

Quantum friction in the presence of a perfectly conducting plate

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A neutral but polarizable particle at rest near a perfectly conducting plate feels a force normal to the surface of the plate, which tends to pull the particle towards the plate. This is the well-known Casimir-Polder force, which has long been theoretically proposed and experimentally observed. In this paper, we explore the transverse frictional force on an atom moving uniformly parallel to a perfectly conducting plate. Although many theoretical predictions can be found for the quantum friction on a particle moving above an imperfect surface, the extreme situation with a perfectly conducting plate seems to have been largely ignored by the theoretical community. We investigate this ideal case as a natural extension of our previous [work](#) on quantum vacuum friction, and conclude that there does exist a quantum frictional force on an atom moving above a perfectly conducting plate, [which we will abbreviate by PCQF. Like quantum vacuum friction, PCQF arises from the interaction between the particle and the surrounding blackbody radiation. But, the behavior of PCQF differs from the quantum vacuum friction, in that the vacuum fields are modified by the perfectly conducting plate.](#) Very interestingly, the distance dependence, the temperature dependence and even the sign of the frictional force can depend on the polarization state of the atom. For an isotropic atom with a static polarizability, the resultant frictional force is found to be negative definite and therefore remains a true drag. Just above the surface of the plate, the magnitude of the frictional force is twice that of the quantum vacuum friction in the absence of the plate.

I. INTRODUCTION

It is well known that, when a neutral but polarizable particle sits near a perfectly conducting (PC) plate, it feels a force normal to the surface, pulling it towards the plate. This attractive force is often named after Casimir and Polder, who predicted it back in 1948 [1]. The Casimir-Polder force was first experimentally confirmed by measuring the deflection of a sodium atom beam passing through a gold cavity in 1993 [2]. [Ever since, there have been many more ingenious experiments, not only reassuring the existence of the Casimir-Polder force, but also detected the force at different \(non-retarded, retarded and thermal\) regimes \[3–6\]. For review of the Casimir-Polder forces or the broader context of Casimir interactions, see, for example, Refs. \[7–10\].](#)

[Here, we ask the question:](#) will a force [tangential](#) to the surface of the PC plate arise when the particle moves parallel to the plate?

Even though the subject of quantum friction (QF) with a dielectric surface has been much discussed in the literature, this more idealized case involving a PC plate seems to have been largely ignored. The lack of discussion of this case may be due to an [“intuition”](#) arising from the image particle picture. One might think that the interaction between the particle and the PC plate can be entirely mimicked by the particle’s interaction with its image. As the particle moves above the plate, the image moves below the plate. Because the plate is perfectly conducting, the image keeps up with the particle and is always located at the mirror position of the particle. Consequently, any interaction between the two would only lie in the direction normal to the surface of the plate and no force in the transverse directions could possibly arise. This reasoning sounds convincing except that it ignores one important aspect: the particle interacts with the blackbody radiation surrounding it even when the plate is taken away. [Taking blackbody radiation into account does result in a frictional force on the moving particle.](#) The nonrelativistic discussion of frictional force on particles moving in free space filled with only blackbody radiation can be traced back to the works of Mkrtchian et al. [11] or even Einstein and Hopf [12]. Ever since, there has been considerable interest in the subject of blackbody friction/quantum vacuum friction (QVF) [13–16]. Recently, we have also investigated such quantum vacuum frictional effects on a particle moving with relativistic velocities, be it a nondissipative atom [17] or an intrinsically dissipative nanoparticle [18]. Now, when a PC plate is added into the configuration, the vacuum field in the vicinity of the plate

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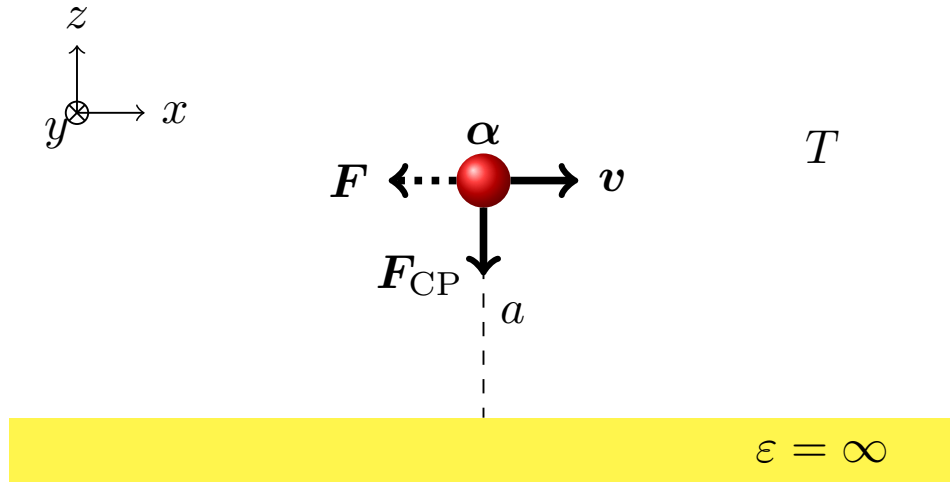


FIG. 1. Illustration of an atom moving parallel to a PC plate.

will be different from that of the free space considered in Refs. [17, 18]. We therefore expect the QVF to be modified and become spatially varying in the normal direction. For convenience of presentation, we will refer to this quantum frictional force on a neutral particle passing above a PC plate as PCQF.

In this paper, we focus on calculating the PCQF for a nondissipative atom. This is somewhat simpler than the calculation of PCQF for a dissipative nanoparticle, where the temperatures of the particle and of the environment enter the problem independently. The discussion of PCQF for the dissipative nanoparticle is postponed to a subsequent paper.

Throughout the paper, we set $k_B = c = \hbar = 1$ in the analytic expressions. SI units are reinstated in the numerical evaluations.

II. GENERAL THEORY

The physical situation we consider is illustrated in Fig. 1. A PC plate lies in the x - y plane. An atom is at a distance a from the plate and moves in the x direction with constant velocity v . The polarizability tensor of the atom is $\alpha(\omega)$, which could be dispersive in frequency and have different components corresponding to different polarization states of the atom. Since the atom we consider is intrinsically nondissipative, $\alpha(\omega)$ is a real quantity. The radiation background is at finite temperature T . We assume the PC plate is in thermal equilibrium with the radiation background. Because of its motion, the atom is not in equilibrium with the radiation. However, it is guaranteed, by the optical theorem [19], to be in the nonequilibrium steady state (NESS) [20], and it does not have an independent temperature [17].

There exists two special frames in the problem: the rest frame of the atom, \mathcal{P} , and the rest frame of radiation, \mathcal{R} , in which the background radiation is isotropic. It is more convenient to work in frame \mathcal{P} and we find the quantum frictional force (tangential component of the force) on the atom to be:¹

$$F = \int \frac{d\omega}{2\pi} \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{d^2\bar{\mathbf{k}}_\perp}{(2\pi)^2} (\bar{k}_x - k_x) \text{tr} [\alpha(\omega) \cdot \Im \mathbf{g}'(\omega, \mathbf{k}_\perp; a, a) \cdot \alpha(\omega) \cdot \Im \mathbf{g}'(\omega, \bar{\mathbf{k}}_\perp; a, a)] \coth \frac{\beta\gamma(\omega + \bar{k}_x v)}{2}, \quad (2.1)$$

where $\gamma = 1/\sqrt{1-v^2}$ is the relativistic dilation factor. Notice that the frequency in the thermal factor is Doppler shifted. Both the atom's polarizability tensor, α , and the Green's dyadic, \mathbf{g}' , in Eq. (2.1) are defined in frame \mathcal{P} . We put a prime on \mathbf{g}' here, because we will also refer to the Green's dyadic defined in frame \mathcal{R} , \mathbf{g} , the explicit form of which is recorded in Appendix B. In the actual calculation, each component of \mathbf{g}' needs to be expressed in terms of a combination of different components of \mathbf{g} , using the Lorentz transformation for the Green's dyadic detailed in Appendix A. The inverse temperature of the radiation, $\beta = 1/T$, is defined in the rest frame of radiation, \mathcal{R} . Finally,

¹ The friction on an atom moving above a general planar surface has actually been calculated in Ref. [17] and tabulated for different polarization states in Appendix D therein. These can be shown to be equivalent to Eq.(2.1) using the transformations listed in Appendix A.

$\Im\mathbf{g}$ refers to the anti-Hermitian part of the Green's dyadic, of which the components are

$$(\Im\mathbf{g})_{ij}(\omega, \mathbf{k}_\perp; z, \tilde{z}) = \frac{g_{ij}(\omega, \mathbf{k}_\perp; z, \tilde{z}) - g_{ji}^*(\omega, \mathbf{k}_\perp; \tilde{z}, z)}{2i} = \frac{g_{ij}(\omega, \mathbf{k}_\perp; z, \tilde{z}) - g_{ji}(-\omega, -\mathbf{k}_\perp; \tilde{z}, z)}{2i}. \quad (2.2)$$

The quantum friction in Eq. (2.1) is in fact the x component of the Lorentz force on a moving dipole quantized using the fluctuation-dissipation theorem (FDT). Because the atom is intrinsically nondissipative, the frictional force is second order in α like that discussed in Ref. [17]. There are two contributions to the force: the \bar{k}_x term comes from the field fluctuations directly while the k_x term comes from the induced dipole fluctuations. Although entering the friction formula with different signs, the two contributions do not cancel each other due to the Doppler shifting of the frequency in the coth factor. **We consider only a reciprocal point particle, that is, one for which the polarizability tensor is symmetric, $\alpha_{ij} = \alpha_{ji}$. Then, a relative velocity between the particle and the surrounding blackbody background is necessary for any transverse force to arise.**² Even at zero temperature, the quantum friction does not vanish in general. But we have learned from our previous investigations that if the background is just free space, the resultant QVF does vanish at zero temperature [17, 18].

The matrix structure under the trace in the integrand is in general complicated. Even for an isotropic atom, there will be contributions to the quantum friction that mix the different diagonal polarization states of the atom and pick up the off-diagonal components of \mathbf{g}' .³ Moreover, each component of \mathbf{g}' is still to be re-expressed as a combination of the different components of \mathbf{g} in the actual calculation. However, the special background indicated in Fig. 1 has several features which greatly help to simplify the calculation. In the presence of the PC plate, \mathbf{g} is found using the general expressions in Appendix B to be

$$\mathbf{g}^{\text{PC}}(\omega, \mathbf{k}_\perp; a, a) = \begin{pmatrix} \frac{\omega^2 - k_x^2}{2\kappa}(1 - e^{-2\kappa a}) & -\frac{k_x k_y}{2\kappa}(1 - e^{-2\kappa a}) & -\frac{i}{2}k_x e^{-2\kappa a} \\ -\frac{k_x k_y}{2\kappa}(1 - e^{-2\kappa a}) & \frac{\omega^2 - k_y^2}{2\kappa}(1 - e^{-2\kappa a}) & -\frac{i}{2}k_y e^{-2\kappa a} \\ +\frac{i}{2}k_x e^{-2\kappa a} & +\frac{i}{2}k_y e^{-2\kappa a} & \frac{k^2}{2\kappa}(1 + e^{-2\kappa a}) \end{pmatrix} \quad (2.3)$$

with $\kappa = \sqrt{k^2 - \omega^2}$. Similar to the vacuum background, the permittivity of a PC plate, $\varepsilon \rightarrow \infty$, is invariant under a Lorentz transformation in the x direction. As a result, \mathbf{g}' is found to be identical to \mathbf{g} when applying the transformations listed in Appendix A:

$$\mathbf{g}'^{\text{PC}}(\omega, \mathbf{k}_\perp; a, a) = \mathbf{g}^{\text{PC}}(\omega, \mathbf{k}_\perp; a, a). \quad (2.4)$$

That is, for this special case, we can replace \mathbf{g}' in Eq. (2.1) with \mathbf{g} in Eq. (2.3). In addition, terms containing the product of g_{xy} and g_{yx} or g_{yz} and g_{zy} do not contribute to the friction due to their oddness in k_y . **For simplicity of analysis, we will assume the polarizability tensor of the atom to be diagonal throughout the paper.** Then, the only contribution to the friction involving the off-diagonal components of the Green's dyadic, which mix different components of the polarizability tensor, is

$$F^{\text{XZ}} = 2 \int \frac{d\omega}{2\pi} \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \frac{d^2 \bar{\mathbf{k}}_\perp}{(2\pi)^2} (\bar{k}_x - k_x) \alpha_{xx}(\omega) (\Im\mathbf{g})_{xz}^{\text{PC}}(\omega, \mathbf{k}_\perp; a, a) \alpha_{zz}(\omega) (\Im\mathbf{g})_{zx}^{\text{PC}}(\omega, \bar{\mathbf{k}}_\perp; a, a) \coth \left[\frac{\beta}{2} \gamma(\omega + \bar{k}_x v) \right]. \quad (2.5)$$

Here and in the rest of the paper, we use a superscript on F to specify contributions from different polarization states of the atom. In fact, F^{XZ} turns out to be the most interesting contribution to the frictional force, because it actually corresponds to a push instead of a drag.

Before presenting the results we obtain for PCQF, let us stress that no PCQF arises at zero temperature. In Appendix C, we prove that the zero temperature QF is absent not only for the vacuum case and the PC case, but also for the broader class of diaphanous materials⁴.

III. EXACT RESULTS AND VARIOUS LIMITS

We see from the starting formula Eq. (2.1) that each contribution to the frictional force is proportional to the product of two nonvanishing components of the polarizability tensor. **For an atom with a diagonal polarizability**

² For a nonreciprocal point particle which is not in thermal equilibrium with the environment, self propulsion can be induced even if the particle is initially at rest. We will explore the fluctuation-induced effects for such a nonreciprocal particle in Ref. [21]. In addition, self propulsion is also possible for extended objects made up of reciprocal but nonuniform materials. See, for example, Ref. [22] by Reid et al.

³ The effective polarizability, $\hat{\alpha} = \alpha \cdot \mathbf{\Gamma} \cdot \alpha$, as defined in Ref. [17], can acquire off-diagonal components through the off-diagonal components of the Green's dyadic, even when the intrinsic polarizability of the atom, α , is diagonal.

⁴ A material is diaphanous if its permittivity, ε , and permeability, μ , satisfy $\varepsilon\mu = 1$.

tensor, most of the contributions that mix the components of the tensor can be ruled out based on the symmetry of the integrand in Eq.(2.1). There are actually only four nonvanishing contributions left. They are proportional to α_{xx}^2 , α_{yy}^2 , α_{zz}^2 , $\alpha_{xx}\alpha_{zz}$ and will be denoted as F^{XX} , F^{YY} , F^{ZZ} and F^{XZ} , respectively.

Crucial to the calculation is finding the anti-Hermitian part of the relevant components of \mathbf{g}^{PC} . It can be seen from Eq. (2.3) that $\Im \mathbf{g}^{\text{PC}} = \mathbf{0}$ unless the propagation wave number κ develops an imaginary part. Since the integrand in Eq. (2.1) involves the product of two Green's dyadics evaluated at $(\omega, \mathbf{k}_\perp)$ and $(\omega, \bar{\mathbf{k}}_\perp)$, respectively, the integration is restricted to regions where the propagation wave numbers associated with both Green's dyadics become imaginary,

$$\kappa \rightarrow -i \operatorname{sgn}(\omega) \sqrt{\omega^2 - k^2}, \quad k^2 < \omega^2, \quad \bar{\kappa} \rightarrow -i \operatorname{sgn}(\omega) \sqrt{\omega^2 - \bar{k}^2}, \quad \bar{k}^2 < \omega^2. \quad (3.1)$$

The branches need to be chosen so that the Green's dyadic is retarded.⁵ The anti-Hermitian parts of the relevant components of the Green's dyadic therefore read

$$(\Im \mathbf{g})_{xx}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{Im} g_{xx}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{sgn}(\omega) \frac{\omega^2 - k_x^2}{2\sqrt{\omega^2 - k^2}} \left[1 - \cos\left(2\sqrt{\omega^2 - k^2}a\right) \right], \quad (3.2a)$$

$$(\Im \mathbf{g})_{yy}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{Im} g_{yy}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{sgn}(\omega) \frac{\omega^2 - k_y^2}{2\sqrt{\omega^2 - k^2}} \left[1 - \cos\left(2\sqrt{\omega^2 - k^2}a\right) \right], \quad (3.2b)$$

$$(\Im \mathbf{g})_{zz}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{Im} g_{zz}(\omega, \mathbf{k}_\perp; a, a) = \operatorname{sgn}(\omega) \frac{k^2}{2\sqrt{\omega^2 - k^2}} \left[1 + \cos\left(2\sqrt{\omega^2 - k^2}a\right) \right], \quad (3.2c)$$

$$(\Im \mathbf{g})_{xz}(\omega, \mathbf{k}_\perp; a, a) = -(\Im \mathbf{g})_{zx}(\omega, \mathbf{k}_\perp; a, a) = -i \operatorname{sgn}(\omega) \frac{k_x}{2} \sin\left(2\sqrt{\omega^2 - k^2}a\right). \quad (3.2d)$$

The off-diagonal components of $\Im \mathbf{g}$ are different from the diagonal components in several respects. First of all, they are purely imaginary. Second, they are odd in k_x . As a result, in Eq. (2.1), F^{XZ} contributes to the total friction through the $-k_x$ term, while F^{XX} , F^{YY} and F^{ZZ} all contribute through the \bar{k}_x term. It is precisely the apparent minus sign in the $-k_x$ term that renders F^{XZ} positive, corresponding to a push instead of a drag.⁶ Third, they do not contain terms independent of the atom-plate separation, a , as those in the diagonal components. These terms reflect the vacuum contributions. So, the off-diagonal components do not contribute to the QVF discussed in Ref. [17].

Without any further assumptions, we insert Eq. (3.2) into Eq. (2.1) and integrate k_x , k_y and \bar{k}_y analytically. With a further change of variable, $\bar{k}_x = \omega u$, we find the contribution to the PCQF from the PQ polarization states can be written as

$$F^{\text{PQ}} = \frac{1}{32\pi^3} \int_0^\infty d\omega \alpha_{pp}(\omega) \alpha_{qq}(\omega) \omega^7 \mathcal{F}^{\text{PQ}}(x, v, b), \quad (3.3)$$

and for each contribution, \mathcal{F}^{PQ} reads

$$\mathcal{F}^{XX}(x, v, z) = \left\{ \frac{4}{3} - \frac{2}{x^3} [x \cos x + (x^2 - 1) \sin x] \right\} \int_{-1}^1 du u (1 - u^2) \left[1 - J_0\left(x\sqrt{1 - u^2}\right) \right] \frac{1}{e^{x\gamma(1+uv)b} - 1}, \quad (3.4a)$$

$$\begin{aligned} \mathcal{F}^{YY}(x, v, z) &= \left\{ \frac{4}{3} - \frac{2}{x^3} [x \cos x + (x^2 - 1) \sin x] \right\} \\ &\times \int_{-1}^1 du u \left[\frac{1}{2}(1 + u^2) - J_0\left(x\sqrt{1 - u^2}\right) + \frac{\sqrt{1 - u^2}}{x} J_1\left(x\sqrt{1 - u^2}\right) \right] \frac{1}{e^{x\gamma(1+uv)b} - 1}, \end{aligned} \quad (3.4b)$$

⁵ Here, we are using the retarded Green's dyadic as opposed to the advanced Green's dyadic. The word "retarded" can also refer to the effect of retardation or a finite velocity of light, which we also incorporate in our calculations.

⁶ Physically, the $-k_x$ term comes from the induced dipole fluctuations, while the \bar{k}_x term comes from the direct field fluctuations. In the QVF case, these two different contributions also occur and they are called F'_{II} and F'_I , respectively, in Ref. [17]. There, F'_{II} vanishes, which reflects the fact that the dipole radiation emitted by the atom is isotropic. Here, the fact that F^{XZ} exists through the $-k_x$ term indicates that the dipole radiation in the x direction is no longer isotropic when the PC plate is present. Furthermore, the positive sign of this contribution reflects that the corresponding dipole radiation emitted backward must exceed that emitted forward.

$$\mathcal{F}^{ZZ}(x, v, z) = \left\{ \frac{4}{3} - \frac{4}{x^3} [x \cos x - \sin x] \right\} \times \int_{-1}^1 du u \left[\frac{1}{2}(1+u^2) + u^2 J_0(x\sqrt{1-u^2}) + \frac{\sqrt{1-u^2}}{x} J_1(x\sqrt{1-u^2}) \right] \frac{1}{e^{x\gamma(1+uv)b} - 1}, \quad (3.4c)$$

$$\mathcal{F}^{XZ} = -2 \left\{ \frac{2}{x^4} [-3x \cos x - (x^2 - 3) \sin x] \right\} \int_{-1}^1 du u \sqrt{1-u^2} J_1(x\sqrt{1-u^2}) \frac{1}{e^{x\gamma(1+uv)b} - 1}. \quad (3.4d)$$

Here, we have introduced a dimensionless frequency scaled by the distance a ,

$$x = 2\omega a, \quad (3.5)$$

as well as a dimensionless inverse temperature also scaled by a ,

$$b = \frac{\beta}{2a} = \frac{1}{2aT}. \quad (3.6)$$

So far, the expressions we have for PCQF in Eq. (3.4) are exact and involve the dynamical polarizability of the atom. For frequencies smaller than the lowest excitation energy of the atom, the dynamical polarizability, $\alpha(\omega)$, can be replaced by the static polarizability [23], $\alpha(0)$. Due to the common exponential factors in Eq. (3.4a)–(3.4d), the high frequency modes with $\beta\omega = xb \gg 1$ will be cut off and do not significantly contribute to the ω integral. Therefore, so long as the temperature is not high enough to excite the atom to its higher energy states, we can work in the static limit, where we substitute the polarizability with its static value. This allows us to take the product of the polarizabilities out of the ω integral in Eq. (3.3):

$$F^{\text{PQ}} = \frac{\alpha_{pp}(0)\alpha_{qq}(0)}{32\pi^3(2a)^8} f^{\text{PQ}}(v, b), \quad f^{\text{PQ}}(v, b) = \int_0^\infty dx x^7 \mathcal{F}^{\text{PQ}}(x, v, b), \quad (3.7)$$

where the dimensionless functions f^{PQ} now characterize contributions to PCQF from different polarization states.

Note the magnitude of b determines the dominating modes of the x integral in Eq. (3.7). For $b \ll 1$, it is dominated by the large x modes, where the complicated x dependences in the integrands become subdominant and drop out, except for the common factor of $\frac{x^7}{e^{x\gamma(1+uv)b} - 1}$. As a result, the diagonal contributions F^{XX} , F^{YY} and F^{ZZ} become distance independent and proportional to T^8 . Indeed, for $b \ll 1$, the diagonal contributions of PCQF precisely reduce to the corresponding contributions of QVF in Ref. [17]. On the other hand, F^{XZ} , which is proportional to T^4/a^4 , becomes completely negligible in comparison to the diagonal contributions. To sum up, the contributions to PCQF in the small b limit read

$$F_{b \ll 1}^{\text{PQ}} = \frac{\alpha_{pp}^2(0)}{32\pi^3(2a)^8} f_{b \ll 1}^{\text{PQ}}(v, b), \quad f_{b \ll 1}^{\text{PQ}}(v, b) = \begin{cases} -\frac{4\Gamma(8)\zeta(8)}{36^8} \frac{32}{105} \gamma^4 v(7 + 3v^2), & \text{PQ} = \text{XX} \\ -\frac{4\Gamma(8)\zeta(8)}{36^8} \frac{32}{105} \gamma^6 v(14 + 37v^2 + 9v^4), & \text{PQ} = \text{YY}, \text{ZZ} \\ \frac{16\Gamma(4)\zeta(4)}{b^4} \frac{v}{\gamma^4}, & \text{PQ} = \text{XZ}. \end{cases} \quad (3.8)$$

As is shown in Eq. (3.8), unlike the diagonal contributions which monotonically increase with velocity, we find F^{XZ} vanishes when the velocity approaches the speed of light. Since the small b limit of F^{XZ} has not been worked out in Ref. [17], we show how to obtain it analytically in Appendix E.

It is not so surprising that the small b limit of PCQF coincides with QVF. Small b values correspond to large distances or high temperatures. When the atom is far away from the PC plate, it is obvious that PCQF should reduce to QVF. In the case of high temperatures (but not so high to ionize the atom), the atom interacts with photons of very high frequency. It therefore mainly probes the very short distances around it and, effectively, does not feel the PC plate. That is, in the high temperature limit, the distribution of energy eigenvalues of photons interacting with the atom is insensitive to the presence of the plate.

Since quantum vacuum friction has been explored for a nondissipative atom in Ref. [17], the new physics really lies in the large b limit, the short-distance or low-temperature behavior of PCQF. For $b \gg 1$, the small x modes dominate the integrals. We can therefore expand the integrands in powers of x before carrying out the integrals. Quite interestingly, the integrands for various polarization states exhibit different leading power behavior in x , which determines the distance and temperature dependences of their contributions to PCQF. After expansion in x , both the

x and u integrals can be done exactly if we keep only the leading in b terms. (Appendix E contains an approach to derive the asymptotic expression for F^{XZ} in the large b limit as well.) For $b \gg 1$, the resultant PCQF is found to be

$$F_{b \gg 1}^{\text{PQ}} = \frac{\alpha_{pp}(0)\alpha_{qq}(0)}{32\pi^3(2a)^8} f_{b \gg 1}^{\text{PQ}}(v, b), \quad f_{b \gg 1}^{\text{PQ}}(v, b) = \begin{cases} -\frac{\Gamma(12)\zeta(12)}{15b^{12}} \frac{64}{3465} \gamma^6 v(99 + 110v^2 + 15v^4), & \text{PQ} = \text{XX} \\ -\frac{\Gamma(12)\zeta(12)}{15b^{12}} \frac{32}{3465} \gamma^8 v(297 + 1034v^2 + 625v^4 + 60v^6), & \text{PQ} = \text{YY} \\ -\frac{8\Gamma(8)\zeta(8)}{3b^8} \frac{64}{105} \gamma^6 v(14 + 37v^2 + 9v^4), & \text{PQ} = \text{ZZ} \\ 2\frac{\Gamma(10)\zeta(10)}{15b^{10}} \frac{8}{63} \gamma^6 v(21 + 30v^2 + 5v^4), & \text{PQ} = \text{XZ}. \end{cases} \quad (3.9)$$

Since the results shown in Eq. (3.9) are for the large z limit, it is apparent that F^{ZZ} dominates over the contributions from the other polarizations. In this limit, F^{ZZ} is independent of distance a and proportional to T^8 , just as is the case for QVF. In fact, we find F^{ZZ} is precisely four times the corresponding QVF contribution shown in Eq. (3.8). The next leading contribution, F^{XZ} , is proportional to $a^2 T^{10}$ with an overall positive sign, suggesting that this particular contribution corresponds to a push instead of a drag. The smallest contributions, F^{XX} and F^{YY} , are both proportional to $a^4 T^{12}$. On closer examination of Eq. (3.9), we also observe that f^{YY} is always greater than f^{XX} , for arbitrary velocities.

Interestingly, these behaviors of PCQF may be easily understood from the image particle picture criticized in the Introduction. In fact, there is nothing wrong with replacing the PC plate by an image particle moving synchronously with the actual particle. We only need to keep in mind that both particles would interact with the surrounding photon bath, so that a frictional force does indeed arise. Following this line of reasoning, the image particle would double the normal component of the fluctuation-induced field, E_z , but eliminate the tangential components, E_x and E_y , at the surface of the PC plate. Since these fluctuation-induced frictional forces are proportional to the product of the relevant fields, F^{ZZ} is therefore quadrupled while the other contributions are all suppressed when the distance between the particle and the PC plate approaches zero.

We have been advised by Matthias Krüger that the physics here is analogous to a classical situation in hydrodynamics. For example, the authors of Ref. [24] studied colloidal particles driven through a suspension of mutually noninteracting Brownian particles and the corresponding frictional force induced by the nonequilibrium fluid structure. (The flow field comoving with the colloidal particles is not in equilibrium with the Brownian particles.) They found that the frictional force on a single colloidal particle traveling along a wall (analogous to the PC plate in our case) is precisely the same as that on two colloidal particles driven side by side. The authors also found an enhancement of the friction due to the wall/image colloidal particle in comparison to the friction on an isolated colloidal particle. From the density plot of the solute Brownian particles, they interpret this increase in friction as the result of more Brownian particles aggregating in front of the colloidal particles when the wall/image particle is present. An analogous interpretation applies to what we see here in this paper. That is, the electromagnetic energy density is stronger near the PC plate.

So far, both the small b results in Eq. (3.8) and the large b results in Eq. (3.9) are exact in velocity. Another question is whether we can obtain the nonrelativistic (NR) limit analytically without assuming anything about b . This is possible as long as the dynamical polarizability is still replaced by its static value. We illustrate the procedure of obtaining the NR limit for F^{XZ} , valid for all b values in Appendix D. It turns out that all contributions to PCQF start with a term linear in v in the NR limit.

IV. NUMERICAL RESULTS

As one of the contributions, F^{XZ} , is positive (a push), while the others are all negative (a drag), a natural question arises: could the overall “frictional” force on an atom ever flip sign and therefore become a push? Of course, from Eq. (3.8) and Eq. (3.9), we can already conclude that the overall PCQF is negative definite in both the small b (vacuum/high-temperature) limit and large b (short-distance/low-temperature) limit. But, there is no convincing argument just from the analytic results suggesting that PCQF cannot switch sign in the intermediate b regime. Therefore, we resort to numerical methods to ascertain the sign of PCQF.

We will here mainly consider atoms in their ground states, the polarizability of which is normally quite isotropic⁷ and can be well approximated by its static value, $\alpha(\omega) = \alpha(0)\mathbf{1}$. For such isotropic atoms, the sign of the PCQF is

⁷ Closed-shell atoms are almost exactly isotropic [25]. Even for open-shell atoms, the anisotropy is typically small compared to the isotropic part of the polarizability. Among the elements in a certain period, the anisotropy is largest when the first p electron is added [23].

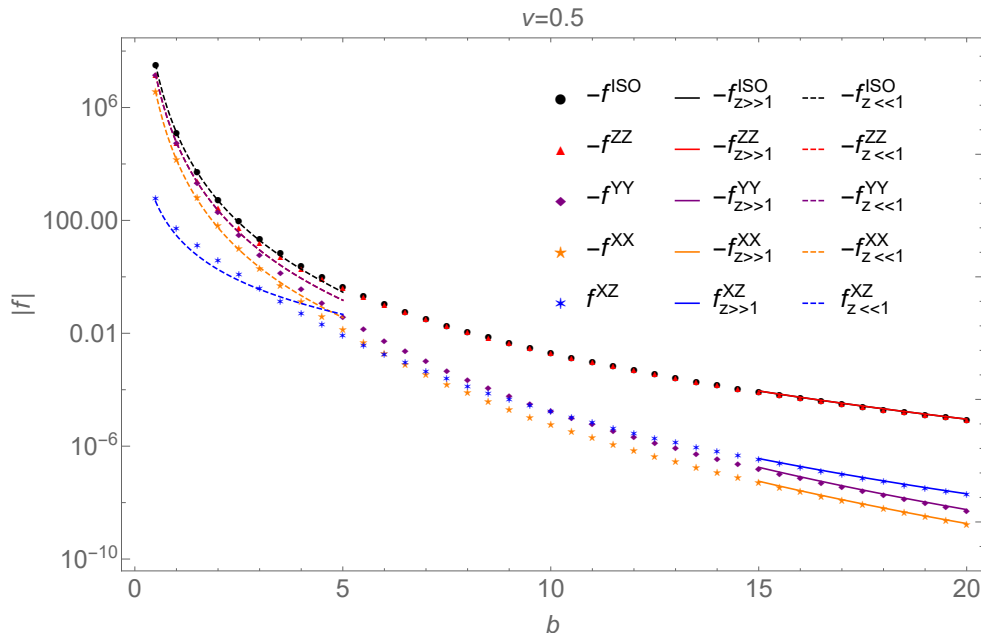


FIG. 2. The absolute values of the dimensionless functions f^{PQ} in Eq. (4.1) are shown as functions of b for fixed velocity ($v = 0.5$). The numerical results are computed directly using Eq. (3.4) and Eq. (3.7). Their small b and large b approximations are obtained from Eq. (3.8) and Eq. (3.9), respectively. Since the small b approximation for f^{ZZ} and f^{YY} is identical, the dashed purple line overlaps with the dashed red line. As is seen, the small b approximation of F^{XZ} cannot give a good description of the numerical data beyond $b = 1$. A further detailed plot is provided in Appendix E, where the agreement between the analytic approximation and the numerical data for F^{XZ} is more clearly demonstrated for smaller b values.

determined by the sum of the dimensionless functions introduced in Eq. (3.7):

$$F^{\text{ISO}} = \frac{\alpha^2(0)}{32\pi^3(2a)^8} f^{\text{ISO}}(v, b), \quad f^{\text{ISO}}(v, b) = f^{\text{XX}}(v, b) + f^{\text{YY}}(v, b) + f^{\text{ZZ}}(v, b) + f^{\text{XZ}}(v, b). \quad (4.1)$$

We show the absolute value of these dimensionless functions across their transition region in Fig. 2. Starting from small b values, the total frictional force on the isotropic particle is dominated almost evenly between the ZZ and YY contributions. But as b grows larger, the weight of the YY contribution decays so that the ZZ contribution solely dominates the entire frictional force. As for the unique positive contribution from the XZ polarization, it is completely negligible when b is small but it eventually surpasses the contributions from the XX and YY polarizations for large b . Nonetheless, it never dominates the ZZ polarization. The asymptotic (in b) expressions in Eq. (3.8) and Eq. (3.9) are consistent with these behaviors and the agreement with the numerical data in their supposedly valid regimes are also clearly illustrated in the figure. So, we can conclude that the total PCQF on an isotropic atom *is always a drag*, since it *cannot* change sign even in the intermediate b regime.

Another interesting aspect of the force is, of course, its magnitude. Fluctuation induced forces are typically small. But, is the PCQF possibly accessible to experiment? Here, we estimate PCQF on a cesium (Cs) atom, which has the largest static polarizability,⁸ according to Ref. [25], $\alpha_{\text{Cs}}(0) = 59.3 \text{ \AA}^3$. Because the expression (3.7) we use for numerical calculation is obtained in the static limit, the corresponding numerical results are only expected to be appropriate when the atom is in its ground state, that is, up to the temperature that corresponds to the *first excitation energy* of the Cs atom, $T_1 = 16\,100 \text{ K}$,⁹ beyond which a model for its dynamical polarizability is needed. In Fig. 3, we show the magnitude of the total frictional force on a Cs atom up to T_1 , fixing velocity and distance. The friction clearly exhibits a power-law dependence on temperature. This is no surprise because we already know that the frictional force should behave as T^8 in both the large b (low T) and small b (high T) regimes.

Of course, PCQF also depends on the distance between the atom and the plate, distinguishing it from QVF. Considering the size of the Cs atom¹⁰, we should keep the distance greater than 1 nm to avoid additional surface

⁸ Within a period, the alkali metal atoms generally have the biggest polarizabilities. They are also supposed to have very tiny anisotropy because their valence electrons are in s states [23]. Cs has the largest polarizability among the alkali metal atoms.

⁹ This temperature and the ionization temperature used later are obtained from the first excitation energy of Cs listed in Ref. [26].

¹⁰ Cesium also has the largest covalent radius (244 pm) among the nonradioactive atoms according to Ref. [27].

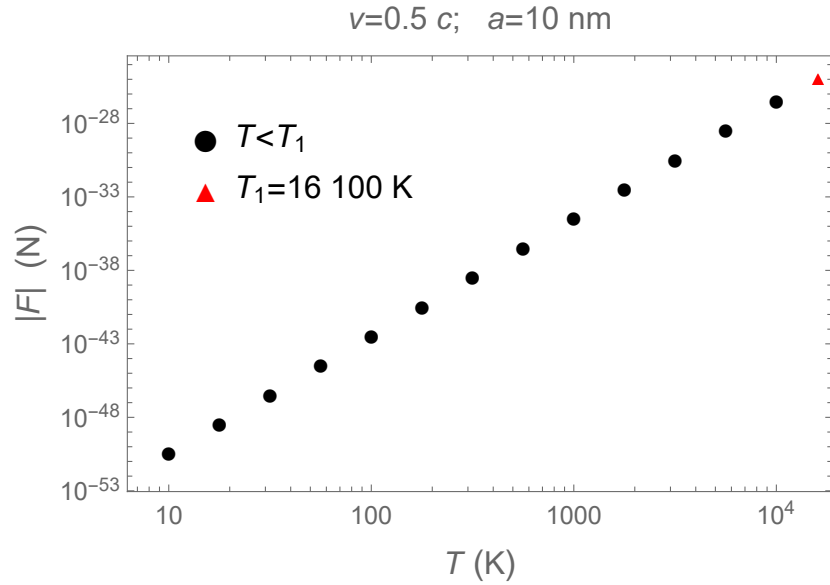


FIG. 3. The magnitude of the total frictional force on a Cs atom moving at $v = 0.5c$ and at a distance $a = 10$ nm from the PC plate is plotted as a function of temperature. The friction at the first excited temperature is indicated by the red triangle, with a magnitude of 1.30×10^{-25} N.

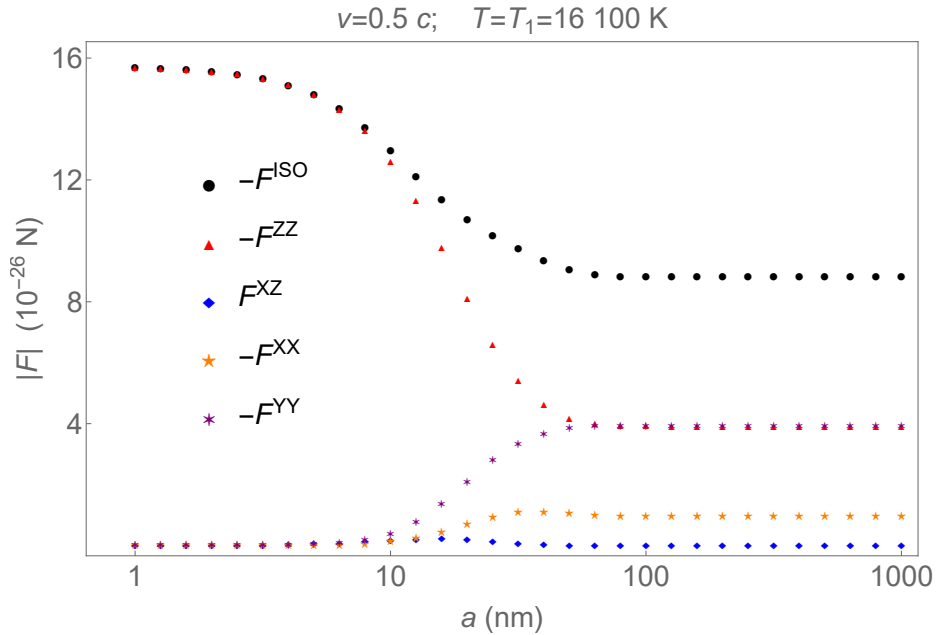


FIG. 4. The magnitude of the total frictional force, along with its contributions from different polarizations, on a Cs atom moving at $v = 0.5c$ and at its first excited temperature $T = 16 100$ K is plotted as a function of distance. The largest magnitude of the total friction shown by the black dots is for $a = 1$ nm, being 1.57×10^{-25} N.

effects. We therefore show the magnitude of PCQF for a Cs atom as a function of distance in Fig. 4, from 1 nm to $1 \mu\text{m}$, fixing the velocity at $v = 0.5$ and temperature at $T = T_1$. It is seen that the total friction is only doubled when the distance is reduced from $1 \mu\text{m}$ to 1 nm. This can be well understood from the asymptotic behavior of the dominant contributions: the ZZ contribution quadruples, yet the YY contribution vanishes at small distances, which is also clearly illustrated in the figure.

Finally, PCQF depends on the velocity of the atom. As is shown in Fig. 5a, the magnitude of the frictional force is linear in v for very small velocities; however, the velocity dependence becomes more prominent for larger velocities. In Fig. 5b, we not only plot the total frictional force at the first excitation temperature, $T_1 = 16 100$ K, but also

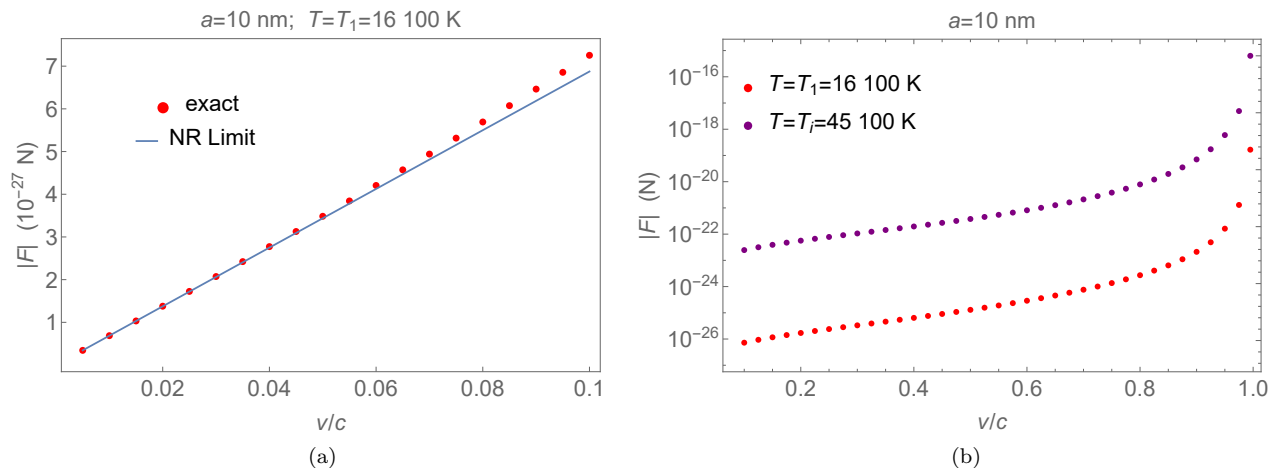


FIG. 5. The velocity dependence of the magnitude of the total frictional force on a cesium atom at a distance of $a = 10$ nm away from the PC plate. (a) At the first excitation temperature, $T_1 = 16\,100$ K, the frictional force is plotted as a function of velocity, for $v/c \in [0.005, 0.100]$. The red dots show the exact numerical results based on Eq. (3.7) and Eq. (3.4). The blue solid line shows the term linear in v obtained using the nonrelativistic approximation detailed in Appendix D. (b) In the more relativistic regime, $v/c \in [0.100, 0.995]$, the red dots show the total frictional force at the first excitation temperature, $T_1 = 16\,100$ K, while the purple dots show the numerical results extrapolated to the ionization temperature, $T_i = 45\,100$ K. For the maximum velocity shown in the figure, $v = 0.995$, the magnitude of the total friction is 1.66×10^{-19} N at T_1 and 6.30×10^{-16} N at T_i .

extrapolate our numerical results to the ionization temperature of the cesium atom, $T_i = 45\,100$ K [26]. Above T_i , the outermost electron will be stripped off the atom so that the cesium atom cannot stay neutral. It is therefore not feasible experimentally to detect the quantum friction on an atom above its ionization temperature. In between T_1 and T_i , the atom can be excited, though not ionized. Now, the frequencies corresponding to the transition of the atom's internal energy levels become important in evaluating PCQF. At these frequencies, the polarizability of the atom develops an imaginary part [28], which results in a PCQF that is first order in the polarizability. This effect is not included in the results we show for $T = T_i$. In addition, by employing the static value for the polarizability, we underestimate the magnitude of the second order PCQF, because atoms in excited states, e.g., Rydberg atoms¹¹, tend to have much larger polarizabilities.

V. CONCLUSIONS AND OUTLOOK

In this paper, we calculate the frictional force induced by fluctuations of the electromagnetic field on a neutral, nondissipative atom moving parallel to a perfectly conducting (PC) plate, which we term PCQF for brevity. This friction exists in second order in the polarizability of the atom and reduces to the quantum vacuum friction previously explored [17] in the limit of large distance from the plate or high temperature. At short distances or low temperatures, however, the PC plate modifies the behavior of the frictional force. For an isotropic atom, the frictional force is found to be negative definite (a drag) and twice the magnitude of the quantum vacuum friction felt by the same atom moving through blackbody radiation without a PC plate. Interestingly, the contribution to PCQF from a particular polarization state of the atom is positive (a push). However, this contribution turns out to be always subdominant to the negative contributions from the other polarization states. As a result, the total transverse force on the moving atom remains a drag. The magnitude of the friction seems to be too tiny to be observed unless the atom is made to move in a very hot background, at very high velocities. It is then fitting to make some comments about the experimental conditions on temperature and velocity.

First, the temperature is absolutely bounded by the ionization temperature of the atom, T_i , because the atom will no longer stay neutral above T_i . In this paper, we have used a static polarizability for the atom in the numerical

¹¹ Even though Rydberg atoms possess much larger polarizabilities, which presumably will enhance the resulting frictional effect, we are unsure whether they could be appropriate candidates for experimental consideration, because blackbody radiation induces transitions to lower n states and reduces the lifetime of the Rydberg states. Even at room temperature, transitions induced by blackbody radiation can contribute more to the decay rate than the spontaneous transitions [29]. At higher temperatures, the transition rate induced by blackbody radiation is even larger.

calculations, which can only be justified if the atom remains in its ground state. The temperature is therefore further bounded by the temperature, T_1 , corresponding to the first excitation energy of the atom. In principle, however, one can calculate PCQF on the atom up to its ionization temperature if its dynamical polarizability is known for a sufficiently wide frequency spectrum.

Of course, for experiments, the material which approximates a PC plate is very likely to give a more restrictive bound on the temperature. For example, the standard candidate, gold, will melt at 1337 K. Even if one could imagine using liquid metals to mimic the perfectly conducting plate, the temperature is still bounded by the boiling point of the metal. Tungsten has the highest boiling point among metals, 6203 K, which is still much lower than the typical ionization temperature, T_i , of an atom. (For the cesium atom discussed in the paper, $T_i = 45\,100$ K.) This reality might motivate us to study PCQF for situations when the plate is not in thermal equilibrium with the radiation background. For the Casimir-Polder force (the force normal to the plate), such a scenario has been studied both theoretically [30] and experimentally [4].

Another apparent challenge to any feasible experiment is accelerating neutral atoms to relativistic velocities. But, in fact, it is possible nowadays to manipulate the conventional ion accelerators so that fast ions can be converted to neutral atoms with little change in momentum. For example, in Ref. [31], the maximum kinetic energy obtained for a copper atom is 1 MeV, which is equivalent to a velocity of 0.0058, after conversion using the relativistic formula for kinetic energy, $K = (\gamma - 1)m$.

This paper only considers a very idealized background with the PC plate. For a surface with a real finite index of refraction, n , there will be induced Cherenkov friction [32] on the moving particle if it moves at a velocity above the Cherenkov threshold, $v > 1/n$. If one further allows the surface to have dissipation, which is unavoidable in reality and perhaps induces an even greater frictional effect, the problem becomes complicated by the presence of several different mechanisms that give rise to friction. Ref. [33] provides a recent overview of this complicated subject with many useful references; however, it mainly focuses on only zero-temperature effects. Other works, like Ref. [34], do include finite-temperature effects but the discussion is restricted to only the nonrelativistic regime. In the future, we intend to calculate the quantum friction associated with a dispersive and dissipative surface fully for arbitrary temperatures and relativistic velocities. Alternatively, one could still assume a perfectly conducting boundary but allow the moving particle itself to be intrinsically dissipative. The resulting PCQF will then be a modification of the quantum vacuum friction studied in Ref. [18], where an independent temperature of the particle comes into play. We will discuss such PCQF in a subsequent paper.

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Appendix A: THE TRANSFORMATION OF THE REDUCED GREEN'S DYADIC

In this appendix, we provide the connection between the reduced Green's dyadic in frame \mathcal{P} , \mathbf{g}' , and that in frame \mathcal{R} , \mathbf{g} . It is straightforwardly obtained by considering the Lorentz transformation of the electromagnetic field and applying the FDT in both frames consistently. Note the transformation of the material properties like $\boldsymbol{\varepsilon}$ or $\boldsymbol{\mu}$ is never invoked because we eventually express the quantum friction in terms of \mathbf{g} instead of \mathbf{g}' .

In writing down the connection between \mathbf{g} and \mathbf{g}' , we will use $(\omega', \mathbf{k}'_{\perp})$ for the frequency and momentum in the atom's rest frame (\mathcal{P}), and $(\omega, \mathbf{k}_{\perp})$ for those transformed into the rest frame of the radiation (\mathcal{R}),

$$\omega = \gamma(\omega' + k'_x v), \quad k_x = \gamma(k'_x + \omega' v), \quad k_y = k'_y. \quad (\text{A1})$$

Below all components of \mathbf{g}' are expressed in terms of components of \mathbf{g} :

$$\begin{aligned}
g'_{xx}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= g_{xx}(\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{yy}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{(\omega' + k'_x v)^2} \left(\frac{\omega'^2}{\gamma^2} g_{yy} + k_y'^2 v^2 g_{xx} + \frac{\omega'}{\gamma} k'_y v g_{xy} + \frac{\omega'}{\gamma} k'_y v g_{yx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{zz}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{(\omega' + k'_x v)^2} \left(\frac{\omega'^2}{\gamma^2} g_{zz} + v^2 \partial_z \partial_{\tilde{z}} g_{xx} + i \frac{\omega'}{\gamma} v \partial_{\tilde{z}} g_{zx} - i \frac{\omega'}{\gamma} v \partial_z g_{xz} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{xy}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{\omega' + k'_x v} \left(\frac{\omega'}{\gamma} g_{xy} + k'_y v g_{xx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{yx}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{\omega' + k'_x v} \left(\frac{\omega'}{\gamma} g_{yx} + k'_y v g_{xx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{zx}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{\omega' + k'_x v} \left(\frac{\omega'}{\gamma} g_{zx} + i v \partial_z g_{xx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{xz}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{\omega' + k'_x v} \left(\frac{\omega'}{\gamma} g_{xz} - i v \partial_{\tilde{z}} g_{xx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{yz}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{(\omega' + k'_x v)^2} \left(\frac{\omega'^2}{\gamma^2} g_{yz} - i k_y' v^2 \partial_{\tilde{z}} g_{xx} + i \frac{\omega'}{\gamma} v \partial_{\tilde{z}} g_{yx} - \frac{\omega'}{\gamma} k_y' v g_{xz} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}), \\
g'_{zy}(\omega', \mathbf{k}'_{\perp}; z, \tilde{z}) &= \frac{1}{(\omega' + k'_x v)^2} \left(\frac{\omega'^2}{\gamma^2} g_{zy} + i k_y' v^2 \partial_z g_{xx} - i \frac{\omega'}{\gamma} v \partial_z g_{xy} - \frac{\omega'}{\gamma} k_y' v g_{zx} \right) (\omega, \mathbf{k}_{\perp}; z, \tilde{z}). \tag{A2}
\end{aligned}$$

Appendix B: THE FORM OF THE REDUCED GREEN'S DYADIC

In this appendix, we give the explicit form of the reduced Green's dyadic used in the paper. The Green's dyadic $\mathbf{\Gamma}(\mathbf{r}, \tilde{\mathbf{r}}; \omega)$ in frequency space satisfies the following differential equation,

$$\left[-\varepsilon(\mathbf{r}; \omega) + \frac{1}{\omega^2} \nabla \times \boldsymbol{\mu}^{-1}(\mathbf{r}; \omega) \cdot \nabla \times \right] \mathbf{\Gamma}(\mathbf{r}, \tilde{\mathbf{r}}; \omega) = \mathbf{1} \delta(\mathbf{r} - \tilde{\mathbf{r}}). \tag{B1}$$

where $\varepsilon(\mathbf{r}; \omega)$ and $\boldsymbol{\mu}(\mathbf{r}; \omega)$ are the permittivity and permeability at the field point \mathbf{r} . In deriving Eq. (B1), we have ignored the spatial dispersion effects so that these susceptibilities are local in space. The geometry of the problem we consider possesses translational symmetry in the x - y plane, which permits us to Fourier transform the Green's dyadic in these spatial directions,

$$\mathbf{\Gamma}(\mathbf{r}, \tilde{\mathbf{r}}; \omega) = \int \frac{d^2 \mathbf{k}_{\perp}}{(2\pi)^2} e^{i \mathbf{k}_{\perp} \cdot (\mathbf{r}_{\perp} - \tilde{\mathbf{r}}_{\perp})} \mathbf{g}(z, \tilde{z}; \omega, \mathbf{k}_{\perp}). \tag{B2}$$

In this paper, we always evaluate the Green's dyadic at the position of the particle, where the permittivity and the permeability become scalars and take the vacuum value, $\varepsilon = \mu = 1$. The reduced Green's dyadic \mathbf{g} then takes the special form

$$\mathbf{g}(z, \tilde{z}; \omega, \mathbf{k}_{\perp}) = \begin{pmatrix} \frac{k_x^2}{k^2} \partial_z \partial_{\tilde{z}} g^H + \frac{k_y^2}{k^2} \omega^2 g^E & \frac{k_x k_y}{k^2} \partial_z \partial_{\tilde{z}} g^H - \frac{k_x k_y}{k^2} \omega^2 g^E & i k_x \partial_z g^H \\ \frac{k_x k_y}{k^2} \partial_z \partial_{\tilde{z}} g^H - \frac{k_x k_y}{k^2} \omega^2 g^E & \frac{k_y^2}{k^2} \partial_z \partial_{\tilde{z}} g^H + \frac{k_x^2}{k^2} \omega^2 g^E & i k_y \partial_z g^H \\ -i k_x \partial_{\tilde{z}} g^H & -i k_y \partial_{\tilde{z}} g^H & k^2 g^H \end{pmatrix}. \tag{B3}$$

The scalar Green's functions that construct the Green's dyadic consist of a bulk part and a scattering part,

$$g^{E,H}(z, \tilde{z}; \omega, k) = \frac{1}{2\kappa} e^{-\kappa|z-\tilde{z}|} + \frac{r^{E,H}}{2\kappa} e^{-\kappa(z+\tilde{z})}, \tag{B4}$$

with the reflection coefficients

$$r^E = \frac{\kappa - \kappa'/\mu}{\kappa + \kappa'/\mu}, \quad r^H = \frac{\kappa - \kappa'/\varepsilon}{\kappa + \kappa'/\varepsilon}. \tag{B5}$$

Here, ε and μ are the permittivity and permeability of the reflecting surface, which is assumed to be homogeneous and isotropic for simplicity, and κ and κ' are the propagation wave numbers associated with the vacuum and the surface, respectively, given by

$$\kappa^2 = k^2 - \omega^2, \quad \kappa'^2 = k^2 - \omega^2 \varepsilon \mu. \quad (\text{B6})$$

In certain regions for ω and \mathbf{k}_\perp , these wave numbers could develop an imaginary part, which is crucial for discussions of dissipative forces like quantum friction. In those regions, the branch is so chosen that the retarded requirement of the Green's dyadic is guaranteed,

$$\kappa \rightarrow -i \operatorname{sgn}(\omega) \sqrt{\omega^2 - k^2}, \quad \omega^2 > k^2; \quad \kappa' \rightarrow -i \operatorname{sgn}(\omega) \sqrt{\omega^2 \varepsilon \mu - k^2}, \quad \omega^2 \varepsilon \mu > k^2. \quad (\text{B7})$$

Note that κ becomes odd in ω in the region where it develops an imaginary part.

In the perfectly (electrically) conducting limit for the surface considered in this paper, the permittivity and permeability take the extreme values [35],

$$\varepsilon \rightarrow \infty, \quad \mu \rightarrow 0 \quad (\text{B8})$$

so that the reflection coefficients simplify to be

$$r^{E,H} = \mp 1. \quad (\text{B9})$$

Appendix C: THE ABSENCE OF ZERO TEMPERATURE QUANTUM FRICTION IN THE PRESENCE OF A DIAPHANOUS MEDIUM

In this appendix, we supply a proof for why no zero temperature QF should arise for the vacuum case and the PC case. Further, we extend the claim to include any diaphanous, nondissipative medium with the property $\varepsilon \mu = 1$.

The general QF for an atom, Eq. (2.1) can be rewritten as the following when the temperature is set to be zero:

$$F = 2 \int_0^\infty \frac{d\omega}{2\pi} \int \frac{d^2 \mathbf{k}_\perp}{(2\pi)^2} \frac{d^2 \bar{\mathbf{k}}_\perp}{(2\pi)^2} \bar{k}_x \operatorname{tr} [\boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \mathbf{k}_\perp; a, a) \cdot \boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \bar{\mathbf{k}}_\perp; a, a)] [\operatorname{sgn}(\omega + \bar{k}_x v) - \operatorname{sgn}(\omega + k_x v)]. \quad (\text{C1})$$

To obtain Eq. (C1), we have exchanged k_x and \bar{k}_x for the second term in Eq. (2.1) and used the evenness of the integrand under the total reflection of its frequency and wave vector arguments $(\omega, \mathbf{k}_\perp, \bar{\mathbf{k}}_\perp) \rightarrow (-\omega, -\mathbf{k}_\perp, -\bar{\mathbf{k}}_\perp)$. In order to make the argument clearer, let us change the \mathbf{k}_\perp and $\bar{\mathbf{k}}_\perp$ into dimensionless variables using ω as a positive scale,

$$k_x = \omega x, \quad k_y = \omega y, \quad \bar{k}_x = \omega \bar{x}, \quad \bar{k}_y = \omega \bar{y}. \quad (\text{C2})$$

The frictional force now reads

$$F = \frac{1}{16\pi^5} \int_0^\infty d\omega \omega^5 \int dx dy d\bar{x} d\bar{y} \bar{x} \operatorname{tr} [\boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega x, \omega y) \cdot \boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega \bar{x}, \omega \bar{y})] [\operatorname{sgn}(1 + \bar{x}v) - \operatorname{sgn}(1 + xv)], \quad (\text{C3})$$

where we have suppressed the spatial z coordinates of the Green's dyadics. The difference in the sgn functions can be translated into limits for the x and \bar{x} integrals, leading to

$$F = \frac{1}{8\pi^5} \int_0^\infty d\omega \omega^5 \int dy d\bar{y} \left[\int_{-\infty}^{-\frac{1}{v}} dx \int_{-\frac{1}{v}}^\infty d\bar{x} - \int_{-\frac{1}{v}}^\infty dx \int_{-\infty}^{-\frac{1}{v}} d\bar{x} \right] \bar{x} \operatorname{tr} [\boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega x, \omega y) \cdot \boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega \bar{x}, \omega \bar{y})]. \quad (\text{C4})$$

By exchanging x and \bar{x} again for the second term inside the bracket of Eq. (C4), we find the frictional force becomes

$$F = \frac{1}{8\pi^5} \int_0^\infty d\omega \omega^5 \int dy d\bar{y} \int_{-\infty}^{-\frac{1}{v}} dx \int_{-\frac{1}{v}}^\infty d\bar{x} (\bar{x} - x) \operatorname{tr} [\boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega x, \omega y) \cdot \boldsymbol{\alpha}(\omega) \cdot \Im \mathbf{g}'(\omega, \omega \bar{x}, \omega \bar{y})]. \quad (\text{C5})$$

Now, the limit on x prevents the vacuum propagation wave number of the first reduced Green's dyadic, κ , from developing an imaginary part, because of

$$\kappa^2 = k^2 - \omega^2 = \omega^2(x^2 + y^2 - 1) > 0, \quad x < -\frac{1}{v}. \quad (\text{C6})$$

For the simplest vacuum situation where only the diagonal components of the Green's dyadic contribute to the integral (see Appendix A of Ref. [17] for a detailed discussion), the anti-Hermitian part reduces to the ordinary imaginary part. But the only possible source of an imaginary part for the first Green's dyadic in Eq. (C5), κ , is now real definite. As a result, the zero temperature QVF vanishes.

For backgrounds other than vacuum, zero temperature quantum friction exists in general because the propagation wave number associated with the medium can become imaginary since

$$\kappa'^2 = k^2 - \omega^2 \varepsilon \mu = \omega^2 (x^2 + y^2 - \varepsilon \mu) \quad (\text{C7})$$

does not have a definite sign. A diaphanous medium with the special property,

$$\varepsilon \mu = 1, \quad (\text{C8})$$

however, is an exception, for which the propagation wave number coincides with the vacuum one, $\kappa' = \kappa$. This nice coincidence renders the reflection coefficients to be real definite as long as ε and μ are real,

$$r^E = \frac{\mu - 1}{\mu + 1} = \frac{1 - \varepsilon}{1 + \varepsilon}, \quad r^H = \frac{\varepsilon - 1}{\varepsilon + 1}. \quad (\text{C9})$$

Therefore, the only source of the imaginary part in the scalar Green's functions Eq. (B4) is still the κ as in the vacuum case. It can be further checked that the anti-Hermitian part of \mathbf{g}' vanishes unless κ develops an imaginary part even though the off-diagonal components of the Green's dyadic and the transformation between \mathbf{g}' and \mathbf{g} needs to be taken into account. Again, recalling Eq. (C6), the zero temperature QF must be absent even if such a diaphanous medium is present in the background.

Now, apparently, both the perfect conductor defined by Eq. (B8) and Eq. (B9) and the vacuum background can be deemed as members of the family of diaphanous materials, for which the total reflection coefficient $r^E + r^H = 0$.

Appendix D: THE NONRELATIVISTIC LIMIT OF PCQF

In this appendix, we obtain the nonrelativistic (NR) limit of PCQF directly from the expressions in Eq. (3.4) and Eq. (3.7), where we have already replaced the dynamical polarizability with the static polarizability. We will use F^{XZ} in particular as an example to illustrate the procedure:

$$F^{\text{XZ}} = \frac{\alpha_{xx}(0)\alpha_{zz}(0)}{8\pi^3(2a)^8} \int_0^\infty dx x^3 [3x \cos x + (x^2 - 3) \sin x] \\ \times \int_{-1}^1 du u \sqrt{1 - u^2} J_1(x\sqrt{1 - u^2}) \frac{1}{e^{xb\gamma(1+uv)} - 1}. \quad (\text{D1})$$

In the NR limit, the exponential factor can be expanded in v . Keeping only up to the term linear in v , we obtain

$$F^{\text{XZ}} = \frac{\alpha_{xx}(0)\alpha_{zz}(0)}{8\pi^3(2a)^8} \int_0^\infty dx x^3 [3x \cos x + (x^2 - 3) \sin x] \\ \times \int_{-1}^1 du u \sqrt{1 - u^2} J_1(x\sqrt{1 - u^2}) \left[\frac{1}{e^{xb} - 1} - uv \frac{xb e^{xb}}{(e^{xb} - 1)^2} \right]. \quad (\text{D2})$$

Note the term constant in v vanishes because of its oddness in u , as there should be no spontaneous quantum propulsion for a reciprocal point particle. See Ref. [21]. We are left with the term linear in v as expected. The u integral can then be easily carried out and we obtain

$$F^{\text{XZ}} = \frac{\alpha_{xx}(0)\alpha_{zz}(0)}{16\pi^3(2a)^8} v I(b), \quad I(b) = b \int_0^\infty dx \frac{[3x \cos x + (x^2 - 3) \sin x]^2}{\sinh^2(xb/2)}. \quad (\text{D3})$$

Now we focus on $I(b)$, which carries all the b dependence of F^{XZ} , and rewrite it as

$$I(b) = -4 \int_0^\infty dx [3x \cos x + (x^2 - 3) \sin x]^2 \frac{d}{dx} \left(\frac{1}{e^{xb} - 1} \right). \quad (\text{D4})$$

Using integration by parts, this becomes

$$\begin{aligned}
I(b) &= 4 \int_0^\infty dx \left[x(2x^2 + 3) + x^2(x^2 - 6) \sin(2x) + x(4x^2 - 3) \cos(2x) \right] \frac{1}{e^{xb} - 1} \\
&= 4 \left\{ \int_0^\infty dx \frac{(2x^3 + 3x)}{e^{xb} - 1} + \left[\frac{d^4}{d\eta^4} - 4 \frac{d^3}{d\eta^3} + 6 \frac{d^2}{d\eta^2} - 3 \frac{d}{d\eta} \right] \int_0^\infty dx \frac{\sin(\eta x)}{e^{xb} - 1} \right\} \Bigg|_{\eta=2} \\
&= 4 \left\{ \int_0^\infty dx \frac{(2x^3 + 3x)}{e^{xb} - 1} + \left[\frac{d^4}{d\eta^4} - 4 \frac{d^3}{d\eta^3} + 6 \frac{d^2}{d\eta^2} - 3 \frac{d}{d\eta} \right] \left[\frac{\pi}{2b} \coth\left(\frac{\eta\pi}{b}\right) - \frac{1}{2\eta} \right] \right\} \Bigg|_{\eta=2} \\
&= \frac{8\pi^4}{15b^4} + \frac{2\pi^2}{b^2} - 9 \\
&\quad + \left\{ \frac{16\pi^5}{b^5} \left[3 \coth^2\left(\frac{2\pi}{b}\right) - 2 \right] \coth\left(\frac{2\pi}{b}\right) + \frac{16\pi^4}{b^4} \left[3 \coth^2\left(\frac{2\pi}{b}\right) - 1 \right] + \frac{24\pi^3}{b^3} \coth\left(\frac{2\pi}{b}\right) + \frac{6\pi^2}{b^2} \right\} \operatorname{csch}^2\left(\frac{2\pi}{b}\right). \tag{D5}
\end{aligned}$$

For $b \ll 1$, the leading term of $I(b)$ is $8\pi^4/15b^4$. In Appendix E, we will show both analytically and numerically that F^{XZ} behaves as $1/b^4$ in the small b limit, even for relativistic velocities. When $I(b)$ is expanded for $b \gg 1$, on the other hand, multiple cancellation occurs and the leading term of $I(b)$ is found to be $1024\pi^{10}/1485b^{10}$. This agrees with the large b limit for arbitrary velocities already obtained in Eq. (3.9).

We have also found the other contributions to the PCQF all have a nonvanishing term linear in v . The procedure outlined in this appendix works to extract the correct NR limits of these other contributions to PCQF as well.

Appendix E: SMALL AND LARGE b LIMITS OF PCQF

In this appendix, we show how to obtain the small and large b limits of PCQF for all v . We will, again, focus on the XZ polarization contribution.

Rewriting Eq. (3.4d) as

$$\mathcal{F}^{\text{XZ}}(x, v, b) = -\sqrt{\pi} 2^{\frac{3}{2}} x^{-\frac{3}{2}} J_{\frac{5}{2}}(x) \int_{-1}^1 du u \sqrt{1-u^2} J_1(x\sqrt{1-u^2}) \frac{1}{e^{x\gamma b(1+uv)} - 1} \tag{E1}$$

and explicitly expanding the thermal occupation factor as a Maclaurin series in the v variable, but retaining the implicit dependence of γ on v ,

$$\frac{1}{e^{x\gamma b(1+uv)} - 1} = \sum_{n=0}^{\infty} \frac{v^n}{n!} \left[\frac{\partial^n}{\partial v^n} \frac{1}{e^{x\gamma b(1+uv)} - 1} \right] \Bigg|_{v=0} = \sum_{n=0}^{\infty} \frac{v^n u^n}{n!} (x\gamma b)^n \frac{\partial^n}{\partial (x\gamma b)^n} \frac{1}{e^{x\gamma b} - 1}, \tag{E2}$$

we obtain

$$\begin{aligned}
\mathcal{F}^{\text{XZ}}(x, v, b) &= -\sqrt{\pi} 2^{\frac{3}{2}} x^{-\frac{3}{2}} J_{\frac{5}{2}}(x) \sum_{n=0}^{\infty} \frac{v^n}{n!} \int_{-1}^1 du u^{n+1} \sqrt{1-u^2} J_1(x\sqrt{1-u^2}) b^n \frac{\partial^n}{\partial b^n} \frac{1}{e^{x\gamma b} - 1} \\
&= -\pi J_{\frac{5}{2}}(x) \sum_{m=0}^{\infty} \frac{v^{2m+1}}{m!} 2^{2-m} x^{-(m+3)} J_{m+\frac{5}{2}}(x) b^{2m+1} \frac{\partial^{2m+1}}{\partial b^{2m+1}} \frac{1}{e^{x\gamma b} - 1}, \tag{E3}
\end{aligned}$$

where we have noticed that the [even](#) n terms vanishes because of the symmetry of the integrand for the u integral.

It follows from Eq. (3.7) and Eq. (E3) that

$$f^{\text{XZ}}(v, b) = -\pi \sum_{m=0}^{\infty} \frac{v^{2m+1}}{m!} 2^{2-m} b^{2m+1} \frac{\partial^{2m+1}}{\partial b^{2m+1}} \int_0^\infty dx x^{4-m} J_{\frac{5}{2}}(x) J_{m+\frac{5}{2}}(x) \frac{1}{e^{x\gamma b} - 1}, \tag{E4}$$

which may be cast in forms suitable for small or large b by employing representations of the integrand (other than the thermal occupation factor) that are appropriate for large or small x , respectively.

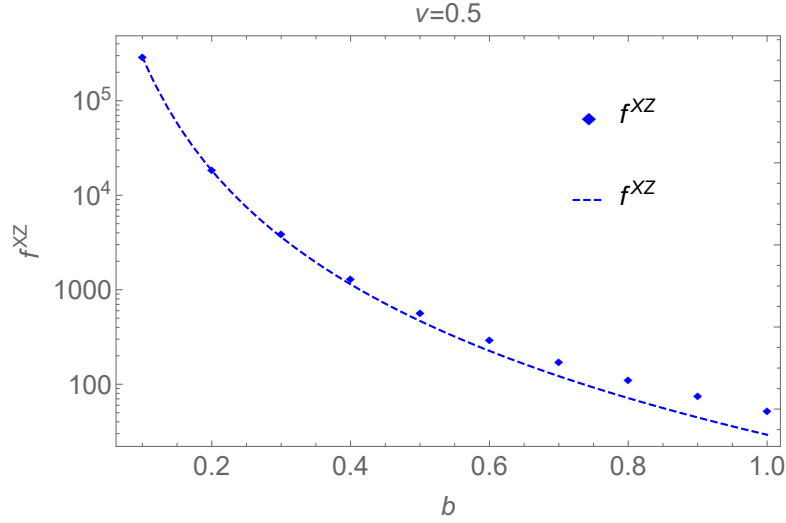


FIG. 6. At fixed velocity $v = 0.5$, the numerical results for f^{XZ} (dots) and its small b approximation (dashed line) obtained in Eq. (E7) are shown for $b \in [0, 1]$.

Thus, the finite series representation

$$J_{n+\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \left[\sin\left(x - \frac{\pi}{2}n\right) \sum_{k=0}^{\lfloor \frac{n}{2} \rfloor} \frac{(-1)^k (n+2k)!}{(2k)!(n-2k)!} (2x)^{-2k} \right. \\ \left. + \cos\left(x - \frac{\pi}{2}n\right) \sum_{k=0}^{\lfloor \frac{n-1}{2} \rfloor} \frac{(-1)^k (n+2k+1)!}{(2k+1)!(n-2k-1)!} (2x)^{-(2k+1)} \right], \quad (\text{E5})$$

appropriate for large x , may be used to generate an expansion for $f^{\text{XZ}}(v, b)$ that is suitable for small b . We will be content to establish the leading-order term for small b , which derives from the leading-order term in the above representation for large x :

$$J_{n+\frac{1}{2}}(x) \sim \sqrt{\frac{2}{\pi x}} \sin\left(x - \frac{\pi}{2}n\right), \quad x \rightarrow \infty. \quad (\text{E6})$$

Using Eq. (E6) in Eq. (E4) and keeping only the $m = 0$ term, corresponding to the leading x -power in the integrand, we readily obtain

$$f^{\text{XZ}}(v, b) \sim -8vb \frac{\partial}{\partial b} \int_0^\infty dx x^3 \sin^2 x \frac{1}{e^{x\gamma b} - 1} \sim -4vb \frac{\partial}{\partial b} \Gamma(4)\zeta(4)(\gamma b)^{-4} = \frac{16\pi^4 v}{15\gamma^4 b^4}, \quad b \rightarrow 0. \quad (\text{E7})$$

It is interesting to note the appearance of the Planck-Einstein transformed temperature, $T_\gamma \equiv \frac{T}{\gamma}$, in this (high-temperature) limit. Note Eq. (E7) captures not only the correct b dependence but also the velocity dependence of f^{XZ} in the small b limit. The agreement of Eq. (E7) with the numerical data for $v = 0.5$ is illustrated in Fig. 6.

Likewise, the infinite series representation

$$J_\mu(x)J_\nu(x) = \sum_{n=0}^{\infty} \frac{(-1)^n (\mu + \nu + n + 1)_n}{n! \Gamma(\mu + n + 1) \Gamma(\nu + n + 1)} \left(\frac{x}{2}\right)^{\mu + \nu + 2n}, \quad (\text{E8})$$

appropriate for small x , may be used to generate an expansion for $f^{\text{XZ}}(v, b)$ that is suitable for large b . In this case,

the leading x -power in the integrand in Eq. (E4) is independent of m , so all terms must be included, resulting in

$$\begin{aligned}
f^{\text{XZ}}(v, b) &\sim -\pi \sum_{m=0}^{\infty} \frac{v^{2m+1}}{m!} \frac{2^{-(3+2m)}}{\Gamma(\frac{7}{2}) \Gamma(m + \frac{7}{2})} b^{2m+1} \frac{\partial^{2m+1}}{\partial b^{2m+1}} \int_0^{\infty} dx x^9 \frac{1}{e^{x\gamma b} - 1} \\
&= -\pi \sum_{m=0}^{\infty} \frac{v^{2m+1}}{m!} \frac{2^{-(3+2m)}}{\Gamma(\frac{7}{2}) \Gamma(m + \frac{7}{2})} b^{2m+1} \frac{\partial^{2m+1}}{\partial b^{2m+1}} \Gamma(10)\zeta(10)(\gamma b)^{-10} \\
&= \pi \sum_{m=0}^{\infty} \frac{v^{2m+1}}{m!} \frac{2^{-(3+2m)} (2m+10)! \zeta(10)}{\Gamma(\frac{7}{2}) \Gamma(m + \frac{7}{2})} \frac{1}{(\gamma b)^{10}} \\
&= \frac{2^8 \zeta(10) v}{15 \gamma^{10} b^{10}} \sum_{m=0}^{\infty} v^{2m} (m+1)(m+2)(m+3)(m+4)(m+5)(2m+7)(2m+9) \\
&= \frac{2^{11} 3 \zeta(10)}{b^{10}} \gamma^6 v (21 + 30v^2 + 5v^4), \quad b \rightarrow \infty,
\end{aligned} \tag{E9}$$

where we have used the identity

$$\gamma^{2n} = \frac{1}{(n-1)!} \frac{d^{n-1}}{d(v^2)^{n-1}} \frac{1}{1-v^2} = \frac{1}{(n-1)!} \sum_{m=0}^{\infty} v^{2m} (m+1)(m+2) \cdots (m+n-1). \tag{E10}$$

The result obtained in Eq. (E9) is precisely that found in Eq. (3.9).

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