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Market Power and Income Distribution: Lessons from Hybrid Industrial-Labour Economics

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Market Power and Income Distribution: Lessons from Hybrid Industrial-Labour Economics*

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Abstract

Over the past thirty years, the income gap between capital and labour has widened, a shift accompanied by an increase in dominant firms' market power. To understand the underlying causes, our study integrates imperfect competition in both product and labour markets, revealing how different labour market rent sharing mechanisms impact income distribution. We show that firms' gross profit margin serves as a consistent measure of overall market power and a crucial determinant of income inequity. We also develop a novel empirical method to estimate production function, markup and markdown powers, which we apply to a panel of UK manufacturing firms. Our findings demonstrate that the root cause of inequality in market power is the large disparity in firms' productivity, leading to both income inequity and inefficiency, not a trade-off between the two. Our research underscores the importance of addressing market power concentration to promote equitable and efficient economic growth.

Keywords: firm heterogeneity, inequality, multifactor productivity, market powers, markup, markdown, oligopsony, rent sharing, income distribution, estimation of production function, identification method

JEL Classification: D21, D33, D43, D6, E24, J2, J3, L4

1 Introduction

Over the last three decades capital income has risen much faster than labour income in most developed market economies, where stagnant wages[1] have not kept pace with rising firms' profit margins (Elsby *et al.*, 2013,

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[1] Although the term wage stagnation often means a lack of growth in real wage rate (e.g., Acemoglu and Autor, 2011), it more specifically refers to the phenomenon that the growth rate of real wage falls below the potential indicated by the growth rate of

Karabarbounis and Neiman, 2014). This macroeconomic trend has been associated with rise of superstar firms (Autor *et al.*, 2020) and increase of market power of firms at the top of its distribution (De Loecker *et al.*, 2020). The presence of market power seems to challenge the conventional view that inequity is the price to pay to achieve sustained efficiency. To understand the causes of these secular trends and ascertain whether they contradict such conventional view, we develop a Joint Oligopoly-Oligopsony Model (JOOM) that integrates imperfect competition in both product and labour markets, featuring diverse labour market rent sharing mechanisms, ranging from oligopsony to collective bargaining[2]. The model unveils the mechanisms that connect inequalities in firm productivity, competitiveness and market powers, to capital and labour income shares, and shows that large inequality in the distribution of gross profit margin can lead to both income inequity and inefficiency, not a trade-off between them. Furthermore, the model lays the theoretical foundation for a novel and simple empirical method for joint estimation of firms' production function and overall market powers.

Our study provides a unifying framework to study a number of phenomena noted in different streams of literature. First, the influential paper by Autor *et al.* (2020) highlights the role of dominant frontier firms – the so called superstar firms – on the fall of labour share. The defining characteristic of the superstar firms is their superiority in productive efficiency relative to their laggard rivals, resulting in above-average profit margins and below-average labour income shares. This calls for a refocus of attention from market concentration to inequality in firm competitiveness, a defining feature of our analysis.[3]

Second, the equally influential work by De Loecker *et al.* (2020) documents how markups have increased over the last few decades. However, their estimation of markup power can either over- or under-estimate firms' overall market power depending on wage markdown power and whether the rent-sharing mechanism is influenced by collective bargaining. To avoid such potential biases, recent studies of market powers using firm or plant level micro panel data, such as Tortarolo and Zarate (2020), Mertens (2022), and Traina (2022), jointly estimate markups and broadly defined markdowns.[4] One of the theoretical pillars that underpin empirical work on measuring market powers is what we call the joint monopolistic-monopsonistic competition model (JMMCM) where firms can have both product price markup and (broadly defined) wage markdown power. While JMMCM assumes strategic independence between firms, JOOM can capture their strategic interaction, a key feature to avoid biases in the joint estimation of firm production function and market powers.

Finally, rent sharing between firms and their employees has long been a central theme in labour economics. For example, the "declining worker power" hypothesis proposed by Stansbury and Summers (2020) asserts that

labour productivity (see, e.g., Mishel, 2012; Bivens and Mishel, 2015). ILO and OECD (2015) went even further to suggest a causal connection: "A falling labour share often reflects more rapid growth in labour productivity than in average labour compensation, and an increase in returns to capital relative to labour."

[2] The inclusion of variation in rent sharing mechanisms is dictated by the evidence in data.

[3] While high inequality (positive skewness) can cause concentration, the converse is not true.

[4] Previous work that have used the production function approach to investigate imperfect competition in both product and labour markets include Bughin (1996), Crépon *et al.* (2002), Dobbelaere (2004), Dobbelaere and Mairesse (2013).

the weakening of unionisation and workers' collective wage bargaining power in the US over the last decades resulted in a redistribution of economic rents from labour to capital owners. While the formalised mechanisms for rent sharing do vary, they can be grouped in two main streams: (i) bargaining power, related either to search fiction, or cost of hiring, training and firing, and (ii) imperfect competition in labour market caused by finitely elastic firm specific labour supply function. The first group also includes[5] models that focus on collective bargaining and unionisation, such as the efficient bargaining (McDonald and Solow, 1981) and the right-to-manage bargaining (Nickell and Andrews, 1983). The second group includes the labour market monopsony[6] power theory (see Manning, 2003, 2011 and 2021).[7] For example, Card *et al.* (2018) develop a monopsonistic competition model, where the source of firms' wage markdown power is the heterogeneity across employees in their valuation of jobs at different firms (causing finite elasticity of labour supply with respect to wage), while firms' inability to discriminate against such worker heterogeneity empowers workers for rent sharing.

Building upon the literature above, we develop a hybrid Industrial-Labour Economics model that integrates imperfect competition in both product and labour markets, allowing for diverse labour market rent sharing mechanisms. Our theory shows that a firm's gross profit margin is not only a suitable measure of overall market power, but also a key determinant of income distribution between labour and capital. We model the strategic interaction between competing firms' input-output choices and show such interaction can introduce biases to most prevalent joint estimations of production function[8] and market powers.[9] To overcome such biases, we propose a novel and simple empirical identification method, based on the factor cost share approach, *a la* Solow (1957), but applied only to the competitive fringe firms, which are firms that have approximately zero market powers. We use our novel approach to estimate annual joint distributions of short-run productivity, markup, markdown, overall market power, and value added share of labour in a panel of UK manufacturing firms for 2003-2019. Our theory and empirical analysis show: (1) The structural variation of labour market rent sharing

[5] Burdett and Mortensen (1998) model a labour market with search frictions, wage posting, and on-the-job search. This line of research does not involve collective bargaining. Gouin-Bonenfant (2022) studies the relationship between firm productivity dispersion and labour share along this line, abstract from product market power.

[6] The term "monopsony", which literally means "sole buyer", was coined by Joan Robinson (1933). The term is also used loosely to mean market power of a small number of buyers, although the more precise term should be oligopsony, the term that we decide to use in this paper, and covers the special case of monopsony.

[7] Empirically, there has been a long-standing body of evidence that, contrary to the standard view of perfectly competitive labour markets, the labour supply functions faced by individual firms are less than perfectly elastic (Boal and Ransom, 1997, Ashenfelter *et al.*, 2010, Manning, 2011). Recent cross-industry studies have also shown that firms operating in more concentrated markets exercise more wage markdown power, to the detriment of workers in terms of suppressed or stagnating wages (Azar, Marinescu and Steinbaum, 2019, Benmelech *et al.*, 2018). In light of both new and classic work in the field of Industrial Organisation, Berry *et al.* (2019) caution this recent literature on some of its limitations: "A main difficulty in this area is that most of the existing studies of monopsony and wages ... proceed to estimate regressions of wages on measures of concentration ... studies like this may provide some interesting descriptions of concentration and wages, but are not ultimately informative about whether monopsony power has grown and is depressing wages." These authors also call for more detailed industry-specific studies to establish the causal relation in imperfect competition in labour markets. In-depth industry-specific studies have also taken off, with the aim to trace the root causes of wage markdown power to economic primitives, such as imperfectly elastic market-level or firm-level labour supply function (Azar, Berry and Marinescu 2019, Kroft *et al.*, 2021).

[8] This affects both the control function approach (Olley and Pakes, 1996, Levinsohn and Petrin, 2003, Ackerberg *et al.*, 2015) and the dynamic panel data approach (Arellano and Bond, 1991, Blundell and Bond, 1998, 2000).

[9] See Appendix C.

mechanisms between oligopsony and efficient bargaining, each including at least 40% of UK manufacturing firms, is a key determinant of wage growth and income distribution . (2) The observed high inequality in firm short-run productivity distribution is a root cause of inequality in the distributions of firm market power, and labour share of value added. (3) The persistent high inequality in firm market power distribution is a measure of both inefficiency and income inequity. Contrary to the conventional view that inequities can be reduced only at the expenses of efficiency, we show that reduction of inequality in firm overall market power distribution, through, for instance, promoting knowledge diffusion, may improve both efficiency and equity.

Main Findings. The extended JOOM theory shows that the overall market power index, measured by the gross profit margin (δ_i), is in general an increasing function of both the markup index (ρ_i) and wage markdown power index (χ_{Li}). This relation justifies the use of δ_i as a suitable measure of overall market power, irrespective of how markup and markdown powers interact, or whether the labour market rent-sharing mechanism is influenced by worker collective bargaining.

Our panel data of UK manufacturing firms show large firm heterogeneity within a typical four-digit sic code industry along the following key dimensions: (1) firm's short-run productivity ω_i, measured by value added per worker, (2) firm's overall market power index δ_i, measured by gross profit margin, (3) firm's variable cost share of labour ψ_{Li}, (4) firm's value added share of labour ν_{Li}. Figure 1 visualises, respectively, the inequality in value added per worker, and gross profit margin, with the histograms of highly positively skewed distributions. The heterogeneity of firms shown in Figure 1, in particular the dispersion of firms between the top and bottom of the distribution, is at the heart of our analysis. Our extended JOOM theory predicts that the inequality in (1) is a root cause of inequality along each of the dimensions (2) - (4), and the inequality in (3) reflects a key structural difference in labour market rent sharing mechanisms between oligopsony and collective bargaining, affecting the wage markdown index χ_{Li}, and how it responds to firm short-run productivity ω_i.

The competitive fringe firms' values of the variable cost share of labour, ψ_{Li}, equal the estimated industry specific elasticity of labour, θ_L, both by theory and in the data. Other firms have values of ψ_{Li} diverging from θ_L in both directions, implying that the estimated broadly defined markdown power index χ_{Li} has both positive and negative values. Since χ_{Li} measures how the wage w_i departs from the marginal revenue product of labour $MRPL_i$ (i.e., $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i}$), $\chi_{Li} > 0$ is consistent with oligopsony, and $\chi_{Li} < 0$ is consistent with the efficient bargaining model *a la* McDonald and Solow (1981). These novel insights call for separate analyses between two χ_{Li}-value-based Rent Sharing (RS) types. Accordingly, in our main empirical analysis we will distinguish between RS type I for $\chi_{Li} > 0$ and RS type III for $\chi_{Li} < 0$, with the residual group RS type II including (but not limited to) fringe firms that have no market powers.

By including these two distinctive rent sharing mechanisms, the extended JOOM produces two testable predictions. The first is about the relationship between firm short-run productivity and markdown power: (i) productivity ω_i is positively (respectively, negatively) correlated with χ_{Li} for firms in RS type I (respectively, RS type III). This prediction implies that regression analysis that is commonly used in the literature to quantitatively attribute the cause of a change of value added share of labour to markup and wage markdown power, but fails to distinguish RS types, tends to bias the results one way or the other. The second prediction is about the relationship between firm's short-run productivity and variable cost share of labour: (ii) productivity ω_i is positively (respectively, negatively) correlated with ψ_{Li} for firms in RS type I (respectively, RS type III). Our empirical evidence supports both of these two predictions, thus setting a challenge to all alternative explanations of why the relationship between ψ_{Li} and ω_i diverges across RS types I and III. For example, it is hard to see why the two groups should systematically differ in technology, a usual suspect in explaining labour share dynamics.

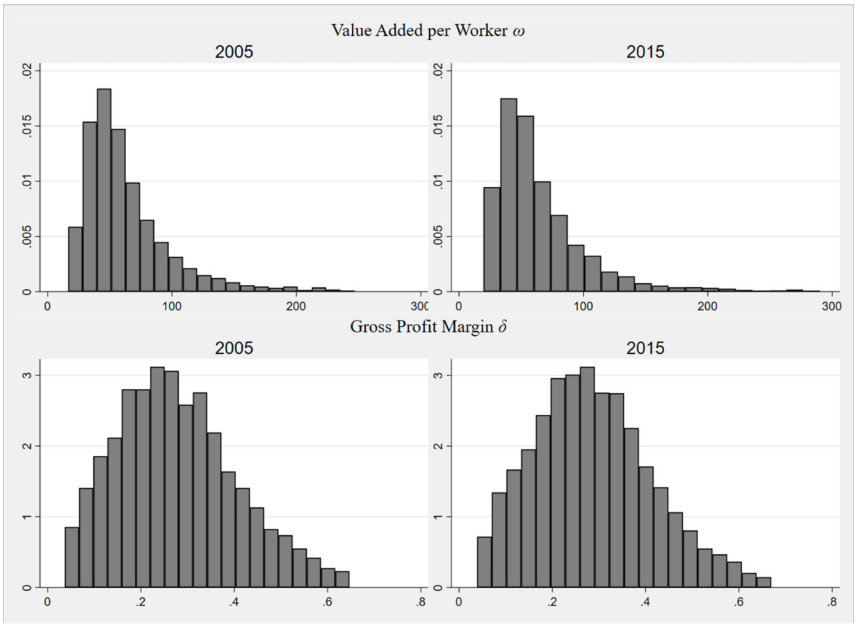

Figure 1: Histograms of Value Added Per Worker ω, and Gross Profit Margin δ

By showing that gross profit margin (δ_i) is both a measure of overall market power and a key determinant of income distribution, our analysis makes clear that sustained large inequality in the distribution of δ_i, caused by sustained inequality in firm short-run productivity, leads to both income inequity and inefficiency, not a trade-off between them. Tackling prolonged high inequality in firm market power can therefore promote equitable and efficient economic growth.[10]

Contribution to Literature. Our first contribution is to the literature on monopsony power, particularly to the wage markdown power, originated from Robinson (1933). The extended JOOM encompasses both product price markup and broadly defined wage markdown powers. Our game-theoretic industrial organisation approach

[10] Section 5 elaborates the point that economic growth marred by wage stagnation is both inequitable and inefficient.

captures the strategic interaction between competing firms, which has been absent in monopolistic and monopsonistic competition models such as Card *et al.* (2018). This feature is particularly relevant to the study of heterogeneous firms which do not fully qualify as monopolistic or monopsonistic, or strategically independent, and possess very dispersed market powers, ranging from dominant superstar firms to competitive fringes. Our theoretical and empirical analyses show the importance of modeling two flexible inputs of production, including labour and intermediate inputs, and investigating the determination of substitution of flexible capital for labour. We show that the broadly defined wage markdown power determines firms' variable cost shares of labour. To the best of our knowledge, we are the first to conduct separate regression analysis by RS types.

Our second contribution is to microeconomics theory, by generalising the foundational notion of marginal cost and extending its definition to imperfectly competitive flexible input markets. When production uses multiple flexible inputs, conventional notion of marginal cost is well defined only if (a) all relevant flexible input markets are perfectly competitive, or without strategic interaction, and (b) the firm minimises cost. Our extended definition marginal cost relaxes both conditions (a) and (b),[11] and allows for more flexible definitions of markup and markdown indices, which capture the effects of labour market rent sharing on marginal cost.

Our third contribution is to the empirical literature on estimation of production function and market powers. By replacing JOOM for the neoclassical perfect competition model, this paper shows that the factor (cost) share approach, pioneered by Solow (1957), should be selectively applied only to the competitive fringe firms, and firms under the influence of right-to-manage collective bargaining.[12,13] An advantage of our approach is to avoid the econometric problem caused by spatial correlation stemming from strategic interaction between competing firms, which invalidates standard econometric assumptions.

The works more closely related to our empirical findings are those by Mertens (2022), Tortarolo and Zarate (2020), and Traina (2022), which jointly estimate both markup and (broadly defined) markdown powers using firm or plant level data in manufacturing sectors. All of the studies find strong evidence of firm labour market power, in addition to product price markup power. Differently from the approach used in these papers, our identification strategy takes full advantage of the rich causal mechanism information from the extended JOOM theory, with the purpose of better dealing with unobservables[14] and the problem of (market, strategic) spatial correlation, as well as some common data problems such as unavailability of firm level product prices.[15] Despite the methodological

[11] For example, it applies to flexible input markets, where the labour market is an oligopsony, which violates (a). It also applies to the efficient bargaining model, where the optimal wage and employment choices in the Nash bargaining solution violate both (a) and (b).

[12] While the application to the competitive fringe firms is generally valid, under the additional assumption that the intermediate input market is perfectly competitive, the applicability can be extended to firms with wage floor (Robinson, 1933), or right-to-manage collective bargaining (Nickel and Andrews, 1983).

[13] Our approach is related to the works by Hall (1988), who pioneered the production function approach and departure from perfect competition, and Bughin (1996), Crépon *et al.* (2002), Dobbelaere (2004), De Loecker and Warzynski (2012), and Dobbelaere and Mairesse (2013), who all in varied ways, advanced the production function approach.

[14] One of the most important unobservables is the boundary of the market, i.e., market definition, within which strategic interaction between firms invalidates the presumption of independence.

[15] See Gandhi *et al.* (2020) and Bond *et al.* (2022) for the non-identification problems that arise for estimating production function

differences, our findings bear much similarity with these works. Namely, (i) both markup and broadly defined markdown powers play fundamental roles in firms' conducts and performances. (ii) markup and markdown powers are negatively correlated (shared with Mertens (2022), and Tortarolo and Zarate (2020)[16]). (iii) Firm productivity (measured by value added per worker) is dispersed and positively correlates with firms' markup power and gross profit margin. Similarly, Tortarolo and Zarate (2020) find value added per worker positively correlated with firms' markup and combined market power. (iv) The correlation between firm productivity and broadly defined markdown power changes from positive to negative when the markdown power index changes from positive to negative. Without distinguishing between rent sharing regimes, Tortarolo and Zarate (2020) document negative correlation between value added per worker and markdown power.

Finally, our investigation contributes to the global debate on the causes of the secular fall of labour share of GDP. A number of researchers have argued that a leading cause is the substitution of capital for labour, but differ in the specific mechanisms involved. For example, Karabarbounis and Neiman (2014) attribute it to a secular decline of the prices of capital goods. Elsby *et al.* (2013) point to offshoring of the labour-intensive component of the U.S. supply chain. Doraszelski and Jaumandreu (2018) propose that the substitution is caused by labour-augmenting (biased) technological change. Along two apparently separate dimensions, De Loecker *et al.* (2020) and Autor *et al.* (2020) argue that the rise of superstar firms' product market power are a main cause of the fall of labour share, while Stansbury and Summers (2020) point to the decline of workers' collective power as a cause of decline in labour share. Our theoretic and empirical analyses identify increasing inequality of firm short-run productivity (associated with the rise of superstar firms) as a root cause of rising aggregate market power and falling aggregate labour share, but also accommodate roles for the substitution of capital for labour. First of all, the rise of superstar firms relies on superior competitiveness, which is likely a result of substituting fixed capital (including knowledge capital) for variable inputs, including flexible labour. Second, superstar firms' have an incentive to substitute flexible capital (i.e., intermediate input) for labour (in the absence of intervention of worker collective bargaining). Such tendency of some, historically contained by the worker collective bargaining mechanism, can accelerate if worker power is weakened as documented by Stansbury and Summers (2020), and result in a fall of labour share even without the superstar firms gaining more overall market power. These two channels can result in secular trends of increasing (broadly defined) markdown power and falling labour shares, similar to what have been documented by Mertens (2022) and Traina (2022).[17]

parameters in the absence of firm level product price data.

[16] A note of caution: the way Tortarolo and Zarate (2020) define wage markdown as $\frac{w_i}{MRPLi}$, instead of $\frac{MRPL_i}{w_i}$, may cause an incorrect interpretation by some commentators of their finding as showing positive correlation between markup and wage markdown powers.

[17] Gouin-Bonenfant (2022) studies the relationship between firm productivity dispersion and labour share on the basis of on-the-job search model allowing between-firm competition in labour market. While he also attributes productivity dispersion as a cause of depression of labour share, he focuses on labour market search-friction rather than collective bargaining and product market power. Kehrig and Vincent (2021) provide micro-level empirical analysis of the labour share decline, and find evidence for productivity dispersion as a driver, and they note that high productivity low labour share establishments enjoy a product price premium relative

The remainder of the paper is organised as follows: Section 2 introduces the JOOM, derives the fundamental equations of market powers, and explores its implications for input mix (i.e., substitution of labour with intermediate inputs), welfare and functional income distribution. Section 3 develops an extension of JOOM that nests three rent sharing mechanisms, including the influences of collective bargaining. Section 4 presents quantitative and empirical analyses. Section 5 discusses policy implications. Section 6 concludes.

2 The Model

A novelty of JOOM is to allow asymmetric strategic interaction between superstar firms and their rivals in both labour and product markets. While the canonical model features Cournot quantity competition in the product market and wage posting competition in the labour market, we will emphasise the general results which do not rely on the particular modelling choices between quantity/price competition in the product and labour markets. In Appendix A, we extend JOOM to all variations to ensure the generality of our main results.

Let each firm $i \in \{1, \cdots, n\}$ face finitely elastic upward-sloping residual labour supply function $L_i(\mathbf{w})$, which depends on the posted wage vector $\mathbf{w} \equiv (w_1, \cdots, w_n)$. The firm specific supply elasticity $\epsilon_{Li} \equiv \frac{\partial L_i(\mathbf{w})}{\partial w_i} \frac{w_i}{L_i(\mathbf{w})} < \infty$, implies imperfect competition in labour market, and also satisfies

$$\frac{\partial \epsilon_{Li}}{\partial w_i} < 0, \tag{1}$$

$$\frac{\partial \epsilon_{Li}}{\partial w_j} > 0. \tag{2}$$

Property (1) means a high-paying (and large) employer faces more inelastic labour supply and departs further from price-taker behavior. Property (2) implies the firm specific labour supply becomes more elastic as a rival firm pays higher wage (and employs more workers). This underlies strategic interaction in labour market competition.

Let the product market demand system be described by residual inverse demand functions $P_i(\mathbf{q})$ for all firms i, which depends on output vector $\mathbf{q} \equiv (q_1, \cdots, q_n)$. Each firm's residual demand elasticity, $\epsilon_i \equiv -\frac{1}{\frac{\partial P_i(\mathbf{q})}{\partial q_i} \frac{q_i}{P_i(\mathbf{q})}} < \infty$, is finite, and has the following properties:

$$\frac{\partial \epsilon_i}{\partial q_i} < 0, \tag{3}$$

$$\frac{\partial \epsilon_i}{\partial q_j} > 0. \tag{4}$$

Property (3) means a high-output and large firm faces more inelastic demand and departs further from price-taker

to their peers. Our contribution complements all these works looking for root cause in productivity dispersion by enriching analysis of relevant mechanisms, covering both markup power and broadly defined markdown power, and allowing for various labour market rent sharing types. Additionally, our welfare analysis identifies inefficiencies stemming from persistent high inequality in firm market power distribution.

behavior. Property (4) means the residual demand becomes more elastic as a rival firm produces more. These properties are satisfied by many commonly used models, such as linear demand functions and homogeneous good market with constant price elasticity of demand.

The notion of short-run production function plays a central role in our analysis. Let the short-run production function be $F_i(x_i, l_i)$ for all i, where x_i is the intermediate input (or flexible capital), and l_i is flexible labour input. $F_i(x_i, l_i)$ depends on the fixed (tangible and intangible) capital that can be changed in the long run, which we do not explicitly model for the static model. Instead, we assume $F_i(x_i, l_i) = A_i f(x_i, l_i)$, where A_i is the Hicks neutral technology coefficient, which measures the short-run multi-factor productivity (MFP).[18] Let the market for intermediate input be perfectly competitive with constant price p_X.

The conditional short-run profit maximisation problem for all i is:

$$\max_{q_i, x_i, w_i, q_i \leq F_i(x_i, L_i(\mathbf{w}))} \pi_i(\mathbf{q}, \mathbf{w}, x_i) = \underbrace{P_i(\mathbf{q})\, q_i}_{R_i} - \underbrace{[w_i L_i(\mathbf{w}) + p_X x_i]}_{C_i}, \tag{5}$$

where R_i and C_i respectively denote revenue and cost. The Nash equilibrium of the game is such that all firms conditionally maximise their short-run profits.

2.1 Definitions of Marginal Cost and Market Power Indices

In the standard Cournot model well-defined cost functions are assumed, and therefore the notion of marginal cost is also well defined. For the canonical JOOM, it is impossible to specify well-defined cost functions because of imperfect competition and strategic interaction in the labour market. For this reason, we define each firm's marginal cost at the Nash equilibrium point, starting from the foundational Lagrangian maximisation problem:

$$\max_{q_i, x_i, w_i, \lambda_i} \mathfrak{L}_i = \underbrace{P_i(\mathbf{q})\, q_i}_{R_i} - \underbrace{[w_i L_i(\mathbf{w}) + p_X x_i]}_{C_i} - \lambda_i\left[q_i - F_i(x_i, L_i(\mathbf{w}))\right], \tag{6}$$

[18] We deliberately choose the term MFP to differentiate from the familiar notion of total factor productivity (TFP) because fixed capital is not an argument of the short-run production function. MFP is a residual of output unexplained by flexible labour and intermediate input, and it captures the contribution of all forms of fixed capital (including knowledge capital intangible or embodied in physical capital) some of which are notoriously difficult to measure directly. The catch-all variable MFP measures their overall effect. Also, MFP is more relevant than TFP to a firm's short-run competitiveness.

and the following four first-order conditions:

$$\begin{aligned}
\frac{\partial \mathfrak{L}_i}{\partial q_i} &= P_i(\mathbf{q}) + \frac{\partial P_i(\mathbf{q})}{\partial q_i} q_i - \lambda_i = 0, && (7)\\
\frac{\partial \mathfrak{L}_i}{\partial x_i} &= -p_X + \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i} = 0, && (8)\\
\frac{\partial \mathfrak{L}_i}{\partial w_i} &= -L_i(\mathbf{w}) - w_i \frac{\partial L_i(\mathbf{w})}{\partial w_i} + \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial L_i} \frac{\partial L_i(\mathbf{w})}{\partial w_i} = 0, && (9)\\
\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} &= q_i - F_i(x_i, L_i(\mathbf{w})) = 0. && (10)
\end{aligned}$$

Let $\mathfrak{L}_i^*$, R_i^* and C_i^* respectively denote maximised profit, and its (optimal) revenue and cost components. The following equation:

$$\frac{\partial \mathfrak{L}_i^*}{\partial q_i} = \frac{\partial R_i^*}{\partial q_i} - \frac{\partial C_i^*}{\partial q_i} = 0, \tag{11}$$

and eq. (7) imply that at the optimum,

$$\lambda_i = MR_i \equiv \frac{\partial R_i}{\partial q_i} = MC_i \equiv \frac{\partial C_i}{\partial q_i}, \tag{12}$$

i.e., Lagrangian multiplier λ_i equals both the (optimal) marginal revenue and marginal cost. Based on these results, we can define or interpret λ_i in eq. (8) and (9) as either marginal revenue or marginal cost. We can then use λ_i in the definition of marginal revenue product of labour, as $MRPL_i = \lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i}$. Now, denote the Lerner index, a standard measure of markup power, by $\rho_i \equiv \frac{p_i - MC_i}{p_i}$, and the wage markdown power index by $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i}$. Consequently,

$$\begin{aligned}
\rho_i &= \frac{p_i - \lambda_i}{p_i}, && (13)\\
\chi_{Li} &= \frac{\lambda_i \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i} - w_i}{w_i}. && (14)
\end{aligned}$$

Finally, we introduce the gross profit margin δ_i, our candidate index of overall market power, defined by:

$$\delta_i \equiv \frac{p_i - AVC_i}{p_i} \equiv \frac{p_i q_i - VC_i}{p_i q_i}, \tag{15}$$

where AVC_i and VC_i respectively denote average variable cost and variable cost. Obvious, δ_i is also the producer surplus to revenue ratio.

2.2 Fundamental Equations of Market Powers

The main insight on the interaction between markup and markdown powers, and how they contribute to the overall market power is summarised by the following theorem:

Theorem 1 *For the joint oligopoly-oligopsony model (JOOM), the markup and markdown power indices ρ_i and χ_{Li} satisfy the following **First Fundamental Equation of Market Powers**:*

$$\rho_i + (1+\chi_{Li})\frac{\phi_{Li}}{\theta_{Li}} = 1, \tag{16}$$

*where $\phi_{Li} \equiv \frac{w_i L_i}{p_i F_i}$ is firm i's revenue share of labour, and $\theta_{Li} \equiv \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial L_i}\frac{L_i}{F_i}$ is the output elasticity of labour. Furthermore, if the production functions $F_i(x_i, l_i)$ are homogeneous of degree 1, i.e., of constant return to scale, then the gross profit margin, δ_i, satisfies the following **Second Fundamental Equation of Market Powers**:*

$$\delta_i = \rho_i + \frac{\theta_{Li}(1-\rho_i)\chi_{Li}}{1+\chi_{Li}}, \tag{17}$$

with

$$\begin{aligned} \frac{\partial \delta_i}{\partial \rho_i} &= 1 - \frac{\theta_{Li}\chi_{Li}}{1+\chi_{Li}} > 0, && (18)\\ \frac{\partial \delta_i}{\partial \chi_{Li}} &= \frac{\theta_{Li}(1-\rho_i)}{(1+\chi_{Li})^2} > 0, && (19)\\ \delta_i &= \rho_i + \chi_{Li}\phi_{Li}. && (20) \end{aligned}$$

Proof. Eq. (7) and (13) imply

$$\rho_i = \frac{1}{\epsilon_i}. \tag{21}$$

Eq. (8) implies

$$\frac{\lambda_i}{p_i} = \frac{\phi_{Xi}}{\theta_{Xi}}, \tag{22}$$

where $\phi_{Xi} \equiv \frac{p_X x_i}{p_i F_i}$ is the revenue share of intermediate input, and $\theta_{Xi} \equiv \frac{\partial F_i(x_i, L_i(\mathbf{w}))}{\partial x_i}\frac{x_i}{F_i}$ is the output elasticity of intermediate input. Eq. (9), (13) and (14) imply

$$\frac{\lambda_i}{p_i} = 1 - \rho_i = \frac{\left(1+\frac{1}{\epsilon_{Li}}\right)\phi_{Li}}{\theta_{Li}}, \tag{23}$$

where $\epsilon_{Li} \equiv \frac{1}{\frac{\partial L_i(\mathbf{w})}{\partial w_i}\frac{w_i}{L_i(\mathbf{w})}}$, and

$$\chi_{Li} = \frac{1}{\epsilon_{Li}}. \tag{24}$$

Eq. (23) and (24) imply (16). Eq. (16) and (22) imply,

$$\begin{aligned}\phi_{Li} &= \frac{\theta_{Li}\left(1-\rho_i\right)}{1+\chi_{Li}}, \\ \phi_{Xi} &= \theta_{Xi}\left(1-\rho_i\right),\end{aligned} \tag{25}$$

and

$$1-\left(\phi_{Li}+\phi_{Xi}\right)=\rho_i+\frac{\theta_{Li}\left(1-\rho_i\right)\chi_{Li}}{\left(1+\chi_{Li}\right)}+\left(1-\theta_{Li}-\theta_{Xi}\right)\left(1-\rho_i\right). \tag{26}$$

Identities in (15) imply

$$\delta_i \equiv 1-\phi_{Li}-\phi_{Xi}. \tag{27}$$

Constant return to scale implies

$$\theta_{Li}+\theta_{Xi}=1, \tag{28}$$

and eq. (17) - (19). Eq. (20) follows from (17) and (25). ■

The market power indices (ρ_i, χ_{Li}) measure how the firm's pricing in product and labour markets departs from the benchmarks of price-taking behaviour. The benchmark for product price is marginal cost. The benchmark for wage is the marginal revenue product of labour. Perfect competition in both markets imply $\rho_i = \chi_{Li} = 0$. Neither of these two market power indices in isolation is generally adequate to measure the relative competitiveness of the firm vis-a-vis its rivals. For example, when $\rho_i > 0$ and $\chi_{Li} > 0$, either of the indices underestimates the firm's relative competitiveness. The second fundamental equation of market powers (17) shows that δ_i is an ideal index because it is an creasing function of both ρ_i and χ_{Li}, so it measures the net departure of firm's pricing behaviour from the price-taking benchmark. An additional merit of index δ_i is that it can be easily computed using firm accounting data. The central role the index δ_i plays both theoretically and empirically will transpire in the following sections of this paper.

Eq. (16) extends beyond the standard restrictions of $\rho_i = \frac{1}{\epsilon_i}$ and $\chi_{Li} = \frac{1}{\epsilon_{Li}}$ (see (21) and (24)), even allowing for $\rho_i < 0$ and $\chi_{Li} < 0$.[19] When the restrictions: $\rho_i = \frac{1}{\epsilon_i}$ and $\chi_{Li} = \frac{1}{\epsilon_{Li}}$ do apply, eq. (16) implies:

$$\frac{1}{\epsilon_i}+\left(1+\frac{1}{\epsilon_{Li}}\right)\frac{\phi_{Li}}{\theta_{Li}}=1. \tag{29}$$

In Appendix A we show eq. (16), (17), and (29) hold for a broad variety of joint oligopoly-oligopsony models, regardless whether the product and labour market decision variables are quantities or prices.

[19] For empirical work, this generality is actually essential. Section 3 presents the extension to $\chi_{Li} \leq 0$. In Section 4 we consider the possibility of limit pricing, which may result in $\rho_i < 0$.

2.3 Wage Markdown Power: A Driver for Substituting Capital for Labour

Each profit maximising firm is necessarily a cost minimiser.[20] In the JOOM, the wage markdown power of a firm provides an incentive to substitute intermediate input for labour, which a price-taking firm in the labour market does not have. The following theorem captures this insight.

Theorem 2 *Denote by ψ_{Li} the variable cost share of labour, i.e.,*

$$\psi_{Li} \equiv \frac{w_i l_i}{p_X x_i + w_i l_i}. \tag{30}$$

Let the production function in the JOOM be $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$ for all $i \in \{1, \cdots, n\}$. Let firms i and i' in the JOOM be such that $\chi_{Li} \geqslant \chi_{Li'} = 0$, then

$$\psi_{Li} = \frac{1-\alpha}{1+\alpha\chi_{Li}} \leqslant \psi_{Li'} = 1-\alpha. \tag{31}$$

Proof. Eq. (8), (9) and (24) imply

$$\frac{\theta_{Li}}{\theta_{Xi}} = \frac{w_i (1+\chi_{Li}) l_i}{p_X x_i}. \tag{32}$$

For the Cob-Douglas production function $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$: $\theta_{Xi} = \alpha$ and $\theta_{Li} = 1-\alpha$, eq. (32) implies

$$\psi_{Li} = \frac{1-\alpha}{1+\alpha\chi_{Li}}, \tag{33}$$

and (31). ■

Corollary 3 *Let the production function in the JOOM be $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$ for all $i \in \{1, \cdots, n\}$. Then the ratio of expenditures of intermediate input and labour is:*

$$\frac{p_X x_i}{w_i l_i} = \frac{\alpha}{1-\alpha} + \frac{\alpha}{1-\alpha}\chi_{Li}, \tag{34}$$

which is equivalent to

$$\chi_{Li} = \frac{1-\alpha}{\alpha}\left(\frac{1}{\psi_{Li}} - \frac{1}{1-\alpha}\right), \tag{35}$$

$$\chi_{Li} = \frac{1-\alpha}{\alpha}\frac{p_X x_i}{w_i l_i} - 1, \tag{36}$$

[20] The results given this section can be extended to constrained cost minimisation, which is treated in Section 3, as long as eq. (16) and (22) hold.

and the markup power index is given by

$$\rho_i = 1 - \frac{(1-\delta_i)(1-\psi_{Li})}{\alpha}, \tag{37}$$

$$\rho_i = 1 - \frac{\phi_{Li}}{\alpha}\frac{p_X x_i}{w_i l_i}. \tag{38}$$

Proof. Eq. (34), (35) and (36) are immediate implications of (33). (37) follows from (17) and (35). (38) is implied by (16) and (36). ■

Eq. (34) predicts that $\frac{p_X x_i}{w_i l_i}$ and ψ_{Li} depend on wage markdown power index χ_{Li} (given that $\frac{p_X x_i}{w_i l_i} \equiv \frac{1}{\psi_{Li}} - 1$). If there is substantial dispersion in χ_{Li} among firms in an industry, then eq. (34) and (35) predict substantial dispersion in $\frac{p_X x_i}{w_i l_i}$ and ψ_{Li}. Thus, $\frac{p_X x_i}{w_i l_i}$ (and equivalently ψ_{Li}) can be used for identifying χ_{Li} if the parameter α has been identified. Similarly, δ_i and ψ_{Li} (and equivalently ϕ_{Li} and $\frac{p_X x_i}{w_i l_i}$) can be used to pin down ρ_i if α has been identified. In the next section these relationships are used to define a novel approach for the identification of α.

2.3.1 A Novel Factor (Cost) Share Approach for Identification of Production Function Parameters

Eq. (35) and (36) are useful to guide empirical measurement of markdown power index χ_{Li}. Conditional on $\chi_{Lj} \geq 0$ and $\rho_j \geq 0$ for all j, the restriction: $\delta_j = 0$ implies $\chi_{Lj} = \rho_j = 0$, i.e., firm j is a price taker in both product and labour market, and $\frac{p_X x_j}{w_j l_j} = \frac{\alpha}{1-\alpha}$ and $\psi_{Lj} = 1 - \alpha$. We call these the competitive fringe firms, and discuss their existence condition in Appendix B.

Index χ_{Li} can be estimated using cross-sectional firm level accounting data. By applying the cost share approach to the competitive fringe firms, we can identify $\frac{\alpha}{1-\alpha}$:

$$\frac{\alpha}{1-\alpha} = mean\left\{\frac{p_X x_j}{w_j l_j} : \delta_j = 0\right\}, \text{ or } \alpha = 1 - mean\left\{\psi_{Lj} : \delta_j = 0\right\}. \tag{39}$$

In practice the set $\left\{\psi_{Lj} : \delta_j = 0\right\}$ could be small or even empty, and the estimation can be replaced by

$$\frac{\alpha}{1-\alpha} = median\left\{\frac{p_X x_j}{w_j l_j}\right\}, \text{ or } \alpha = 1 - median\left\{\psi_{Lj}\right\}, \tag{40}$$

and resulting in

$$\chi_{Li} = \frac{\frac{p_X x_i}{w_i l_i}}{median\left\{\frac{p_X x_j}{w_j l_j}\right\}} - 1, \text{ or } \chi_{Li} = \frac{\frac{1}{\psi_{Li}} - 1}{median\left\{\frac{1}{\psi_{Li}} - 1\right\}} - 1. \tag{41}$$

2.4 Market Powers, Income Distribution and Welfare (Efficiency) Analysis

As shown above, the gross profit margin δ_i is not only a measure of a firm's overall market power, but also reflects a firm's competitiveness in terms of its ability to sustain gross profitability against rival firms. Because δ_i also measures the producer surplus ratio to revenue, it is a key variable in the standard (Marshallian style) welfare analysis which features producer surplus. In section 2.4.1 we show how market power indices δ_i, ρ_i and χ_{Li}, as well as firm short-run productivity, relate to functional income distribution between labour and capital. In section 2.4.2, we introduce the notion of worker surplus, which is an essential part of partial equilibrium welfare analysis when labour market is subject to imperfect competition. Section 2.4.3 goes beyond the familiar linkage between market power and inefficiency (measured by deadweight loss) to explore inefficiency implications of dispersion in firm market power indices.

2.4.1 Value Added Shares and Functional Income Distribution

Define $\nu_{Li} \equiv \frac{\phi_{Li}}{\phi_{Li}+\delta_i}$ (and $\nu_{Ki} \equiv \frac{\delta_i}{\phi_{Li}+\delta_i}$) as the value added share of labour (and capital). Obviously

$$\nu_{Li} = \frac{\phi_{Li}}{1-\phi_{Xi}}, \nu_{Ki} = \frac{1-\phi_{Xi}-\phi_{Li}}{1-\phi_{Xi}}, \nu_{Li}+\nu_{Ki} = 1.$$

Define $\psi_{Li} \equiv \frac{\phi_{Li}}{\phi_{Xi}+\phi_{Li}}$ ($\psi_{Xi} \equiv \frac{\phi_{Xi}}{\phi_{Xi}+\phi_{Li}}$) as the variable cost share of labour (and intermediate input), and $\omega_i \equiv \frac{p_i q_i - p_X x_i}{l_i}$ as the value added per worker[21].

Proposition 4 *The value added share of labour ν_{Li} can be expressed as functions of variables: w_i and ω_i as well as δ_i and ψ_{Li} by the following identities:*

$$\nu_{Li} \equiv \frac{w_i}{\omega_i} \equiv \frac{1}{1+\frac{\delta_i}{1-\delta_i}\frac{1}{\psi_{Li}}}, \tag{42}$$

which imply the following comparative statics: (i) for w_i and ω_i, $\frac{\partial \nu_{Li}}{\partial \omega_i} < 0$ and $\frac{\partial \nu_{Li}}{\partial w_i} > 0$. (ii) for δ_i and ψ_{Li},

$$\frac{\partial \nu_{Li}}{\partial \delta_i} = \frac{-\psi_{Li}}{\left[\delta_i + (1-\delta_i)\,\psi_{Li}\right]^2} < 0, \tag{43}$$

$$\frac{\partial \nu_{Li}}{\partial \psi_{Li}} = \frac{(1-\delta_i)\,\delta_i}{\left[\delta_i + (1-\delta_i)\,\psi_{Li}\right]^2} > 0. \tag{44}$$

Proposition 5 *For the joint oligopoly-oligopsony model, let $b > 0$ be the lower bound of wages. Then*

$$\lim_{\omega_i \to \infty} \nu_{Li} = 0. \tag{45}$$

[21] More precisely this is value added per unit of labour. Here we adopt the term value added per worker, which is commonly used in the empirical literature on rent sharing (Card *et al.*, 2018).

For Cobb-Doglas short-run production functions with constant returns to scale: $F_i(x_i, l_i) = A_i x_i^{\alpha} l_i^{1-\alpha}$, *the variable cost share of labour* ψ_{Li} *can be expressed as a function of* χ_{Li} *by:*

$$\psi_{Li} = \frac{1}{1 + (1 + \chi_{Li}) \frac{\alpha}{1-\alpha}}. \tag{46}$$

ν_{Li} *can be expressed by:*

$$\nu_{Li} = \frac{1}{1 + \frac{\delta_i}{1-\delta_i} \left[1 + (1 + \chi_{Li}) \frac{\alpha}{1-\alpha}\right]}. \tag{47}$$

Given that the gross profit margin δ_i *can be expressed as a function of* ρ_i *and* χ_{Li} *by eq. (17),* ν_{Li} *can be expressed as a function of* ρ_i *and* χ_{Li}*, with the following comparative statics:*

$$\frac{\partial \nu_{Li}}{\partial \rho_i} = \frac{\partial \nu_{Li}}{\partial \delta_i} \frac{\partial \delta_i}{\partial \rho_i} < 0, \tag{48}$$

$$\frac{\partial \nu_{Li}}{\partial \chi_{Li}} = \frac{\partial \nu_{Li}}{\partial \delta_i} \frac{\partial \delta_i}{\partial \chi_{Li}} + \frac{\partial \nu_{Li}}{\partial \psi_{Li}} \frac{\partial \psi_{Li}}{\partial \chi_{Li}} < 0. \tag{49}$$

Proof. A necessary condition for $\omega_i \to \infty$ is $A_i \to \infty$, which, taking rival's firms' MFP as fixed and finite, implies firm i is in a winner-takes-all situation, and the wage is set at $w_i = b$, and $\lim_{\omega_i \to \infty} \nu_{Li} = \lim_{\omega_i \to \infty} \frac{b}{\omega_i} = 0$. ∎

Eq. (45) shows the extent to which wage growth can lag behind the firm productivity growth. For oligopsony labour market, if ω_i is sufficiently large, wage stops growing when ω_i grows.

Eq. (42) provides mechanism information that identifies two pairs of channels through which ν_{Li} can be affected by an exogenous change in firm MFP, A_i. The first pair of channels are through the gross profit margin δ_i, which is also an index of overall market power, and the variable cost share of labour ψ_{Li}, which conversely measures the substitution of intermediate input for labour. The second pair of channels are through market power indices ρ_i and χ_{Li}.

Quantitatively, eq. (42) implies

$$\frac{d\nu_{Li}}{dA_i} = \frac{-\psi_{Li} \frac{d\delta_i}{dA_i} + (1 - \delta_i) \delta_i \frac{d\psi_{Li}}{dA_i}}{\left[\delta_i + (1 - \delta_i) \psi_{Li}\right]^2}. \tag{50}$$

For MFP A_i to have a negative net effect on ν_{Li}, i.e., $\frac{d\nu_{Li}}{dA_i} < 0$, it is necessary and sufficient that

$$\frac{d \ln \delta_i}{d \ln A_i} > (1 - \delta_i) \frac{d \ln \psi_{Li}}{d \ln A_i}. \tag{51}$$

Eq. (42) reveals that the imperfect competition affects the functional income distribution between labour and capital. It implies that firm short-run productivity, measured by A_i, can affect functional income distribution through channels δ_i and ψ_{Li}. Suppose $\frac{d \ln \delta_i}{d \ln A_i} > 0$; if $\frac{d \ln \psi_{Li}}{d \ln A_i} < 0$, then $\frac{d\nu_{Li}}{dA_i} < 0$, i.e., firms at the top (right tail) of MFP distribution tends to have smaller value added share of labour than those at the left tail. In oligopsony labour market $\frac{d \ln \psi_{Li}}{d \ln A_i} < 0$ is plausible because the markdown power incentivises firms to substitute intermediate input for labour.

Finally, we note that the value added per worker $\omega_i \equiv \frac{p_i q_i - p_X x_i}{l_i}$ captures the residual output unexplained by flexible inputs, just as MFP A_i does. It is therefore an alternative measure of firm short-run productivity to MFP, as well as a proxy for MFP. The latter point can be illustrated for the Cobb-Douglas production function with constant returns to scale:

$$\omega_i = p_i A_i \left(\frac{x_i}{l_i} \right)^{\alpha} - p_X \left(\frac{x_i}{l_i} \right),$$

with which ω_i and A_i are positively correlated within a static equilibrium (we validate this claim by simulation in Section 4.1). Note that if A_i in eq. (50) and inequality (51) is replaced by ω_i, the results continue to hold. We extend the applicability of eq. (42)-(50) in Section 3.2.

2.4.2 Worker Surplus

In the conventional partial equilibrium welfare analysis of market power, the stake holders include only the consumers and producers, but not the workers because the labour market is assumed to be perfectly competitive and there is no (non-trivial) worker surplus. This is not the case when the inverse labour supply function is upward sloping.[22] In general, wage determination involves rent sharing between workers and the firm they work for, the outcome of which can certainly make the workers better or worse off. Its welfare consequence can be measured in terms of worker surplus, akin to consumer surplus and producer surplus.

Allowing heterogeneity in workers' preferences over jobs, however, complicates the definition of worker surplus. It appears necessary to distinguish between heterogeneous and homogeneous preferences, as we illustrate below.

Logit Choice-Based Wage Posting Model We follow Card *et al.* (2018) to consider a heterogeneous job market with wage posting, and firm-specific labour supply function:

$$L_i(\mathbf{w}) = \mathcal{L} \frac{(w_i - b)^{\beta} a_i}{\sum_{j=1}^{n} (w_j - b)^{\beta} a_j}, \tag{52}$$

where $\mathcal{L}$ is the to aggregate labour supply, b is the outside option/benefit, a_i is the firm specific amenity parameter. The term $\frac{(w_i - b)^{\beta} a_i}{\sum_{j=1}^{n} (w_j - b)^{\beta} a_j}$ is the logit probability for a worker to work for firm i given the wage vector $\mathbf{w}$. The elasticity of firm-specific labour supply is:

[22] Writing for the Handbook of Labour Economics, Alan Manning (2010) states: "It is increasingly recognized that labor markets are pervasively imperfectly competitive, that there are rents to the employment relationship for both worker and employer". This means, as the author explains, "the loss of the current job makes the worker worse off".

$$\epsilon_{Li} \equiv \frac{\partial L_i(\mathbf{w})}{\partial w_i}\frac{w_i}{L_i(\mathbf{w})} = \frac{\beta(1-s_{Li})w_i}{w_i - b}, \tag{53}$$

where $s_{Li} \equiv \frac{L_i(\mathbf{w})}{\mathcal{L}}$ is the labour market share of firm i, with $\lim_{s_{Li}\to 1}\epsilon_{Li} = 0$. The following partial derivatives are important features of the model: (i)

$$\frac{\partial \epsilon_{Li}}{\partial w_i} = \frac{-\beta\frac{\partial s_i}{\partial w_i}\left(1-\frac{b}{w_i}\right) - \beta(1-s_i)\frac{b}{w_i^2}}{\left(1-\frac{b}{w_i}\right)^2} < 0, \tag{54}$$

with $\lim_{w_i\to b}\epsilon_{Li} = \infty$, that is, large and high wage employers face more inelastic residual labour supplies. (ii) $\frac{\partial\epsilon_{Li}}{\partial w_j} > 0$, i.e., a rise of a rival firm's wage raises the residual labour supply elasticity. (iii) $\frac{\partial\epsilon_{Li}}{\partial b} > 0$, meaning that an increase in outside option/benefit raises every firm's labour supply elasticity.

Quantity Competition Labour Market Model Consider a homogeneous labour market where the firms simultaneously decide on the amounts of labour they employ l_i and let the wage rate to be determined by the market inverse labour supply function $W(L)$, with $W'(\cdot) > 0$, $L = \sum_{i=1}^N l_i$. The market labour supply function can be written as $L = L(w)$ where w is the wage rate. Let $\epsilon_L \equiv \frac{dL(w)}{dw}\frac{w}{L}$ be the elasticity of market level labour supply. Then the firm level elasticity of labour supply is $\epsilon_{Li} = \frac{\epsilon_L}{s_{Li}}$, where $s_{Li} \equiv \frac{l_i}{L}$ is the labour market share of firm i. In this model, $\chi_{Li} = \frac{1}{\epsilon_{Li}} = \frac{s_{Li}}{\epsilon_L}$.

For the model of homogeneous preferences, the upward-sloping inverse labour supply function is a measure of the marginal willingness to accept (WTA), or reservation wage of workers – this is the counterpart of interpreting a downward-sloping inverse demand function as quantifying the willingness to pay (WTP) or reservation price of consumers. Thus the worker surplus, WS, as defined by

$$WS \equiv \int_0^L (W(L) - W(y))\,dy. \tag{55}$$

measures the gain of workers from their current employment relationship. For the model of heterogeneous preferences with a common reservation wage b, we define:

$$WS \equiv \sum_{i=1}^n L_i(\mathbf{w})(w_i - b), \tag{56}$$

which is the employment weighted total of wage differential $(w_i - b)$.

With the introduction of the worker welfare standard, anticompetitive harm (to workers) in the labour market, e.g., caused by a horizontal merger, can be defined by the condition: $\triangle WS < 0$. If $\triangle WS > 0$ then the merger is actually procompetitive in the labour market.

For comparison, we write the standard definition of consumer surplus, CS, for the model of homogeneous good:

$$CS \equiv \int_0^Q \left(P\left(z\right) - P\left(Q\right)\right) dz. \tag{57}$$

2.4.3 Distribution of Market Powers and Efficiency Implications

It is well understood that if input markets are all perfectly competitive, then the markup powers in the product market cause deadweight loss, which includes the part of reduction of consumer surplus, relative to perfectly competitive equilibrium, that is not transferred to producer surplus. To sharpen the idea, consider the case where the short-run production functions have constant returns to scale and firms can differ in their MFP, and the product market is a Cournot oligopoly. Under perfect competition in the product market, only the most efficient firm(s) would produce and the market price would equal the lowest marginal cost among all firms. In the Cournot-Nash equilibrium, the deadweight loss arises from both loss of productive efficiency because some less efficient firms engage in production[23], and loss of allocative efficiency because the most efficient firm produces less than socially desirable. The distribution of Lerner index ρ_i provides information on the deadweight loss. In particular, the statistical measures of inequality in the distribution of ρ_i, such as coefficient of variation or skewness, can be used to measure productive inefficiency, while the revenue share weighted mean of ρ_i can be used to assess allocative inefficiency in the spirit of Banerjee and Duflo (2005) and Hsieh and Klenow (2009).

Now we relax the assumption that the labour market is perfectly competitive. The firms' wage markdown power then becomes another source of deadweight loss, which reduces worker surplus without full transfer to producer surplus or consumer surplus. This part of the deadweight loss can also be decomposed into loss of productive efficiency, because some less efficient firms engage in employment, and loss of allocative efficiency, because the most efficient firm employs less labour than socially desirable. The frontier firms' superior (broadly defined) technology fails to diffuse to all workers employed in this industry through the expansion of frontier firms' employment of labour. Under the labour market oligopsony in the absence of collective bargaining, statistical measures of inequality in the distribution of χ_{Li}, such as coefficient of variation or skewness, can be used to evaluate productive inefficiency, while the employment share weighted mean of χ_{Li} can be used to measure allocative inefficiency. Following the second fundamental equation of market powers (17), analogous measures based on δ_i can be used to quantify overall productive and allocative inefficiencies.[24]

[23] This line of argument can trace its origin to Banerjee and Duflo (2005), and Hsieh and Klenow (2009), who treat the dispersion in firm productivity distribution as a measure of productive inefficiency. In the current context, given the theoretical insight that the dispersion in the distribution of firm market powers is an effect of the dispersion in the distribution of firm productivity, we can treat the former also as a sign of productive inefficiency.

[24] The validity of this type of statements on productive inefficiency of dispersed distribution of market powers relies on the assumption of constant returns to scale for short-run production function. That is, technological frontier firms can expand their output and employment scales without suffering diminishing returns to scale. The cause of their self restraints on expansion therefore lies solely in their self awareness of bigness (relative to market size) and related market power considerations. In this case, if the dispersion in firm market power distribution persists, that can be interpreted as a possible sign of obstacle to knowledge diffusion.

The second fundamental equation of market powers (17) provides the mechanism knowledge which connects the markup (ρ_i) and markdown (χ_{Li}) powers and producer surplus (δ_i). Thus, firm gross profit margin δ_i is both a core variable in the determination of income distribution and a core measure of market power, with its links to productive and allocative inefficiencies. The distribution of gross profit margin δ_i is particularly informative about the overall state of competition and inefficiencies of the actual, imperfectly competitive markets.[25]

It is of interest to note the distribution of δ_i is also informative about functional income distribution because of the relations: $\nu_{Ki} = \frac{\delta_i}{\delta_i + \phi_{Li}}$ and $\nu_{Li} = \frac{\phi_{Li}}{\delta_i + \phi_{Li}}$. These indicate the mechanisms of market powers through which inefficiencies and inequity in functional income distribution are connected.

3 Extension of JOOM to Accommodate Collective Bargaining

A possible effect of collective bargaining on firm-specific wage markdown index χ_{Li} is to extend its domain from $\mathbb{R}_+$ to $\mathbb{R}$, i.e., allowing $\chi_{Li} \leq 0$. In this section we follow Crépon *et al.* (2005) to derive negative broadly defined markdown power index $\chi_{Li} < 0$ for the efficient bargaining model *a la* McDonald and Solow (1983), and theoretically identify its determinants. It is also desirable to let the extension cover Robinson (1933) wage floor model and the right-to-manage model (Nickel and Andrews, 1983), which can be implemented by including models restricted to $\chi_{Li} = 0$. In the full extension, the oligopsony model (OL), the right-to-manage bargaining model (RB) and the efficient bargaining model (EB) together form a partition of all possible relations between wage (W) and marginal revenue product of labour ($MRPL$). They respectively predict the three mutually exclusive possibilities: $W < MRPL$ (OL), $W = MRPL$ (RB) and $W > MRPL$ (EB).[26] The extension let the first and second fundamental equations of market powers (16) and (17) apply more generally.

Consider the workers collectively bargain with the firm over both the level of employment l_i and wage w_i, with the objective to maximise $l_i (w_i - \bar{w}_i)$, where $\bar{w}_i$ is the reservation wage. The firm's objective is to maximise short-run profit $R_i - w_i l_i - p_X x_i$, where $R_i = P_i(\mathbf{q}) q_i$ is the firm revenue, with $q_i = F_i(x_i, l_i)$ representing the output and production function. Therefore the *de facto* objective of decision[27] differs from profit maximisation. The outcome is given by the Pareto efficient Nash bargaining solution that solves the following maximisation problem:

$$\max_{w_i, l_i, x_i} \left[l_i (w_i - \bar{w}_i)\right]^{\eta_i} \left[R_i - w_i l_i - p_X x_i\right]^{1-\eta_i}, \tag{58}$$

[25] This statement becomes even more relevant given the fact that the second fundamental equation of market powers (17) can be extended to accommodate various labour market sharing mechanisms ranging from oligopsony to collective bargaining.

[26] This observation has been inspired by Dobbelaere and Mairesse (2013).

[27] The interpretation of this may go beyond merely collective bargaining. It may involve some form of worker participation in corporate governance.

where η_i is the worker absolute bargaining power coefficient. The first order conditions include:

$$\frac{\partial R_i}{\partial x_i} = p_X, \tag{59}$$

$$w_i = \bar{w}_i + \frac{\eta_i}{1-\eta_i}\frac{R_i - w_i l_i - p_X x_i}{l_i}, \tag{60}$$

$$w_i = \frac{\partial R_i}{\partial l_i} + \frac{\eta_i}{1-\eta_i}\frac{R_i - w_i l_i - p_X x_i}{l_i}. \tag{61}$$

The core of the model extension is extending the definition of marginal cost under relaxed cost minimisation requirement. More precisely, it allows the substitution between intermediate input and labour to violate cost minimisation. We extend the definition of marginal cost to

$$MC_i \equiv \frac{p_X}{\frac{\partial F_i}{\partial x_i}}. \tag{62}$$

This is the ratio between changes of cost and output caused by an infinitesimal dx_i, evaluated at the conditionally optimal level of x_i. The definition has two desirable features: (i) the input price p_X is invariant to input quantity x_i. (ii) The input quantity x_i is optimal conditionally (on w_i and l_i). This is reflected in the following implication from $\frac{\partial R_i}{\partial x_i} = \frac{\partial R_i}{\partial q_i}\frac{\partial F_i}{\partial x_i}$ and eq. (59):

$$MR_i \equiv \frac{\partial R_i}{\partial q_i} = \frac{p_X}{\frac{\partial F_i}{\partial x_i}} = MC_i, \tag{63}$$

i.e., marginal revenue equals marginal cost.

The definitions of markup and markdown power indices $\rho_i \equiv 1 - \frac{MC_i}{P_i}$ and $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i}$ apply to the extended JOOM.

3.1 Extension of the Fundamental Equations of Market Powers

Lemma 6 *For the efficient bargaining model, let marginal cost MC_i be defined by eq. (62). Then the marginal revenue product of labour and wage rate are given by*

$$MRPL_i \equiv \frac{\partial R_i}{\partial l_i} = \bar{w}_i, \tag{64}$$

$$w_i = (1-\eta_i)\,\bar{w}_i + \eta_i \omega_i, \tag{65}$$

i.e., the wage is the weighted average of $\bar{w}_i$ (which equals $MRPL_i$) and ω_i, where $\omega_i \equiv \frac{R_i - p_X x_i}{l_i}$ is the value added per worker, with the weights $(1-\eta_i)$ and η_i respectively. The range of w_i is given by

$$MRPL_i = \bar{w}_i \leq w_i \leq \omega_i.$$

The marginal cost also satisfies:

$$MC_i = \frac{(1+\chi_{Li})\, w_i}{\frac{\partial F_i}{\partial l_i}}. \tag{66}$$

Proof. Eq. (60) and (61) imply (64). $\omega_i \equiv \frac{R_i - p_X x_i}{l_i}$ allows eq. (60) to be rewritten as (65). $\chi_{Li} \equiv \frac{MRPL_i - w_i}{w_i}$ implies

$$\begin{aligned} MRPL_i &= (1+\chi_{Li})\, w_i = \frac{\partial R_i}{\partial l_i} = \frac{\partial R_i}{\partial q_i}\frac{\partial F_i}{\partial l_i}, \\ \frac{\partial R_i}{\partial q_i} &= \frac{(1+\chi_{Li})\, w_i}{\frac{\partial F_i}{\partial l_i}}. \end{aligned} \tag{67}$$

Eq. (63) and (67) imply (66). ■

Theorem 7 (Extension Theorem) *Let the labour market rent sharing mechanism in the extended joint oligopoly-oligopsony model either be an oligopsony as described in the joint oligopoly-oligopsony model, or worker collective bargaining as described by the efficient bargaining model. Let marginal cost MC_i be defined by eq. (62). Then the* ***First Fundamental Equation of Market Power*** *(16) applies to this extended model, i.e.,*

$$\rho_i + (1+\chi_{Li})\frac{\phi_{Li}}{\theta_{L_i}} = 1. \tag{68}$$

Furthermore, if the production functions $F_i(x_i, l_i)$ are homogeneous of degree 1, i.e., of constant return to scale, then the ***Second Fundamental Equation of Market Powers*** *(17) applies to this extended model, i.e.,*

$$\delta_i = \rho_i + \frac{\theta_{Li}(1-\rho_i)\,\chi_{Li}}{1+\chi_{Li}}. \tag{69}$$

Proof. Eq. (62) and $\rho_i \equiv 1 - \frac{MC_i}{P_i}$ imply

$$\frac{1}{1-\rho_i} = \frac{\theta_{Xi}}{\phi_{Xi}}. \tag{70}$$

Eq. (66) and $\rho_i \equiv 1 - \frac{MC_i}{P_i}$ imply (68). Finally, eq. (68) and (70) and constant return to scale imply (69). ■

3.2 Functional Income Distribution

This section mirrors Section 2.4.1 about functional income distribution in JOOM. The value added shares of labour and fixed capital have the equivalent definitions: $\nu_{Li} \equiv \frac{w_i}{\omega_i}$ and $\nu_{Ki} \equiv 1 - \frac{w_i}{\omega_i}$. It is obvious that we can extend the applicability of eq. (42)-(49) to the extended JOOM.

Proposition 8 *In the efficient bargaining model with $\bar{w}_i = \bar{w}$ and $\eta_i = \eta$, eq. (65) and the following hold:*

$$\nu_{Li} = (1-\eta)\frac{\bar{w}}{\omega_i} + \eta, \tag{71}$$
$$\chi_{Li} = \frac{\bar{w}}{\sigma\omega_i + (1-\eta)\bar{w}} - 1 \leqslant 0, \tag{72}$$

implying

$$\frac{\partial w_i}{\partial \omega_i} = \eta > 0, \tag{73}$$
$$\frac{\partial \nu_{Li}}{\partial \omega_i} = -(1-\eta)\frac{\bar{w}}{\omega_i^2} < 0, \tag{74}$$
$$\lim_{\omega_i \to \infty} \nu_{Li} = \eta > 0, \tag{75}$$
$$\frac{\partial \chi_{Li}}{\partial \omega_i} = -\frac{\sigma\bar{w}}{[\sigma\omega_i + (1-\eta)\bar{w}]^2} < 0. \tag{76}$$

The relationship between ν_{Li} and χ_{Li} is determined by

$$\nu_{Li} = \frac{\eta}{1-(1-\eta)(1+\chi_{Li})}, \tag{77}$$

implying

$$\frac{\partial \nu_{Li}}{\partial \chi_{Li}} > 0. \tag{78}$$

For Cobb-Douglas production function: $F_i = A_i x_i^{\alpha} l_i^{1-\alpha}$, the following hold:

$$\psi_{Li} \equiv \frac{w_i l_i}{w_i l_i + p_X x_i} = \frac{1}{1+\frac{\alpha}{1-\alpha}(1+\chi_{Li})}, \tag{79}$$
$$\psi_{Li} = \frac{1}{1+\frac{\alpha}{1-\alpha}\frac{1}{\eta\frac{\omega_i}{\bar{w}}+(1-\eta)}}, \tag{80}$$
$$\frac{\partial \psi_{Li}}{\partial \omega_i} > 0, \tag{81}$$
$$\delta_i = \frac{(1-\eta)\left(\frac{\omega_i}{\bar{w}}-1\right)}{\frac{\omega_i}{\bar{w}}+\frac{\alpha}{1-\alpha}}, \tag{82}$$
$$\frac{\partial \delta_i}{\partial \omega_i} = \frac{1-\eta}{(1-\alpha)\bar{w}\left(\frac{\omega_i}{\bar{w}}+\frac{\alpha}{1-\alpha}\right)^2} > 0, \tag{83}$$
$$\rho_i = 1 - \frac{1}{\alpha + (1-\alpha)\frac{\omega_i}{\bar{w}}}, \tag{84}$$
$$\frac{\partial \rho_i}{\partial \omega_i} = \frac{1-\alpha}{\bar{w}\left[\alpha+(1-\alpha)\frac{\omega_i}{\bar{w}}\right]^2} > 0. \tag{85}$$

For product market with Cournot competition, with $\rho_i = \frac{s_i}{\epsilon}$:

$$\frac{\partial s_i}{\partial \omega_i} > 0. \tag{86}$$

Proof. Eq. (65) implies (71), and then (74) is obvious. Eq. (65) also implies (73). Eq. (72) follows from the definition of χ_{Li} and (64), and then implies (72) and (76). Eq. (77) is implied by (71) and (72). For Cobb-Douglas production function $F_i = A_i x_i^{\alpha} l_i^{1-\alpha}$: Eq. (63) and (66) imply

$$\frac{p_X x_i}{w_i l_i} = \frac{\alpha (1 + \chi_{Li})}{1 - \alpha} \tag{87}$$

and (79). Then (72) and (79) imply (80) and (81). The following equations follow from the definitions of δ_i and ψ_{Li} and (80):

$$\begin{aligned} \delta_i &= \frac{\omega_i l_i - w_i l_i}{\omega_i l_i + p_X x_i} = \frac{\frac{\omega_i}{w_i} - 1}{\frac{\omega_i}{w_i} + \frac{p_X x_i}{w_i l_i}} \\ &= \frac{\frac{\omega_i}{w_i} - 1}{\frac{\omega_i}{w_i} + \frac{1}{\psi_{Li}} - 1} = \frac{(1-\eta)\left(\frac{\omega_i}{\bar{w}} - 1\right)}{\frac{\omega_i}{\bar{w}} + \frac{\alpha}{1-\alpha}}, \end{aligned}$$

and imply (83). Eq. (68), (72), (80) and (82) imply (84) and (85). The assumption of Cournot competition then implies (86). ■

Proposition 8 touches on some distinctive features of the efficient bargaining model (EB) that can be used for its identification against the oligopsony model (OL): (a) χ_{Li} decreases in ω_i for (EB) while the opposite holds for (OL). (b) ψ_{Li} increases in ω_i for (EB), in contrast to the opposite holding for (OL). (c) ψ_{Li} has a positive lower bound in η for (EB) while for (OL), ψ_{Li} converges to zero in the limit, i.e., $\lim_{\omega_i \to \infty} = 0$. Property (c) shows firm level wage stagnation is bounded in (EB) in the sense of $\nu_{Li} \geq \eta > 0$, i.e., ν_{Li} is bounded away from 0, which contrasts to (OL) with unbounded firm level wage stagnation: ν_{Li} is not bounded away from 0. So (c) is a useful feature for mechanism identification.

Proposition 8 further shows that there is a negative effect of ω_i on ν_{Li} through the channel of δ_i, which dominates the positive effect of ω_i on ν_{Li} through the channel of ψ_{Li}. This means that although the efficient bargaining mechanism can prevent more productive firms from substituting intermediate input for labour, it is insufficient to prevent them from gaining higher gross profit margin. The effect of δ_i tends to dominate the effect of ψ_{Li}, and decrease the value added share of labour ν_{Li}. Similarly, although the efficient bargaining mechanism can reduce the firm's markdown power χ_{Li} and prevent more productive firms from substituting intermediate input for labour, it is insufficient to prevent them from gaining higher markup power ρ_i. The effect of ρ_i tends to dominate the effect of χ_{Li}, and decrease the value added share of labour ν_{Li}.

4 Quantitative and Empirical Analyses

In this section, we first simulate or analytically derive theoretically predicted correlation patterns and then compare them to the estimated joint distribution of variables of interest using a panel of UK manufacturing firms. Section 4.1 presents simulations of the canonical JOOM, and Section 4.2 explores causal inference based on theoretical mechanism information. The simulated data is also used in Appendix C to illustrate that strategic interaction between competing firms introduces spatial correlation across observations that can cause biases to conventional methods for joint estimation of production function and market powers. A further strength of our empirical framework is that it avoids problems of measurement errors due to unobservability of fixed capital and prices of variable inputs and outputs, for the variables we use are calculated directly from accounting data. Differently from the control-function approach *a la* Olley-Pakes (1996) and Levenshon-Petrin (2003), our approach does not rest on the strict requirement of the invertibility of the unobserved productivity. In Section 4.3 we implement our novel factor cost share approach to identify industry specific production function parameters and estimate market power indices. Finally, we use regression analysis to validate the model predictions (Section 4.4).

4.1 Simulation: Firm Heterogeneity, Rise of Superstar Firm and Strategic Interaction in JOOM

We begin with a simulation of the canonical JOOM, a model-based thought experiment that permits only a single exogenous variation to the model: the increase of the dominant frontier firm's short-run MFP. This rise of the superstar firm's productivity is the root cause of all endogenous changes between the old and new (unique) Nash equilibrium points. This is the basis for causal interpretation of the correlation pattern we derive through simulation.

The values of the relevant parameters are given in Table 1. We skip the parameters of the canonical model which do not materially affect the variables of interest. (See Appendix B.1 for details).

Table 1: Model Parameters for Canonical JOOM

α	β	ϵ	n_1	n_2	n_3	A_1	A_2	A_3	γ
0.8	0.2	1.01	1	4	5	γ^5	γ^2	1	1.1

Parameter α is the output elasticity of intermediate input. The value of 0.8 is in line with our own estimation. Parameter β affects firm level labour supply elasticity. The value of 0.2 is compatible with those suggested in the literature. Parameter ϵ is the product market level demand elasticity. The value of 1.01 is a low market demand elasticity but is not unusual. We consider the firm population in the market consists of: one superstar (dominant frontier) - tier 1 firm, four tier 2 firms, and 5 (approximately) competitive fringe - tier 3 firms. The initial values of firm MFP are respectively $A_1 = \gamma^5$, $A_2 = \gamma^2$, and $A_3 = 1$, where $\gamma = 1.1$. These values capture (predetermined) firm heterogeneity, which causes inequality in firm performances. These causal links can be partially evidenced by correlation among the firm level variables.

We consider a single exogenous variation that changes A_1 from γ^5 (5 steps ahead of the fringe firms) to γ^6 (6 steps ahead), causing an increase of inequality in firm MFP distribution. Our simulation exercise is a thought experiment, designed to ensure there is a common root cause for all endogenous changes (effects). At the individual firm level, this root cause is the rise of the superstar firm's MFP, A_1. At the firm population distribution level, the root cause is the increase of inequality in firm short-run productivity distribution, as indicated by the increases in the coefficient of variation (CV) and skewness.

The simulation results, presented in Table 2, indicate that when the superstar firm's MFP increases, it causes either the firm's marginal cost to fall or the marginal revenue product of labour to rise, causing the markup (ρ_1) and markdown (χ_{L1}) powers, and the overall market power (δ_1) to rise. The increased competitiveness makes it profitable for the superstar firm to expand output and hence product market share (s_1), as well as value added share (ς_1), and to secure labour supply by offering higher wages (w_1) than the competitive fringe firms.[28] The superior productivity of the superstar firm reduces its rivals' relative competitiveness, and thus causes their market power measures ($\rho_j, \chi_{Lj}, \delta_j$ for $j = 2, 3$) to decrease, their product market shares (s_j) and value added shares (ς_j) to shrink, and their wages (w_j) to fall. The change in competitive pressure in this thought experiment is asymmetric: while the rival firms face an increase, the superstar firm faces a decrease. Accordingly, it is impossible to state whether the market competition has increased or decreased, because of the firm heterogeneity.

Additionally, the rise of the superstar firm's markdown power (χ_{L1}) reduces the firm's variable cost share of labour (ψ_{L1}), and increases substitution of flexible capital for labour, as required by cost minimisation. The rising δ_1 and falling ψ_{L1} cause the value added share of labour (ν_{L1}) to fall. For the rival firms, their falling markdown powers (χ_{Lj}) raise their variable cost shares of labour (ψ_{Lj}). Their falling δ_j and rising ψ_{Lj} cause the value added share of labour (ν_{Lj}) to rise.

These correlation patterns are summarised in Table 16 in Appendix D.1, and later used in Table 5.[29]

As far as firm level wage stagnation is concerned, the canonical JOOM predicts an unbounded firm level wage stagnation in oligopsony labour market (Proposition 5). In contrast for the efficient bargaining version of the

[28] If the superstar firm is much more productive than its rivals, it may not need to pay a substantial wage premium to hire more labour to achieve output and product market share expansions. This possibility therefore does not rule out negative correction between ρ_i and χ_{Li}, and between ω_i and s_{Li}.

[29] We note that the key variable MFP A_i is not observable from the firm level micro panel data, but the value added per worker ω_i is observable. We therefore use average value added of labour (denoted by ω_i) as a proxy for A_i for empirical work. To confirm proxy quality, we look at their simulated correlation coefficient, which is very high at 0.99.

Table 2: Simulation of Canonical JOOM

$t=1$	A	ω	w	s	ς	ρ	χ_L	δ	ϕ_L	ψ_L	ν_L
tier1*1	1.1^5	9.1827	1.3199	.27796	.36787	.275 21	1.4007	.35973	.060386	.094313	.14374
tier2*4	1.1^2	3.3543	1.1476	.12291	.11512	.12170	.72708	.19559	.10172	.12645	.34212
tier3*5	1	1.4338	1.0244	.046077	.03433	.045621	.12954	.067528	.16898	.18122	.71448
mean	1.1451	2.9769	1.1032	0.1	0.1	.099012	.49567	.14797	.13122	.15062	.50846
std	.194	2.3802	.097691	.073263	.10232	.072539	.43507	.097904	.041 67	.033647	.22507
CV	.16942	.79956	.088552	.73263	1.0232	.73263	.87774	.66165	.31756	.22339	.44265
skew	1.1872	1.6377	.86579	1.2461	1.6634	1.246	.66233	.78049	−.26085	−.24093	−.20354
w.mean	1.2432	3.3663	1.1212	.14831	.19423	.14684	.57711	.21171	.10573	.13412	.33306
$t=2$	A	ω	w	s	ς	ρ	χ_L	δ	ϕ_L	ψ_L	ν_L
tier1*1	1.1^6	11.066	1.3289	.32777	.44904	.32453	1.4464	.40454	.055209	.092716	.12009
tier2*4	1.1^2	3.0714	1.1271	.11784	.10305	.11668	.6403	.18576	.10769	.13225	.36697
tier3*5	1	1.3071	1.0129	.04017	.027752	.039772	.068884	.052179	.17966	.18955	.77494
mean	1.1612	2.9887	1.0988	0.1	0.1	.099011	.4352	.14085	.13843	.15695	.54627
std	.23851	2.9704	.10348	.088851	.12822	.087973	.45482	.11398	.046194	.036332	.25203
CV	.205 40	.99388	.094175	.88851	1.2822	.88852	1.0451	.80923	.3337	.23149	.46137
skew	1.5254	1.92	.97737	1.5353	1.9278	1.535 2	.87443	1.0273	−.34689	−.31686	−.25407
w.mean	1.3005	3.5594	1.1131	.17105	.24797	.16936	.54081	.23064	.10494	.1364	.31272

extended JOOM, the wage determination is:

$$w_i = (1-\eta)\,\bar{w} + \eta\omega_i, \tag{88}$$

implying the determination of value added share of labour:

$$\nu_{Li} = \eta + (1-\eta)\,\frac{\bar{w}}{\omega_i}. \tag{89}$$

Proposition 8 predicts that efficient bargaining imposes a positive lower bound to ν_L.

The simulation results in Table 2 and eq. (72) and (77) elucidate the predicted relationship between $\ln \nu_{Li}$ and $\ln(1+\chi_{Li})$, and between $\ln \omega_i$ and $\ln(1+\chi_{Li})$, illustrated by Figures 6 and 7 in Appendix D.2. They provide strong mechanism identifications, which we articulate in the Section 4.2, and use for empirical analysis in Section 4.4.

4.2 Mechanism Identification and Causal Inference Based on Extended JOOM

Now we explore the theoretical mechanism information further, with an emphasis on the difference of labour market rent sharing mechanisms. For that, we turn to the theoretical prediction of the efficient bargaining model of Section 3. The analytical results derived thereof show much similarity with its counterpart in the canonical JOOM of Section 2, but with some interesting distinctions (see point 2 below). The commonality with JOOM prediction is the negative relation between ω_i and ν_{Li}, as shown in Table 5. The intuitive interpretation of this negative correlation is similar between oligopsony and efficient bargaining models. Higher short-run productivity ω_i means stronger relative competitiveness, which is manifested in larger overall market power δ_i, however allocated between ρ_i and χ_{Li}, and causes higher degree of firm level wage stagnation (measured by $\frac{\omega_i}{w_i} \equiv \frac{1}{\nu_{Li}}$) and lower labour share ν_{Li}.

The extended JOOM theory provides two robust transmission mechanisms:

1. **The productivity-competitiveness-market power mechanism**. Higher short-run productivity ω_i causes stronger relative competitiveness and overall market power δ_i, which, *ceteris paribus*, causes lower value added share ν_{Li}. The net effect of ω_i on ν_{Li} (after taking into account of the effect of ω_i on ν_{Li} through ψ_{Li}) is negative.

2. **The labour market rent-sharing mechanism**. A firm's RS type affects the effects of ω_i on ψ_{Li} and ν_{Li}. For RS type I firms with $\chi_{Li} > 0$, higher short-run productivity ω_i causes lower variable cost share of labour ψ_{Li}, which, *ceteris paribus*, causes lower value added share ν_{Li}. For RS type III firms with $\chi_{Li} < 0$, higher short-run productivity ω_i causes higher variable cost share of labour ψ_{Li}, which, *ceteris paribus*, causes higher value added share ν_{Li}.

The above theoretical mechanisms form the basis for our empirical identification and causal interpretation. The predicted correlation patterns support such mechanism information. Additionally, the most distinctive mechanism identification involves the broadly defined markdown power index χ_{Li}. First, χ_{Li} has negative value for the efficient bargaining model (EB), in contrast to the positive value in the oligopsony model (OL). Second, the comparative statics relationship between ω_i and χ_{Li} as well as ψ_{Li} also have opposite signs: for the canonical JOOM (OL), an increase of ω_i causes χ_{Li} to rise and ψ_{Li} to fall, the opposites hold for the efficient bargaining model (EB). The reason for this difference is that the efficient bargaining (EB) based rent sharing mechanism constrains the substitution of flexible capital (i.e., intermediate input) for labour with the effect of $\chi_{Li} < 0$, and imposes an upper bound for firm level wage stagnation: $\frac{\omega_i}{w_i} < \frac{1}{\eta}$ and a positive lower bound for labour share: $\nu_{Li} > \eta > 0$. These bounds and effects do not exist for the oligopsony model (OL).

4.3 Identification of Production Function Parameters, Estimation of Market Powers and Classification of Rent Sharing Types

The theoretical underpinning of our empirical identification of production function rests on the notion of competitive fringe firm, which is treated in Appendix B. The practical empirical estimation of parameter α and χ_L are given by eq. (40) and (41). Let the subscripts ikt stand for firm i in industry k in period t. We estimate α_{kt} by:

$$\alpha_{kt} = median\left\{1 - \psi_{Likt} : \text{all } i \text{ in industry } k \text{ in period } t\right\}. \tag{90}$$

Once α_{kt} is identified, χ_{Likt} is calculated according to:

$$\chi_{Likt} = \frac{1-\alpha_{kt}}{\alpha_{kt}} \frac{p_{Xkt}x_{ikt}}{w_{ikt}l_{ikt}} - 1, \tag{91}$$

where $p_{Xkt}x_{ijt}$ and $w_{ikt}l_{ikt}$ are available from firm level micro panel data. ρ_{ikt} is computed as

$$\rho_{ikt} = 1 - (1 + \chi_{Likt}) \frac{\phi_{Likt}}{1-\alpha_{kt}}, \tag{92}$$

where ϕ_{Likt} is the revenue share of payroll of firm i in industry k in period t.

The estimated distribution of χ_{Likt} (and δ_{ikt}) allows us to empirically classify the RS types. For given industry k in year t, the specification of the boundary of RS type II in the $(\delta_{ikt}, \chi_{Likt})$ space partitions the data set into three subsets. We define:

$$\begin{aligned}
\text{RS type I} &\equiv \left\{(\delta_{ikt}, \chi_{Likt}) : \ln(1+\chi_{Likt}) \geq 0.5 \ln(1+\delta_{ikt}) > 0\right\}, && (93)\\
\text{RS type II} &\equiv \left\{(\delta_{ikt},{}_{Likt}) : \left|\ln(1+\chi_{Likt})\right| < 0.5 \ln(1+\delta_{ikt})\right\}, && (94)\\
\text{RS type III} &\equiv \left\{(\delta_{ikt},{}_{Likt}) : \ln(1+\chi_{Likt}) \leq -0.5 \ln(1+\delta_{ikt}) < 0\right\}. && (95)
\end{aligned}$$

This partition has the following properties: (i) The point $(\delta_{ikt}, \chi_{Likt}) = (0, 0)$, which theoretically represents the competitive fringe firms, is on the common boundary of the three types. This property is desirable because it matches the fact the competitive fringe firms are the limit points for all three RS types. (ii) It allows some margin of error for the values of χ_{Li} so that some firms with $|\ln(1+\chi_{Li})| > 0$ (i.e., $|\chi_{Li}| > 0$) still belong to RS type II. (iii) The margin of error is an increasing function of δ_{ikt}. Figure 2 shows how the RS types classification partitions the $(\ln(1+\delta_{ikt}), \ln(1+\chi_{Likt}))$ space. Table 3 shows the RS types distribution. RS types I and III each has at least 40% of UK manufacturing firms.

The transition matrices reported in Table 18 of Appendix D.3 show that the classification of RS types is rather stable over time: an important requisite for our analysis.

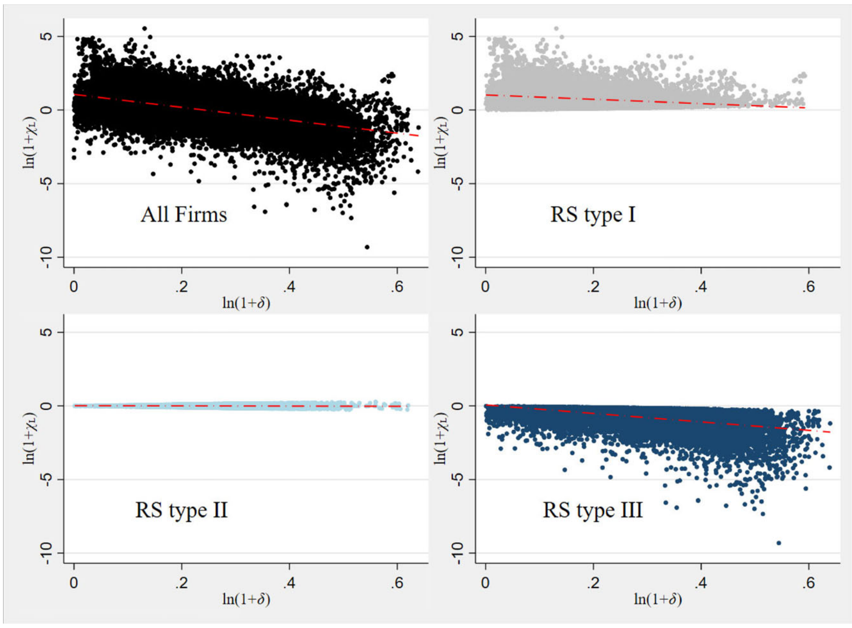

Figure 2: Partition of the $(\ln(1+\delta_{ikt}), \ln(1+\chi_{Likt}))$ space. The full set (of all firms) is partitioned into three mutually exclusive subsets, labelled RS Types I, II and III.

Table 3: RS Types Distribution

	RS type			
	I	II	III	Total
Obs.	25,067	10,406	24,891	60,364
%	42	17	41	100

4.4 Data and Empirical Results

4.4.1 Data Description

Our data are retrieved from the Fame database published by Bureau van Dijk. Fame includes all companies with 5 or more employees registered in the UK and Ireland (over 3 million companies). We focus on 4-digit sic industries in UK manufacturing sector for a 17-year period from 2003 to 2019.

The key variables retrieved from Fame include: turnover (revenue), cost of good sold, pay roll, employment, and 4-digit sic code. The cost of good sold includes both cost of intermediate inputs and labour, and is used as a measure of variable costs. Pay roll is used to measure variable labour cost, and the ratio between pay roll and employment is used for average wage. The difference between cost of good sold and pay roll captures the cost of intermediate inputs. Gross profit is computed as the difference between revenue and cost of good sold. We drop observations with negative gross profits, which constitute merely one percentage of the sample. Finally, value added is obtained as the sum of gross profit margin and pay roll. Value added per worker is used as a measure of the firm's short-run productivity.

Table 4: Summary Statistics

	Pooled Data			RS Type I		
	Obs	Mean	Std dev	Obs	Mean	Std dev
ω	60367	68.72	88.98	25067	82.48	125.6
CV_ω	3156	.52027	.31507	2305	.52436	.31125
δ	60367	.2842	.1370	25067	.2182	.1073
CV_δ	3156	.44670	.18670	2305	.40162	.18401
ν_L	60367	.4140	.1466	25067	.3782	.1491
CV_{ν_L}	3156	.31808	.13793	2305	.34328	.17494
α	3501	.74805	.10986	—	—	—
	RS Type II			**RS Type III**		
	Obs	Mean	Std dev	Obs	Mean	Std dev
ω	10406	66.21	59.70	24891	55.92	38.38
CV_ω	1908	.34503	.21833	2321	.39661	.22348
δ	10406	.2991	.1110	24891	.3444	.1438
CV_δ	1908	.27637	.17014	2321	.36972	.19577
ν_L	10406	.4002	.1257	24891	.4558	.1414
CV_{ν_L}	1908	.23310	.13831	2321	.27601	.16554
α	—	—	—	—	—	—

All the other relevant variables for the empirical analysis, including markups ρ_i and markdowns χ_{Li}, are easily computed from the variables above using the theoretical framework of Section 2, and the formulation in Section 4.3. Table 4 shows descriptive statistics of a few key variables and their coefficients of variation (CV) for different RS types. Complete summary statistics are reported in Tables 19 and 20 in Appendix D.4. The results show that firm heterogeneity and a high level of inequality are typical for 4-digit sic industries in UK manufacturing sector. Also, the mean value of parameter α is 0.75 with standard deviation of 0.11, values in line with those in the literature.

4.4.2 Correlation Evidence

Table 5 show the correlations between ω and some directly observable variables, and compares the theoretically predicted patterns and their counterparts in the data. The intra-equilibrium correlation patterns for both RS type I and RS type III predicted by the extended JOOM model are confirmed by our data.[30] Particularly interesting is the fact that data confirm the different signs of correlation between ω and ψ_L predicted by the oligopoly model (negative correlation) and efficient bargaining model (positive correlation).[31] By confirming statement 2 in Section 4.2, results in Table 5 allow the extended JOOM to pass the first mechanism identification hurdle.

[30] For ς_i, the efficient bargaining model does not have analytically derivable prediction, data show the behaviour is qualitatively similar across RS types I and III.

[31] This divergence is confirmed by regressions of ψ_{Li} on ω_i which are reported in Table 9.

Table 5: Comparison of Correlation Patterns between Models and Data (Part 1)

Correlation with ω	w (1)	s (2)	ς (3)	δ (4)	ψ_L (5)	ν_L (6)
Oligopsony Model RS type I Data	$+$ .4003	$+$ .1231	$+$ .1284	$+$ .3038	$-$ $-.2169$	$-$ $-.3979$
E. Bargainging Model RS type III Data	$+$ .6418	$+$ .0869	? .1165	$+$ .5230	$+$ .0349	$-$ $-.5543$
Correlation with CV_ω	CV_w (7)	CV_s (8)	CV_ς (9)	CV_δ (10)	CV_{ψ_L} (11)	CV_{ν_L} (12)
Oligopsony Model RS type I Data	$+$ .4644	$+$ .3162	$+$ .3487	$+$ .3005	$+$ .3287	$+$ .5276
E. Bargainging Model RS type III Data	$+$ .6255	$+$ .1817	? .2009	$+$ .3659	$+$ .1629	$+$ .5709

4.4.3 Regression Evidence

Column (6) of Table 5 shows that the joint distribution of ν_{Likt} and ω_{ikt} have negative correlations for both RS type I and III. Column (12) further shows that the coefficients of variation of ν_{Likt} and ω_{ikt} calculated for each industry-year observation (k, t), have positive correlation for both RS type I and III. The regressions of ν_L on ω, reported in Table 6, provide further evidence of a negative statistical relation between ν_L and ω. For RS types I and III and pooled data sets, the effects of ω on ν_L are all negative, statistically significant at 1%, and economically large: one upward standard deviation of ω from the mean causes ν_L to fall by 0.06 ($= -0.0005 \times 125.6$), 0.07 ($= -0.0019 \times 38.38$) and 0.05 ($= -0.0006 \times 88.98$) from the mean. These are respectively 0.4, 0.5 and 0.3 of the standard deviations of ν_L. In the regression analysis we control for α and HHI_ς to ensure that the change in ν_L is not caused by industry level change in technology, or concentration. These results in combination with theoretical mechanisms information suggest that the heterogeneity in firm short-run productivity ω is the root cause of the dispersion in firm level value added share of labour ν_L.

To examine the mechanism statements articulated in Section 4.2 empirically, we first regress ν_L on δ and ψ_L, and then δ and ψ_L, respectively, on ω. Tables 7, 8 and 9 report the results. All regression coefficients have the predicted signs and are statistically significant at 1%, thus providing strong support to the mechanism statements. Interestingly, the regression of ν_L on ω is intimately related to wage regressions in the empirical rent sharing literature of labour economics (see Card *et al.*, 2018). We explore this relation next.

4.4.4 Wage Regressions and Rent Sharing Mechanisms

The regressions of ν_{Li} on ω_i reported Table 6 do not control the wage rate w_i. Without loss of generality we consider ν_{Li} to be a function of the vector $\boldsymbol{\omega} \equiv (\omega_j)_{j\in\mathcal{N}}$ and RS type T_i, denoted by $\nu_L(\boldsymbol{\omega}, T_i)$, with $\frac{\partial \nu_L(\boldsymbol{\omega}, T_i)}{\partial \omega_i} < 0$. Thus, the following equation holds:

Table 6: Regression of ν_L on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L
ω	-.0005***	-.0005***	-.0005***	-.0019***	-.0019***	-.0019***	-.0006***	-.0006***	-.0006***
	(.0001)	(.0001)	(.0001)	(.0004)	(.0004)	(.0004)	(.0002)	(.0002)	(.0002)
α		-.1426***	-.1484***		-.0605	-.0679*		-.1199***	-.1197***
		(.0469)	(.0448)		(.0389)	(.0378)		(.0289)	(.0289)
HHI_ς			.0244			.0313**			-.0003
			(.0251)			(.0142)			(.0003)
Con	1.592***	1.710***	1.699***	.5921***	.6426***	.6267***	.9235***	1.023***	1.023***
	(.4029)	(.4066)	(.4092)	(.0202)	(.0384)	(.0388)	(.1766)	(.1817)	(.1817)
R^2	.301	.301	.301	.403	.403	.403	.275	.275	.275
Obs	25300	25300	25300	25308	25308	25308	60367	60367	60367

$$\ln w_i = \ln \nu_L(\boldsymbol{\omega}) + \ln \omega_i,$$

with log-log linear approximation:

$$\ln w_i \approx C + \xi \ln \omega_i, \tag{96}$$

where ξ is the rent sharing elasticity, a key parameter in the empirical rent sharing literature. This approximation implies

$$\nu_{Li} \equiv \frac{w_i}{\omega_i} \approx e^C \omega_i^{\xi-1} \geq 0, \tag{97}$$

and for $\xi \in (0,1)$:

$$\lim_{\omega_i \to \infty} \nu_{Li} = 0,$$

which is characteristic of the oligopsony model. Therefore, we expect this log-log linear approximation to be appropriate for RS type I.

The efficient bargaining model suggests an alterative formulation:

$$w_i = (1-\eta)\bar{w} + \sigma\omega_i, \tag{98}$$

where $\bar{w}$ is the external competitive wage offer, and η is the workers' collective bargaining power parameter. This

Table 7: Regression of ν_L on δ and ψ_L

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L
δ	-1.249***	-1.249***	-1.249***	-1.129***	-1.129***	-1.129***	-1.151***	-1.151***	-1.151***
	(.037)	(.037)	(.037)	(.013)	(.013)	(.013)	(.0185)	(.0185)	(.0185)
ψ_L	1.414***	1.420***	1.420***	.509***	.507***	.507***	.757***	.756***	.756***
	(.044)	(.045)	(.045)	(.013)	(.013)	(.013)	(.016)	(.016)	(.016)
α		.074***	.074***		-.064***	-.052***		-.024	-.023
		(.026)	(.026)		(.019)	(.017)		(.018)	(.018)
HHI_ς						.008			-.0002**
						(.005)			(.0001)
Con	.479***	.404***	.404***	.624***	.677***	.674***	.516***	.535***	.535***
	(.012)	(.027)	(.027)	(.003)	(.017)	(.017)	(.004)	(.017)	(.017)
R^2	.856	.856	.856	.949	.949	.949	.839	.839	.839
Obs	25067	25067	25067	24891	24891	24891	60367	60367	60367

formulation implies

$$\begin{aligned}\nu_{Li} &\equiv \frac{w_i}{\omega_i} = (1-\eta)\frac{\bar{w}}{\omega_i} + \eta > 0, \quad (99)\\ \lim_{\omega_i \to \infty} \nu_{Li} &= \eta > 0,\end{aligned}$$

which is a hallmark of the efficient bargaining model. We expect the linear formulation to be appropriate for RS type III.

Table 10 reports log-linear wage regression results based on (96). The coefficient on $\ln \omega$ is the elasticity of rent sharing. Its estimates for RS types I and III, are respectively 0.39 and 0.55 and the one for the pooled data sits in the middle at 0.44. These are larger than the $(0.2, 0.3)$ range documented by the literature (see Card *et al.*, 2018), although not specifically for manufacturing sector. Our theory shows that the notion of constant elasticity of rent sharing ξ may be applicable to labour market oligopsony, but not to efficient bargaining. Accordingly, we take 0.4 as our mean estimate of rent sharing elasticity for RS type I. Table 11 reports the linear wage regression results based on (98). The estimated values of η for RS types I and III, and the pooled data are: 0.03, 0.13 and 0.04. We take 0.13 as our mean estimated rent share for RS type III since our theory shows that the parameter is meaningful only for this type.

The labour share determination equations for RS types I and III are structurally different. For the former the mean is

$$\nu_{Li} = C\omega_i^{-(1-0.4)} = C\omega_i^{-0.6},$$

Table 8: Regression of δ on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	δ	δ	δ	δ	δ	δ	δ	δ	δ
ω	.0003***	.0003***	.0003***	.0020***	.0020***	.0020***	.0003***	.0003***	.0003***
	(.0001)	(.0001)	(.0001)	(.0003)	(.0003)	(.0003)	(.0001)	(.0001)	(.0001)
α		-.2296***	-.2295***		-.3577***	-.3535***		-.3149***	-.3147***
		(.0349)	(.0349)		(.0417)	(.0413)		(.0238)	(.0238)
HHI_ς			-.0002			-.0155			-.0004*
			(.0001)			(.0119)			(.0002)
Con	-.3292**	-.1406	-.1406	.1638***	.4614***	.4691***	-0.0107	.2499***	.2500***
	(.1659)	(.1725)	(.1725)	(.0172)	(.0381)	(.0387)	(.0826)	(.0867)	(.0867)
R^2	.337	.340	.340	.431	.434	.434	.200	.204	.204
Obs	25067	25067	25067	24891	24891	24891	60367	60367	60367

Table 9: Regression of ψ_L on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ψ_L	ψ_L	ψ_L	ψ_L	ψ_L	ψ_L	ψ_L	ψ_L	ψ_L
ω	-.0001***	-.0001***	-.0001***	.0003***	.0003***	.0003***	-.0003***	-.0003***	-.0003***
	(.0000)	(.0000)	(.0000)	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)	(.0001)
α		-.3599***	-.3598***		-.8148***	-.8109***		-.6130***	-.6128***
		(.0358)	(.0358)		(.0676)	(.0676)		(.0370)	(.0370)
HHI_ς			-.0001***			-.0142			-.0001***
			(.0000)			(.0129)			(.0000)
Con	.3635***	.6592***	.2000***	.3012***	.2149***	.9859***	.1615***	.9690***	.9689***
	(.0873)	(.0947)	(.0042)	(.0080)	(.0116)	(.0561)	(.0075)	(.0887)	(.0887)
R^2	.487	.499	.499	.399	.414	.414	.207	.215	.215
Obs	25067	25067	25067	24920	24920	24920	60855	60855	60855

Table 10: Regression of $\ln w$ on $\ln \omega$

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	$\ln w$	$\ln w$	$\ln w$	$\ln w$	$\ln w$	$\ln w$	$\ln w$	$\ln w$	$\ln w$
$\ln \omega$	.3902***	.3906***		.5524***	.5525***		.4426***	.4428***	
	(.0136)	(.0137)		(.0368)	(.0368)		(.0188)	(.0188)	
Lag $\ln \omega$			.3417***			.4839***			.3922***
			(.0119)			(.0417)			(.0200)
α		-.3927***	-.2729**		-.1199*	-.0407		-.2577***	-.1654**
		(.1201)	(.1253)		(.0705)	(.0731)		(.0568)	(.0610)
HHI_{s_L}		.0213	-.0097		.0029	.0156		.0260	.0090
		(.0312)	(.0384)		(.0226)	(.0224)		(.0209)	(.0229)
Con	.6385***	.9523***	1.168***	.9353***	1.034***	1.2761***	.9328***	1.134***	1.339***
	(.0908)	(.1306)	(.1330)	(.1398)	(.1536)	(.1705)	(.0932)	(.0976)	(.1047)
R^2	.548	.548	.491	.682	.682	.608	.589	.589	.526
Obs	25067	25067	23018	24891	24891	20190	60367	60367	54745

where C is a constant, with $\lim_{\omega_i \to \infty} \nu_{Li} = 0$. For the latter the mean is

$$\nu_{Li} = 0.13 + \frac{(1-\eta)\,\bar{w}}{\omega_i},$$

where $\bar{w}$ is the estimated external wage offer, $\lim_{\omega_i \to \infty} \nu_{Li} = 0.13 > 0$. So for RS types I and III, the firm level wage stagnation patterns are also structurally different, particularly at the limits. Figure 3 shows the scatter plot of $(\ln(1+\chi_{Likt}), \ln \nu_{Likt})$. It is apparent that the RS type III firms behave differently from the RS type I firms. The obvious difference resembles the theoretical relationship depicted in Figure 6 in Appendix D.2 in the following: (i) For RS type I, χ_{Likt} and ν_{Likt} are negatively correlated, in contrast to the positive correlation for RS type III. (ii) $\ln \nu_{Likt}$ as a function of $\ln(1+\chi_{Likt})$ looks more likely to have a finite lower bound for RS type III, compared to RS type I.

So far, our analysis at firm-level has revealed that wage stagnation and low value added share of labour happen at the top of the firm productivity distribution. To understand wage stagnation at the aggregate level, we need to consider that ν_L is negatively correlated with value added share ς, as shown in Table 12. This implies the covariance between ς and ν_L is negative. Consequently,

$$\mu^{\varsigma}_{\nu_L} = \mu_{\nu_L} + cov\,[\varsigma, \nu_L] < \mu_{\nu_L}, \tag{100}$$

where $\mu^{\varsigma}_{\nu_L}$ and μ_{ν_L} are respectively the ς-weighted mean and simple mean of ν_L. Then the link between wage stagnation at the firm level and in the aggregate is that firms at the bottom of the value added share of labour (ν_L) distribution tend to appear at the top of the value added share (ς) distribution. Because ς is the weight for

Table 11: Regression of w on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	w	w	w	w	w	w	w	w	w
ω	.0304***	.0305***		.1316***	.1316***		.0388***	.0388***	
	(.0091)	(.0091)		(.0243)	(.0243)		(.0125)	(.0125)	
Lag ω			.0274***			.1230***			.0348***
			(.0076)			(.0161)			(.0109)
α		-3.653	-4.007		-2.788	-1.325		-4.112***	-3.022**
		(3.452)	(2.611)		(1.741)	(1.919)		(1.407)	(1.354)
HHI_{s_L}		.7478	.6416		.2235	.0682		1.346	1.269
		(1.644)	(1.334)		(0.6546)	(.7176)		(.8545)	(.8917)
Con	-42.99**	-38.46*	-36.61*	14.59***	16.80***	16.76***	-12.20	-9.416	-6.742
	(20.73)	(21.41)	(19.15)	(1.844)	(1.918)	(1.825)	(11.33)	(11.69)	(10.48)
R^2	.326	.327	.305	.508	.508	.471	.321	.321	.303
Obs	25067	25067	23018	24891	24891	22304	60367	60367	54745

aggregation of value added share of labour, other things being equal, more inequality in the distributions of ν_L and ς tend to further depress $\mu^{\varsigma}_{\nu_L}$ below μ_{ν_L}, resulting in more severe aggregate wage stagnation.

Table 12: Comparison of Correlation Patterns between Models and Data (Part 2)

Correlation with ν_L	w (1)	s (2)	ς (3)	δ (4)	ψ_L (5)	ρ (6)	χ_L (7)
Oligopsony Model	$-$	$-$	$-$	$-$	$+$	$-$	$-$
RS type I Data	$-.1671$	$-.0086$	$-.0294$	$-.6013$	.3945	$-.3402$	$-.1250$
E. Bargaining Model	$-$	$-$	?	$-$	$+$	$-$	$+$
RS type III Data	$-.0696$	$-.0586$	$-.0943$	$-.8042$	.0155	$-.4840$	.1097

4.4.5 Attribution of Income Inequity to Market Powers

Existing empirical studies have looked at the effects of market powers on the labour share based on regressions of ν_L on ρ and χ_L (or their proxies commonly used in the literature such as $\frac{1}{1-\rho} \equiv \frac{P}{MC}$ and $1+\chi_L \equiv \frac{MRPL}{w}$), without controlling RS type (see Tortarolo and Zarate, 2020, Mertens, 2022, and Traina, 2022). However, the extended JOOM theory makes clear that it is crucial to control for different labour market rent sharing mechanisms. Furthermore, both our theory and empirical evidence show that the signs and magnitude of markup and markdown power indices depend on the level of substitution of flexible capital (i.e. intermediate input) for labour, which in turn depends on firm productivity ω and is affected by the firm's rent sharing mechanism. In Appendix D.5 we show to what extent: (a) heterogeneity in (ρ, χ_L) can explain heterogeneity in ν_L, and (b) heterogeneity in ω can explain changes in ρ and χ_L. In this section we study to what extent: (c) (ρ, χ_L) can explain the transmission of

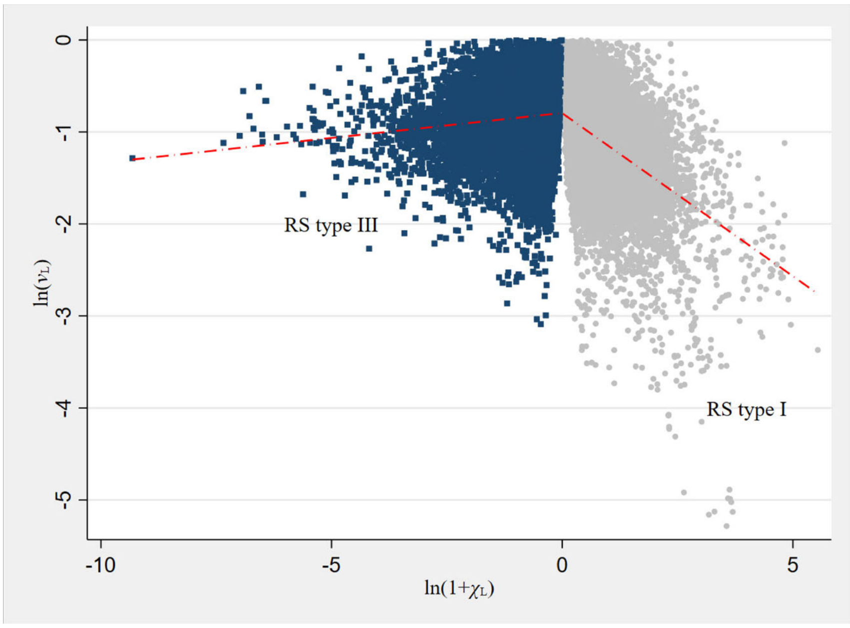

Figure 3: Scatter plot of $(\ln(1+\chi_{Likt}), \ln(1+\nu_{Likt}))$.

changes in ω to changes in ν_L, and (d) the combination of (δ, ψ_L) can outperform (ρ, χ_L) in (c).

We consider one standard deviation increase of ω, and trace its effects on ν_L, through the channels of ρ and χ_L vs δ and ψ_L. This exercise is conducted separately for RS types I and III, and the pooled data. The results, which are reported in Table 13, show that the effects of variation in ω on ν_L, transmitted though the channels of (δ, ψ_L) are all comparable with regressions of ν_L on ω. In contrast, the effects through the channels of (ρ, χ_L) are biased downward in magnitude, even more so for RS type I and the pooled data. This is related to the fact that, as measured by R^2 values, ω does a better job in explaining variation in (δ, ψ_L) than (ρ, χ_L), as shown in Tables 8 and 9, and Tables 22 and 23 in Appendix D.5. From an economic point of view, the reasons for the superior performance of (δ, ψ_L) relative to (ρ, χ_L) are twofold. First, higher short-run productivity ω implies stronger relative competitiveness: the ability to sustain gross profitability under the competitive pressure exerted by rival firms. Second, the variable cost share of labour ψ_L depends both on the interaction between labour market rent sharing mechanism (RS type) and productivity ω. This explains why, as shown in Table 9, ω can explain ψ_L for both RS types I and III, even though the effects of ω on ψ_L have opposite signs. In contrast, (ρ, χ_L) are measurements of deviations from price behaviour in product and labour markets. Each in isolation, cannot measure a firm's relative competitiveness. As explained above, that δ is an increasing function of both ρ and χ_L, and ψ_L is a decreasing function of χ_L. Although ρ and χ_L can be used as proxies for δ and ψ_L, we know these proxies are imperfect because eq. (35) shows that χ_L is a non-linear decreasing function of ψ_L, and eq. (37) shows that ρ is a quadratic function of δ and ψ_L with interaction term.

The way a firm departs from the benchmarks of price taking behaviour in product and labour markets depends not only on the interaction between labour market rent sharing mechanism (RS type) and productivity ω, but can also be affected by response to potential competition, e.g., whether to accommodate or deter entry, an issue which our static JOOM theory has abstracted away.[32] Entry deterrence incentive can lead to limit pricing, and even product price below marginal cost, i.e., $\rho < 0$. Our data provides clear evidence of negative values of ρ for RS type I firms, as is shown in Figure 4. Another piece of evidence which is consistent with the limit pricing hypothesis is the negative correlation between ρ and χ_L even for RS type I, as is shown in Figure 4. This fact does show a particular limitation of the simulation based on the canonical JOOM, namely the assumption of $\rho_i = \frac{1}{\epsilon_i}$. This, however, does not affect the validity of the two fundamental equations of market powers and their extensions, which are independent of this assumption. The limit pricing hypothesis may help explain why the performance of both (δ, ψ_L) and (ρ, χ_L) in explaining the effect of ω on ν_L is worse for RS type I than III. The reason is that limit pricing (or the response to potential competition or entry) may play a more crucial role for RS type I than III, in which case it can be considered an omitted relevant variable for the RS type I group.

Table 13: Effect of $\triangle\omega$ on $\triangle\nu_L$ and Transmission Mechanisms

	$\triangle\omega$*					$\triangle\nu_L$
RS type I	125.6					−.063
RS type III	38.4					−.073
Pooled data	89.0					−.053
	$\triangle\omega$*	$\triangle\rho$	Partial $\triangle\nu_L$	$\triangle\chi_L$	Partial $\triangle\nu_L$	Net $\triangle\nu_L$
RS type I	125.6	.025	−.009	1.193	−.003	−.012
RS type III	36.7	.066	−.087	−.022	.019	−.068
Pooled data	89.0	.000	−.000	.881	−.026	−.026
	$\triangle\omega$*	$\triangle\delta$	Partial $\triangle\nu_L$	$\triangle\psi_L$	Partial $\triangle\nu_L$	Net $\triangle\nu_L$
RS type I	125.6	.038	−.047	−.013	−.018	−.065
RS type III	36.7	.073	−.082	.011	.006	−.076
Pooled data	89.0	.027	−.032	−.027	−.020	−.052

*$\triangle\omega$ refers to one standard deviation change in ω

To conclude this section, we note that Table 13 also illustrates how the difference between RS types I and III affects the signs of $\triangle\chi_L/\triangle\omega$ and $\triangle\psi_L/\triangle\omega$, and contributes to the net $\triangle\nu_L/\triangle\omega$. Among RS type I firms, a superstar firm with superior productivity ω_i has both higher positive values of markup and markdown powers ρ_i and χ_{L_i}, each contributing to lowering the value added share of labour ν_{Li}. Higher χ_{L_i} also reduces ψ_{Li}, i.e., substitutes flexible capital for labour. In contrast, among RS type III firms, a superstar firm with superior productivity ω_i has higher positive value of ρ_i, but lower negative value of χ_{L_i}. The former contributes to lowering

[32] To accommodate entry deterrence behaviour, a Stackelberg type dynamic model may needed. The notion of limit pricing is relevant. An alternative can be a model of contestable market where limit pricing is used by incumbent firms to fend off potential entrant.

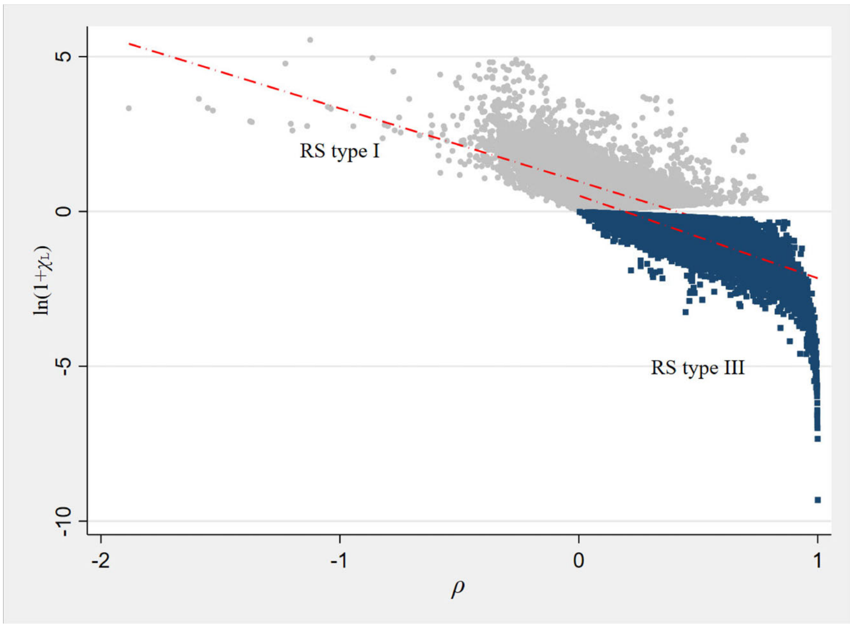

Figure 4: Scatter Plot in $(\rho_{ikt}, \ln(1+\chi_{Likt}))$ space.

the value added share of labour ν_{Li}, and the latter has an offsetting effect of increasing ν_{Li}, as well as increasing ψ_{Li}, i.e., curbing the substitution of flexible capital for labour. The net effect of increasing ω_i on ν_{Li}, through the channels of ρ_i and χ_{L_i} is negative. In light of the extended JOOM theory and the empirical evidence that underpins the results here, it is apparent that the efficient bargaining rent sharing mechanism (i.e., for RS type III firms) curbs wage markdown power and moderates firm level wage stagnation, while the oligopsony rent sharing mechanism (i.e., for RS type I firms) cannot. This implies that if a substantial fraction of RS type III firms transform into RS type I firms (which does not occur in the UK manufacturing during 2003-2019, but could have happened somewhere else, or in a different time period), then this change can exacerbate wage stagnation at the aggregate level (see Stansbury and Summers (2020)).

5 Policy Implications

In this section we explore the policy implications of our analysis. We begin by noting that the simulation of the canonical JOOM, which features labour market oligopsony (see Table 2), shows that the rise of the superstar firm does improve consumer surplus CS (by reducing product price), but not worker surplus WS (because of lowering weighted average wage). The rise of superstar firm's short-run productivity, measured by A and ω, increases the industry average of firm productivity and marginal product of labour. Its benefit passes on to consumers, but not to workers on average, as measured by the average wage.[33] The rise of a superstar firm, in the absence of efficient bargaining intervention, reduces the superstar firm's conditional demand for labour because of both an efficiency effect and effect of substituting flexible capital (i.e., intermediate input) for labour. Although the superstar firm increases wages for its own employees, it forces its competitors to reduce their wage offers as their sales and product market shares are reallocated to the superstar firm, reducing their demand for labour. The weakened competition from rival firms in the labour market moderate the pressure for the superstar firm to raise wage to secure labour supply. The net effect at industry level is a decrease in weighted average wage and fall of aggregate labour share of value added, as documented in the empirical literature.

The insight can also be stated in terms of wage stagnation. Firm level wage stagnation occurs in general with the dominant frontier firms, as they only share a decreasing fraction of gains in value added per worker with their employees, thus also causing a firm level fall of labour share in value added. In the absence of catching up, the laggard rivals of the frontier firms see their gross profit margins and value added shares in the market decline and their firm level labour shares of value added increase. The reallocation of value added share in the market to the dominant frontier firms from their laggard rivals, results in wage stagnation and fall of labour share of value added at the aggregate level, as documented in the empirical literature. This new insight into the causes of wage stagnation makes clear that the concentration of market power in the hands of dominant firms can produce inequitable economic growth (see Proposition 5).

Furthermore, the concentration of market power in dominant firms results in static inefficiencies which may be insufficiently compensated by dynamic efficiency. For static inefficiencies, recall that we argue in Section 2.4.3 that high inequality in δ causes overall productive and allocative inefficiencies. Since the inequality in firm short-run productivity ω is the root cause of inequality in δ, persistent sizable inequality in ω is also an indication of static inefficiency. Figure 1 visualises the fact that the inequality in the distributions of ω and δ is not only sizable, but also persistent over time. Overall, the superstar firms decrease worker welfare and do not capture the full efficiency gains.

Shifting the focus on dynamic efficiency, a relevant policy question is whether knowledge diffusion can help rectify some of these static inefficiencies as well as reduce inequity of functional income distribution caused by inequality in ω. To explore this issue, we first use the canonical JOOM to simulate the effect of knowledge diffusion that allows the rival firms to catch up with the superstar firm. Specifically, in stead of letting the superstar firm move forward one step further (keeping rivals stationary), we simulate the effect of letting all rivals move one step forward (keeping the superstar firm stationary). Results reported in Table 14 show that the knowledge diffusion

[33] According to NYT article: "Inside Amazon's Worst Human Resources Problem" (October 24, 2021) https://nytimes.blog/inside-amazons-worst-human-resources-problem/, "Amazon's workers routinely took a back seat to customers during the company's meteoric rise to retail dominance. Amazon built cutting-edge package processing facilities to cater to shoppers' appetite for fast delivery, far outpacing competitors. But the business did not devote enough resources and attention to how it served employees."

can reduce the coefficient of variation in ω as well as in δ. It increases the weighted means of ω and w and reduces the weighted mean of ν_L. These are achieved in addition to increasing consumer surplus by lowering product price. Therefore, knowledge diffusion improves both consumer and worker welfare, reduces aggregate inequity in income distribution, and enhances efficiency by reducing inequality in the distribution of δ. In comparison with the simulation of rising superstar firm (reported in Table 2), this simulation of knowledge diffusion increases the weighted means of both w and ν_L, reduces CV_A, CV_ω and CV_δ, and decreases product price (which is not reported in Table 2). That means knowledge diffusion can potentially rectify the undesirable effects of the rise of superstar firms.

Table 14: Simulation of Canonical JOOM with Knowledge Diffusion

$t=1$	A	ω	w	P	δ	ψ_L	ν_L
tier1*1	1.1^5	9.1827	1.3199	.28202	.35973	.094313	.14374
tier2*4	1.1^2	3.3543	1.1476	.28202	.19559	.12645	.34212
tier3*5	1	1.4338	1.0244	.28202	.067528	.18122	.71448
mean	1.1451	2.9769	1.1032	.28202	.14797	.15062	.50846
std	.194	2.3802	.097691	0	.097904	.033647	.22507
CV	.16942	.79956	.088552	0	.66165	.22339	.44265
skew	1.1872	1.6377	.86579	0	.78049	$-.24093$	$-.20354$
w.mean	1.2432	3.3663	1.1212	.28202	.21171	.13412	.33306
$t=2$	A	ω	w	P	δ	ψ_L	ν_L
tier1*1	1.1^5	7.3159	1.2914	.26100	.31445	.098323	.17652
tier2*4	1.1^3	3.6279	1.1653	.26100	.2049	.12194	.3212
tier3*5	1.1	1.5609	1.0360	.26100	.080876	.17366	.66370
mean	1.2435	2.9632	1.1133	.26100	.15384	.14544	.47798
std	.17265	1.8423	.089698	0	.083562	.03057	.20046
CV	.13884	.62173	.080569	0	.54317	.21019	.41939
skew	.72632	1.1171	.54191	0	.46834	$-.15437$	$-.13387$
w.mean	1.3181	3.2318	1.1272	.26100	.1983	.13248	.34879

Simulations in Tables 2 and 14 show that the superstar firms play a complex role in market competition. In their infancy, the "would be" superstar firms are innovators who strive for superior productivity. Successful superstar firms gain in sales and product market shares, and they exert more competitive pressure on their rivals. The anticipated rise in their market power is the incentive for their innovation in the first place. This has been recognised as an important driver of innovation and economic development by economists since Schumpeter (1934, 1942).

If the dominance of a superstar firm is transitory, followed by catching up by rival firms, and hence a churn, as envisaged by Schumpeter (1934), the rise of superstar firms has the potential to benefit both workers and consumers, and enhance efficiency in the long run. Figure 5 shows respectively the rank persistence along the dimensions of revenue and value added per worker. The graphs show the weighted and simple means of the number of firms that are in the top 10 rank (in a given metric) in 4-digit sic code manufacturing industries in each year, as well as three years ago.[34] Rank persistence over time is visible along both dimensions of revenue and value added per worker. This evidence is not consistent with the notion that the dominance of frontier firms is only transitory.[35] Therefore, while we have clear evidence of sizable and persistent inequality in firm short-run productivity and overall market power, with the implication of large and persistent static inefficiency, the evidence of churning among firms in size and short-run productivity is weak. In the absence of productivity growth, the argument for the so called trade off between static and dynamic efficiencies thus appears feeble. This new insight underscores the importance of addressing market power concentration to promote equitable and efficient economic growth.

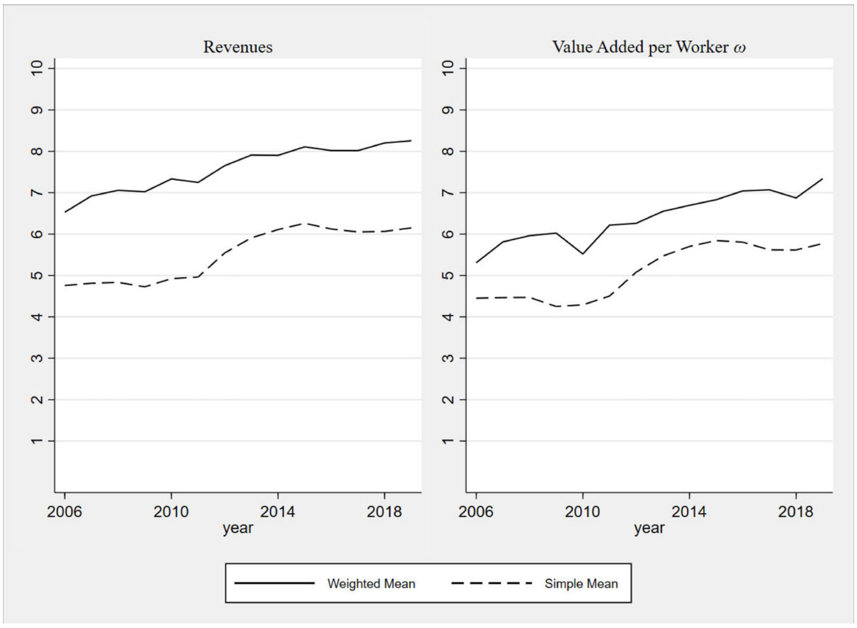

Figure 5: Top 10 Rank Persistency in Revenues and Value Added Per Worker

6 Conclusion

To investigate the relation between market power and income distribution, we develop hybrid industrial-labour economics models with imperfect competition in product market and various labour market rent sharing mechanisms, and show that predetermined differences in short-run productivity are a root cause of heterogeneities in firms' competitiveness, market powers and income distribution between capital and labour.

Using data of UK manufacturing firms, we find strong evidence of both oligopsony and efficient bargaining

[34] Rank persistence in turnover was used by CMA in their The State of UK Competition reports 2020 and 2022.
[35] For evidence of persistent super-normal profits, see Furman and Orszag (2018), Barkai (2020) and Gutiérrez and Philippon (2017).

mechanisms of labour market rent sharing, each accounting for around 40% of the firms in our dataset. We show that failing to distinguish between them in the empirical analysis can cause biases when attributing changes in income distribution to changes in markup or markdown powers, and incorrectly characterising asymptotic properties of firm level wage stagnation.

From the methodological point of view, we provide a novel way to estimate production function parameters and market powers, which relies on competitive fringe firms included in the remaining 20% of the firms. Although competitive fringe firms have little effects on aggregate market outcomes, methodologically they allow us to identify production function parameters, and obtain unbiased estimates of market power indices. Our approach provides the foundation for using widely available counting/financial data to measure firm level short-run productivity, competitiveness, market powers and distribution of value added.

We find that the inequality in firm short-run productivity is large and persistent, which causes persistent sizable inequality in firm competitiveness, market powers and value added share of labour. Our welfare and efficiency analyses show that these inequalities have inefficiency as well as income inequity implications. Dominant firms share a decreasing fraction of gain in value added per worker with their employees, thus causing wage stagnation and a fall of labour share. In the absence of catching up, the laggard rivals of the frontier firms see their gross profit margins and value added shares in the market decline and their firm level labour shares of value added increase. The reallocation of value added share in the market from laggards to frontier firms, results in aggregate wage stagnation and fall of labour share of value added. In short, the rise of superstar firms is detrimental to worker welfare.

Our work has two important policy implications. First, it provides theoretical support to the calls for a reform of antitrust enforcement to protect labour market competition (between firms), as exemplified by CEA (2016), Naidu *et al.* (2018), and Marinescu and Hovenkamp (2018). Second, it shows that factors that affect the inequality in firm short-run productivity, such as knowledge diffusion, are the key to reduce inequality in firm short-run productivity, and concentration of market power, and are therefore drivers for equitable and efficient economic growth. Barriers to knowledge diffusion should then receive more attention in policy making and enforcement.

Our analysis shows that the difference between rent-sharing mechanisms (RS types) plays an important role in affecting firm's behaviour in terms of substitution of flexible capital for labour as well as in affecting the asymptotic property of firm level wage stagnation. Among the UK manufacturing firms, the rent-sharing (RS) types were relatively stable for the period 2003-2019. It is not clear whether this was the case historically, in other sectors, and in other countries. These are certainly worthy questions for future research.

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A Extensions

A.1 JOOM with Price Competition and Wage Posting

In this section we study a variant of the JOOM presented in Section 2, by replacing quantity competition in the product market with price competition. Let the firm-specific demand be $D_i(\mathbf{p})$, and the short-run profit maximisation problem be:

$$\max_{p_i, x_i, w_i, D_i(\mathbf{p}) \leq F_i(x_i, L_i(\mathbf{w}))} \pi_i(\mathbf{p}, \mathbf{w}, x_i) = p_i D_i(\mathbf{p}) - w_i L_i(\mathbf{w}) - p_X x_i. \tag{101}$$

The Lagrangian multiplier method is given by:

$$\max_{p_i, x_i, w_i, \lambda_i} \mathfrak{L}_i = p_i D_i(\mathbf{p}) - w_i L_i(\mathbf{w}) - p_X x_i - \lambda_i (D_i(\mathbf{p}) - F_i(x_i, L_i(\mathbf{w}))). \tag{102}$$

The first order conditions are:

$$\frac{\partial \mathfrak{L}_i}{\partial p_i} = D_i(\mathbf{p}) + p_i \frac{\partial D_i(\mathbf{p})}{\partial p_i} - \lambda_i \frac{\partial D_i(\mathbf{p})}{\partial p_i} = 0, \tag{103}$$

$\frac{\partial \mathfrak{L}_i}{\partial x_i} = 0$ and $\frac{\partial \mathfrak{L}_i}{\partial w_i} = 0$, which are identical to (8) and (9);

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = D_i(\mathbf{p}) - F_i(x_i, L_i(\mathbf{w})) = 0. \tag{104}$$

The marginal revenue is given by

$$\lambda_i = p_i + \frac{D_i(\mathbf{p})}{\frac{\partial D_i(\mathbf{p})}{\partial p_i}} = \left(1 - \frac{1}{\epsilon_i}\right) p_i, \tag{105}$$

where $\epsilon_i \equiv -\frac{\partial D_i(\mathbf{p})}{\partial p_i} \frac{p_i}{D_i(\mathbf{p})}$ is the residual demand elasticity of firm i.

Eq. (22) and (23) remain valid for this variant of JOOM. With the variant definition of ϵ_i noted, eq. (21) can be extended to include the current case. Consequently, eq. (16) and (17) can be similarly extended hereto.

A.2 JOOM with Quantity and Employment Competition

Let the product market demand system be described by $P_i(\mathbf{q})$ for all i, and the labour market supply by wage function $W(L)$, where $L = \sum_{j=1}^{n} l_j$ is the total aggregate labour input and l_j is labour input of firm j. For this variant, the short-run profit maximisation is given by:

$$\max_{q_i,x_i,l_i,q_i\leq F_i(x_i,l_i)} \pi_i\left(\mathbf{q},\mathbf{l},x_i\right) = P_i\left(\mathbf{q}\right)q_i - W\left(L\right)l_i - p_X x_i, \tag{106}$$

with the Lagrange multiplier method:

$$\max_{q_i,x_i,l_i,\lambda_i} \mathfrak{L}_i = P_i\left(\mathbf{q}\right)q_i - W\left(L\right)l_i - p_X x_i - \lambda_i\left(q_i - f_i\left(x_i,l_i\right)\right). \tag{107}$$

The first order conditions that need slightly new treatment are:

$$\frac{\partial \mathfrak{L}_i}{\partial x_i} = -p_X + \lambda_i \frac{\partial F_i\left(x_i,l_i\right)}{\partial x_i} = 0, \tag{108}$$

$$\frac{\partial \mathfrak{L}_i}{\partial l_i} = -W\left(L\right) - W'\left(L\right)l_i + \lambda_i \frac{\partial F_i\left(x_i,l_i\right)}{\partial l_i} = 0, \tag{109}$$

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = q_i - F_i\left(x_i,l_i\right) = 0. \tag{110}$$

Define the residual labour supply elasticity for this variant by $\epsilon_{Li} \equiv \frac{1}{\frac{\partial W}{\partial l_i}\frac{l_i}{W}}$ and note that $\epsilon_{Li} = \frac{\epsilon_L}{s_{Li}}$, where $\epsilon_L \equiv \frac{1}{\frac{\partial W}{\partial L}\frac{L}{W}}$ is the market level labour supply elasticity and $s_{Li} \equiv \frac{l_i}{L}$ is the firm's labour market share. With these minor adjustments in place, eq. (21), (23), (16) and (17) can be extended to the current setting.

A.3 JOOM with Price and Employment Competition

Let the product market demand system be described by $D_i\left(\mathbf{p}\right)$ for all i, and the labour market supply by wage function $W\left(L\right)$, where $L = \sum_{j=1}^{n} l_j$ is the total aggregate labour input and l_j is labour input of firm j. The modified short-run profit maximisation problems are given by:

$$\max_{p_i,x_i,l_i,D_i(\mathbf{p})\leq F_i(x_i,l_i)} \pi_i\left(\mathbf{p},\mathbf{l},x_i\right) = p_i D_i\left(\mathbf{p}\right) - W\left(L\right)l_i - p_X x_i. \tag{111}$$

$$\max_{p_i,x_i,l_i,\lambda_i} \mathfrak{L}_i = p_i D_i\left(\mathbf{p}\right) - W\left(L\right)l_i - p_X x_i - \lambda_i\left(D_i\left(\mathbf{p}\right) - F_i\left(x_i,l_i\right)\right). \tag{112}$$

The first order conditions are the same as equations (103), (108) and (109) and

$$\frac{\partial \mathfrak{L}_i}{\partial \lambda_i} = D_i\left(\mathbf{p}\right) - F_i\left(x_i,l_i\right) = 0. \tag{113}$$

For this setting, we need to redefine: $\epsilon_i \equiv -\frac{\partial D_i(\mathbf{p})}{\partial p_i}\frac{p_i}{D_i(\mathbf{p})}$ and $\epsilon_{Li} \equiv \frac{1}{\frac{\partial W}{\partial l_i}\frac{l_i}{W}}$ with $\epsilon_{Li} = \frac{\epsilon_L}{s_{Li}}$ and $\epsilon_L \equiv \frac{1}{\frac{\partial W}{\partial L}\frac{L}{W}}$. Then eq. (21), (23), (16) and (17) can be extended to the current variant of JOOM.

B Coexistence of Superstar and Competitive Fringe Firms

Superstar firms and competitive fringe firms can coexist only if there is large dispersion in MFP. Studying the coexistence is important in our framework because, as explained in Section 2.3.1, we apply the factor (cost) share approach to competitive fringe firms in order to identify the common production function parameters. In Section B.1 we use the canonical JOOM for deriving the conditions for such coexistence. We study competitive fringe firms and efficient bargaining in Section B.2.

B.1 Canonical JOOM with Three Tiers of Firms

Let the short-run production function be Cobb-Douglas with the output elasticity of intermediate input being α:

$$F_i\left(x_i, l_i\right) = A_i x_i^{\alpha} l_i^{1-\alpha}.$$

The key feature of this model is the dispersion of firm short-run productivity, measured by Hicks-neutral technology coefficient A_i. To strike a balance between parsimony and descriptive realism, we assume all A_i only take three values: $A_1 > A_2 > A_3$. Thus, each firm i belongs to one of three subgroups of $\mathcal{N} \equiv \{1, \cdots, n\}$. Let $\{\mathcal{N}_1, \mathcal{N}_2, \mathcal{N}_3\}$ be a partition of $\mathcal{N}$ and $i \in \mathcal{N}_i$ for $i = 1, 2, 3$. We call $\mathcal{N}_1$ the frontier tier, each member of which is a frontier firm, essentially a clone of firm 1. $\mathcal{N}_2$ is the middling tier, and firm 2 represents a middling firm. $\mathcal{N}_3$ are the fringe firms, who are clones of firm 3. Let $n_i \equiv card\mathcal{N}_i$ be the cardinality of subset $\mathcal{N}_i$, for $i = 1, 2, 3$. So $n = \sum_{j=1}^{3} n_j$.

Let the firms engage in static Cournot competition in homogeneous good market with market inverse demand function $P\left(Q\right) = P_0 Q^{-\frac{1}{\epsilon}}$, where Q is the aggregate output and $\epsilon > 1$ is the constant demand elasticity. Note $\frac{1}{\epsilon_i} = \frac{s_i}{\epsilon}$.

We consider the Nash equilibrium in which there is symmetry within each subgroup $\mathcal{N}_i$, for $i = 1, 2, 3$. The unknowns of the Nash equilibrium simultaneous equation system include, for $i = 1, 2, 3 : w_i, q_i, x_i; l_i, s_i, \epsilon_{Li}, \phi_{Li}; Q, p$.

Using $\rho_i = \frac{s_i}{\epsilon}$ and $\chi_{Li} = \frac{1}{\epsilon_{Li}}$, the respective first fundamental equations of market powers imply, for $i = 1, 2, 3$:

$$\frac{s_i}{\epsilon} + \left(1 + \frac{1}{\epsilon_{Li}}\right) \frac{\phi_{Li}}{1-\alpha} = 1. \tag{114}$$

The other equations include:

$$
\begin{aligned}
l_i &= \mathcal{L}\frac{(w_i-b)^\beta}{\sum_{j=1}^{3} n_j (w_j-b)^\beta}, && (115)\\
\epsilon_{Li} &= \frac{\beta\sum_{j=1,j\neq i}^{3} n_j (w_j-b)^\beta}{\sum_{j=1}^{3} n_j (w_j-b)^\beta}\frac{w_i}{w_i-b}, && (116)\\
x_i &= \frac{\alpha}{1-\alpha}\left(1+\frac{1}{\epsilon_{Li}}\right)\frac{w_i l_i}{p_X}, && (117)\\
q_i &= A_i x_i^\alpha l_i^{1-\alpha}, && (118)\\
Q &= \sum_{j=1}^{3} n_j q_j, && (119)\\
s_i &= \frac{q_i}{Q}, && (120)\\
p &= P_0 Q^{\frac{1}{\epsilon}}, && (121)\\
\phi_{Li} &= \frac{w_i l_i}{p q_i}. && (122)
\end{aligned}
$$

B.1.1 Superstar Firm and Inequality in MFP Distribution

The dispersion of firm productivity is one of the most robust facts documented in the firm heterogeneity literature. In this context the adjective superstar usually carries the connotation of some distance from the nearest competitors. To capture this sense formally and statistically, we confine superstar firms to frontier firms along the short-run MFP A_i (approximated by value added per worker, ω_i) dimension in an industry where the firm level ω_i distribution is highly dispersed relative to mean, or strongly positively skewed. Accordingly, we reserve the term “rise of superstar firm” for the increase of dispersion relative to mean, or positive skewness in firm level ω_i distribution.[36]

So, what is the economic significance of inequality in firm short-run MFP distribution? The answer to this question should begin with recalling that the short-run MFP differs from the more commonly used variable: total factor productivity (TFP). MFP measures the contribution to output from factors other than flexible labour and intermediate inputs. It therefore includes that from all fixed capital (both tangible and intangible). In contrast, TFP does not include that from fixed capital which has been taken into account. Therefore short-run MFP A_i (or ω_i) measures the effective stock of fixed capital, and the inequality in short-run MFP distribution reflects the concentration of effective fixed capital stock at the top end of its distribution. This inequality between firms' MFP is also interesting for the study of inequality in income distribution. To understand the connection between the two, it is important to note that the strategic investment to acquire the fixed capital that underpins the

[36] According to our definition of superstar firm in an industry, because a monopoly industry has no dispersion or positive skewness in the firm MFP distribution, the term does not apply to the monopolist firm. In this sense, the term “winner takes all” cannot be characteristic of superstar firm either, because it means elimination of dispersion and skewness of firm MFP distribution. In contrast, the coexistence of superstar and (approximately) competitive fringe firms may be typical in industries which feature superstar firms.

short-run MFP is sunk. Sunk (predetermined, pre-committed) strategic investment affects the short-run strategic interaction between competing firms, and gives the lead investing firm a first mover advantage. The short-run MFP, not the commonly used TFP, underpins and hence measures the short-run competitiveness of each firm.

B.1.2 Winner Takes All vs Coexistence with Competitive Fringe Firms

In the extreme simplifying model that combines Schumpeterian creative destruction, Bertrand competition and constant marginal cost, winner takes all, even if the productivity gap between the frontier firm and its nearest rivals is very small. The winner-takes-all outcome is much harder to occur in the JOOM if the product market is a Cournot oligopoly product market and the labour market is an oligopsony *a la* Card *et al.* (2018), unless the frontier firm is sufficiently superior than its nearest rivals.

We first consider the winner-takes-all condition, particularly the threshold of the ratio $\frac{A_1}{A_2}$, denoted by g_1, such that if $\frac{A_1}{A_2} > g_1$ then winner takes all, i.e., the frontier firm monopolises the markets. Then we consider the competitive-fringe firm coexistence threshold of the ratio $\frac{A_1}{A_3}$, denoted by g_2, such that, taking A_2 and A_3 as given, for $\frac{A_1}{A_3} \to g_2$ then $\rho_3 \to 0$ and $\chi_3 \to 0$.

Proposition 9 *There exists g_1 such that for $\frac{A_1}{A_2} > g_1$: $s_1 = s_{L1} = 1$.*

Proof. Consider the cost minimisation problem faced by a hypothetical price taker firm with constant wage rate b and intermediate input price p_X, and production function $F(x,l) = Ax^{\alpha}l^{1-\alpha}$. The optimal inputs x, l must solve the following two equations:

$$\begin{aligned} Ax^{\alpha}l^{1-\alpha} &= q, \\ \frac{p_X x}{bl} &= \frac{\alpha}{1-\alpha}. \end{aligned}$$

The conditional input demands are

$$\begin{aligned} l^* &= \left(\frac{(1-\alpha)\, p_X}{\alpha b}\right)^{\alpha} \frac{q}{A}, \\ x^* &= \left(\frac{\alpha b}{(1-\alpha)\, p_X}\right)^{1-\alpha} \frac{q}{A}, \end{aligned}$$

with input ratio:

$$\frac{x^*}{l^*} = \frac{\alpha b}{(1-\alpha)\, p_X}.$$

The cost function for the price taker is

$$\begin{aligned} C(q) &= \left(\left(\frac{1-\alpha}{\alpha} \right)^{\alpha} + \left(\frac{\alpha}{1-\alpha} \right)^{1-\alpha} \right) \frac{b^{1-\alpha} p_X^{\alpha} q}{A} \\ &= \frac{p_X^{\alpha} b^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A} q, \end{aligned}$$

from which we can derive the constant marginal cost for the price taker:

$$MC = \frac{p_X^{\alpha} b^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A}. \tag{123}$$

There is a threshold of product price: $\bar{p} = MC$ such that if $P(Q) < \bar{p} = MC$, then winner-takes-all occurs, and $w_1 = b$. Consider the frontier firm's production function $F_1(x_1, l_1) = A_1 x_1^{\alpha} l_1^{1-\alpha}$. If $l_1 = L$, i.e., $s_{Li} = 1$, then $F_1(x_1, l_1) = A_1 x_1^{\alpha} l_1^{1-\alpha}$, and cost minimisation entails:

$$MC_1 \frac{\partial F_1}{\partial x_1} = \alpha MC_1 A_1 x_1^{\alpha-1} l_1^{1-\alpha} = p_X,$$

which implies that the marginal cost:

$$MC_1 = \frac{p_X}{\alpha A_1} \left(\frac{x_1}{l_1} \right)^{1-\alpha},$$

with

$$\frac{x_1}{l_1} \geq \frac{x^*}{l^*} = \frac{\alpha b}{(1-\alpha) p_X},$$

i.e., the input mix of the monopolising frontier firm tends to substitute intermediate input for labour as the labour supply becomes inelastic. Thus

$$MC_1 \geq \frac{p_X^{\alpha} b^{1-\alpha}}{A_1 \alpha^{\alpha} (1-\alpha)^{1-\alpha}}. \tag{124}$$

Profit maximisation entails

$$\rho_1 = \frac{1}{\epsilon} < 1,$$

and hence

$$P(Q) = \frac{MC_1}{1-\frac{1}{\epsilon}} \geq \frac{\frac{p_X^{\alpha} b^{1-\alpha}}{A_1 \alpha^{\alpha} (1-\alpha)^{1-\alpha}}}{1-\frac{1}{\epsilon}}. \tag{125}$$

The winner-takes-all threshold: $P(Q) < \bar{p} = MC$ implies

$$\begin{aligned} \frac{\frac{p_X^{\alpha} b^{1-\alpha}}{A_1 \alpha^{\alpha} (1-\alpha)^{1-\alpha}}}{1-\frac{1}{\epsilon}} &\leq \frac{p_X^{\alpha} b^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A} \\ &\Leftrightarrow \frac{A_1}{A} \geq \frac{1}{1-\frac{1}{\epsilon}}. \end{aligned} \tag{126}$$

Define $g_1 \equiv \frac{1}{1-\frac{1}{\epsilon}}$. Therefore for $\frac{A_1}{A_2} > g_1$, $s_1 = s_{L1} = 1$. ■

Claim 10 *For each given (A_2, A_3), there exists $g_2 > 0$ such that $A_1 \to g_2 A_3$ implies $\rho_3 \to 0$ and $\chi_{L3} \to 0$, i.e., tier 3 firms are (approximately) price takers in both product and labour markets.*

Analytical and Simulation Evidence. Consider tier 3 firms are approximately price takers in both the product and labour markets. Therefore their marginal cost is given by (123), i.e.,

$$MC_3 = \frac{p_X^{\alpha} b^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_3}. \tag{127}$$

We show, by construction, the existence of threshold: $\tilde{p} = MC_3$ such that if $P(Q) \to \tilde{p}$ then $\rho_3 \to 0$ and $\chi_{L3} \to 0$. Consider the frontier firm's production function $F_1(x_1, l_1) = A_1 x_1^{\alpha} l_1^{1-\alpha}$. The optimal flexible inputs are

$$\begin{aligned} l_1^* &= \left(\frac{(1-\alpha) p_X}{\alpha w_1 (1+\chi_{L1})}\right)^{\alpha} \frac{q_1}{A_1}, \\ x_1^* &= \left(\frac{\alpha w_1 (1+\chi_{L1})}{(1-\alpha) p_X}\right)^{1-\alpha} \frac{q_1}{A_1}, \end{aligned}$$

implying the input mix:

$$\frac{x_1^*}{l_1^*} = \frac{\alpha w_1 (1+\chi_{L1})}{(1-\alpha) p_X}. \tag{128}$$

The marginal cost is

$$MC_1 = \frac{w_1 (1+\chi_{L1})}{\frac{\partial F_1}{\partial l_1}} = \frac{w_1 (1+\chi_{L1})}{(1-\alpha) A_1 \left(\frac{x_1^*}{l_1^*}\right)^{\alpha}} = \frac{p_X^{\alpha} \left((1+\chi_{L1}) w_1\right)^{1-\alpha}}{\alpha^{\alpha} (1-\alpha)^{1-\alpha} A_1}. \tag{129}$$

The threshold price for fringe firms is:

$$\tilde{p} = \frac{MC_1}{1-\rho_1} = MC_3. \tag{130}$$

The optimal demands for the intermediate input satisfy

$$\frac{\frac{((1+\chi_{L1}) w_1)^{1-\alpha}}{b^{1-\alpha}}}{1-\frac{s_1}{\epsilon}} = \frac{A_1}{A_3}. \tag{131}$$

Similarly

$$\begin{aligned} \frac{\frac{((1+\chi_{L2}) w_2)^{1-\alpha}}{b^{1-\alpha}}}{1-\frac{s_2}{\epsilon}} &= \frac{A_2}{A_3}. && (132) \\ s_3 &= 0. && (133) \end{aligned}$$

Simulation evidence (available upon request) shows that for each given (A_2, A_3), the three equations (131), (132) and (133) determine a unique solution to the three unknowns: (w_1^*, w_2^*, A_1^*). Define $g_2 \equiv \frac{A_1^*}{A_3}$. For $\frac{A_1}{A_3} \in (g_2, g_1)$: if $\frac{A_1}{A_3} \to g_2$ then $\rho_3 \to 0$ and $\chi_{L3} \to 0$. ■

B.2 Competitive Fringe Firms and Efficient Bargaining

In this section we characterise the behaviour of competitive fringe firms in the setting of efficient bargaining model.

Eq. (72) implies the following relation between χ_{Li} and δ_i:

$$\chi_{Li} = -\frac{\eta_i \delta_i R_i}{\left[\left(1 - \eta_i\right) \bar{w}_i + \eta_i \omega_i\right] l_i}. \tag{134}$$

From (83) we know δ_i is increasing in ω_i, which is a measure of firm short-run productivity. If we let ω_i to decrease, then δ_i will approach to its infimum (maximal lower bound) which is 0.

Proposition 11 *For the efficient bargaining model with $\bar{w}_i = \bar{w}$ and $\eta_i = \eta$, let the production functions $F_i\left(x_i, l_i\right)$ be homogeneous of degree 1, i.e., of constant return to scale. Then*

$$\lim_{\omega_i \to \bar{w}} \delta_i = 0, \lim_{\omega_i \to \bar{w}} \chi_{Li} = 0, \lim_{\omega_i \to \bar{w}} \rho_i = 0.$$

This proposition shows that the notion of competitive fringe firms can be well defined in the setting of efficient bargaining model. The approximately competitive fringe firms are the least productive among the competing firms and they are approximately price takers in both product and labour markets.

C Spatial Correlation Caused by Strategic Interaction

In this section we use the canonical JOOM simulation results to demonstrate that strategic interaction between heterogeneous competing firms can cause spatial correlation. Such spatial correlation violates some standard assumption that underlie prevalent approaches to joint estimation of production function parameters and market powers, therefore can make the estimators biased.

Consider the following linear regression for estimating production function parameters (θ_L, θ_X) :

$$\ln q_{it} = \theta_L \ln l_{it} + \theta_X \ln x_{it} + \ln A_{it} + \varepsilon_i, \tag{135}$$

where $\ln A_i$ is the natural logarithm of firm MFP, which is unobservable, with

$$\begin{aligned} E[\varepsilon_{it}] &= 0, \\ E[\varepsilon_{it}\varepsilon_{jt}] &= 0, \\ E\left[\begin{pmatrix} \ln l_{it} \\ \ln x_{it} \end{pmatrix} \varepsilon_{it}\right] &= 0, \\ E\left[\begin{pmatrix} \ln l_{it} \\ \ln x_{it} \end{pmatrix} \varepsilon_{jt}\right] &= 0. \end{aligned}$$

Let the flexible input vector $\begin{pmatrix} l_{it} \\ x_{it} \end{pmatrix}$ be determined endogenously and dependent on A_{it}. Thus

$$Cov\left[\begin{pmatrix} \ln l_{it} \\ \ln x_{it} \end{pmatrix}, \ln A_{it}\right] \neq 0. \tag{136}$$

This correlation causes the well known simultaneity bias for OLS estimation of (θ_L, θ_X). The prevalent control function (proxy) approach and the dynamic panel data GMM estimation are designed to address this problem.

Unfortunately the strategic interaction between competing firms makes $\begin{pmatrix} l_{it} \\ x_{it} \end{pmatrix}$ to depend on A_{jt} for all j that is in the same market as firm i, therefore introduces spatial correlation into the large scale cross-section data set:

$$Cov\left[\begin{pmatrix} \ln l_{it} \\ \ln x_{it} \end{pmatrix}, \ln A_{jt}\right] \neq 0 \tag{137}$$

for i and j that in the same markets. We use the simulation results to demonstrate the point.

Table 15 is based on the simulation described in Section 4.1. We assume that the "econometrician" can only observe the firms' outputs and inputs and does not know the change in productivity of the superstar firm. Since the exogenous variation from A_{1t-1} to A_{1t} (for $t = 2$) is the root cause of all endogenous changes in the model, by design, the econometrician faces a non-identification problem under the true model. Under the false assumption that all firms are strategically independent, the inequality (137) is replaced with equality. And, following the control function approach (see Olley and Pakes, 1996, Levinsohn and Petrin, 2003, Ackerberg *et al.*, 2015), the econometrician would estimate (θ_L, θ_X) by assuming that the unobservable A_{it} can be expressed as an unknown function of the inputs (x_{it}, l_{it}), and imposing a certain structure on the dynamics of productivity over time. However, because the endogenous changes from $(q_{it-1}, l_{it-1}, x_{it-1})$ to (q_{it}, l_{it}, x_{it}) are all caused by the unobserved exogenous change from A_{1t-1} to A_{1t}, the relation between (x_{it}, l_{it}) and A_{it} also depends on the unobservable A_{1t}. The control function approach will then produce biased estimates of the production function parameters, since it is not designed to address the endogeneity problems caused by the spacial correlation with competing firms' productivity. Similar problems arise if the econometrician applies the dynamic panel data approach.

Table 15: Simulated Data with Spatial Correlation Caused by Unobserved Rise of Superstar Firm

$t = 1$	A	q	l	x
tier 2 ($\times 4$)	1.1^2	46.217	1.1553	91.592
tier 3 ($\times 5$)	1	17.326	0.80604	37.307
$t = 2$	A	q	l	x
tier 2 ($\times 4$)	1.1^2	45.187	1.1942	88.315
tier 3 ($\times 5$)	1	15.404	0.75575	32.729

D Supplementary Results

D.1 Simulation-Based Correlations

Some remarks on the simulated correlation pattern, reported in Table 16, are in order. To start, we note that the key variable MFP A_i is not observable from the firm level micro panel data, but the value added per worker ω_i is observable. We therefore use ω_i as a proxy for A_i in empirical work. To confirm proxy quality, we look at their simulated correlation coefficient for the Canonical JOOM described in Appendix B.1. Rows (1) and (2) of Table 16 show that not only correlation between ω_i and A_i is very high at 0.99, but that the simulation-based sign pattern of correlation with other relevant variables are the same. Furthermore, rows (3) and (4) show that correlation signs are the same for the corresponding coefficients of variation. Table 16 indicates that firm short-run productivity ω_i is positively correlated with wage w_i, revenue, employment and value added shares s_i, s_{Li} and ς_i, market power indices ρ_i, χ_{Li} and δ_i; and negatively correlated to labour shares in revenue, variable cost, and value added: ϕ_{Li}, ψ_{Li} and ν_{Li}. The coefficient of variation of ω_i is positively correlated with the coefficients of variation of all the above listed endogenous variables.

Table 16: Correlation Patterns from Simulation of Canonical JOOM

	A	ω	w	s	s_L	ς	ρ	χ_L	δ	ϕ_L	ψ_L	ν_L
(1) Corr w. A	1	0.99	+	+	+	+	+	+	+	−	−	−
(2) Corr w. ω	0.99	1	+	+	+	+	+	+	+	−	−	−
	CV_A	CV_ω	CV_w	CV_s	CV_{s_L}	CV_ς	CV_ρ	CV_{χ_L}	CV_δ	CV_{ϕ_L}	CV_{ψ_L}	CV_{ν_L}
(3) Corr w. CV_A	1	+	+	+	+	+	+	+	+	+	+	+
(4) Corr w. CV_ω	+	1	+	+	+	+	+	+	+	+	+	+

D.2 Model Predictions

Table 17: Parameters for Efficient Bargaining Model

$\bar{w}$	η
7	0.13

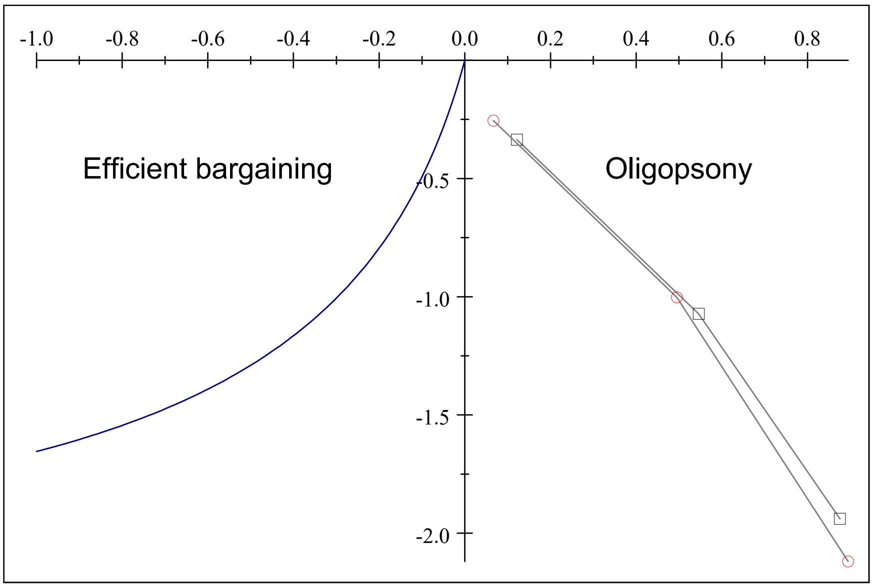

Figure 6: Simulated relationship between horizontal axis: $\ln(1+\chi_{Li})$ and vertical axis: $\ln \nu_{Li}$. The simulation of the oligopsony model uses results from Table 2. The simulation of the efficient bargaining model uses parameters from Table 17.

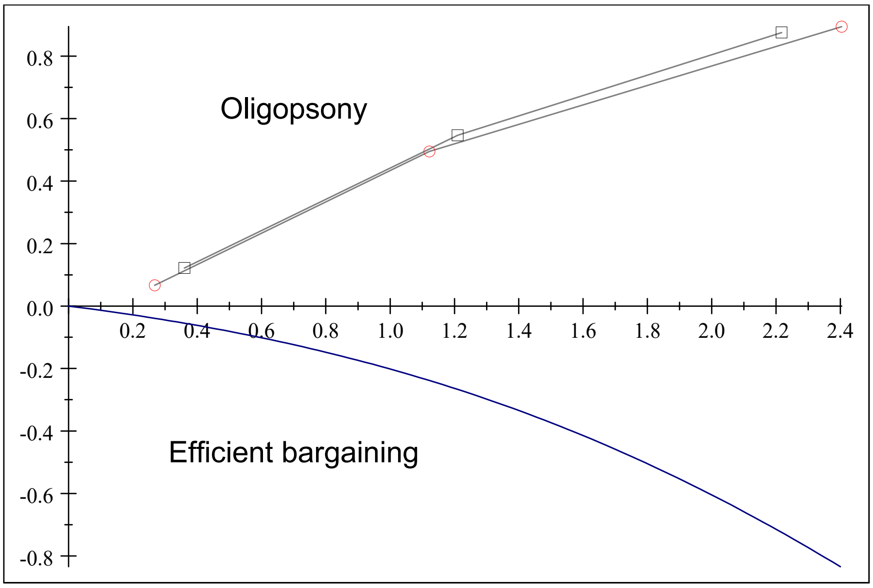

Figure 7: Simulated relationship between horizontal axis: $\ln \omega_i$ and vertical axis: $\ln(1+\chi_{Li})$.

D.3 Stability of RS Types

It is important to investigate if the classification of RS types is stable over time. Table 18, which reports the Markov transition matrices after 5 and 10 years, shows that the probabilities of remaining in RS type I after 5 and 10 years are 83% and 80%, and the probabilities of transition from RS type I to RS type III after 5 and 10 years are 6% and 10%. The probabilities of remaining in RS type III after 5 and 10 years are 74% and 70%, and the probabilities of transition from RS type III to RS type I after 5 and 10 years are 11% and 14%. These figures show that the distinction of labour market rent sharing mechanisms between oligopsony and efficient bargaining is stable in the UK manufacturing during 2003-2019.

Table 18: RS Types Transition Matrix

	RS type 2010			
RS type 2005	I	II	III	Total
I	12,218 (.83)	1,648 (.11)	892 (.06)	14,758
II	2,971 (.41)	2,354 (.32)	1,979 (.27)	7,304
III	1,654 (.11)	2,160 (.15)	10,591 (.74)	14,405
Total	16,843	6,162	13,462	36,467
	RS type 2015			
RS type 2005	I	II	III	Total
I	11,811 (.80)	1,596 (.11)	1,424 (.10)	14,831
II	3,134 (.43)	2,148 (.29)	2,005 (.28)	7,287
III	1,996 (.14)	2,323 (.16)	10,019 (.70)	14,398
Total	16,921	6,067	13,448	36,436

D.4 Complete Descriptive Statistics

Table 19: Summary Statistics

	Pooled Data					RS Type I				
	Obs	Mean	Std dev	Min	Max	Obs	Mean	Std dev	Min	Max
ω	60367	68.72	88.98	.3232	6174	25067	82.48	125.6	1.255	6174
w	60367	30.01	11.45	.0155	485.7	25067	30.96	11.88	.0155	485.7
s	60367	.0575	.1399	.0000	1	25067	.0646	.1373	.0001	.9997
s_L	60367	.0575	.1373	.0001	1	25067	.0470	.1119	.0001	.9962
ς	60367	.0580	.1477	.0000	12.69	25067	.0528	.1441	.0000	12.69
δ	60367	.2842	.1370	0	.9261	25067	.2182	.1073	.0004	.9261
ψ_L	60367	.2962	.1740	.0003	.9997	25067	.1656	.0805	.0003	.5716
ν_L	60367	.4140	.1466	.0009	1	25067	.3782	.1491	.0009	.9976
α	3501	.74805	.10986	.1654	.99523	—	—	—	—	—
ρ	60367	.2900	.2405	−1.883	.9999	25067	.0968	.1550	−1.883	.9151
χ_L	60367	.5535	6.626	−.9999	1378	25067	1.795	10.15	.0127	1378
	RS Type II					RS Type III				
	Obs	Mean	Std dev	Min	Max	Obs	Mean	Std dev	Min	Max
ω	10406	66.21	59.70	1.805	2285	24891	55.92	38.38	.3232	1883
w	10406	29.42	11.20	.6570	219.1	24891	29.31	11.04	.1040	217.5
s	10406	.0955	.2109	.0000	1	24891	.0344	.0932	.0000	.9815
s_L	10406	.0989	.2120	.0001	1	24891	.0506	.1160	.0001	.9816
ς	10406	.1001	.2129	.0000	1	24891	.0457	.1103	.0000	1.176
δ	10406	.2991	.1110	.0013	.8592	24891	.3444	.1438	.0008	.8955
ψ_L	10406	.2777	.0897	.0048	.8346	24891	.4355	.1636	.0421	.9997
ν_L	10406	.4002	.1257	.0387	.9864	24891	.4558	.1414	.0455	.9972
α	—	—	—	—	—	—	—	—	—	—
ρ	10406	.3000	.1128	.0013	.8688	24891	.4806	.1918	.0041	.9999
χ_L	10406	−.0008	.0702	−.2206	.3175	24891	−.4652	.2249	−.9999	−.0122

Table 20: Coefficients of Variation

	Pooled Data					RS Type I				
	Obs	Mean	Std dev	Min	Max	Obs	Mean	Std dev	Min	Max
CV_ω	3156	.52027	.31507	.00008	2.6194	2305	.52436	.31125	.00162	2.7038
CV_w	3156	.27983	.16260	.00090	1.3762	2305	.27595	.15709	.00017	1.6132
CV_s	3156	1.3064	.80413	0	6.0621	2305	1.1136	.63056	0	4.4523
CV_{s_L}	3156	1.2267	.79934	0	7.2609	2305	1.0708	.59337	0	5.3439
CV_ς	3156	1.2478	.81341	.00024	6.8026	2305	1.0963	.63498	.00302	4.8428
CV_δ	3156	.44670	.18670	0	1.3730	2305	.40162	.18401	.00080	1.2656
CV_{ψ_L}	3156	.51993	.21828	.00769	1.9389	2305	.35017	.18749	.00059	1.5150
CV_{ν_L}	3156	.31808	.13793	.00008	1.3606	2305	.34328	.17494	.00148	1.3746
	RS Type II					RS Type III				
	Obs	Mean	Std dev	Min	Max	Obs	Mean	Std dev	Min	Max
CV_ω	1908	.34503	.21833	0	1.9352	2321	.39661	.22348	0	2.4546
CV_w	1908	.21900	.14671	0	1.1129	2321	.25839	.15705	0	1.6156
CV_s	1908	.82537	.48204	0	2.9173	2321	1.0206	.65891	0	4.8986
CV_{s_L}	1908	.80941	.45829	0	2.8657	2321	.97698	.63575	0	4.4202
CV_ς	1908	.82846	.48217	0	2.9516	2321	1.0050	.65651	0	5.0528
CV_δ	1908	.27637	.17014	0	1.2220	2321	.36972	.19577	0	1.3442
CV_{ψ_L}	1908	.04512	.02406	0	.15752	2321	.27601	.16554	0	1.2148
CV_{ν_L}	1908	.23310	.13831	0	.94670	2321	.26232	.12890	0	.89719

D.5 Empirical Results

Table 21 reports regressions of ν_L on ρ and χ_L, separately for RS types I and III, as well as for the pooled data. The ρ and χ_L coefficients all have predicted signs, which are invariant across RS types, but they are quantitatively different. The R^2 of the regressions for RS types I and III are respectively: 0.286 and 0.823, showing that the explanatory power of (ρ, χ_L) for ν_L is very strong in the case of RS type III and more moderate for RS type I. For pooled data, the explanation power is much weaker.

Table 21: Regression of ν_L on ρ and χ_L

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L	ν_L
ρ	-.3411***	-.3430***	-.3430***	-1.2604***	-1.3190***	-1.3191***	-.0732***	-.0730***	-.0730***
	(.0405)	(.0407)	(.0407)	(.0234)	(.0191)	(.0191)	(.0206)	(.0206)	(.0206)
χ_L	-.0025	-.0025	-.0025	-.8078***	-.8566***	-.8567***	-.0030	-.0030	-.0030
	(.0016)	(.0016)	(.0016)	(.0205)	(.0163)	(.0163)	(.0020)	(.0020)	(.0020)
α		.0758	.0759		-1.028***	-1.026***		-.1459***	-.1456***
		(.0707)	(.0707)		(.0647)	(.0644)		(.0305)	(.0304)
HHI_ς			-.0002			-.0109			.017
			(.0001)			(.0090)			(.013)
Con	.2922***	.2297***	.2297***	.5453***	1.397***	1.402***	.3882***	.5094***	.5095***
	(.0391)	(.0666)	(.0666)	(.0069)	(.0533)	(.0546)	(.0186)	(.0344)	(.0344)
R^2	.286	.286	.286	.823	.853	.853	.172	.173	.173
Obs	25067	25067	25067	24891	24891	24891	60367	60367	60367

In contrast, Table 7 shows remarkably high values of R^2: 0.856 for RS type I, 0.949 for RS type III, and 0.839 for the pooled data. Notably, these explanation powers are achieved without firm fixed effect. These dominate their counterparts for (ρ, χ_L).

Tables 22 and 23 report respectively regressions of ρ and χ_L on ω. They show that, although the effects all have the predicted signs, ω has only weak explanation power on χ_L for RS type III ($R^2 = 0.106$), the power diminishes further for RS type I ($R^2 = 0.044$) and the pooled data ($R^2 = 0.028$). ω can explain ρ better for RS type III than type I ($R^2 = 0.295$ and 0.164), but the explanation power remains weak for pooled data ($R^2 = 0.040$).

Table 22: Regression of ρ on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ	ρ
ω	.0002***	.0002***	.0002***	.0018***	.0018***	.0018***	.0000	.0000	.0000
	(.0001)	(.0001)	(.0001)	(.0003)	(.0003)	(.0003)	(.0000)	(.0000)	(.0000)
α		.8296***	.8301***		-.3156***	-.3102***		.1685**	.1691**
		(.1571)	(.1571)		(.0705)	(.0701)		(.0714)	(.0714)
HHI_ς			-.0005***			-.0200			-.0010***
			(.0001)			(.0191)			(.0002)
Con	-.2380*	-.9195***	-.9195***	.3078***	.5703***	.5803***	.2717***	.1323**	.1325**
	(.1300)	(.1874)	(.1873)	(.0171)	(.0607)	(.0616)	(.0389)	(.0664)	(.0664)
R^2	.146	.164	.164	.294	.295	.295	.049	.050	.050
Obs	25067	25067	25067	24891	24891	24891	60367	60367	60367

Table 23: Regression of χ_L on ω

	RS type I			RS type III			Pooled		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	χ_L	χ_L	χ_L	χ_L	χ_L	χ_L	χ_L	χ_L	χ_L
ω	.0094***	.0095***	.0095***	-.0006***	-.0006***	-.0006***	.0099***	.0099***	.0099***
	(.0026)	(.0026)	(.0026)	(.0002)	(.0002)	(.0002)	(.0023)	(.0023)	(.0023)
α		-16.61***	-16.62***		-.6268***	-.6271***		-6.927***	-6.932***
		(3.717)	(3.718)		(.0824)	(.0819)		(1.583)	(1.584)
HHI_ς			.0038***			.0012			.0031***
			(.0013)			(.0271)			(.0008)
Con	-3.093	10.55*	10.55*	-.5172***	.0040	.0034	-2.151	3.583*	3.585*
	(5.927)	(5.800)	(5.800)	(.0112)	(0.0687)	(.0712)	(2.064)	(2.015)	(2.015)
R^2	.043	.044	.044	.101	.106	.106	.027	.028	.028
Obs	25067	25067	25067	24920	24920	24920	60855	60855	60855

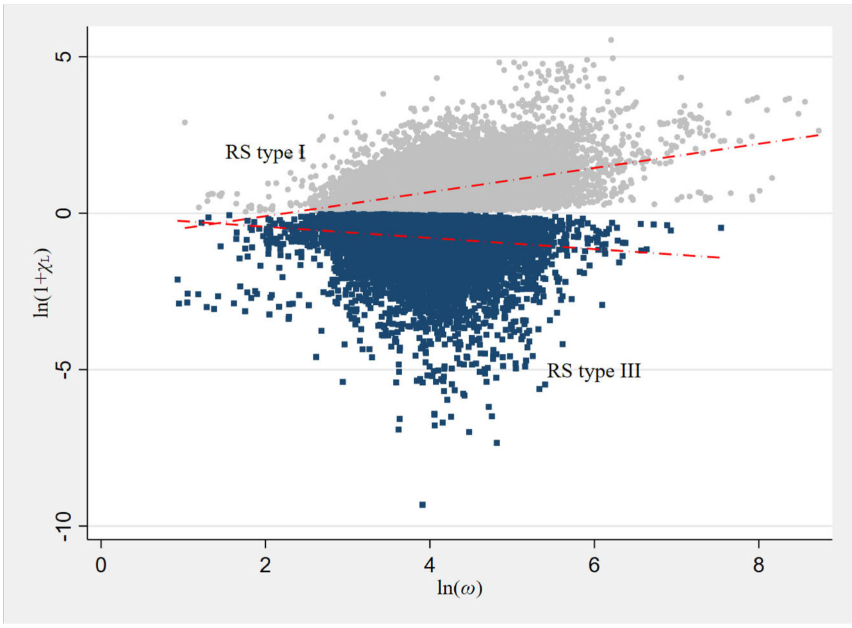

Figure 8: Scatter plot of $(\ln \omega_{Likt}, \ln(1 + \chi_{Likt}))$

Table 23 also shows that the coefficient of ω changes sign from positive to negative when we switch from RS type I to III. Figure 8 is the scatter plot of $(\ln(\omega_{ikt}), \ln(1 + \chi_{Likt}))$. It is apparent that the RS type III firms behave differently from the RS type I firms. The visually clear difference resembles Figure 7.