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Adaptive risk assessments

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ABSTRACT

We model a decision maker who can exert costly effort to adapt her risk assessments, thereby optimizing the value of her risky prospects. We provide an axiomatic characterization of the model and show how costs of adaption can be elicited and compared across individuals. In a moral hazard problem, we show that adapting risk assessments can weaken the effect of monetary incentives for effort provision, which has important implications for agency problems. We provide several examples to illustrate how adapting risk assessments can rationalize many well-known choice anomalies.

1. Introduction

A standard economic agent evaluates a risky prospect by its expected utility. For instance, when considering effort provision, insurance purchase, or portfolio choice, a standard agent assesses her options with a single expected utility function. A direct implication of this idea is that the agent has a stable fixed assessment of risk that she applies whenever she encounters a risky prospect. There are, however, many factors –informational, psychological, or environmental– that can affect the agent's risk assessments upon their realizations. In fact, there is strong evidence that the wealth level, internal stress, or exogenous shocks –among many others– can change an agent's risk assessments.[1] A rational agent would like to mitigate the adverse effects of these factors. For instance, an agent can acquire information to better control wealth prospects; or can exercise introspection to better regulate erratic stress; or can pay attention to environmental cues to better prepare for future shocks, thereby adjusting her risk assessments so as to improve her welfare.[2]

In this paper, we investigate how the ability of adapting risk assessments can affect choice behavior, and how observed choice behavior can reveal the adaption problem. To study these questions, we model a decision maker (DM) who can exert costly effort to adapt her risk assessments, thereby improving her evaluation of risky prospects. We show that our model can provide a general framework to rationalize a wide variety of choice behavior under risk, while having enough structure to elicit meaningful parameters from choice data. In particular, our model provides a rationale to many choice anomalies that have been widely documented in experiments (e.g., the common consequence, certainty, or magnitude effects), which cannot be explained by the standard model.

A key feature of our model is that the DM responds to incentives when choosing her adaption strategy. To illustrate, consider a simple moral hazard problem. Suppose the DM receives a transfer $t > 0$ in case of completion of a risky task. The DM receives utility from the transfer t by a function $v(t, \theta)$, where θ denotes a utility parameter. In this setting, adaption can be viewed as a strategy the DM can follow to change her utility parameter by way of choosing $\theta \in [\underline{\theta}, \bar{\theta}]$, but at a cost $c(\theta)$. Anticipating that the value of her future problem will be $v(t, \theta)$, the DM then faces the following adaption choice problem: $\max_{\theta \in [\underline{\theta}, \bar{\theta}]} [v(t, \theta) - c(\theta)]$.

By adapting her utility, the DM is able to improve her potential benefit from the transaction. However, she must balance this benefit against the adaption cost $c(\theta)$. In particular, the solution to this adaption problem depends on the incentives that she faces. For instance, we argue in Section 2 that adaption can weaken the effect of monetary incentives, which impact the way the DM responds to changes in the transfer. For a wide range of model parameters, the DM *decreases* effort provision in response to an

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1 For a survey article that examines a growing body of literature on the instability of risk preferences, see Schildberg-Horisch (2018).

2 In a similar vein, an agent can purchase durable goods or engage in a mortgage contract to lower the volatility of her wealth, stabilize state of mind, and better absorb possible shocks (see, e.g., Chetty and Szeidl, 2007) or Grossman and Laroque, 1990).

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increase in the transfer level.[3] This negative effect, which is driven by the ability of the DM to adapt her utility, can impact the viability of instruments aimed at encouraging effort provision behavior.

In well-known models of decision making under risk, such as the expected utility model, there is no opportunity for the DM to exert effort to affect her risk assessments. As a result, these models do not generate the type of responses to incentives that arise under the adaption model. Other economic settings where adaption can have important implications for choice behavior include insurance-purchase or portfolio-choice problems. For instance, in an insurance-purchase problem, a change in income may alter the DM's incentives to adapt her risk assessments, thereby affecting the amount of coverage the DM purchases. In a portfolio-choice problem, a change in wealth may alter the DM's incentives to adapt her risk assessments, thereby affecting the share of income she invests in risky assets.

Our general model formalizes the idea of adjusting risk assessments by identifying a novel parameter –the cost of adaption– that determines how a DM is able to adapt her risk assessments. We view these costs as subjective, and possibly encompassing many forms of strategies. A challenge in identifying such costs is that adaption itself may often be a *hidden action*, and therefore not observable; for instance, when it represents a purely mental effort. To overcome this challenge, we study the observable implications of adaption in a framework where the DM chooses between lotteries today, anticipating that she will exercise adaption before an outcome realizes tomorrow. Our identifying assumption is that the DM anticipates the benefits and costs of adaption when she chooses a lottery, so that her preferences over lotteries –which can be revealed by choice behavior– incorporate her adaption problem. In this lottery-choice framework, we give an axiomatic characterization of a general model of costly adaption (Theorem 1), and show how adaption costs can be elicited from choice-data (Theorem 2). We also show that a DM with higher costs of adaption values basic lotteries more, establishing a comparative measure of ability to adapt risk assessments in terms of choice behavior (Theorem 3).

An evidence against using a single expected utility function to represent the DM's preferences for choice under risk is the Allais (1953) paradox type choice behavior which results in a well-known violation of the independence axiom. A strand of experimental literature (e.g., Conlisk, 1989, Huck and Muller, 2012, and Blavatskyy et al., 2021) investigates how robust these types of paradoxes are. A key finding of this literature is that incentives matter such that the subjects tend to violate the independence axiom more when prizes are high, while less violations occur when prizes are low. An important feature of our model is that risk assessments are not necessarily fixed, and rather can be changed by adaption as a response to changes in incentives leading to Allais (1953) type choices (see Section 6).

In terms of preferences over lotteries, this feature of our model induces a key behavior, *Increasing Desire for Basic Lotteries*, which are lotteries that only yield the best or the worst outcome. This behavior reflects the idea that rather than mixing the lotteries themselves, it is better to mix their basic lottery equivalents. The reason for this behavior is that although the DM is certain about the values of the best and worst outcomes (where no adaption is needed), she may not be certain about the relative values of intermediate outcomes (where she needs adaption). As such, when lotteries are mixed, since (i) the number of intermediate outcomes increases, it becomes harder to adapt her risk assessments, and since (ii) the likelihoods of intermediate outcomes decrease, it becomes less appealing to adapt risk assessments. As a result, relative to the mixture of the basic lottery equivalents, the mixture of the lotteries both makes it harder to utilize adapting risk assessments and reduces the potential benefit of adaption.

Our model of adaptive risk assessments belongs to the literature on decision making under risk, where the strong independence axiom is replaced with a weaker counterpart. Axiomatic work on a strand of this literature was pioneered by the works of Dekel (1986), Chew (1989), and Gul (1991) to rationalize observed violations of the independence axiom. A key implication of these models is the betweenness axiom which induces that the DM's risk assessments are locally fixed and therefore cannot be changed along each indifference set. We show that betweenness preferences and our model overlap whenever risk assessments are globally fixed, and so the overlapping preferences assume an expected utility representation (Proposition 2).

A related model to our static model of preferences with adaptive risk assessments is the dynamic mixture-averse preferences of Sarver (2018). Although both his model (in a dynamic setting) and ours (in a static setting) are convex functions of lotteries, the characterizations are significantly different such that while his axioms imply the existence of certainty equivalents, ours imply basic lottery equivalents. Importantly while dynamic mixture-averse preferences are more general and can therefore have multiple equivalent representations, our adaptive risk assessments preferences essentially have a unique representation and its parameters can be elicited from choice data. In particular, benefits and costs of adaption are separated in our model allowing to conduct viable comparative statics, welfare and policy analysis. There are other models in the literature that are concave functions of lotteries and therefore can imply a decrease in desire for mixtures with basic lotteries (e.g., Maccheroni, 2002, Cerreia-Vioglio, 2009, and Cerreia-Vioglio et al., 2015). The overlap of these models and our model implies no desire for basic lotteries and therefore the overlap is equivalent to the expected utility model (Proposition 2). We provide a detailed discussion of the related literature after developing our model (see Section 5).

Several experiments with decision making under risk show that individuals exhibit increasing relative risk aversion (see, e.g., Holt and Laury, 2002). While the expected utility model can explain this behavior with many possible Bernoulli utilities (e.g., the constant absolute risk averse utility function), it typically runs into the problem of inducing very high levels of risk aversion when stakes are scaled up (see, e.g., Rabin, 2000). In Section 6, we argue briefly that our model of adaptive risk assessments can accommodate simultaneously both increasing relative and decreasing absolute risk aversion behavior providing a rationale for experimental choices without yielding absurd levels of risk aversion. Finally, our work is related to an applied literature which try to understand why monetary incentives may backfire when encouraging individuals to modify their choice behavior (see Gneezy et al., 2011 or Kamenica, 2012 for survey articles). A key implication in this literature is that the DM's extrinsic incentives can come into conflict with intrinsic motivations stemming from individual or social concerns (e.g., reputation). In this regard, our work provides a *novel* rationale for the negative reaction of individuals towards monetary incentives as we illustrate with a simple example in the next section.

The paper is organized as follows. Section 2 presents the moral hazard problem in more detail, and shows how ability to

[3] In fact, there is a sizeable body of literature employing lab or field experiments with similar implications documenting that monetary incentives can backfire. For instance, Alfitian et al. (2021) conduct a field experiment showing that monetary bonuses increase employee absenteeism in a retail chain in Germany by 45%; Klor et al. (2014)) show in a lab experiment that an increase in monetary rewards induce agents to exert lower effort in the completion of a joint task; Wagner et al. (2020) conduct a field experiment and show that stronger financial incentives can lead to substantially less effort by community health workers in Uganda.

adapt risk assessments can generate unintended effects on effort provision. In Section 3, we introduce the framework and define preferences over lotteries induced by a general model of adaptive risk assessments. Section 4 presents the axioms and contains our results on representation, elicitation, and comparative statics. We also characterize two special cases of the model where adaption strategy is constrained or fixed. Section 5 provides a review of the related literature. In Section 6, we provide several examples to illustrate some choice implications of the adaptive risk assessments model for decision making under risk. Section 7 concludes. Proofs are given in an Appendix.

2. Application: a principal–agent problem

To illustrate our general model, we consider a simple moral hazard problem with an agent who can adapt her risk assessments. We show that the ability to adapt risk assessments can lead to unintended effects in effort inducing settings that have important implications for agency problems aimed at improving efficiency.

Agent's problem. Suppose that there is a project that can end in either success or failure, which stochastically depends on how much effort $e \in [0, 1]$ an agent exerts. Specifically, let $p(e) = e^{\alpha}$ with $\alpha \in (0, 1)$ be the probability of success when the agent exerts an effort level $e \in [0, 1]$. Suppose exerting effort is costly to the agent which is given by the function $k(e) = \lambda e^{\beta}$ for some $\lambda > 0$ and $\beta > 1$.

The agent receives an amount of transfer $t \in [0, T]$ for some $T > 0$ in case of a success, otherwise there is no transfer made. Suppose the agent receives payoff from any transfer t by an increasing utility function $u_{\theta}(t)$ for some $\theta \in [\underline{\theta}, \bar{\theta}]$, where $0 < \underline{\theta} < \bar{\theta} < 1$. Thus, given an amount of transfer $t > 0$ and a utility function $u_{\theta}(t)$, the agent chooses an optimal level of effort by the following choice problem $\max_{e \in [0,1]} [p(e)\, u_{\theta}(t) - k(e)]$. Given our assumptions, the solution of this problem can be given by an optimal effort function $e^*_{\theta}(t) = [\frac{\alpha}{\lambda\beta} u_{\theta}(t)]^{\frac{1}{\beta-\alpha}}$ which is increasing in t. Let $v(t, \theta)$ be the value of agent's optimal effort choice problem; that is, $v(t, \theta) = p(e^*_{\theta}(t))\, u_{\theta}(t) - k(e^*_{\theta}(t))$.

By adapting her risk assessments $\theta \in [\underline{\theta}, \bar{\theta}]$, the agent can optimize her valuation of the risky transfers subject to some cost given by a function $c(\theta)$, which is assumed to be increasing in θ. As such, given an amount of transfer $t > 0$, the agent faces the following adaption choice problem, $\max_{\theta \in [\underline{\theta}, \bar{\theta}]} [v(t, \theta) - c(\theta)]$, as discussed in the Introduction.

Principal's problem. Now consider the problem of the principal who wants to offer an optimal transfer t in order to incentivize the agent to provide more effort. Suppose that the principal receives a payoff of z when the project is successful and otherwise receives no payoff. Given that the agent exerts effort by $e^*_{\theta}(t)$ as defined above, the principal faces the following transfer choice problem, $\max_{t \in [0,T]} \left[p(e^*_{\theta}(t))(z - t)\right]$. The optimal solution to this problem $t(z, \theta)$ satisfies the equality $\frac{u_{\theta}(t)}{u'_{\theta}(t)} = \frac{\alpha}{\beta-\alpha}[z - t]$. Suppose $u_{\theta}(t)$ is an increasing concave function; that is, $u'_{\theta}(t) > 0$ and $u''_{\theta}(t) \leq 0$. Then the optimality condition implies that as the value of the project increases (i.e., z goes up), the principal should increase the level of optimal transfer $t(z, \theta)$ in order to extract more effort from the agent.

This policy, however, may cause –contrary to its objective– less effort provision (i.e., e^* goes down) when the agent can adapt her risk assessments by way of adjusting θ. The reason is that when t increases the agent becomes relatively well-off. Thus, the marginal benefit of adaption diminishes, and the agent therefore wants to choose a lower level of θ which can, in turn, induce her to provide less effort. In fact, when the optimal effort e^* is elastic enough, the indirect effect of raising t, through e^*, can overcome its direct effect, and can overall lead to a decrease in the level of effort provided.

To make this argument more concrete, suppose without loss of generality that $T = 1$ and assume that $u_{\theta}(t)$ belongs to the following class of piecewise linear utility functions: for each $\theta \in [\underline{\theta}, \bar{\theta}]$, let $u_{\theta}(t) = \theta t$ for all $t < 1$ and $u_{\theta}(1) = 1$. Moreover, suppose that cost of adaption is such that $c(\theta) = r\theta^{\gamma}$ for some $r > 0$ and $\gamma > 1$. Using these functions, we can set the adaption choice problem, $\max_{\theta \in [\underline{\theta}, \bar{\theta}]} [v(t, \theta) - c(\theta)]$, as defined above and find the optimal parameter θ as a function of transfer t as $\theta^*(t) = m\, t^{\frac{\beta}{\gamma(\beta-\alpha)-\beta}}$, where $m > 0$ is a constant which depends only on exogenous parameters of the problem.[4] Thus, whenever $\gamma < \frac{\beta}{\beta-\alpha}$, optimal parameter $\theta^*(t)$ decreases as transfer t increases. But this means, as transfer t increases, likelihood of success $p(e^*_{\theta}(t)) = [\frac{\alpha}{\lambda\beta} m]^{\frac{\alpha}{\beta-\alpha}}\ t^{\frac{\gamma\alpha}{\gamma(\beta-\alpha)-\beta}}$ drops since optimal level of effort $e^*_{\theta}(t) = [\frac{\alpha}{\lambda\beta}\theta t]^{\frac{1}{\beta-\alpha}} = [\frac{\alpha}{\lambda\beta} m\, t^{\frac{\gamma(\beta-\alpha)}{\gamma(\beta-\alpha)-\beta}}]^{\frac{1}{\beta-\alpha}} = [\frac{\alpha}{\lambda\beta} m]^{\frac{1}{\beta-\alpha}}\ t^{\frac{\gamma}{\gamma(\beta-\alpha)-\beta}}$ falls whenever $\gamma < \frac{\beta}{\beta-\alpha}$.

Discussion. To set a viable policy goal, it is therefore important to identify whether the agent can adapt her risk assessments, and how elastic her adaptions are to incentives. A main factor which affects the elasticity of adapting risk assessments is the agent's cost of adaption (e.g., γ is a key parameter in our simple example above, which determines how sensitive the agent's costs are to the changes in adaption levels). Our identification result, Theorem 2, provides a method for eliciting costs under a general adaptive risk assessments model (see Section 4.3). To avoid incurring adaption costs, the agent in the moral hazard problem would be willing to pay a premium for the option to have the maximum payment (e.g., by becoming a residual claimant). Intuitively, the premium the agent would be willing to pay should depend on her costs of adaption, where higher costs should reflect in a willingness to pay a higher premium. In this regard, our comparative statics result, Theorem 3, provides a comparative measure of ability to adapt risk assessments in terms of desire for basic lotteries under a general adaptive risk assessments model (see Section 4.4). Most models of decision making under risk, such as the expected utility model, cannot generate the type of negative effects that arise under the adaptive risk assessments model. In these models, a change in the transfer can only scale up or down the value of the agent's problem without invoking negative effects. Unintended negative effects arise in our model because the DM exhibits an increasing desire for basic lotteries, and our characterization result, Theorem 1, provides a way of testing such behavioral responses under a general adaptive risk assessments model (see Section 4.2).

3. Preliminaries

In this section, we describe the framework and define the preferences induced by a general model of adaptive risk assessments.

3.1. Framework

In the following, X is a finite set of n prizes, with typical elements $x, y, z \in X$ called *outcomes*; P is the set of all probability distributions on X, with typical elements $p, q, r \in P$ called *lotteries*.[5] We also sometimes represent a lottery $p \in P$ more explicitly

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4 To be precise, we have $m = [\frac{(\frac{\alpha}{\lambda\beta})^{\frac{\alpha}{\beta-\alpha}} - \lambda(\frac{\alpha}{\lambda\beta})^{\frac{\beta}{\beta-\alpha}}}{r\gamma} \times \frac{\beta}{\beta-\alpha}]^{\frac{\beta-\alpha}{\gamma(\beta-\alpha)-\beta}}$ which is positive since $\beta > \alpha$ and all exogenous parameters are positive.

5 Our results can be extended to a setting where there is a continuum of outcomes. We consider a finite set of outcomes for ease of exposition.

in the form of $p = (x, [p(x)]\,; \,\ldots\,; \, z, [p(z)])$, where $p(y) \in [0, 1]$ denotes the probability of an outcome $y \in X$ under lottery p. For any given outcome $x \in X$, we denote the degenerate lottery which gives x for sure by δ_x. For any $\alpha \in [0, 1]$, let $\alpha\, p + (1-\alpha)\, q \in P$ denote a *mixed-lottery* $r \in P$, which is the mixture of lotteries p and q, where $r(x) = \alpha\, p(x) + (1-\alpha)\, q(x)$ for all $x \in X$.

Our primitive is a binary relation $\succsim$ on the set of lotteries P, with asymmetric part denoted $\succ$ and symmetric part denoted $\sim$. We interpret the binary relation $\succsim$ as the preference relation of a DM who chooses a lottery, anticipating that she will adapt her risk assessments towards the lottery when consuming it later. We say a functional $V : P \to \mathbb{R}$ represents $\succsim$ when, for all lotteries p and q, $p \succsim q$ if and only if $V(p) \geq V(q)$.

Let $\rhd_1$ denote the first-order stochastic dominance relation associated with $\succsim$, which is a strict partial order such that $p \rhd_1 q$ if $\sum_{\delta_x \succsim \delta_z} p(x) \geq \sum_{\delta_x \succsim \delta_z} q(x)$ for all $z \in X$ and $\sum_{\delta_x \succsim \delta_z} p(x) > \sum_{\delta_x \succsim \delta_z} q(x)$ for some $z \in X$.

Since X is finite, whenever $\succsim$ is complete, there must be a pair of best and worst outcomes $b, w \in X$ such that $\delta_b \succsim \delta_x \succsim \delta_w$ for any $x \neq b, w$. For simplicity, we will assume that $\delta_b \succ \delta_x \succ \delta_w$ for any $x \neq b, w$ and so both b and w will be unique. We call an outcome $x \in X$ *intermediate* if $x \neq b, w$, and denote by X_i the set of intermediate outcomes in X. Let $P_{bw} = \{p \in P : p(x) = 0 \text{ if } x \in X_i\}$ be the set of *basic* lotteries whose supports consist of only the best or worst outcomes. A lottery is *non-basic* if it assigns positive probability to an intermediate outcome. For any given lottery $p \in P$, let $b_p \in P_{bw}$ denote a basic lottery equivalent of it; that is, $p \sim b_p$ and let $b(p)$ denote in short the probability weight on the best outcome in b_p. Notice that, in general, there can be multiple basic lottery equivalents, but whenever $\succsim$ is monotone with respect to first-order stochastic dominance, then the basic lottery equivalent is unique.[6]

For any two vectors $u, v \in \mathbb{R}^n$, let $u \cdot v$ denote the dot product of u and v. An expected utility function on P can be identified with an element of $\mathbb{R}^n$; hence, if $u \in \mathbb{R}^n$ and $p \in P$, we use $u \cdot p$ and $u(p)$ interchangeably. Note that for any $u \in \mathbb{R}^n$ representing $\succsim$ over X, we can let, without loss of generality, that $u(\delta_w) = 0$ and $u(\delta_b) = 1$. Let $\mathcal{U} = \left\{u \in \mathbb{R}^n \,:\, u(\delta_w) = 0,\ u(\delta_b) = 1,\ u(\delta_x) \geq u(\delta_y) \text{ iff } \delta_x \succsim \delta_y\right\}$ be the set of *utilities,* which represent $\succsim$ over X with typical elements $u, v \in \mathcal{U}$.

3.2. The model

We view adaption of risk assessments as a strategy the DM can follow to alter her risk assessments so as to make a better evaluation of risky outcomes. Exercising adaption, however, requires effort, and so the DM must incur the costs in order to exploit the benefits. These costs are represented by a function $c : \mathcal{U} \to [0, \infty]$, where $c(u)$ can be interpreted as a measure of the effort required to adapt risk assessments to u. We say a cost function c is *proper* whenever $c(u) < \infty$ for some $u \in \mathcal{U}$, and $\liminf_{u \to v} c(u) \geq c(v)$ for every $v \in \mathcal{U}$, where convergence is defined with respect to the weak*-topology.

A preference order with adaptive risk assessments reflects the behavior of a DM who acts as if she anticipates exerting an optimal level of adaption effort before consuming a risky outcome.

[6] In the following, we will assume that the DM's preferences are non-trivial weak order, which are monotone with respect to the first order stochastic dominance, and which are mixture-continuous. As such, given that we have a unique pair of best and worst outcomes $b, w \in X$ such that $\delta_b \succ \delta_x \succ \delta_w$ for any $x \in X$ with $x \neq b, w$, by monotonicity, $\delta_b \succsim p \succsim \delta_w$ for any $p \in P$, and so by mixture continuity, the following (non-empty) sets, whose union is equal to $[0, 1]$, must be closed: $\{\alpha \in [0, 1] : \alpha\delta_b + (1-\alpha)\delta_w \succsim p\}$ and $\{\alpha \in [0, 1] : p \succsim \alpha\delta_b + (1-\alpha)\delta_w\}$. Since $[0, 1]$ is connected, these two sets must intersect; that is, there must exist some $\alpha_p \in [0, 1]$ such that $p \sim \alpha_p\delta_b + (1-\alpha_p)\delta_w$. In fact, by monotonicity, α_p is unique. Let $b_p = \alpha_p\delta_b + (1-\alpha_p)\delta_w \in P$ and $b(p) = \alpha_p \in [0, 1]$.

Definition 1 (*Adaptive Risk Assessments*)**.** A binary relation $\succsim$ on lotteries is an *adaptive risk assessments (ARA) preference* if there exists a proper cost function $c : \mathcal{U} \to [0, \infty]$ such that the functional $V : P \to \mathbb{R}$, defined by

$$V(p) = \max_{u \in \mathcal{U}} \left[u(p) - c(u)\right],$$

represents $\succsim$. In this case, we also say that $\succsim$ is represented by c.

The ARA model admits a natural interpretation, in which the utility $V(p)$ represents the anticipated net utility gained from lottery p after an optimal assessment of risk is chosen through a costly cognitive process.[7] Notice that although each $u \in \mathcal{U}$ agrees on the value of best and worst outcomes, they may disagree on the value of intermediate outcomes. As such, the DM's objective is to adjust the evaluation of the intermediate outcomes by way of a costly process. Notice also that, in general, the DM can consider all utilities in $\mathcal{U}$ as part of her optimization problem. However, the DM could also consider only a subset $U \subset \mathcal{U}$. Such constraints can always be incorporated by setting $c(u) = \infty$ for any $u \notin U$, which is observationally equivalent to excluding u in Definition 1.

3.3. Adaption costs

Properness is a minimal property of a cost function to ensure that the adaption problem is well-defined. We impose no other a priori restrictions on the cost function. On the other hand, there are a number of intuitive properties that, without loss of generality, can be imposed on an adaptation cost function (see Corollary 1 in Section 4.3).

Definition 2 (*Canonical Costs*)**.** We say a cost function c is canonical if it is (i) *grounded:* $c(u) = 0$ for some $u \in \mathcal{U}$, (ii) *convex:* $c(\alpha u + (1-\alpha)v) \leq \alpha c(u) + (1-\alpha)c(v)$ for all $u, v \in \mathcal{U}$ and $\alpha \in (0, 1)$, and (iii) *monotone:* $c(u) \geq c(v)$ for all $u, v \in \mathcal{U}$ with $u \geq v$.

Groundedness reflects the idea that the DM has the option not to exercise an adaption strategy, thereby incurring no cost. Convexity means that randomizing between two adaption strategies cannot lower the costs. Monotonicity captures the idea that adapting risk assessments towards a higher valuation is more costly.

4. Analysis

In this section, we provide our analysis of the ARA model. We first discuss behavioral implications of the model. We then provide our main findings: a representation theorem, an elicitation procedure, and a comparative statics result. We also characterize two special cases of the adaption model, where adaption efforts are constrained or fixed.

4.1. Axioms

The following three axioms are standard in the literature.

Axiom 1 (*Non-trivial Weak Order*)**.** For all lotteries $p, q, r \in P$, (i) $p \succsim q$ or $q \succsim p$, and (ii) $p \succsim q$ and $q \succsim r$ implies $p \succsim q$. There exist outcomes, $b, w \in X$, such that $\delta_b \succ \delta_x \succ \delta_w$ for all $x \neq b, w$.

Axiom 2 (*Mixture Continuity*)**.** For all lotteries $p, q, r \in P$, the following sets are closed: $\{\alpha \in [0, 1] : \alpha p + (1-\alpha)q \succsim r\}$ and $\{\alpha \in [0, 1] : r \succsim \alpha p + (1-\alpha)q\}$.

[7] Following Kahneman (1994), one can call $V(p)$ as the "decision utility" reflecting the desirability of lottery p as inferred from the DM's decisions, while each $u(p)$ as the "experience utility" reflecting the DM's realized welfare from lottery p.

Axiom 3 (*Monotonicity*). For all lotteries $p, q \in P$, if $p \rhd_1 q$, then $p \succ q$.

Axiom 1 requires that the preference order is non-trivial, complete, and transitive. Axiom 2 imposes continuity for preferences over mixtures of lotteries. Axiom 3 says that the preference order is monotone with respect to first-order stochastic dominance relation. The following two axioms are key behavioral implications of the adaptive risk assessments model.

Axiom 4 (*Basic Lottery Independence*). For all lotteries $p, q \in P$, basic lotteries $r, s \in P_{bw}$, and $\alpha \in (0, 1)$, if $\alpha p + (1-\alpha)r \succsim \alpha q + (1-\alpha)r$, then $\alpha p + (1-\alpha)s \succsim \alpha q + (1-\alpha)s$.

Axiom 4 reflects the idea that adaption of risk assessments is only relevant for non-basic lotteries and not for basic lotteries. The reason is that possible adaptions of the DM's risk assessments will agree on the value of basic lotteries since they all value best and worst outcomes in the same way. As such, when adapting her risk assessments, the DM only considers non-basic lotteries and disregards the information about basic lotteries. Hence, when non-basic lotteries and their weights are fixed across mixtures, the preferences over lotteries do not alternate.

Axiom 5 (*Increasing Desire for Basic Lotteries*). For all lotteries $p, q \in P$, basic lotteries $r, s \in P_{bw}$, and $\alpha \in (0, 1)$, if $p \sim r$ and $q \sim s$, then $\alpha r + (1-\alpha)s \succsim \alpha p + (1-\alpha)q$.

Axiom 5 reflects the idea that the DM values basic lotteries more when mixing; that is, when lotteries are mixed, her desire for basic lotteries increases. The reason is that when non-basic lotteries are mixed –relative to each lottery– the number of intermediate outcomes increases while their likelihood decreases. But then, since the number of intermediate outcomes increases, it becomes harder to adapt risk assessments and since their likelihood decreases, it becomes less incentivizing to adapt risk assessments. On the other hand, when basic lotteries are mixed, the mixture lottery is still a basic lottery and the mixing does not create any additional difficulty in terms of adapting risk assessments. As a result, the value of mixing basic lotteries stays above the value of mixing equally good non-basic lotteries.

4.2. Characterization

The following theorem shows that Axioms 1–5 characterize the behavior of a DM who chooses among lotteries "as if" she anticipates adapting her risk assessments before consuming a lottery. Thus, Axioms 1–5 are both necessary and sufficient to test the ARA model using preferences over lotteries.

Theorem 1. *A binary relation on lotteries $\succsim$ is an ARA preference if and only if it satisfies Axioms 1–5.*

Proof sketch: Necessity part of the proof is straightforward to show. For sufficiency, Lemma 1 (Appendix A.2) shows that if a binary relation $\succsim$ satisfies Axioms 1–5, then there exist an adaption cost function $c : \mathcal{U} \to [0, \infty]$ such that c represents $\succsim$. In particular, Axioms 1–5 imply that every lottery $p \in P$ has a *basic lottery equivalent* $b_p \in P_{bw}$ such that $p \sim b_p$. Using basic lottery equivalents, then a functional I over the set $\Phi = \{\phi_p : \mathcal{U} \to \mathbb{R} \mid \phi_p(u) = u(p),\ \forall u \in \mathcal{U}, \forall p \in P\}$ can be defined such that, for all lotteries p and q, we have $p \succsim q$ if and only if $I(\phi_p) \geq I(\phi_q)$. The remainder of the proof uses Axioms 1–5 to show that I is monotone, continuous and convex, and employs duality arguments to establish the desired representation.

4.3. Elicitation

In general, it is not possible to identify a unique cost function since, for instance, if an ARA preference $\succsim$ is represented by c, then c_λ also represents $\succsim$ for any $\lambda > 0$, where $c_\lambda(u) = c(u) + \lambda$ for all $u \in \mathcal{U}$. The following result shows that, for each ARA preference, there is a unique *minimal* cost function.

Theorem 2. *Let $\succsim$ be an ARA preference. Then, the function $c^* : \mathcal{U} \to [0, \infty]$, defined by $c^*(u) = \sup_{p \in P} u(p) - b(p)$ for all $p \in P$, is the unique minimal cost function which represents $\succsim$.*

Theorem 2 shows that ARA preferences can always be represented by a unique minimal cost function c^*. In particular, the minimal cost function c^* can be constructed from data on basic lottery equivalents; since $c^*(u) \geq u(p) - b(p)$ for any lottery p, the basic lottery equivalent b_p can be used to determine $u(p) - b(p)$ as a lower bound on the cost of u. Using basic lottery equivalents for other lotteries then leads to a more precise lower bound. Theorem 2 shows that this procedure approximates $c^*(u)$ arbitrarily closely, thereby establishing a direct connection between the adaptation cost function and lottery-choice behavior. As the following corollary shows, c^* satisfies the properties in Definition 2.

Corollary 1. *Let c^* be the minimal cost function for a given ARA preference $\succsim$. Then c^* satisfies groundedness, convexity, and monotonicity.*

We see by this corollary that the minimal cost function c^* is canonical, and therefore, we refer to c^* as the *canonical representation* of $\succsim$.

4.4. Comparative statics

As an application of our identification result, we consider a measure of comparative adaption. We say that DM2 is *less able to adapt* than DM1 when adaption is costlier for DM2 than DM1; that is, $c_2^* \geq c_1^*$. Intuitively, when DM2 is less able to adapt her risk assessments, she should find the option of having a basic lottery –which eliminates the need to adapt– more valuable than DM1. The following comparative defines when DM2 finds basic lotteries more valuable than DM1.

Definition 3 (*Comparative Desire for Basic Lotteries*). Let $\succsim_1$ and $\succsim_2$ be binary relations on the set of lotteries P. Then $\succsim_2$ has a *stronger desire for basic lotteries* than $\succsim_1$ if, for all lotteries $p \in P$ and $q \in P_{bw}$, whenever $q \succ_1 p$, then $q \succ_2 p$.

The following theorem shows that the comparative in Definition 3 characterizes when DM2 is less able to adapt her risk assessments than DM1.

Theorem 3. *Let $\succsim_1$ and $\succsim_2$ be ARA preferences with canonical costs c_1^* and c_2^*, respectively. Then, $\succsim_2$ has a stronger desire for basic lotteries than $\succsim_1$ if and only if $c_2^* \geq c_1^*$.*

Theorem 3 provides a behavioral measure of comparative ability to adapt risk assessments. In particular, Theorem 3 implies that the utility difference between a lottery and its basic lottery equivalent is higher for a DM who is less able to adapt. As such, the DM will be willing to pay a higher premium for the option to have the basic lottery, thereby avoiding higher adaption costs.

4.5. Special cases

We conclude this section by characterizing two special cases of the ARA model.

Constrained adaption. A special case of our model is one where the DM's adaption effort is constrained, rather than costly. This special case is characterized by the following axiom.

Axiom. [Neutral Desire for Basic Lotteries] For each lottery $p \in P$ and $q \in P_{bw}$, if $p \sim q$, then $q \sim \alpha p + (1-\alpha)q$ for all $\alpha \in (0, 1)$.

Neutral Desire for Basic Lotteries (NeDBL) reflects the idea that the DM's desire for basic lotteries does not change when a lottery is mixed with its basic lottery equivalent. The reason is that when a lottery is mixed with a basic lottery, the likelihood of its outcomes change, but not the set of its intermediate outcomes. As such, whenever the DM incurs no cost of adapting risk assessments, but is constrained, she will keep choosing the same risk assessments if the lottery is mixed with a basic lottery. As a result, she will be indifferent between the lottery and its mixtures with a basic lottery equivalent. In fact, the following result shows that ARA preferences satisfy NeDBL if and only if the DM's adaption choices are costless, but constrained.

Proposition 1. *Let $\succsim$ be an ARA preference. Then $\succsim$ satisfies Neutral Desire for Basic Lotteries if and only if there exists a set $U \subset \mathcal{U}$ such that for all $p, q \in P$, $p \succsim q$ if and only if $\max_{u \in U} u(p) \geq \max_{u \in U} u(q)$.*

Relative to the expected utility model where the risk assessment is fixed, for the representation in Proposition 1 adaption choices can vary across lotteries, but are restricted to a constraint set. Note that choice set U above can be taken, without loss of generality, as a closed convex set which can be constructed using the equation given in Theorem 2.

No adaption. Expected utility model is naturally an important special case of our ARA model, where the risk assessment is fixed. This special case is characterized by the following stronger neutral desire for basic lotteries axiom.

Axiom (*No Desire for Basic Lotteries*)**.** For all lotteries $p, q \in P$, if $p \sim q$, then $p \sim \alpha p + (1-\alpha)q$ for all $\alpha \in (0, 1)$.

No Desire for Basic Lotteries (NoDBL) reflects the idea that the DM's desire for basic lotteries does not change when a lottery is mixed with another lottery. When a lottery is mixed with another lottery, the mixture lottery has more intermediate outcomes with lower likelihoods. But if the DM's risk assessment is fixed, then such mixtures do not affect the DM's choice of a risk assessment. As a result, she will be indifferent between a lottery and its mixture with an equivalent lottery, regardless of whether the equivalent lottery is basic or not. In fact, the following result shows that ARA preferences satisfy NoDBL if and only if there is a common solution to the DM's adaption choice problem.

Proposition 2. *Let $\succsim$ be an ARA preference. Then $\succsim$ satisfies No Desire for Basic Lotteries if and only if there exists a unique $u \in \mathcal{U}$ such that for all $p, q \in P$, $p \succsim q$ if and only if $u(p) \geq u(q)$.*

Proposition 2 shows that the expected utility model can be recovered as a special case of the ARA preferences. This result shows that any non-expected utility model satisfying the betweenness axiom can overlap with the ARA preferences only when they agree with the expected utility model. The next section provides a closer inspection of the relation of the ARA preferences with some other non-expected utility models given in the literature.

5. Related literature

Cerreia-Vioglou (2009). The ARA model V given in Definition 1 is a convex function of lotteries; that is, $V(\alpha p + (1-\alpha)q) \leq \alpha V(p) + (1-\alpha)V(q)$ for any $p, q \in P$ and $\alpha \in [0, 1]$. Cerreia-Vioglio (2009) studies a general quasi-concave model of risk preferences. As such, the dual of his model, which can be given as

$$v(p) = \sup_{u \in \mathcal{V}} U(u(p), u),$$

represents a general class of mixture averse preferences under risk. Hence, the dual of Cerreia-Vioglio (2009)'s model naturally contains our model of ARA preferences as a special case.

Sarver (2018). Another model of risk preferences which extends our ARA model is Sarver (2018)'s model of mixture averse preferences. Sarver (2018) considers Epstein and Zin (1989) preferences in a dynamic setting of consumption and saving, and identify an important class of risk preferences by imposing a Mixture Aversion axiom. Restricted to the static setting and focusing on preferences over consumption lotteries, his Optimal Risk Attitude (ORA) model is also a convex function of lotteries which can be written in our static setting as

$$v(p) = \sup_{\phi \in \Phi} \phi(p).$$

Clearly, the ORA model extends our model of ARA preferences in the static setting. To see this, note that for any $u \in \mathcal{U}$ and $p \in P$, we can transform the objective function $u(p) - c(u)$ in Definition 1 to $\phi(p)$, where $\phi(p) = \sum \phi(\delta_x)p(x)$ such that $\phi(\delta_x) = u(\delta_x) - c(u)$ for each $x \in X$. However, this is not the only way of embedding costs into the transformed utilities. For instance, for any two outcomes $x, y \in X$, we can have $\phi_\epsilon(\delta_x) = \phi(\delta_x) - \epsilon$ and also have $\phi_\epsilon(\delta_y) = \phi(\delta_y) + \frac{p_x}{p_y}\epsilon$ for some $\epsilon > 0$, while for any other $z \in X$ we can let $\phi_\epsilon(\delta_z) = \phi(\delta_z)$. If we let $\phi_\epsilon(p) = \sum \phi_\epsilon(\delta_x)p(x)$, then by construction we have $\phi(p) = \phi_\epsilon(p)$ showing that the ARA preferences can have non-unique ORA representations.

While this is the case, our discussions in Section 4.3 show that there is essentially a unique representation for ARA preferences given by the cost function c^*. More importantly, we have shown that this parameter can be elicited from choice data using basic lottery equivalents. As a result, our axiomatic characterization and elicitation results identify an important class of Mixture Averse preferences in the static setting which can be uniquely constructed using choice data. As we discussed with our moral hazard example, unique elicitation of the parameters is important to conduct viable comparative statics, welfare and policy analysis.

Quiggin (1982). Another related non-expected utility model of risk preferences is the rank dependent utility (RDU) model first proposed by Quiggin (1982). According to this model, the value of lottery p is given by the expected utility

$$v(p) = \sum u(x)\pi(p(x)),$$

where $u \in \mathcal{U}$ and $\pi(p(x)) = w(\sum_{\delta_x \succsim \delta_z} p(z)) - w(\sum_{\delta_x \succ \delta_z} p(z))$ for all $x \in X$ with $w : [0, 1] \to [0, 1]$ satisfying $w(0) = 0$ and $w(1) = 1$. Sarver (2018) shows that risk averse RDU model is a special case of his Mixture Averse preferences. In this case, w must be concave (see, Chew et al., 1987). We now argue that the overlap between the risk averse RDU model and the constrained ARA preferences is the expected utility model. To see this, note that our Axioms 1–3 and Quiggin (1982)'s first three axioms coincide, while risk averse RDU implies our Axiom 5. On the other hand, the risk averse RDU model has to agree with the expected utility model whenever it satisfies our Axiom 4 and Axiom NeDBL. To see the validity of this claim, let $p = (w, [p_1]; b, [p_2])$ and $q = (x; [1])$ for some $x \in X_i$ such that $p \sim q$. Then, by Axiom 4 and Axiom NeDBL, $\alpha p + (1-\alpha)\delta_b \sim \alpha q + (1-\alpha)\delta_b$ for any $\alpha \in (0, 1)$. These two indifference relations imply that $u(b)(1-w(p_1)) = u(x)$ and $u(b)(1-w(\alpha p_1)) = u(x)w(\alpha)+u(b)(1-w(\alpha))$, respectively, which together imply $w(\alpha)w(p_1) = w(\alpha p_1)$. Since p_1 and α are arbitrary, we deduce that $w(a.b) = w(a).w(b)$ for all $a, b \in (0, 1)$. Thus, by Aczel (1966, Theorem 3, p.41), for all $a \in (0, 1)$, we must have $w(a) = a^k$ for some $k \geq 0$. Moreover, we must have $k \leq 1$ since w is concave.

Again, $\alpha p + (1-\alpha)\delta_w \sim \alpha q + (1-\alpha)\delta_w$ for any $\alpha \in (0, 1)$ by Axiom 4 and Axiom NeDBL. This indifference implies that $u(b)(1-w(\alpha p_1 + 1 - \alpha)) = u(x)(1-w(1-\alpha))$. Since we also have $u(b)(1-w(p_1)) = u(x)$ from above, we derive that

$$w(\alpha p_1 + 1 - \alpha) + w(p_1)w(1-\alpha) = w(p_1) + w(1-\alpha),$$

for all p_1 and α in $(0, 1)$. Let $p_1 = \frac{1}{2}$ and $\alpha = \frac{1}{2}$ and use the fact that $w(a) = a^k$ in above equation. Then we have $(\frac{3}{4})^k + (\frac{1}{4})^k = 2(\frac{1}{2})^k$. Clearly this equation is true only when $k = 0$ or $k = 1$.[8] Since $w(a)$ is non-constant, we deduce that $k = 1$, and so $w(a) = a$ for all $a \in [0, 1]$ showing that constrained ARA preferences and risk averse RDU preferences overlap only when they both agree with the expected utility model.

Cerreia-Vioglou, Dillenberger, and Ortoleva (2015). A prominent model of preferences under risk is the Cautious Expected Utility (CEU) model of Cerreia-Vioglio et al. (2015). CEU is a quasi-concave model of risk preferences, and it therefore belongs to the class studied in Cerreia-Vioglio (2009). As such, the overlap of CEU and ARA preferences is the expected utility model. While this is the case, the dual of CEU, which satisfies

$$v(p) = \sup_{u \in \mathcal{W}} u^{-1}[\sum u(x)p(x)],$$

is a quasi-convex model of risk preferences, and it can have non-trivial overlap with the ARA preferences. The following argument shows that the two models overlap only for the expected utility model. To see this, note that both the CEU model and its dual implies that for any $p \in P$ and $x \in X$ with $p \sim \delta_x$, one must have $\alpha p + (1-\alpha)\delta_x \sim p$. If we impose this condition on the ARA preferences, we obtain $u(p) - c(u) = v(\delta_x) - c(v)$ and $w(\alpha p + (1-\alpha)\delta_x) - c(w) = u(p) - c(u)$, where u, v, w are optimal utilities for p, δ_x, and $\alpha p + (1-\alpha)\delta_x$, respectively. As a result, we must have $w(p) - c(w) \leq u(p) - c(u)$ and $w(\delta_x) - c(w) \leq v(\delta_x) - c(v)$. But given that $w(\alpha p + (1-\alpha)\delta_x) - c(w) = u(p) - c(u) = v(\delta_x) - c(v)$ is true, each inequality above must be an equality. But since p, x, and α are arbitrary, this means that there must be a unique utility that is feasible and therefore the ARA preferences must have an expected utility representation.

Maccheroni (2002). Another related model to our model of ARA preferences is the Maxmin under Risk (MUR) model of Maccheroni (2002). MUR is also a quasi-concave model of risk preferences and therefore belongs to the class studied by Cerreia-Vioglio (2009). As a result, its overlap with ARA preferences is the expected utility model. The dual of MUR, on the other hand, can be written for each $p \in P$ as:

$$v(p) = \max_{u \in \mathcal{U}} \sum u(x)p(x),$$

which is a quasi-convex model of risk preferences and which has essentially non-trivial overlap with the constrained ARA preferences that we characterized in Proposition 1. Note that the model we characterized in Proposition 1 cannot generate the type of incentive effects discussed in our moral hazard application. The ARA preferences (or its dual model in general) can generate such incentive effects only because of the variable cost parameter. Thus, our Axiom 4 is crucial to allow these richer choice behavior. Importantly, our identification result, Theorem 2, provides a method to elicit these cost parameters.

Maccheroni, Marinacci, and Rustichini (2006). Another related model to our ARA preferences is the Variational Model of Maccheroni et al. (2006). The variational model is a quasi-concave model of preferences under *uncertainty* proposed to study ambiguity aversion. It belongs to the general class of Uncertainty Averse Preferences considered by Cerreia-Vioglio et al. (2011). It is not hard to draw a structural parallel between the variational model and the dual of our ARA preferences, which can be given as follows:

$$\min_{u \in \mathcal{U}} u(p) + c(u).$$

While choice variables in the variational model are belief functions which represent the likelihood of states, choice variables in the ARA preferences (or its dual) are utility transformations which change the value of outcomes. As a result, there are significant differences between our proof methods. For instance, we do not use certainty equivalents unlike Maccheroni et al. (2006) since these equivalences do not reflect the evaluation of outcomes by the same utility function. To establish equivalences, we rather use basic lotteries for which there is no uncertainty about their value. We believe considering basic lottery equivalents can be useful in developing related theories.

6. Choice implications

In this section, we provide examples to illustrate some possible implications of the adaption of risk assessments model for choice between lotteries. These examples show that adaption of risk assessments can lead to systematic violations of the expected utility model, which have been widely documented in experiments. We start our discussion by illustrating that the ability to adapt risk assessments can generate the common consequence effect.

Common consequence effect. Let $w = 0$, $b = 2500$, and $x = 2400$. Suppose the DM has ARA preferences $\succsim$. In particular, let c represent $\succsim$ where $c(u) = 0$ for $u \in \mathcal{U}$ such that $u(\delta_x) = 0.69$, while $c(v) = 0.15$ for $v \in \mathcal{U}$ such that $v(\delta_x) = 0.99$, and $c(v') = \infty$ for all $v' \in \mathcal{U} \setminus \{u, v\}$. Consider lotteries $p = (2400, [1])$ and $q = (2500, [0.33]; 2400, [0.66]; 0, [0.01])$, and lotteries $p' = (2400, [0.34]; 0, [0.66])$ and $q' = (2500, [0.33]; 0, [0.67])$. Kahneman and Tversky (1979) observe that most subjects prefer p over q, and q' over p', exhibiting the common consequence effect as a violation of the independence axiom.

Given the adaption costs, we observe that $p \succ q$ and $q' \succ p'$ verifying the common consequence effect. This is true since $v(\delta_x) - c(v) = 0.84 > 0.69 = u(\delta_x) - c(u)$, the DM adapts her risk preferences to v when evaluating p. Hence, we have $V(p) = 0.84$. We also have $0.33 + 0.66\, v(\delta_x) - c(v) \approx 0.83 > 0.64 = 0.33 + 0.66\, u(\delta_x) - c(u)$. Hence, $V(q) \approx 0.83$ implying that $p \succ q$. When it comes to evaluating p', we observe that $0.34\, u(\delta_x) - c(u) \approx 0.23 > 0.19 \approx 0.34\, v(\delta_x) - c(v)$. Thus, we have $V(p') \approx 0.23$. And finally, we have $V(q') = 0.33$, and so $q' \succ p'$, as desired. □

Next we consider a simple example that illustrates that the ability to adapt risk assessments can generate the common ratio effect.

Common ratio effect. Let $w = 0$, $b = 4000$, and $x = 3000$. Suppose the DM has ARA preferences $\succsim$ with c defined as in common consequence effect example above. Consider lotteries $p = (3000, [1])$ and $q = (4000, [0.8]; 0, [0.2])$, and lotteries $p' = (3000, [0.25]; 0, [0.75])$ and $q' = (4000, [0.2]; 0, [0.8])$. Kahneman and Tversky (1979) observe that most subjects prefer p over

[8] In fact, by Aczel (1966, Theorem 1, p.43), the most general solution to Jensen's Equation, $f(\frac{x+y}{2}) = \frac{f(x)+f(y)}{2}$, is such that $f(x) = cx + d$ for some $c, d \in \mathbb{R}$. This is compatible with $w(a) = a^k$ only when $k = 0$ or $k = 1$.

q, and q' over p', exhibiting the common ratio effect as a violation of the independence axiom.

Given the adaption costs, we observe that $p \succ q$ and $q' \succ p'$ verifying the common ratio effect. This is true since $v(\delta_x)-c(v) = 0.84 > 0.69 = u(\delta_x)-c(u)$, the DM adapts her risk assessments to v when evaluating p. Hence, we have $V(p) = 0.84$. On the other hand, we have $V(q) = 0.8$ implying that $p \succ q$. When it comes to evaluating p', we observe that $0.25\, u(\delta_x) - c(u) \approx 0.17 > 0.10 \approx 0.25\, v(\delta_x)-c(v)$. Thus, we have $V(p') \approx 0.17$. And finally, we have $V(q') = 0.2$ implying that $q' \succ p'$, as desired. □

Next we consider another simple example that illustrates that the ability to adapt risk assessments can generate the magnitude effect.

Magnitude effect. The magnitude effect refers to the choice situations where the DM is risk seeking for small gains, while turning progressively to risk aversion as magnitude of the outcomes increase. More formally, following Prelec and Lowenstein (1991), let $p = (x, [\alpha]\,;\, 0, [1-\alpha])$ and $q = (y, [\beta]\,;\, 0, [1-\beta])$, as well as let $q' = (\lambda y, [\beta]\,;\, 0, [1-\beta])$ and $p' = (\lambda x, [\alpha]\,;\, 0, [1-\alpha])$ where $\lambda > 1$, $x > y > 0$ and $\alpha < \beta$. Prelec and Lowenstein (1991) define the magnitude effect as the choice behavior such that $p \sim q$ implies $q' \succ p'$. Note that, loosely speaking, lottery p is riskier than q and lottery p' is riskier than q'.

Let $w = 0$, $b = 100$. Suppose the DM has ARA preferences $\succsim$ with a cost function c defined as follows: (i) $c(u) = 0$ for $u \in \mathcal{U}$ where $u(\delta_x) = (\frac{x+40}{150})^3$ for $x \in (0, 100)$, (ii) $c(v) = 0.1$ for $v \in \mathcal{U}$ where $v(\delta_x) = (\frac{x}{100})^{0.9}$ for $x \in (0, 100)$, and (iii) $c(v') = \infty$ for all $v' \in \mathcal{U} \setminus \{u, v\}$. Consider lotteries $p = (20, [0.45]\,;\, 0, [0.55])$ and $q = (15, [0.6]\,;\, 0, [0.4])$. Since $0.45\times(0.2)^{0.9}-0.1 < 0.45\times(0.4)^3$, we have $V(p) = 0.45 \times (0.4)^3 \approx 0.029$. Since $0.6 \times (0.15)^{0.9} - 0.1 < 0.6 \times (0.366)^3$, we have $V(q) = 0.6 \times (0.366)^3 \approx 0.029$. Thus, we have $p \sim q$. Now let $\lambda = 4$, and consider lotteries $p' = (80, [0.45]\,;\, 0, [0.55])$ and $q' = (60, [0.6]\,;\, 0, [0.4])$. Since $0.45 \times (0.8)^{0.9} - 0.1 > 0.45 \times (0.8)^3$, we have $V(p') = 0.45 \times (0.8)^{0.9} - 0.1 \approx 0.268$. Since $0.6 \times (0.6)^{0.9} - 0.1 > 0.6 \times (0.66)^3$, we have $V(q') = 0.6 \times (0.6)^{0.9} - 0.1 \approx 0.279$. Hence, we have $q' \succ p'$ demonstrating the magnitude effect. □

In particular, we have just demonstrated that for small risks, the DM might have risk seeking preferences measured by a convex function like u, while for large risks, she may have risk averse preferences measured by a concave function like v.[9] Finally, we consider an argument about the possibility of accommodating both increasing relative and decreasing absolute risk aversion by ARA preferences.

Relative and absolute risk aversion. Holt and Laury (2002) argue that in order to avoid absurd levels of high risk aversion when stakes are high while allowing also for increasing relative risk aversion (as several experiments suggest) one needs to have a model of risk preferences exhibiting decreasing absolute risk aversion. For this purpose, they consider the expected utility model with the hybrid "power-expo" function $U(x) = \frac{1-\exp(-\alpha x^{1-r})}{\alpha}$ and show that this model fits the data well. While this two parameter model can serve their purpose, we now argue that a two parameter specification of our ARA model can provide an alternative to explain the data. For this, suppose that the DM has ARA preferences and receives utility from each outcome $x \in [0, 1]$ by an iso-elastic concave utility function $u(x) = x^{1-\theta(x)}$ where $\theta(x) : [0, 1] \to [0, 1]$ is an increasing function of x such that $\theta(x)/x$ is decreasing. Then, since $\theta(x)$ is determined by an optimization problem, a standard envelope theorem argument implies that we have $-\frac{u''(x)x}{u'(x)} = \theta(x)$ and $-\frac{u''(x)}{u'(x)} = \frac{\theta(x)}{x}$ showing that the ARA model can capture both increasing relative and decreasing absolute risk aversion. Moreover, the cost function c can provide additional freedom to help fit the data better. □

7. Conclusion

In this paper, we analyze the behavior of a DM who can exercise costly effort to adapt her risk assessments, thereby improving the evaluation of her risky prospects. We provide an axiomatic characterization of the ARA preferences and show how the costs of adaption can be elicited and compared across individuals. We also show that the ARA preferences are related to many well-known models of decision making under risk and argue that adaption can also be relevant in a variety of other settings, including decision making about effort provision, insurance purchase, and portfolio choice.

We show that adaption of risk assessments (i) induces an increase in desire for basic lotteries, leading to systematic violations of the independence axiom, (ii) provides a novel source for well documented violations of expected utility model (e.g., the common consequence, common ratio, and magnitude effect), (iii) generates unintended negative effects in a moral hazard problem, and (iv) provides a flexible framework to fit experimental data. The ARA preferences model is therefore sufficiently general to rationalize a wide variety of choices under risk, while having enough structure to identify viable parameters from choice data.

Data availability

No data was used for the research described in the article

Appendix

A.1. Preliminaries

Let Σ denote the Borel sigma-algebra over $\mathcal{U}$, and let $B(\Sigma)$ be the set of bounded Σ-measurable functions mapping $\mathcal{U}$ to $\mathbb{R}$. When endowed with the sup-norm metric, $B(\Sigma)$ is a Banach space. The topological dual of $B(\Sigma)$ is the space $ba(\Sigma)$ of all bounded and finitely-additive set functions $\mu : \Sigma \to \mathbb{R}$, the duality being $\langle \varphi, \mu \rangle = \int_{\mathcal{U}} \varphi(u)\,\mu(du)$ for all $\varphi \in B(\Sigma)$ and all $\mu \in ba(\Sigma)$ (see, e.g., Dunford and Schwartz, 1958, p. 258). For $\varphi, \psi \in B(\Sigma)$, we write $\varphi \geq \psi$ if $\varphi(u) \geq \psi(u)$ for all $u \in \mathcal{U}$. Let $\Delta(\mathcal{U})$ be the set of all finitely-additive Borel probability measures on $\mathcal{U}$, with a typical element $\pi \in \Delta(\mathcal{U})$.

Let Φ be a non-empty subset of $B(\Sigma)$, and Φ_c be the constant functions in Φ. Set Φ is called a *tube* if $\Phi = \Phi + \mathbb{R}$. A functional $I : \Phi \to \mathbb{R}$ is *(i) normalized* if $I(k) = k$ for all $k \in \Phi_c$, *(ii) monotone* if $\varphi \geq \psi$ implies $I(\varphi) \geq I(\psi)$ for all $\varphi, \psi \in \Phi$, *(iii) translation invariant* if $I(\alpha\varphi+(1-\alpha)k) = I(\alpha\varphi)+(1-\alpha)k$ for all $\varphi \in \Phi, k \in \Phi_c$, and $\alpha \in [0, 1]$, such that $\alpha\varphi, \alpha\varphi + (1-\alpha)k \in \Phi$, *(iv) vertically invariant* if $I(\varphi + k) = I(\varphi) + k$ for all $\varphi \in \Phi$ and $k \in \Phi_c$ such that $\varphi + k \in \Phi$, and a *(v) niveloid* if $I(\varphi) - I(\psi) \leq \sup_{u\in\mathcal{U}}(\varphi(u) - \psi(u))$ for all $\varphi, \psi \in \Phi$.[10]

For notational convenience, we denote $\alpha p + (1-\alpha)q$ by $p\alpha q$ for $p, q \in P$ and $\alpha \in [0, 1]$. Let $P^o = \{p \in P : p(w) > 0\}$ be the set of lotteries whose supports consist of the worst outcome w. For $p \in P$, define $\phi_p : \mathcal{U} \to \mathbb{R}$ by $\phi_p(u) = u(p)$ for all $u \in \mathcal{U}$. Since $u(p) \in [0, 1]$ for all $u \in \mathcal{U}$, $\phi_p \in B(\Sigma, [0, 1])$, where $B(\Sigma, [0, 1])$ denotes the functions in $B(\Sigma)$ which assume values in $[0, 1]$. Let $\Phi = \{\phi_p : p \in P\}$ and $\Phi^o = \{\phi_p : p \in P^o\}$. Clearly $0 \in \Phi^o$ and $\Phi^o \subseteq \Phi$. Moreover, since $\phi_{p\alpha q} = \alpha\phi_p + (1-\alpha)\phi_q$ for any $p, q \in P$ and $\alpha \in [0, 1]$, both Φ and Φ^o are convex sets.

9 This is basically a special case of the Hypothesis II phenomenon that Machina (1982) defines using a generalized expected utility model.

10 Clearly, a niveloid is Lipschitz continuous. Moreover, Cerreia-Vioglio et al. (2014) show that a niveloid is a monotone vertically invariant functional, while the converse is true whenever its domain is a tube.

A.2. Lemmas

In this Section, we state and prove two lemmas that are used to establish the results in the text. The first lemma provides a representation for a binary relation satisfying Axioms 1–5. The second lemma establishes that there is a common solution to the adaption choice problem for a collection of lotteries if and only if the DM is neutral with respect to the mixture of these lotteries.

Implications of Axioms 1–5: The following lemma obtains several results which we use in proving Theorems 1 and 2 in the text:

Lemma 1. *Let $\succsim$ be a binary relation on P that satisfies Axioms 1–5. Then:*

(1) Every lottery $p \in P$ has a basic lottery equivalent $b_p \in P_{bw}$ such that $p \succsim q$ if and only if $b(p) \geq b(q)$.
(2) The function c^* defined on $\mathcal{U}$ by $c^*(u) = \sup_{p \in P} u(p) - b(p)$ for all $u \in \mathcal{U}$ is non-negative lower-semicontinuous and $c^*(u) < \infty$ for some $u \in \mathcal{U}$.
(3) The functional $W : P \to \mathbb{R}$, defined by $W(p) = \max_{u \in \mathcal{U}} u(p) - c^*(u)$ for all $p \in P$ represents $\succsim$.

Proof. Let $\succsim$ be a binary relation on $\mathcal{A}$ that satisfies Axioms 1–5.
[**Part (i)**]: Since X is a finite set, we have a unique pair of best and worst outcomes $b, w \in X$ such that $\delta_b \succ \delta_x \succ \delta_w$ for any $x \in X$ with $x \neq b, w$. Clearly, by Axiom 3, $\delta_b \succsim p \succsim \delta_w$ for any $p \in P$, and so by Axiom 2, the following (non-empty) sets, whose union is equal to $[0, 1]$, must be closed: $\{\alpha \in [0, 1] : \delta_b \alpha \delta_w \succsim p\}$ and $\{\alpha \in [0, 1] : p \succsim \delta_b \alpha \delta_w\}$. Since $[0, 1]$ is connected, these two sets must intersect; that is, there must exist some $\alpha_p \in [0, 1]$ such that $p \sim \delta_b \alpha_p \delta_w$. By Axiom 3, α_p must be unique. Let $b_p \in P_{bw}$ be equal to this unique basic lottery $\delta_b \alpha_p \delta_w$ and call the unique mixture weight $b(p)$. Note that by Axiom 3, $p \succsim q$ if and only if $b(p) \geq b(q)$.
[**Part (ii)**]: For any $u \in \mathcal{U}$ and $p \in P_{bw}$, $u(p) - b(p) = 0$, and so c^* is non-negative. Since c^* is the supremum of continuous functions, it is lower semicontinuous. Finally, since $u(p) \in [0, 1]$ for any $u \in \mathcal{U}$ and $p \in P$, it follows that $u(p) - b(p) \in [-1, 1]$, and so $c^*(u) \in [-1, 1]$ showing that $c^*(u) < \infty$ for some $u \in \mathcal{U}$.
[**Part (iii)**]: To establish the desired representation, we show that there is a monotone normalized convex niveloid $I : \Phi \to \mathbb{R}$ such that, for all lotteries p and q, $p \succsim q$ if and only if $I(\phi_p) \geq I(\phi_q)$. Following the approach in Maccheroni et al. (2006), an application of Fenchel–Moreau duality then establishes $I(\phi_p) = \max_{u \in \mathcal{U}} u(p) - c^*(u)$ for all $p \in P$. We start the proof by defining a functional I^o on Φ^o, and then use Axiom 2 to extend the functional to Φ.

Let $I^o : \Phi^o \to \mathbb{R}$ be a functional defined by $I^o(\phi_p) = b(p)$ for all $p \in P$, where $b_p \in P_{bw}$ denotes a basic lottery equivalent of p. For any two lotteries $p, q \in P^o$ with basic lottery equivalents b_p and b_q, $p \succsim q$ if and only if $b_p \succsim b_q$, and so $I^o(\phi_p) \geq I^o(\phi_q)$ if and only if $p \succsim q$ by part (i). Note that for any $p, q \in P$, if $p \neq q$, then we can easily find some $u \in \mathcal{U}$ such that $u(p) \neq u(q)$, and so $\phi_p \neq \phi_q$. Thus, whenever $\phi_p = \phi_q$, we must have $p = q$ implying that I^o is well-defined.

For the rest of the proof, we proceed in steps to establish that I^o is a monotone normalized convex niveloid.
Step 1 (I^o is monotone): Let $p, q \in P$ such that $\phi_p \geq \phi_q$. By definition, $u(p) \geq u(q)$ for all $u \in \mathcal{U}$. But this can happen only when either $p = q$ or $p \rhd_1 q$. In the first case, we have $I^o(\phi_p) = I^o(\phi_q)$ and in the second case, we have $I^o(\phi_p) > I^o(\phi_q)$ by Axiom 3 and the fact that $p \succ q$ if and only if $I^o(\phi_p) > I^o(\phi_q)$. Thus, $\phi_p \geq \phi_q$ implies $I^o(\phi_p) \geq I^o(\phi_q)$.
Step 2 (I^o is normalized): Let $k \in \mathbb{R}$ such that $k \in \Phi^o$. This implies there is a basic lottery $p \in P_{bw}$ such that $k = \phi_p = b(p)$. Hence, $I^o(k) = I^o(\phi_p) = b(p) = k$.
Step 3 (I^o is convex): Let $p, q \in P^o$ and $\alpha \in [0, 1]$. Note that $p\alpha q \in P^o$ and so $\phi_{p\alpha q} \in \Phi^o$. By part (i) there exist some $b_p, b_q \in P_{bw}$. By Axiom 5, we have $b_p \alpha b_q \succsim p\alpha q$, and so

$$\begin{aligned}\alpha I^o(\phi_p) + (1-\alpha) I^o(\phi_q) &= \alpha b(p) + (1-\alpha) b(q) = I^o(\phi_{b_p \alpha b_q}) \\ &\geq I^o(\phi_{p\alpha q}) = I^o(\alpha\phi_p + (1-\alpha)\phi_q).\end{aligned}$$

Step 4 (I^o is translation invariant): Let $p \in P^o$, $q \in P_{bw}$ with $b(q) = k$, and $\alpha \in (0, 1)$. By Axiom 3, we have $\delta_b \alpha \delta_w \succsim p\alpha\delta_w \succsim \delta_w \alpha \delta_w$. The argument used in the proof of part (i) above yields a $\beta \in [0, 1]$ such that $p\alpha\delta_w \sim (\delta_b\alpha\delta_w)\beta(\delta_w\alpha\delta_w)$. Thus, letting $r = \delta_b \beta \delta_w \in P_{bw}$ implies $p\alpha\delta_w \sim r\alpha\delta_w$. By Axiom 4, it follows that $p\alpha q \sim r\alpha q$, and so

$$\begin{aligned} I^o(\alpha\phi_p + (1-\alpha)k) &= I^o(\phi_{p\alpha q}) = I^o(\phi_{r\alpha q}) \\ &= \alpha u(r) + (1-\alpha) u(q) = \alpha u(r) + (1-\alpha) k \\ &= \alpha u(r) + (1-\alpha) u(\delta_w) + (1-\alpha) k \\ &= I^o(\phi_{r\alpha\delta_w}) + (1-\alpha) k = I^o(\phi_{p\alpha\delta_w}) + (1-\alpha) k \\ &= I^o(\alpha\phi_p + (1-\alpha)\phi_{\delta_w}) + (1-\alpha) k \\ &= I^o(\alpha\phi_p) + (1-\alpha) k, \end{aligned}$$

establishing that I^o is translation invariant.
Step 5 (I^o is vertically invariant): The result follows from Step 1 of the proof of Lemma 20 in Maccheroni et al. (2004) once we show that for all $p \in P^o$ and $k \in \mathbb{R}$ such that $\phi_p + k \in \Phi^o$, there exists some $\alpha \in (0, 1)$ satisfying $\frac{\phi_p}{\alpha}, \frac{\phi_p + k}{\alpha} \in \Phi^o$. To see this, let $p^\theta = \theta p + (1-\theta)\delta_w$ for any $p \in P$ and $\theta > 0$. Note that for any given $p \in P^o$ there exists some $\theta > 1$ such that $p^\theta \in P^o$. Clearly if $p^\theta \in P^o$, then $p^{\theta'} \in P^o$ for any $\theta' < \theta$.

Let $p \in P^o$ and $k \in \mathbb{R}$ such that $\phi_p + k \in \Phi^o$. Pick any $\theta > 1$ such that $p^\theta \in P^o$, and call it θ_p. Since $\phi_p + k \in \Phi^o$, there exists some $q \in P^o$ such that $\phi_q = \phi_p + k$. Since $q \in P^o$, there exists some $\theta > 1$ such that $q^\theta \in P^o$. Pick any such $\theta > 1$ for $q \in P^o$, and call it θ_q. Let $\theta_* = \min\{\theta_p, \theta_q\}$. Observe that $u \cdot p^\theta = \theta(u \cdot p)$ for any $u \in \mathcal{U}$, $p \in P$, and $\theta > 0$. Therefore, we have $\phi_{p^{\theta_*}} = \theta_* \cdot \phi_p \in \Phi^o$ and $\phi_{q^{\theta_*}} = \theta_* \cdot \phi_q \in \Phi^o$. Let $\alpha = 1/\theta_*$. We have shown $\frac{\phi_p}{\alpha}, \frac{\phi_p + k}{\alpha} \in \Phi^o$ as desired.
Step 6 (I^o is a niveloid): Since I^o is vertically invariant, functional $I^* : \Phi^o + \mathbb{R} \to \mathbb{R}$, defined by $I^*(\phi + k) = I^o(\phi) + k$ for all $\phi \in \Phi^o$, is the unique vertically invariant extension of I^* to the tube generated by Φ^o (Maccheroni et al., 2004, Lemma 22). Moreover, since Φ^o is a convex set and I^o is a convex functional, the obvious adaption of the arguments in Maccheroni et al. (2004, Lemma 22) establishes that I^* is also convex. We now show that I^* must also be monotone. By the first paragraph in the proof of Maccheroni et al. (2004, Lemma 24), it is sufficient to show that if $\phi, \psi \in \Phi^o$ and $\phi + k \geq \psi$, then $I^*(\phi + k) \geq I^*(\psi)$.

Let $p, q \in P^o$ and $k \in \mathbb{R}$ such that $\phi_p + k \geq \phi_q$. Clearly there exists $\alpha \in (0, 1)$ such that $\alpha(\phi_p + k) + (1-\alpha)\phi_q = \alpha\phi_p + (1-\alpha)\phi_q + \alpha k \in \Phi^o$. Moreover, since $\phi_p + k \geq \phi_q$, $\alpha(\phi_p + k) + (1-\alpha)\phi_q \geq \phi_q$. Now assume, for contradiction, that $I^*(\phi_p + k) < I^*(\phi_q)$. Since I^* is convex, this would imply

$$\begin{aligned} I^o(\phi_q) &= \alpha I^*(\phi_q) + (1-\alpha) I^*(\phi_q) > \alpha I^*(\phi_q + k) + (1-\alpha) I^*(\phi_q) \\ &\geq I^*(\alpha(\phi_p + k) + (1-\alpha)\phi_q) = I^o(\alpha(\phi_p + k) + (1-\alpha)\phi_q), \end{aligned}$$

which contradicts that I^o is monotone, and thus I^* must be monotone.

Since I^o is vertically invariant, and its unique vertically invariant extension to the tube generated by Φ^o, I^*, is monotone, I^o is a niveloid by Maccheroni et al. (2004, Lemma 23). In sum, we have shown that I^o is a normalized convex niveloid.

We now extend I^o to Φ. For any lottery $p \in P$ and number $m \in \mathbb{N}$, define $p^m = p \frac{m-1}{m} \delta_w$ and denote $\varphi_p^m = \varphi_{p^m}$. Note that for all $p \in P$ and $m \in \mathbb{N}$, $p^m \in \mathcal{P}^o$ and $\phi_p^m \to \phi_p$ uniformly as $m \to \infty$. Define a functional $I : \Phi \to \mathbb{R}$ by $I(\phi_p) = \lim_{m \to \infty} I^o(\phi_p^m)$ for all $p \in P$. Since I^o is a niveloid, it is a continuous function, and so I^o preserves convergence. Thus, for any lottery $p \in P$, the sequence $\{I^o(\phi_p^m)\}_{m\in\mathbb{N}}$ converges to a point in [0, 1] showing that I is well-defined. The following arguments show that I preserves the properties of I^o, i.e., it is also a normalized convex niveloid.

Since I^o is a niveloid, we have $I^o(\phi_p^m)-I^o(\phi_q^m) \leq \max\left(\phi_p^m - \phi_q^m\right)$ for any $p, q \in P$, and $m \in \mathbb{N}$. Thus we obtain,

$$\begin{aligned} I(\phi_p) - I(\phi_q) &= \lim_{m\to\infty}\left(I^o(\phi_p^m)\right) - \lim_{m\to\infty}\left(I^o(\phi_q^m)\right) \\ &= \lim_{m\to\infty}\left(I^o(\phi_p^m) - I^o(\phi_q^m)\right) \\ &\leq \lim_{m\to\infty}\left(\max\left(\phi_p^m - \phi_q^m\right)\right) \\ &= \lim_{m\to\infty}\frac{m-1}{m}\left(\max\left(\phi_p - \phi_q\right)\right) \\ &= \max\left(\phi_p - \phi_q\right), \end{aligned}$$

establishing that I is a niveloid.

Clearly I is normalized. Now let $p, q \in P$, and $\alpha \in [0, 1]$. Since Φ is a convex set, $\alpha\phi_p + (1-\alpha)\phi_q \in \Phi$, and so by convexity of I^o we have

$$\begin{aligned} I(\alpha\phi_p + (1-\alpha)\phi_q) &= \lim_{m\to\infty}\left(I^o(\phi_{p\alpha q}^m)\right) \\ &= \lim_{m\to\infty}\left(I^o\alpha\phi_p^m + (1-\alpha)\phi_q^m\right) \\ &\leq \lim_{m\to\infty}\left(\alpha I^o(\phi_p^m) + (1-\alpha)I^o(\phi_q^m)\right) \\ &= \alpha\lim_{m\to\infty} I^o(\phi_p^m) + (1-\alpha)\lim_{m\to\infty} I^o(\phi_q^m) \\ &= \alpha I(\phi_p) + (1-\alpha)I(\phi_q), \end{aligned}$$

showing that I is convex. As a result, I is a normalized convex niveloid which assumes values in [0, 1].

Since Φ is a convex subset of $B(\Sigma, [0, 1])$ and I is a normalized convex niveloid, the obvious adaption of the arguments in the proof of Maccheroni et al. (2004, Lemma 27) establishes that $I(\phi) = \max_{\pi\in\Delta(\mathcal{U})}(\langle\phi, \pi\rangle - c(\pi))$ for all $\varphi \in \Phi$, where $c : \Delta(\mathcal{U}) \to [0, \infty]$ is defined as $c(\pi) = \sup_{p\in P}\left(\langle\phi_p, \pi\rangle - b(p)\right)$. Note that for any given $\pi \in \Delta(\mathcal{U})$, we have $u_\pi(p) = \int u(p)d(\pi)$ for some $u_\pi \in \mathcal{U}$ for all $p \in P$. Thus, for any $p \in P$, we have $I(\phi_p) = \max_{u\in\mathcal{U}}\left(\phi_p(u) - c^*(u)\right)$, where $c^* : \mathcal{U} \to [0, \infty]$ is defined as in part (ii) above which satisfies $c^*(u_\pi) = c(\pi)$.

It remains to show that, for all $p, q \in P$, $p \succsim q$ if and only if $I(\phi_p) \geq I(\phi_q)$. We establish the contrapositive for each direction.

First, suppose that $p \succ q$. Using part (i), we can find $r, s \in P_{bw}$ such that $p \succ r \succ s \succ q$. Then by Axiom 2, there exists some $M \in \mathbb{N}$ such that for all $m \geq M$, $p^m \succsim r \succ s \succsim q^m$.

Otherwise, it must be the case that $r \succsim p$ or $q \succsim s$, a contradiction. Thus, for all $m \geq M$, we must have $I^o(\varphi_p^m) \geq b(r) > b(s) \geq I^o(\varphi_q^m)$. As such, we obtain $I(\varphi_p) \geq b(r) > b(s) \geq I(\varphi_q)$ since weak inequalities are preserved in the limit, and so $I(\phi_p) > I(\phi_q)$.

For the converse, suppose that $I(\phi_p) > I(\phi_q)$. By construction, $1 \geq I(\phi_p)$ and $I(\phi_1) \geq 0$. Hence, there exist $r, s \in P_{bw}$ such that $I(\phi_p) > b(r) > b(s) > I(\phi_q)$.

Since I is continuous, $I^o(\phi_p^m) \geq b(r) > b(s) \geq I^o(\phi_q^m)$ for all $m \geq M$ for some $M \in \mathbb{N}$ implying that for all $m \geq M$, $p^m \succsim r \succ s \succsim q^m$. Hence, by Axiom 2, it follows that $p \succsim r \succ s \succsim q$, and so $p \succ q$.

As a result, the function $W : P \to \mathbb{R}$, defined by $W(p) = I(\phi_p)$ represents $\succsim$. □

Neutral desire for basic lotteries: The next lemma is used in the proofs of Propositions 1 and 2. It characterizes when there is a common solution to the choice of adaption problem for a given finite collection of lotteries.

Let $W : P \to \mathbb{R}$ and $\mathcal{D} : P \twoheadrightarrow \mathcal{U}$ represent, respectively, the value function and policy correspondence of the adaption choice problem with parameters (u, c) as identified in Lemma 1. That is, for any $p \in P$ let $W(p) = \max_{u\in\mathcal{U}}\left(\phi_p(u) - c(u)\right)$ and $\mathcal{D}(p) = \arg\max_{u\in\mathcal{U}}\left(\phi_p(u) - c(u)\right)$. We observe that W has the following convexity property, $W(\sum_i \alpha_i p^i) \leq \sum_i \alpha_i W(p^i)$ for $p^1, \ldots, p^N \in P$ and $\alpha_1, \ldots, \alpha_N \in (0, 1)$ such that $\sum_{i=1}^N \alpha_i = 1$.

Lemma 2. *Let $p^1, \ldots, p^N \in P$ and $\alpha_1, \ldots, \alpha_N \in (0, 1)$ such that $\sum_{i=1}^N \alpha_i = 1$. Then, $W(\sum_i \alpha_i p^i) = \sum_i \alpha_i W(p^i)$ if and only if $\bigcap_i \mathcal{D}(p^i) \neq \emptyset$.*

Proof (*Necessity*).: Let $p^1, \ldots, p^N \in P$ and $\alpha_1, \ldots, \alpha_N \in (0, 1)$ with $\sum_{i=1}^N \alpha_i = 1$ and $W(\sum_i \alpha_i p^i) = \sum_i \alpha_i W(p^i)$. We proceed by induction on N. If $N = 1$, then the result trivially holds. Now suppose that $N > 1$, and the implication holds for $N - 1$.

Without loss of generality, let $\alpha_1 = \min_i \alpha_i$, and set $q = \frac{\alpha_2}{1-\alpha_1}p^2 + \cdots + \frac{\alpha_N}{1-\alpha_1}p^N$. Since $\alpha_i/(1-\alpha_1) \leq 1$ for all $i = 2, \ldots, N$, we have $q \in P$. By the convexity property of W, we have $W(q) \leq \sum_{i=2}^N \left(\frac{\alpha_i}{1-\alpha_1}\right) W(p^i)$ and

$$\begin{aligned} &\sum_i \alpha_i W(p^i) = W\left(\sum_i \alpha_i p^i\right) = W\left(p^1\alpha_1 q\right) \\ &\leq \alpha_1 W(p^1) + (1-\alpha_1)W(q). \end{aligned}$$

Hence, $\sum_{i=2}^N \alpha_i W(p^i) = (1-\alpha_1)W(q)$ and $\alpha_1 W(p^1) + (1-\alpha_1)W(q) = W(p^1\alpha_1 q)$. Now choose some $u \in \mathcal{D}(p^1\alpha_1 q)$. Then,

$$\alpha_1\phi_{p^1}(u) + (1-\alpha_1)\phi_q(u) - W\left(p^1\alpha_1 q\right) = c(u) \geq \phi_{p^1}(u) - W\left(p^1\right).$$

Replacing $W(p^1)$ with $\frac{1}{\alpha_1}W\left(p^1\alpha_1 q\right) - \frac{1-\alpha_1}{\alpha_1}W(q)$, and rearranging, we get

$$(1-\alpha_1)\phi_q(u) - \frac{1-\alpha_1}{\alpha_1}W(q) \geq (1-\alpha_1)\phi_{p^1}(u) - \frac{1-\alpha_1}{\alpha_1}W\left(p^1\alpha_1 q\right).$$

Multiplying both sides of the inequality by $\alpha_1/(1-\alpha_1)$ and adding $(1-\alpha_1)\phi_q(u)$, we get $\phi_q(u) - W(q) \geq \alpha_1\phi_{p^1}(u) + (1-\alpha_1)\phi_q(u) - W\left(p^1\alpha_1 q\right)$ which implies that $\phi_q(u) - W(q) \geq c(u)$, and so $u \in \mathcal{D}(q)$.

By an analogous argument, $u \in \mathcal{D}(p^1)$ and thus, $\mathcal{D}\left(p^1\alpha_1 q\right) \subset \mathcal{D}(p^1) \cap \mathcal{D}(q)$. Since $\sum_{i=2}^N \alpha_i W(p^i) = (1-\alpha_1)W(q)$, by the inductive assumption, $\mathcal{D}(q) \subset \mathcal{D}(p^i)$ for all $i = 2, \ldots, N$, and so $\mathcal{D}\left(\sum_i \alpha_i p^i\right) \subset \mathcal{D}(p^i)$ for all $i = 1, \ldots, N$. Since $\mathcal{D}\left(\sum_i \alpha_i p^i\right) \neq \emptyset$, we have $\bigcap_i \mathcal{D}(p^i) \neq \emptyset$.

[**Sufficiency**]: Let $u \in \bigcap_i \mathcal{D}\left(p^i\right)$. Then $\sum_i \alpha_i W(p^i) = \sum_i \alpha_i\phi_{p^i}(u) - c(u)$ implying $\sum_i \alpha_i W(p^i) \leq W\left(\sum_i \alpha_i p^i\right)$. On the other hand, the convexity property of W implies $\sum_i \alpha_i W(p^i) \geq W\left(\sum_i \alpha_i p^i\right)$, and so $W(\sum_i \alpha_i p^i) = \sum_i \alpha_i W(p^i)$. □

A.3. Proofs for the results in the text

Proof of Theorem 1. It is straightforward to show that an adaptive risk assessment preference satisfies Axioms 1–5. For the converse, let $\succsim$ be a binary relation that satisfies Axioms 1–5. Then by Lemma 1, c^* represents $\succsim$ and so $\succsim$ is an adaptive risk assessment preference. □

Proof of Proposition 1. It is straightforward to prove that a preference $\succsim$ defined in Proposition 1 is an adaptive risk assessment preference which satisfies the NeDBL axiom.

For the converse, suppose $\succsim$ is an adaptive risk assessment preference which satisfies the NeDBL axiom. Let $W : P \to \mathbb{R}$ be given by $W(p) = \max_{u\in\mathcal{U}}\phi_p(u) - c(u)$ which represents $\succsim$ with the minimal cost function c as identified in Lemma 1.

Let $p \in P$ and $q \in P_{bw}$ be such that $p \sim q$. By NeDBL, $p \sim \alpha p + (1-\alpha)q$ for any $\alpha \in (0, 1)$, and so $W(\alpha p + (1-\alpha)q) = \alpha W(p) + (1-\alpha)W(q)$. As such, by Lemma 2, there exists some

$u_p \in \mathcal{D}(p) \cap \mathcal{D}(q)$, where $\mathcal{D} : P \twoheadrightarrow \mathcal{U}$ is the policy correspondence defined as in Lemma 1. Since c is the minimal cost function, it is grounded and so $c(u_p) = 0$. Hence, $W(p) = \phi_p(u_p) - c(u_p) = \max_{u \in U} \phi_p(u)$ where $U = \{u \in \mathcal{U} : c(u) = 0\}$. □

Proof of Proposition 2. Let $\succsim$ be an adaptive risk assessment preference with a representation c. It is straightforward to show that if there is a common solution $u \in \mathcal{U}$ to the adaption problem of $\succsim$ for each lottery $p \in P$, then $\succsim$ satisfies the NoDBL axiom. For the converse, define the value function $W : P \to \mathbb{R}$ as in Lemma 1. Let $p, q \in P$ and $\alpha \in (0, 1)$, and let b_p and b_q be basic lottery equivalents of p and q, respectively. By NoDBL, $b_p \alpha b_q \sim b_p \alpha q \sim p \alpha q$. Thus, $W(p\alpha q) = \alpha W(p) + (1 - \alpha)W(q)$. By induction, for lotteries $p^1, \ldots, p^N \in P$ and $\alpha_1, \ldots, \alpha_N \in [0, 1]$ such that $\sum_i \alpha_i = 1$, $W(\sum_i \alpha_i p^i) = \sum_i \alpha_i W(p^i)$.

Let $\mathcal{D} : P \twoheadrightarrow \mathcal{U}$ be the policy correspondence defined as in Lemma 1. By Lemma 2, it follows that $\bigcap_i \mathcal{D}(p^i) \neq \emptyset$. Hence, the collection of closed sets $\{\mathcal{D}(p) : p \in P\}$ has the finite intersection property. Since $\mathcal{U}$ is compact, it follows that there exists some $u \in \bigcap_{p \in P} \mathcal{D}(p)$, and so $V(p) = \phi_p(u) - c(u)$ for all lotteries $p \in P$. Thus, $p \succsim q$ if and only if $\phi_p(u) \geq \phi_q(u)$ for all lotteries $p, q \in P$. Clearly, u must be unique. Otherwise, there is some $v \in \bigcap_{p \in P} \mathcal{D}(p)$ with $v \neq u$. Hence, there is some $x \in X$ such that $u(\delta_x) \neq v(\delta_x)$ implying that $\delta_x \nsim \delta_x$, a contradiction. □

Proof of Theorem 2. Let $\succsim$ be an adaptive risk assessment preference represented by c. By Lemma 1, c^* also represents $\succsim$. It therefore remains to show that $c \geq c^*$ (establishing c^* as the minimal cost function). By way of contradiction, suppose $c(u) < c^*(u)$ for some $u \in \mathcal{U}$. Then, by definition of c^*, there exists a lottery $p \in P$ such that $\phi_p(u) - b(p) > c(u)$, i.e., $\phi_p(u) - c(u) > b(p)$. Hence, $\phi_p(u) - c(u) > \max_{v \in \mathcal{U}} \phi_p(v) - c(v)$, a contradiction. □

Proof of Corollary 1. Let c^* be a representation for an adaptive risk assessment preference where c^* is the minimal cost function. In the proof of Lemma 1, part (ii), we show that c^* is grounded. Since c^* is the supremum over linear functions, c^* is convex. Finally, to establish monotonicity, let $u, v \in \mathcal{U}$ with $u \geq v$. For all $p \in P$, $\phi_p(u) \geq \phi_p(v)$, and so $\sup_{p \in P} \big(\phi_p(u) - b(p)\big) \geq \sup_{p \in P} \big(\phi_p(v) - b(p)\big)$ implying $c^*(u) \geq c^*(v)$. □

Proof of Theorem 3. Let $\succsim_1$ and $\succsim_2$ be adaptive risk assessment preferences with canonical representations c_1^* and c_2^*, respectively.

[Necessity]: Suppose $\succsim_2$ has a stronger desire for simplicity than $\succsim_1$. Consider a lottery p and let $b_p^1 \sim_1 p$ and $b_p^2 \sim_2 p$. Since $\succsim_2$ has a stronger desire for simplicity than $\succsim_1$, $b_p^1 \succsim_1 b_p^2$ and so $b^1(p) \geq b^2(p)$. As a result, for any $u \in \mathcal{U}$,

$$c_2^*(u) = \sup_{p \in P} \big(\phi_p(u) - b^2(p)\big) \geq \sup_{p \in P} \big(\phi_p(u) - b^1(p)\big) = c_1^*(u).$$

[Sufficiency]: Suppose that $c_1^* \leq c_2^*$. Let $q \succ_1 p$ for some $p \in P$ and $q \in P_{bw}$. Then $q \succ_2 p$ follows since,

$$q(b) > \max_{u \in \mathcal{U}} \big(\varphi_p(u) - c_1(u)\big) \geq \max_{u \in \mathcal{U}} \big(\varphi_p(u) - c_2(u)\big). \quad \square$$

References

Aczel, J., 1966. In: Oser, H. (Ed.), Lectures on Functional Equations and Their Applications. Elsevier.

Alfitian, J., Sliwka, D., Vogelsang, T., 2021. When bonuses backfire: evidence from the workplace. Mimeo.

Allais, M., 1953. Le Comportement de l'Homme Rationnel Devant le Risque: Critique des Postulats et Axiomes de l'École amÉricaine. Econometrica 503–546.

Blavatskyy, P., Ortmann, A., Panchenko, V., 2021. On the experimental robustness of the allais paradox. Am. Econ. J. Microecon.

Cerreia-Vioglio, S., 2009. Maxmin expected utility on a subjective state space: convex preferences under risk. mimeo.

Cerreia-Vioglio, S., Dillenberger, D., Ortoleva, P., 2015. Cautious expected utility and the certainty effect. Econometrica 83 (2), 693–728.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., Montrucchio, L., 2011. Uncertainty averse preferences. J. Econom. Theory 146, 1275–1330.

Cerreia-Vioglio, S., Maccheroni, F., Marinacci, M., Rustichini, A., 2014. Niveloids and their extensions: risk measures on small domains. J. Math. Anal. Appl. 413, 343–360.

Chetty, R., Szeidl, A., 2007. Consumption commitments and risk preferences. Q. J. Econ. 122, 831–877.

Chew, S., 1989. Axiomatic utility theories with the betweenness property. Ann. Oper. Res. 19, 273–298.

Chew, S.H., Karni, E., Safra, Z., 1987. Risk aversion in the theory of expected utility with rank dependent probabilities. J. Econ. Theory 42, 370–381.

Conlisk, J., 1989. Three variants on the allais example. Amer. Econ. Rev. 79, 392–407.

Dekel, E., 1986. An axiomatic characterization of preferences under uncertainty: weakening the independence axiom. J. Econom. Theory 40, 304–318.

Dunford, N., Schwartz, J.T., 1958. Linear Operators: General Theory. Interscience Publishers.

Epstein, L.G., Zin, S., 1989. Substitution, risk aversion, and the temporal behavior of consumption and asset returns: a theoretical framework. Econometrica 57, 937–969.

Gneezy, U., Meier, S., Rey-Biel, P., 2011. When and why incentives (don't) work to modify behavior. J. Econ. Perspect. 25 (4), 191–210.

Grossman, S., Laroque, G., 1990. Asset pricing and optimal portfolio choice in the presence of illiquid durable consumption goods. Econometrica 58, 25–51.

Gul, F., 1991. A theory of disappointment in decision making under uncertainty. Econometrica 59, 667–686.

Holt, C.A., Laury, S.K., 2002. Risk aversion and incentive effects. Amer. Econ. Rev. 92 (5), 1644–1655.

Huck, S., Muller, W., 2012. Allais for all: revisiting the paradox in a large representative sample. J. Risk Uncertain. 44 (2), 261–293.

Kahneman, D., 1994. New challenges to the rationality assumptions. J. Inst. Theor. Econ. 150, 18–36.

Kahneman, D., Tversky, A., 1979. Prospect theory: an analysis of decision under risk. Econometrica 47 (2), 263–291.

Kamenica, E., 2012. Behavioral economics and psychology of incentives. Annu. Rev. Econ. 4, 427–452.

Klor, E.F., Kube, S., Winter, E., Zultan, R., 2014. Can higher rewards lead to less effort? Incentive reversal in teams. J. Econ. Behav. Organ. 97, 72–83.

Maccheroni, F., 2002. Maxmin under risk. Econom. Theory 19, 823–831.

Maccheroni, F., Marinacci, M., Rustichini, A., 2004. Variational representation of preferences under ambiguity. In: International Center for Economic Research Working Papers. 5/2004.

Maccheroni, F., Marinacci, M., Rustichini, A., 2006. Ambiguity aversion, robustness, and the variational representation of preferences. Econometrica 74 (6), 1447–1498.

Machina, M.J., 1982. Expected utility theory without the independence axiom. Econometrica 50, 277–323.

Prelec, D., Lowenstein, G., 1991. Decision making over time and under uncertainty: a common approach. Manage. Sci. 37 (7), 770–786.

Quiggin, J., 1982. A theory of anticipated utility. J. Econ. Behav. Organ. 3, 323–343.

Rabin, M., 2000. Risk aversion and expected utility theory: a calibration theorem. Econometrica 68 (5), 1281–1292.

Sarver, T., 2018. Dynamic mixture-averse preferences. Econometrica 86 (4), 1347–1382.

Schildberg-Horisch, H., 2018. Are risk preferences stable? J. Econ. Perspect. 32, 135–154.

Wagner, Z., Asiimwe, J.B., Levine, D., 2020. When financial incentives backfire: Evidence from a community health worker experiment in Uganda. J. Dev. Econ. 144 (102437).