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Faculty of Social Sciences
School of Mathematical Sciences

Robust Optimization Technique for Hydropower Production

by

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A thesis for the degree of
Doctor of Philosophy

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Abstract

Faculty of Social Sciences
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Robust Optimization Technique for Hydropower Production

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The aim of this thesis is to propose efficient methods for scheduling a long-term hydropower production. Managing hydropower is challenging, as it consists of various variables and constraints. The hydropower studied here consist of storage water, release water, and pumping storage. Moreover, uncertainty regarding the amount of inflow water to the reservoir makes the problem more difficult. Therefore, it is essential to model the problem which taking into account the uncertainty in the problem. Consequently, We model the problem using Robust Optimization (RO) and Stochastic Dynamic Programming (SDP) and compare the results. The result shows that RO performs better than SDP. Hence, we focus on solely using the RO model for the rest of the research.

We also investigate two types of decision making, namely risk averse and risk-neutral, then capture their behaviour in our model. Because of the uncertainty regarding inflow water, we want our decision to be able to adapt to any realisation of inflow. Therefore, we define the decision variables as an affine function of inflow water. These decision variables depend on a series of τ of inflow, where τ is the time window of how far we look back into the history of inflow. The result shows that an increase in τ causes an improvement in the result in training set. However, it is shown that there stops being an improvement at some point of τ and that too large of τ causes a degradation in the quality of the result when it is applied to validation set.

The phenomenon of the decreasing quality of result in validation set when τ is increasing prove that there is an overfitting on the model. Therefore, we add a regularisation to the model to prevent the overfitting. We add the constraints to restrict that the amount inflow water today give more effect to the decision on how much water should be released today rather than the amount of inflow water yesterday. Adding this constraints to the model improves the quality of the solution in the validation set.

Furthermore, we add the spill variable to the problem. This variable is bounded by a function called as evacuation curve. The evacuation curve is a function of the maximum amount of water to be spilled at specific level of storage at a specific time and scenario of inflow. This function is increasing and non-linear. However, we approximate it to an affine function to keep the model linear. The numerical result suggests that the spill variable is not useful when the average water in the storage is not close to the maximum capacity.

Finally, we add the waterhead variable to the model. Previously, the problem assumes that the waterhead is constant. However, in reality, the waterhead is not fixed and depends on the water level in the storage. The variable waterhead then is described as an affine function of storage variable. Thus, this converts the model to a non-linear and non-convex model. We combine this non linear model with a rolling horizon algorithm to solve the problem. We propose two rolling horizon algorithms: Simple Rolling horizon (SRH) and Dynamic Rolling Horizon (DRH). The result shows that considering the rolling horizon algorithms, SRH and DRH, leads to better revenue and shorter time to run than when the problem is solved in full time horizon at once. Moreover, comparing the result of the model considering waterhead and the linear model with constant waterhead, it is found that considering waterhead gives better revenue. Lastly, by comparing two rolling horizon algorithms, it can be shown that DRH produces better result rather than SRH.

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Declaration of Authorship

I declare that this thesis and the work presented in it is my own and has been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission

Signed:.. Date:..................

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Nomenclature

Index	
t	time stage index (hours)
T	set of all time stages in planning period
ω	scenario
Ω	set of all scenarios
i	index of reservoir
Parameters	
η	Efficiency of the turbine
γ	water density (kg/m^3)
g	gravitational constant (m/s^2)
h_i	the distance of waterfall (m)
Variables	
π_t	the electricity price at time t ($Euro/MWh$)
$f_t^{i,\omega}$	inflow into reservoir i at time t under scenario ω (m^3/s)
$o_t^{i,\omega}$	water released through turbines i at time t under scenario ω
p_t^{ω}	water pumped back from downstream reservoir into the upstream reservoir at time t under scenario ω
$s_t^{i,\omega}$	amount of water in basin i at time t under scenario ω
o_{min}, o_{max}	the minimum and maximum amount of water released from the basin
p_{min}, p_{max}	the minimum and maximum amount of water pumped back
s_{min}, s_{max}	the minimum and maximum amount of water in the storage
α, β	affine variables
τ	time window
Pr^{ω}	the probability for a scenario ω to be happened
ψ	epigraph variable
$C_i(s)$	the evacuation curve describes maximum amount of water allowed to be spilled from basin i for a given amount of water in the storage s at time t

Chapter 1

Introduction

The rapid development of the world has caused an increase in the amount of energy needed. In addition, it has become evident that there is a need to produce energy that can be renewed since energy produced from fossil fuel is increasingly becoming scarce and cannot be renewed. Therefore, lots of researches focus on producing energy from hydropower, wind power, geothermal, etc which can be sources of renewable energy. Renewable energy has some advantages such as the fact it can minimize dependency on energy coming from fossil fuels and reduce carbon emissions which contribute to global warming (Baños et al., 2011).

In 2018, carbon emissions reportedly increased by 2.7 percent around the world (Mulvaney, 2019). There will be a number of detrimental effects if global warming continues in this way, such as rising sea levels which are predicted to rise between 10 to 32 inches or perhaps higher by the end of the century. In addition, hurricanes are expected to become stronger, and floods as well as droughts will happen more frequently. Therefore, reducing carbon emissions is urgently required. Indeed, it has been reported that emissions could be reduced by up to 30 percent by 2050 by building more efficient buildings. We can also reduce another 30 percent by moving from fossil fuels to renewable sources (Borunda, 2019).

Hydropower, an example of renewable energy resources, is basically the power generated through the movement of water and may be used for various purposes. Currently, hydropower is the largest renewable source of electricity in the world and contributes to almost 20 percent of the world's electricity production. Given the fact that water is denser than air, even small movements of the water can generate a huge amount of energy. Nowadays, there are several forms of waterpower that are used to produce electricity such as hydroelectricity and ocean energy. Hydroelectricity produces electrical power using the gravitational force of flowing water, while ocean energy is the energy produced from ocean waves (Baños et al., 2011).

Hydropower systems are constructed from various components such as reservoirs, tunnels, pipes, and pumping stations, etc. Managing these systems is demanding because they include many complicated variables such as water inflows and outflows, pumping facility, storage capacity, and water supply demands. This problem becomes harder to solve computationally because of the uncertainty of the variables. This uncertainty is due to the unpredictability of inflows which measures of how much water flows into reservoirs (Rani and Moreira, 2010). Water inflows is hard to predict as it is impacted by the weather conditions. Thus, it is challenging to do so for the long-time, because generally the accuracy of a weather forecast decreases against the time horizon.

The problem of hydropower optimization can be modelled using Linear Programming (LP). In LP, the objective function and all of the constraints should be in a linear form. It is well-known that LP is used to solve problems in various disciplines because it is simple to implement and can be solved with any LP solvers.

The major drawback of employing LP to solve hydropower optimization problems is that LP does not take into account the uncertainty. This may cause a problem because the important aspect of hydropower optimization is inflow which includes the uncertainty. Neglecting uncertainty can result in a solution that is not optimal or even infeasible. Consider the example below.

Example 1.1. *Let $T = 1$ and there are two scenarios for inflow. The water balance constraint (defined later on equation (3.10)) at time $t = 1$ for each scenario is:*

$$\omega = 1 \rightarrow f_1^1 + s_0 = o_1 + s_1 \tag{1.1}$$

$$\omega = 2 \rightarrow f_1^2 + s_0 = o_1 + s_1 \tag{1.2}$$

Assume that $s_0 = 0$. Therefore, we get:

$$s_1 = f_1^1 - o_1 \tag{1.3}$$

$$s_1 = f_1^2 - o_1. \tag{1.4}$$

If $f_1^1 \neq f_1^2$ then the equation (1.3) and (1.4) have different values or in other words, they are inconsistent. The example above shows that neglecting uncertainty leads to infeasibility in the problem with $\omega \geq 2$. Therefore, a method which accommodate uncertainty is needed.

Two well-known methods for handling uncertainty are Robust optimization (RO) and Stochastic Programming (SP). RO works with uncertain sets to capture the uncertainty and obtains the best solution under the worst case scenario. On the other hand, SP tries to obtain a solution with the optimal expected value.

Moreover, RO is also a commonly used method to solve the problem associated with hydropower. Patino (2017) in her thesis studied the performance of two-stage RO on problems related to hydropower problem and found that RO performs well when applied to these issues. Her work showed that the result she obtained is immune to almost all of the realizations of inflow.

The aim of this thesis is to propose efficient methods to determine a long-term hydropower production schedule that maximizes the revenue from energy selling. The revenue is defined by a multiplication between the electricity price and the electricity produced at each time. Furthermore, the rate of electricity production at each time depends on the amount of water released, the waterhead, efficiency of the turbine, water density, and gravitational constant. To simplify the model, the waterhead is usually assumed as a constant such as in the work done by Patino (2017). However, in reality the waterhead is not constant. Gauvin et al. (2018), Zambelli et al. (2011), and De Ladurantaye et al. (2009) considered the variable waterhead into their models. Nonetheless, considering the variable waterhead causes the complexity in the model as the variable waterhead are generally non-linear and non-convex.

Solving non-linear model can be challenging. Moreover, solving non-linear model under long time period is more complicated because the model will be too hard and too large to be solved at once. Therefore, a rolling-horizon algorithm is considered. In the later part of this work, we consider using a rolling horizon algorithm to solve the model. The idea of a rolling horizon algorithm is to divide the entire time period into smaller intervals of time and solve the problem iteratively within the shorter time horizon.

A rolling horizon algorithm is useful when solving either a problem that contains uncertainty or a huge problem which is hard to solve at once. The benefit of this method is that it can solve problems in a faster computational time (Bischi et al., 2019). Because of this advantage, this algorithm has been applied in many fields, such as finance (Jofre et al., 2017), energy storage planning (Wakui et al., 2022), water management (Arena et al., 2017), and hydropower scheduling (Gauvin et al., 2018).

To begin with in this study, we focus on hydropower systems in a single reservoir. We solve this problem using Robust optimization and Stochastic Dynamic Programming. Then, we propose to solve this problem with affine rules. Following that, we extend our study to cover a hydropower system with two-basin, two turbines, and one pumping pipe. Spillage and variable waterhead are considered to make the model more realistic. As mentioned before, a water-head variable is non-convex. Therefore, the combination of non-linear robust affine and rolling horizon is proposed to solve the problem. Thus, our model will allow an understanding of what affects the various variables and different dataset time-window will bring to hydropower production and its profit. Also, this study may give the knowledge on what model is the best for scheduling the hydro system so that it can maximize the profit.

We then do the case study on Mattmark Hydropower System to see the performance of the models discussed. Mattmark is one of the largest hydropower systems in Switzerland. The setup of Mattmark hydropower system is a representative of other hydropower systems in Switzerland which is one of the highest hydroelectricity producers in Europe. The hydropower systems in Switzerland are well-designed, however, they are prone to climate change and socio-economic changes Anghileri, Botter et al. (2018). Therefore, understanding how the strategies studied here can be adapted on Mattmark hydropower system becomes interesting to explore.

The rest of the thesis is organised as follows. In the next chapter (Chapter 2), the literature review on hydropower, Robust Optimization, SDP, and rolling horizon is presented. Then, the main content of the thesis is divided into two parts. The first part discusses single-basin hydropower problems including the model, from robust model, affine model, and SDP model, as well as the numerical results. These are covered in chapter 3.

The second part explains about the problem associated with two-basins hydropower problem and the case study on Mattmark hydropower system and presented in Chapter 4. This chapter covers the model including the linear model, the model with the spill-variables and the variable waterhead. We also propose a two algorithms of rolling horizon to solve the non-linear model considering variable waterhead. The numerical result and discussion related to the result obtained is also reviewed there. The last chapter, Chapter 5, concludes all of the discussion on the thesis.

1.1 Related Work and Positioning

Recently, the interest in the subject of hydropower systems has seen an increase. Based on its objective function, the work on hydropower optimization can be divided into two groups: maximizing the revenue or profit and minimizing operational cost. Research into maximizing profits from hydropower has been done by a number of researchers including Faber, Patino, and De Ladurantaye. Faber and Stedinger (2001) studied maximizing the profit with SDP and ensemble streamflow prediction, while Patino (2017) solved the problem using Robust Optimization method. She solved one-year hydropower production for a two-reservoir system. The model she worked on included storage variables, released water variables, and pumping variables. She modeled her problem with the deterministic model and the robust model, then compared the result obtained. She showed that the quality of the result obtained by RO is better than the result obtained by the deterministic model.

De Ladurantaye et al. (2009) also studied maximizing profit from energy selling using two models, deterministic and stochastic to a short-term hydropower production. Her models included storage variables, released water variables, spillage variables, and waterhead variables. She then compared the result obtained from both models. Her work showed that the result from the stochastic model was better in quality than the result

TABLE 1.1: Related Works

Reference	**Model**	**Method**	**Objective**	**Variables**
Patino, 2017	Linear	Robust	Max Revenue	Volume, outflow, pumping
Zambelli et al., 2011	Non-Linear	Deterministic, SDP	Min Cost	outflow, volume, spill
De Ladurantaye et al., 2009	Non-Linear	Deterministic, Stochastic	Max Profit	Volume, outflow, spill, waterhead
Anghileri, Castelletti et al., 2018	Non-Linear	SDP	Balancing revenue and production	volume, outflow, waterhead
Braaten et al., 2016	Linear	Affine Robust	max profit	volume, outflow, pumping, spill
Gauvin et al., 2017	Linear	Affine Robust	Min risk of flood	volume, outflow, spill
Gauvin et al., 2018	Non-Linear	Successive LP-Rolling Horizon	Min risk of flood	volume, outflow, spill, water-head

from the deterministic model.

Anghileri, Castelletti et al. (2018) studied balancing the revenue and electricity production in hydropower. The hydropower systems they used included two storages, released facilities, and pumping facilities. However, she neglected the pumping facility and did not include it in the model.

Zambelli et al. (2011) attempted to solve a single-hydropower system using Stochastic Dynamic Programming (SDP) and Deterministic Dynamic Programming (DDP). Storage variables, released water variables, spillage variables, and variable waterhead are the variables included in their model. The purpose is to solve a long-term hydropower scheduling with the objective is to minimize the production cost. The problem is solved over one year period in monthly schedule. They later found that SDP tends to discharge more water and maintain the reservoir at a lower level. On the other hand, DDP tried to preserve more storage and it affects the efficiency in the hydropower because the waterhead is maintained higher. In the case of a wet scenario, the result produced by SDP is better because it provides more storage for the water inflows. However, in dry weather, DDP provides better results because it ensured more water in storage, while SDP suffers from water depletion.

Braaten et al. (2016) in their work, studied maximizing the profit from a two-basin hydropower system. The aim is to generate a weekly schedule over one year period. The variables considered in the model are storage variables, released water variables, pumping variables, and spillage variables. They solved the problem with a robust optimization problem and approximated the discharge variable with an affine rule. They found that the results are robust while considering the deviations in inflow and price.

Gauvin et al. (2017) also worked with affine rule to solve a short-term hydropower scheduling problem. The objective of their model is minimizing the risk of flood. They approximated the solutions with standard affine decision rules and lifted decision rules. They, then concluded that affine rule produced good solutions while preserving tractability on the model.

The purpose of employing affine policy to the decision variable is so that the result obtained can adapt to any realisation of inflow. Therefore, the decision variable is defined as an affine function of the previous inflow. In this case, Gauvin et al. (2017) and Braaten et al. (2016) included all the previous data of inflows from the beginning period to the one-time step before. This may work for short-term period or include relatively small data. However, for a long-term period including large data set, it may affect the performance.

Moreover, in his later work, Gauvin et al. (2018) also studied the hydropower problem with waterhead variables. As mentioned before that waterhead variables are generally non-convex and non-linear. Therefore, They employed successive linear programming and combined it with a rolling horizon.

1.2 Contributions

Our study involves solving a long-term hydropower production scheduling problem. The problem is solved over one year period based on 4-hourly schedule and subject to some constraints as well as boundaries. The hydropower systems we work on involve two reservoirs, two storages, and one pumping facility. We model the problems with Robust based affine decision rule and SDP and then compare the performance from both models. For the robust model, we consider two types of decision making strategies, namely risk-averse and risk-neutral in the objective function. Different from other previous works, we study the influence of look back time windows into the model. Moreover, we use several values of time windows instead of choosing just one time window. Therefore, we are able to analyse how this look-back time window may affect to the problem and this yields to our first contribution.

The result obtained from affine rules based on robust optimization shows that the increase in look back time-window affects its performance in the validation set where we test the solution. Based on the result, we find that the model suffers from overfitting. Therefore, we model a new problem by adding a regularization constraint and this leads to our second contribution.

In the next part, we add spill variables and water-head variables. This water-head variable makes model non-linear and non-convex. To solve this model, we use non-linear programming instead of try to linearize it as was the case in Gauvin's work. This model is large and not easy because its non linearity. Therefore, we apply the rolling horizon and non-linear affine robust to solve the problem and it brings to our third contribution.

Chapter 2

Literature Review

2.1 Review of Hydropower Production System

Energy plays an important rule on the world's development. The rapid growth of human population and the acceleration of industrialization around the world affect the increasing demand of energy. While depending solely on fossil fuels as an energy resource is not enough because fossil fuel will become scarce and rare by the time. Also, it affects the environment by releasing pollutants into nature. On the other hand, the renewable energy sources, such as hydropower, wind, and solar power are clearer, non-polluting in nature, more abundant and can be renewed more easily. Therefore, the direction of the development in energy moves toward renewable energy resources. One of the most widely used of renewable energy sources is Hydropower. It contributes to almost 17% of the world energy production (Kumar and Saini, 2022).

Hydropower is, simply put, a conversion of the potential energy of flowing water into electricity. There are 4 types of hydropower: run-of-river (RoR) hydropower, storage hydropower, pumped-storage hydropower, and offshore hydropower. In RoR hydropower, the water from the river flows through a canal into a turbine. It typically either has only a small storage facility or none at all. Storage hydropower is a type of hydropower that has a reservoir for water storage. Electricity is produced by releasing water from the reservoir through a turbine. Pumped storage hydropower connects two or more reservoirs. In this type of hydropower, the water can be pumped back from the lower reservoir into the higher reservoir. Generally, water is discharged when the electricity price is high and pumped back when the price is low. The last type of hydropower, offshore hydropower produces electricity from the power of the waves in the sea (Pelkola, 2018).

Among those types of Hydropower, the pumped hydropower plant is the most widely used because it can store the big amount of potential energy, the efficiency of the energy conversion, and the cost of each unit of power (Pérez-Díaz et al., 2015). The pumped hydropower plant is pumping the water when the demand is low. Therefore, the same water can be used again to produce electricity when the price is high. However, one of three main challenges mentioned by Pérez-Díaz et al. (2015) is the need of the more accurate model for pumped hydropower problem.

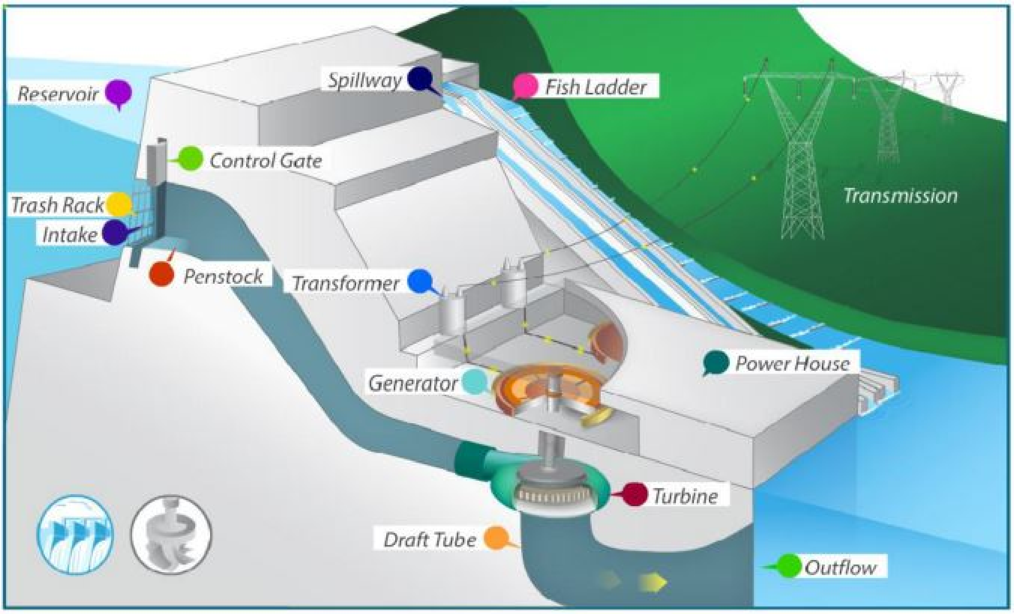


FIGURE 2.1: Illustration of Hydropower Syste (Yaseen et al., 2020)

Hydropower systems usually consist of various component, such as reservoir, the turbine, pumping pipe, spillway, etc. Hydropower utilises the potential energy of flowing water from a higher place to a lower place and transforms it into electricity. The process is illustrated in Figure 2.1. Water is stored in the reservoir and released through a penstock. The force of falling water causes the turbine to spin. At the end of the penstock is a generator which spins when the turbine spins. The generator converts the energy from the turbine into electricity (Pelkola, 2018). The main objective of solving hydropower problem is to schedule the water turbined at each point in time so that it can generate the maximum profit from energy selling. This objective should be maximized while satisfying several constraints. Tahanan et al. (2015) distinguished the constraints of the hydro system as static or dynamic. The typical static constraints of the hydro system are:

1. Reservoir level which indicates the level of the water stored in each reservoir. This has to be maintained to remain between a fixed lower and upper bound.

2. Bounds that limit the turbine and pumps because of their physical constraints.

While the type of dynamic constraints explained by Tahanan et al. (2015) such as:

1. Flow equations which involve the balance constraint of the reservoir level. This reservoir level is updated based on the natural inflow, amount of water that is turbined and spilled, also the amount of water pumped from lower reservoir to upper reservoir. However, spilling and pumping might not be applied for all reservoirs.

2. Flow delay which indicates the time delay for the turbined water to reach the next reservoir. It can be hours or even more.

3. Ramp rate which regulates that each adjacent turbining levels should be close to each other.

4. Smooth turbining. This constraints is needed to avoid extreme strain on the system.

The hydropower scheduling problem is solved under a specific time horizon which usually is divided into three terms, namely short-term, mid-term, and long-term. The short-term scheduling covers planning for hydropower production over 2-14 days, while mid-term includes planning for 3-18 months, and long-term covers planning for 1-5 years (Hammid et al., 2020). This research, will focus on solving a long-term hydropower production problem.

Zambelli et al. (2011) proposed to solve a long-term hydropower scheduling using stochastic dynamic programming and compared the results with deterministic dynamic programming. They solved a single-basin hydropower system under one year period with monthly basis. The hydropower they solved included water storage, waterhead, water discharged, and water spillage. They found that SDP attempted to produce the operation policy which maintain the water storage at the lower level, while DDP tend to preserve more water in the storage. In case of wet scenario, SDP gave a better operation policy because it provided more storage for water inflows. On the other hand, in the dry weather, DDP performs better by maintaining more water in the storage, while SDP suffered from water depletion.

Another work on a long-term hydropower scheduling was done by He et al. (2022) who proposed Improved Dynamic Programming (IDP) with relaxation strategy to solve the model. The result showed that the computational time increased linearly with the increasing of discretization on states variable. However, this method proved to be able to reduce the computational burden and improve the efficiency compared to the usual dynamic programming.

2.1.1 Spillway

Spillway is a structure in a hydropower system that allows water to be spilled through the spillway and not turbinated. The purpose of spillway is to prevent flooding in the reservoir above the plant. For example, if the turbine is not working because it is under maintenance so the water cannot be released to be turbinated, but it can still flow in to the reservoir. If the water in the basin continues to increase and exceeds the maximum storage capacity, then flooding can occur. Therefore, to prevent flooding in the reservoir, a spillway is built to allow some water to be spill out.

Therefore, the spill variable is essentially a function of the storage volume variable. However, water discharged through a spillway does not contribute to energy production. Therefore, it should be avoided if it is possible.

Gauvin et al. (2017) modeled the spillage variable to be bounded by a function called evacuation curve. This function represents maximum amount of water allowed to be spilled from each water storage at a specific time for a given water storage level. He suggested that the maximum water to be spilled is correlated with the structure of the basin. He proposed to model the evacuation curve as a linear function of the storage level to ensure the linearity in the model. On the other hand, Feng et al. (2018) modeled the spillage variables to be bounded by some specific minimum and maximum values.

2.1.2 Variable Waterhead

The amount of the electricity that hydropower plant produce depends on two factors, specifically how far the water falls and how much water falls. The higher the water falls from, the more power it produces. The height of waterfalls (h) is defined as the difference between the reservoir level and the tailrace, while the reservoir level depends on the volume of water stored. Thus, in the last few chapters, we will discuss the problem of water-head sensitive. For this problem, we define the rate of energy production as:

$$P_t = \frac{1}{3600}.10^{-6}\eta\gamma\, g\, h_t\, o_t \tag{2.1}$$

where h_t is the water-head variable at time t.

De Ladurantaye et al. (2009) and Zambelli et al. (2011) considered water head variables on their models and solved them with SDP. Zambelli et al. (2011) modeled waterhead variables as the difference between forebay, tailrace and penstock head loss, where the forebay and tailrace functions are approximated by fourth degree of polynomial functions and the penstock is approximated by a quadratic function. On the contrary, Zambelli defined the waterhead variables by second degree polynomial function of storage volume.

Gauvin et al. (2018), Feng et al. (2018) and Anghileri, Castelletti et al. (2018) also included waterhead variables in their models. Gauvin attempted to solve it using a successive linear programming model. Gauvin formulated the waterhead variables as the division between the net waterhead and the reference waterhead. The net waterhead is calculated by the difference between the forebay and tailrace elevation at each time.

Moreover, the tailrace and forebay elevations are approximated by affine functions of the water storage. While Feng defined the water head variables as the difference between the current storage level now and the storage level one previous point of time divided by a function of released water. On the other hand, Anghileri assumed that the waterhead variable at each time can be computed from the storage-stage rating curve.

2.2 Robust Optimization

The real optimization problem, often includes some uncertain data sets. In hydropower optimization, uncertainty arises regarding the amount of water inflows. The most useful method to deal with issues of uncertainty such as hydropower optimization problem is Robust Optimization (RO). In RO, it is assumed that uncertain data belongs to a certain set and the aim is to create a solution that is feasible for any possible scenarios. One way to obtain a solution that will perform well for any realisation is by implementing the worst case scenario approach (Gabrel et al., 2014).

There are two types of uncertainty: uncertainty on the feasibility solution and uncertainty on the objective value. If uncertainty affects the feasibility solution, robust optimization will endeavour to identify the solution that will be feasible for any realisation. Furthermore, RO focuses on obtaining the solution that is feasible for any realisation from the unknown coefficient within a smaller, realistic set which is called an uncertainty set. This uncertainty set is centred around the nominal value of the uncertain parameters. The aim is to optimize the objective over the set of solutions that are feasible for all of the coefficients in the uncertainty set. The choice of the set plays a crucial role and has to be thought about carefully to ensure the computational tractability of the robust problem and to restrict the deterioration of the objective at optimality. If the uncertainty affects the optimality of the solution, RO tries to find a solution that performs well for any realisation. One common approach is to optimize the worst-case objective, while another approach is by creating a new inequality that brings the objective into the feasible set (Gabrel et al., 2014).

Consider a Linear Programming (LP) problem as shown belows:

$$Minimize \quad f(x) \tag{2.2}$$
$$Subject \quad to: \quad g(x) \leq 0 \tag{2.3}$$
$$\tag{2.4}$$

where $x \in \mathbb{R}^n$ is the vector of decision variables.

The general RO formulation for LP under uncertainty $\mathcal{U}$ is:

$$Minimize \quad f(x) \tag{2.5}$$
$$Subject \quad to: \quad g(x,\zeta) \qquad \forall \zeta \in \mathcal{U} \tag{2.6}$$

There are several types of uncertainty sets ($\mathcal{U}$), such as finite uncertainty, interval uncertainty, bounded uncertainty, or ellipsoidal uncertainty. Finite uncertainty means that the uncertainty sets consist of a finite list of different scenarios. Interval uncertainty set defines intervals for the parameters. While bounded uncertainty set bounds the deviation of the parameters and an ellipsoidal uncertainty set is used to cut off the unlikely corners (Chassein and Goerigk, 2016).

RO has been applied in many fields such as finance, energy, engineering, etc. RO has became very popular because of its tractability on many classes of uncertainty sets and problem types (Gorissen et al., 2015). Moreover, RO creates a solution that is immune to all realisation under uncertainty when the parameter of uncertainty is not stochastics or the distribution is unknown (Bertsimas et al., 2011).

The solutions obtained from RO are designed to be immune to any realisation of inputs. Therefore, the result tends to be conservative particularly for the decisions which are made sequentially. To overcome this issue, a two-stage approach is needed. Because of the difficulty involved in solving multiple-stage problems using RO, many theoretical works focus on two-stage optimization. Two-stage optimization includes two sets of decision variables, which are the 'here and now' variables in the first stage and the 'wait and see' variables in the second stage. However, the very simple two-stage RO can be an NP-hard problem.

2.3 Adjustable Robustness

In two-stage robust optimization, not all of the decision variables are revealed at once. Some of the variables are decided after the realization of other variables. For example, in the study of hydropower optimization, the amount of water discharged today is depended on how much water goes into the system yesterday. This variable is called wait-and-see variables. Therefore, some of the decision variables can be adapted a while later, which is a function of the uncertain data. This is usually called as the Adjustable Robust Counterpart (ARC).

The ARC is defined by Ben-Tal et al. (2004) as below:

$$\min_{x,y(.)}\{c^T x : A\,(\omega)\,x + By\,(\omega) \leq d \qquad \forall \omega \in \Omega\}, \tag{2.7}$$

where $x \in \mathbb{R}^n$ and is the first-stage decision variables (here-and-now) which is decided before ω is realized, whereas y is the second-stage variables (wait-and-see) which can be adjusted according to the realization of the actual data. Notice that y may depend on some or all of the uncertain data ω. (Yanıkoğlu et al., 2019, Ben-Tal et al., 2004)

It can be seen that ARC is more flexible than RO since it yields to more flexible decision that can be adjusted depending on the realization of the uncertain data. ARC also produces a better optimal objective values that at least as good as the one produced by classic RO. It shows that ARC produces more flexible decision that can be adjusted with the realization of the uncertain data at the current stage. (Yanıkoğlu et al., 2019, Ben-Tal et al., 2004)

However, ARC is a complex problem because it is allowed to the variable $y(\omega)$ to adjust themself into the realization of the data. Consider a second-stage decision problem, as seen in the process below.

$$\text{decision}\,(x_1) \rightarrow \text{observation}\,(\eta_2) \rightarrow \text{decision}\,(x_2) \rightarrow \tag{2.8}$$

$$... \rightarrow \text{observation}\,(\eta_T) \rightarrow \text{decision}\,(x_T) \tag{2.9}$$

where η_t represents the data history up to time t. At each time t, we observe the data η_t, however the future data η_{t+1} is uncertain. Therefore, our decision at time t should only depend on the information at that time and should not depend on future observation. This is usually referred as anticipative. In addition, we can apply the affine policies to avoid an anticipative solution.

The use of affine policies for Robust Optimization was introduced by Ben-Tal et al. (2004). The idea behind this is that the second-stage decisions are assumed to be a simple function. For example, suppose there is a two-stage robust optimization problem:

$$\max_{x\in\chi,y(\zeta)} \min_{\zeta\in U} \quad c^T x + d^T y\,(\zeta) \tag{2.10}$$

$$s.t. Ax + By \leq \Psi\,(x)\,\zeta, \quad \forall\zeta \in U \tag{2.11}$$

where x is the first stage variable, while y is the decision under the scenario (ζ) and U is the uncertainty set. The adjustable decisions are forced to become affine functions of the data, which is then called an affine decision rule:

$$y\,(\zeta) = Y\zeta + y \tag{2.12}$$

Therefore, the problem 2.10 can be approximated with:

$$\max_{x\in\chi,y,Y} \min_{\zeta\in U} \quad c^T x + d^T\,(Y\zeta + y) \tag{2.13}$$

$$s.t. \quad Ax + B\,(Y\zeta + y) \leq \Psi\,(x)\,\zeta, \quad \forall\zeta \in U \tag{2.14}$$

The affine policy has been used by many researchers to solve hydropower scheduling problems. The idea of employing affine rules for hydropower problem is that the decision variable will be able to adapt to the realisation of inflow. This function will depend on realisation of inflow of all or some previous data.

Gauvin et al. (2017) in his research applied affine policies based on robust optimization on the decision variables. The problem they solved is a 30-day hydropower scheduling problem. The variables included in the problem are release variables, storage variables, and spillage variables. They found that applying affine rule on the model can produce good solutions and at the same time, can maintain the tractability of the model.

Braaten et al. (2016) also solved hydropower scheduling problem using affine decision rule based on robust optimization. He solved the problem including 1-year horizon in weekly basis (52 weeks). His problem included releasing water pipe, pumping facility, and storage. He also found that a model with affine policy produces a good solution.

2.4 Epigraph Reformulation

However, computing the second-stage optimization problem is not easy. One way to deal with that problem is by transforming a convex problem into an equivalent one. This is useful to find an explicit solution or for algorithmic purpose. There are various transformation approaches and not all of them can preserve the convexity. One of the transformation approaches that can preserve convexity is the Epigraph reformulation (Ghaoui, 2012).

Let $f : \mathbb{R}^n \rightarrow \mathbb{R}$ is a convex function and is defined as:

$$\text{graph} f = \{(x, f(x)) \,|x \in \mathcal{X}\}, \tag{2.15}$$

then, the equivalent epigraph form for those functions is defined as (Wei, 2020):

$$\text{epi} f = \{(x, t) \,|x \in \mathcal{X}, f(x) \leq t\}. \tag{2.16}$$

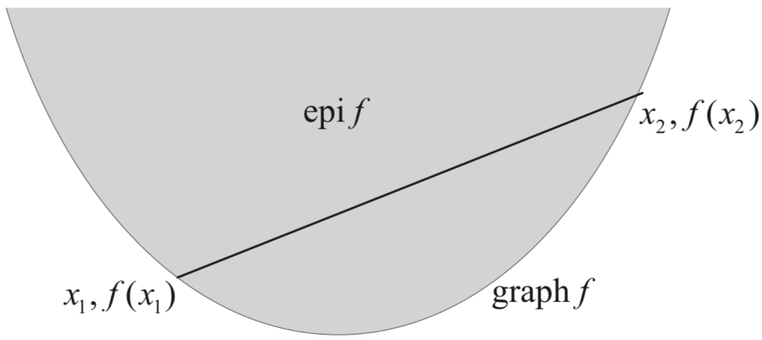


FIGURE 2.2: Illustration of the graph of function $f(x)$ (solid line) and its epigraph reformulation (shaded area) (Wei, 2020)

Epigraph is usually used to reformulate optimization problems. A problem whose objective function is nonlinear can be reformulated by a linear objective and an extra constraint in the epigraph form. Consider the maximization problem as:

$$\max_x f(x), \; x \in \chi \tag{2.17}$$

Moreover, the objective function 2.17 can be reformulated through epigraph reformulation:

$$\max_{\psi,x} \quad \psi \tag{2.18}$$

$$\text{s.t.} \quad \psi \leq f(x) \tag{2.19}$$

$$x \in \chi \tag{2.20}$$

2.5 Stochastic Dynamic Programming

Similarly to RO, stochastic programming is a powerful method used to deal with uncertain data. The difference is that stochastic programming makes it necessary to know or estimate the probability of the uncertain data. Often, this method is applied to a problem that involves making decisions repeatedly on the same occasions.

Stochastic Dynamic Programming (SDP) has been applied to various type of problems, such as asset allocation problems (Infanger, 2016), planning problems (Cristobal et al., 2009), and reservoir optimization problems. Indeed, the popularity of SDP lies in its capability to be applied to a wide range problems.

SDP is usually applied to a problem which requires the decisions to be made sequentially and when the aim is to make a decision that will perform well on average. Moreover, SDP is often applied to two-stage problems. The idea is that an optimal solution will be obtained based on the data available at the time of the decision and that it does not depend on future observation. Please refer to A.1 to see the simple example process of dynamic programming.

The example in A.1 is an application from deterministic dynamic programming. The difference between deterministic and stochastic programming is that stochastic programming needs the probability to be known or assumed. The SDP problem can be formulated as below:

$$\min_{x \in X} \left\{ g(x) = c^T x + \mathbb{E}[Q(x,\zeta)] \right\}, \tag{2.21}$$

where $Q(x,\zeta)$ is the optimal value of the second stage

$$\min_{y} q^T y \quad \text{subject to} \quad Tx + Wy \leq h \tag{2.22}$$

where x represents the first stage decision variables, while y is the second stage decision variables, and $\zeta = (q, T, W, h)$ contains the data from the second stage problem. In the first stage, we have to make a decision x before a realisation of the uncertain data ξ. In this stage, we optimize the cost $c^T x$ of the first stage decision, in addition to the expected cost of the second stage decision. After a realization of ξ, we optimize the behaviour by solving an appropriate problem at the second stage. The problem of this stage can be described as an optimization of the optimal behaviour when uncertain data is revealed. In other words, we can say that the solution of the second stage problem is an action that can be done recursively, where Wy influences a possibility of inconsistency on the $Tx \leq h$ and $q^T y$ is the cost of this action. The formulation above includes the assumption that the second stage data ξ can be modeled as a random vector and its probability distribution is known.

The SDP algorithm is easy to implement. However, the SDP has one main drawback that it cannot be applied to a large problem because the state variables need to be discretized. This problem is usually referred as the curse of dimensionality. The curse of dimensionality is a typical challenge faced when solving dynamic programming. This term was first introduced by Bellman. The curse of dimensionality refers to the exponential increase in the data that has to be observed as the dimension is increased (Chen, 2009). This will affect the computational performance and increase the time needed to find the optimal solution.

SDP has been widely used to solve hydropower production problems because of its capability. SDP can handle problems that are non-linear, constraints act in both the state and the control, and when there is a random disturbance on the problem. However, SDP requires that the sets of state, control, and random disturbance must be finite in time t. Therefore, those variables must be discretized. Moreover, the computing time increases as the number of the state, control, and disturbance variables increases (Castelletti et al., 2008).

De Ladurantaye et al. (2009) solved a hydropower production problem that maximizes profit using the Stochastic model. Their research aimed to find a 24-hour production plan. The problem included released water variables, storage variables, spillage, and water-head variables. They conclude that the solution obtained from the stochastic model outperformed the deterministic one.

Another research done by Zambelli et al. (2011) also studied SDP on solving hydropower scheduling problems. The problem they solved is aimed to minimize the operational cost with several variables including released water variables, spillage, and storage variables. They applied the SDP to a single reservoir system over a 1-year period. They then compared the result gained with Deterministic Dynamic Programming (DDP). They found that the result from SDP maintained the storage at a low level which minimizes the spillage, this, therefore, implied greater discharged water. While DDP tends to produce a solution that preserves the storage to a higher level which affects the performance in wet scenarios. Despite those differences, both SDP and DDP shared similar performances.

2.6 Rolling Horizon Algorithm

The rolling horizon algorithm was first introduced by Le and Day (1982). They presented the method to solve an expansion problem for electric generators (Le and Day, 1982). The idea of a rolling horizon is to split the whole time horizon into multiple and smaller subsets of the horizon and then solve the subproblems sequentially. The solution for the full-time horizon is obtained by connecting the solutions from each of the subproblems. This approach is useful when solving a problem with uncertainty or a large scale problem (Bischi et al., 2019).

Wakui et al. (2022) applied a rolling horizon algorithm to solve long-term operational planning of energy storage and supply systems. At the beginning, they solved the simplified version of the original problem to find the bound. After that, they solved the subproblem within a shorter time frame and used the bound found previously as the terminal condition. This step was done sequentially over each time slots in which the initial and end conditions are determined by the previous time slot result and the simplified problem result. They found that the result obtained by applying a rolling horizon algorithm is better and faster than one obtained using a conventional method (a method without applying rolling horizon algorithm).

Bischi et al. (2019) also applied a rolling-horizon algorithm to solve a long-term scheduling of cogeneration systems. The problem was originally over a yearly basis. They, then split the horizon into weekly basis. The result they obtained proves that a rolling horizon algorithm allows for an improvement in objective function by almost 3%.

In addition, Silvente et al. (2015) also utilised rolling horizon algorithms in their work. They used it to solve the planning of energy supply and demand in microgrids. They also found that a rolling horizon produces a better result more rapidly than other methods.

Moreover, Gauvin et al. (2017) also worked with rolling horizon algorithm to solve hydropower scheduling problem. Their work involved solving a non-convex problem. He combine rolling-horizon and successive linear programming to solve the problem. The result obtained is compared with the historical decisions made and found that the result provides better solution on minimizing the flood.

Chapter 3

Single-Basin Hydropower Optimization Models

In this chapter, we consider a simplified system with a single basin and a linear objective function. A more realistic case in which the Swiss Mattmark hydropower system is analyzed in the next chapter.

3.1 Deterministic Model for Single-Basin Hydropower Optimization Model

The hydropower system which we consider here consists of a basin/water storage facility. Let T be the set of time steps. At each time $t \in T$, o_t represents the amount of water released from the water storage, while f_t represents the units of water flows into basin 1 from the environment. The amount of water in the storage at each time $t \in T$ is denoted by s_t. The typical hydropower situation discussed here can be seen at Figure 3.1.

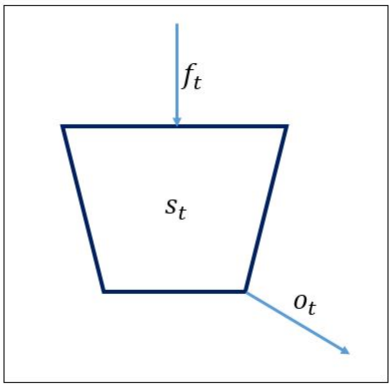


FIGURE 3.1: A Typical Single Basin Hydropower

Hydropower uses the potential energy of water movement from a higher place to a lower place. Those potential energy is then transformed into electricity. The potential energy (E) of the water movement is defined as:

$$E = \gamma\, g\, h\, V \tag{3.1}$$

where:
η: the efficiency of the turbine
γ: the water density $\left(kg/m^3\right)$
g: gravitational constant $\left(m/s^2\right)$
h: the distance of the waterfall (m)
V: the amount of water discharged $\left(m^3\right)$.
Meanwhile, the power is defined as the rate of energy production. Therefore, the power output of the reservoir at each time (t) can be written as:

$$P_t = \eta\gamma\, g\, h\, o_t \tag{3.2}$$

where η is the efficiency of the turbine, o_t is the amount of water turbinated (m^3/s) and therefore, the unit of P_t is $m^2.kg/s^3$ or $Joules$.

The objective of the hydropower optimization is to decide the amount (o_t) of water to be released from the water storage at each point in time $t \in T$ which maximizes the revenue from selling energy to the market during every period. This revenue can be described as the total energy price times the rate of energy production over a period of time. However the price of electricity is defined in *Euro* per *MWh*. Therefore, we have to convert the unit of P_t in (3.2) from Joules to Mwh by multiplying it by $\frac{1}{3600}.10^{-6}$. Hence the total revenue is:

$$r = \sum_{t=1}^{t=T} \pi_t.\frac{1}{3600}.10^{-6} P_t = \sum_{t=1}^{t=T} \pi_t.\frac{1}{3600}.10^{-6}\eta\gamma\, g\, h\, o_t \tag{3.3}$$

where: π_t is the price of energy selling at time t ($Euro/MWh$) and P_t is the rate of energy production at time t in MWh. Therefore, for simplicity, the objective function will be written as:

$$r = \sum_{t=1}^{t=T} \pi_t \xi \eta\gamma\, g\, h\, o_t \tag{3.4}$$

where ξ is the coefficient to convert $Joule$ to Mwh, which is $\frac{1}{3600}10^{-6}$.

Balance Constraints

The volume of water inflows plus that of water available at time t in the water storage must be equal to the volume of water outflows plus that of water still available in that storage at next time period. Therefore, the balance equation can be written as:

$$f_t + s_{t-1} = o_t + s_t, \qquad \forall t \in T \tag{3.5}$$

Boundaries

The water released from each basin is bounded by the maximum capacity of the turbines. This means that in every time periods, the amount of water released from the storage cannot exceed the maximum capacity of the turbines:

$$o_{min} \leq o_t \leq o_{\max} \qquad \forall t \in T \tag{3.6}$$

Additionally, the amount of water available in the storage cannot exceed the maximum volume of the storage and should not go below a certain lower bound:

$$s_{\min} \leq s_t \leq s_{\max} \qquad \forall t \in T \tag{3.7}$$

Initial/Final Storage Conditions

The water stored in the basin at the end of time period $|T|$ must be guaranteed to be enough so that the hydropower plant can keep running for the next cycle.

$$s_{|T|} \geq s_0 \tag{3.8}$$

Therefore, the complete model of deterministic formulation for single-basin hydropower optimization can be written as:

Complete Model

$$\max \quad \sum_{t=1}^{T} \pi_t \xi \eta \, g \gamma \, h \, o_t \tag{3.9}$$

$$\text{s.t} \quad f_t + s_{t-1} = o_t + s_t \qquad \forall t \in T \tag{3.10}$$

$$0 \leq o_t \leq o_{\max} \qquad \forall t \in T \tag{3.11}$$

$$s_{min} \leq s_t \leq s_{\max} \qquad \forall t \in T \tag{3.12}$$

$$s_0 \leq s_{|T|} \tag{3.13}$$

3.2 Robust Model for Single-Basin Hydropower Problem

The model in Eq. (3.9-3.13) does not take into account the uncertainty in inflow water. While in reality, how much water will go into the system is uncertain. We do not know for sure the amount of water flowing to the basin at the time. Therefore, we will consider the uncertainty into the model.

The uncertainty in this problem appears on inflow variables. Therefore, we should consider that the realisation of inflow depends on the scenario. Therefore, in this RO model, the inflow variable f and the storage level s are the second stage variable and depend on the scenarios, while the decision variable o is the first stage variable. The aim of this model is to find the solution which is immune to any realisation of inflow.

We write f_t^ω as the amount of water inflow to the reservoir at time $t \in T$ under scenario $\omega \in \Omega$ and s_t^ω as the amount of water in the storage at time $t \in T$ under the scenario $\omega \in \Omega$, where Ω represents the set of all possible scenarios.

Moreover, the objective function can be written as:

$$\max \sum_{t=1}^{T} \pi_t . P_t \tag{3.14}$$

which aims to maximizing the revenue by multiplying the price at each time $t \in T$ and the rate of electricity production.
Subject to:
(Balance constraints)

$$f_t^\omega + s_{t-1}^\omega = o_t + s_t^\omega, \qquad \forall t \in T; \forall \omega \in \Omega \tag{3.15}$$

This constraint explains that the amount of water in the basin at time t is depend on the scenario. Therefore, the amount of water in the basin at time t for scenario ω is equal to the total of amount of water in the basin at time $t-1$ for related scenario, and the inflow water for the same scenario, minus the amount of water released.
(Boundaries)

$$o_{min} \leq o_t \leq o_{\max} \qquad \forall t \in T \tag{3.16}$$

$$s_{\min} \leq s_t^\omega \leq s_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.17}$$

These constraints bound both the released water and the stored water because of their physical construction. Therefore, both o_t and s_t^ω are bounded by some minimum and maximum number.
(Initial Storage Conditions)

$$s_0 \leq s_{|T|}^\omega \qquad \forall \omega \in \Omega \tag{3.18}$$

This constraints restricts that the amount of water stored in the basin at the end of period should not be less than its amount in the beginning of period. This is to ensure that the system has enough water to run for the next cycle (cyclostationerity). The complete robust model for this problem can be seen on Appendix A.2.

3.3 Affine Decision Rule Model for Single-Basin Hydropower Problem

In the previous section, the decision variable o_t is a first stage variable. o_t does not depend on the scenarios and is aimed to be immune towards any realisations of inflow. However, this may lead to an anticipative solution. Consider the following example.

Example 3.1. *:*
Consider these two scenarios of inflow and its optimal outflows:

TABLE 3.1: Example of an Anticipative Problem

t		1	2	3	4
scenario 1	inflow	1	2	7	9
	outflow	5	9	25	28
scenario 2	inflow	1	2	8	25
	outflow	5	11	27	29

The table above shows an example of two scenarios of inflows and their optimal outflows. For example in the first scenario at time $t = 1$, there is 1 unit of water inflows into the reservoir and the optimal water released at that time is 5. Moreover, at time $t = 2$, there are 2 units of inflow water and the optimal amount of water released is 9 unit, and so on.

If you compare scenario 1 and scenario 2, the inflow in both scenarios at time $t = 1$ and $t = 2$ are similar. Therefore, at time $t = 1$ the optimal released water is the same regardless of the scenario. However, at time $t = 2$, in order to decide how much water we should release, we have to know how much water is flowing to the reservoir at the next period. If the amount of inflow water at time $t = 3$ is 7, it means we are in scenario 1 and the optimal amount of water should be released at time $t = 2$ is 9. While, if the water inflows at time $t = 3$ is 8 units, it means that we are in scenario 2, so the optimal amount of water to be discharged at time $t = 2$ is 11. This is what it is called as anticipativity, that is the optimal solution requires knowledge of the future.

Therefore, to prevent the anticipative solution, in this section we formulate the decision variable o_t to be determined by a non-anticipative function. Here, the decision variable o_t^ω is a two stage variable and depends on the scenario. We want our decision variable to adapt to the realisation of inflow. Hence, the new variables α_t and $\beta_{tt'}$ are introduced, so to:

$$o_t^\omega = \alpha_t + \sum_{t'=1}^{t-1} \beta_{tt'} f_t'^\omega \qquad \forall \omega \in \Omega, t \in T. \tag{3.19}$$

Hereinafter, o_t^ω is an affine function and depends on the scenario. Looking in more details, for each $t \in T$, the outflow variable depends on the summation of variable β and the corresponding inflow from time 1 until $t-1$. Hence, variable β will be of dimension $|T| \times |T|$ which is very large. This may affect the computation time. To overcome this issue, we can restrict o_t to only depend on τ numbers of β variables and the corresponding inflows. In other words, o_t is formed from the summation of β and the corresponding inflow from time $t - \tau$ to time $t - 1$ for some given value of τ. This restriction lead to β with smaller dimension than before. Therefore, the equation (3.20) now is:

$$o_t^{\omega} = \alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t'^{\omega} \qquad \forall \omega \in \Omega, t \in T. \tag{3.20}$$

Based on how the decision makers tolerate the risk, there are three types of risk: risk-averse, risk-neutral, and risk-taking (Harris and Wu, 2014). A risk-averse decision maker tends to avoid any risks. A risk-averse decision makes usually considers the objective function in the worst-case scenario.

$$\max_x \min_\omega f(x;\omega), \tag{3.21}$$

While a risk-neutral decision maker neither takes too much risk or avoid taking risks. The problem for a risk-averse decision maker is modelled by taking expectation in the objective function.

$$\max_{x,\omega} \mathbb{E}[f(x;\omega)] \tag{3.22}$$

Moreover, a risk-taking decision maker is not afraid to accept the risk. The objective function for a risk-taking decision maker is modelled as:

$$\max_x \max_\omega f(x;\omega). \tag{3.23}$$

In this research, we consider two types of objectives, which are risk-averse and risk-neutral. In the risk-averse, the objective is to find the released water schedule which maximizes the revenue under the worst-case scenario.

3.3.1 Affine Decision Rule for Risk-Averse Decisions

In this subsection, the single-basin hydropower optimization is modeled with affine decision rule. Therefore, the decision variable o is defined with an affine function as in Eq. (3.20) and now is a two-stage variable which depends on scenario. The purpose is so that the decision variable is able to adapt to any realization of inflow. The model is basically similar with the robust model shown in A.2 except that now the decision variable is an affine function. Therefore, the o_t in A.2 is substituted with the function (3.20). The complete model of the Affine Decision Rule Model for Single-Basin Hydropower Optimization Problem can bee seen below.

Problem 3.1 (Affine Decision Rule Model for Single-Basin Hydropower Optimization Problem)**.**

$$\max_{\alpha,\beta,s} \quad \min_{\omega\in\Omega} \quad \sum_{t=1}^{|T|} \pi_t \, \xi\eta \, g \, \gamma \, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_{t'}^{\omega} \right) \tag{3.24}$$

$$\text{s.t.} \quad f_t^{\omega} + s_{t-1}^{\omega} = \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_{t'}^{\omega} \right) + s_t^{\omega} \qquad \forall t \in T, \omega \in \Omega \tag{3.25}$$

$$s_{\min} \leq s_t^{\omega} \leq s_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.26}$$

$$o_{min} \leq \alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_{t'}^{\omega} \leq o_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.27}$$

$$s_{|T|}^{\omega} \geq s_0 \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.28}$$

where ξ is the parameter to convert Joules into MWh which is ($\frac{1}{3600}.10^{-6}$), while $o_{\min}$ and $o_{\max}$ represent the minimum and maximum amount of water allowed to be released respectively. s_{min} is the minimum amount of water should remain on the storage. On the other hand, because each basin has maximum capacity, therefore s_{max} describes the maximum capacity of basin.

Notice that if we set $\tau = 0$, then model 3.1 is equivalent to the model in A.2.

The objective function in equation (3.24) includes maximization and minimization problem. Hence, it makes this problem complicated. Therefore, we can convert the equation (3.24) through epigraph reformulation to make the problem easier to solve. Epigraph is one of the transformation approaches used to change a problem into an equivalent one. The epigraph reformulation is supposed to convert a convex problem to its equivalent problem. In other words, we will convert the objective function (3.24) to an equivalent one. To do so, we introduce a new variable ψ for epigraph reformulation.

The objective function (3.24) shows that the model is supposed to maximize the revenue in the worst case scenario or maximizing the minimum revenue for all scenarios. Therefore, we can rewrite it with the epigraph reformulation into:

$$\max \quad \psi \tag{3.29}$$

$$\text{s.t} \quad \psi \leq \min_{\omega\in\Omega} \left(\sum_{t=1}^{|T|} \pi_t \, \xi\eta \, g \, \gamma \, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega} \right) \right) \tag{3.30}$$

Those two equations (3.29)-(3.30) aims to maximize ψ where ψ is less than the minimum revenue for all scenarios $\omega \in \Omega$. Notice that the equation (3.30) still has a 'min' function which thus makes the overall model is difficult to solve. Therefore, we need further reformulation to make the problem easier to be solved.

Equation (3.30) implies that the value of ψ should be no more than the minimum value of $\sum_{t=1}^{|T|} \pi_t \, \xi\eta \, g \, \gamma \, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega} \right)$. We can paraphrase the previous sentence as:

the value of ψ should be no more than the value of $\sum_{t=1}^{|T|} \pi_t\, \xi\eta\, g\, \gamma\, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega}\right)$ over all $\omega \in \Omega$, or it can be written mathematically as:

$$\max_{\psi,\alpha,\beta,s} \quad \psi \tag{3.31}$$

$$\text{s.t} \quad \psi \leq \left(\sum_{t=1}^{|T|} \pi_t\, \xi\eta\, g\, \gamma\, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega}\right)\right) \qquad \forall \omega \in \Omega \tag{3.32}$$

Now, we have a new model by reformulating the equation (3.24) into its equivalent reformulation (3.31)-(3.32) through an epigraph reformulation. We called this new model as the Epigraph Reformulation Model of Affine Decision Rule Model for Single-Basin Hydropower Optimization Problem as follow.

Problem 3.2 (Affine Decision Rule Model for Single-Basin Hydropower Optimization Problem)**.**

$$\max_{\psi,\alpha,\beta,s} \quad \psi \tag{3.33}$$

$$s.t \quad \psi \leq \left(\sum_{t=1}^{|T|} \pi_t\, \xi\eta\, g\, \gamma\, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega}\right)\right) \qquad \forall \omega \in \Omega \tag{3.34}$$

$$f_t^{\omega} + s_{t-1}^{\omega} = \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_{t'}^{\omega}\right) + s_t^{\omega} \qquad \forall t \in T, \omega \in \Omega \tag{3.35}$$

$$s_{\min} \leq s_t^{\omega} \leq s_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.36}$$

$$o_{\min} \leq \alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_{t'}^{\omega} \leq o_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{3.37}$$

$$s_{|T|}^{\omega} \geq s_0 \qquad \forall \omega \in \Omega. \tag{3.38}$$

3.3.2 Affine Decision Rule for Risk Neutral Single-Basin Hydropower Optimization Problem

A risk-neutral decision maker tends to consider the objective function is optimized on its expected value over all scenarios. Therefore, the objective function of this new model is to maximize the expected value of the profit. The overall model is similar to the risk-averse model in 3.24-3.28. The difference lays in the objective, where in the risk-averse model, the objective is to maximize in the worst-case scenario, while in the risk-neutral, the objective is maximizing under the expectation.

Therefore, the objective function in this risk-neutral model is:

$$\max_{\alpha,\beta,s} \quad \sum_{t\in T} \sum_{\omega\in\Omega} p_{\omega} \left(\pi_t\, \xi\eta\, g\gamma\, h \left(\alpha_t + \sum_{t'=t-\tau}^{t-1} \beta_{tt'} f_t^{\omega}\right)\right) \tag{3.39}$$

Where p_{ω} is the probability of scenario ω happening. The complete model of the Affine decision rule for risk-neutral single-basin hydropower optimization can be seen below.

Problem 3.3 (Affine Decision Rule for Risk Neutral Single-Basin Hydropower Optimization Problem)**.**

$$\max_{\alpha,\beta,s} \quad \sum_{t\in T}\sum_{\omega\in\Omega} p_\omega \left(\pi_t\,\xi\eta\,g\gamma\,h\left(\alpha_t + \sum_{t'=t-\tau}^{t-1}\beta_{tt'}f_t^\omega\right)\right) \tag{3.40}$$

$$s.t. \quad f_t^\omega + s_{t-1}^\omega = \left(\alpha_t + \sum_{t'=t-\tau}^{t-1}\beta_{tt'}f_{t'}^\omega\right) + s_t^\omega \qquad \forall t\in T, \omega\in\Omega \tag{3.41}$$

$$s_{\min} \le s_t^\omega \le s_{\max} \qquad \forall t\in T, \forall\omega\in\Omega \tag{3.42}$$

$$o_{\min} \le \alpha_t + \sum_{t'=t-\tau}^{t-1}\beta_{tt'}f_t^\omega \le o_{\max} \qquad \forall t\in T, \forall\omega\in\Omega \tag{3.43}$$

$$s_{|T|}^\omega \ge s_0 \qquad \forall\omega\in\Omega. \tag{3.44}$$

3.4 Stochastic Dynamic Programming (SDP) Approach for Single-Basin Hydropower Problem

In previous sections, we studied the problem where the decision variable depends on the inflow f, while in this chapter, we study another approach that involves the decision variable depends on the storage level s and time t.

Problem 3.4 (SDP Model for single basin hydropower problem)**.** *Let* $s_t \in \{0, ..., s_{\max}\}, \forall t \in |T|$ *and* $r_t(s_t)$ *to be the optimal solution at* $t \in T$*. For all* $t \in \{1, 2, ..., |T|\}$*:*

$$r_t(s_t) = \max_o \ \pi_t\,\xi(\eta\,g\,\gamma\,h\,o_t) + \sum_{\omega\in\Omega} p_\omega r_{t+1}(s_{t+1}) \tag{3.45}$$

$$s.t. \quad s_t - s_{\max} + \max_{\omega\in\Omega}\{f_t^\omega\} \le o_t \qquad \forall t\in T \tag{3.46}$$

$$o_t \le s_t - s_{\min} + \min_{\omega\in\Omega}\{f_t^\omega\} \qquad \forall t\in T \tag{3.47}$$

$$o_t \ge o_{min} \qquad \forall t\in T \tag{3.48}$$

where the first term of (3.45) denotes the profits made at time t, while the second term of (3.45) denotes the expected value of the optimal profit from time $t+1$.

Proof. Before solving Problem 3.4, we have to prove that this problem is feasible and the result is optimal.

Let s_{t+1} be the state variable at time $t+1$. The state variables from time t to $t+1$ should satisfy the mass balance conditions as below:

$$s_{t+1} = s_t + f_t^\omega - o_t \tag{3.49}$$

and the bounds constraints:

$$s_{\min} \leq s_{t+1} \leq s_{\max} \tag{3.50}$$

where f_t^{ω} is inflow at time t and under scenario $\omega \in \Omega$. From equation (3.49) and equation (3.50), we get:

$$s_{\min} \leq s_t + f_t^{\omega} - o_t \leq s_{\max}. \tag{3.51}$$

Therefore, the lower bound for o_t is

$$o_t \geq s_t + f_t^{\omega} - s_{\max} \tag{3.52}$$

and the upper bound is:

$$o_t \leq s_t + f_t^{\omega} - s_{\min} \tag{3.53}$$

If we want o_t to satisfy both bounds for all $\omega \in \Omega$, we have to consider the worst-case scenario that gives the lowest restriction for lower and upper bounds on o_t. Therefore, o_t should satisfy:

$$o_t \geq \max_{\omega \in \Omega}\{s_t + f_t^{\omega} - S_{\max}\} = s_t - s_{\max} + \max_{\omega \in \Omega}\{f_t^{\omega}\} \tag{3.54}$$

and

$$o_t \leq \min_{\omega \in \Omega}\{s_t + f_t^{\omega} - S_{\min}\} = s_t - s_{\min} + \min_{\omega \in \Omega}\{f_t^{\omega}\}. \tag{3.55}$$

Because we derive Equation (3.54) and (3.55) from Equation (3.50), it shows that whatever the realization of o_t is, s_{t+1} is always non-negative. Therefore, the problem is feasible.

Furthermore, we have to prove the optimality. At time $t = |T|$:

$$r_{|T|}\left(s_{|T|}\right) = \max_{o}\left\{\pi_{|T|}\, o\right\} \tag{3.56}$$

is optimal.
Let us assume that $o^* = \left(o_t^*, o_{t+1}^*, ..., o_{|T|}^*\right)$ is optimal for $r_t\left(s_t\right)$ and $\phi_t\left(s, o\right)$ is the solution value of the $r_t\left(s_t\right)$ corresponding to o. Let us also assume that $o^* = \left(o_t^*, o_{t+1}^*, ..., o_{|T|}^*\right)$ is not optimal for $r_{t+1}\left(s_{t+1}\right)$. Therefore, there is some $\hat{o} = \left(\hat{o}_{t+1}, ..., \hat{o}_{|T|}\right)$ such that $\phi_t\left(s, \hat{o}\right)$ is strictly larger than $\phi_t\left(s, o^*\right)$. However, then, the solution $\left(o_t^*, \hat{o}_{t+1}, ..., \hat{o}_{|T|}\right)$ is a better solution to $r_t\left(s_t\right)$. This contradicts our assumption that o^* is optimal for $r_t\left(s_t\right)$. □

The SDP model (3.45) - (3.48) is solved through backward recursion. The computational procedure starts at the end of the time horizon. The pseudocode for SDP model can be seen at Algorithm 1. This approach produces a release decision for every possible state at each stage rather than only a single release schedule. Therefore, the state variable s_t and the control variable o_t should be simplified to make the algorithm is computationally feasible. The state variable will be evaluated on each possible discrete level for every stage. At each stage and state, a released water o_t is decided to maximize current benefit and the expected future benefit $r_{t+1}(s_{t+1}, f_{t+1})$. This function is usually called as cost-to-go function or Bellman function and is described as a recursive function.

The purpose of this approach is to find a set of solutions which can be defined as a set of functions:

$$\left\{\Psi_1(s_1, f_1), \Psi_2(s_2, f_2), ..., \Psi_{|T|}\left(s_{|T|}, f_{|T|}\right)\right\} \tag{3.57}$$

where $o_t = \Psi_t(s_t, f_t)$ is the vector of water discharged at time t that depends on the amount of water in the storage (s_t) and the amount of inflow.

Algorithm 1: Pseudocode for SDP for Single-Basin Hydropower Problem

```
Data: f_t^ω, π_t, ∀ω ∈ Ω_ts, t ∈ T, o_min, o_max, s_min, s_max, step_s, step_o
for t: T, T-1, ..., 1 do
    if t = T-1 then
        for s in range (s_min, s_max, step_s) do
            for o in range (o_min, o_max, step_o) do
                r_t(s,o) = π_t ξ(η g γ h o_t);
            end
            r_t(s) = max(r_t(s,o)) and o(t,s) is o maximizing r_t(s)
        end
    else
        for s in range (s_min, s_max, step_s) do
            o_min = 0;
            o_max = s − s_min + min_{ω∈Ω} f_t^ω;
            r_t(s) = 0;
            for o in range (o_min, o_max, step_o) do
                for ω ∈ Ω do
                    s' = s + f_t^ω − o;
                    r_t(s,o) = π_t ξ(η g γ h o_t) + Σ_{ω∈Ω} p_ω r_{t+1}(s_{t+1});
                    if r_t(s,o) > r_t(s) then
                        r_t(s) = r_t(s,o) and o(t,s) = o
                end
            end
        end
    end
end
Result: Matrix t × s of r, matrix t × s of o
```

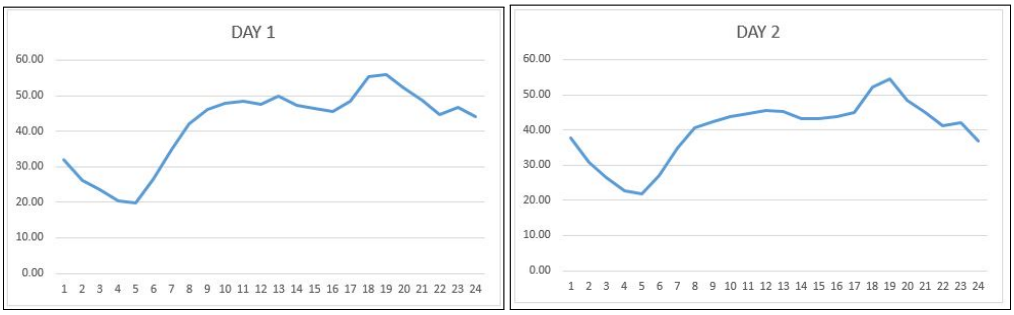
3.5 Numerical Result and Discussion

In this section, we implement the models introduced in the previous sections to solve scheduling problems in a hydropower plant with a single basin. The typical single-basin hydropower system can be seen in figure 3.1. The models are applied into Mattmark hydropower system but with some simplification. This system actually consist of 2 basins and 1 pumping storage and the more detailed discussion can be seen on Chapter

4. However, for this chapter, we only use the inflow data of upstream basin and the electricity price. The input data covers one year data of electricity price and inflow data based on an hourly schedule. We consider 20 scenarios of inflow data which 14 of them are used for training, while the rest is used for validation.

The validation is aimed to observe the performance of the result obtained when it is applied to the data that are not used in the optimization (validation set). The idea is to substitute the result obtained to the original model while observing whether they are violating the constraints or bounds and keeps track the number of violations. The pseudocode for the validation process can be seen on Algorithm 4. To do the optimization, we set the initial storage as $1 \times 10^7\ m^3$.

First, we apply the SDP model and discretize the state and control dimensions. In this case, we discretize the state dimension into several values which are 10, 100, and 1000, while we discretize the control variable into fixed numbers, 19. We try to apply the SDP model with the input data of hourly schedule of inflow. However, it takes very long time to run this model. As we predict, this model suffers from curse of dimensionality. Therefore, to practicality, we apply granularity of the input data and we choose to take 4 hourly data. To do so, we take the average of each 4 consecutive points in the data. Hence, now the data we have is the schedule of inflow water over one year in 4 hourly basis.



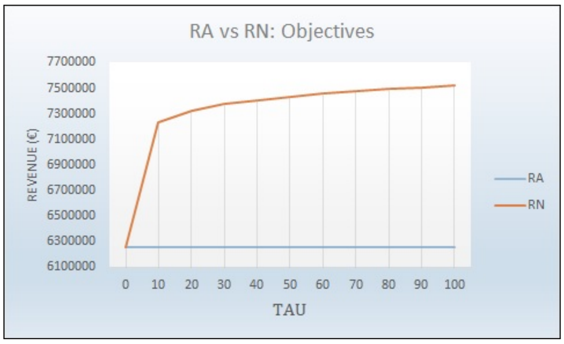
(A) Day 1 (B) Day 2

FIGURE 3.2: The Electricity Price on Day 1 and Day 2

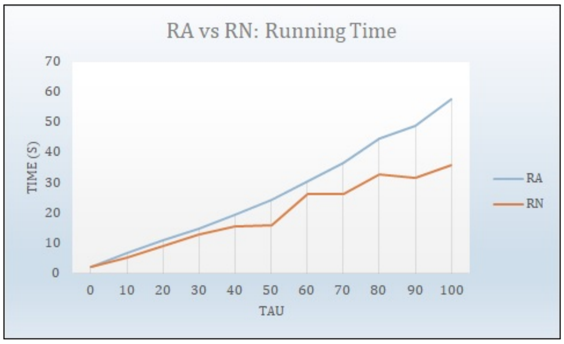
Figure 3.2 shows the typical of the electricity on each day. If you see at those figures, they show that the price behaviour is the same every day. Above, we present the picture for day 1 and day 2 for the illustration. The other days also have typical behaviour with those 2 days we present here. Let take a closer look to Figure 3.2a, in the first 4 hour, the electricity price tend to decrease. Then, the next 4 hours, the price is increasing. From hour 9 to 12, the price seems to steady and start to decrease in the next 4 hour. Then, the price increase in the next 4 hour and then decrease for the rest of the day. We want to preserve this behaviour on the price, therefore we choose the granularity specifically to 4 hours.

For robust models, we test them in Problem 3.2 and 3.3 and run them with τ from 0 to

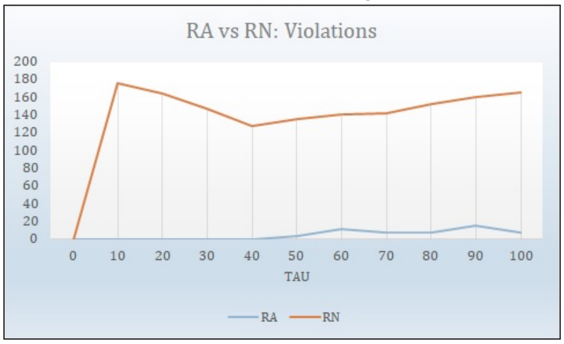
100 in increments of 10. Then, we compare the results and apply the result obtained from each model to the validation set.



(A) RA vs RN: Objectives in Training Set



(B) RA vs RN: Running Time



(C) RA vs RN: Violations in Validation Set

FIGURE 3.3: The Comparison between the result of RA and RN models

First, we compare the performance of RA and RN models. Figure 3.3 shows the comparison between RA and RN models in terms of objective value, running time, and violation. If we refer to Figure 3.3a and Figure 3.3b, RN performs better than RA. It gives better objective value and takes less time to run. Even though the running time for both models increases over τ, but the increasing in RN is not as sharp as the increasing in RA. However, with this better performance, RN suffers from more violations as shown in Figure 3.3c.

In addition, the performance of RA and RN in a training set are compared and they can be seen on Figure 3.4 - 3.7. Figure 3.4 and Figure 3.5 plot the results of RA model in the training set for τ equal to 0 and 100, respectively. While Figure 3.6 and Figure 3.7 are

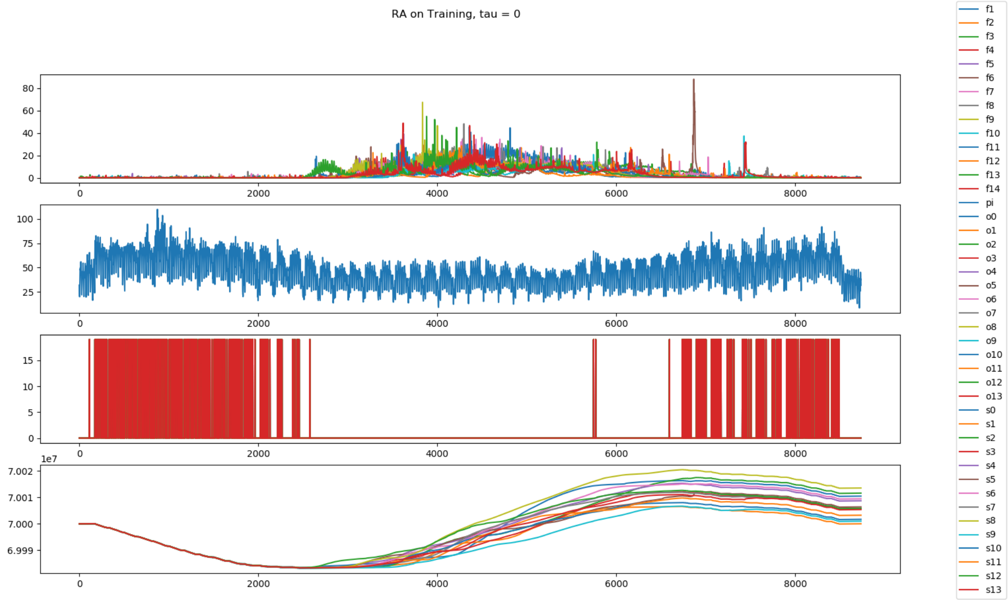


FIGURE 3.4: RA $\tau = 0$ in Training Set

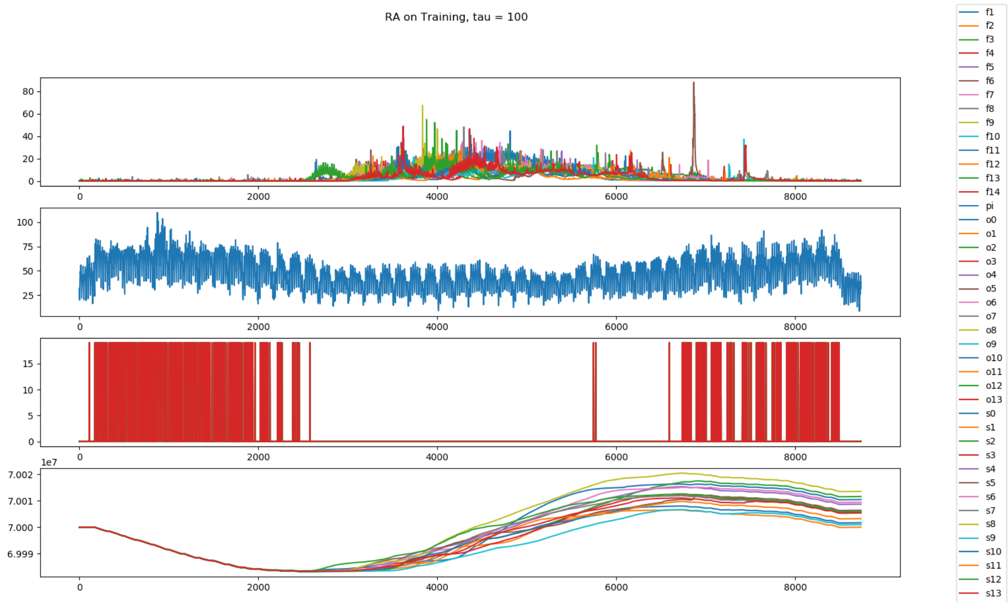


FIGURE 3.5: RA $\tau = 100$ in Training Set

graphic for the RN model in the training set with τ equal to 0 and 100, respectively. In each figure, 4 diagrams are embedded which show the amount of water released, the amount of water in the storage, the inflow, and the price consecutively.

As mentioned before, the model with $\tau = 0$ is the robust model without affine variables. Because with $\tau = 0$ meaning that we force all β to be equal to 0. Therefore, this model does not depend on the scenario. Hence, we can see that Figure 3.4 and 3.6 share similarities.

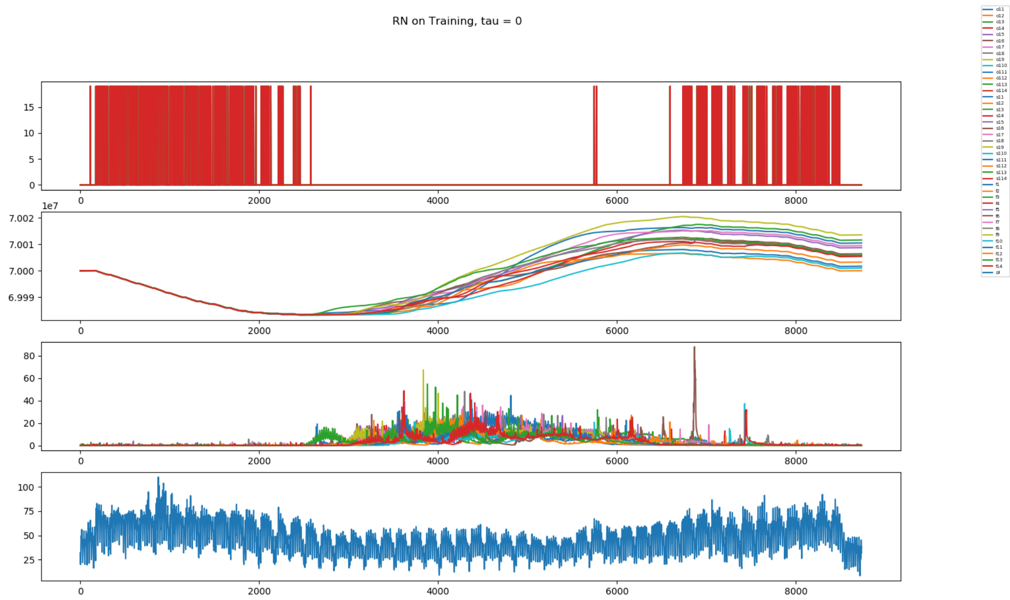


FIGURE 3.6: RN $\tau = 0$ in Training Set

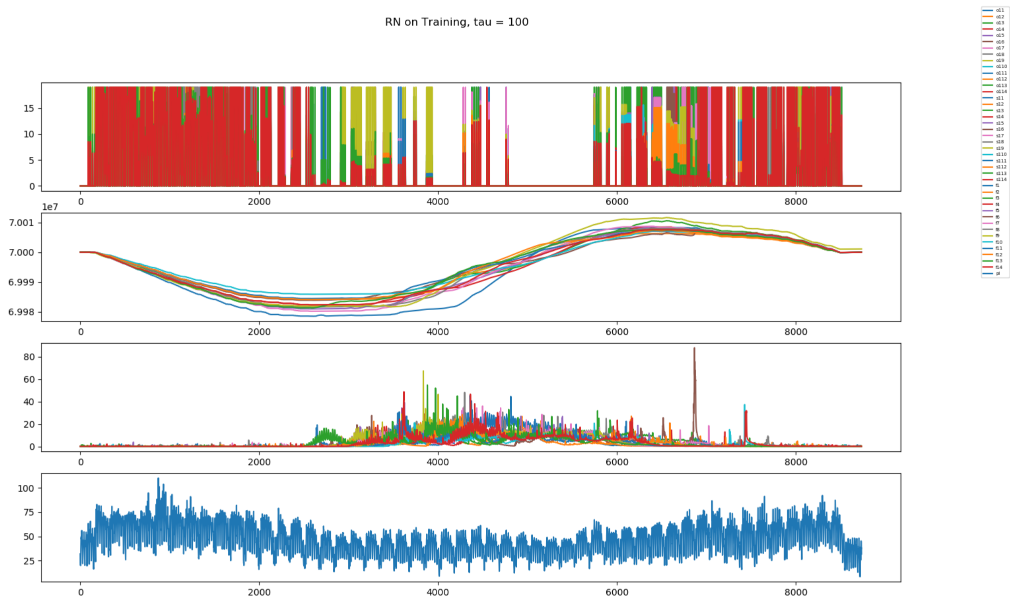


FIGURE 3.7: RN $\tau = 100$ in Training Set

Moreover, comparing the graphic of storage s on Figure 3.5 and Figure 3.7, we can see that RN tends to force the amount of water in the storage to be equal to the initial storage value for all of the scenarios at the end of the horizon. While in RA, at the end of the horizon, the amount of water in the storage for all scenarios is greater than the initial storage value yet they are divergent, not pointing towards the same value in every scenario. However, it is evident that both RA and RN tend to release water when the price is high, for example at the beginning and end of the horizon.

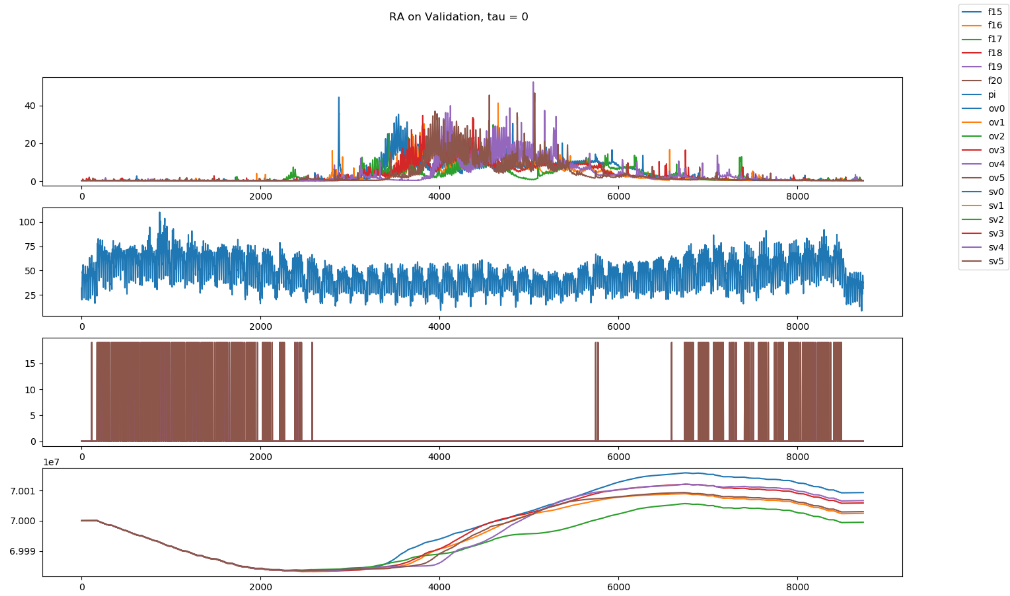


FIGURE 3.8: RA $\tau = 0$ in Validation Set

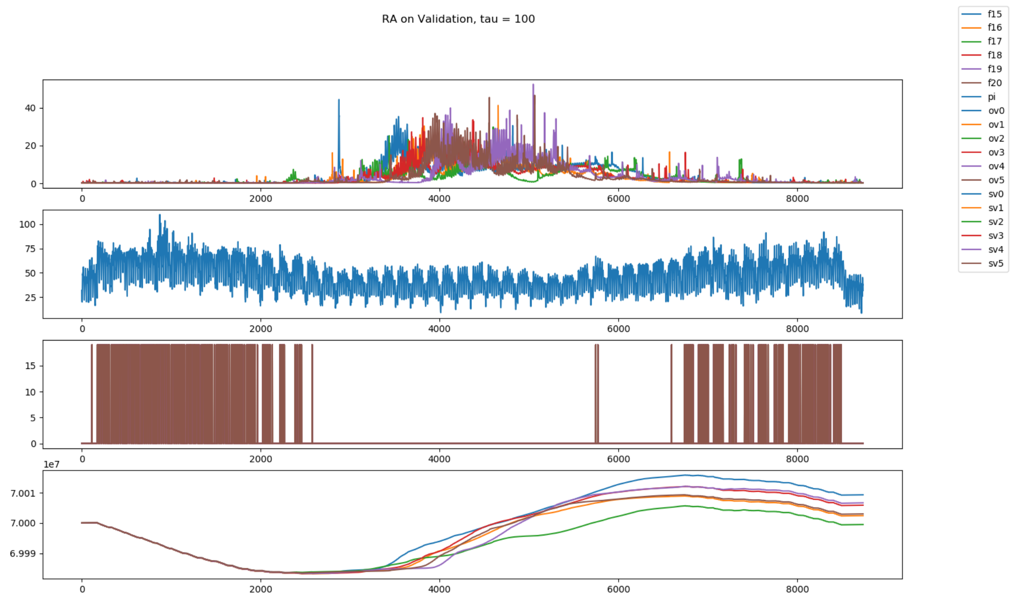


FIGURE 3.9: RA $\tau = 100$ in Validation Set

Figure 3.13 - 3.15 illustrates the performance of SDP in the validation set for the discretization of state variables as 10, 100, and 1000. In Figure 3.13, SDP does not release water at the beginning of the period, but does release a large amount of water in the end of period when the price is high. In addition, it also releases some water during the middle of the period when there is more water flowing into the basin. Figure 3.14 shows the result of validation set for SDP with a discretization of the storage values in 100 values. The result seems to be similar to the result from the robust-affine model

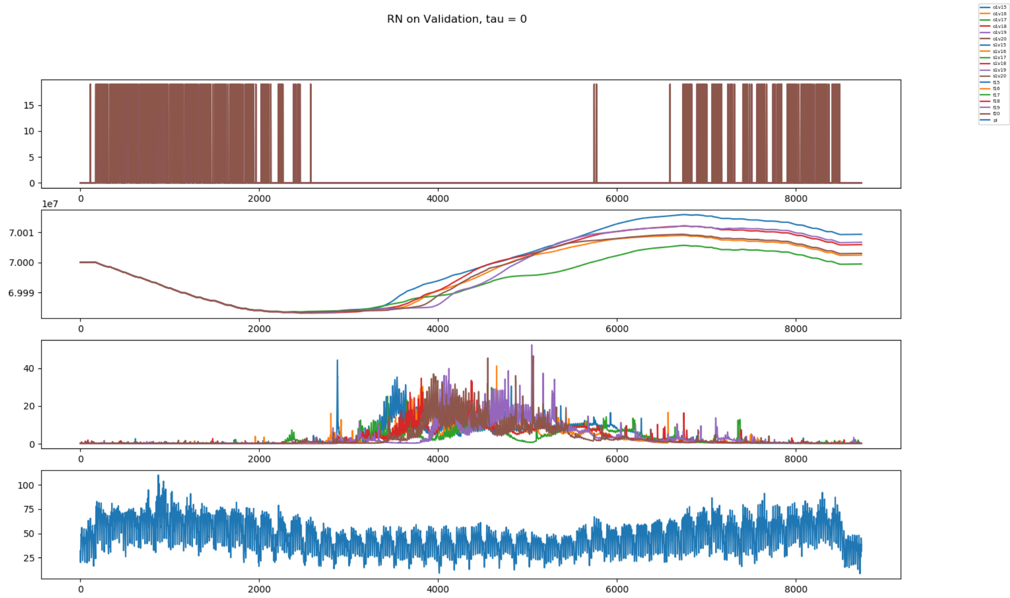


FIGURE 3.10: RN $\tau = 0$ in Validation Set

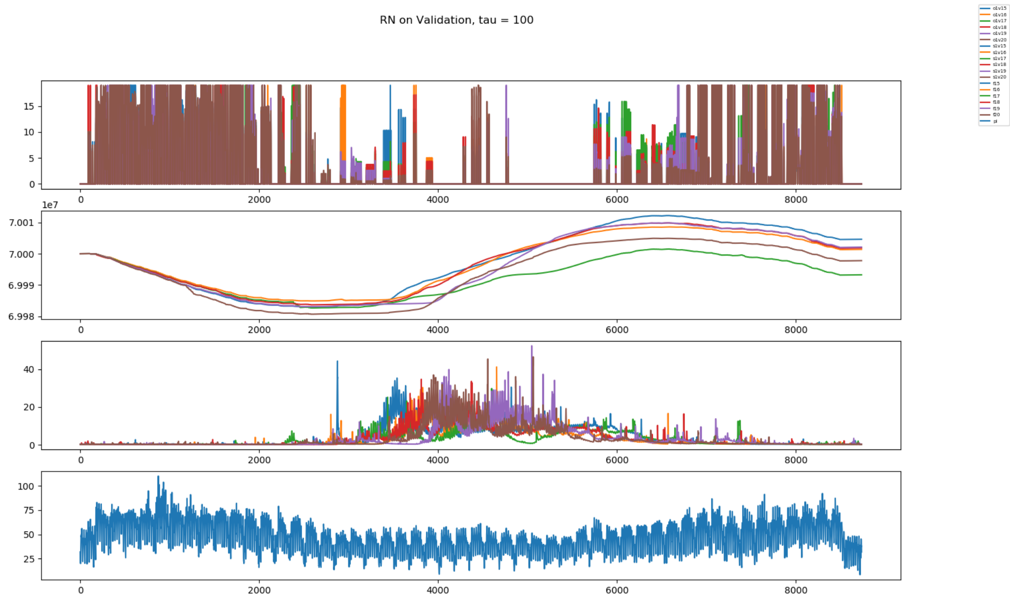
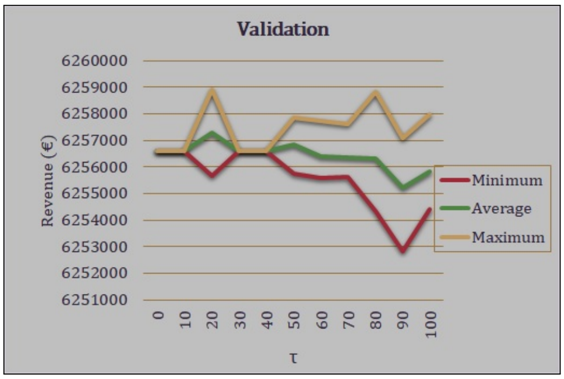


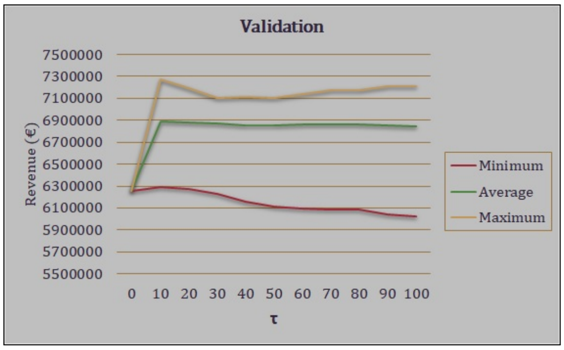
FIGURE 3.11: RN $\tau = 100$ in Validation Set

during which more water is released at the beginning and the end period when the price is high. However, SDP also release less water during the middle period. The water released here is not great in quantity because the price is not high, yet it is enough to ensure the amount of water in the storage is less than maximum capacity during the wet period.

Moreover, for SDP with 1000 discretizations, as captured in Figure 3.15, it shows that a great deal of water is released during the first and second trimester which causes the



(A) Statistics of RA in Validation Set



(B) Statistics of RN in Validation Set

FIGURE 3.12: The Statistics of RA and RN models in Validation Set

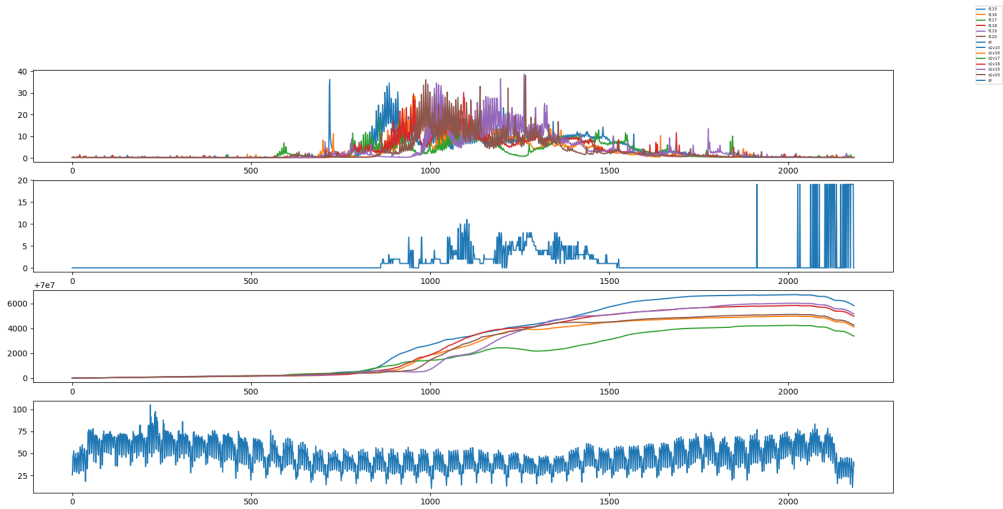


FIGURE 3.13: SDP with 10 discretization in Validation Set

violations in the amount of water in the storage in the end of period. If we refer to the graph, we can see that the amount of water in the storage at the end of period is less than the initial amount of water. This is crucial because this constraint is to ensure that the hydropower saves enough water for the next cycle.

Table 3.2 shows the comparison between the three aforementioned models regarding

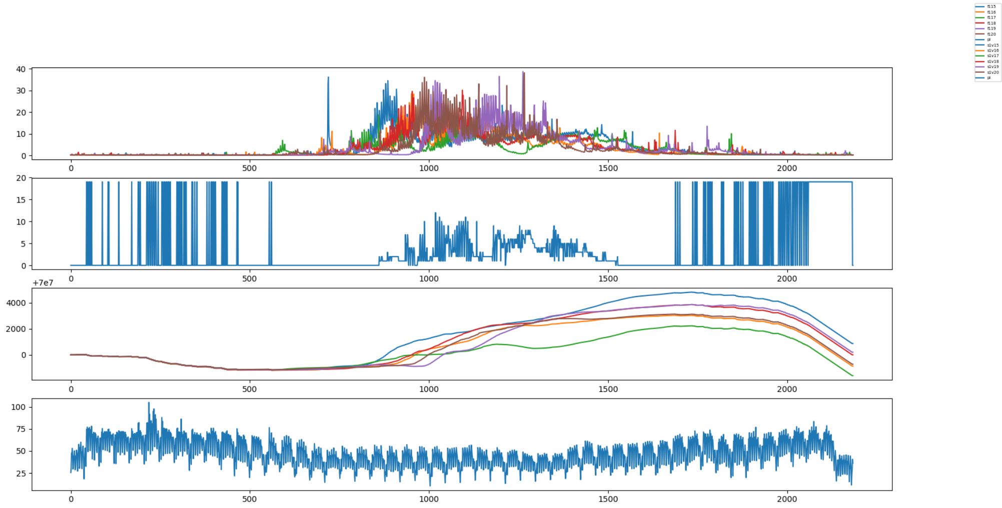


FIGURE 3.14: SDP with 100 discretization in Validation Set

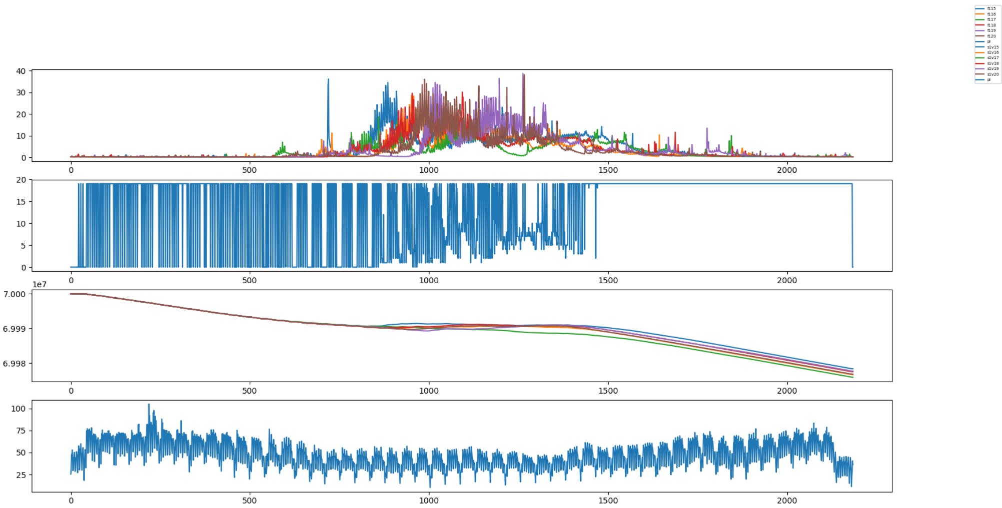


FIGURE 3.15: SDP with 1000 discretization in Validation Set

TABLE 3.2: Comparison between SDP, RA, and RN

	SDP””””””””			RA ($\tau = 10$)	RN ($\tau = 10$)
	10	100	1000		
Time (s)	5.93	58.81	536.55	43.28	42.40
Obj	2342351.17	3036429.17	5061960.0	6256582.56	7234953.48

their running time and objectives. Overall, robust models take less time to run while produce better result than SDP. Moreover, when the discretization of state dimensions of SDP is increased 10 times, the running time also increases with almost the same increment. With the running time needed by SDP, the objective value that is produced is lower than RA or RN. This can be explained by the fact that in SDP, the varieties of variable s to be chosen is limited because of the discretization. For example, in 10 discretizations of state variables, there are only 10 possible value of states of variables. While in RA and RN is more flexible because variable s is continuous. Therefore, with the limitation of SDP, we decide to not to include SDP in our next study.

Chapter 4

Two-Basin Hydropower Optimization Models

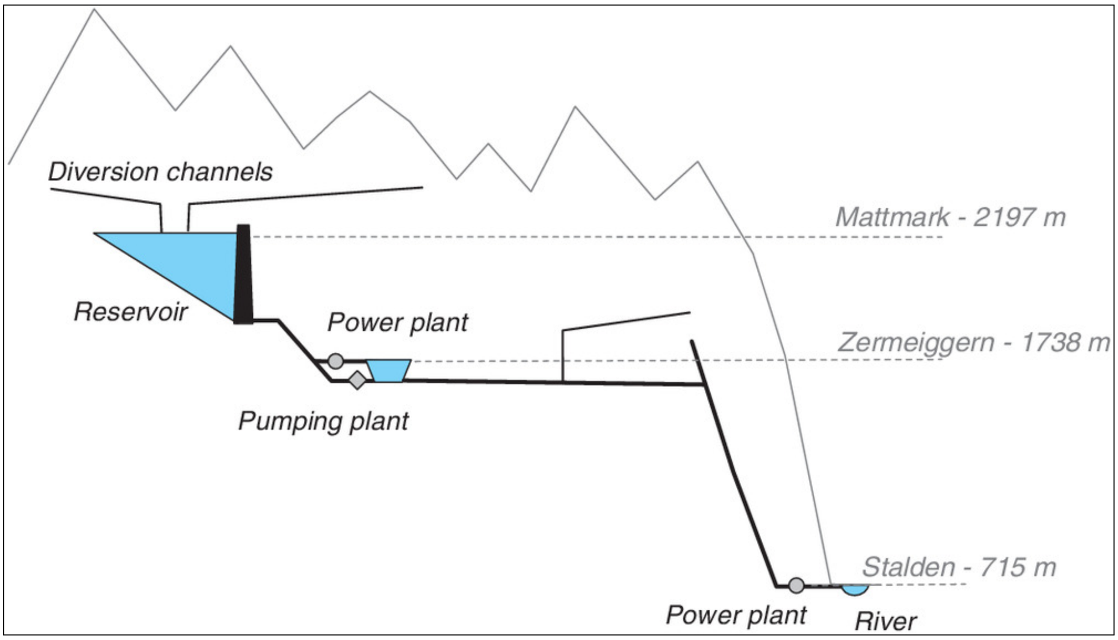


FIGURE 4.1: Mattmark Hydropower System (Anghileri, Castelletti et al., 2018)

In this chapter, we study the more realistic hydropower system in which we consider studying the Switzerland Mattmark hydropower system. The Mattmark Hydropower system can be seen in Figure 4.1. This system consists of two basins, two power plants and a pumping plant. Therefore, we will extend our study to consider two water storage, spillway, waterhead dependancy, and pumping system.

The purpose of pumping in hydropower system is to connect the lower and upper reservoir so that water can be pumped from downstream reservoir back to upstream reservoir. This Mattmark Hydropower system work as follow: The natural inflow water goes Mattmark dam and Zermeiggern dam. Mattmark is the upstraim dam abd the

Zeirmeggern is in the lower place. From Mattmark dam, some water are released and turbinated and then goes to Zermeiggern dam. While from Zermeiggern dam, some water is released and turbinated then goes to the river, also some water is pumped back to Mattmark dam.

The benefit from pumping is that the same water can be used multiple times to produce energy. However, pumping does not produce energy yet requires electricity. Therefore, pumping is usually done in the nighttime when the electricity demand and price is low, while in the daytime when the price is high, the hydropower system produces energy by releasing and turbinating the water (Hunt et al., 2022). Hence, our objective now is to maximize the profit of the hydropower system which is defined as the difference between the revenue from energy selling and the cost for pumping. Thus, we introduce a new variable p_t as the amount of water pumped back from basin 2 to basin 1 at time $t \in T$. Thus, the aim is to find the water release scheduling from both basin and also pumping scheduling with respect to balance constraints and the physical constraints of turbines, pumping pipe, and water storages. The robust formulation for the hydropower problem with two-basin hydropower system is written below.

4.1 Robust Model for Two Basins Hydropower Optimization Problem

Problem 4.1 (Robust model for Two-Basin Hydropower Problem)**.**

$$
\begin{aligned}
\max \quad & \sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 \, g\gamma \, h^1 \, o_t^1 + \eta^2 \, g\gamma \, h^2 \, o_t^2 - \\
& \frac{1}{\eta^p} \, g\gamma \, h^1 \, p_t) && (4.1)\\
s.t. \quad & f_t^{1\omega} + p_t + s_{t-1}^{1\omega} = o_t^1 + s_t^{1\omega} & \forall t \in T \omega \in \Omega \quad & (4.2)\\
& o_t^1 + f_t^{2\omega} + s_{t-1}^{\omega} = o_t^2 + p_t + s_t^{2\omega} & \forall t \in T, \omega \in \Omega \quad & (4.3)\\
& s_{|T|}^{i\omega} \geq s_0^{i\omega} & i = 1,2, \forall \omega \in \Omega \quad & (4.4)\\
& o_{min} \leq o_t^i \leq o_{max}^i & i = 1,2, \forall t \in T, \omega \in \Omega \quad & (4.5)\\
& p_{min} \leq p_t \leq p_{max} & i = 1,2, \forall t \in T, \omega \in \Omega \quad & (4.6)\\
& s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} & i = 1,2, \forall t \in T, \omega \in \Omega \quad & (4.7)
\end{aligned}
$$

The Equation (4.1) is the objective which aims to maximize the profit from the energy selling. This function is defined by multiplying the price and the total energy production where the total energy production is calculated by summing the total outflow water from both basins minus the total water pumped back from basin 2 to basin 1.

Equation (4.2) and (4.3) are the balance constraints for basin 1 and basin 2 respectively. These constraints differ from the balance constrain (3.15) in terms of it now depends on pumping and water released from both basins. Equation (4.2) describes the amount of water in basin 1 at time t as the amount of water in that basin at time $t-1$ plus the amount of water flowing to the basin plus the amount of water pumped back from downstream basin minus the amount of water released at time t. While Equation (4.3) defines the amount of water in basin 2 at time t is equal to the amount of water in the basin at the previous time plus the amount of water flows to the basin minus the amount of water released at that time.

Furthermore, constraint (4.4) ensures that the amount of water in the storage at the end of time horizon is at least the amount of water at the initial period. This constraint is needed to guarantee that the hydropower system has enough water to run for the next cycle. Moreover, constraints (4.5)-(4.7) are the boundary constraints for $o,\ p, and\, s$ accordingly which ensure that those variables must be within the some fixed numbers because of its physical limitation.

4.2 Affine Decision Rule Model for Two-Basin Hydropower Optimization Problem

As mentioned in the previous section, we want our policy to be able to adapt to any realization of the inflows. Therefore, we now model the decision variables o and p as affine functions of inflow water variables.

$$o_t^{i\omega} = \alpha_t^i + \sum_{t'=0}^{T} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=0}^{T} \beta_{tt'}^{i2} f_{t'}^{2\omega} \qquad i = 1, 2, \forall t \in T, \omega \in \Omega \tag{4.8}$$

$$p_t^{\omega} = \alpha_t^3 + \sum_{t'=0}^{T} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=0}^{T} \beta_{tt'}^{32} f_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{4.9}$$

To ensure that the decision variables represent the real implementable decisions that are non-anticipative, it is enough to force $\beta_{tt'} = 0, \forall t' \geq t$. This is because in reality, we never know the future. Hence, our decision variables must be non-anticipative, meaning that they cannot depend on the future.

$$o_t^{i\omega} = \alpha_t^i + \sum_{t'=0}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=0}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \qquad i = 1, 2, \forall t \in T, \omega \in \Omega \tag{4.10}$$

$$p_t^{\omega} = \alpha_t^3 + \sum_{t'=0}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=0}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{4.11}$$

In (4.10) and (4.11) both o and p variables depend on the history data of inflow from the beginning of the cycle to one time step before. This causes variables o and p to be dense and thus causes problems in running time. Therefore, we want to restrict these variables, so that the affine decision variables depend only on the history data of inflow from specific time window τ.

$$o_t^{i\omega} = \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \qquad i = 1, 2, \forall t \in T, \omega \in \Omega \tag{4.12}$$

$$p_t^{\omega} = \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{4.13}$$

Thenceforth, we transform all decision variables o and p in the models to affine decision variables defined in equation 4.12 and 4.13.

4.2.1 Risk-Averse for two-basin Hydropower Problem

Moreover, similarly to what we did in the previous section, we also consider two types of decision-making here, namely risk-averse and risk-neutral decision-making. As mentioned before, the difference between those two models lies in the objective functions. In risk-averse, the objective is to maximize the revenue under the worst-case scenario. From a Mathematical perspective, the objective for risk-averse can be written as:

$$\begin{aligned} \max \quad \min_{\omega \in \Omega} \quad & \sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 \, g\gamma \, h^1(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) \\ & + \eta^2 \, g\gamma \, h^2(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) \\ & - \frac{1}{\eta^p} \, g\gamma \, h^1(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega})) \end{aligned} \tag{4.14}$$

However, this objective includes the max and min functions which makes this function hard to solve. Therefore, with the epigraph reformulation as mentioned in the previous section, we will reformulate the objective into another equivalent function. For that purpose, we introduce a new variable ψ so that:

$$\max \quad \psi \tag{4.15}$$

$$\begin{aligned} \text{s.t} \quad \psi \leq \min_{\omega \in \Omega} \quad & \sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 \, g\gamma \, h^1(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) \\ & + \eta^2 \, g\gamma \, h^2(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) \\ & - \frac{1}{\eta^p} \, g\gamma \, h^1(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega})) \end{aligned} \tag{4.16}$$

This objective aims to maximize ψ where ψ is less than the minimum of the revenue for all $\omega \in \Omega$. Those equations still include min function. Accordingly, we need to rewrite the equations to omit the 'min' function. Thus, the objective in equation 4.15-4.16 can be rewritten as:

$$\max \quad \psi \tag{4.17}$$

$$\begin{aligned} \text{s.t} \quad & \psi - (\min_{\omega \in \Omega} \sum_{t=1}^{|T|} \pi_t \, \xi (\eta^1 \, g\gamma \, h^1 (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) \\ & + \eta^2 \, g\gamma \, h^2 (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) \\ & - \frac{1}{\eta^p} \, g\gamma \, h^1 (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}))) \le 0 \qquad \forall \omega \in \Omega \end{aligned} \tag{4.18}$$

Hence, the risk-averse two-basin hydropower model aims to find the optimal released water and pumping scheduling from both basins which maximize the aforementioned objective and satisfy some constraints:

Balance Constraints

$$\begin{aligned} & f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = \\ & \qquad (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} \quad \forall t \in T, \omega \in \Omega \end{aligned} \tag{4.19}$$

This constraint denotes the balance constraint for the upper basin. It explains that the amount of water in the basin at time t is defined as the total of amount of water inflow and pumped to the basin at that time plus the remaining water in the basin from time $t-1$ minus the water released from the basin.

While the balance constraint for the lower basin describes that the amount of water in the basin at time t is equal to the total amount of water inflow, the water released from the upper basin and the remaining water in the basin at time $t-1$ minus the water released and pumped at time t. Mathematically, it can be written as:

$$\begin{aligned} & (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + f_t^{2\omega} + s_{t-1}^{2\omega} = \\ & \qquad (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) \\ & \qquad + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_t^{2\omega} \quad \forall t \in T, \omega \in \Omega \end{aligned} \tag{4.20}$$

Boundaries

$$o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i \quad i = 1, 2, \forall t \in T, \omega \in \Omega \tag{4.21}$$

$$p_{min} \leq \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} \quad \forall t \in T, \omega \in \Omega \tag{4.22}$$

$$s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} \quad i = 1, 2, \forall t \in T, \omega \in \Omega. \tag{4.23}$$

Those three equations describe the physical limitations of the turbines, pumping pipe, and water storage. Those contraints denote the boundaries for variables o, p, and s respectively. It shows that the decision variables are bounded between some specific values.

Cyclostationary Constraint

$$s_{|T|}^{i\omega} \geq s_0^{i\omega}, \quad i = 1, 2, \forall \omega \in \Omega \tag{4.24}$$

This equation aims to manage the amount of water in each basin at the end of time horizon should not be less than the amount of water in the respective basin at the beginning of the period. This is to ensure that the system has enough water to run for the next cycle.

Furthermore, the complete mathematical model for risk-averse for two-basin hydropower problem can be seen in A.4.

4.2.2 Risk-Neutral for Two-Basin Hydropower Problem

This model differs from the previous model on the objective. In risk-neutral, the objective function is in the form of expectation. Here, we will maximize the revenue under the expectation. Therefore, the objective function for the risk-neutral is as follows.

$$\begin{aligned}
\max \sum_{\omega \in \Omega} (\frac{1}{Pr^\omega} \sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 \, g\gamma \, h^1(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + \\
\eta^2 \, g\gamma \, h^2(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) - \\
\frac{1}{\eta^p} \, g\gamma \, h^1(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega})))
\end{aligned} \tag{4.25}$$

where Pr^ω denotes the probability of scenario ω to be happened. While the constraints for the model are similar to the constraints in the risk-averse model. Therefore, the complete model for the risk-neutral for two-basin hydropower problem is shown in A.5.

4.3 Two Basin Hydropower Model with Spillage Variables

In this section, we consider spillage in our problem. The idea is that the hydropower system will be allowed to spill some water if the amount of water in the storage is above a certain point. The aim of spillage is to prevent flooding caused by excessive water in the storage because, for example the turbine is not working.

Feng et al. (2018) modeled the spill variable to be bounded by a specific minimum and maximum values. While Zambelli et al. (2011) modeled the spill variable to be at least 0 without maximum bound. Moreover, De Ladurantaye et al. (2009) modeled spill variable to be between 0 and a maximum value. Whereas Gauvin et al. (2017) defined that the maximum amount of water can be spilled is bounded by an evacuation curve function $C(t,\omega)$ which follow the specific structure of the basin. Evacuation curve function models the maximum amount of water allowed to be spilled at time t given the specific amount of water stored in the storage. This function is increasing, non-convex, and non-linear function. This non linearity complicates the problem even more. Therefore, he decide to approximate the function using an affine function to preserve the linear structure in the problem

Thus, we introduce a new variable $l_t^{i\omega}$, $\forall t \in T, \omega \in \Omega$, $i \in \{1,2\}$ which defines water spilled from reservoir i at time t and scenario ω. Moreover, we adopt the evacuation curve from Gauvin et al. (2017) which bounds the spillage variable.

$$C_i(t,\omega) = x_i^1 + y_i^1\, s_t^{i\omega}, \quad \forall t \in T, \omega \in \Omega, i \in 1,2, \tag{4.26}$$

where $x_i^1, y_i^1 \in \mathbb{R}$ Therefore, the boundary constraints for spill variables are:

$$l_t^{i\omega} \leq x_i^1 + y_i^1\, s_t^{i\omega}, \qquad \forall t \in T, \omega \in \omega, i \in \{1,2\}. \tag{4.27}$$

Accordingly, the amount of water in the storage now also depends on the amount of water spilled. Thus, the amount of water in the first basin at time t is equal to the total amount of water in the storage at time $t-1$ and the amount of water pumped back as well as the water flows to the basin at time t minus the amount of water released and spilled at time t.

$$\begin{aligned} f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = \\ (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} + l_t^{1\omega}, \forall t \in T, \omega \in \Omega. \end{aligned} \tag{4.28}$$

At the same time, the amount of water in the second basin at time t is equal to the total amount of water in the second basin at time $t-1$, and the amount of water released from the first basin, as well as the amount of water, flows to the basin at time t minus the amount of water released, pumped, and spilled from the basin at time t.

$$(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + f_t^{2\omega} + s_{t-1}^{2\omega} = (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_t^{2\omega} + l_t^{2\omega}, \quad \forall t \in T, \omega \in \Omega. \tag{4.29}$$

We then embed the Equation (4.27) to Problem (A.4) - (A.5) and adjust the balance constraints for the basins according to the aforesaid explanation. The complete model for hydropower problem with spill variable can be seen on A.6 for the risk-averse two-basin hydropower problem with spill variable and A.7 for the risk-neutral two-basin hydropower problem with spill variable.

4.4 Two Basin Hydropower Model with Variable Waterhead

In this section, we extend our model by considering variable waterhead. Unlike in previous sections where the height of the basin is assumed to be constant, here we consider the fact that a variety of waterhead affects the energy production.

The water-head, also known as the hydraulic variable, is defined as the difference between the headwater level in an upstream reservoir where water is released and turbinated. It is measured vertically in meters. Considering this variable is important as it explains the fact that water released from a higher level will produce more energy than the same amount of water released from the lower level.

Therefore, we approximate the waterhead using an affine function of the water storage.

$$h_t^{i\omega} = x_i^2 + y_i^2 s_t^{1\omega}, \quad \forall i \in \{1,2\}, t \in T, \omega \in \Omega, \tag{4.30}$$

where $h_t^{i\omega}$ is the water-head level at basin i at time t under scenario ω and $x_i, y_i \in \mathbb{R}$. Equataion 4.30 shows that the water-head at each time depends on the amount of water in the storage at that time. Therefore, the waterhead varies based on the amount of water stored in the basin. However, if you see on Figure 4.1, it is seen that Zermeiggern dam is relatively small compared to Mattmark dam. Therefore, on this study, we will conside waterhead variable on Mattmark dam and assume that the waterhead is constant in Zermeiggern dam.

Then the variables h_1 on (A.13) and (A.14) are changed to the variable waterhead defined on Equation 4.30. Therefore, the revenue for the hydropower is now defined as:

$$\sum_{t=1}^{|T|} \pi_t \, \xi(\left(x_1^2 + y_1^2 s_t^{1\omega}\right) \eta^1 \, g\gamma(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega}$$
$$+ \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + (\left(x_2^2 + y_2^2 s_t^{1\omega}\right) \eta^2 \, g\gamma(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega}$$
$$+ \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) - (\left(x_1^2 + y_1^2 s_t^{1\omega}\right) \frac{1}{\eta^p} \, g\gamma$$
$$(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega})) \qquad \forall \omega \in \Omega \quad (4.31)$$

Equation 4.31 is the revenue when the variable waterhead is considered. Multiplying $(x_1^2 + y_1^2 s_t^{\omega 1})$ with α_t^1 in the equation above leads to a multiplication between two variables, $s_t^{\omega 1}$ and α_t^1. This multiplication between two variables is usually called as a bilinear term. Because of this bilinear term, the Equation 4.31 becomes non-linear and thus, makes the overall problem no longer linear.

Then, for the hydropower model with waterhead variable, we adjust the revenue formula in (A.13) for risk-averse problem and (A.14) for risk-neutral problem to $h_t^{i\omega}$ in equation (4.30) to Equation 4.31. Therefore, the complete formulation of the Risk-Averse two-basin hydropower problem with variable waterhead can be seen on A.9, while for Risk-Neutral can be seen on A.10.

	Model	Variables	Constraints
Robust Model for Two Basin Hydropower Optimization Problem (4.1)	Linear	$f_t^{i\omega}$ o_t^i p_t $s_t^{i\omega}$	Balance Constraints Cyclostationery Constraints Boundary Constraints
Affine Decision Rule Model for Two Basins Hydropower Optimization Problem (4.2)	Linear	$f_t^{i\omega}$ $o_t^{i\omega}$ p_t^{ω} $s_t^{i\omega}$	Balance Constraints Cyclostationery Constraints Boundary Constraints
Two Basin Hydropower Model with Spillage Variables (4.3)	Linear	$f_t^{i\omega}$ o_t^i p_t $s_t^{i\omega}$ $l_t^{i\omega}$	Balance Constraints Cyclostationery Constraints Boundary Constraints Spillage Boundary
Two Basin Hydropower Model with Variable waterhead (4.4)	Non Linear	$f_t^{i\omega}$ o_t^i p_t $s_t^{i\omega}$ $h_t^{i\omega}$	Balance Constraints Cyclostationery Constraints Boundary Constraints

TABLE 4.1: The Comparison between the Models

4.5 Rolling Horizon

Adding Equation (4.30) to the problem makes the problem non-linear. If we look at (A.29) and (A.30), those equations contain bilinear term wich is the multiplication of storage variables and α as well as β variable. Furthermore, we test these problems into scheduling a one-year hydropower production based on 4 hourly schedule. The problem is quite large and takes very long time to run. Therefore, we apply a rolling horizon algorithm to solve the problem.

We propose two rolling-horizon algorithms which we call as simple rolling horizon (SRH) algorithm and dynamic rolling-horizon (DRH) algorithm.

4.5.1 Simple Rolling Horizon (SRH) Algorithm

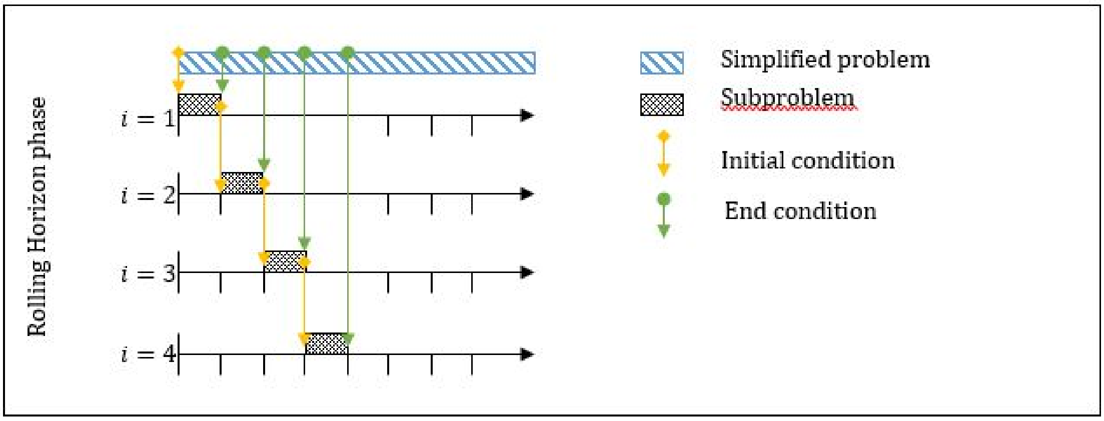


FIGURE 4.2: The Diagram of Simple Rolling-Horizon (SRH) Algorithm

The rolling-horizon algorithm process we propose here can be seen in Figure 4.2. This algorithm consists of two main processes, the Tracking phase and the Optimization phase. Let T_i as the time-window for the subproblem i

Tracking Phase: During this phase, the simplified model is solved and its result is tracked. The simplified model used is Problem A.4 for risk-averse problem and Problem A.5 for risk-neutral problem. Let $s^* = \left(s^*_1, ..., s^*_{|T|}\right)$ be the storage variables obtained from solving simplified model.

Optimization Phase: In this phase, Problem (A.9) for risk-averse and (A.10) for risk-neutral is solved iteratively under the smaller time window.

First Iteration: For the first iteration, the initial and end condition for the subproblem is obtained from the result of the simplified problem. Set $s^1_{init} = s^*_0$ and $s^1_{end} = s^*_{T_1}$ These conditions are essential since if we do not fix the end condition to some values, the algorithm tends to release maximum amount of water by leaving a small amount of water or even empty storage. This should be avoided because we need to ensure that the hydropower system always has enough water to run the production. Let $s'^i = \left(s'^i_1, ..., s'^i_{T_1}\right)$ be the storage variables obtained from solving the variable waterhead model.

Next Iterations: From the second iteration onward, the initial condition is obtained from the last condition of the previous subproblem and the end condition is obtained from the simplified problem. Let $T_i = (T_{i-1} + 1, ..., T_i)$ be the time window of the subproblem on iteration i. Set $s^i_{init} = s'^{i-1}_{T_i}$ and $s^i_{end} = s*^{i-1}_{T_i}$ and solve the model in Problem A.9 or A.10 under those conditions.

We drop constraints (A.45) and (A.54) and change them with the end condition explained above. This algorithm shares similarities with shrinking and reciding horizon approach proposed by Wakui et al. (2022). Our algorithm differs from that proposed by Wakui et al. (2022) in terms of the final horizon. Let us assume that T is the whole horizon and it is divided into several time windows for the subproblems $[t_0, t_0 + K_1], [t_0 + K_1, t_0 + K_2], ..., [t_0 + K_{n-1}, t_0 + K_n]$ where $K_n \geq T$. Wakui consider this condition to ensure the cyclostationarity of the process in the model and use the corresponding time for $[T, t_0 + K_n]$.

Moreover, in SRH algorihtm proposed here, the simplified model solved in initialization phase has to satisfies the initial/final condition stated in Equation (A.9) or Equation (A.17). The purpose of this constraints is to ensure that at the end of period, the amount of water in the storage is at least the amount of water at the beginning period so that the hydropower system has enough water to continue to the next cycle (cyclostationary process). Hence, fixing the end condition of the storage at each time window to be equal to the amount of water in storage at the corresponding time obtained from simplified model is sufficient to ensure the cyclostationary process.

4.5.2 Dynamic Rolling Horizon (DRH) Algorithm

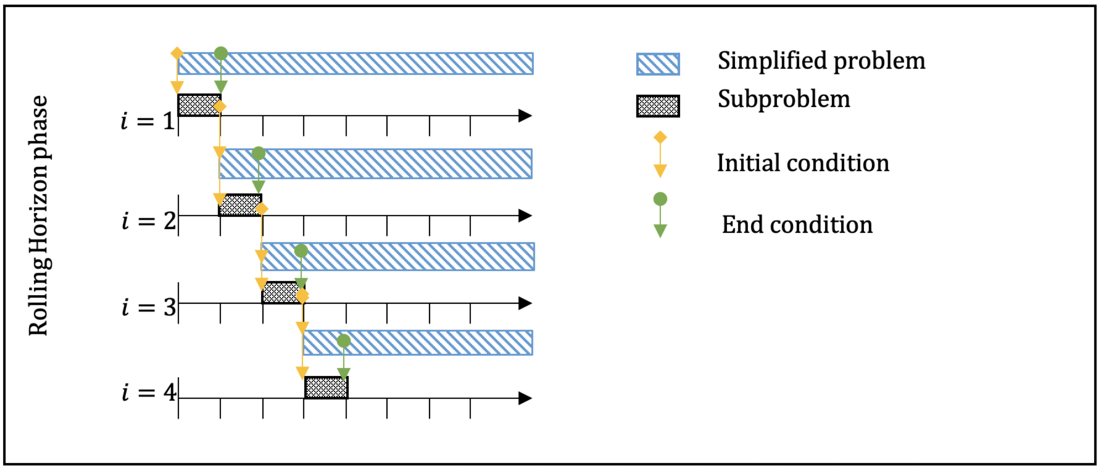


FIGURE 4.3: The Diagram of Rolling-Horizon Algorithm

Algorithm 2: Pseudocode for Simple Rolling Horizon (SRH)

Tracking Phase;
Data: $f_t^{\omega}, \pi_t, \forall \omega \in \Omega_{ts}, t \in T, o_{min}, o_{max}, s_{min}, s_{max}$
Optimize A.4 for Risk-Averse and A.5 for Risk-Neutral
Result: $s_t, \alpha_t, \beta_{t,t'}, \forall t \in T, t' \in (t-\tau, t-1)$
Optimization Phase;
Split T into n subsets so that
$T_1 = \left(T_0, T_{\frac{|T|}{n}}\right), \quad T_2 = \left(T_{\frac{|T|}{n}-1}, T_{2\frac{|T|}{n}}\right), ..., \quad T_n = \left(T_{(n-1)\frac{|T|}{n}-1}, T_{|T|}\right)$
Data: $f_t^{\omega}, \pi_t, \forall \omega \in \Omega_{ts}, t \in T, o_{min}, o_{max}, s_{min}, s_{max}, s^{i*} = \left(s_1^{i*}, s_2^{i*}, ..., s_{|T|}^{i*}\right)$ from the tracking phase for $i = 1, 2$
for *k in range n* **do**
  if $k = 1$ **then**
    Set $s^i_{|T_1|} = s^{i*}_{|T_1|}$
    Optimize A.9 for Risk-Averse and A.10 for Risk-Neutral in range $(0, T-1)$
    Result: $s_t^{i'}, \alpha_t, \beta_{t,t'}, \forall t \in T_1, t' \in (t-\tau, t-1), i = 1, 2$
  else
    Set:
    $s_0^i = si'_{T_{k-1}}$
    $s^i_{|T_k|} = s^{i*}_{|T_k|}$
    Optimize A.9 for Risk-Averse and A.10 for Risk-Neutral in range (T_{k-1}, T_k)
    Result: $s_t^{i'}, \alpha_t, \beta_{t,t'}, \forall t \in T_k, t' \in (t-\tau, t-1), i = 1, 2$
  end
end

This algorithm is basically the extension of the previous algorithm, however the result from simplified model is updated at each iteration. The algorithm consists of two main processes at each iteration which are Tracking phase, and Optimization phase. Let T be the whole horizon and it is divided into n time window where T_i be time window for the subproblem at iteration $i = \{1, .., n\}$.

i=1 **Tracking phase:** In this phase, the simplified model Problem (A.4) or (A.5) is solved for the whole horizon T. Let s^* be the storage variables obtained from the result of the simplified model

Optimization phase: Set $s_{init} = s_0^*$ and $s_{end} = s^*_{T_1}$. Then, the constraint in Equation (A.9) or (A.17) is changed to $s^{i\omega}_{|T|} = s^{i\omega}_{end}$ and the non-linear model with variable waterhead in Problem (A.9) or (A.10) is solved with those conditions. Let s' be the storage variables obtained from the result in this phase.

i=2,...,n **Tracking phase:** the simplified model Problem (A.4) or (A.5) is solved for the horizon $(T_{i-1}+1, ..., T)$ and $s_{init} = s'_{T_{i-1}}$

Optimization phase: Set $s_{init} = s'_{T_{i-1}}$ and $s_{end} = s^*_{T_i}$. Then, the constraint in Equation A.9 or A.17 is changed to $s^{i\omega}_{|}T| = s^{i\omega}_{end}$ and the non-linear model with variable waterhead in Problem A.9 or A.10 is solved with those conditions.

Algorithm 3: Pseudocode for Dynamic Rolling Horizon (DRH)

Split T into n subsets so that

$$T_1 = \left(T_0, T_{\frac{|T|}{n}}\right), \quad T_2 = \left(T_{\frac{|T|}{n}-1}, T_{2\frac{|T|}{n}}\right), ..., \quad T_n = \left(T_{(n-1)\frac{|T|}{n}-1}, T_{|T|}\right)$$

for *k in range n* **do**
- **if** $k = 1$ **then**
  - Tracking Phase;
  - Optimize A.4 for Risk-Averse and A.5 for Risk-Neutral in range $(0, T)$
  - **Result:** $s_t^{i*}, \alpha_t, \beta_{t,t'}, \forall t \in T, t' \in (t-\tau, t-1), i = 1, 2$
  - Otimization Phase;
  - **Data:** $f_t^{\omega}, \pi_t, \forall \omega \in \Omega_{ts}, t \in T, o_{min}, o_{max}, s_{min}, s_{max}, s^{i*} = \left(s_1^{i*}, s_2^{i*}, ..., s_{|T|}^{i*}\right)$ from the tracking phase, for $i = 1, 2$
  - Set:
  - $s_{|T_1|}^{i} = s_{|T_1|}^{i*}$
  - Optimize A.9 for Risk-Averse and A.10 for Risk-Neutral
  - **Result:** $s_t^{i'}, \alpha_t, \beta_{t,t'}, \forall t \in T_1, t' \in (t-\tau, t-1), i = 1, 2$
- **else**
  - Tracking Phase;
  - Set:
  - $s_0^{i} = s_{|T_{k-1}|}^{i'}$
  - Optimize A.4 for Risk-Averse and A.5 for Risk-Neutral in range (T_{k-1}, T)
  - **Result:** $s_t^{i*}, \alpha_t, \beta_{t,t'}, \forall t \in T, t' \in (t-\tau, t-1), i = 1, 2$
  - Optimization Phase;
  - Set:
  - $s_0^{i} = s_{T_{k-1}}^{i'}$
  - $s_{|T_k|}^{i} = s_{|T_k|}^{i*}$
  - Optimize A.9 for Risk-Averse and A.10 for Risk-Neutral
  - **Result:** $s_t^{i'}, \alpha_t, \beta_{t,t'}, \forall t \in T_k, t' \in (t-\tau, t-1), i = 1, 2$
- **end**

end

4.6 Numerical Result and Discussion

We test the models to the Switzerland Mattmark Hydropower system which can be seen on Figure 4.1. This hydropower system consists of 2 basins, and one pumping system. The upper basin, which is Mattmark dam, has maximum capacity of storage around $10^8 \quad m^3$ and its associated power plant, the Zeirmeggern power plant, is located just below the Mattmark dam and has the hydraulic head between 370 and 460 m.The downstream basin which is Zeirmeggern dam has the storage capacity around 100,000 m^3 and its associated power plant, Stalden power plant, is located at the end of the valley and has a hydraulic head close to 1,000 m (Anghileri, Castelletti et al., 2018).

We apply the models to scheduling those hydropower system over a one-year period. The input data consist of inflow data and the electricity data. The model parameters for the hydropower reservoirs and plants were set to values available in the literature

(Anghileri, Castelletti et al., 2018, Anghileri, Botter et al., 2018) and on the website https://www.kraftwerkemattmarkag.ch/anlagen/. Reasonable assumptions whenever information was not publicly available. The inflow data is the data of inflow on two basins on hourly schedule. There are 20 different scenarios of inflow and we use the first 14 scenarios as training set and the rest as the validation set. The inflow scenarios used in the training and validation set were stochastically generated using the model published in Anghileri, Castelletti et al. (2018).

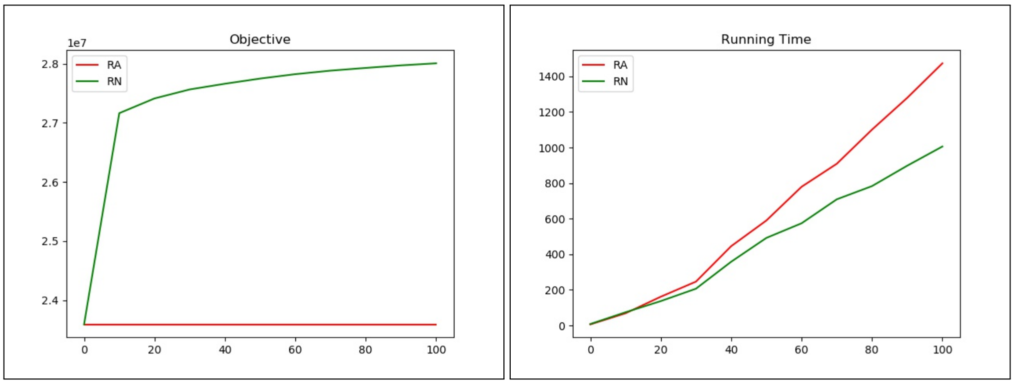
We set the initial water storage on the Mattmark dam as 70,000,000 m^3 and on Zermeiggern dam as 50,066 m^3. For Problem A.4 - A.7, where we assume that the water-head variable is constant, we fix the height for Mattmark power plant as 400 m and as 980 m for Zermeiggern power plant. We gradually run the problem with τ from 0 to 100. The result obtained then is applied to the validation set. Validation is aimed to see the performance of the result obtained with it is applied to the data outside the data used for optimization (training set). It keeps tracking of how many violations made. The process of validation can be seen on A.8.

4.6.1 Robust-Affine Model

In this section, we present the results from the aforementioned data to the model shown in Problem A.4 and A.5. To compare the result obtained from optimizing RA (A.4) and RN (A.5) of two-basin hydropower system, one can look at Figure 4.4.

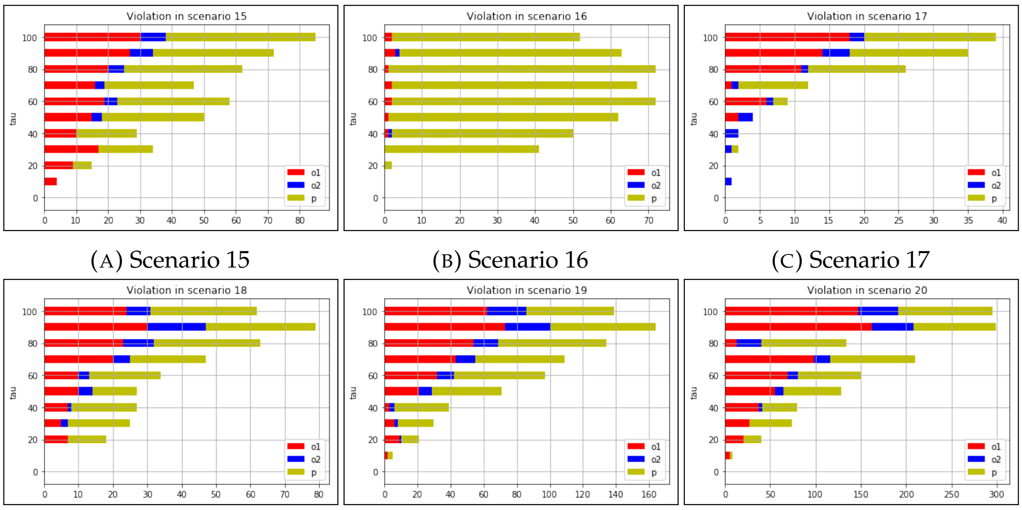
From Figure 4.4a, it can be seen that the objectives of RA remain the same throughout τ. On the other hand, the objective of RN increases in parallel with the increasing in τ. However, the increment on the objective after $\tau = 10$ is not as sharp as before $\tau = 10$.

Regarding the running time, both RA and RN experience a sharp rise along the τ. Furthermore, we can expect that the running time in RA is higher than in RN because RA has one more constraint for each scenario. Therefore, there are 14 more constraints in RA than in RN and it affects the running time.



(A) Objectives (B) Running Time

FIGURE 4.4: Comparison between RA (A.4) and RN (A.5)



(A) Scenario 15 (B) Scenario 16 (C) Scenario 17

(D) Scenario 18 (E) Scenario 19 (F) Scenario 20

FIGURE 4.5: Violation on RA

The result obtained from the optimization, then is applied to the data in the validation set. Furthermore, the number of violations when the result is applied to the validation set is calculated. If the variables violate the maximum bound, then we fix the variables to be equal the maximum bound for practical perspective reasons.

Figure 4.5 shows the number of violations for each scenario in the validation set. As can be seen in the Figure that there is no violation in the storage for all scenarios in all τ. Also, the number of violations increases when τ is increased.

Figures 4.6 - 4.11 show the results from the RA Model. The first two illustrate the result in the training set, while the rest present the application of the result into the validation set. For each graph, there are 6 different plots. The first three plots show the amount of water released from the first basin, from the second basin and the water pumped back each time and across all scenarios in the training set. The next two plots show the amount of water remaining in the first basin and in the second basin, while the last plot illustratess the price of electricity at each point in time.

Figure 4.12c plots the revenue on the validation set for each scenario over each τ from 0 to 100. We can see that from $\tau = 0$ to $\tau = 10$ the revenue for each scenario in the validation set increases. This trend continues until $\tau = 40$ except in scenario 16. This scenario produces the lowest revenue compared to the other scenarios and it starts to decrease from $\tau = 10$, then decreases more rapidly from $\tau = 40$ onwards.

Figure 4.12a and 4.12b show the statistics of the revenue of RA in the training set and validation set respectively. The 'Minimum', 'Average', and 'Maximum' describe the minimum, average, and revenue for all scenarios in the related set, namely training or validation. While the objective shows the objective value obtained from the optimization process.

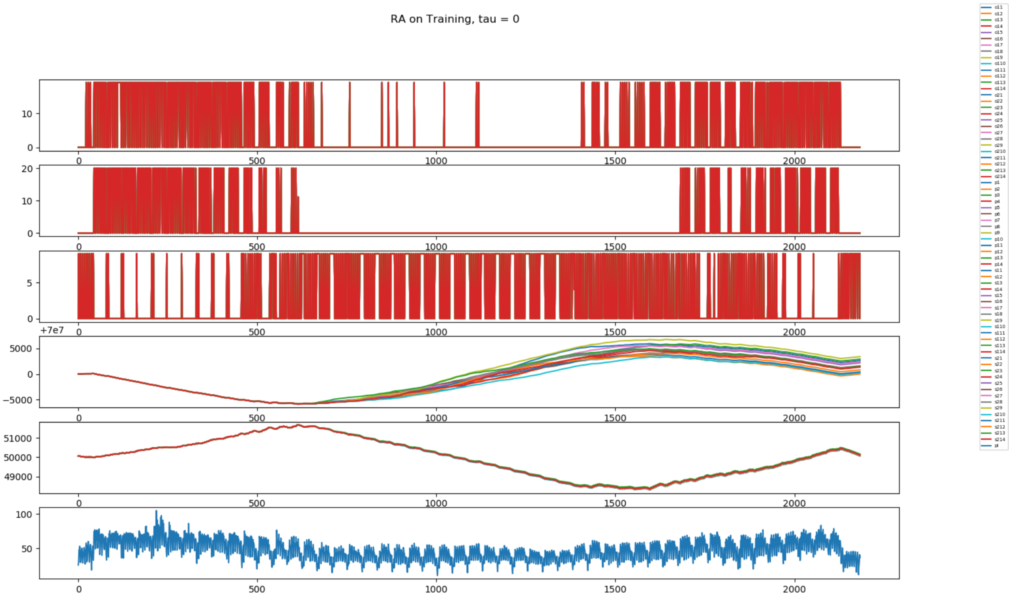


FIGURE 4.6: RA $\tau = 0$ in Training Set

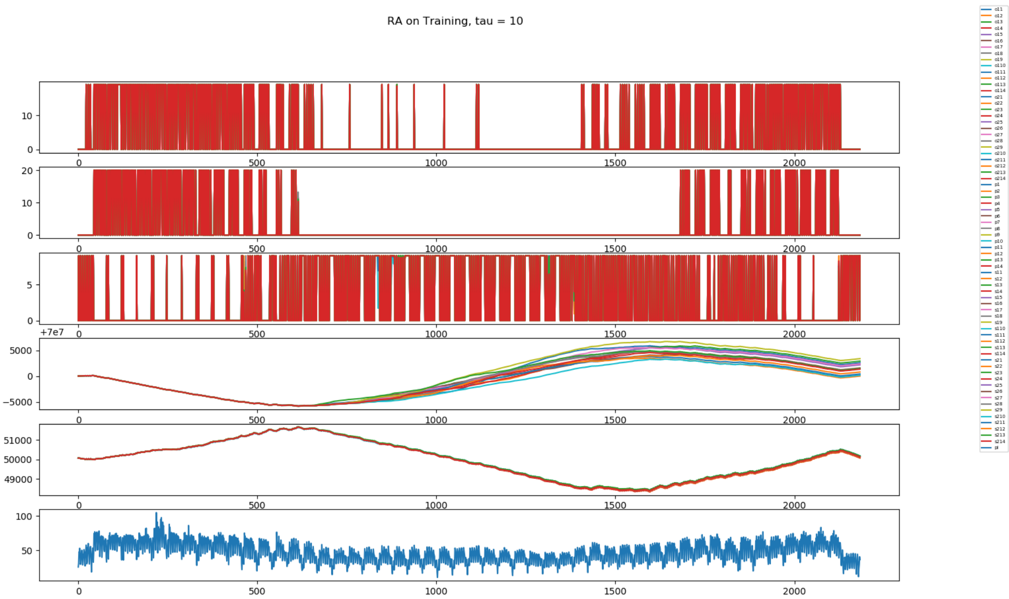


FIGURE 4.7: RA $\tau = 10$ in Training Set

In Figure 4.12a, the objective found is equal to the minimum revenue in the training set. This can be explained because in the RA model, we are looking for the solution for the worst-case scenario. If we look at both in Training and in validation set, in average, the revenue increase until $\tau = 40$ from which point it begins to fall.

If we refer to the violation pictured in Figure 4.5, applying $\tau = 0$ may be good in terms of there is no violation on it. However, if we refer to Figure 4.12c and 4.12b, it shows that fixing the τ to be zero may lead to missing a chance to get better profit. As seen in

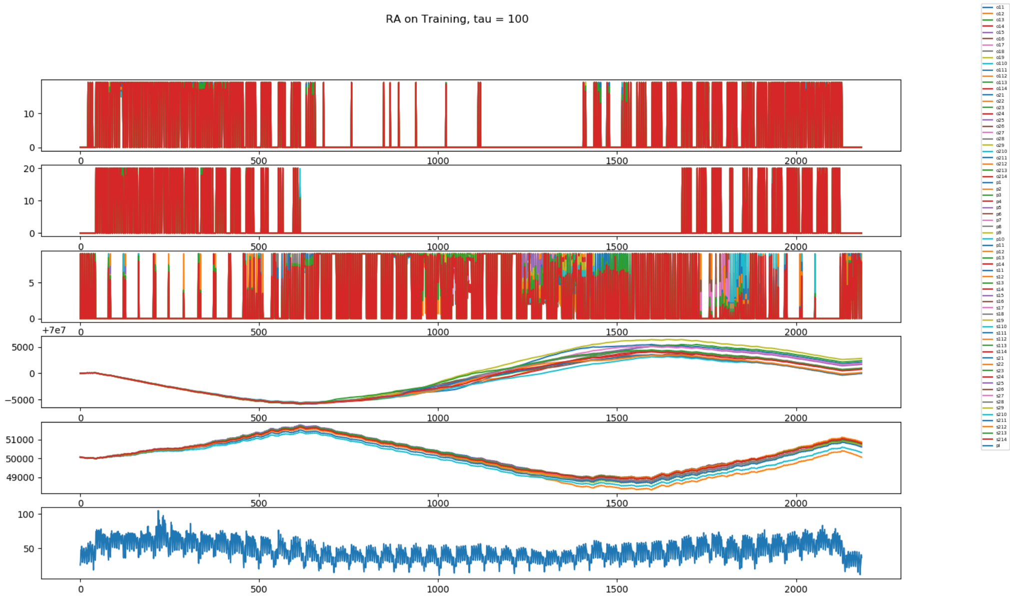


FIGURE 4.8: RA $\tau = 100$ in Training Set

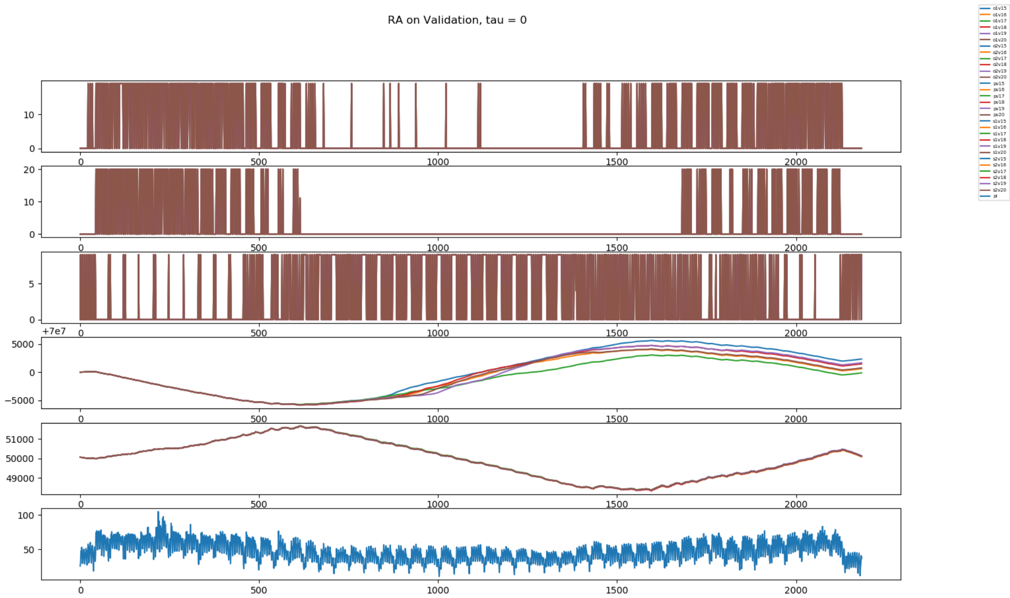


FIGURE 4.9: RA $\tau = 0$ in Validation Set

that figure that we can gain better revenue for fixing τ between 10 to 40.

Compared with RA, RN does better performance. Refer to Figure 4.4, RN produces better objectives with less running time. Figure 4.13 below shows the violations for each scenario when the result is applied to the validation set. It can be seen that there are no violations in the storage variable. The violations only occurs on the decision variables. It seems that the decision variables tend to violate the maximum bounds and it can be fixed by force them to be equal to the maximum amount allowed.

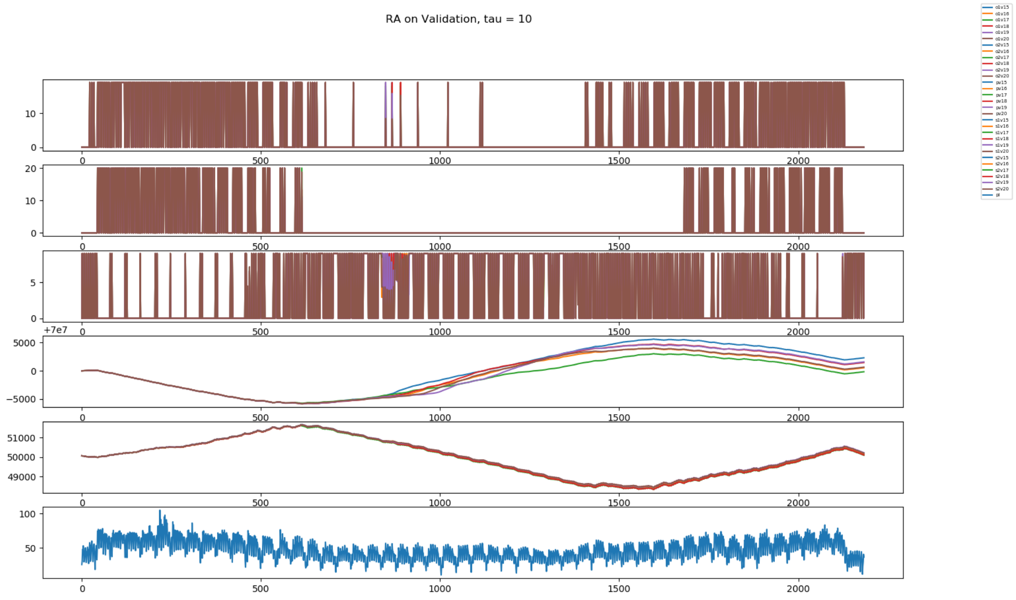


FIGURE 4.10: RA $\tau = 10$ in Validation Set

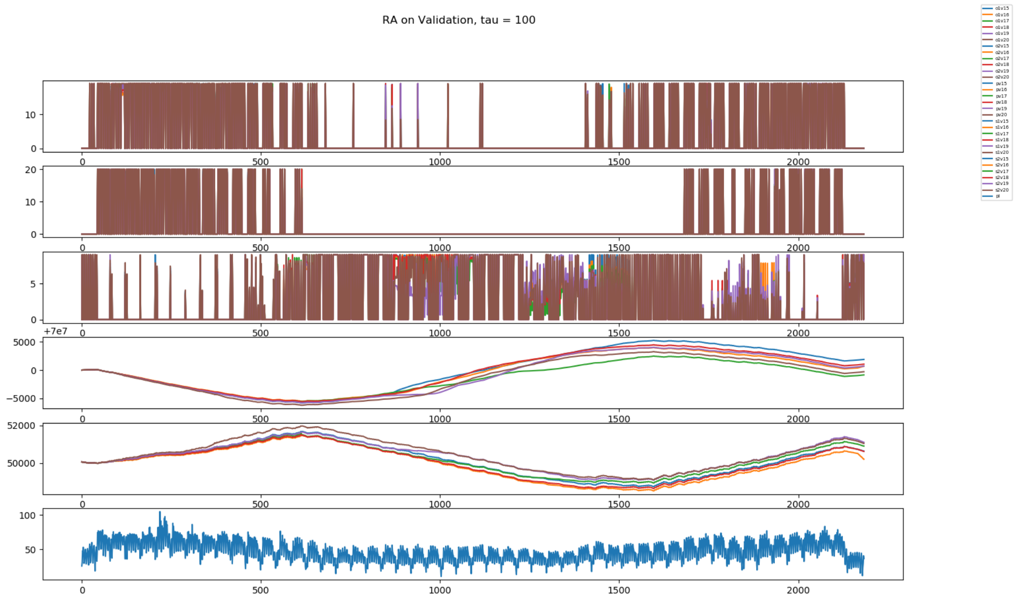
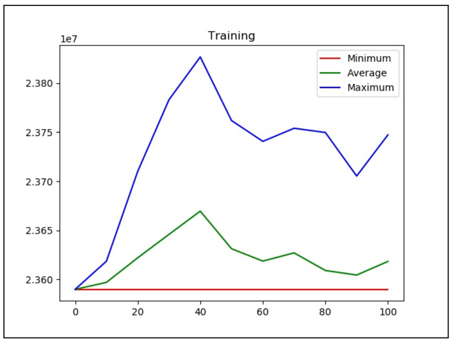


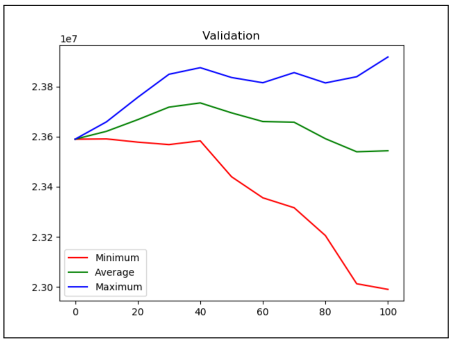
FIGURE 4.11: RA $\tau = 100$ in Validation Set

The violations rise along with the increase in τ. We suspect that the model suffers from an overfitting phenomenon. Therefore, in the next section, we will apply regularization to prevent overfitting and obtain a better performance on the validation set.

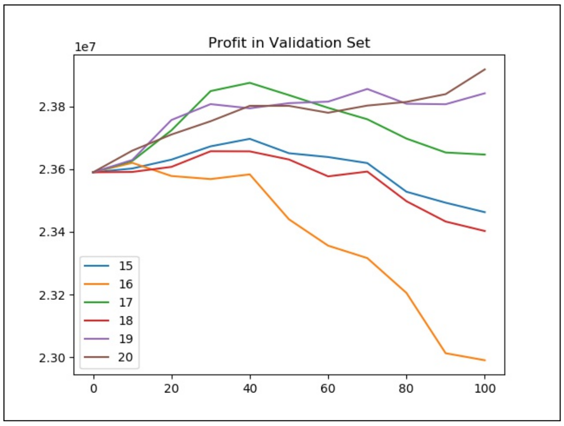
Looking closer at Figure 4.16, it shows that the storage variables is convergent at the end of the horizon. RN tried to force the storage variable for each scenario to be equal to the initial amount of storage. This is very beneficial in practice since this turns the problem into a cyclostationery process. It can guarantee that the result obtained for this



(A) Statistics of RA in The Training Set

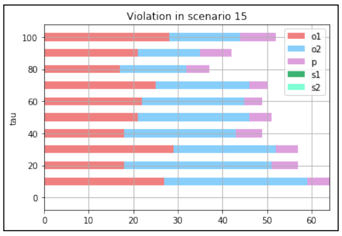


(B) Statistics of RA in The Validation Set

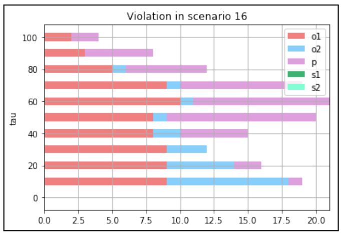


(C) Revenue of RA Model for each scenario in The Validation Set

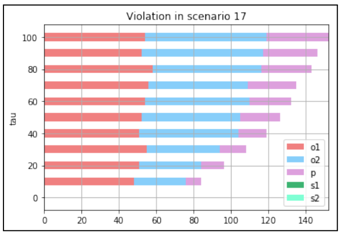
FIGURE 4.12: Performance of RA



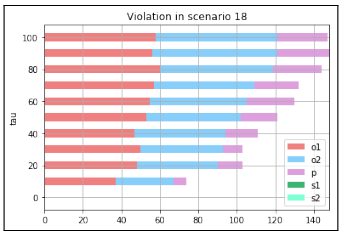
(A) Scenario 15



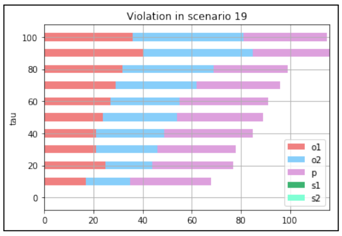
(B) Scenario 16



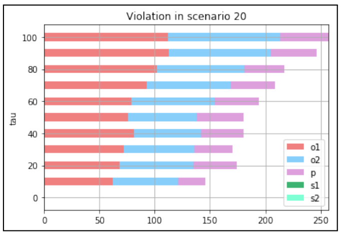
(C) Scenario 17



(D) Scenario 18



(E) Scenario 19



(F) Scenario 20

FIGURE 4.13: Violations for RN Model

horizon can be mimicked to the next time horizon as the storage amount at the end of this horizon means the storage amount at the start of the next horizon.

If we analyse the performance of RN by looking at Figure 4.20, it is evident that the

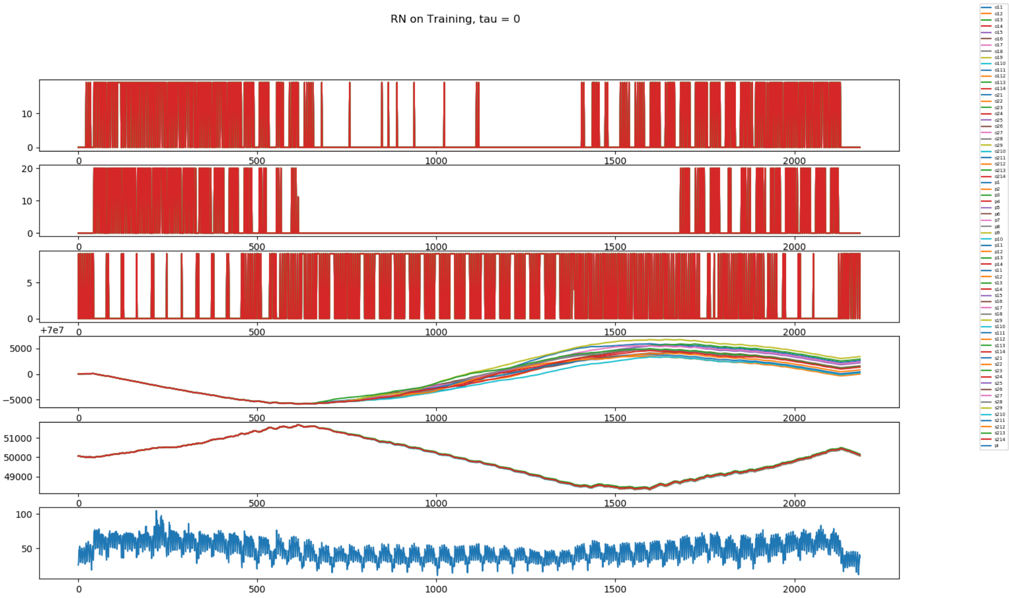


FIGURE 4.14: RN $\tau = 0$ in the Training Set

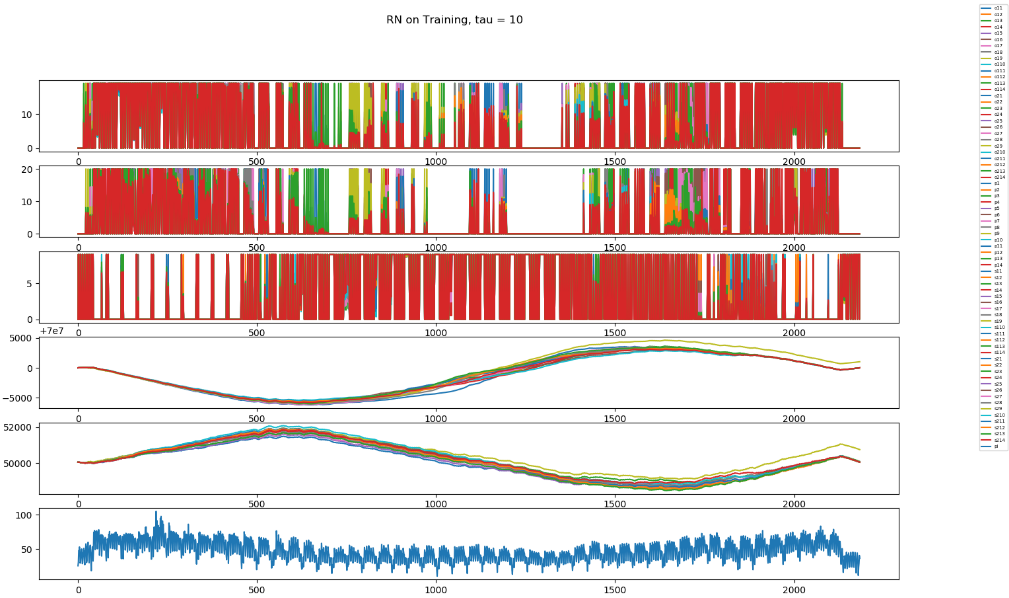


FIGURE 4.15: RN $\tau = 10$ in the Training Set

objective value achieved from the optimization process is equal to the average revenue for all scenarios in the training set. This is because in the RN model, we optimize under the expectation.

Moreover, the revenue gained by applying the RN result to the scenarios in the validation set increased from $\tau = 0$. On average, revenue increases before it starts to decrease at $\tau = 100$.

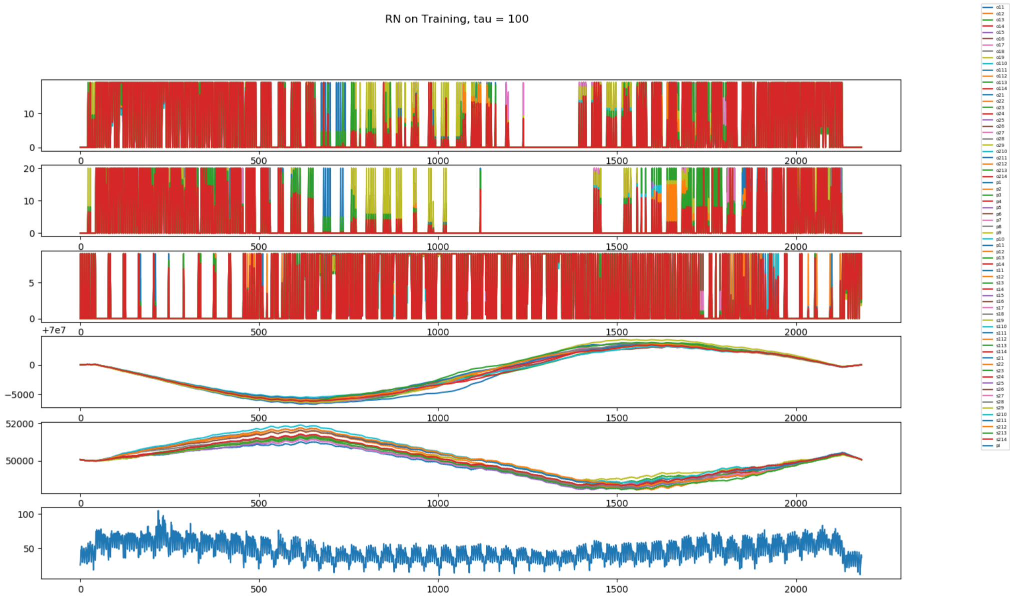


FIGURE 4.16: RN $\tau = 100$ in the Training Set

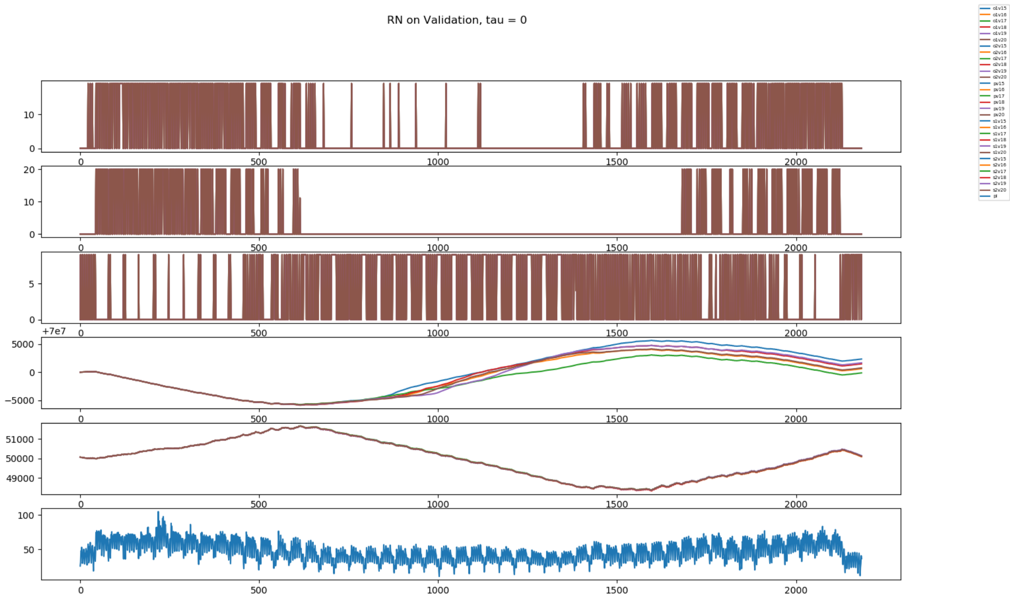


FIGURE 4.17: RN $\tau = 0$ in the Validation Set

4.6.2 Two-Basin Hydropower Model with Spill Variable

This problem is an extension of the previous problem. In this problem, a spill variable is added to the model. The idea of the spill variable is to allow some water to be spilled when the amount of water in the storage reaches some fixed point.

Gauvin et al. (2017) showed that the spill variable is bounded by a function called as

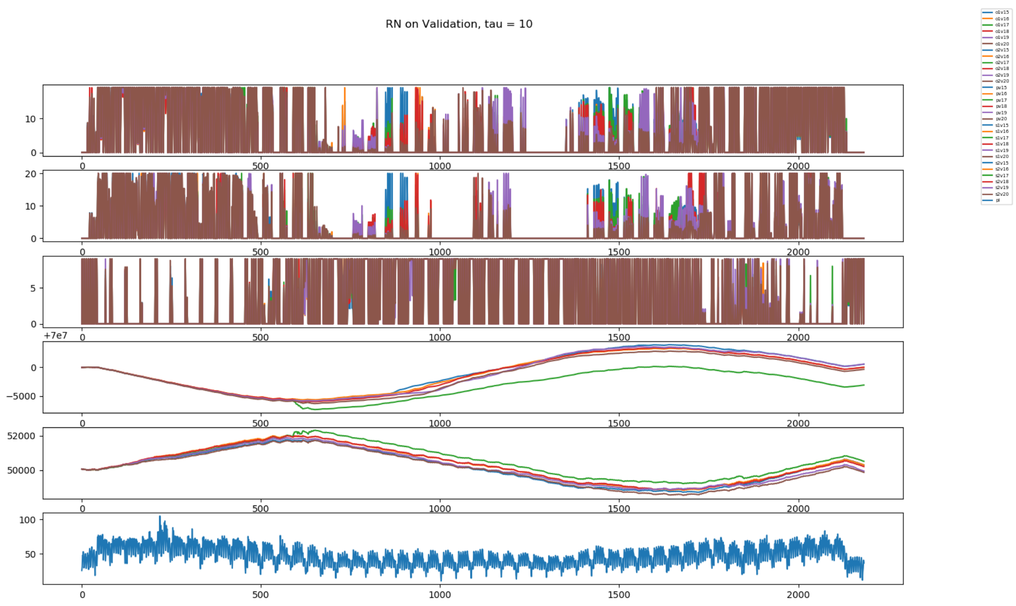


FIGURE 4.18: RN $\tau = 10$ in the Validation Set

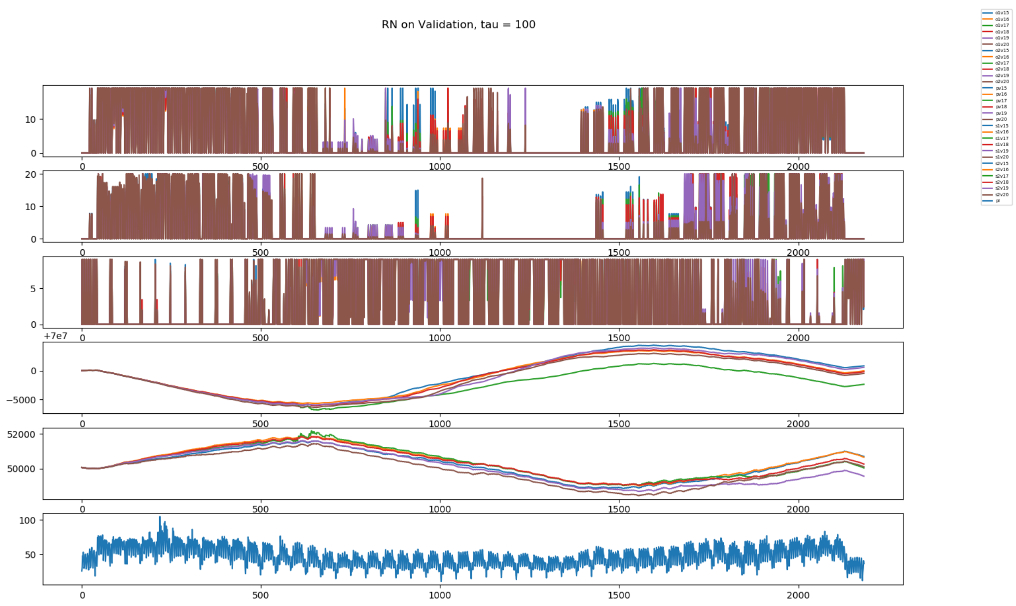
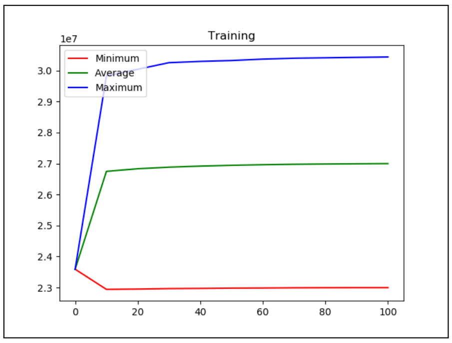


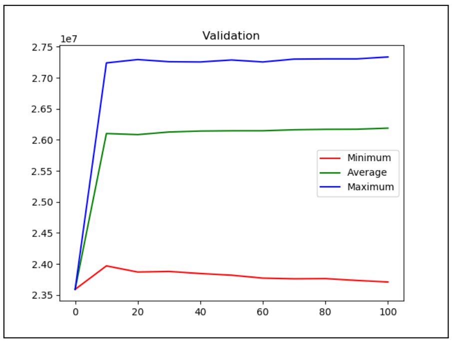
FIGURE 4.19: RN $\tau = 100$ in the Validation Set

Evacuation curve. This function is generally smooth, non-linear, non-convex, and increasing. Therefore, to preserve the linearity in the model, we approximate the curve using an affine function. Thus, this evacuation curve is a function of the amount of water in the storage.

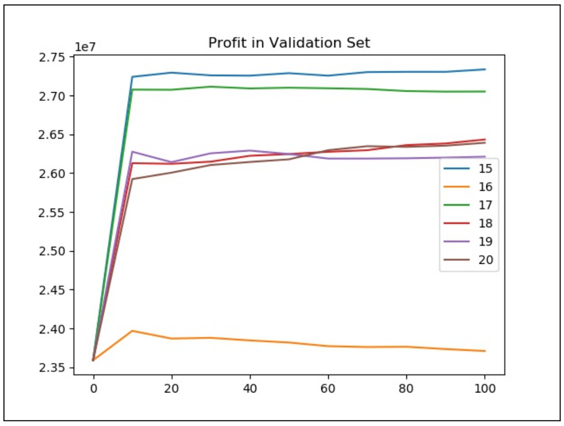
$$C\left(t,\omega\right) = x_i + y_i s_t^{i\omega}, \qquad \forall \omega \in \omega, t \in T, i \in \{1,2\} \tag{4.32}$$



(A) Statistics of RN in the Training Set



(B) Statistics of RN in the Validation Set



(C) Revenue of RN Model for each scenario in the Validation Set

FIGURE 4.20: Performance of RN

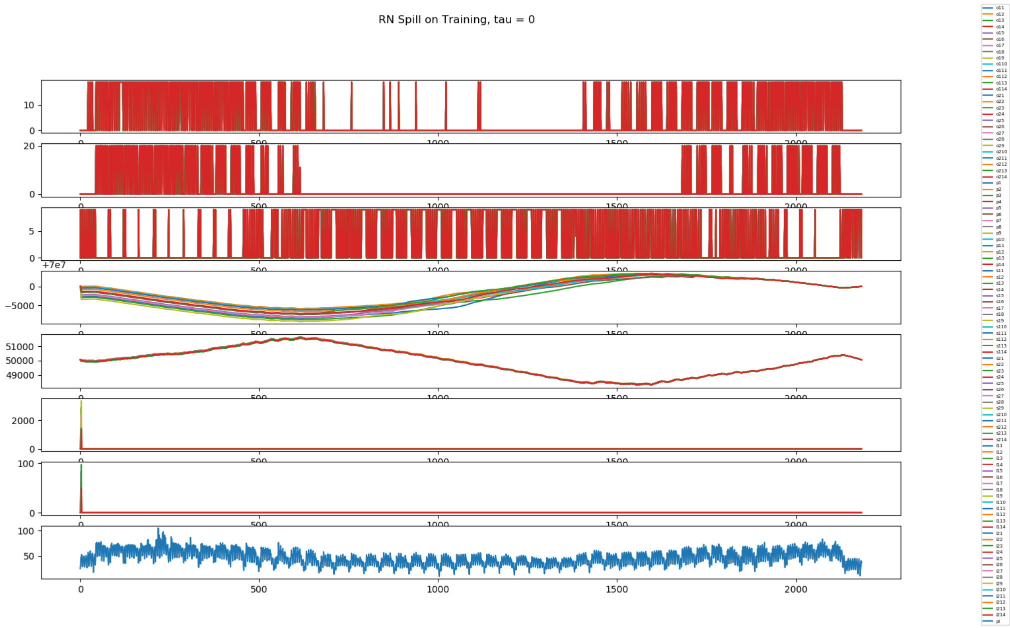


FIGURE 4.21: RN with Spill Variable $\tau = 0$ in the Training Set

Figure 4.22 - 4.24 show the performance of RN with the spill variable where the first

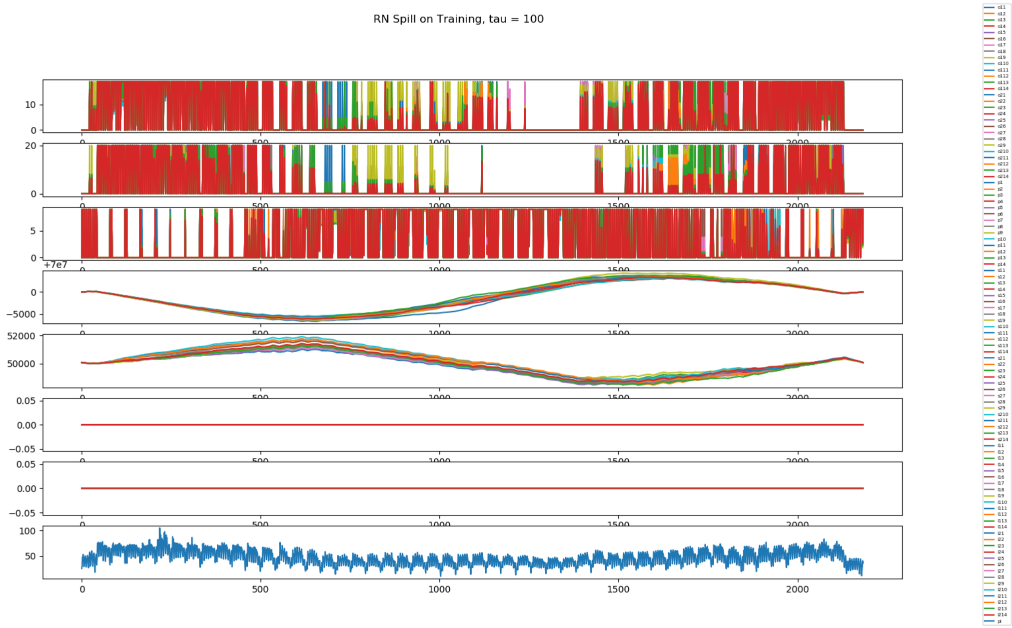


FIGURE 4.22: RN with Spill Variable $\tau = 100$ in the Training Set

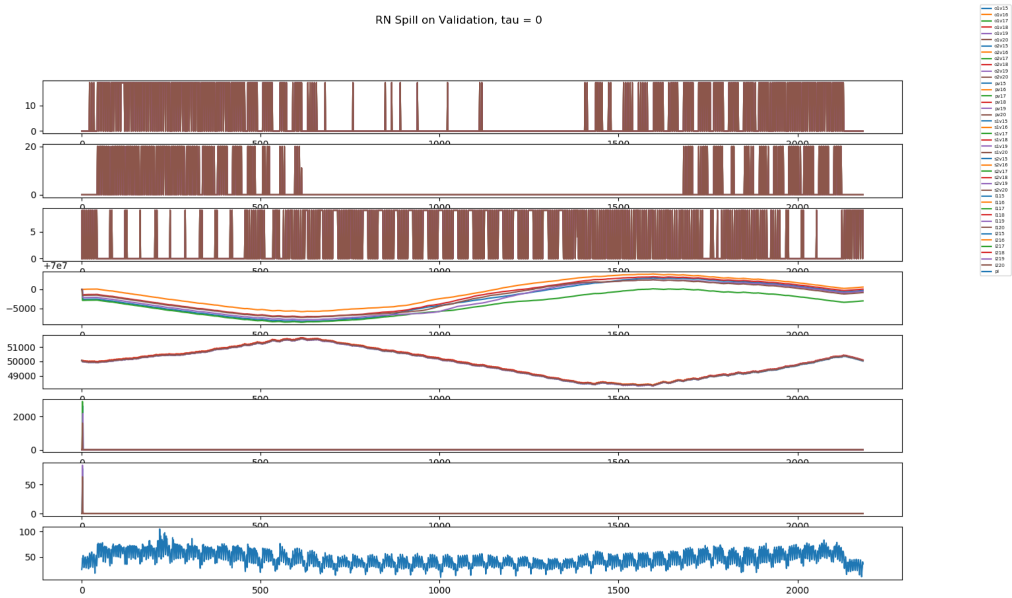


FIGURE 4.23: RN with Spill Variable $\tau = 0$ in the Validation Set

two graphs are the result in the training set, while the remaining is in the validation set. There are 8 plots in each figure. The first three graphs plot the amount of water released from basin 1, basin 2, and water pumped back to the first basin at each time t for all scenarios in the related set. The next two draw the amount of water in basin 1 and 2 respectively, while the next two plots present the amount of water spilled from the related basin. Then, the last plot shows the electricity price over time.

The fifth and sixth plot in Figure 4.22 indicate that there is no water spilled. If the

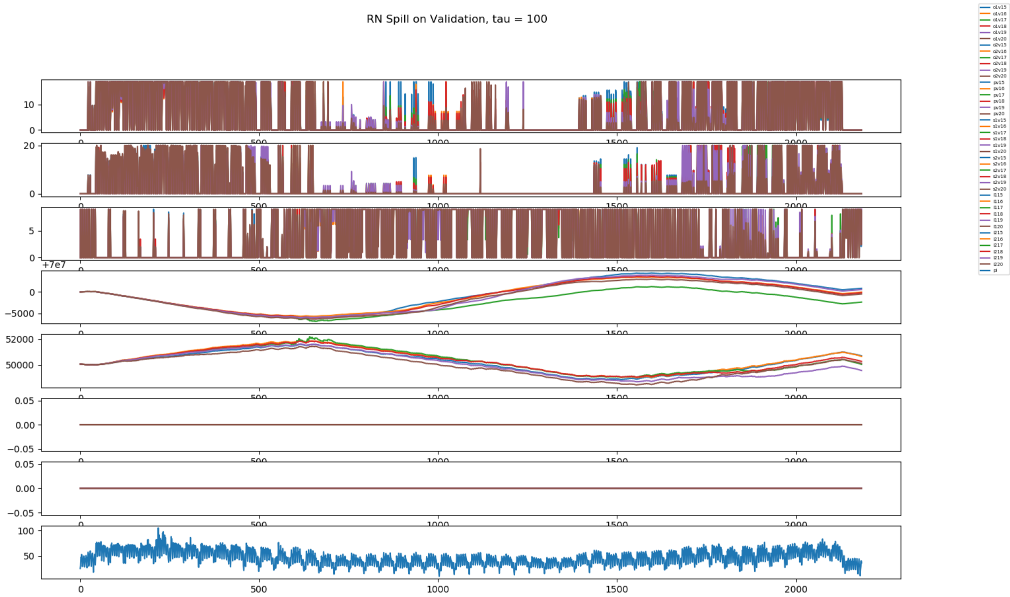
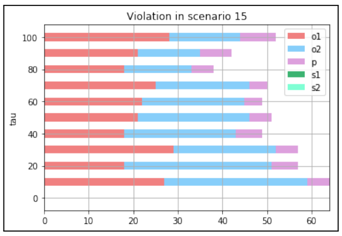
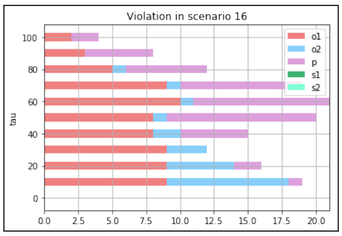


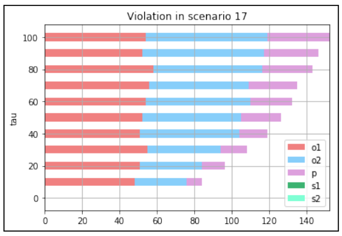
FIGURE 4.24: RN with Spill Variable $\tau = 100$ in the Validation Set



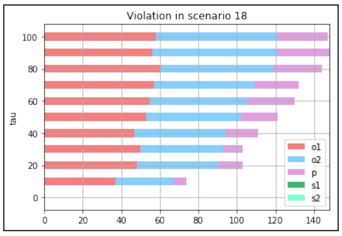
(A) Scenario 15



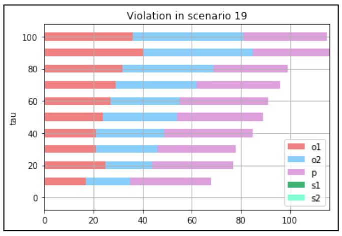
(B) Scenario 16



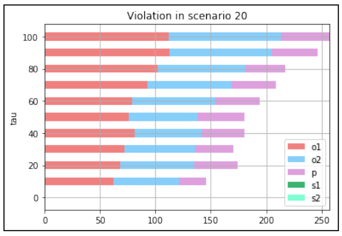
(C) Scenario 17



(D) Scenario 18



(E) Scenario 19



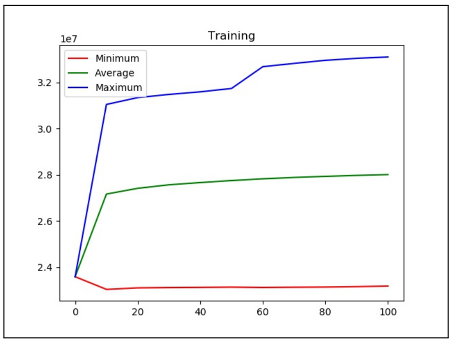
(F) Scenario 20

FIGURE 4.25: Violations for RN with Spill Variable Model

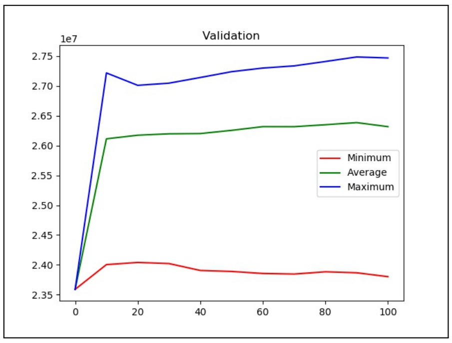
spill variable is not present in the model, then the model is essentially an RN model in Problem A.5. Therefore, we can see that Figure 4.22 shares similarities with Figure 4.16.

Furthermore, we compare the performance of RN for $\tau = 0$ in Figure 4.14 and 4.21 and analyse the result. It shows that in model with spill variable, the storage variable is convergence in the end of the horizon. The spill variable appears to force the convergence in the storage variable.

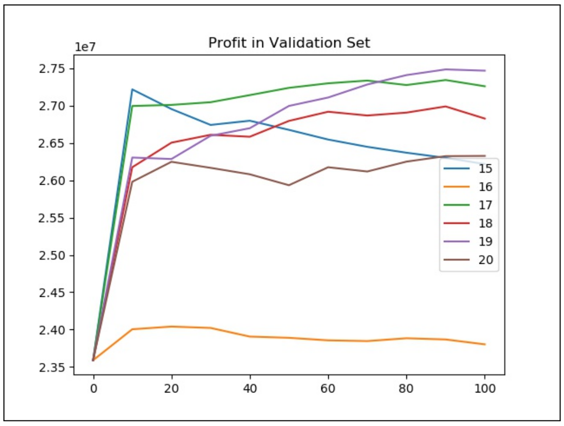
Comparing Figure 4.20c and 4.25, as well as Figure 4.20 and 4.20, we can see that they are similar. Therefore, we can say that the performance of RN with and without the spill variables is identical even in the validation set. Furthermore, Figure 4.27 present



(A) Statistics of RN with Spill Variable in the Training Set



(B) Statistics of RN with Spill Variable in the Validation Set



(C) Revenue of RN with Spill Variable Model for each scenario in the Validation Set

FIGURE 4.26: Performance of RN with Spill Variable

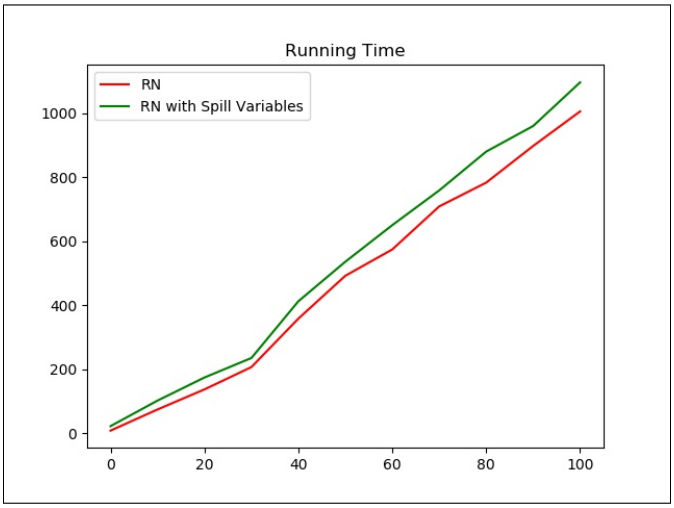


FIGURE 4.27: Running Time of RN without Spill Variable Vs with Spill Variable

the running time needed by both RN with and without spill variables for each τ. In addition, it is evident that RN with spill variables took more time to run compared with a similar model without spill variable. This is to be expected because in the model with the spill variable, there are more constraints and this undoubtedly affects the running

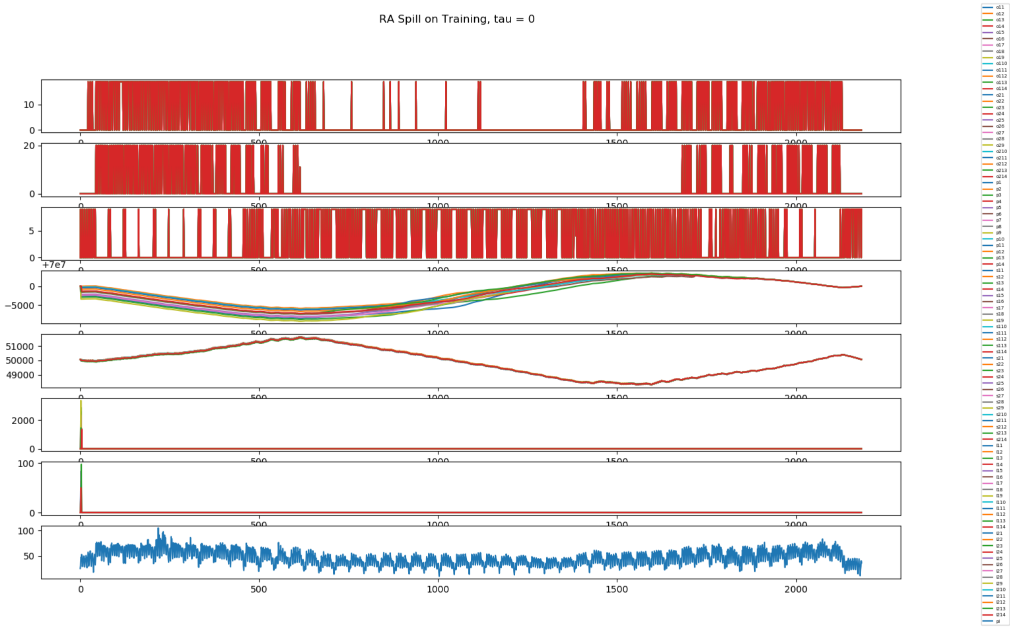


FIGURE 4.28: RA with Spill Variable $\tau = 0$ in the Training Set

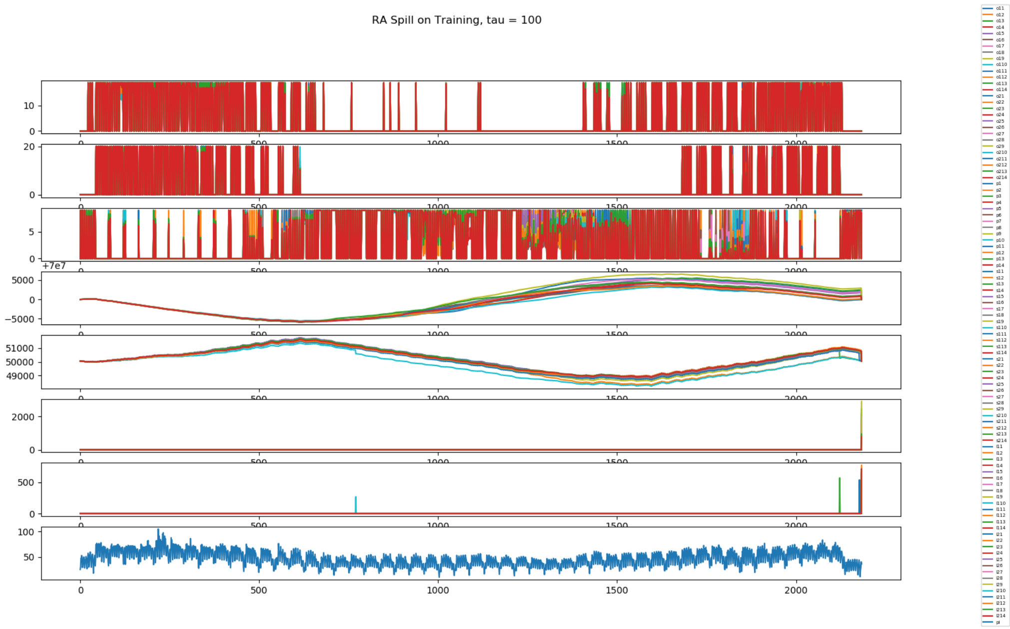


FIGURE 4.29: RA with Spill Variable $\tau = 100$ in the Training Set

time. Although they are different, we can see that the running time for both models follow the same pattern.

Moreover, this result is also similar on the RA model. If we compare Figure 4.29 to 4.8, we can see that they are almost the same. However, it seems that the spill variables tried to force the storage variable to become equal to the initial storage amount at the end of the horizon. Furthermore, if we look at the performance of RA model with spill variable shown in Figure 4.32, it is almost identical to the performance of RA without

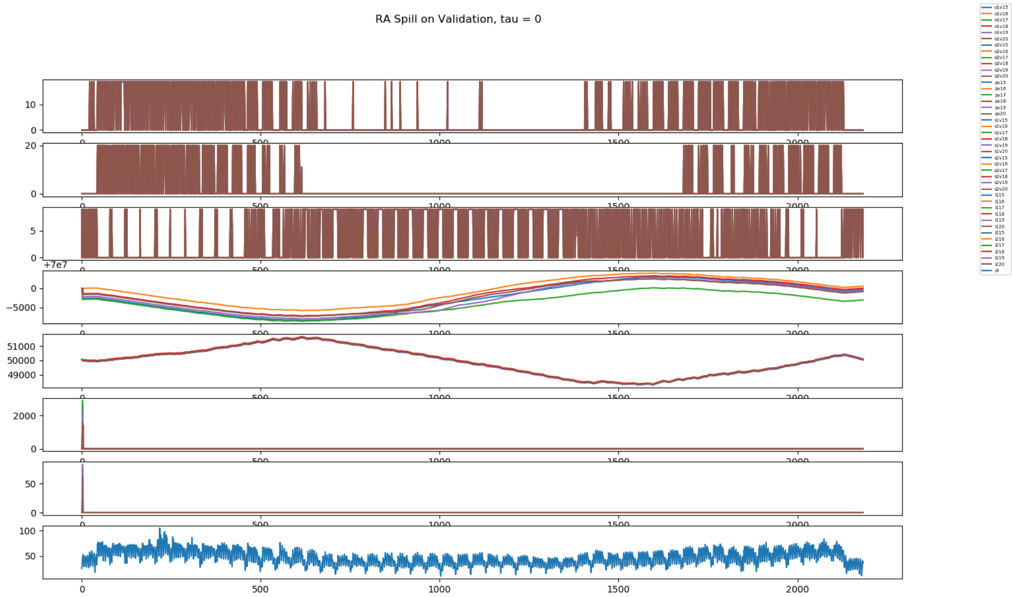


FIGURE 4.30: RA with Spill Variable $\tau = 0$ in the Validation Set

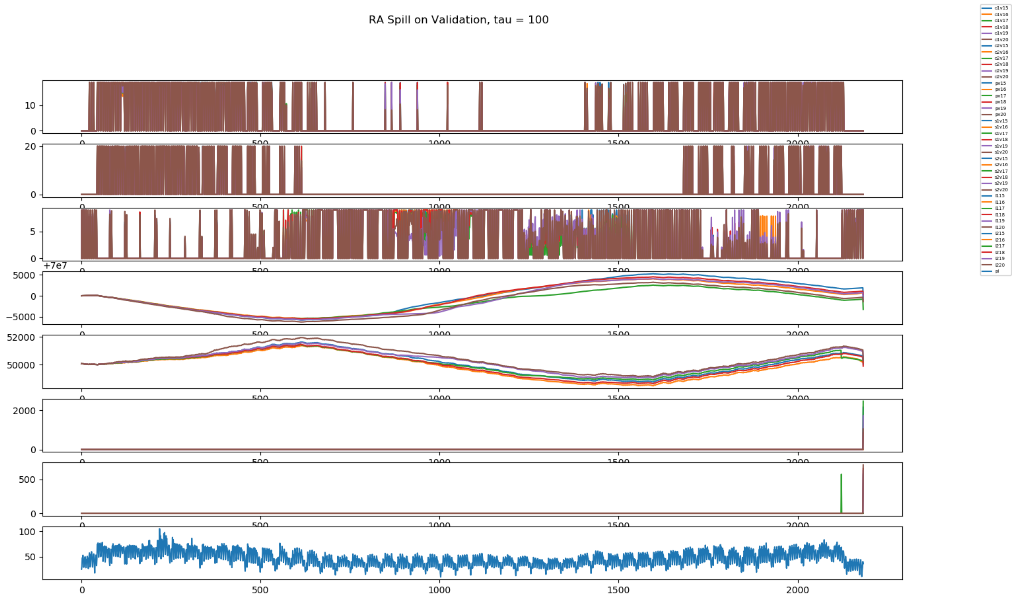
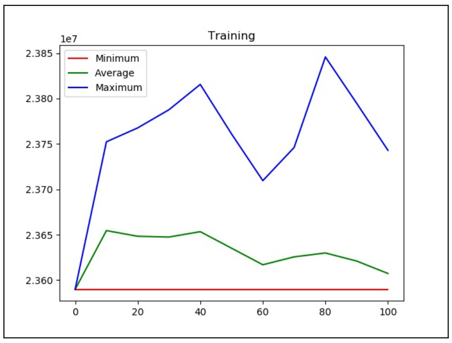


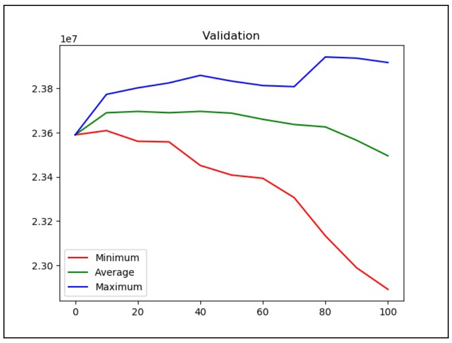
FIGURE 4.31: RA with Spill Variable $\tau = 100$ in the Validation Set

spill variable in Figure 4.12c - 4.12b.

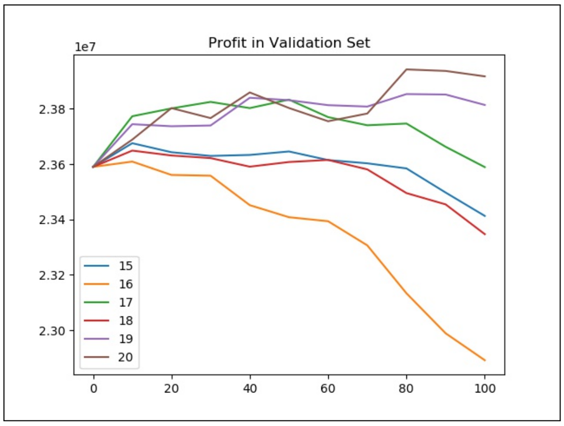
Therefore, we can conclude that in both RA and RN cases, the spill variable does not give much effect to the model. If we look at the average amount of water in the storage, we find that the average amount of water in the first basin is around 70% of maximum capacity, while the average amount of water in the second basin is around 51% of its maximum limit. Thus, we can deduce that the spill variable is not significant in the model because the average amount of water in the storage throughout the year is far



(A) Statistics of RA with Spill Variable in the Training Set



(B) Statistics of RA with Spill Variable in the Validation Set



(C) Revenue of RA with Spill Variable Model for each scenario in the Validation Set

FIGURE 4.32: Performance of RA with Spill Variable

from maximum storage limit, thus meaning that there is no condition for the spill variable to be active.

4.6.3 Regularization

As discussed above, the models suffer from overfitting phenomenon. Therefore, we will apply regularization on β variables to prevent overfitting. We add

$$\beta^i_{t,t'-1} \leq \beta^i_{t,t'} \tag{4.33}$$

as the regularization constraints. The purpose of Equation (4.33) is to restrict β variables. This equation implies that the amount of inflow water today affects more to the decision about how much water to release now rather than yesterday's inflow water.

Figures 4.33-4.34 plots the result of RN in training set and validation set for τ equal to 0 and 100. If we compare them with Figures 4.16 and 4.19, we can see that those figures are quite similar. Moreover, Figure 4.35 shows the performance of RN with regularization. From that figure, we can see that both in the validation and training set, the performance is quite stable after $\tau = 10$.

FIGURE 4.33: RN with Regularization $\tau = 100$ in the Training Set

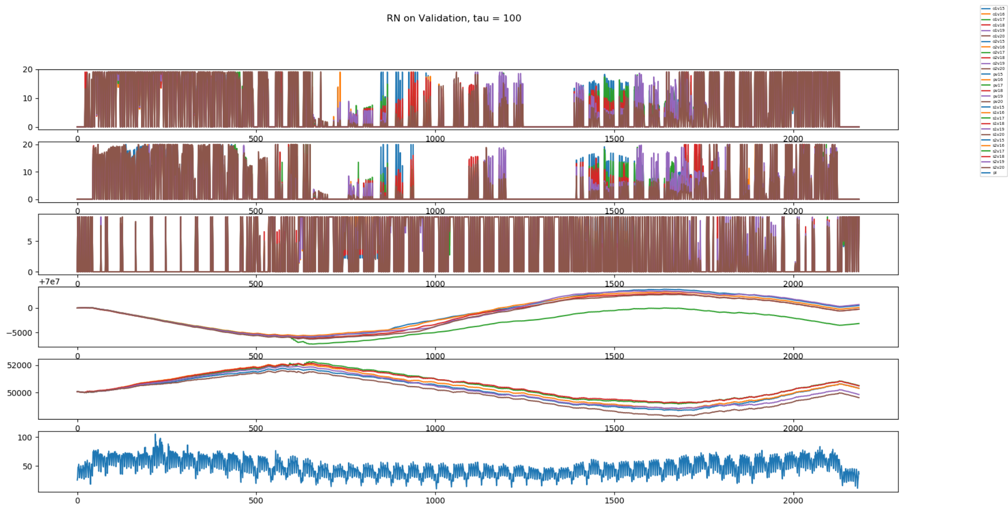
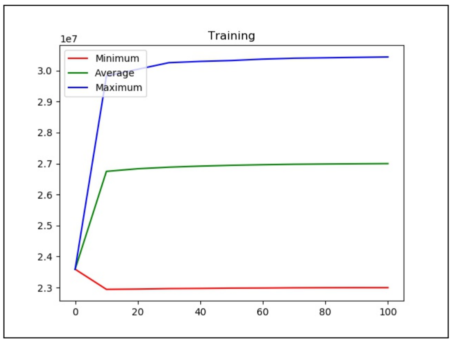


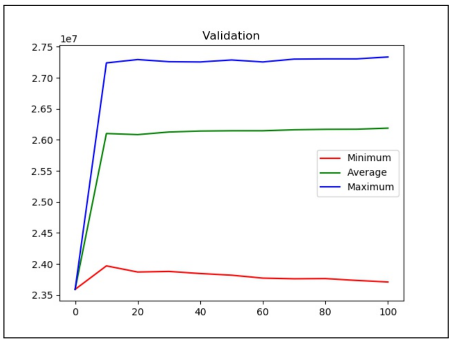
FIGURE 4.34: RN with Regularization $\tau = 100$ in the Validation Set

Table 4.2 presents the comparison between RN without and with regularization in terms of their objectives, violations, and running time. The third column of the table shows the objective obtained from both models for each τ from 0 to 100. We can see that the objective produced from RN is better than from RN with regularization.

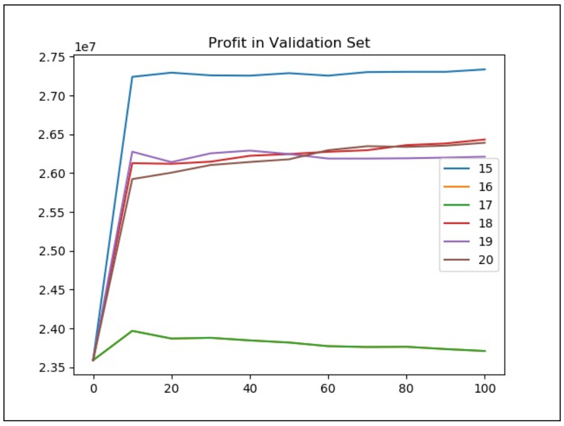
Columns 4-6 of Table 4.2 present the statistics in the validation set from RN and RN with regularization. We can see that the result obtained from RN is still better than from RN with regularization model, but the difference is not that big like the one in the training set. However, if we see column 7 of Table 4.2 which shows the number of violations in each τ, it can be seen that the number of violations produced from RN with regularization is less than from RN model without regularization. Therefore, we can conclude that RN model with regularization performs better in the validation set than



(A) Statistics of RN with Regularization in the Training Set



(B) Statistics of RN with Regularization in the Validation Set



(C) Revenue of RN with Regularization Model for each scenario in the Validation Set

FIGURE 4.35: Performance of RN with regularization

the same model without regularization. In terms of running time shown in column 8 of the Table, we can expect that the running time needed for RN with regularization is longer than it is needed for RN model because the presence of the regularization constraints in Equation 4.33 for the model with regularization.

Table 4.3 shows the results obtained by RA without and with regularization. The third column present the objective value obtained from both models, while columns 4-6 of the table present the minimum, average, and maximum revenue in the validation set for both RA models without and with regularization, and the last two columns show the violations and running time.

Column 3 in Table 4.3 indicates that there is no difference on the performance of RA without and with regularization in the training set, however, if we look further into the validation set, columns 5 of table 4.33 report that in average, the revenue produced by RA with regularization in the validation set is better than that produced by RA without regularization. Moreover, the violations from RA with regularization is less than it is from model without regularization. This indicates that model with regularization produces solution that performs better in the validation set with the cost of longer in running time.

TABLE 4.2: The Comparison between RN without and with Regularization

	τ	**obj**	**Min Val**	**Ave Val**	**Max Val**	**Viol**	**Time**
RN	10	27164674.8	24004677.6	26111734.8	27215686.8	455	74.1
	20	27413848.8	24040778.4	26172426.0	27008107.2	523	136.8
	30	27566100.0	24022335.6	26196035.4	27044208.0	528	206.6
	40	27662630.4	23906577.6	26200090.2	27138776.4	559	358.1
	50	27750135.6	23890096.8	26253652.8	27236484.0	587	492.1
	60	27824691.6	23856350.4	26315913.6	27297306.0	617	574.4
	70	27884336.4	23846540.4	26315194.2	27333799.2	641	709.1
	80	27930247.2	23884210.8	26347763.4	27407178.0	652	783.4
	90	27973411.2	23868514.8	26384191.2	27483696.0	706	898.0
	100	28008334.8	23802591.6	26315521.2	27466822.8	727	1006.1
RN with Reg	10	26747553.6	23968576.8	26102382.6	27240800.4	417	139.1
	20	26831919.6	23869299.6	26084528.4	27294559.2	422	338.4
	30	26882931.6	23878717.2	26126776.8	27260028.0	410	621.9
	40	26917070.4	23845363.2	26141949.6	27256104.0	405	1111.4
	50	26943361.2	23818287.6	26146331.4	27287888.4	416	1437.2
	60	26961804.0	23771199.6	26146658.4	27256104.0	411	1761.2
	70	26976715.2	23760212.4	26162943.0	27302014.8	415	2404.8
	80	26985348.0	23763351.6	26169679.2	27305546.4	429	2976.9
	90	26992018.8	23733921.6	26171248.8	27305546.4	427	3263.4
	100	26997904.8	23708808.0	26189103.0	27336546.0	443	3508.0

TABLE 4.3: The Comparison between RA without and with regularization

	τ	**obj**	**Min Val**	**Ave Val**	**Max Val**	**Viol**	**Time**
RA	10	23589910.8	23591088.0	23621237.4	23658973.2	18	68.4
	20	23589910.8	23578138.8	23667736.8	23756680.8	96	161.4
	30	23589910.8	23568328.8	23717833.2	23848894.8	206	246.4
	40	23589910.8	23583240.0	23734575.6	23875185.6	227	445.6
	50	23589910.8	23440014.0	23694943.2	23835945.6	342	590.0
	60	23589910.8	23356040.4	23660346.6	23815148.4	420	779.6
	70	23589910.8	23316408.0	23657534.4	23855565.6	492	909.0
	80	23589910.8	23205358.8	23591872.8	23814363.6	491	1100.9
	90	23589910.8	23013082.8	23539683.6	23839084.8	712	1279.6
	100	23589910.8	22990716.0	23543738.4	23917957.2	672	1473.7
RA with Reg	10	23589910.8	23613454.8	23621629.8	23629543.2	13	154.4
	20	23589910.8	23606391.6	23621695.2	23644062.0	31	377.4
	30	23589910.8	23593834.8	23630262.6	23684871.6	43	604.7
	40	23589910.8	23603252.4	23636017.8	23689580.4	60	1117.9
	50	23589910.8	23609138.4	23652825.6	23729605.2	72	1450.1
	60	23589910.8	23705276.4	23744254.8	23834376.0	100	1914.3
	70	23589910.8	23606391.6	23652891.0	23715478.8	83	2606.6
	80	23589910.8	23611492.8	23666428.8	23739022.8	90	3149.8
	90	23589910.8	23608353.6	23696839.8	23786110.8	141	3435.1
	100	23589910.8	23649948.0	23729278.2	23818680.0	141	3643.0

From both results in RA and RN model with regularization, therefore, we can say that employing the regularization equation shown in Equation 4.33 to the model can produce the solution which will do better in the validation set. The model with regularization can produce the solution that produces similar revenue in the training set with the model without regularization but with better quality because the model can keep the violations small when it is applied to the validation set.

4.6.4 Two-Basin Hydropower Problem with Variable Waterhead

Refer to figure 4.1, it can be seen that the Zeirmeggern dam is relatively small compared to the Mattmark dam. Therefore, we will consider the waterhead as a variable in basin 1 and ignore its effect in basin 2. In other words, we will assume that waterhead is constant in basin 2.

In this section, we will test the models in Problem (A.9) and (A.10). Before we continue to solve the problem with a variable waterhead, we take solutions obtained from linear model in Problem (A.5) and then reevaluate the profits with assuming a variable waterhead.

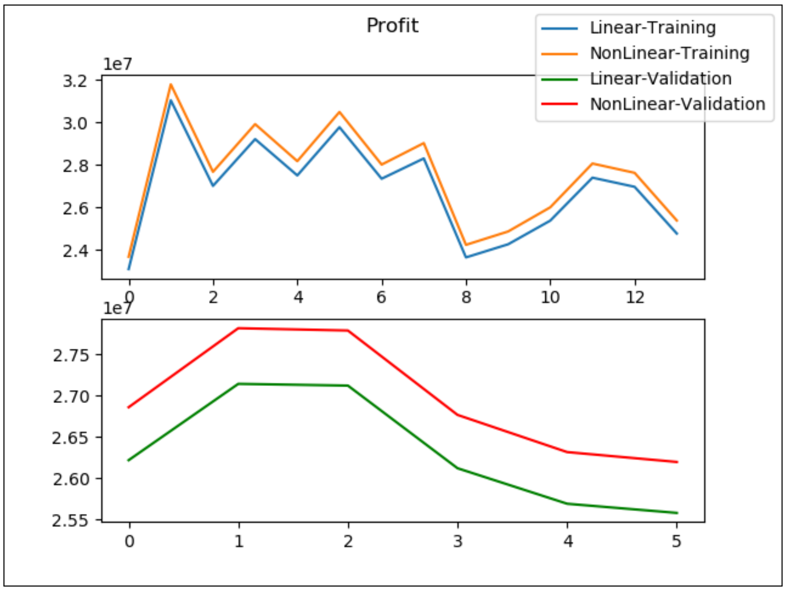


FIGURE 4.36: The comparison of the profit with and without waterhead variables

Figure 4.36 shows the comparison of the profit if the result obtained from the linear model is applied to the objective function without and with variable waterhead. The first plot presents the profit in the training set for all scenarios. The plot shows that the profit is better when the variable waterhead is taken into account. The second plot displays the profit for the scenarios in the validation set. It also shows that when variable waterhead is taken into consideration, the profit goes up by 600,000 *Euro* higher. Therefore, it is evident that with the same amount of water released, considering waterhead variables will lead to better profits than ignoring them. Hence, it becomes important to bring the waterhead variables into the models.

First, we test the proposed algorithms to solve Problem A.9 under $\tau = 2$. We solved the problem A.9 as a non-convex MIP model in a full horizon at once with the help of Gurobi 9.5.0. Then, we compare the result with the result obtained from solving the same model using the proposed rolling horizon algorithm, SRH and DRH. It can be seen from Table 4.4 that both models with rolling horizon produce a better result than when the problem is solved in the full horizon. The rolling horizon algorithms produce $500,000 Euro$ more than the result when the problem is solved at once. Also in terms of running time, the model with a rolling horizon takes less time to run.

Moreover, comparing the result from SRH and DRH, it was found that DRH produces better revenue and fewer violations than SRH. However, DRH takes a longer time to run. It can be expected since at each iteration, DRH solves two problems which are the simplified model and the non-linear model. Furthermore, if we see to Figure 4.37 which shows the comparison of the performance of SRH and DRH in the validation, it shows that DRH makes better profit than SRH for each scenario in the validation set. Therefore, we can conclude that DRH performs better than SRH and henceforth, we will test DRH with $\tau = 10$ and compare the result with the linear model A.4 and A.5.

TABLE 4.4: The Comparison of the Results from SRH and DRH

Model	obj	Viol	Time
Linear	23668905.7	15	5667.23
SRH	24171580.2	14	2603.80
DRH	24180626.9	13	3170.87

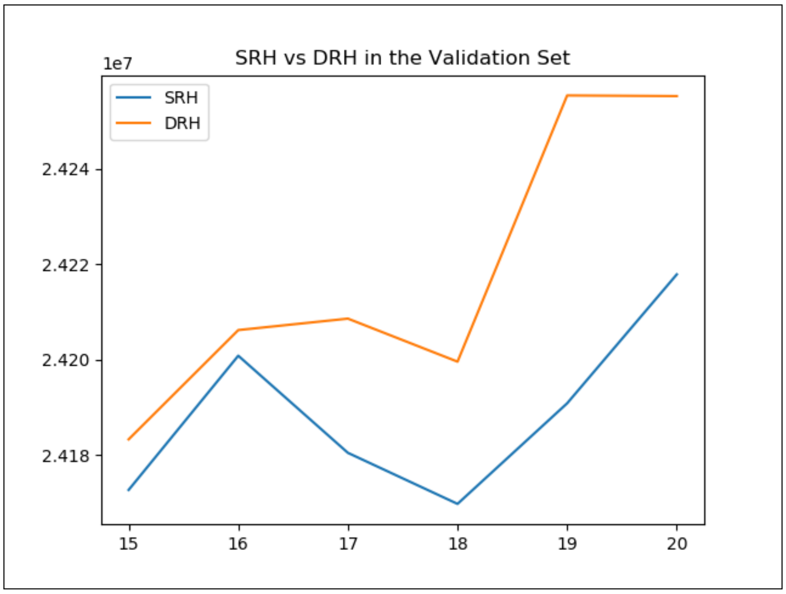


FIGURE 4.37: SRH vs DRH in the Validation Set

Figures 4.38-4.39 show the results from the RA model with variable waterhead solved by DRH in the training set and validation set. Compared with Figure 4.7 which shows the result for the RA model without waterhead variables in the training set, it can be seen that both of them produce a similar policy. However, if we refer to Table 4.5, we can see that RA model with variable waterhead produces a better objective than the model without waterhead variables. Moreover, the RA model with waterhead variables also outperforms the RA model without waterhead variables when the result is applied to the validation set. It produce better profit in the validation set and even less violation. The same goes with RN model. Considering waterhead variables in RN model yields to almost 200,000 *Euro* more in the revenue. And even more, it generates an extra more than 600,000 *Euro* in the revenue when waterhead variable is considered in RA model. However, because of the non-linearity in the constraints, the RA model with variable waterhead takes a longer time to run.

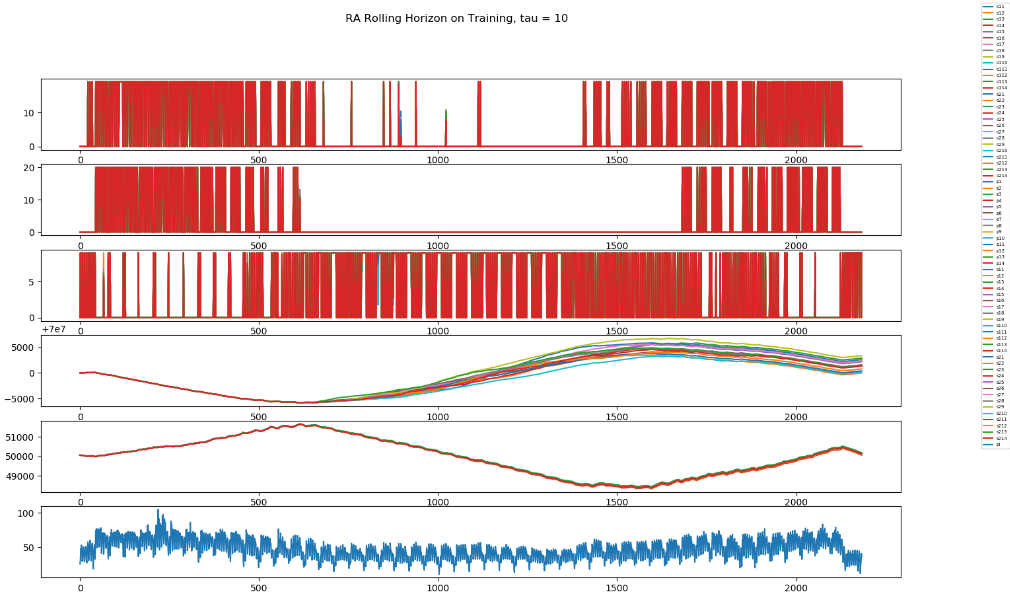


FIGURE 4.38: RA with variable waterhead, $\tau = 10$ in the Training Set

TABLE 4.5: The Comparison between Linear Models and Models with Variable Waterhead

Model	obj	Min Val	Ave Val	Max Val	Viol	Time
RA Linear	23589910.8	23591088.0	23621237.4	23658973.2	18	68.40
RA-DRH	24190771.3	24188409.6	24218043.5	24231471.4	17	5718.60
RN Linear	27164674.8	24004677.6	26111734.8	27215686.8	445	74.10
RN-DRH	27330660.0	24081195.6	26178246.6	27250156.0	346	4409.52

Figures 4.40 and 4.41 show the performance of RN model with variable waterhead solved by DRH algorithm in the training set and validation set. If we compare Figure 4.40 and 4.15 which shows the performance of RN with $\tau = 10$ but without considering variable waterhead, it shows that the result from RN model without variable waterhead tends to release water more frequently even in the time when the price is low. It can be seen around time 1100 to 1500, when the price is relatively low, the linear RN model keeps releasing the water, while the RN model with variable waterhead tends to wait

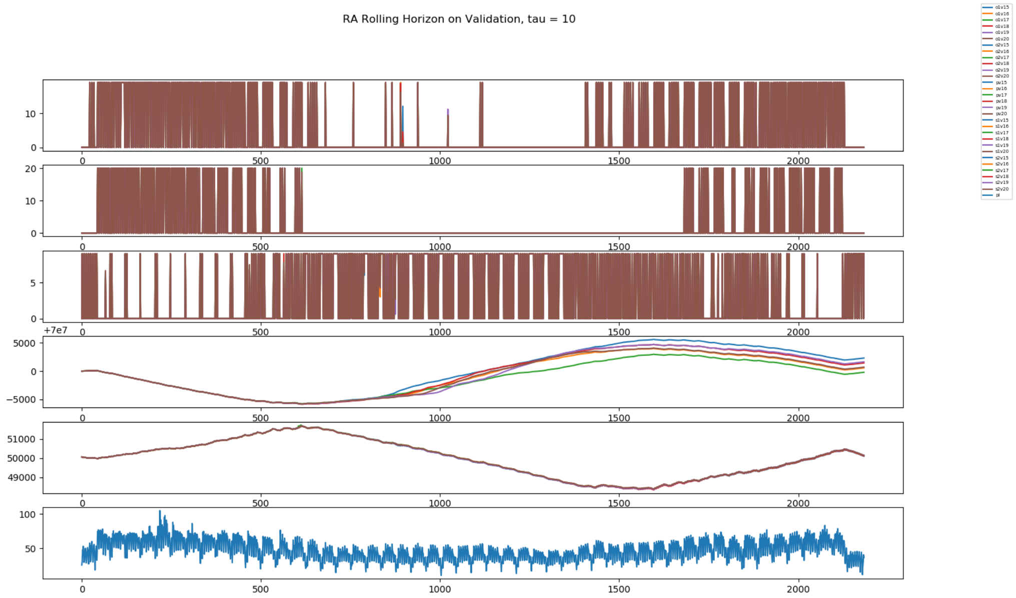


FIGURE 4.39: RA with variable waterhead, $\tau = 10$ in the Validation Set

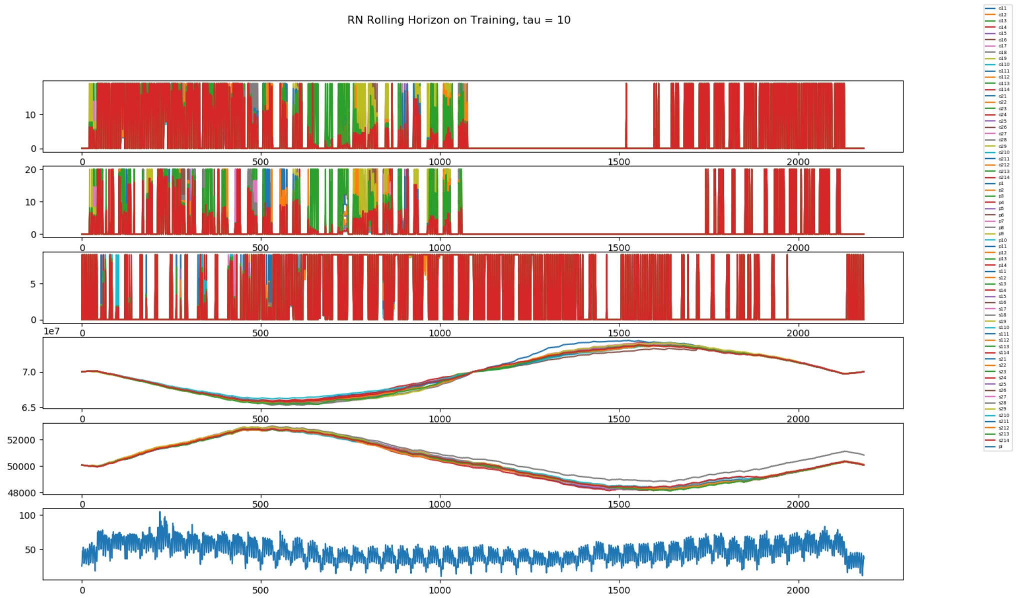


FIGURE 4.40: RN with variable waterhead, $\tau = 10$ in the Training Set

until the basin is full and then releases the water when the basin is full. This is because, when considering variable waterhead, releasing water when the basin is full will produce more energy than releasing it when it is not full.

Moreover, the RN model with variable waterhead produces a higher profit than the model without variable waterhead. Even in the validation set, the RA model with variable waterhead performs better, as it leads to a higher profit and a smaller violation. However, the price for such a better performance is a longer time for solving the

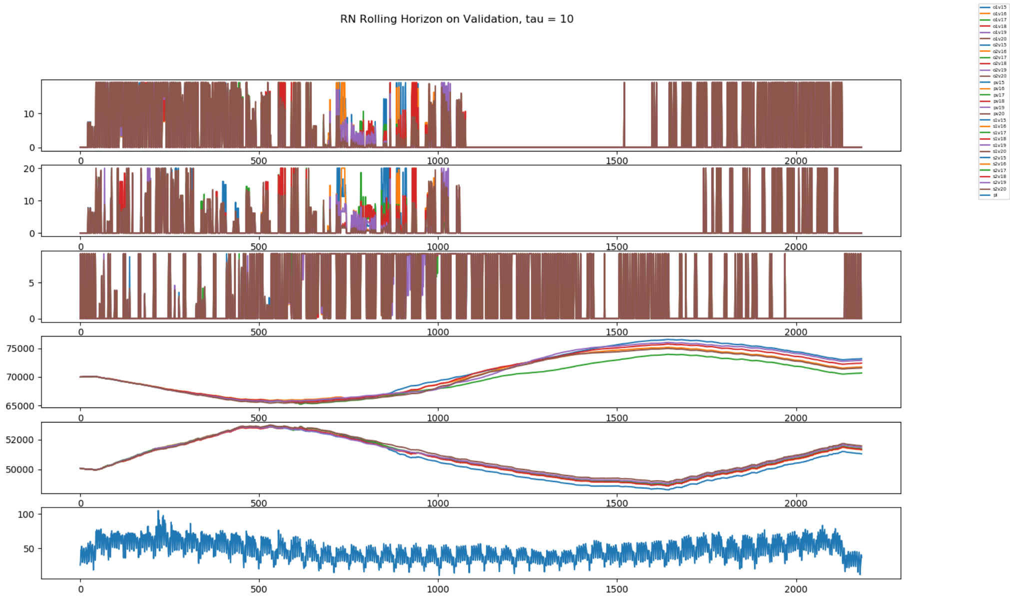


FIGURE 4.41: RN with variable waterhead, $\tau = 10$ in the Validation Set

model. It can be expected since RN model with variable waterhead contains bilinear terms in the objective.

Therefore, from those results, it is shown that considering variable waterhead in hydropower optimization problem is important. It can lead to a better result which also performs better in the validation set.

Chapter 5

Conclusions

At the beginning of this work, we modeled a long-term hydropower production problem that includes a single basin using a robust optimization (RO) model. this model is the simplification of the real-world model we will do later. We modeled the decision variable o to be a second-stage variable and dependent on the inflows up to τ time steps in the past so that the decision variable can adapt to any realization of the inflows. Then, we compared the results with various values of τ and concluded that the performance of the model is increasing in parallel with the increase of τ. However, we also saw from the numerical result that the bigger the τ is, the bigger the violations when the result is applied to the validation set. The results showed that at some point of τ, the performance of the model stops increasing and even starts to decrease. This yields to our first contribution. Therefore, we suspected that the model experienced an overfitting phenomenon.

We also modeled the problem with a stochastic method and solved it using Stochastic Dynamic Programming (SDP). However, as expected, SDP experienced what was referred as the curse of dimensionality which caused a very long running time. We compared the results obtained from SDP and RO and it showed that even with longer running time, the results obtained from SDP are not better than those from RO. Therefore, it is not worth continuing the research on SDP and we focus our research on the robust model.

The research then was extended to the multi-basin hydropower optimization problem. We modeled a hydropower system that consists of two basins, two release water pipes, and one pumping system. The decision variables for the problem were the amount of water released from both basins and the amount of water pumped back to the upstream basin. We modeled them to be second-stage variables and inflow dependent. The result showed that the models suffered from overfitting as the violations increased in parallel with an increment in the time window. Therefore, we applied regularization to prevent the overfitting phenomenon in the model. We added constraints that restrict the effect of water inflow at time $t-1$ to the decision made at time t is less than the effect of the amount of water inflow at time t. Adding this regularization constraint improved the quality of the result in the validation set which is shown by fewer violations. This leads to our second contribution.

Moreover, we added the spill variable to the model which aimed to account for some water being spilled if the water in the storage reaches a certain point. This spill variable is bounded by a function called an evacuation curve. The evacuation curve models the maximum amount of water to be allowed to spill at time t for scenario ω. This function is increasing and non-convex. Therefore, to preserve the linearity of the model we approximated the function by an affine function. However, the result showed that the spill variable is unnecessary when the average amount of water in the storage is far from maximum capacity.

In the aforementioned models, we assumed that the height of the reservoir, also known as waterhead, is constant. However, in reality, the waterhead is not constant as it depends on how much water is in storage. Therefore, in this stage, we modeled the problem with non-linearity in the waterhead. The water head variable is defined as an affine function of the storage level. Adding this variable made the model to be non-convex and non-linear.

Solving the non-linear model is challenging. Moreover, solving the non-linear model for a large horizon is even more difficult. Therefore, we applied a rolling horizon technique to split the problem into smaller subproblems. These problems are then solved sequentially. This leads to our third contribution. We introduce 2 types of Rolling horizons, namely Simple Rolling horizon (SRH) and Dynamic Rolling Horizon (DRH). Both SRH and DRH consist of 2 phases, tracking phase and optimization phase. The difference is that in SRH, the tracking phase is done once at the beginning, whereas in DRH, the tracking phase is done iteratively along with the optimization phase.

First, we compare SRH and DRH for a small value of τ and find that DRH produces a better result but needs a longer time to run rather than SRH. Therefore, we take only DRH for the next experiment. We, then compare the result of affine models and the models with waterhead. The computational result shows that the improvement is significant when the model with variable waterhead is employed rather than ignoring them. Considering variable waterhead generates 100,000*Euro* more in RN and 600,000*Euro* more in RA than when it is ignored. However, as it can be expected, the time to solve the models is much longer.

Appendix A

A.1 Example of the Process of Dynamic Programming

Consider this problem: We want to optimize our profit from energy selling at time $T = 0, 1, 2$ with a unit of water inflow at every time, thus t is $1, 2, 3$ and the price is $10, 5, 8$. The capacity storage at time $t = 0$ is 1, while the minimum capacity of the storage is 0 and maximum capacity of the storage is 3.

To solve this problem, we do a backward recursion. We start from time 2 with the capacity of storage is 0. At time $t = 2$, there are 3 units of water inflows. Therefore, we have 3 total units of water in the basin. We have 4 scenarios for this: (1) we keep all of the water from the basin which means releasing 0 unit of water, (2) we release 1 unit of water, so there are 2 units of water remaining in the basin, (3) we release 2 units of water and keep 1 unit of water in the basin, (4) we release all of the water, so there is no more water in the basin. For scenario (1), we do not release any water so we make 0 profit. For scenario (2) we release 1 unit of water so we make $1 \times 8 = 8$ in profit (8 is the price at time t). For scenario (3), we make $2 \times 8 = 16$ in profit. For scenario (4), we make $3 \times 8 = 24$ in profit. From those 4 scenarios, we will get the highest profit by releasing 3 units of water. We continue to calculate this by assuming the capacity of the water in the storage is 1, 2, and 3. After that, we move to time $t = 1$ with the respective unit of water inflow and the price being 2 and 5. Starting by assuming no water is in the storage, we have 2 units of water in total because there is 2 units of water inflows. In these circumstances, we have 3 scenarios: (1) we keep all of the water and do not release any water, (2) we keep 1 unit of water and release 1 unit, (3) we release all of the water. For scenario (1), by keeping all of the water, we will get 0 profit and have 2 units of water in the storage at time 2. By having 2 units of water at time 2 we will get 40 in profit. In total, we obtain 40 in profit from 0 profit at time 1 and 40 in profit at time 2. For scenario (2), at time $t = 1$ we make $1 \times 5 = 5$ in profit and have 1 water in the storage at time $t = 2$ which means we make 32 in profit. Therefore, we make $5 + 32 = 37$ in profit in total. For scenario (3), we make $2 \times 5 = 10$ in profit at time 1 and have no water in the storage at time 2 thus making 24 in profit. In total, we get $10 + 24 = 34$ profit. From all of those 3 scenarios, the best one is to keep all of the water in order to make 40 in profit. We continue this work until time $t = 0$. The result of this can be seen in Table A.1 and A.2.

TABLE A.1: Table for profit

Capacity	$t = 0$	$t = 1$	$t = 2$
0	50	40	24
1	60	48	32
2	70	53	40
3	80	58	48

TABLE A.2: Table for unit of water outflows

Capacity	$t = 0$	$t = 1$	$t = 2$
0	1	0	3
1	2	0	4
2	3	1	5
3	4	2	6

A.2 Robust Model for Single Basin Hydropower Problem

$$\max \quad \sum_{t=1}^{|T|} \pi_t . P_t \tag{A.1}$$

$$\text{s.t.} \quad f_t^{\omega} + s_{t-1}^{\omega} = o_t + s_t^{\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.2}$$

$$s_{\min} \leq s_t^{\omega} \leq s_{\max} \qquad \forall t \in T, \forall \omega \in \Omega \tag{A.3}$$

$$s_0 \leq s_{|T|}^{\omega} \qquad \forall t \in T, \forall \omega \in \Omega \tag{A.4}$$

$$o_{min} \leq o_t \leq o_{\max} \qquad \forall t \in T \tag{A.5}$$

A.3 Pseudocode for Validation process on Single-Basin Hydropower Problem

Algorithm 4: Pseudocode for Validation process on Single-Basin Hydropower Problem

```
Optimization;
Data: f_t^ω, π_t, ∀ω ∈ Ω_ts, t ∈ T, o_min, o_max, s_min, s_max
if Risk-Averse then
    Solve ??;
else if Risk-Neutral then
    Solve 3.3
end
Result: α_t, β_{t,t'}, ∀t ∈ T, t' ∈ (t − τ, t − 1)
Validation;
Data: f_t^ω ∀t ∈ T, ω ∈ Ω_vs
violation = 0;
for ω ∈ Ω_vs do
    for t ∈ T do
        o_t^ω = α_t + Σ_{t'=t−τ}^{t−1} β_{tt'} f_{t'}^ω;
        if o_t^ω < o_min then
            violation += 1;
            o_t^ω = o_min
        else if o_t^ω > o_max then
            violation += 1;
            o_t^ω = o_max
        end
        s_t^ω = f_t^ω + s_{t−1}^ω − o_t^ω;
        if s_t^ω < s_min then
            violation += 1;
            δ = s_min − s_t^ω o_t^ω = o_t^ω − δ s_t^ω = 0
        else if s_t^ω > s_max then
            violation += 1;
            δ = s_t^ω − s_max o_t^ω = o_t^ω + δ s_t^ω = s_max;
            if o_t^ω > o_max then
                o_t^ω = o_max
            end
        end
    end
    Revenue = Σ_{t∈T} π_t.o_t^ω
end
```

A.4 Affine Decision Rule based Robust Optimization for Risk-Averse two-Basin Hydropower Optimization Problem

$$
\begin{aligned}
\max \quad & \psi && && \text{(A.6)}\\
\text{s.t.} \quad & f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} && \forall t \in T, \omega \in \Omega && \text{(A.7)}\\
& (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + f_t^{2\omega} + s_{t-1}^{2\omega} = (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) && &&\\
& + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_t^{2\omega} && \forall t \in T, \omega \in \Omega && \text{(A.8)}\\
& s_{|T|}^{i\omega} \geq s_0^{i\omega} && i = 1,2, \forall \omega \in \Omega && \text{(A.9)}\\
& o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i && i = 1,2, \forall t \in T, \omega \in \Omega && \text{(A.10)}\\
& p_{min} \leq \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} && \forall t \in T, \omega \in \Omega && \text{(A.11)}\\
& s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} && i = 1,2, \forall t \in T, \omega \in \Omega && \text{(A.12)}\\
& \psi - (\sum_{t=1}^{|T|} \pi_t \xi (\eta^1 g \gamma h^1 (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) && &&\\
& + \eta^2 g \gamma h^2 (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) && &&\\
& - \frac{1}{\eta^p} g \gamma h^1 (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}))) \leq 0. && \forall \omega \in \Omega && \text{(A.13)}
\end{aligned}
$$

A.5 Affine Decision Rule Based Robust Optimization for Risk-Neutral Two-Basin Hydropower Optimization Problem

$$\max \sum_{\omega\in\Omega} \Big(\frac{1}{Pr^{\omega}} \sum_{t=1}^{|T|} \pi_t\, \xi (\eta^1 g\gamma\, h^1(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + \eta^2 g\gamma\, h^2(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega}$$
$$+ \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) - \frac{1}{\eta^p} g\gamma\, h^1(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}))\Big) \tag{A.14}$$

$$\text{s.t.} \quad f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.15}$$

$$(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + f_t^{2\omega} + s_{t-1}^{2\omega} = \sum_{i=2}^{3} \left(\alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega}\right) + s_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.16}$$

$$s_{|T|}^{i\omega} \geq s_0^{i\omega} \qquad i = 1,2, \forall \omega \in \Omega \tag{A.17}$$

$$o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.18}$$

$$p_{min} \leq \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} \qquad \forall t \in T, \omega \in \Omega \tag{A.19}$$

$$s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega. \tag{A.20}$$

where Pr^{ω} is the probability of scenario ω to be happened.

A.6 Affine Decision Rule for Risk-Averse Two-Basin Hydropower Optimization Problem with Spill Variable

$$\max \quad \psi \tag{A.21}$$

$$\text{s.t.} \quad f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} + l_t^{1\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.22}$$

$$\left(\alpha_t^1 + \sum_{i=1}^{2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{1i} f_{t'}^{1\omega} \right)\right) + f_t^{2\omega} + s_{t-1}^{2\omega} = \sum_{i=2}^{3} \left(\alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \right) + s_t^{2\omega} + l_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.23}$$

$$s_{|T|}^{i\omega} \geq s_0^{i\omega} \qquad i = 1,2, \forall \omega \in \Omega \tag{A.24}$$

$$o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.25}$$

$$p_{min} \leq \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} \qquad \forall t \in T, \omega \in \Omega \tag{A.26}$$

$$s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.27}$$

$$l_t^{i\omega} \leq x_i^1 + y_i^1 s_t^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.28}$$

$$\psi - (\sum_{t=1}^{|T|} \pi_t \xi g \gamma (\eta^1 h^1 \left(\alpha_t^1 + \sum_{i=1}^{2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{1i} f_{t'}^{i\omega} \right)\right) + \eta^2 h^2 (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega})$$

$$- \frac{1}{\eta^p} h^1 (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}))) \leq 0 \qquad \forall \omega \in \Omega. \tag{A.29}$$

A.7 Affine Decision Rule for Risk-Neutral Two-Basin Hydropower Optimization Problem with Spill Variable

$$
\begin{aligned}
\max \sum_{\omega\in\Omega} (\frac{1}{Pr^{\omega}} \sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 g\gamma \, h^1(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + \eta^2 g\gamma \, h^2(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} \\
+ \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) - \frac{1}{\eta^p} g\gamma \, h^1(\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega})))
\end{aligned}
\tag{A.30}
$$

$$
\text{s.t.} \quad f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} + l_t^{1\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.31}
$$

$$
\left(\alpha_t^1 + \sum_{i=1}^{2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{1i} f_{t'}^{1\omega}\right)\right) + f_t^{2\omega} + s_{t-1}^{2\omega} = \sum_{i=2}^{3} \left(\alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega}\right) + s_t^{2\omega} + l_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.32}
$$

$$
s_{|T|}^{i\omega} \geq s_0^{i\omega} \qquad i = 1,2, \forall \omega \in \Omega \tag{A.33}
$$

$$
o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.34}
$$

$$
p_{min} \leq \alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} \qquad \forall t \in T, \omega \in \Omega \tag{A.35}
$$

$$
s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.36}
$$

$$
l_t^{i\omega} \leq x_i^1 + y_i^1 \, s_t^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega. \tag{A.37}
$$

A.8 Pseudocode for Validation on Two-Basin Hydropower

Algorithm 5: Pseudocode for Validation on Two-Basin Hydropower Problem

Validation;

Data: $f_t^{\omega}, \pi_t, \forall \omega \in \Omega_{ts}, t \in T, o_{min}^i, o_{max}^i, s_{min}^i, s_{max}^i$ and $\alpha_t^i, \beta_{tt'}^{ik}, \forall t \in T, i = 1,2,3, k = 1,2$

violation = 0;

for $\omega \in \Omega_{vs}$ **do**

  for $t \in T$ **do**

    for *i in range 2* **do**

      $o_t^{i\omega} = \alpha_t^i + \sum_{k=1,2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{ik} f_{t'}^{k\omega} \right);$

      if $o_t^{i\omega} < o_{min}^i$ **then**

        violation += 1;

        $o_t^{\omega} = o_{min}$

      else if $o_t^{i\omega} > o_{max}^i$ **then**

        violation += 1;

        $o_t^{i\omega} = o_{max}^i$

      end

    end

    $p_t^{\omega} = \alpha_t^3 + \sum_{k=1,2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{3k} f_{t'}^{k\omega} \right);$

    if $p_t^{\omega} < p_{min}$ **then**

      violation += 1;

      $p_t^{\omega} = p_{min}$

    else if $p_t^{\omega} > p_{max}$ **then**

      violation += 1;

      $p_t^{\omega} = p_{max}$

    end

    $s_t^{1\omega} = f_t^{1\omega} + s_{t-1}^{1\omega} + \sum_{i=1,3} \left(\alpha_t^i + \sum_{k=1,2} \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{ik} f_{t'}^{k\omega} \right) \right);$

    $s_t^{2\omega} = f_t^{\omega} + s_{t-1}^{\omega} + \sum_{i=1}^{3} \left((-1)^{i=1} \left(\alpha_t^i \left(\sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{ik} f_t^{k\omega} \right) \right) \right);$

    for *i = 1,2* **do**

      if $s_t^{i\omega} < s_{min}^i$ **then**

        violation += 1;

        $\delta = s_{min}^i - s_t^{i\omega}; \quad o_t^{i\omega} = o_t^{i\omega} - \delta; \quad s_t^{i\omega} = s_{min}^i$

      else if $s_t^{i\omega} > s_{max}^i$ **then**

        violation += 1;

        $\delta = s_t^{i\omega} - s_{max}; \quad o_t^{i\omega} = o_t^{i\omega} + \delta; \quad s_t^{i\omega} = s_{max}^i;$

        if $o_t^{\omega} > o_{max}^i$ **then**

          $o_t^{i\omega} = o_{max}^i$

        end

      end

    end

  end

  Revenue = $\sum_{t \in T} \pi_t \left(o_t^{1\omega} + o_t^{2\omega} + p_t^{\omega} \right)$

end

A.9 Affine Risk Averse water-head dependence

$$\max \quad \psi \tag{A.38}$$

$$\text{s.t.} \quad f_t^{1\omega} + (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_{t-1}^{1\omega} = (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + s_t^{1\omega} + l_t^{1\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.39}$$

$$(\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + f_t^{2\omega} + s_{t-1}^{2\omega} = (\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega})$$

$$+ (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}) + s_t^{2\omega} + l_t^{2\omega} \qquad \forall t \in T, \omega \in \Omega \tag{A.40}$$

$$o_{min} \leq \alpha_t^i + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i1} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{i2} f_{t'}^{2\omega} \leq o_{max}^i \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.41}$$

$$p_{min} \leq \alpha_t^3 + \sum_{t'=0}^{t-1} \beta_{tt'}^{31} f_t^{1\omega} + \sum_{t'=0}^{t-1} \beta_{tt'}^{32} f_t^{2\omega} \leq p_{max} \qquad \forall t \in T, \omega \in \Omega \tag{A.42}$$

$$l_t^{i\omega} \leq x_i^1 + y_i^1 s_t^{1\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.43}$$

$$s_{min} \leq s_t^{i\omega} \leq s_{max}^{i\omega} \qquad i = 1,2, \forall t \in T, \omega \in \Omega \tag{A.44}$$

$$s_{|T|}^{i\omega} \geq s_0^{i\omega} \qquad i = 1,2, \forall \omega \in \Omega \tag{A.45}$$

$$\psi - (\sum_{t=1}^{|T|} \pi_t \, \xi(\eta^1 \, g\gamma \left(x_1^2 + y_1^2 s_t^{1\omega}\right) (\alpha_t^1 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{11} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{12} f_{t'}^{2\omega}) + \eta^2 \, g\gamma \left(x_2^2 + y_2^2 s_t^{2\omega}\right)$$

$$(\alpha_t^2 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{21} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{22} f_{t'}^{2\omega}) - \frac{1}{\eta^p} \, g\gamma \left(x_1^2 + y_1^2 s_t^{1\omega}\right) (\alpha_t^3 + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{31} f_{t'}^{1\omega} + \sum_{t'=t-\tau}^{t-1} \beta_{tt'}^{32} f_{t'}^{2\omega}))) \leq 0 \qquad \forall \omega \in \Omega \tag{A.46}$$

A.10 Affine-Risk Neutral water-head dependence

$$\max \quad \sum_{\omega\in\Omega}\Big(\frac{1}{Pr^\omega}\sum_{t=1}^{|T|}\pi_t\,\xi\Big(\Big(x_1^2+y_1^2 s_t^{1\omega}\Big)\,\eta^1\,g\gamma\big(\alpha_t^1+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{11}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{12}f_{t'}^{2\omega}\big)+\Big(x_2^2+y_2^2 s_t^{1\omega}\Big)\,\eta^2\,g\gamma$$

$$\big(\alpha_t^2+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{21}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{22}f_{t'}^{2\omega}\big)-\Big(\Big(x_1^2+y_1^2 s_t^{1\omega}\Big)\,\frac{1}{\eta^p}\,g\gamma\big(\alpha_t^3+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{31}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{32}f_{t'}^{2\omega}\big)\Big) \qquad \text{(A.47)}$$

$$\text{s.t.} \quad f_t^{1\omega}+\big(\alpha_t^3+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{31}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{32}f_{t'}^{2\omega}\big)+s_{t-1}^{1\omega}=\big(\alpha_t^1+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{11}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{12}f_{t'}^{2\omega}\big)+s_t^{1\omega} \qquad \forall t\in T,\omega\in\Omega \quad \text{(A.48)}$$

$$\big(\alpha_t^1+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{11}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{12}f_{t'}^{2\omega}\big)+f_t^{2\omega}+s_{t-1}^{2\omega}=\big(\alpha_t^2+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{21}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{22}f_{t'}^{2\omega}\big)$$

$$+\big(\alpha_t^3+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{31}f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{32}f_{t'}^{2\omega}\big)+s_t^{2\omega} \qquad \forall t\in T,\omega\in\Omega \quad \text{(A.49)}$$

$$o_{min}\le\alpha_t^i+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{i1}\,f_{t'}^{1\omega}+\sum_{t'=t-\tau}^{t-1}\beta_{tt'}^{i2}\,f_{t'}^{2\omega}\le o_{max}^i \qquad i=1,2,\forall t\in T,\omega\in\Omega \quad \text{(A.50)}$$

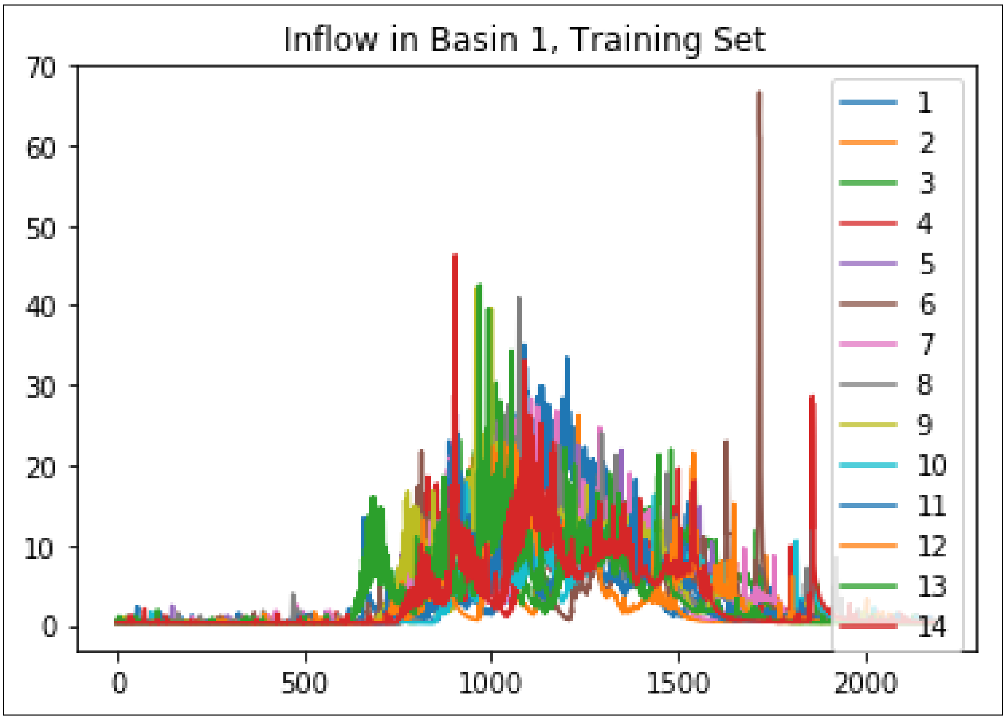
$$p_{min}\le\alpha_t^3+\sum_{t'=0}^{t-1}\beta_{tt'}^{31}\,f_t^{1\omega}+\sum_{t'=0}^{t-1}\beta_{tt'}^{32}\,f_t^{2\omega}\le p_{max} \qquad \forall t\in T,\omega\in\Omega \quad \text{(A.51)}$$

$$l_t^{i\omega}\le x_i^1+y_i^1 s_t^{1\omega} \qquad i=1,2,\forall t\in T,\omega\in\Omega \quad \text{(A.52)}$$

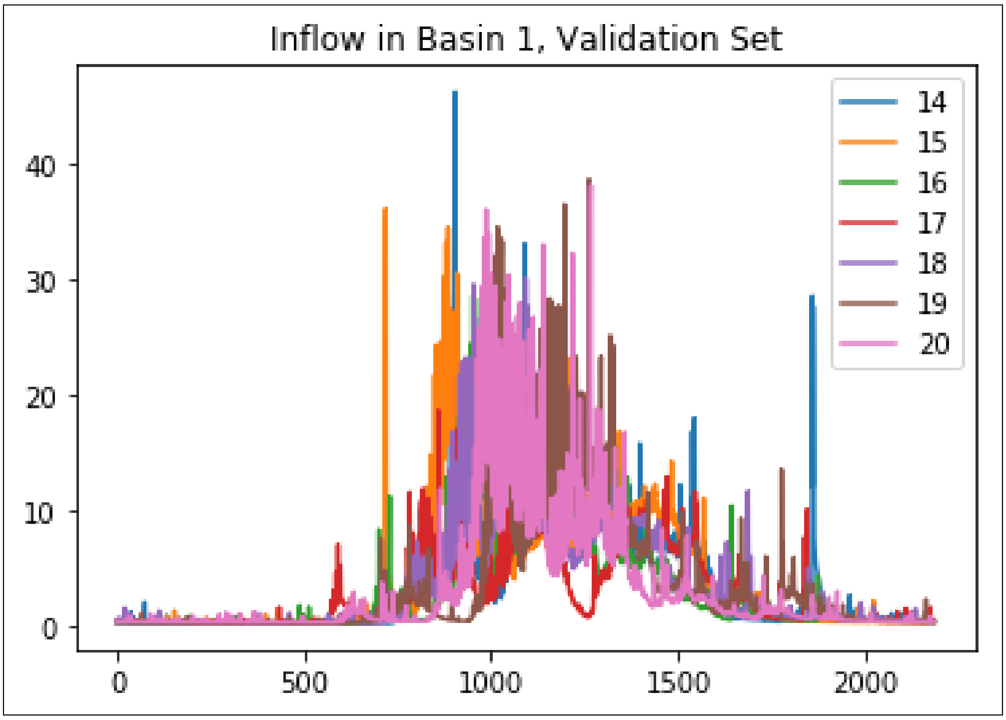
$$s_{min}\le s_t^{i\omega}\le s_{max}^{i\omega} \qquad i=1,2,\forall t\in T,\omega\in\Omega \quad \text{(A.53)}$$

$$s_{|T|}^{i\omega}\ge s_0^{i\omega} \qquad i=1,2,\forall\omega\in\Omega \quad \text{(A.54)}$$

Appendix B

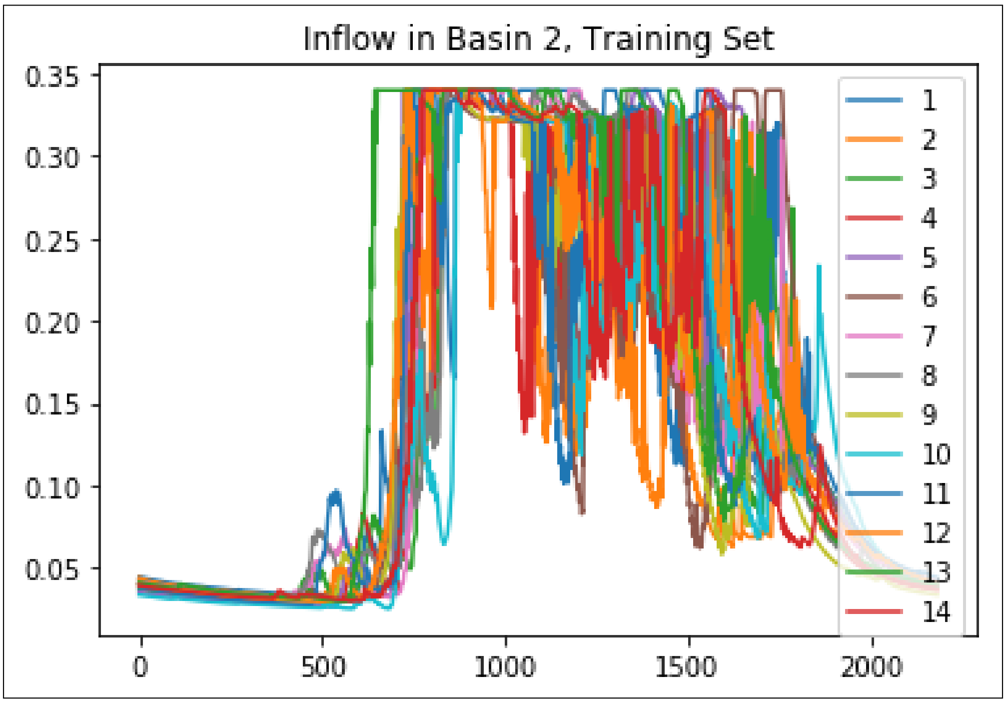


(A) Inflow in Basin 1 in Training Set

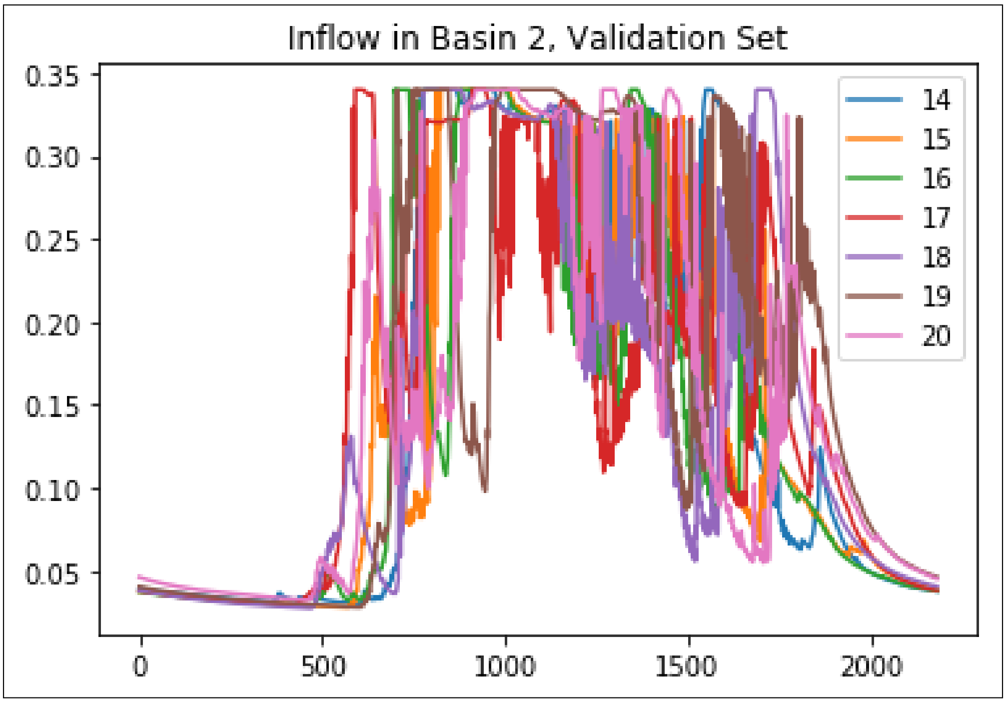


(B) Inflow in Basin 1 in Validation Set

FIGURE B.1: Inflow in Basin 1



(A) Inflow in Basin 2 in Training Set



(B) Inflow in Basin 2 in Validation Set

FIGURE B.2: Inflow in Basin 2

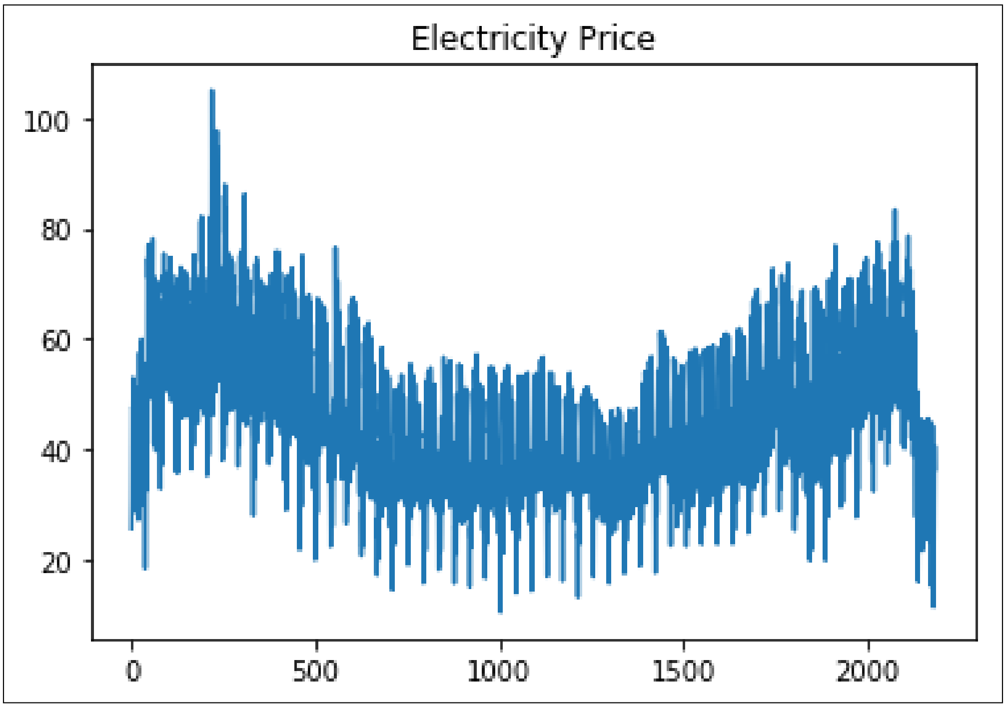


FIGURE B.3: Electricity Price

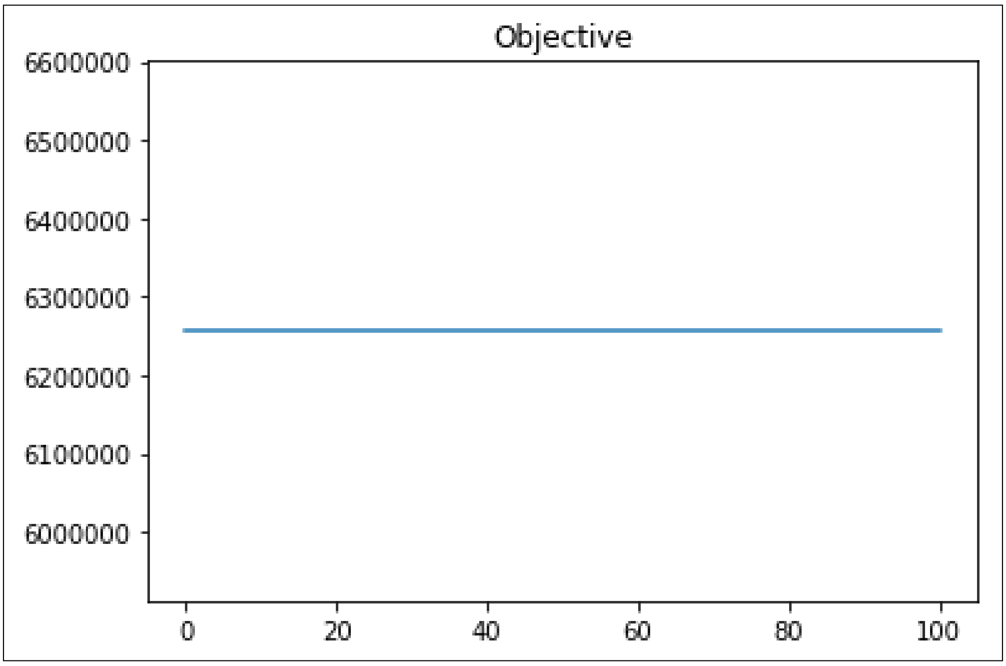


FIGURE B.4: Objective, RA Single

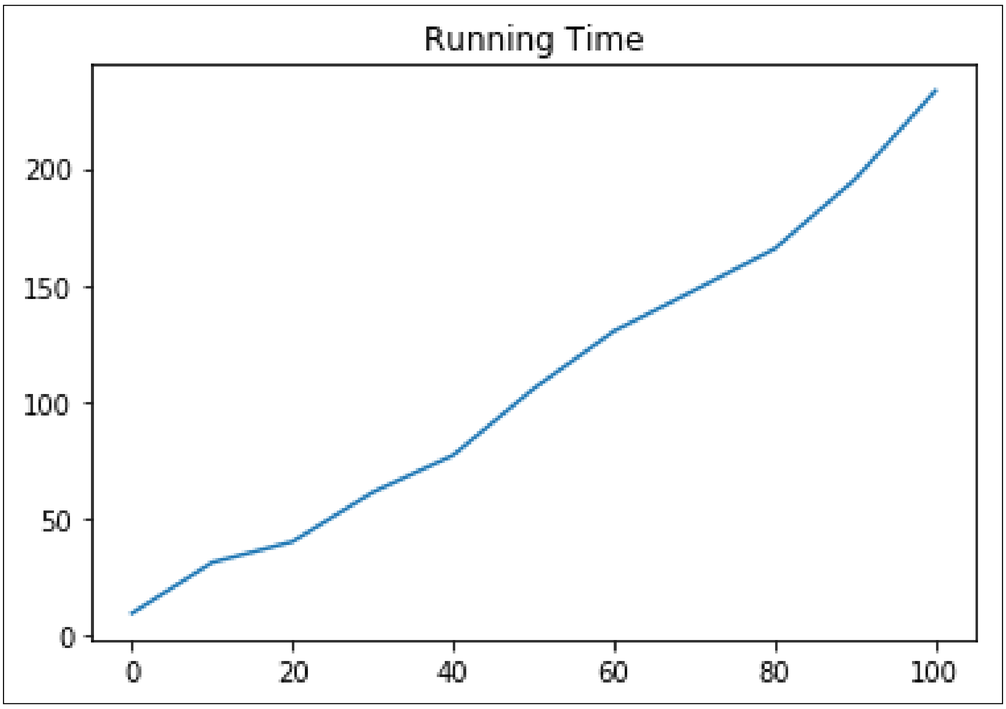


FIGURE B.5: Running Time, RA Single

FIGURE B.6: Revenue in Training Set, RA Single

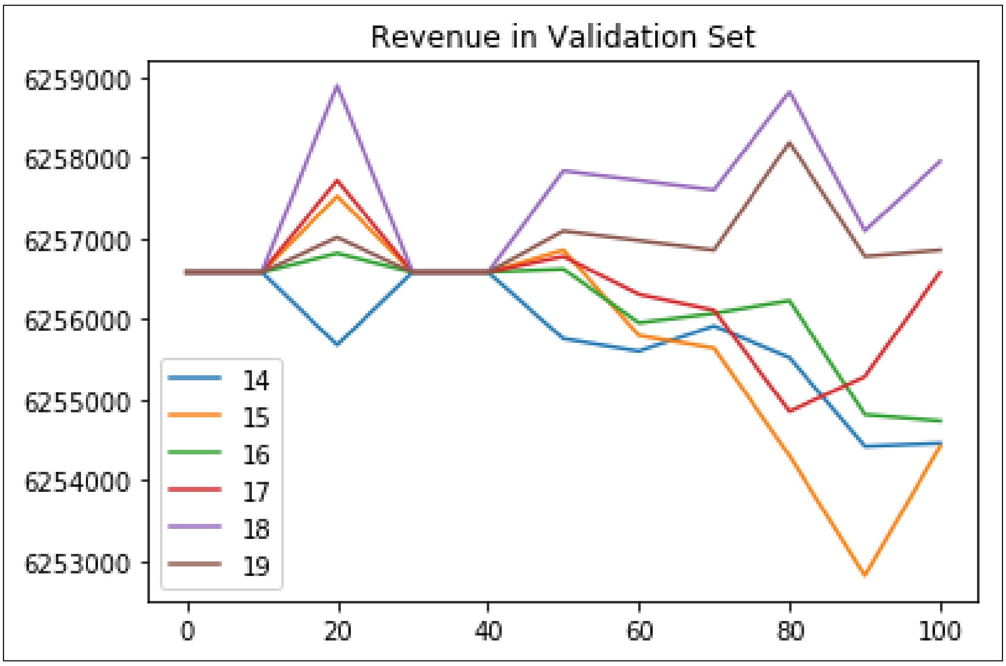


FIGURE B.7: Revenue in Validation Set, RA Single

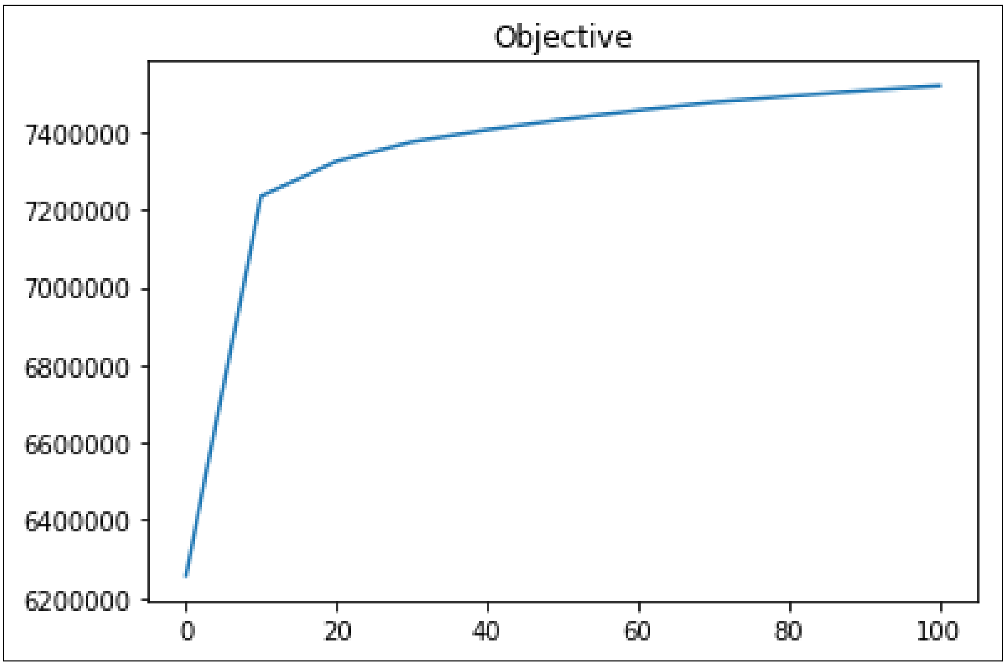


FIGURE B.8: Objective, RN Single

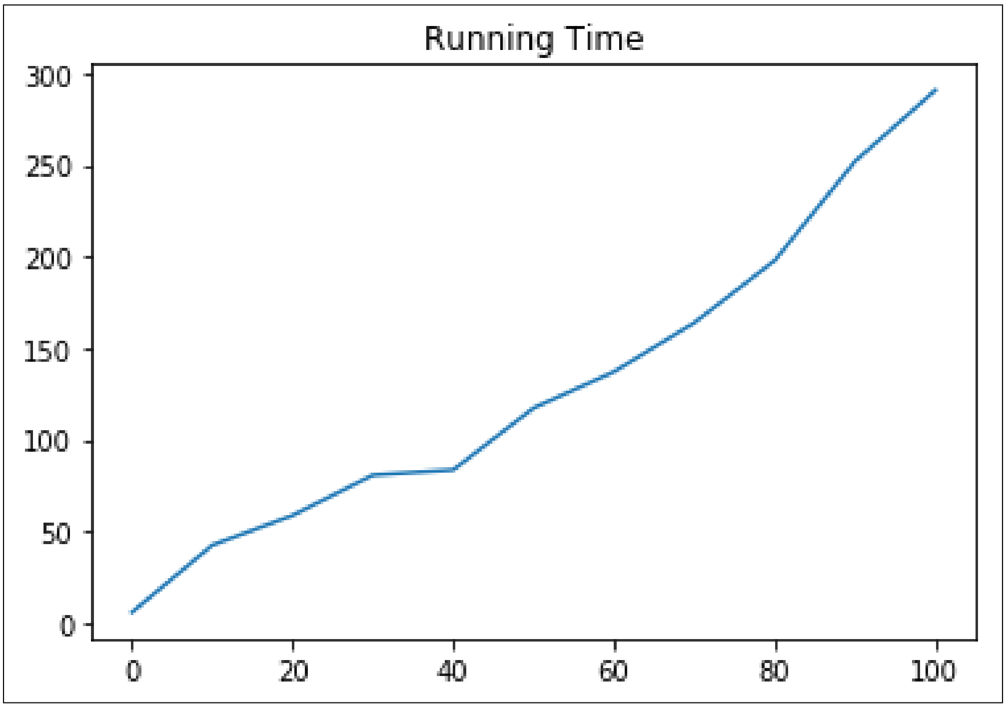


FIGURE B.9: Running Time, RN Single

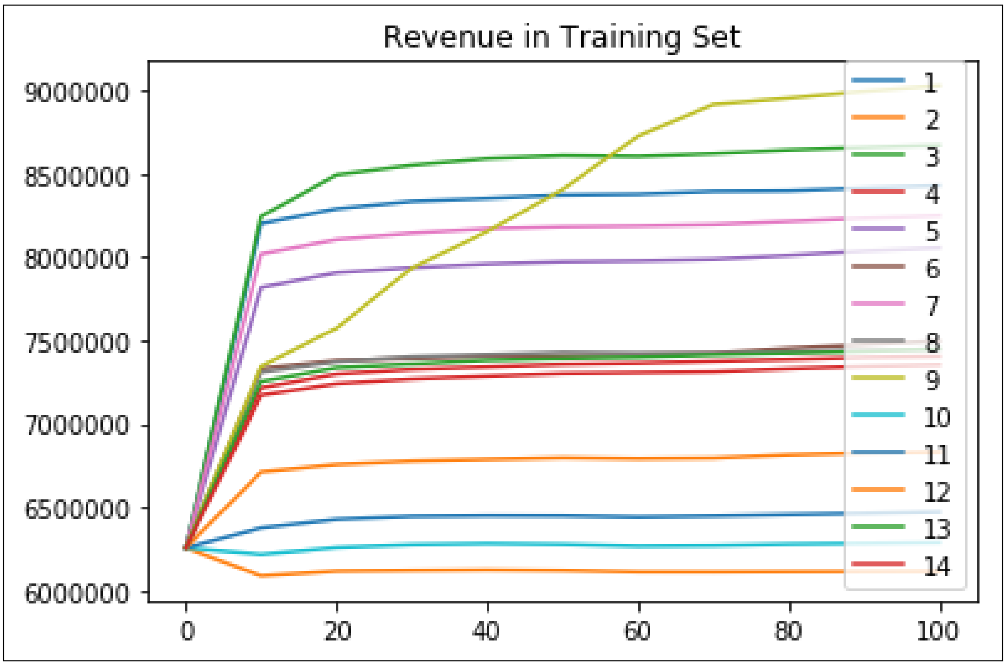


FIGURE B.10: Revenue in Training Set, RN Single

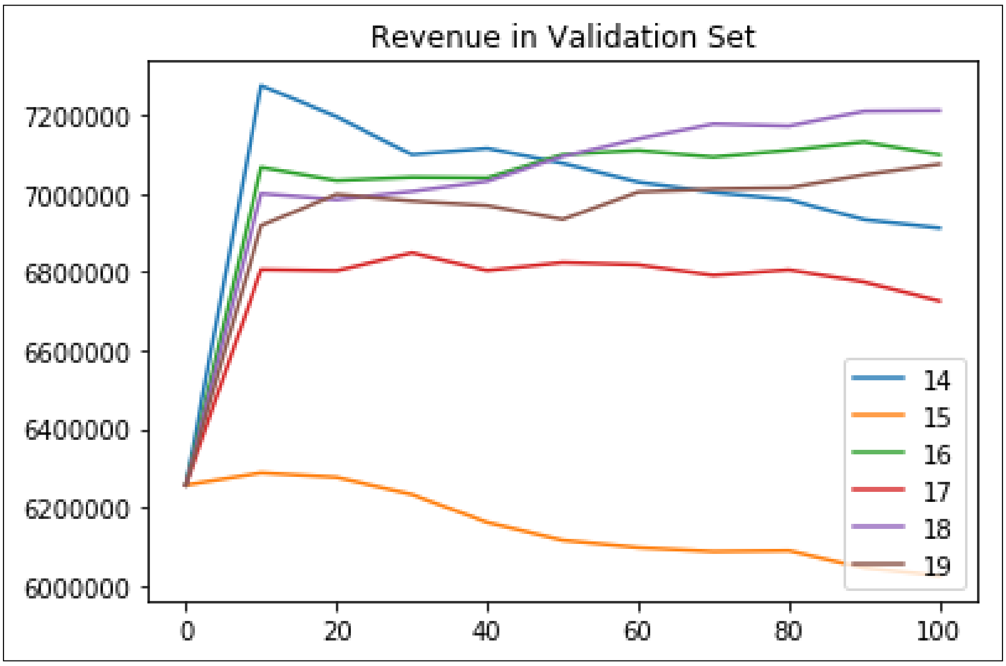


FIGURE B.11: Revenue in Validation Set, RN Single

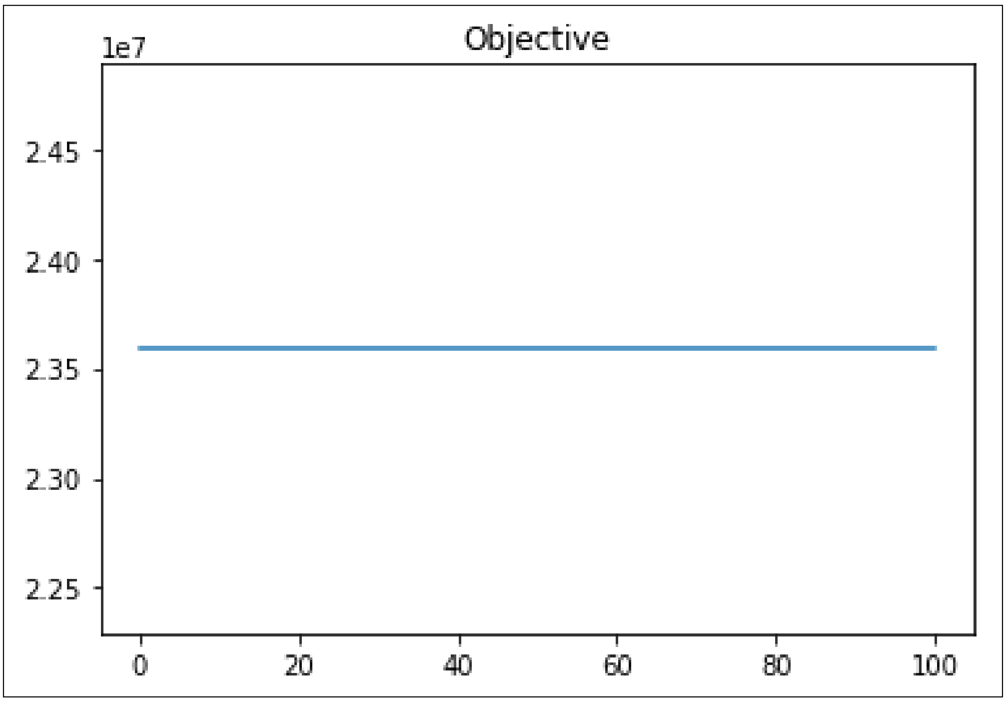


FIGURE B.12: Objective, RA Two-Basin Hydropower problem

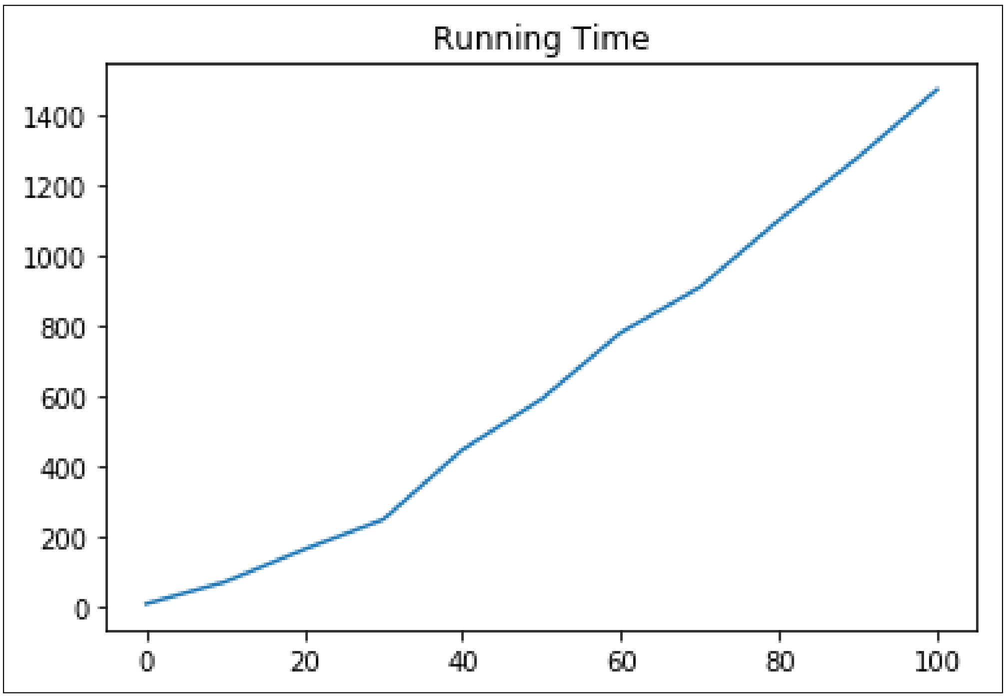


FIGURE B.13: Running Time, RA Two-Basin Hydropower problem

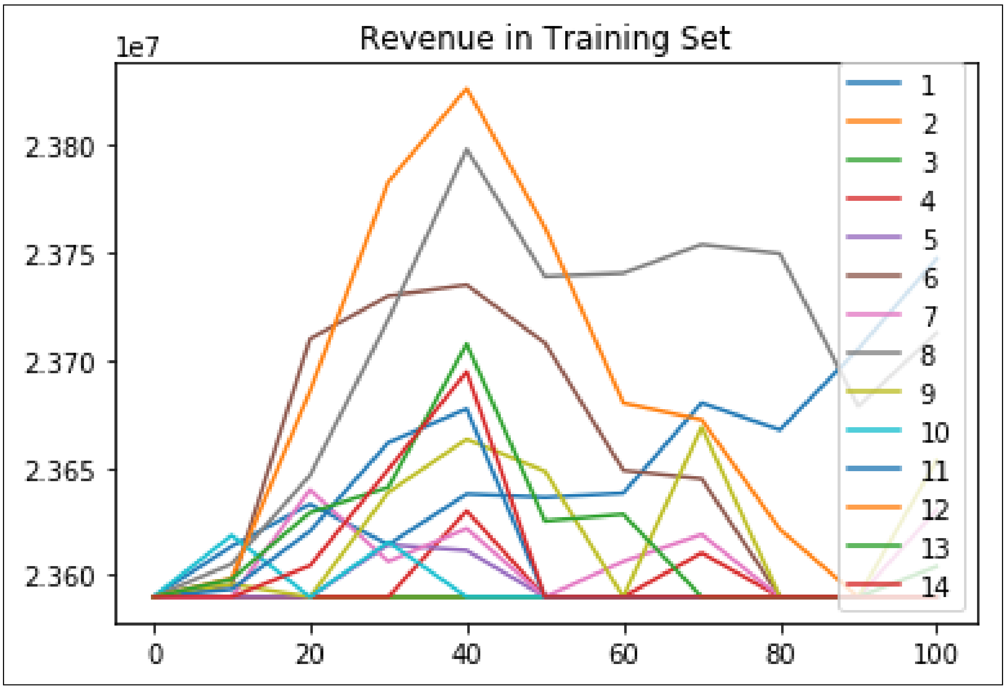


FIGURE B.14: Revenue in Training Set, RA Two-Basin Hydropower problem

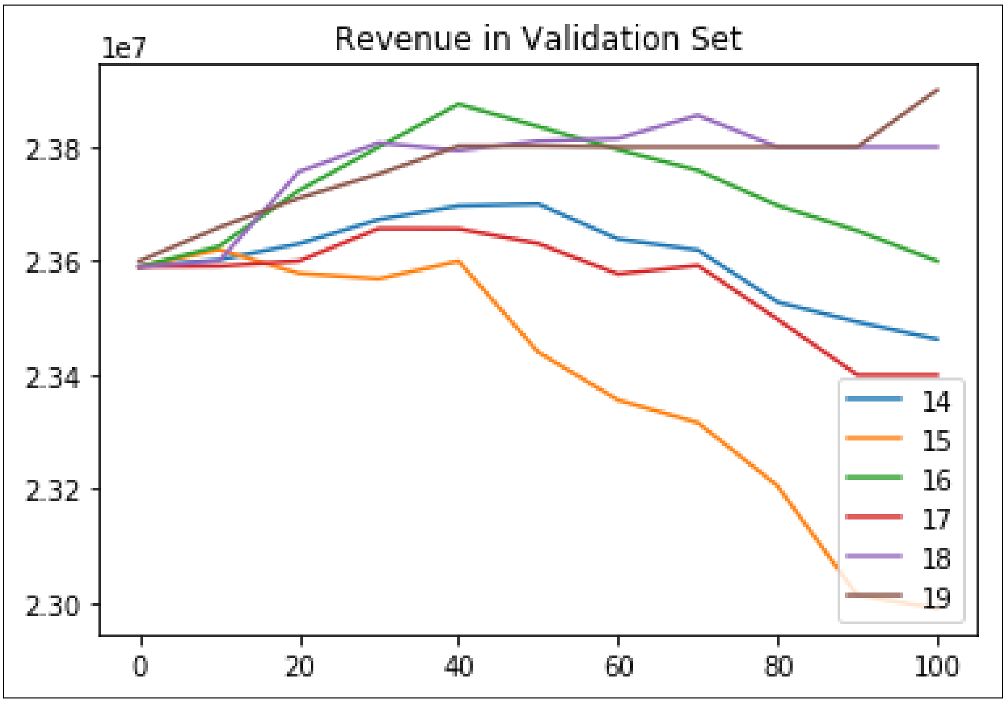


FIGURE B.15: Revenue in Validation Set, RA Two-Basin Hydropower problem

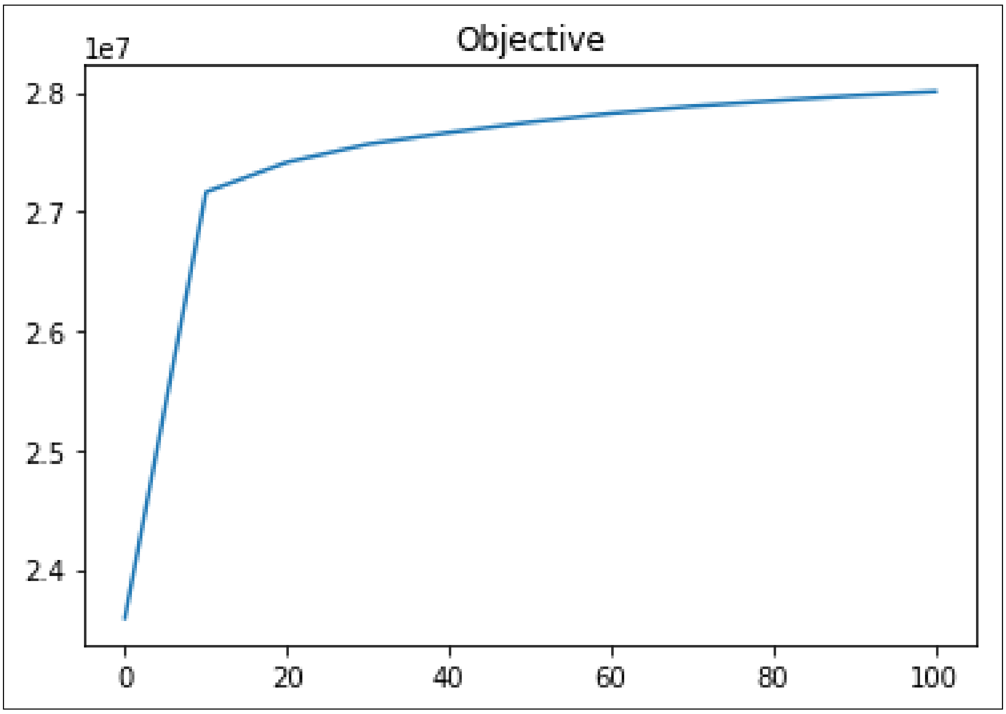


FIGURE B.16: Objective, RN Two-Basin Hydropower problem

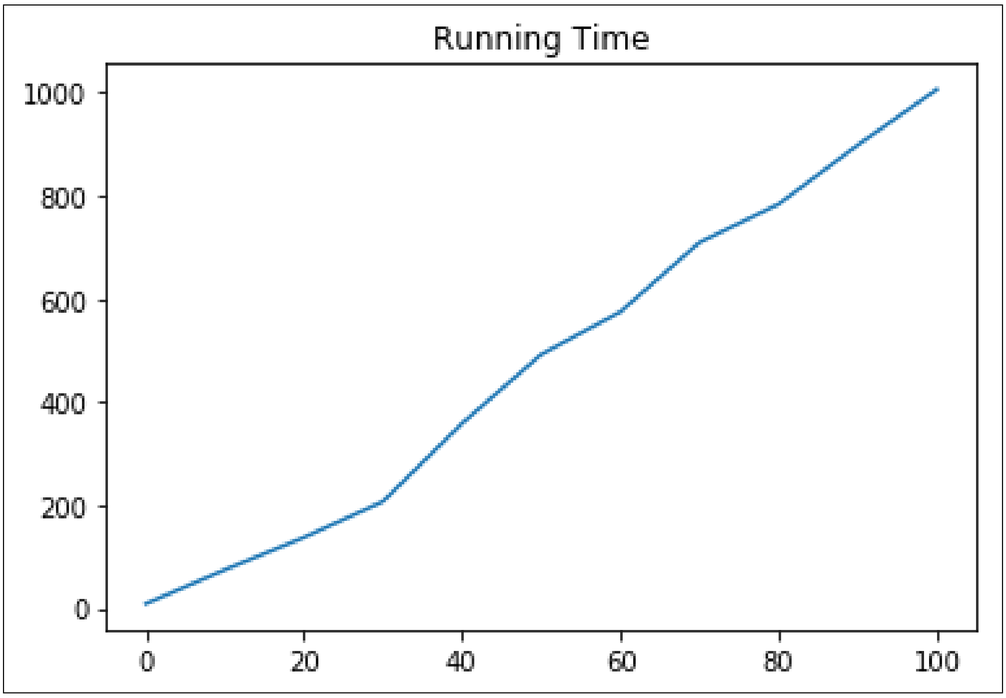


FIGURE B.17: Running Time, RN Two-Basin Hydropower problem

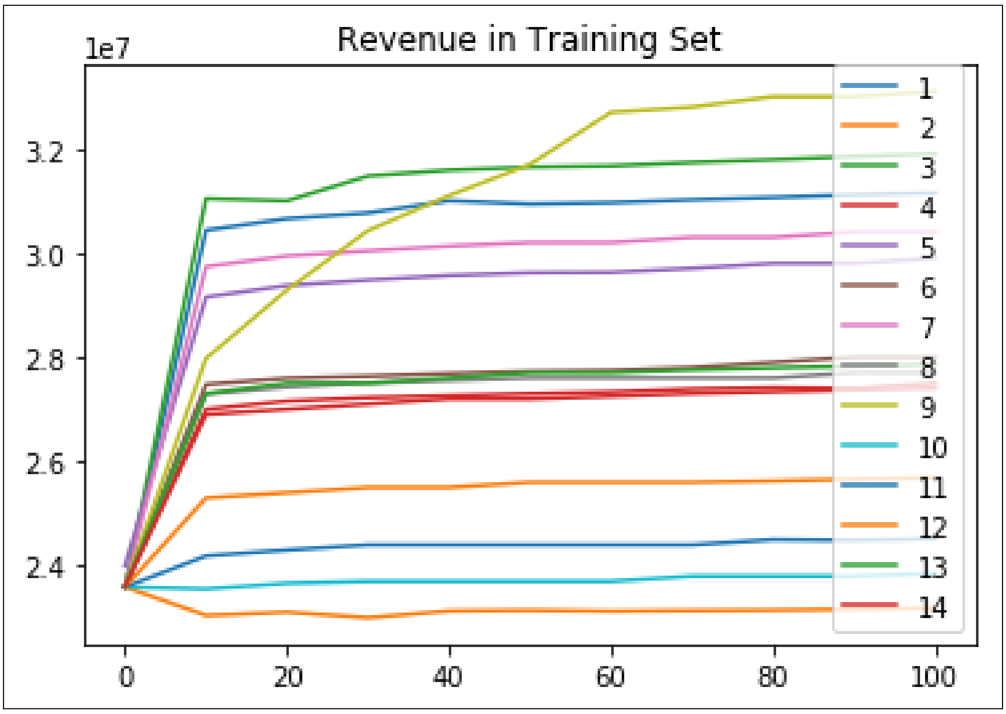


FIGURE B.18: Revenue in Training Set, RN Two-Basin Hydropower problem

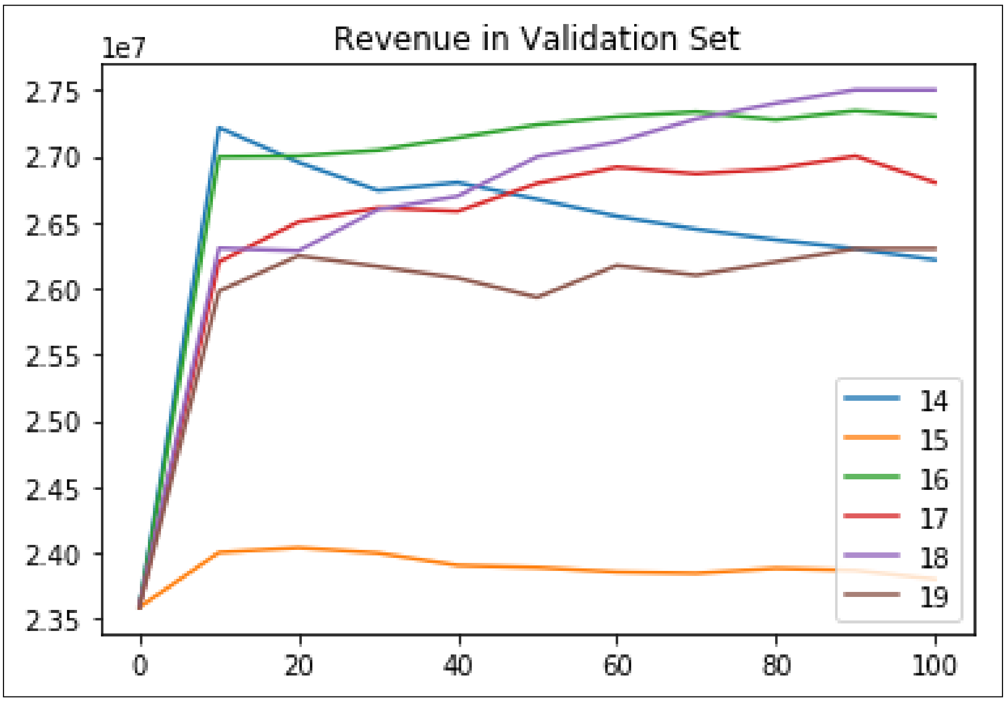


FIGURE B.19: Revenue in Validation Set, RN Two-Basin Hydropower problem

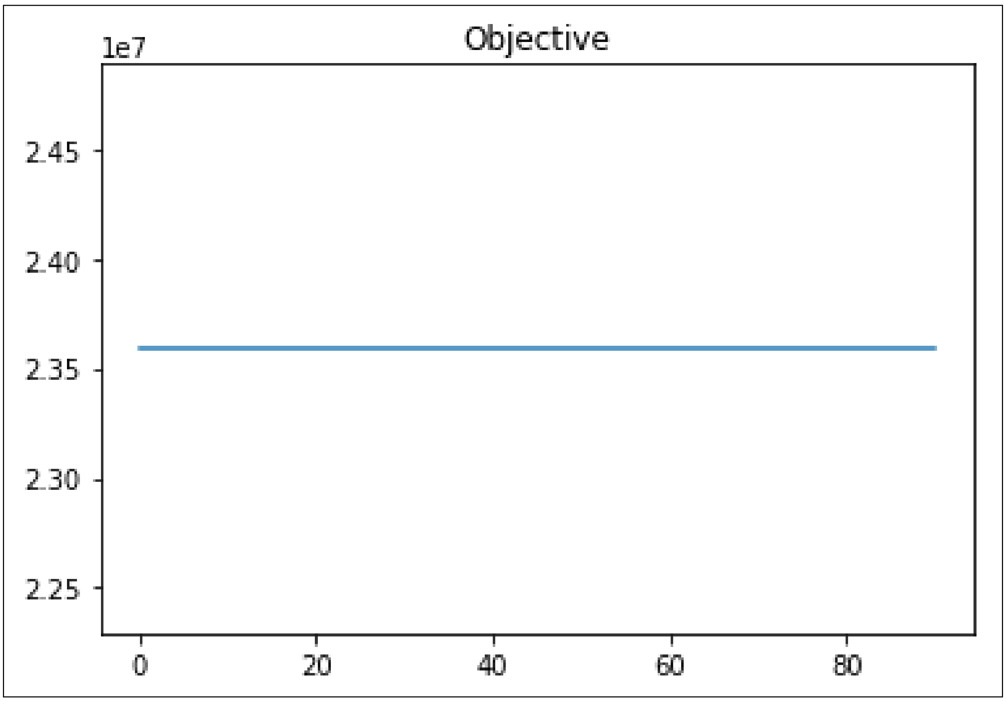


FIGURE B.20: Objective, RA Two-Basin Hydropower problem with regularization

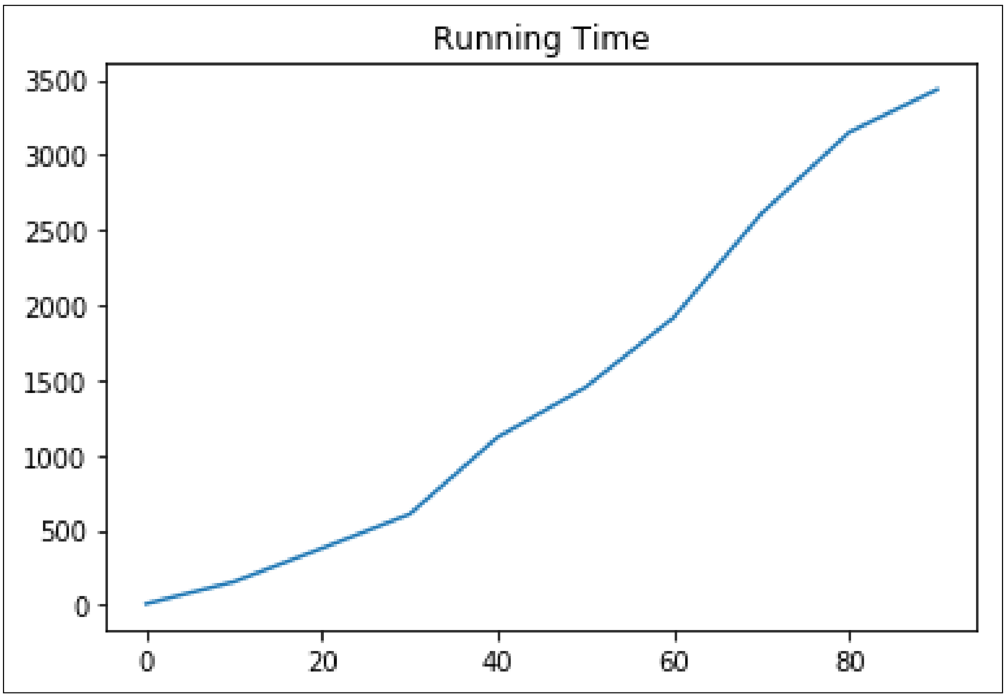


FIGURE B.21: Running Time, RA Two-Basin Hydropower problem with regularization

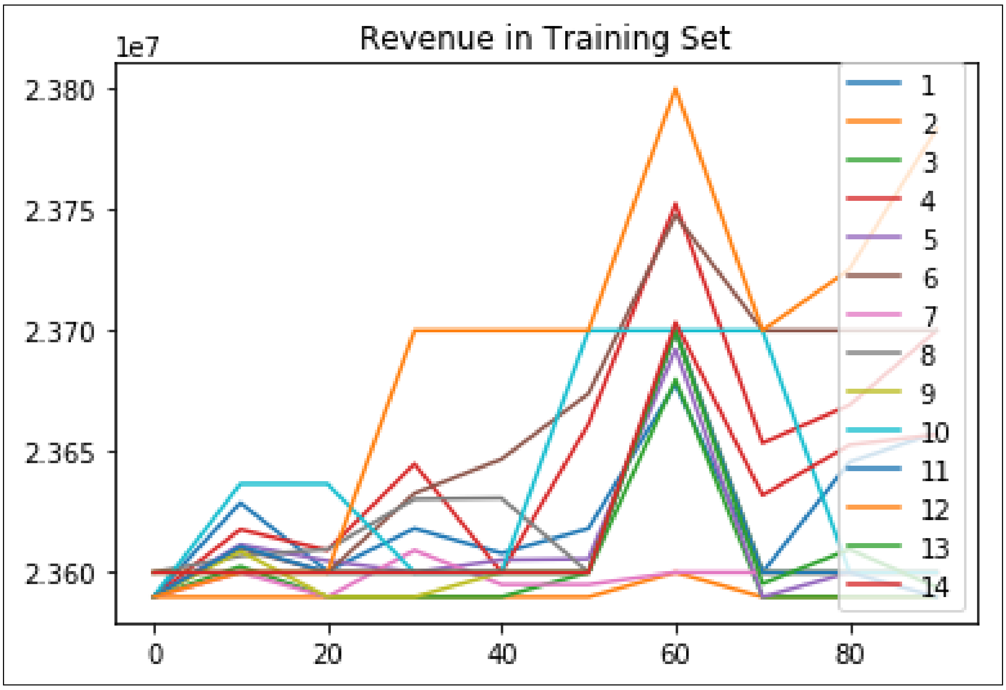


FIGURE B.22: Revenue in Training Set, RA Two-Basin Hydropower problem with regularization

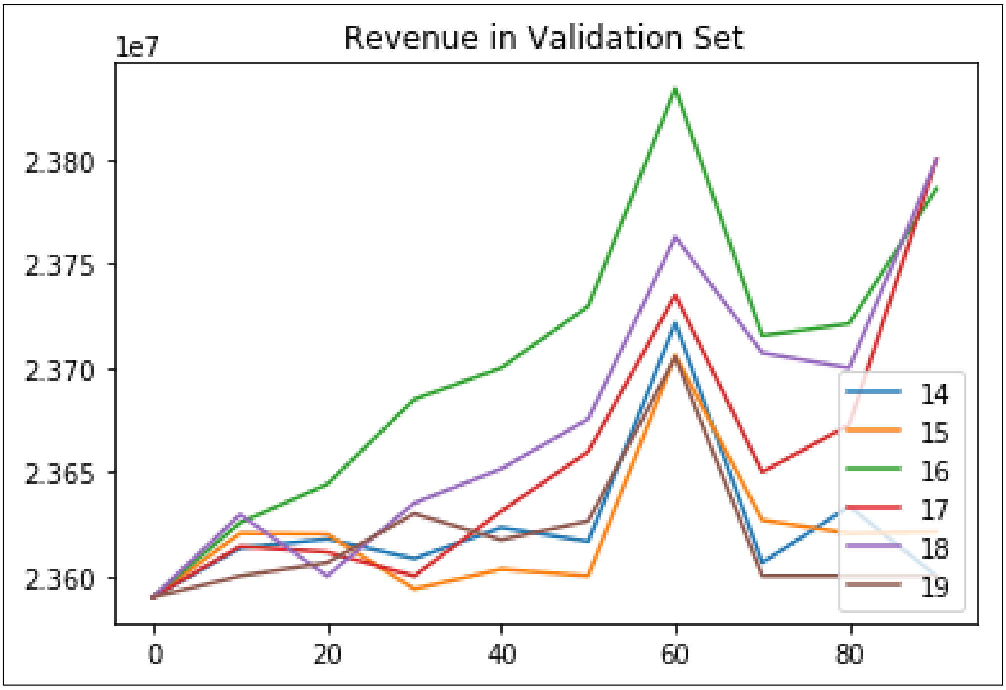


FIGURE B.23: Revenue in Validation Set, RA Two-Basin Hydropower problem with regularization

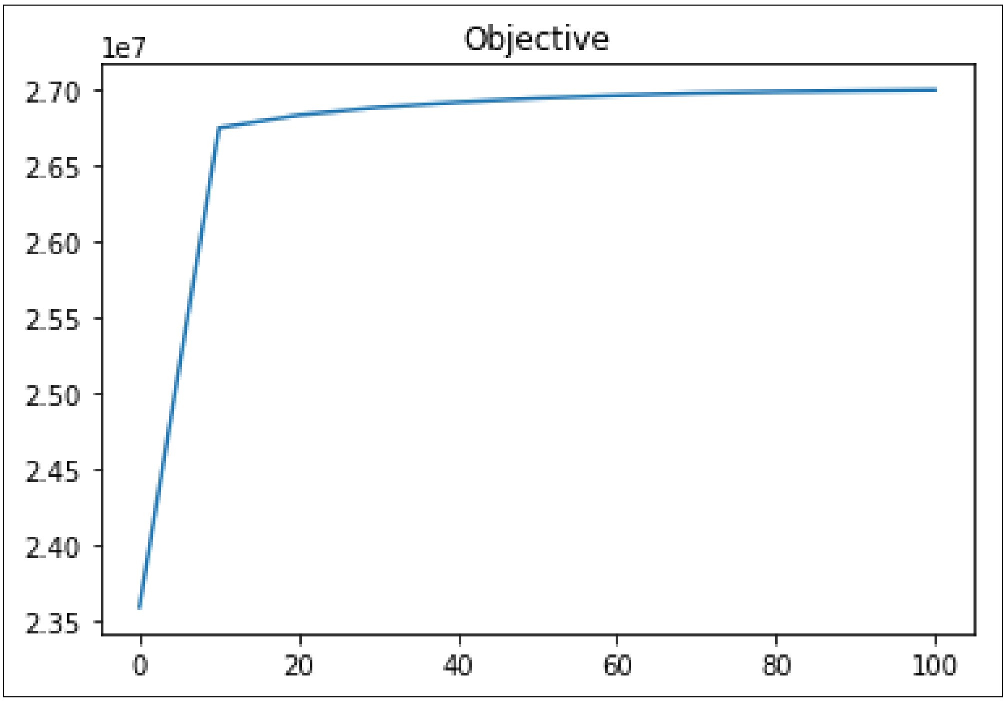


FIGURE B.24: Objective, RN Two-Basin Hydropower problem with regularization

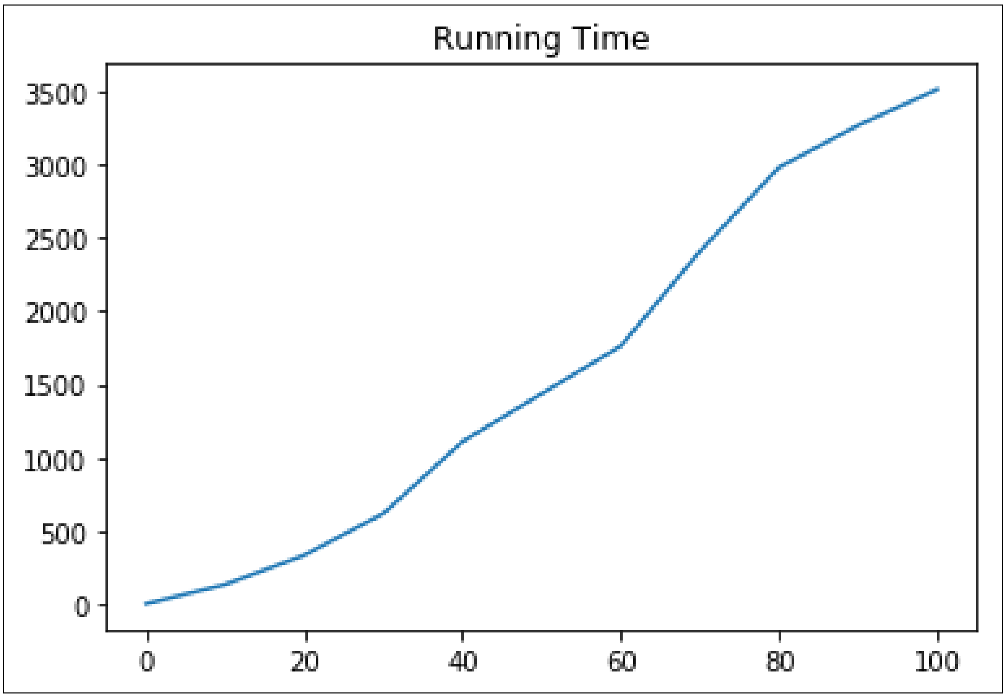


FIGURE B.25: Running Time, RN Two-Basin Hydropower problem with regularization

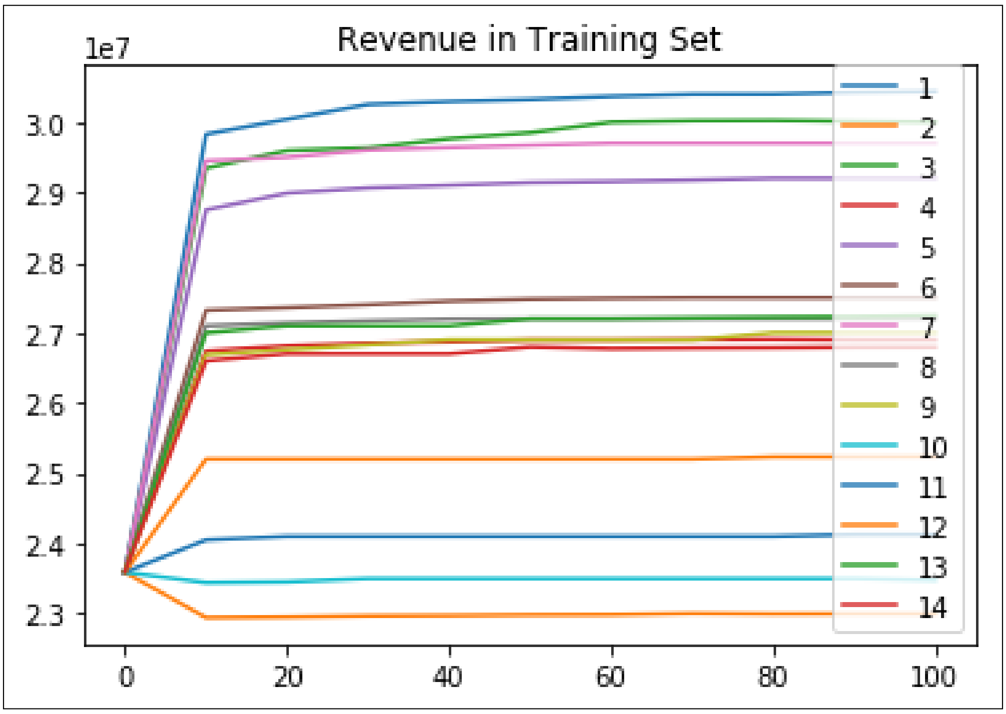


FIGURE B.26: Revenue in Training Set, RN Two-Basin Hydropower problem with regularization

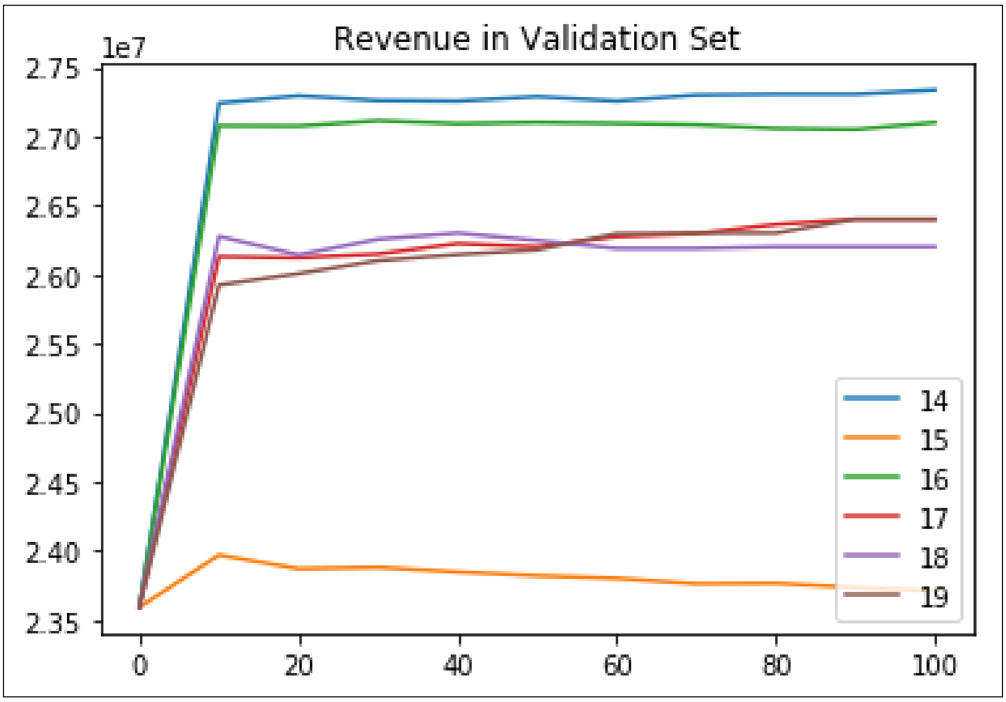


FIGURE B.27: Revenue in Validation Set, RN Two-Basin Hydropower problem with regularization

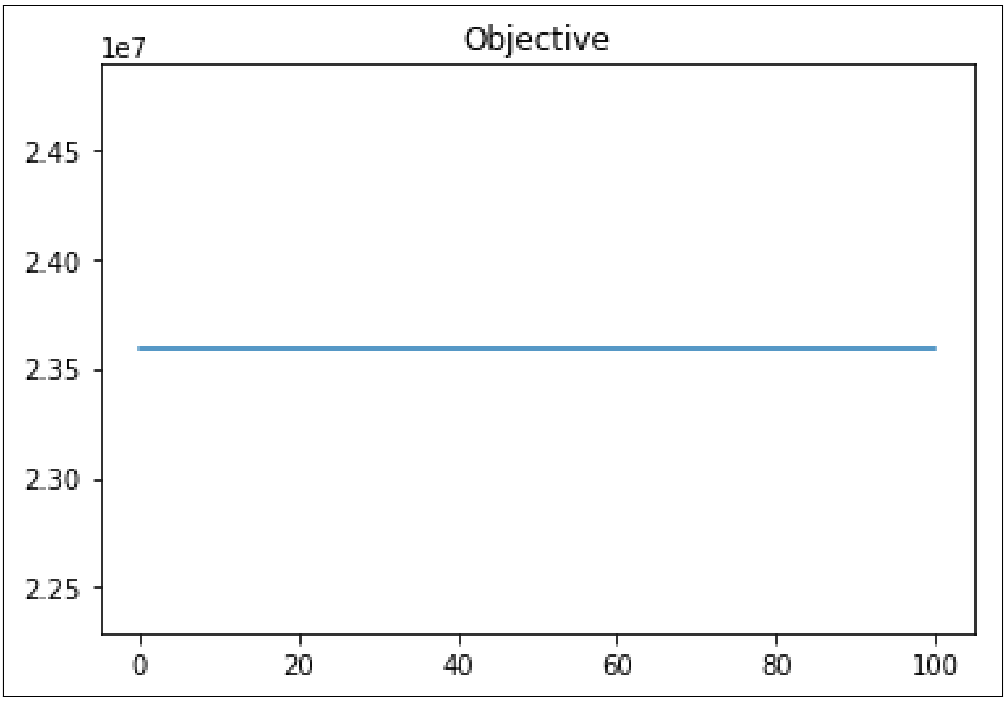


FIGURE B.28: Objective, RA Two-Basin Hydropower problem with Spillage Variable

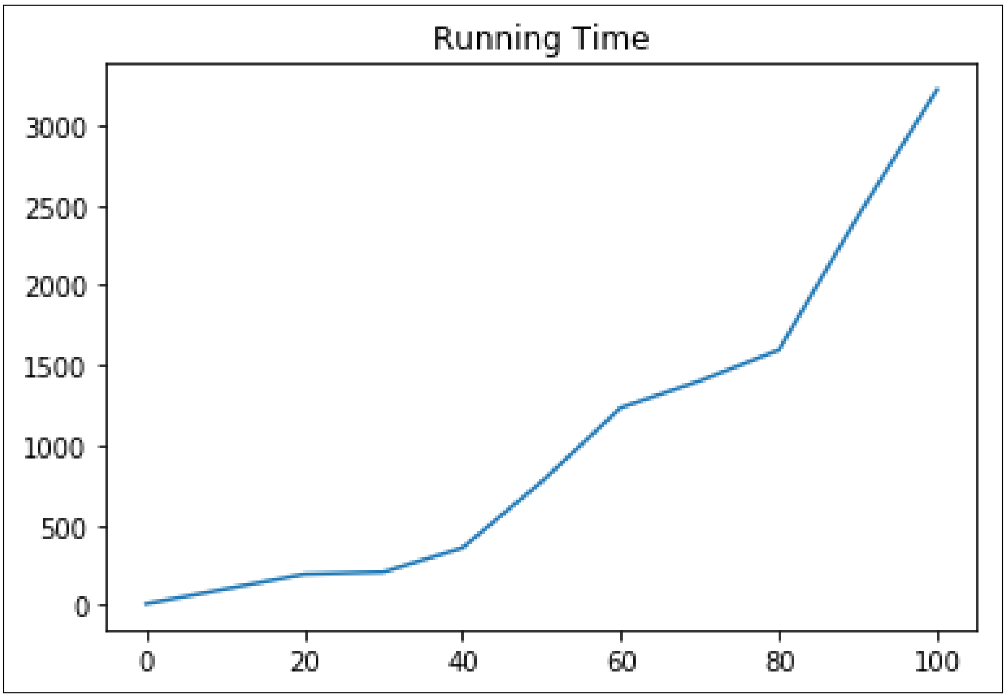


FIGURE B.29: Running Time, RA Two-Basin Hydropower problem with Spillage Variable

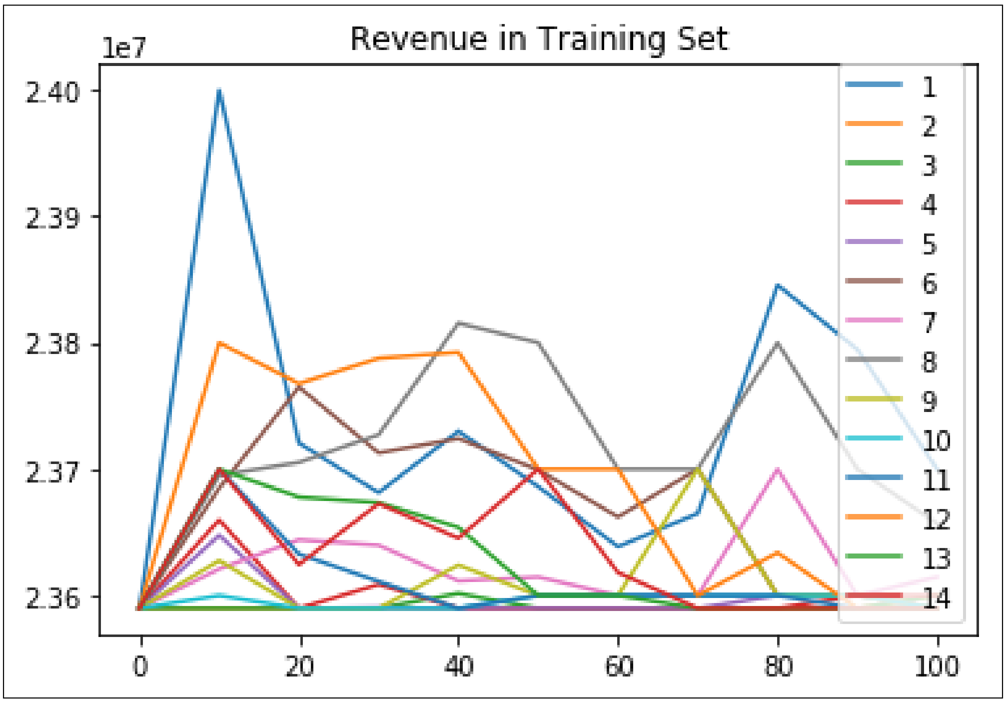


FIGURE B.30: Revenue in Training Set, RA Two-Basin Hydropower problem with Spillage Variable

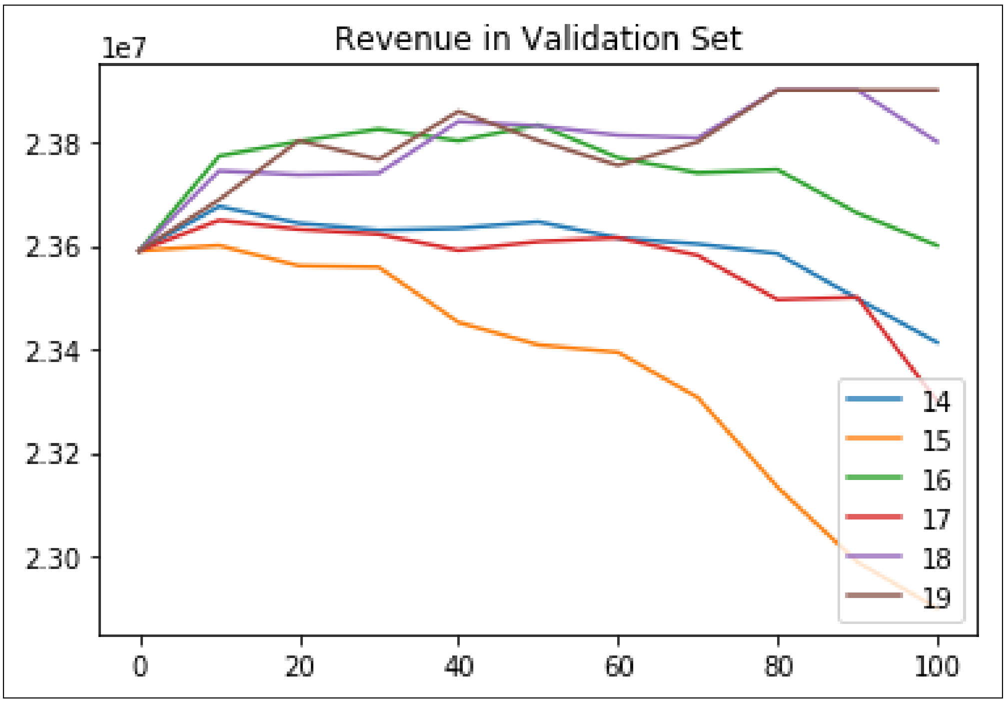


FIGURE B.31: Revenue in Validation Set, RA Two-Basin Hydropower problem with Spillage Variable

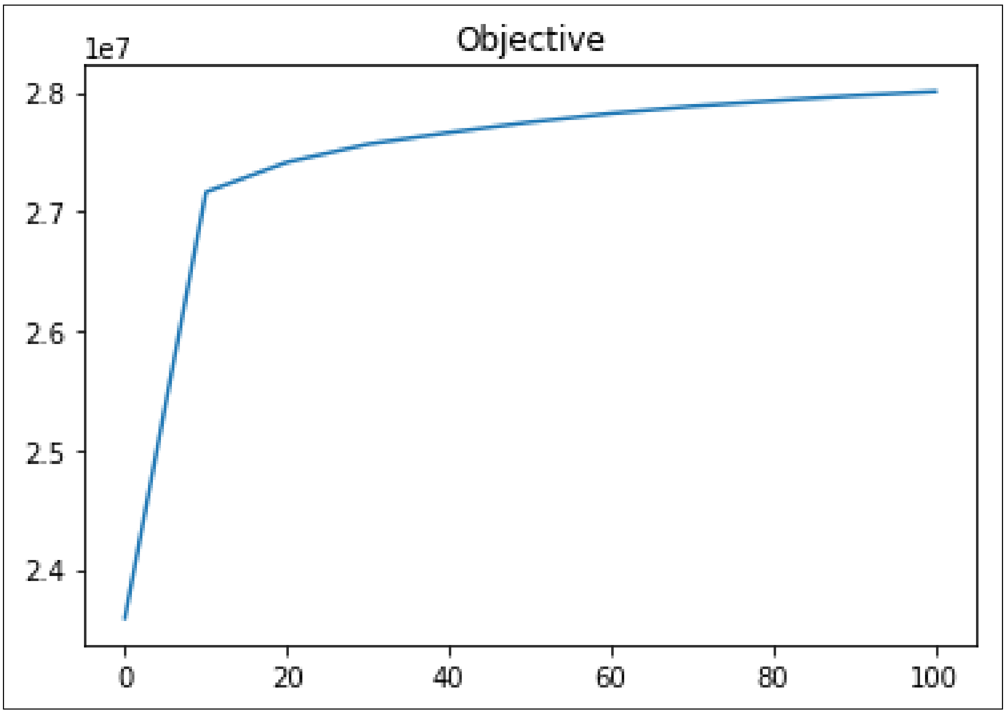


FIGURE B.32: Objective, RN Two-Basin Hydropower problem with Spillage Variable

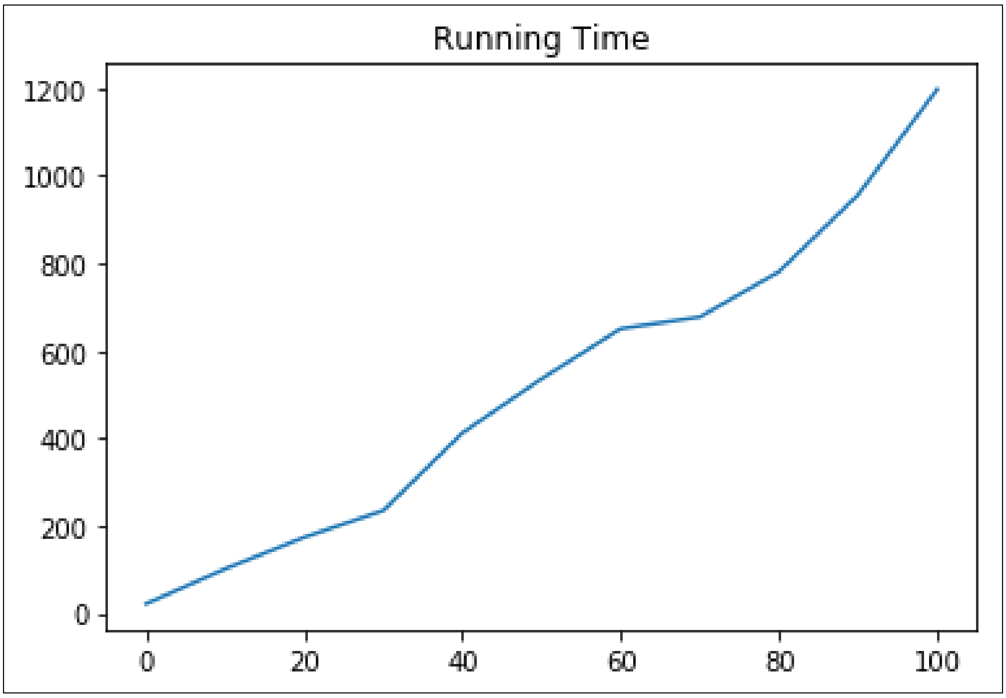


FIGURE B.33: Running Time, RN Two-Basin Hydropower problem with Spillage Variable

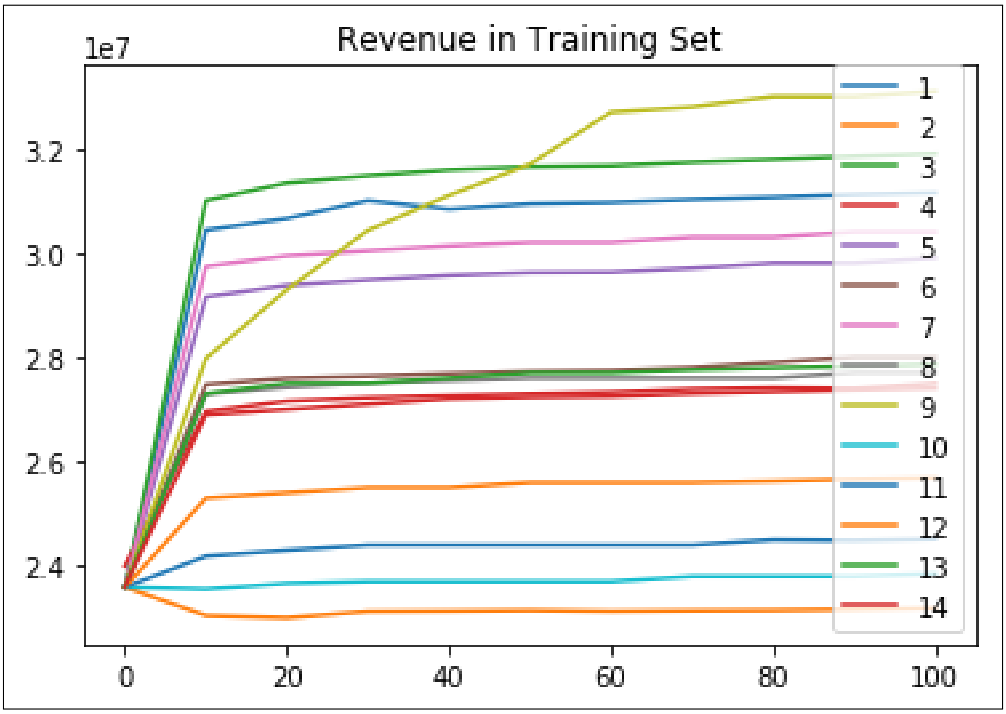


FIGURE B.34: Revenue in Training Set, RN Two-Basin Hydropower problem with Spillage Variable

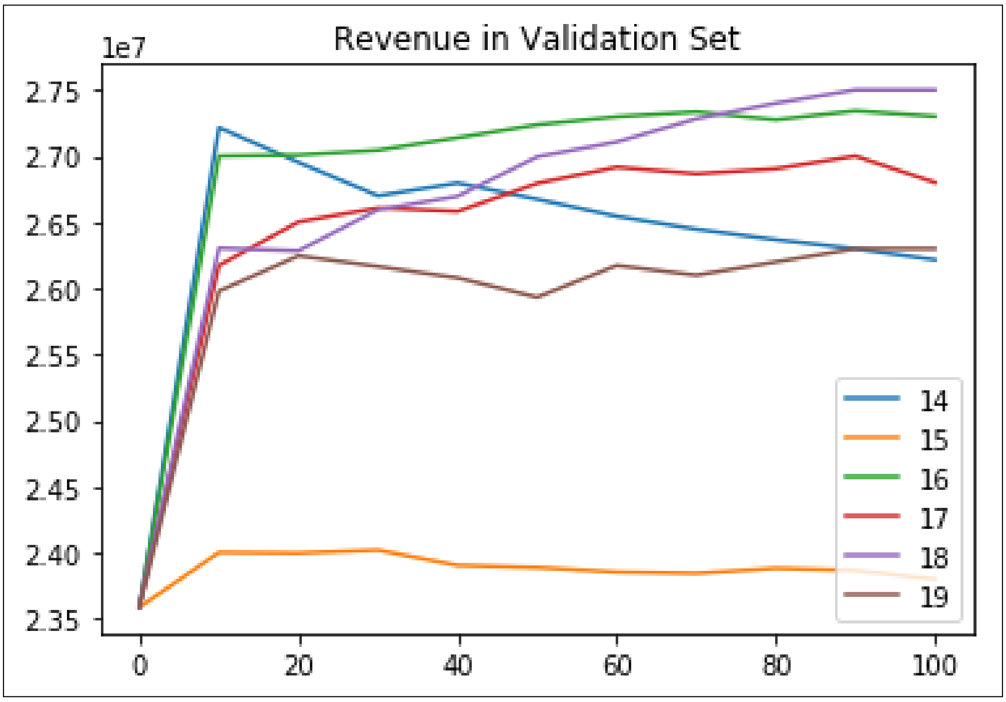


FIGURE B.35: Revenue in Validation Set, RN Two-Basin Hydropower problem with Spillage Variable

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