Testing CP-violation in a Heavy Higgs Sector at CLIC

Kingman Cheung,[1,2,*] Ying-nan Mao,[3,†] Stefano Moretti,[4,5,‡] and Rui Zhang[6,§]

[1]*Department of Physics and CTC, National Tsing Hua University, Hsinchu, Taiwan 300*

[2]*Division of Quantum Phases and Devices, School of Physics,*
Konkuk University, Seoul 143-701, Republic of Korea

[3]*Department of Physics, School of Science,*
Wuhan University of Technology, Wuhan 430070, Hubei, China

[4]*School of Physics and Astronomy, University of Southampton,*
Southampton, SO17 1BJ, United Kingdom

[5]*Department of Physics and Astronomy, Uppsala University, Box 516, SE-751 20 Uppsala, Sweden*

[6]*Theoretical Physics Division, Institute of High Energy Physics, Beijing 100049, China*

We propose a novel method to test CP-violation in the heavy (pseudo)scalar sector of an extended Higgs model, in which we make simultaneous use of the HVV ($V = W^{\pm}, Z$) and $Ht\bar{t}$ interactions of a heavy Higgs state H. This is possible at the Compact Linear Collider (CLIC) by exploiting H production from Vector-Boson Fusion (VBF) and decay to $t\bar{t}$ pairs. We analyze the distribution of the azimuthal angle between the leptons coming from top and antitop quarks, that would allow one to disentangle the CP nature of such a heavy Higgs state. We also show its implications for the 2-Higgs-Doublet Model (2HDM) with CP-violation.

I. INTRODUCTION

CP-violation was first discovered in the long-lived K-meson rare decay channel $K_L \to 2\pi$ in 1964 [1]. More CP-violation effects were also measured in the K-, D- and B-meson sectors in the past several decades [2–4] (see [5] for a historical review). All these measured CP-violation effects are consistent with the explanation given through the Kobayashi-Maskawa (KM) mechanism [6], which represents another success of the Standard Model (SM) of particle physics. However, it is necessary to search for CP-violation sources Beyond the SM (BSM). One important reason to do so is that the amount of CP-violation contained in the SM is not enough to explain the matter-antimatter

asymmetry in the Universe [7–9].

Theoretically, many BSM scenarios can accommodate additional CP-violation sources to remedy such a flaw of the SM. However, the latter are strongly constrained by experiments. Specifically, measurements of the Electric Dipole Moments (EDMs) of, e.g., electron and neutron [10–12] have already set stringent limits on such new sources (or else could reveal their existence) [13–16], as the sensitivities involved are far above the SM predictions [17–19]. However, the EDM measurements, being very inclusive, are only an "indirect" probe of such new CP-violation sources, which means that, even if we discovered herein CP-violation above the SM predictions, it is unlikely that we could determine the actual interactions involved. Conversely, collider experiments, despite having weaker sensitivities to CP-violation in comparison to EDM ones, can afford one, thanks to the vast variety of exclusive observables that one can define herein, with a "direct" probe of CP-violation.

The case for the complementarity of these two experimental settings can easily be made for BSM frameworks with extended Higgs sectors [20–24]. As an example, Ref. [25] studied both EDM and collider effects in a 2-Higgs Doublet Model (2HDM) [23] with explicit CP-violation, in which non-zero EDMs are expected to be the first signal of it with collider effects able to provide additional information[1].

After the discovery of the 125 GeV Higgs boson at the Large Hadron Collider (LHC) [43–45], testing its CP properties is crucial to ascertain the structure of the underlying Higgs sector. On the one hand, current measurements are consistent with the CP-even (or scalar) state of the SM. On the other hand, an additional Higgs state, possibly mixing with it, may have different CP-properties (e.g., being pseudoscalar or a mixture of the two). To stay with 2HDMs, in these BSM scenarios, an effective method to test the CP-properties of the ensuing physical states is trying to test CP-violation effects in the Yukawa interactions between such an additional (heavy) Higgs boson and fermions via the Lagrangian term

$$\mathcal{L} \supset -\bar{f}\left(g_S + \mathrm{i}g_P\gamma^5\right)fH. \tag{1}$$

Usually, $f = t$ or τ, because a top quark or τ lepton decays quickly enough so that the CP-properties and spin information of the decaying object is protected in its final state distributions. In fact, the spin and CP quantum numbers correlate strongly in the Yukawa interaction. Phenomenologically, there are a lot of works in literature trying to test CP-violation in $Ht\bar{t}$ [25, 37–39, 46–59] or $H\tau^+\tau^-$ [27, 35, 40, 41, 60–66] interactions at colliders.

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[1] For this topic, see also other similar phenomenological studies in a variety of alternative BSM scenarios [26–42].

Besides this, one can also probe CP-violation in the purely bosonic sector, through the interactions between a Higgs state and the SM massive gauge bosons. To exploit this approach, we again need such an additional Higgs state H (while the SM-like 125 GeV Higgs boson is denoted as h). The general effective interactions among h, H and $V = W, Z$ are (with θ_W being the weak mixing angle) [2]

$$\mathcal{L} \supset \left(\frac{2m_W^2}{v} W^{+,\mu} W_\mu^- + \frac{m_Z^2}{v} Z^\mu Z_\mu \right) (c_h h + c_H H) + \frac{c_{hH} g}{2c_{\theta_W}} Z^\mu (h\partial_\mu H - H\partial_\mu h) . \tag{2}$$

We already know that $c_h \neq 0$ through current LHC measurements. If both c_H and c_{hH} are non-zero, we will confirm CP-violation in the Higgs sector because in such a case h and H cannot be CP eigenstates at the same time, as was shown in [67–69].

In the SM, as hinted above, the only CP-violation source is the complex phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [6], which means that, if there exists new CP-violation in Higgs interactions, the Higgs sector of the SM must be extended. Consequently, it becomes attractive to search for CP-violation through the dynamics of additional Higgs states, as done recently for 2HDMs, both fundamental and composite, in Refs. [38–41, 70], which indeed exploited either $Ht\bar{t}$ or HVV couplings.

In this paper, we propose a novel method to test CP-violation through such a heavy H state, as we consider its interactions with both massive fermions and gauge bosons simultaneously. The advantage of this approach is that the CP-even component in H is confirmed through HVV ($V = W^\pm, Z$) interactions while the CP-odd component in H is confirmed through $Ht\bar{t}$ interaction. In order to do so, we consider a process in which the heavy Higgs state H is produced through Vector Bosons Fusion (VBF), i.e., W^+W^-- or ZZ-fusion, and decays into top (anti)quark pairs ($t\bar{t}$). As collider setup, we choose an electron-positron one, in preference to a hadronic one, because of the cleanliness of the described signature therein and, amongst the various future options for the latter, we privilege the Compact Linear Collider (CLIC) design [71–77] because its $\sqrt{s}$ can reach $\mathcal{O}(\text{TeV})$, hence, comparable to the LHC reach.

The paper is organized as follows. We describe our method in Sec. II using a model-independent formulation. In Sec. III, as an illustration, we apply it to the 2HDM with CP-violation. Finally, we conclude in Sec. IV.

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[2] In this paper, we use $s_\theta \equiv \sin\theta$, $c_\theta \equiv \cos\theta$, and $t_\theta \equiv \tan\theta$ for any angle θ to simplify notation.

II. MODEL-INDEPENDENT STUDIES

A. Method

Assuming a heavy scalar H is discovered, and in this paper we focus on the CP properties of this particle. Its effective interactions with massive gauge bosons and fermions can be written in general as

$$\mathcal{L} \supset c_V H \left(\frac{2m_W^2}{v} W^{+,\mu} W_\mu^- + \frac{m_Z^2}{v} Z^\mu Z_\mu \right) - \sum_f \frac{m_f}{v} H \bar{f} \left[\mathrm{Re}\left(c_f\right) + \mathrm{i}\mathrm{Im}\left(c_f\right) \gamma^5 \right] f. \qquad (3)$$

We choose the VBF processes $VV \to H \to t\bar{t}$ where $V = W^\pm$ or Z, and the Feynman diagrams are shown in Fig. 1. If such processes can be measured, we have $c_V \neq 0$ and thus the CP-even

FIG. 1: The Feynman diagrams for VBF processes $VV \to H \to t\bar{t}$ where $V = W^\pm, Z$.
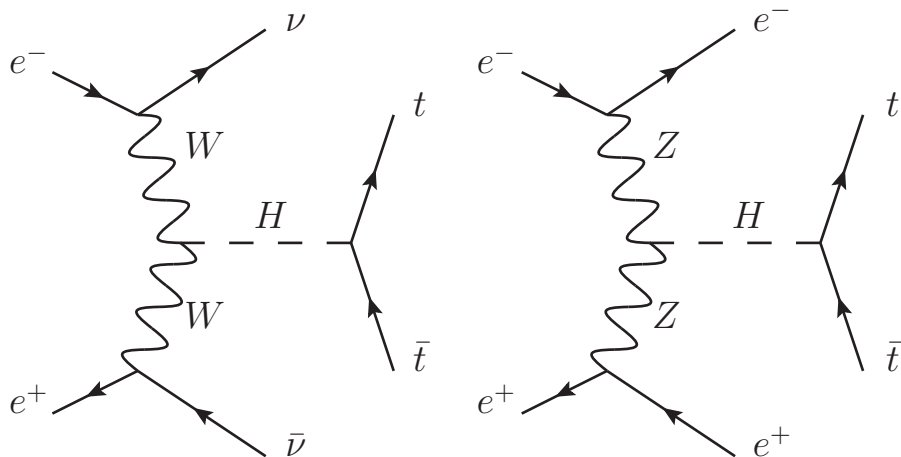
component of H will be confirmed. For the final state $t\bar{t}$, if $\mathrm{Im}\left(c_t\right) = 0$ and $\mathrm{Re}\left(c_t\right) \neq 0$, meaning it is a pure scalar, the $t\bar{t}$ pair will form in a $^3P_0$ state. Instead, if $\mathrm{Re}\left(c_t\right) = 0$ and $\mathrm{Im}\left(c_t\right) \neq 0$, meaning it is a pure pseudoscalar, the $t\bar{t}$ pair will form a $^1S_0$ state. In the CP-violation scenario, there will be both $^3P_0$ and $^1S_0$ types of $t\bar{t}$ final states. Thus, the spin correlation behavior between the top and antitop quarks is sensitive to the CP nature of Higgs states in Yukawa interactions.

We choose semi-leptonic decay channels $t\left(\bar{t}\right) \to b\ell^+\nu \left(\bar{b}\ell^-\bar{\nu}\right)$ $(\ell = e, \mu)$ for both top and antitop quarks. The azimuthal angle between ℓ^+ and ℓ^- (denoted as $\Delta\phi$) is a good observable to measure the spin correlations between top and antitop quarks [78–83], hence, it is helpful to probe CP-violation in the Yukawa sector [3]. For example, we show the normalized distributions

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[3] A similar behavior appears in $t\bar{t}h$ associated production, in which $\Delta\phi$ is one of the best observables to probe CP-violation in $ht\bar{t}$ interaction [25, 38, 39, 81, 82].

of the azimuthal angle $\Delta\phi$ (denoted as $\frac{1}{\sigma}\frac{d\sigma}{d\Delta\phi}$) between the charged leptons from $t\bar{t}$ at CLIC with $\sqrt{s} = 1.5$ TeV in Fig. 2 for both the W^+W^-- and ZZ-fusion channels. It is clear to see that the

FIG. 2: Normalized distributions of the azimuthal angle $\Delta\phi$ (denoted as $\frac{1}{\sigma}\frac{d\sigma}{d(\Delta\phi/\pi)}$) between the charged leptons from $t\bar{t}$ at CLIC with $\sqrt{s} = 1.5$ TeV for both the W^+W^--fusion (the left plot) and ZZ-fusion (the right plot) channels. The blue lines are for a CP-even $Ht\bar{t}$ coupling, while the red lines are for a CP-odd $Ht\bar{t}$ coupling.
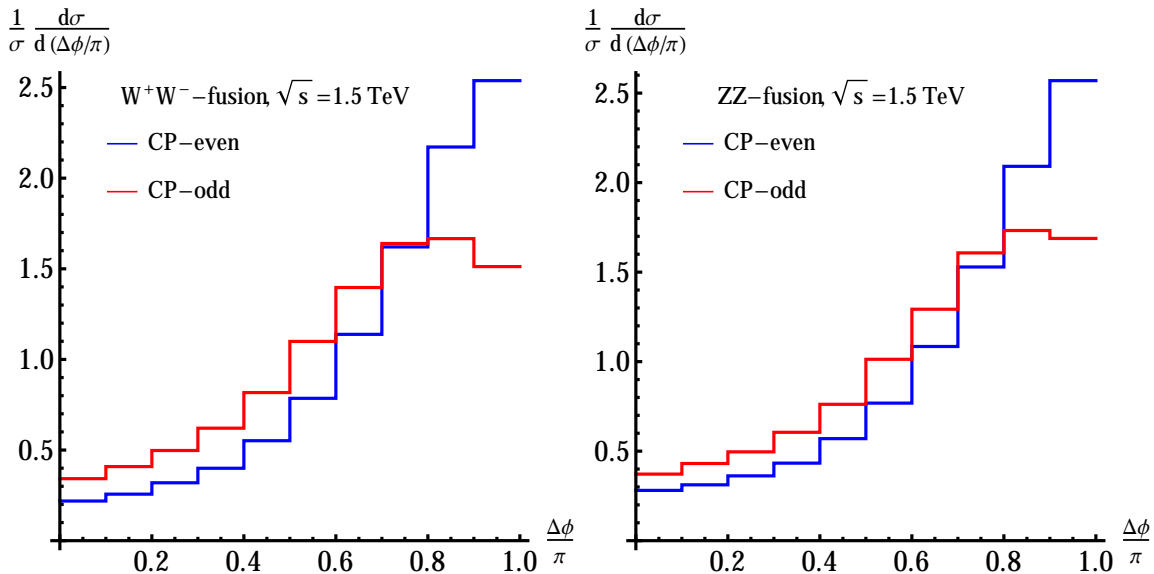
$\Delta\phi$ distributions are different between the cases with CP-even and CP-odd $Ht\bar{t}$ couplings, for both W^+W^-- and ZZ-fusion channels. As discussed above, VBF production implies the existence of the CP-even component in H and, from the $\Delta\phi$ distribution, if we can find the evidence of a non-zero $\text{Im}(c_t)$ (or equivalently the $^1S_0$ type $t\bar{t}$ final state), we can confirm also the CP-odd component in H, and hence the CP-violation effects. Such a method will prove to be more effective for a heavy scalar H mainly containing the CP-odd component, as this is the most different one from the SM background.

B. Simulation Studies at CLIC

We choose two cases: CLIC with $\sqrt{s} = 1.5$ TeV and 3 TeV separately. We further choose c_V not larger than 0.3, because the global-fit for the 125 GeV Higgs boson data implies $c_V \lesssim 0.3$ [4]. The direct LHC search for a heavy scalar H decaying to ZZ final states sets further limits on c_V

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[4] Or else the couplings between the 125 GeV Higgs boson and massive gauge bosons will be too small to satisfy the LHC data, see more detailed analysis in [25].

if $m_H \lesssim 700$ GeV [84], so that we choose the LHC-favored region with a benchmark point having $m_H = 700$ GeV.

In our simulation studies, we consider two VBF processes at CLIC: W^+W^- fusion ($e^+e^- \to \nu\bar{\nu}H$) and ZZ fusion ($e^+e^- \to e^+e^-H$), with the heavy Higgs H decaying to a $t\bar{t}$ pair with top quark and antiquark decaying semileptonically. We assume that the heavy Higgs H can decay via only three channels: $H \to t\bar{t}$, W^+W^- and ZZ. The Branching Ratio (BR) for the $H \to t\bar{t}$ decay channel [5]

$$\mathrm{BR}_{H \to t\bar{t}} \equiv \frac{\Gamma_{H \to t\bar{t}}}{\Gamma_{H \to t\bar{t}} + \Gamma_{H \to W^+W^-} + \Gamma_{H \to ZZ}} \tag{4}$$

thus depends on the couplings c_V and c_t, where [23]

$$\Gamma_{H \to t\bar{t}} = \frac{3m_H}{8\pi}\left(\frac{m_t}{v}\right)^2\left[[\mathrm{Re}(c_t)]^2\left(1 - \frac{4m_t^2}{m_H^2}\right)^{\frac{3}{2}} + [\mathrm{Im}(c_t)]^2\left(1 - \frac{4m_t^2}{m_H^2}\right)^{\frac{1}{2}}\right], \tag{5}$$

$$\Gamma_{H \to W^+W^-} = \frac{m_H^3 c_V^2}{16\pi v^2}\sqrt{1 - \frac{4m_W^2}{m_H^2}}\left(1 - \frac{4m_W^2}{m_H^2} + \frac{12m_W^4}{m_H^4}\right), \tag{6}$$

$$\Gamma_{H \to ZZ} = \frac{m_H^3 c_V^2}{32\pi v^2}\sqrt{1 - \frac{4m_Z^2}{m_H^2}}\left(1 - \frac{4m_Z^2}{m_H^2} + \frac{12m_Z^4}{m_H^4}\right). \tag{7}$$

As we choose $m_H = 700$ GeV in our simulation studies, the $\mathrm{BR}_{H \to t\bar{t}}$ has the following numerical dependence on c_V and c_t:

$$\mathrm{Br}_{H \to t\bar{t}} = \frac{0.174\,[\mathrm{Re}(c_t)]^2 + 0.229\,[\mathrm{Im}(c_t)]^2}{c_V^2 + 0.174\,[\mathrm{Re}(c_t)]^2 + 0.229\,[\mathrm{Im}(c_t)]^2}. \tag{8}$$

In Eq. 5, the term proportional to $[\mathrm{Re}(c_t)]^2$ implies that the partial decay width to $t\bar{t}$ pairs involves a $^3P_0$ state, while the term proportional to $[\mathrm{Im}(c_t)]^2$ implies that the partial decay width to $t\bar{t}$ pairs involves a $^1S_0$ state. The branching ratio of the top quark semileptonic decay is chosen as 21.34%, which is the sum of electron and muon channels, as shown in the Particle Data Group (PDG) review [4]. In our simulation studies, we generate the signal and background events at the Leading Order (LO) order using MadGraph5 [85]. We include bremsstrahlung/Initial State Radiation (ISR) effects through the "isronlyll" option for Parton Distribution Function (PDFs) [86].

In the W^+W^--fusion channel, the main background is the SM s-channel $t\bar{t}$ production because of its large production rate (and the fact that the (anti)neutrinos in the final state of the signal cannot be triggered on) while other background processes are numerically negligible. In the ZZ-fusion channel, the main background is instead SM $t\bar{t}e^+e^-$ production, which comes from both the

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[5] Here we do not consider the ZHh coupling for simplification.

VBF production process of $t\bar{t}$ and the $t\bar{t}Z$ associated production process with the Z boson decaying to an electron pair. To reduce the SM backgrounds, we apply the selection cuts in Table I.

TABLE I: Selection cuts for W^+W^-- and ZZ-fusion processes at CLIC with $\sqrt{s} = 1.5$ TeV (the upper two entries) and $\sqrt{s} = 3$ TeV (the lower two entries).

Process	Selection cuts
W^+W^--fusion ($\sqrt{s} = 1.5$ TeV)	$n_\ell = 2,\ n_b = 2,\ \mid\eta^\ell\mid < 3,\ \mid\eta^b\mid < 5,$ $p_T^{\ell,b} > 10$ GeV, $p_T^{\mathrm{miss}} < 300$ GeV, $\Delta R_{\ell\ell,b\ell,bb} > 0.4,$ $m_{bb\ell\ell} < 600$ GeV, $m_{\ell\ell} < 350$ GeV, $m_{\mathrm{inv}} > 850$ GeV.
ZZ-fusion ($\sqrt{s} = 1.5$ TeV)	$n_\ell \geq 3,\ n_e \geq 1,\ n_{\ell^+} \cdot n_{\ell^-} > 0,\ n_b = 2,$ $\mid\eta^\ell\mid < 3,\ \mid\eta^b\mid < 5,\ \max(\mid\eta^\ell\mid) > 2,\ \Delta R_{\ell\ell,b\ell,bb} > 0.4,$ $p_T^{\ell,b} > 10$ GeV, $m_{bb\ell\ell} > 350$ GeV, $m_{\mathrm{inv}} < 400$ GeV.
W^+W^--fusion ($\sqrt{s} = 3$ TeV)	$n_\ell = 2,\ n_b = 2,\ \mid\eta^\ell\mid < 3,\ \mid\eta^b\mid < 5,$ $p_T^{\ell,b} > 10$ GeV, $p_T^{\mathrm{miss}} < 150$ GeV, $p_T^{\ell\ell} < 200$ GeV, $\Delta R_{\ell\ell,b\ell,bb} > 0.4,$ $m_{bb\ell\ell} < 700$ GeV, $m_{\ell\ell} < 450$ GeV, $m_{\mathrm{inv}} > 1900$ GeV.
ZZ-fusion ($\sqrt{s} = 3$ TeV)	$n_\ell \geq 3,\ n_e \geq 1,\ n_{\ell^+} \cdot n_{\ell^-} > 0,\ n_b = 2,$ $\mid\eta^\ell\mid < 3,\ \mid\eta^b\mid < 5,\ \max(\mid\eta^\ell\mid) > 2,$ $\Delta R_{\ell\ell,b\ell,bb} > 0.4,\ p_T^{\ell,b} > 10$ GeV, $m_{bb\ell\ell} > 350$ GeV.

After performing these selection cuts, we have the cross sections σ_i and the corresponding selection efficiencies ϵ for the signal (denoted as the index "sig") and background (denoted as the index "bkg") processes in Table II together with the discovery potential as a function of the machine luminosity L. For the rates in the table, we know that the signal production cross sections

TABLE II: Cross sections after selection cuts, selection efficiencies and expected significance for W^+W^-- and ZZ-fusion processes at CLIC with $\sqrt{s} = 1.5$ TeV (the upper two entries) and $\sqrt{s} = 3$ TeV (the lower two entries). Here, $N_{\mathrm{sig}} = \sigma_{\mathrm{sig}}L$ and $N_{\mathrm{bkg}} = \sigma_{\mathrm{bkg}}L$ are separately the event rates for signal and background after selection cuts, while L is the integrated luminosity.

Process	σ_{sig} (fb)	ϵ_{sig}	σ_{bkg} (fb)	ϵ_{bkg}	$N_{\mathrm{sig}}/\sqrt{N_{\mathrm{bkg}}}$
W^+W^--fusion ($\sqrt{s} = 1.5$ TeV)	0.94κ	64%	0.34	8.4%	$1.6 \times 10^2 \kappa \sqrt{L/(10\text{ ab}^{-1})}$
ZZ-fusion ($\sqrt{s} = 1.5$ TeV)	0.088κ	61%	0.013	36%	$78\kappa\sqrt{L/(10\text{ ab}^{-1})}$
W^+W^--fusion ($\sqrt{s} = 3$ TeV)	4.14κ	62%	0.041	3.6%	$2.0 \times 10^3 \kappa \sqrt{L/(10\text{ ab}^{-1})}$
ZZ-fusion ($\sqrt{s} = 3$ TeV)	0.28κ	42%	0.019	21%	$2.0 \times 10^2 \kappa \sqrt{L/(10\text{ ab}^{-1})}$

are proportional to the parameter

$$\kappa \equiv \frac{c_V^2 \left(0.174\,[\mathrm{Re}(c_t)]^2 + 0.229\,[\mathrm{Im}(c_t)]^2\right)}{c_V^2 + 0.174\,[\mathrm{Re}(c_t)]^2 + 0.229\,[\mathrm{Im}(c_t)]^2} = c_V^2\,\mathrm{Br}_{H \to t\bar{t}}. \tag{9}$$

Since $c_V \lesssim 0.3$ [25], if we fix $|c_t| = 1$, the largest allowed number for this parameter should be $\kappa_{\max}^+ = 0.059$ for a CP-even $Ht\bar{t}$ coupling and $\kappa_{\max}^- = 0.065$ a for CP-odd $Ht\bar{t}$ coupling. If we choose as typical integrated luminosity $L = 10$ ab^{-1} and the largest allowed $c_V = 0.3$, in all the four cases, the signal $VV \to H \to t(\to b\ell^+\nu)\bar{t}(\to \bar{b}\ell^-\bar{\nu})$ will be discovered with its significance close to or larger than 5σ for both CP-even and CP-odd $Ht\bar{t}$ couplings quite promptly at CLIC, so that we have the basis to further analyze the final state $\Delta\phi$ distributions to probe the CP nature of the H state.

C. Analysis and Results

For all the four cases (both W^+W^-- and ZZ-fusion, with $\sqrt{s} = 1.5$ TeV and 3 TeV), we show the $\Delta\phi$ distribution in Fig. 3, including both signal and background events. We also add the $\Delta\phi$ distribution for pure background events for comparison. The plots clearly show differences between the two CP hypothese in the $\Delta\phi$ distribution even after adding the background events.

We then define the forward-backward asymmetry as

$$A \equiv \frac{N_{\Delta\phi > \pi/2} - N_{\Delta\phi < \pi/2}}{N_{\Delta\phi > \pi/2} + N_{\Delta\phi < \pi/2}} = \frac{N_+ - N_-}{N_+ + N_-}, \tag{10}$$

which is sensitive to the CP nature of the $Ht\bar{t}$ coupling. Its uncertainty can be calculated through

$$\sigma_A = \sqrt{\frac{4N_+N_-}{N^3}}, \tag{11}$$

where $N = N_+ + N_-$ is the total number of events. In our analysis, for all the cases, we must consider the signal and background events together, since they become indistinguishable experimentally even after our selection cuts. Thus, $N_\pm$ contains both signal and background events.

We calculate such a forward-backward asymmetry for each process (denoted through the sub-indices W^+W^- and ZZ in the plots) for both pure CP-even (+) and pure CP-odd (−) $Ht\bar{t}$ couplings, hence, we use $A^\pm$ (in the forthcoming text), together with their $\pm 1\sigma$ uncertainties $\sigma_{A^\pm}$ assuming as integrated luminosity $L = 10$ ab^{-1}, and show the results in Fig. 4. In the calculation, we always fix $[\mathrm{Re}(c_t)]^2 + 1.32[\mathrm{Re}(c_t)]^2 = 1$ as a benchmark point, so that the parameter κ defined in Eq. 9 becomes $\kappa = 0.174c_V^2/(c_V^2 + 0.174)$ and the total cross sections do not depend on $\arg(c_t) = \arctan[\mathrm{Im}(c_t)/\mathrm{Re}(c_t)]$. For the case with CP-mixing $Ht\bar{t}$ coupling, the forward-backward

FIG. 3: Differential cross sections in the azimuthal angle $\Delta\phi$ [denoted as $\frac{d\sigma}{d(\Delta\phi/\pi)}$] between the charged leptons from $t\bar{t}$ at CLIC including both signal and background events: the upper plots are for the case with $\sqrt{s} = 1.5$ TeV and the lower plots are for the case with $\sqrt{s} = 3$ TeV while the left plots are for the W^+W^--fusion process and the right plots are for the ZZ-fusion process. The blue lines are for a CP-even $Ht\bar{t}$ coupling together with background events while the red lines are for a CP-odd $Ht\bar{t}$ coupling together with background events while we also show the distribution for pure background events as black lines.
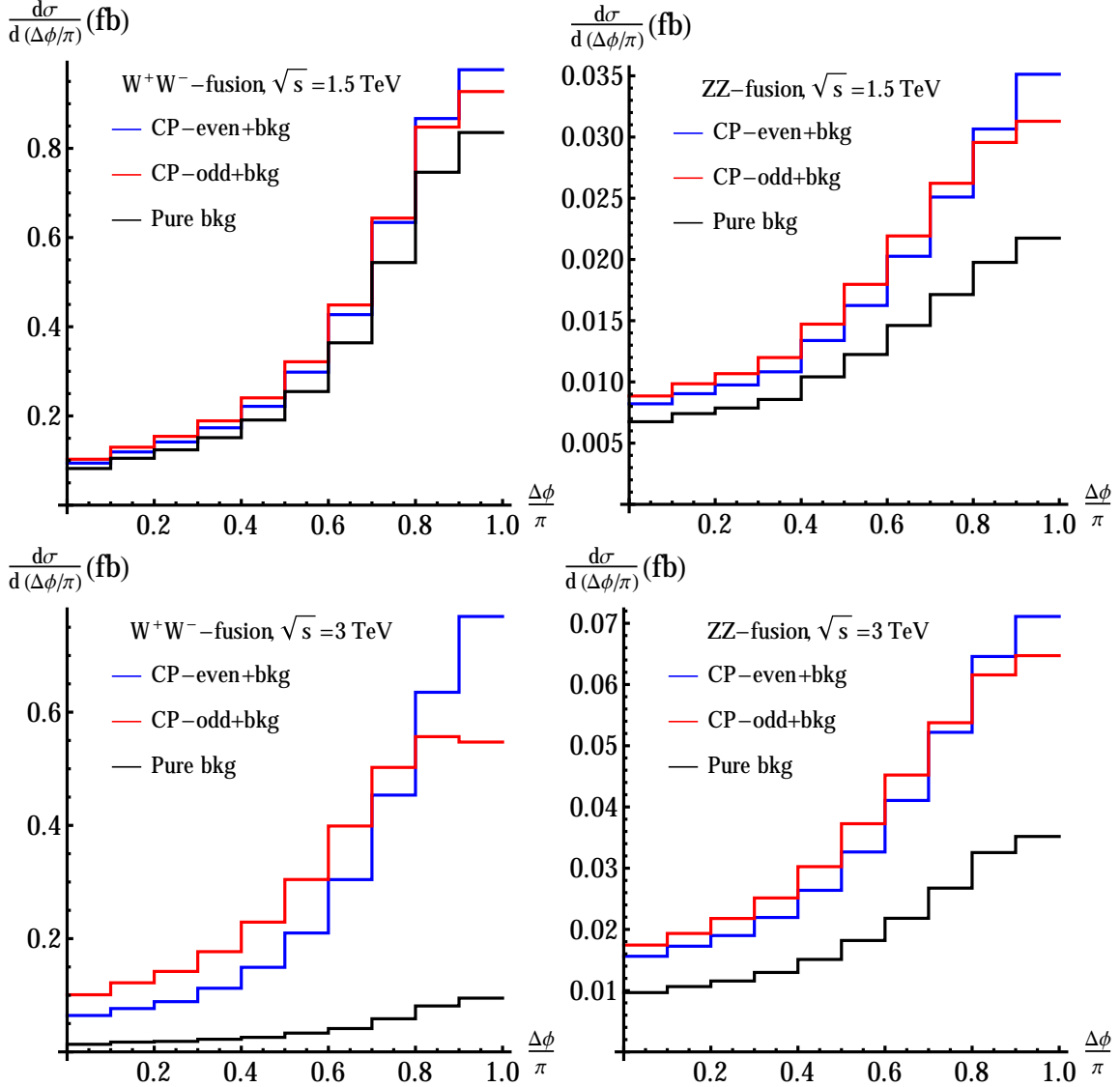
asymmetry will be located between blue and red lines. In our method, a non-zero $\text{Im}(c_t)$ is enough to probe CP-violation, thus we choose a pure CP-odd $Ht\bar{t}$ coupling to find the largest deviation from the CP-conserving case (CP-even). For given experimental conditions and model parameters, the number of standard deviation

$$s_0 = \frac{|A - A^+|}{\sigma_{A^+}} \tag{12}$$

FIG. 4: Forward-backward asymmetries A including both signal and background events: the upper plots are for the case with $\sqrt{s} = 1.5$ TeV and the lower plots are for the case with $\sqrt{s} = 3$ TeV while the left plots are for the W^+W^--fusion process and the right plots are for the ZZ-fusion process. We fix $[\mathrm{Re}(c_t)]^2 + 1.32[\mathrm{Re}(c_t)]^2 = 1$ for all processes. The blue lines are for A^+ including background events while the red lines are for A^- including background events. The solid lines are the central values $A^{\pm}$ while the dashed lines are $A^{\pm} \pm \sigma_{A^{\pm}}$, where $\sigma_{A^{\pm}}$ are the 1σ uncertainties for $A^{\pm}$ for the integrated luminosity $L = 10$ ab^{-1}.
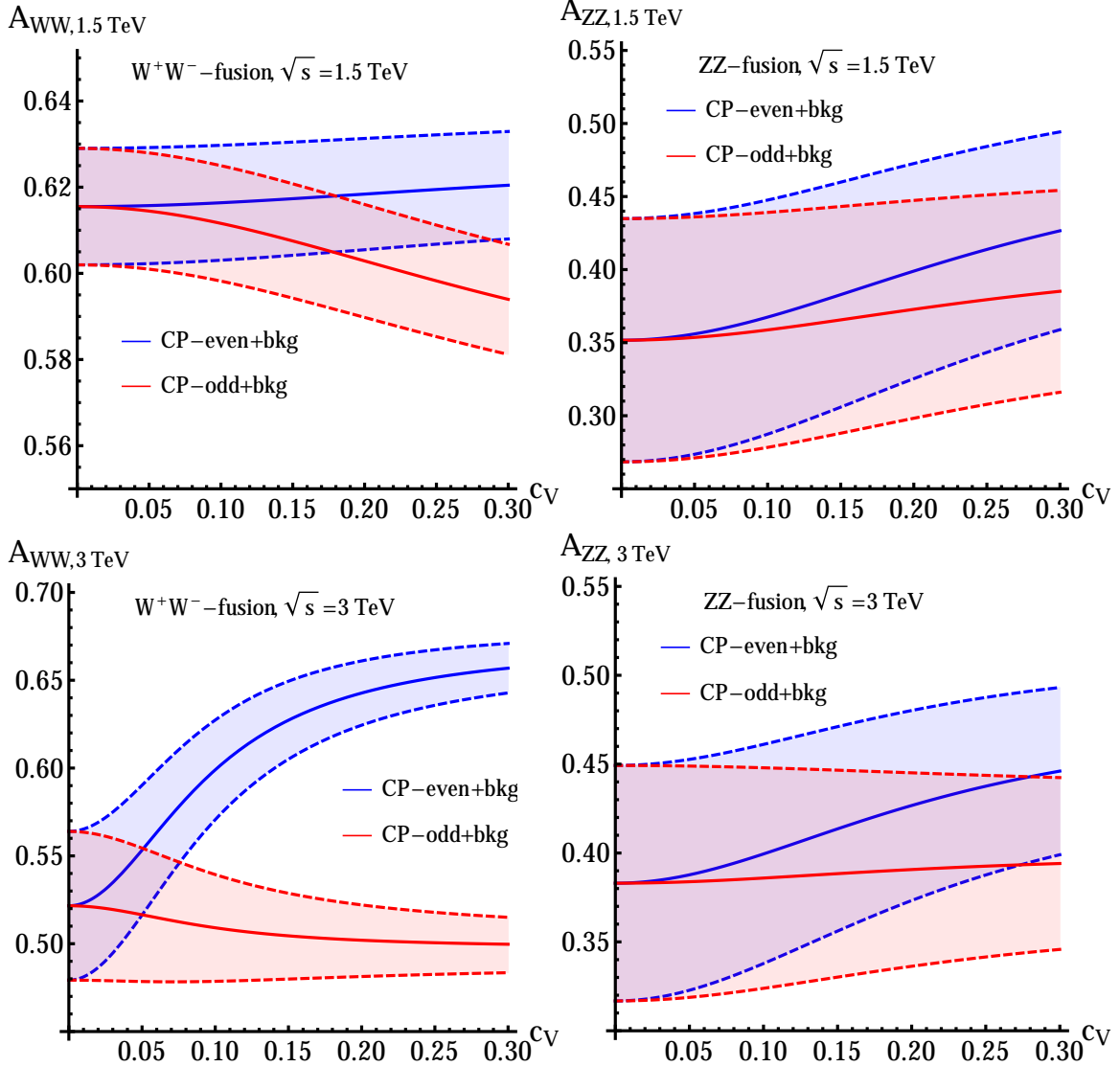
away from the CP-conserving case measures the significance to discover CP-violation, so that, for a pure CP-odd $Ht\bar{t}$ coupling, we have $A = A^-$, which will give us the largest significance for CP-violation.

From the right plots in Fig. 4, it is clear that in the ZZ-fusion channel, it is difficult to distinguish between a CP-even and CP-odd $Ht\bar{t}$ coupling. Even for the largest allowed value

$c_V = 0.3$, we still have s_0 less than or close to 1, meaning that A^- is quite close to A^+. That is mainly because of the small cross section and hence event number of the ZZ-fusion process, in turn affecting adversely the error. Thus, for ZZ-fusion, we do not need further analysis. For W^+W^--fusion with $\sqrt{s} = 1.5$ TeV and upon choosing the largest allowed $c_V = 0.3$, $s_0 \simeq 2.1$ meaning a slight deviation can be found but still not an evidence strong enough to discover CP-violation. That is because at the $\sqrt{s} = 1.5$ TeV CLIC, there is still large $t\bar{t}$ background which cannot be reduced effectively, i.e., $\sigma_{\rm bkg} \gg \sigma_{\rm sig}$. The large background erases the difference between A^+ and A^- and thus only a 2.1σ deviation is left even for $c_V = 0.3$. We do not analyze this case further then. For W^+W^--fusion with $\sqrt{s} = 3$ TeV, from the lower-left plot in Fig. 4, if $c_V \gtrsim 0.1$, we have $s_0 \gtrsim 3$ meaning that A^- and A^+ are significantly different in this case.

Therefore, we discuss the W^+W^--fusion process further at CLIC with $\sqrt{s} = 3$ TeV. Experimentally, the two useful observables are the total cross section $\sigma_{\rm tot} = \sigma_{\rm sig} + \sigma_{\rm bkg}$ and the forward-backward asymmetry A in the $\Delta\phi$ distribution. Notice that $\sigma_{\rm tot}$ depends only on the parameter κ while A depends on both κ and $\xi \equiv \arg(c_t)$. Numerically, we have

$$\sigma_{\rm tot} = \sigma_{\rm sig} + \sigma_{\rm bkg} = (4.14\kappa + 0.041) \text{ fb}, \tag{13}$$

$$A = \frac{1}{\sigma_{\rm tot}} \left(A_{\rm bkg}\sigma_{\rm bkg} + A^+ \sigma_{\rm sig} \frac{c_\xi^2}{c_\xi^2 + 1.32 s_\xi^2} + A^- \sigma_{\rm sig} \frac{1.32 s_\xi^2}{c_\xi^2 + 1.32 s_\xi^2} \right)$$

$$= \frac{1}{\kappa + 9.8 \times 10^{-3}} \left(5.1 \times 10^{-3} + \frac{0.68 c_\xi^2 + 0.65 s_\xi^2}{c_\xi^2 + 1.32 s_\xi^2} \kappa \right). \tag{14}$$

With a luminosity $L = 10$ ab^{-1}, the relative uncertainty of $\sigma_{\rm tot}$ is determined by $\delta\sigma_{\rm tot}/\sigma_{\rm tot} = 1/\sqrt{N_{\rm sig} + N_{\rm bkg}} \sim \mathcal{O}(10^{-2})$ [6], which is ignorable compared with the relative uncertainty of A. In Fig. 5, we show the correlation between asymmetry A and total cross section $\sigma_{\rm tot}$ for different ξ in the left plot, together with the standard deviation away from the CP-conserving case (denoted as $s_0 = |A - A^+|/\sigma_{A^+}$, meaning the discovery potential for CP-violation) for different observed asymmetry A and total cross section $\sigma_{\rm tot}$ values in the right plot. If $\sigma_{\rm tot} \gtrsim 0.08$ fb (corresponding to $\kappa \gtrsim 0.009$ or $c_V \gtrsim 0.1$ if $|c_t| \simeq 1$), a pure CP-odd $Ht\bar{t}$ coupling is expected to be evidenced at the 3σ level; while $\sigma_{\rm tot} \gtrsim 0.11$ fb (corresponding to $\kappa \gtrsim 0.017$ or $c_V \gtrsim 0.14$ if $|c_t| \simeq 1$), a pure CP-odd $Ht\bar{t}$ coupling is expected to be discovered at 5σ level. For the largest allowed $c_V \simeq 0.3$ (corresponding to $\kappa \simeq 0.06$ or $\sigma_{\rm tot} \simeq 0.3$ fb if $|c_t| \simeq 1$), a pure CP-odd $Ht\bar{t}$ coupling corresponding to $\xi = \pi/2$ is expected to be discovered at the 11.5σ level, while the $3(5)\sigma$ evidence (discovery) boundary corresponds to $\xi \simeq 0.15\pi(0.21\pi)$.

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[6] This estimation comes only from our signal process $WW \to H \to t(\to b\ell^+\nu)\bar{t}(\to \bar{b}\ell^-\bar{\nu})$. With the help of other decay channels of H and $t(\bar{t})$, we will obtain a better estimation on the uncertainty of $\sigma_{\rm tot}$.

FIG. 5: In the left plot: we show the expected correlation between the asymmetry A and total cross section σ_{tot} for different $\xi \equiv \arg(c_t)$: the blue line with $\xi = 0$ means pure CP-even $Ht\bar{t}$ coupling corresponding to the CP-conserving case (together with its $\pm 1\sigma$ uncertainty) while the three red lines with $\xi = \pi/6, \pi/3, \pi/2$ correspond to CP-violation cases. In the right plot: we show the standard deviation away from the CP-conserving case (denoted as $s_0 = |A - A^+|/\sigma_{A^+}$, meaning the discovery potential for CP-violation) in the $A - \sigma_{\text{tot}}$ plane, together with the 3σ (dashed black line) and 5σ (solid black line) evidence and discovery boundaries, respectively.
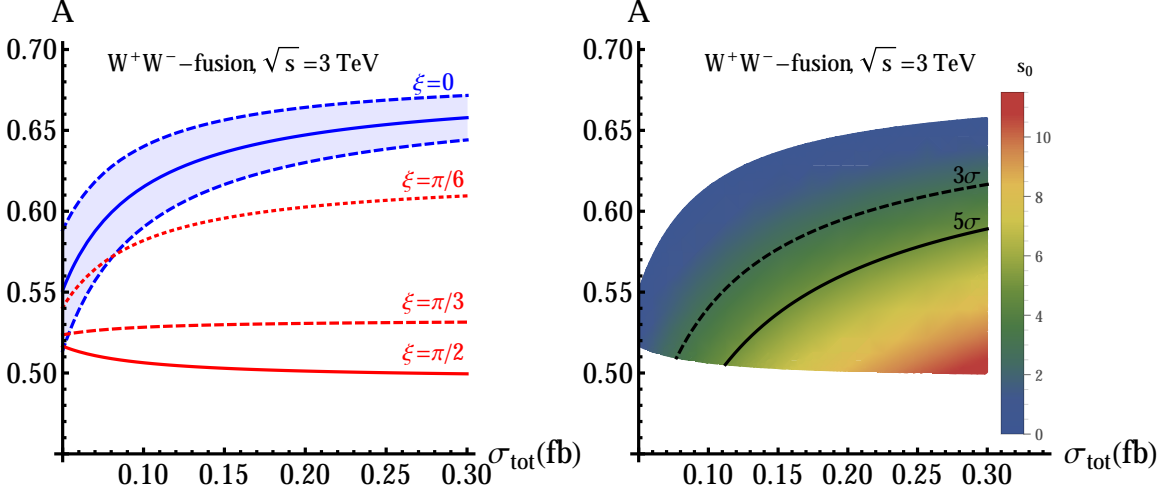
III.   IMPLICATION FOR THE 2HDM WITH CP-VIOLATION

A.   Model Set-up

We choose the 2HDM with CP-violation [23] as a test model in this section. Mainly following the conventions in [25, 87], the Lagrangian in the scalar sector is

$$\mathcal{L} \supset \sum_{i=1,2} (D_\mu \phi_i)^\dagger (D^\mu \phi_i) - V(\phi_1, \phi_2), \qquad (15)$$

where $\phi_1 = \left(\varphi_1^+, \frac{1}{\sqrt{2}}(v_1 + \eta_1 + i\chi_1) \right)^T$ and $\phi_2 = \left(\varphi_2^+, \frac{1}{\sqrt{2}}(v_2 + \eta_2 + i\chi_2) \right)^T$ are two SU(2) doublets. The Vacuum Expected Values (VEVs) $v_{1,2}$ satisfy the relation $v = \sqrt{|v_1|^2 + |v_2|^2} = 246$ GeV. We also define $t_\beta \equiv |v_2/v_1|$ as usual. The scalar potential is given by

$$\begin{aligned}
V(\phi_1, \phi_2) = & -\frac{1}{2} \left[m_1^2 \phi_1^\dagger \phi_1 + m_2^2 \phi_2^\dagger \phi_2 + \left(m_{12}^2 \phi_1^\dagger \phi_2 + \text{H.c.} \right) \right] + \frac{1}{2} \left[\lambda_1 \left(\phi_1^\dagger \phi_1 \right)^2 + \lambda_2 \left(\phi_2^\dagger \phi_2 \right)^2 \right] \\
& + \lambda_3 \left(\phi_1^\dagger \phi_1 \right) \left(\phi_2^\dagger \phi_2 \right) + \lambda_4 \left(\phi_1^\dagger \phi_2 \right) \left(\phi_2^\dagger \phi_1 \right) + \left[\frac{\lambda_5}{2} \left(\phi_1^\dagger \phi_2 \right)^2 + \text{H.c.} \right],
\end{aligned} \qquad (16)$$

where we assumed a softly broken Z_2 symmetry [7] to avoid the possible tree-level Flavour Changing Neutral Current (FCNC) interactions. Here, m_{12}^2, λ_5, and v_2/v_1 can be complex parameters and we can always perform a field rotation to make at least one of them real. We choose v_2/v_1 to be real [8] and thus the vacuum conditions lead us to the relation [25, 87]

$$\text{Im}\left(m_{12}^2\right) = v_1 v_2 \text{Im}\left(\lambda_5\right). \tag{17}$$

If both sides in the equation above are non-zero, there will be CP-violation in the scalar sector. The Goldstone modes G^+ and G^0 can be recovered through a diagonalization procedure as

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \varphi_1^+ \\ \varphi_2^+ \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix} \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix}. \tag{18}$$

If there is no CP-violation, A^0 should be a pure pseudoscalar while, in the CP-violation scenario, A^0 must further mix with $\eta_{1,2}$ to obtain the neutral mass eigenstates as

$$\begin{pmatrix} H_1 \\ H_2 \\ H_3 \end{pmatrix} = R \begin{pmatrix} \eta_1 \\ \eta_2 \\ A^0 \end{pmatrix}. \tag{19}$$

The mixing matrix R is parameterized following the convention in [25] as

$$R = \begin{pmatrix} 1 & & \\ & c_{\alpha_3} & s_{\alpha_3} \\ & -s_{\alpha_3} & c_{\alpha_3} \end{pmatrix} \begin{pmatrix} c_{\alpha_2} & & s_{\alpha_2} \\ & 1 & \\ -s_{\alpha_2} & & c_{\alpha_2} \end{pmatrix} \begin{pmatrix} c_{\beta+\alpha_1} & s_{\beta+\alpha_1} & \\ -s_{\beta+\alpha_1} & c_{\beta+\alpha_1} & \\ & & 1 \end{pmatrix}. \tag{20}$$

With this convention, if $\alpha_{1,2} \to 0$, H_1 becomes the SM Higgs boson. Here, α_2 is an important parameter because it measures the CP-violation mixing corresponding to the SM-like Higgs boson H_1. In the Yukawa sector, a fermion bilinear can couple to only one scalar doublet due to the Z_2 symmetry. Denoting $Q_L \equiv (u,d)_L^T$ and $L_L \equiv (\nu,\ell)_L^T$, we always assume that $\bar{Q}_L u_R$ couples to ϕ_2, and thus the four types of Yukawa interactions are

$$\mathcal{L} \supset \begin{cases} -Y_U \bar{Q}_L \tilde{\phi}_2 U_R - Y_D \bar{Q}_L \phi_2 D_R - Y_\ell \bar{L}_L \phi_2 \ell_R + \text{H.c.}, & (\text{Type I}), \\ -Y_U \bar{Q}_L \tilde{\phi}_2 U_R - Y_D \bar{Q}_L \phi_1 D_R - Y_\ell \bar{L}_L \phi_1 \ell_R + \text{H.c.}, & (\text{Type II}), \\ -Y_U \bar{Q}_L \tilde{\phi}_2 U_R - Y_D \bar{Q}_L \phi_2 D_R - Y_\ell \bar{L}_L \phi_1 \ell_R + \text{H.c.}, & (\text{Type III}), \\ -Y_U \bar{Q}_L \tilde{\phi}_2 U_R - Y_D \bar{Q}_L \phi_1 D_R - Y_\ell \bar{L}_L \phi_2 \ell_R + \text{H.c.}, & (\text{Type IV}). \end{cases} \tag{21}$$

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[7] Under the Z_2 transformation, $\phi_1 \to \phi_1$ and $\phi_2 \to -\phi_2$ and in Eq. 16 only the m_{12}^2-term breaks this symmetry.
[8] Indeed we can then make sure both v_1 and v_2 are real through gauge transformations.

Following our analysis in [25], the Type I and IV models are facing very stringent electron EDM constraints and thus the CP-violation mixings are limited to $\mathcal{O}\left(10^{-3}\right)$. However, in Type II and III models, a possible cancellation between different contributions to the electron EDM leads to a much weaker constraint on the CP-violation mixing α_2 [25, 29, 32–34, 42, 88–94]. As shown in [25], in the Type II model $|\alpha_2| \lesssim 0.1$ mainly due to the neutron EDM constraint while in the Type III model $|\alpha_2| \lesssim 0.3$ mainly due to the global-fit on LHC Higgs data[9]. The cancellation appears around $t_\beta \simeq 1$, depending weakly on $m_{2,3}$ and α_2. Thus, we choose the Type III model as an example. We consider the case for which $H_{2,3}$ have a large mass splitting and $H = H_2$ is dominated by the pseudoscalar component, thus $\alpha_3 \sim \pi/2$ and we have the relation

$$t_{\alpha_3} = \frac{\left(m_3^2 - m_2^2\right) + \sqrt{\left(m_3^2 - m_2^2\right)^2 s_{2\beta+\alpha_1}^2 - 4\left(m_3^2 - m_1^2\right)\left(m_2^2 - m_1^2\right) s_{\alpha_2}^2 c_{2\beta+\alpha_1}^2}}{2\left(m_2^2 - m_1^2\right) s_{\alpha_2} c_{2\beta+\alpha_1}}. \qquad (22)$$

In the Type III 2HDM, when $\alpha_1 \simeq 0$, $\alpha_3 \simeq \pi/2$ and $t_\beta \simeq 1$, the coefficients in Eq. 3 are reduced to

$$c_V \simeq -s_{\alpha_2}, \qquad \text{and} \qquad c_t \simeq -s_{\alpha_2} - \mathrm{i} c_{\alpha_2} = -\mathrm{e}^{\mathrm{i}(\pi/2-\alpha_2)}. \qquad (23)$$

Thus α_2 is a key parameter measuring CP-violation in the (pseudo)scalar sector.

B.   Implications of CP-violation in the 2HDM

If we choose a scenario with the aforementioned cancellations in the electron EDM which allows larger CP-violation angle α_2, we have the expected correlation between the asymmetry A and the total cross section σ_{tot}, as discussed in Sec. II C. Both A and σ_{tot} depends only on the parameter α_2 for a given m_H (and hence the electron EDM cancellation condition will fix $t_\beta \simeq 1$). In this scenario, the $Ht\bar{t}$ coupling is dominated by the CP-odd component and thus the expected asymmetry A should be close to the case with pure CP-odd $Ht\bar{t}$ coupling. The $H \to Zh$ decay channel is negligible here.

In this section, we choose the W^+W^--fusion channel, at CLIC with $\sqrt{s} = 3$ TeV and 10 ab^{-1} luminosity, as above. In the left plot of Fig. 6, we show the predicted asymmetry A depending on the total cross section σ_{tot} in this scenario of a 2HDM with $\pm 1\sigma, \pm 2\sigma, \pm 3\sigma$ uncertainties, together with the prediction from the CP-conserving case for comparison. In the right plot of Fig. 6, we show the discovery potential of CP-violation depending on the total cross section σ_{tot} if an asymmetry A equalling the 2HDM prediction is observed. For the left plot, if an observed (σ_{tot}, A)

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[9] As shown in [25], the difference comes from the neutron EDM calculation. An accidental partial cancellation in the Type III model makes the constraints from the neutron EDM much weaker than in the Type II model.

FIG. 6: In the left plot: we show the predicted asymmetry A versus the total cross section σ_{tot} in the chosen 2HDM with $\pm 1\sigma$ (green), $\pm 2\sigma$ (yellow) and $\pm 3\sigma$ (blue) uncertainties. The thick black line shows the central value of the 2HDM prediction. We also show the CP-conserving prediction as the thick blue line as a comparison. In the right plot: we show the discovery potential of CP-violation versus the total cross section σ_{tot} if an asymmetry A equal to the 2HDM prediction is observed.
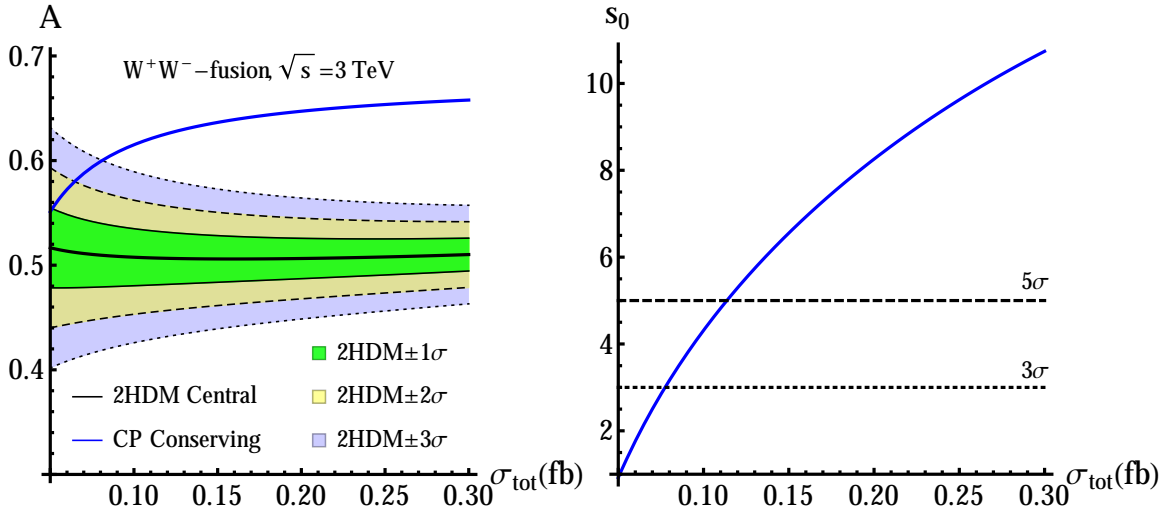
point is located outside the yellow (blue) boundaries, it will mean that the 2HDM scenario we discuss here is excluded at 95%(99.7%) Confidence Level (C.L.) and thus this 2HDM scenario is disfavored. And, if an asymmetry A equal to the prediction by this 2HDM scenario is observed, meaning this 2HDM scenario is favored, we show the discovery potential of CP-violation in the right plot. We can discover CP-violation at $3(5)\sigma$ level if $\sigma_{\text{tot}} \gtrsim 0.08(0.11)$ fb, corresponding to $|\alpha_2| \gtrsim 0.096(0.14)$. Finally, for the largest allowed $|\alpha_2| \simeq 0.3$, corresponding to $\sigma_{\text{tot}} \simeq 0.3$ fb, we can discover CP-violation at the 10.7σ level.

IV. CONCLUSIONS AND DISCUSSION

In this paper, we propose a novel method to test CP-violation in a (pseudo)scalar sector characterized by a heavy scalar boson H with couplings to the heavy gauge bosons and a complex coupling to the top quark. We have studied the physics potential of an electron-positron collider at $\sqrt{s} = 1.5$ and 3 TeV, such as CLIC. At such high energies, the production of a heavy scalar boson is dominated by W^+W^-- and ZZ-fusion. We choose the process in which the heavy scalar H is produced through VBF channel, and decays to $t\bar{t}$ pair. In our method, the CP-even component of H is confirmed through the HVV coupling, while the CP-odd component of H should be confirmed through the CP-odd $Ht\bar{t}$ coupling. The CP nature of $Ht\bar{t}$ coupling is tested through the spin

correlation between t and $\bar{t}$, which is sensitive to the distribution of the azimuthal angle between the leptons decaying from t and $\bar{t}$ quarks.

In our study, we found that the ZZ-fusion channel suffers from the SM background and cannot provide large enough significance to see the effect of CP-violation even under the most favorable scenario of CP-violation at $\sqrt{s} = 1.5$ or 3 TeV. In contrast, the W^+W^--fusion channel provides a reasonable separation of pure CP-even and CP-odd $Ht\bar{t}$ coupling at $\sqrt{s} = 1.5$ TeV and the significant difference (more than 5σ) between the CP-even and CP-odd $Ht\bar{t}$ coupling can be seen at $\sqrt{s} = 3$ TeV CLIC with 10 ab^{-1} luminosity, under a favorable scenario of CP-violation. The physics potential is summarized in Fig. 5, in which one can see that a pure CP-odd $Ht\bar{t}$ coupling can be discovered at 5σ level for $\sigma_{\text{tot}} \simeq 0.11$ fb (corresponding to $c_V \simeq 0.14$ if assuming $|c_t| = 1$), and it can be stretched to 11.5σ for $\sigma_{\text{tot}} \simeq 0.3$ fb (corresponding to the largest allowed $c_V \simeq 0.3$ if assuming $|c_t| = 1$).

Implications for the 2HDM with CP-violation in the Higgs sector were also studied. Type III model affords a fairly large CP-violating angle α_2, such that this scenario can be analyzed similarly to what we did for the model-independent approach. The results are summarized in Fig. 6. Eventually, we showed that at $\sqrt{s} = 3$ TeV CLIC with 10 ab^{-1} luminosity, the 2HDM Type III with a favorable CP-violating set-up can be discovered at 5σ level when $\sigma_{\text{tot}} \simeq 0.11$ fb (corresponding to $|\alpha_2| \simeq 0.14$), and it can be stretched to 10.7σ when $\sigma_{\text{tot}} \simeq 0.3$ fb (corresponding to the largest allowed $|\alpha_2| \simeq 0.3$).

In short, an electron-positron collider operating in the multi-TeV energy range, such as CLIC, is a useful apparatus to study CP-violation effects in the (pseudo)scalar Higgs sector by using VBF production (through the charged current channel) of a heavy Higgs state decaying into a $t\bar{t}$ pair, in turn yielding two (prompt) leptons. We have come to this conclusion by performing a sophisticated Monte Carlo (MC) analysis, albeit limited to the parton level, however, we are confident that our results can be replicated at the full detector level, given that they are driven by inclusive and exclusive observables solely exploiting electron and muon kinematics.

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