European Journal of Operational Research xxx (xxxx) xxx

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Innovative Applications of O.R.

Budget allocation of food procurement for natural disaster response

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ARTICLE INFO

Article history:
Received 8 June 2022
Accepted 8 May 2023
Available online xxx

Keywords:
Humanitarian logistics
Disaster management
Procurement lot sizing
Robust optimisation
Stochastic programming

ABSTRACT

This paper studies a variant of the lot sizing problem that arises in the context of disaster management. In this problem, a fixed budget has to be allocated efficiently over multiple time periods to procure large quantities of a staple food that will be stored and later delivered to people affected by disaster strikes whose numbers are unknown in advance. Starting from the deterministic model where perfect information is assumed, different formulations to address the uncertainties are constructed: classical robust optimisation, risk-minimisation stochastic programming, and adjustable robust optimisation. Experiments conducted using data from West Java, Indonesia allow us to discuss the advantages and drawbacks of each method. Our methods constitute a toolbox to support decision makers with making procurement decisions and answering managerial questions such as which annual budget is fair and safe, or when storage peaks are likely to occur.

1. Introduction

We consider a variant of the lot sizing problem that arises in the context of disaster management. In this problem, a fixed budget has to be allocated efficiently over multiple time periods to procure a staple food that will be stored and later delivered to people affected by disasters. Future demands, prices and availabilities are unknown at the start of the planning horizon but data of past disaster events exists that can be used to estimate probability distributions or simulate scenarios via resampling. Demand variability can be particularly high due to the unpredictable nature of disasters. Our main objective is to minimise the shortage of food, while secondary objectives include minimising the waste resulting from food items perishing, and the budget usage.

To approach the problem, a deterministic model, which assumes future information is available, is first introduced as a deterministic linear program. We then explore three methods to drop the assumption of perfect future information to allow us to find robust solutions to the problem. We assume the information available to the decision maker to make a new procurement decision is a set of historical scenarios, the information that has been revealed so far and the current state of the system. Our first method is risk-minimisation stochastic programming (Birge & Louveaux, 2011; Rockafellar & Uryasev, 2000) based on a two-stage formulation of our problem. The first-stage decision variables are the procurement decisions over the remaining time periods, while the second-stage variables are scenario-adapted measures that evaluate the consequences of the first-stage decisions over the historical scenarios. The second method is classical robust optimisation (Ben-Tal, El Ghaoui, & Nemirovski, 2009); here, we assume that the parameters of the model can vary within specific ranges, defined by historical data. It is straightforward to deduce the robust counterpart formulation from the two-stage formulation in the first method, and we examine ways to reduce the degree of conservatism of the robust solutions. Our final method is adjustable robust optimisation (Ben-Tal et al., 2009; Yanikoglu, Gorissen, & den Hertog, 2019) in which we look for an optimal decision rule to extract from historical data so that it can be applied to make immediate decisions during the planning time horizon of a new scenario. Decision rules can be formalised in various ways in adjustable robust optimisation; however for reasons of tractability we choose to use an affine mapping from the information that has been revealed so far to the immediate here-and-now procurement decision. Experiments conducted on generated data, which simulates real historical events, allow us to discuss the advantages and

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https://doi.org/10.1016/j.ejor.2023.05.015
0377-2217/

drawbacks of each method. Our methods constitute a toolbox to support or to produce automated decisions for annual planning for disaster preparedness. Some methods are tunable according to how much risk the decision makers could take. All methods are able to provide answers to high-level managerial questions such as how large the annual budget should be, the amount of storage capacity that is required, or when storage peaks are likely to occur.

We demonstrate our methods on a case study from West Java province in Indonesia, which has the highest multi-disaster risk in Indonesia. Between 2016 and 2020, West Java province experienced an average of 4 disasters per day, mostly small-to-medium scale disasters. Onggo et al. (2021) showed that during this period, the most common disasters were landslide, tornado, flood and fire (Onggo et al., 2021). West Java province covers 37,000 square kilometre and has a population of over 46 million people, giving it the second highest population density in Indonesia. Hence, when floods strike, they often affect a significant number of people and result in a high number of refugees. Between 2016 and 2020, a total of more than 1.5 million victims were affected, with almost 100,000 of them having to live in shelters and so becoming classified as refugees. We invite readers who are interested in the detail to read (Onggo et al., 2021).

By law, the Indonesian government is required to provide assistance, including food supplies, to those affected by a disaster for up to 14 days after a disaster strikes. This emergency period may be extended when needed. In this case study, we choose rice because it is the main staple food in Indonesia. The Indonesian government sets aside a national budget known as CBP (Cadangan Beras Pemerintah or Government's Rice Reserve) to stock rice to be distributed to refugees during disaster response operations. The rice procurement is carried out by the state logistics bureau (BULOG). Hence, at the beginning of the budget year, the West Java provincial government will sign an availability contract with BULOG that guarantees that CBP rice is always available when needed. BULOG makes the decision on how much rice to buy each month. The challenge is that the demand is seasonal with high variability. Likewise, rice production is seasonal and the price varies. West Java province is one of the main rice producers; hence, availability is not an issue but it is included in the model as this may not be the case in other provinces. Our model is developed to help BULOG West Java division to make the optimal procurement decisions for the provincial government and all municipality governments in West Java province.

The supply for each refugee is delivered as soon as possible by the local disaster relief agency to each shelter from the nearest BULOG warehouse. This demonstrates the importance of optimising rice procurement for disasters. If too little rice is bought then people's lives and their welfare will be affected and if too much is bought, it wastes money that could be used for other government services including those involved in the disaster response and disaster recovery phases. A further issue with over-stocking is that it could lead to food waste. While rice has a long shelf-life, the lack of suitable storage facilities designed specifically for rice (e.g. temperature controlled silos) shortens the shelf-life. The wastage of a significant amount of CBP rice is a known issue in Indonesia (e.g. CNBC Indonesia, 2018[1]). BULOG thus faces what we term the rice procurement problem in which they decide the amount of rice to purchase in each time period in order to minimise the shortage of supply to refugees, within a fixed annual budget. The price of rice varies stochastically during the year and the numbers of disasters and associated refugees also vary randomly through the year following a clear seasonality. Rice is assumed to be perishable and BULOG's policy is to use rice that is not older than six months old. Any rice not used within its allowed lifetime is assumed to be wasted.

This paper focuses on procurement pre-disaster by purchasing the right amount of rice at the right time in a budget year. Procurement affects the efficiency of disaster management operations such as the distribution of relief items. Hence, suitable models are needed to support procurement decisions which, subsequently, can improve the performance of disaster response operations. Despite the importance of procurement in disaster operations management, the current Operational Research/Management Science (ORMS) literature on disaster operations management has focused primarily on problems of facility location, inventory management and transportation (Aghajani & Torabi, 2020). Only a few studies focus on procurement decisions (Balçik & Ak, 2014). Hence, our work contributes to the ORMS literature on procurement in disaster management. In the early years, most ORMS models used synthetic and often unrealistic data to test their models. However, recently, researchers have recognised this as one of the main weaknesses and encouraged the use of real-world data to evaluate how ORMS models perform under real-world cases and to test the robustness of the solutions (e.g. Esposito Amideo, Scaparra, & Kotiadis, 2019; Farahani, Lotfi, Baghaian, Ruiz, & Rezapour, 2020; Kovacs & Moshtari, 2019; Sabbaghtorkan, Batta, & He, 2020). This is especially important for disaster management, where the variation of demand is high. In addition to promoting the use of real-world data, studies that apply ORMS methods to procurement decisions in disaster management, especially for decision makers in regions that experience disasters regularly (daily, weekly or monthly), is lacking. These regions include many Asian countries. Furthermore, research into humanitarian food supply chains is still underdeveloped (Perdana et al., 2022). Hence, this paper contributes to the literature on humanitarian food supply chains. Finally, we also contribute to the literature on stochastic optimisation by providing an empirical study to compare a number of approaches to develop a robust solution for a practical problem, namely classical robust optimisation, adjustable robust optimisation (Ben-Tal et al., 2009; Yanikoglu et al., 2019) and risk-averse stochastic programming (Rockafellar & Uryasev, 2000). It is uncommon to see comparisons of these approaches on real-world problems such as disaster management, since the two fields of robust optimisation and stochastic programming are founded on different assumptions about the knowledge of the underlying distribution of uncertainties; an exception we could find is (Ni, Shu, & Song, 2018).

After reviewing the literature in the next section, we provide a mathematical formulation for the problem and describe each of the approaches used in Section 3. Numerical experiments based on a real case study from West Java province, Indonesia, are presented in Section 4 and are followed by a conclusion and suggestions for future work.

2. Literature review

We divide the literature review into two subsections. First, we review the literature on procurement in disaster management to provide an overview of how ORMS have been used to help. As we propose the use of robust optimisation and of stochastic programming with risk-minimisation, in the second subsection, we review the literature on different optimisation techniques to address uncertainty and give examples on how they have been applied in disaster management and related areas.

2.1. Procurement in disaster management

The procurement of relief items in response to a disaster can be carried out during the mitigation phase (pre-disaster) or during

[1] https://www.cnbcindonesia.com/news/20180626183343-4-20609/buwas-musnahkan-200000-ton-beras-bulog

the response phase (post-disaster). The pre-disaster procurement is typically planned so that relief items can be pre-positioned at strategic locations. The main advantage of this strategy is the quick response time in delivering relief items to victims post-disaster. The main drawbacks include the potentially low utilisation of resources, high waste (for perishable items), and high inventory cost. This is due to the uncertainty in the demand for and supply of relief items, which makes it challenging to match the supply and demand. Developing a pre-positioning strategy is typically more challenging for non-climate induced disasters as they are not as predictable as those resulting from extreme weather. The post-disaster procurement strategy is advantageous in minimising the waste, minimising inventory costs, and maximising resource utilisation; however, the response time to fulfil the demand from victims is typically longer. One of the main challenges for a post-disaster strategy is that information about the demand may be incomplete or inaccurate at the start of a disaster. The post-disaster strategy includes direct procurement and the implementation of various contracts (e.g. option contract Liang, Wang, & Gao, 2012, quantity flexibility contract Balçik & Ak, 2014, and auctions Aghajani & Torabi, 2020). Both pre-disaster and post-disaster strategies can be combined so that a smaller number of relief items are pre-positioned and when the demand exceeds the current safety stock level, the post-disaster strategy is executed. We focus here on pre-disaster procurement but discuss post-disaster strategies in future work.

The characteristics of the procurement problem in disaster management lend itself to stochastic optimisation models in ORMS. What follows is a review of ORMS methods that have been used to support procurement strategies in the preparedness and response stages of disaster management. The strategies are pre-positioning, direct procurement, contract and auction.

Applications of ORMS methods to the development of a pre-positioning strategy are reviewed in Sabbaghtorkan et al. (2020) who consider publications between 2000 and 2018. They group the research into three categories: location, allocation and location-allocation. The objective of methods in the location category is to find the best locations to pre-position facilities while research into allocation aims to determine the best inventory level of relief supplies in the facilities at known locations. Location-allocation research combines these two problems, determining the best combination of location of facilities and inventory level at each facility. In the context of procurement, the latter two categories are relevant. One of the research directions that they identify is the need for more realistic modelling incorporating practical assumptions such as budget limitations and basing work on realistic case studies (or real data). This is the gap that our work is aiming to fill. Another aspect that has not been addressed in the pre-positioning strategy is that in practice, budget usage and monitoring are important for government agencies due to strict budget regulations. Hence, the challenge is not only about where best to locate the relief items but also how much budget needs to be spent each month to procure them. This work specifically addresses this challenge.

The ORMS methods that have been used for allocation and location-allocation problems include two-stage stochastic programming (TSSP), game theory, and stochastic process. For example, TSSP with recourse action was used to pre-position medical supplies for hurricane preparedness (Paul & Zhang, 2019). In the first stage, they optimise the locations of distribution points, the inventory levels of medical supply and the transportation capacity. In the second stage, after the realisation of hurricane landfall, they determine additional shipping capacity to increase the levels of medical supplies at certain distribution points. An example of the use of game theory is shown in Nagurney, Salarpour, Dong, & Nagurney (2020) where the authors combined game theory and TSSP. The TSSP was used to model the decisions made by each humanitarian organisation during the preparedness phase (stage 1) and response phase (stage 2), while game theory was used to capture the competition among humanitarian organisations (i.e. funding and access to transportation service). A stochastic process model was used for a continuous review two substitutable perishable product disaster inventory model in Yadavalli, Sundar, & Udayabaskaran (2015). They carried out steady state analysis and produced the mean number of: satisfied demands; demands in which the products were substituted; and lost demand.

The direct post-disaster procurement strategy has been modelled using TSSP with recourse (e.g. Falasca & Zobel, 2011; Hu, Han, & Meng, 2017; Torabi, Shokr, Tofighi, & Heydari, 2018). In this method, in the first stage, procurement decisions are made under uncertainty (e.g. demand and damages to infrastructure are not known with certainty). In the second stage, the real information is revealed and a recourse action is applied to improve the decisions made in the first stage. A two-period bi-objective mixed-integer non-linear programming model was developed by Aghajani & Torabi (2020) to model the post-disaster procurement decision, which accounts for the unavailability of information at the beginning of a disaster. Game theory has also been used for purchasing decisions. For example, Nagurney, Salarpour, & Daniele (2019) used game theory to model the purchasing decision of humanitarian organisations (i.e. local or non-local) and to capture the competition among humanitarian organisations. These studies focus on a large scale disaster and none of them consider the regular procurement decisions that need to be made throughout a year to ensure an adequate supply of relief items within a fixed budget. Regular procurement decisions are important in cases where disasters happen more frequently such as in West Java where they happen on a daily basis.

Various types of contract have been proposed to manage the uncertainties affecting procurement decisions. In (Liang et al., 2012) Liang and co-authors considered an option contract and showed that it was possible to have a feasible range of prices in which both the buyer and the supplier were profitable. A quantity flexible contract (QFC) is used in Balçik & Ak (2014), which describes a scenario-based stochastic programming model to design a QFC in which the suppliers were required to supply relief items post-disaster according to the contract's requirements. A QFC is used to coordinate relief items procurement activities in a three-echelon relief chain in Nikkhoo, Bozorgi-Amiri, & Heydari (2018), where the chain comprises a relief organisation (i.e. NGO), one relief item supplier, and affected areas. In their proposed QFC, the relief organisation places an order to the supplier to be stored at the supplier's warehouse before a disaster happens. The relief organisation is committed to buy the relief items at an agreed minimum amount, while the supplier is committed to provide the agreed minimum amount and if required, an agreed extra amount. In our case, the government sign a QFC contract with BULOG in which BULOG is required to supply rice whenever the local government needs it, up to a certain amount annually.

Auction is one of the most commonly used post-disaster procurement strategies. In (Ertem, Buyurgan, & Rossetti, 2010), Ertem and co-authors developed two mathematical models to determine the winner in an auction involving multiple bidders and multiple auctioneers to procure relief items post-disaster. In similar work, Ertem & Buyurgan (2011) used two integer programming models for bid construction and bid evaluation in an auction involving multiple bidders and one auctioneer. Following on, Ertem, Buyurgan, & Pohl (2012) used a genetic algorithm, simulated annealing and integer programming model to analyse bid construction of an auction to procure relief items considering product substitution and partial fulfilment. More recently, Shokr & Torabi (2017) describes the development of an enhanced reverse auction framework to procure excess relief items post-disaster. Two novel possibilistic models are built to support the bidders and the auctioneer on the bid construction and bid evaluation, respectively. The models consider the unit prices, quantities, delivery times and transportation modes of relief items. West Java is the main producer of rice; hence, post-disaster procurement of rice is rare and we do not consider post-disaster procurement here; however, this may be important for regions that do not produce enough food for their people as food will need to be procured elsewhere.

Several studies combine pre- and post-disaster procurement strategies. The development of a two-stage scenario-based fuzzy stochastic programming model that combines pre-positioning and QFC is described in Torabi et al. (2018). Hu and Dong used a TSSP that integrates facility location, inventory policy, supplier selection, and distribution of relief items in Hu & Dong (2019), while Hu et al. (2017) uses a TSPP that integrates inventory and procurement policy, and supplier selection.

Overall, we have found the uses of ORMS methods that focus on procurement for disaster management do not cover the specific problem encountered by disaster agencies in Indonesia and other developing countries that are prone to frequent disasters. Our problem belongs to the category of inventory management problems (Ghiani, Laporte, & Musmanno, 2004), more specifically the lot sizing problem that focuses on the procurement and warehousing processes. This is an important problem faced by the decision makers at the disaster agencies with whom we are working because they have to make regular procurement decisions. The literature rarely studies the lot sizing problem for disaster management. This is partly due to the dominance of studies that address a one-off major disaster instead of the continuous frequent small-to-medium scale disasters (Onggo et al., 2021). Furthermore, we focus on an application in humanitarian food supply that is still under-researched (Perdana et al., 2022).

2.2. Addressing uncertainties in disaster management

The high level of uncertainty is one of the main characteristics of disaster management. Regarding our problem of procurement of emergency staple food for West Java, we examine three optimisation methods that can be used to support decision making under uncertainty: risk-averse stochastic programming, classical robust optimisation and adjustable robust optimisation.

We provide an overview of relevant research into these methods, their extensions and how they have been applied in the context of disaster management below.

Robust optimisation finds a strategy that gives the best worst-case performance among all allowed realisations of the inputs, where the inputs can take any value within a defined uncertainty set (Ben-Tal et al., 2009). The approach does not require any assumption about the distribution of the underlying uncertainties, and can be conservative. In the context of our procurement problem, this conservatism can be problematic for non-immediate decisions such as those further in the planning horizon. Thus, Bertsimas & Sim (2004) introduced the notion of Γ-robustness in which deviations are instead allowed within a given *budget of uncertainty* and showed that solving the robust counterpart, under the new robustness definition, is equivalent to finding the solution for the deterministic formulation with modified inputs. In disaster management, it has been applied to finding the optimal locations of facilities and the allocation of resources (e.g. Sun, Wang, & Xue, 2021; Zokaee, Bozorgi-Amiri, & Sadjadi, 2016). Most of the studies focus on single disaster event while our model is designed for multi-disaster events that happen during a financial year. This is applied to a supply chain example in Bertsimas & Thiele (2006b) where some investigation is carried out into how Γ should be selected.

An extension of robust optimisation is the two-stage setting where the decision maker adjusts their strategy based on information revealed over time. For example, in the context of the procurement problem, data on the amount of stock still available in the warehouse can be used to decide on procurement strategies for future months. The first-stage variables are *here-and-now* decisions that are required to be valid across all realisations while the second-stage variables are *wait-and-see* decisions that can be adapted to each realisation. This extension has been applied to pre-position disaster relief items (Ni et al., 2018; Velasquez, Mayorga, & Özaltın, 2020). For example in Velasquez et al. (2020), in the first stage, they optimise the facility locations and the amount of relief items to store at each facility. In the second stage, after a disaster struck, they optimise the number of additional relief items to procure. Unlike ours, their model is designed for one disaster and both row and column generation techniques are used to solve the linear formulation. Another benefit of the two-stage formulation is that it allows the implementation of the so-called adjustable robust optimisation to apply to future realisations or unseen scenarios (Ben-Tal et al., 2009; Yanikoglu et al., 2019). An application of this technique for a problem similar to ours in the context of power production can be found in Coniglio, Koster, & Spiekermann (2018).

In stochastic programming, the general assumption is that the distribution of the uncertainties is known, then the objective function is often formulated as the minimisation of an expected cost. In a risk-averse setting such as disaster preparedness, the expected cost can be replaced by the quantile of a cost distribution. This is equivalent to the concept of Value-at-Risk (VaR) optimisation in finance (Morgan, 1996). VaR or quantile optimisation is computationally expensive and the metrics may have some undesirable property such as non-convexity, thus in optimisation contexts, Conditional Value-at-Risk (CVaR) offers a better alternative (Rockafellar & Uryasev, 2000). CVaR is the expected value beyond the quantile, also known as the expected shortfall. VaR and CVaR have been used in the context of procurement problems (e.g. Alem, Oliveira, & Peinado, 2020; Charwand & Gitizadeh, 2020; Mahmutoğulları, Çavuş, & Aktürk, 2018). When the exact solution for the optimisation problem cannot be expressed analytically, Monte Carlo simulation is used instead and the method is referred to as Sample-Average Approximation (SAA) (Birge & Louveaux, 2011). SAA is particularly useful in our application because it allows the implementation and comparison of robust optimisation and stochastic programming. We generate the scenarios, either for building the optimisation models, or for testing their performance, by resampling real data describing historical disasters in West Java. As far as we know, our work is the first to apply both stochastic programming and robust optimisation to an inventory problem for disaster management and evaluate them using real data.

Distributionally robust optimisation, or data-driven robust optimisation, is an emerging method (Duque, Mehrotra, & Morton, 2022; Esfahani & Kuhn, 2018) and can be seen as a middle ground between stochastic programming and robust optimisation. Similar to stochastic programming, the goal is to optimise the expected performance, but under the worst possible distribution of uncertainties, where the uncertainty set is defined as the space of all probability distributions that validate a set of statistical properties observed in the real data. The method is therefore particularly useful when only partial knowledge about the statistical properties of the uncertain parameters is available or it is difficult to simulate historical scenarios due to the scarcity of available data. This method is relatively new for disaster management. A recent example is given in Zhang, Liu, Yang, & Zhang (2020) who optimised the location of local distribution centres and points of distribution under uncertain travel times, formulating the problem as one of location-allocation. As with the majority of studies, their model was designed for a single disaster event, unlike our model which is designed for multiple disaster events.

Multi-stage stochastic programming also has the potential for addressing a planning problem with uncertainties over multiple time periods (Birge & Louveaux, 2011). Under some general assumptions, such as the ability to model future events as a scenario tree, thus implicitly ensuring the Markov property holds for the process and the transition probabilities are known, general techniques can be used to solve such a problem, e.g. stochastic dual dynamic programming (Pereira & Pinto, 1991). A similar technique has been applied to disaster management in the context of distributing relief items during a hurricane (Siddig & Song, 2021).

3. Methodology

We begin by providing a mathematical description of the procurement problem introduced in Section 1 before formulating several approaches to solving it. The first approach assumes that demand and prices are deterministic and provides a naive approach to procurement that ignores uncertainties. We then go on to describe how stochastic programming, robust optimisation and adjustable robust optimisation can be used as approaches that account for unknown demand and price in the rice procurement problem.

3.1. Preliminaries

The set of real numbers is denoted $\mathbb{R}$ and the integers $\mathbb{Z}$, their half positive subsets including zeros are denoted with a + in superscript, e.g. $\mathbb{Z}^+$. For an integer n, we use $[n]$ to denote the set $\{1, 2, \ldots, n\}$, and for a fractional x, $[x]$ denotes its nearest integer. The indicator function of an event A is denoted $\mathbb{1}_A$, i.e. returns $\mathbb{1}$ if A is true, and 0 otherwise. Our decision variables are indexed in subscript and/or superscript, e.g. x_i, while our indexed problem-parameters are written with parentheses, e.g. $d(i)$. Table 1 provides a reference list for the notation used in this paper.

3.2. Problem description

We consider the following procurement problem under uncertainty. A fixed budget B is available to spend to procure quantities of a perishable item to satisfy demand over a time horizon of n periods. At time period i, the purchase price $p(i)$ and the available market supply or availability $a(i)$ are known, but the demand $d(i)$, and future demands, prices and availabilities $(d(j), p(j), a(j))_{j>i}$ are unknown. Data on demands, prices and availability in previous time periods are recorded and accessible, $\xi^i = \{(d(j), p(j), a(j))_{1\leq j<i}\} \cup \{(p(i), a(i))\}$, along with a fixed set of historical data $\mathcal{H} = \{(d(j), p(j), a(j))_{j\in[n]}\}$ from previously experienced horizons. Quantities of the item that have stayed in the inventory for τ time periods or beyond are considered perished and they are automatically removed. Some initial inventory, described by state x_0, is made available at the beginning of the time horizon and some *safety stock* r may be required at the end of the planning horizon. There may be a requirement that this stock has a particular level of freshness, such that by the end of the time horizon it has not stayed more than $t < \tau$ time periods in the inventory (for $t = \tau - 1$ this requirement is the most relaxed as it only restricts the age of the safety stock to be within the expiry date).

The goal is to construct a decision making routine, denoted by Optimiser, that takes the role of a decision maker. It takes $\mathcal{H}$, ξ^i and the current state of the inventory, i.e. quantities in stock and still usable (not yet perished), at each period $i \in [n]$ as input, then outputs the buy quantity y_i, i.e. a *here-and-now* decision, such that the key performance indicators at the end of the time horizon

Table 1
Notation used in the formulation of the problem.

Term	Description
Symbols used in the basic/deterministic formulation	
i	time index
j	age index
x_{ij}	quantity of stock of age j remaining at the end of period i
y_i	target buy quantity in period i
c_{ij}	amount of stock of age j consumed in period i
s_i	shortage of stock in period i
n	number of time periods
τ	age limit at expiration
B	procurement budget
$I(j)$	initial inventory of age j
r	safety stock requirement
t	safety stock age limit
$d(i)$	demand in period i
$p(i)$	price in period i
$a(i)$	availability in period i
V	storage limit
Symbols used in stochastic programming	
π	occurrence probability of a scenario
α	risk level in VaR/CVaR
η	Value-at-Risk variable
u	positive side in CVaR model
Symbols used in the robust formulation	
$\mathcal{U}$	uncertainty set
Ω	set of scenarios
ξ	a particular scenario
$\mathcal{S}$	the greatest shortage
$\mathcal{B}$	the highest budget spent
ν_0	offset parameter of the affine mapping
ν_i^d	parameter of the affine mapping for the demand in period i
ν_i^p	parameter of the affine mapping for the price in period i
ν_i^a	parameter of the affine mapping for the availability in period i
Symbols used in the dynamic test procedure	
Optimiser	optimiser
Generator	generator
Simulator	simulator
$\mathcal{H}$	set of historical scenarios
$\mathcal{T}$	set of test scenarios
b	current budget
$\mathrm{y}(\xi, i)$	amount bought in period i in scenario ξ
$\mathrm{x}(\xi, i, j)$	inventory of age j in period i in scenario ξ
$\mathrm{s}(\xi, i)$	shortage in period i in scenario ξ
c	amount consumed

are optimised. These are the minimisation of shortage, waste and budget usage in that exact priority order. Algorithm 1 describes how an Optimiser can be used to decide on the amount to purchase in each month based on historical data and observed demand, price and availability. Here, a sequence of full information $\xi = (d(j), p(j), a(j))_{j\in[n]}$ is referred to as a *scenario*, and the partial sequence ξ^i as described above is called a *partial scenario* until time i.

In Algorithm 1, the detailed state of the system including the shortage $\mathrm{s}(\xi, i)$, waste $\mathrm{x}(\xi, i, \tau)$ and remaining budget $\mathrm{b}(\xi, i)$ are collected at the end of each time step. The Optimiser is queried in line 4 for the next decision y_i of the quantity to procure in the current time period i. This line captures the uncertain and dynamic nature of our problem, as only a set $\mathcal{H}$ of historical data, the partial scenario ξ^i, plus the current state of the inventory system are revealed to the Optimiser. The procurement order is then checked with the available budget $\mathrm{b}(\xi, i-1)$ and the actual price $p(\xi, i)$ in line 5 and the actual quantity procured $\mathrm{y}(\xi, i)$ is deduced. The remaining budget is then updated in line 6. Finally, the inventory is updated using a FIFO policy (lines 7–16). At the end of the time horizon (typically 1 year; line 17), $\mathrm{s}(\xi, n+1)$ is computed to take into account the unmet safety stock. We note that quantities of the item stored in the system are assumed to be integers; how-

Algorithm 1: Implementation of an OPTIMISER given historical data $\mathcal{H}$ for a new scenario ξ.

Input: OPTIMISER, $\mathcal{T}$, ξ scenario for demand, price and availability made known to the algorithm month by month, initial inventory x_0, parameters n, B, r, τ, t.
Output: Target buy quantity y_i, and actual purchase quantity $\mathrm{y}(\xi, i)$, shortage $\mathrm{s}(\xi, i)$, waste $\mathrm{x}(\xi, i, \tau)$ and remaining budget $\mathrm{b}(\xi, i)$ for each $i \in [n]$.

1 $\mathrm{b}(\xi, 0) \leftarrow B$;
2 $\mathrm{x}(\xi, 0, \cdot) \leftarrow \mathrm{x}_0$;
3 **foreach** $i \in [n]$ **do**
  // query OPTIMISER for the next decision
4   $y_i \leftarrow$ OPTIMISER$(\mathcal{T}, \xi^i, \mathrm{b}(\xi, i-1), \mathrm{x}(\xi, i-1, \cdot))$;
5   $\mathrm{y}(\xi, i) \leftarrow \min\left(\lceil y_i \rceil, \lfloor \frac{\mathrm{b}(\xi, i-1)}{p(\xi, i)} \rfloor, a(\xi, i)\right)$;
6   $\mathrm{b}(\xi, i) \leftarrow \mathrm{b}(\xi, i-1) - \mathrm{y}(\xi, i) \times p(\xi, i)$;
  // update the inventory using FIFO policy
7   $\mathrm{s}(\xi, i) \leftarrow d(\xi, i)$;
8   $\mathrm{x}(\xi, i, 1) \leftarrow \mathrm{y}(\xi, i)$;
9   **foreach** $j \in [\tau] \setminus \{1\}$ **do**
10     $\mathrm{x}(\xi, i, j) \leftarrow \mathrm{x}(\xi, i-1, j-1)$;
11   **for** $j \leftarrow \tau$ **downto** 1 **do**
12   **if** $\mathrm{s}(\xi, i) = 0$ **then**
13   **break**;
14     $\mathrm{c} \leftarrow \min(\mathrm{s}(\xi, i), \mathrm{x}(\xi, i, j))$;
15     $\mathrm{x}(\xi, i, j) \leftarrow \mathrm{x}(\xi, i, j) - \mathrm{c}$;
16     $\mathrm{s}(\xi, i) \leftarrow \mathrm{s}(\xi, i) - \mathrm{c}$;
17 $\mathrm{s}(\xi, n+1) \leftarrow \max\left(r - \sum_{j=1}^{t} \mathrm{x}(\xi, n, j), 0\right)$;

ever to give more flexibility to the OPTIMISER, we allow it to produce fractional decisions, and this explains the use of the rounding and floor functions in line 5. The output of the algorithm provides a recommendation of the purchase quantities in each time period and allows the computation of the key performance indicators. We wish to minimise three indicators that are listed in a priority order as follows: the first and most important is the total shortage $\sum_{i=1}^{n+1} \mathrm{s}(\xi, i)$; then it is followed by the total waste $\sum_{i=1}^{n} \mathrm{x}(\xi, i, \tau)$; and finally the budget spent $B - \mathrm{b}(\xi, n)$.

In practice, a small set $\mathcal{H}$ is often given, typically data recorded or computed from disaster events and market information from previously operated years, and ξ records what has happened during the planning year. We assume that the scenarios of $\mathcal{H}$ and ξ come from the same distribution.

In order to assert the robustness of an OPTIMISER, multiple future scenarios ξ similar to the historical data need to be generated and we refer to these as the set $\mathcal{T}$ of test scenarios. This generation uses an operator called the GENERATOR. Our GENERATOR and hence its generated scenarios are based on real data and are designed to incorporate the observed seasonality in the demands and prices. Details are given in Section 4.2.

In the following sections we describe OPTIMISER s that could be used to solve the procurement problem, beginning with those that use a deterministic formulation.

3.3. Deterministic formulation

In the deterministic model, we assume that full information $\xi = (d(i), p(i), a(i))_{i\in[n]}$ is available when making the procurement decision y_i for each time period. The results obtained by this formulation therefore represent a lower bound on the performance for any realistic OPTIMISER which only has access to the past and the present. In Section 3.5, we also show that the classical robust formulation is equivalent to a deterministic formulation, in which the parameters are deduced from the uncertainty set.

In addition to the decision variables y_i, we use variables x_{ij} to store the quantity of item at age j remaining at the end of period i; variables s_i to record the shortages (unmet demands) at time period i; and variables c_{ij} for the quantities of item of age j consumed at period $i \in [n]$. The procurement problem can then be formulated as the following linear program, which we discuss below.

$$(\text{BUDGETALLOC-DET}) \ \min \left(\sum_{i\in[n+1]} s_i, \sum_{i\in[n]} x_{i\tau}, \sum_{i\in[n]} y_i p(i) \right) \tag{1}$$

s.t.

$$\mathbb{1}_{\{j>1\}} x_{(i-1)(j-1)} + \mathbb{1}_{\{j=1\}} y_i = c_{i(j-1)} + x_{ij} \quad \forall i \in [n], \forall j \in [\tau] \tag{2}$$

$$s_i + \sum_{j=0}^{\tau-1} c_{ij} = d(i) \quad \forall i \in [n] \tag{3}$$

$$s_{n+1} + \sum_{j=1}^{t} x_{nj} \geq r \tag{4}$$

$$\sum_{i\in[n]} y_i p(i) \leq B \tag{5}$$

$$y_i \in \mathbb{R}^+, \ y_i \leq a(i) \quad \forall i \in [n] \tag{6}$$

$$x_{ij} \in \mathbb{R}^+, \ x_{0j} = l(j), x_{0\tau} = 0 \quad \forall i \in \{0\} \cup [n], \forall j \in [\tau] \tag{7}$$

$$s_i \in \mathbb{R}^+ \quad \forall i \in [n+1] \tag{8}$$

$$c_{ij} \in \mathbb{R}^+ \quad \forall i \in [n], \forall j \in \{0\} \cup [\tau - 1] \tag{9}$$

The objectives (1) are lexicographically ordered: we first minimise the shortage $\sum_{i\in[n+1]} s_i$, then the waste $\sum_{i\in[n]} x_{i\tau}$, and finally the budget usage $\sum_{i\in[n]} y_i p(i)$. The *balance* constraints (2) ensure that the consumable quantity, which is either freshly bought or the leftover from the previous period, equals the actual consumption plus any leftover in the current period. Constraints (3) connect consumed quantities to the demand; here the shortage s_i works like a slack variable. Constraints (4) ensure there is sufficient safety stock with age no more than t at the end of the planning horizon and we note that $t < \tau$. The budget limitation is set in constraint (5). The availability of stock to procure, which may be due to the production level in the region, provides a limit on the amount that can be purchased in constraints (6). The initial inventory levels are set in constraints (7). Note that we do not require input $l(\tau)$, which corresponds to initial stock of age τ, which is due to expire, and instead we set $x_{0\tau} = 0$. This is done because if the model was run for the previous year, then $l(\tau)$ would have already been counted as a waste for that year as the stock is no longer usable. Finally, the variables are defined in (6)–(9).

To solve a multi-objective optimisation problem via the lexicographical method, we first solve the problem with the highest priority objective (the shortage in our case), then fix the obtained optimal value as a constraint before moving to the next highest priority objective and continuing until all objectives have been considered. Some modern Linear Programming solvers have features to support this approach with a minimal effort, e.g. CPLEX since version 12.9. A reduction of the formulation based on the redundancy in the objectives is discussed in Appendix A, which shows that the waste minimisation is redundant when both of the other objectives are included. Some extensions of the model to incorporate other complexities often seen in similar situations are discussed in Appendix B.

3.4. Stochastic programming

In stochastic programming, we look for a solution, in this case the procurement quantities y_i, that works well on average over a set Ω of scenarios (e.g. a set of generated or observed scenarios $\Omega = \mathcal{H}$) and we assume the occurrence probability of each scenario $\xi \in \Omega$, denoted by $\pi(\xi)$, is known. These probabilities are often asserted by experts. In our case, the scenarios either come from real historical data, or are sampled with a Generator described later in Section 4.2 and this is in essence the Sample Average Approximation method (Birge & Louveaux, 2011), thus we will assume the scenarios are equiprobable. This yields the following two-stage stochastic programming formulation of the problem. The first-stage decisions are y_i, the procurement quantities at time period i; while all of the other variables are second-stage as they are scenario-adapted measures. These are indexed with ξ (e.g. $s_{i\xi}$, $x_{i\tau\xi}$, etc) and evaluate the impacts of the first-stage decisions over scenario ξ.

$$(\text{BudgetAlloc-Stoch}) \ \min \left(\sum_{\xi\in\Omega}\pi(\xi)\sum_{i\in[n+1]} s_{i\xi},\ \sum_{\xi\in\Omega}\pi(\xi)\sum_{i\in[n]} x_{i\tau\xi},\ \sum_{\xi\in\Omega}\pi(\xi)\sum_{i\in[n]} y_i p(i,\xi) \right) \quad \text{s.t.} \tag{10}$$

$$\mathbb{1}_{\{j>1\}}x_{(i-1)(j-1)\xi} + \mathbb{1}_{\{j=1\}}y_i = c_{i(j-1)\xi} + x_{ij\xi} \qquad \forall i\in[n], \forall j\in[\tau], \forall\xi\in\Omega \tag{11}$$

$$s_{i\xi} + \sum_{j=0}^{\tau-1} c_{ij\xi} = d(i,\xi) \qquad \forall i\in[n], \forall\xi\in\Omega \tag{12}$$

$$s_{(n+1)\xi} + \sum_{j=1}^{t} x_{nj\xi} \ge r \qquad \forall\xi\in\Omega \tag{13}$$

$$\sum_{i\in[n]} y_i p(i,\xi) \le B \qquad \forall\xi\in\Omega \tag{14}$$

$$y_i \in \mathbb{R}^+,\ y_i \le a(i,\xi) \qquad \forall i\in[n], \forall\xi\in\Omega \tag{15}$$

$$x_{ij\xi}\in\mathbb{R}^+,\ x_{0j\xi}=l(j), x_{0\tau\xi}=0 \qquad \forall i\in\{0\}\cup[n], \forall j\in[\tau], \forall\xi\in\Omega \tag{16}$$

$$s_{i\xi}\in\mathbb{R}^+ \qquad \forall i\in[n+1], \forall\xi\in\Omega \tag{17}$$

$$c_{ij\xi}\in\mathbb{R}^+ \qquad \forall i\in[n], \forall j\in\{0\}\cup[\tau-1], \forall\xi\in\Omega \tag{18}$$

Relative to the deterministic formulation, here we have extended all of the constraints to the scenario dimension, and we have done the same for all of the variables, with the exception of the y_i, which are our first-stage decisions. The objectives (10) minimise the shortage, then waste and budget spent in expectation.

The robustness of the solution in terms of shortage can be improved as follows. Instead of optimising the normal expectation, we optimise the conditional expectation beyond some α-quantile, and this is known as Conditional Value-at-Risk (CVaR) (Rockafellar & Uryasev, 2000), where the value of the α-quantile is the Value-at-Risk, VaR. For example, when this is applied to the first objective of shortage, we have the following linear formulation:

$$(\text{BudgetAlloc-CVaR}) \ \min \left(\eta^s + \frac{1}{1-\alpha}\sum_{\xi\in\Omega}\pi(\xi)u^s_\xi,\ \sum_{\xi\in\Omega}\pi(\xi)\sum_{i\in[n]} x_{i\tau\xi},\ \sum_{\xi\in\Omega}\pi(\xi)\sum_{i\in[n]} y_i p(i,\xi) \right) \quad \text{s.t.} \tag{19}$$

(11), (12), (13), (14), (15), (16), (17), and (18)

$$u^s_\xi \ge \sum_{i\in[n+1]} s_{i\xi} - \eta^s \qquad \forall\xi\in\Omega \tag{20}$$

$$\eta^s\in\mathbb{R}, u^s_\xi\in\mathbb{R}^+ \qquad \forall\xi\in\Omega \tag{21}$$

Here α is the confidence (or inversely risk) level parameter set by the user, e.g. if 5% of the worst objective values are considered in computing VaR/CVaR then α is set to 0.95. If α is set to 0 then BudgetAlloc-CVaR is equivalent to BudgetAlloc-Stoch.

When there is sufficient budget to cover absolutely no shortage over Ω in BudgetAlloc-Stoch, i.e. there exists a solution that allows zero in the first objective, then the two models are also equivalent, since the distribution of the first objective with the given solution is collapsed into a single point. In that case, it is possible to keep the first objective of BudgetAlloc-Stoch and apply CVaR formulation to the second objective in a similar fashion, by introducing variables η^x and u^x_ξ and updating the second objective. We refer to this model as BudgetAlloc-CVaRW.

3.5. Robust optimisation

In robust optimisation, we want to make the best decision in the worst case scenario. The robust formulation can be derived directly from the deterministic one by adding the assumption that the solution should hold for any (uncertain) parameters taken from some uncertainty set $\mathcal{U}$. The uncertainty set is designed to account for seasonality in the price and demand as we discuss further in Section 4.2. In our model, this is equivalent to optimising BudgetAlloc-Det where $(d(i), p(i), a(i))_{i\in[n]} \in \mathcal{U}$.

The need for modelling slack variables such as s_i with equality constraints is unavoidable for our problem since these variables appear in the objectives, therefore it is not possible to produce an equivalent formulation where uncertain inputs only appear in inequality constraints like in Bertsimas & Thiele (2006b). This leads to too strong a degree of conservatism and infeasibility if the model contains only first-stage variables, i.e. equality constraint (3) cannot be satisfied simultaneously for two different values of $d(i)$. A similar issue can be found in Coniglio et al. (2018), and some discussion on this can be found in Chapter 1 of (Ben-Tal et al., 2009). We therefore move directly to the two-stage robust formulation, which shares some similarities with BudgetAlloc-Stoch. For conciseness, here we make use of the result described in Section 3.3 that the waste minimisation objective is redundant and consequently we can optimise over just shortage and budget. The formulation is as follows:

$$(\textsc{BudgetAlloc-Robust}) \ \min\,(\mathcal{S},\mathcal{B}) \quad \text{s.t.} \tag{22}$$

(11), (12), (13), (14), (15), (16), (17), and (18)

$$\sum_{i\in[n+1]} s_{i\xi} \le \mathcal{S} \qquad \forall \xi \in \Omega \tag{23}$$

$$\sum_{i\in[n]} y_i p(i,\xi) \le \mathcal{B} \qquad \forall \xi \in \Omega \tag{24}$$

$$\mathcal{S},\mathcal{B} \in \mathbb{R}^+ \tag{25}$$

Here $\Omega = \{\xi \mid (d(i,\xi), p(i,\xi), a(i,\xi))_{i\in[n]} \in \mathcal{U}\}$. Several types of uncertainty set can be considered for $\mathcal{U}$, such as box, polyhedral or ellipsoidal (Ben-Tal et al., 2009). We have the following observation related to the deterministic formulation. BudgetAlloc-Robust can have an infinite number of constraints, and a formal derivation of a compact counterpart would involve considering the dual formulation; however, it is intuitive to see that the worst case corresponds to the smallest availability, and the largest demands and prices as stated more formally in Proposition 1.

Proposition 1. *BudgetAlloc-Robust with the uncertainty set $\mathcal{U}$ is equivalent to the optimisation problem of (2)–(35) where $(d(i), p(i), a(i))$ is replaced with $(\max_{\xi\in\Omega}\{d(i)\}, \max_{\xi\in\Omega}\{p(i)\}, \min_{\xi\in\Omega}\{a(i)\})$ for each $i \in [n]$.*

Proof. It follows from (12) that

$$c_{i0\xi} = d(i,\xi) - s_{i\xi} - \sum_{j=1}^{\tau-1} c_{ij\xi} \qquad \forall i \in [n], \forall \xi \in \Omega \tag{26}$$

and combining this with (11) for the case $j = 1$ gives

$$y_i = c_{i0\xi} + x_{i1\xi} = d(i,\xi) - s_{i\xi} + x_{i1\xi} - \sum_{j=1}^{\tau-1} c_{ij\xi} \quad \forall i \in [n], \forall \xi \in \Omega \tag{27}$$

Therefore, (14) and (15) are equivalent to

$$\sum_{i\in[n]} \left(d(i,\xi) - s_{i\xi} + x_{i1\xi} - \sum_{j=1}^{\tau-1} c_{ij\xi} \right) p(i,\xi) \le B \qquad \forall \xi \in \Omega \tag{28}$$

$$y_i \in \mathbb{R}^+,\ d(i,\xi) - s_{i\xi} + x_{i1\xi} - \sum_{j=1}^{\tau-1} c_{ij\xi} \le a(i,\xi) \quad \forall i \in [n], \forall \xi \in \Omega \tag{29}$$

Note that $d(i,\xi)$ appears in the LHS of both (28) and (29) with coefficients $p(i,\xi) > 0$ and 1 respectively, thus the larger the value it has, the more dominant the formulation is over other possible values. This also holds true for $p(i,\xi)$ as it appears in the LHS of (29) with coefficient $d(i,\xi) > 0$. Input $a(i,\xi)$ appears on the RHS of (29) with coefficient 1, thus the dominant formulation is obtained with its smallest value. □

Taking these inputs to the maximal (or minimal) values can produce highly conservative solutions, thus in practice when Ω is large, we consider smaller statistics, for example 0.95 or 0.90-quantiles, or even lower if the variation is small. These details are discussed in Section 4.

3.6. Scenario-based adjustable decisions

In adjustable robust optimisation (Ben-Tal et al., 2009; Yanikoglu et al., 2019), we look for an optimal decision rule extracted from historical scenarios so that it can be applied at successive time points during the planning time horizon of a new scenario. In general, a decision rule is a mapping from the recent past data including recently revealed information and possibly the current state of the system to a new decision or recommendation. This allows the recommendations to be adaptive with respect to recent events of a scenario.

There are various ways to formalise a decision rule and each yields a specific approach to adjustable robust optimisation (Yanikoglu et al., 2019). We choose ours to be an affine mapping from the input space of the information that has been revealed so far, to the decision space of the immediate buy quantity. Some advantages of this mapping are that it is natural and requires fewer assumptions about the modelling of the uncertainties, and that if the original problem is a linear program, the resulting formulation will also be a linear program, and hence tractable. An application of the same approach for another variant of the lot sizing problem can be found in Coniglio et al. (2018).

We introduce the parameters, ν_0, ν_i^d, ν_i^p and ν_i^a of the affine mapping as the new first-stage decisions. We then push the old first-stage decisions y_i to the second stage, i.e. make them scenario-dependent $y_{i\xi}$, and set them equal to the outputs of applying the mapping to the data of each scenario used

$$y_{i\xi} = \nu_0 + \sum_{i'=1}^{i-1} d(i',\xi)\nu_{i'}^d + \sum_{i'=1}^{i} \left(p(i',\xi)\nu_{i'}^p + a(i',\xi)\nu_{i'}^a \right). \qquad \forall i \in [n], \forall \xi \in \Omega \tag{30}$$

This gives the following adjustable robust formulation:

$$(\textsc{BudgetAlloc-ARO}) \ \min\,(\mathcal{S},\mathcal{B}) \quad \text{s.t.} \tag{31}$$

$$(12), (13), (16), (17), (18), (23), (25), \text{ and } (30)$$

$$\mathbb{1}_{\{j>1\}} x_{(i-1)(j-1)\xi} + \mathbb{1}_{\{j=1\}} y_{i\xi} = c_{i(j-1)\xi} + x_{ij\xi} \qquad \forall i \in [n], \forall j \in [\tau], \forall \xi \in \Omega \tag{32}$$

$$\sum_{i\in[n]} y_{i\xi} p(i,\xi) \le B \qquad \forall \xi \in \Omega \tag{33}$$

$$\sum_{i\in[n]} y_{i\xi} p(i,\xi) \le \mathcal{B} \qquad \forall \xi \in \Omega \tag{34}$$

Once the model is solved on a set of historical scenarios, the parameters of the affine mapping are stored, and can be applied afterwards, using the same Eq. (30), which is essentially a dot product on a new scenario (either real or simulated) to produce successive decisions as additional information is revealed.

4. Numerical experiments

We evaluate our methods using the rice procurement problem faced by BULOG in West Java, which has an obligation to ensure rice is always available for refugees, as described in Section 1. Using historical refugee data (Onggo et al., 2021) and price data from BULOG, we construct a Generator, described in Section 4.2, that is used to generate sets of scenarios for prices and demand. The optimisation methods described in the previous section are tested using the Simulator to evaluate their robustness and computational performance.

4.1. Case study

West Java province is the centre of rice production and consequently we ignore constraints on availability in our experiments. The province allocated budget equivalent to 100 tonnes of emergency rice for each municipality and there were 27 municipalities in the province. BULOG has 30 warehouses of large capacities in West Java, which are used to store rice for disaster relief as well as for market intervention and poverty reduction. Emergency rice has priority and uses only a small portion of storage compared to rice used for other purposes. As a result, we are able to ignore constraints on storage capacity. We focus on provision of emergency rice for disasters.

Rice is typically bought throughout the year and we use our methods to determine a target buy quantity per month, using a planning horizon of one year, thus $n = 12$. Ideally, rice should be used within 6 months of purchase, thus we set $\tau = 7$ and use $t = 6$. At the beginning of the year, a quantity of 10 tonnes of fresh rice per municipality is assumed available to use, and the same quantity of usable rice is required at the end of the year as safety stock. In the following section we discuss how to construct sets of historical data $\mathcal{H}$ and testing data $\mathcal{T}$ for the different scenarios used in our experiments.

4.2. Scenario generation

The disaster data from 2016 to 2020 was made available to us by the West Java provincial government and comprises 8111 records, each recording disaster event, date, location (municipality and local district), disaster type (e.g. flood, landslide, wildfire), number of refugees and affected infrastructure, etc. for West Java, Indonesia. BULOG provided us with the price of rice data aggregated by month for 2018–2020 inclusive for West Java, Indonesia as well as the minimum allowed purchasing price for rice between 2018 and 2020.

As we do not have the price data for 2016 and 2017, we use prices generated from a log-normal distribution fitted to the price data from 2018 to 2020 as estimates for 2016 and 2017, adjusted to ensure they are above the government's minimum buy price. This gives us our set of (quasi-) real historical data $\mathcal{H}_1$, consisting of 5 scenarios of monthly demands and prices.

Additional sets of scenarios are created using a Generator. These are needed to fully test the robustness of our Optimiser s. In each scenario, we independently sample the price and the number of refugees in each month. Prices are sampled from a set of log-normal distributions for each month of the year fitted to historical price data (2018 to 2020).

The generation of events for a one-year scenario, described below, is designed to reproduce the seasonality observed in demand for rice.

- For each disaster type, the number of events is sampled from a Poisson distribution with mean equal to the annual average number of events of that type. The set of corresponding events are then created.
- The municipalities that are affected by each disaster are then sampled using the empirical distribution. Based on the municipality and disaster type, the number of refugees is generated by resampling from the numbers of refugees of past events of the same type that have occurred in the same municipality.
- Finally, the dates of disasters are sampled for each of the generated events. We match the distribution of events over the years to detailed historical data of real events categorised by disaster type and month.

Aggregating the events for each month gives the total number of refugees for the month. As per regulation, each refugee will receive 400 g of rice per day during the emergency period (fourteen days post-disaster), hence, we can calculate the monthly demand for rice.

We use the Generator to generate two sets $\mathcal{H}_2$ and $\mathcal{T}_2$ of synthetic historical scenarios, each containing 100 scenarios. The former is used as the historical set for modelling while the latter is used to evaluate the solutions. These complement the set of historical data from BULOG in $\mathcal{H}_1$. The sets of scenarios $\mathcal{H}_1$, $\mathcal{H}_2$ and $\mathcal{T}_2$ along with our experimental results are available at https://github.com/stephanong/relief-ops/.

4.3. Experimental set up

The evaluation process Simulator and the optimisation models are implemented in Python using CPLEX 12.10 as the linear solver. To avoid numerical instabilities we set CPLEX to use the dual simplex method, and we multiply the RHS and the coefficients of the budget constraint by 10^{-3}. The experiments were conducted on a Linux cluster of CPU with 2.6 GHz Intel Sandybridge processors and 64 GB of memory where a CPU was allocated to evaluate each Optimiser.

4.4. Optimisation routines

We test the following Optimiser s, built from the models described in Section 3.

- Pct100: based on BudgetAlloc-Robust, which is essentially BudgetAlloc-Det where the demands and prices for each month are taken from the 100-percentile of the historical data $\Omega = \mathcal{H}$.
- Pct90: similar to Pct100 but the 90-percentile of demands and prices of the historical data are used as a way to reduce the degree of conservatism of the solution.
- Stoch: based on BudgetAlloc-Stoch.
- CVaR90 and CVaR95: based on BudgetAlloc-CVaR with α set to 0.90 and 0.95 respectively.
- CVaRW90 and CVaRW95: similar to the above but using BudgetAlloc-CVaRW.
- ARO: based on BudgetAlloc-ARO.

As discussed in Section 3, with the exception of BudgetAlloc-ARO whose decisions are adapted as new data are revealed, all of the other models make fixed decisions. More specifically, the models can be fully constructed based on the set of historical data $\Omega = \mathcal{H}$ and the decisions can then be fixed for the whole year, and in this case line 4 of Algorithm 1 essentially only returns a precomputed value. Setting purchasing decisions for each month at the start of the year and not allowing updates based on observed demand is likely to make these methods less effective compared with BudgetAlloc-ARO. To provide a fair test we instead allow each of these Optimiser s to update their purchasing recommendations each month. This *remodelling* approach, which is clearly described in Bertsimas & Thiele (2006a), proceeds as follows. In the query on line 4 of Algorithm 1, the model is reconstructed for the remaining time periods $n - i$ as the new planning horizon (so the input $\Omega = \mathcal{H}$ is only considered for these periods); the new budget is $\mathrm{b}(\xi, i-1)$; and the initial inventory is $\mathrm{x}(\xi, i-1, \cdot)$.

When presenting the results for an Optimiser over multiple scenarios, since we are interested in robust solutions for a minimisation problem, our main focus will be on the quantiles above the median, e.g. 0.9 (denoted q90) or 1.0 (the maximum). We will also show a classical performance metric for stochastic programming, the so-called *expected value of perfect information* (EVPI)

Table 2
Summary of the results for methods modelled on $\mathcal{H}_1 \setminus \{\xi\}$ then tested on $\{\xi\}$. Median values are presented alongside the minimum, maximum values, and EVPIs for each objective.

Method	Shortage (10^3 kilograms)				Waste (10^3 kilograms)				Budget spent (10^6 Rupiahs)			
	med	min	max	EVPI	med	min	max	EVPI	med	min	max	EVPI
Pct100	0.0	0.0	0.96	0.19	249.72	116.96	359.30	45.78	3860.74	3216.56	3902.38	776.37
Stoch	0.0	0.0	0.00	0.00	245.29	13.66	251.58	2.32	3651.19	3216.56	3680.49	544.79
ARO	0.0	0.0	133.85	26.77	245.32	2.07	251.58	0.01	3405.90	2342.30	4130.29	475.78

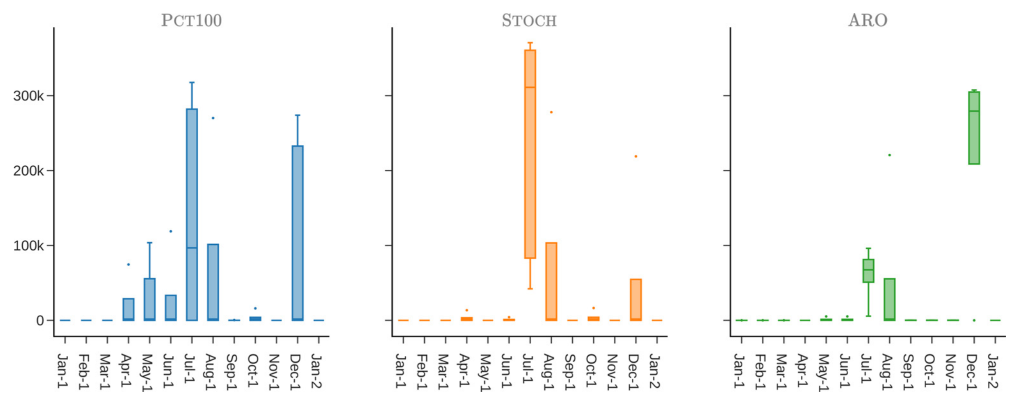

Fig. 1. Procurement quantities in kilograms when modelled and tested on $\mathcal{H}_1$.

(Birge & Louveaux, 2011). EVPI measures the gap between the average performance of the Optimiser and that of an ideal method, often called the *clairvoyant*, that has access to future information. Since the clairvoyant solution requires the decisions to be adapted to each scenario, it represents a lower bound for their expected performance for all Optimiser s. Therefore, lower numbers of EVPI suggest better expected performance. Recall that our problem has multiple objectives which are lexicographically ordered, hence the clairvoyant solution is computed once for each scenario but EVPIs will be shown separately for each objective.

4.5. Robustness over real scenarios

We examine how the proposed approaches behave with a small data set $\mathcal{H}_1$; more specifically, the approaches are tested on $\mathcal{T} = \{\xi\}$ for each $\xi \in \mathcal{H}_1$ while being modelled with $\mathcal{H} = \mathcal{H}_1 \setminus \{\xi\}$ as the historical data. The set $\mathcal{H}$ here is sparse (contains only 4 scenarios), thus the only relevant Optimiser s are Pct100, Stoch and ARO.

A summary of results is given in Table 2. Regarding the primary objective of shortage, Pct100 and Stoch can achieve zero shortage in the majority of scenarios, with a small exception that Pct100 sometimes has a minor issue with the safety stock. On the other hand, while the median shortages produced by ARO are zero, the relative average shortage indicated by EVPI is non-negligible and the maximum shortage is very high. While this is partially due to the large variance in the sparse data set, particularly the large chain of disasters during 2016, these results also hint that Pct100, Stoch can make quite conservative decisions, while ARO has not seen enough data to construct a robust model. Considering the total budget spent, Table 2 shows that in terms of median values Pct100 is the most expensive approach, followed by Stoch and then ARO. However, ARO can have high variability in the budget spent, and this is likely due to its adaptation to the unseen large disasters while modelled on a sparse set of data.

The procurement activities differ between the models as shown in Fig. 1. The majority of procurement using Pct100 takes place in April to August, while Stoch and ARO both carry out purchasing in July and August when the original stock starts to expire and the prices are low. All three methods also buy more rice in December to meet the safety stock requirement.

4.6. Robustness over simulated future scenarios

In this section, we analyse how the proposed models perform when tested on $\mathcal{T} = \mathcal{H}_2$ while using either the limited real scenarios in $\mathcal{H} = \mathcal{H}_1$ or the 100 simulated scenarios in $\mathcal{H} = \mathcal{H}_2$ as the historical data to train the models.

4.6.1. Modelling with limited historical data

We first consider the case $\mathcal{H} = \mathcal{H}_1$ where only limited information is used in modelling. Again, because of the sparsity of this set the only relevant Optimiser s are Pct100, Stoch and ARO. Results show that shortages do occur but typically only in a few scenarios. A summary of results is given in Table 3. Among the methods, Stoch produces the least shortage while ARO produces the most. Regardless of the methods, the largest shortage is due to the unmet safety stock requirement at the end of the planning horizon. Due to the increased variability in demand compared with the small number of historical scenarios considered in Section 4.5, the total spend is greater, even though this change to the median and average cases (columns med and EVPI) are relatively small. On the other hand, the maximum shortage is significantly increased but waste is reduced.

4.6.2. Modelling with simulated historical data

We consider $\mathcal{H} = \mathcal{H}_2$ for modelling and $\mathcal{T} = \mathcal{T}_2$ for testing, and recall that both sets have 100 scenarios. All Optimiser s are relevant in this case, allowing us to fully evaluate the difference in their performance and to consider different values for their parameters. Table 4 includes a summary of the results.

We first consider Pct100 and Pct90, and notice that in terms of the first objective of shortage, in most of the 100 scenarios the shortage can be maintained at zero, except for 4 scenarios where a shortage of up to 30 tonnes can occur for Pct90. On the other hand, by reducing the degree of conservatism, the method allows a significant improvement in terms of waste and budget spent compared to Pct100. For example, in the average cases the EVPI of

Table 3
Summary of the results for methods modelled on $\mathcal{H}_1$ then tested on $\mathcal{T}_2$. Median values are presented alongside the 90th quantiles, maximum values, and EVPIs for each objective.

Method	Shortage (10^3 kilograms)				Waste (10^3 kilograms)				Budget spent (10^6 Rupiahs)			
	med	q90	max	EVPI	med	q90	max	EVPI	med	q90	max	EVPI
Pct100	0.0	14.41	106.25	5.54	234.58	327.78	357.15	37.97	3779.08	3956.63	4579.29	580.17
Stoch	0.0	0.00	72.73	1.41	215.54	246.05	261.44	0.00	3654.92	3672.04	4256.14	589.59
ARO	0.0	68.73	269.73	26.60	215.54	246.05	261.44	0.01	3386.78	3685.13	5008.68	171.40

Table 4
Summary of the results for methods modelled on $\mathcal{H}_2$ then tested on $\mathcal{T}_2$. Median values are presented alongside the 90th quantiles, maximum values, and EVPIs for each objective.

Method	Shortage (10^3 kilograms)				Waste (10^3 kilograms)				Budget spent (10^6 Rupiahs)			
	med	q90	max	EVPI	med	q90	max	EVPI	med	q90	max	EVPI
Pct90	0.0	0.0	30.25	0.80	231.76	279.78	306.56	17.89	3629.72	3766.92	4805.54	684.26
Pct100	0.0	0.0	0.00	0.00	513.15	837.21	1080.14	288.16	8154.50	10933.46	12192.22	5577.01
Stoch	0.0	0.0	0.00	0.00	215.54	246.05	261.44	3.25	4180.42	4504.38	5422.03	1278.14
CVaR90	0.0	0.0	0.00	0.00	215.54	246.05	261.44	3.25	4180.42	4504.38	5422.03	1278.14
CVaR95	0.0	0.0	0.00	0.00	215.54	246.05	261.44	3.25	4180.42	4504.38	5422.03	1278.14
CVaRW90	0.0	0.0	0.00	0.00	215.54	246.05	261.44	3.28	4180.36	4504.38	5422.03	1278.24
CVaRW95	0.0	0.0	0.00	0.00	215.54	246.05	261.44	3.30	4180.28	4504.38	5422.03	1278.26
ARO	0.0	0.0	21.21	0.44	215.56	246.05	261.44	0.04	4060.99	4214.73	4521.35	1055.38

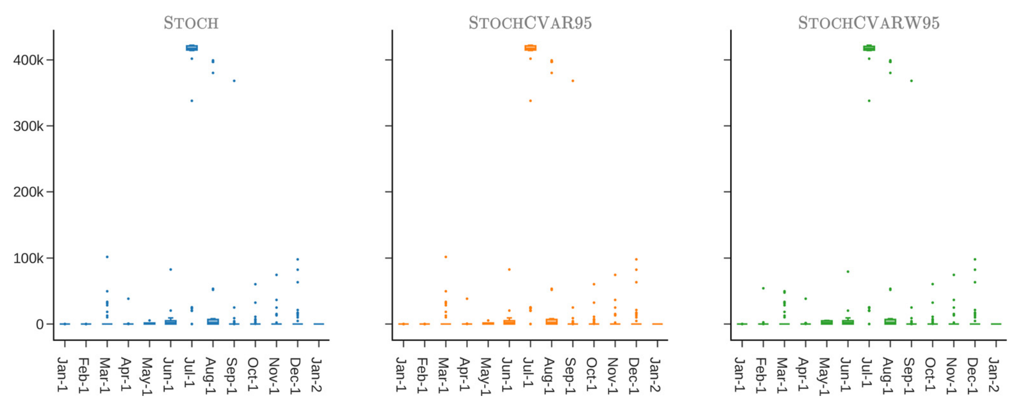

Fig. 2. Procurement activities of Stoch, CVaR95 and CVaRW95 when modelled and tested with $\mathcal{H}_2, \mathcal{T}_2$ respectively.

waste for Pct90 is fifteen times less than that for Pct100, and this factor is still two and three times less in the median and worst cases respectively.

Regarding the stochastic programming methods, we observe that Stoch and CVaR95 make very similar procurement decisions which differ to those produced by CVaRW95. This can be seen in Fig. 2. We note that all these methods can maintain a net zero shortage, and this confirms our theory that when the budget is sufficient, it is not necessary to model the first objective with CVaR.

In the remainder of the paper, we will mainly compare the three methods Pct90, CVaRW95 and ARO. Table 4 and Figs. 3 and 4 show the performance of these methods. We observe that the methods are able to produce near zero shortage even though they were modelled and tested on different sets of scenarios, with very few exceptions. The waste peak is still around 260 tonnes for the month of July, and because the approaches are modelled and tested with more scenarios, larger budget spending is expected in the worst case, e.g. up to 5.5 billion Rupiahs for the case of CVaRW95. The storage peak of usable rice differs between these methods, e.g. CVaRW95 may requires to store up to 500 tonnes in the month of July while this requirement is lower for the other methods and can occur in a different month.

4.6.3. The role of safety stock

From the previous experiments, we can see that there is a large waste in the month of July when the initial stock expires, thus we also experimented with a zero initial stock and safety stock requirement but with a planning horizon of 13 months. A summary of the results is shown in Table 5.

In this setting, positive shortages remain outliers with Pct90 suffering the most. Both methods CVaRW95 and Pct90 maintain very similar levels of waste and budget usage despite the initial stock no longer being available. ARO on the other hand, manages to reduce the peak of waste down to 100 tonnes, but at the expense of having a large budget usage, up to 8 billion Rupiahs with the third quartile of the distribution or 12 billion Rupiahs in the worst case, and also a high variability.

4.6.4. Computational performance

Recall that all models are tractable as we only use linear programming, however due to working with large numbers we instructed the LP-solver to use the dual simplex method, which in theory can be slower than point interior methods such as barrier for large linear models. Table 6 shows the average computational times for Pct, Stoch, CVaR, CVaRW and ARO methods when modelled and tested on $\mathcal{H}_2, \mathcal{T}_2$. ARO is a fast method because only a single linear model is required to be solved at the beginning; and the stochastic programming methods are found to be the slowest.

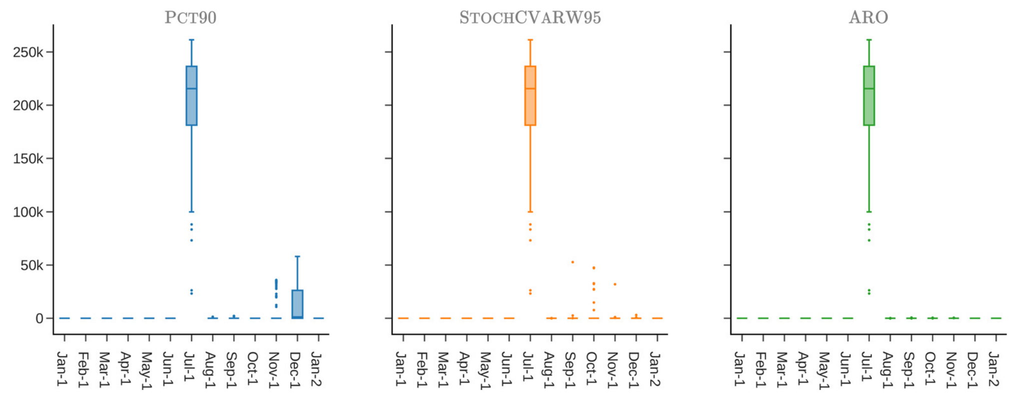

Fig. 3. Waste of Pct90, CVaRW95 and ARO when modelled and tested with $\mathcal{H}_2, \mathcal{T}_2$ respectively.

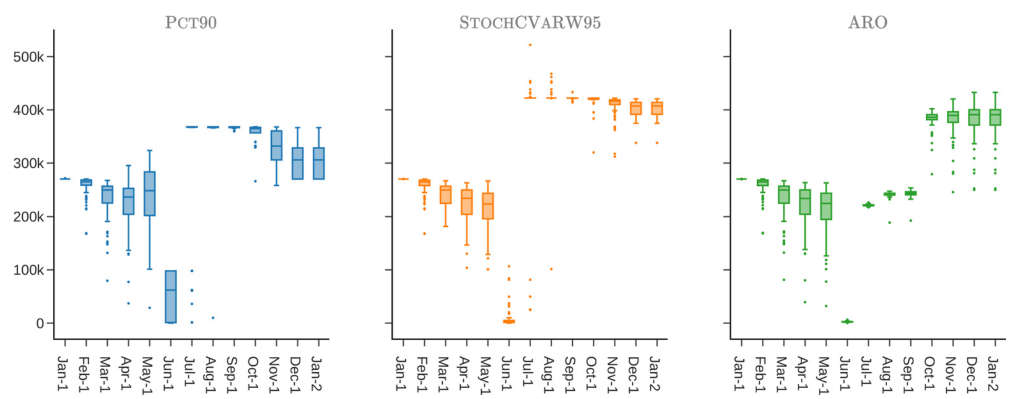

Fig. 4. Useable quantities in store of Pct90, CVaRW95 and ARO when modelled and tested with $\mathcal{H}_2, \mathcal{T}_2$ respectively.

Table 5
Summary of the results for methods modelled on $\mathcal{H}_2$ then tested on $\mathcal{T}_2$, and under the setting that the safety stock is set to zero but there is an extra month in the planning horizon. Median values are presented alongside the 90th quantiles, maximum values, and EVPIs for each objective.

Method	Shortage (10^3 kilograms)				Waste (10^3 kilograms)				Budget spent (10^6 Rupiahs)			
	med	q90	max	EVPI	med	q90	max	EVPI	med	q90	max	EVPI
Pct90	0.0	9.85	145.57	5.78	128.82	222.37	270.59	128.57	3012.65	3415.44	3791.28	1750.99
Pct100	0.0	0.00	1.25	0.01	627.91	944.82	1183.40	598.13	9379.94	11937.53	13174.75	8344.79
Stoch	0.0	0.00	7.14	0.08	155.43	170.13	174.79	144.81	3808.39	4574.29	5403.12	2742.45
CVaR90	0.0	0.00	7.14	0.08	155.43	170.13	174.79	144.81	3808.39	4574.29	5403.12	2742.45
CVaR95	0.0	0.00	7.14	0.08	155.43	170.13	174.79	144.81	3808.39	4574.29	5403.12	2742.45
CVaRW90	0.0	0.00	2.81	0.03	162.06	205.05	215.50	160.11	3985.19	4639.29	5394.45	2891.72
CVaRW95	0.0	0.00	2.81	0.03	162.37	205.53	215.50	160.13	3985.12	4639.29	5394.42	2891.69
ARO	0.0	0.00	13.76	0.14	180.64	200.38	220.15	171.00	4579.07	7086.56	12774.98	3913.76

Table 6
Computational time in seconds for modelling/testing with 100 scenarios and 12 month planning horizon.

Method	CPU time (seconds)
ARO	3.753
Pct	35.538
Stoch	1604.733
CVaR	1580.924
CVaRW	1493.099

4.7. Discussion

The results show that using a wider range of scenarios for setting model parameters results in more robust solutions and consequently our first recommendation to someone solving a similar problem would be to supplement sparse historical data on demands and prices by generating a set of pseudo-scenarios with similar characteristics to those seen in practice. Sparse data also limits the methods available as Pct90 and CVaR90/95 and CVaRW90/95 methods are only feasible with a substantial set of scenarios.

The correct provision of safety stock has a significant influence on the waste produced by the procurement system, particularly for the ARO approach. It also appears to push the waste later in the year and spread it out over a few months rather than seeing all of the waste occur in July. In terms of managing the waste rice, which is typically sold at a low price on the open market, spreading out the waste over several months is beneficial. Insisting on stock being available at the end of each year also places constraints on the procurement strategies and to fit in with annual budgeting, using a 13 month time window seems more appropriate than insisting on a fixed safety stock at the end of the year.

In terms of the methods used, we can see the following general characteristics. The rather crude approach of optimising the 90th percentile of demand seen in Pct90 appears to result in significant reductions in the budget used but poor performance on both shortage and waste, despite the remodelling approach we use to account for observed demand during the year. The ARO approach performs poorly when only sparse historical data are available but better when a more substantial dataset can be used for fitting, while the stochastic programming methods using conditional value at risk (CVaR) perform well in both cases. Nonetheless, the ARO approach has a significantly shorter computational time than the computationally expensive Stoch approaches. These methods are most likely to be applied on a monthly basis and in this case, computational times for all methods are reasonable. If the optimisation needs to be run more regularly, then it may be better to use a method such as ARO which returns results close to instantaneously.

5. Conclusion

We have presented a study that is based on real procurement decisions faced by agencies involved in disaster preparedness and response in West Java Indonesia. The structure of the relief food procurement problem faced by West Java matches those seen in other regions that suffer from frequent disasters. The problem has the following characteristics: (i) it operates based on an annual budget cycle; (ii) the timing of the procurement decision is important and the decision needs to be updated regularly; (iii) some relief food items are perishable; (iv) the availability and the price of food items are seasonal and stochastic; and (v) demand is seasonal and highly variable. These characteristics match those of other relief items such as medicines and blood. Consequently the need for determining optimal purchasing decisions for perishable relief items is widespread and the innovative stochastic optimisation approach presented here could be used in a wide range of different situations.

When modelling natural disasters it is vital to take into account the inherent variability of future information such as demands, prices, and availabilities. But this is also true in areas outside disaster relief. Therefore, the comparisons of different methods for generating robust solutions for a stochastic optimisation problem in which decisions must be made periodically over a finite time window should also be valuable outside of this application. Using a real case with the aforementioned characteristics, the comparisons of robust optimisation, stochastic programming and adjustable robust optimisation show that the adjustable decision approaches have similar results to those of stochastic programming in which the conditional value at risk is optimised, but significantly shorter computation times. From the experimental results, we observe that despite each approach making different procurement decisions, shortages are observed only rarely for the majority of approaches, suggesting the solutions produced are robust with respect to this prioritised objective. The consequence of prioritising the minimisation of shortage is that waste is unavoidable, but this can be further optimised as a second objective using the lexicographical approach adopted here.

Further analysis of the results provides managerial insights into the procurement process when a specific method/formulation is used. These include the possibility of predicting when and how high the peaks of waste, budget spending, and inventory storage will be. We also discussed the role of the safety stock in annual planning with our results suggesting that better outcomes can be obtained by optimising over 13 months with no safety stock than by insisting on a final safety stock.

In summary, we have considered the procurement of emergency food items for disaster management as being isolated from the process of delivering the items to the refugees, explicitly assuming that all deliveries can be fulfilled successfully. We have demonstrated that there are several optimisation techniques that are able to generate robust solutions for the procurement of emergency food for natural disasters. The solutions they produce have different characteristics but whichever approach is used, it is still important to conduct simulation experiments in order to understand the likely impacts of the approach on the logistics of the food distribution system, as well as budget spending, storage capacities and potential waste.

Future work should consider the impacts of integrating the procurement and delivery processes in order to better understand the importance of making the right decisions both temporally and spatially, i.e. when to procure the items and where and how to distribute the items into the warehouses so as to fulfil the demands efficiently. The data used our case study are sufficiently detailed to allow for that extension. Regarding the context of disaster management in Indonesia, we have also isolated the procurement process for emergency rice from rice bought for market intervention and poverty reduction. It could be interesting in future to consider these three demands together in order to produce better informed decisions on the procurement. The consequence of the shortage minimisation for perishable items is waste. One alternative to reduce the amount of waste is by combining pre- and post-disaster strategies. Hence, future work is needed to investigate the right balance between pre-disaster and post-disaster strategies.

In terms of methodology, we have noticed that some of the approaches applied here may not perform well when the number of historical scenarios is too small. This is not an issue for our case study, because disaster events occur often in West Java thus we have sufficient data to simulate additional scenarios. Nevertheless, for the sake of making the methods more widely applicable, future work should look into improving the robustness of the approach when the available data are too scarce for simulation; for example by using distributionally robust optimisation (Duque et al., 2022; Esfahani & Kuhn, 2018). It could also be interesting to address our problem from the perspective of multi-stage stochastic programming because the procurement decisions are made over multiple time periods. The challenges with such an approach include how to address the continuous state space, and additionally how to justify the Markov property, which is often assumed in such formulations (Pereira & Pinto, 1991; Siddig & Song, 2021; 2022).

Acknowledgements

This research is funded by the UK EPSRC GCRF grant number EP/T00360X/1, the support and availability of Prof. Anna Nagurney and from the other members of the advisory board of this project are fully acknowledged. The authors would like to thank Dr. Stefano Coniglio and colleagues from CORMSIS at the University of Southampton for fruitful discussions. The use of the IRIDIS High Performance Computing Facility, and associated support services at the University of Southampton, in the completion of this work is fully acknowledged. The aggregated historical data and scenarios are available from https://github.com/stephanong/relief-ops/. The data which were used in simulating these scenarios are provided by the West Java Government, the West Java Regional Disaster Management Agency (BPBD), the Bureau of Logistics (BULOG) of West Java, and the Indonesian National Agency for Disaster Countermeasure (BNPB), their support is fully acknowledged.

Appendix A. Redundancy in the objectives of the deterministic model

We have the following remark regarding the objectives. There is some redundancy in the objectives (1). Specifically it suffices to consider:

$$\min\left(\sum_{i\in[n+1]} s_i, \sum_{i\in[n]} y_i p(i)\right) \tag{35}$$

which is covered in Proposition 2. However, we still keep the objectives (1) in our experiments because the computational time to solve the linear programs are fast enough to not notice any significant difference. Furthermore, in practice, there is a political pressure to reduce food waste. Hence, we need to have a model that explicitly considers waste minimisation. Last but not least, the expired rice can still be used for other purposes that have some monetary values such processing as rice flour or selling it on the open market at a lower price.

Proposition 2. *A solution of (2)–(9) which is optimal for objectives (35) is also optimal for objectives (1).*

Proof. By contradiction, assume after optimising (35), we can still minimise the waste in (1) by some additional quantity δ. This implies that at some point in time we have spent the budget on δ that was subsequently wasted. This would mean that the budget was not optimal and this contradicts our assumption. □

Appendix B. Extensions of the deterministic model

The following extensions of BudgetAlloc-Det are possible. Firstly, we do not model the warehouse capacity because this was not considered a constraint in our problem situation, as we discuss in Section 4.1. Nevertheless, adding a limit V on the storage quantity would be relatively straightforward using the following constraint:

$$\sum_{j=1}^{\tau-1} x_{ij} \le V. \qquad \forall i \in n \tag{36}$$

Secondly, we assume that the lead time for delivery of rice is zero, because only locally produced rice is considered. Java Island is the center of rice production in Indonesia and consequently this is a reasonable assumption for the use case we discuss later. A more general model can be constructed where rice of a different profile is also stored, where the other rice has a non-zero lead time δ, a different age limit τ' and different prices $p'(i)$. In this case it suffices to add a new set of variables (y'_i, x'_{ij}, c'_{ij}), additional constraints similar to (2) and to adapt (3) and (5) as follows:

$$\mathbb{1}_{\{j>1\}} x'_{(i-1)(j-1)} + \mathbb{1}_{\{j=1\}} y'_{i-\delta} = c'_{i(j-1)} + x_{ij} \qquad \forall i \in [n], \forall j \in [\tau] \tag{37}$$

$$s_i + \sum_{j=0}^{\tau-1} c_{ij} + \sum_{j=0}^{\tau'-1} c'_{ij} = d(i) \qquad \forall i \in [n] \tag{38}$$

$$\sum_{i\in[n]} y_i p(i) + \sum_{i\in[n]} y'_i p'(i) \le B \tag{39}$$

This could be useful where rice is frequently imported for disaster relief.

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