Educational Studies in Mathematics
https://doi.org/10.1007/s10649-023-10232-1

Preschool children's conceptions of the meanings and use of written numerals in everyday life: a phenomenographic study of the nature and structure of qualitative variation

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Accepted: 2 April 2023
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Abstract
Supporting children's understanding of the everyday, cultural use of written numerals is highly significant, as it is this understanding that gives meaning to classroom conversations on the purposes of written numbers. This paper presents findings from a phenomenographic study of the qualitatively different ways in which 3–5-year-old children interpret the meanings and use of numerals in everyday contexts. The study involved a volunteer sample of 37 preschool children. With their family's support, children played a Number Spotting game, taking photographs of numerals in their environments. These photographs were supplemented with other photographs selected by the researchers and used in individual photo-elicitation interviews with children. We collected data on children's interpretations of a range of examples of numerals used to denote quantity, order and measurement, and numerals used as labels/identifiers. The findings document qualitatively different categories that capture the range of children's expressed conceptions as well as the critical aspects of variation that underpin *how* qualitatively different categories of conceptions differ or relate to each other. The study provides original insights into the nature and structure of children's awareness of the cultural uses of written numerals. The findings can support early mathematics teaching to make meaningful connections between the knowledge that children develop outside of school and the new knowledge about written numbers that they develop in formal education.

Keywords Written numerals · Early mathematics · Cultural rules of written numbers · Phenomenography · Qualitative research · Preschool education

1 Introduction

Children's understanding of the rules and conventions that underlie the use of written numbers is a constitutive element of early mathematics competency and essential for accessing mathematics learning (Worthington et al., 2019). The term numeral refers to "a symbol

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or string of symbols representing a number" (McCloskey & Macaruso, 1995, p. 351). In this study, we use the term written numerals or written numbers to refer to Hindu-Arabic numerals in digit form (e.g., 6, 181). Written numerals are cultural artefacts and form part of a broader system of mathematical notations that is used to represent and communicate mathematical relations and different quantitative and non-quantitative ideas (Brizuela, 2004; Tolchinsky Landsmann & Karmiloff-Smith, 1992).

In their family home, the preschool environment and in public spaces, young children encounter written numbers on clocks, books, games, the TV, on houses, and various kinds of digital displays. Bialystok (2000) notes that to understand any conventional representational system and use it, children need to recognize that the notational forms are symbols that convey information as well as understand the rules that govern the notational system itself. In the case of written numerals as representational symbols, children do not only need to understand the mathematical conventions that govern the written number system, but they also need to recognize that numerals communicate specific information that varies, within everyday situations.

Munn and Kleinberg (2003) distinguish between primary and cultural rules related to early number learning. Primary rules refer to the mathematical rules of number and the written number system itself. These are usually taught at school as part of formal education and in relation to arithmetic, while cultural rules "tell us how to use a system and what its role is in our culture" (p. 51). Cultural rules are about what numbers are used for, how they are used and who uses them.

Examining how young children interpret and understand the cultural rules that underlie the use of numerals in everyday life is highly significant and in the early years, "cultural rules are possibly the most important thing for children to learn" (Munn & Kleinberg, 2003, p. 53), as they provide the fundamental basis upon which school learning is built. While children's understanding of the primary rules, i.e., the conventions underlying the written number system has been researched (e.g., Brizuela, 2004; Johansson, 2005; Mix et al., 2014; Tolchinsky, 2003; Zhou & Wang, 2004) and theorized for mathematics pedagogy (Coles & Sinclair, 2017; Hiebert, 1988), there is limited research about children's understanding of the cultural uses of written numerals in everyday contexts (e.g., Sinclair & Sinclair, 1984, 1986).

Sinclair and Sinclair's (1984, 1986) research revealed age-related differences in responses provided by 4–6-year-old children about the meaning of environmental numerical print in photographs selected by the researchers and the frequency of different types of responses across age groups. Notwithstanding the significant contribution that their research has made, that study did not provide a detailed insight on the qualitative nature and variation of different responses.

The current study aims to address this research gap by adopting a phenomenographic approach to the research design and analysis, combined with photo-elicitation interviews with children for the collection of data. Marton (1986) describes phenomenography as a research approach "for mapping the qualitative different ways in which people experience, conceptualise, perceive, and understand various aspects of, and phenomena in, the world around them" (p. 31). Therefore, in phenomenography, a phenomenon is "an aspect of the world," as experienced and conceived by people (Marton, 1986, p. 33). Bishop (1988, p. 182) has argued that there is a need for mathematics education research to view and study mathematics as "an example of a cultural phenomenon," highlighting that written language and mathematical symbolism are conceptual tools that constitute an important "technological" component when engaging with mathematics as a cultural product.

Our study does not examine children's engagement with mathematical activities and problems. However, in line with the above ideas that emphasize the social and cultural dimension of written mathematical symbolism, we align with the view that written numerals, as part of a system of symbols used to convey number-related meanings, are social and cultural constructions (Brandt & Tiedemann, 2009). Beyond their use in the mathematics classroom, written numerals are used in everyday life, to communicate specific, socially and culturally developed number-related meanings and information (e.g., the numeral 20 on a speed sign denotes the speed limit, in miles per hour, that needs to be adhered to in a particular section of a road). Information that written numerals convey in everyday life is related to notions that may have mathematics-related meanings (e.g., measurement, order, quantity) or not (e.g., numeral-identifier on a bus or t-shirt). Such information and the rules that underpin it determine how aspects of everyday life within societies and cultures are organized and function. Therefore, in the context of this study, we consider that the uses of written numerals in everyday life constitute a conceived and experienced sociocultural "phenomenon."

Based on the phenomenographic assumption that people conceive phenomena in the world around them in different ways, the study is original and significant in aiming to provide evidence for the range of qualitatively different conceptions that preschool children express about written numerals and their everyday uses, as a phenomenon that they experience in the world around them (before they go to school). In line with phenomenographic research, where the focus is on capturing the "collective sum of ways of experiencing" (Åkerlind, 2008, p. 635), our aim is to examine the qualitative range, and variation within the range of children's collective interpretations, as expressed within our participant group. Thus, an examination of the mechanisms through which individual participants' prior sociocultural experiences, contexts, and factors, such as home interactions, curriculum content, and socioeconomic or cultural background may have shaped their conceptions of the meanings of numerals in everyday life, and an analysis of individual differences, are beyond the scope of the analysis presented here.

In addition to mapping the qualitatively varied ways in which people conceive the same phenomenon, the goal of phenomenographic research is to examine and depict "the very structure of the variation itself" (Marton, 1986, p. 41). Therefore, we consider that this methodology is appropriate and will enable us to apply an original analytical lens to document the range of qualitatively varied ways in which children conceive the possible meanings and uses of written numerals in everyday life, but to also reveal the structure of children's conceptions. That is, to unearth the underpinning aspects of knowledge and kinds of awareness that our research participants draw from (Collier-Reed & Ingerman, 2013), in making sense of written numbers in their environments. The study is significant in aiming to provide an evidence-based account for *how* qualitatively different conceptions of written numerals in everyday life may differ or relate to each other. This is necessary for developing evidence-informed pedagogical action in the early years that can help children develop an understanding of written numerals that goes beyond, but also connects with, their mathematics learning in the classroom.

The study presented here seeks to answer the following research questions:

1. How do preschool children interpret the meaning and uses of written numerals in their everyday environments?
2. What qualitative variations can be identified in children's expressed interpretations and what is the underpinning structure that delineates these variations?

2 Young children's engagement with number and written numerals in their everyday environments

In this section, we present examples of previous research on contexts that contribute to young children's introduction to numbers with everyday meanings and purposes, outside of the classroom. We also summarize previous research that has examined specifically young children's attention to and understandings of the purposes of *written numerals* outside the classroom context. We explain the positioning of our study and how it extends the existing field of knowledge.

2.1 Contexts that contribute to children's introduction to number in their everyday life

In the early years, play and informal activities that young children participate in with their families, such as everyday chores, meals, and shopping, as well as children's interactions with others as part of daily life, provide extensive numerical information and support the development of children's understanding of number-related ideas outside the classroom (Black et al., 2019; Ramani & Siegler, 2015; Saxe, 2004).

An ethnographic study conducted by Worthington and van Oers (2016) provided evidence on the culturally embedded nature of number-related ideas that 3–4-year-old children form, by examining the cultural mathematical understandings and communications that emerge in children's pretend play. The study showed that children's cultural, everyday knowledge provided contextual and mathematical meanings in their spontaneous play and influenced their mathematical thinking. In this domain of the everyday concrete and empirical, spontaneous, everyday concepts develop, which can later lay the foundation for developing scientific concepts through instruction (Daniels, 2007).

However, research has also shown that the everyday, contextual knowledge that contributes to children's introduction to numbers in their everyday lives can diverge from the meanings that children are introduced to by school mathematics instruction and early mathematics curricula. One indicative example is research conducted by Brenner (1998), who explored the meaningfulness of the everyday mathematical topic of money for American preschool and elementary school children. The findings indicated that children began school with already-developed (although not mathematically sophisticated) everyday common sense and values about money. However, as part of the school curriculum, children encountered money-related activities and kinds of money that were not meaningful for them, creating a disconnection between everyday life and school. In particular, while in everyday life and shopping contexts children were exposed to quarters and dollars, classroom activities focused on practice with pennies, towards which children had a derisive attitude "calling them 'junk' and delegating them to the toy box with the play money" (Brenner, 1998, p. 148). The study thus provided evidence for a marked difference between knowledge and values stemming from children's everyday experiences and their learning in school.

Parental values related to everyday knowledge about numbers and their influence on number-related practices at home have also been explored. For example, in a study on numeracy practices and interactions in relation to measurement concepts at school and at home, Meaney (2011) showed how the increased frequency of discussions within a 6–7-year-old child's home environment focusing on telling the time was impelled by the need to adhere to societal expectations (being at school on time), and the mother's values

and beliefs about how telling the time should be learnt. Similarly, the influence of parental beliefs and values on everyday mathematics in the home environment was discussed in a study by Lembrér (2020). The findings showed that parents' perspectives converged on emphasizing the value of mathematics-related activities at home that had everyday, societal value. Using interview methods similar to those we use in our study, Lembrér asked Norwegian parents to take photographs of their children's engagement with mathematical activities at home. With the use of these photographs in photo-elicitation interviews, the study showed that parents most valued opportunities offered by everyday activities for the development of children's knowledge of number names, their writing of numerical symbols and their skills in measuring and using money.

Studies such as the above have shown that the understandings that young children develop about numbers relating to everyday life are socially and culturally influenced. They also highlight that children's mathematical experiences at home vary and may differ from the mathematics-related experiences that children have at school. Therefore, in this study, we accept the position that knowledge is "embedded in cultural forms of thinking entangled with a symbolic and material reality that provides the basis for interpreting, understanding and transforming the world of the individuals and the concepts and ideas they form about it" (Radford, 2018, p. 4067).

Building on this, the current research moves beyond an examination of sociocultural influences on children's understandings of *number* meanings in their everyday life, to an exploration of children's conceptions of the meanings that *written numerals* have and communicate in everyday life, as the children themselves share and express these. The phenomenographic approach that we adopt for our study "agrees that experience is historically, socially and culturally situated, but it doesn't investigate the situated nature of experience" (Åkerlind, 2022, p. 7). Rather, phenomenography has "a focus on collective experience, not individual experience, and expects different ways of understanding a phenomenon to vary with sociocultural context" (Åkerlind, 2022, p. 7).

While previous research, such as the studies discussed here, has focused on everyday number-related meanings (e.g., money, time), very few studies have explored children's understandings of the use of *written numerals* for the communication of concepts such as money, time, speed, size, etc. in everyday environments and this is where our study's contribution is situated. The following section summarizes previous research that has focused specifically on children's attention on *written numerals* in everyday contexts.

2.2 Children's attention to written numerals and understanding of the purposes of numerals in everyday contexts

In everyday life and before school, children encounter written numerals in their environment that may have quantitative purposes, to denote, for example, cardinality, but they also encounter numbers in the environment that serve "social uses" and "nonquantitative purposes" (Tolchinsky, 2003, p. 110) and which are underpinned by social knowledge, such as, numbers on buses, phone numbers, and barcode numbers. Tolchinsky (2003) further notes that the meanings and purposes of written numerals in everyday life constitute part of the cultural and social knowledge that is "embodied in the many cultural artefacts used in daily life and transmitted through social uses" (p. 110). In their attempt to make sense of these different social and cultural uses of written numerals, children actively create their

own understandings and interpretations of mathematical notations rather than receiving and copying information from their environments (Brizuela, 2004).

Sinclair and Sinclair's (1984, 1986) work is a rare example of research exploring how children describe the meanings of written numbers in everyday situations. Notably, they highlight that the different uses of numerals in children's environment, where numbers can refer not only to quantitative ideas but also (and more frequently) to order (e.g., floor numbers), measurement (e.g., weight), or labels (e.g., football shirts), are "in marked contrast with the written numerical material children are presented with in school where written numerals represent cardinalities and where their use is restricted to more or less relevant or interesting (but generally artificial) computations" (p. 68).

They asked 4–6-year-old children to describe the meaning of written numerals in photographs depicting examples of numerical environmental print (e.g., bus numbers, price tags). They found that as children construct their understanding of the written number system, they "work on the form of the symbols, the rules for their combination and use, linking them with their verbal counterparts, and integrating and coordinating information provided by other people on the function and meaning of particular environmental numerals" (Sinclair & Sinclair, 1986, p. 70).

Children's responses were classified into five categories: (1) *No response*; (2) *Description of numeral* (comments on the numeral itself but not on its function); (3) *Global function* (attributing a function that is very general or unclear without specifying one possibility); (4) *Specific function* (indicating the specific information that the numeral provides — amount, order etc. — and determining one possibility among others); (5) *Tag* (interpretation of the numerals as the name of the object on which they appeared) (Sinclair & Sinclair, 1984, p. 177). The researchers conducted an analysis of the frequency of different types of response for each age group (4, 5, and 6 year olds) and found that *Specific function* responses were associated with age, thus indicating a developmental trend. They also found that while *Global function* and *Specific function* categories overlapped, solely *Global function* responses were given earlier, before they were combined with *Specific function* responses and gradually disappeared with age. Sinclair and Sinclair (1986) noted that when responses are more precise, "they specify the types of information that is indeed transmitted by numerals in the environment, even when the response may be called "incorrect" from an adult point of view" (p. 70).

Ewers-Rogers and Cowan (1996) examined 3–4-year-old children's understanding of the social uses of written numbers in the environment by exploring whether children noticed the absence of written numbers in photographs of objects that normally have written numbers on them. They subsequently revealed the same photographs with the written numeral on the objects to children and asked them about the meaning and purpose of the numerals. They found that children rarely noticed the absence of numerals in photographs. However, in line with Sinclair and Sinclair (1984, 1986), their findings suggested that preschool children have considerable knowledge about numerals in everyday situations and know that numerals are not only used to represent quantity. For the written numbers that children provided more specific explanations for, they concluded, like Sinclair and Sinclair (1986), that this was in cases where the children appeared to know about these numerals through discussions with adults.

Recent research (Rathé et al., 2019, 2020) has also examined young children's spontaneous noticing and focusing on Arabic number symbols (SFONS) but with a focus on the role that this has on children's early mathematics development. SFONS is defined as "children's tendency to spontaneously focus their attention on Arabic number symbols in their everyday environment without being explicitly prompted to do so" (Rathé et al., 2020, p.

39). SFONS relates to children's tendency to focus on numerals in everyday contexts and not to their capability to recognize and use numerals within mathematics-specific tasks. A 3-year longitudinal study with 4–5-year-old children in Belgium found that high SFONS was positively related to children's skills in counting and mathematics achievement and children's performance in the SFONS task was not confounded by their knowledge of Arabic numerals. Based on these findings, it was concluded that tendencies to spontaneously focus their attention on numerals in their everyday environment is a contributing factor to early mathematics development differences among individuals.

The studies conducted by Sinclair and Sinclair (1984) and Ewers-Rogers and Cowan (1996) on the social uses of written numbers, and Rathé and colleagues' (2019, 2020) studies on children's spontaneous attention to Arabic numerals, are framed within differing paradigms and research purposes. However, they have all highlighted the importance of understanding how young children conceive the purposes of numerals in everyday contexts and their tendencies for focusing their attention on numerals in their environment. Research on children's understanding of the social purposes of numerals has not provided an in-depth analysis of the range of qualitative variations in how preschool children interpret the different uses of written numerals in the world around them, including in their own everyday activities. To this end, the current study aims to adopt a qualitative methodology that involves children in the collection of visual material that is subsequently used for the collection of data, bringing their own experiences closer to the centre of the research.

3 Theoretical and methodological underpinnings

Phenomenography aims at examining people's ideas about a phenomenon and experiences of it, thus focusing on describing qualitative variations in people's interpretations of reality (Marton, 1981). Within phenomenography, learning is viewed as a change in awareness (Marton & Booth, 1997), that is, the new ways in which an individual comes to see, becomes aware of, or understands a phenomenon, "coming to see new features and relate them to one another and to the whole, as well as to the wider world" (Booth, 2004, p. 13). Therefore, conceptions of a phenomenon are constructed and reconstructed through individuals' social activity and experiences within their everyday contexts.

Different individuals understand the same phenomenon differently in different situations and may be simultaneously aware of certain aspects and parts of the phenomenon rather than the whole, resulting in different patterns of awareness (Åkerlind, 2018). Therefore, by examining different patterns of awareness and variation in these, phenomenography aims to present a holistic, and collective description of how a phenomenon is experienced and understood rather than to describe individual experiences (Orgill, 2012).

In phenomenographic analysis, conceptions are considered as "*categories of description* to be used in facilitating the grasp of concrete cases of human functioning" (Marton, 1981, p. 177, emphasis in the original). Marton (1981) specifies that individuals move between categories on different occasions. Therefore, categories of description or the varying forms through which individuals perceive reality "should not be considered as categories of classifying individuals, but as categories for describing ways of perceiving the world around us" (Marton, 1981, p. 195).

Marton and Pong (2005) indicate that "a conception can be characterised as composed of both a referential aspect — i.e., a particular meaning of an individual object (anything delimited and attended to by subjects) — and a structural aspect — i.e., the combination of

features discerned and focused upon by the subject" (p. 336). The referential aspect focuses on *what* is experienced while the structural aspect refers to *how* individuals think about the phenomenon. The structural aspect of a phenomenon is defined by the internal and external horizons (Taylor & Booth, 2015). The internal horizon is the internal relationship between the different parts of a phenomenon, "how the parts are distinct from each other, and how the parts jointly form a cohesive entity" while the external horizon, "the 'discernment of the whole from the context' enables the phenomenon to be distinguished from its context and background" (Marton & Booth, 1997, p. 87). Through experience of the parts, the whole and the relationships between them, further degrees of meaning can be discerned (Marton & Booth, 1997).

In our study, we operationalize these notions as follows: The "phenomenon" is the use of written numerals in everyday life. The referential aspect is depicted in the *categories of description* that will emerge from the analysis of data and which encapsulate the meaning of written numerals that children are aware of and express. The structural aspect is *how* children are aware, it refers to the structure of their interpretations, the elements of the phenomenon that children discern and focus on, and the critical aspects that inform the structure of their awareness, with regards to the written numerals. The external horizon, "the specific context or contexts in which the phenomenon can be found" (Hajar, 2021, p. 1424), is the contexts and situations within which numerals are used and which make their meanings vary. The internal horizon can comprise of the digit(s) of the written numeral, the object on which it appears, other symbols that may accompany the number (e.g., £) or the format of the number (e.g., car number plates).

Both the referential and structural aspects are depicted in the results of phenomenographic research which are presented in the form of an *outcome space*. The *outcome space* is usually presented as a diagram that depicts the qualitatively different categories of description and the structural, logical relationship between categories (Han & Ellis, 2019). Collier-Reed and Ingerman (2013) note that categories of description within the outcome space have both structure and meaning and should depict the *critical aspects*, that is, the fundamental differences between different ways in which people make sense of a phenomenon.

A phenomenographic approach has been used in previous mathematics education research to examine how children experience numbers within the context of arithmetic and how these experiences underpin the development of arithmetic skills (Marton & Booth, 1997). Neuman (1999) used a phenomenographic analysis of primary school children's awareness and understanding of division, providing a model that depicts the increasingly sophisticated ways in which primary school children experience simple partitive and quotative division problems. Research by Herbert et al. (2015) illustrated different ways in which teachers perceive mathematical reasoning and the dimensions of variation that separated the categories from each other, providing, thus, a framework for assessing and tracking teacher's perceptions of mathematical reasoning as part of professional learning programmes.

Our study differs from previous phenomenographic studies in mathematics education as it does not focus on children's understanding of the concept of number within an arithmetic context. Rather, the current study focuses on how children perceive the multiple meanings that written numerals represent outside the context of arithmetic, and the purposes that they serve in life, beyond their computational use (Sinclair & Sinclair, 1984).

Recognizing the limitations of an empirical paper such as this in addressing the full complexity of the theoretical notions that we draw upon, in this section, we have presented and operationalized the key ideas that underpin our methodology and research design that

will be presented in the following section. In a separate reflective methodological discussion (Stott & Voutsina, 2023), we have discussed in further detail the complexity of key phenomenographic notions and implications for their application in research.

4 Research design and methods

4.1 Involving children: Number Spotting game

This study included a volunteer sample of 37 preschool children (mean age: 4.01 years) from 28 preschools located in two cities in the South of England. With their parents' support, children were invited to play a Number Spotting game over 5 days, noticing written numerals in their everyday environment. Parents were asked to take pictures of the written numbers that children noticed using their mobile phones and send them to the researchers with a brief note on where they were when they spotted the number and what they were doing. We asked parents not to change their everyday routines but to prompt their children to play the game as part of their usual everyday activities inside and outside the home environment. As the game was embedded within various instances of the families' everyday life, we did not collect data related to the parent–child interaction during the game. The researchers created a digital diary of "Experience Snapshots" for each child-participant. The use of mobile-phone snapshots of everyday activities has been employed successfully by Plowman (2014). In the present study, the use of this visual technique enabled the collection of authentic examples of numerals that children encountered in their own lives. A selection of each child's images of numerals spotted in the environment, supplemented by photographs of written numerals on everyday objects and situations that the researchers had sourced, was used as a tool to produce interview data of children's interpretations of the meanings of written numbers in everyday life. This is in line with the position that visual methods can be used to elicit other forms of data (Banks, 2001).

4.2 Individual interviews with photo-elicitation element

The "Experience Snapshots" from the Number Spotting game were used to engage children in conversation during individual, video-recorded interviews. The use of photographs as stimuli follows the methodology of visual elicitation as a qualitative interview practice that helps participants in recollecting and reflecting upon their experiences and has successfully been used in studies aiming at engaging young children in research (e.g., Clark, 2005; Pyle, 2013).

We adopted a completely open approach for the Number Spotting game and did not provide instructions on specific kinds or contexts of numerals that families would look for. One limitation of this approach was that the number of photographs that the families sent us varied and did not necessarily cover a range of number meanings. One family sent in 38 photographs, the majority sent between 10 and 15 and one family did not send any at all. From all the photographs that were sent in by parents, the most common type (59.6%) included examples of numerals as identifiers (i.e., numbers on buses, bus stop, car licence plate, house number, car parking number). Measurement-related numerals (e.g., numerals showing price, weight, capacity, clock/watch numbers, numerals on height charts and speed signs) were the second most common type (26.1%).

Therefore, the interview protocol was tailored for each participant, to ensure that it included photographs sent by the family and supplemented with additional photographs selected by the researchers. On average, two thirds of the photographs discussed per interview were photographs of numerals from the researchers' photograph bank and one-third were photographs sent by the parents. This enabled us to collect data on children's interpretations of a range of examples of numerals used to denote quantity, order and measurement, and numerals used as labels/identifiers.

For each interview protocol, we ensured that the combination of photographs selected from those sent by the family and photographs supplemented from our photo bank included: eight photographs depicting numerals with a measurement-related meaning (speed limit, time, size, length, price, age, capacity, calendar date), two photographs of numerals depicting quantity (e.g., number of items in food packaging), two photographs with numerals as identifiers (bus number, car number plate), and two photographs with numerals depicting order (e.g., order of athletes completing a race, house numbers as ordering/identifying numerals). Following the above selection, we included additional photographs from those sent by the family, showing numerals of any of the above meanings that would be interesting to discuss based on the child's experience (e.g., points won when playing a game, hotel room number when on holidays, a number on a building or bus stop).

Prior to the main data collection, we had trialled our interview approach with children who did not participate in the main study. This trial suggested that in conversations that focused on more than fifteen photographs, some children were likely to lose their engagement. However, this varied across individual children. For the main study, we prepared an interview protocol for each child that included more than fifteen photographs for potential discussion, however, across interviews, we discussed fifteen to seventeen examples of numerals, on average.

The inclusion of photographs sent by families meant that across interviews, the discussion with each child was not focused on an identical range of photographs. This is not a limitation in the current study as the aim is not to compare and categorize individual children's conceptions. In this study, "conceptions and ways of understanding are not seen as individual qualities" (Marton, 1981, p. 177). Rather, the research aimed at describing the full range of variation in the collective ways in which children interpret the phenomenon of the use of written numerals and its purpose in the world around them. The categories of description emerging in this study constitute the interpretation of key aspects of the collective voice of children in our sample, as derived from individual, contextual, expressed experiences (Collier-Reed & Ingerman, 2013).

Therefore, we anticipated that including in the interview photographs sent to us by the family and photographs from our own photo bank, that the children had not seen before, would allow us to have conversations that had the potential to reveal the full range of variation in our participants' patterns of awareness, based on more and less familiar instances of numerals within our participants' experienced everyday life. This aligns with the research goal of a phenomenographic approach (Åkerlind, 2018) and our research aims.

The interviews were conducted by the second author. They took place at the authors' academic institution, in a room with a large internal glass wall. We were acutely aware that the university research environment was new and unfamiliar to children; an "adult space where children have less control" (Punch, 2002, p. 326). The interview location enabled children and their parents to maintain visual contact with each other throughout the session. Children could bring a favourite item with them, if they wished, and a conversation between the child, parent and researcher started on the walk to the interview room, with a focus on the child's favourite item, to establish a positive relationship and make

children feel as comfortable as possible within the environment unfamiliar to them. The study strictly followed the British Educational Research Association ethical guidelines for research with young children (BERA, 2018).

The interview was semi-structured. It included a sequence of questions and prompts but also allowed us to follow up children's responses and explore spontaneous comments. The interviewer used a laptop to show one photograph of a written numeral at a time, from the pre-prepared protocol, and asked questions such as: What do you think is written there? What does it say? If the child referred to the symbol as a number, the interviewer followed up with questions such as: Do you know the name of this number? What does this number tell us? Why do you think this number was there? In most cases, the children recognized the written numeral as a number and named it directly. When another symbol (e.g., £, ml) appeared in the photograph, some children referred to these as either a number or letter. In such cases, the interviewer continued the conversation by referring to the symbol as the child had expressed. When the interviewer considered that the exchange related to each example had reached a closing point, she changed the photograph shown and the conversation continued with a focus on another written numeral. The decision about when to move to another example was informed by the interviewer's conscious attention to and monitoring of the interview duration, so that the children would not get tired or lose engagement. The interview duration was between 15–20 min. As an example, we include extracts below from one transcript. "R" refers to the Researcher conducting the interview and "C" stands for Child.

R. Can you see what's written there?
C. A 3, 8.
R. A 3, 8.
R. And where are those numbers?
C. On the bus.
R. On the bus.
C. On the bus stop.
R. At the bus stop, okay. Where was the bus stop?
C. [no response]
R. Was it at the university here? Or was it somewhere else?
C. Yeah [gets distracted by something on the floor and chats about it]
R. So why are those numbers there? Do you know on that wall?
C. I don't know [continuing to play with toy dinosaur].

R. What's that?
C. A birthday cake!
R. A birthday cake and what's written on the top?
C. Four, four! [clearly and excitedly]
R. Why is there a 4 on top of the cake?
C. Because someone's turning 4.
R. Someone's turning 4.
C. [slight nod] that's what my birthday cake looks like…. [inaudible] my shape number.
R. Aaah.

R. And what about that? Can you see something written at the top of the tee shirt there?
C. Nine.
R. Why do you think there's a nine there?
C. Because, because, it's for 9-year-olds [traces a shape on the table with his finger]
R. It's for 9-year-olds. Okay.

Ashworth and Lucas (2000) highlight that researchers need to set aside anything that may draw away from the participant's experience. This process is referred to as 'bracketing'. Bracketing, prevents assumptions and opinions from "distorting an experience into an experience of another kind" (Stolz, 2020, p. 1080). As our phenomenon is based on culturally shared understandings and rules, we adopted the practice of reflexivity (Lincoln & Guba, 1985), allowing us to identify our own preconceptions and take measures to minimize their influence on the research process. The interviewer adopted a curious and open approach to interviewing. No judgements were made on anything that children said about a numeral, and they were not corrected in any way. The interviewer did not rephrase what children said based on her own understanding of the meanings of written numbers (Ashworth & Lucas, 1998).

Capturing children's responses and expressed meanings that were communicated in both verbal and non-verbal ways, constituted a challenge for the transcription process. Ashworth and Lucas (2000) emphasize that in phenomenography, the transcription needs to include "anything that is likely to affect the interpretation of meaning" (p. 304). The transcripts included gestures, pointing actions, and disengaged actions such as lying on the chair. We did not analyse the child's nonverbal communication separately, but we referred to both verbal and non-verbal behaviour, using both audio and video recordings (Herbert & Pierce, 2013), to support our interpretation and gain insight into the points where the child was not able to express themselves fully verbally (Barron & Engle, 2007).

4.3 Data analysis

Phenomenography takes an inductive and iterative approach to analysis. Both authors read the same randomly selected transcripts several times to familiarize themselves with the data (Ashworth & Lucas, 2000), before commencing the analysis. The photographs of written numerals used in the interviews were typically on an object and had a context (location, situation in which object is used, when and where the photograph was taken). Based on our initial readings and immersion in a sample of transcripts, it gradually became apparent that children's expressed and shared conceptions of the meaning and purpose of the depicted numerals drew from these dimensions in different ways. As noted in the theoretical section, we operationalized the internal horizon as including the digits of the written numeral, the object on which it appears, and, if appropriate, any symbol accompanying the written numeral. The external horizon is operationalized here as the contexts and situations within which written numerals are used.

We coded a total of 437 response extracts from all participants in relation to whether the children's responses suggested that they made links between their expressed meaning of the written numeral in the photograph and their recognition of and reference to the *object* on which the numeral appeared, the *context* depicted in the photograph or the child's experience of relevant contexts and situations associated with the object and the numeral, and any other *symbol* that appeared next to the numeral. Based on this, we developed codes that depicted a spectrum of object, context and symbol links. Table 1 provides a list of the link codes that we developed, with examples from our data. Some link codes (e.g., No link) were common across the dimensions of object (O) and context (C). Other codes (e.g., Linked and situation/activity-specific, Implicit link/awareness of symbol unclear)

were only relevant to one of the three dimensions, for example, Context (C) or Symbol (S) respectively.

Extracts were assigned more than one link code. As one example of this, in Table 1, the extract that is connected to a child's response in relation to a £1 sticker on a bag of sweets is repeated three times. This is to provide an indicative example of how the same extract was assigned three separate codes in relation to three dimensions: context, object and symbol, as relevant. To capture the *context* dimension, seven types of link codes emerged from the data, to elucidate the nuanced differences between context situations and experiences that children drew from, to express the meaning of the written numeral (e.g., Linked and situation/activity-specific, Linked to personal occurrence, or Linked and universal: link of a numeral with the broader context of an everyday life situation that may not necessarily be directly associated with experience of a personal event and activity in the child's own life). The 437 extracts coded with object, context and/or symbol links do not include extracts

Table 1 Object (O)/context (C)/symbol (S) link codes with examples from data

No link (O/C) Child made no link to the object or context to deduce (or express) the meaning and / or purpose of the written number.	[Discussing photograph showing numeral 8 on food packaging] R. So why do you think there's an 8 there next to those lollies? C. Because, **because that's the way people do stuff.**	
Misaligned (O/C) The child draws from broader experience to make a guess or deduce and express a possible meaning and / or purposes of the number that is in line with cultural rules, though misaligned with meaning of numeral in photograph.	[Discussing photograph showing numeral 9 on t-shirt size label] R. Why is there a 9 on the t-shirt? C. **Because it is 40, 40 pounds for kids**.	
Loose link (O/C) Response indicates some recognition / vague reference to the number in reference to the object and / or context.	[Discussing photograph showing several numerals on car number plate] R. Why does the car have numbers on it do you think? C. Because **that's what number it is**.	
Linked (O) Response indicates expressed meaning and / or purpose of written number that is explicitly linked and aligned with the object on which the written number appears, in line with culturally shared rules. If the photograph includes a symbol, the child recognises and refers to the symbol when providing a response that indicates awareness of the meaning and / or purpose of the number.	[Discussing photograph showing numeral 10 on road] R. And what is that written there, do you know? [points at the 10]. C. Yeah, it's 10. R. What does that 10 tell us? C. **How many miles you go on there.** [Discussing photograph showing price tag with £1 on packet of sweets] C. Aah, **it's one Pound.** R. Ha, how do you know that? C. Because I do. R. Okay, what is one pound? C. **Give the person one pound.**	 
Linked to personal occurrence (C) Response indicates meaning and / or purpose of written number associated with the particular context of a personal, experienced event, occurrence or personal perspective.	[Discussing photograph with numeral 4 on a birthday cake] C. It's **my** birthday cake. R. Is it your birthday cake? C. **Four!** R. Four! Really? Is there something written here? [indicates the number on the cake] C. It's a number 4. R. So what does that mean? C. **It's a number 4 cake for me**.	
Linked and situation or activity-specific (C) Response indicates explicit meaning and / or purpose of written number, in line with culturally shared rules and with recognition and reference to the context of a particular situation or activity from child's experience.	[Discussing photograph showing numeral 070 on a game screen] R. Ok, what does it tell you? When you're playing the game, what does that say to you? C. **That's the points**.	

Table 1 (continued)

Linked and universal (C) Response indicates explicit meaning and / or purpose of written number, in line with culturally shared rules, with reference to and aligned with context.	[Discussing photograph showing numeral 50 on speed sign] R. What is this thing? C. It's a sign… [makes a circling gesture with his left hand] R. Where? C. In the grass. R. In the grass and who might look at that sign, what does it tell us? C. **It tells … which … which fast … speed you are going.** _____ [Discussing photograph showing price tag with £1 on packet of sweets] C. Aah, **it's one Pound.** R. Ha, how do you know that? C. Because I do. R. Okay, what is one pound? C. **Give the person one pound.**	 
Linked and universal with examples (C) Response indicates explicit meaning and / or purpose of written number, in line with culturally shared rules, with reference to and aligned with context. Response indicates universal / broad meaning and / or purpose and includes at least one additional example of purpose.	[Discussing photograph showing numerals on clock face] C. Hmm, it's an alarm clock **and there's numbers round it** [makes circling gesture with left hand], **because the hands can get to them** [pointing gestures] **to tell you what time it is.** … **for instance, if the hand is at 12 o'clock, it's 12 o'clock, lunch time [pointing gestures].**	
Implicit link / unclear (S) Response indicates culturally accepted and shared meaning and / or purpose of the number. They may recognise the symbol and may draw meaning and / or purpose from this but they do not explicitly refer to it, so we do not have explicit indication of this.	[Discussing photograph showing numeral 2 followed by KG] C. And it's got a 2 on [points to the number]. R. Ok, tell me why. Do you know? C. **So she knows what kilo it is.** R. Sorry? C. Kilo it is. **So she knows it's two kilos. When she does her exercises, when she wants to pick 3 kilo weights, she picks one with a 3 on.** _____ [Discussing photograph showing price tag with £1 on packet of sweets] C. Aah, **it's one Pound.** R. Ha, how do you know that? C. Because I do. R. Okay, what is one pound? C. **Give the person one pound.**	 

that were coded as "No data" ($n = 118$), when the child did not provide any response at all (i.e., instances where the child remained silent and did not give a response of "I don't know") or if we skipped a photograph when the child appeared tired or losing engagement.

Alongside and parallel to the assignment of link codes, we started grouping extracts from a limited number of transcripts, on the basis of differences and similarities that arose in the fragments in terms of the expressed meaning and purpose of the numeral, thus engaging with the contrasting, sorting and grouping processes that Han and Ellis (2019) outline. We gradually expanded the pool of transcripts that we analyzed and applied a process of dialogic dependability checking (Collier-Reed et al., 2009), whereby each of us coded and grouped new transcripts individually, before meeting to discuss and critique any differences in our coding and grouping, by reviewing evidence from the transcript and the video data, until agreement was reached.

In expanding our analysis of extracts from gradually added sets of transcripts, we started laying out descriptive group headings which emerged from the data. These gradually developed into emergent, categories of description. Through our contrasting and sorting process, differences and similarities between groupings arose related to factors such as the completeness of the expressed meaning and purpose of the numeral and the alignment of the expressed meaning and purpose with the particular instance of a written numeral depicted in a photograph and with wider, culturally shared understandings. As more data were added in the pool, groupings were reconsidered, and new descriptive group headings were created or merged. This iterative process required what Bowden (2005) refers to as "conscious vigilance" (p. 27), repeatedly examining the detail of the scripts and stepping back to consider the broader pool of data and emerging groupings.

5 Findings

At the end of the analysis, groupings of interpretations and inferred conceptions based on children's responses were crystalized into categories of description (Table 2). The categories of description in Table 2 capture all the qualitatively different ways in which the children in our sample conceived and interpreted the meaning and purpose of different types of written numerals on everyday objects and within different contexts. We did not use these categories to classify individual children, but to capture their collective voice in describing the way they conceive numerals in the world around them. One way to think about this is as a "map of the collective mind" (Orgill, 2012, p. 2609) — an ordered list of categories with documented, through data examples and category descriptions, relationships between these categories of the participants' expressed experiences.

The outcome space brings together the categories of description (Fig. 1, first column) and the spectrum of links (top row) that children made between the numeral and the object, context and accompanying symbols, depicting the overall structure of children's increasingly more expanded and explicit (i.e., expressed) conceptions and interpretations of the meaning and purpose of written numbers in everyday life.

As we navigate down and across the outcome space, its structure indicates children's *expanding awareness* (Åkerlind, 2008), and thus, a move towards more complete and fully expressed conceptions of the meanings and purposes of written numbers in everyday life, that are informed by stronger or better aligned links with the context, object and, where relevant, symbol dimensions. The structure of this outcome space is not hierarchically *inclusive* as defined by Han and Ellis (2019). This means that, previous categories are not included in subsequent categories. Some of the categories, for example, 7a, 7b, and 8, are of the same standing in terms of meaning, but still distinct, because interpretations captured in these categories draw from different connections of discerned and recognized elements associated with the written number (i.e., either object and context or symbol), as illustrated by the intersections between categories and the spectrum of links. The hierarchical structure of our outcome space is depicted by the increasing complexity in how children draw from connections with different dimensions associated with the phenomenon, to express its meaning. As Åkerlind (2022) notes, the ordering of categories "indicates the way in which human awareness of any phenomenon collectively unfolds or expands" (p. 6).

The outcome space is not developmental in nature. The categories of description and spectrum of context, object, and symbol links do not depict a developmental trajectory through which children expand their awareness of the meanings and purposes of written numbers. Children do not need to pass through a previous category to reach another. Rather, as mentioned earlier, the same child may express interpretations and conceptions that fall under more than one category, for different numerals in everyday contexts. The variation of colour at the intersections between categories of description and the spectrum of links provides an indication of the density of coding under each intersection, i.e., the darkest blocks indicate the densest coding. Therefore, for each category, the coloured blocks depict *the range* of coded references for that category in relation to the kinds of links that children made between the numeral and the context, object, and symbol dimensions.

In all the extracts that were classified under Category 1 (see examples in Table 2), we found that the children did not make any expressed links between the written numeral and the context or object to deduce the meaning and purpose of the written number (Fig. 1). This included responses where it was unclear if the children recognized the

Table 2 Categories of description with data extracts

Categories of Description: Children's expressed meaning and / or purpose of numerals		Detailed description of category	Extract examples and photograph	Extract examples and photograph
1	No expressed meaning and / or purpose of numeral.	Child's response does not indicate awareness of meaning and / or purpose of the numeral. No link to object / context / symbol (where relevant) to extract meaning and / or purpose. This includes responses such as: "I don't know."	R. Why is that number 4 on the cake?  C. Because **it has a hot, hot, hot, candle**.	R. Why is this girl standing on the one, do you think? Why is this one standing on two?  C. **Because it's lovely**.
2	Description of actions associated with the context and object, rather than number-focused meaning and / or purpose of numeral	The child attempts a response that includes description of actions (of people that occur in that context) loosely associated with the object and / or context. The child tries to communicate the message that the number gives through a loose description of actions associated with object and / or context.	R. Ok, what do they mean? [referring to the numbers] Why are they there?  C. **Because they're running.** R. Because they're running. Why is this little girl standing on the one? C. Because we're doing a race.	R. Why is that there, do you think on the t-shirt, that 6?  C. Because **you have to wear it**. C. And there's more numbers there [points the numbers next to the barcode] R. There is, isn't there, loads of them. What do you think they might say, looking at them?  C. [points to the barcode] **that means I need to put it and to scan it and go home and with the milk.**
3	Meaning and / or purpose of numeral misaligned with object and context.	Child's response is misaligned with object and context in the photograph. Response includes culturally accepted possible meanings and / or purpose of the numeral using broader personal experience to attempt sense making.	R. And that one [pointing to the 20] do you know anything about that?  C. Uh, it's 20… **there might be 20 things in it or if it's a book or something, there's 20 pages.**	R. What are those numbers there in the kitchen?  C. **Because it's safe, for Mummy.** R. Why is it safe for Mummy? What do the numbers tell her? C. Um… um… **don't touch them.**

Table 2 (continued)

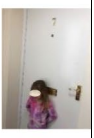
Categories of Description: Children's expressed meaning and / or purpose of numerals		Detailed description of category	Extract examples and photograph	Extract examples and photograph
4	Meaning and / or purpose of numeral linked to the object but misaligned with context.	Child's response links to the object but the context is misaligned. However, the context inferred in the child's response is probable, and in line with broader culturally shared rules.	 R. Do you know what those numbers tell us on the milk bottle? C. Well, there's … sometimes stuff round [makes a circling gesture] a milk bottle, so [pause] it … **might just be telling you something, um about it, you can give it to a 3-year old boy or a 4-year old boy.**	 R. What's that? C. Candy. R. And what's that written there? [points to the yellow price label] C. [points to the label, in a very soft voice says] 1 and 2 R. Why is that number on the candy, do you think? C. **Because there's a lot.**
5	Recognition that the numeral has *a* meaning and / or purpose, but the specific communicative message of the numeral is not expressed.	Child's response suggests that they recognise that numerals have different meanings and uses / functions e.g., as labels, quantities etc., however the child has not yet begun to express that communicative message explicitly.	 R. So why is that 6 there on the bus? C. **Because that's the number of it.**	 R. Where were you? C. It's at a hotel. R. A hotel, ok. Why is this seven on the door? Why is that important? C. **Because that's what number you are.**
6	Meaning and / or purpose not fully expressed but there is expressed recognition of a specific communicative message in line with culturally shared rules.	Child's response suggests that they recognise that the number communicates a message. The meaning and purpose is not fully communicated, but in line with culturally shared rules. There is some recognition / reference to the number in the response and the response is generally linked to object and loosely linked to context.	 R. Why has the boat got numbers? C. Because **they know the people it's called** … it's called a fish and chip boat. R. A fish and chip boat, okay. The people … tell me about the people … on the fish and chip boat. C. They know it's a fish and chip boat and **they know what boats they're going on**.	 R. A 5! Why do cars have number s like this? C. **So we know it's car … that's ours.**
7a	Fully expressed meaning and / or purpose implicitly drawn from symbol accompanying numeral, in line with culturally	The child's response appears to be implicitly drawn from the symbol (i.e., for photographs showing £, ml, kg, cm, etc) to express meaning and / or purpose of the	 C. Err, well, there might be seven because… I… it **might be 7 because it's 7 pounds or something**. R. How do you know it's 7 pounds?	 R. What do these tell us? [indicates the capacity numbers on the label] C. 3, 4, uh, zero, 8 R. Do you

Table 2 (continued)

Categories of Description: Children's expressed meaning and / or purpose of numerals		Detailed description of category	Extract examples and photograph	Extract examples and photograph
	shared rules.	numeral. However, we do not have explicit evidence of reference to the symbol.	C. Well it might not, well I, um, I don't really know. R. Well it might be. Is there something here that tells you that it's 7 pounds? C. Um, there might be, or there might … [inaudible]	know anything about why those numbers are there? C. **To know what... to know how full it is, I think?**
7b	Fully expressed meaning and / or purpose of numeral in line with culturally shared rules.	The child's response includes explicitly expressed meaning and / or purpose (in line with culturally shared rules), that is linked and aligned with context and object.	R. Why is this boy standing on one? And why is that boy standing on two? [indicating the boys on 1 and 2].  C. Because, **he came first, second place, third place [indicating each block as he speaks].**	R. What does that tell us, that eight? Why is it there?  C. **That means there's eight rocket lollies.**
8	Fully expressed meaning and / or purpose of numeral, in line with culturally shared rules, with examples provided.	The child's response includes explicit meaning and / or purpose (in line with culturally shared meaning), fully expressed, linked and aligned with context and object or implicitly linked to symbol. Child's response reveals examples that go beyond those in the photograph.	C. Hmm, it's an alarm clock and there's numbers round it [makes circling gesture with left hand], because **the hands can get to them to tell you what time it is**. **… for instance, if the hand is at 12 o'clock, it's 12 o'clock, lunch time.** 	C. Well, there are lots of numbers [makes a movement gesture up the side of the ruler] **and they're numbers to measure** [with confidence]  R. To measure what? C. Well, **if you were drawing [makes hand gestures] something, and you wanted to measure it [slides hands together] and make it as long as the ruler, then you will know how long that would be.** C. It's got a 2 on [points to the number]  R. Ok, tell me why. Do you know why? C. **So she knows what kilo it is.** R. Sorry? C. **Kilo it is. So she knows it's two kilos.... When she does her exercises, when she wants to pick 3 kilo weights, she picks one with a 3 on.**

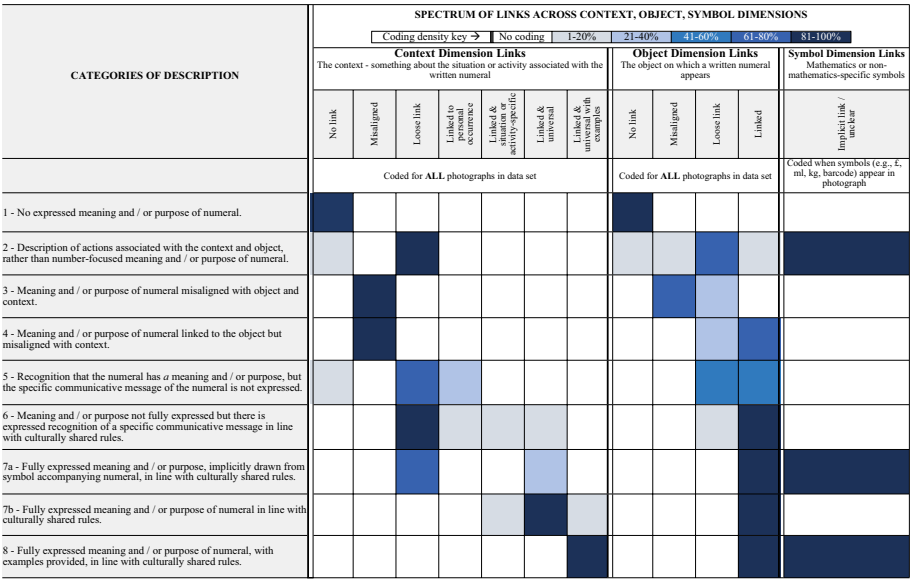


Fig. 1 Outcome space depicting the structure of children's conceptions of the meaning and purpose of written numerals in everyday environments

object and/or context depicted in the photograph, as well as "I can't remember," "I don't know," and head shaking.

For expressed meanings under Category 2, children largely drew on their recognition of the object depicted in the photograph, making loose or no link with the context dimension (Fig. 1). Responses under this category (Table 2) were focused on the actions of the object (e.g., "It ticks you with the time" or people in relation to the object or context (e.g., running or "somebody's wearing it as just clothes") rather than the purpose and meaning of the written number. For all responses in this category that were related to numerals accompanied by a symbol in the photograph, the child may have drawn implicitly from recognition of the symbol, but this was not always clear. The extract example in Table 2 related to numerals on the milk bottle is indicative of a response that is a description of actions (scanning) that may be implicitly drawn from the child's recognition of the barcode symbol that accompanied the numerals.

Category 3 (Table 2) included interpretations of the meaning and purposes of written numbers that were based on misaligned or loose links to the object depicted in the photograph and/or misaligned links to the context dimension (Fig. 1). Children drew from their broader experience to attempt sense making and their attempted interpretations in this category included culturally shared, *possible* meanings of the written numbers in the photograph. In the example of the digital clock of the oven photograph (Table 2, Category 3) the response suggests that the meaning and purpose of the numeral is to indicate temperature, which is a feasible use of numerals, in line with broader culturally shared rules. Although this is misaligned with the numerals that show the time on the clock (object), the response draws from the child's wider experience, recognition of the object (kitchen oven) on which the digital clock is positioned, and the associated context of baking/cooking.

Responses in Category 4 (Table 2) are mostly linked with the object on which the numeral appears (Fig. 1) but they are misaligned with the context dimension. Responses are based on possible meanings of numerals that are feasible and aligned with broader cultural understandings, drawn from children's life experiences (Category 4, Table 2). For example, the child who, in relation to numerals indicating capacity on a family milk bottle, responds that they may mean that it is a for a 3- or 4-year-old boy, attempts an interpretation that is drawn from their wider experience of age-appropriate food products for young children and their experience of what the numerals on such products may mean. Although the response is misaligned with the particular example, it is nevertheless a reasonable interpretation and in line with wider culturally shared understandings related to what numbers on food products might indicate.

Although both Categories 3 and 4 depict what could be seen as a misalignment between the expressed conception and the wider, culturally shared understanding of the meaning and purpose of the particular example of a numeral, we consider these categories to be particularly interesting and noteworthy. This is because they capture attempted interpretations of meaning and purpose that are feasible, indicating that in the process of meaning making, children draw from their broader experienced world. The two categories are distinct, depicting a nuanced difference in the nature of the misalignment observed in children's interpretations.

Category 5 captures interpretations that indicate that children recognize that written numerals have different meanings and uses or functions, for example as labels, but the specific communicative message of the numeral is not fully expressed (Table 2). Figure 1 shows that interpretations include mostly direct or loose links with the object. Loose context links are predominant, but there are also no context links as well as a few context links associated with personal events. For example, responses that suggest recognition that the number on the bus has the function of labelling, "that's the number of it." Similarly, the number on a hotel door is recognized as showing "What number you are," suggesting some recognition that the numeral has a purpose, even though the child does not express the specific communicative message of the written number.

Category 6 interpretations include a clearer recognition that the written numeral gives a specific message to those who read it, often using words like "to know," "to make sure," or that the numeral is loosely associated to the context such as fast/slow (speed), tall/long (length), late/early (time), and full (capacity). For example, the extracts included in Category 6 (Table 2), indicate that the child recognizes that the number on the boat gives a message to people so that *they know* which boat to go on and the number plate of the car is for people *to know* that this car belongs to them. For such interpretations, children draw from a wider range of links with contexts that they are aware of, from their experiences, and the expressed meaning and purpose of the numeral is clearly linked with the object (Fig. 1).

It is interesting to observe that for both Categories 5 and 6, responses indicate recognition that the numeral has a specific meaning, however, this is not fully formulated. Responses in these categories appear to be underpinned by a greater variety of context links, with children drawing on personal events (e.g., birthday) and specific activities (e.g., numerals depicting the score when playing a game) (Fig. 1).

Categories 7a, 7b, and 8 capture fully expressed interpretations of the meaning and purpose of the written numeral that are aligned with culturally shared rules (Table 2) but illustrate nuanced differences related to the varied links that children make with contexts and/or the object or symbols that appear next to the numeral (Fig. 1). Category 7 is split into two sub-categories to differentiate responses to photographs of numerals that were

accompanied by another symbol (Category 7a), from responses to photographs that did not include a symbol next to the numeral (Category 7b). For interpretations under Category 7a (Table 2), children may have drawn only from their awareness and recognition of the symbol that accompanies the numeral (for example, recognizing that the pound symbol appearing next to a numeral on an advertising poster denotes price, in the context of paying for something, and not necessarily the price of the poster itself), or, they may have drawn from the pound symbol as well as the context and object in the photograph (e.g., £1 label on a bag of sweets). In this category, the responses included fully expressed meaning of the numeral but without an explicit reference to the symbol that may have informed its deduced meaning, even after a prompt for the child to explain their response (see examples in Table 2). For this reason, Category 7a includes responses that may have been *implicitly drawn from the symbol that accompanies the numeral* (i.e., for photographs showing £, ml, kg, cm, etc.), to express meaning and/or purpose of the numeral, but which do not include an explicit reference to the symbol.

For interpretations under Category 7b, children drew from links that they made to the context and object dimensions specifically (Fig. 1), as, in these cases, the photograph did not include another symbol next to the numeral (e.g., number of lollies on food packaging). Category 7b interpretations are fully expressed and may be quantified with the number, e.g., "there are 25 pieces," "she came 1st." Particularly with measurement type photographs, the response may include words that are directly related to the context and numeral, for example miles, mph (speed), o'clock, centimetres, £, kilograms, etc.

Category 8 (Table 2) is distinct, in that, the expressed meaning and purpose of the numerals are in line with culturally shared rules, but also, drawing from their broader experiences of context, children's interpretations are extended to include examples beyond what is shown in the photograph ("for instance if the hand is at 12 o'clock, it's 12 o'clock, lunch time"). All interpretations under this category include links to the object and universal links to the context (Fig. 1).

We found that the emerging intersections between the categories of description and the range of context/object/symbol links, as depicted in the outcome space, reveal that the categories of description are underpinned by four key *critical aspects*. These critical aspects define how categories are different and distinct and the logical relationship between them. The critical aspects that we identified in our analysis and which define the structural connection and distinction between the categories of description are:

a) Whether the child *recognizes* the written numeral as a number and *recognizes and is aware of* the object, context and/or accompanying symbol associated with the numeral.
b) Whether the child *makes a connection* between the written numeral, object, context and/or symbol *and uses this connection* to deduce or express meaning and purpose.
c) Whether the expressed meaning and connections that children make are *aligned with* the object, context or symbol dimensions at the *specific* occurrence of the written numeral (in the photograph under discussion).
d) Whether the expressed meaning and connections that children make are *aligned with* the *broader, culturally shared* rules that underpin the meaning and purpose of the written numeral in everyday life.

While expressed *recognition and awareness* are at the core of the first critical aspect, the second critical aspect is centred on the *connections* that children make between the elements that they recognize (numeral, object, context, symbol). The core of the last two

critical aspects above is the alignment between the expressed meaning of the numeral and the specific depiction within the photograph and the alignment with the broader, cultural rules associated with the use of the written numeral in everyday situations.

6 Discussion and conclusion

This study reveals the structure that underpins the qualitative variation in children's conceptions of the use of written numerals in everyday life and shows that this is informed by the nature of links that children make between the numeral and their awareness of the context, object and symbol associated with the numeral. Durden (2018) notes that the more people expand their awareness of the internal and external horizon of a phenomenon, the more sophisticated their conceptions of the phenomenon are. Our findings show that as children's awareness of the context, object and symbol dimensions expand, that is, when children are aware of more elements within the internal and external horizon of the phenomenon, the nature of links between these dimensions and the written numeral changes, reflecting an expanded awareness of the meanings and purposes of written numerals in everyday environments.

It is this underpinning structure that has allowed us to identify the four critical aspects of variation that differentiate the categories of description (Collier-Reed & Ingerman, 2013) and determine key differences between our participants' expressed meanings. As we navigate down the structured outcome space, which is the full range of variation of conceptions emerging from our data (Taylor & Booth, 2015), categories include more of the critical aspects of variation (Durden, 2018), demonstrating children's expanded awareness of the meanings and purposes of written numerals in everyday environments.

Björklund et al. (2021) outlined key skills for the learning of arithmetic in terms of the critical aspects that children need to discern, to apply flexible strategies for solving arithmetic tasks. In our study, through a phenomenographic research approach, we add a new dimension to existing research by highlighting the critical aspects that, based on our data, formed the core of the participating children's conceptions of the social and cultural uses of written numerals in everyday contexts, outside of formal arithmetic.

Phenomenography has been criticized for a tendency to equate participants' experiences with their accounts of those experiences (Orgill, 2012). Throughout our research process, we have remained acutely aware of the fact that our participants were young children who were still developing their verbal expression. As mentioned earlier in the paper, we recorded and captured verbal as well as non-verbal communication (e.g., gestures, hesitations, pointing actions) and referred to these continually to support and check our interpretations. The emerging categories of description, our definition and naming of these, intentionally capture differences related to fully expressed conceptions and conceptions that may not be fully expressed. Our coding of data also accounted for the possibility of implicit awareness where the data have not provided us with explicit, that is, expressed evidence of awareness (for example, through assigning a code for unclear/implicit awareness of other symbols that may accompany a numeral and determine its meaning). This acknowledges that a not expressed, or not fully developed expression of awareness, should not be necessarily interpreted as lack of awareness and experience. This is in line with Saljo's (1997, p. 178) recommendation that the data must be approached as "indicative of accounting practices" of phenomena, that is, "ways of understanding, talking, arguing and in general, ways of bringing the world into language in order to be able to communicate," rather than as a direct record of people's experiences. Phenomenography is a research approach that lies within the interpretative paradigm

(Collier-Reed et al., 2009) and on this basis, we acknowledge the "inevitably relational nature of researchers' interpretation of the data" (Åkerlind, 2022, p. 6).

Brenner (1998) supports the view that individuals receive feedback about "socially accepted" meanings through their participation in relevant discourses (p. 151). The critical aspects of variation, as revealed in this study, provide evidence related to the key aspects of knowledge and awareness that differentiate the qualitatively varied conceptions that children express about the use of written numbers in their environment. Therefore, the critical aspects that depict the variation underpinning the categories of description provide a guide for elements of knowledge that number-related discourses at home, as well as discussions and activities at preschool (Brandt & Tiedemann, 2009), need to focus on, in order to foster and develop children's awareness of the multiple meanings and uses of written numerals, outside the context of arithmetic.

In this study, we did not examine directly, or through data collected from parents or early years educators, children's interactions with others that may have affected their interpretations of the phenomenon under focus. However, our exploration has been built on the position that children are "from the beginning of their life a member of a community that extensively employs embodiments of mathematical knowledge" (van Oers, 2001, pp. 59–60). Furthermore, the purposes of numerals in life constitute part of the cultural and social knowledge that is "embodied in the many cultural artefacts used in daily life and transmitted through social uses" (Tolchinsky, 2003, p. 110). Therefore, we consider that the richness and range of variation in children's conceptions that we mapped out in relation to the meanings and uses of numerals in life, is undoubtedly influenced by social and cultural experiences and children's participation in everyday activities in their family, preschool, and community environments. We did not collect data related to whether certain types of written numerals (e.g., numerals indicating time, price, or age) were more likely to be experienced and discussed at home or at preschool than other types of numerals. Future research could usefully examine relations between the nature of children's experiences and interactions that they have with others, at home or at preschool, and their expressed conceptions. It would also be interesting to explore potential links between children's expressed conceptions and socioeconomic or ethnicity-related factors that may relate to the frequency and nature of children's experiences about the meanings of written numbers in everyday life.

In relation to previous research conducted on the everyday meanings of written numbers (Sinclair & Sinclair, 1984, 1986), our study provides an expanded and qualitatively nuanced depiction of the variation and nature of young children's conceptions of the meanings of the uses of written numerals in everyday contexts, beyond identifying loose versus specific responses. We have done this through detailed consideration of the kinds of links and connections that children draw from, which reveal the underpinning structure of children's conceptions. In our analysis, we have also considered links that children may make between the written numeral and other symbols that may accompany the numeral, as a separate dimension that appears to inform children's interpretations alongside to, or irrespectively of, the object on which the numeral appears. In our ongoing research, we focus on the examination of whether the emerging categories of description could be used in a future study, as a framework of analysis for exploring distribution of responses within and across categories in relation to what a numeral signifies (quantity, order, numeral as label).

In line with previous research by Sinclair and Sinclair (1984, 1986) and Ewers-Rogers and Cowan (1996), we found that very young children have rich knowledge about written numerals around them, and they also know that written numerals are used in everyday life to represent varied ideas and not only quantities. Ewers-Rogers and Cowan (1996) noted: "we do not know whether this supports their early number work at school or instead

hinders them: they may be confused by their knowledge and teachers may fail to consider the different uses of numbers the child knows" (p. 24). We consider the concern expressed by Ewers-Rogers and Cowan to be still relevant today.

Phenomenography focuses on revealing and describing the qualitative different ways in which people interpret or conceptualize aspects of reality. This is a significant research goal because learners have various ways of understanding a particular concept that often differ from the teacher's or other adults' conceptions and the conceptions that are considered as "correct" or "authorized" (Marton, 1981). From an education point of view, focusing on understanding how children think about different phenomena is essential, as this understanding can inform decisions on what is to be learnt and on what could constitute a fruitful pedagogical approach that builds on previous understandings.

This study contributes to this important educational goal by offering new evidence on the nature and structure of the varied patterns of awareness that children develop about the use of written numerals in the world around them, and by revealing the critical aspects and elements of knowledge that underpin this variation. Early teaching of number-related ideas could build on this evidence, and make meaningful connections with the varied knowledge that children develop through their everyday experiences and bring with them to school, for purposeful classroom conversations.

Acknowledgements We are grateful to all children and families who participated in this project. We also thank the reviewers of our paper for their insightful and useful comments.

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Funding This study was funded by The Leverhulme Trust, in the UK (RPG-2019–330).

Data Availability Raw data not available due to ethical restrictions. The participants of this study did not give written consent for their data to be shared publicly.

Declarations

The research followed the British Educational Research Association Ethical Guidelines for Educational Research (2018).

Conflict of interest The authors declare no competing interests.

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