A Memory in the Bond: Green Bond and Sectoral Investment Interdependence in a Fractionally Cointegrated VAR Framework

Tapas Mishra [*], Donghyun Park, Mamata Parhi, Gazi Salah Uddin and Shu Tian

Abstract

The urgency surrounding environmental sustainability has triggered an innovation of financing channels for climate and environmental projects. Green bond as one such channel has garnered immense interest from investors, with an implicit view that this fixed-income instrument is a relatively safer choice as an investment portfolio. Yet, the uncomfortable spread of greenwashing as a marketing spin has subjected green bonds to significant market volatility, at least as much as other financial assets or sectoral indices if not more. Whether green bonds as a financial instrument may incur losses to the extent of the loss in various sector indices, can be gauged by studying the nature of their contemporaneous growth. In this paper, we use daily data on green bonds and several S&P sectoral indices and a fractionally cointegrated vector autoregression framework (FCVAR) to study the extent to which green bonds dynamically co-move with various sectoral indices. Such a co-movement, if any, would elicit the extent to which a variation of uncertainty would determine an investor's inclination to the diversification of a portfolio between an investment in a sectoral index and a green bond. The identifying mechanism is the shock-dissipation speed, which also informs a policymaker before choosing the right instrument to stabilise the system. We show that the system-wide shocks indeed dissipate slower than could be predicted by a conventional cointegrated VAR system. Further, the property of the slow error correction within the dynamic system of Green Bond and sectoral S&P indices, for instance, may demonstrate the speed of adjustment of the global economy to sudden shocks. Rigorous predictions exercises complement our baseline conclusions.

Key Words: Green bond; S&P sectoral performance; Long memory error corrections; Fractionally cointegrated VAR

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1 Introduction

Financing of climate and environmentally sustainable projects requires huge funds[1], firstly because, firms are often slow to adapt to a new technology despite realising the huge potential of gains from such a strategic investment. Secondly, the spread of greenwashing[2] as a deceptive marketing spin by various corporations has subjected this asset-backed financial instrument to a significant degree of volatility. A study of the extent the green bond incurs contemporaneous losses or gains with other financial assets or indices (such as S&P 500 sectoral indices) can unravel the inherent nature of volatility of green bond. To an important extent, it can also establish how, under persistent uncertainty, disequilibrium shocks in the green bond index correct (with slow or greater speed) while co-moving with various sectoral indices (if at all there is any co-movement). This paper introduces a co-movement mechanism of green bond with sectoral indices in an environment where the shocks dissipate more slowly than the extant literature on the subject assumes. Within such an environment we model cointegration relationships in a way that shows nonlinear disequilibrium error corrections. A forecasting exercise is carried out to lend deeper insights into the way green bond and sectoral prices are set to co-move together.

The emergence of green-bond is largely a product of the supply-side and financing constraints of corporations towards new environmentally sustainable projects. Pham (2021) finds that the dependence between green bond and green equity is relatively small under normal market conditions once movements in the general stock, energy and fixed-income markets are controlled for. Contrarily, the author finds that green bond and green equity are more connected during extreme market movements, where they boom and bust together. An important finding concerns significant short-lived spillover effects between green bond and green equity. Russo et al. (2020) in recent research examines the main drivers of green bond and find the instrumental role of firm-specific corporate sustainability-oriented strategy as a major determinant. Undoubtedly, the demand-push from investors to invest in those climate and environmental projects can make up for the financial deficits towards a desired equilibrium. An imposing argument for the development of the green bond market is that it can act as a bridge between capital providers (i.e. institutional investors) and sustainable assets (i.e. renewable energy).

One way to finance those projects is by investing in green bonds. If a corporation is issuing a green bond to finance their climate and environmental projects[3], there are two motives for this; (i) banks often give better interest rates to corporations which are environmentally conscious and hence it carries a reputation-value persistence, and (ii) the corporation can also expect environmentally conscientious investors to fund those projects who expect the 'long-term' return from

[1] Going by the estimates of the S&P, global issuance of sustainable bonds that includes social, green and sustainability-linked bonds, can together exceed 1 trillion USD in 2021, a near-five fold increase over 2018 levels.

[2] Greenwashing refers to a situation when corporations mislead investors or consumers by instilling a false belief that the product or service they offer or wish to create, or even the organisation itself is environmentally friendly or sustainable. (https://www.globalcitizen.org/en/content/greenwashing-what-is-it-and-how-to-avoid-it/).

[3] Lately, companies are making more noise about 'sustainability' aspects of their growth, with 129 firms on the S&P500 citing ESG in their fourth quarter 2020 earnings calls. From a mere 14 firms that touched on ESG in their Q4 earnings call two years ago to the current 129 firms is a huge leap.

these investments as profitable and stable. The S&P 500 Green Bond Index has been designed to track the global green bond market. While there is no doubt that there is a strong demand for sustainable financing instruments that are also likely to increase, serious concerns around the accuracy of issuer sustainability claims can have deep impacts on the integrity and development of the sustainable finance market.

A huge growth in Environmental, Social, and Governance (ESG) listed companies in the S&P 500, the great volume of ESG marketing and labelling, together with non-uniform sustainability commitments and reporting, have also expounded the difficulty for stakeholders to identify trustworthy claims - ones that are reliable and unreliable. In other words, this 'greenwashing' has become the biggest concern for about 44% of investors when selecting ESG investment. Historically, whenever a socially and environmentally driven innovation is made, its widespread misrepresentation of potential benefits from a particular innovation can sway investors' sentiment so much so that the expected long-term gains from those environmentally sustainable projects may disappear. In other words, contrary to common beliefs and perhaps for the right reason, the growth in S&P 500 green bond index may not represent investors' true sentiments and the fluctuations can perhaps reflect on the dangers of 'greenwashing', embedding an inherent degree of volatility in the index which is rather supposed to be long-run stable.

The question is, to what extent observed volatility in the green bond index co-moves with various sectoral indices, such as health, energy, finance, and information technology[4]? These sectoral indices are regularly subject to common shocks and their degree of fluctuations can be used by investors to compare the extent of exposure to factors (Figure 1 presents the relative weights of various sectoral indices in the S&P 500). In the present study, we aim to investigate whether and to what extent the integration of green bonds into the investor's portfolio provides superior returns or if it implies a trade-off between sustainability concerns and financial performance, through the analysis of green and conventional bond indices. Since shock propagation has an inherent memory feature, our investigation is within the long-memory framework.

In this study, we rigorously model dynamic co-movement between green bond prices and sectoral markets such as the renewable energy stock market, and health and financial sectors. Our modelling paradigm is founded on the real-life shock propagation mechanism within a system, in that shocks converge slowly, thanks to complex interaction within the system. Our investigation exploits the rich features of cointegration, which can be used in our context to identify assets which share a common equilibrium. We shed further light on a well-known violation of the expectations hypothesis - the assumption of a unit root in the sectoral prices and green bonds. We premise that the nonstationarity (in our case, long-memory) stems from the holding premium. This may be cointegrated with the spread. An important characteristic of our green bond and sectoral price systems is that shocks may dissipate slowly and these can make disequilibrium corrections slower than expected, limiting thus the possibility of the system converging quickly to the long-run mean.

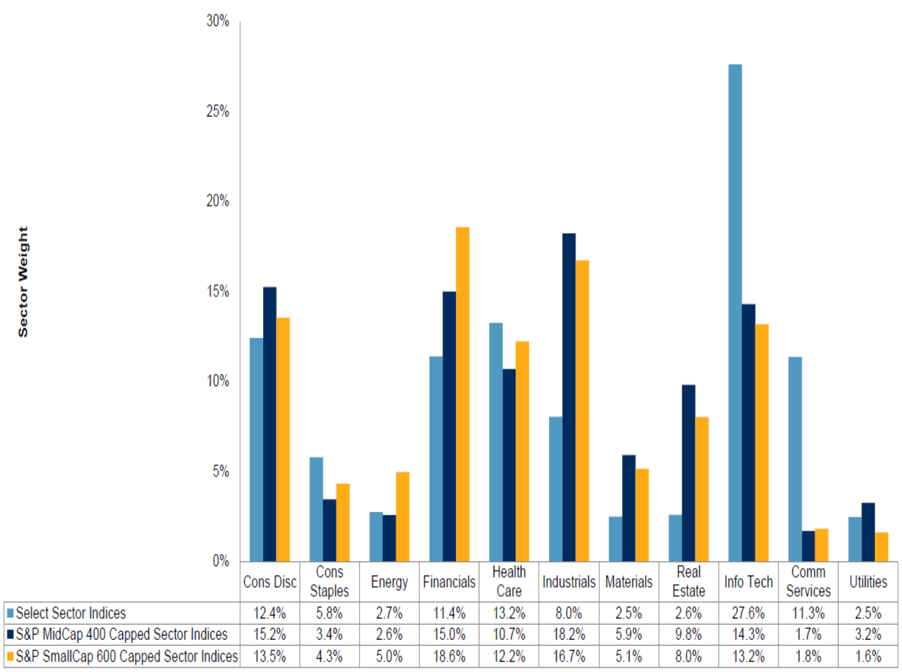
Given the current circumstance of high dimensional nonlinearity in economic systems around

[4]There are eleven sectoral indices in the S&P 500. See Appendix for a description of representative sectors.

the world, it is only reasonable to expect that the green bond market and sectoral indices may co-move but the speed at which the joint system will be long-run stable will differ due to the strong memory of shocks within the system. Finding a co-movement pattern between the green bond and sectoral indices have policy relevance because one can predict the growth of the green bond market vis-a-vis sectoral indices. In other words, it will be interesting to study whether there are rooms for both sectors to grow sustainably.

Further, an in-depth study of co-movement patterns of the green bond index and representative sectoral indices can unravel important information on the future of green bonds and especially the stability in investors' sentiment as the latter is being rewarded by a high and sustainable return from an investment. Le et al. (2021) in a recent work, study spillover connectedness among green bonds, Fintech and cryptocurrencies. Using time and frequency domain measures, the authors find that the overall connectedness of technology assets and traditional common stocks is very high, concluding further that the probability of contemporaneous losses is very high during turbulence in the economy. Pineiro-Chousa et al. (2021), in another interesting piece of research, investigates how investors' sentiment (extracted from social networks) determines price movements in the green bond market. The authors find positive effects of social networks in the trajectory of green bond growth because increasing awareness about environmental and social causes are modelled here in the form of diffusion and strength of the network.

Figure 1: **Comparison of S&P 500 Sectoral Weights**



	Cons Disc	Cons Staples	Energy	Financials	Health Care	Industrials	Materials	Real Estate	Info Tech	Comm Services	Utilities
Select Sector Indices	12.4%	5.8%	2.7%	11.4%	13.2%	8.0%	2.5%	2.6%	27.6%	11.3%	2.5%
S&P MidCap 400 Capped Sector Indices	15.2%	3.4%	2.6%	15.0%	10.7%	18.2%	5.9%	9.8%	14.3%	1.7%	3.2%
S&P SmallCap 600 Capped Sector Indices	13.5%	4.3%	5.0%	18.6%	12.2%	16.7%	5.1%	8.0%	13.2%	1.8%	1.6%

Source - https://www.spglobal.com/spdji/en/landing/investment-themes/sectors/

A study of dynamic interdependence in a memory-driven environment holds policy values, because evidence of a long memory (system-wise or individual-series-wise) would indicate that the system or the series there is a significant degree of inefficiency and this inefficiency can be used by investors as a tool for hedging or arbitrage opportunities in the market. This is a leading reason, why the dynamic interdependence between the green bond and other sectoral indices needs to be studied in a memory-embedded environment.

The rest of the paper is planned as follows. Section 2 provides a brief review of the literature. Section 3 presents our methodological design. Section 4 discusses data and elaborates on the results. Section 5 presents conclusions and summarizes the policy relevance of our work.

2 Literature

2.1 Traditional issue with investment

A critical issue with investments in recent times concerns the fact that incumbents are primarily the organizations that are diverting their resources from traditional sources of energy towards cleaner energy sources. Therefore, they have significantly higher sources of financial availability. The incumbents may seek alternative financing options such as bond and equity issues. On the other hand, the SMEs are newcomers with primarily small- and medium-sized operations and services, lacking the availability of such investment opportunities. Furthermore, due to the relatively less age of the SMEs compared with the incumbents, raising finances to support R&D activities is challenging. Therefore, it is necessary for governmental organizations and regulatory agencies, especially in developing economies, to introduce new means of financing options for both SMEs and the incumbents that may facilitate the transition process towards cleaner sources of energy. Green Equity (GE) and Green Loan (GL), in this regard, are of remarkable interest for humans, organizations, and institutions to expand their investments dedicated to mitigating the impact of climate uncertainty and the prospective healthcare and economic costs.

The development of new green investments across developed and emerging economies is clearly indicative of a significant intensity of new investments in renewables in developing economies. More specifically, the growth of new investments is higher in Asia with both China and India among the forefronts of emerging economies and in the transition process towards cleaner sources of energy. Partly due to the requirement of conformity to the United Nations' Sustainable Development Growth plan and partly due to their realisation of their own social responsibility, lately, organizations have been incorporating sustainable and green approaches, assimilating them into their sustainable development commitments. Cortellini and Panetta (2021) provide an excellent systematic review of the literature on green bonds.

2.2 Green bond types and investible opportunity

Depending on the form of green bonds, such as the use-of-proceeds bonds (or plain vanilla bonds), project bonds, and securitized bonds, there are implications for legal recourse in case the issuer defaults (Jones et al., 2020). The distinction among types of green bonds is also central to understanding why many corporations recourse to greenwashing practices. For instance, there is a difference between use-of-proceeds revenue bonds and securitized bond, where for the former, the proceeds are normally earmarked for green projects in the portfolio of an issuer. The recourse is limited to the issuer's revenue generation. For the latter, the bond is collateralized by one or more revenue-producing green projects. In this case, project revenue is used to repay the bond.

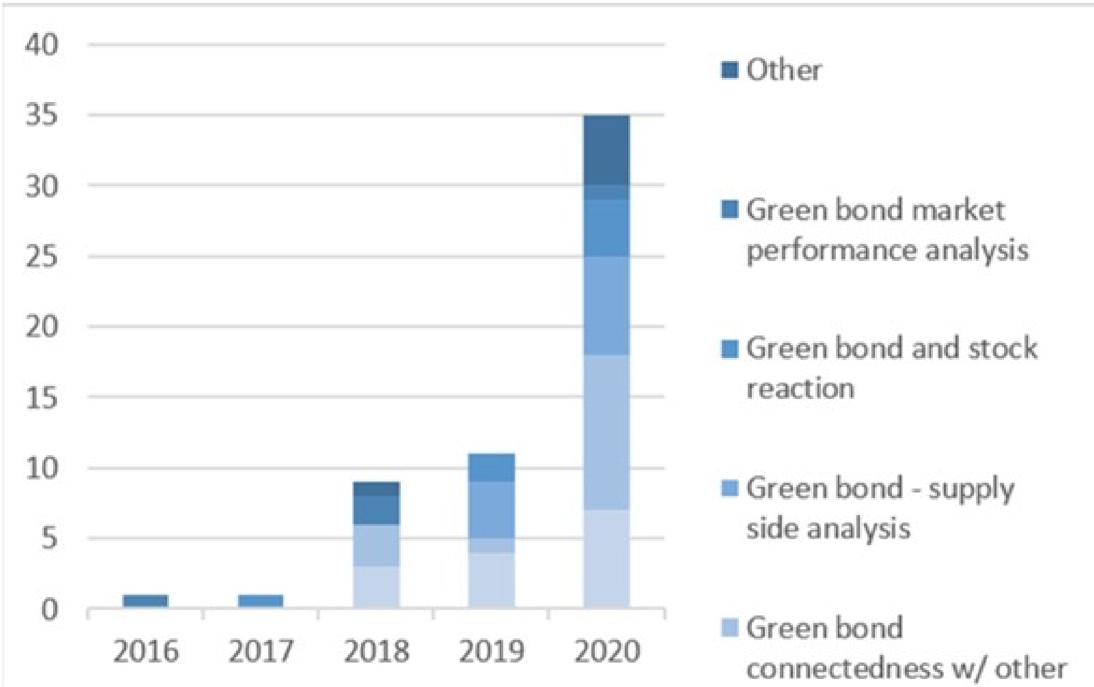
When one looks at an investible opportunity in green bonds and compares it against other available asset types such as the conventional financial stock markets and health-related stocks, a natural question arises on whether and to what extent shareholders benefit from green bonds. Tang and Zhang (2020) have shown that green bonds have experienced significant geographical diversification spreading from Europe to numerous emerging economies (in particular, China and Malaysia). The authors have found that stock prices positively respond to green bond issuance, although there was no evidence of a significant premium for green bonds. This suggests that the positive stock returns around green bond announcements are not fully driven by the lower cost of debt. The authors conclude from their investigation that a firm's issuance of green bonds is generally beneficial to its existing shareholders. Zerbib (2019), Larcker and Watts (2020), and Jones et al. (2020) among others, also emphasize the central and growing importance of green bonds as an imperative investible proposition. The research in these and related work shows that there is an inclination of corporations to treat ecological deficit with debt as their pro-environmental preferences on bond prices drive them to charge higher premiums from green bond investments.

2.3 Extant research taxonomy in green bonds and the context of our contribution

Since research on green bonds is still developing, in the past five years or so the research has mainly focused on four thematic areas: *(i)* the performance of the green bond market, *(ii)* the reaction of the stock market to green bond investment, *(iii)* the supply side dynamics of green bonds, and *(iv)* the interdependence or connectedness of green bonds with various sectors. Following Cortellini and Panetta (2021), in Figure 2, we have represented a research taxonomy of green bonds since 2016. An impressive discussion of contributions about each thematic area can be found in Cortellini and Panetta (2021).

The current paper is heavily inclined towards the theme of the interconnectedness of the green bond market with other sectors. It also contributes to the strand of literature that focuses on the *reaction* of the stock market to green bond investment and growth. Extant research on methodological implementation while studying dynamic interconnectedness between green bond and other sectors, regularly employ multivariate generalized autoregressive conditional heteroscedasticity (GARCH) or dynamic conditional correlations GARCH, a number of research also focuses on

Figure 2: **Whither the trend?**



Source: Cortellini, G. and I.C. Panetta (2021). Green Bond: A Systematic Literature Review for Future Research Agendas. Journal of Risk and Financial Management 14: 589.

copula-based methods, wavelet correlation approach and structural vector autoregression (VAR) estimation. A number of recent research have investigated the relationship between green bonds and sectors such as renewable energy. (Liu et al., 2020), for instance, employ a time-varying copula model to estimate the dynamic relationship between the two, whereas Hammoudeh et al. (2020) employs a time-varying Granger causality test to model the relationship between green bonds, clean energy, CO2 emission allowance. In related research, Nguyen et al. (2020) use a rolling window wavelet estimation to demonstrate that the interdependence between green bonds and clean energy is high, but it is weak with respect to stocks and commodities markets. Overall, the existing research shows that green bonds can serve as an effective hedging tool in the financial market.

An innovative feature of our approach is to design a memory-embedded VAR approach to model dynamic interdependence between green bond and various sectors. This way we offer new insights into the nature of connectedness between these sectors as well as the reaction of stock market to green bond - the two important thematic areas of research in green bond literature. The rigorous empirical dissection of green bonds and their dynamic co-movement pattern with various financial instruments across sectors is rather sparse. The nascent literature has begun to understand the efficacy of green bond data, identifying to the extent greenwashing has more or less deceived investors' sentiments. A key proposition which underlines our empirical investigation is that greenwashing might have infused nonzero mean noises and greater standard deviations in green bonds to an extent that the observed volatility may be contemporaneously correlated with other financial instruments, which are not environmentally sustainable. Another possible leading reason for the lack of a robust body of literature on green bonds' co-movement is a general lack of an underlying theory. Why would green bonds co-move with other financial instruments and what this would mean for the future of green bonds or for that matter for non-environmentally sustainable financial assets, will continue to remain a debatable theoretical and empirical question. Yet, some emerging empirical work showing the spillover effects of various financial assets and green bonds, provide good arguments that the green bond and the chosen assets may alternately work as net giver and net receiver of shocks (Pineiro-Chousa et al., 2021). In our work, we elicit the fact that downplaying the significance of memory may bias our inferences on the direction and magnitude of interdependence between green bond and other sectors. A measurable part of policy ineffectiveness can be attributed to this missing memory - a feature that can also mask the true response of a stock market to green bond investment.

3 Methodology

Since our objective is to investigate if there are co-movement patterns between the green bond and sectoral indices and to the extent, the level of uncertainty and long-memory control of this pattern, a natural way to study this is to employ a cointegration technique within a memory-driven framework. While the conventional approach is to treat a time series to be either stationary (with an integration order (d)) or nonstationary ($d \geq 1$), we premise that shocks do not die out so fast given the complexity of the interaction of a variable within a system as well as the arrival of new shocks (the timing of which are unknown). In other words, a time series can be allowed to be fractionally integrated, representing various degrees of 'memory' of shock persistence. Such a differential convergence speed of shocks can impact our inferences on 'disequilibrium correction' as the speed at which errors are corrected from the system, can determine the nature of stability of the system as well. Keeping this important trait in mind, we advance the use of a fractionally cointegrated VAR (FCVAR) model to have a profound understanding of the dynamics of the cointegration relationship of green bonds with sectoral indices and a measure of uncertainty.

Before we employ FCVAR, we need to check first whether the individual series in question display long memory. We briefly present the methodology in the ensuing section.

(a) Long-memory: Source and Estimation

(i) Source of long-memory in Green Bond: Although there is a robust body of work related to the measurement of long-memory and its implications in economics and financial time series, it is not a common parlance in environmental time series, such as the applications to Green Bond. An imposing question is what theoretical justification one can provide for long memory in the Green Bond series? Apart from the econometric methodological interests, identification of the source of long memory in an economic, financial and/or environmental series such as the Green Bond, is very important.

By its very nature of construction, the S&P Green Bond index tracks the green bond market at the global level. The index adopts stringent standards while incorporating those bonds whose proceeds are actually used to fund environmentally friendly projects. Econometrically, the nature of aggregation of various corporations' bonds while creating the index can be a potential source of long memory.

Let's denote the by z_i, the $i > 1$ a number of corporations' bonds. Over time, t, y_i evolves following a path-dependent or an autoregressive (AR) process:

$$y_{i,t} = \alpha_{i,1} + \alpha_{i,2} y_{i,t-1} + u_{i,t} \tag{1}$$

$\alpha_{i,2}$ is either 0 or 1.. If we allow $\alpha_{i,2}$ to follow a $\beta(u, v)$ distribution, then $\frac{1}{N}\sum_1^N y_{i,t} = Y_t \sim I(d)$, that is the aggregate Y_t can be distributed as an *I(d)* process. Note however that the AR coefficient, $\alpha_{i,2}$ varies over i. For when $\alpha_{i,2} \approx 0$, this can be referred to as a 'random' component, whereas $\alpha_{i,2} \approx 1$, we can call it a 'regular' component. In case the distribution of $\alpha_{i,2}$ follows a $Beta(u, v)$ distribution across various components, then the Green Bond index, $y_t = N^{-1}\sum_{i=1}^N y_{i,t}$ can be shown to be fractionally integrated of order $d = 1 - v$ if N is large.

(ii) Estimation of Long-memory:

Let's denote the green bond index time series (or other sectoral indices) as y_t (for $t = 1, \ldots, T$). We model y_t as an $I(d)$ process or integration of order *d* in the sense that we may need to difference (a possible non-stationary) y_t series (Green Bond series, for instance) d^{th} times to make it stationary:

$$(1 - L)^d y_t = \psi(L)\varepsilon_t \tag{2}$$

In equation (2) $(1 - L)^d$ is the difference operator of order d with time lag denoted by L, such that when $d = 1$, $(1 - L)^1 y_t = y_t - y_{t-1} = \Delta y_t$. $\psi(L^j)$ is the coefficient of the error term (ε) at each specific time period $t - j$ with $\sum_{j=0}^{\infty} |\psi(L^j)| < \infty$, $j = 0, 1, 2, \ldots$. ε_t, the error term is assumed to follow a distribution with zero mean and constant variance: $\varepsilon_t \sim iid(0, \sigma^2)$. We tread beyond convention and allow d to be of fractional, rather than of integrated order. Granger and Joyeux (1980) discusses details of the rich dynamics of shock convergence of this assumption - one that approximates real-life dynamics robustly. the shock propagation mechanism can be visualised by allowing a power series expansion (see equation 4) based on $(1 - L)^{-d}$ in equation (3):

$$y_t = (1 - L)^{-d}\psi(L)\varepsilon_t \tag{3}$$

where $\gamma_0 \equiv 1$ and

$$\gamma_j = \frac{(d + j - 1)(d + j - 2)\cdots(d + 2)(d + 1)(d)}{j!} \tag{4}$$

where $\gamma_j \cong (j + 1)^{d-1}$ given that $d < 1$ and j is large.

A fractionally integrated process can be represented as an infinite order moving average (MA(∞))

process such that

$$y_t = (1-L)^{-d}\varepsilon_t = \gamma_0\varepsilon_t + \gamma_1\varepsilon_{t-1} + \gamma_2\varepsilon_{t-2} + \cdots \tag{5}$$

In equation (5), the impulse response coefficients of y_t, i.e. γ_j, reveal how fast or slow shocks dissipate over time depending on the magnitude of the coefficient, γ. Table 1 describes various shock convergence patterns and their implications for the stability of the time series in question. In

We estimate d using Shimotsu (2010) (details in the empirical section). For dynamic estimates of memory, we perform a rolling window estimation of Shimotsu (2010).

Table 1: **Implications of d values for stability of a series**

d	Type of Memory	(Non)Stationarity	Convergence	Variance	Shock Duration
$d < 0$	Long	Stationary	Mean-convergence	Finite	Long
$d = 0$	Short	Stationary	Mean-convergence	Finite	Short
$0 < d < 0.5$	Long	Stationary	Mean-convergence	Finite	Long
$0.5 \leq d < 1$	Long	Non-stationary	Mean-convergence	Infinite	Long
$d = 1$	Permanent	Non-stationary, unit root process	Non-convergence	Infinite	Permanent
$d > 1$	Permanent	Non-stationary	Non-convergence	Infinite	Permanent, growing effects over time

(b) Understanding dynamic interdependence of green bond with sectoral indices: A memory-driven approach

Does green bond cointegrate with various sectoral indices? If so, at what rate do disequilibrium errors correct to produce a stable co-moving relationship? To this effect, a fractionally cointegrated vector autoregressive (FCVAR) model, proposed by Johansen (2008) and Johansen and Nielsen (2012), is very useful. The FCVAR is designed to detect equilibrium relationships between fractionally integrated variables. A restricted version, the cointegrated vector autoregressive (CVAR) model (Johansen, 1995) can detect an equilibrium relationship between variables that are integrated of order one, i.e. exhibit unit root behaviour, where deviations from this relationship are not integrated. In contrast, the fractionally cointegrated VAR model can detect relationships between variables that are integrated of a fractional order, with deviations that can also be fractionally integrated but of a lower order than the variables themselves. This allows for the study of relationships with deviations that correct more slowly than with the CVAR.

The FCVAR model is formulated as follows:

$$\Delta^d(Y_t - \rho) = \alpha\beta' L_d(Y_t - \rho) + \sum_{i=1}^{p}\Gamma_i \Delta^d L_d^i (Y_t - \rho) + \varepsilon_t \tag{6}$$

where Y_t is a K-dimensional $I(d)$ time series at time t of green bond and other sectoral indice. In the above, i stands for numbers of short-run dynamics with $i = 1, 2, \ldots, p$; Γ_i is the coefficient of each temporal lagged Y_t; Π is a parameter matrix identified by two parameters, viz. $\Pi = \alpha\beta'$. α and β are $K \times r$ matrices, β identifies the cointegrating relationship(s) among variables in Y_t, and α defines the adjustment speed towards the long-run equilibrium of each variable in Y_t. r is the rank of Y_t, and its value indicates the number of cointegration(s) in the model with $0 \leq r \leq K$. Δ^d and L_b represent fractional difference operator with order d and the fractional lag operator with b, respectively, where $\Delta^d = 1 - L_d = (1 - L)^d$ and $L_b = 1 - \Delta^b$. d and b could be either integer or fractional values.[5] ε_t is a K-dimensional identically independent distributed error term with zero mean and constant variance-covariance matrix ($\varepsilon_t \sim iid(0, \Omega)$). The inclusion of a constant, i.e. $\beta^{'}\rho$, in the long-run relationship(s) in $\beta^{'}L_b\Delta^{d-b}Y_t$ captures unobserved explanatory powers in the identified relationship(s). An innovative feature of the FCVAR is that it allows multiple time series to be fractionally integrated with order d and fractionally cointegrated to order $d - b$. A simple assumption one can follow is to set $d = b$ to ensure short-memory stationarity in the cointegrating relationship(s). For estimation, maximum likelihood (ML) estimators can provide reliable estimates of the FCVAR model parameters (Johansen and Nielsen, 2012).[6]

Similar to the CVAR model, the significance of FCVAR model parameters can be tested by hypothesis testing (Jones et al., 2014). The framework of hypothesis testing on long-run parameters, i.e., β and α, can be formulated as

$$\beta = \omega\lambda \tag{7}$$

$$\alpha = \tau\theta \tag{8}$$

With respect to the test on β (Equation (7)), ω is a $K \times q$ matrix of identifying restriction(s) on the cointegrating relationship(s), and λ is a $q \times r$ matrix defining free varying parameter(s). q is the number of restriction(s) associated with β-related hypothesis tests. In a context when each cointegrating relationship is imposed with the same restriction, the degree of freedom of the hypothesis test is equal to $(K - q)r$. If the number of cointegrating relationships is greater than one, viz. $r > 1$, different restrictions could be imposed on different columns of β. β can then be re-expressed as a row vector, i.e., $\beta = (\omega_1\lambda_1, \omega_2\lambda_2, \ldots, \omega_r\lambda_r)$. Each column of β is the product between ω_i and λ_i, where ω_i is a $K \times q_i$ matrix and defines the imposed restriction on the column i of β; λ_i is a $q_i \times 1$ matrix and defines the free varying parameter on the column i of β. In that case, the degrees of freedom of the hypothesis test is $\sum_{i=1}^{r}(K - r - q_i + 1)$. Concerning the test on α as in Equation (8), τ is a $K \times l$ matrix that defines restriction(s) on disequilibrium error corrections of target variables, and θ is a $l \times r$ matrix representing free varying parameter(s) with $l \geq r$. l stands for the number of restriction(s) associated with α-related hypothesis tests. Its degree of freedom is $(K - l)r$.

The FCVAR model can also deal with the endogeneity concern that arises from simultaneity issues in the form of bi-directional relationships in green bonds and other sectors' price systems.

[5]Their values should be positive to ensure that the integration order of target series would not be affected by applying the fractional lag operator (L_d) (Tschernig et al., 2013).

[6]See Jones et al. (2014) for details.

Further, the presence of a constant term while obtaining the long-run relationship in the system can as well address the problem of omitted variables (although partially). There may also be a problem of *overidentification* in the cointegration relationship. This issue can be addressed by studying the significance of model parameters and wherever required, imposing zero restrictions on insignificant ones. In our estimation, we test for weak-exogeneity in the long-run parameters of the FCVAR system, i.e., α and β, by testing for zero restriction on its feedback coefficient in the α-matrix. If α coefficient of the variable is not significantly different from zero, we treat it as weakly-exogenous, indicating that it contributes nothing to restore the long-run equilibrium after disequilibrium has pervaded the system.

At the same time, whether a variable in the system forms long-run cointegrating relationship(s) is concluded by testing for zero restrictions on its feedback coefficient in the β-matrix. If β coefficient of the variable is restricted to zero, the variable would not enter the cointegrating relation(s). In addition, the FCVAR estimation is conducted by using a grid search in our case, through which the (parameter) identification problem discussed in Carlini and Santucci de Magistris (2019) is resolved (Nielsen and Popiel, 2018).

4 Data and preliminary observations

4.1 Data

We use a daily dataset (28/07/2009 - 28/07/2021) for green bonds as well as some of the selected S&P 500 sectoral indices, viz., Energy, Health, Financials, and Information Technology.[7] The S&P Green Bond US Dollar Index is designed to measure the performance of US dollar-denominated, green-labelled bonds from the S&P Green Bond Index. Although the market size of green bonds is relatively small compared with the boom of cryptocurrencies since 2013, both are evoking the immense interests of investors. Among sectoral indices, for instance, the financial technology index tracks the performance of financial technology companies that are publicly traded in the U.S. Hence, this proxy represents the performance of an asset in the 4th industrial revolution. We also use Bloomberg Barclays's MSCI green bond index as an alternative measure for Green Bond to reflect investment in environmental sustainability projects.[8] A description of the data is in the Appendix. Important to note that due to data availability issues, the starting date for this data is April 1, 2014. The US Economic Policy Uncertainty has also been used to study, how the co-movement patterns between the green bond and various sector indices differ over variations in uncertainty. The latter is one important source of information asymmetry in the market and the use of the uncertainty index in our estimation enables an informed inference on co-movement heterogeneity and investment choice between the green bond and sectoral indices.

[7]There are 11 S&P 500 sectoral indices, viz., Health Care, Consumer Discretionary, Energy, Financials, Industrials, Commodity Services, Materials, Consumer Staples, Information Technology, Utilities, and Real estate. In the Appendix, we have presented detailed definitions of these and other sectoral indices.

[8]Many thanks to a referee for suggesting this variable.

In Table 2 we present descriptive statistics. It appears that the mean price index for green bond is 108.289 with a standard deviation of 6.494, where a significant difference exists between the minimum (96.553) and maximum (124.305). The differences are larger for the Bloomberg MSCI Green Bond index (standard deviation of 7.79; minimum = 81.916 and maximum = 122.306). Table 2 also summarises descriptive statistics for other sectoral indices plus a measure of policy uncertainty (usepupo) to paint the broader picture of underlying uncertainty in the economy. Specifically, S&P Financials comprises those companies included in the S&P 500 that are classified as members of the Global Industry Classification Standard (GICS) financials sector. All components of the S&P 500 are assigned to one of the eleven Select Sector Indices, which seek to track major economic segments and are highly liquid benchmarks.

Table 2: **Descriptive statistics**

Variable	Mean	p50	Std. Dev	Min	Max	p25	p50	p75	Interquartile Range
Green Bond	108.289	108.494	6.494	96.553	124.305	102.596	108.494	113.768	11.172
MSCI Green Bond	103.999	102.672	7.790	81.916	122.306	97.944	102.672	108.116	353.070
Sector: Health	767.871	803.990	309.749	318.500	1534.570	444.555	803.990	1002.730	558.175
Sector: Energy	495.086	501.970	99.253	179.940	737.090	438.580	501.970	558.110	119.530
Sector: Financials	331.026	318.120	112.854	151.850	637.750	217.590	318.120	427.165	209.575
Sector: Info. Technology	896.463	700.270	561.705	307.860	2713.400	467.635	700.270	1199.870	732.235
US Policy Uncertainty	121.626	97.210	88.085	3.320	807.660	65.770	97.210	146.690	80.920

4.2 Mitigating the effects of cycles and trends

The presence of periodic fluctuations, such as seasonality and cyclicality, can mask the true nature of the movement of the data, in our case, green bond and other sectoral indices (Canepa et al., 2020). The repeated fluctuations, defined as *business cycles* can describe the periodic behaviour of the data (Hodrick and Prescott, 1997). To free our variables from possible cyclical movements in the mid/long-runs, which may otherwise obscure the true nature of the dynamics of time series and its persistence, we remove business cycles from our raw data using the recently developed Hamilton filter method (Hamilton, 2018). The Hamilton filter method decomposes a time series into cyclical and trend components.[9] Despite the popularity of the Hodrick-Prescott (H-P) filter in this direction (Hodrick and Prescott, 1997), Hamilton criticises the technique to be flawed misrepresenting the underlying data-generating process.

Figure 3 (upper panel) presents a time series plot of the green bond (and Hamilton-filtered data of the green bond). Similarly, The lower panel in Figure 3 similarly presents time series plots for the clean energy index of S&P 500 along with the plot using the Hamilton filtered data. Both Figures reveal significant variations over time, although the clean energy index shows a promising trend of growth from 2014. To what extent does the green bond (or clean energy index - a proxy for environmental sustainability) co-moves with various sectoral indices such as health, energy, finance, and information technology? Figure 4 (upper panel) shows that the sectoral energy price

[9] We use these transformed data for the rest of the empirical analyses.

index appears to strongly co-move with the green bond index, while the lower panel of Figure 4 depicts a similar pattern although there is a substantial difference in the price variations. Figure 5 presents the pattern of co-movement between S&P green bond and Bloomberg MSCI green bond indices. Once again, we find a visible pattern of strong connectedness in their trends.

5 Empirical Results

Our findings are mainly presented in the following two aspects. First, the presence of the long-memory feature of our empirical dataset is demonstrated by the fractional integration order of our included variables. Having the long memory in univariate series is a prerequisite to verify whether the FCVAR model is appropriate to our data. The second part of the results discusses the determination of equilibrium green bonds and sectoral indices.

5.1 Which integration order works better?

(i) Stationarity and unit root tests

Before one estimates a fractionally cointegrated VAR system, it is important to ensure that our data are truly characterised by long-memory. It is possible that the series in question may be fractionally integrated if it rejects the null hypothesis of both stationary and unit root tests at the same time. In inherent reason is that a fractionally integrated series does not have a unit root, although it is still likely to be non-stationary (Jones et al., 2014). To ensure this, we have conducted Kwiatkowski-Phillips-Schmidt-Shin (KPSS) stationarity test and the Augmented Dickey-Fuller (ADF) for unit root of each series, respectively. Table 3 reports these results. We observe that all series reject the null hypothesis of stationarity using the KPSS test. Further, using the ADF test we reject the null hypothesis of a unit root. From Table 3, we can conclude that all series in our system are of fractional integration order (d).

Table 3: **Stationarity and unit root tests**

	Green Bond	Clean Energy	Health care	Energy	Financial	Industrial	Information Tech	EPU
KPSS Test	0.208**	0.162**	0.250***	0.171**	0.677***	0.229***	0.131*	1.255***
ADF Test	-1.311	-3.755**	-4.102***	-3.886**	-3.148*	1.708	-3.222*	-3.664**

Note: (i) *: 10% significance level; **: 5% significance level; ***: 1% significance level. (ii) Information criteria (IC) have been used to select lags.

(ii) Are Green Bond and Sectoral Indices inefficient? Evidence using long-memory features

(a) ***Static estimation of memory: inefficiency in a static sense***

To lend quantitative evidence that our green bond index and various sectoral indices are inefficient markets, we do not demonstrate that these indices possess long-memory features (that is, characterised by fractional integration order, d). Accordingly, we have estimated the fractional

Table 4: **Robinson's semiparametric estimates of 'memory' (d)**

Greenbond				MSCI Index		All Other Variables		
Power	Ords	Est d	Std Err	Est d	Std Err	Variable	Est d	Std Err
0.50	55	0.959	0.080	0.502	0.071	Health Sector	0.975	0.023
0.55	83	0.999	0.067	0.622	0.059			
0.60	125	0.957	0.052	0.422	0.051	Energy Sector	0.874	0.023
0.65	187	0.928	0.041	0.402	0.044	Financial Sector	0.960	0.023
0.70	279	0.954	0.035	0.275	0.033			
0.75	419	0.979	0.031	0.215	0.029			
0.80	627	0.966	0.025	0.210	0.022			
0.85	937	0.968	0.020	0.222	0.022			
0.90	1401	0.923	0.017	0.198	0.015	Info Tech Sector	0.970	0.023
						US: Policy Uncertainty	0.587	0.023

Note: (a) The standard errors are calculated by $(4\psi)^{-1/2}$ where $\psi = N^B$ and N is the number of observations.

integration parameter, d, of the individual series by using (Robinson, 1994) semiparametric procedure as well as the two-step exact local Whittle estimator (2ELW) and '2ELW' estimator with demeaned and detrended data (Shimotsu, 2010). In Table 4, we have presented Robinson's estimates for various bandwidths (0.5 to 0.9) and find that the estimated value of d for the green bond index is less than 1. Given the length of the data and a preliminary simulation exercise (results of which are available upon request), we have chosen 0.7 as the optimal bandwidth. The estimated d for this bandwidth is 0.954, which is also statistically significant. For several sectoral indices and uncertainty index, we have also estimated d at this bandwidth and the results show that except for the energy index, for all variables, the $d < 1$.

In a nutshell, our static d estimation is consistent with the findings in the last section. A fractionally integrated series would be non-stationary (i.e., rejection of the KPSS test) if its d is greater than 0.5, while it would not have a unit root (i.e. rejection of the ADF test) if it's d value is also less than 1. At the same time, even if a series does not reject the null hypothesis of the ADF test, it can still be fractionally integrated when it's d value is either greater than 1 or very close but not equal to 1.[10]

(b) ***Dynamic inefficiency? Dynamic rolling window estimation***

A policy-relevant question then arises: is the evidence of inefficiency, demonstrated by long-memory parameters, dynamic in nature? In other words, inefficiency may not strictly be a static phenomenon. As new information arrives in the market, for instance, the rising level of policy uncertainty or important data on the seriousness of climate disasters where policymakers are keen to reverse the trend of the impending environmental problems, this may make the nature of inefficiency dynamic. That is, with the arrival of more information, it is possible that the estimated degree of inefficiency 'in the static sense' may dynamically converge to a more efficient state,

[10]The ADF test previously employed is built based on a standard left-sided unit root test where the null hypothesis suggests unit root (i.e., $d = 1$) against the alternative hypothesis of $d < 1$. Corresponding inferences may be unreliable in the condition when $d > 1$.

something that the conventional financial theoretic literature floats the idea about the efficiency of a market; with the arrival of new information, the current value of the system may capture the relevant information about the past, one that will be useful to predict the future. Further, to study if, over our period of study, the Green Bond market has been asymptotically efficient. One way to estimate this is to employ a rolling window estimation of the memory parameter. Arguably, this is also an innovative feature of our empirical exploration. We formalise our theory in the following way.

Recall that 'memory', M is a function of some known factors (Z) and some unknown or unobserved factors (ϵ) at the time, t. Assuming time to be finite (t tends to T, a finite (large) number), then temporal change in 'memory' (to broadly represent, the inefficiency in our system), would converge in probability, to ($\bar{M}$), holding other things constant:

$$M_t = f(X_t, \epsilon_t) \tag{9}$$

$$\lim_{t \to T} \frac{d(M_t)}{dt} \to \bar{M} \tag{10}$$

To lend insights into the above proposition, we have estimated d for Green Bond, in a rolling-window environment.[11] We perform this estimation using a two-step exact local Whittle estimator of Shimotsu et al. (2005). An initial 3-year window setting was used although a smaller window setting also produces similar results thus showing invariance of our finding of long memory, on average, for all years.[12] For the green bond index, the d value is estimated on a rolling basis until approaching the end of the sample. We accordingly generate d series for this variable with the daily frequency where each observation denotes the d estimate of the corresponding window. A complete illustration of the time-varying d for all variables using different estimators is reported in Figure 6. It is clear that dynamic estimates show long memory over a different time period and for all bandwidths, although the estimates hobble around 0.5 to 1.5 depending on the low and higher bandwidth (a lower bandwidth reflects more noise and a higher bandwidth removes those noises by trading-off more data, in terms of averaging). To summarise, our rolling window estimation does not provide any evidence of asymptotic stability of inefficiency over time, despite some periods, where we find estimates of memory are fairly stable but start diverging for other periods. It is not difficult to imagine that such a nature of dynamic inefficiency often results from market-related noises, which are not randomly distributed with zero mean and constant variance. From the rolling-window estimates of d, the average d or $\bar{M}$ (memory) is found to be 0.78 and the 'memory of the memory' estimation (that is, the relative speed of acceleration or deceleration to $\bar{M}$ is found to be at 0.65. This implies that the innate tendency of the green bond series predicts a stable pattern, although shocks will take time to taper off or converge to the mean.

[11]Estimation for other variables are available with the authors. We do not present the results here to save space.
[12]The results are available upon request.

5.2 FCVAR Results: The System Dynamics

So far, we have discussed whether a fractional integration order (or short/long memory) describes the nature of growth trajectories of green bonds and various other sectoral indices. It was clear, from both static and dynamic analyses of long-memory estimates, that a shock in these variables would take a long time to taper off. This implies, for instance, if the green bond series is combined with other variables in a system to describe dynamic interdependence/co-movement, the shocks will take a long time to stabilise, making the system vulnerable to the arrival of new shocks into the system.

In the context of our research, this would imply that investors who are looking to hedge for green bonds or other sectoral indices as a hedge against the green bond would require a deeper understanding of the shock propagation mechanism within the system. Since the way shocks move and prolong determine the welfare costs in a society, the technologies (in other words, policy instruments) that are needed to control such nature of shocks, need to be contextually designed. To shed deeper insights into this, in this section, we present and discuss results from FCVAR estimation of a number of models.

We begin with a baseline model that comprises green bonds, energy sectoral index and economic policy uncertainty. We undertake a forecasting exercise to depict the predictive power of the model we have estimated. Further analyses are carried out on various FCVAR estimations involving other sectoral indices, such as health, financials and information technology price index. Finally, we perform sensitivity analyses by first using an alternative indicator of the green bond, viz., Bloomberg Barclays MSCI Global Green Bond Index and second, by comparing results between pre- and post-COVID samples. The out-of-sample forecasting exercise is performed for each category of the FCVAR model pertaining to baseline and various other sectors (from (i) to (iv). Table 9 summarises the forecast evaluation for the baseline FCVAR model, whereas Table 15 summarises the results for the rest of the sector-specific FCVAR run.

(i) Green Bond, Sectoral Energy Price Index and Policy Uncertainty interdependence

The primary step for the FCVAR estimation involves the selection of the system lag order and the model rank. To determine the number of system lag order (p), we follow Jones et al. (2014) to select the optimal number using a series of Likelihood Ratio (LR) tests through a *'general to specific'* strategy. Specifically, the LR test starts from a generous lag order, viz. $p = 8$, by assuming that the short-run dynamics of housing prices and macroeconomic factors exist within eight quarters. For each LR test, the null hypothesis is that the coefficient of the highest lag order (p) is not significant ($H_0 : \Gamma_p = 0$), against the alternative hypothesis in favour of the significance of Γ_p ($H_1 : \Gamma_p \neq 0$). If H_0 associated with a specified p is accepted, that p should be dropped, and the model will then be re-estimated with a smaller p until H_0 of the new p can be significantly rejected. In each LR test, the Ljung-Box Q-test is applied to examine if the residuals are serially correlated.[13] If its null

[13]The number of lags in the Ljung-Box Q test is chosen as 12. We also tried other lag orders such as 4, 8, and 16, and the test results are qualitatively the same.

hypothesis of no autocorrelation is rejected, we will also have to drop that specified p and move one step back in the model specification. To confirm the p that we finally choose is the best among all qualified ones, which should both have a significant coefficient and no autocorrelation in the residuals, we use the information criteria (IC) where the optimal p should have a minimum IC.

After choosing an optimal p, we determine the number of ranks ($rank$) in the FCVAR system, i.e. the number of long-run cointegrating relationships. Following the literature (e.g., Johansen, 1995), $rank$ is selected using a series of Likelihood Ratio (LR) tests where we sequentially test null hypotheses $H_0^r : rank = k$ for $k = 0, 1, \ldots, K$ against the same alternative hypothesis implying the *full rank*, i.e. $H_1^r : rank = K$. K is the total number of variables and equals to the *full rank* in the system. The selected rank order is the one that first accepts its corresponding H_0^r. Moreover, it is known that parameters of cointegrating relationship(s), viz. α and β, cannot be separately identified without normalization restrictions for the matrix Π in Equation (6). To uncover the determination of equilibrium in the green bond market, we impose an identification restriction that normalizes β with regard to green bond prices.

To specify the green bond price equation of the FCVAR model, we first select the system lag order (k). The results are reported in Table 5, suggesting that the optimal $k = 3$ given its significant coefficient, no serial correlation in the corresponding residuals, and the lowest Akaike information criteria (AIC) value. For this chosen lag of the system, the estimated memory ($d = 0.606$), implies that the system has a long memory that is mean-convergent or can be long-run stable, given other things are constant. With the selected system lag order, we then test the rank in the green bond function by conducting a series of LR tests. The results are presented in Table 6 where the first two null hypotheses (i.e., $rank = 0$ and $rank = 1$) are significantly rejected against the same alternative hypothesis of $rank = 3$, viz. the *full rank*. Then, updated null hypotheses with higher ranks continue to be tested. Given that our main focus is the determination of green bond prices, we would like to keep as many factors as possible in the cointegrating relationship normalized by $Greenbond$. We eventually accept the null hypothesis of $rank = 2$ with $P - value = 0.354$, indicating two cointegrating relationships in the green bond equation. Thus, the FCVAR model for the green bond function is specified as 3 short-run terms and 2 ranks. The corresponding estimates are reported in Equation (15) with the cointegrating relations identified by Equations 12 and 13.

Table 5: **Lag-order selection - FCVAR**

k	r	d	LogL	LR	pv	AIC	BIC	pmvQ
4	3	0.597	31077.24	11.43	0.247	-62056.48	-61760.61	0
3	3	0.606	31071.52	24.71	0.003	-62063.04*	-61821.52	0
2	3	0.571	31059.17	215.09	0	-62056.33	-61869.15*	0
1	3	0.983	30951.62	132.62	0	-61859.25	-61726.41	0
0	3	0.965	30885.31	0	0	-61744.63	-61666.13	0

Specifically, the estimated parameters of each variable for the error adjustment speed (α) in

Table 6: **Rank tests - FCVAR**

$Rank$	$\hat{d}$	$Log-L$	$LRstatistic$	$P-value$
0	0.430	31062.711	17.623	0.04
1	0.402	31066.477	10.09	0.039
2	0.574	31070.363	2.318	0.354
3	0.606	31071.522	—-	—-

the cointegrating relationship are shown in a matrix form on the right-hand side of Equation (11). The column vector ν_t, i.e., $[\nu_{1t} \ \ \nu_{2t}]'$, stands for the two long-run cointegrating relations, which are normalized with regard to clean energy prices. The two relations are defined by $\nu_t = \beta' L_d(Y_t - \rho) = 0$.[14] A striking feature of our FCVAR estimation is that the cointegration order, i.e. system d, is a fraction of 0.574 with a standard error of 0.029, demonstrating gradual adjustment of the green bond index over time. The fractional d in the green bond price equation further confirms the inefficiency of the green bond market, showing a predictable pattern of its price dynamics with strong autocorrelation (e.g., Case and Shiller, 1989; Larsen and Weum, 2008). Based on the Ljung-Box Q-test statistic ($Q_{\hat{\varepsilon}}$), it appears that the residuals are white noise, indicating that the FCVAR estimation of the green bond function is correctly specified.

FCVAR estimation:

$$\Delta^{\hat{d}} \left(\begin{bmatrix} greenbond \\ energy \\ EPU \end{bmatrix} - \begin{bmatrix} 1.549 \\ 1.795 \\ 1.179 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.001 & -0.001 \\ 0.011 & -0.002 \\ 0.238 & 0.094 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{11}$$

$$\hat{d} = \underset{(0.029)}{0.574},\ Q_{\varepsilon}(12) = \underset{(0.996)}{358.611},\ LogL = 31070.363$$

$$greenbond^* = 1.101 - 0.007 \times energyindex_t + 0.063 \times EPU_t + \nu_{1t} \tag{12}$$

$$EnergyPrice^* = 1.111 + 0.379 \times EPU_t + \nu_{1t} \tag{13}$$

The cointegrating relations normalized in Equations 12 and 13 respectively demonstrate the driving force for variations in green bond prices by energy sector price fluctuations and uncertainty levels in the long-run steady state. As shown in Equation 12, changes in the sectoral energy index exert a negative impact (0.007) on green bonds, while the coefficient for uncertainty is positive, (0.063). Further, equation 13 presents a cointegrating equation for the impact of EPU on

[14]Estimated coefficients of the short-run terms ($\{\hat{\Gamma}_i\}_{i=1}^4$) are suppressed as our research focus is the long-run relationships. They are available from the authors upon request.

energy prices, which is positive, as expected. The implications of these cointegration relations are that with a rising level of uncertainty (EPU), energy prices may increase, thanks to the persistence of asymmetric information and negative signal to the market. As well, we find that as EPU rises, people tend to invest more in green bonds, as traditional assets become too risky as a safe investment alternative. The negative effects of the sectoral energy index on green bonds demonstrate that they are substitutes. In other words, if there is a unit rise in prices of sectoral energy, investors may like to invest more in the index to leverage profit and hence will be less inclined to invest in green bonds, keeping the level of uncertainty constant. Our results resonate with the extant literature, such as Reboredo (2018) and Reboredo and Ugolini (2020).

Table 7: **Hypothesis tests**

H_S^d	The fractional order, d, equals to one.	H_{S1}^{α}	Green bond is weakly exogenous.
H_{S1}^{β}	Green bond does not enter the cointegrating relationship.	H_{S2}^{α}	Energy price is weakly exogenous.
H_{S2}^{β}	Energy price cointegrating relationship.	H_{S3}^{α}	EPU is weakly exogenous.
H_{S3}^{β}	Energy price does not enter the cointegrating relationship.	H_{S4}^{α}	Energy price is weakly exogenous.
H_{S4}^{β}	EPU does not enter the cointegrating relationship.	H_{S5}^{α}	EPU is weakly exogenous.

Our next strategy is to test various hypotheses for this model. The results are shown in the hypothesis test table (Table 8). First, as a test of model specification check, we test whether our VAR system is characterised by long memory. In other words, we check if FCVAR instead of CVAR (with d=1) is adequate. We strongly reject the hypothesis suggesting that the FCVAR model is a more appropriate specification than the CVAR model. We next test for the absence of a green bond, which is a hypothesis that imposes zero restriction on the coefficient of the green bond index in the long-run equilibrium. The LR test statistic is 3.968, which is statistically significant. Thus, we strongly reject the null hypothesis that the green bond is absent from the long-run equilibrium. Further, because there are no other β hypotheses that are relevant to this model, we move to tests of weak exogeneity on the α coefficients to determine whether or not the variables respond to long-run disequilibrium errors. As an example, consider the test H^{α} that green bond, in this case, the first variable, is weakly exogenous. As evident from Table 8, the high LR statistic of 15.738 and p-value = 0, we strongly reject this hypothesis.

With the non-rejected hypotheses, we have re-estimated the model. The results are in the appendix. The signs and magnitudes of the estimated coefficients are similar to the ones obtained in the unrestricted model. Furthermore, the residuals appear to be white noise with a p-value value of 0.152 for the Ljung-Box Q test. The interpretation of the long-run equilibrium is the same as in the unrestricted model. Namely, we find that green bond price decreases in response to an increase in the sectoral index of energy and increases with a rise in uncertainty.

(a) Forecast Evaluation

Table 8: **Results of a hypothesis test: Green bond, energy index and uncertainty**

	H_S^d	H_{S1}^β	H_{S2}^β	H_{S3}^β	H_{S4}^β	H_{S5}^β	H_{S6}^β	H_{S7}^β
df	1	1	5	1	1	1	1	1
LR Statistic	15.333	3.968	22.124	2.101	38.325	67.830	71.107	118.443
P-Value	0.000***	0.081*	0.000***	0.120	0.000***	0.000***	0.000***	0.000***
	H_{S1}^α	H_{S2}^α	H_{S3}^α	H_{S4}^α	H_{S5}^α	H_{S6}^α		
df	1	1	1	1	1	1		
LR Statistic	15.738	6.844	35.301	17.400	2.604	36.467		
P-Value	0.000***	0.032**	0.000***	0.000***	0.250	0.000***		

Note: (a) *: significant at the 10% level, **: significant at the 5% level, ***: significant at 1% level; (b) df denotes the degree of freedom; (c) LR is the abbreviation for the Likelihood Ratio test;

To what extent, do the results obtained using our FCVAR model possess good predictive power? To assess the forecast performance of FCVAR and CVAR models, the data was partitioned into two parts, $estimation\ period$ and $test\ period$. $Estimation\ period$ was based on the sample span between 28.07.2009 and 28.07.2019 and $test\ period$ for forecast performance evaluation was set to period between 29.07.2019 and 28.07.2021. Out-of-sample forecasting is a dynamic form of forecast methodology where posteriors are used as priors in generating next-step ahead forecast so that we can compare results that establish the reliability of the estimated FCVAR models relative to the conventional Cointegrated VAR (CVAR) model. As is well-known statistical tests of a model's forecast performance are commonly conducted by splitting a given data set into an *in-sample* period, used for the initial parameter estimation and model selection, and an *out-of-sample* period, used to evaluate forecasting performance. Modellers often exploit empirical evidence based on out-of-sample forecast performance as it is considered more reliable than evidence based on *in-sample performance*. This is because the *in-sample* forecast can be more sensitive to outliers and data mining. Further, *out-of-sample* forecasts also better reflect the information available to the forecaster in 'real time'.

The steps taken are as follows. Once FCVAR and CVAR models have been estimated, a set of consecutive forecasts with 30-day horizon are generated on each day in the $test\ period$. Forecasting errors are then calculated for each forecast horizon considered. Relative forecasting performance (RFE) of each **Green Bond Model 1** (*Green bond, energy price, uncertainty*) and **Green Bond Model 2** (*Green bond, health price and uncertainty*) FCVAR and CVAR models were then assessed by calculating the percentage change between their RMSFE values as defined below:

$$RFE = 100 \times \left\{ \frac{FCVAR_{RMSFE}}{CVAR_{RMSFE}} - 1 \right\} \tag{14}$$

Here a negative RFE indicates the relative forecasting performance superiority of the FCVAR model with respect to CVAR model, and a positive value indicates the superiority of the CVAR model over the FCVAR counterpart. Six out-of-sample forecast windows were considered for forecast evaluation of Model 1 and Model 2: 1-step/1-day ahead, 5-step/1-week ahead, 10-step/2-weeks ahead, 20-step/1-month ahead, 30-step/1.5-month ahead and 40-step/2-months ahead forecasts.

Table 9: : **RMSFE values and relative performance of FCVAR and CVAR models for Green Bond Model 1 and Green Bond Model 2**

Model		*Forecast Horizon*					
		T+1	*T+5*	*T+10*	*T+20*	*T+30*	*T+40*
a) Magnitudes of FCVAR RMSFE values							
	Model 1	1.286	2.386	3.221	4.433	5.245	5.287
	Model 2	1.321	2.363	3.198	4.055	4.599	4.671
b) Magnitudes of CVAR RMSFE values							
	Model 1	1.310	2.528	3.541	5.155	6.714	8.087
	Model 2	1.336	2.561	3.696	5.571	7.592	9.431
c) RFE - FCVAR vs CVAR							
	Model 1	-1.77%	-5.61%	-9.06%	-14.01%	-21.88%	-34.61%
	Model 2	-1.13%	-7.74%	-13.50%	-27.21%	-39.42%	-50.48%

Note: (i) Model's forecasting performance is measured by the RMSFE values. (ii) Section (a) reports the RMSFE values for the multivariate FCVAR models of Model 1 and Model 2. (iii) Section (b) reports the RMSFE values for the multivariate CVAR models of Model 1 and Model 2. (iv) Section (c) reports the relative performance of the FCVAR with respect to CVAR model in terms of RMSFE values of Model 1 and Model 2; negative values signify FCVAR model superiority and vice versa.

The magnitudes of RMSFE values of Model 1 and Model 2 reported in section (a) and (b) of Table 9 which report goodness of fit between forecasted and actual values to suggest that FCVAR methodology provides superior forecast estimation compared to CVAR approach for all forecast horizons considered in this study. Results also demonstrate that Model 2 benefits from FCVAR modelling the most out of the two variables considered for forecast evaluation. Looking at section (a) of the table, the accuracy of the Model 2 FCVAR model does not deteriorate at the same rate it does in the case of Model 1. For example, while RMSFE values for the 1-step ahead forecast of Model 1 and Model 2 are 1.286 and 1.321 respectively the gap widens in the case of the 40-steps ahead forecast where they are 5.287 and 4.671. The increasing spread between the two models indicates that using proposed market variables in the FCVAR framework is able to replicate Model 2 dynamics better than the dynamics in Model 1.

Relative forecasting performance presented in section (c) of Table 9 demonstrates the level of improvement FCVAR methodology delivers over its CVAR counterpart. In both Model 1 and Model 2 cases, FCVAR models yield more accurate forecasts compared to CVAR models. Results indicate that the accuracy increases with the forecast horizon, suggesting that the FCVAR approach is better suited to replicate the long-run equilibrium relationship that exists amongst the variables in the system. Overall, our results indicate that the CVAR model performs significantly worse in forecasting. In other words, the FCVAR methodology is superior in capturing long-term green bond index trends. This is due to the model's ability to better approximate the long-run equilibrium that exists between the variables compared to the CVAR technique. This leads to the accuracy in forecasts, as demonstrated using the Green Bond index.

(ii) Interdependence in Green Bond, Sectoral Health Price Index, and Policy Uncertainty

We begin with the system where Green Bond is assumed to co-move with Sectoral Health Price and Policy Uncertainty. We find that this system elicits one cointegration rank where the estimated value of the system 'memory' (*d*) is 0.478. This is a mean-reverting long-memory process, demonstrating that in the long run, this dynamic system is likely to stabilise; a further predictive implication that the system can co-move with stability. The unrestricted FCVAR estimation also provides the value of $d = 0.766 < 1$ and equation (17) provides the cointegration relationship. We observe that in the cointegration relationship, a rise in health index price will have a negative impact on the Green Bond price - which is natural as Green Bonds and Other sectoral indices are not complementary. An investment in a green project will potentially diversify an investor's fund from another financial instrument (in this case, the health index). Further, with rising uncertainty (EPU), we also observe that investors will be more inclined to invest in green projects; further eliciting a general investment sentiment that during uncertain times, stick to an investment with an expected stable or secured return.

Table 10: **Rank selection - FCVAR**

Rank	d	Log-likelihood	LR Stat	P-value
0	0.487	32509.04	19.734	0.02
1	0.478	32516.355	5.103	0.277
2	0.490	32518.572	0.668	0.414
3	0.501	32518.906	—-	—-

FCVAR estimation:

$$\Delta^{\hat{d}}\left(\begin{bmatrix} greenbond \\ health \\ EPU \end{bmatrix} - \begin{bmatrix} 1.549 \\ 1.680 \\ 1.275 \end{bmatrix}\right) = L_{\hat{d}}\begin{bmatrix} -0.001 \\ 0.005 \\ 0.559 \end{bmatrix}\begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4}\hat{\Gamma}_i\Delta^{\hat{d}}L^i_{\hat{d}}(Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{15}$$

$\hat{d} = \underset{(0.029)}{0.766}, Q_{\varepsilon}(12) = \underset{(0.804)}{307.211}, LogL = -2298.015$

$$greenbond^{*} = 1.133 - 0.241 \times healthindex_t + 0.075 \times EPU_t + \nu_{1t} \tag{16}$$

(iii) Interdependence in Green Bond, Sectoral Financial Price Index, and Policy Uncertainty

With respect to the relationship between green bonds, the sectoral financial index and uncertainty, we also observe interesting patterns. Recall that the S&P 500 Financials comprises those companies included in the S&P 500 that are classified as members of the Global Industry Classifications financials sector. Our premise is that the green bond index would co-move with the price variations in the financial index and the impact magnitude can be moderated by the presence of uncertainty.

Tables 11 and 12 present lag and rank selection of the FCVAR model comprising of the green bond, financials and uncertainty variables. Using BIC criteria we select 2 lags of the FCVAR system (Table 11) where the estimated memory parameter is $d = 0.778$. Table 12 reports rank selection results. We find that our system has one cointegrating rank as the p-value for rank = 1 is 0.751 and thus we reject rank = 0 against rank = 1. Finally, we perform unrestricted FCVAR using the selected lags and the rank of the system. Equations 14 and 15 present these results. The system $d = 0.769$ with a p-value = 0.021, implying significance of the long memory of the FCVAR system. The cointegration equation shows that green bond is positively affected by price variations in the financial index as well as uncertainty, similar to the results we obtained for the energy index.

Table 11: **Lag-order selection - FCVAR**

k	r	d	b	LogL	LR	pv	AIC	BIC	pmvQ
4	3	0.943	0.943	39080.22	51.26	0	-78062.44*	-77766.57	0
3	3	0.859	0.859	39054.59	63.98	0	-78029.18	-77787.66	0
2	3	0.778	0.778	39022.6	232.57	0	-77983.2	-77796.02*	0
1	3	0.856	0.856	38906.32	544.8	0	-77768.64	-77635.8	0
0	3	0.863	0.863	38633.92	0	0	-77241.84	-77163.34	0

Table 12: **Rank selection - FCVAR**

Rank	d	b	Log-likelihood	LR	P-value
0	0.802	0.802	39005.644	33.917	0.018
1	0.769	0.769	39019.624	5.957	0.751
2	0.779	0.779	39022.318	0.57	0.901
3	0.778	0.778	39022.602	—-	—-

FCVAR estimation:

$$\Delta^{\hat{d}}\left(\begin{bmatrix} greenbond \\ finance \\ EPU \end{bmatrix} - \begin{bmatrix} 1.550 \\ 1.666 \\ 1.443 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} 0.001 \\ -0.005 \\ 0.026 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^{i} (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{17}$$

$\hat{d} = \underset{(0.021)}{0.769}$, $Q_\varepsilon(12) = \underset{(0.904)}{327.221}$, $LogL = -2400.019$

$$greenbond^{*} = 1.152 - 0.331 \times financeindex_t + 0.262 \times EPU_t + \nu_{1t} \tag{18}$$

(iv) Green bond, sectoral Information Technology index and uncertainty

We also use sectoral information technology index to form cointegrating relationship with green bond and uncertainty. The lag selection of the FCVAR shows four lags are optimal (Table 14) and those lags help us choose one cointegration rank (Table 15). The results of the unrestricted FCVAR estimation with four lags and 1 cointegrating rank for this FCVAR system is summarised in equation 16 and 17. The system memory ($d = 0.415$) and the cointegration relationship show that a rise in information technology price index increases green bond prices but an increase in uncertainty increases green bond prices.

Table 13: **Lag-order selection - FCVAR**

k	r	d	b	LogL	LR	pv	AIC	BIC	PmvQ
4	3	0.444	0.444	-31678.62	36.88	0	63455.25*	63751.12	0
3	3	0.410	0.41	-31697.06	32.79	0	63474.13	63715.65	0
2	3	0.590	0.59	-31713.46	109.87	0	63488.92	63676.10*	0
1	3	0.457	0.457	-31768.4	465.01	0	63580.79	63713.63	0
0	3	0.885	0.885	-32000.9	0	0	64027.8	64106.3	0

Table 14: **Rank selection - FCVAR**

Rank	d	b	Log-likelihood	LR statistic	P-value
0	0.271	0.271	-31700.634	44.019	0
1	0.425	0.425	-31683.62	9.99	0.041
2	0.415	0.415	-31679.319	1.389	0.239
3	0.444	0.444	-31678.625	—-	—-

FCVAR estimation:

$$\Delta^{\hat{d}} \left(\begin{bmatrix} greenbond \\ Infotech \\ EPU \end{bmatrix} - \begin{bmatrix} 110.934 \\ 331.670 \\ 29.358 \end{bmatrix} \right) = L_{\hat{d}} \begin{bmatrix} -0.010 & -0.001 \\ -0.235 & 0.017 \\ 0.223 & 0.026 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L^i_{\hat{d}} (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{19}$$

$\hat{d} = \underset{(0.030)}{0.415}$, $Q_{\varepsilon}(12) = \underset{(0.980)}{366.520}$, $LogL = -2598.020$

$$greenbond^{*} = 1.122 - 0.001 \times infotechindex_t + 0.38 \times EPU_t + \nu_{1t} \tag{20}$$

$$Infotech^{*} = 1.250 - 2.143 \times EPU_t + \nu_{1t} \tag{21}$$

In Table 15, we have summarized the forecast performances for various models, viz., the FCVAR estimation with the health price index, financial price index, and information technology price index, respectively. It is evident (by looking at the negative values and comparing the detailed specification in Table 9) that FCVAR models - across specifications of sector inclusion - significantly outweigh the competing CVAR models for all time horizons (one day to 40 days ahead forecasts).

Table 15: : **RMSFE values and relative performance of FCVAR and CVAR models for different systems**

Model	*Forecast Horizon*					
	T+1	*T+5*	*T+10*	*T+20*	*T+30*	*T+40*
(a) RFE - FCVAR vs CVAR (Financial Price)						
Model 3	-1.59%	-4.98%	-8.11%	-12.15%	-18.80%	-31.60%
(b) RFE - FCVAR vs CVAR (Information Tech.)						
Model 4	-1.87%	-5.91%	-8.29%	-13.86%	-20.64%	-33.66%
(c) RFE - FCVAR vs CVAR (MSCI Green Bond)						
Model 5	-1.60%	-4.87%	-7.32%	-14.62%	-21.55%	-34.09%

Note: (i) We report the relative performance of the FCVAR with respect to the CVAR model in terms of RMSFE values of Model 3 (model with financial price index) and Model 4 (model with information technology price index); negative values signify FCVAR model superiority and vice versa. (ii) Model 5 corresponds to the results for MSCI green bond as an alternative proxy for a green bond.

5.3 Robustness

In this section, we study the robustness of our results by employing two changes to our baseline regression setting. First, we use an alternative proxy for green bonds. In particular, we use Bloomberg Barclays MSCI Global Green Bond Index for this purpose.[15] The MSCI Green Bond Index is a robust measure of the global market for fixed-income securities issued to fund projects with direct environmental benefits. To adhere to established green bond principles, an independent research-driven methodology is used to evaluate index-eligible green bonds. The index was created in November 2014, with index history backfilled to January 1, 2014. Our second strategy is to compare results between the pre- and post-COVID periods.

(i) Using an alternative measure of green bond: the Bloomberg Barclays MSCI Global Green Bond Index

As an alternative proxy for S&P's green bond index, we have used Bloomberg Barclays MSCI Global Green Bond Index to study the robustness of our results. Accordingly, we design the following FCVAR system: a system made of the MSCI green bond index, the sectoral energy index and uncertainty. As is the rule, we first select the system lag order (k). We select $k = 2$ using BIC criteria. Next, the rank test provides us with one rank of the FCVAR system as the p-value for rank = 1 is 0.350. At this rank, the d=0.501 shows a long memory but with the mean-convergent property. The unrestricted FCVAR results are not significantly different from the baseline results (equation 11) with the S&P green bond index. We find that the estimated system-wide d=0.509 shows the persistence of shocks and information asymmetry but a possibility that the system will stabilise within a short time duration. Further, the sign and impact magnitudes of the sectoral energy price index and that of the policy uncertainty on MSCI green bond are not significantly quantitatively different from the baseline regression. An out-of-sample forecasting exercise has also been carried out for this FCVAR run. The results are reported in Table 15 (Model 5). The negative values, which appear to grow in longer time horizons, clearly prove that FCVAR is the better method than the competing CVAR model to describe the dynamics of such an interdependent system.

Table 16: **Lag-order selection - FCVAR**

k	r	d	LogL	LR	pv	AIC	BIC	PmvQ
4	3	0.472	27734.10	23.07	0.006	-56697.62*	-57060.79	0
3	3	0.537	27710.25	38.32	0	-56539.33	-57110.06	0
2	3	0.515	27536.32	222.24	0	-56445.22	-57144.08*	0
1	3	0.918	27266.11	159.17	0	-56238.01	-56994.18	0
0	3	1.001	27149.38	0	0	-55933.78	-56907.36	0

[15] Many thanks to an anonymous referee to suggest the use of this measure as a robustness exercise.

Table 17: **Rank tests - FCVAR**

Rank	d	Log-likelihood	LR	P-value
0	0.498	27455.138	25.332	0.035
1	0.501	27638.605	6.397	0.350
2	0.522	27745.362	2.400	0.402
3	0.539	28091.113	—-	—-

FCVAR estimation:

$$\Delta^{\hat{d}}\left(\begin{bmatrix} MSCIgreenbond \\ energy \\ EPU \end{bmatrix} - \begin{bmatrix} 1.320 \\ 1.622 \\ 1.074 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.004 & -0.004 \\ 0.016 & -0.004 \\ 0.208 & 0.087 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^{i} (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{22}$$

$\hat{d} = \underset{(0.021)}{0.509}$, $Q_{\varepsilon}(12) = \underset{(0.887)}{333.514}$, $LogL = 32408.214$

$$MSCIgreenbond^{*} = 1.133 - 0.009 \times energyindex_t + 0.074 \times EPU_t + \nu_{1t} \tag{23}$$

$$EnergyPrice^{*} = 1.129 + 0.298 \times EPU_t + \nu_{1t} \tag{24}$$

(i) Sub-sample analyses: Pre- and Post-COVID period comparison

In this section, we present results from a sub-sample regression for the baseline model (S&P 500 green bond, sectoral energy, and EPU) for pre-COVID (29/07/2008 to 29/01/2020) and post-COVID (30/01/2020 - 28/07/2021). We followed the World Health Organization (WHO) declaration of the outbreak as a public health emergency of international concern on 30 January 2020). For the sake of brevity and minimising repetition, we have presented below the cointegration equations and the system representation of the unrestricted FCVAR for both pre-COVID (equations 25-27) and post-COVID (equations 28-30). Cautions need to be taken to lend reliable interpretability of results given the length of samples for the two periods: we have 2742 daily observations in the pre-COVID period and 390 daily observations in the post-COVID period. Considering equations 25 and 28 for pre- and post-COVID estimates of $\hat{d}$, an indicator of system-wide memory or persistence of shocks, we detect a distinguishable pattern: the pre-COVID estimates of 0.422 is significantly smaller than the post-COVID estimate (0.814). In other words, the post-COVID system estimation depicts the presence of greater depth of information asymmetry and instability, a feature that might help investors to diversify their portfolio from a risky asset to green bond. In addition, we also notice from equations 26 and 29, respectively for the two pre and post-COVID period cointegration equations, that policy uncertainty (EPU) has greater positive effects in the post-COVID on green bond than it is for the pre-COVID sample. This is expected because policy uncertainty is observational high in the post-COVID period. This state of proliferated uncertainty would have larger effects on green bond investment in the form of diversification of an investor's portfolio.

(a) FCVAR estimation (Pre-COVID sample):

$$\Delta^{\hat{d}}\left(\begin{bmatrix} greenbond \\ energy \\ EPU \end{bmatrix} - \begin{bmatrix} 1.102 \\ 1.014 \\ 0.908 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.001 & -0.001 \\ 0.012 & -0.003 \\ 0.175 & 0.064 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{25}$$

$\hat{d} = \underset{(0.021)}{0.422}$, $Q_{\varepsilon}(12) = \underset{(0.887)}{333.514}$, $LogL = 32408.214$

$$greenbond^* = 1.111 - 0.004 \times energyindex_t + 0.065 \times EPU_t + \nu_{1t} \tag{26}$$

$$EnergyPrice^* = 1.001 + 0.121 \times EPU_t + \nu_{1t} \tag{27}$$

(b) FCVAR estimation (Post-COVID sample):

$$\Delta^{\hat{d}}\left(\begin{bmatrix} MSCIgreenbond \\ energy \\ EPU \end{bmatrix} - \begin{bmatrix} 1.446 \\ 1.747 \\ 1.286 \end{bmatrix}\right) = L_{\hat{d}} \begin{bmatrix} -0.008 & -0.008 \\ 0.023 & -0.004 \\ 0.301 & 0.080 \end{bmatrix} \begin{bmatrix} \nu_{1t} \\ \nu_{2t} \end{bmatrix} + \sum_{i=1}^{4} \hat{\Gamma}_i \Delta^{\hat{d}} L_{\hat{d}}^i (Y_t - \hat{\rho}) + \hat{\varepsilon}_t \tag{28}$$

$\hat{d} = \underset{(0.033)}{0.814}$, $Q_{\varepsilon}(12) = \underset{(0.802)}{415.501}$, $LogL = 41409.110$

$$greenbond^* = 1.687 - 0.012 \times energyindex_t + 0.097 \times EPU_t + \nu_{1t} \tag{29}$$

$$EnergyPrice^* = 1.774 + 0.303 \times EPU_t + \nu_{1t} \tag{30}$$

5.4 Non-technical exposition of results

We present in this section, a non-technical implication of the estimated FCVAR results by eliciting the interpretability of results in two important aspects. First, the implications of system-wide memory (a result of unrestricted FCVAR estimation) and the meaning of various cointegrating relationships. Second, we distinguish our results across sectors (viz., health, information technology, and financial sector indices).

The main idea of our memory-embedded VAR estimation was to model short and long-run relationships among green bonds, sector-specific indices and policy uncertainty. The results are intended to help us understand the magnitude of system-wide shocks, so much so that their rates of dissipation (i.e., the time to be taken to completely disappear from the system) will inform policymakers to chart out the effectiveness of intervention plans. In our baseline FCVAR model (viz., with S&P green bond, sectoral energy price index, and economic policy uncertainty), we have a system memory ($\hat{d}$=0.574, which means that this 3-variables system will require a policy intervention (technically, an error-correction term), to stabilise the system in the long-run. The estimates of $\hat{d}$=0.574, which is clearly less than 1, implies that it is indeed possible to regain the stability of this system in a way that can help the long-term growth of green bonds. The magnitude of this shock is not very different from our re-estimation of the same 3-variable system but now replacing S&P green bond with Bloomberg Barclays' MSCI green bond index. The results are presented in Section 5.3 (equation 22), which shows that the estimated $\hat{d}$=0.509. This estimate depicts a bit smaller persistence than that of the baseline model, yet both are within the range between 0.5 to 1, depicting the presence of longer-term memory but the ability of the system to attain stability with proper policy interventions.

How different are the results across sectors? A primary assumption of the green bond dynamic system is that this environmentally sustainable investment strategy will experience greater growth at a time when there is higher policy uncertainty. Because uncertainty can infuse greater air of volatility in various sectoral indices, the estimates of FCVAR across sectors are expected to be characteristically affine, albeit with some differences in magnitudes. It is indeed the case, for instance, the estimation of system-wide memory with respect to the sectoral health price index, financial price index, and information technology is 0.766, 0.769, and 0.415, respectively. Whilst we observe greater persistence, in other words, the possibility of the system taking a longer period of time to stabilise (for health and financial sectors), we find a relatively shorter time period to stability for the information technology sector. This result is also commensurate with the larger literature (Reboredo, 2018).

6 Concluding remarks and policy implications

Sustainability agenda, in a number of policy forums, have reinforced the need for rigorous empirical investigations to enable academics, investors and policy practitioners with a clear pathway of risk-return trade-off. More importantly, financing 'green' projects preserve value for the future bequest of welfare. In the face of nascent literature on the identification of the dynamic relationship between green bonds and various sectoral performances, this paper proposes a robust empirical mechanism and argues for its implications for investment strategy and policies. Indeed, investment strategy in green bonds has implications for portfolio diversification and risk management.

Our contribution embeds the dynamic role of uncertainty - an imposing concern in investment decisions in modern-day times, in shaping the green bond and sectoral performance interdependence.

A primary hypothesis, the first one to test in the extant literature, is that the green bond index can co-move with sectoral indices as the latter are subject to market forces, and so is the green bond, despite its sustainable image. As uncertainty grows, the expectation is that the green bond index will react positively, because investors can take green bonds as a safe hedge against uncertainty rise. Because uncertainty may exert negative effects on sectoral indices, such as health care, financials, and the energy sector, among others, we expect that volatility in these indices may also increase investment in green bonds, producing a positive relationship. Our FCVAR estimation for a green bond, energy price and uncertainty depicted expected patterns. A suit of robustness exercise lent validity to our claims. An important finding of our work is also that in the green bond and the VAR system of green bond with sectoral indices and uncertainty, in both cases, we find strong long-memory features, although they depict the possibility of long-run mean convergence. The conclusion is that the system is inefficient and an investor can still use this inefficiency degree to choose among a portfolio of investments - with the sectoral indices or with the green bond.

What implications do our results have for policy and practitioners, such as investors? With regard to the policy, there are two immediate implications. First, a non-accountability of the magnitude of memory in an interactive system such as ours downplays the convergence speed of shocks within the system. Eventually, such an omission costs the timing requirement to stabilise shocks with policy interventions, such as governmental incentives for proactive investment in green bonds. Just because the growth of green bonds is tightly interlinked with the growth of various assets or stocks as attractive investible opportunities, a true understanding of the nature of this interdependence, in particular, the dissipation rate of system-wide shocks is central to effective strategic policy planning. An investor will be very keen to know the number of days or months a shock within such a system is going to disappear so that he can plan his investment portfolio by diversification needs.

Further, the fact that the price of green bonds is supposedly less volatile than that of other sectoral indices (non-green sectors), investment in a green bond can be an important hedging opportunity. Indeed, if one finds a long-run equilibrium relationship between green bonds and other sectoral indices, it may raise a few concerns on the likely volatility of green bonds - one that literature has shown, is due to extensive greenwashing. If we consider the interdependence between green bonds and other sectoral indices (such as health, information technology, and others) investors who intend to invest at various time horizons can engage in diverse investment portfolios and hedging choices. Our results, especially the sub-sample analyses have shed light on the differential magnitude of system-wide shock persistence considering before and after COVID, showing the greater degree of shock persistence (relative to pre-COVID scenario) will create intense information asymmetry, encouraging risk-averse investors to consider green bond investment seriously.

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7 Appendix: Alternative measure of Green Bond and S&P Sectoral Indices

We present here the definition of an alternative measure of Green Bond and selected sectoral indices. For the latter, we have used definitions of S&P 500.

- Bloomberg Barclays MSCI Green Bond Index

The Bloomberg Barclays MSCI Green Bond Index is an objective and robust measure of the global market for fixed-income securities. To measure it, an independent research-driven methodology is used to evaluate index-eligible green bonds with the objective to ensure that they adhere to established Green Bond Principles and to classify bonds by their environmental use of proceeds.

- Information Technology

This sector comprises companies that develop or distribute technological items or services (such as computers, microprocessors, and operating systems) and includes internet companies. This sector has experienced a paradigmatic change in recent years because of the rapid rise in technology-based companies.

- Health care

This sector consists of companies such as medical supply, pharmaceutical, and scientific-based operations or services that aim to improve the human body or mind. Familiar names include Johnson & Johnson, a medical device and pharmaceutical company that owns Tylenol, and Abiomed, which manufactures medical implant devices. Of course, Cannabis companies are a new, but rapidly growing, part of the healthcare sector. Currently, the more well-known ones include Canopy Growth Corp. and Aurora Cannabis, with market caps of $23 billion and $12 billion, respectively.

- Financials

The financial sector includes all companies involved in finance, investing, and the movement or storage of money. This covers banks, credit card issuers, credit unions, insurance companies, and mortgage real estate investment trusts (REITs). Companies within this sector are usually relatively stable, as many are mature, well-established firms. Banks in this sector include Bank of America Corp, JPMorgan Chase & Co., and Goldman Sachs. Other notable sector names include Berkshire Hathaway, American Express, and Aon plc.

- Energy

The energy sector consists of all companies that play a part in the oil, gas, and consumable fuels business. This includes companies that find, drill, and extract the commodity. It also includes the companies that refine the material and companies that provide or manufacture the equipment used in the refinement process. Companies such as Exxon Mobil and Chevron extract and refine gas, while companies like Kinder Morgan transport fuel to gas stations.

Figure 3: **Temporal Pattern of Green Bond and sectoral price for energy and financials**

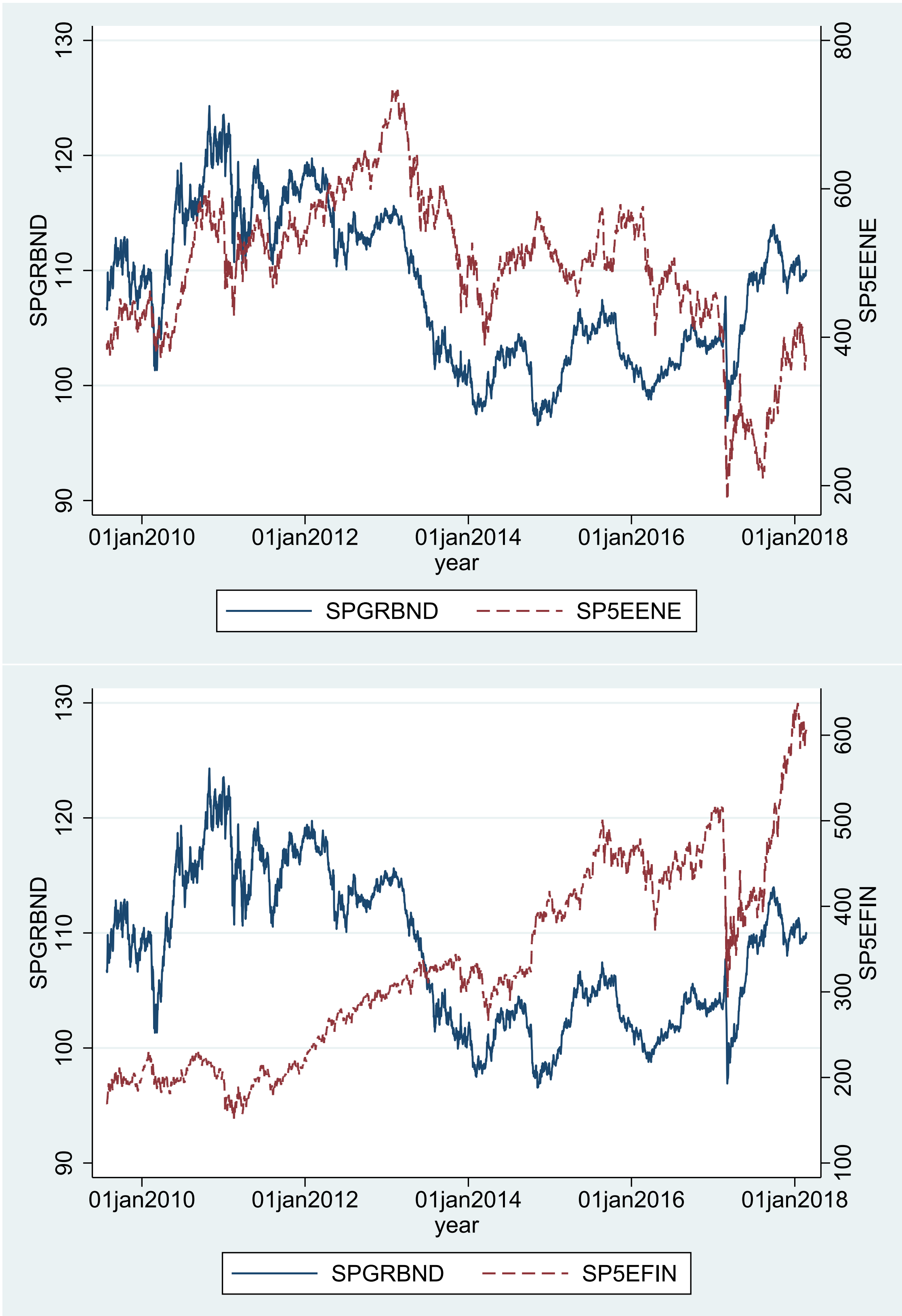

Figure 4: **Temporal Pattern of Green Bond and sectoral price for health and information technology**

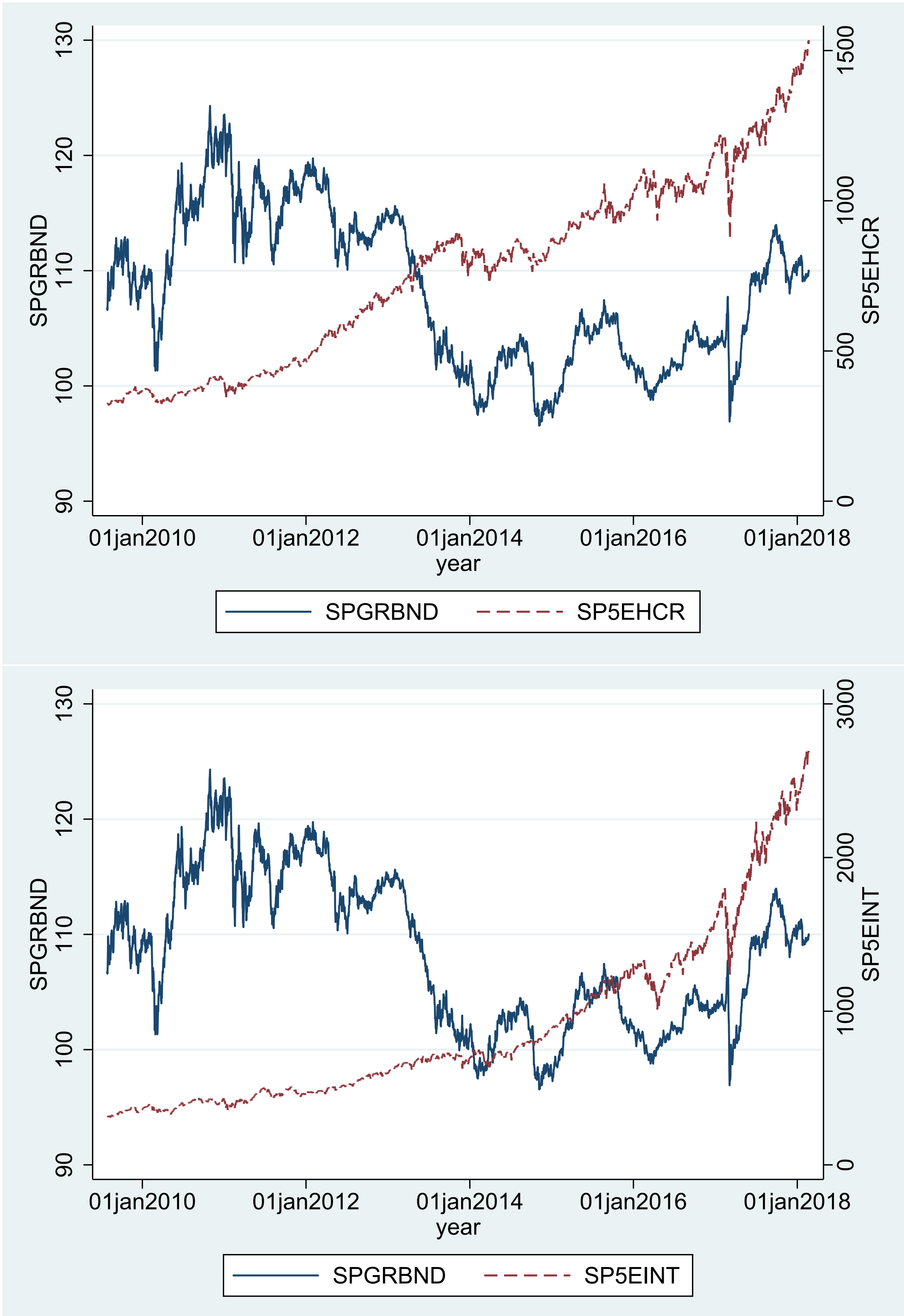

Figure 5: **Temporal Pattern of Green Bond and MSCI Green Bond**

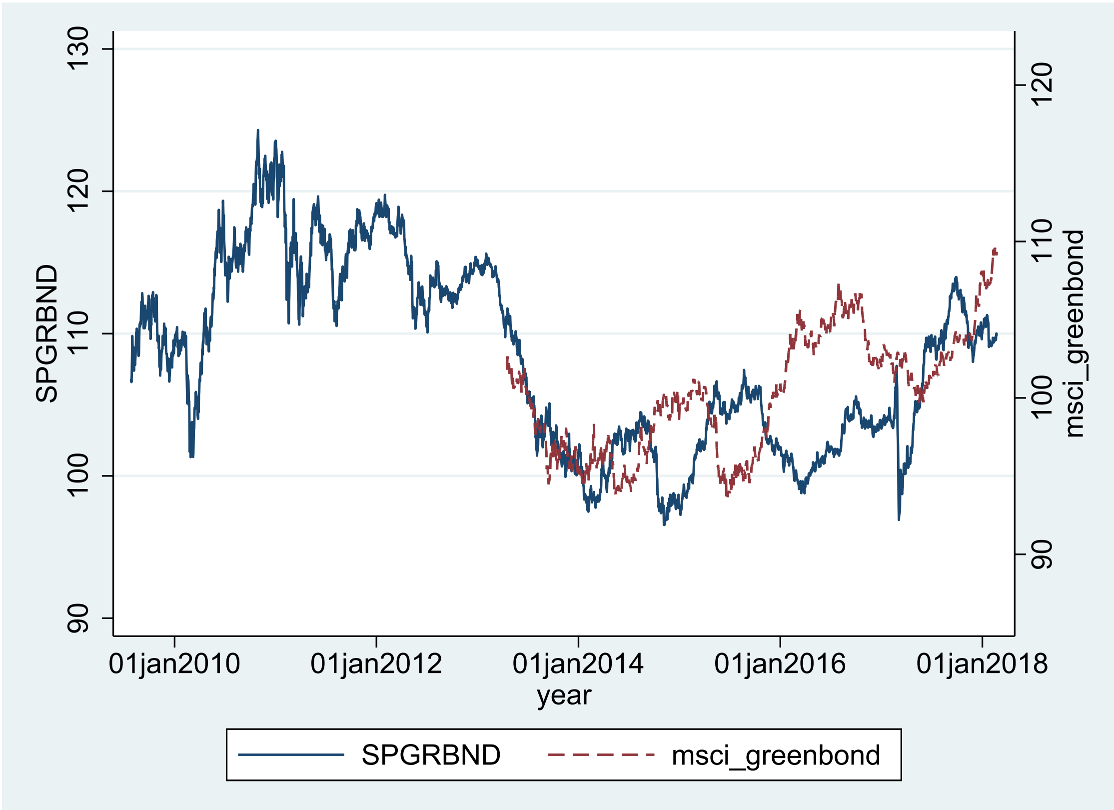

Figure 6: **Dynamics of Rolling-window d Estimates Green Bond**