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# An Asymptotic Framework for Fox's H-Fading Channel with Application to Diversity-Combining Receivers

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Abstract—Unified statistics are valuable in the performance analysis of communication systems. In this context, Fox's Hfunction has been shown to be eminently suitable for diverse scenarios. Another pivotal requirement is to have a low computational complexity, which is often hard to achieve for generalized models. Given this motivation, in this article we have presented high-power, low-complexity solutions for the outage probability (OP) and the average symbol error probability (SEP). Additionally, diversity techniques are harnessed for mitigating the effect of fading, which are then analysed based on the results derived. The entire methodology is governed by the origin probability density function rather than approximating the performance metrics under high-power. All the presented mathematical expressions are compared and validated through computer simulations (Monte-Carlo) to verify the accuracy of the proposed framework.

#### I. Introduction

Given the availability of generalised distributions like  $\alpha$ - $\mu$ ,  $\eta$ - $\mu$ ,  $\kappa$ - $\mu$ ,  $\alpha$ - $\eta$ - $\mu$ ,  $\alpha$ - $\kappa$ - $\mu$ , etc., the need for having unified statistics is increasing day by day. In this context, Fox's H-function has grown in popularity because of its versatile nature and elegant form in terms of representing most of the earlier proposed fading channels [1]. The distribution offers significant flexibility in characterizing numerous fading models [2, Table I]. The measurement and modeling campaigns have evidenced that the multipath fading of the V2V channel at 5 GHz and 5.2 GHz can be closely modelled by the generalized Fox's H-function distribution. The model encompasses the various other typical models of vehicular and D2D propagation scenarios such as Rayleigh, Nakagami-m, Fisher-F (at 5.8 GHz for D2D application), Weibull etc. [3], [4], [5].

Various researchers have proposed novel frame-works for analysing the performance of wireless systems relying on Fox's H-fading [1], [2], [6]–[10]. In this regard, Yilmaz *et al.* [6] has presented a methodology for deriving the closed-form

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expressions of the average BEP and average capacity for transmission over the Hyper Fox's H-fading model expressed in terms of Fox's H-function. The evaluation of the average BEP and average capacity expressions routinely utilize the MGF based approach for both single and multiple links. In [7], [8], the channel capacity is derived both with and without delay constraints for different adaptive schemes relying on optimal rate adaptation, on optimal power, on channel inversion, and on truncated channel inversion at a fixed rate over Fox's Hfading channel. Abo Rahama et al. [1] derived novel results for the PDF and CDF over the sum of Fox's H-function utilizing the MGF-based approach. They also utilized these results for evaluating the OP, average BEP, and ergodic capacity of wireless system. Physical layer statistics have been presented for transmission over Fox's H wiretap fading channel in [9], where the eavesdropper may be colluding or non-colluding. In case of a colluding scenario, the eavesdropper is equipped with MRC and SC schemes. The validity of the proposed formulation has been verified under different fading environments. The results presented in [9] subsumed the results presented in previous treatises. Furthermore, El Ayadi [2] and Chauhan et al. [10] derived cognitive radio performance metrics for transmission over Fox's H-fading under different propagation environments. In [1], [2], [6]–[10] the performance metrics relied either on univariate, bivariate, or multivariate Fox's H-functions. However, the earlier analytics may limit the operating range due to instabilities, while evaluating these expressions by numerical integration. Furthermore, they may fail to provide a visual interpretation of how the system reacts to time-variant changes in channel and system characteristics. Therefore, it is usually desired to find simpler expressions for the quantities of interest.

To address these concerns, the research community turned to asymptotic analysis at high SNR [11]–[14]. This approach results in computational simplicity and allows us to analyse the variations of statistics by relying on the average SNR. It also helps in comparing the performance of various modulation schemes. This approach has been frequently invoked [1], [2], [10]–[14]. Specifically, the asymptotic results derived for various metrics are discussed both for SISO and SIMO scenarios [1], [2], [10]. The high-power expressions of the average bit error or symbol error probabilities are evaluated in [1]. Similarly, the average probability of detection and average area under the receiver's operating characteristic curve are also derived in [1].

TABLE I NOMENCLATURE.

5G	Fifth Generation
A2G	Air-to-Ground
ALN	Additive Laplacian Noise
AWGN	Additive White Gaussian Noise
$\alpha$ - $\mu$ /G	$\alpha$ - $\mu$ /Gamma
BEP	Bit Error Probability
BFSK	Binary Frequency Shift Keying
BPSK	Binary Phase-Shift Keying
CDF	Cumulative Distribution Function
D2D	Device-to-Device
DSS	Double Shadowed Scenario
EGC	Equal Gain Combining
EGK	Extended Generalized- $K$
EG-G	Extended Generalized Gamma
F-S	Fisher-Snedecor
G2A	Ground-to-Air
GBK	Generalised Bessel-K
GK	Generalised- $K$
i.i.d.	Independent and Identically Distributed
i.n.i.d.	Independent and Non-Identically Distributed
LT	Laplace Transform
MED	Minimum Euclidian Distance
MGF	Moment-Generating Function
MIMO	Multiple-Input Multiple-Output
MRC	Maximum-Ratio Combining
OP	Outage Probability
PAM	Pulse Amplitude Modulation
PDF	Probability Density Function
PSK	Phase-Shift Keying
QoS	Quality-of-Service
QPSK	Quadrature Phase Shift Keying
RMS	Root-Mean-Square
Rx	Receiver
SAGIN	Space-Air-Ground Integrated Network
SC	Selection Combining
SEP	Symbol Error Probability
SIMO	Single-Input Multiple-Output
SISO	Single-Input Single-Output
SNR	Signal-to-Noise Ratio
SSS	Single Shadowed Scenario
Tx	Transmitter
V2V	Vehicle-to-Vehicle

The recent works on the asymptotic analysis over Fox Hfading channels have been listed in Table 1. Although, the asymptotic analysis of different system performance metrics was presented for Fox H-fading channels, most of the analysis was carried out for single-branch Rxs. Only [1] has considered the asymptotic analysis of Fox H-fading channels for MRC diversity, but the results derived are not in generic form. Explicitly, the average SEP expressions were found using Eq. (13) of [15], which is only applicable for BPSK and BFSK. The asymptotic outage and ergodic capacity expressions do not rely on simple functions, but due to the presence of multiple summation terms required for MRC diversity, substantial computation is required for their evaluation. Furthermore, it is noted from Table 1, that none of the previous contributions have analysed EGC and SC diversity for Fox H-fading channels. Although EGC is suboptimal, it benefits from simpler channel estimation, which only requires estimation of the channel phase, but not the channel amplitude [16]. Due to the complex circuitry of the MRC, the EGC is often preferred in 5G applications, including V2V and D2D communication. Hence the asymptotic analysis of Fox H-fading channel is the need of the hour. Furthermore, none of the previous works

TABLE II NOTATIONS

$\alpha$	Non-linearity parameter				
$\mu$	Number of multipath cluster				
$\gamma$	Instantaneous received SNR				
$ar{\gamma}$	Average received SNR				
$egin{array}{c} \gamma \ ar{\gamma} \ \hat{r} \ \xi \ \hat{r}_s \end{array}$	Fading figure of EG-G distribution				
ξ	Shaping factor of EG-G distribution				
$\hat{\hat{r}}_s$	Shadowing severity of EG-G distribu				
	tion				
$\xi_s$	Shadowing inhomogeneity of EG-G				
\$8	distribution				
$\Gamma(.)$	Gamma function				
$\hat{n}$	Fading severity of F-S F distribution				
$\hat{m}$	Shadowing severity of F-S F distribu-				
116	tion				
e î					
$n_G, m_G, \& \lambda$	Shaping factors of GBK distribution				
$\delta$	Path loss exponent				
$\stackrel{D}{}$	Tx and Rx separation				
$\hat{k}$	Shape parameter for Gamma distribu-				
	tion				
$\hat{\Omega}$	Average power of $\alpha$ - $\mu$ distribution				
$m_i, \{i=1,2\}$	Nakagami- <i>m</i> fading parameters				
$a_i, \{i = 1, 2\}$	Shaping parameter of Inverse Gamma				
, ,	distribution				
$\tilde{m}_i$ & $\tilde{m}_{si}$ , $\{i =$	Shape parameters of GK distribution				
1,2}	• •				
$P_t$	Transmitted power				
$\vec{E}$	Instantaneous propagation loss				
$N_0$	Noise power				
$\Omega_{si}, \{i=1,2\}$	Mean power of GK distribution				
$\tilde{\epsilon}$	Ratio of equivalent beam radius and				
•	pointing error displacement standard				
	deviation at the Rx				
r	Detection parameter for Gamma-				
,	Gamma distribution				
ς & π	Scintillation parameters of the atmo-				
, 66 00	spheric turbulence				
	spheric tarbatenee				
ILD -	average SNR for Gamma-Gamma dis-				
$\mu_{R_D}$	average SNR for Gamma-Gamma dis- tribution				
$\mu_{R_D}$	tribution				
L	tribution Number of independent parallel paths				
$L \ \gamma_r$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR				
L	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous re-				
$L \\ \gamma_r \\ M_{\gamma_r}(s)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR				
$L \ \gamma_r$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the out-				
$L \\ \gamma_r \\ M_{\gamma_r}(s) \\ \gamma_{MRC}$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector				
$L \\ \gamma_r \\ M_{\gamma_r}(s)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the out-				
$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector				
$L \\ \gamma_r \\ M_{\gamma_r}(s) \\ \gamma_{MRC}$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the Output One EGC detector Instantaneous received SNR at the Output One EGC detector Instantaneous Received SNR at the Output One EGC detector Instantaneous Receive				
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$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope				
$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$ $f_Y(\gamma)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope PDF of received SNR				
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$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$ $f_Y(\gamma)$ $F_{\gamma_r}(\gamma)$ $\gamma_{th}$ $P_{out}(.)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope PDF of received SNR $r^{th}$ branch CDF Threshold SNR Probability of outage				
$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$ $f_Y(\gamma)$ $F_{\gamma_r}(\gamma)$ $\gamma_{th}$ $P_{out}(.)$ $erfc(.)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope PDF of received SNR $r^{th}$ branch CDF Threshold SNR Probability of outage Complementary error function				
$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$ $f_Y(\gamma)$ $F_{\gamma_r}(\gamma)$ $\gamma_{th}$ $P_{out}(.)$ $erf_C(.)$ $P_e(.)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope PDF of received SNR $r^{th}$ branch CDF Threshold SNR Probability of outage Complementary error function Instantaneous SEP				
$L$ $\gamma_r$ $M_{\gamma_r}(s)$ $\gamma_{MRC}$ $\gamma_{EGC}$ $\gamma_{SC}$ $M_{X_r}(s)$ $f_Y(\gamma)$ $F_{\gamma_r}(\gamma)$ $\gamma_{th}$ $P_{out}(.)$ $erfc(.)$	tribution Number of independent parallel paths $r^{th}$ branch instantaneous received SNR MGF of $r^{th}$ branch instantaneous received SNR Instantaneous received SNR Instantaneous received SNR at the output of MRC detector Instantaneous received SNR at the output of EGC detector Instantaneous received SNR at the output of SC detector MGF of $r^{th}$ branch envelope PDF of received SNR $r^{th}$ branch CDF Threshold SNR Probability of outage Complementary error function				

have considered Laplacian noise in their analysis, which is essentially used for satisfying the differential privacy that prevents information leakage in secure applications, indoor, outdoor, free-space optics and undersea communication environments [17]–[21].

In an attempt to fill these knowledge gaps, we have carried out the asymptotic analysis of Fox-H fading channels for various diversity combining techniques. The application of the resultant formulas in deriving the generic expressions of the performance metrics has been demonstrated. We have considered both the ubiquitous AWGN and ALN. By contrast, the

Parameters	[1]	[2]	[9]	[10]	[22]	[23]	[24]	[25]	[26]	This work
i.i.d. Branches					<b>√</b>			<b>√</b>	<b>√</b>	<b>√</b>
i.n.i.d. Branches	<b>√</b>									<b>√</b>
MRC diversity	<b>√</b>							<b>√</b>	<b>√</b>	<b>√</b>
Outage probability	<b>√</b>				<b>√</b>		<b>√</b>			<b>√</b>
Average SEP	<b>√</b>						<b>√</b>		<b>√</b>	<b>√</b>
Effective capacity						<b>√</b>				
Ergodic capacity	<b>√</b>						<b>√</b>		<b>√</b>	
Physical Layer Security parameters			<b>√</b>		<b>√</b>					
Spectrum sensing parameters		<b>√</b>		<b>√</b>				<b>√</b>		
Diversity gain				<b>√</b>						✓
AWGN	<b>√</b>	✓								
Laplacian noise										✓
Origin PDF-based approach				<b>√</b>				<b>√</b>	<b>√</b>	✓
Function approximation under High SNR-based approach	<b>√</b>	<b>√</b>	<b>√</b>		<b>√</b>	<b>√</b>	<b>√</b>			
EGC diversity										✓
SC diversity										<b>√</b>

TABLE III
CONTRASTING OUR CONTRIBUTIONS TO THE LITERATURE OF FOX H-FADING CHANNELS

system considered here has multiple Rx antennas along with diverse combining schemes such as MRC, EGC, and SC. In this treatise, we have focused on a PDF-based approach. The main contributions of this work are summarised as follows:

- New expressions are derived for different diversity combining techniques, relying on both i.i.d and independent as well as on non-identically distributed branches.
- Explicitly, analytical expressions are derived for both the OP and for the average SEP. Furthermore, unified error probability statistics are obtained for both coherent and noncoherent schemes in AWGN scenarios. Additionally, the error probability of coherent modulation schemes is evaluated in the face of ALN.
- The final BER and OP expressions are articulated with the aid of computationally convenient asymptotes. Simulations are conducted to validate the accuracy of the results produced, which are accurate at high-SNRs.

The results derived here are beneficial as they (i) yield a unified mechanism for the average SEP and OP associated with multiple branches; (ii) serve as an efficient potential tool for analysing the system performance, when the channel characteristics are acquired by Fox's H module or in the analysis of special cases, which have hitherto not been taken into consideration. For easy referencing, all the acronyms and variables used throughout the work are defined in Table I and Table II, respectively.

The rest of the paper is organised as follows: In Section 2, the general Fox channel model is covered in detail, and a few unique Fox H-fading channel examples are taken into consideration. The methodology of handling different diversity schemes is elaborated on in Section 3. In Section 4, asymptotic results are provided for the OP and the average SEP under both AWGN and ALN scenarios. Section 5 verifies the accuracy of the results. Finally, Section 6 summarizes the paper.

## II. FOX'S H-FADING CHANNEL

In the literature, Fox's H representation is often used to characterize the channels in wireless propagation environments [1], [2], [10], because it subsumes well-known distributions modelling the practical propagation channels [6]. These channels

include the Rayleigh, Maxwell, Weibull, Gamma, Weibull, Nakagami-m, GK, EGK, EG-G, F-S-F, Nakagami-m/Gamma, scenarios as their special cases [2], [6], [9]. The model's practical utility can be found in a variety of applications, including vehicle-to-vehicle and keyhole MIMO systems [3], [27], G2A, and A2G unmanned aerial vehicle communications [28], millimeter-wave (i.e.  $\geq$  60 GHz) & free space optical communication [29], as well as D2D scenarios of indoor and outdoor environments [5]. Let the statistical variation of these real-world wireless communication environments follow Fox's H representation. Then the instantaneous power PDF is expressed as [2, equation (10)]

$$f_Y(\gamma) = \mathcal{R}H_{\hat{p},\hat{q}}^{\hat{m},\hat{n}} \left[ \mathcal{T}\gamma \Big|_{(f_j,F_j)_{j=1..\hat{q}}}^{(e_j,E_j)_{j=1..\hat{q}}} \right], \ \gamma > 0,$$
 (1)

where the parameters  $\hat{m}$ ,  $\hat{n}$ ,  $\hat{p}$ ,  $\hat{q}$  posses positive integer values, and follow  $0 \le \hat{m} \le \hat{q}$ ,  $0 \le \hat{n} \le \hat{p}$ . The constants  $\mathcal{R}$  and  $\mathcal{T}$  in (1) are adjusted to satisfy that  $\int_{0}^{\infty} f_{Y}(\gamma)d\gamma=1$ . Equation (1) can alternatively be written in integral form as [30, equations. (1.2) & (1.3)]

$$f_{Y}(\gamma) = \frac{\mathcal{R}}{2\pi i} \int_{\mathcal{L}} \Theta_{\hat{p},\hat{q}}^{\hat{m},\hat{n}} \left[ s \Big|_{(f_{j},E_{j})_{j=1..\hat{p}}}^{(e_{j},E_{j})_{j=1..\hat{p}}} \right] (\mathcal{T}\gamma)^{-s} ds,$$

$$= \frac{\mathcal{R}}{2\pi i} \int_{\mathcal{L}} (\mathcal{T}\gamma)^{-s}$$

$$\times \frac{\prod_{j=1}^{\hat{m}} \Gamma(f_{j} + F_{j}S) \prod_{j=1}^{\hat{n}} \Gamma(1 - e_{j} - E_{j}S)}{\prod_{j=\hat{m}+1}^{\hat{p}} \Gamma(e_{j} + E_{j}S) \prod_{j=\hat{m}+1}^{\hat{q}} \Gamma(1 - f_{j} - F_{j}S)} ds,$$

$$(2)$$

where  $E_j>0$  for  $j=1,...\hat{p}$  and  $F_j>0$  for  $j=1,...\hat{q}$  and the path of integration will be governed by the function's parameters. The  $f_Y(\gamma)$  expressions are illustrated in Table IV for the Maxwell,  $\alpha$ - $\mu$ , EG-G [6], F-S- $\mathcal{F}$  [9], GBK,  $\alpha$ - $\mu$ /G, SSS, DSS, cascaded G-K, Gamma-Gamma, N\*Rayleigh, N\*Nakagami-m, N\*GK, N\*Fisher, and N\* $\alpha$ - $\mathcal{F}$  models for ground-to-ground and A2G channels. The values of  $\mathcal{R}$  and  $\mathcal{T}>0$  from the table may be determined for different fading

models. Explicitly, in Table IV, we can see some special cases of Fox's H-fading models, which have already been introduced in the literature, such as the Maxwell,  $\alpha$ - $\mu$  [9, Table I], E G-G [6, Table II], F-S  $\mathcal{F}$  [9, Table I], and N\* $\alpha$ - $\mathcal{F}$  [31] distributions. But others, such as, GBK is deduced by utilizing [32, equation (8.4.23.1)] and [30, equations (1.59-1.60)] on [33, equation (1)]. Fox's H-function is derived for  $\alpha$ - $\mu$ /G distributions by following [30, equation (1.60)] and [34, equation (7)], for DSS from [36, equation (7)], for SSS from [36, equation (10)], for cascaded G-K from [35, equation (18)], while for N\*Rayleigh from [38], for N\*Nakagami-*m* from [39], for N\*GK from [40], and for N\*Fisher distributions from [41], which are derived with the aid of [32, equation (8.3.2.21)] and then [30, Property 1.3 & 1.5]. Finally, Fox's H for Gamma-Gamma distribution [37] is deduced with the aid of [32, equation (8.3.2.21)] and [30, Property 1.4, 1.3, & 1.5].

#### III. ASYMPTOTIC ANALYSIS

Asymptotic analysis has gained special attention during the past few years in system optimization [11]. In the literature, different methodologies are proposed for analysing the system performance attained at both low and high power, as detailed below:

In [11], Zhang *et al.* proposed a new asymptotic analysis framework, in which the PDF is first approximated around the origin and then different performance metrics are evaluated. This approach is well suited for most distributions, but it faces difficulties in diversity analysis associated with distributions having summation terms, as exemplified by the Mixture-Gamma [42], Weibull/Log-normal [43], and Mixture-Inverse Gaussian [44], [45] distributions. Peppas *et al.* [46] proposed asymptotic solutions for cognitive radio scenarios by approximating the results at high power, but their solutions exhibited limitations both when the PDF has a complex mathematical structure, and for spatial diversity techniques.

Additionally, asymptotic frameworks are capable of reducing the analytical complexity of diversity reception. The conventional approaches rely on the PDF at the origin, followed by averaging and using the AWGN and ALN contamination to derive the asymptotic expressions of the OP, average SEP, and average probability of detection. As a benefit, Fox's H-fading model will resolve the issues stated above. When  $Z \to 0$  the expression given by (1), will have the structure given as [47, Corollary 3]

$$\mathcal{R}H_{\hat{p},\hat{q}}^{\hat{m},\hat{n}}\left[\mathcal{T}Z\right]_{Z\to 0} = \mathcal{R}h_{j}(\mathcal{T}Z)^{\Omega},\tag{3}$$

where  $\Omega=min_{1\leq j\leq \hat{m}}\left[\frac{Re(f_j)}{F_j}\right]$  and  $h_j$  is evaluated using [47, Corollary 2] and is defined as

$$h_{j} = \frac{\prod_{i=1, i \neq j}^{\hat{m}} \Gamma\left(f_{i} - \frac{f_{j}F_{i}}{F_{j}}\right) \prod_{i=1}^{\hat{n}} \Gamma\left(1 - e_{i} + \frac{f_{j}E_{i}}{F_{j}}\right)}{F_{j} \prod_{i=\hat{n}+1}^{\hat{p}} \Gamma\left(e_{i} - \frac{f_{j}E_{i}}{F_{j}}\right) \prod_{i=\hat{m}+1}^{\hat{q}} \Gamma\left(1 - f_{i} - \frac{f_{j}F_{i}}{F_{j}}\right)},$$
(4)

where  $h_j$  is listed in Table V for various Fox's H-fading channels. In the cases, where  $\hat{m}>1$ ,  $h_j$  has numerous values, which hence requires the selection of the most appropriate

parameters. The CDF of the expression given by (3) is evaluated by employing the identity  $F_Y(\gamma) = \int\limits_0^\gamma f_Y(\gamma) d\gamma$  and it is given by

$$F_Y(\gamma) = \frac{\mathcal{R}h_j \mathcal{T}^{\Omega}}{\Omega + 1} \gamma^{\Omega + 1}.$$
 (5)

# A. Diversity Analysis of Fox's H-Distribution

In a communication system, the spurious effect of fading is often compensated by the inclusion of multiple antennas at the Tx or at the Rx or both, so that the QoS can be maintained. Various signal combining techniques are utilised in the open literature to improve the QoS. Among them, MRC, EGC, and SC are the most popular diversity schemes. The most complex MRC yields the best improvements, followed by EGC and SC. In this subsection, we highlight the PDF expressions of the above diversity schemes.

1) MRC diversity PDF: The MRC scheme combines the signal received from independent paths, which are first cophased, then weighted by a corresponding weighting factor inversely proportional to their SNR, and finally added improve the instantaneous SNR of the combiner output. The MRC output may then be expressed as [48]

$$\gamma_{MRC} = \sum_{r=1}^{L} \gamma_r. \tag{6}$$

Taking the LT of (6), yields

$$M_{\gamma_{MRC}}(s) = \prod_{r=1}^{L} M_{\gamma_r}(s). \tag{7}$$

The MGF for the individual branch SNR is calculated by taking the LT of (3) with the aid of [49, Eq. 3.381.4], as given by

$$M_{\gamma}(s) = \mathcal{R}h_{j}\mathcal{T}^{\Omega}\frac{\Gamma(\Omega+1)}{s^{\Omega+1}}.$$
 (8)

Substituting (8) into (7) and after some further algebraic manipulations, we arrive at:

$$M_{\gamma_{MRC}}(s) = \left\{ \prod_{r=1}^{L} \mathcal{R}_r h_j \mathcal{T}^{\Omega_r} \Gamma(\Omega_r + 1) \right\} \frac{1}{\left(\sum_{s=1}^{L} \Omega_r\right) + L}.$$
(9)

Upon multiplying the numerator and denominator of (9) by  $\Gamma\left(\left(\sum_{r=1}^L\Omega_r\right)+L\right)$  and then taking the inverse LT, we get the desired origin PDF expression of

$$f_{\gamma_{MRC}}^{inid}(\gamma) = \mathcal{K}^{MRC} \gamma^{\Psi^{MRC} - 1}, \tag{10}$$

where the parameters  $\mathcal{K}^{MRC}$  and  $\Psi^{MRC}$  are defined in Table VI. Upon assuming L i.i.d. branches, and  $\bar{\gamma}_1 = \bar{\gamma}_2 = ..... = \bar{\gamma}_L = \bar{\gamma}$ , (7) may be reformulated as

$$M_{\gamma_{MRC}}^{iid}(s) = \left[M_{\gamma_r}(s)\right]^L. \tag{11}$$

Upon substituting (8) into (11) and then employing the inverse LT, one obtains the origin PDF under the i.i.d. case associated with the same structure as given in (10). The respective parameters are illustrated in Table VI. In Fig. 1 the Fisher-Snedecor F origin power PDF is compared to the exact power

 $\begin{tabular}{ll} TABLE~IV\\ Unique~distribution~examples~of~Fox's~H-function. \end{tabular}$ 

Channel	$f_Y(\gamma)$
Maxwell [2, Table I]	$f_Y(\gamma) = \frac{3}{\sqrt{\pi}\tilde{\gamma}} H_{0,1}^{1,0} \begin{bmatrix} \frac{3\gamma}{2\tilde{\gamma}} \Big _{(\frac{1}{2};1)} \end{bmatrix}$
$\alpha$ -μ [9, Table I]	$f_{Y}(\gamma) = \frac{3}{\sqrt{\pi \bar{\gamma}}} H_{0,1}^{1,0} \begin{bmatrix} \frac{3\gamma}{2\bar{\gamma}} \Big _{(\frac{1}{2};1)} \end{bmatrix}$ $f_{Y}(\gamma) = \frac{\mu^{\frac{2}{\alpha}}}{\Gamma(\mu)\bar{\gamma}} H_{0,1}^{1,0} \begin{bmatrix} \frac{\mu^{\frac{2}{\alpha}}\gamma}{\bar{\gamma}} \Big _{(\mu-\frac{2}{\alpha};\frac{2}{\alpha})} \end{bmatrix},  \alpha > 0 \& \mu > 0$
EG-G [6, Table II]	$f_{Y}(\gamma) = \frac{\beta \beta_{s}}{\Gamma(\hat{r})\Gamma(\hat{r}_{s})\bar{\gamma}} H_{0,2}^{2,0} \begin{bmatrix} -\frac{\beta \beta_{s}}{\bar{\gamma}} \gamma \Big _{(\hat{r} - \frac{1}{\xi}, \frac{1}{\xi}), (\hat{r}_{s} - \frac{1}{\xi_{s}}, \frac{1}{\xi_{s}})} \end{bmatrix},  \beta = \Gamma(\hat{r} + 1/\xi)/\Gamma(\hat{r}) \& \beta_{s} = \Gamma(\hat{r}_{s} + 1/\xi)/\Gamma(\hat{r}_{s} + 1/$
	$1/\xi_s)/\Gamma(\hat{r}_s)$
F-S $\mathcal{F}$ [9, Table I]	$f_Y(\gamma) = \frac{\hat{n}}{\hat{m}\Gamma(\hat{n})\Gamma(\hat{m})\bar{\gamma}} H_{1,1}^{1,1} \begin{bmatrix} \hat{n}\gamma \\ \hat{m}\bar{\gamma} \end{bmatrix} \begin{pmatrix} -(\hat{m},1) \\ \hat{m}\bar{\gamma} \end{pmatrix} \begin{pmatrix} (\hat{n},1) \\ (\hat{n}-1,1) \end{pmatrix}$
GBK [33]	$f_Y(\gamma) = \frac{1}{\Gamma(m_G)\Gamma(n_G)\Xi} H_{0,2}^{2,0} \begin{bmatrix} \gamma & - & - \\ \Xi \Big _{(m_G - \frac{1}{\lambda}; \frac{1}{\lambda}) \ (n_G - \frac{1}{\lambda}; \frac{1}{\lambda})} \end{bmatrix},  \Xi = \frac{\bar{\gamma}\Gamma(m_G)\Gamma(n_G)}{\Gamma(m_G + 1/\hat{\lambda})\Gamma(n_G + 1/\hat{\lambda})}$
$\alpha$ - $\mu$ /G [34]	$1 \cdot \left[ \frac{1}{2} p \delta \right]$
	$f_Y(\gamma) = \frac{\mu^{\frac{1}{\alpha}} D^{\delta}}{\Gamma(\mu)\Gamma(\hat{k})v\hat{\Omega}} H_{0,2}^{2,0} \left[ \frac{\mu^{\alpha} D^{\delta} \gamma}{v\hat{\Omega}} \Big _{(\mu - \frac{1}{\alpha}, 1)(\hat{k} - 1, 1)} \right], 2 < \delta < 6$
SSS [36]	$f_{Y}(\gamma) = \frac{\mu^{\frac{1}{\alpha}}D^{\delta}}{\Gamma(\mu)\Gamma(\hat{k})v\hat{\Omega}} H_{0,2}^{2,0} \left[ \frac{\mu^{\frac{1}{\alpha}}D^{\delta}\gamma}{v\hat{\Omega}} \Big _{(\mu - \frac{1}{\alpha}, 1)(\hat{k} - 1, 1)} \right], 2 < \delta < 6$ $f_{Y}(\gamma) = \frac{A_{1}m_{1}m_{2}}{\hat{\gamma}} H_{1,2}^{2,1} \left[ \frac{m_{1}m_{2}\gamma}{\hat{\gamma}} \Big _{(m_{1} - 1, 1)(m_{2} - 1, 1)}^{(-a_{1}, 1)} \right],  A_{1} = \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(a_{1})}$ $f_{Y}(\gamma) = \frac{A_{2}m_{1}m_{2}}{\hat{\gamma}} H_{2,2}^{2,2} \left[ \frac{m_{1}m_{2}\gamma}{\hat{\gamma}} \Big _{(m_{1} - 1, 1)(m_{2} - 1, 1)}^{(-a_{2}, 1)(-a_{1}, 1)} \right],  A_{2} = \frac{1}{\Gamma(m_{1})\Gamma(m_{2})\Gamma(a_{1})\Gamma(a_{2})}$ $f_{Y}(\gamma) = A_{3}A_{4}H_{0,4}^{4,0} \left[ A_{4}\gamma \Big _{(\tilde{m}_{s_{1}} - 1, 1)(\tilde{m}_{1} - 1)(\tilde{m}_{s_{2}} - 1, 1)(\tilde{m}_{2} - 1)}^{(\tilde{m}_{s_{1}} - 1, 1)(\tilde{m}_{s_{2}} - 1, 1)(\tilde{m}_{s_{2}} - 1)(\tilde{m}_{s_{2}} - 1, 1)(\tilde{m}_{s_{2}} - $
DSS [36]	$f_Y(\gamma) = \frac{A_2 m_1 m_2}{\bar{\gamma}} H_{2,2}^{2,2} \left[ \frac{m_1 m_2 \gamma}{\bar{\gamma}} \Big _{(m_1 - 1, 1)(m_2 - 1, 1)}^{(-a_2, 1)(-a_1, 1)} \right],  A_2 = \frac{1}{\Gamma(m_1) \Gamma(m_2) \Gamma(a_1) \Gamma(a_2)}$
Cascaded G-K [35]	$f_Y(\gamma) = \mathcal{A}_3 \mathcal{A}_4 H_{0,4}^{4,0} \left[ \mathcal{A}_4 \gamma \Big _{(\tilde{m}_{s1} - 1, 1)(\tilde{m}_1 - 1)(\tilde{m}_{s2} - 1, 1)(\tilde{m}_2 - 1)}^{} \right],  \mathcal{A}_3 = \frac{1}{\Gamma(\tilde{m}_1)\Gamma(\tilde{m}_2)\Gamma(\tilde{m}_{s1})\Gamma(\tilde{m}_{s2})}$
	$\mathcal{A}_4 = \frac{m_1 m_2 m_{s1} m_{s2}}{\tilde{\gamma}}$ & $\bar{\gamma} = \frac{P_t E}{N_c} \Omega_{s1} \Omega_{s2}$
Gamma-Gamma [37]	$f_Y(\gamma) = \frac{(\varsigma \varpi)^r \tilde{\xi}^2}{\mu_{R_D} \Gamma(\varsigma) \Gamma(\varpi)} H_{1,3}^{3,0} \left[ \frac{(\varsigma \varpi)^r}{\mu_{R_D}} \gamma \Big _{(\tilde{\xi}^2 - r, r)(\varsigma - r, r)(\varpi - r, r)}^{(\tilde{\xi}^2 + 1 - r, r)} \right]$
N*Rayleigh [38]	$f_Y(\gamma) = \frac{1}{\bar{\gamma}} H_{0,N}^{N,0} \begin{bmatrix} \gamma & - \\ \bar{\gamma} &   \\ (0,1)(0,1) (0,1) \end{bmatrix}$
N*Nakagami-m [39]	$f_Y(\gamma) = \frac{1}{\bar{\gamma}} H_{0,N}^{N,0} \begin{bmatrix} \frac{\gamma}{\bar{\gamma}} \prod_{i=1}^{N} \Gamma(m_i) \Big _{(m_1-1,1)(m_2-1,1)-(m_N-1,1)} \\ \end{bmatrix}$
N*GK [40]	$f_{Y}(\gamma) = \frac{\prod\limits_{i=1}^{N} \tilde{m}_{i}\tilde{m}_{si}}{\bar{\gamma}\prod\limits_{i=1}^{N} \Gamma(\tilde{m}_{i})\Gamma(\tilde{m}_{si})} H_{0,2N}^{2N,0} \left[\prod\limits_{i=1}^{N} \frac{\Gamma(\tilde{m}_{i})\Gamma(\tilde{m}_{si})\gamma}{\bar{\gamma}}\Big _{(\tilde{m}_{1}-1,1)(\tilde{m}_{s1}-1,1)(\tilde{m}_{N}-1,1)(\tilde{m}_{sN-1},1)}\right]$
N*Fisher [41]	$f_Y(\gamma) = \frac{1}{\bar{\gamma} \prod_{i=1}^{N} \Gamma(\tilde{n}_i) \Gamma(\hat{m}_i - 1)} H_{N,N}^{N,N} \left[ \frac{\gamma}{\bar{\gamma}} \prod_{i=1}^{N} \frac{\Gamma(\hat{n}_i)}{\Gamma(\hat{m}_i - 1)} \Big _{(\hat{n}_1 - 1, 1)(\hat{n}_2 - 1, 1) - \dots - (\hat{n}_N - 1, 1)}^{(\hat{n}_1, 1)(\hat{n}_2 - 1, 1) - \dots - (\hat{n}_N - 1, 1)} \right]$
N*α-F [31]	$f_Y(\gamma) = \frac{1}{\prod\limits_{i=1}^N \Theta_i^{\frac{2}{\alpha_i}} \bar{\gamma}_i \Gamma(\mu_i) \Gamma(\hat{m}_i)} H_{N,N}^{N,N} \begin{bmatrix} \frac{\gamma}{N} \frac{2}{\Theta_i^{\alpha_i}} \bar{\gamma}_i \\ \prod\limits_{i=1}^N \Theta_i^{\frac{2}{\alpha_i}} \bar{\gamma}_i \end{bmatrix}_{\Upsilon_1, \Upsilon_2,\Upsilon_N}^{\Phi_1, \Phi_2,\Phi_N}, \Phi_i = (1 - \hat{m}_i - 2/\alpha_i, 2/\alpha_i)$
	$\Theta_i = rac{\hat{m}_i - 1}{\mu_i}$ and $\Upsilon_i = (\mu_i - 2/lpha_i, 2/lpha_i)$

PDF for different fading parameters. It is observed that the plots follow the exact PDF for a wide range of instantaneous SNRs. The multipath parameter n determines the slope of the power PDF at  $\gamma \to 0$ . However, as the shadowing parameter  $m_s$  and the average SNR are changed, there is a parallel shift in the figure.

2) EGC diversity PDF: Similar to MRC, the signals gleaned from L paths are co-phased, weighted with a factor of unity and then are summed to get the combiner's output SNR as given below

$$\gamma_{EGC} = \left(\frac{1}{\sqrt{L}} \sum_{r=1}^{L} \sqrt{\gamma_r}\right)^2. \tag{12}$$

Taking the square root of both sides, yields

$$X_{EGC} = \sum_{r=1}^{L} \frac{X_r}{\sqrt{L}}.$$
 (13)

Similarly to MRC, the MGF is expressed as

$$M_{X_{EGC}}(s) = \prod_{r=1}^{L} M_{X_r} \left(\frac{s}{\sqrt{L}}\right). \tag{14}$$

The envelope PDF for (3) is deduced by using the relationship of  $P_Y(\gamma) = \frac{P_R\left(\sqrt{\frac{\hat{r}^2\gamma}{\hat{\gamma}}}\right)}{2\sqrt{\frac{\hat{r}^2\gamma}{\hat{r}^2}}}$  [50, Eq. 2.3], yielding

$$f_X(x) = \frac{2\mathcal{R}h_j \mathcal{T}^{\Omega} \bar{\gamma}^{\Omega+1}}{\hat{x}^{2\Omega+2}} x^{2\Omega+1}.$$
 (15)

Taking the LT of (15) results in

$$M_X\left(\frac{s}{\sqrt{L}}\right) = \frac{2\mathcal{R}h_j \mathcal{T}^{\Omega} \bar{\gamma}^{\Omega+1} \Gamma(2\Omega+2)}{\hat{x}^{2\Omega+2} \left(\frac{s}{\sqrt{L}}\right)^{2\Omega+2}}.$$
 (16)

Channel	$\hat{m}$	$h_j$				
Maxwell	1	$h_1 = 1$				
$\alpha - \mu$	1	$h_1 = \frac{\alpha}{2}$				
E G-G	2	$h_1 = \xi \Gamma\left(\frac{\hat{r}_s \xi_s - \hat{r}\xi}{\xi_s}\right), h_2 = \xi_s \Gamma\left(\frac{\hat{r}\xi - \hat{r}_s \xi_s}{\xi}\right)$				
F-S $\mathcal{F}$	1	$h_1$ = $\Gamma(\hat{n}+\hat{m})$				
GBK	2	$h_1$ = $\hat{\lambda}\Gamma(n_G-m_G)$ , $h_2$ = $\hat{\lambda}\Gamma(m_G-n_G)$				
α-μ/G	2	$h_1$ = $\Gamma\left(\hat{k}-\mu+\frac{1}{\alpha}-1\right)$ ), $h_2$ = $\Gamma\left(\mu-\hat{k}-\frac{1}{\alpha}+1\right)$ )				
SSS	2	$h_1 = \Gamma(m_2 - m_1)\Gamma(a_1 + m_1), h_2 = \Gamma(m_1 - m_2)\Gamma(a_1 + m_2)$				
DSS	2	$h_1 = \Gamma(m_2 - m_1)\Gamma(a_2 + m_1)\Gamma(a_1 + m_1), \ h_2 = \Gamma(m_1 - m_2)\Gamma(a_2 + m_2)\Gamma(a_1 + m_2)$				
Cascaded G-K	4	$\begin{array}{lll} h_1 = & \Gamma(\tilde{m}_1 - \tilde{m}_{s1}) \Gamma(\tilde{m}_{s2} - \tilde{m}_{s1}) \Gamma(\tilde{m}_2 - \tilde{m}_{s1}), & h_2 = & \Gamma(\tilde{m}_{s1} - \tilde{m}_1) \Gamma(\tilde{m}_{s2} - \tilde{m}_1) \Gamma(\tilde{m}_2 - \tilde{m}_1), & h_3 = & \Gamma(\tilde{m}_{s1} - \tilde{m}_{s2}) \Gamma(\tilde{m}_1 - \tilde{m}_{s2}) \Gamma(\tilde{m}_2 - \tilde{m}_{s2}), & h_4 = & \Gamma(\tilde{m}_{s1} - \tilde{m}_2) \Gamma(\tilde{m}_1 - \tilde{m}_2) \Gamma(\tilde{m}_{s2} - \tilde{m}_2) \end{array}$				
Gamma-Gamma	3	$h_1 = \Gamma(\varsigma - \tilde{\xi}^2)\Gamma(\varpi - \tilde{\xi}^2), \ h_2 = \frac{\Gamma(\tilde{\xi}^2 - \varsigma)\Gamma(\varpi - \varsigma)}{\Gamma(\tilde{\xi}^2 + 1 - \varsigma)}, \ h_3 = \frac{\Gamma(\tilde{\xi}^2 - \varpi)\Gamma(\varsigma - \varpi)}{\Gamma(\tilde{\xi}^2 + 1 - \varpi)}$				
N-Rayleigh	N	$h_i$ =1; $1 \le j \le N$				
N-Nakagami- $m$	N	$h_i = \prod_{i=1, i \neq j}^{N} \Gamma(m_i - m_j); \qquad \qquad 1 \leq j \leq N$				
N-GK	2N	$\begin{array}{l} h_1 = \Gamma(m_1 \ - \ \tilde{m}_{s1}) \Gamma(m_2 \ - \ \tilde{m}_{s1}) \Gamma(\tilde{m}_{s2} \ - \ \tilde{m}_{s1}) \Gamma(\tilde{m}_{sN} \ - \ \tilde{m}_{s1}) \Gamma(m_N \ - \ \tilde{m}_{s1}); \\ h_2 = \Gamma(\tilde{m}_{s1} \ - \ m_1) \Gamma(m_2 \ - \ m_1) \Gamma(\tilde{m}_{s2} \ - \ m_1) \Gamma(\tilde{m}_{sN} \ - \ m_1) \Gamma(m_N \ - \ m_1); \\ h_3 = \Gamma(\tilde{m}_{s1} \ - \ \tilde{m}_{s2}) \Gamma(m_1 \ - \ \tilde{m}_{s2}) \Gamma(m_2 \ - \ \tilde{m}_{s2}) \Gamma(\tilde{m}_{sN} \ - \ \tilde{m}_{s2}) \Gamma(m_N \ - \ \tilde{m}_{s2}); \\ h_{2N-1} = \Gamma(\tilde{m}_{s1} \ - \ \tilde{m}_{sN}) \Gamma(m_1 \ - \ \tilde{m}_{sN}) \Gamma(\tilde{m}_{s2} \ - \ \tilde{m}_{sN}) \Gamma(m_{N-1} \ - \ \tilde{m}_{sN}) \Gamma(m_N \ - \ \tilde{m}_{sN}); \\ h_{2N} = \Gamma(\tilde{m}_{s1} \ - \ m_N) \Gamma(\tilde{m}_{s1} \ - \ m_N) \Gamma(\tilde{m}_{s2} \ - \ m_N) \Gamma(m_{N-1} \ - \ m_N) \Gamma(\tilde{m}_{sN} \ - \ m_N) \Gamma(\tilde{m}$				
N-Fisher	N	$h_j = \prod_{i=1, i \neq j}^{N} \Gamma(\hat{n}_i - \hat{n}_j) \prod_{i=1}^{N} \Gamma(\hat{n}_j - \hat{m}_i); \qquad 1 \leq j \leq N$				
$N$ - $lpha$ - $\mathcal{F}$	N	$h_j = \prod_{i=1, i \neq j}^{N} \Gamma(\Upsilon_i - \Upsilon_j) \prod_{i=1}^{N} \Gamma(\Upsilon_j - \Phi_i); \qquad 1 \leq j \leq N$				

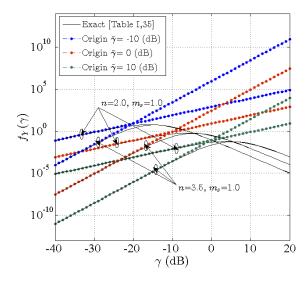


Fig. 1. Fisher-Snedecor F distribution's origin and exact PDF.

Substituting (16) into (14) and carrying out some further algebraic manipulations yields

$$M_{X_{EGC}}(s) = \left\{ \prod_{r=1}^{L} \left( \frac{2h_j \mathcal{R}_r \mathcal{T}^{\Omega_r} \bar{\gamma_r}^{\Omega+1} \Gamma(2\Omega_r + 2)}{\hat{x}^{2\Omega_r + 2}} \right) \right\}$$

$$\times \frac{L^{\sum_{r=1}^{L} (\Omega_r + 1)} \Gamma\left(\sum_{r=1}^{L} (2\Omega_r + 2)\right)}{\Gamma\left(\sum_{r=1}^{L} (2\Omega_r + 2)\right) s_{r=1}^{\sum_{r=1}^{L} (2\Omega_r + 2)}} .$$
(1)

(17)

Taking the inverse LT of (16) yields the origin envelope PDF of EGC reception, where the independent paths are non-identical

$$f_{X_{EGC}}^{inid}(x) = \left\{ \prod_{r=1}^{L} \left( \frac{2h_j \mathcal{R}_r \mathcal{T}^{\Omega_r} \bar{\gamma}_r^{\Omega+1} \Gamma(2\Omega_r + 2)}{\hat{x}^{2\Omega_r + 2}} \right) \right\} \times \frac{L_{r=1}^{\sum_{r=1}^{L} (\Omega_r + 1) \sum_{x=1}^{L} (2\Omega_r + 2) - 1}}{\Gamma\left(\sum_{r=1}^{L} (2\Omega_r + 2)\right)}$$
(18)

Finally, upon substituting the relationship given by [50, Eq. 2.3] into (18), the origin power PDF of EGC diversity reception is deduced similar by (10), where the parameters are defined in Table VI. Similarly to MRC, for i.i.d. the origin power PDF of EGC diversity reception will have a similar structure to (10) and the associated parameters are given in Table VI.

3) SC diversity PDF: In SC spatial diversity reception, the path having the highest SNR will be selected, thus the CDF is given by

$$F_{\gamma_{SC}}(\gamma) = \prod_{r=1}^{L} F_{\gamma_r}(\gamma). \tag{19}$$

Substituting (5) into (19) gives

$$F_{\gamma_{SC}}^{inid}(\gamma) = \left\{ \prod_{r=1}^{L} \left( \frac{\mathcal{R}_r h_j \mathcal{T}^{\Omega_r}}{\Omega_r + 1} \right) \right\} \gamma_{r=1}^{L} (\Omega_r + 1)^{-1}. \tag{20}$$

To derive the PDF expression, the above relationship has to be differentiated with respect to the parameter  $\gamma$ . It yields the origin PDF of SC diversity similar to the origin MRC PDF given by (10). For i.i.d. scenarios, the PDF of SC diversity is given by

$$f_{Y_{SC}}^{iid}(\gamma) = L\left[F_Y(\gamma)\right]^{L-1} f_Y(\gamma). \tag{21}$$

Upon substituting (3) and (5) into (21), one can obtain the origin PDF expression for SC diversity similar to (10). In Table VI, the fading parameters are illustrated both for i.n.i.d. and i.i.d. cases.

## IV. PERFORMANCE METRICS

Wireless communication systems have diverse performance metrics, such as the OP, the average SEP, the probability of missed detection, etc. Our research primarily focuses on high-power approximations of the performance metrics using the origin PDF. Therefore, we have evaluated the OP and average SEP performance metrics for both SIMO and SISO systems.

## A. Outage Probability

The OP describes the probability that the received SNR is below the minimum threshold ( $\gamma_{th}$ ), which is formulated as [43]

$$P_{out}(\gamma_{th}) = P[\gamma < \gamma_{th}] = \int_{0}^{\gamma_{th}} f_Y(\gamma) d\gamma.$$
 (22)

Upon substituting (10) into (22), after some manipulations, we get

$$P_{out}(\gamma_{th}) = \frac{\mathcal{K}^D \gamma_{th}^{\Psi^D}}{\Psi^D},\tag{23}$$

where D represents the MRC, EGC, and SC diversity parameters.

# B. Average SEP

Noise is one of the main sources of signal impediments in the communication system. The level of impact on the data transmission can be quantified by the average SEP. In this subsection, we will presented the analytical expressions of results for both the coherent and non-coherent detection based average SEP. The analytical representation of SEP under a fading environment is deduced by averaging i.e. integrating  $P_e(\gamma)$  across the entire dynamic range of the fading PDF  $f_Y(\gamma)$  [43, Eq. (12)]

$$P(e) = \int_{0}^{\infty} P_{e}(\gamma) f_{Y}(\gamma) d\gamma, \qquad (24)$$

where  $P_e(\gamma)$  is the SEP of non-fading environments.

- 1) Coherent average SEP: Here, we will consider the scenario where phase recovery at the Rx is possible in the face of both AWGN and ALN.
- a) AWGN average SEP: Under AWGN the SEP of a non-faded environment is given by [43, Eq. (13)]

$$P_e(\gamma) = \hat{\mathcal{A}}erfc(\sqrt{\hat{\mathcal{B}}\gamma}),$$
 (25)

where the constant parameters  $\hat{A}$  and  $\hat{B}$  are associated with different coherent modulation schemes defined in [51, Table I]. Upon substituting (25) and (10) into (24) and setting  $\sqrt{\hat{B}\gamma} = r$ , we get

$$P(e) = \frac{2\hat{\mathcal{A}}\mathcal{K}^D}{\hat{\mathcal{B}}^{\Psi^D}} \int_{0}^{\infty} t^{2\Psi^D - 1} erfc(t) dt.$$
 (26)

Evidently, by the inclusion of [52, Eq. (2.8.2.1)], (26) can be simplified to

$$\bar{P(e)} = \frac{\hat{\mathcal{A}}\mathcal{K}^D}{\hat{\mathcal{B}}^{\Psi^D}\Psi^D\sqrt{\pi}}\Gamma\left(\frac{2\Psi^D + 1}{2}\right). \tag{27}$$

b) ALN average SEP: Under ALN, the average SEP statistics discussed here are of BPSK, QPSK, and M-ary PSK. The transmitted signal is detected using the MED based detector, rather than a maximum likelihood detector. The MED detector is simple and its SEP expressions are of low complexity. The conditional SEP of BPSK and QPSK is formulated as

$$P_e(\gamma) = (a + b\sqrt{\gamma})e^{-c\sqrt{\gamma}},\tag{28}$$

where the constants a and b are defined in [20] and for the sake of completeness are reproduced here in Table VII. Upon substituting (10) and (28) into (24), as well as setting  $\sqrt{\gamma} = t$ , with the aid of [49, Eq. (3.381.4)], we have

$$P(e) = \frac{2aK^D\Gamma(2\Psi^D)}{c^{2\Psi^D}} + \frac{2bK^D\Gamma(2\Psi^D + 1)}{c^{2\Psi^D + 1}}.$$
 (29)

TABLE VI DISTRIBUTION PARAMETERS FOR MRC, EGC, AND SC DIVERSITY SCHEMES

Distribution Parameters	i.n.i.d.	i.i.d.
KMRC	$\frac{\prod\limits_{r=1}^{L}\left(\mathcal{R}_{r}h_{j}\mathcal{T}_{r}^{\Omega_{r}}\Gamma(\Omega_{r}+1)\right)}{\Gamma\left(\left(\sum\limits_{r=1}^{L}\Omega_{r}\right)+L\right)}$	$\frac{\mathcal{R}^{L}h_{j}^{L}\mathcal{T}^{L\Omega}\left(\Gamma(\Omega+1)\right)^{L}}{\Gamma\left(\Omega L+L\right)}$
$\Psi^{MRC}$	$\left(\sum_{r=1}^{L} \Omega_r\right) + L$	$L\left(\Omega+1 ight)$
$\mathcal{K}^{EGC}$	$\frac{\left\{\prod_{r=1}^{L} \left(2h_j \mathcal{R}_r \mathcal{T}_r^{\Omega_r} \Gamma(2\Omega_r + 2)\right)\right\} L^{\sum\limits_{r=1}^{L} (\Omega_r + 1)}}{2\Gamma\left(\sum\limits_{r=1}^{L} (2\Omega_r + 2)\right)}$	$\frac{\left(2h_{j}\mathcal{R}\mathcal{T}^{\Omega}\Gamma(2\Omega+2)\right)^{L}L^{L(\Omega+1)}}{2\Gamma((2\Omega+2)L)}$
$\Psi^{EGC}$	$\sum_{r=1}^{L} (\Omega_r + 1)$	$L(\Omega+1)$
$\mathcal{K}^{SC}$	$\left\{\sum_{r=1}^{L}\left(\Omega_{r}+1\right)\right\}\left\{\prod_{r=1}^{L}\left(\frac{\mathcal{R}_{r}h_{j}\mathcal{T}_{r}^{\Omega_{r}}}{\Omega_{r}+1}\right)\right\}$	$L\frac{\left(\mathcal{R}h_{j}\mathcal{T}^{\Omega}\right)^{L}}{(\Omega+1)^{L-1}}$
$\Psi^{SC}$	$\sum_{r=1}^{L} (\Omega_r + 1)$	$L(\Omega+1)$

TABLE VII
SEP PARAMETERS FOR ADDITIVE LAPLACIAN NOISE

Modulation Scheme	a	b	c
BPSK	$\frac{1}{2}$	0	2
QPSK	$\frac{3}{4}$	1	2

The conditional SEP of M-ary PSK for  $M \ge 8$  is given as [20, Eq. (7)]

$$P_e(\gamma) = \frac{8}{M} \sum_{r=0}^{\frac{M}{4}-1} \mathcal{G}(r,\gamma) + \frac{2\tan\left(\frac{\pi}{M}\right)^2}{M\left(1 - \tan\left(\frac{\pi}{M}\right)^2\right)} e^{-2\sqrt{\gamma}}, (30)$$

where

$$G(r,\gamma) = \frac{1}{2W_1} \left( W_1^2 e^{-K_1\sqrt{\gamma}} - W_2^2 e^{-K_2\sqrt{\gamma}} \right) - \frac{\sin\left(\frac{2\pi}{M}\right)}{8\left(\cos\left(\frac{2\pi}{M}\right) + \sin\left(\frac{4r\pi}{M}\right)\right)} e^{-K_3\sqrt{\gamma}},$$

(31

with 
$$W_1 = cos\left((2r+1)\frac{\pi}{M}\right)$$
,  $W_2 = sin\left((2r+1)\frac{\pi}{M}\right)$ ,  $\mathcal{K}_1 = \frac{-2sin\left(\frac{\pi}{M}\right)}{e^{cos\left((2r+1)\frac{\pi}{M}\right)}}$ ,  $\mathcal{K}_2 = \frac{-2sin\left(\frac{\pi}{M}\right)}{e^{sin\left((2r+1)\frac{\pi}{M}\right)}}$ , and  $\mathcal{K}_3 = 2\sqrt{2}cos\left(\frac{2r\pi}{M} - \frac{\pi}{M}\right)$ . Utilizing the methodology of BPSK and

QPSK, the SEP of M-ary PSK is deduced as follows

$$P(\bar{e}) = \frac{8\mathcal{K}}{M} \sum_{r=0}^{\frac{M}{4} - 1} \frac{\cos\left((2r+1)\frac{\pi}{M}\right)^{2} \Gamma(2\Psi)}{\cos\left((2r+1)\frac{2\pi}{M}\right) \mathcal{K}_{1}^{2\Psi}}$$

$$- \frac{8\mathcal{K}}{M} \sum_{r=0}^{\frac{M}{4} - 1} \frac{\sin\left((2r+1)\frac{\pi}{M}\right)^{2} \Gamma(2\Psi)}{\cos\left((2r+1)\frac{2\pi}{M}\right) \mathcal{K}_{2}^{2\Psi}}$$

$$- \frac{8\mathcal{K}}{M} \sum_{r=0}^{\frac{M}{4} - 1} \frac{\sin\left(\frac{2\pi}{M}\right) \Gamma(2\Psi)}{4\left(\cos\left(\frac{2\pi}{M}\right) + \sin\left(\frac{4r\pi}{M}\right) \mathcal{K}_{3}^{2\Psi}\right)}$$

$$+ \frac{\tan\left(\frac{\pi}{M}\right)^{2} \Gamma(2\Psi)}{M\left(1 - \tan\left(\frac{\pi}{M}\right)^{2}\right) 2^{2\Psi - 1}}.$$
(32)

2) Non-Coherent average SEP: Coherent detection requires perfect phase and channel state information at the Rx, which is hard to acquire. Therefore, non-coherent detection emerges as a low-complexity design for wireless sensor and relay networks [53]. Additionally, it is particularly beneficial for high Doppler SAGIN scenarios [54]. Specifically, it is beneficial because it reduces the system complexity and cost by eliminating channel estimation. Hence, we have derived the analytical expression of binary frequency shift keying, differential phase shift keying, and M-ary phase shift keying. The conditional SEP expression of AWGN is given by [43, Eq. (18)]

$$P_e(\gamma) = \mathcal{A}_n e^{-\mathcal{B}_n \gamma},\tag{33}$$

where the constants  $A_n$  and  $B_n$  are given in [43, Table 2]. Upon substituting (10) and (33) into (24), with the aid of [49,

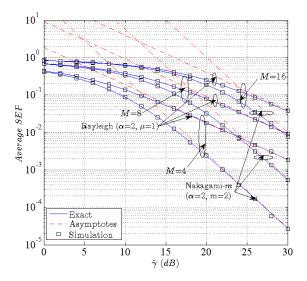


Fig. 2. Average SEP of M-ary PAM for special cases of  $\alpha$ - $\mu$  fading channel.

Eq. (3.381.4)], we arrive at

$$P(e) \approx \frac{\mathcal{A}_n \mathcal{K}^D \Gamma(2\Psi^D)}{\mathcal{A}_n^{2\Psi^D}}.$$
 (34)

## C. Diversity gain and SNR gain

Both the diversity gain and the SNR gain are important performance metrics that provide insights concerning the family of the diversity combining systems at high SNRs. The diversity gain determines the slope of the SEP curve, while the SNR gain measures the shift of the SEP curve relative to a benchmark SEP curve without changing its slope [11], [55], [56]. The SNR and diversity gains affect the OP and the average SEP as follows:

$$P_{out}(\gamma_{th}) \approx \left(\mathcal{G}^{C_{op}}\bar{\gamma}\right)^{\mathcal{G}^{D_{op}}}$$

$$\bar{P}(e) \approx \left(\mathcal{G}^{C_{Pe}}\bar{\gamma}\right)^{\mathcal{G}^{D_{Pe}}},$$
(35)

where the SNR and diversity gain parameters for the OP and average SEP are given by  $\mathcal{G}^{C_{op}}$  &  $\mathcal{G}^{C_{Pe}}$  and  $\mathcal{G}^{D_{op}}$  &  $\mathcal{G}^{D_{Pe}}$ , respectively.

## V. RESULTS AND DISCUSSIONS

The accuracy of the mathematical analysis presented in the previous sections requires validation. Hence, all the theoretical plots are compared to the Monte-Carlo simulation results with  $\geq \! 10^6$  samples. Additionally, we also examine the impact of system parameters through different performance metrics, such as the OP and average SEP/BEP for a few special cases of Fox's H-functions, namely for the  $\alpha\text{-}\mu$ , Fisher-Snedecor F, GBK, and single shadowed scenarios.

In Fig. 2, the average SEP of coherent M-ary PAM has been plotted for the  $\alpha$ - $\mu$  fading model. The figure contains exact (generated through integral simulation), asymptotic, and simulation results. To elaborate, Fig. 2 includes two special cases, namely the Rayleigh ( $\alpha$ =2 &  $\mu$ ) and Nakagami-m ( $\alpha$ =2 & m=2 $\mu$ ) scenarios, for M=4, 8, and 16 conveying 2, 3, and

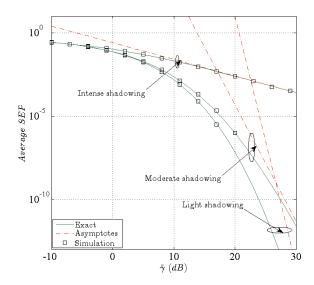


Fig. 3. Average BEP of BPSK for Fisher- ${\cal F}$  fading under different shadowing with Additive Laplacian noise.

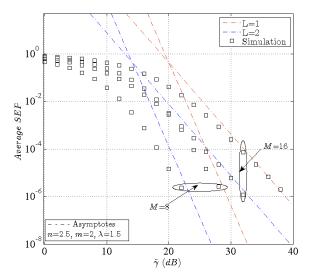


Fig. 4. Average SEP of M-ary PSK for GBK distribution with MRC diversity.

4 bits. The figure clearly demonstrates that as expected, an increase in the constellation size increases the SEP, the intraconstellation distance is reduced. It is further noted that the asymptotes coincide with the exact and simulation results.

Figure 3 shows the average BEP for binary phase shift keying in the face of ALN for transmission over Fisher-Snedecor F distributed fading channels. The set of values considered for the Fisher-Snedecor F parameters n and  $m_s$  are in the range of  $1 \rightarrow 15$ . The figure includes plots for three special shadowing cases, such as intense shadowing  $(n=1, m_s=1)$ , moderate shadowing  $(n=7, m_s=7)$ , and light shadowing  $(n=15, m_s=15)$ . As expected, shadowing has an adverse effect on the system. The SEP of  $10^{-5}$  is achieved at about 16 (dB) for light shadowing, nearly 18 (dB) for moderate shadowing, and at a very high SNR beyond 30 (dB) for intense shadowing.

The average SEP plots for MRC and SC diversity are illustrated in Figs. 4 and 5. Explicitly, Fig. 4 plots the SEP of

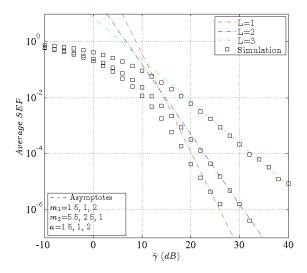


Fig. 5. Average SEP of QPSK for SSS distribution with i.n.i.d. distributed branches.

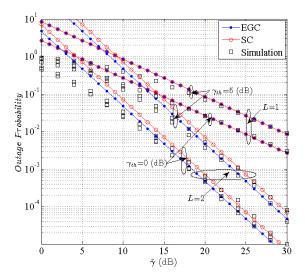


Fig. 6. Outage probability for N cascaded Nakagami-m fading.

M-ary phase shift keying for transmission over GBK channels. Here, two i.i.d. branches are considered, and the parameters are as follows: n=2.5, m=2,  $\lambda$ =1.5, for constellation sizes of M=8 & 16. As expected, the increasing number of diversity branches improves the system's reliability. For example, to achieve SEP of  $10^{-4}$ , SNRs of 25 (dB) and 18 (dB) are required for diversity orders of L=1 and L=2 respectively at M=8. Similarly, SNRs of 32 (dB) and 27 (dB) are required for diversity orders of L=1 and L=2 at M=16. It is further noted in Fig. 4 that a comparatively higher shift in SNR ( $\sim$  7 (dB)) is achieved at M=8, while  $\sim$  5 (dB) will be observed for M=16.

In Fig. 5, the average SEP plot for QPSK modulation scheme versus average SNR is plotted for the transmission over SSS model using SC diversity. Under this case, the individual branches are considered to be i.n.i.d., where  $m_1$ ={1.5,1,2},  $m_2$ ={5.5,2.5,1}, a={1.5,1,2}, and L={1,2,3}. We observe that the error probability decreases

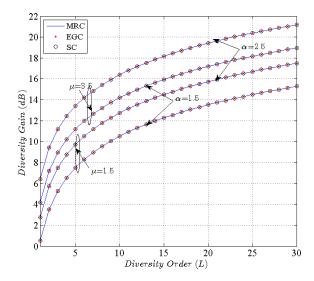


Fig. 7. Diversity gain of OP versus L for  $\alpha$ - $\mu$  fading with varying fading parameters.

upon increasing the diversity order. This is because as the number diversity paths increases, the combiner selects the specific path having the highest SNR. For example, to arrive at a SEP of  $10^{-4}$ , the system having a single branch requires an average SNR of 28 (dB), while a dual-branch system requires an SNR of  $\sim$ 22 (dB), and a triple-branch system needs 20 (dB). This explains why the initial diversity order increment has a significant contribution to the performance enhancement attained.

EGC and SC diversity schemes. The parameters considered here are as follows:  $m_1$ =1.5,  $m_2$ =1, threshold SNR, i.e.,  $\gamma_{th}$ =0 (dB) & 5 (dB), and the number of diversity paths is L=2 & 3. The OP is typically reduced as the number of antennas at the Rx increases. This is due to the fact that systems with multiple antennas have higher SNR. It is further observed that the OP of EGC diversity is lower than that of SC diversity and it increases as the threshold SNR increases. For example, at an average SNR of 30 (dB), the probability of outage is  $9 \times 10^{-3}$  with  $\gamma_{th}$ =5 (dB) and  $2.9 \times 10^{-3}$  with  $\gamma_{th}$ =0 (dB), respectively. In all the plots, it has been verified that the asymptotes are in good agreement with the simulations at high SNRs, but depends on the choice of the fading parameters.

Fig. 7 depicts the plot of diversity gain against diversity order. The analytical expression for the diversity gain is given by  $\mathcal{G}^{D_{op}}=L(\Omega+1)$ . It is observed from the plot and the expressions that all the diversity combining schemes have the same representation of diversity gain and are also independent of the OP threshold. It is further observed that the diversity gain increases as the fading parameters  $\alpha$  and  $\mu$  increase.

# VI. SUMMARY AND CONCLUSION

Unified average SEP and OP results have been presented for generalised Fox's H-fading with spatial diversity. The entire body of work presented is based on the origin PDF, which produces performance results at a reduced complexity. The work also addressed the parameters of the origin PDF for some special cases of Fox's H-fading that are well suited for different wireless applications, namely, inter-vehicle communication, unmanned aerial communication, millimetre wave communication, etc. All the formulations derived are compared to numerical simulation results to quantify their precision.

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