

Optical mode localization sensing based on fibre-coupled ring resonators

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Abstract: Mode localization is widely used in coupled micro-electro-mechanical system (MEMS) resonators for ultra-sensitive sensing. Here, for the first time, we experimentally demonstrate the phenomenon of optical mode localization in fibre-coupled ring resonators. For an optical system, resonant mode splitting happens when multiple resonators are coupled. Localized external perturbation applied to the system will cause uneven energy distributions of the split modes to the coupled rings, this phenomenon is called the optical mode localization. In this paper, two fibre-ring resonators are coupled. The perturbation is generated by two thermoelectric heaters. We define the normalized amplitude difference between the two split modes as: $(T_{M1} - T_{M2})/T_{M1} \times 100\%$. It is found that this value can be varied from 2.5% to 22.5% when the temperature are changed by the value from 0K to 8.5K. This brings a $\sim 2.4\%/K$ variation rate, which is three orders of magnitude greater than the variation rate of the frequency over temperature changes of the resonator due to thermal perturbation. The measured data reach good agreement with theoretical results, which demonstrates the feasibility of optical mode localization as a new sensing mechanism for ultra-sensitive fibre temperature sensing.

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1. Introduction

The phenomenon of the mode localization is widely studied in coupled micro-electro-mechanical system (MEMS) resonators [1]. When two identical resonators are weakly coupled, the perturbation of a small stiffness or mass in one resonator will result in the uneven distribution of the vibration energy over the whole system [2, 3]. The unevenly-distributed energy is reflected by the resonant amplitude ratio between resonant modes, which is strongly dependent on the magnitude of the perturbation. Compared with measuring the resonant frequency shifts in resonators, measuring the amplitudes variations of eigenstates caused by vibration localization in weakly coupled resonators provides two unique advantages for sensing applications. Firstly, The parametric sensitivity in terms of variation rate is enhanced by at least three orders of magnitudes [2–7]. Secondly, the intrinsic common mode rejection due to the differential measurement techniques significantly reduces the impact of the environmental perturbations, which leads to better temperature stability [7, 8].

Compared with MEMS resonators, optical resonators exhibit several advantages. Optical ring or disk resonators can achieve high quality factors without using vibration structures and vacuum conditions [9–11]. These devices show great diversity in scale, with a variety of sizes from micrometers (waveguide) [12] to meters (fibres) [9], which play a key role in integrated photonic circuits [13–15], sensing [16] and optical communication [17].

For optical resonators, resonant modes will be split when two resonators are weakly coupled [18]. By a localized perturbation, the induced changes in the refractive index and coupling coefficient will change the energy distribution to all resonant modes. These changes thereby result in asymmetrical splitting of the resonant modes. The symmetry feature of the mode splitting can be evaluated by the modal power ratio between two split modes, which can indicate the magnitude of the perturbation. Therefore the localized perturbation can be quantitatively measured by examining the modal power ratio between the split modes [3–6].

46 This sensing mechanism can be embedded in optical waveguide/fibre systems to develop
 47 ultra-sensitive sensors. In this paper, we experimentally investigate, for the first time, the optical
 48 mode localization in fibre-coupled ring resonators. In the experiment, two different coupled ring
 49 resonators (Device A with the coupling ratio $\kappa = 0.1$ and Device B with $\kappa = 0.5$. κ refers to the
 50 strength of the coupling between the resonator and the input/output fibres.) are fabricated. By
 51 tuning the thermal perturbation, the variations of the amplitude difference between the split modes
 52 are measured. For Device A, the difference can be tuned from -6% to -2% as the temperature
 53 variation increases from 0K to 8.5K, the corresponding variation rates are $\sim -0.41\%/K$ and \sim
 54 $0.39\%/K$ when the upper and bottom rings are heated, respectively. In contrast, For Device B,
 55 the difference can be tuned from 2.5% to 22.5% when the temperature variation increases from
 56 0K to 8.5K, the corresponding variation rates are $\sim -2.2\%/K$ and $\sim 2.4\%/K$ when the upper and
 57 bottom rings are heated, respectively. The demonstrated results exhibit three orders of magnitude
 58 improvement in the sensitivity in terms of variation rate to the temperature changes comparing
 59 with the variation rate of the thermally-induced frequency shift. Combining the advantages of
 60 optical fibres, we provide a new way for ultra-sensitive fibre temperature sensing.

61 2. Experiment setup

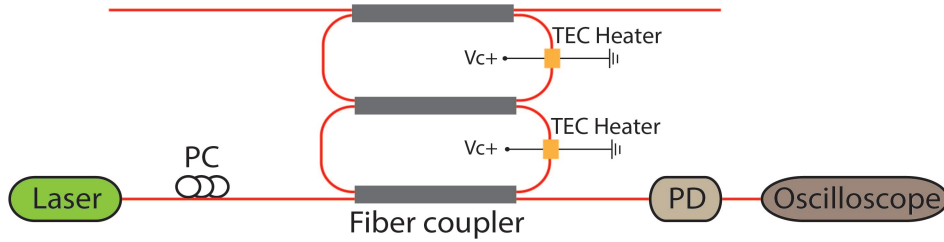


Fig. 1. Schematic of the system. The device consists of two coupled ring resonators, which are formed by three fibre couplers. Two thermoelectric (TEC) heaters are placed on the upper and bottom rings, respectively. The uneven distribution of optical energy will occur when only one of the heaters is utilized.

62 Figure. 1 is the schematic of the system. The coupled ring resonators are produced by
 63 fusion-splicing the output ports of 2×2 single-mode fibre couplers to their input ports. The
 64 circumference of each ring resonator is around 50 cm. A tunable laser with the linewidth of 1
 65 MHz is used to measure the resonance response of the coupled ring resonator. A polarization
 66 controller is placed after the laser to ensure only one eigenmode is excited to the device [9].
 67 The light signal from the output of the coupled ring resonator is captured and measured by a
 68 photodetector and an oscilloscope. In this experiment, the critical coupling is desired to ensure a
 69 high extinction ratio (ER). However, due to the unavoidable losses induced by the fibres, couplers
 70 and fusion splices, it is hard to achieve critical coupling by using fibre couplers with fixed
 71 coupling coefficients. Here, two coupling coefficients (0.1 and 0.5) are used to form two devices
 72 (Device A and B), and their performances are compared in this paper.

73 3. Transmission spectrum for the devices with different coupling coefficients

74 The spectrum of the devices without thermal perturbation is shown in figure. 2. The split notches
 75 M1 and M2 in the spectrum indicate two split resonant modes, which are called symmetric and
 76 antisymmetric modes [18, 19]. For one mode, the ER is defined as the difference between the
 77 on-resonance and off-resonance transmissions [12, 20]. To maximise ER, the coupling coefficient

78 κ and attenuation coefficient α have to be equal to reach the critical coupling. Figure. 2 (a)
79 displays the transmission spectrum of Device A. In this situation, the maximum ER is 0.15, which
80 indicates a massive gap between the κ and the α . Device B is also examined, which is shown in
81 figure. 2 (b). It shows that when the κ is increased, the maximum ER increases from 0.15 to
82 0.4, which enlarges the initial ER by 160%. It is also found that the enlarged ER significantly
83 improves the device's sensitivity to the temperature changes, which will be discussed later. It
84 should be noticed that the initial ERs of the split modes shown in figure. 2 are different. This
85 is because the two fabricated fibre-coupled ring resonators are not identical due to fabrication
86 imperfection. Nevertheless, this will not affect the measurement because each time before the
87 temperature is measured, the response without temperature perturbation is measured first and
88 it will be used as an initial benchmark. When temperature perturbation is induced, the actual
89 temperature change will be measured by the difference between the response and the pre-set
90 benchmark.

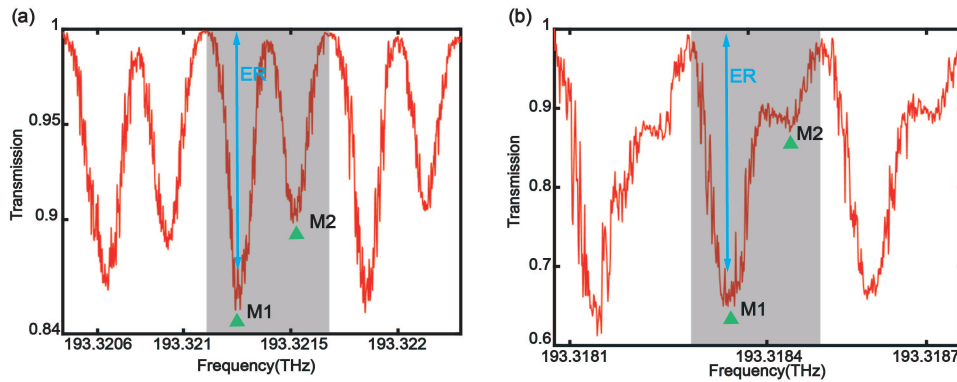


Fig. 2. Measured spectrum of the devices with different coupling coefficients: (a) Device A with $\kappa = 0.1$ and (b) Device B with $\kappa = 0.5$. The split notches M1 and M2 are the two split resonant modes. The gray area represents one period.

91 4. Simulation results

92 4.1. Fitting of the measured data

93 Figure. 3 (a) and (b) display the theoretical fits to the measured spectrum. The fitting functions
94 are calculated by using the transfer matrix method [21], and also verified by the Lumerical
95 interconnect module simulation. Before doing further analysis, a smoothing function is utilized
96 to remove the glitches of the signal. Compare with the raw data shown in figure. 2, most glitches
97 are removed after smoothing by a Gaussian window and the processed signals show good fits
98 with the theoretical models.

99 4.2. The ERs of the split modes when the device is heated

100 In this experiment, the thermal perturbation is induced by two thermoelectric (TEC) heaters,
101 placed on the upper and bottom rings, respectively. The heating length that the TEC heaters
102 can cover is 1 cm. Due to the thermal-optical effect, the refractive index of the covered coupled
103 ring resonators will change according to the induced temperature variation. The thermal-optic
104 coefficient is $1.33 \times 10^{-5} K^{-1}$ for silica optical fibre; therefore there will be 0.000133 refractive
105 index changes per K. We simulate the transmission responses with temperature variations, and
106 the results are shown in figure. 3 (c) and (d). It can be seen that the variations of the two split

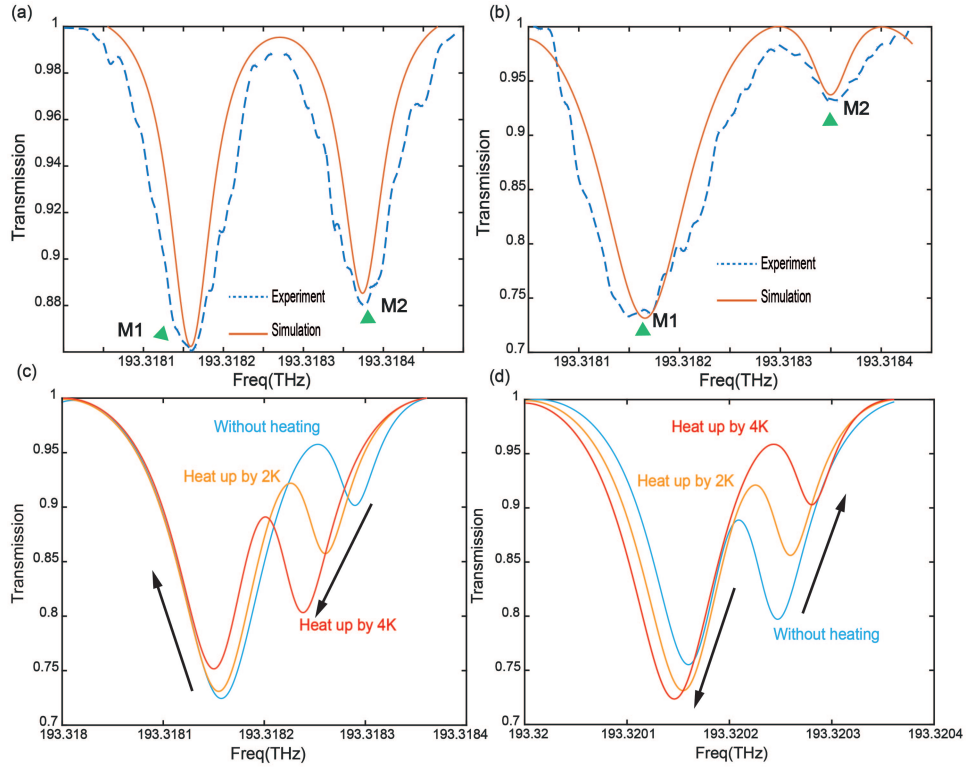


Fig. 3. (a) The theoretical fits to the measured spectrum for Device A. (b) The theoretical fits to the measured spectrum for Device B. (c) and (d) The changes of two split modes when temperature perturbation is induced, the simulation is based on $\kappa = 0.5$. (c) when the upper ring is heated. (d) when the bottom ring is heated.

107 modes show opposite trends when different rings are heated up, respectively. In figure. 3 (c), only
 108 the upper ring is heated. When the temperature is increased by the value from 0K (Blue line) to
 109 4k (Red line), the ERs of M2 increase from 0.1 to 0.22 while the ERs of M1 drop from 0.28 to
 110 0.25. Comparing with only heating the upper ring, the variations of M1 and M2 will be different
 111 if only the bottom ring is heated, which is shown in figure. 3 (d). The ERs of M2 reduce from
 112 0.22 to 0.1 and the ERs of M1 increase from 0.24 to 0.28 as the bottom ring is heated. These
 113 opposite changes are due to the optical mode localization effect, it induces unevenly distributed
 114 energy to the split modes, which will be discussed in the next section.

115 4.3. The energy distributions of the split modes when the device is heated

116 After comparing the transmissions between different heating positions, it is found that heating the
 117 upper ring will cause opposite changes on the ERs of the two split modes regarding to heating the
 118 lower ring. This is due to the induced uneven optical energy distributions of the split modes to
 119 the two rings when different rings are heated. Based on our previous work, when the temperature
 120 of the upper ring increases, as shown in figure. 4 (a), the ER of M1 drops while the ER of M2
 121 rises. This indicates that the energy of the mode M1 is more localized into the bottom ring.
 122 In contrast, for the mode M2, the optical energy is more localized into the upper ring. While
 123 heating the bottom ring will give opposite distributions, as shown in figure. 4 (b), the ER of
 124 M1 raises and the ER of M2 declines. In this case, the energy is more allocated into the upper
 125 ring when the mode M1 is pumped, and into the bottom ring when the mode M2 is pumped. In

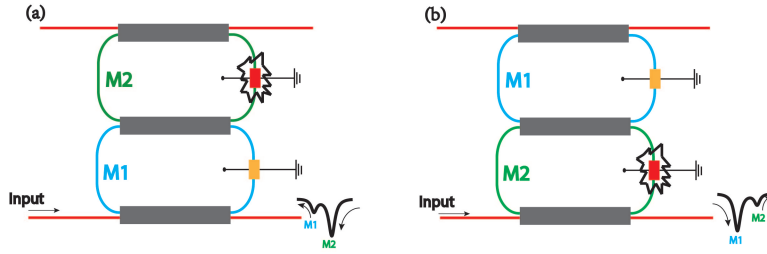


Fig. 4. The optical energy distributions of M1 and M2 when two rings are heated, respectively. (a) When the upper ring is heated. (b) When the bottom ring is heated.

126 conclusion, the energy distributions of the resonant modes are significantly influenced by the
 127 location of the heating source. Also, the energy concentration of M2 and M1 are proportional
 128 and inversely proportional to the temperature, respectively. In this way, we correlate the energy
 129 localization of the split modes to the measured ER changes, enabling us to utilize the measured
 130 thermally-induced ERs changes to quantify the energy distribution of the resonance modes.

131 **4.4. The comparison between M1 and M2 when the temperature is changing**

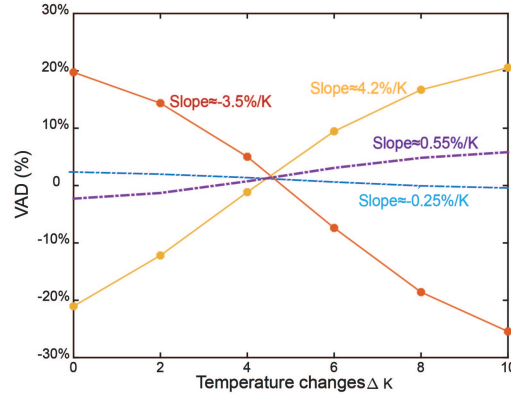


Fig. 5. The simulated results for the variation of VAD when the temperature is changing. Yellow line: the upper ring is heated, $\kappa = 0.5$. Red line: the bottom ring is heated, $\kappa = 0.5$. Purple line: the upper ring is heated, $\kappa = 0.1$. blue line: the bottom ring is heated, $\kappa = 0.1$.

132 In this experiment, the variation of the normalized amplitude difference (VAD) between
 133 M1 and M2 is used to quantify the induced thermal perturbation. VAD is defined as $VAD =$
 134 $((T_{M1} - T_{M2})/T_{M1}) \times 100\%$, where T_{M1} and T_{M2} are the normalized amplitudes of the mode M1
 135 and M2, respectively. When the upper ring or the bottom ring is heated, the VAD will vary with
 136 temperature. In figure. 5, Device A and Device B are evaluated, respectively. When Device A is
 137 tested, the significant difference between the attenuation coefficient α and coupling coefficient κ
 138 decreases ER, which limits the total range of amplitude changes. When the upper ring is heated
 139 (Blue line), the VAD is gradually decreasing from 2% to -1% as the temperature increases from
 140 0k to 10k, the corresponding variation rate is $\sim -0.25\%/K$. In contrast, the VAD exhibits opposite
 141 trend when the bottom ring is heated (Purple line), which is gradually increasing from -2% to
 142 5%, the corresponding variation rate is $\sim 0.55\%/K$. However, when the κ increases, enabling the

143 coupled rings to approach critical coupling, the maximum ERs of both modes will increase. As
 144 shown by the yellow and red lines in figure. 5, the VAD is gradually changing between -25%
 145 to 20% as the temperature is increasing. The variation rates are $\sim -3.5\%/K$ and $\sim 4.2\%/K$
 146 for the upper and bottom heating positions, respectively. The results show that under the same
 147 temperature changing range ΔK , the variation rate of VAD for Device B is four times higher
 148 than the Device A. In conclusion, the VAD variation rate is significantly relied on the maximum
 149 ER that the system can reach. Heating different positions gives opposite VAD variations, the
 150 variation is 0 when the transmissions of M1 and M2 are equal, which agrees with the theory.

151 4.5. Comparing with thermally-induced frequency-shift

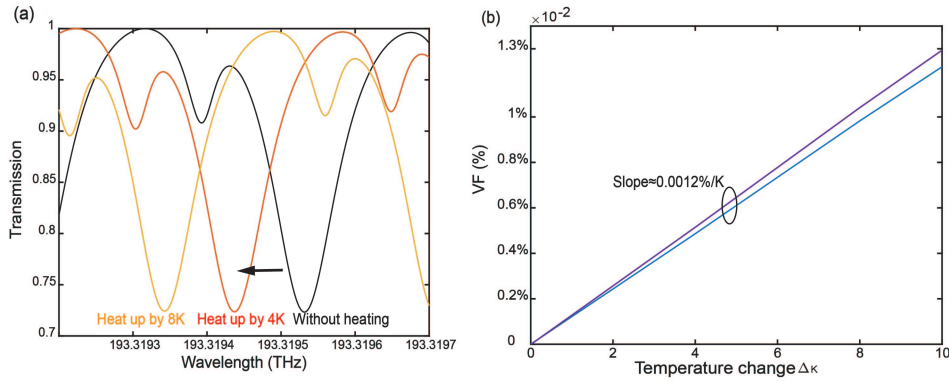


Fig. 6. (a) The simulated frequency shift when two rings are heated at the same time.
 (b) The variation of the VF when ΔT increases from 0K to 10K, purple line: M1, blue
 line: M2.

152 As the coupled ring resonator is heated, the frequency of the resonant mode will shift due to
 153 the thermal-optical effect. However, this shift is small as the thermal-optical coefficient is small.
 154 In theory, the resonant frequency shift that is due to the thermal-optical effect can be expressed
 155 as [22]:

$$f_r - f_0 = \frac{N}{n_g} \cdot f_r \cdot \Delta T \quad (1)$$

156 where f_0 and f_r are the resonant frequencies before and after thermal perturbation, respectively.
 157 N is the thermal-optical coefficient and n_g is the group index. Figure. 6 (a) displays the
 158 resonant frequency shift when both rings are heated. Both split modes are blue-shifted when the
 159 temperature increases. The resonant frequencies of modes M1 and M2 are shifted by 72MHz
 160 and 90MHz, respectively when the temperature is increasing by 8K. To evaluate the temperature
 161 sensitivity of the thermally-induced-frequency-shift-based sensing, we define the variation of the
 162 resonant frequency as $VF = ((f_r - f_0)/f_0) \times 100\%$, where f_0 and f_r are the resonant frequencies
 163 before and after the temperature changes. As shown in figure. 6 (b), when the temperature is
 164 increased by the value from 0K to 10K, both resonant frequencies shift at a similar rate. The
 165 variation rate of the VF is around 0.0012%/K. Comparing the VAD and the VF, it is found that the
 166 variation rate of the optical-mode-localization-effect-based VAD is about 3 orders of magnitude
 167 greater than the variation rate of the frequency-shift-based VF, and this can be further enhanced if
 168 the critical coupling is reached. Traditionally, fibre ring resonators are rarely used as temperature
 169 sensor because thermal optical effects are immaterial when using large cavities [22], only on-chip
 170 micro-resonators are eligible for temperature sensing [23, 24]. With the mode localization

171 enhancement, the sensitivity to temperature changes improves by 3 orders of magnitude, which
172 makes fibre-coupled ring resonators a potential ultra-high sensitive temperature sensor.

173 5. Experimental results

174 5.1. The temperature response of TEC heater

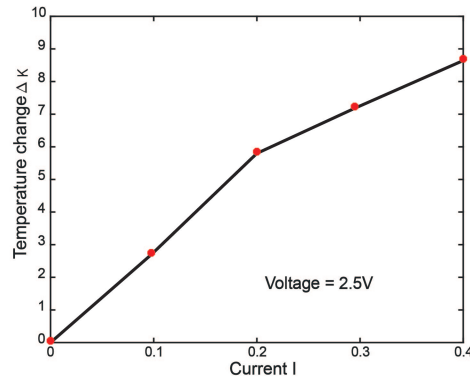


Fig. 7. The temperature response of TEC heater

175 Firstly, two TEC heaters are tested. The load voltage is fixed to 2.5V, as we gradually increase
176 the current, the temperature responses are shown in figure. 7. The temperatures at $I=0A$, 0.1A,
177 0.2A and 0.4A will be used to test our devices.

178 5.2. Transmission responses when the devices are heated

179 Figure. 8 displays the measured transmission responses when the upper or bottom ring is heated.
180 In the experiment, both devices are heated up by the value from 0K to 8.5K. For Device A, The
181 ER of M1 decreases from 0.16 to 0.13 when the upper ring is heated and increases from 0.09
182 to 0.12 when the bottom ring is heated, respectively. In contrast, the ER of M2 increases from
183 0.09 to 0.12 when the upper ring is heated and decreases from 0.16 to 0.13 when the bottom
184 ring is heated, respectively. The results are shown in figure. 8 (a) and (b). When Device B is
185 tested, the coupling coefficient κ increases from 0.1 to 0.5. As shown in figure. 8 (c) and (d), the
186 variations of M1 and M2 show the same trends as Device A, but as the temperature increases, the
187 steps of the variations become larger. the ER of M1 decreases from 0.12 to 0.04 when the upper
188 ring is heated and increases from 0.06 to 0.15 when the bottom ring is heated, respectively. In
189 contrast, the ER of M2 increases from 0.17 to 0.23 when the upper ring is heated and decreases
190 from 0.27 to 0.16 when the bottom ring is heated, respectively. Overall, increasing the coupling
191 coefficient κ from 0.1 to 0.5 enables the coupled rings to approach critical coupling, which
192 improves the maximum ERs of both modes. The measured opposite ERs changes of two modes
193 with temperature also show good agreement with the theoretical results shown in figure. 3 (c)
194 and (d).

195 5.3. The variation of VAD during temperature changes

196 To evaluate the device's sensitivity to the temperature changes, the variation of VAD is calculated
197 based on the recorded transmissions. Figure. 9 (a) displays the VAD variation at 0K, 2.5K, 5.8K
198 and 8.5K heated temperature. For Device B, the variation of the VAD ranges from 2.5% to 22.5%.
199 The corresponding variation rate is $\sim -2.2\%/K$ when the upper ring is heated and $\sim 2.6\%/K$
200 when the bottom ring is heated. In contrast, for Device A, the variation range of VAD is limited

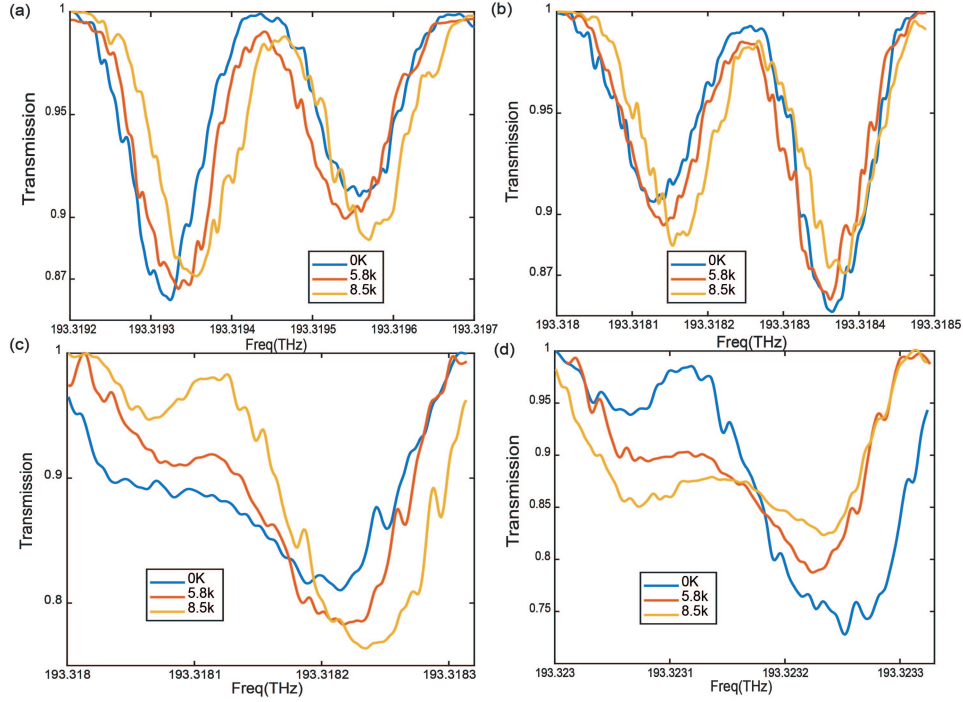


Fig. 8. The transmission response of the device with (a) $\kappa = 0.1$ when the upper ring is heated, (b) $\kappa = 0.1$ when the bottom ring is heated, (c) $\kappa = 0.5$ when the upper ring is heated (d) $\kappa = 0.5$ when the bottom ring is heated. The inset is the temperature changes induced by TEC heaters.

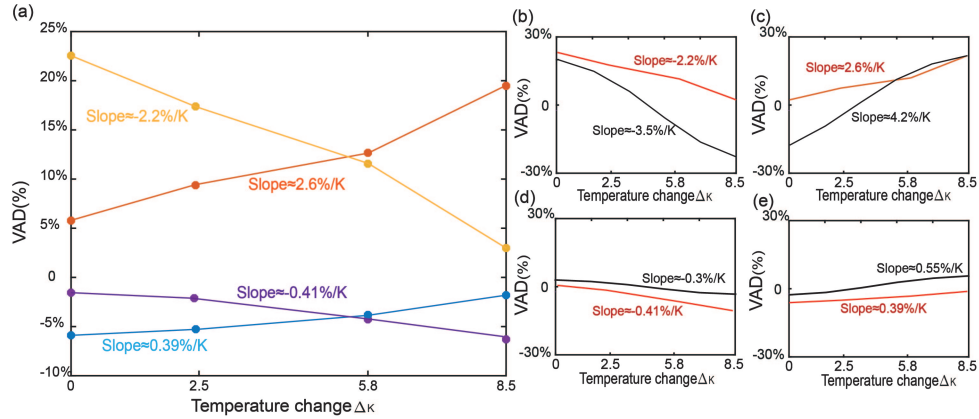


Fig. 9. (a) The results for the variation of VAD with temperature changes. Yellow line: $\kappa = 0.5$, the upper ring is heated. Orange line: $\kappa = 0.5$, the bottom ring is heated. Purple line: $\kappa = 0.1$, the upper ring is heated. Blue line: $\kappa = 0.1$, the bottom ring is heated. (b)-(e) The comparisons between the simulated (Black line) and the measured (Red line) VADs: (b) $\kappa = 0.5$, the upper ring is heated. (c) $\kappa = 0.5$, the bottom ring is heated. (d) $\kappa = 0.1$, the upper ring is heated. (e) $\kappa = 0.1$, the bottom ring is heated.

201 within -6% to -2% . The variation rates are $\sim -0.41\%/K$ and $\sim 0.39\%/K$ for the upper and bottom

202 heating positions, respectively, which is around six times lower than Device A. The result is
203 similar to our simulated one shown in figure. 5, where the variation rate of VAD improves by
204 four times when increasing κ from 0.1 to 0.5. Figure. 5 (b)-(e) show the comparisons of VADs
205 between the simulations and the measured results. It is found that the variation rates of the VADs
206 of the measured results are slightly lower than the simulation results except when $\kappa = 0.1$ and the
207 upper ring is heated, where the variation rate is $\sim -0.41\%/K$ in the experiment and $\sim -0.3\%$
208 in the simulation. In addition, the maximum error occurs when $\kappa = 0.5$ and the upper ring is
209 heated, which is shown in figure. 5 (c), the variation rate in the experiment is $\sim 1.6\%$ lower than
210 the simulation. Nevertheless, the experiment results exhibit at least three orders of magnitude
211 higher variation rate if compared with the thermally-induced-frequency-shift-based sensing.

212 6. Discussion

213 Through theoretical and experimental studies, the phenomenon of optical mode localization
214 due to the temperature perturbation in fibre-coupled ring resonators has been demonstrated.
215 The device's sensitivity to temperature changes is improved by at least 3 orders of magnitude
216 compared with frequency-shift-based sensing. In the experiment, the couplings of the fibre
217 couplers are fixed so that it's hard to balance between the loss-induced attenuation factor α
218 and coupling coefficient κ . Therefore the critical coupling is hard to reach to maximize ER. If
219 replacing them by tunable couplers, the sensitivity could be even higher.

220 In this experiment, the data for each measurement is obtained by averaging three replicate
221 measurements. This approach effectively reduces the experimental errors caused by optical noise
222 originated from the detector, laser power, external vibration, etc [25]. Due to the limited size of
223 the fibre ring resonator, where a minimum circumference of 30cm has been reported [9], the FSR
224 is within the range of hundreds megahertz. In this case, laser with good frequency resolution
225 (narrow linewidth) is required. In this experiment, the measured minimum FSR of our device is
226 400 MHz, which is shown in figure. 3. The linewidth of the laser is 1MHz, which means 400
227 points can be measured for each period. In addition, random de-tuning will also happen because
228 the laser has reached its maximum tunable linewidth. Therefore, laser with narrower linewidth
229 can be used to improve the accuracy of the measurement.

230 The system can be potentially used for ultra-sensitive temperature fibre sensing. In addition,
231 comparing with on-chip integrated photonic sensors, fibre-coupled ring resonators offer larger
232 scale for sensing range. Also, due to the flexibility of optical fibres, the sensing objects can be
233 more than just regular shapes, which enables sensing under various conditions.

234 7. Conclusion

235 In summary, we experimentally demonstrate the optical mode localization in fibre-coupled ring
236 resonators and its feasibility for high-sensitive sensing. Two fibre-coupled ring resonators with
237 different coupling coefficients (Device A and Device B) are fabricated by fusion-spliced fibre
238 couplers, the thermal perturbations are induced by two TEC heaters covered on the upper and the
239 bottom ring resonators, respectively. The VAD variation rate of $\sim 2.3\%/K$ is achieved when the
240 coupling coefficient is 0.5, which is three orders of magnitude greater than the variation rate of
241 the frequency-shift-based VF. The value can be further enhanced if the system reaches critical
242 coupling. This result shows that optical mode localization-based sensing increases the sensitivity
243 of a large cavity to the changes in refractive index, resulting in a larger shift in the amplitude of a
244 resonant mode for a given change in refractive index, which provide a new way for ultra-sensitive
245 fibre temperature sensing.

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249 **Disclosures.** The authors declare no conflicts of interest

250 **Data availability.** The data that support the findings of this study are openly available at the University of
251 Southampton ePrints research repository [26].

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