{q}_2)|\sigma] \approx \left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2),$$

so that the quality follower's profit is strictly positive and strictly increases in q_2, if $\Delta(\sigma, q_1^0, q_2^0)$ is strictly positive, that is, if q_1^0 and q_2^0 are sufficiently close or if σ is sufficiently informative (in the sense of dispersion in the posterior expected qualities). This observation implies the following lemma.

Lemma 7. *For any X^*, there are q_1^0, q_2^0 such that the quality follower will fully degrade quality if no information is generated, but neither firm will degrade quality if information is generated by at least one firm.*

The lemma states that information generation can prevent harmful quality degradation. Indeed, a sufficiently low cost k will guarantee some information generation in equilibrium, which implies the following corollary.

Corollary 4. *There are q_1^0, q_2^0 and k such that information generation and no quality degradation constitute a subgame perfect Nash equilibrium of the quality degradation and information generation game.*

If the case described in Lemma 7 and the following corollary do not apply, the quality follower may partially degrade, even though information is generated in equilibrium. For example, if the signals σ_1 and σ_2 are discrete, then the probability of a quality ranking reversal may be discontinuous in the amount of quality degradation by the quality follower. That is, this probability may be very small when the quality follower degrades by a small amount, but jump discontinuously if the amount of quality degradation passes a given threshold. If, at the same time, the benefit of increasing vertical distance is large, the quality follower may prefer to partially degrade quality. We will not consider this possibility here.

Turning to social welfare, assume the case described in Lemma 7, i.e. information generation prevents quality degradation. The social benefit is:

$$E[S(\hat{q}_1, \hat{q}_2)] - S(q_1^0, \underline{s}) = E[S(\hat{q}_1, \hat{q}_2)] - S(q_1^0, q_2^0) + S(q_1^0, q_2^0) - S(q_1^0, \underline{s})$$
$$= (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\, \Delta((\emptyset, \sigma_i), q_1^0, q_2^0) + (q_2^0 - \underline{s})F(X^*)E[\theta|\theta < X^*]. \tag{10}$$

The first term of this expression is the benefit of information generation given initial quality levels. The second term stems from (5) and is the benefit from preventing quality degradation. It increases in $q_2^0 - \underline{s}$ (the amount of quality degradation prevented by generating information), in the market share of the quality follower and in the average valuation of these consumers. The following proposition summarizes these observations.

Proposition 6. *There are q_1^0, q_2^0 and k such that the social benefit of information generation with endogenous quality is strictly greater than the one with exogenous qualities.*

Proof. Immediate from Lemma 7, (10) and the following discussion. □

Since information generation can have an additional social benefit when quality is endogenous rather than exogenous, Proposition 5 may no longer apply. That is, when quality is exogenous, the efficient number of signals may be zero, but in equilibrium at least one firm generates information. Under endogenous quality choice the fact that a firm is expected to generate information prevents quality degradation. If the social benefit of preventing quality degradation (given by the second part of 10) is larger than the net social cost of an additional signal (given by the second part of 10 minus k), then one firm generating information is the socially optimal outcome with endogenous quality levels. Similarly, with exogenous quality levels there are situations in which the efficient number of signals is zero, which is also the equilibrium outcome. With endogenous quality levels, however, the absence of information generation leads to quality degradation and may, therefore, be inefficient. The following corollary summarizes these observations.

Corollary 5. *Suppose the case described in Lemma 7 holds. There are cases in which there is over-investment in information generation with exogenous quality, but the efficient level of information generation when quality levels are endogenous. Similarly, there are cases in which there is the efficient level of information generation with exogenous quality, but under-investment in information generation when quality levels are endogenous.*

6 Conclusion

We consider a standard duopoly with vertically differentiated products and study firms' incentives to generate information. Our main result is that firms will under- or overinvest in information generation, depending on the distribution of market shares in the pricing equilibrium. Taste distributions that generate a low market share for the quality follower are associated with under-provision of information, and vice versa. Since a higher market share of the quality follower implies higher deadweight loss in the pricing the inefficiencies in the pricing equilibrium and in information generation are connected. We provide an ordering of taste distribution by the induced deadweight loss, and show that higher deadweight loss is associated to overinvestment in information. We also show that information generation has a positive externality on the other firm's profit and thus firms benefit from coordinating their information generation activities. Finally, we introduce the possibility of quality degradation and show that quality degradation and information generation are substitutes for increasing vertical product differentiation. Thus the possibility to generate information may reduce harmful quality degradation.

This last result implies that there are situations in which information generation should be discouraged, if quality levels are exogenous—possibly via a tax—but information generation should be encouraged, if quality levels are endogenous—possibly via a subsidy. This insight carries over to whether cooperation and coordination of competing firms ought to be allowed. In some situations coordination in information generation should be prevented, if quality levels are exogenous, but be allowed or even encouraged, if quality levels are endogenous. This, however, suggests that the optimal policy may be time inconsistent, because the policymaker may want to revise the policy after quality levels are set; this is an intriguing question for future research.

Our analysis assumed a covered market: in equilibrium all consumers purchase some product. Removing our Assumption 2 would potentially allow for equilibria in which some consumers do not purchase at all. If firms' quality levels are sufficiently close, however, their profits are close to zero and the market is covered. The logic laid out above continues to apply: information generation by firms is privately valuable, because it increases expected vertical distance and profits. From the social point of view, information generation may cause some consumers to stop consuming, which generates an additional source of inefficiency relative to the case of a covered market considered above. If the initial quality distance between firms is large, so that not all consumers purchase, our results may no longer apply: e.g. firms' benefits from information generation may become negative. A thorough analysis of this case is deferred to future work.

Appendix

Proof of Lemma 1

We start by deriving a useful lemma:

Lemma 8. *In any pure strategy Nash equilibrium:*

- *The quality leader faces strictly positive demand:* $\overline{\theta} > X \geq Y$.
- *If not all consumers purchase from the quality leader* ($X > \underline{\theta}$)*, then there is positive demand for the quality follower* ($X > Y \geq \underline{\theta}$).

Proof. Note first that $X = Y = \overline{\theta}$ quickly leads to a contradiction: if both prices are so high that no consumers purchase, then each firm can earn strictly positive payoff by deviating to a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$.

Suppose that $X = \overline{\theta}$, i.e. the quality leader faces zero demand. Then, by the argument above, $Y < \overline{\theta}$ and the quality follower faces positive demand. This cannot be an equilibrium because the quality leader can set its price equal that of the quality follower, generate positive demand and earn positive profits.

Finally, suppose that $\underline{\theta} < X < \overline{\theta}$, i.e. the quality leader faces positive demand, but does not capture the entire market. Then $Y < X$. To see this suppose the contrary, i.e. $Y = X$. This cannot be an equilibrium because the quality follower will earn strictly positive profits by setting a small, but positive price, which will attract a positive measure of consumers because $F(\theta)$ is continuous and $\overline{\theta} > 0$. □

Hence, in any pure strategy Nash equilibrium the quality leader faces demand $1 - F(X)$ and the quality follower faces demand $F(X) - F(Y)$. Profits are given by:

$$\pi_L(p_L, p_F) = p_L \cdot (1 - F(X)) \text{ and } \pi_F(p_L, p_F) = p_F \cdot (F(X) - F(Y)).$$

The best responses are defined as:

$$p_L(p_F) = \text{argmax}_{p_L} \{\pi_L(p_L, p_F)\}$$
$$p_F(p_L) = \text{argmax}_{p_F} \{\pi_F(p_L, p_F)\}.$$

We first compute the leader's best response and then move to the follower's best response. As we will see, computing the leader's best response is quite straightforward, while computing the follower's best response is complicated by a kink in the profit function.

Quality leader's best response. Consider first the quality leader's problem. For given p_F the leader can out-price the follower and set $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, so that $X = \underline{\theta}$. In this case the leader serves the entire market and its profit equals p_L. Hence, conditional on $X = \underline{\theta}$ the quality leader maximizes profits by setting $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$. If instead $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F$, the leader serves only a fraction of the total market, and $X > Y \geq \underline{\theta}$. Using the definition of X the quality leader's problem becomes:

$$\max_{p_L \geq \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F} \left\{ p_L \left(1 - F\left(\frac{p_L - p_F}{\hat{q}_L - \hat{q}_F} \right) \right) \right\}.$$

The first derivative of the objective function is

$$1 - F(X) - \frac{p_L f(X)}{\hat{q}_L - \hat{q}_F}$$

or

$$\left(\frac{1 - F(X)}{f(X)} - \frac{p_L}{\hat{q}_L - \hat{q}_F} \right) f(X),$$

and equals zero at

$$p_L = \frac{1 - F(X)}{f(X)} (\hat{q}_L - \hat{q}_F), \tag{11}$$

which is unique due to log-concavity. Log-concavity also implies that the second derivative of the objective function is negative at $p_L = \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. The quality leader's objective function therefore strictly increases for $p_L < \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$ and strictly decreases for $p_L > \frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

Quality follower's best response. We now turn to the quality follower F's best response. We first deal with the trivial case where the leader corners the market. Suppose the quality leader chooses $p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F)$. Then, for any p_F, the quality leader covers the entire market and the quality follower's profit is zero for any p_L. Thus the quality follower's best response is

$$p_F(p_L) = [0, \infty) \text{ if } p_L \leq \underline{\theta}(\hat{q}_L - \hat{q}_F).$$

This establishes the first part of $p_F(p_L)$ in the lemma.

Suppose now that $p_L > \underline{\theta}(\hat{q}_L - \hat{q}_F)$ instead. Then there are $p_F > 0$ such that the follower has positive demand and profit. Note that the follower's profit function has a kink at price $p_F = \underline{\theta}\hat{q}_F$, but is well-behaved above and below, which allows us to characterize the follower's best response distinguishing the cases of $p_F \leq \underline{\theta}\hat{q}_F$ (in which case $Y = \overline{\theta}$ and the market is covered) and $p_F > \underline{\theta}\hat{q}_F$ (in which case $Y > \overline{\theta}$ and the market is not covered).

Covered market. If $p_F \leq \underline{\theta}\hat{q}_F$, then all consumers purchase one of the goods and $Y = \underline{\theta}$, so that a change in p_F only affects X. Conditional on $Y = \underline{\theta}$, the follower's profit function is

$$\max_{p_F \leq \underline{\theta}\hat{q}_F} \{p_F F(X)\}.$$

The objective function's first derivative is

$$\left(\frac{F(X)}{f(X)} - \frac{p_F}{\hat{q}_L - \hat{q}_F} \right) f(X).$$

The first derivative equals zero at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which is unique by log-concavity. Again, conditional on $Y = \underline{\theta}$, the follower's profit function is strictly concave at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$, which in turns imply that profits conditional on $Y = \underline{\theta}$ are first increasing then decreasing in p_F, reaching a maximum at $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$.

Non-covered market. If instead $p_F > \underline{\theta}\hat{q}_F$ some consumers will not purchase, so that a change in p_F will affect both X and Y. Conditional on

$\underline{\theta} \leq Y < X$, the follower's profit function is now

$$\max_{p_F \geq \underline{\theta}\hat{q}_F} \{p_F(F(X) - F(Y))\}.$$

The objective function's first derivative is

$$F(X) - F(Y) - p_F \left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Now Condition (A2) becomes useful: it implies that the above expression is always negative, which implies that the quality follower always sets a price so that $Y = \underline{\theta}$ and the market is covered.

To see why, note that 11 implies that $X < \frac{1-F(X)}{f(X)}$ so that $X \leq \theta^*$. Hence, by the definition of m (see Assumption 2) $f(X) < m$ and $f(Y) < m$. Recall that the first order condition for the case $Y > \underline{\theta}$ is

$$F(X) - F(Y) - p_F \left(\frac{f(X)}{\hat{q}_L - \hat{q}_F} + \frac{f(Y)}{\hat{q}_F} \right).$$

Because $F(X) - F(Y) \leq 1$, $p_F > \underline{\theta}\hat{q}_F$ (whenever $Y > \underline{\theta}$), $\frac{\hat{q}_F}{\hat{q}_L} \geq \frac{\underline{s}}{\overline{s}}$, and $f(X), f(Y) \geq m$, the above expression is always smaller than

$$1 - m\underline{\theta} \left(1 - \frac{\underline{s}}{\overline{s}}\right)^{-1},$$

which is negative under (A2). Hence the first order condition for the case $Y > \underline{\theta}$ is always negative, and the quality follower is always better off by setting p_F such that $Y = \underline{\theta}$.

Hence, $p_F > \underline{\theta}\hat{q}_F$ cannot occur, implying the second part of $p_F(p_L)$ in the lemma.

Proof of Proposition 1

As a preliminary step, note that (A2) is equivalent to

$$\frac{1}{m\underline{\theta}} - 1 \leq \left(\frac{\overline{s}}{\overline{s} - \underline{s}} \right) - 1 \Leftrightarrow \left(\frac{1}{m\underline{\theta}} - 1 \right) \left(\frac{\overline{s}}{\underline{s}} - 1 \right) \leq 1. \quad (12)$$

We prove each part of the proposition separately.

(i) Recall the quality leader's best reply as derived above:

$$p_L(p_F) = \max\left\{\frac{1-F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}(\hat{q}_L - \hat{q}_F) + p_F\right\}.$$

By log-concavity $\frac{1-F(X)}{f(X)}$ decreases and is thus maximal for $x = \underline{\theta}$. Hence, if

$$\left(\underline{\theta} - \frac{1}{f(\underline{\theta})}\right)(\hat{q}_L - \hat{q}_F) + p_F \geq 0$$

the quality leader's captures the entire market. If $p_F > 0$, this cannot be an equilibrium because the quality follower should lower its price and earn positive profits. If instead $p_F = 0$ and $p_L = \underline{\theta}(\hat{q}_L - \hat{q}_F)$ then no firm can make a profitable deviation, and these prices constitute a Nash equilibrium. If $\underline{\theta} f(\underline{\theta}) \geq 1$, therefore, in equilibrium the leader captures the entire market.

(ii) Suppose instead $1 > \underline{\theta} f(\underline{\theta})$ from now on. The observations made in the text above imply that in this case the quality leader's best reply to $p_F = 0$ is $p_L = \frac{1-F(p_L/(\hat{q}_L - \hat{q}_F))}{f(p_L/(\hat{q}_L - \hat{q}_F))}(\hat{q}_L - \hat{q}_F)$. Hence, by Lemma 8 the Nash equilibrium will necessarily have $X > \underline{\theta}$ (implying $f(X) > 0$) and $p_F = \min\left\{\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F), \underline{\theta}\hat{q}_F\right\} > 0$. Therefore there are two possible cases, depending on whether the quality follower's best response is a corner solution ($p_F = \underline{\theta}\hat{q}_F$) or an interior solution ($p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$).

Suppose first that the quality follower's best response has an interior solution:

$$\frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F) \leq \underline{\theta}\hat{q}_F, \tag{13}$$

so that $p_F = \frac{F(X)}{f(X)}(\hat{q}_L - \hat{q}_F)$. In this case, by definition of X, the equilibrium cutoff X solves

$$X = \frac{1 - 2F(X)}{f(X)}. \tag{14}$$

This equation has a unique solution because, by log concavity, its RHS is decreasing in X and we have assumed $1 > \underline{\theta} f(\underline{\theta})$. This constitutes a Nash equilibrium if indeed the solution X of equation (14) satisfies condition (13). Condition (13) can be rewritten as

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L}{\hat{q}_F}-1\right)\leq 1.$$

Note that $\frac{\hat{q}_L}{\hat{q}_F}$ is at most $\frac{\overline{s}}{\underline{s}}$, and that by (14) $\frac{F(X)}{f(X)}=\frac{1}{2}\left(\frac{1}{f(X)}-X\right)$, which is at most $\frac{1}{2}\left(\frac{1}{m}-\underline{\theta}\right)$.[22] Therefore,

$$\frac{F(X)}{f(X)\underline{\theta}}\left(\frac{\hat{q}_L}{\hat{q}_F}-1\right)<\frac{1}{2}\left(\frac{1}{m\underline{\theta}}-1\right)\left(\frac{\overline{s}}{\underline{s}}-1\right)<1,$$

where the last inequality follows by (12). Hence, (13) holds and thus $p_F=\frac{F(X)}{f(X)}(\hat{q}_L-\hat{q}_F)$ and $p_L=\frac{1-F(X)}{f(X)}(\hat{q}_L-\hat{q}_F)$, with X defined implicitly by (14) is a Nash equilibrium.

To conclude the proof, we show that there is no equilibrium in which $1>\underline{\theta}f(\underline{\theta})$, and hence the quality leader's best response has an interior solution:

$$p_L=\frac{1-F(X)}{f(X)}(\hat{q}_L-\hat{q}_F),$$

but at the same time (13) is violated, and hence the quality follower's best response has a corner solution:

$$p_F=\underline{\theta}\hat{q}_F.$$

If such equilibrium exists, then by definition

$$X=\frac{1-F(X)}{f(X)}-\underline{\theta}\left(\frac{\hat{q}_L}{\hat{q}_F}-1\right)^{-1} \tag{15}$$

This is consistent with a Nash equilibrium if indeed for this X (13) is violated.

Note that by (15) $\frac{F(X)}{f(X)}$ is smaller than $\frac{1}{f(X)}-X$ which, in turn, is smaller

[22] Here we make use again of a fact established in the proof of Lemma 1 (see its last paragraph): that $f(X)\geq m$.

than $\frac{1}{m} - \underline{\theta}$. Also, $\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}$ must be smaller than $\left(\frac{\overline{s}}{\underline{s}} - 1\right)$. It follows that

$$\frac{F(X)}{f(X)\underline{\theta}} \left(\frac{\hat{q}_L - \hat{q}_F}{\hat{q}_F}\right) \leq \left(\frac{1}{m\underline{\theta}} - 1\right) \left(\frac{\overline{s}}{\underline{s}} - 1\right) \leq 1,$$

where the last inequality follows by (12). Hence, (13) must hold and there cannot be a Nash equilibrium with $p_F = \underline{\theta}\hat{q}_F$.

Proof of Lemma 2

Suppose that no firm acquires information; then $\hat{q}_i = q_i$ for $i = 1, 2$ and (by assumption) firm 1 is the quality leader. The ex ante expected social welfare is then $S(q_1, q_2) = E[\theta]\, q_1 = E[\theta]\, E[\hat{q}_1|\sigma]$, where the last equality follows from the law of iterated expectation and holds for any σ. The social benefit of acquiring information is therefore given by the difference between expected social welfare given a chosen signal configuration σ and expected social welfare when no information is acquired:

$$\begin{aligned} E[S(\hat{q}_1, \hat{q}_2)|\sigma] - S(q_1, q_2) &= E[\theta]\, E[\max\{\hat{q}_1, \hat{q}_2\}|\sigma] - E[\theta]\, E[\hat{q}_1|\sigma] \\ &= E[\theta]\{E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} + E[\hat{q}_2|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \\ &\quad - E[\hat{q}_1|\hat{q}_1 \geq \hat{q}_2, \sigma]\mathrm{pr}\{\hat{q}_1 \geq \hat{q}_2|\sigma\} - E[\hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\}\} \\ &= E[\theta]\, E[\hat{q}_2 - \hat{q}_1|\hat{q}_2 \geq \hat{q}_1, \sigma]\mathrm{pr}\{\hat{q}_2 \geq \hat{q}_1|\sigma\} \equiv E[\theta]\, \Delta(\sigma, q_1, q_2). \end{aligned}$$

The proposition follows by writing

$$E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - E[S(\hat{q}_1, \hat{q}_2)|\sigma'] = (E[S(\hat{q}_1, \hat{q}_2)|\sigma''] - S(q_1, q_2)) + (S(q_1, q_2) - E[S(\hat{q}_1, \hat{q}_2)|\sigma'])$$

Proof of Lemma 3

Consider the random variable $x \equiv \hat{q}_2 - \hat{q}_1$. For any signal structure σ it must hold that

$$\Delta(\sigma, q_1, q_2) = E[\max\{x, 0\}|\sigma].$$

That is, the expected quality gain is equal to the expected value of a convex function. Therefore, for any σ and σ', such that the distribution of x given

σ' second order stochastically dominates the one given σ

$$\Delta(\sigma, q_1, q_2) \geq \Delta(\sigma', q_1, q_2).$$

Note that the distribution of x for signal configuration (σ_1, σ_2) is a mean preserving spread of the distribution of x for $(\emptyset, \sigma_i)$. This is a consequence of the fact that beliefs are martingales: the expected value of the posterior is always equal to the prior. To see this, note that given a realization of the first signal yields ex-post expected value $E[x|\hat{\sigma}_1]$. By the law of iterated expectations $E[x|\hat{\sigma}_1, \sigma_2] = E[x|\hat{\sigma}_1]$. Hence, once the first signal is drawn, drawing a second signal keeps constant the expected value of x, but only adds noise and spreads out the distribution of x. This establishes the lemma.

Proof of Proposition 3

We distinguish three cases:

1. It is socially optimal to generate no information, that is

$$2k > E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) \text{ and } k > E[\theta]\Delta((\emptyset, \sigma_i), q_1, q_2) \quad i \in \{1, 2\}.$$

By Proposition 2 each firm's best reply to the other firm not generating information is to not generate information either. Likewise, each firm i's best reply to the other firm $-i$ generating information is not to generate information, if

$$k \geq \underline{\theta}\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_{-i}), q_1, q_2)\right),$$

which is always true, because

$$\begin{aligned}&\underline{\theta}(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))\\ &\leq E[\theta]\Delta((\sigma_1, \sigma_2), q_1, q_2) - \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq 2k - \underline{\theta}\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k.\end{aligned}$$

Hence, in the case when no information generation is socially optimal there is a unique Nash equilibrium, in which neither firm generates any information.

2. It is socially optimal for firm i to generate information, but not firm $-i$, that is

$$E[\theta]\Delta((\emptyset,\sigma_{-i}),q_1,q_2) \leq E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2) \equiv \hat{k}_1 \text{ and}$$
$$\hat{k}_4 \equiv E[\theta](\Delta((\sigma_1,\sigma_2)q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)) < k < E[\theta]\Delta((\emptyset,\sigma_i),q_1,q_2). \tag{16}$$

The second inequality immediately implies

$$k > \underline{\theta}\left[\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right].$$

This means that if firm i generates information, then firm $-i$'s best reply is to not generate information. Hence, there is no Nash equilibrium, in which both firms generate information.

Suppose that

$$k > \underline{\theta}\Delta((\emptyset,\sigma_i),q_1,q_2) \equiv \hat{k}_0. \tag{17}$$

Then neither firm finds it profitable to generate information if the other firm does not. Hence, in the unique Nash equilibrium there is no information generation.

If instead

$$\underline{\theta}\Delta((\emptyset,\sigma_{-i}),q_1,q_2) < k \leq \underline{\theta}\Delta((\emptyset,\sigma_i),q_1,q_2),$$

then there is a unique equilibrium in which firm i generates information. Finally, if

$$k \leq \underline{\theta}\Delta((\emptyset,\sigma_{-i}),q_1,q_2) \equiv \hat{k}_5,$$

then there are multiple equilibria, in which each firm may generate information, while the other one does not. In one of these equilibria firm $-i$ generates information, but not firm i. This is inefficient, because by assumption $\Delta((\emptyset,\sigma_i),q_1,q_2) < \Delta((\emptyset,\sigma_{-i}),q_1,q_2)$, i.e. firm i's signal generates more information (as measured by the dispersion of the posteriors) and higher social welfare than firm $-i$'s signal.

3. It is socially optimal for both firms to generate information, that is

$$2k \leq E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2) \text{ and}$$
$$E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2)-2k \geq E[\theta]\max\{\Delta((\emptyset,\sigma_1),q_1,q_2),\Delta((\sigma_1,\sigma_2),q_1,q_2)\}-k.$$

That is, the net social benefit of drawing both signals is positive, and exceeds the net social benefit of drawing either individual signal. The above inequalities can be rewritten as

$$k \leq E[\theta]\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)- \max\left\{\frac{1}{2}\Delta((\sigma_1,\sigma_2),q_1,q_2),\Delta((\emptyset,\sigma_1),q_1,q_2),\Delta((\emptyset,\sigma_2),q_1,q_2)\right\}\right) \equiv \hat{k}_3.$$

A necessary condition for both firms to generate information in a Nash equilibrium (including the case of multiple equilibria) is

$$k < \underline{\theta}\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \Delta((\emptyset,\sigma_i),q_1,q_2)\right),$$

for both firms $i = 1,2$, or

$$k \leq \underline{\theta}\left(\Delta((\sigma_1,\sigma_2),q_1,q_2) - \max\left\{\Delta((\emptyset,\sigma_1),q_1,q_2),\Delta((\emptyset,\sigma_2),q_1,q_2)\right\}\right) \equiv \hat{k}_2.$$

Therefore, if

$$k > \hat{k}_2 \text{ and } k \leq \hat{k}_3, \tag{18}$$

then the number of signals drawn in a Nash equilibrium is strictly less than in the social optimum. Otherwise, there will be a (possibly unique) Nash equilibrium that is efficient.

We therefore established that the number of signals drawn in equilibrium is always smaller than the socially optimal number of signals, strictly so if either both conditions (16) and (17) hold, or both conditions in (18) hold.

Note also that, in both cases, the set of such k for which fewer signals than optimal are drawn expands with $E[\theta]-\underline{\theta}$ and with the first difference of $\Delta(.)$.

We also established the possibility of a coordination failure: when the efficient number of signals is 1, either firm generating one signal may be a Nash equilibrium, and in particular only the firm with the less informative signal generating information may be an equilibrium, which is inefficient.

Proof of Lemma 4

Simply note that social welfare can be written as

$$\begin{aligned} S(\hat{q}_1,\hat{q}_2) &= \max\{\hat{q}_1,\hat{q}_2\}E[\theta] - |\hat{q}_1-\hat{q}_2|F(X^*)E[\theta|\theta<X^*] \\ &= \max\{\hat{q}_1,\hat{q}_2\}E[\theta] - (2\max\{\hat{q}_1,\hat{q}_2\}-\hat{q}_1-\hat{q}_2)\,F(X^*)E[\theta|\theta<X^*] \\ &= \max\{\hat{q}_1,\hat{q}_2\}\,(E[\theta]-2F(X^*)E[\theta|\theta<X^*]) + (\hat{q}_1+\hat{q}_2)\,F(X^*)E[\theta|\theta<X^*]. \end{aligned}$$

The statement follows from the same derivations detailed in the proof of Lemma 2.

Proof of Proposition 4

For given σ, write

$$\begin{aligned} E[\pi_i(\hat{q}_i,\hat{q}_{-i})|\sigma] =& \frac{(1-F(X^*))^2}{f(X^*)}\mathrm{pr}\{\hat{q}_i\geq\hat{q}_{-i}|\sigma\}E[\hat{q}_i-\hat{q}_{-i}|\hat{q}_i\geq\hat{q}_{-i},\sigma'']+ \\ & \frac{F(X^*)^2}{f(X^*)}\mathrm{pr}\{\hat{q}_{-i}\geq\hat{q}_i|\sigma\}E[\hat{q}_{-i}-\hat{q}_i|\hat{q}_{-i}\geq\hat{q}_i,\sigma] \end{aligned}$$

Suppose $\hat{q}_i\geq\hat{q}_{-i}$. By using the law of iterated expectation, write

$$\begin{aligned} &\pi_i(q_i,q_{-i}) = E\left[\hat{q}_i-\hat{q}_{-i}|\sigma\right]\frac{(1-F(X^*))^2}{f(X^*)} = \\ &\frac{(1-F(X^*))^2}{f(X^*)}\left(\mathrm{pr}\{\hat{q}_i\geq\hat{q}_{-i}|\sigma\}E[\hat{q}_i-\hat{q}_{-i}|\hat{q}_i\geq\hat{q}_{-i},\sigma]+\mathrm{pr}\{\hat{q}_{-i}\geq\hat{q}_i|\sigma\}E[\hat{q}_i-\hat{q}_{-i}|\hat{q}_{-i}\geq\hat{q}_i,\sigma]\right) \end{aligned}$$

Similarly, if $\hat{q}_i \leq \hat{q}_{-i}$ write

$$\pi_i(q_i, q_{-i}) = E\left[\hat{q}_i - \hat{q}_{-i}|\sigma\right] \frac{F(X^*)^2}{f(X^*)} =$$

$$\frac{F(X^*)^2}{f(X^*)} \left(\text{pr}\{\hat{q}_i \geq \hat{q}_{-i}|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_i \geq \hat{q}_{-i}, \sigma] + \text{pr}\{\hat{q}_{-i} \geq \hat{q}_i|\sigma\}E[\hat{q}_i - \hat{q}_{-i}|\hat{q}_{-i} \geq \hat{q}_i, \sigma]\right)$$

Both when $\hat{q}_i \geq \hat{q}_{-i}$ and when $\hat{q}_i \leq \hat{q}_{-i}$ we can then write

$$E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma] - \pi_i(q_i, q_{-i}) = \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2)$$

Writing

$$\begin{aligned} & E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] = \\ & \quad (E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma''] - \pi_i(q_i, q_{-i})) - (E[\pi_i(\hat{q}_i, \hat{q}_{-i})|\sigma'] - \pi_i(q_i, q_{-i})) = \\ & \quad \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta(\sigma'', q_1, q_2) - \Delta(\sigma', q_1, q_2)), \end{aligned}$$

Concludes the proof.

Proof of Proposition 5

The pure strategy Nash equilibria of the information generation game in the case of a duopoly are similar to the ones derived for the case of a monopoly, modulo the different expression for the private benefit of information generation. We have:

- If $k > \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is an equilibrium in which only firm i generates information.
- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$ and $\left(X^* + 2 \cdot \frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \geq k$ for at least one $i \in \{1, 2\}$, then there is a unique equilibrium in which both firms generate information.

- if $k \leq \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2))$,
but $\left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right)\Delta((\emptyset, \sigma_i), q_1, q_2) \leq k$ for both $i = 1, 2$, then there are multiple equilibria: one in which no firm generates information, and one in which both firms generate information.

- Otherwise there is no information generation in equilibrium.

Following the structure of the proof of Proposition 3 we consider different cases. For ease of notation define the social value of information generation as

$$S \cdot \Delta(\sigma, q_1, q_2) \equiv (E[\theta] - 2F(X^*)E[\theta|\theta < X^*])\, \Delta(\sigma, q_1, q_2),$$

and the private value of information generation as

$$P \cdot \Delta(\sigma, q_1, q_2) \equiv \left(X^* + 2\frac{F(X^*)^2}{f(X^*)}\right) \Delta(\sigma, q_1, q_2).$$

Condition (8) implies that $P > S$, so that the private benefit of information generation is higher than the social benefit. We distinguish three cases.

1. No information generation is socially optimal:

   $$k > S\Delta((\emptyset, \sigma_i), q_1, q_2) \text{ and } 2k > S\Delta((\sigma_1, \sigma_2), q_1, q_2).$$

   At least one firm i will invest if $k < P\Delta((\emptyset, \sigma_i), q_1, q_2)$, and thus the number of signals generated in equilibrium is higher than socially optimal if

   $$\hat{k}_0 \equiv S \max\{\Delta((\emptyset, \sigma_i), q_1, q_2), \Delta((\sigma_1, \sigma_2), q_1, q_2)\} < k$$
   $$< P\Delta((\emptyset, \sigma_i), q_1, q_2) \equiv \hat{k}_1.$$

   Otherwise, if the above condition does not hold, there may be an equilibrium, in which both firms invest, if

   $$k \leq P\,(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)) \quad \forall i \in \{1, 2\}.$$

   In this case there are two equilibria: one with both firms investing and one with neither firm investing.

2. It is socially optimal that firm i but not firm $-i$ generates information:

   $S\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)\right)<k<S\Delta((\emptyset,\sigma_i),q_1,q_2)$ and $\Delta((\emptyset,\sigma_{-i}),q_1,q_2)<\Delta((\emptyset,\sigma_i),q_1,q_2)$.

   Since $P>S$, at least one firm will invest in any Nash equilibrium, so the number of signals is at least the socially optimal one. For both firms to invest to be the unique Nash equilibrium it is necessary that

   $$k<P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)\right) \text{ and } k<P\Delta((\emptyset,\sigma_i),q_1,q_2),$$

   Since $S<P$ the second condition holds. Hence, both firms will invest and there will be overinvestment if

   $$\begin{aligned}\hat{k}_2 &\equiv S\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)\right)<k\\ &<P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)\right)\equiv\hat{k}_3.\end{aligned}$$

   If the above condition is violated, but

   $$\hat{k}_4\equiv P\left(\Delta((\sigma_1,\sigma_2),q_1,q_2)-\Delta((\emptyset,\sigma_i),q_1,q_2)\right)<k<P\Delta((\emptyset,\sigma_{-i}),q_1,q_2)\equiv\hat{k}_5,$$

   then in equilibrium only one firm invests. If firm i invests then the equilibrium is efficient. If firm $-i$ invests, then the equilibrium is inefficient. In this last case, in equilibrium the information generated in equilibrium is *less* then the social optimum, because the firm with the least informative signal generates information in equilibrium.

3. It is socially optimal that both firms to generate information:

   $2k<E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2)$ and
   $E[\theta]\Delta((\sigma_1,\sigma_2),q_1,q_2)-2k>E[\theta]\max\{\Delta((\emptyset,\sigma_1),q_1,q_2),\Delta((\emptyset,\sigma_2),q_1,q_2)\}-k$.

   A necessary and sufficient condition for a Nash equilibrium, in which both firms generate information, is:

$$k < P\left(\Delta((\sigma_1, \sigma_2), q_1, q_2) - \Delta((\emptyset, \sigma_i), q_1, q_2)\right),$$

for both firms $i = 1, 2$. Because $P > S$, there is always an equilibrium in which both firms generate information. There may also be another equilibrium, in which no firm generates information. But, as discussed in the text, if both equilibria exist, the one in which both firms generate information Pareto dominates the other.

Restricting our attention to equilibria that are not Pareto dominated, we established that the number of signals drawn in equilibrium is always above the efficient one, strictly so in some cases. It is again possible that both the efficient and equilibrium number of signals is one, but the "wrong" firm generates information in equilibrium.

Computations for Lemma 5

Recall that for a uniform distribution $X^* = (\underline{\theta} + \overline{\theta})/3$, and the conditional expectations are $E[\theta|\theta < X^*] = (4\underline{\theta} + \overline{\theta})/6$ and $E[\theta|\theta > X^*] = (\underline{\theta} + 4\overline{\theta})/6$. The private benefit (LHS of (8)) becomes:

$$\frac{5}{9}(\overline{\theta} - \underline{\theta}) + \frac{2\overline{\theta}\underline{\theta}}{9(\overline{\theta} - \underline{\theta})}. \tag{19}$$

The social benefits (its RHS) are given by:

$$\frac{7}{18}(\overline{\theta} - \underline{\theta}) + \frac{5\overline{\theta}\underline{\theta}}{9(\overline{\theta} - \underline{\theta})}, \tag{20}$$

An increase in the mean keeping the variance, i.e. $\overline{\theta} - \underline{\theta}$, constant increases the private incentive to generate information by (19). However, by (20) the social value of information increases faster than its private value.

Consider a mean-preserving spread (increasing $\overline{\theta}$ and keeping expected value $\mu = (\overline{\theta} + \underline{\theta})/2$ constant). Rewriting the private benefit (19) in terms of mean μ and upper bound $\overline{\theta}$ yields

$$\frac{10}{9}(\overline{\theta} - \mu) + \frac{2\overline{\theta}(2\mu - \overline{\theta})}{18(\overline{\theta} - \mu)}.$$

The social benefit (20) becomes:

$$\frac{7}{9}(\overline{\theta} - \mu) + \frac{5\overline{\theta}(2\mu - \overline{\theta})}{18(\overline{\theta} - \mu)}.$$

Increasing $\underline{\theta}$ while keeping μ constant will increase the private benefit more (or decrease less) than the social benefit.

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