Coalition Formation and Cost Sharing for Truck Platooning

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Abstract

Truck platooning consists of one or several trucks driving very closely behind the platoon leader with the help of technology. Platooning reduces fuel consumption, carbon emissions and congestion while increasing road safety and the productivity of trucks and drivers. This article focuses on the advance planning of platoons. First, we study platoon formation from a system-wide optimization perspective. We formalize the underlying optimization problem and we propose exact and approximate solution approaches that appear to perform very well in instances of practical size. Second, we posit that truck platooning is much more likely to develop efficiently among multiple operators. This involves a shift in business relations between freight operators through cost sharing. We make use of cooperative game theory to study cost allocations among players. Our analysis shows that a compromise is needed among existence, stability and computational efficiency. However, we propose cost allocation rules for cooperative platooning games that perform very well in practice with regard to their stability. Finally, we propose an illustrative example based on the settings of the Port of Rotterdam and we provide a series of insights.

KEYWORDS: Platooning, Logistics, Cost Sharing, Cooperative Game Theory.

1 Introduction

Freight transportation is the backbone of global trade and, accordingly, the competitiveness of an economy is strongly correlated with the efficiency of its logistics sector (Hausman et al., 2013). The global freight volume is expected to double from 2015 to 2050 according to the projections of the International Transport Forum (ITF, 2021). Therefore, efficient decision-making is required at all stages of the transport chain to tackle the manifold challenges related to the management of freight transportation systems. Road freight plays a specific role among all transport modes. For instance, the US trucking industry alone generated more than $700 billion in revenue in 2021. Trucking companies are constantly looking for technological and operational advancements to improve the efficiency of road freight transportation and automation appears to be driving the forthcoming revolution. This disruptive breakthrough is expected to provide substantial benefits in terms of safety, cost reduction and traffic efficiency. However, we posit here that reaching fully autonomous heavy freight vehicles is a journey that needs to be achieved via several milestones. This is in line with the different levels of automation proposed by the US National Highway Traffic Safety Administration, from level 0 (i.e., no autonomy) to level 5 (i.e., full autonomy) (NHTSA, 2019).

The level of automation for trucks is currently lower than that for cars and the journey towards self-driving cars is likely to differ from the journey towards self-driving trucks. One of the most discernible examples of this difference is the recent surge in interest in *truck platooning*. The latter consists of one or several trucks driving very closely behind the platoon leader with the help of technology that creates a virtual link between trucks. The leading truck is driven manually and the following trucks can automatically accelerate, brake and steer to follow the actions of the platoon leader. Driver-assistive truck platooning (i.e., with drivers in all trucks of a platoon) appears nowadays as one of the most straightforward milestones (level 3 autonomy) towards full automation. Truck platooning has attracted a lot of attention during recent years and several real-life tests are currently taking place. MAN and DB Schenker announced the world's first practical use of platooning in June 2018 in Germany. Subsequently, Volvo Trucks North America and FedEx followed with a test in the US two days later. Peloton Technology, a Silicon Valley-based start-up dedicated to the development of truck platooning technologies, has raised more than $78 million since its inception in 2011. We explain this rise in interest in terms of the benefits that truck platooning might bring. These benefits include fuel savings due to reduced aerodynamic drag, increased road safety, reduced congestion and possibly higher productivity of trucks and drivers.

A side benefit of the reduction in fuel consumption is the reduction in carbon emissions, which is a major challenge in the fight against global warming. The truck platooning market size is projected to grow from $1.417 billion in 2020 to $7.345 billion by 2026 (MarketWatch, 2020).

In parallel with the recent attention from the industry towards truck platooning, the academic literature has expanded rapidly. The next section provides an overview of the related literature. We highlight here that the full benefits of truck platooning will require advance planning. Some trucks might be required to wait for a short time to form a platoon. Accordingly, Muratori et al. (2017) highlight that approximately two-thirds of truck miles in the US are *platoonable* when using a 15-minute waiting threshold. This shows the importance of proper planning. In reviewing the related literature, we conclude that the contributions so far mainly consider a centralized setting. This perspective might make sense for the planning of platooning operations of a single company. However, this is not consistent with the fragmented structure of the trucking industry, which is characterized by many small players (Cassidy, 2020). Therefore, the benefits of platooning will be fully reaped only if several companies join forces to form platoons. This will require trucks belonging to different operators to be *platoonable*. Accordingly, the concept of multi-brand platooning is developing (see, for example, the Horizon 2020 project titled ENSEMBLE (ENabling SafE Multi-Brand pLatooning for Europe) financed by the European Union). This led Martin Flach, Iveco UK alternative fuels director, to state during the Microlise 2018 conference that: "The real challenge is multi-brand platooning. You have to set up protocols that can be shared between truck manufacturers. And you need a mechanism for sharing the benefits between operators. The challenge should not be underestimated." Bhoopalam et al. (2018) similarly highlight the network effect associated with truck platooning, as an insufficient number of participants will limit the chances of platoon formation.

In this article, we aim to analyze coalition formation and cost sharing for the planning of platooning operations. We consider a set of trucks controlled by different operators, each having a specific ready time and starting location. First, we study the scheduling of truck movements related to platoons' formation. We formalize the underlying optimization problem and we show strong connections with lot-sizing models. These connections inspires different solution approaches, exact and approximate, that appear to perform very well in instances with practical sizes. Second, we study how the costs might be shared among players to ensure that the solution obtained will be implementable in practice, as unfair cost sharing will surely impede the collaboration among operators. We formalize the cooperative platooning game and we introduce an approximation game referred to as the *consecutive platooning game*. We show that while the core (Shapley, 1955)[1] of the platooning game might be empty, the core of the consecutive platooning game is non-empty. We also show how to compute an allocation in the core of the consecutive platooning game in polynomial time. We additionally focus on an alternative approach based on the Shapley allocation (Shapley, 1953). Although calculations of the Shapley allocation in general are computationally challenging, we provide expressions for the Shapley allocation of the game under certain assumptions. Third, we derive additional insights from some numerical examples inspired by a case study of the Port of Rotterdam. To the best of our knowledge, this paper is the first to fully assess the implications of truck collaboration via platooning.

2 Literature Review and Contributions

We detail below the three main streams of literature related to our work. First, we review the main results on technical aspects of platooning together with results related to the energy savings and related costs of this technology. Second, we review the literature dealing with the planning of platooning operations. Third, we provide an overview of the contributions related to platooning among multiple operators.

There is a large body of literature related to the technical aspects of platooning. We provide a brief overview of the main results here. Platooning requires investing in several types of sensor (mainly radars, cameras and lidars) and vehicle-to-vehicle communication technologies (such as dedicated short-range communications). In return, platooning enables trucks to drive very closely behind each other, and therefore, the most straightforward benefit of truck platooning is fuel saving due to reduced aerodynamic drag. Many studies try to quantify those savings (see, for example, Lu and Shladover (2011); Liang et al. (2014); Tsugawa et al. (2016); Vahidi and Sciarrzetta (2018); McAuliffe et al. (2018)), and estimates vary greatly, ranging from 3% to 25%. Platooning can also induce other types of benefits that are generally not accounted for in traditional return-on-investment calculations, such as improved safety (Axelsson, 2016), reduced carbon emissions (Alam, 2014), reduced congestion (Ploeg et al., 2011), and better use of road infrastructure. In addition to those benefits, platooning is often considered as a necessary step towards full automation from a public acceptance point of view. Overall, this highlights that truck platooning is likely to develop in the near future despite the fact that the business case is not yet well defined (Sivanandham

[1]It is worth mentioning that although the literature often associates the definition of the core to Gillies (1953), it was Shapley (1955) who first defined the core in its current form (Zhao, 2018).

and Gajanand, 2020). Some contributions also focus on the operational challenges related to intra-platoon management (Durey et al., 2020; Yuan and Lo, 2020; Zhao and Zhang, 2020) as well as on macro-level impact of autonomous vehicles on infrastructure utilization (see e.g., Mirzaeian et al. (2020) and Ghiasi et al. (2017)).

The literature on the planning of platooning operations has developed rapidly in recent years. We refer to Bhoopalam et al. (2018) for an in-depth review and we highlight below some key contributions. One of the most studied problem is the problem of finding the optimal routing of a set of trucks (including departure time) so as to optimize the total travelling costs. Depending on the assumptions, the underlying variants and models might reveal polynomially solvable or NP-hard and sometimes the complexity is unknown. Baskar et al. (2013) focus on the optimal routing of truck platoons on a motorway. They approximate the problem via a mixed-integer linear formulation and via (continuous) nonlinear programming. Larsson et al. (2015) show that some variants of the problem are NP-hard even when all trucks start at the same location and at the same time and when deadlines are ignored. Zhang et al. (2017) consider soft time windows. They formulate the problem as a mixed-integer linear programming problem and solve it with exact solutions. Boysen et al. (2018) study the problem along a single path. They consider trucks that start from the same location and that have the same destination. Moreover, each truck has a starting time window. The authors show that several of these simplified settings can be solved in polynomial time. Larsen et al. (2019) present a heuristic for hub-based platooning and analyze the performance as compared to exact resolutions via a solver. Nourmohammadzadeh and Hartmann (2019) propose a meta-heuristic solution methodology inspired by ant colony optimization. Bhoopalam et al. (2020) considered vehicle routing formulations with time windows. They show how to solve two special cases in polynomial time and they propose some heuristics to solve the general version of the problem. Most contributions above assume fixed speeds but some authors additionally consider speed optimization: see, for example, Luo et al. (2018), van de Hoef (2018) and Chen et al. (2021).

Truck platooning is very likely to involve several operators. Yet, platooning among multiple operators has been only studied in a few papers. Farokhi and Johansson (2013) develop an atomic congestion game including both cars and trucks to model the traffic flow on the road. The authors identify the pure strategy Nash equilibria of the game. Johansson et al. (2018) consider a non-cooperative platooning game for a set of trucks with the same origin, but different destinations. The authors show that this game is an exact potential game and they derive an algorithm to identify a Nash equilibrium based on best response dynamics. Sun and Yin (2021a) seek to develop an auction mechanism to allocate profit among platoon participants. The authors specifically account for the difference in fuel savings between the platoon leader and following trucks. They prove that there is no allocation that guaranty individual rationality and budget balance and they propose an approximation for an efficient auction mechanism. Sun and Yin (2021b) study a multi-agent system that accounts for fuel savings that vary over different platoon positions. The authors propose two mechanisms based on the bilateral trade model and one-sided matching and they additionally study information visibility. Johansson et al. (2021) study a multi-fleet platoon coordination system where each of the fleets aims to maximize their own profit. They propose a cross-fleet Pareto-improving coordination strategy that can be implemented in real-time.

These five contributions in the last paragraph are particularly helpful in understanding the dynamics of truck platooning when multiple actors interact. Complementary to these results, we aim at taking a cooperative game theory approach in this article. Cooperative game theory is relevant to study cost (or benefit) allocation among multiple actors in case groups of players can interact. This is particularly relevant in our setting as we consider that platooning is coordinated through a platooning platform. To the best of our knowledge, the only contribution to this problem is the work of Sun and Yin (2019). Assuming common starting/ending times and locations, the authors formulate an optimization problem to find the best driving speed which maximizes the trucks' utilities in platoons. This contribution is noteworthy, however, speed optimization is not the key practical issue for the planning of truck platoons. Heterogeneous starting times and locations are the main practical issues faced by trucks who are willing to form platoons. In this article, we incorporate 'waiting" as a crucial parameter in a time-sensitive freight delivery context and we formulate an optimization problem that exhibits strong connections to the lot sizing literature (Karimi et al., 2003; Pochet and Wolsey, 2006; Jans and Degraeve, 2008; Buschkühl et al., 2010; Brahimi et al., 2017). We also address the questions of stability in platooning coalitions and fair allocations. Our article contributes in particular to the field of cooperative game theory applications related to logistics and transportation (Göthe-Lundgren et al., 1996; Engevall et al., 2004; Krajewska et al., 2007; Özener and Ergun, 2008; Lozano et al., 2013; Hezarkhani et al., 2016; Verdonck et al., 2016; Hezarkhani, 2016; Defryn et al., 2016; Guajardo et al., 2018). We refer to Guajardo and Rönnqvist (2016) for an extensive review of the literature in this area. Our models exhibit strong connections (and might create interesting bridges) with economic lot-sizing games as well (Van den Heuvel et al., 2007; Xu and Yang, 2009; Gopaladesikan et al, 2012; Toriello and Uhan, 2014; Chen and Zhang, 2016).

Our contribution is threefold. First, we study system-wide optimization for the planning of trucking platoons. We propose exact and approximate solution approaches and we assess their performances through numerical experiments. The results show that our approximate solution approaches are rather close to optimal and can be computed rapidly for instances with practical sizes. Second, we study cost sharing mechanisms for collaborative truck platooning. We focus on two fundamental solution concepts in cooperative game theory: the core and the Shapley allocation. We show that the core of the cooperative platooning game can be empty in general but highlight an approximation game with a non-empty core. Also, we show how to compute an allocation, in polynomial time, in the core of this approximation game. We additionally show that despite computational challenges in calculating the Shapley allocation in general, compact expressions can be formulated under certain assumptions. Third, the paper also contributes to a better understanding of the implications of truck collaboration via platooning. We propose an illustrative example based on the settings of the Port of Rotterdam and derive key insights. Among others, we show that platoons are likely to form in the Port of Rotterdam and that if properly planned, truck platooning could be a quite effective strategy for reducing fuel consumption for a low average waiting time among participating trucks. Furthermore, our proposed allocation rule appears to work well and can help to glean the benefits of platooning in practice.

3 Model Formulation

Truck platooning among multiple operators is coordinated through a platooning platform[2] which collects the relevant information from the trucks and communicate the plans back to them. After the platform announces the "rules of engagement", individual trucks with platooning technology decide whether to join the platform or not. Note that we assume that each truck is controlled independently as this is in line with the fragmented structure of the trucking industry highlighted in the introduction section. We refer to Figure 1 for an illustration of the interactions between the players and the platform. We provide a summary of our notation in Table 1.

Let $N = \{1, 2, ..., n\}$ be the set of trucks in the system equipped with platooning technology. We focus on the possibility of platooning along a common path, such as a major highway, starting at point 0 and ending at point E. For simplicity we assume that each truck wants to travel to E.

[2]The platooning platform can operate either as a non-for-profit entity managed by a public authority, or as a commission-based intermediary which receives a percentage of the generated savings for its users. In both cases, the platform's objective would be to maximize the generated savings in the system.

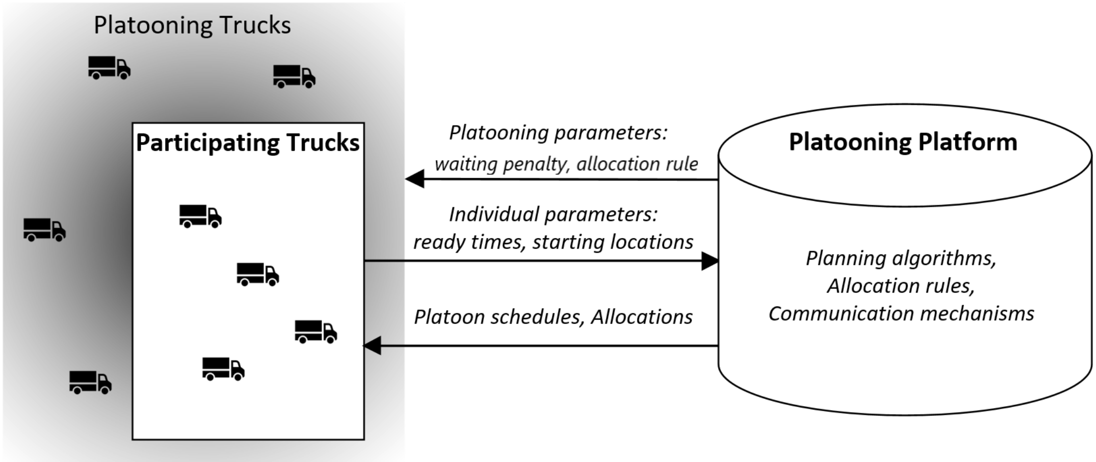

Figure 1: The schematic of the platooning platform

Trucks start their journey towards the destination point E at locations off the common path but join the common path at specific points. Let $x_i \in [0, E)$ be the starting location of truck $i \in N$ on the common path and let $d_i = E - x_i$ be its distance to the destination. Let $r_i \in \mathbb{R}$ be the expected earliest time at which truck $i \in N$ can be present at location x_i. We call r_i the *ready time* of i. We assume that moving trucks maintain a uniform average speed of $v > 0$ and that the trucks do not go backward. While speed adjustment might be valuable at the execution phase of truck platooning, adjusting starting times is a more reasonable tactic at the planning phase which is our main focus. Adjusting starting time also induces less traffic disturbance (Johansson et al., 2021). We assume that when a truck starts its journey, it will not stop and wait along the way. This is motivated by practical considerations—although, relaxing this assumption may lead to additional savings. It is usually much cheaper to start a journey later than have an interim stop along the way (for instance, drivers can perform other tasks before the start of the journey). In some cases it might even be infeasible to stop on a motorway. For each truck $i \in N$, let $t_i = r_i + \frac{d_i}{v}$ be the *earliest arrival time* of i at E. Figure 2 depicts a platooning example in our context.

As stated in the introduction, a platoon consists of one or several trucks driving very closely behind the platoon leader with the help of technology that creates a virtual link between them. Since trucks do not wait after they start their journey, trucks that form a platoon will arrive at E simultaneously[3]. The notion of a platoon, assuming constant speed and no stopovers, is thus closely related to the definition of (feasible) scheduled arrival times $a_i \geq t_i$ for each truck $i \in N$,

[3] we neglect here the time separating the first and the last truck in a platoon as at the tactical level and in practical instances, it is very unlikely that an optimal solution will synchronize more than a handful of trucks

N	Set of trucks
S	A subset of players $S \subseteq N$
E	Location of the final destination
x_i	Starting location of truck i
d_i	Distance of truck i from the destination
r_i	Ready time of truck i
v	Average driving speed
t_i	Earliest arrival time of truck i at E
a_i	Scheduled arrival time of truck i at E
$(a_i)_{i\in N}$	A schedule for N
L	Maximum permitted length of a platoon
f_k	Joint traveling cost per unit of distance for a k-truck platoon
Q_S	Minimum traveling cost of platoon S
$w_{i,j}$	Minimum waiting time of truck i for truck j to join
p	Waiting cost rate (nominal)
$c_{i,j}$	Minimum waiting cost of truck i for truck j to join
W_S	Minimum waiting cost for platoon S
Z_S	Minimum cost for platoon S
$C(S)$	Optimal platooning cost for S
y_T	Set partitioning variable for platoon $T \subseteq N$
F_k	Minimum traveling cost per unit of distance for a k-truck platoon, split into subplatoons of size at most L
(N, C)	A cooperative platooning game
(N, H)	A cooperative consecutive platooning game
(N, W)	A cooperative platoon waiting game
$\boldsymbol{\varphi}$	An allocation
$\boldsymbol{\Phi}$	Shapley allocation
$[k]$	Notation for the set $\{1, ..., k\}$

Table 1: List of notations

which, collectively, comprises a *schedule* of the trucks $(a_i)_{i\in N}$. We are now ready to provide the formal definition of a platoon in our context.

Definition 1 (Platoon)**.** *Let $(a_i)_{i\in N}$, $a_i \geq t_i$ for all $i \in N$, be a schedule of the trucks. Let $S \subseteq N$ be a non-empty set of trucks. S forms a platoon if there is A_S such that $a_i = A_S$ for all $i \in S$.*

As stated in the above definition, trucks form a platoon if they reach E at the same arrival time A_S. By definition, a subset of a platoon is also considered as a (sub)platoon. The above definition also allows us to consider single-truck platoons which simplifies the exposition of the paper. Note that this definition is a relaxation of the notion of (physical) platoon but it is convenient mathematically as this does not require detailed specification of the organization process of physical platoons of S over time. In practice there might be limitations regarding the maximum size of a platoon which we denote with L. We will see in Subsection 3.1 how to deal with such a case without

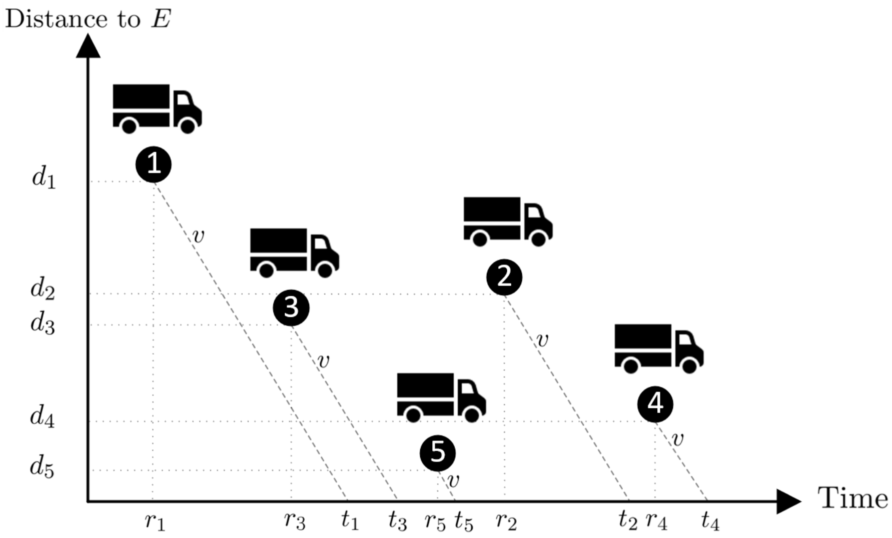

Figure 2: A platooning situation with five trucks

loss of generality.

Platooning allows trucks to save on fuel and travel costs. If some trucks want to form a platoon, however, they might have to wait for others to catch up. Therefore, there are two types of costs in our context: traveling costs and waiting costs.

Traveling costs: Traveling costs incorporate fuel costs as well as driver and maintenance costs. Traveling costs depend on whether trucks are traveling together or not. When some trucks travel together, their fuel consumption rate improves due to better aerodynamics. For $k \in N \cup \{0\}$, we denote by f_k the *joint traveling cost* per unit of distance[4] when k trucks travel together in a platoon. As including a new truck in a platoon reduces only a fraction of the traveling costs of the corresponding truck, we assume that f_k is non-decreasing in k and we set $f_0 = 0$ by convention (this ensures that $f_k \geq 0$ for all $k \geq 0$). Assume that a set $S \subseteq N$ forms a platoon and assume without loss of generality that the trucks in S are numbered $1, ..., |S|$ with $d_1 \geq ... \geq d_{|S|}$. The traveling costs for platoon S are (with the convention $d_{|S|+1} = 0$):

$$Q_S = \sum_{k=1}^{|S|} (d_k - d_{k+1})\, f_k. \tag{1}$$

Note that the travelling cost of a platoon is independent of the schedule of the trucks in the platoon.

[4]We can also express traveling cost per unit of time. Since the speed is fixed, we can turn one into another.

Waiting costs: Trucks may have to wait for each other in order to catch up and travel together in platoons. Waiting, however, is costly. For a pair of trucks $i, j \in N$, we define $w_{i,j} := t_j - t_i$. $|w_{i,j}| = |w_{j,i}|$ represents the minimum waiting time for truck i and j to meet up. Whenever $w_{i,j} > 0$, truck i must wait for truck j if they want to travel together. Otherwise, if $w_{i,j} < 0$, truck j must wait for i. When $w_{i,j} = 0$, no waiting is required for the two trucks to join. If a truck waits while another is joining, the waiting truck incurs the waiting cost. Trucks may not incur waiting costs at the same rate—which is dependent on the urgency of their deliveries. However, in order to avoid problems with strategic announcement of waiting cost rates by the players, the platform chooses a uniform *nominal* waiting penalty per unit of time $p \geq 0$.[5] This approach is reasonable in any cooperative system that works based on unobservable attributes. In what follows, we assume accordingly that p is predefined and we refer to the trucks who decide to join the platform as *participating trucks*. The platoon schedules recommended by the platform are binding for the participating trucks. In what follows, we assume without loss of generality that all trucks in N are participating trucks. We additionally discuss the choice of p and its impact on the number of participating trucks in Section 6. Assume that a set $S \subseteq N$ forms a platoon. The waiting costs of a platoon S scheduled to arrive at time $A_S \geq \max\{t_i, i \in S\}$ is $p \sum_{i \in S}(A_S - t_i)$. The schedule which minimizes the waiting cost for platoon S is obtained by setting $a_i = \max\{t_j, j \in S\}$ for all $i \in S$. Hence, the minimum waiting costs of a platoon S are:

$$W_S = p \sum_{i \in S} \left(\max\{t_j, j \in S\} - t_i\right). \tag{2}$$

We focus, in the remaining part of this section and in the next section, on the problem of finding a schedule of the trucks (and thus a grouping of the trucks into platoons) that minimizes the sum of the travelling and waiting costs. Since Q_S is independent of A_S, the optimal schedule for a platoon S which minimizes the sum of travelling and waiting costs is obtained by setting $a_i = \max\{t_j, j \in S\}$ for all $i \in S$. We denote by Z_S the total cost of a platoon S, that is, $Z_S := Q_S + W_S$ where Q_S and W_S are given by formula (1) and (2) respectively.

The optimization problem associated with a platooning situation is, thus, to find the best combination of platoons to be formed. Formally, we want to solve the following problem.

Problem 1. *Input: n trucks with earliest arrival times $t_1, \dots, t_n \geq 0$ and starting locations $x_1, \dots, x_n$ in $[0, E)$; $p \geq 0$; and a non-negative and non-decreasing function f_k for $k \in \{1, \dots, n\}$. Output: a*

[5] The choice of p could even be optimized to maximize the expected benefits from platooning if we have access to (or good guess on) the minimum value, ρ_i, of the waiting cost rate for which truck i would agree to participate.

partition of the n trucks into platoons $S_1, ..., S_m$*, for some* $m \geq 1$*, that minimizes* $\sum_{i=1}^{m} Z_{S_m}$.

For future analysis, it is convenient to look at subproblems defined over a subset of trucks. We define $C(S)$ to be the cost of an optimal solution to Problem 1 when we restrict to the trucks in S. The natural formulation for $C(S)$ is in the form of a set partitioning problem as given by:

$$
\begin{aligned}
C(S) = \min \quad & \textstyle\sum_{T \subseteq S} y_T Z_T \\
s.t. \quad & \textstyle\sum_{T \subseteq S: T \ni i} y_T = 1 \quad \forall i \in S \\
& y_T \in \{0,1\} \qquad\quad \forall T \subseteq S
\end{aligned} \tag{3}
$$

The set partitioning problem above finds the best combination of platoons among the trucks in S which results in the minimum cost for S. While Formulation (3) is important for investigating game-theoretic properties later on in the paper, we propose an alternative formulation in Section 4 that is more promising as a solution platform.

3.1 A useful observation

We show here that we can easily integrate limitations on the size of the platoons in the cost function. Assume that the maximum permissible size of a platoon is $L \geq 1$. This does not proscribe, in principle, (almost) simultaneous arrivals of more than L trucks at the destination. In fact, we can always split a large platoon into smaller sub-groups of size no greater than L. We can even do this optimally. Let F_k be the minimum traveling cost for a platoon made of k trucks when it is split into subplatoons of size at most L (note that $F_0 := 0$). F_k can be computed inductively (by dynamic programming) as follows (we set, for convenience, $f_k = +\infty$ for $k > L$):

$$F_k := \min\{f_i + F_{k-i},\ i = 1, \ldots, k\},\ \text{for all } k = 1, \ldots, n \tag{4}$$

Note that each time a truck joins a platoon along the way, the optimal subgroups may change but it can easily be operationalized as there is no need to change the order of the trucks. For instance, imagine that $F_3 = f_3$ and that $F_4 = f_2 + f_2$. A truck joining a platoon of 3 trucks will join the tail of the convoy and the group of 4 trucks will split into 2 groups of 2. The function F_k exhibits some nice properties

Lemma 1. *The function* F_k *is non-decreasing and sub-additive in* k*.*

All proofs appear in Appendix A. Note that we can define (and work with) F_k also in the case where there is no limit on the size of a platoon. So for the rest of the paper we will assume without loss of generality that the joint travelling costs f_k, in addition of being non-decreasing, are also sub-additive (if not, we substitute f_k with F_k).

Observation 1. *We can assume without loss of generality that the joint travelling cost function f_k is non-negative, non-decreasing and sub-additive.*

4 Optimization Problem

We focus in this section on solving Problem 1 'efficiently'. We start with exhibiting an interesting connection to lot-sizing problems and we then provide different exact and approximate methods (with no performance guarantees) inspired from this connection.[6]

4.1 Connection to lot-sizing

The problem has rather a straightforward lot-sizing flavor if we reverse the time arrow. Indeed, it can be cast as a dynamic (multi-item) lot-sizing problem (with no backorder) as follows. Let $T := \max\{t_i, i = 1, ..., n\}$. Consider the problem where a warehouse located at location E has to serve a (unit) demand for different items $i = 1, ..., n$, and the demand for item i occurs at time $T - t_i$. An item i comes from the location $x_i \in [0, E)$. The warehouse might order several items in the same period. We assume that orders are served instantaneously, say, overnight. If a set S of items are ordered in the same period, the cost of ordering is Q_S as defined in equation (1). Items might arrive earlier than their demand period. In this case, they can be held in stock at a per unit cost of p per unit of time (no backorder is allowed). When a set S of items is ordered in the same period (with no other items), the holding cost is then exactly W_S as defined in equation (2). The original Problem 1 is thus equivalent to determining, in the lot-sizing setting, when to order the items so as to minimize the sum of the ordering and the holding costs.

There is a large body of literature on multi-item lot-sizing problems, see, e.g., Karimi et al. (2003) and Diaz-Madroñero et al. (2014). However, to the best of our knowledge, the corresponding problem is neither known to be polynomially solvable nor to be NP-hard. The most general polynomial time algorithm relating to our problem seems to be due to Anily et al. (2009). It allows to solve the case where all d_is are identical and the transportation cost function is of the form

[6] The reader unfamiliar with lot-sizing problems may skip next subsection as it serves mainly as motivation for the different algorithmic approaches considered here and to create connections with the corresponding literature, but the next subsections are self-contained (except for a few references to some relevant literature).

$f_k = \lceil \frac{k}{C} \rceil K + ak$ for some constant $K, a, C \geq 0$. In Subsection 4.3, we provide a generalization of this result for non-identical d_is and $f_k = K + ak$.

One of the usual properties to check when facing a new lot-sizing problem is the validity of the Zero-Inventory-Ordering (ZIO) assumption, that is, no order will take place if there is remaining inventory. In our platooning context, a schedule is ZIO if all platoons are formed with trucks that have consecutive earliest arrival times. Unfortunately, the ZIO property does not hold here. This rules out the possibility of applying a simple dynamic programming approach *à la Wagner-Within* (Wagner and Whitin, 1958). The following example shows that even for only three items, the best ZIO policy can be as far as 20% away from the optimal.

Example 1. *Consider a three-item problem with* $t_1 = 0$, $t_2 = 0.5$, $t_3 = 1$, $d_1 = 1$, $d_2 = \epsilon$, $d_3 = 1$, $f = [1, 1, 2]$, *and* $p = \frac{2}{3}$, *for some* $0 < \epsilon << 1$. *The optimal solution is to order items 1 and 3 together in period* $T - t_3$ *and to order item 2 in period* $T - t_2$. *The cost of the corresponding solution tends to* $5/3$ *as* ϵ *tends to* 0, *while the value of the best ZIO policy (that consists in ordering the three items separately) tends to* 2 *as* ϵ *tends to* 0. △

It might be possible, however, that the best ZIO policy provides a good approximation with a performance guarantee. This happens in some contexts, see e.g., Chan et al (2002a) and Chan et al (2002b), where it is proven that the best ZIO policy is never more than 33% away from optimal. Although the proof techniques used in the corresponding papers do not seem to extend to our setting, it does not rule out such a result. Nevertheless, we believe that ZIO policies are of interest in practice as they are extremely fast to compute and they seem to be very close to optimal for non-extreme instances as we will show later (see Section 4.5). We investigate this in the next subsection.

4.2 Dynamic programming approaches *à la Wagner-Within*

We assume here that the trucks are ordered such that $t_1 \leq ... \leq t_n$[7] and we define a ZIO solution to Problem 1 as a solution such that for all $1 \leq r \leq m - 1$, and for all $(i, j) \in S_r \times S_{r+1}$, we have $i < j$. We can compute an optimal ZIO solution as follows. We call $ZIO(j)$ the optimal ZIO solution to the problem if we restrict to items $1, ..., j$. We can compute $ZIO(j)$ inductively by dynamic

[7] It is not difficult to see that ties do not cause any problem in the definition: breaking ties arbitrarily might remove some solutions but the fact that the joint traveling cost do not depend on the specific trucks, but only on their number, suffices to guarantee that we can always find alternative ZIO solutions of the same cost.

programming by setting $ZIO(0) = 0$, and by computing $ZIO(j)$ from $j = 1$ to n using:

$$ZIO(j) := \min\{K(i,j) + ZIO(i),\ i = 0,\dots,j-1\}, \tag{5}$$

where $K(i,j)$ represents the cost of grouping all trucks $i+1,...,j$ in a platoon arriving at E at time t_j, that is, $K(i,j) := Z_{\{i+1,...,j\}}$. The next result follows immediately.

Lemma 2. *$ZIO(n)$ returns the optimal ZIO solution to the problem and can be computed in time $O(n^2)$.*

Although Example 1 above shows that ZIO policies are not optimal in general, there are situations where this is the case.

Theorem 1. *Suppose $d_i = d_j$ for all $i,j \in N$. $ZIO(n)$ returns the optimal solution to Problem 1.*

It is possible to use the ZIO solution as a heuristic solution in the general case. We will analyze the performance of this heuristic in Section 4.5. It is also tempting to try to exploit Theorem 1 further to design another interesting heuristic to this problem. Theorem 1 suggests to group trucks by 'similar' locations first and then apply a ZIO policy in each group. Such a heuristic naturally solves the problem in Example 1 and we know that the policy is close to optimal in each group from Theorem 1. We can even choose how to group the locations optimally (given that we use ZIO within each group) if we restrict to contiguous groups of locations (in the x-space). The corresponding solution can be computed, again, by dynamic programming. We assume here that the trucks are arranged such that $d_1 \le ... \le d_n$. For convenience we let $d_0 = 0$. Let us denote by $HEUR(i)$ the optimal solution of the heuristic described above to the subproblem restricted to trucks in $\{1,...,i\}$. We set $HEUR(0) = 0$. The value of $HEUR(j)$ can be computed inductively by dynamic programming for $j = 1,...,n$ using:

$$HEUR(j) := \min\{OPT\_ZIO(i,j) + HEUR(i),\ i = 0,\dots,j-1\}, \tag{6}$$

where $OPT\_ZIO(i,j)$ is the value of the optimal ZIO solution to the problem restricted to trucks in $\{i+1,...,j\}$ (which need to be rearranged by t_i and can be computed by the previous algorithm).

Lemma 3. *The solution of the heuristic, $HEUR(n)$, can be computed in time $O(n^4)$.*

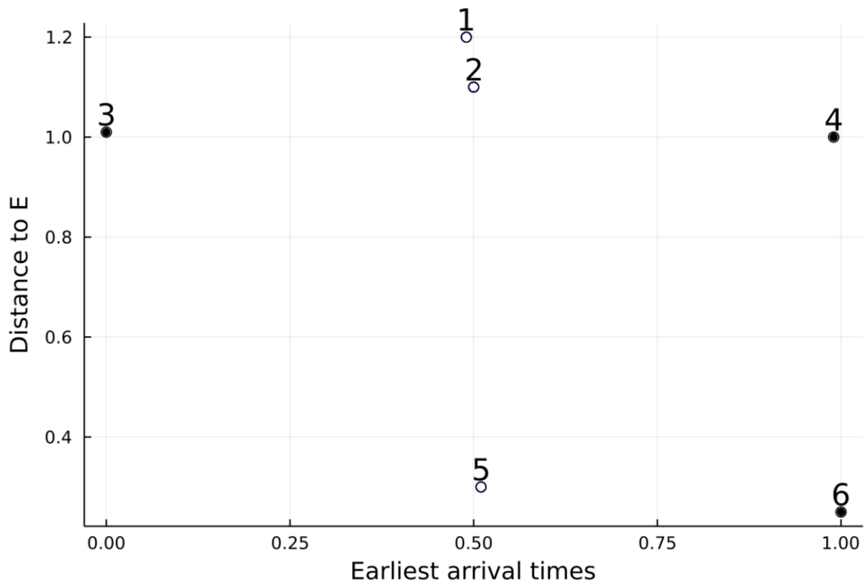

Figure 3: A 6-trucks example. The two optimal platoons are $\{1, 2, 5\}$ and $\{3, 4, 6\}$ which are distinguished by two different colors.

Note that this naïve implementation of the heuristic would be impractical for large values of n. It might be possible to reduce the complexity to $O(n^3)$ or beyond by exploiting the structure. We did not investigate this possibility as the ZIO heuristic is very attractive computationally for large scale instances as we discuss later.

Besides, unfortunately, the heuristic does not solve the problem to optimality in general as illustrated by the following example.

Example 2. *Consider a problem with six trucks where the t_is are $[0.49, 0.5, 0, 0.99, 0.51, 1]$, the d_is are $[1.2, 1.1, 1.01, 1, 0.3, 0.25]$, $f = [0.7, 0.7, 1, 1.4, 1.7, 2]$ and $p = 0.4$ (see Figure 3 for an illustration). The total cost of the optimal solution is 2.128 (it groups $\{1, 2, 5\}$ and $\{3, 4, 6\}$) while the total cost for the solution given by the heuristic is 2.183 which is the same as the optimal ZIO solution (it groups $\{1, 2, 3, 5\}$ and $\{4, 6\}$).* △

The above example also illustrates that optimal groups may have complex intersecting geometries. One question that we leave open is the complexity of Problem 1. Some features of the underlying multi-item lot-sizing problem suggest that the problem could be solved in polynomial time (e.g., same waiting penalty for all trucks, ordering cost independent of the period, unit demands) and some features suggest it could be hard (e.g., the general shape of the ordering cost function, and the geometry of the optimal solution(s)). Although we could not solve the general case, we show how to solve a non-trivial special case with dynamic programming in the next section.

4.3 A polynomial time algorithm when f_k is an affine function of k.

We focus here on the special case where $f_k = K + ak$ for some fixed $K, a \geq 0$. Given that the linear part of the joint traveling cost will be paid in any solution to Problem 1 (any partition of the trucks into platoons will incur a linear traveling cost of $a \sum_{k=1}^{n} d_k$), we restrict w.l.o.g. to the case where $f_k = K$ for some fixed $K \geq 0$. We assume that no two trucks have identical earliest arrival time and distance, i.e., there does not exist $k \neq k'$ such that $t_k = t_{k'}$ and $d_k = d_{k'}$; otherwise, we consider that there is a 'group' of q_k trucks traveling together (the algorithm below obviously generalizes to this setting). Let I be the number of distinct t_is and $t'_1 < ... < t'_I$ their values (for convenience we assume that $t'_1 > 0$ and we set $t'_0 = 0$). Let J be the number of distinct d_js and $0 < d'_1 < ... < d'_J$ their values (for convenience we set $d'_0 = 0$). We can, uniquely, represent each (group of) truck(s) $k \in \{1, ..., n\}$ by a pair $(i, j) \in \mathcal{P} = \{1, ..., I\} \times \{1, ..., J\}$, thus, with a slight abuse of notation, in what follows we distinguish a truck with the pair (i, j). For all $\underline{i} \leq \bar{i} \in \{0, ..., I\}$ and $\bar{j} \in \{0, ..., J\}$, we define:

- $OPT^R(\underline{i}, \bar{i}, \bar{j})$ to be the optimal value to Problem 1 when we restrict to trucks in $T(\underline{i}, \bar{i}, \bar{j}) := \{(i, j) \in P \text{ with } \underline{i} < i \leq \bar{i} \text{ and } j \leq \bar{j}\}$.
- $OPT^L(\underline{i}, \bar{i}, \bar{j})$ to be the optimal value to Problem 1 when we restrict to trucks in $T(\underline{i}, \bar{i}, \bar{j})$ and, additionally, impose that the platoon's scheduled arrival time is $t'_{\bar{i}}$.

We want to determine $OPT^R(0, I, J)$ and we can compute this by dynamic programming as shown by the next theorem. To do so, we define $j(\underline{i}, \bar{i}, \bar{j})$ as the largest j over all trucks (i, j) such that $\underline{i} < i \leq \bar{i}$ and $j \leq \bar{j}$ (for convenience we set $j(\underline{i}, \bar{i}, \bar{j}) = 0$ when the set of trucks is empty). We also define $K(\underline{i}, \bar{i}, \underline{j}, \bar{j})$ to be the waiting cost of the platoon made of all trucks (i, j) such that $\underline{i} < i \leq \bar{i}$ and $\underline{j} < j \leq \bar{j}$ if they are scheduled at time $t'_{\bar{i}}$ plus the share of the ordering cost associated with the corresponding leg, that is:

$$K(\underline{i}, \bar{i}, \underline{j}, \bar{j}) = K(d'_{\bar{j}} - d'_{\underline{j}}) + p \sum_{(i,j) \in P: \underline{i} < i \leq \bar{i}, \underline{j} < j \leq \bar{j}} (t'_{\bar{i}} - t'_i). \tag{7}$$

Theorem 2. *For all $\underline{i} < \bar{i}$ and all $\bar{j} \neq 0$, $OPT^L(\underline{i}, \bar{i}, \bar{j})$ and $OPT^R(\underline{i}, \bar{i}, \bar{j})$ can be formulated as following:*

- $OPT^L(\underline{i}, \bar{i}, \bar{j}) = \min_{(i,j):(\underline{i} < i < \bar{i}, 0 < j \leq \bar{j}) \vee (i = \underline{i}, j = 0)} K(\underline{i}, \bar{i}, j, \bar{j}) + OPT^L(i, \bar{i}, j) + OPT^L(\underline{i}, i, j)$
- $OPT^R(\underline{i}, \bar{i}, \bar{j}) = \min_{i: \underline{i} < i \leq \bar{i}} OPT^L(\underline{i}, i, j(\underline{i}, \bar{i}, \bar{j})) + OPT^R(i, \bar{i}, j(\underline{i}, \bar{i}, \bar{j}))$

with $OPT^L(\underline{i},\underline{i},\bar{j}) = OPT^R(\underline{i},\underline{i},\bar{j}) = 0$ *for all* $\underline{i},\bar{j}$ *and* $OPT^L(\underline{i},\bar{i},0) = OPT^R(\underline{i},\bar{i},0) = 0$ *for all* $\underline{i} \leq \bar{i}$*. Moreover, we can compute* $OPT^R(0, I, J)$ *in time* $O(n^5)$*.*

Note that the time complexity of the corresponding algorithm is prohibitive for large size instances. However Theorem 2 proves that this special case is polynomial (albeit non-trivial), which leaves hope for more efficient polynomial time algorithms for this variant - exploiting additional structures - and for polynomial time algorithms for extensions. For instance, in practice, the transportation costs are close to be affine if we omit restrictions in platoon size (see table 2). One might exploit ideas from Anily et al. (2009) to extend our complexity result to affine functions with size restrictions. This would certainly yield very good approximations in practice. We leave these challenging questions for future research.

The algorithm described above generalizes naturally to the following case as well; there are multiple demands for each 'truck', the fixed costs are period-dependent and there are different waiting penalties for each truck. It is thus a proper generalization of the uncapacitated multi-item lot-sizing problem. To the best of our knowledge, our algorithm is new and solves a case that, while interesting and relevant, has not been studied in the lot-sizing literature. We therefore believe that this problem and its generalizations might raise interest from the lot-sizing community.

4.4 Integer programming formulation

We now consider an integer programming (IP) formulation of the problem that can be used to solve the problem exactly. We assume w.l.o.g. in this subsection that the trucks are arranged by non-increasing order of distance from E, that is, $d_1 \geq d_2 \geq ... \geq d_n$. For convenience we let $d_{n+1} = 0$.

We consider a natural facility location reformulation of the problem. The basic idea is to identify a 'representative' truck for each platoon S in our solution, that is, a truck $i \in N$ with $a_i = \max\{t_j, j \in S\}$, and to assign the other trucks in S to this representative.

Let y_{jj} be a binary variable indicating whether truck j is a representative of a platoon, that is, the scheduled arrival time of the platoon is t_j. Let y_{ij} be binary variables indicating whether truck i is assigned to the platoon represented by truck j (for each $i, j \in N$). We denote by c_{ij} the waiting cost of truck i if it is assigned to j, that is, $c_{ij} := pw_{i,j}$ for each $i, j \in N$ with $t_i \leq t_j$ (c_{ij} can be set arbitrary for each $i, j \in N$ with $t_i > t_j$, as we forbid such assignments in any feasible solution to the problem, see below). For all $1 \leq j, k \leq n$, the number of trucks traveling together from d_k to d_{k+1} in the platoon represented by j is $\sum_{i=1}^{k} y_{ij}$. Because the joint travelling cost function is discrete (and not much structured), we introduce binary variables z^l_{kj} that indicate whether the number of trucks assigned to platoon represented by j from d_k to d_{k+1} is l.

Problem 1, thus, can be represented via the following IP[8]:

$$\min \sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}y_{ij} + \sum_{j=1}^{n}\sum_{k=1}^{n}\sum_{l=1}^{n} f_l(d_k - d_{k+1})z^l_{kj} \tag{8}$$

$$y_{ij} \leq y_{jj} \qquad \forall i,j \in N \tag{9}$$

$$\sum_{j=1}^{n} y_{ij} = 1 \qquad \forall i \in N \tag{10}$$

$$y_{ij} = 0 \qquad \forall i,j \in N : t_i > t_j \tag{11}$$

$$\sum_{i=1}^{k} y_{ij} = \sum_{l=1}^{n} l z^l_{kj} \qquad \forall j,k \in N \tag{12}$$

$$\sum_{l=0}^{n} z^l_{kj} = 1 \qquad \forall j,k \in N \tag{13}$$

$$y_{ij}, z^l_{kj} \in \{0,1\} \qquad \forall i,j,k \in N, \quad \forall l \in N \tag{14}$$

The cost function (8) follows formulae (1) and (2). Inequalities in (9) ensure that a truck cannot be assigned to a non-representative truck. Inequalities in (10) require that each truck is assigned to a representative truck unless it is a representative truck itself. Inequalities in (11) states that we cannot assign a truck to a representative truck whose arrival time is earlier than the earliest arrival time of the former. Inequalities in (12) and (13) ensure that z^l_{kj} is one if and only if the number of trucks assigned to the platoon represented by j from d_k to d_{k+1} (that is, $\sum_{i=1}^{k} y_{ij}$) is l. It is straightforward to check that any optimal solution to Problem 1 can be turned into an optimal solution (of same cost) to this formulation and vice-versa.

4.5 Numerical experiments

In order to assess the performances of the best ZIO solution and of the HEUR heuristic, and to understand the computational performances of the IP formulation, we have implemented the corresponding algorithms and the IP formulation in Julia. The IP formulation was solved using CPLEX 20.1, in the JuMP environment, with a precision (CPX_PARAM_EPGAP) set to 10^{-3}. All computations were ran on a MacBook Pro equipped with a 2 GHz Quad-Core Intel Core i5 processor and a RAM memory of 32 GB.

[8] we could remove the variables set to zero and sum over the appropriate indices.

We built the instances for the numerical experiments as follows. We have considered a highway section of 100 km, and trucks planning to arrive at destination E in the next hour. We have considered waiting costs of €$20, 40, 60, 80$ and 100 and we have limited our scope to $10, 30$ or 50 trucks arriving in the period. The key challenge for this numerical analysis relates to the estimation of travel costs. Our model assumes generic features for the joint traveling cost function and this allows for a high degree of flexibility in building the instances. Therefore, we decided to work with be best estimate available for constructing f_k. We assume that $f_1 =$€2/km, of which 30% relates to fuel cost, in accordance with discussions with trucking industry experts in Europe. As highlighted in Section 2, the estimates for fuel savings when traveling in platoons vary greatly. Therefore, we consider two options for fuel savings. For the first option, we assume that the leading truck reduces its fuel consumption by 18%, while the second truck saves 24% and the third truck saves 23%, in line with Lu and Shladover (2011). For the second option, we assume fuel saving of 1.6% for the lead truck, 7.6% for the second truck and 10.5% for the final truck in accordance with McAuliffe et al. (2018). Note that fuel savings for platoons formed with more than three trucks are not available in the literature. We have limited the permissible number of trucks in a platoon to $2, 3$ or 5. For instances with $L = 5$, we assume that the savings are similar for all trucks in a platoon except for the first and the last trucks. The two corresponding functions are given in Table 2: f^1 is the estimation according to Lu and Shladover (2011) and f^2 is the estimation according to McAuliffe et al. (2018).

Table 2: Joint traveling cost functions

k	1	2	3	4	5
f_k^1	2	3.754	5.610	7.466	9.322
f_k^2	2	3.9274	5.8818	7.8362	9.7906

For each choice of the above parameters (90 possibilities), we have generated 10 instances at random—hence, a total of 900 instances. The distance to E was generated uniformly at random in $(0, 100)$ and the earliest arrival time uniformly at random in $(0, 1)$. We report the empirical computational and approximation performances in Tables 3, 4, 5, 6 and 7. The approximation ratio tables report statistics on the value of the optimal ZIO (resp. HEUR) solution over the value of the solution found by CPLEX. Note that the ratio can be below 1 as the precision was set to 10^{-3} in CPLEX.

For the anticipated use of the tool at the planning stage (see Figure 2), quick response time is fundamental. Hence, the IP formulation introduced here does not provide an appropriate solution

Table 3: Computational time: IP formulation (in seconds)

n	mean	median	max	min
10	0.0438371	0.040341	0.134955	0.0276781
30	4.2121475	4.013390	7.622500	3.4200000
50	83.5907503	77.994550	255.933000	65.8844000

Table 4: Computational time: ZIO (in seconds)

n	mean	median	max	min
10	0.0001889	0.0001768	0.0004750	0.0001600
30	0.0018873	0.0016390	0.0306523	0.0013901
50	0.0097530	0.0065614	0.1649130	0.0055678

Table 5: Computational time: HEUR (in seconds)

n	mean	median	max	min
10	0.0017876	0.0016221	0.0349589	0.0014217
30	0.1201306	0.1132710	0.1453110	0.1011410
50	1.2857464	1.2087700	2.2335900	1.0755000

Table 6: Approximation ratio: ZIO

n	mean	median	max	min
10	1.000500	1.000000	1.012324	0.9992583
30	1.000507	1.000234	1.005211	0.9990998
50	1.000435	1.000188	1.004378	0.9991025

Table 7: Approximation ratio: HEUR

n	mean	median	max	min
10	1.0000389	1.0000000	1.003628	0.9992120
30	1.0000166	1.0000000	1.001976	0.9990998
50	0.9999606	0.9999119	1.001738	0.9990697

for instances with 50 trucks or more. Although one could try to use more advanced IP models and algorithmic techniques to solve the problem exactly, the performances offered by the two heuristics considered here would be more than enough in practice. Our numerical experiments show a small optimality gap on all 900 instances: the worst gap is 1.23% for the ZIO solution and 0.36% for the HEUR heuristic (observe that sometimes the heuristics perform better than the IP model; this is because the optimality tolerance was set to 10^{-3} in CPLEX; the computational performances would have been even worst for a smaller tolerance). Given the level of approximation of the tactical model (assuming constant speed, simultaneous arrival of trucks in a platoon, etc.), the gap can be considered negligible for both heuristics and, given the computational complexity (see Lemmas 2 and 3), we recommend the use of the ZIO solution. Moreover, in the practical situation where the number of trucks in a platoon is limited to a small L, as it is the case here, a (close to[9]) optimal ZIO solution can be computed in time $O(nL)$ by simply restricting the ZIO solution to consider only the next L trucks (at most) and thus it is linear in n. We have tested the corresponding implementation and it solves instances with 10,000 trucks in less than a second, which would be very suitable for practical situations.

For illustrating the structure of optimal solutions, we provide an example associate with an instance with 30 trucks in Figure 4 ($L = 5$ in this example).

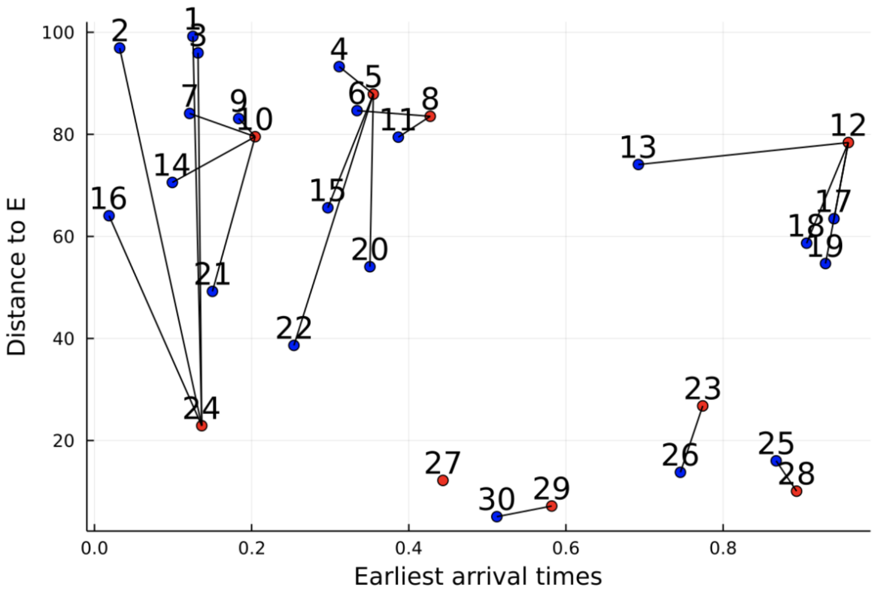

Figure 4: An example of a close-to-optimal solution to the IP formulation (with a tolerance of 10^{-3} in CPLEX). The red points represents the representative trucks and the lines linking blue points to a red point show the assignment of the trucks to a representative truck.

5 Cooperative Platooning Games and Cost Sharing

In this section we address the cost sharing problems that arise naturally in platooning situations that involve multiple operators. Our main focus is the problem of redistributing the cost of the system-wide solution computed in the previous section in a 'fair' way. It falls into the scope of collaborative (cost) games. We focus on two fundamental concepts in collaborative game theory: the core and the Shapley allocation. We provide some basic definitions and properties related to

[9]It is close to optimal as in this implementation, we do not consider the possibility to reorganize subplatoons according to the optimal F_k: if, for instance, two platoons of L consecutive trucks arrive at the same time, we only exploit observation in Subsection 3.1 in each subplatoon, but not over the union of the two.

cooperative games and the latter concepts in Appendix B.

Given a platooning situation, we focus on the associated game and refer to it as the *cooperative platooning game*. In this case, given the set of trucks N, we consider the game (N, C) where for each subset $S \subseteq N$, the characteristic function C is defined as the cost of an optimal solution to the platooning problem (see Problem 1) restricted to S.

One of the challenges in dealing with the cooperative platooning games through the concepts of core and the Shapley allocation is the complexity of computing $C(S)$ for all $S \subseteq N$. As discussed in the previous section, the computational complexity is open and we do not yet have an exact algorithm that enables us to solve large-size instances. Hence, if computational tractability is of importance, it is reasonable to restrict attention to variants of these games defined with respect to approximate characteristic functions. We consider here the variants defined with respect to the ZIO heuristic. Remember that the ZIO heuristic builds upon an arrangement of trucks by non-decreasing order of earliest arrival times, that is, $t_1 \leq ... \leq t_n$. We define, subsequently, the *consecutive platooning game* (N, H) where for every $S \subseteq N$, $H(S)$ represents the minimum cost that coalition S can pay (that is, without involving other players) if the platform uses the ZIO heuristic to solve the optimization problem. More formally, assume S is of the form $S_1 \cup ... \cup S_\mu$, for some $\mu \geq 1$, where $S_1, ..., S_\mu$ contains only consecutive elements and $\max\{i \in S_\nu\} < min\{i \in S_{\nu+1}\}$ for all $\nu = 1, ..., \mu - 1$. We define $H(S) = ZIO(S_1) + ... + ZIO(S_\mu)$, where $ZIO(S_\nu)$ represents the cost of the best ZIO solution when we restrict to elements in S_ν for $\nu = 1, ..., \mu$ (note that the ZIO heuristic may create subplatoons in each set S_ν). Accordingly, we study this approximation game in conjunction with the original cooperative platooning game.

5.1 The core

We have seen in Section 3 that the optimization problem associated with a platooning situation can be represented by a set partitioning formulation, i.e., program (3). Subsequently, the cooperative platooning games are special cases of set partitioning games (Deng et al., 1999). The fundamental result regarding the core of partitioning games is presented by Deng et al. (1999) where they show that the core is non-empty if and only if the integrality gap of the associated optimization problem is zero (that is, the linear relaxation has the same optimal value). This result would apply to platooning games and their associated set partitioning formulations. Nevertheless, as the following example shows, the platooning games may have empty cores in general.

Example 3. *Consider a platooning situation with three trucks. Suppose $d_1 = d_3 = 210$ and $d_2 = 140$. Earliest arrival times are $t_1 = 3$ and $t_2 = r_3 = 3$. Waiting cost is $p = 20$ per unit of time. The joint traveling cost is $f_1 = 2$, $f_2 = 3.7$, and $f_3 = 5.5$. Slikker and Hezarkhani (2018) show that the core of (N, C) is empty whenever $\sum_{i \in N} C(N \setminus \{i\}) < (|N| - 1)C(N)$. In this example we have $C(\{1,2\}) + C(\{1,3\}) + C(\{2,3\}) < 2C(N)$, thus the core is empty.* △

Interestingly, this is not the case for the consecutive platooning game, as shown in our next result.

Theorem 3. *The core of a consecutive platooning game is non-empty and we can find a point in the core in time $O(n^2)$. Assuming the trucks are ordered so that $t_1 \leq \ldots \leq t_n$ and with the convention that $ZIO(0) = 0$, this point is:*

$$\varphi_i(N, H) = ZIO(i) - ZIO(i-1) \qquad \forall i \in N \tag{15}$$

Allocation (15) is a by-product of the ZIO heuristics which we devised earlier. That is, there is no need to formulate and solve a separate optimization problem for obtaining the cost allocations in the core (a problem which is burdensome to construct due to the large number of constraints associated with all possible sub-coalitions). In this case, the cost allocations can be obtained directly by finding, for each player, the marginal costs of adding this player in the dynamic program. We refer to this allocation as the ZIO allocation.

5.2 The Shapley allocation

The Shapley allocation provides an alternative mechanism to the core for 'fair' distribution of costs among the players. Although we do not know how to compute the Shapley allocation efficiently in general for the different games considered in this paper in general, yet we provide the Shapley allocation for the platooning games under a certain assumption (see Assumption 1). This is an interesting result on its own as it also solves a question raised in Gopaladesikan et al (2012) for lot-sizing games.

We start with the special case of platooning games with no waiting costs. We assume in the following that a set $S \subseteq N$ forms a platoon and assume w.o.l.g. that the trucks are arranged such that $d_1 \geq \ldots \geq d_n$. In the absence of waiting costs, we have $C(S) = Z_S = Q_S$—since the optimal solution is to always group as many trucks as possible given the sub-additivity of f. The cost of a coalition S can thus be re-written (with the convention $d_{n+1} = 0$) as:

$$C(S) = \sum_{k=1}^{n} (d_k - d_{k+1})\, f_{|S\cap\{1,\dots,k\}|}. \tag{16}$$

The platooning game (N, C) in this case is a (non-negative) weighted sum of games (N, C_k) for $k \in N$ with $C_k(S) = (d_k - d_{k+1})\, f_{|S\cap\{1,\dots,k\}|}$. By linearity of the Shapley allocation, it is enough to focus on individual games (N, C_k) for a given $k \in N$. Each player $i \in N$, $i \le k$, is symmetric in game (N, C_k), as for any $i, j \le k$ and for any S with $S \cap \{i, j\} = \emptyset$, we have $C_k(S \cup \{i\}) = C_k(S \cup \{j\})$. Moreover, each player $i \in N$ with $i > k$, is a dummy player in game (N, C_k), as for any $i > k$ and for any S we have $C_k(S \cup \{i\}) = C_k(S)$. It follows that in the Shapley allocation, the cost $C_k(N)$ will be split equally between the players $i \le k$. Observing that $C_k(N) = (d_k - d_{k+1})\, f_k$, we get the following lemma.

Lemma 4. *In the absence of waiting costs, and assuming w.l.o.g. that the players are arranged such that* $d_1 \ge \dots \ge d_n \ge d_{n+1} = 0$*, the Shapley allocation for the platooning game* (N, C) *is:*

$$\Phi_i(N, C) = \sum_{k=i}^{n} (d_k - d_{k+1})\, \frac{f_k}{k}, \qquad \forall i \in N \tag{17}$$

The result above is similar to the expression of the Shapley allocation for the airport games as described by Littlechild and Owen (1973). Although the Shapley allocation is not necessarily situated in the core, our next result indicates special cases when this is the case.

Remark 1. *In the absence of waiting costs and assuming, w.l.o.g., a sub-additive* f*, the following statements hold:*

- *If* f *is concave, the game is concave and the Shapley allocation is in the core;*
- *If* $f_{k'} \le \frac{k'}{k} f_k$ *for all* $k' \ge k$*, the Shapley allocation is in the core.*

Note that by Observation 1, f_k can always be assumed to be sub-additive in k. When the joint traveling cost function is concave, the cooperative platooning game would be concave and, subsequently, the Shapley allocation is in the core. The Shapley allocation is also in the core for a slight relaxation of the concavity assumption. The second condition in Remark 1 is sometime referred to as non-negative homogeneity. Our result in this case prove particularly interesting in practice as the functions described in Table 2 are not concave but satisfy this property. We next show how to go beyond Remark 1 and deal with non-zero waiting costs. The following assumption and observation are crucial.

Assumption 1. *Given a platooning game* (N, C)*, for any subset* $S \subseteq N$*,* S *is an optimal solution to the platooning problem associated with* S*, that is, the solution to the set partitioning problem* $C(S)$ *in (3) is* $y^*_S = 1$*.*

The above assumption applies to the situations where it is always optimal to group as many trucks as possible given any subset of N. Assumption 1 holds in the absence of waiting costs, but other situations also fall into the scope of this assumption. For instance, platoon games associated with a traveling cost function of the form $f_k = K + ak$, where the trucks all share a common starting location, and where N is a *minimal optimal platoon* (that is, there is no optimal platooning solution to N made of two or more platoons) satisfy the assumption: if a subset of trucks $S \subset N$ would benefit from using two or more platoons, and say S_1 (resp. S_2) is the first (resp. second) platoon scheduled, the grand coalition N would also benefit from using platoon S_1. Merging S_1 and S_2 in the solution associated with S leads to an additional cost (by optimality of the solution using S_1 and S_2 for S) of $p|S_1|(\max\{t_i, i \in S_2\} - \max\{t_i, i \in S_1\}) - K \geq 0$ and thus, vice-versa, using platoon S_1 in the solution to N would lead to a reduction of cost of $p|S_1|(\max\{t_i, i \in S\} - \max\{t_i, i \in S_1\}) - K \geq p|S_1|(\max\{t_i, i \in S_2\} - \max\{t_i, i \in S_1\}) - K \geq 0$, which is a contradiction to the optimality of platoon N or its minimality. As discussed earlier, this special case is interesting in the lot-sizing literature as it tackles a question from Gopaladesikan et al (2012) regarding the Shapley allocation and Corollary 1 offers some partial answer.

Observation 2. *Given the platooning game* (N, C)*, and under Assumption 1, for any* $S \subseteq N$*, we have:*

$$C(S) = \sum_{k=1}^{n} (d_k - d_{k+1}) f_{|S \cap \{1,\ldots,k\}|} + p \sum_{j \in S} (\max\{t_i, i \in S\} - t_j) \tag{18}$$

Drawing upon Observation 2, linearity of the Shapley allocation, and Lemma 4, we can compute the Shapley allocation for any platooning game satisfying Assumption 1, if we can compute the Shapley allocation for the *platoon waiting game* (N, W), where for every $S \subseteq N$ we have $W(S) = \sum_{j \in S} (\max\{t_i, i \in S\} - t_j)$, efficiently. The next lemma shows that this is the case.

Lemma 5. *The Shapley allocation of the game (N,W) is, for every $i \in N$:*

$$\Phi_i(N,W) = \sum_{j:t_j<t_i} \frac{(t_i - t_j)}{n!} \sum_{k=0}^{n_j-1} (k+1)!(n-k-2)!(k+1)\binom{n_j-1}{k} + \sum_{j:t_j>t_i} \frac{(t_j - t_i)}{n!} \sum_{k=0}^{n_j-2} (k+1)!(n-k-2)!\binom{n_j-2}{k} \tag{19}$$

where for all $j \geq 1$, $n_j := |\{k : t_k \leq t_j\}|$.

By linearity of the Shapley allocation, we get the following corollary by combining Lemma 4 and Lemma 5.

Corollary 1. *Under Assumption 1, and assuming w.l.o.g. that the players are arranged such that $d_1 \geq \ldots \geq d_n \geq d_{n+1} = 0$, the Shapley allocation of the platooning game (N,C) is:*

$$\Phi_i(N,C) = \sum_{k=i}^{n} (d_k - d_{k+1}) \frac{f_k}{k} + \sum_{j:t_j<t_i} \frac{(t_i - t_j)}{n!} \sum_{k=0}^{n_j-1} (k+1)!(n-k-2)!(k+1)\binom{n_j-1}{k} + \sum_{j:t_j>t_i} \frac{(t_j - t_i)}{n!} \sum_{k=0}^{n_j-2} (k+1)!(n-k-2)!\binom{n_j-2}{k} \quad \forall i \in N \tag{20}$$

where for all $j \geq 1$, $n_j := |\{k : t_k \leq t_j\}|$.

5.3 On a fair cost-sharing mechanism for the platooning game

In the previous two sections, we have proposed different mechanisms to share costs among the different players in the platooning game and its approximation, i.e., consecutive platooning game. No solution is a panacea and we need to compromise on some aspects (e.g., stability, existence, efficiency of calculation) to implement some solution in practice.

To design a practical solution, we need in particular to decide on both how to solve the optimization problem and how to share the costs associated with the corresponding solution. As already discussed in the previous section, we advocate for solving the optimization problem with the ZIO heuristic in practice as its computational and optimality performances are excellent. Given this choice we would then advocate to choose the allocation proposed by Theorem 3 as is has several virtues. First it is a by-product of the optimization. Second, it is in the core of the consecutive platooning game and thus no coalition that can arise from solving the optimization problem with the ZIO heuristic would benefit from leaving the proposed platoon. A key issue with the ZIO allocation is that there are still some players that would benefit from leaving their coalition and join a new one. It might even happen inside a platoon, as shown by Example 4.

Example 4. *Consider a three-item problem with* $t_1 = 0$, $t_2 = 0.5$, $t_3 = 1$, $d_1 = 1$, $d_2 = \epsilon$, $d_3 = 1$, $f = [1, 1, 1]$, *and* $p = \frac{2}{3}$, *for some* $0 < \epsilon << 1$ *(note that this resembles the situation in Example 1, except for the joint travelling cost). The ZIO solution is to make a platoon of all players at the total cost of 2. The allocation* $\boldsymbol{\varphi} = (1, \epsilon, 1 - \epsilon)$ *is in the core of the consecutive platooning game. However, for any* $0 < \epsilon < 1/3$, *the coalition* $\{1, 3\}$ *could do better together.* △

In practice this can happen only if the platoons are of small size (in this case a subgroup of players could identify that they can be better off by simply enumerating over all possibilities) as otherwise players would face computational difficulties to identify such situations too. When L is small, we can easily detect such critical situations by solving the optimization problem over each platoon (created by the ZIO heuristic) with the IP model in Section 4.4 (or simply by enumeration if L is very small). Moreover, for such small L, it is also possible in principle to check if the core of the platoon game is empty - and to compute a solution in the core (if it is non-empty) by enumerating all subsets and by solving a linear program with L variables and $O(2^L)$ constraints. Actually, if the core is empty, it is also possible to compute the Nucleolus in such a case by using a sequence of linear programs (the so-called Maschler's scheme Maschler et al. (1979)). Computing the Shapley allocation can be done in a similar way. We could then choose one of these mechanisms to guarantee more fairness inside each platoon.

6 Application and Insights

We apply our model to an illustrative example derived from the settings of the Port of Rotterdam. The Port of Rotterdam Authority has developed an ambitious program to foster the development of truck platooning. Accordingly, we expect the Port of Rotterdam to be one of the first places in Europe to allow for implementation of large-scale truck platooning. Note that truck platooning is considered as particularly attractive for drayage operations (You et al., 2020). A key area for such a development is the motorway connecting the extensions of the port (Maasvlakte 1 and 2) to the ring of the city of Rotterdam, that is, the A15. The objective of the Port Authority is "to have at least a hundred truck platoons a day driving on the A15, comprising at least three hundred trucks" (Port of Rotterdam Authority , 2018). We assume that platoons can form at Maasvlakte and that all trucks need to reach the ring of Rotterdam, the final point in their journey where platooning is allowed. The distance between Maasvlakte and the ring is $E = 43$ km. We assume that some trucks become available at the Euromax terminal in Massvlakte ($x = 0$ km) while some others become available at the Plaza, a large truck stop where drivers can take their obligatory rest periods ($x = 14$ km). The average speed of trucks on Dutch motorways is $v = 80$ km/h.

Recall that participating trucks equipped with platooning technology are required to signal their starting locations and their ready times to the platooning platform prior to operations in order to allow proper planning of platoons. On the one hand, this cannot be done too far in advance as trucks would find it difficult to accurately provide the required information. On the other end, too short a notice would restrict the ability to plan operations properly. In what follows, we accordingly model one hour of operations as this is considered as a fair trade-off based on discussions with trucking industry experts in Europe. Note that we additionally discuss below the sensitivity of our results to the planning frequency.

It is quite challenging to estimate the market penetration of truck platooning technology. We take the current market penetration of adaptive cruise control technology in the USA as a benchmark. According to the American Trucking Association, there are about 3.43 million Class 8 trucks in operation in the USA, of which about 100,000 are equipped with adaptive cruise control technology, leading to a market penetration of 2.92%. According to Gerrits et al. (2019), 155 trucks per hour travel from Maasvlakte to the ring of Rotterdam via the A15. Therefore, we can estimate that about five trucks per hour ($155 \times 2.92\%$) equipped with platooning technology will travel from Maasvlakte to the ring of Rotterdam via the A15. Accordingly, we consider instances with $n = 5$ trucks in what follows. Then, we extend our analysis to larger market penetrations by considering examples with $n = 10$, $n = 30$ and $n = 50$ trucks.

We refer to Subsection 4.5 for a detailed explanation on how we constructed traveling cost functions and to Table 2 for related data. Through this case study, we consider maximum physical platoon length $L = 2$ and $L = 3$ as this is in line with current practice for pilot trials. We additionally consider three values for the nominal waiting cost proposed by the platform, that is, $p =$€10/h, $p =$€20/h and $p =$€30/h. This corresponds to a practically reasonable range for nominal waiting costs as truck drivers have to stop for 45 minutes after every 4.5 hours of driving, and consequently, they have some flexibility over their departure times.

We conduct a full factorial analysis by considering all possible combinations of the above mentioned parameters and we consider 10 instances for each combination. This leads to 480 instances

$(4n \times 2f_i \times 3d \times 2L \times 10)$. For illustrative purposes, we select starting locations $x_i = 0$ or $x_i = 14$ randomly with equal probabilities. Ready times (expressed in hour) are generated uniformly at random in $[0, 1]$. The minimum value of the waiting cost for which a truck i would agree to participate, i.e., ρ_i, is generated uniformly at random in $[0, 30]$. Accordingly, we assume that all trucks for which $\rho_i \leq p$ will join the platform. Recall that we define these trucks as *participating trucks*. The number of participating trucks depends on n, p, and the distribution of ρ_i. For all instances, we identify an optimal ZIO solution and a ZIO allocation in accordance with our analysis and recommendations in Sections 4 and 5. In what follows, we present and analyze the results and derive some managerial insights. Note that through this section, we assume that a platoon is composed by at least two trucks.

To begin with, out of the 480 instances examined, there were 450 instances (90%) which include at least one platoon. Besides, 48% of all trucks and 70% of the participating trucks on average form a platoon. These results are very encouraging and they show the potential extent of development for platooning initiatives in the Port of Rotterdam area. Note that platoons are formed with trucks that do not depart either at the same time or from the same location. This implies that planning of truck platooning is key. If we limit our analysis to the 120 instances for which $n = 5$, then 79 instances (66%) include at least one platoon. This shows that even with a low market penetration of the platooning technology, platoons will easily form in the Port of Rotterdam if platooning is planned properly. We highlight that half of our instances assume low fuel savings due to platooning in accordance with the results of McAuliffe et al. (2018) which prescribe particularly low fuel savings. Note that at least one platoon is formed for 50% of the instances with $n = 5$ and low fuel savings. Our results also enable us to estimate the market penetration of the platooning technology required to achieve the objective of the Port of Rotterdam Authority. If we extend our results to a full day of operations, we expect the Port of Rotterdam Authority to meet its objective in terms of the number of platooned trucks for a market penetration of about 15% ($n = 23$), and in terms of the number of platoons for a market penetration of about 12% ($n = 19$).

Observation 3. *Platoons are likely to form in the Port of Rotterdam even with a low market penetration of the platooning technology and low fuel savings, if properly planned. The objectives of the Port of Rotterdam Authority would be met if about 15% of the trucks are equipped with the platooning technology.*

We study the percentage of fuel savings due to platooning. To do so, we compare the fuel consumption of the obtained ZIO solution with the fuel consumption in case all trucks travel alone. This enables us to evaluate the practical efficiency of platooning in terms of fuel savings. We refer to the latter as *effective* fuel savings. This can be viewed as the average fuel savings among all trucks, independently of whether they join a platoon or not. In comparison, fuel savings obtained in the literature accounts for savings only during the platooned leg of travel. We refer to the latter as *nominal* fuel savings. Note that we include all trucks in our calculations, that is, the average takes into account non-participating trucks as well. The results appear as box plots in Figure 5 (with the average represented by a cross). We additionally represent box plots for subsets of instances as follows. We consider instances with different values of n, p and L separately as well as instances with the fuel savings of Lu and Shladover (2011) only (referred to as as "fuel 1" instances), and instances with the fuel savings of McAuliffe et al. (2018) only (referred to as "fuel 2" instances).

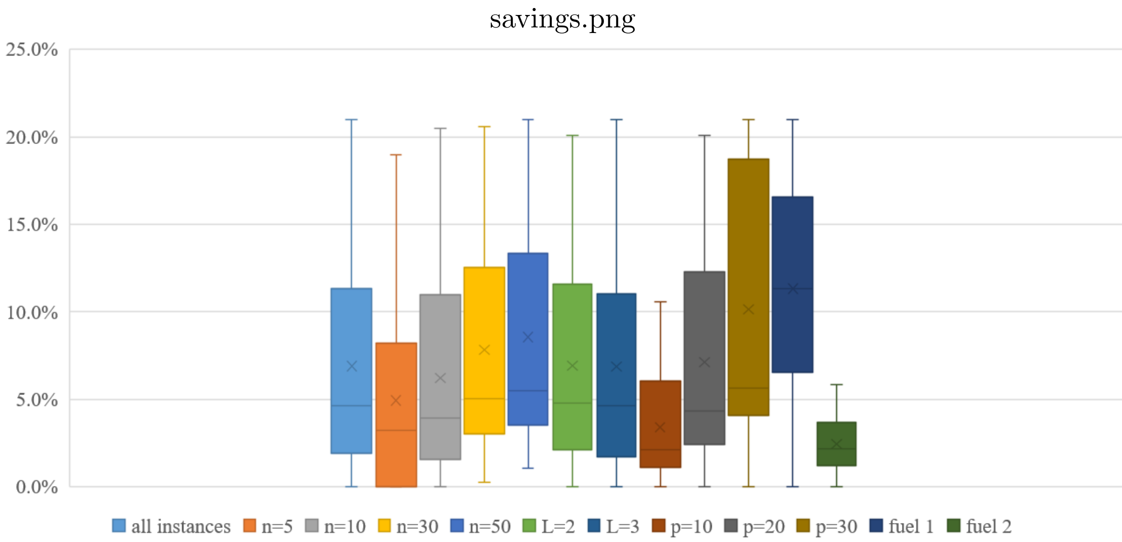

Figure 5: Percentage of effective fuel savings due to platooning

We can derive several observations from Figure 5. Firstly, the average effective fuel saving is 6.9%. This is a promising result which highlights the value of platooning for sustainable logistics. Secondly, the variation of n has a moderate effect on fuel savings (average fuel savings increase from 4.9% to 8.5% when n increases from $n = 5$ to $n = 50$). This implies that platooning can be efficient for reducing fuel consumption even when the market penetration of the technology is low. Third, the largest variation in fuel savings is observed when comparing the fuel 1 against fuel 2 instances. The average effective fuel savings varies from 11.3% to 2.4%. There are indeed two effects when nominal fuel savings decrease. This directly influences the efficiency of the system in terms of savings, but this also indirectly reduces the formation of platoons as well as the distance traveled in platoons. On average, 81% of the participating trucks form a platoon for the fuel 1 instances as compared to 59% for the fuel 2 instances.

Observation 4. *Truck platooning is quite effective to reduce fuel consumption in practice. However, a reduction in nominal fuel savings has a strong effect on the number of trucks involved and the distance traveled in platoons.*

We further study the cost savings obtained from truck platooning. To this end, we compare the costs of the ZIO solution obtained with the costs incurred if all trucks traveled alone in each instance. Note that we include only participating trucks in these calculations. The results appear in Figure 6. We can observe that the average cost savings is 2.4%. Note that this calculation underestimates the real savings as it is based on nominal waiting penalty chosen by the platooning platform. Our calculations, therefore, serve as a lower bound on the real costs savings. There are also indirect benefits not accounted for in this article such as an improved company image by being perceived as environmentally friendly and by being viewed as technologically more advanced. Additionally, a 2.4% reduction in operating costs can create a competitive advantage. Finally, the technology will become available with more advanced developments of truck automation (level 4 and level 5), and, subsequently, platooning will become a cost-effective and easy to implement practice. We can also observe from Figure 6 that the largest variation in cost savings is observed when comparing the fuel 1 instances with the fuel 2 instances. The average saving is 4.0% for fuel 1 instances versus 0.7% for fuel 2 instances.

savings.png

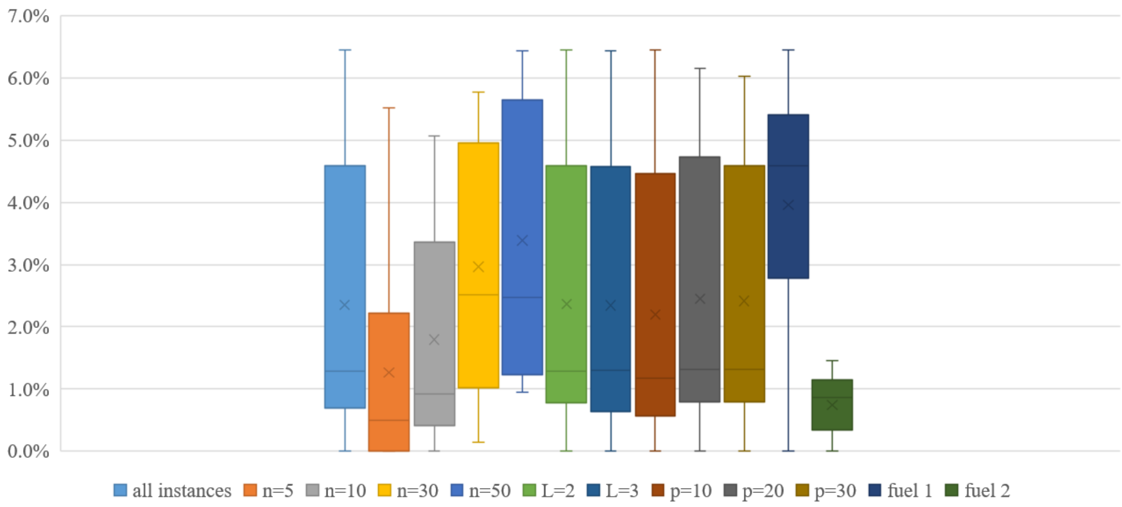

Figure 6: Percentage of cost savings due to platooning

Observation 5. *Platooning enables a slight decrease in operating costs even when calculated based on a lower bound of savings and with low market penetration. Therefore, this is likely that truck platooning will become a widespread practice with the advancement of truck automation.*

We also assess the influence of the nominal waiting penalty proposed by the platform. At first, we can observe from Figure 5 that p plays quite a significant role in terms of effective fuel savings. The optimal choice is $p = 30$ leading to effective fuel savings of 10.1% (as compared to 3.4% fuel savings for $p = 10$). When p is small, the number of participating trucks is low (due to the reservation for not being compensated sufficiently for the waiting) and this reduces effective fuel savings. The percentage of participating trucks varies from 37% for $p = 10$ to 100% for $p = 30$. On the other hand, we can observe from Figure 6 that average cost savings are less sensitive to variations in waiting penalty. Moreover, the highest average cost savings of 2.45% are obtained for $p = 20$ (with average fuel savings of 7.1%). Even if the increase in p positively influences the number of participating trucks (which can positively influence platoon formation and cost reduction), this increase in nominal waiting penalty reduces the advantages of platooning. This shows a potential tension between the cost-optimal and the environmentally-optimal choices of nominal waiting cost by the platform. Yet, the cost saving is 2.42% for $p = 30$. This shows that a marginal increase in costs might lead to a large decrease in fuel consumption.

Observation 6. *Medium nominal waiting penalties proposed by the platform lead to higher cost savings. However, higher nominal waiting penalties would enable an increase in the number of participating trucks. This positively affects fuel savings with a very limited impact on costs.*

We can also notice that participating trucks wait, on average, for about 1.5 minutes in the instances studied. The maximum time that a truck needs to wait is less than 5 minutes for more than 96% of the instances studied. We can conclude that the increase in waiting time seems very reasonable for practical applications. This result is driven by two reasons. Firstly, fuel cost is only a small share of travel costs and therefore, moderate fuel savings lead to small savings in travel costs. Consequently, trucks would not be willing to wait for too long even for $p = 10$. Accordingly, average waiting for participating trucks is about 1.7 minutes in the instances with $p = 10$. Secondly, due to limits on the length of physical platoons, many platoons of small size tend to be formed. This reduces waiting time. These low waiting times also translate into a very moderate effect on the planning horizon chosen under reasonable boundaries. Recall that we selected a planning horizon of one hour as a fair trade off. An increase in this planning horizon would have a marginal impact on the performance of the system, however, this would reduce the motivation to participate for trucking companies and trucks drivers.

Observation 7. *The average waiting time for the participating trucks is very low irrespective of the waiting penalty chosen. Moreover, short planning frequencies are close to optimal. These two characteristics strengthen the applicability of truck platooning.*

We next focus on the issues related to sharing the costs of platoons in the associated cooperative games. For all instances, we calculate a ZIO allocation, which, as shown in Theorem 3, is always in the core of the consecutive platooning games, and can be found in polynomial time. We then test whether these allocations would raise any 'objection' by the players with regard to their stability. This requires checking the solution to the integer relaxation of the associated partitioning game to see whether the solution to the relaxation satisfies the integrality constraints. Accordingly, we found out that in 281 instances out of 450 (62%) the allocations are actually raise no objections. In fact, in more than 75% of cases the allocated costs are at most 3% higher than the optimal coalition costs.

Observation 8. *The core of the platooning game can be empty in real-life applications. Still, the proposed ZIO allocation serves as good substitute as this one is highly likely to produce small deviations from stable allocations.*

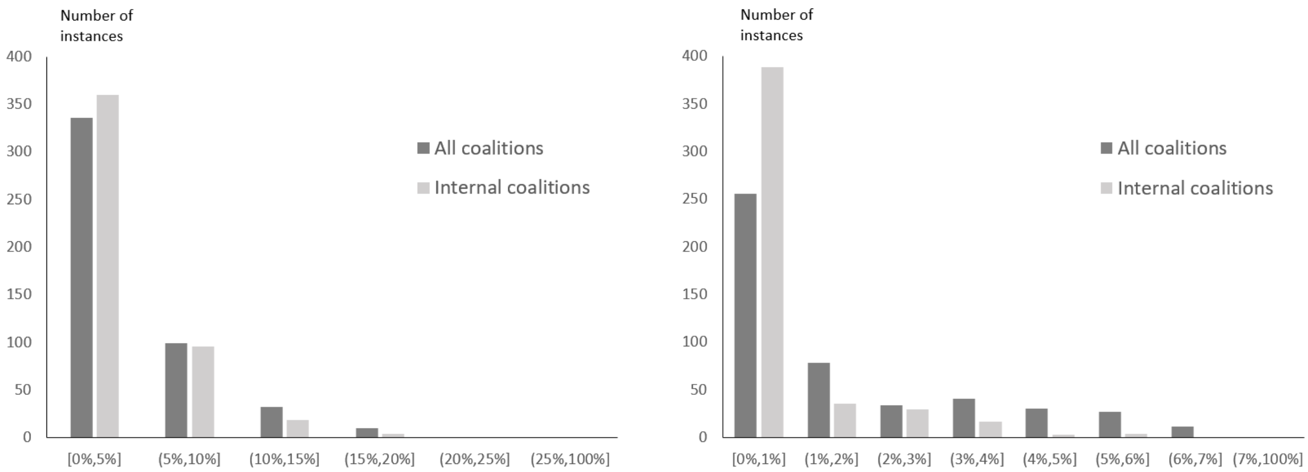

Figure 7: Histogram representation of stability metrics: [Left] Percentage of coalitions for whom the cost allocation violates stability; [Right] Maximum ratio of allocated cost to optimal coalition costs

Figure 7 further illustrates the behaviour of allocated costs in our instances across two metrics. On the left hand side, we check the number of coalitions which would have an objection to their allocated costs, that is, the coalitions for whom the allocations violates the stability constraint. On the right hand side, we examine the magnitude of such objections upon their existence. We analyze these metrics for two types of coalitions: *all coalitions*, which allows players to compare their performance with players outside of their platoons, and *internal coalitions* where players are only able to form sub-coalitions within their determined platoons. Our numerical examples show that violation of stability often happens at exceptional rate, that is, only in 9% of instances the stability conditions are violated for more than 10% (and at most 20%) of all coalitions. This is even less pronounced if we restrict to internal coalitions (only in 5% of instances we see instability for more than 10% of coalitions). The magnitude of violations are also often negligible. That is, in 92% of instances the maximum violation of stability is within 5%. Naturally, if we only consider the possibility of colluding with players within the determined platoons, in 95% of instances the players can improve their total cost savings by at most 3%. We believe that these small deviations would render our allocations viable in practice.

Observation 9. *The proposed ZIO allocation is a reasonable choice in practice as instability of coalitions are uncommon and the magnitude of such objections are often negligible. Objections to ZIO allocations would be even smaller if players can only communicate with players within their assigned platoons.*

7 Conclusions

Truck platooning among multiple operators is the most likely future for truck platooning and a key milestone towards full truck automation. In this paper we addressed two key problems in the planning of truck platooning. First, we studied the optimization of platoon formation. We formalized the related optimization problem that exhibits strong connections with lot-sizing models. This allowed us to propose exact and approximate solution approaches. Among them, our numerical experiments emphasize the practical advantages of the ZIO solution. Second, we addressed cost sharing issues based on cooperative game theory. Our results show that a compromise is needed between existence, stability and computational complexity. However, our new ZIO allocation appears to perform very well in practice. Finally, we provided a series of insights which can be used as guiding principles for dealing with real-life situations.

We foresee three main avenues for future research. At first, the computational complexity of the optimization problem related to the planning of platoon formation is still open. We were able to show that a non-trivial special case can be solved in polynomial time. However, we were not able to conclude for the general case despite our efforts. Our ZIO solution also appears to perform very well for practical size instance. We believe that future research could aim to identify under which conditions the best ZIO policy provides a good approximation with performance guarantee. Second, we consider a sub-additive joint traveling costs in this article, as this is the most general way to model economies of scale. Future research could consider stronger assumptions such as elastic costs—see, for example, Bouchery et al. (2020) for an application to intermodal transportation—or concave costs. Note that the traveling cost data obtained though the illustrative example we presented do not meet the concavity assumption. However, we believe that concave traveling cost could be encountered under specific settings such as driverless platooning. We believe that additional theoretical results could be derived under these assumptions, both on the scheduling and on the cost sharing side. Third, deciding how to arbitrate between the corresponding cost sharing mechanisms (or even proposing some kind of compromise) is a non-trivial task and we believe that it would deserve an in-depth analysis. This can also be done by defining more specific fairness criteria and test existing and new allocation rules for ascertaining the achievement of such requirements. We leave this for future research.

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