

Tri-hypercharge: a separate gauged weak hypercharge for each fermion family as the origin of flavour

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ABSTRACT: We propose a tri-hypercharge (TH) extension of the Standard Model (SM) in which a separate gauged weak hypercharge is associated with each fermion family. In this way, every quark and lepton multiplet carries unique gauge quantum numbers under the extended gauge group, providing the starting point for a theory of flavour. If the Higgs doublets only carry third family hypercharge, then only third family renormalisable Yukawa couplings are allowed. However, non-renormalisable Yukawa couplings may be induced by the high scale Higgs fields (hyperons) which break the three hypercharges down to the SM hypercharge, providing an explanation for fermion mass hierarchies and the smallness of CKM quark mixing. Following a similar methodology, we study the origin of neutrino masses and mixing in this model. Due to the TH gauge symmetry, the implementation of a seesaw mechanism naturally leads to a low scale seesaw, where the right-handed neutrinos in the model may be as light as the TeV scale. We present simple examples of hyperon fields which can reproduce all quark and lepton (including neutrino) masses and mixing. After a preliminary phenomenological study, we conclude that one of the massive Z' bosons can be as light as a few TeV, with implications for flavour-violating observables, LHC physics and electroweak precision observables.

Contents

1	Introduction	1
2	The Tri-hypercharge gauge theory	3
3	Charged fermion masses and mixing	4
3.1	Lessons from the spurion formalism	4
3.2	From spurions to hyperons	6
3.3	Model 1: Minimal case with three hyperons	8
3.4	Model 2: Five hyperons for a more predictive setup	10
4	Neutrino masses and mixing	11
4.1	General considerations and spurion formalism	11
4.2	Example of successful neutrino mixing from the seesaw mechanism	12
5	Phenomenology	14
5.1	Couplings of the heavy Z' bosons to fermions	14
5.2	The high scale boson Z'_{12}	15
5.3	The low scale boson Z'_{23}	17
6	Conclusions	20
A	General formalism for the seesaw mechanism	21
B	High scale symmetry breaking	23
C	Low scale symmetry breaking	24

1 Introduction

The existence of the three families of quarks and leptons is one of the fundamental mysteries of the Standard Model (SM). Indeed many of the parameters of the SM are associated with the resulting quark and lepton mass patterns and mixings. The low energy masses of quarks and charged leptons may be expressed approximately as [1]

$$m_t \sim \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_c \sim \lambda^{3.3} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_u \sim \lambda^{7.5} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad (1.1)$$

$$m_b \sim \lambda^{2.5} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_s \sim \lambda^{5.0} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_d \sim \lambda^{7.0} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad (1.2)$$

$$m_\tau \sim \lambda^{3.0} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_\mu \sim \lambda^{4.9} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad m_e \sim \lambda^{8.4} \frac{v_{\text{SM}}}{\sqrt{2}}, \quad (1.3)$$

where $v_{\text{SM}} \simeq 246$ GeV is the SM vacuum expectation value (VEV) and $\lambda \simeq 0.224$ is the Wolfenstein parameter which parameterises the CKM matrix as

$$V_{us} \sim \lambda, \quad V_{cb} \sim \lambda^2, \quad V_{ub} \sim \lambda^3. \quad (1.4)$$

The hierarchical patterns of masses and CKM mixing, and the possibility that they might be understood in the form of a more fundamental *theory of flavour* beyond the SM, has classically been denoted as the *flavour puzzle*. The discovery of neutrino oscillations, which proves that at least two neutrinos have non-zero mass, has made the flavour puzzle difficult to ignore, enlarging the flavour sector with extra neutrino mixing angles [2, 3]

$$\tan \theta_{23} \sim 1, \quad \tan \theta_{12} \sim \frac{1}{\sqrt{2}}, \quad \theta_{13} \sim \frac{\lambda}{\sqrt{2}}, \quad (1.5)$$

plus very tiny neutrino masses which follow either a normal or inverted ordering. The need of a theory of flavour for fermion masses and mixings is now more urgent than ever.

A successful theory of flavour usually involves new gauge structures beyond the SM group, broken at some high scale down to the SM. Traditionally, such gauge structures have been considered to be flavour universal, such as grand unified theories (GUTs) which embed all three families in an identical way, extended GUTs which embed all three families, along with extra exotic fermions, or family non-universal theories involving family symmetries which commute with the SM gauge group, and are then spontaneously broken, leading to family structure. However, there are other less explored ways in which the SM gauge group could be embedded into a larger gauge structure in a flavour non-universal way. In particular, *non-universal family decomposition* of the SM gauge group (including a hierarchical symmetry breaking pattern down to the SM) was first proposed during the 80s and 90s, with the purpose of motivating lepton non-universality [4–7] or assisting technicolor model building [8–10]. However, the natural origin of flavour hierarchies in such a framework was not highlighted until more recently in [11, 12]. Here it was proposed that the flavour non-universality of Yukawa couplings in the SM might well find its origin in a flavour non-universal gauge sector, broken in a hierarchical way down to the SM, where such a hierarchy between scales might find a

natural origin in extra dimensional setups [13]. Interestingly, model building in this direction has received particular attention in recent years [14–19]. With the exception of Ref. [19], the remaining recent attempts have been motivated by the need to obtain a TeV scale vector leptoquark from Pati-Salam unification¹ in order to address the so-called B -anomalies. Therefore, all these setups share a similar feature: a low scale $SU(4)$ gauge group under which only the third family of SM fermions transforms in a non-trivial way. Instead, here we want to explore the capabilities of flavour non-universality to address the flavour puzzle in a more minimal, simple and bottom-up approach.

In this paper, we propose that the SM gauge group is embedded at high energy into a larger gauge group which includes three separate weak hypercharge gauge groups,

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}, \quad (1.6)$$

which we will denote as the *tri-hypercharge* (TH) $U(1)_Y^3$ gauge group. We will associate each of the three hypercharge gauge groups with a separate SM family, such that each fermion family i only carries hypercharge under a corresponding $U(1)_{Y_i}$ factor. This ensures that each family transforms differently under the gauge group $U(1)_Y^3$, which avoids the family repetition of the SM, and provides the starting point for a theory of flavour. For example, assuming that a single Higgs doublet only carries third family hypercharge, then only the third family Yukawa couplings are allowed at renormalisable level. With two Higgs doublets carrying third family hypercharge, we show that the naturalness of the scheme increases. This simple and economical framework naturally explains the heaviness of the third family, the smallness of V_{cb} and V_{ub} , and delivers an accidental and global $U(2)^5$ flavour symmetry acting on the light families, which is known to provide a good first order description of the SM spectrum plus an efficient suppression of flavour-violating effects for new physics [23]. Remarkably, this appears to be the simplest way to provide the $U(2)^5$ flavour symmetry². The masses of first and second family fermions, along with the CKM mixing, then appear as *soft* $U(2)^5$ -breaking terms that arise after the cascade spontaneous symmetry breaking of $U(1)_Y^3$ down to SM hypercharge, which can be parameterised in a model-independent way in terms of spurions. In a realistic model, the spurions will be realised by a choice of “hyperon” scalars which transform under the different family hypercharge groups, breaking the tri-hypercharge symmetry.

The paper is organised as follows. In Section 2 we introduce the TH gauge group, along with the fermion and Higgs content of the model. We discuss the implications for third family fermion masses along with the mass hierarchy between the top and bottom/tau fermions. In Section 3 we study the origin of charged fermion masses and mixing in the TH model, firstly via a spurion formalism which reveals model-independent considerations, and secondly by introducing example models with hyperons. In Section 4 we study the origin of neutrino masses and mixing. In particular, we discuss the impact of the $U(2)^5$ flavour symmetry over the dimension-5 Weinberg operator, and afterwards we provide an example seesaw model where neutrino masses and mixing can be accommodated. In Section 5 we perform a preliminary exploration of the phenomenological implications and discovery prospects of the $U(1)_Y^3$ theory of flavour. Finally, Section 6 outlines our main conclusions.

¹Remarkably, flavour non-universality is not the only way to connect the TeV-scale Pati-Salam vector leptoquark addressing the B -anomalies with the origin of flavour hierarchies, see the flavour universal theory of flavour in [20–22].

²An alternative (less simple) way to deliver $U(2)^5$ consists of decomposing $SU(2)_L$ and either the vector-like or right-handed components of hypercharge (see the complete review of Ref. [24]). Another example [25] considered an extension of the SM by a $U(1)_{Y_3}$ gauge group under which only third family fermions (and the Higgs) are hypercharge-like charged, where $U(1)_{Y_3}$ commutes with SM hypercharge, leading to an accidental $U(2)^5$.

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_{Y_1}$	$U(1)_{Y_2}$	$U(1)_{Y_3}$
Q_1	$\mathbf{3}$	$\mathbf{2}$	1/6	0	0
u_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	-2/3	0	0
d_1^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	1/3	0	0
L_1	$\mathbf{1}$	$\mathbf{2}$	-1/2	0	0
e_1^c	$\mathbf{1}$	$\mathbf{1}$	1	0	0
Q_2	$\mathbf{3}$	$\mathbf{2}$	0	1/6	0
u_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	-2/3	0
d_2^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	1/3	0
L_2	$\mathbf{1}$	$\mathbf{2}$	0	-1/2	0
e_2^c	$\mathbf{1}$	$\mathbf{1}$	0	1	0
Q_3	$\mathbf{3}$	$\mathbf{2}$	0	0	1/6
u_3^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	-2/3
d_3^c	$\bar{\mathbf{3}}$	$\mathbf{1}$	0	0	1/3
L_3	$\mathbf{1}$	$\mathbf{2}$	0	0	-1/2
e_3^c	$\mathbf{1}$	$\mathbf{1}$	0	0	1

Table 1: Charge assignments of the SM fermions under the TH gauge group. Q_i and L_i (where $i = 1, 2, 3$) are left-handed (LH) $SU(2)_L$ doublets of chiral quarks and leptons, while u_i^c , d_i^c and e_i^c are the CP conjugated right-handed (RH) quarks and leptons (so that they become left-handed).

2 The Tri-hypercharge gauge theory

The TH extended gauge group is based on assigning a separate gauged weak hypercharge to each fermion family,

$$SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3}, \quad (2.1)$$

in such a way that the i th fermion family only carries Y_i hypercharge, with the other hypercharges set equal to zero (see Table 1), where $Y = Y_1 + Y_2 + Y_3$ is equal to SM weak hypercharge. Anomalies cancel separately for each family, as in the SM, but without family replication. The TH gauge group is broken down to the SM via appropriate SM singlet scalars, which however carry family hypercharges. We denote these fields linking the family hypercharges as *hyperons*. A common assumption in non-universal theories of flavour is that the link scalars develop hierarchical VEVs, in such a way that the family specific gauge groups are broken in a cascade symmetry breaking down to the SM. In particular, we motivate the following symmetry breaking pattern

$$\begin{aligned}
& SU(3)_c \times SU(2)_L \times U(1)_{Y_1} \times U(1)_{Y_2} \times U(1)_{Y_3} \\
& \xrightarrow{v_{12}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3} \\
& \xrightarrow{v_{23}} SU(3)_c \times SU(2)_L \times U(1)_{Y_1+Y_2+Y_3}.
\end{aligned} \quad (2.2)$$

We will see that a mild hierarchy $v_{12} > v_{23}$ can play a role for accommodating the fermion mass hierarchies. Moreover, this setup has the advantage that the massive Z' arising from the 12-breaking is naturally heavier, suppressing the dangerous 1-2 flavour-changing neutral currents (FCNCs).

Instead, the Z' arising from 23-breaking can be lighter, leading to a more interesting low energy phenomenology and discovery prospects, as we will see in Section 5.

Provided that the SM Higgs only carries third family hypercharge $H(\mathbf{1}, \mathbf{2})_{(0,0,-\frac{1}{2})}$, then only the third family Yukawa couplings are allowed at renormalisable level and an accidental $U(2)^5$ flavour symmetry acting on the light families emerges,

$$\mathcal{L} = y_t Q_3 \tilde{H} u_3^c + y_b Q_3 H d_3^c + y_\tau L_3 H e_3^c + \text{h.c.} \quad (2.3)$$

where \tilde{H} is the CP conjugate of H . This setup already provides an explanation of the smallness of light fermions masses with respect to the third family, along with the smallness of quark mixing, as they all must arise from non-renormalisable operators which provide small $U(2)^5$ -breaking effects. Although this is a good first order description of the SM spectrum, the question of why the bottom and tau fermions are much lighter than the top remains unanswered, and assuming only a single Higgs doublet, a tuning of order 2% for the bottom coupling and of 1% for the tau coupling would be required. Given that $m_{s,\mu} \propto \lambda^5 m_t$ while $m_c \propto \lambda^3 m_t$, this setup also requires to generate a stronger fermion hierarchy in the down and charged lepton sectors with respect to the up sector, unless the tuning in the bottom and tau couplings is extended to the second family. As we shall see shortly, the $U(1)_Y^3$ model (and very likely a more general set of non-universal theories of flavour) predicts a similar mass hierarchy for all charged sectors.

Due to the above considerations, it seems natural to consider a type II two Higgs doublet model (2HDM), where both Higgs doublets only carry third family hypercharge,

$$H_u(\mathbf{1}, \mathbf{2})_{(0,0,\frac{1}{2})}, \quad H_d(\mathbf{1}, \mathbf{2})_{(0,0,-\frac{1}{2})}, \quad (2.4)$$

where as usual for a type II 2HDM, FCNCs can be forbidden by either a softly broken Z_2 discrete symmetry or by supersymmetry (not necessarily low scale), which we however do not specify in order to preserve the bottom-up spirit of this work. In any case, $\tan\beta = v_u/v_d \sim \lambda^{-2} \approx 20$, which is compatible with current data (see e.g. [26, 27]), will provide the hierarchy between the top and bottom/tau masses with all dimensionless couplings being natural. Such an overall hierarchy between the down and charged lepton sector with respect to the up sector is extended to all families, providing a better description of second family charged fermion masses as we shall see.

3 Charged fermion masses and mixing

3.1 Lessons from the spurion formalism

In all generality, we introduce $U(2)^5$ -breaking *spurions* Φ in the Yukawa matrices of charged fermions

$$\begin{aligned} \mathcal{L} = & \left(Q_1 \ Q_2 \ Q_3 \right) \begin{pmatrix} \Phi(\frac{1}{2}, 0, -\frac{1}{2}) & \Phi(-\frac{1}{6}, \frac{2}{3}, -\frac{1}{2}) & \Phi(-\frac{1}{6}, 0, \frac{1}{6}) \\ \Phi(\frac{2}{3}, -\frac{1}{6}, -\frac{1}{2}) & \Phi(0, \frac{1}{2}, -\frac{1}{2}) & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ \Phi(\frac{2}{3}, 0, -\frac{2}{3}) & \Phi(0, \frac{2}{3}, -\frac{2}{3}) & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u \\ & + \left(Q_1 \ Q_2 \ Q_3 \right) \begin{pmatrix} \Phi(-\frac{1}{2}, 0, \frac{1}{2}) & \Phi(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2}) & \Phi(-\frac{1}{6}, 0, \frac{1}{6}) \\ \Phi(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2}) & \Phi(0, -\frac{1}{2}, \frac{1}{2}) & \Phi(0, -\frac{1}{6}, \frac{1}{6}) \\ \Phi(-\frac{1}{3}, 0, \frac{1}{3}) & \Phi(0, -\frac{1}{3}, \frac{1}{3}) & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d \end{aligned} \quad (3.1)$$

$$+ \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(-\frac{1}{2}, 0, \frac{1}{2}) & \Phi(\frac{1}{2}, -1, \frac{1}{2}) & \Phi(\frac{1}{2}, 0, -\frac{1}{2}) \\ \Phi(-1, \frac{1}{2}, \frac{1}{2}) & \Phi(0, -\frac{1}{2}, \frac{1}{2}) & \Phi(0, \frac{1}{2}, -\frac{1}{2}) \\ \Phi(-1, 0, 1) & \Phi(0, -1, 1) & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d + \text{h.c.},$$

where each spurion carries non-trivial charge assignments under $U(1)_Y^3$. In an effective field theory (EFT) approach, each spurion above can be matched to specific ratios of hyperons ϕ_i over EFT cut-off scales Λ_i , i.e.

$$\Phi = \frac{\phi_1 \dots \phi_n}{\Lambda_1 \dots \Lambda_n}, \quad (3.2)$$

where we have suppressed dimensionless couplings. The choice of hyperons and $\Lambda(s)$ above carries all the model dependence.

Assuming that the cut-off scales Λ_i of the EFT are universal, i.e. that all Λ_i (s) are common to all charged sectors, then the spurion formalism reveals some general considerations about the origin of charged fermion masses and mixing:

- The same spurions (up to conjugation) appear in the diagonal entries of all matrices. Therefore, unless texture zeros are introduced in specific models, this means that the masses of second family fermions are likely to be degenerate up to dimensionless couplings, and the same discussion applies to first family fermions. This motivates again the addition of the second Higgs doublet or an alternative mechanism in order to generate the hierarchy between the charm mass and the lighter strange and muon masses.
- The same spurions appear in the (2,3) entries of the up and down matrices. Therefore, the 2-3 mixing in both the up and down sectors is expected to be of similar size, giving no predictions about the alignment of the CKM element V_{cb} . The similar argument applies to 1-3 mixing and V_{ub} .
- The spurions in the (1,2) entry of the up and down matrices are different. Therefore, specific models have the potential to give predictions about the alignment of the CKM element V_{us} .
- The same spurion (up to conjugation) that enters in all (2,2) entries also populates the (2,3) entry of the charged lepton Yukawa matrix. Similarly, the same spurion (up to conjugation) that enters in all (1,1) entries also populates the (1,3) entry of the charged lepton Yukawa matrix. In general, this predicts left-handed $\mu - \tau$ ($e - \tau$) mixing of $\mathcal{O}(m_2/m_3)$ ($\mathcal{O}(m_1/m_3)$), unless texture zeros are introduced in specific models (see Section 3.2). This leads to a sizable enhancement of LFV $\tau \rightarrow \mu$ and $\tau \rightarrow e$ transitions above the SM predictions, mediated by heavy Z' bosons in the model (see Section 5).
- The spurions in the lower off-diagonal entries of the matrices all carry independent charge assignments, so right-handed fermion mixing is highly model-dependent and can be different in all charged sectors.

In the following, we go beyond the spurion formalism and introduce different sets of hyperons. As we shall see, the hyperons will provide small $U(2)^5$ -breaking effects via non-renormalisable operators, leading to the masses of first and second family charged fermions, along with CKM mixing. In the next few subsections, we will describe simple scenarios which provide a good description of charged fermion masses and mixing.

3.2 From spurions to hyperons

The physical origin of the spurions of the previous subsection will correspond to new Higgs scalar fields that break the $U(1)_Y^3$ symmetry, which we call hyperons. The hyperons induce small $U(2)^5$ -breaking effects at the non-renormalisable level that will lead to the masses and mixings of charged fermions. As the most straightforward scenario, we could promote the spurions in the diagonal entries of the matrices in Eq. (3.1) to hyperons, along with the off-diagonal spurions in the upper half of the down matrix³. In an EFT approach, the set of hyperons that we have assume will eventually generate the following Yukawa matrices,

$$\mathcal{L}^{d \leq 5} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \phi_{\ell 13}^{(\frac{1}{2}, 0, -\frac{1}{2})} & 0 & \phi_{q 13}^{(-\frac{1}{6}, 0, \frac{1}{6})} \\ 0 & \phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})} & \phi_{q 23}^{(0, -\frac{1}{6}, \frac{1}{6})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{H_u}{\Lambda} \quad (3.3)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2}, 0, \frac{1}{2})} & \phi_{d 12}^{(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2})} & \phi_{q 13}^{(-\frac{1}{6}, 0, \frac{1}{6})} \\ 0 & \tilde{\phi}_{\ell 23}^{(0, -\frac{1}{2}, \frac{1}{2})} & \phi_{q 23}^{(0, -\frac{1}{6}, \frac{1}{6})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \frac{H_d}{\Lambda} \quad (3.4)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} \tilde{\phi}_{\ell 13}^{(-\frac{1}{2}, 0, \frac{1}{2})} & 0 & \phi_{\ell 13}^{(\frac{1}{2}, 0, -\frac{1}{2})} \\ 0 & \tilde{\phi}_{\ell 23}^{(0, -\frac{1}{2}, \frac{1}{2})} & \phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \frac{H_d}{\Lambda}. \quad (3.5)$$

where the universal scale Λ is the high cut-off of the EFT and we ignore the $\mathcal{O}(1)$ dimensionless couplings of each entry. Although we have chosen only the specific set of hyperons shown, leaving some zeros in the matrices, these zeros may be filled in by higher order operators with dimension larger than 5, which however we are neglecting

When the hyperons develop VEVs, assumed to be smaller than the cut-off scale Λ , then each entry of the matrix will receive a suppressed numerical effective coupling given by $\langle \phi \rangle / \Lambda$, whose values can be assumed arbitrarily. Having the freedom to choose arbitrary VEVs for each spurion, the Yukawa matrices above could provide a good first order description of charged fermion masses and CKM mixing. We choose to fix the ratios of VEV over Λ in terms of powers of the Wolfenstein parameter $\lambda \simeq 0.224$, obtaining

$$\mathcal{L} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \lambda^6 & 0 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.6)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \lambda^6 & \lambda^4 & \lambda^3 \\ 0 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.7)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} \lambda^6 & 0 & \lambda^5 \\ 0 & \lambda^3 & \lambda^3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}}. \quad (3.8)$$

³Notice that the same spurions enter in both the (1,3) and (2,3) entries of the up and down matrices of Eq. (3.1).

As anticipated from the spurion formalism, the alignment of V_{cb} and V_{ub} is not predicted by the model. In contrast, the model predicts a relevant LH $\mu - \tau$ ($e - \tau$) mixing connected to the same hyperon that provides the second family (first family) effective Yukawa coupling. Thanks to the addition of the second Higgs doublet, the model successfully explains third and second family fermion masses with $\mathcal{O}(1)$ dimensionless couplings. The down-quark and electron mass are also reasonably explained, although the up-quark mass is a factor $\mathcal{O}(\lambda^{1.5})$ larger than expected. Notice that so far we are only assuming one universal cut-off scale Λ , while in realistic models several cut-off scales Λ may be obtained from e.g. vector-like fermions in the UV theory, which could provide a larger suppression for the up-quark effective coupling. Therefore, within the limitations of our EFT approach, the description of charged fermion masses given by the set of Eqs. (3.6-3.8) is very successful.

As another example, one could also consider a model where the (1,1) spurion in Eq. (3.1) is not promoted to hyperon, but instead all the spurions in the (1,2) and (2,1) entries are promoted, so that the Yukawa matrices look like

$$\mathcal{L}^{d \leq 5} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} 0 & \phi_{u12}^{(-\frac{1}{6}, \frac{2}{3}, -\frac{1}{2})} & \phi_{q13}^{(-\frac{1}{6}, 0, \frac{1}{6})} \\ \phi_{u21}^{(\frac{2}{3}, -\frac{1}{6}, -\frac{1}{2})} & \phi_{\ell23}^{(0, \frac{1}{2}, -\frac{1}{2})} & \phi_{q23}^{(0, -\frac{1}{6}, \frac{1}{6})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{H_u}{\Lambda} \quad (3.9)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} 0 & \phi_{d12}^{(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2})} & \phi_{q13}^{(-\frac{1}{6}, 0, \frac{1}{6})} \\ \phi_{d21}^{(-\frac{1}{3}, -\frac{1}{6}, \frac{1}{2})} & \tilde{\phi}_{\ell23}^{(0, -\frac{1}{2}, \frac{1}{2})} & \phi_{q23}^{(0, -\frac{1}{6}, \frac{1}{6})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \frac{H_d}{\Lambda} \quad (3.10)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} 0 & \phi_{e12}^{(\frac{1}{2}, -1, \frac{1}{2})} & 0 \\ \phi_{e21}^{(-1, \frac{1}{2}, \frac{1}{2})} & \tilde{\phi}_{\ell23}^{(0, -\frac{1}{2}, \frac{1}{2})} & \phi_{\ell23}^{(0, \frac{1}{2}, -\frac{1}{2})} \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \frac{H_d}{\Lambda}. \quad (3.11)$$

The VEV over Λ assignments of the new hyperons can be fixed by the requirement of addressing first family fermion masses, obtaining Yukawa matrices with texture zeros given by

$$\mathcal{L} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} 0 & \lambda^5 & \lambda^3 \\ \lambda^{5.5} & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.12)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} 0 & \lambda^4 & \lambda^3 \\ \lambda^4 & \lambda^3 & \lambda^2 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.13)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} 0 & \lambda^5 & 0 \\ \lambda^{4.4} & \lambda^3 & \lambda^3 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}}, \quad (3.14)$$

which provide an even better description of first family fermion masses than the original simplified model. Notice that in this scenario, a sizable LH $e - \tau$ mixing is no longer predicted.

We conclude that the most straightforward choices of hyperons, motivated by the spurion

formalism, already provide a good description of charged fermion masses and mixings. However, these simplified models leave some questions unanswered. Given that we are assuming the symmetry breaking highlighted in Eq. (2.2), we notice that there are no hyperons breaking the first and second hypercharges down to their diagonal subgroup, and we would expect those to play some role in the origin of fermion hierarchies and mixing. Moreover, in the simplified models introduced so far, several hyperons display unexplained large hierarchies of VEVs whose values are assumed *a posteriori* to fit the fermion masses. Given that all these hyperons participate in the 23-breaking step of Eq. (2.2), we would expect all of them to develop VEVs at a similar scale, rather than the hierarchical scales assumed. This motivates further model building. In the following subsections we discuss a couple of example models which address these issues.

3.3 Model 1: Minimal case with three hyperons

We introduce here the following set of three hyperons,

$$\phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})}, \quad \phi_{q 23}^{(0, -\frac{1}{6}, \frac{1}{6})}, \quad \phi_{q 12}^{(-\frac{1}{6}, \frac{1}{6}, 0)}. \quad (3.15)$$

Following the EFT approach of the previous subsection, we now analyse the effective Yukawa textures obtained by combining the SM charged fermions, the Higgs doublets and the hyperons, in a tower of non-renormalisable operators preserving the $U(1)_Y^3$ gauge symmetry,

$$\mathcal{L} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \tilde{\phi}_{q 12}^3 \phi_{\ell 23} & \phi_{q 12} \phi_{\ell 23} & \phi_{q 12} \phi_{q 23} \\ \tilde{\phi}_{q 12}^4 \phi_{\ell 23} & \phi_{\ell 23} & \phi_{q 23} \\ \tilde{\phi}_{q 12}^4 \phi_{\ell 23} \tilde{\phi}_{q 23} & \phi_{\ell 23} \tilde{\phi}_{q 23} & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u \quad (3.16)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \phi_{q 12}^3 \tilde{\phi}_{\ell 23} & \phi_{q 12} \tilde{\phi}_{\ell 23} & \phi_{q 12} \phi_{q 23} \\ \phi_{q 12}^2 \tilde{\phi}_{\ell 23} & \tilde{\phi}_{\ell 23} & \phi_{q 23} \\ \phi_{q 12}^2 \phi_{q 23}^2 & \phi_{q 23}^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d \quad (3.17)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} \phi_{q 12}^3 \tilde{\phi}_{\ell 23} & \tilde{\phi}_{q 12}^3 \tilde{\phi}_{\ell 23} & \tilde{\phi}_{q 12}^3 \phi_{\ell 23} \\ \phi_{q 12}^6 \tilde{\phi}_{\ell 23} & \tilde{\phi}_{\ell 23} & \phi_{\ell 23} \\ \phi_{q 12}^6 \tilde{\phi}_{\ell 23}^2 & \tilde{\phi}_{\ell 23}^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d, \quad (3.18)$$

where the powers of Λ in the denominator and the dimensionless couplings of each entry are not shown. Once the hyperons above develop VEVs, we obtain very economical and efficient Yukawa textures for modeling the observed pattern of SM Yukawa couplings. In particular, the mass of second family fermions arise at dimension 5 in the EFT, while first family masses have an extra suppression as they arise from dimension 8 operators. Regarding CKM mixing, 2-3 quark mixing leading to V_{cb} arises from dimension 5 operators, while V_{ub} has an extra mild suppression as it arises from dimension 6 operators. In all cases, right-handed fermion mixing is suppressed with respect to left-handed mixing. This is a highly desirable feature, given the strong phenomenological constraints on right-handed flavour-changing currents [28, 29], which may be mediated by heavy Z' bosons arising from the symmetry breaking of $U(1)_Y^3$.

In good approximation, quark mixing leading to V_{us} arises as the ratio of the (1,2) and (2,2) entries of the quark matrices above, therefore we expect

$$\frac{\langle \phi_{q 12} \rangle}{\Lambda} \sim V_{us} \simeq \lambda, \quad (3.19)$$

where $\lambda = \sin \theta_C \simeq 0.224$. In a similar manner, we can fix the ratio $\langle \phi_{q23} \rangle / \Lambda$ by reproducing the observed V_{cb}

$$\frac{\langle \phi_{q23} \rangle}{\Lambda} \sim V_{cb} \simeq \lambda^2. \quad (3.20)$$

Given that both $\langle \phi_{q23} \rangle$ and $\langle \phi_{\ell 23} \rangle$ play a role in the last step of the symmetry breaking cascade (see Eq. (2.2)), it is expected that both VEVs live at the same scale. This way, we choose,

$$\frac{\langle \phi_{\ell 23} \rangle}{\Lambda} \sim \lambda^{2.5}, \quad (3.21)$$

which, given that $\langle H_d \rangle$ provides an extra suppression of $\mathcal{O}(\lambda^2)$ for down and charged lepton Yukawas, allows to predict all second family masses with natural dimensionless couplings. In contrast to the simplified models of Section 3.2, this model provides all the 23-breaking VEVs at a similar scale, plus a larger 12-breaking VEV, following a mild hierarchy given by $v_{23}/v_{12} \sim \lambda$. This way, the symmetry breaking of the $U(1)_Y^3$ gauge group proceeds just like in Eq. (2.2).

Having fixed all the hyperon VEVs with respect to Λ , now we are able to write the full mass matrices for each sector in terms of the Wolfenstein parameter λ ,

$$\mathcal{L} = \begin{pmatrix} u_1 & u_2 & u_3 \end{pmatrix} \begin{pmatrix} \lambda^{5.5} & \lambda^{3.5} & \lambda^3 \\ \lambda^{6.5} & \lambda^{2.5} & \lambda^2 \\ \lambda^{8.5} & \lambda^{4.5} & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.22)$$

$$+ \begin{pmatrix} d_1 & d_2 & d_3 \end{pmatrix} \begin{pmatrix} \lambda^{5.5} & \lambda^{3.5} & \lambda^3 \\ \lambda^{4.5} & \lambda^{2.5} & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.23)$$

$$+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^{5.5} & \lambda^{5.5} & \lambda^{5.5} \\ \lambda^{8.5} & \lambda^{2.5} & \lambda^{2.5} \\ \lambda^{11} & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}}. \quad (3.24)$$

We can see that this setup provides a reasonable description of charged fermion masses and mixing. Although the up-quark mass seems a bit large, we remember that we are only assuming one universal cut-off scale Λ , while in realistic models several scales Λ may be provided by e.g. vector-like fermions in the UV theory, as discussed Section 3.2. All things considered, the description of fermion masses seems very efficient, considering the limitations of our EFT framework.

However, the model does not predict the alignment of V_{us} . Moreover, we also notice that RH $s - d$ mixing is just mildly suppressed as $s_{12}^{dR} \simeq \mathcal{O}(\lambda^2)$ in this model. Given the stringent bounds over left-right scalar operators contributing to $K^0 - \bar{K}^0$ meson mixing [28, 29] (which will arise in our model as we shall see in Section 5), the scale v_{12} can be pushed far above the TeV if V_{us} originates from the down sector. From a phenomenological point of view, it would be interesting to find models which give clear predictions about the alignment of V_{us} , and ideally provide a more efficient suppression of RH quark mixing. We shall see in the next subsection that this can be achieved by minimally extending the set of hyperons of this model.

3.4 Model 2: Five hyperons for a more predictive setup

Model 1 proposed in the previous section, despite its simplicity and minimality, does not give clear predictions about the alignment of the CKM matrix. Heavy Z' bosons arising from the symmetry breaking of $U(1)_Y^3$ have the potential to mediate contributions to $K^0 - \bar{K}^0$ meson mixing, which could set a lower bound over the scale of $U(1)_Y^3$, but such contributions depend on the alignment of V_{us} . Moreover, the largest contributions to $K^0 - \bar{K}^0$ mixing depend both on the alignment of V_{us} and on RH $s - d$ mixing, which is just mildly suppressed in Model 1. Therefore, we propose here a similar model with slightly extended hyperon content that can account for a clear prediction about the alignment of V_{us} , plus a more efficient suppression of RH fermion mixing. We consider here the hyperons

$$\phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})}, \quad \phi_{q 23}^{(0, -\frac{1}{6}, \frac{1}{6})}, \quad \phi_{q 13}^{(-\frac{1}{6}, 0, \frac{1}{6})}, \quad \phi_{d 12}^{(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{2})}, \quad \phi_{e 12}^{(\frac{1}{4}, -\frac{1}{4}, 0)}. \quad (3.25)$$

With this set of hyperons, the effective Yukawa couplings in the EFT are (suppressing as usual powers of Λ and dimensionless couplings)

$$\mathcal{L} = (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \phi_{e 12}^2 \tilde{\phi}_{\ell 23} & \phi_{q 13} \tilde{\phi}_{q 23} \phi_{\ell 23} & \phi_{q 13} \\ \phi_{e 12}^2 \tilde{\phi}_{q 13} \phi_{q 23} \phi_{\ell 23} & \phi_{\ell 23} & \phi_{q 23} \\ \phi_{e 12}^2 \tilde{\phi}_{q 13} \phi_{\ell 23} & \phi_{\ell 23} \tilde{\phi}_{q 23} & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} H_u \quad (3.26)$$

$$+ (Q_1 \ Q_2 \ Q_3) \begin{pmatrix} \phi_{e 12}^2 \tilde{\phi}_{\ell 23} & \phi_{d 12} & \phi_{q 13} \\ \phi_{q 13}^2 \phi_{q 23} & \tilde{\phi}_{\ell 23} & \phi_{q 23} \\ \phi_{q 13}^2 & \phi_{q 23}^2 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} H_d \quad (3.27)$$

$$+ (L_1 \ L_2 \ L_3) \begin{pmatrix} \tilde{\phi}_{e 12}^2 \tilde{\phi}_{\ell 23} & \phi_{e 12}^2 \tilde{\phi}_{\ell 23} & \phi_{e 12}^2 \phi_{\ell 23} \\ \tilde{\phi}_{e 12}^4 \tilde{\phi}_{\ell 23} & \tilde{\phi}_{\ell 23} & \phi_{\ell 23} \\ \tilde{\phi}_{e 12}^4 \tilde{\phi}_{\ell 23}^2 & \tilde{\phi}_{\ell 23}^2 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} H_d. \quad (3.28)$$

Following the same approach as with Model 1, we assign the following powers of λ to the VEV over Λ ratios in order to reproduce fermion masses and CKM mixing,

$$\frac{\langle \phi_{\ell 23} \rangle}{\Lambda} = \frac{\langle \phi_{q 13} \rangle}{\Lambda} \simeq \lambda^3, \quad \frac{\langle \phi_{q 23} \rangle}{\Lambda} \simeq \lambda^2, \quad \frac{\langle \phi_{d 12} \rangle}{\Lambda} \simeq \lambda^4, \quad \frac{\langle \phi_{e 12} \rangle}{\Lambda} \simeq \lambda. \quad (3.29)$$

Although it would seem that in this scenario there exists a mild hierarchy between 23 VEVs of $\mathcal{O}(\lambda^2)$, since $\phi_{d 12}$ only appears in the 12 entry of the down matrix, it would be very reasonable that the dimensionless coupling in that entry provides a factor λ suppression, such that all 23-breaking VEVs live at the same scale. The largest VEV is still the 12-breaking one, which is now associated to the hyperon $\phi_{e 12}$, and the mild hierarchy between scales remains as $v_{12}/v_{23} \simeq \lambda$. With these assignments of VEVs over Λ , the Yukawa textures are given by

$$\mathcal{L} = (u_1 \ u_2 \ u_3) \begin{pmatrix} \lambda^5 & \lambda^8 & \lambda^3 \\ \lambda^{10} & \lambda^3 & \lambda^2 \\ \lambda^7 & \lambda^5 & 1 \end{pmatrix} \begin{pmatrix} u_1^c \\ u_2^c \\ u_3^c \end{pmatrix} \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.30)$$

$$+ (d_1 \ d_2 \ d_3) \begin{pmatrix} \lambda^5 & \lambda^4 & \lambda^3 \\ \lambda^8 & \lambda^3 & \lambda^2 \\ \lambda^6 & \lambda^4 & 1 \end{pmatrix} \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}} \quad (3.31)$$

$$+ \begin{pmatrix} e_1 & e_2 & e_3 \end{pmatrix} \begin{pmatrix} \lambda^5 & \lambda^5 & \lambda^5 \\ \lambda^7 & \lambda^3 & \lambda^3 \\ \lambda^{10} & \lambda^6 & 1 \end{pmatrix} \begin{pmatrix} e_1^c \\ e_2^c \\ e_3^c \end{pmatrix} \lambda^2 \frac{v_{\text{SM}}}{\sqrt{2}}. \quad (3.32)$$

Just like in Model 1, this model provides a compelling description of all charged fermion masses and mixing. Notice that this scenario provides a very efficient suppression of RH fermion mixing. Moreover, it is clear that here V_{us} mixing originates from the down sector, providing a more predictive setup than Model 1, which will be useful phenomenological purposes as discussed in Section 5.

4 Neutrino masses and mixing

4.1 General considerations and spurion formalism

The origin of neutrino masses and mixing requires a dedicated analysis due to their particular properties. We start by introducing $U(2)^5$ -breaking spurions (carrying inverse of mass dimension) for the Weinberg operator

$$\mathcal{L}_{\text{Weinberg}} = \begin{pmatrix} L_1 & L_2 & L_3 \end{pmatrix} \begin{pmatrix} \Phi(1, 0, -1) & \Phi(\frac{1}{2}, \frac{1}{2}, -1) & \Phi(\frac{1}{2}, 0, -\frac{1}{2}) \\ \Phi(\frac{1}{2}, \frac{1}{2}, -1) & \Phi(0, 1, -1) & \Phi(0, \frac{1}{2}, -\frac{1}{2}) \\ \Phi(\frac{1}{2}, 0, -\frac{1}{2}) & \Phi(0, \frac{1}{2}, -\frac{1}{2}) & 1 \end{pmatrix} \begin{pmatrix} L_1 \\ L_2 \\ L_3 \end{pmatrix} H_u H_u, \quad (4.1)$$

which reveals that, as expected, the $U(2)^5$ flavour symmetry is also present in the neutrino sector. As a consequence, generally one expects one neutrino to be much heavier than the others, displaying tiny mixing with the other neutrinos. In the spirit of the seesaw mechanism, one could think of adding a $U(1)_Y^3$ singlet neutrino as $N(0, 0, 0)$. Such a singlet neutrino can only couple to the third family active neutrino at renormalisable level, i.e. $\mathcal{L}_N \supset L_3 H_u N + m_N N N$, where all fermion fields are written in a left-handed convention. This way, the coupling $L_2 H_u N$, which is required for large atmospheric neutrino mixing, necessarily arises from non-renormalisable operators. Therefore, it is expected to be suppressed with respect to $L_3 H_u N$. This seems to be inconsistent with large atmospheric neutrino mixing, at least within the validity of our EFT framework. As anticipated before, this is a consequence of the accidental $U(2)^5$ flavour symmetry delivered by the TH model.

Given such general considerations, we conclude that in order to obtain neutrino masses and mixing from the seesaw mechanism, it is required to add SM singlet neutrinos that carry tri-hypercharges (but whose hypercharges add up to zero). The latter have to be vector-like in order to cancel gauge anomalies. As a consequence, they can obtain their mass from the unspecified vector-like mass terms and from the VEVs of hyperons in the specific model.

Remarkably, if some SM singlet neutrino N_{atm} carrying non-trivial hypercharges provides the mass terms $\mathcal{L}_N \supset L_3 H_u N_{\text{atm}} + L_2 H_u \bar{N}_{\text{atm}}$, as required to explain atmospheric mixing, then the conjugate neutrino will always couple to L_3 as $L_3 H_u \bar{N}_{\text{atm}}$, but not necessarily to L_2 . As shown in Appendix A, when the terms $L_3 H_u \bar{N}_{\text{atm}}$ enter the seesaw mechanism, they lead to a hierarchical effective neutrino mass matrix proportional to the vector-like mass terms, while the terms involving SM singlet neutrinos like N_{atm} lead to an anarchic neutrino mass matrix proportional to the VEVs of the hyperons. Therefore, the simplest way to explain the observed pattern of anarchic neutrino mixing requires that the vector-like terms are of similar or smaller order than the VEVs of the

hyperons, leading to a *low scale* seesaw mechanism if the VEVs of the hyperons are not very large (which is consistent with current data as discussed in Section 5).

4.2 Example of successful neutrino mixing from the seesaw mechanism

In the following, we provide an example scenario which reproduces the observed patterns of neutrino mixing, as a proof of principle. According to the discussion in the previous subsection, in order to implement a seesaw mechanism that delivers large neutrino mixing, we need to add vector-like neutrinos that carry tri-hypercharges (but whose hypercharges add up to zero). We also need to introduce hyperons that will provide small Dirac mass terms for the active neutrinos in the form of non-renormalisable operators. Under these considerations, we start by adding the following vector-like neutrino⁴ and hyperon

$$N_{\text{atm}}^{(0, \frac{1}{4}, -\frac{1}{4})}, \quad \bar{N}_{\text{atm}}^{(0, -\frac{1}{4}, \frac{1}{4})}, \quad \phi_{\text{atm}}^{(0, \frac{1}{4}, -\frac{1}{4})} \quad (4.2)$$

where the charge assignments are chosen to provide large *atmospheric neutrino mixing*. This way, we can write the following non-renormalisable operators along with the Majorana and vector-like mass of N_{atm} ,

$$\begin{aligned} \mathcal{L}_{N_{\text{atm}}} \supset & \frac{1}{\Lambda_{\text{atm}}} (\phi_{\text{atm}} L_2 + \tilde{\phi}_{\text{atm}} L_3) H_u N_{\text{atm}} + \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} L_3 H_u \bar{N}_{\text{atm}} \\ & + \phi_{\ell 23} N_{\text{atm}} N_{\text{atm}} + \tilde{\phi}_{\ell 23} \bar{N}_{\text{atm}} \bar{N}_{\text{atm}} + M_{N_{\text{atm}}} \bar{N}_{\text{atm}} N_{\text{atm}}, \end{aligned} \quad (4.3)$$

where we have ignored the $\mathcal{O}(1)$ dimensionless couplings, and the hyperon $\phi_{\ell 23}^{(0, \frac{1}{2}, -\frac{1}{2})}$ is already present in both Model 1 and Model 2 for the charged fermion sector. In a similar spirit, we introduce another vector-like neutrino and hyperons in order to obtain large *solar neutrino mixing*

$$N_{\text{sol}}^{(\frac{1}{4}, \frac{1}{4}, -\frac{1}{2})}, \quad \bar{N}_{\text{sol}}^{(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{2})}, \quad \phi_{\text{sol}}^{(-\frac{1}{2}, -\frac{1}{2}, 1)}, \quad \phi_{\nu 13}^{(-\frac{1}{4}, -\frac{1}{4}, \frac{1}{2})} \quad (4.4)$$

which provide the following non-renormalisable operators and mass terms,

$$\begin{aligned} \mathcal{L}_{N_{\text{sol}}} \supset & \frac{1}{\Lambda_{\text{sol}}} (\phi_{e 12} L_1 + \tilde{\phi}_{e 12} L_2 + \phi_{\nu 13} L_3) H_u N_{\text{sol}} + \frac{\phi_{\nu 13}}{\Lambda_{\text{sol}}} L_3 H_u \bar{N}_{\text{sol}} \\ & + \phi_{\text{sol}} N_{\text{sol}} N_{\text{sol}} + \tilde{\phi}_{\text{sol}} \bar{N}_{\text{sol}} \bar{N}_{\text{sol}} + M_{N_{\text{sol}}} \bar{N}_{\text{sol}} N_{\text{sol}}, \end{aligned} \quad (4.5)$$

where we have ignored again the $\mathcal{O}(1)$ dimensionless couplings, and the hyperon $\phi_{e 12}^{(\frac{1}{2}, -\frac{1}{2}, 0)}$ is already present in Model 2 for the charged fermion sector. The hyperon $\phi_{\nu 13}$ (which eventually will populate the (1,3) entry of the effective neutrino mass matrix) is not necessarily connected to reactor mixing, which would already arise from the other operators, but it is required in order to have enough free parameters to fit all observed neutrino mixing angles and mass splittings.

Notice that the vector-like neutrinos N_{atm} and N_{sol} get contributions to their masses from the VEVs of the hyperons $\phi_{\ell 23}$ and ϕ_{sol} , respectively, which we denote generically as v_{23} since they both take part in the 23-breaking step of Eq. (2.2). In addition, N_{atm} and N_{sol} get contributions to their

⁴Beware that we keep working in our left-handed convention, where both fermion fields N_{atm} and \bar{N}_{atm} are left-handed.

masses from the unspecified vector-like mass terms, that we generically denote as M_{VL} . As shown in Appendix A (see Eq. (A.5)), given that the conjugate neutrinos \bar{N}_{atm} and \bar{N}_{sol} only couple to the third family, the seesaw formula reveals that the effective neutrino mass matrix m_ν receives two main contributions:

- A contribution proportional to v_{23} which populates all entries of m_ν with $\mathcal{O}(1)$ terms.
- A contribution proportional to the vector-like masses M_{VL} , which populates only the third row and column entries of m_ν with $\mathcal{O}(1)$ terms and the others are zero.

Therefore, if $M_{\text{VL}} \gg v_{23}$, then in good approximation m_ν will have only the third row and column being non-zero, which is inconsistent with the observed pattern of neutrino mixing and mass splittings. Instead, if $M_{\text{VL}} \lesssim v_{23}$, then the contribution proportional v_{23} dominates the seesaw mechanism. In this case, the conjugate neutrinos \bar{N}_{atm} and \bar{N}_{sol} become irrelevant for the seesaw mechanism, and m_ν can be obtained by considering only the presence of the SM singlet neutrinos N_{atm} and N_{sol} and applying the seesaw formula. We construct the Dirac and Majorana matrices (ignoring $\mathcal{O}(1)$ dimensionless couplings) as

$$m_D = \begin{pmatrix} N_{\text{sol}} & N_{\text{atm}} \\ L_1 | \frac{\phi_{e12}}{\Lambda_{\text{sol}}} & 0 \\ L_2 | \frac{\phi_{e12}}{\Lambda_{\text{sol}}} & \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} \\ L_3 | \frac{\phi_{\nu 13}}{\Lambda_{\text{sol}}} & \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} \end{pmatrix} H_u, \quad M_N = \begin{pmatrix} N_{\text{sol}} & N_{\text{atm}} \\ N_{\text{sol}} | \frac{\phi_{\text{sol}}}{\Lambda_{\text{sol}}} & 0 \\ N_{\text{atm}} | 0 & \phi_{\ell 23} \end{pmatrix}. \quad (4.6)$$

We could have included the $U(1)_Y^3$ singlet neutrino $N(0,0,0)$, but as discussed in the previous subsection, ultimately its contributions to the Weinberg operator are always suppressed by its indefinite mass m_N , having little implications for the seesaw mechanism. We therefore assume that such neutrino, if exists, is in any case decoupled from the seesaw, as in sequential dominance [30, 31]. After applying the seesaw formula, we obtain the Weinberg operator (ignoring again $\mathcal{O}(1)$ dimensionless couplings) as

$$m_\nu = m_D M_N^{-1} m_D^T = \begin{pmatrix} \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{e12}^2}{\phi_{\text{sol}}} & \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{e12} \tilde{\phi}_{e12}}{\phi_{\text{sol}}} & \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{e12} \phi_{\nu 13}}{\phi_{\text{sol}}} \\ \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{e12} \tilde{\phi}_{e12}}{\phi_{\text{sol}}} & \frac{1}{\Lambda_{\text{sol}}^2} \frac{\tilde{\phi}_{e12}^2}{\phi_{\text{sol}}} + \frac{1}{\Lambda_{\text{atm}}^2} \frac{\phi_{\text{atm}}^2}{\phi_{\ell 23}} & \frac{1}{\Lambda_{\text{atm}}^2} \frac{\phi_{\text{atm}} \phi_{\text{atm}}}{\phi_{\ell 23}} + \frac{1}{\Lambda_{\text{sol}}^2} \frac{\tilde{\phi}_{e12} \phi_{\nu 13}}{\phi_{\text{sol}}} \\ \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{e12} \phi_{\nu 13}}{\phi_{\text{sol}}} & \frac{1}{\Lambda_{\text{atm}}^2} \frac{\phi_{\text{atm}} \tilde{\phi}_{e12}}{\phi_{\ell 23}} + \frac{1}{\Lambda_{\text{sol}}^2} \frac{\tilde{\phi}_{e12} \phi_{\nu 13}}{\phi_{\text{sol}}} & \frac{1}{\Lambda_{\text{atm}}^2} \frac{\phi_{\text{atm}}^2}{\phi_{\ell 23}} + \frac{1}{\Lambda_{\text{sol}}^2} \frac{\phi_{\nu 13}^2}{\phi_{\text{sol}}} \end{pmatrix} H_u H_u. \quad (4.7)$$

Motivated by our discussion of the charged fermion sector (see Section 3), we assume that $\langle \phi_{e12} \rangle \simeq v_{12}$ and $\langle \phi_{\ell 23} \rangle \approx \langle \phi_{\text{atm}} \rangle \approx \langle \phi_{\text{sol}} \rangle \simeq v_{23}$, with v_{12} and v_{23} related by the mild hierarchy $v_{23}/v_{12} \simeq \lambda$. This way, we obtain

$$m_\nu \simeq \begin{pmatrix} \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda^2} & \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda^2} & \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda} \\ \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda^2} & 1 + \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda^2} & 1 + \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda} \\ \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda} & 1 + \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda} & 1 + \frac{\Lambda_{\text{atm}}^2}{\Lambda_{\text{sol}}^2} \frac{1}{\lambda} \end{pmatrix} v_{23} \frac{H_u H_u}{\Lambda_{\text{atm}}^2}. \quad (4.8)$$

If $\Lambda_{\text{sol}} = \Lambda_{\text{atm}}$, we observe that there exists a mild hierarchy of order λ^2 between the 12 and 23 sectors in the matrix above. Considering the dimensionless coefficients that we have ignored so far,

the numerical diagonalisation of m_ν would require some parameters of $\mathcal{O}(0.01)$ in order to explain the observed neutrino mixing angles and mass splittings [2, 3]. This description can be further improved if we assume a mild hierarchy between cut-off scales $\Lambda_{\text{atm}}/\Lambda_{\text{sol}} \simeq \lambda$, obtaining to leading order for each entry (and ignoring dimensionless coefficients),

$$m_\nu \simeq \begin{pmatrix} 1 & 1 & \lambda \\ 1 & 1 & 1 \\ \lambda & 1 & 1 \end{pmatrix} v_{23} \frac{v_{\text{SM}}^2}{\Lambda_{\text{atm}}^2}, \quad (4.9)$$

where we have introduced the SM VEV as $\langle H_u \rangle = v_{\text{SM}}$ (neglecting the factor $1/\sqrt{2}$). Considering now the dimensionless coefficients in the matrix above, we find that numerical diagonalisation can accommodate all the observed neutrino mixing angles and mass splittings [2, 3] with $\mathcal{O}(1)$ parameters, and we are able to reproduce both normal and inverted ordered scenarios.

Notice that we have been driven to a scenario where the vector-like neutrinos get Majorana masses from the VEVs of hyperons in the model. Furthermore, the vector-like masses necessarily have to be of the same or smaller order than the VEVs of the hyperons in order to explain the observed pattern of neutrino mixing. Therefore, in the particular example included in this section, the vector-like neutrinos get a mass at the scale v_{23} of the 23-breaking step in Eq. (2.2), which could happen at a relatively low scale as we shall see in Section 5. As a consequence, the vector-like neutrinos involved in the seesaw mechanism are expected to be relatively light, and the high energy cut-offs of the EFT Λ_{atm} and Λ_{sol} are expected to provide most the suppression of light neutrino masses. As anticipated before, due to the $U(2)^5$ flavour symmetry provided by the TH model, we have been driven to a low scale seesaw in order to predict the observed pattern of neutrino mixing.

5 Phenomenology

5.1 Couplings of the heavy Z' bosons to fermions

In Sections 3 and 4, we have discussed examples of $U(1)_Y^3$ models which provide a compelling description of all fermion masses and mixings, and we have highlighted model-independent features which are intrinsic to the $U(1)_Y^3$ framework. Under well-motivated arguments, we have assumed that the symmetry breaking pattern of the TH group down to the SM is described by Eq. (2.2), in such a way that at a high scale v_{12} , the group $U(1)_{Y_1} \times U(1)_{Y_2}$ is broken down to its diagonal subgroup. The remaining group $U(1)_{Y_1+Y_2} \times U(1)_{Y_3}$ is broken down to SM hypercharge at a lower scale v_{23} . In specific models, a mild hierarchy of scales $v_{23}/v_{12} \simeq \lambda$ plays a role on the origin of flavour hierarchies.

A massive gauge boson Z'_{12} is predicted to live at the higher scale v_{12} , displaying *intrinsically* flavour non-universal couplings to the first two generations of SM fermions. Similarly, another massive boson Z'_{23} lives at the lower scale v_{23} . The pattern of symmetry breaking is such that Z'_{23} has flavour universal couplings to first and second family fermions, while the couplings to the third family are intrinsically different. In the following, we include the coupling matrices in family space (from the covariant derivatives in Appendices B and C), ignoring fermion mass mixing,

$$\mathcal{L}_{Z'_{12}} \supset Y_{\psi_{L,R}} \bar{\psi}_{L,R} \gamma^\mu \begin{pmatrix} -g_1 \sin \theta_{12} & 0 & 0 \\ 0 & g_2 \cos \theta_{12} & 0 \\ 0 & 0 & 0 \end{pmatrix} \psi_{L,R} Z'_{12\mu}, \quad \sin \theta_{12} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}, \quad (5.1)$$

$$\mathcal{L}_{Z'_{23}} \supset Y_{\psi_{L,R}} \bar{\psi}_{L,R} \gamma^\mu \begin{pmatrix} -g_{12} \sin \theta_{23} & 0 & 0 \\ 0 & -g_{12} \sin \theta_{23} & 0 \\ 0 & 0 & g_3 \cos \theta_{23} \end{pmatrix} \psi_{L,R} Z'_{23\mu}, \quad \sin \theta_{23} = \frac{g_{12}}{\sqrt{g_{12}^2 + g_3^2}}, \quad (5.2)$$

where $Y_{\psi_{L,R}}$ is the SM hypercharge of $\psi_{L,R}$ ⁵, where ψ is a 3-component column vector containing the three families $\psi = u^i, d^i, e^i, \nu^i$. Explicitly, $\psi_L = u_L^i, d_L^i, e_L^i, \nu_L^i$ with $Y_{\psi_L} = 1/6, 1/6, -1/2, -1/2$, and $\psi_R = u_R^i, d_R^i, e_R^i$ with $Y_{\psi_R} = 2/3, -1/3, -1$, respectively, ignoring couplings to the SM singlet neutrinos discussed in the previous section⁶.

Including fermion mass mixing, we would have $\psi_{L,R} = V_{\psi_{L,R}} \hat{\psi}_{L,R}$ with $\hat{\psi}_{L,R}$ containing the mass eigenstates and $V_{\psi_{L,R}}$ being the *non-generic* mixing matrices obtained after diagonalising the Yukawa matrices for a given model. Notice that the couplings to *right-handed* fermions are expected to be larger since their hypercharges are generally larger in magnitude than left-handed fermions. The SM hypercharge gauge coupling $g_Y(M_Z) \simeq 0.36$ is entangled to the g_i couplings via the relations

$$g_Y = \frac{g_1 g_3}{\sqrt{g_{12}^2 + g_3^2}}, \quad g_{12} = \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}}. \quad (5.3)$$

The expressions above reveal a lower bound on the gauge couplings $g_i \gtrsim g_Y$. Throughout this work we have considered a bottom-up approach where the $U(1)_Y^3$ model is just the next step in our understanding of nature, which reveals information about the origin of flavour, but nevertheless is an EFT remnant of a more UV complete theory. In this spirit, we have studied the RGE evolution of the gauge couplings g_i , obtaining that for $g_i(\text{TeV}) \simeq 1$ the model can be extrapolated to the Planck scale (and beyond). Instead, for $g_i(\text{TeV}) \simeq 2$, a Landau pole is found at a scale $\mathcal{O}(10^4 \text{ TeV})$, which anyway seems like a reasonable scale for a UV embedding, given that we expect the cut-off scale of the neutrino EFT (see Section 4) to be around $\mathcal{O}(10^6 \text{ TeV})$ in order to explain the tiny neutrino masses. Therefore, in order to protect the perturbativity of the model, we avoid to consider $g_i > 2$ in the phenomenological analysis. Nevertheless, we highlight a natural scenario where the three gauge couplings have a similar size $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3} g_Y$, which could be connected to a possible *gauge unification*. This benchmark is depicted as a dashed horizontal line in Figs. 1 and 2.

5.2 The high scale boson Z'_{12}

In any implementation of the $U(1)_Y^3$ model, Z'_{12} is expected to mediate sizable tree-level transitions between first and second generation left-handed quarks, either in the up or down sector depending on the alignment of the CKM matrix predicted by the specific model. Furthermore, our analysis in Section 3 reveals that $U(1)_Y^3$ models generally predict non-vanishing charged lepton mixing and RH quark mixing. This way, contributions to $K^0 - \bar{K}^0$ and $D^0 - \bar{D}^0$ meson mixing [28, 29], along with CLFV processes such as $\mu \rightarrow e \gamma$ [1], have the potential to push the scale v_{12} far above the TeV.

Being more specific, for Model 1 described in Section 3.3, we find the stringent bounds over v_{12} to come from the scalar and coloured operator $(\bar{s}_L^\alpha d_R^\beta)(\bar{s}_R^\beta d_L^\alpha)$ obtained after integrating out Z'_{12} at tree-level, which contributes to $K^0 - \bar{K}^0$ mixing. Model 1 predicts the up and down left-handed mixing to be similar up to dimensionless couplings, which must therefore play some role in the

⁵Note that we have departed from our purely LH convention, used in the rest of the paper, to use instead a LH and RH convention, which is more familiar in phenomenological studies.

⁶Note that such low scale SM singlet neutrinos may be observable at colliders via their gauge couplings to Z'_{23} , which can be obtained from the covariant derivative in Eq. (C.23).

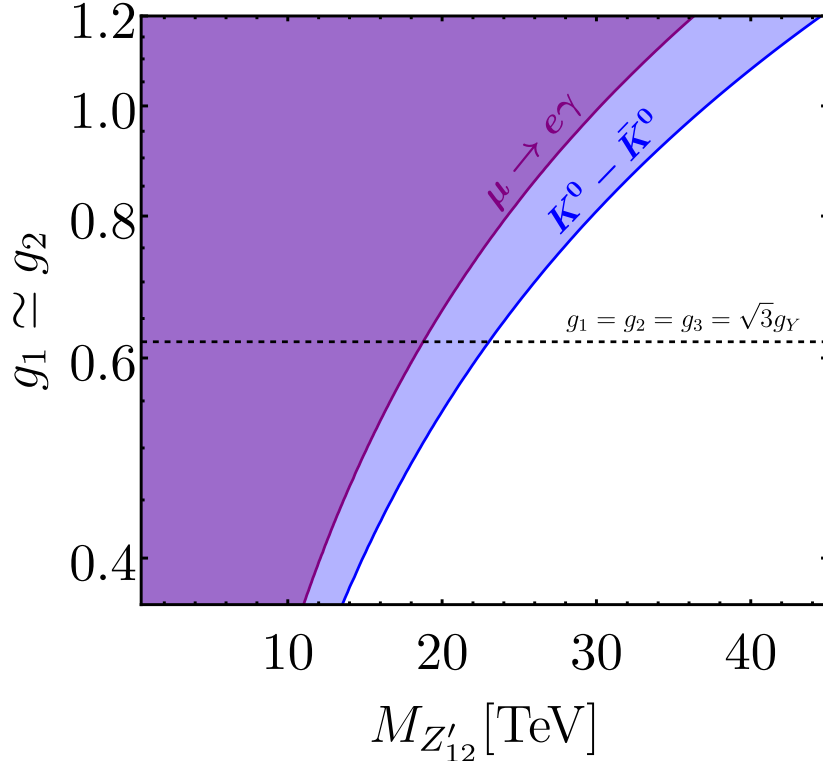


Figure 1: Parameter space of the high scale breaking, where $M_{Z'_{12}}$ is the mass of the heavy Z'_{12} gauge boson and g_1, g_2 are the gauge couplings of the $U(1)_{Y_1}$ and $\tilde{U}(1)_{Y_2}$ groups, respectively. For simplicity, we assume g_1 and g_2 to be similar, and the non-generic fermion mixing predicted by Model 2 in Section 3.4. Shaded regions in the plot depict 95% CL exclusions over the parameter space. The dashed line represents the natural benchmark $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3}g_Y$ motivated in the main text.

alignment of the CKM. In either case, mildly suppressed RH $s-d$ mixing $s_{12}^{dR} \simeq \mathcal{O}(\lambda^2)$ is predicted. If V_{us} originates mostly from the down sector, then $K^0 - \bar{K}^0$ mixing imposes the stringent bound $M_{Z'_{12}} > 170$ TeV for gauge couplings of $\mathcal{O}(0.5)$. Instead, if the dimensionless coupling provides a mild suppression of $\mathcal{O}(0.1)$ in left-handed $s-d$ mixing, such that V_{us} originates mostly from the up sector, then the bound is relaxed to $M_{Z'_{12}} > 55$ TeV. We find bounds from $D^0 - \bar{D}^0$ mixing to be always weaker, even if V_{us} originates from the up sector, since RH up mixing is strongly suppressed.

In contrast with Model 1, Model 2 described in Section 3.4 provides a more predictive scenario where V_{us} originates unambiguously from the down sector. Here RH quark mixing is more suppressed, obtaining $s_{12}^{dR} \simeq \mathcal{O}(\lambda^5)$. Nevertheless, $K^0 - \bar{K}^0$ mixing still imposes the strongest bounds over the parameter case, as can be seen in Fig. 1. In this case, the lower bound over the mass of Z'_{12} can be as low as 10-50 TeV, depending on the values of the gauge couplings. We find the CLFV process $\mu \rightarrow e\gamma$ to provide a slightly weaker bound over the parameter space, because charged lepton mixing is generally suppressed with respect to quark mixing. We find the bound from $\mu \rightarrow 3e$ to be very similar to the bound from $\mu \rightarrow e\gamma$.

5.3 The low scale boson Z'_{23}

Given that the high scale symmetry breaking can be as low as 10-20 TeV for reasonable values of the gauge couplings, and considering the hierarchy of scales $v_{23}/v_{12} \simeq \lambda$, it is possible to find the low scale breaking v_{23} near the TeV scale. Since Z'_{23} features universal couplings to the first and second families, the stringent bounds from $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow e\gamma$ are avoided, in the spirit of the *GIM mechanism*. This way, Z'_{23} can live at the TeV scale, within the reach of LHC and future colliders.

Any implementation of the $U(1)_Y^3$ model predicts small *mixing* between Z'_{23} and the SM Z boson given by the mixing angle (see Appendix C)

$$\sin\theta_{Z-Z'_{23}} = \frac{g_3 \cos\theta_{23}}{\sqrt{g_Y^2 + g_L^2}} \left(\frac{M_Z^0}{M_{Z'_{23}}^0} \right)^2, \quad (5.4)$$

where M_Z^0 and $M_{Z'_{23}}^0$ are the masses of the Z and Z'_{23} bosons in the absence of mixing, respectively, and g_L is the gauge coupling of $SU(2)_L$. This mixing leads to a *small shift* on the mass of the Z boson, which has an impact on the so-called ρ parameter

$$\rho = \frac{M_W^2}{M_Z^2 \cos^2 \theta_W} = \frac{1}{1 - g_3^2 \cos^2 \theta_{23} \left(\frac{v_{\text{SM}}}{2M_{Z'}^0} \right)^2}, \quad (5.5)$$

which is predicted as $\rho = 1$ in the SM. The fact that M_Z is always shifted to smaller values leads to $\rho > 1$. Given that M_Z is commonly an input experimental parameter of the SM used in the determination of g_Y and g_L , the downward shift of M_Z with respect to the SM prediction would be seen from the experimental point of view as an upward shift of M_W with respect to the SM prediction. Nevertheless, the experimental picture of M_W is puzzling after the recent measurement by CDF [32]. This measurement points towards M_W being larger than the SM prediction with high significance, but it is in tension with the combination of measurements by LHC, LEP and Tevatron D0 [1]. Neglecting the recent CDF measurement for the moment, current data⁷ provides $\rho = 1.0003 \pm 0.0005$ [1] (assuming both the oblique parameters T and S are non-zero, as we expect in our model). We obtain the approximate bound $g_3/M_{Z'} < 3.1$ TeV at 95% CL, which translates to an approximate bound over the mixing angle of $\sin\theta_{Z-Z'_{23}} < 0.001$.

$Z - Z'_{23}$ mixing also shifts the couplings of the Z boson to fermions, leading to an important impact over Z -pole electroweak precision observables (EWPOs) if Z'_{23} lives at the TeV scale. We find bounds coming from tests of Z boson lepton universality and flavour-violating Z decays to be not competitive with the bound from ρ . The electron asymmetry parameter A_e , which already deviates from the SM by almost 2σ [36], is expected to deviate further in our model. Nevertheless, we expect our model to improve the fit of A_b^{FB} , which is in tension with the SM prediction by more than 2σ [36]. In conclusion, the global effect of our model over EWPOs can only be captured by performing a global fit, which we leave for future work. In this direction, a dedicated phenomenological analysis along the lines of [21, 37] would be much needed. Global fits of EWPOs in the context of other

⁷The current world average (without the latest CDF measurement) of M_W does not consider the very recent M_W update by ATLAS [33]. Given that the central value and the uncertainty of this measurement are just slightly reduced with respect to the 2017 measurement [34], we do not expect a big impact over the world average.

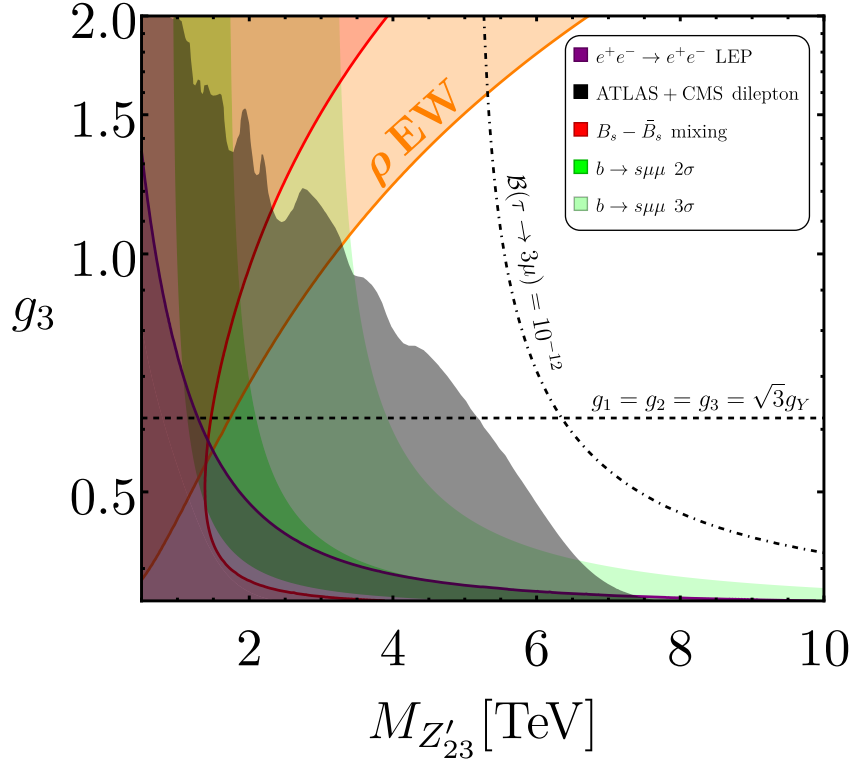


Figure 2: Parameter space of the low scale breaking, where $M_{Z'_{23}}$ is the mass of the heavy Z'_{23} gauge boson and g_3 is the gauge coupling of the $U(1)_{Y_3}$ group. The gauge coupling g_{12} is fixed in terms of g_3 and g_Y via Eq. (5.3), and we consider the non-generic fermion mixing predicted by Model 2 in Section 3.4. Shaded regions in the plot depict 95% CL exclusions over the parameter space, with the exception of the green (light green) region which is preferred by a global fit to $b \rightarrow s\mu\mu$ data at 2σ (3σ) [35]. The dashed line represents the natural benchmark $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3}g_Y$ motivated in the main text. The dashed-dotted line represents the contour where $\mathcal{B}(\tau \rightarrow 3\mu) = 10^{-12}$.

Z' models have been performed in the literature, see e.g. [38–40], which obtain 95% CL maximum values of $\sin\theta_{Z-Z'}$ ranging from 0.002 to 0.0006 depending on the model. We expect our model to lie on the more restrictive side of that range. In principle, our model can explain the anomalous CDF M_W measurement if it is confirmed, however in this case we expect the contributions to other EWPOs to be intolerably large, worsening the global fit.

The massive boson Z'_{23} has sizable couplings to light quarks and light charged leptons unless g_3 is very large, which we do not expect based on naturalness arguments and also to protect the extrapolation of the model in the UV, as mentioned before. Consequently, on general grounds we expect a large production of a TeV-scale Z'_{23} at LHC, plus a large branching fraction to electrons and muons. We have prepared the UFO model of Z'_{23} using `Feynrules` [41], and then we have computed the Z'_{23} production cross section from 13 TeV pp collisions using `Madgraph5` [42] with the default PDF `NNPDF23L0`. We estimated analytically the branching fraction to electrons and muons, and we computed the total decay width via the narrow width approximation. We confront our results with the limits from the most recent dilepton resonance searches by ATLAS [43] and CMS [44] in order to obtain 95% CL exclusion bounds. The bounds from ditau [45] and ditop [46] searches turn out

Z'_{23} decay mode	\mathcal{B}
$\bar{t}t$	~ 0.28
$\bar{u}u + \bar{c}c$	~ 0.14
$\bar{t}c + \bar{c}t$	$\sim 10^{-4}$
$\bar{b}b$	~ 0.08
$\bar{d}d + \bar{s}s$	~ 0.04
$\bar{b}s + \bar{s}b$	$\sim 10^{-4}$
$\tau^+\tau^-$	~ 0.25
$e^+e^- = \mu^+\mu^-$	~ 0.06
$\tau^+\mu^- + \tau^-\mu^+$	$\sim 10^{-5}$
$\bar{\nu}\nu$	~ 0.08

Table 2: Main decay modes of Z'_{23} for the natural benchmark $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3}g_Y$. We assume that decays into heavy SM singlet neutrinos are kinematically forbidden or suppressed.

to be not competitive even for the region of large g_3 , where the bound from the ρ EW parameter is stronger. Our results are depicted as the black shaded region in Fig. 2. As expected, the bounds become weaker in the region $g_3 > 1$ where the couplings to light fermions become mildly suppressed. In the region of small g_3 we find the opposite behavior, such that LHC limits can exclude Z'_{23} as heavy as 6-7 TeV. After combining the LHC exclusion with the bounds coming from the ρ EW parameter, we conclude that we can find Z'_{23} as light as 3.4 TeV for $g_3 = 1$.

Given that Z'_{23} has sizable couplings to electrons, we have studied the bounds over contact interactions obtained at LEP [47]. For our model, the most competitive bounds arise from contact interactions involving only electrons. Assuming vector-like interactions, the bounds by LEP are only sensitive to regions with very small g_3 , but can exclude Z'_{23} masses beyond 10 TeV. However, we expect this bound to be slightly overestimated for our model, since the interactions of Z'_{23} are not exactly vector-like due to the different hypercharge of e_L and e_R , as depicted in Eq. (5.2). Nevertheless, the bounds over chiral operators are much weaker than the bound over the vector-like operator, and a dedicated reanalysis of the data would be required in order to obtain the proper bound for our model, which is beyond the scope of this work. Therefore, we prefer to be conservative and depict the largest bound of the vector-like operator as the purple region in Fig. 2.

We have also consider implications for B -physics. The heavy boson Z'_{23} has a sizable left-handed $b_L s_L$ coupling and an approximately vector-like and universal coupling to electron and muon pairs. Given these features, a Z'_{23} with a mass of 2 TeV mediates a meaningful contribution to the effective operator $\mathcal{O}_9^{\ell\ell}$ (with $\ell = e, \mu$), where sizable NP contributions are preferred according to the most recent global fits [35], without contributing to the SM-like $R_{K^{(*)}}$ ratios [48]. However, as depicted in Fig. 2, the region where the model could address the anomalies in $b \rightarrow s\mu\mu$ data are in tension with the bounds obtained by dilepton searches, as expected for a Z' which has sizable couplings to light quarks. Nevertheless, we can see that a relevant $C_9^{\ell\ell} \sim 0.1$ can be obtained for a heavier Z'_{23} in the region where $g_3 < 0.5$, as in this region couplings to muons are enhanced.

The flavour-violating structure of Z'_{23} couplings leads to sizable contributions to $B_s - \bar{B}_s$ meson mixing [49] and LFV processes involving $\tau \rightarrow \mu$ transitions and $\tau \rightarrow e$ transitions, although well below existing experimental limits [1]. $\tau \rightarrow \mu(e)$ transitions arise from mixing angles connected to the flavour hierarchy $\mathcal{O}(m_2/m_3)$ ($\mathcal{O}(m_1/m_3)$), see Section 3.1. As an example, in Fig. 2 we depict

the contour for $\mathcal{B}(\tau \rightarrow 3\mu) = 10^{-12}$. We find $\tau \rightarrow \mu\gamma$ to be more competitive than $\tau \rightarrow 3\mu$ only in the region $g_3 > 1$.

Beyond indirect detection, in the near future Z'_{23} could be directly produced at LHC, HL-LHC and future colliders such as FCC or a muon collider. The particular pattern of Z'_{23} couplings would allow to disentangle our model from all other proposals. For the natural benchmark $g_1 \simeq g_2 \simeq g_3 \simeq \sqrt{3}g_Y$, Z' preferentially decays to top pairs and ditaus, as can be seen in Table 2. Furthermore, Z'_{23} preferentially couples and decays to *right-handed* charged fermions, given their larger hypercharge with respect to left-handed charged fermions. Similarly, decays to down-type quarks are generally suppressed with respect to (right-handed) up-type quarks and charged leptons, given the smaller hypercharge of the former. An alternative way of discovery would be the detection of the hyperon scalars breaking the $U(1)_Y^3$ group down to SM hypercharge, however we leave a study about the related phenomenology for future work. In the same spirit, the model naturally predicts SM singlet neutrinos which could be as light as the TeV scale (see Section 4.2), with phenomenological implications yet to be explored in a future work.

6 Conclusions

We have proposed a tri-hypercharge (TH) extension of the Standard Model (SM), based on assigning a separate gauged weak hypercharge to each family. The idea is that each fermion family i only carries hypercharge under a corresponding $U(1)_{Y_i}$ factor. This ensures that each family transforms differently under the TH gauge group $U(1)_Y^3$, which avoids the family repetition of the SM, and provides the starting point for a theory of flavour.

The three family specific hypercharge groups are spontaneously broken in a cascade symmetry breaking down to the SM hypercharge. We have motivated a particular symmetry breaking pattern, where in a first step $U(1)_{Y_1} \times U(1)_{Y_2}$ is broken down to its diagonal subgroup at a high scale v_{12} . The remaining group $U(1)_{Y_1+Y_2} \times U(1)_{Y_3}$ is broken down to SM hypercharge at a scale v_{23} . The hierarchy of scales v_{23}/v_{12} generally plays a role in the origin of flavour hierarchies, although we have found that a mild hierarchy $v_{23}/v_{12} \simeq \lambda$ is enough for specific implementations of the model, where $\lambda \simeq 0.224$ is the Wolfenstein parameter.

Assuming that the SM Higgs only carries third family hypercharge, then only the third family Yukawa couplings are allowed at renormalisable level. This explains the heaviness of the third family, the smallness of V_{cb} and V_{ub} quark mixing, and delivers an accidental and global $U(2)^5$ flavour symmetry acting on the light families, which provides a reasonable first order description of the SM spectrum. However, $U(2)^5$ does not explain the hierarchical heaviness of the top quark with respect to the bottom and tau fermions. Furthermore, we have proven that such a hierarchy between different charged sectors is translated to the light families, worsening the description of second family fermion masses. We have motivated the addition of a second Higgs doublet as a natural and elegant solution, which allows a more natural description of the hierarchies between the different charged fermion sectors.

We have explored the capabilities of the $U(1)_Y^3$ model to explain the observed hierarchies and mixing in the charged fermion sector, via the addition of non-renormalisable operators containing $U(1)_Y^3$ -breaking scalars which act as small breaking effects of $U(2)^5$. After extracting model-independent considerations from the spurion formalism, we have presented example models where all charged fermion masses and mixings are addressed. Following a similar methodology, we have

studied the origin of neutrino masses and mixing in the TH model. We have shown that due to the $U(1)_Y^3$ gauge symmetry, the implementation of a seesaw mechanism naturally leads to a low scale seesaw, where the SM singlet neutrinos in the model may be as light as the TeV scale. We have provided an example model compatible with the observed pattern of neutrino mixing.

Finally, we have performed a preliminary exploration of the phenomenological implications and discovery prospects of the $U(1)_Y^3$ theory of flavour. The heavy gauge boson Z'_{12} arising from the 12-breaking displays completely flavour non-universal couplings to fermions, and generally contributes to $\Delta F = 2$ and CLFV processes. The size of the most dangerous contributions are however model-dependent. In selected specific models provided in this manuscript, we have found that the most dangerous contributions to $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow e\gamma$ are strongly suppressed, allowing for Z'_{12} to be as light as 10-50 TeV. Therefore, the lightest gauge boson Z'_{23} arising from the 23-breaking can live at the TeV scale, within the reach of LHC and future colliders, since Z'_{23} avoids bounds from $K^0 - \bar{K}^0$ mixing and $\mu \rightarrow e\gamma$ thanks to an accidental GIM mechanism for light fermions.

We find the gauge boson Z'_{23} to have a rich low energy phenomenology: mixing with the SM Z boson leads to implications for the W boson mass and EWPOs, plus we expect sizable contributions to flavour-violating processes involving the third family, such as $\tau \rightarrow 3\mu(e)$ and $B_s - \bar{B}_s$ meson mixing. After our preliminary analysis, we find that current data allows Z'_{23} to be as light as 3-4 TeV in some regions of the parameter space, the strongest bounds coming from dilepton searches at LHC along with the contribution to the ρ EW parameter. In the case of discovery, the particular pattern of Z'_{23} couplings and decays to fermions will allow to disentangle our model from all other proposals. However, most of the phenomenological consequences are yet to be explored in detail: a global fit to EWPOs and flavour observables will allow to properly confront our model versus current data. An alternative way of discovery would be the detection of the Higgs scalars (hyperons) breaking the $U(1)_Y^3$ down to SM hypercharge, however we leave a discussion about the related phenomenology for future work. In the same spirit, the model naturally predicts SM singlet neutrinos which could be as light as TeV scale, with phenomenological implications yet to be explored.

Acknowledgements

We would like to thank the CERN Theory group for hospitality and financial support during an intermediate stage of this work. This work has received funding from the European Union's Horizon 2020 Research and Innovation Programme under Marie Skłodowska-Curie grant agreement HIDDeN European ITN project (H2020-MSCA-ITN-2019//860881-HIDDeN). SFK acknowledges the STFC Consolidated Grant ST/L000296/1.

A General formalism for the seesaw mechanism

In Section 4 we have motivated that in order to explain the observed pattern of neutrino mixing, it is required to add SM singlet neutrinos which carry family hypercharges (but are SM singlets). Therefore, the latter have to be vector-like in order to cancel gauge anomalies. In all generality, such vector-like neutrinos can get contributions to their mass from the unspecified vector-like mass terms and from Majorana masses given by VEVs of hyperons breaking the $U(1)_Y^3$ group. Both the SM singlet neutrinos N and the conjugate partners \bar{N} (both LH in our convention) can couple to the active neutrinos ν through non-renormalisable operators involving by the hyperons. This way,

one can write the general neutrino matrix,

$$M_\nu = \left(\begin{array}{c|ccc} & \nu & \bar{N} & N \\ \hline \nu & 0 & m_{DL} & m_{DR} \\ \bar{N} & m_{DL}^T & M_L & M_{LR} \\ N & m_{DR}^T & M_{LR}^T & M_R \end{array} \right) \equiv \begin{pmatrix} 0 & m_D \\ m_D^T & M_N \end{pmatrix}. \quad (\text{A.1})$$

We briefly mention that in the limit $m_{DL} = 0$, the matrix above looks similar to that of the standard *inverse seesaw* (see e.g. [50, 51]). However, in inverse seesaw M_L would be associated to a small parameter μ that provides most of the suppression of neutrino masses, while in our model M_L is in any case above the EW scale because it originates from the VEVs of hyperons, and the suppression of neutrino masses comes mostly from the smallness of $m_{DL,R}$ (which arise from non-renormalisable operators).

In order to match Eq. (A.1) to our example seesaw model of Section 4.2, we define ν as a 3-component vector containing the weak eigenstates of active neutrinos, while N and \bar{N} are 2-component vectors containing the SM singlet N and conjugate neutrinos \bar{N} , respectively. Similarly, in the following we define each one of the sub-matrices of Eq. (A.1) in terms of the matter content of Section 4.2, ignoring the $\mathcal{O}(1)$ dimensionless couplings:

$$m_{DL} = \begin{pmatrix} & \bar{N}_{\text{sol}} & \bar{N}_{\text{atm}} \\ L_1 & 0 & 0 \\ L_2 & 0 & 0 \\ L_3 & \frac{\tilde{\phi}_{\nu 13}}{\Lambda_{\text{sol}}} & \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} \end{pmatrix} H_u, \quad m_{DR} = \begin{pmatrix} & N_{\text{sol}} & N_{\text{atm}} \\ L_1 & \frac{\phi_{e12}}{\Lambda_{\text{sol}}} & 0 \\ L_2 & \frac{\phi_{e12}}{\Lambda_{\text{sol}}} & \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} \\ L_3 & \frac{\phi_{\nu 13}}{\Lambda_{\text{sol}}} & \frac{\phi_{\text{atm}}}{\Lambda_{\text{atm}}} \end{pmatrix} H_u, \quad (\text{A.2})$$

$$M_L = \begin{pmatrix} & \bar{N}_{\text{sol}} & \bar{N}_{\text{atm}} \\ \bar{N}_{\text{sol}} & \tilde{\phi}_{\text{sol}} & 0 \\ \bar{N}_{\text{atm}} & 0 & \tilde{\phi}_{\ell 23} \end{pmatrix} \approx v_{23} \mathbb{I}_{2 \times 2}, \quad M_R \approx \begin{pmatrix} & N_{\text{sol}} & N_{\text{atm}} \\ N_{\text{sol}} & \phi_{\text{sol}} & 0 \\ N_{\text{atm}} & 0 & \phi_{\ell 23} \end{pmatrix} \approx v_{23} \mathbb{I}_{2 \times 2}, \quad (\text{A.3})$$

$$M_{LR} = \begin{pmatrix} & N_{\text{sol}} & N_{\text{atm}} \\ \bar{N}_{\text{sol}} & M_{N_{\text{sol}}} & 0 \\ \bar{N}_{\text{atm}} & 0 & M_{N_{\text{atm}}} \end{pmatrix} \approx M_{\text{VL}} \mathbb{I}_{2 \times 2}, \quad (\text{A.4})$$

where $\mathbb{I}_{2 \times 2}$ is the 2×2 identity matrix. In Eqs. (A.3) and (A.4) above, we have considered the following approximations and assumptions:

- We have neglected $\mathcal{O}(1)$ dimensionless couplings generally present for each non-zero entry of Eq. (A.3). With this consideration, we find $M_L = M_R$ after the hyperons develop their VEVs. Furthermore, since the two hyperons appearing in M_L and M_R participate in the 23-breaking step of Eq. (2.2), we have assumed that they both develop a similar VEV $\langle \phi_{\ell 23} \rangle \approx \langle \phi_{\text{sol}} \rangle \simeq v_{23}$, since any other choice would be unnatural.
- For simplicity, we have assumed a similar vector-like mass for both neutrinos in Eq. (A.4), i.e. $M_{N_{\text{sol}}} \approx M_{N_{\text{atm}}} \equiv M_{\text{VL}}$.

Provided that the condition $m_D \ll M_N$ is fulfilled in Eq. (A.5), we can apply the seesaw formula as

$$m_\nu = m_D M_N^{-1} m_D^T = (m_{D_L} \ m_{D_R}) \begin{pmatrix} v_{23} & -M_{\text{VL}} \\ -M_{\text{VL}} & v_{23} \end{pmatrix} \begin{pmatrix} m_{D_L}^T \\ m_{D_R}^T \end{pmatrix} \frac{1}{v_{23}^2 - M_{\text{VL}}^2} \quad (\text{A.5})$$

$$= \left[m_{D_L} m_{D_L}^T v_{23} - m_{D_L} m_{D_R}^T M_{\text{VL}} - m_{D_R} m_{D_L}^T M_{\text{VL}} + m_{D_R} m_{D_R}^T v_{23} \right] \frac{1}{v_{23}^2 - M_{\text{VL}}^2}.$$

Given the structure of m_{D_L} and m_{D_R} in Eq. (A.2), and assuming the mild hierarchies $v_{23}/v_{12} \simeq \lambda$ and $\Lambda_{\text{sol}}/\Lambda_{\text{atm}} \simeq \lambda$ motivated in the main text, the product $m_{D_R} m_{D_R}^T$ provides an anarchic matrix with $\mathcal{O}(1)$ coefficients in each entry. Instead, the products involving m_{D_L} lead to a hierarchical matrix where only the entries in the third row and column are populated with $\mathcal{O}(1)$ terms and the others are zero. Therefore, if $M_{\text{VL}} \gg v_{23}$, then the effective neutrino matrix becomes hierarchical, rendering impossible to explain the observed pattern of neutrino mixing and mass splittings with $\mathcal{O}(1)$ parameters. Instead, if M_{VL} is of the same order or smaller than v_{23} , i.e. $M_{\text{VL}} \lesssim v_{23}$, then the resulting matrix is in any case an anarchic matrix with $\mathcal{O}(1)$ coefficients in each entry, and the final results match those presented in Section 4.2.

This argument holds as long as m_{D_L} is populated by zeros in at least some of the entries involving L_1 and L_2 , like in our example model of Section 4.2. Instead, in the very particular case where both m_{D_L} and m_{D_R} are similar matrices and of the same order, then the terms proportional to M_{VL} in Eq. (A.5) can provide an anarchic matrix and $M_{\text{VL}} > v_{23}$ is possible. Nevertheless, even in this case we expect M_{VL} not to be very large, since the smallness of m_{D_L} and m_{D_R} (that arise from non-renormalisable operators) already provides most of the suppression for the neutrino masses. Furthermore, this scenario involves the addition of several extra hyperons with very particular charges, making the model very complicated, so we do not consider it.

B High scale symmetry breaking

Assuming that the 12-breaking scale is far above the electroweak scale, at very high energies the theory is well described by $U(1)_{Y_1} \times U(1)_{Y_2}$ with renormalisable Lagrangian (neglecting fermion content and any kinetic mixing⁸ for simplicity),

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^{(1)} F^{\mu\nu(1)} - \frac{1}{4} F_{\mu\nu}^{(2)} F^{\mu\nu(2)} + (D_\mu \phi_{12})^* D^\mu \phi_{12} - V(\phi_{12}), \quad (\text{B.1})$$

where for simplicity we assume only one hyperon $\phi_{12}(q, -q)$, which develops a VEV $\langle \phi_{12} \rangle = v_{12}/\sqrt{2}$ spontaneously breaking $U(1)_{Y_1} \times U(1)_{Y_2}$ down to its diagonal subgroup. The covariant derivative reads

$$D_\mu = \partial_\mu - ig_1 Y_1 B_{1\mu} - ig_2 Y_2 B_{2\mu}. \quad (\text{B.2})$$

Expanding the kinetic term of ϕ_{12} , we obtain mass terms for the gauge bosons as

⁸Considering kinetic mixing in the Lagrangian of Eq. (B.1) only leads to a redefinition of either the g_1 or g_2 couplings in the canonical basis (where the kinetic terms are diagonal), with no phenomenological implications.

$$\mathcal{M}^2 = \frac{q^2 v_{12}^2}{2} \begin{pmatrix} & B_1^\mu & B_2^\mu \\ B_{1\mu} & g_1^2 & -g_1 g_2 \\ B_{2\mu} & -g_1 g_2 & g_2^2 \end{pmatrix}. \quad (\text{B.3})$$

The diagonalisation of the matrix above reveals

$$\hat{\mathcal{M}}^2 = \frac{q^2 v_{12}^2}{2} \begin{pmatrix} & Y_{12}^\mu & Z'_{12}^\mu \\ Y_{12\mu} & 0 & 0 \\ Z'_{12\mu} & 0 & g_1^2 + g_2^2 \end{pmatrix}, \quad (\text{B.4})$$

in the basis of mass eigenstates given by

$$\begin{pmatrix} Y_{12\mu} \\ Z'_{12\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} \\ -\sin \theta_{12} & \cos \theta_{12} \end{pmatrix} \begin{pmatrix} B_{1\mu} \\ B_{2\mu} \end{pmatrix}, \quad \sin \theta_{12} = \frac{g_1}{\sqrt{g_1^2 + g_2^2}}. \quad (\text{B.5})$$

Therefore, we obtain a massive gauge boson $Z'_{12\mu}$ at the scale v_{12} , while $Y_1 + Y_2$ associated to the gauge boson $Y_{12\mu}$ remains unbroken. These results are trivially generalised for the case of more hyperons. The covariant derivative in the new basis is given by

$$\begin{aligned} D_\mu &= \partial_\mu - i \frac{g_1 g_2}{\sqrt{g_1^2 + g_2^2}} (Y_1 + Y_2) Y_{12\mu} - i \left(-\frac{g_1^2}{\sqrt{g_1^2 + g_2^2}} Y_1 + \frac{g_2^2}{\sqrt{g_1^2 + g_2^2}} Y_2 \right) Z'_\mu \\ &= \partial_\mu - i g_{12} (Y_1 + Y_2) Y_{12\mu} - i (-g_1 \sin \theta_{12} Y_1 + g_2 \cos \theta_{12} Y_2) Z'_\mu. \end{aligned} \quad (\text{B.6})$$

The couplings to fermions in Eq. (5.1) are readily extracted by expanding the fermion kinetic term applying Eq. (B.6).

C Low scale symmetry breaking

The renormalisable Lagrangian of a theory $SU(2)_L \times U(1)_{Y_1+Y_2} \times U(1)_{Y_3}$ with $H(\mathbf{2})_{(0, \frac{1}{2})}$ and $\phi_{23}(\mathbf{1})_{(q, -q)}$ reads (neglecting fermion content and any kinetic mixing⁹ for simplicity),

$$\begin{aligned} \mathcal{L}_{\text{ren}} &= -\frac{1}{4} F_{\mu\nu}^{(12)} F^{\mu\nu(12)} - \frac{1}{4} F_{\mu\nu}^{(3)} F^{\mu\nu(3)} - \frac{1}{4} W_{\mu\nu}^a W_a^{\mu\nu} \\ &+ (D_\mu H)^\dagger D^\mu H + (D_\mu \phi_{23})^* D^\mu \phi_{23} \\ &- V(H, \phi_{23}), \end{aligned} \quad (\text{C.1})$$

where the covariant derivatives read

$$D_\mu H = \left(\partial_\mu - i g_L \frac{\sigma^a}{2} W_\mu^a - i \frac{g_3}{2} B_{3\mu} \right) H, \quad (\text{C.2})$$

$$D_\mu \phi_{23} = \left(\partial_\mu - i g_{12} q B_{12\mu} + i g_3 q B_{3\mu} \right) \phi_{23}, \quad (\text{C.3})$$

⁹If kinetic mixing is considered, then it will lead to a further term in $\sin \theta_{Z-Z'_{23}}$ of Eq. (C.19) suppressed by $(M_Z^0/M_{Z'_{23}}^0)^2$ as well. This new term will be proportional to the kinetic mixing parameter $\sin \chi$, so it is expected to be extra suppressed with respect to the leading mass mixing term and we can safely neglect it.

and σ^a with $a = 1, 2, 3$ are the Pauli matrices. The Higgs doublet develops the usual EW symmetry breaking VEV as

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_{\text{SM}} \end{pmatrix}, \quad (\text{C.4})$$

while the hyperon develops a higher scale VEV as

$$\langle \phi_{23} \rangle = \frac{v_{23}}{\sqrt{2}}, \quad (\text{C.5})$$

which spontaneously breaks the group $U(1)_{Y_1+Y_2} \times U(1)_{Y_3}$ down to its diagonal subgroup.

Expanding the following kinetic terms with the expressions of the covariant derivatives of Eqs. (C.2) and (C.3), we obtain

$$(D_\mu H)^\dagger D^\mu H + (D_\mu \phi_{23})^* D^\mu \phi_{23} = \frac{g_L^2}{4} W_\mu W^{\mu\dagger} + \frac{q^2 v_{23}^2}{2} \begin{pmatrix} W_3^\mu & B_{12}^\mu & B_3^\mu \\ W_{3\mu} | & g_L^2 r^2 & 0 & -g_L g_3 r^2 \\ B_{12\mu} | & 0 & g_{12}^2 & -g_{12} g_3 \\ B_{3\mu} | & -g_L g_3 r^2 & -g_{12} g_3 & g_3^2 + g_3^2 r^2 \end{pmatrix}. \quad (\text{C.6})$$

where $r = \frac{v_{\text{SM}}}{2q v_{23}} \ll 1$, and we denote M_{gauge}^2 as the off-diagonal matrix above. Given the two different scales in the mass matrix, we apply first the following transformation

$$\begin{pmatrix} W_3^\mu \\ Y^\mu \\ X^\mu \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B_{12}^\mu \\ B_3^\mu \end{pmatrix} = \begin{pmatrix} W_3^\mu \\ \cos \theta_{23} B_{12}^\mu + \sin \theta_{23} B_3^\mu \\ -\sin \theta_{23} B_{12}^\mu + \cos \theta_{23} B_3^\mu \end{pmatrix}, \quad (\text{C.7})$$

where

$$\sin \theta_{23} = \frac{g_{12}}{\sqrt{g_{12}^2 + g_3^2}}, \quad (\text{C.8})$$

obtaining

$$V_{\theta_{23}} M_{\text{gauge}}^2 V_{\theta_{23}}^\dagger = \frac{q^2 v_{23}^2}{2} \begin{pmatrix} W_3^\mu & Y^\mu & X^\mu \\ W_{3\mu} | & g_L^2 r^2 & -g_L g_Y r^2 & -g_L g_X r^2 \\ Y_\mu | & -g_L g_Y r^2 & g_Y^2 r^2 & g_Y g_X r^2 \\ X_\mu | & -g_L g_X r^2 & g_Y g_X r^2 & g_F^2 + g_X^2 r^2 \end{pmatrix}, \quad (\text{C.9})$$

where Y^μ is associated to the SM hypercharge gauge boson with gauge coupling

$$g_Y = \frac{g_{12} g_3}{\sqrt{g_{12}^2 + g_3^2}} \simeq 0.36, \quad (\text{C.10})$$

where the numeric value depicted is evaluated at the EW scale, and X^μ can be seen as an effective gauge boson with effective couplings

$$g_X = \frac{g_3^2}{\sqrt{g_{12}^2 + g_3^2}}, \quad g_F = \sqrt{g_{12}^2 + g_3^2}, \quad (\text{C.11})$$

to the Higgs boson and to ϕ_{23} , respectively. In this basis the covariant derivatives read

$$D_\mu H = (\partial_\mu - ig_L \frac{\sigma^a}{2} W_\mu^a - i \frac{g_Y}{2} Y_\mu - i \frac{g_X}{2} X_\mu) H, \quad (\text{C.12})$$

$$D_\mu \phi_{23} = (\partial_\mu - iq g_F X_\mu) \phi_{23}. \quad (\text{C.13})$$

The mass matrix in this basis can be block-diagonalised by applying the following transformation

$$\begin{pmatrix} A^\mu \\ (Z^0)^\mu \\ X^\mu \end{pmatrix} = \begin{pmatrix} \sin \theta_W & \cos \theta_W & 0 \\ \cos \theta_W & -\sin \theta_W & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ Y^\mu \\ X^\mu \end{pmatrix} = \begin{pmatrix} \cos \theta_W Y^\mu + \sin \theta_W W_3^\mu \\ -\sin \theta_W Y^\mu + \cos \theta_W W_3^\mu \\ X^\mu \end{pmatrix}, \quad (\text{C.14})$$

where the mixing angle is identified with the usual weak mixing angle as

$$\sin \theta_W = \frac{g_Y}{\sqrt{g_Y^2 + g_L^2}}, \quad (\text{C.15})$$

obtaining

$$V_{\theta_W} V_{\theta_{23}} M_{\text{gauge}}^2 (V_{\theta_W} V_{\theta_{23}})^\dagger = \frac{q^2 v_{23}^2}{2} \begin{pmatrix} A^\mu & (Z^0)^\mu & X^\mu \\ A_\mu | 0 & 0 & 0 \\ (Z^0)_\mu | 0 & (g_L^2 + g_Y^2) r^2 & -g_X \sqrt{g_Y^2 + g_L^2} r^2 \\ X_\mu | 0 & -g_X \sqrt{g_Y^2 + g_L^2} r^2 & g_F^2 + g_X^2 r^2 \end{pmatrix}, \quad (\text{C.16})$$

where we have already identified the massless photon. Now we diagonalise the remaining 2×2 sub-block in the limit of small r^2 . We obtain

$$Z_\mu = \cos \theta_{Z-Z'_{23}} (-\sin \theta_W Y_\mu + \cos \theta_W W_{3\mu}) + \sin \theta_{Z-Z'_{23}} X_\mu, \quad (\text{C.17})$$

$$Z'_{23\mu} = -\sin \theta_{Z-Z'_{23}} (-\sin \theta_W Y_\mu + \cos \theta_W W_{3\mu}) + \cos \theta_{Z-Z'_{23}} X_\mu, \quad (\text{C.18})$$

where to leading order in r^2

$$\sin \theta_{Z-Z'_{23}} \approx \frac{\sqrt{g_Y^2 + g_L^2} g_X}{g_F^2} r^2 = \frac{g_3 \cos \theta_{23}}{\sqrt{g_Y^2 + g_L^2}} \left(\frac{M_Z^0}{M_{Z'_{23}}^0} \right)^2. \quad (\text{C.19})$$

We can see that the SM Z boson carries a small admixture of the X_μ boson, which provides a small shift to its mass as

$$M_Z^2 \approx q^2 v_{23}^2 (g_Y^2 + g_L^2) \left(r^2 - \frac{g_X^2}{g_F^2} r^4 \right) = (M_Z^0)^2 \left[1 - \frac{g_3^2 \cos^2 \theta_{23}}{(g_Y^2 + g_L^2)} \left(\frac{M_Z^0}{M_{Z'_{23}}^0} \right)^2 \right] \quad (\text{C.20})$$

$$M_{Z'_{23}}^2 \approx q^2 v_{23}^2 g_F^2 \left(1 + \frac{g_X^2}{g_F^2} r^2 \right) = (M_{Z'_{23}}^0)^2 \left[1 + \frac{g_3^2 \cos^2 \theta_{23}}{(g_Y^2 + g_L^2)} \left(\frac{M_Z^0}{M_{Z'_{23}}^0} \right)^2 \right] \quad (\text{C.21})$$

where

$$M_Z^0 = \frac{v_{\text{SM}}}{2} \sqrt{g_Y^2 + g_L^2}, \quad M_{Z'_{23}}^0 = qv_{23} \sqrt{g_{12}^2 + g_3^2}. \quad (\text{C.22})$$

are the masses of the Z boson in the SM and the mass of the Z'_{23} boson in absence of $Z - Z'_{23}$ mass mixing, respectively. All these results can be easily generalised for the case of more hyperons or more Higgs doublets.

As expected, the SM Z boson mass arises at order r^2 , with a leading correction from $Z - Z'_{23}$ mixing arising at order r^4 . Instead, the Z'_{23} boson arises at leading order in the power expansion, with the leading correction from $Z - Z'_{23}$ mixing arising at order r^2 . Remarkably, the presence of $Z - Z'_{23}$ mixing always shifts the mass of the Z boson downward with respect to the SM prediction.

The equations obtained match general results in the literature [25, 52, 53], which consider scenarios where the starting point is a matrix such as Eq. (C.9) with $g_F = g_X$. Our equations match those of these papers when $g_F = g_X$ (and taking into account that we need to perform an extra rotation θ_{23} to arrive to Eq. (C.9)). Neglecting the small $Z - Z'_{23}$ mixing, the couplings of the Z'_{23} gauge boson given in Eq. (5.2) are obtained by expanding the fermion kinetic terms in the usual way, using the covariant derivative (where T_3 is the third-component $SU(2)_L$ isospin)

$$D_\mu = \partial_\mu - i \left[eQA_\mu + (T_3 g_L \cos \theta_W - g_Y \sin \theta_W (Y_1 + Y_2 + Y_3)) Z'_\mu{}^0 + (-g_{12} \sin \theta_{23} (Y_1 + Y_2) + g_3 \cos \theta_{23} Y_3) Z'_{23\mu} \right], \quad (\text{C.23})$$

which is an excellent approximation for all practical purposes other than precision Z boson phenomenology. In that case, one has to consider that the couplings of the Z boson to fermions are shifted due to $Z - Z'_{23}$ mixing as

$$g_Z^{f_L f_L} = \left(g_{Z'}^{f_L f_L} \right)^0 + \sin \theta_{Z-Z'_{23}} g_{Z'_{23}}^{f_L f_L}, \quad (\text{C.24})$$

where $g_{Z'}^{f_L f_L}$ are the fermion couplings of Z'_{23} in absence of $Z - Z'_{23}$ mixing, as given in Eq. (5.2), and similarly for right-handed fermions by just replacing L by R everywhere. We can see that in any case, the shift in the Z boson couplings is suppressed by the small ratio $(M_Z^0/M_{Z'_{23}}^0)^2$.

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