UNIVERSITY OF SOUTHAMPTON

Faculty of Social Science

School of Economic, Social and Political Sciences

# Essays on Quantitative Macroeconomics and International Finance

*by*

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Abstract

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This thesis investigates the impact of male abundance on the marriage market matching outcomes in China and its implication on household income inequality, it also studies how capital controls affect the promotion of Chinese currency internationalization. In the first chapter, a brief introduction of the topics in this thesis is presented and I also highlight the significance of the research questions. Chapter 2 provides an empirical analysis of the impact of the rising male abundance on the degree of assortative matching in the Chinese marriage market, and investigates its influence on household income inequality. I find that the sex ratio in the marriage market is negatively correlated with the degree of assortative matching, and this negatively affects household income inequality, an instrumental variable analysis is also presented. In Chapter 3, we develop a general equilibrium model to structurally interpret the impact of the sex ratio in the marriage market on the degree of assortative matching, the quantitative results highlight that sex ratio negatively associates with household sorting and negatively affects household income inequality, these results are consistent with data sample and the literature's finding. We also endogenize the education decision for males and females before they enter the marriage market, the result indicates that males and females are more likely to increase education investment with the rising male abundance in the marriage market. Chapter 4, looks at the open economy issues in China, in which we quantitatively study how capital account restriction affects Chinese currency usage in the rest of the world countries, the result points out that an increase in capital control limits the promotion of Chinese currency internationalization.

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# Declaration of Authorship

I declare that this thesis and the work presented in it is my own and has been generated by me as the result of my own original research.

I confirm that:

1. This work was done wholly or mainly while in candidature for a research degree at this University;
2. Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated;
3. Where I have consulted the published work of others, this is always clearly attributed;
4. Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work;
5. I have acknowledged all main sources of help;
6. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself;
7. None of this work has been published before submission

Signed:..... Date:.....

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*To Meichun Yang*

# Chapter 1

# General Introduction

The increasing role of China in the world economy has arisen the attention of macroeconomists. Over the past 40 years, China has changed from a planned economy to a market economy and a series of economic policies have been implemented to accelerate this transformation, such as monetary policy reform (interest rate liberalization, capital account liberalization), population policy reform (entirely abandon the One Child Policy and change to three children policy now). These policy reforms have been widely discussed by macroeconomists and enable policymakers to better consider the policy design and reform (Liu et al. (2021b); Song et al. (2011)). The features of the Chinese economy in past years have been described as fast economic growth, sustained capital accumulation, growing urbanization, and gradual development of financial markets. Before 2010, the average GDP growth rate reaches 10% per year, the capital return rate remains above 20%, and the saving rate was higher than the investment rate, additionally, China experienced a growing net foreign surplus, which makes it become the largest U.S. dollar debt holder in the world (Song and Xiong (2018)). Excepting the fast-changing of macroeconomic indices, the demographic structures in China also experience huge changes. China now has a low fertility rate and imbalanced sex ratio, also an aging population, these generate significant impacts on labor market outcomes and eventually affect aggregate output, consumption, investment, and so on.

Research on the Chinese economy is essential at present. China has generated huge

impacts on the global economy, which is the second largest economy, the largest exporter, the largest creditor, and possesses the largest population and foreign exchange reserves in the world. Between 2010 and 2019, China contributed to world economic growth with an average share of 30%. The shocks of the Chinese economy impose significant influences on the rest of the world. The studies on the Chinese economy involve several topics, such as population control policy, economic growth, labor market outcomes, income inequality, monetary policy, capital account liberalization, and Chinese currency internationalization Meng (2012); Song et al. (2014); Naughton (2006). In this thesis, we focus on the discussion of the implications of the Chinese marriage market matching outcomes on income inequality, and human capital accumulation, and also looking at an open economy perspective, we study the implications of capital account restriction on Chinese currency internationalization.

This thesis consists of three chapters. The first chapter and second chapter belong to the same project, which documents the increasing trend of male abundance in China and studies its impact on the Chinese marriage market matching outcomes, and also evaluates the implications on household income inequality both empirically and theoretically. By using the population census data, I find that the trend line of sex ratio at birth (the ratio of sons to daughters in the region) in China is increasing across the urban region and rural region, previous studies focus on the quantitative evaluation of this rising male abundance on household saving rate, crime rate, and housing prices. In the first project, we pioneered in investigating how this rising male abundance affects the Chinese marriage market matching outcome, which is measured by the degree of assortative matching, importantly, we construct the education correlation coefficient as the index in measuring the degree of assortative matching. The data sample is restricted to the household level and controls the age range and Hukou (an indicator that demonstrates whether you are born in a rural area or an urban area in China, it affects the migration outcomes of households) type of cohorts.

In the first chapter, we empirically investigate the effect of sex ratio in the marriage market on the degree of assortative matching and present several robust results, then we evaluate how this pattern affects household income inequality. The employment of

instrumental variables in this chapter enables us to resolve the endogeneity issue. The second chapter, which comes from the same project, employs a general equilibrium model to quantitatively explore the impact of sex ratio in the marriage market on the degree of assortative matching, and further investigate how sex ratio in the marriage market affects household income inequality, and we endogenize the education decision of males and females to show the human capital investment difference between men and women in response of rising male abundance.

The third chapter of this thesis looks at the open economy issues. We focus on the role of China in international finance, specifically, we study the impact of capital controls on the progress of Chinese currency internationalization. Compared with the dominant status of China in global trade, its influence on international finance is poorly studied, and this could be attributed to its capital account restrictions (Horn et al. (2021)), which were tightly restricted before its entrance into World Trade Organization (WTO). Furthermore, its currency in the international monetary system and international price system has negligible performance before the People's Bank of China (PBoC) starts the internationalization of Chinese currency (renminbi, hence forcing the RMB). In 2009, the central bank of China initiated the programs of RMB internationalization, which intend to promote the role of the Chinese currency in the international monetary system and international price system. The channels of realization of this objective include the encouragement of trade invoicing with RMB, central bank cooperation to build currency swap lines, and capital account liberalization to increase the entrance of foreign investment in the Chinese asset market. These programs significantly promote RMB usage in the international monetary system and price systems. The third chapter documents these patterns and illustrates the performance of other currencies (the U.S. dollar, and euro). We intend to use the framework of Chahrour and Valchev (2022) to quantitatively evaluate how capital controls in China affect the progress of RMB internationalization, the calibrated model demonstrates that capital controls negatively affect renminbi usage in the rest of the world, a higher degree of capital controls is associated with a lower degree of renminbi usage. The model can be generalized and made several extensions in further research, such as evaluating different exchange rate regimes, studying the optimal

level of capital account restrictions, and so on.

The three chapters of the thesis employ different methodologies, including empirical investigation and structural models. This enables us to form a quantitative analysis of research topics. Chapters one and two discuss how the demographic structure changes affect aggregate economic outcomes, which focuses on household formation in macroeconomic research. The papers of Becker (1973, 1974) first employ economic theories to explain households formation and develop a new branch of macroeconomics: family economics, which is important for policymakers to design policy, such as pension system, public expenditures, and human capital accumulation. Chapter three discusses the rising player in the international monetary system and the international price system, the renminbi, which is also significant for policymakers to design policy, especially how this change in the international monetary system implicates monetary policy design, central bank communications, and responses to foreign shocks. In summary, the research in this thesis provides strong policy implications and fills the gap in the corresponding literature.

# Chapter 2

# Gender Imbalance, Assortative Matching and Household Income Inequality in China–Evidence

*This paper evaluates how the male abundance in China affects the marriage market matching outcomes and its implications on household income inequality. By using the household-level data we first examine the effect of the sex ratio on the degree of assortative matching, which is measured by the education correlation between couples in regions. We restrict the sample cohorts whose ages are between 20 to 30, which is the age range that most first marriage takes place. The sex ratio in the study is defined at the prefecture level. The regression results indicate that there is a negative correlation between the sex ratio in the marriage market and the degree of assortative matching between couples. This relationship is robust when we use alternative model specifications, in which we investigate how the sex ratio affects the education gap between couples. Then we examine the effect of the sex ratio on household income inequality, the results demonstrate that the sex ratio is negatively related to household income inequality in China. We resolve the potential endogeneity issues of our key independent variable (the sex ratio in the marriage market) by using instrumental variables, which we collect data from the monetary penalties for violating the birth quotas that are restricted by the One Child Policy (OCP), to form measures that could reflect the OCP enforcement intensity.*

*We find that the introduction of the OCP distorted gender balance in China and amplify the negative effect of the sex ratio on the degree of assortative matching.*

## 2.1 Introduction

Marriage market matching outcomes have important implications for economic development. Sorting in marriages affects the next generation's education attainment, human capital accumulation, income inequality, and labour force participation decisions (see Fernandez et al. (2005); Du et al. (2015); Becker (1973, 1974)). An important determinant of marriage market outcomes is the sex ratio, i.e., the ratio of males to females in the population. Studies of Abramitzky et al. (2011); Becker (1973, 1974); Du et al. (2015) indicated that the position of men (women) in the marriage market improves (worsens) in the presence of a reduction of sex ratio (female abundance). In this study, we aim to investigate the matching pattern in the Chinese marriage market in the presence of a rising sex ratio and study its implications for household income inequality.

Many papers have documented an increasing trend of assortative matching in the marriage market for most developed countries (US, UK, Germany, Denmark, etc.). The widespread view comes from the empirical fact that the proportion of couples who have the same degree of education (i.e., educational homogamy) keep growing over the last few decades (Eika et al. (2019); Greenwood et al. (2014)). The increasing degree of assortative matching is associated with a rising household income inequality (increased Gini coefficients for most countries like the US, UK, Germany, and Latin American countries). Previous studies have developed theories to explain the mechanism that why increasing assortative matching results in rising income inequality, in general, two principal approaches are used to model and understand the dynamics of marriage patterns: search and matching-based models highlight market-level forces and treat heterogeneous perceptions over partners (love) as primitives; another approach is collective models which focuses on the influence of intrahousehold bargaining (Galichon et al. (2019)). However, most of them fail to consider the marriage dynamics in an unbalanced number of participants in the market, and they always assume that there are equal numbers of males and females in the marriage market.

The demographic structure in China has greatly changed since the 1980s, which is featured by declining fertility, an aging population, and rising male abundance. The cause of these changes is the introduction of the One-Child Policy (OCP), which aims to control population growth. In 1979, China introduced its unprecedented One-Child Policy, under which households exceeding the birth quota were penalized. Until the end of 2015, the OCP has been fully abolished but changed to a twin policy (each spouse could have two children). Studies like Ebenstein (2010); Edlund et al. (2013); Li et al. (2011) pointed out that the OCP distorted the sex ratio in China. Under the OCP, the son preference of parents has made pre-birth selection a common practice, owing to the widespread adoption of Ultrasound-B technology (Wei and Zhang (2011)). Since the implementation of the OCP in the 1980s, the ratio of boys to girls at birth has increased dramatically, especially for the second and third birth. Using population census data from the National Bureau of Statistics in China, we inferred the sex ratio at birth for the Chinese population from 1973 to 2015, which is illustrated by Fig. 2.1.

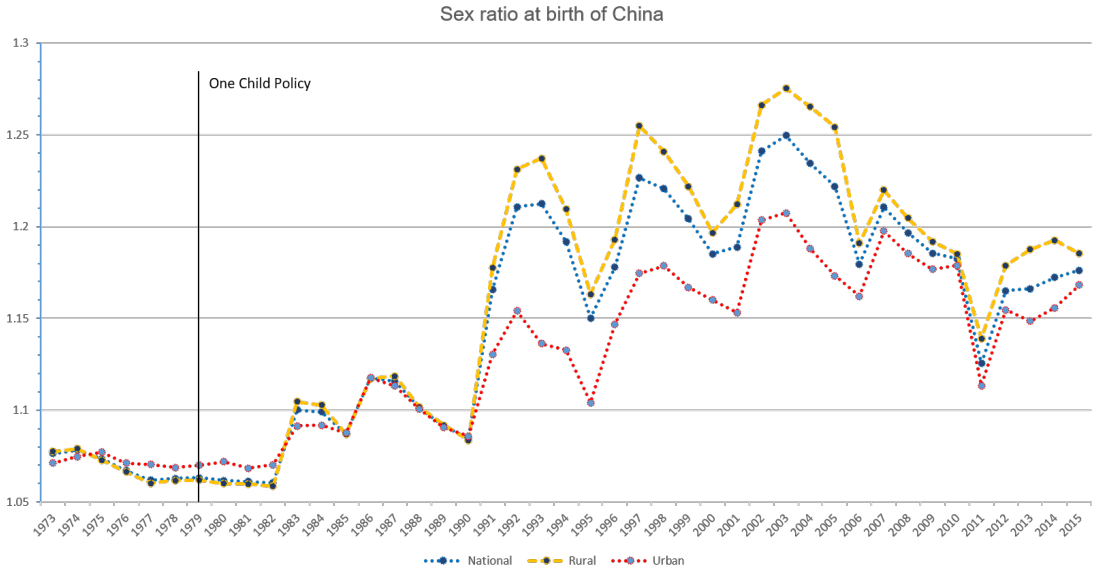



FIGURE 2.1: Sex ratio at birth in China

Source: Chinese Population Census(1982,1985,1990,1995...2015)

What will be the consequences of male abundance on the real economy? Previous studies have focused on evaluating the impact of gender imbalance on household saving rate, crime, and marital matching (Abramitzky et al. (2011); Wei and Zhang

(2011); Edlund et al. (2013)). They find that the sex ratio is positively correlated with the Chinese household saving rate and crime rate, they also find that the rising sex ratio results in female hypergamy. In this study, we firstly employed household-level data to empirically investigate how the rising sex ratio in the marriage market affects the degree of assortative matching, and how it implicates the variation of household income inequality in China. The data we have used comes from three sources, which are China Household Income Projects (CHIPs), the Chinese Population Census, and the China City Statistical Yearbook respectively. We employed three waves of CHIPs (2002, 2007, and 2013) and restrict the sample to the urban area, in which we can capture more variations of individual education levels as the education level in the rural area are less heterogeneous compared with the urban region. From the regression results, we find that the sex ratio at the prefecture level is negatively related to the couple's education correlation, which we employ education correlation coefficients to reflect the degree of assortative matching. This relationship is robust when we use other model specifications. When examining the relationship between the sex ratio and household income inequality, we find that the results implicate a positive correlation between these two. To study the mechanism that why the sex ratio is negatively correlated with the degree of assortative matching, we also develop a structural model to study the marriage market pattern with the rising male abundance in the following chapter.

### 2.1.1 Existing Literature

This study contributes to three strands of the literature. First, it contributes to the research that evaluates the impacts of the One-Child Policy on the Chinese economy. Many studies have investigated the impact of the One-Child Policy on household saving rates (İmrohoroğlu and Zhao (2020, 2018)), education attainment (Qin et al. (2017); Wang and Zhang (2018); Bairoliya and Miller (2021); Rosenzweig and Zhang (2009)) , female labor market outcome (Zhang (2017); Edlund et al. (2013)), and sex ratio distortions (Ebenstein (2011)). Choukhmane et al. (2013), developed a quantitative OLG model to illustrate that the OCP has significantly increased the

human capital of the only child generation and accounts for 30% of rising in aggregate savings. They pointed out that the fertility restrictions reduced parents' expected old-age support coming from the next generations, which result in parents raising savings and education investment in their children. Liao (2013) used an overlapping generations model with the fertility constraint to investigate how the OCP affects human capital accumulation, output per capita, and welfare differences across generations and skill groups. She highlighted that the OCP promoted human capital and GDP per capita in China. Huang et al. (2021) studied the how the OCP impact social and economic outcomes over a life-cycle. They found that exposure to stricter enforcement of the OCP at a young age results in higher human capital, more white-collar jobs, household income, consumption, and savings. In this study, we consider the OCP as an exogenous shock that affects the gender imbalance in China (Ebenstein (2010); Li et al. (2011)), which impacts couples' matching outcome and implicate variations of income inequality.

Second, this study contributes to the literature that investigates the interaction between household assortative matching and income inequality. Becker (1973, 1974) had indicated that males tend to marry females with similar socioeconomic conditions, and assortative matching is positively related to household income inequality. Greenwood et al. (2014, 2016, 2017) use several waves of US census data to analyze the trend of positive assortative mating, they found that there is a rising in the positive assortative mating in the US between 1960 and 2005, and positive assortative mating contributed the increase of income inequality across households. The same conclusion is drawn from Fernandez et al. (2005), which uses cross countries' data to illustrate that assortative matching is positively correlated with inequality for 33 nations, the analysis of their developed structural model also supports this augment. Fernández and Rogerson (2001) developed a dynamic model of inter-generational education acquisition and marital sorting to demonstrate that increased sorting will significantly increase income inequality. Eika et al. (2019) employ data from Denmark, Germany, Norway, the UK, and the US to document the degree of educational assortative matching, they find that assortative matching accounts for a certain portion of household income inequality in each nation, but the variations in the

degree of assortative matching weakly move the time trends of household income inequality over time. The previous papers seldom included the analysis of unequal numbers of men and women when developing the structural model of the marriage pattern, this paper studies how the assortative matching evolves with unequal numbers of marriageable age men and women in the marriage market.

Third, we contribute to the literature on evaluating the effect of the sex ratio on economic outcomes (household saving rate, housing price, crime rate, etc.). Bhaskar and Hopkins (2016) theoretically indicated that male imbalance will cause parents over-investment in sons but under-investment in daughters. Angrist (2002); Abramitzky et al. (2011) illustrated that the decreasing of sex ratios caused male hypergamy. Lafortune (2013) empirically shows that sex imbalance increases pre-marriage investment in education. Wei and Zhang (2011); Wei et al. (2017) indicated from their studies that the rising sex ratio in China caused families with sons to save more and purchase more expensive houses, where the savings and family housing value are used to denote male's attractiveness in the marriage market. Edlund et al. (2013) found that an increasing sex ratio is positively associated with the rising crime rate. Du et al. (2015) concluded from their findings that sex imbalance in China resulted in female hypergamy in terms of the personal income gap, parents' wealth gap, and occupation within the government system. Previous papers seldom investigate the impact of male abundance on the degree of household assortative matching, and the implications on income inequality and education investment decision, this study contributes to the literature on this question both empirically and theoretically (in Chapter 3).

The rest of the paper is organized as follows. Section 2.2 presents brief descriptions of the institutional background for the OCP and the Chinese marriage market. Section 2.3 discuss the data source and the sample restrictions in this study. Section 2.4 presents the empirical results about the correlation between the sex ratio in the marriage market and the degree of assortative matching. Section 2.5 reports the empirical results about how the sex ratio affect the household income inequality. Section 2.6 employs the instrumental variables to resolve the endogeneity issues in the

regressions, in which we use the intensity of the OCP enforcement as one of the instrumental variables for the sex ratio. Section 2.7 draws the conclusions.

## 2.2 Background on the One Child Policy and the Chinese Marriage Market

This section presents some institutional background for the One Child Policy and introduces the tradition of the marriage market in China.

### 2.2.1 One Child Policy in China

The One-Child Policy (OCP) in China was enacted in 1979, which lasted for over 30 years and affected millions of people. China had already enacted an aggressive family planning policy in the early 1970s, and the fertility rates had already dropped sharply before the initiation of the OCP. Fig. 2.2 demonstrates that the fertility rate keeps declining since 1965, before the starting of the OCP, the fertility rate dropped. Comparing the fertility rate in China, the U.S., and at the world level, the study finds that after the implementation of the OCP for several years, China has the lowest fertility rate. The principal content of the OCP is that each couple was restricted to having only one child. But the implementation of this policy in practice varied by ethnicity, region, hukou types, and time. In terms of ethnicity, the OCP restrictions focused on Han people, which on average make up 92 percent of the population. As illustrated in Table 2.1, the Han ethnicity people are the majority population in China, where minorities have a share of less than 10 percent. In all provinces, minorities were allowed to have a second child without penalties. But the Han people in China were restricted to have a second baby. In terms of region, before the OCP was implemented, the fertility rate was significantly higher in the less developed western regions when compared with the more developed eastern regions. Due to this fact, the enforcement of the OCP in the western part of China was more difficult because it means a significant change in social and cultural tradition, which result in a stronger resistance

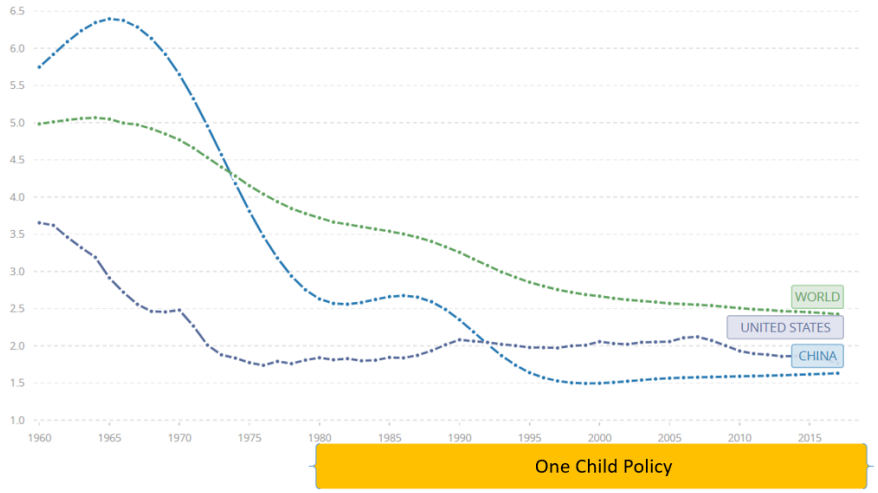



FIGURE 2.2: Fertility Rate

Source: World Bank

by local people (Huang et al. (2021); Zhang (2017)).

TABLE 2.1: The proportion of different types of population

| Census | Han-share | Urban-share |
|---|---|---|
| 1982 | 93.30 | 21.13 |
| 1990 | 91.89 | 26.41 |
| 2000 | 91.21 | 36.22 |
| 2010 | 91.51 | 49.95 |

Source: China's Population Census

The enforcement of the OCP varied by hukou type. The actual penalties for the excess births varied across rural and urban hukou types. In practice, the government regulated the behavior of urban residents more easily compared with rural residents. The urban residents in China can be directly and easily affected by the policies of governments. In the regulations of the penalties for violating the birth quota in the urban area, the residents who worked in state-owned enterprises could lose their jobs and social welfare benefits if they have more than one baby. For rural residents, the common punishment for the family who have above-quota births was a one-time fine. The distinguishable enforcement resulted in a two-tier policy, the urban family were only allowed to have a baby whereas rural families were often allowed to have a second child. Huang et al. (2021); Ebenstein (2010, 2011); Edlund et al. (2013) pointed

out that rural families were allowed to have a second child if the firstborn was a girl, which led to the sex imbalance in China. Also, when examining the trend from Fig. 2.1, we find that the sex ratio at birth in the rural area has a higher magnitude compared with the urban area. This data fact is consistent with the analysis of the violation of the OCP for rural residents.

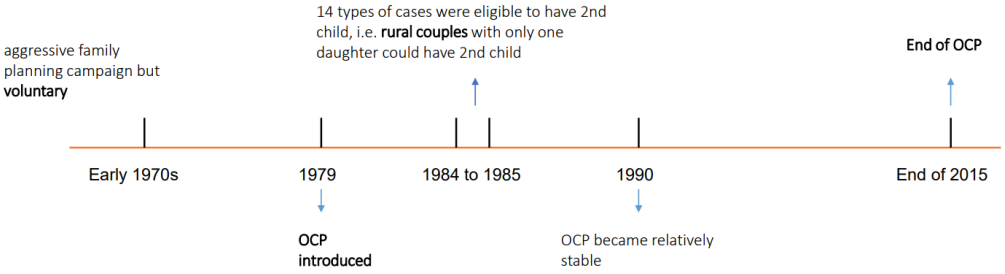



FIGURE 2.3: Timeline of One Child Policy

The enforcement of the OCP also varied across time spans. In Fig. 2.3, the timeline of the OCP has been illustrated. Before the initiation of the OCP, in the early 1970s, there were aggressive family planning campaigns, which proposed that households ought to have fewer children to manage population growth in China. The campaign was voluntary and each family was encouraged to follow government's guidance (Huang et al. (2021); Zhang (2017)). In 1979, the OCP was officially implemented throughout China, the central government proposed that it would be essential to impose legislation and penalties on above-quota births. Substantial policy regulations were made and delivered to the local government in each province. However, during the mid of 1980s, there was strong resistance from the rural family in China, especially those families who have only one female child. Considering the practical difficulties, the OCP was relaxed to make it more feasible for rural residents. From 1984 to 1985, there were 14 types of cases that are eligible for having a second child, and the most important rule indicated that a rural family with one girl could have a second child (Zhang (2017)). In 1990, the OCP became stable in terms of enforcement and resident compliance. By the end of 2015, the OCP was fully abolished in China, whereas a

two-children policy had been issued (each family can have two children), and following up, now, China initiates the three-children policy (each family can have three children).

### 2.2.2 The Marriage Market

According to Wei and Zhang (2011); Wei et al. (2017), the marriage market is "local" for most Chinese people, that is: most marriages are formed among couples who come from the same cities or counties. Based on the results of the 2000 census, 92 percent of rural residents live in their county (there were over 2000 counties in China) of birth and about 62 percent of urban residents live in the city where they were born. In addition, the results from the 2000 census indicate that 89 percent of marriages are formed between couples from the same counties. The China household and income projects (CHIPs) in 2002 reports that about 82 percent of migrant workers get married to their spouses who come from the same province. In summary, the mobility that is driven by the incentive of residents in forming marriage is limited in China.

This study focuses on the 'young marriage' between couples whose ages are between 20 to 30. The reasons on why choose this age range are as follow: firstly, Chinese couples mostly have their first marriage at the age of 20 to 30, which is illustrated by Fig. 2.4. The 2005 census reports that the central tendencies of the first marriage in urban China, where males mostly ranged from 20 to 35 for their first marriage, and females mostly ranged from 18 to 32. Secondly, choosing 'young marriage' can better reflect how the sex ratio in the marriage market affects couples matching outcomes, since older agents' marriage decisions may involve personal income, children, health conditions, housing, etc. Thirdly, the study proposes a model to demonstrate the matching pattern of agents, in which agents decide to form a household depending on the expected earnings in the future and the match quality, choosing 'young marriage' enable the empirical analysis as close as the spirit of the model.

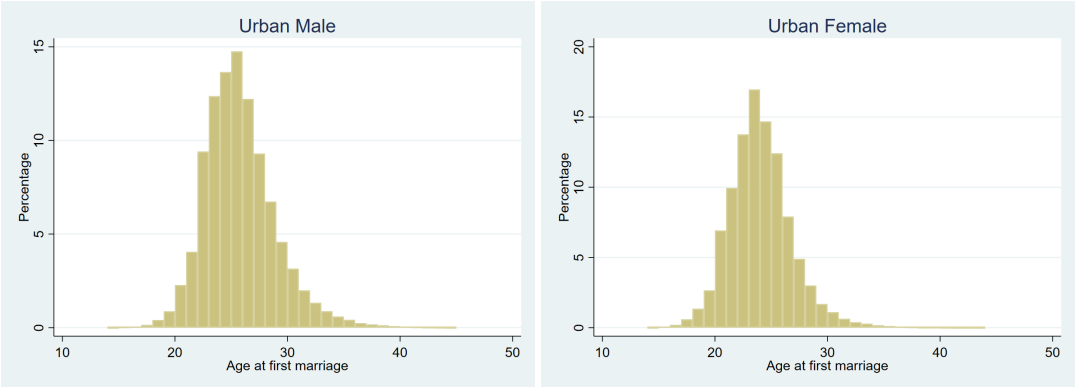



FIGURE 2.4: Age of first marriage in China

Source: 2005 Chinese Population Census

## 2.3 Data and Sample

The data mainly comes from three sources, which are Chinese Household Income Projects (CHIPs), the Chinese Population Census, and the China City Statistical Yearbook. We use three waves of CHIPs (2002, 2007, and 2013) to collect household-level data, which enable us to measure the regional education correlation between couples. The study restricts the couple's age from 20 to 30. There are two reasons that we restrict this range of cohort's ages. Firstly, as illustrated in Fig. 2.4, the age of first marriage among Chinese couples concentrates from 20 to 30, we restrict the first marriage because a second marriage or multiple times of marriage may involve the consideration of children, housing, wealth, and other factors that are hard to capture, which may result in a biased estimation between the relationship of sex ratio in the marriage market and the degree of household sorting. Secondly, we restrict the first marriage, which is close to the spirit of model design in chapter 3, where we model the payoff of marriage comes from the agent's life-cycle income, agents randomly meet and infer the partner's future income.

There are many methods in using the sex ratio to reflect the marriage market competition. Wei and Zhang (2011) employed the sex ratio of age 0 to 9 cohort in the 1990 census to infer the sex ratio of age 12 to 21 cohorts in 2002, where they used 2002 CHIPs data to study the competitive saving for son family and daughter family, the sex ratio is measured with the proportion of pre-marital males to pre-marital females.

Wei et al. (2017) employed the sex ratio of cohort aged 0 to 14 from the 2000 census to reflect the marriage market competition intensity. Edlund et al. (2013) defined the sex ratio over the region, cohort, and gender, which they employed six provinces' data from the Urban Household Survey (UHS) to study the effect of sex ratio on individual education attainment and labor market outcomes. In this study, we define the sex ratio in marriage at the prefecture level, which we infer from the population census of 2000 and 2010. The sex ratio is the share of aged 20 to 30 males to aged 20 to 30 females. The detailed selection methods can be found in Table. 2.2.

The study uses the CHIPs and the China City Statistical Yearbook to form control variables. In the regional level regression, the controls are collected from the China City Statistical Yearbook, to match the waves of CHIPs (2002, 2007, and 2013). We compute the mean values of each control variable, then use these means as the covariates in the regression. As the degree of household sorting can be affected by regional GDP per capita, employment status, population density, and so on for several years ago. For instance, the controls we used in the 2007 waves, are means of each covariate collected from 2003, 2004, 2005, 2006, and 2007. We employ the setting for the other two waves. The controls at the regional level include GDP per capita, local government education expenditure, the volume of FDI, population density, number of unemployed workers, local average wages, number of colleges, number of high schools, number of college teachers, and number of high school teachers.

TABLE 2.2: Cohort age selection

| Wave | Cohort age | Inferred from | Corresponding age group in the census |
|---|---|---|---|
| 2002 | 20 to 30 | Census 2000 | 18 to 28 |
| 2007 | 20 to 30 | Census 2010 | 23 to 33 |
| 2013 | 20 to 30 | Census 2010 | 17 to 27 |

Source: Population Census of 2000, 2010

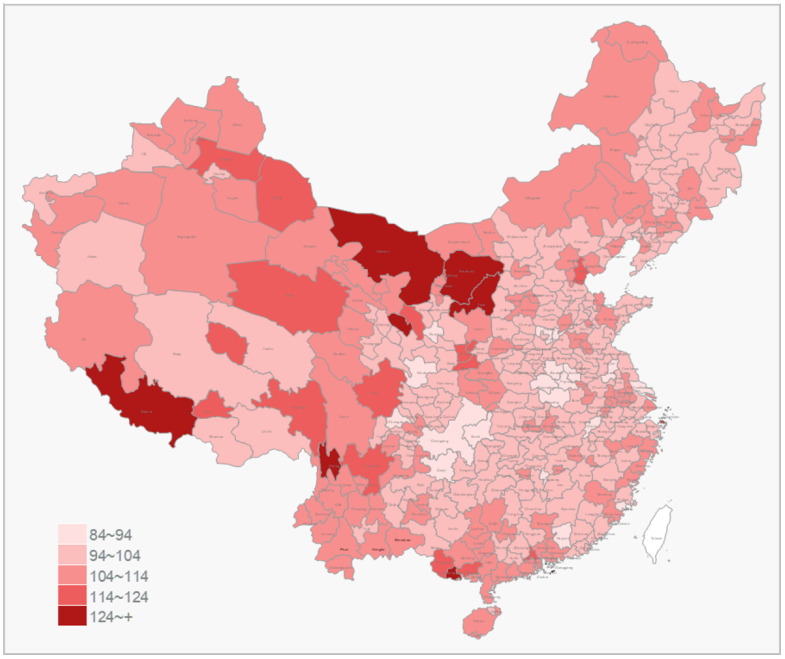



FIGURE 2.5: Sex ratio of age 20 to 30 cohort at prefectures

## 2.4 Sex Ratio and the Assortative Matching

We firstly study the effect of sex ratio on assortative matching at the regional level by proposing the following regression:

$$EduCorr_{jt} = \alpha + \beta SR_{jt} + \gamma Z_{jt} + \tau_t + \varepsilon_{jt} \tag{2.1}$$

Equation 2.1 is used to study the effect of sex ratio ($SR_{jt}$) on couples' education correlation ($EduCorr_{jt}$) at the regional level, where $j$, $t$ denotes region, CHIPs wave respectively. In this setting, we use the education correlation in the regions to measure the degree of assortative matching in the marriage market. $Z_{jt}$ are control variables which are the mean value of each variable over the past five years, including population density, the share of age 20 to 30, GDP per capita, regional average wage, FDI, the number of colleges and the number of high schools. We also controlled time fix effect ($\tau_t$). The results are illustrated in the Table. 2.3.

Table 2.3 illustrates that the sex ratio at the regional level is negatively correlated with the education correlation between couples, where the variable "srpref" denotes the

TABLE 2.3: Sex ratio and education correlation at the prefecture level

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| srpref | -2.12** | -2.11** | -2.11** | -2.11** |
| | (0.93) | (0.93) | (0.93) | (0.91) |
| Time FE | No | No | Yes | Yes |
| Controls of regional characteristics | Yes | Yes | Yes | Yes |
| Cluster robust standard errors | No | Yes | No | Yes |
| Observations | 101 | 101 | 101 | 101 |
| $R^2_{adj}$ | 0.05 | 0.05 | 0.03 | 0.03 |
| F-stat | 1.18 | 1.22 | 1.09 | 1.13 |
| Prob > F | 0.32 | 0.30 | 0.38 | 0.35 |

Standard errors in parentheses

Source: CHIPs, China City Statistical Yearbook, Census 2000, 2010

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

sex ratio at the prefecture level. It illustrates that a one-unit increase in the sex ratio is associated with a 2.12 unit decrease in the degree of assortative matching. The results are all statistically significant at the 5% level. Time fix effects have also been controlled, the standard errors in regression are optional in robust standard error or not. The negative correlation between the sex ratio in the marriage market and household sorting is consistent with our hypothesis that a higher sex ratio in the marriage market is associated with a lower degree of household assortative matching.

To examine whether the negative relationship between the sex ratio and household sorting is robust, we proposed an alternative regression.

$$wedu_{ijt} = \alpha + \beta hedu_{ijt} + \gamma SR_{jt} + \psi hedu_{ijt} \times SR_{jt} + \theta X_{ijt} + \rho Z_{jt} + \delta_j + \tau_t + \varepsilon_{ijt} \quad (2.2)$$

Regression 2.2 is used to illustrate how husbands' education ($hedu_{ijt}$) levels change in the presence of a rising sex ratio in the marriage market, we employ the interaction term here $\psi hedu_{ijt} \times SR_{jt}$ to demonstrate the marginal effect of the wifes' education ($wedu_{ijt}$) on husbands' education by interacting the sex ratio. This is to see how this marginal value changes with the variation of the sex ratio in the marriage market. Where $X_{ijt}$ capture individual-level control variables which include husband's age, wife's age, husband's age square, wife's age square, husband's status (work or not),

wife's status (work or not), husband's Hukou type (urban or rural), wife's Hukou type (urban or rural), husband's minority (dummy variable, yes or no), wife's minority (yes or no). And $Z_{jt}$ stands for control variables at the regional level, same as the setting in the regression 2.1, we use the mean value of each variable over the past five years to reflect the influences of regional characteristics on individual education attainment. The regional control variables include GDP per capita, population size, the share of aged 20 to 30 people in the total population, the share of aged 15 to 64 people in the total population (working-age people), and the number of colleges and the number of high schools. $\delta_j$ control for region fixed effect and $\tau_t$ control for time fixed effect, where $\varepsilon_{ijt} \sim N(0, \sigma^2)$.

TABLE 2.4: Interaction term

| | (1) wedu | (2) wedu | (3) wedu | (4) wedu | (5) wedu |
|---|---|---|---|---|---|
| hedu | 1.33*** | 1.36*** | 1.35*** | 1.35*** | 1.36*** |
| | (0.45) | (0.48) | (0.46) | (0.46) | (0.53) |
| srpref | 6.89 | 7.74 | 7.43 | 7.45 | 6.41 |
| | (5.51) | (6.00) | (5.90) | (5.95) | (8.51) |
| hedu × srpref | -0.59 | -0.63* | -0.65* | -0.65* | -0.67* |
| | (0.44) | (0.46) | (0.45) | (0.45) | (0.51) |
| Controls of regional characteristics | No | Yes | Yes | Yes | Yes |
| Controls of individual characteristics | No | No | Yes | Yes | Yes |
| Time FE | No | No | No | Yes | Yes |
| Region FE | No | No | No | No | Yes |
| Observations | 847 | 754 | 754 | 754 | 754 |
| $R^2$ | 0.52 | 0.53 | 0.57 | 0.57 | 0.61 |

OLS estimation, Standard errors in parentheses
Source: CHIPs, China City Statistical Yearbook, Census 2000, 2010
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table. 2.4 shows that the wife's education level (*wedu*) is positively correlated with the husband's education level (*hedu*), where the interaction term $\psi$ between the husband's education level and the sex ratio in the region has negative coefficients; the coefficients are statistically significant at 10% level for other four regressions, excepting the first

regression that doesn't control for regional characteristics, individual characteristics, time fixed effects and region fixed effects. When looking at the marginal effect of the husband's education on the wife's education in response to the sex ratio, we see that: $ME = \frac{\Delta wedu_{ijt}}{\Delta hedu_{ijt}} = \beta + \psi SR_{jt}$=1.36-0.67$SR$. This illustrates that with the rising sex ratio, there is a decreasing marginal effect of the wife's education level on the husband's education level. The coefficient of interaction term $\psi$ in column (5) is statistically significant at 10% level, The interpretation here is that the increase of the sex ratio in the marriage market is negatively associated with the household sorting, as one unit increase in the sex ratio is associated with 1.36-0.67=0.69 unit decrease of the marginal effect of *wedu* on *hedu*. The decreasing marginal effect could be led by the decline of the wife's years of schooling when keeping the husband's years of schooling unchanged, by the rise of the husband's education level when keeping the wife's education level unchanged, or by the rise of husband's education level when there is declining of wife's education level. In all these scenarios, the decreasing marginal effect implies that household sorting is decreasing with the rise of the sex ratio in the marriage market.

We also proposed regression 2.3 to study the effect of the sex ratio on household sorting.

$$EduGap_{ijt} = \alpha + \beta SR_{jt} + \lambda X_{ijt} + \gamma Z_{jt} + \delta_j + \tau_t + \varepsilon_{ijt} \tag{2.3}$$

By using regression 2.3, where the degree of household sorting is measured by the education gap between husband and wife ($EduGap$), we would expect that $\beta$ is positive, which indicates that the rising of the sex ratios increases the education gap between couples. Du et al. (2015) employed a similar regression and found that there is a positive relationship between the sex ratio and the couple's education gap, which indicated female hypergamy in China. In regression 2.3, $X_{ijt}$ are control variables which include the age gap of the couple, age gap square, a dummy for husband's work status, and minority difference between spouses. $Z_{jt}$ stands for control variables at the regional level, the same as the setting in regression 2.1, we use the mean value of each variable over the past five years to capture the regional characteristics in the past,

these controls include GDP per capita, population size, the share of 20 to 30 age people in total population, the share of 15 to 64 age people in total population (working-age people), the number of colleges and the number of high schools. $\delta_j$ controls for regional fixed effect and $\tau_t$ controls for time fixed effect, where $\varepsilon_{ijt} \sim N(0, \sigma^2)$.

TABLE 2.5: Education gap between spouse

| | (1) edugap | (2) edugap | (3) edugap | (4) edugap | (5) edugap |
|---|---|---|---|---|---|
| srpref | 0.59 | 3.34** | 3.49** | 3.91** | 5.42** |
| | (0.98) | (1.51) | (1.50) | (1.57) | (2.33) |
| Controls of regional characteristics | No | Yes | Yes | Yes | Yes |
| Controls of individual characteristics | No | No | Yes | Yes | Yes |
| Time FE | No | No | No | Yes | Yes |
| Region FE | No | No | No | No | Yes |
| Observations | 847 | 754 | 754 | 754 | 754 |
| $R^2$ | 0.00 | 0.03 | 0.05 | 0.06 | 0.18 |

OLS estimation, $EduGap^s = |Edu^h - Edu^w|$, Standard errors in parentheses
Standard errors are clustered at prefecture level
Source: CHIPs, China City Statistical Yearbook, Census 2000, 2010
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table. 2.5 demonstrates that there is a positive and significant relationship between the sex ratio at the regional level and the couple's education distance. With the rising sex ratio, there is a rising in education distance between husband and wife. The results are statistically significant at the 5% level. The implications of these results are consistent with the output of regression 2.1 and regression 2.2.

In summary, the empirical evidence points out that the sex ratio in the marriage market is negatively correlated with the degree of household sorting. The mechanism behind this relationship will be investigated in the model part. The following section will employ instrumental variables to resolve the endogeneity concerns and discuss how the sex ratio affects household income inequality.

## 2.5 Sex Ratio and the Household Income Inequality

This part presents the results of the analysis of how the sex ratio in the marriage market affects household income inequality. The study previously investigates that sex ratio is negatively correlated with the degree of assortative matching, which implies that sex ratio should negatively correlate with household income inequality. As sorting and household income inequality presents a positive relationship in most of the empirical evidence (Eika et al. (2019); Fernandez et al. (2005); Greenwood et al. (2014)). In this study, we first form regression to examine the correlation between household sorting and income inequality. Two variables are used to measure income inequality, which is the Gini coefficient and wage percentile ratio (80 to 20) respectively. The measurement is illustrated below in equation 2.4.

$$Pwage = log(\frac{Couple\ wage_{80}}{Couple\ wage_{20}}) \tag{2.4}$$

To examine the relationship between household sorting and household income inequality, a series of Tobit estimations are implemented, as the household incomes are left-censored due to the existence of unemployed families. When we construct the household income inequality measurement, some observations are zero and thus we employ the Tobit model to form regressions. Table. 2.6 demonstrates the results on how household sorting affects income inequality, the sorting in this part is measured by education correlation at the regional level, and the control variables include GDP per capita, population density, the numbers of colleges and the numbers of the working-age population (age 15 to 64). The results of Table. 2.6 indicate that sorting is positively and significantly correlated with household income inequality. The results are consistent with the literature's finding that household sorting is positively related to household income inequality.

TABLE 2.6: Sorting and income inequality

| | (1) Gini | (2) Pwage | (3) Gini | (4) Pwage | (5) Gini | (6) Pwage |
|---|---|---|---|---|---|---|
| Sorting | 0.03* | 0.23** | 0.03 | 0.21** | 0.03* | 0.21** |
| | (0.02) | (0.10) | (0.02) | (0.09) | (0.02) | (0.10) |
| Controls of regional characteristics | No | No | Yes | Yes | Yes | Yes |
| Time FE | No | No | No | No | Yes | Yes |
| Observations | 115 | 114 | 105 | 105 | 105 | 105 |
| *Pseudo* $R^2$ | -0.01 | 0.01 | -0.08 | 0.02 | -0.10 | 0.03 |

Tobit I model, Standard errors in parentheses
Source: CHIPs, China City Statistical Yearbook, Census 2000, 2010
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

TABLE 2.7: Sex ratio and income inequality

| | (1) Gini | (2) Pwage | (3) Gini | (4) Pwage | (5) Gini | (6) Pwage |
|---|---|---|---|---|---|---|
| srpref | -0.01 | -0.53 | -0.00 | -0.59 | -0.10 | -1.24 |
| | (0.20) | (1.21) | (0.19) | (1.23) | (0.20) | (1.25) |
| Controls of regional characteristics | No | No | Yes | Yes | Yes | Yes |
| Time FE | No | No | No | No | Yes | Yes |
| Observations | 159 | 158 | 147 | 147 | 147 | 147 |
| *Pseudo* $R^2$ | -0.00 | 0.00 | -0.56 | 0.02 | -1.07 | 0.04 |

Tobit I model, Standard errors in parentheses
Source: CHIPs, China City Statistical Yearbook, Census 2000, 2010
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

At the same time, this study examines how the sex ratio affects household income inequality, which presents the results below in Table. 2.7. Similarly, we employ a series of Tobit estimations to study the correlation between the sex ratio and income inequality. The estimated coefficients are all negative, but they are statistically insignificant, this might be the reason that there is a mediation relationship between sex ratio and household sorting, and household sorting affects household income inequality.

## 2.6 Instrument Variables

The examination of the correlation between the sex ratio in the marriage market and couples' education correlation may encounter endogeneity problems. The degree of the household sorting in our main regression (regression 2.1) is measured by the

couples' education correlation at the prefecture level. There may exist some of the missing regressors that are simultaneously correlated with local couples' education correlation and local sex ratios. In addition, the sex ratio in the marriage market is measured by the ratio of the number of marriageable age males over the number of marriageable age females in a prefecture. There may be some measurement errors of it, for instance, some amount of males or females are homosexual, also, some amount of migration for marriage purpose increase the noise to prefecture sex ratios (the indicator of marriage market competition intensity). Generally speaking, the local sex ratio in the study may be endogenous, which could generate biased estimates.

This part presents an instrumental variable approach to resolve the endogeneity concerns. Many studies have pointed out that China's One-Child Policy (OCP) has significantly affected the gender balance, which exogenously determined the sex ratio at birth and hence the sex ratio in the marriage market (Ebenstein (2010); Zhang (2017); Ebenstein (2011); Wei and Zhang (2011)). The IVs we employed in this study follow the idea of Ebenstein (2010), who first used the monetary penalties of violation on One-Child Policy to examine how does One-Child Policy distort the sex ratio. Even though the goals of this policy are national, the enforcement is local and the monetary penalties are various across provinces. Here we proposed three potential instrumental variables, which are:

- ($Fine_{p,w}$) Monetary penalties for violating the birth quotas in provinces, which take the form of a percentage deduction of local yearly income, and last for several years. For instance, Shanghai reported that the unauthorized birth was imposed a 10 percent wage deduction from both parents for 16 years. Ebenstein (2010) developed an algorithm to compute the present value of fine rates across provinces and years which assume a 2 percent discount rate. The plotted fine rates can be found in Appendix. This study collects the data of the fine rates from his paper as the instrumental variable. The fine rates in each year ($t$) and each province ($p$) is denoted by ($Punishment_{pt}$).

- ($Premium_{p,w}$) Dummy for the existence of extra fines for violations at higher-order births. For instance, additional penalties reduce the incentive of

families to have a third or fourth child. These two-family planning variables are averaged over the corresponding birth years of cohorts aged 20 to 30 in different CHIPs waves. A detailed description can be found in Table. 2.8.

- ($PD$) Provinces dummy: as the implemented penalties varied across provinces.
- ($log(wage)$) Regional average wages.

The fines ($Fine_{p,w}$) of different provinces ($p$) at different waves of CHIPs data sample are computed as:

$$Fine_{p,w} = \frac{1}{T}\sum_{t}^{T} Punishment_{pt} \tag{2.5}$$

Where the $Punishment_{pt}$ is the fine rates in each province $p$ and year $t$. We average the fine rates to generate a single amount fine, which is used to illustrate the average intensity of the penalty for violating family planning policy. Table. 2.8 demonstrates the detailed computation methods. For each wave of CHIPs, we restrict the cohort

TABLE 2.8: The construction of IVs

| Wave | Cohort age | Birth year | $Fine_{pw}$ |
|---|---|---|---|
| 2002 | 20 to 30 | 1972 to 1982 | $\frac{1}{T}\sum_{t=1972}^{T=1982} Punishment_{pt}$ |
| 2007 | 20 to 30 | 1977 to 1987 | $\frac{1}{T}\sum_{t=1977}^{T=1987} Punishment_{pt}$ |
| 2013 | 20 to 30 | 1983 to 1993 | $\frac{1}{T}\sum_{t=1983}^{T=1993} Punishment_{pt}$ |

age between 20 to 30, the table shows the corresponding birth year, and during the birth year interval, we compute the average value of fine rates. After the formalization of instrumental variables, we specify the regression below to have the two-stage least square (2SLS) estimation:

$$IV: \begin{cases} SR_{jt} = \alpha_1 + \psi V_{jt} + \theta Z_{jt} + \tau_t + u_{jt} \\ \\ EduCorr_{jt} = \alpha_2 + \beta \hat{SR}_{jt} + \gamma Z_{jt} + \tau_t + \varepsilon_{jt} \end{cases} \tag{2.6}$$

Where $V_{jt}$ are the instrumental variables, including average fine rates, average premium, province dummy, and regional average wages. $Z_{jt}$ are the same control variables for the two stages of regression.

TABLE 2.9: IV results

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| First Stage | | | | |
| Fine | 0.01 | 0.01 | 0.02 | 0.02 |
| | (0.03) | (0.03) | (0.04) | (0.04) |
| Premium | | 0.05 | | 0.03 |
| | | (0.04) | | (0.04) |
| Wage(log) | | | 0.07 | 0.06 |
| | | | (0.05) | (0.05) |
| Province dummies | Yes | Yes | Yes | Yes |
| Controls | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |
| Observations | 139 | 139 | 138 | 138 |
| $R^2$ | 0.53 | 0.54 | 0.55 | 0.55 |
| | (1) | (2) | (3) | (4) |
| Second Stage | | | | |
| srpref | -2.25 | -2.04 | -3.22* | -2.91* |
| | (1.78) | (1.72) | (1.73) | (1.69) |
| Controls | Yes | Yes | Yes | Yes |
| Year FE | Yes | Yes | Yes | Yes |
| Observations | 101 | 101 | 101 | 101 |
| $R^2$ | 0.13 | 0.13 | 0.12 | 0.12 |
| Hansen's $J$ statistic (p-value) | 0.49 | 0.56 | 0.47 | 0.53 |
| Kleibergen-Paap rk Wald F statistic | 4.80 | 6.35 | 11.99 | 13.23 |

2SLS estimation, sorting is measured with Pearson correlation, Standard errors in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table. 2.9 demonstrates the IV regression results in both the first stage and second stage. In the first stage regressions, the coefficients on all the instrumental variables we used are positive, but not statistically significant. The positive sign indicates that violation of the OCP increases the sex ratio imbalance, which implies that with a more severe penalty for violation of the family planning policy, there is an increasing trend for parents aggressively aborting girls, which results in a higher sex ratio. In the second stage regressions, the coefficient of sex ratio on household sorting is negative and statistically significant in columns 3 and column 4.

Hansen's $J$ statistics in all four regressions do not reject the null hypothesis that the overidentifying restrictions are valid, the test effectively checks whether the estimates obtained from each possible just-identified model are significantly different. If different instruments identify different local average treatment effects (LATE), the null will be rejected even if each instrument is valid for a particular LATE. From the table, one can conclude that different IVs used in the model don't identify different local average treatment effects, the combination of the IVs which have been used in these four models are valid.

The Kleibergen-Paap rk Wald F statistics are related to the second stage of 2SLS regression, which is used to examine the weak IV problems. In column 3 and column 4 are 11.99 and 13.23 respectively, we can reject the weak IV null hypothesis.

Additionally, the point estimates in Table. 2.9 are generally larger than the OLS estimators in Table. 2.3. This result implies the downward bias in Table. 2.3, which are generated from missing regressors or measurement errors. The larger magnitude of estimated coefficients in columns 3 and 4 indicate that once we include the information on the OCP intensity in the regression, there is a higher impact on the effect of sex ratio in the marriage market on household sorting. A higher sex ratio results in a lower degree of assortative matching among couples.

In summary, in the empirical evidence part, the research has investigated the relationship between the sex ratio and assortative matching, where the presented negative correlation between the two variables is robust in all three regressions. We also examined the relationship between sorting and income inequality, the results are consistent with other literature's finding that household sorting increase the degree of income inequality, however, we do not find significant evidence that the sex ratio affects household income inequality. We further use the IVs to study the causality and endogeneity issues between the sex ratio and household sorting, in the meantime, this approach provides a policy evaluation on the One-Child Policy, where the results indicate that the OCP increased the gender imbalance in China, also amplify the magnitude of sex ratio's impact on household sorting.

## 2.7 Conclusion

The rising gender imbalance in China has caused the attention of economists. This study is motivated by the fact that after the introduction of the One-Child Policy, there is a rising trend in sex ratio at the national level. This demographic change induced intensive competition in the marriage market, which is documented by many studies. In this research, we propose that rising gender imbalance would reduce the degree of assortative matching for Chinese couples, which results in a decrease in household income inequality in China. This argument has been examined by both the empirical analysis and the theoretical model (in Chapter 3).

From the empirical evidence, we find that the sex ratio in regions is negatively correlated with the educational correlation between husband and wife. This result is robust when we use other regression specifications. We further examined the relationship between household sorting and income inequality, the result is consistent with other literature's finding that there exists a positive correlation between them. We then study how the sex ratio affects household income inequality, and the results illustrate a negative relationship. We also employ IVs to resolve the potential endogeneity issues in our key independent variable (the sex ratio in the marriage market), which come from the penalty of violating the One-Child Policy restrictions, taking the forms of punishment ratio, province dummies, and so on, to illustrate that the One-Child Policy distorted gender balance in China and amplify the effect of the sex ratio on the degree of assortative matching. To further extend our analysis, a structural interpretation is proposed in Chapter 3, in which we employ a model to demonstrate the marriage market dynamics and matching outcomes in China.

# Chapter 3

# Gender Imbalance, Assortative Matching and Household Income Inequality in China–Theory

*In light of chapter 2's robust empirical evidence that the sex ratio in the marriage market is negatively correlated with household sorting, this chapter intends to study the mechanism that how sex ratio affects household sorting, its implication on household income inequality, and further evaluate how the agents make education decision with the rising of male abundance. We propose a general equilibrium model to understand marriage patterns and provide a structural interpretation of the empirical results. The model has featured four types of agents with two opposite genders and two skill types, there are two rounds of matching in the marriage market, and the simulated results suggest that the sex ratio in the marriage market is negatively correlated with the degree of assortative matching, and it implicates a decline of household income inequality in China. In addition, the model predicts an increasing proportion of skilled males and females in China when sex ratios keep rising.*

## 3.1 Introduction

The male abundance in China has caused a surge in the studies that analyze its effect on the Chinese economy. Many of the studies have focused on evaluating its impact on family outcomes (intrahousehold bargaining, female labor force participation, etc.), crime rate, household saving rate, and inter-generational investment. Few of them have looked at how male abundance affects household sorting and its implications on household income inequality. Most of the relevant research has employed reduced-form analysis to interpret the impact of male abundance on household sorting, this study first uses a structural framework to undertake the quantitative analysis.

The main objective of this paper is to quantitatively assess how male abundance affects household sorting and study its implications on income inequality by employing a search and matching-based model. This framework follows the work of Fernandez et al. (2005), which proposed a two rounds of the matching model to reflect the marriage market dynamics, however, they failed to consider the dynamics in the presence of unbalanced numbers of males and females in the marriage market. In light of the previous chapter's empirical evidence that the sex ratio in the marriage market is negatively correlated with the degree of assortative matching between couples, we modified the framework of Fernandez et al. (2005) to accommodate male abundance in the marriage market. We introduce the sex ratio exogenously in the model which is calibrated from the data sample, the marriage market includes four types of agents which are skilled males, unskilled males, skilled females, and unskilled females. Two rounds of matching allow us to develop the linkage between the assortative matching and unbalanced numbers of participants in the marriage market, in which agents face the trade-off between accepting an unsatisfactory match in the first round and rejecting that match and going to the second round but with a lower likelihood to meet a 'good' partner. The structural model enables us to understand why the sex ratio is negatively related to household sorting and how it implicates household income inequality.

The model can quantitatively match the empirical evidence we have presented in chapter 2 and the previous literature's findings. First of all, the generated quantitative results from the model also indicate that the degree of assortative matching decrease with the increase of the sex ratio, the reason behind it is that the males are less picky with the rising sex ratio. Secondly, the model points out that there is a decline in household income inequality in China with the increase in the sex ratio, this result is robust when we use alternative income inequality measurements. Thirdly, we further extend the model to allow agents to make education decisions before they enter the marriage market in the second period, in which they can decide to become skilled or unskilled agents. The result highlights that there is an increasing proportion of skilled males and skilled females when the sex ratio keeps rising, this implies that males and females are more likely to become skilled agents with the rising in male abundance, this result provides an explanation for the rising education level that has been observed from data in China, it indicates that rising male abundance also contributes the education expansion in China in recent years, though it's just one of many.

### 3.1.1 Existing Literature

There are two principal approaches to model and interpret marriage patterns, which are matching models and collective models respectively (Galichon et al. (2019); Chiappori (2020)). The matching-based models highlight market-level forces and treat agents' heterogeneous tastes over partners as one part of the essential building block. The collective models emphasize the effects of intra-household bargaining. Pioneered by Becker (1973); Shapley and Shubik (1971), the matching models have been widely used to model the marriage market pattern, in which males and females with heterogeneous tastes are able to form households, this framework highlights the sharing of the marriage surplus within a transferable utility setting (Chiappori and Mazzocco (2017); Mourifié and Siow (2021); Greinecker and Kah (2021)). The collective model approach proposed by Chiappori (1992) highlights the intra-household bargaining in a complex feasible utility set, there is nonlinearity in the household bargaining process due to the imperfect substitutability. Excepting these

two principal approaches, Galichon et al. (2019) develop an imperfectly transferable utility matching model with random utility, which unite the marriage matching models and the collective models. In their framework, couples participate in a one-to-one bilateral matching market, the utility transfers within the matching market are not additive, they also prove the existence and uniqueness of the equilibrium which includes taste heterogeneity.

Some structural models have also been used to interpret the interaction between assortative matching and household income inequality. Becker (1973, 1974) had indicated that males tend to marry females with similar socioeconomic conditions when looking at the matching patterns of couples, and assortative matching is positively related to household income inequality. Greenwood et al. (2014, 2016, 2017) use several waves of US census data to analyze the trend of positive assortative mating, they found that there is a rising in the positive assortative mating in the US between 1960 and 2005, and positive assortative mating contributed the increase of income inequality across households. The same conclusion is drawn from Fernandez et al. (2005), which uses cross countries' data to illustrate that assortative matching is positively correlated with inequality for 33 nations, the analysis of their developed structural model also supports this augment. In addition, Greenwood et al. (2016) developed a unified model of marriage, divorce, educational attainment, and married female labor-force participation to demonstrate that positive assortative mating positively affects household income inequality. Fernandez et al. (2005) also introduced a two rounds of matching model to understand the marriage market pattern, in which they show that household sorting increases inequality. Fernández and Rogerson (2001) developed a dynamic model of inter-generational education acquisition and marital sorting to demonstrate that increased sorting will significantly increase income inequality. Eika et al. (2019) employ data from Denmark, Germany, Norway, the UK, and the US to document the degree of educational assortative matching, they find that assortative matching accounts for a certain portion of household income inequality in each nation, but the variations in the degree of assortative matching weakly move the time trends of household income inequality over time. The previous papers seldom included the analysis of unequal numbers of men and women when developing the

structural model of the marriage pattern, this paper studies how the assortative matching evolves with unequal numbers of marriageable age men and women in the marriage market.

The rest of this chapter is organized as follows. Section 3.2 describes the details of the construction of the model, after that, the model is been calibrated by using a data sample, and then the corresponding quantitative analysis is presented. The study also makes extensions by allowing agents to choose education investment before they enter the marriage market, and relevant quantitative results are reported. Section 3.5 draws the conclusions.

## 3.2 The Model

To inspect the dynamics and mechanism that why the rising sex ratio is associated with declining household sorting, a model is proposed in this section. The model is following Fernandez et al. (2005), in which we propose a general equilibrium model to study the effect of male abundance on household assortative matching and its implication on household income inequality.

### 3.2.1 Setting

The economy is populated by two opposite genders of agents, and each gender is characterized by two types (either skilled '$s$' or unskilled '$u$'). Thus, there are four types of agents. There are two rounds of matching in the marriage market, before all types of agents enter into the marriage, they make their skill choice, either skilled or unskilled, and after the education decision is made, all types of agents enter the marriage market and meet randomly and observe the opposite gender's skill types, in which they infer the partner's future income and draw a match-specific quality (love).

Agents are assumed to derive utility from the consumption of household income (husband's wage plus wife's wage) and match-specific quality. The indirect utility function for couples with match quality $q$ and household income $I$ is:

$$V(I_{ij}, q) = U(I_{ij}) + q = U[e(w_m^z + w_f^z)] + q \tag{3.1}$$

Where $U$ is a continuous and strictly increasing function of $I$, $i$ denotes the types of males, which $i \in \{m_s, m_u\}$, $j$ denotes the types of females, which $j \in \{f_s, f_u\}$. $g$ stands for gender and $z$ denotes an individual's skill type, which is $z \in \{s, u\}$, $e$ is used to measure the household equivalence scale, which is a measure of the cost of living for a household given family size and demographic composition. Household income composes by:

$$I_{ij} = w_i + w_j \tag{3.2}$$

For $i \in \{m_s, m_u\}$ and $j \in \{f_s, f_u\}$. Where $w_m$ denotes the wage of males and $w_f$ denotes the wage of females, all the wages are exogenously been calibrated by using CHIPs 2013 urban data sample, where we construct measures to compute the agent's lifetime income.

In population, the number of males is massed to 1 ($N^m = 1$), where the number of females ($N^f < 1$) is set to smaller than males. In this setting, $\frac{N^m}{N^f} = \frac{1}{N^f}$ denotes the sex ratio in the marriage market. Let $\psi_m$ denotes the share of skilled male in all males, and $\psi_f$ denotes the share of skilled female in all females.

### 3.2.2 Matching

There are two rounds of matching in the marriage market, where in the first round, agents meet randomly and draw a random match-specific quality $q$, we assume that $q$ is non-negative and follows the continuous distribution, with the mean value and support of $\mu$ and $[q_{\min}, q_{\max}]$ respectively, we also restrict that $q_{\min}$ and $q_{\max}$ are non-negative. The cumulative distribution function (CDF) of $q$ is $Q$. The match can be accepted by both agents (resulting in a marriage) or rejected by at least one of the agents (both agents go to the second round of matching).

In the second round, agents can only match with their own skill group and draw a

new random match quality, assuming this quality is match specific i.i.d. draws from the same continuous CDF $Q$. Individuals who remain single can only obtain utility from his or her own income.

Let $\bar{q}_g^{zz}$ be the cutoff match quality for an agent with gender $g$ and match type $zz$, where $g \in \{m, f\}$, and $zz \in \{ss, su, us, uu\}$. Below the cutoff match quality, the agent will not accept the match. Thus, the probability of agents getting married is: Prob $\{q \geq \bar{q}_m^{zz}\} \wedge$ Prob $\{q \geq \bar{q}_f^{zz}\}$, that is

$$1 - Q\left[\max(\bar{q}_m^{zz}, \bar{q}_f^{zz})\right] = (1 - Q(\bar{q}_m^{zz}))(1 - Q(\bar{q}_f^{zz}))$$

In the first round, since $N^m > N^f$, some men will not meet potential partners, and the total number of meetings is $N^f$. The number of meetings that take place in the first round is:

| | |
|---|---|
| $N^f \psi_m \psi_f$ : | the number of meetings between skilled male and skilled female |
| $N^f \psi_m (1 - \psi_f)$ : | the number of meetings between skilled male and unskilled female |
| $N^f (1 - \psi_m) \psi_f$ : | the number of meetings between unskilled male and skilled female |
| $N^f (1 - \psi_m)(1 - \psi_f)$ : | the number of meetings between unskilled male and unskilled female |

(3.3)

In the second round, remaining unmarried skilled males match (and get married) with skilled females, and remaining unmarried unskilled males match (and get married) with unskilled females. Let $\theta_g^z$ stand for the number of the unmarried agent after the first round of the match with gender $g$ and skill type $z$. Then, the number of unmarried agents after the first round of matching will be:

$$\theta_m^s = (1-N^f)\psi_m + N^f\psi_m\psi_f Q\left[\max(\bar{q}_m^{ss}, \bar{q}_f^{ss})\right] + N^f\psi_m(1-\psi_f)Q\left[\max(\bar{q}_m^{su}, \bar{q}_f^{su})\right]$$
the number of unmarried skilled males after round 1

$$\theta_m^u = (1-N^f)(1-\psi_m) + N^f(1-\psi_m)\psi_f Q\left[\max(\bar{q}_m^{us}, \bar{q}_f^{us})\right] + N^f(1-\psi_m)(1-\psi_f)Q\left[\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})\right]$$
the number of unmarried unskilled males after round 1

$$\theta_f^s = N^f\psi_m\psi_f Q\left[\max(\bar{q}_m^{ss}, \bar{q}_f^{ss})\right] + N^f(1-\psi_m)\psi_f Q\left[\max(\bar{q}_m^{us}, \bar{q}_f^{us})\right]$$
the number of unmarried skilled females after round 1

$$\theta_f^u = N^f\psi_m(1-\psi_f)Q\left[\max(\bar{q}_m^{su}, \bar{q}_f^{su})\right] + N^f(1-\psi_m)(1-\psi_f)Q\left[\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})\right]$$
the number of unmarried unskilled females after round 1 (3.4)

There is a trade-off between accepting the first round of the match and rejecting it which agent goes to the second round of the match. In particular, a skilled male who meets an unskilled female in the first round and draws a high match quality will encounter a trade-off between forming a low-income household with high match quality, or rejecting this match and forming a high-income household with a skilled female in the second round but with an expected match quality that is $\mu$.

### 3.2.3 Equilibrium

Given $V(I_{ij}, q) = U(I_{ij}) + q$, a skilled male will be indifferent between accepting a first-round match with an unskilled female and rejecting that match and entering into the second round if:

$$V(I_{m_s,f_u}, \bar{q}_m^{su}) = E[V(I_{m_s,f_s}, \mu)]$$

Where $\bar{q}_m^{su}$ denotes the reservation match quality for the skilled male when he meets with an unskilled female. The first character in the superscript '$s$' is used to denote the male's skill type, whereas the second character '$u$' is used to denote the female's skill

type. In the following part, we use this notation to represent the reservation match quality for each type of agent. Thus, there is eight reservation match quality in total. Hence, the equilibrium is determined by eight match quality cutoffs in the first round. Solving for the value of $q$ at the equilibrium yields the reservation quality of $\bar{q}_g^{zz}$.

In round 1, a skilled male matches with a skilled female will be indifferent if:

$$U[e(w_m^s + w_f^s)] + \bar{q}_m^{ss} = \min(\frac{\theta_f^s}{\theta_m^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_f^s}{\theta_m^s}, 1)\right] U(w_m^s)$$

And if

$$U[e(w_m^s + w_f^s)] + q_{\min} > \min(\frac{\theta_f^s}{\theta_m^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_f^s}{\theta_m^s}, 1)\right] U(w_m^s)$$

then all the skilled males that are matched with skilled females will want to get married in round 1.

If

$$U[e(w_m^s + w_f^s)] + q_{\max} < \min(\frac{\theta_f^s}{\theta_m^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_f^s}{\theta_m^s}, 1)\right] U(w_m^s)$$

then no skilled male matched with a skilled female will want to get married in round 1. Where $q_{\max}$, $q_{\min}$ represent the upper bound and lower bound of match quality.

In summary, eight indifference conditions determine the equilibrium, which are illustrated as follows:

The indifference condition for a skilled male matched with a skilled female:

$$U[e(w_m^s + w_f^s)] + \bar{q}_m^{ss} = \min(\frac{\theta_f^s}{\theta_m^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_f^s}{\theta_m^s}, 1)\right] U(w_m^s)$$

The indifference condition for a skilled female matched with a skilled male:

$$U[e(w_m^s + w_f^s)] + \bar{q}_f^{ss} = \min(\frac{\theta_m^s}{\theta_f^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_m^s}{\theta_f^s}, 1)\right] U(w_f^s)$$

The indifference condition for an unskilled male matched with a skilled female:

$$U[e(w_m^u + w_f^s)] + \bar{q}_m^{us} = \min(\frac{\theta_f^u}{\theta_m^u}, 1) \left[U[e(w_m^u + w_f^u)] + \mu\right] + \left[1 - \min(\frac{\theta_f^u}{\theta_m^u}, 1)\right] U(w_m^u)$$

The indifference condition for a skilled female matched with an unskilled male:

$$U[e(w_m^u + w_f^s)] + \bar{q}_f^{us} = \min(\frac{\theta_m^s}{\theta_f^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_m^s}{\theta_f^s}, 1)\right] U(w_f^s)$$

The indifference condition for a skilled male matched with an unskilled female:

$$U[e(w_m^s + w_f^u)] + \bar{q}_m^{su} = \min(\frac{\theta_f^s}{\theta_m^s}, 1) \left[U[e(w_m^s + w_f^s)] + \mu\right] + \left[1 - \min(\frac{\theta_f^s}{\theta_m^s}, 1)\right] U(w_m^s)$$

The indifference condition for an unskilled female matched with a skilled male:

$$U[e(w_m^s + w_f^u)] + \bar{q}_f^{su} = \min(\frac{\theta_m^u}{\theta_f^u}, 1) \left[U[e(w_m^u + w_f^u)] + \mu\right] + \left[1 - \min(\frac{\theta_m^u}{\theta_f^u}, 1)\right] U(w_f^u)$$

The indifference condition for an unskilled male matched with an unskilled female:

$$U[e(w_m^u + w_f^u)] + \bar{q}_m^{uu} = \min(\frac{\theta_f^u}{\theta_m^u}, 1) \left[U[e(w_m^u + w_f^u)] + \mu\right] + \left[1 - \min(\frac{\theta_f^u}{\theta_m^u}, 1)\right] U(w_m^u)$$

The indifference condition for an unskilled female matched with an unskilled male:

$$U[e(w_m^u + w_f^u)] + \bar{q}_f^{uu} = \min(\frac{\theta_m^u}{\theta_f^u}, 1) \left[U[e(w_m^u + w_f^u)] + \mu\right] + \left[1 - \min(\frac{\theta_m^u}{\theta_f^u}, 1)\right] U(w_f^u)$$

### 3.2.4 Equilibrium Distribution of Households

Since we have solved the reservation match quality for different types of matches, we can generate the equilibrium distribution of household types. Let $\pi_{zz}$ denote the number of different types of households in the population, thus we have:

The number of skilled husbands with skilled wives in the population:

$$\pi_{ss} = \underbrace{N^f \psi_m \psi_f \left(1 - Q[\max(\bar{q}_m^{ss}, \bar{q}_f^{ss})]\right)}_{\text{marriage at the first round}} + \underbrace{\min(\theta_m^s, \theta_f^s)}_{\text{marriage at the second round}}$$

The number of skilled husbands with unskilled wives in the population:

$$\pi_{su} = N^f \psi_m (1 - \psi_f) \left(1 - Q[\max(\bar{q}_m^{su}, \bar{q}_f^{su})]\right)$$

The number of unskilled husbands with skilled wives in the population:

$$\pi_{us} = N^f (1 - \psi_m) \psi_f \left(1 - Q[\max(\bar{q}_m^{us}, \bar{q}_f^{us})]\right)$$

The number of unskilled husbands with unskilled wives in the population:

$$\pi_{uu} = N^f (1 - \psi_m)(1 - \psi_f) \left(1 - Q[\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})]\right) + \min(\theta_m^u, \theta_f^u)$$

The number of unmarried skilled males after two rounds of matching in the population:

$$\pi_s = \max(\theta_m^s - \theta_f^s, 0)$$

The number of unmarried unskilled males after two rounds of matching in the population:

$$\pi_u = \max(\theta_m^u - \theta_f^u, 0)$$

Note that we have:

$$\pi_{ss} + \pi_{su} + \pi_{us} + \pi_{uu} = N^f$$

Where the total number of marriages equals the number of females in the population.

Also have:

$$\pi_{ss} + \pi_{su} + \pi_{us} + \pi_{uu} + \pi_s + \pi_u = 1$$

### 3.2.5 Education Correlation Between Husband and Wife

Education correlation between husband and wife $\rho$ is been defined as:

$$\rho = \frac{Cov(X,Y)}{\sqrt{Var(X)}\sqrt{Var(Y)}} = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E(X)^2}\sqrt{E(Y^2) - E(Y)^2}}$$

Where $X$ and $Y$ stands for events $\{husband\}$ and $\{wife\}$ respectively, let:

$$X = \begin{cases} 1 & \text{husband is skilled} \\ 0 & \text{husband is unskilled} \end{cases} \qquad Y = \begin{cases} 1 & \text{wife is skilled} \\ 0 & \text{wife is unskilled} \end{cases} \tag{3.5}$$

Hence, the expectations of $X$ and $Y$ are:

$$E(X) = \frac{\pi_{ss}+\pi_{su}}{N^f} \qquad E(Y) = \frac{\pi_{ss}+\pi_{us}}{N^f} \tag{3.6}$$

And we have:

$$E(X^2) = \frac{\pi_{ss}+\pi_{su}}{N^f} \qquad E(Y^2) = \frac{\pi_{ss}+\pi_{us}}{N^f} \tag{3.7}$$

$$E(XY) = \frac{\pi_{ss}}{N^f}$$

Thus:

$$\rho = \frac{E(XY) - E(X)E(Y)}{\sqrt{E(X^2) - E(X)^2}\sqrt{E(Y^2) - E(Y)^2}} = \frac{\frac{\pi_{ss}}{N^f} - \frac{\pi_{ss}+\pi_{su}}{N^f}\frac{\pi_{ss}+\pi_{us}}{N^f}}{\sqrt{(\frac{\pi_{ss}+\pi_{su}}{N^f}) - (\frac{\pi_{ss}+\pi_{su}}{N^f})^2}\sqrt{(\frac{\pi_{ss}+\pi_{us}}{N^f}) - (\frac{\pi_{ss}+\pi_{us}}{N^f})^2}} \tag{3.8}$$

### 3.2.6 Calibration and Functional Forms

The model is calibrated to match the empirical evidence and reproduce quantitative properties. The utility function takes the form of:

$$U(I_{ij}) = log(I_{ij})$$

We assume match quality follows a continuous uniform distribution. The support of it is $[0, 1]$, where the $\mu = 0.5$ in this setting. The wages for different types of agents are exogenously been calibrated. We employ CHIPs 2013 data to project the lifetime wages for four types of agents. The life cycle for individuals is divided into five-year intervals, starting from 25 for both males and females. The mandatory retirement age in China is different for males and females, for males, the restricted age is 65, for females, the restricted age is 60. For the projections of lifetime wage of males, the interval is 25-30, 31-36,..., and 60-65. For females, it's up to 60. The wage we used is the before-tax net labor wage, as tax may play the role of redistributing incomes. The discount factor used in the projection is 0.96. We compute the average wage over the five-years interval and discount it. The wage of the unskilled female is normalized to 1, we use the wage premiums to obtain those for skilled females, unskilled males, and skilled males. Where the wage premium of a skilled female is equal to the ratio of the projected lifetime wage for a skilled female over an unskilled female, and the wage premium of an unskilled male is equal to the ratio of the projected lifetime wage for an unskilled male over unskilled female, the same setting applies to the wage premium of skilled male.

The number of females in the model is exogenously computed from the Chinese population census in 2000 and 2010, the age range selection is consistent with empirical analysis (20 to 30). Where the mean value of $N^f$ is 0.97. Education shares of skilled males and skilled females are projected from three waves of CHIPs urban sample data (2002, 2007, 2013), we compute the mean education value for age group 20 to 30 for both males and females in each wave of CHIPs respectively, then take the mean value of those three waves to get the value of $\psi^m$ and $\psi^f$.

TABLE 3.1: Calibration

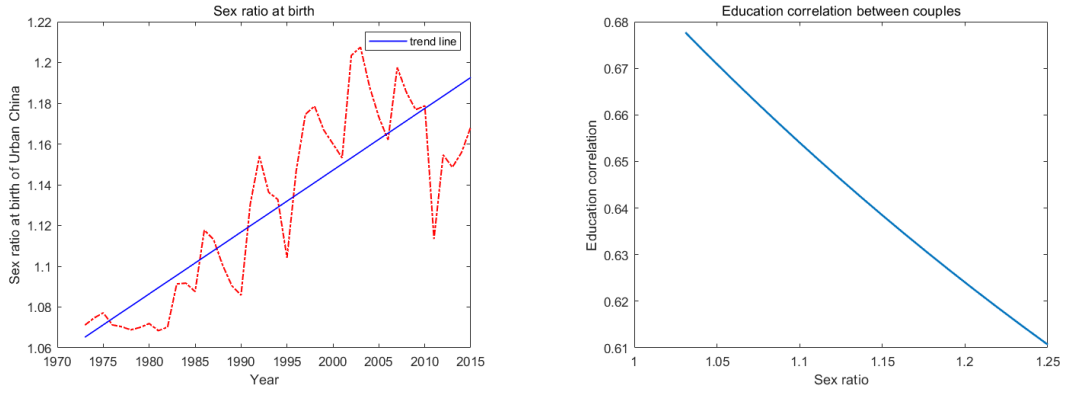
| Parameters | Value | Source |
|---|---|---|
| $N^f$ | 0.80 to 0.98 | Census 2000, 2010 |
| $w_u^f$ | 1 | CHIPs 2013 |
| $w_u^m$ | 1.3592 | CHIPs 2013 |
| $w_s^f$ | 1.6162 | CHIPs 2013 |
| $w_s^m$ | 2.0579 | CHIPs 2013 |
| $\psi^m$ | 0.5132 | CHIPs 2002, 2007, 2013 |
| $\psi^f$ | 0.4883 | CHIPs 2002, 2007, 2013 |

## 3.3 Quantitative Analysis

This section presents the key results of quantitative analysis. By using the eight indifference conditions and calibrated parameters, we could solve eight reservation match quality in the model. After then we could generate some quantitative experiments. The section is organized as follows. First, we present results on the degree of assortative matching (measured by education correlation between couples in the model) and illustrate how it evolves with the rise in sex ratio. Second, we show the implication of changing education assortative matching on household income inequality by constructing the Lorenz curve and Gini coefficient.

TABLE 3.2: Reservation match quality $\bar{q}_g^{zz}$

| $N^f$ | Sex ratio | $\bar{q}_m^{ss}$ | $\bar{q}_f^{ss}$ | $\bar{q}_m^{us}$ | $\bar{q}_f^{us}$ | $\bar{q}_m^{su}$ | $\bar{q}_f^{su}$ | $\bar{q}_m^{uu}$ | $\bar{q}_f^{uu}$ |
|---|---|---|---|---|---|---|---|---|---|
| 0.80 | 1.25 | 0.33 | 0.50 | 0.00 | 0.71 | 0.51 | 0.24 | 0.24 | 0.50 |
| 0.81 | 1.23 | 0.33 | 0.50 | 0.02 | 0.71 | 0.52 | 0.24 | 0.25 | 0.50 |
| 0.82 | 1.22 | 0.34 | 0.50 | 0.03 | 0.71 | 0.53 | 0.24 | 0.26 | 0.50 |
| 0.83 | 1.20 | 0.35 | 0.50 | 0.04 | 0.71 | 0.54 | 0.24 | 0.27 | 0.50 |
| 0.84 | 1.19 | 0.36 | 0.50 | 0.05 | 0.71 | 0.54 | 0.24 | 0.29 | 0.50 |
| 0.85 | 1.18 | 0.37 | 0.50 | 0.07 | 0.71 | 0.55 | 0.24 | 0.30 | 0.50 |
| 0.86 | 1.16 | 0.37 | 0.50 | 0.08 | 0.71 | 0.56 | 0.24 | 0.31 | 0.50 |
| 0.87 | 1.15 | 0.38 | 0.50 | 0.09 | 0.71 | 0.57 | 0.24 | 0.32 | 0.50 |
| 0.88 | 1.14 | 0.39 | 0.50 | 0.10 | 0.71 | 0.58 | 0.24 | 0.34 | 0.50 |
| 0.89 | 1.12 | 0.40 | 0.50 | 0.12 | 0.71 | 0.58 | 0.24 | 0.35 | 0.50 |
| 0.90 | 1.11 | 0.41 | 0.50 | 0.13 | 0.71 | 0.59 | 0.24 | 0.36 | 0.50 |
| 0.91 | 1.10 | 0.41 | 0.50 | 0.15 | 0.71 | 0.60 | 0.24 | 0.38 | 0.50 |
| 0.92 | 1.09 | 0.42 | 0.50 | 0.16 | 0.71 | 0.61 | 0.24 | 0.39 | 0.50 |
| 0.93 | 1.08 | 0.43 | 0.50 | 0.17 | 0.71 | 0.62 | 0.24 | 0.41 | 0.50 |
| 0.94 | 1.06 | 0.44 | 0.50 | 0.19 | 0.71 | 0.62 | 0.24 | 0.42 | 0.50 |
| 0.95 | 1.05 | 0.45 | 0.50 | 0.20 | 0.71 | 0.63 | 0.24 | 0.44 | 0.50 |
| 0.96 | 1.04 | 0.45 | 0.50 | 0.22 | 0.71 | 0.64 | 0.24 | 0.45 | 0.50 |
| 0.97 | 1.03 | 0.46 | 0.50 | 0.23 | 0.71 | 0.65 | 0.24 | 0.47 | 0.50 |




(A) (B)

FIGURE 3.1: Household sorting across time

The solved reservation match quality illustrates that with the rising sex ratio in the marriage market, all types of males are less picky relative to females. For all genders, in the match type of a skilled female pair with an unskilled male, the reservation match quality is highest among the other seven, which indicates that there is the highest probability that a female will reject the match when she meets with an unskilled male (about 71.1%). Additionally, the reservation match quality for the unskilled male is lowest when he meets a skilled female, with the increasing sex ratio, this type of reservation match quality remains the lowest one and approaches zero.

### 3.3.1 Education Correlation Between Couples

By using the solved reservation match quality, we could compute the education correlation between husband and wife with different values of sex ratio. The idea is to trace the trend of the degree of household sorting when there is a rising gender imbalance. Using equation 3.8, the degree of assortative matching is measured with the education correlation between couples in the marriage market. The simulated results have been illustrated below:

As illustrated in Fig. 3.1, where the left panel of it is data plotting, to demonstrate the increasing trend of sex ratio at birth in China; and the right panel is from the model output, which illustrates the education correlation between husband and wife is decreasing with the rise in the sex ratio. This is in line with our empirical evidence

that the sex ratio is negatively correlated with the degree of assortative matching. The intuition here is that the rise of the number of males in the marriage market makes men less likely to find a partner, thus males are less picky and resulting in a decrease in education sorting. When looking at the sex ratio at birth from the data sample, the plotted trend line indicates that the sex ratio at birth is increasing as time forward, this implies that the sex ratio will increase when the young cohorts enter the marriage market as time forward. Also, it implies that there will be decreasing in household sorting when time forward, which means a declining trend of household income inequality.

Additionally, comparing the moments which are generated from the solved model with data, we see that model can 95.74% match with the sample mean in terms of the mean value of education correlation. As for the standard deviation of education correlation, the model generates a much smaller value compared with the data sample, this is because there is more heterogeneity in regional education correlation when looking at the data sample. One potential way to address the underestimation of the volatility of education correlation is to introduce idiosyncratic shock in the eight indifference conditions, to capture more heterogeneity across agents.

TABLE 3.3: Fit of the model, education correlation between couples

| Moment | Model | Data (sample) |
|---|---|---|
| Education correlation (mean) | 0.66 | 0.69 |
| Education correlation (Std.dev.) | 0.02 | 0.10 |

Source: CHIPs 2002, 2007, 2013

### 3.3.2 Implication for Household Income Inequality

Marriage market matching outcomes have important implications on household income inequality. The increase in the degree of assortative marriage is associated with the rise in household income inequality. In this section, we present results on how male abundance affects household income inequality in urban China. Multiple indexes can be used to measure household income inequality. We first employ the

Gini coefficient to show the relationship between the sex ratio in the marriage market and income inequality. By constructing the Lorenz curve (formulating the cumulative household share in population and cumulative income share), the Gini coefficient can be derived analytically, which can be found in Appendix.

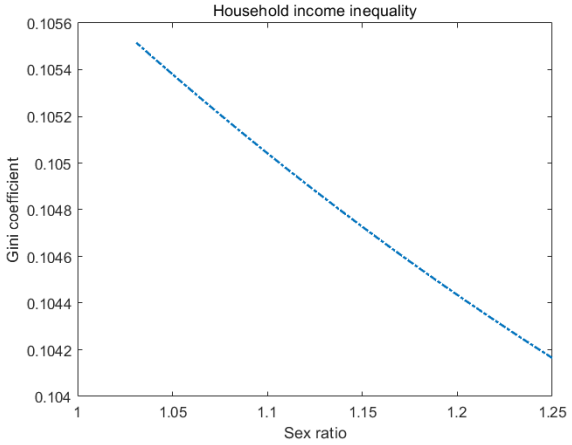
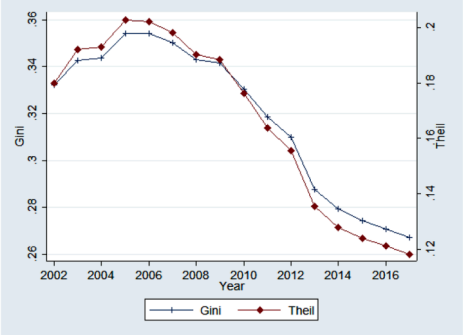


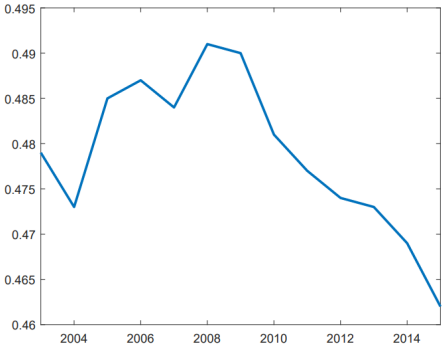

FIGURE 3.2: Gini coefficient and sex ratio

Fig. 3.2 shows that the sex ratio in the marriage market is negatively correlated with household income inequality in China when using the Gini coefficient. It also indicates that the Gini coefficient of Chinese households decreases as time forward. The intuition here is that due to the decline of household sorting, household income is more equally distributed as males are less picky in the marriage market. When comparing the results with the literature, we find that the declining trend of household income inequality is consistent with the latest paper's finding, which is the declining trend of household income inequality as time forward. The results from Fig. 3.3 illustrate that inequality declines substantially after a certain year in China. We then use other income inequality measures to see whether this result is robust. The plotted results point out that household income inequality is negatively associated with the sex ratio in the marriage market with other inequality measures being used (see Fig. 3.4).




(A) Regional inequality in income per capita

Source: Kanbur et al. (2021)




(B) Gini coefficient

Source: Bai et al. (2020)

FIGURE 3.3: Income inequality in China

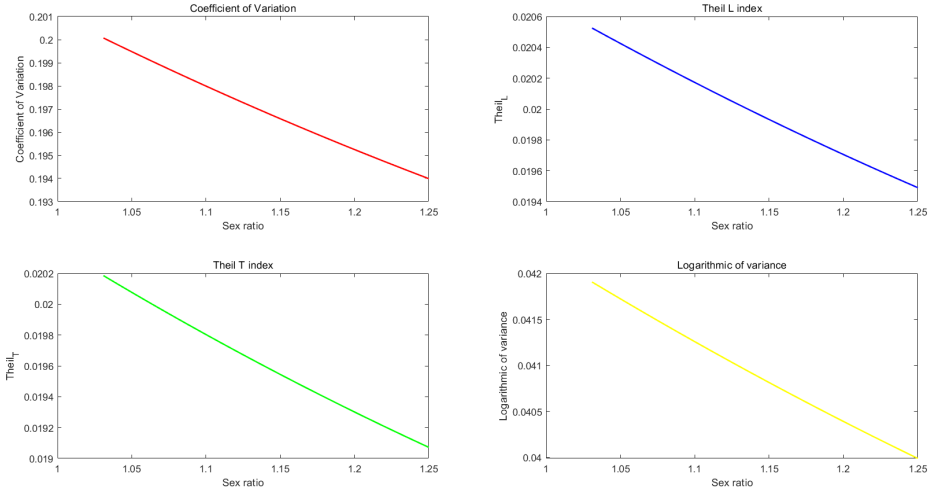



FIGURE 3.4: Sex ratio and other household income inequality measures

## 3.4 Education Decision

In the previous sections, the skilled shares for both males and females are calibrated from the data sample, rather, investigating how agents choose their education level is interesting, the objective to have this extension is to have a counterfactual experiment on how would male choose when he expects the marriage market competition will be more intensive with the rise of sex ratio. To answer this question, we allow agents to choose their education investment before they enter the marriage market, and once finish their education, agents will enter the marriage market competition.

Agents form expectations on utility gain received from matching in the marriage

market, in which they decide to whether become a skilled agent or an unskilled agent. Education investment decision could generate two sources of returns, which come from the labor market (this section still assume wages for different types of agent are exogenously determined) and the marriage market (e.g. determine the probability that unskilled females meet with skilled males). The expected value function thus changes to:

$$E_t V_{t+1}(\psi^g_{t+1}) = E_t \left\{ U \left[ e(w^z_m + w^z_f) \right] + q^{zz}_g \right\} \tag{3.9}$$

The decision to become a skilled or an unskilled agent involves several scenarios, we write the expected value function explicitly, which is illustrated below. For different agents, the decision to become either skilled or unskilled is plotted in Appendix B.3. The decision tree explicitly demonstrates the outcomes of each step of the decision, and we can compute the corresponding expected utility.

The introduction of time subscripts in the equation 3.9 is to help one recognize the forward-looking behavior of agents, in which males and females form expected payoff in getting married. The sex ratio used in this part is exogenous and calibrated by using the data samples, each time, we input the sex ratio into the model and solve the fraction of skilled males $\psi^m$ and skilled females $\psi^f$, the sex ratio is used to trace out the time path and increasing sex ratio corresponds with time forward, thus we are able to explain the increasing education level which is observed in census data.

The expected utility of being a skilled male given the fraction of $\psi^f_{t+1}$ skilled female is:

$$
\begin{aligned}
\mathrm{E}_t V^s_m(\psi^m_{t+1}) =& \psi^f_{t+1} N^f_{t+1} \left\{ \int_{\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_m \right] dq \right. \\
& + Q[\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)] \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_m \right] dq \\
& \left. + Q[\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)] \left[ 1 - \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \right] U[w^s_m] \right\} \\
& + (1 - \psi^f_{t+1}) N^f_{t+1} \left\{ \int_{\max(\bar{q}^{su}_m, \bar{q}^{su}_f)}^{q_{\max}} \left[ U\left( e[w^s_m + w^u_f] \right) + q^{su}_m \right] dq \right. \\
& + Q[\max(\bar{q}^{su}_m, \bar{q}^{su}_f)] \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_m \right] dq \\
& \left. + Q[\max(\bar{q}^{su}_m, \bar{q}^{su}_f)] \left[ 1 - \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \right] U\left[ w^s_m \right] \right\} \\
& + (1 - N^f_{t+1}) \left\{ \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_m \right] dq \right. \\
& \left. + \left[ 1 - \min\left( \frac{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}, 1 \right) \right] U\left[ w^s_m \right] \right\}
\end{aligned}
\tag{3.10}
$$

The expected utility of being a skilled female given the fraction of $\psi^m_{t+1}$ skilled male is:

$$
\begin{aligned}
\mathrm{E}_t V^s_f(\psi^f_{t+1}) = \min\left(\frac{\psi^m_{t+1}}{N^f_{t+1}}, 1\right) \Bigg\{ & \int_{\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_f \right] dq \\
& + Q[\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)] \min\left(\frac{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_f \right] dq \\
& + Q[\max(\bar{q}^{ss}_m, \bar{q}^{ss}_f)] \left[ 1 - \min\left(\frac{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \right] U\left[ w^s_f \right] \Bigg\} \\
+ \min\left(\frac{1 - \psi^m_{t+1}}{N^f_{t+1}}, 1\right) \Bigg\{ & \int_{\max(\bar{q}^{us}_m, \bar{q}^{us}_f)}^{q_{\max}} \left[ U\left( e[w^u_m + w^s_f] \right) + q^{us}_f \right] dq \\
& + Q[\max(\bar{q}^{us}_m, \bar{q}^{us}_f)] \min\left(\frac{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w^s_m + w^s_f] \right) + q^{ss}_f \right] dq \\
& + Q[\max(\bar{q}^{us}_m, \bar{q}^{us}_f)] \left[ 1 - \min\left(\frac{\theta^s_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^s_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \right] U\left[ w^s_f \right] \Bigg\}
\end{aligned}
\tag{3.11}
$$

The expected utility of being an unskilled male given the fraction of $\psi_{t+1}^f$ skilled female is:

$$
\begin{aligned}
\mathrm{E}_t V_m^u(\psi_{t+1}^m) =& \psi_{t+1}^f N_{t+1}^f \left\{ \int_{\max(\bar{q}_m^{us}, \bar{q}_f^{us})}^{q_{\max}} \left[ U\left( e[w_m^u + w_f^s] \right) + q_m^{us} \right] dq \right. \\
& + Q[\max(\bar{q}_m^{us}, \bar{q}_f^{us})] \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w_m^u + w_f^u] \right) + q_m^{uu} \right] dq \\
& \left. + Q[\max(\bar{q}_m^{us}, \bar{q}_f^{us})] \left[ 1 - \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \right] U[w_m^u] \right\} \\
& + (1 - \psi_{t+1}^f) N_{t+1}^f \left\{ \int_{\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})}^{q_{\max}} \left[ U\left( e[w_m^u + w_f^u] \right) + q_m^{uu} \right] dq \right. \\
& + Q[\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})] \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w_m^u + w_f^u] \right) + q_m^{uu} \right] dq \\
& \left. + Q[\max(\bar{q}_m^{uu}, \bar{q}_f^{uu})] \left[ 1 - \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \right] U[w_m^u] \right\} \\
& + (1 - N_{t+1}^f) \left\{ \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[ U\left( e[w_m^u + w_f^u] \right) + q_m^{uu} \right] dq \right. \\
& \left. + \left[ 1 - \min\left(\frac{\theta_f^u(\psi_{t+1}^m, \psi_{t+1}^f)}{\theta_m^u(\psi_{t+1}^m, \psi_{t+1}^f)}, 1\right) \right] U[w_m^u] \right\}
\end{aligned}
\tag{3.12}
$$

The expected utility of being an unskilled female given the fraction of $\psi^m_{t+1}$ skilled male is:

$$
\begin{aligned}
\mathrm{E}_t V^u_f\left(\psi^f_{t+1}\right) = \min\left(\frac{\psi^m_{t+1}}{N^f_{t+1}}, 1\right) \Bigg\{ & \int_{\max(\bar{q}^{su}_m, \bar{q}^{su}_f)}^{q_{\max}} \left[U\left(e[w^s_m + w^u_f]\right) + q^{su}_f\right] dq \\
& + Q[\max(\bar{q}^{su}_m, \bar{q}^{su}_f)] \min\left(\frac{\theta^u_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^u_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[U\left(e[w^u_m + w^u_f]\right) + q^{uu}_f\right] dq \\
& + Q[\max(\bar{q}^{su}_m, \bar{q}^{su}_f)] \left[1 - \min\left(\frac{\theta^u_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^u_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right)\right] U\left[w^u_f\right] \Bigg\} \\
+ \min\left(\frac{1-\psi^m_{t+1}}{N^f_{t+1}}, 1\right) \Bigg\{ & \int_{\max(\bar{q}^{uu}_m, \bar{q}^{uu}_f)}^{q_{\max}} \left[U\left(e[w^u_m + w^u_f]\right) + q^{uu}_f\right] dq \\
& + Q[\max(\bar{q}^{uu}_m, \bar{q}^{uu}_f)] \min\left(\frac{\theta^u_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^u_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right) \int_{q_{\min}}^{q_{\max}} \left[U\left(e[w^u_m + w^u_f]\right) + q^{uu}_f\right] dq \\
& + Q[\max(\bar{q}^{uu}_m, \bar{q}^{uu}_f)] \left[1 - \min\left(\frac{\theta^u_m(\psi^m_{t+1}, \psi^f_{t+1})}{\theta^u_f(\psi^m_{t+1}, \psi^f_{t+1})}, 1\right)\right] U\left[w^u_f\right] \Bigg\}
\end{aligned}
\tag{3.13}
$$

### 3.4.1 Education Decision and the Sex Ratio

Becoming a skilled agent requires some efforts $\phi_g$ ($g \in \{male, female\}$), which is $\in [0, +\infty)$, the effort is i.i.d. for all males and females, with the CDF of $\Phi_g$. An agent $i$ wants to become skilled if:

$$EV_i^s - EV_i^u \geq \phi_i \tag{3.14}$$

The skilled-unskilled payoff indifference generates when the fraction of $\psi$ is skilled:

$$\begin{aligned} \bar{\phi}(\psi_{t+1}^m) &\equiv E_t V_m^s(\psi_{t+1}^m) - E_t V_m^u(\psi_{t+1}^m) \\ \bar{\phi}(\psi_{t+1}^f) &\equiv E_t V_f^s(\psi_{t+1}^f) - E_t V_f^u(\psi_{t+1}^f) \end{aligned} \tag{3.15}$$

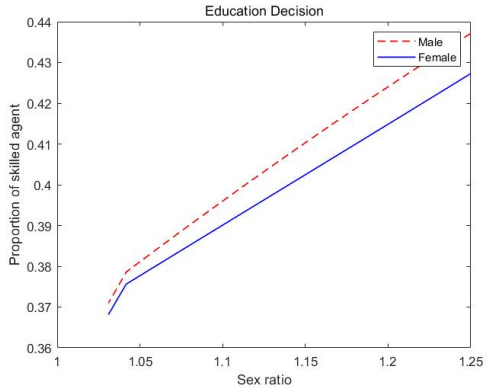
For any individual $i$: when $\phi_i \leq \bar{\phi}$ (the effort that has to be devoted to becoming a skilled individual is smaller or equal to the reservation effort $\bar{\phi}$, an agent wants to become skilled).

Without considering the education costs, agents can borrow freely, and contingent on their efforts, the young agent will make the same decision irrespective of their family type (family provides the funding for education). This implies that in equilibrium: a fraction $\Phi(\bar{\phi}_g)$ of individuals would become skilled, then the equilibrium distribution of education fraction for two genders are:

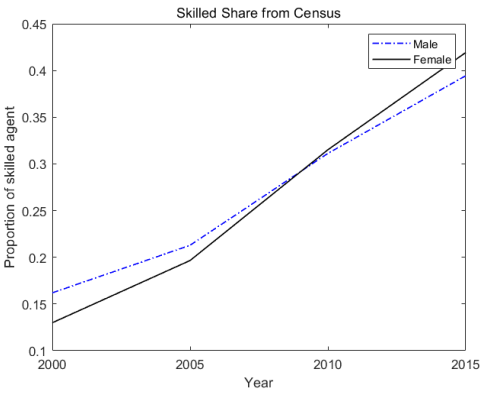
$$\begin{aligned} \psi_{t+1}^m &= \Phi(\bar{\phi}_m) \\ \psi_{t+1}^f &= \Phi(\bar{\phi}_f) \end{aligned} \tag{3.16}$$

Thus:

$$\begin{aligned} \bar{\phi}(\Phi(\bar{\phi}_m)) &= E_t V_m^s(\Phi(\bar{\phi}_m)) - E_t V_m^u(\Phi(\bar{\phi}_m)) \\ \bar{\phi}(\Phi(\bar{\phi}_f)) &= E_t V_f^s(\Phi(\bar{\phi}_f)) - E_t V_f^u(\Phi(\bar{\phi}_f)) \end{aligned} \tag{3.17}$$




(A) Share of the skilled agent from model




(B) Census 2000, 2005, 2010, 2015

FIGURE 3.5: Education decision and sex ratio

We first assume that $\phi_g$ is a uniform distribution for both males and females, which is $\in [0, 1]$. Under this assumption, we have:

$$\begin{aligned} \psi^m_{t+1} &= \mathrm{E}_t V^s_m(\psi^m_{t+1}) - \mathrm{E}_t V^u_m(\psi^m_{t+1}) \\ \psi^f_{t+1} &= \mathrm{E}_t V^s_f(\psi^f_{t+1}) - \mathrm{E}_t V^u_f(\psi^f_{t+1}) \end{aligned} \tag{3.18}$$

By taking integration on both sides of equation 3.15 and evaluating it between the whole range of $\phi$ (that is $\in [0, 1]$), we can get equation 3.18. After solving it, we generate the results plotted in Fig. 3.5a. As illustrated in it, we find that there is an increasing trend in the share of skilled agents for both males and females, with the rise in the sex ratio. For males, the incentive of becoming a skilled individual comes from the expectation of the rising intensity of marriage market competition in the future, as males are less picky and there is decreasing degree of household sorting, to compensate for the payoff gap in a match with unskilled females, males are more willing to increase education investment to become a skilled agent to earn higher wages. For females, the incentive for them to become skilled individuals with a rise in sex ratio is that due to the decline of household sorting, becoming a skilled female is more likely to get married to an unskilled male, to compensate for the payoff gap in a match with unskilled males, females are more willing to increase education

investment to gain higher wages. Comparing the results generated from the model with data facts, we find that the trend of the share of the skilled agent as the sex ratio increase is consistent with the trend plotted by census data. This result provides one possible explanation for the rising education level in China, which is observed by analyzing census data, the rising male abundance implies an increase in education attainment for both males and females. Studies like Li et al. (2017); Bhaskar et al. (2022) also highlight the rising education attainment in China, which indicates that parents increasing children's education investment with the rising of male abundance. Our results support this finding and highlight the impact of marriage market competition on education attainment in the presence of rising male abundance.

## 3.5 Conclusion

Motivated by chapter 2's empirical evidence that the sex ratio in the marriage market is negatively correlated with the degree of assortative matching, it implicates a decline in household income inequality. This chapter develops a general equilibrium model to structurally interpret the marriage market pattern with the rising number of males in the marriage market, it highlights the mechanism that why the sex ratio is negatively correlated with household sorting and how it implicates household income inequality in China. The study also extends the model to allow agents to make education investment decisions before they enter the marriage market in the presence of rising male abundance.

The results from the model indicate that females are more picky in the marriage market and males are less picky with the increase in the number of males in the marriage market. The rising sex ratio is associated with a decline in the degree of assortative matching. The quantitative results are consistent with the evidence that the rising sex ratio implies a decreasing household sorting. Also, the sex ratio in the marriage market is negatively correlated with household income inequality, the increase in the sex ratio implies a forward of time, demonstrating that household income inequality declines in China as time forward, this relationship is robust when

we use alternative income inequality measurements. The negative correlation is consistent with the literature's finding that household income inequality declined in China as time forward. Extensionally, we endogenize human capital investment decisions for males and females before they enter the marriage market, the quantitative results point out that males and females are more willing to become skilled agents with the increasing sex ratio, this pattern is consistent with the trend of the census data we have plotted, that is as time forward, the proportion of skilled males and skilled females increase in China. This simulated result provides one possible explanation for the rising education level in China, which is described by census data. Male abundance in China results in not only decreasing in household assortative marriage but also decreasing household income inequality, it also contributes to the increasing education level for both males and females, though it's one of many reasons, these findings complement a gap in the literature, and contribute the understanding of whether demographic structure changes affect the aggregate economy and human capital accumulation.

# Chapter 4

# Capital Controls and Chinese Currency Internationalization

*This paper develops a model to study the role of China's capital controls in affecting the progress of Chinese currency internationalization. In the model, the world consists of two big countries, the U.S. and China, and a continuum of small open economies making up the rest of the world. We highlight that the emergence of multiple international currencies is driven by the demand to finance international trade. Households from the rest of the world decide which types of assets to invest and firms decide which assets to use as collateral for financing trade. The persistence of the regime is driven by positive interactions between the saving decisions of households and the financing decisions of firms. Given China's different degrees of capital controls, the model generates an equilibrium distribution of Chinese currency in the international monetary market, with an increase in capital control, Chinese currency usage declines in the rest of the world countries. The model can quantitatively match the dollar share and RMB share in the world. The welfare gains of becoming the dominant currency are substantial.*

## 4.1 Introduction

There are over 180 currencies in the world, but only a small number of them are used as international currencies, which are dominant in international trade invoicing, issuance of financial assets, and central bank foreign exchange reserves. The dominant currencies in the world are the U.S. dollar and euro, which are heavily used in international trade activities and financial activities. Other currencies such as the Japanese yen, pound sterling, and the Chinese yuan (Renminbi) play a minor role in international presence. Many studies have investigated the benefits of one nation's currency becoming dominant, but few of them have investigated how a currency can achieve this status and even become an international currency. The benefits of currency dominance include enhancement of political power, seigniorage revenues, safety premium in its financial assets, and favorable exchange rate movements following a shock (Bahaj and Reis (2020)). Figure 4.1 demonstrates that the U.S. dollar plays the biggest role in the current international monetary system, which is dominant in exchange reserves, financial accounts, trade invoicing, and international payments.

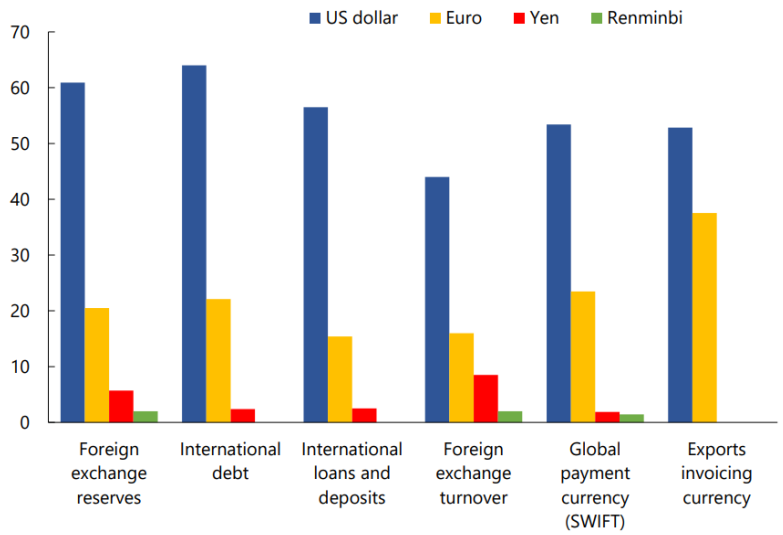



FIGURE 4.1: The international monetary system

Source: Gopinath and Itskhoki (2021)

Historically, there were several dominant currencies in the world. Before World War I, the British pound was the dominant currency, which was frequently used as a unit of

account in trade invoicing, and also as the denomination of safe assets. Before this, the Dutch Guilder was dominant in the 18th century. This persistence largely attributes to the prosperous cross-border trade activities among countries (Eichengreen (2011)). Going back to 1912, the U.S. was the largest exporter in the world, but its currency was not an international currency, firms and banks had to access London financial markets to acquire funding for trade activities, which was denominated in the British pound. After the enacting of the Federal Reserve Act in 1913, U.S. banks were allowed to build branches in foreign nations, following this, the U.S. started to internationalize the dollar. By 1925, the U.S. dollar had become an international currency, and by World War II it had become a dominant currency. The establishment of the Bretton Woods system in 1944 reinforced the dominance of the U.S. dollar, which required countries to convert their currencies into U.S. dollars, with the dollar convertible to gold (Ilzetzki et al. (2021); Reinhart and Rogoff (2004)). The collapse of this system in 1971 enable the emergence of a free-floating exchange rate regime, from this time onward, there are multiple international currencies (Japanese yen, the UK pound, French franc, and German DM) were extensively used in international presence whereas the U.S. dollar is still the dominant one. The emergence of the euro in 1999 challenged the dominant status of the dollar, which nowadays takes up the second largest usage in the international monetary system (see Figure 4.1).

In the middle of 2009, the Chinese government implemented a series of policies to initiate the internationalization of the Renminbi (RMB). During that time, China was the largest exporter and creditor in the world (Horn et al. (2021)), however, the tight capital controls severely limited the RMB usage in foreign countries. The RMB internationalization started with the trade settlement by allowing for goods and serves to be settled in RMB, and this scheme continues up to now, the establishment of an offshore market in Hong Kong accelerated the implementation of trade settlement in RMB, as Hong Kong has the free trade agreements and lacks the capital controls. The RMB-denominated assets can be freely traded in Hong Kong. In addition, the People's Bank of China (PBoC, central bank of China) created currency swap lines with other nations' central banks, which effectively lend RMB to banks in these nations, often with collateral tied to international trade credits (Bahaj and Reis (2020)). Furthermore,

there are increasing volumes of Chinese foreign direct investments denominated by RMB, which is included in the capital account. By 2016, the RMB was included in the Special Drawing Rights (SDR) basket with a weight of 10.92%. As of 2022Q3, the Chinese renminbi takes up 2.76% of the world's foreign currency reserves.

In this paper, we depart from a framework of Chahrour and Valchev (2022) to evaluate the general equilibrium effect of capital controls on the progress of RMB internationalization. We developed a dynamic general equilibrium model with two big countries (the U.S. and China) and a continuum of small open economies in the rest of the world, featuring capital controls in the household portfolio investment, to study how different degrees of capital controls affect the equilibrium usage of Chinese RMB in the rest of world. The framework embedded two frictions in international trade, which are trade contract commitment friction (creating the demands on collateral to guarantee enforcement) and financial friction (the friction of obtaining the collateral). Households gain extra benefits by holding collateral assets (two types of bonds issued by the U.S. government and the Chinese government) and trading firms use the collateral assets to ensure the execution of trade contracts, which generate profits for them. The interaction between households assets saving decisions and trading firms' collateral asset demand decisions gives rise to a persistent currency dominance. The detailed interpretation of model can be found in section 4.3.

We make two contributions to the literature. First, there are massive studies that discuss the U.S. dollar and euro dominance in the international monetary system and the international price system. However, the attention pays to Chinese currency internationalization is inadequate. Formal analysis that is either empirically or theoretically about the rising of the renminbi as an international currency lacks, this study supplements this. Second, the formal evaluation of how capital control quantitatively affects the progress of RMB internationalization is missing, most studies investigate the impact of capital controls on financial markets, and capital misallocations, we first quantitatively evaluate the impact of capital controls on currency internationalization.

The rest of the paper is organized as follows. Section 4.1.1 provides a synthetic

literature review on relevant research topics. Section 4.2 discusses the capital account restrictions in China. Section 4.3 employs the structural model to quantitatively evaluate how capital controls affect the usage of Chinese RMB. Section 4.4 introduces the exogenously fixed parameters and calibration which are following the paper of Chahrour and Valchev (2022), and then form quantitative analysis and provide policy implications. Section 4.5 concludes.

### 4.1.1 Related Literature

This paper relates to three strands of literature. First, growing studies have investigated the reasons, patterns, and consequences for the dominant status of the U.S. dollar and euro, which explain the large-scale use of a currency in international trade, financial contracts, and foreign exchange reserves. Mukhin (2022) documents the dominance of the U.S. dollar in the international price system and proposes a quantitative general equilibrium model where different forms of complementarities in price setting (producer currency pricing, local currency pricing, and dominant currency pricing) across firms influence currency choice decisions, which makes exporters coordinate on the same currency of invoicing. The dominance of the dollar comes from its historical dependence and currency peg on it. Similarly, Amiti et al. (2022) investigates how firms choose the invoicing currency and its implications for exchange rate pass-through into export prices and quantities, which illustrates that an exporting firm's invoicing currency choice is determined by its competitors' currency choice and its importing currency choice, the U.S. dollar maintains strength in dominant status as most invoicing prices are sticky in the dollar. Gopinath and Stein (2021) highlight that a currency's role as a unit of account is complementary to its function as a store of value and this complementarity can result in the emergence of a single dominant currency. The role of a currency in a unit of account and a store of value reinforce each other, as large usage of dollar invoicing in international trade triggers an increasing demand for safe dollar deposits, and the relative low dollar-denominated assets borrowing cost attracts exporters to use the dollar as invoicing currency. Furthermore, Gopinath et al. (2020) presents a dominant currency

paradigm that highlights the implications of dominant currency monetary policy shocks on exchange rates and fluctuations. Egorov and Mukhin (2020) evaluate the policy implications of dollar stickiness in international prices, including the optimal monetary policy for the U.S. and non-U.S. countries. They highlight that inflation targeting is optimal for non-U.S. economies and the U.S. gains higher welfare owing to dollar dominance. Farhi and Maggiori (2018) starts from the role of the store of value of a currency, which proposes that the international monetary system is featured with three characteristics: reserve assets supply and demand, exchange rate regime, and international monetary institutions. Similarly, Gourinchas et al. (2019) investigates the implications of currency hegemony on external imbalance and spillover effects. Chahrour and Valchev (2022) highlight that the persistence of dollar dominance is attributed to the interaction between an instrument of saving and an instrument of trade finance.

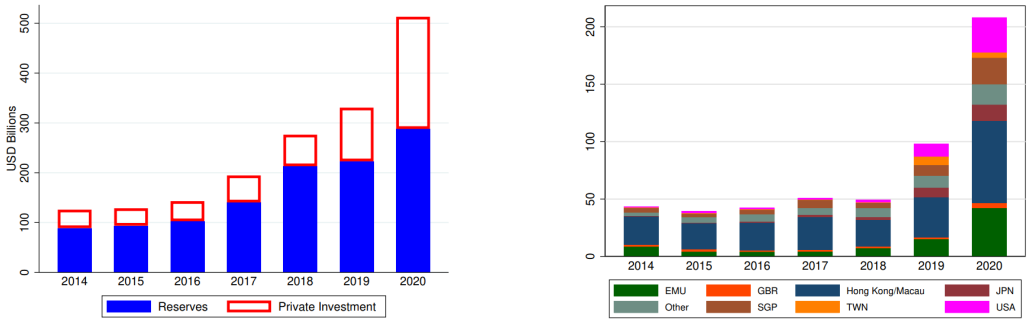
Second, this study relates to literature that formally discusses Chinese currency internationalization. Comparing to the investigations on the U.S. dollar dominance and the role of the U.S. dollar in the international monetary system and in the international price system. Formal analysis has been inadequate in discussing renminbi internationalization in an either empirical or theoretical way. Farhi and Maggiori (2019) developed a model of the international monetary system to analyze the potential of the renminbi in replacing the U.S. dollar, which indicates that the rising of the renminbi in the role of reserve assets and trade invoicing is likely to erode the dollar dominance. Bahaj and Reis (2020) discussed the implication of currency swap lines building between the central bank of China and other economies, they point out that the jump-start of RMB as an international currency is attributed to the creation of currency swap lines, where RMB serves as the instrument to obtain working capital and trade credit. Clayton et al. (2022) empirically characterize the pattern of RMB internationalization by looking at the gradual opening of the Chinese bond market to foreign investors, which proposes a dynamic reputation model to discuss the rationale of this openness. Jermann et al. (2019) analyzed the exchange rate regime of the People's Bank of China (central bank of China), which highlights that the introduction of managed floating exchange rate policy in 2015 aided the

promotion of RMB internationalization.

Third, this paper evaluates the impact of capital controls on the progress of RMB internationalization, which relates to literature that has studied capital account restrictions. Edwards (1999) evaluate the costs and benefits of capital control in Chile. Erten et al. (2021) highlight that capital controls induce private investors to internalize their contributions to financial fragility and aggregate demands, and capital control is effective counter-cyclically. Chang et al. (2015) studied optimal monetary policy in a framework that accounts for the characteristics of the Chinese economy (capital controls, managed exchange rates, and sterilized interventions). Moreover, related to capital controls and their implications. Liu et al. (2021a) develop a general equilibrium model to illustrate that there is a trade-off between productivity and allocation efficiency from capital account liberalization with a less developed financial market in China, which highlight that gradual liberalization of capital account benefits welfare. Prasad et al. (2006) concluded that, under a weak financial system, the liberalization of the capital account would generate significant risks for the Chinese economy. Also, Farhi and Werning (2012) showed that the effects of excess movements of foreign capital caused by risk premium shocks can be mitigated with capital controls. In contrast to these literature strands, De Paoli and Lipinska (2013) concludes that capital controls have the potential to trigger adverse responses from other economies, which counteract the welfare effects that these policies generate on the individual country. This paper adds to this literature by showing how capital controls interact with the rest of the world's demand for Chinese currency.

## 4.2 Capital Controls in China

There still exist capital controls in China during the progress of RMB internationalization. The term "capital controls" refers to the restrictions on the quantity or price of financial assets or liability buying and selling between domestic and foreign (Rebucci and Ma (2019)). There are two directions of capital flow restrictions in China, which are capital inflow restriction and outflow restriction. For




(A) Foreign ownership of Chinese-issued RMB bonds

(B) Regions of private holders of RMB bonds

FIGURE 4.2: Foreign investment of RMB bonds

the capital outflow restriction, domestic residents are limited in accessing international assets markets. In 2006, the Qualified Domestic Institutional Investor (QDII) program was introduced to allow selected domestic investors to transact foreign listed equities and debt securities, which were subject to quotas, and the volume was small. Furthermore, China's State Administration of Foreign Exchange (SAFE) limits the volume of foreign currencies that each individual resident can exchange, which is up to 50,000 dollars per year, and any amount that exceeds this limit has to be permitted by the SAFE. For the capital inflow restriction, the Chinese government encourages foreign direct investment but the size is small, which accounts for about 2% of total fixed investment since 2015. Similarly, there are the Qualified Foreign Institutional Investor (QFII) programs for foreign investors, which restrict their access to domestic asset markets and are subject to quotas and investment scope (Liu et al. (2021a); Chang et al. (2015); Hu and Yuan (2021); Song et al. (2014)).

Capital account restrictions have been eased to aid the progress of RMB internationalization. Unlike the U.S. and Euro-zone bond markets, foreign investors are restricted in accessing the Chinese bond market, which severely limits the use of the Chinese Renminbi (RMB) as an international currency. The capital account has been liberalized in recent years, some arguments indicate that gradual liberalization is more appropriate than rapid openness, as the financial system is distorted in China (Brunnermeier et al. (2017, 2020)). The liberalization of capital inflow involves the selection of investor bases, which allows certain types of investment to flow into China's domestic bond market. Following the introduction of QFII programs in 2002,

in the 2010s, the Chinese government significantly liberalized the direct access to the domestic bond market for foreign investors, by allowing them to participate in the China Interbank Bond Market (CIBM direct), which the majority of players are central banks and sovereign wealth funds. And in 2015, long-term investors such as central banks and sovereign wealth funds are fully allowed to access the Chinese bond market without quotas. Additionally, the Bond Connect program was introduced in 2017, which is based offshore in Hong Kong and can be participated in via trading platforms such as Bloomberg in the absence of registration of QFII or CIBM direct. All of these programs have significantly liberalized the restrictions on accessing the Chinese bond market. Recently, China is included in global bond indices such as the Bloomberg Global Aggregate Index and the JP Morgan Government Bond Index. However, the gradual steps of bond market liberalization do not imply the free of capital controls, the size of foreign investors in the Chinese bond market is not large enough and there are still restrictions on the type of investors, which are significantly composed of central banks and sovereign wealth funds. The private investment is growing in recent years (Clayton et al. (2022)). Figure 4.2a reports that private investment in RMB bonds has increased in recent years, whereas the investment is largely driven by central banks in the past. Figure 4.2b reports that the largest private RMB bondholders are the European Monetary Union, the U.S., Singapore, and Japan.

## 4.3 The Model

To quantitatively evaluate how capital account restrictions in China affect the progress of RMB internationalization. We propose a dynamic general equilibrium model in this section by following the framework of Chahrour and Valchev (2022), in which we introduce capital control tax rate $\tau$ in the budget constraint for households, and also introduce uneven exogenous supplies of two assets which are issued by the Chinese government and the U.S. government. The model highlights that the interaction between household saving decisions and trading firms' collateral assets demanding decisions determines the equilibrium usage of currency in the world, which shed light on the roles in the stores of value and medium of exchange of a currency.

### 4.3.1 Setting

The world consists of two big countries, the U.S. ($US$) and China ($CN$) of equal size in terms of GDP ($\mu_{US} = \mu_{CN}$), and a continuum of small open economies making up the rest of the world ($RoW$) with a total mass of $\mu_{RoW}$. There are several reasons that the study set the rest of the world as a continuum of small open economies, first, as documented in Chahrour and Valchev (2022), international trades take place between many country pairs (Japan to Australia, etc.) in the rest of the world are settled with US dollar, to demonstrate the transactions between $RoW$ and $RoW$, the study set a continuum of small open economies making up the rest of the world, then the endogenous demand for dollars is the endogenous choice of the $RoW$ countries of which currency to use when transacting with others. Second, if the study sets the rest of the world as one representative world, then there is no demonstration of $RoW$ to $RoW$ trade, the endogenous decision on currency choice from the $RoW$, in this case, is exogenous and fails to show the emergence of dominant currency steady state which is driven by $RoW$ to $RoW$ trade transactions.

Thus we have:

$$\mu_{US} + \mu_{CN} + \mu_{RoW} \equiv 1 \tag{4.1}$$

There are two assets in the model, which are bonds issued by the U.S. government and bonds issued by the Chinese government. Each of them is recognized as a safe asset and available in the exogenous supply of ($\bar{B}^{USD}$, $\bar{B}^{CNY}$). Both assets serve as the saving instrument for households and also the collateral asset used by trading firms to guarantee the enforcement of trading contracts. The assumptions of the exogenous supply of $\bar{B}^{USD}$ and $\bar{B}^{CNY}$ are made due to the following reasons, first, the focus of the model is around the currency usage of $RoW$, since there are fixed supply of these two assets, the internationalization of one currency is determined by the $RoW$, where households decide which asset to invest and trading firms decide which asset to be used as transaction collateral, it highlights that the emergence of international

currency is a coordinating outcome between households and trading firm in the $RoW$. Second, one can also endogenize bonds supply, but this would introduce a fiscal authority problem, since in the model, increasing the supply of dollar bond always increase the attractiveness of dollar dominant equilibrium, then the optimal policy, in this case, is to let the US government issues infinite amount of dollar bond, which is not real and fail to demonstrate the model mechanism.

Following the setting of Chahrour and Valchev (2022), we use $j \in \{US, CN, [0, \mu_{RoW}]\}$ to index countries in the model, and in each country $j$, there is a representative household and a continuum of risk-neutral trading firms, as there is a continuum of small open economies in the rest of the world, the notation on this is indexed by an interval ($[0, \mu_{RoW}]$). Households in each country $j$ consume and allocate their saving between two types of assets by using their endowment income $Y_{jt}$. Trading firms in each country decide which type of assets to be used as a collateral instrument (either RMB bonds or dollar bonds). Given the capital controls of the Chinese government, Chinese households purchasing dollar bonds have to pay an additional cost, which is levied by a tax at a rate of $\tau$. Also, Non-Chinese households purchasing RMB bonds have to pay an additional cost, which is also levied by a tax at a rate of $\tau$. We employ $\tau$ to measure the degree of capital control of the Chinese government, which is inspired by the setting of Uribe and Schmitt-Grohé (2017).

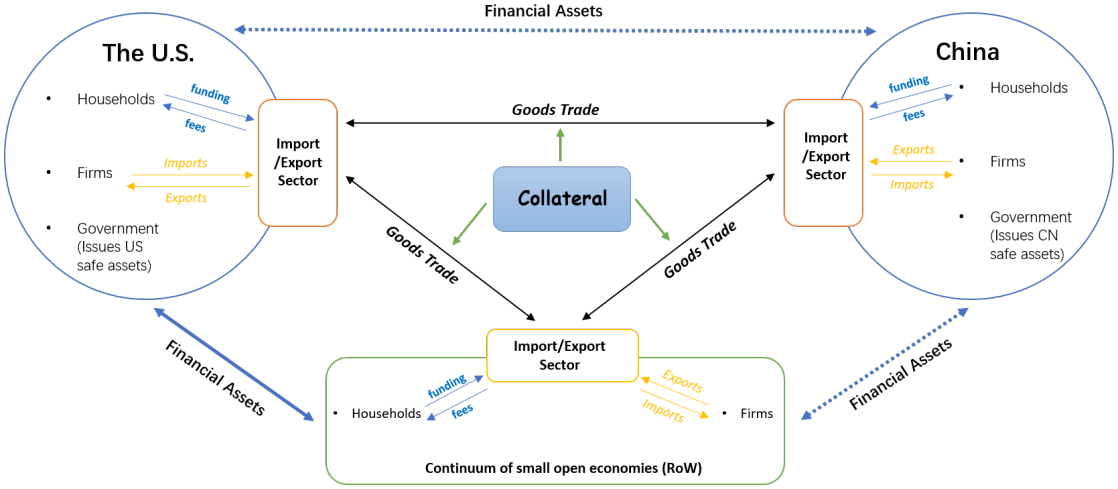



FIGURE 4.3: Graphical Presentation of the Model

Figure 4.3 graphically demonstrates the dynamics of the model, which highlights the friction of assets transaction internationally (as illustrated by the dashed line between China and other countries) and the interactions between household saving decisions and trading firms' funding decisions.

### 4.3.2 Households

Households in each country solve consumption and saving problems, deciding the allocation of their endowment income between consumption and savings in the two assets. At this stage, we assume that the goods in each country are perfectly substitutable, so the consumption bundles only consist of one single good. Both types of bonds promise one unit of that good.

The currencies $z$ in the world are $\{USD, CNY\}$. $Q_t^z$ denotes the price of a U.S. bond or Chinese bond, and $B_{jt}^z$ is the quantity of bond held by households. The amount of bonds has to be non-negative, which implies there is no short position for households. And $Y_{jt}$ is exogenous endowment income. Excepting the interest rate, each bond earns an additional return which comes from the collateral-use fees that firms pay to households, which are named liquidity premiums and denoted by $\Delta_{jt}^z$. Following the original setting of Chahrour and Valchev (2022), the short position on the asset is ruled out, where households anticipate that these assets are safe and holding returns will incur at maturity. The bonds in the model are long-term government bonds, with lower returns but lower risks of holding.

For the U.S. households $j = US$, we have:

$$\begin{gathered}
\max_{C_{US,t}, B_{US,t}^{USD}, B_{US,t}^{CNY}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{US,t}^{1-\sigma}}{1-\sigma} \\
s.t.: \\
C_{US,t} + (Q_t^{USD} - \Delta_{US,t}^{USD}) B_{US,t}^{USD} \\
+ (Q_t^{CNY} - \Delta_{US,t}^{CNY}) B_{US,t}^{CNY} [1+\tau] = B_{US,t-1}^{USD} + B_{US,t-1}^{CNY} + Y_{US,t}
\end{gathered} \tag{4.2}$$

Solving the U.S. household problem and we get the optimal asset holdings $B_{US,t}^{USD}$ and $B_{US,t}^{CNY}$, then the Euler equation is:

$$\frac{1}{\beta} = E_t \left\{ \frac{C_{US,t+1}^{-\sigma}}{C_{US,t}^{-\sigma}} \frac{1}{Q_t^{USD} - \Delta_{US,t}^{USD}} \right\} = E_t \left\{ \frac{C_{US,t+1}^{-\sigma}}{C_{US,t}^{-\sigma}} \frac{1}{(Q_t^{CNY} - \Delta_{US,t}^{CNY})\,[1+\tau]} \right\} \tag{4.3}$$

In the steady state we have:

$$\frac{1}{\beta} = \frac{1}{Q^{USD} - \Delta_{US}^{USD}} = \frac{1}{(Q^{CNY} - \Delta_{US}^{CNY})\,[1+\tau]} \tag{4.4}$$

For the Chinese households $j = CN$, we have:

$$\begin{gathered} \max_{C_{CN,t}, B_{CN,t}^{USD}, B_{CN,t}^{CNY}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{CN,t}^{1-\sigma}}{1-\sigma} \\ s.t.: \\ C_{CN,t} + (Q_t^{USD} - \Delta_{CN,t}^{USD}) B_{CN,t}^{USD}\,[1+\tau] \\ +(Q_t^{CNY} - \Delta_{CN,t}^{CNY}) B_{CN,t}^{CNY} = B_{CN,t-1}^{USD} + B_{CN,t-1}^{CNY} + Y_{CN,t} + T_t \end{gathered} \tag{4.5}$$

The Chinese government imposes a capital control tax on foreign asset purchasing for Chinese households, and foreign households have to pay capital control tax when purchasing Chinese bonds, the tax is rebated to Chinese households, thus we have $T_t$ in the Chinese household budget constraint.

The optimal asset holdings $B_{CN,t}^{USD}$ and $B_{CN,t}^{CNY}$ implies the Euler equations:

$$\frac{1}{\beta} = E_t \left\{ \frac{C_{CN,t+1}^{-\sigma}}{C_{CN,t}^{-\sigma}} \frac{1}{(Q_t^{USD} - \Delta_{CN,t}^{USD})\,[1+\tau]} \right\} = E_t \left\{ \frac{C_{CN,t+1}^{-\sigma}}{C_{CN,t}^{-\sigma}} \frac{1}{Q_t^{CNY} - \Delta_{CN,t}^{CNY}} \right\} \tag{4.6}$$

In the steady state we have:

$$\frac{1}{\beta} = \frac{1}{(Q^{USD} - \Delta_{CN}^{USD})\,[1+\tau]} = \frac{1}{Q^{CNY} - \Delta_{CN}^{CNY}} \tag{4.7}$$

For the households in the country $j \in [0, \mu_{RoW}]$, we have:

$$\begin{gathered} \max_{C_{jt}, B_{jt}^{USD}, B_{jt}^{CNY}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma} \\ s.t.: \\ C_{jt} + (Q_t^{USD} - \Delta_{jt}^{USD}) B_{jt}^{USD} \\ + (Q_t^{CNY} - \Delta_{jt}^{CNY}) B_{jt}^{CNY} [1+\tau] = B_{jt-1}^{USD} + B_{jt-1}^{CNY} + Y_{jt} \end{gathered} \tag{4.8}$$

The optimal asset holdings $B_{jt}^{USD}$ and $B_{jt}^{CNY}$ implies the Euler equations:

$$\frac{1}{\beta} = E_t \left\{ \frac{C_{jt+1}^{-\sigma}}{C_{jt}^{-\sigma}} \frac{1}{Q_t^{USD} - \Delta_{jt}^{USD}} \right\} = E_t \left\{ \frac{C_{jt+1}^{-\sigma}}{C_{jt}^{-\sigma}} \frac{1}{(Q_t^{CNY} - \Delta_{jt}^{CNY})[1+\tau]} \right\} \tag{4.9}$$

In the steady state we have:

$$\frac{1}{\beta} = \frac{1}{Q^{USD} - \Delta_j^{USD}} = \frac{1}{(Q^{CNY} - \Delta_j^{CNY})[1+\tau]} \tag{4.10}$$

Thus we have:

$$\frac{1}{\beta} = \frac{1}{Q^{USD} - \Delta_{US}^{USD}} = \frac{1}{(Q^{USD} - \Delta_{CN}^{USD})[1+\tau]} = \frac{1}{Q^{USD} - \Delta_j^{USD}} \tag{4.11}$$

$$\frac{1}{\beta} = \frac{1}{(Q^{CNY} - \Delta_{US}^{CNY})[1+\tau]} = \frac{1}{Q^{CNY} - \Delta_{CN}^{CNY}} = \frac{1}{(Q^{CNY} - \Delta_j^{CNY})[1+\tau]} \tag{4.12}$$

Equations 4.11 and 4.12 imply that with different degrees of capital control $\tau$, the bond holding net payoff is smaller than the net payoff without capital control, that is to say,

the liquidity premium paid to the Chinese households with capital controls is larger than other countries when holding the U.S. bonds.

### 4.3.3 Trading Firms

#### 4.3.3.1 Decision

Within each country, the trading firms make profitable transactions with a randomly-matched foreign partner. If the transaction is executed, a joint surplus of $2\pi$ incurs and is evenly split between the two counterparties. The trading surplus $\pi$ is exogenous at this stage. There is some probability that the transaction contract will not be enforced, one side of the pair may choose to default, thus before the transaction is executed, each firm has to post collateral to guarantee their side of contract enforcement. Both U.S. assets and Chinese assets can be used as collateral. Trading firms make a binary decision in collateral asset choice, either the dollar bonds or RMB bonds, they seek an intra-period loan of one of them in domestic bond-specific search and matching credit markets. On the other side of the domestic bond-specific search and matching credit markets, the portfolio holdings of households provide the source of the safe assets used as collateral. The amount of collateral assets that trading firms intend to use is fixed and normalized to one.

We define the probability that a firm in the country $j$ successfully borrows the asset which is denominated by currency $z$ is $\mathcal{P}_j^z$, where the $z$ is $\in \{USD, CNY\}$. The successful borrowing of one unit of collateral asset requires firms to pay a fee, that is $r^z$, the fee is paid to households for the use of the asset and then firms proceed to international trade. If the firm fails to borrow assets in the credit market, then it continues the trade but uses a backup funding plan which still provides its chosen collateral but absorbs all surplus from the transaction. The fees that trading firms paid to the households have to be smaller than the contract execution profit $\pi$, we simply assume that the fees are fixed at this moment, which is:

$$r^{USD} = r^{CNY} = r < \pi$$

Once trading firms are successfully funded, they randomly match with a trading counterparty from another country. In the matching, the pair executes trading contracts by using their collateral asset to clear any payments and split the gross transaction surplus $2\pi$. If the two counterparties' collateral is mismatched, for instance, one side of the match uses dollar assets and the other side uses RMB assets, this transaction's surplus is reduced by a collateral asset mismatch cost of $2\phi$. This study assumes that:

$$\phi < \pi - r$$

So even in the case of collateral asset mismatch, the execution of trade deals is still profitable.

#### 4.3.3.2 Payoff

Given the above setting, we develop the differential payoff of using a U.S. asset over an RMB-denominated asset to have the trade financing as:

$$V_j^{USD} = \mathcal{P}_j^{USD}\left[\pi - r_j^{USD} - \phi(1-\bar{S})\right] - \mathcal{P}_j^{CNY}\left[\pi - r_j^{CNY} - \phi\bar{S}\right] \tag{4.13}$$

A country $j$ firm would choose to apply for the USD trade financing if the expected payoff of using dollar financing is greater than using RMB financing. Where in the equation 4.13, $\mathcal{P}_j^{USD}$ denotes the probability that a firm in country $j$ successfully borrows a U.S. bond, $\mathcal{P}_j^{CNY}$ represents the probability that a firm in country $j$ successfully borrow a Chinese bond. $\bar{S}$ is the proportion of all trading firms in the world that choose the U.S. asset as collateral, which can also be interpreted as the probability that a firm matches with a counterparty that uses U.S. assets as collateral. If $V_j^{USD} > 0$, then all the firms prefer to use the dollar asset as collateral.

This study assumes that firms in the U.S. and China always seek to be funded by their

respective domestic assets, so the objective is to explain the optimal currency choice of the RoW trading firms. Then the average use of the U.S. asset as collateral in the world is:

$$\bar{S} = \mu_{US} S_{US} + \mu_{CN} S_{CN} + \int_0^{\mu_{RoW}} S_j dj = \mu_{US} + \int_0^{\mu_{RoW}} S_j dj \tag{4.14}$$

Where in the equation 4.14, $S_{US}$ and $S_{CN}$ denote the fraction of firms in the U.S. and China that seek the U.S. asset as collateral respectively, under the assumption, we have $S_{US} = 1$ and $S_{CN} = 0$. The $S_j$ is the proportion of firms in country $j \in [0, \mu_{RoW}]$ use the U.S. asset as the collateral, where $S_j \in [0, 1]$. Thus $1 - S_j$ denotes the share of firms in the country $j \in [0, \mu_{RoW}]$ use the Chinese asset as the collateral.

The number of matches in country $j$'s asset credit market is governed by the constant returns to scale matching function, which is:

$$M^F(B, S) = \frac{BS}{B + S} \tag{4.15}$$

The $B$ in the above equation is the units of the asset on offer, and $S$ denotes the number of trading firms demanding the asset. Under this setting, a firm in country $j$ searching for the U.S. asset as collateral succeeds with the probability of:

$$\mathcal{P}_j^{USD} = \frac{M^F(B_j^{USD}, S_j)}{S_j} = \frac{B_j^{USD}}{B_j^{USD} + S_j}$$

Using this expression and substituting it into equation 4.15, we can get:

$$V_j^{USD} = \frac{B_j^{USD}}{B_j^{USD} + S_j} [\pi - r - \phi(1 - \bar{S})] - \frac{B_j^{CNY}}{B_j^{CNY} + 1 - S_j} [\pi - r - \phi\bar{S}] \tag{4.16}$$

Equation 4.16 demonstrates that within the domestic credit market, the choice of the collateral asset is substitutable, a trading firm in country $j$ could choose a U.S. asset or

a Chinese asset as collateral, with a large share of the other country $j' \neq j$ apply the U.S. asset for collateral, country $j$'s credit market becomes more congested, which would lower the probability that a trading firm in the country $j$ successfully seek the dollar asset as collateral, thus reducing the relative payoff of using this kind of asset for funding. In addition, equation 4.16 highlights the interaction between the collateral choice of trading firms and the households savings decision, the presence of $B_j^{USD}$ suggests that a trading firm's expected payoff of using dollar asset as collateral increase with the increasing of household's holdings on $B_j^{USD}$. This implies that an increasing share of U.S. asset holding raises the probability that a trading firm successfully obtains this collateral.

This study focuses on symmetric equilibria. For all $j \in [0, \mu_{RoW}]$, we impose the conditions of:

$$S_j = S_{RoW}, \quad B_j^{USD} = B_{RoW}^{USD}, \quad B_j^{CNY} = B_{RoW}^{CNY}$$

Under this setting, we have equation 4.14 changed to:

$$\bar{S} = \mu_{US} + \int_0^{\mu_{RoW}} S_j dj = \mu_{US} + \mu_{RoW} S_{RoW}$$

Where the $S_{RoW}$ denotes the fraction of trading firms in the rest of the world that seek U.S. assets as collateral.

Following Chahrour and Valchev (2022), there are multiple equilibria of the fraction of the dollar asset used as the collateral in the rest of the world ($S_{RoW}$), which then is named quasi-equilibrium, and specified as follows: given the household asset holdings in rest of world countries $\{B_{RoW}^{USD}, B_{RoW}^{CNY}\}$, a symmetric quasi-equilibrium in collateral asset choice is: there is one $S_{RoW}$ in the state space that makes no trading firms have incentives to change its funding decision, which is the function of household's asset holdings. Implied by equation 4.16 and using the above definition on the quasi-equilibrium of $S_{RoW}$, we can characterize the equilibrium of $S_{RoW}$ as:

$$V_{RoW}^{USD} S_{RoW}(1 - S_{RoW}) = 0 \tag{4.17}$$

Equation 4.17 illustrates three possible allocations of $S_{RoW}$. First, with $V_{RoW}^{USD} > 0$, only if $S_{RoW} = 1$, this implies that the payoff of using dollar assets as collateral is always profitable, thus all trading firms in the rest of world choose to use the U.S. asset for financing, which they have no incentives to change their asset selection decision under this scenario. Second, with $V_{RoW}^{USD} < 0$, only if $S_{RoW} = 0$, this implies that all the trading firms in the rest of the world choose to use the Chinese asset as collateral. Third, if there is a unique value of $S_{RoW}$ that satisfy equation 4.17, then $S_{RoW}$ becomes a function of $\{B_{RoW}^{USD}, B_{RoW}^{CNY}\}$.

PROPOSITION 1: *Given the household bonds portfolio holdings, there is a unique currency quasi-equilibrium for any feasible bonds allocation if and only if*

$$\phi < \min(\phi^{sunspot\ 1}, \phi^{sunspot\ 2})$$

$$\phi^{sunspot\ 1} \equiv \frac{\pi - r}{\bar{B}^{USD} + 0.5\mu_{RoW} + 0.5}$$

$$\phi^{sunspot\ 2} \equiv \frac{\pi - r}{\bar{B}^{CNY} + 0.5\mu_{RoW} + 0.5}$$

PROOF:
We provide the proof in the appendix.

Proposition 1 demonstrates that the currency choice quasi-equilibrium does not always exist, under the condition of this proposition, the currency choice in the use of the collateral assets is the coordination outcome among trading firms rather than the knife-edge case (either $S_{RoW} = 1$ or $S_{RoW} = 0$). Consistent with empirical facts that the supply of renminbi-denominated bonds is smaller than the supply of USD-denominated bonds, which is $\bar{B}^{CNY} < \bar{B}^{USD}$, so we know that $\phi^{sunspot\ 1} < \phi^{sunspot\ 2}$ under this case.

For any $\phi < \min(\phi^{sunspot\ 1}, \phi^{sunspot\ 2})$, we can get a unique interior solution that is between 0 and 1 by solving equation 4.16.

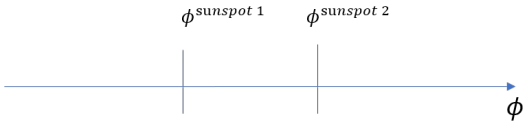



FIGURE 4.4: Different values of $\phi$

#### 4.3.3.3 Bond Holding Premium

How do the trading firms' funding decisions affect household saving decisions? In this part, we introduce the bond holding premium, which is equal to the expected collateral use fees that trading firms pay to households.

$$\Delta_j^{USD} = \underbrace{\frac{M^F(B_j^{USD}, S_j)}{B_j^{USD}}}_{\text{the prob. of successful lending to firms}} \times \quad r = \frac{S_j}{B_j^{USD} + S_j} r \tag{4.18}$$

$$\Delta_j^{CNY} = \frac{M^F(B_j^{CNY}, 1 - S_j)}{B_j^{CNY}} \times r = \frac{1 - S_j}{B_j^{CNY} + 1 - S_j} r \tag{4.19}$$

The above equations describe the expected premium of holding bonds when households make saving decisions on the bonds portfolio. The expected premium is equal to the probability that households successfully lend the asset to trading firms times the asset using fee $r$.

#### 4.3.3.4 Equilibrium Household Portfolios Holdings

PROPOSITION 2: *In equilibrium, the portfolios of bond holding are functions of trading firms' currency choice, which are:*

$$B_j^{USD} = \bar{B}^{USD} \frac{S_j}{\int_0^{\mu_{RoW}} S_j dj + \mu_{US}}$$

$$B_j^{CNY} = \Gamma(\bar{B}^{CNY}, 1 - S_j)$$

$$\bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega(1 - S_j)}{(1 - S_j) - \Omega(1 - S_j) - \Omega B_j^{CNY}} + \int_0^{\mu_{RoW}} (1 - S_j) dj \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega(1 - S_j)}{(1 - S_j) + \Omega B_j^{CNY} + \Omega(1 - S_j)}$$

$$\Omega = \frac{\beta\tau}{(1 + \tau)r}$$

PROOF:

We provide the proof in the appendix.

In the above proposition, the equilibrium allocation of renminbi-denominated bonds in the rest of the world $B_j^{CNY}$ is a nonlinear function of $\bar{B}^{CNY}$ and $S_j$, which is explicitly determined by the third line. Where the $\bar{B}^{USD}$ and $\bar{B}^{CNY}$ are exogenously been determined. The detailed proof can be found in the appendix, where the $\tau$ is used to reflect the degree of capital control in China.

The equilibrium allocation of U.S. dollar-denominated bonds in the rest of the world demonstrates an upward-sloping trend, that is higher dollar asset usage $S_j$ implies higher $B_j^{USD}$. This highlights that an asset that is frequently used as collateral to guarantee international transactions generates a higher liquidity premium, which in turn motivates households to increase their saving on that asset. Additionally, Proposition 2 indicates that there is a nonlinear relationship between renminbi-denominated assets being used in the rest of the world and the degree of capital controls.

### 4.3.4 Government

The Chinese government taxes domestic households when they purchase foreign bonds, and taxes foreign households when they purchase Chinese bonds. These revenues are rebated to Chinese households as transfers so the budget constraint is balanced. This condition is given by:

$$\tau\left[\left(Q_t^{CNY}-\Delta_{RoW,t}^{CNY}\right)B_{RoW,t}^{CNY}+\left(Q_t^{CNY}-\Delta_{US,t}^{CNY}\right)B_{US,t}^{CNY}+\left(Q_t^{USD}-\Delta_{CN,t}^{USD}\right)B_{CN,t}^{USD})\right]=T_t \tag{4.20}$$

### 4.3.5 Equilibrium

This study focuses on the steady-state equilibrium, which consists of: the rest of world currency usage $S_{RoW}$, the allocation on the bond holdings $\{B_{RoW}^{USD}, B_{RoW}^{CNY}, B_{US}^{USD}, B_{US}^{CNY}, B_{CN}^{USD}, B_{CN}^{CNY}\}$, the bond prices $\{Q^{USD}, Q^{CNY}\}$ and bond holding premiums $\{\Delta^{USD}, \Delta^{CNY}\}$. Listed by the following conditions:

- There exists a quasi-equilibrium in currency usage under the conditions in Proposition 1.
- The optimal conditions of household bond holdings are satisfied, which is indicated in Proposition 2.
- The bonds market clear conditions are satisfied:

$$\begin{aligned}\mu_{US}B_{US}^{USD}+\mu_{CN}B_{CN}^{USD}+\mu_{RoW}B_{RoW}^{USD}&=\bar{B}^{USD}\\ \mu_{US}B_{US}^{CNY}+\mu_{CN}B_{CN}^{CNY}+\mu_{RoW}B_{RoW}^{CNY}&=\bar{B}^{CNY}\end{aligned} \tag{4.21}$$

- The bond holding premium are satisfied as per equations 4.18 and 4.19.

## 4.4 Quantitative Analysis

A quantitative analysis is presented in this section. The parameters are calibrated in the model, we also exogenously fix part parameters, which is in line with the work of Chahrour and Valchev (2022). The objective of this section is to quantitatively evaluate how different degrees of capital control (measured by capital control tax $\tau$) affect RMB usage in the rest of the world. And then the policy implications proposed here.

### 4.4.1 Calibration

Some parameters in the model are collected from the study of Chahrour and Valchev (2022), as this study departs from their work with some part modifications. Then for some parameters, we calibrate them to match the observed moments in Chinese data. Table 4.1 reports the exogenously fixed parameters, part of the parameters are

TABLE 4.1: Exogenously fixed parameters

| Parameters | Value | Interpretation | Target or Source |
|---|---|---|---|
| $\beta$ | 0.96 | Discount factor | Annual frequency |
| $\phi$ | 0.01 | Collateral mismatch cost | Chahrour and Valchev (2022) |
| $\pi$ | 1 | Trade surplus | Normalization to 1 |
| $\mu_{US} = \mu_{CN}$ | 0.20 | Country GDP size | IMF |
| $r$ | 0.005 | Collateral use fees | Chahrour and Valchev (2022) |

borrowed from the literature. To be specific, $\mu_{US} = \mu_{CN}$ is measured by the International Monetary Fund in recent years, which are the ratios of each country's GDP to the global GDP, we take the average ratio over the past 5 years by using the purchasing power parity computed GDP, so we get an equal size of the U.S. and China. The discount factor is 0.96, which is consistent with most literature. $\phi$ is the collateral asset mismatch cost, which is equal to 1% of transaction profit, and it comes from the paper of Chahrour and Valchev (2022). $\pi$ is normalized to one to reflect the trade contract execution profit. $r$ is also from literature, which is the collateral use fees that firms pay to households, generally, it can be understood as the convenience yield by holding a bond.

TABLE 4.2: Calibration strategy

| Parameters | Value | Description | Target or Source |
|---|---|---|---|
| $\bar{B}^{USD}$ | 1.4710 | The U.S. bonds supply in the world | Chahrour and Valchev (2022) |
| $\bar{B}^{CNY}$ | 1.0297 | Chinese bonds supply in the world | Proportional of $\bar{B}^{USD}$ |
| $\tau$ | 0 to 0.25 | Capital control tax | Liu et al. (2021a) |

Table 4.2 reports the calibration strategy we used, the exogenous supply of dollar bonds is calibrated from literature, which they set this value to match the U.S. government debt of GDP, and it is consistent with the U.S. average. For the exogenous supply of renminbi bonds, we collected the Chinese bonds market data and estimated the size of the Chinese bond issuance, and take the ratio of it to the U.S. bonds supply. $\tau$ is inspired by the study of Liu et al. (2021a), which uses the data of China's international investment position report published by the SAFE. The ratio of China's foreign liabilities to its annual GDP ranges from 0 to 25% from 2006 to 2016.

### 4.4.2 Quantitative Results

How does capital control tax impact the progress of RMB usage in the rest of the world? After the calibration of some parameters and exogenous variables, we conduct the numerical experiments in this subsection. Figure 4.5 reports that with the increase of capital control tax in China, Chinese currency usage in the rest of the world declined. The solutions are evaluated around the steady state of the model, thus the renminbi usage fixes at a stable level, which is consistent with Chahrour and Valchev (2022)'s result. The intuition here is that given any level of capital control, the Chinese currency usage will be severely limited, and once entirely removal the capital control ($\tau = 0$), the renminbi will compose a significant share in the international monetary system and price system with a share of about 29%.

The marginal impact of rising $\tau$ fall to 5% as captured in Fig 4.5, the model at this stage is evaluated around the steady state, thus the renminbi usage in the rest of the world fixed at 5% when capital control tax increasing, that is to say, above a certain level of capital control in China, the U.S. dollar will be the dominant currency in the rest of world with a share of 95%. The reasons why this marginal impact falls to 5%

are as follows: first, as demonstrated in the equation 4.17, there are three steady state equilibriums, which are dollar dominant steady state ($S_{RoW} = 1$), renminbi dominant steady steady ($S_{RoW} = 0$), and multipolar steady state with $S_{RoW}$ be an interior solution. After introducing calibration in the model, we solve the interior solution of $S_{RoW}$, in which we get the relation between $1 - S_{RoW}$ and $\tau$, the value of $1 - S_{RoW}$ depends on the parameters and exogenous variables we have introduced. Second, intuitively, a higher additional cost has been paid to purchase renminbi bonds reducing the incentives for households to invest in this type of bond, which results in substitution for dollar bonds and making the dollar bond the dominant asset in the world, thus we have $1 - S_{Row} = 0.05$ and $S_{RoW} = 0.95$.

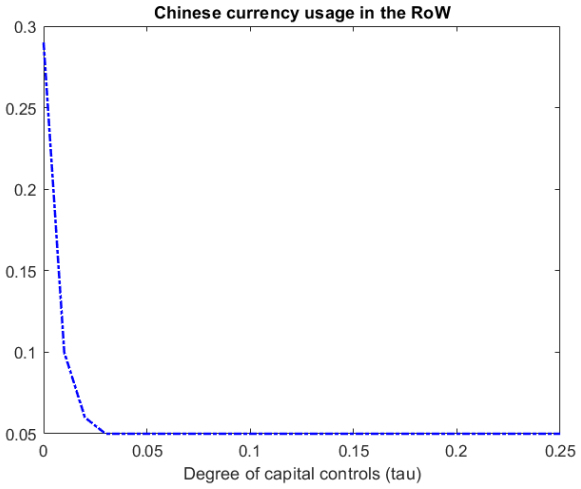



FIGURE 4.5: Renminbi usage in the rest of the world countries

Excepting the negative impact of capital account restriction on Renminbi usage, the model also provides additional insight into the following aspects. First, in the household problem, the steady state Euler equations demonstrate that in the absence of capital control in China, the Chinese bonds holding return is higher than the US bonds holding return ($\frac{1}{Q^{CNY} - \Delta_{US}^{CNY}} > \frac{1}{Q^{USD} - \Delta_{US}^{USD}}$), this implies that the Chinese bond holding premium $\Delta^{CNY}$ is higher enough to compensate the additional cost that is induced by capital control in China, it also implies that the net return in investing Chinese bonds is higher than the US bond, which is captured by Chang et al. (2015). Second, the model highlights the interaction between household portfolio investment

decisions and firms' trade financing decision drives the emergence of international currency, which provide policy implications for the central bank of China, to enhance the accessibility of Chinese bonds and also the gradual opening of the financial market, as mentioned in Clayton et al. (2022).

## 4.5 Conclusion

This paper departs from a framework of Chahrour and Valchev (2022) to evaluate the general equilibrium effect of capital controls on the progress of RMB internationalization. We developed a dynamic general equilibrium model with two big countries (the U.S. and China) and a continuum of small open economies in the rest of the world, featuring capital controls in the household portfolio investment, to study how different degrees of capital controls affect the equilibrium usage of Chinese RMB in the rest of world. The framework embedded two frictions in international trade, which are trade contract commitment friction (creating the demands on collateral to guarantee enforcement) and financial friction (the friction of obtaining the collateral). Households gain extra benefits by holding collateral assets (two types of bonds issued by the U.S. government and the Chinese government) and trading firms use the collateral assets to ensure the execution of trade contracts, which generate profits for them. The interaction between households assets saving decisions and trading firms' collateral asset demand decisions gives rise to a persistent currency dominance.

We first evaluate how capital control in China affects the progress of renminbi internationalization by illustrating the quantitative results. After the calibration, the model result indicates that there is a nonlinear effect between the capital controls and currency international usage, with an increase in capital control tax, there is decreasing in renminbi usage in the rest of the world, which is consistent with data fact that even in the presence of the capital controls in China at present, there is still a small proportion of renminbi usage in the rest of the world. The entire lifting of capital control makes Chinese usage increase to a significant level, which is equivalent to the

euro's status.

There are several extensions that can be made within this framework. Further research intends to introduce more dynamics in the model such as adding price, consumption bundles, and idiosyncratic risk in the payoff differential function, the object is to study the transition dynamics of renminbi usage with the change of capital controls, this would allow one to investigate the policy implications, such as announced vs unannounced capital account liberalization policy. Further extension is in progress.

# Chapter 5

# Conclusions

In this thesis, I intend to contribute to the understanding of the Chinese economy in both the labor economics field and the international finance field. I highlight the demographic structure changes in China and quantitatively evaluate their impact on the Chinese marriage market matching outcomes and household income inequality. I also highlight the capital account restriction in China and RMB internationalization programs, in which I employ an existing framework to quantitatively study the influence of capital controls in China on the promotion of RMB internationalization.

This thesis consists of three chapters, in the first chapter, I empirically investigate how the male abundance in the marriage market affects the marriage market matching outcomes, Notably, I employ three waves of household-level data, and two rounds of population census data as well as the China City Statistical Yearbook to conduct empirical research design. I find that an increase in the sex ratio in the marriage market is negatively associated with the degree of assortative matching, the relationship is robust once I use alternative model specifications. Also, I empirically find that the sex ratio in the marriage market negatively affects household income inequality. Furthermore, the study also uses instrument variables to address the endogeneity issues.

Chapter 3 provides a structural interpretation of the relationship between the sex ratio in the marriage market and the degree of assortative matching. I develop a general

equilibrium model to quantitatively show that the sex ratio is negatively associated with household sorting and also household income inequality. The results are consistent with the literature's findings. I then endogenize the education investment decision to see how males and females make education decisions in response to a rising male abundance.

In chapter 4, I employ a framework developed by Chahrour and Valchev (2022) to quantitatively evaluate the impact of capital account restrictions on the progress of RMB internationalization. I find that the increase in capital control reduces Chinese currency usage in the rest of the world countries. The developed model can be generalized and allow for several extensions, adding more ingredients and studying the transition dynamics of renminbi usage with the change of capital controls, this would allow one to compare the implications of announced vs unannounced capital account liberalization policy.

The findings in this thesis leave much future research, with one direction that studies the impact of male abundance on wealth inequality in China, and another one studies the optimal way to ease the capital account restrictions or evaluates the transition path with a gradual liberalization on capital account.

# Appendix A

# Appendix to Chapter 2

## A.1 Data Sources

The empirical evidence collects data from three sources, which include China Household Income Projects (CHIPs), the Chinese Population census, and China City Statistical Yearbook. We use three waves of CHIPs (2002, 2007, and 2013) to form a household-level data sample, which is restricted to urban areas as the education level in rural areas is less heterogeneous. Using the household-level data, the study is able to measure the degree of assortative matching. Restricting the cohort ages between 20 to 30, the study measures the degree of assortative matching by using education correlation between couples in a region.

The sex ratio in the marriage market is measured by the ratio of the number of males to the number of females in a region. We infer the sex ratio from the population census. The details of how we infer the sex ratio in the marriage market can be found in Table 2.2.

The study employs the data from China City Statistical Yearbook (1999 to 2013 year by year) to construct the control variables, for the CHIPs wave of 2002, we compute the mean value of each control variable to reflect the local region's characteristics, including GDP per capita, population density, government education expenditure, the number of unemployed people, the number of high schools, and the number of

colleges. All of these variables are computed by taking the mean value over the past five years, as not only the current regional characteristics determine the education level of males and females in the marriage market, but also the past information in a region affects the migration flow of males and females, education investment of males and females.

## A.2 Fertility Fine Across Provinces

Figure A.1 shows the fertility fine rate across provinces. The fine rate for violating the

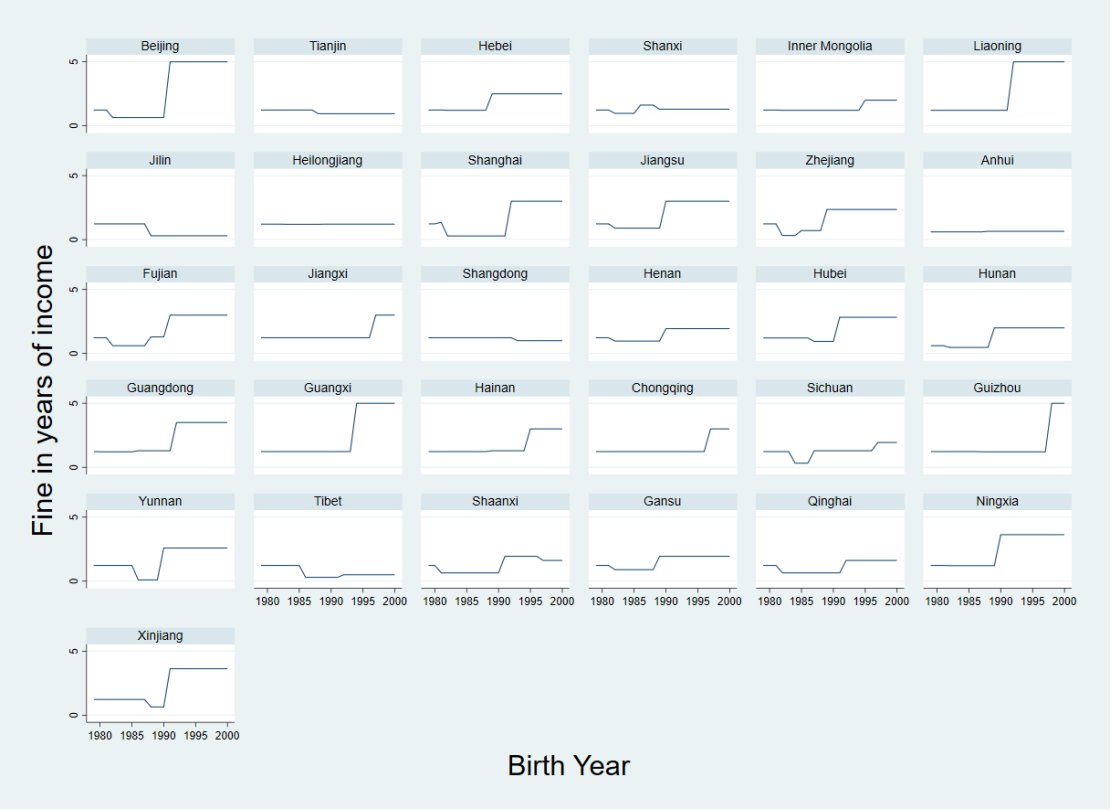



FIGURE A.1: Fertility fine rates across provinces

birth quotas in a province takes the form of a percentage deduction of local yearly income and lasts for several years. As demonstrated in Figure A.1, the fine rate is not quite volatile, as the penalty lasts for several years, once there are updates from the central government's policy, the fine rate changes accordingly. The highest level of fine rate is 5%, which is in Beijing, Liaoning, Guangxi, and Guizhou.

# Appendix B

# Appendix to Chapter 3

## B.1 Equilibrium Distribution of Households

TABLE B.1: Equilibrium distribution of households

| $N^f$ | Sex ratio | $\pi_{ss}$ | $\pi_{su}$ | $\pi_{us}$ | $\pi_{uu}$ | $\pi_s$ | $\pi_u$ |
|---|---|---|---|---|---|---|---|
| 0.80 | 1.25 | 0.336 | 0.103 | 0.055 | 0.306 | 0.075 | 0.125 |
| 0.81 | 1.23 | 0.340 | 0.102 | 0.056 | 0.312 | 0.071 | 0.119 |
| 0.82 | 1.22 | 0.344 | 0.102 | 0.056 | 0.318 | 0.067 | 0.113 |
| 0.83 | 1.20 | 0.348 | 0.101 | 0.057 | 0.323 | 0.064 | 0.106 |
| 0.84 | 1.19 | 0.352 | 0.101 | 0.058 | 0.329 | 0.060 | 0.100 |
| 0.85 | 1.18 | 0.357 | 0.100 | 0.058 | 0.335 | 0.056 | 0.094 |
| 0.86 | 1.16 | 0.361 | 0.099 | 0.059 | 0.341 | 0.053 | 0.087 |
| 0.87 | 1.15 | 0.365 | 0.099 | 0.060 | 0.346 | 0.049 | 0.081 |
| 0.88 | 1.14 | 0.369 | 0.098 | 0.060 | 0.352 | 0.046 | 0.074 |
| 0.89 | 1.12 | 0.373 | 0.097 | 0.061 | 0.358 | 0.042 | 0.068 |
| 0.90 | 1.11 | 0.378 | 0.097 | 0.062 | 0.364 | 0.039 | 0.061 |
| 0.91 | 1.10 | 0.382 | 0.096 | 0.063 | 0.370 | 0.036 | 0.054 |
| 0.92 | 1.09 | 0.386 | 0.095 | 0.063 | 0.376 | 0.032 | 0.048 |
| 0.93 | 1.08 | 0.390 | 0.094 | 0.064 | 0.382 | 0.029 | 0.041 |
| 0.94 | 1.06 | 0.394 | 0.093 | 0.065 | 0.388 | 0.026 | 0.034 |
| 0.95 | 1.05 | 0.399 | 0.092 | 0.065 | 0.394 | 0.022 | 0.028 |
| 0.96 | 1.04 | 0.403 | 0.091 | 0.066 | 0.400 | 0.019 | 0.021 |
| 0.97 | 1.03 | 0.407 | 0.090 | 0.067 | 0.406 | 0.016 | 0.014 |

Table B.1 reports the equilibrium distribution of households, by solving the 8 cutoff match quality, we compute the numbers of different types of households. With the increase of the sex ratio in the marriage market, the couple type of skilled husband and skilled wife decreases, the same trend can be found in the couple type of unskilled husband and unskilled wife. For other types of couples, the numbers increase with a rising sex ratio. For single males, both the number of skilled males and unskilled

males increases. The couples of skilled husband and skilled wife, and the couples of unskilled husband and unskilled wife compose the largest proportion of couples in all types of marriage. Starting from 81.3% to 64.22% with the increase in sex ratio in the marriage market from 1.031 to 1.250.

## B.2 Constructing the Gini Coefficient and Lorenz Curve

To construct the Gini coefficient and Lorenz curve analytically, we present the formula here. Following Greenwood et al. (2014), we first compute the cumulative share of different types of households by using the results from Table B.1, then the corresponding cumulative share of household income is computed. Figure B.1 graphically illustrates the computation of the Gini coefficient, where the horizontal axis is the cumulative share of households, and the vertical axis is the corresponding cumulative share of household income. For the couple types of unskilled husband and unskilled wife, the corresponding household income is $\pi_{uu}(w_m^u + w_f^u)$. Equations B.1 and B.2 describe the computation methods of cumulative share of household income. For each type of household in population, the household income is computed as the number of this type of household times the corresponding income.

$$
\begin{aligned}
I_1 &= I_{ss} = \textstyle\sum_i^{\pi_{ss}} w_{h,i}^s + \sum_j^{\pi_{ss}} w_{w,j}^s = \pi_{ss}\bar{w}_h^s + \pi_{ss}\bar{w}_w^s = \pi_{ss}(\bar{w}_h^s + \bar{w}_w^s) = \pi_{ss}(w_m^s + w_f^s) \\
I_2 &= I_{su} = \textstyle\sum_i^{\pi_{su}} w_{h,i}^s + \sum_j^{\pi_{su}} w_{w,j}^u = \pi_{su}\bar{w}_h^s + \pi_{su}\bar{w}_w^u = \pi_{su}(\bar{w}_h^s + \bar{w}_w^u) = \pi_{su}(w_m^s + w_f^u) \\
I_3 &= I_{us} = \textstyle\sum_i^{\pi_{us}} w_{h,i}^u + \sum_j^{\pi_{us}} w_{w,j}^s = \pi_{us}\bar{w}_h^u + \pi_{us}\bar{w}_w^s = \pi_{us}(\bar{w}_h^u + \bar{w}_w^s) = \pi_{us}(w_m^u + w_f^s) \\
I_4 &= I_{uu} = \textstyle\sum_i^{\pi_{uu}} w_{h,i}^u + \sum_j^{\pi_{uu}} w_{w,j}^u = \pi_{uu}\bar{w}_h^u + \pi_{uu}\bar{w}_w^u = \pi_{uu}(\bar{w}_h^u + \bar{w}_w^u) = \pi_{uu}(w_m^u + w_f^u) \\
I_5 &= I_s = \textstyle\sum_i^{\pi_s} w_{single,i}^s = \pi_s\bar{w}_{single}^s = \pi_s w_m^s \\
I_6 &= I_u = \textstyle\sum_i^{\pi_u} w_{single,i}^u = \pi_u\bar{w}_{single}^u = \pi_u w_m^u \\
I &= \textstyle\sum I_i
\end{aligned}
\tag{B.1}
$$

Let $\chi_{zz}$ denote the share of income for household type $zz$ over the total population, then we have:

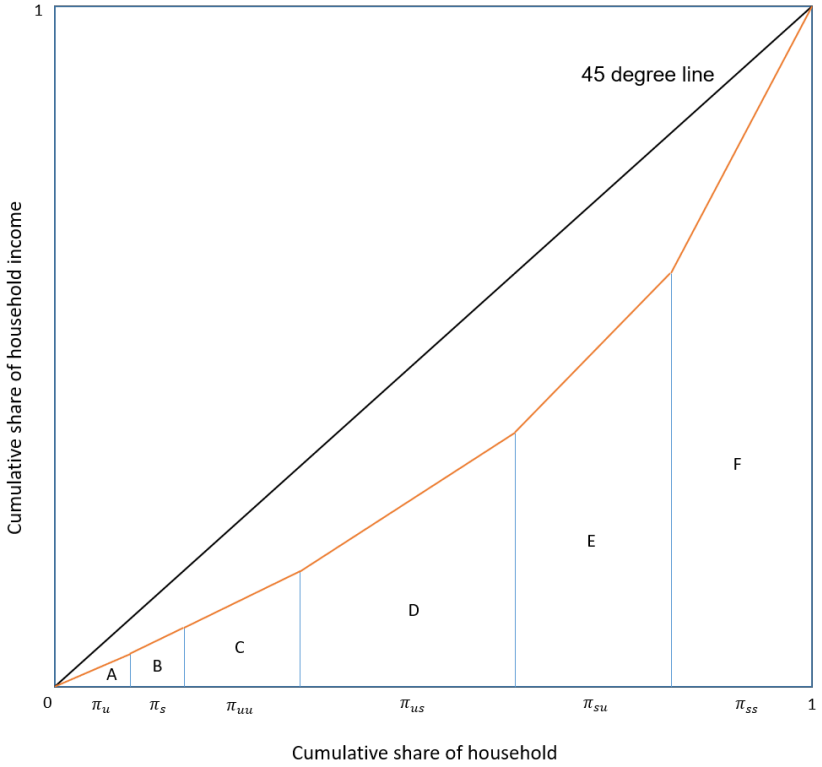



FIGURE B.1: Gini coefficient and Lorenz curve

$$\begin{aligned}
\chi_{ss} &= \frac{I_1}{I} \\
\chi_{su} &= \frac{I_2}{I} \\
\chi_{us} &= \frac{I_3}{I} \\
\chi_{uu} &= \frac{I_4}{I} \\
\chi_{s} &= \frac{I_5}{I} \\
\chi_{u} &= \frac{I_6}{I}
\end{aligned} \tag{B.2}$$

For the share of different types of households, we have:

$$\pi_{ss} + \pi_{su} + \pi_{us} + \pi_{uu} + \pi_s + \pi_u = 1$$

Let $\lambda_{zz}$ denotes the share of different types of households over the total population, we set:

$$\begin{aligned}
\lambda_{ss} &= \pi_{ss} \\
\lambda_{su} &= \pi_{su} \\
\lambda_{us} &= \pi_{us} \\
\lambda_{uu} &= \pi_{uu} \\
\lambda_{s} &= \pi_{s} \\
\lambda_{u} &= \pi_{u}
\end{aligned} \tag{B.3}$$

The Gini coefficient associates with the Lorenz curve equals twice the area between the Lorenz curve and the 45-degree line. Or the Gini coefficient can be computed as $1 - 2\Delta$, where $\Delta$ is the area below the Lorenz curve (the yellow line in Fig B.1). In this research, we divide $\Delta$ into six areas, which are area A, B, C, D, E, F respectively (which can be found from Fig B.1). Thus, the Gini coefficient ($g$) is:

$$\begin{aligned}
g &= 1 - 2\sum_{k \in K} area_k \\
where\ K &= \{A, B, C, D, E, F\}
\end{aligned} \tag{B.4}$$

And we have the computation of each area below:

$$\begin{aligned}
area_A &= \tfrac{\lambda_u \chi_u}{2} \\
area_B &= \tfrac{\lambda_s[\chi_u + (\chi_u + \chi_s)]}{2} \\
area_C &= \tfrac{\lambda_{uu}[(\chi_u + \chi_s) + (\chi_u + \chi_s + \chi_{uu})]}{2} \\
area_D &= \tfrac{\lambda_{us}[(\chi_u + \chi_s + \chi_{uu}) + (\chi_u + \chi_s + \chi_{uu} + \chi_{us})]}{2} \\
area_E &= \tfrac{\lambda_{su}[(\chi_u + \chi_s + \chi_{uu} + \chi_{us}) + (\chi_u + \chi_s + \chi_{uu} + \chi_{us} + \chi_{su})]}{2} \\
area_F &= \tfrac{\lambda_{ss}[(\chi_u + \chi_s + \chi_{uu} + \chi_{us} + \chi_{su}) + (\chi_u + \chi_s + \chi_{uu} + \chi_{us} + \chi_{su} + \chi_{ss})]}{2}
\end{aligned} \tag{B.5}$$

## B.3 Education Decision of Males and Females

This section plots the decision tree for males and females in making their education investment decision. Males and females can decide whether they want to become skilled or unskilled agents before they enter the marriage market. Fig B.2 demonstrates the decision tree for males who want to become skilled, there are three outcomes for him in the first round of matching, meeting with a skilled female $f_s$ or an unskilled female $f_u$, or meeting with no one '$-$' (due to the number of males $N^m = 1$ is larger than the number of females $N^f$). In the meeting with a skilled female $f_s$, he could accept this match and form a marriage ('Yes' in the bracket) or either reject this match and go to the second round of matching ('No' in the bracket), where he chooses to accept this type of match or reject and remain single. In the meeting with an unskilled female $f_u$, he could accept this match ('Yes' in the bracket) and form households with this type of female or either reject it ('No' in the bracket) and enter into the second round of matching, where he chooses to accept a match with a skilled female $f_s$ or reject it and remain single ('Single' in the bracket). In the meeting with no one in the first round ('$-$' in the bracket), this type of male directly goes to the second round of matching and chooses either to accept the match with a skilled female $f_s$ or reject it and remain single ('Single' in the bracket).

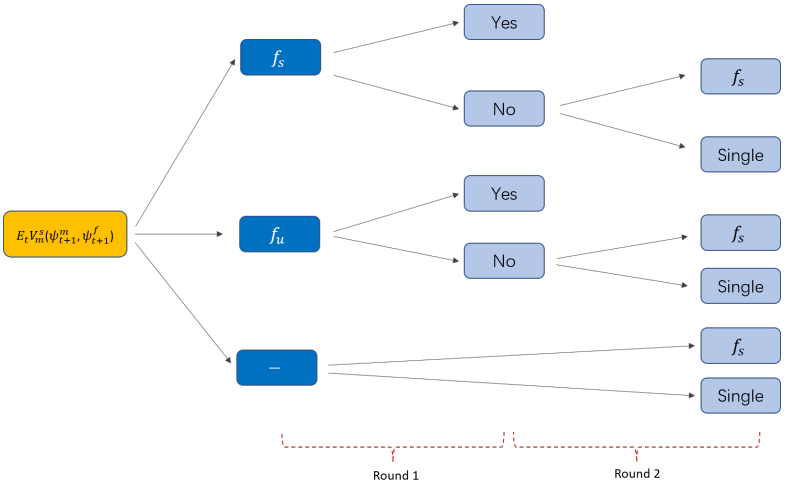



FIGURE B.2: Decision tree for the male who wants to become skilled

For the females, due to the number of males is larger than the number of females, every woman will meet with one type of male in the first round (either $m_s$ or $m_u$), and

then she makes the marriage decisions. Figure B.3 plots the decision tree for females who want to become skilled. In the first round the matching, a female can meet with a skilled male $m_s$ or an unskilled male $m_u$, in the meeting with a skilled male $m_s$, she could accept this match and form household ('Yes' in the bracket) or reject this match ('No' in the bracket) and she goes to the second round match, where she meets with a skilled male $m_s$ and decides to get married or not if she rejects it, then she will remain single ('Single' in the bracket). In the meeting with an unskilled male $m_u$, she could accept this match and form a household ('Yes' in the bracket) or reject this match ('No' in the bracket) and she goes to the second round match, where she meets with a skilled male $m_s$ and decides to get married or not if she rejects it, then she will remain single ('Single' in the bracket).

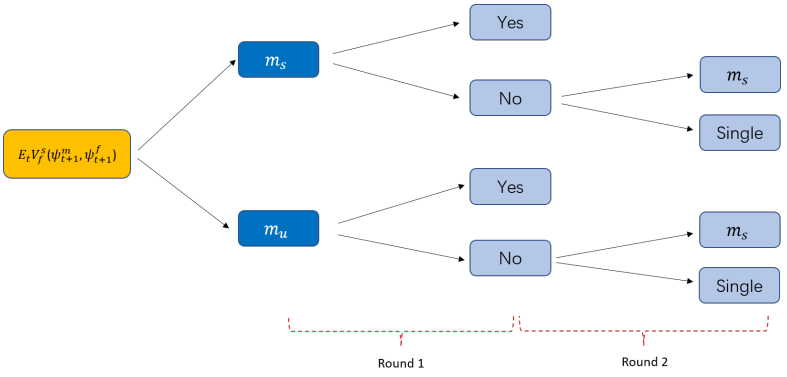



FIGURE B.3: Decision tree for the female who wants to become skilled

For the analysis of male who wants to become unskilled and female who wants to become unskilled, the decision trees are illustrated in Figure B.4 and Figure B.5. The same analysis procedures apply to these two cases.

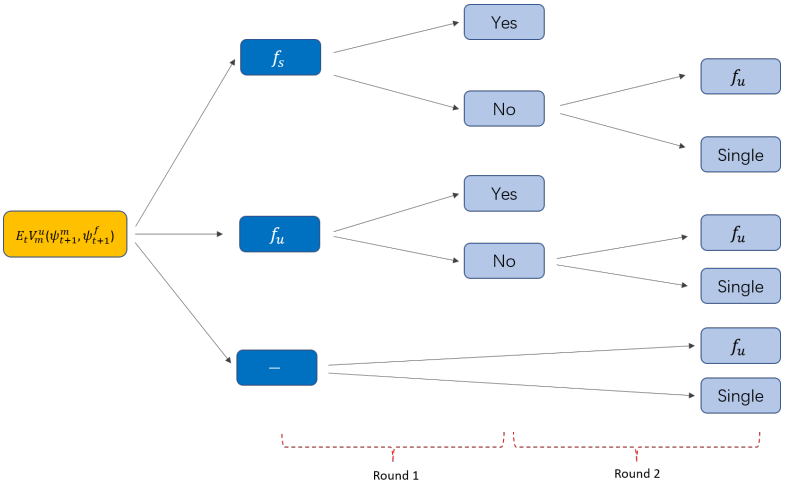



FIGURE B.4: Decision tree for the male who wants to become unskilled

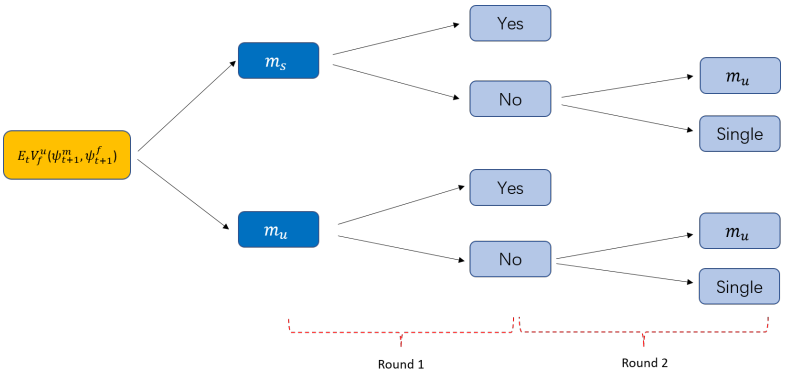



FIGURE B.5: Decision tree for the female who wants to become unskilled

# Appendix C

# Appendix to Chapter 4

## C.1 Household Problem

For the households in the country $j \in [0, \mu_{RoW}]$, we have:

$$\max_{C_{jt}, B_{jt}^{USD}, B_{jt}^{CNY}} E_0 \sum_{t=0}^{\infty} \beta^t \frac{C_{jt}^{1-\sigma}}{1-\sigma}$$

$$s.t.:$$

$$C_{jt} + (Q_t^{USD} - \Delta_{jt}^{USD}) B_{jt}^{USD}$$

$$+(Q_t^{CNY} - \Delta_{jt}^{CNY}) B_{jt}^{CNY} [1+\tau] = B_{jt-1}^{USD} + B_{jt-1}^{CNY} + Y_{jt}$$

The Lagrangian of the household problems is set by:

$$\mathcal{L} = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{C_{jt}^{1-\sigma}}{1-\sigma} - \lambda_{jt} \left[ C_{jt} + (Q_t^{USD} - \Delta_{jt}^{USD}) B_{jt}^{USD} + (Q_t^{CNY} - \Delta_{jt}^{CNY}) B_{jt}^{CNY} [1+\tau] - B_{jt-1}^{USD} - B_{jt-1}^{CNY} - Y_{jt} \right] \right\}$$

The first order conditions with respect to all the endogenous variables and Lagrangian

multiplier are:

$$\frac{\partial \mathcal{L}}{\partial C_{jt}} = 0 \Rightarrow C_{jt}^{-\sigma} = \lambda_{jt}$$

$$\frac{\partial \mathcal{L}}{\partial B_{jt}^{USD}} = 0 \Rightarrow \lambda_{jt}(Q_t^{USD} - \Delta_{jt}^{USD}) = \beta E_t \left\{\lambda_{jt+1}\right\}$$

$$\frac{\partial \mathcal{L}}{\partial B_{jt}^{CNY}} = 0 \Rightarrow \lambda_{jt}(Q_t^{CNY} - \Delta_{jt}^{CNY})\left[1+\tau\right] = \beta E_t \left\{\lambda_{jt+1}\right\}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda_{jt}} = 0 \Rightarrow C_{jt} + (Q_t^{USD} - \Delta_{jt}^{USD})B_{jt}^{USD} + (Q_t^{CNY} - \Delta_{jt}^{CNY})B_{jt}^{CNY}\left[1+\tau\right] = B_{jt-1}^{USD} + B_{jt-1}^{CNY} + Y_{jt} + T_{jt} \tag{C.1}$$

We can get the Euler equations as:

$$\frac{1}{\beta} = E_t\left\{\frac{C_{jt+1}^{-\sigma}}{C_{jt}^{-\sigma}}\frac{1}{Q_t^{USD} - \Delta_{jt}^{USD}}\right\} = E_t\left\{\frac{C_{jt+1}^{-\sigma}}{C_{jt}^{-\sigma}}\frac{1}{(Q_t^{CNY} - \Delta_{jt}^{CNY})\left[1+\tau\right]}\right\} \tag{C.2}$$

In the steady state we have:

$$\frac{1}{\beta} = \frac{1}{Q^{USD} - \Delta_j^{USD}} = \frac{1}{(Q^{CNY} - \Delta_j^{CNY})\left[1+\tau\right]} \tag{C.3}$$

## C.2 Proof of Proposition 1

The supply of each asset is $\bar{B}^{USD}$ and $\bar{B}^{CNY}$ respectively, using the market clear conditions, which are:

$$\bar{B}^{USD} = \mu_{US}B_{US}^{USD} + \mu_{CN}B_{CN}^{USD} + \mu_{RoW}B_{RoW}^{USD}$$

$$\bar{B}^{CNY} = \mu_{US}B_{US}^{CNY} + \mu_{CN}B_{CN}^{CNY} + \mu_{RoW}B_{RoW}^{CNY}$$

This implied an upper bound on the feasible rest of world bond holdings:

$$B_{RoW}^{USD} < \frac{\bar{B}^{USD}}{\mu_{RoW}}$$

$$B_{RoW}^{CNY} < \frac{\bar{B}^{CNY}}{\mu_{RoW}}$$

When $B_{RoW}^{USD} = 0$ or $B_{RoW}^{CNY} = 0$, the quasi equilibrium is unique. For example, when $B_{RoW}^{CNY} = 0$, equation 4.16 changes to:

$$V^{USD}(S_{RoW}) = \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + S_{RoW}} [\pi - r - \phi(1 - \bar{S})] = \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + S_{RoW}} [\pi - r - \phi(1 - \mu_{US} - \mu_{RoW} S_{RoW})] > 0$$

Where we have used the condition of:

$$\bar{S} = \mu_{US} + \int_0^{\mu_{RoW}} S_j dj = \mu_{US} + \mu_{RoW} S_{RoW}$$

Under this case, since $\phi < \pi - r$, the only equilibrium value of $S_{RoW}$ is 1, which implies that all the trading firms in the rest of the world seek the U.S. asset as collateral. When $B_{RoW}^{USD} = 0$, $V^{USD}(S_{RoW}) < 0$, the only equilibrium is $S_{RoW} = 0$.

For any pair of bond holdings, which means $B_{RoW}^{USD} \in (0, \frac{\bar{B}^{USD}}{\mu_{RoW}})$ and $B_{RoW}^{CNY} \in (0, \frac{\bar{B}^{CNY}}{\mu_{RoW}})$. By using the condition of:

$$\mu_{US} = \mu_{CN} = \frac{1 - \mu_{RoW}}{2}$$

, we rewrite equation 4.16 as:

$$\begin{aligned} V^{USD}(S_{RoW}) = & \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + S_{RoW}} [\pi - r - \phi(1 - \mu_{US} - \mu_{RoW} S_{RoW})] \\ & - \frac{B_{RoW}^{CNY}}{B_{RoW}^{CNY} + 1 - S_{RoW}} [\pi - r - \phi(\mu_{CN} + \mu_{RoW} S_{RoW})] \end{aligned} \tag{C.4}$$

Evaluating equation C.4 at $S_{RoW} = 1$ we have:

$$V^{USD}(1) = \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + 1} [\pi - r - \phi(1 - \mu_{US} - \mu_{RoW})] - [\pi - r - \phi(\mu_{CN} + \mu_{RoW})]$$

$$\Rightarrow V^{USD}(1) = \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + 1} \left[\pi - r - \phi(1 - \frac{1 - \mu_{RoW}}{2} - \mu_{RoW})\right] - \left[\pi - r - \phi(\frac{1 - \mu_{RoW}}{2} + \mu_{RoW})\right]$$

$$\Rightarrow V^{USD}(1) = \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + 1} \left[\pi - r - \phi(\frac{1}{2} - \frac{\mu_{RoW}}{2})\right] - \left[\pi - r - \phi(\frac{1}{2} + \frac{\mu_{RoW}}{2})\right]$$

$$\Rightarrow V^{USD}(1) = -\frac{\pi - r}{B_{RoW}^{USD} + 1} - \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + 1} \frac{\phi}{2} + \frac{B_{RoW}^{USD}}{B_{RoW}^{USD} + 1} \frac{\phi \mu_{RoW}}{2} + \frac{\phi}{2} + \frac{\phi \mu_{RoW}}{2}$$

$$\Rightarrow V^{USD}(1) = -\frac{\pi - r}{B^{USD}_{RoW} + 1} - \frac{B^{USD}_{RoW}}{B^{USD}_{RoW} + 1}\frac{\phi}{2} + \frac{B^{USD}_{RoW}}{B^{USD}_{RoW} + 1}\frac{\phi\mu_{RoW}}{2} + \frac{B^{USD}_{RoW} + 1}{B^{USD}_{RoW} + 1}\frac{\phi}{2} + \frac{B^{USD}_{RoW} + 1}{B^{USD}_{RoW} + 1}\frac{\phi\mu_{RoW}}{2}$$

$$\Rightarrow V^{USD}(1) = -\frac{\pi - r}{B^{USD}_{RoW} + 1} - \frac{\phi}{2}\left[\frac{B^{USD}_{RoW} - B^{USD}_{RoW}\mu_{RoW} - B^{USD}_{RoW} - 1 - B^{USD}_{RoW}\mu_{RoW} - \mu_{RoW}}{B^{USD}_{RoW} + 1}\right]$$

$$\Rightarrow V^{USD}(1) = -\frac{\pi - r}{B^{USD}_{RoW} + 1} - \frac{\phi}{2}\left[\frac{-2B^{USD}_{RoW}\mu_{RoW} - 1 - \mu_{RoW}}{B^{USD}_{RoW} + 1}\right]$$

$$\Rightarrow V^{USD}(1) = -\frac{\pi - r}{B^{USD}_{RoW} + 1} + \phi\left[\frac{B^{USD}_{RoW}\mu_{RoW} + \frac{1}{2} + \frac{\mu_{RoW}}{2}}{B^{USD}_{RoW} + 1}\right]$$

If $V^{USD}(1) < 0$, then all the trading firms in the rest of the world would choose to use Chinese bonds as collateral, thus $S_{RoW} = 1$ is not a quasi equilibrium. If $V^{USD}(1) > 0$, $S_{RoW} = 1$ is a quasi equilibrium. When $V^{USD}(1) < 0$, we have:

$$V^{USD}(1) = -\frac{\pi - r}{B^{USD}_{RoW} + 1} + \phi\left[\frac{B^{USD}_{RoW}\mu_{RoW} + \frac{1}{2} + \frac{\mu_{RoW}}{2}}{B^{USD}_{RoW} + 1}\right] < 0$$

$$\Rightarrow \phi < \frac{\pi - r}{B^{USD}_{RoW}\mu_{RoW} + \frac{1}{2} + \frac{\mu_{RoW}}{2}}$$

Thus, $V^{USD}(1)$ is strictly negative if and only if:

$$\phi < \frac{\pi - r}{\mu_{RoW}(B^{USD}_{RoW} + \frac{1}{2}) + \frac{1}{2}}$$

Since $B^{USD}_{RoW} < \frac{\bar{B}^{USD}}{\mu_{RoW}}$, we have:

$$\frac{\pi - r}{\mu_{RoW}(B^{USD}_{RoW} + \frac{1}{2}) + \frac{1}{2}} > \frac{\pi - r}{\bar{B}^{USD} + \frac{\mu_{RoW}}{2} + \frac{1}{2}}$$

Here we define $\phi^{sunspot\ 1}$ as:

$$\phi^{sunspot\ 1} = \frac{\pi - r}{\bar{B}^{USD} + \frac{\mu_{RoW}}{2} + \frac{1}{2}}$$

Thus, for any $\phi < \phi^{sunspot\ 1}$, $V^{USD}(1)$ is strictly negative, hence, $S_{RoW} = 1$ is not a quasi-equilibrium.

Similarly, if we evaluate equation C.4 at $S_{RoW} = 0$ we have:

$$V^{USD}(0) = \left[\pi - r - \phi(1 - \frac{1-\mu_{RoW}}{2})\right] - \frac{B_{RoW}^{CNY}}{B_{RoW}^{CNY}+1}\left[\pi - r - \phi(\frac{1-\mu_{RoW}}{2})\right]$$

$$\Rightarrow V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} - \frac{\phi}{2} - \frac{\phi\mu_{RoW}}{2} + \frac{B_{RoW}^{CNY}}{B_{RoW}^{CNY}+1}\frac{\phi}{2} - \frac{B_{RoW}^{CNY}}{B_{RoW}^{CNY}+1}\frac{\phi\mu_{RoW}}{2}$$

$$\Rightarrow V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} - \phi(\frac{1}{2} + \frac{1}{2}\mu_{RoW}) + \phi\frac{B_{RoW}^{CNY}}{B_{RoW}^{CNY}+1}(\frac{1}{2} - \frac{1}{2}\mu_{RoW})$$

$$\Rightarrow V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} + \phi\left[\frac{\frac{1}{2}B_{RoW}^{CNY} - \frac{1}{2}B_{RoW}^{CNY}\mu_{RoW} - (\frac{1}{2} + \frac{1}{2}\mu_{RoW})(B_{RoW}^{CNY}+1)}{B_{RoW}^{CNY}+1}\right]$$

$$\Rightarrow V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} + \phi\frac{-\frac{1}{2} - B_{RoW}^{CNY}\mu_{RoW} - \frac{1}{2}\mu_{RoW}}{B_{RoW}^{CNY}+1}$$

$$\Rightarrow V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} - \phi\left[\frac{B_{RoW}^{CNY}\mu_{RoW} + \frac{1}{2} + \frac{1}{2}\mu_{RoW}}{B_{RoW}^{CNY}+1}\right]$$

If $V^{USD}(0) > 0$, then all the trading firms in the rest of the world would choose to use the U.S. bonds as collateral, thus $S_{RoW} = 0$ is not a quasi equilibrium. If $V^{USD}(0) < 0$, $S_{RoW} = 0$ is a quasi equilibrium. When $V^{USD}(0) > 0$, we have:

$$V^{USD}(0) = \frac{\pi - r}{B_{RoW}^{CNY}+1} - \phi\left[\frac{B_{RoW}^{CNY}\mu_{RoW} + \frac{1}{2} + \frac{1}{2}\mu_{RoW}}{B_{RoW}^{CNY}+1}\right] > 0$$

$$\Rightarrow \phi < \frac{\pi - r}{B_{RoW}^{CNY}\mu_{RoW} + \frac{1}{2} + \frac{\mu_{RoW}}{2}}$$

Thus, $V^{USD}(0)$ is strictly positive if and only if:

$$\phi < \frac{\pi - r}{\mu_{RoW}(B_{RoW}^{CNY} + \frac{1}{2}) + \frac{1}{2}}$$

Since $B_{RoW}^{CNY} < \frac{\bar{B}^{CNY}}{\mu_{RoW}}$, we have:

$$\frac{\pi - r}{\mu_{RoW}(B_{RoW}^{CNY} + \frac{1}{2}) + \frac{1}{2}} > \frac{\pi - r}{\bar{B}^{CNY} + \frac{\mu_{RoW}}{2} + \frac{1}{2}}$$

Here we define $\phi^{sunspot\ 2}$ as:

$$\phi^{sunspot\ 2} = \frac{\pi - r}{\bar{B}^{CNY} + \frac{\mu_{RoW}}{2} + \frac{1}{2}}$$

Thus, for any $\phi < \phi^{sunspot\ 2}$, $V^{USD}(0)$ is strictly positive, hence, $S_{RoW} = 0$ is not a quasi-equilibrium.

Here we have defined two sunspot equilibria, which illustrate the conditions that the quasi-equilibrium of $S_{RoW}$ does not hold. Thus, for any:

$$\phi < \min(\phi^{sunspot\ 1}, \phi^{sunspot\ 2})$$

$S_{RoW} = 1$ and $S_{RoW} = 0$ are not the quasi-equilibrium for any feasible allocation of bond holdings when $B^{USD} > 0$ and $B^{CNY} > 0$. Under this condition, all existing quasi-equilibrium must be the interior solution of equation 4.16 and solve it at $V^{USD}(S_{RoW}) = 0$.

We have proposed sufficient and necessary conditions that guarantee the existence of quasi-equilibrium, and now we prove the uniqueness of the quasi-equilibrium. We have equation C.4

$$V^{USD}(S_{RoW}) = \frac{B^{USD}_{RoW}}{B^{USD}_{RoW} + S_{RoW}} \left[\pi - r - \phi(1 - \mu_{US} - \mu_{RoW} S_{RoW})\right]$$

$$- \frac{B^{CNY}_{RoW}}{B^{CNY}_{RoW} + 1 - S_{RoW}} \left[\pi - r - \phi(\mu_{CN} + \mu_{RoW} S_{RoW})\right]$$

Setting the above equation equal to zero, and multiplying by $\frac{1}{\phi}(B^{USD}_{RoW} + S_{RoW})(B^{CNY}_{RoW} + 1 - S_{RoW})$, we have:

$$\frac{1}{\phi} B^{USD}_{RoW}(B^{CNY}_{RoW} + 1 - S_{RoW}) \left[\pi - r - \phi(1 - \mu_{US} - \mu_{RoW} S_{RoW})\right]$$

$$- \frac{1}{\phi} B^{CNY}_{RoW}(B^{USD}_{RoW} + S_{RoW}) \left[\pi - r - \phi(\mu_{CN} + \mu_{RoW} S_{RoW})\right] = 0$$

$$\Rightarrow \quad (\frac{1}{\phi}B^{USD}_{RoW}B^{CNY}_{RoW} + \frac{1}{\phi}B^{USD}_{RoW} - \frac{1}{\phi}B^{USD}_{RoW}S_{RoW})\,[\pi - r - \phi(1 - \mu_{US} - \mu_{RoW}S_{RoW})]$$

$$- (\frac{1}{\phi}B^{CNY}_{RoW}B^{USD}_{RoW} + \frac{1}{\phi}B^{CNY}_{RoW}S_{RoW})\,[\pi - r - \phi(\mu_{CN} + \mu_{RoW}S_{RoW})] = 0$$

$$\begin{aligned}
\Rightarrow \quad & \frac{1}{\phi}B^{USD}_{RoW}B^{CNY}_{RoW}\pi + \frac{1}{\phi}B^{USD}_{RoW}\pi - \frac{1}{\phi}B^{USD}_{RoW}S_{RoW}\pi \\
& - \frac{1}{\phi}B^{USD}_{RoW}B^{CNY}_{RoW}r - \frac{1}{\phi}B^{USD}_{RoW}r + \frac{1}{\phi}B^{USD}_{RoW}S_{RoW}r \\
& - B^{USD}_{RoW}B^{CNY}_{RoW} - B^{USD}_{RoW} + B^{USD}_{RoW}S_{RoW} \\
& + B^{USD}_{RoW}B^{CNY}_{RoW}\mu_{US} + B^{USD}_{RoW}\mu_{US} - B^{USD}_{RoW}\mu_{US}S_{RoW} \\
& + B^{USD}_{RoW}B^{CNY}_{RoW}\mu_{RoW}S_{RoW} + B^{USD}_{RoW}\mu_{RoW}S_{RoW} - B^{USD}_{RoW}\mu_{RoW}S^2_{RoW} \\
& - \frac{1}{\phi}B^{CNY}_{RoW}B^{USD}_{RoW}\pi - \frac{1}{\phi}B^{CNY}_{RoW}S_{RoW}\pi \\
& + \frac{1}{\phi}B^{CNY}_{RoW}B^{USD}_{RoW}r + \frac{1}{\phi}B^{CNY}_{RoW}S_{RoW}r \\
& + B^{CNY}_{RoW}B^{USD}_{RoW}\mu_{CN} + B^{CNY}_{RoW}S_{RoW}\mu_{CN} \\
& + B^{CNY}_{RoW}B^{USD}_{RoW}\mu_{RoW}S_{RoW} + B^{CNY}_{RoW}\mu_{RoW}S^2_{RoW} = 0
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & (B^{CNY}_{RoW} - B^{USD}_{RoW})\mu_{RoW}S^2_{RoW} \\
& + \Big(B^{CNY}_{RoW}B^{USD}_{RoW}\mu_{RoW} + B^{CNY}_{RoW}\mu_{CN} + \frac{1}{\phi}B^{CNY}_{RoW}r - \frac{1}{\phi}B^{CNY}_{RoW}\pi \\
& + B^{USD}_{RoW}\mu_{RoW} + B^{USD}_{RoW}B^{CNY}_{RoW}\mu_{RoW} - B^{USD}_{RoW}\mu_{US} + B^{USD}_{RoW} + \frac{1}{\phi}B^{USD}_{RoW}r - \frac{1}{\phi}B^{USD}_{RoW}\pi\Big)\,S_{RoW} \\
& + B^{CNY}_{RoW}B^{USD}_{RoW}\mu_{CN} + \frac{1}{\phi}B^{CNY}_{RoW}B^{USD}_{RoW}r - \frac{1}{\phi}B^{CNY}_{RoW}B^{USD}_{RoW}\pi + B^{USD}_{RoW}\mu_{US} + B^{USD}_{RoW}B^{CNY}_{RoW}\mu_{US} \\
& - B^{USD}_{RoW}B^{CNY}_{RoW} - B^{USD}_{RoW} - \frac{1}{\phi}B^{USD}_{RoW}B^{CNY}_{RoW}r - \frac{1}{\phi}B^{USD}_{RoW}r + \frac{1}{\phi}B^{USD}_{RoW}B^{CNY}_{RoW}\pi + \frac{1}{\phi}B^{USD}_{RoW}\pi
\end{aligned}$$

$$\begin{aligned}
\Rightarrow \quad & (B^{CNY}_{RoW} - B^{USD}_{RoW})\mu_{RoW}S^2_{RoW} \\
& + \Big(2B^{CNY}_{RoW}B^{USD}_{RoW}\mu_{RoW} + B^{CNY}_{RoW}\frac{1-\mu_{RoW}}{2} - \frac{\pi - r}{\phi}B^{CNY}_{RoW} \\
& + B^{USD}_{RoW}\mu_{RoW} - B^{USD}_{RoW}\frac{1-\mu_{RoW}}{2} + B^{USD}_{RoW} - \frac{\pi - r}{\phi}B^{USD}_{RoW}\Big)\,S_{RoW} \\
& + B^{CNY}_{RoW}B^{USD}_{RoW}(1 - \mu_{RoW}) + B^{USD}_{RoW}\frac{1-\mu_{RoW}}{2} \\
& - B^{USD}_{RoW}B^{CNY}_{RoW} - B^{USD}_{RoW} + \frac{\pi - r}{\phi}B^{USD}_{RoW}
\end{aligned}$$

Thus we get a quadratic polynomial function of $S_{RoW}$, after rearranging we get:

$$
\begin{aligned}
&(B_{RoW}^{CNY} - B_{RoW}^{USD})\mu_{RoW} S_{RoW}^2 \\
&+ \left[ B_{RoW}^{USD} \left( 0.5 + 1.5\mu_{RoW} - 2\frac{\pi - r}{\phi} \right) + B_{RoW}^{CNY} \left( 0.5 - 0.5\mu_{RoW} + 2B_{RoW}^{USD}\mu_{RoW} - \frac{\pi - r}{\phi} \right) \right] S_{RoW} \\
&+ B_{RoW}^{USD} \left[ \frac{\pi - r}{\phi} - \left( 0.5 + \mu_{RoW}(B_{RoW}^{CNY} + 1) \right) \right]
\end{aligned}
\tag{C.5}
$$

Evaluating equation C.5 at $S_{RoW} = 0$ we have:

$$
B_{RoW}^{USD} \left[ \frac{\pi - r}{\phi} - \left( 0.5 + \mu_{RoW}(B_{RoW}^{CNY} + 1) \right) \right]
$$

Since the condition that guarantees the existence of quasi-equilibrium of equation 4.16 is:

$$
\phi < \frac{\pi - r}{\bar{B}^{CNY} + \frac{\mu_{RoW}}{2} + \frac{1}{2}}
$$

Rearranging this we have:

$$
\bar{B}^{CNY} + 0.5\mu_{RoW} + 0.5 < \frac{\pi - r}{\phi}
$$

We also know that:

$$
\mu_{RoW}(B_{RoW}^{CNY} + 0.5) + 0.5 < \bar{B}^{CNY} + 0.5\mu_{RoW} + 0.5
$$

$$
\mu_{RoW}(B_{RoW}^{CNY} + 0.5) + 0.5 < \mu_{RoW}(B_{RoW}^{CNY} + 1) + 0.5
$$

Compare the value of:

$$
\bar{B}^{CNY} - \mu_{RoW} B_{RoW}^{CNY} - 0.5\mu_{RoW}
$$

When evaluating Evaluating equation C.5 at $S_{RoW} = 1$, we have:

$$
\begin{aligned}
& B_{RoW}^{CNY}\mu_{RoW} - B_{RoW}^{USD}\mu_{RoW} \\
& + B_{RoW}^{USD}\left(0.5 + 1.5\mu_{RoW} - 2\frac{\pi - r}{\phi}\right) + B_{RoW}^{CNY}\left(0.5 - 0.5\mu_{RoW} + 2B_{RoW}^{USD}\mu_{RoW} - \frac{\pi - r}{\phi}\right) \\
& + B_{RoW}^{USD}\left[\frac{\pi - r}{\phi} - \left(0.5 + \mu_{RoW}(B_{RoW}^{CNY} + 1)\right)\right]
\end{aligned}
$$

$$
\begin{aligned}
\Rightarrow \quad & B_{RoW}^{CNY}\mu_{RoW} - B_{RoW}^{USD}\mu_{RoW} \\
& + 0.5B_{RoW}^{USD} + 1.5B_{RoW}^{USD}\mu_{RoW} - 2B_{RoW}^{USD}\frac{\pi - r}{\phi} + 0.5B_{RoW}^{CNY} - 0.5\mu_{RoW}B_{RoW}^{CNY} + 2B_{RoW}^{USD}B_{RoW}^{CNY}\mu_{RoW} - \frac{\pi - r}{\phi}B_{RoW}^{CNY} \\
& + B_{RoW}^{USD}\frac{\pi - r}{\phi} - 0.5B_{RoW}^{USD} - \mu_{RoW}B_{RoW}^{CNY}B_{RoW}^{USD} - \mu_{RoW}B_{RoW}^{USD}
\end{aligned}
$$

$$
\Rightarrow \quad 0.5B_{RoW}^{CNY}\mu_{RoW} - 0.5B_{RoW}^{USD}\mu_{RoW} - B_{RoW}^{USD}\frac{\pi - r}{\phi} + 0.5B_{RoW}^{CNY} - B_{RoW}^{CNY}\frac{\pi - r}{\phi} + \mu_{RoW}B_{RoW}^{CNY}B_{RoW}^{USD}
$$

$$
\Rightarrow \quad B_{RoW}^{CNY}\left(0.5\mu_{RoW} + 0.5 - \frac{\pi - r}{\phi}\right) - B_{RoW}^{USD}\left(\frac{\pi - r}{\phi} - \mu_{RoW}\left(B_{RoW}^{CNY} - 0.5\right)\right)
$$

Since

$$
0.5\mu_{RoW} + 0.5 < \bar{B}^{CNY} + 0.5\mu_{RoW} + 0.5 < \frac{\pi - r}{\phi}
$$

And:

$$
\bar{B}^{CNY} + 0.5\mu_{RoW} + 0.5 - \mu_{RoW}B_{RoW}^{CNY} + 0.5\mu_{RoW}
$$

$$
\Rightarrow \quad \bar{B}^{CNY} - \mu_{RoW}B_{RoW}^{CNY} + \mu_{RoW} + 0.5 > 0
$$

Thus:

$$\mu_{RoW} B_{RoW}^{CNY} - 0.5\mu_{RoW} < \frac{\pi - r}{\phi}$$

Hence we find that the value of the polynomial function of $S_{RoW}$ is negative at $S_{RoW} = 1$.

The sign of quadratic function of $S_{RoW}$ are different at $S_{RoW} = 1$ and $S_{RoW} = 0$, thus we conclude that there is a unique quasi-equilibrium when $S_{RoW} \in (0, 1)$ that makes equation 4.16 equal to zero.

## C.3 Proof of Proposition 2

Using equations 4.11 and 4.12 respectively:

$$\frac{1}{\beta} = \frac{1}{Q^{USD} - \Delta_{US}^{USD}} = \frac{1}{(Q^{USD} - \Delta_{CN}^{USD})\,[1+\tau]} = \frac{1}{Q^{USD} - \Delta_{j}^{USD}}$$

$$\frac{1}{\beta} = \frac{1}{(Q^{CNY} - \Delta_{US}^{CNY})\,[1+\tau]} = \frac{1}{Q^{CNY} - \Delta_{CN}^{CNY}} = \frac{1}{(Q^{CNY} - \Delta_{j}^{CNY})\,[1+\tau]}$$

For the U.S. dollar-denominated asset, we have:

$$\Delta_{US}^{USD} = \Delta_{j}^{USD}$$

$$\Rightarrow \frac{S_{US}}{B_{US}^{USD} + S_{US}} r = \frac{S_j}{B_j^{USD} + S_j} r$$

$$\Rightarrow \frac{S_{US}}{B_{US}^{USD} + S_{US}} = \frac{S_j}{B_j^{USD} + S_j}$$

$$\Rightarrow B_{US}^{USD} = B_j^{USD} \frac{S_{US}}{S_j} \tag{C.6}$$

Using bond market clear conditions, we have:

$$\bar{B}^{USD} = \mu_{US} B_{US}^{USD} + \mu_{CN} B_{CN}^{USD} + \mu_{RoW} B_{RoW}^{USD}$$

Since $B_{CN}^{USD} = B_{US}^{CNY} = 0$, we know that:

$$\bar{B}^{USD} = \mu_{US} B_{US}^{USD} + \mu_{RoW} B_{RoW}^{USD} = \mu_{US} B_{US}^{USD} + \int_0^{\mu_{RoW}} B_i^{USD} di$$

Using equation C.6, we can get:

$$\bar{B}^{USD} = \mu_{US} B_{US}^{USD} + \int_0^{\mu_{RoW}} \frac{B_{US}^{USD}}{S_{US}} S_i di$$

$$\Rightarrow \bar{B}^{USD} = \mu_{US} B_j^{USD} \frac{S_{US}}{S_j} + \int_0^{\mu_{RoW}} \frac{B_{US}^{USD}}{S_{US}} S_i di$$

Since we know that: $S_{US} = 1$, and we pick a specific country $j$ and replace the $US$, thus we have:

$$\Rightarrow \bar{B}^{USD} = \mu_{US} \frac{B_j^{USD}}{S_j} + \frac{B_j^{USD}}{S_j} \int_0^{\mu_{RoW}} S_i di$$

$$\Rightarrow B_j^{USD} = \bar{B}^{USD} \frac{S_j}{\mu_{US} + \int_0^{\mu_{RoW}} S_i di}$$

For the renminbi-denominated asset, we have:

$$Q^{CNY} - \Delta_{CN}^{CNY} = (Q^{CNY} - \Delta_j^{CNY})[1 + \tau] = \beta$$

$$\Rightarrow \Delta_{CN}^{CNY} = \Delta_j^{CNY} - \frac{\beta\tau}{1+\tau}$$

$$\Rightarrow \frac{1 - S_{CN}}{B_{CN}^{CNY} + 1 - S_{CN}} r = \frac{1 - S_j}{B_j^{CNY} + 1 - S_j} r - \frac{\beta\tau}{1+\tau}$$

$$\Rightarrow \frac{1}{B_{CN}^{CNY} + 1} r = \frac{1 - S_j}{B_j^{CNY} + 1 - S_j} r - \frac{\beta\tau}{1+\tau}$$

$$\Rightarrow \frac{1}{B_{CN}^{CNY} + 1} = \frac{1 - S_j}{B_j^{CNY} + 1 - S_j} - \frac{\beta\tau}{(1+\tau)r}$$

Let:

$$\Omega = \frac{\beta\tau}{(1+\tau)r}$$

$$P_j = 1 - S_j$$

Thus we have:

$$\Rightarrow \frac{1}{B_{CN}^{CNY} + 1} = \frac{P_j}{B_j^{CNY} + P_j} - \Omega$$

$$\Rightarrow (B_{CN}^{CNY}+1)(B_j^{CNY}+P_j)\frac{1}{B_{CN}^{CNY}+1}=(B_{CN}^{CNY}+1)(B_j^{CNY}+P_j)\left[\frac{P_j}{B_j^{CNY}+P_j}-\Omega\right]$$

$$\Rightarrow B_j^{CNY}+P_j=(B_{CN}^{CNY}+1)P_j-\Omega(B_{CN}^{CNY}+1)(B_j^{CNY}+P_j)$$

$$\Rightarrow B_j^{CNY}+P_j=B_{CN}^{CNY}P_j+P_j-\Omega\left(B_{CN}^{CNY}B_j^{CNY}+B_{CN}^{CNY}P_j+B_j^{CNY}+P_j\right)$$

$$\Rightarrow B_j^{CNY}=B_{CN}^{CNY}P_j-\Omega B_{CN}^{CNY}B_j^{CNY}-\Omega B_{CN}^{CNY}P_j-\Omega B_j^{CNY}-\Omega P_j$$

For $B_j^{CNY}$, we have:

$$\Rightarrow B_j^{CNY}+\Omega B_{CN}^{CNY}B_j^{CNY}+\Omega B_j^{CNY}=B_{CN}^{CNY}P_j-\Omega B_{CN}^{CNY}P_j-\Omega P_j$$

$$\Rightarrow B_j^{CNY}(1+\Omega B_{CN}^{CNY}+\Omega)=B_{CN}^{CNY}P_j-\Omega B_{CN}^{CNY}P_j-\Omega P_j$$

$$\Rightarrow B_j^{CNY}=P_j\frac{B_{CN}^{CNY}-\Omega B_{CN}^{CNY}-\Omega}{1+\Omega B_{CN}^{CNY}+\Omega} \tag{C.7}$$

For $B_{CN}^{CNY}$, we have:

$$\Rightarrow B_{CN}^{CNY}P_j-\Omega B_{CN}^{CNY}P_j-\Omega B_{CN}^{CNY}B_j^{CNY}=B_j^{CNY}+\Omega B_j^{CNY}+\Omega P_j$$

$$\Rightarrow B_{CN}^{CNY}(P_j-\Omega P_j-\Omega B_j^{CNY})=B_j^{CNY}+\Omega B_j^{CNY}+\Omega P_j$$

$$\Rightarrow B_{CN}^{CNY}=\frac{B_j^{CNY}+\Omega B_j^{CNY}+\Omega P_j}{P_j-\Omega P_j-\Omega B_j^{CNY}} \tag{C.8}$$

Using bond market clear conditions, we have:

$$\bar{B}^{CNY}=\mu_{US}B_{US}^{CNY}+\mu_{CN}B_{CN}^{CNY}+\mu_{RoW}B_{RoW}^{CNY}$$

Since $B_{CN}^{USD}=B_{US}^{CNY}=0$, we know that:

$$\bar{B}^{CNY}=\mu_{CN}B_{CN}^{CNY}+\mu_{RoW}B_{RoW}^{CNY}=\mu_{CN}B_{CN}^{CNY}+\int_0^{\mu_{RoW}}B_i^{CNY}di$$

Using equations C.7 and C.8, we can get:

$$\bar{B}^{CNY} = \mu_{CN} B_{CN}^{CNY} + \mu_{RoW} B_{RoW}^{CNY} = \mu_{CN} B_{CN}^{CNY} + \frac{B_{CN}^{CNY} - \Omega B_{CN}^{CNY} - \Omega}{1 + \Omega B_{CN}^{CNY} + \Omega} \int_0^{\mu_{RoW}} P_i di$$

$$\Rightarrow \bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \frac{B_{CN}^{CNY} - \Omega B_{CN}^{CNY} - \Omega}{1 + \Omega B_{CN}^{CNY} + \Omega} \int_0^{\mu_{RoW}} P_i di$$

$$\Rightarrow \bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \frac{\frac{B_{CN}^{CNY}}{P_{CN}} - \Omega \frac{B_{CN}^{CNY}}{P_{CN}} - \Omega}{1 + \Omega \frac{B_{CN}^{CNY}}{P_{CN}} + \Omega} \int_0^{\mu_{RoW}} P_i di$$

Where $P_{CN} = 1$.

Picking a specific country $j$ and replacing the $CN$, we have:

$$\Rightarrow \bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \frac{\frac{B_j^{CNY}}{P_j} - \Omega \frac{B_j^{CNY}}{P_j} - \Omega}{1 + \Omega \frac{B_j^{CNY}}{P_j} + \Omega} \int_0^{\mu_{RoW}} P_i di$$

$$\Rightarrow \bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega P_j}{P_j + \Omega B_j^{CNY} + \Omega P_j} \int_0^{\mu_{RoW}} P_i di$$

Let $\Psi = \int_0^{\mu_{RoW}} P_i di$, thus we have:

$$\Rightarrow \bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \Psi \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega P_j}{P_j + \Omega B_j^{CNY} + \Omega P_j}$$

$$\Rightarrow \quad \bar{B}^{CNY} \left( P_j - \Omega P_j - \Omega B_j^{CNY} \right) \left( P_j + \Omega B_j^{CNY} + \Omega P_j \right) =$$

$$\mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} \left( P_j - \Omega P_j - \Omega B_j^{CNY} \right) \left( P_j + \Omega B_j^{CNY} + \Omega P_j \right)$$

$$+ \Psi \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega P_j}{P_j + \Omega B_j^{CNY} + \Omega P_j} \left( P_j - \Omega P_j - \Omega B_j^{CNY} \right) \left( P_j + \Omega B_j^{CNY} + \Omega P_j \right)$$

$$\Rightarrow \quad \bar{B}^{CNY}\left(P_j-\Omega P_j-\Omega B_j^{CNY}\right)\left(P_j+\Omega B_j^{CNY}+\Omega P_j\right)=$$

$$\mu_{CN}\left(B_j^{CNY}+\Omega B_j^{CNY}+\Omega P_j\right)\left(P_j+\Omega B_j^{CNY}+\Omega P_j\right)$$

$$+\Psi\left(B_j^{CNY}-\Omega B_j^{CNY}-\Omega P_j\right)\left(P_j-\Omega P_j-\Omega B_j^{CNY}\right)$$

$$\Rightarrow \quad \bar{B}^{CNY}\left[P_j^2+\Omega P_j B_j^{CNY}+\Omega P_j^2-\Omega P_j^2-\Omega^2 B_j^{CNY}P_j-\Omega^2 P_j^2-\Omega P_j B_j^{CNY}-\Omega^2(B_j^{CNY})^2-\Omega^2 P_j B_j^{CNY}\right]=$$

$$\mu_{CN}\left[P_j B_j^{CNY}+\Omega(B_j^{CNY})^2+\Omega P_j B_j^{CNY}+\Omega P_j B_j^{CNY}+\Omega^2(B_j^{CNY})^2+\Omega^2 P_j B_j^{CNY}+\Omega P_j^2+\Omega^2 P_j B_j^{CNY}+\Omega^2 P_j^2\right]$$

$$+\Psi\left(B_j^{CNY}P_j-\Omega B_j^{CNY}P_j-\Omega(B_j^{CNY})^2-\Omega B_j^{CNY}P_j+\Omega^2 B_j^{CNY}P_j+\Omega^2(B_j^{CNY})^2-\Omega P_j^2+\Omega^2 P_j^2+\Omega^2 B_j^{CNY}P_j\right)$$

$$\Rightarrow \quad \bar{B}^{CNY}\left[P_j^2-2\Omega^2 B_j^{CNY}P_j-\Omega^2 P_j^2-\Omega^2(B_j^{CNY})^2\right]=$$

$$\mu_{CN}\left[P_j B_j^{CNY}+\Omega(B_j^{CNY})^2+2\Omega P_j B_j^{CNY}+\Omega^2(B_j^{CNY})^2+2\Omega^2 P_j B_j^{CNY}+\Omega P_j^2+\Omega^2 P_j^2\right]$$

$$+\Psi\left[P_j B_j^{CNY}-\Omega(B_j^{CNY})^2-2\Omega P_j B_j^{CNY}+\Omega^2(B_j^{CNY})^2+2\Omega^2 P_j B_j^{CNY}-\Omega P_j^2+\Omega^2 P_j^2\right]$$

$$\Rightarrow \quad \bar{B}^{CNY}P_j^2-2\bar{B}^{CNY}\Omega^2 B_j^{CNY}P_j-\bar{B}^{CNY}\Omega^2 P_j^2-\bar{B}^{CNY}\Omega^2(B_j^{CNY})^2=$$

$$(\mu_{CN}+\Psi)P_j B_j^{CNY}+(\mu_{CN}-\Psi)\Omega(B_j^{CNY})^2+2(\mu_{CN}-\Psi)\Omega P_j B_j^{CNY}+(\mu_{CN}+\Psi)\Omega^2(B_j^{CNY})^2$$

$$+2(\mu_{CN}+\Psi)\Omega^2 P_j B_j^{CNY}+(\mu_{CN}-\Psi)\Omega P_j^2+(\mu_{CN}+\Psi)\Omega^2 P_j^2$$

$$\Rightarrow \quad \bar{B}^{CNY}\Omega^2(B_j^{CNY})^2 + \left[(\mu_{CN}-\Psi)\Omega(B_j^{CNY})^2 + (\mu_{CN}+\Psi)\Omega^2(B_j^{CNY})^2\right]$$

$$+ 2\bar{B}^{CNY}\Omega^2 P_j B_j^{CNY} + (\mu_{CN}+\Psi)P_j B_j^{CNY} + 2(\mu_{CN}-\Psi)\Omega P_j B_j^{CNY} + 2(\mu_{CN}+\Psi)\Omega^2 P_j B_j^{CNY}$$

$$+ \bar{B}^{CNY}\Omega^2 P_j^2 - \bar{B}^{CNY}P_j^2 + (\mu_{CN}-\Psi)\Omega P_j^2 + (\mu_{CN}+\Psi)\Omega^2 P_j^2 = 0$$

$$\Rightarrow \quad \left[\bar{B}^{CNY}\Omega^2 + (\mu_{CN}-\Psi)\Omega + (\mu_{CN}+\Psi)\Omega^2\right](B_j^{CNY})^2$$

$$+ \left[2\bar{B}^{CNY}\Omega^2 P_j + (\mu_{CN}+\Psi)P_j + 2(\mu_{CN}-\Psi)\Omega P_j + 2(\mu_{CN}+\Psi)\Omega^2 P_j\right] B_j^{CNY} \tag{C.9}$$

$$+ \bar{B}^{CNY}\Omega^2 P_j^2 - \bar{B}^{CNY}P_j^2 + (\mu_{CN}-\Psi)\Omega P_j^2 + (\mu_{CN}+\Psi)\Omega^2 P_j^2 = 0$$

Hence we reach a quadratic polynomial function of $B_j^{CNY}$ and $P_j$. And this function is explicitly expressed as:

$$B_j^{CNY} = \Gamma(\bar{B}^{CNY}, P_j)$$

$$\bar{B}^{CNY} = \mu_{CN}\frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega P_j}{P_j - \Omega P_j - \Omega B_j^{CNY}} + \int_0^{\mu_{RoW}} P_j dj \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega P_j}{P_j + \Omega B_j^{CNY} + \Omega P_j}$$

$$P_j = 1 - S_j$$

$$\Omega = \frac{\beta\tau}{(1+\tau)r}$$

Thus we have:

$$B_j^{CNY} = \Gamma(\bar{B}^{CNY}, 1 - S_j)$$

$$\bar{B}^{CNY} = \mu_{CN} \frac{B_j^{CNY} + \Omega B_j^{CNY} + \Omega(1 - S_j)}{(1 - S_j) - \Omega(1 - S_j) - \Omega B_j^{CNY}} + \int_0^{\mu_{RoW}} (1 - S_j) dj \frac{B_j^{CNY} - \Omega B_j^{CNY} - \Omega(1 - S_j)}{(1 - S_j) + \Omega B_j^{CNY} + \Omega(1 - S_j)}$$

$$\Omega = \frac{\beta\tau}{(1 + \tau)r} \tag{C.10}$$

To get the value of equilibrium renminbi-denominated bond holding, we apply numerical methods which firstly generate grids of $S_j$, and using the equation C.10 we can get the value of $B_j^{CNY}$.

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