A KKT Conditions Based Transceiver Optimization Framework for RIS-Aided Multi-User MIMO Networks

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***Abstract*—In many core problems of signal processing and wireless communications, Karush-Kuhn-Tucker (KKT) conditions based optimization plays a fundamental role. Hence we investigate the KKT conditions in the context of optimizing positive semidefinite matrix variables under nonconvex rank constraints. More explicitly, based on the properties of KKT conditions, we optimize a reconfigurable intelligent surface (RIS) aided multi-user multi-input multi-output (MU-MIMO) network. Specifically, we consider the capacity maximization and sum mean square error (MSE) minimization problems of both the RIS-aided MU-MIMO uplink (UL) and downlink (DL) under multiple weighted power constraints and rank constraints. As for the RIS-aided MU-MIMO UL, the optimal structures of the signal covariance matrices are derived based on the KKT conditions. Furthermore, an efficient procedure is designed for solving the capacity maximization and sum mean square error (MSE) minimization problems. Then the UL-DL dualities are exploited for solving the capacity maximization and MSE minimization problems of the RIS-aided MU-MIMO DL based on the results of the UL optimization. Hence in the proposed framework, the phase shifting matrix of the RIS is jointly optimized with the signal covariance matrices for both the UL and DL. Our simulation results demonstrate the performance advantages of the proposed framework.**

***Index terms*—** Duality, KKT conditions, matrix variables, covariance optimization, RIS, MU-MIMO.

I. Introduction

MULTI-INPUT multi-output (MIMO) techniques [1]–[6] have become an integral component in wireless communications. Indeed, in the 4G, 5G and 6G eras they constitute one of the most important innovations [3]–[12]. To elaborate from an information theoretic and signal processing perspective, the covariance matrix is the most salient statistical characteristic of the received signal [1], [4], [7], [9]. The corresponding covariance matrix optimization problems of MIMO communications are much more challenging than that of their single-antenna counterparts because matrix variables have to be optimized [2], [7]. Hence these optimization problems play a critical role [2], [4], [7]. Moreover, a covariance matrix must be a positive semi-definite matrix, which is also subject to certain structural constraints. This further exacerbates the grade of challenge [13]–[15].

For matrix variables, generally speaking, there are four fundamental categories of optimization frameworks. Firstly, the Karush-Kuhn-Tucker (KKT) conditions based methods constitute the most popular optimization framework [1], [6], [7], [10], [13], [16], [17]. The wide adoption of KKT conditions accrues from the fact that for convex optimization the KKT conditions constitute both the necessary and sufficient conditions for finding the optimal solutions [18]. Based on the KKT conditions, one can readily derive the most widely used solutions, namely the water-filling solutions [7], [13], [19]–[22]. This is the reason for the remarkable success of the KKT conditions based optimization framework [7]. Secondly, the majorization theory based methods also form an important optimization framework, which exploits the intricate matrix inequalities associated with the diagonal elements and the eigenvalues [23] for deriving optimal structures of the matrix variables [2]. Thirdly, the standard optimization programming based methods also form a widely used framework of computing matrix variables [3], [5], [24]. Finally, the recent matrix-monotonic optimization framework has also been found beneficial for solving diverse optimization problems in MIMO communications [4], [25], [26].

Naturally, there is no general-purpose mathematical tool that can solve *all* the problems encountered in signal processing and wireless communications. Different methods or designs have different pros and cons as well as limitations. Elegant but specific methods might have limited applicability. When the optimization problems of MIMO systems are complicated and nonconvex, it becomes a challenge to glean crisp insights from the KKT conditions often relying on complicated equations and inequalities. Moreover, for these nonconvex optimization problems, the KKT conditions are only the necessary conditions for optimality. Hence many researchers believe that

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the KKT conditions based methods are not particularly useful when dealing with positive semidefinite matrix variables. On the other hand, the majorization theory based methods are usually limited to single-user MIMO scenarios [2]. Additionally, the majorization theory based methods have strict limitations on the mathematical formulas of the objective functions. Furthermore, the majorization theory based methods are only applicable to the transceiver designs under sum power constraints [2]. From a practical implementation perspective, each antenna has its own power amplifier, hence the per-antenna power constraints are more practical than the sum power constraints [20], [21], [27], [28]. Although it was proved in [10] that for per-antenna power constraints at high signal-to-noise ratios (SNR) the capacity-achieving covariance matrices also admit closed-form solutions, the majorization theory based methods do not seem to work in this case. By contrast, the standard optimization programming based methods are readily applicable to diverse MIMO systems. Both semidefinite programming (SDP), as well as second order cone programming (SOCP) and geometric programming (GP) have enjoyed substantial success [18]. Unfortunately, however, the physical interpretation of the optimal solutions found by these methods remains unclear and they also suffer from high computational complexity. Recently, the matrix-monotonic framework has been shown to be eminently suitable for optimizing the matrix variables of MIMO systems, including multi-hop amplify-and-forward MIMO scenarios [4], [25], [26]. However, the matrix-monotonic optimization framework relies on numerous strict mathematical limitations and complicated mathematical expressions.

A lesser-known benefit of the KKT conditions based methods is that since they are implemented based on matrix derivatives [14], they have fewer limitations than either the family of majorization theory based methods, or the standard optimization programming based methods, and the matrix-monotonic optimization methods [4], [25], [26]. As a further compelling benefit, the KKT conditions based methods usually result in closed-form solutions rather than relying on numerical results. For example, the KKT conditions based methods are more amenable to deriving closed-form solutions than standard convex programming methods, as shown in [7]. Therefore, the family of KKT conditions based methods deserves careful reconsideration. In our previous work [7], it is shown that the KKT conditions based methods are extremely useful even in challenging optimization problems, provided that they are used appropriately. Explicitly, upon specifically reformulating the KKT conditions with respect to matrix variables, very useful structures and results can be derived based on a small number of KKT conditions. Specifically, the framework of [7] offers a wide range of applications, including both the cases of perfect channel state information (CSI) and imperfect CSI. Several existing solutions are subsumed as its special cases.

However, in [7], the KKT conditions based optimization framework is only proposed for point-to-point MIMO communications without the assistance of RISs and in the absence of rank constraints. Hence the result of [7] is not applicable to DL MU-MIMO communications. In this paper, we take a further step of investigating KKT conditions based optimization in the context of multi-user MIMO (MU-MIMO) communications under rank constraints. Specifically, in contrast to [7], we consider the family of optimization problems involving multiple positive semidefinite matrix variables. Moreover, the rank constraints imposed on the signal covariance matrices are also taken into account. In other words, the number of data streams at each transmitter is constrained by a predefined threshold. To the best of our knowledge, the rank constraints have not been taken into account for KKT based methods in the open literature [11], [28]. Furthermore, we also extend the traditional MIMO systems to the family of reconfigurable intelligent surface (RIS) aided MIMO communications [29]–[35]. By contrast, we consider multiple performance metrics and exploit the uplink-downlink duality under a more general multi-user RIS-aided MIMO scenario. This demonstrates that the family of KKT conditions based methods is indeed applicable to sophisticated scenarios, including RIS-aided MU-MIMO systems. Our new contributions are boldly and explicitly contrasted to the state-of-the-art in Table I, which are further detailed as follows:

- The KKT conditions of the optimization of positive semidefinite matrix variables are investigated. It is proved that when rank constraints are considered, the corresponding Lagrange multiplier is a Hermitian matrix instead of the widely used positive semidefinite matrix. Moreover, further fundamental results concerning the KKT conditions are derived and based on these a diverse variety of nonconvex rank constraints can also be considered. Under a mild condition, the optimal structures of the matrix variables can also be derived.
- The covariance optimization problems of diverse MU-MIMO networks are investigated under rank constraints, including: 1/ the capacity maximization of the RIS-aided MU-MIMO UL (UL) under multiple weighted power constraints and rank constraints; 2/ the MSE minimization of the RIS-aided MU-MIMO UL under multiple weighted power constraints and rank constraints; 3/ the capacity maximization of the RIS-aided MU-MIMO downlink (DL) under multiple weighted power constraints and rank constraints, and finally; 4/ the MSE minimization of the RIS-aided MU-MIMO downlink under multiple weighted power constraints and rank constraints.
- The covariance matrices and the phase shifting matrix of the RIS are jointly optimized alternatively based on the KKT conditions. By appropriately reformulating the KKT conditions, the optimal structures of the covariance matrices can be derived. The optimization of the phase shifting matrix at the RIS can be transferred into a quadratic optimization problem under constant modulus constraints, which can be solved effectively based on popular iterative algorithms, such as the majorization-minimization (MM) framework and the alternating direction method of multipliers (ADMM) algorithm. The global optimality of the solution obtained by the AO method is proved under a simplified scenario.
- The corresponding covariance matrices are optimized for the MU-MIMO DL by exploiting the uplink-downlink

TABLE I
BOLDLY AND EXPLICITLY CONTRASTING OUR CONTRIBUTIONS TO THE STATE-OF-THE-ART IN MIMO SYSTEM OPTIMIZATION.

	[7]	[26]	[29]	[30]	[31]	[41]	Proposed
Multi-user MIMO		✓	✓	✓		✓	✓
Per-antenna power constraints	✓	✓					✓
Different noise covariance matrices of the users							✓
Rank constraints							✓
Uplink-downlink duality			✓	✓		✓	✓
RIS-aided scenario			✓	✓	✓		✓

duality. Based on the capacity or MSE dualities, the DL optimization problems of RIS-aided MIMO systems under multiple weighted power constraints are transformed into the corresponding virtual UL optimization problems. Then, the optimal solutions can be derived based on KKT conditions. A modified weighted mean square error minimization (WMMSE) algorithm is also devised as an alternative to our duality based design constructed for DL optimization problems.

- The simulation results show that the proposed optimization framework achieves the same or better performance as the traditional numerical optimization algorithms and the modified WMMSE algorithm, while offering clearer physical insights at a reduced computational complexity.

The rest of the paper is organized as follows. Section II presents our fundamental results for the KKT conditions associated with positive semidefinite matrix variables. In Section III, the capacity maximization of the RIS-aided MU-MIMO UL is investigated under multiple weighted power constraints, while Section IV discusses the MSE minimization of the RIS-aided MU-MIMO UL under multiple weighted power constraints. In Section V, the capacity maximization of the RIS-aided MU-MIMO DL is considered under multiple weighted power constraints, and Section VI investigates the MSE minimization of the RIS-aided MU-MIMO DL under multiple weighted power constraints. Our numerical results are discussed in Section VII, and our conclusions are offered in Section VIII.

II. PRELIMINARY RESULTS ON KKT CONDITIONS

In this section, some fundamental results on the KKT conditions are firstly derived, which then form the theoretical basis of the ensuing transceiver optimization. Viewing the covariance matrices as optimization variables has a pair of advantages. Firstly, the order of the associated function is reduced, for example second-order terms become first-order terms. Secondly, the hidden convexity is revealed. However, there is also an intrinsic disadvantage. Specifically, rank constraints are nonconvex in nature, which limits the applications of covariance-based optimization. Later an efficient algorithm is conceived for demonstrating, how to overcome this impediment.

A. Lagrange Multipliers for Positive Semidefinite Matrix Variables

Let us consider a general optimization problem associated with a positive semidefinite matrix variable $\boldsymbol{Q} \in \mathbb{C}^{M\times M}$, formulated as follows

$$\textbf{P.1}: \quad \min_{\boldsymbol{Q}} f(\boldsymbol{Q}) \text{ s.t. } g_l(\boldsymbol{Q}) \leq 0,\ 1 \leq l \leq L,\ \boldsymbol{Q} \succeq \boldsymbol{0}, \quad (1)$$

where $f(\cdot)$ and $g_l(\cdot)$s are all real-valued functions. In order to reformulate the positive semidefinite constraint, **P. 1** is equivalent to the following optimization problem

$$\begin{aligned}&\textbf{P.2}: \ \min_{\boldsymbol{Q}} \ f(\boldsymbol{Q}),\\ &\text{s.t. } g_l(\boldsymbol{Q}) \leq 0,\ 1 \leq l \leq L,\ \lambda_m(\boldsymbol{Q}) \geq \boldsymbol{0},\ 1 \leq m \leq M, \quad (2)\end{aligned}$$

where $\lambda_m(\boldsymbol{Q})$ is the mth largest eigenvalue of $\boldsymbol{Q}$. The corresponding Lagrangian function [18] of **P. 2** is :

$$\begin{aligned}&\mathcal{L}\left(\boldsymbol{Q}, \{\mu_l\}_{l=1}^L, \{\omega_m\}_{m=1}^M\right)\\ &= f(\boldsymbol{Q}) + \sum\nolimits_{l=1}^{L} \mu_l g_l(\boldsymbol{Q}) - \sum\nolimits_{m=1}^{M} \omega_m \lambda_m(\boldsymbol{Q}), \quad (3)\end{aligned}$$

where the nonnegative real scalars μ_l and ω_m are the Lagrange multipliers corresponding to the constraints in **P. 2**. Note that it is very difficult to derive KKT conditions based on (3). This is because the derivative of the mth largest eigenvalue $\lambda_m(\boldsymbol{Q})$ with respect to $\boldsymbol{Q}$ is very difficult to be expressed in a closed-form. To overcome this difficulty, we show that the term $\sum_{m=1}^{M}\omega_m\lambda_m(\boldsymbol{Q})$ is equivalent to the following simple matrix function

$$\sum\nolimits_{m=1}^{M} \omega_m \lambda_m(\boldsymbol{Q}) = \mathrm{Tr}(\boldsymbol{\Psi}\boldsymbol{Q}), \quad (4)$$

where $\boldsymbol{\Psi} \in \mathbb{C}^{M\times M}$ satisfies

$$\left[\boldsymbol{U}_{\boldsymbol{Q}}^{\mathrm{H}} \boldsymbol{\Psi} \boldsymbol{U}_{\boldsymbol{Q}}\right]_{m,m} = \omega_m,\ 1 \leq m \leq M, \quad (5)$$

in which the unitary matrix $\boldsymbol{U}_{\boldsymbol{Q}}$ is defined based on the eigenvalue decomposition (EVD):

$$\boldsymbol{Q} = \boldsymbol{U}_{\boldsymbol{Q}} \boldsymbol{\Lambda}_{\boldsymbol{Q}} \boldsymbol{U}_{\boldsymbol{Q}}^{\mathrm{H}},\ \text{with } \boldsymbol{\Lambda}_{\boldsymbol{Q}} \searrow . \quad (6)$$

where $\searrow$ means that the diagonal elements of a matrix are in descending order. Since the Lagrange multipliers ω_m are always nonnegative and independent of the unitary matrix $\boldsymbol{U}_{\boldsymbol{Q}}$, it can then be concluded from (5) that $\boldsymbol{\Psi}$ must be positive semidefinite. This property is widely exploited to derive the optimal structures of the matrix variables.

Moreover, in the corresponding KKT conditions of **P. 2**, we always have $\sum_{m=1}^{M}\omega_m\lambda_m(\boldsymbol{Q}) = 0$, and therefore together with (4) the following equality always holds as well

$$\mathrm{Tr}(\boldsymbol{\Psi}\boldsymbol{Q}) = 0. \quad (7)$$

This equality has two-fold meanings as elaborated below.

Property 1 The positive semidefinite matrices $\boldsymbol{\Psi}$ and $\boldsymbol{Q}$ have the same EVD unitary matrix. In other words, there exists a unitary matrix $\boldsymbol{U}$ such that the following equalities hold

$$\boldsymbol{U}^{\rm H}\boldsymbol{Q}\boldsymbol{U} = \boldsymbol{\Lambda}_{\boldsymbol{Q}},\ \boldsymbol{U}^{\rm H}\boldsymbol{\Psi}\boldsymbol{U} = \boldsymbol{\Lambda}_{\boldsymbol{\Psi}}, \tag{8}$$

where $\boldsymbol{\Lambda}_{\boldsymbol{Q}}$ and $\boldsymbol{\Lambda}_{\boldsymbol{\Psi}}$ are diagonal matrices, but there is no ordering of their diagonal elements.

Property 2 Based on (8), if $\lambda_m(\boldsymbol{Q}) \neq 0$, the corresponding diagonal element of $\boldsymbol{\Lambda}_{\boldsymbol{\Psi}}$ also equals to zero, i.e., $\lambda_m(\boldsymbol{\Psi}) = 0$.

When the rank constraint on a positive semidefinite $\boldsymbol{Q}$ is considered, i.e., $\mathrm{Rank}\{\boldsymbol{Q}\} \leq N_{\rm C}$, the constraint can be written in the following form

$$\lambda_m(\boldsymbol{Q}) \geq 0, 1 \leq m \leq N_{N_{\rm C}} \text{ and } \lambda_m(\boldsymbol{Q}) = 0, m > N_{N_{\rm C}}, \tag{9}$$

and the corresponding Lagrange multipliers $\{\lambda_m(\boldsymbol{\Psi}), m > N_{N_{\rm C}}\}$ can have any value.

Based on (9) and (5), the corresponding Lagrange multiplier $\boldsymbol{\Psi}$ satisfies

$$\boldsymbol{Q}\boldsymbol{\Psi} = \boldsymbol{0}, \tag{10}$$

where $\boldsymbol{\Psi}$ is a Hermitian matrix instead of a positive semidefinite matrix and the resultant KKT conditions are only necessary conditions for optimality. Observe that without ordering diagonal elements, we arrive at a Hermitian $\boldsymbol{\Psi}$ rather than a positive semidefinite one.

B. Derivation of KKT Conditions Based Optimal Solution

Again, for convex optimization, the KKT conditions constitute necessary and sufficient conditions for finding the optimal solutions. Therefore, to solve a convex optimization problem, such as the capacity maximization for point-to-point MIMO communications, we only need to find the solutions satisfying the KKT conditions, which are guaranteed to be the optimal solutions. However, for a generic optimization problem when Slater's condition [18] is satisfied, the KKT conditions are only necessary conditions for optimal solutions. In this paper, we focus on optimization problems associated with semidefinite matrix variables. It is worth noting that even for nonconvex optimization, the KKT conditions are still very useful. For nonconvex optimization problems, there are usually two kinds of logic to derive the optimal solutions. The first one is to prove that all the solutions satisfying the KKT conditions have the same structure. The other logic is to reveal the hidden convexity, and then try to find the solutions satisfying the corresponding KKT conditions. The conclusion given below forms the basis for the following mathematical derivations.

Conclusion 1 In the following two equalities

$$\begin{cases} \boldsymbol{H}^{\rm H}\big(\boldsymbol{I}+\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{\Sigma}\big(\boldsymbol{I}+\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{H} = \mu\boldsymbol{\Phi} - \boldsymbol{\Psi}, \\ \boldsymbol{Q}\boldsymbol{\Psi} = \boldsymbol{0}, \end{cases} \tag{11}$$

$\boldsymbol{\Phi}$ is a positive definite matrix, $\boldsymbol{\Psi}$ is a Hermitian matrix, and both $\boldsymbol{\Sigma}$ and $\boldsymbol{\Phi}$ can be functions of $\boldsymbol{Q}$, while the parameter μ is an arbitrary nonnegative scalar. When $\boldsymbol{\Sigma}$ and $\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\rm H}$ have the same EVD unitary matrix, i.e., there exists a unitary matrix $\boldsymbol{U}_{\boldsymbol{\Sigma}}$ such that

$$\begin{aligned} &\boldsymbol{U}_{\boldsymbol{\Sigma}}^{\rm H}\boldsymbol{\Sigma}\boldsymbol{U}_{\boldsymbol{\Sigma}} = \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}} \text{ with } \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}} \searrow \text{ and} \\ &\boldsymbol{U}_{\boldsymbol{\Sigma}}^{\rm H}\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\rm H}\boldsymbol{U}_{\boldsymbol{\Sigma}} = \boldsymbol{\Lambda} \text{ with } \boldsymbol{\Lambda} \searrow, \end{aligned} \tag{12}$$

it can be concluded that $\boldsymbol{Q}$ satisfies the following structure

$$\boldsymbol{Q} = \boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}\boldsymbol{\Lambda}_{\boldsymbol{Q}}\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}^{\rm H}\boldsymbol{\Phi}^{-\frac{1}{2}}, \tag{13}$$

in which $\boldsymbol{\Lambda}_{\boldsymbol{Q}}$ is a diagonal matrix and the unitary matrix $\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}$ is defined based on the following singular value decomposition (SVD)

$$\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}} = \boldsymbol{U}_{\boldsymbol{\mathcal{H}}}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}}\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}^{\rm H} \text{ with } \boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}} \searrow. \tag{14}$$

Proof: Note that $\boldsymbol{\Phi}$ is positive definite. By defining

$$\widetilde{\boldsymbol{Q}} = \boldsymbol{\Phi}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{\Phi}^{\frac{1}{2}}, \tag{15}$$

the two equalities in (11) are equivalent to

$$\begin{cases} \boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big(\boldsymbol{I}+\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{\Sigma}(\boldsymbol{I} \\ \quad +\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}} = \mu\boldsymbol{I} - \boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{\Psi}\boldsymbol{\Phi}^{-\frac{1}{2}}, \\ \widetilde{\boldsymbol{Q}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{\Psi}\boldsymbol{\Phi}^{-\frac{1}{2}} = \boldsymbol{0}. \end{cases} \tag{16}$$

Left multiplying $\widetilde{\boldsymbol{Q}}^{\frac{1}{2}}$ and right multiplying $\widetilde{\boldsymbol{Q}}^{\frac{1}{2}}$ on the first equality in (16), we have

$$\begin{aligned} &\widetilde{\boldsymbol{Q}}^{\frac{1}{2}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big(\boldsymbol{I}+\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{\Sigma}(\boldsymbol{I} \\ &\quad +\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{H}^{\rm H}\big)^{-1}\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}^{\frac{1}{2}} = \mu\widetilde{\boldsymbol{Q}}. \end{aligned} \tag{17}$$

Based on (17) it can be concluded that the following equality holds

$$\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}^{\frac{1}{2}} = \boldsymbol{U}_{\boldsymbol{\Sigma}}\boldsymbol{\Lambda}_{\boldsymbol{A}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}^{\rm H}, \tag{18}$$

where $\boldsymbol{\Lambda}_{\boldsymbol{A}}$ is a diagonal matrix and the unitary matrix $\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}$ is defined as follows

$$\widetilde{\boldsymbol{Q}} = \boldsymbol{\Phi}^{\frac{1}{2}}\boldsymbol{Q}\boldsymbol{\Phi}^{\frac{1}{2}} = \boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}^{\rm H}. \tag{19}$$

The equation (18) can be rewritten as

$$\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}^{\frac{1}{2}} = \boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}^{\frac{1}{2}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}^{\rm H} = \boldsymbol{U}_{\boldsymbol{\Sigma}}\boldsymbol{\Lambda}_{\boldsymbol{A}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}^{\rm H} \tag{20}$$

based on which we have

$$\boldsymbol{U}_{\boldsymbol{\Sigma}}^{\rm H}\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}^{\frac{1}{2}} = \boldsymbol{\Lambda}_{\boldsymbol{A}}. \tag{21}$$

(i) $\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}$ is a full-rank diagonal matrix: From (21), it is readily concluded that $\boldsymbol{U}_{\boldsymbol{\Sigma}}^{\rm H}\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}$ is a diagonal matrix. This together with the definition of $\widetilde{\boldsymbol{Q}}$ directly leads to (13).

(ii) $\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}$ is a general diagonal matrix with some diagonal elements being zeros: Based on (21) it can still be concluded that the columns of $\boldsymbol{U}_{\widetilde{\boldsymbol{Q}}}$ corresponding to the nonzero values of $\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}$ are the eigenvectors of the right SVD unitary matrix of $\boldsymbol{H}\boldsymbol{\Phi}^{-\frac{1}{2}}$. As there are no constraints on the other eigenvectors corresponding to the zero diagonal elements of $\boldsymbol{\Lambda}_{\widetilde{\boldsymbol{Q}}}$, (13) is still applicable in this case. ■

Remark 1 We would like to point out that for **Conclusion 1** when $\boldsymbol{\Sigma} = \boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}}$, the conclusion reduces to the optimization of capacity maximization. On the other hand, when $\boldsymbol{\Sigma} = \boldsymbol{I}$, the conclusion reduces to the optimization of MSE minimization.

Conclusion 2 Assume that there is a rank constraint on the positive semidefinite matrix $\boldsymbol{Q}$, i.e., $\mathrm{Rank}\{\boldsymbol{Q}\} \leq N_{\mathrm{C}}$, and the objective function is monotonically decreasing with respect to $\boldsymbol{\lambda}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}})$, where $\boldsymbol{\lambda}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}})$ denotes the vector consisting of the eigenvalues of $\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}}$. The optimal $\boldsymbol{Q}$ satisfying (11) processes the following diagonalizable structure

$$\boldsymbol{Q} = \boldsymbol{\Phi}^{-\frac{1}{2}}[\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}]_{:,1:N_{\mathrm{C}}}\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}}\left([\boldsymbol{V}_{\boldsymbol{\mathcal{H}}}]_{:,1:N_{\mathrm{C}}}\right)^{\mathrm{H}}\boldsymbol{\Phi}^{-\frac{1}{2}}, \tag{22}$$

where $\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}}$ is an $N_{\mathrm{C}} \times N_{\mathrm{C}}$ diagonal matrix, while $[\boldsymbol{V}]_{:,1:N_{\mathrm{C}}}$ denotes the sub-matrix consisting of the first N_{C} columns of $\boldsymbol{V}$.

Based on **Conclusion 1** and the fact that the rank constraint does not change the KKT conditions, when $\mathrm{Rank}\{\boldsymbol{Q}\} \leq N_{\mathrm{C}}$, there are multiple $\boldsymbol{Q}$ satisfying (13). When the objective function is monotonically decreasing with respect to $\boldsymbol{\lambda}(\boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}})$, it is obvious that the first N_{C} largest eigenchannels should be chosen.

III. Capacity Maximization of the MU-MIMO UL

This section investigates the capacity maximization of our RIS-aided MU-MIMO UL system seen in Fig. 1(a). For RIS-aided MU-MIMO uplink, the signal model is given by

$$\boldsymbol{y} = \sum\nolimits_{k=1}^{K}\left(\boldsymbol{H}_{\mathrm{SD},k} + \boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k}\right)\boldsymbol{s}_k + \boldsymbol{n}. \tag{23}$$

In (23), $\boldsymbol{y}$ is the received signal vector at the BS, and $\boldsymbol{H}_{\mathrm{SD},k} \in \mathbb{C}^{Nt \times I_k}$ is the direct channel matrix between the BS and the kth mobile user, while $\boldsymbol{H}_{\mathrm{RD}}$ and $\boldsymbol{H}_{\mathrm{SR},k}$ are the channel matrices between the BS and the RIS and between the RIS and the kth mobile terminal, respectively. Furthermore, the diagonal matrix $\boldsymbol{\Theta}$ represents the phase shifting matrix at the RIS, the vector $\boldsymbol{s}_k$ is the signal vector transmitted from the kth user with covariance matrix $\mathbb{E}\{\boldsymbol{s}_k\boldsymbol{s}_k^{\mathrm{H}}\} = \boldsymbol{Q}_{\mathrm{U},k}$, and $\boldsymbol{n}$ is the additive white Gaussian noise vector at the BS, whose covariance matrix is $\mathbb{E}\{\boldsymbol{n}\boldsymbol{n}^{\mathrm{H}}\} = \boldsymbol{R}_{\mathrm{n}}$.

Based on the signal model (23), the optimization problem of capacity maximization is formulated as

$$\textbf{P.3}: \begin{cases} \max\limits_{\{\boldsymbol{Q}_{\mathrm{U},k}\},\boldsymbol{\Theta}} \log\det\left(\boldsymbol{I} + \sum_{k=1}^{K}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-1}\right), \\ \mathrm{s.t.} \quad \boldsymbol{H}_k = \boldsymbol{H}_{\mathrm{SD},k} + \boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k}, \\ \qquad \mathrm{Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k}) \leq P_{k,i}, 1 \leq i \leq I_k, \ 1 \leq k \leq K, \\ \qquad \boldsymbol{Q}_{\mathrm{U},k} \succeq \boldsymbol{0}, \ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k}) \leq N_{\mathrm{C},k}, \ 1 \leq k \leq K, \end{cases} \tag{24}$$

where the positive semidefinite matrices $\boldsymbol{\Omega}_{k,i}$ are the weighting matrices in the multiple linear power constraints at the kth mobile user or terminal [7], [11], [20], and $P_{k,i}$ are the corresponding power limits. In **P.3** there are two kinds of optimization variables to solve, i.e., $\{\boldsymbol{Q}_{\mathrm{U},k}\}$ and $\boldsymbol{\Theta}$. The phase shifting matrix $\boldsymbol{\Theta}$ is a diagonal matrix and each diagonal element of $\boldsymbol{\Theta}$ has a constant modulus. The full KKT conditions [18] of **P.3** with respect to $\boldsymbol{Q}_{\mathrm{U},k}$ are given by (25) shown at the bottom of this page, where the Hermitian matrix $\boldsymbol{\Psi}_k$ is the Lagrangian multiplier corresponding to the positive semidefinite constraint $\boldsymbol{Q}_{\mathrm{U},k} \succeq \boldsymbol{0}$ with $\mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k}) \leq N_{\mathrm{C},k}$.

A. *Optimization of* $\boldsymbol{Q}_{\mathrm{U},k}$

First introduce the scalars μ_k and define

$$\widetilde{\mu}_{k,i} = \mu_{k,i}/\mu_k, \ \forall k, i. \tag{26}$$

Then based on **Conclusion 2**, the optimal transmission covariance matrices for **P.3** have the water-filling structure (27) shown at the bottom of this page, where $(x)^+ \triangleq \max(x, 0)$ and the positive definite matrix $\widetilde{\boldsymbol{\Omega}}_k$ is defined as

$$\widetilde{\boldsymbol{\Omega}}_k = \sum\nolimits_{i=1}^{I_k}\widetilde{\mu}_{k,i}\boldsymbol{\Omega}_{k,i}, \tag{28}$$

and the unitary matrix $\mathbf{V}_{\boldsymbol{\mathcal{H}}_k}$ is defined by the following SVD

$$\boldsymbol{\Pi}_k^{-\frac{1}{2}}\boldsymbol{H}_k\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}} = \boldsymbol{U}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}^{\mathrm{H}} \text{ with } \boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k} \searrow, \tag{29}$$

in which positive definite matrix $\boldsymbol{\Pi}_k$ is defined as

$$\boldsymbol{\Pi}_k = \boldsymbol{R}_{\mathrm{n}} + \sum\nolimits_{j \neq k}\boldsymbol{H}_j\boldsymbol{Q}_{\mathrm{U},j}\boldsymbol{H}_j^{\mathrm{H}}. \tag{30}$$

Computation of μ_k and $\widetilde{\mu}_{k,i}$, $\forall k, i$: The scalars μ_k can be computed based on the following equality

$$\mathrm{Tr}\left(\sum\nolimits_{i=1}^{I_k}\widetilde{\mu}_{k,i}\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k}\right) = \sum\nolimits_{i=1}^{I_k}P_{k,i} = P_k, 1 \leq k \leq K. \tag{31}$$

$$\begin{cases} \boldsymbol{H}_k^{\mathrm{H}}\left(\boldsymbol{R}_{\mathrm{n}} + \sum_{j\neq k}\boldsymbol{H}_j\boldsymbol{Q}_{\mathrm{U},j}\boldsymbol{H}_j^{\mathrm{H}} + \boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\right)^{-1}\boldsymbol{H}_k = \sum_{i=1}^{I_k}\mu_{k,i}\boldsymbol{\Omega}_{k,i} - \boldsymbol{\Psi}_k, 1 \leq k \leq K, \\ \mu_{k,i} \geq 0, \ \mu_{k,i}(\mathrm{Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k}) - P_{k,i}) = 0, \mathrm{Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k}) \leq P_{k,i}, \ 1 \leq i \leq I_k, \\ \boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{\Psi}_k = \boldsymbol{0}, \ \boldsymbol{Q}_{\mathrm{U},k} \succeq \boldsymbol{0}, \ 1 \leq k \leq K, \end{cases} \tag{25}$$

$$\boldsymbol{Q}_{\mathrm{U},k} = \widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}\left[\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}\right]_{:,1:N_{\mathrm{C},k}}\left(\mu_k^{-1}\boldsymbol{I} - \left[\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\right]_{1:N_{\mathrm{C},k},1:N_{\mathrm{C},k}}^{-2}\right)^{+}\left[\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}\right]_{:,1:N_{\mathrm{C},k}}^{\mathrm{H}}\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}, \ 1 \leq k \leq K, \tag{27}$$

$$\log\det\left(\boldsymbol{I} + \sum_{k=1}^{K}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-1}\right) = \log\det\left(\boldsymbol{I} + \left(\sum_{k=1}^{K}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\left(\boldsymbol{H}_{\mathrm{SD},k} + \boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k}\right)\boldsymbol{Q}_{\mathrm{U},k}\left(\boldsymbol{H}_{\mathrm{SD},k} + \boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k}\right)^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\right)\right), \tag{33}$$

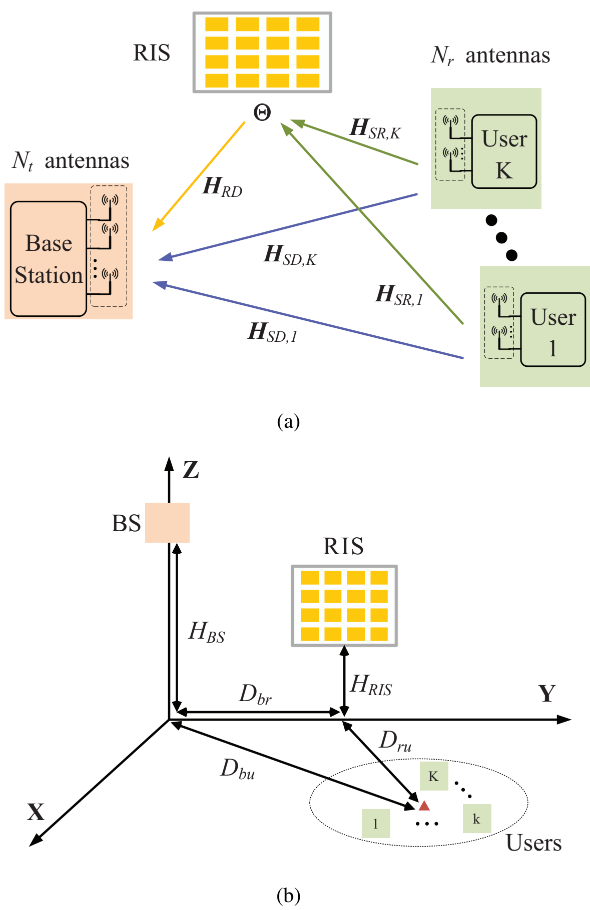

Fig. 1: (a) The single-cell RIS-aided uplink MU-MIMO system; (b) Simulation setup of the RIS-aided MU-MIMO system.

Hence the computation of μ_k is a standard water-level computation for iterative water-filling solution. In addition, $\widetilde{\mu}_{k,i}$ can be effectively computed using a modified subgradient algorithm shown in **Algorithm 1** [7].

B. Optimization of RIS Diagonal Matrix $\boldsymbol{\Theta}$

Based on the definition of $\boldsymbol{H}_k$ in **P. 3**, it is obvious that $\boldsymbol{H}_k$ is a function of the phase shifting matrix $\boldsymbol{\Theta}$, which can be written in the following form

$$\boldsymbol{H}_k = \boldsymbol{H}_{\mathrm{SD},k} + \boldsymbol{H}_{\mathrm{RD}} \boldsymbol{\Theta} \boldsymbol{H}_{\mathrm{SR},k} = \boldsymbol{H}_{\mathrm{SD},k} + \sum_{m=1}^{M} \theta_m \boldsymbol{\mathcal{H}}_{k,m}. \tag{32}$$

The objective function of the optimization problem **P. 3** can be rewritten as (33), which can be maximized via optimizing the phase shifting matrix $\boldsymbol{\Theta}$ element by element, and at each iteration a closed-form optimal solution can be derived [30]. In order to reduce the computational complexity, the objective function (33) can be approximated leading to the following optimization **P. 4**:

$$\mathbf{P.4}: \min_{\boldsymbol{W},\boldsymbol{G},\boldsymbol{\Theta}} \mathrm{Tr}\big(\boldsymbol{W}\big((\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}} - \boldsymbol{I})(\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}} - \boldsymbol{I})^{\mathrm{H}} + \boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}\big)\big) - \log\det(\boldsymbol{W}) + c, \tag{34}$$

Algorithm 1 The Modified Subgradient Method [7]

Initialize: Initialize the dual variables $\mu_{k,i}^{(0)}$, $\forall k, i$; iteration index $t = 0$; maximum iteration number $T_{\max}$; positive scalars a, b, c for step size; sufficiently small threshold $\epsilon > 0$.

repeat

Calculate $P_k = \sum_{l=1}^{I_k} P_{k,l}$, and $\widetilde{\mu}_{k,i}^{(t)} = \mu_{k,i}^{(t)} P_k \Big/ \left(\sum_{l=1}^{I_k} \mu_{k,l}^{(t)} P_{k,l}\right)$, $1 \le i \le I_k$.

Given $\widetilde{\boldsymbol{\Omega}}_k^{(t)} = \sum_{i=1}^{I_k} \widetilde{\mu}_{k,i}^{(t)} \boldsymbol{\Omega}_{k,i}$, solve optimization problem **P. 3** to obtain $\boldsymbol{Q}_{\mathrm{U},k}^{(t)}$ using (27).

Set the step size $a_i^{(t)} = \frac{a}{b \cdot t + c}$, $1 \le i \le I_k$, where $a, b, c > 0$..

Update $\mu_{k,i}^{(t+1)} = \left[\mu_{k,i}^{(t)} + a_i^{(t)} \left(\mathrm{Tr}\left(\boldsymbol{\Omega}_{k,i} \boldsymbol{Q}_{\mathrm{U},k}^{(t)}\right) - P_{k,i}\right)\right]^{+}$, $1 \le i \le I_k$.

Update $t = t + 1$.

until $\left|\mu_{k,i}^{(t)} \left(\mathrm{Tr}\left(\boldsymbol{\Omega}_{k,i} \boldsymbol{Q}_{\mathrm{U},k}^{(t)}\right) - P_{k,i}\right)\right| \le \epsilon$, $\forall i$, or $t = T_{\max}$.

return The optimal $\boldsymbol{Q}_{\mathrm{U},k}^{\star} = \boldsymbol{Q}_{\mathrm{U},k}^{(t)}$ to the optimization problem for user k.

where c is a constant independent of the optimization variables, and $\boldsymbol{H}_{\mathrm{V}}$ can be understood as a virtual channel matrix, which is defined as

$$\boldsymbol{H}_{\mathrm{V}} = \left[\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}} \boldsymbol{H}_1 \boldsymbol{Q}_{\mathrm{U},1}^{\frac{1}{2}}\right)^{*} \cdots \left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}} \boldsymbol{H}_K \boldsymbol{Q}_{\mathrm{U},K}^{\frac{1}{2}}\right)^{*}\right]^{\mathrm{T}}. \tag{35}$$

Note that in each iteration the optimal $\boldsymbol{G}$ is derived in the following closed-form

$$\boldsymbol{G} = (\boldsymbol{H}_{\mathrm{V}}^{\mathrm{H}} \boldsymbol{H}_{\mathrm{V}} + \boldsymbol{I})^{-1} \boldsymbol{H}_{\mathrm{V}}^{\mathrm{H}}. \tag{36}$$

It is worth noting that the matrix inversion operation is only performed on a very low-dimensional matrix. Also in each iteration, $\boldsymbol{W}$ equals

$$\boldsymbol{W}^{-1} = (\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}} - \boldsymbol{I})(\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}} - \boldsymbol{I})^{\mathrm{H}} + \boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}. \tag{37}$$

Based on (36), the complexity of computing $\boldsymbol{W}$ is also low as $\boldsymbol{G}\boldsymbol{G}^{\mathrm{H}}$ is a low dimension matrix. In each iteration, the optimization of $\boldsymbol{\Theta}$ is formulated as the following optimization problem

$$\mathbf{P.5}: \min_{\boldsymbol{\Theta}} \mathrm{Tr}\big(\boldsymbol{H}_{\mathrm{V}}^{\mathrm{H}} \boldsymbol{G}^{\mathrm{H}} \boldsymbol{W} \boldsymbol{G} \boldsymbol{H}_{\mathrm{V}}\big) - 2\Re\{\mathrm{Tr}(\boldsymbol{W}\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}})\}. \tag{38}$$

Based on the definition of $\boldsymbol{H}_{\mathrm{V}}$ given in (35) and defining

$$\left[\boldsymbol{W}^{\frac{1}{2}} \boldsymbol{G}\right]_k = \left[\boldsymbol{W}^{\frac{1}{2}} \boldsymbol{G}\right]_{:, N_R(k-1)+1:N_R k}, \tag{39}$$

we have (40) shown at the top of the next page, based on which the optimization problem (38) is further transferred into the following compact form

$$\mathbf{P.6}: \min_{\boldsymbol{\theta}} \boldsymbol{\theta}^{\mathrm{H}} \boldsymbol{A} \boldsymbol{\theta} - 2\Re\{\boldsymbol{\theta}^{\mathrm{H}} \boldsymbol{b}\}, \tag{41}$$

in which the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{b}$ are defined respectively as (42) and (43). The optimization problem (41) can be solved using existing iterative algorithms, such as the MM algorithm [36]–[38] and the ADMM algorithm [39].

$$\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\boldsymbol{H}_{\mathrm{V}}=\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}=\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}+\sum_{m=1}^{M}\theta_m^*\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\mathcal{H}}_{k,m}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}, \tag{40}$$

$$[\boldsymbol{A}]_{m,n}=\mathrm{Tr}\left(\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\mathcal{H}}_{k,m}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\mathcal{H}}_{k,n}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}\right)^{\mathrm{H}}\right), \tag{42}$$

$$[\boldsymbol{b}]_{m}=\mathrm{Tr}\left(\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\mathcal{H}}_{k,m}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}\left(\boldsymbol{W}^{\frac{1}{2}}-\sum_{k=1}^{K}\left[\boldsymbol{W}^{\frac{1}{2}}\boldsymbol{G}\right]_k\left(\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{Q}_{\mathrm{U},k}^{\frac{1}{2}}\right)^{\mathrm{H}}\right)^{\mathrm{H}}\right). \tag{43}$$

C. Discussions

In this part, we discuss both the efficiency and convergence of this alternative optimization (AO) method. It is indeed difficult to find a globally optimal solution for MU-MIMO communications via the AO method. Instead, a simplified but meaningful SU-MIMO scenario associated with rank-1 LoS channels is considered, where the global optimality of the solutions obtained from our AO method can be proved. We consider the UL capacity maximization problem as an example, see **Lemma 1.** In each step, one of the two block variables, e.g. $\{\boldsymbol{Q}_{\mathrm{U},k}\}$, is optimized with the other one fixed and they are not coupled in the constraints. The popular MM algorithm is used for RIS phase shift matrix optimization by successively minimizing a sequence of surrogate functions. Because the function's value is non-decreasing during iterations and the capacity is upper-bounded by a finite number, the convergence of the AO method is guaranteed. Furthermore, since the optimal $\{\boldsymbol{Q}_{\mathrm{U},k}\}$ and a stationary solution to $\boldsymbol{\Theta}$ are derived in each iteration, the AO method converges to a set of locally optimal points.

Lemma 1. For the UL SU-MIMO scenario associated with the rank-1 LoS channels for the BS-RIS as well as RIS-User links and no direct BS-User link - which implies that $K=1$ and $\boldsymbol{H}_{\mathrm{RD}}=\boldsymbol{\alpha}_{\mathrm{B}}(v_b)\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^b,\theta^b)$, $\boldsymbol{H}_{\mathrm{SR}}=\boldsymbol{\alpha}_{\mathrm{P}}(\psi^u,\theta^u)\boldsymbol{\alpha}_{\mathrm{U}}^{\mathrm{H}}(v_u)$ - the AO method used in this paper can indeed find the globally optimal solution $\{\{\widetilde{\boldsymbol{Q}}^{'}\},\boldsymbol{\Theta}^{'}\}$ of the UL capacity maximization problem (24).

Proof: Please refer to Appendix B for proof and notations. ■

Note that this lemma can also be applied to the other three problems under the same SU-MIMO scenario. The convergence of the AO method is guaranteed for MU-MIMO systems.

IV. MSE Minimization of the MU-MIMO UL

Based on the signal model (23), the sum-MSE minimization in RIS-aided MU-MIMO UL can be formulated as follows

$$\mathbf{P.7}:\begin{cases}\min\limits_{\{\boldsymbol{Q}_{\mathrm{U},k}\},\boldsymbol{\Theta}}\mathrm{Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\right)^{-1}\right),\\ \text{s.t.}\quad \boldsymbol{H}_k=\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k},\\ \qquad \mathrm{Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k})\le P_{k,i},\ 1\le i\le I_k,\ 1\le k\le K,\\ \qquad \boldsymbol{Q}_{\mathrm{U},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k})\le N_{\mathrm{C},k},\ 1\le k\le K.\end{cases} \tag{44}$$

In the following, **P. 7** is solved in an iterative manner. Specifically, the iterative procedure consists of two phases. In the first phase, $\boldsymbol{\Theta}$ is fixed, and $\boldsymbol{Q}_{\mathrm{U},k}$ are optimized iteratively. In the second phase, $\boldsymbol{Q}_{\mathrm{U},k}$ are fixed, and $\boldsymbol{\Theta}$ is optimized. It is obvious that without the rank constraints $\mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k})\le N_{\mathrm{C},k}$, **P. 7** is a convex optimization problem with respect to $\boldsymbol{Q}_{\mathrm{U},k}$, which can be solved by using standard optimization software toolboxes, such as CVX [40]. In this work, in order to avoid high computational complexity, the objective function is replaced by its lower bound and thus we aim at solving the following approximated optimization problem

$$\mathbf{P.8}:\begin{cases}\min\limits_{\substack{\{\boldsymbol{Q}_{\mathrm{U},k}\},\\ \{\boldsymbol{U}_k\},\boldsymbol{\Theta}}}\mathrm{Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{U}_k\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{U}_k^{\mathrm{H}}\right)^{-1}\right),\\ \text{s.t.}\quad \boldsymbol{H}_k=\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k},\boldsymbol{U}_k=\widetilde{\boldsymbol{\Sigma}}_k^{\frac{1}{2}}\boldsymbol{\Pi}_k\boldsymbol{\Sigma}_k^{-\frac{1}{2}},\\ \qquad \mathrm{Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{\mathrm{U},k})\le P_{k,i},\ 1\le i\le I_k,\ 1\le k\le K,\\ \qquad \boldsymbol{Q}_{\mathrm{U},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k})\le N_{\mathrm{C},k},\ 1\le k\le K,\end{cases} \tag{45}$$

where $\boldsymbol{\Pi}_k$ is a unitary matrix, the matrices $\widetilde{\boldsymbol{\Sigma}}_k$ and $\boldsymbol{\Sigma}_k$ are defined respectively by

$$\widetilde{\boldsymbol{\Sigma}}_k=\boldsymbol{I}+\sum\nolimits_{j\neq k}\boldsymbol{U}_j\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_j\boldsymbol{Q}_{\mathrm{U},j}\boldsymbol{H}_j^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{U}_j^{\mathrm{H}}, \tag{46}$$

$$\boldsymbol{\Sigma}_k=\boldsymbol{I}+\sum\nolimits_{j\neq k}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_j\boldsymbol{Q}_{\mathrm{U},j}\boldsymbol{H}_j^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}. \tag{47}$$

With fixed $\{\boldsymbol{U}_k\}$ and $\boldsymbol{\Theta}$, the corresponding KKT conditions of **P. 8** with respect to $\boldsymbol{Q}_{\mathrm{U},k}$ are given by (48) at the top of the next page. Based the first KKT condition in (48), the optimization problem **P. 8** is equivalent to the following problem

$$\mathbf{P.9}:\begin{cases}\min\limits_{\substack{\{\boldsymbol{Q}_{\mathrm{U},k}\},\\ \{\boldsymbol{U}_k\},\boldsymbol{\Theta}}}\mathrm{Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{U}_k\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{U},k}\boldsymbol{H}_k^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{U}_k^{\mathrm{H}}\right)^{-1}\right),\\ \text{s.t.}\quad \boldsymbol{H}_k=\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD}}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR},k},\ \boldsymbol{U}_k=\widetilde{\boldsymbol{\Sigma}}_k^{\frac{1}{2}}\boldsymbol{\Pi}_k\boldsymbol{\Sigma}_k^{-\frac{1}{2}},\\ \qquad \mathrm{Tr}(\widetilde{\boldsymbol{\Omega}}_k\boldsymbol{Q}_{\mathrm{U},k})\le P_k,\ 1\le k\le K,\\ \qquad \boldsymbol{Q}_{\mathrm{U},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{U},k})\le N_{\mathrm{C},k},\ 1\le k\le K,\end{cases} \tag{49}$$

where the positive definite matrix $\widetilde{\boldsymbol{\Omega}}_k$ is defined as

$$\widetilde{\boldsymbol{\Omega}}_k=\sum\nolimits_{i=1}^{I}\widetilde{\mu}_i\boldsymbol{\Omega}_{k,i}, \tag{50}$$

and $\widetilde{\mu}_i$ are given in (26) which can be computed by **Algorithm 1**. In the following, **P. 9** is optimized in an alternating manner among $\{\boldsymbol{Q}_{\mathrm{U},k}\}$, $\{\boldsymbol{U}_k\}$, and $\boldsymbol{\Theta}$.

$$\begin{cases} \boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{U}_k^{\rm H}\left(\boldsymbol{I}+\sum_{j\neq k}\boldsymbol{U}_j\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_j\boldsymbol{Q}_{{\rm U},j}\boldsymbol{H}_j^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{U}_j^{\rm H}+\boldsymbol{U}_k\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{{\rm U},k}\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{U}_k^{\rm H}\right)^{-2} \\ \times\boldsymbol{U}_k\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k=\sum_{i=1}^{I_k}\mu_{k,i}\boldsymbol{\Omega}_{k,i}-\boldsymbol{\Psi}_k,\ 1\le k\le K, \\ \mu_{k,i}\ge 0,\ \mu_{k,i}({\rm Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{{\rm U},k})-P_{k,i})=0, {\rm Tr}(\boldsymbol{\Omega}_{k,i}\boldsymbol{Q}_{{\rm U},k})\le P_{k,i},\ 1\le i\le I_k, \\ \boldsymbol{Q}_{{\rm U},k}\boldsymbol{\Psi}_k=\boldsymbol{0},\ \boldsymbol{Q}_{{\rm U},k}\succeq\boldsymbol{0},\ 1\le k\le K. \end{cases} \tag{48}$$

$$\mathbf{P.10}:\begin{cases} \min\limits_{\boldsymbol{Q}_{{\rm U},k},\boldsymbol{\Pi}_k} & {\rm Tr}\left(\widetilde{\boldsymbol{\Sigma}}_k^{-1}\left(\boldsymbol{I}+\boldsymbol{\Pi}_k\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}\widetilde{\boldsymbol{Q}}_{{\rm U},k}\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{\Pi}_k^{\rm H}\right)^{-1}\right), \\ {\rm s.t.} & \boldsymbol{H}_k=\boldsymbol{H}_{{\rm SD},k}+\boldsymbol{H}_{\rm RD}\boldsymbol{\Theta}\boldsymbol{H}_{{\rm SR},k}, \\ & {\rm Tr}(\widetilde{\boldsymbol{Q}}_{{\rm U},k})\le P_k, {\rm Rank}(\widetilde{\boldsymbol{Q}}_{{\rm U},k})\le N_{{\rm C},k},\ 1\le k\le K. \end{cases} \tag{52}$$

$$\begin{aligned} &\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{\Pi}^{\rm H}\left(\boldsymbol{I}+\boldsymbol{\Pi}_k\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{{\rm U},k}\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{\Pi}_k^{\rm H}\right)^{-1}\widetilde{\boldsymbol{\Sigma}}_k^{-1} \\ &\times\left(\boldsymbol{I}+\boldsymbol{\Pi}_k\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{{\rm U},k}\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{\Pi}_k^{\rm H}\right)^{-1}\boldsymbol{\Pi}\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k=\sum_{i=1}^{I_k}\mu_{k,i}\boldsymbol{\Omega}_{k,i}-\boldsymbol{\Psi}_k. \end{aligned} \tag{56}$$

$$\begin{aligned} &{\rm Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\boldsymbol{Q}_{{\rm U},k}\boldsymbol{H}_k^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\right)^{-1}\right) \\ &={\rm Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\left(\boldsymbol{H}_{{\rm SD},k}+\boldsymbol{H}_{\rm RD}\boldsymbol{\Theta}\boldsymbol{H}_{{\rm SR},k}\right)\boldsymbol{Q}_{{\rm U},k}\left(\boldsymbol{H}_{{\rm D},k}+\boldsymbol{H}_{\rm RD}\boldsymbol{\Theta}\boldsymbol{H}_{{\rm SR},k}\right)^{\rm H}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\right)^{-1}\right), \end{aligned} \tag{60}$$

A. Optimization of $\boldsymbol{Q}_{{\rm U},k}$

First define

$$\widetilde{\boldsymbol{Q}}_{{\rm U},k}=\widetilde{\boldsymbol{\Omega}}_k^{\frac{1}{2}}\boldsymbol{Q}_{{\rm U},k}\widetilde{\boldsymbol{\Omega}}_k^{\frac{1}{2}}. \tag{51}$$

In the kth iteration with fixed $\{\boldsymbol{U}_j\}$ and $\boldsymbol{Q}_{{\rm U},j}$ for $j\neq k$, the optimization problem **P. 9** becomes **P. 10** at the top of this page. For **P. 10**, the optimal unitary matrix $\boldsymbol{\Pi}_k$ is given by

$$\boldsymbol{\Pi}_k=\boldsymbol{U}_{\boldsymbol{\Sigma}_k}\boldsymbol{U}_{\boldsymbol{\mathcal{H}}_k}^{\rm H}, \tag{53}$$

where the unitary matrices $\boldsymbol{U}_{\boldsymbol{\Sigma}_k}$ and $\boldsymbol{U}_{\boldsymbol{\mathcal{H}}_k}$ are defined respectively based on the following EVD and SVD

$$\widetilde{\boldsymbol{\Sigma}}_k^{-1}=\boldsymbol{U}_{\boldsymbol{\Sigma}_k}\boldsymbol{\Lambda}_{\boldsymbol{\Sigma}_k}^{-1}\boldsymbol{U}_{\boldsymbol{\Sigma}_k}^{\rm H},\ \text{with}\ \boldsymbol{\Lambda}_{\boldsymbol{\Sigma}_k}^{-1}\searrow, \tag{54}$$

$$\boldsymbol{\Sigma}_k^{-\frac{1}{2}}\boldsymbol{R}_{\rm n}^{-\frac{1}{2}}\boldsymbol{H}_k\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}=\boldsymbol{U}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}^{\rm H},\ \text{with}\ \boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\searrow. \tag{55}$$

Based on (53), the first KKT condition in (48) can be rewritten into the formula (56). Then according to **Conclusion 2**, the optimal $\widetilde{\boldsymbol{Q}}_k$ satisfies the following structure

$$\widetilde{\boldsymbol{Q}}_k=\widetilde{\boldsymbol{\Omega}}_k^{\frac{1}{2}}[\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}]_{:,1:N_{\rm C,k}}\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}_k}[\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k}]_{:,1:N_{{\rm C},k}}^{\rm H}\widetilde{\boldsymbol{\Omega}}_k^{-\frac{1}{2}}. \tag{57}$$

Based on (57), **P. 11** can be rewritten as

$$\mathbf{P.11}:\begin{cases} \min\limits_{\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}_k}} & {\rm Tr}\left(\boldsymbol{\Lambda}_{\boldsymbol{\Sigma}_k}^{-1}\left(\boldsymbol{I}+\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}_k}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}^{\rm T}\right)^{-1}\right), \\ {\rm s.t.} & {\rm Tr}(\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}_k})\le P_k. \end{cases} \tag{58}$$

The optimal solution of **P. 11** is derived to be the following water-filling solution

$$[\widetilde{\boldsymbol{\Lambda}}_{\boldsymbol{Q}_k}]_{n,n}=\left(\sqrt{\frac{1}{\mu[\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}^{\rm T}]_{n,n}[\boldsymbol{\Lambda}_{\boldsymbol{\Sigma}_k}]_{n,n}}}-\frac{1}{[\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}\boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k}^{\rm T}]_{n,n}}\right)^{+}, \quad 1\le n\le N_{{\rm C},k}. \tag{59}$$

where μ is the Lagrange multiplier corresponding to the power constraint in **P. 9**.

B. Optimization of $\boldsymbol{\Theta}$

With all the $\boldsymbol{Q}_{{\rm U},k}$ fixed, the phase shifting matrix $\boldsymbol{\Theta}$ at the RIS is optimized. The objective function of **P. 7** can be reformulated as (60), which can be minimized via optimizing $\boldsymbol{\Theta}$ element-by-element and at each iteration there are closed-form optimal solutions. In order to reduce the computational complexity, **P 7** is equivalent to the following optimization problem

$$\mathbf{P.12}:\ \min_{\boldsymbol{G},\boldsymbol{\Theta}}\ {\rm Tr}\big(((\boldsymbol{G}\boldsymbol{H}_{\rm V}-\boldsymbol{I})(\boldsymbol{G}\boldsymbol{H}_{\rm V}-\boldsymbol{I})^{\rm H}+\boldsymbol{G}\boldsymbol{G}^{\rm H})\big), \tag{61}$$

which can be solved in an iterating manner based on (36) and (38).

V. Capacity Maximization of the MU-MIMO DL

In RIS-aided MU-MIMO downlink, the BS transmits the user-related information to all the K users. Under multiple weighted power constraints and with perfect CSI, the optimization problem of the transmission covariance matrices for the sum-capacity maximization is formulated as

$$\mathbf{P.13}:\begin{cases} \max\limits_{\{\boldsymbol{Q}_{{\rm D},k}\},\boldsymbol{\Theta}} \sum_{k=1}^{K}\log\left|\boldsymbol{I}+\boldsymbol{\Sigma}_{{\rm D},k}^{-1}\boldsymbol{H}_k\boldsymbol{Q}_{{\rm D},k}\boldsymbol{H}_k^{\rm H}\right|, \\ {\rm s.t.}\quad \boldsymbol{H}_k=\boldsymbol{H}_{{\rm SD},k}+\boldsymbol{H}_{{\rm RD},k}\boldsymbol{\Theta}\boldsymbol{H}_{\rm SR}, \\ \qquad \boldsymbol{\Sigma}_{{\rm D},k}=\boldsymbol{R}_{{\rm D},{\rm n}_k}+\boldsymbol{H}_k\sum_{j<k}\boldsymbol{Q}_{{\rm D},j}\boldsymbol{H}_k^{\rm H}, \\ \qquad {\rm Tr}\Big(\boldsymbol{\Omega}_{{\rm D},i}\sum_{k=1}^{K}\boldsymbol{Q}_{{\rm D},k}\Big)\le P_i,\ 1\le i\le I, \\ \qquad \boldsymbol{Q}_{{\rm D},k}\succeq\boldsymbol{0},\ {\rm Rank}(\boldsymbol{Q}_{{\rm D},k})\le N_{{\rm C},k}, 1\le k\le K, \end{cases} \tag{62}$$

where $\mathbf{\Omega}_{\mathrm{D},i}$ is the ith constraint's weighting matrix and P_i is the corresponding power limit. In addition, $\boldsymbol{R}_{\mathrm{D,n}_k}$ is the noise covariance matrix at the kth user, and the matrix $\boldsymbol{H}_{\mathrm{SD},k}$ denotes the channel matrix between the BS and the kth user, while $\boldsymbol{H}_{\mathrm{RD},k}$ is the channel matrix from the RIS to the kth user and $\boldsymbol{H}_{\mathrm{SR}}$ is the channel matrix from the BS to the RIS. Dirty paper coding is used at the BS.

Based on the KKT conditions with respect to $\mathbf{Q}_{D,k}$ with $\mathbf{\Theta}$ fixed, the optimization problem **P. 13** is equivalent to

$$\textbf{P. 14}: \begin{cases} \max\limits_{\{\boldsymbol{Q}_{\mathrm{D},k}\},\mathbf{\Theta}} \sum_{k=1}^K \log\left|\boldsymbol{I}+\mathbf{\Sigma}_{\mathrm{D,k}}^{-1}\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{D,k}}\boldsymbol{H}_k^{\mathrm{H}}\right|, \\ \text{s.t.} \quad \boldsymbol{H}_k=\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\mathbf{\Theta}\boldsymbol{H}_{\mathrm{SR}}, \\ \qquad \mathbf{\Sigma}_{\mathrm{D},k}=\boldsymbol{R}_{\mathrm{D,n}_k}+\boldsymbol{H}_k\sum_{j<k}\boldsymbol{Q}_{\mathrm{D},j}\boldsymbol{H}_k^{\mathrm{H}}, \\ \qquad \mathrm{Tr}\Big(\widetilde{\mathbf{\Omega}}_{\mathrm{D}}\sum_{k=1}^K\boldsymbol{Q}_{\mathrm{D},k}\Big)\le P, \\ \qquad \boldsymbol{Q}_{\mathrm{D},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{D},k})\le N_{\mathrm{C},k},\ 1\le k\le K, \end{cases} \tag{63}$$

where P is the sum-antenna power of the BS and the positive definite matrix $\widetilde{\mathbf{\Omega}}_{\mathrm{D}}$ is defined as

$$\widetilde{\mathbf{\Omega}}_{\mathrm{D}}=\sum_{i=1}^{I}\widetilde{\mu}_i\mathbf{\Omega}_{\mathrm{D},i}. \tag{64}$$

Defining the following auxiliary variables

$$\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}=\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{\frac{1}{2}}\boldsymbol{Q}_{\mathrm{D},k}\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{\frac{1}{2}}, \quad \boldsymbol{H}_{\mathrm{D},k}=\boldsymbol{R}_{\mathrm{D,n}_k}^{-\frac{1}{2}}\boldsymbol{H}_k\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}, \tag{65}$$

the optimization problem **P. 14** is further equivalent to

$$\textbf{P. 15}: \begin{cases} \max\limits_{\{\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\},\mathbf{\Theta}} \sum_{k=1}^K \log\left|\boldsymbol{I}+\widetilde{\mathbf{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\right|, \\ \text{s.t.} \quad \boldsymbol{H}_{\mathrm{D},k}=\boldsymbol{R}_{\mathrm{D,n}_k}^{-\frac{1}{2}}(\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\mathbf{\Theta}\boldsymbol{H}_{\mathrm{SR}})\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}, \\ \qquad \widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}=\boldsymbol{I}+\boldsymbol{H}_{\mathrm{D},k}\sum_{j<k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},j}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}, \\ \qquad \mathrm{Tr}\Big(\sum_{k=1}^K\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\Big)\le P, \\ \qquad \widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\widetilde{\boldsymbol{Q}}_{\mathrm{D},k})\le N_{\mathrm{C},k},\ 1\le k\le K. \end{cases} \tag{66}$$

Given the complex mathematical formulation of $\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}$ in **P. 15**, it remains an open challenge to solve **P. 15** directly. Instead, some mathematical transformations may be invoked by exploiting the uplink-downlink duality. Specifically, for an arbitrary positive definite matrix $\widetilde{\mathbf{\Sigma}}_{\mathrm{U,k}}$, the objective function of **P. 15** can be rewritten in the form [41] as (67) shown at the top of the next page, where the unitary matrices $\boldsymbol{U}_{\mathrm{D},k}$ and $\boldsymbol{V}_{\boldsymbol{D},k}$ are defined based on the following SVD

$$\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}=\boldsymbol{U}_{\mathrm{D},k}\mathbf{\Lambda}_{\mathrm{D},k}\boldsymbol{V}_{\mathrm{D},k}^{\mathrm{H}}, \text{ with } \mathbf{\Lambda}_{\mathrm{D},k}\searrow. \tag{68}$$

Based on (67) and defining the auxiliary matrix variables of (69),(70) at the top of the next page, the objective function of **P. 15** is equivalent to

$$\begin{aligned}&\sum_{k=1}^K\log\left|\boldsymbol{I}+\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\right|\\ &=\sum_{k=1}^K\log\left|\boldsymbol{I}+\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}\right|.\end{aligned} \tag{71}$$

It is worth noting that the equality in (71) holds for arbitrary positive matrix $\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}$. When

$$\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}=\boldsymbol{I}+\sum_{j>k}\boldsymbol{H}_{\mathrm{U},j}\widetilde{\boldsymbol{Q}}_{\mathrm{U},j}\boldsymbol{H}_{\mathrm{U},j}^{\mathrm{H}}, \tag{72}$$

and together with (71), the objective function of **P. 15** is finally equivalent to

$$\begin{aligned}&\sum_{k=1}^K\log\left|\boldsymbol{I}+\widetilde{\mathbf{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\right|\\ &=\sum_{k=1}^K\log\left|\boldsymbol{I}+\widetilde{\mathbf{\Sigma}}_{\mathrm{U,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}\right|\\ &=\log\left|\boldsymbol{I}+\sum_{k=1}^K\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right|.\end{aligned} \tag{73}$$

On the other hand, based on the definition of $\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}$ in (69), we have

$$\begin{aligned}&\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}=\\ &\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}\boldsymbol{U}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{\frac{1}{2}}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{\frac{1}{2}}\boldsymbol{U}_{\mathrm{D},k}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}.\end{aligned} \tag{74}$$

Substituting (74) into the power constraint in **P. 15**, we have

$$\mathrm{Tr}\Big(\sum_{k=1}^K\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\Big)=\mathrm{Tr}\Big(\sum_{k=1}^K\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\Big)\le P. \tag{75}$$

Based on (73) and (75), the optimization problem **P. 15** is equivalent to the following optimization

$$\textbf{P. 16}: \begin{cases} \max\limits_{\{\boldsymbol{Q}_{\mathrm{U,k}}\},\mathbf{\Theta}} \log\left|\boldsymbol{I}+\sum_{k=1}^K\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right|, \\ \text{s.t.} \quad \boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}=\boldsymbol{R}_{\mathrm{D,n}_k}^{-\frac{1}{2}}(\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\mathbf{\Theta}\boldsymbol{H}_{\mathrm{SR}})\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}, \\ \qquad \mathrm{Tr}\Big(\sum_{k=1}^K\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\Big)\le P, \\ \qquad \widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\widetilde{\boldsymbol{Q}}_{\mathrm{U},k})\le N_{\mathrm{C},k},\ 1\le k\le K. \end{cases} \tag{76}$$

It is obvious that the optimization problem **P. 16** can be solved effectively [9]. After solving **P. 16**, the key task is to how to derive the optimal $\{\boldsymbol{Q}_{\mathrm{D,k}}\}$ from the optimal $\{\boldsymbol{Q}_{\mathrm{U,k}}\}$ obtained. Based on the definitions of $\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}$ in **P. 15** and $\widetilde{\mathbf{\Sigma}}_{\mathrm{U},k}$ in (72) together with (68), $\boldsymbol{Q}_{\mathrm{D,k}}$ can be computed from $k=1$ to $k=K$ in a recursion manner [41].

VI. MSE MINIMIZATION OF THE MU-MIMO DL

In this section, the covariance matrix optimization of MSE minimization for RIS-aided MU-MIMO DL communications is investigated. This optimization problem is defined by

$$\textbf{P. 17}: \begin{cases} \min\limits_{\{\boldsymbol{Q}_{\mathrm{D},k}\},\mathbf{\Theta}} \sum_{k=1}^K \mathrm{Tr}\left(\left(\boldsymbol{I}+\boldsymbol{H}_k\boldsymbol{Q}_{\mathrm{D},k}\boldsymbol{H}_k^{\mathrm{H}}\mathbf{\Sigma}_{\mathrm{D},k}^{-1}\right)^{-1}\right), \\ \text{s.t.} \quad \boldsymbol{H}_k=\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\mathbf{\Theta}\boldsymbol{H}_{\mathrm{SR}}, \\ \qquad \mathbf{\Sigma}_{\mathrm{D},k}=\boldsymbol{R}_{\mathrm{n}_k}+\boldsymbol{H}_k\sum_{j\neq k}\boldsymbol{Q}_{\mathrm{D},j}\boldsymbol{H}_k^{\mathrm{H}}, \\ \qquad \mathrm{Tr}\Big(\mathbf{\Omega}_{\mathrm{D},i}\sum_{k=1}^K\boldsymbol{Q}_{\mathrm{D},k}\Big)\le P_i,\ 1\le i\le I, \\ \qquad \boldsymbol{Q}_{\mathrm{D},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\boldsymbol{Q}_{\mathrm{D},k})\le N_{\mathrm{C},k},\ 1\le k\le K. \end{cases} \tag{77}$$

Based on the KKT conditions with respect to $\boldsymbol{Q}_{\mathrm{D},k}$, **P. 17** is equivalent to

$$\textbf{P. 18}: \begin{cases} \min\limits_{\{\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\},\mathbf{\Theta}} \sum_{k=1}^K \mathrm{Tr}\Big(\Big(\boldsymbol{I}_{\mathrm{B_k}}+\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\Big)^{-1}\Big), \\ \text{s.t.} \quad \boldsymbol{H}_{\mathrm{D},k}=\boldsymbol{R}_{\mathrm{n}_k}^{-\frac{1}{2}}(\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\mathbf{\Theta}\boldsymbol{H}_{\mathrm{SR}})\widetilde{\mathbf{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}, \\ \qquad \widetilde{\mathbf{\Sigma}}_{\mathrm{D},k}=\boldsymbol{I}+\boldsymbol{H}_{\mathrm{D},k}\sum_{j\neq k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},j}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}, \\ \qquad \mathrm{Tr}\Big(\sum_{k=1}^K\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\Big)\le P, \\ \qquad \widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\succeq\boldsymbol{0},\ \mathrm{Rank}(\widetilde{\boldsymbol{Q}}_{\mathrm{D},k})\le N_{\mathrm{C},k},\ 1\le k\le K. \end{cases} \tag{78}$$

$$\sum\nolimits_{k=1}^{K}\log\left|\boldsymbol{I}+\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\right|=\sum\nolimits_{k=1}^{K}\log\left|\boldsymbol{I}+\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{-\frac{1}{2}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{\frac{1}{2}}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{\frac{1}{2}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\right|$$
$$=\sum_{k=1}^{K}\log\left|\boldsymbol{I}+\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{U}_{\mathrm{D},k}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{\frac{1}{2}}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U},k}^{\frac{1}{2}}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}\boldsymbol{U}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D,k}}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U},k}^{-\frac{1}{2}}\right|, \tag{67}$$

$$\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}=\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\boldsymbol{U}_{\mathrm{D},k}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{\frac{1}{2}}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{U,k}}^{\frac{1}{2}}[\boldsymbol{V}_{\mathrm{D},k}]_{:,1:N_R}\boldsymbol{U}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}, \tag{69}$$
$$\boldsymbol{H}_{\mathrm{U},k}=\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}, \tag{70}$$

$$\sum\nolimits_{k=1}^{K}\mathrm{Tr}\left(\left(\boldsymbol{I}_{\mathrm{B_k}}+\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\boldsymbol{H}_{\mathrm{D},k}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\widetilde{\boldsymbol{\Sigma}}_{\mathrm{D},k}^{-\frac{1}{2}}\right)^{-1}\right)$$
$$=\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{I}_{N_k}-\boldsymbol{G}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}-\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}+\boldsymbol{G}_{\mathrm{D},k}\left(\boldsymbol{I}+\boldsymbol{H}_{\mathrm{D},k}\sum_{j}\boldsymbol{P}_{\mathrm{D},j}\boldsymbol{P}_{\mathrm{D},j}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\right)\boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}\right) \tag{80}$$

$$\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{I}_{N_k}-\boldsymbol{G}_{\mathrm{D},k}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}-\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}+\boldsymbol{G}_{\mathrm{D},k}\left(\boldsymbol{I}+\boldsymbol{H}_{\mathrm{D},k}\sum_{j}\boldsymbol{P}_{\mathrm{D},j}\boldsymbol{P}_{\mathrm{D},j}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\right)\boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}\right)$$
$$=\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{I}_{N_k}-\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}-\boldsymbol{G}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}+\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\left(\frac{1}{\alpha_k^2}\boldsymbol{I}+\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\sum_{j}\frac{\alpha_j^2}{\alpha_k^2}\boldsymbol{G}_{\mathrm{U},j}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},j}\boldsymbol{H}_{\mathrm{U},k}\right)\boldsymbol{P}_{\mathrm{U},k}\right). \tag{82}$$

$$\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{I}_{N_k}-\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}-\boldsymbol{G}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}+\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\left(\frac{1}{\alpha_k^2}\boldsymbol{I}+\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\sum_{j}\frac{\alpha_j^2}{\alpha_k^2}\boldsymbol{G}_{\mathrm{U},j}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},j}\boldsymbol{H}_{\mathrm{U},k}\right)\boldsymbol{P}_{\mathrm{U},k}\right)$$
$$=\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{I}_{N_k}-\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}-\boldsymbol{G}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}+\boldsymbol{G}_{\mathrm{U},k}\left(\boldsymbol{I}+\sum_{j}\boldsymbol{H}_{\mathrm{U},j}\boldsymbol{P}_{\mathrm{U},j}\boldsymbol{P}_{\mathrm{U},j}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},j}^{\mathrm{H}}\right)\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}\right)$$
$$=\mathrm{Tr}\left(\boldsymbol{I}-\sum_{k=1}^{K}\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}-\sum_{k=1}^{K}\boldsymbol{G}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}+\boldsymbol{G}_{\mathrm{U}}\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right)\boldsymbol{G}_{\mathrm{U}}^{\mathrm{H}}\right)$$
$$=\mathrm{Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right)^{-1}\right), \tag{86}$$

It is challenging to directly derive optimal solutions in closed-form for the optimization problem (77) or (78). Therefore, **P. 18** is first transferred into a virtual UL optimization problem. By defining

$$\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}=\boldsymbol{P}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}, \tag{79}$$

where $\boldsymbol{P}_{\mathrm{D},k}$ is an $N_T\times N_{\mathrm{C},k}$ matrix, the objective function in **P. 17** equals (80) with $\boldsymbol{G}_{\mathrm{D},k}=\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\left(\boldsymbol{I}+\boldsymbol{H}_{\mathrm{D},k}\sum_j\boldsymbol{P}_{\mathrm{D},j}\boldsymbol{P}_{\mathrm{D},j}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\right)^{-1}$. Introducing the following new auxiliary variables

$$\boldsymbol{H}_{\mathrm{U},k}=\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}},\ \boldsymbol{G}_{\mathrm{U},k}=\frac{1}{\alpha_k}\boldsymbol{P}_{\mathrm{D,k}}^{\mathrm{H}},$$
$$\boldsymbol{P}_{\mathrm{U},k}=\alpha_k\boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}},\ 1\le k\le K, \tag{81}$$

where α_k are the scaling factors, the objective function in (80) is further equivalent to (82). When the following equations are satisfied [42]

$$\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}=\boldsymbol{P}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}, \tag{83}$$
$$\boldsymbol{Z}\left[\alpha_1^2\cdots\alpha_K^2\right]^{\mathrm{T}}=\left[\mathrm{Tr}\left(\widetilde{\boldsymbol{Q}}_{\mathrm{U},1}\right)\cdots\mathrm{Tr}\left(\widetilde{\boldsymbol{Q}}_{\mathrm{U},K}\right)\right]^{\mathrm{T}}, \tag{84}$$

$$[\boldsymbol{Z}]_{k,j}=\begin{cases}\sum_{i=1,i\neq k}^{K}\mathrm{Tr}\left(\boldsymbol{H}_{\mathrm{U},i}\widetilde{\boldsymbol{Q}}_{\mathrm{U},i}\boldsymbol{H}_{\mathrm{U},i}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}\right) & \\ \qquad+\mathrm{Tr}\left(\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}\right), & k=j,\\ -\mathrm{Tr}\left(\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U,k}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},j}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},j}\right), & k\neq j,\end{cases} \tag{85}$$

the objective function in (82) equals (86), where for the final two equalities the following definition is used

$$\boldsymbol{G}_{\mathrm{U}}=\left[\boldsymbol{G}_{\mathrm{U},1}^{\mathrm{T}}\cdots\boldsymbol{G}_{\mathrm{U},K}^{\mathrm{T}}\right]^{\mathrm{T}}\text{ with}$$
$$\boldsymbol{G}_{\mathrm{U},k}=\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{H}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}\boldsymbol{P}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right)^{-1},1\le k\le K. \tag{87}$$

Summing up all the rows of (84), the constraint in **P. 18** satisfies

$$\sum_{K=1}^{K}\alpha_k^2\mathrm{Tr}\left(\boldsymbol{G}_{\mathrm{U},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{U},k}\right)=\sum_{k=1}^{K}\mathrm{Tr}\left(\boldsymbol{P}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}\right)$$
$$=\mathrm{Tr}\left(\sum_{K=1}^{K}\widetilde{\boldsymbol{Q}}_{\mathrm{D},K}\right)=\mathrm{Tr}\left(\sum_{K=1}^{K}\widetilde{\boldsymbol{Q}}_{\mathrm{U},K}\right). \tag{88}$$

Based on (86) and (88), the optimization problem **P.18** is equivalent to the following one

$$\mathbf{P.19}: \begin{cases} \min\limits_{\{\widetilde{\boldsymbol{Q}}_{\mathrm{U},k},\boldsymbol{\Theta}\}} \operatorname{Tr}\left(\left(\boldsymbol{I}+\sum_{k=1}^{K}\boldsymbol{H}_{\mathrm{U},k}\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}\boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}}\right)^{-1}\right), \\ \text{s.t.} \quad \boldsymbol{H}_{\mathrm{U},k}^{\mathrm{H}} = \boldsymbol{R}_{\mathrm{n}_k}^{-\frac{1}{2}}(\boldsymbol{H}_{\mathrm{SD},k}+\boldsymbol{H}_{\mathrm{RD},k}\boldsymbol{\Theta}\boldsymbol{H}_{\mathrm{SR}})\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}, \\ \qquad \operatorname{Tr}(\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}) \leq P_k, \\ \qquad \widetilde{\boldsymbol{Q}}_{\mathrm{U},k} \succeq \boldsymbol{0}, \ \operatorname{Rank}(\widetilde{\boldsymbol{Q}}_{\mathrm{U},k}) \leq N_{\mathrm{C},k}, \ 1 \leq k \leq K, \end{cases} \tag{89}$$

which is a special case of **P.7**. In other words, **P.18** can be solved effectively based on the proposed algorithm for the UL case. Note that the power allocation P_k for each user can be optimally determined by the dual decomposition technique [9]. In the simulation, we simply assume that the whole power P is equally allocated to all users, which can largely reduce the complexity with marginal performance loss.

For the MU-MIMO DL without the assistance of RIS, we also devise the modified WMMSE algorithm as an alternative to the above proposed duality-based approach, for solving the capacity maximization and MSE minimization problems. This modified WMMSE algorithm is detailed in Appendix A.

VII. Simulation Results and Discussions

In this section, we evaluate the performance of the proposed transceiver optimization algorithms presented in Sections III to VI.

A. Simulation System Setup

We consider a single-cell MU-MIMO system, where the BS is equipped with N_t antennas to support K users, and all the users have the same number of antennas N_r, while the RIS deploys a uniform planar array with 4 elements per dimension, i.e., the number of antennas at the RIS is $N_{\mathrm{RIS}} = 4 \times 4$. The BS antenna array is placed at the height of $H_{\mathrm{BS}} = 15\,\mathrm{m}$, the mobile user nodes are placed on the ground of $H_{\mathrm{MU}} = 0\,\mathrm{m}$, and the RIS antenna array is placed at the height of $H_{\mathrm{RIS}} = 10\,\mathrm{m}$. All the users are uniformly distributed within a circle of radius $10\,\mathrm{m}$. We set the horizontal distances between the BS-RIS, BS-Users, and RIS-Users to $D_{\mathrm{br}} = 85\,\mathrm{m}$, $D_{\mathrm{bu}} = 120\,\mathrm{m}$, and $D_{\mathrm{ru}} = 50\,\mathrm{m}$, respectively, with D_{bu} and D_{ru} measuring distances from the BS and the RIS to the center of the users' circle, respectively. The corresponding 3D view is shown in Fig. 1(b). We assume the Rician fading channel model with distance-dependent pathloss, i.e.,

$$\boldsymbol{H} = \sqrt{\gamma}\left(\sqrt{\beta}\boldsymbol{H}_{\mathrm{LoS}} + \sqrt{1-\beta}\boldsymbol{H}_{\mathrm{NLoS}}\right). \tag{90}$$

The pathloss coefficient is $\gamma = \gamma_0 \mathrm{D}^{-\alpha}$, where D denotes the distance between transmitter and receiver, and γ_0 is the pathloss for unit distance given by $\gamma_0 = -30\,\mathrm{dB}$. The pathloss exponent α is set to be $\alpha_{\mathrm{bu}} = 3.6$, $\alpha_{\mathrm{br}} = 1.9$ and $\alpha_{\mathrm{ru}} = 1.6$ for the BS-Users, BS-RIS and RIS-Users links, respectively. Except for the BS-Users channel whose Rician factor is set to 0, we set $\beta = 0.95$ for both the BS-RIS and RIS-Users channels. Assuming the isotropic antennas area is $\frac{\lambda^2}{4\pi}$ with operation frequency $f = 2.4\mathrm{GHz}$, we denote the total transmit power as PTx for the users of UL and for the BS

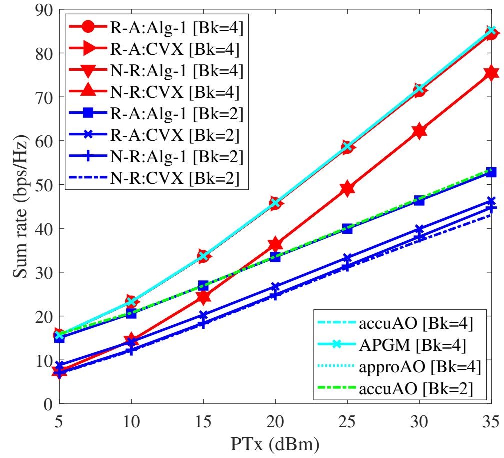

Fig. 2: Sum-rates of the proposed and CVX algorithms versus SNR at the BS in the UL system with $K = 2$, $N_t = 16$, $N_r = 4$, and two values of B_k, where 'R-A' abbreviates for RIS-aided, and 'N-R' for no RIS.

of DL, and define the average receiving signal-to-noise ratio as $\mathrm{SNR} = \frac{\mathrm{PTx}(\mathrm{PL}_{\mathrm{D}}+\mathrm{PL}_{\mathrm{R}})}{\sigma^2}$, where $\mathrm{PL}_{\mathrm{D}}, \mathrm{PL}_{\mathrm{R}}$ representing the pathloss for the BS-Users, BS-RIS-Users links respectively with $\mathrm{PL}_{\mathrm{D}} = 10\log_{10}(\frac{\lambda^2}{4\pi^2}) - 10\log_{10}(\mathrm{D}_{\mathrm{bu}}^{\alpha_{\mathrm{bu}}})$ dB, and σ^2 representing the noise power at the receiver. All the results are obtained by averaging over 100 MIMO channel realizations.

B. Uplink Transceiver Optimization Performance

In the UL capacity and MSE optimization based communication scenarios, two cases of the covariance matrices are considered, namely, the full-rank covariance matrix case with $N_r = B_k = 4$ and the rank-deficient case with $N_r = 4$ and $B_k = 2$. All the users are assumed to have the same transmit power, with the ratio of the maximum per-antenna power of each user as $P_1 : P_2 : \cdots : P_{N_r} = 4 : 3 : 2 : 1$. We set the noise power σ^2 at the BS to -110 dBm and vary PTx of the users. We compare the proposed algorithm with the numerical CVX optimization [40]. The CVX algorithm for the full-rank case directly uses the matlab toolbox for the optimal covariance matrix solver [40]. However, the optimization objective function over the rank-deficient covariance matrices is non-convex. We use the traditional WMMSE algorithm for the numerical CVX based approach in the rank-deficient case, with the precoding matrix solved via the CVX toolbox. In the RIS-aided scenario, we alternatively optimize the covariance matrices and the RIS phase elements until convergence with the two-hop transmission channel. In the scenario without RIS, only the direct channel exists between the BS and the users, and we only need to optimize the covariance matrices.

Fig. 2 compares the sum-rates as the functions of SNR at the BS, achieved by the proposed Algorithm 1 and the numerical CVX optimization. For RIS-aided uplink, 'R-A:Alg-1' uses Algorithm 1 to optimize the covariance matrices followed by optimizing the RIS phase elements, and 'R-A:CVX' adopts the CVX numerical optimization of the covariance matrices followed by optimizing the RIS phase elements, while for UL without RIS, 'N-R:Alg-1' applies Algorithm 1 to optimize the covariance matrices, and 'N-R:CVX' performs the numerical

CVX optimization for the covariance matrices. Besides, the 'accuAO' uses the greedy block coordinate maximization (GBCM) method proposed in [29] for optimizing the covariance matrices followed by optimizing the RIS elements via an optimal solution proposed in [31]. They are activated in an alternative fashion. The 'APGM' method of [29] uses the alternating projected gradient method for the covariance matrices and the RIS phase shift optimization respectively. Furthermore, the 'approAO' applies the projected gradient method of [29] for approximately optimizing the covariance matrices which is combined with the optimal solution proposed in [31] for RIS optimization activated alternatively. The covariance matrix optimizations of 'APGM' and 'approAO' involve approximation, and 'accuAO' gives accurate solutions for both types of the variables, but at a high computational complexity especially on RIS. '[$B_k = 4$]' and '[$B_k = 2$]' represent the full-rank case and the rank-deficient case, respectively. As expected, employing RIS enhances the achievable capacity, as RIS increases the rank of the overall channel. Observe in Fig. 2 that for the full-rank scenario, Algorithm 1 attains the same optimal performance as other benchmarks and the numerical CVX optimization. For the rank-deficient scenario, particularly in the RIS-aided uplink, Algorithm 1 attains the same performance as 'accuAO', and all of which outperform the CVX since the former ones are combined with dirty paper coding while the latter one is not. Fig. 3 plots the convergence speed of both the proposed AO algorithm and of the three benchmarks. It can be observed that for each number of users, our proposed algorithm converges faster to a similar objective function value as the three benchmarks.

Fig. 4 compares the sum-MSEs as the functions of SNR, attained by the proposed MMSE solution (57) and the numerical CVX algorithm. Specifically, for the UL without RIS, 'N-R:CVX' uses the numerical CVX algorithm for the covariance matrix optimization, and 'N-R:Prop' applies the proposed MMSE solution (57) for the covariance matrix optimization, while for the RIS-aided uplink, 'R-A:CVX' carries out the numerical CVX optimization for the covariance matrices followed by optimizing the RIS phase elements, and 'R-A:Prop' uses the proposed MMSE solution for the covariance matrix optimization followed by optimizing the RIS phase elements. The results of Fig. 4 show that the proposed MMSE solution (57) and the numerical CVX optimization attain the same optimal sum-MSE performance.

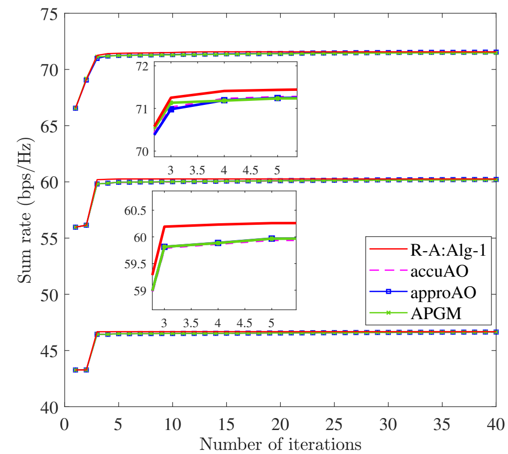

Fig. 3: Sum-rates versus the number of AO iterations with $N_t = 16, B_k = N_r = 4$.

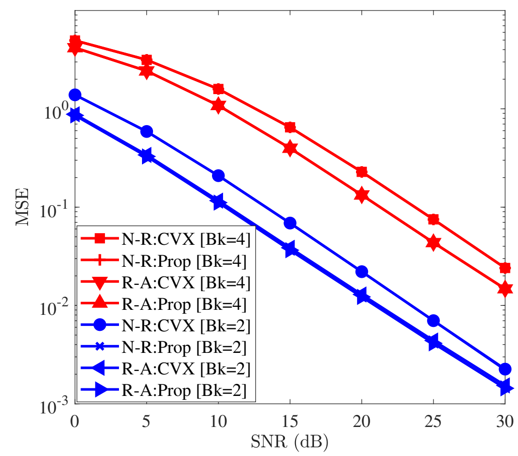

Fig. 4: Sum-MSEs of the proposed and CVX algorithms versus SNR at the BS in the UL system with $K = 2$, $N_t = 16$, $N_r = 4$, and two values of B_k, where 'R-A' abbreviates for RIS-aided, and 'N-R' for no RIS.

C. Downlink Transceiver Optimization Performance

For the DL capacity and MSE optimization based communication scenarios, we consider $B_k = N_r$ with two values of N_r for the users. The ratio of the maximum per-antenna powers at the BS is set to $P_1 : P_2 : \cdots : P_{N_t} = 4 : 3 : 2 : 1 : \cdots : 4 : 3 : 2 : 1$. The noise power σ^2 at each user is set to be -110 dBm and we vary the PTx of the BS. For our proposed duality-based approach, the DL capacity and MSE optimization solutions are obtained based on the equivalent uplink-dual result, which is clearly applicable to the RIS-aided downlink, denoted as 'Duality-RISop', as well as to the DL without the assistance of RIS, denoted as 'Duality-noRISop'. We also use a traditional optimization method, the modified weighted MMSE algorithm of Appendix A, as a benchmark. This modified WMMSE algorithm only optimizes the covariance matrices, and therefore it is only applicable to the DL without RIS, which is denoted as 'WMMSE-noRISop'.

Fig. 5 compares the DL sum-rate performance of the proposed duality method and the traditional modified WMMSE algorithm. Observe that for the MU-MIMO DL without the assistance of RIS, our duality-based approach outperforms the modified WMMSE algorithm considerably, particularly in the case where the total number of antennas of all the users ($N_r \times K = 8 \times 2$) approach the number of BS antennas ($N_t = 16$). The performance gain of 'Duality-noRISop' over 'WMMSE-noRISop' results from utilizing the dirty paper coding at the BS, which is not used in the modified WMMSE algorithm. Fig. 6 depicts the DL MSE performance of the proposed duality method and the modified WMMSE algorithm. It can be seen from Fig. 6 that for the MU-MIMO DL without RIS, both the duality method and the modified WMMSE algorithm attain the same optimal performance. By comparing the curves of 'Duality-RISop' and 'Duality-

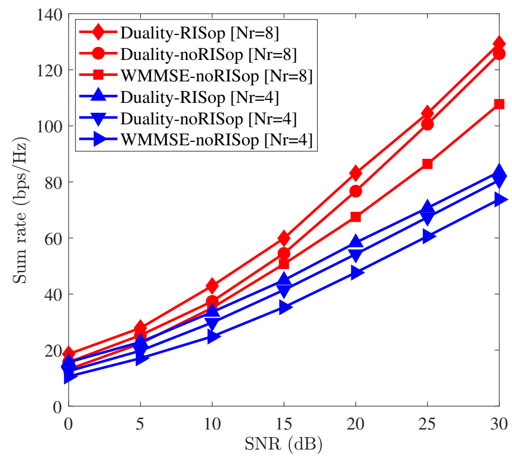

Fig. 5: Sum-rates of the proposed duality approach and the modified WMMSE versus SNR at the users in the DL system with $K = 2$, $N_t = 16$, $B_k = N_r$, and two values of N_r.

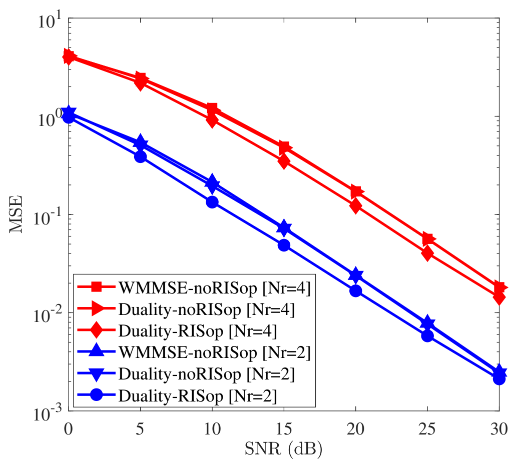

Fig. 6: Sum-MSEs of the proposed duality approach and the modified WMMSE versus SNR at the users in the DL system with $K = 2$, $N_t = 16$, $B_k = N_r$, and two values of N_r.

noRISop' in both figures, it can also be seen that the achievable performance is enhanced with the help of RIS.

VIII. Conclusions

We have investigated the fundamental properties of KKT conditions in the context of optimization problems associated with positive semidefinite matrix variables under rank constraints. Based on the properties derived, the signal covariance optimization problems formulated for capacity maximization and sum-MSE minimization have been solved and the corresponding RIS phase shifting matrix optimization has been transferred into a quadratic optimization problem associated with unit-modulus constraints in the context of RIS-aided MU-MIMO UL systems under rank constraints and multiple weighted power constraints. Moreover, by exploiting the uplink-downlink dualities for both capacity maximization and MSE minimization, the transceiver optimization problem of the RIS-aided MU-MIMO DL has also been solved. Our numerical results have demonstrated that the proposed MU-MIMO transceiver optimizations attain the same or better sum-rate and sum-MSE performance than the numerical CVX optimization algorithm and the traditional modified WMMSE algorithm.

Appendix A
Modified WMMSE Algorithm

To solve the optimization problems for the DL without RIS using the WMMSE, we modify this traditional algorithm to adapt to the multiple weighted power constraints [35]. In general, we still use the WMMSE steps to update the covariance matrix variables alternatively until the objective function converges. The key of our modified WMMSE algorithm is to involve **Algorithm 1** at the step of precoding matrix updation for conversion from per-antenna power constraints to a total-antenna power constraint, that is, we process every iteration while satisfying the per-antenna power constraint. This modified WMMSE is detailed in **Algorithm 2**.

Appendix B
Proof of Lemma 1

We take the UL SU-MIMO case for the proof and omit the subscript k. For the rank-1 LoS channels, we define the steering vectors at the BS, RIS and User as $\boldsymbol{\alpha}_{\mathrm{B}}(\cdot), \boldsymbol{\alpha}_{\mathrm{P}}(\cdot,\cdot), \boldsymbol{\alpha}_{\mathrm{U}}(\cdot)$ respectively, AODs v_u, ψ^b, θ^b and AOAs v_b, ψ^u, θ^u, which satisfy the properties as: $|[\boldsymbol{\alpha}_{\mathrm{B}}]_i| = \frac{1}{\sqrt{N_t}}|, |[\boldsymbol{\alpha}_{\mathrm{U}}]_i| = \frac{1}{\sqrt{I_k}}, |[\boldsymbol{\alpha}_{\mathrm{P}}]_i| = \frac{1}{\sqrt{N_{\mathrm{RIS}}}}$, $\boldsymbol{\alpha}_{\mathrm{B}}^{\mathrm{H}}\boldsymbol{\alpha}_{\mathrm{B}} = 1, \boldsymbol{\alpha}_{\mathrm{U}}^{\mathrm{H}}\boldsymbol{\alpha}_{\mathrm{U}} = 1, \boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}\boldsymbol{\alpha}_{\mathrm{P}} = 1$.

Defining the noise covariance matrix as $\boldsymbol{R}_{\mathrm{n}}$ and total transmit power as P, the objective function of **P.3** can be reformulated as

$$\begin{aligned}&\log\det\left(\boldsymbol{I} + \boldsymbol{H}\boldsymbol{Q}\boldsymbol{H}^{\mathrm{H}}\boldsymbol{R}_{\mathrm{n}}^{-1}\right)\\&= \log\det\Big(1 + e^2 * \tilde{\boldsymbol{\alpha}}_{\mathrm{B}}(v_b)\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^b,\theta^b)\boldsymbol{\Theta}\boldsymbol{\alpha}_{\mathrm{P}}(\psi^u,\theta^u)\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u)\\&\qquad \widetilde{\boldsymbol{Q}}\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}(v_u)\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^u,\theta^u)\boldsymbol{\Theta}^{\mathrm{H}}\boldsymbol{\alpha}_{\mathrm{P}}(\psi^b,\theta^b)\tilde{\boldsymbol{\alpha}}_{\mathrm{B}}^{\mathrm{H}}(v_b)\Big),\end{aligned}$$

where $\widetilde{\boldsymbol{\Omega}}$, $\boldsymbol{R}_{\mathrm{n}}$ are diagonal matrices and

$$\begin{aligned}&\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u) = \frac{\boldsymbol{\alpha}_{\mathrm{U}}^{\mathrm{H}}(v_u)\widetilde{\boldsymbol{\Omega}}^{-\frac{1}{2}}}{\|\boldsymbol{\alpha}_{\mathrm{U}}^{\mathrm{H}}(v_u)\widetilde{\boldsymbol{\Omega}}^{-\frac{1}{2}}\|} = e^{j\angle\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u)},\ \widetilde{\boldsymbol{Q}} = \widetilde{\boldsymbol{\Omega}}^{\frac{1}{2}}\boldsymbol{Q},\\&\tilde{\boldsymbol{\alpha}}_{\mathrm{B}}(v_b) = \frac{\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\alpha}_{\mathrm{B}}(v_b)}{\|\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\alpha}_{\mathrm{B}}(v_b)\|} = e^{j\angle\tilde{\boldsymbol{\alpha}}_{\mathrm{B}}(v_b)},\\&e = \|\boldsymbol{R}_{\mathrm{n}}^{-\frac{1}{2}}\boldsymbol{\alpha}_{\mathrm{B}}(v_b)\| * \|\boldsymbol{\alpha}_{\mathrm{U}}^{\mathrm{H}}(v_u)\widetilde{\boldsymbol{\Omega}}^{-\frac{1}{2}}\|.\end{aligned}$$

In contrast to (29), the matrices for rank-1 LoS channels can be rewritten as

$$\begin{aligned}&\boldsymbol{\Pi}\boldsymbol{H}\widetilde{\boldsymbol{\Omega}}^{-\frac{1}{2}} = e * \tilde{\boldsymbol{\alpha}}_{\mathrm{B}}(v_b)\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^b,\theta^b)\boldsymbol{\Theta}\boldsymbol{\alpha}_{\mathrm{P}}(\psi^u,\theta^u)\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u),\\&\boldsymbol{U}_{\boldsymbol{\mathcal{H}}_k} = \tilde{\boldsymbol{\alpha}}_{\mathrm{B}}(v_b),\ \boldsymbol{\Lambda}_{\boldsymbol{\mathcal{H}}_k} = e * \boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^b,\theta^b)\boldsymbol{\Theta}\boldsymbol{\alpha}_{\mathrm{P}}(\psi^u,\theta^u),\\&\boldsymbol{V}_{\boldsymbol{\mathcal{H}}_k} = \tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u).\end{aligned}$$

Using the property $\det(\boldsymbol{I} + \boldsymbol{A}\boldsymbol{B}) = \det(\boldsymbol{I} + \boldsymbol{B}\boldsymbol{A})$, the optimal solutions to $\{\{\widetilde{\boldsymbol{Q}}^{'}\}, \boldsymbol{\Theta}^{'}\}$ can be easily obtained as $\widetilde{\boldsymbol{Q}}^{'} = e^{j\angle\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}(v_u)} * P * e^{j\angle\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u)}$, $\boldsymbol{\Theta}^{'} = \mathrm{diag}(\boldsymbol{\alpha}_{\mathrm{P}}(\psi^b,\theta^b))\mathrm{diag}(\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^u,\theta^u))$, $\boldsymbol{\theta}^{'} = \mathrm{diag}(\boldsymbol{\Theta}) = \mathrm{diag}(\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^u,\theta^u))\boldsymbol{\alpha}_{\mathrm{P}}(\psi^b,\theta^b)$. Treating the $e^{j\angle\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}(v_u)}$ and P respectively as the eigenvector and eigenvalue of $\widetilde{\boldsymbol{Q}}$, the solutions above coincide with (27) whose optimality is confirmed.

Algorithm 2 The Modified WMMSE Method

Initialize: Random variables $\boldsymbol{P}_{\mathrm{D},k}$, $\forall k$, such that $\mathrm{Tr}\left(\sum_{k=1}^{K}\boldsymbol{P}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}^{\mathrm{H}}\right) = \mathrm{Tr}\left(\sum_{k=1}^{K}\widetilde{\boldsymbol{Q}}_{\mathrm{D},k}\right) = P$; Set $\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}} = \boldsymbol{I}$ and define $\boldsymbol{H}_{\mathrm{D},k} = \boldsymbol{H}_k\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}$, where $\boldsymbol{H}_k$ is the original channel.

repeat
- Set $\boldsymbol{H}_{\mathrm{D},k} = \boldsymbol{H}_k\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{-\frac{1}{2}}$.
- $\boldsymbol{G}_{\mathrm{D},k} := \left(\sum_{i=1}^{K}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},i}\boldsymbol{P}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}} + \boldsymbol{R}_{nk}\right)^{-1}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}, \forall k$.
- $\boldsymbol{W}_k := \boldsymbol{I}$ or $\boldsymbol{W}_k := \left(\boldsymbol{I} - \boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}\right)^{-1}$, $\forall k$, respectively for DL MSE and capacity optimization.
- $\forall k$, $\boldsymbol{P}_{\mathrm{D},k} := \left(\sum_{i=1}^{K}\boldsymbol{H}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},i}\boldsymbol{W}_i\boldsymbol{G}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},i}+\lambda^*\boldsymbol{I}\right)^{-1}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},k}\boldsymbol{W}_k$.
- Use **Algorithm 1**: Random dual variables $\mu_i^{(0)}$, $\forall i$; iteration index $t=0$; maximum iteration number $T_{\max}$; sufficiently small threshold $\epsilon>0$.
- **repeat**
  - Calculate $\widetilde{\mu}_i^{(t)} = \mu_i^{(t)}P/\left(\sum_{l=1}^{I}\mu_l^{(t)}P_l\right)$, $\forall i$, and $\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{(t)} = \sum_{i=1}^{I}\widetilde{\mu}_i^{(t)}\boldsymbol{\Omega}_{\mathrm{D},i}$; set $\boldsymbol{H}_{\mathrm{D},k} = \boldsymbol{H}_k\big(\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{(t)}\big)^{-\frac{1}{2}}$, $1 \le k \le K$.
  - $\boldsymbol{G}_{\mathrm{D},k} := \left(\sum_{i=1}^{K}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},i}\boldsymbol{P}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}+\boldsymbol{R}_{nk}\right)^{-1}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}, \forall k$.
  - $\boldsymbol{W}_k := \boldsymbol{I}$ or $\boldsymbol{W}_k := \left(\boldsymbol{I} - \boldsymbol{G}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},k}\boldsymbol{P}_{\mathrm{D},k}\right)^{-1}$, $\forall k$, respectively for DL MSE and capacity optimization.
  - $\forall k$, $\boldsymbol{P}_{\mathrm{D},k} := \left(\sum_{i=1}^{K}\boldsymbol{H}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},i}\boldsymbol{W}_i\boldsymbol{G}_{\mathrm{D},i}^{\mathrm{H}}\boldsymbol{H}_{\mathrm{D},i}+\lambda^*\boldsymbol{I}\right)^{-1}\boldsymbol{H}_{\mathrm{D},k}^{\mathrm{H}}\boldsymbol{G}_{\mathrm{D},k}\boldsymbol{W}_k$.
  - Calculate $\boldsymbol{P}_k^{(t)} = \big(\widetilde{\boldsymbol{\Omega}}_{\mathrm{D}}^{(t)}\big)^{-\frac{1}{2}}\boldsymbol{P}_{\mathrm{D},k}$ and $\boldsymbol{Q}_{\mathrm{D},k}^{(t)} = \boldsymbol{P}_k^{(t)}\big(\boldsymbol{P}_k^{(t)}\big)^{\mathrm{H}}$, $\forall k$.
  - Set the step size $a_i^{(t)} = \frac{a}{b\cdot t+c}$, $1 \le i \le I$, where $\{a,b,c\}>0$.
  - Update $\mu_i^{(t+1)} = \left[\mu_i^{(t)}+a_i^{(t)}\left(\mathrm{Tr}\left(\boldsymbol{\Omega}_{\mathrm{D},i}\sum_{l=1}^{K}\boldsymbol{Q}_{\mathrm{D},l}^{(t)}\right)-P_i\right)\right]^+$, $1 \le i \le I$.
  - Update $t = t+1$.
- **until** $\left|\mu_i^{(t)}\left(\mathrm{Tr}\left(\boldsymbol{\Omega}_{\mathrm{D},i}\sum_{i=1}^{K}\boldsymbol{Q}_{\mathrm{D},i}^{(t)}\right) - P_i\right)\right| \le \epsilon$, $\forall i$, or $t = T_{\max}$.

until MSE or capacity converge.

return The optimal $\boldsymbol{Q}_{\mathrm{D},k}^{\star} = \boldsymbol{P}_k\boldsymbol{P}_k^{\mathrm{H}}$ to the optimization problem.

Next, we will show the solution $\boldsymbol{\theta}^{'}$ also coincides with the one to **P. 6**.

Based on the optimal $\widetilde{\boldsymbol{Q}}^{'}$, the equations (35-37,42-43) can be rewritten as

$$\boldsymbol{H}_{\mathrm{V}} = \boldsymbol{a},\ \boldsymbol{G} = \frac{\boldsymbol{a}^{\mathrm{H}}}{\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}}+1},\ \boldsymbol{W} = \boldsymbol{a}\boldsymbol{a}^{\mathrm{H}}+1,$$
$$\boldsymbol{A} = \frac{|c|^2}{d^2} * \boldsymbol{g}\boldsymbol{g}^{\mathrm{H}},\ \boldsymbol{b} = c * \boldsymbol{g},$$

and these terms are all based on the obtained solution $\boldsymbol{\theta}_k$ from the k^{th} iteration, where

$$\begin{aligned}
&\boldsymbol{g} = \mathrm{diag}(\boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^u,\theta^u))\boldsymbol{\alpha}_{\mathrm{P}}(\psi^b,\theta^b), \quad \boldsymbol{g}^{\mathrm{H}}\boldsymbol{g} = 1,\\
&\boldsymbol{a} = \left[e * \boldsymbol{\alpha}_{\mathrm{P}}^{\mathrm{H}}(\psi^b,\theta^b)\boldsymbol{\Theta}\boldsymbol{\alpha}_{\mathrm{P}}(\psi^u,\theta^u)\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}^{\mathrm{H}}(v_u)\widetilde{\boldsymbol{Q}}^{-\frac{1}{2}}\right]^{\mathrm{H}}\\
&\quad = e * P * \tilde{\boldsymbol{\alpha}}_{\mathrm{U}}(v_u)\boldsymbol{\theta}_k^{\mathrm{H}}\boldsymbol{g},\\
&c = \boldsymbol{a}^{\mathrm{H}}\tilde{\boldsymbol{\alpha}}_{\mathrm{U}}(v_u), \quad d = (\boldsymbol{a}\boldsymbol{a}^{\mathrm{H}}+1).
\end{aligned}$$

It can be observed that $\boldsymbol{A}$ is a rank-1 positive semidefinite matrix. By discarding constant terms and using the MM algorithm [37], the majorization problem at the $(k+1)^{th}$ iteration for **P. 6** is

$$\min_{\boldsymbol{\theta}}\ \Re\{\boldsymbol{\theta}^{\mathrm{H}}\boldsymbol{u}\}, \tag{91}$$

where $\boldsymbol{u} = (\boldsymbol{A} - \lambda_{max}(\boldsymbol{A})\boldsymbol{I})\boldsymbol{\theta}_k - \boldsymbol{b} = \boldsymbol{g} * \frac{|c|^2}{d} * \boldsymbol{g}^{\mathrm{H}}\boldsymbol{\theta}_k - \boldsymbol{\theta}_k * \frac{|c|^2}{d} - \boldsymbol{g} * c$ is constant since $\boldsymbol{\theta}_k$ is known from the k^{th} iteration. Let $\boldsymbol{u} = \left[|u_1|e^{j\phi_1}, ..., |u_{N_{\mathrm{RIS}}}|e^{j\phi_{N_{\mathrm{RIS}}}}\right]^{\mathrm{H}}$, the solution to problem (91) is given by $\boldsymbol{\theta}_{k+1} = \left[e^{j(\phi_1+\pi)}, ..., e^{j(\phi_{N_{\mathrm{RIS}}}+\pi)}\right]$. Let $\boldsymbol{\theta}_0 = \boldsymbol{g}$ and we can get $\boldsymbol{u} = -(eP) * \boldsymbol{g} = |eP|e^{\pi+\angle\boldsymbol{g}}$ and $\boldsymbol{\theta}_1 = e^{2\pi+\angle\boldsymbol{g}} = \boldsymbol{g} = \boldsymbol{\theta}^{'}$. It can be observed that $\boldsymbol{g}$ is a stationary point to a series of majorization problems. This completes the proof that the AO method is guaranteed to find a globally optimal solution to the UL SU-MIMO capacity maximization problem. The effectiveness of the AO method for the other three problems under the same rank-1 LoS SU-MIMO scenario setting can be proved similarly.

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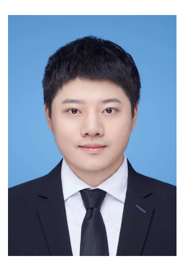

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