Nano-optical Metrology with Phase Singularities

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We show how the exploitation of phase singularities in topologically structured light fields can enable dramatic sensitivity improvements in diffraction-based optical displacement and dimensional metrology. We reveal the mechanisms by which diffraction of structured light on an arbitrary scatterer can be orders of magnitude more sensitive to the shape and/or position of the object when it intersects with a phase singularity in the incident field.

Recent experiments (arxiv:2205.01475v; doi:10.1002/advs.202002886) have shown empirically that topologically structured illumination can improve optical measurement and imaging resolution over plane wave or unstructured (e.g. Gaussian) illumination. Picometre scale precision has been achieved and it has been inferred that this is due principally to the presence of the high phase gradients found within topologically structured fields.

Here we present an analytical description of this phenomenon, derived from scalar diffraction theory, which explains why topological light scattering is so sensitive, and which can thus enable the rational design of highly sensitive (sub-)nano-optical metrologies.

The diffraction pattern resulting from interaction of an arbitrary field $A(x)e^{i\phi(x)}$ with an arbitrary object can be written as the sum of three integrals:

$$U_1(x_0) = A(x)\exp\big(j\phi(x)\big)\int CW(x, x_0, z)dx,$$

$$U_2(x_0) = \int \frac{dA}{dx}\exp\big(j\phi(x)\big)\left[\int CW(x, x_0, z)dx\right]dx,$$

$$U_3(x_0) = -j\int \frac{d\phi}{dx}A(x)\exp\big(j\phi(x)\big)\left[\int CW(x, x_0, z)dx\right]dx.$$

Where $CW(x, x_0, z)$ represents a cylindrical wavelet emanating from each point in the incident plane. From here, it is seen that the total diffraction pattern depends *independently* on the incident field magnitude $A(x)$, the magnitude of the field derivative dA/dx and the magnitude of the phase derivative $d\phi/dx$. The second and third of these are negligible or zero in unstructured fields but can dominate in case of highly structured fields containing singularities and/or sub-diffraction energy hot-spots.

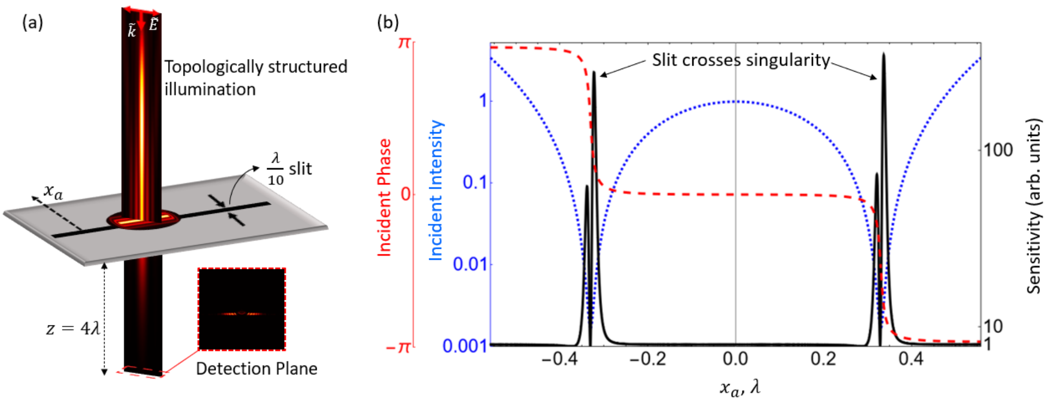

Fig. 1 – (a) Diffraction of a superoscillatory light field by a sub-wavelength slit in a PEC screen. (b) Correlation analysis of diffraction patterns obtained as the slit is scanned across the incident field. Incident field intensity and phase at the centre of the slit are plotted respectively as blue dotted and red dashed lines. The solid black line shows the magnitude of change in the transmission diffraction pattern, observed in a plane 4λ behind the slit, due to lateral translation of the slit by a distance $\lambda/5000$, as a measure of the sensitivity of scattering to the position of the slit with respect to structural features of the incident field, evaluated using the Pearson correlation coefficient.

This is illustrated in Fig. 1, which considers the case of a sub-wavelength slit (width 0.1λ) that is scanned across a super-oscillatory intensity hotspot flanked by a pair of phase singularities. By calculating the correlation coefficient between diffraction patterns for adjacent positions (between which the slit is translated by 0.0002λ) we find that scattering intensity is orders of magnitude more sensitive to sub-wavelength changes in the aperture position when the aperture interacts with a phase singularity of the incident field, as opposed to the regions of constant phase, and as compared to the plane wave case. Our work provides a quantitative explanation for the way in which deeply subwavelength resolution can be achieved with topologically structured light and will inform the further development of technologies for optical metrology and imaging.